ON ANISOTROPIC MOTION OF HOMOGENEOUS MATTER

Yakov Zeldovich

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In a recent paper Narlikar (1) raised the problem of the anisotropic motion of homogeneous matter with a linear velocity distribution:

$$u_i = H_{ik} x_k \tag{1}$$

with a view to cosmological models of this type.

In his opinion, the solution of the Newtonian problem in this case is identical with the cosmological solution given by general relativity theory, and so are the relations obtained by Friedman, on the one hand, and Milne and McCrea on the other.

Within the framework of Newtonian theory, the equation of motion contains the second derivatives of the gravitational potential ϕ with respect to the coordinates. The Poisson equation gives:

$$\Delta\phi = 2\phi_{ii} = 4\pi G\rho. \tag{2}$$

Narlikar assumes that ϕ_{ik} is an isotropic tensor:

$$\phi_{ik} = 2\pi G\rho\delta_{ik}/3. \tag{3}$$

From equation (12) in (1) it follows that

$$\ddot{x} = -\frac{4\pi G\rho}{3}\ddot{x}. \tag{4}$$

This assumption is arbitrary and disagrees with the character of the problem raised by equation (1).

Let us examine an ellipsoid filled with a fluid of constant density. Inside the ellipsoid the values of ϕ_{ik} will be independent of position but will depend on i and k. The principal axes of ϕ_{ik} coincide with the principal axes of the ellipsoid, and the least characteristic value of ϕ_{ik} is associated with the direction of the longest axis of the ellipsoid. With the velocity varying linearly, the ellipsoid will change in shape with time; but it will remain an ellipsoid, and ϕ_{ik} will vary by a definite law.

With the problem thus stated, even if we assume that originally we have a sphere to which Narlikar's relations (3) and (4) apply, they will be upset by the anisotropic distribution of velocity.

An analysis of the motion of an ellipsoid in a general case will show that when the total energy $E < 0$, gravitational collapse to infinite density will inevitably take place, and at a definite moment of time, t_0, the ellipsoid will become a flat ellipse, with $\rho(t_0 - t)$ tending to a limit as $t \rightarrow t_0 - 0$. The gravitational energy will in this case remain finite, as will velocities and accelerations.

In Newtonian theory, the equations for $\rho(t)$, $H_{ik}(t)$ and $\phi_{ik}(t)$ remain unchanged when an ellipsoid increases in size, and one may go over to the limit of infinite space filled with matter. In such a case, however, the initial values of $\phi_{ik}(t=0)$ and, consequently, the relation as a whole will depend on the shape of the ellipsoid which has been allowed an unlimited increase in size for a transition to infinite space.

This solution (and the one given by Narlikar) disagrees with the anisotropic cosmological relations obtained by Heckmann and Schücking (2). This conclusion is borne out by the fact that in the latter case it is possible, for example, that $\phi_{11}>0$; $\phi_{22}>0$; $\phi_{33}<0$, although, as is given in (3), the condition:

$$2(\phi_{11}+\phi_{22}+\phi_{33})=4\pi G\rho,$$

is satisfied.

When $\rho\to\infty$, the right-hand side increases as $(t_0-t)^{-1}$, while $|\phi_{11}|$, $|\phi_{22}|$ and $|\phi_{33}|$ grow at a higher rate to $(t_0-t)^{-2}$.

In Newtonian cosmology, both in an ellipsoid and in a sphere, all values of ϕ_{ik} are positive and do not grow faster than does ρ.

This disagreement between the Newtonian and relativistic cosmologies implies that a solution will be independent of the absolute size of an ellipsoid only within the framework of Newtonian theory, and will not when, with an increase in the size of the ellipsoid, velocities approach that of light and the Newtonian gravitational potential approaches c^2.

A more detailed paper on the subject is to appear in the *Astronomical Journal* (*USSR*).

2 *Vorobyovskoye Highway,*
Apartment 47,
Moscow, U.S.S.R.:

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References

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(3) Novikov, I. D., *Astronomical Journal* (*USSR*), **38**, p. 961, 1961.