Chapter 1

Introduction

1.1 Definition of an Ideal Turbulence Model

Turbulence modeling is one of three key elements in Computational Fluid Dynamics (CFD). Very precise mathematical theories have evolved for the other two key elements, viz., grid generation and algorithm development. By its nature — in creating a mathematical model that approximates the physical behavior of turbulent flows — far less precision has been achieved in turbulence modeling. This is not really a surprising event since our objective has been to approximate an extremely complicated phenomenon.

The field is, to some extent, a throwback to the days of Prandtl, Taylor, von Kármán and all the many other clever engineers who spent a good portion of their time devising engineering approximations and models describing complicated physical flows. Simplicity combined with physical insight seems to have been a common denominator of the work of these great men. Using their work as a gauge, **an ideal model should introduce the minimum amount of complexity while capturing the essence of the relevant physics**. This description of an ideal model serves as the main keystone of this text.

1.2 How Complex Does a Turbulence Model Have to Be?

Aside from any physical considerations, turbulence is inherently three dimensional and time dependent. Thus, an enormous amount of information is required to completely describe a turbulent flow. Fortunately, we usually require something less than a complete time history over all spatial coordinates for every flow property. Thus, for a given turbulent-flow application, we must pose the following question. Given a set of initial and/or boundary conditions, how do we predict the physically meaningful properties of the flow? What properties of a given flow are meaningful is generally dictated by the application. For the simplest applications, we may require only the skin friction and heat-transfer coefficients. More esoteric applications may require detailed knowledge of energy spectra, turbulence fluctuation magnitudes and scales.

Certainly, we should expect the complexity of the mathematics needed for a given application to increase as the amount of required flowfield detail increases. On the one hand, if all we require is skin friction for an attached flow, a simple mixing-length model (Chapter 3) may suffice. Such models are well developed and can be implemented with very little specialized knowledge. On the other hand, if we desire a complete time history of every aspect of a turbulent flow, only a solution to the complete Navier-Stokes equation will suffice. Such a solution requires an extremely accurate numerical solver and may require use of subtle transform techniques, not to mention vast computer resources. Most engineering problems fall somewhere between these two extremes.

Thus, once the question of how much detail we need is answered, the level of complexity of the model follows, qualitatively speaking. In the spirit of Prandtl, Taylor and von Kármán, the conscientious engineer will strive to use as conceptually simple an approach as possible to achieve his ends. Overkill is often accompanied by unexpected difficulties that, in CFD applications, almost always manifest themselves as numerical difficulties!

1.3 Comments on the Physics of Turbulence

Before plunging into the mathematics of turbulence, it is worthwhile to first discuss physical aspects of the phenomenon. The following discussion is not intended as a complete description of this complex topic. Rather, we focus upon a few features of interest in engineering applications, and in construction of a mathematical model. For a more-complete introduction, refer to a basic text on the physics of turbulence such as those by Tennekes and Lumley (1983) or Landahl and Mollo-Christensen (1992).

In 1937, Taylor and von Kármán [see Goldstein (1938)] proposed the following definition of turbulence: "Turbulence is an irregular motion which in general makes its appearance in fluids, gaseous or liquid, when they flow past solid surfaces or even when neighboring streams of the same fluid flow past or over one another." It is characterized by the presence of a large range of excited length and time scales. The irregular nature of turbulence stands in contrast to laminar motion, so called historically, because the fluid was imagined to flow in smooth laminae, or layers. Virtually all flows of practical engineering interest are turbulent. Turbulent flows always occur when the Reynolds number is large. For slightly viscous fluids such as water and air, large Reynolds number corresponds to anything stronger than a small swirl or a puff of wind. Careful analysis of solutions to the Navier-Stokes equation, or more typically to its boundary-layer form, show that turbulence develops as an instability of laminar flow.

To analyze the stability of laminar flows, virtually all methods begin by linearizing the equations of motion. Although some degree of success can be achieved in predicting the onset of instabilities that ultimately lead to turbulence with linear theories, the inherent nonlinearity of the Navier-Stokes equation precludes a complete analytical description of the actual transition process, let alone the fully-turbulent state. For a real (i.e., viscous) fluid, the instabilities result from interaction between the Navier-Stokes equation's nonlinear inertial terms and viscous terms. The interaction is very complex because it is rotational, fully three dimensional and time dependent.

The strongly rotational nature of turbulence goes hand-in-hand with its three dimensionality. Vigorous stretching of vortex lines is required to maintain the ever-present fluctuating vorticity in a turbulent flow. Vortex stretching is absent in two-dimensional flows so that turbulence must be three dimensional. This inherent three dimensionality means there are no satisfactory two-dimensional approximations and this is one of the reasons turbulence remains the most noteworthy unsolved scientific problem of the twentieth century.

The time-dependent nature of turbulence also contributes to its intractability. The additional complexity goes beyond the introduction of an additional dimension. Turbulence is characterized by random fluctuations thus obviating a deterministic approach to the problem. Rather, we must use statistical methods. On the one hand, this aspect is not really a problem from the engineer's view. Even if we had a complete time history of a turbulent flow, we would usually integrate the flow properties of interest over time to extract time-averages. On the other hand, time averaging operations lead to statistical correlations in the equations of motion that cannot be determined a priori. This is the classical closure problem, which is the primary focus of this text.

In principle, the time-dependent, three-dimensional Navier-Stokes equation contains all of the physics of a given turbulent flow. That this is true follows from the fact that turbulence is a continuum phenomenon. As noted by Tennekes and Lumley (1983), "Even the smallest scales occurring in a turbulent flow are ordinarily far larger than any molecular length scale." Nevertheless, the smallest scales of turbulence are still extremely small. They are generally many orders of magnitude smaller than the largest scales of turbulence, the latter being of the same order of magnitude as the dimension of the object about which the fluid is flowing. Furthermore, the ratio of smallest to largest scales decreases rapidly as the Reynolds number increases. To make an accurate numerical simulation (i.e., a full time-dependent three-dimensional solution) of a turbulent flow, all physically relevant scales must be resolved. While more and more progress is being made with such simulations, computers of the early 1990's have insufficient memory and speed to solve any turbulent flow problem of practical interest. To underscore the magnitude of the problem, Speziale (1985) notes that a numerical simulation of turbulent pipe flow at a Reynolds number of 500,000 would require a computer 10 million times faster than a Cray Y/MP. However, the results are very useful in developing and testing turbulence models in the limit of low Reynolds number.

Turbulence consists of a continuous spectrum of scales ranging from largest to smallest, as opposed to a discrete set of scales. In order to visualize a turbulent flow with a spectrum of scales we often refer to turbulent eddies. A turbulent eddy can be thought of as a local swirling motion whose characteristic dimension is the local turbulence scale (Figure 1.1). Eddies overlap in space, large ones carrying smaller ones. Turbulence features a cascading process whereby, as the turbulence decays, its kinetic energy transfers from larger eddies to smaller eddies. Ultimately, the smallest eddies dissipate into heat through the action of molecular viscosity. Thus, we observe that turbulent flows are always dissipative.

Perhaps the most important feature of turbulence from an engineering point of view is its enhanced diffusivity. Turbulent diffusion greatly enhances the transfer of mass, momentum and energy. Apparent stresses often develop in turbulent flows that are several orders of magnitude larger than in corresponding laminar flows.

The nonlinearity of the Navier-Stokes equation leads to interactions between fluctuations of differing wavelengths and directions. As discussed above, the wavelengths of the motion usually extend all the way from a maximum comparable to the width of the flow to a minimum fixed by viscous dissipation of energy. The main physical process that spreads the motion over a wide range of wavelengths is vortex stretching. The turbulence gains energy if the vortex elements are primarily oriented in a direction in which the mean velocity gradients can stretch them. Most importantly, wavelengths that are not too small compared to the mean-flow width interact most strongly with the mean flow. Consequently, the larger-scale

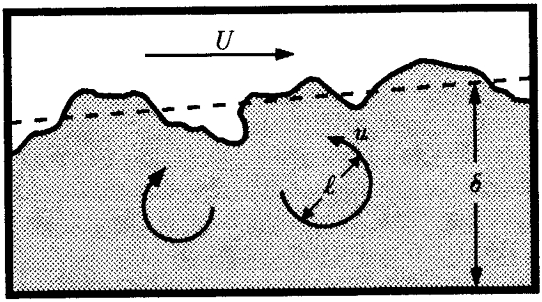

Figure 1.1: Large eddies in a turbulent boundary layer. The flow above the boundary layer has a steady velocity U; the eddies move at randomly-fluctuating velocities of the order of a tenth of U. The largest eddy size (ℓ) is comparable to the boundary-layer thickness (δ). The interface and the flow above the boundary is quite sharp [Corrsin and Kistler (1954)].

turbulent motion carries most of the energy and is mainly responsible for the enhanced diffusivity and attending stresses. In turn, the larger eddies randomly stretch the vortex elements that comprise the smaller eddies, cascading energy to them.

An especially striking feature of a turbulent shear flow is the way large bodies of fluid migrate across the flow, carrying smaller-scale disturbances with them. The arrival of these large eddies near the interface between the turbulent region and nonturbulent fluid distorts the interface into a highly convoluted shape (Figure 1.1). In addition to migrating across the flow, they have a lifetime so long that they persist for distances as much as 30 times the width of the flow [Bradshaw (1972)]. Hence, the turbulent stresses at a given position depend upon upstream history and cannot be uniquely specified in terms of the local strain-rate tensor as in laminar flow.

As we progress through the following chapters, we will introduce more specific details of turbulence properties for common flows on an as-needed basis.

1.4 A Brief History of Turbulence Modeling

The primary emphasis in this book is upon the time-averaged Navier-Stokes equation. The origin of this approach dates back to the end of the nineteenth century when Reynolds (1895) published results of his research on turbulence. His pioneering work proved to have such profound importance for all future developments that we refer to the standard time-averaging process as one type of Reynolds averaging.

The earliest attempts at developing a mathematical description of turbulent stresses sought to mimic the molecular gradient-diffusion process. In this spirit, Boussinesq (1877) introduced the concept of an eddy viscosity. As with Reynolds, Boussinesq has been immortalized in turbulence literature. The Boussinesq eddy-viscosity approximation is so widely known that few authors find a need to reference his original paper.

Neither Reynolds nor Boussinesq attempted solution of the Reynolds-averaged Navier-Stokes equation in any systematic manner. Much of the physics of viscous flows was a mystery in the nineteenth century, and further progress awaited Prandtl's discovery of the boundary layer in 1904. Focusing upon turbulent flows, Prandtl (1925) introduced the mixing length (an analog of the mean-free path of a gas) and a straightforward prescription for computing the eddy viscosity in terms of the mixing length. The mixing-length hypothesis, closely related to the eddy-viscosity concept, formed the basis of virtually all turbulence-modeling research for the next twenty years. Important early contributions were made by several researchers, most notably by von Kármán (1930). In modern terminology, we refer to a mixing-length model as an **algebraic model** or a **zero-equation model of turbulence**. By definition, an **n-equation model** signifies a model that requires solution of **n** additional differential transport equations in addition to those expressing conservation of mass, momentum and energy.

To improve the ability to predict properties of turbulent flows and to develop a more realistic mathematical description of the turbulent stresses, Prandtl (1945) postulated a model in which the eddy viscosity depends upon the kinetic energy of the turbulent fluctuations, k. He proposed a modeled differential equation approximating the exact equation for k. This improvement, on a conceptual level, takes account of the fact that the eddy viscosity is affected by where the flow has been, i.e., upon flow history. Thus was born the concept of the so-called **one-equation model of turbulence**.

While having an eddy viscosity that depends upon flow history provides a more physically realistic model, the need to specify a turbulence length scale remains. Since the length scale can be thought of as a characteristic eddy size and since such scales are different for each flow, turbulence models that do not provide a length scale are **incomplete**. That is, we must know something about the flow, other than initial and boundary conditions, in advance in order to obtain a solution. Such models are not without merit and, in fact, have proven to be of great value in many engineering applications.

To elaborate a bit further, an incomplete model generally defines a turbulence length in a prescribed manner from the mean flow, e.g. the displacement thickness, δ^*, for an attached boundary layer. However, a different length in this example would be needed when the boundary layer separates since δ^* may be negative. Yet another length might be needed for free shear flows, etc. In essence, incomplete models usually define quantities that may vary more simply or more slowly than the Reynolds stresses (e.g. eddy viscosity and mixing length). Presumably such quantities are easier to correlate than the actual stresses.

A particularly desirable type of turbulence model would be one that can be applied to a given turbulent flow by prescribing at most the appropriate boundary and/or initial conditions. Ideally, no advance knowledge of any property of the turbulence should be required to obtain a solution. We define such a model as being **complete**. Note that our definition implies nothing regarding the accuracy or universality of the model, only that it can be used to determine a flow with no prior knowledge of any flow details.

Kolmogorov (1942) introduced the first **complete** model of turbulence. In addition to having a modeled equation for k, he introduced a second parameter ω that he referred to as "the rate of dissipation of energy in unit volume and time." The reciprocal of ω serves as a turbulence time scale, while $k^{1/2}/\omega$ serves as the analog of the mixing length. In this model, known as a k-ω model, ω satisfies a differential equation similar to the equation for k. The model is thus termed a **two-equation model of turbulence**. While this model offered great promise, it went with virtually no applications for the next quarter century because of the unavailability of computers to solve its nonlinear differential equations.

Rotta (1951) laid the foundation for turbulence models that obviate use of the Boussinesq approximation. He devised a plausible model for the differential equation governing evolution of the tensor that represents the turbulent stresses, i.e., the Reynolds-stress tensor. This approach is called **second-order** or **second-moment closure**. The primary conceptual advantage of second-order closure is the natural manner in which nonlocal and history effects are incorporated. Such models automatically accommodate complicating effects such as streamline curvature, rigid-body rotation, and body forces. This stands in contrast to eddy-viscosity models that fail to properly account for these effects. For a three-dimensional flow, a second-order closure model introduces seven equations, one for the turbulence scale and six for the components of the Reynolds-stress tensor. As with Kolmogorov's k-ω model, second-order closure models awaited adequate computer resources.

Thus, by the early 1950's, four main categories of turbulence models had evolved, viz.,

1. Algebraic (Zero-Equation) Models
2. One-Equation Models
3. Two-Equation Models
4. Second-Order Closure Models

With the coming of the age of computers since the 1960's, further development of all four classes of turbulence models has occurred. The following overview lists a few of the most important modern developments for each of the four classes.

Algebraic Models. Van Driest (1956) devised a viscous damping correction for the mixing-length model that is included in virtually all algebraic models in use today. Cebeci and Smith (1974) refined the eddy-viscosity/mixing-length model to a point that it can be used with great confidence for most attached boundary layers. To remove some of the difficulties in defining the turbulence length scale from the shear-layer thickness, Baldwin and Lomax (1978) have proposed an alternative algebraic model that enjoys widespread use.

One-Equation Models. Of the four types of turbulence models described above, the one-equation model has enjoyed the least popularity and success. Perhaps the most successful model of this type was formulated by Bradshaw, Ferriss and Atwell (1967). In the 1968 Stanford Conference on Computation of Turbulent Boundary Layers [Coles and Hirst (1969)] the best turbulence models of the day were tested against the best experimental data of the day. In this author's opinion, of all the models used, the Bradshaw-Ferriss-Atwell model most faithfully reproduced measured flow properties. There has been some renewed interest in one-equation models [c.f. Baldwin and Barth (1990), Goldberg (1991) and Spalart and Allmaras (1992)], motivated primarily by the ease with which such model equations can be solved numerically, relative to two-equation models and second-order closure models.

Two-Equation Models. While Kolmogorov's k-ω model was the first of this type, it remained in obscurity until the coming of the computer. By far, the most extensive work on two-equation models has been done by Launder and Spalding (1972) and a continuing succession of students and colleagues. Launder's k-ϵ model, where ϵ is proportional to the product of k and ω, is as well known as the mixing-length model and is the most widely used two-equation model. Even the model's demonstrable inadequacy for flows with adverse pressure gradient [c.f. Rodi and Scheuerer (1986) and Wilcox (1988a)] has done little to discourage its widespread use. With no prior knowledge of Kolmogorov's work, Saffman (1970) formulated a k-ω

model that enjoys advantages over the k-ϵ model, especially for integrating through the viscous sublayer and for predicting effects of adverse pressure gradient. Wilcox and Alber (1972), Saffman and Wilcox (1974), Wilcox and Traci (1976), Wilcox and Rubesin (1980), and Wilcox (1988a), for example, have pursued further development and application of k-ω models. As pointed out by Lakshminarayana (1986), k-ω models are the second most widely used type of two-equation turbulence model.

Second-Order Closure Models. By the 1970's, sufficient computer resources became available to permit serious development of this class of model. The most noteworthy efforts were those of Donaldson [Donaldson and Rosenbaum (1968)], Daly and Harlow (1970) and Launder, Reece and Rodi (1975). The latter has become the baseline second-order closure model: more recent contributions by Lumley (1978), Speziale (1985, 1987a) and Reynolds (1987) have added mathematical rigor to the closure process. However, because of the large number of equations and complexity involved in second-order closure models, they have thus far found their way into a relatively small number of applications compared to algebraic and two-equation models.

This book investigates all four classes of turbulence models. The primary emphasis is upon examining the underlying physical foundation and upon developing the mathematical tools for analyzing and testing the models. **The text is not intended to be a catalog of all turbulence models.** Rather, we approach each class of models in a generic sense. Detailed information is provided for models that have stood the test of time; additionally, references are given for most models.