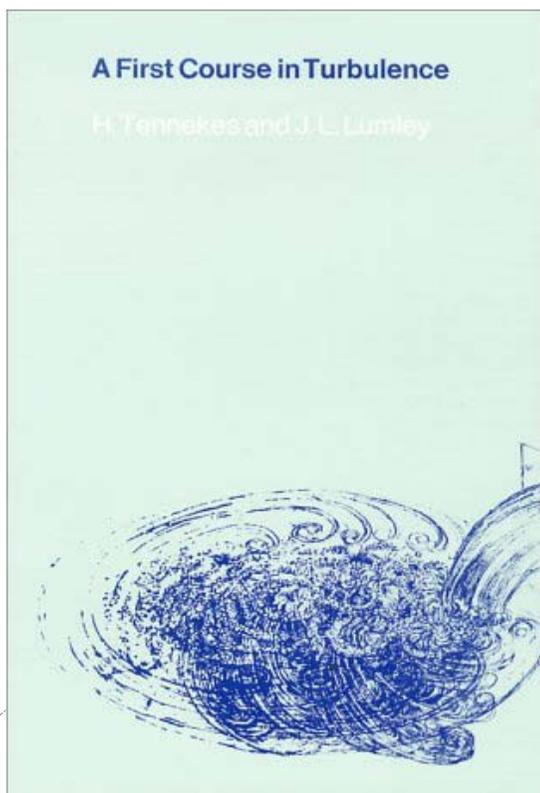


# **A FIRST COURSE IN TURBULENCE**

**H. Tennekes and J. L. Lumley**



**The MIT Press**

**Cambridge, Massachusetts, and London, England**

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ISBN 0 262 20019 8 (hardcover)

Library of Congress catalog card number: 77-165072

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## PREFACE

In the customary description of turbulence, there are always more unknowns than equations. This is called the closure problem; at present, the gap can be closed only with models and estimates based on intuition and experience. For a newcomer to turbulence, there is yet another closure problem: several dozen introductory texts in general fluid dynamics exist, but the gap between these and the monographs and advanced texts in turbulence is wide. This book is designed to bridge the second closure problem by introducing the reader to the tools that must be used to bridge the first.

A basic tool of turbulence theory is dimensional analysis; it is always used in conjunction with an appeal to the idea that turbulent flows should be independent of the Reynolds number if they are scaled properly. These tools are sufficient for a first study of most problems in turbulence; those requiring sophisticated mathematics have been avoided wherever possible. Of course, dimensional reasoning is incapable of actually solving the equations governing turbulent flows. A direct attack on this problem, however, is beyond the scope of this book because it requires advanced statistics and Fourier analysis. Also, even the most sophisticated studies, so far, have met with relatively little success. The purpose of this book is to introduce its readers to turbulence; it is neither a research monograph nor an advanced text.

Some understanding of viscous-flow and boundary-layer theory is a prerequisite for a successful study of much of the material presented here. On the other hand, we assume that the reader is not familiar with stochastic processes and Fourier transforms. Because the Reynolds stress is a second-rank tensor, the use of tensor notation could not be avoided; however, very little tensor analysis is needed to understand elementary operations on the equations of motion in Cartesian coordinate systems.

We use most of the material in this book in an introductory turbulence course for college seniors and first-year graduate students. We feel that this book can also serve as a supplementary text for courses in general fluid dynamics. We have attempted to avoid a bias toward any specific discipline, in the hope that the material will be useful for meteorologists, oceanographers, and astrophysicists, as well as for aerospace, mechanical, chemical, and pollution control engineers.

The scope of this book did not permit us to describe the experimental methods used in turbulence research. Also, because this is an introduction to

turbulence, we have not attempted to give an exhaustive list of references. The bibliography lists the books devoted to turbulence as well as some major papers. The most comprehensive of the recent books is Monin and Yaglom's *Statistical Fluid Mechanics* (Monin and Yaglom, 1971); it contains a complete bibliography of the current journal literature.

The manuscript was read by Dr. S. Corrsin and Dr. J. A. B. Wills; they offered many valuable comments. Miss Constance Hazuda typed several drafts and the final manuscript. A preliminary set of lecture notes was compiled in 1967 by Mr. A. S. Chaplin. Several generations of students contributed to the development of the presentation of the material. While writing this book, the authors received research support from the Atmospheric Sciences Section, National Science Foundation, under grants GA-1019 and GA-18109.

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June 1970

## BRIEF GUIDE ON THE USE OF SYMBOLS

The theory of turbulence contains many, often crude, approximations. Many relations (except the equations of motion and their formal consequences) therefore do not really permit the use of the equality sign. We adopt the following usage. If the error involved in writing an equation is smaller than about 30%, we use the approximate equality sign  $\cong$ . For crude approximations the symbol  $\sim$  is employed. This generally means that the nondimensional coefficient that would make the relation an equation is not greater than 5 and not smaller than  $1/5$ . If the value of the coefficient is of interest (for example, if the theory is to be compared with experimental data or if a statement about the coefficient is in order), the equality sign is used and the coefficient is entered explicitly. If the problem discussed is the selection of the dominant terms in an equation of motion, the order symbol  $\mathcal{O}$ , which does not make any commitment on the value of the coefficient, is employed. After the dominant terms have been selected, the equality sign is used in the resulting simplified equation, with the understanding that the error involved can be made arbitrarily small by increasing the parameter in the problem (often a Reynolds number) without limit. We do not claim that we have been completely consistent, but in most cases the meaning of the symbols is made clear in the text.

Though it may sometimes seem confusing, this usage serves as a continuing reminder that relatively few accurate statements can be made about a turbulent flow without recourse to experimental evidence on that flow. If one has to study a flow for which no data are available, all one can do is to find the characteristic parameters (velocity, length, time, and other scales) and to make crude (say within a factor of two) estimates of the properties of the flow. This is no mean accomplishment; it allows one to design an experiment in a sensible way and to select the appropriate nondimensional form in which the experimental data should be presented.