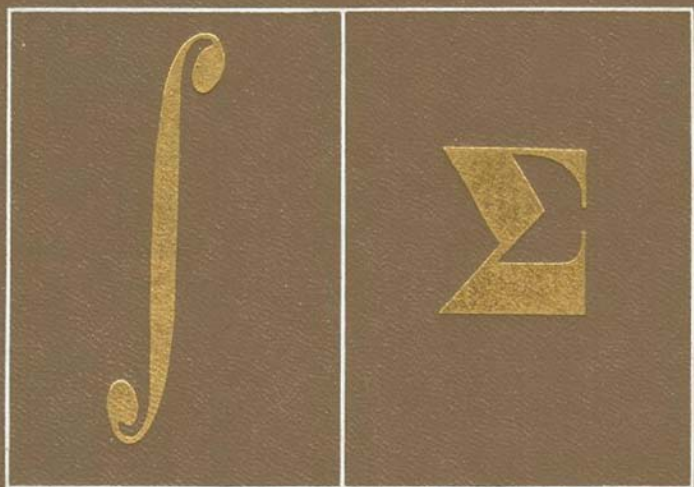


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ИНТЕГРАЛЫ И РЯДЫ

СПЕЦИАЛЬНЫЕ
ФУНКЦИИ



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Книга содержит неопределенные и определенные интегралы, конечные суммы и ряды со специальными функциями. Она является наиболее полным справочным руководством, включает результаты, изложенные в аналогичных изданиях, а также в научной и периодической литературе, опубликованной в последние годы. Некоторые результаты публикуются впервые.

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ПРЕДИСЛОВИЕ

Разнообразие задач, при решении которых возникает необходимость вычисления интегралов и суммирования рядов, вызвало появление большого числа справочных руководств в этой области классического анализа. К числу таких справочников, наиболее полно отражающих основные результаты на период составления, принадлежат известные «Таблицы интегралов, сумм, рядов и произведений» И. С. Градштейна и И. М. Рыжика. Эти таблицы содержат формулы, которые были получены до конца 40-х годов. С тех пор интерес к этой области математического анализа не уменьшился, а требования практических приложений привели к необходимости издания более полного справочного руководства, в котором были бы отражены новые результаты. В связи с этим в 1981 году появилась наша книга «Интегралы и ряды Элементарные функции», содержащая интегралы и ряды с элементарными функциями и существенно перекрывающая соответствующие разделы упомянутого справочника.

Предлагаемая вниманию читателя книга содержит неопределенные и определенные, в том числе кратные, интегралы, конечные суммы и ряды со специальными функциями. Она написана с целью удовлетворения запросов широкого круга специалистов в различных областях науки и техники и включает результаты, содержащиеся в аналогичных изданиях, а также в научной и периодической литературе, изданной в последнее время. Ряд результатов публикуется впервые.

Основному тексту предшествует достаточно подробное оглавление, с помощью которого можно отыскать нужные формулы. Используемые обозначения, как правило, общеприняты в математической литературе и приводятся в указателях в конце книги. При ссылках запись вида 2.2.4.3 обозначает формулу 3 из пункта 2.2.4. Во всех главах $k, l, m, n = 0, 1, 2, 3, \dots$, если не указаны другие ограничения.

Для компактности изложения используется сокращенная запись. Например, формула

$$\int_0^{\infty} x^{\alpha-1} \begin{cases} \sin cx \\ \cos cx \end{cases} J_{\nu}(cx) dx = \frac{2^{\nu-1}}{c^{\alpha}} \Gamma \left[\begin{matrix} (2\alpha + 2\nu + 1 \pm 1)/4, & 1/2 - \alpha \\ (3 \pm 1 - 2\alpha - 2\nu)/4, & 1 + \nu - \alpha \end{matrix} \right]$$

$$[c > 0; -\begin{Bmatrix} 1 \\ 0 \end{Bmatrix} - \operatorname{Re} \nu < \operatorname{Re} \alpha < 1/2]$$

представляет собой сокращенную запись двух формул:

$$\int_0^{\infty} x^{\alpha-1} \sin cx J_{\nu}(cx) dx = \frac{2^{\nu-1}}{c^{\alpha}} \Gamma \left[\begin{matrix} (\alpha + \nu + 1)/2, & 1/2 - \alpha \\ 1 - (\alpha + \nu)/2, & 1 + \nu - \alpha \end{matrix} \right]$$

$$[c > 0; -1 - \operatorname{Re} \nu < \operatorname{Re} \alpha < 1/2]$$

(берутся только верхние знаки и верхние выражения в фигурных скобках) и

$$\int_0^{\infty} x^{\alpha-1} \cos cx J_{\nu}(cx) dx = \frac{2^{\nu-1}}{c^{\alpha}} \Gamma \left[\begin{matrix} (\alpha+\nu)/2, 1/2-\alpha \\ (1-\alpha-\nu)/2, 1+\nu-\alpha \end{matrix} \right]$$

$$[c > 0; -\operatorname{Re} \nu < \operatorname{Re} \alpha < 1/2]$$

(берутся только нижние знаки и нижние выражения в фигурных скобках); по

определению $\Gamma \left[\begin{matrix} a_1, \dots, a_m \\ b_1, \dots, b_n \end{matrix} \right] = \prod_{k=1}^m \Gamma(a_k) / \prod_{l=1}^n \Gamma(b_l)$.

Различные представления одного и того же интеграла записываются под разными номерами, но без повторения левой части.

В приложениях содержится справочный материал, который может быть применен при вычислении интегралов и суммировании рядов, а также при преобразовании их к виду, содержащемуся в книге.

Список основных литературных источников, использованных при составлении настоящего справочного руководства, помещен в конце книги.

Мы будем весьма признательны всем читателям, которые обратят наше внимание на неизбежные в работе такого объема недосмотры

Авторы

Глава 1. НЕОПРЕДЕЛЕННЫЕ ИНТЕГРАЛЫ

1.1. ВВЕДЕНИЕ

В этой главе содержатся неопределенные интегралы от специальных функций а также определенные интегралы, у которых один из пределов интегрирования является переменным, в случаях, когда подынтегральная функция от него не зависит; постоянная интегрирования для краткости опущена

Некоторые формулы при определенных значениях параметров теряют смысл. Если эти значения следуют из структуры формулы, то соответствующие разъяснения опускаются. Выражения для интеграла при этих значениях параметров, как правило, даются в последующих формулах.

1.2. НЕПОЛНЫЕ ГАММА-ФУНКЦИИ $\gamma(\alpha, x)$, $\Gamma(\alpha, x)$ И БЕТА-ФУНКЦИЯ $B_x(\alpha, \beta)$

Условие: $\operatorname{Re} \alpha, \operatorname{Re} \alpha > 0$.

1.2.1. Интегралы вида $\int x^\lambda \left\{ \frac{\gamma(\alpha, ax^v)}{\Gamma(\alpha, ax^v)} \right\} dx$.

$$1. \int_0^x x^\lambda \left\{ \frac{\gamma(\alpha, ax^v)}{\Gamma(\alpha, ax^v)} \right\} dx = \frac{x^{\lambda+1}}{\lambda+1} \left\{ \frac{\gamma(\alpha, ax^v)}{\Gamma(\alpha, ax^v)} \right\} -$$

$$- \frac{1}{(\lambda+1) a^{(\lambda+1)/v}} \left\{ \frac{\gamma((\lambda+\alpha v+1)/v, ax^v)}{\Gamma((\lambda+\alpha v+1)/v, ax^v)} \right\} + \frac{\Gamma(\alpha + (\lambda+1)/v)}{(\lambda+1) a^{(\lambda+1)/v}} \left\{ \frac{0}{1} \right\}$$

$$\left[\left\{ \begin{array}{l} \operatorname{Re}(\lambda + \alpha v) > -1 \\ \operatorname{Re}(\lambda + \alpha v), \operatorname{Re} \lambda > -1 \end{array} \right\}, v > 0 \right].$$

$$2. \int_0^x \left\{ \frac{\gamma(\alpha, ax)}{\Gamma(\alpha, ax)} \right\} dx = x \left\{ \frac{\gamma(\alpha, ax)}{\Gamma(\alpha, ax)} \right\} - \frac{1}{a} \left\{ \frac{\gamma(\alpha+1, ax)}{\Gamma(\alpha+1, ax)} \right\} + \frac{\Gamma(\alpha+1)}{a} \left\{ \frac{0}{1} \right\}.$$

$$3. \int x^n \left\{ \frac{\gamma(\alpha, (ax+b)^v)}{\Gamma(\alpha, (ax+b)^v)} \right\} dx = \frac{1}{a^{n+1}} \sum_{k=0}^n (-1)^k \binom{n}{k} \frac{b^{n-k}}{k+1} \times$$

$$\times \left[(ax+b)^{k+1} \left\{ \frac{\gamma(\alpha, (ax+b)^v)}{\Gamma(\alpha, (ax+b)^v)} \right\} \mp \gamma\left(\alpha + \frac{k+1}{v}, (ax+b)^v\right) \right]$$

$$\left[\left\{ \begin{array}{l} b, \operatorname{Re}(n + \alpha v + 1) > 0 \\ b > 0 \end{array} \right\}, v > 0 \right].$$

1.2.2. Интегралы вида $\int f(x) \gamma(\alpha, \phi(x)) dx$.

$$1. \int_0^x e^{-bx} \gamma(\alpha, ax) dx = -\frac{e^{-bx}}{b} \gamma(\alpha, ax) + \frac{1}{b} \left(\frac{a}{a+b} \right)^\alpha \gamma(\alpha, ax+bx).$$

$$2. \int e^{-bx} \gamma\left(\frac{n+1}{2}, a^2 x^2\right) dx = \\ = \frac{1}{b} \left\{ e^{b^2/(4a^2)} \sum_{k=0}^n (-1)^{n-k} \binom{n}{k} \left(\frac{b}{2a}\right)^{n-k} \gamma\left(\frac{k+1}{2}, \left(ax + \frac{b}{2a}\right)^2\right) - e^{-bx} \gamma\left(\frac{n+1}{2}, a^2 x^2\right) \right\}.$$

$$3. \int x e^{a^2 x^2} \gamma(\alpha, (ax+b)^2) dx = \\ = \frac{1}{2a^2} e^{a^2 x^2} \gamma(\alpha, (ax+b)^2) - \frac{1}{a^2 (2b)^{2\alpha}} e^{b^2} \gamma(2\alpha, 2abx + 2b^2).$$

$$4. \int e^x \gamma^2(\alpha, x) dx = e^x \gamma^2(\alpha, x) - \frac{2}{\alpha} x^\alpha \gamma(\alpha, x) + \frac{2}{\alpha} \gamma(2\alpha, x).$$

1.2.3. Интегралы вида $\int f(x) B_x(\alpha, \beta) dx$.

$$1. \int x^\lambda B_x(\alpha, \beta) dx = \frac{x^{\lambda+1}}{\lambda+1} B_x(\alpha, \beta) - \frac{1}{\lambda+1} B_x(\alpha + \lambda + 1, \beta).$$

$$2. \int x^{n-\alpha} B_x(\alpha, \beta) dx = \\ = \frac{x^{n-\alpha+1}}{n-\alpha+1} B_x(\alpha, \beta) + \frac{1}{n-\alpha+1} \sum_{k=0}^n (-1)^k \binom{n}{k} \frac{(1-x)^{k+\beta}}{k+\beta}.$$

$$3. \int (1-x)^\lambda B_x(\alpha, \beta) dx = -\frac{(1-x)^{\lambda+1}}{\lambda+1} B_x(\alpha, \beta) + \frac{1}{\lambda+1} B_x(\alpha, \beta + \lambda + 1).$$

$$4. \int (1-x)^{n-\beta} B_x(\alpha, \beta) dx = \\ = -\frac{(1-x)^{n-\beta+1}}{n-\beta+1} B_x(\alpha, \beta) + \frac{1}{n-\beta+1} \sum_{k=0}^n (-1)^k \binom{n}{k} \frac{x^{k+\alpha}}{k+\alpha}.$$

$$5. \int x^{\alpha-1} (1-x)^{\beta-1} B_x(\alpha, \beta) dx = \frac{1}{2} B_x^2(\alpha, \beta).$$

$$6. \int \frac{1}{x} B_x(\alpha, \beta) dx = \ln x B_x(\alpha, \beta) - \frac{\partial}{\partial \alpha} B_x(\alpha, \beta).$$

$$7. \int \frac{1}{1-x} B_x(\alpha, \beta) dx = -\ln(1-x) B_x(\alpha, \beta) + \frac{\partial}{\partial \beta} B_x(\alpha, \beta).$$

1.3. ИНТЕГРАЛЬНАЯ ПОКАЗАТЕЛЬНАЯ ФУНКЦИЯ $Ei(x)$

1.3.1. Интегралы вида $\int x^\lambda Ei(-ax-b) dx$.

Условие: $a, b > 0$.

$$1. \int_0^x x^\lambda Ei(-ax) dx = \frac{x^{\lambda+1}}{\lambda+1} Ei(-ax) - \frac{1}{(\lambda+1)a^{\lambda+1}} \gamma(\lambda+1, ax)$$

[$\operatorname{Re} \lambda > -1$]

$$2. \int_x^\infty x^\lambda Ei(-ax) dx = -\frac{x^{\lambda+1}}{\lambda+1} Ei(-ax) - \frac{1}{(\lambda+1)a^{\lambda+1}} \Gamma(\lambda+1, ax).$$

$$3. \int_x^\infty x^{-\lambda-n-1} Ei(-ax) dx = \frac{x^{-n-\lambda}}{n+\lambda} Ei(-ax) + (-1)^{n+1} \frac{a^{n+\lambda}}{(n+\lambda)} \Gamma\left[\lambda+n+1\right] \times \\ \times \Gamma(1-\lambda, ax) + \frac{x^{-n-\lambda} e^{-ax}}{(n+\lambda) \Gamma(\lambda+n+1)} \sum_{k=0}^n \Gamma(n+\lambda-k) (-ax)^k.$$

$$4. \int x^n \operatorname{Ei}(-ax) dx = \frac{x^{n+1}}{n+1} \operatorname{Ei}(-ax) + \frac{n!}{(n+1)a^{n+1}} e^{-ax} \sum_{k=0}^n \frac{(ax)^k}{k!}.$$

$$5. \int x^n \operatorname{Ei}(-ax-b) dx = \\ = \operatorname{Ei}(-ax-b) \sum_{k=0}^n (-1)^{n-k} \frac{n!}{(n-k)!} \frac{(b/a)^{n-k}}{(k+1)!} \left(x + \frac{b}{a}\right)^{k+1} + \\ + \frac{1}{a^{n+1}} e^{-ax-b} \sum_{k=0}^n \frac{(-1)^{n-k} n! b^{n-k}}{(n-k)! (k+1)!} \sum_{m=0}^k \frac{(ax+b)^m}{m!}.$$

$$6. \int_x^\infty \frac{1}{x^n} \operatorname{Ei}(-ax) dx = \frac{1}{(n-1)! (n-1) x^{n-1}} \left\{ [(n-1)! - (-ax)^{n-1}] \operatorname{Ei}(-ax) + \right. \\ \left. + e^{-ax} \sum_{k=0}^{n-2} (n-k-2)! (-ax)^k \right\} \quad [n \geq 2].$$

$$7. \int \frac{1}{x^n} \operatorname{Ei}(-ax-b) dx = -\frac{1}{n-1} \left[\frac{1}{x^{n-1}} - \left(-\frac{a}{b}\right)^{n-1} \right] \operatorname{Ei}(-ax-b) - \\ - \frac{1}{n-1} \left(-\frac{a}{b}\right)^{n-1} e^{-b} \operatorname{Ei}(-ax) \sum_{k=0}^{n-2} \frac{b^k}{k!} + \frac{1}{n-1} \left(-\frac{a}{b}\right)^{n-1} e^{-ax-b} \times \\ \times \sum_{k=1}^{n-2} \frac{1}{k!} \left(-\frac{b}{ax}\right)^k \sum_{m=0}^{k-1} (k-m-1)! (-ax)^m \quad [n \geq 2].$$

$$8. \int \operatorname{Ei}(-ax) dx = x \operatorname{Ei}(-ax) + \frac{1}{a} e^{-ax}.$$

$$9. \int x \operatorname{Ei}(-ax) dx = \frac{x^2}{2} \operatorname{Ei}(-ax) + \frac{1}{2a^2} (1+ax) e^{-ax}.$$

$$10. \int \frac{1}{x} \operatorname{Ei}(-ax) dx = \ln x \operatorname{Ei}(-ax) - \int \frac{1}{x} e^{-ax} \ln x dx.$$

$$11. \int_x^\infty \frac{1}{x} \operatorname{Ei}(-ax) dx = -\frac{1}{2} [(C + \ln ax)^2 + \zeta(2)] - \sum_{k=1}^\infty \frac{(-ax)^k}{k! k^2}.$$

$$12. \int_x^\infty \frac{1}{x^2} \operatorname{Ei}(-ax) dx = \frac{1}{x} [(1+ax) \operatorname{Ei}(-ax) + e^{-ax}].$$

$$13. \int \frac{1}{x^2} \operatorname{Ei}(-ax-b) dx = \frac{1}{b} \left[ae^{-b} \operatorname{Ei}(-ax) - \frac{1}{x} (ax+b) \operatorname{Ei}(-ax-b) \right].$$

1.3.2. Интегралы вида $\int x^\lambda e^{cx} \operatorname{Ei}(-ax-b) dx$.

Условие. $a, b, c > 0$

$$1. \int x^\lambda e^{-cx} \operatorname{Ei}(-ax) dx = -\frac{1}{c} x^\lambda e^{-cx} \operatorname{Ei}(-ax) - \\ - \frac{1}{c} (a+c)^{-\lambda} \Gamma(\lambda, ax+cx) + \frac{\lambda}{c} \int x^{\lambda-1} e^{-cx} \operatorname{Ei}(-ax) dx.$$

2. $\int x^n e^{-cx} \operatorname{Ei}(-ax) dx =$

$$= \frac{n!}{c^{n+1}} \operatorname{Ei}(-ax - cx) - \frac{n!}{c^{n+1}} e^{-cx} \operatorname{Ei}(-ax) \sum_{k=0}^n \frac{(cx)^k}{k!} -$$

$$- \frac{n!}{c^{n+1}} e^{-(a+c)x} \sum_{k=1}^n \frac{1}{k} \left(\frac{c}{a+c} \right)^k \sum_{m=0}^{k-1} \frac{(a+c)^m x^m}{m!}.$$
3. $\int e^{-cx} \operatorname{Ei}(-ax) dx = \frac{1}{c} [\operatorname{Ei}(-ax - cx) - e^{-cx} \operatorname{Ei}(-ax)] \quad [a \neq -c].$
4. $\int e^{\pm cx} \operatorname{Ei}(\mp ax) dx = \mp \frac{1}{c} [\operatorname{Ei}(\mp ax \pm cx) - e^{\pm cx} \operatorname{Ei}(\mp ax)].$
5. $\int_0^x e^{cx} \operatorname{Ei}(-ax) dx = -\frac{1}{c} [\operatorname{Ei}(cx - ax) - e^{cx} \operatorname{Ei}(-ax) - \ln \left(1 - \frac{c}{a} \right)]$
 $[a > c].$
6. $\int_0^x e^{\pm ax} \operatorname{Ei}(\mp ax) dx = \pm \frac{1}{a} [e^{\pm ax} \operatorname{Ei}(\mp ax) - \ln ax - C].$
7. $\int_0^x x^\lambda e^{\pm ax} \operatorname{Ei}(\mp ax) dx =$

$$= -x^{\lambda+1} \sum_{k=0}^{\infty} \frac{(\pm ax)^k}{k! (k+\lambda+1)^2} [1 + (k+\lambda+1)(\psi(k+1) - \ln ax)]$$

$$\left[\operatorname{Re} \lambda > -1, \left\{ \begin{array}{l} |\arg a| < \pi \\ a > 0 \end{array} \right\} \right].$$
8. $\int_0^x x e^{ax} \operatorname{Ei}(-ax) dx = \frac{1}{a^2} [C - ax + \ln ax - (1 - ax) e^{ax} \operatorname{Ei}(-ax)].$
9. $\int x e^{ax} \operatorname{Ei}(-ax - b) dx =$

$$= \frac{1}{a} \left(x - \frac{1}{a} \right) e^{ax} \operatorname{Ei}(-ax - b) - \frac{e^{-b}}{a} \left[x - \frac{1}{a} (1+b) \ln(ax+b) \right].$$
10. $\int x e^{cx} \operatorname{Ei}(-ax - b) dx = \frac{1}{c} \left(x - \frac{1}{c} \right) e^{cx} \operatorname{Ei}(-ax - b) +$

$$+ \frac{1}{c(a-c)} e^{-(a-c)x-b} + \frac{1}{ac^2} (a+bc) e^{-bc/a} \operatorname{Ei} \left[\frac{(c-a)(ax+b)}{a} \right]$$
 $[a > c].$
11. $\int \frac{1}{x} e^{-cx} \operatorname{Ei}(-ax) dx = \operatorname{Ei}(-ax) \operatorname{Ei}(-cx) - \int \frac{1}{x} e^{-ax} \operatorname{Ei}(-cx) dx.$
12. $\int_x^\infty \frac{1}{x} e^{-cx} \operatorname{Ei}(-ax) dx = [C + \ln cx - \operatorname{Ei}(-cx)] \operatorname{Ei}(-ax) -$

$$- \frac{1}{2} [\zeta(2) + (C + \ln ax)^2] - e^{-ax} \sum_{k=0}^{\infty} \frac{1}{(k+1)^2} \left(-\frac{c}{a} \right)^{k+1} \sum_{m=0}^k \frac{(ax)^m}{m!} - \sum_{k=1}^{\infty} \frac{(-ax)^k}{k! k^2}.$$

$$13. \int \frac{1}{x} e^{-ax} \operatorname{Ei}(-ax) dx = \frac{1}{2} \operatorname{Ei}^2(-ax).$$

1.3.3. Интегралы вида $\int f(x) \operatorname{Ei}(-ax) dx$.

Условие: $a, b > 0$.

$$1. \int x^\lambda \ln x \operatorname{Ei}(-ax) dx = \frac{1}{(\lambda+1)a^{\lambda+1}} \left(\ln x - \frac{1}{\lambda+1} \right) \times \\ \times [(ax)^{\lambda+1} \operatorname{Ei}(-ax) - \gamma(\lambda+1, ax)] + \frac{x^{\lambda+1}}{\lambda+1} \sum_{k=0}^{\infty} \frac{(-ax)^k}{k!(k+\lambda+1)^2} \\ [\operatorname{Re} \lambda > -1].$$

$$2. \int x^n \ln x \operatorname{Ei}(-ax) dx = \\ = \frac{n! e^{-ax}}{(n+1)a^{n+1}} \left[\sum_{k=0}^{n-1} \frac{1}{k+1} \sum_{m=0}^k \frac{(ax)^m}{m!} - \left(\frac{1}{n+1} - \ln x \right) \sum_{k=0}^n \frac{(ax)^k}{k!} \right] + \\ + \frac{1}{n+1} \left[x^{n+1} \ln x - \frac{x^{n+1}}{n+1} - \frac{n!}{a^{n+1}} \right] \operatorname{Ei}(-ax).$$

$$3. \int \ln x \operatorname{Ei}(-ax) dx = -\frac{1}{a} [(1 - \ln x) e^{-ax} + (1 + ax - ax \ln x) \operatorname{Ei}(-ax)].$$

$$4. \int x \ln x \operatorname{Ei}(-ax) dx = \frac{1}{2a^2} \left[1 + (1 + ax) \ln x - \frac{1}{2} (1 + ax) \right] e^{-ax} - \\ - \frac{1}{2a^2} \left(1 + \frac{1}{2} a^2 x^2 - a^2 x^2 \ln x \right) \operatorname{Ei}(-ax).$$

$$5. \int e^{-bx} \ln x \operatorname{Ei}(-ax) dx = \frac{1}{b} \ln x [\operatorname{Ei}(-ax - bx) - \ln(ax + bx) - C] - \\ - \frac{1}{b} [e^{-bx} \ln x - \operatorname{Ei}(-bx)] \operatorname{Ei}(-ax) + \frac{1}{2b} \ln^2 x - \frac{1}{b} \sum_{k=1}^{\infty} \frac{(-1)^k (a+b)^k x^k}{k! k^2} - \\ - \frac{1}{b} \int \frac{1}{x} e^{-ax} \operatorname{Ei}(-bx) dx \quad [\text{см. 1.3.2.11—12}].$$

$$6. \int \begin{Bmatrix} \operatorname{sh} bx \\ \operatorname{ch} bx \end{Bmatrix} \operatorname{Ei}(-ax) dx = \\ = \frac{1}{b} \begin{Bmatrix} \operatorname{ch} bx \\ \operatorname{sh} bx \end{Bmatrix} \operatorname{Ei}(-ax) - \frac{1}{2b} [\operatorname{Ei}(bx - ax) \pm \operatorname{Ei}(-bx - ax)].$$

$$7. \int \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} \operatorname{Ei}(-ax) dx = \\ = \mp \frac{1}{b} \begin{Bmatrix} \cos bx \\ \sin bx \end{Bmatrix} \operatorname{Ei}(-ax) + \frac{i^{(1 \mp 1)/2}}{2b} [\operatorname{Ei}(-ax - ibx) \pm \operatorname{Ei}(-ax + ibx)]$$

1.3.4. Интегралы, содержащие произведения функций $\operatorname{Ei}(-ax)$.

Условие: $a, b > 0$.

$$1. \int x^\lambda \operatorname{Ei}(-ax) \operatorname{Ei}(-bx) dx = \frac{x^{\lambda+1}}{\lambda+1} \operatorname{Ei}(-ax) \operatorname{Ei}(-bx) - \\ - \frac{1}{\lambda+1} \int x^\lambda e^{-ax} \operatorname{Ei}(-bx) dx - \frac{1}{\lambda+1} \int x^\lambda e^{-bx} \operatorname{Ei}(-ax) dx \\ [\text{см. 1.3.2.1}].$$

- $$2. \int x^n \operatorname{Ei}(-ax) \operatorname{Ei}(-bx) dx = \frac{x^{n+1}}{n+1} \operatorname{Ei}(-ax) \operatorname{Ei}(-bx) -$$
- $$- \frac{n!}{n+1} \left(\frac{1}{a^{n+1}} + \frac{1}{b^{n+1}} \right) \operatorname{Ei}(-ax-bx) + \frac{n!}{(n+1)a^{n+1}} e^{-ax} \operatorname{Ei}(-bx) \sum_{k=0}^n \frac{(ax)^k}{k!} +$$
- $$+ \frac{n!}{(n+1)b^{n+1}} e^{-bx} \operatorname{Ei}(-ax) \sum_{k=0}^n \frac{(bx)^k}{k!} +$$
- $$+ \frac{n!}{n+1} e^{-(a+b)x} \sum_{k=1}^n \frac{1}{k(a+b)^k} (a^{k-n-1} + b^{k-n-1}) \sum_{m=0}^{k-1} \frac{(a+b)^m x^m}{m!}.$$
- $$3. \int \operatorname{Ei}(-ax) \operatorname{Ei}(bx) dx = x \operatorname{Ei}(-ax) \operatorname{Ei}(bx) + \left(\frac{1}{b} - \frac{1}{a} \right) \operatorname{Ei}(bx-ax) +$$
- $$+ \frac{1}{a} e^{-ax} \operatorname{Ei}(bx) - \frac{1}{b} e^{bx} \operatorname{Ei}(-ax) \quad [a > b].$$
- $$4. \int \operatorname{Ei}(-ax) \operatorname{Ei}(ax) dx = x \operatorname{Ei}(-ax) \operatorname{Ei}(ax) + \frac{1}{a} [e^{-ax} \operatorname{Ei}(ax) - e^{ax} \operatorname{Ei}(-ax)].$$
- $$5. \int x \operatorname{Ei}(-ax) \operatorname{Ei}(-bx) dx = \frac{x^2}{2} \operatorname{Ei}(-ax) \operatorname{Ei}(-bx) -$$
- $$- \frac{a^2 + b^2}{2a^2b^2} \operatorname{Ei}(-ax-bx) + \frac{1}{2a^2} (1+ax) e^{-ax} \operatorname{Ei}(-bx) +$$
- $$+ \frac{1}{2b^2} (1+bx) e^{-bx} \operatorname{Ei}(-ax) + \frac{1}{2ab} e^{-(a+b)x}.$$
- $$6. \int \operatorname{Ei}^3(-ax) dx = x \operatorname{Ei}^3(-ax) + \frac{3}{a} e^{-ax} \operatorname{Ei}(-ax) - \frac{6}{a} \int \frac{1}{x} e^{-2ax} \operatorname{Ei}(-ax) dx.$$

1.4. ИНТЕГРАЛЬНЫЕ СИНУС $\operatorname{si}(x)$ И КОСИНУС $\operatorname{ci}(x)$

1.4.1. Интегралы вида $\int x^\lambda \begin{Bmatrix} \operatorname{si}(ax) \\ \operatorname{ci}(ax) \end{Bmatrix} dx$.

Условие. $a, b > 0$.

- $$1. \int x^\lambda \begin{Bmatrix} \operatorname{si}(ax) \\ \operatorname{ci}(ax) \end{Bmatrix} dx = \frac{x^{\lambda+1}}{\lambda+1} \begin{Bmatrix} \operatorname{si}(ax) \\ \operatorname{ci}(ax) \end{Bmatrix} + \frac{1}{\lambda+1} \begin{Bmatrix} S(x, \lambda+1) \\ C(x, \lambda+1) \end{Bmatrix}.$$
- $$2. \int x^n \begin{Bmatrix} \operatorname{si}(ax) \\ \operatorname{ci}(ax) \end{Bmatrix} dx =$$
- $$= \frac{x^{n+1}}{n+1} \begin{Bmatrix} \operatorname{si}(ax) \\ \operatorname{ci}(ax) \end{Bmatrix} + \frac{n!}{a^{n+1}(n+1)} \sum_{k=0}^n \frac{(ax)^k}{k!} \begin{Bmatrix} \sin[ax + (n-k+1)\pi/2] \\ \cos[ax + (n-k+1)\pi/2] \end{Bmatrix}.$$
- $$3. \int_x^\infty \frac{1}{x^n} \begin{Bmatrix} \operatorname{si}(ax) \\ \operatorname{ci}(ax) \end{Bmatrix} dx =$$
- $$= \frac{1}{(n-1)! (n-1) x^{n-1}} \left[(n-1)! - (-ax)^{n-1} \cos \frac{n-1}{2} \pi \right] \begin{Bmatrix} \operatorname{si}(ax) \\ \operatorname{ci}(ax) \end{Bmatrix} \pm$$
- $$\pm (-ax)^{n-1} \sin \frac{n-1}{2} \pi \begin{Bmatrix} \operatorname{ci}(ax) \\ \operatorname{si}(ax) \end{Bmatrix} + \sum_{k=0}^{n-2} (n-k-2)! (ax)^k \begin{Bmatrix} \sin(ax + k\pi/2) \\ \cos(ax + k\pi/2) \end{Bmatrix}.$$
- $$4. \int \begin{Bmatrix} \operatorname{si}(ax) \\ \operatorname{ci}(ax) \end{Bmatrix} dx = x \begin{Bmatrix} \operatorname{si}(ax) \\ \operatorname{ci}(ax) \end{Bmatrix} \pm \frac{1}{a} \begin{Bmatrix} \cos ax \\ \sin ax \end{Bmatrix}.$$
- $$5. \int x \begin{Bmatrix} \operatorname{si}(ax) \\ \operatorname{ci}(ax) \end{Bmatrix} dx = \frac{x^2}{2} \begin{Bmatrix} \operatorname{si}(ax) \\ \operatorname{ci}(ax) \end{Bmatrix} - \frac{1}{2a^2} \left[\begin{Bmatrix} \sin ax \\ \cos ax \end{Bmatrix} \mp ax \begin{Bmatrix} \cos ax \\ \sin ax \end{Bmatrix} \right].$$

$$6. \int \frac{1}{x} \left\{ \begin{matrix} \text{si} (ax) \\ \text{ci} (ax) \end{matrix} \right\} dx = \ln x \left\{ \begin{matrix} \text{si} (ax) \\ \text{ci} (ax) \end{matrix} \right\} - \int \frac{1}{x} \ln x \left\{ \begin{matrix} \sin ax \\ \cos ax \end{matrix} \right\} dx.$$

$$7. \int \frac{1}{x^2} \left\{ \begin{matrix} \text{si} (ax + b) \\ \text{ci} (ax + b) \end{matrix} \right\} dx = \\ = \frac{a}{b} \left[\left\{ \begin{matrix} \sin b \\ \cos b \end{matrix} \right\} \text{ci} (ax) \pm \left\{ \begin{matrix} \cos b \\ \sin b \end{matrix} \right\} \text{si} (ax) \right] - \frac{1}{bx} (ax + b) \left\{ \begin{matrix} \text{si} (ax + b) \\ \text{ci} (ax + b) \end{matrix} \right\}.$$

$$8. \int_1^{\infty} \frac{1}{x^2} \left\{ \begin{matrix} \text{si} (ax) \\ \text{ci} (ax) \end{matrix} \right\} dx = \frac{1}{x} \left[\left\{ \begin{matrix} \text{si} (ax) \\ \text{ci} (ax) \end{matrix} \right\} + \left\{ \begin{matrix} \sin ax \\ \cos ax \end{matrix} \right\} \right] \mp a \left\{ \begin{matrix} \text{ci} (ax) \\ \text{si} (ax) \end{matrix} \right\}.$$

1.4.2. Интегралы вида $\int f(x) \left\{ \begin{matrix} \text{si} (ax) \\ \text{ci} (ax) \end{matrix} \right\} dx$

$$1. \int \frac{1}{x} e^{-bx} \left\{ \begin{matrix} \text{si} (ax) \\ \text{ci} (ax) \end{matrix} \right\} dx = \text{Ei} (-bx) \left\{ \begin{matrix} \text{si} (ax) \\ \text{ci} (ax) \end{matrix} \right\} - \int \frac{1}{x} \text{Ei} (-bx) \left\{ \begin{matrix} \sin ax \\ \cos ax \end{matrix} \right\} dx$$

$$2. \int_0^x \left\{ \begin{matrix} \sin bx \\ \cos bx \end{matrix} \right\} \text{ci} (ax) dx = \\ = \mp \frac{1}{b} \left\{ \begin{matrix} \cos bx \\ \sin bx \end{matrix} \right\} \text{ci} (ax) \pm \frac{1}{2b} \left\{ \begin{matrix} \text{ci} (ax + bx) + \text{ci} | ax - bx | \\ \text{si} (ax + bx) + \text{si} | ax - bx | \end{matrix} \right\} - \frac{1}{2b} \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \ln \left| 1 - \frac{b^2}{a^2} \right| \\ \left[\begin{matrix} a, b > 0; a \neq b \\ a, b > 0 \end{matrix} \right].$$

$$3. \int_0^x \left\{ \begin{matrix} \sin bx \\ \cos bx \end{matrix} \right\} \text{si} (ax) dx = \\ = \mp \frac{1}{b} \left\{ \begin{matrix} \cos bx \\ \sin bx \end{matrix} \right\} \text{si} (ax) + \frac{1}{2b} \left\{ \begin{matrix} \text{si} (ax + bx) - \text{si} | ax - bx | \\ \text{ci} (ax + bx) - \text{ci} | ax - bx | \end{matrix} \right\} + \frac{1}{2b} \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \ln \left| \frac{a-b}{a+b} \right| \\ \left[\begin{matrix} a, b > 0 \\ a, b > 0; a \neq b \end{matrix} \right].$$

$$4. \int_0^x \left\{ \begin{matrix} \cos ax \text{ si} (ax) \\ \sin ax \text{ ci} (ax) \end{matrix} \right\} dx = \frac{1}{2a} \left[\text{ci} (2ax) \pm 2 \left\{ \begin{matrix} \sin ax \text{ si} (ax) \\ \cos ax \text{ ci} (ax) \end{matrix} \right\} \mp \ln ax - \ln 2 \mp C \right]$$

$[a > 0].$

1.4.3. Интегралы, содержащие произведения функций $\text{si} (ax)$ и $\text{ci} (bx)$.

$$1. \int \text{ci} (ax) \text{ci} (bx) dx = x \text{ci} (ax) \text{ci} (bx) + \frac{1}{2a} [\text{si} (ax + bx) + \text{si} (ax - bx)] + \\ + \frac{1}{2b} [\text{si} (ax + bx) + \text{si} (bx - ax)] - \frac{1}{a} \sin ax \text{ci} (bx) - \frac{1}{b} \sin bx \text{ci} (ax) \\ [a, b > 0].$$

$$2. \int \text{si} (ax) \text{si} (bx) dx = x \text{si} (ax) \text{si} (bx) - \frac{1}{2b} [\text{si} (ax + bx) + \text{si} (ax - bx)] - \\ - \frac{1}{2a} [\text{si} (ax + bx) + \text{si} (bx - ax)] + \frac{1}{a} \cos ax \text{si} (bx) + \frac{1}{b} \cos bx \text{si} (ax) \\ [a, b > 0].$$

$$3. \int \text{si} (ax) \text{ci} (bx) dx = x \text{si} (ax) \text{ci} (bx) + \frac{1}{a} \cos ax \text{ci} (bx) - \\ - \frac{1}{b} \sin bx \text{si} (ax) - \frac{a+b}{2ab} \text{ci} (ax + bx) + \frac{a-b}{2ab} \text{ci} (ax - bx) \\ [a > b > 0].$$

1.5. ИНТЕГРАЛЫ ВЕРОЯТНОСТИ $\operatorname{erf}(x)$, $\operatorname{erfc}(x)$

1.5.1. Интегралы вида $\int x^\lambda \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx$

$$1. \int x^\lambda \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx = \frac{\lambda(\lambda-1)}{2(\lambda+1)a^2} \int x^{\lambda-2} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx + \\ + \left(x^2 - \frac{\lambda}{2a^2} \right) \frac{x^{\lambda-1}}{\lambda+1} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} \pm \frac{1}{a\sqrt{\pi}(\lambda+1)} x^\lambda e^{-a^2 x^2}.$$

$$2. = \frac{x^{\lambda+1}}{\lambda+1} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} \mp \frac{1}{(\lambda+1)a^{\lambda+1}\sqrt{\pi}} \gamma\left(\frac{\lambda}{2}+1, a^2 x^2\right)$$

[$\operatorname{Re} \lambda > -(3 \pm 1)/2$; $\lambda \neq -1$].

$$3. = \frac{x^{\lambda+1}}{\lambda+1} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} \pm \frac{1}{(\lambda+1)a^{\lambda+1}\sqrt{\pi}} \Gamma\left(\frac{\lambda}{2}+1, a^2 x^2\right).$$

$$4. \int x^{2n} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx = \frac{x^{2n+1}}{2n+1} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} \pm \frac{n!}{a^{2n+1}\sqrt{\pi}(2n+1)} e^{-a^2 x^2} \sum_{k=0}^n \frac{(ax)^{2k}}{k!}.$$

$$5. \int x^{2n+1} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx = \frac{x^{2n+2}}{2n+2} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} \pm \\ \pm \frac{(2n+1)!! x}{2^{n+1} a^{2n+1} (n+1) \sqrt{\pi}} e^{-a^2 x^2} \sum_{k=0}^n \frac{(2a^2 x^2)^k}{(2k+1)!!} \mp \frac{\Gamma(n+3/2)}{2(n+1)a^{2n+2}\sqrt{\pi}} \operatorname{erf}(ax).$$

$$6. \int (ax+b)^n \operatorname{erf}(x) dx = \\ = \frac{1}{(n+1)a} (ax+b)^{n+1} \operatorname{erf}(x) - \frac{b^{n+1}}{\sqrt{\pi}(n+1)a} \sum_{k=0}^{n+1} \binom{n+1}{k} \left(\frac{a}{b}\right)^k \gamma\left(\frac{k+1}{2}, x^2\right).$$

$$7. \int \frac{1}{x^{2n}} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx = -\frac{1}{(2n-1)x^{2n-1}} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} \pm \\ \pm \frac{x^{-2n}}{a\sqrt{\pi}(n-1)!(2n-1)} e^{-a^2 x^2} \sum_{k=1}^{n-1} (-1)^k (n-k-1)! (ax)^{2k} \pm \\ \pm \frac{(-1)^{n-1} a^{2n-1}}{\sqrt{\pi}(n-1)!(2n-1)} \operatorname{Ei}(-a^2 x^2).$$

$$8. \int \frac{1}{x^{2n+1}} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx = -\frac{1}{2nx^{2n}} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} \pm \\ \pm \frac{1}{2an\sqrt{\pi}\Gamma(n+1/2)} e^{-a^2 x^2} \sum_{k=1}^n (-1)^k \Gamma\left(n-k+\frac{1}{2}\right) a^{2k} x^{2k-2n-1} \pm \\ \pm \frac{(-1)^n a^{2n} \sqrt{\pi}}{2n\Gamma(n+1/2)} \operatorname{erf}(ax).$$

$$9. \int \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx = x \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} \pm \frac{1}{a\sqrt{\pi}} e^{-a^2 x^2}.$$

$$10. \int x \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx = \frac{x^2}{2} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} \mp \frac{1}{4a^2} \operatorname{erf}(ax) \pm \frac{x}{2a\sqrt{\pi}} e^{-a^2 x^2}.$$

$$11. \int \frac{1}{x} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx = \ln x \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} \mp \frac{2a}{\sqrt{\pi}} \int \ln x e^{-a^2 x^2} dx.$$

$$12. \int \frac{x^2-b}{(x^2+b)^2} \operatorname{erf}(ax) dx = -\frac{x}{x^2+b} \operatorname{erf}(ax) + \frac{a}{\sqrt{\pi}} e^{a^2 b} \operatorname{Ei}(-a^2 x^2 - a^2 b).$$

1.5.2. Интегралы вида $\int x^\lambda e^{bx} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx$.

$$1. \int x^\lambda e^{bx} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx = \frac{1}{b} x^\lambda e^{bx} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} - \\ - \frac{\lambda}{b} \int x^{\lambda-1} e^{bx} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx \mp \frac{2a}{b\sqrt{\pi}} \int x^\lambda e^{-a^2x^2+bx} dx.$$

$$2. \int x^n e^{bx} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx = (-1)^n \frac{n!}{b^{n+1}} e^{bx} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} \sum_{k=0}^n \frac{(-bx)^k}{k!} \mp \\ \mp (-1)^n \frac{2an!}{\sqrt{\pi} b^{n+1}} \sum_{k=0}^n \frac{(-b)^k}{k!} \int x^k e^{-a^2x^2+bx} dx.$$

$$3. \int e^{bx} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx = \frac{1}{b} e^{bx} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} \mp \frac{1}{b} e^{b^2/(4a^2)} \left\{ \begin{matrix} \operatorname{erf}[ax - b/(2a)] \\ \operatorname{erfc}[ax - b/(2a)] \end{matrix} \right\}.$$

$$4. \int x e^{bx} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx = \frac{1}{b} \left(x - \frac{1}{b} \right) e^{bx} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} \mp \\ \mp \frac{1}{b} e^{b^2/(4a^2)} \left[\left(\frac{b}{2a^2} - \frac{1}{b} \right) \operatorname{erf} \left(ax - \frac{b}{2a} \right) - \frac{1}{a\sqrt{\pi}} e^{-(2ax - b/a)^2/4} \right].$$

1.5.3. Интегралы вида $\int x^\lambda e^{-b^2x^2} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx$.

$$1. \int x^\lambda e^{-b^2x^2} \operatorname{erf}(ax) dx = -\frac{1}{2b^2} x^{\lambda-1} e^{-b^2x^2} \operatorname{erf}(ax) + \\ + \frac{a}{b^2\sqrt{\pi}} \int x^{\lambda-1} e^{-b^2x^2} dx + \frac{\lambda-1}{2b^2} \int x^{\lambda-2} e^{-b^2x^2} \operatorname{erf}(ax) dx.$$

$$2. \int_0^x x^\lambda e^{a^2x^2} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx = \\ = \pm \frac{2ax^{\lambda+2}}{\sqrt{\pi}(\lambda+2)} {}_2F_2 \left(1, \frac{\lambda}{2} + 1; \frac{3}{2}, \frac{\lambda}{2} + 2; a^2x^2 \right) + \begin{cases} 0 \\ 1 \end{cases} \frac{x^{\lambda+1}}{\lambda+1} {}_1F_1 \left(\frac{\lambda+1}{2}; \frac{\lambda+3}{2}; a^2x^2 \right) \\ [\operatorname{Re} \lambda > -(3 \pm 1)/2].$$

$$3. \int_x^\infty x^\lambda e^{\mp a^2x^2} \left\{ \begin{matrix} \operatorname{erfi}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx = \\ = \mp \frac{2ax^{\lambda+2}}{\sqrt{\pi}(\lambda+2)} {}_2F_2 \left(1, \frac{\lambda}{2} + 1; \frac{3}{2}, \frac{\lambda}{2} + 2; \mp a^2x^2 \right) - \frac{\Gamma((\lambda+1)/2)}{2a^{\lambda+1}} \left\{ \begin{matrix} \operatorname{ctg}(\lambda\pi/2) \\ \operatorname{cosec}(\lambda\pi/2) \end{matrix} \right\} - \\ - \begin{cases} 0 \\ 1 \end{cases} \frac{x^{\lambda+1}}{\lambda+1} {}_1F_1 \left(\frac{\lambda+1}{2}; \frac{\lambda+3}{2}; a^2x^2 \right) \quad [\operatorname{Re} \lambda < 0; |\arg a| < \pi/4].$$

$$4. \int_0^x x^\lambda e^{-a^2x^2} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfi}(ax) \end{matrix} \right\} dx = \frac{2a}{\sqrt{\pi}(\lambda+2)} x^{\lambda+2} {}_2F_2 \left(1, \frac{\lambda+2}{2}; \frac{3}{2}, \frac{\lambda}{2} + 2; \pm a^2x^2 \right) \\ [\operatorname{Re} \lambda > -2].$$

$$5. \int x^{2n} e^{-a^2x^2} \operatorname{erf}(ax) dx = \\ = \frac{(2n)!}{n!} \frac{\sqrt{\pi}}{(2a)^{2n+1}} \left[\frac{1}{2} \operatorname{erf}^2(ax) - \frac{2a}{\sqrt{\pi}} x e^{-a^2x^2} \operatorname{erf}(ax) \sum_{k=0}^{n-1} \frac{k!}{(2k+1)!} (4a^2x^2)^k - \right. \\ \left. - \frac{1}{\pi} e^{-2a^2x^2} \sum_{k=0}^{n-1} \frac{2^k (k!)^2}{(2k+1)!} \sum_{m=0}^k \frac{(2a^2x^2)^m}{m!} \right].$$

$$\begin{aligned}
6. \int x^{2n+1} e^{-a^2 x^2} \operatorname{erf}(ax) dx &= \frac{n!}{a^{2n+2}} \left[\frac{1}{2^{3/2}} \operatorname{erf}(ax \sqrt{2}) \sum_{k=0}^n \binom{2k}{k} \frac{1^k}{8} - \right. \\
&\quad \left. - \frac{1}{2} e^{-a^2 x^2} \operatorname{erf}(ax) \sum_{k=0}^n \frac{(a^2 x^2)^k}{k!} - \frac{ax}{\sqrt{\pi}} e^{-2a^2 x^2} \sum_{k=1}^n \binom{2k}{k} \frac{1}{8^k} \sum_{m=0}^{k-1} \frac{m!}{(2m+1)!} (8a^2 x^2)^m \right]. \\
7. \int e^{-a^2 x^2} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx &= \pm \frac{\sqrt{\pi}}{4a} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\}^2. \\
8. \int x e^{a^2 x^2} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx &= \frac{1}{2a^2} e^{a^2 x^2} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} \mp \frac{x}{a \sqrt{\pi}}. \\
9. \int x e^{-b^2 x^2} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx &= \pm \frac{a}{2b^2 \sqrt{a^2 + b^2}} \operatorname{erf}(\sqrt{a^2 + b^2} x) - \frac{e^{-b^2 x^2}}{2b^2} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\}. \\
10. \int x e^{-b^2 x^2} \left\{ \begin{matrix} \operatorname{erf}(ax+c) \\ \operatorname{erfc}(ax+c) \end{matrix} \right\} dx &= \\
&= \pm \frac{a}{2b^2 \sqrt{a^2 + b^2}} e^{-b^2 c^2 / (a^2 + b^2)} \operatorname{erf} \left(x \sqrt{a^2 + b^2} + \frac{ac}{\sqrt{a^2 + b^2}} \right) - \\
&\quad - \frac{1}{2b^2} e^{-b^2 x^2} \left\{ \begin{matrix} \operatorname{erf}(ax+c) \\ \operatorname{erfc}(ax+c) \end{matrix} \right\}. \\
11. \int \frac{1}{x^2} e^{-a^2 x^2} \operatorname{erf}(ax) dx &= -\frac{1}{x} e^{-a^2 x^2} \operatorname{erf}(ax) - \frac{a \sqrt{\pi}}{2} \operatorname{erf}^2(ax) + \frac{a}{\sqrt{\pi}} \operatorname{Ei}(-2a^2 x^2). \\
12. \int [a e^{-a^2 x^2} \operatorname{erf}(bx) + b e^{-b^2 x^2} \operatorname{erf}(ax)] dx &= \frac{\sqrt{\pi}}{2} \operatorname{erf}(ax) \operatorname{erf}(bx).
\end{aligned}$$

1.5.4. Интегралы, содержащие гиперболические и тригонометрические функции и $\left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\}$.

$$\begin{aligned}
1. \int \operatorname{sh} bx \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx &= \\
&= \frac{1}{b} \operatorname{ch} bx \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} \mp \frac{1}{2b} e^{b^2 / (4a^2)} \left[\operatorname{erf} \left(ax + \frac{b}{2a} \right) + \operatorname{erf} \left(ax - \frac{b}{2a} \right) \right]. \\
2. \int \operatorname{ch} bx \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx &= \\
&= \frac{1}{b} \operatorname{sh} bx \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} \pm \frac{1}{2b} e^{b^2 / (4a^2)} \left[\operatorname{erf} \left(ax + \frac{b}{2a} \right) - \operatorname{erf} \left(ax - \frac{b}{2a} \right) \right]. \\
3. \int \sin bx \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx &= \\
&= -\frac{1}{b} \cos bx \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} \pm \frac{1}{2b} e^{-b^2 / (4a^2)} \left[\operatorname{erf} \left(ax - i \frac{b}{2a} \right) + \operatorname{erf} \left(ax + i \frac{b}{2a} \right) \right]. \\
4. \int \cos bx \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx &= \\
&= \frac{1}{b} \sin bx \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} \pm \frac{i}{2b} e^{-b^2 / (4a^2)} \left[\operatorname{erf} \left(ax - \frac{ib}{2a} \right) - \operatorname{erf} \left(ax + \frac{ib}{2a} \right) \right]. \\
5. \int e^{-bx} \sin cx \operatorname{erf}(ax) dx &= -\frac{1}{b^2 + c^2} e^{-bx} (c \cos cx + b \sin cx) \operatorname{erf}(ax) + \\
&\quad + \frac{1}{b^2 + c^2} e^{(b^2 - c^2) / (4a^2)} \left[\frac{c + ib}{2} e^{ibc / (2a^2)} \operatorname{erf} \left(ax + \frac{b + ic}{2a} \right) \right] + \\
&\quad + \frac{c - ib}{2} e^{-ibc / (2a^2)} \operatorname{erf} \left(ax + \frac{b - ic}{2a} \right) \right].
\end{aligned}$$

$$\begin{aligned}
6. \int x e^{-bx^2} \left\{ \begin{matrix} \sin cx^2 \\ \cos cx^2 \end{matrix} \right\} \operatorname{erf}(ax) dx = \\
= \frac{1}{4(b-ic)} \left[\frac{a}{\sqrt{a^2+b-ic}} \operatorname{erf}(x\sqrt{a^2+b-ic}) - e^{-(b-ic)x^2} \operatorname{erf}(ax) \right] \left\{ \begin{matrix} -i \\ 1 \end{matrix} \right\} \mp \\
\mp \frac{1}{4(b+ic)} \left[\frac{a}{\sqrt{a^2+b+ic}} \operatorname{erf}(x\sqrt{a^2+b+ic}) - e^{-(b+ic)x^2} \operatorname{erf}(ax) \right].
\end{aligned}$$

1.5.5. Интегралы вида $\int x^\lambda \ln x \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx$.

$$\begin{aligned}
1. \int x^\lambda \ln x \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx = \frac{1}{\lambda+1} \left[x^{\lambda+1} \ln x \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} \pm \right. \\
\pm \frac{1}{a\sqrt{\pi}} x^\lambda e^{-a^2x^2} \ln x - \int x^\lambda \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx \mp \frac{1}{a\sqrt{\pi}} \int x^{\lambda-1} e^{-a^2x^2} dx \mp \\
\left. \mp \frac{\lambda}{a\sqrt{\pi}} \int x^{\lambda-1} e^{-a^2x^2} \ln x dx \right] \left\{ \begin{matrix} -i \\ 1 \end{matrix} \right\}.
\end{aligned}$$

$$\begin{aligned}
2. \int x^\lambda \ln x \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx = \\
= \frac{\lambda^2+2\lambda-1}{2a^2(\lambda+1)^2} \int x^{\lambda-2} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx + \frac{\lambda(\lambda-1)}{2a^2(\lambda+1)} \int x^{\lambda-2} \ln x \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx + \\
+ x^{\lambda-1} \frac{(\lambda+1) \ln x - 1}{(\lambda+1)^2} \left[\left(x^2 - \frac{\lambda}{2a^2} \right) \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} \pm \frac{x}{a\sqrt{\pi}} e^{-a^2x^2} \right] - \\
- \frac{x^{\lambda+1}}{2a^2(\lambda+1)} \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} - \frac{x^\lambda}{2a^2(\lambda+1)} e^{-a^2x^2}.
\end{aligned}$$

$$\begin{aligned}
3. \int \ln x \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx = \\
= (\ln x - 1) \left[x \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} \pm \frac{1}{a\sqrt{\pi}} e^{-a^2x^2} \right] \mp \frac{1}{2a\sqrt{\pi}} \operatorname{Ei}(-a^2x^2).
\end{aligned}$$

$$\begin{aligned}
4. \int x \ln x \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} dx = \frac{x^2}{4} (2 \ln x - 1) \left\{ \begin{matrix} \operatorname{erf}(ax) \\ \operatorname{erfc}(ax) \end{matrix} \right\} \pm \\
\pm \frac{x}{4a\sqrt{\pi}} e^{-a^2x^2} (2 \ln x - 1) \mp \frac{1}{8a^2} \operatorname{erf}(ax) \mp \frac{1}{2a\sqrt{\pi}} \int e^{-a^2x^2} \ln x dx.
\end{aligned}$$

1.5.6. Интегралы вида $\int x^\lambda \operatorname{erf}(ax) \operatorname{erf}(bx) dx$.

$$\begin{aligned}
1. \int x^\lambda \operatorname{erf}(ax) \operatorname{erf}(bx) dx = \frac{1}{\lambda+1} x^{\lambda+1} \operatorname{erf}(ax) \operatorname{erf}(bx) - \\
- \frac{2b}{(\lambda+1)\sqrt{\pi}} \int x^{\lambda+1} e^{-b^2x^2} \operatorname{erf}(ax) dx - \frac{2a}{(\lambda+1)\sqrt{\pi}} \int x^{\lambda+1} e^{-a^2x^2} \operatorname{erf}(bx) dx \\
\text{[см. 1.5.3].}
\end{aligned}$$

$$\begin{aligned}
2. \int \operatorname{erf}(ax) \operatorname{erf}(bx) dx = x \operatorname{erf}(ax) \operatorname{erf}(bx) - \\
- \frac{\sqrt{a^2+b^2}}{ab\sqrt{\pi}} \operatorname{erf}(x\sqrt{a^2+b^2}) + \frac{1}{a\sqrt{\pi}} e^{-a^2x^2} \operatorname{erf}(bx) + \frac{1}{b\sqrt{\pi}} e^{-b^2x^2} \operatorname{erf}(ax).
\end{aligned}$$

$$3. \int \operatorname{erf}^2(ax) dx = x \operatorname{erf}^2(ax) - \frac{1}{a} \sqrt{\frac{2}{\pi}} \operatorname{erf}(ax\sqrt{2}) + \frac{2}{a\sqrt{\pi}} e^{-a^2x^2} \operatorname{erf}(ax)$$

$$4. \int x^{2n} \operatorname{erf}^2(ax) dx = \frac{x^{2n+1}}{2n+1} \operatorname{erf}^2(ax) - \frac{4(n!)}{(2n+1)\sqrt{\pi} a^{2n+1}} \left[\frac{1}{2\sqrt{2}} \operatorname{erf}(ax\sqrt{2}) \sum_{k=0}^n \binom{2k}{k} \frac{1}{8^k} - \frac{1}{2} e^{-a^2 x^2} \operatorname{erf}(ax) \sum_{k=0}^n \frac{(a^2 x^2)^k}{k!} - \frac{ax}{\sqrt{\pi}} e^{-2a^2 x^2} \sum_{k=0}^n \binom{2k}{k} \frac{1}{8^k} \sum_{m=0}^{k-1} \frac{m!}{(2m+1)!} (8a^2 x^2)^m \right].$$

$$5. \int x^{2n+1} \operatorname{erf}^2(ax) dx = \left[x^{2n+2} - \frac{2(2n+1)!}{(2a)^{2n+2} n!} \right] \frac{\operatorname{erf}^2(ax)}{2n+2} + \frac{(2n+1)!}{(2a)^{2n+1} (n+1)!} \left[\frac{2x}{\sqrt{\pi}} e^{-a^2 x^2} \operatorname{erf}(ax) \sum_{k=0}^n \frac{k!}{(2k+1)!} (4a^2 x^2)^k + \frac{1}{\pi a} e^{-2a^2 x^2} \sum_{k=0}^n \frac{2^k (k!)^2}{(2k+1)!} \sum_{m=0}^k \frac{(2a^2 x^2)^m}{m!} \right].$$

$$6. \int x \operatorname{erf}^2(ax) dx = \left(\frac{x^2}{2} - \frac{1}{4a^2} \right) \operatorname{erf}^2(ax) + \frac{x}{a\sqrt{\pi}} e^{-a^2 x^2} \operatorname{erf}(ax) + \frac{1}{2\pi a^2} e^{-2a^2 x^2}.$$

$$7. \int \operatorname{erf}(ax) \operatorname{erf}(iax) dx = x \operatorname{erf}(ax) \operatorname{erf}(iax) + \frac{1}{a\sqrt{\pi}} e^{-a^2 x^2} \operatorname{erf}(iax) - \frac{i}{a\sqrt{\pi}} e^{a^2 x^2} \operatorname{erf}(ax)$$

$$8. \int_0^x x^\lambda \operatorname{erf}(ax) \operatorname{erf}(iax) dx = \frac{4ia^2 x^{\lambda+3}}{\pi(\lambda+3)} {}_3F_4\left(\frac{1}{2}, 1, \frac{\lambda+3}{4}; \frac{3}{4}, \frac{5}{4}, \frac{3}{2}, \frac{\lambda+7}{4}; \frac{a^4 x^4}{4}\right) \quad [\operatorname{Re} \lambda > -3]$$

$$9. \int e^{-a^2 x^2} \operatorname{erf}^\mu(ax) dx = \frac{\sqrt{\pi}}{2a(\mu+1)} \operatorname{erf}^{\mu+1}(ax).$$

$$10. \int \frac{e^{-a^2 x^2}}{\operatorname{erf}(ax)} dx = \frac{\sqrt{\pi}}{2a} \ln[\operatorname{erf}(ax)].$$

1.6. ИНТЕГРАЛЫ ФРЕНЕЛЯ $S(x)$ И $C(x)$

1.6.1. Интегралы вида $\int x^\lambda \left\{ \begin{matrix} S(ax) \\ C(ax) \end{matrix} \right\} dx$.

$$1. \int x^\lambda \left\{ \begin{matrix} S(ax) \\ C(ax) \end{matrix} \right\} dx = \frac{x^{\lambda+1}}{\lambda+1} \left\{ \begin{matrix} S(ax) \\ C(ax) \end{matrix} \right\} + \frac{1}{(\lambda+1)a^{\lambda+1}\sqrt{2\pi}} \left\{ \begin{matrix} S(ax, \lambda+3/2) \\ C(ax, \lambda+3/2) \end{matrix} \right\}.$$

$$2. \int x^{n-1/2} \left\{ \begin{matrix} S(ax) \\ C(ax) \end{matrix} \right\} dx = \frac{2x^{n+1/2}}{2n+1} \left\{ \begin{matrix} S(ax) \\ C(ax) \end{matrix} \right\} \pm \frac{n!}{2n+1} \sqrt{\frac{2}{\pi a}} \sum_{k=0}^n \frac{1}{(n-k)!} \frac{x^{n-k}}{a^k} \left\{ \begin{matrix} \cos(ax+k\pi/2) \\ \sin(ax+k\pi/2) \end{matrix} \right\}.$$

$$3. \int x^n \left\{ \begin{matrix} S(ax) \\ C(ax) \end{matrix} \right\} dx = \frac{x^{n+1}}{n+1} \left\{ \begin{matrix} S(ax) \\ C(ax) \end{matrix} \right\} - \frac{\sqrt{a}}{(n+1)\sqrt{2\pi}} \int x^{n+1/2} \left\{ \begin{matrix} \sin ax \\ \cos ax \end{matrix} \right\} dx.$$

$$4. \int \left\{ \begin{matrix} S(ax) \\ C(ax) \end{matrix} \right\} dx = x \left\{ \begin{matrix} S(ax) \\ C(ax) \end{matrix} \right\} \mp \frac{1}{2a} \left\{ \begin{matrix} C(ax) \\ S(ax) \end{matrix} \right\} \pm \sqrt{\frac{x}{2\pi a}} \left\{ \begin{matrix} \cos ax \\ \sin ax \end{matrix} \right\}.$$

$$5. \int \frac{1}{\sqrt{x}} \left\{ \begin{matrix} S(ax) \\ C(ax) \end{matrix} \right\} dx = 2\sqrt{x} \left\{ \begin{matrix} S(ax) \\ C(ax) \end{matrix} \right\} \pm \sqrt{\frac{2}{\pi a}} \left\{ \begin{matrix} \cos ax \\ \sin ax \end{matrix} \right\}.$$

1.6.2. Интегралы вида $\int f(x) \begin{Bmatrix} S(ax) \\ C(ax) \end{Bmatrix} dx$.

1. $\int \frac{1}{\sqrt{x}} e^{-b\sqrt{x}} \begin{Bmatrix} S(ax) \\ C(ax) \end{Bmatrix} dx =$
 $= -\frac{2}{b} e^{-b\sqrt{x}} \begin{Bmatrix} S(ax) \\ C(ax) \end{Bmatrix} + \frac{1}{b} \left[S\left(ax + ib\sqrt{x} - \frac{b^2}{4a}\right) + S\left(ax - ib\sqrt{x} - \frac{b^2}{4a}\right) \pm \right.$
 $\pm iC\left(ax - ib\sqrt{x} - \frac{b^2}{4a}\right) \mp iC\left(ax + ib\sqrt{x} - \frac{b^2}{4a}\right) \left. \begin{Bmatrix} \cos[b^2/(4a)] \\ \sin[b^2/(4a)] \end{Bmatrix} + \right.$
 $\left. + \frac{1}{b} \left[C\left(ax + ib\sqrt{x} - \frac{b^2}{4a}\right) + C\left(ax - ib\sqrt{x} - \frac{b^2}{4a}\right) + \right.$
 $\left. + iS\left(ax + ib\sqrt{x} - \frac{b^2}{4a}\right) - iS\left(ax - ib\sqrt{x} - \frac{b^2}{4a}\right) \right] \begin{Bmatrix} \sin[b^2/(4a)] \\ \cos[b^2/(4a)] \end{Bmatrix}.$
2. $\int \sin bx \begin{Bmatrix} S(ax) \\ C(ax) \end{Bmatrix} dx = -\frac{\cos bx}{b} \begin{Bmatrix} S(ax) \\ C(ax) \end{Bmatrix} +$
 $+ \frac{\sqrt{a}}{2b} \left[\frac{1}{\sqrt{a+b}} \begin{Bmatrix} S(ax+bx) \\ C(ax+bx) \end{Bmatrix} + \frac{\operatorname{sgn}(a-b)}{\sqrt{|a-b|}} \begin{Bmatrix} S(|a-b|x) \\ C(|a-b|x) \end{Bmatrix} \right].$
3. $\int \cos bx \begin{Bmatrix} S(ax) \\ C(ax) \end{Bmatrix} dx = \frac{\sin bx}{b} \begin{Bmatrix} S(ax) \\ C(ax) \end{Bmatrix} \pm$
 $\pm \frac{\sqrt{a}}{2b} \left[\frac{1}{\sqrt{a+b}} \begin{Bmatrix} C(ax+bx) \\ S(ax+bx) \end{Bmatrix} - \frac{\operatorname{sgn}(a-b)}{\sqrt{|a-b|}} \begin{Bmatrix} C(|a-b|x) \\ S(|a-b|x) \end{Bmatrix} \right].$
4. $\int \frac{1}{\sqrt{x}} \sin b\sqrt{x} \begin{Bmatrix} S(ax) \\ C(ax) \end{Bmatrix} dx = -\frac{2}{b} \cos b\sqrt{x} \begin{Bmatrix} S(ax) \\ C(ax) \end{Bmatrix} +$
 $+ \frac{1}{b} \cos \frac{b^2}{4a} \begin{Bmatrix} S(ax+b\sqrt{x}+b^2/(4a)) + S(ax-b\sqrt{x}+b^2/(4a)) \\ C(ax+b\sqrt{x}+b^2/(4a)) + C(ax-b\sqrt{x}+b^2/(4a)) \end{Bmatrix} \mp$
 $\mp \frac{1}{b} \sin \frac{b^2}{4a} \begin{Bmatrix} C(ax+b\sqrt{x}+b^2/(4a)) + C(ax-b\sqrt{x}+b^2/(4a)) \\ S(ax+b\sqrt{x}+b^2/(4a)) + S(ax-b\sqrt{x}+b^2/(4a)) \end{Bmatrix}.$
5. $\int \frac{1}{\sqrt{x}} \cos b\sqrt{x} \begin{Bmatrix} S(ax) \\ C(ax) \end{Bmatrix} dx = \frac{2}{b} \sin b\sqrt{x} \begin{Bmatrix} S(ax) \\ C(ax) \end{Bmatrix} +$
 $+ \frac{1}{b} \sin \frac{b^2}{4a} \begin{Bmatrix} S(ax+b\sqrt{x}+b^2/(4a)) - S(ax-b\sqrt{x}+b^2/(4a)) \\ C(ax+b\sqrt{x}+b^2/(4a)) - C(ax-b\sqrt{x}+b^2/(4a)) \end{Bmatrix} \pm$
 $\pm \frac{1}{b} \cos \frac{b^2}{4a} \begin{Bmatrix} C(ax+b\sqrt{x}+b^2/(4a)) - C(ax-b\sqrt{x}+b^2/(4a)) \\ S(ax+b\sqrt{x}+b^2/(4a)) - S(ax-b\sqrt{x}+b^2/(4a)) \end{Bmatrix}.$
6. $\int \frac{1}{\sqrt{x}} \ln x \begin{Bmatrix} S(ax) \\ C(ax) \end{Bmatrix} dx = 2\sqrt{x} \ln x \begin{Bmatrix} S(ax) \\ C(ax) \end{Bmatrix} - 4\sqrt{x} \begin{Bmatrix} S(ax) \\ C(ax) \end{Bmatrix} \mp$
 $\mp 2\sqrt{\frac{2}{\pi a}} \begin{Bmatrix} \cos ax \\ \sin ax \end{Bmatrix} \pm \sqrt{\frac{2}{\pi a}} \begin{Bmatrix} \cos ax \\ \sin ax \end{Bmatrix} \ln x \mp \sqrt{\frac{2}{\pi a}} \begin{Bmatrix} \operatorname{ci}(ax) \\ \operatorname{si}(ax) \end{Bmatrix}.$

1.6.3. Интегралы содержащие произведения функций $S(ax)$ и $C(bx)$.

1. $\int \frac{1}{\sqrt{x}} \begin{Bmatrix} S(ax) \\ C(ax) \end{Bmatrix}^2 dx = 2\sqrt{x} \begin{Bmatrix} S(ax) \\ C(ax) \end{Bmatrix}^2 \pm 2\sqrt{\frac{2}{\pi a}} \begin{Bmatrix} \cos ax S(ax) \\ \sin ax C(ax) \end{Bmatrix} \mp \frac{1}{\sqrt{\pi a}} S(2ax)$
2. $\int \frac{1}{\sqrt{x}} S(ax) S(bx) dx = 2\sqrt{x} S(ax) S(bx) + \sqrt{\frac{2}{\pi a}} \cos ax S(bx) +$
 $+ \sqrt{\frac{2}{\pi b}} \cos bx S(ax) - \frac{(a+b)^{1/2}}{\sqrt{2\pi ab}} S(ax+bx) - \frac{|a-b|^{1/2}}{\sqrt{2\pi ab}} S(|a-b|x)$

$$3. \int \frac{1}{\sqrt{x}} C(ax) C(bx) dx = 2\sqrt{x} C(ax) C(bx) - \sqrt{\frac{2}{\pi a}} \sin ax C(bx) - \\ - \sqrt{\frac{2}{\pi b}} \sin bx C(ax) + \frac{(a+b)^{1/2}}{\sqrt{2\pi ab}} S(ax+bx) - \frac{|a-b|^{1/2}}{\sqrt{2\pi ab}} S(|a-b|x).$$

$$4. \int \frac{1}{\sqrt{x}} S(ax) C(bx) dx = 2\sqrt{x} S(ax) C(bx) + \sqrt{\frac{2}{\pi a}} \cos ax C(bx) - \\ - \sqrt{\frac{2}{\pi b}} \sin bx S(ax) - \frac{(a+b)^{1/2}}{\sqrt{2\pi ab}} C(ax+bx) + \operatorname{sgn}(a-b) \frac{|a-b|^{1/2}}{\sqrt{2\pi ab}} C(|a-b|x)$$

$$5. \int \frac{1}{\sqrt{x}} S(ax) C(ax) dx = \\ = 2\sqrt{x} S(ax) C(ax) + \sqrt{\frac{2}{\pi a}} [\cos ax C(ax) - \sin ax S(ax)] - \frac{1}{\sqrt{\pi a}} C(2ax)$$

1.7. ФУНКЦИЯ ПАРАБОЛИЧЕСКОГО ЦИЛИНДРА $D_\nu(x)$

1.7.1. Интегралы вида $\int x^\lambda e^{\pm a^2 x^2/4} D_\nu(ax) dx$

$$1. \int x^\lambda e^{\pm x^2/4} D_\nu(x) dx = \left\{ \frac{1}{-1} \right\} x^{\lambda+1} e^{\pm x^2/4} D_{\nu-1 \pm 1}(x) \mp \\ \mp \left\{ \frac{\nu(\nu-1)/(\lambda+\nu+1)}{\lambda-\nu+2} \right\} \int x^\lambda e^{\pm x^2/4} D_{\nu-2}(x) dx$$

$$2. \int_0^x x^\lambda e^{\pm a^2 x^2/4} D_\nu(ax) dx = \\ = \frac{2^{\nu/2} \sqrt{\pi} x^{\lambda+1}}{(\lambda+1) \Gamma(1/2 - \nu/2)} {}_2F_2 \left(\frac{\lambda+2}{2}, \frac{1 \mp 1}{4} \mp \frac{\nu}{2}; \frac{1}{2}, \frac{\lambda+3}{2}; \pm \frac{a^2 x^2}{2} \right) - \\ - \frac{2^{(\nu+1)/2} \sqrt{\pi} x^{\lambda+2}}{(\lambda+2) \Gamma(-\nu/2)} {}_2F_2 \left(\frac{\lambda+2}{2}, \frac{3 \mp 1}{4} \mp \frac{\nu}{2}; \frac{3}{2}, \frac{\lambda+4}{2}; \pm \frac{a^2 x^2}{4} \right) \\ [\operatorname{Re} \lambda > -1]$$

$$3. \int_x^\infty x^\lambda e^{\pm a^2 x^2/4} D_\nu(ax) dx = \\ = - \frac{2^{\nu/2} \sqrt{\pi} x^{\lambda+1}}{(\lambda+1) \Gamma(1/2 - \nu/2)} {}_2F_2 \left(\frac{\lambda+2}{2}, \frac{1 \mp 1}{4} \mp \frac{\nu}{2}; \frac{1}{2}, \frac{\lambda+3}{2}; \pm \frac{a^2 x^2}{4} \right) + \\ + \frac{2^{(\nu+1)/2} \sqrt{\pi} x^{\lambda+2}}{(\lambda+2) \Gamma(-\nu/2)} {}_2F_2 \left(\frac{\lambda+2}{2}, \frac{3 \mp 1}{4} \mp \frac{\nu}{2}; \frac{3}{2}, \frac{\lambda+4}{2}; \pm \frac{a^2 x^2}{4} \right) + (V2a)^{-\lambda-1} A_\pm \\ \left[\left\{ \begin{array}{l} \operatorname{Re}(\lambda+\nu) < -1, |\arg a| < 3\pi/4 \\ |\arg a| < \pi/4 \end{array} \right\}, A_+ = 2^{-\nu/2-1} \Gamma \left[\frac{\lambda+1}{-\nu}, -(\lambda+\nu+1)/2 \right], \right. \\ \left. A_- = 2^{\nu/2} \sqrt{\pi} \Gamma \left[\frac{\lambda+1}{(\lambda-\nu+2)/2} \right] \right].$$

$$4. \int x^n e^{\pm x^2/4} D_\nu(x) dx = \\ = - \frac{1}{n+1} e^{\pm x^2/4} \sum_{k=1}^{n+1} (-n-1)_k \left\{ \frac{1/(\nu+1)_k}{(-1)^k} \right\} x^{n-k+1} D_{\nu \pm k}(x)$$

$$5. \int e^{\pm x^2/4} D_\nu(x) dx = \left\{ \frac{1/(\nu+1)}{-1} \right\} e^{\pm x^2/4} D_{\nu \pm 1}(x).$$

$$6. \int x e^{\pm x^2/4} D_\nu(x) dx = \left\{ \frac{1}{\nu+2} \right\} e^{\pm x^2/4} \left[D_\nu(x) \pm \left\{ \frac{1}{\nu} \right\} x D_{\nu \pm 1}(x) \right].$$

$$7. \int x^2 e^{\pm x^2/4} D_\nu(x) dx = \\ = \left\{ \frac{1}{3+\nu} \right\} e^{\pm x^2/4} \left[\pm 2x D_\nu(x) + \left\{ \frac{x^2 - 2/(\nu+1)}{\nu x^2 - 2} \right\} D_{\nu \pm 1}(x) \right].$$

$$8. \int x^{-\nu-2} e^{x^2/4} D_\nu(x) dx = \frac{1}{(\nu+1)(\nu+2)} x^{-\nu-2} e^{x^2/4} D_{\nu+2}(x).$$

$$9. \int x^{\nu-2} e^{-x^2/4} D_\nu(x) dx = -x^{\nu-1} e^{-x^2/4} D_{\nu-2}(x)$$

1.7.2. Интегралы вида $\int f(x) D_\nu^2(x) dx$.

$$1. \int D_\nu^2(x) dx = \nu \int D_{\nu-1}^2(x) dx - D_{\nu-1}(x) D_\nu(x).$$

$$2. \int x^{2n+1} e^{\pm x^2/2} D_\nu^2(x) dx = \\ = \pm \frac{x^{2n}}{2} e^{\pm x^2/2} \left[D_\nu^2(x) + \left\{ \frac{1}{\nu} \right\} D_{\nu \pm 1}^2(x) \right] \mp n \int x^{2n-1} e^{\pm x^2/2} D_\nu^2(x) dx \mp \\ \mp \left\{ \frac{n/(\nu+1)}{2n} \right\} \int x^{2n-1} e^{\pm x^2/2} D_{\nu \pm 1}^2(x) dx$$

1.8. ФУНКЦИЯ БЕССЕЛЯ $J_\nu(x)$

1.8.1. Интегралы вида $\int x^\lambda J_\nu(x) dx$

$$1. \int_0^x x^\lambda J_\nu(x) dx = \frac{x^{\lambda+\nu+1}}{2^\nu (\lambda+\nu+1) \Gamma(\nu+1)} {}_1F_2 \left(\frac{\lambda+\nu+1}{2}; \frac{\lambda+\nu+3}{2}; \nu+1, -\frac{x^2}{4} \right) \\ [\operatorname{Re}(\lambda+\nu) > -1].$$

$$2. \int_x^\infty x^\lambda J_\nu(x) dx = 2^\lambda \Gamma \left[\frac{(\nu+\lambda+1)/2}{(\nu-\lambda+1)/2} \right] - \\ - \frac{x^{\lambda+\nu+1}}{2^\nu (\lambda+\nu+1) \Gamma(\nu+1)} {}_1F_2 \left(\frac{\lambda+\nu+1}{2}; \nu+1, \frac{\lambda+\nu+3}{2}; -\frac{x^2}{4} \right) \\ [\operatorname{Re} \lambda < 1/2].$$

$$3. \int x^\lambda J_\nu(x) dx = x^\lambda J_{\nu+1}(x) - (\lambda - \nu - 1) \int x^{\lambda-1} J_{\nu+1}(x) dx$$

$$4. = x^\lambda J_{\nu+1}(x) + (\lambda - \nu - 1) x^{\lambda-1} J_\nu(x) - [(\lambda-1)^2 - \nu^2] \int x^{\lambda-2} J_\nu(x) dx$$

$$5. = x [(\lambda + \nu - 1) J_\nu(x) s_{\lambda-1, \nu-1}(x) - J_{\nu-1}(x) s_{\lambda, \nu}(x)].$$

$$6. \int x^{n+1/2} J_{m+1/2}(ax) dx =$$

$$= \frac{1}{a^{n+3/2}} \sqrt{\frac{2}{\pi}} \left[\sin \left(ax - \frac{m\pi}{2} \right) \sum_{k=0}^{[(n-1)/2]} C_{n-2k-1} (ax)^{n-2k-1} + \right. \\ \left. + \cos \left(ax - \frac{m\pi}{2} \right) \sum_{k=0}^{[n/2]} C_{n-2k} (ax)^{n-2k} \right] \\ \left[n \geq m; C_n = -1, C_{n-k-1} = (-1)^k [P_{k+1} - (n-k) C_{n-k}], k=0, 1, \dots, n-1; \right. \\ \left. P_k = (-1)^{[k/2]} \frac{(m+k)!}{k! (m-k)! 2^k} \text{ при } k=0, 1, \dots, m, P_k = 0 \text{ при } k > m \right].$$

$$\begin{aligned}
7. \quad &= \frac{1}{2^{n+3/2}} \sqrt{\frac{2}{\pi}} \left[\sin \left(ax - \frac{m\pi}{2} \right) \sum_{k=[(n+1)/2]}^{[(m-2)/2]} C_{n-2k-1} (ax)^{n-2k-1} + \right. \\
&\quad \left. + \cos \left(ax - \frac{m\pi}{2} \right) \sum_{k=[(n+2)/2]}^{[(m-1)/2]} C_{n-2k} (ax)^{n-2k} + \right. \\
&\quad \left. + (-1)^{[n/2]} C_{-1} \left(\sin \frac{n-m}{2} \pi \operatorname{si}(ax) - \cos \frac{n-m}{2} \pi \operatorname{ci}(ax) \right) \right] + \\
&\quad + \delta_{-n,1} (-1)^{[n/2]} \sqrt{\frac{2}{\pi a}} \sin \frac{n\pi}{2} \left(\cos \frac{m\pi}{2} \operatorname{si}(ax) - \sin \frac{m\pi}{2} \operatorname{ci}(ax) \right) \\
&\quad \left[n < 0, m \neq 0; C_{n-m+1} = \frac{P_m}{n-m+1}, C_{n-m+k+1} = \frac{1}{n-m+k+1} ((-1)^{k+m} C_{n-m+k} + P_{m-k}), \right. \\
&\quad \left. k=1, 2, \dots, m-n-2; P_k \text{ см в 1.8.1.6} \right].
\end{aligned}$$

$$\begin{aligned}
8. \quad &= \frac{1}{a^{n+3/2}} \sqrt{\frac{2}{\pi}} \left[\sin \left(ax - \frac{m\pi}{2} \right) \sum_{k=1}^{[m/2]} C_{n-2k+1} (ax)^{n-2k+1} + \right. \\
&\quad \left. + \cos \left(ax - \frac{m\pi}{2} \right) \sum_{k=0}^{[(m-1)/2]} C_{n-2k} (ax)^{n-2k} + \right. \\
&\quad \left. + (-1)^{[n/2]} (P_{n+1} + (-1)^{n+1} C_{-1}) \left(\sin \frac{m+n}{2} \pi \operatorname{si}(ax) + \cos \frac{m+n}{2} \pi \operatorname{ci}(ax) \right) \right] \\
&\quad \left[0 \leq n \leq m, C_n = -1, C_{n-k-1} = (-1)^k (P_{k+1} - (n-k) C_{n-k}), k=0, 1, \dots, n-1, \right. \\
&\quad \left. C_{n-m+1} = \frac{P_m}{n-m+1}, C_{n-m+k+1} = \frac{1}{n-m+k+1} ((-1)^{k+m} C_{n-m+k} + P_{m-k}), \right. \\
&\quad \left. k=1, 2, \dots, m-n-2; C_{-1} = 0 \text{ при } n=m-1; P_k \text{ см. в 1.8.1.6} \right].
\end{aligned}$$

$$9. \quad \int_0^x J_\nu(x) dx = 2 \sum_{k=0}^{\infty} J_{\nu+2k+1}(x) \quad [\operatorname{Re} \nu > -1].$$

$$10. \quad \int J_\nu(x) dx = \int J_{\nu+2n}(x) dx + 2 \sum_{k=0}^{n-1} J_{\nu+2k+1}(x).$$

$$11. \quad \int_0^x J_n(x) dx = (n-1) x J_n(x) s_{-1, n-1}(x) - x J_{n-1}(x) s_{0, n}(x).$$

$$12. \quad \int_0^x J_{2n}(x) dx = \int_0^x J_0(x) dx - 2 \sum_{k=0}^{n-1} J_{2k+1}(x) \quad [\text{см. 1.8.1.14}]$$

$$13. \quad \int_0^x J_{2n+1}(x) dx = 1 - J_0(x) - 2 \sum_{k=1}^n J_{2k}(x).$$

$$14. \quad \int_0^x J_0(x) dx = x J_0(x) + \frac{\pi x}{2} [J_1(x) H_0(x) - J_0(x) H_1(x)].$$

$$15. \quad \int_0^x J_{\pm 1/2}(x) dx = 2 \begin{Bmatrix} S(x) \\ C(x) \end{Bmatrix}.$$

$$16. \int_{x_1}^{x_2} J_1(x) dx = \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \mp J_0(x) \quad \left[\left\{ \begin{matrix} x_1=0, & x_2=x \\ x_1=x, & x_2=\infty \end{matrix} \right\} \right].$$

$$17. \int_0^x x^{\nu+3} J_\nu(x) dx = x^{\nu+3} J_{\nu+1}(x) - 2x^{\nu+2} J_{\nu+2}(x) \quad [\operatorname{Re} \nu > -2].$$

$$18. \int \frac{1}{x} J_{2n}(x) dx = \frac{1}{2n} \left[1 - \frac{2}{x} \sum_{k=0}^{n-1} (2k+1) J_{2k+1}(x) \right].$$

$$19. \int_0^x \frac{1}{x} J_{2n+1}(x) dx = \frac{1}{2n+1} \left[\int_0^x J_0(x) dx - J_1(x) - \frac{4}{x} \sum_{k=1}^n k J_{2k}(x) \right] \quad [\text{см } 1.8.1.14].$$

$$20. \int_x^\infty \frac{1}{x} J_0(x) dx = \frac{x^2}{8} {}_2F_3 \left(1, 1; 2, 2, 2; -\frac{x^2}{4} \right) - \ln \frac{x}{2} - C \quad [x > 0].$$

$$21. \int_0^x x^{1 \pm \nu} J_\nu(x) dx = \pm x^{1 \pm \nu} J_{\nu \pm 1}(x) + \frac{1}{2^{\nu-1} \Gamma(\nu)} \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \quad \left[\left\{ \begin{matrix} \operatorname{Re} \nu > -1 \\ \nu - \text{любое} \end{matrix} \right\} \right].$$

$$22. \int_0^x x^\nu J_\nu(x) dx = 2^{\nu-1} \sqrt{\pi} \Gamma\left(\frac{1}{2} + \nu\right) x [J_\nu(x) H_{\nu-1}(x) - J_{\nu-1}(x) H_\nu(x)] \quad [\operatorname{Re} \nu > -1/2].$$

$$23. \int_0^x x^\lambda [1 - J_0(x)] dx = \frac{x^{\lambda+3}}{4(\lambda+3)} {}_2F_3 \left(1, \frac{\lambda+3}{2}; 2, 2, \frac{\lambda+5}{2}; -\frac{x^2}{4} \right) \quad [\operatorname{Re} \lambda > -3].$$

1.8.2. Интегралы, содержащие степенную, показательную, тригонометрические функции и $J_\nu(x)$.

$$1. \int_0^x x^\lambda e^{ix} J_\nu(x) dx = \frac{x^{\lambda+\nu+1}}{2^\nu (\lambda+\nu+1) \Gamma(\nu+1)} {}_2F_3 \left(\nu + \frac{1}{2}, \lambda + \nu + 1; 2\nu + 1, \lambda + \nu + 2; 2ix \right) \quad [\operatorname{Re}(\lambda + \nu) > -1].$$

$$2. \int_0^x x^\lambda e^{\pm ix} J_\nu(x) dx = x e^{\pm i\lambda\pi/2} \left[\frac{\lambda - \nu}{\lambda + \nu + 1} J_\nu(x) h_{\lambda, \nu+1}(x e^{\mp i\pi/2}) \pm i J_{\nu+1}(x) h_{\lambda, \nu}(x e^{\mp i\pi/2}) \right] + \frac{x^{\lambda+1}}{\lambda + \nu + 1} e^{\pm ix} J_\nu(x) \quad [\operatorname{Re}(\lambda + \nu) > -1].$$

$$3. \int_0^x x^{\pm \nu} e^{ix} J_\nu(x) dx = \frac{x^{1 \pm \nu} e^{ix}}{1 \pm 2\nu} [J_\nu(x) \mp i J_{\nu \pm 1}(x)] + \frac{i}{2^{\nu-1} (2\nu-1) \Gamma(\nu)} \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \quad \left[\left\{ \begin{matrix} \operatorname{Re} \nu > -1/2 \\ \nu \neq 1/2 \end{matrix} \right\} \right].$$

$$4. \int_0^x x^{v+1} e^{ax^2} J_v(bx) dx = \frac{b^v e^{ax^2}}{(2ia)^{v+1}} [U_{v+1}(2iax^2, bx) - iU_{v+2}(2iax^2, bx)]$$

[Re v > -1].

$$5. \int x^v \left\{ \frac{\sin x}{\cos x} \right\} J_v(x) dx = \frac{x^{v+1}}{2v+1} \left[\left\{ \frac{\sin x}{\cos x} \right\} J_v(x) \mp \left\{ \frac{\cos x}{\sin x} \right\} J_{v+1}(x) \right].$$

$$6. \int x^{v+1} \left\{ \frac{\sin x}{\cos x} \right\} J_v(x) dx = x^{v+1} \left\{ \frac{\sin x}{\cos x} \right\} J_{v+1}(x) - \\ - \frac{x^{v+2}}{2v+3} \left[\left\{ \frac{\sin x}{\cos x} \right\} J_{v+2}(x) \pm \left\{ \frac{\cos x}{\sin x} \right\} J_{v+1}(x) \right].$$

$$7. \int_0^x x^{-3/2} \left\{ \frac{\sin x}{\cos x} \right\} J_v(x) dx = \mp \frac{2\sqrt{x}}{2v^2-1} \left\{ \frac{\cos x}{\sin x} \right\} J_v(x) + \\ + \left\{ \frac{\sin x}{\cos x} \right\} \left[\frac{2}{(2v-1)\sqrt{x}} J_v(x) - \left(v^2 - \frac{1}{4} \right) \sqrt{x} J_v(x) \right]$$

[Re v > \mp 1/2]

$$8. \int_0^x x^{v+1} \left\{ \frac{\sin ax^2}{\cos ax^2} \right\} J_v(bx) dx = \\ = (2a)^{-v-1} b^v \left[U_{v+1}(2ax^2, bx) \left\{ \frac{\sin ax^2}{\cos ax^2} \right\} \mp U_{v+2}(2ax^2, bx) \left\{ \frac{\cos ax^2}{\sin ax^2} \right\} \right]$$

[Re v > -(3 \pm 1)/2]

1.8.3. Интегралы вида $\int x^\lambda J_\mu(ax) J_\nu(bx) dx$

$$1. \int_0^x x^\lambda J_\mu(x) J_\nu(x) dx = \frac{x^{\lambda+\mu+\nu+1}}{2^{\mu+\nu} (\lambda+\mu+\nu+1) \Gamma(\mu+1) \Gamma(\nu+1)} \times \\ \times {}_3F_4 \left(\frac{\mu+\nu+1}{2}, \frac{\mu+\nu+2}{2}, \frac{\lambda+\mu+\nu+1}{2}; \mu+1, \nu+1, \mu+\nu+1, \right. \\ \left. \frac{\lambda+\mu+\nu+3}{2}; -x^2 \right)$$

[Re(\lambda + \mu + \nu) > -1]

$$2. \int x^\lambda J_\mu(x) J_\nu(x) dx = \frac{x^{\lambda+1}}{\lambda-\mu-\nu+1} [J_{\mu-1}(x) J_{\nu-1}(x) + J_\mu(x) J_\nu(x)] - \\ - \frac{\lambda+\mu+\nu-1}{\lambda-\mu-\nu+1} \int x^\lambda J_{\mu-1}(x) J_{\nu-1}(x) dx$$

$$3. \int x^{\pm\mu \pm \nu + 1} J_\mu(x) J_\nu(x) dx = \\ = \pm \frac{x^{\pm\mu \pm \nu + 1}}{2(\mu + \nu \pm 1)} [J_\mu(x) J_\nu(x) + J_{\mu \pm 1}(x) J_{\nu \pm 1}(x)]$$

$$4. \int x^\lambda J_\nu^2(x) dx = \frac{\lambda-1}{\lambda} \left[v^2 - \frac{(\lambda-1)^2}{4} \right] \int x^{\lambda-2} J_\nu^2(x) dx + \\ + \frac{1}{2\lambda} x^{\lambda-1} \left\{ \left[x J'_\nu(x) - \frac{\lambda-1}{2} J_\nu(x) \right]^2 + \left[x^2 - v^2 + \frac{(\lambda-1)^2}{4} \right] J_\nu^2(x) \right\}.$$

$$5. \int x^{\pm 2\nu + 1} J_\nu^2(x) dx = \frac{1}{2(1 \pm 2\nu)} x^{2 \pm 2\nu} [J_\nu^2(x) + J_{\nu \pm 1}^2(x)].$$

$$6. \int_0^x J_{3/2}^2(x) dx = \frac{1}{\pi} \left[\frac{\sin 2x}{x} - \frac{\sin^2 x}{x^2} + \ln 2x - \text{ci}(2x) + C - 1 \right].$$

7. $\int J_\nu(x) J_{\nu+1}(x) dx = \sum_{k=0}^{\infty} J_{\nu+k+1}^2(x)$
8. $\int_0^x J_n(x) J_{n+1}(x) dx = \frac{1}{2} - \frac{1}{2} J_0^2(x) - \sum_{k=1}^n J_k^2(x).$
9. $\int_0^x J_0(x) J_1(x) dx = \frac{1}{2} - \frac{1}{2} J_0^2(x).$
10. $\int x J_\nu(ax) J_\nu(bx) dx = \frac{x}{a^2 - b^2} [a J_{\nu+1}(ax) J_\nu(bx) - b J_\nu(ax) J_{\nu+1}(bx)].$
11. $\int_0^x x J_\nu^2(ax) dx = \frac{x^2}{2} [J_\nu'(ax)]^2 + \frac{1}{2} \left(x^2 - \frac{\nu^2}{a^2} \right) J_\nu^2(ax) \quad [\operatorname{Re} \nu > -1].$
12. $\int x J_\nu^2(ax) dx = \frac{x^2}{2} [J_\nu^2(ax) - J_{\nu-1}(ax) J_{\nu+1}(ax)].$
13. $\int_0^x x J_0^2(x) dx = \frac{x^2}{2} [J_0^2(x) + J_1^2(x)].$
14. $\int x [J_{\nu-1}^2(x) - J_{\nu+1}^2(x)] dx = 2\nu J_\nu^2(x).$
15. $\int_0^x x^2 J_{3/2}^2(x) dx = \frac{1}{\pi} \left[\frac{x^2}{2} + \frac{x}{2} \sin 2x - \frac{5}{2} \sin^2 x + \ln 2x - \operatorname{ci}(2x) + C \right].$
16. $\int \frac{1}{x} J_\nu^2(x) dx = \frac{1}{2\nu} J_\nu^2(x) + \frac{x}{2\nu} \left[J_{\nu+1}(x) \frac{\partial}{\partial \nu} J_\nu(x) - J_\nu(x) \frac{\partial}{\partial \nu} J_{\nu+1}(x) \right].$
17. $\int_0^x \frac{1}{x} J_n^2(x) dx = \frac{1}{2n} \left[1 + J_0^2(x) + J_n^2(x) - 2 \sum_{k=0}^n J_k^2(x) \right] \quad [n \geq 1]$
18. $\int \frac{1}{x} J_\mu(ax) J_\nu(ax) dx =$
 $= -\frac{ax}{\mu^2 - \nu^2} [J_{\mu+1}(ax) J_\nu(ax) - J_\mu(ax) J_{\nu+1}(ax)] + \frac{1}{\mu + \nu} J_\mu(ax) J_\nu(ax)$
19. $\int \frac{1}{x} J_{\mu+n}(x) J_{\nu+n}(x) dx = \frac{1}{\mu + \nu + 2n} \left[(\mu + \nu) \int \frac{1}{x} J_\mu(x) J_\nu(x) dx - \right.$
 $\left. - J_\mu(x) J_\nu(x) - J_{\mu+n}(x) J_{\nu+n}(x) - 2 \sum_{k=1}^{n-1} J_{\mu+k}(x) J_{\nu+k}(x) \right].$
20. $\int \frac{1}{x^2} J_\nu^2(x) dx = \frac{2}{(4\nu^2 - 1)x} \left\{ \left[x J_\nu'(x) + \frac{1}{2} J_\nu(x) \right]^2 + \left(x^2 - \nu^2 + \frac{1}{4} \right) J_\nu^2(x) \right\}.$
21. $\int \frac{1}{x^2} J_n^2(x) dx = -\frac{2}{4n^2 - 1} J_{n-1}(x) J_n(x) +$
 $+ \frac{2x}{4n^2 - 1} [J_{n-1}^2(x) - J_{n-2}(x) J_n(x)] - \frac{1}{(2n+1)x} J_n^2(x)$
22. $\int_0^x \frac{1}{x^2} J_{3/2}^2(x) dx = \frac{1}{2\pi} - \frac{1}{4x} [J_{1/2}^2(x) + J_{3/2}^2(x)].$

$$23. \int \frac{1}{x^2} J_{n-1}(x) J_n(x) dx = -\frac{1}{2x} J_{n-1}(x) J_n(x) - \\ - \frac{1}{4n(n-1)} \left[J_0^2(x) + 2 \sum_{k=1}^{n-2} J_k^2(x) - (n-2) J_{n-1}^2(x) - (n-1) J_n^2(x) \right].$$

$$24. \int \frac{1}{x^2} J_m(x) J_n(x) dx = \\ = -\frac{1}{(m+n+1)x} J_m(x) J_n(x) - \frac{1}{(n+m)^2-1} [J_{m-1}(x) J_n(x) + J_m(x) J_{n-1}(x)] - \\ - \frac{2x}{[(m+n)^2-1][(m-n)^2-1]} J_{m-1}(x) J_{n-1}(x) - \\ - \frac{x J_m(x) J_{n-2}(x)}{(m+n+1)[m^2-(n-1)^2]} - \frac{x J_{m-2}(x) J_n(x)}{(m+n+1)[n^2-(m-1)^2]}.$$

$$25. \int \frac{1}{x^3} J_n^2(x) dx = -\frac{1}{n(n^2-1)x^2} \left[\frac{n(n-1)}{2} J_n^2(x) + \sum_{k=1}^{n-1} k^2 J_k^2(x) \right].$$

$$26. \int \frac{1}{x^3} J_m(x) J_n(x) dx = \\ = -\frac{1}{m+n+2} \left\{ \frac{J_m(x) J_n(x)}{x^2} + \frac{1}{m+n} \frac{J_{m-1}(x) J_n(x) + J_m(x) J_{n-1}(x)}{x} + \right. \\ + \frac{1}{(m+n-1)^2-1} [J_{m-2}(x) J_n(x) + 2J_{m-1}(x) J_{n-1}(x) + J_m(x) J_{n-2}(x)] + \\ + \frac{x}{m+n} \left[\frac{J_{n-3}(x) J_m(x)}{m^2-(n-2)^2} + \frac{n-m-4}{(n-m)[m^2-(n-2)^2]} J_{n-2}(x) J_{m-1}(x) + \right. \\ \left. + \frac{n-m+4}{(n-m)[n^2-(m-2)^2]} J_{n-1}(x) J_{m-2}(x) + \frac{J_n(x) J_{m-3}(x)}{n^2-(m-2)^2} \right] \Big\}.$$

1.8.4. Интегралы вида $\int \frac{f(x)}{J_\mu(x) J_\nu(x)} dx$.

$$1. \int \frac{dx}{x J_\nu^2(x)} = \frac{\pi}{2} \frac{Y_\nu(x)}{J_\nu(x)}.$$

$$2. \int \frac{dx}{x J_\nu(x) J_{-\nu}(x)} = \frac{\pi}{2 \sin \nu \pi} \ln \frac{J_\nu(x)}{J_{-\nu}(x)}.$$

$$3. \int \frac{(1-\nu^2/x^2) dx}{x [J'_\nu(x)]^2} = \frac{\pi}{2} \frac{Y'_\nu(x)}{J'_\nu(x)}.$$

1.9. ФУНКЦИЯ НЕЙМАНА $Y_\nu(x)$

1.9.1. Интегралы вида $\int x^\lambda Y_\nu(x) dx$.

$$1. \int_{x_1}^{x_2} x^\lambda Y_\nu(x) dx = \frac{\cos \nu \pi \Gamma(-\nu)}{2^{\nu \pi} (\lambda + \nu + 1)} x^{\lambda + \nu + 1} \times \\ \times {}_1F_2 \left(\frac{\lambda + \nu + 1}{2}; 1 + \nu, \frac{\lambda + \nu + 3}{2}; -\frac{x^2}{4} \right) - \\ - \frac{2^\nu \Gamma(\nu)}{\lambda - \nu + 1} x^{\lambda - \nu + 1} {}_1F_2 \left(\frac{\lambda - \nu + 1}{2}; 1 - \nu, \frac{\lambda - \nu + 3}{2}; -\frac{x^2}{4} \right) - \\ - \begin{cases} 0 \\ 1 \end{cases} \frac{2^\lambda}{\pi} \cos \frac{\lambda - \nu + 1}{2} \pi \Gamma \left(\frac{\lambda + \nu + 1}{2} \right) \Gamma \left(\frac{\lambda - \nu + 1}{2} \right) \\ \left[\begin{cases} x_1 = 0, x_2 = x; \operatorname{Re} \lambda > |\operatorname{Re} \nu| - 1 \\ x_1 = x, x_2 = \infty; \operatorname{Re} \lambda < 1/2 \end{cases} \right].$$

$$2. \int_0^x x^\lambda Y_\nu(x) dx = (\lambda + \nu - 1) x Y_\nu(x) s_{\lambda-1, \nu-1}(x) - x Y_{\nu-1}(x) s_{\lambda, \nu}(x)$$

$$3. \int_0^x x^\nu Y_\nu(x) dx = 2^{\nu-1} \sqrt{\pi} \Gamma\left(\nu + \frac{1}{2}\right) x [Y_\nu(x) H_{\nu-1}(x) - Y_{\nu-1}(x) H_\nu(x)]$$

[Re $\nu > -1/2$].

$$4. \int_0^x x^{1 \pm \nu} Y_\nu(x) dx = \pm x^{1 \pm \nu} Y_{1 \pm \nu}(x) + \frac{2^{1 \pm \nu} \Gamma(1 \pm \nu)}{\pi} \left\{ \frac{1}{\cos \nu \pi} \right\}$$

[$\pm \operatorname{Re} \nu > -1$].

$$5. \int_x^\infty x^{\nu+1} Y_\nu(x) dx = -x^{\nu+1} Y_{\nu+1}(x) + \frac{2^{\nu+1} \Gamma(\nu+1)}{\pi}$$

[Re $\nu < -1/2$].

$$6. \int Y_\nu(x) dx = 2 \sum_{k=0}^{\infty} Y_{\nu+2k+1}(x).$$

$$7. \int_0^x Y_0(x) dx = x Y_0(x) + \frac{\pi x}{2} [Y_1(x) H_0(x) - Y_0(x) H_1(x)].$$

1.9.2. Интегралы вида $\int x^\lambda e^{\pm ix} Y_\nu(x) dx$.

$$1. \int_0^x x^\lambda e^{\pm ix} Y_\nu(x) dx = x e^{\pm i\lambda\pi/2} \left[\frac{\lambda - \nu}{\lambda + \nu + 1} Y_\nu(x) h_{\lambda, \nu+1}(x e^{\mp i\pi/2}) \pm \right.$$

$$\left. \pm i Y_{\nu+1}(x) h_{\lambda, \nu}(x e^{\mp i\pi/2}) \right] + \frac{x^{\lambda+1}}{\lambda + \nu + 1} e^{\pm ix} Y_\nu(x) \quad [\operatorname{Re} \lambda > |\operatorname{Re} \nu| - 1].$$

$$2. \int_0^x x^\nu e^{ix} Y_\nu(x) dx = \frac{x^{\nu+1} e^{ix}}{2\nu+1} [Y_\nu(x) - i Y_{\nu+1}(x)] - \frac{2^{\nu+1} i \Gamma(\nu+1)}{\pi (2\nu+1)}$$

[Re $\nu > -1/2$].

1.9.3. Интегралы вида $\int x^\lambda Y_\mu(ax) Y_\nu(bx) dx$.

$$1. \int x^{\pm \mu \pm \nu + 1} Y_\mu(x) Y_\nu(x) dx = \pm \frac{x^{\pm \mu \pm \nu + 2}}{2(\mu + \nu \pm 1)} [Y_\mu(x) Y_\nu(x) + Y_{\mu \pm 1}(x) Y_{\nu \pm 1}(x)].$$

$$2. \int x Y_\nu(ax) Y_\nu(bx) dx = \frac{x}{a^2 - b^2} [a Y_{\nu+1}(ax) Y_\nu(bx) - b Y_\nu(ax) Y_{\nu+1}(bx)].$$

$$3. \int \frac{1}{x} Y_\mu(ax) Y_\nu(ax) dx =$$

$$= -\frac{ax}{\mu^2 - \nu^2} [Y_{\mu+1}(ax) Y_\nu(ax) - Y_\mu(ax) Y_{\nu+1}(ax)] + \frac{1}{\mu + \nu} Y_\mu(ax) Y_\nu(ax).$$

$$4. \int \frac{1}{x} Y_\nu^2(ax) dx = \frac{ax}{2\nu} \left[Y_{\nu+1}(ax) \frac{\partial Y_\nu(ax)}{\partial \nu} - Y_\nu(ax) \frac{\partial Y_{\nu+1}(ax)}{\partial \nu} \right] + \frac{1}{2\nu} Y_\nu^2(ax).$$

$$5. \int x^\lambda Y_\nu^2(x) dx = \frac{\lambda-1}{\lambda} \left[\nu^2 - \left(\frac{\lambda-1}{2} \right)^2 \right] \int x^{\lambda-2} Y_\nu^2(x) dx +$$

$$+ \frac{1}{2\lambda} x^{\lambda-1} \left\{ \left[x Y'_\nu(x) - \frac{\lambda-1}{2} Y_\nu(x) \right]^2 + \left[x^2 - \nu^2 + \left(\frac{\lambda-1}{2} \right)^2 \right] Y_\nu^2(x) \right\}.$$

$$6. \int x^{1 \pm 2\nu} Y_\nu^2(x) dx = \frac{1}{2(1 \pm 2\nu)} x^{2 \pm 2\nu} [Y_\nu^2(x) + Y_{\nu \pm 1}^2(x)].$$

$$7. \int_0^x x Y_v^2(ax) dx = \frac{x^2}{2} [Y_v^2(ax) - Y_{v-1}(ax) Y_{v+1}(ax)] \quad [|\operatorname{Re} v| < 2].$$

$$8. \int \frac{1}{x} Y_n^2(x) dx = -\frac{1}{2n} \left[Y_0^2(x) + Y_n^2(x) + 2 \sum_{k=1}^{n-1} Y_k^2(x) \right].$$

$$9. \int \frac{1}{x^2} Y_v^2(x) dx = \frac{2}{(4v^2-1)x} \left\{ \left[x J_v'(x) + \frac{1}{2} Y_v(x) \right]^2 + \left(x^2 - v^2 + \frac{1}{4} \right) Y_v^2(x) \right\}.$$

1.9.4. Интегралы, содержащие $\int \frac{f(x)}{Y_v(x)} dx$.

$$1. \int \frac{dx}{x Y_v^2(x)} = -\frac{\pi}{2} \frac{J_v(x)}{Y_v(x)}.$$

$$2. \int \frac{(1-v^2/x^2) dx}{x [Y_v'(x)]^2} = -\frac{\pi}{2} \frac{J_v'(x)}{Y_v'(x)}.$$

$$3. \int \frac{(1-v^2/x^2) dx}{x J_v'(x) Y_v'(x)} = \frac{\pi}{2} \ln \frac{Y_v'(x)}{J_v'(x)}.$$

1.9.5. Интегралы, содержащие $J_\mu(ax) Y_v(bx)$.

$$1. \int x^{\pm \mu \pm v + 1} J_\mu(x) Y_v(x) dx = \pm \frac{x^{\pm \mu \pm v + 2}}{2(\mu + v \pm 1)} [J_\mu(x) Y_v(x) + J_{\mu \pm 1}(x) Y_{v \pm 1}(x)].$$

$$2. \int x J_v(ax) Y_v(bx) dx = \frac{x}{b^2 - a^2} [b J_v(ax) Y_{v+1}(bx) - a J_{v+1}(ax) Y_v(bx)].$$

$$3. \int x J_v(ax) Y_v(ax) dx = \\ = \frac{x^2}{4} [2 J_v(ax) Y_v(ax) - J_{v+1}(ax) Y_{v-1}(ax) - J_{v-1}(ax) Y_{v+1}(ax)].$$

$$4. \int_0^x x J_0(ax) Y_0(bx) dx = \frac{2}{\pi(b^2 - a^2)} + \frac{x}{b^2 - a^2} [b J_0(ax) Y_1(bx) - a Y_0(bx) J_1(ax)].$$

$$5. \int \frac{1}{x} J_\mu(ax) Y_v(ax) dx = \\ = -\frac{ax}{\mu^2 - v^2} [J_{\mu+1}(ax) Y_v(ax) - J_\mu(ax) Y_{v+1}(x)] + \frac{1}{\mu + v} J_\mu(ax) Y_v(ax).$$

$$6. \int \frac{1}{x} J_n(x) Y_n(x) dx = -\frac{1}{2n} \left[J_0(x) Y_0(x) + J_n(x) Y_n(x) + 2 \sum_{k=1}^{n-1} J_k(x) Y_k(x) \right].$$

$$7. \int \frac{dx}{x J_v(x) Y_v(x)} = \frac{\pi}{2} \ln \frac{Y_v(x)}{J_v(x)}.$$

1.10. ФУНКЦИИ ГАНКЕЛЯ $H_v^{(j)}(x)$ И $H_v^{(2)}(x)$

Обозначение: $j=1$ или 2 .

1.10.1. Интегралы вида: $\int x^\lambda H_v^{(j)}(x) dx$.

$$1. \int_0^x x^\lambda H_v^{(j)}(x) dx = \frac{2^\lambda}{\pi} e^{(-1)^j + 1} i^{(\lambda - v) \pi / 2} \Gamma\left(\frac{\lambda - v + 1}{2}\right) \Gamma\left(\frac{\lambda + v + 1}{2}\right) + \\ + (\lambda + v - 1) x H_v^{(j)}(x) S_{\lambda-1, v-1}(x) - x H_{v-1}^{(j)}(x) S_{\lambda, v}(x) \\ [|\operatorname{Re} \lambda| > |\operatorname{Re} v| - 1].$$

$$2. \int_0^x H_0^{(j)}(x) dx = x H_0^{(j)}(x) + \frac{\pi x}{2} [H_1^{(j)}(x) H_0(x) - H_0^{(j)}(x) H_1(x)].$$

$$3. \int_0^x x^\nu H_\nu^{(j)}(x) dx = 2^{\nu-1} \sqrt{\pi} \Gamma\left(\nu + \frac{1}{2}\right) x [H_\nu^{(j)}(x) H_{\nu-1}(x) - H_{\nu-1}^{(j)}(x) H_\nu(x)]$$

【Re $\nu > -1/2$ 】.

$$4. \int x^{1 \pm \nu} H_\nu^{(j)}(x) dx = \pm x^{1 \pm \nu} H_{\nu \pm 1}^{(j)}(x).$$

$$5. \int H_\nu^{(j)}(x) dx = \int H_{\nu+2n}^{(j)}(x) dx + 2 \sum_{k=0}^{n-1} H_{\nu+2k+1}^{(j)}(x).$$

$$6. \int_x^\infty \frac{1}{x} H_0^{(j)}(x) dx = 2x H_0^{(j)}(x) S_{-2, 1}(x) - x H_1^{(j)}(x) S_{-1, 0}(x).$$

1.10.2. Интегралы вида $\int x^\lambda e^{px} H_\nu^{(j)}(x) dx$.

$$1. \int_0^x x^\lambda e^{ix} H_\nu^{(j)}(x) dx =$$

$$= \frac{(-\sec \lambda \pi \cos \nu \pi)^{j-1}}{2^\lambda \sqrt{\pi} \Gamma(\lambda + 3/2)} e^{(-1)^{j-1} i (\lambda - \nu) \pi/2} \Gamma(\lambda + \nu + 1) \Gamma(\lambda - \nu + 1) +$$

$$+ x e^{i\lambda \pi/2} \left[\frac{\lambda - \nu}{\lambda + \nu + 1} H_\nu^{(j)}(x) H_{\lambda, \nu+1}(x e^{-i\pi/2}) + i H_{\nu+1}^{(j)}(x) H_{\lambda, \nu}(x e^{-i\pi/2}) \right] +$$

$$+ \frac{x^{\lambda+1}}{\lambda + \nu + 1} e^{ix} H_\nu^{(j)}(x) \quad [\text{Re } \lambda > |\text{Re } \nu| - 1]$$

1.10.3. Интегралы вида $\int \frac{f(x)}{H_\nu^{(j)}(x)} dx$.

$$1. \int \frac{dx}{x [H_\nu^{(1)}(x)]^2} = -\frac{\pi}{4i} \frac{H_\nu^{(2)}(x)}{H_\nu^{(1)}(x)}.$$

$$2. \int \frac{dx}{x [H_\nu^{(2)}(x)]^2} = \frac{\pi}{4i} \frac{H_\nu^{(1)}(x)}{H_\nu^{(2)}(x)}.$$

$$3. \int \frac{dx}{x H_\nu^{(1)}(x) H_\nu^{(2)}(x)} = -\frac{\pi}{4i} \ln \frac{H_\nu^{(2)}(x)}{H_\nu^{(1)}(x)}.$$

1.11. МОДИФИЦИРОВАННАЯ ФУНКЦИЯ БЕССЕЛЯ $I_\nu(x)$

1.11.1. Интегралы вида $\int x^\lambda I_\nu(x) dx$

$$1. \int_0^x x^\lambda I_\nu(x) dx = \frac{x^{\lambda+\nu+1}}{2^\nu (\lambda + \nu + 1) \Gamma(\nu + 1)} {}_1F_2\left(\frac{\lambda + \nu + 1}{2}; \frac{\lambda + \nu + 3}{2}, \nu + 1; \frac{x^2}{4}\right)$$

【Re $(\lambda + \nu) > -1$ 】.

$$2. \int x^\lambda I_\nu(x) dx =$$

$$= e^{-i\lambda \pi/2} [(\lambda + \nu - 1) x I_\nu(x) S_{\lambda-1, \nu-1}(x e^{i\pi/2}) + i x I_{\nu-1}(x) S_{\lambda, \nu}(x e^{i\pi/2})].$$

$$3. \int_0^x x^\nu I_\nu(x) dx = 2^{\nu-1} \sqrt{\pi} \Gamma\left(\nu + \frac{1}{2}\right) x [I_\nu(x) L_{\nu-1}(x) - I_{\nu-1}(x) L_\nu(x)]$$

[Re $\nu > -1/2$].

$$4. \int_0^x I_0(x) dx = x I_0(x) + \frac{\pi x}{2} [I_0(x) L_1(x) - I_1(x) L_0(x)].$$

$$5. \int_0^x x^{1 \pm \nu} I_\nu(x) dx = x^{1 \pm \nu} I_{\nu \pm 1}(x) - \frac{1}{2^{\nu-1} \Gamma(\nu)} \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$$

[$\{\text{Re } \nu > -1\}$]
[$\nu - \text{любое}$]].

$$6. \int_0^x I_\nu(x) dx = 2 \sum_{k=0}^{\infty} (-1)^k I_{\nu+2k+1}(x)$$

[Re $\nu > -1$].

1.11.2. Интегралы вида $\int x^\lambda e^{\pm x} I_\nu(x) dx$.

$$1. \int_0^x x^\lambda e^{\pm x} I_\nu(x) dx =$$

$$= \frac{x^{\lambda+\nu+1}}{2^\nu (\lambda + \nu + 1) \Gamma(\nu + 1)} {}_2F_2\left(\nu + \frac{1}{2}, \lambda + \nu + 1; 2\nu + 1, \lambda + \nu + 2; \pm 2x\right).$$

$$2. = \frac{(\lambda - \nu)x}{\lambda + \nu + 1} e^{-i\delta\lambda\pi} I_\nu(x) h_{\lambda, \nu+1}(xe^{i\delta\pi}) \pm$$

$$\pm x e^{-i\delta\lambda\pi} I_{\nu+1}(x) h_{\lambda, \nu}(xe^{i\delta\pi}) + \frac{x^{\lambda+1}}{\lambda + \nu + 1} e^{\pm x} I_\nu(x)$$

[Re $(\lambda + \nu) > -1$, $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$].

$$3. \int_0^x x^\nu e^{\pm x} I_\nu(x) dx = \frac{x^{\nu+1} e^{\pm x}}{2\nu + 1} [I_\nu(x) \mp I_{\nu+1}(x)]$$

[Re $\nu > -1/2$].

$$4. \int_0^x e^{-x} I_n(x) dx = x e^{-x} [I_n(x) + I_{n+1}(x)] + n [e^{-x} I_0(x) - 1] + 2n e^{-x} \sum_{k=1}^n I_k(x).$$

$$5. \int_0^x x^{-\nu} e^{\pm x} I_\nu(x) dx = -\frac{x^{-\nu+1} e^{\pm x}}{2\nu - 1} [I_\nu(x) \mp I_{\nu-1}(x)] \mp \frac{2^{1-\nu}}{(2\nu - 1) \Gamma(\nu)}.$$

1.11.3. Интегралы вида $\int x^\lambda I_\mu(ax) I_\nu(bx) dx$.

$$1. \int_0^x I_{\nu+1}(ax) I_\nu(ax) dx = \frac{1}{a} \sum_{k=0}^{\infty} (-1)^k I_{k+\nu+1}^2(ax)$$

[Re $\nu > -1$]

$$2. \int_0^x x^\lambda I_\mu(ax) I_\nu(ax) dx = \frac{(a/2)^{\mu+\nu} x^{\lambda+\mu+\nu+1}}{(\lambda + \mu + \nu + 1) \Gamma(\mu + 1) \Gamma(\nu + 1)} \times$$

$$\times {}_3F_4\left(\frac{\lambda + \mu + \nu + 1}{2}, \frac{\mu + \nu + 1}{2}, \frac{\mu + \nu}{2} + 1;$$

$$\mu + 1, \nu + 1, \mu + \nu + 1, \frac{\lambda + \mu + \nu + 3}{2}; a^2 x^2\right)$$

[Re $(\lambda + \mu + \nu) > -1$].

$$3. \int_0^x x I_\nu(ax) I_\nu(bx) dx = \frac{x}{a^2 - b^2} [a I_{\nu+1}(ax) I_\nu(bx) - b I_\nu(ax) I_{\nu+1}(bx)]$$

[Re $\nu > -1$].

$$4. \int_0^x x I_\nu^2(ax) dx = -\frac{x^2}{2} [I'_\nu(ax)]^2 + \frac{1}{2} \left(x^2 + \frac{\nu^2}{a^2} \right) I_\nu^2(ax)$$

[Re $\nu > -1$].

1.11.4. Интегралы вида $\int \frac{f(x)}{I_\nu(x)} dx$.

$$1. \int \frac{dx}{x I_\nu^2(x)} = -\frac{K_\nu(x)}{I_\nu(x)}.$$

$$2. \int \frac{(1 + \nu^2/x^2) dx}{x [I'_\nu(x)]^2} = \frac{K'_\nu(x)}{I'_\nu(x)}.$$

1.11.5. Интегралы вида $\int x^\lambda J_\nu(ax) I_\nu(bx) dx$.

$$1. \int_0^x x^\lambda J_\nu(x) I_\nu(x) dx =$$

$$= \frac{x^{\lambda+2\nu+1}}{2^{2\nu} (\lambda+2\nu+1) \Gamma^2(\nu+1)} {}_1F_4 \left(\frac{\lambda+2\nu+1}{4}; \nu+1, \frac{\nu+1}{2}, \frac{\nu+2}{2}, \frac{\lambda+2\nu+5}{4}; -\frac{x^4}{64} \right)$$

[Re $(\lambda+2\nu) > -1$].

$$2. \int_0^x x J_\nu(ax) I_\nu(bx) dx = \frac{x}{a^2 + b^2} [a J_{\nu+1}(ax) I_\nu(bx) + b J_\nu(ax) I_{\nu+1}(bx)]$$

[Re $\nu > -1$].

1.12. ФУНКЦИЯ МАКДОНАЛЬДА $K_\nu(x)$

1.12.1. Интегралы вида $\int x^\lambda K_\nu(x) dx$.

$$1. \int x^\lambda K_\nu(x) dx = e^{-i\lambda\pi/2} (\lambda + \nu - 1) x K_\nu(x) s_{\lambda-1, \nu-1}(xe^{i\pi/2}) +$$

$$+ e^{-i(\lambda+1)\pi/2} x K_{\nu-1}(x) s_{\lambda, \nu}(xe^{i\pi/2}).$$

$$2. \int_0^x x^\lambda K_\nu(x) dx = \frac{2^{\nu-1} \Gamma(\nu)}{\lambda - \nu + 1} x^{\lambda-\nu+1} {}_1F_2 \left(\frac{\lambda - \nu + 1}{2}; 1 - \nu, \frac{\lambda - \nu + 3}{2}; \frac{x^2}{4} \right) +$$

$$+ \frac{2^{-\nu-1} \Gamma(-\nu)}{\lambda + \nu + 1} x^{\lambda+\nu+1} {}_1F_2 \left(\frac{\lambda + \nu + 1}{2}; 1 + \nu, \frac{\lambda + \nu + 3}{2}; \frac{x^2}{4} \right)$$

[Re $\lambda > |\text{Re } \nu| - 1$].

$$3. \int_0^x x^\nu K_\nu(x) dx = 2^{\nu-1} \sqrt{\pi} \Gamma \left(\nu + \frac{1}{2} \right) x [K_\nu(x) L_{\nu-1}(x) + K_{\nu-1}(x) L_\nu(x)]$$

[Re $\nu > -1/2$].

$$4. \int_0^x K_0(x) dx = x K_0(x) + \frac{\pi x}{2} [K_0(x) L_1(x) - K_1(x) L_0(x)].$$

$$5. \int_0^x x^{1 \pm \nu} K_\nu(x) dx = -x^{1 \pm \nu} K_{1 \pm \nu}(x) + 2^{\pm \nu} \Gamma(1 \pm \nu)$$

[$\pm \text{Re } \nu > -1$]

$$6. \int_0^x x^n K_0(x) dx = \frac{x^{n+1}}{n+1} K_0(x) {}_1F_2\left(1; \frac{n+1}{2}, \frac{n+3}{2}; \frac{x^2}{4}\right) + \\ + \frac{x^{n+2}}{(n+1)^2} K_1(x) {}_1F_2\left(1; \frac{n+3}{2}, \frac{n+3}{2}; \frac{x^2}{4}\right).$$

1.12.2. Интегралы вида $\int x^\lambda e^{\pm x} K_\nu(x) dx$.

$$1. \int_0^x x^\lambda e^{\pm x} K_\nu(x) dx = \frac{(\lambda - \nu)x}{\lambda + \nu + 1} e^{-i\delta\lambda\pi} K_\nu(x) h_{\lambda, \nu+1}(xe^{i\delta\pi}) \mp \\ \mp xe^{-i\delta\lambda\pi} K_{\nu+1}(x) h_{\lambda, \nu}(xe^{i\delta\pi}) + \frac{x^{\lambda+1}}{\lambda + \nu + 1} e^{\pm x} K_\nu(x) \\ \left[\operatorname{Re} \lambda > |\operatorname{Re} \nu| - 1, \delta = \begin{cases} 1 \\ 0 \end{cases} \right]$$

$$2. \int_0^x x^\nu e^{\pm x} K_\nu(x) dx = \frac{x^{\nu+1} e^{\pm x}}{2\nu + 1} [K_\nu(x) \pm K_{\nu+1}(x)] \mp \frac{2^\nu \Gamma(\nu + 1)}{2\nu + 1} \\ [\operatorname{Re} \nu > -1/2]$$

$$3. \int_x^\infty x^{-\nu} e^x K_\nu(x) dx = \frac{x^{-\nu+1} e^x}{2\nu - 1} [K_\nu(x) + K_{\nu-1}(x)] \\ [\operatorname{Re} \nu > 1/2]$$

1.12.3. Интегралы вида $\int x^\lambda K_\mu(ax) K_\nu(bx) dx$.

$$1. \int_x^\infty x K_\nu(ax) K_\nu(bx) dx = \frac{x}{a^2 - b^2} [a K_{\nu-1}(ax) K_\nu(bx) - b K_\nu(ax) K_{\nu-1}(bx)] \\ [\operatorname{Re}(a + b) > 0]$$

$$2. \int_x^\infty x K_\nu^2(ax) dx = \frac{x^2}{2} [K_\nu'(ax)]^2 - \frac{1}{2} \left(x^2 + \frac{\nu^2}{a^2}\right) K_\nu^2(ax) \\ [\operatorname{Re} a > 0]$$

$$3. \int_x^\infty \frac{1}{x} K_\mu(x) K_\nu(x) dx = \frac{x}{\mu^2 - \nu^2} [K_\mu(x) K_\nu'(x) - K_\nu(x) K_\mu'(x)].$$

1.12.4. Интегралы, содержащие $J_\mu(ax) K_\nu(bx)$ или $I_\mu(ax) K_\nu(bx)$.

$$1. \int_0^x x J_\nu(ax) K_\nu(bx) dx = \\ = \frac{x}{a^2 + b^2} \{a J_{\nu+1}(ax) K_\nu(bx) - b J_\nu(ax) K_{\nu+1}(bx)\} + \frac{(a/b)^\nu}{a^2 + b^2} \\ [\operatorname{Re} \nu > -1].$$

$$2. \int_0^x x I_\nu(ax) K_\nu(bx) dx = \frac{(a/b)^\nu}{b^2 - a^2} - \frac{x}{b^2 - a^2} [a I_{\nu+1}(ax) K_\nu(bx) + b I_\nu(ax) K_{\nu+1}(bx)] \\ [\operatorname{Re} \nu > -1]$$

$$3. \int_0^x x I_\nu(ax) K_\nu(ax) dx = \frac{x^2}{2} \left[\left(1 + \frac{\nu^2}{a^2 x^2}\right) I_\nu(ax) K_\nu(ax) - I_\nu'(ax) K_\nu'(ax) \right] - \frac{\nu}{2a^2} \\ [\operatorname{Re} \nu > -1]$$

$$4. \int_0^x \frac{1}{x} I_\mu(ax) K_\nu(ax) dx = \frac{ax}{\nu^2 - \mu^2} [I_\mu(ax) K'_\nu(ax) - K_\nu(ax) I'_\mu(ax)]$$

$[\operatorname{Re} \mu > |\operatorname{Re} \nu|]$.

1.12.5. Интегралы вида $\int \frac{f(x)}{K_\nu(x)} dx$.

$$1. \int \frac{dx}{x I_\nu(x) K_\nu(x)} = \ln \frac{I_\nu(x)}{K_\nu(x)}.$$

$$2. \int \frac{dx}{x K_\nu^2(x)} = \frac{I_\nu(x)}{K_\nu(x)}.$$

1.13. ЦИЛИНДРИЧЕСКИЕ ФУНКЦИИ $Z_\nu(x)$

Обозначения:

$$Z_\nu(x) = c_1 J_\nu(x) + c_2 Y_\nu(x) + c_3 H_\nu^{(1)}(x) + c_4 H_\nu^{(2)}(x),$$

$$Z_\nu^{(1)}(x) = c_1^{(1)} J_\nu(x) + c_2^{(1)} Y_\nu(x) + c_3^{(1)} H_\nu^{(1)}(x) + c_4^{(1)} H_\nu^{(2)}(x),$$

$$Z_\nu^{(2)}(x) = c_1^{(2)} J_\nu(x) + c_2^{(2)} Y_\nu(x) + c_3^{(2)} H_\nu^{(1)}(x) + c_4^{(2)} H_\nu^{(2)}(x),$$

$c_i^{(j)}$ — комплексные постоянные.

1.13.1. Интегралы вида $\int x^\lambda Z_\nu(x) dx$.

$$1. \int x^\lambda Z_\nu(x) dx = [\nu^2 - (\lambda - 1)^2] \int x^{\lambda-2} Z_\nu(x) dx + \\ + x^{\lambda-1} [x Z_{\nu+1}(x) + (\lambda - \nu - 1) Z_\nu(x)].$$

$$2. \int Z_\nu(x) dx = 2 \sum_{k=0}^{\infty} Z_{\nu+2k+1}(x).$$

1.13.2. Интегралы вида $\int x^\lambda Z_\mu^{(1)}(ax) Z_\nu^{(2)}(bx) dx$.

$$1. \int x^\lambda Z_\mu^{(1)}(x) Z_\nu^{(2)}(x) dx = \\ = \frac{1}{\lambda - \mu - \nu + 1} \left\{ x^{\lambda+1} [Z_{\mu-1}^{(1)}(x) Z_{\nu-1}^{(2)}(x) + Z_\mu^{(1)}(x) Z_\nu^{(2)}(x)] - \right. \\ \left. - (\lambda + \mu + \nu - 1) \int x^\lambda Z_{\mu-1}^{(1)}(x) Z_{\nu-1}^{(2)}(x) dx \right\}.$$

$$2. \int x^{\pm \mu \pm \nu + 1} Z_\mu^{(1)}(x) Z_\nu^{(2)}(x) dx = \\ = \pm \frac{x^{\pm \mu \pm \nu + 2}}{2(\mu + \nu \pm 1)} [Z_\mu^{(1)}(x) Z_\nu^{(2)}(x) + Z_{\mu \pm 1}^{(1)}(x) Z_{\nu \pm 1}^{(2)}(x)].$$

$$3. \int x Z_\nu^{(1)}(ax) Z_\nu^{(2)}(bx) dx = \frac{x}{a^2 - b^2} [a Z_{\nu+1}^{(1)}(ax) Z_\nu^{(2)}(bx) - b Z_\nu^{(1)}(ax) Z_{\nu+1}^{(2)}(ax)].$$

$$4. \int x Z_\nu^{(1)}(ax) Z_\nu^{(2)}(ax) dx = \\ = \frac{x^2}{4} [2 Z_\nu^{(1)}(ax) Z_\nu^{(2)}(ax) - Z_{\nu-1}^{(1)}(ax) Z_{\nu+1}^{(2)}(ax) - Z_{\nu+1}^{(1)}(ax) Z_{\nu-1}^{(2)}(ax)].$$

$$5. \int \frac{1}{x} Z_\mu^{(1)}(ax) Z_\nu^{(2)}(ax) dx = \\ = - \frac{ax}{\mu^2 - \nu^2} [Z_{\mu+1}^{(1)}(ax) Z_\nu^{(2)}(ax) - Z_\mu^{(1)}(ax) Z_{\nu+1}^{(2)}(ax)] + \frac{1}{\mu + \nu} Z_\mu^{(1)}(ax) Z_\nu^{(2)}(ax).$$

$$6. \int \frac{1}{x} Z_v^{(1)}(ax) Z_v^{(2)}(ax) dx = \\ = \frac{ax}{2v} \left[Z_{v+1}^{(1)}(ax) \frac{\partial Z_v^{(2)}(ax)}{\partial v} - Z_v^{(1)}(ax) \frac{\partial Z_{v+1}^{(2)}(ax)}{\partial v} \right] + \frac{1}{2v} Z_v^{(1)}(ax) Z_v^{(2)}(ax).$$

$$7. \int \frac{1}{x} Z_n^{(1)}(x) Z_n^{(2)}(x) dx = \\ = -\frac{1}{2n} \left[Z_0^{(1)}(x) Z_0^{(2)}(x) + Z_n^{(1)}(x) Z_n^{(2)}(x) + 2 \sum_{k=1}^{n-1} Z_k^{(1)}(x) Z_k^{(2)}(x) \right].$$

$$8. \int \frac{1}{x} Z_{\mu+n}^{(1)}(x) Z_{\nu+n}^{(2)}(x) dx = \\ = \frac{1}{\mu+\nu+2n} \left[(\mu+\nu) \int \frac{1}{x} Z_{\mu}^{(1)}(x) Z_{\nu}^{(2)}(x) dx - Z_{\mu}^{(1)}(x) Z_{\nu}^{(2)}(x) - \right. \\ \left. - Z_{\mu+n}^{(1)}(x) Z_{\nu+n}^{(2)}(x) - 2 \sum_{k=1}^{n-1} Z_{\mu+k}^{(1)}(x) Z_{\nu+k}^{(2)}(x) \right].$$

$$9. \int \left[(a^2 - b^2)x - \frac{\mu^2 - \nu^2}{x} \right] Z_{\mu}^{(1)}(ax) Z_{\nu}^{(2)}(bx) dx = \\ = bx Z_{\mu}^{(1)}(ax) Z_{\nu-1}^{(2)}(bx) - ax Z_{\mu-1}^{(1)}(ax) Z_{\nu}^{(2)}(bx) + (\mu - \nu) Z_{\mu}^{(1)}(ax) Z_{\nu}^{(2)}(bx).$$

1.13.3. Интегралы вида $\int x^{\lambda} Z_{\nu}^2(x) dx$.

$$1. \int x^{\lambda} Z_{\nu}^2(x) dx = \frac{\lambda-1}{\lambda} \left[\nu^2 - \frac{(\lambda-1)^2}{4} \right] \int x^{\lambda-2} Z_{\nu}^2(x) dx + \\ + \frac{x^{\lambda-1}}{2\lambda} \left\{ \left[x Z'_{\nu}(x) - \frac{\lambda-1}{2} Z_{\nu}(x) \right]^2 + \left[x^2 - \nu^2 + \frac{(\lambda-1)^2}{4} \right] Z_{\nu}^2(x) \right\}.$$

$$2. \int x^{\pm 2\nu+1} Z_{\nu}^2(x) dx = \pm \frac{x^{\pm 2\nu+2}}{2(2\nu \pm 1)} [Z_{\nu \pm 1}^2(x) + Z_{\nu}^2(x)].$$

$$3. \int \frac{1}{x} Z_n^2(x) dx = -\frac{1}{2n} \left[Z_0^2(x) - Z_n^2(x) + 2 \sum_{k=1}^n Z_k^2(x) \right].$$

$$4. \int \frac{1}{x^2} Z_{\nu}^2(x) dx = \frac{2}{(4\nu^2-1)x} \left\{ \left[x Z'_{\nu}(x) + \frac{1}{2} Z_{\nu}(x) \right]^2 + \left(x^2 - \nu^2 + \frac{1}{4} \right) Z_{\nu}^2(x) \right\}.$$

1.14. ОРТОГОНАЛЬНЫЕ МНОГОЧЛЕНЫ ЛЕЖАНДРА $P_n(x)$,

ЧЕБЫШЕВА $T_n(x)$, $U_n(x)$, ЛАГЕРРА $L_n(x)$, $L_n^{\alpha}(x)$, ЭРМИТА $H_n(x)$
ГЕГЕНБАУЭРА $C_n^{\nu}(x)$ И ЯКОБИ $P_n^{(\alpha, \beta)}(x)$

1.14.1. Интегралы, содержащие $P_n(x)$
(см. также 1.14.5—6; $P_n(x) = C_n^{1/2}(x) = P_n^{(0,0)}(x)$).

$$1. \int P_n(x) dx = -\frac{1}{n(n+1)} (1-x^2) P'_n(x) = \frac{1}{2n+1} [P_{n+1}(x) - P_{n-1}(x)] \\ [n=1, 2, 3, \dots].$$

$$2. \int (1-x^2)^{(n-2)/2} P_n(x) dx = -\frac{1}{n} (1-x^2)^{n/2} P_{n-1}(x) \\ [n=1, 2, 3, \dots].$$

$$3. \int (1-x^2)^{-(n+3)/2} P_n(x) dx = \frac{1}{n+1} (1-x^2)^{-(n+1)/2} P_{n+1}(x).$$

$$4. \int \ln \left| \frac{1+x}{1-x} \right| P_n(x) dx = -\frac{1}{n(n+1)} \left[(1-x^2) P'_n(x) \ln \left| \frac{1+x}{1-x} \right| - 2P_n(x) \right].$$

1.14.2. Интегралы, содержащие $T_n(x)$ и $U_n(x)$
См. также 1.14.5—6;

$$T_n(x) = 2^{2n} \frac{(n!)^2}{(2n)!} P_n^{(-1/2, -1/2)}(x),$$

$$U_n(x) = C_n^1(x) = 2^{2n+1} \frac{[(n+1)!]}{(2n+2)!} P_n^{(1/2, 1/2)}(x).$$

$$1. \int T_n(x) dx = \frac{1}{2} \left[\frac{1}{n+1} T_{n+1}(x) - \frac{1}{n-1} T_{n-1}(x) \right] \quad [n \geq 1].$$

$$2. \quad = 2^{2n+1} \frac{(n!)^2}{(2n)! (n-1)!} P_{n+1}^{(-3/2, -3/2)}(x).$$

$$3. \int (1-x^2)^{(n-3)/2} T_n(x) dx = -\frac{1}{n-1} (1-x^2)^{(n-1)/2} T_{n-1}(x) \quad [n \geq 1].$$

$$4. \int (1-x^2)^{-(n+3)/2} T_n(x) dx = \frac{1}{n+1} (1-x^2)^{-(n+1)/2} T_{n+1}(x).$$

$$5. \int U_n(x) dx = \frac{1}{2(n+1)} [U_{n+1}(x) - U_{n-1}(x)] \quad [n \geq 1].$$

$$6. \quad = \frac{1}{n+1} T_{n+1}(x).$$

$$7. \int (1-x^2)^{(n-1)/2} U_n(x) dx = -\frac{1}{n} (1-x^2)^{(n+1)/2} U_{n-1}(x) \quad [n \geq 1].$$

$$8. \int (1-x^2)^{-(n+3)/2} U_n(x) dx = \frac{1}{n+2} (1-x^2)^{-(n+1)/2} U_{n+1}(x).$$

1.14.3. Интегралы, содержащие $L_n(x)$ и $L_n^\alpha(x)$.

$$1. \int_0^x L_n(x) dx = \frac{x}{n+1} [L_n(x) - L'_n(x)] = L_n(x) - L_{n+1}(x).$$

$$2. \int_0^x L_n^\alpha(x) dx = -L_{n+1}^{\alpha-1}(x) + \frac{(\alpha)_{n+1}}{(n+1)!}.$$

$$3. \int_{x_1}^{x_2} x^\lambda L_n^\alpha(ax) dx = \pm \frac{(\alpha+1)_n}{n! (\lambda+1)} x^{\lambda+1} {}_2F_2(-n, \lambda+1; \alpha+1, \lambda+2; ax) \\ \left[\begin{matrix} \{x_1=0, x_2=x; \operatorname{Re} \lambda > -1\} \\ \{x_1=x, x_2=\infty; \operatorname{Re} \lambda < -n-1\} \end{matrix} \right].$$

$$4. \int_0^x x^\alpha L_n^\alpha(x) dx = \frac{1}{n+\alpha+1} x^{\alpha+1} L_{n+1}^{\alpha+1}(x).$$

$$5. \int x^{-n-2} L_n^\alpha(x) dx = -\frac{1}{n+\alpha+1} x^{-n-1} L_{n+1}^\alpha(x).$$

$$6. \int_x^\infty e^{-x} L_n^\alpha(x) dx = e^{-x} [L_n^\alpha(x) - L_{n-1}^\alpha(x)], \quad [n=1, 2, 3, \dots].$$

$$7. \int_{x_1}^{x_2} x^\lambda e^{-ax} L_n^\alpha(ax) dx = \\ = \pm \frac{(\alpha+1)_n}{n! (\lambda+1)} x^{\lambda+1} {}_2F_2(\alpha+n+1, \lambda+1; \alpha+1, \lambda+2; -ax) + \frac{(\alpha-\lambda)_n}{n! a^{\lambda+1}} \Gamma(\lambda+1) \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \\ \left[\begin{matrix} \{x_1=0, x_2=x; \operatorname{Re} \lambda > -1\} \\ \{x_1=x, x_2=\infty; \operatorname{Re} a > 0\} \end{matrix}, x > 0 \right].$$

$$8. \int x^\alpha e^{-x} L_n^\alpha(x) dx = \frac{1}{n} x^{\alpha+1} e^{-x} L_{n+1}^{\alpha+1}(x) \bullet \quad [n=1, 2, 3, \dots].$$

$$9. \int x^{\alpha+n-1} e^{-x} L_n^\alpha(x) dx = \frac{1}{n} x^{n+\alpha} e^{-x} L_{n-1}^\alpha(x) \quad [n=1, 2, 3, \dots].$$

$$10. \int \left(1 - \frac{1}{x}\right) x^{-2(n+1)} [L_n^\alpha(x)]^2 dx = \\ = \frac{1}{2} x^{-2(n+1)} [L_n^\alpha(x)]^2 + \frac{n+1}{2(n+\alpha+1)} x^{-2(n+1)} [L_{n+1}^\alpha(x)]^2.$$

$$11. \int \left(1 - \frac{2n+\alpha}{x}\right) x^{2(n+\alpha)} e^{-2x} [L_n^\alpha(x)]^2 dx = \\ = -\frac{1}{2} x^{2(n+\alpha)} e^{-2x} [L_n^\alpha(x)]^2 - \frac{n+\alpha}{2n} x^{2(n+\alpha)} e^{-2x} [L_{n-1}^\alpha(x)]^2$$

[n = 1, 2, 3, ...].

1.14.4. Интегралы, содержащие $H_n(x)$

(см. также 1.14.3; $H_{2n+\varepsilon}(x) = (-1)^n 2^{2n+\varepsilon} n! x^\varepsilon L_n^{\varepsilon-1/2}(x^2)$ [$\varepsilon = 0$ или 1]).

$$1. \int_{x_1}^{x_2} x^\lambda H_{2n+\varepsilon}(ax) dx = \pm (-1)^n \frac{2^{2n+\varepsilon} a^\varepsilon x^{\lambda+\varepsilon+1}}{\lambda+\varepsilon+1} \left(\varepsilon + \frac{1}{2}\right)_n \times \\ \times {}_2F_2\left(-n, \frac{\lambda+\varepsilon+1}{2}; \varepsilon + \frac{1}{2}, \frac{\lambda+\varepsilon+3}{2}; a^2 x^2\right) \\ \left[\varepsilon = 0 \text{ или } 1, \begin{cases} x_1 = 0, & x_2 = x; & \operatorname{Re} \lambda > -\varepsilon - 1 \\ x_1 = x, & x_2 = \infty, & \operatorname{Re} \lambda < -2n - \varepsilon - 1 \end{cases}\right].$$

$$2. \int H_n(x) dx = \frac{1}{2(n+1)} H_{n+1}(x).$$

$$3. \int x H_n(x) dx = \frac{x}{2(n+1)} H_{n+1}(x) - \frac{1}{4(n+1)(n+2)} H_{n+2}(x).$$

$$4. \int x^{-n-2} H_n(x) dx = \frac{x^{-n-2}}{2(n+1)(n+2)} H_{n+2}(x).$$

$$5. \int_{x_1}^{x_2} x^\lambda e^{-a^2 x^2} H_{2n+\varepsilon}(ax) dx = \pm (-1)^n \frac{2^{2n+\varepsilon} a^\varepsilon x^{\lambda+\varepsilon+1}}{\lambda+\varepsilon+1} \left(\varepsilon + \frac{1}{2}\right)_n \times \\ \times {}_2F_2\left(\frac{2\varepsilon+2n+1}{2}, \frac{\lambda+\varepsilon+1}{2}; \varepsilon + \frac{1}{2}, \frac{\lambda+\varepsilon+3}{2}; -a^2 x^2\right) + \\ + (-1)^n \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} 2^{2n+\varepsilon-1} a^{-\lambda-1} \left(\frac{\varepsilon-\lambda}{2}\right)_n \Gamma\left(\frac{\varepsilon+\lambda+1}{2}\right) \\ \left[\varepsilon = 0 \text{ или } 1, \begin{cases} x_1 = 0, & x_2 = x; & \operatorname{Re} \lambda > -\varepsilon - 1 \\ x_1 = x, & x_2 = \infty; & |\arg a| < \pi/4 \end{cases}\right].$$

$$6. \int e^{-x^2} H_n(x) dx = -e^{-x^2} H_{n-1}(x).$$

$$7. \int x^{n-2} e^{-x^2} H_n(x) dx = -2x^{n-1} e^{-x^2} H_{n-2}(x) \quad [n \geq 2]$$

$$8. \int \operatorname{erf}(ax) H_n(ax) dx = \frac{1}{2a(n+1)} \left[\operatorname{erf}(ax) H_{n+1}(ax) + \frac{2}{\sqrt{\pi}} e^{-a^2 x^2} H_n(ax) \right].$$

$$9. \int e^{-x^2} \operatorname{erfi}(x) H_n(x) dx = \frac{1}{n\sqrt{\pi}} H_n(x) - e^{-x^2} \operatorname{erfi}(x) H_{n-1}(x).$$

$$10. \int x H_n^2(x) dx = \frac{1}{4} H_n^2(x) + \frac{1}{8(n+1)} H_{n+1}^2(x).$$

$$11. \int x e^{-2x^2} H_n^2(x) dx = -\frac{1}{4} e^{-2x^2} H_n^2(x) - \frac{n}{2} e^{-2x^2} H_{n-1}^2(x).$$

1.14.5. Интегралы, содержащие $C_n^\nu(x)$

(см. также 1.14.6; $C_n^\nu(x) = \frac{(2\nu)_n}{(\nu+1/2)_n} P_n^{\nu-1/2, \nu-1/2}(x)$).

$$1. \int C_n^\nu(x) dx = \frac{1}{2(n+\nu)} [C_{n+1}^\nu(x) - C_{n-1}^\nu(x)] \quad [n \geq 1].$$

$$2. \quad = \frac{1}{2(v-1)} C_{n+1}^{v-1}(x).$$

$$3. \quad \int C_n^{3/2}(x) dx = P_{n+1}(x).$$

$$4. \quad \int C_n^2(x) dx = \frac{1}{2} U_{n+1}(x).$$

$$5. \quad \int (1-x^2)^{(n+2v-3)/2} C_n^v(x) dx = -\frac{1}{n} (1-x^2)^{(n+2v-1)/2} C_{n-1}^v(x) \quad [n \geq 1]$$

$$6. \quad \int (1-x^2)^{-(n+3)/2} C_n^v(x) dx = \frac{1}{n+2v} (1-x^2)^{-(n+1)/2} C_{n+1}^v(x).$$

$$7. \quad \int_0^x (1-x^2)^{v-1/2} C_n^v(x) dx = -\frac{2v(1-x^2)^{v+1/2}}{n(n+2v)} C_{n-1}^{v+1}(x).$$

1.14.6. Интегралы, содержащие $P_n^{(\alpha, \beta)}(x)$.

$$1. \quad \int P_n^{(\alpha, \beta)}(x) dx = \frac{2}{\alpha + \beta + n} P_{n+1}^{(\alpha-1, \beta-1)}(x).$$

$$2. \quad \int (1-x)^\alpha (1+x)^\beta P_n^{(\alpha, \beta)}(x) dx = -\frac{1}{2n} (1-x)^{\alpha+1} (1+x)^{\beta+1} P_{n-1}^{(\alpha+1, \beta+1)}(x) \quad [n=1, 2, 3, \dots]$$

$$3. \quad \int x^m (1-x)^\alpha (1+x)^\beta P_n^{(\alpha, \beta)}(x) dx = \\ = \frac{1}{m+1} \sum_{k=1}^N (-1)^k (-m-1)_k (1-x)^{\alpha+k} (1+x)^{\beta+k} P_{n-k}^{(\alpha+k, \beta+k)}(x) + \\ + \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} (-1)^n (-m)_{n-1} \int x^{m-n} (1-x)^{\alpha+n} (1+x)^{\beta+n} dx \\ \left[\begin{Bmatrix} m \geq n; N=n \\ m < n; N=m+1 \end{Bmatrix} \right].$$

$$4. \quad \int_{x_1}^{x_2} (x-1)^{\mu-1} P_n^{(\alpha, \beta)}(x) dx = \\ = \pm \frac{(\alpha+1)_n}{n! \mu} (x-1)^\mu {}_3F_2 \left(-n, \alpha + \beta + n + 1, \mu; \alpha + 1, \mu + 1; \frac{1-x}{2} \right) \\ \left[\begin{Bmatrix} x_1=1, x_2=x; \operatorname{Re} \mu > 0 \\ x_1=x, x_2=\infty; \operatorname{Re} \mu < -n \end{Bmatrix}, x > 1 \right].$$

$$5. \quad \int_{x_1}^{x_2} (x-1)^{\mu-1} (x+1)^\beta P_n^{(\alpha, \beta)}(x) dx = \\ = \pm \frac{(\alpha+1)_n}{n! \mu} 2^\mu (x-1)^\mu {}_3F_2 \left(\alpha + n + 1, -\beta - n, \mu; \beta + 1, \mu + 1; \frac{1-x}{2} \right) + \\ + \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \frac{2^{\beta+\mu}}{n!} \Gamma \left[\begin{matrix} \alpha - \mu + n + 1, \mu, -\beta - \mu - n \\ \alpha - \mu + 1, -\beta - n \end{matrix} \right] \\ \left[\begin{Bmatrix} x_1=1, x_2=x; \operatorname{Re} \mu > 0 \\ x_1=x, x_2=\infty; \operatorname{Re}(\beta + \mu) < -n \end{Bmatrix}, x > 1 \right].$$

$$6. \quad \int_{-1}^x (x+1)^{\mu-1} P_n^{(\alpha, \beta)}(x) dx = \\ = \frac{(-1)^n (\beta+1)_n}{n! \mu} (x+1)^\mu {}_3F_2 \left(-n, \alpha + \beta + n + 1, \mu; \beta + 1, \mu + 1; \frac{x+1}{2} \right) \\ [x > -1; \operatorname{Re} \mu > 0].$$

$$7. \int_x (x+1)^{\mu-1} P_n^{(\alpha, \beta)}(x) dx = -\frac{(x+1)^{n+\mu}}{2^n n! (n+\mu)} \Gamma \left[\frac{\alpha+\beta+2n+1}{\alpha+\beta+n+1} \right] \times \\ \times {}_3F_2 \left(-n, -n-\beta, -n-\mu; -2n-\alpha-\beta, 1-n-\mu; \frac{2}{x+1} \right) \\ [x > -1; \operatorname{Re} \mu < -n].$$

$$8. \int_{x_1}^{x_2} (1-x)^\alpha (1+x)^{\mu-1} P_n^{(\alpha, \beta)}(x) dx = \\ = \pm (-1)^n \frac{(\beta+1)_n}{n! \mu} 2^\alpha (1+x)^\mu {}_3F_2 \left(-\alpha-n, n+\beta+1, \mu; \beta+1, \mu+1; \frac{1+x}{2} \right) + \\ + \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} (-1)^n \frac{(\beta-\mu+1)_n}{n!} 2^{\alpha+\mu} B(\alpha+n+1, \mu) \\ \left[\begin{Bmatrix} x_1=-1, x_2=x; \operatorname{Re} \mu > 0 \\ x_1=x, x_2=1; \operatorname{Re} \alpha > -1 \end{Bmatrix}, -1 < x < 1 \right].$$

$$9. \int_{x_1}^{x_2} (1-x)^{\mu-1} (1+x)^{-\mu-n-1} P_n^{(\alpha, \beta)}(x) dx = \\ = \mp \frac{(\alpha+1)_n}{n! \mu} 2^{-n-1} \left(\frac{1-x}{1+x} \right)^\mu {}_3F_2 \left(-n, -n-\beta, \mu; \alpha+1, \mu+1; \frac{x-1}{x+1} \right) \\ \left[\begin{Bmatrix} x_1=-1, x_2=x; \operatorname{Re} \mu < -n \\ x_1=x, x_2=1; \operatorname{Re} \mu > 0 \end{Bmatrix}, -1 < x < 1 \right].$$

$$10. \int_{x_1}^{x_2} (x-1)^\alpha (x+1)^{\mu-1} P_n^{(\alpha, \beta)}(x) dx = \\ = \pm \frac{2^{-n}}{n! (\alpha+\mu+n)} \Gamma \left[\frac{\alpha+\beta+2n+1}{\alpha+\beta+n+1} \right] (x+1)^{\alpha+\mu+n} {}_3F_2 \left(-\alpha-n, \right. \\ \left. -\alpha-\beta-n, -\alpha-\mu-n; -\alpha-\beta-2n, -\alpha-\mu-n-1; \frac{2}{x+1} \right) + \\ + \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{(\beta-\mu+1)_n}{n!} 2^{\alpha+\mu} B(\alpha+1, -\alpha-\mu-n) \\ \left[\begin{Bmatrix} x_1=1, x_2=x; \operatorname{Re} \alpha > -1 \\ x_1=x, x_2=\infty; \operatorname{Re} (\alpha+\mu) < -n \end{Bmatrix}, x > 1 \right].$$

$$11. \int_{x_1}^{x_2} (1-x)^{\mu-1} (1+x)^{\alpha+\beta-\mu+n} P_n^{(\alpha, \beta)}(x) dx = \pm \frac{(\alpha+1)_n}{n! \mu} 2^{\alpha+\beta+n} \times \\ \times \left(\frac{1-x}{1+x} \right)^\mu {}_3F_2 \left(\alpha+n+1, \alpha+\beta+n+1, \mu; \alpha+1, \mu+1; \frac{x-1}{x+1} \right) + \\ + \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \frac{2^{\alpha+\beta+n}}{n!} (\alpha-\mu+1)_n B(\mu, \alpha+\beta-\mu+n-1) \\ \left[\begin{Bmatrix} x_1=x, x_2=1; \operatorname{Re} \mu > 0 \\ x_1=-1, x_2=x; \operatorname{Re} (\mu-\alpha-\beta) < n+1 \end{Bmatrix}, -1 < x < 1 \right].$$

$$12. \int_{x_1}^{x_2} (1-x)^{\mu-1} (1+x)^\beta P_n^{(\alpha, \beta)}(x) dx = \\ = \pm \frac{(\alpha+1)_n}{n! \mu} 2^\beta (1-x)^\mu {}_3F_2 \left(\alpha+n+1, -\beta-n, \mu; \alpha+1, \mu+1; \frac{1-x}{2} \right) + \\ + \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \frac{(\alpha-\mu+1)_n}{n!} 2^{\beta+\mu} B(\beta+n+1, \mu) \\ \left[\begin{Bmatrix} x_1=x, x_2=1; \operatorname{Re} \mu > 0 \\ x_1=-1, x_2=x; \operatorname{Re} \beta > -1 \end{Bmatrix}, -1 < x < 1 \right].$$

Глава 2. ОПРЕДЕЛЕННЫЕ ИНТЕГРАЛЫ

2.1. ВВЕДЕНИЕ

В этой главе содержатся определенные интегралы от специальных функций. Интегралы, у которых один из пределов интегрирования является переменным, в случаях, когда подынтегральная функция от него не зависит, помещены в главе 1. Интегралы, содержащие произвольные функции, имеются в [12] (п. 2.1.2). Отметим, что многие определенные интегралы могут быть вычислены с помощью преобразования Меллина [10]. Указанные в формулах условия сходимости интегралов обеспечивают их существование либо в обычном смысле, либо как несобственных интегралов, либо в смысле главного значения.

Некоторые формулы при частных значениях параметров теряют смысл, так как в их правых частях возникают неопределенности; раскрытие неопределенностей позволяет получить формулы, справедливые при этих значениях параметров.

В начале разделов, как правило, помещены интегралы общего вида, у которых аргумент одной из подынтегральных функций зависит от параметра $r > 0$. При рациональных значениях этого параметра правые части равенств выражаются через обобщенные гипергеометрические функции. Частные значения интегралов общего вида помещены среди формул последующих пунктов.

В качестве примера найдем некоторые частные значения интеграла общего вида

$$I(r) = \int_0^{\infty} x^{\alpha-1} e^{-px^r} J_{\nu}(cx) dx.$$

В силу формулы 2.12.1.5 при $r > 1$ имеем

$$I(r) = \frac{c^{\nu} p^{-(\alpha+\nu)/r}}{2^{\nu} \Gamma(\nu+1)} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (\nu+1)_k} \Gamma\left(\frac{2k+\alpha+\nu}{r}\right) \left(\frac{c}{2p^{1/r}}\right)^{2k} \\ [\operatorname{Re} p, \operatorname{Re}(\alpha+\nu) > 0; |\arg c| < \pi].$$

После перехода к пределу при $r \rightarrow 1+0$ с учетом формулы для $\Gamma(z+2k)$ из приложения II.1 получим

$$I(1) = I(1+0) = \\ = \left(\frac{c}{2}\right)^{\nu} p^{-\alpha-\nu} \frac{\Gamma(\alpha+\nu)}{\Gamma(\nu+1)} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (\nu+1)_k} \left(\frac{\alpha+\nu}{2}\right)_k \left(\frac{\alpha+\nu+1}{2}\right)_k 2^{2k} \left(\frac{c^2}{4p^2}\right)^k;$$

используя определение гипергеометрической функции ${}_2F_1(a, b; c; z)$, последнему выражению можно придать следующий вид:

$$I(1) = \frac{c^v}{2^v p^{\alpha+v}} \Gamma \left[\frac{\alpha+v}{v+1} \right] {}_2F_1 \left(\frac{\alpha+v}{2}, \frac{\alpha+v+1}{2}; v+1; -\frac{c^2}{p^2} \right)$$

(см. 2.12.8.3); при этом условия сходимости интеграла $\operatorname{Re} p > 0$, $|\arg c| < \pi$ следует заменить на $\operatorname{Re} p > |\operatorname{Im} c|$. Отметим, что то же выражение для рассматриваемого интеграла можно вывести и исходя из формулы 2.12.1.5 при $r < 1$ с помощью перехода к пределу при $r \rightarrow 1-0$.

Преобразовав теперь гамма-функцию по формуле для $\Gamma(z+k)$ из приложения 11.1 и воспользовавшись определением вырожденной гипергеометрической функции, можно получить

$$\begin{aligned} I(2) &= \frac{c^v \Gamma((\alpha+v)/2)}{2^{v+1} p^{(\alpha+v)/2} \Gamma(v+1)} \sum_{k=0}^{\infty} \frac{((\alpha+v)/2)_k}{k! (v+1)_k} \left(-\frac{c^2}{4p} \right)^k = \\ &= \frac{c^v}{2^{v+1} p^{(\alpha+v)/2}} \Gamma \left[\frac{(\alpha+v)/2}{v+1} \right] {}_1F_1 \left(\frac{\alpha+v}{2}; v+1; -\frac{c^2}{4p} \right) \end{aligned}$$

(см. 2.12.9.3).

Рассмотрим еще случай, когда $r = \frac{2m}{n}$, где m, n — взаимно простые целые числа и $2m > n > 0$. Представим индекс суммирования k в виде $k = ml + j$; $j = 0, 1, 2, \dots, m-1$; $l = 0, 1, 2, \dots$, и разобьем сумму по k на m бесконечных сумм по l :

$$\begin{aligned} I \left(\frac{2m}{n} \right) &= \frac{nc^v p^{-(\alpha+v)n/(2m)}}{2^{v+1} m} \sum_{j=0}^{m-1} \sum_{l=0}^{\infty} \frac{(-1)^{ml+j}}{(ml+j)! \Gamma(1+v+ml+j)} \times \\ &\quad \times \Gamma \left(\frac{\alpha+v+2j}{2m} n + ln \right) \left(\frac{c}{2p^{n/(2m)}} \right)^{2ml+2j}. \end{aligned}$$

С помощью формулы для $\Gamma(z+nk)$ из приложения 11.1 и формулы для $(nk+m)!$ из приложения 1.2 получаем соотношение

$$\begin{aligned} I \left(\frac{2m}{n} \right) &= \frac{nc^v p^{-(\alpha+v)n/(2m)}}{2^{v+1} m} \sum_{j=0}^{m-1} \left(-\frac{c^2}{4p^{n/m}} \right)^j \frac{1}{j! \Gamma(1+v+j)} \times \\ &\quad \times \Gamma \left(\frac{\alpha+v+2j}{2m} n \right) \sum_{l=0}^{\infty} \frac{((\alpha+v+2j)/(2m))_l ((\alpha+v+2j)/(2m) + 1/n)_l \dots}{((j+1)/m)_l ((j+2)/m)_l \dots ((j+m)/m)_l m^{ml}} \times \\ &\quad \times \frac{\dots ((\alpha+v+2j)/(2m) + (n-1)/n)_l n^{nl}}{((1+v+j)/m)_l ((2+v+j)/m)_l \dots ((m+v+j)/m)_l m^{ml}} \left(-\frac{c^2}{4p^{n/m}} \right)^{ml}. \end{aligned}$$

Используя теперь символ обобщенной гипергеометрической функции, приходим к окончательному выражению

$$\begin{aligned} I \left(\frac{2m}{n} \right) &= \frac{nc^v}{2^{v+1} m p^{(\alpha+v)n/(2m)}} \sum_{j=0}^{m-1} \left(-\frac{c^2}{4p^{n/m}} \right)^j \frac{1}{j! \Gamma(v+j+1)} \times \\ &\quad \times \Gamma \left(\frac{\alpha+v+2j}{2m} n \right) {}_{n+1}F_{2m} \left(1, \Delta(n, (\alpha+v+2j)n/(2m)); \Delta(m, j+1), \Delta(m, v+j+1); (-1)^m \left(\frac{c}{2m} \right)^{2m} \left(\frac{n}{p} \right)^n \right), \end{aligned}$$

где

$$\Delta(n, a) = \frac{a}{n}, \frac{a+1}{n}, \dots, \frac{a+n-1}{n}.$$

2.2. ГАММА-ФУНКЦИЯ $\Gamma(x)$

Таблицу интегралов по прямой $(\gamma - i\infty, \gamma + i\infty)$, $\gamma = \operatorname{Re} s$, см в [10, § 10].

2.2.1. Интегралы, содержащие $\Gamma(c+x)$ ($0 < x < \infty$).

$$1. \int_a^\infty (x-a)^{\alpha-1} \frac{b^x}{\Gamma(1+x)} dx = \Gamma(\alpha) \mu(b, \alpha-1, a) \quad [a, b, \operatorname{Re} \alpha > 0].$$

$$2. \int_0^\infty \frac{b^x}{\Gamma(1+x)} dx = e^b - \int_{-\infty}^\infty \frac{1}{t^2 + \pi^2} e^{-be^t} dt = \nu(b) \quad [b > 0].$$

$$3. \int_0^\infty \frac{b^x}{\Gamma(c+x)} dx = b^{1-c} \nu(b, c-1) \quad [b, \operatorname{Re} c > 0].$$

$$4. \int_0^\infty \frac{x^{\alpha-1} b^x}{\Gamma(1+x)} dx = \Gamma(\alpha) \mu(b, \alpha-1) \quad [b, \operatorname{Re} \alpha > 0].$$

$$5. \int_0^\infty \frac{x^{\alpha-1} b^x}{\Gamma(c+x)} dx = \Gamma(\alpha) b^{1-c} \mu(b, \alpha-1, c-1) \quad [b, \operatorname{Re} c, \operatorname{Re} \alpha > 0].$$

$$6. \int_0^\infty \frac{dx}{\Gamma(c+x) \Gamma(c-x)} = \frac{2^{2c-3}}{\Gamma(2c-1)} \quad [\operatorname{Re} c > 1/2].$$

$$7. \int_0^\infty \frac{\sin \pi n x}{\sin \pi x} \frac{dx}{\Gamma(c+x) \Gamma(c-x)} = \frac{1 - (-1)^n}{2} \frac{2^{2c-3}}{\Gamma(2c-1)} \quad [\operatorname{Re} c > 1/2].$$

$$8. \int_0^\infty \frac{dx}{[\Gamma(c+x) \Gamma(c-x)]^2} = \frac{\Gamma(4c-3)}{2\Gamma^4(2c-1)} \quad [\operatorname{Re} c > 3/4].$$

$$9. \int_0^\infty \frac{\cos \pi x}{[\Gamma(c+x) \Gamma(c-x)]^2} dx = \frac{1}{4\Gamma[2c-1, c, c]} \quad [\operatorname{Re} c > 1/2].$$

2.2.2. Интегралы, содержащие $\Gamma(a \pm x)$ ($-\infty < x < \infty$).

$$1. \int_{-\infty}^\infty \Gamma(a+x) \Gamma(b-x) dx = \mp 2^{1-a-b} \pi i \Gamma(a+b) \quad \left[\operatorname{Re}(a+b) < 1, \begin{cases} \operatorname{Im} a, \operatorname{Im} b > 0 \\ \operatorname{Im} a, \operatorname{Im} b < 0 \end{cases} \right].$$

$$2. = 0 \quad [\operatorname{Re}(a+b) < 1, \operatorname{Im} a \cdot \operatorname{Im} b < 0].$$

$$3. \int_{-\infty}^\infty \Gamma \left[\frac{a+x}{c+x} \right] dx = 0 \quad [\operatorname{Im} a \neq 0, \operatorname{Re}(c-a) > 1].$$

$$4. \int_{-\infty}^\infty \frac{dx}{\Gamma(c+x) \Gamma(d-x)} = \frac{2^{c+d-2}}{\Gamma(c+d-1)} \quad [\operatorname{Re}(c+d) > 1].$$

$$5. \int_{-\infty}^\infty \Gamma \left[\frac{a_1+x}{c_1+x}, \frac{a_2+x}{c_2+x} \right] dx = \pm \frac{2\pi^2 i}{\sin(a_2-a_1)\pi} \Gamma \left[\frac{c_1+c_2-a_1-a_2-1}{c_1-a_1, c_1-a_2, c_2-a_1, c_2-a_2} \right] \cdot \left[\operatorname{Re}(c_1+c_2-a_1-a_2) > 1, \begin{cases} \operatorname{Im} a_1 < \operatorname{Im} a_2 < 0 \\ \operatorname{Im} a_2 < \operatorname{Im} a_1 < 0 \end{cases} \right].$$

$$6. \quad = 0 \quad [\operatorname{Re}(c_1 + c_2 - a_1 - a_2) > 1; \operatorname{Im} a_1, \operatorname{Im} a_2 > 0].$$

$$7. \quad \int_{-\infty}^{\infty} \Gamma[a_1 + x, a_2 + x, b_1 - x, b_2 - x] dx = \\ = 2\pi i \Gamma \left[\begin{matrix} a_1 + b_1, a_1 + b_2, a_2 + b_1, a_2 + b_2 \\ a_1 + a_2 + b_1 + b_2 \end{matrix} \right] \quad [\operatorname{Re}(a_1 + a_2 + b_1 + b_2) < 1]$$

$$8. \quad \int_{-\infty}^{\infty} \Gamma \left[\begin{matrix} a + x \\ c_1 + x, c_2 + x, d - x \end{matrix} \right] dx = \frac{\pi \exp [\pm \pi (a - c_1) i/2]}{\Gamma[d + c_1 - 1, (d - c_2)/2, (c_1 - a + 1)/2]} \\ \left[a = c_2 - c_1 - d + 1, \operatorname{Im} a \neq 0; \operatorname{Re}(d + c_1) > 3/2, \begin{cases} \operatorname{Im} a > 0 \\ \operatorname{Im} a < 0 \end{cases} \right].$$

$$9. \quad \int_{-\infty}^{\infty} \frac{dx}{\Gamma[c_1 + x, c_2 + x, d_1 - x, d_2 - x]} = \\ = \Gamma \left[\begin{matrix} c_1 + c_2 + d_1 + d_2 - 3 \\ c_1 + d_1 - 1, c_1 + d_2 - 1, c_2 + d_1 - 1, c_2 + d_2 - 1 \end{matrix} \right] \\ [\operatorname{Re}(c_1 + c_2 + d_1 + d_2) > 3]$$

$$10. \quad \int_{-\infty}^{\infty} \frac{R(x) dx}{\Gamma[c_1 + x, c_2 + x, d_1 - x, d_2 - x]} = \\ = \Gamma \left[\begin{matrix} c_1 + c_2 + d_1 + d_2 - 3 \\ c_1 + d_1 - 1, c_1 + d_2 - 1, c_2 + d_1 - 1, c_2 + d_2 - 1 \end{matrix} \right] \int_0^1 R(t) dt \\ [\operatorname{Re}(c_1 + c_2 + d_1 + d_2) > 3; R(x) = R(x + 1)]$$

$$11. \quad = \frac{1}{\Gamma[(c_1 + d_1)/2, (c_2 + d_2)/2, c_1 + d_2 - 1]} \int_0^1 R(t) \cos \frac{2t + c_1 - d_1}{2} \pi dt \\ [c_1 + d_2 = c_2 + d_1; \operatorname{Re}(c_1 + c_2 + d_1 + d_2) > 2; R(x + 1) = -R(x)]$$

$$12. \quad \int_{-\infty}^{\infty} \exp[2(\pi n + \varphi)xi] \Gamma(a + x) \Gamma(b - x) dx = \\ = 2\pi i \Gamma(a + b) (2 \cos \varphi)^{-a-b} e^{(b-a)\varphi i} [\eta_n(b) e^{2\pi n b i} - \eta_n(-a) e^{-2\pi n a i}] \\ \left[\operatorname{Re}(a + b) < 1; -\pi/2 < \varphi < \pi/2; n - \text{целое}, \right. \\ \left. \begin{cases} \eta_n(z) = 0, \text{ если } (1/2 - n) \operatorname{Im} z > 0 \\ \eta_n(z) = \operatorname{sgn}(1/2 - n), \text{ если } (1/2 - n) \operatorname{Im} z < 0 \end{cases} \right].$$

$$13. \quad \int_{-\infty}^{\infty} \exp[(2\pi n + \pi - 2\varphi)xi] \Gamma \left[\begin{matrix} a + x \\ c + x \end{matrix} \right] dx = \\ = \begin{cases} 1 \\ 0 \end{cases} 2\pi i \operatorname{sgn} \left(n + \frac{1}{2} \right) \frac{(2 \cos \varphi)^{c-a-1}}{\Gamma(c-a)} \exp[-(2\pi n + \pi - \varphi)ai + \varphi i(c-1)] \\ [\operatorname{Re}(c-a) > 0; -\pi/2 < \varphi < \pi/2; n = \dots -2, -1, 0, 1, 2, \dots; \\ \pm(n + 1/2) \operatorname{Im} a < 0].$$

$$14. \quad \int_{-\infty}^{\infty} \frac{e^{i\varphi x}}{\Gamma(c+x) \Gamma(d-x)} dx = \begin{cases} 1 \\ 0 \end{cases} \frac{[2 \cos(\varphi/2)]^{c+d-2}}{\Gamma(c+d-1)} e^{(d-c)\varphi i/2} \\ \left[\begin{cases} -\pi < \varphi < \pi \\ \varphi \geq \pi \text{ или } \varphi \leq -\pi \end{cases}, \operatorname{Re}(c+d) > 1 \right].$$

$$15. \int_{-\infty}^{\infty} \frac{R(x) e^{(2\pi n + \varphi)x}}{\Gamma(c+x) \Gamma(d-x)} dx = \frac{[2 \cos(\varphi/2)]^{c+d-2}}{\Gamma(c+d-1)} e^{(d-c)\varphi/2} \int_0^1 R(t) e^{2\pi n t} dt$$

[Re(c+d) > 1; $-\pi < \varphi < \pi$; n — целое; $R(x+1) = R(x)$]

$$16. \int_{-\infty}^{\infty} \frac{e^{i\pi ax}}{\Gamma[c_1+x, c_2+gx, d_1-x, d_2-gx]} dx = 0$$

[Re(c₁ + c₂ + d₁ + d₂) > 2; Im a = Im g = 0; |a| > |g| + 1]

$$17. \int_{-\infty}^{\infty} \frac{e^{i\pi x}}{\Gamma[c_1+x, c_2+2x, d_1-x, d_2-2x]} dx =$$

$$= \frac{2^{c_1+c_2+d_1+d_2-5}}{\sqrt{\pi}} e^{i\pi(d_1-c_1)/2} \Gamma\left[\frac{(c_1+c_2+d_1+d_2-3)/2}{(c_1+d_1)/2, c_2+d_2-1, 2c_1+d_2-2}\right]$$

[2(c₁ - d₁) = c₂ - d₂; Re(c₁ + c₂ + d₁ + d₂) > 3]

$$18. \int_{-\infty}^{\infty} \frac{1}{\Gamma(c+x) \Gamma(d-x)} \left\{ \frac{\sin bx}{\cos bx} \right\} dx = \frac{[2 \cos(b/2)]^{c+d-2}}{\Gamma(c+d-1)} \left\{ \frac{\sin \varphi}{\cos \varphi} \right\}$$

[$\varphi = b(d-c)/2$; $-\pi < b < \pi$; Re(c+d) > 1].

$$19. = 0 \quad [b > \pi \text{ или } b < -\pi, \text{ Re}(c+d) > 1]$$

$$20. \int_{-\infty}^{\infty} \frac{\sin(n+1)\pi x}{\sin \pi x} \frac{dx}{\Gamma(c+x) \Gamma(d-x)} = \frac{1+(-1)^n}{2} \frac{2^{c+d-2}}{\Gamma(c+d-1)}$$

[Re(c+d) > 1]

$$21. \int_{-\infty}^{\infty} \frac{1}{\Gamma[c_1+x, c_2+x, d_1-x, d_2-x]} \left\{ \frac{\sin \pi x}{\cos \pi x} \right\} dx =$$

$$= \frac{1}{2\Gamma[(c_1+d_1)/2, (c_2+d_2)/2, c_1+d_2-1]} \left\{ \frac{\sin \varphi}{\cos \varphi} \right\}$$

[$\varphi = \pi(d_1-c_1)/2$, $c_1+d_2 = c_2+d_1$; Re(c₁ + c₂ - d₁ - d₂) > 2].

$$22. \int_{-\infty}^{\infty} \frac{\cos(c_2+d_1+x)\pi}{\Gamma[c_1+x, c_2+2x, d_1-x, d_2-2x]} dx =$$

$$= \frac{1}{2\Gamma[c_2+d_2, 2c_1+d_2-2, 2d_1+c_2-2]}$$

[c₁ + c₂ + d₁ + d₂ = 4].

$$23. \int_{-\infty}^{\infty} \frac{\sin(c_1-d_1+2x)\pi}{\Gamma[c_1+x, c_2+2x, d_1-x, d_2-2x]} dx =$$

$$= \pm \frac{2^{2c_1-c_2-3}}{\sqrt{\pi} \Gamma(c_2-c_1+d_1+1/2) \Gamma(2c_1+d_2-2)}$$

[2(c₁ - d₁) = c₂ - d₂ + k, k = ±1, ±2]

$$24. \int_{-\infty}^{\infty} \frac{\sin(c_1-d_1+2x)\pi}{\Gamma[c_1+x, c_2+3x, d_1-x, d_2-3x]} dx =$$

$$= \pm \frac{3^{3c_1+d_2-4}}{4\pi} \Gamma\left[\frac{2c_1-d_1+d_2-2}{c_2+d_2-1, 3c_1+d_2-3}\right]$$

[3(c₁ - d₁) = c₂ - d₂ + k, k = ±1, ±2]

2.2.3. Интегралы, содержащие $\ln \Gamma(a+x)$.

$$1. \int_0^1 \ln \Gamma(x) dx = \frac{\ln(2\pi)}{2}.$$

$$2. \int_a^{a+1} \ln \Gamma(x) dx = a \ln a - a + \frac{\ln(2\pi)}{2} \quad [a \geq 0].$$

$$3. \int_0^n \ln \Gamma(a+x) dx = \sum_{k=0}^{n-1} (a+k) \ln(a+k) - na + \frac{n}{2} \ln(2\pi) - \frac{n(n-1)}{2} \quad [a \geq 0].$$

$$4. \int_0^\infty x^{\alpha-1} \ln \left(\sqrt{x} \Gamma \left[x + \frac{1}{2} \right] \right) dx = \frac{\Gamma(\alpha) (2^{\alpha+1} - 1)}{2^{2\alpha+1} \pi^\alpha \cos(\alpha\pi/2)} \zeta(\alpha+1) \quad [0 < \operatorname{Re} \alpha < 1].$$

$$5. \int_0^1 e^{2\pi n x i} \ln \Gamma(a+x) dx = \frac{1}{2\pi n i} [\ln a - e^{-2\pi n a i} \operatorname{Ei}(2\pi n a i)] \quad [a > 0; n = \pm 1, \pm 2, \pm 3, \dots].$$

$$6. \int_0^1 \left\{ \frac{\sin \pi n x}{\cos \pi n x} \right\} \ln \Gamma(x) dx = \frac{1}{2\pi n} \left\{ \frac{2 \ln(\pi n) + 2C}{\pi} \right\} \quad [n = 2, 4, 6, \dots].$$

$$7. = \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{1}{\pi n} \left[\ln \frac{\pi}{2} + 2 \sum_{k=0}^{[n/2]-1} \frac{1}{2k+1} + \frac{1}{n} \right] \quad [n = 1, 3, 5, \dots].$$

$$8. \int_0^1 \left\{ \frac{\sin 2\pi n x}{\cos 2\pi n x} \right\} \ln \Gamma(x) dx = \frac{1}{4\pi n} \left\{ \frac{2C + 2 \ln(2\pi n)}{\pi} \right\} \quad [n = 1, 2, 3, \dots].$$

$$9. \int_0^1 \left\{ \frac{\sin 2\pi n x}{\cos 2\pi n x} \right\} \ln \Gamma(a+x) dx = -\frac{1}{2\pi n} \left[\left\{ \begin{matrix} \ln a \\ 0 \end{matrix} \right\} + \left\{ \frac{\cos 2\pi n a}{\sin 2\pi n a} \right\} \operatorname{ci}(2\pi n a) \mp \left\{ \frac{\sin 2\pi n a}{\cos 2\pi n a} \right\} \operatorname{si}(2\pi n a) \right] \quad [a > 0; n = 1, 2, 3, \dots]$$

2.2.4. Интегралы, содержащие $\Gamma(a+ix)$.

$$1. \int_0^\infty |\Gamma(a+ix)|^2 dx = \frac{\pi}{2^{2a}} \Gamma(2a) \quad [a > 0].$$

$$2. \int_{-\infty}^\infty \Gamma(1+ix) dx = \frac{2\pi}{e}.$$

$$3. \int_0^\infty |\Gamma(a_1+ix) \Gamma(a_2+ix)|^2 dx = 2^{1-2a_1-2a_2} \pi^{3/2} \Gamma \left[\frac{2a_1, 2a_2, a_1+a_2}{a_1+a_2+1/2} \right] \quad [a_1, a_2 > 0].$$

$$4. \int_0^\infty \left| \Gamma \left[\frac{a+ix}{c+ix} \right] \right|^2 dx = 2^{2c-2a-2} \sqrt{\pi} \Gamma \left[\frac{2a, c-a-1/2}{2c-1, c-a} \right] \quad [0 < a < c-1/2].$$

$$5. \int_0^{\infty} \cos bx \cdot |\Gamma(a + ix)|^2 dx = \frac{\pi}{2^{2a}} \Gamma(2a) \operatorname{sech}^{2a} \frac{b}{2} \quad [a, b > 0].$$

$$6. \int_0^{\infty} \frac{1}{\operatorname{ch} 2\pi x} \left| \Gamma\left(\frac{1}{4} + ix\right) \right|^4 dx = 4\pi G.$$

$$7. \int_0^{\infty} x^2 \operatorname{ch} \pi x |\Gamma(a + ix)|^4 dx = \frac{\pi}{2} \Gamma[a+1, a+1, 2a] \quad [\operatorname{Re} a > 0].$$

2.3. ПСИ-ФУНКЦИЯ $\psi(x)$

Интегралы, содержащие $\psi(x)$, могут быть также получены путем дифференцирования по параметру соответствующих формул из раздела 2.2, так как имеет место равенство

$$\frac{d}{da} \ln \Gamma(a+x) = \psi(a+x).$$

2.3.1. Интегралы от $\psi(x)$.

$$1. \int_a^b \psi(x+c) dx = \ln \Gamma \left[\frac{b+c}{a+c} \right] \quad [a, b, c > 0].$$

$$2. \int_a^{a+1} x\psi(x) dx = a + \ln \Gamma(a+1) - \frac{1}{2} \ln(2\pi) \quad [a \geq 0].$$

$$3. \int_0^n x\psi(x+a) dx = n \ln \Gamma(n+a) - \sum_{k=0}^{n-1} (a+k) \ln(a+k) + \\ + na - \frac{n}{2} \ln(2\pi) + \frac{n(n-1)}{2} \quad [a \geq 0; n=1, 2, 3, \dots].$$

$$4. \int_0^{\infty} x^{\alpha-1} [C + \psi(x+1)] dx = -\frac{\pi}{\sin \pi \alpha} \zeta(1-\alpha) \quad [-1 < \operatorname{Re} \alpha < 0].$$

$$5. \int_0^{\infty} x^{\alpha-1} [\psi(x+a) - \psi(x+b)] dx = \frac{\pi}{\sin \alpha \pi} [\zeta(1-\alpha, b) - \zeta(1-\alpha, a)] \\ [a, b > 0; 0 < \operatorname{Re} \alpha < 1].$$

$$6. \int_0^1 e^{2\pi n x i} \psi(x+a) dx = e^{-2\pi n a i} \operatorname{Ei}(2\pi n a i) \quad [a > 0; n = \pm 1, \pm 2, \dots].$$

$$7. \int_0^1 \sin bx \psi(x) dx = -\frac{\pi}{2} \begin{cases} 1 \\ 0 \end{cases} \quad \left[\begin{cases} b = 2\pi n, n = 1, 2, 3, \dots \\ b = \pi \end{cases} \right].$$

$$8. \int_0^1 \begin{Bmatrix} \sin 2\pi n x \\ \cos 2\pi n x \end{Bmatrix} \psi(x+a) dx = \sin(2\pi n a) \begin{Bmatrix} \operatorname{ci}(2\pi n a) \\ \operatorname{si}(2\pi n a) \end{Bmatrix} \pm \cos(2\pi n a) \begin{Bmatrix} \operatorname{si}(2\pi n a) \\ \operatorname{ci}(2\pi n a) \end{Bmatrix} \\ [a \geq 0; n = 1, 2, 3, \dots].$$

$$9. \int_0^1 \sin \pi x \sin \pi n x \psi(x) dx = \frac{1 - (-1)^n}{4} \ln \frac{n-1}{n+1} \quad [n \neq 1].$$

$$10. \quad = -\frac{1}{2} [C + \ln(2\pi)] \quad [n = 1].$$

$$11. \int_0^1 x(1-x) \cos \pi x \psi(\pm x) dx = \mp \frac{7}{2\pi^2} \zeta(3) + \frac{2}{\pi^2} \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\}.$$

$$12. \int_0^1 x(1-x) \cos \pi x \psi\left(x \pm \frac{1}{2}\right) dx = \frac{5 \pm 1}{\pi^2} - \frac{1}{2} \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \text{Si}\left(\frac{\pi}{2}\right).$$

$$13. \int_0^{1/2} x\left(\frac{1}{2} - x\right) \cos 2\pi x \left[\psi\left(x + \frac{1}{2}\right) - \psi(x) \right] dx = \frac{1}{2\pi^2} - \frac{7}{8\pi^2} \zeta(3).$$

$$14. \int_0^\infty \sin bx [\psi(a+ix) - \psi(a-ix)] dx = \pi i e^{-ab} (1 - e^{-b})^{-1} \quad [a, b > 0].$$

$$15. \int_0^\infty x^{\alpha-1} [\ln x - \psi(x+1)] dx = \frac{\pi}{\sin \alpha\pi} \zeta(1-\alpha) \quad [0 < \text{Re } \alpha < 1].$$

$$16. \int_0^\infty x^{\alpha-1} \left[\ln x - \psi\left(x + \frac{1}{2}\right) \right] dx = \frac{2^{1-\alpha} - 1}{\sin \alpha\pi} \zeta(\alpha) \quad [0 < \text{Re } \alpha < 1].$$

$$17. \int_0^\infty x^{\alpha-1} [\ln(x+1) - \psi(x+1)] dx = \frac{\pi}{\sin \alpha\pi} \left[\zeta(1-\alpha) + \frac{1}{\alpha} \right] \quad [0 < \text{Re } \alpha < 1].$$

$$18. \int_0^\infty \cos bx [\ln x - \psi(x+1)] dx = \frac{1}{2} \left[\ln \frac{b}{2\pi} - \psi\left(\frac{b}{2\pi} + 1\right) \right] \quad [b > 0].$$

$$19. \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} [\ln z - \psi(z+1)] z^{-s} dz = -2\pi i e^{-s\pi i} \zeta(s) \quad [-1 < \gamma < 0].$$

2.3.2. Интегралы от $\psi^{(n)}(x)$.

$$1. \int_0^\infty x^{\alpha-1} \left[\frac{1}{x} - \psi'(x+1) \right] dx = \frac{\pi(\alpha-1)}{\sin \alpha\pi} \zeta(2-\alpha) \quad [1 < \text{Re } \alpha < 2].$$

$$2. \int_0^\infty x^{\alpha-1} \left[\frac{1}{x+1} - \psi'(x+1) \right] dx = \frac{\pi(\alpha-1)}{\sin \alpha\pi} \left[\zeta(2-\alpha) + \frac{1}{\alpha-1} \right] \quad [0 < \text{Re } \alpha < 2]$$

$$3. \int_a^{a+1} x^{n+1} \psi^{(n)}(x) dx = I_n \quad [a \geq 0].$$

$$I_n = [(a+1)^{n+1} - a^{n+1}] \psi^{(n-1)}(a+1) - (-1)^n (n-1)! a - (n+1) I_{n-1},$$

$$I_0 = \ln \Gamma(a+1) + a - \frac{1}{2} \ln 2\pi.$$

$$4. \int_0^m x^{n+1} \psi^{(n)}(x+a) dx = J_n \quad [a \geq 0; m=1, 2, 3, \dots]$$

$$J_n = m^{n+1} \psi^{(n-1)}(m) - (n+1) J_{n-1},$$

$$J_0 = m \ln \Gamma(m+a) - \sum_{k=0}^m (a+k) \ln(a+k) + ma - \frac{m}{2} \ln(2\pi) + \frac{m(m-1)}{2}.$$

$$5. \int_0^\infty x^{\alpha-1} \psi^{(n)}(x+1) dx = (-1)^{n-1} \frac{\pi(1-\alpha)_n}{\sin \alpha\pi} \zeta(1+n-\alpha)$$

$$[0 < \operatorname{Re} \alpha < n; n=1, 2, 3, \dots]$$

2.4. ДЗЕТА-ФУНКЦИЯ РИМАНА $\zeta(x)$

2.4.1. Интегралы, содержащие $\zeta(x)$ ($0 < x < \infty$).

$$1. \int_0^\infty x^{2n} \pi^{-ix} \operatorname{ch} bx \Gamma\left(\frac{1}{4} + ix\right) \zeta\left(\frac{1}{2} + 2ix\right) dx = \\ = \pi^{5/4} \left[\frac{(-1)^{n+1}}{2^{4n+1}} e^{-b/4} + \frac{d^{2n}}{db^{2n}} \left(e^{ib/4} \sum_{k=1}^\infty \exp(-\pi k^2 e^{ib}) \right) \right].$$

$$2. \int_0^\infty \pi^{-ix} \cos bx \Gamma\left(\frac{1}{4} + ix\right) \zeta\left(\frac{1}{2} + 2ix\right) dx = \\ = -\frac{\pi^{5/4}}{2} \left[e^{b/4} - 2e^{-b/4} \sum_{k=1}^\infty \exp(-\pi k^2 e^{-b}) \right].$$

2.4.2. Интегралы, содержащие $\zeta(s)$, по прямой ($\gamma - i\infty, \gamma + i\infty$).

Условие: $|\arg z| < \pi$.

$$1. \int_{\gamma-i\infty}^{\gamma+i\infty} \Gamma(s) \zeta(s) z^{-s} ds = \frac{2\pi}{e^z - 1} \quad [\gamma > 1].$$

$$2. \int_{\gamma-i\infty}^{\gamma+i\infty} \Gamma(s) \zeta(2s) z^{-s} ds = \pi i \theta_3(0, e^z) - \pi i \quad [\gamma > 1/2].$$

$$3. = \pi i \theta_3(0, e^z) - \frac{\pi^{3/2} i}{\sqrt{z}} - \pi i \quad [\gamma = 1/4].$$

$$4. \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{\zeta(1-s)}{\sin \pi s} z^{-s} ds = 2i [\ln z - \psi'(1+z)] \quad [0 < \gamma < 1].$$

$$5. \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{\zeta(s) z^{-s}}{s \sin \pi s} ds = 2i [z \ln z - z - \ln \Gamma(1+z)] \quad [0 < \gamma < 1].$$

$$6. \int_{\gamma-i\infty}^{\gamma+i\infty} \Gamma(s) \Gamma(b-s) \zeta(s) \zeta(b-s) ds = 2\pi i \Gamma(b) [\zeta(b-1) - \zeta(b)]$$

$$[1 < \gamma < \operatorname{Re} b - 1]$$

$$7. \int_{\gamma-i\infty}^{\gamma+i\infty} \Gamma(s) \Gamma(b-s) \zeta(2s) \zeta(2b-2s) ds = 2\pi i \Gamma(b) [(1-2^{-b}) \zeta^2(b) - \zeta(2b)]$$

[1/2 < γ < $\operatorname{Re} b - 1/2$]

2.5. ИНТЕГРАЛЬНАЯ ПОКАЗАТЕЛЬНАЯ ФУНКЦИЯ $Ei(x)$

2.5.1. Интегралы общего вида.

$$1. \int_0^a x^{\alpha-1} (a^r - x^r)^{\beta-1} Ei(-cx) dx =$$

$$= a^{\alpha+r\beta-r} \Gamma(\beta) \sum_{k=1}^{\infty} \frac{(-1)^k}{k!k} \Gamma\left[\frac{(\alpha+k)/r}{\beta + (\alpha+k)/r}\right] (ac)^k +$$

$$+ \frac{a^{\alpha+\beta r-r}}{r^2} B\left(\beta, \frac{\alpha}{r}\right) \left[\psi\left(\frac{\alpha}{r}\right) - \psi\left(\beta + \frac{\alpha}{r}\right) + rC + r \ln(ac)\right]$$

[$a, r, \operatorname{Re} \alpha, \operatorname{Re} \beta > 0; |\arg c| < \pi$].

$$2. \int_a^{\infty} x^{\alpha-1} (x^r - a^r)^{\beta-1} Ei(-cx) dx =$$

$$= a^{\alpha+r\beta-r} \Gamma(\beta) \sum_{k=1}^{\infty} \frac{(-1)^k}{k!k} \Gamma\left[\frac{1-\beta-(\alpha+k)/r}{1-(\alpha+k)/r}\right] (ac)^k +$$

$$+ c^{r-r\beta-\alpha} \sum_{k=0}^{\infty} \frac{(1-\beta)_k \Gamma(\alpha+\beta r-r-kr)}{k! (r-\beta r-\alpha+kr)} (ac)^{rk} +$$

$$+ \frac{a^{\alpha+\beta r-r}}{r^2} B\left(\beta, 1-\beta-\frac{\alpha}{r}\right) \left[\psi\left(1-\frac{\alpha}{r}\right) - \psi\left(1-\beta-\frac{\alpha}{r}\right) + rC + r \ln(ac)\right]$$

[$a, r, \operatorname{Re} \beta, \operatorname{Re} c > 0$].

$$3. \int_0^{\infty} \frac{x^{\alpha-1}}{(x^r + z^r)^\rho} Ei(-cx) dx = \frac{z^{\alpha-r\rho}}{\Gamma(\rho)} \sum_{k=1}^{\infty} \frac{(-1)^k}{k!k} \times$$

$$\times \Gamma\left(\frac{\alpha+k}{r}\right) \Gamma\left(\rho - \frac{\alpha+k}{r}\right) (cz)^k + c^{r\rho-\alpha} \sum_{k=0}^{\infty} \frac{(-1)^k (\rho)_k \Gamma(\alpha-r\rho-rk)}{k! (r\rho-\alpha+rk)} (cz)^{rk} +$$

$$+ \frac{z^{\alpha-r\rho}}{r^2} B\left(\frac{\alpha}{r}, \rho - \frac{\alpha}{r}\right) \left[\psi\left(\frac{\alpha}{r}\right) - \psi\left(\rho - \frac{\alpha}{r}\right) + rC + r \ln(cz)\right]$$

[$r, \operatorname{Re} \alpha, \operatorname{Re} c > 0; r | \arg z| < \pi$].

$$4. \int_0^{\infty} \frac{x^{\alpha-1}}{x^r - y^r} Ei(-cx) dx =$$

$$= -\pi y^{\alpha-r} \sum_{k=1}^{\infty} \frac{(-1)^k}{k!k} \operatorname{ctg}\left(\frac{\alpha+k}{r}\pi\right) (cy)^k + c^{r-\alpha} \sum_{k=0}^{\infty} \frac{\Gamma(\alpha-r-rk)}{r-\alpha+rk} (cy)^{rk} -$$

$$- \frac{\pi y^{\alpha-r}}{r^2} \operatorname{ctg} \frac{\alpha\pi}{r} \left[rC + r \ln(cy) - 2\pi \operatorname{cosec} \frac{2\alpha\pi}{r}\right]$$

[$r, y, \operatorname{Re} \alpha, \operatorname{Re} c > 0$].

$$5. \int_0^{\infty} x^{\alpha-1} e^{-px^r} Ei(-cx) dx = U(r)$$

[$r, \operatorname{Re} \alpha, \operatorname{Re} c, \operatorname{Re} p > 0$].

$$U(r) = \frac{p^{-\alpha/r}}{r} \sum_{k=1}^{\infty} \Gamma\left(\frac{\alpha+k}{r}\right) \frac{(-1)^k}{k!k} \left(\frac{c^r}{p}\right)^{k/r} +$$

$$+ \frac{p^{-\alpha/r}}{r^2} \Gamma\left(\frac{\alpha}{r}\right) \left[\psi\left(\frac{\alpha}{r}\right) + rC - \ln \frac{p}{c^r} \right] \quad |r > 1|,$$

$$U(r) = -c^{-\alpha} \sum_{k=0}^{\infty} \frac{\Gamma(\alpha + rk)}{k! (\alpha + rk)} \left(-\frac{p}{c^r}\right)^k \quad |r < 1|,$$

$U(1)$ см. в 2.5.3 1—12.

$$6. \int_0^{\infty} x^{\alpha-1} e^{-px-r} \operatorname{Ei}(-cx) dx = \frac{p^{\alpha/r}}{r} \sum_{k=1}^{\infty} \Gamma\left(-\frac{\alpha+k}{r}\right) \frac{(-p^{1/r}c)^k}{k!k} -$$

$$- c^{-\alpha} \sum_{k=0}^{\infty} \frac{\Gamma(\alpha - kr)}{k! (\alpha - kr)} (-pc^r)^k + \frac{p^{\alpha/r}}{r^2} \Gamma\left(-\frac{\alpha}{r}\right) \left[\ln(pc^r) - \psi\left(-\frac{\alpha}{r}\right) + rC \right]$$

$|r, \operatorname{Re} c, \operatorname{Re} p > 0|.$

$$7. \int_0^a x^{\alpha-1} (a^r - x^r)^{\beta-1} e^{\pm cx} \operatorname{Ei}(\mp cx) dx =$$

$$= \frac{a^{\alpha+r\beta-r}}{r^2} \Gamma(\beta) \sum_{k=0}^{\infty} \Gamma\left[\frac{(\alpha+k)/r}{(\alpha+k)/r + \beta}\right] \left[\psi\left(\frac{\alpha+k}{r}\right) - \right.$$

$$\left. - \psi\left(\frac{\alpha+k}{r} + \beta\right) - r\psi(k+1) + r \ln(ac) \right] \frac{(\pm ac)^k}{k!}$$

$[a, r, \operatorname{Re} \alpha, \operatorname{Re} \beta > 0, \{|\arg a| < \pi\}, \{|\arg c| < \pi\}, \{a > 0\}].$

$$8. \int_a^{\infty} x^{\alpha-1} (x^r - a^r)^{\beta-1} e^{\pm cx} \operatorname{Ei}(\mp cx) dx =$$

$$= \frac{a^{\alpha+r\beta-r}}{r^2} \Gamma(\beta) \sum_{k=0}^{\infty} \Gamma\left[\frac{1-\beta-(\alpha+k)/r}{1-(\alpha+k)/r}\right] \left[\psi\left(1 - \frac{\alpha+k}{r}\right) - \right.$$

$$\left. - \psi\left(1 - \beta - \frac{\alpha+k}{r}\right) - r\psi(k+1) + r \ln(ac) \right] \frac{(\pm ac)^k}{k!} +$$

$$+ \pi c^{r-\alpha-\beta r} \sum_{k=0}^{\infty} \frac{(1-\beta)_k}{k!} \Gamma(\alpha + \beta r - r - kr) \left\{ \begin{matrix} \operatorname{cosec} [(kr+r-\alpha-\beta r)\pi] \\ \operatorname{ctg} [(kr+r-\alpha-\beta r)\pi] \end{matrix} \right\} (ac)^{rk}$$

$[a, r, \operatorname{Re} \beta > 0; \operatorname{Re}(\alpha + \beta r) < r+1, \{|\arg a| < \pi\}, \{|\arg c| < \pi\}, \{a > 0\}].$

$$9. \int_0^{\infty} \frac{x^{\alpha-1}}{(x^r + z^r)^{\rho}} e^{\pm cx} \operatorname{Ei}(\mp cx) dx =$$

$$= \frac{z^{\alpha-r\rho}}{r^2 \Gamma(\rho)} \sum_{k=0}^{\infty} \Gamma\left(\frac{\alpha+k}{r}\right) \Gamma\left(\rho - \frac{\alpha+k}{r}\right) \left[\psi\left(\frac{\alpha+k}{r}\right) - \right.$$

$$\left. - \psi\left(\rho - \frac{\alpha+k}{r}\right) - r\psi(k+1) + r \ln(cz) \right] \frac{(\pm cz)^k}{k!} +$$

$$+ \pi c^{r\rho-\alpha} \sum_{k=0}^{\infty} \frac{(-1)^k (\rho)_k}{k!} \Gamma(\alpha - r\rho - rk) \left\{ \begin{matrix} \operatorname{cosec} [(r\rho + rk - \alpha)\pi] \\ \operatorname{ctg} [(r\rho + rk - \alpha)\pi] \end{matrix} \right\} (cz)^{rk}$$

$[r > 0; 0 < \operatorname{Re} \alpha < r \operatorname{Re} \rho + 1; r|\arg z| < \pi, \{|\arg c| < \pi\}, \{c > 0\}].$

$$\begin{aligned}
 10. \int_0^{\infty} \frac{x^{\alpha-1}}{x^r - y^r} e^{\pm cx} \operatorname{Ei}(\mp cx) dx = \\
 = \frac{y^{\alpha-r}}{r^2} \sum_{k=0}^{\infty} \frac{(\pm cy)^k}{k!} \left[\pi \operatorname{cosec}^2 \frac{\alpha+k}{r} \pi + r \psi(k+1) - r \ln(cy) \right] + \\
 + \pi c^{r-\alpha} \sum_{k=0}^{\infty} \Gamma(\alpha-r-rk) \left\{ \begin{matrix} \operatorname{cosec} [(rk+r-\alpha)\pi] \\ \operatorname{ctg} [(rk+r-\alpha)\pi] \end{matrix} \right\} (cy)^{rk} \\
 \left[r, y > 0; 0 < \operatorname{Re} \alpha < r+1, \left\{ \begin{matrix} |\arg c| < \pi \\ c > 0 \end{matrix} \right\} \right].
 \end{aligned}$$

$$11. \int_0^{\infty} x^{\alpha-1} e^{-px^r} \pm cx \operatorname{Ei}(\mp cx) dx = V(r) \quad \left[r, \operatorname{Re} \alpha, \operatorname{Re} p > 0, \left\{ \begin{matrix} |\arg c| < \pi \\ c > 0 \end{matrix} \right\} \right],$$

$$V(r) = \frac{p^{-\alpha/r}}{r^2} \sum_{k=0}^{\infty} \Gamma\left(\frac{\alpha+k}{r}\right) \left[\psi\left(\frac{\alpha+k}{r}\right) - r \psi(k+1) + \ln \frac{c^r}{p} \right] \frac{1}{k!} \left(\pm \frac{c}{p^{1/r}} \right)^k \quad [r > 1],$$

$$\begin{aligned}
 V(r) = -\frac{\pi}{c^{\alpha}} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \Gamma(\alpha+rk) \left\{ \begin{matrix} \operatorname{cosec} [(\alpha+rk)\pi] \\ \operatorname{ctg} [(\alpha+rk)\pi] \end{matrix} \right\} \left(\frac{p}{c^r} \right)^k - \\
 - \frac{p^{-\alpha/r}}{r} \sum_{k=0}^{\infty} (-1)^k \Gamma\left(\frac{\alpha-k-1}{r}\right) k! \left(\frac{p}{c^r} \right)^{(k+1)/r} \quad [r < 1],
 \end{aligned}$$

$V(1)$ см в 2.5.3 1-12.

$$\begin{aligned}
 12. \int_0^{\infty} x^{\alpha-1} e^{-px^{-r}} \pm cx \operatorname{Ei}(\mp cx) dx = \\
 = -\frac{\pi}{c^{\alpha}} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \Gamma(\alpha-kr) (c^r p)^k \left\{ \begin{matrix} \operatorname{cosec} [(\alpha-kr)\pi] \\ \operatorname{ctg} [(\alpha-kr)\pi] \end{matrix} \right\} + \\
 + \frac{p^{\alpha/r}}{r} \sum_{k=0}^{\infty} \frac{(\pm cp^{1/r})^k}{k!} \Gamma\left(-\frac{\alpha+k}{r}\right) \left[\ln(cp^{1/r}) - \psi(k+1) - \frac{1}{r} \psi\left(-\frac{\alpha+k}{r}\right) \right] \\
 \left[r, \operatorname{Re} p > 0; \operatorname{Re} \alpha < 1, \left\{ \begin{matrix} |\arg c| < \pi \\ c > 0 \end{matrix} \right\} \right].
 \end{aligned}$$

$$13. \int_0^{\infty} x^{\alpha-1} \left\{ \begin{matrix} \sin bx^r \\ \cos bx^r \end{matrix} \right\} \operatorname{Ei}(-cx) dx = W(r) \quad \left[b, r, \operatorname{Re} c > 0; \operatorname{Re} \alpha > -r\delta; \delta = \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \right],$$

$$\begin{aligned}
 W(r) = \frac{b^{-\alpha/r}}{r} \left[\sum_{k=1}^{\infty} \frac{1}{k!k} \Gamma\left(\frac{\alpha+k}{r}\right) \left\{ \begin{matrix} \sin [(\alpha+k)\pi/(2r)] \\ \cos [(\alpha+k)\pi/(2r)] \end{matrix} \right\} \left(-\frac{c}{b^{1/r}} \right)^k + \right. \\
 + \left(c + \frac{1}{2r} \psi\left(\frac{\delta}{2} + \frac{\alpha}{2r}\right) + \frac{1}{2r} \psi\left(\frac{1+\delta}{2} - \frac{\alpha}{2r}\right) + \right. \\
 \left. \left. + \ln c - \frac{1}{r} \ln \frac{b}{2} \right) \Gamma\left(\frac{\alpha}{r}\right) \left\{ \begin{matrix} \sin [\alpha\pi/(2r)] \\ \cos [\alpha\pi/(2r)] \end{matrix} \right\} \right] \quad [r > 1],
 \end{aligned}$$

$$W(r) = -b^{\delta} c^{-\alpha-\delta r} \sum_{k=0}^{\infty} \frac{(-1)^k \Gamma(\alpha+r\delta+2rk)}{(1/2+\delta)_k k! (\alpha+r\delta+2rk)} \left(\frac{b}{2c^r} \right)^{2k} \quad [r < 1],$$

$W(1)$ см. в 2.5.7.4.

$$14. \int_0^{\infty} x^{\alpha-1} e^{-px^r} \begin{Bmatrix} \sin bx^r \\ \cos bx^r \end{Bmatrix} \text{Ei}(-cx) dx = X(r)$$

$$[b, r, \text{Re} c, \text{Re} p > 0; \text{Re} \alpha > -r\delta; \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}],$$

$$X(r) = \frac{1}{r(\sqrt{b^2+p^2})^{\alpha/r}} \left\{ \sum_{k=1}^{\infty} \frac{(-1)^k}{k!k} \Gamma\left(\frac{\alpha+k}{r}\right) \times \right. \\ \times \cos\left(\frac{\delta\pi}{2} - \frac{\alpha+k}{r} \arccos \frac{p}{\sqrt{b^2+p^2}}\right) \frac{c^k}{(\sqrt{b^2+p^2})^{k/r}} + \\ + \cos\left(\frac{\delta\pi}{2} - \frac{\alpha}{r} \arccos \frac{p}{\sqrt{b^2+p^2}}\right) \left[C + \frac{1}{r} \psi\left(\frac{\alpha}{r}\right) + \right. \\ \left. + \ln \frac{c}{(\sqrt{b^2+p^2})^{1/r}} + \frac{1}{r} \text{tg}\left(\frac{\delta\pi}{2} - \frac{\alpha}{r} \arccos \frac{p}{\sqrt{b^2+p^2}}\right) \arccos \frac{p}{\sqrt{b^2+p^2}} \right] \Big\}$$

$$[r > 1] \text{ или } [r = 1; |c|^2 < |b^2 + p^2|],$$

$$X(r) = -\frac{1}{c^{\alpha}} \sum_{k=0}^{\infty} \frac{(-1)^k \Gamma(\alpha + kr)}{k! (\alpha + kr)} \cos\left(\frac{\delta\pi}{2} + k \arccos \frac{p}{\sqrt{b^2+p^2}}\right) \left(\frac{\sqrt{b^2+p^2}}{c^r}\right)^k$$

$$[r < 1] \text{ или } [r = 1; |c|^2 > |b^2 + p^2|].$$

$$15. \int_0^{\infty} x^{\alpha-1} e^{\pm cx} \sin bx^r \text{Ei}(\mp cx) dx = Y(1, r)$$

$$[b, r > 0; -r < \text{Re} \alpha < r+1, \begin{Bmatrix} |\arg c| < \pi \\ c > 0 \end{Bmatrix}, \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}],$$

$$Y(\delta, r) = -\frac{b^{-\alpha/r}}{r} \sum_{k=0}^{\infty} \frac{1}{k!} \Gamma\left(\frac{\alpha+k}{r}\right) \cos \frac{\alpha+k-r\delta}{2r} \pi \times \\ \times \left(\pm \frac{c}{b^{1/r}}\right)^k \left[\psi(k+1) - \frac{1}{2r} \psi\left(\frac{\delta}{2} + \frac{\alpha+k}{2r}\right) - \right. \\ \left. - \frac{1}{2r} \psi\left(\frac{1+\delta}{2} - \frac{\alpha+k}{2r}\right) - \frac{1}{r} \ln\left(\frac{2c^r}{b}\right) \right] \quad [r > 1],$$

$$Y(\delta, r) = \frac{b^{(1-\alpha)/r}}{c^r} \sum_{k=0}^{\infty} (\mp 1)^{k+1} k! \Gamma\left(\frac{\alpha-k-1}{r}\right) \cos \frac{\alpha-k-r\delta-1}{2r} \pi \left(\frac{2c^r}{b}\right)^{-k/r} - \\ - \frac{b^0 \pi}{c^{\alpha+r\delta}} \sum_{k=0}^{\infty} \frac{(-1)^k \Gamma(\alpha+r\delta+2rk)}{k! (\delta+1/2)_k} \left\{ \text{cosec}(\alpha+r\delta+2rk)\pi \right\} \left(\frac{b}{2c^r}\right)^{2k} \\ [r < 1],$$

$$Y(\delta, 1) \text{ см. в 2.5.8.4—5.}$$

$$16. \int_0^{\infty} x^{\alpha-1} e^{\pm cx} \cos bx^r \text{Ei}(\mp cx) dx = Y(0, r)$$

$$[b, r > 0; 0 < \text{Re} \alpha < r+1; \begin{Bmatrix} |\arg c| < \pi \\ c > 0 \end{Bmatrix}; Y(\delta, r) \text{ см. в 2.5.1.15}].$$

$$17. \int_0^{\infty} x^{\alpha-1} \left\{ \frac{\ln(x^r + z^r)}{\ln|x^r - z^r|} \right\} \text{Ei}(-cx) dx =$$

$$\pi z^{\alpha} \sum_{k=1}^{\infty} \frac{(-cz)^k}{k!k(\alpha+k)} \left\{ \text{cosec}[(\alpha+k)\pi/r] \right\} +$$

$$+ \frac{\pi z^{\alpha}}{\alpha r} \text{cosec} \frac{\alpha\pi}{r} \left[rC - \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \pi \sin \frac{\alpha\pi}{r} - \pi \text{ctg} \frac{\alpha\pi}{r} - \frac{r}{\alpha} + r \ln(cz) \right] +$$

$$+ c^{-\alpha} \sum_{k=1}^{\infty} \frac{\Gamma(\alpha - kr)}{k(\alpha - kr)} (\mp c^r z^r)^k + \frac{r\Gamma(\alpha)}{\alpha c^{\alpha}} \left[\frac{1}{\alpha} - \psi(\alpha) + \ln(c) \right]$$

[r, Re c, Re \alpha > 0; r | arg z| < \pi].

$$18. \int_0^{\infty} x^{\alpha-1} \text{Ei}(-bx^r) \text{Ei}(-cx) dx = -b^{-\alpha/r} \sum_{k=1}^{\infty} \frac{(-1)^k}{k!k(\alpha+k)} \Gamma\left(\frac{\alpha+k}{r}\right) \times$$

$$\times \left(\frac{c^r}{b}\right)^{k/r} + \frac{b^{-\alpha/r}}{\alpha^2 r} \Gamma\left(\frac{\alpha}{r}\right) \left[r - \alpha r C - \alpha \psi\left(\frac{\alpha}{r}\right) + \alpha \ln \frac{b}{c^r} \right]$$

[r > 1 (или r = 1, |c| < |b|); Re \alpha, Re b > 0;
|arg c| < \pi; при r < 1 — замена x^r = t].

$$19. \int_0^{\infty} x^{\alpha-1} \text{Ei}\left(-\frac{b}{x^r}\right) \text{Ei}(-cx) dx =$$

$$= b^{\alpha/r} \sum_{k=1}^{\infty} \frac{1}{k!k(\alpha+k)} \Gamma\left(-\frac{\alpha+k}{r}\right) (-b^{1/r}c)^k - c^{-\alpha} \sum_{k=1}^{\infty} \frac{\Gamma(\alpha - kr)}{k!k(\alpha - kr)} (-bc^r)^k +$$

$$+ \frac{b^{\alpha/r}}{\alpha^2 r} \Gamma\left(-\frac{\alpha}{r}\right) \left[\alpha r C - r + \alpha \ln(bc^r) - \alpha \psi\left(-\frac{\alpha}{r}\right) \right] -$$

$$- \frac{\Gamma(\alpha)}{\alpha^2 c^{\alpha}} [\alpha C - \alpha r + r - \alpha r \psi(\alpha) + \alpha \ln(bc^r)]$$

[r, Re b, Re c > 0].

$$20. \int_0^{\infty} x^{\alpha-1} e^{\pm cx} \text{Ei}(-bx^r) \text{Ei}(\mp cx) dx = Z(r)$$

[r, Re \alpha, Re b > 0, \left\{ \begin{matrix} |\arg c| < \pi \\ c > 0 \end{matrix} \right\}],

$$Z(r) = b^{-\alpha/r} \sum_{k=0}^{\infty} \frac{(\pm 1)^k}{k!(\alpha+k)} \Gamma\left(\frac{\alpha+k}{r}\right) \left[\psi(k+1) - \frac{1}{r} \psi\left(\frac{\alpha+k}{r}\right) + \right.$$

$$\left. + \frac{1}{r} \ln \frac{b}{c^r} + \frac{1}{\alpha+k} \right] \left(\frac{c^r}{b}\right)^{k/r} \quad [r > 1],$$

$$Z(r) = \pm \frac{b^{(1-\alpha)/r}}{c} \sum_{k=0}^{\infty} \frac{(\mp 1)^k k!}{\alpha - k - 1} \Gamma\left(\frac{\alpha - k - 1}{r}\right) \left(\frac{b}{c^r}\right)^{k/r} -$$

$$- \pi c^{-\alpha} \sum_{k=1}^{\infty} \frac{(-1)^k}{k!k} \Gamma(\alpha + kr) \left\{ \text{cosec}[(\alpha + kr)\pi] \right\} \left(\frac{b}{c^r}\right)^k -$$

$$- \frac{\pi \Gamma(\alpha)}{c^{\alpha}} \left[C + r \psi(\alpha) - r \pi \left\{ \frac{\text{ctg} \alpha \pi}{2 \text{cosec} 2\alpha \pi} \right\} + \ln \frac{b}{c^r} \right] \left\{ \frac{\text{cosec} \alpha \pi}{\text{ctg} \alpha \pi} \right\}$$

[r < 1].

$$\begin{aligned}
21. \int_0^{\infty} x^{\alpha-1} e^{\pm bx^r \pm cx} \operatorname{Ei}(\mp bx^r) \operatorname{Ei}(\mp cx) dx = \\
= \frac{\pi b^{-\alpha/r}}{r} \sum_{k=0}^{\infty} \frac{(\pm 1)^k}{k!} \Gamma\left(\frac{\alpha+k}{r}\right) \left\{ \operatorname{cosec}[(\alpha+k)\pi/r] \right\} \times \\
\times \left[\psi(k+1) - \frac{1}{r} \psi\left(\frac{\alpha+k}{r}\right) + \frac{\pi}{r} \left\{ \operatorname{ctg}[(\alpha+k)\pi/r] \right\} + \frac{1}{r} \ln \frac{b}{c^r} \right] \left(\frac{c^r}{b}\right)^{k/r} - \\
- \frac{\pi c^{r-\alpha}}{b} \sum_{k=0}^{\infty} (\mp 1)^{k+1} k! \Gamma(\alpha-r-kr) \left\{ \operatorname{cosec}[(\alpha-r-kr)\pi] \right\} \left(\frac{c^r}{b}\right)^k \\
\left[r > 1 \text{ (или } r=1, |c| < |b|); 0 < \operatorname{Re} \alpha < r+1, \right. \\
\left. \left\{ |\arg b|, |\arg c| < \pi \right\}, \text{ при } r < 1 - \text{замена } x^r = t \right].
\end{aligned}$$

$$22. \int_0^{\infty} x^{\alpha-1} e^{-bx^r + cx} \operatorname{Ei}(bx^r) \operatorname{Ei}(-cx) dx = U(r)$$

[b, r > 0; 0 < Re α < r+1; |arg c| < π],

$$\begin{aligned}
U(r) = \frac{\pi b^{-\alpha/r}}{r} \sum_{k=0}^{\infty} \frac{1}{k!} \Gamma\left(\frac{\alpha+k}{r}\right) \operatorname{ctg} \frac{\alpha+k}{r} \pi \left[\psi(k+1) - \right. \\
\left. - \frac{1}{r} \psi\left(\frac{\alpha+k}{r}\right) + \frac{2\pi}{r} \operatorname{cosec} \frac{2(\alpha+k)}{r} \pi + \frac{1}{r} \ln \frac{b}{c^r} \right] \left(\frac{c^r}{b}\right)^{k/r} + \\
+ \frac{\pi c^{r-\alpha}}{b} \sum_{k=0}^{\infty} \frac{k! \Gamma(\alpha-r-kr)}{\sin(\alpha-r-kr)\pi} \left(-\frac{c^r}{b}\right)^k \quad [r > 1].
\end{aligned}$$

$$\begin{aligned}
U(r) = \pi c^{-\alpha} \sum_{k=0}^{\infty} \frac{(-1)^k \Gamma(\alpha+kr)}{k! \sin(\alpha+kr)\pi} \left[\psi(k+1) - r\psi(\alpha+kr) + \right. \\
\left. + \pi r \operatorname{ctg}(\alpha+kr)\pi + \ln \frac{c^r}{b} \right] \left(\frac{b}{c^r}\right)^k + \\
+ \frac{\pi b^{(1-\alpha)/r}}{c^r} \sum_{k=0}^{\infty} (-1)^k k! \operatorname{ctg} \frac{\alpha-k-1}{r} \pi \Gamma\left(\frac{\alpha-k-1}{r}\right) \left(\frac{b}{c^r}\right)^{k/r} \\
[r < 1]
\end{aligned}$$

2.5.2. Интегралы от $A(x) \operatorname{Ei}(cx+d)$.

$$1. \int_0^{\infty} \operatorname{Ei}(-cx) dx = -\frac{1}{c} \quad [\operatorname{Re} c > 0]$$

$$2. \int_0^{\infty} x^{\alpha-1} \operatorname{Ei}(-cx) dx = -\frac{\Gamma(\alpha)}{c c^{\alpha}} \quad [\operatorname{Re} \alpha, \operatorname{Re} c > 0]$$

$$3. \int_0^{\infty} x^n \operatorname{Ei}(-cx) dx = -\frac{n!}{(n+1) c^{n+1}} \quad [\operatorname{Re} c > 0].$$

$$4. \int_0^{\infty} x^{-1/2} \operatorname{Ei}(-cx) dx = -2 \sqrt{\frac{\pi}{c}} \quad [\operatorname{Re} c > 0].$$

$$5. \int_0^{\infty} \frac{\text{Ei}(-cx)}{(x+z)^2} dx = \frac{1}{z} e^{cz} \text{Ei}(-cz) \quad [\text{Re} c > 0; |\arg z| < \pi]$$

$$6. \int_0^a x^{\alpha-1} (a-x)^{\beta-1} \text{Ei}(-cx) dx = \\ = -a^{\alpha+\beta} c B(\alpha+1, \beta) {}_2F_2(\alpha+1, 1, 1; \alpha+\beta+1, 2, 2; -ac) + \\ + a^{\alpha+\beta-1} B(\alpha, \beta) [\psi(\alpha) - \psi(\alpha+\beta) + \ln(ac) + C] \\ [\alpha, \text{Re} \alpha, \text{Re} \beta > 0; |\arg c| < \pi]$$

$$7. \int_a^{\infty} x^{\alpha-1} (x-a)^{\beta-1} \text{Ei}(-cx) dx = \\ = -a^{\alpha+\beta} c B(\beta, -\alpha-\beta) {}_2F_2(\alpha+1, 1, 1; \alpha+\beta+1, 2, 2; -ac) + \\ + \frac{c^{1-\alpha-\beta} \Gamma(\alpha+\beta-1)}{1-\alpha-\beta} {}_2F_2(1-\beta, 1-\alpha-\beta; 2-\alpha-\beta, 2-\alpha-\beta; -ac) + \\ + a^{\alpha+\beta-1} B(\beta, 1-\alpha-\beta) [\psi(1-\alpha) - \psi(1-\alpha-\beta) + \ln(ac) + C] \\ [\alpha, \text{Re} \beta, \text{Re} c > 0]$$

$$8. \int_0^{\infty} \frac{x^{\alpha-1}}{(x+z)^{\rho}} \text{Ei}(-cx) dx = \\ = -cz^{1+\alpha-\rho} B(\alpha+1, \rho-\alpha-1) {}_2F_2(\alpha+1, 1, 1; 2+\alpha-\rho, 2, 2; cz) + \\ + \frac{c^{\rho-\alpha} \Gamma(\alpha-\rho)}{\rho-\alpha} {}_2F_2(\rho, \rho-\alpha; 1+\rho-\alpha, 1+\rho-\alpha; cz) + \\ + z^{\alpha-\rho} B(\alpha, \rho-\alpha) [\psi(\alpha) - \psi(\rho-\alpha) + \ln(cz) + C] \\ [\text{Re} \alpha, \text{Re} c > 0; |\arg z| < \pi]$$

$$9. \int_0^{\infty} \frac{x^{\alpha-1}}{(x+z)^{\alpha+1}} \text{Ei}(-cx) dx = -\frac{\Gamma(\alpha)}{\alpha z^{3/2} \sqrt{c}} e^{cz/2} W_{1/2-\alpha, 0}(cz) \\ [\text{Re} \alpha, \text{Re} c > 0; |\arg z| < \pi]$$

$$10. \int_0^{\infty} \frac{x^{\alpha-1}}{x-y} \text{Ei}(-cx) dx = \pi y^{\alpha-1} \text{ctg} \alpha \pi \left[\frac{2\pi}{\sin 2\alpha \pi} - \text{Ei}(-cy) \right] + \\ + \frac{c^{1-\alpha} \Gamma(\alpha-1)}{1-\alpha} {}_2F_2(1-\alpha, 1; 2-\alpha, 2-\alpha; -cy) \quad [y, \text{Re} \alpha, \text{Re} c > 0]$$

$$11. \int_0^{\infty} x^{\alpha-1} \text{Ei}(-cx-b) dx = -b^{(\alpha-1)/2} c^{-\alpha} \Gamma(\alpha) e^{-b/2} W_{-(\alpha+1)/2, -\alpha/2}(b) \\ [\text{Re} c, \text{Re} \alpha > 0; |\arg b| < \pi]$$

$$12. \int_0^{\infty} x^n \text{Ei}(-cx-b) dx = \frac{1}{(n+1)c^{n+1}} \left[(-b)^{n+1} \text{Ei}(-b) - e^{-b} \sum_{k=0}^n (n-k)! (-b)^k \right] \\ [\text{Re} c > 0; |\arg b| < \pi]$$

$$13. \int_a^{\infty} x^n \text{Ei}(a-x) dx = -\sum_{k=0}^n \binom{n}{k} \frac{k!}{k+1} a^{n-k} \quad [a \geq 0]$$

$$14. \int_0^{\infty} x^n \operatorname{Ei}(-|x-y|) dx = -2 \sum_{k=0}^{[n/2]} \binom{n}{2k} \frac{(2k)!}{2k+1} y^{n-2k} +$$

$$+ e^{-y} \sum_{k=0}^n (-1)^k \binom{n}{k} \frac{k!}{k+1} y^{n-k} \sum_{j=0}^k \frac{y^j}{j!} + \frac{y^{n+1}}{n+1} \operatorname{Ei}(-y) \quad [\operatorname{Im} y = 0].$$

$$15. \int_0^{\infty} \frac{x^{\rho-1}}{(x+b)^{\rho}} \operatorname{Ei}(-x-b) dx = -\frac{\Gamma(\rho-1)}{b^{3/2}} e^{-b/2} W_{1/2-\rho, 0}(b)$$

$$[\operatorname{Re} \rho > 1; |\arg b| < \pi].$$

$$16. \int_0^{\infty} x^{\alpha-1} [e^{bx} \operatorname{Ei}(-b\sqrt{x^2+z^2}-bz) + e^{-bx} \operatorname{Ei}(b\sqrt{x^2+z^2}-bz)] dx =$$

$$= -\sqrt{\pi} \left(\frac{b}{2}\right)^{(1-\alpha)/2} z^{(\alpha+1)/2} \operatorname{ctg} \frac{\alpha\pi}{2} \Gamma\left(\frac{\alpha}{2}\right) K_{(\alpha+1)/2}(bz)$$

$$[b, \operatorname{Re} z > 0; 0 < \operatorname{Re} \alpha < 1]$$

2.5.3. Интегралы, содержащие $A(x)$, e^{px} и $\operatorname{Ei}(cx+d)$.

$$1. \int_0^{\infty} e^{-px} \operatorname{Ei}(-cx) dx = -\frac{1}{p} \ln\left(1 + \frac{p}{c}\right) \quad [\operatorname{Re}(p+c) > 0; |\arg c| < \pi]$$

$$2. \int_0^{\infty} x^{\alpha-1} e^{\mp px} \operatorname{Ei}(-cx) dx = -\frac{\Gamma(\alpha)}{\alpha(c \pm p)^{\alpha}} {}_2F_1\left(\alpha, 1; \alpha+1; \frac{p}{p \pm c}\right) =$$

$$= -\frac{\Gamma(\alpha)}{(c \pm p)^{\alpha}} \Phi\left(\frac{p}{p \pm c}, 1, \alpha\right)$$

$$\left[\operatorname{Re} \alpha > 0, \begin{cases} \operatorname{Re}(p+c) > 0; |\arg c| < \pi \\ \operatorname{Re}(c-p) > 0 \end{cases}\right].$$

$$3. \int_0^{\infty} x^{-1/2} e^{px} \operatorname{Ei}(-cx) dx = -2\sqrt{\frac{\pi}{p}} \arcsin \sqrt{\frac{p}{c}} \quad [c > p].$$

$$4. \int_0^{\infty} x^{-1/2} e^{-px} \operatorname{Ei}(-cx) dx = -2\sqrt{\frac{\pi}{p}} \ln \frac{\sqrt{p+c} + \sqrt{p}}{\sqrt{c}}$$

$$[\operatorname{Re}(p+c) > 0; |\arg c| < \pi]$$

$$5. \int_0^{\infty} x e^{-px} \operatorname{Ei}(-cx) dx = \frac{1}{p(p+c)} - \frac{1}{p^2} \ln\left(1 + \frac{p}{c}\right)$$

$$[\operatorname{Re}(p+c) > 0; |\arg c| < \pi]$$

$$6. \int_0^{\infty} x^{\alpha-1} e^{cx} \operatorname{Ei}(-cx) dx = -\frac{\pi \Gamma(\alpha)}{c^{\alpha} \sin \alpha\pi}$$

$$[0 < \operatorname{Re} \alpha < 1; |\arg c| < \pi]$$

$$7. \int_0^{\infty} x^n e^{-px} \operatorname{Ei}(\pm cx) dx = \frac{n!}{p^{n+1}} \left[\sum_{k=1}^n \frac{1}{k} \left(\frac{p}{p \mp c}\right)^k - \ln\left(\frac{p}{c} \mp 1\right) \right]$$

$$\left[\begin{cases} \operatorname{Re} p > c > 0 \\ \operatorname{Re}(p+c) > 0, |\arg c| < \pi \end{cases}\right].$$

$$8. \int_0^{\infty} e^{-px} \operatorname{Ei}(cx) dx = -\frac{1}{p} \ln\left(\frac{p}{c} - 1\right) \quad [\operatorname{Re} p > c > 0]$$

$$9. \int_0^{\infty} x^{\alpha-1} e^{-px} \operatorname{Ei}(cx) dx = -\frac{\pi}{p^{\alpha}} \operatorname{ctg} \alpha\pi \Gamma(\alpha) + \frac{\Gamma(\alpha-1)}{c(p-c)^{\alpha-1}} {}_2F_1\left(1, 1; 2-\alpha; \frac{c-p}{c}\right) \\ [\operatorname{Re} \alpha > 0; \operatorname{Re} p > c > 0]$$

$$10. \int_0^{\infty} x^{\alpha-1} e^{-cx} \operatorname{Ei}(cx) dx = -\frac{\pi \operatorname{ctg} \alpha\pi}{c^{\alpha}} \Gamma(\alpha) \quad [0 < \operatorname{Re} \alpha < 1; c > 0].$$

$$11. \int_0^{\infty} x^{-1/2} e^{-px} \operatorname{Ei}(cx) dx = -\sqrt{\frac{\pi}{p}} \ln \left[2 \frac{p + \sqrt{p(p-c)}}{c} - 1 \right] \\ [\operatorname{Re} p > c > 0]$$

$$12. \int_0^{\infty} x e^{-px} \operatorname{Ei}(cx) dx = -\frac{1}{p^2} \left[\ln \left(\frac{p}{c} - 1 \right) - \frac{p}{p-c} \right] \quad [\operatorname{Re} p > c > 0]$$

$$13. \int_{-\infty}^{\infty} e^{-(c+ib)x} \operatorname{Ei}(cx) dx = -\frac{\pi \operatorname{sgn} b}{b-ic} \quad [\operatorname{Im} b = \operatorname{Im} c = 0]$$

$$14. \int_0^{\infty} x^{\alpha-1} e^{cx} \operatorname{Ei}(-cx-b) dx = \frac{\pi b^{(\alpha-1)/2} c^{-\alpha}}{\sin \alpha\pi} e^{-b/2} W_{(\alpha-1)/2, \alpha/2}(b) \\ [0 < \operatorname{Re} \alpha < 1]$$

$$15. \int_0^{\infty} \frac{1-e^{-px}}{x} \operatorname{Ei}(-cx) dx = \operatorname{Li}_2\left(-\frac{p}{c}\right) \quad [c \geq p > 0]$$

$$16. \int_0^{\infty} \frac{1}{1 \mp b e^{\mp px}} \operatorname{Ei}(-cx) dx = -\left\{ \begin{matrix} c^{-1} \\ 0 \end{matrix} \right\} \mp \frac{1}{p} \sum_{k=1}^{\infty} \frac{(\pm b)^{\pm k}}{k} \ln \left(1 + \frac{kp}{c} \right) \\ \left[\operatorname{Re} c, \operatorname{Re} p > 0, \left\{ \begin{matrix} -1 \leq b < 1 \\ b > 1 \end{matrix} \right\} \right]$$

2.5.4. Интегралы от $x^{\alpha} e^{f(x)} \operatorname{Ei}(cx)$.

$$1. \int_0^{\infty} x^{\alpha-1} e^{-p/x} \operatorname{Ei}(-cx) dx = \Gamma(-\alpha) p^{\alpha} \left[\frac{cp}{\alpha+1} {}_2F_3(1, 1; 2, 2, \alpha+2; cp) - \right. \\ \left. - \psi(-\alpha) + C + \ln(cp) \right] - \frac{\Gamma(\alpha)}{\alpha c^{\alpha}} {}_1F_2(-\alpha; 1-\alpha, 1-\alpha; cp) \\ [\operatorname{Re} c, \operatorname{Re} p > 0]$$

$$2. \int_0^{\infty} x^{-1/2} e^{-p/x} \operatorname{Ei}(-cx) dx = -2 \sqrt{\frac{\pi}{c}} e^{-2\sqrt{cp}} - 4 \sqrt{p\pi} \operatorname{Ei}(-2\sqrt{cp}) \\ [\operatorname{Re} c, \operatorname{Re} p > 0].$$

$$3. \int_0^{\infty} x^{-3/2} e^{-p/x} \operatorname{Ei}(-cx) dx = 2 \sqrt{\frac{\pi}{p}} \operatorname{Ei}(-2\sqrt{cp}) \quad [\operatorname{Re} c, \operatorname{Re} p > 0]$$

$$4. \int_0^{\infty} x^{-2} e^{-p/x} \operatorname{Ei}(-cx) dx = -\frac{2}{p} K_0(2\sqrt{cp}) \quad [\operatorname{Re} c, \operatorname{Re} p > 0]$$

$$5. \int_0^{\infty} x^{-3/2} e^{-px-b/x} \operatorname{Ei}(\pm cx) dx = \sqrt{\frac{\pi}{b}} \left[e^{2\sqrt{bp}} \operatorname{Ei}(-2\sqrt{bp} - 2\sqrt{b(p \mp c)}) + \right. \\ \left. + e^{-2\sqrt{bp}} \operatorname{Ei}(2\sqrt{bp} - 2\sqrt{b(p \mp c)}) \right] \\ \left[\operatorname{Re} b > 0, \begin{cases} \operatorname{Re} p > c > 0 \\ \operatorname{Re}(p+c) > 0; |\arg c| < \pi \end{cases} \right].$$

$$6. \int_0^{\infty} x^{-1/2} e^{cx-b/x} \operatorname{Ei}(-cx) dx = \\ = 2 \sqrt{\frac{\pi}{c}} [\cos 2\sqrt{bc} \operatorname{si}(2\sqrt{bc}) - \sin 2\sqrt{bc} \operatorname{ci}(2\sqrt{bc})] \\ [\operatorname{Re} b > 0; |\arg c| < \pi]$$

$$7. \int_0^{\infty} x^{-3/2} e^{cx-b/x} \operatorname{Ei}(-cx) dx = \\ = 2 \sqrt{\frac{\pi}{b}} [\cos 2\sqrt{bc} \operatorname{ci}(2\sqrt{bc}) + \sin 2\sqrt{bc} \operatorname{si}(2\sqrt{bc})] \\ [\operatorname{Re} b > 0; |\arg c| < \pi].$$

2.5.5. Интегралы, содержащие $e^{t(x)}$ и $\varphi(x) \operatorname{Ei}(bx) + \chi(x) \operatorname{Ei}(cx)$.

$$1. \int_0^{\infty} x^{\alpha-1} e^{cx} [\operatorname{Ei}(-2cx) - \operatorname{Ei}(-cx)] dx = \frac{\Gamma(\alpha)}{2c^{\alpha}} \left[\psi\left(1 - \frac{\alpha}{2}\right) - \psi\left(\frac{1-\alpha}{2}\right) \right] \\ [0 < \operatorname{Re} \alpha < 1; |\arg c| < \pi].$$

$$2. \int_0^{\infty} x^{\alpha-1} e^{cx} [\operatorname{Ei}(-bx) - \operatorname{Ei}(-cx)] dx = \frac{(b-c)^{1-\alpha}}{(1-\alpha)c} \Gamma(\alpha) \Phi\left(\frac{c-b}{c}, 1, 1-\alpha\right) \\ [0 < \operatorname{Re} \alpha < 1; \operatorname{Re}(b-c) > 0, |\arg c| < \pi].$$

$$3. \int_0^{\infty} \left[\frac{e^{-cx}}{x-y} \operatorname{Ei}(cx) - \frac{e^{cx}}{x+y} \operatorname{Ei}(-cx) \right] dx = \begin{cases} 1 \\ 0 \end{cases} \pi^2 e^{-cy} \quad \left[c > 0, \begin{cases} y > 0 \\ y < 0 \end{cases} \right].$$

$$4. \int_0^{\infty} e^{-px^2} [\operatorname{Ei}(-cx) + \operatorname{Ei}(cx)] dx = \frac{1}{2} \sqrt{\frac{\pi}{p}} \operatorname{Ei}\left(\frac{c^2}{4p}\right) \quad [\operatorname{Re} p > 0].$$

$$5. \int_0^{\infty} e^{-px^2} [e^{cx} \operatorname{Ei}(-cx) + e^{-cx} \operatorname{Ei}(cx)] dx = \frac{1}{2} \sqrt{\frac{\pi}{p}} \exp\left(\frac{c^2}{4p}\right) \operatorname{Ei}\left(-\frac{c^2}{4p}\right) \\ [\operatorname{Re} p > 0].$$

$$6. \int_0^{\infty} e^{-px^2} \{ e^{(b+c)x/2} [\operatorname{Ei}(-bx) + \operatorname{Ei}(-cx)] + e^{-(b+c)x/2} [\operatorname{Ei}(bx) + \operatorname{Ei}(cx)] \} dx = \\ = \sqrt{\frac{\pi}{p}} \exp\left[\frac{(b+c)^2}{4p}\right] \operatorname{Ei}\left(-\frac{bc}{p}\right) \quad [\operatorname{Re} p > 0].$$

2.5.6. Интегралы, содержащие гиперболические функции и $\operatorname{Ei}(-cx)$.

$$1. \int_0^{\infty} \operatorname{ch} bx \operatorname{Ei}(-cx) dx = \frac{1}{2b} \ln \frac{c-b}{c+b} \quad [c > b > 0]$$

$$2. \int_0^{\infty} \frac{1}{x} \operatorname{sh} bx \operatorname{Ei}(-cx) dx = \frac{1}{2} \left[\operatorname{Li}_2\left(-\frac{b}{c}\right) - \operatorname{Li}_2\left(\frac{b}{c}\right) \right] \quad [c > b > 0].$$

2.5.7. Интегралы, содержащие тригонометрические функции и $\operatorname{Ei}(cx)$.

Обозначение: $\delta = \begin{cases} 1 \\ 0 \end{cases}$.

$$1. \int_0^{\infty} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} \operatorname{Ei}(-cx) dx = -\frac{1}{2b} \begin{Bmatrix} \ln(1+b^2c^{-2}) \\ 2 \operatorname{arctg}(b/c) \end{Bmatrix} \quad [b, \operatorname{Re} c > 0].$$

$$2. \int_0^{\infty} \frac{\sin 2nx}{\sin x} \operatorname{Ei}(-cx) dx = -2 \sum_{k=0}^{n-1} \frac{1}{2k+1} \operatorname{arctg} \frac{2k+1}{c} \quad [\operatorname{Re} c > 0].$$

$$3. \int_0^{\infty} \frac{\sin (2n+1)x}{\sin x} \operatorname{Ei}(-cx) dx = -\frac{1}{c} - \sum_{k=1}^n \frac{1}{k} \operatorname{arctg} \frac{2k}{c} \quad [\operatorname{Re} c > 0].$$

$$4. \int_0^{\infty} x^{\alpha-1} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} \operatorname{Ei}(-cx) dx =$$

$$= -\frac{b^{\delta} \Gamma(\alpha+\delta)}{(\alpha+\delta) c^{\alpha+\delta}} {}_2F_2\left(\frac{\alpha+1}{2}, \frac{\alpha+\delta}{2}, \frac{\alpha}{2}+\delta; \frac{1}{2}+\delta, 1+\frac{\alpha+\delta}{2}; -\frac{b^2}{c^2}\right)$$

[b, $\operatorname{Re} c > 0$; $\operatorname{Re} \alpha > -\delta$]

$$5. \int_0^{\infty} x \cos bx \operatorname{Ei}(-cx) dx = \frac{1}{2b^2} \ln\left(1+\frac{b^2}{c^2}\right) - \frac{1}{b^2+c^2} \quad [b, \operatorname{Re} c > 0]$$

$$6. \int_0^{\infty} \sin b \sqrt{x} \operatorname{Ei}(-cx) dx = -\frac{2\pi}{b^2} \operatorname{erf}\left(\frac{b}{2\sqrt{c}}\right) + \frac{2}{b} \sqrt{\frac{\pi}{c}} \exp\left(-\frac{b^2}{4c}\right)$$

[b, $\operatorname{Re} c > 0$].

$$7. \int_0^{\infty} x^{\alpha-1} \begin{Bmatrix} \sin b \sqrt{x} \\ \cos b \sqrt{x} \end{Bmatrix} \operatorname{Ei}(-cx) dx =$$

$$= -\frac{2b^{\delta} \Gamma(\alpha+\delta/2)}{c^{\alpha+\delta/2} (2\alpha+\delta)} {}_2F_2\left(\alpha+\frac{\delta}{2}, \alpha+\frac{\delta}{2}; \frac{1}{2}+\delta, \alpha+\frac{\delta}{2}+1; -\frac{b^2}{4c}\right)$$

[b, $\operatorname{Re} c > 0$; $\operatorname{Re} \alpha > -\delta/2$]

$$8. \int_0^{\infty} x^{-1/2} \cos b \sqrt{x} \operatorname{Ei}(-cx) dx = -\frac{2\pi}{b} \operatorname{erf}\left(\frac{b}{2\sqrt{c}}\right) \quad [b, \operatorname{Re} c > 0].$$

2.5.8. Интегралы, содержащие e^{-px} , тригонометрические функции и $\operatorname{Ei}(cx)$.

$$1. \int_0^{\infty} e^{-px} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} \operatorname{Ei}(-cx) dx = -\frac{1}{p^2+b^2} \left[\begin{Bmatrix} b \\ p \end{Bmatrix} \ln \frac{\sqrt{(p+c)^2+b^2}}{c} \mp \begin{Bmatrix} p \\ b \end{Bmatrix} \operatorname{arctg} \frac{b}{p+c} \right]$$

[b, $\operatorname{Re}(p+c) > 0$; $|\arg c| < \pi$]

$$2. \int_0^{\infty} e^{-px} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} \operatorname{Ei}(cx) dx = -\frac{1}{p^2+b^2} \left[\begin{Bmatrix} b \\ p \end{Bmatrix} \ln \frac{\sqrt{(p-c)^2+b^2}}{c} \mp \begin{Bmatrix} p \\ b \end{Bmatrix} \operatorname{arctg} \frac{b}{p-c} \right]$$

[b ≥ 0 ; $\operatorname{Re} p > c > 0$].

$$3. \int_0^{\infty} e^{cx} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} \text{Ei}(-cx) dx = -\frac{1}{b^2 + c^2} \left[\frac{\pi}{2} \begin{Bmatrix} c \\ b \end{Bmatrix} \pm \begin{Bmatrix} b \\ c \end{Bmatrix} \ln \frac{b}{|c|} \right]$$

[$b > 0$; $|\arg c| < \pi$].

$$4. \int_0^{\infty} x^{\alpha-1} e^{\pm cx} \sin bx \text{Ei}(\mp cx) dx = U(1)$$

[$b > 0$; $-1 < \text{Re } \alpha < 2$, $\begin{Bmatrix} |\arg c| < \pi \\ c > 0 \end{Bmatrix}$].

$$U(\delta) = \mp \frac{b^{1-\alpha}}{c} \Gamma(\alpha-1) \sin \frac{\alpha-\delta}{2} \pi {}_3F_2 \left(1, 1, \frac{1}{2}; 1-\frac{\alpha}{2}, \frac{3-\alpha}{2}; -\frac{b^2}{c^2} \right) -$$

$$- \frac{b^{2-\alpha}}{c^2} \Gamma(\alpha-2) \cos \frac{\alpha-\delta}{2} \pi {}_3F_2 \left(1, 1, \frac{3}{2}; 2-\frac{\alpha}{2}, \frac{3-\alpha}{2}; -\frac{b^2}{c^2} \right) -$$

$$- \frac{\pi b^{\delta}}{c^{\alpha+\delta}} \begin{Bmatrix} \text{cosec } \alpha\pi \\ \text{ctg } \alpha\pi \end{Bmatrix} \Gamma(\alpha+\delta) {}_2F_1 \left(\frac{\alpha+1}{2}, \frac{\alpha}{2} + \delta; \delta + \frac{1}{2}; -\frac{b^2}{c^2} \right).$$

$$5. \int_0^{\infty} x^{\alpha-1} e^{\pm cx} \cos bx \text{Ei}(\mp cx) dx = U(0)$$

[$b > 0$; $0 < \text{Re } \alpha < 2$, $\begin{Bmatrix} |\arg c| < \pi \\ c > 0 \end{Bmatrix}$, $U(\delta)$ см. в 2.5.8.4].

2.5.9. Интегралы от $x^{\alpha} \ln^n x \text{Ei}(cx)$.

$$1. \int_0^{\infty} \ln x \text{Ei}(-cx) dx = \frac{1}{c} (1 + \bar{C} + \ln c) \quad [\text{Re } c > 0].$$

$$2. \int_0^{\infty} x^{\alpha-1} \ln x \text{Ei}(-cx) dx = I_{\alpha}$$

$$I_{\alpha} = \frac{\Gamma(\alpha)}{\alpha^2 c^{\alpha}} [1 + \alpha \ln c - \alpha \psi(\alpha)],$$

$$I_2 = \frac{1}{4c^2} (-1 + 2\bar{C} + 2 \ln c),$$

$$I_{n+1} = \frac{n!}{(n+1) c^{n+1}} \left(\frac{1}{n+1} + \bar{C} + \ln c - \sum_{k=1}^n \frac{1}{k} \right).$$

$$3. \int_0^{\infty} x^{\alpha-1} \ln^n x \text{Ei}(-cx) dx = -\frac{d^n}{d\alpha^n} \left(\frac{\Gamma(\alpha)}{\alpha c^{\alpha}} \right) \quad [\text{Re } \alpha, \text{Re } c > 0].$$

2.5.10. Интегралы от $x^{\alpha} e^{-px} \ln^n x \text{Ei}(cx)$.

$$1. \int_0^{\infty} e^{-cx} \ln x \text{Ei}(-cx) dx = \frac{1}{2c} [\zeta(2) + (\bar{C} + \ln c) \ln 4 + \ln^2 2] \quad [\text{Re } c > 0].$$

$$2. \int_0^{\infty} x^{\alpha-1} e^{\pm cx} \ln^n x \text{Ei}(\mp cx) dx = I_n \quad \left[0 < \text{Re } \alpha < 1, \begin{Bmatrix} |\arg c| < \pi \\ c > 0 \end{Bmatrix} \right].$$

$$I_n = -\pi \frac{d^n}{d\alpha^n} \left(\frac{\Gamma(\alpha)}{c^{\alpha}} \begin{Bmatrix} \text{cosec } \alpha\pi \\ \text{ctg } \alpha\pi \end{Bmatrix} \right),$$

$$I_1 = -\pi \frac{\Gamma(\alpha)}{c^{\alpha}} \begin{Bmatrix} \text{cosec } \alpha\pi \\ \text{ctg } \alpha\pi \end{Bmatrix} \left[\psi(\alpha) - \ln c - \pi \begin{Bmatrix} \text{ctg } \alpha\pi \\ 2 \text{ cosec } 2\alpha\pi \end{Bmatrix} \right].$$

$$3. \int_0^{\infty} e^{-px} \ln x \operatorname{Ei}(-cx) dx = \frac{1}{p} \left[(\ln(p+c) + C) \ln \left(1 + \frac{p}{c} \right) + \frac{p}{p+c} \Phi \left(\frac{p}{p+c}, 2, 1 \right) \right] \\ [\operatorname{Re}(p+c) > 0; |\arg c| < \pi].$$

$$4. \int_0^{\infty} e^{-px} \ln x \operatorname{Ei}(cx) dx = \frac{1}{2p} \left[2 \operatorname{Li}_2 \left(-\frac{c}{p} \right) + \ln^2 \frac{c}{p} + \ln^2 (Cp - Cc) - \ln^2 (Cc) + \frac{\pi^2}{3} \right] \\ [\operatorname{Re} p > c > 0].$$

$$5. \int_0^{\infty} x^{\alpha-1} e^{-px} \ln^n x \operatorname{Ei}(-cx) dx = -\frac{d^n}{d\alpha^n} \left[\frac{\Gamma(\alpha)}{(c+p)^\alpha} \Phi \left(\frac{p}{p+c}, 1, \alpha \right) \right] \\ [\operatorname{Re} \alpha, \operatorname{Re}(p+c) > 0; |\arg c| < \pi]$$

$$6. \int_0^{\infty} x^{\alpha-1} e^{-px} \ln x \operatorname{Ei}(-cx) dx = I_\alpha \quad [\operatorname{Re} \alpha, \operatorname{Re}(p+c) > 0; |\arg c| < \pi].$$

$$I_\alpha = \frac{\Gamma(\alpha)}{(p+c)^\alpha} \left\{ [\ln(p+c) - \psi(\alpha)] \Phi \left(\frac{p}{p+c}, 1, \alpha \right) + \Phi \left(\frac{p}{p+c}, 2, \alpha \right) \right\}, \\ I_{n+1} = \frac{n!}{p^{n+1}} \left[\ln \left(1 + \frac{p}{c} \right) - \sum_{k=1}^n \frac{1}{k} \left(\frac{p}{p+c} \right)^k \right] \left[C + \ln(p+c) - \sum_{k=1}^n \frac{1}{k} \right] + \\ + \frac{n!}{(p+c)^{n+1}} \Phi \left(\frac{p}{p+c}, 2, n+1 \right).$$

2.5.11. Интегралы от $x^\alpha \operatorname{Ei}(bx^{\pm 1} + d) \operatorname{Ei}(cx)$.

$$1. \int_0^{\infty} \operatorname{Ei}(-cx) \operatorname{Ei}(cx) dx = 0 \quad [c > 0]$$

$$2. \int_0^{\infty} x^{\alpha-1} \operatorname{Ei}(-cx) \operatorname{Ei}(cx) dx = \frac{2^{\alpha-2} \sqrt{\pi}}{c^\alpha} \operatorname{ctg} \frac{\alpha\pi}{2} \Gamma^2 \left(\frac{\alpha}{2} \right) \\ [0 < \operatorname{Re} \alpha < 2; c > 0].$$

$$3. \int_0^{\infty} \operatorname{Ei}(-bx) \operatorname{Ei}(\mp cx) dx = \frac{c \pm b}{bc} \ln(b \pm c) - \frac{1}{b} \ln c \mp \frac{1}{c} \ln b \\ \left[\left\{ \begin{array}{l} \operatorname{Re}(b+c) > 0 \\ \operatorname{Re} b > c > 0 \end{array} \right\} \right]$$

$$4. \int_0^{\infty} x^{\alpha-1} \operatorname{Ei}^2(-cx) dx = \frac{\Gamma(\alpha)}{2^{\alpha-1} \alpha c^\alpha} \Phi \left(\frac{1}{2}, 1, \alpha \right) \quad [\operatorname{Re} \alpha, \operatorname{Re} c > 0]$$

$$5. \int_0^{\infty} x^{\alpha-1} \operatorname{Ei}(-bx) \operatorname{Ei}(-cx) dx = I_\alpha \quad [\operatorname{Re}(b+c), \operatorname{Re} \alpha > 0].$$

$$I_\alpha = \frac{\Gamma(\alpha)}{b^\alpha} \left\{ \frac{\alpha c}{(\alpha+1)b} {}_4F_3 \left(1, 1, \alpha+1, \alpha+1; 2, 2, \alpha+2; -\frac{c}{b} \right) + \right. \\ \left. + \frac{1}{\alpha} \left[\frac{1}{\alpha} - \psi(\alpha) - C + \ln \frac{b}{c} \right] \right\},$$

$$I_2 = \frac{b^2 + c^2}{2b^2c^2} \ln(b+c) - \frac{\ln b}{2c^2} - \frac{\ln c}{2b^2} - \frac{1}{2bc},$$

$$I_3 = \frac{2}{3} \left[\frac{b^3 + c^3}{b^3c^3} \ln(b+c) - \frac{\ln b}{c^3} - \frac{\ln c}{b^3} - \frac{b^2 - bc + c^2}{b^2c^2(b+c)} \right],$$

$$I_{n+1} = -\frac{n!}{n+1} \left\{ \frac{1}{b^{n+1}} \left[\ln \frac{c}{b+c} + \sum_{k=1}^n \frac{1}{k} \left(\frac{b}{b+c} \right)^k \right] + \right. \\ \left. + \frac{1}{c^{n+1}} \left[\ln \frac{b}{b+c} + \sum_{k=1}^n \frac{1}{k} \left(\frac{c}{b+c} \right)^k \right] \right\}.$$

$$6. \int_0^a \text{Ei}(x-a) \text{Ei}(-x) dx = 2(C + \ln a) e^{-a} - 2(1 - aC - a \ln a) \text{Ei}(-a) - \\ - a [\zeta(2) + (C + \ln a)^2] - 2a \sum_{k=1}^{\infty} \frac{(-a)^k}{k! k^2} \quad [a > 0]$$

$$7. \int_a^{\infty} \text{Ei}(a-x) \text{Ei}(-x) dx = e^{-a} [\ln 2 + e^{2a} \text{Ei}(-2a)] + \frac{a}{2} [(C + \ln a)^2 - 2\zeta(2)] - \\ - a(C + \ln a) \text{Ei}(a) + a \ln 2 [\text{Ei}(-a) - \text{Ei}(a)] - \text{Ei}(-a) + a \sum_{k=1}^{\infty} b_k \frac{a^k}{k! k} \\ \left[b_k = \frac{1}{k} + 2 \sum_{j=1}^{[(k+1)/2]} \frac{1}{2j-1}; a > 0 \right]$$

$$8. \int_0^{\infty} \text{Ei}\left(-\frac{b}{x}\right) \text{Ei}(-cx) dx = \frac{1}{c} \sum_{k=0}^{\infty} \frac{(bc)^k}{(k!)^2 (k+1)} \left[\frac{1}{1-k} - 2\psi(k+1) + \ln(bc) \right] \\ [\text{Re } b, \text{Re } c > 0]$$

$$9. \int_0^{\infty} x^{-1/2} \text{Ei}\left(-\frac{b}{x}\right) \text{Ei}(-cx) dx = -4 \sqrt{\frac{\pi}{c}} [(1+2\sqrt{bc}) \text{Ei}(-2\sqrt{bc}) + e^{-2\sqrt{bc}}] \\ [\text{Re } b, \text{Re } c > 0]$$

2.5.12. Интегралы от $x^\alpha e^{f(x)} \text{Ei}(bx^{\pm 1}) \text{Ei}(cx)$.

$$1. \int_0^{\infty} e^{kcx} \text{Ei}^2(-cx) dx = \frac{\pi^2}{6kc} \quad [k=1, 2; \text{Re } c > 0].$$

$$2. \int_0^{\infty} e^{-cx} \text{Ei}^2(-cx) dx = \frac{1}{c} \left[\frac{\pi^2}{6} - 2 \text{Li}_2\left(\frac{1}{3}\right) + 2 \ln 2 \ln 3 - \ln^2 3 \right] = c^{-1} \cdot 1,228558... \\ [\text{Re } c > 0]$$

$$3. \int_0^{\infty} e^{c(c-1)x} \text{Ei}^2(-cx) dx = \frac{1}{c-1} \left[\frac{\pi^2}{12} + \text{Li}_2\left(\frac{c-2}{c}\right) \right] \quad [|\arg c| < \pi]$$

$$4. \int_0^{\infty} e^{-2cx} \text{Ei}^2(cx) dx = \frac{\pi^2}{4c} \quad [c > 0]$$

$$5. \int_0^{\infty} e^{-cx} \text{Ei}(-cx) \text{Ei}(cx) dx = \frac{\pi^2}{12c} \quad [c > 0]$$

$$\begin{aligned}
6. \int_0^\infty x^{\alpha-1} e^{-px} \operatorname{Ei}(-cx) \operatorname{Ei}(cx) dx = \\
= 2^{\alpha-2} \sqrt{\pi} c^{-\alpha} \operatorname{ctg} \frac{\alpha\pi}{2} \Gamma^2\left(\frac{\alpha}{2}\right) {}_2F_1\left(\frac{\alpha}{2}, \frac{\alpha}{2}; \frac{1}{2}; \frac{p^2}{c^2}\right) + \\
+ \frac{2^{\alpha-1} p \sqrt{\pi}}{c^{\alpha+1}} \operatorname{tg} \frac{\alpha\pi}{2} \Gamma^2\left(\frac{\alpha+1}{2}\right) {}_2F_1\left(\frac{\alpha+1}{2}, \frac{\alpha+1}{2}; \frac{3}{2}; \frac{p^2}{c^2}\right) - \\
- \frac{2p^{\alpha-2} \Gamma(\alpha-2)}{\sqrt{\pi} c^2} {}_3F_2\left(1, 1, 1; 2-\frac{\alpha}{2}, \frac{3-\alpha}{2}; \frac{p^2}{c^2}\right) \quad [c, \operatorname{Re} \alpha, \operatorname{Re} p > 0].
\end{aligned}$$

$$\begin{aligned}
7. \int_0^\infty x^{\alpha-1} e^{cx} \operatorname{Ei}^2(-cx) dx = \frac{\Gamma(\alpha)}{2} \left[\frac{4\pi \operatorname{ctg} \alpha\pi}{\sin \alpha\pi} - \psi'\left(1-\frac{\alpha}{2}\right) + \psi'\left(\frac{1-\alpha}{2}\right) \right] \\
[\operatorname{Re} \alpha, \operatorname{Re} c > 0].
\end{aligned}$$

$$\begin{aligned}
8. \int_0^\infty x e^{-cx} \operatorname{Ei}^2(-cx) dx = \frac{1}{c^2} \left[\frac{\pi^2}{6} - 2 \operatorname{Li}_2\left(\frac{1}{3}\right) + 2 \ln 2 \ln 3 - \ln^2 3 - \ln 3 \right] = \\
= c^{-2} \cdot 0,129946... \quad [\operatorname{Re} c > 0]
\end{aligned}$$

$$9. \int_0^\infty [e^{-cx} + cx \operatorname{Ei}(-cx)]^2 dx = \frac{2}{3c} (1 - \ln 2) \quad [\operatorname{Re} c > 0]$$

$$\begin{aligned}
10. \int_0^\infty e^{-(b+c)x} \operatorname{Ei}(-bx) \operatorname{Ei}(cx) dx = \\
= \frac{1}{b+c} \left[\frac{\pi^2}{12} + \frac{1}{2} \ln^2 \frac{c}{b} - \ln \frac{2b+c}{b} \ln \frac{c}{b} + \operatorname{Li}_2\left(\frac{c}{2b+c}\right) \right] \quad [c, \operatorname{Re} b > 0].
\end{aligned}$$

$$\begin{aligned}
11. \int_0^\infty e^{(b-c)x} \operatorname{Ei}(-bx) \operatorname{Ei}(-cx) dx = \\
= \frac{1}{c-b} \left[\frac{\pi^2}{12} + \frac{1}{2} \ln^2 2 - \operatorname{Li}_2\left(1-\frac{c}{2b}\right) + \ln 2 \ln \frac{c}{2b} \right] \quad [\operatorname{Re} c > 0; |\arg b| < \pi].
\end{aligned}$$

$$12. \int_0^\infty x^n e^{-px} \operatorname{Ei}(-bx) \operatorname{Ei}(-cx) dx = I_n \quad [\operatorname{Re}(b+c+p) > 0],$$

$$\begin{aligned}
I_n = \frac{n!}{p^{n+1}} \left\{ \frac{\pi^2}{6} - \operatorname{Li}_2\left(\frac{b}{b+c+p}\right) - \operatorname{Li}_2\left(\frac{c}{b+c+p}\right) + \ln b \ln c - \ln^2(b+c+p) + \right. \\
+ \left[\ln(b+p) - \sum_{k=1}^n \frac{1}{k} \left(\frac{p}{b+p}\right)^k \right] \ln \frac{b+c+p}{c} + \\
+ \left[\ln(c+p) - \sum_{k=1}^n \frac{1}{k} \left(\frac{p}{c+p}\right)^k \right] \ln \frac{b+c+p}{b} + \\
+ \sum_{k=2}^n \frac{1}{k} \left\{ \sum_{l=1}^{k-1} \frac{1}{k-l} \left[\left(\frac{b+c+p}{b+p}\right)^l + \left(\frac{b+c+p}{c+p}\right)^l \right] \right\} \left(\frac{p}{b+c+p}\right)^k \Big\}.
\end{aligned}$$

$$\begin{aligned}
13. \int_0^\infty x^{-1/2} e^{cx+b/x} \operatorname{Ei}\left(-\frac{b}{x}\right) \operatorname{Ei}(-cx) dx = -\frac{2\pi^{3/2}}{\sqrt{c}} e^{2\sqrt{bc}} \operatorname{Ei}(-2\sqrt{bc}) \\
[|\arg b|, |\arg c| < \pi].
\end{aligned}$$

2.5.13. Интегралы от $x^\alpha \ln^n x \operatorname{Ei}^2(-cx)$.

$$1. \int_0^\infty \ln x \operatorname{Ei}^2(-x) dx = -\zeta(2) - 2(\gamma + 1) \ln 2 - \ln^2 2.$$

$$2. \int_0^\infty x^{\alpha-1} \ln^n x \operatorname{Ei}^2(-cx) dx = \frac{d^n}{d\alpha^n} \left[\frac{\Gamma(\alpha)}{2^{\alpha-1} \alpha c^\alpha} \Phi\left(\frac{1}{2}, 1, \alpha\right) \right]$$

[$\operatorname{Re} \alpha, \operatorname{Re} c > 0$].

$$3. \int_0^\infty x^{\alpha-1} \ln x \operatorname{Ei}^2(-x) dx = I_\alpha$$

[$\operatorname{Re} \alpha > 0$].

$$I_\alpha = \frac{2^{1-\alpha}}{\alpha} \Gamma(\alpha) \left\{ \left[\psi(\alpha) - \frac{1}{\alpha} - \ln 2 \right] \sum_{k=0}^\infty \frac{1}{2^k (k+\alpha)} - \sum_{k=0}^\infty \frac{1}{2^k (k+\alpha)^2} \right\},$$

$$I_{n+1} = -\frac{n!}{n+1} \left\{ \zeta(2) + \ln^2 2 - 2\gamma \sum_{k=1}^n \frac{1}{2^k k} + \right. \\ \left. + 2 \left[\gamma + \frac{1}{n+1} - \sum_{k=1}^n \frac{1}{k} \left(1 + \frac{1}{2^k} \right) \right] \ln 2 - 2 \sum_{k=1}^n \frac{1}{2^k k} \left[\frac{1}{n+1} + \frac{1}{k} - \sum_{j=1}^n \frac{1}{j} \right] \right\}.$$

2.6. ИНТЕГРАЛЬНЫЕ СИНУС $\operatorname{si}(x)$ И КОСИНУС $\operatorname{ci}(x)$

2.6.1. Интегралы общего вида.

Обозначение: $\delta = \begin{cases} 1 \\ 0 \end{cases}$.

$$1. \int_0^a x^{\alpha-1} (a^r - x^r)^{\beta-1} \begin{Bmatrix} \operatorname{si}(cx) \\ \operatorname{ci}(cx) \end{Bmatrix} dx = \\ = \frac{a^{\alpha+r\beta+\delta-r} c^\delta}{2r} \Gamma(\beta) \left\{ \sum_{k=1}^\infty \frac{(-a^2 c^2/4)^k}{(1/2+\delta)_k (\delta/2+k) k!} \Gamma\left[\frac{(\alpha+\delta+2k)/r}{(\alpha+\delta+2k)/r+\beta}\right] + \right. \\ \left. + 2\Gamma\left[\frac{(\alpha+\delta)/r}{(\alpha+\delta)/r+\beta}\right] \left[\delta + (1-\delta) \left(\gamma + \ln(ac) + \frac{1}{r} \psi\left(\frac{\alpha}{r}\right) - \frac{1}{r} \psi\left(\frac{\alpha}{r} + \beta\right) \right) \right] \right\} - \\ - \frac{\delta \pi a^{\alpha+r\beta-r}}{2r} B\left(\beta, \frac{\alpha}{r}\right) \quad [a, r, \operatorname{Re} \alpha, \operatorname{Re} \beta > 0]$$

$$2. \int_a^\infty x^{\alpha-1} (x^r - a^r)^{\beta-1} \begin{Bmatrix} \operatorname{si}(cx) \\ \operatorname{ci}(cx) \end{Bmatrix} dx = \\ = \frac{a^{\alpha+r\beta+\delta-r} c^\delta}{2r} \Gamma(\beta) \left\{ \sum_{k=1}^\infty \frac{(-a^2 c^2/4)^k}{(\delta/2+k) (1/2+\delta)_k k!} \Gamma\left[\frac{1-\beta-(\alpha+\delta+2k)/r}{1-(\alpha+\delta+2k)/r}\right] + \right. \\ \left. + 2\Gamma\left[\frac{1-\beta-(\alpha+\delta)/r}{1-(\alpha+\delta)/r}\right] \left[\delta + (1-\delta) \left(\gamma + \ln(ac) + \frac{1}{r} \psi\left(1-\frac{\alpha}{r}\right) - \right. \right. \right. \\ \left. \left. - \frac{1}{r} \psi\left(1-\beta-\frac{\alpha}{r}\right) \right) \right] \right\} - \frac{\delta \pi a^{\alpha+r\beta-r}}{2r} B\left(\beta, 1-\beta-\frac{\alpha}{r}\right) \pm \\ \pm c^{r-r\beta-\alpha} \sum_{k=0}^\infty \frac{(1-\beta)_k \Gamma(r\beta+\alpha-r-rk)}{k! (r\beta+\alpha-r-rk)} \begin{Bmatrix} \sin[(1-\beta-\alpha/r+k)r\pi/2] \\ \cos[(1-\beta-\alpha/r+k)r\pi/2] \end{Bmatrix} (ac)^{kr} \\ [a, c, r, \operatorname{Re} \beta > 0; \operatorname{Re}(\alpha+\beta r) < r+2]$$

$$\begin{aligned}
3. \int_0^\infty \frac{x^{\alpha-1}}{(x^r + z^r)^\rho} \begin{Bmatrix} \text{si}(cx) \\ \text{ci}(cx) \end{Bmatrix} dx = \\
= \frac{c^\delta z^{\alpha-r\rho+\delta}}{2r\Gamma(\rho)} \left\{ \sum_{k=1}^\infty \frac{(-c^2 z^2/4)^k}{(\delta/2+k)(1/2+\delta)_k k!} \Gamma\left(\frac{\alpha+\delta+2k}{r}\right) \Gamma\left(\rho - \frac{\alpha+\delta+2k}{r}\right) + \right. \\
+ 2\Gamma\left(\frac{\alpha+\delta}{r}\right) \Gamma\left(\rho - \frac{\alpha+\delta}{r}\right) \left[\delta + (1-\delta) \left(C + \ln(cz) + \frac{1}{r} \psi\left(\frac{\alpha}{r}\right) - \right. \right. \\
\left. \left. - \frac{1}{r} \psi\left(\rho - \frac{\alpha}{r}\right) \right) \right] \right\} - \frac{\delta\pi z^{\alpha-r\rho}}{2r} B\left(\frac{\alpha}{r}, \rho - \frac{\alpha}{r}\right) \pm \\
\pm c^{\rho-r-\alpha} \sum_{k=0}^\infty \frac{(\rho)_k (-c^r z^r)^k}{k! (\alpha-r\rho-rk)} \Gamma(\alpha-r\rho-rk) \begin{Bmatrix} \sin[(\rho - \alpha/r + k)r\pi/2] \\ \cos[(\rho - \alpha/r + k)r\pi/2] \end{Bmatrix} \\
[c, r > 0; 0 < \text{Re } \alpha < r \text{Re } \rho + 2; r|\arg z| < \pi].
\end{aligned}$$

$$\begin{aligned}
4. \int_0^\infty \frac{x^{\alpha-1}}{x^r - y^r} \begin{Bmatrix} \text{si}(cx) \\ \text{ci}(cx) \end{Bmatrix} dx = \\
= -\frac{\pi c^\delta y^{\alpha-r+\delta}}{2r} \left\{ \sum_{k=1}^\infty \frac{(-c^2 y^2/4)^k}{(\delta/2+k)(1/2+\delta)_k k!} \text{ctg} \frac{\alpha+\delta+2k}{r} \pi + \right. \\
+ 2 \text{ctg} \frac{\alpha+\delta}{r} \pi \left[\delta + (1-\delta) \left(C + \ln(cy) - \frac{2\pi}{r} \text{cosec} \frac{2\alpha\pi}{r} \right) \right] \right\} + \frac{\delta\pi^2 y^{\alpha-r}}{2r} \text{ctg} \frac{\alpha\pi}{r} \mp \\
\mp \frac{c^{r-\alpha}}{r} \sum_{k=0}^\infty \frac{(yc)^{rk} \Gamma(\alpha-r-k)}{(1-\alpha/r+k)} \begin{Bmatrix} \sin[(1-\alpha/r+k)r\pi/2] \\ \cos[(1-\alpha/r+k)r\pi/2] \end{Bmatrix} \\
[c, r, y > 0; 0 < \text{Re } \alpha < r+2].
\end{aligned}$$

$$5. \int_0^\infty x^{\alpha-1} e^{-px^r} \begin{Bmatrix} \text{si}(cx) \\ \text{ci}(cx) \end{Bmatrix} dx = U(r) \quad [c, r, \text{Re } \alpha, \text{Re } \rho > 0],$$

$$\begin{aligned}
U(r) = \frac{c^\delta}{2r\rho^{(\alpha+\delta)/r}} \left\{ \sum_{k=1}^\infty \frac{(-1)^k}{(k+\delta/2)(1/2+\delta)_k} \Gamma\left(\frac{\alpha+\delta+2k}{r}\right) \left(\frac{c}{2\rho^{1/r}}\right)^{2k} + \right. \\
+ 2\Gamma\left(\frac{\alpha+\delta}{r}\right) \left[\delta + (1-\delta) \left(C + \ln c - \frac{1}{r} \ln \rho + \frac{1}{r} \psi\left(\frac{\alpha}{r}\right) \right) \right] \right\} - \frac{\delta\pi}{2r\rho^{\alpha/r}} \Gamma\left(\frac{\alpha}{r}\right) \\
[r > 1].
\end{aligned}$$

$$U(r) = -c^{-\alpha} \sum_{k=0}^\infty \frac{\Gamma(\alpha+rk)}{k! (\alpha+rk)} \left(-\frac{\rho}{c^r}\right)^k \begin{Bmatrix} \sin[(\alpha+rk)\pi/2] \\ \cos[(\alpha+rk)\pi/2] \end{Bmatrix} \quad [r < 1].$$

$U(1)$ см. в. 2.6.3 3.

$$\begin{aligned}
6. \int_0^\infty x^{\alpha-1} e^{-px^r} \begin{Bmatrix} \text{si}(cx) \\ \text{ci}(cx) \end{Bmatrix} dx = \\
= \frac{c^\delta \rho^{(\alpha+\delta)/r}}{2r} \left\{ \sum_{k=1}^\infty \frac{(-1)^k}{(k+\delta/2)(1/2+\delta)_k} \Gamma\left(-\frac{\alpha+\delta+2k}{r}\right) \left(\frac{cp^{1/r}}{2}\right)^{2k} + \right. \\
+ 2\Gamma\left(-\frac{\alpha+\delta}{r}\right) \left[\delta + (1-\delta) \left(C + \ln c + \frac{1}{r} \ln \rho - \frac{1}{r} \psi\left(-\frac{\alpha}{r}\right) \right) \right] \right\} + \\
+ \frac{c^{-\alpha}}{r} \sum_{k=0}^\infty \frac{(-\rho c^r)^k}{(k-\alpha/r) k!} \Gamma(\alpha-rk) \begin{Bmatrix} \sin[(\alpha-rk)\pi/2] \\ \cos[(\alpha-rk)\pi/2] \end{Bmatrix} + \frac{\delta\pi\rho^{\alpha/r}}{2r} \Gamma\left(-\frac{\alpha}{r}\right) \\
[c, r, \text{Re } \rho > 0; \text{Re } \alpha < 2].
\end{aligned}$$

$$7. \int_0^{\infty} x^{\alpha-1} \sin bx^r \left\{ \begin{matrix} \text{si} (cx) \\ \text{ci} (cx) \end{matrix} \right\} dx = V(1, r) \quad [b, c, r > 0; -r < \text{Re} \alpha < 2],$$

$$V(\gamma, r) = \frac{c^\delta}{2b^{(\alpha+\delta)/r}} \left\{ \sum_{k=1}^{\infty} \frac{(-1)^k}{(k+\delta/2)(1/2+\delta)_k k!} \Gamma\left(\frac{2k+\alpha+\delta}{r}\right) \times \right. \\ \times \left(\frac{c}{2b^{1/r}}\right)^{2k} \cos \frac{2k+\alpha+\delta-r\gamma}{2r} \pi + 2\Gamma\left(\frac{\alpha+\delta}{r}\right) \cos \frac{\alpha+\delta-r\gamma}{2r} \pi \times \\ \times \left[\delta + (1-\delta) \left(C + \ln \frac{c}{b^{1/r}} + \frac{\pi}{2r} \operatorname{tg} \frac{r\gamma-\alpha}{2r} \pi + \frac{1}{r} \psi\left(\frac{\alpha}{r}\right) \right) \right] \Bigg\} - \\ - \frac{\delta\pi}{2b^{\alpha/r}} \Gamma\left(\frac{\alpha}{r}\right) \cos \frac{\alpha-r\gamma}{2r} \pi \quad [r > 1],$$

$$V(\gamma, r) = -\frac{b\gamma}{c^{\alpha+\gamma r}} \sum_{k=0}^{\infty} \frac{(-1)^k \Gamma(\alpha+r\gamma+2rk)}{(1/2+\gamma)_k k! (\alpha+r\gamma+2rk)} \times \\ \times \left\{ \begin{matrix} \sin [(\alpha+r\gamma+2rk)\pi/2] \\ \cos [(\alpha+r\gamma+2rk)\pi/2] \end{matrix} \right\} \left(\frac{b}{2c^r}\right)^{2k} \quad [r < 1],$$

$V(\gamma, 1)$ см. в 2.6.4.5–6.

$$8. \int_0^{\infty} x^{\alpha-1} \cos bx^r \left\{ \begin{matrix} \text{si} (cx) \\ \text{ci} (cx) \end{matrix} \right\} dx = V(0, r) \quad [b, c, r > 0; 0 < \text{Re} \alpha < 2; V(\gamma, r) \text{ см. в 2.6.1.7}],$$

$$9. \int_0^{\infty} x^{\alpha-1} e^{-px^r} \sin bx^r \left\{ \begin{matrix} \text{si} (cx) \\ \text{ci} (cx) \end{matrix} \right\} dx = W(1) \quad [b, c, r, \operatorname{Re} p > 0; \operatorname{Re} \alpha > -r],$$

$$W(\gamma) = \frac{c^\delta (b^2 + p^2)^{-(\alpha+\delta)/(2r)}}{2r} \left\{ \sum_{k=1}^{\infty} \frac{(-1)^k}{(k+\delta/2)(1/2+\delta)_k k!} \times \right. \\ \times \Gamma\left(\frac{\alpha+\delta+2k}{r}\right) \cos\left(\frac{\gamma\pi}{2} - \frac{\alpha+\delta+2k}{r} \arccos \frac{p}{\sqrt{b^2+p^2}}\right) \frac{c^{2k}}{4^k (b^2+p^2)^{k/r}} + \\ + 2\Gamma\left(\frac{\alpha+\delta}{r}\right) \cos\left(\frac{\gamma\pi}{2} - \frac{\alpha+\delta}{r} \arccos \frac{p}{\sqrt{b^2+p^2}}\right) \left[\delta + (1-\delta) \left(C + \frac{1}{r} \psi\left(\frac{\alpha}{r}\right) + \right. \right. \\ \left. \left. + \frac{1}{2r} \ln \frac{c^{2r}}{b^2+p^2} + \frac{1}{r} \arccos \frac{p}{\sqrt{b^2+p^2}} \operatorname{tg} \left(\frac{\gamma\pi}{2} - \frac{\alpha}{r} \arccos \frac{p}{\sqrt{b^2+p^2}} \right) \right) \right] \Bigg\} \\ [r > 1] \text{ или } [r=1; |c|^2 < |b^2+p^2|],$$

$$W(\gamma) = -c^{-\alpha} \sum_{k=0}^{\infty} \frac{(-1)^k \Gamma(\alpha+rk)}{k! (\alpha+rk)} \left\{ \begin{matrix} \sin [(\alpha+rk)\pi/2] \\ \cos [(\alpha+rk)\pi/2] \end{matrix} \right\} \times \\ \times \cos\left(\frac{\gamma\pi}{2} + k \arccos \frac{p}{\sqrt{b^2+p^2}}\right) \frac{(b^2+p^2)^{k/2}}{c^{rk}} \\ [r < 1] \text{ или } [r=1; |c|^2 > |b^2+p^2|].$$

$$10. \int_0^{\infty} x^{\alpha-1} e^{-px^r} \cos bx^r \left\{ \begin{matrix} \text{si} (cx) \\ \text{ci} (cx) \end{matrix} \right\} dx = W(0) \quad [b, c, r, \operatorname{Re} p, \operatorname{Re} \alpha > 0; W(\gamma) \text{ см. в 2.6.1.9}].$$

$$11. \int_0^{\infty} x^{\alpha-1} \ln(x^r + z^r) \left\{ \begin{matrix} \text{si} (cx) \\ \text{ci} (cx) \end{matrix} \right\} dx = Y(1) \quad [c, r > 0; 0 < \operatorname{Re} \alpha < 2; r|\arg z| < \pi].$$

$$Y(\gamma) = -\frac{1}{c^\alpha} \sum_{k=1}^{\infty} \frac{(-1)^{\gamma k} (cz)^{rk} \Gamma(\alpha - rk)}{k(rk - \alpha)} \left\{ \begin{array}{l} \sin [(\alpha - rk) \pi/2] \\ \cos [(\alpha - rk) \pi/2] \end{array} \right\} +$$

$$+ \pi c^\delta z^{\alpha+\delta} \left\{ \sum_{k=1}^{\infty} \frac{(-1)^k (cz/2)^{2k} \cos^{1-\gamma} [(\alpha + \delta + 2k) \pi/r]}{k! (2k + \delta) (1/2 + \delta)_k (\alpha + \delta + 2k) \sin [(\alpha + \delta + 2k) \pi/r]} + \right.$$

$$+ \frac{\cos^{1-\gamma} [(\alpha + \delta) \pi/r]}{(\alpha + \delta) \sin [(\alpha + \delta) \pi/r]} \left[\delta + (1 - \delta) \left(C - \frac{1}{\alpha} + \ln(cz) - \frac{\pi}{r} \operatorname{ctg} \frac{\alpha\pi}{r} - \right. \right.$$

$$\left. \left. - \frac{(1 - \gamma) \pi}{r} \operatorname{tg} \frac{\alpha\pi}{r} \right) \right] \left. \right\} - \frac{\delta \pi^2 z^\alpha \cos^{1-\gamma} (\alpha\pi/r)}{2\alpha \sin (\alpha\pi/r)} + \frac{r \Gamma(\alpha)}{\alpha c^\alpha} \times$$

$$\times \left\{ \begin{array}{l} \sin (\alpha\pi/2) \\ \cos (\alpha\pi/2) \end{array} \right\} \left[\frac{1}{\alpha} + \ln(c) - \psi(\alpha) \mp \frac{\pi}{2} \operatorname{tg}^{\mp 1} \frac{\alpha\pi}{2} \right]$$

$$12. \int_0^\infty x^{\alpha-1} \ln |x^r - z^r| \left\{ \begin{array}{l} \operatorname{si}(cx) \\ \operatorname{ci}(cx) \end{array} \right\} dx = Y(0)$$

$[c, r, z > 0; 0 < \operatorname{Re} \alpha < 2; Y(\gamma) \text{ см. в 2.6.1.11}]$

$$13. \int_0^\infty x^{\alpha-1} \operatorname{si}(bx^r) \left\{ \begin{array}{l} \operatorname{si}(cx) \\ \operatorname{ci}(cx) \end{array} \right\} dx = Z(1, r)$$

$[b, c, r > 0; 0 < \operatorname{Re} \alpha < r + 2; \text{ при } 0 < r < 1 - \text{замена } x^r = t]$

$$Z(\gamma, r) = \frac{c^\delta}{b^{(\alpha+\delta)/r}} \left\{ \sum_{k=1}^{\infty} \frac{(-1)^k \Gamma((2k + \alpha + \delta)/r)}{(2k + \delta) (2k + \alpha + \delta) (\delta + 1/2)_k k!} \frac{c^{2k}}{2^{2k} b^{2k/r}} \times \right.$$

$$\times \cos \frac{\alpha + \delta + 2k - r\gamma}{2r} \pi + \frac{4}{\alpha + \delta} \Gamma\left(\frac{\alpha + \delta}{r}\right) \cos \frac{\alpha + \delta - r\gamma}{2r} \pi \times$$

$$\times \left[\delta + (1 - \delta) \left(C - \frac{1}{\alpha} + \ln \frac{c}{b^{1/r}} + \frac{1}{r} \psi\left(\frac{\alpha}{r}\right) + \frac{\pi}{2r} \operatorname{tg} \frac{r\gamma - \alpha}{2r} \pi \right) \right] \left. \right\} -$$

$$- \frac{\delta \pi}{2\alpha b^{\alpha/r}} \Gamma\left(\frac{\alpha}{r}\right) \cos \frac{r\gamma - \alpha}{2r} \pi \quad [r > 1],$$

$Z(\gamma, 1) \text{ см. в 2.6.7.1—5.}$

$$14. \int_0^\infty x^{\alpha-1} \operatorname{ci}(bx^r) \left\{ \begin{array}{l} \operatorname{si}(cx) \\ \operatorname{ci}(cx) \end{array} \right\} dx = Z(0, r)$$

$[b, c, r > 0; 0 < \operatorname{Re} \alpha < r + 2; Z(\gamma, r) \text{ см. в 2.6.1.13}]$

2.6.2. Интегралы от $A(x) \left\{ \begin{array}{l} \operatorname{si}(cx) \\ \operatorname{ci}(cx) \end{array} \right\}$ и $A(x) \operatorname{Si}(cx)$.

Обозначение: $\delta = \left\{ \begin{array}{l} 1 \\ 0 \end{array} \right\}$.

$$1. \int_0^\infty \left\{ \begin{array}{l} \operatorname{si}(cx) \\ \operatorname{ci}(cx) \end{array} \right\} dx = -\frac{1}{c} \left\{ \begin{array}{l} 1 \\ 0 \end{array} \right\} \quad [c > 0]$$

$$2. \int_0^\infty x^{\alpha-1} \left\{ \begin{array}{l} \operatorname{si}(cx) \\ \operatorname{ci}(cx) \end{array} \right\} dx = -\frac{\Gamma(\alpha)}{\alpha c^\alpha} \left\{ \begin{array}{l} \sin(\alpha\pi/2) \\ \cos(\alpha\pi/2) \end{array} \right\} \quad [c > 0; 0 < \operatorname{Re} \alpha < 2]$$

$$3. \int_0^\infty x^{\alpha-1} \operatorname{Si}(cx) dx = -\frac{\Gamma(\alpha)}{\alpha c^\alpha} \sin \frac{\alpha\pi}{2} \quad [c > 0; -1 < \operatorname{Re} \alpha < 0]$$

4. $\int_0^{\infty} \frac{\text{ci}(cx)}{x+z} dx = -\frac{1}{2} \text{si}^2(cz) - \frac{1}{2} \text{ci}^2(cz)$ [$c > 0$; $|\arg z| < \pi$]
5. $\int_{-\infty}^{\infty} \frac{\text{sgn } x}{x-y} \text{si}(c|x|) dx = \pi \text{ci}(c|y|)$ [$c > 0$; $y \neq 0$].
6. $\int_{-\infty}^{\infty} \frac{\text{ci}(c|x|)}{x-y} dx = -\pi \text{sgn } y \text{si}(c|y|)$ [$c > 0$; $y \neq 0$].
7. $\int_0^a x^{\alpha-1} (a^2 - x^2)^{\beta-1} \begin{Bmatrix} \text{si}(cx) \\ \text{ci}(cx) \end{Bmatrix} dx = -\frac{a^{\alpha+2\beta+\delta} c^{\delta+2}}{4(\delta+2)(1+2\delta)} \times$
 $\times B\left(\beta, \frac{\alpha+\delta}{2} + 1\right) {}_3F_4\left(1, 1+\delta/2, (\alpha+\delta)/2+1; 2, 3/2+\delta, 2+\delta/2, (\alpha+\delta)/2+\beta+1; -a^2 c^2/4\right) +$
 $+ \frac{a^{\alpha+2\beta+\delta-2} c^{\delta}}{2} B\left(\frac{\alpha+\delta}{2}, \beta\right) \left[\delta + (1-\delta) \left(C + \frac{1}{2} \psi\left(\frac{\alpha}{2}\right) - \frac{1}{2} \psi\left(\frac{\alpha}{2} + \beta\right) + \ln(ac) \right) \right] -$
 $- \frac{\delta \pi a^{\alpha+2\beta-2}}{4} B\left(\beta, \frac{\alpha}{2}\right)$ [$a, c, \text{Re } \alpha, \text{Re } \beta > 0$].
8. $\int_a^{\infty} x^{\alpha-1} (x^2 - a^2)^{\beta-1} \begin{Bmatrix} \text{si}(cx) \\ \text{ci}(cx) \end{Bmatrix} dx = -\frac{a^{\alpha+2\beta+\delta} c^{\delta+2}}{4(1+2\delta)(2+\delta)} \times$
 $\times B\left(\beta, -\beta - \frac{\alpha+\delta}{2}\right) {}_3F_4\left(1, 1+\delta/2, 1+(\alpha+\delta)/2; 2, 3/2+\delta, 2+\delta/2, 1+\beta+(\alpha+\delta)/2; -a^2 c^2/4\right) +$
 $+ \frac{a^{\alpha+2\beta+\delta-2} c^{\delta}}{2} B\left(\beta, 1-\beta - \frac{\alpha+\delta}{2}\right) \left[\delta + (1-\delta) \left(C + \frac{1}{2} \psi\left(1 - \frac{\alpha}{2}\right) - \right. \right.$
 $\left. - \frac{1}{2} \psi\left(1 - \frac{\alpha}{2} - \beta\right) + \ln(ac) \right) \right] - \frac{\delta \pi a^{\alpha+2\beta-2}}{4} B\left(\beta, 1-\beta - \frac{\alpha}{2}\right) -$
 $- \frac{c^{\alpha-2\beta-\alpha}}{2-2\beta-\alpha} \Gamma(\alpha-2+2\beta) \begin{Bmatrix} \sin(\beta+\alpha/2)\pi \\ \cos(\beta+\alpha/2)\pi \end{Bmatrix} \times$
 $\times {}_2F_3\left(1-\beta, 1-\beta-\alpha/2; 2-\beta-\alpha/2, 2-\beta-\alpha/2, (3-\alpha)/2-\beta; -a^2 c^2/4\right)$ [$a, c, \text{Re } \beta > 0$; $\text{Re } (\alpha+2\beta) < 4$].
9. $\int_0^{\infty} \frac{x^{\alpha-1}}{(x^2+z^2)^{\rho}} \begin{Bmatrix} \text{si}(cx) \\ \text{ci}(cx) \end{Bmatrix} dx = -\frac{c^{\alpha+\delta} z^{\alpha+2-2\rho+\delta}}{4(1+2\delta)(2+\delta)} \times$
 $\times B\left(\frac{\alpha+\delta}{2}+1, \rho - \frac{\alpha+\delta}{2} - 1\right) {}_3F_4\left(1, 1+\delta/2, (\alpha+\delta)/2+1; 2, 3/2+\delta, 2+\delta/2, 2+(\alpha+\delta)/2-\rho; c^2 z^2/4\right) +$
 $+ \frac{z^{\alpha-2\rho+\delta} c^{\delta}}{2} B\left(\frac{\alpha+\delta}{2}, \rho - \frac{\alpha+\delta}{2}\right) \left[\delta + (1-\delta) \left(C + \frac{1}{2} \psi\left(\frac{\alpha}{2}\right) - \frac{1}{2} \psi\left(\rho - \frac{\alpha}{2}\right) + \right. \right.$
 $\left. + \ln(cz) \right) \right] - \frac{\delta z^{\alpha-2\rho} \pi}{4} B\left(\frac{\alpha}{2}, \rho - \frac{\alpha}{2}\right) + \frac{c^{2\rho-\alpha}}{2\rho-\alpha} \Gamma(\alpha-2\rho) \begin{Bmatrix} \sin(\alpha/2-\rho)\pi \\ \cos(\alpha/2-\rho)\pi \end{Bmatrix} \times$
 $\times {}_2F_3\left(\rho-\alpha/2, \rho; 1+\rho-\alpha/2, 1+\rho-\alpha/2, (1-\alpha)/2+\rho; c^2 z^2/4\right)$ [$c, \text{Re } z > 0$; $0 < \text{Re } \alpha < 2 \text{Re } \rho + 2$].
10. $\int_0^{\infty} \frac{1}{x^2+z^2} \begin{Bmatrix} x \text{si}(cx) \\ \text{ci}(cx) \end{Bmatrix} dx = \frac{\pi}{2} \begin{Bmatrix} 1 \\ z^{-1} \end{Bmatrix} \text{Ei}(-cz)$ [$c, \text{Re } z > 0$].

$$11. \int_0^{\infty} \frac{x^{-1}}{x^2 + z^2} \operatorname{Si}(cx) dx = \frac{\pi}{2z^2} [C + \ln(cz) - \operatorname{Ei}(-cz)] \quad [c, \operatorname{Re} z > 0].$$

$$12. \int_0^{\infty} \frac{x^{\alpha-1}}{x^2 - y^2} \left\{ \begin{matrix} \operatorname{si}(cx) \\ \operatorname{ci}(cx) \end{matrix} \right\} dx = \frac{\pi c^{\alpha+\delta} y^{\alpha+\delta}}{4(1+2\delta)(2+\delta)} \operatorname{ctg} \frac{\alpha+\delta}{2} \pi \times$$

$$\times {}_2F_3 \left(\begin{matrix} 1, 1+\delta/2; \\ 2, 3/2+\delta, 2+\delta/2 \end{matrix} \middle| -c^2 y^2/4 \right) - \frac{\pi c^{\delta} y^{\alpha+\delta-2}}{2} \times$$

$$\times \operatorname{ctg} \frac{\alpha+\delta}{2} \pi [\delta + (1-\delta)(C + \ln(cy) - \pi \operatorname{cosec} \alpha\pi)] + \frac{\delta \pi^2 y^{\alpha-2}}{4} \operatorname{ctg} \frac{\alpha\pi}{2} +$$

$$+ \frac{c^{2-\alpha} \Gamma(\alpha-2)}{\alpha-2} \left\{ \begin{matrix} \sin(\alpha\pi/2) \\ \cos(\alpha\pi/2) \end{matrix} \right\} {}_2F_3 \left(\begin{matrix} 1, 1-\alpha/2; \\ 2-\alpha/2, 2-\alpha/2, (3-\alpha)/2 \end{matrix} \middle| -c^2 y^2/4 \right)$$

$$[c, y > 0; 0 < \operatorname{Re} \alpha < 4].$$

$$13. \int_0^{\infty} \frac{1}{x^2 - y^2} \left\{ \begin{matrix} x \operatorname{si}(cx) \\ \operatorname{ci}(cx) \end{matrix} \right\} dx = \pm \frac{\pi}{2} \left\{ \begin{matrix} \operatorname{ci}(cy) \\ y^{-1} \operatorname{si}(cy) \end{matrix} \right\} \quad [c, y > 0].$$

2.6.3. Интегралы от $x^{\alpha} e^{-px} \left\{ \begin{matrix} \operatorname{si}(cx) \\ \operatorname{ci}(cx) \end{matrix} \right\}$ и $x^{\alpha} e^{-px} \operatorname{Si}(cx)$.

$$1. \int_0^{\infty} e^{-px} \left\{ \begin{matrix} \operatorname{si}(cx) \\ \operatorname{ci}(cx) \end{matrix} \right\} dx = -\frac{1}{p} \left\{ \begin{matrix} \operatorname{arctg}(p/c) \\ \ln \sqrt{1+p^2/c^2} \end{matrix} \right\} \quad [c, \operatorname{Re} p > 0]$$

$$2. \int_0^{\infty} e^{-px} \operatorname{Si}(cx) dx = \frac{1}{p} \operatorname{arctg} \frac{p}{c} \quad [c, \operatorname{Re} p > 0].$$

$$3. \int_0^{\infty} x^{\alpha-1} e^{-px} \left\{ \begin{matrix} \operatorname{si}(cx) \\ \operatorname{ci}(cx) \end{matrix} \right\} dx =$$

$$= \pm \frac{p \Gamma(\alpha+1)}{(\alpha+1) c^{\alpha+1}} \left\{ \begin{matrix} \cos(\alpha\pi/2) \\ \sin(\alpha\pi/2) \end{matrix} \right\} {}_3F_2 \left(\begin{matrix} \frac{\alpha+1}{2}, \frac{\alpha+1}{2}, \frac{\alpha+2}{2}; \frac{3}{2}, \frac{\alpha+3}{2}; -\frac{p^2}{c^2} \right) -$$

$$- \frac{\Gamma(\alpha)}{\alpha c^{\alpha}} \left\{ \begin{matrix} \sin(\alpha\pi/2) \\ \cos(\alpha\pi/2) \end{matrix} \right\} {}_3F_2 \left(\begin{matrix} \frac{\alpha}{2}, \frac{\alpha}{2}, \frac{\alpha+1}{2}; \frac{1}{2}, \frac{\alpha}{2}+1; -\frac{p^2}{c^2} \right)$$

$$[c, \operatorname{Re} \alpha, \operatorname{Re} p > 0].$$

$$4. \int_0^{\infty} x e^{-px} \operatorname{ci}(cx) dx = \frac{1}{p^2 + c^2} - \frac{1}{2p^2} \ln \left(1 + \frac{p^2}{c^2} \right) \quad [c, \operatorname{Re} p > 0].$$

$$5. \int_0^{\infty} e^{-px^2} \operatorname{ci}(cx) dx = \frac{1}{4} \sqrt{\frac{\pi}{p}} \operatorname{Ei} \left(-\frac{c^2}{4p} \right) \quad [c, \operatorname{Re} p > 0].$$

$$6. \int_0^{\infty} x^{\alpha-1} e^{-px^2} \left\{ \begin{matrix} \operatorname{si}(cx) \\ \operatorname{ci}(cx) \end{matrix} \right\} dx = -\frac{c^{\delta+2}}{4p^{(\alpha+\delta)/2+1} (\delta+2) (1+2\delta)} \times$$

$$\times \Gamma \left(\frac{\alpha+\delta}{2} + 1 \right) {}_3F_3 \left(\begin{matrix} 1, \delta/2+1, (\alpha+\delta)/2+1; \\ 2, \delta/2+2, 3/2+\delta \end{matrix} \middle| -c^2/(4p) \right) +$$

$$+ \frac{c^{\delta}}{4p^{(\alpha+\delta)/2}} \Gamma \left(\frac{\alpha+\delta}{2} \right) \left[2\delta + (1-\delta) \left(2C + \psi \left(\frac{\alpha}{2} \right) + \ln \frac{c^2}{p} \right) \right] - \frac{\delta \pi}{4p^{\alpha/2}} \Gamma \left(\frac{\alpha}{2} \right)$$

$$[c, \operatorname{Re} \alpha, \operatorname{Re} p > 0; \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}].$$

$$7. \int_0^{\infty} x e^{-p x^2} \operatorname{si}(cx) dx = -\frac{\pi}{4p} \operatorname{erfc}\left(\frac{c}{2\sqrt{p}}\right) \quad [\operatorname{Re} p > 0]$$

$$8. \int_0^{\infty} x^{-1/2} e^{-p\sqrt{x}} \operatorname{si}(cx) dx = -2\sqrt{\frac{2\pi}{c}} \left\{ \left[\frac{1}{2} - S\left(\frac{p^2}{4c}\right) \right]^2 + \left[\frac{1}{2} - C\left(\frac{p^2}{4c}\right) \right]^2 \right\} \\ [c, \operatorname{Re} p > 0].$$

$$9. \int_0^{\infty} \frac{1}{x^2} e^{-p/x} \begin{Bmatrix} \operatorname{si}(cx) \\ \operatorname{ci}(cx) \end{Bmatrix} dx = \pm \frac{2}{p} \begin{Bmatrix} \operatorname{kei}(2\sqrt{pc}) \\ \operatorname{ker}(2\sqrt{pc}) \end{Bmatrix} \quad [c, \operatorname{Re} p > 0].$$

2.6.4. Интегралы, содержащие степенную, тригонометрические функции, $\begin{Bmatrix} \operatorname{si}(cx) \\ \operatorname{ci}(cx) \end{Bmatrix}$ или $\operatorname{Si}(cx)$

Обозначение: $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$.

$$1. \int_0^{\infty} \begin{Bmatrix} \sin bx \operatorname{si}(cx) \\ \cos bx \operatorname{ci}(cx) \end{Bmatrix} dx = 0 \quad [0 < b < c].$$

$$2. \int_0^{\infty} \begin{Bmatrix} \sin bx \operatorname{si}(cx) \\ \cos bx \operatorname{ci}(cx) \end{Bmatrix} dx = -\frac{\pi}{2b} \quad [0 < c < b].$$

$$3. \int_0^{\infty} \begin{Bmatrix} \sin cx \operatorname{si}(cx) \\ \cos cx \operatorname{ci}(cx) \end{Bmatrix} dx = -\frac{\pi}{4c} \quad [c > 0].$$

$$4. \int_0^{\infty} \begin{Bmatrix} \cos bx \operatorname{si}(cx) \\ \sin bx \operatorname{ci}(cx) \end{Bmatrix} dx = -\frac{1}{2b} \left\{ \ln |(b+c)(b-c)^{-1}|, \right. \\ \left. \ln [1 - \min^2(b, c)/\max^2(b, c)] \right\} \\ [b, c > 0; b \neq c].$$

$$5. \int_0^{\infty} x^{\alpha-1} \sin bx \begin{Bmatrix} \operatorname{si}(cx) \\ \operatorname{ci}(cx) \end{Bmatrix} dx = U(1),$$

$$U(\gamma) = \frac{c^{\delta+2} \Gamma(2+\alpha+\delta)}{2(\delta+2)(1+2\delta)b^{2+\alpha+\delta}} \cos \frac{\alpha+\delta-\gamma}{2} \pi \times \\ \times {}_4F_3 \left(\begin{matrix} 1, 1+\delta/2, (\alpha+3)/2, \alpha/2+\delta+1 \\ 2, 2+\delta/2, 3/2+\delta \end{matrix}; c^2/b^2 \right) + \\ + \frac{c^{\delta}}{b^{\alpha+\delta}} \Gamma(\alpha+\delta) \cos \frac{\alpha+\delta-\gamma}{2} \pi \left[\delta + (1-\delta) \left(\mathbf{C} + \psi(\alpha) + \right. \right. \\ \left. \left. + \frac{\pi}{2} \operatorname{tg} \frac{\gamma-\alpha}{2} \pi + \ln \frac{c}{b} \right) \right] - \frac{\delta \pi \Gamma(\alpha)}{2b^{\alpha}} \cos \frac{\alpha-\gamma}{2} \pi \quad [0 < c < b; -1 < \operatorname{Re} \alpha < 2],$$

$$U(\gamma) = -\frac{b^{\gamma} \Gamma(\alpha+\gamma)}{c^{\alpha+\gamma} (\alpha+\gamma)} \begin{Bmatrix} \sin [(\alpha+\gamma) \pi/2] \\ \cos [(\alpha+\gamma) \pi/2] \end{Bmatrix} \times \\ \times {}_3F_2 \left(\begin{matrix} (\alpha+1)/2, (\alpha+\gamma)/2, \alpha/2+\gamma \\ (\alpha+\gamma)/2+1, \gamma+1/2 \end{matrix}; b^2/c^2 \right) \quad [0 < b < c; -1 < \operatorname{Re} \alpha < 2].$$

$$6. \int_0^{\infty} x^{\alpha-1} \cos bx \begin{Bmatrix} \text{si} (cx) \\ \text{ci} (cx) \end{Bmatrix} dx = U(0)$$

[$b, c > 0$; $b \neq c$; $0 < \text{Re } \alpha < 2$; $U(\gamma)$ см. в 2.6.4.5].

$$7. \int_0^{\infty} \frac{1}{x} \sin bx \text{Si} (cx) dx = \frac{1}{2} \left[\text{Li}_2 \left(\frac{c}{b} \right) - \text{Li}_2 \left(-\frac{c}{b} \right) \right] \quad [b, c > 0]$$

$$8. \int_0^{\infty} \frac{1}{x^2} \sin \frac{b}{x} \text{Si} (cx) dx = \frac{\pi}{2b} [1 - J_0(2\sqrt{bc})] \quad [b, c > 0]$$

$$9. \int_0^{\infty} x^{\alpha-1} \left[\begin{Bmatrix} \sin cx \\ \cos cx \end{Bmatrix} \text{ci} (cx) \mp \begin{Bmatrix} \cos cx \\ \sin cx \end{Bmatrix} \text{si} (cx) \right] dx = \pm \frac{\pi \Gamma(\alpha)}{2c^\alpha} \begin{Bmatrix} \sec(\alpha\pi/2) \\ \text{cosec}(\alpha\pi/2) \end{Bmatrix}$$

[$c > 0$, $0 < \text{Re } \alpha < 1$]

$$10. \int_{-\infty}^{\infty} \frac{1}{x-y} \left[\begin{Bmatrix} \cos cx \\ \sin cx \end{Bmatrix} \text{ci} (c|x|) \pm \text{sgn } x \begin{Bmatrix} \sin cx \\ \cos cx \end{Bmatrix} \text{si} (c|x|) \right] dx =$$

$$= -\pi \left[\text{sgn } y \begin{Bmatrix} \cos cy \\ \sin cy \end{Bmatrix} \text{si} (c|y|) \mp \begin{Bmatrix} \sin cy \\ \cos cy \end{Bmatrix} \text{ci} (c|y|) \right]$$

[$c > 0$; $\text{Im } y = 0$, $y \neq 0$]

$$11. \int_0^{\infty} \frac{1}{x^2+z^2} \begin{Bmatrix} \sin bx \text{si} (cx) \\ \cos bx \text{ci} (cx) \end{Bmatrix} dx = \frac{\pi}{2z} \begin{Bmatrix} \text{sh } bz \\ \text{ch } bz \end{Bmatrix} \text{Ei} (-cz)$$

[$0 < b \leq c$; $\text{Re } z > 0$]

$$12. = \pm \frac{\pi}{4z} e^{-bz} [\text{Ei} (bz) - \text{Ei} (-cz) - \text{Ei} (cz)] + \frac{\pi}{4z} e^{bz} \text{Ei} (-bz)$$

[$0 < c \leq b$; $\text{Re } z > 0$]

$$13. \int_0^{\infty} \frac{x}{x^2+z^2} \sin bx \text{ci} (cx) dx = -\frac{\pi}{2} \text{sh } bz \text{Ei} (-cz)$$

[$0 < b \leq c$; $\text{Re } z > 0$]

$$14. = \frac{\pi}{4} e^{-bz} [\text{Ei} (-bz) + \text{Ei} (bz) - \text{Ei} (-cz) - \text{Ei} (cz)] - \frac{\pi}{2} \text{sh } bz \text{Ei} (-bz)$$

[$0 < c \leq b$; $\text{Re } z > 0$]

$$15. \int_0^{\infty} \frac{1}{x^2+z^2} \begin{Bmatrix} \sin bx \\ x \cos bx \end{Bmatrix} \text{Si} (cx) dx =$$

$$= \frac{\pi}{4} \begin{Bmatrix} z^{-1} \\ 1 \end{Bmatrix} \{ e^{bz} [\text{Ei} (-cz) - \text{Ei} (-bz)] \pm e^{-bz} [\text{Ei} (bz) - \text{Ei} (-cz)] \}$$

[$0 < b \leq c$; $\text{Re } z > 0$]

$$16. = \pm \frac{\pi}{4} \begin{Bmatrix} z^{-1} \\ 1 \end{Bmatrix} e^{-bz} [\text{Ei} (cz) - \text{Ei} (-cz)] \quad [0 < c \leq b; \text{Re } z > 0]$$

$$17. \int_0^{\infty} \frac{\sin bx}{a - \cos x} \text{si} (cx) dx = -\frac{\pi(r+1)r}{2b(r-1)(r^2-1)} \begin{Bmatrix} r^n \\ 2-r^n \end{Bmatrix}$$

[$b = \sigma \mp n$; $r = a + \sqrt{a^2 - 1}$; $a > 1$]

$$18. = -\frac{\pi r}{b(r-1)(r^2-1)} \left\{ 1 + r - r^{n+1} \right\} \left[\begin{matrix} c-n-1 < b < c-n \\ c+n < b < c+n+1 \end{matrix} \right], \quad r \text{ см. в } 2.6.17.2].$$

$$19. \int_a^\infty (x^2 - a^2)^{\beta-1} \left[\begin{matrix} \cos cx \\ \sin cx \end{matrix} \right] \text{ci}(cx) \pm \begin{matrix} \sin cx \\ \cos cx \end{matrix} \text{si}(cx) \Big] dx =$$

$$= -a^{\beta-1/2} c^{1/2-\beta} \Gamma(\beta) \left\{ \begin{matrix} \pi^{-1/2} \Gamma(3/2-\beta) S_{\beta-3/2, \beta-1/2}(ac) \\ 2^{\beta-5/2} \pi^{3/2} \operatorname{cosec} \beta \pi [Y_{\beta-1/2}(ac) - H_{\beta-1/2}(ac)] \end{matrix} \right\}$$

$$[a, c > 0; \quad 0 < \operatorname{Re} \beta < (5 \pm 1)/4].$$

$$20. \int_a^\infty x(x^2 - a^2)^{\beta-1} \left[\begin{matrix} \cos cx \\ \sin cx \end{matrix} \right] \text{ci}(cx) \pm \begin{matrix} \sin cx \\ \cos cx \end{matrix} \text{si}(cx) \Big] dx =$$

$$= \mp \frac{a^{\beta+1/2} c^{1/2-\beta}}{2\sqrt{\pi}} \left\{ \begin{matrix} 2\pi \operatorname{cosec} \beta \pi S_{\beta-3/2, \beta+1/2}(ac) \\ \Gamma(\beta) \Gamma(1/2-\beta) S_{\beta-1/2, \beta+1/2}(ac) \end{matrix} \right\}$$

$$[a, c > 0, \quad 0 < \operatorname{Re} \beta < (3 \pm 1)/4].$$

2.6.5. Интегралы, содержащие показательную, тригонометрические функции, $\begin{Bmatrix} \text{si}(cx) \\ \text{ci}(cx) \end{Bmatrix}$ или $\text{Si}(cx)$.

$$1. \int_0^\infty e^{-px} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} \text{ci}(cx) dx =$$

$$= \frac{1}{2(b^2 + p^2)} \left[\pm \begin{Bmatrix} p \\ b \end{Bmatrix} \operatorname{arctg} \frac{2bp}{p^2 + c^2 - b^2} - \frac{1}{2} \begin{Bmatrix} b \\ p \end{Bmatrix} \ln \frac{(p^2 + c^2 - b^2)^2 + 4b^2 p^2}{c^4} \right]$$

$$[c > b > 0; \quad \operatorname{Re} p > 0].$$

$$2. \int_0^\infty e^{-px} \cos bx \text{si}(cx) dx =$$

$$= -\frac{1}{2(b^2 + p^2)} \left[\frac{b}{2} \ln \frac{p^2 + (b+c)^2}{p^2 + (b-c)^2} + p \operatorname{arctg} \frac{2pc}{c^2 - b^2 - p^2} \right] =$$

$$= -\frac{1}{2(p+ib)} \operatorname{arctg} \frac{p+ib}{c} - \frac{1}{2(p-ib)} \operatorname{arctg} \frac{p-ib}{c} \quad [b, c, \operatorname{Re} p > 0].$$

$$3. \int_0^\infty e^{-px} \sin cx \text{si}(cx) dx = \frac{1}{2(p^2 + c^2)} \left(c \operatorname{arctg} \frac{2c}{p} + p \ln \frac{\sqrt{p^2 + 4c^2}}{p} \right)$$

$$[c, \operatorname{Re} p > 0]$$

$$4. \int_0^\infty e^{-px} \left[\pm \begin{Bmatrix} \sin cx \\ \cos cx \end{Bmatrix} \text{ci}(cx) + \begin{Bmatrix} \cos cx \\ \sin cx \end{Bmatrix} \text{si}(cx) \right] dx =$$

$$= \frac{1}{p^2 + c^2} \left(\begin{Bmatrix} p \\ c \end{Bmatrix} \operatorname{arctg} \frac{2c}{p} \mp \begin{Bmatrix} c \\ p \end{Bmatrix} \ln \frac{\sqrt{p^2 + 4c^2}}{c} \right)$$

$$[c, \operatorname{Re} p > 0]$$

$$5. \int_0^\infty e^{-px} \left[\begin{Bmatrix} \sin cx \\ \cos cx \end{Bmatrix} \text{ci}(cx) \mp \begin{Bmatrix} \cos cx \\ \sin cx \end{Bmatrix} \text{si}(cx) \right] dx = \frac{1}{p^2 + c^2} \begin{Bmatrix} c \\ p \end{Bmatrix} \ln \frac{c}{p}$$

$$[c, \operatorname{Re} p > 0]$$

$$\begin{aligned}
6. \int_0^{\infty} x^{\alpha-1} e^{-px} \left[\begin{matrix} \sin cx \\ \cos cx \end{matrix} \right] \text{ci}(cx) \mp \begin{matrix} \cos cx \\ \sin cx \end{matrix} \text{si}(cx) \Big] dx = \\
= \frac{\pi p \Gamma(\alpha+1)}{2c^{\alpha+1}} \left\{ \frac{\text{cosec}(\alpha\pi/2)}{\sec(\alpha\pi/2)} \right\} {}_2F_1 \left(\frac{\alpha+1}{2}, \frac{\alpha}{2}+1; \frac{3}{2}; -\frac{p^2}{c^2} \right) \pm \\
\pm \frac{\pi \Gamma(\alpha)}{2c^{\alpha}} \left\{ \frac{\sec(\alpha\pi/2)}{\text{cosec}(\alpha\pi/2)} \right\} {}_2F_1 \left(\frac{\alpha}{2}, \frac{\alpha+1}{2}; \frac{1}{2}; -\frac{p^2}{c^2} \right) \pm \\
\pm c^{\delta-2} p^{\delta-\alpha-\delta} \Gamma(\alpha+\delta-2) {}_3F_2 \left(1, 1, \frac{3}{2}-\delta; \frac{3-\alpha}{2}, 2-\delta-\frac{\alpha}{2}; -\frac{p^2}{c^2} \right) \\
\left[\alpha, \text{Re } \alpha, \text{Re } p > 0, \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right].
\end{aligned}$$

$$\begin{aligned}
7. \int_0^{\infty} x^{-1/2} e^{-px} \left[\begin{matrix} \sin(cx/2) \\ \cos(cx/2) \end{matrix} \right] \text{ci}(cx) \mp \begin{matrix} \cos(cx/2) \\ \sin(cx/2) \end{matrix} \text{Si}(cx) \Big] dx = \\
= - \sqrt{\frac{\pi(\sqrt{4p^2+c^2} \mp 2p)}{(4p^2+c^2)}} \ln \frac{\sqrt{4p^2+c^2} \mp 2p}{c} \quad [c, \text{Re } p > 0].
\end{aligned}$$

$$\begin{aligned}
8. \int_0^{\infty} e^{-px^2} [\cos cx \text{ci}(cx) + \sin cx \text{si}(cx)] dx = \\
= \sqrt{\frac{\pi}{8p}} \exp\left(-\frac{c^2}{4p}\right) \left[\pi i \operatorname{erf}\left(\frac{ic}{2\sqrt{p}}\right) + \operatorname{Ei}\left(\frac{c^2}{4p}\right) \right] \\
[\text{Re } p > 0; |\arg c| < \pi].
\end{aligned}$$

$$\begin{aligned}
9. \int_0^{\infty} e^{-px^2} [\cos cx \text{ci}(cx) + \sin cx \text{Si}(cx)] dx = \frac{1}{4} \sqrt{\frac{\pi}{p}} \exp\left(-\frac{c^2}{4p}\right) \operatorname{Ei}\left(\frac{c^2}{4p}\right) \\
[\text{Re } p > 0, |\arg c| < \pi].
\end{aligned}$$

$$\begin{aligned}
10. \int_0^{\infty} e^{-px^2} \left\{ \cos \frac{b \pm c}{2} x [\text{ci}(bx) + \text{ci}(cx)] + \sin \frac{b \pm c}{2} x [\text{Si}(bx) \pm \text{Si}(cx)] \right\} dx = \\
= \frac{1}{2} \sqrt{\frac{\pi}{p}} \exp\left(-\frac{(b \pm c)^2}{16p}\right) \operatorname{Ei}\left(\pm \frac{bc}{4p}\right) \quad [\text{Re } p > 0, |\arg c| < \pi].
\end{aligned}$$

$$\begin{aligned}
11. \int_0^{\infty} x^{-3/2} e^{-px-b/x} \left[\begin{matrix} \sin(cx/2) \\ \cos(cx/2) \end{matrix} \right] \text{ci}(cx) \mp \begin{matrix} \cos(cx/2) \\ \sin(cx/2) \end{matrix} \text{Si}(cx) \Big] dx = \\
= \sqrt{\frac{\pi}{b}} \left\{ e^{-z_+} \left[\begin{matrix} \sin z_- \\ \cos z_- \end{matrix} \right] \text{ci}(2z_+) \mp \begin{matrix} \cos z_- \\ \sin z_- \end{matrix} \text{Si}(2z_+) \right] \mp e^{z_+} \begin{matrix} \sin z_- \\ \cos z_- \end{matrix} \operatorname{Ei}(-2z_+) \right\} \\
[z_{\pm} = \sqrt{b/2} (\sqrt{4p^2+c^2} \pm 2p); c, \text{Re } b > 0; \text{Re } p \geq 0].
\end{aligned}$$

2.6.6. Интегралы от $x^{\alpha} \ln^n x \begin{Bmatrix} \text{si}(cx) \\ \text{ci}(cx) \end{Bmatrix}$.

$$1. \int_0^{\infty} x^{\alpha-1} \ln^n x \begin{Bmatrix} \text{si}(cx) \\ \text{ci}(cx) \end{Bmatrix} dx = I_n \quad [c > 0; 0 < \text{Re } \alpha < 1].$$

$$I_n = -\frac{d^n}{d\alpha^n} \left[\frac{\Gamma(\alpha)}{\alpha c^{\alpha}} \begin{Bmatrix} \sin(\alpha\pi/2) \\ \cos(\alpha\pi/2) \end{Bmatrix} \right],$$

$$I_1 = \frac{\Gamma(\alpha)}{\alpha c^{\alpha}} \begin{Bmatrix} \sin(\alpha\pi/2) \\ \cos(\alpha\pi/2) \end{Bmatrix} \left[\frac{1}{\alpha} + \ln c - \psi(\alpha) \mp \frac{\pi}{2} \begin{Bmatrix} \operatorname{ctg}(\alpha\pi/2) \\ \operatorname{tg}(\alpha\pi/2) \end{Bmatrix} \right].$$

2.6.7. Интегралы, содержащие $\begin{Bmatrix} \text{si}(bx) & \text{si}(cx) \\ \text{ci}(bx) & \text{ci}(cx) \end{Bmatrix}$ или $\text{si}(bx) \text{ci}(cx)$.

Обозначение: $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$.

$$1. \int_0^{\infty} \text{si}(bx) \text{ci}(cx) dx = \frac{1}{2c} \ln \left| \frac{b+c}{b-c} \right| + \frac{1}{2b} \ln \left| \frac{b^2-c^2}{c^2} \right| \quad [b, c > 0; b \neq c].$$

$$2. \quad = \frac{1}{c} \ln 2 \quad [b = c > 0].$$

$$3. \int_0^{\infty} \begin{Bmatrix} \text{si}(bx) & \text{si}(cx) \\ \text{ci}(bx) & \text{ci}(cx) \end{Bmatrix} dx = \frac{\pi}{2b} \quad [b \geq c > 0].$$

$$4. \int_0^{\infty} x^{\alpha-1} \text{si}(bx) \begin{Bmatrix} \text{si}(cx) \\ \text{ci}(cx) \end{Bmatrix} dx = U(1) \quad [0 < c < b; 0 < \text{Re } \alpha < 2].$$

$$U(\gamma) = \frac{c^{2+\delta} \Gamma(\alpha + \delta + 2)}{2b^{\alpha+\delta+2} (\delta + 2) (2\delta + 1) (\alpha + \delta + 2)} \begin{Bmatrix} \sin [(\gamma - \alpha) \pi/2] \\ \cos [(\gamma - \alpha) \pi/2] \end{Bmatrix} \times \\ \times {}_3F_4 \left(\begin{matrix} 1, \delta/2 + 1, (\alpha + \delta)/2 + 1, \alpha/2 + \delta + 1, (\alpha + 3)/2; \\ 2, \delta/2 + 2, (\alpha + \delta)/2 + 2, \delta + 3/2 \end{matrix}; \frac{c^2}{b^2} \right) + \\ + \frac{c^{\delta} \Gamma(\alpha + \delta)}{(\alpha + \delta) b^{\alpha+\delta}} \begin{Bmatrix} \sin [(\gamma - \alpha) \pi/2] \\ \cos [(\gamma - \alpha) \pi/2] \end{Bmatrix} \left[\delta + (1 - \delta) \left(C + \psi(\alpha) - \right. \right. \\ \left. \left. - \frac{1}{\alpha} + \frac{\pi}{2} \text{tg} \frac{\gamma - \alpha}{2} \pi + \ln \frac{c}{b} \right) \right] - \frac{\delta \pi \Gamma(\alpha)}{2\alpha b^{\alpha}} \cos \frac{\gamma - \alpha}{2} \pi.$$

$$5. \int_0^{\infty} x^{\alpha-1} \text{ci}(bx) \begin{Bmatrix} \text{si}(cx) \\ \text{ci}(cx) \end{Bmatrix} dx = U(0) \quad [0 < c < b; 0 < \text{Re } \alpha < 2; U(\gamma) \text{ см. в 2.6.7.4}].$$

$$6. \int_0^{\infty} x^{\alpha-1} [\text{si}^2(cx) + \text{ci}^2(cx)] dx = \frac{\pi \Gamma(\alpha)}{\alpha c^{\alpha}} \text{cosec} \frac{\alpha \pi}{2} \quad [0 < \text{Re } \alpha < 2].$$

$$7. \int_0^{\infty} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} [\text{si}^2(cx) + \text{ci}^2(cx)] dx = \\ = \begin{Bmatrix} -2 \ln(b/c) \ln(1 - b^2 c^{-2}) - \text{Li}_2(b^2 c^{-2}) \\ \pi b^{-1} \ln(1 + bc^{-1}) \end{Bmatrix} \quad [b, c > 0].$$

$$8. \int_0^{\infty} x^{\alpha-1} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} [\text{si}^2(cx) + \text{ci}^2(cx)] dx = \\ = -\frac{b^{2-\alpha}}{c^2} \Gamma(\alpha - 2) \begin{Bmatrix} \sin(\alpha \pi/2) \\ \cos(\alpha \pi/2) \end{Bmatrix} {}_4F_3 \left(\begin{matrix} 1, 1, 1, 3/2; \\ 2, 2 - \alpha/2, (3 - \alpha)/2 \end{matrix}; \frac{b^2}{c^2} \right) + \\ + \frac{\pi b^{\delta} \Gamma(\alpha + \delta)}{(\alpha + \delta) c^{\alpha+\delta}} \begin{Bmatrix} \sec(\alpha \pi/2) \\ \text{cosec}(\alpha \pi/2) \end{Bmatrix} {}_3F_2 \left(\begin{matrix} \frac{\alpha + \delta}{2}, \frac{\alpha + 1}{2}, \frac{\alpha}{2} + \delta; \\ \delta + \frac{1}{2}, \frac{\alpha + \delta}{2} + 1; \end{matrix}; \frac{b^2}{c^2} \right) \quad [b, c > 0; -\delta < \text{Re } \alpha < 2].$$

$$9. \int_0^{\infty} [\cos cx \text{ci}(cx) + \sin cx \text{si}(cx)]^2 dx = \frac{\pi}{2c} \quad [c > 0].$$

$$10. \int_0^{\infty} \left[\begin{Bmatrix} \cos bx \\ \sin bx \end{Bmatrix} \text{ci}(bx) \pm \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} \text{si}(bx) \right] \left[\begin{Bmatrix} \cos cx \\ \sin cx \end{Bmatrix} \text{ci}(cx) \pm \begin{Bmatrix} \sin cx \\ \cos cx \end{Bmatrix} \text{si}(cx) \right] dx = \frac{\pi}{2(b-c)} \ln \frac{b}{c} \quad [b, c > 0; b \neq c].$$

2.6.8. Интегралы от $x^\alpha \text{Ei}(-bx^n) \begin{Bmatrix} \text{si}(cx) \\ \text{ci}(cx) \end{Bmatrix}$.

Обозначение: $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$.

$$1. \int_0^{\infty} \text{Ei}(-bx) \text{si}(cx) dx = \frac{1}{b} \text{arctg} \frac{b}{c} + \frac{1}{2c} \ln \left(1 + \frac{c^2}{b^2} \right) \quad [c, \text{Re } b > 0]$$

$$2. \int_0^{\infty} \text{Ei}(-cx) \text{ci}(cx) dx = \frac{1}{4c} (\pi + 2 \ln 2) \quad [c > 0].$$

$$3. \int_0^{\infty} x^{\alpha-1} \text{Ei}(-bx) \begin{Bmatrix} \text{si}(cx) \\ \text{ci}(cx) \end{Bmatrix} dx = \frac{c^{\alpha+\delta} \Gamma(\alpha+\delta+2)}{2(\delta+2)(1+2\delta)(\alpha+\delta+2)b^{\alpha+\delta+2}} \times \\ \times {}_4F_4 \left(\begin{matrix} 1, \delta/2+1, (\alpha+\delta)/2+1, (\alpha+3)/2, \alpha/2+\delta+1 \\ 2, 3/2+\delta, \delta/2+2, (\alpha+\delta)/2+2 \end{matrix}; -c^2/b^2 \right) - \\ - \frac{c^\delta \Gamma(\alpha+\delta)}{(\alpha+\delta)b^{\alpha+\delta}} \left[\delta + (1-\delta) \left(\text{C} + \psi(\alpha) - \frac{1}{\alpha} + \ln \frac{c}{b} \right) \right] + \frac{\delta \pi \Gamma(\alpha)}{2\alpha b^\alpha} \quad [c, \text{Re } b, \text{Re } \alpha > 0].$$

$$4. \int_0^{\infty} x^{\alpha-1} \text{Ei}(-bx^2) \begin{Bmatrix} \text{si}(cx) \\ \text{ci}(cx) \end{Bmatrix} dx = \frac{c^{\alpha+\delta}}{2(\delta+2)(1+2\delta)(\alpha+\delta+2)b^{(\alpha+\delta)/2+1}} \times \\ \times \Gamma\left(\frac{\alpha+\delta}{2}+1\right) {}_4F_4 \left(\begin{matrix} 1, \delta/2+1, (\alpha+\delta)/2+1, (\alpha+\delta)/2+1 \\ 2, 3/2+\delta, \delta/2+2, (\alpha+\delta)/2+2 \end{matrix}; -c^2/(4b) \right) - \\ - \frac{c^\delta}{(\alpha+\delta)b^{(\alpha+\delta)/2}} \Gamma\left(\frac{\alpha+\delta}{2}\right) \left[\delta + (1-\delta) \left(\text{C} + \frac{1}{2} \psi\left(\frac{\alpha}{2}\right) - \frac{1}{\alpha} + \ln \frac{c}{b^{1/2}} \right) \right] + \\ + \frac{\delta \pi}{2\alpha b^{\alpha/2}} \Gamma\left(\frac{\alpha}{2}\right) \quad [\text{Re } b, \text{Re } \alpha > 0].$$

2.7. ИНТЕГРАЛЬНЫЕ ГИПЕРБОЛИЧЕСКИЕ СИНУС $\text{shi}(x)$ И КОСИНУС $\text{chi}(x)$

2.7.1. Интегралы общего вида.

Обозначение: $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$.

$$1. \int_0^a x^{\alpha-1} (a^r - x^r)^{\beta-1} \begin{Bmatrix} \text{shi}(cx) \\ \text{chi}(cx) \end{Bmatrix} dx = \\ = \frac{a^{\alpha+r\beta+\delta-r} c^\delta}{2r} \Gamma(\beta) \left\{ \sum_{k=1}^{\infty} \frac{(a^2 c^2 / 4)^k}{(1/2+\delta)_k (\delta/2+k) k!} \Gamma\left[\frac{(\alpha+\delta+2k)/r}{(\alpha+\delta+2k)/r+\beta}\right] + \right. \\ \left. + 2\Gamma\left[\frac{(\alpha+\delta)/r}{(\alpha+\delta)/r+\beta}\right] \left[\delta + (1-\delta) \left(\ln(ac) + \frac{1}{r} \psi\left(\frac{\alpha}{r}\right) - \frac{1}{r} \psi\left(\frac{\alpha}{r}+\beta\right) + \text{C} \right) \right] \right\} \quad [a, r, \text{Re } \alpha, \text{Re } \beta > 0]$$

$$2. \int_0^{\infty} x^{\alpha-1} e^{-px^r} \begin{Bmatrix} \text{shi}(cx) \\ \text{chi}(cx) \end{Bmatrix} dx =$$

$$= \frac{c^{\delta}}{2r p^{(\alpha+\delta)/r}} \left\{ \sum_{k=1}^{\infty} \frac{1}{(k+\delta/2)(1/2+\delta)_k k!} \Gamma\left(\frac{\alpha+\delta+2k}{r}\right) \left(\frac{c}{2p^{1/r}}\right)^{2k} + \right.$$

$$\left. + 2\Gamma\left(\frac{\alpha+\delta}{r}\right) \left[\delta + (1-\delta) \left(\ln c - \frac{1}{r} \ln p + \frac{1}{r} \psi\left(\frac{\alpha}{r}\right) + C \right) \right] \right\}$$

[$r > 1$; $\text{Re } p, \text{Re } \alpha > 0$] или
[$r = 1$; $|p| > |c|$; $\text{Re } p > |\text{Re } c|$; $\text{Re } \alpha > 0$].

$$3. \int_0^{\infty} x^{\alpha-1} e^{-px^r} \sin bx^r \begin{Bmatrix} \text{shi}(cx) \\ \text{chi}(cx) \end{Bmatrix} dx = U(1)$$

[$r > 1$; $\text{Re } p > |\text{Im } b|$; $\text{Re } \alpha > -r$] или [$r = 1$; $|c|^2 < |b^2 + p^2|$;
 $\text{Re } p > |\text{Re } c| + |\text{Im } b|$; $\text{Re } \alpha > -1$].

$$U(\gamma) = \frac{c^{\delta}}{2r (b^2 + p^2)^{(\alpha+\delta)/(2r)}} \left\{ \sum_{k=1}^{\infty} \frac{1}{(k+\delta/2)(1/2+\delta)_k k!} \times \right.$$

$$\times \Gamma\left(\frac{\alpha+\delta+2k}{r}\right) \cos\left(\frac{\gamma\pi}{2} - \frac{\alpha+\delta+2k}{r} \arccos \frac{p}{\sqrt{b^2+p^2}}\right) \frac{c^{2k}}{2^{2k} (b^2 + p^2)^{k/r}} +$$

$$+ 2\Gamma\left(\frac{\alpha+\delta}{r}\right) \cos\left(\frac{\gamma\pi}{2} - \frac{\alpha+\delta}{r} \arccos \frac{p}{\sqrt{b^2+p^2}}\right) \left[\delta + (1-\delta) \left(\frac{1}{r} \psi\left(\frac{\alpha}{r}\right) + \right. \right.$$

$$\left. \left. + \frac{1}{2r} \ln \frac{c^{2r}}{b^2 + p^2} + \frac{1}{r} \arccos \frac{p}{\sqrt{b^2+p^2}} \text{tg}\left(\frac{\gamma\pi}{2} - \frac{\alpha}{r} \arccos \frac{p}{\sqrt{b^2+p^2}}\right) + C \right) \right] \right\}.$$

$$4. \int_0^{\infty} x^{\alpha-1} e^{-px^r} \cos bx^r \begin{Bmatrix} \text{shi}(cx) \\ \text{chi}(cx) \end{Bmatrix} dx = U(0)$$

[$r > 1$; $\text{Re } p > |\text{Im } b|$; $\text{Re } \alpha > 0$] или [$r = 1$; $|c|^2 < |b^2 + p^2|$;
 $\text{Re } p > |\text{Re } c| + |\text{Im } b|$; $\text{Re } \alpha > 0$; $U(\gamma)$ см. в. 2.7.1.3].

2.7.2. Интегралы от $A(x) \begin{Bmatrix} \text{shi}(cx) \\ \text{chi}(cx) \end{Bmatrix}$.

$$1. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\beta-1} \begin{Bmatrix} \text{shi}(cx) \\ \text{chi}(cx) \end{Bmatrix} dx = \frac{a^{\alpha+2\beta+\delta} c^{2+\delta}}{4(\delta+2)(1+2\delta)} \times$$

$$\times B\left(\beta, \frac{\alpha+\delta}{2} + 1\right) {}_3F_4\left(1, 1+\delta/2, (\alpha+\delta)/2+1; 2, 3/2+\delta, 2+\delta/2, (\alpha+\delta)/2+\beta+1\right) +$$

$$+ \frac{a^{\alpha+2\beta+\delta-2} c^{\delta}}{2} B\left(\frac{\alpha+\delta}{2}, \beta\right) \left[\delta + (1-\delta) \left(\ln(ac) + \right. \right.$$

$$\left. \left. + \frac{1}{2} \psi\left(\frac{\alpha}{2}\right) - \frac{1}{2} \psi\left(\frac{\alpha}{2} + \beta\right) + C \right) \right] \quad \left[a, \text{Re } \alpha, \text{Re } \beta > 0; \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right].$$

2.7.3. Интегралы от $x^{\alpha} e^{-px^m} \begin{Bmatrix} \text{shi}(cx) \\ \text{chi}(cx) \end{Bmatrix}$.

Обозначение: $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$.

$$1. \int_0^{\infty} e^{-px} \begin{Bmatrix} \text{shi}(cx) \\ \text{chi}(cx) \end{Bmatrix} dx = \frac{1}{2p} \begin{Bmatrix} \ln[(p+c)/(p-c)] \\ \ln[c^2/(p^2 - c^2)] \end{Bmatrix} \quad [\text{Re } p > |\text{Re } c|; c \neq 0].$$

$$2. \int_0^{\infty} x^{\alpha-1} e^{-px} \left\{ \begin{matrix} \text{shi} (cx) \\ \text{chi} (cx) \end{matrix} \right\} dx = \frac{c^{\alpha+\delta} \Gamma(\alpha+\delta+2)}{2(\delta+2)(1+2\delta)p^{\alpha+\delta+2}} \times \\ \times {}_4F_3 \left(\begin{matrix} 1, \delta/2+1, (\alpha+3)/2, \alpha/2+\delta+1; c^2/p^2 \end{matrix} \right) + \\ + \frac{c^{\delta} \Gamma(\alpha+\delta)}{p^{\alpha+\delta}} \left[\delta + (1-\delta) \left(\ln \frac{c}{p} + \psi(\alpha) + \mathcal{C} \right) \right] \quad [\text{Re } \alpha > 0; \text{Re } p > |\text{Re } c|].$$

$$3. \int_0^{\infty} x e^{-px} \text{chi} (cx) dx = \frac{1}{p^2 - c^2} - \frac{1}{2p^2} \ln \left(\frac{p^2}{c^2} - 1 \right) \quad [\text{Re } p > |\text{Re } c|].$$

$$4. \int_0^{\infty} e^{-px^2} \text{chi} (cx) dx = \frac{1}{4} \sqrt{\frac{\pi}{p}} \left[\text{Ei} \left(\frac{c^2}{4p} \right) - \pi i \right] \quad [\text{Re } p > 0; |\arg c| < \pi]$$

$$5. \int_0^{\infty} x^{\alpha-1} e^{-px^2} \left\{ \begin{matrix} \text{shi} (cx) \\ \text{chi} (cx) \end{matrix} \right\} dx = \frac{c^{\delta+2}}{4(\delta+2)(1+2\delta)p^{(\alpha+\delta)/2+1}} \times \\ \times \Gamma \left(\frac{\alpha+\delta}{2} + 1 \right) {}_3F_3 \left(\begin{matrix} 1, \delta/2+1, (\alpha+\delta)/2+1; c^2/(4p) \end{matrix} \right) + \\ + \frac{c^{\delta}}{2p^{(\alpha+\delta)/2}} \Gamma \left(\frac{\alpha+\delta}{2} \right) \left[\delta + (1-\delta) \left(\mathcal{C} + \frac{1}{2} \psi \left(\frac{\alpha}{2} \right) + \ln \frac{c}{\sqrt{p}} \right) \right] \\ [\text{Re } \alpha, \text{Re } p > 0, |\arg c| < \pi].$$

2.7.4. Интегралы, содержащие e^{-px^m} , гиперболические функции и $\left\{ \begin{matrix} \text{shi} (cx) \\ \text{chi} (cx) \end{matrix} \right\}$.

$$1. \int_0^{\infty} e^{-px} \left[\left\{ \begin{matrix} \text{sh } cx \\ \text{ch } cx \end{matrix} \right\} \text{chi} (cx) \mp \left\{ \begin{matrix} \text{ch } cx \\ \text{sh } cx \end{matrix} \right\} \text{shi} (cx) \right] dx = \frac{1}{p^2 - c^2} \left\{ \frac{c}{p} \right\} \ln \frac{c}{p} \\ [\text{Re } p > 2|\text{Re } c|]$$

$$2. \int_0^{\infty} x^{(1 \mp 1)/2} e^{-px^2} \left[\left\{ \begin{matrix} \text{sh } cx \\ \text{ch } cx \end{matrix} \right\} \text{chi} (cx) \right] \mp \left\{ \begin{matrix} \text{ch } cx \\ \text{sh } cx \end{matrix} \right\} \text{shi} (cx) dx = \\ = \mp \frac{1}{4} \sqrt{\frac{\pi}{p}} \left\{ \frac{1}{c/p} \right\} \exp \left(\frac{c^2}{4p} \right) \text{Ei} \left(-\frac{c^2}{4p} \right) \quad [\text{Re } p > 0; |\arg c| < \pi]$$

2.7.5. Интегралы, содержащие e^{-px} , тригонометрические функции и $\left\{ \begin{matrix} \text{shi} (cx) \\ \text{chi} (cx) \end{matrix} \right\}$.

$$1. \int_0^{\infty} e^{-px} \left\{ \begin{matrix} \sin bx \\ \cos bx \end{matrix} \right\} \text{chi} (cx) dx = \frac{1}{2(b^2 + p^2)} \left[\pm \left\{ \frac{p}{b} \right\} \arctg \frac{2bp}{p^2 - b^2 - c^2} - \pi i \left\{ \frac{b}{p} \right\} - \right. \\ \left. - \frac{1}{2} \left\{ \frac{b}{p} \right\} \ln \frac{(p^2 - b^2 - c^2)^2 + 4b^2 p^2}{c^4} \right] \quad [b > 0; \text{Re } p > |\text{Re } c|, c \neq 0]$$

$$2. \int_0^{\infty} e^{-px} \cos bx \text{shi} (cx) dx = \\ = \frac{i}{2(p+ib)} \arctg \frac{b-ip}{c} - \frac{i}{2(p-ib)} \arctg \frac{b+ip}{c} - \frac{\pi ip}{2(p^2 + b^2)} \\ [b > 0; \text{Re } p > |\text{Re } c|, c \neq 0]$$

2.8. ИНТЕГРАЛЫ ВЕРОЯТНОСТИ $\operatorname{erf}(x)$ И $\operatorname{erfc}(x)$.

Некоторые интегралы, содержащие функции $\operatorname{erf}(x)$, $\operatorname{erfc}(x)$, можно получить также из формул раздела 2.10, поскольку

$$\left\{ \begin{matrix} \gamma(1/2, x^2) \\ \Gamma(1/2, x^2) \end{matrix} \right\} = \sqrt{\pi} \left\{ \begin{matrix} \operatorname{erf}(x) \\ \operatorname{erfc}(x) \end{matrix} \right\}.$$

2.8.1. Интегралы общего вида.

$$\begin{aligned} 1. \int_0^a x^{\alpha-1} (a^r - x^r)^{\beta-1} \left\{ \begin{matrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{matrix} \right\} dx = \\ = \pm \frac{a^{\alpha+r\beta+1-r} c \Gamma(\beta)}{r \sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-a^2 c^2)^k}{k! (k+1/2)} \Gamma \left[\frac{(2k+\alpha+1)/r}{(2k+\alpha+1)/r + \beta} \right] + \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{a^{\alpha+r\beta-r}}{r} B \left(\frac{r}{\alpha}, \beta \right) \\ [a, r, \operatorname{Re} \beta > 0; \operatorname{Re} \alpha > -(1 \pm 1)/2]. \end{aligned}$$

$$\begin{aligned} 2. \int_a^{\infty} x^{\alpha-1} (x^r - a^r)^{\beta-1} \left\{ \begin{matrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{matrix} \right\} dx = \\ = \pm \frac{a^{\alpha+r\beta+1-r} c \Gamma(\beta)}{r \sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-a^2 c^2)^k}{k! (k+1/2)} \Gamma \left[\frac{1-\beta-(2k+\alpha+1)/r}{1-(2k+\alpha+1)/r} \right] \pm \\ \pm \frac{c^{r-r\beta-\alpha}}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(1-\beta)_k (ac)^{rk}}{k! (rk+r-r\beta-\alpha)} \Gamma \left(\frac{1+\alpha-r+r\beta-rk}{2} \right) + \\ + \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{a^{\alpha+r\beta-r}}{r} B \left(1-\beta-\frac{\alpha}{r}, \beta \right) \\ [r, \operatorname{Re} \beta > 0; |\arg c| < \pi/4, \left\{ \begin{matrix} a > 0 \\ a > 0 \end{matrix} \right\}; \operatorname{Re}(\alpha+r\beta) < r]. \end{aligned}$$

$$\begin{aligned} 3. \int_0^{\infty} \frac{x^{\alpha-1}}{(x^r + z^r)^{\rho}} \left\{ \begin{matrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{matrix} \right\} dx = \\ = \pm \frac{c z^{\alpha-r\rho+1}}{r \sqrt{\pi} \Gamma(\rho)} \sum_{k=0}^{\infty} \frac{(-c^2 z^2)^k}{k! (k+1/2)} \Gamma \left(\frac{2k+\alpha+1}{r} \right) \Gamma \left(\rho - \frac{2k+\alpha+1}{r} \right) \pm \\ \pm \frac{c^{r\rho-\alpha}}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(\rho)_k (-c^r z^r)^k}{k! (rk+r\rho-\alpha)} \Gamma \left(\frac{1+\alpha-r\rho-rk}{2} \right) + \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{z^{\alpha-r\rho}}{r} B \left(\frac{\alpha}{r}, \rho - \frac{\alpha}{r} \right) \\ [r > 0; r |\arg z| < \pi; |\arg c| < \pi/4, \left\{ \begin{matrix} -1 < \operatorname{Re} \alpha < r \operatorname{Re} \rho \\ \operatorname{Re} \alpha > 0 \end{matrix} \right\}]. \end{aligned}$$

$$\begin{aligned} 4. \int_0^{\infty} \frac{x^{\alpha-1}}{x^r - y^r} \left\{ \begin{matrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{matrix} \right\} dx = \mp \frac{\sqrt{\pi} c y^{\alpha-r+1}}{r} \sum_{k=0}^{\infty} \frac{(-c^2 y^2)^k}{k! (k+1/2)} \operatorname{ctg} \frac{rk+\alpha+1}{r} \pi \pm \\ \pm \frac{c^{r-\alpha}}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(cy)^{rk}}{rk+r-\alpha} \Gamma \left(\frac{1+\alpha-r-rk}{2} \right) - \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{\pi y^{\alpha-r}}{r} \operatorname{ctg} \frac{\alpha\pi}{r} \\ [r, y > 0; |\arg c| < \pi/4, \left\{ \begin{matrix} -1 < \operatorname{Re} \alpha < r \\ \operatorname{Re} \alpha > 0 \end{matrix} \right\}]. \end{aligned}$$

$$5. \int_0^{\infty} x^{\alpha-1} e^{-px^r} \left\{ \begin{matrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{matrix} \right\} dx = U(r)$$

$$[r, \operatorname{Re} p > 0; |\arg c| < \pi/4; \operatorname{Re} \alpha > -(1 \pm 1)/2].$$

$$U(r) = \pm \frac{c}{\sqrt{\pi} p^{(\alpha+1)/r}} \sum_{k=0}^{\infty} \frac{(-1)^k}{(k+1/2) k!} \Gamma \left(\frac{2k+\alpha+1}{r} \right) \left(\frac{c}{p^{1/r}} \right)^{2k} + \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{1}{p^{\alpha/r}} \Gamma \left(\frac{\alpha}{r} \right) \\ [r > 2].$$

$$U(r) = \mp \frac{1}{c^\alpha \sqrt{\pi}} \sum_{k=0}^{\infty} \frac{1}{k! (rk + \alpha)} \Gamma\left(\frac{1 + \alpha + rk}{2}\right) \left(-\frac{p}{c^r}\right)^k + \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{1}{p^{\alpha/r} r} \Gamma\left(\frac{\alpha}{r}\right) \quad [r < 2].$$

$U(2)$ см. в 2.8.5.6.

$$\begin{aligned} 6. \int_0^{\infty} x^{\alpha-1} e^{-px-r} \left\{ \begin{matrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{matrix} \right\} dx = \\ = \pm \frac{cp^{(\alpha+1)/r}}{r \sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (k+1/2)} \Gamma\left(-\frac{2k+\alpha+1}{r}\right) (cp^{1/r})^{2k} \pm \\ \pm \frac{1}{c^\alpha \sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-c^r p)^k}{k! (rk - \alpha)} \Gamma\left(\frac{1 + \alpha - rk}{2}\right) + \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{p^{\alpha/r}}{r} \Gamma\left(-\frac{\alpha}{r}\right) \\ [r > 0; |\arg c| < \pi/4, \left\{ \begin{matrix} \operatorname{Re} p > 0; \operatorname{Re} \alpha < 0 \\ \operatorname{Re} p > 0 \end{matrix} \right\}]. \end{aligned}$$

$$\begin{aligned} 7. \int_0^a x^{\alpha-1} (a^r - x^r)^{\beta-1} e^{c^2 x^2} \left\{ \begin{matrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{matrix} \right\} dx = \\ = \pm \frac{2a^{1+\alpha+r\beta-r} c}{r \sqrt{\pi}} \Gamma(\beta) \sum_{k=0}^{\infty} \frac{(ac)^{2k}}{(3/2)_k} \Gamma\left[\frac{(2k+\alpha+1)/r}{(2k+\alpha+1)/r + \beta}\right] + \\ + \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{a^{\alpha+r\beta-r}}{r} \Gamma(\beta) \sum_{k=0}^{\infty} \frac{(ac)^{2k}}{k!} \Gamma\left[\frac{(2k+\alpha)/r}{(2k+\alpha)/r + \beta}\right] \\ [a, r, \operatorname{Re} \beta > 0; \operatorname{Re} \alpha > -(1 \pm 1)/2]. \end{aligned}$$

$$\begin{aligned} 8. \int_a^{\infty} x^{\alpha-1} (a^r - x^r)^{\beta-1} e^{\mp c^2 x^2} \left\{ \begin{matrix} \operatorname{erf}(icx) \\ \operatorname{erfc}(cx) \end{matrix} \right\} dx = \\ = \left\{ \begin{matrix} i \\ -1 \end{matrix} \right\} \frac{2a^{1+\alpha+r\beta-r} c}{r \sqrt{\pi}} \Gamma(\beta) \sum_{k=0}^{\infty} \frac{(\mp a^2 c^2)^k}{(3/2)_k} \Gamma\left[\frac{1 - \beta - (2k + \alpha + 1)/r}{1 - (2k + \alpha + 1)/r}\right] + \\ + \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{a^{\alpha+r\beta-r}}{r} \Gamma(\beta) \sum_{k=0}^{\infty} \frac{(ac)^{2k}}{k!} \Gamma\left[\frac{1 - \beta - (2k + \alpha)/r}{1 - (2k + \alpha)/r}\right] - \left\{ \begin{matrix} i \\ -1 \end{matrix} \right\} \frac{c^{r-\beta-\alpha}}{2} \times \\ \times \sum_{k=0}^{\infty} \frac{(1-\beta)_k}{k!} \frac{\sin^{(1 \pm 1)/2} [(r - \alpha - r\beta + rk) \pi/2]}{\cos [(r - \alpha - r\beta + rk) \pi/2]} \Gamma\left(\frac{\alpha - r + r\beta - rk}{2}\right) (ac)^{rk} \\ [a, r, \operatorname{Re} \beta > 0; \operatorname{Re}(\alpha + r\beta) < r + 1; |\arg c| < \pi/4]. \end{aligned}$$

$$\begin{aligned} 9. \int_0^{\infty} \frac{x^{\alpha-1}}{(x^r + z^r)^\rho} e^{\mp c^2 x^2} \left\{ \begin{matrix} \operatorname{erf}(icx) \\ \operatorname{erfc}(cx) \end{matrix} \right\} dx = \\ = \left\{ \begin{matrix} i \\ -1 \end{matrix} \right\} \frac{2cz^{\alpha-r\rho+1}}{r \sqrt{\pi} \Gamma(\rho)} \sum_{k=0}^{\infty} \frac{(\mp c^2 z^2)^k}{(3/2)_k} \Gamma\left(\frac{2k + \alpha + 1}{r}\right) \Gamma\left(\rho - \frac{2k + \alpha + 1}{r}\right) + \\ + \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{z^{\alpha-r\rho}}{r \Gamma(\rho)} \sum_{k=0}^{\infty} \frac{(cz)^{2k}}{k!} \Gamma\left(\frac{2k + \alpha}{r}\right) \Gamma\left(\rho - \frac{2k + \alpha}{r}\right) \mp \\ \mp \left\{ \begin{matrix} i \\ 1 \end{matrix} \right\} \frac{c^{r\rho-\alpha}}{2} \sum_{k=0}^{\infty} \frac{(\rho)_k}{k!} \frac{\sin^{(1 \pm 1)/2} [(r\rho + rk - \alpha) \pi/2]}{\cos [(r\rho + rk - \alpha) \pi/2]} \Gamma\left(\frac{\alpha - r\rho - rk}{2}\right) (-c^r z^r)^k \\ [r > 0; -(1 \pm 1)/2 < \operatorname{Re} \alpha < r \operatorname{Re} \rho + 1; |\arg c| < \pi/4; r |\arg z| < \pi]. \end{aligned}$$

$$\begin{aligned}
10. \int_0^{\infty} \frac{x^{\alpha-1}}{x^r - y^r} e^{\mp c^2 x^2} \begin{Bmatrix} \operatorname{erf}(icx) \\ \operatorname{erfc}(cx) \end{Bmatrix} dx = \\
= \left\{ \begin{matrix} -1 \\ 1 \end{matrix} \right\} \frac{2}{r} c \sqrt{\pi} y^{\alpha-r+1} \sum_{k=0}^{\infty} \frac{(\mp c^2 y^2)^k}{(3/2)_k} \operatorname{ctg} \frac{2k+\alpha+1}{r} \pi - \\
- \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{\pi y^{\alpha-r}}{r} \sum_{k=0}^{\infty} \frac{(cy)^{2k}}{k!} \operatorname{ctg} \frac{\alpha+2k}{r} \pi \mp \left\{ \begin{matrix} i \\ 1 \end{matrix} \right\} \frac{c^{-\alpha}}{2} \times \\
\times \sum_{k=0}^{\infty} \frac{\sin^{(1 \pm 1)/2} [(r-\alpha+rk)\pi/2]}{\cos [(r-\alpha+rk)\pi/2]} \Gamma\left(\frac{\alpha-r-rk}{2}\right) (cy)^{rk} \\
[r, y > 0; -(1 \pm 1)/2 < \operatorname{Re} \alpha < r+1; |\arg c| < \pi/4].
\end{aligned}$$

$$\begin{aligned}
11. \int_0^{\infty} x^{\alpha-1} e^{-px^r \mp c^2 x^2} \begin{Bmatrix} \operatorname{erf}(icx) \\ \operatorname{erfc}(cx) \end{Bmatrix} dx = U(r) \\
[r, \operatorname{Re} p > 0; |\arg c| < \pi/4; \operatorname{Re} \alpha > -(1 \pm 1)/2].
\end{aligned}$$

$$\begin{aligned}
U(r) = \left\{ \begin{matrix} -1 \\ 1 \end{matrix} \right\} \frac{2c}{r \sqrt{\pi} p^{(\alpha+1)/r}} \sum_{k=0}^{\infty} \frac{1}{(3/2)_k} \Gamma\left(\frac{\alpha+1+2k}{r}\right) \left(\mp \frac{c^2}{p^{2/r}}\right)^k + \\
+ \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{1}{p^{\alpha/r}} \sum_{k=0}^{\infty} \frac{1}{k!} \Gamma\left(\frac{2k+\alpha}{r}\right) \left(\frac{c}{p^{1/r}}\right)^{2k} \quad [r > 2],
\end{aligned}$$

$$\begin{aligned}
U(r) = \left\{ \begin{matrix} i \\ 1 \end{matrix} \right\} \frac{p^{(1-\alpha)/r}}{cr \sqrt{\pi}} \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)_k \operatorname{F}\left(\frac{\alpha-1-2k}{r}\right) \left(\pm \frac{p^{2/r}}{c^2}\right)^k + \\
+ \left\{ \begin{matrix} i \\ 1 \end{matrix} \right\} \frac{1}{2c^{\alpha}} \sum_{k=0}^{\infty} \frac{1}{k!} \Gamma\left(\frac{\alpha+rk}{2}\right) \frac{\sin^{(1 \pm 1)/2} [(\alpha+rk)\pi/2]}{\cos [(\alpha+rk)\pi/2]} \left(-\frac{p}{c^r}\right)^k \quad [r < 2].
\end{aligned}$$

$U(2)$ см. в 2.8.5.6.

$$\begin{aligned}
12. \int_0^{\infty} x^{\alpha-1} e^{-px^{-r} \mp c^2 x^2} \begin{Bmatrix} \operatorname{erf}(icx) \\ \operatorname{erfc}(cx) \end{Bmatrix} dx = \\
= \left\{ \begin{matrix} -1 \\ 1 \end{matrix} \right\} \frac{2c}{r \sqrt{\pi}} p^{(\alpha+1)/r} \sum_{k=0}^{\infty} \frac{1}{(3/2)_k} \Gamma\left(-\frac{2k+\alpha+1}{r}\right) (\mp c^2 p^{2/r})^k + \\
+ \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{p^{\alpha/r}}{r} \sum_{k=0}^{\infty} \frac{1}{k!} \Gamma\left(-\frac{2k+\alpha}{r}\right) (cp^{1/r})^{2k} + \\
+ \left\{ \begin{matrix} i \\ 1 \end{matrix} \right\} \frac{1}{2c^{\alpha}} \sum_{k=0}^{\infty} \frac{1}{k!} \Gamma\left(\frac{\alpha-rk}{2}\right) \frac{\sin^{(1 \pm 1)/2} [(\alpha-rk)\pi/2]}{\cos [(\alpha-rk)\pi/2]} (-c^r p)^k \\
[r, \operatorname{Re} p > 0; |\arg c| < \pi/4; \operatorname{Re} \alpha < 1].
\end{aligned}$$

$$\begin{aligned}
13. \int_0^{\infty} x^{\alpha-1} \begin{Bmatrix} \sin bx^r \\ \cos bx^r \end{Bmatrix} \operatorname{erf}(cx) dx = U(1) \\
[b, r > 0; |\arg c| < \pi/4; -(r \pm r)/2 - 1 < \operatorname{Re} \alpha < r].
\end{aligned}$$

$$\begin{aligned}
U(r) = (-1)^{r-1} \frac{c}{r \sqrt{\pi} b^{(\alpha+1)/r}} \sum_{k=0}^{\infty} \frac{1}{k! (k+1/2)} \Gamma\left(\frac{2k+\alpha+1}{r}\right) \times \\
\times \begin{Bmatrix} \sin [(2k+\alpha+1)\pi/(2r)] \\ \cos [(2k+\alpha+1)\pi/(2r)] \end{Bmatrix} \left(-\frac{c^2}{b^{2/r}}\right)^k + \frac{1-r}{b^{\alpha/r}} \Gamma\left(\frac{\alpha}{r}\right) \begin{Bmatrix} \sin [\alpha\pi/(2r)] \\ \cos [\alpha\pi/(2r)] \end{Bmatrix} \\
[r > 2].
\end{aligned}$$

$$U(\gamma) = (-1)^\gamma \frac{b^\delta}{c^{\alpha+r\delta} \sqrt{\pi}} \sum_{k=0}^{\infty} \frac{1}{k! (\delta + 1/2)_k (2rk + \alpha + r\delta)} \times \\ \times \Gamma\left(\frac{1 + \alpha + r\delta}{2} + rk\right) \left(-\frac{b^2}{4c^{2r}}\right)^k + \frac{\gamma}{b^{\alpha/r} r} \Gamma\left(\frac{\alpha}{r}\right) \begin{cases} \sin [\alpha\pi/(2r)] \\ \cos [\alpha\pi/(2r)] \end{cases} \\ \left[r < 2; \delta = \begin{cases} 1 \\ 0 \end{cases}\right],$$

$U(\gamma)$ при $r=2$ см. в 2.8.11.18.

$$14. \int_0^\infty x^{\alpha-1} \begin{cases} \sin bx^r \\ \cos bx^r \end{cases} \operatorname{erfc}(cx) dx = U(0) \\ [b, r > 0; |\arg c| < \pi/4; \operatorname{Re} \alpha > -(r \pm r)/2; U(\gamma) \text{ см. в 2.8.1.13}].$$

$$15. \int_0^\infty x^{\alpha-1} e^{-px^r} \begin{cases} \sin bx^r \\ \cos bx^r \end{cases} \operatorname{erf}(cx) dx = V(1) \\ [b, r, \operatorname{Re} p > 0; |\arg c| < \pi/4; \operatorname{Re} \alpha > -(r \pm r)/2 - 1].$$

$$V(\gamma) = (-1)^{\gamma-1} \frac{c}{r \sqrt{\pi} (\sqrt{b^2 + p^2})^{(\alpha+1)/r}} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (k + 1/2)} \times \\ \times \Gamma\left(\frac{2k + \alpha + 1}{r}\right) \cos\left(\frac{1 \pm 1}{4} \pi - \frac{2k + \alpha + 1}{r} \arccos \frac{p}{\sqrt{b^2 + p^2}}\right) \frac{c^{2k}}{(b^2 + p^2)^{k/r}} + \\ + (1 - \gamma) \frac{\Gamma(\alpha/r)}{r (\sqrt{b^2 + p^2})^{\alpha/r}} \cos\left(\frac{1 \pm 1}{4} \pi - \frac{\alpha}{r} \arccos \frac{p}{\sqrt{b^2 + p^2}}\right) \\ [r > 2] \text{ или } [r = 2, |c|^4 < |b^2 + p^2|].$$

$$V(\gamma) = \frac{(-1)^\gamma}{c^\alpha \sqrt{\pi}} \sum_{k=0}^{\infty} \frac{1}{k! (rk + \alpha)} \Gamma\left(\frac{rk + \alpha + 1}{2}\right) \times \\ \times \cos\left(\frac{1 \pm 1}{4} \pi + k \arccos \frac{p}{\sqrt{b^2 + p^2}}\right) \left(-\frac{\sqrt{b^2 + p^2}}{c}\right)^k + \\ + \frac{\gamma \Gamma(\alpha/r)}{r (\sqrt{b^2 + p^2})^{\alpha/r}} \cos\left(\frac{1 \pm 1}{4} \pi - \frac{\alpha}{r} \arccos \frac{p}{\sqrt{b^2 + p^2}}\right) \\ [r < 2] \text{ или } [r = 2, |c|^4 > |b^2 + p^2|].$$

$$16. \int_0^\infty x^{\alpha-1} e^{-px^r} \begin{cases} \sin bx^r \\ \cos bx^r \end{cases} \operatorname{erfc}(cx) dx = V(0) \\ [b, r, \operatorname{Re} p > 0, |\arg c| < \pi/4; \operatorname{Re} \alpha > -(r \pm r)/2; V(\gamma) \text{ см. в 2.8.1.15}].$$

$$17. \int_0^\infty x^{\alpha-1} e^{-c^2 x^2} \begin{cases} \sin bx^r \\ \cos bx^r \end{cases} \operatorname{erf}(l cx) dx = W(1) \\ [b, r > 0; -1 - \delta r < \operatorname{Re} \alpha < r + 1; |\arg c| < \pi/4; \delta = \begin{cases} 1 \\ 0 \end{cases}],$$

$$W(\gamma) = -(-i)^\gamma \frac{2c}{r \sqrt{\pi} b^{(\alpha+1)/r}} \sum_{k=0}^{\infty} \frac{(-1)^{\gamma k}}{(3/2)_k} \Gamma\left(\frac{2k + \alpha + 1}{r}\right) \times \\ \times \begin{cases} \sin [(2k + \alpha + 1) \pi/(2r)] \\ \cos [(2k + \alpha + 1) \pi/(2r)] \end{cases} \left(\frac{c}{b^{1/r}}\right)^{2k} + \\ + \frac{1 - \gamma}{b^{\alpha/r} r} \sum_{k=0}^{\infty} \frac{1}{k!} \Gamma\left(\frac{2k + \alpha}{r}\right) \begin{cases} \sin [(2k + \alpha) \pi/(2r)] \\ \cos [(2k + \alpha) \pi/(2r)] \end{cases} \left(\frac{c}{b^{1/r}}\right)^{2k} \\ [r > 2].$$

$$W(\gamma) = \frac{i^\gamma b^{(1-\alpha)/r}}{c^r \sqrt{\pi}} \sum_{k=0}^{\infty} (-1)^{(\gamma+1)k} \left(\frac{1}{2}\right)_k \Gamma\left(\frac{\alpha-2k-1}{r}\right) \times$$

$$\times \left\{ \frac{\sin [(\alpha-2k-1)\pi/(2r)]}{\cos [(\alpha-2k-1)\pi/(2r)]} \left(\frac{b^{1/r}}{c}\right)^{2k} + \frac{i^\gamma b^\delta}{2c^{\alpha+r\delta}} \right\} \times$$

$$\times \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (1/2+\delta)_k} \Gamma\left(\frac{\alpha+\delta r+2rk}{2}\right) \frac{\sin^\gamma [(\alpha+\delta r+2rk)\pi/2]}{\cos [(\alpha+\delta r+2rk)\pi/2]} \left(\frac{b}{2c^r}\right)^{2k}$$

[r < 2].

$W(\gamma)$ при $r=2$ см. в 2.8.14.13—14.

$$18. \int_0^\infty x^{\alpha-1} e^{c^2 x^2} \left\{ \frac{\sin bx^r}{\cos bx^r} \right\} \operatorname{erfc}(cx) dx = W(0)$$

$$\left[b, r > 0; -\delta r < \operatorname{Re} \alpha < r+1; |\arg c| < \pi/4; W(\gamma) \text{ см в 2.8.1.17; } \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right].$$

$$19. \int_0^\infty x^{\alpha-1} e^{-c^2 x^2 - px^r} \left\{ \frac{\sin bx^r}{\cos bx^r} \right\} \operatorname{erf}(icx) dx = X(1)$$

$$\left[b, r, \operatorname{Re} p > 0; \operatorname{Re} \alpha > -1 - r\delta; |\arg c| < \pi/4, \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right],$$

$$X(\gamma) = -\frac{2(-i)^\gamma c}{\sqrt{\pi} r (\sqrt{b^2+p^2})^{(\alpha+1)/r}} \sum_{k=0}^{\infty} \frac{(-1)^{\gamma k}}{(3/2)_k} \Gamma\left(\frac{2k+\alpha+1}{r}\right) \times$$

$$\times \cos\left(\frac{\delta\pi}{2} - \frac{2k+\alpha+1}{r} \arccos \frac{p}{\sqrt{b^2+p^2}}\right) \frac{c^{2k}}{(b^2+p^2)^{k/r}} +$$

$$+ \frac{1-\gamma}{r (\sqrt{b^2+p^2})^{\alpha/r}} \sum_{k=0}^{\infty} \frac{1}{k!} \Gamma\left(\frac{2k+\alpha}{r}\right) \cos\left(\frac{\delta\pi}{2} - \frac{2k+\alpha}{r} \arccos \frac{p}{\sqrt{b^2+p^2}}\right) \frac{c^{2k}}{(b^2+p^2)^{k/r}}$$

[r > 2] или [r = 2; |c|^2 < |b^2 + p^2|].

$$X(\gamma) = \frac{i^\gamma}{2c^\alpha} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \Gamma\left(\frac{\alpha+rk}{2}\right) \cos\left(\frac{\delta\pi}{2} + k \arccos \frac{p}{\sqrt{b^2+p^2}}\right) \times$$

$$\times \frac{\sin^\gamma [(\alpha+rk)\pi/2]}{\cos [(\alpha+rk)\pi/2]} \left(\frac{\sqrt{b^2+p^2}}{c}\right)^k + \frac{i^\gamma}{\sqrt{\pi} c^r (\sqrt{b^2+p^2})^{(\alpha-1)/r}} \sum_{k=0}^{\infty} (-1)^{k(\gamma+1)} \left(\frac{1}{2}\right)_k \times$$

$$\times \Gamma\left(\frac{\alpha-2k-1}{r}\right) \cos\left(\frac{\delta\pi}{2} + \frac{1-\alpha+2k}{r} \arccos \frac{p}{\sqrt{b^2+p^2}}\right) \frac{(b^2+p^2)^{k/r}}{c^{2k}}$$

[r < 2] или [r = 2; |c|^2 > |b^2 + p^2|].

$$20. \int_0^\infty x^{\alpha-1} e^{c^2 x^2 - px^r} \left\{ \frac{\sin bx^r}{\cos bx^r} \right\} \operatorname{erfc}(cx) dx = X(0)$$

$$\left[b, r, \operatorname{Re} p > 0; \operatorname{Re} \alpha > -r\delta; |\arg c| < \pi/4; X(\gamma) \text{ см в 2.8.1.19; } \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right].$$

$$21. \int_0^\infty x^{\alpha-1} \left\{ \frac{\ln(x^r+z^r)}{\ln|x^r-z^r|} \right\} \operatorname{erf}(cx) dx = Y(1)$$

$$\left[r > 0; -1 < \operatorname{Re} \alpha < 0; |\arg c| < \pi/4, \left\{ \begin{array}{l} r |\arg z| < \pi \\ z > 0 \end{array} \right\} \right],$$

$$Y(\gamma) = -(-1)^\gamma \sqrt{\pi} c z^{\alpha+1} \times$$

$$\times \sum_{k=0}^{\infty} \frac{(-c^2 z^2)^k}{(k+1/2)(2k+\alpha+1)k!} \left\{ \operatorname{cosec} [(2k+\alpha+1)\pi/r] \right\} +$$

$$+ \frac{(-1)^\gamma}{\sqrt{\pi} c^\alpha} \sum_{k=1}^{\infty} \frac{(\mp c^r z^r)^k}{k(rk-\alpha)} \Gamma\left(\frac{1+\alpha-rk}{2}\right) -$$

$$- \frac{(-1)^\gamma r}{2\alpha \sqrt{\pi} c^\alpha} \Gamma\left(\frac{\alpha+1}{2}\right) \left[\frac{2}{\alpha} - \psi\left(\frac{\alpha+1}{2}\right) + 2 \ln c \right] + (1-\gamma) \frac{\pi z^\alpha}{\alpha} \left\{ \operatorname{cosec} (\alpha\pi/r) \right\}.$$

$$22. \int_0^\infty x^{\alpha-1} \left\{ \frac{\ln(x^r + z^r)}{\ln |x^r + z^r|} \right\} \operatorname{erfc}(cx) dx = Y(0)$$

$$26. \int_0^{\infty} x^{\alpha-1} \left\{ \begin{array}{l} \operatorname{erf}(bx^r) \\ \operatorname{erfc}(bx^r) \end{array} \right\} \operatorname{erfc}(cx) dx = U(0)$$

$[r > 0; |\arg b|, |\arg c| < \pi/4; \operatorname{Re} \alpha > -(r \pm r)/2; U(\gamma) \text{ см. в 2.8.1.25}]$

$$27. \int_0^{\infty} x^{\alpha-1} e^{-c^2 x^2} \left\{ \begin{array}{l} \operatorname{erf}(bx^r) \\ \operatorname{erfc}(bx^r) \end{array} \right\} \operatorname{erf}(icx) dx = V(1)$$

$\left[r > 0; |\arg b|, |\arg c| < \pi/4, \left\{ \begin{array}{l} -1-r < \operatorname{Re} \alpha < 1 \\ \operatorname{Re} \alpha > -1 \end{array} \right\} \right],$

$$V(\gamma) = \pm \frac{2(-i)^{\gamma} c}{\pi b^{(\alpha+1)/r}} \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma}}{(3/2)_k (2k+\alpha+1)} \Gamma\left(\frac{2k+\alpha+r+1}{2r}\right) \left(\frac{c}{b^{1/r}}\right)^{2k} \mp$$

$$\mp \frac{1-\gamma}{\sqrt{\pi} b^{\alpha/r}} \sum_{k=0}^{\infty} \frac{1}{k! (2k+\alpha)} \Gamma\left(\frac{2k+\alpha+r}{2r}\right) \left(\frac{c}{b^{1/r}}\right)^{2k} +$$

$$+ \left\{ \begin{array}{l} 1 \\ 0 \end{array} \right\} \frac{i^{\gamma} \sin^{\gamma}(\alpha\pi/2)}{2c^{\alpha} \cos(\alpha\pi/2)} \Gamma\left(\frac{\alpha}{2}\right) \quad [r > 1]$$

$$V(\gamma) = \pm \frac{i^{\gamma}}{\pi c b^{(\alpha-1)/r}} \sum_{k=0}^{\infty} \frac{(-1)^{k(\gamma+1)} (1/2)_k}{2k-\alpha+1} \Gamma\left(\frac{r+\alpha-1-2k}{2r}\right) \left(\frac{b^{1/r}}{c}\right)^{2k} \pm$$

$$\pm \frac{i^{\gamma} b}{\sqrt{\pi} c^{\alpha+r}} \sum_{k=0}^{\infty} \frac{\Gamma(rk+(\alpha+r)/2)}{k! (2k+1)} \frac{\sin^{\gamma}[(2rk+\alpha+r)\pi/2]}{\cos[(2rk+\alpha+r)\pi/2]} \left(-\frac{b^2}{c^{2r}}\right)^k +$$

$$+ \left\{ \begin{array}{l} 0 \\ 1 \end{array} \right\} \frac{i^{\gamma} \sin^{\gamma}(\alpha\pi/2)}{2c^{\alpha} \cos(\alpha\pi/2)} \Gamma\left(\frac{\alpha}{2}\right) \quad [r < 1]$$

$V(\gamma)$ при $r=1$ см. в 2.8.19.5, 2.8.20.12, 2.8.21.14—15.

$$28. \int_0^{\infty} x^{\alpha-1} e^{c^2 x^2} \left\{ \begin{array}{l} \operatorname{erf}(bx^r) \\ \operatorname{erfc}(bx^r) \end{array} \right\} \operatorname{erfc}(cx) dx = V(0)$$

$[r > 0; |\arg b|, |\arg c| < \pi/4, \left\{ \begin{array}{l} -r < \operatorname{Re} \alpha < 1 \\ \operatorname{Re} \alpha > 0 \end{array} \right\}, V(\gamma) \text{ см. в 2.8.1.27}]$

$$29. \int_0^{\infty} x^{\alpha-1} e^{-c^2 x^2 + b^2 x^{2r}} \left\{ \begin{array}{l} \operatorname{erf}(ibx^r) \\ \operatorname{erfc}(ibx^r) \end{array} \right\} \operatorname{erf}(icx) dx = W(1)$$

$[r > 0; |\arg b|, |\arg c| < \pi/4, \left\{ \begin{array}{l} |\operatorname{Re} \alpha| < 1+r \\ -1 < \operatorname{Re} \alpha < 1+r \end{array} \right\}]$

$$W(\gamma) = - \left\{ \begin{array}{l} i \\ 1 \end{array} \right\} \frac{(-i)^{\gamma} c}{\sqrt{\pi} b^{(\alpha+1)/r}} \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma}}{(3/2)_k} \Gamma\left(\frac{2k+\alpha+1}{2r}\right) \times$$

$$\times \left\{ \begin{array}{l} \operatorname{tg}[(2k+\alpha+1)\pi/(2r)] \\ \sec[(2k+\alpha+1)\pi/(2r)] \end{array} \right\} \left(\frac{c}{b^{1/r}}\right)^{2k} + \left\{ \begin{array}{l} i \\ 1 \end{array} \right\} \frac{1-\gamma}{2b^{\alpha/r}} \times$$

$$\times \sum_{k=0}^{\infty} \frac{1}{k!} \Gamma\left(\frac{2k+\alpha}{2r}\right) \left\{ \begin{array}{l} \operatorname{tg}[(\alpha+2k)\pi/(2r)] \\ \sec[(\alpha+2k)\pi/(2r)] \end{array} \right\} \left(\frac{c}{b^{1/r}}\right)^{2k} +$$

$$+ \left\{ \begin{array}{l} i \\ 1 \end{array} \right\} \frac{i^{\gamma} c^{\alpha-\alpha}}{2b \sqrt{\pi}} \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)_k \Gamma\left(\frac{\alpha-r}{2} - rk\right) \frac{\sin^{\gamma}[(\alpha-r-2rk)\pi/2]}{\cos[(\alpha-r-2rk)\pi/2]} \left(\mp \frac{c^{2r}}{b^2}\right)^k$$

$[r > 1]$

$$\begin{aligned}
W(\gamma) = & \left\{ \begin{matrix} i \\ 1 \end{matrix} \right\} \frac{i^\gamma b^{(1-\alpha)/r}}{2cr \sqrt{\pi}} \sum_{k=0}^{\infty} (-1)^{k(\gamma+1)} \left(\frac{1}{2} \right)_k \Gamma \left(\frac{\alpha-2k-1}{2r} \right) \times \\
& \times \left\{ \begin{matrix} \operatorname{tg} [(\alpha-2k-1) \pi/(2r)] \\ \sec [(\alpha-2k-1) \pi/(2r)] \end{matrix} \right\} \left(\frac{b^{1/r}}{c} \right)^{2k} - \left\{ \begin{matrix} -i \\ 1 \end{matrix} \right\} \frac{i^\gamma b}{\sqrt{\pi} c^{\alpha+r}} \times \\
& \times \sum_{k=0}^{\infty} \frac{1}{(3/2)_k} \Gamma \left(kr + \frac{\alpha+r}{2} \right) \frac{\sin^\gamma [(\alpha+r+2rk) \pi/2]}{\cos [(\alpha+r+2rk) \pi/2]} \left(\mp \frac{b^2}{c^{2r}} \right)^k + \\
& + \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{i^\gamma}{2c^\alpha} \sum_{k=0}^{\infty} \frac{1}{k!} \Gamma \left(rk + \frac{\alpha}{2} \right) \frac{\sin^\gamma [(\alpha+2rk) \pi/2]}{\cos [(\alpha+2rk) \pi/2]} \left(\frac{b}{c^r} \right)^{2k} \quad |r < 1,
\end{aligned}$$

$W(\gamma)$ при $r=1$ см в 2.8.19.7, 2.8.20.14.

$$\begin{aligned}
30. \quad & \int_0^\infty x^{\alpha-1} e^{c^2 x^2 \mp b^2 x^{2r}} \left\{ \begin{matrix} \operatorname{erf}(ibx^r) \\ \operatorname{erfc}(bx^r) \end{matrix} \right\} \operatorname{erfc}(cx) dx = W(0) \\
& |r > 0; \arg c = 0, |\arg c| < \pi/4; -(r \pm 1)/2 < \operatorname{Re} \alpha < r+1; W(\gamma) \text{ см в 2.8.1.29}
\end{aligned}$$

$$\begin{aligned}
31. \quad & \int_0^\infty x^{\alpha-1} \operatorname{Ei}(-bx^r) \left\{ \begin{matrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{matrix} \right\} dx = X(r) \\
& |r, \operatorname{Re} b > 0; |\arg c| < \pi/4; \operatorname{Re} \alpha > -(1 \pm 1)/2,
\end{aligned}$$

$$\begin{aligned}
X(r) = & \mp \frac{c}{\sqrt{\pi} b^{(\alpha+1)/r}} \sum_{k=0}^{\infty} \frac{1}{k! (k+1/2) (2k+\alpha+1)} \Gamma \left(\frac{2k+\alpha+1}{r} \right) \left(-\frac{c^2}{b^{2/r}} \right)^k - \\
& - \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{1}{\alpha b^{\alpha/r}} \Gamma \left(\frac{\alpha}{r} \right) \quad |r > 2,
\end{aligned}$$

$$\begin{aligned}
X(r) = & \mp \frac{1}{\sqrt{\pi} c^\alpha} \sum_{k=1}^{\infty} \frac{1}{k! k (\alpha+rk)} \Gamma \left(\frac{1+\alpha+rk}{2} \right) \left(-\frac{b}{c^r} \right)^k - \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{1}{\alpha b^{\alpha/r}} \Gamma \left(\frac{\alpha}{r} \right) \mp \\
& \mp \frac{1}{\alpha \sqrt{\pi} c^\alpha} \Gamma \left(\frac{\alpha+1}{2} \right) \left[c - \frac{r}{2} + \frac{r}{2} \psi \left(\frac{\alpha+1}{2} \right) - \ln \frac{c^r}{b} \right] \quad |r < 2,
\end{aligned}$$

$X(2)$ см в 2.8.25.3.

$$\begin{aligned}
32. \quad & \int_0^\infty x^{\alpha-1} e^{\mp c^2 x^2} \operatorname{Ei}(-bx^r) \left\{ \begin{matrix} \operatorname{erf}(icx) \\ \operatorname{erfc}(cx) \end{matrix} \right\} dx = Y(r) \\
& |r, \operatorname{Re} b > 0; |\arg c| < \pi/4; \operatorname{Re} \alpha > -(1 \pm 1)/2,
\end{aligned}$$

$$\begin{aligned}
Y(r) = & \mp \left\{ \begin{matrix} i \\ 1 \end{matrix} \right\} \frac{2c}{\sqrt{\pi} b^{(\alpha+1)/r}} \sum_{k=0}^{\infty} \frac{1}{(3/2)_k (2k+\alpha+1)} \Gamma \left(\frac{2k+\alpha+1}{r} \right) \left(\mp \frac{c^2}{b^{2/r}} \right)^k - \\
& - \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} b^{-\alpha/r} \sum_{k=0}^{\infty} \frac{1}{k! (2k+\alpha)} \Gamma \left(\frac{2k+\alpha}{r} \right) \left(\frac{c}{b^{1/r}} \right)^{2k} \quad |r > 2,
\end{aligned}$$

$$\begin{aligned}
Y(r) = & \left\{ \begin{matrix} i \\ 1 \end{matrix} \right\} \frac{b^{(1-\alpha)/r}}{c \sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(1/2)_k}{2k-\alpha+1} \Gamma \left(\frac{\alpha-2k-1}{r} \right) \left(\pm \frac{b^{2/r}}{c^2} \right)^k + \\
& + \left\{ \begin{matrix} i \\ 1 \end{matrix} \right\} \frac{1}{2c^\alpha} \sum_{k=1}^{\infty} \frac{1}{k! k} \Gamma \left(\frac{\alpha+rk}{2} \right) \left\{ \begin{matrix} \operatorname{tg} [(\alpha+rk) \pi/2] \\ \sec [(\alpha+rk) \pi/2] \end{matrix} \right\} \left(-\frac{b}{c^r} \right)^k + \\
& + \left\{ \begin{matrix} i \\ 1 \end{matrix} \right\} \frac{1}{4c^\alpha} \left\{ \begin{matrix} \operatorname{tg} (\alpha\pi/2) \\ \sec (\alpha\pi/2) \end{matrix} \right\} \Gamma \left(\frac{\alpha}{2} \right) \left[2c + r \psi \left(\frac{\alpha}{2} \right) + \frac{\pi \sin^{\mp 1} (\alpha\pi/2)}{\cos (\alpha\pi/2)} + 2 \ln \frac{b}{c^r} \right] \quad |r < 2,
\end{aligned}$$

$Y(2)$ см в 2.8.25.4.

$$33. \int_0^{\infty} x^{\alpha-1} \begin{Bmatrix} \text{si} (bx^r) \\ \text{ci} (bx^r) \end{Bmatrix} \text{erf} (cx) dx = Z(1)$$

[b, r > 0; -1 < Re α < 2r; |arg c| < π/4]

$$Z(\gamma) = \frac{(-1)^{\gamma} c}{\sqrt{\pi} b^{(\alpha+1)/r}} \sum_{k=0}^{\infty} \frac{1}{k! (k+1/2) (2k+\alpha+1)} \Gamma\left(\frac{2k+\alpha+1}{r}\right) \times$$

$$\times \begin{Bmatrix} \sin [(2k+\alpha+1) \pi/(2r)] \\ \cos [(2k+\alpha+1) \pi/(2r)] \end{Bmatrix} \left(-\frac{c^2}{b^{2/r}}\right)^k - \frac{1-\gamma}{\alpha b^{\alpha/r}} \Gamma\left(\frac{\alpha}{r}\right) \begin{Bmatrix} \sin [\alpha \pi/(2r)] \\ \cos [\alpha \pi/(2r)] \end{Bmatrix}$$

[r > 2]

$$Z(\gamma) = \frac{(-1)^{\gamma} b^{\delta}}{\sqrt{\pi} c^{\alpha+r\delta}} \sum_{k=1}^{\infty} \frac{\Gamma((\alpha+1+r\delta)/2+kr)}{k! (2rk+\alpha+\delta r) (2k+\delta) (1/2+\delta)_k} \left(-\frac{b^2}{4c^{2r}}\right)^k +$$

$$+ \frac{(-1)^{\gamma} b^{\delta}}{\sqrt{\pi} c^{\alpha+r\delta} (\alpha+\delta r)} \Gamma\left(\frac{\alpha+1+r\delta}{2}\right) \begin{Bmatrix} 1 \\ R \end{Bmatrix} - \frac{\gamma}{\alpha b^{\alpha/r}} \Gamma\left(\frac{\alpha}{r}\right) \begin{Bmatrix} \sin [\alpha \pi/(2r)] \\ \cos [\alpha \pi/(2r)] \end{Bmatrix} -$$

$$- \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{(-1)^{\gamma} \sqrt{\pi}}{2\alpha c^{\alpha}} \Gamma\left(\frac{\alpha+1}{2}\right) \quad \left[r < 2; R = \frac{r}{2} \psi\left(\frac{\alpha+1}{2}\right) + \gamma - \frac{r}{\alpha} - \ln \frac{c^r}{b}\right],$$

Z(γ) при r=2 см. в 2.8.26.6—7.

$$34. \int_0^{\infty} x^{\alpha-1} \begin{Bmatrix} \text{si} (bx^r) \\ \text{ci} (bx^r) \end{Bmatrix} \text{erfc} (cx) dx = Z(0)$$

[b, r, Re α > 0, |arg c| < π/4, Z(γ) см. в 2.8.1.33]

2.8.2. Интегралы от $x^{\alpha} (z \pm x)^{\beta} \begin{Bmatrix} \text{erf} (cx) \\ \text{erfc} (cx) \end{Bmatrix}$.

$$1. \int_0^{\infty} x^{\alpha-1} \begin{Bmatrix} \text{erf} (cx) \\ \text{erfc} (cx) \end{Bmatrix} dx = \mp \frac{1}{\alpha c^{\alpha} \sqrt{\pi}} \Gamma\left(\frac{\alpha+1}{2}\right)$$

[|arg c| < π/4, $\begin{Bmatrix} -1 < \text{Re } \alpha < 0 \\ \text{Re } \alpha > 0 \end{Bmatrix}$]

$$2. \int_a^{\infty} x^{\alpha-1} (x-a)^{\beta-1} \begin{Bmatrix} \text{erf} (cx) \\ \text{erfc} (cx) \end{Bmatrix} dx = \pm \frac{2a^{\alpha+\beta} c}{\sqrt{\pi}} B(-\alpha-\beta, \beta) \times$$

$$\times {}_3F_3\left(\frac{\alpha}{2}+1, \frac{\alpha+1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{\alpha+\beta}{2}+1, \frac{\alpha+\beta+1}{2}; -a^2 c^2\right) \pm$$

$$\pm \frac{c^{1-\alpha-\beta}}{\sqrt{\pi} (1-\alpha-\beta)} \Gamma\left(\frac{\alpha+\beta}{2}\right) {}_3F_3\left(\frac{1-\beta}{2}, 1-\frac{\beta}{2}, \frac{1-\alpha-\beta}{2}; \frac{3-\alpha-\beta}{2}, \frac{1}{2}, 1-\frac{\alpha+\beta}{2}; -a^2 c^2\right) \pm$$

$$\pm \frac{ac^{2-\alpha-\beta} (1-\beta)}{\sqrt{\pi} (2-\alpha-\beta)} \Gamma\left(\frac{\alpha+\beta-1}{2}\right) \times$$

$$\times {}_3F_3\left(1-\frac{\beta}{2}, \frac{3-\beta}{2}, 1-\frac{\alpha+\beta}{2}; \frac{3}{2}, 2-\frac{\alpha+\beta}{2}, \frac{3-\alpha-\beta}{2}; -a^2 c^2\right) +$$

$$+ \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} a^{\alpha+\beta-1} B(1-\alpha-\beta, \beta)$$

[Re β > 0; |arg c| < π/4, $\begin{Bmatrix} a > 0; \text{Re}(\alpha+\beta) < 1 \\ a > 0 \end{Bmatrix}$]

$$3. \int_0^a x^{\alpha-1} (a-x)^{\beta-1} \begin{Bmatrix} \text{erf} (cx) \\ \text{erfc} (cx) \end{Bmatrix} dx = \pm \frac{2a^{\alpha+\beta} c}{\sqrt{\pi}} B(\alpha+1, \beta) \times$$

$$\times {}_3F_3\left(\frac{\alpha+1}{2}, \frac{\alpha}{2}+1, \frac{1}{2}; \frac{\alpha+\beta+1}{2}, \frac{\alpha+\beta}{2}+1, \frac{3}{2}; -a^2 c^2\right) +$$

$$+ \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} a^{\alpha+\beta-1} B(\alpha, \beta)$$

[a, Re β > 0; Re α > -(1 ± 1)/2]

$$\begin{aligned}
4. \int_0^\infty \frac{x^{\alpha-1}}{(x+z)^\rho} \left\{ \begin{matrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{matrix} \right\} dx &= \pm \frac{2cz^{1+\alpha-\rho}}{\sqrt{\pi}} B(\alpha+1, \rho-\alpha-1) \times \\
&\times {}_3F_2\left(\frac{1}{2}, \frac{\alpha+1}{2}, \frac{\alpha}{2}+1, \frac{3+\alpha-\rho}{2}, 1+\frac{\alpha-\rho}{2}, \frac{3}{2}; -c^2z^2\right) \pm \\
&\pm \frac{c^{\rho-\alpha}}{\sqrt{\pi}(\rho-\alpha)} \Gamma\left(\frac{\alpha+1-\rho}{2}\right) {}_3F_2\left(\frac{\rho}{2}, \frac{\rho+1}{2}, \frac{\rho-\alpha}{2}; \frac{1}{2}, \frac{\rho-\alpha}{2}+1, \frac{1+\rho-\alpha}{2}; -c^2z^2\right) \mp \\
&\mp \frac{\rho c^{1+\rho-\alpha}}{\sqrt{\pi}(1+\rho-\alpha)} \Gamma\left(\frac{\alpha-\rho}{2}\right) {}_3F_2\left(\frac{\rho+1}{2}, \frac{\rho}{2}+1, \frac{1+\rho-\alpha}{2}; \frac{3}{2}, \frac{3+\rho-\alpha}{2}, 1+\frac{\rho-\alpha}{2}; -c^2z^2\right) + \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} z^{\alpha-\rho} B(\alpha, \rho-\alpha, \\
&\left[|\arg z| < \pi, |\arg c| < \pi/4, \left\{ \begin{matrix} -1 < \operatorname{Re} \alpha < \operatorname{Re} \rho \\ \operatorname{Re} \alpha > 0 \end{matrix} \right\} \right].
\end{aligned}$$

$$\begin{aligned}
5. \int_0^\infty \frac{x^{\alpha-1}}{x-y} \left\{ \begin{matrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{matrix} \right\} dx &= \mp \frac{\pi y^{\alpha-1}}{c} \operatorname{ctg} \alpha \pi \operatorname{erf}(cy) \pm \\
&\pm \frac{c^{1-\alpha}}{\sqrt{\pi}(1-\alpha)} \Gamma\left(\frac{\alpha}{2}\right) {}_3F_2\left(\frac{1-\alpha}{2}, 1; \frac{3-\alpha}{2}, 1-\frac{\alpha}{2}; -c^2y^2\right) \pm \\
&\pm \frac{c^{\alpha-\alpha}}{\sqrt{\pi}(2-\alpha)} \Gamma\left(\frac{\alpha-1}{2}\right) {}_3F_2\left(1-\frac{\alpha}{2}, 1; 2-\frac{\alpha}{2}, \frac{3-\alpha}{2}; -c^2y^2\right) - \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \pi y^{\alpha-1} \operatorname{ctg} \alpha \pi \\
&\left[y > 0; |\arg c| < \pi/4, \left\{ \begin{matrix} |\operatorname{Re} \alpha| < 1 \\ \operatorname{Re} \alpha > 0 \end{matrix} \right\} \right].
\end{aligned}$$

$$6. \int_{-\infty}^\infty \frac{\operatorname{erf}(cx)}{(x+z)^2} dx = -2i \sqrt{\pi} c e^{-c^2z^2} \operatorname{erf}(icz) \quad \{\operatorname{Im} z \neq 0; c > 0\}$$

2.8.3. Интегралы от $x^\alpha (z^2 \pm x^2)^\beta \left\{ \begin{matrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{matrix} \right\}$.

$$\begin{aligned}
1. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\beta-1} \left\{ \begin{matrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{matrix} \right\} dx &= \\
&= \pm \frac{a^{\alpha+2\beta-1} c}{\sqrt{\pi}} B\left(\frac{\alpha+1}{2}, \beta\right) {}_3F_2\left(\frac{\alpha+1}{2}, \frac{1}{2}; \frac{\alpha+1}{2} + \beta, \frac{3}{2}; -a^2c^2\right) + \\
&+ \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{a^{\alpha+2\beta-2}}{2} B\left(\frac{\alpha}{2}, \beta\right) \quad [a, \operatorname{Re} \beta > 0; \operatorname{Re} \alpha > -(1 \pm 1)/2]
\end{aligned}$$

$$\begin{aligned}
2. \int_a^\infty x^{\alpha-1} (x^2 - a^2)^{\beta-1} \left\{ \begin{matrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{matrix} \right\} dx &= \\
&= \pm \frac{a^{\alpha+2\beta-1} c}{\sqrt{\pi}} B\left(\beta, \frac{1-\alpha}{2} - \beta\right) {}_3F_2\left(\frac{1}{2}, \frac{\alpha+1}{2}; \frac{\alpha+1}{2} + \beta, \frac{3}{2}; -a^2c^2\right) \pm \\
&\pm \frac{c^{\alpha-2\beta-\alpha}}{\sqrt{\pi}(2-2\beta-\alpha)} \Gamma\left(\frac{\alpha-1}{2} + \beta\right) {}_3F_2\left(1-\beta - \frac{\alpha}{2}, 1-\beta; 2-\beta - \frac{\alpha}{2}, \frac{3-\alpha}{2} - \beta; -a^2c^2\right) + \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{a^{\alpha+2\beta-2}}{2} B\left(1-\beta - \frac{\alpha}{2}, \beta\right) \\
&\left[\operatorname{Re} \beta > 0; |\arg c| < \pi/4, \left\{ \begin{matrix} a > 0, \operatorname{Re}(\alpha + 2\beta) < 2 \\ a > 0 \end{matrix} \right\} \right].
\end{aligned}$$

$$3. \int_a^{\infty} x^{-2\beta-1} (x^2 - a^2)^{\beta-1} \operatorname{erf}(cx) dx =$$

$$= \frac{1}{\beta a^2} \left[1 - \frac{\Gamma(\beta+1)}{\sqrt{\pi} a c} \exp\left(-\frac{a^2 c^2}{2}\right) W_{-\beta-1/4, 1/4}(a^2 c^2) \right]$$

[a, Re $\beta > 0$; |arg c| < $\pi/4$]

$$1. \int_0^{\infty} x^{\alpha-1} (a + bx^2)^n \operatorname{erfc}(cx) dx =$$

$$= \frac{a^n}{\sqrt{\pi} \alpha c^\alpha} \Gamma\left(\frac{\alpha+1}{2}\right) {}_3F_1\left(-n, \frac{\alpha}{2}, \frac{\alpha+1}{2}; \frac{\alpha}{2}+1; -\frac{b}{ac^2}\right)$$

[Re $\alpha > 0$; |arg c| < $\pi/4$]

$$5. \int_0^{\infty} \frac{x^{\alpha-1}}{(x^2 + z^2)^\rho} \left\{ \begin{matrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{matrix} \right\} dx =$$

$$= \pm \frac{c^{2\rho+\alpha-2\rho}}{\sqrt{\pi}} B\left(\frac{\alpha+1}{2}, \rho - \frac{\alpha+1}{2}\right) {}_2F_2\left(\frac{\alpha+1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3+\alpha}{2} - \rho; c^2 z^2\right) \pm$$

$$\pm \frac{c^{2\rho-\alpha}}{\sqrt{\pi} (2\rho-\alpha)} \Gamma\left(\frac{\alpha+1}{2} - \rho\right) {}_2F_2\left(\rho, \rho - \frac{\alpha}{2}; 1 + \rho - \frac{\alpha}{2}, \frac{1-\alpha}{2} + \rho; c^2 z^2\right) +$$

$$+ \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{z^{\alpha-2\rho}}{2} B\left(\frac{\alpha}{2}, \rho - \frac{\alpha}{2}\right)$$

[Re $z > 0$; |arg c| < $\pi/4$, $\left\{ \begin{matrix} -1 < \operatorname{Re} \alpha < 2 \operatorname{Re} \rho \\ \operatorname{Re} \alpha > 0 \end{matrix} \right\}$].

$$6. \int_0^{\infty} \frac{\operatorname{erf}(cx)}{(x^2 + z^2)^{3/2}} dx = \frac{1}{z^2} [1 - e^{c^2 z^2} \operatorname{erfc}(cz)]$$

[Re $z > 0$; |arg c| < $\pi/4$]

$$7. \int_0^{\infty} \frac{x \operatorname{erf}(cx)}{(x^2 + z^2)^2} dx = \frac{\sqrt{\pi} c}{2z} e^{c^2 z^2} \operatorname{erfc}(cz)$$

[Re $z > 0$, |arg c| < $\pi/4$]

$$8. \int_0^{\infty} \frac{x}{(x^2 + z^2)^{3/2}} \operatorname{erf}(cx) dx = \frac{c}{\sqrt{\pi}} \exp\left(\frac{c^2 z^2}{2}\right) K_0\left(\frac{c^2 z^2}{2}\right)$$

[Re $z > 0$; |arg c| < $\pi/4$]

$$9. \int_0^{\infty} \frac{x^{\alpha-1}}{x^2 - y^2} \left\{ \begin{matrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{matrix} \right\} dx = \pm \frac{\pi y^{\alpha-2}}{2} \operatorname{tg} \frac{\alpha\pi}{2} \operatorname{erf}(cy) \pm$$

$$\pm \frac{c^{\alpha-\alpha}}{\sqrt{\pi} (2-\alpha)} \Gamma\left(\frac{\alpha-1}{2}\right) {}_2F_2\left(1 - \frac{\alpha}{2}, 1; \frac{3-\alpha}{2}, 2 - \frac{\alpha}{2}; -c^2 y^2\right) -$$

$$- \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{\pi y^{\alpha-2}}{2} \operatorname{ctg} \frac{\alpha\pi}{2}$$

[y > 0; |arg c| < $\pi/4$, $\left\{ \begin{matrix} -1 < \operatorname{Re} \alpha < 2 \\ \operatorname{Re} \alpha > 0 \end{matrix} \right\}$].

2.8.4. Интегралы содержащие x^α и $\left\{ \begin{matrix} \operatorname{erf}(\varphi(x)) \\ \operatorname{erfc}(\varphi(x)) \end{matrix} \right\}$.

$$1. \int_0^{\infty} x^{\alpha-1} [\operatorname{erf}(ax+b) - \operatorname{erf}(cx+d)] dx =$$

$$= \frac{\Gamma(\alpha)}{2^{(\alpha-1)/2} \sqrt{\pi}} [c^{-\alpha} e^{-d^2/2} D_{-\alpha-1}(\sqrt{2}d) - a^{-\alpha} e^{-b^2/2} D_{-\alpha-1}(\sqrt{2}b)]$$

[Re $\alpha > 0$; |arg a|, |arg c| < $\pi/4$]

$$2. \int_0^{\infty} \frac{\operatorname{erf}(ax+b) - \operatorname{erf}(cx+b)}{x} dx = \ln \frac{a}{c} \operatorname{erfc}(b) \quad [|\arg a|, |\arg c| < \pi/4].$$

$$3. \int_0^{\infty} x^{\alpha-1} \operatorname{erfc}\left(cx \pm \frac{b}{x}\right) dx = \\ = \frac{2b}{\alpha \sqrt{\pi}} \left(\frac{b}{c}\right)^{(\alpha-1)/2} e^{\mp 2bc} [K_{(\alpha+1)/2}(2bc) \mp K_{(\alpha-1)/2}(2bc)] \\ [b > 0; |\arg c| < \pi/4].$$

2.8.5. Интегралы от $x^{\alpha} e^{px \pm m} \begin{Bmatrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{Bmatrix}$.

$$1. \int_0^{\infty} e^{-px} \begin{Bmatrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{Bmatrix} dx = \frac{1}{p} \left[\begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \pm \exp\left(\frac{p^2}{4c^2}\right) \operatorname{erfc}\left(\frac{p}{2c}\right) \right] \\ \left[\begin{Bmatrix} |\arg c| < \pi/4; \operatorname{Re} p > 0 \\ |\arg c| < \pi/4 \end{Bmatrix} \right].$$

$$2. \int_0^{\infty} x^{\alpha-1} e^{-px} \begin{Bmatrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{Bmatrix} dx = \\ = \mp \frac{1}{c^{\alpha} \sqrt{\pi} \alpha} \Gamma\left(\frac{\alpha+1}{2}\right) {}_2F_2\left(\frac{\alpha}{2}, \frac{\alpha+1}{2}; \frac{\alpha}{2}+1, \frac{1}{2}; \frac{p^2}{4c^2}\right) \pm \\ \pm \frac{p}{c^{\alpha+1} \sqrt{\pi} (\alpha+1)} \Gamma\left(\frac{\alpha}{2}+1\right) {}_2F_2\left(\frac{\alpha+1}{2}, \frac{\alpha}{2}+1; \frac{\alpha+3}{2}, \frac{3}{2}; \frac{p^2}{4c^2}\right) + \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{\Gamma(\alpha)}{p^{\alpha}} \\ [|\arg c| < \pi/4, \left\{ \begin{array}{l} \operatorname{Re} p > 0; \operatorname{Re} \alpha > -1 \\ \operatorname{Re} \alpha > 0 \end{array} \right\}].$$

$$3. \int_0^{\infty} \frac{1}{x} e^{-px} \operatorname{erf}(cx) dx = -\frac{1}{2} \operatorname{Ei}\left(\frac{p^2}{4c^2}\right) - \frac{p}{\sqrt{\pi} c} {}_2F_2\left(\frac{1}{2}, 1; \frac{3}{2}, \frac{3}{2}; \frac{p^2}{4c^2}\right) \\ [\operatorname{Re} p > 0; |\arg c| < \pi/4]$$

$$4. \int_0^{\infty} x^n e^{-px} \begin{Bmatrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{Bmatrix} dx = I_n \quad \left[\begin{Bmatrix} \operatorname{Re} p > 0; |\arg c| < \pi/4 \\ |\arg c| < \pi/4 \end{Bmatrix} \right],$$

$$I_1 = \pm \frac{2c^2 - p'}{2c^2 p^2} \exp\left(\frac{p^2}{4c^2}\right) \operatorname{erfc}\left(\frac{p}{2c}\right) + \begin{Bmatrix} 0 \\ p^2 \end{Bmatrix} \pm \frac{1}{\sqrt{\pi} c p},$$

$$I_n = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \frac{n!}{p^{n+1}} \pm \sqrt{\frac{2}{\pi}} \sum_{k=0}^n \frac{n!}{p^{k+1}} (V 2 c)^{k-n} \exp\left(\frac{p^2}{8c^2}\right) D_{k-n-1}\left(\frac{p}{V 2 c}\right) = \\ = \pm (-1)^n \frac{\partial^n}{\partial p^n} \left[\frac{1}{p} \exp\left(\frac{p^2}{4c^2}\right) \operatorname{erfc}\left(\frac{p}{2c}\right) - \begin{Bmatrix} 0 \\ p^{-1} \end{Bmatrix} \right].$$

$$5. \int_0^{\infty} e^{px^2} \operatorname{erfc}(cx) dx = \frac{1}{2 \sqrt{\pi p}} \ln \frac{c + \sqrt{p}}{c - \sqrt{p}} \\ [|\arg c| < \pi/4; \operatorname{Re}(c^2 - p) > 0].$$

$$6. \int_0^{\infty} x^{\alpha-1} e^{-px^2} \begin{Bmatrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{Bmatrix} dx = \\ = \pm \frac{1}{\sqrt{\pi} p^{(\alpha+1)/2}} \Gamma\left(\frac{\alpha+1}{2}\right) {}_2F_1\left(\frac{1}{2}, \frac{\alpha+1}{2}; \frac{3}{2}; -\frac{c^2}{p}\right) + \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \frac{\Gamma(\alpha/2)}{2p^{\alpha/2}} \\ [\operatorname{Re} p > 0, \operatorname{Re} \alpha > -(1 \pm 1)/2; |\arg c| < \pi/4]$$

$$7. \int_0^{\infty} x^{\alpha-1} e^{-px^2} \operatorname{erfc}(cx) dx = \frac{c^{-\alpha}}{\alpha \sqrt{\pi}} \Gamma\left(\frac{\alpha+1}{2}\right) {}_2F_1\left(\frac{\alpha}{2}, \frac{\alpha+1}{2}; \frac{\alpha}{2}+1; \frac{p}{c^2}\right)$$

[Re α , Re $(c^2 - p) > 0$].

$$8. \int_0^{\infty} \frac{1}{x} e^{-px^2} \operatorname{erf}(cx) dx = \frac{1}{2} \ln \frac{\sqrt{c^2+p}+c}{\sqrt{c^2+p}-c}$$

[Re $p > 0$; |arg $c| < \pi/4$].

$$9. \int_0^{\infty} x^n e^{-px^2} \begin{Bmatrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{Bmatrix} dx = I_n$$

[{Re p , Re $(c^2 + p) > 0$ }]

$$I_1 = \frac{1}{2p} \left(\begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \pm \frac{c}{\sqrt{c^2+p}} \right),$$

$$I_2 = \pm \frac{1}{2\sqrt{\pi} p^{3/2}} \left(\frac{c\sqrt{p}}{c^2+p} - \operatorname{arctg} \frac{\sqrt{p}}{c} \right) + \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{\sqrt{\pi}}{4p^{3/2}},$$

$$I_{2m} = \frac{\sqrt{\pi} (2m-1)!}{2^{m+1} p^{m+1/2}} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \mp \frac{(-1)^m}{\sqrt{\pi}} \frac{\partial^m}{\partial p^m} \left(\frac{1}{\sqrt{p}} \operatorname{arctg} \frac{\sqrt{p}}{c} \right) =$$

$$= \frac{(-1)^m}{\sqrt{\pi}} \frac{\partial^m}{\partial p^m} \left[\frac{1}{\sqrt{p}} \operatorname{arctg} \left(\frac{c}{\sqrt{p}} \right)^{\pm 1} \right],$$

$$I_{2m+1} = \frac{(-1)^m m!}{2p^{m+1}} \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \pm \frac{(-1)^m c}{2} \frac{\partial^m}{\partial p^m} \left(\frac{1}{p\sqrt{p+c^2}} \right).$$

$$10. \int_0^{\infty} x^{\alpha-1} e^{-c^2 x^2} \operatorname{erf}(icx) dx = \frac{\pi i}{2c^{\alpha} \Gamma(1-\alpha/2)} \sec \frac{\alpha\pi}{2}$$

[|Re $\alpha| < 1$; |arg $c| < \pi/4$].

$$11. \int_0^{\infty} x^{\alpha-1} e^{c^2 x^2} \operatorname{erfc}(cx) dx = \frac{\Gamma(\alpha/2)}{2c^{\alpha}} \sec \frac{\alpha\pi}{2}$$

[0 < Re α < 1]

$$12. \int_1^{\infty} \frac{1}{x^3} e^{c^2 x^2} \operatorname{erfc}(cx) dx = \frac{1+2c^2}{2} e^{c^2} \operatorname{erfc}(c) - \frac{c}{\sqrt{\pi}}$$

[|arg $c| < \pi/4$].

$$13. \int_0^{\infty} x^3 e^{-px^4} \operatorname{erf}(cx) dx = \frac{c^2}{8\sqrt{\pi} p^{3/2}} \exp\left(\frac{c^4}{8p}\right) K_{1/4}\left(\frac{c^4}{8p}\right)$$

[Re $p > 0$; |arg $c| < \pi/4$].

$$14. \int_0^{\infty} x^{\alpha-1} e^{-p/x} \begin{Bmatrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{Bmatrix} dx =$$

$$= \pm \frac{2cp^{\alpha+1}}{\sqrt{\pi}} \Gamma(-\alpha-1) {}_1F_3\left(\frac{1}{2}; \frac{3}{2}, \frac{3+\alpha}{2}, 1+\frac{\alpha}{2}; -\frac{c^2 p^2}{4}\right) \mp$$

$$\mp \frac{1}{c^{\alpha} \sqrt{\pi} \alpha} \Gamma\left(\frac{\alpha+1}{2}\right) {}_1F_3\left(-\frac{\alpha}{2}; 1-\frac{\alpha}{2}, \frac{1}{2}, \frac{1-\alpha}{2}; -\frac{c^2 p^2}{4}\right) \mp$$

$$\mp \frac{p}{c^{\alpha-1} \sqrt{\pi} (1-\alpha)} \Gamma\left(\frac{\alpha}{2}\right) {}_1F_3\left(\frac{1-\alpha}{2}; \frac{3-\alpha}{2}, 1-\frac{\alpha}{2}, \frac{3}{2}; -\frac{c^2 p^2}{4}\right) +$$

$$+ \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} p^{\alpha} \Gamma(-\alpha)$$

[|arg $c| < \pi/4$, {Re $p > 0$; Re $\alpha < 0$ }]

$$15. \int_0^{\infty} x^{\alpha-1} e^{-p/x^2} \begin{Bmatrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{Bmatrix} dx =$$

$$= \pm \frac{c p^{(\alpha+1)/2}}{\sqrt{\pi}} \Gamma\left(-\frac{\alpha+1}{2}\right) {}_1F_2\left(\frac{1}{2}; \frac{3}{2}, \frac{3+\alpha}{2}; c^2 p\right) \mp$$

$$\mp \frac{1}{c^{\alpha} \sqrt{\pi} \alpha} \Gamma\left(\frac{\alpha+1}{2}\right) {}_1F_2\left(-\frac{\alpha}{2}; 1-\frac{\alpha}{2}, \frac{1-\alpha}{2}; c^2 p\right) + \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \frac{p^{\alpha/2}}{2} \Gamma\left(-\frac{\alpha}{2}\right)$$

$$\left[|\arg c| < \pi/4 \quad \begin{cases} \operatorname{Re} p > 0; \operatorname{Re} \alpha < 0 \\ \operatorname{Re} p > 0 \end{cases} \right].$$

$$16. \int_0^{\infty} x e^{-p/x^2} \operatorname{erfc}(cx) dx = \frac{1+2c\sqrt{p}}{4c^2} e^{-2c\sqrt{p}} + p \operatorname{Ei}(-2c\sqrt{p})$$

[Re $p > 0$; $|\arg c| < \pi/4$].

$$17. \int_0^{\infty} \frac{1}{x} e^{-p/x^2} \operatorname{erfc}(cx) dx = -\operatorname{Ei}(-2c\sqrt{p})$$

[Re $p > 0$; $|\arg c| < \pi/4$]

$$18. \int_0^{\infty} \frac{1}{x^3} e^{-p/x^2} \begin{Bmatrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{Bmatrix} dx = \frac{1}{2p} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \mp e^{-2c\sqrt{p}}$$

[Re $p > 0$; $|\arg c| < \pi/4$]

$$19. \int_0^{\infty} x^{-n} e^{-p/x^2} \operatorname{erf}(cx) dx = I_n$$

[Re $p > 0$; $|\arg c| < \pi/4$]

$$I_2 = \frac{2c}{\sqrt{\pi}} K_0(2c\sqrt{p}) + \sqrt{\pi} c [K_0(2c\sqrt{p}) L_1(2c\sqrt{p}) + K_1(2c\sqrt{p}) L_0(2c\sqrt{p})],$$

$$I_4 = \frac{1}{2p^2} [1 - (1 + c\sqrt{p}) e^{-2c\sqrt{p}}],$$

$$I_{2m+2} = \frac{m!}{2p^{m+1}} \left[1 - \frac{2\sqrt[4]{c^2 p}}{\sqrt{\pi}} \sum_{k=0}^m \frac{(c\sqrt{p})^k}{k!} K_{k-1/2}(2c\sqrt{p}) \right].$$

2.8.6. Интегралы от $x^{\alpha} e^{i/cx} \begin{Bmatrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{Bmatrix}$.

$$1. \int_0^{\infty} e^{-px + c^2 x^2} \operatorname{erfc}(cx) dx = -\frac{1}{2c\sqrt{\pi}} \exp\left(-\frac{p^2}{4c^2}\right) \left[\pi i \operatorname{erf}\left(\frac{ip}{2c}\right) + \operatorname{Ei}\left(\frac{p^2}{4c^2}\right) \right]$$

[Re $p > 0$]

$$2. \int_0^{\infty} e^{-px - c^2 x^2} \operatorname{erf}(cx) dx = \frac{\sqrt{\pi}}{4c} \exp\left(\frac{p^2}{4c^2}\right) \operatorname{erfc}^2\left(\frac{p}{2\sqrt{2}c}\right)$$

[$|\arg c| < \pi/4$]

$$3. \int_0^{\infty} x^{\alpha-1} e^{-px \mp c^2 x^2} \begin{Bmatrix} \operatorname{erf}(icx) \\ \operatorname{erfc}(cx) \end{Bmatrix} dx =$$

$$= \begin{Bmatrix} i \\ 1 \end{Bmatrix} \frac{p^{1-\alpha} \Gamma(\alpha-1)}{c\sqrt{\pi}} {}_2F_2\left(1, \frac{1}{2}; \frac{3-\alpha}{2}, 1-\frac{\alpha}{2}; \pm \frac{p^2}{4c^2}\right) +$$

$$+ \begin{Bmatrix} i \\ 1 \end{Bmatrix} \frac{\Gamma(\alpha/2)}{2c^{\alpha}} \begin{Bmatrix} \operatorname{tg}(\alpha\pi/2) \\ \operatorname{sec}(\alpha\pi/2) \end{Bmatrix} {}_1F_1\left(\frac{\alpha}{2}; \frac{1}{2}; \pm \frac{p^2}{4c^2}\right) +$$

$$+ \begin{Bmatrix} -i \\ 1 \end{Bmatrix} \frac{p}{2c^{\alpha+1}} \Gamma\left(\frac{\alpha+1}{2}\right) \begin{Bmatrix} \operatorname{ctg}(\alpha\pi/2) \\ \operatorname{cosec}(\alpha\pi/2) \end{Bmatrix} {}_1F_1\left(\frac{\alpha+1}{2}; \frac{3}{2}; \pm \frac{p^2}{4c^2}\right)$$

[Re $p > 0$; $|\arg c| < \pi/4$; Re $\alpha > -(1 \pm 1)/2$]

$$4. \int_0^{\infty} x^n e^{-px - c^2 x^2} \operatorname{erf}(icx) dx = I_n \quad [\operatorname{Re} p > 0; |\arg c| < \pi/4].$$

$$I_n = \frac{(-1)^n}{2ic\sqrt{\pi}} \frac{\partial^n}{\partial p^n} \left[\exp\left(\frac{p^2}{4c^2}\right) \operatorname{Ei}\left(-\frac{p^2}{4c^2}\right) \right],$$

$$I_1 = \frac{i}{4pc^3\sqrt{\pi}} \left[4c^3 + p^2 \exp\left(\frac{p^2}{4c^2}\right) \operatorname{Ei}\left(-\frac{p^2}{4c^2}\right) \right].$$

$$5. \int_0^{\infty} x^{\alpha-1} e^{-px - bx^2} \operatorname{erf}(cx) dx =$$

$$= \frac{c}{\sqrt{\pi} b^{(\alpha+1)/2}} \Gamma\left(\frac{\alpha+1}{2}\right) \Psi_1\left(\frac{\alpha+1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}; -\frac{c^2}{b}, \frac{p^2}{4b}\right) -$$

$$- \frac{cp}{\sqrt{\pi} b^{\alpha/2+1}} \Gamma\left(\frac{\alpha}{2}+1\right) \Psi_1\left(\frac{\alpha}{2}+1, \frac{1}{2}, \frac{3}{2}, \frac{3}{2}; -\frac{c^2}{b}, \frac{p^2}{4b}\right)$$

$$[\operatorname{Re} \alpha > -1; \operatorname{Re} b, \operatorname{Re}(b+c^2) > 0]$$

$$\text{или } [\operatorname{Re} \alpha > -1; \operatorname{Re} b = \max(0, -\operatorname{Re} c^2); \operatorname{Re} p = 0]$$

$$\text{или } [-1 < \operatorname{Re} \alpha < 0; |\arg c| < \pi/4; \operatorname{Re} b = \max(0, -\operatorname{Re} c^2); \operatorname{Re} p = 0]$$

$$\text{или } [|\operatorname{Re} \alpha| < 0; \operatorname{Re} c^2 < 0, \operatorname{Re} p = \max(0, -\operatorname{Re} c^2); \operatorname{Re} p = 0].$$

$$6. \int_0^{\infty} x^{\alpha-1} e^{-c^2 x^2 - b/x^2} \left\{ \begin{array}{l} \operatorname{erf}(icx) \\ \operatorname{erfc}(cx) \end{array} \right\} dx =$$

$$= \left\{ \begin{array}{l} -i \\ 1 \end{array} \right\} \frac{\pi b^{\alpha/4}}{2c^{\alpha/2}} \sec \frac{\alpha\pi}{2} \left[\begin{array}{l} L_{\alpha/2}(2c\sqrt{b}) - I_{-\alpha/2}(2c\sqrt{b}) \\ H_{\alpha/2}(2c\sqrt{b}) - Y_{\alpha/2}(2c\sqrt{b}) \end{array} \right]$$

$$[\operatorname{Re} p > 0; |\arg c| < \pi/4, \operatorname{Re} \alpha < 1, \alpha \neq -1, -3, \dots]$$

$$7. \int_0^{\infty} x^{-n} e^{c^2 x^2 - b/x^2} \operatorname{erfc}(cx) dx = I_n$$

$$[\operatorname{Re} p > 0; |\arg c| < \pi/4].$$

$$I_1 = \frac{\pi}{2} [H_0(2c\sqrt{b}) - Y_0(2c\sqrt{b})],$$

$$I_2 = \frac{1}{\sqrt{\pi b}} [\sin(2c\sqrt{b}) \operatorname{ci}(2c\sqrt{b}) - \cos(2c\sqrt{b}) \operatorname{si}(2c\sqrt{b})],$$

$$I_3 = \frac{\pi c}{2\sqrt{b}} [H_1(2c\sqrt{b}) - Y_1(2c\sqrt{b})] - \frac{c}{\sqrt{b}}.$$

2.8.7. Интегралы от $x^\alpha (z \pm x)^\beta e^{px^2} \left\{ \begin{array}{l} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{array} \right\}$.

$$1. \int_0^a x^{\alpha-1} (a-x)^{\beta-1} e^{c^2 x^2} \left\{ \begin{array}{l} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{array} \right\} dx =$$

$$= \pm \frac{2a^{\alpha+\beta} c}{\sqrt{\pi}} B(\alpha+1, \beta) {}_3F_3\left(1, \frac{\alpha+1}{2}, \frac{\alpha}{2}+1; \frac{3}{2}, \frac{\alpha+\beta+1}{2}, \frac{\alpha+\beta}{2}+1; a^2 c^2\right) +$$

$$+ \left\{ \begin{array}{l} 0 \\ 1 \end{array} \right\} a^{\alpha+\beta-1} B(\alpha, \beta) {}_2F_2\left(\frac{\alpha}{2}, \frac{\alpha+1}{2}; \frac{\alpha+\beta}{2}, \frac{\alpha+\beta+1}{2}; a^2 c^2\right)$$

$$[a, \operatorname{Re} \beta > 0; \operatorname{Re} \alpha > -(1 \pm 1)/2]$$

$$2. \int_0^{\infty} \frac{x^{\alpha-1}}{(x+z)^{\rho}} e^{\mp c^2 x^2} \left\{ \begin{matrix} \operatorname{erf}(icx) \\ \operatorname{erfc}(cx) \end{matrix} \right\} dx = \left\{ \begin{matrix} i \\ -1 \end{matrix} \right\} \frac{2c^2 z^{\alpha-\rho+1}}{\sqrt{\pi}} \times$$

$$\times B(\alpha+1, \rho-\alpha-1) {}_3F_3 \left(1, \frac{\alpha+1}{2}, \frac{\alpha}{2}+1; \frac{3}{2}, \frac{3+\alpha-\rho}{2}, 1+\frac{\alpha-\rho}{2}; \mp c^2 z^2 \right) \mp$$

$$\mp \left\{ \begin{matrix} i \\ 1 \end{matrix} \right\} \frac{c^{\rho-\alpha}}{2} \left\{ \begin{matrix} \operatorname{tg}[(\rho-\alpha)\pi/2] \\ \operatorname{sech}[(\rho-\alpha)\pi/2] \end{matrix} \right\} \Gamma\left(\frac{\alpha-\rho}{2}\right) {}_2F_2 \left(\frac{\rho}{2}, \frac{\rho+1}{2}; \frac{1}{2}, 1+\frac{\rho-\alpha}{2}; \mp c^2 z^2 \right) \pm$$

$$\pm \left\{ \begin{matrix} i \\ 1 \end{matrix} \right\} \frac{c^{\rho-\alpha-1} z^{\rho}}{2} \Gamma\left(\frac{\alpha-\rho-1}{2}\right) \left\{ \begin{matrix} \operatorname{ctg}[(\alpha-\rho)\pi/2] \\ \operatorname{cosec}[(\alpha-\rho)\pi/2] \end{matrix} \right\} {}_2F_2 \left(\frac{\rho+1}{2}, \frac{\rho}{2}+1; \frac{3}{2}, \right.$$

$$\left. \frac{3+\rho-\alpha}{2}; \mp c^2 z^2 \right) + \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} z^{\alpha-\rho} B(\alpha, \rho-\alpha) {}_2F_2 \left(\frac{\alpha}{2}, \frac{\alpha+1}{2}; 1+\frac{\alpha-\rho}{2}, \right.$$

$$\left. \frac{1+\alpha-\rho}{2}; c^2 z^2 \right)$$

$$[-(1 \pm 1)/2 < \operatorname{Re} \alpha < \operatorname{Re} \rho + 1; |\arg c| < \pi/4; |\arg z| < \pi]$$

$$3. \int_0^{\infty} \frac{x^{\alpha-1}}{x+z} e^{-c^2 x^2} \operatorname{erf}(icx) dx = \frac{z^{\alpha-1}}{2} e^{-c^2 z^2} \left[i\alpha \operatorname{ctg} \frac{\alpha\pi}{2} \Gamma\left(\frac{\alpha+1}{2}\right) \gamma\left(\frac{1-\alpha}{2}, -c^2 z^2\right) + \right.$$

$$\left. + i\alpha^{-1} \operatorname{tg} \frac{\alpha\pi}{2} \Gamma\left(\frac{\alpha}{2}\right) \gamma\left(1-\frac{\alpha}{2}, -c^2 z^2\right) - \frac{\pi}{\sin \alpha\pi} \operatorname{erf}(icz) \right]$$

$$[-1 < \operatorname{Re} \alpha < 2; |\arg z| < \pi; |\arg c| < \pi/4]$$

$$4. \int_0^{\infty} \frac{x^{\alpha-1}}{x-y} e^{\mp c^2 x^2} \left\{ \begin{matrix} \operatorname{erf}(icx) \\ \operatorname{erfc}(cx) \end{matrix} \right\} dx = -\pi y^{\alpha-1} \operatorname{ctg} \alpha\pi e^{\mp c^2 y^2} \left\{ \begin{matrix} \operatorname{erf}(icy) \\ \operatorname{erfc}(cy) \end{matrix} \right\} \mp$$

$$\mp \left\{ \begin{matrix} i \\ 1 \end{matrix} \right\} \frac{c^{1-\alpha}}{2} \Gamma\left(\frac{\alpha-1}{2}\right) \left\{ \begin{matrix} \operatorname{ctg}(\alpha\pi/2) \\ \operatorname{cosec}(\alpha\pi/2) \end{matrix} \right\} {}_1F_1 \left(1; \frac{3-\alpha}{2}; \mp c^2 y^2 \right) \pm$$

$$\pm \left\{ \begin{matrix} i \\ 1 \end{matrix} \right\} \frac{c^{2-\alpha} y}{2} \Gamma\left(\frac{\alpha}{2}-1\right) \left\{ \begin{matrix} \operatorname{tg}(\alpha\pi/2) \\ \cos(\alpha\pi/2) \end{matrix} \right\} {}_1F_1 \left(1; 2-\frac{\alpha}{2}; \mp c^2 y^2 \right)$$

$$[y > 0; -(1 \pm 1)/2 < \operatorname{Re} \alpha < 2; |\arg c| < \pi/4]$$

$$5. \int_{-\infty}^{\infty} \frac{e^{-c^2 x^2}}{x+z} \operatorname{erf}(icx) dx = \pi i e^{-c^2 z^2} \quad [|\arg c| < \pi/4; \operatorname{Im} z \neq 0]$$

$$6. \int_{-\infty}^{\infty} \frac{e^{-c^2 x^2}}{x(x-y)} \operatorname{erf}(icx) dx = \frac{\pi i}{y} (e^{-c^2 y^2} - 1) \quad [y > 0; |\arg c| < \pi/4]$$

2.8.8. Интегралы от $x^{\alpha} (z^2 \pm x^2)^{\beta} e^{\rho x^2} \left\{ \begin{matrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{matrix} \right\}$.

$$1. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\beta-1} e^{c^2 x^2} \left\{ \begin{matrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{matrix} \right\} dx =$$

$$= \pm \frac{a^{\alpha+2\beta-1} c}{\sqrt{\pi}} B\left(\frac{\alpha+1}{2}, \beta\right) {}_2F_2 \left(1, \frac{\alpha+1}{2}; \frac{3}{2}, \beta + \frac{\alpha+1}{2}; a^2 c^2 \right) +$$

$$+ \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{a^{\alpha+2\beta-2}}{2} B\left(\frac{\alpha}{2}, \beta\right) {}_1F_1 \left(\frac{\alpha}{2}; \frac{\alpha}{2} + \beta; a^2 c^2 \right)$$

$$[a, \operatorname{Re} \beta > 0; \operatorname{Re} \alpha > -(1 \pm 1)/2]$$

$$2. \int_0^a \frac{1}{x \sqrt{a^2 - x^2}} e^{-c^2 x^2} \operatorname{erf}(icx) dx = \frac{\pi i}{2a} \operatorname{erf}(ac) \quad [a > 0]$$

$$8. \int_0^a x (a^2 - x^2)^{\beta-1} e^{-c^2 x^2} \operatorname{erf}(icx) dx =$$

$$= \frac{ia^{\beta-1/2}}{2c^{\beta+1/2}} \Gamma(\beta) e^{-a^2 c^2/2} M_{(1-2\beta)/4, (1+2\beta)/4}(a^2 c^2) \quad [a, \operatorname{Re} \beta > 0]$$

$$4. \int_0^a \frac{x}{\sqrt{a^2 - x^2}} e^{c^2 x^2} \left\{ \begin{matrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{matrix} \right\} dx = \pm \frac{\sqrt{\pi}}{2c} \left[e^{a^2 c^2} \left\{ \begin{matrix} 1 \\ \operatorname{erfc}(ac) \end{matrix} \right\} - 1 \right]$$

$$[a > 0]$$

$$5. \int_0^a x^{-2\beta} (a^2 - x^2)^{\beta-1} e^{c^2 x^2} \operatorname{erfc}(cx) dx = \frac{\Gamma(1-2\beta)}{2^{1/2-\beta} \sqrt{\pi} a} \Gamma(\beta) e^{a^2 c^2/2} D_{2\beta-1}(ac \sqrt{2})$$

$$[a > 0; 0 < \operatorname{Re} \beta < 1/2].$$

$$6. \int_a^\infty x^{\alpha-1} (x^2 - a^2)^{\beta-1} e^{\mp c^2 x^2} \left\{ \begin{matrix} \operatorname{erf}(icx) \\ \operatorname{erfc}(cx) \end{matrix} \right\} dx =$$

$$= \left\{ \begin{matrix} i \\ -1 \end{matrix} \right\} \frac{a^{\alpha+2\beta-1} c}{\sqrt{\pi}} B\left(\frac{1-\alpha}{2} - \beta, \beta\right) {}_2F_2\left(\frac{1+\alpha}{2}, 1; \frac{3}{2}, \frac{1+\alpha}{2} + \beta; \mp a^2 c^2\right) +$$

$$+ \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{a^{\alpha+2\beta-2}}{2} B\left(1 - \beta - \frac{\alpha}{2}, \beta\right) {}_1F_1\left(\frac{\alpha}{2}; \frac{\alpha}{2} + \beta; a^2 c^2\right) +$$

$$+ \left\{ \begin{matrix} i \\ -1 \end{matrix} \right\} \frac{c^{2-2\beta-\alpha}}{2} \left\{ \frac{\operatorname{tg}(\alpha/2 + \beta)\pi}{\sec(\alpha/2 + \beta)\pi} \right\} \Gamma\left(\frac{\alpha}{2} + \beta - 1\right) {}_1F_1\left(1 - \beta; 2 - \beta - \frac{\alpha}{2}; \mp a^2 c^2\right)$$

$$[a, \operatorname{Re} \beta > 0; \operatorname{Re}(\alpha + 2\beta) < 3; |\arg c| < \pi/4].$$

$$7. \int_0^\infty \frac{x^{\alpha-1}}{(x^2 + z^2)^\rho} e^{\mp c^2 x^2} \left\{ \begin{matrix} \operatorname{erf}(icx) \\ \operatorname{erfc}(cx) \end{matrix} \right\} dx =$$

$$= \left\{ \begin{matrix} i \\ -1 \end{matrix} \right\} \frac{c^2 z^{\alpha-2\rho+1}}{\sqrt{\pi}} B\left(\frac{\alpha+1}{2}, \rho - \frac{\alpha+1}{2}\right) {}_2F_2\left(1, \frac{\alpha+1}{2}; \frac{3}{2}, \frac{3+\alpha}{2} - \rho; \pm c^2 z^2\right) \mp$$

$$\mp \left\{ \begin{matrix} i \\ 1 \end{matrix} \right\} \frac{c^{2\rho-\alpha}}{2} \left\{ \frac{\operatorname{tg}[(\rho - \alpha/2)\pi]}{\sec[(\rho - \alpha/2)\pi]} \right\} \Gamma\left(\frac{\alpha}{2} - \rho\right) {}_1F_1\left(\rho; 1 + \rho - \frac{\alpha}{2}; \pm c^2 z^2\right) +$$

$$+ \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{z^{\alpha-2\rho}}{2} B\left(\frac{\alpha}{2}, \rho - \frac{\alpha}{2}\right) {}_1F_1\left(\frac{\alpha}{2}; 1 + \frac{\alpha}{2} - \rho; -c^2 z^2\right)$$

$$[\operatorname{Re} z > 0; -(1 \pm 1)/2 < \operatorname{Re} \alpha < 2 \operatorname{Re} \rho + 1; |\arg c| < \pi/4].$$

$$8. \int_0^\infty \frac{x^{\alpha-1}}{x^2 + z^2} e^{-c^2 x^2} \operatorname{erf}(icx) dx =$$

$$= \frac{\pi i z^{\alpha-2}}{2 \cos(\alpha\pi/2)} e^{c^2 z^2} \left[\operatorname{erf}(cz) - \frac{1}{\Gamma(1-\alpha/2)} \gamma\left(1 - \frac{\alpha}{2}, c^2 z^2\right) \right]$$

$$[-1 < \operatorname{Re} \alpha < 3; \operatorname{Re} z > 0; |\arg c| < \pi/4].$$

$$9. \int_0^\infty \frac{x^{\alpha-1}}{x^2 + z^2} e^{c^2 x^2} \operatorname{erfc}(cx) dx =$$

$$= \frac{\pi z^{\alpha-2}}{2 \cos(\alpha\pi/2)} e^{-c^2 z^2} \left[i \operatorname{erf}(icz) + \frac{i^{\alpha-2}}{\pi} \Gamma\left(\frac{\alpha}{2}\right) \gamma\left(1 - \frac{\alpha}{2}, -c^2 z^2\right) + \operatorname{ctg} \frac{\alpha\pi}{2} \right]$$

$$[0 < \operatorname{Re} \alpha < 3; \operatorname{Re} z > 0; |\arg c| < \pi/4]$$

$$10. \int_0^\infty \frac{x^k}{x^2 + z^2} e^{-c^2 x^2} \operatorname{erf}(icx) dx = I_k \quad [\operatorname{Re} z > 0; |\arg c| < \pi/4].$$

$$I_{-1} = \frac{\pi i}{2z^2} [1 - e^{c^2 z^2} \operatorname{erfc}(cz)],$$

$$I_0 = -\frac{ci}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{2 \ln(cz) - \psi(3/2+k)}{(3/2)_k} (cz)^{2k},$$

$$I_1 = \frac{\pi i}{2} e^{c^2 z^2} \operatorname{erfc}(cz).$$

$$11. \int_0^{\infty} \frac{x^k}{(x^2+z^2)^2} e^{-c^2 x^2} \operatorname{erf}(icx) dx = J_k \quad [\operatorname{Re} z > 0; |\arg c| < \pi/4].$$

$$J_{-1} = \frac{\pi i}{2z^4} [1 - (1 - c^2 z^2) e^{c^2 z^2} \operatorname{erfc}(cz)] - \frac{\sqrt{\pi} ic}{2z^3},$$

$$J_1 = \frac{\pi ic^2}{2} e^{c^2 z^2} \operatorname{erf}(cz) - \frac{\sqrt{\pi} ic}{2z},$$

$$J_3 = \frac{\pi i}{2} (1 + c^2 z^2) e^{c^2 z^2} \operatorname{erfc}(cz) - \frac{\sqrt{\pi} i}{2} cz.$$

$$12. \int_0^{\infty} \frac{x^{\alpha-1}}{x^2-y^2} e^{\mp c^2 x^2} \left\{ \begin{array}{l} \operatorname{erf}(icx) \\ \operatorname{erfc}(cx) \end{array} \right\} dx =$$

$$= \frac{\pi}{2} y^{\alpha-2} \operatorname{tg} \frac{\alpha\pi}{2} e^{\mp c^2 y^2} \left\{ \begin{array}{l} \operatorname{erf}(icy) \\ \operatorname{erfc}(cy) \end{array} \right\} - \left\{ \begin{array}{l} 0 \\ 1 \end{array} \right\} \frac{\pi y^{\alpha-2}}{\sin \alpha\pi} e^{c^2 y^2} \pm$$

$$\pm \left\{ \begin{array}{l} i \\ 1 \end{array} \right\} \frac{c^{2-\alpha}}{2} \Gamma\left(\frac{\alpha}{2}-1\right) \left\{ \begin{array}{l} \operatorname{tg}(\alpha\pi/2) \\ \sec(\alpha\pi/2) \end{array} \right\} {}_1F_1\left(1; 2-\frac{\alpha}{2}; \mp c^2 y^2\right)$$

[$y > 0; |\arg c| < \pi/4; -(1 \pm 1)/2 < \operatorname{Re} \alpha < 3$].

2.8.9. Интегралы от $A(x) e^{f(x)} \left\{ \begin{array}{l} \operatorname{erf}(\varphi(x)) \\ \operatorname{erfc}(\varphi(x)) \end{array} \right\}$.

$$1. \int_0^{\infty} x^n e^{-px} \left\{ \begin{array}{l} \operatorname{erf}(cx+b) \\ \operatorname{erfc}(cx+b) \end{array} \right\} dx =$$

$$= (-1)^n \frac{\partial^n}{\partial p^n} \left[\frac{1}{p} \left\{ \begin{array}{l} \operatorname{erf}(b) \\ \operatorname{erfc}(b) \end{array} \right\} \right] \pm \frac{1}{p} \exp\left(\frac{p^2+4pbc}{4c^2}\right) \operatorname{erfc}\left(b + \frac{p}{2c}\right)$$

[$\operatorname{Re} p > 0; |\arg c| < \pi/4$]

$$2. \int_0^{\infty} x^{2n+1} e^{-px^2} \left\{ \begin{array}{l} \operatorname{erf}(cx+b) \\ \operatorname{erfc}(cx+b) \end{array} \right\} dx =$$

$$= \left\{ \begin{array}{l} 0 \\ 1 \end{array} \right\} \frac{n!}{2p^{n+1}} \pm \frac{(-1)^n}{2} \frac{\partial^n}{\partial p^n} \left[\frac{1}{p} \operatorname{erf}(b) + \frac{c}{p\sqrt{c^2+p}} \exp\left(-\frac{pb^2}{c^2+p}\right) \operatorname{erfc}\left(\frac{bc}{\sqrt{c^2+p}}\right) \right]$$

[$\operatorname{Re} p > 0; |\arg c| < \pi/4$]

$$3. \int_0^{\infty} x^{\alpha-1} e^{c^2 x^2} \operatorname{erfc}(cx+b) dx = \frac{\Gamma(\alpha)}{(2c)^\alpha \sqrt{\pi}} \Gamma\left(\frac{1-\alpha}{2}, b^2\right)$$

$$[0 < \operatorname{Re} \alpha < 1; |\arg c| < \pi/4]$$

$$4. \int_0^{\infty} x e^{-px^2} [\operatorname{erf}(cx+b) \pm \operatorname{erf}(cx-b)] dx =$$

$$= \pm \frac{c}{p\sqrt{c^2+p}} \exp\left(-\frac{b^2 p}{c^2+p}\right) \left\{ \begin{array}{l} 1 \\ B \end{array} \right\} + \frac{1}{p} \left\{ \begin{array}{l} 0 \\ 1 \end{array} \right\} \operatorname{erf}(b)$$

[$B = \operatorname{erf}(bc/\sqrt{c^2+p}); \operatorname{Re} p > 0; |\arg c| < \pi/4$]

$$5. \int_0^{\infty} x^2 e^{-px^2} [\operatorname{erf}(cx+b) + \operatorname{erf}(cx-b)] dx =$$

$$= \frac{c}{p^2 (c^2+p)^{5/2}} \left[(c^2+p)^2 - \frac{p}{2} (1-2b^2)(c^2+p) - b^2 p^2 \right] \exp\left(-\frac{b^2 p}{c^2+p}\right),$$

[Re p > 0; |arg c| < π/4]

$$6. \int_0^{\infty} x e^{-px^2} \operatorname{erf}\left(cx \pm \frac{b}{x}\right) dx = \frac{1}{2p} \exp\left[-2b(\sqrt{c^2+p} \pm c)\right] \left(\frac{c}{\sqrt{c^2+p}} \mp 1\right) \pm \frac{1}{2p}$$

[b, Re p > 0; |arg c| < π/4]

$$7. \int_0^{\infty} x^{2n+1} e^{-px^2} \operatorname{erfc}\left(cx \pm \frac{b}{x}\right) dx =$$

$$= \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \frac{n!}{2p^{n+1}} \pm \frac{n! \sqrt{b} (c^2+p)^{1/4}}{\sqrt{\pi} p^{n+1}} e^{\mp 2bc} \sum_{k=0}^n \frac{p^k}{k!} \left(\frac{b^2}{c^2+p}\right)^{k/2} \times$$

$$\times \left[K_{k-1/2}(2b\sqrt{c^2+p}) \mp \frac{c}{\sqrt{c^2+p}} K_{k+1/2}(2b\sqrt{c^2+p}) \right]$$

[b, Re(c^2+p) > 0]

2.8.10. Интегралы, содержащие e^{px^m} , гиперболические функции и $\begin{Bmatrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{Bmatrix}$.

$$1. \int_0^{\infty} e^{-px} \begin{Bmatrix} \operatorname{sh} bx \\ \operatorname{ch} bx \end{Bmatrix} \operatorname{erf}(cx) dx =$$

$$= \frac{1}{2} \left[\frac{1}{p-b} \exp\left(\frac{(p-b)^2}{4c^2}\right) \operatorname{erfc}\left(\frac{p-b}{2c}\right) \mp \frac{1}{p+b} \exp\left(\frac{(p+b)^2}{4c^2}\right) \operatorname{erfc}\left(\frac{p+b}{2c}\right) \right]$$

[Re p > |Re b|; |arg c| < π/4]

$$2. \int_0^{\infty} e^{-px^2} \operatorname{sh} bx \operatorname{erf}(cx) dx = \frac{\sqrt{\pi}}{2\sqrt{p}} \exp\left(\frac{b^2}{4p}\right) \operatorname{erf}\left(\frac{bc}{2\sqrt{c^2 p + p^2}}\right)$$

[Re p > 0; |arg c| < π/4]

$$3. \int_0^{\infty} e^{-px^2} \operatorname{sh}^n bx^2 \operatorname{erf}(cx) dx = \frac{2^{-n}}{\sqrt{\pi}} \sum_{k=0}^n \binom{n}{k} \frac{(-1)^k}{\sqrt{p-nb+2kb}} \operatorname{arctg} \frac{c}{\sqrt{p-nb+2kb}}$$

[Re p > n, Re b; |arg c| < π/4]

2.8.11. Интегралы, содержащие тригонометрические функции и $\begin{Bmatrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{Bmatrix}$.

Обозначение: $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$.

$$1. \int_0^{\infty} x^{\alpha-1} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} \operatorname{erf}(cx) dx = U(l)$$

$$U(\gamma) = \frac{(-1)^{\gamma} b^{\delta}}{\sqrt{\pi} (\alpha+\delta) c^{\alpha+\delta}} \Gamma\left(\frac{1+\alpha+\delta}{2}\right) {}_2F_2\left(\frac{1+\alpha}{2}, \frac{\alpha}{2}+\delta; \delta+\frac{1}{2}, \frac{\alpha+\delta}{2}+1; -\frac{b^2}{4c^2}\right) +$$

$$+ \frac{\gamma \Gamma(\alpha)}{b^{\alpha}} \begin{Bmatrix} \sin(\alpha\pi/2) \\ \cos(\alpha\pi/2) \end{Bmatrix}$$

[b > 0; |arg c| < π/4; -1-δ < Re α < 1]

$$2. \int_0^{\infty} x^{\alpha-1} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} \operatorname{erfc}(cx) dx = U(0)$$

$[b > 0; |\arg c| < \pi/4; \operatorname{Re} \alpha > -\delta; U(\gamma) \text{ см. в 2.8.11.1}]$

$$3. \int_0^{\infty} \begin{Bmatrix} x^{-2} \sin bx \\ x^{-1} \cos bx \end{Bmatrix} \operatorname{erf}(cx) dx = -\frac{b^{(1 \pm 1)/2}}{2} \operatorname{Ei}\left(-\frac{b^2}{4c^2}\right) + \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} c \sqrt{\pi} \operatorname{erf}\left(\frac{b}{2c}\right)$$

$[b > 0; |\arg c| < \pi/4]$

$$4. \int_0^{\infty} x \cos bx \operatorname{erfc}(cx) dx = \frac{1}{2c^2} \exp\left(-\frac{b^2}{4c^2}\right) - \frac{1}{b^2} \left[1 - \exp\left(-\frac{b^2}{4c^2}\right)\right]$$

$[b > 0; |\arg c| < \pi/4]$

$$5. \int_0^{\infty} \begin{Bmatrix} \sin^{2n} bx \\ \cos^{2n} bx \end{Bmatrix} \operatorname{erfc}(cx) dx =$$

$$= \frac{1}{2^{2n} b} \left[\binom{2n}{n} \frac{b}{\sqrt{\pi} c} - i \sum_{k=0}^{n-1} \frac{(-1)^{n-k}}{n-k} \binom{2n}{k} \exp\left(-b^2 \frac{(n-k)^2}{c^2}\right) \operatorname{erf}\left(ib \frac{n-k}{c}\right) \right]$$

$[b > 0; |\arg c| < \pi/4, n = 1, 2, 3, \dots]$

$$6. \int_0^{\infty} \begin{Bmatrix} \sin^{2n+1} bx \\ \cos^{2n+1} bx \end{Bmatrix} \operatorname{erfc}(cx) dx = \frac{1}{2^{2n} b} \sum_{k=0}^n \frac{(-1)^{n+k}}{2n-2k+1} \binom{2n+1}{k} \begin{Bmatrix} A \\ B \end{Bmatrix}$$

$$\left[A = 1 - \exp\left(-\left(n-k+\frac{1}{2}\right)^2 \frac{b^2}{c^2}\right), \right.$$

$$\left. B = -i \exp\left(-\left(n-k+\frac{1}{2}\right)^2 \frac{b^2}{c^2}\right) \operatorname{erf}\left(i\left(n-k+\frac{1}{2}\right) \frac{b}{c}\right); b > 0; |\arg c| < \pi/4 \right].$$

$$7. \int_0^{\infty} x^{\alpha-1} \begin{Bmatrix} \sin^{2n} bx \\ \cos^{2n} bx \end{Bmatrix} \operatorname{erfc}(cx) dx = \frac{1}{2^{2n} \sqrt{\pi} \alpha c^{\alpha}} \Gamma\left(\frac{\alpha+1}{2}\right) \times$$

$$\times \left[2 \sum_{k=0}^{n-1} (-1)^{n-k} \binom{2n}{k} {}_2F_2\left(\frac{\alpha}{2}, \frac{\alpha+1}{2}; \frac{1}{2}, \frac{\alpha}{2}+1; -(n-k)^2 \frac{b^2}{c^2}\right) + \binom{2n}{n} \right]$$

$[b > 0; \operatorname{Re} \alpha > -n \mp n; |\arg c| < \pi/4, n = 1, 2, 3, \dots]$

$$8. \int_0^{\infty} x^{\alpha-1} \begin{Bmatrix} \sin^{2n+1} bx \\ \cos^{2n+1} bx \end{Bmatrix} \operatorname{erfc}(cx) dx =$$

$$= \frac{b^{\delta}}{2^{2n} \sqrt{\pi} (\alpha+\delta) c^{\alpha+\delta}} \Gamma\left(\frac{\alpha+\delta+1}{2}\right) \sum_{k=0}^n (-1)^{n+k} \binom{2n+1}{k} \begin{Bmatrix} 2n-2k+1 \\ 1 \end{Bmatrix} \times$$

$$\times {}_2F_2\left(\frac{\alpha+1}{2}, \frac{\alpha}{2}+\delta; \frac{1}{2}+\delta, \frac{\alpha+\delta}{2}+1; -\left(n-k+\frac{1}{2}\right)^2 \frac{b^2}{c^2}\right)$$

$$\left[b > 0; \operatorname{Re} \alpha > \begin{Bmatrix} -2n-3 \\ 0 \end{Bmatrix}; |\arg c| < \pi/4 \right].$$

$$9. \int_0^{\infty} \frac{1}{x} \cos^{2n+1} bx \operatorname{erf}(cx) dx = -2^{-2n-1} \sum_{k=0}^n \binom{2n+1}{k} \operatorname{Ei}\left[-\left(n-k+\frac{1}{2}\right)^2 \frac{b^2}{c^2}\right]$$

$[b > 0; |\arg c| < \pi/4]$

$$10. \int_0^{\infty} \frac{1}{x^2} \sin^{2n+1} bx \operatorname{erf}(cx) dx = \frac{(-1)^{n+1}}{2^{2n}} \sum_{k=0}^n (-1)^{k+1} \times$$

$$\times \binom{2n+1}{k} \left\{ \sqrt{\pi} c \operatorname{erf}\left(\frac{2n-2k+1}{2} \frac{b}{c}\right) - \left(n-k+\frac{1}{2}\right) b \operatorname{Ei}\left[-\left(n-k+\frac{1}{2}\right)^2 \frac{b^2}{c^2}\right] \right\}$$

$$[b > 0, |\arg c| < \pi/4]$$

$$11. \int_0^{\infty} \frac{\sin^2 bx}{x} \operatorname{erfc}(cx) dx = \frac{1}{4} \left[C + 2 \ln \frac{b}{c} - \operatorname{Ei} \left(-\frac{b^2}{c^2} \right) \right]$$

$$[b > 0; |\arg c| < \pi/4]$$

$$12. \int_0^{\infty} \left\{ \begin{matrix} \sin (bx+a) \\ \cos (bx+a) \end{matrix} \right\} \operatorname{erfc}(cx) dx =$$

$$= \frac{1}{b} \left[\left\{ \begin{matrix} \cos a \\ \sin a \end{matrix} \right\} \mp \exp \left(-\frac{b^2}{4c^2} \right) \left(\left\{ \begin{matrix} \cos a \\ \sin a \end{matrix} \right\} \pm i \left\{ \begin{matrix} \sin a \\ \cos a \end{matrix} \right\} \operatorname{erf} \left(\frac{ib}{2c} \right) \right) \right] \quad [a, b, c > 0]$$

$$13. \int_{-\infty}^{\infty} \frac{1}{x} \left\{ \begin{matrix} \sin (bx+a) \\ \cos (bx+a) \end{matrix} \right\} \operatorname{erf}(cx) dx = -\operatorname{Ei} \left(-\frac{b^2}{4c^2} \right) \left\{ \begin{matrix} \sin a \\ \cos a \end{matrix} \right\}$$

$$[a, b > 0; |\arg c| < \pi/4]$$

$$14. \int_0^{\infty} x^{-n} (\cos ax - \cos bx) \operatorname{erf}(cx) dx = I_n$$

$$[a, b > 0; |\arg c| < \pi/4]$$

$$I_2 = \frac{1}{2\sqrt{\pi}c} \left[b^2 {}_3F_3 \left(\frac{1}{2}, 1, 1; \frac{3}{2}, \frac{3}{2}, 2; -\frac{b^2}{4c^2} \right) - \right. \\ \left. - a^2 {}_3F_3 \left(\frac{1}{2}, 1, 1; \frac{3}{2}, \frac{3}{2}, 2; -\frac{a^2}{4c^2} \right) \right]$$

$$I_2 = \sqrt{\pi}c \left[b \operatorname{erf} \left(\frac{b}{2c} \right) - a \operatorname{erf} \left(\frac{a}{2c} \right) \right] + c^2 \left[\exp \left(-\frac{b^2}{4c^2} \right) - \right. \\ \left. - \exp \left(-\frac{a^2}{4c^2} \right) \right] - \frac{1}{4} \left[b^2 \operatorname{Ei} \left(-\frac{b^2}{4c^2} \right) - a^2 \operatorname{Ei} \left(-\frac{a^2}{4c^2} \right) \right].$$

$$15. \int_0^{\infty} \frac{1}{x} \left\{ \begin{matrix} \sin ax \sin bx \\ \cos ax \cos bx \end{matrix} \right\} \operatorname{erf}(cx) dx = -\frac{1}{4} \left[\operatorname{Ei} \left(-\frac{(b-a)^2}{4c^2} \right) \mp \operatorname{Ei} \left(-\frac{(b+a)^2}{4c^2} \right) \right]$$

$$[a, b > 0; a \neq b; |\arg c| < \pi/4]$$

$$16. \int_0^{\infty} \left\{ \begin{matrix} \sin bx^2 \\ \cos bx^2 \end{matrix} \right\} \operatorname{erf}(cx) dx = \frac{1}{4\sqrt{2\pi}b} \left(\pm \ln \frac{b+c^2+c\sqrt{2b}}{b+c^2-c\sqrt{2b}} + 2 \operatorname{arctg} \frac{c\sqrt{2b}}{b-c^2} \right)$$

$$[b > 0; |\arg c| < \pi/4]$$

$$17. \int_0^{\infty} \left\{ \begin{matrix} \sin c^2 x^2 \\ \cos c^2 x^2 \end{matrix} \right\} \operatorname{erf}(cx) dx = \frac{1}{4c\sqrt{2\pi}} [\pi + 2 \ln (3 \pm 2\sqrt{2})]$$

$$[c > 0]$$

$$18. \int_0^{\infty} x^{\alpha-1} \left\{ \begin{matrix} \sin bx^2 \\ \cos bx^2 \end{matrix} \right\} \operatorname{erf}(cx) dx =$$

$$= -\frac{b^0}{\sqrt{\pi}(\alpha+2\delta)c^{\alpha+2\delta}} \Gamma \left(\frac{1+\alpha}{2} + \delta \right) {}_3F_3 \left(\frac{\alpha+3}{4}, \frac{\alpha+2\delta}{4}, \delta + \frac{\alpha+1}{4}; \right. \\ \left. \frac{\alpha+2\delta}{4} + 1, \delta + \frac{1}{2}; -\frac{b^2}{c^4} \right) + \frac{1}{2b^{\alpha/2}} \Gamma \left(\frac{\alpha}{2} \right) \left\{ \begin{matrix} \sin (\alpha\pi/4) \\ \cos (\alpha\pi/4) \end{matrix} \right\}$$

$$[b > 0, -(2 \pm 1) < \operatorname{Re} \alpha < 2; |\arg c| < \pi/4]$$

$$19. \int_0^{\infty} \frac{1}{x^2} \sin bx^2 \operatorname{erf}(cx) dx =$$

$$= \frac{c}{\sqrt{\pi}} \operatorname{arctg} \frac{b}{c^2} + \sqrt{\frac{b}{2\pi}} \operatorname{arctg} \frac{c\sqrt{2b}}{b-c^2} + \frac{1}{2} \sqrt{\frac{b}{2\pi}} \ln \frac{b+c^2-c\sqrt{2b}}{b+c^2+c\sqrt{2b}}$$

$$[b > 0; |\arg c| < \pi/4]$$

$$20. \int_0^{\infty} x \begin{Bmatrix} \sin bx^2 \\ \cos bx^2 \end{Bmatrix} \operatorname{erfc}(cx) dx = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{1}{2b} \mp \frac{c(\sqrt{b^2 + c^4} \mp c^2)^{-1/2}}{2\sqrt{2}(b^2 + c^4)}$$

[$b > 0, |\arg c| < \pi/4$]

$$21. \int_0^{\infty} x \cos^{2n+1} bx^2 \operatorname{erfc}(cx) dx =$$

$$= \frac{c}{2^{2n+3/2}} \sum_{k=0}^n \frac{\binom{2n+1}{k}}{\sqrt{c^4 + (2n-2k+1)^2 b^2} \sqrt{c^2 + \sqrt{c^4 + (2n-2k+1)^2 b^2}}}$$

[$b > 0, |\arg c| < \pi/4$]

$$22. \int_0^{\infty} \frac{1}{x^3} \begin{Bmatrix} \sin(b/x^2) \\ \cos(b/x^2) \end{Bmatrix} \operatorname{erf}(cx) dx = \frac{1}{2b} \left[\begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \mp e^{-c\sqrt{2b}} \begin{Bmatrix} \cos(c\sqrt{2b}) \\ \sin(c\sqrt{2b}) \end{Bmatrix} \right]$$

[$b > 0; |\arg c| < \pi/4$]

$$23. \int_0^{\infty} \operatorname{tg} bx \operatorname{erfc}(cx) dx = \frac{1}{b} \left[\sum_{k=1}^{\infty} \frac{(-1)^k}{k} \exp\left(-\frac{k^2 b^2}{c^2}\right) + \ln 2 \right]$$

[$b > 0; |\arg c| < \pi/4$]

$$24. \int_0^{\infty} \frac{\sin nbx}{\sin bx} \operatorname{erfc}(cx) dx = \frac{1}{\sqrt{\pi}c} \left(n - 2 \left[\frac{n}{2} \right] \right) - \frac{2i}{b} \sum_{k=1}^{\lfloor n/2 \rfloor} \frac{1}{l_k} \exp\left(-\frac{l_k^2 b^2}{4c^2}\right) \operatorname{erf}\left(\frac{il_k b}{2c}\right)$$

[$l_k = \begin{cases} 2k, & n = 1, 3, 5, \dots, \\ 2k-1, & n = 2, 4, 6, \dots, \end{cases} \quad b > 0; |\arg c| < \pi/4$]

$$25. \int_0^{\infty} \frac{\sin 2nbx}{\cos bx} \operatorname{erfc}(cx) dx = \frac{2}{b} \sum_{k=1}^n \frac{(-1)^{n-k}}{2k-1} \left[1 - \exp\left(-\left(k - \frac{1}{2}\right)^2 \frac{b^2}{c^2}\right) \right]$$

[$b > 0; |\arg c| < \pi/4$]

$$26. \int_0^{\infty} \frac{\cos(2n+1)bx}{\cos bx} \operatorname{erfc}(cx) dx = \frac{(-1)^n}{\sqrt{\pi}c} - \frac{i}{b} \sum_{k=1}^n \frac{(-1)^{n-k}}{k} \exp\left(-\frac{k^2 b^2}{c^2}\right) \operatorname{erf}\left(\frac{ikb}{c}\right)$$

[$b > 0; |\arg c| < \pi/4$]

$$27. \int_0^{\infty} \frac{\sin(2n+1)bx}{x \sin bx} [\operatorname{erf}(ax) - \operatorname{erf}(cx)] dx =$$

$$= \ln \frac{a}{c} + \sum_{k=1}^n \left[\operatorname{Ei}\left(-\frac{k^2 b^2}{c^2}\right) - \operatorname{Ei}\left(-\frac{k^2 b^2}{a^2}\right) \right]$$

[$b > 0; |\arg a|, |\arg c| < \pi/4$]

$$28. \int_0^{\infty} \frac{\sin(4n+1)bx}{x \cos bx} [\operatorname{erf}(ax) - \operatorname{erf}(cx)] dx =$$

$$= \ln \frac{a}{c} + \sum_{k=1}^{2n} (-1)^k \left[\operatorname{Ei}\left(-\frac{k^2 b^2}{c^2}\right) - \operatorname{Ei}\left(-\frac{k^2 b^2}{a^2}\right) \right]$$

[$b > 0; |\arg a|, |\arg c| < \pi/4$]

$$29. \int_0^{\infty} \frac{1}{x} \begin{Bmatrix} \sin^2 bx \\ \cos^2 bx \end{Bmatrix} [\operatorname{erf}(ax) - \operatorname{erf}(cx)] dx = \frac{1}{2} \ln \frac{a}{c} \pm \frac{1}{4} \left[\operatorname{Ei}\left(-\frac{b^2}{a^2}\right) - \operatorname{Ei}\left(-\frac{b^2}{c^2}\right) \right]$$

[$b > 0; |\arg a|, |\arg c| < \pi/4$]

2.8.12. Интегралы, содержащие тригонометрические функции и $\begin{Bmatrix} \operatorname{erf}(cx+d) \\ \operatorname{erfc}(cx+d) \end{Bmatrix}$.

$$1. \int_0^{\infty} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} \operatorname{erfc}(cx+a) dx = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{1}{b} \operatorname{erfc}(a) - \\ - \begin{Bmatrix} 1 \\ i \end{Bmatrix} \frac{1}{2b} \exp\left(-\frac{b^2}{4c^2}\right) \left[\pm e^{iab/c} \operatorname{erfc}\left(a+i\frac{b}{2c}\right) + e^{-iab/c} \operatorname{erfc}\left(a-i\frac{b}{2c}\right) \right] \\ [b, c > 0]$$

$$2. \int_0^{\infty} \begin{Bmatrix} \sin^{2n} bx \\ \cos^{2n} bx \end{Bmatrix} [\operatorname{erf}(cx+a) - \operatorname{erf}(cx-a)] dx = \\ = \frac{1}{2^{2n-1}b} \left[\binom{2n}{n} \frac{ab}{c} + \sum_{k=0}^{n-1} \frac{(-1)^{n-k}}{n-k} \binom{2n}{k} \exp\left(-\frac{(n-k)^2 b^2}{c^2}\right) \sin \frac{2(n-k)ab}{c} \right] \\ [a, b, c > 0].$$

$$3. \int_0^{\infty} \cos^{2n+1} bx [\operatorname{erf}(cx+a) - \operatorname{erf}(cx-a)] dx = \\ = \frac{1}{2^{2n-1}b} \sum_{k=0}^n \frac{1}{2n-2k+1} \binom{2n+1}{k} \exp\left[-\frac{(2n-2k+1)^2 b^2}{4c^2}\right] \sin \frac{(2n-2k+1)ab}{c} \\ [a, b, c > 0].$$

2.8.13. Интегралы, содержащие $\begin{Bmatrix} \operatorname{erf}(\varphi(\sin x, \cos x)) \\ \operatorname{erfc}(\varphi(\sin x, \cos x)) \end{Bmatrix}$.

$$1. \int_0^{\pi/2} \sin(2n+1)x \operatorname{erf}(c \sin x) dx = \frac{\sqrt{\pi} c}{2(2n+1)} e^{-c^2/2} \left[I_n\left(\frac{c^2}{2}\right) + I_{n+1}\left(\frac{c^2}{2}\right) \right].$$

$$2. \int_0^{\pi/2} \begin{Bmatrix} \sin 2x \\ \cos 2x \end{Bmatrix} \operatorname{erf}(c \operatorname{tg} x) dx = \pm \pi^{\pm 1/2} c e^{c^2} \begin{Bmatrix} \operatorname{erfc}(c) \\ \operatorname{Ei}(-c^2) \end{Bmatrix}.$$

$$3. \int_0^{\pi/2^{\sigma}} \cos bx \operatorname{erf}(c \sin x) dx = \\ = \frac{2^{2-\sigma} c \cos^{2-\sigma}(b\pi/2)}{\sqrt{\pi} (1-b^2)} {}_2F_2\left(\frac{1}{2}, 1; \frac{3-b}{2}, \frac{3+b}{2}; -c^2\right) \quad [\sigma=0 \text{ или } 1].$$

$$4. \int_0^{\pi} \sin nx \operatorname{erf}(c \sin x) dx = \frac{1-(-1)^n}{2} \frac{c^n}{n!} \Gamma\left(\frac{n}{2}\right) {}_1F_1\left(\frac{n}{2}; n+1; -c^2\right).$$

$$5. \int_0^{\pi} \frac{1}{\sin x} \operatorname{erf}\left(\frac{c \sin x}{a+b \cos x}\right) dx = \frac{2c \sqrt{\pi}}{\sqrt{a^2-b^2}} {}_2F_2\left(\frac{1}{2}, \frac{1}{2}; 1, \frac{3}{2}; -\frac{c^2}{b^2-a^2}\right) \\ [a > b > 0; c > 0]$$

$$6. \int_0^{n\pi} x \operatorname{erf}(c \sin x) dx = (-1)^{n+1} 2 \sqrt{\pi} n c {}_2F_2\left(\frac{1}{2}, 1; \frac{3}{2}, \frac{3}{2}; -c^2\right).$$

$$7. \int_{m\pi}^{n\pi} \frac{x}{\cos x} \operatorname{erf}(c \cos x) dx = \pi^{3/2} (n^2-m^2) c {}_2F_2\left(\frac{1}{2}, \frac{1}{2}; 1, \frac{3}{2}; -c^2\right).$$

2.8.14. Интегралы, содержащие x^α , e^{px^2} , тригонометрические функции и $\left\{ \begin{matrix} \operatorname{erf}(cx+d) \\ \operatorname{erfc}(cx+d) \end{matrix} \right\}$.

$$1. \int_0^\infty e^{px^2} \sin bx \operatorname{erfc}(cx) dx = \frac{1}{2} \sqrt{\frac{\pi}{p}} \exp\left(\frac{b^2}{4p}\right) \left[\operatorname{erf}\left(\frac{bc}{2\sqrt{c^2p-p^2}}\right) - \operatorname{erf}\left(\frac{b}{2\sqrt{p}}\right) \right]$$

[$b, \operatorname{Re}(c^2 - p) > 0; |\arg c| < \pi/4$].

$$2. \int_0^\infty x e^{-px^2} \cos bx \operatorname{erf}(cx) dx =$$

$$= \frac{c}{2p\sqrt{c^2+p}} \exp\left(-\frac{c^2}{4c^2+4p}\right) + \frac{ib\sqrt{\pi}}{4p^{3/2}} \exp\left(-\frac{b^2}{4p}\right) \operatorname{erf}\left(\frac{ibc}{2\sqrt{c^2p+p^2}}\right)$$

[$b, \operatorname{Re} p > 0; |\arg c| < \pi/4$].

$$3. \int_0^\infty x^{2n} e^{-px^2} \sin bx \operatorname{erf}(cx) dx =$$

$$= (-1)^{n+1} i \frac{\sqrt{\pi}}{2\sqrt{p}} \frac{\partial^{2n}}{\partial b^{2n}} \left[\exp\left(-\frac{b^2}{4p}\right) \operatorname{erf}\left(\frac{ibc}{2\sqrt{c^2p+p^2}}\right) \right]$$

[$b, \operatorname{Re} p > 0; |\arg c| < \pi/4$].

$$4. \int_0^\infty e^{-c^2x^2} \sin bx \operatorname{erf}(icx) dx = \frac{\sqrt{\pi} i}{2c} \exp\left(-\frac{b^2}{4c^2}\right)$$

[$b > 0; |\arg c| < \pi/4$]

$$5. \int_0^\infty e^{c^2x^2} \left\{ \begin{matrix} \sin bx \\ \cos bx \end{matrix} \right\} \operatorname{erfc}(cx) dx = \pm \frac{\pi^{\pm 1/2}}{2c} \exp\left(\frac{b^2}{4c^2}\right) \left\{ \begin{matrix} \operatorname{erfc}[b/(2c)] \\ \operatorname{Ei}[-b^2/(4c^2)] \end{matrix} \right\}$$

[$b > 0; |\arg c| < \pi/4$].

$$6. \int_0^\infty x^{\alpha-1} e^{-c^2x^2} \left\{ \begin{matrix} \sin bx \\ \cos bx \end{matrix} \right\} \operatorname{erf}(icx) dx = U(1)$$

[$b > 0; -1-\delta < \operatorname{Re} \alpha < 2; |\arg c| < \pi/4, \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$].

$$U(\gamma) = \mp \frac{i\gamma b^{1-\alpha} \Gamma(\alpha-1)}{c\sqrt{\pi}} \left\{ \begin{matrix} \cos(\alpha\pi/2) \\ \sin(\alpha\pi/2) \end{matrix} \right\} {}_2F_2\left(1, \frac{1}{2}; \frac{3-\alpha}{2}, 1-\frac{\alpha}{2}; (-1)^\gamma \frac{b^2}{4c^2}\right) \mp$$

$$\mp \frac{i\gamma b^\delta}{2c^{\alpha+\delta}} \Gamma\left(\frac{\alpha+\delta}{2}\right) \left\{ \begin{matrix} \cos(\alpha\pi/2) \\ \sin(\alpha\pi/2) \end{matrix} \right\}^\gamma \left\{ \begin{matrix} \operatorname{cosec}(\alpha\pi/2) \\ \sec(\alpha\pi/2) \end{matrix} \right\} {}_1F_1\left(\frac{\alpha+\delta}{2}; \delta+\frac{1}{2}; (-1)^\gamma \frac{b^2}{4c^2}\right).$$

$$7. \int_0^\infty x^{\alpha-1} e^{c^2x^2} \left\{ \begin{matrix} \sin bx \\ \cos bx \end{matrix} \right\} \operatorname{erfc}(cx) dx = U(0)$$

[$b > 0; -\delta < \operatorname{Re} \alpha < 2; |\arg c| < \pi/4; \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}, U(\gamma) \text{ см. в 2.8.14.6}].$

$$8. \int_0^\infty \frac{1}{x} e^{-kc^2x^2} \cos bx \operatorname{erf}(icx) dx = \frac{\pi i}{2k} \operatorname{erfc}\left(\frac{b}{2\sqrt{k}c}\right)$$

[$k = 1, 2; b > 0; |\arg c| < \pi/4$].

$$9. \int_0^\infty \frac{1}{x^2} e^{-kc^2x^2} \sin bx \operatorname{erf}(icx) dx = I_k$$

[$b > 0; |\arg c| < \pi/4$].

$$I_1 = i \sqrt{\pi} c \left[1 - \exp \left(-\frac{b^2}{4c^2} \right) \right] + \frac{\pi i b}{2} \operatorname{erfc} \left(\frac{b}{2c} \right),$$

$$I_2 = \frac{\pi i b}{4} \left[1 - \operatorname{erf}^2 \left(\frac{b}{2 \sqrt{2} c} \right) \right] + \sqrt{\pi} i c \operatorname{erf} \left(\frac{b}{2c} \right) - i c \exp \left(-\frac{b^2}{8c^2} \right) \operatorname{erf} \left(\frac{b}{2 \sqrt{2} c} \right).$$

$$10. \int_0^{\infty} e^{-px^2} \sin^{2n+1} bx \operatorname{erf}(cx) dx =$$

$$= \frac{i \sqrt{\pi}}{2^{2n+1} \sqrt{p}} \sum_{k=0}^n (-1)^{n+k-1} \binom{2n+1}{k} \exp \left[-\left(n-k+\frac{1}{2} \right)^2 \frac{b^2}{p} \right] \operatorname{erf} \left(i b c \frac{2n-2k+1}{2 \sqrt{c^2 p + p^2}} \right)$$

[b, Re p > 0; |arg c| < π/4].

$$11. \int_0^{\infty} e^{-c^2 x^2} \sin^{2n+1} bx \operatorname{erf}(icx) dx =$$

$$= \frac{i \sqrt{\pi}}{2^{2n+1} c} \sum_{k=0}^n (-1)^{n+k} \binom{2n+1}{k} \exp \left[-\left(n-k+\frac{1}{2} \right)^2 \frac{b^2}{c^2} \right]$$

[b > 0; |arg c| < π/4].

$$12. \int_0^{\infty} \frac{1}{x} e^{-c^2 x^2} \sin^2 bx \operatorname{erf}(icx) dx = \frac{\pi i}{4} \operatorname{erf} \left(\frac{b}{c} \right)$$

[b > 0; |arg c| < π/4].

$$13. \int_0^{\infty} x^{\alpha-1} e^{-c^2 x^2} \left\{ \begin{matrix} \sin bx^2 \\ \cos bx^2 \end{matrix} \right\} \operatorname{erf}(icx) dx = V(1)$$

$$\left[b > 0; -1-2\delta < \operatorname{Re} \alpha < 3; |\arg c| < \pi/4, \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right].$$

$$V(\gamma) = \frac{i \gamma b^{(1-\alpha)/2}}{2c \sqrt{\pi}} \Gamma \left(\frac{\alpha-1}{2} \right) \left\{ \begin{matrix} \sin [(\alpha-1) \pi/4] \\ \cos [(\alpha-1) \pi/4] \end{matrix} \right\} \times$$

$$\times {}_3F_2 \left(1, \frac{1}{4}, \frac{3}{4}; \frac{5-\alpha}{4}, \frac{3-\alpha}{4}; -\frac{b^2}{c^4} \right) +$$

$$+ \frac{(-i) \gamma b^{(3-\alpha)/2}}{4c^3 \sqrt{\pi}} \Gamma \left(\frac{\alpha-3}{2} \right) \left\{ \begin{matrix} \sin [(\alpha+1) \pi/4] \\ \cos [(\alpha+1) \pi/4] \end{matrix} \right\} \times$$

$$\times {}_3F_2 \left(1, \frac{3}{4}, \frac{5}{4}; \frac{5-\alpha}{4}, \frac{7-\alpha}{4}; -\frac{b^2}{c^4} \right) \mp$$

$$\mp \frac{(\mp i) \gamma b^{\delta}}{2c^{\alpha+2\delta}} \Gamma \left(\frac{\alpha}{2} + \delta \right) \frac{\sin^{\gamma}(\alpha\pi/2)}{\cos(\alpha\pi/2)} {}_2F_1 \left(\frac{\alpha+2}{4}, \frac{\alpha}{4} + \delta; \delta + \frac{1}{2}; -\frac{b^2}{c^4} \right).$$

$$14. \int_0^{\infty} x^{\alpha-1} e^{c^2 x^2} \left\{ \begin{matrix} \sin bx^2 \\ \cos bx^2 \end{matrix} \right\} \operatorname{erfc}(cx) dx = V(0)$$

$$\left[b > 0; -2\delta < \operatorname{Re} \alpha < 3; |\arg c| < \pi/4, \quad = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}, V(\gamma) \text{ см. в 2.8.14.13} \right].$$

$$15. \int_0^{\infty} x e^{-px^2} \left\{ \begin{matrix} \sin bx^2 \\ \cos bx^2 \end{matrix} \right\} \operatorname{erf}(cx) (dx) =$$

$$= \frac{c}{2 \sqrt{2}} \frac{b \sqrt{V(c^2+p)^2 + b^2} \pm c^2 \pm p \mp p \sqrt{V(c^2+p)^2 + b^2} \mp c^2 \mp p}{(c^2+p) \sqrt{(c^2+p)^2 + b^2}}$$

$$[b, \operatorname{Re} p > 0; |\arg c| < \pi/4].$$

$$16. \int_0^{\infty} e^{c^2 x^2} \cos bx \operatorname{erfc}(cx + a) dx = -\frac{1}{2\sqrt{\pi}c} \exp\left(\frac{b^2}{4c^2}\right) \operatorname{Ei}\left(-a^2 - \frac{b^2}{4c^2}\right) \\ [a, b > 0; |\arg c| < \pi/4]$$

$$17. \int_0^{\infty} e^{c^2 x^2} \sin bx \operatorname{erfc}\left(cx + \frac{b}{2c}\right) dx = \frac{\sqrt{\pi}}{4c} \left[1 - \operatorname{erf}^2\left(\frac{b}{2c}\right)\right] \\ [b > 0; |\arg c| < \pi/4]$$

2.8.15. Интегралы, содержащие гиперболические, тригонометрические функции и $\operatorname{erfc}(cx + d)$

$$1. \int_0^{\infty} \left\{ \frac{\operatorname{sh} bx \sin bx}{\operatorname{ch} bx \cos bx} \right\} \operatorname{erfc}(cx) dx = \frac{1}{b} \left[\left\{ \frac{\sin \varphi}{\cos \varphi} \right\} C(\varphi) \mp \left\{ \frac{\cos \varphi}{\sin \varphi} \right\} S(\varphi) \right] \\ [\varphi = b^2/(2c^2) \quad |\arg c| < \pi/4]$$

$$2. \int_0^{\infty} \left\{ \frac{\operatorname{sh} ax \cos bx}{\operatorname{ch} ax \sin bx} \right\} \operatorname{erfc}(cx) dx = \\ = \frac{1}{a^2 + b^2} \left[\exp\left(\frac{a^2 - b^2}{4c^2}\right) \left(\left\{ \frac{b}{a} \right\} \sin \frac{ab}{2c} \pm \left\{ \frac{a}{b} \right\} \cos \frac{ab}{2c} \right) \mp \left\{ \frac{a}{b} \right\} \right] \\ [|\arg c| < \pi/4]$$

$$3. \int_0^{\infty} \left\{ \frac{\operatorname{sh} bx \sin bx}{\operatorname{ch} bx \cos bx} \right\} \operatorname{erfc}(cx) dx = \\ = \frac{1}{2b} \left[\frac{b^2}{c^2} \left\{ \frac{\sin [b^2/(2c^2)]}{\cos [b^2/(2c^2)]} \right\} \pm \left\{ \frac{\cos [b^2/(2c^2)]}{\sin [b^2/(2c^2)]} \right\} - \left\{ \frac{1}{0} \right\} \right] \\ [|\arg c| < \pi/4]$$

$$4. \int_0^{\infty} \left\{ \frac{\operatorname{sh} bx}{\operatorname{ch} bx} \right\} \operatorname{erfc}(cx + a) dx = \\ = -\frac{1}{b} \left\{ \frac{1}{0} \right\} \operatorname{erfc}(a) + \frac{1}{2b} \exp\left(\frac{b^2}{4c^2}\right) \left[\pm e^{ab/c} \operatorname{erfc}\left(a + \frac{b}{2c}\right) + e^{-ab/c} \operatorname{erfc}\left(a - \frac{b}{2c}\right) \right] \\ [|\arg c| < \pi/4]$$

2.8.16. Интегралы, содержащие логарифмическую функцию и $\left\{ \frac{\operatorname{erf}(cx)}{\operatorname{erfc}(cx)} \right\}$.

$$1. \int_0^{\infty} x^{\alpha-1} \ln^n x \operatorname{erf}(cx) dx = \\ = \frac{(-1)^n n!}{(\alpha+1)^{n+1}} \frac{2c}{\sqrt{\pi}} {}_{n+2}F_{n+2} \left(\frac{1}{2}, \frac{\alpha+1}{2}, \dots, \frac{\alpha+1}{2}; \frac{3}{2}, \frac{\alpha+3}{2}, \dots, \frac{\alpha+3}{2}; -c^2 \right) \\ [\operatorname{Re} \alpha > -1]$$

$$2. \int_0^{\infty} x^{\alpha-1} \ln^n x \operatorname{erf}(cx) dx = -\frac{1}{\sqrt{\pi}} \frac{\partial^n}{\partial \alpha^n} \left[\frac{c^{-\alpha}}{\alpha} \Gamma\left(\frac{\alpha+1}{2}\right) \right] \\ [-1 < \operatorname{Re} \alpha < 0; |\arg c| < \pi/4]$$

$$3. \int_0^{\infty} \ln^n x \operatorname{erfc}(cx) dx = I_n \\ [|\arg c| < \pi/4]$$

$$I_1 = -\frac{1}{\sqrt{\pi}c} \left(1 + \frac{C}{2} + \ln c \right),$$

$$I_2 = \frac{1}{\sqrt{\pi}c} \left[\frac{C^2}{4} + C(\ln c + 1) + \frac{\pi^2}{24} + 2 + \ln c(2 + \ln c) \right],$$

$$I_3 = -\frac{1}{8\sqrt{\pi}c} \left[C^3 + 6C^2 + C \left(\frac{\pi^2}{2} + 24 \right) + \pi^2 + 48 - \psi''(1) + \right. \\ \left. + (6C^2 + 24C + \pi^2 + 48) \ln c + 12(C+2) \ln^2 c + 8 \ln^3 c \right]$$

$$4. \int_0^\infty x^{\alpha-1} \ln(x^2 + z^2) \left\{ \begin{matrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{matrix} \right\} dx = U(1)$$

$$\left[\operatorname{Re} z > 0; |\arg c| < \pi/4, \left\{ \begin{matrix} -1 < \operatorname{Re} \alpha < 0 \\ 0 < \operatorname{Re} \alpha \end{matrix} \right\} \right]$$

$$U(\gamma) = \mp \frac{(-1)^\gamma 2\sqrt{\pi} c z^{\alpha+1} \sin^{1-\gamma}(\alpha\pi/2)}{(\alpha+1) \cos(\alpha\pi/2)} {}_2F_2 \left(\frac{1}{2}, \frac{\alpha+1}{2}; \frac{3}{2}, \frac{\alpha+3}{2}; (-1)^{1-\gamma} c^2 z^2 \right) \mp \\ \mp \frac{(-1)^\gamma c^{2-\alpha} z^2}{(2-\alpha)\sqrt{\pi}} \Gamma\left(\frac{\alpha-1}{2}\right) {}_3F_3 \left(1 - \frac{\alpha}{2}, 1, 1; 2, 2 - \frac{\alpha}{2}, \frac{3-\alpha}{2}; (-1)^{1-\gamma} c^2 z^2 \right) \pm \\ \pm \frac{1}{\alpha c^\alpha \sqrt{\pi}} \Gamma\left(\frac{\alpha+1}{2}\right) \left[\frac{2}{\alpha} - \psi\left(\frac{\alpha+1}{2}\right) + 2 \ln c \right] + \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{\pi z^\alpha}{\alpha \sin(\alpha\pi/2)}.$$

$$5. \int_0^\infty x^{\alpha-1} \ln(x^2 - z^2) \left\{ \begin{matrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{matrix} \right\} dx = U(0)$$

$$\left[z > 0, \arg c < \pi/4, \left\{ \begin{matrix} -1 < \operatorname{Re} \alpha < 0 \\ 0 < \operatorname{Re} \alpha \end{matrix} \right\}, U(\gamma) \text{ см в 2.8.16.4} \right]$$

$$6. \int_0^\infty \ln \left| \frac{x^2 - y^2}{x^2 + y^2} \right| \operatorname{erfc}(cx) dx = \frac{2cy^2}{\sqrt{\pi}(c^2 y^2 + 1)} - \\ - \frac{2y}{\sqrt{\pi}} \operatorname{arctg} \frac{1}{cy} - \frac{2}{\sqrt{\pi}c} [\operatorname{sh}_1(c^2 y^2) \operatorname{ch}(c^2 y^2) - \operatorname{chi}(c^2 y^2) \operatorname{sh}(c^2 y^2)]$$

$$[y > 0; |\arg c| < \pi/4]$$

$$7. \int_0^\infty x^{\alpha-1} e^{-px^2} \ln(bx) \operatorname{erf}(cx) dx = \\ = -\frac{c^{-\alpha}}{2\sqrt{\pi}\alpha} \Gamma\left(\frac{\alpha+1}{2}\right) \left[\psi\left(\frac{\alpha+1}{2}\right) {}_2F_1\left(\frac{\alpha+1}{2}, \frac{\alpha}{2}; \frac{\alpha}{2} + 1; -\frac{p}{c^2}\right) - \right. \\ \left. - \frac{c^\alpha}{(c^2 + p)^{\alpha/2}} \ln \frac{c^2 + p}{b^2} {}_2F_1\left(\frac{1}{2}, \frac{\alpha}{2}; \frac{\alpha}{2} + 1; \frac{p}{c^2 + p}\right) - \right. \\ \left. - \frac{2c^\alpha}{\alpha(c^2 + p)^{\alpha/2}} {}_3F_2\left(\frac{1}{2}, \frac{\alpha}{2}, \frac{\alpha}{2}; \frac{\alpha}{2} + 1, \frac{\alpha}{2} + 1; \frac{p}{c^2 + p}\right) \right]$$

$$[\operatorname{Re} p, \operatorname{Re} \alpha > 0, |\arg b| < \pi, |\arg c| < \pi/4]$$

$$8. \int_0^\infty x e^{-px^2} \ln(cx) \operatorname{erfc}(cx) dx = \\ = -\frac{1}{2p} \left[\frac{C}{2} + 2 \ln \frac{c + \sqrt{c^2 + p}}{c} - c \frac{C + \ln 4 - 2 \ln c + \ln(c^2 + p)}{\sqrt{c^2 + p}} \right]$$

$$[\operatorname{Re} p > 0, |\arg c| < \pi/4]$$

2.8.17. Интегралы, содержащие обратные тригонометрические функции и $\left\{ \begin{matrix} \operatorname{erf}(\varphi(x)) \\ \operatorname{erfc}(\varphi(x)) \end{matrix} \right\}$.

$$1. \int_0^1 \left\{ \begin{matrix} \arcsin x \\ \arccos x \end{matrix} \right\} \operatorname{erf}(cx) dx =$$

$$= \frac{\sqrt{\pi}}{2} c \left[\left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} {}_2F_2\left(\frac{1}{2}, 1; \frac{3}{2}, 2; -c^2\right) \mp \frac{1}{2} {}_2F_2\left(\frac{1}{2}, 1; 2, 2; -c^2\right) \right]$$

$$2. \int_0^1 x^{\alpha-1} \begin{cases} \arcsin x \\ \arccos x \end{cases} \operatorname{erf}(cx) dx = \begin{cases} 1 \\ 0 \end{cases} \frac{c \sqrt{\pi}}{\alpha+1} {}_2F_2 \left(\frac{1}{2}, \frac{\alpha+1}{2}; \frac{3}{2}, \frac{\alpha+3}{2}; -c^2 \right) \mp \\ \mp \frac{c}{\alpha+1} \Gamma \left[\frac{\alpha/2+1}{(\alpha+3)/2} \right] {}_3F_3 \left(\frac{1}{2}, \frac{\alpha+1}{2}, \frac{\alpha}{2}+1; \frac{3}{2}, \frac{\alpha+3}{2}, \frac{\alpha+3}{2}; -c^2 \right) \\ [\operatorname{Re} \alpha > -(3 \pm 1)/2]$$

$$3. \int_0^1 x^{\alpha-1} e^{c^2 x^2} \begin{cases} \arcsin x \\ \arccos x \end{cases} \operatorname{erf}(cx) dx = \begin{cases} 1 \\ 0 \end{cases} \frac{c \sqrt{\pi}}{\alpha+1} {}_2F_2 \left(1, \frac{\alpha+1}{2}; \frac{3}{2}, \frac{\alpha+3}{2}; c^2 \right) \mp \\ \mp \frac{c}{\alpha+1} \Gamma \left[\frac{\alpha/2+1}{(\alpha+3)/2} \right] {}_3F_3 \left(1, \frac{\alpha+1}{2}, \frac{\alpha}{2}+1; \frac{3}{2}, \frac{\alpha+3}{2}, \frac{\alpha+3}{2}; c^2 \right) \\ [\operatorname{Re} \alpha > -(3 \pm 1)/2].$$

$$4. \int_{-1}^1 \arccos x \operatorname{erf}(c \sqrt{1-x^2}) dx = \frac{\pi^{3/2} c}{2} {}_1F_1 \left(\frac{1}{2}; 2; -c^2 \right).$$

$$5. \int_{-1}^1 \frac{\arccos x}{(1-x^2)^\sigma} \operatorname{erf}(c \sqrt{1-x^2}) dx = \\ = 2^{2-2\sigma} \pi^{2\sigma-1/2} c {}_2F_2 \left(\frac{1}{2}, \frac{3}{2}-\sigma; 2-\sigma, \frac{3}{2}; -c^2 \right) \quad [\sigma = 1/2 \text{ или } 1].$$

$$6. \int_{-1}^1 e^{-c^2 x^2} \arccos x \operatorname{erf}(c \sqrt{1-x^2}) dx = \frac{\pi^{3/2}}{2c} (1 - e^{-c^2}).$$

$$7. \int_0^\infty x^{\alpha-1} \operatorname{arctg} x \operatorname{erf}(cx) dx = \frac{\sqrt{\pi} c}{2 \sin(\alpha\pi/2)} {}_2F_2 \left(\frac{1}{2}, \frac{\alpha+1}{2}; \frac{3}{2}, \frac{\alpha+3}{2}; c^2 \right) \\ [-2 < \operatorname{Re} \alpha < 0; |\arg c| < \pi/4]$$

2.8.18. Интегралы, содержащие $A(x)$ и $\prod_k \operatorname{erf}(\varphi_k(x))$.

$$1. \int_0^\infty x^{\alpha-1} \operatorname{erf}^2(cx) dx = \\ = -\frac{2}{(\alpha+1)\pi c^\alpha} \Gamma \left(\frac{\alpha}{2}+1 \right) {}_3F_2 \left(\frac{1}{2}, \frac{\alpha+1}{2}, \frac{\alpha}{2}+1; \frac{3}{2}, \frac{\alpha+3}{2}; -1 \right) - \\ - \frac{1}{\alpha \sqrt{\pi} c^\alpha} \Gamma \left(\frac{\alpha+1}{2} \right) \quad [-2 < \operatorname{Re} \alpha < 0; |\arg c| < \pi/4].$$

$$2. \int_0^\infty \frac{1}{x^2} \operatorname{erf}^2(cx) dx = \frac{4c}{\sqrt{\pi}} \ln(\sqrt{2}+1) \quad [|\arg c| < \pi/4]$$

$$3. \int_0^\infty x^{\alpha-1} \operatorname{erf}(bx) \operatorname{erf}(cx) dx = \\ = -\frac{2c}{\pi(\alpha+1)b^{\alpha+1}} \Gamma \left(\frac{\alpha}{2}+1 \right) {}_3F_2 \left(\frac{1}{2}, \frac{\alpha+1}{2}, \frac{\alpha}{2}+1; \frac{3}{2}, \frac{\alpha+3}{2}; -\frac{c^2}{b^2} \right) - \\ - \frac{1}{\alpha \sqrt{\pi} c^\alpha} \Gamma \left(\frac{\alpha+1}{2} \right) \quad [-2 < \operatorname{Re} \alpha < 0; |\arg b|, |\arg c| < \pi/4].$$

$$4. \int_0^{\infty} \frac{1}{x^2} \operatorname{erf}(bx) \operatorname{erf}(cx) dx = \frac{2}{\sqrt{\pi}} \left(c \ln \frac{b + \sqrt{b^2 + c^2}}{c} + \ln \frac{c + \sqrt{b^2 + c^2}}{b} \right) \\ [|\arg b|, |\arg c| < \pi/4]$$

$$5. \int_0^a x^{\alpha-1} (a^4 - x^4)^{\beta-1} \operatorname{erf}(cx) \operatorname{erf}(icx) dx = \\ = \frac{ia^{\alpha+4\beta-2}c^2}{\pi} B\left(\frac{\alpha+2}{4}, \beta\right) {}_3F_4\left(\frac{1}{2}, 1, \frac{\alpha+2}{4}; \frac{3}{4}, \frac{5}{4}, \frac{3}{2}, \beta + \frac{\alpha+2}{4}; \frac{a^4c^4}{4}\right) \\ [a, \operatorname{Re} \beta > 0; \operatorname{Re} \alpha > -2].$$

$$6. \int_0^1 x \operatorname{erf}(\sqrt{1-x^2}) \operatorname{erf}(x) dx = \frac{1}{2e}.$$

$$7. \int_0^{\infty} [\operatorname{erf}(ax) - \operatorname{erf}(bx)] [\operatorname{erf}(cx) - \operatorname{erf}(dx)] dx = \\ = \frac{1}{\sqrt{\pi}} \left(\frac{\sqrt{b^2 + c^2}}{bc} + \frac{\sqrt{a^2 + d^2}}{ad} - \frac{\sqrt{a^2 + c^2}}{ac} - \frac{\sqrt{b^2 + d^2}}{bd} \right) \\ [|\arg a|, |\arg b|, |\arg c|, |\arg d| < \pi/4]$$

$$8. \int_0^{\infty} x^{\alpha-1} [1 - \operatorname{erf}^2(cx)] dx = \frac{2}{\pi c^{\alpha}} \Gamma\left(\frac{\alpha}{2}\right) {}_2F_1\left(\frac{1}{2}, \frac{\alpha}{2} + 1; \frac{3}{2}; -1\right) \\ [\operatorname{Re} \alpha > 0; |\arg c| < \pi/4]$$

$$9. \int_0^{\infty} [1 - \operatorname{erf}^n(cx)] dx = I_n \quad [|\arg c| < \pi/4].$$

$$I_1 = \frac{1}{\sqrt{\pi}c}, \quad I_2 = \frac{1}{c} \sqrt{\frac{2}{\pi}}, \quad I_3 = \frac{6\sqrt{2}}{\pi^{3/2}c} \operatorname{arctg} \frac{\sqrt{2}}{2},$$

$$I_4 = \frac{12\sqrt{2}}{\pi^{3/2}c} \operatorname{arctg} \frac{\sqrt{2}}{4}.$$

$$10. \int_0^{\infty} [1 - \operatorname{erf}^2(cx + d)] dx = \\ = \frac{2e^{-d^2} \operatorname{erf}(d)}{\sqrt{\pi}c} - \frac{d}{c} [1 - \operatorname{erf}^2(d)] + \frac{\sqrt{2}}{\sqrt{\pi}c} \operatorname{erfc}(\sqrt{2}d) \quad [|\arg c| < \pi/4]$$

$$11. \int_0^{\infty} [\operatorname{erf}(ax) - \operatorname{erf}(bx)] \operatorname{erf}\left(\frac{c}{x}\right) dx = \\ = \frac{1}{\sqrt{\pi}} \left[\frac{1}{b} - \frac{1}{a} + \frac{1}{a} e^{-2ac} - \frac{1}{b} e^{-2bc} + 2c \operatorname{Ei}(-2ac) - 2c \operatorname{Ei}(-2bc) \right] \\ [|\arg a|, |\arg b|, |\arg c| < \pi/4].$$

$$12. \int_0^{\infty} \frac{1}{x^2} \operatorname{erf}(ax) \left[\operatorname{erf}\left(\frac{b}{x}\right) - \operatorname{erf}\left(\frac{c}{x}\right) \right] dx = \\ = \frac{1}{\sqrt{\pi}} \left[\frac{1}{c} - \frac{1}{b} + \frac{1}{b} e^{-2ab} - \frac{1}{c} e^{-2ac} + 2a \operatorname{Ei}(-2ab) - 2a \operatorname{Ei}(-2ac) \right] \\ [|\arg a|, |\arg b|, |\arg c| < \pi/4]$$

$$13. \int_0^{\infty} \frac{1}{x^3} \operatorname{erf}^3(cx) dx = \frac{12c^2}{\pi} \ln \frac{\sqrt{3}+1}{\sqrt{2}} - c^2 \quad [|\arg c| < \pi/4].$$

$$14. \int_0^{\infty} \frac{1}{x^3} \operatorname{erf}(ax) \operatorname{erf}(bx) \operatorname{erf}(cx) dx = \\ = \frac{4}{\pi} \left(ab \operatorname{Arsh} \frac{c}{\sqrt{a^2+b^2}} + ac \operatorname{Arsh} \frac{b}{\sqrt{a^2+c^2}} + bc \operatorname{Arsh} \frac{a}{\sqrt{b^2+c^2}} \right) - \\ - \frac{2}{\sqrt{\pi}} \left(a^2 \operatorname{arctg} \frac{bc}{a\Delta} + b^2 \operatorname{arctg} \frac{ac}{b\Delta} + c^2 \operatorname{arctg} \frac{ab}{c\Delta} \right) \\ [\Delta = \sqrt{a^2+b^2+c^2}; |\arg a|, |\arg b|, |\arg c| < \pi/4].$$

$$15. \int_0^{\infty} [1 - \operatorname{erf}^2(cx)]^2 dx = \frac{2\sqrt{2}}{\pi^{3/2}c} \left(\pi - 6 \operatorname{arctg} \frac{\sqrt{2}}{4} \right) \quad [|\arg c| < \pi/4]$$

$$16. \int_0^{\infty} \operatorname{erf}(icx) [1 - \operatorname{erf}^n(cx)] dx = I_n \quad [|\arg c| < \pi/4],$$

$$I_2 = \frac{i}{\sqrt{\pi}c} + \frac{2\sqrt{2}i}{\pi^{3/2}c} \ln \frac{2+\sqrt{2}}{2-\sqrt{2}}, \quad I_3 = \frac{i}{\sqrt{\pi}c} + \frac{3\sqrt{2}i}{\pi^{3/2}c} \ln 3.$$

2.8.19. Интегралы, содержащие e^{px^m} и $\prod_k \operatorname{erf}(\varphi_k(x))$.

$$1. \int_0^{\infty} x^n e^{-px} \operatorname{erf}^2(cx) dx = (-1)^n \frac{\partial^n}{\partial p^n} \left[\frac{1}{p} \exp\left(\frac{p^2}{4c^2}\right) \operatorname{erfc}^2\left(\frac{p}{2\sqrt{2}c}\right) \right] \\ [\operatorname{Re} p > 0; |\arg c| < \pi/4].$$

$$2. \int_0^{\infty} x^{2n+1} e^{-px^2} \operatorname{erf}^2(cx) dx = \frac{(-1)^n c}{\pi} \frac{\partial^n}{\partial p^n} \left(\frac{1}{p\sqrt{c^2+p}} \operatorname{arctg} \frac{c}{\sqrt{c^2+p}} \right) \\ [\operatorname{Re} p > 0, |\arg c| < \pi/4]$$

$$3. \int_0^{\infty} x^{\alpha-1} e^{-px^2} \operatorname{erf}(icx) \operatorname{erf}(cx) dx = \\ = \frac{2ic^2}{\pi p^{\alpha/2+1}} {}_4F_3\left(\frac{1}{2}, 1, \frac{\alpha+2}{4}, \frac{\alpha}{4}+1; \frac{3}{2}, \frac{3}{4}, \frac{5}{4}; \frac{c^4}{p^2}\right) \\ [\operatorname{Re} \alpha > -2; |\arg c| < \pi/4; \operatorname{Re}(p-c^2) > 0].$$

$$4. \int_0^{\infty} x^{2n} e^{-px^2} \operatorname{erf}(icx) \operatorname{erf}(cx) dx = \frac{(-1)^n i}{\sqrt{\pi}} \frac{\partial^n}{\partial p^n} \left(\frac{1}{\sqrt{p}} \operatorname{Arth} \frac{c^2}{p} \right) \\ [|\arg c| < \pi/4; \operatorname{Re}(p-c^2) > 0]$$

$$5. \int_0^{\infty} x^{\alpha-1} e^{-b^2x^2} \operatorname{erf}(ibx) \operatorname{erf}(cx) dx = \\ = -\frac{2ib}{\pi(\alpha+1)c^{\alpha+1}} \Gamma\left(\frac{\alpha}{2}+1\right) {}_3F_3\left(1, \frac{\alpha+1}{2}, \frac{\alpha}{2}+1; \frac{\alpha+3}{2}, \frac{3}{2}; -\frac{b^2}{c^2}\right) + \\ + \frac{i}{2b^{\alpha}} \operatorname{tg} \frac{\alpha\pi}{2} \Gamma\left(\frac{\alpha}{2}\right) \quad [-2 < \operatorname{Re} \alpha < 1; |\arg b|, |\arg c| < \pi/4]$$

$$6. \int_0^{\infty} e^{-2c^2x^2} \operatorname{erf}^2(icx) dx = -\frac{1}{2c} \sqrt{\frac{\pi}{2}} \quad [|\arg c| < \pi/4].$$

$$7. \int_0^{\infty} x^{\alpha-1} e^{-(b^2+c^2)x^2} \operatorname{erf}(ibx) \operatorname{erf}(icx) dx =$$

$$= \frac{c}{\sqrt{\pi} b^{\alpha+1}} \Gamma\left(\frac{\alpha+1}{2}\right) \operatorname{ctg} \frac{\alpha\pi}{2} {}_2F_1\left(\frac{\alpha+1}{2}, 1; \frac{3}{2}; -\frac{c^2}{b^2}\right) +$$

$$+ \frac{c^{1-\alpha}}{2b\sqrt{\pi}} \Gamma\left(\frac{\alpha-1}{2}\right) \operatorname{ctg} \frac{\alpha\pi}{2} {}_2F_1\left(1, \frac{1}{2}; \frac{3-\alpha}{2}; -\frac{c^2}{b^2}\right)$$

[|Re α| < 2; |arg b|, |arg c| < π/4]

$$8. \int_0^{\infty} x^n e^{-px^2} \operatorname{erf}(bx) \operatorname{erf}(cx) dx = I_n$$

[Δ = √(b² + c² + p); Re p > 0; |arg b|, |arg c| < π/4],

$$I_{-2} = \frac{2}{\sqrt{\pi}} \left(c \ln \frac{b+\Delta}{\sqrt{p+c^2}} + b \ln \frac{c+\Delta}{\sqrt{p+b^2}} - \sqrt{p} \operatorname{arctg} \frac{bc}{\Delta\sqrt{p}} \right),$$

$$I_0 = \frac{1}{\sqrt{\pi p}} \operatorname{arctg} \frac{bc}{\Delta\sqrt{p}},$$

$$I_1 = \frac{1}{\pi p} \left(\frac{b}{\sqrt{b^2+p}} \operatorname{arctg} \frac{c}{\sqrt{b^2+p}} + \frac{c}{\sqrt{c^2+p}} \operatorname{arctg} \frac{b}{\sqrt{c^2+p}} \right).$$

$$9. \int_0^{\infty} x^{\alpha-1} e^{-px^2} \operatorname{erf}(icx) \operatorname{erf}(cx) dx =$$

$$= \frac{ic^2}{\pi p^{(\alpha+2)/4}} \Gamma\left(\frac{\alpha+2}{4}\right) {}_3F_3\left(\frac{1}{2}, 1, \frac{\alpha+2}{4}; \frac{3}{4}, \frac{5}{4}, \frac{3}{2}; \frac{c^4}{4p}\right)$$

[Re α > -2, Re p > 0; |arg c| < π/4]

$$10. \int_0^{\infty} \frac{1}{x} e^{-4c^2x^2} \operatorname{erf}(i\sqrt{2}cx) \operatorname{erf}^2(icx) dx = -\frac{i}{2\sqrt{\pi}}$$

[|arg c| < π/4]

$$11. \int_0^{\infty} x e^{-px^2} \operatorname{erf}(ax) \operatorname{erf}(bx) \operatorname{erf}(cx) dx =$$

$$= \frac{1}{\pi p} \left(\frac{a}{\sqrt{a^2+p}} \operatorname{arctg} \frac{bc}{\Delta\sqrt{a^2+p}} + \frac{b}{\sqrt{b^2+p}} \operatorname{arctg} \frac{ac}{\Delta\sqrt{b^2+p}} + \frac{c}{\sqrt{c^2+p}} \operatorname{arctg} \frac{ab}{\Delta\sqrt{c^2+p}} \right)$$

[Δ = √(a² + b² + c² + p); Re p > 0; |arg a|, |arg b|, |arg c| < π/4]

2.8.20. Интегралы, содержащие $\prod_k \operatorname{erfc}(\varphi_k(x))$

$$1. \int_0^{\infty} \operatorname{erfc}^n(cx) dx = I_n$$

[|arg c| < π/4]

$$I_1 = \frac{1}{\sqrt{\pi}c}, \quad I_2 = \frac{2-\sqrt{2}}{\sqrt{\pi}c},$$

$$I_3 = \frac{3}{\pi^{3/2}c} \left[\pi(1-\sqrt{2}) + 2\sqrt{2} \operatorname{arctg} \frac{\sqrt{2}}{2} \right],$$

$$I_4 = \frac{2}{\pi^{3/2}c} \left[\pi(2-3\sqrt{2}) + 6\sqrt{2} \left(2 \operatorname{arctg} \frac{\sqrt{2}}{2} - \operatorname{arctg} \frac{\sqrt{2}}{4} \right) \right].$$

$$2. \int_0^{\infty} \operatorname{erfc}(bx) \operatorname{erfc}(cx) dx = \frac{b+c-\sqrt{b^2+c^2}}{\sqrt{\pi}bc}$$

[|arg b|, |arg c| < π/4]

$$3. \int_0^{\infty} x^2 \operatorname{erfc}^2(cx) dx = \frac{1}{3\sqrt{\pi} c^3} \quad [|\arg c| < \pi/4].$$

$$4. \int_0^{\infty} x^{\alpha-1} \operatorname{erfc}^2(cx) dx = \\ = -\frac{2}{\pi(\alpha+1)c^{\alpha}} \Gamma\left(\frac{\alpha}{2}+1\right) {}_3F_2\left(\frac{1}{2}, \frac{\alpha+1}{2}, \frac{\alpha}{2}+1; \frac{3}{2}, \frac{\alpha+3}{2}; -1\right) + \\ + \frac{1}{\alpha\sqrt{\pi}c^{\alpha}} \Gamma\left(\frac{\alpha+1}{2}\right) \quad [\operatorname{Re} \alpha > 0, |\arg c| < \pi/4]$$

$$5. \int_0^{\infty} x^{\alpha-1} \operatorname{erfc}(bx) \operatorname{erfc}(cx) dx = \\ = -\frac{2c}{\pi(\alpha+1)b^{\alpha+1}} \Gamma\left(\frac{\alpha}{2}+1\right) {}_3F_2\left(\frac{1}{2}, \frac{\alpha+1}{2}, \frac{\alpha}{2}+1; \frac{3}{2}, \frac{\alpha+3}{2}; -\frac{c^2}{b^2}\right) + \\ + \frac{1}{\alpha b^{\alpha}\sqrt{\pi}} \Gamma\left(\frac{\alpha+1}{2}\right) \quad [\operatorname{Re} \alpha > 0, |\arg b|, |\arg c| < \pi/4]$$

$$6. \int_0^{\infty} \prod_{k=1}^4 \operatorname{erfc}(c_k x) dx = \frac{1}{\sqrt{\pi}} \left(\frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3} + \frac{1}{c_4} \right) - \\ - \frac{2}{\pi^{3/2}} \sum_{k=1}^4 c_k^{-1} \sum_{\substack{j=1 \\ j \neq k}}^4 \frac{c_j}{\sqrt{c_k^2 + c_j^2}} \left(\frac{\pi}{2} - \operatorname{arctg} \frac{c_l}{\sqrt{c_k^2 + c_j^2}} - \right. \\ \left. - \operatorname{arctg} \frac{c_m}{\sqrt{c_k^2 + c_j^2}} + \operatorname{arctg} \frac{c_l c_m}{\sqrt{c_k^2 + c_j^2} \sqrt{c_l^2 + c_m^2 + c_k^2 + c_j^2}} \right) \\ [l, m \neq 1, l, m \neq k, |\arg c_k| < \pi/4]$$

$$7. \int_0^{\infty} \operatorname{erfc}^2(cx+a) dx = \frac{2}{\sqrt{\pi}c} \operatorname{erf}(a) e^{-a^2} - \frac{a}{c} \operatorname{erfc}^2(a) - \frac{1}{c} \sqrt{\frac{2}{\pi}} \operatorname{erfc}(a\sqrt{2}) \\ [|\arg c| < \pi/4]$$

$$8. \int_0^{\infty} \operatorname{erfc}\left(\frac{b}{x}\right) \operatorname{erfc}(cx) dx = \frac{1}{\sqrt{\pi}c} e^{-2bc} + \frac{2b}{\sqrt{\pi}} \operatorname{Ei}(-2be) \\ [|\arg b|, |\arg c| < \pi/4]$$

$$9. \int_0^{\infty} x^2 \operatorname{erfc}\left(\frac{b}{x}\right) \operatorname{erfc}(cx) dx = \frac{1}{6c^3\sqrt{\pi}} [e^{-2bc}(2+bc-2b^2c^2) - 4b^3c^3 \operatorname{Ei}(-2bc)] \\ [|\arg b|, |\arg c| < \pi/4]$$

$$10. \int_0^{\infty} e^{px} \operatorname{erfc}^2(cx) dx = \\ = \frac{1}{p} - \frac{1}{p} \exp\left(\frac{p^2}{4c^2}\right) \left[1 - 2 \operatorname{erf}\left(\frac{p}{2c}\right) + 2 \operatorname{erf}\left(\frac{p}{2\sqrt{2}c}\right) - \operatorname{erf}^2\left(\frac{p}{2\sqrt{2}c}\right) \right] \\ [|\arg c| < \pi/4]$$

$$11. \int_0^{\infty} e^{c^2 x^2} \operatorname{erfc}(bx) \operatorname{erfc}(cx) dx = \frac{1}{c\sqrt{\pi}} \ln\left(1 + \frac{c}{b}\right) \\ [|\arg b|, |\arg c| < \pi/4].$$

$$\begin{aligned}
 12. \quad \int_0^{\infty} x^{\alpha-1} e^{c^2 x^2} \operatorname{erfc}(bx) \operatorname{erfc}(cx) dx = \\
 = -\frac{2c}{\pi(\alpha+1)b^{\alpha+1}} \Gamma\left(\frac{\alpha}{2}+1\right) {}_3F_2\left(1, \frac{\alpha+1}{2}, \frac{\alpha}{2}+1; \frac{\alpha+3}{2}, \frac{3}{2}; \frac{c^2}{b^2}\right) + \\
 + \frac{1}{\alpha \sqrt{\pi} b^{\alpha}} \Gamma\left(\frac{\alpha+1}{2}\right) {}_2F_1\left(\frac{\alpha}{2}, \frac{\alpha+1}{2}; \frac{\alpha}{2}+1; \frac{c^2}{b^2}\right) \\
 [\operatorname{Re} \alpha > 0, |\arg b|, |\arg c| < \pi/4]
 \end{aligned}$$

$$\begin{aligned}
 13. \quad \int_0^{\infty} e^{-px^2} \operatorname{erfc}(bx) \operatorname{erfc}(cx) dx = \\
 = \frac{1}{\sqrt{\pi p}} \left(\frac{\pi}{2} - \operatorname{arctg} \frac{b}{\sqrt{p}} - \operatorname{arctg} \frac{c}{\sqrt{p}} + \operatorname{arctg} \frac{bc}{\sqrt{p(b^2 + c^2 + p)}} \right) \\
 [|\arg b|, |\arg c| < \pi/4, \operatorname{Re}(p + c^2 + b^2) > 0]
 \end{aligned}$$

$$\begin{aligned}
 14. \quad \int_0^{\infty} x^{\alpha-1} e^{(b^2+c^2)x^2} \operatorname{erfc}(bx) \operatorname{erfc}(cx) dx = \\
 = \frac{c}{\sqrt{\pi} b^{\alpha+1}} \Gamma\left(\frac{\alpha+1}{2}\right) \operatorname{cosec}\left(\frac{\alpha\pi}{2}\right) {}_2F_1\left(\frac{\alpha+1}{2}, 1; \frac{3}{2}; -\frac{c^2}{b^2}\right) + \\
 + \frac{\Gamma(\alpha/2)}{2(b^2+c^2)^{\alpha/2}} \sec\left(\frac{\alpha\pi}{2}\right) + \frac{c^{1-\alpha}}{2b \sqrt{\pi}} \Gamma\left(\frac{\alpha-1}{2}\right) \operatorname{cosec} \frac{\alpha\pi}{2} {}_2F_1\left(1, \frac{1}{2}; \frac{3-\alpha}{2}; -\frac{c^2}{b^2}\right) \\
 [0 < \operatorname{Re} \alpha < 2, |\arg b|, |\arg c| < \pi/4]
 \end{aligned}$$

$$\begin{aligned}
 15. \quad \int_0^{\infty} x e^{c^2 x^2} \operatorname{erfc}^n(cx) dx = I_n \quad [|\arg c| < \pi/4], \\
 I_2 = \frac{4-\pi}{2\pi c^2}, \quad I_3 = \frac{3(2-\sqrt{2})}{\pi c^2}, \\
 I_4 = \frac{12(1-\sqrt{2})}{\pi c^2} + \frac{24\sqrt{2}}{\pi^2 c^2} \operatorname{arctg} \frac{\sqrt{2}}{2}, \\
 I_5 = \frac{10(2-3\sqrt{2})}{\pi c^2} + \frac{60\sqrt{2}}{\pi^2 c^2} \left(2 \operatorname{arctg} \frac{\sqrt{2}}{2} - \operatorname{arctg} \frac{\sqrt{2}}{4} \right).
 \end{aligned}$$

2.8.21 Интегралы, содержащие $\prod_{k,l} \operatorname{erf}(\varphi_k(x)) \operatorname{erfc}(\psi_l(x))$

$$1. \quad \int_0^{\infty} \frac{1}{x} \operatorname{erf}(cx) \operatorname{erfc}(cx) dx = \frac{2}{\pi} G \quad [|\arg c| < \pi/4]$$

$$\begin{aligned}
 2. \quad \int_0^{\infty} x^{\alpha-1} \operatorname{erf}(icx) \operatorname{erfc}(cx) dx = \frac{i}{\alpha \sqrt{\pi} c^{\alpha}} \Gamma\left(\frac{\alpha+1}{2}\right) \operatorname{tg} \frac{\alpha\pi}{4} \\
 [-1 < \operatorname{Re} \alpha < 2; |\arg c| < \pi/4]
 \end{aligned}$$

$$3. \quad \int_0^{\infty} \operatorname{erf}(bx) \operatorname{erfc}(cx) dx = \frac{\sqrt{b^2+c^2}-c}{\sqrt{\pi} bc} \quad [|\arg b|, |\arg c| < \pi/4]$$

$$4. \int_0^{\infty} x^{\alpha-1} \operatorname{erf}(bx) \operatorname{erfc}(cx) dx = \\ = \frac{2c}{\pi(\alpha+1)b^{\alpha+1}} \Gamma\left(\frac{\alpha+1}{2}\right) {}_3F_2\left(\frac{1}{2}, \frac{\alpha+1}{2}, \frac{\alpha}{2}+1; \frac{3}{2}, \frac{\alpha+3}{2}; -\frac{c^2}{b^2}\right) + \\ + \frac{1}{\alpha\sqrt{\pi}} \Gamma\left(\frac{\alpha+1}{2}\right) \left(\frac{1}{c^{\alpha}} - \frac{1}{b^{\alpha}}\right) \quad [\operatorname{Re} \alpha > -1; |\arg b|, |\arg c| < \pi/4]$$

$$5. \int_0^{\infty} \frac{1}{x} \operatorname{erf}(bx) \operatorname{erfc}(cx) dx = \frac{2b}{\pi c} {}_3F_2\left(\frac{1}{2}, \frac{1}{2}, 1; \frac{3}{2}, \frac{3}{2}; -\frac{b^2}{c^2}\right) \\ [|\arg b|, |\arg c| < \pi/4]$$

$$6. \int_0^{\infty} \operatorname{erf}^n(cx) \operatorname{erfc}(cx) dx = I_n \quad [|\arg c| < \pi/4],$$

$$I_1 = \frac{\sqrt{2}-1}{\sqrt{\pi}c}, \quad I_2 = -\frac{\sqrt{2}}{\pi^{3/2}c} \left(\pi - 6 \operatorname{arctg} \frac{\sqrt{2}}{2}\right),$$

$$I_3 = \frac{6\sqrt{2}}{\pi^{3/2}c} \left(2 \operatorname{arctg} \frac{\sqrt{2}}{4} - \operatorname{arctg} \frac{\sqrt{2}}{2}\right).$$

$$7. \int_0^{\infty} \operatorname{erf}(cx) \operatorname{erfc}^2(cx) dx = \frac{2\sqrt{2}-1}{\sqrt{\pi}c} - \frac{6\sqrt{2}}{\pi^{3/2}c} \operatorname{arctg} \frac{\sqrt{2}}{2} \quad [|\arg c| < \pi/4]$$

$$8. \int_0^{\infty} \operatorname{erf}^2(cx) \operatorname{erfc}^2(cx) dx = -\frac{\sqrt{2}}{\pi^{3/2}c} \left(\pi + 12 \operatorname{arctg} \frac{\sqrt{2}}{4} - 12 \operatorname{arctg} \frac{\sqrt{2}}{2}\right) \\ [|\arg c| < \pi/4]$$

$$9. \int_0^{\infty} \operatorname{erf}(icx) \operatorname{erfc}^n(cx) dx = I_n \quad [|\arg c| < \pi/4],$$

$$I_1 = \frac{i}{\sqrt{\pi}c}, \quad I_2 = \frac{i}{\sqrt{\pi}c} - \frac{i\sqrt{2}}{\pi^{3/2}c} \ln \frac{2+\sqrt{2}}{2-\sqrt{2}},$$

$$I_3 = \frac{i}{\sqrt{\pi}c} - \frac{3i\sqrt{2}}{\pi^{3/2}c} \ln \frac{2+\sqrt{2}}{3(2-\sqrt{2})}.$$

$$10. \int_0^{\infty} x^{\alpha-1} \operatorname{erf}(ibx) \operatorname{erf}(bx) \operatorname{erfc}(cx) dx = \frac{4ib^2}{\pi^{3/2}(\alpha+2)c^{\alpha+2}} \times \\ \times \Gamma\left(\frac{\alpha+3}{2}\right) {}_5F_4\left(\frac{1}{2}, 1, \frac{\alpha+2}{4}, \frac{\alpha+3}{4}, \frac{\alpha}{4}+1; \frac{3}{4}, \frac{5}{4}, \frac{3}{2}, \frac{\alpha+6}{4}; \frac{b^4}{4c^4}\right) \\ [\operatorname{Re} \alpha > -2; |\arg b|, |\arg c| < \pi/4, \operatorname{Re}(c^2 - b^2) > 0]$$

$$11. \int_0^{\infty} \operatorname{erf}\left(\frac{b}{x}\right) \operatorname{erfc}(cx) dx = \frac{1-e^{-2bc}}{\sqrt{\pi}c} - \frac{2b}{\sqrt{\pi}} \operatorname{Ei}(-2bc) \\ [|\arg b|, |\arg c| < \pi/4]$$

$$12. \int_0^{\infty} \frac{1}{x^2} \left[\operatorname{erf}\left(\frac{a}{x}\right) - \operatorname{erf}\left(\frac{b}{x}\right) \right] \operatorname{erfc}(cx) dx = \\ = \frac{1}{\sqrt{\pi}} \left[\frac{1}{b} e^{-2bc} - \frac{1}{a} e^{-2ac} + 2c \operatorname{Ei}(-2bc) - 2c \operatorname{Ei}(-2ac) \right] \\ [|\arg b|, |\arg c| < \pi/4]$$

$$13. \int_0^{\infty} e^{b^2 x^2} \operatorname{erf}(bx) \operatorname{erfc}(cx) dx = -\frac{1}{2b\sqrt{\pi}} \ln\left(1 - \frac{b^2}{c^2}\right) \quad [\operatorname{Re}(c^2 - b^2) > 0; |\arg c| < \pi/4]$$

$$14. \int_0^{\infty} x^{\alpha-1} e^{b^2 x^2} \operatorname{erf}(bx) \operatorname{erfc}(cx) dx = \frac{2b}{\pi(\alpha+1)c^{\alpha+1}} \Gamma\left(\frac{\alpha}{2} + 1\right) {}_3F_2\left(1, \frac{\alpha+1}{2}, \frac{\alpha}{2} + 1; \frac{3}{2}, \frac{\alpha+3}{2}; \frac{b^2}{c^2}\right) \\ [\operatorname{Re} \alpha > -1; \operatorname{Re}(c^2 - b^2) > 0; |\arg c| < \pi/4]$$

$$15. \int_0^{\infty} x^{\alpha-1} e^{c^2 x^2} \operatorname{erf}(bx) \operatorname{erfc}(cx) dx = \frac{2c}{\pi(\alpha+1)b^{\alpha+1}} \Gamma\left(\frac{\alpha}{2} + 1\right) {}_3F_2\left(1, \frac{\alpha+1}{2}, \frac{\alpha}{2} + 1; \frac{\alpha+3}{2}, \frac{3}{2}; \frac{c^2}{b^2}\right) - \\ - \frac{1}{\alpha\sqrt{\pi}b^{\alpha}} \Gamma\left(\frac{\alpha+1}{2}\right) {}_2F_1\left(\frac{\alpha}{2}, \frac{\alpha+1}{2}; \frac{\alpha}{2} + 1; \frac{c^2}{b^2}\right) + \frac{\Gamma(\alpha/2)}{2c^{\alpha}} \sec \frac{\alpha\pi}{2} \\ [|\operatorname{Re} \alpha| < 1; |\arg b|, |\arg c| < \pi/4]$$

2.8.22. Интегралы, содержащие гиперболические функции

и $\left\{ \begin{matrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{matrix} \right\}^2$.

$$1. \int_0^{\infty} \left\{ \begin{matrix} \operatorname{sh} bx \\ \operatorname{ch} bx \end{matrix} \right\} [1 - \operatorname{erf}^2(cx)] dx = \frac{1}{b} \exp\left(\frac{b^2}{4c^2}\right) \left\{ \begin{matrix} A \\ B \end{matrix} \right\} \\ \left[A = 1 - \exp\left(-\frac{b^2}{4c^2}\right) + \operatorname{erf}^2\left(\frac{b}{2\sqrt{2}c}\right), B = 2 \operatorname{erf}\left(\frac{b}{2\sqrt{2}c}\right); |\arg c| < \pi/4 \right].$$

$$2. \int_0^{\infty} \left\{ \begin{matrix} \operatorname{sh} bx \\ \operatorname{ch} bx \end{matrix} \right\} \operatorname{erfc}^2(cx) dx = \frac{1}{b} \exp\left(\frac{b^2}{4c^2}\right) \left\{ \begin{matrix} A \\ B \end{matrix} \right\} \\ \left[A = 1 - \exp\left(-\frac{b^2}{4c^2}\right) - \operatorname{erf}^2\left(\frac{b}{2\sqrt{2}c}\right), B = 2 \operatorname{erf}\left(\frac{b}{2c}\right) - 2 \operatorname{erf}\left(\frac{b}{2\sqrt{2}c}\right); |\arg c| < \pi/4 \right].$$

$$3. \int_0^{\infty} \left\{ \begin{matrix} \operatorname{sh} bx \\ \operatorname{ch} bx \end{matrix} \right\}^2 \operatorname{erfc}^2(cx) dx = \pm \frac{1 - \sqrt{2}}{\sqrt{2\pi}c} + \frac{1}{2b} e^{b^2/c^2} \left[\operatorname{erf}\left(\frac{b}{c}\right) - \operatorname{erf}\left(\frac{b}{\sqrt{2}c}\right) \right] \\ [|\arg c| < \pi/4].$$

2.8.23. Интегралы, содержащие тригонометрические функции и произведения функций $\operatorname{erf}(bx)$ $\operatorname{erfc}(cx)$

$$1. \int_0^{\infty} \left\{ \begin{matrix} \sin bx \\ \cos bx \end{matrix} \right\}^n [1 - \operatorname{erf}^2(cx)] dx = I_n \quad [b > 0; |\arg c| < \pi/4].$$

$$I_{2m} = \frac{1}{2^{2m}} \left[\binom{2m}{m} \frac{\sqrt{2}}{\sqrt{\pi}c} - \frac{2i}{b} \sum_{k=0}^{m-1} \frac{(-1)^{m-k}}{m-k} \binom{2m}{k} \exp\left(-\frac{(m-k)^2 b^2}{c^2}\right) \operatorname{erf}\left(ib \frac{m-k}{\sqrt{2}c}\right) \right],$$

$$I_{2m+1} = \frac{1}{2^{2m}b} \sum_{k=0}^m \frac{(-1)^{m-k}}{2m-2k+1} \binom{2m+1}{k} \left\{ \begin{matrix} A_k \\ B_k \end{matrix} \right\} \\ \left[A_k = 1 - \exp\left(-\left(m-k+\frac{1}{2}\right)^2 \frac{b^2}{c^2}\right) \operatorname{erfc}\left(-ib \frac{2m-2k+1}{2\sqrt{2}c}\right), \right. \\ \left. B_k = -2i \exp\left(-\left(m-k+\frac{1}{2}\right)^2 \frac{b^2}{c^2}\right) \operatorname{erf}\left(ib \frac{2m-2k+1}{2\sqrt{2}c}\right) \right].$$

$$2. \int_0^{\infty} \cos bx [1 - \operatorname{erf}(ax) \operatorname{erf}(cx)] dx =$$

$$= -\frac{i}{b} \left[\exp\left(-\frac{b^2}{4c}\right) \operatorname{erf}\left(\frac{iab}{2c\sqrt{a^2+c^2}}\right) + \operatorname{erf}\left(-\frac{b^2}{4a}\right) \operatorname{erf}\left(\frac{ibc}{2a\sqrt{a^2+c^2}}\right) \right]$$

$[b > 0, |\arg a|, |\arg c| < \pi/4]$

$$3. \int_0^{\infty} \cos^{2n+1} bx [1 - \operatorname{erf}(ax) \operatorname{erf}(cx)] dx =$$

$$= -\frac{i}{2^{2n}b} \sum_{k=0}^n \frac{1}{2n-2k+1} \binom{2n+1}{k} \left\{ \exp\left[-\left(n-k+\frac{1}{2}\right)^2 \frac{b^2}{a^2}\right] \times \right.$$

$$\times \operatorname{erf}\left(\frac{ibc}{2a\sqrt{a^2+c^2}} \frac{2n-2k+1}{2}\right) + \exp\left[-\left(n-k+\frac{1}{2}\right)^2 \frac{b^2}{c^2}\right] \operatorname{erf}\left(\frac{iab}{2c\sqrt{a^2+c^2}} \frac{2n-2k+1}{2}\right) \Big\}$$

$[b > 0, |\arg a|, |\arg c| < \pi/4]$

$$4. \int_0^{\infty} \left\{ \frac{\sin bx}{\cos bx} \right\}^n \operatorname{erfc}^2(cx) dx = I_n \quad [b > 0; |\arg c| < \pi/4]$$

$$I_{2m} = \binom{2m}{m} \frac{2-\sqrt{2}}{2^{2m}c\sqrt{\pi}} - \frac{i}{2^{2m-1}b} \sum_{k=0}^{m-1} \frac{(-1)^{m-k}}{m-k} \times$$

$$\times \binom{2m}{k} \exp\left[-(m-k)^2 \frac{b^2}{c^2}\right] \left[\operatorname{erf}\left(ib \frac{m-k}{\sqrt{2}c}\right) - \operatorname{erf}\left(ib \frac{m-k}{c}\right) \right],$$

$$I_{2m+1} = \frac{1}{2^{2m}b} \sum_{k=0}^m \frac{(-1)^{m-k}}{2m-2k+1} \binom{2m+1}{k} \{A\}$$

$$\left[A = 1 - \exp\left[-\left(m-k+\frac{1}{2}\right)^2 \frac{b^2}{c^2}\right] \operatorname{erfc}\left(\frac{ib}{2\sqrt{2}c} \frac{2m-2k+1}{2}\right), \right.$$

$$\left. B = -2i \exp\left[-\left(m-k+\frac{1}{2}\right)^2 \frac{b^2}{c^2}\right] \left[\operatorname{erf}\left(\frac{ib}{2c} \frac{2m-2k+1}{2}\right) - \operatorname{erf}\left(\frac{ib}{2\sqrt{2}c} \frac{2m-2k+1}{2}\right) \right] \right].$$

$$5. \int_0^{\infty} \cos ax \left\{ \frac{\operatorname{erf}(bx)}{\operatorname{erfc}(bx)} \right\} \operatorname{erfc}(cx) dx =$$

$$= \mp \frac{i}{a} \left\{ \exp\left(-\frac{a^2}{4b^2}\right) \left[\operatorname{erf}\left(\frac{iac}{2b\sqrt{b^2+c^2}}\right) - \operatorname{erf}\left(\frac{ia}{2b}\right) \right] + \right.$$

$$\left. + \exp\left(-\frac{a^2}{4c^2}\right) \left[\operatorname{erf}\left(\frac{iab}{2c\sqrt{b^2+c^2}}\right) - \{0\} \operatorname{erf}\left(\frac{ia}{2c}\right) \right] \right\}$$

$[a > 0; |\arg b|, |\arg c| < \pi/4]$

$$6. \int_0^{\infty} \left\{ \frac{\sin ax}{\cos ax} \right\}^2 \operatorname{erfc}(bx) \operatorname{erfc}(cx) dx =$$

$$= \frac{1}{2\sqrt{\pi}} \left\{ \frac{b+c-\sqrt{b^2+c^2}}{bc} \pm \frac{i\sqrt{\pi}}{2a} e^{-a^2/b^2} \left[\operatorname{erf}\left(\frac{ia}{b}\right) - \operatorname{erf}\left(\frac{iac}{b\sqrt{b^2+c^2}}\right) \right] \pm \right.$$

$$\left. \pm \frac{i\sqrt{\pi}}{2a} e^{-a^2/c^2} \left[\operatorname{erf}\left(\frac{ia}{c}\right) - \operatorname{erf}\left(\frac{iao}{c\sqrt{b^2+c^2}}\right) \right] \right\}$$

$[a > 0; |\arg b|, |\arg c| < \pi/4]$

$$\begin{aligned}
7. \int_0^{\infty} \left\{ \begin{array}{l} \operatorname{sh} bx \cos bx \\ \operatorname{ch} bx \sin bx \end{array} \right\} \operatorname{erfc}^2(cx) dx = \\
= \frac{1}{2b} (\sin \varphi \pm \cos \varphi - 1) + \frac{\sqrt{2}}{2b} \left[(\sin \varphi \mp \cos \varphi) \left(C^2 \left(\frac{\varphi}{2} \right) - S^2 \left(\frac{\varphi}{2} \right) \right) - \right. \\
\left. - 2 (\cos \varphi \pm \sin \varphi) C \left(\frac{\varphi}{2} \right) S \left(\frac{\varphi}{2} \right) \right] \quad [\varphi = b^2/(2c^2), \quad |\arg c| < \pi/4].
\end{aligned}$$

2.8.24. Интегралы, содержащие $\ln x$ и произведения функций $\operatorname{erf}(bx)$, $\operatorname{erfc}(cx)$

$$\begin{aligned}
1. \int_0^1 x^{\alpha-1} \ln x \operatorname{erf}(icx) \operatorname{erf}(cx) dx = \\
= -\frac{4ic^2}{\pi(\alpha+2)^2} {}_4F_5 \left(\frac{1}{2}, 1, \frac{\alpha+2}{4}, \frac{\alpha+2}{4}; \frac{3}{4}, \frac{5}{4}, \frac{3}{2}, \frac{\alpha+6}{4}, \frac{\alpha+6}{4}; \frac{c^4}{4} \right) \\
[\operatorname{Re} \alpha > -2].
\end{aligned}$$

$$2. \int_0^{\infty} x e^{x^2} \ln x \operatorname{erfc}^2(x) dx = \frac{2}{\pi} \left(G - 1 + \frac{\pi-4}{8} C \right).$$

2.8.25. Интегралы, содержащие $\operatorname{Ei}(-bx^{\pm 2}) \left\{ \begin{array}{l} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{array} \right\}$.

$$\begin{aligned}
1. \int_0^{\infty} \operatorname{Ei}(-bx^2) \left\{ \begin{array}{l} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{array} \right\} dx = \pm \frac{1}{\sqrt{\pi}c} \ln \left(1 + \frac{c^2}{b} \right) - \frac{2}{\sqrt{\pi}b} \operatorname{arctg} \left(\frac{c}{\sqrt{b}} \right)^{\pm 1} \\
[\operatorname{Re} b > 0; \quad |\arg c| < \pi/4].
\end{aligned}$$

$$\begin{aligned}
2. \int_0^{\infty} x \operatorname{Ei}(-bx^2) \operatorname{erf}(cx) dx = \frac{\sqrt{b+c^2}}{2bc} - \frac{1}{4c^2} \ln \frac{\sqrt{b+c^2}-c}{\sqrt{b+c^2}+c} \\
[\operatorname{Re} b > 0, \quad |\arg c| < \pi/4]
\end{aligned}$$

$$\begin{aligned}
3. \int_0^{\infty} x^{\alpha-1} \operatorname{Ei}(-bx^2) \operatorname{erf}(cx) dx = -\frac{2c}{\sqrt{\pi}(\alpha+1)b^{(\alpha+1)/2}} \times \\
\times \Gamma \left(\frac{\alpha+1}{2} \right) {}_2F_3 \left(\frac{1}{2}, \frac{\alpha+1}{2}, \frac{\alpha+1}{2}; \frac{3}{2}, \frac{\alpha+3}{2}; -\frac{c^2}{b} \right) \\
[\operatorname{Re} \alpha > -1; \quad \operatorname{Re} b > 0; \quad |\arg c| < \pi/4].
\end{aligned}$$

$$\begin{aligned}
4. \int_0^{\infty} x^{\alpha-1} e^{c^2 x^2} \operatorname{Ei}(-bx^2) \operatorname{erf}(cx) dx = \\
= -\frac{2c}{\sqrt{\pi}(\alpha+1)b^{(\alpha+1)/2}} \Gamma \left(\frac{\alpha+1}{2} \right) {}_2F_3 \left(1, \frac{\alpha+1}{2}, \frac{\alpha+1}{2}; \frac{3}{2}, \frac{\alpha+3}{2}; \frac{c^2}{b} \right) \\
[\operatorname{Re} \alpha > -1; \quad \operatorname{Re}(b-c^2) > 0; \quad |\arg c| < \pi/4].
\end{aligned}$$

$$\begin{aligned}
5. \int_0^{\infty} x e^{-px^2} \operatorname{Ei}(-bx^2) \operatorname{erf}(cx) dx = \\
= \frac{1}{p} \ln \frac{c + \sqrt{c^2 + b + p}}{\sqrt{b+p}} - \frac{c}{p\sqrt{c^2+b}} \ln \frac{\sqrt{c^2+p} + \sqrt{c^2+b+p}}{\sqrt{b}} \\
[\operatorname{Re}(b+p) > 0; \quad |\arg c| < \pi/4].
\end{aligned}$$

$$6. \int_0^{\infty} x^{-n} \operatorname{Ei} \left(-\frac{b}{x^2} \right) \operatorname{erfc}(cx) dx = I_n \quad [\operatorname{Re} b > 0; |\arg c| < \pi/4].$$

$$I_1 = -\zeta(2) - [C + 2 \ln(2c\sqrt{b})]^2 - 2 \sum_{k=1}^{\infty} \frac{(-2c\sqrt{b})^k}{k! k^2},$$

$$I_2 = \frac{2c\sqrt{b}-1}{2b} e^{-2c\sqrt{b}} + 2c^2 \operatorname{Ei}(-2c\sqrt{b}).$$

$$7. \int_0^{\infty} \operatorname{Ei}(-bx^2) \operatorname{erf}^2(cx) dx = \frac{4}{\sqrt{\pi}(c^2+b)} \ln \left(c \sqrt{\frac{2}{b}} + \sqrt{\frac{2c^2+b}{b}} \right) -$$

$$- \frac{4}{\sqrt{\pi}c} \ln \frac{c + \sqrt{2c^2+b}}{\sqrt{c^2+b}} - \frac{2}{\sqrt{\pi}b} \operatorname{arctg} \frac{c^2}{\sqrt{2bc^2+b^2}}$$

$$[\operatorname{Re} b > 0; |\arg c| < \pi/4].$$

$$8. \int_0^{\infty} \operatorname{Ei}(-bx^2) \operatorname{erf}(icx) \operatorname{erf}(cx) dx =$$

$$= \frac{i}{\sqrt{\pi}} \left[c \ln \frac{\sqrt{b}+c}{\sqrt{b}-c} - 2c \operatorname{arctg} \frac{c}{\sqrt{b}} - \frac{1}{\sqrt{b}} \ln \frac{b+c^2}{b-c^2} \right]$$

$$[\operatorname{Re}(b-c^2) > 0; |\arg c| < \pi/4]$$

$$9. \int_0^{\infty} \operatorname{Ei}(-ax^2) \operatorname{erf}(bx) \operatorname{erf}(cx) dx =$$

$$= \frac{2}{\sqrt{\pi}b} \left(\frac{c}{\sqrt{a+c^2}} \ln \frac{\sqrt{b^2+c^2} + \sqrt{a+b^2+c^2}}{\sqrt{a}} - \ln \frac{c + \sqrt{a+b^2+c^2}}{\sqrt{a+b^2}} \right) +$$

$$+ \frac{2}{\sqrt{\pi}c} \left(\frac{b}{\sqrt{a+b^2}} \ln \frac{\sqrt{b^2+c^2} + \sqrt{a+b^2+c^2}}{\sqrt{a}} - \ln \frac{b + \sqrt{a+b^2+c^2}}{\sqrt{a+c^2}} \right) -$$

$$- \frac{2}{\sqrt{\pi}a} \operatorname{arctg} \frac{bc}{\sqrt{a(a+b^2+c^2)}} \quad [\operatorname{Re} a > 0, \arg b, \arg c < \pi/4]$$

2.8.26. Интегралы, содержащие $\begin{Bmatrix} \operatorname{si}(bx^n) \\ \operatorname{ci}(bx^n) \end{Bmatrix} \begin{Bmatrix} \operatorname{erf}(cx) \\ \operatorname{erfc}(cx) \end{Bmatrix}$.

$$1. \int_0^{\infty} x^{\alpha-1} \begin{Bmatrix} \operatorname{si}(bx) \\ \operatorname{ci}(bx) \end{Bmatrix} \operatorname{erf}(cx) dx = U(1)$$

$$[b > 0; -1 < \operatorname{Re} \alpha < 2; |\arg c| < \pi/4, \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}].$$

$$U(\gamma) = \frac{(-1)^{\gamma+1} b^{2+\delta}}{2\sqrt{\pi} c^{\alpha+2+\delta} (\alpha+2+\delta) (2+\delta) (1+2\delta)} \Gamma\left(\frac{\alpha+3+\delta}{2}\right) \times$$

$$\times {}_4F_4\left(1, \frac{\alpha+3}{2}, \frac{\alpha}{2} + \delta + 1, \frac{\delta}{2} + 1; 2, \frac{\alpha+\delta}{2} + 2, \frac{\delta}{2} + 2, \frac{3}{2} + \delta; -\frac{b^2}{4c^2}\right) +$$

$$+ \frac{(-1)^{\gamma} b^0}{\sqrt{\pi} c^{\alpha+\delta} (\alpha+\delta)} \Gamma\left(\frac{\alpha+1+\delta}{2}\right) \begin{Bmatrix} 1 \\ R \end{Bmatrix} - \frac{\gamma \Gamma(\alpha)}{\alpha b a} \begin{Bmatrix} \sin(\alpha\pi/2) \\ \cos(\alpha\pi/2) \end{Bmatrix} +$$

$$+ \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{(-1)^{\gamma-1} \sqrt{\pi}}{2\alpha c^{\alpha}} \Gamma\left(\frac{\alpha+1}{2}\right) \quad \left[R = \frac{1}{2} \psi\left(\frac{\alpha+1}{2}\right) + C - \frac{1}{\alpha} - \ln \frac{c}{b}\right].$$

$$2. \int_0^{\infty} x^{\alpha-1} \begin{Bmatrix} \operatorname{si}(bx) \\ \operatorname{ci}(bx) \end{Bmatrix} \operatorname{erfc}(cx) dx = U(0)$$

$[b, \operatorname{Re} \alpha > 0; |\arg c| < \pi/4, U(\gamma) \text{ см. в 2.8.26.1}].$

$$3. \int_0^{\infty} \frac{1}{x} \operatorname{ci}(bx) \operatorname{erf}(cx) dx =$$

$$= -\frac{1}{8} \left[\zeta(2) + \left(c - 2 \ln \frac{2c}{b} \right)^2 \right] - \frac{1}{4} \sum_{k=0}^{\infty} \left(-\frac{b^2}{4c^2} \right)^{k+1} \frac{1}{k!(k+1)^3}$$

[$b > 0; |\arg c| < \pi/4$]

$$4. \int_0^{\infty} \frac{1}{x^2} [\operatorname{si}(ax) - \operatorname{si}(bx)] \operatorname{erf}(cx) dx =$$

$$= {}_2F_2 \left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -\frac{a^2}{4c^2} \right) - {}_2F_2 \left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -\frac{b^2}{4c^2} \right) -$$

$$- \frac{a}{2} \operatorname{Ei} \left(-\frac{a^2}{4c^2} \right) + \frac{b}{2} \operatorname{Ei} \left(-\frac{b^2}{4c^2} \right) + \sqrt{\pi} c \left[\operatorname{erf} \left(\frac{a}{2c} \right) - \operatorname{erf} \left(\frac{b}{2c} \right) \right]$$

[$a, b > 0; |\arg c| < \pi/4$]

$$5. \int_0^{\infty} \operatorname{si}(bx) \operatorname{erfc}(cx) dx = \frac{1}{b} \left[\exp \left(-\frac{b^2}{4c^2} \right) - 1 \right] - \frac{\sqrt{\pi}}{2c} \operatorname{erfc} \left(\frac{b}{2c} \right)$$

[$b > 0; |\arg c| < \pi/4$]

$$6. \int_0^{\infty} x^{\alpha-1} \begin{Bmatrix} \operatorname{si}(bx^2) \\ \operatorname{ci}(bx^2) \end{Bmatrix} \operatorname{erf}(cx) dx = V(1)$$

[$b > 0; -1 < \operatorname{Re} \alpha < 4; |\arg c| < \pi/4$]

$$V(\gamma) = \frac{2(-1)^{\gamma} c}{\sqrt{\pi} b^{(\alpha+1)/2} (\alpha+1)} \Gamma \left(\frac{\alpha+1}{2} \right) \begin{Bmatrix} \sin [(\alpha+1)\pi/4] \\ \cos [(\alpha+1)\pi/4] \end{Bmatrix} \times$$

$$\times {}_4F_3 \left(\frac{1}{4}, \frac{\alpha+1}{4}, \frac{\alpha+1}{4}, \frac{\alpha+3}{4}; \frac{1}{2}, \frac{5}{4}, \frac{\alpha+5}{4}; \frac{c^4}{b^2} \right) +$$

$$+ \frac{2(-1)^{\gamma} c^3}{\sqrt{\pi} b^{(\alpha+3)/2} (\alpha+1)} \Gamma \left(\frac{\alpha+3}{2} \right) \begin{Bmatrix} \sin [(\alpha-1)\pi/4] \\ \cos [(\alpha-1)\pi/4] \end{Bmatrix} \times$$

$$\times {}_4F_3 \left(\frac{3}{4}, \frac{\alpha+3}{4}, \frac{\alpha+3}{4}, \frac{\alpha+5}{4}; \frac{3}{2}, \frac{\alpha+7}{4}, \frac{7}{4}; \frac{c^4}{b^2} \right) -$$

$$- \frac{1-\gamma}{\alpha b^{\alpha/2}} \Gamma \left(\frac{\alpha}{2} \right) \begin{Bmatrix} \sin (\alpha\pi/4) \\ \cos (\alpha\pi/4) \end{Bmatrix}.$$

$$7. \int_0^{\infty} x^{\alpha-1} \begin{Bmatrix} \operatorname{si}(bx^2) \\ \operatorname{ci}(bx^2) \end{Bmatrix} \operatorname{erfc}(cx) dx = V(0)$$

[$b, \operatorname{Re} \alpha > 0; |\arg c| < \pi/4; V(\gamma)$ см. в 2.8.26.6]

$$8. \int_0^{\infty} e^{-c^2 x^2} \operatorname{si}(bx) \operatorname{erf}(icx) dx = \frac{\sqrt{\pi} i}{4c} \operatorname{Ei} \left(-\frac{b^2}{4c^2} \right)$$

[$b > 0; |\arg c| < \pi/4$]

$$9. \int_0^{\infty} \frac{1}{x} e^{-c^2 x^2} \operatorname{ci}(bx) \operatorname{erf}(icx) dx =$$

$$= \frac{\pi i}{2} \left[\frac{C}{2} + \ln \frac{b}{c} - \frac{b}{c\sqrt{\pi}} {}_2F_2 \left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -\frac{b^2}{4c^2} \right) \right]$$

[$b > 0; |\arg c| < \pi/4$]

2.9. ИНТЕГРАЛЫ ФРЕНЕЛЯ $S(x)$, $C(x)$

2.9.1. Интегралы общего вида.

Обозначение: $\delta = \begin{cases} 1 \\ 0 \end{cases}$.

$$1. \int_0^a x^{\alpha-1} (a^r - x^r)^{\beta-1} \left\{ \begin{matrix} S(cx) \\ C(cx) \end{matrix} \right\} dx = \frac{\sqrt{2} a^{\alpha+r\beta+\delta-r+1/2} c^{\delta+1/2}}{r\sqrt{\pi}} \times$$

$$\times \Gamma(\beta) \sum_{k=0}^{\infty} \frac{1}{(4k+2\delta+1)(\delta+1/2)_k k!} \Gamma \left[\frac{(2k+\alpha+\delta+1/2)/r}{(2k+\alpha+\delta+1/2)/r+\beta} \right] \left(-\frac{a^2 c^2}{4} \right)^k$$

[$a, r, \operatorname{Re} \beta > 0; \operatorname{Re} \alpha > -\delta - 1/2$].

$$2. \int_a^{\infty} x^{\alpha-1} (x^r - a^r)^{\beta-1} \left\{ \begin{matrix} S(cx) \\ C(cx) \end{matrix} \right\} dx = \frac{\sqrt{2} a^{\alpha+r\beta+\delta-r+1/2} c^{\delta+1/2}}{r\sqrt{\pi}} \times$$

$$\times \Gamma(\beta) \sum_{k=0}^{\infty} \frac{1}{(4k+2\delta+1)(\delta+1/2)_k k!} \Gamma \left[\frac{1-\beta-(2k+\alpha+\delta+1/2)/r}{1-(2k+\alpha+\delta+1/2)/r} \right] \times$$

$$\times \left(-\frac{a^2 c^2}{4} \right)^k + \frac{c^{r-\beta-\alpha}}{r\sqrt{2\pi}} \sum_{k=0}^{\infty} \frac{(1-\beta)_k (ac)^{rk}}{k! (k+1-\beta-\alpha/r)} \times$$

$$\times \Gamma \left(\alpha - r - rk + r\beta + \frac{1}{2} \right) \begin{cases} \cos [(r+rk-r\beta-\alpha+1/2)\pi/2] \\ \sin [(r+rk-r\beta-\alpha+1/2)\pi/2] \end{cases}$$

[$a, c, r, \operatorname{Re} \beta > 0; \operatorname{Re}(\alpha+r\beta) < r$].

$$3. \int_0^{\infty} \frac{x^{\alpha-1}}{(x^r + z^r)^{\rho}} \left\{ \begin{matrix} S(cx) \\ C(cx) \end{matrix} \right\} dx = \frac{\sqrt{2} c^{\delta+1/2} z^{\alpha+\delta-r\rho+1/2}}{r\sqrt{\pi} \Gamma(\rho)} \times$$

$$\times \sum_{k=0}^{\infty} \frac{(-c^2 z^2)^k}{2^{2k} (4k+2\delta+1)(\delta+1/2)_k k!} \Gamma \left(\frac{4k+2\alpha+2\delta+1}{2r} \right) \times$$

$$\times \Gamma \left(\rho - \frac{4k+2\alpha+2\delta+1}{2r} \right) + \frac{c^{r\rho-\alpha}}{\sqrt{2\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k (\rho)_k (cz)^{rk}}{k! (rk+r\rho-\alpha)} \times$$

$$\times \Gamma \left(\alpha - rk - r\rho + \frac{1}{2} \right) \begin{cases} \cos [(rk+r\rho-\alpha+1/2)\pi/2] \\ \sin [(rk+r\rho-\alpha+1/2)\pi/2] \end{cases}$$

[$a, r > 0; r|\arg z| < \pi; -\delta - 1/2 < \operatorname{Re} \alpha < r \operatorname{Re} \rho$]

$$4. \int_0^{\infty} \frac{x^{\alpha-1}}{x^r - y^r} \left\{ \begin{matrix} S(cx) \\ C(cx) \end{matrix} \right\} dx = -\frac{\sqrt{2\pi} c^{\delta+1/2} y^{\alpha+\delta-r+1/2}}{r} \times$$

$$\times \sum_{k=0}^{\infty} \frac{(-c^2 y^2)^k}{2^{2k} (4k+2\delta+1)(\delta+1/2)_k k!} \operatorname{ctg} \frac{4k+2\alpha+2\delta+1}{2r} \pi +$$

$$+ \frac{c^{r-\alpha}}{\sqrt{2\pi}} \sum_{k=0}^{\infty} \frac{(cy)^{rk}}{rk+r-\alpha} \Gamma \left(\alpha - rk - r + \frac{1}{2} \right) \begin{cases} \cos [(rk+r-\alpha+1/2)\pi/2] \\ \sin [(rk+r-\alpha+1/2)\pi/2] \end{cases}$$

[$c, r, y > 0; -\delta - 1/2 < \operatorname{Re} \alpha < r$]

$$5. \int_0^{\infty} x^{\alpha-1} e^{-px^r} \left\{ \begin{matrix} S(cx) \\ C(cx) \end{matrix} \right\} dx = U(r)$$

[$c, r, \operatorname{Re} p > 0; \operatorname{Re} \alpha > -\delta - 1/2$].

$$U(r) = \frac{\sqrt{2} c^{\delta+1/2}}{r\sqrt{\pi} p^{(\alpha+\delta+1/2)/r}} \sum_{k=0}^{\infty} \frac{(-1)^k}{(1/2+\delta)_k (4k+2\delta+1) k!} \times$$

$$\times \Gamma \left(\frac{4k+2\alpha+2\delta+1}{2r} \right) \left(\frac{c}{2p^{1/r}} \right)^{2k}$$

[$r > 1$].

$$U(r) = -\frac{1}{c^\alpha \sqrt{2\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k \Gamma(rk + \alpha + 1/2)}{(rk + \alpha) k!} \begin{cases} \cos[(1/2 - \alpha - rk)\pi/2] \\ \sin[(1/2 - \alpha - rk)\pi/2] \end{cases} \left(\frac{p}{c^r}\right)^k + \\ + \frac{1}{2p^{\alpha/r} r} \Gamma\left(\frac{\alpha}{r}\right) \quad [r < 1],$$

$U(1)$ см. в 2.9.3.2.

$$6. \int_0^{\infty} x^{\alpha-1} e^{-px-r} \begin{Bmatrix} S(cx) \\ C(cx) \end{Bmatrix} dx = \frac{\sqrt{2} c^{\delta+1/2} p^{\alpha+\delta+1/2}/r}{r\sqrt{\pi}} \times \\ \times \sum_{k=0}^{\infty} \frac{(-1)^k}{(1/2+\delta)_k (4k+2\delta+1) k!} \Gamma\left(-\frac{4k+2\alpha+2\delta+1}{2r}\right) \left(\frac{cp^{1/r}}{2}\right)^{2k} + \\ + \frac{1}{\sqrt{2\pi} c^\alpha} \sum_{k=0}^{\infty} \frac{(-1)^k \Gamma(\alpha - rk + 1/2)}{(rk - \alpha) k!} \begin{cases} \cos[(rk - \alpha + 1/2)\pi/2] \\ \sin[(rk - \alpha + 1/2)\pi/2] \end{cases} (cp)^k \\ [c, r, \operatorname{Re} p > 0; \operatorname{Re} \alpha < 0]$$

$$7. \int_0^{\infty} x^{\alpha-1} \sin bx^r \begin{Bmatrix} S(cx) \\ C(cx) \end{Bmatrix} dx = V(1) \quad [b, c, r > 0; -r - \delta - 1/2 < \operatorname{Re} \alpha < r].$$

$$V(\gamma) = \frac{\sqrt{2} c^{\delta+1/2}}{r\sqrt{\pi} b^{\alpha+\delta+1/2}/r} \sum_{k=0}^{\infty} \frac{(-1)^k}{(4k+2\delta+1) (\delta+1/2)_k k!} \times \\ \times \Gamma\left(\frac{4k+2\alpha+2\delta+1}{2r}\right) \cos \frac{4k+2\alpha+2\delta-2r\gamma+1}{4r} \pi \left(\frac{c}{2b^{1/r}}\right)^{2k} \quad [r > 1],$$

$$V(\gamma) = -\frac{b\gamma}{\sqrt{2\pi} c^{\alpha+r\gamma}} \sum_{k=0}^{\infty} \frac{(-1)^k}{(2rk + \alpha + r\gamma) (\gamma+1/2)_k k!} \times \\ \times \Gamma\left(\frac{4rk+2\alpha+2r\gamma+1}{2}\right) \begin{cases} \cos[(4rk+2\alpha+2r\gamma-1)\pi/4] \\ \sin[(4rk+2\alpha+2r\gamma-1)\pi/4] \end{cases} \left(\frac{b}{2c^r}\right)^{2k} + \\ + \frac{1}{2b^{\alpha/r} r} \Gamma\left(\frac{\alpha}{r}\right) \cos \frac{\alpha-r\gamma}{2r} \pi \quad [r < 1].$$

$V(\gamma)$ при $r=1$ см в 2.9.4.3—4

$$8. \int_0^{\infty} x^{\alpha-1} \cos bx^r \begin{Bmatrix} S(cx) \\ C(cx) \end{Bmatrix} dx = V(0) \\ [b, c, r > 0; -\delta - 1/2 < \operatorname{Re} \alpha < r; V(\gamma) \text{ см. в 2.9.1.7}]$$

$$9. \int_0^{\infty} x^{\alpha-1} e^{-px-r} \sin bx^r \begin{Bmatrix} S(cx) \\ C(cx) \end{Bmatrix} dx = W(1) \\ [b, c, r, \operatorname{Re} p > 0; \operatorname{Re} \alpha > -r - \delta - 1/2].$$

$$W(\gamma) = \frac{\sqrt{2} c^{\delta+1/2}}{r\sqrt{\pi} (b^2 + p^2)^{(2\alpha+2\delta+1)/(4r)}} \times \\ \times \sum_{k=0}^{\infty} \frac{(-1)^k}{(4k+2\delta+1) (\delta+1/2)_k k!} \Gamma\left(\frac{4k+2\alpha+2\delta+1}{2r}\right) \times \\ \times \cos\left(\frac{\gamma\pi}{2} - \frac{4k+2\alpha+2\delta+1}{2r} \arccos \frac{p}{\sqrt{b^2+p^2}}\right) \frac{c^{2k}}{2^{2k} (b^2+p^2)^{k/r}} \\ [r > 1] \text{ или } [r=1; |c|^2 < |b^2+p^2|].$$

$$W(\gamma) = -\frac{1}{\sqrt{2}c^\alpha} \sum_{k=0}^{\infty} \frac{(-1)^k \Gamma(rk + \alpha + 1/2)}{k! (rk + \alpha)} \times \\ \times \left\{ \begin{array}{l} \cos [(1 - 2rk - 2\alpha) \pi/4] \\ \sin [(1 - 2rk - 2\alpha) \pi/4] \end{array} \right\} \cos \left(\frac{\gamma\pi}{2} + k \arccos \frac{p}{\sqrt{b^2 + p^2}} \right) \times \\ \times \left(\frac{\sqrt{b^2 + p^2}}{c^r} \right)^k + \frac{(b^2 + p^2)^{-\alpha/(2r)}}{2r} \Gamma\left(\frac{\alpha}{r}\right) \cos \left(\frac{\gamma\pi}{2} - \frac{\alpha}{r} \arccos \frac{p}{\sqrt{b^2 + p^2}} \right) \\ [r < 1] \text{ или } [r = 1; |c|^2 > |b^2 + p^2|]$$

$$10. \int_0^{\infty} x^{\alpha-1} e^{-px^r} \cos bx^r \left\{ \begin{array}{l} S(cx) \\ C(cx) \end{array} \right\} dx = W(0)$$

$[b, c, r, \operatorname{Re} p > 0; \operatorname{Re} \alpha > -\delta - 1/2; W(\gamma) \text{ см. в 2.9.1.9}]$

$$11. \int_0^{\infty} x^{\alpha-1} \ln(x^r + z^r) \left\{ \begin{array}{l} S(cx) \\ C(cx) \end{array} \right\} dx = X(1)$$

$[c, r > 0; -\delta - 1/2 < \operatorname{Re} \alpha < 0; r |\arg z| < \pi]$

$$X(\gamma) = -2\sqrt{2\pi} c^{\delta+1/2} z^{\alpha+\delta+1/2} \times \\ \times \sum_{k=0}^{\infty} \frac{(-c^2 z^2)^k \cos^{1-\gamma} \frac{2k+\alpha+\delta+1/2}{r} \pi \operatorname{cosec} \frac{2k+\alpha+\delta+1/2}{r} \pi}{2^{2k} (4k+2\delta+1) (4k+2\alpha+2\delta+1) (\delta+1/2)_k k!} - \\ - \frac{1}{\sqrt{2\pi} c^\alpha} \sum_{k=1}^{\infty} \frac{(-1)^k \gamma (cz)^{rk} \Gamma(\alpha - kr + 1/2)}{k(kr - \alpha)} \left\{ \begin{array}{l} \cos [(2kr - 2\alpha + 1) \pi/4] \\ \sin [(2kr - 2\alpha + 1) \pi/4] \end{array} \right\} + \\ + \frac{r \Gamma(\alpha + 1/2)}{\sqrt{2\pi} \alpha c^\alpha} \left\{ \begin{array}{l} \cos [(1 - 2\alpha) \pi/4] \\ \sin [(1 - 2\alpha) \pi/4] \end{array} \right\} \left[\frac{1}{\alpha} - \psi\left(\alpha + \frac{1}{2}\right) + \frac{\pi}{2} \operatorname{tg} \frac{2\alpha - 1}{4} \pi + \ln c \right].$$

$$12. \int_0^{\infty} x^{\alpha-1} \ln |x^r - z^r| \left\{ \begin{array}{l} S(cx) \\ C(cx) \end{array} \right\} dx = X(0)$$

$[c, r, z > 0; -\delta - 1/2 < \operatorname{Re} \alpha < 0; X(\gamma) \text{ см. в 2.9.1.11}]$

$$13. \int_0^{\infty} x^{\alpha-1} S(bx^r) \left\{ \begin{array}{l} S(cx) \\ C(cx) \end{array} \right\} dx = Y(1)$$

$[b, c, r > 0; -\delta - (3r + 1)/2 < \operatorname{Re} \alpha < 0; \\ \text{при } r < 1 - \text{замена } x = t^{1/r}].$

$$Y(\gamma) = \frac{c^{\delta+1/2}}{8\pi b^{(2\alpha+2\delta+1)/(2r)}} \times$$

$$\times \sum_{k=0}^{\infty} \frac{(-1)^k}{(1/2 + \delta)_k (k + (2\delta + 1)/4) (k + (1 + 2\alpha + 2\delta)/4) k!} \times \\ \times \Gamma\left(\frac{4k + 2\alpha + 2\delta + 1 + r}{2r}\right) \sin \left[\left(\frac{2\gamma + 1}{4} - \frac{4k + 2\alpha + 2\delta + 1}{4r} \right) \pi \right] \times \\ \times \left(\frac{c}{2b^{1/r}} \right)^{2k} + \frac{\Gamma(\alpha + 1/2)}{2\alpha \sqrt{2\pi} c^\alpha} \left\{ \begin{array}{l} \cos [(1 - 2\alpha) \pi/4] \\ \sin [(1 - 2\alpha) \pi/4] \end{array} \right\} \quad [r > 1].$$

$Y(\gamma) \text{ при } r=1 \text{ см в 2.9.6.1--2.}$

$$14. \int_0^{\infty} x^{\alpha-1} C(bx^r) \left\{ \begin{array}{l} S(cx) \\ C(cx) \end{array} \right\} dx = Y(0)$$

$[b, c, r > 0; -\delta - (r + 1)/2 < \operatorname{Re} \alpha < 0; Y(\gamma) \text{ см. в 2.9.1.13}].$

$$15. \int_0^{\infty} x^{\alpha-1} \operatorname{erf}(bx^r) \begin{Bmatrix} S(cx) \\ C(cx) \end{Bmatrix} dx = Z(1)$$

[$c, r > 0; -1 \mp 1/2 - r < \operatorname{Re} \alpha < 0; |\arg b| < \pi/4$],

$$Z(\gamma) = (-1)^{\gamma} \frac{c^{\delta+1/2}}{4\pi \sqrt{2} b^{(\alpha+\delta+1/2)/r}} \sum_{k=0}^{\infty} \frac{1}{k! (\delta+1/2)_k} \times \\ \times \Gamma\left(\frac{2k+r+\alpha+\delta+1/2}{2r}\right) \left(-\frac{c^2}{4b^{2/r}}\right)^k - \gamma \frac{\Gamma(\alpha+1/2)}{\sqrt{2\pi} \alpha c^{\alpha}} \begin{Bmatrix} \sin[(2\alpha+1)\pi/4] \\ \cos[(2\alpha+1)\pi/4] \end{Bmatrix} \\ [r > 1/2],$$

$$Z(\gamma) = \frac{(-1)^{\gamma} \sqrt{2} b}{\pi c^{\alpha+r}} \sum_{k=0}^{\infty} \frac{\Gamma(2rk+\alpha+r+1/2)}{k! (2k+1) (2rk+\alpha+r)} \begin{Bmatrix} \sin[(4rk+2\alpha+2r+1)\pi/4] \\ \cos[(4rk+2\alpha+2r+1)\pi/4] \end{Bmatrix} + \\ + \frac{(-1)^{\gamma}}{2\alpha \sqrt{\pi} b^{\alpha/r}} \Gamma\left(\frac{\alpha+r}{2r}\right) - (1-\gamma) \frac{\Gamma(\alpha+1/2)}{\sqrt{2\pi} \alpha c^{\alpha}} \begin{Bmatrix} \sin[(2\alpha+1)\pi/4] \\ \cos[(2\alpha+1)\pi/4] \end{Bmatrix} \\ [r < 1/2],$$

$Z(\gamma)$ при $r=1/2$ см. в 2.9.8.1—2.

$$16. \int_0^{\infty} x^{\alpha-1} \operatorname{erfc}(bx^r) \begin{Bmatrix} S(cx) \\ C(cx) \end{Bmatrix} dx = Z(0)$$

[$c, r > 0; \operatorname{Re} \alpha > -1 \mp 1/2; |\arg b| < \pi/4; Z(\gamma)$ см. в 2.9.1.15].

2.9.2. Интегралы от $A(x) \begin{Bmatrix} S(cx) \\ C(cx) \end{Bmatrix}$.

Обозначение. $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$.

$$1. \int_0^{\infty} x^{\alpha-1} \begin{Bmatrix} S(cx) \\ C(cx) \end{Bmatrix} dx = -\frac{\Gamma(\alpha+1/2)}{\sqrt{2\pi} \alpha c^{\alpha}} \begin{Bmatrix} \sin[(2\alpha+1)\pi/4] \\ \cos[(2\alpha+1)\pi/4] \end{Bmatrix}$$

[$c > 0; -1 \mp 1/2 < \operatorname{Re} \alpha < 0; \alpha \neq -1/2$]

$$2. \int_0^{\infty} x^{-3/2} S(cx) dx = \sqrt{\frac{\pi c}{2}}$$

[$c > 0$].

$$3. \int_0^{\infty} x^{\alpha-1} \left[\frac{1}{2} - \begin{Bmatrix} S(cx) \\ C(cx) \end{Bmatrix} \right] dx = \frac{\Gamma(\alpha+1/2)}{\sqrt{2\pi} \alpha c^{\alpha}} \begin{Bmatrix} \sin[(2\alpha+1)\pi/4] \\ \cos[(2\alpha+1)\pi/4] \end{Bmatrix}$$

[$c > 0; 0 < \operatorname{Re} \alpha < 3/2$]

$$4. \int_0^{\infty} \frac{1}{x} \begin{Bmatrix} S(bx) - S(cx) \\ C(bx) - C(cx) \end{Bmatrix} dx = \frac{1}{2} \ln \frac{b}{c}$$

[$b, c > 0$].

$$5. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\beta-1} \begin{Bmatrix} S(cx) \\ C(cx) \end{Bmatrix} dx =$$

$$\mp \frac{a^{\alpha+2\beta} - \frac{3}{2} c^{\delta+1/2}}{\sqrt{2\pi} (2\delta+1)} B\left(\beta, \frac{\alpha+\delta}{2} + \frac{1}{4}\right) {}_2F_3\left(\frac{\alpha+\delta}{2} + \frac{1}{4}, \frac{2\delta+1}{4}; \delta + \frac{1}{2}, \frac{5+2\delta}{4}, \frac{\alpha+\delta}{2} + \frac{1}{4} + \beta; -\frac{a^2 c^2}{4}\right) \\ [a, \operatorname{Re} \beta > 0; \operatorname{Re} \alpha > -\delta - 1/2]$$

$$\begin{aligned}
6. \int_a^\infty x^{\alpha-1} (x^2 - a^2)^{\beta-1} \left\{ \frac{S(cx)}{C(cx)} \right\} dx = \\
= \frac{a^{\alpha+2\beta+\delta-3/2} c^{\delta+1/2}}{\sqrt{2\pi} (1+2\delta)} B\left(\beta, \frac{3}{4} - \frac{\alpha+\delta}{2} - \beta\right) \times \\
\times {}_2F_3\left(\frac{1}{4} + \frac{\alpha+\delta}{2}, \frac{1+2\delta}{4}; \frac{5+2\delta}{4}, \frac{1}{2} + \delta, \frac{1}{4} + \beta + \frac{\alpha+\delta}{2}; -\frac{a^2 c^2}{4}\right) + \\
+ \frac{c^{2-\alpha-2\beta}}{(2\beta+\alpha-2)\sqrt{2\pi}} \Gamma\left(\alpha+2\beta - \frac{3}{2}\right) \left\{ \frac{\sin [((2\alpha+1)/4 + \beta)\pi]}{\cos [((2\alpha+1)/4 + \beta)\pi]} \right\} \times \\
\times {}_2F_3\left(1-\beta, 1-\beta - \frac{\alpha}{2}; 2-\beta - \frac{\alpha}{2}, \frac{5-2\alpha}{4} - \beta, \frac{7-2\alpha}{4} - \beta; -\frac{a^2 c^2}{4}\right) \\
[a, c, \operatorname{Re} \beta > 0; \operatorname{Re}(\alpha+2\beta) < 2]
\end{aligned}$$

$$\begin{aligned}
7. \int_0^\infty \frac{x^{\alpha-1}}{(x^2+z^2)^\rho} \left\{ \frac{S(cx)}{C(cx)} \right\} dx = \\
= \frac{c^{\delta+1/2} z^{\alpha+\delta-2\rho+1/2}}{(2\delta+1)\sqrt{2\pi}} B\left(\rho - \frac{\alpha+\delta}{2} - \frac{1}{4}, \frac{\alpha+\delta}{2} + \frac{1}{4}\right) \times \\
\times {}_2F_3\left(\frac{1+2\delta}{4}, \frac{\alpha+\delta}{2} + \frac{1}{4}; \frac{5+2\delta}{4}, \frac{1}{2} + \delta, \frac{5}{4} + \frac{\alpha+\delta}{2} - \rho; \frac{c^2 z^2}{4}\right) + \\
+ \frac{c^{2\rho-\alpha}}{\sqrt{2\pi} (2\rho-\alpha)} \left\{ \frac{\sin [((2\alpha+1)/4 - \rho)\pi]}{\cos [((2\alpha+1)/4 - \rho)\pi]} \right\} \Gamma\left(\frac{1}{2} + \alpha - 2\rho\right) \times \\
\times {}_2F_3\left(\rho - \frac{\alpha}{2}, \rho, 1 + \rho - \frac{\alpha}{2}, \frac{3-2\alpha}{4} + \rho, \frac{1-2\alpha}{4} + \rho; \frac{c^2 z^2}{4}\right) \\
[c, \operatorname{Re} z > 0; -\delta - 1/2 < \operatorname{Re} \alpha < 2 \operatorname{Re} \rho]
\end{aligned}$$

$$8. \int_0^\infty \frac{x^{\pm 1/2}}{x^2 + z^2} \left\{ \frac{S(cx)}{C(cx)} \right\} dx = \frac{\pi}{2\sqrt{2z}} \left\{ \frac{1}{z^{-1}} \right\} \operatorname{erf}(\sqrt{cz}) \quad [c, \operatorname{Re} z > 0]$$

$$9. \int_0^\infty \frac{x}{(x^2+z^2)^{3/2}} \left\{ \frac{S(cx)}{C(cx)} \right\} dx = \frac{c}{2} I_{\pm 1/4} \left(\frac{cz}{2} \right) K_{1/4} \left(\frac{cz}{2} \right) \quad [c, \operatorname{Re} z > 0]$$

$$\begin{aligned}
10. \int_0^\infty \frac{x^{\alpha-1}}{x^2+y^2} \left\{ \frac{S(cx)}{C(cx)} \right\} dx = \frac{c^{\alpha-1}}{(\alpha-2)\sqrt{2\pi}} \Gamma\left(\alpha - \frac{3}{2}\right) \times \\
\times \left\{ \frac{\sin [(2\alpha+1)\pi/4]}{\cos [(2\alpha+1)\pi/4]} \right\} {}_2F_3\left(1, 1 - \frac{\alpha}{2}; 2 - \frac{\alpha}{2}, \frac{7-2\alpha}{4}, \frac{5-2\alpha}{4}; -\frac{c^2 y^2}{4}\right) \pm \\
\pm \sqrt{\frac{\pi c}{2}} \frac{c^\delta y^{\alpha+\delta-3/2}}{2\delta+1} \left(\operatorname{tg} \frac{2\alpha+1}{4} \pi \right)^{\pm 1} {}_1F_2\left(\frac{2\delta+1}{4}; \delta + \frac{1}{2}, \frac{2\delta+5}{4}; -\frac{c^2 y^2}{4}\right) \\
[c, y > 0; -1/2 - \delta < \operatorname{Re} \alpha < 2]
\end{aligned}$$

$$\begin{aligned}
11. \int_0^\infty \frac{x^{-1/2}}{\sqrt{x^2+z^2}} (\sqrt{x^2+z^2} \mp z)^{1/2} \left[\frac{1}{2} - \left\{ \frac{S(cx)}{C(cx)} \right\} \right] dx = -\frac{1}{2} \operatorname{Ei}(-cz) \\
[c, \operatorname{Re} z > 0]
\end{aligned}$$

$$\begin{aligned}
12. \int_0^\infty \frac{C(cx)}{(x^2+z^2)(x^2+3^2 z^2) \dots (x^2+(2n+1)^2 z^2)} dx = \\
= \frac{(-1)^n \pi}{(2n+1)! 2^{2n+3/2} z^{2n+1}} \sum_{k=0}^n (-1)^k \binom{2n+1}{k} \frac{\operatorname{erf}(\sqrt{(2n-2k+1)cz})}{\sqrt{2n-2k+1}} \\
[c, \operatorname{Re} z > 0]
\end{aligned}$$

2.9.3. Интегралы от $x^\alpha e^{-px^r} \left\{ \begin{matrix} S(cx) \\ C(cx) \end{matrix} \right\}$.

Обозначение: $\delta = \begin{cases} 1 \\ 0 \end{cases}$.

$$1. \int_0^\infty e^{-px} \left\{ \begin{matrix} S(cx) \\ C(cx) \end{matrix} \right\} dx = \frac{\sqrt{c}}{2p \sqrt{p^2 + c^2}} \sqrt{p^2 + c^2} \mp p \quad [c, \operatorname{Re} p > 0].$$

$$2. \int_0^\infty x^{\alpha-1} e^{-px} \left\{ \begin{matrix} S(cx) \\ C(cx) \end{matrix} \right\} dx = \sqrt{\frac{2}{\pi}} \frac{c^{\delta+1/2} \Gamma(\alpha + \delta + 1/2)}{p^{\alpha+\delta+1/2} (2\delta+1)} \times \\ \times {}_3F_2 \left(\frac{2\delta+1}{4}, \frac{2\alpha+3}{4}, \frac{2\alpha+1}{4} + \delta; \frac{2\delta+5}{4}, \delta + \frac{1}{2}; -\frac{c^2}{p^2} \right) \\ [c, \operatorname{Re} p > 0; \operatorname{Re} \alpha > -\delta - 1/2].$$

$$3. \int_0^\infty x^{-1/2} e^{-px} \left\{ \begin{matrix} S(cx) \\ C(cx) \end{matrix} \right\} dx = \frac{1}{4 \sqrt{\pi p}} \left(2 \operatorname{arctg} \frac{\sqrt{2cp}}{p-c} \pm \ln \frac{c - \sqrt{2cp} + p}{c + \sqrt{2cp} + p} \right) \\ [c, \operatorname{Re} p > 0].$$

$$4. \int_0^\infty x e^{-px} S(cx) dx = \frac{\sqrt{c}}{2p^2} \left(\frac{p}{2 \sqrt{p^2 + c^2}} + \frac{p^2}{p^2 + c^2} + 1 \right) \left(\frac{\sqrt{p^2 + c^2} - p}{p^2 + c^2} \right)^{1/2} \\ [c, \operatorname{Re} p > 0].$$

$$5. \int_0^\infty x^{\alpha-1} e^{-px^2} \left\{ \begin{matrix} S(cx) \\ C(cx) \end{matrix} \right\} dx = \frac{c^{\delta+1/2}}{\sqrt{2\pi} (2\delta+1) p^{(2\alpha+2\delta+1)/4}} \times \\ \times \Gamma \left(\frac{2\alpha+2\delta+1}{4} \right) {}_3F_2 \left(\frac{2\delta+1}{4}, \frac{2\alpha+2\delta+1}{4}; \frac{2\delta+5}{4}, \delta + \frac{1}{2}; -\frac{c^2}{4p} \right) \\ [\operatorname{Re} p > 0; \operatorname{Re} \alpha > -\delta - 1/2].$$

$$6. \int_0^\infty x e^{-px^2} \left\{ \begin{matrix} S(cx) \\ C(cx) \end{matrix} \right\} dx = \frac{\sqrt{\pi} c}{8p^{3/2}} \exp \left(-\frac{c^2}{8p} \right) I_{\pm 1/4} \left(\frac{c^2}{8p} \right) \\ [\operatorname{Re} p > 0].$$

$$7. \int_0^\infty x^{-1/2} e^{-px^2} C(cx) dx = \frac{1}{4 \sqrt[4]{p}} \gamma \left(\frac{1}{4}, \frac{c^2}{8p} \right) \\ [\operatorname{Re} p > 0].$$

$$8. \int_0^\infty x^{-1/2} e^{-p \sqrt{x}} \left\{ \begin{matrix} S(cx) \\ C(cx) \end{matrix} \right\} dx = \frac{2}{p} \left[\left(\frac{1}{2} - S(\varphi) \right) \left\{ \begin{matrix} \sin \varphi \\ \cos \varphi \end{matrix} \right\} \pm \left(\frac{1}{2} - C(\varphi) \right) \left\{ \begin{matrix} \cos \varphi \\ \sin \varphi \end{matrix} \right\} \right] \\ [\varphi = p^2/(4c); \operatorname{Re} p > 0].$$

$$9. \int_0^\infty x^{-2} e^{-p/x} \left\{ \begin{matrix} S(cx) \\ C(cx) \end{matrix} \right\} dx = \frac{1}{2p} [1 - e^{-\sqrt{2pc}} (\cos \sqrt{2pc} \pm \sin \sqrt{2pc})] \\ [c, \operatorname{Re} p > 0].$$

2.9.4. Интегралы, содержащие тригонометрические функции и $\left\{ \begin{matrix} S(cx) \\ C(cx) \end{matrix} \right\}$.

Обозначение: $\delta = \begin{cases} 1 \\ 0 \end{cases}$.

$$\Gamma. \int_0^\infty \sin bx \left[\frac{1}{2} - \left\{ \begin{matrix} S(cx) \\ C(cx) \end{matrix} \right\} \right] dx = \frac{1}{2b} - \frac{\sqrt{c}}{4b} \left[\frac{1}{\sqrt{b+c}} + \frac{1}{\sqrt{|b-c|}} \left\{ \begin{matrix} \operatorname{sgn}(c-b) \\ 1 \end{matrix} \right\} \right] \\ [b, c > 0; b \neq c].$$

$$2. \int_0^{\infty} \cos bx \left[\frac{1}{2} - \left\{ \frac{S(cx)}{C(cx)} \right\} \right] dx = \frac{\sqrt{c}}{4b} \left[\frac{1}{\sqrt{|b-c|}} \left\{ \operatorname{sgn}(b-c) \right\} \mp \frac{1}{\sqrt{b+c}} \right]$$

[b, c > 0; b ≠ c]

$$3. \int_0^{\infty} x^{\alpha-1} \sin bx \left\{ \frac{S(cx)}{C(cx)} \right\} dx = U(1)$$

[b, c > 0; -3/2 - δ < Re α < 1 при b ≠ c, -3/2 - δ < Re α < 1/2 при b = c]

$$U(\gamma) = \sqrt{\frac{2}{\pi}} \frac{c^{\delta+1/2} \Gamma(\alpha+\delta+1/2)}{(2\delta+1) b^{\alpha+\delta+1/2}} \left\{ \frac{\sin[(2\gamma-2\alpha-1)\pi/4]}{\cos[(2\gamma-2\alpha-1)\pi/4]} \right\} \times$$

$$\times {}_2F_2\left(\frac{2\delta+1}{4}, \frac{2\alpha+3}{4}, \frac{2\alpha+1}{4} + \delta; \frac{2\delta+5}{4}, \delta + \frac{1}{2}; \frac{c^2}{b^2}\right) \quad [0 < c \leq b].$$

$$U(\gamma) = -\frac{b\gamma\Gamma(\alpha+\gamma+1/2)}{\sqrt{2\pi}(\alpha+\gamma)c^{\alpha+\gamma}} \left\{ \frac{\cos[(1-2\alpha-2\gamma)\pi/4]}{\sin[(1-2\alpha-2\gamma)\pi/4]} \right\} \times$$

$$\times {}_2F_2\left(\frac{\alpha+\gamma}{2}, \frac{2\alpha+3}{4}, \frac{2\alpha+1}{4} + \gamma; \frac{\alpha+\gamma}{2} + 1, \frac{1}{2} + \gamma; \frac{b^2}{c^2}\right) +$$

$$+ \frac{1}{2b\alpha} \Gamma(\alpha) \cos \frac{\gamma-\alpha}{2} \pi \quad [0 < b < c]$$

$$4. \int_0^{\infty} x^{\alpha-1} \cos bx \left\{ \frac{S(cx)}{C(cx)} \right\} dx = U(0)$$

[b, c > 0; -1/2 - δ < Re α < 1 при b ≠ c; -1/2 - δ < Re α < 1/2 при b = c; U(γ) см. в 2.9.4.3]

$$5. \int_0^{\infty} x^{-1/2} \left\{ \frac{\sin bx S(cx)}{\cos bx C(cx)} \right\} dx = \frac{1}{2} \sqrt{\frac{\pi}{2b}}$$

[c > b > 0]

$$= \frac{1}{4} \sqrt{\frac{\pi}{2c}} \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \quad \left[\begin{Bmatrix} b > c > 0 \\ b = c > 0 \end{Bmatrix} \right].$$

$$6. \int_0^{\infty} \frac{1}{x} \left\{ \frac{\cos bx S(cx)}{\sin bx C(cx)} \right\} dx = \frac{1}{2} \left(\ln \frac{\sqrt{c} + \sqrt{b+c}}{\sqrt{b}} \mp \arcsin \sqrt{\frac{c}{b}} \right)$$

[b > c > 0]

$$= \frac{1}{2} \left(\ln \frac{\sqrt{c} + \sqrt{c+b}}{\sqrt{c} + \sqrt{c-b}} \mp \frac{\pi}{2} \right) \quad [c > b > 0]$$

$$7. \int_0^{\infty} \frac{1}{x} \sin cx \left\{ \frac{S(cx)}{C(cx)} \right\} dx = \frac{1}{2} \left[\frac{\pi}{2} \mp \ln(\sqrt{2} + 1) \right]$$

[c > 0]

$$8. \int_0^{\infty} \frac{1}{x} \cos cx \left\{ \frac{S(cx)}{C(cx)} \right\} dx = \frac{1}{2} \ln(\sqrt{2} + 1) \mp \frac{\pi}{4}$$

[c > 0]

$$9. \int_0^{\infty} x \sin bx^2 \left[\frac{1}{2} - \left\{ \frac{S(cx)}{C(cx)} \right\} \right] dx = \frac{1}{2b} - \frac{\sqrt{\pi} c}{8b^{3/2}} J_{\pm 1/4} \left(\frac{c^2}{8b} \right) \cos \left(\frac{c^2}{8b} - \frac{2 \pm 1}{8} \pi \right)$$

[b, c > 0]

$$10. \int_0^{\infty} x \cos bx^2 \left[\frac{1}{2} - \left\{ \frac{S(cx)}{C(cx)} \right\} \right] dx = -\frac{\sqrt{\pi} c}{8b^{3/2}} J_{\pm 1/4} \left(\frac{c^2}{8b} \right) \sin \left(\frac{c^2}{8b} - \frac{2 \pm 1}{8} \pi \right)$$

[b, c > 0]

$$11. \int_0^{\infty} \frac{1}{x} \cos b \sqrt{x} \left\{ \begin{matrix} S(cx) \\ C(cx) \end{matrix} \right\} dx = -\frac{1}{2} \left[\text{ci} \left(\frac{b^2}{4c} \right) \mp \text{si} \left(\frac{b^2}{4c} \right) \right] \quad [b, c > 0].$$

$$12. \int_0^{\infty} x^{-1/2} \sin b \sqrt{x} \left[\frac{1}{2} - \left\{ \begin{matrix} S(cx) \\ C(cx) \end{matrix} \right\} \right] dx = \frac{1}{b} \left(1 - \cos \frac{b^2}{4c} \pm \sin \frac{b^2}{4c} \right) \quad [b, c > 0].$$

$$13. \int_0^{\infty} x^{-1/2} \cos b \sqrt{x} \left[\frac{1}{2} - \left\{ \begin{matrix} S(cx) \\ C(cx) \end{matrix} \right\} \right] dx =$$

$$= \frac{2}{b} \left[C \left(\frac{b}{2\sqrt{c}} \right) \left\{ \begin{matrix} \cos [b^2/(4c)] \\ \sin [b^2/(4c)] \end{matrix} \right\} \pm S \left(\frac{b}{2\sqrt{c}} \right) \left\{ \begin{matrix} \sin [b^2/(4c)] \\ \cos [b^2/(4c)] \end{matrix} \right\} \right] \quad [b, c > 0].$$

$$14. \int_0^{\infty} \frac{1}{x} \sin \frac{b}{x} \left[\frac{1}{2} - \left\{ \begin{matrix} S(cx) \\ C(cx) \end{matrix} \right\} \right] dx =$$

$$= -\frac{1}{2} [\text{si}(2\sqrt{bc}) \mp \text{ci}(2\sqrt{bc}) + \text{Ei}(-2\sqrt{bc})] \quad [b, c > 0].$$

$$15. \int_0^{\infty} \frac{1}{x} \cos \frac{b}{x} \left[\frac{1}{2} - \left\{ \begin{matrix} S(cx) \\ C(cx) \end{matrix} \right\} \right] dx =$$

$$= -\frac{1}{2} [\text{ci}(2\sqrt{bc}) \pm \text{si}(2\sqrt{bc}) + \text{Ei}(-2\sqrt{bc})] \quad [b, c > 0].$$

$$16. \int_0^{\infty} x^{\alpha-1} \left[\left\{ \begin{matrix} \sin cx \\ \cos cx \end{matrix} \right\} C(cx) \mp \left\{ \begin{matrix} \cos cx \\ \sin cx \end{matrix} \right\} S(cx) \right] dx =$$

$$= \frac{\pi}{2^{3/2} c^{\alpha} \Gamma(1-\alpha)} \left\{ \begin{matrix} \text{cosec} [(1-2\alpha)\pi/4] \\ \sec [(1-2\alpha)\pi/4] \end{matrix} \right\} \quad [c > 0; -1 \mp 1/2 < \text{Re} \alpha < 1/2].$$

$$17. \int_0^{\infty} x^{\alpha-1} \left[\left\{ \begin{matrix} \sin cx \\ \cos cx \end{matrix} \right\} \left(\frac{1}{2} - S(cx) \right) \pm \left\{ \begin{matrix} \cos cx \\ \sin cx \end{matrix} \right\} \left(\frac{1}{2} - C(cx) \right) \right] dx =$$

$$= \frac{\Gamma(\alpha)}{2^{3/2} c^{\alpha}} \left\{ \begin{matrix} \sec [(1-2\alpha)\pi/4] \\ \text{cosec} [(1-2\alpha)\pi/4] \end{matrix} \right\} \quad [c > 0; 0 < \text{Re} \alpha < 1 \pm 1/2].$$

$$18. \int_a^{\infty} (x^2 - a^2)^{\beta-1} \left[\left\{ \begin{matrix} \sin cx \\ \cos cx \end{matrix} \right\} \left(\frac{1}{2} - S(cx) \right) \pm \left\{ \begin{matrix} \cos cx \\ \sin cx \end{matrix} \right\} \left(\frac{1}{2} - C(cx) \right) \right] dx =$$

$$= \frac{a^{\beta-1/2} c^{1/2-\beta}}{2^{1+\delta} \sqrt{2\pi}} B \left(\beta, \frac{3+2\delta}{4} - \beta \right) S_{\beta-\delta, \beta-1/2}(ac) \quad [a, c > 0; 0 < \text{Re} \beta < (3+2\delta)/4].$$

$$19. \int_a^{\infty} x (x^2 - a^2)^{\beta-1} \left[\left\{ \begin{matrix} \sin cx \\ \cos cx \end{matrix} \right\} \left(\frac{1}{2} - S(cx) \right) \pm \left\{ \begin{matrix} \cos cx \\ \sin cx \end{matrix} \right\} \left(\frac{1}{2} - C(cx) \right) \right] dx =$$

$$= \frac{a^{\beta+3/2-\delta} c^{3/2-\beta-\delta}}{2^{5/2} \pi^{3/2-\delta}} B \left(\beta, \frac{1+2\delta}{4} - \beta \right) S_{\beta-\delta, \beta+1/2}(ac) \quad [a, c > 0; 0 < \text{Re} \beta < (2 \pm 1)/4]$$

2.9.5. Интегралы, содержащие e^{-px} , тригонометрические функции и $\begin{Bmatrix} S(cx) \\ C(cx) \end{Bmatrix}$.

$$1. \int_0^{\infty} x^{-1/2} e^{-b\sqrt{x}} \begin{Bmatrix} \sin b\sqrt{x} S(cx) \\ \cos b\sqrt{x} C(cx) \end{Bmatrix} dx = \frac{1}{2b} \exp\left(-\frac{b^2}{2c}\right) \quad [b, c > 0].$$

$$2. \int_0^{\infty} x^{-1/2} e^{-b\sqrt{x}} \begin{Bmatrix} \cos b\sqrt{x} S(cx) \\ \sin b\sqrt{x} C(cx) \end{Bmatrix} dx = \\ = \frac{1}{2b} \left[\exp\left(\frac{b^2}{2c}\right) \operatorname{erfc}\left(\frac{b}{\sqrt{2c}}\right) \mp \frac{2}{\sqrt{\pi}} \exp\left(-\frac{b^2}{2c}\right) \operatorname{erfi}\left(\frac{b}{\sqrt{2c}}\right) \right] \quad [b, c > 0].$$

$$3. \int_0^{\infty} x^{-1/2} e^{-px} [\cos cx C(cx) \pm \sin cx S(cx)] dx = \\ = \frac{p}{4c^{3/4} \sqrt{\pi(p+c)}} \left[\frac{1}{\sqrt{V p+c \mp V c}} \left(\frac{\pi}{2} - \operatorname{arctg} \frac{V p+c - V c}{\sqrt[4]{4c} \sqrt{V p+c \pm V c}} \right) - \right. \\ \left. - \frac{1}{2 \sqrt{V p+c \pm V c}} \ln \frac{V c - \sqrt[4]{4c} \sqrt{V p+c + V c} + V p+c}{V c + \sqrt[4]{4c} \sqrt{V p+c \pm V c} + V p+c} \right] \\ [c, \operatorname{Re} p > 0].$$

$$4. \int_0^{\infty} x^{-1/2} e^{-p\sqrt{x}} \left[\cos cx \begin{Bmatrix} C(cx) \\ S(cx) \end{Bmatrix} \pm \sin cx \begin{Bmatrix} S(cx) \\ C(cx) \end{Bmatrix} \right] dx = \\ = \sqrt{\frac{\pi}{2c}} \left[\operatorname{ci}(\varphi) \begin{Bmatrix} \sin \varphi \\ \cos \varphi \end{Bmatrix} \mp \operatorname{si}(\varphi) \begin{Bmatrix} \cos \varphi \\ \sin \varphi \end{Bmatrix} \right] \quad [\varphi = p^2/(4c); c, \operatorname{Re} p > 0].$$

2.9.6. Интегралы, содержащие произведения функций $S(bx), C(cx)$

Обозначение: $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$.

$$1. \int_0^{\infty} x^{\alpha-1} S(bx) \begin{Bmatrix} S(cx) \\ C(cx) \end{Bmatrix} dx = U(1) \quad [b, c > 0; -2 - \delta < \operatorname{Re} \alpha < 0].$$

$$U(\gamma) = - \frac{2c^{\delta+1/2} \Gamma(\alpha + \delta + 1)}{\pi(2\delta + 1)(2\delta + \alpha + 1)b^{\alpha + \delta + 1/2}} \begin{Bmatrix} \cos[(1 + 2\alpha - 2\gamma)\pi/4] \\ \sin[(1 + 2\alpha - 2\gamma)\pi/4] \end{Bmatrix} \times \\ \times {}_4F_3\left(\frac{2\delta + 1}{4}, \frac{2\alpha + 2\delta + 1}{4}, \frac{\alpha + \delta + 1}{4}, \frac{\alpha + \delta + 3}{4}; \delta + \frac{1}{2}, \frac{2\delta + 5}{4}, \right. \\ \left. \frac{2\alpha + 2\delta + 5}{4}; \frac{c^2}{b^2}\right) + \frac{\Gamma(\alpha + 1/2)}{2\alpha \sqrt{2\pi} c^{\alpha}} \begin{Bmatrix} \cos[(1 - 2\alpha)\pi/4] \\ \sin[(1 - 2\alpha)\pi/4] \end{Bmatrix}.$$

$$2. \int_0^{\infty} x^{\alpha-1} C(bx) \begin{Bmatrix} S(cx) \\ C(cx) \end{Bmatrix} dx = U(0)$$

$[b, c > 0; -\delta - 1 < \operatorname{Re} \alpha < 0; U(\gamma)$ см. в 2.9.6.1]

$$3. \int_0^{\infty} x^{-3/2} C(bx) S(cx) dx =$$

$$= \frac{1}{\sqrt{2\pi}} \left[\sqrt{b} \left(\ln \frac{V c + V b + c}{V b} - \operatorname{arcsin} \sqrt{\frac{c}{b}} \right) + \sqrt{c} \left(\ln \frac{V b + V b + c}{V b + V b - c} + \frac{\pi}{2} \right) \right] \\ [b > c > 0].$$

$$4. \quad = \frac{1}{\sqrt{2\pi}} \left[\sqrt{c} \left(\ln \frac{\sqrt{b} + \sqrt{b+c}}{\sqrt{c}} + \arcsin \sqrt{\frac{b}{c}} \right) + \right. \\ \left. + \sqrt{b} \cdot \left(\ln \frac{\sqrt{c} + \sqrt{c+b}}{\sqrt{c} + \sqrt{c-b}} - \frac{\pi}{2} \right) \right] \quad [c > b > 0]$$

$$5. \quad \int_0^\infty x^{-3/2} C(cx) S(cx) dx = \sqrt{\frac{2c}{\pi}} \ln(\sqrt{2} + 1) \quad [c > 0]$$

$$6. \quad \int_0^\infty x^{-3/2} \left\{ \frac{S(cx)}{C(cx)} \right\}^2 dx = \sqrt{\frac{2c}{\pi}} \left[\frac{\pi}{2} \mp \ln(\sqrt{2} + 1) \right] \quad [c > 0]$$

$$7. \quad \int_0^\infty x^{\alpha-1} [C^2(cx) - S^2(cx)] dx = \frac{2\Gamma(\alpha)}{\pi c^\alpha} \sin \frac{\alpha\pi}{2} {}_2F_1\left(\alpha+1, \frac{1}{2}; \frac{3}{2}; -1\right) \\ [c > 0; 0 < \operatorname{Re} \alpha < 1]$$

$$8. \quad \int_0^\infty x^{-1/2} [C^2(cx) - S^2(cx)] dx = \frac{1}{\sqrt{\pi c}} \quad [c > 0]$$

$$9. \quad \int_0^\infty \frac{1}{x} [C^2(cx) - S^2(cx)] dx = \frac{\pi}{4} \quad [c > 0]$$

$$10. \quad \int_0^\infty \frac{x}{x^2 + z^2} [C^2(cx) - S^2(cx)] dx = \frac{\pi}{4} [1 - \operatorname{erf}^2(cz)] \quad [c, \operatorname{Re} z > 0]$$

$$11. \quad \int_0^\infty x^{-1/2} S(bx) [C^2(cx) - S^2(cx)] dx = \sqrt{\frac{b-c}{2\pi bc}} \quad [0 \leq c \leq b < 2c]$$

$$12. \quad \int_0^\infty x^{\alpha-1} \left\{ \left[\frac{1}{2} - C(cx) \right]^2 + \left[\frac{1}{2} - S(cx) \right]^2 \right\} dx = \frac{\Gamma(\alpha+1/2)}{2\sqrt{\pi} \alpha c^\alpha} \sec \frac{\alpha\pi}{2} \\ [c > 0; 0 < \operatorname{Re} \alpha < 1]$$

$$13. \quad \int_0^\infty x^{-1/2} \left[\frac{1}{2} - \left\{ \frac{S(cx)}{C(cx)} \right\} \right]^2 dx = \frac{1}{2\sqrt{\pi c}} \left\{ \frac{2\sqrt{2}-1}{1} \right\} \quad [c > 0]$$

$$14. \quad \int_0^\infty \sin bx \left[\frac{1}{2} - C(cx) \right]^2 dx = \frac{1}{2b} - \frac{\sqrt{c}}{4b\sqrt{b+c}} - \frac{\sqrt{c}}{4b\sqrt{|b-c|}} \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \\ \left[\left\{ \begin{matrix} b > 2c \\ b < 2c \end{matrix} \right\}; b, c > 0 \right]$$

$$15. \quad \int_0^\infty \frac{1}{x} \left\{ \frac{S(bx)}{C(bx)} \right\} \left[\frac{1}{2} - \left\{ \frac{S(cx)}{C(cx)} \right\} \right] dx = \frac{1}{4} \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \ln \frac{b}{c} \quad \left[\left\{ \begin{matrix} 0 < c < b \\ 0 < b < c \end{matrix} \right\} \right]$$

$$16. \quad \int_0^\infty x^{-1/2} \left[\frac{1}{2} - \left\{ \frac{S(bx)}{C(bx)} \right\} \right] \left[\frac{1}{2} - \left\{ \frac{S(cx)}{C(cx)} \right\} \right] dx = \\ = \frac{\sqrt{b} + \sqrt{c}}{\sqrt{2\pi bc}} \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} - \frac{1}{2\sqrt{2\pi bc}} (\sqrt{|b-c|} \pm \sqrt{b+c}) \quad [b, c > 0]$$

$$17. \int_0^{\infty} x^{-1/2} \left[\frac{1}{2} - C(bx) \right] \left[\frac{1}{2} - S(cx) \right] dx = \\ = \frac{1}{\sqrt{2\pi c}} - \frac{1}{2\sqrt{2\pi bc}} [\sqrt{b+c} + \operatorname{sgn}(b-c) \sqrt{|b-c|}] \quad [b, c > 0]$$

2.9.7. Интегралы, содержащие $\begin{Bmatrix} \operatorname{si}(bx^r) \\ \operatorname{ci}(bx^r) \end{Bmatrix} \begin{Bmatrix} S(cx) \\ C(cx) \end{Bmatrix}$.

Обозначение $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$.

$$1. \int_0^{\infty} x^{\alpha-1} \operatorname{si}(bx) \begin{Bmatrix} S(cx) \\ C(cx) \end{Bmatrix} dx = V(1) \quad [b, c > 0; -\delta - 1/2 < \operatorname{Re} \alpha < 2],$$

$$V(\gamma) = \frac{2^{3/2} c^{\delta+1/2} \Gamma(\alpha + \delta + 1/2)}{\sqrt{\pi} (2\delta + 1) (2\alpha + 2\delta + 1) b^{\alpha+\delta+1/2}} \begin{Bmatrix} \sin[(2\gamma - 2\alpha - 1)\pi/4] \\ \cos[(2\gamma - 2\alpha - 1)\pi/4] \end{Bmatrix} \times \\ \times {}_4F_3\left(\frac{2\delta+1}{4}, \frac{2\alpha+2\delta+1}{4}, \frac{2\alpha+3}{4}, \frac{2\alpha+1}{4} + \delta, \delta + \frac{1}{2}, \frac{2\delta+5}{4}, \frac{2\alpha+2\delta+5}{4}; \frac{c^2}{b^2}\right) \\ [c < b] \text{ или } [c = b \quad \operatorname{Re} \alpha < 1],$$

$$V(\gamma) = -\frac{b^{\gamma+2} \Gamma(\alpha + \gamma + 5/2)}{2\sqrt{2\pi} (\gamma + 2) (\alpha + \gamma + 2) (2\gamma + 1) c^{\alpha+\gamma+2}} \begin{Bmatrix} \cos[(1-2\alpha-2\gamma)\pi/4] \\ \sin[(1-2\alpha-2\gamma)\pi/4] \end{Bmatrix} \times \\ \times {}_5F_4\left(\frac{\gamma}{2} + 1, \frac{\alpha + \gamma}{2} + 1, \frac{2\alpha+7}{4}, \frac{2\alpha+5}{4} + \gamma, 1, \right. \\ \left. \frac{\gamma}{2} + 2, \frac{\alpha + \gamma}{2} + 2, \frac{3}{2} + \gamma, 2, \frac{b^2}{c^2}\right) \pm \frac{\gamma b \Gamma(\alpha + 3/2)}{\sqrt{2\pi} (\alpha + 1) c^{\alpha+1}} \begin{Bmatrix} \cos[(2\alpha+1)\pi/4] \\ \sin[(2\alpha+1)\pi/4] \end{Bmatrix} + \\ + \frac{\Gamma(\alpha + 1/2)}{\sqrt{2\pi} \alpha c^{\alpha}} \begin{Bmatrix} \cos[(1-2\alpha)\pi/4] \\ \sin[(1-2\alpha)\pi/4] \end{Bmatrix} \left\{ \frac{\gamma\pi}{2} + (1-\gamma) \left[C - \frac{1}{\alpha} + \psi\left(\alpha + \frac{1}{2}\right) - \right. \right. \\ \left. \left. - \frac{\pi}{2} \operatorname{tg} \frac{2\alpha-1}{4} \pi - \ln \frac{c}{b} \right] \right\} + \frac{\Gamma(\alpha)}{2\alpha b^{\alpha}} \cos \frac{\alpha-\gamma}{2} \pi \quad [c > b]$$

$$2. \int_0^{\infty} x^{\alpha-1} \operatorname{ci}(bx) \begin{Bmatrix} S(cx) \\ C(cx) \end{Bmatrix} dx = V(0) \quad [b, c > 0, -\delta - 1/2 < \operatorname{Re} \alpha < 2 \quad V(\gamma) \text{ см в 2.9.7.1}]$$

$$3. \int_0^{\infty} x^{-1/2} \operatorname{si}(b\sqrt{x}) \left[\frac{1}{2} - \begin{Bmatrix} S(cx) \\ C(cx) \end{Bmatrix} \right] dx = 2 \sqrt{\frac{4 \pm 2\sqrt{2}}{\pi c}} \left[\frac{1}{2} - S\left(\frac{b^2}{2c}\right) \right] \quad [b, c > 0]$$

2.9.8. Интегралы, содержащие $\begin{Bmatrix} \operatorname{erf}(bx^r) \\ \operatorname{erfc}(bx^r) \end{Bmatrix} \begin{Bmatrix} S(cx) \\ C(cx) \end{Bmatrix}$.

$$1. \int_0^{\infty} x^{\alpha-1} \operatorname{erf}(b\sqrt{x}) \begin{Bmatrix} S(cx) \\ C(cx) \end{Bmatrix} dx = U(1) \quad [c > 0, -(3 \pm 1)/2 < \operatorname{Re} \alpha < 0, |\arg b| < \pi/4, \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}],$$

$$U(\gamma) = \frac{(-1)^{\gamma} 2^{3/2} c^{\delta+1/2} \Gamma(\alpha + \delta + 1)}{\pi (2\delta + 1) (2\alpha + 2\delta + 1) b^{2\alpha+2\delta+1}} {}_4F_3\left(\frac{\alpha}{2} + 1, \frac{\alpha+1}{2} + \delta, \frac{2\delta+1}{4}, \right. \\ \left. \frac{2\alpha+2\delta+1}{4}; \delta + \frac{1}{2}, \frac{2\delta+5}{4}, \frac{2\alpha+2\delta+5}{4}, -\frac{c^2}{b^4}\right) - \\ - \gamma \frac{\Gamma(\alpha + 1/2)}{\sqrt{2\pi} \alpha c^{\alpha}} \begin{Bmatrix} \sin[(2\alpha+1)\pi/4] \\ \cos[(2\alpha+1)\pi/4] \end{Bmatrix}.$$

$$2. \int_0^{\infty} x^{\alpha-1} \operatorname{erfc}(b \sqrt{x}) \begin{Bmatrix} S(cx) \\ C(cx) \end{Bmatrix} dx = U(0)$$

$[c > 0, \operatorname{Re} \alpha > -1 \mp 1/2 \quad |\arg b| < \pi/4 \quad U(\gamma) \text{ см в } 2981]$

$$3. \int_0^{\infty} \operatorname{erfc}(cx) \begin{Bmatrix} S(cx) \\ C(cx) \end{Bmatrix} dx = \frac{1}{2\sqrt{2\pi}c} \left(\sqrt{\sqrt{2}+1} \pm \sqrt{\sqrt{2}-1} - \begin{Bmatrix} 2 \\ 0 \end{Bmatrix} \right)$$

$[c > 0]$

$$4. \int_0^{\infty} \operatorname{erfc}(bx) \begin{Bmatrix} S(cx) \\ C(cx) \end{Bmatrix} dx =$$

$$= \frac{1}{2\sqrt{\pi} \sqrt{b^4 + c^4}} \left(\frac{c}{b} \sqrt{\sqrt{b^4 + c^4} \mp b^2} \pm \frac{b}{c} \sqrt{\sqrt{b^4 + c^4} \pm b^2} \right) - \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{1}{\sqrt{2\pi}c}$$

$[c > 0, |\arg b| < \pi/4]$

2.10 НЕПОЛНЫЕ ГАММА-ФУНКЦИИ $\Gamma(v, x)$, $\gamma(v, x)$

2.10.1. Интегралы общего вида

$$1. \int_0^a x^{\alpha-1} (a^r - x^r)^{\beta-1} \begin{Bmatrix} \Gamma(v, cx) \\ \gamma(v, cx) \end{Bmatrix} dx =$$

$$= \mp \frac{a^{\alpha+r\beta-r+v} c^v}{r} \Gamma(\beta) \sum_{k=0}^{\infty} \frac{(-ac)^k}{k! (v+k)} \Gamma \left[\frac{(\alpha+v+k)/r}{(\alpha+v+k)/r + \beta} \right] +$$

$$+ \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{a^{\alpha+r\beta-r}}{r} \Gamma(v) B \left(\frac{\alpha}{r}, \beta \right) \quad \left[a, r, \operatorname{Re}(\alpha+v), \operatorname{Re} \beta > 0, \begin{Bmatrix} \operatorname{Re} \alpha > 0 \\ \operatorname{Re} v > 0 \end{Bmatrix} \right]$$

$$2. \int_a^{\infty} x^{\alpha-1} (x^r - a^r)^{\beta-1} \begin{Bmatrix} \Gamma(v, cx) \\ \gamma(v, cx) \end{Bmatrix} dx =$$

$$= \mp \frac{a^{\alpha+r\beta-r+v} c^v}{r} \Gamma(\beta) \sum_{k=0}^{\infty} \frac{(-ac)^k}{k! (v+k)} \Gamma \left[\frac{1-\beta-(\alpha+v+k)/r}{1-(\alpha+v+k)/r} \right] \mp c^{r-\beta-r-\alpha} \times$$

$$\times \sum_{k=0}^{\infty} \frac{(1-\beta)_k \Gamma(v+\alpha-r+r\beta-rk)}{k! (kr+r-\beta r-\alpha)} (ac)^{rk} + \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{a^{\alpha+r\beta-r}}{r} \Gamma(v) B \left(1-\beta-\frac{\alpha}{r}, \beta \right)$$

$\left[a, r, \operatorname{Re} \beta > 0, \begin{Bmatrix} \operatorname{Re} c > 0 \\ \operatorname{Re} \alpha, \operatorname{Re} v > 0, \operatorname{Re}(\alpha+r\beta) < r \end{Bmatrix} \right]$

$$3. \int_0^{\infty} \frac{x^{\alpha-1}}{(x^r + z^r)^{\rho}} \begin{Bmatrix} \Gamma(v, cx) \\ \gamma(v, cx) \end{Bmatrix} dx =$$

$$= \mp \frac{c^v z^{\alpha-r\rho+v}}{r \Gamma(\rho)} \sum_{k=0}^{\infty} \frac{(-cz)^k}{k! (v+k)} \Gamma \left(\frac{\alpha+v+k}{r} \right) \Gamma \left(\rho - \frac{\alpha+v+k}{r} \right) \mp$$

$$\mp c^{r-\rho} \sum_{k=0}^{\infty} \frac{(-1)^k (\rho)_k \Gamma(v+\alpha-r\rho-rk)}{k! (kr+\rho r-\alpha)} (cz)^{rk} + \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{z^{\alpha-r\rho}}{r} \Gamma(v) B \left(\frac{\alpha}{r}, \rho - \frac{\alpha}{r} \right)$$

$\left[r, \operatorname{Re}(\alpha+v), \operatorname{Re} c > 0, r |\arg z| < \pi, \begin{Bmatrix} \operatorname{Re} \alpha > 0 \\ \operatorname{Re} v, \operatorname{Re}(r\rho-\alpha) > 0 \end{Bmatrix} \right]$

$$4. \int_0^{\infty} \frac{x^{\alpha-1}}{x^r - y^r} \left\{ \frac{\Gamma(v, cx)}{\gamma(v, cx)} \right\} dx = \pm \frac{\pi c^v y^{\alpha+v-r}}{r} \sum_{k=0}^{\infty} \frac{(-cy)^k}{k! (v+k)} \operatorname{ctg} \frac{\alpha+v+k}{r} \pi \mp$$

$$\mp c^{r-\alpha} \sum_{k=0}^{\infty} \frac{\Gamma(v+\alpha-r-kr)}{kr+r-\alpha} (cy)^{kr} + \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{\pi y^{\alpha-r} \Gamma(v)}{r} \operatorname{ctg} \frac{\alpha\pi}{r}$$

$$\left[r, y, \operatorname{Re}(\alpha+v), \operatorname{Re} c > 0, \left\{ \begin{matrix} \operatorname{Re} \alpha > 0 \\ \operatorname{Re} v > 0; \operatorname{Re} \alpha < r \end{matrix} \right\} \right].$$

$$5. \int_0^{\infty} x^{\alpha-1} e^{-px^r} \left\{ \frac{\Gamma(v, cx)}{\gamma(v, cx)} \right\} dx = U(r)$$

$$\left[r, \operatorname{Re} c, \operatorname{Re} p, \operatorname{Re}(\alpha+v) > 0, \left\{ \begin{matrix} \operatorname{Re} \alpha > 0 \\ \operatorname{Re} v > 0 \end{matrix} \right\} \right].$$

$$U(r) = \mp \frac{c^v}{p^{(v+\alpha)/r}} \sum_{k=0}^{\infty} \frac{1}{k! (k+v)} \Gamma\left(\frac{\alpha+v+k}{r}\right) \left(-\frac{c}{p^{1/r}}\right)^k + \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{\Gamma(v)}{p^{\alpha/r}} \Gamma\left(\frac{\alpha}{r}\right)$$

$$[r > 1].$$

$$U(r) = \pm c^{-\alpha} \sum_{k=0}^{\infty} \frac{\Gamma(\alpha+v+kr)}{k! (kr+\alpha)} \left(-\frac{p}{c^r}\right)^k + \frac{\Gamma(v)}{r p^{\alpha/r}} \Gamma\left(\frac{\alpha}{r}\right) \quad [r < 1].$$

$U(1)$ см в 2.10.3.2.

$$6. \int_0^a x^{\alpha-1} (a^r - x^r)^{\beta-1} e^{cx} \Gamma(v, cx) dx =$$

$$= \frac{a^{\alpha+\beta r-r}}{r} \Gamma(\beta) \Gamma(v) \sum_{k=0}^{\infty} \frac{(ac)^k}{k!} \Gamma\left[\frac{(\alpha+k)/r}{(\alpha+k)/r + \beta}\right] -$$

$$- \frac{a^{\alpha+v+\beta r-r} c^v}{rv} \Gamma(\beta) \sum_{k=0}^{\infty} \frac{(ac)^k}{(1+v)_k} \Gamma\left[\frac{(\alpha+v+k)/r}{(\alpha+v+k)/r + \beta}\right]$$

$$[a, r, \operatorname{Re} \alpha, \operatorname{Re} \beta, \operatorname{Re}(\alpha+v) > 0; |\arg c| < \pi]$$

$$7. \int_a^{\infty} x^{\alpha-1} (x^r - a^r)^{\beta-1} e^{cx} \Gamma(v, cx) dx =$$

$$= \frac{a^{\alpha+\beta r-r}}{r} \Gamma(\beta) \Gamma(v) \sum_{k=0}^{\infty} \frac{(ac)^k}{k!} \Gamma\left[\frac{1-\beta-(\alpha+k)/r}{1-(\alpha+k)/r}\right] -$$

$$- \frac{a^{\alpha+v+\beta r-r} c^v}{rv} \Gamma(\beta) \sum_{k=0}^{\infty} \frac{(ac)^k}{(v+1)_k} \Gamma\left[\frac{1-\beta-(\alpha+v+k)/r}{1-(\alpha+v+k)/r}\right] +$$

$$+ \frac{\pi c^{r-\beta r-\alpha}}{\Gamma(1-v)} \sum_{k=0}^{\infty} \frac{(1-\beta)_k \Gamma(\alpha-r+\beta r-rk)}{k! \sin(\alpha+v-r+\beta r-rk) \pi} (ac)^{rk}$$

$$[a, r, \operatorname{Re} \beta > 0, \operatorname{Re}(\alpha+v+r\beta) < r+1; |\arg c| < \pi].$$

$$8. \int_0^{\infty} \frac{x^{\alpha-1}}{(x^r + z^r)^{\rho}} e^{cx} \Gamma(v, cx) dx = \frac{z^{\alpha-r\rho}}{r} \Gamma\left[\frac{v}{\rho}\right] \sum_{k=0}^{\infty} \frac{(cz)^k}{k!} \Gamma\left(\frac{\alpha+k}{r}\right) \Gamma\left(\rho - \frac{\alpha+k}{r}\right) -$$

$$- \frac{c^v z^{\alpha+v-r\rho}}{rv \Gamma(\rho)} \sum_{k=0}^{\infty} \frac{(cz)^k}{(1+v)_k} \Gamma\left(\frac{\alpha+v+k}{r}\right) \Gamma\left(\rho - \frac{\alpha+v+k}{r}\right) +$$

$$+ \frac{\pi c^{r-\alpha}}{\Gamma(1-v)} \sum_{k=0}^{\infty} \frac{(-1)^k (\rho)_k \Gamma(\alpha-r\rho-rk) (cz)^{rk}}{k! \sin(\alpha+v-r\rho-rk) \pi}$$

$$[r, \operatorname{Re} \alpha, \operatorname{Re}(\alpha+v) > 0; \operatorname{Re}(\alpha+v-r\rho) < 1; |\arg c|, r|\arg z| < \pi].$$

$$9. \int_0^{\infty} \frac{x^{\alpha-1}}{x^r - y^r} e^{cx} \Gamma(v, cx) dx =$$

$$= -\frac{\pi y^{\alpha-r}}{r} \Gamma(v) \sum_{k=0}^{\infty} \frac{(cy)^k}{k!} \operatorname{ctg} \frac{\alpha+k}{r} \pi + \frac{\pi c^v y^{\alpha+v-r}}{rv} \sum_{k=0}^{\infty} \frac{(cy)^k}{(1+v)_k} \operatorname{ctg} \frac{\alpha+v+k}{r} \pi +$$

$$+ \frac{\pi c^{r-\alpha}}{\Gamma(1-v)} \sum_{k=0}^{\infty} \frac{\Gamma(\alpha-r-rk) (cy)^{rk}}{\sin(v+\alpha-r-rk) \pi}$$

$[r, y, \operatorname{Re} \alpha > 0; 0 < \operatorname{Re}(\alpha+v) < r+1; |\arg c| < \pi]$

$$10. \int_0^{\infty} x^{\alpha-1} e^{-px^r + cx} \Gamma(v, cx) dx = V(r)$$

$[r, \operatorname{Re} \alpha, \operatorname{Re}(\alpha+v), \operatorname{Re} p > 0; |\arg c| < \pi],$

$$V(r) = \frac{\Gamma(v)}{p^{\alpha/r} r} \sum_{k=0}^{\infty} \frac{1}{k!} \Gamma\left(\frac{\alpha+k}{r}\right) \left(\frac{c}{p^{1/r}}\right)^k -$$

$$- \frac{c^v}{vp^{(\alpha+v)/r}} \sum_{k=0}^{\infty} \frac{1}{(1+v)_k} \Gamma\left(\frac{\alpha+v+k}{r}\right) \left(\frac{c}{p^{1/r}}\right)^k \quad [r > 1].$$

$$V(r) = \frac{c^{v-1}}{p^{(\alpha+v-1)/r}} \sum_{k=0}^{\infty} (1-v)_k \Gamma\left(\frac{\alpha+v-1-k}{r}\right) \left(-\frac{p^{1/r}}{c}\right)^k +$$

$$+ \frac{\pi}{c^{\alpha} \Gamma(1-v)} \sum_{k=0}^{\infty} \frac{\Gamma(\alpha-rk)}{k! \sin(\alpha+v-rk) \pi} \left(-\frac{p}{c^r}\right)^k \quad [r < 1].$$

$V(1)$ см. в 2.10.3.2

$$11. \int_0^{\infty} x^{\alpha-1} e^{-px^{-r} + cx} \Gamma(v, cx) dx = \frac{\Gamma(v)}{r} p^{\alpha/r} \sum_{k=0}^{\infty} \frac{1}{k!} \Gamma\left(-\frac{\alpha+k}{r}\right) (cp^{1/r})^k -$$

$$- \frac{c^v p^{(\alpha+v)/r}}{rv} \sum_{k=0}^{\infty} \frac{1}{(1+v)_k} \Gamma\left(-\frac{\alpha+v+k}{r}\right) (cp^{1/r})^k +$$

$$+ \frac{\pi}{c^{\alpha} \Gamma(1-v)} \sum_{k=0}^{\infty} \frac{\Gamma(\alpha-rk)}{k! \sin(\alpha+v-rk) \pi} (-c^r p)^k$$

$r, \operatorname{Re} p > 0; \operatorname{Re}(\alpha+v) < 1; |\arg c| < \pi].$

$$12. \int_0^{\infty} x^{\alpha-1} e^{-px^{-r}} \left\{ \begin{matrix} \Gamma(v, cx) \\ \gamma(v, cx) \end{matrix} \right\} dx =$$

$$= \mp \frac{c^v p^{(\alpha+v)/r}}{r} \sum_{k=0}^{\infty} \frac{1}{k! (k+v)} \Gamma\left(-\frac{\alpha+v+k}{r}\right) (-cp^{1/r})^k +$$

$$+ \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{p^{\alpha/r}}{r} \Gamma(v) \Gamma\left(-\frac{\alpha}{r}\right) \mp c^{-\alpha} \sum_{k=0}^{\infty} \frac{\Gamma(\alpha+v-rk)}{k! (rk-\alpha)} (-c^r p)^k$$

$\left[r, \operatorname{Re} p > 0, \left\{ \begin{matrix} \operatorname{Re} c > 0 \\ \operatorname{Re} c, \operatorname{Re} v > 0; \operatorname{Re} \alpha < 0 \end{matrix} \right\} \right]$

$$13. \int_0^{\infty} x^{\alpha-1} \sin bx^r \left\{ \begin{matrix} \Gamma(v, cx) \\ \gamma(v, cx) \end{matrix} \right\} dx = W(1)$$

$\left[b, r, \operatorname{Re} c > 0; \operatorname{Re}(\alpha+v) > -r, \left\{ \begin{matrix} \operatorname{Re} \alpha > -r \\ \operatorname{Re} v > 0; \operatorname{Re} \alpha < r \end{matrix} \right\} \right]$

$$W(\gamma) = \mp \frac{c^v}{r} b^{-(\alpha+v)/r} \sum_{k=0}^{\infty} \frac{1}{k! (k+v)} \Gamma\left(\frac{\alpha+v+k}{r}\right) \times \\ \times \cos \frac{\gamma r - \alpha - v - k}{2r} \pi \left(-\frac{c}{b^{1/r}}\right)^k + \begin{cases} 1 \\ 0 \end{cases} \frac{\Gamma(v) \Gamma(\alpha/r)}{b^{\alpha/r} r} \cos \frac{\gamma r - \alpha}{2r} \pi \quad [r > 1],$$

$$W(\gamma) = \pm \frac{b\gamma}{c^{\alpha+r\gamma}} \sum_{k=0}^{\infty} \frac{\Gamma(\alpha+v+r\gamma+2rk)}{(1/2+\gamma)_k (\alpha+r\gamma+2rk) k!} \left(-\frac{b^2}{4c^{2r}}\right)^k + \\ + \begin{cases} 0 \\ 1 \end{cases} \frac{\Gamma(v) \Gamma(\alpha/r)}{b^{\alpha/r} r} \cos \frac{\gamma r - \alpha}{2r} \pi \quad [r < 1],$$

$W(\gamma)$ при $r=1$ см. в 2.10.4.1—2

$$14. \int_0^{\infty} x^{\alpha-1} \cos bx^r \left\{ \frac{\Gamma(v, cx)}{\gamma(v, cx)} \right\} dx = W(0) \\ [b, r, \operatorname{Re} c, \operatorname{Re}(\alpha+v) > 0, \left\{ \begin{array}{l} \operatorname{Re} \alpha > 0 \\ \operatorname{Re} v > 0; \operatorname{Re} \alpha < r \end{array} \right\}, W(\gamma) \text{ см. в 2.10.1.13}].$$

$$15. \int_0^{\infty} x^{\alpha-1} e^{-px^r} \sin bx^r \left\{ \frac{\Gamma(v, cx)}{\gamma(v, cx)} \right\} dx = X(1) \\ [b, r, \operatorname{Re} p, \operatorname{Re} c > 0; \operatorname{Re}(\alpha+v) > -r, \left\{ \begin{array}{l} \operatorname{Re} \alpha > -r \\ \operatorname{Re} v > 0 \end{array} \right\}].$$

$$X(\gamma) = \mp \frac{c^v}{r (\sqrt{b^2+p^2})^{(\alpha+v)/r}} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (k+v)} \Gamma\left(\frac{\alpha+v+k}{r}\right) \times \\ \times \cos \left(\frac{\gamma\pi}{2} - \frac{\alpha+v+k}{r} \arccos \frac{p}{\sqrt{b^2+p^2}} \right) \frac{c^k}{(\sqrt{b^2+p^2})^{k/r}} + \\ + \begin{cases} 1 \\ 0 \end{cases} \frac{\Gamma(v) \Gamma(\alpha/r)}{r (\sqrt{b^2+p^2})^{\alpha/r}} \cos \left(\frac{\gamma\pi}{2} - \frac{\alpha}{r} \arccos \frac{p}{\sqrt{b^2+p^2}} \right) \\ [r > 1] \text{ или } [r=1; |c|^2 < |b^2+p^2|],$$

$$X(\gamma) = \pm c^{-\alpha} \sum_{k=0}^{\infty} \frac{\Gamma(\alpha+v+rk)}{k! (\alpha+rk)} \cos \left(\frac{\gamma\pi}{2} + k \arccos \frac{p}{\sqrt{b^2+p^2}} \right) \left(-\frac{\sqrt{b^2+p^2}}{c^r} \right)^k + \\ + \begin{cases} 0 \\ 1 \end{cases} \frac{\Gamma(v) \Gamma(\alpha/r)}{r (\sqrt{b^2+p^2})^{\alpha/r}} \cos \left(\frac{\gamma\pi}{2} - \frac{\alpha}{r} \arccos \frac{p}{\sqrt{b^2+p^2}} \right) \\ [r < 1] \text{ или } [r=1; |c|^2 > |b^2+p^2|]$$

$$16. \int_0^{\infty} x^{\alpha-1} e^{-px^r} \cos bx^r \left\{ \frac{\Gamma(v, cx)}{\gamma(v, cx)} \right\} dx = X(0) \\ [b, r, \operatorname{Re} c, \operatorname{Re} p, \operatorname{Re}(\alpha+v) > 0, \left\{ \begin{array}{l} \operatorname{Re} \alpha > 0 \\ \operatorname{Re} v > 0 \end{array} \right\}; X(\gamma) \text{ см. в 2.10.1.15}].$$

$$17. \int_0^{\infty} x^{\alpha-1} e^{cx} \left\{ \frac{\sin bx^r}{\cos bx^r} \right\} \Gamma(v, cx) dx = Y(r) \\ [b, r > 0, \operatorname{Re} \alpha > -r\delta, -r\delta < \operatorname{Re}(\alpha+v) < r+1; |\arg c| < \pi, \delta = \begin{cases} 1 \\ 0 \end{cases}],$$

$$Y(r) = \frac{\Gamma(v)}{b^{\alpha/r} r} \sum_{k=0}^{\infty} \frac{1}{k!} \Gamma\left(\frac{\alpha+k}{r}\right) \left\{ \frac{\sin [(\alpha+k)\pi/(2r)]}{\cos [(\alpha+k)\pi/(2r)]} \right\} \frac{c^k}{b^{k/r}} - \\ - \frac{c^v}{vb^{(\alpha+v)/r} r} \sum_{k=0}^{\infty} \frac{1}{(1+v)_k} \Gamma\left(\frac{\alpha+v+k}{r}\right) \left\{ \frac{\sin [(\alpha+v+k)\pi/(2r)]}{\cos [(\alpha+v+k)\pi/(2r)]} \right\} \frac{c^k}{b^{k/r}} \\ [r > 1],$$

$$Y(r) = \frac{\pi b^\delta}{\Gamma(1-\nu) c^{\alpha+\delta r}} \sum_{k=0}^{\infty} \frac{\Gamma(\alpha+\delta r+2rk)}{(\delta+1/2)_k k!} \operatorname{cosec}(\alpha+\nu+\delta r+2rk)\pi \left(-\frac{b^2}{4c^{2r}}\right)^k +$$

$$+ \frac{b^{(1-\alpha-\nu)/r} c^{\nu-1}}{r} \sum_{k=0}^{\infty} (1-\nu)_k \Gamma\left(\frac{\alpha+\nu-k-1}{r}\right) \left\{ \frac{\sin[(\alpha+\nu-k-1)\pi/(2r)]}{\cos[(\alpha+\nu-k-1)\pi/(2r)]} \right\} \left(-\frac{b^{1/r}}{c}\right)^k$$

$[r < 1],$

$Y(1)$ см. в 2.10.5.1.

$$18. \int_0^\infty x^{\alpha-1} \ln(x^r + z^r) \left\{ \frac{\Gamma(\nu, cx)}{\gamma(\nu, cx)} \right\} dx = Z(1)$$

$$\left[r, \operatorname{Re} c, \operatorname{Re}(\alpha+\nu) > 0; r|\arg z| < \pi, \begin{cases} \operatorname{Re} \alpha > 0 \\ \operatorname{Re} \nu > 0; \operatorname{Re} \alpha < 0 \end{cases} \right],$$

$$Z(\gamma) = \mp \pi c^\nu z^{\alpha+\nu} \sum_{k=0}^{\infty} \frac{(-cz)^k}{k! (k+\nu) (\alpha+\nu+k)} \frac{\cos^{1-\gamma}[(\alpha+\nu+k)\pi/r]}{\sin[(\alpha+\nu+k)\pi/r]} \pm$$

$$\pm \frac{1}{c^\alpha} \sum_{k=1}^{\infty} \frac{(-1)^{\nu k} \Gamma(\alpha+\nu-rk)}{k(rk-\alpha)} (cz)^{rk} \pm$$

$$\pm \frac{r\Gamma(\alpha+\nu)}{\alpha c^\alpha} \left[\psi(\alpha+\nu) - \frac{1}{\alpha} - \ln c \right] + \begin{cases} 1 \\ 0 \end{cases} \frac{\pi \Gamma(\nu) \cos^{1-\gamma}(\alpha\pi/r)}{\alpha \sin(\alpha\pi/r)} z^\alpha.$$

$$19. \int_0^\infty x^{\alpha-1} \ln |x^r - z^r| \left\{ \frac{\Gamma(\nu, cx)}{\gamma(\nu, cx)} \right\} dx = Z(0)$$

$$\left[r, z, \operatorname{Re} c, \operatorname{Re}(\alpha+\nu) > 0, \begin{cases} \operatorname{Re} \alpha > 0 \\ \operatorname{Re} \nu > 0; \operatorname{Re} \alpha < 0 \end{cases}, Z(\gamma) \text{ см. в 2.10.1.18} \right].$$

$$20. \int_0^\infty x^{\alpha-1} e^{cx} \left\{ \frac{\ln(x^r + z^r)}{\ln |x^r - z^r|} \right\} \Gamma(\nu, cx) dx =$$

$$= \pi \Gamma(\nu) z^\alpha \sum_{k=0}^{\infty} \frac{(cz)^k}{k! (k+\alpha)} \left\{ \frac{\operatorname{cosec}[(\alpha+k)\pi/r]}{\operatorname{ctg}[(\alpha+k)\pi/r]} \right\} -$$

$$- \frac{\pi}{\nu} c^\nu z^{\alpha+\nu} \sum_{k=0}^{\infty} \frac{(cz)^k}{(1+\nu)_k (k+\alpha+\nu)} \left\{ \frac{\operatorname{cosec}[(\alpha+\nu+k)\pi/r]}{\operatorname{ctg}[(\alpha+\nu+k)\pi/r]} \right\} -$$

$$- \frac{\pi}{c^\alpha \Gamma(1-\nu)} \sum_{k=1}^{\infty} \frac{(-1)^k \Gamma(\alpha-rk)}{k \sin(\alpha+\nu-rk)\pi} (cz)^{rk} +$$

$$+ \frac{\pi r}{c^\alpha \sin(\alpha+\nu)\pi} \Gamma\left[\frac{\alpha}{1-\nu}\right] [\psi(\alpha) - \pi \operatorname{ctg}(\alpha+\nu)\pi - \ln c]$$

$$\left[r, \operatorname{Re} \alpha, \operatorname{Re}(\alpha+\nu) > 0, \operatorname{Re}(\alpha+\nu) < 1; |\arg c| < \pi, \begin{cases} r|\arg z| < \pi \\ z > 0 \end{cases} \right].$$

$$21. \int_0^\infty x^{\alpha-1} \Gamma(\mu, bx^r) \left\{ \frac{\Gamma(\nu, cx)}{\gamma(\nu, cx)} \right\} dx = P(1)$$

$$\left[r, \operatorname{Re} b, \operatorname{Re} c, \operatorname{Re}(\alpha+\nu), \operatorname{Re}(\alpha+r\mu+\nu) > 0, \begin{cases} \operatorname{Re} \alpha, \operatorname{Re}(\alpha+r\mu) > 0 \\ \operatorname{Re} \mu, \operatorname{Re} \nu > 0 \end{cases}; \text{ при } r < 1 - \text{замена } x = t^{1/r} \right],$$

$$P(\gamma) = \pm (-1)^\gamma \frac{c^\gamma}{b^{(\alpha+\gamma)/r}} \sum_{k=0}^{\infty} \frac{1}{k! (k+\gamma) (k+\alpha+\gamma)} \times \\ \times \Gamma\left(\mu + \frac{\alpha+\gamma+k}{r}\right) \left(-\frac{c}{b^{1/r}}\right)^k - (-1)^\gamma \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{\Gamma(\gamma) \Gamma(\mu+\alpha/r)}{\alpha b^{\alpha/r}} \pm \\ \pm (1-\gamma) \frac{\Gamma(\mu) \Gamma(\gamma+\alpha)}{\alpha c^\alpha} \quad [r > 1].$$

$P(\gamma)$ при $r=1$ см в 2.10.6.1—2.

$$22. \int_0^\infty x^{\alpha-1} \gamma(\mu, bx^r) \begin{Bmatrix} \Gamma(\gamma, cx) \\ \gamma(\gamma, cx) \end{Bmatrix} dx = P(0) \\ [r, \operatorname{Re} b, \operatorname{Re} c, \operatorname{Re} \mu, \operatorname{Re}(\alpha+r\mu+\gamma) > 0, \begin{Bmatrix} \operatorname{Re}(\alpha+\mu r) > 0 \\ \operatorname{Re} \gamma > 0; \operatorname{Re} \alpha < 0 \end{Bmatrix}, P(\gamma) \text{ см. в 2.10.1.21}].$$

$$23. \int_0^\infty x^{\alpha-1} \operatorname{Ei}(-bx^r) \begin{Bmatrix} \Gamma(\gamma, cx) \\ \gamma(\gamma, cx) \end{Bmatrix} dx = Q(r) \\ [r, \operatorname{Re} b, \operatorname{Re} c, \operatorname{Re}(\alpha+\gamma) > 0, \begin{Bmatrix} \operatorname{Re} \alpha > 0 \\ \operatorname{Re} \gamma > 0 \end{Bmatrix}],$$

$$Q(r) = \pm \frac{c^\gamma}{b^{(\alpha+\gamma)/r}} \sum_{k=0}^{\infty} \frac{1}{k! (k+\gamma) (k+\alpha+\gamma)} \Gamma\left(\frac{\alpha+\gamma+k}{r}\right) \left(-\frac{c}{b^{1/r}}\right)^k - \\ - \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{\Gamma(\gamma) \Gamma(\alpha/r)}{\alpha b^{\alpha/r}} \quad [r > 1].$$

$$Q(r) = \pm c^{-\alpha} \sum_{k=1}^{\infty} \frac{\Gamma(\alpha+\gamma+kr)}{k! k (\alpha+kr)} \left(-\frac{b}{c^r}\right)^k \pm \\ \pm \frac{\Gamma(\alpha+\gamma)}{\alpha c^\alpha} \left[r\psi(\alpha+\gamma) - \frac{r}{\alpha} + C - \ln \frac{c^r}{b}\right] \pm \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \frac{\Gamma(\gamma) \Gamma(\alpha/r)}{\alpha b^{\alpha/r}} \quad [r < 1].$$

$Q(1)$ см. в 2.10.7.1.

$$24. \int_0^\infty x^{\alpha-1} \operatorname{erf}(bx^r) \begin{Bmatrix} \Gamma(\gamma, cx) \\ \gamma(\gamma, cx) \end{Bmatrix} dx = R(1) \\ [r, \operatorname{Re} c > 0; \operatorname{Re}(\alpha+\gamma) > -r; |\arg b| < \pi/4, \begin{Bmatrix} \operatorname{Re} \alpha > -r \\ \operatorname{Re} \gamma > 0; \operatorname{Re} \alpha < 0 \end{Bmatrix}],$$

$$R(\gamma) = \mp \frac{(-1)^\gamma c^\gamma}{\sqrt{\pi} b^{(\alpha+\gamma)/r}} \sum_{k=0}^{\infty} \frac{1}{k! (k+\gamma) (k+\alpha+\gamma)} \Gamma\left(\frac{r+\alpha+\gamma+k}{2r}\right) \left(-\frac{c}{b^{1/r}}\right)^k + \\ + \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{(-1)^\gamma \Gamma(\gamma)}{\alpha \sqrt{\pi} b^{\alpha/r}} \Gamma\left(\frac{\alpha+r}{2r}\right) \pm \gamma \frac{\Gamma(\alpha+\gamma)}{\alpha c^\alpha} \quad [r > 1/2].$$

$$R(\gamma) = \mp \frac{(-1)^\gamma 2b}{\sqrt{\pi} c^{\alpha+r}} \sum_{k=0}^{\infty} \frac{\Gamma(\alpha+\gamma+r+2rk)}{k! (2k+1) (2rk+r+\alpha)} \left(-\frac{b^2}{c^{2r}}\right)^k + \\ + \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \frac{(-1)^\gamma \Gamma(\gamma)}{\alpha \sqrt{\pi} b^{\alpha/r}} \Gamma\left(\frac{\alpha+r}{2r}\right) \mp (\gamma-1) \frac{\Gamma(\alpha+\gamma)}{\alpha c^\alpha} \quad [r < 1/2].$$

$R(\gamma)$ при $r=1/2$ см. в 2.10.8.1—2.

$$25. \int_0^\infty x^{\alpha-1} \operatorname{erfc}(bx^r) \begin{Bmatrix} \Gamma(\gamma, cx) \\ \gamma(\gamma, cx) \end{Bmatrix} dx = R(0) \\ [r, \operatorname{Re} c, \operatorname{Re}(\alpha+\gamma) > 0; |\arg b| < \pi/4, \begin{Bmatrix} \operatorname{Re} \alpha > 0 \\ \operatorname{Re} \gamma > 0 \end{Bmatrix}, R(\gamma) \text{ см. в 2.10.1.24}].$$

2.10.2. Интегралы от $A(x) \left\{ \frac{\Gamma(v, cx)}{\gamma(v, cx)} \right\}$.

$$1. \int_0^{\infty} x^{\alpha-1} \left\{ \frac{\Gamma(v, cx)}{\gamma(v, cx)} \right\} dx = \pm \frac{\Gamma(\alpha+v)}{ac^{\alpha}} \quad [\operatorname{Re}(\alpha+v), \operatorname{Re}c, \pm \operatorname{Re}\alpha > 0]$$

$$2. \int_0^a x^{\alpha-1} (a-x)^{\beta-1} \left\{ \frac{\Gamma(v, cx)}{\gamma(v, cx)} \right\} dx = \mp \frac{a^{\alpha+\beta+v-1} c^v}{v} B(\beta, \alpha+v) \times$$

$$\times {}_2F_2(v, \alpha+v; v+1, \alpha+\beta+v; -ac) + \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} a^{\alpha+\beta-1} \Gamma(v) B(\alpha, \beta)$$

$$\left[a, \operatorname{Re}\beta, \operatorname{Re}(\alpha+v) > 0, \left\{ \begin{matrix} \operatorname{Re}\alpha > 0 \\ \operatorname{Re}v > 0 \end{matrix} \right\} \right]$$

$$3. \int_a^{\infty} x^{\alpha-1} (x-a)^{\beta-1} \left\{ \frac{\Gamma(v, cx)}{\gamma(v, cx)} \right\} dx =$$

$$= \mp \frac{a^{\alpha+\beta+v-1} c^v}{v} B(\beta, 1-\alpha-\beta-v) {}_2F_2(\alpha+v, v; v+1, \alpha+\beta+v; -ac) \mp$$

$$\mp \frac{c^{1-\alpha-\beta}}{1-\alpha-\beta} \Gamma(\alpha+\beta+v-1) {}_2F_2(1-\alpha-\beta, 1-\beta; 2-\alpha-\beta-v, 2-\alpha-\beta; -ac) +$$

$$+ \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} a^{\alpha+\beta-1} \Gamma(v) B(\beta, 1-\alpha-\beta)$$

$$\left[a, \operatorname{Re}\beta > 0, \left\{ \begin{matrix} \operatorname{Re}c > 0 \\ \operatorname{Re}v > 0; \operatorname{Re}(\alpha+\beta) < 1 \end{matrix} \right\} \right].$$

$$4. \int_0^{\infty} \frac{x^{\alpha-1}}{(x+z)^{\rho}} \left\{ \frac{\Gamma(v, cx)}{\gamma(v, cx)} \right\} dx =$$

$$= \mp \frac{c^v z^{\alpha+v-\rho}}{v} B(\alpha+v, \rho-\alpha-v) {}_2F_2(v, \alpha+v; v+1, 1+\alpha+v-\rho; cz) \mp$$

$$\mp \frac{c^{\rho-\alpha}}{\rho-\alpha} \Gamma(\alpha+v-\rho) {}_2F_2(\rho-\alpha, \rho; 1+\rho-\alpha, 1+\rho-\alpha-v; cz) +$$

$$+ \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} z^{\alpha-\rho} \Gamma(v) B(\alpha, \rho-\alpha)$$

$$\left[\operatorname{Re}(\alpha+v) > 0; |\arg z| < \pi, \left\{ \begin{matrix} \operatorname{Re}\alpha, \operatorname{Re}c > 0 \\ \operatorname{Re}v, \operatorname{Re}c, \operatorname{Re}(\rho-\alpha) > 0 \end{matrix} \right\} \right].$$

$$5. \int_0^{\infty} \frac{x^{\alpha-1}}{x-y} \left\{ \frac{\Gamma(v, cx)}{\gamma(v, cx)} \right\} dx = \pm \pi y^{\alpha-1} \operatorname{ctg}(\alpha+v) \pi \gamma(v, cy) \mp$$

$$\mp \frac{c^{1-\alpha}}{1-\alpha} \Gamma(\alpha+v-1) {}_2F_2(1-\alpha, 1; 2-\alpha, 2-\alpha-v; -cy) -$$

$$- \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \pi \Gamma(v) y^{\alpha-1} \operatorname{ctg} \alpha \pi \quad \left[y, \operatorname{Re}(\alpha+v) > 0, \left\{ \begin{matrix} \operatorname{Re}\alpha, \operatorname{Re}c > 0 \\ \operatorname{Re}c, \operatorname{Re}v > 0; \operatorname{Re}\alpha < 1 \end{matrix} \right\} \right].$$

2.10.3. Интегралы от $A(x) e^{f(x)} \left\{ \frac{\Gamma(v, cx)}{\gamma(v, cx)} \right\}$.

$$1. \int_0^{\infty} e^{-\rho x} \left\{ \frac{\Gamma(v, cx)}{\gamma(v, cx)} \right\} dx = \frac{\Gamma(v)}{\rho} \left[\left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \mp \left(\frac{c}{\rho+c} \right)^v \right]$$

$$[\operatorname{Re}c, \operatorname{Re}\rho > 0; \operatorname{Re}v > -(1 \pm 1)/2]$$

$$2. \int_0^{\infty} x^{\alpha-1} e^{-px} \left\{ \frac{\Gamma(v, cx)}{\gamma(v, cx)} \right\} dx =$$

$$= \mp \frac{c^v \Gamma(\alpha+v)}{v p^{\alpha+v}} {}_2F_1(v, \alpha+v; v+1; -\frac{c}{p}) + \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{\Gamma(v) \Gamma(\alpha)}{p^{\alpha}}$$

$$\left[\operatorname{Re} c, \operatorname{Re} p, \operatorname{Re}(\alpha+v) > 0, \left\{ \begin{matrix} \operatorname{Re} \alpha > 0 \\ \operatorname{Re} v > 0 \end{matrix} \right\} \right].$$

$$3. \int_0^{\infty} x^{\alpha-1} e^{cx} \Gamma(v, cx) dx = \frac{\pi}{c^{\alpha} \sin(\alpha+v) \pi} \Gamma \left[\begin{matrix} \alpha \\ 1-v \end{matrix} \right]$$

$$[\operatorname{Re} \alpha, \operatorname{Re} c > 0; 0 < \operatorname{Re}(\alpha+v) < 1]$$

$$4. \int_0^a x^{-v} (a-x)^{\beta-1} e^{cx} \gamma(v, cx) dx = B(\beta, v) a^{\beta} c^v e^{ac} \gamma(\beta+v, ac)$$

$$[a, \operatorname{Re} \beta > 0; 0 < \operatorname{Re} v < 1].$$

$$5. \int_0^a x^{\alpha-1} (a-x)^{\beta-1} e^{cx} \Gamma(v, cx) dx = a^{\alpha+\beta-1} \Gamma(v) B(\alpha, \beta) {}_1F_1(\alpha; \alpha+\beta; ac) -$$

$$- \frac{a^{\alpha+\beta+v-1} c^v}{v} B(\beta, \alpha+v) {}_2F_2(\alpha+v, 1; v+1, \alpha+\beta+v; ac)$$

$$[a, \operatorname{Re} \alpha, \operatorname{Re} \beta, \operatorname{Re}(\alpha+v) > 0; |\arg c| < \pi].$$

$$6. \int_a^{\infty} x^{\alpha-1} (x-a)^{\beta-1} e^{cx} \Gamma(v, cx) dx =$$

$$= a^{\alpha+\beta-1} \Gamma(v) B(\beta, 1-\alpha-\beta) {}_1F_1(\alpha; \alpha+\beta; ac) -$$

$$- \frac{a^{\alpha+\beta+v-1} c^v}{v} B(\beta, 1-\alpha-\beta-v) {}_2F_2(\alpha+v, 1; v+1, \alpha+\beta+v; ac) -$$

$$- \frac{\pi c^{1-\alpha-\beta}}{\sin(\alpha+\beta+v) \pi} \Gamma \left[\begin{matrix} \alpha+\beta-1 \\ 1-v \end{matrix} \right] {}_1F_1(1-\beta; 2-\alpha-\beta; ac)$$

$$[a, \operatorname{Re} \beta > 0; \operatorname{Re}(\alpha+\beta+v) < 2; |\arg c| < \pi].$$

$$7. \int_0^{\infty} \frac{x^{\alpha-1}}{(x+z)^{\rho}} e^{cx} \Gamma(v, cx) dx =$$

$$= z^{\alpha-\rho} \Gamma(v) B(\alpha, \rho-\alpha) {}_1F_1(\alpha; 1+\alpha-\rho; -cz) -$$

$$- \frac{c^v z^{\alpha+v-\rho}}{v} B(\alpha+v, \rho-\alpha-v) {}_2F_2(\alpha+v, 1; v+1, 1-\rho+\alpha+v; -cz) +$$

$$+ \frac{\pi c^{\rho-\alpha}}{\sin(\alpha+v-\rho) \pi} \Gamma \left[\begin{matrix} \alpha-\rho \\ 1-v \end{matrix} \right] {}_1F_1(\rho; 1+\rho-\alpha; -cz)$$

$$[\operatorname{Re} \alpha, \operatorname{Re}(\alpha+v) > 0; \operatorname{Re}(\alpha+v-\rho) < 1; |\arg c|, |\arg z| < \pi]$$

$$8. \int_0^{\infty} \frac{x^{\alpha-1}}{x-y} e^{cx} \Gamma(v, cx) dx =$$

$$= -\pi y^{\alpha-1} e^{cy} \Gamma(v) \operatorname{ctg} \alpha \pi + \pi y^{\alpha-1} e^{-cy} \operatorname{ctg}(\alpha+v) \pi \gamma(v, cy) -$$

$$- \frac{\pi}{\sin(\alpha+v) \pi} \Gamma \left[\begin{matrix} \alpha \\ 1-v \end{matrix} \right] y^{\alpha-1} e^{-cy} \gamma(1-\alpha, cy)$$

$$[y, \operatorname{Re} \alpha > 0; 0 < \operatorname{Re}(\alpha+v) < 2; |\arg c| < \pi].$$

$$\begin{aligned}
9. \int_0^{\infty} x^{\alpha-1} e^{-px^2} \left\{ \frac{\Gamma(v, cx)}{\gamma(v, cx)} \right\} dx = \\
= \mp \frac{c^v}{2v} p^{-(\alpha+v)/2} \Gamma\left(\frac{\alpha+v}{2}\right) {}_2F_2\left(\frac{v}{2}, \frac{\alpha+v}{2}; \frac{1}{2}, \frac{v}{2}+1; \frac{c^2}{4p}\right) \pm \\
\pm \frac{c^{v+1}}{2(v+1)} p^{-(\alpha+v+1)/2} \Gamma\left(\frac{\alpha+v+1}{2}\right) {}_2F_2\left(\frac{v+1}{2}, \frac{\alpha+v+1}{2}; \frac{3}{2}, \frac{v+3}{2}; \frac{c^2}{4p}\right) + \\
+ \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{\Gamma(v) \Gamma(\alpha/2)}{2p^{\alpha/2}} \\
\left[\operatorname{Re} c, \operatorname{Re} p, \operatorname{Re}(\alpha+v) > 0, \left\{ \begin{matrix} \operatorname{Re} \alpha > 0 \\ \operatorname{Re} v > 0 \end{matrix} \right\} \right].
\end{aligned}$$

$$10. \int_0^{\infty} x e^{-px^2} \gamma(v, cx) dx = \frac{c^v \Gamma(v)}{(2p)^{v/2+1}} \exp\left(\frac{c^2}{8p}\right) D_{-v}\left(\frac{c}{\sqrt{2p}}\right)$$

[$\operatorname{Re} c, \operatorname{Re} p > 0; \operatorname{Re} v > -2$].

$$\begin{aligned}
11. \int_0^{\infty} x^{\alpha-1} e^{-p\sqrt{x}} \left\{ \frac{\Gamma(v, cx)}{\gamma(v, cx)} \right\} dx = \pm \frac{\Gamma(v+\alpha)}{\alpha c^{\alpha}} {}_2F_2\left(\alpha, \alpha+v; \alpha+1, \frac{1}{2}; \frac{p^2}{4c}\right) \mp \\
\mp \frac{2p\Gamma(\alpha+v+1/2)}{(2\alpha+1)c^{\alpha+1/2}} {}_2F_2\left(\alpha+\frac{1}{2}, \alpha+v+\frac{1}{2}; \frac{3}{2}, \alpha+\frac{3}{2}; \frac{p^2}{4c}\right) \\
\left[\operatorname{Re} c, \operatorname{Re} p, \operatorname{Re}(\alpha+v) > 0, \left\{ \begin{matrix} \operatorname{Re} \alpha > 0 \\ \operatorname{Re} v > 0 \end{matrix} \right\} \right].
\end{aligned}$$

$$12. \int_0^{\infty} x^{-1/2} e^{-p\sqrt{x}} \gamma(v, cx) dx = \frac{\Gamma(2v)}{2^v p} \exp\left(\frac{p^2}{8c}\right) D_{-2v}\left(\frac{p}{\sqrt{2c}}\right)$$

[$\operatorname{Re} c, \operatorname{Re} p, \operatorname{Re} v > 0$].

$$\begin{aligned}
13. \int_0^{\infty} x^{-1/2} e^{-p\sqrt{x}-cx} \gamma(v, e^{\pi i} cx) dx = \\
= \frac{2^{1-2v} \Gamma(2v)}{\sqrt{c}} \exp\left(v\pi i + \frac{p^2}{4c}\right) \Gamma\left(\frac{1}{2}-v, \frac{p^2}{4c}\right) \\
\left[\operatorname{Re} c, \operatorname{Re} p, \operatorname{Re} v > 0 \right].
\end{aligned}$$

$$\begin{aligned}
14. \int_0^{\infty} x^{\alpha-1} e^{-p/x} \left\{ \frac{\Gamma(v, cx)}{\gamma(v, cx)} \right\} dx = \\
= \mp \frac{c^v p^{\alpha+v}}{v} \Gamma(-\alpha-v) {}_1F_2(v; v+1, 1+\alpha+v; cp) \pm \\
\pm \frac{\Gamma(\alpha+v)}{\alpha c^{\alpha}} {}_1F_2(-\alpha; 1-\alpha, 1-\alpha-v; cp) + \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} p^{\alpha} \Gamma(v) \Gamma(-\alpha) \\
\left[\operatorname{Re} p > 0, \left\{ \begin{matrix} \operatorname{Re} c > 0 \\ \operatorname{Re} c, \operatorname{Re} v > 0; \operatorname{Re} \alpha < 0 \end{matrix} \right\} \right].
\end{aligned}$$

$$15. \int_0^{\infty} \frac{1}{x^2} e^{-p/x} \Gamma(v, cx) dx = 2c^{v/2} p^{v/2-1} K_v(2\sqrt{cp})$$

[$\operatorname{Re} c, \operatorname{Re} p > 0$].

$$16. \int_0^{\infty} x^{-v-3/2} e^{-p/x} \left\{ \frac{\Gamma(v, cx)}{\gamma(v, cx)} \right\} dx = \frac{2^{1-2v} \sqrt{\pi}}{p^{v+1/2}} \left\{ \frac{\Gamma(2v, 2\sqrt{cp})}{\gamma(2v, 2\sqrt{cp})} \right\}$$

[$\operatorname{Re} p > 0, \left\{ \begin{matrix} \operatorname{Re} c > 0 \\ \operatorname{Re} c, \operatorname{Re} v > 0 \end{matrix} \right\}$].

$$17. \int_0^{\infty} x^{-\nu-1/2} e^{-p/x} \left\{ \begin{matrix} \Gamma(\nu, cx) \\ \gamma(\nu, cx) \end{matrix} \right\} dx = \frac{2^{-2\nu} \sqrt{\pi}}{c p^{\nu-1/2}} \left\{ \begin{matrix} \Gamma(2\nu-1, 2\sqrt{cp}) \\ \gamma(2\nu-1, 2\sqrt{cp}) \end{matrix} \right\} \\ \left[\operatorname{Re} p > 0, \left\{ \begin{matrix} \operatorname{Re} c > 0 \\ \operatorname{Re} c > 0; \operatorname{Re} \nu > 1/2 \end{matrix} \right\} \right].$$

$$18. \int_0^{\infty} x^{\alpha-1} e^{cx-p/x} \Gamma(\nu, cx) dx = 2^{2-\alpha-2\nu} \Gamma(1-\alpha-\nu) c^{-\alpha/2} p^{\alpha/2} S_{2\nu+\alpha-1, -\alpha}(2\sqrt{cp}) \\ [\operatorname{Re} p > 0; \operatorname{Re}(\alpha+\nu) < 1; |\arg c| < \pi]$$

2.10.4. Интегралы, содержащие тригонометрические функции и $\left\{ \begin{matrix} \Gamma(\nu, cx) \\ \gamma(\nu, cx) \end{matrix} \right\}$.

$$1. \int_0^{\infty} x^{\alpha-1} \sin bx \left\{ \begin{matrix} \Gamma(\nu, cx) \\ \gamma(\nu, cx) \end{matrix} \right\} dx = U(1)$$

$$\left[b, \operatorname{Re} c > 0; \operatorname{Re}(\alpha+\nu) > -1, \left\{ \begin{matrix} \operatorname{Re} \alpha > -1 \\ \operatorname{Re} \nu > 0; \operatorname{Re} \alpha < 1 \end{matrix} \right\} \right], \\ U(\gamma) = \pm \frac{b \nu \Gamma(\alpha+\nu+\gamma)}{(\alpha+\gamma) c^{\alpha+\gamma}} {}_2F_2 \left(\frac{\alpha+\gamma}{2}, \frac{\alpha+\nu+1}{2}, \frac{\alpha+\nu}{2} + \gamma, \right. \\ \left. \frac{\alpha+\gamma}{2} + 1, \gamma + \frac{1}{2}; -\frac{b^2}{c^2} \right) + \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{\Gamma(\alpha) \Gamma(\nu)}{b^{\alpha}} \cos \frac{\gamma-\alpha}{2} \pi$$

$$2. \int_0^{\infty} x^{\alpha-1} \cos bx \left\{ \begin{matrix} \Gamma(\nu, cx) \\ \gamma(\nu, cx) \end{matrix} \right\} dx = U(0)$$

$$\left[b, \operatorname{Re} c, \operatorname{Re}(\alpha+\nu) > 0, \left\{ \begin{matrix} \operatorname{Re} \alpha > 0 \\ \operatorname{Re} \nu > 0; \operatorname{Re} \alpha < 1 \end{matrix} \right\} \right], \\ U(\gamma) \text{ см. в 2.10.4.1}].$$

$$3. \int_0^{\infty} x^{\alpha-1} \sin b \sqrt{x} \left\{ \begin{matrix} \Gamma(\nu, cx) \\ \gamma(\nu, cx) \end{matrix} \right\} dx = V(1)$$

$$\left[b, \operatorname{Re} c > 0; \operatorname{Re}(\alpha+\nu) > -1/2, \left\{ \begin{matrix} \operatorname{Re} \alpha > -1/2 \\ \operatorname{Re} \nu > 0, \operatorname{Re} \alpha < 1/2 \end{matrix} \right\} \right] \\ V(\gamma) = \pm \frac{2b \nu \Gamma(\alpha+\nu+\gamma/2)}{(2\alpha+\gamma) c^{\alpha+\gamma/2}} {}_2F_2 \left(\alpha+\nu+\frac{\gamma}{2}, \alpha+\frac{\gamma}{2}; \alpha+\frac{\gamma}{2}+1, \gamma+\frac{1}{2}; -\frac{b^2}{4c} \right) + \\ + \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{2\Gamma(\nu) \Gamma(2\alpha)}{b^{2\alpha}} \cos \frac{\gamma-2\alpha}{2} \pi$$

$$4. \int_0^{\infty} x^{\alpha-1} \cos b \sqrt{x} \left\{ \begin{matrix} \Gamma(\nu, cx) \\ \gamma(\nu, cx) \end{matrix} \right\} dx = V(0)$$

$$\left[b, \operatorname{Re} c, \operatorname{Re}(\alpha+\nu) > 0, \left\{ \begin{matrix} \operatorname{Re} \alpha > 0 \\ \operatorname{Re} \nu > 0; \operatorname{Re} \alpha < 1/2 \end{matrix} \right\}, V(\gamma) \text{ см. в 2.10.4.3} \right]$$

2.10.5. Интегралы, содержащие e^{cx} , тригонометрические функции и $\left\{ \begin{matrix} \Gamma(\nu, cx) \\ \gamma(\nu, cx) \end{matrix} \right\}$.

Обозначение: $\delta = \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\}$.

$$1. \int_0^{\infty} x^{\alpha-1} e^{cx} \left\{ \begin{matrix} \sin bx \\ \cos bx \end{matrix} \right\} \Gamma(\nu, cx) dx =$$

$$= \mp \frac{b^{\delta} \pi}{c^{\alpha+\delta} \sin(\alpha+\nu) \pi} \Gamma \left[\begin{matrix} \alpha+\delta \\ 1-\nu \end{matrix} \right] {}_4F_3 \left(\frac{\alpha+1}{2}, \frac{\alpha}{2} + \delta, \right.$$

$$\begin{aligned} & \frac{\alpha+\nu+1}{2}, \frac{\alpha+\nu}{2}+\delta; \delta+\frac{1}{2}, \frac{\alpha+\nu+1}{2}, \frac{\alpha+\nu}{2}+\delta; -\frac{b^2}{c^2} \Big) \mp \\ & \mp b^{1-\alpha-\nu} c^{\nu-1} \Gamma(\alpha+\nu-1) \left\{ \frac{\cos[(\alpha+\nu)\pi/2]}{\sin[(\alpha+\nu)\pi/2]} \right\} {}_3F_2 \left(\frac{1-\nu}{2}, 1-\frac{\nu}{2}, 1; 1-\frac{\alpha+\nu}{2}, \right. \\ & \left. \frac{3-\alpha-\nu}{2}; -\frac{b^2}{c^2} \right) + \frac{(1-\nu) c^{\nu-2} \Gamma(\alpha+\nu-2)}{b^{\alpha+\nu-2}} \left\{ \frac{\sin[(\alpha+\nu)\pi/2]}{\cos[(\alpha+\nu)\pi/2]} \right\} \times \\ & \times {}_3F_2 \left(1-\frac{\nu}{2}, \frac{3-\nu}{2}, 1; \frac{3-\alpha-\nu}{2}, 2-\frac{\alpha+\nu}{2}; -\frac{b^2}{c^2} \right) \\ & [b>0; \operatorname{Re} \alpha > -\delta; -\delta < \operatorname{Re}(\alpha+\nu) < r+1; |\arg c| < \pi] \end{aligned}$$

$$\begin{aligned} 2. \int_0^\infty x^{\alpha-1} e^{cx} \left\{ \frac{\sin b \sqrt{x}}{\cos b \sqrt{x}} \right\} \Gamma(\nu, cx) dx = \\ = \frac{\pi b^\delta \Gamma(\alpha+\delta/2)}{\Gamma(1-\nu) c^{\alpha+\delta/2}} \operatorname{cosec} \frac{2\alpha+2\nu+\delta}{2} {}_1F_1 \left(\alpha+\frac{\delta}{2}; \delta+\frac{1}{2}; \frac{b^2}{4c} \right) - \\ - 2b^{2-\alpha-\nu} c^{\nu-1} \Gamma(2\alpha+2\nu-2) \left\{ \frac{\sin(\alpha+\nu)\pi}{\cos(\alpha+\nu)\pi} \right\} \times \\ \times {}_2F_2 \left(1-\nu, 1; 2-\alpha-\nu, \frac{3}{2}-\alpha-\nu; -\frac{b^2}{4c} \right) \\ [b>0; \operatorname{Re} \alpha > -\delta/2; -\delta/2 < \operatorname{Re}(\alpha+\nu) < 3/2; |\arg c| < \pi]. \end{aligned}$$

$$\begin{aligned} 3. \int_0^\infty x^{-\nu} e^{cx} \sin b \sqrt{x} \Gamma(\nu, cx) dx = \\ = 2^{1-\nu} \sqrt{\pi} c^{\nu-1} \Gamma\left(\frac{3}{2}-\nu\right) \exp\left(\frac{b^2}{8c}\right) D_{2\nu-2}\left(\frac{b}{\sqrt{2c}}\right) \\ [b>0; \operatorname{Re} \nu < 3/2; |\arg c| < \pi]. \end{aligned}$$

2.10.6. Интегралы, содержащие произведения функций $\Gamma(\mu, bx)$ и $\gamma(\nu, cx)$

$$\begin{aligned} 1. \int_0^\infty x^{\alpha-1} \Gamma(\mu, bx) \left\{ \frac{\Gamma(\nu, cx)}{\gamma(\nu, cx)} \right\} dx = U(1) \\ \left[\operatorname{Re} b, \operatorname{Re} c, \operatorname{Re}(\alpha+\nu), \operatorname{Re}(\alpha+\mu+\nu) > 0, \left\{ \begin{array}{l} \operatorname{Re} \alpha, \operatorname{Re}(\alpha+\mu) > 0 \\ \operatorname{Re} \mu, \operatorname{Re} \nu > 0 \end{array} \right\} \right], \end{aligned}$$

$$\begin{aligned} U(\gamma) = \pm (-1)^\nu \frac{c^\nu \Gamma(\alpha+\mu+\nu)}{\nu(\alpha+\nu) b^{\alpha+\nu}} {}_3F_2 \left(\alpha+\mu+\nu, \nu, \alpha+\nu; \right. \\ \left. \nu+1, \alpha+\nu+1; -\frac{c}{b} \right) - (-1)^\nu \left\{ \frac{1}{0} \right\} \frac{\Gamma(\nu) \Gamma(\alpha+\mu)}{ab^\alpha} \pm (1-\gamma) \frac{\Gamma(\mu) \Gamma(\alpha+\nu)}{ac^\alpha}. \end{aligned}$$

$$\begin{aligned} 2. \int_0^\infty x^{\alpha-1} \gamma(\mu, bx) \left\{ \frac{\Gamma(\nu, cx)}{\gamma(\nu, cx)} \right\} dx = U(0) \\ \left[\operatorname{Re} b, \operatorname{Re} c, \operatorname{Re} \mu, \operatorname{Re}(\alpha+\mu+\nu) > 0, \left\{ \begin{array}{l} \operatorname{Re}(\alpha+\mu) > 0 \\ \operatorname{Re} \nu > 0; \operatorname{Re} \alpha < 0 \end{array} \right\}, U(\gamma) \text{ см. в 2.10.6.1} \right]. \end{aligned}$$

$$\begin{aligned} 3. \int_0^\infty x^{\alpha-1} e^{-px} \gamma(\mu, bx) \gamma(\nu, cx) dx = \\ = \frac{\Gamma(\alpha+\mu+\nu) b^\mu c^\nu}{\mu \nu p^{\alpha+\mu+\nu}} F_2 \left(\alpha+\mu+\nu, \mu, \nu; \nu+1, \nu+1; -\frac{b}{p}, -\frac{c}{p} \right) \\ [\operatorname{Re} b, \operatorname{Re} c, \operatorname{Re} p, \operatorname{Re} \mu, \operatorname{Re} \nu, \operatorname{Re}(\alpha+\mu+\nu) > 0]. \end{aligned}$$

2.10.7. Интегралы, содержащие $Ei(bx) \left\{ \frac{\Gamma(v, cx)}{\gamma(v, cx)} \right\}$.

$$1. \int_0^{\infty} x^{\alpha-1} Ei(-bx) \left\{ \frac{\Gamma(v, cx)}{\gamma(v, cx)} \right\} dx =$$

$$= \pm \frac{c^v \Gamma(\alpha+v)}{v(\alpha+v) b^{\alpha+v}} {}_3F_2 \left(v, \alpha+v, \alpha+v; v+1, \alpha+v+1; -\frac{c}{b} \right) - \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{\Gamma(v) \Gamma(\alpha)}{ab^{\alpha}}$$

$$\left[\operatorname{Re} b, \operatorname{Re} c, \operatorname{Re}(\alpha+v) > 0, \left\{ \begin{matrix} \operatorname{Re} \alpha > 0 \\ \operatorname{Re} v > 0 \end{matrix} \right\} \right]$$

$$2. \int_0^{\infty} e^{cx} Ei(-bx) \gamma(v, cx) dx = -\frac{\Gamma(v) c^v}{b^{v+1}} \Phi \left(\frac{c}{b}, 1, v+1 \right)$$

$$[\operatorname{Re}(b-c), \operatorname{Re} v > 0]$$

$$3. \int_0^{\infty} e^{cx} Ei(-bx) \gamma(n, cx) dx = \frac{(n-1)!}{c} \left[\ln \frac{b-c}{c} + \sum_{k=1}^{n-2} \frac{1}{k} \left(\frac{c}{b} \right)^k \right]$$

$$[\operatorname{Re}(b-c) > 0; n=1, 2, 3, \dots]$$

2.10.8. Интегралы, содержащие $\left\{ \frac{\operatorname{erf}(b\sqrt{x})}{\operatorname{erfc}(b\sqrt{x})} \right\} \left\{ \frac{\Gamma(v, cx)}{\gamma(v, cx)} \right\}$.

$$1. \int_0^{\infty} x^{\alpha-1} \operatorname{erf}(b\sqrt{x}) \left\{ \frac{\Gamma(v, cx)}{\gamma(v, cx)} \right\} dx = U(1)$$

$$\left[\operatorname{Re} c > 0; \operatorname{Re}(\alpha+v) > -1/2; \arg b < \pi/4, \left\{ \begin{matrix} \operatorname{Re} \alpha > -1/2 \\ \operatorname{Re} v > 0 \end{matrix} \right\} \right],$$

$$U(\gamma) = \mp \frac{(-1)^{\gamma} \Gamma(\alpha+v+1/2) c^v}{v(\alpha+v) \sqrt{\pi} b^{2\alpha+2v}} {}_3F_2 \left(v, \alpha+v, \alpha+v+\frac{1}{2}; v+1, \right.$$

$$\left. \alpha+v+1; -\frac{c}{b^2} \right) + \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{(-1)^{\gamma} \Gamma(v) \Gamma(\alpha+1/2)}{\alpha \sqrt{\pi} b^{2\alpha}} \pm \gamma \frac{\Gamma(\alpha+v)}{\alpha c^{\alpha}}.$$

$$2. \int_0^{\infty} x^{\alpha-1} \operatorname{erfc}(b\sqrt{x}) \left\{ \frac{\Gamma(v, cx)}{\gamma(v, cx)} \right\} dx = U(0)$$

$$[\operatorname{Re} c, \operatorname{Re}(\alpha+v) > 0; \arg b < \pi/4, \left\{ \begin{matrix} \operatorname{Re} \alpha > 0 \\ \operatorname{Re} v > 0 \end{matrix} \right\}, U(\gamma) \text{ см в 2.10.8.1}].$$

$$3. \int_0^{\infty} x^{\alpha-1} e^{cx} \operatorname{erfc}(b\sqrt{x}) \gamma(v, cx) dx =$$

$$= \frac{c^v \Gamma(\alpha+v+1/2)}{\sqrt{\pi} v(\alpha+v) b^{2\alpha+2v}} {}_3F_2 \left(1, \alpha+v, \alpha+v+\frac{1}{2}; v+1, \alpha+v+1; \frac{c}{b^2} \right)$$

$$[\operatorname{Re}(\alpha+v), \operatorname{Re} v, \operatorname{Re}(b^2-c) > 0; \arg b < \pi/4]$$

$$4. \int_0^{\infty} x^{\alpha-1} e^{h^2 x} \operatorname{erf}(b\sqrt{x}) \Gamma(v, cx) dx =$$

$$= \frac{4b \Gamma(\alpha+v+1/2)}{\sqrt{\pi} (2\alpha+1) c^{\alpha+1/2}} {}_3F_2 \left(1, \alpha+\frac{1}{2}, \alpha+v+\frac{1}{2}; \frac{3}{2}, \alpha+\frac{3}{2}; \frac{b^2}{c} \right)$$

$$[\operatorname{Re} \alpha, \operatorname{Re}(\alpha+v) > -1/2; \operatorname{Re}(c-b^2) > 0; \arg b < \pi/4]$$

$$\begin{aligned}
5. \int_0^{\infty} x^{\alpha-1} \operatorname{erfi}(b \sqrt{x}) \operatorname{erf}(b \sqrt{x}) \Gamma(v, cx) dx = \\
= \frac{4b^2 \Gamma(\alpha+v+1)}{\pi(\alpha+1)c^{\alpha+1}} {}_5F_4 \left(\frac{1}{2}, 1, \frac{\alpha+1}{2}, \frac{\alpha+v+1}{2}, \frac{\alpha+v}{2}+1; \right. \\
\left. \frac{3}{4}, \frac{5}{4}, \frac{3}{2}, \frac{\alpha+3}{2}; \frac{b^4}{c^2} \right) \\
[\operatorname{Re} \alpha, \operatorname{Re}(\alpha+v) > -1; b, \operatorname{Re} c > 0].
\end{aligned}$$

2.11. ФУНКЦИЯ ПАРАБОЛИЧЕСКОГО ЦИЛИНДРА $D_v(x)$

2.11.1. Интегралы общего вида.

$$\begin{aligned}
1. \int_0^a x^{\alpha-1} (a^r - x^r)^{\beta-1} e^{\pm c^2 x^2/4} D_v(cx) dx = \frac{2^{v/2} \sqrt{\pi}}{r} \Gamma(\beta) a^{\alpha+r\beta-r} \times \\
\times \sum_{k=0}^{\infty} \frac{[(-v)_k]^{(1 \pm 1)/2}}{k!} \Gamma \left[\frac{(\alpha+k)/r}{(\alpha+k)/r + \beta}, (1-v \pm k)/2 \right] (-\sqrt{2 \mp 1} ac)^k \\
[a, r, \operatorname{Re} \alpha, \operatorname{Re} \beta > 0]
\end{aligned}$$

$$\begin{aligned}
2. \int_a^{\infty} x^{\alpha-1} (x^r - a^r)^{\beta-1} e^{\pm c^2 x^2/4} D_v(cx) dx = \frac{2^{v/2} \sqrt{\pi}}{r} \Gamma(\beta) a^{\alpha+r\beta-r} \times \\
\times \sum_{k=0}^{\infty} \frac{[(-v)_k]^{(1 \pm 1)/2}}{k!} \Gamma \left[\frac{1-\beta-(\alpha+k)/r}{1-(\alpha+k)/r}, (1-v \pm k)/2 \right] (-\sqrt{2 \mp 1} ac)^k + \\
+ 2^{(v+r\beta-\alpha)/2} \sqrt{\pi} [2^{v+1} \sqrt{\pi} \Gamma(-v)]^{(\mp 1-1)/2} c^{r\beta-\alpha} \times \\
\times \sum_{k=0}^{\infty} \frac{(1-\beta)_k \Gamma(\alpha-r+r\beta-rk)}{k!} \Gamma^{\pm 1} \left(\frac{1 \mp 1}{4} - \frac{v \pm \alpha \mp r(1-\beta+k)}{2} \right) (\sqrt{2} ac)^{rk} \\
\left[r, \operatorname{Re} \beta > 0; \arg c | < (2 \pm 1)\pi/4, \begin{cases} a > 0; \operatorname{Re}(\alpha+v+r\beta) < r \\ a > 0 \end{cases} \right].
\end{aligned}$$

$$\begin{aligned}
3. \int_0^{\infty} \frac{x^{\alpha-1}}{(x^r + z^r)^{\rho}} e^{\pm c^2 x^2/4} D_v(cx) dx = \frac{2^{v/2} \sqrt{\pi}}{r \Gamma(\rho)} z^{\alpha-r\rho} \times \\
\times \sum_{k=0}^{\infty} \frac{[(-v)_k]^{(1 \pm 1)/2}}{k!} \Gamma \left[\frac{(\alpha+k)/r, \rho-(\alpha+k)/r}{(1-v \pm k)/2} \right] (-\sqrt{2 \mp 1} cz)^k + \\
+ 2^{(v+r\rho-\alpha)/2} \sqrt{\pi} [2^{v+1} \sqrt{\pi} \Gamma(-v)]^{(\mp 1-1)/2} c^{r\rho-\alpha} \times \\
\times \sum_{k=0}^{\infty} \frac{(-1)^k (\rho)_k \Gamma(\alpha-r\rho-rk)}{k!} \Gamma^{\pm 1} \left(\frac{1 \mp 1}{4} - \frac{v \pm \alpha \mp r(\rho+k)}{2} \right) (\sqrt{2} cz)^{rk} \\
\left[r > 0, |\arg c| < (2 \pm 1)\pi/4, r, |\arg z| < \pi, \begin{cases} 0 < \operatorname{Re} \alpha < \operatorname{Re}(r\rho-v) \\ \operatorname{Re} \alpha > 0 \end{cases} \right]
\end{aligned}$$

$$\begin{aligned}
4. \int_0^\infty \frac{x^{\alpha-1}}{x^r - y^r} e^{\pm c^2 x^2/4} D_\nu(cx) dx = \\
= -\frac{2^{\nu/2} \pi^{3/2}}{r} y^{\alpha-r} \sum_{k=0}^\infty \frac{[(-\nu)_k]^{(1\pm 1)/2}}{k!} \Gamma^{-1}\left(\frac{1-\nu \pm k}{2}\right) \operatorname{ctg} \frac{\alpha+k}{r} \pi (-\sqrt{2^{\mp 1}} cy)^k + \\
+ 2^{(\nu+r-\alpha)/2} \sqrt{\pi} [2^{\nu+1} \sqrt{\pi} \Gamma(-\nu)]^{(\mp 1-1)/2} c^{r-\alpha} \times \\
\times \sum_{k=0}^\infty \Gamma(\alpha-r-rk) \Gamma^{\pm 1}\left(\frac{1 \mp 1}{4} - \frac{\nu \pm \alpha \mp r(1+k)}{2}\right) (\sqrt{2} cy)^{rk} \\
\left[r, y > 0; |\arg c| < (2 \pm 1) \pi/4, \begin{cases} 0 < \operatorname{Re} \alpha < r - \operatorname{Re} \nu \\ \operatorname{Re} \alpha > 0 \end{cases} \right].
\end{aligned}$$

$$\begin{aligned}
5. \int_0^\infty x^{\alpha-1} e^{-px^r \pm c^2 x^2/4} D_\nu(cx) dx = U(r) \\
[r, \operatorname{Re} p, \operatorname{Re} \alpha > 0; |\arg c| < (2 \pm 1) \pi/4, \\
U(r) = \frac{2^{\nu/2} \sqrt{\pi}}{p^{\alpha/r} r} \sum_{k=0}^\infty \frac{[(-\nu)_k]^{(1\pm 1)/2}}{k!} \Gamma\left[\frac{(\alpha+k)/r}{(1-\nu \pm k)/2}\right] \left(-\frac{\sqrt{2^{\mp 1}} c}{p^{1/r}}\right)^k \\
[r > 2],
\end{aligned}$$

$$\begin{aligned}
U(r) = \frac{2^{(\nu-\alpha)/2} \sqrt{\pi}}{c^\alpha} [2^{\nu+1} \sqrt{\pi} \Gamma(-\nu)]^{-(1\pm 1)/2} \sum_{k=0}^\infty \frac{(-1)^k \Gamma(\alpha+rk)}{k!} \times \\
\times \Gamma^{\pm 1}\left(\frac{-\nu \mp \alpha \mp rk}{2} + \frac{1 \mp 1}{4}\right) \left(\frac{p}{2^{r/2} c^r}\right)^k + \begin{cases} 1 \\ 0 \end{cases} \frac{c^\nu}{p^{(\alpha+\nu)/r}} \times \\
\times \sum_{k=0}^\infty \frac{(-\nu)_{2k}}{k!} \Gamma\left(\frac{\alpha+\nu-2k}{r}\right) \left(-\frac{p^{2/r}}{2c^2}\right)^k \\
[r < 2].
\end{aligned}$$

$U(2)$ см. в 2.11 3.2

$$\begin{aligned}
6. \int_0^\infty x^{\alpha-1} e^{-px^{-r} \pm c^2 x^2/4} D_\nu(cx) dx = \frac{2^{\nu/2} \sqrt{\pi} p^{\alpha/r}}{r} \times \\
\times \sum_{k=0}^\infty \frac{[(-\nu)_k]^{(1\pm 1)/2}}{k!} \Gamma\left[-\frac{(\alpha+k)/r}{(1-\nu \pm k)/2}\right] (-\sqrt{2^{\mp 1}} c p^{1/r})^k + \\
+ \frac{2^{(\nu-\alpha)/2} \sqrt{\pi}}{c^\alpha} [2^{\nu+1} \sqrt{\pi} \Gamma(-\nu)]^{-(1\pm 1)/2} \sum_{k=0}^\infty \frac{(-1)^k \Gamma(\alpha-rk)}{k!} \times \\
\times \Gamma^{\pm 1}\left(\frac{-\nu \mp \alpha \pm rk}{2} + \frac{1 \mp 1}{4}\right) (2^{r/2} c^r p)^k \\
\left[r > 0; |\arg c| < (2 \pm 1) \pi/4, \begin{cases} \operatorname{Re} p > 0; \operatorname{Re}(\alpha+\nu) < 0 \\ \operatorname{Re} p > 0 \end{cases} \right].
\end{aligned}$$

$$\begin{aligned}
7. \int_0^\infty x^{\alpha-1} e^{\pm c^2 x^2/4} \sin bx^r D_\nu(cx) dx = V(1) \\
\left[b, r > 0; |\arg c| < (2 \pm 1) \pi/4, \begin{cases} -r < \operatorname{Re} \alpha < r - \operatorname{Re} \nu \\ \operatorname{Re} \alpha > -r \end{cases} \right],
\end{aligned}$$

$$\begin{aligned}
V(\gamma) = \frac{2^{\nu/2} \sqrt{\pi}}{b^{\alpha/r} r} \sum_{k=0}^\infty \frac{[(-\nu)_k]^{(1\pm 1)/2}}{k!} \Gamma\left[\frac{(\alpha+k)/r}{(1-\nu \pm k)/2}\right] \times \\
\times \cos \frac{\gamma r - \alpha - k}{2r} \pi \left(-\frac{\sqrt{2^{\mp 1}} c}{b^{1/r}}\right)^k \\
[r > 2],
\end{aligned}$$

$$V(\gamma) = \frac{2^{(\nu-\alpha-\gamma r)/2} b^\gamma \sqrt{\pi}}{c^{\alpha+\gamma r}} [2^{\nu+1} \sqrt{\pi} \Gamma(-\nu)]^{-(1\pm 1)/2} \times$$

$$\times \sum_{k=0}^{\infty} \frac{\Gamma(\alpha+r\gamma+2rk)}{(1/2+\gamma)_k k!} \Gamma^{\pm 1} \left(\frac{-\nu \mp \alpha \mp r\gamma}{2} \mp rk + \frac{1 \mp 1}{4} \right) \left(-\frac{b^2}{2^{r+2} c^{2r}} \right)^k +$$

$$+ \begin{cases} 1 \\ 0 \end{cases} \frac{c^\nu}{b^{(\alpha+\nu)/r}} \sum_{k=0}^{\infty} \frac{(-\nu)_{2k}}{k!} \Gamma \left(\frac{\alpha+\nu-2k}{r} \right) \cos \frac{r\gamma-\alpha-\nu+2k}{2r} \pi \left(-\frac{b^{2/r}}{2c^2} \right)^k$$

$$[r < 2].$$

$V(\gamma)$ при $r=2$ см. в 2.11.6.4—5.

$$8. \int_0^\infty x^{\alpha-1} e^{\pm c^2 x^2/4} \cos bx^r D_\nu(cx) dx = V(0)$$

$$[b, r > 0; |\arg c| < (2 \pm 1)\pi/4, \{0 < \operatorname{Re} \alpha < r - \operatorname{Re} \nu\}, \operatorname{Re} \alpha > 0], V(\gamma) \text{ см. в 2.11.1.7}].$$

$$9. \int_0^\infty x^{\alpha-1} e^{-px^r \pm c^2 x^2/4} \sin bx^r D_\nu(cx) dx = W(1)$$

$$[b, r, \operatorname{Re} p > 0; \operatorname{Re} \alpha > -r; |\arg c| < (2 \pm 1)\pi/4].$$

$$W(\gamma) = \frac{2^{\nu/2} \sqrt{\pi}}{r (\sqrt{b^2+p^2})^{\alpha/r}} \sum_{k=0}^{\infty} \frac{(-1)^k [(-\nu)_k]^{(1\pm 1)/2}}{k!} \times$$

$$\times \Gamma \left[\frac{(\alpha+k)/r}{(1-\nu \pm k)/2} \right] \cos \left(\frac{\gamma\pi}{2} - \frac{\alpha+k}{r} \arcsin \frac{b}{\sqrt{b^2+p^2}} \right) \frac{2^{\mp k/2} c^k}{(\sqrt{b^2+p^2})^{k/r}}$$

$$[r > 2] \text{ или } [r=2; |c|^4 < 4|b^2+p^2|]$$

$$W(\gamma) = \frac{2^{(\nu-\alpha)/2} \sqrt{\pi}}{c^\alpha} [2^{\nu+1} \sqrt{\pi} \Gamma(-\nu)]^{-(1\pm 1)/2} \sum_{k=0}^{\infty} \frac{\Gamma(\alpha+rk)}{k!} \times$$

$$\times \Gamma^{\pm 1} \left(\frac{-\nu \mp \alpha \mp rk}{2} + \frac{1 \mp 1}{4} \right) \cos \left(\frac{\gamma\pi}{2} + k \arcsin \frac{b}{\sqrt{b^2+p^2}} \right) \left(-\frac{\sqrt{b^2+p^2}}{2^{r/2} c^r} \right)^k +$$

$$+ \begin{cases} 1 \\ 0 \end{cases} \frac{c^\nu}{r (\sqrt{b^2+p^2})^{(\alpha+\nu)/r}} \sum_{k=0}^{\infty} \frac{(-1)^k (-\nu)_{2k}}{k!} \Gamma \left(\frac{\alpha+\nu-2k}{r} \right) \times$$

$$\times \cos \left(\frac{\gamma\pi}{2} + \frac{2k-\alpha-\nu}{r} \arcsin \frac{b}{\sqrt{b^2+p^2}} \right) \frac{(b^2+p^2)^{k/r}}{2^k c^{2k}}$$

$$[r < 2] \text{ или } [r=2; |c|^4 > 4|b^2+p^2|].$$

$$10. \int_0^\infty x^{\alpha-1} e^{-px^r \pm c^2 x^2/4} \cos bx^r D_\nu(cx) dx = W(0)$$

$$[b, r, \operatorname{Re} p, \operatorname{Re} \alpha > 0; |\arg c| < (2 \pm 1)\pi/4; W(\gamma) \text{ см в 2.11.1.9}].$$

$$11. \int_0^\infty x^{\alpha-1} \ln(x^r+z^r) e^{\pm c^2 x^2/4} D_\nu(cx) dx = X(1)$$

$$[r > 0; |\arg c| < (2 \pm 1)\pi/4; r|\arg z| < \pi, \{0 < \operatorname{Re} \alpha < -\operatorname{Re} \nu\}, \operatorname{Re} \alpha > 0],$$

$$\begin{aligned}
X(\gamma) = & -2^{v/2} \pi^{3/2} z^\alpha \times \\
& \times \sum_{k=0}^{\infty} \frac{[(-v)_k]^{(1 \pm 1)/2} (-\sqrt{2 \mp 1} cz)^k \cos^{1-\gamma}[(\alpha+k)\pi/r]}{k! (k-\alpha) \Gamma((1-v \pm k)/2) \sin[(\alpha+k)\pi/r]} - \\
& - \frac{2^{(v-\alpha)/2} \sqrt{\pi}}{c^\alpha} [2^{v+1} \sqrt{\pi} \Gamma(-v)]^{-(1 \pm 1)/2} \sum_{k=1}^{\infty} (-1)^{\gamma k} \frac{\Gamma(\alpha - rk)}{k} \times \\
& \times \Gamma^{\pm 1} \left(\frac{-v \mp \alpha \pm rk}{2} + \frac{1 \mp 1}{4} \right) (\sqrt{2} cz)^{rk} + \\
& + \frac{2^{(v-\alpha)/2-1} r \sqrt{\pi} \Gamma(\alpha)}{c^\alpha} [2^{v+1} \sqrt{\pi} \Gamma(-v)]^{-(1 \pm 1)/2} \times \\
& \times \Gamma^{\pm 1} \left(\frac{-v \mp \alpha}{2} + \frac{1 \mp 1}{4} \right) \left[2\psi(\alpha) - \psi\left(\frac{1 \mp 1}{4} - \frac{v \pm \alpha}{2}\right) - \ln(2c^2) \right].
\end{aligned}$$

$$\begin{aligned}
12. \quad & \int_0^\infty x^{\alpha-1} \ln |x^r - z^r| e^{\pm c^2 x^2/4} D_\nu(cx) dx = X(0) \\
& [r, z > 0; |\arg c| < (2 \pm 1)\pi/4 \quad \left\{ \begin{array}{l} 0 < \operatorname{Re} \alpha < -\operatorname{Re} v \\ \operatorname{Re} \alpha > 0 \end{array} \right\}, X(\gamma) \text{ см. в 2.11.1.11}].
\end{aligned}$$

$$\begin{aligned}
13. \quad & \int_0^\infty x^{\alpha-1} e^{b^2 x^{2r}/4 \pm c^2 x^2/4} D_\mu(bx^r) D_\nu(cx) dx = Y(1) \\
& [r > 0; |\arg b| < 3\pi/4; |\arg c| < (2 \pm 1)\pi/4, \\
& \left\{ \begin{array}{l} 0 < \operatorname{Re} \alpha < -\operatorname{Re}(v + r\mu) \\ \operatorname{Re} \alpha > 0 \end{array} \right\}; \text{ при } r < 1 - \text{замена } x = t^r],
\end{aligned}$$

$$\begin{aligned}
Y(\gamma) = & \frac{\pi}{h^{\alpha/r} r} 2^{-(\mu+v+\alpha/r)/2 + \alpha\gamma/r} \Gamma^{-\gamma}(-\mu) \times \\
& \times \sum_{k=0}^{\infty} \frac{[(-v)_k]^{(1 \pm 1)/2}}{k!} \Gamma \left[\frac{(\alpha+k)/r}{(1 \pm k - v)/2, [r - \mu r + (-1)^\gamma (k + \alpha)]/(2r)} \right] \times \\
& \times \Gamma^\gamma \left(-\mu - \frac{\alpha+k}{r} \right) \left[-\frac{2^{\mp \frac{1}{2}} - \frac{(-1)^\gamma}{2r} c}{b^{1/r}} \right]^k + \frac{\gamma 2^\varphi b^\mu [\Gamma(-v)]^{-(1 \pm 1)/2} \Gamma^{1-\gamma}(-\mu)}{\pi^{\gamma/2} c^{\alpha+\mu r}} \times \\
& \times \sum_{k=0}^{\infty} \frac{(-\mu)_{2k}}{k!} \Gamma \left[\frac{\alpha + \mu r - 2rk}{(1 - v \mp \alpha \mp \mu r)/2 \pm kr} \right] \Gamma^{(1 \pm 1)/2} (2rk - \alpha - v - \mu r) \left(-\frac{2^{\mp r-1} c^{2r}}{b^2} \right)^k \\
& [r > 1; \varphi = (\alpha + \mu r)(\delta - 1/2) + \mu(1 - \gamma) - \gamma + \gamma/2 + 1],
\end{aligned}$$

$Y(\gamma)$ при $r=1$ см. в 2.11.8.2—3.

$$\begin{aligned}
14. \quad & \int_0^\infty x^{\alpha-1} e^{-b^2 x^{2r}/4 \pm c^2 x^2/4} D_\mu(bx^r) D_\nu(cx) dx = Y(0) \\
& [r, \operatorname{Re} \alpha > 0; |\arg b| < \pi/4; |\arg c| < (2 \pm 1)\pi/4; Y(\gamma) \text{ см. в 2.11.1.13}].
\end{aligned}$$

$$\begin{aligned}
15. \quad & \int_0^\infty x^{\alpha-1} e^{\pm c^2 x^2/4} \operatorname{erf}(bx^r) D_\nu(cx) dx = Z(1) \\
& [r, \operatorname{Re} b > 0; |\arg c| < (2 \pm 1)\pi/4, \left\{ \begin{array}{l} -r < \operatorname{Re} \alpha < -\operatorname{Re} v \\ \operatorname{Re} \alpha > -r \end{array} \right\}],
\end{aligned}$$

$$Z(\gamma) = (-1)^{\gamma} 2^{\gamma/2} b^{-\alpha/r} \sum_{k=0}^{\infty} \frac{[(-\gamma)_k]^{(1 \pm 1)/2}}{k! (k + \alpha)} \Gamma \left[\frac{(\alpha + r + k)/(2r)}{(1 - \gamma \pm k)/2} \right] \left(-\frac{\sqrt{2 \mp 1} c}{b^{1/r}} \right)^k +$$

$$+ \gamma \frac{2^{(\gamma-\alpha)/2} \sqrt{\pi}}{c^{\alpha}} \Gamma(\alpha) \Gamma^{\pm 1} \left(\frac{-\gamma \mp \alpha}{2} + \frac{1 \mp 1}{4} \right) [2^{\gamma+1} \sqrt{\pi} \Gamma(-\gamma)]^{-(1 \pm 1)/2}$$

[r > 1],

$$Z(\gamma) = -(-1)^{\gamma} \frac{2^{(\gamma-\alpha-r)/2} b}{c^{\alpha+r}} [2^{\gamma+1} \sqrt{\pi} \Gamma(-\gamma)]^{-(1 \pm 1)/2} \times$$

$$\times \sum_{k=0}^{\infty} \frac{\Gamma(\alpha + r + 2rk)}{k! (k + 1/2)} \Gamma^{\pm 1} \left(\frac{-\gamma \mp (\alpha + r + 2rk)}{2} + \frac{1 \mp 1}{4} \right) \left(-\frac{b^2}{2r c^{2r}} \right)^k -$$

$$- \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} (-1)^{\gamma} \frac{c^{\gamma}}{2 \sqrt{\pi} b^{(\alpha+\gamma)/r}} \sum_{k=0}^{\infty} \frac{(-\gamma)_{2k}}{k! [k - (\alpha + \gamma)/2]} \Gamma \left(\frac{\alpha + \gamma + r - 2k}{2r} \right) \left(-\frac{b^{2/r}}{2c^2} \right)^k +$$

$$+ (1 - \gamma) \frac{2^{(\gamma-\alpha)/2} \sqrt{\pi}}{c^{\alpha}} \Gamma(\alpha) \Gamma^{\pm 1} \left(\frac{-\gamma \mp \alpha}{2} + \frac{1 \mp 1}{4} \right)$$

[r < 1].

$Z(\gamma)$ при $r=1$ см. в 2.11.9.1-2

$$16. \int_0^{\infty} x^{\alpha-1} e^{\pm c^2 x^2/4} \operatorname{erfc}(bx^r) D_{\gamma}(cx) dx = Z(0)$$

[r, Re α > 0; |arg b| < π/4; |arg c| < (2 ± 1)π/4; Z(γ) см. в 2.11.1.15]

2.11.2. Интегралы от $A(x) D_{\gamma}(cx)$

$$1. \int_0^{\infty} x^{\alpha-1} D_{\gamma}(cx) dx = 2^{(\gamma-\alpha)/2} \sqrt{\pi} c^{-\alpha} \Gamma \left[\frac{\alpha}{(1 + \alpha - \gamma)/2} \right] \times$$

$$\times {}_2F_1 \left(\frac{\alpha}{2}, \frac{\alpha+1}{2}; \frac{1 + \alpha - \gamma}{2}; \frac{1}{2} \right)$$

[Re α > 0; |arg c| < π/4].

$$2. \int_0^{\infty} x^{-1} {}^2D_{\gamma}(cx) dx = \frac{2^{\gamma} \pi^{3/2}}{\sqrt{c} \Gamma((3-\gamma)/4) \Gamma((2-\gamma)/4)}$$

[|arg c| < π/4]

$$3. \int_0^{\infty} \frac{x}{x^2 + z^2} [D_{\gamma}(-cx) - D_{\gamma}(cx)] dx = -\sqrt{2\pi} \Gamma(\gamma+1) \sin \frac{\gamma\pi}{2} e^{c^2 z^2/4} D_{-\gamma-1}(cz)$$

[Re z > 0; |arg c| < π/4].

2.11.3. Интегралы от $A(x) e^{px^2} D_{\gamma}(cx)$.

$$1. \int_0^{\infty} e^{-px^2} D_{2n}(cx) dx = \frac{\sqrt{\pi} (2n)! (c^2 - 4p)^n}{2^n n! (c^2 + 4p)^{n+1/2}}$$

[R(c^2 + 4p) > 0]

$$2. \int_0^{\infty} x^{\alpha-1} e^{-px^2} D_{\gamma}(cx) dx =$$

$$= 2^{(\gamma-\alpha)/2} \sqrt{\pi} c^{-\alpha} \Gamma \left[\frac{\alpha}{(1 + \alpha - \gamma)/2} \right] {}_2F_1 \left(\frac{\alpha}{2}, \frac{\alpha+1}{2}; \frac{1 + \alpha - \gamma}{2}; \frac{c^2 - 4p}{2c^2} \right)$$

[Re α, Re(c^2 + 4p) > 0] или [Re(c^2 + 4p) = 0; 0 < Re α < -Re γ].

$$3. \int_0^{\infty} x^{-1/2} e^{-px^2} D_{\gamma}(cx) dx =$$

$$= \frac{2^{(2\gamma+1)/4} \sqrt{\pi} c e^{-i\pi/4}}{(16p^2 - c^4)^{1/4}} \left(\frac{4p + c^2}{4p - c^2} \right)^{-(2\gamma+1)/8} Q_{(2\gamma+3)/4}^{1/4} \left(\sqrt{\frac{8p}{16p^2 - c^4}} \right)$$

[Re(c^2 + 4p) > 0]

$$4. \int_0^{\infty} x^{-\nu} e^{-px^2} D_{\nu}(cx) dx = \frac{2^{-\nu/2} \sqrt{\pi}}{\sqrt{c^2 + 4p}} (\sqrt{c^2 + 4p} + \sqrt{2} c)^{\nu}$$

[Re(c² + 4p) > 0; Re ν < 1]

$$5. \int_0^{\infty} x^{\nu-2n} e^{-px^2} D_{\nu}(cx) dx =$$

$$= \frac{\sqrt{\pi} \Gamma(\nu+1) (c^2 - 4p)^n}{2^{2n+1/2} n! c^{\nu+1}} {}_2F_1\left(\frac{\nu+1}{2}, \frac{\nu}{2} + 1; n+1; \frac{c^2 - 4p}{2c^2}\right)$$

[Re(c² + 4p) > 0; Re ν > 2n - 1]

$$6. \int_0^{\infty} x^{-\nu-2} e^{-px^2} D_{\nu}(cx) dx = - \frac{\sqrt{\pi} (\sqrt{c^2 + 4p} + \sqrt{2} c)^{\nu+1}}{2^{\nu/2+1} (\nu+1)}$$

[Re(c² + 4p) > 0; Re ν < -1]

$$7. \int_0^{\infty} x e^{-px^2} D_{2n+1}(cx) dx = \frac{2^{n+3/2} c}{(c^2 + 4p)^{3/2}} \Gamma\left(n + \frac{3}{2}\right) \left(\frac{c^2 - 4p}{c^2 + 4p}\right)^n$$

[Re(c² + 4p) > 0]

$$8. \int_0^{\infty} x^{\nu-1} e^{-c^2 x^2/4} D_{\nu}(cx) dx = \frac{\Gamma(\nu) \cos(\nu\pi/4)}{2^{\nu/2} c^{\nu}}$$

[Re ν > 0; |arg c| < π/4]

$$9. \int_0^{\infty} x^{\nu+1} e^{-c^2 x^2/4} D_{\nu}(cx) dx = \frac{\Gamma(\nu+1)}{2^{(\nu+3)/2} c^{\nu+2}} \sin\left(\frac{\nu+1}{4}\pi\right)$$

[Re ν > -2; |arg c| < π/4]

$$10. \int_0^{\infty} x^{\alpha-1} e^{c^2 x^2/4} D_{\nu}(cx) dx = \frac{c^{-\alpha}}{2^{(\alpha+\nu)/2+1}} \Gamma\left[\begin{matrix} \alpha, -(\alpha+\nu)/2 \\ -\nu \end{matrix}\right]$$

[0 < Re α < -Re ν; |arg c| < 3π/4]

$$11. \int_0^{\infty} x^{\alpha-1} e^{-c^2 x^2/4} D_{\nu}(cx) dx = \frac{2^{(\nu-\alpha)/2} \sqrt{\pi}}{c^{\alpha}} \Gamma\left[\begin{matrix} \alpha \\ (1+\alpha-\nu)/2 \end{matrix}\right]$$

[Re α > 0; |arg c| < π/4]

$$12. \int_0^{\infty} x^{(1 \mp 1)/2} e^{-px^2} [D_{\nu}(-cx) \pm D_{\nu}(cx)] dx =$$

$$= \frac{2^{\nu/2+1} \pi}{\Gamma((1 \pm 1 - 2\nu)/4)} \left\{ \frac{1}{2^{3/2} c} \right\} (4p \pm c^2)^{-(1 \pm \nu)/2} (4p \mp c^2)^{\pm(\nu+1 \mp 1)/2}$$

[Re(4p + c²) > 0]

$$13. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\beta-1} e^{\pm c^2 x^2/4} D_{\nu}(cx) dx =$$

$$= \frac{2^{\nu/2-1} \sqrt{\pi} a^{\alpha+2\beta-2}}{\Gamma(1/2 - \nu/2)} B\left(\frac{\alpha}{2}, \beta\right) {}_2F_3\left(\mp \frac{\nu}{2} + \frac{1 \mp 1}{4}, \frac{\alpha}{2}; \frac{1}{2}, \frac{\alpha}{2} + \beta; \pm \frac{a^2 c^2}{2}\right) -$$

$$- \frac{2^{(\nu-1)/2} \sqrt{\pi} a^{\alpha+2\beta-1} c}{\Gamma(-\nu/2)} B\left(\frac{\alpha+1}{2}, \beta\right) {}_2F_3\left(\mp \frac{\nu}{2} + \frac{3 \mp 1}{4}, \frac{\alpha+1}{2}; \frac{3}{2}, \frac{\alpha+1}{2} + \beta; \pm \frac{a^2 c^2}{2}\right)$$

[a, Re α, Re β > 0]

$$14. \int_0^a x^{-\nu-2\beta-1} (a^2-x^2)^{\beta-1} e^{c^2 x^2/4} D_\nu(cx) dx = \\ = 2^{\beta-1} \Gamma \left[\begin{matrix} \beta, & -\nu-2\beta \\ & -\nu \end{matrix} \right] a^{-\nu-2} e^{a^2 c^2/4} D_{\nu+2\beta}(ac) \quad [a>0; 0<\operatorname{Re} \beta < -\operatorname{Re} \nu/2]$$

$$15. \int_a^\infty x^{\alpha-1} (x^2-a^2)^{\beta-1} e^{\pm c^2 x^2/4} D_\nu(cx) dx = \frac{2^{\nu/2-1} \sqrt{\pi} a^{\alpha+2\beta-2}}{\Gamma(1/2-\nu/2)} \times \\ \times B \left(\beta, 1-\beta-\frac{\alpha}{2} \right) {}_2F_2 \left(\mp \frac{\nu}{2} + \frac{1 \mp 1}{4}, \frac{\alpha}{2}; \frac{1}{2}, \frac{\alpha}{2} + \beta; \pm \frac{a^2 c^2}{2} \right) - \\ - \frac{2^{(\nu-1)/2} \sqrt{\pi} a^{\alpha+2\beta-1} c}{\Gamma(-\nu/2)} B \left(\beta, \frac{1-\alpha}{2} - \beta \right) \times \\ \times {}_2F_2 \left(\mp \frac{\nu}{2} + \frac{3 \mp 1}{4}, \frac{\alpha+1}{2}; \frac{3}{2}, \frac{\alpha+1}{2} + \beta; \pm \frac{a^2 c^2}{2} \right) + \\ + 2^{1-\beta-(\alpha \pm \nu)/2} \sqrt{\pi} \Gamma^{\pm 1} \left(\frac{1 \pm 3}{4} \mp \beta - \frac{\alpha \pm \nu}{2} \right) [2 \sqrt{\pi} \Gamma(-\nu)]^{-(1 \pm 1)/2} \times \\ \times \Gamma(\alpha+2\beta-2) c^{\alpha-2\beta-\alpha} {}_2F_2 \left(1-\beta, \frac{5 \mp 1}{4} - \beta - \frac{\alpha \pm \nu}{2}; 2-\beta-\frac{\alpha}{2}, \frac{3-\alpha}{2} - \beta; \pm \frac{a^2 c^2}{2} \right) \\ \left[\operatorname{Re} \beta > 0; |\arg c| < (2 \pm 1) \pi/4, \begin{cases} a>0; \\ a>0 \end{cases} \operatorname{Re}(\alpha+\nu+2\beta) < 2 \right].$$

$$16. \int_a^\infty x^{\nu-2\beta} (x^2-a^2)^{\beta-1} e^{\pm c^2 x^2/4} D_\nu(cx) dx = 2^{\beta-1} a^{\nu-1} \Gamma(\beta) e^{\pm a^2 c^2/4} D_{\nu-2\beta}(ac) \\ \left[\operatorname{Re} \beta > 0; |\arg c| < (2 \pm 1) \pi/4, \begin{cases} a>0; \\ a>0 \end{cases} \operatorname{Re} \nu < 1/2 \right].$$

$$17. \int_0^\infty \frac{x^{\alpha-1}}{(x^2+z^2)^\rho} e^{\pm c^2 x^2/4} D_\nu(cx) dx = \frac{2^{\nu/2-1} \sqrt{\pi} z^{\alpha-2\rho}}{\Gamma(1/2-\nu/2)} B \left(\frac{\alpha}{2}, \rho - \frac{\alpha}{2} \right) \times \\ \times {}_2F_2 \left(\mp \frac{\nu}{2} + \frac{1 \mp 1}{4}, \frac{\alpha}{2}; \frac{1}{2}, 1 + \frac{\alpha}{2} - \rho; \mp \frac{c^2 z^2}{2} \right) - \\ - \frac{2^{(\nu-1)/2} \sqrt{\pi} c z^{\alpha-2\rho+1}}{\Gamma(-\nu/2)} B \left(\frac{\alpha+1}{2}, \rho - \frac{\alpha+1}{2} \right) \times \\ \times {}_2F_2 \left(\mp \frac{\nu}{2} + \frac{3 \mp 1}{4}, \frac{\alpha+1}{2}; \frac{3}{2}, \frac{3+\alpha}{2} - \rho; \mp \frac{c^2 z^2}{2} \right) + \\ + 2^{\rho-(\alpha \pm \nu)/2} \sqrt{\pi} [2 \sqrt{\pi} \Gamma(-\nu)]^{-(1 \pm 1)/2} \Gamma^{\pm 1} \left(\frac{-\nu \mp \alpha}{2} \pm \rho + \frac{1 \mp 1}{4} \right) \times \\ \times \Gamma(\alpha-2\rho) c^{2\rho-\alpha} {}_2F_2 \left(\rho, \frac{1 \mp 1}{4} + \rho - \frac{\alpha \pm \nu}{2}; 1+\rho-\frac{\alpha}{2}, \frac{1-\alpha}{2} + \rho; \mp \frac{c^2 z^2}{2} \right) \\ \left[|\arg c| < (2 \pm 1) \pi/4; \operatorname{Re} z > 0, \begin{cases} 0 < \operatorname{Re} \alpha < \operatorname{Re}(2\rho-\nu) \\ \operatorname{Re} \alpha > 0 \end{cases} \right].$$

$$18. \int_0^\infty \frac{x^\nu}{x^2+z^2} e^{-c^2 x^2/4} D_\nu(cx) dx = \sqrt{\frac{\pi}{2}} \Gamma(\nu+1) z^{\nu-1} e^{c^2 z^2/4} D_{-\nu-1}(cz) \\ [\operatorname{Re} z > 0; \operatorname{Re} \nu > -1; |\arg c| < \pi/4]$$

$$19. \int_0^\infty \frac{x^{\nu-1}}{\sqrt{x^2+z^2}} e^{-c^2 x^2/4} D_\nu(cx) dx = \Gamma(\nu) z^{\nu-1} e^{c^2 z^2/4} D_{-\nu}(cz) \\ [\operatorname{Re} \nu, \operatorname{Re} z > 0; |\arg c| < \pi/4]$$

$$20. \int_0^{\infty} \frac{x^{\alpha-1}}{x^2-y^2} e^{\pm c^2 x^2/4} D_{\nu}(cx) dx = -\frac{2^{v/2-1} \pi^{3/2}}{\Gamma(1/2-v/2)} \operatorname{ctg} \frac{\alpha\pi}{2} y^{\alpha-2} {}_1F_1\left(\mp \frac{v}{2} + \frac{1 \mp 1}{4}; \frac{1}{2}; \pm \frac{c^2 y^2}{2}\right) - \frac{2^{(v-1)/2} \pi^{3/2}}{\Gamma(-v/2)} \operatorname{tg} \frac{\alpha\pi}{2} c y^{\alpha-1} {}_1F_1\left(\mp \frac{v}{2} + \frac{3 \mp 1}{4}; \frac{3}{2}; \pm \frac{c^2 y^2}{2}\right) + 2^{1-(\alpha \pm v)/2} \sqrt{\pi} c^{\alpha-\alpha} \Gamma^{\pm 1}\left(\frac{-v \mp \alpha}{2} + \frac{1 \pm 3}{4}\right) [2 \sqrt{\pi} \Gamma(-v)]^{-(1 \pm 1)/2} \times \\ \times \Gamma(\alpha-2) {}_2F_2\left(\frac{5 \mp 1}{4} - \frac{\alpha \pm v}{2}, 1; 2 - \frac{\alpha}{2}, \frac{3-\alpha}{2}; \pm \frac{c^2 y^2}{2}\right) \\ \left[\nu > 0; |\arg c| < (2 \pm 1) \pi/4, \begin{cases} 0 < \operatorname{Re} \alpha < 2 - \operatorname{Re} \nu \\ \operatorname{Re} \alpha > 0 \end{cases} \right].$$

$$21. \int_{-\infty}^{\infty} \frac{1}{x-z} e^{-c^2 x^2/4} D_n(cx) dx = \pm (-1)^n i \sqrt{2\pi} n! e^{-c^2 z^2/4} D_{-n-1}(\mp icz) \\ \left[|\arg c| < \pi/4, \begin{cases} \operatorname{Im} z > 0 \\ \operatorname{Im} z < 0 \end{cases} \right].$$

2.11.4. Интегралы от $A(x) e^{f(x)} D_{\nu}(cx)$.

$$1. \int_0^{\infty} x^{\alpha-1} e^{-px \pm c^2 x^2/4} D_{\nu}(cx) dx = \\ = 2^{-(\alpha \pm v)/2} \sqrt{\pi} \Gamma(\alpha) [2 \sqrt{\pi} \Gamma(-v)]^{-(1 \pm 1)/2} \Gamma^{\pm 1}\left(\frac{\mp \alpha - v}{2} + \frac{1 \mp 1}{4}\right) c^{-\alpha} \times \\ \times {}_2F_2\left(\frac{\alpha}{2}, \frac{\alpha+1}{2}; \frac{1}{2}, \frac{\alpha \pm v}{2} + \frac{3 \pm 1}{4}; \mp \frac{p^2}{2c^2}\right) - 2^{-(\alpha \pm v + 1)/2} \sqrt{\pi} \times \\ \times \Gamma(\alpha+1) [2 \sqrt{\pi} \Gamma(-v)]^{-(1 \pm 1)/2} \Gamma^{\pm 1}\left(\frac{\mp \alpha - v}{2} + \frac{1 \mp 3}{4}\right) p c^{-\alpha-1} \times \\ \times {}_2F_2\left(\frac{\alpha+1}{2}, \frac{\alpha}{2} + 1; \frac{3}{2}, \frac{\alpha \pm v}{2} + \frac{5 \pm 1}{4}; \mp \frac{p^2}{2c^2}\right) + \\ + \begin{cases} 1 \\ 0 \end{cases} \Gamma(\alpha+v) c^v p^{-\alpha-v} {}_2F_2\left(-\frac{v}{2}, \frac{1-v}{2}; \frac{1-\alpha-v}{2}, 1 - \frac{\alpha+v}{2}; -\frac{p^2}{2c^2}\right) \\ \left[|\arg c| < (2 \pm 1) \pi/4, \begin{cases} \operatorname{Re} \alpha, \operatorname{Re} p > 0 \\ \operatorname{Re} \alpha > 0 \end{cases} \right].$$

$$2. \int_0^{\infty} x^{\alpha-1} e^{-p/x \pm c^2 x^2/4} D_{\nu}(cx) dx = \\ = 2^{v/2} \sqrt{\pi} p^{\alpha} \Gamma\left[\frac{-\alpha}{(1-v)/2}\right] {}_1F_2\left(\mp \frac{v}{2} + \frac{1 \mp 1}{4}; \frac{\alpha}{2} + 1, \frac{\alpha+1}{2}, \frac{1}{2}; \pm \frac{c^2 p^2}{8}\right) - \\ - 2^{(v+1)/2} \sqrt{\pi} p^{\alpha+1} c \Gamma\left[\frac{-\alpha-1}{-v/2}\right] {}_1F_2\left(\mp \frac{v}{2} + \frac{3 \mp 1}{4}; \frac{\alpha}{2} + 1, \frac{\alpha+3}{2}, \frac{3}{2}; \pm \frac{c^2 p^2}{8}\right) + 2^{-(\alpha \pm v)/2} \sqrt{\pi} \Gamma(\alpha) [2 \sqrt{\pi} \Gamma(-v)]^{-(1 \pm 1)/2} \times \\ \times \Gamma^{\pm 1}\left(\frac{\mp \alpha - v}{2} + \frac{1 \mp 1}{4}\right) c^{-\alpha} {}_1F_2\left(\frac{-\alpha \mp v}{2} + \frac{1 \mp 1}{4}; 1 - \frac{\alpha}{2}, \frac{1-\alpha}{2}, \frac{1}{2}; \pm \frac{c^2 p^2}{8}\right) - 2^{(1-\alpha \mp v)/2} \sqrt{\pi} \Gamma(\alpha-1) \times \\ \times [2 \sqrt{\pi} \Gamma(-v)]^{-(1 \pm 1)/2} \Gamma^{\pm 1}\left(\frac{\mp \alpha - v}{2} + \frac{1 \pm 1}{4}\right) c^{1-\alpha} p \times \\ \times {}_1F_2\left(\frac{-\alpha \mp v}{2} + \frac{3 \mp 1}{4}; \frac{3-\alpha}{2}, 1 - \frac{\alpha}{2}, \frac{3}{2}; \pm \frac{c^2 p^2}{8}\right) \\ \left[|\arg c| < (2 \pm 1) \pi/4, \begin{cases} \operatorname{Re} p > 0, \operatorname{Re}(\alpha+v) < 0 \\ \operatorname{Re} p > 0 \end{cases} \right].$$

$$\begin{aligned}
3. \int_0^{\infty} x^{\alpha-1} e^{-px^2 \pm c^2 x^2/4} D_{\nu}(cx) dx = \\
= 2^{\nu/2-1} \sqrt{\pi} \Gamma\left[\frac{-\alpha/2}{(1-\nu)/2}\right] p^{\alpha/2} {}_1F_2\left(\mp \frac{\nu}{2} + \frac{1 \mp 1}{4}; 1 + \frac{\alpha}{2}, \frac{1}{2}; \mp \frac{c^2 p}{2}\right) - \\
- 2^{(\nu-1)/2} \sqrt{\pi} \Gamma\left[\frac{-(\alpha+1)/2}{-\nu/2}\right] c p^{(1+\alpha)/2} {}_1F_2\left(\mp \frac{\nu}{2} + \frac{3 \mp 1}{4}; \frac{3+\alpha}{2}, \frac{3}{2}; \mp \frac{c^2 p}{2}\right) + \\
+ 2^{-(\alpha \pm \nu)/2} \sqrt{\pi} [2 \sqrt{\pi} \Gamma(-\nu)]^{-(1 \pm 1)/2} \Gamma^{\pm 1}\left(\frac{-\nu \mp \alpha}{2} + \frac{1 \mp 1}{4}\right) \Gamma(\alpha) c^{-\alpha} \times \\
\times {}_1F_2\left(\frac{-\alpha \mp \nu}{2} + \frac{1 \mp 1}{4}; 1 - \frac{\alpha}{2}, \frac{1-\alpha}{2}; \mp \frac{c^2 p}{2}\right) \\
\left[|\arg c| < (2 \pm 1) \pi/4, \begin{cases} \operatorname{Re} p > 0, \operatorname{Re}(\alpha + \nu) < 0 \\ \operatorname{Re} p > 0 \end{cases}\right].
\end{aligned}$$

$$\begin{aligned}
4. \int_0^{\infty} x^{\nu-2} e^{-px^2 - c^2 x^2/4} D_{\nu}(cx) dx = 2^{\nu/2-1} \sqrt{\pi} p^{(\nu-1)/2} e^{-c \sqrt{2p}} \\
[\operatorname{Re} p > 0; |\arg c| < \pi/4]
\end{aligned}$$

$$\begin{aligned}
5. \int_0^{\infty} e^{-px - c^2 x^2/4} [D_{\nu}(-cx) - D_{\nu}(cx)] dx = \\
= \frac{\sqrt{2\pi} p^{\nu}}{c^{\nu+1} \Gamma(-\nu/2)} \exp\left(\frac{p^2}{2c^2}\right) \Gamma\left(-\frac{\nu}{2}, \frac{p^2}{2c^2}\right) \\
[\operatorname{Re} p > 0, |\arg c| < \pi/4].
\end{aligned}$$

$$\begin{aligned}
6. \int_a^{\infty} x^{\nu-2\beta} (x-a)^{\beta-1} e^{-(x-a)^2 c^2/4} D_{\nu}(cx) dx = 2^{2\beta-2} a^{\nu-1} c^{\beta-1} \Gamma(\beta) D_{\nu-2\beta}(ac) \\
[a, \operatorname{Re} \beta > 0; |\arg c| < \pi/4]
\end{aligned}$$

$$\begin{aligned}
7. \int_{-\infty}^{\infty} e^{-b(x-y)^2 + c^2 x^2/4} D_{\nu}(cx) dx = \\
= \sqrt{2\pi} (2b)^{-(\nu+1)/2} (2b - c^2)^{\nu/2} \exp\left[\frac{bc^2 y^2}{2(2b - c^2)}\right] D_{\nu}\left(\frac{cy \sqrt{2b}}{\sqrt{2b - c^2}}\right) \\
[b, \operatorname{Re} b > 0; |\arg c| < 3\pi/4].
\end{aligned}$$

2.11.5. Интегралы, содержащие гиперболические функции и $D_{\nu}(\varphi(x))$

$$\begin{aligned}
1. \int_0^{\infty} \operatorname{ch} bx e^{c^2 \operatorname{sh}^2 x} D_{\nu}(2c \operatorname{ch} x) dx = \\
= 2^{-(\nu+5)/2} c^{-1} \mathbb{B}\left(\frac{b-y}{2}, -\frac{b+y}{2}\right) \mathbb{W}_{(\nu+1)/2, b/2}(2c^2) \\
[\operatorname{Re} \nu < -|\operatorname{Re} b|; |\arg c| < 3\pi/4]
\end{aligned}$$

$$\begin{aligned}
2. \int_0^{\infty} \operatorname{ch} bx e^{-c^2 \operatorname{sh}^2 x} D_{\nu}(2c \operatorname{ch} x) dx = 2^{(\nu-3)/2} \sqrt{\pi} c^{-1} \mathbb{W}_{\nu/2, b/2}(2c^2) \\
[|\arg c| < \pi/4].
\end{aligned}$$

2.11.6. Интегралы от $x^\alpha e^{\pm c^2 x^2} \begin{Bmatrix} \sin bx^n \\ \cos bx^n \end{Bmatrix} D_\nu(cx)$.

$$1. \int_0^\infty x^{\alpha-1} e^{\pm c^2 x^2/4} \sin bx D_\nu(cx) dx = U(1)$$

$$\left[b > 0; |\arg c| < (2 \pm 1) \pi/4, \begin{Bmatrix} -1 < \operatorname{Re} \alpha < 1 - \operatorname{Re} \nu \\ \operatorname{Re} \alpha > -1 \end{Bmatrix} \right],$$

$$U(\gamma) = \frac{2^{(\nu-\alpha-\gamma)/2} b^\gamma \sqrt{\pi}}{c^{\alpha+\gamma}} [2^{\nu+1} \sqrt{\pi} \Gamma(-\nu)]^{-(1 \pm 1)/2} \Gamma(\alpha + \gamma) \times \\ \times \Gamma^{\pm 1} \left(\frac{-\nu \mp \alpha \mp \gamma}{2} + \frac{1 \mp 1}{4} \right) {}_2F_2 \left(\frac{\alpha + \gamma}{2}, \frac{\alpha + \gamma + 1}{2}; \frac{\alpha + \gamma \pm \nu}{2} + \right. \\ \left. + \frac{3 \pm 1}{4}, \frac{1}{2} + \gamma; \pm \frac{b^2}{2c^2} \right) + \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{c^\nu \Gamma(\alpha + \nu)}{b^{\alpha+\nu}} \cos \frac{\gamma - \alpha - \nu}{2} \pi \times \\ \times {}_2F_2 \left(-\frac{\nu}{2}, \frac{1-\nu}{2}; 1 - \frac{\alpha + \nu}{2}, \frac{1 - \alpha - \nu}{2}; \frac{b^2}{2c^2} \right).$$

$$2. \int_0^\infty x^{\alpha-1} e^{\pm c^2 x^2/4} \cos bx D_\nu(cx) dx = U(0)$$

$$\left[b > 0; |\arg c| < (2 \pm 1) \pi/4, \begin{Bmatrix} 0 < \operatorname{Re} \alpha < 1 - \operatorname{Re} \nu \\ \operatorname{Re} \alpha > 0 \end{Bmatrix}, U(\gamma) \text{ см. в 2.11.6.1} \right].$$

$$3. \int_0^\infty e^{-c^2 x^2/4} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} D_{2n+(1 \pm 1)/2}(cx) dx =$$

$$= (-1)^n \sqrt{\frac{\pi}{2}} b^{2n+(1 \pm 1)/2} c^{-2n-(3 \pm 1)/2} \exp \left(-\frac{b^2}{2c^2} \right) \\ [b > 0; |\arg c| < \pi/4]$$

$$4. \int_0^\infty x^{\alpha-1} e^{\pm c^2 x^2/4} \sin bx^2 D_\nu(cx) dx = V(1)$$

$$\left[b > 0; |\arg c| < (2 \pm 1) \pi/4, \begin{Bmatrix} -2 < \operatorname{Re} \alpha < 2 - \operatorname{Re} \nu \\ \operatorname{Re} \alpha > -2 \end{Bmatrix} \right],$$

$$V(\gamma) = \frac{2^{\nu/2-1} \sqrt{\pi}}{b^{\alpha/2}} \Gamma \left[\frac{\alpha/2}{(1-\nu)/2} \right] \cos \frac{2\gamma-\alpha}{2} \pi {}_2F_1 \left(\mp \frac{\nu}{2} + \frac{1 \mp 1}{4}, \frac{\alpha}{2}; \frac{1}{2}; \mp \frac{c^2}{2b} \right) - \\ - \frac{2^{(\nu+1)/2} \sqrt{\pi} c}{b^{(\alpha+1)/2}} \Gamma \left[\frac{(\alpha+1)/2}{-\nu/2} \right] \sin \frac{2\gamma-\alpha}{2} \pi {}_2F_1 \left(\mp \frac{\nu}{2} + \frac{3 \mp 1}{4}, \frac{\alpha+1}{2}; \frac{3}{2}; \mp \frac{c^2}{2b} \right).$$

$$5. \int_0^\infty x^{\alpha-1} e^{\pm c^2 x^2/4} \cos bx^2 D_\nu(cx) dx = V(0)$$

$$\left[b > 0; |\arg c| < (2 \pm 1) \pi/4, \begin{Bmatrix} 0 < \operatorname{Re} \alpha < 2 - \operatorname{Re} \nu \\ \operatorname{Re} \alpha > 0 \end{Bmatrix}, V(\gamma) \text{ см. в 2.11.6.4} \right].$$

2.11.7. Интегралы от $A(x) [D_\mu(\varphi(x)) D_\nu(cx)]^{\pm 1}$.

$$1. \int_0^\infty D_\mu(cx) D_\nu(cx) dx =$$

$$= \frac{2^{-(\mu+\nu+3)/2}}{c(\mu-\nu) \Gamma(-\mu) \Gamma(-\nu)} \left[\Gamma \left(-\frac{\mu}{2} \right) \Gamma \left(\frac{1-\nu}{2} \right) - \Gamma \left(-\frac{\nu}{2} \right) \Gamma \left(\frac{1-\mu}{2} \right) \right] \\ [|\arg c| < \pi/4]$$

$$2. \int_0^{\infty} D_{\nu}^2(cx) dx = \frac{\sqrt{\pi} 2^{-3/2}}{c \Gamma(-\nu)} \left[\psi\left(\frac{1-\nu}{2}\right) - \psi\left(-\frac{\nu}{2}\right) \right] \quad [|\arg c| < \pi/4]$$

$$3. \int_0^{\infty} D_{\mu}(-cx) D_{\nu}(cx) dx = \frac{2^{-(\mu+\nu+3)/2}}{c(\mu-\nu) \Gamma(-\mu) \Gamma(-\nu)} \left[\Gamma\left(-\frac{\mu}{2}\right) \Gamma\left(\frac{1-\nu}{2}\right) + \Gamma\left(-\frac{\nu}{2}\right) \Gamma\left(\frac{1-\mu}{2}\right) \right] \\ [\operatorname{Re}(\mu-\nu) > 0; |\arg c| < \pi/4]$$

$$4. \int_0^{\infty} x^{\alpha-1} D_{-\nu-1}(cx) D_{\nu}(cx) dx = 2^{-\alpha-1/2} \pi c^{-\alpha} \Gamma \left[\frac{\alpha}{(2+\alpha-2\nu)/4}, \frac{(\alpha+2\nu)/4+1}{} \right] \\ [\operatorname{Re} \alpha > 0; |\arg c| < \pi/4]$$

$$5. \int_0^{\infty} x^{\alpha-1} D_{\nu}(e^{-\pi i/4} cx) D_{\nu}(e^{\pi i/4} cx) dx = \frac{\sqrt{\pi}}{2^{\alpha+1} c^{\alpha}} \Gamma \left[\begin{matrix} \alpha, & -(\alpha+2\nu)/4 \\ -\nu, & (2+\alpha-2\nu)/4 \end{matrix} \right] \\ [0 < \operatorname{Re} \alpha < -2 \operatorname{Re} \nu; \operatorname{Re} c > 0].$$

$$6. \int_0^{\infty} x^{\alpha-1} [D_{-n}^2(-icx) - D_{-n}^2(icx)] dx = \frac{(-1)^{n-1} 2^{\alpha/2} \pi i}{[(n-1)!]^2 c^{\alpha}} \Gamma \left[\begin{matrix} n + (\alpha-1)/2, & (1-\alpha)/2 \\ 1-\alpha/2 \end{matrix} \right] \\ [n > 0; 1-2n < \operatorname{Re} \alpha < 1, |\arg c| < \pi/4].$$

$$7. \int_{-\infty}^{\infty} D_m(cx) D_n(cx) dx = \frac{n!}{c} \sqrt{2\pi} \delta_{m,n} \quad [|\arg c| < \pi/4]$$

$$8. \int_0^a x^{-\nu} (a^2 - x^2)^{-(\mu+3)/2} D_{\mu}(c \sqrt{a^2 - x^2}) D_{\nu}(cx) dx = - \sqrt{\frac{\pi}{2}} \frac{a^{-\mu-\nu-2}}{\mu+1} D_{\mu+\nu+1}(ac) \\ [a > 0; \operatorname{Re} \mu < -1; \operatorname{Re} \nu < 1].$$

$$9. \int_0^{\infty} \frac{1}{x^2 + z^2} |D_{\nu}(icx)|^{-2} dx = \sqrt{\frac{\pi}{2}} \frac{\Gamma(1-\nu)}{z} D_{\nu-1}(cz) / D_{\nu}(cz) \\ [c, \operatorname{Re} z > 0; \nu < 0]$$

$$10. \int_0^{\infty} \frac{x^2}{x^2 + z^2} |D_{\nu}(icx)|^{-2} dx = \sqrt{\frac{\pi}{2}} \frac{\Gamma(1-\nu)}{c} [1 - cz D_{\nu-1}(cz) / D_{\nu}(cz)] \\ c, \operatorname{Re} z > 0; \nu < 0].$$

2.11.8. Интегралы от $f(x) D_{\mu}(bx) D_{\nu}(cx)$.

$$1. \int_0^{\infty} e^{-px^2} D_n(bx) D_n(cx) dx = \frac{\sqrt{\pi} n!}{\sqrt{b^2 + c^2 + 4p}} \left(\frac{b^2 + c^2 - 4p}{b^2 + c^2 + 4p} \right)^{n/2} P_n \left(\frac{2bc}{\sqrt{(b^2 + c^2)^2 - 16p^2}} \right) \\ [\operatorname{Re}(b^2 + c^2 + 4p) > 0, |\arg b|, |\arg c| < \pi/4]$$

$$2. \int_0^{\infty} x^{\alpha-1} e^{(b^2 \pm c^2) x^2/4} D_{\mu}(bx) D_{\nu}(cx) dx = U(1)$$

$$\left[|\arg b| < 3\pi/4; |\arg c| < (2 \pm 1)\pi/4, \begin{cases} 0 < \operatorname{Re} \alpha < -\operatorname{Re}(\mu + \nu) \\ \operatorname{Re} \alpha > 0 \end{cases} \right],$$

$$U(\gamma) = \frac{2^{(\mu+\nu-\alpha)/2} \pi}{b^{\alpha}} \Gamma\left[\frac{\alpha}{(1-\nu)/2}\right] [2^{\mu+1} \sqrt{\pi} \Gamma(-\mu)]^{-\gamma} \times \\ \times \Gamma^{2\gamma-1}\left(\frac{(-1)^{\gamma} \alpha - \mu + 1 - \gamma}{2}\right) {}_3F_2\left(\frac{\alpha}{2}, \frac{\alpha+1}{2}, \mp \frac{\nu}{2} + \frac{1 \mp 1}{4}; \right. \\ \left. \frac{1+\gamma+\alpha-(-1)^{\gamma} \mu}{2}, \frac{1}{2}; \pm (-1)^{\gamma} \frac{c^2}{b^2}\right) - \frac{2^{(\mu+\nu-\alpha)/2} \pi c}{b^{\alpha+1}} \times \\ \times \Gamma\left[\frac{\alpha+1}{-\nu/2}\right] [2^{\mu+1} \sqrt{\pi} \Gamma(-\mu)]^{-\gamma} \Gamma^{2\gamma-1}\left(\frac{2-3\gamma-\mu+(-1)^{\gamma} \alpha}{2}\right) \times \\ \times {}_3F_2\left(\frac{\alpha+1}{2}, \frac{\alpha}{2}+1, \mp \frac{\nu}{2} + \frac{3 \mp 1}{4}; \frac{2+\gamma+\alpha-(-1)^{\gamma} \mu}{2}, \frac{3}{2}; \pm (-1)^{\gamma} \frac{c^2}{b^2}\right) + \\ + \gamma \frac{2^{(\nu-\mu-\alpha)/2} \sqrt{\pi} \Gamma(\alpha+\mu)}{c^{\alpha+\mu}} [2^{\nu+1} \sqrt{\pi} \Gamma(-\nu)]^{-(1 \pm 1)/2} b^{\mu} \Gamma^{\pm 1}\left(\frac{-\nu \mp \mu \mp \alpha}{2} + \right. \\ \left. + \frac{1 \mp 1}{4}\right) {}_3F_2\left(-\frac{\mu}{2}, \frac{1-\mu}{2}, -\frac{\alpha+\mu \pm \nu}{2} + \frac{1 \mp 1}{4}; 1 - \frac{\alpha+\mu}{2}, \right. \\ \left. \frac{1-\alpha-\mu}{2}; \pm (-1)^{\gamma} \frac{c^2}{b^2}\right).$$

$$3. \int_0^{\infty} x^{\alpha-1} e^{(-b^2 \pm c^2) x^2/4} D_{\mu}(bx) D_{\nu}(cx) dx = U(0)$$

$$[\operatorname{Re} \alpha > 0, |\arg b| < \pi/4, |\arg c| < (2 \pm 1)\pi/4; U(\gamma) \text{ см. в 2.11.8.2}]$$

$$4. \int_0^{\infty} x e^{-\rho x^2} [D_{-n}^2(-icx) - D_{-n}^2(icx)] dx = \frac{(-1)^{n-1}}{(n-1)!} \sqrt{\frac{2}{\rho}} \frac{\pi i c^{2n-1}}{(2\rho + c^2)^n}$$

$$[n, \operatorname{Re} \rho > 0; |\arg c| < \pi/4].$$

$$5. \int_0^{\infty} \sin bx [D_{-n}^2(-icx) - D_{-n}^2(icx)] dx =$$

$$= \frac{(-1)^{n-1} \pi i \sqrt{2}}{(n-1)! c} \exp\left(-\frac{b^2}{2c^2}\right) L_{n-1}\left(\frac{b^2}{2c^2}\right) \quad [b, n > 0; |\arg c| < \pi/4].$$

$$6. \int_0^{\infty} x \begin{Bmatrix} \sin bx^2 \\ \cos bx^2 \end{Bmatrix} [D_{-\nu-1}(-cx) \mp D_{-\nu-1}(cx)] D_{\nu}(cx) dx =$$

$$= -2^{-\nu-(1 \mp 1)/4} \sqrt{\pi} b^{-\nu-1} c \begin{Bmatrix} \cos(\nu\pi/2) \\ \sin(\nu\pi/2) \end{Bmatrix} \frac{(c^2 + \sqrt{c^4 + 4b^2})^{\nu+1/2}}{\sqrt{c^4 + 4b^2}} \\ [b, c > 0]$$

2.11.9. Интегралы, содержащие $\begin{Bmatrix} \operatorname{erf}(bx) \\ \operatorname{erfc}(bx) \end{Bmatrix}$ и $D_{\nu}(cx)$.

$$1. \int_0^{\infty} x^{\alpha-1} e^{\pm c^2 x^2/4} \operatorname{erf}(bx) D_{\nu}(cx) dx = U(1)$$

$$\left[\operatorname{Re} b > 0, \arg c < (2 \pm 1)\pi/4, \begin{cases} -1 < \operatorname{Re} \alpha < -\operatorname{Re} \nu \\ \operatorname{Re} \alpha > -1 \end{cases} \right],$$

$$U(\gamma) = (-1)^{\gamma-1} \frac{2^{(\gamma-\alpha+1)/2} b}{c^{\alpha+1}} [2^{\gamma+1} \sqrt{\pi} \Gamma(-\gamma)]^{-(1 \pm 1)/2} \Gamma(\alpha+1) \times \\ \times \Gamma^{\pm 1} \left(\frac{-\gamma \mp \alpha}{2} + \frac{1 \mp 3}{4} \right) {}_3F_2 \left(\frac{1}{2}, \frac{\alpha+1}{2}, \frac{\alpha}{2} + 1; \frac{3}{2}, \frac{\alpha \pm \gamma}{2} + \frac{5 \pm 1}{4}; \pm \frac{2b^2}{c^2} \right) + \\ + \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{(-1)^{\gamma} c^{\gamma}}{\sqrt{\pi} (\alpha+\gamma) b^{\alpha+\gamma}} \Gamma \left(\frac{\alpha+\gamma+1}{2} \right) {}_3F_2 \left(-\frac{\gamma}{2}, \frac{1-\gamma}{2}, -\frac{\alpha+\gamma}{2}; 1 - \frac{\alpha+\gamma}{2}, \right. \\ \left. \frac{1-\alpha-\gamma}{2}; \frac{2b^2}{c^2} \right) + (1-\gamma) \frac{2^{(\gamma-\alpha)/2} \sqrt{\pi}}{c^{\alpha}} \Gamma(\alpha) \Gamma^{\pm 1} \left(\frac{-\gamma \mp \alpha}{2} + \frac{1 \mp 1}{4} \right).$$

$$2. \int_0^{\infty} x^{\alpha-1} e^{\pm c^2 x^2/4} \operatorname{erfc}(bx) D_{\nu}(cx) dx = U(0)$$

$$[\operatorname{Re} \alpha > 0; |\arg b| < \pi/4; |\arg c| < (2 \pm 1) \pi/4, U(\gamma) \text{ см. в 2.11.9.1}].$$

$$3. \int_0^{\infty} e^{-c^2 x^2/4} \operatorname{erf}(bx) D_{2n+1}(cx) dx = (-1)^n \frac{(2n-1)!!}{c} \left(\frac{2b^2}{2b^2 + c^2} \right)^{n+1/2}$$

$$[\operatorname{Re} b > 0; |\arg c| < \pi/4].$$

$$4. \int_0^{\infty} e^{c^2 x^2/4} \operatorname{erfc} \left(\frac{cx}{\sqrt{2}} \right) D_{\nu}(\pm cx) dx = \frac{2^{(\nu+1)/2}}{(\nu+1)c} \left[\frac{1}{\Gamma(1/2 - \nu/2)} \mp \frac{\sqrt{\pi}}{\Gamma(-\nu/2)} \right]$$

$$[c > 0, \{ \operatorname{Re} \nu \neq -1 \}, \{ \operatorname{Re} \nu > -1 \}].$$

$$5. \int_0^{\infty} x^{\alpha-1} e^{-c^2 x^2/4} \operatorname{erfc}(bx) [D_{\nu}(-cx) - D_{\nu}(cx)] dx =$$

$$= - \frac{2^{(\nu+3)/2} c \sin(\nu\pi/2)}{\pi (\alpha+1) b^{\alpha+1}} \Gamma \left(\frac{\alpha}{2} + 1 \right) \Gamma \left(\frac{\nu}{2} + 1 \right) \times \\ \times {}_3F_2 \left(\frac{\alpha+1}{2}, \frac{\alpha}{2} + 1, \frac{\nu}{2} + 1; \frac{3}{2}, \frac{\alpha+3}{2}; -\frac{c^2}{2b^2} \right) \\ [\operatorname{Re} \alpha, \operatorname{Re}(2b^2 + c^2) > 0, |\arg b| < \pi/4]$$

$$6. \int_0^{\infty} x e^{b^2 x^2 - c^2 x^2/4} \operatorname{erfc}(bx) [D_{\nu}(-cx) - D_{\nu}(cx)] dx =$$

$$= - \frac{2^{(\nu+1)/2} c}{(2+\nu) \Gamma(1-\nu/2) b^3} {}_2F_1 \left(2, \frac{\nu}{2} + 1; \frac{\nu}{2} + 2; 1 - \frac{c^2}{2b^2} \right) \\ [|\arg b|, |\arg c| < \pi/4].$$

$$7. \int_0^{\infty} e^{-c^2 x^2/4} \operatorname{erf}(bx) [D_{\nu}(cx) - D_{\nu}(-cx)] dx = - \frac{2^{\nu+1/2} \sqrt{\pi} b^{\nu} (2b^2 + c^2)^{-\nu/2}}{c \Gamma(\nu/2 + 1)}$$

$$[|\arg b|, |\arg c| < \pi/4].$$

$$8. \int_0^{\infty} e^{-c^2 x^2/2} \operatorname{erf} \left(\frac{icx}{\sqrt{2}} \right) D_{\nu-1}(cx) D_{\nu}(cx) dx = i \frac{\psi(1-\nu/2) - \psi(1/2 - \nu/2)}{4c \Gamma(1-\nu)}$$

$$[c > 0; \operatorname{Re} \nu < 0].$$

$$9. \int_0^{\infty} e^{-b^2 x^2} \operatorname{erf}(ibx) [D_{-n}^2(-icx) - D_{-n}^2(icx)] dx =$$

$$= \frac{(-1)^n}{(n-1)!} \frac{(2\pi)^{3/2}}{\sqrt{2b^2 + c^2}} {}_2F_1 \left(1-n, \frac{1}{2}; 1; \frac{2b^2}{2b^2 + c^2} \right) \\ [|\arg b|, |\arg c| < \pi/4, n = 1, 2, 3, \dots]$$

2.11.10. Интегралы по индексу, содержащие $D_x(c)$.

$$1. \int_0^{\infty} \cos bx D_{x-1/2}(c) D_{-x-1/2}(c) dx = \frac{1}{2} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \sqrt{\frac{\pi}{\cos b}} \exp\left(-\frac{c^2 \cos b}{2}\right) \left[\begin{Bmatrix} 0 < b < \pi/2 \\ b > \pi/2 \end{Bmatrix} \right].$$

$$2. \int_{\gamma-i\infty}^{\gamma+i\infty} a^x \Gamma(-x) D_x(c) dx = 2\pi i \exp\left(-\frac{a^2}{2} - \frac{c^2}{4} - ac\right) [\gamma < 0; |\arg a| < \pi/4].$$

$$3. \int_{-1/2-i\infty}^{-1/2+i\infty} \frac{a^x}{\sin \pi x} D_{-x-1}(b) D_x(c) dx = -\frac{2i}{\sqrt{a^2+1}} D_0\left(\frac{ab-c}{\sqrt{a^2+1}}\right) D_{-1}\left(\frac{ac+b}{\sqrt{a^2+1}}\right).$$

$$4. \int_{\gamma-i\infty}^{\gamma+i\infty} \Gamma\left(-\frac{x}{2}\right) \Gamma\left(\frac{x+1}{2}\right) D_{-x-1}(b) D_x(c) dx = 2\sqrt{2}\pi i K_0\left(\frac{b^2+c^2}{4i}\right) [-2 < \gamma < 0].$$

$$5. \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{a^x}{\sin \pi x} [D_{-x-1}(-b) D_x(-ic) + D_{-x-1}(b) D_x(ic)] dx = -\frac{2i\sqrt{2\pi}}{\sqrt{a^2+1}} \exp\left[-\frac{(a^2-1)(b^2-c^2)-4iabc}{4(a^2+1)}\right] [-1 < \gamma < 0; |\arg a| < \pi/2]$$

2.12. ФУНКЦИЯ БЕССЕЛЯ $J_\nu(x)$

При $\nu = \pm \frac{1}{2}, \pm \frac{3}{2}, \dots$ функция $J_\nu(x)$ сводится к элементарным, в частности,

$$J_{\pm 1/2}(cx) = \sqrt{\frac{2}{\pi cx}} \begin{Bmatrix} \sin cx \\ \cos cx \end{Bmatrix};$$

соответствующие интегралы см в [12]

2.12.1. Интегралы общего вида.

Обозначение. $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$.

$$1. \int_0^a x^{\alpha-1} (a^r - x^r)^{\beta-1} J_\nu(cx) dx = U(0) \quad [a, r, \operatorname{Re} \beta, \operatorname{Re}(\alpha + \nu) > 0].$$

$$U(e) = \frac{a^{\alpha+\nu+r\beta-r} c^\nu}{2^\nu r} \Gamma\left[\begin{matrix} \beta \\ \nu+1 \end{matrix}\right] \sum_{k=0}^{\infty} \frac{1}{k! (\nu+1)_k (2k+\nu)^e} \times \times \Gamma\left[\frac{(2k+\alpha+\nu)/r}{(2k+\alpha+\nu)/r+\beta}\right] \left(-\frac{a^2 c^2}{4}\right)^k.$$

$$2. \int_a^\infty x^{\alpha-1} (x^r - a^r)^{\beta-1} J_\nu(cx) dx = V(0)$$

$$[a, c, r, \operatorname{Re} \beta > 0; \operatorname{Re}(\alpha + r\beta) < r + 3/2].$$

$$V(\varepsilon) = \frac{a^{\alpha+\nu+r\beta-r} c^\nu}{2^\nu r} \Gamma\left[\frac{\beta}{\nu+1}\right] \sum_{k=0}^{\infty} \frac{1}{k! (\nu+1)_k (2k+\nu)^{\varepsilon}} \times \\ \times \Gamma\left[\frac{1-\beta-(2k+\alpha+\nu)/r}{1-(2k+\alpha+\nu)/r}\right] \left(-\frac{a^2 c^2}{4}\right)^k + \frac{1}{2} \left(\frac{c}{2}\right)^{r-\alpha-r\beta} \times \\ \times \sum_{k=0}^{\infty} \frac{(1-\beta)_k}{k! (r-r\beta+r\beta-\alpha)^{\varepsilon}} \Gamma\left[\frac{(\nu+\alpha-rk+r\beta-r)/2}{(\nu-\alpha+rk-r\beta+r)/2+1}\right] \left(\frac{ac}{2}\right)^{rk}.$$

$$3. \int_0^{\infty} \frac{x^{\alpha-1}}{(x^r+z^r)^\rho} J_\nu(cx) dx = W(0)$$

$$[c, r > 0; -\operatorname{Re} \nu < \operatorname{Re} \alpha < r \operatorname{Re} \rho + 3/2; r |\arg z| < \pi],$$

$$W(\varepsilon) = \frac{c^\nu z^{\alpha+\nu-r\rho}}{2^\nu r \Gamma(\rho) \Gamma(\nu+1)} \sum_{k=0}^{\infty} \frac{1}{k! (1+\nu)_k (2k+\nu)^{\varepsilon}} \Gamma\left(\frac{2k+\alpha+\nu}{r}\right) \times \\ \times \Gamma\left(\rho - \frac{2k+\alpha+\nu}{r}\right) \left(-\frac{c^2 z^2}{4}\right)^k + \frac{1}{2} \left(\frac{c}{2}\right)^{r\rho-\alpha} \times \\ \times \sum_{k=0}^{\infty} \frac{(-1)^k (\rho)_k}{k! (r\rho+rk-\alpha)^{\varepsilon}} \Gamma\left[\frac{(\nu+\alpha-r\rho-rk)/2}{(\nu-\alpha+r\rho+rk)/2+1}\right] \left(\frac{cz}{2}\right)^{rk}.$$

$$4. \int_0^{\infty} \frac{x^{\alpha-1}}{x^r-y^r} J_\nu(cx) dx = X(0)$$

$$[c, r, y > 0; -\operatorname{Re} \nu < \operatorname{Re} \alpha < r + 3/2].$$

$$X(\varepsilon) = -\frac{\pi c^\nu y^{\alpha+\nu-r}}{2^\nu r \Gamma(\nu+1)} \sum_{k=0}^{\infty} \frac{1}{k! (\nu+1)_k (2k+\nu)^{\varepsilon}} \operatorname{ctg} \frac{2k+\alpha+\nu}{r} \pi \left(-\frac{c^2 y^2}{4}\right)^k + \\ + \frac{1}{2} \left(\frac{2}{c}\right)^{\alpha-r} \sum_{k=0}^{\infty} \frac{1}{(r+rk-\alpha)^{\varepsilon}} \Gamma\left[\frac{(\nu+\alpha-r-rk)/2}{(\nu-\alpha+r+rk)/2+1}\right] \left(\frac{cy}{2}\right)^{rk}.$$

$$5. \int_0^{\infty} x^{\alpha-1} e^{-px^r} J_\nu(cx) dx = Y(0)$$

$$[r, \operatorname{Re} p, \operatorname{Re}(\alpha+\nu) > 0; c > 0 \\ \text{при } r \leq 1, |\arg c| < \pi \text{ при } r > 1],$$

$$Y(\varepsilon) = \frac{c^\nu p^{-(\alpha+\nu)/r}}{2^\nu r \Gamma(\nu+1)} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (\nu+1)_k (2k+\nu)^{\varepsilon}} \Gamma\left(\frac{2k+\alpha+\nu}{r}\right) \left(\frac{c}{2p^{1/r}}\right)^{2k}$$

$$[r > 1],$$

$$Y(\varepsilon) = \frac{2^{\alpha-1}}{c^\alpha} \sum_{k=0}^{\infty} \frac{1}{k! (rk+\alpha)^{\varepsilon}} \Gamma\left[\frac{(\nu+\alpha+rk)/2}{(\nu-\alpha-rk)/2+1}\right] \left(-\frac{2rp}{c^r}\right)^k$$

$$[r < 1],$$

$Y(0)$ при $r=1$ см. в 2.12.8.4.

$$6. \int_0^{\infty} x^{\alpha-1} e^{-px^{-r}} J_\nu(cx) dx = Z(0)$$

$$[c, r, \operatorname{Re} p > 0; \operatorname{Re} \alpha < 3/2].$$

$$Z(\varepsilon) = \frac{c^\nu p^{(\alpha+\nu)/r}}{2^\nu r \Gamma(\nu+1)} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (\nu+1)_k (2k+\nu)^{\varepsilon}} \Gamma\left(-\frac{2k+\alpha+\nu}{r}\right) \left(\frac{cp^{1/r}}{2}\right)^{2k} + \\ + \frac{2^{\alpha-1}}{c^\alpha} \sum_{k=0}^{\infty} \frac{1}{k! (rk-\alpha)^{\varepsilon}} \Gamma\left[\frac{(\nu+\alpha-rk)/2}{(\nu-\alpha+rk)/2+1}\right] \left(-\frac{c^r p}{2r}\right)^k.$$

$$7. \int_0^{\infty} x^{\alpha-1} \left\{ \frac{\sin bx^r}{\cos bx^r} \right\} J_{\nu}(cx) dx = P(0)$$

$[b, c, r > 0; -r\delta - \operatorname{Re} \nu < \operatorname{Re} \alpha < 1/2 + \max(r, 1);$
 $-r\delta - \operatorname{Re} \nu < \operatorname{Re} \alpha < 1/2$ при $r = 1, b = c]$,

$$P(s) = \frac{b^{-(\alpha+\nu)/r} c^{\nu}}{2^{\nu} r \Gamma(\nu+1)} \sum_{k=0}^{\infty} \frac{1}{k! (\nu+1)_k (2k+\nu)^s} \Gamma\left(\frac{2k+\alpha+\nu}{r}\right) \times$$

$$\times \left\{ \frac{\sin [(2k+\alpha+\nu) \pi/(2r)]}{\cos [(2k+\alpha+\nu) \pi/(2r)]} \right\} \left(-\frac{c^2}{4b^{2/r}}\right)^k \quad [r > 1].$$

$$P(s) = \frac{2^{\alpha+r\delta-1} b^{\delta}}{c^{\alpha+r\delta}} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (\delta+1/2)_k (\delta r + \alpha + 2rk)^s} \times$$

$$\times \Gamma\left[\frac{(\nu+\alpha+r\delta)/2 + rk}{(\nu-\alpha-r\delta)/2 + 1 - rk}\right] \left(\frac{2^{r-1} b}{c^r}\right)^{2k} \quad [r < 1].$$

$P(0)$ при $r=1$ см. в 2.12.15.3—4.

$$8. \int_0^{\infty} x^{\alpha-1} e^{-px^r} \left\{ \frac{\sin bx^r}{\cos bx^r} \right\} J_{\nu}(cx) dx = Q(0)$$

$[b, r, \operatorname{Re} p > 0; \operatorname{Re}(\alpha+\nu) > -\delta r; c > 0$
 при $r \leq 1, |\arg c| < \pi$ при $r > 1]$,

$$Q(s) = \frac{2^{-\nu} c^{\nu}}{r (\sqrt{b^2 + p^2})^{(\alpha+\nu)/r} \Gamma(\nu+1)} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (\nu+1)_k (2k+\nu)^s} \times$$

$$\times \Gamma\left(\frac{2k+\alpha+\nu}{r}\right) \cos\left(\frac{\delta\pi}{2} - \frac{2k+\alpha+\nu}{r} \arccos \frac{p}{\sqrt{b^2 + p^2}}\right) \frac{c^{2k}}{2^{2k} (b^2 + p^2)^{k/r}}$$

$[r > 1]$ или $[r=1; |c|^2 < |b^2 + p^2|]$,

$$Q(s) = \frac{2^{\alpha-1}}{c^{\alpha}} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (rk+\alpha)^s} \Gamma\left[\frac{(\nu+\alpha+rk)/2}{(\nu-\alpha-rk)/2 + 1}\right] \times$$

$$\times \cos\left(\frac{\delta\pi}{2} + k \arccos \frac{p}{\sqrt{b^2 + p^2}}\right) \left(\frac{2^r \sqrt{b^2 + p^2}}{c^r}\right)^k$$

$[r < 1]$ или $[r=1; |c|^2 > |b^2 + p^2|]$.

$$9. \int_0^{\infty} x^{\alpha-1} \left\{ \frac{\ln(x^r + z^r)}{\ln|x^r - z^r|} \right\} J_{\nu}(cx) dx = R(0)$$

$[c, r, \operatorname{Re}(\alpha+\nu) > 0; \operatorname{Re} \alpha < 3/2, \left\{ \begin{array}{l} r |\arg z| < \pi \\ z > 0 \end{array} \right\}]$,

$$R(s) = (-1)^s \frac{\pi c^{\nu} z^{\alpha+\nu}}{2^{\nu} \Gamma(\nu+1)} \sum_{k=0}^{\infty} \frac{(-c^2 z^2/4)^k}{k! (\nu+1)_k (2k+\nu)^s (2k+\alpha+\nu)} \times$$

$$\times \left\{ \operatorname{cosec} [(2k+\alpha+\nu) \pi/r] \right\} + \frac{2^{\alpha-2r}}{\alpha^s c^{\alpha}} \Gamma\left[\frac{(\nu+\alpha)/2}{(\nu-\alpha)/2 + 1}\right] \times$$

$$\times \left[\psi\left(\frac{\nu+\alpha}{2}\right) + \psi\left(\frac{\nu-\alpha}{2} + 1\right) - \frac{2s}{\alpha} - 2 \ln \frac{c}{2} \right] -$$

$$- \frac{2^{\alpha-1}}{c^{\alpha}} \sum_{k=1}^{\infty} \frac{(-1)^k}{k (\alpha - rk)^s} \Gamma\left[\frac{(\nu+\alpha-rk)/2}{1 + (\nu-\alpha+rk)/2}\right] \left(\frac{cz}{2}\right)^{rk}$$

$$10. \int_0^{\infty} x^{\alpha-1} J_{\mu}(bx^r) J_{\nu}(cx) dx = S(0)$$

$$[b, c, r, \operatorname{Re}(\alpha + \nu + r\mu) > 0; \operatorname{Re} \alpha < (3r + 1)/2 \text{ при } r > 1,$$

$$\operatorname{Re} \alpha < (3 + r)/2 \text{ при } r < 1; \operatorname{Re} \alpha < 2 \text{ при } r = 1, b \neq c,$$

$$\operatorname{Re} \alpha < 1 \text{ при } r = 1, b = c; \text{ при } r < 1 - \text{замена } x = t^{1/r},$$

$$S(e) = \frac{2^{(\alpha + \nu)/r - \nu - 1} c^{\nu}}{b^{(\alpha + \nu)/r} r \Gamma(\nu + 1)} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (\nu + 1)_k (\nu + 2k)^e} \times \\ \times \Gamma \left[\frac{(\mu r + \alpha + \nu + 2k)/(2r)}{1 + (\mu r - \alpha - \nu - 2k)/(2r)} \right] \left(\frac{2^{1/r-1} c}{b^{1/r}} \right)^{2k} \quad [r > 1],$$

$$S(0) \text{ при } r=1 \text{ см. в 2.12.31.1.}$$

$$11. \int_0^{\infty} x^{\alpha-1} J_{\mu} \left(\frac{b}{x^r} \right) J_{\nu}(cx) dx = T(0)$$

$$[b, c, r > 0; -3r/2 - \operatorname{Re} \nu < \operatorname{Re} \alpha < 3/2 + r \operatorname{Re} \mu],$$

$$T(e) = \frac{b^{(\alpha + \nu)/r} c^{\nu}}{2^{(\alpha + \nu)/r + \nu + 1} r \Gamma(\nu + 1)} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (\nu + 1)_k (2k + \nu)^e} \times \\ \times \Gamma \left[\frac{(\mu r - \alpha - \nu - 2k)/(2r)}{1 + (\mu r + \alpha + \nu + 2k)/(2r)} \right] \left(\frac{b^{1/r} c}{2^{1/r+1}} \right)^{2k} + \frac{2^{\alpha - \mu - \mu r - 1} b^{\mu}}{c^{\alpha - \mu} r \Gamma(\mu + 1)} \times \\ \times \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (\mu + 1)_k (2kr + \mu r - \alpha)^e} \Gamma \left[\frac{(\nu + \alpha - \mu r)/2 - rk}{1 + (\nu - \alpha + \mu r)/2 + rk} \right] \left(\frac{bc^r}{2^{r+1}} \right)^{2k}.$$

$$12. \int_0^{\infty} x^{\alpha-1} \operatorname{Ei}(-bx^r) J_{\nu}(cx) dx = X(0) \quad [c, r, \operatorname{Re} b, \operatorname{Re}(\alpha + \nu) > 0],$$

$$X(e) = -\frac{c^{\nu}}{2^{\nu} b^{(\alpha + \nu)/r} r \Gamma(\nu + 1)} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (\nu + 1)_k (2k + \nu)^e (2k + \alpha + \nu)} \times \\ \times \Gamma \left(\frac{\alpha + \nu + 2k}{r} \right) \left(\frac{c}{2b^{1/r}} \right)^{2k} \quad [r > 1],$$

$$X(e) = \frac{2^{\alpha-1}}{c^{\alpha}} \sum_{k=1}^{\infty} \frac{1}{k! k (\alpha + rk)^e} \Gamma \left[\frac{(\nu + \alpha + rk)/2}{(\nu - \alpha - rk)/2 + 1} \right] \times \\ \times \left(-\frac{2^r b}{c^r} \right)^k + \frac{2^{\alpha-2}}{\alpha^e c^{\alpha}} \Gamma \left[\frac{(\nu + \alpha)/2}{(\nu - \alpha)/2 + 1} \right] \left[2C - \frac{2er}{\alpha} + \right. \\ \left. + r\psi \left(\frac{\alpha + \nu}{2} \right) + r\psi \left(\frac{\nu - \alpha}{2} + 1 \right) + 2 \ln \frac{2^r b}{c^r} \right] \quad [r < 1],$$

$$X(0) \text{ при } r=1 \text{ см в 2.12.47.2.}$$

$$13. \int_0^{\infty} x^{\alpha-1} e^{\pm bx^r} \operatorname{Ei}(\mp bx^r) J_{\nu}(cx) dx = Y(0)$$

$$[c, r, \operatorname{Re} b > 0; -\operatorname{Re} \nu < \operatorname{Re} \alpha < r + 3/2],$$

$$Y(e) = -\frac{\pi (c/2)^v}{b^{(\alpha+v)/r} \Gamma(v+1)} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (v+1)_k (2k+v)^s} \times \\ \times \Gamma\left(\frac{2k+\alpha+v}{r}\right) \left\{ \begin{array}{l} \operatorname{cosec} [(2k+\alpha+v) \pi/r] \\ \operatorname{ctg} [(2k+\alpha+v) \pi/r] \end{array} \right\} \left(\frac{c}{2b^{1/r}}\right)^{2k} \mp \\ \mp \left(\frac{c}{2}\right)^{r-\alpha} \frac{1}{2b} \sum_{k=0}^{\infty} \frac{k!}{(rk+r-\alpha)^s} \Gamma\left[\frac{(v+\alpha-r-rk)/2}{(v-\alpha+r+rk)/2+1}\right] \left(\mp \frac{c^r}{2rb}\right)^k \\ [r > 1],$$

$$Y(e) = -\frac{2^{\alpha-2}}{c^\alpha} \sum_{k=0}^{\infty} \frac{1}{k! (\alpha+rk)^s} \Gamma\left[\frac{(v+\alpha+rk)/2}{(v-\alpha-rk)/2+1}\right] \times \\ \times \left[2\psi(k+1) - r\psi\left(\frac{v+\alpha+rk}{2}\right) - r\psi\left(\frac{v-\alpha-rk}{2}+1\right) + \right. \\ \left. + \frac{2er}{\alpha+rk} - 2 \ln \frac{2rb}{c^r} \right] \left(\pm \frac{2rb}{c^r}\right)^k \quad [r < 1].$$

$Y(0)$ при $r=1$ см. в 2.12.47.11.

$$14. \int_0^\infty x^{\alpha-1} \left\{ \begin{array}{l} \operatorname{si}(bx^r) \\ \operatorname{ci}(bx^r) \end{array} \right\} J_\nu(cx) dx = Z(0)$$

$[b, c, r, \operatorname{Re}(\alpha+v) > 0; \operatorname{Re} \alpha < r + 1/2 + \max(r, 1)]$.

$$Z(e) = -\frac{b^{-(\alpha+v)/r}}{\Gamma(v+1)} \left(\frac{c}{2}\right)^v \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (v+1)_k (2k+v)^s (2k+\alpha+v)} \times \\ \times \Gamma\left(\frac{2k+\alpha+v}{r}\right) \left\{ \begin{array}{l} \sin [(2k+\alpha+v) \pi/(2r)] \\ \cos [(2k+\alpha+v) \pi/(2r)] \end{array} \right\} \left(\frac{c}{2b^{1/r}}\right)^{2k} \quad [r > 1],$$

$$Z(e) = \frac{2^{\alpha+\delta r-1} b^\delta}{c^{\alpha+\delta r}} \sum_{k=1}^{\infty} \frac{(-1)^k}{k! (2k+\delta) (\delta+1/2)_k (\delta r+2rk+\alpha)^s} \times \\ \times \Gamma\left[\frac{(v+\alpha+\delta r)/2+kr}{1+(v-\alpha-\delta r)/2-kr}\right] \left(\frac{b}{2^{1-r} c^r}\right)^{2k} + \\ + \frac{2^{\alpha+\delta r-1} b^\delta}{(\delta r+\alpha)^s c^{\alpha+\delta r}} \Gamma\left[\frac{(v+\alpha+\delta r)/2}{(v-\alpha-\delta r)/2+1}\right] \left\{ \begin{array}{l} 1 \\ R \end{array} \right\} - \left\{ \begin{array}{l} 1 \\ 0 \end{array} \right\} \frac{2^{\alpha-2} \pi}{\alpha^s c^\alpha} \Gamma\left[\frac{(v+\alpha)/2}{(v-\alpha)/2+1}\right] \\ \left[r < 1; R = c + \frac{r}{2} \psi\left(\frac{v+\alpha}{2}\right) + \frac{r}{2} \psi\left(\frac{v-\alpha}{2}+1\right) - \frac{r^2}{2} - \ln \frac{c^r}{2^r b} \right],$$

$Z(0)$ при $r=1$ см. в 2.12.48.2.

$$15. \int_0^\infty x^{\alpha-1} \left\{ \begin{array}{l} \operatorname{erf}(bx^r) \\ \operatorname{erfc}(bx^r) \end{array} \right\} J_\nu(cx) dx = U(0)$$

$[c, r > 0; |\arg b| < \pi/4, \left\{ \begin{array}{l} -r - \operatorname{Re} v < \operatorname{Re} \alpha < 3/2 \\ \operatorname{Re}(\alpha+v) > 0 \end{array} \right\}]$,

$$U(e) = \mp \frac{b^{-(\alpha+v)/r}}{\sqrt{\pi} \Gamma(v+1)} \left(\frac{c}{2}\right)^v \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (v+1)_k (2k+v)^s (2k+\alpha+v)} \times \\ \times \Gamma\left(\frac{2k+\alpha+v+r}{2r}\right) \left(\frac{c}{2b^{1/r}}\right)^{2k} + \left\{ \begin{array}{l} 1 \\ 0 \end{array} \right\} \frac{2^{\alpha-1}}{(-\alpha)^s c^\alpha} \Gamma\left[\frac{(v+\alpha)/2}{(v-\alpha)/2+1}\right] \\ [r > 1/2].$$

$$U(\varepsilon) = \pm \frac{b}{\sqrt{\pi}} \left(\frac{2}{c}\right)^{\alpha+r} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (2k+1) (r+\alpha+2rk)^{\varepsilon}} \times \\ \times \Gamma \left[\frac{(\nu+\alpha+r)/2 + kr}{(\nu-\alpha-r)/2 + 1 - rk} \right] \left(\frac{2rb}{c^r}\right)^{2k} + \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{2^{\alpha-1}}{\alpha^{\varepsilon} c^{\alpha}} \Gamma \left[\frac{(\nu+\alpha)/2}{(\nu-\alpha)/2 + 1} \right] \\ [r < 1/2].$$

$U(0)$ при $r=1/2$ см. в 2.12.49.8.

$$16. \int_0^{\infty} x^{\alpha-1} e^{-\frac{1}{2} b^2 x^{2r}} \begin{cases} \operatorname{erf}(ibx^r) \\ \operatorname{erfc}(bx^r) \end{cases} J_{\nu}(cx) dx = V(0)$$

$[c, r > 0; |\arg b| < \pi/4; -\operatorname{Re} \nu - (1 \pm 1)r/2 < \operatorname{Re} \alpha < r + 3/2].$

$$V(\varepsilon) = \left\{ \begin{matrix} i \\ 1 \end{matrix} \right\} \frac{c^{\nu}}{2^{\nu+1} b^{(\alpha+\nu)/r} r \Gamma(\nu+1)} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (\nu+1)_k (2k+\nu)^{\varepsilon}} \times \\ \times \Gamma \left(\frac{2k+\alpha+\nu}{2r} \right) \left\{ \begin{matrix} \operatorname{tg} [(2k+\alpha+\nu) \pi/(2r)] \\ \sec [(2k+\alpha+\nu) \pi/(2r)] \end{matrix} \right\} \left(\frac{c}{2b^{1/r}} \right)^{2k} + \\ + \left\{ \begin{matrix} i \\ 1 \end{matrix} \right\} \frac{2^{\alpha-r-1} c^{r-\alpha}}{\sqrt{\pi} b} \sum_{k=0}^{\infty} \frac{(\pm 1)^k (1/2)_k}{(2rk+r-\alpha)^{\varepsilon}} \Gamma \left[\frac{(\nu+\alpha-r)/2 - kr}{(\nu-\alpha+r)/2 + 1 + kr} \right] \left(\frac{c^r}{2rb} \right)^{2k} \\ [r \geq 1/2].$$

$$V(\varepsilon) = \left\{ \begin{matrix} i \\ -1 \end{matrix} \right\} \left(\frac{2}{c}\right)^{\alpha+r} \frac{b}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k}{(\alpha+r+2rk)^{\varepsilon} (3/2)_k} \times \\ \times \Gamma \left[\frac{(\nu+\alpha+r)/2 + rk}{(\nu-\alpha-r)/2 - rk + 1} \right] \left(\frac{2rb}{c^r}\right)^{2k} + \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{2^{\alpha-1}}{c^{\alpha}} \times \\ \times \sum_{k=0}^{\infty} \frac{1}{k! (\alpha+2rk)^{\varepsilon}} \Gamma \left[\frac{(\nu+\alpha)/2 + rk}{(\nu-\alpha)/2 + 1 - rk} \right] \left(\frac{2rb}{c^r}\right)^{2k} \\ [r < 1/2].$$

$V(0)$ при $r=1/2$ см. в 2.12.49.18.

$$17. \int_0^{\infty} x^{\alpha-1} \begin{cases} S(bx^r) \\ C(bx^r) \end{cases} J_{\nu}(cx) dx = W(0)$$

$[b, c, r > 0; -(2 \pm 1)r/2 - \operatorname{Re} \nu < \operatorname{Re} \alpha < 3/2].$

$$W(\varepsilon) = - \frac{c^{\nu}}{2^{\nu+1/2} \sqrt{\pi} b^{(\alpha+\nu)/r} r \Gamma(\nu+1)} \times \\ \times \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (\nu+1)_k (2k+\nu)^{\varepsilon} (2k+\alpha+\nu)} \Gamma \left(\frac{1}{2} + \frac{2k+\alpha+\nu}{r} \right) \times \\ \times \left\{ \begin{matrix} \cos [(r/2 - \alpha - \nu - 2k) \pi/(2r)] \\ \sin [(r/2 - \alpha - \nu - 2k) \pi/(2r)] \end{matrix} \right\} \left(\frac{c}{2b^{1/r}} \right)^{2k} + \frac{2^{\alpha-2}}{(-\alpha)^{\varepsilon} c^{\alpha}} \Gamma \left[\frac{(\nu+\alpha)/2}{(\nu-\alpha)/2 + 1} \right] \\ [r > 1].$$

$$W(\varepsilon) = \frac{2^{\alpha+\delta r + (r-1)/2} b^{\delta+1/2}}{\sqrt{\pi} c^{\alpha+\delta r + r/2}} \times \\ \times \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (\delta+1/2)_k (4k+2\delta+1) (\alpha+r\delta+r/2+2rk)^{\varepsilon}} \times \\ \times \Gamma \left[\frac{(\nu+\alpha+\delta r+1/2)/2 + rk}{(\nu-\alpha-\delta r+3/2)/2 - rk} \right] \left(\frac{b}{2^{1-r} c^r} \right)^{2k} \\ [r < 1].$$

$W(0)$ при $r=1$ см. в 2.12.50.1.

$$18. \int_0^{\infty} x^{\alpha-1} \left\{ \frac{\gamma(\mu, bx')}{\Gamma(\mu, bx')} \right\} J_\nu(cx) dx = X(0)$$

$$\left[c, r, \operatorname{Re} b, \operatorname{Re}(\alpha + \nu + \mu r) > 0, \left\{ \begin{array}{l} \operatorname{Re} \mu > 0, \operatorname{Re} \alpha < 3/2 \\ \operatorname{Re}(\alpha + \nu) > 0 \end{array} \right\} \right],$$

$$X(\varepsilon) = \mp \frac{b^{-(\alpha+\nu)/r}}{\Gamma(\nu+1)} \left(\frac{c}{2} \right)^\nu \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (\nu+1)_k (2k+\nu)^\varepsilon (\alpha+\nu+2k)} \times \\ \times \Gamma\left(\mu + \frac{\alpha+\nu+2k}{r}\right) \left(\frac{c}{2b^{1/r}} \right)^{2k} + \left\{ \begin{array}{l} 1 \\ 0 \end{array} \right\} \frac{2^{\alpha-1}}{(-\alpha)^\varepsilon c^\alpha} \Gamma\left[\mu, (\alpha+\nu)/2\right] \\ [r > 1],$$

$$X(\varepsilon) = \pm \frac{2^{\alpha+\mu r-1} b^\mu}{c^{\alpha+\mu r}} \sum_{k=0}^{\infty} \frac{1}{k! (\mu+k) (\alpha+r\mu+rk)^\varepsilon} \times \\ \times \Gamma\left[\frac{(\nu+\alpha+\mu r+rk)/2}{(\nu-\alpha-\mu r-rk)/2+1}\right] \left(-\frac{2^r b}{c^r}\right)^k + \left\{ \begin{array}{l} 0 \\ 1 \end{array} \right\} \frac{2^{\alpha-1}}{\alpha^\varepsilon c^\alpha} \Gamma\left[\mu, (\nu+\alpha)/2\right] \\ [r < 1],$$

$X(0)$ при $r=1$ см в 2.12.51.1

$$19. \int_0^{\infty} x^{\alpha-1} e^{\pm b^2 x^{2r}/4} D_\mu(bx^r) J_\nu(cx) dx = Y(0)$$

$$\left[c, r > 0; |\arg b| < (2 \pm 1)\pi/4, \left\{ \begin{array}{l} -\operatorname{Re} \nu < \operatorname{Re} \alpha < 3/2 - r \operatorname{Re} \mu \\ \operatorname{Re}(\alpha + \nu) > 0 \end{array} \right\} \right],$$

$$Y(\varepsilon) = \frac{2^{-(\alpha+\nu)/(2r) - \nu \mp (\mu+1 \pm 1)/2} c^\nu}{b^{(\alpha+\nu)/r} r \Gamma(\nu+1)} \left\{ \frac{\Gamma^{-1}(-\mu)}{\sqrt{\pi}} \right\} \times \\ \times \sum_{k=0}^{\infty} \frac{1}{k! (\nu+1)_k (2k+\nu)^\varepsilon} \Gamma\left(\frac{2k+\alpha+\nu}{r}\right) \Gamma^{\pm 1}\left(\frac{1 \mp 1}{4} \mp \frac{2k+\alpha+\nu \pm r\mu}{2r}\right) \times \\ \times \left(-\frac{c^2}{2^2 + 1/r b^{2/r}}\right)^k + \left\{ \begin{array}{l} 1 \\ 0 \end{array} \right\} \frac{2^{\mu r + \alpha - 1} b^\mu}{c^{\alpha+\mu r}} \sum_{k=0}^{\infty} \frac{(-\mu)_{2k}}{k! (2rk - \mu r - \alpha)^\varepsilon} \times \\ \times \Gamma\left[\frac{(\nu+\alpha+r\mu)/2 - rk}{(\nu-\alpha-r\mu)/2+1+rk}\right] \left(-\frac{c^{2r}}{2^{2r+1} b^2}\right)^k \quad [r > 1/2]$$

$$Y(\varepsilon) = \frac{2^{\alpha \mp (\mu+1 \pm 3)/2}}{c^\alpha} \left\{ \frac{\Gamma^{-1}(-\mu)}{\sqrt{\pi}} \right\} \sum_{k=0}^{\infty} \frac{1}{k! (\alpha+rk)^\varepsilon} \times \\ \times \Gamma\left[\frac{(\alpha+\nu+rk)/2}{(\nu-\alpha-rk)/2+1}\right] \Gamma^{\pm 1}\left(\frac{1 \mp 1}{4} - \frac{\mu \mp k}{2}\right) \left(-\frac{2^{r+1/2} b}{c^r}\right)^k \quad [r < 1/2]$$

2.12.2. Интегралы от $x^\alpha J_\nu(cx)$.

$$1. \int_0^{\infty} J_\nu(cx) dx = \frac{1}{c} \quad [c > 0; \operatorname{Re} \nu > -1]$$

$$2. \int_0^{\infty} x^{\alpha-1} J_\nu(cx) dx = 2^{\alpha-1} c^{-\alpha} \Gamma\left[\frac{(\alpha+\nu)/2}{1+(\nu-\alpha)/2}\right]$$

$$[c > 0, -\operatorname{Re} \nu < \operatorname{Re} \alpha < 3/2],$$

$$3. \int_0^{\infty} x^{\alpha-1} [1 - J_0(cx)] dx = -\frac{2^{\alpha-1}}{c^\alpha} \Gamma\left[\frac{\alpha/2+1}{1-\alpha/2}\right] \quad [c > 0, -2 < \operatorname{Re} \alpha < 0]$$

2.12.3. Интегралы от $x^\alpha (z \pm x)^\beta J_\nu(cx)$.

$$1. \int_0^a x^{\alpha-1} (a-x)^{\beta-1} J_\nu(cx) dx = a^{\alpha+\beta+\nu-1} \left(\frac{c}{2}\right)^\nu \Gamma\left[\begin{matrix} \beta, \alpha+\nu \\ \nu+1, \alpha+\beta+\nu \end{matrix}\right] \times \\ \times {}_2F_3\left(\frac{\alpha+\nu}{2}, \frac{\alpha+\nu+1}{2}; \nu+1, \frac{\alpha+\beta+\nu}{2}, \frac{\alpha+\beta+\nu+1}{2}; -\frac{a^2c^2}{4}\right) \\ [a, \operatorname{Re} \beta, \operatorname{Re}(\alpha+\nu) > 0]$$

$$2. \int_0^a \frac{1}{\sqrt{a-x}} J_\nu(cx) dx = \pi \sqrt{\frac{a}{2}} J_{(2\nu+1)/4}\left(\frac{ac}{2}\right) J_{(2\nu-1)/4}\left(\frac{ac}{2}\right) \\ [a > 0; \operatorname{Re} \nu > -1].$$

$$3. \int_0^a \frac{x^n}{\sqrt{a-x}} J_\nu(cx) dx = \\ = \pi \left(\frac{a}{2}\right)^{n+1/2} \sum_{k=0}^n \binom{n}{k} J_{(2\nu+2n+1)/4-k}\left(\frac{ac}{2}\right) J_{(2\nu-2n-1)/4+k}\left(\frac{ac}{2}\right) \\ [a > 0; \operatorname{Re} \nu > -n-1].$$

$$4. \int_a^\infty x^{\alpha-1} (x-a)^{\beta-1} J_\nu(cx) dx = \frac{2^{\alpha+\beta-2}}{c^{\alpha+\beta-1}} \Gamma\left[\begin{matrix} (\alpha+\beta+\nu-1)/2 \\ (3+\nu-\alpha-\beta)/2 \end{matrix}\right] \times \\ \times {}_2F_3\left(1-\frac{\beta}{2}, \frac{1-\beta}{2}; \frac{1}{2}, \frac{3-\alpha-\beta-\nu}{2}, \frac{3+\nu-\alpha-\beta}{2}; -\frac{a^2c^2}{4}\right) - \frac{2^{\alpha+\beta-2}a}{c^{\alpha+\beta-2}} (\beta-1) \times \\ \times \Gamma\left[\begin{matrix} (\alpha+\beta+\nu)/2-1 \\ 2+(\nu-\alpha-\beta)/2 \end{matrix}\right] {}_2F_3\left(1-\frac{\beta}{2}, \frac{3-\beta}{2}; \frac{3}{2}, 2-\frac{\alpha+\beta+\nu}{2}, \right. \\ \left. 2+\frac{\nu-\alpha-\beta}{2}; -\frac{a^2c^2}{4}\right) + a^{\alpha+\beta+\nu-1} \left(\frac{c}{2}\right)^\nu \Gamma\left[\begin{matrix} \beta, 1-\beta-\alpha-\nu \\ 1-\alpha-\nu, \nu+1 \end{matrix}\right] \times \\ \times {}_2F_3\left(\frac{1+\alpha+\nu}{2}, \frac{\alpha+\nu}{2}; \nu+1, \frac{1+\alpha+\beta+\nu}{2}, \frac{\alpha+\beta+\nu}{2}; -\frac{a^2c^2}{4}\right) \\ [a, c, \operatorname{Re} \beta > 0, \operatorname{Re}(\alpha+\beta) < 5/2]$$

$$5. \int_0^\infty \frac{1}{x+z} J_\nu(cx) dx = \frac{\pi}{\sin \nu\pi} [J_\nu(cz) - J_\nu(cz)] \\ [c > 0, \operatorname{Re} \nu > -1, |\arg z| < \pi].$$

$$6. \int_0^\infty \frac{1}{x+z} J_0(cx) dx = \frac{\pi}{2} [H_0(cz) - Y_0(cz)] \\ [c > 0, |\arg z| < \pi]$$

$$7. \int_0^\infty \frac{x^{\alpha-1}}{x+z} J_\nu(cx) dx = (2z)^{\alpha-1} \left\{ \Gamma\left[\begin{matrix} (\alpha+\nu)/2 \\ 1+(\nu-\alpha)/2 \end{matrix}\right] S_{1-\alpha, \nu}(cz) - \right. \\ \left. - 2\Gamma\left[\begin{matrix} (1+\alpha+\nu)/2 \\ (1+\nu-\alpha)/2 \end{matrix}\right] S_{-\alpha, \nu}(cz) \right\} + \frac{\pi z^{\alpha-1}}{\sin(\alpha+\nu)\pi} J_\nu(cz) \\ [c > 0, -\operatorname{Re} \nu < \operatorname{Re} \alpha < 5/2, |\arg z| < \pi]$$

$$8. \int_0^\infty \frac{x^\nu}{x+z} J_\nu(cx) dx = \frac{\pi z^\nu}{2 \cos \nu\pi} [H_{-\nu}(cz) - Y_{-\nu}(cz)] \\ [c > 0, -1/2 < \operatorname{Re} \nu < 3/2, |\arg z| < \pi]$$

$$9. \int_0^{\infty} \frac{x^{\alpha-1}}{(x+z)^{\rho}} J_{\nu}(cx) dx = \left(\frac{c}{2}\right)^{\nu} z^{\alpha-\rho+\nu} \Gamma\left[\begin{matrix} \rho-\alpha-\nu, \alpha+\nu \\ \rho, \nu+1 \end{matrix}\right] \times \\ \times {}_2F_3\left(\frac{\alpha+\nu}{2}, \frac{\alpha+\nu+1}{2}; 1+\frac{\alpha+\nu-\rho}{2}, \frac{1+\alpha+\nu-\rho}{2}, \nu+1; -\frac{c^2 z^2}{4}\right) + \\ + 2^{\alpha-\rho-1} c^{\rho-\alpha} \Gamma\left[\frac{(\alpha+\nu-\rho)/2}{1+(\rho+\nu-\alpha)/2}\right] {}_2F_3\left(\frac{\rho}{2}, \frac{\rho+1}{2}; \frac{1}{2}, 1+\frac{\rho-\nu-\alpha}{2}, \right. \\ \left. 1+\frac{\rho+\nu-\alpha}{2}; -\frac{c^2 z^2}{4}\right) - 2^{\alpha-\rho-2} \rho c^{1+\rho-\alpha} {}_2\Gamma\left[\frac{(\alpha+\nu-\rho-1)/2}{(3+\rho+\nu-\alpha)/2}\right] \times \\ \times {}_2F_3\left(\frac{\rho+1}{2}, \frac{\rho}{2}+1; \frac{3}{2}, \frac{3-\alpha-\nu+\rho}{2}, \frac{3+\rho+\nu-\alpha}{2}; -\frac{c^2 z^2}{4}\right) \\ [c > 0; \operatorname{Re}(\alpha+\nu) > 0; \operatorname{Re}(\alpha-\rho) < 3/2, |\arg z| < \pi].$$

$$10. \int_0^{\infty} \frac{x^{\alpha-1}}{x-y} J_{\nu}(cx) dx = 2^{\alpha-2} c^{1-\alpha} \Gamma\left[\frac{(\alpha+\nu-1)/2}{(\nu-\alpha+3)/2}\right] \times \\ \times {}_1F_2\left(1; \frac{3-\alpha-\nu}{2}, \frac{3+\nu-\alpha}{2}; -\frac{c^2 y^2}{4}\right) + 2^{\alpha-2} c^{\alpha-\nu} y \Gamma\left[\frac{(\nu+\alpha)/2-1}{2+(\nu-\alpha)/2}\right] \times \\ \times {}_1F_2\left(1; 2-\frac{\alpha+\nu}{2}, 2+\frac{\nu-\alpha}{2}; -\frac{c^2 y^2}{4}\right) - \pi y^{\alpha-1} \operatorname{ctg}(\alpha+\nu) \pi J_{\nu}(cy) \\ [c, y > 0; -\operatorname{Re} \nu < \operatorname{Re} \alpha < 5/2].$$

$$11. \int_0^{\infty} \frac{x^{\nu+1}}{x-y} J_{\nu}(cx) dx = \\ = \frac{\pi y^{\nu+1}}{2 \cos \nu \pi} [\sin \nu \pi J_{\nu}(cy) - \cos \nu \pi Y_{\nu}(cy) - H_{-\nu}(cy)] + \frac{2^{\nu} \Gamma(\nu+1/2)}{\sqrt{\pi} c^{\nu+1}} \\ [c, y > 0; -1 < \operatorname{Re} \nu < 1/2]$$

$$12. \int_0^{\infty} \frac{x^{\nu}}{x-y} J_{\nu}(cx) dx = \frac{\pi y^{\nu}}{2 \cos \nu \pi} [\sin \nu \pi J_{\nu}(cy) - \cos \nu \pi Y_{\nu}(cy) - H_{-\nu}(cy)] \\ [c, y > 0; -1/2 < \operatorname{Re} \nu < 3/2].$$

2.12.4. Интегралы от $x^{\alpha} (z^2 \pm x^2)^{\beta} J_{\nu}(cx)$.

$$1. \int_0^a \frac{1}{\sqrt{a^2-x^2}} J_{\nu}(cx) dx = \frac{\pi}{2} J_{\nu/2}^2\left(\frac{ac}{2}\right) \quad [a > 0; \operatorname{Re} \nu > -1].$$

$$2. \int_0^a \frac{1}{\sqrt{a^2-x^2}} J_1(cx) dx = \frac{1 - \cos ac}{ac} \quad [a > 0].$$

$$3. \int_0^a x^{\alpha-1} (a^2-x^2)^{\beta-1} J_{\nu}(cx) dx = \\ = \frac{a^{2\beta+\alpha+\nu-2} c^{\nu}}{2^{\nu+1}} \Gamma\left[\begin{matrix} \beta, (\alpha+\nu)/2 \\ \beta+(\alpha+\nu)/2, \nu+1 \end{matrix}\right] {}_1F_2\left(\frac{\alpha+\nu}{2}; \frac{\alpha+\nu}{2}+\beta, \nu+1; -\frac{a^2 c^2}{4}\right) \\ [a, \operatorname{Re} \beta, \operatorname{Re}(\alpha+\nu) > 0].$$

$$4. \int_0^a x^{\nu+2n+1} (a^2-x^2)^{\beta-1} J_{\nu}(cx) dx = \\ = \frac{1}{2} (\nu+1)_n \Gamma(\beta) a^{2n+2\beta+\nu} \left(\frac{2}{ac}\right)^{n+\beta} \sum_{k=0}^n \frac{(-1)^k}{(\nu+1)_k} \binom{n}{k} \left(\frac{ac}{2}\right)^k J_{k+n+\beta+\nu}(ac) \\ [a, \operatorname{Re} \beta > 0; \operatorname{Re} \nu > -n-1].$$

$$5. \int_0^a x^{1-n} (a^2 - x^2)^{\beta-1} J_n(cx) dx =$$

$$= (-1)^n 2^{\beta-1} \Gamma(\beta) \frac{a^{\beta-n}}{c^\beta} \left[J_{\beta-n}(ac) - \sum_{k=0}^{n-1} \frac{(-1)^k (ac/2)^{\beta-n+2k}}{k! \Gamma(\beta-n+k+1)} \right]$$

[a, Re $\beta > 0$].

$$6. \int_0^a x^{\nu+1} (a^2 - x^2)^{\beta-1} J_\nu(cx) dx = \frac{2^{\beta-1} a^{\beta+\nu}}{c^\beta} \Gamma(\beta) J_{\beta+\nu}(ac)$$

[a, Re $\beta > 0$; Re $\nu > -1$].

$$7. \int_0^a x^{1-\nu} (a^2 - x^2)^{\beta-1} J_\nu(cx) dx = \frac{2^{1-\nu} a^{\beta-\nu}}{c^\beta \Gamma(\nu)} s_{\nu+\beta-1, \beta-\nu}(ac)$$

[a, Re $\beta > 0$]

$$8. \int_0^a x^\nu (a^2 - x^2)^{\nu-1/2} J_\nu(cx) dx = 2^{\nu-1} \sqrt{\pi} \left(\frac{a^2}{c} \right)^\nu \Gamma\left(\nu + \frac{1}{2}\right) J_\nu^2\left(\frac{ac}{2}\right)$$

[a > 0; Re $\nu > -1/2$]

$$9. \int_0^a x^\nu (a^2 - x^2)^{\nu-3/2} J_\nu(cx) dx =$$

$$= \frac{\sqrt{\pi}}{2^{2-\nu}} a^{2\nu-1} c^{1-\nu} \Gamma\left(\nu - \frac{1}{2}\right) J_{\nu-1}\left(\frac{ac}{2}\right) J_\nu\left(\frac{ac}{2}\right)$$

[a > 0; Re $\nu > 1/2$].

$$10. \int_0^a x^{-\nu} (a^2 - x^2)^{-\nu-1/2} J_\nu(cx) dx = \frac{c^\nu \sqrt{\pi}}{2^{\nu+1} a^{2\nu}} \Gamma\left(\frac{1}{2} - \nu\right) J_\nu\left(\frac{ac}{2}\right) J_{-\nu}\left(\frac{ac}{2}\right)$$

[a > 0; Re $\nu < 1/2$]

$$11. \int_0^a \frac{x^{1-\nu}}{\sqrt{a^2 - x^2}} J_\nu(cx) dx = a^{1/2-\nu} \sqrt{\frac{\pi}{2c}} H_{\nu-1/2}(ac)$$

[a > 0]

$$12. \int_0^a \frac{x}{\sqrt{a^2 - x^2}} J_\nu(cx) dx = \frac{\pi a}{2} J_{(\nu+1)/2}\left(\frac{ac}{2}\right) J_{(\nu-1)/2}\left(\frac{ac}{2}\right)$$

[a > 0; Re $\nu > -2$].

$$13. \int_0^a \frac{x}{\sqrt{a^2 - x^2}} J_0(cx) dx = \frac{\sin ac}{c}$$

[a > 0].

$$14. \int_a^\infty \frac{1}{\sqrt{x^2 - a^2}} J_\nu(cx) dx = -\frac{\pi}{2} J_{\nu/2}\left(\frac{ac}{2}\right) Y_{\nu/2}\left(\frac{ac}{2}\right)$$

[a, c > 0].

$$15. \int_a^\infty \frac{1}{\sqrt{x^2 - a^2}} J_1(cx) dx = \frac{\sin ac}{c}$$

[a, c > 0]

$$\begin{aligned}
16. \int_a^\infty x^{\alpha-1} (x^2 - a^2)^{\beta-1} J_\nu(cx) dx &= \frac{a^{\alpha+2\beta+\nu-2} c^\nu}{2^{\nu+1}} \times \\
&\times \Gamma \left[\begin{matrix} \beta, 1-\beta-(\alpha+\nu)/2 \\ \nu+1, 1-(\alpha+\nu)/2 \end{matrix} \right] {}_1F_2 \left(\frac{\alpha+\nu}{2}; \nu+1, \frac{\alpha+\nu}{2} + \beta; -\frac{a^2 c^2}{4} \right) + \\
&+ 2^{\alpha+2\beta-3} c^{2-\alpha-2\beta} \Gamma \left[\begin{matrix} \beta + (\alpha+\nu)/2 - 1 \\ 2-\beta + (\nu-\alpha)/2 \end{matrix} \right] \times \\
&\times {}_1F_2 \left(1-\beta; 2-\beta - \frac{\alpha+\nu}{2}, 2-\beta + \frac{\nu-\alpha}{2}; -\frac{a^2 c^2}{4} \right) \\
&[a, c, \operatorname{Re} \beta > 0, \operatorname{Re} (\alpha + 2\beta) < 7/2].
\end{aligned}$$

$$\begin{aligned}
17. \int_a^\infty x^{1 \pm \nu} (x^2 - a^2)^{\beta-1} J_\nu(cx) dx &= \\
&= 2^{\beta-1} a^\beta \pm \nu c^{-\beta} \Gamma(\beta) \left\{ \begin{matrix} [\cos \beta \pi J_{\nu+\beta}(ac) - \sin \beta \pi Y_{\nu+\beta}(ac)] \\ J_{\nu-\beta}(ac) \end{matrix} \right\} \\
&[a, c, \operatorname{Re} \beta > 0, \operatorname{Re} (2\beta \pm \nu) < 3/2]
\end{aligned}$$

$$\begin{aligned}
18. \int_a^\infty x^{\pm \nu} (x^2 - a^2)^{\pm \nu - 1/2} J_\nu(cx) dx &= \\
&= \pm 2^{\pm \nu - 2} \sqrt{\pi} a^{\pm 2\nu} c^{\mp \nu} \Gamma \left(\frac{1}{2} \pm \nu \right) \left\{ \begin{matrix} [J_\nu(z) Y_{-\nu}(z) + Y_\nu(z) J_{-\nu}(z)] \\ 2J_\nu(z) Y_\nu(z) \end{matrix} \right\} \\
&[a, c > 0, |\operatorname{Re} \nu| < 1/2, z = ac/2]
\end{aligned}$$

$$19. \int_a^\infty \frac{1}{x \sqrt{x^2 - a^2}} J_0(cx) dx = \frac{\pi}{2a} - \frac{1}{a} \operatorname{Si}(ac) \quad [a, c > 0].$$

$$\begin{aligned}
20. \int_0^\infty \frac{x^{\alpha-1}}{x^2 + z^2} J_\nu(cx) dx &= \frac{\pi z^{\alpha-2}}{2} \operatorname{cosec} \frac{(\nu+\alpha)\pi}{2} I_\nu(cz) - \\
&- 2^{\alpha-3} e^{i\pi/2} (2+\nu-\alpha)(2-\nu-\alpha) \Gamma \left[\begin{matrix} (\alpha+\nu)/2 - 1 \\ 2 + (\nu-\alpha)/2 \end{matrix} \right] z^{\alpha-2} s_{1-\alpha, \nu}(icz) \\
&[c, \operatorname{Re} z > 0; -\operatorname{Re} \nu < \operatorname{Re} \alpha < 7/2].
\end{aligned}$$

$$\begin{aligned}
21. \int_0^\infty \frac{x^{\pm \nu}}{x^2 + z^2} J_\nu(cx) dx &= \frac{\pi}{2} z^{\pm \nu - 1} \left\{ \begin{matrix} \cos \nu \pi \\ 1 \end{matrix} \right\} [I_\nu(cz) - L_{\mp \nu}(cz)] \\
&[c, \operatorname{Re} z > 0, \left\{ \begin{matrix} -1/2 < \operatorname{Re} \nu < 5/2 \\ \operatorname{Re} \nu > -5/2 \end{matrix} \right\}].
\end{aligned}$$

$$\begin{aligned}
22. \int_0^\infty \frac{x}{x^2 + z^2} J_\nu(cx) dx &= \frac{\pi}{2 \sin \nu \pi} \left[J_\nu(icz) + J_\nu(-icz) - 2 \cos \frac{\nu \pi}{2} I_\nu(cz) \right] \\
&[c, \operatorname{Re} z > 0; \operatorname{Re} \nu > -2].
\end{aligned}$$

$$23. \int_0^\infty \frac{x}{x^2 + z^2} J_0(cx) dx = K_0(cz) \quad [c, \operatorname{Re} z > 0]$$

$$\begin{aligned}
24. \int_0^\infty \frac{1}{x(x^2 + z^2)} J_{2n+1}(cx) dx &= \frac{(-1)^n \pi}{2z^2} [I_{2n+1}(cz) - L_{2n+1}(cz)] \\
&[c, \operatorname{Re} z > 0]
\end{aligned}$$

$$25. \int_0^{\infty} \frac{1}{\sqrt{x^2 + z^2}} J_{\nu}(cx) dx = I_{\nu/2} \left(\frac{cz}{2} \right) K_{\nu/2} \left(\frac{cz}{2} \right) \quad [c, \operatorname{Re} z > 0; \operatorname{Re} \nu > -1]$$

$$26. \int_0^{\infty} \frac{1}{\sqrt{x^2 + z^2}} J_1(cx) dx = \frac{1}{cz} (1 - e^{-cz}) \quad [c, \operatorname{Re} z > 0]$$

$$27. \int_0^{\infty} \frac{x^{\alpha-1}}{(x^2 + z^2)^{\rho}} J_{\nu}(cx) dx = 2^{-\nu-1} c^{\nu} z^{\alpha+\nu-2\rho} \Gamma \left[\begin{matrix} (\alpha+\nu)/2, \rho-(\alpha+\nu)/2 \\ \rho, \nu+1 \end{matrix} \right] \times \\ \times {}_1F_2 \left(\frac{\alpha+\nu}{2}; \frac{\alpha+\nu}{2} + 1 - \rho, \nu+1; \frac{c^2 z^2}{4} \right) + 2^{\alpha-2\rho-1} c^{2\rho-\alpha} \Gamma \left[\begin{matrix} (\alpha+\nu)/2 - \rho \\ 1+\rho + (\nu-\alpha)/2 \end{matrix} \right] \times \\ \times {}_1F_2 \left(\rho; 1+\rho + \frac{\nu-\alpha}{2}, \rho+1 - \frac{\nu+\alpha}{2}; \frac{c^2 z^2}{4} \right) \quad [c, \operatorname{Re} z > 0; -\operatorname{Re} \nu < \operatorname{Re} \alpha < 2\operatorname{Re} \rho + 3/2]$$

$$28. \int_0^{\infty} \frac{x^{\nu+1}}{(x^2 + z^2)^{\rho}} J_{\nu}(cx) dx = \frac{c^{\rho-1} z^{\nu-\rho+1}}{2^{\rho-1} \Gamma(\rho)} K_{\nu-\rho+1}(cz) \quad [c, \operatorname{Re} z > 0; -1 < \operatorname{Re} \nu < 2\operatorname{Re} \rho - 1/2]$$

$$29. \int_0^{\infty} \frac{x^{1-\nu}}{(x^2 + z^2)^{\rho}} J_{\nu}(cx) dx = c^{\rho-1} z^{1-\rho-\nu} \left\{ 2^{-\rho} \Gamma \left[\begin{matrix} 1-\rho \\ 1-\nu \end{matrix} \right] I_{\nu+\rho-1}(cz) - \right. \\ \left. - \frac{2^{1-\nu}}{\Gamma(\nu)} \exp \left[-\frac{i\pi}{2} (\nu-\rho+1) \right] s_{\nu-\rho, 1-\nu-\rho}(icz) \right\} \quad [c, \operatorname{Re} z > 0; \operatorname{Re}(2\rho+\nu) > 1/2]$$

$$30. \int_0^{\infty} \frac{x^{-\nu}}{(x^2 + z^2)^{\nu+1/2}} J_{\nu}(cx) dx = (2c)^{\nu} z^{-2\nu} \Gamma \left[\begin{matrix} \nu+1 \\ 2\nu+1 \end{matrix} \right] I_{\nu} \left(\frac{cz}{2} \right) K_{\nu} \left(\frac{cz}{2} \right) \quad [c, \operatorname{Re} z > 0; \operatorname{Re} \nu > -1/2]$$

$$31. \int_0^{\infty} \frac{x^{1-\nu}}{\sqrt{x^2 + z^2}} J_{\nu}(cx) dx = \sqrt{\frac{\pi}{2c}} z^{1/2-\nu} [I_{\nu-1/2}(cz) - L_{\nu-1/2}(cz)] \quad [c, \operatorname{Re} z > 0; \operatorname{Re} \nu > -1/2]$$

$$32. \int_0^{\infty} \frac{1}{x^2 - y^2} J_{\nu}(cx) dx = \frac{\pi}{2y} \left\{ \operatorname{tg} \frac{\nu\pi}{2} [J_{\nu}(cy) - J_{\nu}(cy)] + E_{\nu}(cy) \right\} \quad [c, y > 0; \operatorname{Re} \nu > -1]$$

$$33. \int_0^{\infty} \frac{x^{\alpha-1}}{x^2 - y^2} J_{\nu}(cx) dx = 2^{\alpha-3} c^{2-\alpha} \Gamma \left[\begin{matrix} (\alpha+\nu)/2 - 1 \\ (\nu-\alpha)/2 + 2 \end{matrix} \right] \times \\ \times {}_1F_2 \left(1; 2 - \frac{\alpha+\nu}{2}, 2 - \frac{\alpha-\nu}{2}; -\frac{c^2 y^2}{4} \right) - \frac{\pi y^{\alpha-3}}{2} \operatorname{ctg} \frac{\alpha+\nu}{2} \pi J_{\nu}(cy) \quad [c, y > 0; -\operatorname{Re} \nu < \operatorname{Re} \alpha < 7/2]$$

$$34. \int_0^{\infty} \frac{x^{\nu}}{x^2 - y^2} J_{\nu}(cx) dx = \frac{\pi y^{\nu-1}}{2 \cos \nu\pi} [\sin \nu\pi J_{\nu}(cy) - H_{-\nu}(cy)] \quad [c, y > 0; -1/2 < \operatorname{Re} \nu < 5/2]$$

$$35. \int_0^{\infty} \frac{x^{\nu+1}}{x^2 - y^2} J_{\nu}(cx) dx = -\frac{\pi}{2} y^{\nu} Y_{\nu}(cy) \quad [c, y > 0; -1 < \operatorname{Re} \nu < 3/2]$$

$$36. \int_0^{\infty} \frac{x^{-\nu}}{x^2 - y^2} J_{\nu}(cx) dx = -\frac{\pi}{2} y^{-\nu-1} H_{\nu}(cy) \quad [c, y > 0; \operatorname{Re} \nu > -5/2].$$

$$37. \int_0^{\infty} \frac{x^{n-1}}{x^2 - y_{m,n}^2} J_n(x) dx = \frac{1 - J_0(y_{m,n})}{y_{m,n} \alpha_{m,n}}$$

$[n = 0, 1, 2; y_{m,n}$ — корни уравнения $J_n(y) = 0, m = 1, 2, 3, \dots;$
 $\alpha_{m,0} = -J_1(y_{m,0}), \alpha_{m,1} = y_{m,1} J_0(y_{m,1}), \alpha_{m,2} = J_1(y_{m,2})].$

2.12.5. Интегралы от $x^{\alpha} (x^4 \pm z^4)^{\beta} J_{\nu}(cx)$

$$1. \int_0^{\infty} \frac{x^{\alpha-1}}{(x^4 + z^4)^{\rho}} J_{\nu}(cx) dx = 2^{\alpha-4\rho-1} c^{4\rho-\alpha} \Gamma \left[\frac{(\alpha+\nu)/2 - 2\rho}{(\nu-\alpha)/2 + 2\rho + 1} \right] \times \\ \times {}_1F_4 \left(\rho; 1+\rho - \frac{\alpha+\nu}{4}, \rho + \frac{1-\alpha-\nu}{4}, \rho + \frac{2+\nu-\alpha}{4}, \rho + 1 + \frac{\nu-\alpha}{4}; -\frac{c^4 z^4}{256} \right) - \\ - \frac{c^{\nu+2} z^{2+\nu+\alpha-4\rho}}{2^{\nu+4}} \Gamma \left[\begin{matrix} \rho - (\alpha+\nu+2)/4, (2+\alpha+\nu)/4 \\ \rho, \nu+2 \end{matrix} \right] \times \\ \times {}_1F_4 \left(\frac{\alpha+\nu+2}{4}; \frac{3}{2}, \frac{3+\nu+\alpha}{4} - \rho, \frac{\nu}{2} + 1, \frac{\nu+3}{2}; -\frac{c^4 z^4}{256} \right) + \\ + \frac{c^{\nu} z^{\alpha+\nu-4\rho}}{2^{\nu+2}} \Gamma \left[\begin{matrix} \rho - (\alpha+\nu)/4, (\alpha+\nu)/4 \\ \rho, \nu+1 \end{matrix} \right] \times \\ \times {}_1F_4 \left(\frac{\alpha+\nu}{4}; \frac{1}{2}, 1-\rho + \frac{\alpha+\nu}{4}, \frac{\nu+1}{2}, \frac{\nu}{2} + 1; -\frac{c^4 z^4}{256} \right) \\ [c > 0; |\arg z| < \pi/4; -\operatorname{Re} \nu < \operatorname{Re} \alpha < 4\operatorname{Re} \rho + 3/2].$$

$$2. \int_0^{\infty} \frac{x^{\nu+2\pm 1}}{(x^4 + z^4)^{\nu+1/2}} J_{\nu}(cx) dx = \\ = \frac{\sqrt{\pi} c^{\nu} z^{-2\nu+1\pm 1}}{2^{(1\mp 1)\nu} \Gamma(\nu+1/2)} J_{\nu-(1\pm 1)/2} \left(\frac{cz}{\sqrt{2}} \right) K_{\nu-(1\pm 1)/2} \left(\frac{cz}{\sqrt{2}} \right) \\ [c > 0; |\arg z| < \pi/4; \operatorname{Re} \nu > -(1\mp 2)/6].$$

$$3. \int_0^{\infty} \frac{x^{2\pm 1}}{x^4 + z^4} J_0(cx) dx = \left\{ \begin{matrix} \ker(cz) \\ -z^{-3} \operatorname{kei}(cz) \end{matrix} \right\} \quad [c > 0; |\arg z| < \pi/4].$$

$$4. \int_0^{\infty} \frac{x^{\alpha-1}}{x^4 - y^4} J_{\nu}(cx) dx = 2^{\alpha-5} c^4 - \alpha \Gamma \left[\frac{(\alpha+\nu)/2 - 2}{(\nu-\alpha)/2 + 3} \right] \times \\ \times {}_1F_4 \left(1; 2 - \frac{\nu+\alpha}{4}, \frac{6-\nu-\alpha}{4}, \frac{6+\nu-\alpha}{4}, 2 + \frac{\nu-\alpha}{4}; \frac{c^4 y^4}{256} \right) - \\ - \frac{\pi c^{\nu+2} y^{\alpha+\nu-2}}{2^{\nu+4} \Gamma(\nu+2)} \operatorname{tg} \frac{\alpha+\nu}{4} {}_0F_3 \left(\frac{3}{2}, \frac{\nu}{2} + 1, \frac{\nu+3}{2}; \frac{c^4 y^4}{256} \right) - \\ - \frac{\pi c^{\nu} y^{\alpha+\nu-4}}{2^{\nu+2} \Gamma(\nu+1)} \operatorname{ctg} \frac{\alpha+\nu}{4} {}_0F_3 \left(\frac{1}{2}, \frac{\nu+1}{2}, \frac{\nu}{2} + 1; \frac{c^4 y^4}{256} \right) \\ [c, y > 0; -\operatorname{Re} \nu < \operatorname{Re} \alpha < 11/2]$$

$$5. \int_0^{\infty} \frac{x^3}{x^4 - y^4} J_0(cx) dx = \frac{1}{2} K_0(cy) - \frac{\pi}{4} Y_0(cy) \quad [c, y > 0]$$

2.12.6. Интегралы от $A(x) J_\nu(cx)$.

$$1. \int_a^b x^{1-\nu} [(b^2 - x^2)(x^2 - a^2)]^{\nu-1/2} J_\nu(cx) dx =$$

$$= 2^{\nu-1} \sqrt{\pi} \left(\frac{b^2 - a^2}{c} \right)^\nu \Gamma\left(\nu + \frac{1}{2}\right) J_\nu\left(\frac{b+a}{2}c\right) J_\nu\left(\frac{b-a}{2}c\right)$$

[$0 < a < b$; $\operatorname{Re} \nu > -1/2$; $|\arg c| < \pi$].

$$2. \int_0^\infty x^{\alpha-1} \frac{(Vx^2 + z^2 \pm z)^\mu}{(x^2 + z^2)^r} J_\nu(cx) dx = \frac{2^{\alpha+\mu-2r-1}}{c^{\alpha+\mu-2r}} \Gamma\left[\frac{(\alpha+\nu+\mu)/2-r}{1+r+(\nu-\alpha-\mu)/2}\right] \times$$

$$\times {}_2F_3\left(r + \frac{\mu}{2}, r - \frac{\mu}{2}; \frac{1}{2}, \frac{1-\alpha-\nu-\mu}{2} + r, 1+r + \frac{\nu-\alpha-\mu}{2}; \frac{c^2 z^2}{4}\right) \pm$$

$$\pm \frac{2^{\alpha+\mu-2r-2}\mu z}{c^{\alpha+\mu-2r-1}} \Gamma\left[\frac{(\alpha+\mu+\nu-1)/2-r}{(\nu-\alpha-\mu+3)/2+r}\right] \times$$

$$\times {}_2F_3\left(\frac{1+\mu}{2} + r, \frac{1-\mu}{2} + r; \frac{3}{2}, \frac{3-\alpha-\mu-\nu}{2} + r, \frac{3+\nu-\alpha-\mu}{2} + r; \frac{c^2 z^2}{4}\right) +$$

$$+ 2^{\alpha+\mu-1} (\mp \mu)^{1-2r} c^\nu z^{\alpha+\mu+\nu-2r} \Gamma\left[\frac{2r-\mu-\alpha-\nu, (\alpha+\nu+\mu \mp \mu)/2}{\nu+1, 1-(\mu \pm \mu+\alpha+\nu)/2}\right] \times$$

$$\times {}_2F_3\left(\mu + \frac{\alpha+\nu}{2}, \frac{\alpha+\nu}{2}; \nu+1, 1 + \frac{\alpha+\nu+\mu}{2} - 2r, \frac{1+\alpha+\mu+\nu}{2}; \frac{c^2 z^2}{4}\right)$$

[$r=0$ или $1/2$; $c, \operatorname{Re} z > 0$; $\operatorname{Re}(\pm \mu - \mu - \nu) < \operatorname{Re} \alpha < 2r + 3/2 - \operatorname{Re} \mu$].

$$3. \int_0^\infty x^{-\mu} \frac{(Vx^2 + z^2 \pm z)^\mu}{Vx^2 + z^2} J_\nu(cx) dx =$$

$$= \frac{1}{cz} \Gamma\left[\frac{(1+\nu \mp \mu)/2}{\nu+1}\right] W_{\pm \mu/2, \nu/2}(cz) M_{\mp \mu/2, \nu/2}(cz)$$

[$c, \operatorname{Re} z > 0$; $\operatorname{Re}(\nu \mp \mu) > -1$].

$$4. \int_0^\infty x^{1 \pm \nu} \frac{(Vx^2 + z^2 \pm z)^{\mp \nu}}{Vx^2 + z^2} J_\nu(cx) dx = \frac{1}{c} e^{-cz}$$

[$c, \operatorname{Re} z > 0$; $\operatorname{Re} \nu > -1$].

$$5. \int_0^\infty x^{\alpha-1} \frac{(Vx^2 + z^2 \pm x)^\mu}{(x^2 + z^2)^r} J_\nu(cx) dx = \frac{c^{2r \mp \mu - \alpha} z^{\mu \mp \mu}}{2^{2r \mp 2\mu - \alpha + 1}} \Gamma\left[\frac{(\alpha+\nu \pm \mu)/2-r}{1+r+(\nu-\alpha \mp \mu)/2}\right] \times$$

$$\times {}_2F_3\left(\frac{1 \mp \mu}{2}, 2r \mp \frac{\mu}{2}; 1 \mp \mu, 1+r - \frac{\alpha+\nu \pm \mu}{2}, 1+r + \frac{\nu-\alpha \mp \mu}{2}; \frac{c^2 z^2}{4}\right) +$$

$$+ (\mp \mu)^{1-2r} c^\nu \frac{z^{\alpha+\mu+\nu-2r}}{2^{\alpha+2\nu-2r+1}} \Gamma\left[\frac{\alpha+\nu, r-(\alpha+\nu \pm \mu)/2}{\nu+1, 1-r+(\alpha+\nu \mp \mu)/2}\right] \times$$

$$\times {}_2F_3\left(\frac{\alpha+\nu}{2}, \frac{\alpha+\nu+1}{2}; 1 + \frac{\alpha+\nu+\mu}{2} - r, 1-r + \frac{\alpha+\nu-\mu}{2}, \nu+1; \frac{c^2 z^2}{4}\right)$$

[$r=0$ или $1/2$; $c, \operatorname{Re} z > 0$; $-\operatorname{Re} \nu < \operatorname{Re} \alpha < 2r + 3/2 \mp \operatorname{Re} \mu$].

$$6. \int_0^\infty \frac{(Vx^2 + z^2 \pm x)^\mu}{Vx^2 + z^2} J_\nu(cx) dx = z^\mu I_{(\nu \mp \mu)/2}\left(\frac{cz}{2}\right) K_{(\nu \pm \mu)/2}\left(\frac{cz}{2}\right)$$

[$c, \operatorname{Re} z > 0$; $\operatorname{Re} \nu > -1$; $\pm \operatorname{Re} \mu < 3/2$].

$$7. \int_0^\infty (Vx^2 + z^2 - x)^2 J_0(cx) dx = \frac{z^2}{c} + \frac{2}{c^3} (cze^{-cz} + e^{-cz} - 1)$$

[$c, \operatorname{Re} z > 0$].

$$8. \int_0^{\infty} x (\sqrt{x^2 + z^2} - x)^2 J_0(cx) dx = -\frac{2z^2}{c} \frac{\partial}{\partial c} \left[I_1\left(\frac{cz}{2}\right) K_1\left(\frac{cz}{2}\right) \right]$$

[$c, \operatorname{Re} z > 0$].

$$9. \int_0^{\infty} x \frac{(\sqrt{x^2 + z^2} + x^2)^{\mu}}{\sqrt{x^2 + z^2}} J_0(cx) dx = z^{2\mu} J_{-\mu}\left(\frac{cz}{\sqrt{2}}\right) K_{\mu}\left(\frac{cz}{\sqrt{2}}\right)$$

[$c > 0$; $|\arg z| < \pi/4$; $\operatorname{Re} \mu < 3/4$].

$$10. \int_0^{\infty} \frac{x}{\sqrt{x^4 + 2a^2x^2 + b^4}} J_0(cx) dx = I_0\left(c \sqrt{\frac{a^2 - b^2}{2}}\right) K_0\left(c \sqrt{\frac{a^2 + b^2}{2}}\right)$$

[$0 \leq b < a$; $c > 0$].

$$11. \int_0^{\infty} \frac{x}{\sqrt{x^4 + 2a^2x^2 + b^4}} J_0(cx) dx = J_0\left(c \sqrt{\frac{b^2 - a^2}{2}}\right) K_0\left(c \sqrt{\frac{b^2 + a^2}{2}}\right)$$

[$0 \leq a < b$; $c > 0$].

$$12. \int_0^{\infty} \frac{x^{\nu+1} [b^2 + x^2 + \sqrt{x^4 \pm 2a^2x^2 + b^4}]^{-2\nu}}{\sqrt{x^4 \pm 2a^2x^2 - b^4}} J_{\nu}(cx) dx =$$

$$= \frac{(b^2 \mp a^2)^{-\nu}}{2^{4\nu} a^{\nu}} J_{2\nu}\left(c \sqrt{\frac{b^2 \mp a^2}{2}}\right) K_{\nu}\left(c \sqrt{\frac{b^2 \pm a^2}{2}}\right)$$

[$0 < a < b$; $c > 0$; $\operatorname{Re} \nu > -1/2$].

$$13. \int_0^a x^{-\mu} \frac{(a + \sqrt{a^2 - x^2})^{\mu} + (a - \sqrt{a^2 - x^2})^{\mu}}{\sqrt{a^2 - x^2}} J_{\nu}(cx) dx =$$

$$= \frac{1}{ac} \Gamma\left[\frac{(1 + \nu + \mu)/2, (1 + \nu - \mu)/2}{1 + \nu, 1 + \nu}\right] M_{\mu/2, \nu/2}(iac) M_{\mu/2, \nu/2}(-iac)$$

[$a > 0$; $\operatorname{Re}(\mu - \nu) < 1$].

$$14. \int_0^a x^{\mu} \frac{(x + i\sqrt{a^2 - x^2})^{\mu} + (x - i\sqrt{a^2 - x^2})^{\mu}}{\sqrt{a^2 - x^2}} J_{\nu}(cx) dx =$$

$$= \pi a^{2\mu} J_{(\nu+\mu)/2}\left(\frac{ac}{2}\right) J_{(\nu-\mu)/2}\left(\frac{ac}{2}\right)$$

[$a > 0$; $\operatorname{Re}(\mu + \nu) > -1$].

$$15. \int_a^{\infty} \frac{(x + \sqrt{x^2 - a^2})^{\mu} + (x - \sqrt{x^2 - a^2})^{\mu}}{\sqrt{x^2 - a^2}} J_{\nu}(cx) dx =$$

$$= -\frac{\pi a^{\mu}}{2} \left[J_{(\nu+\mu)/2}\left(\frac{ac}{2}\right) Y_{(\nu-\mu)/2}\left(\frac{ac}{2}\right) + J_{(\nu-\mu)/2}\left(\frac{ac}{2}\right) Y_{(\nu+\mu)/2}\left(\frac{ac}{2}\right) \right]$$

[$a, c > 0$; $\operatorname{Re} \mu < 3/2$].

$$16. \int_a^{\infty} \frac{(x + \sqrt{x^2 - a^2})^{\nu \pm 1} + (x - \sqrt{x^2 - a^2})^{\nu \pm 1}}{\sqrt{x^2 - a^2}} J_{\nu}(cx) dx =$$

$$= \mp \sqrt{\frac{\pi}{c}} a^{\nu \pm 1 - 1/2} \left[\begin{matrix} \sin(ac/2) \\ \cos(ac/2) \end{matrix} \right] J_{\nu \pm 1/2}\left(\frac{ac}{2}\right) \pm \begin{matrix} \cos(ac/2) \\ \sin(ac/2) \end{matrix} Y_{\nu \pm 1/2}\left(\frac{ac}{2}\right)$$

[$a, c > 0$; $\operatorname{Re} \nu < 3/2 \mp 1$].

2.12.7. Интегралы от $A(x) J_\nu(\Phi(x))$.

$$1. \int_0^1 x^{\alpha-1} J_\nu \left(\frac{c}{x} - cx \right) dx = I_{(\nu+\alpha)/2}(c) K_{(\nu-\alpha)/2}(c)$$

$$[c > 0; \operatorname{Re} \nu > -1; \operatorname{Re} \alpha > -3/2].$$

$$2. \int_1^\infty x^{\alpha-1} J_\nu \left(cx - \frac{c}{x} \right) dx = I_{(\nu-\alpha)/2}(c) K_{(\nu+\alpha)/2}(c)$$

$$[c > 0; \operatorname{Re} \nu > -1; \operatorname{Re} \alpha < 3/2].$$

$$3. \int_0^\infty x^{\alpha-1} J_\nu \left(\left| cx - \frac{c}{x} \right| \right) dx = I_{(\nu-\alpha)/2}(c) K_{(\nu+\alpha)/2}(c) + I_{(\nu+\alpha)/2}(c) K_{(\nu-\alpha)/2}(c)$$

$$[c > 0; \operatorname{Re} \nu > -1; |\operatorname{Re} \alpha| < 3/2].$$

$$4. \int_0^\infty x^{\alpha-1} J_\nu \left(cx + \frac{c}{x} \right) dx = -\frac{\pi}{2} [J_{(\nu-\alpha)/2}(c) Y_{(\nu+\alpha)/2}(c) + J_{(\nu+\alpha)/2}(c) Y_{(\nu-\alpha)/2}(c)]$$

$$[c > 0; |\operatorname{Re} \alpha| < 3/2]$$

$$5. \int_0^\infty x^{\alpha-1} \left(x + \frac{1}{x} \right)^{-1} J_\nu \left(cx + \frac{c}{x} \right) dx =$$

$$= \frac{1}{c} \Gamma \left[\frac{(1+\nu+\alpha)/2, (1+\nu-\alpha)/2}{1+\nu, 1+\nu} \right] M_{\alpha/2, \nu/2} \left(\frac{c}{2} \right) M_{-\alpha/2, \nu/2} \left(\frac{c}{2} \right)$$

$$[c > 0; |\operatorname{Re} \alpha| < |1 + \operatorname{Re} \nu|].$$

$$6. \int_0^\infty x^{\alpha-1} \left(\frac{a+bx}{b+ax} \right)^{\nu/2} J_\nu \left(\sqrt{a^2+b^2+ab(x+x^{-1})} \right) dx =$$

$$= -\pi [J_{\nu/2-\alpha}(a) Y_{\nu/2+\alpha}(b) + J_{\nu/2+\alpha}(b) Y_{\nu/2-\alpha}(a)]$$

$$[a, b > 0; |\operatorname{Re} \alpha| < 3/2].$$

2.12.8. Интегралы от $A(x, e^{px}, J_\nu(cx))$.

$$1. \int_0^1 \frac{dx}{J_0(x)} = 1,101140\dots$$

$$2. \int_0^1 \frac{x dx}{J_1(x)} = 1,418865\dots$$

$$3. \int_0^\infty e^{-px} J_\nu(cx) dx = \frac{c^\nu}{\sqrt{p^2+c^2}} (p + \sqrt{p^2+c^2})^{-\nu}$$

$$[\operatorname{Re} p > |\operatorname{Im} c|; \operatorname{Re} \nu > -1].$$

$$4. \int_0^\infty x^{\alpha-1} e^{-px} J_\nu(cx) dx = I_\nu^\alpha$$

$$[\operatorname{Re}(\alpha + \nu) > 0; \operatorname{Re} p > |\operatorname{Im} c|; k = 2^{-1/2} (1 - p/\sqrt{p^2+c^2})^{1/2}],$$

$$I_\nu^\alpha = \frac{c^\nu}{2^\nu p^{\alpha+\nu}} \Gamma \left[\frac{\nu+\alpha}{\nu+1} \right] {}_2F_1 \left(\frac{\alpha+\nu}{2}, \frac{\alpha+\nu+1}{2}; \nu+1; -\frac{c^2}{p^2} \right) =$$

$$= (p^2+c^2)^{-\alpha/2} \Gamma(\alpha+\nu) P_{\alpha, -1}^{-\nu} \left(\frac{p}{\sqrt{p^2+c^2}} \right),$$

$$I_\nu^{\nu+1} = \frac{(2c)^\nu}{\sqrt{\pi}} (p^2+c^2)^{-\nu-1/2} \Gamma \left(\nu + \frac{1}{2} \right)$$

$$[\operatorname{Re} \nu > -1/2],$$

$$I_\nu^{\nu+2} = \frac{2p(2c)^\nu \Gamma(\nu+3/2)}{\sqrt{\pi} (p^2+c^2)^{\nu+3/2}}$$

$$[\operatorname{Re} \nu > -1],$$

$$I_v^{-1} = \frac{c}{2v} \left[\frac{1}{v-1} \left(\frac{c}{p + \sqrt{p^2 + c^2}} \right)^{v-1} + \frac{1}{v+1} \left(\frac{c}{p + \sqrt{p^2 + c^2}} \right)^{v+1} \right]$$

[Re v > 1].

$$I_v^0 = \frac{c^v}{v} (p + \sqrt{p^2 + c^2})^{-v}$$

[Re v > 0].

$$I_v^n = (-1)^{n-1} c^{-v} \frac{\partial^{n-1}}{\partial p^{n-1}} \frac{(\sqrt{p^2 + c^2} - p)^v}{\sqrt{p^2 + c^2}}$$

[n = 1, 2, 3, ...; Re v > -n].

$$I_0^{1/2} = 2 \sqrt{\frac{1-2k^2}{\pi p}} K(k),$$

$$I_0^{3/2} = \sqrt{\frac{(1-2k^2)^3}{\pi p^3}} [2E(k) - K(k)],$$

$$I_0^{5/2} = \frac{1}{2} \sqrt{\frac{(1-2k^2)^5}{\pi p^5}} [8(1-2k^2)E(k) - (5-8k^2)K(k)],$$

$$I_0^{7/2} = \frac{1}{4} \sqrt{\frac{(1-2k^2)^7}{\pi p^7}} [2(23-128k^2+128k^4)E(k) - (31-144k^2+128k^4)K(k)],$$

$$I_1^{-1/2} = \frac{4\sqrt{p}}{3k\sqrt{\pi(1-k^2)(1-2k^2)}} [(1-k^2)K(k) - (1-2k^2)E(k)],$$

$$I_1^{1/2} = \frac{2}{k} \sqrt{\frac{1-2k^2}{\pi p(1-k^2)}} [E(k) - (1-k^2)K(k)],$$

$$I_1^{3/2} = \frac{1}{k} \sqrt{\frac{(1-2k^2)^3}{\pi p^3(1-k^2)}} [(1-k^2)K(k) - (1-2k^2)E(k)],$$

$$I_1^{5/2} = \frac{1}{2k} \sqrt{\frac{(1-2k^2)^5}{\pi p^5(1-k^2)}} [(1-k^2)(1-8k^2)K(k) - (1-16k^2+16k^4)E(k)],$$

$$I_2^{-1/2} = \frac{4\sqrt{p}}{15k^2(1-k^2)\sqrt{\pi(1-2k^2)}} [2(1-k^2+k^4)E(k) - (1-k^2)(2-k^2)K(k)],$$

$$I_2^{1/2} = \frac{2\sqrt{1-2k^2}}{3k^2(1-k^2)\sqrt{\pi p}} [(1-k^2)(2-3k^2)K(k) - 2(1-2k^2)E(k)],$$

$$I_2^{3/2} = \frac{\sqrt{(1-2k^2)^3}}{k^2(1-k^2)\sqrt{\pi p^3}} [2(1-k^2+k^4)E(k) - (1-k^2)(2-k^2)K(k)],$$

$$I_2^{5/2} = \frac{\sqrt{(1-2k^2)^5}}{2k^2(1-k^2)\sqrt{\pi p^5}} [(2+5k^2-8k^4)(1-k^2)K(k) - \\ - 2(1-2k^2)(1+4k^2-4k^4)E(k)],$$

$$I_2^{7/2} = \frac{\sqrt{(1-2k^2)^7}}{4k^2(1-k^2)\sqrt{\pi p^7}} [(2+15k^2-144k^4+128k^6)(1-k^2)K(k) - \\ - 2(1+7k^2-135k^4+256k^6-128k^8)E(k)],$$

$$I_3^{-1/2} = \frac{4\sqrt{p}}{105k^3\sqrt{\pi(1-k^2)^3(1-2k^2)}} [(1-k^2)(8-15k^2+3k^4)K(k) - \\ - (8-19k^2+9k^4-6k^6)E(k)],$$

$$I_3^{1/2} = \frac{2}{15k^3} \sqrt{\frac{1-2k^2}{\pi p(1-k^2)^3}} [(8-23k^2+23k^4)E(k) - \\ - (1-k^2)(8-19k^2+15k^4)K(k)],$$

$$I_3^{3/2} = \frac{1}{3k^3} \sqrt{\frac{(1-2k^2)^3}{\pi p^3(1-k^2)^3}} [(8-15k^2+3k^4)(1-k^2)K(k) - \\ - (8-19k^2+9k^4-6k^6)E(k)].$$

$$5. \int_0^{\infty} \frac{1}{x} (1 - e^{-px}) J_0(cx) dx = \ln \frac{p + \sqrt{p^2 + c^2}}{c} \quad [c, \operatorname{Re} p > 0]$$

$$6. \int_0^{\infty} \frac{1}{x} e^{-px} [1 - J_0(cx)] dx = -\ln \frac{2p}{p + \sqrt{p^2 + c^2}} \quad [\operatorname{Re} p > 0].$$

$$7. \int_0^{\infty} e^{ipx} J_0(cx) dx = \begin{Bmatrix} 1 \\ i \end{Bmatrix} \frac{1}{\sqrt{c^2 - p^2}} \quad \left[\begin{Bmatrix} 0 \leq p < c \\ 0 < c < p \end{Bmatrix} \right].$$

$$8. \int_0^{\infty} x^{-1/2} e^{ipx} J_{n+1/2}(cx) dx = i^{n+1} \sqrt{\frac{2}{\pi c}} \left[Q_n\left(\frac{p}{c}\right) - \frac{i\pi}{2} P_n\left(\frac{p}{c}\right) \right] \quad [-c \leq p \leq c].$$

$$9. \int_0^{\infty} x^{\alpha-1} e^{\pm icx} J_{\nu}(cx) dx = \frac{1}{\pi^{3/2} (2c)^{\alpha}} e^{\pm i\pi(\alpha+\nu)/2} \Gamma\left[\alpha+\nu, \alpha-\nu, \frac{1}{2}-\alpha\right] \sin(\nu-\alpha)\pi$$

$[c, \operatorname{Re}(\alpha+\nu) > 0, \operatorname{Re} \alpha < 1/2]$

$$10. \int_{-\infty}^{\infty} e^{ipx} J_n(cx) dx = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{2i^n}{\sqrt{c^2 - p^2}} T_n\left(\frac{p}{c}\right) \quad \left[\begin{Bmatrix} -c < p < c \\ |p| > c \end{Bmatrix} \right].$$

$$11. \int_{-\infty}^{\infty} \frac{1}{x} e^{ipx} J_n(cx) dx = -\begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{2i^{n-1}}{nc} \sqrt{c^2 - p^2} U_{n-1}\left(\frac{p}{c}\right) \quad \left[\begin{Bmatrix} -c < p < c \\ |p| > c \end{Bmatrix} \right].$$

$$12. \int_{-\infty}^{\infty} x^{-1/2} e^{ipx} J_{n+1/2}(cx) dx = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} i^n \sqrt{\frac{2\pi}{c}} P_n\left(\frac{p}{c}\right) \quad \left[\begin{Bmatrix} -c < p < c \\ |p| > c \end{Bmatrix} \right].$$

$$13. \int_0^{\infty} \frac{e^{icz}}{x+z} J_0(cx) dx = \frac{\pi i}{2} e^{-ics} H_0^{(1)}(cz) \quad [c > 0; |\arg z| < \pi]$$

$$14. \int_0^{\infty} \frac{1}{x} [e^{-ax} - J_0(cx)] dx = \ln \frac{c}{2a} \quad [a, c > 0].$$

$$15. \int_0^a x^{\alpha-1} (a-x)^{\beta-1} e^{\pm icx} J_{\nu}(cx) dx = a^{\alpha+\beta+\nu-1} \left(\frac{c}{2}\right)^{\nu} \Gamma\left[\alpha+\nu, \beta, \alpha+\nu+\beta, \nu+1\right] {}_2F_3\left(\nu+\frac{1}{2}, \alpha+\nu; 2\nu+1, \alpha+\beta+\nu; \pm 2iac\right)$$

$[a, c, \operatorname{Re} \beta, \operatorname{Re}(\alpha+\nu) > 0]$

$$16. \int_a^{\infty} x^{-\nu} (x-a)^{\beta-1} e^{icx} J_{\nu}(cx) dx = \frac{e^{i\beta\pi/2}}{\sqrt{\pi}} (2c)^{\nu-\beta} \Gamma\left[\beta, \nu-\beta+1/2, 1+2\nu-\beta\right] {}_1F_1\left(\frac{1}{2}+\nu-\beta; 1+2\nu-\beta; 2iac\right)$$

$[a, c > 0; 0 < \operatorname{Re} \beta < \operatorname{Re} \nu + 1/2]$

$$17. \int_0^{\infty} \frac{x^{\nu} J_{\nu}(cx)}{e^{ax} - 1} dx = \frac{(2c)^{\nu}}{\sqrt{\pi}} \Gamma\left(\nu + \frac{1}{2}\right) \sum_{k=1}^{\infty} \frac{1}{(a^2 k^2 + c^2)^{\nu+1/2}}$$

[Re $\nu > 0$; Re $a > |\operatorname{Im} c|$].

2.12.9. Интегралы от $x^{\alpha} e^{-px \pm i r} J_{\nu}(cx)$

$$1. \int_0^{\infty} e^{-px^2} J_{\nu}(cx) dx = \frac{1}{2} \sqrt{\frac{\pi}{p}} \exp\left(-\frac{c^2}{8p}\right) I_{\nu/2}\left(\frac{c^2}{8p}\right)$$

[Re $p > 0$; Re $\nu > -1$; $|\arg c| < \pi$]

$$2. \int_0^{\infty} e^{-px^2} J_1(cx) dx = \frac{1}{c} \left[1 - \exp\left(-\frac{c^2}{4p}\right)\right]$$

[Re $p > 0$; $|\arg c| < \pi$]

$$3. \int_0^{\infty} x^{\alpha-1} e^{-px^2} J_{\nu}(cx) dx = A_{\nu}^{\alpha}$$

[Re p , Re $(\alpha + \nu) > 0$; $|\arg c| < \pi$]

$$A_{\nu}^{\alpha} = c^{\nu} p^{-(\alpha+\nu)/2} 2^{-\nu-1} \Gamma\left[\frac{(\alpha+\nu)/2}{\nu+1}\right] {}_1F_1\left(\frac{\alpha+\nu}{2}; \nu+1; -\frac{c^2}{4p}\right),$$

$$A_{\nu}^{\nu} = 2^{\nu-1} c^{-\nu} \gamma\left(\nu, \frac{c^2}{4p}\right)$$

[Re $\nu > 0$]

$$A_{\nu}^{2n+\nu+2} = \frac{n! c^{\nu}}{2^{\nu+1} p^{n+\nu+1}} \exp\left(-\frac{c^2}{4p}\right) L_n^{\nu}\left(\frac{c^2}{4p}\right)$$

[Re $\nu > -n-1$]

$$A_{\nu}^2 = \frac{c \sqrt{\pi}}{8p^{3/2}} \exp\left(-\frac{c^2}{8p}\right) \left[I_{(\nu-1)/2}\left(\frac{c^2}{8p}\right) - I_{(\nu+1)/2}\left(\frac{c^2}{8p}\right)\right]$$

[Re $\nu > -2$]

$$A_1^{2n+1} = \frac{(-1)^{n-1} n!}{c p^n} \exp\left(-\frac{c^2}{8p}\right) k_{2n}\left(\frac{c^2}{8p}\right).$$

$$4. \int_0^{\infty} x^{\alpha-1} e^{-p\sqrt{x}} J_{\nu}(cx) dx =$$

$$\begin{aligned} &= \frac{2^{\alpha-1}}{c^{\alpha}} \Gamma\left[\frac{(\alpha+\nu)/2}{1+(\nu-\alpha)/2}\right] {}_2F_3\left(\frac{\alpha-\nu}{2}, \frac{\alpha+\nu}{2}; \frac{1}{4}, \frac{1}{2}, \frac{3}{4}; -\frac{p^4}{64c^2}\right) - \\ &- \frac{2^{\alpha-1/2} p}{c^{\alpha+1/2}} \Gamma\left[\frac{(1+2\nu+2\alpha)/4}{(3+2\nu-2\alpha)/4}\right] {}_2F_3\left(\frac{1+2\alpha-2\nu}{4}, \frac{1+2\alpha+2\nu}{4}; \frac{1}{2}, \frac{3}{4}, \frac{5}{4}; -\frac{p^4}{64c^2}\right) + \\ &+ \frac{2^{\alpha-1} p^2}{c^{\alpha+1}} \Gamma\left[\frac{(1+\alpha+\nu)/2}{(1+\nu-\alpha)/2}\right] {}_2F_3\left(\frac{1+\alpha-\nu}{2}, \frac{1+\alpha+\nu}{2}; \frac{3}{4}, \frac{5}{4}, \frac{3}{2}; -\frac{p^4}{64c^2}\right) - \\ &- \frac{2^{\alpha-1/2} p^3}{3c^{\alpha+3/2}} \Gamma\left[\frac{(3+2\alpha+2\nu)/4}{(1-2\alpha+2\nu)/4}\right] {}_2F_3\left(\frac{3+2\alpha-2\nu}{4}, \frac{3+2\alpha+2\nu}{4}; \frac{5}{4}, \frac{3}{2}, \frac{7}{4}; -\frac{p^4}{64c^2}\right) \end{aligned}$$

[c , Re p , Re $(\alpha + \nu) > 0$]

$$5. \int_0^{\infty} x^{-1/2} e^{-p\sqrt{x}} J_{\nu}(cx) dx =$$

$$= \sqrt{\frac{2}{c\pi}} \Gamma\left(\nu + \frac{1}{2}\right) D_{-\nu-1/2}\left(\frac{p}{\sqrt{2c}} e^{\pi i/4}\right) D_{-\nu-1/2}\left(\frac{p}{\sqrt{2c}} e^{-\pi i/4}\right)$$

[c , Re $p > 0$, Re $\nu > -1/2$]

$$6. \int_0^{\infty} x^{-1/2} e^{-p\sqrt{x}} J_0(cx) dx = \frac{\pi p}{8c} \left[J_{1/4}^2\left(\frac{p^2}{8c}\right) + Y_{1/4}^2\left(\frac{p^2}{8c}\right)\right]$$

[c , Re $p > 0$]

$$7. \int_0^{\infty} x^{-1/2} e^{-p\sqrt{x}} J_{3/2}(cx) dx = \frac{p^2}{8\sqrt{2\pi c^3}} \Gamma\left(-\frac{1}{2}, \frac{ip^2}{4c}\right) \Gamma\left(-\frac{1}{2}, -\frac{ip^2}{4c}\right)$$

[c, Re p > 0]

$$8. \int_0^{\infty} x^{-1/4} e^{-p\sqrt{x}} J_{\pm 1/4}(cx) dx = \frac{\sqrt{\pi p}}{2c} \left[H_{\mp 1/4}\left(\frac{p^2}{4c}\right) - Y_{\mp 1/4}\left(\frac{p^2}{4c}\right) \right]$$

[c, Re p > 0].

$$9. \int_0^{\infty} x^{1/4} e^{-p\sqrt{x}} J_{-(2\pm 1)/4}(cx) dx =$$

$$= \pm \frac{\sqrt{\pi}}{4c^2} p^{3/2} \left[H_{-(2\mp 1)/4}\left(\frac{p^2}{4c}\right) - Y_{-(2\mp 1)/4}\left(\frac{p^2}{4c}\right) \right]$$

[c, Re p > 0]

$$10. \int_0^{\infty} x^{\alpha-1} e^{-p/x} J_{\nu}(cx) dx = \frac{2^{\alpha-1}}{c^{\alpha}} \Gamma\left[\frac{(\nu+\alpha)/2}{1+(\nu-\alpha)/2}\right] \times$$

$$\times {}_0F_3\left(\frac{1}{2}, 1-\frac{\nu+\alpha}{2}, 1+\frac{\nu-\alpha}{2}; -\frac{c^2 p^2}{16}\right) -$$

$$- 2^{\alpha-2} c^{1-\alpha} p \Gamma\left[\frac{(\alpha+\nu-1)/2}{(3+\nu-\alpha)/2}\right] {}_0F_3\left(\frac{3}{2}, \frac{3-\nu-\alpha}{2}, \frac{3+\nu-\alpha}{2}; -\frac{c^2 p^2}{16}\right) +$$

$$+ \frac{c^{\nu} p^{\alpha+\nu}}{2^{\nu}} \Gamma\left[\frac{-\alpha-\nu}{\nu+1}\right] {}_0F_3\left(1+\frac{\alpha+\nu}{2}, \frac{1+\alpha+\nu}{2}, \nu+1; -\frac{c^2 p^2}{16}\right)$$

[c, Re p > 0; Re α < 3/2]

$$11. \int_0^{\infty} \frac{1}{x} e^{-p/x} J_{\nu}(cx) dx = 2J_{\nu}(\sqrt{2cp}) K_{\nu}(\sqrt{2cp})$$

[c, Re p > 0]

$$12. \int_0^{\infty} \frac{1}{x^2} e^{-p/x} J_0(cx) dx = \sqrt{\frac{2c}{p}} [J_0(\sqrt{2cp}) K_1(\sqrt{2cp}) + J_1(\sqrt{2cp}) K_0(\sqrt{2cp})]$$

[c, Re p > 0].

$$13. \int_0^{\infty} \frac{1}{x^2} e^{-p/x} J_1(cx) dx = \frac{2c}{p} J_2(\sqrt{2cp}) K_2(\sqrt{2cp})$$

[c, Re p > 0].

$$14. \int_0^{\infty} x^{\alpha-1} e^{-p/x^2} J_{\nu}(cx) dx =$$

$$= \frac{2^{\alpha-1}}{c^{\alpha}} \Gamma\left[\frac{(\alpha+\nu)/2}{1+(\nu-\alpha)/2}\right] {}_0F_2\left(1-\frac{\nu+\alpha}{2}, 1+\frac{\nu-\alpha}{2}; \frac{c^2 p}{4}\right) +$$

$$+ \frac{c^{\nu} p^{(\alpha+\nu)/2}}{2^{\nu+1}} \Gamma\left[\frac{-(\alpha+\nu)/2}{\nu+1}\right] {}_0F_2\left(1+\frac{\alpha+\nu}{2}, \nu+1; \frac{c^2 p}{4}\right)$$

[c, Re p > 0; Re α < 3/2]

$$15. \int_0^{\infty} \frac{1}{x^2} e^{-p/x^2} J_{\pm 1/2}(cx) dx = \frac{c}{4} \left(\frac{c^2 p}{4}\right)^{1/2} [e^{-z} + \varepsilon^{1\pm 1} e^{-\varepsilon z} + \varepsilon^{(1\pm 1)/2} e^{-\varepsilon^2 z}]$$

[z = e^{2\pi i/3}, z = -3\left(\frac{c^2 p}{4}\right)^{1/2}, c, Re p > 0].

2.12.10. Интегралы от $A(x) e^{f(x)} J_\nu(cx)$.

$$1. \int_0^\infty \frac{1}{x} e^{-px-b/x} J_\nu(cx) dx = 2J_\nu(z_-) K_\nu(z_+)$$

$$[z_\pm = \sqrt{2b} (\sqrt{p^2 + c^2} \pm p)^{1/2}; \operatorname{Re} p > |\operatorname{Im} c|; \operatorname{Re} b > 0].$$

$$2. \int_0^\infty \frac{1}{x^2} e^{-px-b/x} J_0(cx) dx = 2c [z_+^{-1} J_1(z_-) K_0(z_+) + z_-^{-1} J_0(z_-) K_1(z_+)]$$

$$[z_\pm = \sqrt{2b} (\sqrt{p^2 + c^2} \pm p)^{1/2}; \operatorname{Re} p > |\operatorname{Im} c|; \operatorname{Re} b > 0].$$

$$3. \int_0^a \frac{x}{\sqrt{a^2 - x^2}} \exp(ip \sqrt{a^2 - x^2}) J_0(cx) dx = I(p, c),$$

$$I(p, c) = (p^2 + c^2)^{-1/2} [\exp(ia \sqrt{p^2 + c^2}) + 2iU_0(a \sqrt{p^2 + c^2} - ap, ac) - iJ_0(ac)]$$

[a, p > 0].

$$4. \int_0^a x^{n+1} (a^2 - x^2)^{(m-1)/2} \exp(ip \sqrt{a^2 - x^2}) J_n(cx) dx =$$

$$= (-1)^n c^n e^{-m\pi i/2} \left(\frac{\partial}{\partial p}\right)^m \left(\frac{\partial}{c \partial c}\right)^n I(p, c) \quad [a, c, p > 0; I(p, c) \text{ см. в 2.12.10.3}].$$

$$5. \int_0^\infty \exp(-p \sqrt{x^2 + z^2}) J_1(cx) dx = \frac{1}{c} [e^{-pz} - p(p^2 + c^2)^{-1/2} \exp(-z \sqrt{p^2 + c^2})]$$

$$[\operatorname{Re} p > |\operatorname{Im} c|; \operatorname{Re} z > 0].$$

$$6. \int_0^\infty x^{\nu+1} \exp(-p \sqrt{x^2 + z^2}) J_\nu(cx) dx =$$

$$= \sqrt{\frac{2}{\pi}} c^\nu p z^{3/2} (p^2 + c^2)^{-\nu-3/4} K_{\nu+3/2}(z \sqrt{p^2 + c^2})$$

$$[\operatorname{Re} p > |\operatorname{Im} c|; \operatorname{Re} z > 0; \operatorname{Re} \nu > -1].$$

$$7. \int_0^\infty x \exp(-p \sqrt{x^2 + z^2}) J_0(cx) dx =$$

$$= p(p^2 + c^2)^{-3/2} (1 + z \sqrt{p^2 + c^2}) \exp(-z \sqrt{p^2 + c^2})$$

$$[\operatorname{Re} p > |\operatorname{Im} c|; \operatorname{Re} z > 0].$$

$$8. \int_0^\infty \frac{\exp(-p \sqrt{x^2 + z^2})}{\sqrt{x^2 + z^2}} J_\nu(cx) dx = A_\nu$$

$$[\operatorname{Re} p > |\operatorname{Im} c|; \operatorname{Re} z > 0; \operatorname{Re} \nu > -1; 2z_\pm = z(\sqrt{p^2 + c^2} \pm p)],$$

$$A_\nu = I_{\nu/2}(z_-) K_{\nu/2}(z_+),$$

$$A_0 = \frac{\exp(-z \sqrt{p^2 + c^2})}{\sqrt{p^2 + c^2}},$$

$$A_1 = \frac{1}{c^2} [e^{-pz} - \exp(-z \sqrt{p^2 + c^2})].$$

$$9. \int_0^{\infty} x^{\alpha-1} \frac{\exp(-p \sqrt{x^2+z^2})}{\sqrt{x^2+z^2}} J_{\nu}(cx) dx =$$

$$= \frac{z^{\alpha+\nu-1} c^{\nu}}{2(\nu-\alpha+1)/2 \sqrt{\pi} p^{(\alpha+\nu-1)/2}} \sum_{k=0}^{\infty} \frac{(-1)^k (cz)^{2k}}{(2p)^k k!} \Gamma \left[\frac{(\alpha+\nu)/2+k}{\nu+k+1} \right] K_{(\alpha+\nu-1)/2+k}(pz)$$

[Re $p > |\operatorname{Im} c|$; Re z , Re $(\alpha+\nu) > 0$].

$$10. \int_0^{\infty} x^{\nu+1} \frac{\exp(-p \sqrt{x^2+z^2})}{\sqrt{x^2+z^2}} J_{\nu}(cx) dx = B_{\nu}$$

[Re $p > |\operatorname{Im} c|$; Re $z > 0$; Re $\nu > -1$].

$$B_{\nu} = \sqrt{\frac{2}{\pi}} c^{\nu} z^{\nu+1/2} (p^2+c^2)^{-(2\nu+1)/4} K_{\nu+1/2}(z \sqrt{p^2+c^2}),$$

$$B_0 = (p^2+c^2)^{-1/2} \exp(-z \sqrt{p^2+c^2}),$$

$$B_1 = c (p^2+c^2)^{-3/2} (1+z \sqrt{p^2+c^2}) \exp(-z \sqrt{p^2+c^2}).$$

$$11. \int_0^{\infty} x \frac{\exp(-p \sqrt{x^2-z^2})}{\sqrt{x^2-z^2}} J_0(cx) dx = (p^2+c^2)^{-1/2} \exp(-iz \sqrt{p^2+c^2})$$

[Re $p > |\operatorname{Im} c|$; $\arg \sqrt{x^2-z^2} = \pi/2$ при $x < z$].

$$12. \int_0^{\infty} \frac{x^{-\mu}}{\sqrt{x^2+z^2}} (\sqrt{x^2+z^2} \pm z)^{\mu} \exp(-p \sqrt{x^2+z^2}) J_{\nu}(cx) dx =$$

$$= \frac{1}{cz} \Gamma \left[\frac{(1+\nu \mp \mu)/2}{1+\nu} \right] W_{\pm \mu/2, \nu/2}(z \sqrt{p^2+c^2} + pz) M_{\mp \mu/2, \nu/2}(z \sqrt{p^2+c^2} - pz)$$

[Re $p > |\operatorname{Im} c|$; Re $z > 0$; Re $(\nu \mp \mu) > -1$].

$$13. \int_0^{\infty} \frac{x^{1 \pm \nu}}{\sqrt{x^2+z^2}} (\sqrt{x^2+z^2} \pm z)^{\mp \nu} \exp(-p \sqrt{x^2+z^2}) J_{\nu}(cx) dx =$$

$$= c^{\nu} (\sqrt{p^2+c^2} + p)^{-\nu} (p^2+c^2)^{-1/2} \exp(-z \sqrt{p^2+c^2})$$

[Re $p > |\operatorname{Im} c|$; Re $z > 0$; Re $\nu > -1$].

$$14. \int_0^{\infty} \frac{x^{\nu-1}}{\sqrt{x^2+z^2}} (\sqrt{x^2+z^2} + z)^{1-\nu} \exp(-p \sqrt{x^2+z^2}) J_{\nu}(cx) dx =$$

$$= (cz)^{-\nu} e^{-pz} \gamma(\nu, z \sqrt{p^2+c^2} - pz)$$

[Re $p > |\operatorname{Im} c|$; Re z , Re $\nu > 0$].

2.12.11. Интегралы от $A(x) e^{-px} J_{\nu}(\varphi(x))$

$$1. \int_0^{\infty} e^{-px} J_0(c \sqrt{x^2+xz}) dx = (p^2+c^2)^{-1/2} \exp\left(\frac{pz - z \sqrt{p^2+c^2}}{2}\right)$$

[Re $p > |\operatorname{Im} c|$; $|\arg z| < \pi$].

$$2. \int_0^{\infty} \frac{x^{\rho-1}}{(x+z)^{\rho}} e^{-px} J_{\nu}(c \sqrt{x^2+xz}) dx =$$

$$= \frac{2}{cz} \Gamma \left[\frac{\rho+\nu/2}{\nu+1} \right] e^{pz/2} W_{1/2-\rho, \nu/2}(2_+) M_{\rho-1/2, \nu/2}(2_-)$$

[$2z_{\pm} = z(\sqrt{p^2+c^2} \pm p)$, Re $p > |\operatorname{Im} c|$; Re $(\nu+2\rho) > 0$; $|\arg z| < \pi$].

$$\begin{aligned}
3. \int_0^{\infty} \frac{x^{-\rho}}{(x+z)^{\rho}} e^{-px} J_{\nu}(c \sqrt{x^2+xz}) dx = \\
= \left(\frac{c}{2}\right)^{\nu} \frac{z^{(\nu+1)/2-\rho}}{\sqrt{\pi}} e^{pz/2} \Gamma(\nu/2-\rho+1) (p^2+c^2)^{(2\rho-\nu-1)/4} \times \\
\times \sum_{k=0}^{\infty} \frac{(\nu/2+\rho)_k}{k! \Gamma(\nu+k+1)} \left(\frac{ic}{2\sqrt{p^2+c^2}}\right)^k K_{(\nu+1)/2+k-\rho} \left(\frac{z}{2} \sqrt{p^2+c^2}\right) \\
[\operatorname{Re} p > |\operatorname{Im} c|; \operatorname{Re}(\nu-2\rho) > -2, |\arg z| < \pi]
\end{aligned}$$

$$\begin{aligned}
4. \int_0^{\infty} \frac{x^{\nu/2-1}}{(x+z)^{\nu/2}} e^{-px} J_{\nu}(c \sqrt{x^2+xz}) dx = \left(\frac{2}{cz}\right)^{\nu} \gamma\left(\nu, \frac{z}{2} \sqrt{p^2+c^2} - \frac{zp}{2}\right) \\
[\operatorname{Re} p > |\operatorname{Im} c|; \operatorname{Re} \nu > 0; |\arg z| < \pi]
\end{aligned}$$

$$\begin{aligned}
5. \int_0^{\infty} \frac{x^{\nu/2}}{(x+z)^{\nu/2}} e^{-px} J_{\nu}(c \sqrt{x^2+xz}) dx = \\
= \frac{c^{\nu}}{\sqrt{p^2+c^2}} (p + \sqrt{p^2+c^2})^{-\nu} \exp\left(\frac{pz - z\sqrt{p^2+c^2}}{2}\right) \\
[\operatorname{Re} p > |\operatorname{Im} c|; \operatorname{Re} \nu > -1; |\arg z| < \pi]
\end{aligned}$$

$$\begin{aligned}
6. \int_0^{\infty} (x^2+xz)^{\nu/2} e^{-px} J_{\nu}(c \sqrt{x^2+xz}) dx = \\
= \frac{1}{\sqrt{\pi}} \left(\frac{c}{2}\right)^{\nu} \left(\frac{z}{\sqrt{p^2+c^2}}\right)^{\nu+1/2} e^{pz/2} K_{\nu+1/2}\left(\frac{z}{2} \sqrt{p^2+c^2}\right) \\
[\operatorname{Re} p > |\operatorname{Im} c|; \operatorname{Re} \nu > -1; |\arg z| < \pi]
\end{aligned}$$

$$\begin{aligned}
7. \int_0^{\infty} \frac{e^{-px}}{\sqrt{x^2+xz}} J_{\nu}(c \sqrt{x^2+xz}) dx = e^{pz/2} I_{\nu/2}(z_-) K_{\nu/2}(z_+) \\
[z_{\pm} = z(\sqrt{p^2+c^2} \pm p); \operatorname{Re} p > |\operatorname{Im} c|; \operatorname{Re} \nu > -1; |\arg z| < \pi]
\end{aligned}$$

$$\begin{aligned}
8. \int_0^{\infty} \frac{e^{-px}}{\sqrt{x^2+xz}} J_1(c \sqrt{x^2+xz}) dx = \frac{2}{cz} \left[1 - \exp\left(\frac{pz - z\sqrt{p^2+c^2}}{2}\right)\right] \\
[\operatorname{Re} p > |\operatorname{Im} c|; |\arg z| < \pi]
\end{aligned}$$

$$\begin{aligned}
9. \int_0^{\infty} (x^2+xz)^{1/2} e^{-px} J_1(c \sqrt{x^2+xz}) dx = \\
= \frac{c}{(p^2+c^2)^{3/2}} \left(1 + \frac{z}{2} \sqrt{p^2+c^2}\right) \exp\left(\frac{pz - z\sqrt{p^2+c^2}}{2}\right) \\
[\operatorname{Re} p > |\operatorname{Im} c|; |\arg z| < \pi]
\end{aligned}$$

2.12.12. Интегралы от $A(x) e^{f(x)} J_{\nu}(\varphi(x))$.

$$\begin{aligned}
1. \int_0^a \frac{x^{\alpha-1}}{x^2-a^2} \exp\left(-b \frac{a^2+x^2}{a^2-x^2}\right) J_{\nu}\left(\frac{2cx}{a^2-x^2}\right) dx = \\
= -\frac{a^{\alpha-1}}{2c} \Gamma\left[\frac{(\nu+\alpha)/2}{\nu+1}\right] M_{(\alpha-1)/2, \nu/2}(z_-) W_{(1-\alpha)/2, \nu/2}(z_+) \\
[z_{\pm} = a^{-1}(\sqrt{a^2b^2+c^2} \pm ab), a, b, c, \operatorname{Re}(\alpha+\nu) > 0].
\end{aligned}$$

$$2. \int_a^\infty \frac{x^{\alpha-1}}{x^2-a^2} \exp\left(-b \frac{x^2+a^2}{x^2-a^2}\right) J_\nu\left(\frac{2cx}{x^2-a^2}\right) dx =$$

$$= \frac{a^{\alpha-1}}{2c} \Gamma\left[\frac{1+(\nu-\alpha)/2}{\nu+1}\right] M_{(1-\alpha)/2, \nu/2}(z_-) W_{(\alpha-1)/2, \nu/2}(z_+)$$

$$[z_\pm = a^{-1}(\sqrt{a^2b^2+c^2} \pm ab), \quad a, b, c > 0; \operatorname{Re} \alpha < \operatorname{Re} \nu + 2].$$

$$3. \int_0^\infty \frac{x^{\alpha-1}}{x^2+z^2} \exp\left(\pm b \frac{z^2-x^2}{z^2+x^2}\right) J_\nu\left(\frac{2cx}{x^2+z^2}\right) dx =$$

$$= \frac{z^{\alpha-1}}{2c} \Gamma\left[\frac{1+(\nu-\alpha)/2}{1+\nu}, \frac{(\nu+\alpha)/2}{1+\nu}\right] M_{\mp(1-\alpha)/2, \nu/2}(z_-) M_{\mp(1-\alpha)/2, \nu/2}(z_+)$$

$$[z_\pm = z^{-1}(bz \pm \sqrt{b^2z^2-c^2} \pm bz); \quad b, c, \operatorname{Re} z > 0; \quad -\operatorname{Re} \nu < \operatorname{Re} \alpha < \operatorname{Re} \nu + 2].$$

2.12.13. Интегралы, содержащие гиперболические функции и $J_\nu(cx)$.

$$1. \int_0^\infty x^{\nu+1} \frac{\operatorname{sh} ax}{\operatorname{sh} bx} J_\nu(cx) dx = \frac{2}{b} \left(\frac{\pi}{b}\right)^{\nu+1} \sum_{k=1}^\infty (-1)^{k-1} k^{\nu+1} \sin \frac{ak\pi}{b} K_\nu\left(\frac{ck\pi}{b}\right)$$

$$[\operatorname{Re} \nu > -1; \operatorname{Re} b > |\operatorname{Re} a| + |\operatorname{Im} c|].$$

$$2. \int_0^a x^{n+2} \operatorname{sh}(c \sqrt{a^2-x^2}) J_n(cx) dx = \frac{a^{2n+3} c^{n+1}}{(2n+3)!!} \quad [a > 0].$$

$$3. \int_0^a \frac{x^{n+1}}{\sqrt{a^2-x^2}} \operatorname{ch}(c \sqrt{a^2-x^2}) J_n(cx) dx = \frac{a^{2n+1} c^n}{(2n+1)!!} \quad [a > 0].$$

$$4. \int_0^\infty e^{-px} \left\{ \frac{\operatorname{sh} bx}{\operatorname{ch} bx} \right\} J_0(cx) dx = \frac{u_+ \mp u_-}{2u_+ u_-}$$

$$[c > 0; \operatorname{Re} p > |\operatorname{Re} b|; \quad a_\pm = \sqrt{c^2 + (\rho \pm b)^2}].$$

$$5. \int_0^\infty \frac{1}{x} e^{-bx} \operatorname{sh} bx J_0(cx) dx = \frac{1}{2} \ln \frac{2b + \sqrt{c^2 + 4b^2}}{c} \quad [c, \operatorname{Re} b > 0].$$

$$6. \int_0^\infty x e^{-b/x} \operatorname{sh} \frac{b}{x} J_0(cx) dx = \frac{2b}{c} J_1(2\sqrt{bc}) K_1(2\sqrt{bc}) \quad [c, \operatorname{Re} b > 0].$$

$$7. \int_0^\infty e^{-px^2} \operatorname{sh} bx^2 J_1(cx) dx = \frac{1}{c} \exp\left(-\frac{c^2 p}{4p^2 - 4b^2}\right) \operatorname{sh} \frac{bc^2}{4p^2 - 4b^2}$$

$$[\operatorname{Re}(p-b) > 0].$$

2.12.14. Интегралы, содержащие $J_\nu(c \operatorname{sh} x)$ и $J_\nu(c \operatorname{ch}^\pm x)$.

$$1. \int_0^\infty \left\{ \frac{\operatorname{sh} bx}{\operatorname{ch} bx} \right\} J_\nu(c \operatorname{sh} x) dx =$$

$$= \frac{1}{2} \left[I_{(\nu-b)/2}\left(\frac{c}{2}\right) K_{(\nu+b)/2}\left(\frac{c}{2}\right) \mp I_{(\nu+b)/2}\left(\frac{c}{2}\right) K_{(\nu-b)/2}\left(\frac{c}{2}\right) \right]$$

$$[c > 0; \operatorname{Re} \nu > -(3 \pm 1)/2; |\operatorname{Re} b| < 1/2]$$

$$2. \int_0^{\infty} \operatorname{sh} bx J_0(c \operatorname{sh} x) dx = \frac{1}{\pi} \sin \frac{b\pi}{2} K_{b/2}^2 \left(\frac{c}{2} \right)$$

$$[c > 0; |\operatorname{Re} b| < 1/2].$$

$$3. \int_0^{\infty} \operatorname{ch} bx J_{\nu}(c \operatorname{ch} x) dx = -\frac{\pi}{4} \left[J_{(\nu+b)/2} \left(\frac{c}{2} \right) Y_{(\nu-b)/2} \left(\frac{c}{2} \right) + \right. \\ \left. + J_{(\nu-b)/2} \left(\frac{c}{2} \right) Y_{(\nu+b)/2} \left(\frac{c}{2} \right) \right]$$

$$[c > 0; |\operatorname{Re} b| < 1/2].$$

$$4. \int_{-\infty}^{\infty} e^{-px - b \operatorname{th} x} J_{\nu} \left(\frac{c}{\operatorname{ch} x} \right) \frac{dx}{\operatorname{ch} x} = \frac{1}{c} \Gamma \left[\frac{(1+\nu+p)/2}{1+\nu}, \frac{(1+\nu-p)/2}{1+\nu} \right] \times \\ \times M_{-p/2, \nu/2} (b + \sqrt{b^2 - c^2}) M_{-p/2, \nu/2} (b - \sqrt{b^2 - c^2})$$

$$[b, c > 0; |\operatorname{Re} p| < \operatorname{Re} \nu + 1].$$

2.12.15. Интегралы от $x^{\alpha} \left\{ \begin{smallmatrix} \sin bx \\ \cos bx \end{smallmatrix} \right\} J_{\nu}(cx)$.

Обозначение: $\delta = \left\{ \begin{smallmatrix} 1 \\ 0 \end{smallmatrix} \right\}$.

$$1. \int_0^1 x^{\alpha-1} \left\{ \begin{smallmatrix} \sin cx \\ \cos cx \end{smallmatrix} \right\} J_{\nu}(cx) dx = \\ = \left\{ \begin{smallmatrix} 0 \\ 1 \end{smallmatrix} \right\} \frac{J_{\nu}(c)}{\alpha + \nu} + 2 \sum_{k=0}^{\infty} \frac{(\nu - \alpha + 1)_{k-1}}{(\nu + \alpha)_{k+1}} (\nu + k) \left\{ \begin{smallmatrix} \sin (c - k\pi/2) \\ \cos (c - k\pi/2) \end{smallmatrix} \right\} J_{\nu+k}(c)$$

$$[\operatorname{Re}(\alpha + \nu) > -\delta].$$

$$2. \int_0^{\infty} \left\{ \begin{smallmatrix} \sin bx \\ \cos bx \end{smallmatrix} \right\} J_{\nu}(cx) dx = (c^2 - b^2)^{-1/2} \left\{ \begin{smallmatrix} \sin [\nu \arcsin (b/c)] \\ \cos [\nu \arcsin (b/c)] \end{smallmatrix} \right\}$$

$$[0 < b < c; \operatorname{Re} \nu > -1 - \delta].$$

$$= \pm \frac{c^{\nu}}{\sqrt{b^2 - c^2}} (b + \sqrt{b^2 - c^2})^{-\nu} \left\{ \begin{smallmatrix} \cos (\nu\pi/2) \\ \sin (\nu\pi/2) \end{smallmatrix} \right\}$$

$$[0 < c < b; \operatorname{Re} \nu > -1 - \delta].$$

$$3. \int_0^{\infty} x^{\alpha-1} \left\{ \begin{smallmatrix} \sin bx \\ \cos bx \end{smallmatrix} \right\} J_{\nu}(cx) dx =$$

$$= \frac{2^{\alpha+\delta-1} b^{\delta}}{c^{\alpha+\delta}} \Gamma \left[\frac{(\alpha + \nu + \delta)/2}{1 + (\nu - \alpha - \delta)/2} \right] {}_2F_1 \left(\frac{\alpha + \delta + \nu}{2}, \frac{\alpha + \delta - \nu}{2}; \delta + \frac{1}{2}; \frac{b^2}{c^2} \right)$$

$$[0 < b < c; -\delta - \operatorname{Re} \nu < \operatorname{Re} \alpha < 3/2].$$

$$4. = \frac{c^{\nu}}{2^{\nu} b^{\nu+\alpha}} \left\{ \begin{smallmatrix} \sin [(\alpha + \nu) \pi/2] \\ \cos [(\alpha + \nu) \pi/2] \end{smallmatrix} \right\} \Gamma \left[\frac{\nu + \alpha}{\nu + 1} \right] {}_2F_1 \left(\frac{\alpha + \nu}{2}, \frac{\alpha + \nu + 1}{2}; \nu + 1; \frac{c^2}{b^2} \right)$$

$$[0 < c < b; -\delta - \operatorname{Re} \nu < \operatorname{Re} \alpha < 3/2].$$

$$5. \int_0^{\infty} x^{\nu+(1 \pm 1)/2} \left\{ \begin{smallmatrix} \sin bx \\ \cos bx \end{smallmatrix} \right\} J_{\nu}(cx) dx =$$

$$= \frac{2^{\nu+\delta}}{\sqrt{\pi}} (-b)^{\delta} c^{\nu} \Gamma \left(\nu + \delta + \frac{1}{2} \right) (c^2 - b^2)^{-\nu-\delta-1/2}$$

$$[0 < b < c; -\delta - 1/2 < \operatorname{Re} \nu < 1/2 - \delta].$$

6.
$$= -\frac{2^{v+\delta}}{\sqrt{\pi}} b^\delta c^v \Gamma\left(v+\delta+\frac{1}{2}\right) \sin v\pi (b^2-c^2)^{-v-\delta-1/2}$$

 $[0 < c < b; -\delta-1/2 < \operatorname{Re} v < 1/2-\delta]$
7.
$$\int_0^\infty x^{v+(1\mp 1)/2} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} J_v(cx) dx = 2^{v+1-\delta} \frac{\sqrt{\pi} c^v b^{1-\delta}}{\Gamma(\delta-v-1/2)} (b^2-c^2)_+^{\delta-v-3/2}$$

 $[b, c > 0; b \neq c; -1 < \operatorname{Re} v < \pm 1/2]$
8.
$$\int_0^\infty \frac{1}{x^2} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} J_v(cx) dx = \frac{\sqrt{c^2-b^2}}{v^2-1} \begin{Bmatrix} \sin t \\ \cos t \end{Bmatrix} \mp \frac{b}{v(v^2-1)} \begin{Bmatrix} \cos t \\ \sin t \end{Bmatrix}$$

 $[0 < b < c; \operatorname{Re} v > 1-\delta; t = v \arcsin(b/c)].$
9.
$$= \mp \frac{c^v (b + v \sqrt{b^2-c^2})}{v(v^2-1)(b + \sqrt{b^2-c^2})^v} \begin{Bmatrix} \cos(v\pi/2) \\ \sin(v\pi/2) \end{Bmatrix}$$

 $[0 < c < b; \operatorname{Re} v > 1-\delta]$
10.
$$\int_0^\infty \frac{1}{x} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} J_v(cx) dx = \frac{1}{v} \begin{Bmatrix} \sin[v \arcsin(b/c)] \\ \cos[v \arcsin(b/c)] \end{Bmatrix}$$

 $[0 < b \leq c; \operatorname{Re} v > -\delta].$
11.
$$= \frac{c^v}{v(b + \sqrt{b^2-c^2})^v} \begin{Bmatrix} \sin(v\pi/2) \\ \cos(v\pi/2) \end{Bmatrix}$$

 $[0 < c \leq b; \operatorname{Re} v > -\delta].$
12.
$$\int_0^\infty x^{1-v} \sin bx J_v(cx) dx = \frac{2^{1-v} \sqrt{\pi} b}{\Gamma(v-1/2)} c^v (c^2-b^2)_+^{v-3/2}$$

 $[b, c > 0; b \neq c; \operatorname{Re} v > 1/2]$
13.
$$\int_0^\infty \frac{\sin bx}{x} J_0(cx) dx = \begin{cases} \arcsin(b/c) \\ \pi/2 \end{cases}$$

 $\left[\begin{cases} 0 < b < c \\ 0 < c < b \end{cases} \right].$
14.
$$\int_0^\infty x^{-3/2} \sin bx J_0(cx) dx =$$

$$= 4 \sqrt{\frac{c}{\pi}} [E(k) - (1-k^2) K(k) + k^2 K(\sqrt{1-k^2}) - E(\sqrt{1-k^2})]$$

 $[0 < b < c; k = \sqrt{b+c}/\sqrt{2c}]$

$$= 4k \sqrt{\frac{c}{\pi}} E\left(\frac{1}{k}\right)$$

 $[0 < c < b; k = \sqrt{b+c}/\sqrt{2c}].$
15.
$$\int_0^\infty x^{-5/2} \sin bx J_1(cx) dx = \frac{16c^{3/2}}{15\sqrt{\pi}} [2(1-k^2+k^4) E(k) -$$

 $-(1-k^2)(2-k^2) K(k) + k^2(1+k^2) K(\sqrt{1-k^2}) - 2(1-k^2+k^4) E(\sqrt{1-k^2})]$
 $[0 < b < c; k = \sqrt{b+c}/\sqrt{2c}]$
16.
$$= \frac{16kc^{3/2}}{15\sqrt{\pi}} \left[2(1-k^2+k^4) E\left(\frac{1}{k}\right) - (1-k^2)(1-2k^2) K\left(\frac{1}{k}\right) \right]$$

 $[0 < c < b, k = \sqrt{b+c}/\sqrt{2c}]$
17.
$$\int_0^\infty x^{-n-1/2} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} J_n(cx) dx = A_n^m$$

 $[b, c > 0; k = \sqrt{b+c}/\sqrt{2c}],$

$$A_0^0 = \frac{1}{\sqrt{\pi c}} [K(k) \mp K(\sqrt{1-k^2})] \quad [b < c],$$

$$A_0^0 = \frac{1}{k\sqrt{\pi c}} K\left(\frac{1}{k}\right) \quad [c < b],$$

$$A_1^0 = \frac{1}{\sqrt{\pi c}} [2E(\sqrt{1-k^2}) - K(\sqrt{1-k^2}) \mp 2E(k) \pm K(k)] \quad [b < c],$$

$$A_{1,1}^0 = \frac{\pm 1}{k\sqrt{\pi c}} \left[(2k^2 - 1) K\left(\frac{1}{k}\right) - 2k^2 E\left(\frac{1}{k}\right) \right] \quad [c < b],$$

$$A_1^1 = \frac{4}{3} \sqrt{\frac{c}{\pi}} [(1-k^2) K(k) - (1-2k^2) E(k) \mp (1-2k^2) E(\sqrt{1-k^2}) \mp k^2 K(\sqrt{1-k^2})] \quad [b < c],$$

$$A_1^1 = \frac{4k}{3} \sqrt{\frac{c}{\pi}} \left[(2k^2 - 1) E\left(\frac{1}{k}\right) + 2(1-k^2) K\left(\frac{1}{k}\right) \right] \quad [c < b],$$

$$A_2^0 = -\frac{1}{3\sqrt{\pi c}} [8(1-2k^2)(E(k) \pm E(\sqrt{1-k^2})) \mp (3-8k^2) K(\sqrt{1-k^2}) - (5-8k^2) K(k)] \quad [b < c],$$

$$A_2^0 = -\frac{1}{3k\sqrt{\pi c}} \left[(1-4k^2)(3-4k^2) K\left(\frac{1}{k}\right) + 8(1-2k^2) E\left(\frac{1}{k}\right) \right] \quad [c < b],$$

$$A_2^1 = \mp \frac{4\sqrt{c}}{15\sqrt{\pi}} [(1-k^2)(1-8k^2) K(k) \pm k^2(7-8k^2) K(\sqrt{1-k^2}) - (1-16k^2+16k^4)(E(k) \mp E(\sqrt{1-k^2}))] \quad [b < c],$$

$$A_2^1 = \pm \frac{4k\sqrt{c}}{15\sqrt{\pi}} \left[(1-16k^2+16k^4) E\left(\frac{1}{k}\right) - 8(1-k^2)(1-2k^2) K(\sqrt{1-k^2}) \right] \quad [c < b],$$

$$A_3^0 = \frac{16c^{3/2}}{105\sqrt{\pi}} [(1-k^2)(2+5k^2-8k^4) K(k) \pm k^2(1-11k^2+8k^4) K(\sqrt{1-k^2}) - 2(1-2k^2)(1+4k^2-4k^4)(E(k) \mp E(\sqrt{1-k^2}))] \quad [b < c],$$

$$A_3^0 = \frac{16c^{3/2}k}{105\sqrt{\pi}} \left[(1-k^2)(1+16k^2-16k^4) K\left(\frac{1}{k}\right) - 2(1-2k^2)(1+4k^2-4k^4) E\left(\frac{1}{k}\right) \right] \quad [c < b],$$

$$A_3^1 = \frac{1}{15\sqrt{\pi c}} [\pm 2(23-128k^2+128k^4)(E(k) \mp E(\sqrt{1-k^2})) \mp (31-144k^2+128k^4) K(k) + (15-112k^2+128k^4) K(\sqrt{1-k^2})] \quad [b < c],$$

$$A_3^1 = \pm \frac{1}{15k\sqrt{\pi c}} \left[(1-2k^2)(15-128k^2+128k^4) K\left(\frac{1}{k}\right) + 2(23-128k^2+128k^4) E\left(\frac{1}{k}\right) \right] \quad [c < b],$$

$$A_4^1 = -\frac{4}{105} \sqrt{\frac{c}{\pi}} [(1-k^2)(3-80k^2+128k^4) K(k) \mp k^2(51-176k^2+128k^4) K(\sqrt{1-k^2}) - (3-134k^2+384k^4-256k^6)(E(k) \pm E(\sqrt{1-k^2}))] \quad [b < c],$$

$$A_3^1 = \frac{4k\sqrt{c}}{105\sqrt{\pi}} \left[(1-2k^2)(3-128k^2+128k^4) E\left(\frac{1}{k}\right) - \right. \\ \left. - 2(1-k^2)(27-128k^2+128k^4) K\left(\frac{1}{k}\right) \right] \quad [c < b].$$

$$A_3^2 = \frac{16c^{3/2}}{945\sqrt{\pi}} \left[\mp (1-k^2)(2+15k^2-144k^4+128k^6) K(k) + \right. \\ \left. + k^2(1-103k^2+232k^4-128k^6) K(\sqrt{1-k^2}) \pm \right. \\ \left. \pm 2(1+7k^2-135k^4+256k^6-128k^8)(E(k) \mp E(\sqrt{1-k^2})) \right] \quad [b < c].$$

$$A_3^3 = \mp \frac{16c^{3/2}k}{945\sqrt{\pi}} \left[(1-k^2)(1-2k^2)(1+128k^2-128k^4) K\left(\frac{1}{k}\right) - \right. \\ \left. - (2+14k^2-270k^4+512k^6-256k^8) E\left(\frac{1}{k}\right) \right] \quad [c < b].$$

$$19. \int_0^\infty x^{\alpha-1} \begin{Bmatrix} \sin cx \\ \cos cx \end{Bmatrix} J_\nu(cx) dx = \frac{1}{(2c)^\alpha \sqrt{\pi}} \Gamma \left[\begin{matrix} \nu+\alpha, 1/2-\alpha \\ 1+\nu-\alpha \end{matrix} \right] \begin{Bmatrix} \sin [(\alpha+\nu)\pi/2] \\ \cos [(\alpha+\nu)\pi/2] \end{Bmatrix} \\ [c > 0; -\delta - \operatorname{Re} \nu < \operatorname{Re} \alpha < 1/2].$$

$$20. \int_0^\infty x^{-\nu-1} \sin cx J_\nu(cx) dx = \frac{\pi c^\nu}{2^{\nu+1} \Gamma(\nu+1)} \quad [c > 0; \operatorname{Re} \nu > -1/2].$$

2.12.16. Интегралы, содержащие $A(x)$, $\begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix}$ и $J_\nu(cx)$

Обозначение $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$.

$$1. \int_0^a \frac{1}{\sqrt{a^2-x^2}} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} J_\nu(cx) dx = \\ = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \frac{\pi}{2} J_0(ab) J_{\nu/2}^2\left(\frac{ac}{2}\right) + \pi \sum_{k=1}^\infty (-1)^k J_{2k+\delta}(ab) J_{k+(\nu+\delta)/2}\left(\frac{ac}{2}\right) J_{(\nu-\delta)/2-k}\left(\frac{ac}{2}\right) \\ [a > 0; \operatorname{Re} \nu > 1-\delta]$$

$$2. \int_0^\infty \frac{x^{(1\pm 1)/2-\nu}}{x^2+z^2} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} J_\nu(cx) dx = \frac{\pi}{2} z^{(\pm 1-1)/2-\nu} e^{-bz} I_\nu(cz) \\ [0 < c \leq b; \operatorname{Re} z > 0; \operatorname{Re} \nu > -(2\mp 1)/2]$$

$$3. \int_0^\infty \frac{x^{\nu+2n+(1\mp 1)/2}}{x^2+z^2} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} J_\nu(cx) dx = (-1)^n z^{\nu+2n-(1\pm 1)/2} \begin{Bmatrix} \operatorname{sh} bz \\ \operatorname{ch} bz \end{Bmatrix} K_\nu(cz) \\ [0 < b \leq c; \operatorname{Re} z > 0; -n-1 < \operatorname{Re} \nu < 2\pm 1/2-2n]$$

$$4. \int_0^\infty \frac{x^{2n-\nu-(1\pm 1)/2}}{x^2+z^2} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} J_\nu(cx) dx = (-1)^{n+\delta} \frac{\pi}{2} z^{2n-\nu-\delta-1} e^{-bz} I_\nu(cz) \\ [0 < c \leq b; \operatorname{Re} z > 0; \operatorname{Re} \nu > 2n-\delta-5/2]$$

$$5. \int_0^\infty \frac{1-\cos bx}{x} J_0(cx) dx = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \ln \frac{b+\sqrt{b^2-c^2}}{c} \quad \left[\begin{Bmatrix} 0 < c \leq b \\ 0 < b < c \end{Bmatrix} \right].$$

6. $\int_0^{\infty} \frac{1 - \cos bx}{x^2} J_0(cx) dx = \frac{\pi b}{2} - c$ [0 < c ≤ b].
7. $= \sqrt{c^2 - b^2} - c + b \arcsin \frac{b}{c}$ [0 < b < c].
8. $\int_0^{\infty} \frac{1 - \cos cx}{x^2} J_{4n+2\pm 1}(cx) dx = \frac{c}{2(2n+1)(4n+2\pm 1)}$ [c > 0]
9. $\int_0^{\infty} \frac{bx - \sin bx}{x^3} J_0(cx) dx = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \left(b \ln \frac{b + \sqrt{b^2 - c^2}}{c} - \sqrt{b^2 - c^2} \right)$ [{ 0 < c ≤ b }
{ 0 < b < c }].
10. $\int_0^{\infty} \frac{bx - \sin bx}{x^3} J_0(cx) dx = \frac{\pi b^2}{4} - bc + \frac{\pi c^2}{8}$ [0 < c ≤ b].
11. $= \frac{3}{4} b \sqrt{c^2 - b^2} - bc + \frac{2b^2 + c^2}{4} \arcsin \frac{b}{c}$ [0 < b < c].
12. $\int_0^{\infty} \frac{\sin bx}{x} [1 - J_0(cx)] dx = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \arccos \frac{b}{c}$ [{ 0 < c ≤ b }
{ 0 < b < c }].
13. $\int_0^{\infty} \frac{\cos bx}{x} [1 - J_0(cx)] dx = \ln \frac{b + \sqrt{b^2 - c^2}}{2b}$ [0 < c ≤ b]
14. $= \ln \frac{c}{2b}$ [0 < b < c]
15. $\int_0^{\infty} \frac{\sin bx}{x^2} [1 - J_0(cx)] dx = b - \sqrt{b^2 - c^2} + b \ln \frac{b + \sqrt{b^2 - c^2}}{2b}$ [0 < c ≤ b].
16. $= b + b \ln \frac{c}{2b}$ [0 < b < c].
17. $\int_0^{\infty} \frac{\cos bx}{x^2} [1 - J_0(cx)] dx = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \left(\arccos \frac{b}{c} - \sqrt{c^2 - b^2} \right)$ [{ 0 < c ≤ b }
{ 0 < b < c }].
18. $\int_0^{\infty} \frac{\sin bx}{x^3} [1 - J_0(cx)] dx = \frac{\pi c^2}{8}$ [0 < c ≤ b]
19. $= \frac{1}{4} \left[3b \sqrt{c^2 - b^2} - \pi b^2 + (2b^2 + c^2) \arcsin \frac{b}{c} \right]$ [0 < b < c]
20. $\int_0^{\infty} \frac{1}{x} [\cos cx - J_0(cx)] dx = \ln \frac{c}{2b}$ [b < c < ∞]

2.12.17. Интегралы от $A(x) \begin{Bmatrix} \sin(a+bx) \\ \cos(a+bx) \end{Bmatrix} J_\nu(cx)$.

$$1. \int_0^a x^{\alpha-1} \begin{Bmatrix} \sin(a-x) \\ \cos(a-x) \end{Bmatrix} J_\nu(x) dx = B_\nu^\alpha \quad \left[a, \operatorname{Re}(\alpha+\nu) > 0; \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right].$$

$$B_\nu^\alpha = \pm 2a^\alpha \sum_{k=0}^{\infty} \frac{(-1)^k (1+\nu-\alpha)_{2k+1-\delta}}{(\nu+\alpha)_{2k+\delta-\delta}} (\nu+2k+2-\delta) J_{\nu+2k+\delta-\delta}(a) + \\ + \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \frac{a^\alpha}{\alpha+\nu} J_{\nu+\delta}(a),$$

$$B_\nu^1 = a J_{\nu+\delta}(a) - 2\nu \sum_{k=0}^{\infty} (-1)^k J_{\nu+2k+\delta+1}(a),$$

$$B_{2n}^1 = a J_{2n+\delta}(a) \pm (-1)^n 2n \left[\begin{Bmatrix} \cos a - J_0(a) \\ \sin a \end{Bmatrix} \pm 2 \sum_{k=0}^{n-1} (-1)^k J_{2k+1+\delta}(a) \right],$$

$$B_{2n+1}^1 = a J_{2n+1+\delta}(a) \pm (-1)^n (2n+1) \left[\begin{Bmatrix} \sin a \\ J_0(a) - \cos a \end{Bmatrix} \mp 2 \sum_{k=1-\delta}^n (-1)^k J_{2k+\delta}(a) \right],$$

$$B_\nu^{\nu+1} = \frac{a^{\nu+1}}{2\nu+1} J_{\nu+\delta}(a),$$

$$B_\nu^{1-\nu} = \pm \frac{a^{1-\nu}}{2\nu-1} J_{\nu-\delta}(a) \mp \frac{2^{1-\nu}}{(2\nu-1)\Gamma(\nu)} \begin{Bmatrix} \cos a \\ \sin a \end{Bmatrix} \quad [\nu \neq 1/2].$$

$$B_\nu^0 = \frac{1+\delta}{\nu} J_{\nu+\delta}(a) + \frac{2}{\nu} \sum_{k=1}^{\infty} (-1)^k J_{\nu+2k+\delta}(a).$$

$$2. \int_0^a \sin(a-x) J_1(x) dx = \sin a - a J_0(a) \quad [a > 0].$$

$$3. \int_a^{a+\pi} x^{-3/2} \sin(x-a) J_0(x) dx = -4 \sqrt{a+\pi} J_0(a+\pi) \quad [J_0(a)=0; a > 0].$$

$$4. \int_0^\infty \frac{x^\nu}{x+z} \begin{Bmatrix} \sin(x+z) \\ \cos(x+z) \end{Bmatrix} J_\nu(x) dx = \pm \frac{\pi z^\nu}{2 \cos \nu\pi} \begin{Bmatrix} J_{-\nu}(z) \\ Y_{-\nu}(z) \end{Bmatrix} \quad [|\arg z| < \pi; |\operatorname{Re} \nu| < 1/2].$$

$$5. \int_0^\infty \frac{\sin(x-y)}{x-y} J_0(x) dx = \frac{\pi}{2} J_0(y) \quad [y > 0].$$

$$6. \int_{-\infty}^\infty \frac{|x|^{2n-\nu}}{x+z} \sin(bx+bz) J_\nu(cx) dx = \pi z^{2n-\nu} J_\nu(cz) \quad [0 < c \leq b; \operatorname{Re} \nu > 2n-3/2; \operatorname{Im} z \neq 0].$$

$$7. \int_{-\infty}^\infty \frac{|x|}{x+z} \sin(bx+bz) J_0(cx) dx = 0 \quad [0 \leq b < c; \operatorname{Im} z \neq 0].$$

$$8. \int_{-\infty}^\infty |x|^{-\nu} \frac{\sin(x-y)}{x-y} J_\nu(x) dx = \frac{\pi}{y^\nu} J_\nu(y) \quad [y > 0; \operatorname{Re} \nu > -3/2].$$

2.12.18. Интегралы от $x^\alpha \left\{ \begin{matrix} \sin (a+bx^2) \\ \cos (a+bx^2) \end{matrix} \right\} J_\nu(cx)$.

$$1. \int_0^a x^{1-n} \left\{ \begin{matrix} \sin bx^2 \\ \cos bx^2 \end{matrix} \right\} J_n(cx) dx = \\ = \frac{(2b)^{n-1} a^{n-2}}{c^n} \left[\mp \left\{ \begin{matrix} \cos a^2 b \\ \sin a^2 b \end{matrix} \right\} U_n \left(\frac{c^2}{2b}, ac \right) - \left\{ \begin{matrix} \sin a^2 b \\ \cos a^2 b \end{matrix} \right\} U_{n+1} \left(\frac{c^2}{2b}, ac \right) + \right. \\ \left. + U_{n+(1 \mp 1)/2} \left(\frac{c^2}{2b}, ac \right) \right] \quad [a > 0]$$

$$2. \int_0^\infty \left\{ \begin{matrix} \sin bx^2 \\ \cos bx^2 \end{matrix} \right\} J_\nu(cx) dx = \frac{1}{2} \sqrt{\frac{\pi}{b}} \left\{ \begin{matrix} \sin t \\ \cos t \end{matrix} \right\} J_{\nu/2} \left(\frac{c^2}{8b} \right) \\ [t = (\nu+1)\pi/4 - c^2/(8b); b, c > 0; \operatorname{Re} \nu > -2 \mp 1].$$

$$3. \int_0^\infty \left\{ \begin{matrix} \sin bx^2 \\ \cos bx^2 \end{matrix} \right\} J_1(cx) dx = \frac{2}{c} \sin \frac{c^2}{8b} \left\{ \begin{matrix} \cos [c^2/(8b)] \\ \sin [c^2/(8b)] \end{matrix} \right\} \quad [b, c > 0]$$

$$4. \int_0^\infty x^{\alpha-1} \left\{ \begin{matrix} \sin bx^2 \\ \cos bx^2 \end{matrix} \right\} J_\nu(cx) dx = \\ = 2^{-\nu-1} b^{-(\alpha+\nu)/2} c^\nu \left\{ \begin{matrix} \sin [(\alpha+\nu)\pi/4] \\ \cos [(\alpha+\nu)\pi/4] \end{matrix} \right\} \Gamma \left[\frac{(\alpha+\nu)/2}{\nu+1} \right] \times \\ \times {}_2F_3 \left(\frac{\alpha+\nu}{4}, \frac{2+\alpha+\nu}{4}; \frac{1}{2}, \frac{\nu+1}{2}, \frac{\nu}{2}+1; -\frac{c^4}{64b^2} \right) \mp \\ \mp 2^{-\nu-3} b^{-(\alpha+\nu)/2-1} c^{\nu+2} \left\{ \begin{matrix} \cos [(\alpha+\nu)\pi/4] \\ \sin [(\alpha+\nu)\pi/4] \end{matrix} \right\} \Gamma \left[\frac{(\alpha+\nu)/2+1}{\nu+2} \right] \times \\ \times {}_2F_3 \left(\frac{2+\alpha+\nu}{4}, 1+\frac{\alpha+\nu}{4}; \frac{3}{2}, \frac{\nu}{2}+1, \frac{\nu+3}{2}; -\frac{c^4}{64b^2} \right) \\ [b, c > 0; -1 \mp 1 - \operatorname{Re} \nu < \operatorname{Re} \alpha < 5/2].$$

$$5. \int_0^\infty x^{\nu+1} \left\{ \begin{matrix} \sin bx^2 \\ \cos bx^2 \end{matrix} \right\} J_\nu(cx) dx = \frac{c^\nu}{(2b)^{\nu+1}} \left\{ \begin{matrix} \cos \varphi \\ \sin \varphi \end{matrix} \right\} \\ [\varphi = c^2/(4b) - \nu\pi/2; b, c > 0; -(3 \pm 1)/2 < \operatorname{Re} \nu < 1/2].$$

$$6. \int_0^\infty x \left\{ \begin{matrix} \sin bx^2 \\ \cos bx^2 \end{matrix} \right\} J_\nu(cx) dx = \frac{c}{8b} \sqrt{\frac{\pi}{b}} \left[\left\{ \begin{matrix} \cos \varphi \\ \sin \varphi \end{matrix} \right\} J_{(\nu-1)/2} \left(\frac{c^2}{8b} \right) \mp \right. \\ \left. \mp \left\{ \begin{matrix} \sin \varphi \\ \cos \varphi \end{matrix} \right\} J_{(\nu+1)/2} \left(\frac{c^2}{8b} \right) \right] \quad [\varphi = c^2/(8b) - \pi\nu/4; b, c > 0; \operatorname{Re} \nu > -3 \mp 1].$$

$$7. \int_0^\infty x \left\{ \begin{matrix} \sin bx^2 \\ \cos bx^2 \end{matrix} \right\} J_0(cx) dx = \frac{1}{2b} \left\{ \begin{matrix} \cos [c^2/(4b)] \\ \sin [c^2/(4b)] \end{matrix} \right\} \quad [b, c > 0]$$

$$8. \int_0^\infty \frac{\sin bx^2}{x} J_0(cx) dx = -\frac{1}{2} \operatorname{si} \left(\frac{c^2}{4b} \right) \quad [b, c > 0].$$

$$9. \int_0^\infty \sin^2 bx^2 J_1(cx) dx = \frac{1}{2c} \cos \frac{c^2}{8b} \quad [b, c > 0].$$

$$10. \int_0^a x^{\nu+1} \left\{ \begin{matrix} \sin b(a^2 - x^2) \\ \cos b(a^2 - x^2) \end{matrix} \right\} J_\nu(cx) dx = (2b)^{-\nu-1} c^\nu U_{\pm(3 \pm 1)/2}(2a^2b, ac)$$

$$[a > 0; \operatorname{Re} \nu > -1].$$

$$11. \int_0^a x^{1-\nu} \left\{ \begin{matrix} \sin b(a^2 - x^2) \\ \cos b(a^2 - x^2) \end{matrix} \right\} J_\nu(cx) dx = \\ = (2b)^{\nu-1} c^{-\nu} \left[\pm U_{\nu+(1 \mp 1)/2} \left(\frac{c^2}{2b}, ac \right) + \sin a^2b U_{\nu+(1 \pm 1)/2} \left(\frac{c^2}{2b}, 0 \right) \mp \right. \\ \left. \mp \cos a^2b U_{\nu-(1 \mp 1)/2} \left(\frac{c^2}{2b}, 0 \right) \right] \quad [a > 0].$$

$$12. \int_a^\infty x^{\nu+1} \left\{ \begin{matrix} \sin b(x^2 - a^2) \\ \cos b(x^2 - a^2) \end{matrix} \right\} J_\nu(cx) dx = \pm (2b)^{-\nu-1} c^\nu V_{(3 \pm 1)/2 - \nu}(2a^2b, ac)$$

$$[b, c > 0; \operatorname{Re} \nu > 1/2].$$

$$13. \int_0^\infty x^{\nu+1} \left\{ \begin{matrix} \sin(b - bx^2) \\ \cos(b - bx^2) \end{matrix} \right\} J_\nu(cx) dx = \mp \frac{c^\nu}{(2b)^{\nu+1}} \left\{ \begin{matrix} \cos \varphi \\ \sin \varphi \end{matrix} \right\}$$

$$[\varphi = b + c^2/(4b) - \nu\pi/2; -1 < \operatorname{Re} \nu < 1/2; b, c > 0].$$

2.12.19. Интегралы, содержащие $x^\alpha \left\{ \begin{matrix} \sin b \sqrt{x} \\ \cos b \sqrt{x} \end{matrix} \right\} J_\nu(cx)$.

$$1. \int_0^\infty x^{\alpha-1} \left\{ \begin{matrix} \sin b \sqrt{x} \\ \cos b \sqrt{x} \end{matrix} \right\} J_\nu(cx) dx = \\ = b^\delta \left(\frac{c}{2} \right)^{\alpha+\delta/2} \Gamma \left[\frac{(\nu+\alpha)/2 + \delta/4}{1 + (\nu-\alpha)/2 - \delta/4} \right] {}_2F_3 \left(\frac{\alpha-\nu}{2} + \frac{\delta}{4}, \frac{\alpha+\nu}{2} + \frac{\delta}{4}; \frac{1}{2}, \right. \\ \left. \frac{1}{4} + \delta, \frac{3}{4}; -\frac{b^4}{64c^2} \right) - \frac{b^{0+2}}{3\delta c^2} \left(\frac{2}{c} \right)^{\alpha+\delta/2-1} \Gamma \left[\frac{(\nu+\alpha-1)/2 - \delta/4}{(1+\nu-\alpha)/2 - \delta/4} \right] \times \\ \times {}_2F_3 \left(\frac{1-\nu+\alpha}{2} + \frac{\delta}{4}, \frac{\nu+\alpha-1}{2} - \frac{\delta}{4}; \frac{3}{2}, \frac{3}{4} + \delta, \frac{5}{4}; -\frac{b^4}{64c^2} \right) \\ [b, c > 0; -\delta/2 - \operatorname{Re} \nu < \operatorname{Re} \alpha < 3/2, \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}].$$

$$2. \int_0^\infty x^{1/4} \left\{ \begin{matrix} \sin b \sqrt{x} \\ \cos b \sqrt{x} \end{matrix} \right\} J_{\mp 1/4}(cx) dx = \frac{\sqrt{\pi}}{c^2} \left(\frac{b}{2} \right)^{3/2} J_{\pm 3/4} \left(\frac{b^2}{4c} \right) \quad [b, c > 0]$$

$$3. \int_0^\infty x^{1/4} \left\{ \begin{matrix} \sin b \sqrt{x} \\ \cos b \sqrt{x} \end{matrix} \right\} J_{\pm 3/4}(cx) dx = \frac{\sqrt{\pi}}{c^2} \left(\frac{b}{2} \right)^{3/2} J_{\mp 1/4} \left(\frac{b^2}{4c} \right) \quad [b, c > 0].$$

$$4. \int_0^\infty x^{-1/4} \left\{ \begin{matrix} \sin b \sqrt{x} \\ \cos b \sqrt{x} \end{matrix} \right\} J_{\pm 1/4}(cx) dx = \frac{1}{c} \sqrt{\frac{\pi b}{2}} J_{\pm 1/4} \left(\frac{b^2}{4c} \right) \quad [b, c > 0]$$

$$5. \int_0^\infty x^{-1/2} \sin b \sqrt{x} J_0(cx) dx = \frac{\pi b}{4c} \left[J_{1/4} \left(\frac{b^2}{8c} \right) - Y_{1/4} \left(\frac{b^2}{8c} \right) \right] \quad [b, c > 0]$$

$$6. \int_0^{\infty} x^{-1/2} \cos b \sqrt{x} J_0(cx) dx =$$

$$= \frac{\pi b}{8c} \{Y_{1/4}(z) [Y_{1/4}(z) - J_{1/4}(z)] - J_{1/4}(z) [J_{1/4}(z) + Y_{1/4}(z)]\}$$

$$[z = b^2/(8c); b, c > 0]$$

$$7. \int_0^{\infty} x^{-1/2} \sin b \sqrt{x} \left\{ \begin{matrix} \sin cx \\ \cos cx \end{matrix} \right\} J_0(cx) dx = \sqrt{\frac{\pi}{2c}} \cos \left(\frac{b^2}{16c} \pm \frac{\pi}{4} \right) J_0 \left(\frac{b^2}{16c} \right)$$

$$[b, c > 0].$$

$$8. \int_0^{\infty} x^{-1/2} \cos b \sqrt{x} \left\{ \begin{matrix} \sin cx \\ \cos cx \end{matrix} \right\} J_0(cx) dx =$$

$$= -\frac{1}{2} \sqrt{\frac{\pi}{2c}} \left[\left\{ \begin{matrix} \cos t \\ \sin t \end{matrix} \right\} J_0 \left(\frac{b^2}{16c} \right) \mp \left\{ \begin{matrix} \sin t \\ \cos t \end{matrix} \right\} Y_0 \left(\frac{b^2}{16c} \right) \right]$$

$$[t = b^2/(16c) - \pi/4; b, c > 0].$$

2.12.20. Интегралы, содержащие $x^{\alpha} \left\{ \begin{matrix} \sin (bx + a/x) \\ \cos (bx + a/x) \end{matrix} \right\} J_{\nu}(cx)$.

$$1. \int_0^{\infty} x^{\alpha-1} \left\{ \begin{matrix} \sin (b/x) \\ \cos (b/x) \end{matrix} \right\} J_{\nu}(cx) dx = 2^{\alpha-\delta-1} b^{\delta} c^{\delta-\alpha} \Gamma \left[\frac{(\nu + \alpha - \delta)/2}{1 + (\delta + \nu - \alpha)/2} \right] \times$$

$$\times {}_0F_3 \left(1 - \frac{\nu + \alpha - \delta}{2}, \frac{1}{2} + \delta, 1 + \frac{\delta + \nu - \alpha}{2}; \frac{b^2 c^2}{16} \right) \mp$$

$$\mp b^{\alpha+\nu} \left(\frac{c}{2} \right)^{\nu} \left\{ \begin{matrix} \sin [(\alpha + \nu) \pi/2] \\ \cos [(\alpha + \nu) \pi/2] \end{matrix} \right\} \left[-\frac{\nu - \alpha}{1 + \nu} \right] {}_0F_3 \left(1 + \frac{\alpha + \nu}{2}, 1 + \nu, \frac{1 + \alpha + \nu}{2}; \frac{b^2 c^2}{16} \right)$$

$$[b, c > 0; -1 - \operatorname{Re} \nu < \operatorname{Re} \alpha < \delta + 3/2; \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}].$$

$$2. \int_0^{\infty} \frac{1}{x} \sin bx \left\{ \begin{matrix} \sin (a/x) \\ \cos (a/x) \end{matrix} \right\} J_{\nu}(cx) dx =$$

$$= \frac{\pi}{2} J_{\nu}(z_+ \sqrt{a}) \left[\left\{ \begin{matrix} \sin (\nu\pi/2) \\ \cos (\nu\pi/2) \end{matrix} \right\} J_{\nu}(z_- \sqrt{a}) \pm \left\{ \begin{matrix} \cos (\nu\pi/2) \\ \sin (\nu\pi/2) \end{matrix} \right\} Y_{\nu}(z_- \sqrt{a}) \right] +$$

$$+ \left\{ \begin{matrix} \cos (\nu\pi/2) \\ \sin (\nu\pi/2) \end{matrix} \right\} I_{\nu}(z_+ \sqrt{a}) K_{\nu}(z_- \sqrt{a}) \quad [0 < c < b; a > 0; z_{\pm} = \sqrt{b+c} \pm \sqrt{b-c}].$$

$$3. \int_0^{\infty} \frac{1}{x} \cos bx \left\{ \begin{matrix} \sin (a/x) \\ \cos (a/x) \end{matrix} \right\} J_{\nu}(cx) dx =$$

$$= \pm \frac{\pi}{2} J_{\nu}(z_+ \sqrt{a}) \left[\left\{ \begin{matrix} \cos (\nu\pi/2) \\ \sin (\nu\pi/2) \end{matrix} \right\} J_{\nu}(z_- \sqrt{a}) \mp \left\{ \begin{matrix} \sin (\nu\pi/2) \\ \cos (\nu\pi/2) \end{matrix} \right\} Y_{\nu}(z_- \sqrt{a}) \right] \mp$$

$$\mp \left\{ \begin{matrix} \sin (\nu\pi/2) \\ \cos (\nu\pi/2) \end{matrix} \right\} I_{\nu}(z_+ \sqrt{a}) K_{\nu}(z_- \sqrt{a}) \quad [0 < c < b; a > 0; z_{\pm} = \sqrt{b+c} \pm \sqrt{b-c}].$$

$$4. \int_0^{\infty} \frac{1}{x} \left\{ \begin{matrix} \sin (bx - a/x) \\ \cos (bx - a/x) \end{matrix} \right\} J_{\nu}(cx) dx = 2 \left\{ \begin{matrix} \sin (\nu\pi/2) \\ \cos (\nu\pi/2) \end{matrix} \right\} I_{\nu}(z_+ \sqrt{a}) K_{\nu}(z_- \sqrt{a})$$

$$[0 < c < b; a > 0; z_{\pm} = \sqrt{b+c} \pm \sqrt{b-c}].$$

$$5. \int_0^{\infty} \frac{1}{x} \left\{ \frac{\sin (bx + a/x)}{\cos (bx + a/x)} \right\} J_{\nu}(cx) dx =$$

$$= J_{\nu}(z_+ \sqrt{a}) \left[\pm \left\{ \frac{\cos (\nu \pi / 2)}{\sin (\nu \pi / 2)} \right\} J_{\nu}(z_- \sqrt{a}) - \left\{ \frac{\sin (\nu \pi / 2)}{\cos (\nu \pi / 2)} \right\} Y_{\nu}(z_- \sqrt{a}) \right]$$

$$[0 < c < b, a > 0; z_{\pm} = \sqrt{b+c} \pm \sqrt{b-c}].$$

2.12.21. Интегралы от $A(x) \left\{ \frac{\sin (b \sqrt{a^2 - x^2})}{\cos (b \sqrt{a^2 - x^2})} \right\} J_{\nu}(cx)$.

$$1. \int_0^a x^{\nu+1} \sin (b \sqrt{a^2 - x^2}) J_{\nu}(cx) dx =$$

$$= \sqrt{\frac{\pi}{2}} a^{\nu+3/2} b c^{\nu} (b^2 + c^2)^{-(2\nu+3)/4} J_{\nu+3/2}(a \sqrt{b^2 + c^2})$$

$$[a > 0; \operatorname{Re} \nu > -1].$$

$$2. \int_0^a x \sin (b \sqrt{a^2 - x^2}) J_0(cx) dx = \frac{ab}{b^2 + c^2} \left[\frac{\sin (a \sqrt{b^2 + c^2})}{a \sqrt{b^2 + c^2}} - \cos (a \sqrt{b^2 + c^2}) \right]$$

$$[a > 0].$$

$$3. \int_0^a \frac{\cos (b \sqrt{a^2 - x^2})}{\sqrt{a^2 - x^2}} J_{\nu}(cx) dx = \frac{\pi}{2} J_{\nu/2} \left(\frac{a \sqrt{b^2 + c^2} + ab}{2} \right) J_{\nu/2} \left(\frac{a \sqrt{b^2 + c^2} - ab}{2} \right)$$

$$[a > 0; \operatorname{Re} \nu > -1].$$

$$4. \int_0^a \frac{\cos (b \sqrt{a^2 - x^2})}{\sqrt{a^2 - x^2}} J_1(cx) dx = \frac{1}{ac} \cos ab - \frac{1}{ac} \cos (a \sqrt{b^2 + c^2})$$

$$[a > 0].$$

$$5. \int_0^a x^{\nu+1} \frac{\cos (b \sqrt{a^2 - x^2})}{\sqrt{a^2 - x^2}} J_{\nu}(cx) dx =$$

$$= \sqrt{\frac{\pi}{2}} a^{\nu+1/2} c^{\nu} (b^2 + c^2)^{-(2\nu+1)/4} J_{\nu+1/2}(a \sqrt{b^2 + c^2})$$

$$[a > 0; \operatorname{Re} \nu > -1].$$

$$6. \int_0^a x \frac{\cos (b \sqrt{a^2 - x^2})}{\sqrt{a^2 - x^2}} J_0(cx) dx = \frac{\sin (a \sqrt{b^2 + c^2})}{\sqrt{b^2 + c^2}}$$

$$[a > 0].$$

2.12.22. Интегралы от $A(x) \left\{ \frac{\sin (b \sqrt{x^2 - a^2})}{\cos (b \sqrt{x^2 - a^2})} \right\} J_{\nu}(cx)$.

$$1. \int_a^{\infty} \sin (b \sqrt{x^2 - a^2}) J_1(cx) dx = \frac{b}{c} (c^2 - b^2)_+^{-1/2} \cos (a \sqrt{c^2 - b^2})$$

$$[a, b, c > 0; b \neq c].$$

$$2. \int_a^{\infty} x^{1-\nu} \sin (b \sqrt{x^2 - a^2}) J_{\nu}(cx) dx =$$

$$= \sqrt{\frac{\pi}{2}} a^{3/2-\nu} b c^{-\nu} (c^2 - b^2)_+^{(2\nu-3)/4} J_{\nu-3/2}(a \sqrt{c^2 - b^2})$$

$$[a > 0; b \neq c; \operatorname{Re} \nu > -1/2].$$

$$3. \int_a^\infty x^{\nu+1} \sin(b \sqrt{x^2 - a^2}) J_\nu(cx) dx =$$

$$= \pm \left(\frac{\pi}{2}\right)^{\pm 1/2} a^{\nu+3/2} c^\nu \left\{ \begin{matrix} b \\ \sin \nu\pi \end{matrix} \right\} |b^2 - c^2|^{-(2\nu+3)/4} \begin{cases} Y_{\nu+3/2}(a \sqrt{c^2 - b^2}) \\ K_{\nu+3/2}(a \sqrt{b^2 - c^2}) \end{cases}$$

$$\left[\begin{cases} 0 < b < c \\ 0 < c < b \end{cases}, a > 0; \operatorname{Re} \nu < -1/2 \right].$$

$$4. \int_a^\infty \frac{\cos(b \sqrt{x^2 - a^2})}{\sqrt{x^2 - a^2}} J_\nu(cx) dx =$$

$$= \cos \frac{\nu\pi}{2} I_{\nu/2} \left(\frac{ab - a \sqrt{b^2 - c^2}}{2} \right) K_{\nu/2} \left(\frac{ab + a \sqrt{b^2 - c^2}}{2} \right)$$

$$[0 < c < b; a > 0].$$

$$5. \int_a^\infty \frac{\cos(b \sqrt{x^2 - a^2})}{\sqrt{x^2 - a^2}} J_1(cx) dx = \begin{cases} 1 \\ 0 \end{cases} \frac{1}{ac} \sin(a \sqrt{c^2 - b^2})$$

$$\left[\begin{cases} 0 < b < c \\ 0 < c < b \end{cases}, a > 0 \right].$$

$$6. \int_a^\infty \frac{x}{\sqrt{x^2 - a^2}} \cos(b \sqrt{x^2 - a^2}) J_0(cx) dx = (c^2 - b^2)_+^{-1/2} \cos(a \sqrt{c^2 - b^2})$$

$$[a, b, c > 0; c \neq b].$$

$$7. \int_a^\infty x^{1-\nu} \frac{\cos(b \sqrt{x^2 - a^2})}{\sqrt{x^2 - a^2}} J_\nu(cx) dx =$$

$$= \sqrt{\frac{\pi}{2}} a^{1/2-\nu} c^{-\nu} (c^2 - b^2)_+^{(2\nu-1)/4} J_{\nu-1/2}(a \sqrt{c^2 - b^2})$$

$$[a, b, c > 0; b \neq c; \operatorname{Re} \nu > -1/2].$$

$$8. \int_a^\infty \frac{x^{1-\nu}}{x^2 + z^2} \left\{ \frac{\sin(b \sqrt{x^2 - a^2})}{(x^2 - a^2)^{-1/2} \cos(b \sqrt{x^2 - a^2})} \right\} J_\nu(cx) dx =$$

$$= \frac{\pi}{2} z^{-\nu} (z^2 + a^2)^{-(1 \mp 1)/4} \exp(-b \sqrt{z^2 + a^2}) I_\nu(cz)$$

$$[0 < c < b; a, \operatorname{Re} z > 0; \operatorname{Re} \nu > -(4 \mp 1)/2]$$

2.12.23. Интегралы от $A(x) \begin{cases} \sin(b \sqrt{x^2 + z^2}) \\ \cos(b \sqrt{x^2 + z^2}) \end{cases} J_\nu(cx).$

$$1. \int_0^\infty x^{\nu+1} \begin{cases} \sin(b \sqrt{x^2 + z^2}) \\ \cos(b \sqrt{x^2 + z^2}) \end{cases} J_\nu(cx) dx = \sqrt{\frac{\pi}{2}} b c^\nu z^{\nu+3/2} (b^2 - c^2)^{-(2\nu+3)/4} \times$$

$$\times \left[\begin{cases} \sin \nu\pi \\ \cos \nu\pi \end{cases} J_{\nu+3/2}(z \sqrt{b^2 - c^2}) \pm \begin{cases} \cos \nu\pi \\ \sin \nu\pi \end{cases} Y_{\nu+3/2}(z \sqrt{b^2 - c^2}) \right]$$

$$[0 < c < b, \operatorname{Re} z > 0; -1 < \operatorname{Re} \nu < -1/2].$$

$$2. = \begin{cases} -1 \\ 0 \end{cases} \sqrt{\frac{2}{\pi}} b c^\nu z^{\nu+3/2} (c^2 - b^2)^{-(2\nu+3)/4} K_{\nu+1/2}(z \sqrt{c^2 - b^2})$$

$$[0 < b < c, \operatorname{Re} z > 0; -1 < \operatorname{Re} \nu < -1/2].$$

$$3. \int_0^{\infty} \frac{1}{\sqrt{x^2+z^2}} \left\{ \begin{matrix} \sin (b \sqrt{x^2+z^2}) \\ \cos (b \sqrt{x^2+z^2}) \end{matrix} \right\} J_{\nu}(cx) dx = \pm \frac{\pi}{2} J_{\nu/2}(\xi_{\pm}) \left\{ \begin{matrix} J_{-\nu/2}(\xi_{\pm}) \\ Y_{-\nu/2}(\xi_{\pm}) \end{matrix} \right\}$$

$$[2\xi_{\pm} = z(b \pm \sqrt{b^2 - c^2}); 0 < c < b; \operatorname{Re} z > 0; \operatorname{Re} \nu > -1].$$

$$4. \int_0^{\infty} \frac{1}{\sqrt{x^2+z^2}} \left\{ \begin{matrix} \sin (b \sqrt{x^2+z^2}) \\ \cos (b \sqrt{x^2+z^2}) \end{matrix} \right\} J_1(cx) dx = \frac{1}{cz} \left\{ \begin{matrix} \sin bz - \sin (z \sqrt{b^2 - c^2}) \\ \cos bz - \cos (z \sqrt{b^2 - c^2}) \end{matrix} \right\}$$

$$[0 < c < b; \operatorname{Re} z > 0].$$

$$5. = \frac{1}{cz} \left[\left\{ \begin{matrix} \sin bz \\ \cos bz \end{matrix} \right\} - \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \exp(-z \sqrt{c^2 - b^2}) \right]$$

$$[0 < b \leq c; \operatorname{Re} z > 0].$$

$$6. \int_0^{\infty} \frac{x^{\nu+1}}{\sqrt{x^2+z^2}} \left\{ \begin{matrix} \sin (b \sqrt{x^2+z^2}) \\ \cos (b \sqrt{x^2+z^2}) \end{matrix} \right\} J_{\nu}(cx) dx =$$

$$= \pm \sqrt{\frac{\pi}{2}} c^{\nu} z^{\nu+1/2} (b^2 - c^2)^{-(2\nu+1)/4} \left\{ \begin{matrix} J_{-\nu-1/2}(z \sqrt{b^2 - c^2}) \\ Y_{-\nu-1/2}(z \sqrt{b^2 - c^2}) \end{matrix} \right\}$$

$$[0 < c < b; \operatorname{Re} z > 0; -1 < \operatorname{Re} \nu < 1/2]$$

$$7. = \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \sqrt{\frac{2}{\pi}} c^{\nu} z^{\nu+1/2} (c^2 - b^2)^{-(2\nu+1)/4} K_{\nu+1/2}(z \sqrt{c^2 - b^2})$$

$$[0 < b < c; \operatorname{Re} z > 0; -1 < \operatorname{Re} \nu < 1/2].$$

$$8. \int_0^{\infty} \frac{x}{\sqrt{x^2+z^2}} \left\{ \begin{matrix} \sin (b \sqrt{x^2+z^2}) \\ \cos (b \sqrt{x^2+z^2}) \end{matrix} \right\} J_0(cx) dx = \pm \frac{1}{\sqrt{b^2 - c^2}} \left\{ \begin{matrix} \cos (z \sqrt{b^2 - c^2}) \\ \sin (z \sqrt{b^2 - c^2}) \end{matrix} \right\}$$

$$[0 < c < b; \operatorname{Re} z > 0]$$

$$9. = \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{1}{\sqrt{c^2 - b^2}} \exp(-z \sqrt{c^2 - b^2})$$

$$[0 < b < c; \operatorname{Re} z > 0].$$

$$10. \int_0^{\infty} \frac{x^{\nu+1}}{(x^2+z^2)^{(5 \pm 1)/4}} \left\{ \begin{matrix} \sin (b \sqrt{x^2+z^2}) \\ \cos (b \sqrt{x^2+z^2}) \end{matrix} \right\} dx = \left\{ \begin{matrix} b \\ 1 \end{matrix} \right\} z^{\nu} K_{\nu}(cz)$$

$$[0 < b < c; \operatorname{Re} z > 0; -1 < \operatorname{Re} \nu < (4 \pm 1)/2].$$

$$11. \int_0^{\infty} \frac{x^{1-\nu}}{\sqrt{x^2+z^2}} (\sqrt{x^2+z^2} - z)^{\nu} \left\{ \begin{matrix} \sin (b \sqrt{x^2+z^2}) \\ \cos (b \sqrt{x^2+z^2}) \end{matrix} \right\} J_{\nu}(cx) dx =$$

$$= \pm \frac{c^{\nu}}{\sqrt{b^2 - c^2}} (\sqrt{b^2 - c^2} + b)^{-\nu} \left\{ \begin{matrix} \cos \varphi \\ \sin \varphi \end{matrix} \right\}$$

$$[\varphi = z \sqrt{b^2 - c^2} + \nu\pi/2; 0 < c < b; \operatorname{Re} z > 0; \operatorname{Re} \nu > -1]$$

$$12. = \frac{1}{c \sqrt{c^2 - b^2}} \exp(-z \sqrt{c^2 - b^2}) \left\{ \begin{matrix} \sin \psi \\ \cos \psi \end{matrix} \right\}$$

$$[\psi = \nu \arcsin(b/c); 0 < b < c; \operatorname{Re} z > 0; \operatorname{Re} \nu > -1].$$

$$13. \int_0^{\infty} \frac{x^{\nu+1}}{x^2 + \xi^2} \frac{\sin(b \sqrt{x^2+z^2})}{\sqrt{x^2+z^2}} J_{\nu}(cx) dx = \xi^{\nu} \left\{ \begin{matrix} (\xi^2 - z^2)^{-1/2} \operatorname{sh}(b \sqrt{\xi^2 - z^2}) \\ (z^2 - \xi^2)^{-1/2} \sin(b \sqrt{z^2 - \xi^2}) \end{matrix} \right\} K_{\nu}(\xi c)$$

$$[0 < b < c; \operatorname{Re} z, \operatorname{Re} \xi > 0; -1 < \operatorname{Re} \nu < 5/2, \left\{ \begin{matrix} |\arg(\xi^2 - z^2)| < \pi \\ \xi < z \end{matrix} \right\}].$$

$$14. \int_0^{\infty} \frac{x^{\nu+1}}{x^2 + \xi^2} \cos(b \sqrt{x^2 + z^2}) J_{\nu}(cx) dx = \xi^{\nu} \begin{cases} \operatorname{ch}(b \sqrt{\xi^2 - z^2}) \\ \cos(b \sqrt{z^2 - \xi^2}) \end{cases} K_{\nu}(\xi c) \\ \left[0 < b < c; \operatorname{Re} z, \operatorname{Re} \xi > 0; -1 < \operatorname{Re} \nu < 3/2, \left\{ \begin{array}{l} |\arg(\xi^2 - z^2)| < \pi \\ \xi < z \end{array} \right\} \right].$$

2.12.24. Интегралы от $A(x) \sin(a+bx) J_{\nu}(\varphi(x))$.

$$1. \int_{-\infty}^{\infty} \frac{\sin b(x-y)}{x-y} J_0(c \sqrt{x^2 + z^2}) dx = \pi J_0(c \sqrt{y^2 + z^2}) \\ [b, c, y, \operatorname{Re} z > 0]$$

2.12.25. Интегралы, содержащие e^{px} , тригонометрические функции и $J_{\nu}(cx)$

$$1. \int_0^{\infty} e^{-px} \begin{cases} \sin bx \\ \cos bx \end{cases} J_0(cx) dx = \frac{1}{\sqrt{(p^2 + (b+c)^2)(p^2 + (b-c)^2)}} \begin{cases} A^2 - c^2 - p^2 \\ A^2 - b^2 \end{cases}^{1/2} \\ [A = (\sqrt{p^2 + (b+c)^2} + \sqrt{p^2 + (b-c)^2})/2, \operatorname{Re} p > |\operatorname{Im} b| + |\operatorname{Im} c|].$$

$$2. \int_0^{\infty} e^{-px} \cos bx J_1(cx) dx = \frac{1}{c} \left[1 - \frac{\sqrt{b^2 + p^2}}{R} \cos(\theta - \eta) \right] \\ [R^2 = (c^2 + p^2 - b^2)^2 + 4b^2 p^2, \operatorname{tg} \theta = b/p, 2bp \operatorname{ctg} 2\eta = c^2 + p^2 - b^2, \operatorname{Re} p > |\operatorname{Im} b| + |\operatorname{Im} c|].$$

$$3. \int_0^{\infty} x^{\alpha-1} e^{-px} \begin{cases} \sin bx \\ \cos bx \end{cases} J_{\nu}(cx) dx = I_{\alpha} \\ \left[\operatorname{Re} p > |\operatorname{Im} b| + |\operatorname{Im} c|; \operatorname{Re}(\alpha + \nu) > -\delta, \delta = \begin{cases} 1 \\ 0 \end{cases} \right],$$

$$I_{\nu+1} = \frac{2^{2\nu} c^{\nu} r^{\nu+3}}{\sqrt{\pi}} \Gamma\left(\nu + \frac{1}{2}\right) (r^2 - 16b^2 c^2)^{-\nu-1/2} \begin{cases} \sin \varphi \\ \cos \varphi \end{cases} \\ [\operatorname{Re} \nu > -(1+\delta)/2].$$

$$I_{\nu+2} = \frac{2^{2\nu+1}}{\sqrt{\pi}} c^{\nu} r^{\nu+6} \Gamma\left(\nu + \frac{3}{2}\right) (r^2 - 16b^2 c^2)^{-\nu-3/2} \begin{cases} \sin \psi \\ \cos \psi \end{cases} \\ [\operatorname{Re} \nu > -\delta/2 - 1].$$

$$I_0 = \frac{(2c)^{\nu}}{\nu} (r + \sqrt{r^2 - 4c^2})^{-\nu} \begin{cases} \sin \theta \\ \cos \theta \end{cases} \quad [\operatorname{Re} \nu > -\delta].$$

$$[r = \sqrt{(b+c)^2 + p^2} + \sqrt{(b-c)^2 + p^2}, \varphi = (2\nu + 2\delta + 1) \operatorname{arctg} \frac{2b \sqrt{r^2 - 4c^2}}{r \sqrt{r^2 - 4b^2}}, \\ \psi = (2\nu + 3) \operatorname{arctg} \frac{2b \sqrt{r^2 - 4c^2}}{r \sqrt{r^2 - 4b^2}}, \theta = \nu \arcsin \frac{2b}{r}].$$

$$4. \int_0^{\infty} \frac{1}{x} e^{-px} \sin bx J_0(cx) dx = \arcsin \frac{2b}{r} \\ [r = \sqrt{(b+c)^2 + p^2} + \sqrt{(b-c)^2 + p^2}; \operatorname{Re} p > |\operatorname{Im} b| + |\operatorname{Im} c|].$$

$$5. \int_0^{\infty} \frac{1}{x} e^{-px} \begin{cases} \sin bx \\ \cos bx \end{cases} J_1(cx) dx = \frac{1}{c} \begin{cases} b - R \sin \eta \\ R \cos \eta - p \end{cases} \\ [R, \eta \text{ см. в 2.12.25.2; } \operatorname{Re} p > |\operatorname{Im} b| + |\operatorname{Im} c|].$$

$$6. \int_0^{\infty} \frac{1}{x^2} e^{-px} \sin bx J_1(cx) dx = \frac{1}{2b^2 c^2} [R \sqrt{b^2 + p^2} \sin(\theta + \varphi) - (b^2 + p^2) \sin 2\theta] +$$

$$+ \frac{c}{2b^2} \operatorname{arctg} \left(\frac{R \sin \varphi + \sqrt{b^2 + p^2} \sin \theta}{R \cos \varphi + \sqrt{b^2 + p^2} \cos \theta} \right)$$

[R, η, θ см. в 2.12.25.2; Re p > |Im b| + |Im c|]

$$7. \int_0^{\infty} x^{\alpha-1} e^{-px} \begin{Bmatrix} \sin cx \\ \cos cx \end{Bmatrix} J_\nu(cx) dx = \frac{(2c)^{\nu+\delta}}{p^{\nu+\alpha+\delta} \sqrt{\pi}} \Gamma \left[\begin{matrix} \alpha + \nu + \delta, \nu + \delta + 1/2 \\ 2\nu + \delta + 1 \end{matrix} \right] \times$$

$$\times {}_4F_3 \left(\frac{\alpha + \nu + 1}{2}, \frac{\alpha + \nu}{2} + \delta, \frac{2\nu + 3}{4}, \frac{2\nu + 1}{4} + \delta; \delta + \frac{1}{2}, \frac{1}{2} + \delta + \nu, 1 + \nu; -\frac{4c^2}{p^2} \right)$$

$$\left[c, \operatorname{Re} p > 0; \operatorname{Re}(\alpha + \nu) > -\delta; \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right].$$

$$8. \int_0^{\infty} x^{-1/2} e^{-p\sqrt{x}} \begin{Bmatrix} \sin cx \\ \cos cx \end{Bmatrix} J_0(cx) dx =$$

$$= \frac{1}{2} \sqrt{\frac{\pi}{2c}} \left[J_0 \left(\frac{p^2}{16c} \right) \begin{Bmatrix} \sin \varphi \\ \cos \varphi \end{Bmatrix} \mp Y_0 \left(\frac{p^2}{16c} \right) \begin{Bmatrix} \cos \varphi \\ \sin \varphi \end{Bmatrix} \right]$$

[φ = p²/(16c) - π/4; c, Re p > 0]

$$9. \int_0^{\infty} e^{-px^2} \sin bx^2 J_1(cx) dx = \frac{1}{c} \exp \left(-\frac{pc^2}{4b^2 + 4p^2} \right) \sin \frac{bc^2}{4(b^2 + p^2)}$$

[b, Re p > 0].

$$10. \int_0^{\infty} x^{\alpha-1} e^{-b\sqrt{x}} \begin{Bmatrix} \sin b\sqrt{x} \\ \cos b\sqrt{x} \end{Bmatrix} J_\nu(cx) dx = \frac{2^{\alpha+1/2} b^3}{3c^{\alpha+3/2}} \Gamma \left[\begin{matrix} (2\nu + 2\alpha + 3)/4 \\ (2\nu - 2\alpha + 1)/4 \end{matrix} \right] \times$$

$$\times {}_2F_3 \left(\frac{2\alpha + 2\nu + 3}{4}, \frac{2\alpha - 2\nu + 3}{4}; \frac{3}{2}, \frac{5}{4}, \frac{7}{4}; \frac{b^4}{16c^2} \right) \pm$$

$$\pm \frac{2^{\alpha-1/2} b}{c^{\alpha+1/2}} \Gamma \left[\begin{matrix} (2\nu + 2\alpha + 1)/4 \\ (2\nu - 2\alpha + 3)/4 \end{matrix} \right] {}_2F_3 \left(\frac{2\alpha + 2\nu + 1}{4}, \frac{2\alpha - 2\nu + 1}{4}; \frac{1}{2}, \frac{3}{4}, \frac{5}{4}; \frac{b^4}{16c^2} \right) \mp$$

$$\mp \frac{2^{\alpha+\delta-1} b^2 \delta}{c^{\alpha+\delta}} \Gamma \left[\begin{matrix} (\alpha + \nu + \delta)/2 \\ (\nu - \alpha - \delta)/2 + 1 \end{matrix} \right] \times$$

$$\times {}_2F_3 \left(\frac{\alpha + \nu + \delta}{2}, \frac{\alpha + \delta - \nu}{2}; \delta + \frac{1}{2}, \frac{3}{4}, \frac{1}{4} + \delta; \frac{b^4}{16c^2} \right)$$

$$\left[c > 0; \operatorname{Re}(\alpha + \nu) > -\delta/2, |\arg b| < \pi/4, \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right].$$

$$11. \int_0^{\infty} x^{-1/2} e^{-b\sqrt{x}} \begin{Bmatrix} \sin b\sqrt{x} \\ \cos b\sqrt{x} \end{Bmatrix} J_0(cx) dx = \frac{b}{2c} I_{\pm 1/4} \left(\frac{b^2}{4c} \right) K_{1/4} \left(\frac{b^2}{4c} \right)$$

[c > 0; |arg b| < π/4].

2.12.26. Интегралы, содержащие тригонометрические функции и $J_\nu(c \operatorname{sh} x)$ или $J_\nu(c \operatorname{ch} x)$.

$$1. \int_0^{\infty} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} J_\nu(c \operatorname{sh} x) dx =$$

$$= \frac{1}{2} \begin{Bmatrix} i \\ 1 \end{Bmatrix} \left[I_{(\nu+ib)/2} \left(\frac{c}{2} \right) K_{(\nu-ib)/2} \left(\frac{c}{2} \right) \mp I_{(\nu-ib)/2} \left(\frac{c}{2} \right) K_{(\nu+ib)/2} \left(\frac{c}{2} \right) \right]$$

[b, c > 0; Re ν > -(3 ± 1)/2].

$$2. \int_0^{\infty} \sin bx J_0(c \operatorname{sh} x) dx = \frac{1}{\pi} \operatorname{sh} \frac{b\pi}{2} K_{ib/2}^2\left(\frac{c}{2}\right) \quad [b, c > 0].$$

$$3. \int_0^{\infty} \cos bx J_{\nu}(c \operatorname{ch} x) dx = \\ = -\frac{\pi}{4} \left[J_{(\nu+ib)/2}\left(\frac{c}{2}\right) Y_{(\nu-ib)/2}\left(\frac{c}{2}\right) + J_{(\nu-ib)/2}\left(\frac{c}{2}\right) Y_{(\nu+ib)/2}\left(\frac{c}{2}\right) \right] \\ [b, c > 0]$$

2.12.27. Интегралы, содержащие показательную, тригонометрические функции и $J_{\nu}(\varphi(\sin x, \cos x))$

$$1. \int_0^{\pi} \left\{ \begin{matrix} \sin bx \\ \cos bx \end{matrix} \right\} J_{\nu}(c \sin x) dx = \pi \left\{ \begin{matrix} \sin(b\pi/2) \\ \cos(b\pi/2) \end{matrix} \right\} J_{(\nu-b)/2}\left(\frac{c}{2}\right) J_{(\nu+b)/2}\left(\frac{c}{2}\right) \\ [b, c > 0; \operatorname{Re} \nu > (-3 \mp 1)/2].$$

$$2. \int_0^{\pi} \cos 2nx J_0\left(c \left\{ \begin{matrix} \sin x \\ \cos x \end{matrix} \right\}\right) dx = (\pm 1)^n \pi J_n^2\left(\frac{c}{2}\right) \quad [c > 0].$$

$$3. \int_0^{\pi} \rho^p \cos x \operatorname{tg}^{2m} \frac{x}{2} J_{2n}(c \sin x) dx = \\ = \frac{1}{c} \Gamma\left[\begin{matrix} m+n+1/2, n-m+1/2 \\ 2n+1, 2n+1 \end{matrix} \right] M_{m,n}(z_+) M_{m,n}(z_-) \\ [z_{\pm} = \rho \pm \sqrt{\rho^2 - c^2}].$$

$$4. \int_0^{\pi} \cos 2nx J_0(\sqrt{a^2 - b^2} \sin^2 x) dx = (-1)^n \pi J_n\left(\frac{a + \sqrt{a^2 - b^2}}{2}\right) J_n\left(\frac{a - \sqrt{a^2 - b^2}}{2}\right) \\ [0 < b \leq a].$$

$$5. \int_0^{\pi} \frac{\cos 2nx}{\sqrt{a^2 - b^2} \sin^2 x} J_1(\sqrt{a^2 - b^2} \sin^2 x) dx = \\ = \frac{(-1)^n \pi}{a \sqrt{a^2 - b^2}} [u_+ J_n(u_-) J_{n+1}(u_+) - u_- J_{n+1}(u_-) J_n(u_+)] \\ [2u_{\pm} = a \pm \sqrt{a^2 - b^2}; 0 < b < a].$$

$$6. \int_0^{2\pi} \cos nx J_0(\sqrt{a^2 + b^2 + 2ab \cos x}) dx = (-1)^n 2\pi J_n(a) J_n(b).$$

$$7. \int_0^{2\pi} \frac{1}{\sqrt{a^2 + b^2 + 2ab \cos x}} \left\{ \begin{matrix} \sin(x/2) \sin[(2n+1)x/2] \\ \cos(x/2) \cos[(2n+1)x/2] \end{matrix} \right\} J_1(\sqrt{a^2 + b^2 + 2ab \cos x}) dx = \\ = \frac{(-1)^n \pi}{b \mp a} [J_n(a) J_{n+1}(b) \mp J_{n+1}(a) J_n(b)] \quad [a \neq \pm b].$$

$$8. \int_0^{2\pi} \frac{\sin x \sin(n+2)x}{a^2 + b^2 + 2ab \cos x} J_2(\sqrt{a^2 + b^2 + 2ab \cos x}) dx = \\ = \frac{(-1)^n \pi}{b^2 - a^2} [J_n(a) J_{n+2}(b) - J_{n+2}(a) J_n(b)] \quad [a \neq \pm b].$$

$$9. \int_0^{2\pi} \left(\frac{a + be^{ix}}{\sqrt{a^2 + b^2 + 2ab \cos x}} \right)^n e^{-imx} J_n(\sqrt{a^2 + b^2 + 2ab \cos x}) dx = \\ = 2\pi J_{n-m}(a) J_m(b) \quad [a, b > 0].$$

$$10. \int_{-\pi/2}^{\pi/2} [\varphi(x)]^{-\mu-\nu} e^{i(\mu-\nu)x} \cos^{\mu+\nu} x J_{\mu+\nu}(c\varphi(x)) dx = \pi (2a)^{-\mu} (2b)^{-\nu} J_{\mu}(ac) J_{\nu}(bc) \\ [\varphi(x) = \sqrt{2(a^2 e^{ix} + b^2 e^{-ix}) \cos x}; \operatorname{Re}(\mu + \nu) > -1].$$

2.12.28. Интегралы от $A(x) \ln \varphi(x) J_{\nu}(cx)$.

$$1. \int_a^b x \ln \frac{b}{x} J_0(cx) dx = \frac{1}{c^2} [J_0(ac) - J_0(bc)] - \frac{a}{c} J_1(ac) \ln \frac{b}{a} \quad [0 < a < b]$$

$$2. \int_0^1 x \ln x J_0(cx) dx = \frac{1}{c^2} [J_0(c) - 1].$$

$$3. \int_0^{\infty} \ln x J_{\nu}(cx) dx = \frac{1}{c} \left[\ln \frac{2}{c} + \psi\left(\frac{\nu+1}{2}\right) \right] \quad [c > 0; \operatorname{Re} \nu > -1].$$

$$4. \int_0^{\infty} \ln x J_0(cx) dx = -\frac{1}{c} [\ln(2c) + C] \quad [c > 0].$$

$$5. \int_0^{\infty} \ln x J_1(cx) dx = -\frac{1}{c} \left(\ln \frac{c}{2} + C \right) \quad [c > 0].$$

$$6. \int_0^{\infty} x^{\alpha-1} \ln^n x J_{\nu}(cx) dx = I_n \quad [c > 0; -\operatorname{Re} \nu < \operatorname{Re} \alpha < 3/2],$$

$$I_n = \frac{1}{2} \frac{\partial^n}{\partial \alpha^n} \left\{ \left(\frac{2}{c} \right)^{\alpha} \Gamma \left[\frac{(\alpha + \nu)/2}{(\nu - \alpha)/2 + 1} \right] \right\},$$

$$I_1 = 2^{\alpha-2} c^{-\alpha} \Gamma \left[\frac{(\alpha + \nu)/2}{(\nu - \alpha)/2 + 1} \right] \left\{ \psi \left(\frac{\alpha + \nu}{2} \right) + \psi \left(\frac{\nu - \alpha}{2} + 1 \right) + 2 \ln \frac{2}{c} \right\},$$

$$I_2 = 2^{\alpha-3} c^{-\alpha} \Gamma \left[\frac{(\alpha + \nu)/2}{(\nu - \alpha)/2 + 1} \right] \times \\ \times \left\{ \left[\psi \left(\frac{\alpha + \nu}{2} \right) + \psi \left(\frac{\nu - \alpha}{2} + 1 \right) + 2 \ln \frac{2}{c} \right]^2 + 2\psi' \left(\frac{\alpha + \nu}{2} \right) - 2\psi' \left(\frac{\nu - \alpha}{2} + 1 \right) \right\}.$$

$$7. \int_0^{\infty} x^5 \ln^3 \frac{x}{a} J_0(cx) dx = -\frac{128}{c^6} \left(\frac{3}{2} + \ln \frac{2}{ac} - C \right) \quad [a, c > 0].$$

$$8. \int_0^{\infty} \ln(x^2 + z^2) J_1(cx) dx = \frac{z}{c} [K_0(cz) + \ln z] \quad [c, \operatorname{Re} z > 0]$$

$$9. \int_0^{\infty} x \ln \left(1 + \frac{z^2}{x^2} \right) J_0(cx) dx = \frac{2}{c} \left[\frac{1}{c} - zK_1(cz) \right] \quad [c, \operatorname{Re} z > 0].$$

$$10. \int_0^{\infty} \ln(1+x^2 z^2) J_1(cx) dx = \frac{4}{c} \ker\left(\frac{c}{z}\right) \quad [c > 0; |\arg z| < \pi/4].$$

$$11. \int_0^{\infty} \ln(xz + \sqrt{1+x^2 z^2}) J_1(cx) dx = \frac{1}{c} I_0\left(\frac{c}{2z}\right) K_0\left(\frac{c}{2z}\right) \quad [c, \operatorname{Re} z > 0].$$

$$12. \int_0^{\infty} x \ln \frac{z + \sqrt{x^2 + z^2}}{x} J_0(cx) dx = \frac{1}{c^2} (1 - e^{-cz}) \quad [c, \operatorname{Re} z > 0].$$

$$13. \int_0^{\infty} \frac{\ln(x + \sqrt{x^2 + z^2})}{\sqrt{x^2 + z^2}} J_0(cx) dx = \frac{1}{2} K_0\left(\frac{cz}{2}\right) + \ln z I_0\left(\frac{cz}{2}\right) K_0\left(\frac{cz}{2}\right) \quad [c, \operatorname{Re} z > 0]$$

$$14. \int_0^{\infty} \frac{1}{\sqrt{x^2 + z^2}} \ln \frac{\sqrt{x^2 + z^2} + x}{\sqrt{x^2 + z^2} - x} J_0(cx) dx = K_0\left(\frac{cz}{2}\right) \quad [c, \operatorname{Re} z > 0]$$

$$15. \int_0^{\infty} \frac{x}{\sqrt{x^4 + z^4}} \ln \frac{\sqrt{x^4 + z^4} + x^2}{z^2} J_0(cx) dx = -\frac{\pi}{2} Y_0\left(\frac{cz}{\sqrt{2}}\right) K_0\left(\frac{cz}{\sqrt{2}}\right) \quad [c > 0; |\arg z| < \pi/4]$$

$$16. \int_0^{\infty} \frac{x}{\sqrt{\varphi^2 - 1}} \ln(\sqrt{\varphi^2 - 1} + \varphi) J_0(cx) dx = 2ab K_0(ac) K_0(bc) \quad [\varphi = (2ab)^{-1}(a^2 + b^2 + x^2); a, b, c > 0].$$

2.12.29. Интегралы, содержащие показательную, тригонометрические и логарифмическую функции и $J_\nu(\varphi(x))$.

$$1. \int_0^{\infty} e^{-px} \ln x J_0(cx) dx = (p^2 + c^2)^{-1/2} [\ln(p + \sqrt{p^2 + c^2}) - \ln(p^2 + c^2) - \ln 2 - C] \quad [\operatorname{Re} p > |\operatorname{Im} c|].$$

$$2. \int_0^{\infty} x^{(1 \pm 1)/2} e^{-px^2} \ln x J_0(cx) dx = \frac{2^{\mp 1}}{p} \exp\left(-\frac{c^2}{4p}\right) \left[\ln \frac{c}{2p} - \frac{1}{2} \operatorname{Ei}\left(\frac{c^2}{4p}\right)\right] \quad [\operatorname{Re} p > 0].$$

$$3. \int_0^{\infty} x^2 e^{-px^2} \ln(px^2) J_0(cx) dx = \frac{c^2 - 4p^2}{8p^3} \exp\left(-\frac{c^2}{4p}\right) \left[\operatorname{Ei}\left(\frac{c^2}{4p}\right) - \ln \frac{c^2}{4p} + 16 \exp\left(-\frac{c^2}{4p}\right) - 8\right] \quad [\operatorname{Re} p > 0].$$

$$4. \int_0^{\infty} x \frac{\exp(-p\sqrt{x^2 + z^2})}{\sqrt{x^2 + z^2}} \ln(\sqrt{x^2 + z^2} \pm z) J_0(cx) dx = \frac{1}{r} \left[e^{-rz} \ln \frac{rz \pm pz}{r} - e^{rz} \operatorname{Ei}(-2rz) \right] + \frac{1}{r} \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} [e^{rz} \operatorname{Ei}(-rz - pz) - e^{-rz} \operatorname{Ei}(rz - pz)] \quad [r = \sqrt{p^2 + c^2}; \operatorname{Re} p > |\operatorname{Im} c|; \operatorname{Re} z > 0].$$

$$5. \int_0^{\infty} e^{-px} \ln x J_0(c \sqrt{x^2 + xz}) dx =$$

$$= \frac{1}{r} e^{pz/2} \left\{ e^{-rz/2} \ln \frac{rz - pz}{2r} - e^{-rz/2} \operatorname{Ei} \left(\frac{rz - pz}{2} \right) + e^{rz/2} \left[\operatorname{Ei} \left(-\frac{rz + pz}{2} \right) - \operatorname{Ei}(-rz) \right] \right\}$$

$$[r = \sqrt{c^2 + p^2}; \operatorname{Re} p > |\operatorname{Im} c|; |\arg z| < \pi].$$

$$6. \int_0^{\infty} e^{-px} \ln(x+z) J_0(c \sqrt{x^2 + xz}) dx =$$

$$= \frac{1}{r} \left[\exp \left(\frac{pz - rz}{2} \right) \ln \frac{pz + rz}{2r} - \exp \left(\frac{pz + rz}{2} \right) \operatorname{Ei}(-rz) \right]$$

$$[r = \sqrt{c^2 + p^2}; \operatorname{Re} p > |\operatorname{Im} c|; |\arg z| < \pi].$$

$$7. \int_0^{\infty} \cos bx \ln ax J_0(cx) dx = -\frac{\pi}{2} (b^2 - c^2)^{-1/2} \quad [0 < c < b].$$

$$8. = -(c^2 - b^2)^{-1/2} \left[C - \ln \frac{ac}{2} + \ln(c^2 - b^2) \right] \quad [0 < b < c].$$

$$9. \int_0^{\infty} \frac{1}{x^2} \left[v \ln \left(\frac{x}{2} \right) - \ln \Gamma(v+1) - \ln J_v(x) \right] dx = \frac{\pi}{2} \quad [\operatorname{Re} v > -1/2].$$

$$10. \int_0^1 \ln J_v(x) dx = A_v, \quad A_0 = -0,086734 \dots,$$

$$A_1 = -1,735351 \dots, \quad A_2 = -4,108958 \dots$$

2.12.30. Интегралы, содержащие обратные тригонометрические функции и $J_v(cx)$.

$$1. \int_0^a \frac{x^{\alpha-1}}{\sqrt{a^2 - x^2}} \cos \left(\mu \arccos \frac{x}{a} \right) J_v(cx) dx =$$

$$= \frac{\pi a^{\alpha+v-1} c^v}{2^{\alpha+v}} \Gamma \left[\begin{matrix} \alpha+v \\ v+1, (\alpha+\mu+v+1)/2, (\alpha-\mu+v+1)/2 \end{matrix} \right] \times$$

$$\times {}_1F_2 \left(\alpha+v; v+1, \frac{\alpha+\mu+v+1}{2}, \frac{\alpha-\mu+v+1}{2}; -\frac{a^2 c^2}{16} \right)$$

$$[a, \operatorname{Re}(\alpha+v) > 0].$$

$$2. \int_0^a \frac{\cos [\mu \arccos (x/a)]}{\sqrt{a^2 - x^2}} J_v(cx) dx = \frac{\pi}{2} J_{(v+\mu)/2} \left(\frac{ac}{2} \right) J_{(v-\mu)/2} \left(\frac{ac}{2} \right)$$

$$[a > 0; \operatorname{Re} v > -1].$$

$$3. \int_0^{\infty} \operatorname{arctg} x^2 J_1(cx) dx = -\frac{2}{c} \operatorname{kei}(c) \quad [c > 0].$$

2.12.31. Интегралы от $x^\alpha J_\mu(bx) J_\nu(cx)$.

$$1. \int_0^{\infty} x^{\alpha-1} J_\mu(bx) J_\nu(cx) dx = A_{\mu, \nu}^{\alpha}$$

$$[b, c, \operatorname{Re}(\alpha + \mu + \nu) > 0; \operatorname{Re} \alpha < 2; k = 2\sqrt{bc}/(b+c)],$$

$$A_{\mu, \nu}^{\alpha} = 2^{\alpha-1} b^{-\nu-\alpha} c^{\nu} \Gamma \left[\frac{(\nu+\mu+\alpha)/2}{(\mu-\nu-\alpha)/2+1}, \nu+1 \right] {}_2F_1 \left(\frac{\nu+\mu+\alpha}{2}, \frac{\nu-\mu+\alpha}{2}; \nu+1; \frac{c^2}{b^2} \right) \quad [0 < c < b];$$

$$A_{\mu, \nu}^{\alpha} = 2^{\alpha-1} b^{\mu} c^{-\mu-\alpha} \Gamma \left[\frac{(\alpha+\mu+\nu)/2}{(\nu-\mu-\alpha)/2+1}, \mu+1 \right] \times \\ \times {}_2F_1 \left(\frac{\alpha+\mu+\nu}{2}, \frac{\alpha+\mu-\nu}{2}; \mu+1; \frac{b^2}{c^2} \right) \quad [0 < b < c];$$

$$A_{\mu, \nu}^{1-\mu} = \frac{2^{1-\mu}}{c} e^{-i\mu\pi} \Gamma \left[\frac{(1+\nu)/2+\mu}{(1-\nu)/2+\mu} \right] (b^2-c^2)^{\mu/2} Q_{(\nu-1)/2}^{\mu} \left(\frac{2b^2-c^2}{c^2} \right) \\ [0 < c < b; \operatorname{Re} \mu, \operatorname{Re} \nu > -1];$$

$$A_{\mu, \nu}^{1-\mu} = \frac{2^{-\mu}}{c} (c^2-b^2)^{\mu/2} P_{(\nu-1)/2}^{-\mu} \left(\frac{c^2-2b^2}{c^2} \right) \quad [0 < b < c; \operatorname{Re} \mu; \operatorname{Re} \nu > -1]$$

$$A_{\mu, \nu}^{1+\mu} = \frac{2^{\mu} \cos(\nu\pi/2)}{\sqrt{\pi b}} \Gamma \left(\frac{\nu+1}{2} + \mu \right) (b^2-c^2)^{-(2\mu+1)/4} P_{\mu-1/2}^{-\nu/2} \left(\frac{2b^2-c^2}{b \sqrt{b^2-c^2}} \right) = \\ = \frac{2^{\mu+1}}{\pi c} \cos \frac{\nu\pi}{2} e^{-i\mu\pi} (b^2-c^2)^{-\mu/2} Q_{(\nu-1)/2}^{\mu} \left(\frac{2b^2-c^2}{c^2} \right) \\ [0 < c < b; \operatorname{Re} \mu < 1; \operatorname{Re}(2\mu+\nu) > -1];$$

$$A_{\mu, \nu}^{1+\mu} = \frac{2^{\mu}}{c} \Gamma \left[\frac{\nu+(1+\nu)/2}{(1+\nu)/2-\mu} \right] (c^2-b^2)^{-\mu/2} P_{(\nu-1)/2}^{-\mu} \left(\frac{c^2-2b^2}{c^2} \right) \\ [0 < b < c; \operatorname{Re} \mu < 1; \operatorname{Re}(2\mu+\nu) > -1];$$

$$A_{\mu, \nu}^{\mu-\nu+2} = 2^{\mu+1-\nu} b^{\mu} \frac{c^{-\nu}}{\Gamma(\nu-\mu)} (c^2-b^2)^{\nu-\mu-1} \quad [b \neq c; -1 < \operatorname{Re} \mu < \operatorname{Re} \nu];$$

$$A_{\mu, \nu}^{\mu+\nu+2} = -\frac{2^{\mu+\nu+1}}{\pi} \left\{ \frac{\sin \mu\pi}{\sin \nu\pi} \right\} \Gamma(\mu+\nu+1) b^{\mu} c^{\nu} |b^2-c^2|^{-\mu-\nu-1} \\ \left[\begin{cases} 0 < b < c \\ 0 < c < b \end{cases}, -1 < \operatorname{Re}(\mu+\nu) < 0 \right];$$

$$A_{\mu, \nu}^{1/2} = \frac{2^{\nu-1/2}}{\sqrt{b}} \Gamma \left[\frac{1/4+(\mu+\nu)/2}{3/4+(\mu-\nu)/2} \right] P_{\mu-1/2}^{-\nu} \left(\frac{\sqrt{b^2-c^2}}{b} \right) \\ [0 < c < b; \operatorname{Re}(\mu+\nu) > -1/2];$$

$$A_{\mu, \nu}^{3/2} = \frac{2^{\nu+1/2}}{\sqrt{b}} \Gamma \left[\frac{(\mu+\nu)/2+3/4}{(\mu-\nu)/2+1/4} \right] (b^2-c^2)^{-1/2} P_{\mu-1/2}^{-\nu} \left(\frac{\sqrt{b^2-c^2}}{b} \right) \\ [0 < c < b; \operatorname{Re}(\mu+\nu) > -3/2];$$

$$A_{\nu, \nu}^{\alpha} = \frac{2^{\alpha-1} (bc)^{\nu}}{(b+c)^{2\nu+\alpha}} \Gamma \left[\frac{\nu+\alpha/2}{\nu+1}, 1-\alpha/2 \right] {}_2F_1 \left(\nu+\frac{\alpha}{2}, \nu+\frac{1}{2}; 2\nu+1; \frac{4bc}{(b+c)^2} \right) = \\ = 2^{\alpha-1} \Gamma \left[\frac{\nu+\alpha/2}{1-\alpha/2} \right] |b^2-c^2|^{-\alpha/2} P_{-\alpha/2}^{-\nu} \left(\frac{b^2+c^2}{|b^2-c^2|} \right) = \\ = 2^{\alpha} \Gamma \left[\frac{\nu+(\alpha+1)/2}{(1-\alpha)/2} \right] |b^2-c^2|^{-(\alpha+1)/2} P_{-(\alpha+1)/2}^{-\nu} \left(\frac{b^2+c^2}{|b^2-c^2|} \right) = \\ = \frac{2^{\alpha} e^{-\alpha\pi i/2} \sqrt{c}}{\sqrt{b\pi} \Gamma((1-\alpha)/2)} |b^2-c^2|^{-\alpha/2} Q_{\nu-1/2}^{\alpha/2} \left(\frac{b^2+c^2}{2bc} \right) \\ [b \neq c; -2 \operatorname{Re} \nu < \operatorname{Re} \alpha < 2];$$

$$A_{-\nu, \nu}^{\alpha} = 2^{\alpha-1} |c^2-b^2|^{-\alpha/2} \Gamma \left[\frac{\alpha/2}{1 \pm \nu - \alpha/2} \right] P_{-\alpha/2}^{\pm \nu} \left(\frac{c^2+b^2}{|c^2-b^2|} \right) \\ [b \neq c; 0 < \operatorname{Re} \alpha < 2];$$

$$A_{\nu-1, \nu}^1 = \frac{b^{\nu-1}}{c^{\nu}} \begin{cases} 1 \\ 0 \end{cases} \quad \left[\begin{cases} 0 < b < c \\ 0 < c < b \end{cases}, \operatorname{Re} \nu > 0 \right];$$

$$A_{\nu, \nu}^0 = \frac{1}{2^{\nu}} \left(\frac{b}{c} \right)^{\nu} \quad [0 < b < c; \operatorname{Re} \nu > 0];$$

$$A_{1,1}^{-1} = \frac{b+c}{3\pi bc} [(b^2+c^2) E(k) - (b-c)^2 K(k)];$$

$$A_{1,0}^0 = \frac{2}{\pi} E\left(\frac{c}{b}\right) \quad [0 < c < b],$$

$$A_{1,0}^0 = \frac{2c}{\pi b} \left[K\left(\frac{b}{c}\right) - \frac{c^2-b^2}{c^2} E\left(\frac{b}{c}\right) \right] \quad [0 < b < c];$$

$$A_{2,1}^0 = \frac{1}{3\pi b^2 c} [(b-c)(b^2+2c^2) K(k) - (b+c)(b^2-2c^2) E(k)];$$

$$A_{3,2}^0 = \frac{1}{15\pi b^3 c^2} [(b-c)(2b^4+5b^2c^2+8c^4) K(k) - (b+c)(2b^4+3b^2c^2-8c^4) E(k)];$$

$$A_{\nu, \nu+2n+1}^1 = (-1)^n \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \left(\frac{c}{b}\right)^\nu P_n^{(0, \nu)}\left(\frac{2c^2}{b^2}-1\right) \left[\begin{matrix} 0 < c < b \\ 0 < b < c \end{matrix} \right], \quad \text{Re } \nu > -n-1.$$

$$A_{0,0}^1 = \frac{2}{\pi c} K\left(\frac{b}{c}\right) \quad [0 < b < c];$$

$$A_{1,1}^1 = \frac{2b}{\pi c^2} D\left(\frac{b}{c}\right) \quad [0 < b < c];$$

$$A_{3,1}^1 = \frac{1}{3\pi b^3 c (b+c)} [(b^4+b^2c^2-8c^4) K(k) - (b+c)^2 (b^2-8c^2) E(k)];$$

$$A_{4,2}^1 = \frac{2}{15\pi b^4 c^2 (b+c)} [(b^6+4b^4c^2+4b^2c^4-24c^6) K(k) - (b+c)^2 (b^4+4b^2c^2-24c^4) E(k)];$$

$$A_{0,0}^{1 \pm 1/2} = \frac{2^{1 \pm 1/2}}{\pi \sqrt{b}} \Gamma \left[\frac{(2 \pm 1)/4}{(2 \mp 1)/4} \right] (b^2-c^2)^{\mp 1/2} K \left[\left(\frac{b - \sqrt{b^2-c^2}}{2b} \right)^{1/2} \right] \quad [0 < c < b].$$

$$2. \int_0^\infty x^{\alpha-1} J_\mu(cx) J_\nu(cx) dx = 2^{\alpha-1} c^{-\alpha} \Gamma \left[\frac{1-\alpha, (\alpha+\nu+\mu)/2}{1+(\mu-\nu-\alpha)/2, 1+(\nu+\mu-\alpha)/2, 1-(\nu-\mu-\alpha)/2} \right] \quad [\text{Re } \alpha < 1; c, \text{Re } (\alpha + \nu + \mu) > 0]$$

$$3. \int_0^\infty \frac{1}{x} J_\mu(cx) J_\nu(cx) dx = \frac{2}{\pi (\nu^2 - \mu^2)} \sin \frac{\nu - \mu}{2} \pi \quad [c, \text{Re } (\mu + \nu) > 0].$$

$$4. \int_{-\infty}^\infty \frac{1}{x} J_{m+1/2}(cx) J_{n+1/2}(cx) dx = \frac{2}{2n+1} \delta_{m,n} \quad [c > 0].$$

$$5. \int_0^\infty \frac{1}{x} [1 - J_0(bx)] J_0(cx) dx = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \ln \frac{b}{c} \quad \left[\begin{matrix} 0 < b < c \\ 0 < c < b \end{matrix} \right].$$

$$6. \int_0^\infty \frac{1}{x^2} [1 - J_0(bx)] J_1(cx) dx = \frac{c}{4} \left(1 + 2 \ln \frac{b}{c} \right) \quad [0 < c < b],$$

$$= \frac{b^2}{4c} \quad [0 < b < c]$$

$$7. \int_0^{\infty} \frac{1}{x} [J_0(bx) - J_0(cx)]^2 dx = \ln \frac{b}{c} \quad (b > c > 0).$$

2.12.32. Интегралы от $A(x) J_{\mu}(bx) J_{\nu}(cx)$.

$$1. \int_0^{\infty} \frac{1}{x \pm z} J_{\nu}^2(cx) dx = -\frac{\pi}{2} J_{\nu}(cz) Y_{\nu}(cz) \mp$$

$$\mp \frac{cz}{\pi} \Gamma\left[\frac{\nu-1/2}{\nu+3/2}\right] {}_2F_3\left(1, 1; \frac{3}{2}, \frac{3}{2} + \nu, \frac{3}{2} - \nu; -c^2 z^2\right) \mp$$

$$\mp \frac{\pi (cz)^{2\nu} \operatorname{tg} \nu\pi}{2^{2\nu+1} \Gamma^2(\nu+1)} {}_1F_2\left(\nu + \frac{1}{2}; \nu+1, 2\nu+1; -c^2 z^2\right)$$

$$\left[c > 0, \left\{ \begin{array}{l} \arg z < \pi \\ z > 0 \end{array} \right\}\right].$$

$$2. \int_0^a x^{\alpha-1} (a-x)^{\beta-1} J_{\mu}(cx) J_{\nu}(cx) dx =$$

$$= a^{\alpha+\beta+\mu+\nu-1} \left(\frac{c}{2}\right)^{\mu+\nu} \Gamma\left[\begin{matrix} \beta, \alpha+\mu+\nu \\ \mu+1, \nu+1, \alpha+\beta+\mu+\nu \end{matrix}\right] \times$$

$$\times {}_4F_6\left(\frac{\mu+\nu+1}{2}, \frac{\mu+\nu+2}{2}, \frac{\mu+\nu+\alpha}{2}, \frac{\mu+\nu+\alpha+1}{2}; \mu+1, \nu+1, \right.$$

$$\left. \mu+\nu+1, \frac{\mu+\nu+\alpha+\beta}{2}, \frac{\mu+\nu+\alpha+\beta+1}{2}; -a^2 c^2\right)$$

$$[a, \operatorname{Re} \beta, \operatorname{Re}(\alpha + \mu + \nu) > 0].$$

$$3. \int_0^a x^{\alpha-1} (a^2-x^2)^{\beta-1} J_{\mu}(cx) J_{\nu}(cx) dx =$$

$$= \frac{1}{2} a^{\alpha+2\beta+\mu+\nu-2} \left(\frac{c}{2}\right)^{\mu+\nu} \Gamma\left[\begin{matrix} \beta, (\alpha+\mu+\nu)/2 \\ \beta+(\alpha+\mu+\nu)/2, 1+\mu, 1+\nu \end{matrix}\right] \times$$

$$\times {}_3F_4\left(\frac{\alpha+\mu+\nu}{2}, \frac{\mu+\nu+1}{2}, \frac{\mu+\nu}{2}+1; \beta+\frac{\alpha+\mu+\nu}{2}, \mu+1, \mu+\nu+1, \nu+1; -a^2 c^2\right)$$

$$[a, \operatorname{Re} \beta, \operatorname{Re}(\alpha + \mu + \nu) > 0]$$

$$4. \int_0^a \frac{x^{\pm 2\nu+1}}{(a^2-x^2)^{\pm \nu+3/2}} J_{-\nu}(cx) J_{\nu}(cx) dx = \frac{(ac)^{\pm \nu}}{2a \sqrt{\pi}} \Gamma\left(\mp \nu - \frac{1}{2}\right) J_{-\nu}(2ac)$$

$$[a > 0; -1 < \pm \operatorname{Re} \nu < -1/2]$$

$$5. \int_0^a x^{2\nu+2} (a^2-x^2)^{-\nu-3/2} J_{\nu+1}(cx) J_{\nu}(cx) dx = \frac{1}{2 \sqrt{\pi}} \Gamma\left(-\nu - \frac{1}{2}\right) (ac)^{\nu} J_{\nu+1}(2ac)$$

$$[a > 0; -1 < \operatorname{Re} \nu < -1/2]$$

$$6. \int_0^a \frac{1}{x \sqrt{a^2-x^2}} J_1^2(cx) dx = \frac{1}{2a} - \frac{1}{2a^2 c} J_1(2ac) \quad [a > 0]$$

$$7. \int_0^a \frac{1}{x^3 \sqrt{a^2-x^2}} J_{\frac{3}{2}}^2(cx) dx = \frac{1}{4a^3 c^2} \left[\frac{a^2 c^2}{2} + \frac{a^4 c^4}{6} - J_2(2ac) - 2ac J_3(2ac) \right] \quad [a > 0]$$

$$\begin{aligned}
8. \int_a^\infty x^{\alpha-1} (x^2 - a^2)^{\beta-1} J_\mu(cx) J_\nu(cx) dx &= \frac{1}{2} \left(\frac{2}{c}\right)^{\alpha+2\beta-2} \times \\
&\times \Gamma \left[\frac{(\alpha+\mu+\nu)/2 + \beta - 1, 3-\alpha-2\beta}{2-\beta+(\mu-\nu-\alpha)/2, 2-\beta+(\mu+\nu-\alpha)/2, 2-\beta+(\nu-\mu-\alpha)/2} \right] \times \\
&\times {}_3F_4 \left(1-\beta, \frac{3-\alpha}{2}-\beta, 2-\beta-\frac{\alpha}{2}; 2-\beta-\frac{\mu+\nu+\alpha}{2}, 2-\beta+\frac{\mu-\nu-\alpha}{2}, \right. \\
&\quad \left. 2-\beta+\frac{\mu+\nu-\alpha}{2}, 2-\beta+\frac{\nu-\mu-\alpha}{2}; -a^2c^2 \right) + \\
&\quad + \frac{a^{\mu+\nu+\alpha+2\beta-2} c^{\mu+\nu}}{2^{\mu+\nu+1}} \Gamma \left[\beta, 1-\beta-(\alpha+\mu+\nu)/2 \right. \\
&\quad \left. 1-(\alpha+\mu+\nu)/2, 1+\mu, 1+\nu \right] \times \\
&\times {}_3F_4 \left(\frac{\alpha+\mu+\nu}{2}, \frac{1+\mu+\nu}{2}, 1+\frac{\mu+\nu}{2}; \beta+\frac{\alpha+\mu+\nu}{2}, 1+\mu, 1+\nu, 1+\mu+\nu; -a^2c^2 \right) \\
&\quad [a, c, \operatorname{Re} \beta > 0; \operatorname{Re} (\alpha + 2\beta) < 3].
\end{aligned}$$

$$\begin{aligned}
9. \int_a^\infty x^{1-2\nu} (x^2 - a^2)^{\nu-3/2} J_\nu^2(cx) dx &= \frac{\Gamma(\nu-1/2)}{2a^{\nu+1}c^\nu \sqrt{\pi}} H_\nu(2ac) \\
&\quad [a, c > 0; \operatorname{Re} \nu > 1/2].
\end{aligned}$$

$$\begin{aligned}
10. \int_0^\infty \frac{x^{\alpha-1}}{(x^2+z^2)^\rho} J_\mu(cx) J_\nu(cx) dx &= \frac{1}{2} \left(\frac{c}{2}\right)^{2\rho-\alpha} \times \\
&\times \Gamma \left[\frac{(\mu+\nu+\alpha)/2 - \rho, 1+2\rho-\alpha}{(\mu-\nu-\alpha)/2 + \rho + 1, (\mu+\nu-\alpha)/2 + \rho + 1, (\nu-\mu-\alpha)/2 + \rho + 1} \right] \times \\
&\times {}_3F_4 \left(\frac{1-\alpha}{2} + \rho, 1 - \frac{\alpha}{2} + \rho, \rho; \rho + 1 - \frac{\mu+\nu+\alpha}{2}, \rho + 1 + \frac{\mu-\nu-\alpha}{2}, \right. \\
&\quad \left. \rho + 1 + \frac{\mu+\nu-\alpha}{2}, \rho + 1 + \frac{\nu-\mu-\alpha}{2}; c^2z^2 \right) + \frac{z^{\alpha-2\rho}}{2} \left(\frac{cz}{2}\right)^{\mu+\nu} \times \\
&\times \Gamma \left[\rho - (\alpha+\mu+\nu)/2, (\alpha+\mu+\nu)/2 \right] {}_3F_4 \left(\frac{1+\mu+\nu}{2}, 1 + \frac{\mu+\nu}{2}, \right. \\
&\quad \left. \frac{\alpha+\mu+\nu}{2}; 1 - \rho + \frac{\alpha+\mu+\nu}{2}, \mu+1, \nu+1, \mu+\nu+1; c^2z^2 \right) \\
&\quad [c, \operatorname{Re} z, \operatorname{Re} (\alpha + \mu + \nu) > 0; \operatorname{Re} (\alpha - 2\rho) < 1]
\end{aligned}$$

$$\begin{aligned}
11. \int_0^\infty \frac{x^{\mu-\nu+2n+1}}{x^2+z^2} J_\mu(bx) J_\nu(cx) dx &= (-1)^n z^{\mu-\nu+2n} I_\nu(cz) K_\mu(bz) \\
&\quad [0 < c < b; \operatorname{Re} z > 0; -1-n < \operatorname{Re} \mu < \operatorname{Re} \nu + 1 - 2n].
\end{aligned}$$

$$\begin{aligned}
12. \int_0^\infty \frac{x^{1-2n}}{x^2+z^2} J_\nu(bx) J_\nu(cx) dx &= (-1)^n z^{-2n} I_\nu(cz) K_\nu(bz) \\
&\quad [0 < c < b; \operatorname{Re} z > 0; \operatorname{Re} \nu > n-1].
\end{aligned}$$

$$\begin{aligned}
13. \int_0^\infty \left[\frac{1}{c(x+z)} - \pi J_\nu^2(cx) \right] dx &= \frac{1}{c} \left[\ln(2cz) - \psi \left(\nu + \frac{1}{2} \right) \right] \\
&\quad [c > 0; \operatorname{Re} \nu > -1/2, |\arg z| < \pi].
\end{aligned}$$

2.12.33. Интегралы от $A(x) J_\mu(a \pm cx) J_\nu(cx+d)$.

$$\begin{aligned}
1. \int_0^a J_\mu(ca-cx) J_\nu(cx) dx &= \frac{2}{c} \sum_{k=0}^\infty (-1)^k J_{\mu+\nu+2k+1}(ac) \\
&\quad [a > 0; \operatorname{Re} \mu, \operatorname{Re} \nu > -1].
\end{aligned}$$

2. $\int_0^a J_0(ca - cx) J_\nu(cx) dx = \frac{2}{c} U_{\nu+1}(ac, ac) \quad [a > 0; \operatorname{Re} \nu > -1].$
3. $\int_0^a J_{-\nu}(ca - cx) J_\nu(cx) dx = \frac{\sin ac}{c} \quad [a > 0; |\operatorname{Re} \nu| < 1/2].$
4. $\int_0^a J_{1-\nu}(ca - cx) J_\nu(cx) dx = \frac{1}{c} [J_0(ac) - \cos ac] \quad [a > 0; -1 < \operatorname{Re} \nu < 2].$
5. $\int_0^a x^{\alpha-1} J_\mu(ca - cx) J_\nu(cx) dx =$

$$= \left(\frac{2}{c}\right)^\alpha \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} (\alpha)_k \Gamma\left[\frac{\alpha + \nu + k}{\nu + k + 1}\right] J_{\alpha + \mu + \nu + 2k}(ac)$$

$$[a, \operatorname{Re}(\alpha + \nu) > 0; \operatorname{Re} \mu > -1].$$
6. $\int_0^a \frac{1}{x} J_\mu(ca - cx) J_\nu(cx) dx = \frac{1}{\nu} J_{\mu+\nu}(ac) \quad [a, \operatorname{Re} \nu > 0; \operatorname{Re} \mu > -1]$
7. $\int_0^a \frac{x^{\alpha-1}}{a-x} J_\mu(ca - cx) J_\nu(cx) dx =$

$$= \left(\frac{2}{c}\right)^\alpha \frac{1}{a\mu} \sum_{k=0}^{\infty} \frac{(\alpha)_k (-1)^k}{k!} \Gamma\left[\frac{\alpha + \nu + k}{\nu + k + 1}\right] (\alpha + \mu + \nu + 2k) J_{\alpha + \mu + \nu + 2k}(ac)$$

$$[a, \operatorname{Re} \mu, \operatorname{Re}(\alpha + \nu) > 0]$$
8. $\int_0^a \frac{1}{x(a-x)} J_\mu(ca - cx) J_\nu(cx) dx = \frac{\mu + \nu}{a\mu\nu} J_{\mu+\nu}(ac) \quad [a, \operatorname{Re} \nu, \operatorname{Re} \mu > 0].$
9. $\int_0^a x^\nu (a-x)^{\mu+(1 \pm 1)/2} J_\mu(ca - cx) J_\nu(cx) dx =$

$$= \frac{a^{\mu+\nu+1 \pm 1/2}}{\sqrt{2\pi c}} B\left(\mu + 1 \pm \frac{1}{2}, \nu + \frac{1}{2}\right) J_{\mu+\nu+1/2}(ac)$$

$$[a > 0, \operatorname{Re} \nu > -1/2, \operatorname{Re} \mu > (-3 \mp 1)/4]$$
10. $\int_0^a x^{\nu+1} (a-x)^{\mu+1} J_\mu(ca - cx) J_\nu(cx) dx = \frac{1}{\sqrt{2\pi}} \Gamma\left[\frac{\mu+3/2, \nu+3/2}{\mu+\nu+2}\right] \times$

$$\times a^{\mu+\nu+3/2} c^{-3/2} \left[J_{\mu+\nu+3/2}(ac) - \frac{ac}{2(\mu+\nu+2)} J_{\mu+\nu+5/2}(ac) \right]$$

$$[a > 0; \operatorname{Re} \mu, \operatorname{Re} \nu > -1]$$
11. $\int_0^a x^{\nu+1} (a-x)^{-\nu-2} J_\mu(ca - cx) J_\nu(cx) dx =$

$$= \frac{2^{2\nu+1}}{\sqrt{\pi}} \Gamma\left[\frac{\nu+3/2, \mu-\nu-1}{\mu+\nu+1}\right] \left(\frac{ac}{2}\right)^\nu \left[J_\mu(ac) - \frac{ac}{\mu+\nu+1} J_{\mu+1}(ac) \right]$$

$$[a > 0; \operatorname{Re}(\mu - \nu) > 1; \operatorname{Re} \nu > -1].$$

$$12. \int_0^a x^\nu (a-x)^{-\nu-1} J_\mu(ca-cx) J_\nu(cx) dx = \frac{(2ac)^\nu}{\sqrt{\pi}} \Gamma \left[\begin{matrix} \nu+1/2, \mu-\nu \\ \mu+\nu+1 \end{matrix} \right] J_\mu(ac)$$

[$a > 0; -1/2 < \operatorname{Re} \nu < \operatorname{Re} \mu$].

$$13. \int_0^a \frac{a-x}{x^2} J_0(ca-cx) J_\nu(cx) dx = \frac{1}{\nu(\nu-1)} \left[J_\nu(ac) - \frac{ac}{\nu+1} J_{\nu+1}(ac) \right]$$

[$a > 0; \operatorname{Re} \nu > 1$].

$$14. \int_{-\infty}^{\infty} \frac{J_\mu(ab+ax) J_\nu(ac+ax)}{(b+x)^\mu (c+x)^\nu} dx =$$

$$= \sqrt{\frac{2\pi}{a}} \Gamma \left[\begin{matrix} \mu+\nu \\ \mu+1/2, \nu+1/2 \end{matrix} \right] (b-c)^{1/2-\mu-\nu} J_{\mu+\nu-1/2}(ab-ac)$$

[$a, \operatorname{Re}(\mu+\nu) > 0, b > c > 0$].

2.12.34. Интегралы от $x^\alpha J_\mu(bx^\pm) J_\nu(cx)$.

$$1. \int_0^\infty J_{2\nu}(b\sqrt{x}) J_\nu(cx) dx = \frac{1}{c} J_\nu\left(\frac{b^2}{4c}\right)$$

[$b, c > 0; \operatorname{Re} \nu > -1/2$].

$$2. \int_0^\infty x^{\alpha-1} J_\mu(b\sqrt{x}) J_\nu(cx) dx =$$

$$= \frac{2^{\alpha-\mu/2-1} b^\mu}{c^{\alpha+\mu/2}} \Gamma \left[\begin{matrix} (\nu+\alpha)/2 + \mu/4 \\ (\nu-\alpha)/2 - \mu/4 + 1, \mu+1 \end{matrix} \right] \times$$

$$\times {}_2F_3 \left(\begin{matrix} \frac{\mu}{4} + \frac{\alpha-\nu}{2}, \frac{\mu}{4} + \frac{\alpha+\nu}{2}; \frac{1}{2}, \frac{\mu+1}{2}, \frac{\mu}{2} + 1, -\frac{b^4}{64c^2} \end{matrix} \right) -$$

$$- \frac{2^{\alpha-\mu/2-2} b^{\mu+2}}{c^{\alpha+\mu/2+1}} \Gamma \left[\begin{matrix} (\nu+\alpha+1)/2 + \mu/4 \\ (\nu-\alpha+1)/2 - \mu/4, \mu+2 \end{matrix} \right] \times$$

$$\times {}_2F_3 \left(\begin{matrix} \frac{1+\alpha-\nu}{2} + \frac{\mu}{4}, \frac{\nu+\alpha+1}{2} + \frac{\mu}{4}; \frac{3}{2}, \frac{\mu}{2} + 1, \frac{\mu+3}{2}; -\frac{b^4}{64c^2} \end{matrix} \right)$$

[$b, c > 0; -\operatorname{Re}(\nu + \mu/2) < \operatorname{Re} \alpha < 7/4$]

$$3. \int_0^\infty \sqrt{x} J_{2\nu \pm 1}(b\sqrt{x}) J_\nu(cx) dx = \frac{b}{2c^2} J_{\nu \pm 1}\left(\frac{b^2}{4c}\right)$$

[$b, c > 0; \operatorname{Re} \nu > (\mp 1 - 3)/4$]

$$4. \int_0^\infty \frac{1}{x^{1 \pm 1}} J_\nu\left(\frac{b}{x}\right) J_\nu(cx) dx = \left\{ \frac{b^{-1}}{c^{-1}} \right\} J_{2\nu}(2\sqrt{bc})$$

[$b, c > 0; \operatorname{Re} \nu > -1/2$]

$$5. \int_0^\infty \frac{1}{x} J_{\nu-1}\left(\frac{b}{x}\right) J_\nu(cx) dx = \frac{1}{\sqrt{bc}} J_{2\nu-1}(2\sqrt{bc})$$

[$b, c > 0; \operatorname{Re} \nu > -1/2$].

$$6. \int_0^{\infty} x^{\alpha-1} J_{\mu} \left(\frac{b}{x} \right) J_{\nu}(cx) dx = \frac{b^{\mu} c^{\mu-\alpha}}{2^{\mu-\alpha+1}} \Gamma \left[\frac{(\nu+\alpha-\mu)/2}{\mu+1, (\mu+\nu-\alpha)/2+1} \right] \times \\ \times {}_0F_3 \left(\mu+1, \frac{\mu-\nu-\alpha}{2}+1, \frac{\mu+\nu-\alpha}{2}+1; \frac{b^2 c^2}{16} \right) + \\ + \frac{b^{\nu+\alpha} c^{\nu}}{2^{\nu+\alpha+1}} \Gamma \left[\frac{(\mu-\nu-\alpha)/2}{\nu+1, (\mu+\nu+\alpha)/2+1} \right] \times \\ \times {}_0F_3 \left(\nu+1, \frac{\nu-\mu+\alpha}{2}+1, \frac{\mu+\nu+\alpha}{2}+1; \frac{b^2 c^2}{16} \right) \\ [b, c > 0; -\operatorname{Re} \nu - 3/2 < \operatorname{Re} \alpha < \operatorname{Re} \mu + 3/2].$$

$$7. \int_0^{\infty} x^{1/2-2\nu} J_{1/2-\nu} \left(\frac{b}{x} \right) J_{\nu}(cx) dx = \\ = -\frac{ib^{1/2-\nu} c^{\nu-1}}{2 \sin 2\nu\pi} [e^{2\pi i \nu} J_{1-2\nu}(u_+) J_{2\nu-1}(u_-) - e^{-2\pi i \nu} J_{2\nu-1}(u_+) J_{1-2\nu}(u_-)] \\ [u_{\pm} = \sqrt{bc/2} e^{\pm \pi i/4}, b, c > 0; -1/2 < \operatorname{Re} \nu < 3].$$

2.12.35. Интегралы от $A(x) J_{\mu}(b \sqrt{z^2 \pm x^2}) J_{\nu}(cx)$.

$$1. \int_0^a x^{\nu} J_0(c \sqrt{a^2 - x^2}) J_{\nu}(cx) dx = \\ = \frac{2^{\nu} \Gamma(\nu+1/2)}{\sqrt{\pi} \Gamma(2\nu+2)} a^{2\nu+1} c^{\nu} {}_1F_2 \left(\frac{1}{2}; \nu+1, \frac{2\nu+3}{2}; \frac{a^2 c^2}{4} \right) \\ [a > 0; \operatorname{Re} \nu > -1/2]$$

$$2. \int_0^a x^{\nu+2n+1} (a^2 - x^2)^{m+\mu/2} J_{\mu}(b \sqrt{a^2 - x^2}) J_{\nu}(cx) dx = \\ = a^{\mu+\nu-m-n+1} b^{\mu} c^{\nu} \left(\frac{\partial}{b \partial b} \right)^m \left(\frac{\partial}{c \partial c} \right)^n [(b^2 + c^2)^{-(\mu+\nu+m+n+1)/2} J_{\mu+\nu+m+n+1}(a \sqrt{b^2 + c^2})] \\ [a > 0; \operatorname{Re} \nu > -n-1; \operatorname{Re} \mu > -m-1].$$

$$3. \int_a^{\infty} x^{\nu+1} (x^2 - a^2)^{\mu/2} J_{\mu}(b \sqrt{x^2 - a^2}) J_{\nu}(cx) dx = \\ = -\frac{2}{\pi} a^{\mu+\nu+1} b^{\mu} c^{\nu} (b^2 - c^2)^{-(\mu+\nu+1)/2} K_{\mu+\nu+1}(a \sqrt{b^2 - c^2}) \\ [0 < c < b; a > 0; \operatorname{Re} \mu > -1; \operatorname{Re}(\mu + \nu) < -1].$$

$$4. = a^{\mu+\nu+1} b^{\mu} c^{\nu} (c^2 - b^2)^{-(\mu+\nu+1)/2} [\sin \mu\pi Y_{\mu+\nu+1}(a \sqrt{c^2 - b^2}) - \\ - \cos \mu\pi J_{\mu+\nu+1}(a \sqrt{c^2 - b^2})] \\ [0 < b < c; a > 0; \operatorname{Re} \mu > -1; \operatorname{Re}(\mu + \nu) < -1].$$

$$5. \int_a^{\infty} x (x^2 - a^2)^{n/2} J_n(a \sqrt{x^2 - a^2}) J_{2n+1}(cx) dx = (c^2 - a^2)^{n/2} J_n(a \sqrt{c^2 - a^2}) \\ [a, c > 0; a \neq c].$$

$$6. \int_a^{\infty} x^{1-\nu} (x^2 - a^2)^{\mu/2} J_{\mu}(b \sqrt{x^2 - a^2}) J_{\nu}(cx) dx = \\ = a^{\mu-\nu+1} b^{\mu} c^{-\nu} (c^2 - b^2)^{(\nu-\mu-1)/2} J_{\nu-\mu-1}(a \sqrt{c^2 - b^2}) \\ [a, b, c > 0; b \neq c; \operatorname{Re} \mu > -1; \operatorname{Re}(\mu - \nu) < -1].$$

$$7. \int_a^\infty \frac{x^{1-\nu} (x^2 - a^2)^{\mu/2 + n - 1}}{x^2 + z^2} J_\mu (b \sqrt{x^2 - a^2}) J_\nu (cx) dx =$$

$$= (-1)^{n+1} z^{-\nu} (a^2 + z^2)^{\mu/2 + n - 1} K_\mu (b \sqrt{a^2 + z^2}) I_\nu (cz)$$

[0 < c < b; a, Re z > 0; -n < Re μ < 4 - 2n + Re ν].

$$8. \int_0^\infty (x^2 + z^2)^{-\nu/2} J_\nu (b \sqrt{x^2 + z^2}) J_1 (cx) dx =$$

$$= \frac{1}{c} z^{-\nu} J_\nu (bz) - \frac{(b^2 - c^2)^{\nu/2}}{(bz)^\nu c} J_\nu (z \sqrt{b^2 - c^2})$$

[0 < c < b; Re z > 0; Re ν > -1].

$$9. \int_0^\infty x^{\nu-1} (x^2 + z^2)^{-\mu/2} J_\mu (b \sqrt{x^2 + z^2}) J_\nu (cx) dx = \frac{2^{\nu-1} \Gamma(\nu)}{c^\nu z^\mu} J_\mu (bz)$$

[0 < b < c; Re z > 0; 0 < Re ν < Re μ + 2].

$$10. \int_0^\infty x^{\nu+1} (x^2 + z^2)^{\mu/2} J_\mu (b \sqrt{x^2 + z^2}) J_\nu (cx) dx =$$

$$= b^\mu c^\nu z^{\mu+\nu+1} (b^2 - c^2)^{-(\mu+\nu+1)/2} [\sin \nu \pi Y_{\mu+\nu+1} (z \sqrt{b^2 - c^2}) -$$

$$- \cos \nu \pi J_{\mu+\nu+1} (z \sqrt{b^2 - c^2})]$$

[0 < c < b; Re z > 0; -1 < Re ν < -Re μ].

$$11. = \frac{2 \sin \mu \pi}{\pi} b^\mu c^\nu z^{\mu+\nu+1} (c^2 - b^2)^{-(\mu+\nu+1)/2} K_{\mu+\nu+1} (z \sqrt{c^2 - b^2})$$

[0 < b < c; Re z > 0; -1 < Re ν < -Re μ].

$$12. \int_0^\infty x^{\nu+1} (x^2 + z^2)^{-\mu/2} J_\mu (b \sqrt{x^2 + z^2}) J_\nu (cx) dx =$$

$$= \frac{c^\nu z^{1+\nu-\mu}}{b^\mu} (b^2 - c^2)^{(\mu-\nu-1)/2} J_{\mu-\nu-1} (z \sqrt{b^2 - c^2})$$

[b, c, Re z > 0; b ≠ c; -1 < Re ν < Re μ].

$$13. \int_0^\infty x^{\nu+1} (x^2 + z^2)^{-(\mu+3)/2} J_\mu (b \sqrt{x^2 + z^2}) J_\nu (cx) dx =$$

$$= \frac{b^\mu}{2^{\mu-1/2}} \sqrt{\frac{c}{\pi}} \frac{z^{\nu-1/2}}{\Gamma(\mu+1)} K_{\nu-1/2} (cz)$$

[0 < b < c; Re z > 0; -1 < Re ν < Re μ + 3].

$$14. \int_0^\infty x^{\nu+2n-1} \frac{J_\mu (b \sqrt{x^2 + z^2}) J_\nu (cx)}{(x^2 + z^2)^{\mu/2} (x^2 + \zeta^2)} dx =$$

$$= (-1)^{n+1} \zeta^{\nu+2n-2} \left\{ \frac{(z^2 - \zeta^2)^{-\mu/2} J_\mu (b \sqrt{z^2 - \zeta^2})}{(\zeta^2 - z^2)^{-\mu/2} I_\mu (b \sqrt{\zeta^2 - z^2})} \right\} K_\nu (c\zeta)$$

[0 < b ≤ c; Re ζ, Re z > 0; -n < Re ν < Re μ + 2, { |arg(z^2 - ζ^2)| < π }
z < ζ].

2.12.36. Интегралы от $x^\alpha J_\mu (\varphi(x)) J_\nu (\chi(x))$.

$$1. \int_0^\infty x^{\alpha-1} J_{\pm \nu} (b \sqrt{x^2 + z^2} + bz) J_\nu (b \sqrt{x^2 + z^2} - bz) dx =$$

$$= \frac{1}{2 \sqrt{\pi}} \left(\frac{z}{b} \right)^{\alpha/2} \Gamma \left[\frac{\nu + \alpha/2, (1 - \alpha)/2}{1 + \nu - \alpha/2} \right] \left(\left\{ \begin{matrix} 1 \\ \cos \nu \pi \end{matrix} \right\} J_{-\alpha/2} (2bz) - \right.$$

$$\left. - \left\{ \begin{matrix} 0 \\ \sin \nu \pi \end{matrix} \right\} Y_{-\alpha/2} (2bz) \right)$$

[b, Re z > 0; -2 Re ν < Re α < 1].

2.12.37. Интегралы, содержащие $J_\mu(cx)/J_\nu(cx)$.

$$1. \int_0^1 \frac{x^k J_m(x)}{J_n(x)} dx = A_{m,n}^k,$$

$$A_{0,1}^1 = 1,914485 \dots, \quad A_{1,0}^0 = 0,279773 \dots, \quad A_{2,1}^0 = 0,127718 \dots$$

$$2. \int_0^\infty \frac{J_{\nu+1}(cx)}{x J_\nu(cx)} dx = \frac{\pi}{2}$$

$$[c > 0].$$

$$3. \int_0^\infty \frac{J_{\nu+1}(cx)}{x(x^2 + z^2) J_\nu(cx)} dx = \frac{\pi J_{\nu+1}(cz)}{2z^2 J_\nu(cz)}$$

$$[c, \operatorname{Re} z > 0]$$

2.12.38. Интегралы от $x^\alpha e^{-px} J_\mu(bx) J_\nu(cx)$

$$1. \int_0^\infty e^{-px} J_\mu(bx) J_\nu(cx) dx = I_{\mu,\nu}$$

$$[\operatorname{Re} p > |\operatorname{Im} b| + |\operatorname{Im} c|; \operatorname{Re}(\mu + \nu) > -1;$$

$$k = \sin \varphi = 2\sqrt{bc}/\sqrt{p^2 + (b+c)^2}, \quad \sin \psi = p/\sqrt{p^2 + (b-c)^2},$$

$$I_{\nu,\nu} = \frac{1}{\pi\sqrt{bc}} Q_{\nu-1/2} \left(\frac{p^2 + b^2 + c^2}{2bc} \right)$$

$$[\operatorname{Re} \nu > -1/2],$$

$$I_{0,0} = \frac{k}{\pi\sqrt{bc}} K(k),$$

$$I_{1,0} = -\frac{pk}{2\pi(b^3c)^{1/2}} \left[K(k) \pm \frac{1}{2b} \Lambda_0(\varphi, \psi) \right] + \left\{ \begin{matrix} 0 \\ b^{-1} \end{matrix} \right\}$$

$$\left[\left\{ \begin{matrix} b < c \\ b > c \end{matrix} \right\} \right],$$

$$I_{1,1} = \frac{1}{\pi k \sqrt{bc}} [(2 - k^2) K(k) - 2E(k)],$$

$$I_{2,0} = \frac{4\sqrt{c}}{\pi b^{3/2} k} E(k) - \frac{kc^{3/2}}{\pi b^{5/2}} K(k) \mp \frac{p}{b^2} \Lambda_0(\varphi, \psi) - \left\{ \begin{matrix} 0 \\ 2pb^{-2} \end{matrix} \right\}$$

$$\left[\left\{ \begin{matrix} b < c \\ b > c \end{matrix} \right\} \right],$$

$$I_{2,1} = \frac{2p}{\pi k (b^3c)^{1/2}} E(k) - \frac{pk(p^2 + b^2 + 2c^2)}{2\pi b^{5/2} c^{3/2}} K(k) \pm \frac{c}{2b^2} \Lambda_0(\varphi, \psi) + \left\{ \begin{matrix} c^{-1} \\ b^{-2}c \end{matrix} \right\}$$

$$\left[\left\{ \begin{matrix} b < c \\ b > c \end{matrix} \right\} \right],$$

$$I_{2,2} = \frac{1}{3\pi k^3 \sqrt{bc}} [(4 - k^2)(4 - 3k^2) K(k) - 8(2 - k^2) E(k)],$$

$$I_{3,2} = \frac{1}{15\pi k^5 \sqrt{bc}} [(128 - 128k^2 + 15k^4)(2 - k^2) K(k) - 2(128 - 128k^2 + 23k^4) E(k)],$$

$$I_{4,4} = \frac{1}{105\pi k^7 \sqrt{bc}} [(6144 - 12288k^2 + 8000k^4 -$$

$$-1856k^6 + 105k^8) K(k) - 32(2 - k^2)(96 - 96k^2 + 11k^4) E(k)].$$

$$2. \int_0^\infty x^{\alpha-1} e^{-px} J_\mu(bx) J_\nu(cx) dx = A_{\mu,\nu}^\alpha$$

$$[\operatorname{Re}(\alpha + \mu + \nu) > 0; \operatorname{Re} p > |\operatorname{Im} b| + |\operatorname{Im} c|]$$

$$k, \varphi, \psi \text{ см. в 2.12.38 1; } 2k_b = (1 - \sqrt{1 - b^2 \gamma^2})^{1/2}, \quad 2k_c = (1 - \sqrt{1 - c^2 \gamma^2})^{1/2},$$

$$2\gamma = \sqrt{p^2 + (b+c)^2} + \sqrt{p^2 + (b-c)^2},$$

$$A_{\mu, \nu}^{\alpha} = \frac{b^{\mu} c^{\nu}}{2^{\mu+\nu} p^{\alpha+\mu+\nu}} \Gamma \left[\begin{matrix} \alpha + \mu + \nu \\ \mu + 1, \nu + 1 \end{matrix} \right] \times$$

$$\times F_4 \left(\frac{\alpha + \mu + \nu}{2}, \frac{\alpha + \mu + \nu + 1}{2}; \mu + 1, \nu + 1; -\frac{b^2}{p^2}, -\frac{c^2}{p^2} \right) =$$

$$= \frac{b^{\mu} c^{\nu}}{2^{\mu+\nu} (p + ib + ic)^{\alpha+\mu+\nu}} \Gamma \left[\begin{matrix} \alpha + \mu + \nu \\ \mu + 1, \nu + 1 \end{matrix} \right] \times$$

$$\times F_2 \left(\alpha + \mu + \nu, \mu + \frac{1}{2}, \nu + \frac{1}{2}; 2\mu + 1, 2\nu + 1; \frac{2ib}{p + ib + ic}, \frac{2ic}{p + ib + ic} \right) =$$

$$= \frac{b^{\mu} c^{\nu}}{2^{\mu+\nu} p^{\alpha+\mu+\nu} \Gamma(\nu + 1)} \sum_{k=0}^{\infty} \frac{1}{k!} \Gamma \left[\begin{matrix} \alpha + \mu + \nu + 2k \\ \mu + k + 1 \end{matrix} \right] \times$$

$$\times {}_2F_1 \left(-k, -\mu - k; \nu + 1; \frac{c^2}{b^2} \right) \left(-\frac{b^2}{4p^2} \right)^k,$$

$$A_{\mu, \nu}^{1/2} = \gamma^{-1/2} \Gamma \left(\mu + \nu + \frac{1}{2} \right) P_{\nu-1/2}^{-\mu} (\sqrt{1 - b^2 \gamma^2}) P_{\mu-1/2}^{-\nu} (\sqrt{1 - c^2 \gamma^2}) =$$

$$= \frac{b^{\mu} c^{\nu}}{2^{\mu+\nu} \gamma^{\mu+\nu+1/2}} \Gamma \left[\begin{matrix} \mu + \nu + 1/2 \\ \mu + 1, \nu + 1 \end{matrix} \right] {}_2F_1 \left(\frac{2\mu + 2\nu + 1}{4}, \frac{2\mu - 2\nu + 1}{4}; \right.$$

$$\left. \mu + 1; \frac{b^2}{\gamma^2} \right) {}_2F_1 \left(\frac{2\mu + 2\nu + 1}{4}, \frac{2\nu - 2\mu + 1}{4}; \nu + 1; \frac{c^2}{\gamma^2} \right)$$

$$[\operatorname{Re}(\mu + \nu) > -1/2],$$

$$A_{\mu, \nu}^{\mu+\nu+(3\pm 1)/2} = \frac{b^{\mu} c^{\nu}}{2^{\mu+\nu} p^{2\mu+2\nu+(3\pm 1)/2}} \Gamma \left[\begin{matrix} 2\mu + 2\nu + (3\pm 1)/2 \\ \mu + 1, \nu + 1 \end{matrix} \right] \times$$

$$\times F_2 \left(\mu + \nu + 1, \mu + \nu + 1 \pm \frac{1}{2}, \mu + \nu + 1 \pm \frac{1}{2}; \mu + 1, \nu + 1; \right.$$

$$\left. 1 + \frac{c^2 - \gamma^2}{p^2}, 1 + \frac{b^2 - \gamma^2}{p^2} \right) [\operatorname{Re}(\mu + \nu) > -(3\pm 1)/4],$$

$$A_{\mu, \nu}^{\mu-\nu+(3\pm 1)/2} = \frac{b^{\mu} c^{\nu}}{2^{\nu-\mu-(1\pm 1)/2} \sqrt{\pi}} \Gamma \left[\begin{matrix} \mu + 1 \pm 1/2 \\ \nu + 1 \end{matrix} \right] \times$$

$$\times (\gamma^2 - c^2)^{-\mu-1\pm 1/2} F_1 \left(\mu + 1 \pm \frac{1}{2}, \nu \mp \frac{1}{2}, \mu + \nu + 1 \pm \frac{1}{2}; \nu + 1; \right.$$

$$\left. \frac{c^2}{c^2 - \gamma^2}, \frac{\gamma^2 - p^2 - b^2}{c^2 - \gamma^2} \right) [\operatorname{Re} \mu > -(3\pm 1)/4],$$

$$A_{\mu, \nu}^{(m+n)/2} \Big|_{2\nu-2\mu-m-n} = \frac{b^{\mu} c^{\nu}}{2^{\mu+\nu+1} (2\mu+1)_{m-1} (2\nu+1)_{n-1}} \times$$

$$\times \Gamma \left[\begin{matrix} \mu + \nu + (m+n)/2 \\ \mu + 1, \nu + 1 \end{matrix} \right] \sum_{k=0}^{m-1} \sum_{l=0}^{n-1} \binom{m-1}{k} \binom{n-1}{l} \left(\mu + \frac{1}{2} \right)_k \times$$

$$\times \left(\mu + \frac{1}{2} \right)_{m-k-1} \left(\nu + \frac{1}{2} \right)_l \left(\nu + \frac{1}{2} \right)_{n-l-1} [(p + ib + ic)^{k+l-(m+n)/2+1} \times$$

$$\times (p + ib - ic)^{k-l+(n-m)/2} + (p - ib - ic)^{k+l-(m+n)/2+1} \times$$

$$\times (p - ib + ic)^{k-l+(n-m)/2}] \left(\frac{\sin \psi}{p} \right)^{2k+2\mu+1} {}_2F_1 \left(\mu + k + \frac{1}{2}, \right.$$

$$\left. \nu + l + \frac{1}{2}; 2\nu + n; 1 - \frac{4\ell \sin^2 \psi}{p^2 \sin^2 \varphi} \right)$$

$$[\operatorname{Re}(\mu + \nu) > -(m+n)/2; m, n = 1, 2, 3, \dots],$$

$$A_{\nu, \nu}^{2\nu+n+1} = \frac{(bc)^\nu}{2^{2\nu-n} p^{n+4\nu+1}} \Gamma \left[\begin{matrix} 4\nu+1 \\ \nu+1, \nu+1 \end{matrix} \right] \sum_{k=0}^{[n/2]} \frac{(-1)^k}{k!} (-n)_{2k} \times \\ \times \left(2\nu + \frac{1}{2} \right)_{n-k} 2^{-2k} F_1 \left(2\nu + n - k + \frac{1}{2}, \nu + \frac{1}{2}, \nu + \frac{1}{2}; \nu + 1; \right. \\ \left. 1 - \frac{4bc}{p^2 \sin^2 \varphi}, -\operatorname{ctg}^2 \psi \right) \quad [\operatorname{Re} \nu > -(n+1)/4],$$

$$A_{\nu, \nu}^{n+1} = \frac{2^n (bc)^\nu \sin^{2n+2\nu+1} \psi}{\sqrt{\pi} p^{n+2\nu+1}} \Gamma \left[\begin{matrix} \nu+1/2 \\ \nu+1 \end{matrix} \right] \times \\ \times \sum_{k=0}^{[n/2]} \frac{(-1)^k}{k!} (-n)_{2k} \left(\nu + \frac{1}{2} \right)_{n-k} (2 \sin \psi)^{-2k} \times \\ \times {}_2F_1 \left(\nu + n - k + \frac{1}{2}, \nu + \frac{1}{2}; 2\nu + 1; 1 - \frac{4bc \sin^2 \psi}{p^2 \sin^2 \varphi} \right) = \\ = \frac{(-1)^n}{\pi \sqrt{bc}} \frac{\partial^n}{\partial p^n} Q_{\nu-1/2} \left(\frac{2-k^2}{k^2} \right) \quad [\operatorname{Re} \nu > -(n+1)/2],$$

$$A_{\nu, \nu}^2 = -\frac{pk^2 (bc)^{-3/2}}{2\pi \sqrt{1-k^2}} Q_{\nu-1/2}^1 \left(\frac{2-k^2}{k^2} \right) \quad [\operatorname{Re} \nu > -1],$$

$$A_{\nu, \nu+1/2}^{3/2} = \sqrt{\frac{2}{\pi}} b^\nu c^{\nu+1/2} \frac{\gamma^{-2\nu-1} \sqrt{\gamma^2 - b^2}}{\sqrt{(p^2 + (b+c)^2) (p^2 + (b-c)^2)}} \quad [\operatorname{Re} \nu > -1],$$

$$A_{1,0}^0 = \frac{2\sqrt{c}}{\pi k \sqrt{b}} E(k) + \frac{k(b^2 - c^2)}{2\pi (b^3 c)^{1/2}} K(k) \mp \frac{p}{2b} \Lambda_0(\varphi, \psi) - \left\{ \begin{matrix} 0 \\ pb^{-1} \end{matrix} \right\} \\ \left[\begin{matrix} \{b < c\} \\ \{b > c\} \end{matrix} \right],$$

$$A_{1,1}^0 = \frac{p}{\pi k \sqrt{bc}} E(k) - \frac{pk(p^2 + 2b^2 + 2c^2)}{4\pi (bc)^{3/2}} K(k) \mp \\ \mp \frac{b^2 - c^2}{4bc} \Lambda_0(\varphi, \psi) + \frac{1}{2} \left(\frac{b}{c} \right)^{\pm 1} \quad \left[\begin{matrix} \{b < c\} \\ \{b > c\} \end{matrix} \right],$$

$$A_{0,0}^2 = \frac{pk^3}{4\pi (1-k^2) (bc)^{3/2}} E(k),$$

$$A_{1,0}^2 = \frac{k}{8\pi b^{5/2} c^{3/2} (1-k^2)} [k^2 (b^2 - c^2 - p^2) E(k) + 4bc (1-k^2) K(k)],$$

$$A_{1,1}^2 = \frac{pk}{4\pi (1-k^2) (bc)^{3/2}} [(2-k^2) E(k) - 2(1-k^2) K(k)],$$

$$A_{2,0}^2 = -\frac{pk}{\pi b^{5/2} c^{1/2}} K(k) - \frac{pk^3}{4\pi (1-k^2) (bc)^{3/2}} E(k) \pm \frac{1}{b^2} \Lambda_0(\varphi, \psi) + \left\{ \begin{matrix} 0 \\ 2b^{-2} \end{matrix} \right\} \\ \left[\begin{matrix} \{b < c\} \\ \{b > c\} \end{matrix} \right],$$

$$A_{1,0}^4 = \frac{pk^3}{32\pi b^{7/2} c^{5/2} (1-k^2)^2} [(2k^4 (b^2 - c^2 - p^2) + \\ + 12bc (1-k^2)) E(k) - k^2 (1-k^2) (b^2 - c^2 - p^2) K(k)],$$

$$A_{1,1}^4 = \frac{k}{16\pi (bc)^{5/2} (1-k^2)^2} [2k^2 (k^4 p^2 - (1-k^2) (b^2 + c^2)) E(k) + \\ + (1-k^2) (8bc (1-k^2) - k^2 (2-k^2) p^2) K(k)],$$

$$A_{0,0}^{1/2} = \frac{4}{\sqrt{\pi^3 \gamma}} K(k_b) K(k_c),$$

$$A_{1,0}^{1/2} = \frac{4}{k_b \sqrt{\pi^3 \gamma (1 - k_b^2)}} [E(k_b) - (1 - k_b^2) K(k_b)] [2E(k_c) - K(k_c)],$$

$$A_{1,1}^{1/2} = \frac{4}{3k_b k_c \sqrt{\pi^3 \gamma (1 - k_b^2) (1 - k_c^2)}} [(1 - 2k_b^2) E(k_b) - (1 - k_b^2) K(k_b)] [(1 - 2k_c^2) E(k_c) - (1 - k_c^2) K(k_c)],$$

$$A_{2,0}^{1/2} = \frac{4}{9k_b^2 (1 - k_b^2) \sqrt{\pi^3 \gamma}} [(1 - k_b^2) (2 - 3k_b^2) K(k_b) - 2(1 - 2k_b^2) E(k_b)] [8(1 - 2k_c^2) E(k_c) - (5 - 8k_c^2) K(k_c)],$$

$$A_{2,1}^{1/2} = \frac{4}{15k_b^2 k_c (1 - k_b^2) \sqrt{\pi^3 \gamma (1 - k_c^2)}} [2(1 - k_b^2 + k_b^4) E(k_b) - (1 - k_b^2) (2 - k_b^2) K(k_b)] [(1 - k_c^2) (1 - 8k_c^2) K(k_c) - (1 - 16k_c^2 + 16k_c^4) E(k_c)],$$

$$A_{2,2}^{1/2} = \frac{4}{105k_b^2 k_c^2 (1 - k_b^2) (1 - k_c^2) \sqrt{\pi^3 \gamma}} [(1 - k_b^2) (2 + 5k_b^2 - 8k_b^4) K(k_b) - 2(1 - 2k_b^2) (1 + 4k_b^2 - 4k_b^4) E(k_b)] \times \\ \times [(1 - k_c^2) (2 + 5k_c^2 - 8k_c^4) K(k_c) - 2(1 - 2k_c^2) (1 + 4k_c^2 - 4k_c^4) E(k_c)].$$

$$3. \int_0^\infty e^{-px} J_\mu(cx) J_\nu(cx) dx = B_{\mu, \nu}$$

$$[\operatorname{Re}(\mu + \nu) > -1; \operatorname{Re} p > 2 + |\operatorname{Im} c|; k = 2c/\sqrt{p^2 + 4c^2}],$$

$$B_{\nu+n, \nu} = -B_{\nu+n-2, \nu} + \frac{2(\nu+n-1)}{n-1} \left[B_{\nu+n-2, \nu} - B_{\nu+n-1, \nu+1} - \frac{p}{c} B_{\nu+n-1, \nu} \right],$$

$$B_{\nu, \nu} = \frac{1}{\pi c} Q_{\nu-1/2} \left(1 + \frac{p^2}{2c^2} \right) \quad [\operatorname{Re} \nu > -1/2],$$

$$B_{\nu+1, \nu} = \frac{p}{\pi c^2} \sum_{k=1}^{\infty} Q_{\nu+k-1/2} \left(1 + \frac{p^2}{2c^2} \right) \quad [\operatorname{Re} \nu > -1],$$

$$B_{n+1, n} = \frac{1}{2c} - \frac{p}{2\pi c^2} Q_{-1/2} \left(1 + \frac{p^2}{2c^2} \right) - \frac{p}{\pi c^2} \sum_{k=1}^n Q_{k-1/2} \left(1 + \frac{p^2}{2c^2} \right),$$

$$B_{n+2, n} = -\frac{p}{c^2} (n+1) + \frac{p^2}{\pi c^3} (n+1) Q_{-1/2} \left(1 + \frac{p^2}{2c^2} \right) + \frac{2n+1}{\pi c} Q_{n-1/2} \left(1 + \frac{p^2}{2c^2} \right) - \\ - \frac{p}{\pi c} (n+1) Q_{n+1/2} \left(1 + \frac{p^2}{2c^2} \right) + \frac{2p^2}{\pi c^3} (n+1) \sum_{k=1}^n Q_{k-1/2} \left(1 + \frac{p^2}{2c^2} \right),$$

$$B_{0,0} = \frac{k}{\pi c} K(k), \quad B_{1,0} = \frac{1}{2c} - \frac{pk}{2\pi c^2} K(k)$$

$$B_{1,1} = \frac{1}{\pi ck} [(2 - k^2) K(k) - 2E(k)],$$

$$B_2 = \frac{4}{\pi ck} E(k) - \frac{k}{\pi c} K(k) - \frac{p}{2},$$

$$B_{2,1} = \frac{2p}{\pi c^2 k} E(k) - \frac{pk(p^2 + 3c^2)}{2\pi c^3} K(k) + \frac{1}{2c},$$

$$B_{2,2} = \frac{1}{2\pi ck^2} [(32 - 33k^2 + 3k^4) K(k) - 4(10 - 23k^2) E(k)] - \frac{2p}{c^2},$$

$$B_{n+1/2, n-1/2} = \frac{1}{\pi c} \operatorname{arctg} \frac{p}{2c} - \frac{p}{\pi c^2} \sum_{k=1}^n Q_{k-1} \left(1 + \frac{p^2}{2c^2}\right),$$

$$B_{n+3/2, n-1/2} = -\frac{p}{\pi c^2} (2n+1) \operatorname{arctg} \frac{p}{2c} + \frac{2n}{\pi c} Q_{n-1} \left(1 + \frac{p^2}{2c^2}\right) - \\ - \frac{2n+1}{\pi c} Q_n \left(1 + \frac{p^2}{2c^2}\right) + \frac{p^2}{\pi c^3} (2n+1) \sum_{k=1}^n Q_{k-1} \left(1 + \frac{p^2}{2c^2}\right),$$

$$B_{5/2, -1/2} = \frac{1}{2\pi c^3} (3p^2 - c^2) \operatorname{arctg} \frac{p}{2c} + \frac{3p}{\pi c^2} \left[Q_0 \left(1 + \frac{p^2}{2c^2}\right) - 1 \right].$$

$$4. \int_0^\infty x^{\alpha-1} e^{-px} J_\mu(cx) J_\nu(cx) dx = C_{\mu, \nu}^\alpha$$

$$[\operatorname{Re}(\alpha + \mu + \nu) > 0; \operatorname{Re} p > 2 |\operatorname{Im} c|; \\ k = 2c/\sqrt{p^2 + 4c^2}, z = \sqrt{p^2 + 4c^2}/p],$$

$$C_{\mu, \nu}^\alpha = \left(\frac{c}{2}\right)^{\mu+\nu} p^{-\alpha-\mu-\nu} \Gamma \left[\frac{\alpha + \mu + \nu}{2} \right] {}_4F_3 \left(\frac{\mu + \nu + 1}{2}, \right. \\ \left. \frac{\mu + \nu}{2} + 1, \frac{\alpha + \mu + \nu}{2}, \frac{\alpha + \mu + \nu + 1}{2}; \mu + \nu + 1, \mu + 1, \nu + 1; -\frac{4c^2}{p^2} \right),$$

$$C_{\nu, \nu}^{2\nu+1} = \frac{c^{4\nu} c^{2\nu}}{\pi p^{4\nu+1}} \Gamma \left[\frac{\nu + 1/2}{2}, \frac{2\nu + 1/2}{2} \right] {}_2F_1 \left(\nu + \frac{1}{2}, 2\nu + \frac{1}{2}; \nu + 1; -\frac{4c^2}{p^2} \right) \\ [\operatorname{Re} \nu > -1/4]$$

$$C_{\nu, \nu}^{1/2} = \frac{\Gamma(2\nu + 1/2)}{2^{2\nu} \sqrt{p}} [P_{-1/4}^{-\nu}(z)]^2 \\ [\operatorname{Re} \nu > -1/4],$$

$$C_{\nu, -\nu}^{1/2} = \sqrt{\frac{\pi}{p}} P_{-1/4}^{\nu}(z) P_{-1/4}^{-\nu}(z),$$

$$C_{\nu, \nu}^{3/2} = \frac{\sqrt{1-k^2}}{2^{2\nu} p^{3/2}} \Gamma \left(2\nu + \frac{3}{2} \right) P_{1/4}^{-\nu}(z) P_{-1/4}^{-\nu}(z) \\ [\operatorname{Re} \nu > -3/4],$$

$$C_{\nu, \nu+1}^{3/2} = \frac{c \sqrt{1-k^2}}{2^{2\nu+1} p^{3/2}} \Gamma \left(2\nu + \frac{5}{2} \right) P_{-1/4}^{-\nu}(z) P_{-1/4}^{-\nu-1}(z) = \\ = \frac{\sqrt{2} \Gamma(\nu + 3/4)}{\pi^{3/2} \sqrt{p(1-k^2)}} Q_{\nu-1/2}^{1/4}(k^{-1}) Q_{\nu+1/2}^{-1/4}(k^{-1}) \\ [\operatorname{Re} \nu > -5/4]$$

$$C_{\nu, -\nu}^{3/2} = \frac{\sqrt{\pi(1-k^2)}}{p^{3/2}} \left\{ \left(\nu + \frac{1}{4} \right) P_{-1/4}^{\nu}(z) P_{1/4}^{-\nu}(z) - \left(\nu - \frac{1}{4} \right) P_{1/4}^{\nu}(z) P_{-1/4}^{-\nu}(z) \right\},$$

$$C_{\nu, \nu}^2 = \frac{2^{2\nu} (2\nu+1) c^{2\nu}}{\pi p^{2\nu+2}} B \left(\nu + \frac{1}{2}, \nu + \frac{1}{2} \right) {}_2F_1 \left(\nu + \frac{1}{2}, \nu + \frac{3}{2}; 2\nu + 1; -\frac{4c^2}{p^2} \right) \\ [\operatorname{Re} \nu > -1],$$

$$C_{\nu, \nu+1}^2 = \frac{2\nu+1}{\pi(2c^2+p^2)} \left[Q_{\nu-1/2} \left(1 + \frac{p^2}{2c^2}\right) + Q_{\nu+1/2} \left(1 + \frac{p^2}{2c^2}\right) \right] \\ [\operatorname{Re} \nu > -3/2],$$

$$C_{1,1}^{-1} = \frac{4c}{3\pi k^3} [(1-k^2) K(k) - (1-2k^2) E(k)] - \frac{p}{2},$$

$$C_{2,2}^{-1} = \frac{4c}{15\pi k^5} [(1-k^2)(4+3k^2) K(k) - (4+k^2-6k^4) E(k)] - \frac{p}{4},$$

$$C_{1,0}^0 = \frac{2}{\pi k} E(k) - \frac{p}{2c},$$

$$C_{1,1}^0 = \frac{p}{\pi c k} E(k) - \frac{pk(p^2+4c^2)}{4\pi c^3} K(k) + \frac{1}{2},$$

$$C_{2,1}^0 = \frac{2}{3\pi k^3} [4(1-k^2) K(k) + (4-5k^2) E(k)] - \frac{p}{2c},$$

$$C_{3,2}^0 = \frac{2}{15\pi k^5} [4(1-k^2)(8+k^2) K(k) - (32-12k^2-23k^4) E(k)] - \frac{p}{2c},$$

$$C_{0,0}^2 = \frac{k}{\pi c p} E(k), \quad C_{1,0}^2 = \frac{k}{4\pi c^2} [K(k) - E(k)],$$

$$C_{2,0}^2 = \frac{1}{c^2} - \frac{pk}{\pi c^3} K(k) - \frac{pk^3}{4\pi(1-k^2)c^3} E(k),$$

$$C_{2,2}^2 = \frac{1}{\pi c p k^3} [(16-16k^2+k^4) E(k) - 8(1-k^2)(2-k^2) K(k)],$$

$$C_{0,0}^3 = \frac{k}{\pi c p^2} [(3-2k^2) E(k) - (1-k^2) K(k)],$$

$$C_{1,1}^3 = \frac{k}{\pi c p^2} [(1-k^2) K(k) - (1-2k^2) E(k)],$$

$$C_{-1/4, 1/4}^{3/2} = \sqrt{\frac{2}{\pi p(p^2+4c^2)}}, \quad C_{-1/4, -3/4}^{3/2} = \frac{p + \sqrt{p^2+4c^2}}{c\sqrt{2\pi p(p^2+4c^2)}}.$$

2.12.39. Интегралы от $A(x) e^{J(x)} J_\mu(\varphi(x)) J_\nu(cx)$

$$1. \int_0^\infty e^{-px^2} J_0(bx) J_1(cx) dx = \frac{1}{c} \left[1 - J\left(\frac{c^2}{4p}, \frac{b^2}{4p}\right) \right] \quad [b > c]$$

$$2. \int_0^\infty x^{\alpha-1} e^{-px^2} J_\mu(bx) J_\nu(cx) dx = \frac{b^\mu c^\nu p^{-(\alpha+\mu+\nu)/2}}{2^{\mu+\nu+1} \Gamma(\nu+1)} \times \\ \times \sum_{k=0}^\infty \Gamma\left[\frac{k+(\alpha+\mu+\nu)/2}{\mu+k+1}\right] \frac{1}{k!} \left(-\frac{b^2}{4p}\right)^k {}_2F_1\left(-k, -\mu-k, \nu+1, \frac{c^2}{b^2}\right) \\ [\operatorname{Re} p, \operatorname{Re}(\alpha+\mu+\nu) > 0].$$

$$3. \int_0^\infty x e^{-px^2} J_\nu(bx) J_\nu(cx) dx = \frac{1}{2p} \exp\left(-\frac{b^2+c^2}{4p}\right) I_\nu\left(\frac{bc}{2p}\right) \\ [\operatorname{Re} p > 0; \operatorname{Re} \nu > -1].$$

$$4. \int_0^\infty x^3 e^{-px^2} J_{n+2}(bx) J_n(cx) dx = \\ = \frac{1}{8p^3 b^2} \left\{ bc(4p-b^2) I_{n+1}\left(\frac{bc}{2p}\right) - bc(4pn+8p+b^2) I_{n-1}\left(\frac{bc}{2p}\right) + \right. \\ \left. + [16p^2 n(n+2) + 4pn+c^2] b^2 + b^4 \right\} I_n\left(\frac{bc}{2p}\right) \exp\left(-\frac{b^2+c^2}{4p}\right) \quad [\operatorname{Re} p > 0].$$

$$5. \int_0^\infty e^{-px^2} J_\nu(ce^{-\pi i/4} x) J_\nu(ce^{-\pi i/4} x) dx = \\ = \frac{2^{-\nu}}{c^2 \sqrt{p}} \Gamma\left[\frac{\nu+1/2}{\nu+1, \nu+1}\right] M_{1/4, \nu/2}\left(\frac{c^2}{2p}\right) M_{-1/4, \nu/2}\left(\frac{c^2}{2p}\right) \\ [\operatorname{Re} c, \operatorname{Re} p > 0, \operatorname{Re} \nu > -1/2].$$

$$6. \int_0^\infty x^{\alpha-1} e^{-px^2} J_\mu(cx) J_\nu(cx) dx = 2^{-\mu-\nu-1} c^{\mu+\nu} p^{-(\alpha+\mu+\nu)/2} \Gamma\left[\frac{(\alpha+\mu+\nu)/2}{\mu+1, \nu+1}\right] \times \\ \times {}_3F_3\left(\frac{\mu+\nu+1}{2}, \frac{\mu+\nu}{2}+1, \frac{\alpha+\mu+\nu}{2}; \mu+1, \nu+1, \mu+\nu+1; -\frac{c^2}{p}\right) \\ [\operatorname{Re} p, \operatorname{Re}(\alpha+\mu+\nu) > 0].$$

$$7. \int_0^{\infty} \frac{1}{x} e^{-px^2} J_{\nu}^2(cx) dx = \frac{1}{2\nu} \exp\left(-\frac{c^2}{2p}\right) \left[I_{\nu}\left(\frac{c^2}{2p}\right) + 2 \sum_{k=1}^{\infty} I_{\nu+k}\left(\frac{c^2}{2p}\right) \right]$$

[Re p , Re $\nu > 0$]

$$8. \int_0^{\infty} x^{-1/4} e^{-p\sqrt{x}} J_{\nu+1/4}(cx) J_{\nu}(cx) dx =$$

$$= \left(\frac{2}{\pi p}\right)^{3/2} \Gamma\left(\nu + \frac{1}{2}\right) \Gamma\left(\nu + \frac{3}{4}\right) W_{-\nu-1/8, 1/8}\left(\frac{ip^2}{8c}\right) W_{-\nu-1/8, 1/8}\left(\frac{-ip^2}{8c}\right)$$

[c , Re $p > 0$; Re $\nu > -1/2$]

$$9. \int_0^{\infty} x^{-1/4} e^{-p\sqrt{x}} J_{1/8}(cx) J_{-1/8}(cx) dx = \frac{\sqrt{\pi p}}{16c} \sec \frac{\pi}{8} H_{1/8}^{(1)}\left(\frac{p^2}{16c}\right) H_{1/8}^{(2)}\left(\frac{p^2}{16c}\right)$$

[c , Re $p > 0$]

$$10. \int_0^{\infty} e^{-px} J_{2\nu}(b\sqrt{x}) J_{\nu}(cx) dx = \frac{1}{\sqrt{p^2 + c^2}} \exp\left(-\frac{b^2 p}{4p^2 + 4c^2}\right) J_{\nu}\left(\frac{b^2 c}{4p^2 + 4c^2}\right)$$

[Re $\nu > -1/2$; Re $p > |\operatorname{Im} c|$]

$$11. \int_0^{\infty} \frac{x}{\sqrt{x^2 + z^2}} \exp\left(-\frac{b^2 z}{x^2 + z^2}\right) J_{\nu}\left(\frac{b^2 x}{x^2 + z^2}\right) J_{\nu}(cx) dx = \frac{e^{-cz}}{c} J_{2\nu}(2b\sqrt{c})$$

[b, c , Re $z > 0$; Re $\nu > -1/2$]

2.12.40. Интегралы, содержащие тригонометрические функции и $J_{\mu}(bx^{\pm r}) J_{\nu}(cx)$.

$$1. \int_0^{\infty} \begin{cases} \sin ax \\ \cos ax \end{cases} J_n(bx) J_n(cx) dx = I_n$$

$$[a, b, c > 0; 2k = \sqrt{\pm a^2 \mp (b \mp c)^2 / bc}],$$

$$I_0 = \begin{cases} 0 \\ 1 \end{cases} \frac{1}{\pi k \sqrt{bc}} K\left(\frac{1}{k}\right) \quad [0 < a < b - c],$$

$$I_0 = \frac{1}{\pi \sqrt{bc}} K(k) \quad [b - c < a < b + c],$$

$$I_0 = \begin{cases} 1 \\ 0 \end{cases} \frac{1}{\pi k \sqrt{bc}} K\left(\frac{1}{k}\right) \quad [a > b + c],$$

$$I_1 = \begin{cases} 0 \\ 1 \end{cases} \frac{1}{\pi k \sqrt{bc}} \left[(2k^2 - 1) K\left(\frac{1}{k}\right) - 2k^2 E\left(\frac{1}{k}\right) \right]$$

[$0 < a < b - c$],

$$I_1 = \mp \frac{1}{\pi \sqrt{bc}} [K(k) - 2E(k)] \quad [b - c < a < b + c],$$

$$I_1 = \begin{cases} 1 \\ 0 \end{cases} \frac{1}{\pi k \sqrt{bc}} \left[2k^2 E\left(\frac{1}{k}\right) + (1 - 2k^2) K\left(\frac{1}{k}\right) \right]$$

[$a > b + c$],

$$I_2 = \begin{cases} 0 \\ 1 \end{cases} \frac{1}{3\pi k \sqrt{bc}} \left[(3 - 16k^2 + 16k^4) K\left(\frac{1}{k}\right) + 8(1 - 2k^2) k^2 E\left(\frac{1}{k}\right) \right]$$

[$0 < a < b - c$],

$$I_2 = \frac{1}{3\pi \sqrt{bc}} [8(1 - 2k^2) E(k) - (5 - 8k^2) K(k)]$$

[$b - c < a < b + c$],

$$I_2 = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{1}{3\pi k \sqrt{bc}} \left[(3 - 16k^2 + 16k^4) K\left(\frac{1}{k}\right) + 8(1 - 2k^2) k^2 E\left(\frac{1}{k}\right) \right] \quad [a > b + c];$$

$$I_3 = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \frac{1}{15\pi k \sqrt{bc}} \left[(15 - 128k^2 + 128k^4) (2k^2 - 1) K\left(\frac{1}{k}\right) - \right. \\ \left. - 2k^2 (23 - 128k^2 + 128k^4) E\left(\frac{1}{k}\right) \right] \quad [0 < a < b - c].$$

$$I_3 = \mp \frac{1}{15\pi \sqrt{bc}} [(31 - 144k^2 + 128k^4) K(k) - 2(23 - 128k^2 + 128k^4) E(k)] \quad [b - c < a < b + c].$$

$$I_3 = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{1}{15\pi k \sqrt{bc}} \left[2k^2 (23 - 128k^2 + 128k^4) E\left(\frac{1}{k}\right) + \right. \\ \left. + (15 - 128k^2 + 128k^4) (1 - 2k^2) K\left(\frac{1}{k}\right) \right] \quad [a > b + c].$$

$$2. \int_0^\infty x^{\nu-\mu+(1\pm 1)/2} \begin{Bmatrix} \sin ax \\ \cos ax \end{Bmatrix} J_\mu(bx) J_\nu(cx) dx = 0 \quad [0 < a < c - b; b > 0; -1 < \operatorname{Re} \nu < \operatorname{Re} \mu + (1 \pm 1)/2]$$

$$3. \int_0^\infty x^{\nu-\mu-(3\pm 1)/2} \begin{Bmatrix} \sin ax \\ \cos ax \end{Bmatrix} J_\mu(bx) J_\nu(cx) dx = 2^{\nu-\mu-1} a^{(1\pm 1)/2} b^\mu c^{-\nu} \Gamma \left[\begin{matrix} \nu \\ \mu + 1 \end{matrix} \right] \\ [0 < a < c - b; b > 0; 0 < \operatorname{Re} \nu < \operatorname{Re} \mu + (5 \pm 1)/2]$$

$$4. \int_0^\infty \sin ax J_\nu(bx) J_\nu(cx) dx = \frac{1}{2\sqrt{bc}} P_{\nu-1/2} \left(\frac{b^2 + c^2 - a^2}{2bc} \right) \quad [b, c > 0; |c - b| < a < b + c; \operatorname{Re} \nu > -1]$$

$$5. = -\frac{\cos \nu\pi}{\pi \sqrt{bc}} Q_{\nu-1/2} \left(\frac{a^2 - b^2 - c^2}{2bc} \right) \quad [0 < b < c; a > b + c; \operatorname{Re} \nu > -1]$$

$$6. \int_0^\infty \cos ax J_1(bx) J_0(cx) dx = J \quad [a, b, c > 0; 2k = \sqrt{a^2 - (b - c)^2} / \sqrt{bc}],$$

$$J = \frac{1}{b} \left[1 - \Lambda_0 \left(\arcsin \sqrt{\frac{2c}{a+b+c}}, k \right) + \frac{c-a}{\pi \sqrt{bc}} K(k) \right] \quad [0 < |c - b| < a < c + b].$$

$$J = \frac{1}{b} \left[1 - \Lambda_0 \left(\arcsin \sqrt{\frac{a+c-b}{a+b+c}}, \frac{1}{k} \right) - \frac{1}{\pi k} \sqrt{\frac{b}{c}} K\left(\frac{1}{k}\right) \right] \quad [b, c > 0; a > b + c].$$

$$J = b^{-1} \quad [0 < |a - c| < a + c < b].$$

$$7. \int_0^\infty \begin{Bmatrix} \sin ax \\ \cos ax \end{Bmatrix} J_\nu^2(cx) dx = \pm \frac{1}{\pi c} \begin{Bmatrix} \cos \nu\pi \\ \sin \nu\pi \end{Bmatrix} Q_{\nu-1/2} \left(\frac{a^2}{2c^2} - 1 \right) \quad [0 < 2c < a; \operatorname{Re} \nu > -(3 \pm 1)/4].$$

$$8. \int_0^\infty \cos ax J_{-\nu}(cx) J_\nu(cx) dx = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{1}{2c} P_{\nu-1/2} \left(\frac{a^2}{2c^2} - 1 \right) \quad \left[\begin{matrix} 0 < a < 2c \\ 0 < 2c < a \end{matrix} \right].$$

$$9. \int_0^{\infty} \sin ax J_1(cx) J_3(cx) dx = \frac{2a}{c^3} - \frac{1}{6\pi c^4 (a+2c)} \times \\ \times [(4a^4 - 17a^2c^2 + 16c^4) K(k) - (4a^2 + 7c^2)(a+2c)^2 E(k)] \\ [a, c > 0; k = \sqrt{8ac/(a+2c)}].$$

$$10. \int_0^{\infty} \cos ax J_m(cx) J_n(cx) dx = I_{m,n} \quad [a, c > 0; k = \sqrt{8ac/(a+2c)}],$$

$$I_{0,1} = \frac{1}{2c} - \frac{1}{\pi c (a+2c)} K(k),$$

$$I_{1,2} = \frac{1}{2c} + \frac{a}{2\pi c^3} \left[\frac{a^2 - 2c^2}{a+2c} K(k) - (a+2c) E(k) \right],$$

$$I_{2,3} = \frac{1}{2c} - \frac{a}{2\pi c^5} [(2a^4 - a^2c^2 + 2c^4) K(k) - c(a+2c)(2a^2 + 7c^2) E(k)].$$

$$11. \int_0^{\infty} x^{\alpha-1} \begin{Bmatrix} \sin ax \\ \cos ax \end{Bmatrix} J_{\mu}(cx) J_{\nu}(cx) dx = I_{\mu,\nu}^{\alpha}$$

$$[a, c > 0; \operatorname{Re}(\alpha + \mu + \nu) > -\delta; \operatorname{Re} \alpha < 2; \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix};$$

$$k = 2^{1/2} (\sqrt{a+2c} - \sqrt{a-2c})^{1/2} (2\sqrt{a} + \sqrt{a+2c} - \sqrt{a-2c})^{-1/2},$$

$$I_{\mu,\nu}^{\alpha} = \left(\frac{c}{2}\right)^{\mu+\nu} a^{-\alpha-\mu-\nu} \Gamma \left[\begin{matrix} \alpha + \mu + \nu \\ \mu + 1, \nu + 1 \end{matrix} \right] \begin{Bmatrix} \sin [(\alpha + \mu + \nu)\pi/2] \\ \cos [(\alpha + \mu + \nu)\pi/2] \end{Bmatrix} \times \\ \times {}_4F_3 \left(\frac{1 + \mu + \nu + \alpha}{2}, \frac{\mu + \nu + \alpha}{2}, \frac{1 + \mu + \nu}{2}, 1 + \frac{\mu + \nu}{2}; \right.$$

$$\left. 1 + \mu + \nu, 1 + \nu, 1 + \mu; \frac{4c^2}{a^2} \right)$$

$$[0 < 2c < a] \text{ или } [a = 2c; \operatorname{Re} \alpha < 1].$$

$$I_{\mu,\nu}^{\alpha} = \frac{a^{\delta}}{2} \left(\frac{2}{c}\right)^{\alpha+\delta} \times$$

$$\times \Gamma \left[\begin{matrix} 1 - \alpha - \delta, (\mu + \nu + \alpha + \delta)/2 \\ 1 + (\mu + \nu - \alpha - \delta)/2, 1 + (\nu - \mu - \alpha - \delta)/2, 1 + (\mu - \nu - \alpha - \delta)/2 \end{matrix} \right] \times$$

$$\times {}_4F_3 \left(\frac{\alpha + \delta - \mu - \nu}{2}, \frac{\mu + \alpha + \delta - \nu}{2}, \frac{\nu + \alpha + \delta - \mu}{2}, \frac{\mu + \nu + \alpha + \delta}{2}; \right.$$

$$\left. \frac{1 + \alpha}{2}, \frac{\alpha}{2} + \delta, \frac{1}{2} + \delta; \frac{a^2}{4c^2} \right) \mp \frac{a^{1-\alpha}}{c} \begin{Bmatrix} \cos (\alpha\pi/2) \\ \sin (\alpha\pi/2) \end{Bmatrix} \Gamma \left[\begin{matrix} \alpha - 1 \\ (1 + \nu - \mu)/2, (1 + \mu - \nu)/2 \end{matrix} \right] \times$$

$$\times {}_4F_3 \left(\frac{1 - \mu - \nu}{2}, \frac{1 - \nu + \mu}{2}, \frac{1 - \mu + \nu}{2}, \frac{\mu + \nu + 1}{2}; \frac{1}{2}, \frac{3 - \alpha}{2}, 1 - \frac{\alpha}{2}; \frac{a^2}{4c^2} \right) +$$

$$+ (\mu + \nu) a^{2-\alpha} c^{-2} \begin{Bmatrix} \sin (\alpha\pi/2) \\ \cos (\alpha\pi/2) \end{Bmatrix} \Gamma \left[\begin{matrix} \alpha - 2 \\ (\mu - \nu)/2, (\nu - \mu)/2 \end{matrix} \right] \times$$

$$\times {}_4F_3 \left(1 - \frac{\mu + \nu}{2}, 1 + \frac{\mu - \nu}{2}, 1 + \frac{\nu - \mu}{2}, 1 + \frac{\mu + \nu}{2}; \frac{3}{2}, 2 - \frac{\alpha}{2}, \frac{3 - \alpha}{2}; \frac{a^2}{4c^2} \right)$$

$$[0 < a < 2c].$$

$$I_{\nu,\nu}^{2\nu+1} = \pm \frac{(4c)^{2\nu}}{a^{2\nu+1} \cos \nu\pi} \Gamma \left[\begin{matrix} 2\nu + 1/2 \\ \nu + 1, 1/2 - \nu \end{matrix} \right] \begin{Bmatrix} \cos 2\nu\pi \\ \sin 2\nu\pi \end{Bmatrix} \times$$

$$\times {}_2F_1 \left(\nu + \frac{1}{2}, \frac{1}{2} + 2\nu; 1 + \nu; \frac{4c^2}{a^2} \right)$$

$$[0 < 2c < a; -(\delta + 1)/4 < \operatorname{Re} \nu < 1/2].$$

$$I_{\nu, \nu}^{2\nu+1} = \frac{a^{-2\nu} 2^{2\nu-1}}{c \sqrt{\pi}} \Gamma \left[\frac{\nu + \delta/2}{(1 + \delta)/2 - \nu} \right] {}_2F_1 \left(\nu + \frac{1}{2}, \frac{1}{2}; 1 - \nu; \frac{a^2}{4c^2} \right) +$$

$$+ \begin{cases} 0 \\ 1 \end{cases} \frac{c^{-2\nu-1}}{2\pi} \Gamma \left[\frac{1/2 + 2\nu}{1/2 - \nu} \right] {}_2F_1 \left(\frac{1}{2} + \nu, \frac{1}{2} + 2\nu; 1 + \nu; \frac{a^2}{4c^2} \right)$$

$$[0 < a < 2c; -(\delta + 1)/4 < \operatorname{Re} \nu < 1/2];$$

$$I_{-\nu \pm 1/2, \nu=0}^{3/2} [0 < 2c < a],$$

$$I_{-\nu \pm 1/2, \nu}^{3/2} = \sqrt{\frac{2}{\pi a (4c^2 - a^2)}} \cos \left[\left(2\nu - \frac{1}{2} \right) \arccos \frac{a}{2c} \right]$$

$$[0 < a < 2c];$$

$$I_{0,0}^{1/2} = \frac{2^{3/2} k^2}{\pi^{3/2} (\sqrt{a+2c} - \sqrt{a-2c})} K^2(k)$$

$$[0 < 2a < a];$$

$$I_{1,1}^{1/2} = -\frac{2^{7/2} c^2 k^2}{3\pi^{3/2} (1 - k^2)^2} (\sqrt{a+2c} - \sqrt{a-2c})^{-5} [(2 - k^2) E(k) - 2(1 - k^2) K(k)]^2$$

$$[0 < 2c < a];$$

$$I_{2,2}^{1/2} = \frac{32 \sqrt{2} c^4 k^2}{105\pi^{3/2} (1 - k^2)^4} (\sqrt{a+2c} - \sqrt{a-2c})^{-9} \times$$

$$\times [(1 - k^2) (16 - 16k^2 - k^4) K(k) - 2(2 - k^2) (4 - 4k^2 - k^4) E(k)]^2$$

$$[0 < 2c < a].$$

$$12. \int_0^\infty x^{l-1} \sin ax J_m(cx) J_n(cx) dx = A_{m,n}^l \quad [a, c > 0; k = \sqrt{8ac/(a+2c)}],$$

$$A_{1,1}^{-1,1} = \frac{a}{2} - \frac{a+2c}{12\pi c^2} [(a^2 + 4c^2) E(k) - (a-2c)^2 K(k)],$$

$$A_{2,2}^{-1,2} = \frac{a}{4} - \frac{a+2c}{60\pi c^4} [(a-2c)^2 (a^2 - c^2) K(k) + (4c^4 + 9a^2 c^2 - a^4) E(k)],$$

$$A_{2,1}^{0,1} = \frac{a}{2c} + \frac{1}{6\pi c^3} [(a-2c) (a^2 - c^2) K(k) - (a+2c) (a^2 + c^2) E(k)],$$

$$A_{2,2}^{0,2} = \frac{a}{2c} + \frac{1}{30\pi c^5} [(a-2c) (3c^4 - 5a^2 c^2 + 2a^4) K(k) +$$

$$+ (a+2c) (3c^4 + 13a^2 c^2 - 2a^4) E(k)].$$

$$13. \int_{-\infty}^\infty \frac{\sin(ax+az)}{x+z} J_{\pm n \pm 1/2}(cx) J_{n+1/2}(cx) dx = \pi J_{n+1/2}(cz) J_{\pm n \pm 1/2}(cz)$$

$$[0 < 2c \leq a; \operatorname{Im} z \neq 0].$$

$$14. \int_0^\infty x \begin{Bmatrix} \sin ax^2 \\ \cos ax^2 \end{Bmatrix} J_\nu(bx) J_\nu(cx) dx = \frac{1}{2a} J_\nu\left(\frac{bc}{2a}\right) \begin{Bmatrix} \cos \varphi \\ \sin \varphi \end{Bmatrix}$$

$$[a, b, c > 0; \varphi = (b^2 + c^2)/(4a) - \nu\pi/2; \operatorname{Re} \nu > (\mp 1 - 3)/2].$$

$$15. \int_0^\infty x^{(2-\nu)/3} \begin{Bmatrix} \sin bx^2 \\ \cos bx^2 \end{Bmatrix} J_{(2\nu-1)/6}(bx^2) J_\nu(cx) dx =$$

$$= (4b)^{(\nu-2)/3} c^{-(\nu+1)/3} \begin{Bmatrix} \sin \varphi \\ \cos \varphi \end{Bmatrix} J_{(2\nu-1)/6}\left(\frac{c^2}{16b}\right)$$

$$[\varphi = (\nu+1)\pi/6 - c^2/(16b); b, c > 0; \operatorname{Re} \nu > -(7 \pm 3)/4].$$

$$16. \int_0^{\infty} x^{-1/4} \begin{Bmatrix} \sin a \sqrt{x} \\ \cos a \sqrt{x} \end{Bmatrix} J_{-\nu \pm 1/4}(cx) J_{\nu}(cx) dx = \\ = \sqrt{\frac{8}{\pi}} a^{-3/2} \left[e^{-\pi i/8} W_{-\nu \pm 1/8, \pm 1/8} \left(-\frac{ia^2}{8c} \right) W_{\nu \mp 1/8, \pm 1/8} \left(-\frac{ia^2}{8c} \right) + \right. \\ \left. + e^{\pi i/8} W_{-\nu \pm 1/8, \pm 1/8} \left(\frac{ia^2}{8c} \right) W_{\nu \mp 1/8, \pm 1/8} \left(\frac{ia^2}{8c} \right) \right] \quad [a, c > 0].$$

$$17. \int_0^{\infty} \begin{Bmatrix} \sin ax \\ \cos ax \end{Bmatrix} J_{2\nu}(b\sqrt{x}) J_{\nu}(cx) dx = \frac{1}{\sqrt{|c^2 - a^2|}} R \left(\frac{ab^2}{4|c^2 - a^2|} \right) J_{\nu} \left(\frac{b^2 c}{4|c^2 - a^2|} \right) \\ [a, b, c > 0; a \neq c; \operatorname{Re} \nu > -(3 \pm 1)/4, \\ R(y) = \begin{cases} \sin y \\ \cos y \end{cases} \text{ при } a < c, \quad R(y) = \begin{cases} \cos y \\ \sin y \end{cases} \text{ при } a > c].$$

$$18. \int_0^1 \left(1 + \frac{1}{x^2} \right) \cos \left(ax - \frac{a}{x} \right) J_{\nu} \left(\frac{c}{x} \right) J_{\nu}(cx) dx = (c^2 - a^2)^{-1/2}_+ J_{2\nu}(2\sqrt{c^2 - a^2}) \\ [a, c > 0; a \neq c, \operatorname{Re} \nu > -1/2]$$

2.12.41. Интегралы, содержащие $J_{\nu}(c \sin x)$.

$$1. \int_0^{\pi/2} \sin^{2\alpha-1} x \cos^{2\beta-1} x J_{\mu}(c \sin 2x) J_{\nu}(c \sin 2x) dx = \\ = \frac{1}{2} \Gamma \left[\frac{\alpha + (\mu + \nu)/2}{\mu + 1}, \frac{\beta + (\mu + \nu)/2}{\nu + 1}, \frac{\alpha + \beta + \mu + \nu}{\alpha + \beta + \mu + \nu} \right] {}_4F_5 \left(\alpha + \frac{\mu + \nu}{2}, \beta + \frac{\mu + \nu}{2}, \right. \\ \left. \frac{\mu + \nu + 1}{2}, \frac{\mu + \nu}{2} + 1; \mu + 1, \nu + 1, \mu + \nu + 1, \frac{\alpha + \beta + \mu + \nu}{2}, \frac{\alpha + \beta + \mu + \nu + 1}{2}; -c^2 \right) \\ [\operatorname{Re}(2\alpha + \mu + \nu), \operatorname{Re}(2\beta + \mu + \nu) > 0]$$

$$2. \int_0^{\pi/2} \operatorname{tg}^{\mu-\nu+1} x J_{\mu}(c \sin 2x) J_{\nu}(c \sin 2x) dx = (2c)^{\mu-\nu} \int_0^{2c} x^{\nu-\mu-1} J_{\mu+\nu}(x) dx \\ [\operatorname{Re}(\mu + 1), \operatorname{Re} \nu > 0; \text{ см. 1.8.1}].$$

2.12.42. Интегралы от $x^{\alpha} J_{\lambda}(ax) J_{\mu}(bx) J_{\nu}(cx)$.

$$1. \int_0^{\infty} J_{\lambda}(ax) J_{\mu}(bx) J_{\nu}(cx) dx = \frac{a^{\lambda} b^{\mu}}{c^{\lambda+\mu+1}} \Gamma \left[\frac{(\mu + \lambda + \nu + 1)/2}{\lambda + 1}, \frac{(\mu + \lambda + \nu + 1)/2}{\mu + 1}, \frac{(1 + \nu - \lambda - \mu)/2}{(1 + \nu - \lambda - \mu)/2} \right] \times \\ \times {}_2F_1 \left(\frac{1 + \lambda + \mu - \nu}{2}, \frac{1 + \lambda + \mu + \nu}{2}; 1 + \lambda; z_+ \right) \times \\ \times {}_2F_1 \left(\frac{1 + \lambda + \mu - \nu}{2}, \frac{1 + \lambda + \mu + \nu}{2}; 1 + \mu; z_- \right) \\ [c > a + b; a, b > 0; \operatorname{Re}(\lambda + \mu + \nu) > -1; 2c^2 z_{\pm} = \pm a^2 \mp b^2 + c^2 + \sqrt{(a^2 + b^2 - c^2)^2 - 4a^2 b^2}].$$

$$2. \int_0^{\infty} J_0(ax) J_0(bx) J_1(cx) dx = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \frac{1}{c} \quad \left[\begin{cases} 0 < c < |a - b| \\ c > a + b \end{cases}, a, b > 0 \right]$$

$$3. = \frac{1}{\pi c} \arccos \frac{a^2 + b^2 - c^2}{2ab} \quad [|a - b| < c < a + b; a, b > 0]$$

$$4. \int_0^{\infty} J_m(ax) J_{n-m}(bx) J_n(cx) dx = I \quad [a, b, c > 0]$$

$$I = \frac{2e^{-im\varphi} (b + ae^{i\varphi})^n}{\pi c^{n-1} \sqrt{4a^2b^2 - (c^2 - a^2 - b^2)^2}}$$

$$\left[|a-b| < c < a+b; \varphi = \arccos \frac{c^2 - a^2 - b^2}{2ab} \right];$$

$$I = 0$$

[в остальных случаях]

$$5. \int_0^{\infty} x^{\alpha-1} J_{\lambda}(ax) J_{\mu}(bx) J_{\nu}(cx) dx = I_{\alpha}$$

$$[a, b, \operatorname{Re}(\alpha + \lambda + \mu + \nu) > 0; c > a + b; \operatorname{Re} \alpha \leq 5/2]$$

$$I_{\alpha} = 2^{\alpha-1} a^{\lambda} b^{\mu} c^{-\alpha-\lambda-\mu} \Gamma \left[\frac{(\alpha + \lambda + \mu + \nu)/2}{\lambda + 1, \mu + 1, 1 - (\alpha + \lambda + \mu - \nu)/2} \right] \times$$

$$\times F_4 \left(\frac{\alpha + \lambda + \mu - \nu}{2}, \frac{\alpha + \lambda + \mu + \nu}{2}; \lambda + 1, \mu + 1; \frac{a^2}{c^2}, \frac{b^2}{c^2} \right),$$

$$I_{\nu-\lambda-\mu+2} = 0$$

$$[-1 < \operatorname{Re} \nu < \operatorname{Re}(\lambda + \mu) + 1/2]$$

$$I_{\nu-\lambda-\mu} = 2^{\nu-\lambda-\mu-1} a^{\lambda} b^{\mu} c^{-\nu} \Gamma \left[\frac{\nu}{\lambda + 1, \mu + 1} \right]$$

$$[0 < \operatorname{Re} \nu < \operatorname{Re}(\lambda + \mu) + 5/2]$$

$$6. \int_0^{\infty} x J_{\mu+\nu}(ax) J_{\mu}(bx) J_{\nu}(cx) dx = \frac{2 \sin \mu \pi}{\pi c} (1 - z_+)^{-\mu/2} (1 - z_-)^{-(\mu+\nu)/2} \times$$

$$\times \arcsin^{\mu+\nu} \sqrt{z_+} \arcsin^{\lambda} \sqrt{z_-} (\sqrt{c^2 - (a+b)^2} - \sqrt{c^2 - (a-b)^2})^{-1}$$

$$[c > a + b, a, b > 0; \operatorname{Re}(\mu + \nu) > -1, 2c^2 z_{\pm} = \pm a^2 \mp b^2 + c^2 \pm \sqrt{(a^2 + b^2 - c^2)^2 - 4a^2 b^2}].$$

$$7. \int_0^{\infty} x^{\nu+1} J_{\mu}(ax) J_{\mu}(bx) J_{\nu}(cx) dx =$$

$$= - \frac{\sqrt{2} c^{\nu} \sin \nu \pi}{\pi^{3/2} (ab)^{\nu+1}} \operatorname{sh}^{-\nu-1/2} v e^{-(2\nu+1) \pi i/2} Q_{\mu-1/2}^{\nu+1/2}(\operatorname{ch} v)$$

$$[0 < c < |a-b|; a, b > 0; 2ab \operatorname{ch} v = a^2 + b^2 - c^2; \operatorname{Re}(\mu + \nu) > -1; \operatorname{Re} \nu < 1/2]$$

$$8. = \frac{c^{\nu} \sin^{-\nu-1/2} v}{\sqrt{2\pi} (ab)^{\nu+1}} \left[\cos \nu \pi P_{\mu-1/2}^{\nu+1/2}(\cos v) - \frac{2}{\pi} \sin \nu \pi Q_{\mu-1/2}^{\nu+1/2}(\cos v) \right]$$

$$[|a-b| < c < a+b, a, b > 0, 2ab \cos v = a^2 + b^2 - c^2; \operatorname{Re}(\mu + \nu) > -1, \operatorname{Re} \nu < 1/2]$$

$$9. = - \frac{\sqrt{2} c^{\nu} \sin \mu \pi}{\pi^{3/2} (ab)^{\nu+1}} e^{-(2\nu+1) \pi i/2} \operatorname{sh}^{-\nu-1/2} u Q_{\mu-1/2}^{\nu+1/2}(\operatorname{ch} u)$$

$$[c > a + b; a, b > 0; 2ab \operatorname{ch} u = c^2 - a^2 - b^2; \operatorname{Re}(\mu + \nu) > -1; \operatorname{Re} \nu < 1/2]$$

$$10. \int_0^{\infty} x^{2-\nu} J_{\mu}(ax) J_{\mu}(bx) J_{\nu}(cx) dx = \frac{\pi}{4abc} \left(\frac{ab}{c} \sin \varphi \right)^{\nu} P_{\mu}^{-\nu}(\cos \varphi)$$

$$[a, b, c > 0; c < a + b, \operatorname{Re} \nu > 1/2, \operatorname{Re} \mu > -3/2, \varphi] - \text{угол треугольника стороны которого равны } a, b, c, \text{ заключенный между сторонами } a \text{ и } b]$$

$$11. \int_0^{\infty} x^{1-\nu} J_{\mu}(ax) J_{\mu}(bx) J_{\nu}(cx) dx = 0$$

$$[0 < c < |a-b|; a, b > 0; \operatorname{Re} \mu > -1; \operatorname{Re} \nu > -1/2]$$

$$12. = \frac{(ab)^{\nu-1}}{\sqrt{2\pi} c^{\nu}} \sin^{\nu-1/2} v P_{\mu-1/2}^{1/2-\nu}(\cos v)$$

$$[|a-b| < c < a+b, a, b > 0, 2ab \cos v = a^2 + b^2 - c^2, \operatorname{Re} \mu > -1, \operatorname{Re} \nu > -1/2]$$

$$13. \quad = -\frac{\sqrt{2} (ab)^{\nu-1}}{\pi^{3/2} c^\nu} \operatorname{sh}^{\nu-1/2} u \sin [(\mu - \nu) \pi] e^{(2\nu-1) \pi i/2} Q_{\mu-1/2}^{1/2-\nu} (\operatorname{ch} u) \\ [c > a+b; a, b > 0, 2ab \operatorname{ch} u = c^2 - a^2 - b^2; \operatorname{Re} \mu > -1; \operatorname{Re} \nu > -1/2]$$

$$14. \quad \int_0^\infty x^{1-\nu} J_\nu(ax) J_\nu(bx) J_\nu(cx) dx = 0$$

$$[0 < c < |a-b| \text{ или } c > a+b; a, b, c > 0; \operatorname{Re} \nu > -1/2]$$

$$15. \quad = \frac{2^{1-3\nu}}{\sqrt{\pi} (abc)^\nu \Gamma(\nu+1/2)} [c^2 - (a-b)^2]^{\nu-1/2} [(a+b)^2 - c^2]^{\nu-1/2} = \\ = \frac{2^{2\nu-1} \Delta^{2\nu-1}}{\sqrt{\pi} (abc)^\nu \Gamma(\nu+1/2)} \quad [|a-b| < c < a+b, a, b, c > 0, \operatorname{Re} \nu > -1/2, \Delta - \text{площадь треугольника, стороны которого равны } a, b, c]$$

$$16. \quad \int_0^\infty x J_\mu(ax) J_\mu(bx) J_0(cx) dx = I \quad [r_\pm = \pm(a^2 + b^2 - c^2)/(2ab)],$$

$$I = \begin{cases} 0 \\ 1 \end{cases} \frac{1}{\pi ab} (1 - r_+^2)^{-1/2} \cos(\mu \arccos r_+)$$

$$\left[\begin{cases} 0 < c < |a-b| \\ |a-b| < c < a+b \end{cases}, a, b > 0; \operatorname{Re} \mu > -1 \right],$$

$$I = \frac{1}{\pi ab} (r_-^2 - 1)^{-1/2} \sin \mu \pi (r_- + \sqrt{r_-^2 - 1})^{-\mu}$$

$$[c > a+b, a, b > 0, \operatorname{Re} \mu > -1]$$

$$17. \quad \int_0^\infty J_\lambda(bx) J_\mu(bx) J_\nu(cx) dx = A(\lambda, \mu, \nu)$$

$$[0 < 2b < c; \operatorname{Re}(\mu + \lambda + \nu) > -1; k = (c - \sqrt{c^2 - 4b^2})^{1/2}/\sqrt{2c}, z = \sqrt{c^2 - 4b^2}/c],$$

$$A(\mu, \mu, \nu) = b^{2\mu} c^{-2\mu-1} \Gamma \left[\begin{matrix} \mu + (\nu+1)/2 \\ \mu+1, \mu+1, (\nu+1)/2 - \mu \end{matrix} \right] \times \\ \times \left[{}_2F_1 \left(\mu + \frac{1-\nu}{2}, \mu + \frac{1+\nu}{2}; \mu+1; k^2 \right) \right]^2 = \frac{1}{c} \Gamma \left[\begin{matrix} (1+\nu)/2 + \mu \\ (1+\nu)/2 - \mu \end{matrix} \right] [P_{(\nu-1)/2}^{-\mu}(z)]^2 \\ [\operatorname{Re}(\nu + 2\mu) > -1]$$

$$A(-\mu, \mu, \nu) = \frac{1}{c} P_{(\nu-1)/2}^{-\mu}(z) P_{(\nu-1)/2}^{\mu}(z) \quad [\operatorname{Re} \nu > -1]$$

$$A(0, 0, 0) = \frac{4}{\pi^2 c} K^2(k), \quad A(0, 0, 1) = \frac{1}{c},$$

$$A(1, 1, 0) = -\frac{4c}{\pi^2 b^2} [E(k) - (1-k^2) K(k)]^2,$$

$$A(1, 1, 1) = 0, \quad A(0, 0, 2) = \frac{4}{\pi^2 c} [2E(k) - K(k)]^2,$$

$$A(1, 1, 2) = \frac{4c}{3\pi^2 b^2} [(1-k^2) K(k) - (1-2k^2) E(k)]^2,$$

$$A(2, 2, 0) = \frac{4c^3}{9\pi^2 b^4} [(2-3k^2)(1-k^2) K(k) - 2(1-2k^2) E(k)]^2,$$

$$A(2, 2, 2) = -\frac{4c^3}{15\pi^2 b^4} [2(1-k^2+k^4) E(k) - (1-k^2)(2-k^2) K(k)]^2,$$

$$A(2, 2, 4) = \frac{4c^3}{105\pi^2 b^4} [(2+5k^2-8k^4)(1-k^2) K(k) - 2(1-2k^2)(1+4k^2-4k^4) E(k)]^2,$$

$$A(3, 3, 0) = -\frac{4c^5}{225\pi^2 b^6} [(8-23k^2+23k^4) E(k) - (1-k^2)(8-19k^2+15k^4) K(k)]^2,$$

$$A(3, 3, 2) = \frac{4c^5}{315\pi^2 b^6} [(8-15k^2+3k^4)(1-k^2) K(k) - (8-19k^2+9k^4-6k^6) E(k)]^2$$

$$18. \int_0^{\infty} J_{\nu}^2(cx) dx = \frac{2}{3c} \Gamma \left[\frac{(3\nu+1)/6}{(3\nu+5)/6, 2/3, 2/3} \right] \quad [c > 0; \operatorname{Re} \nu > -1/3]$$

$$19. \int_0^{\infty} J_{-\nu}^2(cx) J_{\nu}(cx) dx = \frac{4}{3^{3/2}c} \sin \frac{1-3\nu}{3} \pi \Gamma \left[\frac{(1-3\nu)/6}{(5-3\nu)/6, 2/3, 2/3} \right] \\ [c > 0; \operatorname{Re} \nu < 1]$$

$$20. \int_0^{\infty} x^{\alpha-1} J_{\lambda}(bx) J_{\mu}(bx) J_{\nu}(cx) dx = 2^{\alpha-1} b^{\mu+\lambda} c^{-\alpha-\mu-\lambda} \times \\ \times \Gamma \left[\frac{(\alpha+\mu+\lambda+\nu)/2}{1+\mu, 1+\lambda, 1+(\nu-\alpha-\mu-\lambda)/2} \right] {}_4F_3 \left(\frac{\alpha+\mu+\lambda-\nu}{2}, \right. \\ \left. \frac{\alpha+\mu+\lambda+\nu}{2}, \frac{1+\mu+\lambda}{2}, 1+\frac{\mu+\lambda}{2}; 1+\mu+\lambda, 1+\mu, 1+\lambda; \frac{4b^2}{c^2} \right) \\ [0 < 2b < c; -\operatorname{Re}(\mu+\nu+\lambda) < \operatorname{Re} \alpha < 5/2]$$

$$21. = \frac{2^{\alpha-2} c^{1-\alpha}}{b} \Gamma \left[\frac{(\nu+\alpha-1)/2}{(1+\mu-\lambda)/2, (1+\lambda-\mu)/2, (\nu+3-\alpha)/2} \right] \times \\ \times {}_4F_3 \left(\frac{1-\lambda-\mu}{2}, \frac{1+\lambda-\mu}{2}, \frac{1-\lambda+\mu}{2}, \frac{1+\lambda+\mu}{2}; \right. \\ \left. \frac{1}{2}, \frac{3-\alpha-\nu}{2}, \frac{3-\alpha+\nu}{2}; \frac{c^2}{4b^2} \right) + 2^{\alpha-1} b^{-\alpha-\nu} c^{\nu} \times \\ \times \Gamma \left[\frac{1-\alpha-\nu, (\alpha+\lambda+\mu+\nu)/2}{1+(\mu+\lambda-\alpha-\nu)/2, 1+(\mu-\lambda-\alpha-\nu)/2, 1+(\lambda-\mu-\alpha-\nu)/2, 1+\nu} \right] \times \\ \times {}_4F_3 \left(\frac{\alpha+\nu-\mu-\lambda}{2}, \frac{\alpha+\nu+\lambda-\mu}{2}, \frac{\alpha+\nu+\mu-\lambda}{2}, \frac{\alpha+\nu+\mu+\lambda}{2}; \right. \\ \left. \frac{1+\alpha+\nu}{2}, \frac{\alpha+\nu}{2}, 1+\nu; \frac{c^2}{4b^2} \right) - \frac{2^{\alpha-2} c^{2-\alpha} (\mu+\lambda)}{b^2} \times \\ \times \Gamma \left[\frac{(\nu+\alpha)/2-1}{(\mu-\lambda)/2, (\lambda-\mu)/2, 2+(\nu-\alpha)/2} \right] {}_4F_3 \left(1-\frac{\mu+\lambda}{2}, 1+\frac{\lambda-\mu}{2}, \right. \\ \left. 1+\frac{\mu-\lambda}{2}, 1+\frac{\mu+\lambda}{2}; \frac{3}{2}, 2-\frac{\nu+\alpha}{2}, 2+\frac{\nu-\alpha}{2}; \frac{c^2}{4b^2} \right) \\ [0 < c < 2b; -\operatorname{Re}(\mu+\nu+\lambda) < \operatorname{Re} \alpha < 5/2]$$

$$22. \int_0^{\infty} x^{1-\mu} J_{\mu}(bx) J_{\nu}(bx) J_{\nu}(cx) dx = \frac{c^{\mu-1} (4b^2-c^2)_+^{(2\mu-1)/4}}{2^{\mu} \sqrt{\pi} b^{\mu+1/2}} P_{\nu-1/2}^{1/2-\mu} \left(\frac{c}{2b} \right) \\ [b, c > 0, c \neq 2b, \operatorname{Re} \mu > -1/2, \operatorname{Re}(\mu+\nu) > -1]$$

$$23. \int_0^{\infty} x^{\nu+1} J_{\mu}(bx) J_{-\mu}(bx) J_{\nu}(cx) dx = \frac{2^{\nu}}{b \sqrt{\pi} c} (4b^2-c^2)_+^{(2\nu+1)/4} P_{\mu-1/2}^{\nu+1/2} \left(\frac{c^2}{2b^2} - 1 \right) \\ [b, c > 0, c \neq 2b, -1 < \operatorname{Re} \nu < 1/2]$$

$$24. \int_0^{\infty} x^{\nu+1} J_{-\nu}(bx) J_{\nu}(bx) J_{\nu}(cx) dx = \frac{2^{3\nu+1} b^{2\nu} c^{-\nu-1}}{\sqrt{\pi} \Gamma(1/2-\nu)} (4b^2-c^2)_+^{\nu-1/2} \\ [b, c > 0; c \neq 2b; -1 < \operatorname{Re} \nu < 1/2]$$

$$25. \int_0^{\infty} x^{\nu+1} J_{\nu}^2(bx) J_{\nu}(cx) dx = \pm \frac{2^{3\nu+1} b^{2\nu}}{\pi^{3/2} c^{\nu+1}} \Gamma \left(\nu + \frac{1}{2} \right) \left\{ \frac{1}{\sin \nu \pi} \right\} |4b^2 - c^2|^{-\nu-1/2} \\ \left[\left\{ \begin{array}{l} 0 < c < 2b \\ 0 < 2b < c \end{array} \right\}, |\operatorname{Re} \nu| < 1/2 \right]$$

$$\begin{aligned}
26. \quad \int_0^\infty x J_{\nu/2+\mu}(bx) J_{\nu/2-\mu}(bx) J_\nu(cx) dx &= \\
&= \frac{(2b)^{-2\mu}}{\pi c} (4b^2 - c^2)_+^{-1/2} [(c+i\sqrt{4b^2-c^2})^{2\mu} + (c-i\sqrt{4b^2-c^2})^{2\mu}] = \\
&= \frac{2}{\pi c} (4b^2 - c^2)_+^{-1/2} \cos\left(2\mu \arccos \frac{c}{2b}\right) \quad [b, c > 0; c \neq 2b; \operatorname{Re} \nu > -1].
\end{aligned}$$

$$\begin{aligned}
27. \quad \int_0^\infty x J_{(\nu+n)/2}(bx) J_{(\nu-n)/2}(bx) J_\nu(cx) dx &= \frac{2}{\pi c} (4b^2 - c^2)_+^{-1/2} T_n\left(\frac{c}{2b}\right) \\
& \quad [b, c > 0; c \neq 2b; \operatorname{Re} \nu > -1].
\end{aligned}$$

$$\begin{aligned}
28. \quad \int_0^\infty x J_\mu^2(bx) J_0(cx) dx &= \frac{2}{\pi c} (4b^2 - c^2)^{-1/2} \cos\left[\mu \arccos\left(1 - \frac{c^2}{2b^2}\right)\right] \\
& \quad [0 < c < 2b; \operatorname{Re} \mu > -1].
\end{aligned}$$

$$\begin{aligned}
29. \quad &= -\frac{2^{\mu+1} b^{2\mu}}{\pi c} \sin \mu \pi (c^2 - 4b^2)^{-1/2} (c^2 - 2b^2 + c\sqrt{c^2 - 4b^2})^{-\mu} \\
& \quad [0 < 2b < c; \operatorname{Re} \mu > -1]
\end{aligned}$$

$$\begin{aligned}
30. \quad \int_0^\infty x J_\mu(bx) J_0(bx) J_0(cx) dx &= \frac{2}{\pi c} (4b^2 - c^2)_+^{-1/2} \cos\left(\mu \arccos \frac{c}{2b}\right) \\
& \quad [b, c > 0; c \neq 2b].
\end{aligned}$$

$$\begin{aligned}
31. \quad \int_0^\infty \frac{1}{x} J_1^2(bx) J_0(cx) dx &= \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \left(\frac{1}{\pi} \arccos \frac{c}{2b} - \frac{c}{4b^2} \sqrt{4b^2 - c^2} \right) \\
& \quad \left[\begin{Bmatrix} 0 < c < 2b \\ 0 < 2b < c \end{Bmatrix} \right].
\end{aligned}$$

$$\begin{aligned}
32. \quad \int_0^\infty x^{-1/2} J_1^2(bx) J_0(cx) dx &= \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{1}{\pi \sqrt{c}} \left(\arccos \frac{c}{2b} - \frac{c}{4b^2} \sqrt{4b^2 - c^2} \right) \\
& \quad \left[\begin{Bmatrix} 0 < c < 2b \\ 0 < 2b < c \end{Bmatrix} \right].
\end{aligned}$$

$$\begin{aligned}
33. \quad \int_0^\infty x^{1-\nu} [J_{-\mu}^2(bx) - J_\mu^2(bx)] J_\nu(cx) dx &= \\
&= \frac{2^{1-\nu}}{b \sqrt{\pi c}} \sin \mu \pi (c^2 - 4b^2)_+^{(2\nu-1)/4} p_{\mu-1/2}^{1/2-\nu} \left(\frac{c^2}{2b^2} - 1 \right) \\
& \quad [b, c > 0; c \neq 2b; \operatorname{Re} \mu < 1; \operatorname{Re} \nu > -1/2]
\end{aligned}$$

2.12.43. Интегралы от $A(x) J_\lambda(\varphi(x)) J_\mu(\chi(x)) J_\nu(cx)$.

$$\begin{aligned}
1. \quad \int_0^\infty \frac{x^{2n-\mu-1}}{x^2+z^2} J_\nu(ax) J_\mu(bx) J_\nu(cx) dx &= (-1)^{n+1} z^{2n-\mu-2} I_\mu(az) I_\nu(cz) K_\nu(bz) \\
& \quad [a > b+c; b, c, \operatorname{Re} z > 0; \operatorname{Re} \nu > -n; \operatorname{Re} \mu > 2n-9/2]
\end{aligned}$$

$$\begin{aligned}
2. \quad \int_0^\infty x J_{\nu/4-\mu}(bx^2) J_{\nu/4+\mu}(bx^2) J_\nu(cx) dx &= \\
&= \frac{2}{\pi c^2} [e^{v\pi i/4} W_{\mu, \nu/4}(u_+) W_{-\mu, \nu/4}(u_+) + e^{-v\pi i/4} W_{\mu, \nu/4}(u_-) W_{-\mu, \nu/4}(u_-)] \\
& \quad [u_\pm = \pm ic^2/(8b), b, c > 0; \operatorname{Re} \nu > -1/2]
\end{aligned}$$

$$3. \int_0^{\infty} x J_{\nu/4}^2(bx^2) J_{\nu}(cx) dx = -\frac{1}{4b} J_{\nu/4}\left(\frac{c^2}{16b}\right) Y_{\nu/4}\left(\frac{c^2}{16b}\right)$$

$$[b, c > 0; \operatorname{Re} \nu > -1]$$

$$4. \int_0^{\infty} x^{\alpha-1} J_{\lambda}\left(\frac{b}{x}\right) J_{\mu}\left(\frac{b}{x}\right) J_{\nu}(cx) dx =$$

$$= \frac{b^{\lambda+\mu} c^{\lambda+\mu-\alpha}}{2^{2\lambda+2\mu-\alpha+1}} \Gamma\left[\frac{(\nu+\alpha-\lambda-\mu)/2}{(\nu-\alpha+\lambda+\mu)/2+1, \lambda+1, \mu+1}\right] {}_2F_3\left(\frac{\lambda+\mu+1}{2}, \frac{\lambda+\mu}{2}+1; \frac{\lambda+\mu-\alpha-\nu}{2}+1, \frac{\lambda+\mu-\alpha+\nu}{2}+1, \lambda+1, \mu+1, \lambda+\mu+1; \frac{b^2 c^2}{4}\right) + \frac{b^{\alpha+\nu} c^{\nu}}{2^{\alpha+2\nu+1}} \times$$

$$\times \Gamma\left[\frac{\alpha+\nu+1, (\lambda+\mu-\alpha-\nu)/2}{\nu+1, (\alpha+\lambda+\mu+\nu)/2+1, (\alpha-\lambda+\mu+\nu)/2+1, (\alpha+\lambda-\mu+\nu)/2+1}\right] \times$$

$$\times {}_2F_3\left(\frac{\alpha+\nu+1}{2}, \frac{\alpha+\nu}{2}+1; \frac{\alpha+\lambda+\mu+\nu}{2}+1, \frac{\alpha-\lambda-\mu+\nu}{2}+1, \frac{\alpha-\lambda+\mu+\nu}{2}+1, \frac{\alpha+\lambda-\mu+\nu}{2}+1, \nu+1; \frac{b^2 c^2}{4}\right)$$

$$[b, c > 0; \operatorname{Re}(\alpha+\nu) > -1; \operatorname{Re}(\alpha-\lambda-\mu) < 3/2]$$

$$5. \int_0^{\infty} \left[J_{\mu}^2\left(\frac{b}{x}\right) - J_{-\mu}^2\left(\frac{b}{x}\right) \right] J_0(cx) dx =$$

$$= \frac{\sec \mu \pi}{c} [J_{2\mu}(e^{i\pi/4} \sqrt{bc}) J_{2\mu}(e^{-i\pi/4} \sqrt{bc}) - J_{-2\mu}(e^{i\pi/4} \sqrt{bc}) J_{-2\mu}(e^{-i\pi/4} \sqrt{bc})]$$

$$[b, c > 0; |\operatorname{Re} \mu| < 1/4]$$

$$6. \int_0^{\infty} x^{\nu-1} (x^2+z^2)^{-\mu} J_{\mu}(a\sqrt{x^2+z^2}) J_{\mu}(b\sqrt{x^2+z^2}) J_{\nu}(cx) dx =$$

$$= 2^{\nu-1} \Gamma(\nu) c^{-\nu} z^{-2\mu} J_{\mu}(az) J_{\mu}(bz)$$

$$[a, b > 0; c > a+b; 0 < \operatorname{Re} \nu < 2\operatorname{Re} \mu + 5/2]$$

$$7. \int_0^{\infty} x^{\nu+1} J_{\mu}(b\sqrt{x^2+z^2}-bx) J_{\mu}(b\sqrt{x^2+z^2}+bx) J_{\nu}(cx) dx =$$

$$= \left(\frac{z}{4b}\right)^{\mu} \frac{c^{\nu}}{\Gamma(\mu+1) \Gamma(\mu-\nu)} (4b^2-c^2)_{+}^{\mu-\nu-1} {}_1F_2\left(\mu+\frac{1}{2}; 2\mu+1, \mu-\nu; -\frac{4b^2-c^2}{4b} z\right)$$

$$[b, c, \operatorname{Re} z > 0; c \neq 2b; -1 < \operatorname{Re} \nu < \operatorname{Re} \mu]$$

$$8. \int_0^{\infty} x J_{\nu/2}(b\sqrt{x^2+z^2}-bz) J_{\nu/2}(b\sqrt{x^2+z^2}+bz) J_{\nu}(cx) dx =$$

$$= \frac{2}{\pi c} (4b^2-c^2)_{+}^{-1/2} \cos(z\sqrt{4b^2-c^2})$$

$$[b, c, \operatorname{Re} z > 0; c \neq 2b; \operatorname{Re} \nu > -1]$$

2.12.44. Интегралы, содержащие $\prod_{k=1}^n J_{\nu_k}(\varphi_k(x))$, $n \geq 4$.

$$1. \int_0^{\infty} x J_0(ax) J_0(bx) J_0(cx) J_0(dx) dx = I$$

$$[a, b, c, d > 0; 16\Delta^2 = (a+b+c-d)(a+b+d-c)(a+c+d-b)(b+c+d-a)].$$

$$I = \frac{1}{\pi^2 \Delta} K\left(\frac{\sqrt{abcd}}{\Delta}\right) \quad [\Delta^2 > abcd].$$

$$I = \frac{1}{\pi^2 \sqrt{abcd}} K\left(\frac{\Delta}{\sqrt{abcd}}\right) \quad [\Delta^2 < abcd]$$

$$2. \int_0^{\infty} x J_0^2(bx) J_0^2(cx) dx =$$

$$= \frac{2b^2}{\pi \sqrt{(b^2 + c^2)^3 (3b^2 - c^2)}} {}_2F_1\left(\frac{1}{2}, \frac{1}{2}; 1; \frac{16b^2c^2}{(b^2 + c^2)^3 (3b^2 - c^2)}\right) \quad [0 < c < b].$$

$$3. = \frac{1}{2\pi bc} {}_2F_1\left(\frac{1}{2}, \frac{1}{2}; 1; \frac{(b^2 + c^2)^3 (3b^2 - c^2)}{16b^2c^2}\right) \quad [0 < b < c < b\sqrt{3}].$$

$$4. \int_0^{\infty} x^{1-2\nu} J_{\nu}^2(bx) J_{\nu}^2(cx) dx =$$

$$= \frac{b^{2\nu-1}}{2\pi c} \Gamma\left[\nu, \nu+1/2, 2\nu+1/2\right] {}_2F_1\left(\nu, \frac{1}{2}-\nu; 2\nu+\frac{1}{2}; \frac{b^2}{c^2}\right) \quad [0 < b < c, \operatorname{Re} \nu > 0]$$

$$5. \int_0^{\infty} x^{1-2\nu} J_{\nu}^4(cx) dx = \frac{(2c)^{2\nu-2}}{\pi^{3/2}} \Gamma\left[\nu, \nu, 3\nu, \nu+1/2\right] \quad [c, \operatorname{Re} \nu > 0]$$

$$6. \int_0^{\infty} x^{1/2} J_{1/4}^2(bx) J_{1/4}^2(cx) dx = \frac{\Gamma^2(1/4)}{\pi^3 c \sqrt{2b}} K\left(\sqrt{\frac{c - \sqrt{c^2 - b^2}}{2c}}\right) \quad [0 < b < c]$$

$$7. \int_0^{\infty} x^{\alpha-1} \prod_{k=1}^n J_{\nu_k}(c_k x) dx = I_{\alpha}$$

$$[n=2, 3, 4, \dots; c_k > 0, k=1, 2, 3, \dots, n; c_n > c_1 + \dots + c_{n-1}; \\ -\operatorname{Re}(\nu_1 + \dots + \nu_n) < \operatorname{Re} \alpha < n/2 + 1; \mu = \alpha + \nu_1 + \dots + \nu_n],$$

$$I_{\alpha} = 2^{\alpha-1} c_n^{\nu_n - \mu} \Gamma\left[\frac{\mu}{2}, \nu_n - \mu/2 + 1\right] \prod_{k=1}^{n-1} \frac{c_k^{\nu_k}}{\Gamma(\nu_k + 1)} F_c^{(n-1)}\left(\frac{\mu}{2}, \frac{\mu}{2} - \nu_n; \nu_1 + 1, \dots \right. \\ \left. \dots, \nu_{n-1} + 1; \frac{c_1^2}{c_n^2}, \dots, \frac{c_{n-1}^2}{c_n^2}\right),$$

$$I_{\nu_n - \nu_1 - \dots - \nu_{n-1}} = 2^{\nu_n - \nu_1 - \dots - \nu_{n-1} - 1} \frac{\Gamma(\nu_n)}{c_n^{\nu_n}} \prod_{k=1}^{n-1} \frac{c_k^{\nu_k}}{\Gamma(\nu_k + 1)} \\ [0 < \operatorname{Re} \nu_n < \operatorname{Re}(\nu_1 + \dots + \nu_{n-1}) + n/2 + 1],$$

$$I_{\alpha} = 0$$

$$[\alpha = \nu_n - \nu_1 - \dots - \nu_{n-1} + 2m + 2; -m - 1 < \operatorname{Re} \nu_n < \operatorname{Re}(\nu_1 + \dots + \nu_{n-1}) + n/2 - 2m - 1].$$

$$8. \int_0^{\infty} x^{\alpha-1} \prod_{k=1}^n (x^2 + z_k^2)^{-\mu_k/2} J_{\mu_k}(b_k \sqrt{x^2 + z_k^2}) J_{\nu}(cx) dx = A_{\alpha}$$

$$\left[b_k, \operatorname{Re} z_k > 0; k=1, 2, \dots, n; c > \sum_{k=1}^n b_k\right],$$

$$A_{\nu} = 2^{\nu-1} \Gamma(\nu) c^{-\nu} \prod_{k=1}^n z_k^{-\mu_k} J_{\mu_k}(b_k z_k)$$

$$\left[0 < \operatorname{Re} \nu < \sum_{k=1}^n \operatorname{Re} \mu_k + (n+3)/2\right],$$

$$A_{\nu+2} = 0 \quad \left[-1 < \operatorname{Re} \nu < \sum_{k=1}^n \operatorname{Re} \mu_k + (n-1)/2\right].$$

$$9. \int_0^{\infty} x^{\nu-1} (x^2 + z^2)^{-n\mu/2} \prod_{k=1}^n J_{\mu}(b_k \sqrt{x^2 + z^2}) J_{\nu}(cx) dx =$$

$$= 2^{\nu-1} c^{-\nu} z^{-n\mu} \Gamma(\nu) \prod_{k=1}^n J_{\mu}(b_k z)$$

$$\left[b_k, \operatorname{Re} z > 0; k=1, 2, \dots, n; c > \sum_{k=1}^n b_k; 0 < \operatorname{Re} \nu < n \operatorname{Re} \mu + (n+1)/2 \right].$$

2.12.45. Интегралы по индексу, содержащие $J_{\nu \pm x}(c)$ или $J_{ix}(c)$.

$$1. \int_{-\infty}^{\infty} e^{iax} J_x(c) dx = e^{ic \sin a} \begin{cases} 1 \\ 0 \end{cases} \quad \left[\begin{cases} -\pi < a < \pi \\ a < -\pi \text{ или } a > \pi \end{cases}, c > 0 \right].$$

$$2. \quad = \frac{1}{2} \quad [a = \pi; c > 0]$$

$$3. \int_{-\infty}^{\infty} \begin{cases} \operatorname{cosech}(\pi x/2) \\ \operatorname{sech}(\pi x/2) \end{cases} \cos bx J_{ix}(c) dx = \begin{cases} -2i \cos(c \operatorname{ch} b) \\ 2 \sin(c \operatorname{ch} b) \end{cases} \quad [b \geq 0; c > 0]$$

$$4. \int_{-\infty}^{\infty} e^{\pi x/2} \frac{\cos bx}{\operatorname{sh} \pi x} J_{ix}(c) dx = -i e^{ic \operatorname{ch} b} \quad [b, c > 0].$$

$$5. \int_0^{\infty} \begin{cases} \operatorname{cosech}(\pi x/2) \\ \operatorname{sech}(\pi x/2) \end{cases} \cos bx [J_{ix}(c) \mp J_{-ix}(c)] dx = \mp 2 \begin{cases} i \cos(c \operatorname{ch} b) \\ \sin(c \operatorname{ch} b) \end{cases} \quad [b, c > 0]$$

$$6. \int_0^{\infty} \cos \pi x J_x(cx) J_{-x}(cx) dx = \frac{1}{4 \sqrt{1-c^2}} \quad [0 < c < 1].$$

$$7. \int_0^{\infty} \frac{1}{x^2} [\cos cx J_x(cx) J_{-x}(cx) - 1] dx = -\frac{\pi^2}{2} \quad [c > 0].$$

$$8. \int_{-\infty}^{\infty} J_{\mu+x}(c) J_{\nu-x}(c) dx = J_{\mu+\nu}(2c) \quad [\operatorname{Re}(\mu + \nu) > 1; c > 0].$$

$$9. \int_{-\infty}^{\infty} e^{ipx} b^{-x} c^x J_{\mu+x}(b) J_{\nu-x}(c) dx =$$

$$= \begin{cases} 1 \\ 0 \end{cases} b^{\mu} c^{\nu} \left(2 \cos \frac{p}{2} \right)^{(\mu+\nu)/2} (b^2 e^{-ip/2} + c^2 e^{ip/2})^{-(\mu+\nu)/2} \times$$

$$\times \exp \left(ip \frac{\nu - \mu}{2} \right) J_{\nu+\mu} \left(\sqrt{2 \cos \frac{p}{2} (b^2 e^{-ip/2} + c^2 e^{ip/2})} \right)$$

$$\left[b, c > 0; \operatorname{Re}(\mu + \nu) > 1, \begin{cases} -\pi < p < \pi \\ p \geq \pi \text{ или } p \leq -\pi \end{cases} \right].$$

$$10. \int_{-\infty}^{\infty} \cos bx J_{\mu+x}(c) J_{\nu-x}(c) dx = \begin{cases} 1 \\ 0 \end{cases} \cos \frac{\mu - \nu}{2} b J_{\mu+\nu} \left(2c \cos \frac{b}{2} \right)$$

$$\left[\begin{cases} |b| < \pi \\ |b| \geq \pi \end{cases}, \operatorname{Im} b = 0; \operatorname{Re}(\mu + \nu) > 0 \text{ при } |b| = \pi; \operatorname{Re}(\mu + \nu) > -1 \text{ при } |b| \neq \pi \right].$$

$$\begin{aligned}
 11. \quad & \int_{-\infty}^{\infty} J_{\kappa+x}(c) J_{\lambda-x}(c) J_{\mu+x}(c) J_{\nu-x}(c) dx = \\
 & = \Gamma \left[\begin{matrix} \kappa + \lambda + \mu + \nu + 1 \\ \kappa + \lambda + 1, \lambda + \mu + 1, \mu + \nu + 1, \nu + \kappa + 1 \end{matrix} \right] \times \\
 & \times {}_4F_3 \left(\begin{matrix} \frac{\kappa + \lambda + \mu + \nu + 1}{2}, \frac{\kappa + \lambda + \mu + \nu + 1}{2}, \frac{\kappa + \lambda + \mu + \nu}{2} + 1, \frac{\kappa + \lambda + \mu + \nu}{2} + 1; \\ \kappa + \lambda + \mu + \nu + 1, \kappa + \lambda + 1, \lambda + \mu + 1, \mu + \nu + 1, \nu + \kappa + 1; -4c^2 \end{matrix} \right) \\
 & [c > 0; \operatorname{Re}(\kappa + \lambda + \mu + \nu) > -1].
 \end{aligned}$$

2.12.46. Интегралы, содержащие $\Gamma(x)$ и $J_\nu(cx)$.

$$1. \quad \int_0^\infty \frac{x^{\nu+2n-1/2}}{\Gamma(a+x)\Gamma(a-x)} J_\nu(cx) dx = 0 \quad [c \geq \pi; -1 < \operatorname{Re} \nu < 2a - 2n - 7/2].$$

2.12.47. Интегралы, содержащие $\operatorname{Ei}(\varphi(x)) J_\nu(cx)$.

$$1. \quad \int_0^\infty \operatorname{Ei}(-bx) J_0(cx) dx = -\frac{1}{c} \ln \frac{c + \sqrt{b^2 + c^2}}{b} \quad [c, \operatorname{Re} b > 0].$$

$$\begin{aligned}
 2. \quad & \int_0^\infty x^{\alpha-1} \operatorname{Ei}(-bx) J_\nu(cx) dx = -\frac{c^\nu}{2^\nu (\alpha + \nu) b^{\alpha+\nu}} \Gamma \left[\begin{matrix} \alpha + \nu \\ \nu + 1 \end{matrix} \right] \times \\
 & \times {}_3F_2 \left(\begin{matrix} \frac{\alpha + \nu}{2}, \frac{\alpha + \nu}{2}, \frac{\alpha + \nu + 1}{2} \\ \frac{\alpha + \nu}{2} + 1, \nu + 1 \end{matrix}; -\frac{c^2}{b^2} \right) \\
 & [c, \operatorname{Re} b > 0; \operatorname{Re}(\alpha + \nu) > 0].
 \end{aligned}$$

$$3. \quad \int_0^\infty x \operatorname{Ei}(-bx) J_0(cx) dx = \frac{1}{c^2} \left(\frac{b}{\sqrt{b^2 + c^2}} - 1 \right) \quad [c, \operatorname{Re} b > 0].$$

$$4. \quad \int_0^\infty \operatorname{Ei}\left(-\frac{b}{x}\right) J_1(cx) dx = -\frac{2}{c} J_0(\sqrt{2bc}) K_0(\sqrt{2bc}) \quad [c, \operatorname{Re} b > 0].$$

$$5. \quad \int_0^\infty \operatorname{Ei}(-bx^2) J_1(cx) dx = -\frac{1}{c} \left[C + \ln \frac{c^2}{4b} - \operatorname{Ei}\left(\frac{c^2}{4b}\right) \right] \quad [\operatorname{Re} b > 0].$$

$$\begin{aligned}
 6. \quad & \int_0^\infty x^{\alpha-1} \operatorname{Ei}(-bx^2) J_\nu(cx) dx = -\frac{c^\nu}{2^\nu (\alpha + \nu) b^{(\alpha+\nu)/2}} \times \\
 & \times \Gamma \left[\begin{matrix} (\alpha + \nu)/2 \\ \nu + 1 \end{matrix} \right] {}_2F_2 \left(\begin{matrix} \frac{\alpha + \nu}{2}, \frac{\alpha + \nu}{2} \\ \nu + 1, \frac{\alpha + \nu}{2} + 1 \end{matrix}; -\frac{c^2}{4b} \right) \\
 & [\operatorname{Re} b, \operatorname{Re}(\alpha + \nu) > 0]
 \end{aligned}$$

$$7. \quad \int_0^\infty x^{\nu+1} \operatorname{Ei}(-bx^2) J_\nu(cx) dx = -\frac{2^{\nu+1}}{c^{\nu+2}} \gamma \left(\nu + 1, \frac{c^2}{4b} \right) \quad [\operatorname{Re} b > 0; \operatorname{Re} \nu > -1].$$

$$8. \quad \int_0^\infty x^{n+1} \operatorname{Ei}(-bx^2) J_n(cx) dx = -\frac{2^{n+1} n!}{c^{n+2}} \left[1 - \exp\left(-\frac{c^2}{4b}\right) \sum_{k=0}^n \frac{1}{k!} \left(\frac{c^2}{4b}\right)^k \right] \quad [\operatorname{Re} b > 0].$$

$$9. \int_0^{\infty} x \operatorname{Ei}(-bx - b\sqrt{x^2 + z^2}) J_0(cx) dx = -\frac{\exp(-zb - z\sqrt{b^2 + c^2})}{(b + \sqrt{b^2 + c^2})\sqrt{b^2 + c^2}} \quad [c, \operatorname{Re} b, \operatorname{Re} z > 0]$$

$$10. \int_0^{\infty} e^{-cx} \operatorname{Ei}(-bx) J_0(cx) dx = -\frac{1}{\sqrt{2c}} \ln \frac{b + 2c + \sqrt{(b + 2c)^2 + b^2}}{b(1 + \sqrt{2})} \quad [c, \operatorname{Re} b > 0]$$

$$11. \int_0^{\infty} x^{\alpha-1} e^{\pm bx} \operatorname{Ei}(\mp bx) J_\nu(cx) dx = \\ = -\frac{\pi}{b^{\alpha+\nu}} \left(\frac{c}{2}\right)^\nu \Gamma\left[\frac{\alpha+\nu}{2}\right] \left\{ \begin{matrix} \operatorname{cosec}(\frac{\alpha+\nu}{2}\pi) \\ \operatorname{ctg}(\frac{\alpha+\nu}{2}\pi) \end{matrix} \right\} {}_2F_1\left(\frac{\alpha+\nu}{2}, \frac{\alpha+\nu+1}{2}; \nu+1; -\frac{c^2}{b^2}\right) \mp \\ \mp \frac{c^{1-\alpha}}{2^{2-\alpha}b} \Gamma\left[\frac{(\alpha+\nu-1)/2}{(\nu-\alpha+3)/2}\right] {}_3F_2\left(1, 1, \frac{1}{2}; \frac{3-\alpha-\nu}{2}, \frac{3+\nu-\alpha}{2}; -\frac{c^2}{b^2}\right) + \\ + \frac{c^{2-\alpha}}{2^{3-\alpha}b^2} \Gamma\left[\frac{(\alpha+\nu)/2-1}{(\nu-\alpha)/2+2}\right] {}_3F_2\left(1, 1, \frac{3}{2}; 2-\frac{\alpha+\nu}{2}, 2+\frac{\nu-\alpha}{2}; -\frac{c^2}{b^2}\right) \quad [c, \operatorname{Re} b > 0; -\operatorname{Re} \nu < \operatorname{Re} \alpha < 5/2].$$

$$12. \int_0^{\infty} x^{\alpha-1} e^{\pm bx^2} \operatorname{Ei}(\mp bx^2) J_\nu(cx) dx = \\ = -\frac{\pi}{b^{(\alpha+\nu)/2}} \left(\frac{c}{2}\right)^\nu \Gamma\left[\frac{(\alpha+\nu)/2}{\nu+1}\right] \left\{ \begin{matrix} \operatorname{cosec}[(\alpha+\nu)\pi/2] \\ \operatorname{ctg}[(\alpha+\nu)\pi/2] \end{matrix} \right\} {}_1F_1\left(\frac{\alpha+\nu}{2}; \nu+1; \pm \frac{c^2}{4b}\right) \mp \\ \mp \frac{c^{2-\alpha}}{2^{3-\alpha}b} \Gamma\left[\frac{(\alpha+\nu)/2-1}{(\nu-\alpha)/2+2}\right] {}_2F_2\left(1, 1; 2-\frac{\alpha+\nu}{2}, \frac{\nu-\alpha}{2}+2; \pm \frac{c^2}{4b}\right) \quad [c, \operatorname{Re} b > 0; -\operatorname{Re} \nu < \operatorname{Re} \alpha < 5/2]$$

$$13. \int_0^{\infty} x^{\nu+1} e^{bx^2} \operatorname{Ei}(-bx^2) J_\nu(cx) dx = -(2b)^{-\nu-1} c^\nu \Gamma(\nu+1) \exp\left(\frac{c^2}{4b}\right) \Gamma\left(-\nu, \frac{c^2}{4b}\right) \quad [c, \operatorname{Re} b > 0; -1 < \operatorname{Re} \nu < 1/2]$$

$$14. \int_0^{\infty} x e^{bx^2} \operatorname{Ei}(-bx^2) J_0(cx) dx = \frac{1}{2b} \exp\left(\frac{c^2}{4b}\right) \operatorname{Ei}\left(-\frac{c^2}{4b}\right) \quad [c, \operatorname{Re} b > 0].$$

$$15. \int_0^{\infty} x \ln x \operatorname{Ei}(-bx^2) J_0(cx) dx = \\ = \frac{2}{c^2} \left[\ln \frac{c^2}{4\sqrt{b}} - \operatorname{Ei}\left(-\frac{c^2}{4b}\right) + \frac{3}{2} C \right] - \frac{1}{c^2} \exp\left(-\frac{c^2}{4b}\right) \left[\ln \frac{4b^2}{c^2} + \operatorname{Ei}\left(\frac{c^2}{4b}\right) \right] \quad [\operatorname{Re} b > 0]$$

$$16. \int_0^{\infty} [C + \ln(bx^2) - 2 \operatorname{Ei}(-bx^2)] J_1(cx) dx = \frac{1}{c} \left[\ln \frac{c^2}{4b} + C - 2 \operatorname{Ei}\left(-\frac{c^2}{4b}\right) \right] \quad [\operatorname{Re} b > 0]$$

$$17. \int_0^{\infty} x^{\alpha-1} \operatorname{Ei}(-bx^2) J_\mu(cx) J_\nu(cx) dx = -\frac{b^{-(\alpha+\mu+\nu)/2}}{\alpha+\mu+\nu} \left(\frac{c}{2}\right)^{\mu+\nu} \Gamma\left[\frac{(\alpha+\mu+\nu)/2}{\mu+1, \nu+1}\right] \times \\ \times {}_4F_4\left(\frac{\mu+\nu+1}{2}, \frac{\mu+\nu}{2}+1, \frac{\alpha+\mu+\nu}{2}, \frac{\alpha+\mu+\nu}{2}; \mu+1, \nu+1, \right. \\ \left. \mu+\nu+1, \frac{\alpha+\mu+\nu}{2}+1; -\frac{c^2}{b}\right) \quad [\operatorname{Re} b, \operatorname{Re}(\alpha+\mu+\nu) > 0]$$

2.12.48. Интегралы от $x^\alpha \begin{Bmatrix} \text{si} (bx^n) \\ \text{ci} (bx^n) \end{Bmatrix} J_\nu (cx)$.

$$1. \int_0^\infty \text{si} (bx) J_0 (cx) dx = - \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{1}{c} \arcsin \frac{c}{b} \quad \left[\begin{Bmatrix} 0 < c < b \\ 0 < b < c \end{Bmatrix} \right].$$

$$2. \int_0^\infty x^{\alpha-1} \begin{Bmatrix} \text{si} (bx) \\ \text{ci} (bx) \end{Bmatrix} J_\nu (cx) dx = U$$

$[b, c > 0; -\text{Re } \nu < \text{Re } \alpha < 5/2 \text{ при } b \neq c, -\text{Re } \nu < \text{Re } \alpha < 3/2 \text{ при } b = c].$

$$U = - \frac{b^{-\alpha-\nu}}{\alpha+\nu} \left(\frac{c}{2} \right)^\nu \Gamma \left[\begin{matrix} \alpha+\nu \\ \nu+1 \end{matrix} \right] \begin{Bmatrix} \sin [(\alpha+\nu) \pi/2] \\ \cos [(\alpha+\nu) \pi/2] \end{Bmatrix} \times \\ \times {}_3F_2 \left(\frac{\alpha+\nu}{2}, \frac{\alpha+\nu}{2}, \frac{\alpha+\nu+1}{2}; \nu+1, \frac{\alpha+\nu}{2} + 1; \frac{c^2}{b^2} \right) \quad [0 < c < b].$$

$$U = - \frac{2^{\alpha+\delta} b^{\delta+2}}{(2\delta+1)(2+\delta) c^{\alpha+\delta+2}} \Gamma \left[\begin{matrix} (\alpha+\nu+\delta)/2 + 1 \\ (\nu-\alpha-\delta)/2 \end{matrix} \right] \times \\ \times {}_4F_3 \left(1, \frac{\delta}{2} + 1, \frac{\alpha+\nu+\delta}{2} + 1, 1 + \frac{\alpha+\delta-\nu}{2}; 2, \frac{\delta}{2} + 2, \delta + \frac{3}{2}; \frac{b^2}{c^2} \right) + \\ + \frac{2^{\alpha+\delta-1} b^\delta}{c^{\alpha+\delta}} \Gamma \left[\begin{matrix} (\alpha+\nu+\delta)/2 \\ (\nu-\alpha-\delta)/2 + 1 \end{matrix} \right] \begin{Bmatrix} 1 \\ R \end{Bmatrix} - \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{2^{\alpha-2} \pi}{c^\alpha} \Gamma \left[\begin{matrix} (\alpha+\nu)/2 \\ (\nu-\alpha)/2 + 1 \end{matrix} \right] \\ \left[0 < b < c; R = c + \frac{1}{2} \psi \left(\frac{\alpha+\nu}{2} \right) + \frac{1}{2} \psi \left(\frac{\nu-\alpha}{2} + 1 \right) - \ln \frac{c}{2b} \right].$$

$$3. \int_0^\infty \text{si} (bx^2) J_1 (cx) dx = - \frac{1}{c} \text{Si} \left(\frac{c^2}{4b} \right) \quad [b, c > 0].$$

$$4. \int_0^\infty \text{ci} (bx^2) J_{(1 \pm 1)/2} (cx) dx = \frac{1}{c} \left[\text{ci} \left(\frac{c^2}{4b} \right) + \frac{1 \mp 3}{1} C \mp \ln \frac{c^2}{4b} \right] \quad [b, c > 0].$$

$$5. \int_0^\infty x^{\alpha-1} \begin{Bmatrix} \text{si} (bx^2) \\ \text{ci} (bx^2) \end{Bmatrix} J_\nu (cx) dx = - \frac{c^\nu}{2^\nu (\alpha+\nu) b^{(\alpha+\nu)/2}} \Gamma \left[\begin{matrix} (\alpha+\nu)/2 \\ \nu+1 \end{matrix} \right] \times \\ \times \begin{Bmatrix} \sin [(\alpha+\nu) \pi/4] \\ \cos [(\alpha+\nu) \pi/4] \end{Bmatrix} {}_3F_4 \left(\frac{\alpha+\nu}{4}, \frac{\alpha+\nu}{4}, \frac{\alpha+\nu+2}{4}; \frac{1}{2}, \frac{\nu+1}{2}, \frac{\nu}{2} + 1, \right. \\ \left. \frac{\alpha+\nu}{4} + 1; - \frac{c^4}{64b^2} \right) \pm \frac{c^{\nu+2}}{2^{\nu+2} (\alpha+\nu+2) b^{(\alpha+\nu)/2+1}} \Gamma \left[\begin{matrix} (\alpha+\nu)/2 + 1 \\ \nu+2 \end{matrix} \right] \times \\ \times \begin{Bmatrix} \cos [(\alpha+\nu) \pi/4] \\ \sin [(\alpha+\nu) \pi/4] \end{Bmatrix} {}_3F_4 \left(\frac{\alpha+\nu+2}{4}, \frac{\alpha+\nu+2}{4}, \frac{\alpha+\nu}{4} + 1; \frac{3}{2}, \frac{\nu}{2} + 1, \right. \\ \left. \frac{\nu+3}{2}, \frac{\alpha+\nu+6}{4}; - \frac{c^4}{64b^2} \right) \quad [b, c > 0; -\text{Re } \nu < \text{Re } \alpha < 5/2].$$

$$6. \int_0^\infty x \begin{Bmatrix} \text{si} (bx^2) \\ \text{ci} (bx^2) \end{Bmatrix} J_0 (cx) dx = - \frac{2}{c^2} \begin{Bmatrix} \sin \varphi \\ 1 - \cos \varphi \end{Bmatrix} \quad [\varphi = c^2/(4b); b, c > 0].$$

2.12.49. Интегралы, содержащие $\begin{Bmatrix} \text{erf} (\varphi(x)) \\ \text{erfc} (\varphi(x)) \end{Bmatrix}$ и $J_\nu (\chi(x))$.

$$1. \int_0^\infty \begin{Bmatrix} \text{erf} (bx) \\ \text{erfc} (bx) \end{Bmatrix} J_0 (cx) dx = \frac{1}{c} \begin{Bmatrix} \text{erfc} (\varphi) \\ \text{erf} (\varphi) \end{Bmatrix}$$

$$\left[\varphi = \frac{c}{2b}, \begin{Bmatrix} c > 0; |\arg b| < \pi/4 \\ |\arg b| < \pi/4 \end{Bmatrix} \right].$$

$$2. \int_0^{\infty} x^{\alpha-1} \left\{ \begin{matrix} \operatorname{erf}(bx) \\ \operatorname{erfc}(bx) \end{matrix} \right\} J_{\nu}(cx) dx = \mp \frac{b^{-\alpha-\nu}}{(\alpha+\nu)\sqrt{\pi}} \left(\frac{c}{2}\right)^{\nu} \times$$

$$\times \Gamma\left[\frac{(\alpha+\nu+1)/2}{\nu+1}\right] {}_2F_2\left(\frac{\alpha+\nu}{2}, \frac{\alpha+\nu+1}{2}; \nu+1, \frac{\alpha+\nu}{2}+1; -\frac{c^2}{4b^2}\right) +$$

$$+ \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{2^{\alpha-1}}{c^{\alpha}} \Gamma\left[\frac{(\alpha+\nu)/2}{(\nu-\alpha)/2+1}\right]$$

$$\left[|\arg b| < \pi/4, \begin{cases} c > 0; -1 - \operatorname{Re} \nu < \operatorname{Re} \alpha < 3/2 \\ \operatorname{Re}(\alpha+\nu) > 0 \end{cases} \right].$$

$$3. \int_0^{\infty} x^{\nu} \left\{ \begin{matrix} \operatorname{erf}(bx) \\ \operatorname{erfc}(bx) \end{matrix} \right\} J_{\nu}(cx) dx = \frac{2^{\nu}}{\sqrt{\pi} c^{\nu+1}} \left\{ \begin{matrix} \Gamma(\nu+1/2, z) \\ \gamma(\nu+1/2, z) \end{matrix} \right\}$$

$$\left[z = \frac{c^2}{4b^2}; |\arg b| < \frac{\pi}{4}, \begin{cases} c > 0; -1 < \operatorname{Re} \nu < 1/2 \\ \operatorname{Re} \nu > -1/2 \end{cases} \right].$$

$$4. \int_0^{\infty} x^{1-\nu} \operatorname{erf}(bx) J_{\nu}(cx) dx = \frac{c^{\nu-2}}{2^{\nu-1}\Gamma(\nu)} {}_1F_1\left(\frac{1}{2}; \nu; -\frac{c^2}{4b^2}\right)$$

$$[c > 0; \operatorname{Re} \nu > 1/2; |\arg b| < \pi/4].$$

$$5. \int_0^{\infty} x^{\nu+1} \operatorname{erfc}(bx) J_{\nu}(cx) dx =$$

$$= \frac{2b^{-\nu}}{\sqrt{\pi} c^2} \Gamma\left[\frac{\nu+3/2}{\nu+2}\right] \exp\left(-\frac{c^2}{8b^2}\right) M_{-(\nu+1)/2, (\nu+1)/2}\left(\frac{c^2}{4b^2}\right)$$

$$[\operatorname{Re} \nu > -1; |\arg b| < \pi/4].$$

$$6. \int_0^{\infty} x^{-1/2} \operatorname{erf}(bx) J_{3/2}(cx) dx = \frac{b\sqrt{2}}{c^{3/2}} \operatorname{erf}\left(\frac{c}{2b}\right) \quad [c > 0; |\arg b| < \pi/4].$$

$$7. \int_0^{\infty} x^n \operatorname{erf}(bx) J_1(cx) dx = I_n \quad [c > 0; |\arg b| < \pi/4].$$

$$I_0 = \frac{1}{c} \exp\left(-\frac{c^2}{8b^2}\right) I_0\left(\frac{c^2}{8b^2}\right),$$

$$I_1 = \frac{1}{c^2} \operatorname{erfc}\left(\frac{c}{2b}\right) + \frac{1}{\sqrt{\pi} bc} \exp\left(-\frac{c^2}{4b^2}\right),$$

$$I_{-1} = \operatorname{erfc}\left(\frac{c}{2b}\right) + \frac{2b}{\sqrt{\pi} c} \left[1 - \exp\left(-\frac{c^2}{4b^2}\right)\right].$$

$$8. \int_0^{\infty} x^{\alpha-1} \left\{ \begin{matrix} \operatorname{erf}(b\sqrt{x}) \\ \operatorname{erfc}(b\sqrt{x}) \end{matrix} \right\} J_{\nu}(cx) dx = \mp \frac{b^{-2\alpha-2\nu}}{(\alpha+\nu)\sqrt{\pi}} \left(\frac{c}{2}\right)^{\nu} \Gamma\left[\frac{\alpha+\nu+1/2}{\nu+1}\right] \times$$

$$\times {}_2F_2\left(\frac{\alpha+\nu}{2}, \frac{2\alpha+2\nu+1}{4}, \frac{2\alpha+2\nu+3}{4}; \nu+1, \frac{\alpha+\nu}{2}+1; -\frac{c^2}{b^4}\right) +$$

$$+ \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{2^{\alpha-1}}{c^{\alpha}} \Gamma\left[\frac{(\alpha+\nu)/2}{(\nu-\alpha)/2+1}\right]$$

$$\left[c > 0; |\arg b| < \frac{\pi}{4}, \begin{cases} -1/2 - \operatorname{Re} \nu < \operatorname{Re} \alpha < 3/2 \\ \operatorname{Re}(\alpha+\nu) > 0 \end{cases} \right].$$

$$9. \int_0^{\infty} x^{\nu-1/2} \operatorname{erf}(b\sqrt{x}) J_{\nu}(cx) dx = \frac{2^{\nu+1}b}{\pi c^{\nu+1}} \Gamma\left(\nu+\frac{1}{2}\right) {}_2F_1\left(\nu+\frac{1}{2}, \frac{1}{4}; \frac{5}{4}; -\frac{b^4}{c^2}\right)$$

$$[c > 0; -1/2 < \operatorname{Re} \nu < 1; |\arg b| < \pi/4].$$

$$10. \int_0^{\infty} x^{\nu-1/2} \operatorname{erfc}(b\sqrt{x}) J_{\nu}(cx) dx =$$

$$= \frac{2^{\nu+1} c^{\nu} \Gamma(\nu+1/2)}{(4\nu+1) \pi b^{4\nu+1}} {}_2F_1\left(\nu+\frac{1}{2}, \nu+\frac{1}{4}; \nu+\frac{5}{4}; -\frac{c^2}{b^4}\right)$$

$$[c>0; \operatorname{Re} \nu > -1/4; |\arg b| < \pi/4].$$

$$11. \int_0^{\infty} \frac{1}{x} \left[\frac{\sqrt{\pi}}{2cx} \operatorname{erf}(cx) - J_0(x^2) \right] dx = 1 - \ln(\sqrt{2} c)$$

$$[|\arg c| < \pi/4].$$

$$12. \int_0^{\infty} x^{\alpha-1} \operatorname{erf}\left(\frac{b}{x}\right) J_{\nu}(cx) dx =$$

$$= \frac{b^{\alpha+\nu} c^{\nu}}{2^{\nu} (\alpha+\nu) \sqrt{\pi}} \Gamma\left[\frac{(1-\alpha-\nu)/2}{\nu+1}\right] {}_1F_2\left(\frac{\alpha+\nu}{2}; \nu+1, \frac{\alpha+\nu+1}{2}, \frac{\alpha+\nu}{2}+1; \frac{b^2 c^2}{4}\right) +$$

$$+ \frac{2^{\alpha-1} b}{\sqrt{\pi} c^{\alpha-1}} \Gamma\left[\frac{(\nu+\alpha-1)/2}{(3+\nu-\alpha)/2}\right] {}_1F_2\left(\frac{1}{2}; \frac{3}{2}, \frac{3-\alpha-\nu}{2}, \frac{\nu-\alpha+3}{2}; \frac{b^2 c^2}{4}\right)$$

$$[c>0; |\arg b| < \pi/4; -\operatorname{Re} \nu < \operatorname{Re} \alpha < 5/2].$$

$$13. \int_0^{\infty} e^{\mp b^2 x^2} \begin{Bmatrix} \operatorname{erf}(ibx) \\ \operatorname{erfc}(bx) \end{Bmatrix} J_0(cx) dx = \begin{Bmatrix} i \\ 1 \end{Bmatrix} \frac{1}{2b\sqrt{\pi}} \exp\left(\mp \frac{c^2}{8b^2}\right) K_0\left(\frac{c^2}{8b^2}\right)$$

$$[c>0; |\arg b| < \pi/4].$$

$$14. \int_0^{\infty} x^{\alpha-1} e^{\mp b^2 x^2} \begin{Bmatrix} \operatorname{erf}(ibx) \\ \operatorname{erfc}(bx) \end{Bmatrix} J_{\nu}(cx) dx =$$

$$= \begin{Bmatrix} i \\ 1 \end{Bmatrix} \frac{c^{\nu}}{2^{\nu+1} b^{\alpha+\nu}} \Gamma\left[\frac{(\alpha+\nu)/2}{\nu+1}\right] \begin{Bmatrix} \operatorname{tg}[(\alpha+\nu)\pi/2] \\ \sec[(\alpha+\nu)\pi/2] \end{Bmatrix} {}_1F_1\left(\frac{\alpha+\nu}{2}; \nu+1; \mp \frac{c^2}{4b^2}\right) +$$

$$+ \begin{Bmatrix} i \\ 1 \end{Bmatrix} \frac{2^{\alpha-2} c^{1-\alpha}}{\sqrt{\pi} b} \Gamma\left[\frac{(\alpha+\nu-1)/2}{(\nu-\alpha+3)/2}\right] {}_2F_2\left(\frac{1}{2}, 1; \frac{3-\alpha-\nu}{2}, \frac{3-\alpha+\nu}{2}; \mp \frac{c^2}{4b^2}\right)$$

$$[c>0; |\arg b| < \pi/4; -\operatorname{Re} \nu - (1 \pm 1)/2 < \operatorname{Re} \alpha < 5/2].$$

$$15. \int_0^{\infty} x e^{b^2 x^2} \operatorname{erfc}(bx) J_0(cx) dx = \frac{1}{4b^2} \exp\left(\frac{c^2}{4b^2}\right) \operatorname{erfc}\left(\frac{c}{2b}\right)$$

$$[c>0; |\arg b| < \pi/4].$$

$$16. \int_0^{\infty} x^{\nu} e^{b^2 x^2} \operatorname{erfc}(bx) J_{\nu}(cx) dx = \frac{1}{\sqrt{\pi} b^{\nu} c} \Gamma\left(\nu+\frac{1}{2}\right) \exp\left(\frac{c^2}{8b^2}\right) W_{\nu/2, -\nu/2}\left(\frac{c^2}{4b^2}\right)$$

$$[c>0; -1/2 < \operatorname{Re} \nu < 3/2; |\arg b| < \pi/4].$$

$$17. \int_0^{\infty} x^{\nu+1} e^{b^2 x^2} \operatorname{erfc}(bx) J_{\nu}(cx) dx =$$

$$= \frac{c^{\nu}}{2^{\nu+1} \pi b^{2\nu+2}} \Gamma\left(\nu+\frac{3}{2}\right) \exp\left(\frac{c^2}{4b^2}\right) \Gamma\left(-\nu-\frac{1}{2}, \frac{c^2}{4b^2}\right)$$

$$[c>0; -1 < \operatorname{Re} \nu < 1/2; |\arg b| < \pi/4]$$

$$\begin{aligned}
18. \int_0^\infty x^{\alpha-1} e^{-b^2 x} \left\{ \begin{array}{l} \operatorname{erf}(ib\sqrt{x}) \\ \operatorname{erfc}(b\sqrt{x}) \end{array} \right\} J_\nu(cx) dx = \\
= \left\{ \begin{array}{l} i \\ 1 \end{array} \right\} \frac{c^\nu}{2^\nu b^{2\nu+2}} \Gamma\left[\frac{\alpha+\nu}{\nu+1}\right] \left\{ \begin{array}{l} \operatorname{tg}(\alpha+\nu)\pi \\ \sec(\alpha+\nu)\pi \end{array} \right\} {}_2F_1\left(\frac{\alpha+\nu}{2}, \frac{\alpha+\nu+1}{2}; \nu+1; -\frac{c^2}{b^2}\right) + \\
+ \left\{ \begin{array}{l} i \\ 1 \end{array} \right\} \frac{2^{\alpha-3/2} c^{1/2-\alpha}}{\sqrt{\pi} b} \Gamma\left[\frac{(2\nu+2\alpha-1)/4}{(2\nu-2\alpha+5)/4}\right] {}_3F_2\left(\frac{1}{4}, \frac{3}{4}, 1; \frac{5-2\nu-2\alpha}{4}, \frac{5+2\nu-2\alpha}{4}; -\frac{c^2}{b^2}\right) + \\
+ \left\{ \begin{array}{l} i \\ -1 \end{array} \right\} \frac{2^{\alpha-7/2} c^{3/2-\alpha}}{\sqrt{\pi} b^3} \Gamma\left[\frac{(2\nu+2\alpha-3)/4}{(2\nu-2\alpha+7)/4}\right] \times \\
\times {}_3F_2\left(\frac{3}{4}, \frac{5}{4}, 1; \frac{7-2\nu-2\alpha}{4}, \frac{7+2\nu-2\alpha}{4}; -\frac{c^2}{b^2}\right) \\
[c > 0; -\operatorname{Re} \nu - (1 \pm 1)/4 < \operatorname{Re} \alpha < 2; |\arg b| < \pi/4].
\end{aligned}$$

$$19. \int_0^\infty \frac{1}{x} e^{-b^2 x^2} [\operatorname{erf}(ibx+d) + \operatorname{erf}(ibx-d)] J_0(cx) dx = \pi i \operatorname{erfc}(d)$$

[2bd > c > 0].

$$\begin{aligned}
20. \int_0^\infty \frac{x^{1-\nu}}{x^2+z^2} e^{-b^2 x^2} [\operatorname{erf}(ibx+d) + \operatorname{erf}(ibx-d)] J_\nu(cx) dx = \\
= \frac{\pi i}{z^\nu} e^{b^2 z^2} \operatorname{erfc}(bz+d) I_\nu(cz) \\
[2bd > c > 0; \operatorname{Re} z > 0; \operatorname{Re} \nu > 5/2].
\end{aligned}$$

$$\begin{aligned}
21. \int_0^\infty x^{\alpha-1} \left\{ \begin{array}{l} \sin cx \\ \cos cx \end{array} \right\} \operatorname{erfc}(bx) J_\nu(cx) dx = \\
= \frac{(2c)^{\nu+\delta}}{(\alpha+\nu+\delta) \pi b^{\alpha+\nu+\delta}} \Gamma\left[\frac{\nu+\delta+1/2}{2\nu+\delta+1}, \frac{(\alpha+\nu+\delta+1)/2}{2}\right] \times \\
\times {}_4F_4\left(\frac{2\nu+3}{4}, \frac{2\nu+1}{4}+\delta, \frac{\alpha+\nu+1}{2}, \frac{\alpha+\nu}{2}+\delta; \delta+\frac{1}{2}, \nu+1, \nu+\delta+\frac{1}{2}, \frac{\alpha+\nu+\delta}{2}+1; -\frac{4c^2}{b^2}\right) \\
[\operatorname{Re}(\alpha+\nu) > -(1 \pm 1)/2; |\arg b| < \pi/4]
\end{aligned}$$

$$22. \int_0^\infty \operatorname{erf}(bx) J_0(cx) J_1(cx) dx = \frac{1}{2c} {}_2F_2\left(\frac{1}{2}, \frac{1}{2}; 1, 1; -\frac{c^2}{b^2}\right)$$

[c > 0; |\arg b| < \pi/4].

$$\begin{aligned}
23. \int_0^\infty \operatorname{erfc}(bx) J_\nu^2(cx) dx = \\
= \frac{b^{-2\nu-1} c^{2\nu}}{2^{2\nu} \sqrt{\pi} (2\nu+1) \Gamma(\nu+1)} {}_2F_2\left(\nu+\frac{1}{2}, \nu+\frac{1}{2}; \nu+\frac{3}{2}, 2\nu+1; -\frac{c^2}{b^2}\right) \\
[\operatorname{Re} \nu > -1/2; |\arg b| < \pi/4].
\end{aligned}$$

$$\begin{aligned}
24. \int_0^\infty x^{2\nu} \operatorname{erfc}(bx) J_\nu^2(cx) dx = \\
= \frac{c^{2\nu}}{(4\nu+1) \pi b^{4\nu+1}} \Gamma\left[\frac{\nu+1/2}{\nu+1}\right] {}_2F_2\left(\nu+\frac{1}{2}, 2\nu+\frac{1}{2}; \nu+1, 2\nu+\frac{3}{2}; -\frac{c^2}{b^2}\right) \\
[\operatorname{Re} \nu > -1/4; |\arg b| < \pi/4].
\end{aligned}$$

$$25. \int_0^{\infty} x^{\alpha-1} \operatorname{erfc}(bx) J_{\mu}(cx) J_{\nu}(cx) dx =$$

$$= \frac{1}{(\alpha + \mu + \nu) \sqrt{\pi} b^{\alpha + \mu + \nu}} \left(\frac{c}{2}\right)^{\mu + \nu} \Gamma\left[\frac{(\alpha + \mu + \nu + 1)/2}{\mu + 1, \nu + 1}\right] \times$$

$$\times {}_4F_4\left(\frac{\mu + \nu}{2} + 1, \frac{\mu + \nu + 1}{2}, \frac{\alpha + \mu + \nu + 1}{2}, \frac{\alpha + \mu + \nu}{2}; \mu + 1, \nu + 1, \mu + \nu + 1, \frac{\alpha + \mu + \nu}{2} + 1; -\frac{c^2}{b^2}\right)$$

$$[\operatorname{Re}(\alpha + \mu + \nu) > 0; |\arg b| < \pi/4].$$

$$26. \int_0^{\infty} x^{\alpha-1} \operatorname{erf}\left(\frac{b}{x}\right) J_{\nu}^2(cx) dx = \frac{bc^{1-\alpha}}{\sqrt{\pi}} \Gamma\left[\frac{1-\alpha/2, \mu + (\alpha-1)/2}{(3-\alpha)/2, (3-\alpha)/2 + \nu}\right] \times$$

$$\times {}_2F_4\left(\frac{1}{2}, 1 - \frac{\alpha}{2}; \frac{3}{2}, \frac{3-\alpha}{2} - \nu, \frac{3-\alpha}{2}, \frac{3-\alpha}{2} + \nu; b^2 c^2\right)$$

$$[c > 0, -2 \operatorname{Re} \nu < \operatorname{Re} \alpha < 2, |\arg b| < \pi/4]$$

2.12.50. Интегралы от $x^{\alpha} \left\{ \begin{matrix} S(bx^n) \\ C(bx^n) \end{matrix} \right\} J_{\nu}(cx)$.

$$1. \int_0^{\infty} x^{\alpha-1} \left\{ \begin{matrix} S(bx) \\ C(bx) \end{matrix} \right\} J_{\nu}(cx) dx = U$$

$$[b, c > 0; -(2 \pm 1)/2 - \operatorname{Re} \nu < \operatorname{Re} \alpha < 3/2 \text{ при } b \neq c, -(2 \pm 1)/2 - \operatorname{Re} \nu < \operatorname{Re} \alpha < 1 \text{ при } b = c].$$

$$U = -\frac{c^{\nu}}{2^{\nu+1/2} (\alpha + \nu) \sqrt{\pi} b^{\alpha + \nu}} \Gamma\left[\frac{\alpha + \nu + 1/2}{\nu + 1}\right] \left\{ \begin{matrix} \cos[(1 - 2\alpha - 2\nu)\pi/4] \\ \sin[(1 - 2\alpha - 2\nu)\pi/4] \end{matrix} \right\} \times$$

$$\times {}_2F_2\left(\frac{2\alpha + 2\nu + 1}{4}, \frac{2\alpha + 2\nu + 3}{4}, \frac{\alpha + \nu}{2}; \nu + 1, \frac{\alpha + \nu}{2} + 1; \frac{c^2}{b^2}\right) +$$

$$+ \frac{2^{\alpha-2}}{c^{\alpha}} \Gamma\left[\frac{(\alpha + \nu)/2}{(\nu - \alpha)/2 + 1}\right] \quad [0 < c \leq b].$$

$$U = \frac{2^{\alpha+\delta} b^{\delta+1/2}}{3^{\delta} \sqrt{\pi} c^{\alpha+\delta+1/2}} \Gamma\left[\frac{(2\nu+2\alpha+2\delta+1)/4}{(2\nu-2\alpha-2\delta+3)/4}\right] \times$$

$$\times {}_2F_2\left(\frac{2\delta+1}{4}, \frac{2\alpha+2\delta+2\nu+1}{4}, \frac{2\alpha+2\delta-2\nu+1}{4}; \frac{2\delta+3}{4}, \delta + \frac{1}{2}; \frac{b^2}{c^2}\right)$$

$$[0 < b < c].$$

$$2. \int_0^{\infty} x^{-1/2} C(bx) J_0(cx) dx = \frac{1}{4\pi \sqrt{c}} \Gamma^2\left(\frac{1}{4}\right) - \frac{1}{\sqrt{\pi c}} F\left(\arccos \sqrt{\frac{b}{c}}, \frac{1}{\sqrt{2}}\right)$$

$$[0 < b < c]$$

$$3. \int_0^{\infty} \left\{ \begin{matrix} S(bx^2) \\ C(bx^2) \end{matrix} \right\} J_0(cx) dx = \frac{1}{2c} - \frac{1}{c} \left\{ \begin{matrix} C(\varphi) \\ S(\varphi) \end{matrix} \right\}$$

$$[\varphi = c^2/(4b), b, c > 0]$$

$$4. \int_0^{\infty} x^{\alpha-1} \left\{ \begin{matrix} S(bx^2) \\ C(bx^2) \end{matrix} \right\} J_{\nu}(cx) dx =$$

$$= -\frac{b^{-(\alpha+\nu)/2} c^{\nu}}{2^{\nu+1/2} (\alpha + \nu) \sqrt{\pi}} \Gamma\left[\frac{(\alpha + \nu + 1)/4}{\nu + 1}\right] \left\{ \begin{matrix} \cos[(1 - \alpha - \nu)\pi/4] \\ \sin[(1 - \alpha - \nu)\pi/4] \end{matrix} \right\} \times$$

$$\times {}_3F_4\left(\frac{\alpha + \nu}{4}, \frac{\alpha + \nu + 1}{4}, \frac{\alpha + \nu + 3}{4}; \frac{1}{2}, \frac{\nu + 1}{2}, \frac{\nu}{2} + 1, \frac{\alpha + \nu}{4} + 1; -\frac{c^4}{64b^2}\right) \pm$$

$$\pm \frac{b^{-(\alpha+\nu)/2-1} c^{\nu+2}}{2^{\nu+5/2} (\alpha+\nu+2) \sqrt{\pi}} \Gamma \left[\frac{(\alpha+\nu+3)/4}{\nu+2} \right] \left\{ \begin{array}{l} \cos [(1+\alpha+\nu) \pi/4] \\ \sin [(1+\alpha+\nu) \pi/4] \end{array} \right\} \times$$

$$\times {}_3F_4 \left(\frac{\alpha+\nu+2}{4}, \frac{\alpha+\nu+3}{4}, \frac{\alpha+\nu+5}{4}; \frac{3}{2}, \frac{\nu}{2}+1, \frac{\nu+3}{2}, \frac{\alpha+\nu+6}{4}; -\frac{c^2}{64b^2} \right)$$

[$b, c > 0; -2 \mp 1 - \operatorname{Re} \nu < \operatorname{Re} \alpha < 3/2$].

$$5. \int_0^\infty x \left\{ \begin{array}{l} S(bx^2) \\ C(bx^2) \end{array} \right\} J_1(cx) dx = \frac{1}{2c^2} - \frac{1}{c^2} \left\{ \begin{array}{l} C(\varphi) \\ S(\varphi) \end{array} \right\} + \frac{1}{\sqrt{2\pi b} c} \left\{ \begin{array}{l} \cos \varphi \\ \sin \varphi \end{array} \right\}$$

[$\varphi = c^2/(4b); b, c > 0$].

2.12.51. Интегралы, содержащие $\left\{ \begin{array}{l} \gamma(\mu, bx^n) \\ \Gamma(\mu, bx^n) \end{array} \right\} J_\nu(cx)$.

$$1. \int_0^\infty x^{\alpha-1} \left\{ \begin{array}{l} \gamma(\mu, bx) \\ \Gamma(\mu, bx) \end{array} \right\} J_\nu(cx) dx =$$

$$= \mp \frac{b^{-\alpha-\nu}}{\alpha+\nu} \left(\frac{c}{2} \right)^\nu \Gamma \left[\frac{\alpha+\mu+\nu}{\nu+1} \right] {}_3F_2 \left(\frac{\alpha+\nu}{2}, \frac{\alpha+\mu+\nu}{2}, \frac{\alpha+\mu+\nu+1}{2}; \right.$$

$$\left. \frac{\alpha+\nu}{2}+1, \nu+1; -\frac{c^2}{b^2} \right) + \left\{ \begin{array}{l} 1 \\ 0 \end{array} \right\} \frac{2^{\alpha-1}}{c^\alpha} \Gamma \left[\begin{array}{l} \mu, (\alpha+\nu)/2 \\ (\nu-\alpha)/2+1 \end{array} \right]$$

[$c, \operatorname{Re} b, \operatorname{Re}(\alpha+\mu+\nu) > 0, \left\{ \begin{array}{l} \operatorname{Re} \mu > 0, \operatorname{Re} \alpha < 3/2 \\ \operatorname{Re}(\alpha+\nu) > 0 \end{array} \right\}$].

$$2. \int_0^\infty x^{\alpha-1} \left\{ \begin{array}{l} \gamma(\mu, bx^2) \\ \Gamma(\mu, bx^2) \end{array} \right\} J_\nu(cx) dx = \mp \frac{b^{-(\alpha+\nu)/2}}{\alpha+\nu} \left(\frac{c}{2} \right)^\nu \Gamma \left[\frac{\mu+(\alpha+\nu)/2}{\nu+1} \right] \times$$

$$\times {}_2F_2 \left(\frac{\alpha+\nu}{2}, \mu + \frac{\alpha+\nu}{2}; \nu+1, \frac{\alpha+\nu}{2}+1; -\frac{c^2}{4b} \right) + \left\{ \begin{array}{l} 1 \\ 0 \end{array} \right\} \frac{2^{\alpha-1}}{c^\alpha} \Gamma \left[\begin{array}{l} \mu, (\alpha+\nu)/2 \\ (\nu-\alpha)/2+1 \end{array} \right]$$

[$\operatorname{Re} b, \operatorname{Re}(\alpha+\nu+2\mu) > 0, \left\{ \begin{array}{l} c, \operatorname{Re} \mu > 0, \operatorname{Re} \alpha < 3/2 \\ \operatorname{Re}(\alpha+\nu) > 0 \end{array} \right\}$].

$$3. \int_0^\infty x^{-\nu} \gamma \left(\nu + \frac{1}{2}, bx^2 \right) J_\nu(cx) dx = \frac{\sqrt{\pi}}{2^\nu} c^{\nu-1} \operatorname{erfc} \left(\frac{c}{2\sqrt{b}} \right)$$

[$c, \operatorname{Re} b > 0; \operatorname{Re} \nu > -1/2$].

$$4. \int_0^\infty x^{-\nu-1} \gamma(\nu+1, bx^2) J_\nu(cx) dx = -2^{-\nu-1} c^\nu \operatorname{Ei} \left(-\frac{c^2}{4b} \right)$$

[$c, \operatorname{Re} b > 0; \operatorname{Re} \nu > -1$].

$$5. \int_0^\infty x^{\nu+1} e^{bx^2} \Gamma(-\nu, bx^2) J_\nu(cx) dx = -\frac{(2b)^{-\nu-1} c^\nu}{\Gamma(\nu+1)} \exp \left(\frac{c^2}{4b} \right) \operatorname{Ei} \left(-\frac{c^2}{4b} \right)$$

[$c, \operatorname{Re} b > 0; \operatorname{Re} \nu > -1$].

2.12.52. Интегралы, содержащие $D_\mu(\varphi(x)) J_\nu(cx)$.

$$1. \int_0^\infty x^{\alpha-1} e^{\pm b^2 x^2/4} D_\mu(bx) J_\nu(cx) dx =$$

$$= \frac{2^{-(\alpha+3\nu \pm \mu \pm 1+1)/2} c^\nu}{b^{\alpha+\nu}} \left\{ \frac{\Gamma^{-1}(-\mu)}{\sqrt{\pi}} \right\} \Gamma \left[\frac{\alpha+\nu}{\nu+1} \right] \Gamma^{\pm 1} \left(\frac{1 \mp 1}{4} \mp \frac{\alpha+\nu \pm \mu}{2} \right) \times$$

$$\times {}_2F_2 \left(\frac{\alpha+\nu}{2}, \frac{\alpha+\nu+1}{2}; \frac{3 \pm 1}{4} + \frac{\alpha+\nu \pm \mu}{2}, \nu+1; \pm \frac{c^2}{2b^2} \right) + \left\{ \begin{array}{l} 1 \\ 0 \end{array} \right\} \frac{2^{\mu+\alpha-1} b^\mu}{c^{\alpha+\mu}} \times$$

$$\times \Gamma \left[\frac{(\alpha+\nu+\mu)/2}{(\nu-\alpha-\mu)/2+1} \right] {}_2F_2 \left(-\frac{\mu}{2}, \frac{1-\mu}{2}; 1 - \frac{\alpha+\mu+\nu}{2}, 1 + \frac{\nu-\alpha-\mu}{2}; \frac{c^2}{2b^2} \right)$$

[$|\arg b| < (2 \pm 1) \pi/4, \left\{ \begin{array}{l} c > 0, -\operatorname{Re} \nu < \operatorname{Re} \alpha < 3/2 - \operatorname{Re} \mu \\ \operatorname{Re}(\alpha+\nu) > 0 \end{array} \right\}$].

$$2. \int_0^{\infty} x^{\nu} e^{b^2 x^2/4} D_{\mu}(bx) J_{\nu}(cx) dx = I_{\mu} \quad [-1/2 < \operatorname{Re} \nu < 1/2 - \operatorname{Re} \mu; |\arg b| < 3\pi/4],$$

$$I_{\mu} = \frac{2^{(2\nu+3\mu+1)/4} b^{(1+\mu)/2-\nu}}{c^{(\mu+3)/2}} \Gamma\left[\frac{\nu+1/2}{(1-\mu)/2}\right] \exp\left(\frac{c^2}{4b^2}\right) W_{(1+\mu-2\nu)/4, (1+\mu+2\nu)/4}\left(\frac{c^2}{2b^2}\right),$$

$$I_{-2\nu-3} = \frac{c^{\nu}}{(2\nu+1)b^{2\nu+1}} \exp\left(\frac{c^2}{4b^2}\right) D_{-2\nu-1}\left(\frac{c}{b}\right) \quad [\operatorname{Re} \nu > -1/2],$$

$$I_{-2\nu} = \frac{c^{\nu-1}}{b^{2\nu}} \exp\left(\frac{c^2}{4b^2}\right) D_{-2\nu}\left(\frac{c}{b}\right) \quad [\operatorname{Re} \nu > -1/2],$$

$$I_{2\nu-1} = \frac{2^{1/2-\nu} \pi \sin \nu\pi}{bc^{\nu}} \Gamma(2\nu) \exp\left(\frac{c^2}{4b^2}\right) K_{\nu}\left(\frac{c^2}{4b^2}\right) \quad [|\operatorname{Re} \nu| < 1/2].$$

$$3. \int_0^{\infty} x^{\nu+1} e^{b^2 x^2/4} D_{\mu}(bx) J_{\nu}(cx) dx = J_{\mu}$$

$$[-1 < \operatorname{Re} \nu < -\operatorname{Re} \mu - 1/2; |\arg b| < 3\pi/4],$$

$$J_{\mu} = 2^{(3\mu+2\nu)/4+1} b^{\mu/2-\nu} c^{-\mu/2-2} \Gamma\left[\frac{\nu+3/2}{-\mu/2}\right] \exp\left(\frac{c^2}{4b^2}\right) W_{(\mu-2\nu-2)/4, (\mu+2\nu+2)/4}\left(\frac{c^2}{2b^2}\right),$$

$$J_{-2\nu-3} = \frac{c^{\nu}}{b^{2\nu+2}} \exp\left(\frac{c^2}{4b^2}\right) D_{-2\nu-3}\left(\frac{c}{b}\right) \quad [\operatorname{Re} \nu > -1],$$

$$J_{-2\nu-1} = \frac{(2\nu+1)c^{\nu-1}}{b^{2\nu+1}} \exp\left(\frac{c^2}{4b^2}\right) D_{-2\nu-3}\left(\frac{c}{b}\right) \quad [\operatorname{Re} \nu > -1/2],$$

$$J_{2\nu+3} = \frac{\sin \nu\pi \Gamma(2\nu+3)}{\pi c^{\nu+2}} \exp\left(\frac{c^2}{4b^2}\right) K_{\nu+1}\left(\frac{c^2}{4b^2}\right) \quad [-1 < \operatorname{Re} \nu < -5/6]$$

$$4. \int_0^{\infty} x^{\nu} e^{-b^2 x^2/4} D_{\mu}(bx) J_{\nu}(cx) dx = I_{\mu}$$

$$[\operatorname{Re} \nu > -1/2; |\arg b| < \pi/4],$$

$$I_{\mu} = \frac{2^{(\mu+2\nu)/4} c^{\mu/2-1}}{b^{\nu+\mu/2}} \Gamma\left[\frac{\nu+1/2}{\nu-\mu/2+1}\right] \exp\left(-\frac{c^2}{4b^2}\right) M_{(2\nu+\mu)/4, (2\nu-\mu)/4}\left(\frac{c^2}{2b^2}\right),$$

$$I_{-2\nu} = \sqrt{\frac{\pi}{2}} \frac{1}{bc^{\nu}} \exp\left(-\frac{c^2}{4b^2}\right) I_{\nu}\left(\frac{c^2}{4b^2}\right),$$

$$I_{2\nu-1} = -\frac{c^{\nu-1}}{2b^{2\nu} \cos \nu\pi} \exp\left(-\frac{c^2}{4b^2}\right) \left[D_{2\nu-1}\left(\frac{c}{b}\right) - D_{2\nu-1}\left(-\frac{c}{b}\right)\right],$$

$$I_{2\nu} = \frac{2^{\nu-1/2} c^{\nu}}{b^{2\nu+1}} \Gamma\left(\nu+\frac{1}{2}\right) {}_1F_1\left(\nu+\frac{1}{2}; 1; -\frac{c^2}{2b^2}\right),$$

$$I_{2\nu+1} = \frac{c^{\nu}}{2b^{2\nu+1} \cos \nu\pi} \exp\left(-\frac{c^2}{4b^2}\right) \left[D_{2\nu}\left(\frac{c}{b}\right) + D_{2\nu}\left(-\frac{c}{b}\right)\right].$$

$$5. \int_0^{\infty} x^{\nu+1} e^{-b^2 x^2/4} D_{\mu}(bx) J_{\nu}(cx) dx = J_{\mu}$$

$$[\operatorname{Re} \nu > -1; |\arg b| < \pi/4],$$

$$J_{\mu} = \frac{2^{(2\nu+\mu+3)/4} c^{(\mu-3)/2}}{b^{\nu+(\mu+1)/2}} \Gamma\left[\frac{\nu+3/2}{(3-\mu)/2+\nu}\right] \times \\ \times \exp\left(-\frac{c^2}{4b^2}\right) M_{(2\nu+\mu+3)/4, (2\nu-\mu+1)/4}\left(\frac{c^2}{2b^2}\right),$$

$$J_{-2\nu-3} = \sqrt{\frac{\pi}{2}} c^{-\nu-2} \exp\left(-\frac{c^2}{4b^2}\right) I_{\nu+1}\left(\frac{c^2}{4b^2}\right),$$

$$J_{2\nu} = \frac{c^{\nu-1}}{\sqrt{2} b^{2\nu+1} \cos \nu\pi} \exp\left(-\frac{c^2}{4b^2}\right) \left[D_{2\nu+1}\left(\frac{c}{b}\right) - D_{2\nu+1}\left(-\frac{c}{b}\right) \right],$$

$$J_{2\nu+1} = \frac{2^{\nu+1/2} c^\nu}{b^{2\nu+2}} \Gamma\left(\nu + \frac{3}{2}\right) {}_1F_1\left(\nu + \frac{3}{2}; 1; -\frac{c^2}{2b^2}\right),$$

$$J_{2\nu+2} = -\frac{c^\nu \sec \nu\pi}{2b^{2\nu+2}} \exp\left(-\frac{c^2}{4b^2}\right) \left[D_{2\nu+2}\left(\frac{c}{b}\right) + D_{2\nu+2}\left(-\frac{c}{b}\right) \right].$$

$$\begin{aligned} 6. \int_0^\infty x^{\alpha-1} e^{\pm b^2 x^2/4} D_\mu\left(\frac{b}{x}\right) J_\nu(cx) dx = \\ = 2^{(\alpha \mp \mu - \nu - 1 \mp 1)/2} b^{\alpha+\nu} c^\nu \Gamma\left[\frac{-\alpha-\nu}{\nu+1}\right] \left\{ \frac{\Gamma^{-1}(-\mu)}{\sqrt{\pi}} \right\} \Gamma^{\pm 1}\left(\frac{1 \mp 1}{4} \pm \frac{\alpha+\nu \mp \mu}{2}\right) \times \\ \times {}_1F_3\left(\frac{1 \mp 1}{4} + \frac{\alpha+\nu \mp \mu}{2}; \frac{\alpha+\nu}{2} + 1, \frac{\alpha+\nu+1}{2}, \nu+1, \pm \frac{b^2 c^2}{8}\right) + \\ + 2^{\alpha+\mu/2-1} \sqrt{\pi} c^{-\alpha} \Gamma\left[\frac{(\alpha+\nu)/2}{(\nu-\alpha)/2+1}, \frac{(1-\mu)/2}\right] \times \\ \times {}_1F_3\left(\frac{1 \mp 1 - 2\mu}{4}; \frac{1}{2}, 1 - \frac{\alpha+\nu}{2}, 1 + \frac{\nu-\alpha}{2}; \pm \frac{b^2 c^2}{8}\right) - \\ - 2^{\alpha+(\mu-3)/2} \sqrt{\pi} b c^{1-\alpha} \Gamma\left[\frac{(\nu+\alpha-1)/2}{-\mu/2}, \frac{(3+\nu-\alpha)/2}\right] \times \\ \times {}_1F_3\left(\frac{3 \mp 1 - 2\mu}{4}; \frac{3}{2}, \frac{3-\alpha-\nu}{2}, \frac{3+\nu-\alpha}{2}; \pm \frac{b^2 c^2}{8}\right) \\ \left[\operatorname{Re} \alpha < \frac{3}{2}; |\arg b| < \frac{(2 \pm 1)\pi}{4}, \begin{cases} c > 0 \\ c > 0; \operatorname{Re}(\alpha+\nu-\mu) > 0 \end{cases} \right]. \end{aligned}$$

$$7. \int_0^\infty x^\nu e^{-b^2 x^2/4} [(1 \pm 2 \cos \nu\pi) D_\mu(bx) - D_\mu(-bx)] J_\nu(cx) dx = I_\mu$$

$$[\operatorname{Re} \nu > -1/2; |\arg b| < \pi/4],$$

$$I_{2\nu-1} = \mp \frac{c^{\nu-1}}{b^{2\nu}} \exp\left(-\frac{c^2}{4b^2}\right) \left[(1 \pm 2 \cos \nu\pi) D_{2\nu-1}\left(\frac{c}{b}\right) - D_{2\nu-1}\left(-\frac{c}{b}\right) \right],$$

$$I_{2\nu+1} = \pm \frac{c^\nu}{b^{2\nu+1}} \exp\left(-\frac{c^2}{4b^2}\right) \left[(1 \pm 2 \cos \nu\pi) D_{2\nu}\left(\frac{c}{b}\right) + D_{2\nu}\left(-\frac{c}{b}\right) \right].$$

$$8. \int_0^\infty x^{\nu+1} e^{-b^2 x^2/4} [(1 \pm 2 \cos \nu\pi) D_\mu(bx) + D_\mu(-bx)] J_\nu(cx) dx = J_\mu$$

$$[\operatorname{Re} \nu > -1; |\arg b| < \pi/4],$$

$$J_{2\nu} = \pm \frac{c^{\nu-1}}{b^{2\nu+1}} \exp\left(-\frac{c^2}{4b^2}\right) \left[(1 \pm 2 \cos \nu\pi) D_{2\nu+1}\left(\frac{c}{b}\right) - D_{2\nu+1}\left(-\frac{c}{b}\right) \right],$$

$$J_{2\nu+2} = \mp \frac{c^\nu}{b^{2\nu+2}} \exp\left(-\frac{c^2}{4b^2}\right) \left[(1 \pm 2 \cos \nu\pi) D_{2\nu+2}\left(\frac{c}{b}\right) + D_{2\nu+2}\left(-\frac{c}{b}\right) \right].$$

$$9. \int_0^\infty D_{\nu-1}(bx) D_\nu(bx) J_0(cx) dx = \frac{1}{2c} \left[D_\nu\left(\frac{c}{b}\right) D_{\nu-1}\left(-\frac{c}{b}\right) - D_{\nu-1}\left(\frac{c}{b}\right) D_\nu\left(-\frac{c}{b}\right) \right]$$

$$[|\arg b| < \pi/4]$$

$$10. \int_0^\infty D_{\nu-1}(\mp bx) D_\nu(\pm bx) J_0(cx) dx =$$

$$= \frac{1}{2} \left\{ \frac{c^{-1}}{b^{-1}} \right\} \left[D_\nu\left(\frac{c}{b}\right) D_{\nu-1}\left(-\frac{c}{b}\right) + D_\nu\left(-\frac{c}{b}\right) D_{\nu-1}\left(\frac{c}{b}\right) \pm 2 D_\nu\left(\frac{c}{b}\right) D_{\nu-1}\left(\frac{c}{b}\right) \right]$$

$$[b > 0]$$

$$11. \int_0^{\infty} D_n(bx) D_{n+\sigma}(bx) J_{1-\sigma}(cx) dx = \frac{(-1)^{n+\sigma+1}}{c} D_n\left(\frac{c}{b}\right) D_{n+\sigma}\left(\frac{c}{b}\right)$$

[$\sigma = 0$ или 1 ; $|\arg b| < \pi/4$].

$$12. \int_0^{\infty} x^{\alpha-1} D_{-\mu-1}(b\sqrt{x}) D_{\mu}(b\sqrt{x}) J_{\nu}(cx) dx =$$

$$= \frac{2^{1/2-2\alpha-3\nu} \pi c^{\nu}}{b^{2\alpha+2\nu}} \Gamma \left[\begin{matrix} 2\alpha+2\nu \\ \nu+1, (\alpha+\mu+\nu)/2+1, (\alpha+\nu-\mu+1)/2 \end{matrix} \right] \times$$

$$\times {}_4F_3 \left(\frac{\alpha+\nu}{2}, \frac{\alpha+\nu+1}{2}, \frac{2\alpha+2\nu+1}{4}, \frac{2\alpha+2\nu+3}{4}; \nu+1, \right.$$

$$\left. \frac{\alpha+\mu+\nu}{2}+1, \frac{\alpha+\nu-\mu+1}{2}; -\frac{4c^2}{b^4} \right)$$

[$c, \operatorname{Re}(\alpha+\nu) > 0$; $|\arg b| < \pi/4$]

$$13. \int_0^{\infty} x^{-1/2} D_{-\nu-1}(b\sqrt{x}) D_{\nu}(b\sqrt{x}) J_0(cx) dx =$$

$$= \frac{\pi}{2^{3/2}b} P_{-1/4}^{(2\nu+1)/4} \left(\frac{\sqrt{b^4+4c^2}}{b^2} \right) P_{-1/4}^{-(2\nu+1)/4} \left(\frac{\sqrt{b^4+4c^2}}{b^2} \right) =$$

$$= \frac{ib}{2^{3/2}c\Gamma(-\nu/2)\Gamma((\nu+1)/2)} Q_{-(2\nu+3)/4}^{-1/4} \left(\frac{\sqrt{b^4+4c^2}}{2c} \right) Q_{(2\nu-1)/4}^{-1/4} \left(\frac{\sqrt{b^4+4c^2}}{2c} \right)$$

[$b, c > 0$].

$$14. \int_0^{\infty} x^{1/2} D_{-\nu-1/2}(b\sqrt{x} e^{\pi i/4}) D_{-\nu-1/2}(b\sqrt{x} e^{-\pi i/4}) J_{\nu}(cx) dx =$$

$$= \frac{2^{-\nu} \sqrt{\pi} c^{-\nu-1}}{\Gamma(\nu+1/2) \sqrt{b^2+2c}} (\sqrt{b^2+2c}-b)^{2\nu}$$

[$b, c > 0$; $\operatorname{Re} \nu > -1/2$]

$$15. \int_0^{\infty} D_{-\nu-1/2} \left(\frac{b}{\sqrt{x}} e^{\pi i/4} \right) D_{-\nu-1/2} \left(\frac{b}{\sqrt{x}} e^{-\pi i/4} \right) J_{\nu}(cx) dx = \frac{\sqrt{2\pi}}{c\Gamma(\nu+1/2)} e^{-b\sqrt{2c}}$$

[$b, c > 0$; $\operatorname{Re} \nu > -3/4$]

$$16. \int_0^{\infty} x^{1/2} D_{\nu-1/2} \left(\frac{b}{\sqrt{x}} \right) D_{-\nu-1/2} \left(\frac{b}{\sqrt{x}} \right) J_{\nu}(cx) dx =$$

$$= c^{-3/2} e^{-b\sqrt{c}} \cos \left(b\sqrt{c} + \frac{1-2\nu}{4} \pi \right)$$

[$c > 0$; $|\arg b| < \pi/4$; $\operatorname{Re} \nu > -2$]

2.13. ФУНКЦИЯ НЕЙМАНА $Y_{\nu}(x)$

При $\nu = \pm \frac{1}{2}, \pm \frac{3}{2}, \dots$ функция $Y_{\nu}(x)$ сводится к элементарным, в частности,

$$Y_{\pm 1/2}(cx) = \mp \sqrt{\frac{2}{\pi cx}} \begin{Bmatrix} \cos cx \\ \sin cx \end{Bmatrix}.$$

При вычислении интегралов от $Y_{\nu}(x)$ можно использовать формулу

$$Y_{\nu}(cx) = \frac{1}{\sin \nu \pi} [J_{\nu}(cx) \cos \nu \pi - J_{-\nu}(cx)]$$

[$\nu \neq 0, \pm 1, \pm 2, \dots$]

2.13.1. Интегралы общего вида, содержащие $\left\{ \begin{matrix} Y_v(cx) \\ K_v(cx) \end{matrix} \right\}$.

$$1. \int_0^a x^{\alpha-1} (a^r - x^r)^{\beta-1} \left\{ \begin{matrix} Y_v(cx) \\ K_v(cx) \end{matrix} \right\} dx = U_v(0) \quad [a, r, \operatorname{Re} \beta > 0; \operatorname{Re} \alpha > |\operatorname{Re} v|],$$

$$U_v(\varepsilon) = \mp \frac{a^{\alpha+r\beta-r}}{2\pi r} \Gamma(\beta) \left\{ \frac{2}{\pi} \right\} \sum_{k=0}^{\infty} \frac{(\mp 1)^k}{k!} \left[A_k(v) + \left\{ \begin{matrix} \cos v\pi \\ 1 \end{matrix} \right\} A_k(-v) \right] \left(\frac{ac}{2} \right)^{2k},$$

$$A_k(v) = \left(\frac{2}{ac} \right)^v \frac{\Gamma(v)}{(1-v)_k (2k-v)^{\varepsilon}} \Gamma \left[\frac{(\alpha-v+2k)/r}{\beta + (\alpha-v+2k)/r} \right] \\ [v \neq \dots, -2, -1, 0, 1, 2, \dots],$$

$$U_n(\varepsilon) = \mp \frac{a^{\alpha+r\beta-r}}{2\pi r} \Gamma(\beta) \left\{ \frac{2}{\pi} \right\} \left\{ \sum_{k=0}^{n-1} (\mp 1)^k \frac{A_k(n)}{k!} \left(\frac{ac}{2} \right)^{2k} + \right. \\ \left. + \left(\pm \frac{ac}{2} \right)^n \sum_{k=0}^{\infty} \frac{(\mp 1)^k}{k! (n+k)! (2k+n)^{\varepsilon}} \Gamma \left[\frac{(\alpha+n+2k)/r}{\beta + (\alpha+n+2k)/r} \right] \times \right. \\ \left. \times \left[\psi(k+1) + \psi(n+k+1) + \frac{2}{r} \psi \left(\beta + \frac{\alpha+n+2k}{r} \right) - \right. \right. \\ \left. \left. - \frac{2}{r} \psi \left(\frac{\alpha+n+2k}{r} \right) + \frac{2\varepsilon}{n+2k} - 2 \ln \frac{ac}{2} \right] \left(\frac{ac}{2} \right)^{2k} \right\}.$$

$$2. \int_a^{\infty} x^{\alpha-1} (x^r - a^r)^{\beta-1} \left\{ \begin{matrix} Y_v(cx) \\ K_v(cx) \end{matrix} \right\} dx = V_v(0)$$

$$[a, r, \operatorname{Re} \beta > 0, \left\{ \begin{matrix} \operatorname{Re}(\alpha+r\beta) < r+3/2; \\ \operatorname{Re} c > 0 \end{matrix} \right\}],$$

$$V_v(\varepsilon) = \mp \frac{a^{\alpha+r\beta-r}}{2\pi r} \Gamma(\beta) \left\{ \frac{2}{\pi} \right\} \sum_{k=0}^{\infty} \frac{(\mp 1)^k}{k!} \left[B_k(v) + \left\{ \begin{matrix} \cos v\pi \\ 1 \end{matrix} \right\} B_k(-v) \right] \left(\frac{ac}{2} \right)^{2k} \mp \sum_{l=0}^{\infty} C_l(v),$$

$$B_k(v) = \left(\frac{2}{ac} \right)^v \frac{\Gamma(v)}{(1-v)_k (2k-v)^{\varepsilon}} \Gamma \left[\frac{1-\beta-(\alpha-v+2k)/r}{1-(\alpha-v+2k)/r} \right],$$

$$C_l(v) = \frac{1}{2\pi} \left(\frac{c}{2} \right)^{r-r\beta-\alpha} \frac{(1-\beta)_l}{l! (r-r\beta+rl-\alpha)^{\varepsilon}} \left\{ \begin{matrix} \cos [(v-\alpha+r-r\beta+rl)\pi/2] \\ \pi/2 \end{matrix} \right\} \times \\ \times \Gamma \left(\frac{v+\alpha-r(1-\beta+l)}{2} \right) \Gamma \left(\frac{\alpha-v-r(1-\beta+l)}{2} \right) \left(\frac{ac}{2} \right)^{rl} \\ [v \neq \dots, -2, -1, 0, 1, 2, \dots; rl-2k-\alpha-\beta r+r \neq v, -v; \\ k, l=0, 1, 2, \dots],$$

$$V_n(\varepsilon) = \mp \frac{a^{\alpha+r\beta-r}}{2\pi r} \Gamma(\beta) \left\{ \frac{2}{\pi} \right\} \left[\sum_{k=0}^{n-1} \frac{(\mp 1)^k B_k(n)}{k!} \left(\frac{ac}{2} \right)^{2k} + \sum_{m=0}^{\infty} D_m \right] \mp \sum_{l=0}^{\infty} C_l(n),$$

$$D_m = \left(\pm \frac{ac}{2} \right)^n \frac{(\mp 1)^m}{m! (n+m)! (2m+n)^{\varepsilon}} \Gamma \left[\frac{1-\beta-(\alpha+n+2m)/r}{1-(\alpha+n+2m)/r} \right] \times \\ \times \left[\psi(m+1) + \psi(n+m+1) + \frac{2}{r} \psi \left(1-\beta - \frac{\alpha+n+2m}{r} \right) - \right. \\ \left. - \frac{2}{r} \psi \left(1 - \frac{\alpha+n+2m}{r} \right) + \frac{2\varepsilon}{n+2m} - 2 \ln \frac{ac}{2} \right] \left(\frac{ac}{2} \right)^{2m} \\ [rl-2k-\alpha-\beta r+r \neq n, -n; k, l, n=0, 1, 2, \dots].$$

$$3. \int_0^{\infty} \frac{x^{\alpha-1}}{(x^r+z^r)^\rho} \left\{ \frac{Y_\nu(cx)}{K_\nu(cx)} \right\} dx = W_\nu(0)$$

$$\left[r > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu|; r |\arg z| < \pi, \left\{ \begin{array}{l} \operatorname{Re}(\alpha - r\rho) < 3/2; c > 0 \\ \operatorname{Re} c > 0 \end{array} \right\} \right],$$

$$W_\nu(z) =$$

$$= \mp \frac{z^{\alpha-r\rho}}{2\pi r \Gamma(\rho)} \left\{ \frac{2}{\pi} \right\} \sum_{k=0}^{\infty} \frac{(\mp 1)^k}{k!} \left[E_k(\nu) + \left\{ \frac{\cos \nu\pi}{1} \right\} E_k(-\nu) \right] \left(\frac{cz}{2} \right)^{2k} \mp \sum_{l=0}^{\infty} F_l(\nu),$$

$$E_k(\nu) = \left(\frac{2}{cz} \right)^\nu \frac{\Gamma(\nu)}{(1-\nu)_k (2k-\nu)^e} \Gamma\left(\frac{\alpha-\nu+2k}{r}\right) \Gamma\left(\rho - \frac{\alpha-\nu+2k}{r}\right),$$

$$F_l(\nu) = \left(\frac{2}{c} \right)^{\alpha-r\rho} \frac{(-1)^l (\rho)_l}{2\pi l! (r\rho+rl-\alpha)^e} \Gamma\left(\frac{\nu+\alpha-r\rho-rl}{2}\right) \times \\ \times \Gamma\left(\frac{\alpha-\nu-r\rho-rl}{2}\right) \left\{ \frac{\cos[(\nu-\alpha+r\rho+rl)\pi/2]}{\pi/2} \right\} \left(\frac{cz}{2} \right)^{rl} \\ [\nu \neq \dots, -2, -1, 0, 1, 2, \dots; rl-2k-\alpha+r\rho \neq \nu, -\nu; \\ k, l=0, 1, 2, \dots],$$

$$W_n(z) = \mp \frac{z^{\alpha-r\rho}}{2\pi r \Gamma(\rho)} \left\{ \frac{2}{\pi} \right\} \left[\sum_{k=0}^{n-1} \frac{(\mp 1)^k E_k(n)}{k!} \left(\frac{cz}{2} \right)^{2k} + \sum_{m=0}^{\infty} G_m \right] \mp \sum_{l=0}^{\infty} F_l(n),$$

$$G_m = \left(\pm \frac{ca}{2} \right)^n \frac{(\mp 1)^m}{m! (n+m)! (2m+n)^e} \Gamma\left(\frac{\alpha+2m+n}{r}\right) \Gamma\left(\rho - \frac{\alpha+2m+n}{r}\right) \times \\ \times \left[\psi(m+1) + \psi(m+n+1) + \frac{2e}{2m+n} - \frac{2}{r} \psi\left(\frac{\alpha+2m+n}{r}\right) + \right. \\ \left. + \frac{2}{r} \psi\left(\rho - \frac{\alpha+2m+n}{r}\right) - 2 \ln \frac{cz}{2} \right] \left(\frac{cz}{2} \right)^{2m} \\ [rl-2k-\alpha+r\rho \neq n, -n; k, l, n=0, 1, 2, \dots],$$

$$4. \int_0^{\infty} \frac{x^{\alpha-1}}{x^r-y^r} \left\{ \frac{Y_\nu(cx)}{K_\nu(cx)} \right\} dx = X_\nu(0)$$

$$\left[r, y > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu|, \left\{ \begin{array}{l} \operatorname{Re} \alpha < r+3/2; c > 0 \\ \operatorname{Re} c > 0 \end{array} \right\} \right],$$

$$X_\nu(z) = \pm \frac{y^{\alpha-r}}{2r} \left\{ \frac{2}{\pi} \right\} \sum_{k=0}^{\infty} \frac{(\mp 1)^k}{k!} \left[\hat{B}_k(\nu) + \left\{ \frac{\cos \nu\pi}{1} \right\} \hat{B}_k(-\nu) \right] \left(\frac{cy}{2} \right)^{2k} \mp \sum_{l=0}^{\infty} \hat{C}_l(\nu),$$

$$\hat{B}_k(\nu) = \left(\frac{2}{cy} \right)^\nu \frac{\Gamma(\nu)}{(1-\nu)_k (2k-\nu)^e} \operatorname{ctg} \frac{\alpha-\nu+2k}{r} \pi,$$

$$\hat{C}_l(\nu) = \frac{1}{2\pi (r+rl-\alpha)^e} \left(\frac{2}{c} \right)^{\alpha-r} \Gamma\left(\frac{\nu+\alpha-r-rl}{2}\right) \times \\ \times \Gamma\left(\frac{\alpha-\nu-r-rl}{2}\right) \left\{ \frac{\cos[(\nu-\alpha+r+rl)\pi/2]}{\pi/2} \right\} \left(\frac{cy}{2} \right)^{rl} \\ [\nu \neq \dots, -2, -1, 0, 1, 2, \dots; rl-2k-\alpha+r \neq \nu, -\nu; \\ k, l=0, 1, 2, \dots],$$

$$X_n(z) = \pm \frac{y^{\alpha-r}}{2r} \left\{ \frac{2}{\pi} \right\} \left[\sum_{k=0}^{n-1} \frac{(\mp 1)^k \hat{B}_k(n)}{k!} \left(\frac{cy}{2} \right)^{2k} + \sum_{m=0}^{\infty} \hat{D}_m \right] \mp \sum_{l=0}^{\infty} \hat{C}_l(n),$$

$$\hat{D}_m = \left(\pm \frac{cy}{2} \right)^n \frac{(\mp 1)^m}{m! (m+n)! (2m+n)^e} \operatorname{ctg} \frac{\alpha+n+2m}{r} \pi \times \\ \times \left[\psi(m+1) + \psi(n+m+1) + \frac{2e}{n+2m} - \frac{4\pi}{r} \operatorname{cosec} 2 \frac{\alpha+n+2m}{r} \pi - 2 \ln \frac{cy}{2} \right] \left(\frac{cy}{2} \right)^{2m} \\ [rl-2k-\alpha+r \neq n, -n; k, l, n=0, 1, 2, \dots],$$

$$5. \int_0^{\infty} x^{\alpha-1} e^{-px^r} \left\{ \begin{matrix} Y_v(cx) \\ K_v(cx) \end{matrix} \right\} dx = Y_v(0)$$

$$\left[r, \operatorname{Re} p > 0; \operatorname{Re} \alpha > |\operatorname{Re} v|, \left\{ \begin{matrix} c > 0 \\ \operatorname{Re} c > 0 \end{matrix} \right\} \right],$$

$$Y_v(\varepsilon) = \mp \frac{p^{-\alpha/r}}{2r\pi} \left\{ \begin{matrix} 2 \\ \pi \end{matrix} \right\} \sum_{k=0}^{\infty} \frac{(\mp 1)^k}{k!} \left[\hat{A}_k(v) + \left\{ \begin{matrix} \cos v\pi \\ 1 \end{matrix} \right\} \hat{A}_k(-v) \right] \left(\frac{c}{2p^{1/r}} \right)^{2k},$$

$$\hat{A}_k(v) = \left(\frac{2p^{1/r}}{c} \right)^v \frac{\Gamma(v)}{(1-v)_k (2k-v)^{\varepsilon}} \Gamma\left(\frac{\alpha-v+2k}{r}\right) \\ [r > 1; v \neq \dots, -2, -1, 0, 1, 2, \dots],$$

$$Y_n(\varepsilon) = \mp \frac{p^{-\alpha/r}}{2r\pi} \left\{ \begin{matrix} 2 \\ \pi \end{matrix} \right\} \left\{ \sum_{k=0}^{n-1} \frac{(\mp 1)^k \hat{A}_k(n)}{k!} \left(\frac{c}{2p^{1/r}} \right)^{2k} + \right. \\ \left. + \left(\pm \frac{c}{2p^{1/r}} \right)^n \sum_{k=0}^{\infty} \frac{(\mp 1)^k}{k! (n+k)! (2k+n)^{\varepsilon}} \Gamma\left(\frac{\alpha+n+2k}{r}\right) \times \right. \\ \left. \times \left[\psi(k+1) + \psi(n+k+1) + \frac{2\varepsilon}{2k+n} - \frac{2}{r} \psi\left(\frac{\alpha+n+2k}{r}\right) - 2 \ln \frac{c}{2p^{1/r}} \right] \left(\frac{c}{2p^{1/r}} \right)^{2k} \right\} \\ [r > 1; n = 0, 1, 2, \dots],$$

$$Y_v(\varepsilon) = \mp \frac{2^{\alpha-1}}{c^{\alpha}\pi} \sum_{k=0}^{\infty} \frac{1}{k! (kr+\alpha)^{\varepsilon}} \Gamma\left(\frac{\alpha+v+rk}{2}\right) \times \\ \times \Gamma\left(\frac{\alpha-v+rk}{2}\right) \left\{ \begin{matrix} \cos[(\alpha-v+rk)\pi/2] \\ \pi/2 \end{matrix} \right\} \left(-\frac{2rp}{c^r} \right)^k \quad [r < 1],$$

$Y_v(0)$ при $r=1$ см. в 2.13.6.3, 2.16.6.3.

$$6. \int_0^{\infty} x^{\alpha-1} e^{-px^{-r}} \left\{ \begin{matrix} Y_v(cx) \\ K_v(cx) \end{matrix} \right\} dx = Z_v(0)$$

$$\left[r, \operatorname{Re} p > 0, \left\{ \begin{matrix} \operatorname{Re} \alpha < 3/2; c > 0 \\ \operatorname{Re} c > 0 \end{matrix} \right\} \right],$$

$$Z_v(\varepsilon) = \mp \frac{p^{\alpha/r}}{2r\pi} \left\{ \begin{matrix} 2 \\ \pi \end{matrix} \right\} \sum_{k=0}^{\infty} \frac{(\mp 1)^k}{k!} \left[\hat{E}_k(v) + \left\{ \begin{matrix} \cos v\pi \\ 1 \end{matrix} \right\} \hat{E}_k(-v) \right] \left(\frac{cp^{1/r}}{2} \right)^{2k} \mp \sum_{l=0}^{\infty} \hat{F}_l(v),$$

$$\hat{E}_k(v) = \left(\frac{2}{cp^{1/r}} \right)^v \frac{\Gamma(v)}{(1-v)_k (2k-v)^{\varepsilon}} \Gamma\left(\frac{v-\alpha-2k}{r}\right),$$

$$\hat{F}_l(v) = \frac{2^{\alpha-1}}{\pi c^{\alpha}} \frac{(-1)^{\varepsilon}}{l! (rl-\alpha)^{\varepsilon}} \Gamma\left(\frac{\alpha+v-rl}{2}\right) \Gamma\left(\frac{\alpha-v-rl}{2}\right) \times \\ \times \left\{ \begin{matrix} \cos[(\alpha-v-rl)\pi/2] \\ \pi/2 \end{matrix} \right\} \left(-\frac{c^r p}{2r} \right)^l$$

$$[v \neq \dots, -2, -1, 0, 1, 2, \dots; rl-2k-\alpha \neq v, -v; k, l = 0, 1, 2, \dots],$$

$$Z_n(\varepsilon) = \mp \frac{p^{\alpha/r}}{2r\pi} \left\{ \begin{matrix} 2 \\ \pi \end{matrix} \right\} \left[\sum_{k=0}^{n-1} \frac{(\mp 1)^k}{k!} \hat{E}_k(n) \left(\frac{cp^{1/r}}{2} \right)^{2k} + \sum_{m=0}^{\infty} \hat{G}_m \right] \mp \sum_{l=0}^{\infty} \hat{F}_l(n),$$

$$\hat{G}_m = \left(\pm \frac{cp^{1/r}}{2} \right)^n \frac{(\mp 1)^m}{m! (n+m)! (2k+n)^{\varepsilon}} \Gamma\left(-\frac{\alpha+n+2m}{r}\right) \times$$

$$\times \left[\psi(m+1) + \psi(n+m+1) + \frac{2}{r} \psi\left(-\frac{\alpha+n+2m}{r}\right) + \right.$$

$$\left. + \frac{2\varepsilon}{n+2m} - 2 \ln \frac{cp^{1/r}}{2} \right] \left(\frac{cp^{1/r}}{2} \right)^{2m}$$

$$[rl-2k-\alpha \neq n, -n; k, l, n = 0, 1, 2, \dots]$$

$$7. \int_0^\infty x^{\alpha-1} \begin{Bmatrix} \sin bx^r \\ \cos bx^r \end{Bmatrix} Y_\nu(cx) dx = U_\nu(1, 0)$$

$$[b, c, r > 0; |\operatorname{Re} \nu| - \delta r < \operatorname{Re} \alpha < 1/2 + \max(r, 1), \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}],$$

$$U_\nu(\gamma, \varepsilon) = -\frac{b^{-\alpha/r}}{\pi r} \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma}}{k!} [A_k(\nu) + \cos^\gamma \nu \pi A_k(-\nu)] \left(\frac{c}{2b^{1/r}}\right)^{2k},$$

$$A_k(\nu) = \left(\frac{2b^{1/r}}{c}\right)^\nu \frac{\Gamma(\nu)}{(1-\nu)_k (2k-\nu)^\varepsilon} \Gamma\left(\frac{\alpha+2k-\nu}{r}\right) \begin{Bmatrix} \sin [(\alpha+2k-\nu) \pi/(2r)] \\ \cos [(\alpha+2k-\nu) \pi/(2r)] \end{Bmatrix}$$

[r > 1; \nu \neq \dots, -2, -1, 0, 1, 2, \dots],

$$U_n(\gamma, \varepsilon) = -\frac{b^{-\alpha/r}}{\pi r} \left\{ \sum_{k=0}^{n-1} \frac{(-1)^{k\gamma} A_k(n)}{k!} \left(\frac{c}{2b^{1/r}}\right)^{2k} + \right. \\ \left. + \left(\frac{(-1)^{1-\gamma} c}{2b^{1/r}}\right)^n \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma}}{k! (n+k)! (2k+n)^\varepsilon} \Gamma\left(\frac{\alpha+n+2k}{r}\right) \times \right. \\ \left. \times \begin{Bmatrix} \sin [(\alpha+2k+n) \pi/(2r)] \\ \cos [(\alpha+2k+n) \pi/(2r)] \end{Bmatrix} \left[\psi(k+1) + \psi(n+k+1) + \frac{2\varepsilon}{2k+n} - \right. \right. \\ \left. \left. - \frac{2}{r} \psi\left(\frac{\alpha+n+2k}{r}\right) \mp \frac{\pi}{r} \operatorname{tg}^\mp 1 \frac{\alpha+2k+n}{2r} \pi - 2 \ln \frac{c}{2b^{1/r}} \right] \left(\frac{c}{2b^{1/r}}\right)^{2k} \right\}$$

[r > 1; n = 0, 1, 2, \dots],

$$U_\nu(\gamma, \varepsilon) = -\frac{2\alpha+\delta r-1b^\delta}{\pi c^{\alpha+\delta r}} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (1/2+\delta)_k (\alpha+\delta r+2rk)^\varepsilon} \times \\ \times \Gamma\left(\frac{\alpha+\nu+\delta r}{2} + rk\right) \Gamma\left(\frac{\alpha-\nu+\delta r}{2} + rk\right) \cos^\gamma \left(\frac{\nu-\alpha-\delta r-2rk}{2} \pi\right) \left(\frac{b}{2^{1-r} c^r}\right)^{2k}$$

[r < 1],

$U_\nu(\gamma, 0)$ при $r=1$ см. в 2.13.8.2, 2.16.14.3.

$$8. \int_0^\infty x^{\alpha-1} e^{-px^r} \begin{Bmatrix} \sin bx^r \\ \cos bx^r \end{Bmatrix} Y_\nu(cx) dx = V_\nu(1, 0)$$

$$[b, c, r, \operatorname{Re} p > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu| - r\delta, \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}],$$

$$V_\nu(\gamma, \varepsilon) = -\frac{1}{\pi r} \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma}}{k!} [\tilde{A}_k(\nu) + \cos^\gamma \nu \pi \tilde{A}_k(-\nu)] \frac{c^{2k}}{2^{2k} (b^2 + p^2)^{k/r}},$$

$$\tilde{A}_k(\nu) = \frac{2^\nu (b^2 + p^2)^{(\nu-\alpha)/(2r)} \Gamma(\nu)}{c^\nu (1-\nu)_k (2k-\nu)^\varepsilon} \Gamma\left(\frac{\alpha-\nu+2k}{r}\right) \times \\ \times \cos\left(\frac{\alpha-\nu+2k}{r} \arccos \frac{p}{\sqrt{b^2 + p^2}} - \frac{\delta\pi}{2}\right)$$

[r > 1; |c|^2 < |b^2 + p^2| при r=1; \nu \neq \dots, -2, -1, 0, 1, 2, \dots],

$$V_n(\gamma, \varepsilon) = -\frac{1}{\pi r} \left\{ \sum_{k=0}^{n-1} \frac{(-1)^{k\gamma} \tilde{A}_k(n)}{k!} \frac{c^{2k}}{2^{2k} (b^2 + p^2)^{k/r}} + \right. \\ \left. + (-1)^{(1-\gamma)n} (b^2 + p^2)^{-(\alpha+n)/(2r)} \left(\frac{c}{2}\right)^n \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma}}{k! (n+k)! (2k+n)^\varepsilon} \times \right. \\ \left. \times \Gamma\left(\frac{\alpha+n+2k}{r}\right) \cos\left(\frac{\alpha+n+2k}{r} \arccos \frac{p}{\sqrt{b^2 + p^2}} - \frac{\delta\pi}{2}\right) \right\}$$

$$\times \left[\psi(k+1) + \psi(n+k+1) - \frac{2}{r} \psi\left(\frac{\alpha+n+2k}{r}\right) + \frac{2\varepsilon}{2k+n} + \right. \\ \left. + \frac{2}{r} \arccos \frac{p}{\sqrt{b^2+p^2}} \operatorname{tg} \left(\frac{\alpha+n+2k}{r} \arccos \frac{p}{\sqrt{b^2+p^2}} - \frac{\delta\pi}{2} \right) - \right. \\ \left. - \ln \frac{c^2}{4(b^2+p^2)^{1/r}} \right] \left(\frac{c^2}{4(b^2+p^2)^{1/r}} \right)^k \\ [r > 1; |c|^2 < |b^2+p^2| \text{ при } r=1; n=0, 1, 2, \dots],$$

$$V_v(\gamma, \varepsilon) = -\frac{2^{\alpha-1}}{\pi c^\alpha} \sum_{k=0}^{\infty} \frac{1}{k! (\alpha+rk)^\varepsilon} \Gamma\left(\frac{\alpha+v+rk}{2}\right) \Gamma\left(\frac{\alpha-v+rk}{2}\right) \times \\ \times \cos^\gamma \frac{v-\alpha-rk}{2} \pi \cos\left(k \arccos \frac{p}{\sqrt{b^2+p^2}} + \frac{\delta\pi}{2}\right) \left(-\frac{2r\sqrt{b^2+p^2}}{c^r}\right)^k \\ [r < 1; |c|^2 > |b^2+p^2| \text{ при } r=1].$$

$$9. \int_0^\infty x^{\alpha-1} \left\{ \frac{\ln(x^r+z^r)}{\ln|x^r-z^r|} \right\} Y_v(cx) dx = W_v(1, 0)$$

$$\left[\alpha, r > 0; |\operatorname{Re} v| < \operatorname{Re} \alpha < 3/2; r! - 2k - \alpha \neq v, -v; \right. \\ \left. k, l = 0, 1, 2, \dots, \left\{ \begin{array}{l} r |\arg z| < \pi \\ z > 0 \end{array} \right\} \right],$$

$$W_v(\gamma, \varepsilon) = -z^\alpha \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma}}{k!} [B_k(v) + \cos^\gamma v \pi B_k(-v)] \times \\ \times \left(\frac{cz}{2}\right)^{2k} + \sum_{l=0}^{\infty} C_l(v) - C^*(v)$$

$$B_k(v) = \left(\frac{cz}{2}\right)^{-v} \frac{\Gamma(v)}{(1-v)_k (\alpha-v+2k) (v-2k)^\varepsilon} \left\{ \operatorname{cosec} [(\alpha-v+2k)\pi/r] \right\} \left\{ \operatorname{ctg} [(\alpha-v+2k)\pi/r] \right\},$$

$$C_l(v) = \frac{(\mp 1)^{k+1} 2^{\alpha-r-1} z^r}{(\alpha-r-rk)^\varepsilon \pi c^{\alpha-r} (k+1)} \cos^\gamma \frac{v-\alpha+r+rk}{2} \pi \times \\ \times \Gamma\left(\frac{v+\alpha-r-rk}{2}\right) \Gamma\left(\frac{\alpha-v-r-rk}{2}\right) \left(\frac{cz}{2}\right)^{rk},$$

$$C^*(v) = \frac{2^{\alpha-2r}}{\alpha^\varepsilon \pi c^\alpha} \Gamma\left(\frac{\alpha+v}{2}\right) \Gamma\left(\frac{\alpha-v}{2}\right) \cos^\gamma \frac{v-\alpha}{2} \pi \times \\ \times \left[\psi\left(\frac{\alpha+v}{2}\right) + \psi\left(\frac{\alpha-v}{2}\right) + \gamma \pi \operatorname{tg} \frac{v-\alpha}{2} \pi - \frac{2\varepsilon}{\alpha} - 2 \ln \frac{c}{2} \right] \\ [v \neq \dots, -2, -1, 0, 1, 2, \dots],$$

$$W_n(\gamma, \varepsilon) = -z^\alpha \left\{ \sum_{k=0}^{n-1} \frac{(-1)^{k\gamma} B_k(n)}{k!} \left(\frac{cz}{2}\right)^{2k} + \sum_{m=0}^{\infty} D_m \right\} + \sum_{l=0}^{\infty} C_l(n) - C^*(n),$$

$$D_m = \frac{(-1)^{n+(m+n)\gamma+\varepsilon} (cz/2)^n}{m! (m+n)! (2m+n+\alpha) (2m+n)^\varepsilon} \times \\ \times \left\{ \operatorname{cosec} [(\alpha+2m+n)\pi/r] \right\} \left\{ \operatorname{ctg} [(\alpha+2m+n)\pi/r] \right\} \left[\psi(m+1) + \psi(m+n+1) + \frac{2}{\alpha+2m+n} + \frac{2\varepsilon}{2m+n} + \right. \\ \left. + \frac{2\pi}{r} \left\{ \operatorname{ctg} [(\alpha+2m+n)\pi/r] \right\} - 2 \ln \frac{cz}{2} \right] \left(\frac{cz}{2}\right)^{2m} \\ [n=0, 1, 2, \dots]$$

$$10. \int_0^\infty x^{\alpha-1} J_\mu(bx^r) Y_\nu(cx) dx = \tilde{U}_\nu(1, 0)$$

$$[b, c, r > 0; \operatorname{Re} \nu - r \operatorname{Re} \mu < \operatorname{Re} \alpha < (r+1)/2 + \max(r, 1)],$$

$$\tilde{U}_\nu(\gamma, \varepsilon) = -\frac{2^{\alpha/r-1}}{\pi r b^{\alpha/r}} \sum_{k=0}^{\infty} \frac{(-1)^{\gamma k}}{k!} [E_k(\nu) + \cos^{\gamma} \nu \pi E_k(-\nu)] \left(\frac{2^{1/r-1} c}{b^{1/r}}\right)^{2k},$$

$$E_k(\nu) = \left(\frac{2^{1-1/r} b^{1/r}}{c}\right)^{\nu} \frac{\Gamma(\nu)}{(1-\nu)_k (2k-\nu)^{\varepsilon}} \Gamma\left[\frac{(\mu r + \alpha + 2k - \nu)/(2r)}{1 + (\mu r + \nu - \alpha - 2k)/(2r)}\right]$$

$$[r > 1; \nu \neq \dots, -2, -1, 0, 1, 2, \dots],$$

$$\tilde{U}_n(\gamma, \varepsilon) = -\frac{2^{\alpha/r-1}}{\pi r b^{\alpha/r}} \left\{ \sum_{k=0}^{n-1} \frac{(-1)^{k\gamma} E_k(n)}{k!} \left(\frac{2^{1/r-1} c}{b^{1/r}}\right)^{2k} + \right.$$

$$+ \left(\frac{(-1)^{1-\gamma} c}{2^{1-1/r} b^{1/r}}\right)^n \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma}}{k! (n+k)! (2k+n)^{\varepsilon}} \Gamma\left[\frac{(\mu r + \alpha + 2k + n)/(2r)}{1 + (\mu r - n - \alpha - 2k)/(2r)}\right] \times$$

$$\times \left[\psi(k+1) + \psi(k+n+1) + \frac{2\varepsilon}{2k+n} - \frac{1}{r} \psi\left(\frac{\mu r + \alpha + 2k + n}{2r}\right) - \right.$$

$$\left. - \frac{1}{r} \psi\left(1 + \frac{\mu r - n - \alpha - 2k}{2r}\right) - 2 \ln \frac{2^{1/r-1} c}{b^{1/r}} \right] \left(\frac{2^{1/r-1} c}{b^{1/r}}\right)^{2k} \Big\}$$

$$[r > 1; n = 0, 1, 2, \dots],$$

$$\tilde{U}_\nu(\gamma, \varepsilon) = U_\nu^-(\gamma, \varepsilon),$$

$$U_\nu^\mp(\gamma, \varepsilon) = -\frac{2^{\alpha+\mu r-\mu-1} b^\mu}{\pi c^{\alpha+\mu r} \Gamma(\mu+1)} \sum_{k=0}^{\infty} \frac{(\mp 1)^k}{k! (1+\mu)_k (\alpha + \mu r + 2rk)^{\varepsilon}} \times$$

$$\times \cos^{\gamma} \frac{\alpha - \nu + \mu r + 2rk}{2} \pi \Gamma\left(\frac{\alpha + \nu + \mu r}{2} + rk\right) \Gamma\left(\frac{\alpha - \nu + \mu r}{2} + rk\right) \left(\frac{2^{r-1} b}{c^r}\right)^{2k}$$

$$[r < 1],$$

$$\tilde{U}_\nu(\gamma, 0) \text{ при } r=1 \text{ см. в 2.3.15.4, 2.16.21.1.}$$

$$11. \int_0^\infty x^{\alpha-1} \left\{ \frac{Y_\mu(bx^r)}{K_\mu(bx^r)} \right\} Y_\nu(cx) dx = U_{\mu, \nu}(1, 0)$$

$$\left[c, r > 0; \operatorname{Re} \alpha > r |\operatorname{Re} \mu| + |\operatorname{Re} \nu|, \right. \\ \left. \left\{ b > 0, \operatorname{Re} \alpha < (r+1)/2 + \max(r, 1) \right\} \right], \\ \operatorname{Re} b > 0$$

$$U_{\mu, \nu}(\gamma, \varepsilon) = \pm \frac{2^{\alpha/r-2}}{\pi^2 r b^{\alpha/r}} \left\{ \frac{2}{\pi} \right\} \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma}}{k!} [F_k(\nu) + \cos^{\gamma} \nu \pi F_k(-\nu)] \left(\frac{2^{1/r-1} c}{b^{1/r}}\right)^{2k},$$

$$F_k(\nu) = \left(\frac{2^{1-1/r} b^{1/r}}{c}\right)^{\nu} \frac{\Gamma(\nu)}{(1-\nu)_k (2k-\nu)^{\varepsilon}} \left\{ \cos [(\alpha + 2k - \mu r - \nu) \pi / (2r)] \right\} \times$$

$$\times \Gamma\left(\frac{\alpha + 2k + \mu r - \nu}{2r}\right) \Gamma\left(\frac{\alpha + 2k - \mu r - \nu}{2r}\right)$$

$$[r > 1; \nu \neq \dots, -2, -1, 0, 1, 2, \dots],$$

$$U_{\mu, n}(\gamma, \varepsilon) = \pm \frac{2^{\alpha/r-2}}{\pi^2 r b^{\alpha/r}} \left\{ \frac{2}{\pi} \right\} \left\{ \sum_{k=0}^{n-1} \frac{(-1)^{k\gamma} F_k(n)}{k!} \left(\frac{2^{1/r-1} c}{b^{1/r}}\right)^{2k} + \right.$$

$$+ \left(\frac{(-1)^{1-\gamma} c}{2^{1-1/r} b^{1/r}}\right)^n \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma}}{k! (n+k)! (2k+n)^{\varepsilon}} \Gamma\left(\frac{\alpha + 2k + \mu r + n}{2r}\right) \times$$

$$\times \Gamma \left(\frac{\alpha + 2k - \mu r + n}{2r} \right) \left\{ \cos [(\alpha + 2k - \mu r + n) \pi / (2r)] \right\} \times$$

$$\times \left[\psi(k+1) + \psi(k+n+1) + \frac{2\varepsilon}{2k+n} - \frac{1}{r} \psi \left(\frac{\alpha + 2k + \mu r + n}{2r} \right) - \right.$$

$$\left. - \frac{1}{r} \psi \left(\frac{\alpha + 2k - \mu r + n}{2r} \right) + \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{\pi}{r} \operatorname{tg} \frac{\alpha + 2k - \mu r + n}{2r} \pi - 2 \ln \frac{2^{1/r-1} c}{b^{1/r}} \right] \left(\frac{2^{1/r-1} c}{b^{1/r}} \right)^{2k}$$

[$r > 1$; $n = 0, 1, 2, \dots$],

$$U_{\mu, \nu}(\gamma, \varepsilon) = \pm \frac{2^{\alpha-2}}{\pi^2 c^\alpha} \left\{ \begin{matrix} 2 \\ \pi \end{matrix} \right\} \sum_{k=0}^{\infty} \frac{(\mp 1)^k}{k!} \left[G_k(\mu) + \left\{ \begin{matrix} \cos \mu \pi \\ 1 \end{matrix} \right\} G_k(-\mu) \right] \left(\frac{2^{r-1} b}{c^r} \right)^{2k},$$

$$G_k(\mu) = \left(\frac{c^r}{2^{r-1} b} \right)^\mu \frac{\Gamma(\mu)}{(1-\mu)_k (\alpha - \mu r + 2rk)^\varepsilon} \Gamma \left(\frac{\alpha + \nu - \mu r}{2} + rk \right) \times$$

$$\times \Gamma \left(\frac{\alpha - \nu - \mu r}{2} + rk \right) \cos \nu \frac{\alpha - \nu - \mu r + 2rk}{2} \pi$$

[$r < 1$; $\mu \neq \dots, -2, -1, 0, 1, 2, \dots$],

$$U_{m, \nu}(\gamma, \varepsilon) = \pm \frac{2^{\alpha-2}}{\pi^2 c^\alpha} \left\{ \begin{matrix} 2 \\ \pi \end{matrix} \right\} \left\{ \sum_{k=0}^{m-1} \frac{(\mp 1)^k G_k(m)}{k!} \left(\frac{2^{r-1} b}{c^r} \right)^{2k} + \right.$$

$$+ \left(\pm \frac{2^{r-1} b}{c^r} \right)^m \sum_{k=0}^{\infty} \frac{(\mp 1)^k}{k! (m+k)! (\alpha + mr + 2rk)^\varepsilon} \Gamma \left(\frac{\alpha + \nu + mr}{2} + rk \right) \times$$

$$\times \Gamma \left(\frac{\alpha - \nu + mr}{2} + rk \right) \cos \nu \frac{\alpha - \nu + mr + 2rk}{2} \pi \left[\psi(k+1) + \psi(k+m+1) + \right.$$

$$+ \frac{2\varepsilon r}{2rk + mr + \alpha} - r \psi \left(\frac{\alpha + \nu + mr}{2} + rk \right) - r \psi \left(\frac{\alpha - \nu + mr}{2} + rk \right) +$$

$$\left. + \gamma r \pi \operatorname{tg} \frac{\alpha - \nu + mr + 2rk}{2} \pi + 2 \ln \frac{2^{1-r} c^r}{b} \right] \left(\frac{2^{r-1} b}{c^r} \right)^{2k} \Big\}$$

[$r < 1$; $m = 0, 1, 2, \dots$],

$U_{\mu, \nu}(\gamma, 0)$ при $r=1$ см. в 2.13.20.2, 2.16.27.2, 2.16.33.1.

$$12. \int_0^\infty x^{\alpha-1} \operatorname{Ei}(-bx^r) \left\{ \begin{matrix} Y_\nu(cx) \\ K_\nu(cx) \end{matrix} \right\} dx = \hat{U}_\nu(0)$$

$$\left[r, \operatorname{Re} b > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu|, \left\{ \begin{matrix} c > 0 \\ \operatorname{Re} c > 0 \end{matrix} \right\} \right],$$

$$\hat{U}_\nu(\varepsilon) = \mp \frac{b^{-\alpha/r}}{2\pi} \left\{ \begin{matrix} 2 \\ \pi \end{matrix} \right\} \sum_{k=0}^{\infty} \frac{(\mp 1)^k}{k!} \left[\hat{A}_k(\nu) + \left\{ \begin{matrix} \cos \nu \pi \\ 1 \end{matrix} \right\} \hat{A}_k(-\nu) \right] \left(\frac{c}{2b^{1/r}} \right)^{2k},$$

$$A_k(\nu) = \left(\frac{2b^{1/r}}{c} \right)^\nu \frac{\Gamma(\nu)}{(1-\nu)_k (\nu - \alpha - 2k) (2k - \nu)^\varepsilon} \Gamma \left(\frac{\alpha + 2k - \nu}{r} \right)$$

[$r > 1$; $\nu \neq \dots, -2, -1, 0, 1, 2, \dots$],

$$\hat{U}_n(\varepsilon) = \mp \frac{b^{-\alpha/r}}{2\pi} \left\{ \begin{matrix} 2 \\ \pi \end{matrix} \right\} \left\{ \sum_{k=0}^{n-1} \frac{(\mp 1)^k \hat{A}_k(n)}{k!} \left(\frac{c}{2b^{1/r}} \right)^{2k} - \right.$$

$$- \left(\pm \frac{c}{2b^{1/r}} \right)^n \sum_{k=0}^{\infty} \frac{(\mp 1)^k}{k! (n+k)! (\alpha + n + 2k) (2k + n)^\varepsilon} \Gamma \left(\frac{\alpha + n + 2k}{r} \right) \times$$

$$\times \left[\psi(k+1) + \psi(n+k+1) + \frac{2}{\alpha + n + 2k} + \frac{2\varepsilon}{n + 2k} - \frac{2}{r} \psi \left(\frac{\alpha + n + 2k}{r} \right) - \right.$$

$$\left. - 2 \ln \frac{c}{2b^{1/r}} \right] \left(\frac{c}{2b^{1/r}} \right)^{2k} \Big\}$$

[$r > 1$; $n = 0, 1, 2, \dots$],

$$\begin{aligned} \hat{U}_v(\varepsilon) = & \mp \frac{2^{\alpha-1}}{\pi c^\alpha} \sum_{k=1}^{\infty} \frac{1}{k! k (\alpha + rk)^{\varepsilon}} \Gamma\left(\frac{\alpha + v + rk}{2}\right) \times \\ & \times \Gamma\left(\frac{\alpha - v + rk}{2}\right) \left\{ \frac{\cos[(\alpha - v + rk)\pi/2]}{\pi/2} \right\} \left(-\frac{2rb}{c^r}\right)^k \mp \\ & \mp \frac{2^{\alpha-2}}{\alpha^{\varepsilon} \pi c^\alpha} \Gamma\left(\frac{\alpha + v}{2}\right) \Gamma\left(\frac{\alpha - v}{2}\right) \left\{ \frac{\cos[(\alpha - v)\pi/2]}{\pi/2} \right\} \left[r\psi\left(\frac{\alpha + v}{2}\right) + \right. \\ & \left. + r\psi\left(\frac{\alpha - v}{2}\right) - r\pi \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \operatorname{tg} \frac{\alpha - v}{2} \pi + 2C - \frac{2\varepsilon r}{\alpha} + 2 \ln \frac{2rb}{c^r} \right] \quad [r < 1], \end{aligned}$$

$\hat{U}_v(0)$ при $r=1$ см. в 2.13.28.1, 2.16.57.1.

$$13. \int_0^{\infty} x^{\alpha-1} e^{\pm bx^r} \operatorname{Ei}(\mp bx^r) Y_v(cx) dx = \hat{U}_v(1, 0)$$

$[c, r, \operatorname{Re} b > 0; |\operatorname{Re} v| < \operatorname{Re} \alpha < r + 3/2]$.

$$\hat{U}_v(\gamma, \varepsilon) = -\frac{b^{-\alpha/r}}{r} \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma}}{k!} [\hat{B}_k(v) + \cos \gamma v \pi \hat{B}_k(-v)] \left(\frac{c}{2b^{1/r}}\right)^{2k} + \hat{C}_v(\gamma, \varepsilon),$$

$$\hat{B}_k(v) = \left(\frac{2b^{1/r}}{c}\right)^v \frac{\Gamma(v)}{(1-v)_k (2k-v)^{\varepsilon}} \Gamma\left(\frac{\alpha + 2k - v}{r}\right) \left\{ \frac{\operatorname{cosec}[(v - \alpha - 2k)\pi/r]}{\operatorname{ctg}[(v - \alpha - 2k)\pi/r]} \right\},$$

$$\begin{aligned} \hat{C}_v(\gamma, \varepsilon) = & \pm \frac{2^{\alpha-r-1}}{\pi b c^{\alpha-r}} \sum_{k=0}^{\infty} \frac{k!}{(rk + r - \alpha)^{\varepsilon}} \Gamma\left(\frac{\alpha + v - r - rk}{2}\right) \times \\ & \times \Gamma\left(\frac{\alpha - v - r - rk}{2}\right) \cos \gamma \frac{\alpha - v - r - rk}{2} \pi \left(\mp \frac{c^r}{2rb}\right)^k \end{aligned}$$

$[r > 1; v \neq \dots, -2, -1, 0, 1, 2, \dots; rl - 2k - \alpha + r \neq v, -v; k, l = 0, 1, 2, \dots]$.

$$\begin{aligned} \hat{U}_n(\gamma, \varepsilon) = & -\frac{b^{-\alpha/r}}{r} \left\{ \sum_{k=0}^{n-1} \frac{(-1)^{k\gamma} \hat{B}_k(n)}{k!} \left(\frac{c}{2b^{1/r}}\right)^{2k} - \right. \\ & - \left. \left(\frac{(-1)^{1-\gamma} c}{2b^{1/r}}\right)^n \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma}}{k! (n+k)! (n+2k)^{\varepsilon}} \Gamma\left(\frac{\alpha + n + 2k}{r}\right) \times \right. \\ & \times \left\{ \frac{\operatorname{cosec}[(\alpha + n + 2k)\pi/r]}{\operatorname{ctg}[(\alpha + n + 2k)\pi/r]} \left[\psi(k+1) + \psi(n+k+1) + \frac{2\varepsilon}{n+2k} - \frac{2}{r} \psi\left(\frac{\alpha + n + 2k}{r}\right) + \right. \right. \\ & \left. \left. + \frac{2\pi}{r} \left\{ \operatorname{ctg}[(\alpha + n + 2k)\pi/r] \right\} - 2 \ln \frac{c}{2b^{1/r}} \right] \left(\frac{c}{2b^{1/r}}\right)^{2k} \right\} + \hat{C}_n(\gamma, \varepsilon) \end{aligned}$$

$[r > 1; n = 0, 1, 2, \dots; rl - 2k - \alpha + r \neq n, -n; k, l = 0, 1, 2, \dots]$.

$$\begin{aligned} \hat{U}_v(\gamma, \varepsilon) = & \frac{2^{\alpha-2}}{\pi c^\alpha} \sum_{k=0}^{\infty} \frac{1}{k! (\alpha + rk)^{\varepsilon}} \Gamma\left(\frac{\alpha + v + rk}{2}\right) \Gamma\left(\frac{\alpha - v + rk}{2}\right) \times \\ & \times \cos \gamma \frac{\alpha - v + rk}{2} \pi \left[2\psi(k+1) + \frac{2\varepsilon r}{\alpha + rk} - r\psi\left(\frac{\alpha + v + rk}{2}\right) - r\psi\left(\frac{\alpha - v + rk}{2}\right) + \right. \\ & \left. + \gamma r \pi \operatorname{tg} \frac{\alpha - v + rk}{2} \pi + 2 \ln \frac{c^r}{2rb} \right] \left(\pm \frac{2rb}{c^r}\right)^k \quad [r < 1]. \end{aligned}$$

$\hat{U}_v(\gamma, 0)$ при $r=1$ см. в 2.13.28.6, 2.16.57.4.

$$14. \int_0^{\infty} x^{\alpha-1} \begin{Bmatrix} \operatorname{si}(bx^r) \\ \operatorname{ci}(bx^r) \end{Bmatrix} Y_v(cx) dx = \hat{V}_v(1, 0)$$

$[b, c, r > 0; |\operatorname{Re} v| < \operatorname{Re} \alpha < 1/2 + r + \max(r, 1); \delta = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}]$,

$$\hat{V}_v(\gamma, \varepsilon) = -\frac{b^{-\alpha/r}}{\pi} \sum_{k=0}^{\infty} \frac{(-1)^{kv}}{k!} [\hat{D}_k(v) + \cos v \pi \hat{D}_k(-v)] \left(\frac{c}{2b^{1/r}}\right)^{2k},$$

$$\hat{D}_k(v) = \left(\frac{2b^{1/r}}{c}\right)^v \frac{\Gamma(v)}{(1-v)_k (v-\alpha-2k) (2k-v)^s} \Gamma\left(\frac{\alpha-v+2k}{r}\right) \times \\ \times \begin{cases} \sin [(\alpha+2k-v) \pi/(2r)] \\ \cos [(\alpha+2k-v) \pi/(2r)] \end{cases} \quad [r > 1; v \neq \dots, -2, -1, 0, 1, 2, \dots],$$

$$\hat{V}_n(\gamma, \varepsilon) = -\frac{b^{-\alpha/r}}{\pi} \left\{ \sum_{k=0}^{n-1} \frac{(-1)^{kv}}{k!} \hat{D}_k(n) \left(\frac{c}{2b^{1/r}}\right)^{2k} - \right. \\ \left. - \left(\frac{(-1)^{1-v} c}{2b^{1/r}}\right)^n \sum_{k=0}^{\infty} \frac{(-1)^{kv}}{k! (n+k)! (\alpha+n+2k) (2k+n)^s} \times \right. \\ \times \Gamma\left(\frac{\alpha+n+2k}{r}\right) \begin{cases} \sin [(\alpha+2k+n) \pi/(2r)] \\ \cos [(\alpha+2k+n) \pi/(2r)] \end{cases} \left[\psi(k+1) + \right. \\ \left. + \psi(k+n+1) + \frac{2\varepsilon}{2k+n} + \frac{2}{\alpha+n+2k} - \frac{2}{r} \psi\left(\frac{\alpha+n+2k}{r}\right) \mp \right. \\ \left. \mp \frac{\pi}{r} \operatorname{tg}^{\mp 1} \frac{\alpha+2k+n}{2r} \pi - 2 \ln \frac{c}{2b^{1/r}} \right] \left(\frac{c}{2b^{1/r}}\right)^{2k} \Big\} \\ [r > 1; n = 0, 1, 2, \dots],$$

$$\hat{V}_v(\gamma, \varepsilon) = -\frac{2\alpha+\delta r-1b^0}{\pi c^{\alpha+\delta r}} \sum_{k=1}^{\infty} \frac{(-1)^k}{k! (1/2+\delta)_k (2k+\delta) (\alpha+\delta r+2rk)^s} \times \\ \times \cos v \frac{\alpha-v+\delta r+2rk}{2} \pi \Gamma\left(\frac{\alpha+v+\delta r}{2} + rk\right) \Gamma\left(\frac{\alpha-v+\delta r}{2} + rk\right) \times \\ \times \left(\frac{b}{2^{1-r} c^r}\right)^{2k} - \frac{2\alpha+\delta r-1b^0}{\pi c^{\alpha+\delta r} (\alpha+\delta r)^s} \Gamma\left(\frac{\alpha+v+\delta r}{2}\right) \Gamma\left(\frac{\alpha-v+\delta r}{2}\right) \times \\ \times \cos v \left(\frac{\alpha-v+\delta r}{2} \pi\right) \begin{cases} 1 \\ R \end{cases} + \begin{cases} 1 \\ 0 \end{cases} \frac{2^{\alpha-2}}{c^{\alpha} \alpha^s} \Gamma\left(\frac{\alpha+v}{2}\right) \Gamma\left(\frac{\alpha-v}{2}\right) \cos v \frac{\alpha-v}{2} \pi \\ \left[r < 1; R = c - \frac{rs}{\alpha} + \frac{r}{2} \psi\left(\frac{\alpha+v}{2}\right) + \frac{r}{2} \psi\left(\frac{\alpha-v}{2}\right) - \frac{vr\pi}{2} \operatorname{tg} \frac{\alpha-v}{2} \pi - \ln \frac{c^r}{2^r b} \right],$$

$\hat{V}_v(\gamma, 0)$ при $r=1$ см. в 2.13.29.1, 2.16.58.1.

$$15. \int_0^{\infty} x^{\alpha-1} \begin{cases} \operatorname{erf}(bx^r) \\ \operatorname{erfc}(bx^r) \end{cases} Y_v(cx) dx = \hat{W}_v(1, 0)$$

$$\left[r > 0; |\arg b| < \pi/4; \operatorname{Re} \alpha > |\operatorname{Re} v| - (1 \pm 1)r/2, \left\{ \begin{matrix} \operatorname{Re} \alpha < 3/2; c > 0 \\ c > 0 \end{matrix} \right\} \right],$$

$$\hat{W}_v(\gamma, \varepsilon) = \mp \frac{b^{-\alpha/r}}{c^{3/2}} \sum_{k=0}^{\infty} \frac{(-1)^{kv}}{k!} [\hat{E}_k(v) + \cos v \pi \hat{E}_k(-v)] \left(\frac{c}{2b^{1/r}}\right)^{2k} - \hat{F}_v,$$

$$\hat{E}_k(v) = \left(\frac{2b^{1/r}}{c}\right)^v \frac{\Gamma(v)}{(1-v)_k (v-\alpha-2k) (2k-v)^s} \Gamma\left(\frac{r+\alpha+2k-v}{2r}\right),$$

$$\hat{F}_v = \begin{cases} 1 \\ 0 \end{cases} \frac{2^{\alpha-1} (-\alpha)^{-s}}{\pi c^{\alpha}} \Gamma\left(\frac{\alpha+v}{2}\right) \Gamma\left(\frac{\alpha-v}{2}\right) \cos v \frac{\alpha-v}{2} \pi$$

$$[r > 1/2; v \neq \dots, -2, -1, 0, 1, 2, \dots],$$

$$\hat{W}_n(\gamma, \varepsilon) = \mp \frac{b^{-\alpha/r}}{\pi^{3/2}} \left\{ \sum_{k=0}^{n-1} \frac{(-1)^{k\gamma} \hat{E}_k(n)}{k!} \left(\frac{c}{2b^{1/r}} \right)^{2k} - \right. \\ \left. - \left(\frac{(-1)^{1-\gamma} c}{2b^{1/r}} \right)^n \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma}}{k! (n+k)! (n+\alpha+2k) (2k+n)^{\varepsilon}} \times \right. \\ \left. \times \Gamma \left(\frac{r+\alpha+n+2k}{2r} \right) \left[\psi(k+1) + \psi(k+n+1) + \frac{2\varepsilon}{2k+n} + \frac{2}{\alpha+n+2k} - \right. \right. \\ \left. \left. - \frac{1}{r} \psi \left(\frac{r+\alpha+n+2k}{2r} \right) - 2 \ln \frac{c}{2b^{1/r}} \right] \left(\frac{c}{2b^{1/r}} \right)^{2k} \right\} - \hat{F}_n \\ [r > 1/2; \quad n = 0, 1, 2, \dots].$$

$$\hat{W}_v(\gamma, \varepsilon) = \mp \frac{2^{\alpha+rb}}{\pi^{3/2} c^{\alpha+r}} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (2k+1) (\alpha+r+2rk)^{\varepsilon}} \times \\ \times \Gamma \left(\frac{\alpha+v+r}{2} + rk \right) \Gamma \left(\frac{\alpha-v+r}{2} + rk \right) \cos^{\gamma} \frac{\alpha-v+r+2rk}{2} \pi \left(\frac{2rb}{c^r} \right)^{2k} - \\ - \begin{cases} 0 \\ 1 \end{cases} \frac{2^{\alpha-1}}{\alpha^{\varepsilon} \pi c^{\alpha}} \Gamma \left(\frac{\alpha+v}{2} \right) \Gamma \left(\frac{\alpha-v}{2} \right) \cos^{\gamma} \left(\frac{\alpha-v}{2} \pi \right) \quad [r < 1/2].$$

$\hat{W}_v(\gamma, 0)$ при $r=1/2$ см. в 2.13.30.1, 2.16.59.1.

$$16. \int_0^{\infty} x^{\alpha-1} \begin{Bmatrix} S(bx^r) \\ C(bx^r) \end{Bmatrix} Y_v(cx) dx = X_v(1, 0)$$

$$[b, c, r > 0; \quad |\operatorname{Re} v| - (2 \pm 1) r/2 < \operatorname{Re} \alpha < 3/2; \quad \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}],$$

$$X_v(\gamma, \varepsilon) = \mp \frac{b^{-\alpha/r}}{\sqrt{2\pi^3}} \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma}}{k!} [\hat{G}_k(v) + \cos^{\gamma} v \pi \hat{G}_k(-v)] \left(\frac{c}{2b^{1/r}} \right)^{2k} - \hat{A}_v,$$

$$\hat{G}_k(v) = \left(\frac{2b^{1/r}}{c} \right)^v \frac{\Gamma(v)}{(1-v)_k (v-\alpha-2k) (2k-v)^{\varepsilon}} \Gamma \left(\frac{1}{2} + \frac{\alpha-v+2k}{r} \right) \times \\ \times \begin{cases} \sin [(r+2\alpha-2v+4k) \pi/(4r)] \\ \cos [(r+2\alpha-2v+4k) \pi/(4r)] \end{cases},$$

$$\hat{A}_v = \frac{2^{\alpha-2}}{(-\alpha)^{\varepsilon} \pi c^{\alpha}} \Gamma \left(\frac{\alpha+v}{2} \right) \Gamma \left(\frac{\alpha-v}{2} \right) \cos^{\gamma} \frac{\alpha-v}{2} \pi \\ [r > 1; \quad v \neq \dots, -2, -1, 0, 1, 2 \dots].$$

$$X_n(\gamma, \varepsilon) = - \frac{b^{-\alpha/r}}{\sqrt{2\pi^3}} \left\{ \sum_{k=0}^{n-1} \frac{(-1)^{k\gamma} \hat{G}_k(n)}{k!} \left(\frac{c}{2b^{1/r}} \right)^{2k} - \right. \\ \left. - \left(\frac{(-1)^{1-\gamma} c}{2b^{1/r}} \right)^n \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma}}{k! (n+k)! (\alpha+n+2k) (2k+n)^{\varepsilon}} \times \right. \\ \left. \times \Gamma \left(\frac{1}{2} + \frac{\alpha+n+2k}{r} \right) \begin{cases} \sin [(r+2\alpha+2n+4k) \pi/(4r)] \\ \cos [(r+2\alpha+2n+4k) \pi/(4r)] \end{cases} \times \right. \\ \left. \times \left[\psi(k+1) + \psi(k+n+1) + \frac{2\varepsilon}{2k+n} + \frac{2}{\alpha+n+2k} - \frac{2}{r} \psi \left(\frac{1}{2} + \frac{\alpha+n+2k}{r} \right) \mp \right. \right. \\ \left. \left. \mp \frac{\pi}{r} \operatorname{tg} \mp 1 \frac{r+2\alpha+2n+4k}{4r} \pi - 2 \ln \frac{c}{2b^{1/r}} \right] \left(\frac{c}{2b^{1/r}} \right)^{2k} \right\} - \hat{A}_n \\ [r > 1; \quad n = 0, 1, 2, \dots].$$

$$X_v(\gamma, \varepsilon) = - \frac{2^{\alpha+r\delta+(r-1)/2} b^{1/2+\delta}}{\pi^{3/2} c^{\alpha+r\delta+r/2}} \sum_{k=0}^{\infty} \frac{(-1)^k (\alpha+r\delta+r/2+2rk)^{-\varepsilon}}{k! (1/2+\delta)_k (1+2\delta+4k)} \times \\ \times \cos \gamma \left(\frac{\alpha-v+r\delta}{2} + \frac{r}{4} + rk \right) \pi \Gamma \left(\frac{\alpha+v+r\delta}{2} + \frac{r}{4} + rk \right) \times \\ \times \Gamma \left(\frac{\alpha-v+r\delta}{2} + \frac{r}{4} + rk \right) \left(\frac{b}{2^{1-r} c^r} \right)^{2k} \quad r < 1,$$

$X_v(\gamma, 0)$ при $r=1$ см. в 2.13.31.1, 2.16.60.1.

$$17. \int_0^{\infty} x^{\alpha-1} \left\{ \frac{\gamma(\mu, bx^r)}{\Gamma(\mu, bx^r)} \right\} Y_v(cx) dx = Y_v(1, 0)$$

$$\left[c, r, \operatorname{Re} b > 0; \operatorname{Re}(\alpha + r\mu) > |\operatorname{Re} v|, \left\{ \begin{array}{l} \operatorname{Re} \mu > 0; \operatorname{Re} \alpha < 3/2 \\ \operatorname{Re} \alpha > |\operatorname{Re} v| \end{array} \right\} \right],$$

$$Y_v(\gamma, \varepsilon) = \mp \frac{b^{-\alpha/r}}{\pi} \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma}}{k!} [A_k(v) + \cos \gamma v \pi A_k(-v)] \left(\frac{c}{2b^{1/r}} \right)^{2k} - B_v$$

$$A_k(v) = \left(\frac{2b^{1/r}}{c} \right)^v \frac{\Gamma(v)}{(1-v)_k (v-\alpha-2k) (2k-v)^{\varepsilon}} \Gamma \left(\mu + \frac{\alpha+2k-v}{r} \right),$$

$$B_v = \begin{cases} 1 \\ 0 \end{cases} \frac{2^{\alpha-1}}{(-\alpha)^{\varepsilon} \pi c^{\alpha}} \Gamma \left[\mu, \frac{\alpha+v}{2}, \frac{\alpha-v}{2} \right] \cos \gamma \frac{\alpha-v}{2} \pi$$

$$[r > 1; v \neq \dots, -2, -1, 0, 1, 2, \dots],$$

$$Y_n(\gamma, \varepsilon) = \mp \frac{b^{-\alpha/r}}{\pi} \left\{ \sum_{k=0}^{n-1} \frac{(-1)^{k\gamma} A_k(n)}{k!} \left(\frac{c}{2b^{1/r}} \right)^{2k} - \right. \\ \left. - \left(\frac{(-1)^{1-\gamma} c}{2b^{1/r}} \right)^n \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma}}{k! (n+k)! (\alpha+n+2k) (2k+n)^{\varepsilon}} \times \right. \\ \left. \times \Gamma \left(\mu + \frac{\alpha+n+2k}{r} \right) \left[\psi(k+1) + \psi(k+n+1) + \frac{2\varepsilon}{2k+n} + \frac{2}{\alpha+n+2k} - \right. \right. \\ \left. \left. - \frac{2}{r} \psi \left(\mu + \frac{\alpha+n+2k}{r} \right) - 2 \ln \frac{c}{2b^{1/r}} \right] \left(\frac{c}{2b^{1/r}} \right)^{2k} \right\} - B_n$$

$$[r > 1; n = 0, 1, 2, \dots]$$

$$Y_v(\gamma, \varepsilon) = \mp \frac{2^{\alpha+\mu r-1} b^{\mu}}{\pi c^{\alpha+\mu r}} \sum_{k=0}^{\infty} \frac{(\alpha+\mu r+rk)^{-\varepsilon}}{k! (\mu+k)} \Gamma \left(\frac{\alpha+v+\mu r+kr}{2} \right) \times \\ \times \Gamma \left(\frac{\alpha-v+\mu r+kr}{2} \right) \cos \gamma \frac{\alpha-v+\mu r+kr}{2} \pi \left(-\frac{2rb}{c^r} \right)^k - \\ - \begin{cases} 0 \\ 1 \end{cases} \frac{2^{\alpha-1}}{\alpha^{\varepsilon} \pi c^{\alpha}} \Gamma \left[\mu, \frac{\alpha+v}{2}, \frac{\alpha-v}{2} \right] \cos \gamma \frac{\alpha-v}{2} \pi$$

$Y_v(\gamma, 0)$ при $r=1$ см. в 2.13.32.1, 2.16.61.1

$$18. \int_0^{\infty} x^{\alpha-1} e^{\pm b^2 x^{2r}/4} D_{\mu}(bx^r) Y_v(cx) dx = Z_v(1, 0)$$

$$\left[c, r > 0, \left\{ \begin{array}{l} \operatorname{Re} v, < \operatorname{Re} \alpha < 3/2 - r \operatorname{Re} \mu \\ |\operatorname{Re} v| < \operatorname{Re} \alpha \end{array} \right\}, |\arg b| < (2 \pm 1) \pi/4 \right],$$

$$Z_v(\gamma, \varepsilon) = - \frac{2^{\mp(\mu \pm 1 + 1)/2 - \alpha/(2r)}}{\pi b^{\alpha/r} r} \left\{ \frac{\Gamma^{-1}(-\mu)}{V_{\mu}} \right\} \times \\ \times \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma}}{k!} [C_k(v) + \cos \gamma v \pi C_k(-v)] \left(\frac{c^2}{2^{1/r+2} b^{2/r}} \right)^k - D_v$$

$$C_k(v) = \left(\frac{2^{1+1/(2r)} b^{1/r}}{c} \right)^v \frac{\Gamma(v)}{(1-v)_k (2k-v)^2} \Gamma\left(\frac{\alpha+2k-v}{r}\right) \times \\ \times \Gamma^{\pm 1} \left(\frac{1 \mp 1}{4} \mp \frac{\alpha-v+2k \pm r\mu}{2r} \right),$$

$$D_v = \begin{cases} 1 \\ 0 \end{cases} \frac{2^{\alpha+\mu r-1} b^\mu}{\pi c^{\alpha+\mu r}} \sum_{k=0}^{\infty} \frac{(-\mu)_{2k}}{k! (2rk-\mu r-\alpha)^2} \times \\ \times \cos^v \frac{\alpha-v+\mu r-2rk}{2} \pi \Gamma\left(\frac{\alpha+v+\mu r}{2}-rk\right) \Gamma\left(\frac{\alpha-v+\mu r}{2}-rk\right) \left(\frac{c^2}{2^{1+2r} b^2} \right)^k \\ [r > 1/2; \quad v \neq \dots, -2, -1, 0, 1, 2, \dots]$$

$$Z_n(\gamma, \varepsilon) = - \frac{2^{\mp (\mu \pm 1 + 1)/2 - \alpha/(2r)}}{\pi b^{\alpha/r}} \left\{ \frac{\Gamma^{-1}(-\mu)}{\sqrt{\pi}} \right\} \times \\ \times \left\{ \sum_{k=0}^{n-1} \frac{(-1)^{k\gamma} C_k(n)}{k!} \left(\frac{c^2}{2^{1/r+2} b^{2/r}} \right)^k + \left(\frac{(-1)^{1-\gamma} c}{2^{1/(2r)+1} b^{1/r}} \right)^n \times \right. \\ \times \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma}}{k! (n+k)! (2k+n)^2} \Gamma\left(\frac{\alpha+n+2k}{r}\right) \Gamma^{\pm 1} \left(\frac{1 \mp 1}{4} \mp \frac{\alpha+n+2k \pm r\mu}{2r} \right) \times \\ \times \left[\psi(k+1) + \psi(n+k+1) + \frac{2\varepsilon}{2k+n} - \frac{2}{r} \psi\left(\frac{\alpha+n+2k}{r}\right) + \right. \\ \left. + \frac{1}{r} \psi\left(\frac{1 \mp 1}{4} \mp \frac{\alpha+n+2k \pm r\mu}{2r}\right) - \ln \frac{c^2}{2^{1/r+2} b^{2/r}} \right] \left(\frac{c^2}{2^{1/r+2} b^{2/r}} \right)^k \Big\} - D_n \\ [r > 1/2; \quad n = 0, 1, 2, \dots]$$

$$Z_v(\gamma, \varepsilon) = - \frac{2^{\alpha \mp (\mu \pm 3 + 1)/2}}{\pi c^\alpha} \left\{ \frac{\Gamma^{-1}(-\mu)}{\sqrt{\pi}} \right\} \sum_{k=0}^{\infty} \frac{1}{k! (\alpha+rk)^2} \times \\ \times \Gamma\left(\frac{\alpha+v+rk}{2}\right) \Gamma\left(\frac{\alpha-v+rk}{2}\right) \cos^v \frac{\alpha-v+rk}{2} \pi \Gamma^{\pm 1} \left(\frac{1 \mp 1}{4} - \frac{\mu \mp k}{2} \right) \left(- \frac{2^{1/2+r} b}{c} \right)^k \\ [r < 1/2]$$

$Z_v(\gamma, 0)$ при $r=1/2$ см. в 2.13.33.3, 2.16.62.3.

2.13.2. Интегралы от $x^\alpha (x+z)^\beta Y_v(cx)$.

$$1. \int_0^\infty Y_v(cx) dx = - \frac{1}{c} \operatorname{tg} \frac{v\pi}{2} \quad [c > 0; \quad |\operatorname{Re} v| < 1].$$

$$2. \int_0^\infty x^{\alpha-1} Y_v(cx) dx = 2^{\alpha-1} c^{-\alpha} \operatorname{ctg} \frac{v-\alpha}{2} \pi \Gamma \left[\frac{(\alpha+v)/2}{(v-\alpha)/2+1} \right] \\ [c > 0; \quad |\operatorname{Re} v| < \operatorname{Re} \alpha < 3/2]$$

$$3. \int_0^\infty x^v Y_v(cx) dx = 0 \quad [c > 0; \quad |\operatorname{Re} v| < 1/2]$$

$$4. \int_0^\infty \frac{1}{x+z} Y_v(cx) dx = \frac{\pi}{\sin v\pi} [E_v(cz) + Y_v(cz)] + \operatorname{ctg} v\pi [J_v(cz) - J_v(cz)] \\ [c > 0; \quad |\operatorname{Re} v| < 1; \quad \arg z < \pi]$$

$$5. \int_0^\infty \frac{1}{x+z} Y_0(cx) dx = i I_0(cz) - Y_0(cz) \quad [c > 0; \quad |\arg z| < \pi]$$

$$6. \int_0^{\infty} \frac{x^{\alpha-1}}{x+z} Y_v(cx) dx = -\frac{(2z)^{\alpha-1}}{\pi} \left\{ \cos \frac{v-\alpha}{2} \pi \Gamma\left(\frac{\alpha+v}{2}\right) \times \right. \\ \left. \times \Gamma\left(\frac{\alpha-v}{2}\right) S_{1-\alpha, v}(cz) + 2 \sin \frac{\alpha-v}{2} \pi \Gamma\left(\frac{\alpha+v+1}{2}\right) \Gamma\left(\frac{\alpha-v+1}{2}\right) S_{-\alpha, v}(cz) \right\} \\ [c > 0; |\operatorname{Re} v| < \operatorname{Re} \alpha < 5/2; |\arg z| < \pi]$$

$$7. \int_0^{\infty} \frac{x^{-v}}{x+z} Y_v(cx) dx = \\ = z^{-v} \left\{ \frac{\pi}{2} \operatorname{tg} v\pi [Y_v(cz) - H_v(cz)] - \frac{2^{1-v}}{\pi} \cos v\pi \Gamma(1-v) S_{v-1, v}(cz) \right\} \\ [c > 0; -3/2 < \operatorname{Re} v < 1/2; |\arg z| < \pi]$$

$$8. \int_0^{\infty} \frac{x^{\alpha-1}}{(x+z)^{\rho}} Y_v(cx) dx = -\left(\frac{c}{2}\right)^v \frac{z^{\alpha+v-\rho}}{\pi} \cos v\pi \Gamma\left[\begin{matrix} -v, \rho-\alpha-v, \alpha+v \\ \rho \end{matrix}\right] \times \\ \times {}_2F_3\left(\frac{\alpha+v}{2}, \frac{\alpha+v+1}{2}; v+1, 1+\frac{\alpha+v-\rho}{2}, \frac{1+\alpha+v-\rho}{2}; -\frac{c^2 z^2}{4}\right) - \\ - \left(\frac{2}{c}\right)^v \frac{z^{\alpha-v-\rho}}{\pi} \Gamma\left[\begin{matrix} v, \rho-\alpha+v, \alpha-v \\ \rho \end{matrix}\right] {}_2F_3\left(\frac{\alpha-v}{2}, \frac{\alpha-v+1}{2}; \right. \\ \left. 1-v, 1+\frac{\alpha-\rho-v}{2}, \frac{1+\alpha-\rho-v}{2}; -\frac{c^2 z^2}{4}\right) - \\ - \frac{1}{2\pi} \left(\frac{c}{2}\right)^{\rho-\alpha} \cos \frac{v+\rho-\alpha}{2} \pi \Gamma\left(\frac{\alpha+v-\rho}{2}\right) \Gamma\left(\frac{\alpha-v-\rho}{2}\right) \times \\ \times {}_2F_3\left(\frac{\rho}{2}, \frac{\rho+1}{2}; \frac{1}{2}, 1+\frac{\rho-v-\alpha}{2}, 1+\frac{\rho+v-\alpha}{2}; -\frac{c^2 z^2}{4}\right) - \\ - \left(\frac{c}{2}\right)^{\rho+1-\alpha} \frac{z\rho}{2\pi} \sin \frac{v+\rho-\alpha}{2} \pi \Gamma\left(\frac{v+\alpha-\rho-1}{2}\right) \Gamma\left(\frac{\alpha-v-\rho-1}{2}\right) \times \\ \times {}_2F_3\left(\frac{\rho+1}{2}, \frac{\rho+2}{2}; \frac{3}{2}, \frac{3+\rho-v-\alpha}{2}, \frac{3+\rho+v-\alpha}{2}; -\frac{c^2 z^2}{4}\right) \\ [c > 0; |\operatorname{Re} v| < \operatorname{Re} \alpha < 3/2 + \operatorname{Re} \rho; |\arg z| < \pi]$$

$$9. \int_0^{\infty} \frac{1}{x-y} Y_v(cx) dx = \\ = \pi \operatorname{ctg} v\pi [E_v(cy) + Y_v(cy)] + 2\pi \operatorname{ctg}^2 v\pi [J_v(cy) - J_v(cy)] + \pi J_v(cy) \\ [c, y > 0; |\operatorname{Re} v| < 1]$$

$$10. \int_0^{\infty} \frac{x^{\alpha-1}}{x-y} Y_v(cx) dx = \\ = \pi y^{\alpha-1} J_v(cy) + \frac{(2y)^{\alpha-1}}{\pi} \left[\cos \frac{\alpha-v}{2} \pi \Gamma\left(\frac{\alpha-v}{2}\right) \Gamma\left(\frac{\alpha+v}{2}\right) S_{1-\alpha, v}(cy) - \right. \\ \left. - \sin \frac{\alpha-v}{2} \pi \Gamma\left(\frac{\alpha-v+1}{2}\right) \Gamma\left(\frac{\alpha+v+1}{2}\right) S_{-\alpha, v}(cy) \right] \\ [c, y > 0; |\operatorname{Re} v| < \operatorname{Re} \alpha < 5/2]$$

$$11. \int_0^{\infty} \frac{x^{-v}}{x-y} Y_v(cx) dx = y^{-v} \left\{ \frac{\pi}{2} \operatorname{tg} v\pi [H_v(cy) - Y_v(cy)] + \right. \\ \left. + \pi J_v(cy) - \frac{2^{1-v}}{\pi} \cos v\pi \Gamma(1-v) S_{v-1, v}(cy) \right\} \\ [c, y > 0; -3/2 < \operatorname{Re} v < 1/2]$$

$$12. \int_0^{\infty} \frac{x^v}{x-y} Y_v(cx) dx = y^v \left[\pi J_v(cy) - \frac{2^{v+1}}{\pi} \Gamma(v+1) S_{-v-1, v}(cy) \right]$$

[c, y > 0; -1/2 < Re v < 3/2]

2.13.3. Интегралы от $x^\alpha (z^2 \pm x^2)^\beta Y_v(cx)$.

$$1. \int_0^a \frac{1}{\sqrt{a^2-x^2}} Y_v(cx) dx = \frac{\pi}{2} \left[\operatorname{ctg} v\pi J_{v/2}^2\left(\frac{ac}{2}\right) - \operatorname{cosec} v\pi J_{-v/2}^2\left(\frac{ac}{2}\right) \right]$$

[a > 0; |Re v| < 1]

$$2. \int_0^a x^{\alpha-1} (a^2-x^2)^{\beta-1} Y_v(cx) dx = \frac{a^{\alpha+v+2\beta-2} c^v}{2^{v+1}} \operatorname{ctg} v\pi \times$$

$$\times \Gamma\left[\frac{(\alpha+v)/2, \beta}{1+v, \beta+(\alpha+v)/2}\right] {}_1F_2\left(\frac{\alpha+v}{2}; 1+v, \beta+\frac{\alpha+v}{2}; -\frac{a^2 c^2}{4}\right) -$$

$$- \frac{2^{v-1} a^{\alpha-v+2\beta-2}}{c^v \pi} \Gamma\left[\frac{\beta, v, (\alpha-v)/2}{\beta+(\alpha-v)/2}\right] {}_1F_2\left(\frac{\alpha-v}{2}; 1-v, \beta+\frac{\alpha-v}{2}; -\frac{a^2 c^2}{4}\right)$$

[a, Re β > 0; Re α > |Re v|]

$$3. \int_0^a x^{v+1} (a^2-x^2)^{\beta-1} Y_v(cx) dx =$$

$$= \frac{a^{\beta+v}}{c^\beta} \left[2^{\beta-1} \Gamma(\beta) Y_{v+\beta}(ac) + \frac{2^{v+1}}{\pi} \Gamma(v+1) S_{\beta-v-1, \beta+v}(ac) \right]$$

[a, Re β > 0; Re v > -1]

$$4. \int_0^a x^{1-v} (a^2-x^2)^{\beta-1} Y_v(cx) dx =$$

$$= a^{\beta-v} c^{-\beta} \left[\frac{2^{1-v}}{\pi} \cos v\pi \Gamma(1-v) S_{\beta+v-1, \beta-v}(ac) - \frac{2^{\beta-1} \Gamma(\beta)}{\sin v\pi} J_{\beta-v}(ac) \right]$$

[a, Re β > 0, Re v < 1]

$$5. \int_0^a x^v (a^2-x^2)^{v-1/2} Y_v(cx) dx = 2^{v-1} a^{2v} c^{-v} \sqrt{\pi} \Gamma\left(v+\frac{1}{2}\right) J_v\left(\frac{ac}{2}\right) Y_v\left(\frac{ac}{2}\right)$$

[a > 0; Re v > -1/2]

$$6. \int_0^a \frac{x^{1-v}}{\sqrt{a^2-x^2}} Y_v(cx) dx =$$

$$= a^{1/2-v} \sqrt{\frac{\pi}{2c}} \{ \operatorname{ctg} v\pi [H_{v-1/2}(ac) - Y_{v-1/2}(ac)] - J_{v-1/2}(ac) \}$$

[a > 0; Re v < 1]

$$7. \int_0^a \frac{x^{v+1}}{\sqrt{a^2-x^2}} Y_v(cx) dx = \frac{a^{v+1/2}}{\sin v\pi} \sqrt{\frac{\pi}{2c}} [\cos v\pi J_{v+1/2}(ac) - H_{-v-1/2}(ac)]$$

[a > 0; Re v > -1]

$$8. \int_0^{\infty} \frac{1}{\sqrt{x^2-a^2}} Y_v(cx) dx = \frac{\pi}{4} \left[J_{v/2}^2\left(\frac{ac}{2}\right) - Y_{v/2}^2\left(\frac{ac}{2}\right) \right]$$

[a, c > 0]

$$\begin{aligned}
 9. \int_a^\infty x^{\alpha-1} (x^2 - a^2)^{\beta-1} Y_\nu(cx) dx &= \frac{a^{\alpha+\nu+2\beta-2} c^\nu}{2^{\nu+1}} \operatorname{ctg} \nu\pi \times \\
 &\times \Gamma \left[\begin{matrix} \beta, 1-\beta-(\alpha+\nu)/2 \\ 1+\nu, 1-(\alpha+\nu)/2 \end{matrix} \right] {}_1F_2 \left(\frac{\alpha+\nu}{2}; 1+\nu, \beta+\frac{\alpha+\nu}{2}; -\frac{a^2 c^2}{4} \right) - \\
 &- \left(\frac{2}{c} \right)^\nu \frac{a^{\alpha-\nu+2\beta-2}}{2\pi} \Gamma \left[\begin{matrix} \nu, \beta, 1-\beta+(\nu-\alpha)/2 \\ 1+(\nu-\alpha)/2 \end{matrix} \right] {}_1F_2 \left(\frac{\alpha-\nu}{2}; 1-\nu, \beta+\frac{\alpha-\nu}{2}; -\frac{a^2 c^2}{4} \right) + \\
 &+ \frac{1}{2\pi} \left(\frac{2}{c} \right)^{\alpha+2\beta-2} \cos \left(\frac{\nu-\alpha}{2} - \beta \right) \pi \Gamma \left(\beta-1+\frac{\alpha+\nu}{2} \right) \times \\
 &\times \Gamma \left(\beta-1+\frac{\alpha-\nu}{2} \right) {}_1F_2 \left(1-\beta; 2-\beta-\frac{\alpha+\nu}{2}, 2-\beta+\frac{\nu-\alpha}{2}; -\frac{a^2 c^2}{4} \right) \\
 &[a, c, \operatorname{Re} \beta > 0; \operatorname{Re}(\alpha+2\beta) < 7/2].
 \end{aligned}$$

$$\begin{aligned}
 10. \int_a^\infty x^{\nu+1} (x^2 - a^2)^{\beta-1} Y_\nu(cx) dx &= \\
 &= \frac{2^\beta a^{\beta+\nu}}{c^\beta} \Gamma(\beta) [\sin \beta\pi J_{\nu+\beta}(ac) + \cos \beta\pi Y_{\nu+\beta}(ac)] \\
 &[a, c > 0; 0 < \operatorname{Re} \beta < (3 - 2 \operatorname{Re} \nu)/4].
 \end{aligned}$$

$$\begin{aligned}
 11. \int_a^\infty x^{1-\nu} (x^2 - a^2)^{\beta-1} Y_\nu(cx) dx &= \frac{2^{\beta-1} a^{\beta-\nu}}{c^\beta} \Gamma(\beta) Y_{\nu-\beta}(ac) \\
 &[a, c > 0; 0 < \operatorname{Re} \beta < (2 \operatorname{Re} \nu + 3)/4].
 \end{aligned}$$

$$\begin{aligned}
 12. \int_a^\infty x^{1-\nu} (x^2 - a^2)^{\nu-1/2} Y_\nu(cx) dx &= \frac{2^\nu}{\sqrt{\pi}} \Gamma \left(\nu + \frac{1}{2} \right) c^{-\nu-1} \sin ac \\
 &[a, c > 0; |\operatorname{Re} \nu| < 1/2].
 \end{aligned}$$

$$\begin{aligned}
 13. \int_a^\infty x^\nu (x^2 - a^2)^{\nu-1/2} Y_\nu(cx) dx &= \\
 &= 2^{\nu-2} a^{2\nu} c^{-\nu} \sqrt{\pi} \Gamma \left(\nu + \frac{1}{2} \right) \left[J_\nu \left(\frac{ac}{2} \right) J_{-\nu} \left(\frac{ac}{2} \right) - Y_\nu \left(\frac{ac}{2} \right) Y_{-\nu} \left(\frac{ac}{2} \right) \right] \\
 &[a, c > 0; |\operatorname{Re} \nu| < 1/2].
 \end{aligned}$$

$$\begin{aligned}
 14. \int_a^\infty x^{-\nu} (x^2 - a^2)^{-\nu-1/2} Y_\nu(cx) dx &= \\
 &= 2^{-\nu-2} \sqrt{\pi} a^{-2\nu} c^\nu \Gamma \left(\frac{1}{2} - \nu \right) \left[J_\nu \left(\frac{ac}{2} \right) - Y_\nu \left(\frac{ac}{2} \right) \right] \\
 &[a, c > 0; |\operatorname{Re} \nu| < 1/2].
 \end{aligned}$$

$$\begin{aligned}
 15. \int_a^\infty \frac{x^{\nu+1}}{\sqrt{x^2 - a^2}} Y_\nu(cx) dx &= \sqrt{\frac{\pi}{2c}} a^{\nu+1/2} J_{\nu+1/2}(ac) \\
 &[a, c > 0; \operatorname{Re} \nu < 1/2]
 \end{aligned}$$

$$\begin{aligned}
 16. \int_0^\infty \frac{1}{x^2 + z^2} Y_\nu(cx) dx &= -\sec \frac{\nu\pi}{2} \left[\frac{\pi}{2z} \operatorname{tg} \frac{\nu\pi}{2} I_\nu(cz) + \frac{1}{z} K_\nu(cz) - \right. \\
 &- \left. \frac{c}{1-\nu^2} \sin \frac{\nu\pi}{2} {}_1F_2 \left(1; \frac{3-\nu}{2}, \frac{3+\nu}{2}; \frac{c^2 z^2}{4} \right) \right] \\
 &[c, \operatorname{Re} z > 0; |\operatorname{Re} \nu| < 1]
 \end{aligned}$$

$$17. \int_0^{\infty} \frac{1}{\sqrt{x^2+z^2}} Y_\nu(cx) dx = -\frac{1}{\pi} \sec \frac{\nu\pi}{2} K_{\nu/2}\left(\frac{cz}{2}\right) \left[K_{\nu/2}\left(\frac{cz}{2}\right) + \pi \sin \frac{\nu\pi}{2} I_{\nu/2}\left(\frac{cz}{2}\right) \right] \\ [c, \operatorname{Re} z > 0; |\operatorname{Re} \nu| < 1]$$

$$18. \int_0^{\infty} \frac{x^{\alpha-1}}{x^2+z^2} Y_\nu(cx) dx = \frac{1}{2\pi} \left(\frac{c}{2}\right)^{\alpha-\nu} \cos \frac{\alpha-\nu}{2} \pi \Gamma\left(\frac{\alpha+\nu}{2}-1\right) \times \\ \times \Gamma\left(\frac{\alpha-\nu}{2}-1\right) {}_1F_2\left(1; 2-\frac{\alpha+\nu}{2}, 2+\frac{\nu-\alpha}{2}; \frac{c^2 z^2}{4}\right) - \\ - \frac{\pi z^{\alpha-2}}{2} \operatorname{cosec} \frac{\alpha+\nu}{2} \pi \operatorname{ctg} \frac{\alpha-\nu}{2} \pi I_\nu(cz) - z^{\alpha-2} \operatorname{cosec} \frac{\alpha-\nu}{2} \pi K_\nu(cz) \\ [c, \operatorname{Re} z > 0; |\operatorname{Re} \nu| < \operatorname{Re} \alpha < 7/2]$$

$$19. \int_0^{\infty} \frac{x^{-\nu}}{x^2+z^2} Y_\nu(cx) dx = \frac{z^{-\nu-1}}{2 \cos \nu\pi} \{ \pi \sin \nu\pi [L_\nu(cz) - I_\nu(cz)] - 2K_\nu(z) \} \\ [c, \operatorname{Re} z > 0; -5/2 < \operatorname{Re} \nu < 1/2]$$

$$20. \int_0^{\infty} \frac{x^{\nu+1 \pm 1}}{x^2+z^2} Y_\nu(cx) dx = \pm z^{\nu \pm 1} K_\nu(cx) \\ [c, \operatorname{Re} z > 0; -(2 \pm 1)/2 < \operatorname{Re} \nu < (3 \mp 2)/2]$$

$$21. \int_0^{\infty} \frac{x^{\alpha-1}}{(x^2+z^2)^\rho} Y_\nu(cx) dx = \left(\frac{c}{2}\right)^\nu \frac{z^{\alpha+\nu-2\rho}}{2} \operatorname{ctg} \nu\pi \times \\ \times \Gamma\left[\rho - (\alpha+\nu)/2, (\alpha+\nu)/2\right] {}_1F_2\left(\frac{\alpha+\nu}{2}; \nu+1, 1-\rho + \frac{\alpha+\nu}{2}; \frac{c^2 z^2}{4}\right) - \\ - \left(\frac{2}{c}\right)^\nu \frac{z^{\alpha-\nu-2\rho}}{2 \sin \nu\pi} \Gamma\left[\rho + (\nu-\alpha)/2, (\alpha-\nu)/2\right] {}_1F_2\left(\frac{\alpha-\nu}{2}; 1-\nu, \\ 1-\rho + \frac{\alpha-\nu}{2}; \frac{c^2 z^2}{4}\right) - \left(\frac{c}{2}\right)^{2\rho-\alpha} \frac{1}{2\pi} \cos\left(\frac{\nu-\alpha}{2} + \rho\right) \pi \Gamma\left(\frac{\alpha+\nu}{2}-\rho\right) \times \\ \times \Gamma\left(\frac{\alpha-\nu}{2}-\rho\right) {}_1F_2\left(\rho; 1+\rho - \frac{\nu+\alpha}{2}, 1+\rho + \frac{\nu-\alpha}{2}; \frac{c^2 z^2}{4}\right) \\ [c, \operatorname{Re} z > 0; |\operatorname{Re} \nu| < \operatorname{Re} \alpha < 3/2 + 2 \operatorname{Re} \rho]$$

$$22. \int_0^{\infty} \frac{x^{\nu+1}}{(x^2+z^2)^\rho} Y_\nu(cx) dx = \\ = \frac{2^{\nu-1} c^{-\nu}}{\pi (1-\rho)} \Gamma(\nu) z^{2-2\rho} {}_1F_2\left(1; 1-\nu, 2-\rho; \frac{c^2 z^2}{4}\right) - \\ - \frac{c^{\rho-1} z^{\nu+1-\rho}}{2^\rho \sin \nu\pi} \Gamma(1-\rho) [I_{\nu-\rho+1}(cz) - 2 \cos \rho\pi K_{\nu-\rho+1}(cz)] \\ [c, \operatorname{Re} z > 0; -1 < \operatorname{Re} \nu < 2 \operatorname{Re} \rho - 1/2]$$

$$23. \int_0^{\infty} \frac{x^{1-\nu}}{(x^2+z^2)^\rho} Y_\nu(cx) dx = \\ = \frac{c^{\rho-1} z^{1-\rho-\nu}}{2^\rho \pi} \cos \nu\pi \Gamma(\nu) \Gamma(1-\rho) I_{\nu+\rho-1}(cz) - \left(\frac{c}{2}\right)^{\rho-1} \frac{z^{1-\rho-\nu}}{\sin \nu\pi \Gamma(\rho)} K_{\nu+\rho-1}(cz) - \\ - \frac{c^\nu z^{2-2\rho} \operatorname{ctg} \nu\pi}{2^{\nu+1} (1-\rho) \Gamma(\nu+1)} {}_1F_2\left(1; 1+\nu, 2-\rho; \frac{c^2 z^2}{4}\right) \\ [c, \operatorname{Re} z > 0; 1/2 - 2 \operatorname{Re} \rho < \operatorname{Re} \nu < 1]$$

$$24. \int_0^{\infty} \frac{x}{\sqrt{x^2+z^2}} Y_0(cx) dx = \frac{1}{\pi c} [e^{-cz} \operatorname{Ei}(cz) - e^{cz} \operatorname{Ei}(-cz)] \quad [c, \operatorname{Re} z > 0]$$

$$25. \int_0^{\infty} \frac{1}{x^2-y^2} Y_\nu(cx) dx = \\ = \frac{\pi}{2y} \left\{ J_\nu(cy) + \operatorname{tg} \frac{\nu\pi}{2} [J_\nu(cy) - J_{-\nu}(cy) - E_\nu(cy) - Y_\nu(cy)] \right\} \\ [c, y > 0; |\operatorname{Re} \nu| < 1]$$

$$26. \int_0^{\infty} \frac{x^{\alpha-1}}{x^2-y^2} Y_\nu(cx) dx = \frac{1}{2\pi} \left(\frac{c}{2} \right)^{2-\alpha} \cos \frac{\nu-\alpha}{2} \pi \Gamma \left(\frac{\alpha+\nu}{2} - 1 \right) \times \\ \times \Gamma \left(\frac{\alpha-\nu}{2} - 1 \right) {}_1F_2 \left(1; 2 - \frac{\alpha+\nu}{2}, 2 - \frac{\alpha-\nu}{2}; -\frac{c^2 y^2}{4} \right) - \\ - \frac{\pi}{2} y^{\alpha-2} \operatorname{ctg} \nu\pi \operatorname{ctg} \frac{\alpha+\nu}{2} \pi J_\nu(cy) + \frac{\pi y^{\alpha-2}}{2 \sin \nu\pi} \operatorname{ctg} \frac{\alpha-\nu}{2} \pi J_{-\nu}(cy) \\ [c, y > 0; |\operatorname{Re} \nu| < \operatorname{Re} \alpha < 7/2]$$

$$27. \int_0^{\infty} \frac{x^{\nu+\sigma}}{x^2-y^2} Y_\nu(cx) dx = \frac{\pi y^{\nu+\sigma-1}}{2} J_\nu(cy) - 2^\nu \frac{1-(-1)^\sigma}{\pi} \Gamma(\nu+1) S_{-\nu-1, \nu}(cy) \\ [\sigma = 0 \text{ или } 1; c, y > 0; -(1+\sigma)/2 < \operatorname{Re} \nu < (5-2\sigma)/2]$$

$$28. \int_0^{\infty} \frac{x^{-\nu}}{x^2-y^2} Y_\nu(cx) dx = \frac{\pi y^{-\nu-1}}{2 \cos \nu\pi} [J_{-\nu}(cy) + \sin \nu\pi H_\nu(cy)] \\ [c, y > 0; -5/2 < \operatorname{Re} \nu < 1/2]$$

2.13 4. Интегралы от $A(x) Y_\nu(cx)$.

$$1. \int_a^b x^{1-\nu} [(b^2-x^2)(x^2-a^2)]^{\nu-1/2} Y_\nu(cx) dx = \\ = \frac{2^{\nu-1}}{c^\nu} \sqrt{\pi} (b^2-c^2)^\nu \Gamma \left(\nu + \frac{1}{2} \right) J_\nu \left(\frac{bc+ac}{2} \right) J_\nu \left(\frac{bc-ac}{2} \right) \\ [a, b, c > 0; \operatorname{Re} \nu > -1/2]$$

$$2. \int_0^{\infty} x^{\alpha-1} \frac{(\sqrt{x^2+z^2} \pm z)^\mu}{(x^2+z^2)^r} Y_\nu(cx) dx = \\ = \left(\frac{c}{2} \right)^{1-\alpha-\mu} \frac{(\pm \mu z)^{1-2r}}{2\pi} \sin \frac{\nu-\mu-\alpha}{2} \pi \Gamma \left(\frac{\alpha+\mu-\nu-1}{2} \right) \Gamma \left(\frac{\alpha+\mu+\nu-1}{2} \right) \times \\ \times {}_2F_3 \left(\frac{1+\mu}{2}, \frac{1-\mu}{2}; \frac{3-\mu-\alpha-\nu}{2}, \frac{3-\mu-\alpha+\nu}{2}, \frac{3}{2}-2r; \frac{c^2 z^2}{4} \right) - \\ - \left(\frac{c}{2} \right)^{1-\mu-\alpha} \frac{(\mp \mu z)^{2r}}{2\pi} \cos \frac{\nu-\mu-\alpha}{2} \pi \Gamma \left(\frac{\nu+\mu+\alpha}{2} - 2r \right) \times \\ \times \Gamma \left(\frac{\mu+\alpha-\nu}{2} - 2r \right) {}_2F_3 \left(2r + \frac{\mu}{2}, 2r - \frac{\mu}{2}; 1+2r - \frac{\nu+\mu+\alpha}{2}, \right. \\ \left. 1+2r - \frac{\mu+\alpha-\nu}{2}, \frac{1}{2} + 2r; \frac{c^2 z^2}{4} \right) - 2^{\alpha+\mu-1} (-\mu)^{1-2r} c^\nu \times \\ \times 2^{\alpha+\nu+\mu-2r} \cos \nu\pi \operatorname{cosec} \frac{\mu \mp \mu+\alpha+\nu}{2} \pi \times \\ \times \Gamma \left[-\nu, 2r-\alpha-\mu-\nu; 1-\mu-(\alpha+\nu)/2, 1-(\alpha+\nu)/2 \right] {}_2F_3 \left(\mu + \frac{\alpha+\nu}{2}, \frac{\alpha+\nu}{2}; 1+\nu, \right.$$

$$\frac{1+\mu+\alpha+v}{2}, 1-2r+\frac{\mu+\alpha+v}{2}; \frac{c^2 z^2}{4}) -$$

$$-2^{\alpha+\mu-1} (\mp \mu)^{1-2r} c^{-v} z^{\alpha+\mu-v-2r} \operatorname{cosec} \frac{\mu \mp \mu+\alpha-v}{2} \pi \times$$

$$\times \Gamma \left[\begin{matrix} v, 2r-\mu-\alpha+v \\ 1-\mu+(v-\alpha)/2, 1+(v-\alpha)/2 \end{matrix} \right] {}_2F_3 \left(\mu + \frac{\alpha-v}{2}, \frac{\alpha-v}{2}; \right.$$

$$\left. 1-v, \frac{1+\alpha+\mu-v}{2}, 1-2r+\frac{\alpha+\mu-v}{2}; \frac{c^2 z^2}{4} \right)$$

$[r=0 \text{ или } 1/2; c, \operatorname{Re} z > 0; |\operatorname{Re} v| < (\operatorname{Re} \alpha + \mu \mp \mu); \operatorname{Re}(\alpha + \mu) < 2r+3/2]$

$$3. \int_0^\infty x^{-\mu} \frac{(V x^2 + z^2 \pm z)^\mu}{V x^2 + z^2} Y_v(cx) dx =$$

$$= -\frac{1}{c^2} W_{\pm \mu/2, v/2}(cz) \left\{ \operatorname{tg} \frac{v \pm \mu}{2} \pi \Gamma \left[\frac{(1+v \mp \mu)/2}{v+1} \right] M_{\mp \mu/2, v/2}(cz) + \right.$$

$$\left. + \sec \frac{v \pm \mu}{2} \pi W_{\mp \mu/2, v/2}(cz) \right\} \quad [c, \operatorname{Re} z > 0; |\operatorname{Re} v| < 1 \mp |\operatorname{Re} \mu|].$$

$$4. \int_0^\infty x^{\alpha-1} \frac{(V x^2 + z^2 \pm x)^\mu}{(x^2 + z^2)^r} Y_v(cx) dx = -\frac{2^{\alpha \pm 2\mu - 2r - 1}}{\pi} c^{2r - \alpha \mp \mu} \times$$

$$\times z^{\mu \mp \mu} \cos \frac{v - \alpha \mp \mu + 2r}{2} \pi \Gamma \left(\frac{v \pm \alpha \pm \mu}{2} - r \right) \Gamma \left(\frac{\alpha - v \pm \mu}{2} - r \right) \times$$

$$\times {}_2F_3 \left(r \mp \frac{\mu}{2}, \frac{1 \mp \mu}{2} + r; 1+r - \frac{v + \alpha \pm \mu}{2}, 1+r - \frac{\alpha - v \pm \mu}{2}, \right.$$

$$\left. 1 \mp \mu, \frac{c^2 z^2}{4} \right) - 2^{2r-2v-\alpha-1} (\mp \mu)^{1-2r} c^v z^{\alpha+v+\mu-2r} \times$$

$$\times \cos v\pi \operatorname{cosec} \frac{2r - \alpha - v \mp \mu}{2} \pi \Gamma \left[\begin{matrix} \alpha+v, -v \\ 1-r+(\alpha+v \mp \mu)/2, 1-r+(\alpha+v \pm \mu)/2 \end{matrix} \right] \times$$

$$\times {}_2F_3 \left(\frac{\alpha+v}{2}, \frac{\alpha+v+1}{2}; 1+v, 1-r+\frac{\alpha+v \pm \mu}{2}, 1-r+\frac{\alpha+v \mp \mu}{2}; \frac{c^2 z^2}{4} \right) -$$

$$-2^{2v-\alpha-2r-1} (\mp \mu)^{1-2r} c^{-v} z^{\alpha+\mu-2r-v} \operatorname{cosec} \frac{2r+v-\alpha \mp \mu}{2} \pi \times$$

$$\times \Gamma \left[\begin{matrix} v, \alpha-v \\ 1-r+(\alpha-v \mp \mu)/2, 1-r+(\alpha-v \pm \mu)/2 \end{matrix} \right] {}_2F_3 \left(\frac{\alpha-v}{2}, \right.$$

$$\left. \frac{\alpha+1-v}{2}; 1-v, 1-r+\frac{\alpha-v \pm \mu}{2}, 1-r+\frac{\alpha-v \mp \mu}{2}; \frac{c^2 z^2}{4} \right)$$

$[r=0 \text{ или } 1/2; c, \operatorname{Re} z > 0, |\operatorname{Re} v| < \operatorname{Re} \alpha < 3/2 \mp \operatorname{Re} \mu + 2r]$

$$5. \int_0^\infty \frac{(V x^2 + z^2 \pm x)^\mu}{V x^2 + z^2} Y_v(cx) dx = z^\mu \operatorname{ctg} v\pi / (v \mp \mu)/2 \left(\frac{cz}{2} \right) \times$$

$$\times K_{(-v \mp \mu)/2} \left(\frac{cz}{2} \right) - \frac{z^\mu}{\sin v\pi} I_{(-v \mp \mu)/2} \left(\frac{cz}{2} \right) K_{(v \mp \mu)/2} \left(\frac{cz}{2} \right)$$

$[c, \operatorname{Re} z > 0; |\operatorname{Re} v| < 1; \pm \operatorname{Re} \mu < 3/2]$

$$6. \int_a^\infty \frac{(x + V x^2 - a^2)^\mu + (x - V x^2 - a^2)^\mu}{V x^2 - a^2} Y_v(cx) dx =$$

$$= \frac{\pi}{2} a^\mu \left[J_{(v+\mu)/2} \left(\frac{ac}{2} \right) J_{(v-\mu)/2} \left(\frac{ac}{2} \right) - Y_{(v+\mu)/2} \left(\frac{ac}{2} \right) Y_{(v-\mu)/2} \left(\frac{ac}{2} \right) \right]$$

$[a, c > 0; |\operatorname{Re} \mu| < 3/2].$

2.13.5. Интегралы от $A(x) Y_\nu(\varphi(x))$

$$1. \int_0^\infty x^{\alpha-1} Y_0 \left(c \left| x - \frac{1}{x} \right| \right) dx = \frac{2}{\pi} \cos \frac{\alpha\pi}{2} K_{\alpha/2}^2(c) \quad [c > 0; \operatorname{Re} \alpha < 3/2]$$

$$2. \int_0^\infty x^{\alpha-1} Y_\nu \left(cx + \frac{c}{x} \right) dx = \frac{\pi}{2} [J_{(\nu-\alpha)/2}(c) J_{(\nu+\alpha)/2}(c) - Y_{(\nu-\alpha)/2}(c) Y_{(\nu+\alpha)/2}(c)]$$

$$[c > 0; |\operatorname{Re} \alpha| < 3/2]$$

$$3. \int_0^\infty x^{\alpha-1} \left(\frac{a+bx}{b+ax} \right)^{\nu/2} Y_\nu \left(\sqrt{a^2 + b^2 + abx + \frac{ab}{x}} \right) dx =$$

$$= \pi [J_{\nu/2-\alpha}(a) J_{\nu/2+\alpha}(b) - Y_{\nu/2-\alpha}(a) Y_{\nu/2+\alpha}(b)]$$

$$[a, b > 0; |\operatorname{Re} \alpha| < 3/2]$$

2.13.6. Интегралы от $x^\alpha e^{-px^n} Y_\nu(cx)$

$$1. \int_0^\infty e^{-px} Y_\nu(cx) dx =$$

$$= (p^2 + c^2)^{-1/2} \left[c^\nu \operatorname{ctg} \nu\pi (\sqrt{p^2 + c^2} + p)^{-\nu} - \frac{c^{-\nu}}{\sin \nu\pi} (\sqrt{p^2 + c^2} + p)^\nu \right]$$

$$[\operatorname{Re} p > |\operatorname{Im} c|, |\operatorname{Re} \nu| < 1, \nu \neq 0]$$

$$2. \int_0^\infty e^{-px} Y_0(cx) dx = -\frac{2}{\pi \sqrt{p^2 + c^2}} \ln \frac{p + \sqrt{p^2 + c^2}}{c} \quad [\operatorname{Re} p > |\operatorname{Im} c|]$$

$$3. \int_0^\infty x^{\alpha-1} e^{-px} Y_\nu(cx) dx = I_\nu^\alpha$$

$$[\operatorname{Re} \alpha > |\operatorname{Re} \nu|, \operatorname{Re} p > |\operatorname{Im} c|, k = 2^{-1/2} (\sqrt{p^2 + c^2} + p)^{1/2} / (p^2 + c^2)^{1/4},$$

$$I_\nu^\alpha = -\left(\frac{c}{2}\right)^\nu \frac{\cos \nu\pi}{\pi p^{\alpha+\nu}} \Gamma(\alpha + \nu) \Gamma(-\nu) {}_2F_1\left(\frac{\alpha + \nu}{2}, \frac{\alpha + \nu + 1}{2}; 1 + \nu, -\frac{c^2}{p^2}\right) -$$

$$- \left(\frac{2}{c}\right)^\nu \frac{p^{\nu-\alpha}}{\pi} \Gamma(\nu) \Gamma(\alpha - \nu) {}_2F_1\left(\frac{\alpha - \nu}{2}, \frac{\alpha - \nu + 1}{2}; 1 - \nu, -\frac{c^2}{p^2}\right) =$$

$$= -\frac{2}{\pi} \Gamma(\alpha + \nu) (p^2 + c^2)^{-\alpha/2} Q_{\alpha-1}^{-\nu} \left(\frac{p}{\sqrt{p^2 + c^2}} \right) =$$

$$= -\frac{\sin \alpha\pi}{\pi \sin(\alpha + \nu)\pi} \frac{\Gamma(\alpha - \nu)}{(p^2 + c^2)^{\alpha/2}} \left[e^{\nu\pi i/2} Q_{\alpha-1}^\nu \left(\frac{p}{\sqrt{p^2 + c^2}} + i0 \right) + \right.$$

$$\left. + e^{-\nu\pi i/2} Q_{\alpha-1}^\nu \left(\frac{p}{\sqrt{p^2 + c^2}} - i0 \right) \right] = (p^2 + c^2)^{-\alpha/2} \left[\Gamma(\alpha + \nu) \operatorname{ctg} \nu\pi P_{\alpha-1}^{-\nu} \left(\frac{p}{\sqrt{p^2 + c^2}} \right) - \right.$$

$$\left. - \frac{\Gamma(\alpha - \nu)}{\sin \nu\pi} P_{\alpha-1}^\nu \left(\frac{p}{\sqrt{p^2 + c^2}} \right) \right],$$

$$I_0^2 = \frac{2}{\pi (p^2 + c^2)} \left[1 - \frac{p}{\sqrt{p^2 + c^2}} \ln \frac{p + \sqrt{p^2 + c^2}}{c} \right],$$

$$I_1^2 = -\frac{2}{\pi (p^2 + c^2)} \left[\frac{p}{c} + \frac{c}{\sqrt{p^2 + c^2}} \ln \frac{p + \sqrt{p^2 + c^2}}{c} \right],$$

$$I_0^{1/2} = -2 \sqrt{\frac{2k^2 - 1}{\pi p}} K(k), \quad I_0^{3/2} = \sqrt{\frac{(2k^2 - 1)^3}{\pi p^3}} [2E(k) - K(k)],$$

$$I_0^{5/2} = -\frac{1}{2} \sqrt{\frac{(2k^2 - 1)^5}{\pi p^5}} [8(1 - 2k^2)E(k) - (5 - 8k^2)K(k)],$$

$$I_0^{7/2} = \frac{1}{4} \sqrt{\frac{(2k^2-1)^7}{\pi p^7}} [2(23-128k^2+128k^4) E(k) - (31-144k^2+128k^4) K(k)]$$

$$I_1^{3/2} = -\frac{1}{4} \sqrt{\frac{(2k^2-1)^3}{\pi p^3(1-k^2)}} [(1-k^2) K(k) - (1-2k^2) E(k)],$$

$$I_1^{5/2} = \frac{1}{2k} \sqrt{\frac{(2k^2-1)^5}{\pi p^5(1-k^2)}} [(1-k^2)(1-8k^2) K(k) - (1-16k^2+16k^4) E(k)],$$

$$I_1^{7/2} = -\frac{1}{4k} \sqrt{\frac{(2k^2-1)^7}{\pi p^7(1-k^2)}} [(3-80k^2+128k^4)(1-k^2) K(k) - (3-134k^2+384k^4-256k^6) E(k)],$$

$$I_2^{3/2} = -\frac{1}{2k(1-k^2)} \sqrt{\frac{(2k^2-1)^5}{\pi p^5}} [(2+5k^2-8k^4)(1-k^2) K(k) - 2(1-2k^2)(1+4k^2-4k^4) E(k)],$$

$$I_2^{7/2} = \frac{1}{4k^2(1-k^2)} \sqrt{\frac{(2k^2-1)^7}{\pi p^7}} [(2+15k^2-144k^4+128k^6)(1-k^2) \times \\ \times K(k) - 2(1+7k^2-135k^4+256k^6-128k^8) E(k)]$$

$$4. \int_0^\infty e^{ipx} Y_0(cx) dx = \frac{2i}{\pi \sqrt{c^2-p^2}} \arcsin \frac{p}{c} \quad [0 \leq p < c]$$

$$5. = -\frac{1}{\sqrt{p^2-c^2}} + \frac{2i}{\pi \sqrt{p^2-c^2}} \ln \frac{p - \sqrt{p^2-c^2}}{c} \quad [0 < c < p]$$

$$6. \int_0^\infty x^{\alpha-1} e^{\pm icx} Y_\nu(cx) dx = -\pi^{-3/2} (2c)^{-\alpha} e^{\pm(\alpha+\nu)\pi i/2} \times \\ \times [2 \cos \alpha\pi \cos \nu\pi \mp i \sin(\alpha+\nu)\pi] \Gamma\left[\alpha+\nu, \alpha-\nu, \frac{1}{2}-\alpha\right] \\ [c > 0, \operatorname{Re} \alpha > |\operatorname{Re} \nu|, \operatorname{Re} \alpha < 1/2]$$

$$7. \int_0^\infty e^{-px^2} Y_\nu(cx) dx = \\ = -\frac{1}{2} \sqrt{\frac{\pi}{p}} \sec \frac{\nu\pi}{2} \exp\left(-\frac{c^2}{8p}\right) \left[\sin \frac{\nu\pi}{2} I_{\nu/2}\left(\frac{c^2}{8p}\right) + \frac{1}{\pi} K_{\nu/2}\left(\frac{c^2}{8p}\right) \right] \\ [\operatorname{Re} p > 0, |\operatorname{Re} \nu| < 1]$$

$$8. \int_0^\infty x^{\alpha-1} e^{-px^2} Y_\nu(cx) dx = -\frac{2^{-\nu-1} c^\nu}{\pi p^{(\alpha+\nu)/2}} \cos \nu\pi \Gamma(-\nu) \Gamma\left(\frac{\alpha+\nu}{2}\right) \times \\ \times {}_1F_1\left(\frac{\alpha+\nu}{2}; 1+\nu; -\frac{c^2}{4p}\right) - \frac{2^{\nu-1} c^{-\nu}}{\pi p^{(\alpha-\nu)/2}} i(\nu) \Gamma\left(\frac{\alpha-\nu}{2}\right) {}_1F_1\left(\frac{\alpha-\nu}{2}; 1-\nu; -\frac{c^2}{4p}\right) \\ [\operatorname{Re} p > 0, \operatorname{Re} \alpha > |\operatorname{Re} \nu|]$$

$$9. \int_0^\infty x e^{-px^2} Y_\nu(cx) dx = \frac{c \sqrt{\pi}}{8p^{3/2}} \exp\left(-\frac{c^2}{8p}\right) \left\{ \operatorname{ctg} \nu\pi \left[I_{(\nu-1)/2}\left(\frac{c^2}{8p}\right) - \right. \right. \\ \left. \left. - I_{(\nu+1)/2}\left(\frac{c^2}{8p}\right) \right] - \frac{1}{\sin \nu\pi} \left[I_{-(\nu+1)/2}\left(\frac{c^2}{8p}\right) + I_{(1-\nu)/2}\left(\frac{c^2}{8p}\right) \right] \right\} \\ [\operatorname{Re} p > 0, |\operatorname{Re} \nu| < 2, \nu \neq 0, \pm 1]$$

$$10. \int_0^\infty \lambda e^{-px^2} Y_0(cx) dx = \frac{1}{2\pi p} \exp\left(-\frac{c^2}{4p}\right) E_1\left(\frac{c^2}{4p}\right) \quad [\operatorname{Re} p > 0]$$

2.13.7. Интегралы от $A(x) e^{f(x)} Y_\nu(\varphi(x))$

Обозначения $u_\pm = \sqrt{2b} (\sqrt{p^2 + c^2} \pm p)^{1/2}$, $z_\pm = z (\sqrt{p^2 + c^2} \pm p)$.

$$1. \int_0^\infty \frac{1}{x} e^{-px-b/x} Y_\nu(cx) dx = 2Y_\nu(u_-) K_\nu(u_+)$$

$[\operatorname{Re} p > |\operatorname{Im} c|, \operatorname{Re} b > 0]$ или $[p = 0; c, \operatorname{Re} b > 0]$

$$2. \int_0^\infty \frac{1}{x^2} e^{-px-b/x} Y_0(cx) dx = 2c \left[\frac{1}{u_-} Y_0(u_-) K_1(u_+) + \frac{1}{u_+} Y_1(u_-) K_0(u_+) \right]$$

$[\operatorname{Re} p > |\operatorname{Im} c|; \operatorname{Re} b > 0]$ или $[p = 0, c, \operatorname{Re} b > 0]$.

$$3. \int_0^\infty \frac{\exp(-p\sqrt{x^2+z^2})}{\sqrt{x^2+z^2}} Y_\nu(cx) dx =$$

$$= -\frac{1}{\pi} \sec \frac{\nu\pi}{2} K_{\nu/2}\left(\frac{z_+}{2}\right) \left[K_{\nu/2}\left(\frac{z_-}{2}\right) + \pi \sin \frac{\nu\pi}{2} I_{\nu/2}\left(\frac{z_-}{2}\right) \right]$$

$[\operatorname{Re} p > |\operatorname{Im} c|; \operatorname{Re} z > 0, |\operatorname{Re} \nu| < 1]$

$$4. \int_0^\infty \frac{x}{\sqrt{x^2+z^2}} \exp(-p\sqrt{x^2+z^2}) Y_0(cx) dx =$$

$$= \frac{(p^2+c^2)^{-1/2}}{\pi} [\exp(-z\sqrt{p^2+c^2}) \operatorname{Ei}(z_-) - \exp(z\sqrt{p^2+c^2}) \operatorname{Ei}(-z_+)]$$

$[\operatorname{Re} p > |\operatorname{Im} c|; \operatorname{Re} z > 0]$

$$5. \int_0^\infty x^\mu \frac{(\sqrt{x^2+z^2} \pm z)^\mu}{\sqrt{x^2+z^2}} \exp(-p\sqrt{x^2+z^2}) Y_\nu(cx) dx =$$

$$= -\frac{1}{cz} \sec \frac{(\nu \pm \mu)}{2} \pi W_{\pm \mu/2, \nu/2}(z_+) W_{\mp \mu/2, \nu/2}(z_-) -$$

$$-\frac{1}{cz} \operatorname{tg} \frac{\nu \pm \mu}{2} \pi \Gamma \left[\frac{(1+\nu \mp \mu)/2}{\nu+1} \right] W_{\pm \mu/2, \nu/2}(z_+) M_{\mp \mu/2, \nu/2}(z_-)$$

$[\operatorname{Re} p > |\operatorname{Im} c|, \operatorname{Re} z > 0; |\operatorname{Re} \nu| < 1 \mp \operatorname{Re} \mu]$.

$$6. \int_0^\infty e^{-px} Y_0(c\sqrt{x^2+xz}) dx = \frac{(p^2+c^2)^{-1/2}}{\pi} \left[e^{-z_-/2} \operatorname{Ei}\left(\frac{z_-}{2}\right) - e^{z_+/2} \operatorname{Ei}\left(-\frac{z_+}{2}\right) \right]$$

$[\operatorname{Re} p > |\operatorname{Im} c|; |\arg z| < \pi]$

$$7. \int_0^\infty \frac{e^{-px}}{\sqrt{x^2+xz}} Y_0(c\sqrt{x^2+xz}) dx = -\frac{e^{pz/2}}{\pi} K_0\left(\frac{z_+}{4}\right) K_0\left(\frac{z_-}{4}\right)$$

$[\operatorname{Re} p > |\operatorname{Im} c|; |\arg z| < \pi]$

2.13.8. Интегралы, содержащие тригонометрические функции и $Y_\nu(cx)$

$$1. \int_0^\infty \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} Y_\nu(cx) dx = I_\nu$$

$[b, c > 0; |\operatorname{Re} \nu| < (3 \pm 1)/2]$.

$$I_\nu = \pm (c^2 - b^2)^{-1/2} \operatorname{tg}^{\mp 1} \frac{\nu\pi}{2} \begin{Bmatrix} \sin \varphi \\ \cos \varphi \end{Bmatrix}$$

$[\varphi = \nu \arcsin(b/c), 0 < b < c; \nu \neq 0]$

$$I_v = -\frac{c^{-v}}{2\sqrt{b^2-c^2}} \left\{ \begin{matrix} \operatorname{cosec} (v\pi/2) \\ \sec (v\pi/2) \end{matrix} \right\} [(b + \sqrt{b^2-c^2})^v \mp \cos v\pi (b - \sqrt{b^2-c^2})^v] \\ [0 < c < b; v \neq 0]$$

$$I_0 = \begin{cases} 1 \\ 0 \end{cases} \frac{2}{\pi} (c^2 - b^2)^{-1/2} \arcsin \frac{b}{c} \quad [0 < b < c]$$

$$I_0 = -\begin{cases} A \\ 1 \end{cases} (b^2 - c^2)^{-1/2} \\ \left[0 < c < b; A = \frac{2}{\pi} \ln \frac{b + \sqrt{b^2 - c^2}}{c} \right].$$

$$2. \int_0^\infty x^{\alpha-1} \left\{ \begin{matrix} \sin bx \\ \cos bx \end{matrix} \right\} Y_v(cx) dx = I_v^\alpha$$

$$[b, c > 0; |\operatorname{Re} v| - \delta < \operatorname{Re} \alpha < 3/2, \delta = \begin{cases} 1 \\ 0 \end{cases}],$$

$$I_v^\alpha = -\frac{2^{\alpha+\delta-1} b^0}{c^{\alpha+\delta} \pi} \left\{ \begin{matrix} \sin [(v-\alpha)\pi/2] \\ \cos [(v-\alpha)\pi/2] \end{matrix} \right\} \Gamma\left(\frac{\alpha+v+\delta}{2}\right) \Gamma\left(\frac{\alpha-v+\delta}{2}\right) \times \\ \times {}_2F_1\left(\frac{\alpha+v+\delta}{2}, \frac{\alpha-v+\delta}{2}; \delta + \frac{1}{2}; \frac{b^2}{c^2}\right) \quad [0 < b < c]$$

$$I_v^\alpha = -\frac{c^v \cos v\pi}{2^{v+1} b^{\alpha+v}} \left\{ \begin{matrix} \sec [(\alpha+v)\pi/2] \\ \operatorname{cosec} [(\alpha+v)\pi/2] \end{matrix} \right\} \Gamma\left[\frac{-v}{1-\alpha-v}\right] \times \\ \times {}_2F_1\left(\frac{1+\alpha+v}{2}, \frac{\alpha+v}{2}; 1+v; \frac{c^2}{b^2}\right) - \frac{2^{v-1} b^v}{c^v} \left\{ \begin{matrix} \sec [(\alpha-v)\pi/2] \\ \operatorname{cosec} [(\alpha-v)\pi/2] \end{matrix} \right\} \times \\ \times \Gamma\left[\frac{v}{1+v-\alpha}\right] {}_2F_1\left(\frac{1+\alpha-v}{2}, \frac{\alpha-v}{2}; 1-v; \frac{c^2}{b^2}\right) \quad [0 < c < b]$$

$$I_v^{v+\delta+1} = \pm \frac{2^{v+\delta} b^\delta c^v \sqrt{\pi}}{\Gamma(\mp 1/2 - v)} (b^2 - c^2)^{\mp v - \delta - 1/2}$$

$$[b \neq c; |\operatorname{Re} v| < 1/2 + \delta];$$

$$I_v^{1+\delta-v} = -A \frac{(2b)^\delta}{\sqrt{\pi} (2c)^v} \Gamma\left(\delta + \frac{1}{2} - v\right) c^2 - b^2 v - \delta - 1/2 \\ \left[\begin{cases} A = \mp \sin v\pi; 0 < b < c \\ A = 1 & 0 < c < b \end{cases}, -1/2 < \operatorname{Re} v < \delta + 1/2 \right].$$

$$3. \int_0^\infty \frac{1}{x} \sin bx Y_v(cx) dx = -\frac{1}{v} \operatorname{tg} \frac{v\pi}{2} \sin\left(v \arcsin \frac{b}{c}\right)$$

$$[0 < b < c; |\operatorname{Re} v| < 1]$$

$$4. = \frac{c^{-v}}{2v} \sec \frac{v\pi}{2} [\cos v\pi (b - \sqrt{b^2 - c^2})^v - (b + \sqrt{b^2 - c^2})^v]$$

$$[0 < c < b; |\operatorname{Re} v| < 1]$$

$$5. \int_0^\infty \frac{\sin bx}{x} Y_v(cx) dx = \begin{cases} 0 \\ 1 \end{cases} \ln \frac{b - \sqrt{b^2 - c^2}}{c} \quad \left[\begin{cases} 0 < b < c \\ 0 < c < b \end{cases} \right].$$

$$6. \int_0^\infty x^{\alpha-1} \left\{ \begin{matrix} \sin cx \\ \cos cx \end{matrix} \right\} Y_v(cx) dx =$$

$$= -\frac{2^{\alpha-1}}{\alpha \sqrt{\pi}} \left\{ \begin{matrix} \sin [(v-\alpha)\pi/2] \\ \cos [(v-\alpha)\pi/2] \end{matrix} \right\} \Gamma\left[\frac{(\alpha+v+\delta)/2, (\alpha-v+\delta)/2, 1/2-\alpha}{(1+\delta-v-\alpha)/2, (1+\delta+v-\alpha)/2}\right]$$

$$[c > 0; |\operatorname{Re} v| - \delta < \operatorname{Re} \alpha < 1/2, \delta = \begin{cases} 1 \\ 0 \end{cases}].$$

7. $\int_0^\infty \left\{ \begin{matrix} \sin bx^2 \\ \cos bx^2 \end{matrix} \right\} Y_\nu(cx) dx =$

$$= \frac{1}{4} \sqrt{\frac{\pi}{b}} \sec \frac{\nu\pi}{2} \left[\left\{ \begin{matrix} \sin \varphi \\ \cos \varphi \end{matrix} \right\} Y_{\nu/2} \left(\frac{c^2}{8b} \right) \mp \left\{ \begin{matrix} \cos \psi \\ \sin \psi \end{matrix} \right\} J_{\nu/2} \left(\frac{c^2}{8b} \right) \right]$$

$$\left[\varphi = \frac{c^2}{8b} + \frac{\nu-1}{4} \pi, \psi = \frac{c^2}{8b} - \frac{3\nu+1}{4} \pi; b, c > 0; |\operatorname{Re} \nu| < 2 \pm 1 \right].$$
8. $\int_0^\infty x \cos bx^2 Y_0(cx) dx = \frac{1}{2\pi b} \left\{ \operatorname{ci} \left(\frac{c^2}{4b} \right) \sin \frac{c^2}{4b} - \left[\pi + \operatorname{si} \left(\frac{c^2}{4b} \right) \right] \cos \frac{c^2}{4b} \right\}$

$$[b, c > 0].$$
9. $\int_0^\infty x^{-1/4} \left\{ \begin{matrix} \sin b \sqrt{x} \\ \cos b \sqrt{x} \end{matrix} \right\} Y_{\pm 1/4}(cx) dx = -\frac{1}{c} \sqrt{\frac{\pi b}{2}} H_{\pm 1/4} \left(\frac{b^2}{4c} \right)$

$$[b, c > 0].$$
10. $\int_0^\infty x^{-1/2} \cos b \sqrt{x} \left\{ \begin{matrix} \sin cx \\ \cos cx \end{matrix} \right\} Y_0(cx) dx =$

$$= -\frac{1}{2} \sqrt{\frac{\pi}{2c}} \left[\left\{ \begin{matrix} \cos \varphi \\ \sin \varphi \end{matrix} \right\} Y_0 \left(\frac{b^2}{16c} \right) \pm 3 \left\{ \begin{matrix} \sin \varphi \\ \cos \varphi \end{matrix} \right\} J_0 \left(\frac{b^2}{16c} \right) \right]$$

$$[\varphi = b^2/(16c) - \pi/4; b, c > 0].$$
11. $\int_0^\infty \frac{1}{x} \sin bx \left\{ \begin{matrix} \sin (a/x) \\ \cos (a/x) \end{matrix} \right\} Y_\nu(cx) dx =$

$$= \frac{\pi}{2} Y_\nu(u_+) \left[\left\{ \begin{matrix} \sin (\nu\pi/2) \\ \cos (\nu\pi/2) \end{matrix} \right\} J_\nu(u_-) \pm \left\{ \begin{matrix} \cos (\nu\pi/2) \\ \sin (\nu\pi/2) \end{matrix} \right\} Y_\nu(u_-) \right] \mp$$

$$\mp K_\nu(u_-) \left[\left\{ \begin{matrix} \sin (\nu\pi/2) \\ \cos (\nu\pi/2) \end{matrix} \right\} I_\nu(u_+) + \frac{2}{\pi} \left\{ \begin{matrix} \cos (\nu\pi/2) \\ \sin (\nu\pi/2) \end{matrix} \right\} K_\nu(u_+) \right]$$

$$[a > 0; b > c > 0; u_\pm = \sqrt{a}(\sqrt{b+c} \pm \sqrt{b-c}); |\operatorname{Re} \nu| < 2].$$
12. $\int_0^\infty \frac{1}{x} \cos bx \left\{ \begin{matrix} \sin (a/x) \\ \cos (a/x) \end{matrix} \right\} Y_\nu(cx) dx =$

$$= -\frac{\pi}{2} Y_\nu(u_+) \left[\left\{ \begin{matrix} \cos (\nu\pi/2) \\ \sin (\nu\pi/2) \end{matrix} \right\} J_\nu(u_-) + \left\{ \begin{matrix} \sin (\nu\pi/2) \\ \cos (\nu\pi/2) \end{matrix} \right\} Y_\nu(u_-) \right] -$$

$$- K_\nu(u_-) \left[\left\{ \begin{matrix} \cos (\nu\pi/2) \\ \sin (\nu\pi/2) \end{matrix} \right\} I_\nu(u_+) + \frac{2}{\pi} \left\{ \begin{matrix} \sin (\nu\pi/2) \\ \cos (\nu\pi/2) \end{matrix} \right\} K_\nu(u_+) \right]$$

$$[a > 0; b > c > 0; u_\pm = \sqrt{a}(\sqrt{b+c} \pm \sqrt{b-c}); |\operatorname{Re} \nu| < 1].$$
13. $\int_0^a \frac{x}{\sqrt{a^2-x^2}} \cos (b \sqrt{a^2-x^2}) Y_0(cx) dx =$

$$= \frac{1}{\pi} (b^2+c^2)^{-1/2} \{ \sin \varphi [\operatorname{ci}(z_+) + \operatorname{ci}(z_-)] - \cos \varphi [\operatorname{Si}(z_+) + \operatorname{Si}(z_-)] \}$$

$$[\varphi = a \sqrt{b^2+c^2}, z_\pm = \varphi \pm ab; a > 0].$$
14. $\int_a^\infty \frac{x^{1-\nu}}{\sqrt{x^2-a^2}} \cos (b \sqrt{x^2-a^2}) Y_\nu(cx) dx =$

$$= \pm \left(\frac{\pi}{2} \right)^{\pm 1/2} a^{1/2-\nu} c^{-\nu} |c^2-b^2|^{(2\nu-1)/4} \left\{ \begin{matrix} Y_{\nu-1/2}(a \sqrt{c^2-b^2}) \\ K_{\nu-1/2}(a \sqrt{b^2-c^2}) \end{matrix} \right\}$$

$$\left[\begin{matrix} 0 < b < c \\ 0 < c < b \end{matrix} \right], a > 0; \operatorname{Re} \nu > -1/2].$$

$$15. \int_a^\infty \frac{\cos(b\sqrt{x^2-a^2})}{\sqrt{x^2-a^2}} Y_1(cx) dx = -\frac{1}{ac} \left\{ \begin{array}{l} \cos(a\sqrt{c^2-b^2}) \\ \exp(-a\sqrt{b^2-c^2}) \end{array} \right\} \\ [a > 0, \{0 \leq b \leq c\}].$$

$$16. \int_a^\infty x \frac{\cos(b\sqrt{x^2-a^2})}{\sqrt{x^2-a^2}} Y_0(cx) dx = \pm \frac{1}{\sqrt{|c^2-b^2|}} \left\{ \begin{array}{l} \sin(a\sqrt{c^2-b^2}) \\ \exp(-a\sqrt{b^2-c^2}) \end{array} \right\} \\ [a > 0, \{0 \leq b \leq c\}].$$

$$17. \int_0^\infty \frac{1}{\sqrt{x^2+z^2}} \left\{ \begin{array}{l} \sin(b\sqrt{x^2+z^2}) \\ \cos(b\sqrt{x^2+z^2}) \end{array} \right\} Y_\nu(cx) dx = \\ = \pm \frac{\pi}{4} \sec \frac{\nu\pi}{2} \left[Y_{\nu/2}(u_-) \left\{ \begin{array}{l} J_{\nu/2}(u_+) \\ Y_{\nu/2}(u_+) \end{array} \right\} - \right. \\ \left. - J_{\nu/2}(u_-) \left(\left\{ \begin{array}{l} \cos \nu\pi \\ \sin \nu\pi \end{array} \right\} Y_{\nu/2}(u_+) \mp \left\{ \begin{array}{l} \sin \nu\pi \\ \cos \nu\pi \end{array} \right\} J_{\nu/2}(u_+) \right) \right] \\ [Re z > 0; 0 < c < b; |Re \nu| < 1; 2u_\pm = z(b \pm \sqrt{b^2-c^2})].$$

2.13.9. Интегралы, содержащие $exp x^r$, тригонометрические функции и $Y_\nu(cx)$.

$$1. \int_0^\infty x^{-1/2} e^{-p\sqrt{x}} \sin cx Y_0(cx) dx = \\ = \frac{1}{2} \sqrt{\frac{\pi}{2c}} \left[J_0\left(\frac{p^2}{16c}\right) \sin\left(\frac{p^2}{16c} - \frac{\pi}{4}\right) - Y_0\left(\frac{p^2}{16c}\right) \cos\left(\frac{p^2}{16c} - \frac{\pi}{4}\right) \right] \\ [c, Re p > 0]$$

2.13.10. Интегралы, содержащие $Y_\nu(c \operatorname{sh} x)$ или $Y_\nu(c \operatorname{ch} x)$.

$$1. \int_0^\infty \left\{ \begin{array}{l} \operatorname{sh} bx \\ \operatorname{ch} bx \end{array} \right\} Y_0(c \operatorname{sh} x) dx = -\frac{1}{\pi} \cos \frac{b\pi}{2} K_{b/2}\left(\frac{c}{2}\right) + \\ + \left\{ \begin{array}{l} 1 \\ 0 \end{array} \right\} \frac{2}{\pi} \left[I_{b/2}\left(\frac{c}{2}\right) \frac{\partial}{\partial b} K_{b/2}\left(\frac{c}{2}\right) - K_{b/2}\left(\frac{c}{2}\right) \frac{\partial}{\partial b} I_{b/2}\left(\frac{c}{2}\right) \right] \\ [c > 0; |Re b| < 1/2]$$

$$2. \int_0^\infty \operatorname{ch} bx Y_\nu(c \operatorname{ch} x) dx = \\ = \frac{\pi}{4} \left[J_{(\nu+b)/2}\left(\frac{c}{2}\right) J_{(\nu-b)/2}\left(\frac{c}{2}\right) - Y_{(\nu+b)/2}\left(\frac{c}{2}\right) Y_{(\nu-b)/2}\left(\frac{c}{2}\right) \right] \\ [c > 0, |Re b| < 1/2]$$

$$3. \int_0^\infty \cos bx Y_0(c \operatorname{sh} x) dx = -\frac{1}{\pi} \operatorname{ch} \frac{b\pi}{2} K_{ib/2}\left(\frac{c}{2}\right) \quad [b, c > 0]$$

2.13.11. Интегралы, содержащие тригонометрические функции и $Y_\nu(\varphi(\sin x, \cos x))$

$$1. \int_0^\pi \cos 2nx Y_0(\sqrt{a^2-b^2} \sin^2 x) dx = (-1)^n \pi J_n\left(\frac{a-\sqrt{a^2-b^2}}{2}\right) Y_n\left(\frac{a+\sqrt{a^2-b^2}}{2}\right) \\ [0 < b \leq a]$$

$$2. \int_0^\pi \frac{\cos 2nx}{\sqrt{a^2 - b^2 \sin^2 x}} Y_1(\sqrt{a^2 - b^2 \sin^2 x}) dx =$$

$$= \frac{(-1)^n}{2a \sqrt{a^2 - b^2}} \left[z_+ J_n \left(\frac{z_-}{2} \right) Y_{n+1} \left(\frac{z_+}{2} \right) - z_- J_{n+1} \left(\frac{z_-}{2} \right) Y_n \left(\frac{z_+}{2} \right) \right]$$

$$[z_\pm = a \pm \sqrt{a^2 - b^2}; 0 < b \leq a].$$

$$3. \int_0^{2\pi} \cos nx Y_0(\sqrt{a^2 + b^2 + 2ab \cos x}) dx = (-1)^n 2\pi J_n(a) Y_n(b)$$

$$[0 < a < b]$$

$$4. \int_0^{2\pi} \frac{1}{\sqrt{a^2 + b^2 + 2ab \cos x}} \left\{ \frac{\sin(x/2) \sin[(2n+1)x/2]}{\cos(x/2) \cos[(2n+1)x/2]} \right\} \times$$

$$\times Y_1(\sqrt{a^2 + b^2 + 2ab \cos x}) dx = \frac{(-1)^n \pi}{b \mp a} [J_n(a) Y_{n+1}(b) \mp J_{n+1}(a) Y_n(b)]$$

$$[0 < a < b]$$

$$5. \int_0^{2\pi} \frac{\sin x \sin(n+1)x}{a^2 + b^2 + 2ab \cos x} Y_2(\sqrt{a^2 + b^2 + 2ab \cos x}) dx =$$

$$= \frac{(-1)^n \pi}{b^2 - a^2} [J_n(a) Y_{n+2}(b) - J_{n+2}(a) Y_n(b)]$$

$$[0 < a < b].$$

2.13.12. Интегралы, содержащие $\ln ax Y_\nu(cx)$.

$$1. \int_0^\infty \ln x Y_\nu(cx) dx = \frac{\pi}{2c} - \frac{1}{2c} \operatorname{tg} \frac{\nu\pi}{2} \left[2 \ln \frac{2}{c} + \pi \operatorname{tg} \frac{\nu\pi}{2} + 2\psi \left(\frac{1-\nu}{2} \right) \right]$$

$$[c > 0; |\operatorname{Re} \nu| < 1].$$

$$2. \int_0^\infty x^{\alpha-1} \ln x Y_\nu(cx) dx = \frac{2^{\alpha-2}}{c^\alpha} \Gamma \left(\frac{\alpha+\nu}{2} \right) \Gamma \left(\frac{\alpha-\nu}{2} \right) \times$$

$$\times \left\{ \sin \frac{\alpha-\nu}{2} \pi - \frac{1}{\pi} \cos \frac{\alpha-\nu}{2} \pi \left[\psi \left(\frac{\alpha+\nu}{2} \right) + \psi \left(\frac{\alpha-\nu}{2} \right) - 2 \ln \frac{c}{2} \right] \right\}$$

$$[c > 0, |\operatorname{Re} \nu| < \operatorname{Re} \alpha < 3/2]$$

$$3. \int_0^\infty \frac{\ln x}{x^2 + z^2} Y_0(cx) dx = -\frac{\pi^2}{4z} [L_0(cz) - I_0(cz)] - \frac{1}{z} \ln z K_0(cz)$$

$$[c, \operatorname{Re} z > 0].$$

$$4. \int_0^\infty \cos bx \ln ax Y_0(cx) dx = \frac{\pi}{2} (c^2 - b^2)^{-1/2}$$

$$[0 < b < c; |\arg a| < \pi].$$

$$5. = (b^2 - c^2)^{-1/2} \left[C + \ln(b^2 - c^2) - \ln \frac{ac}{2} \right]$$

$$[0 < c < b; |\arg a| < \pi].$$

2.13.13. Интегралы, содержащие обратные тригонометрические функции и $Y_\nu(cx)$

$$1. \int_0^1 \frac{\cos(b \arccos x)}{\sqrt{1-x^2}} Y_\nu(cx) dx =$$

$$= \frac{\pi}{2} \left[\operatorname{ctg} \nu\pi J_{(\nu+b)/2} \left(\frac{c}{2} \right) J_{(\nu-b)/2} \left(\frac{c}{2} \right) - \frac{1}{\sin \nu\pi} J_{(b-\nu)/2} \left(\frac{c}{2} \right) J_{-(b+\nu)/2} \left(\frac{c}{2} \right) \right]$$

$$[|\operatorname{Re} \nu| < 1]$$

$$2. \int_0^1 \frac{\cos(2n \arcsin x)}{\sqrt{1-x^2}} Y_0(cx) dx = \frac{\pi}{2} J_n\left(\frac{c}{2}\right) Y_n\left(\frac{c}{2}\right).$$

$$3. \int_a^\infty \arccos \frac{a}{x} Y_0(cx) dx = \frac{1}{c} \cos ac \quad [a, c > 0]$$

2.13.14. Интегралы, содержащие $\varphi(x) J_\nu(cx) + \chi(x) Y_\nu(cx)$:

$$1. \int_0^\infty x^{-3/2} e^{-p/x} \left[\begin{Bmatrix} \sin cx \\ \cos cx \end{Bmatrix} J_\nu(cx) \pm \begin{Bmatrix} \cos cx \\ \sin cx \end{Bmatrix} Y_\nu(cx) \right] dx =$$

$$= -\frac{4}{\sqrt{\pi p}} \left[\begin{Bmatrix} \cos(\nu\pi/2) \\ \sin(\nu\pi/2) \end{Bmatrix} \text{ker}_{2\nu}(\sqrt{8pc}) \mp \begin{Bmatrix} \sin(\nu\pi/2) \\ \cos(\nu\pi/2) \end{Bmatrix} \text{kei}_{2\nu}(\sqrt{8pc}) \right]$$

[c, Re p > 0]

$$2. \int_0^\infty \frac{1}{x} e^{-p/x} \left[\begin{Bmatrix} \sin cx \\ \cos cx \end{Bmatrix} J_\nu(cx) \pm \begin{Bmatrix} \cos cx \\ \sin cx \end{Bmatrix} Y_\nu(cx) \right] dx =$$

$$= \frac{4}{\pi} \begin{Bmatrix} \cos(\nu\pi/2) \\ \sin(\nu\pi/2) \end{Bmatrix} (\text{kei}_\nu^2(\sqrt{2pc}) - \text{ker}_\nu^2(\sqrt{2pc})) \pm$$

$$\pm 2 \begin{Bmatrix} \sin(\nu\pi/2) \\ \cos(\nu\pi/2) \end{Bmatrix} \text{ker}_\nu(\sqrt{2pc}) \text{kei}_\nu(\sqrt{2pc}) \quad [c, \text{Re } p > 0]$$

$$3. \int_0^\infty \frac{1}{x} \begin{Bmatrix} \sin(b/x) \\ \cos(b/x) \end{Bmatrix} \left[\begin{Bmatrix} \sin cx \\ \cos cx \end{Bmatrix} J_0(cx) \pm \begin{Bmatrix} \cos cx \\ \sin cx \end{Bmatrix} Y_0(cx) \right] dx = \pi J_0(\sqrt{2bc}) Y_0(\sqrt{2bc})$$

[b, c > 0]

2.13.15. Интегралы, содержащие $J_\mu(bx) Y_\nu(cx)$.

$$1. \int_0^\infty J_{\nu+n}(cx) Y_{\nu-n}(cx) dx = \frac{(-1)^{n+1}}{2c} \quad [c > 0; \text{Re } \nu > -1/2]$$

$$2. \int_0^\infty J_0(bx) Y_0(cx) dx = -\begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \frac{2}{\pi b} K\left(\sqrt{1-\frac{c^2}{b^2}}\right) \quad \left[\begin{Bmatrix} 0 < b < c \\ 0 < c < b \end{Bmatrix}\right].$$

$$3. \int_0^\infty J_1(bx) Y_0(cx) dx = -\frac{1}{\pi b} \ln\left(1 - \frac{b^2}{c^2}\right) \quad [0 < b < c]$$

$$4. \int_0^\infty x^{\alpha-1} J_\mu(bx) Y_\nu(cx) dx = A_{\mu, \nu}^\alpha$$

$$A_{\mu, \nu}^\alpha = -\frac{2^{\alpha-1} b^\mu}{\pi c^{\alpha+\mu}} \cos \frac{\alpha+\mu-\nu}{2} \pi \Gamma\left[\begin{matrix} (\alpha+\mu+\nu)/2, (\alpha+\mu-\nu)/2 \\ \mu+1 \end{matrix}\right] \times$$

$$\times {}_2F_1\left(\frac{\alpha+\mu+\nu}{2}, \frac{\alpha+\mu-\nu}{2}; \mu+1; \frac{b^2}{c^2}\right) \quad [0 < b < c]$$

$$A_{\mu, \nu}^\alpha = -\frac{2^{\alpha-1} c^\nu}{\pi b^{\alpha+\nu}} \cos \nu\pi \Gamma\left[\begin{matrix} -\nu, (\alpha+\mu+\nu)/2 \\ 1-(\alpha+\nu-\mu)/2 \end{matrix}\right] \times$$

$$\times {}_2F_1\left(\frac{\alpha-\mu+\nu}{2}, \frac{\alpha+\mu+\nu}{2}; 1+\nu; \frac{c^2}{b^2}\right) - \frac{2^{\alpha-1} b^{\nu-\alpha}}{\pi c^\nu} \times$$

$$\times \Gamma\left[\begin{matrix} \nu, (\alpha+\mu-\nu)/2 \\ 1+(\nu+\mu-\alpha)/2 \end{matrix}\right] {}_2F_1\left(\frac{\alpha-\mu-\nu}{2}, \frac{\alpha+\mu-\nu}{2}; 1-\nu; \frac{c^2}{b^2}\right) \quad [0 < c < b];$$

$$A_{\nu, \nu}^{\alpha} = -\frac{2^{\alpha-1}}{\pi} \cos \frac{\alpha\pi}{2} \Gamma\left(\frac{\alpha}{2}\right) \Gamma\left(\nu + \frac{\alpha}{2}\right) (c^2 - b^2)^{-\alpha/2} P_{-\alpha/2}^{-\nu} \left(\frac{c^2 + b^2}{c^2 - b^2}\right) \\ [0 < b < c; -2 \operatorname{Re} \nu < \operatorname{Re} \alpha < 2; \operatorname{Re} \alpha > 0],$$

$$A_{\nu, \nu}^{\alpha} = -\frac{2^{\alpha-1}}{\pi} (b^2 - c^2)^{-\alpha/2} \Gamma\left(\frac{\alpha}{2}\right) \Gamma\left(\nu + \frac{\alpha}{2}\right) \times \\ \times \left[\cos \frac{\alpha\pi}{2} P_{-\alpha/2}^{-\nu} \left(\frac{b^2 + c^2}{b^2 - c^2}\right) + \frac{2}{\pi} e^{-i\nu\pi} \sin \frac{2\nu + \alpha}{2} \pi Q_{-\alpha/2}^{\nu} \left(\frac{b^2 + c^2}{b^2 - c^2}\right) \right] \\ [0 < c < b; -2 \operatorname{Re} \nu < \operatorname{Re} \alpha < 2; \operatorname{Re} \alpha > 0];$$

$$A_{-\nu, \nu}^{\alpha} = -\frac{2^{\alpha-1}}{\pi} \cos \frac{\alpha - 2\nu}{2} \pi \Gamma\left(\frac{\alpha}{2}\right) \Gamma\left(\frac{\alpha}{2} - \nu\right) (c^2 - b^2)^{-\alpha/2} P_{-\alpha/2}^{\nu} \left(\frac{c^2 + b^2}{c^2 - b^2}\right) \\ [0 < b < c; 2 \operatorname{Re} \nu < \operatorname{Re} \alpha < 2; \operatorname{Re} \alpha > 0]$$

$$A_{-\nu, \nu}^{\alpha} = -\frac{2^{\alpha-1}}{\pi} \Gamma\left(\frac{\alpha}{2}\right) \Gamma\left(\frac{\alpha}{2} - \nu\right) (b^2 - c^2)^{-\alpha/2} \left[\cos \frac{\alpha - 2\nu}{2} \pi P_{-\alpha/2}^{\nu} \left(\frac{b^2 + c^2}{b^2 - c^2}\right) + \right. \\ \left. + \frac{2}{\pi} e^{i\nu\pi} \cos \nu\pi \sin \frac{\alpha - 2\nu}{2} \pi Q_{-\alpha/2}^{-\nu} \left(\frac{b^2 + c^2}{b^2 - c^2}\right) \right] \\ [0 < c < b; 2 \operatorname{Re} \nu < \operatorname{Re} \alpha < 2; \operatorname{Re} \alpha > 0]$$

$$A_{\mu, \nu}^{\mu + \nu + 2} = \pm \frac{2^{\mu + \nu + 1}}{\pi} \begin{Bmatrix} \cos \mu\pi \\ \cos \nu\pi \end{Bmatrix} \Gamma(\mu + \nu + 1) b^{\mu} c^{\nu} |b^2 - c^2|^{-\mu - \nu - 1} \\ \left[\begin{Bmatrix} 0 < b < c \\ 0 < c < b \end{Bmatrix}, \operatorname{Re} \mu > -1; \operatorname{Re}(\mu + \nu) > -1 \right];$$

$$A_{\mu, \nu}^{\mu - \nu + 2} = \frac{2^{\mu - \nu + 1}}{\pi} \begin{Bmatrix} \cos(\mu - \nu)\pi \\ -1 \end{Bmatrix} b^{\mu} c^{-\nu} \Gamma(\mu - \nu + 1) |b^2 - c^2|^{\nu - \mu - 1} \\ \left[\begin{Bmatrix} 0 < b < c \\ 0 < c < b \end{Bmatrix}, \operatorname{Re} \mu > -1; \operatorname{Re}(\nu - \mu) < 1 \right];$$

$$A_{0,0}^{1/2} = -\frac{1}{\pi^2 \sqrt{c}} \Gamma^2\left(\frac{1}{4}\right) K(k)$$

[здесь и ниже $0 < b < c$].

$$A_{0,0}^{3/2} = \frac{2\Gamma^2(3/4)}{\pi^2 \sqrt{c} (c^2 - b^2)} K(k),$$

$$A_{1,0}^{1/2} = \frac{8\sqrt{c}}{\pi^2 b} \Gamma^2\left(\frac{3}{4}\right) [E(k) - (1 - k^2) K(k)],$$

$$A_{2,0}^{1/2} = -\frac{4c^{3/2}}{9\pi^2 b^2} \Gamma^2\left(\frac{1}{4}\right) [(2 - 3k^2)(1 - k^2) K(k) - 2(1 - 2k^2) E(k)],$$

$$A_{1,1}^{1/2} = -\frac{2\sqrt{c}}{3\pi^2 b} \Gamma^2\left(\frac{1}{4}\right) [(1 - k^2) K(k) - (1 - 2k^2) E(k)],$$

$$A_{1,1}^{3/2} = \frac{4\sqrt{c}}{\pi^2 b \sqrt{c^2 - b^2}} \Gamma^2\left(\frac{3}{4}\right) [(1 - k^2) K(k) - (1 - 2k^2) E(k)],$$

$$A_{2,1}^{1/2} = \frac{16c^{3/2}}{5\pi^2 b^2} \Gamma^2\left(\frac{3}{4}\right) [2(1 - k^2 + k^4) E(k) - (2 - k^2)(1 - k^2) K(k)],$$

$$A_{2,2}^{1/2} = -\frac{4c^{3/2}}{21\pi^2 b^2} \Gamma^2\left(\frac{1}{4}\right) [(2 + 5k^2 - 8k^4)(1 - k^2) K(k) - \\ - 2(1 - 2k^2)(1 + 4k^2 - 4k^4) E(k)]$$

$$5. \int_0^{\infty} x^{\alpha-1} J_{\mu}(cx) Y_{\nu}(cx) dx =$$

$$= -\frac{2^{\alpha-1}}{\pi c^{\alpha}} \cos \frac{\alpha + \mu - \nu}{2} \pi \Gamma\left[\frac{(\alpha + \mu + \nu)/2, (\alpha + \mu - \nu)/2, 1 - \alpha}{1 + (\mu + \nu - \alpha)/2, 1 + (\mu - \nu - \alpha)/2}\right] \\ [c > 0; |\operatorname{Re} \nu| - \operatorname{Re} \mu < \operatorname{Re} \alpha < 1]$$

$$\begin{aligned}
6. \int_0^\infty x^{\alpha-1} [J_\nu(bx) Y_{-\nu}(cx) + J_{-\nu}(bx) Y_\nu(cx)] dx = \\
= -\frac{2^{\alpha-1}}{\sqrt{\pi bc}} \Gamma \left[\frac{\alpha/2 + \nu}{1 - \alpha/2}, \frac{\alpha/2 - \nu}{1 - \alpha/2} \right] |b^2 - c^2|^{(1-\alpha)/2} P_{-(1+\nu)/2}^{(1-\alpha)/2} \left(\frac{b^2 + c^2}{2bc} \right) = \\
= -\frac{2^\alpha}{\pi} |b^2 - c^2|^{-\alpha/2} e^{-i\pi\nu} \Gamma \left[\frac{\alpha/2 - \nu}{1 - \alpha/2} \right] Q_{\alpha/2-1}^\nu \left(\frac{b^2 + c^2}{|b^2 - c^2|} \right) \\
[b, c > 0; 2|\operatorname{Re} \nu| < \operatorname{Re} \alpha < 2]
\end{aligned}$$

$$\begin{aligned}
7. \int_0^\infty \frac{x}{x^2 + z^2} J_{\nu+2n+1}(bx) Y_\nu(cx) dx = (-1)^n I_{\nu+2n+1}(bz) K_\nu(cz) \\
[0 < b \leq c; \operatorname{Re} \nu > -n - 3/2].
\end{aligned}$$

$$\begin{aligned}
8. \int_0^\infty \frac{x^{\alpha-1}}{x^2 + z^2} J_\mu(bx) \left[\cos \frac{\alpha + \mu - \nu}{2} \pi J_\nu(cx) + \sin \frac{\alpha + \mu - \nu}{2} \pi Y_\nu(cx) \right] dx = \\
= -z^{\alpha-2} I_\mu(bz) K_\nu(cz) \quad [0 < b \leq c; |\operatorname{Re} \nu| - \operatorname{Re} \mu < \operatorname{Re} \alpha < 3]
\end{aligned}$$

2.13.16. Интегралы от $A(x) J_\mu(\varphi(x)) Y_\nu(\chi(x))$.

$$\begin{aligned}
1. \int_0^\infty x J_{\nu/2}(bx^2) Y_\nu(cx) dx = \\
= \frac{\sec(\nu\pi/2)}{4b} \left[2 \cos \frac{\nu\pi}{2} Y_{\nu/2} \left(\frac{c^2}{4b} \right) - Y_{-\nu/2} \left(\frac{c^2}{4b} \right) + H_{-\nu/2} \left(\frac{c^2}{4b} \right) \right] \\
[b, c > 0; \operatorname{Re} \nu > -1]
\end{aligned}$$

$$2. \int_0^\infty J_{2\nu}(b\sqrt{x}) Y_\nu(cx) dx = -\frac{1}{c} H_\nu \left(\frac{b^2}{4c} \right) \quad [b, c > 0, \operatorname{Re} \nu > -1/2]$$

$$\begin{aligned}
3. \int_0^\infty x^{1/2} J_{2\nu-1}(b\sqrt{x}) Y_\nu(cx) dx = -\frac{b}{2c^2} H_{\nu-1} \left(\frac{b^2}{4c} \right) \\
[b, c > 0; \operatorname{Re} \nu > -1/2]
\end{aligned}$$

$$\begin{aligned}
4. \int_0^\infty x^{\pm 1-1} J_\nu \left(\frac{b}{x} \right) Y_\nu(cx) dx = \left\{ \begin{matrix} c^{-1} \\ b^{-1} \end{matrix} \right\} \left[Y_{2\nu}(2\sqrt{bc}) \pm \frac{2}{\pi} K_{2\nu}(2\sqrt{bc}) \right] \\
[b, c > 0, -1/2 < \operatorname{Re} \nu < (3 \pm 2)/2]
\end{aligned}$$

$$\begin{aligned}
5. \int_a^\infty x^{1-\nu} (x^2 - a^2)^{\mu/2} J_\mu(b\sqrt{x^2 - a^2}) Y_\nu(cx) dx = \\
= \frac{a^{\mu-\nu+1} b^\mu}{c^\nu} |c^2 - b^2|^{(\nu-\mu-1)/2} \left\{ \begin{matrix} Y_{\nu-\mu-1}(a\sqrt{c^2 - b^2}) \\ - (2/\pi) K_{\nu-\mu-1}(a\sqrt{b^2 - c^2}) \end{matrix} \right\} \\
\left[\left\{ \begin{matrix} 0 < b < c \\ 0 < c < b \end{matrix} \right\}, a > 0, -1 < \operatorname{Re} \mu < \operatorname{Re} \nu \right]
\end{aligned}$$

$$\begin{aligned}
6. \int_a^\infty x^{\nu+1} (x^2 - a^2)^{\mu/2} J_\mu(b\sqrt{x^2 - a^2}) Y_\nu(cx) dx = \\
= -a^{\mu+\nu+1} b^\mu c^\nu (c^2 - b^2)^{-(\mu+\nu+1)/2} \left[\sin \nu\pi J_{\mu+\nu+1}(a\sqrt{c^2 - b^2}) + \right. \\
\left. + \cos \mu\pi Y_{\mu+\nu+1}(a\sqrt{c^2 - b^2}) \right] \quad [0 < b < c; a > 0; \operatorname{Re} \mu > -1, \operatorname{Re}(\mu + \nu) < 0]
\end{aligned}$$

$$\begin{aligned}
7. \quad = -\frac{2}{\pi} a^{\mu+\nu+1} b^\mu c^\nu (b^2 - c^2)^{-(\mu+\nu+1)/2} K_{\mu+\nu+1}(a\sqrt{b^2 - c^2}) \\
[0 < c < b; a > 0; \operatorname{Re} \mu > -1, \operatorname{Re}(\mu + \nu) < 0]
\end{aligned}$$

$$8. \int_0^{\infty} \frac{x^{\nu+2n}}{(x^2 + \xi^2)(x^2 + z^2)^{\mu/2}} J_{\mu}(b\sqrt{x^2 + z^2}) Y_{\nu}(cx) dx =$$

$$= (-1)^{n+1} \xi^{\nu+2n-1} (\pm \xi^2 \mp z^2)^{-\mu/2} \left\{ \begin{matrix} I_{\mu}(b\sqrt{\xi^2 - z^2}) \\ J_{\mu}(b\sqrt{z^2 - \xi^2}) \end{matrix} \right\} K_{\nu}(c\xi)$$

[$b, c, \operatorname{Re} z, \operatorname{Re} \xi > 0; -1/2 - n < \operatorname{Re} \nu < 3 - 2n + \operatorname{Re} \mu; |\arg(\pm \xi^2 \mp z^2)| < \pi$].

$$9. \int_0^{\infty} x^{\alpha-1} J_{\nu}(b\sqrt{x^2 + z^2} \pm bz) Y_{\nu}(b\sqrt{x^2 + z^2} \mp bz) dx =$$

$$= \frac{1}{2\sqrt{\pi}} \left(\frac{z}{b}\right)^{\alpha/2} \Gamma\left(\frac{1-\alpha}{2}\right) \left\{ \Gamma\left[\frac{\nu+\alpha/2}{1+\nu-\alpha/2}\right] Y_{-\alpha/2}(2bz) - \right.$$

$$\left. - \begin{cases} 1 \\ 0 \end{cases} \frac{2 \cos \nu\pi}{\pi} \Gamma\left(\frac{\alpha}{2} - \nu\right) \Gamma\left(\frac{\alpha}{2} + \nu\right) J_{\alpha/2}(2bz) \right\}$$

[$b, \operatorname{Re} z > 0; 2 \begin{cases} |\operatorname{Re} \nu| \\ -\operatorname{Re} \nu \end{cases} < \operatorname{Re} \alpha < 1$].

$$10. \int_0^{\infty} x^{\alpha-1} J_{-\nu}(b\sqrt{x^2 + z^2} - bz) Y_{\nu}(b\sqrt{x^2 + z^2} + bz) dx =$$

$$= -\frac{1}{2\sqrt{\pi}} \left(\frac{z}{b}\right)^{\alpha/2} \Gamma\left[\frac{\alpha/2 - \nu, (1-\alpha)/2}{1 - \nu - \alpha/2}\right] [\sin \nu\pi J_{-\alpha/2}(2bz) - \cos \nu\pi Y_{-\alpha/2}(2bz)]$$

[$b, \operatorname{Re} z > 0; 2 \operatorname{Re} \nu < \operatorname{Re} \alpha < 1$].

$$11. \int_0^{\infty} x^{\alpha-1} J_{-\nu}(b\sqrt{x^2 + z^2} + bz) Y_{\nu}(b\sqrt{x^2 + z^2} - bz) dx =$$

$$= -\frac{1}{2\pi^{3/2}} \left(\frac{z}{b}\right)^{\alpha/2} \Gamma\left[\frac{\alpha}{2} + \nu, \frac{\alpha}{2} - \nu, \frac{1-\alpha}{2}\right] \left[\left(\cos \nu\pi \cos\left(\frac{\alpha}{2} + \nu\right)\pi + \cos \frac{\alpha\pi}{2} \right) \times \right.$$

$$\left. \times J_{-\alpha/2}(2bz) + \cos \nu\pi \sin\left(\frac{\alpha}{2} + \nu\right)\pi Y_{-\alpha/2}(2bz) \right]$$

[$b, \operatorname{Re} z > 0; 2|\operatorname{Re} \nu| < \operatorname{Re} \alpha < 1$].

2.13.17. Интегралы от $\varphi(x) [aJ_{\nu}^2(cx) + bY_{\nu}^2(cx)]^{-1}$.

$$1. \int_0^{\infty} \frac{J_{\nu}(cx) Y_{\nu}(bx) - J_{\nu}(bx) Y_{\nu}(cx)}{x [J_{\nu}^2(cx) + Y_{\nu}^2(cx)]} dx = \frac{\pi}{2} \left(\frac{c}{b}\right)^{\nu}$$

[$0 < c < b$].

2.13.18. Интегралы, содержащие e^{px^n} и $J_{\mu}(bx^r) Y_{\nu}(cx)$.

$$1. \int_0^{\infty} x^n e^{-px} J_0(cx) Y_0(cx) dx = I_n$$

[$\operatorname{Re} p > 2|\operatorname{Im} c|; k = p/\sqrt{p^2 + 4c^2}$],

$$I_0 = -\frac{2k}{\pi p} K(k), \quad I_1 = -\frac{2k}{\pi p^2} [K(k) - E(k)],$$

$$I_2 = -\frac{2k}{\pi p^3} [(1+k^2) K(k) - (1+2k^2) E(k)],$$

$$I_3 = -\frac{2k}{\pi p^4} [2(1-2k^4) K(k) + (2+k^2+8k^4) E(k)].$$

$$2. \int_0^{\infty} e^{-px} [J_{\nu}(cx) Y_{-\nu}(bx) + J_{-\nu}(bx) Y_{\nu}(cx)] dx = -\frac{\sec \nu\pi}{\sqrt{bc}} P_{\nu-1/2}\left(\frac{b^2 + c^2 + p^2}{2bc}\right)$$

[$\operatorname{Re} p > |\operatorname{Im} b| + |\operatorname{Im} c|$].

$$3. \int_0^{\infty} e^{-px} [J_0(cx) Y_0(bx) + J_0(bx) Y_0(cx)] dx =$$

$$= -\frac{4}{\pi \sqrt{p^2 + (b+c)^2}} K \left(\sqrt{\frac{p^2 + (b-c)^2}{p^2 + (b+c)^2}} \right) \quad [\operatorname{Re} p > |\operatorname{Im} b| + |\operatorname{Im} c|].$$

$$4. \int_0^{\infty} x^{2\nu+1} e^{-px^2} J_{\nu}(cx) Y_{\nu}(cx) dx = -\frac{c^{\nu-1}}{2\sqrt{\pi}} p^{-(3\nu+1)/2} \exp\left(-\frac{c^2}{2p}\right) W_{\nu/2, \nu/2}\left(\frac{c^2}{p}\right)$$

$$[c, \operatorname{Re} p > 0; \operatorname{Re} \nu > -1/2].$$

$$5. \int_0^{\infty} x e^{-px^2} [J_0(cx) Y_0(bx) + J_0(bx) Y_0(cx)] dx = -\frac{1}{\pi p} \exp\left(-\frac{b^2+c^2}{4p}\right) K_0\left(\frac{bc}{2p}\right)$$

$$[\operatorname{Re} p > 0].$$

$$6. \int_0^{\infty} e^{-px} [J_0(b\sqrt{x}) Y_0(cx) + 2J_0(cx) Y_0(b\sqrt{x})] dx =$$

$$= (p^2 + c^2)^{-1/2} \exp\left(-\frac{b^2 p}{4p^2 + 4c^2}\right) Y_0\left(\frac{b^2 c}{4p^2 + 4c^2}\right) \quad [\operatorname{Re} p > |\operatorname{Im} c|].$$

2.13.19. Интегралы, содержащие тригонометрические функции и $J_{\mu}(bx^{\pm 1}) Y_{\nu}(cx)$.

$$1. \int_0^{\infty} \sin ax J_0(bx) Y_0(cx) dx =$$

$$= \frac{l}{\pi \sqrt{bc}} \left[F\left(\arcsin \sqrt{\frac{b+c-a}{2b}}, l\right) - F\left(\arcsin \sqrt{\frac{a+b+c}{2b}}, l\right) \right]$$

$$[0 < c < b-a; a > 0; l = \sqrt{4bc}/\sqrt{(b+c)^2 - a^2}].$$

$$= \frac{1}{\pi k \sqrt{bc}} \left[F\left(\arcsin \sqrt{\frac{a+c-b}{a+b+c}}, t\right) - \right.$$

$$\left. - F\left(\arcsin \sqrt{\frac{a+b-c}{a+b+c}}, t\right) - K(t) \right]$$

$$[0 < c < a-b; b > 0; t = \sqrt{a^2 - (b+c)^2}/\sqrt{a^2 - (b-c)^2}, k = \sqrt{a^2 - (b-c)^2}/\sqrt{4bc}].$$

$$3. = \frac{1}{\pi \sqrt{bc}} \left[F\left(\arcsin \sqrt{\frac{2c}{a+b+c}}, \frac{1}{l}\right) - F\left(\arcsin \sqrt{\frac{2b}{a+b+c}}, \frac{1}{l}\right) \right]$$

$$[0 < |a-b| < c < a+b; l \text{ см. выше}].$$

$$4. = \frac{l}{\pi \sqrt{bc}} \left[F\left(\arcsin \sqrt{\frac{a+b+c}{2c}}, l\right) - F\left(\arcsin \sqrt{\frac{b+c-a}{2c}}, l\right) \right]$$

$$[c > a+b; a, b > 0; l \text{ см. выше}].$$

$$5. \int_0^{\infty} \sin ax [J_0(cx) Y_0(bx) + J_0(bx) Y_0(cx)] dx =$$

$$= -\begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \frac{4}{\pi \sqrt{a^2 - (b-c)^2}} K\left(\sqrt{\frac{a^2 - (b+c)^2}{a^2 - (b-c)^2}}\right) \quad \left[\begin{Bmatrix} 0 < a < b+c \\ a > b+c \end{Bmatrix}, b, c > 0\right].$$

$$6. \int_0^{\infty} \cos ax J_n(bx) Y_n(cx) dx = I_n \quad [a, b, c > 0; k = \sqrt{a^2 - (b-c)^2} / \sqrt{4bc}],$$

$$I_0 = - \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{2}{\pi \sqrt{bc}} K(k) \quad \left[\begin{matrix} \{0 < a < b-c\} \\ \{0 < a < c-b\} \end{matrix} \right],$$

$$I_0 = - \frac{k^{-(1 \mp 1)/2}}{\pi \sqrt{bc}} K(k^{\pm 1})$$

$$\left[\begin{matrix} \{|b-c| < a < b+c\} \\ \{a > b+c\} \end{matrix} \right];$$

$$I_1 = \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{2}{\pi \sqrt{bc}} [K(k) - 2E(k)]$$

$$\left[\begin{matrix} \{0 < a < b-c\} \\ \{0 < a < c-b\} \end{matrix} \right],$$

$$I_1 = \frac{1}{\pi \sqrt{bc}} [K(k) - 2E(k)]$$

$$[|b-c| < a < b+c],$$

$$I_1 = \frac{1}{\pi k \sqrt{bc}} \left[(2k^2 - 1) K\left(\frac{1}{k}\right) - 2k^2 E\left(\frac{1}{k}\right) \right]$$

$$\{a > b+c\};$$

$$I_2 = \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{2}{3\pi \sqrt{bc}} [8(2k^2 - 1)E(k) + (8k^2 - 5)K(k)]$$

$$\left[\begin{matrix} \{0 < a < b-c\} \\ \{0 < a < c-b\} \end{matrix} \right],$$

$$I_2 = \frac{1}{3\pi \sqrt{bc}} [(5 - 8k^2)K(k) - 8(1 - 2k^2)E(k)]$$

$$[|b-c| < a < b+c],$$

$$I_2 = - \frac{1}{3\pi k \sqrt{bc}} \left[(3 - 16k^2 + 16k^4) K\left(\frac{1}{k}\right) + 8k^2(1 - 2k^2) E\left(\frac{1}{k}\right) \right]$$

$$\{a > b+c\};$$

$$I_3 = \left\{ \begin{matrix} 2 \\ 1 \end{matrix} \right\} \frac{1}{15\pi \sqrt{bc}} [(31 - 144k^2 + 128k^4)K(k) - 2(23 - 128k^2 + 128k^4)E(k)]$$

$$\left[\begin{matrix} \{0 < a < b-c\} \\ \{|b-c| < a < b+c\} \\ \{0 < a < c-b\} \end{matrix} \right],$$

$$I_3 = 0$$

$$I_3 = \frac{1}{15\pi k \sqrt{bc}} \left[(15 - 128k^2 + 128k^4)(2k^2 - 1) K\left(\frac{1}{k}\right) - \right.$$

$$\left. - 2k^2(23 - 128k^2 + 128k^4) E\left(\frac{1}{k}\right) \right]$$

$$\{a > b+c\}$$

$$7. \int_0^{\infty} \sin ax [J_1(cx) Y_0(cx) + J_0(cx) Y_1(cx)] dx = - \frac{a}{\pi c^2} K\left(\frac{a}{2c}\right)$$

$$[0 < a < 2c]$$

$$8. \int_0^{\infty} x^{\alpha-1} \left\{ \begin{matrix} \sin ax \\ \cos ax \end{matrix} \right\} J_\nu(cx) Y_\nu(cx) dx = I$$

$$[a, c > 0; -2 \operatorname{Re} \nu - \delta, -\delta < \operatorname{Re} \alpha < 2; \delta = \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\}],$$

$$I = - \frac{a^\delta c^{-\alpha-\delta}}{2 \sqrt{\pi}} \Gamma \left[\begin{matrix} (\alpha + \delta)/2, \nu + (\alpha + \delta)/2 \\ 1 + \nu - (\alpha + \delta)/2, (1 + \alpha + \delta)/2 \end{matrix} \right] \times$$

$$\times {}_2F_2 \left(\frac{\alpha + \delta}{2} - \nu, \frac{\alpha + \delta}{2}, \frac{\alpha + \delta}{2} + \nu, \frac{1 + \alpha + \delta}{2}, \frac{1}{2} + \delta; \frac{a^2}{4c^2} \right)$$

$$[0 < a < 2c],$$

$$I = -\frac{\Gamma(\alpha)}{\pi v a^\alpha} \left\{ \frac{\sin(\alpha\pi/2)}{\cos(\alpha\pi/2)} \right\} {}_3F_2 \left(\frac{1}{2}, \frac{1+\alpha}{2}, \frac{\alpha}{2}; 1-v, 1+v; \frac{4c^2}{a^2} \right) -$$

$$- \left(\frac{c}{2} \right)^{2v} \frac{\cos v\pi}{\pi a^{\alpha+2v}} \left\{ \frac{\sin[(\alpha+2v)\pi/2]}{\cos[(\alpha+2v)\pi/2]} \right\} \Gamma \left[\frac{-v, \alpha+2v}{1+v} \right] \times$$

$$\times {}_3F_2 \left(\frac{1}{2}+v, \frac{1+\alpha}{2}+v, v+\frac{\alpha}{2}; 1+v, 1+2v; \frac{4c^2}{a^2} \right) \quad [0 < 2c < a]$$

$$9 \int_0^1 \left(1 + \frac{1}{x^2} \right) \sin \left(\frac{a}{x} - ax \right) \left[J_v(cx) Y_v \left(\frac{c}{x} \right) - J_v \left(\frac{c}{x} \right) Y_v(cx) \right] dx =$$

$$= \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \frac{4 \cos v\pi}{\pi \sqrt{a^2 - c^2}} K_{2v} (2 \sqrt{a^2 - c^2}) \quad \left[\begin{Bmatrix} 0 < a < c \\ 0 < c < a \end{Bmatrix}, |\operatorname{Re} v| < 1/2 \right]$$

2.13.20. Интегралы от $x^\alpha Y_\mu(\varphi(x)) Y_\nu(\chi(x))$.

$$1. \int_0^\infty Y_0(bx) Y_0(cx) dx = \frac{4}{\pi(b+c)} K \left(\frac{2\sqrt{bc}}{b+c} \right) - \frac{2}{\pi c} K \left(\frac{b}{c} \right) \quad [0 < b < c]$$

$$2. \int_0^\infty x^{\alpha-1} Y_\mu(bx) Y_\nu(cx) dx = \frac{2^{\alpha-1} b^\mu}{\pi^2 c^{\alpha+\mu}} \cos \mu\pi \cos \frac{\alpha+\mu-\nu}{2} \pi \times$$

$$\times \Gamma \left[-\mu, \frac{\alpha+\mu+\nu}{2}, \frac{\alpha+\mu-\nu}{2} \right] {}_2F_1 \left(\frac{\alpha+\mu+\nu}{2}, \frac{\alpha+\mu-\nu}{2}; 1+\mu; \frac{b^2}{c^2} \right) +$$

$$+ \frac{2^{\alpha-1} c^{\mu-\alpha}}{\pi^2 b^\mu} \cos \frac{\alpha-\mu-\nu}{2} \pi \Gamma \left[\mu, \frac{\alpha+\nu-\mu}{2}, \frac{\alpha-\nu-\mu}{2} \right] \times$$

$$\times {}_2F_1 \left(\frac{\alpha+\nu-\mu}{2}, \frac{\alpha-\nu-\mu}{2}; 1-\mu; \frac{b^2}{c^2} \right) \quad [0 < b < c; |\operatorname{Re} \mu| + |\operatorname{Re} \nu| < \operatorname{Re} \alpha < 2]$$

$$3. \int_0^\infty Y_{2v}(b\sqrt{x}) Y_\nu(cx) dx =$$

$$= \frac{1}{2c} \left[\frac{1}{\cos v\pi} J_{-\nu} \left(\frac{b^2}{4c} \right) + \frac{1}{\sin v\pi} H_{-\nu} \left(\frac{b^2}{4c} \right) - 2 \operatorname{ctg} 2v\pi H_\nu \left(\frac{b^2}{4c} \right) \right]$$

$$[b, c > 0; |\operatorname{Re} v| < 1/2]$$

$$4. \int_0^\infty x^{\pm 1-1} Y_\nu \left(\frac{b}{x} \right) Y_\nu(cx) dx = - \left\{ \frac{c^{-1}}{b^{-1}} \right\} J_{2v}(2\sqrt{bc})$$

$$[b, c > 1; |\operatorname{Re} v| < 1/2]$$

$$5. \int_0^\infty \frac{1}{x} Y_{\nu+1} \left(\frac{b}{x} \right) Y_\nu(cx) dx = - \frac{1}{\sqrt{bc}} J_{2\nu+1}(2\sqrt{bc})$$

$$[b, c > 0; -3/2 < \operatorname{Re} v < 1/2]$$

$$6. \int_0^\infty x^{\alpha-1} Y_\nu(b\sqrt{x^2+z^2}+bz) Y_\nu(b\sqrt{x^2+z^2}-bz) dx =$$

$$= - \frac{1}{2\sqrt{\pi}} \left(\frac{z}{b} \right)^{\alpha/2} \Gamma \left(\frac{1-\alpha}{2} \right) \left\{ \frac{2 \cos v\pi}{\pi} Y_{\alpha/2}(2bz) + \Gamma \left[\frac{\nu+\alpha/2}{1+\nu-\alpha/2} \right] J_{-\alpha/2}(2bz) \right\}$$

$$[b, \operatorname{Re} z > 0; 2|\operatorname{Re} v| < \operatorname{Re} \alpha < 1]$$

2.13.21. Интегралы, содержащие тригонометрические функции и $Y_\mu(bx^{\pm 1}) Y_\nu(cx)$

$$1. \int_0^\infty \cos ax Y_0(bx) Y_0(cx) dx = I \quad [a, b, c > 0; k = \sqrt{(b+c)^2 - a^2} / \sqrt{4bc}]$$

$$I = \frac{1}{\pi k \sqrt{bc}} K\left(\frac{1}{k}\right) \quad [0 < a < |b - c|].$$

$$I = \begin{cases} 1 \\ 2 \end{cases} \frac{1}{\pi \sqrt{bc}} K(k) \quad \left[\begin{cases} |b - c| < a < b + c \\ a > b + c \end{cases} \right].$$

$$2. \int_0^\infty \cos ax [J_0(bx) J_0(cx) + Y_0(bx) Y_0(cx)] dx =$$

$$= \frac{4}{\pi \sqrt{(b+c)^2 - a^2}} K\left(\sqrt{\frac{4bc}{(b+c)^2 - a^2}}\right) \quad [0 < a < |b - c|; b, c > 0].$$

$$3. = \frac{8}{\pi} (\sqrt{a^2 - (b-c)^2} + 2\sqrt{bc})^{-1} K\left(\left|\frac{\sqrt{a^2 - (b-c)^2} - 2\sqrt{bc}}{\sqrt{a^2 - (b-c)^2} + 2\sqrt{bc}}\right|\right) \quad [a > |b - c|; b, c > 0].$$

$$4. \int_0^1 \left(1 + \frac{1}{x^2}\right) \cos\left(a \frac{1-x^2}{x}\right) Y_\nu(cx) Y_\nu\left(\frac{c}{x}\right) dx = -\frac{1}{\sqrt{c^2 - a^2}} J_{2\nu}(2\sqrt{c^2 - a^2})$$

$$[0 < a < c; |\operatorname{Re} \nu| < 1]$$

$$5. = \frac{4 \cos \nu \pi}{\pi \sqrt{a^2 - c^2}} K_{2\nu}(2\sqrt{a^2 - c^2}) \quad [0 < c < a; |\operatorname{Re} \nu| < 1].$$

2.13.22. Интегралы от $A(x) J_\lambda(ax) J_\mu(bx) Y_\nu(cx)$.

$$1. \int_0^\infty x^{\mu+1} J_\nu(ax) J_\mu(bx) Y_\nu(cx) dx = I$$

$$[a, b, c > 0; \operatorname{Re}(\mu + \nu) > -1; -1 < \operatorname{Re} \mu < 1/2; 2acu_\pm = \pm(a^2 + c^2 - b^2)],$$

$$I = \frac{\sqrt{2} b^\mu \cos \mu \pi}{\pi^{3/2} (ac)^{\mu+1}} e^{-(2\mu+1)\pi i/2} (u_+^2 - 1)^{-(2\mu+1)/4} Q_{\nu-1/2}^{\mu+1/2}(u_+)$$

$$[b < c - a].$$

$$I = -\frac{b^\mu}{\sqrt{2\pi} (ac)^{\mu+1}} (u_+^2 - 1)^{-(2\mu+1)/4} \left[2P_{\nu-1/2}^{\mu+1/2}(u_+) - \right.$$

$$\left. - \frac{2}{\pi} \cos \mu \pi e^{-(2\mu+1)\pi i/2} Q_{\nu-1/2}^{\mu+1/2}(u_+) \right] \quad [b < a - c].$$

$$I = \frac{b^\mu}{\sqrt{2\pi} (ac)^{\mu+1}} (1 - u_+^2)^{-(2\mu+1)/4} \left[\sin \mu \pi P_{\nu-1/2}^{\mu+1/2}(u_+) + \frac{2}{\pi} \cos \mu \pi Q_{\nu-1/2}^{\mu+1/2}(u_+) \right]$$

$$[0 < c - a < b < a + c].$$

$$I = -\frac{\sqrt{2} b^\mu \cos \nu \pi}{\pi^{3/2} (ac)^{\mu+1}} (u_-^2 - 1)^{-(2\mu+1)/4} e^{-(2\mu+1)\pi i/2} Q_{\nu-1/2}^{\mu+1/2}(u_-)$$

$$[a < c, b > a + c]$$

$$2. \int_0^\infty x^{\nu+1} J_\mu(ax) J_\mu(bx) Y_\nu(cx) dx = I$$

$$[a, b, c > 0; \operatorname{Re} \nu < 1/2; \operatorname{Re} \mu, \operatorname{Re}(\mu + \nu) > -1; 2abz_\pm = \pm(a^2 + b^2 - c^2)].$$

$$I = -\frac{\sqrt{2} c^\nu}{\pi^{3/2} (ab)^{\nu+1}} \begin{Bmatrix} \cos \nu\pi \\ \cos \mu\pi \end{Bmatrix} (z_+^2 - 1)^{-(2\nu+1)/4} e^{-(2\nu+1)\pi i/2} Q_{\mu-1/2}^{\nu+1/2}(z_+)$$

$$\left\{ \begin{array}{l} \{c < |a-b|\} \\ \{c > a+b\} \end{array} \right\},$$

$$I = -\frac{c^\nu}{\sqrt{2\pi} (ab)^{\nu+1}} (1 - z_+^2)^{-(2\nu+1)/4} \left[\sin \nu\pi P_{\mu-1/2}^{\nu+1/2}(z_+) + \right. \\ \left. + \frac{2}{\pi} \cos \nu\pi Q_{\mu-1/2}^{\nu+1/2}(z_+) \right] \quad \{|a-b| < c < a+b\}$$

$$3. \int_0^\infty x^{1-\nu} J_\mu(ax) J_\mu(bx) Y_\nu(cx) dx = I$$

$$\{a, b, c > 0 \quad \operatorname{Re} \nu > -1/2, \operatorname{Re} \mu > -1 \quad \operatorname{Re}(\nu - \mu) < 1, 2abz_\pm = \pm(a^2 + b^2 - c^2)\},$$

$$I = -\frac{\sqrt{2} (ab)^{\nu-1}}{\pi^{3/2} c^\nu} (z_+^2 - 1)^{(2\nu-1)/4} e^{(2\nu-1)\pi i/2} Q_{\mu-1/2}^{1/2-\nu}(z_+)$$

$$\{c < |a-b|\},$$

$$I = -\frac{\sqrt{2} (ab)^{\nu-1}}{\pi^{3/2} c^\nu} (1 - z_+^2)^{(2\nu-1)/4} Q_{\mu-1/2}^{1/2-\nu}(z_+)$$

$$\{|a-b| < c < a+b\}$$

$$I = \frac{\sqrt{2} (ab)^{\nu-1}}{\pi^{3/2} c^\nu} \cos(\mu - \nu) \pi e^{(2\nu-1)\pi i/2} (z_-^2 - 1)^{(2\nu-1)/4} Q_{\mu-1/2}^{1/2-\nu}(z_-)$$

$$\{c > a+b\}$$

$$4. \int_0^\infty x J_0(ax) J_0(bx) Y_\nu(cx) dx = -\begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{1}{\pi ab} (z^2 - 1)^{-1/2}$$

$$\left\{ \begin{array}{l} \{c < |a-b| \text{ или } c > a+b\} \\ \{|a-b| < c < a+b\} \end{array} \right\} \quad a, b, c > 0, |\operatorname{Re} \nu| < 2, 2abz = a^2 + b^2 - c^2.$$

$$5. \int_0^\infty x J_\nu(ax) J_0(bx) Y_\nu(cx) dx = I$$

$$\{a, b, c > 0 \quad \operatorname{Re} \nu > -1 \quad 2acu_\pm = \pm(a^2 + c^2 - b^2)\},$$

$$I = \pm \frac{1}{\pi ac} (u_\pm^2 - 1)^{-1/2} (u_\pm + \sqrt{u_\pm^2 - 1})^{\mp \nu}$$

$$\left\{ \begin{array}{l} \{0 < b < c-a\} \\ \{0 < b < a-c\} \end{array} \right\},$$

$$I = -\frac{1}{\pi ac} (1 - u_+^2)^{-1/2} \sin(\nu \arccos u_+)$$

$$\{0 < |a-c| < b < a+c\},$$

$$I = -\frac{\cos \nu\pi}{\pi ac} (u_-^2 - 1)^{-1/2} (u_- + \sqrt{u_-^2 - 1})^{-\nu}$$

$$\{b > a+c\}$$

$$6. \int_0^\infty J_\mu^2(bx) Y_\nu(cx) dx = \frac{1}{c} \operatorname{tg} \frac{2\mu - \nu}{2} \pi \Gamma \left[\frac{(1+\nu)/2 + \mu}{(1+\nu)/2 - \mu} \right] \left[P_{(\nu-1)/2}^{-\mu} \left(\frac{\sqrt{c^2 - 4b^2}}{c} \right) \right]^2$$

$$\{0 < 2b < c, 2\operatorname{Re} \mu > \operatorname{Re} \nu - 1\}$$

$$7. \int_0^\infty J_{-\mu}(bx) J_\mu(bx) Y_\nu(cx) dx =$$

$$= -\frac{1}{c} \operatorname{tg} \frac{\nu\pi}{2} P_{(\nu-1)/2}^\mu \left(\frac{\sqrt{c^2 - 4b^2}}{c} \right) P_{(\nu-1)/2}^{-\mu} \left(\frac{\sqrt{c^2 - 4b^2}}{c} \right)$$

$$\{0 < 2b < c, |\operatorname{Re} \mu| < 1\}$$

$$8. \int_0^{\infty} x^{\alpha-1} J_{\lambda}(bx) J_{\mu}(bx) Y_{\nu}(cx) dx = I$$

$$[b, c > 0, |\operatorname{Re} \nu - \operatorname{Re}(\lambda + \mu)| < \operatorname{Re} \alpha < 5/2]$$

$$I = -\frac{2^{\alpha-1} b^{\lambda+\mu}}{\pi c^{\alpha+\lambda+\mu}} \cos \frac{\alpha+\lambda+\mu-\nu}{2} \pi \Gamma \left[\frac{(\alpha+\lambda+\mu+\nu)/2, (\alpha+\lambda+\mu-\nu)/2}{1+\lambda, 1+\mu} \right] \times \\ \times {}_4F_3 \left(\frac{1+\lambda+\mu}{2}, 1+\frac{\lambda+\mu}{2}, \frac{\alpha+\lambda+\mu+\nu}{2}, \frac{\alpha+\lambda+\mu-\nu}{2}; 1+\lambda, 1+\mu, 1+\lambda+\mu, \frac{4b^2}{c^2} \right) \\ [0 < 2b < c],$$

$$I = \frac{2^{\alpha-2} c^{1-\alpha}}{\pi b} \sin \frac{\nu-\alpha}{2} \pi \Gamma \left[\frac{(\alpha+\nu-1)/2, (\alpha-\nu-1)/2}{(1+\lambda-\mu)/2, (1+\mu-\lambda)/2} \right] \times \\ \times {}_4F_3 \left(\frac{1-\mu-\lambda}{2}, \frac{1+\mu-\lambda}{2}, \frac{1+\lambda-\mu}{2}, \frac{1+\mu+\lambda}{2}; \frac{1}{2}, \frac{3-\nu-\alpha}{2}, \frac{3+\nu-\alpha}{2}; \frac{c^2}{4b^2} \right) - \\ - \frac{2^{\alpha}}{\pi b^2} (\lambda+\mu) \cos \frac{\nu-\alpha}{2} \pi \Gamma \left[\frac{(\alpha+\nu)/2-1, (\alpha-\nu)/2-1}{(\mu-\lambda)/2, (\lambda-\mu)/2} \right] \times \\ \times {}_4F_3 \left(1-\frac{\lambda+\mu}{2}, 1+\frac{\lambda-\mu}{2}, 1+\frac{\mu-\lambda}{2}, 1+\frac{\mu+\lambda}{2}; \frac{3}{2}, \right. \\ \left. 2-\frac{\alpha+\nu}{2}, 2-\frac{\alpha-\nu}{2}, \frac{c^2}{4b^2} \right) - \frac{2^{\alpha-1} c^{\nu}}{\pi b^{\alpha+\nu}} \times \\ \times \cos \nu \pi \Gamma \left[\frac{1-\alpha-\nu-\nu, (\alpha+\lambda+\mu+\nu)/2}{1+(\lambda+\mu-\nu-\alpha)/2, 1+(\lambda-\mu-\nu-\alpha)/2, 1+(\mu-\lambda-\nu-\alpha)/2} \right] \times \\ \times {}_4F_3 \left(\frac{\alpha+\nu-\mu-\lambda}{2}, \frac{\alpha+\nu+\mu-\lambda}{2}, \frac{\alpha+\nu+\lambda-\mu}{2}, \frac{\alpha+\nu+\lambda+\mu}{2}; \right. \\ \left. 1+\nu, \frac{\alpha+\nu}{2}, \frac{1+\alpha+\nu}{2}, \frac{c^2}{4b^2} \right) - \frac{2^{\alpha-1} b^{\nu-\alpha}}{c^{1+\nu}} \times \\ \times \Gamma \left[\frac{1+\nu-\alpha, \nu, (\lambda+\mu+\alpha-\nu)/2}{1+(\lambda+\mu+\nu-\alpha)/2, 1+(\lambda+\nu-\mu-\alpha)/2, 1+(\mu+\nu-\lambda-\alpha)/2} \right] \times \\ \times {}_4F_3 \left(\frac{\alpha-\lambda-\mu-\nu}{2}, \frac{\alpha+\mu-\nu-\lambda}{2}, \frac{\alpha+\lambda-\mu-\nu}{2}, \frac{\alpha+\lambda+\mu-\nu}{2}; \right. \\ \left. 1-\nu, \frac{1+\alpha-\nu}{2}, \frac{\alpha-\nu}{2}; \frac{c^2}{4b^2} \right) \quad [0 < c < 2b]$$

$$9. \int_0^{\infty} x^{\nu+1} J_{\nu}^2(bx) Y_{\nu}(cx) dx = \frac{2^{3\nu+1} b^{2\nu} c^{-\nu-1}}{\sqrt{\pi} \Gamma(1/2-\nu)} (c^2-4b^2)^{-\nu-1/2}_+$$

$$[b, c > 0; 2b \neq c; |\operatorname{Re} \nu| < 1/2]$$

$$10. \int_0^{\infty} x^{\alpha-1} J_{\lambda}(ax) J_{\mu}(cx) Y_{\nu}(cx) dx = I$$

$$[a, c > 0; |\operatorname{Re} \nu| - \operatorname{Re}(\lambda + \mu) < \operatorname{Re} \alpha < 5/2],$$

$$I = \frac{2^{\alpha-1} a^{\nu-\mu-\alpha}}{\pi c^{\nu-\mu}} \Gamma \left[\frac{\nu, (\alpha+\lambda+\mu-\nu)/2}{\mu+1, 1+(\lambda+\nu-\mu-\alpha)/2} \right] \times \\ \times {}_4F_3 \left(\frac{\alpha+\mu-\lambda-\nu}{2}, \frac{1+\mu-\nu}{2}, 1+\frac{\mu-\nu}{2}, \frac{\alpha+\lambda+\mu-\nu}{2}; 1-\nu, 1+\mu, 1+\mu-\nu, \frac{4c^2}{a^2} \right) - \\ - \frac{2^{\alpha-1} c^{\mu+\nu}}{\pi a^{\mu+\nu+\alpha}} \cos \nu \pi \Gamma \left[\frac{-\nu, (\alpha+\lambda+\mu+\nu)/2}{1+(\lambda-\mu-\nu-\alpha)/2, 1+\mu} \right] \times \\ \times {}_4F_3 \left(\frac{\alpha+\mu+\nu-\lambda}{2}, \frac{\mu+\nu+1}{2}, \frac{\mu+1}{2}, \frac{\alpha+\lambda+\mu+\nu}{2}; 1+\mu, 1+\mu+\nu, 1+\nu, \frac{4c^2}{a^2} \right) \\ [0 < 2c < a]$$

$$I = \frac{2^{\alpha-2} a^{1-\alpha}}{\pi c} \sin \frac{\mu-\nu}{2} \pi \Gamma \left[\frac{(\lambda+\alpha-1)/2}{(3+\lambda-\alpha)/2} \right] {}_4F_3 \left(\frac{1-\mu-\nu}{2}, \right. \\ \left. \frac{1+\nu-\mu}{2}, \frac{1+\mu+\nu}{2}, \frac{1+\mu-\nu}{2}; \frac{1}{2}, \frac{3-\lambda-\alpha}{2}, \frac{3+\lambda-\alpha}{2}; \frac{a^2}{4c^2} \right) + \\ + \left(\frac{a}{2} \right)^{2-\alpha} \frac{\nu^2-\mu^2}{\pi c^4} \cos \frac{\mu-\nu}{2} \pi \Gamma \left[\frac{(\lambda+\alpha)/2-1}{2+(\lambda-\alpha)/2} \right] \times \\ \times {}_4F_3 \left(1-\frac{\mu+\nu}{2}, 1+\frac{\nu-\mu}{2}, 1+\frac{\mu+\nu}{2}, 1+\frac{\mu-\nu}{2}; \frac{3}{2}, \right. \\ \left. 2-\frac{\lambda+\alpha}{2}, 2+\frac{\lambda-\alpha}{2}; \frac{a^2}{4c^2} \right) - \frac{2^{\alpha-1} a^\lambda}{\pi c^{\alpha+\lambda}} \times \\ \times \cos \frac{\alpha+\lambda+\mu-\nu}{2} \pi \Gamma \left[\frac{1-\alpha-\lambda, (\alpha+\lambda+\mu+\nu)/2, (\alpha+\lambda+\mu-\nu)/2}{1+\lambda, 1+(\mu+\nu-\lambda-\alpha)/2, 1+(\mu-\lambda-\nu-\alpha)/2} \right] \times \\ \times {}_4F_3 \left(\frac{\alpha+\lambda-\mu-\nu}{2}, \frac{\alpha+\lambda+\nu-\mu}{2}, \frac{\alpha+\lambda+\mu+\nu}{2}, \frac{\alpha+\lambda+\mu-\nu}{2}; \right. \\ \left. \frac{1+\alpha+\lambda}{2}, \frac{\alpha+\lambda}{2}, 1+\lambda; \frac{a^2}{4c^2} \right) \quad [0 < a < 2c].$$

$$11. \int_0^\infty x^{\nu+1} J_\nu(ax) J_\nu(cx) Y_\nu(cx) dx = - \frac{2^{3\nu+1} c^{2\nu} a^{-\nu-1}}{\sqrt{\pi} \Gamma(1/2-\nu)} (a^2-4c^2)_+^{-\nu-1/2} \\ [a, c > 0; a \neq 2c; |\operatorname{Re} \nu| < 1/2].$$

$$12. \int_0^\infty x J_{2\nu}(ax) J_\nu(cx) Y_\nu(cx) dx = - \frac{2}{\pi a} (a^2-4c^2)_+^{-1/2} \\ [a, c > 0; a \neq 2c; \operatorname{Re} \nu > -1/2].$$

$$13. \int_0^\infty x J_0(ax) J_\nu(cx) Y_\nu(cx) dx = \\ = - \frac{2^{\nu+1} c^{2\nu}}{\pi a} \cos \nu \pi (a^2-4c^2)^{-1/2} (a^2-2c^2+a\sqrt{a^2-4c^2})^{-\nu} \\ [0 < 2c < a; \operatorname{Re} \nu > -1]$$

$$14. = - \frac{2}{\pi a} (4c^2-a^2)^{-1/2} \sin \left[\nu \arccos \left(1 - \frac{a^2}{2c^2} \right) \right] \\ [0 < a < 2c; \operatorname{Re} \nu > -1]$$

$$15. \int_0^\infty \frac{x^{\alpha-1}}{x^2+z^2} J_\mu(ax) J_\nu(bx) \left[\cos \frac{\alpha+\mu}{2} \pi J_\nu(cx) + \sin \frac{\alpha+\mu}{2} \pi Y_\nu(cx) \right] dx = \\ = - z^{\alpha-2} J_\mu(az) I_\nu(bz) K_\nu(cz) \\ [c > a+b; a, b > 0; \operatorname{Re} \alpha < 9/2; \operatorname{Re}(\alpha+\mu), \operatorname{Re}(\alpha+\mu+2\nu) > 0].$$

2.13.23. Интегралы от $x^\alpha J_\lambda(\varphi(x)) J_\mu(\chi(x)) Y_\nu(cx)$

$$1. \int_0^\infty x J_{\nu/4}(bx^2) I_{-\nu/4}(bx^2) Y_\nu(cx) dx = \frac{1}{16b} \sec \frac{\nu\pi}{4} \times \\ \times \left[2 \cos^2 \frac{\nu\pi}{2} J_{\nu/4}^2 \left(\frac{c^2}{16b} \right) + 2 \sin \frac{\nu\pi}{2} J_{\nu/4} \left(\frac{c^2}{16b} \right) Y_{\nu/4} \left(\frac{c^2}{16b} \right) - Y_{\nu/4}^2 \left(\frac{c^2}{16b} \right) \right] \\ [b, c > 0; |\operatorname{Re} \nu| < 2]$$

$$2. \int_0^\infty J_{2\nu}(a\sqrt{x}) J_\nu(cx) Y_\nu(cx) dx = - \frac{1}{2c} J_\nu^2 \left(\frac{a^2}{16c} \right) \\ [a, c > 0; \operatorname{Re} \nu > -1/4].$$

$$3. \int_0^{\infty} J_{4\nu}(a\sqrt{x}) J_{\nu}(cx) [\sin \nu\pi J_{\nu}(cx) - \cos \nu\pi Y_{\nu}(cx)] dx = \frac{1}{2c} J_{\nu}\left(\frac{a^2}{16c}\right) J_{-\nu}\left(\frac{a^2}{16c}\right) \\ [a, c > 0; \operatorname{Re} \nu > -1].$$

$$4. \int_0^{\infty} x J_{2\nu}(ax) J_{\nu}(b\sqrt{x^2+z^2}-bz) Y_{\nu}(b\sqrt{x^2+z^2}+bz) dx = \\ = \frac{2}{\pi a} (4b^2 - a^2)^{-1/2} \sin(z\sqrt{4b^2 - a^2}) \quad [0 < a < 2b; \operatorname{Re} z > 0; \operatorname{Re} \nu > -1/2].$$

$$5. = -\frac{2}{\pi a} (a^2 - 4b^2)^{-1/2} \exp(-z\sqrt{a^2 - 4b^2}) \\ [0 < 2b < a; \operatorname{Re} z > 0; \operatorname{Re} \nu > -1/2].$$

2.13.24. Интегралы, содержащие $A(x) \prod_k J_{\mu_k}(b_k x) Y_{\nu}(cx)$.

$$1. \int_0^{\infty} x J_{\nu}^2(bx) J_{\nu}(cx) Y_{\nu}(cx) dx = 0 \quad [0 < b < c; \operatorname{Re} \nu > -1/2].$$

$$2. = -\frac{1}{2\pi bc} \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \quad \left[\left\{ \begin{matrix} 0 < b < c \\ 0 < c < b \end{matrix} \right\}, \operatorname{Re} \nu > -1/2 \right].$$

$$3. \int_0^{\infty} \frac{x^{\alpha-1}}{x^2+z^2} \prod_{k=1}^n J_{\mu_k}(b_k x) \left[\cos \frac{\alpha+\lambda-\nu}{2} \pi J_{\nu}(cx) + \sin \frac{\alpha+\lambda-\nu}{2} \pi Y_{\nu}(cx) \right] dx = \\ = -z^{\alpha-2} K_{\nu}(cz) \prod_{k=1}^n I_{\mu_k}(b_k z) \\ \left[\lambda = \sum_{k=1}^n \mu_k; c > \sum_{k=1}^n b_k; b_k > 0; 2 \operatorname{Re} \alpha < n+7; |\operatorname{Re} \nu| < \operatorname{Re}(\alpha+\lambda) \right].$$

2.13.25. Интегралы, содержащие $x^{\alpha} J_{\lambda}(ax) Y_{\mu}(bx) Y_{\nu}(cx)$.

$$1. \int_0^{\infty} x J_0(ax) Y_0^2(cx) dx = \frac{4}{\pi^2 a} (4c^2 - a^2)^{-1/2} \arcsin\left(1 - \frac{a^2}{2c^2}\right) \quad [0 < a < 2c]$$

$$2. = \frac{4}{\pi^2 a} (a^2 - 4c^2)^{-1/2} \ln \frac{a^2 - 2c^2 + a\sqrt{a^2 - 4c^2}}{2c^2} \quad [0 < 2c < a].$$

$$3. \int_0^{\infty} x^{\nu+1} J_{\nu}(ax) [J_0(bx) J_0(cx) + Y_0(bx) Y_0(cx)] dx = \\ = \frac{4a^{\nu}}{\pi^{3/2} [a^2 - (b-c)^2]^{\nu+1}} \Gamma\left[\begin{matrix} \nu+1, \nu+1 \\ \nu+3/2 \end{matrix}\right] {}_2F_1\left(\nu+1, \frac{1}{2}; \nu+\frac{3}{2}; \frac{a^2 - (b+c)^2}{a^2 - (b-c)^2}\right) \\ [a > |b-c|].$$

$$4. \int_0^{\infty} x J_0(ax) [J_0(bx) J_0(cx) + Y_0(bx) Y_0(cx)] dx = \\ = \frac{1}{\sqrt{(b-c)^2 - a^2} \sqrt{(b+c)^2 - a^2}} \{ \ln 2\sqrt{bc} - \ln [\sqrt{(b-c)^2 - a^2} + \sqrt{(b+c)^2 - a^2}] \} \\ [a < |b-c|]$$

$$5. = \frac{1}{\sqrt{a^2 - (b-c)^2} \sqrt{(b+c)^2 - a^2}} \arcsin \sqrt{\frac{(b+c)^2 - a^2}{4bc}} \\ [|b-c| < a < b+c]$$

$$6. \quad = \frac{1}{\sqrt{a^2 - (b-c)^2} \sqrt{a^2 - (b+c)^2}} \operatorname{Arsh} \sqrt{\frac{a^2 - (b+c)^2}{4bc}} \quad [a > b+c].$$

$$7. \quad \int_0^\infty J_\mu(ax) [J_\nu^2(cx) + Y_\nu^2(cx)] dx = \\ = \frac{2 \operatorname{cosec} \nu\pi}{\pi a} \left[P_{(\mu-1)/2}^\nu \left(\frac{\sqrt{a^2 - 4c^2}}{a} \right) Q_{(\mu-1)/2}^{-\nu} \left(\frac{\sqrt{a^2 - 4c^2}}{a} \right) - \right. \\ \left. - P_{(\mu-1)/2}^{-\nu} \left(\frac{\sqrt{a^2 - 4c^2}}{a} \right) Q_{(\mu-1)/2}^\nu \left(\frac{\sqrt{a^2 - 4c^2}}{a} \right) \right] \\ [0 < 2c < a; \operatorname{Re} \mu > 2 | \operatorname{Re} \nu | - 1]$$

$$8. \quad \int_0^\infty x^{\mu+1} J_\mu(ax) [J_\nu^2(cx) + Y_\nu^2(cx)] dx = \\ = \frac{2^{\mu+1}}{\pi^{3/2} c \sqrt{a}} \Gamma(\mu - \nu + 1) \Gamma(\mu + \nu + 1) (a^2 - 4c^2)^{-(2\mu+1)/4} P_{\nu-1/2}^{-\mu-1/2} \left(\frac{a^2}{2c^2} - 1 \right) \\ [0 < 2c < a; 2 | \operatorname{Re} \nu | - 2 < \operatorname{Re} \mu < 1/2]$$

$$9. \quad \int_0^\infty x J_0(ax) [J_\nu^2(cx) + Y_\nu^2(cx)] dx = \\ = \frac{2^{2\nu+1} c^{2\nu}}{\pi a \sin \nu\pi \sqrt{a^2 - 4c^2}} [(a - \sqrt{a^2 - 4c^2})^{-2\nu} - (a + \sqrt{a^2 - 4c^2})^{-2\nu}] \\ [0 < 2c < a; | \operatorname{Re} \nu | < 1]$$

2.13.26. Интегралы от $x^\alpha J_\nu(bx) J_\nu(cx) Y_\nu(bx) Y_\nu(cx)$.

$$1. \quad \int_0^\infty x^{2\nu+1} J_\nu(bx) J_\nu(cx) Y_\nu(bx) Y_\nu(cx) dx = \\ = \frac{b^{2\nu}}{2\pi c^{4\nu+2}} \Gamma \left[\begin{matrix} 3\nu+1 \\ 1/2-\nu, 2\nu+3/2 \end{matrix} \right] {}_2F_1 \left(\nu + \frac{1}{2}, 3\nu+1; 2\nu + \frac{3}{2}; \frac{b^2}{c^2} \right) \\ [0 < b < c; -1/3 < \operatorname{Re} \nu < 1/2]$$

$$2. \quad \int_0^\infty x J_0(bx) J_0(cx) Y_0(bx) Y_0(cx) dx = \frac{1}{2\pi^2 bc} \ln \frac{c+b}{c-b} \quad [0 < b < c].$$

$$3. \quad \int_0^\infty x^{1/2} J_{-1/4}(bx) J_{-1/4}(cx) Y_{-1/4}(bx) Y_{-1/4}(cx) dx = \frac{\Gamma^2(1/4)}{\pi^3 c \sqrt{2b}} K(k) \\ [0 < b < c; k = (c - \sqrt{c^2 - b^2})^{1/2} / \sqrt{2c}].$$

$$4. \quad \int_0^\infty x^{3/2} J_{1/4}(bx) J_{1/4}(cx) Y_{1/4}(bx) Y_{1/4}(cx) dx = \\ = \frac{\Gamma^2(3/4) \sqrt{b}}{2 \sqrt{2} \pi^3 c^3 k^2 (1-k^2) (1-2k^2)} [(1-k^2) K(k) - (1-2k^2) E(k)] \\ [0 < b < c; k \text{ см. в 2.13.26.3}]$$

2.13.27. Интегралы по индексу, содержащие $Y_{\nu+ix}(c)$.

$$1. \quad \int_0^\infty \frac{1}{\operatorname{ch} \pi x} [J_{ix}^2(c) + Y_{ix}^2(c)] dx = H_0(2c) - Y_0(2c) \quad [c > 0].$$

$$2. \int_0^{\infty} \frac{\cos bx}{\operatorname{ch} \pi x} [J_{ix}^2(c) + Y_{ix}^2(c)] dx = H_0\left(2c \operatorname{ch} \frac{b}{2}\right) - Y_0\left(2c \operatorname{ch} \frac{b}{2}\right) \quad [c > 0]$$

$$3. \int_{-\infty}^{\infty} e^{ipx} \left[J_{\nu-ix}(b) \begin{Bmatrix} Y_{\nu+ix}(c) \\ J_{\nu+ix}(c) \end{Bmatrix} \pm Y_{\nu-ix}(b) \begin{Bmatrix} J_{\nu+ix}(c) \\ Y_{\nu+ix}(c) \end{Bmatrix} \right] dx = \\ = \mp 2 \left(\frac{z_{\pm}}{z_{-}} \right)^{\nu} \begin{Bmatrix} J_{2\nu}(\sqrt{z_{+}z_{-}}) \\ Y_{2\nu}(\sqrt{z_{+}z_{-}}) \end{Bmatrix} \quad [z_{\pm} = be^{\mp p/2} + ce^{\pm p/2}; b, c > 0; \operatorname{Im} p = 0].$$

2.13.28. Интегралы, содержащие $\operatorname{Ei}(bx^n) Y_{\nu}(cx)$

$$1. \int_0^{\infty} x^{\alpha-1} \operatorname{Ei}(-bx) Y_{\nu}(cx) dx = -\frac{2^{\alpha} b}{\pi c^{\alpha+1}} \sin \frac{\alpha-\nu}{2} \pi \Gamma\left(\frac{\alpha+\nu+1}{2}\right) \times \\ \times \Gamma\left(\frac{\alpha-\nu+1}{2}\right) {}_3F_2\left(\frac{\alpha+\nu+1}{2}, \frac{\alpha-\nu+1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -\frac{b^2}{c^2}\right) + \\ + \frac{2^{\alpha-1} b^2}{\pi c^{\alpha+2}} \Gamma\left(\frac{\alpha+\nu}{2}+1\right) \Gamma\left(\frac{\alpha-\nu}{2}+1\right) \cos \frac{\alpha-\nu}{2} \pi {}_4F_3\left(\frac{\alpha+\nu}{2}+1, \frac{\alpha-\nu}{2}+1, \right. \\ \left. 1, 1; 2, 2, \frac{3}{2}; -\frac{b^2}{c^2}\right) - \frac{2^{\alpha-2}}{\pi c^{\alpha}} \Gamma\left(\frac{\alpha+\nu}{2}\right) \Gamma\left(\frac{\alpha-\nu}{2}\right) \times \\ \times \cos \frac{\alpha-\nu}{2} \pi \left[\psi\left(\frac{\alpha+\nu}{2}\right) + \psi\left(\frac{\alpha-\nu}{2}\right) - \pi \operatorname{tg} \frac{\alpha-\nu}{2} \pi + 2C + 2 \ln \frac{2b}{c} \right] \\ [c, \operatorname{Re} b > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu|]$$

$$2. \int_0^{\infty} \operatorname{Ei}(-x^2) Y_1(cx) dx = \frac{1}{c} \left(\operatorname{Ei}\left(-\frac{c^2}{4b}\right) - C - \ln \frac{c^2}{4b} \right) \quad [c, \operatorname{Re} b > 0]$$

$$3. \int_0^{\infty} x^{\alpha-1} \operatorname{Ei}(-bx^2) Y_{\nu}(cx) dx = \frac{2^{\nu} b^{(\nu-\alpha)/2} \Gamma(\nu)}{(\alpha-\nu) \pi c^{\nu}} \times \\ \times \Gamma\left(\frac{\alpha-\nu}{2}\right) {}_2F_2\left(\frac{\alpha-\nu}{2}, \frac{\alpha-\nu}{2}; \frac{\alpha-\nu}{2}+1, 1-\nu; -\frac{c^2}{4b}\right) + \\ + \frac{c^{\nu} \cos \nu \pi \Gamma(-\nu)}{2^{\nu} (\alpha+\nu) \pi b^{(\alpha+\nu)/2}} \Gamma\left(\frac{\alpha+\nu}{2}\right) {}_2F_2\left(\frac{\alpha+\nu}{2}, \frac{\alpha+\nu}{2}; \frac{\alpha+\nu}{2}+1, 1+\nu; -\frac{c^2}{4b}\right) \\ [c, \operatorname{Re} b > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu|]$$

$$4. \int_0^{\infty} x \operatorname{Ei}(-bx^2) Y_0(cx) dx = -\frac{2}{\pi c^2} \left[C + \ln \frac{c^2}{4b} - \exp\left(-\frac{c^2}{4b}\right) \operatorname{Ei}\left(\frac{c^2}{4b}\right) \right] \\ [c, \operatorname{Re} b > 0]$$

$$5. \int_0^{\infty} \operatorname{Ei}\left(-\frac{b}{x}\right) Y_1(cx) dx = -\frac{2}{c} Y_0(\sqrt{2bc}) K_0(\sqrt{2bc}) \quad [c, \operatorname{Re} b > 0]$$

$$6. \int_0^{\infty} x^{\alpha-1} e^{\pm bx} \operatorname{Ei}(\mp bx) Y_{\nu}(cx) dx = U(\nu) \quad [c, \operatorname{Re} b > 0; |\operatorname{Re} \nu| < \operatorname{Re} \alpha < 5/2]$$

$$U(\nu) = -\frac{2^{\nu} \Gamma(\nu) \Gamma(\alpha-\nu)}{b^{\alpha-\nu} c^{\nu}} \begin{Bmatrix} \operatorname{cosec}(\nu-\alpha) \pi \\ \operatorname{ctg}(\nu-\alpha) \pi \end{Bmatrix} \times \\ \times {}_2F_1\left(\frac{\alpha-\nu}{2}, \frac{\alpha-\nu+1}{2}; 1-\nu; \frac{(-1)^{\nu} c^2}{b^2}\right) +$$

$$\begin{aligned}
& + \frac{c^v \Gamma(-v)}{2^v b^{\alpha+v}} \Gamma(\alpha+v) \left\{ \begin{matrix} \operatorname{cosec}(\alpha+v)\pi \\ \operatorname{ctg}(\alpha+v)\pi \end{matrix} \right\} {}_2F_1\left(\frac{\alpha+v}{2}, \frac{\alpha+v+1}{2}; v+1; \right. \\
& \left. \frac{(-1)^v c^2}{b^2} \right) \pm \frac{2^{\alpha-2}}{\pi b c^{\alpha-1}} \sin v \frac{\alpha-v}{2} \pi \Gamma\left(\frac{\alpha+v-1}{2}\right) \Gamma\left(\frac{\alpha-v-1}{2}\right) \times \\
& \times {}_2F_2\left(\frac{1}{2}, 1, 1; \frac{3-\alpha-v}{2}, \frac{3-\alpha+v}{2}; \frac{(-1)^v c^2}{b^2}\right) - \\
& - \frac{(-1)^v 2^{\alpha-3}}{\pi b^2 c^{\alpha-2}} \cos v \frac{\alpha-v}{2} \pi \Gamma\left(\frac{\alpha+v}{2}-1\right) \Gamma\left(\frac{\alpha-v}{2}-1\right) \times \\
& \times {}_2F_2\left(1, 1, \frac{3}{2}; 2-\frac{\alpha+v}{2}, 2-\frac{\alpha-v}{2}; \frac{(-1)^v c^2}{b^2}\right).
\end{aligned}$$

$$7. \int_0^\infty x^{v+3\pm 1} e^{bx^2} \operatorname{Ei}(-bx^2) Y_v(cx) dx =$$

$$= \pm \frac{3^{(1\pm 1)/2} \sqrt{\pi}}{2^{(3\pm 1)/2} b^{(v+3\pm 1)/2} c} \Gamma\left(v + \frac{4\pm 1}{2}\right) \exp\left(\frac{c^2}{8b}\right) W_{-(v+3\pm 1)/2, v/2}\left(\frac{c^2}{4b}\right)$$

[c, Re b > 0; -(4±1)/2 < Re v < -(1±2)/2]

2.13.29. Интегралы от $x^\alpha \left\{ \begin{matrix} \operatorname{si}(bx) \\ \operatorname{ci}(bx) \end{matrix} \right\} Y_v(cx)$.

$$1. \int_0^\infty x^{\alpha-1} \left\{ \begin{matrix} \operatorname{si}(bx) \\ \operatorname{ci}(bx) \end{matrix} \right\} Y_v(cx) dx = V(1)$$

$$\begin{aligned}
V(\gamma) = & \frac{2^v b^{v-\alpha} \Gamma(v) \Gamma(\alpha-v)}{(\alpha-v) \pi c^v} \left\{ \begin{matrix} \sin[(\alpha-v)\pi/2] \\ \cos[(\alpha-v)\pi/2] \end{matrix} \right\} \times \\
& \times {}_2F_2\left(\frac{\alpha-v}{2}, \frac{\alpha-v}{2}, \frac{\alpha-v+1}{2}; \frac{\alpha-v}{2}+1, 1-v; \frac{(-1)^{v+1} c^2}{b^2}\right) + \\
& + \frac{c^v \Gamma(-v) \Gamma(\alpha+v)}{2^v (\alpha+v) \pi b^{\alpha+v}} \cos v \pi \left\{ \begin{matrix} \sin[(\alpha+v)\pi/2] \\ \cos[(\alpha+v)\pi/2] \end{matrix} \right\} \times \\
& \times {}_2F_2\left(\frac{\alpha+v}{2}, \frac{\alpha+v}{2}, \frac{\alpha+v+1}{2}; \frac{\alpha+v}{2}+1, 1+v; \frac{(-1)^{v+1} c^2}{b^2}\right).
\end{aligned}$$

2.13.30. Интегралы, содержащие $\left\{ \begin{matrix} \operatorname{erf}(bx) \\ \operatorname{erfc}(bx) \end{matrix} \right\} Y_v(cx)$.

$$1. \int_0^\infty x^{\alpha-1} \left\{ \begin{matrix} \operatorname{erf}(bx) \\ \operatorname{erfc}(bx) \end{matrix} \right\} Y_v(cx) dx = U(1)$$

$$\begin{aligned}
U(\gamma) = & \mp \frac{2^v b^{v-\alpha}}{(\gamma-\alpha) \pi^{3/2} c^v} \Gamma(v) \Gamma\left(\frac{\alpha-v+1}{2}\right) \times \\
& \times {}_2F_2\left(\frac{\alpha-v}{2}, \frac{\alpha-v+1}{2}; \frac{\alpha-v}{2}+1, 1-v; \frac{(-1)^v c^2}{4b^2}\right) \pm \frac{c^v \cos v \pi}{2^v (\alpha+v) \pi^{3/2} b^{\alpha+v}} \times \\
& \times \Gamma(-v) \Gamma\left(\frac{\alpha+v+1}{2}\right) {}_2F_2\left(\frac{\alpha+v}{2}, \frac{\alpha+v+1}{2}; \frac{\alpha+v}{2}+1, 1+v; \frac{(-1)^v c^2}{4b^2}\right) - \\
& - \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{2^{\alpha-1}}{\pi c^\alpha} \Gamma\left(\frac{\alpha+v}{2}\right) \Gamma\left(\frac{\alpha-v}{2}\right) \cos v \frac{\alpha-v}{2} \pi
\end{aligned}$$

$$2. \int_0^\infty x^{\alpha-1} \left\{ \begin{matrix} \operatorname{erf}(b\sqrt{x}) \\ \operatorname{erfc}(b\sqrt{x}) \end{matrix} \right\} Y_v(cx) dx = V(1)$$

$$\left[|\arg b| < \pi/4; |\operatorname{Re} v| < \operatorname{Re} \alpha + (1\pm 1)/4, \left\{ \begin{matrix} \operatorname{Re} \alpha < 3/2, c > 0 \\ c > 0 \end{matrix} \right\} \right],$$

$$V(\gamma) = \mp \frac{2^{\alpha+1/2} b}{\pi^{3/2} c^{\alpha+1/2}} \Gamma\left(\frac{2\alpha+2\nu+1}{4}\right) \Gamma\left(\frac{2\alpha-2\nu+1}{4}\right) \cos \gamma \frac{2\alpha-2\nu+1}{4} \pi \times \\ \times {}_3F_2\left(\frac{2\alpha+2\nu+1}{4}, \frac{2\alpha-2\nu+1}{4}, \frac{1}{4}; \frac{5}{4}, \frac{1}{2}; \frac{(-1)^\gamma b^4}{c^2}\right) \pm \\ \pm (-1)^\gamma \frac{2^{\alpha+3/2} b^3}{3\pi^{3/2} c^{\alpha+3/2}} \Gamma\left(\frac{2\alpha+2\nu+3}{4}\right) \Gamma\left(\frac{2\alpha-2\nu+3}{4}\right) \sin \gamma \frac{2\alpha-2\nu+1}{4} \pi \times \\ \times {}_3F_2\left(\frac{2\alpha+2\nu+3}{4}, \frac{2\alpha-2\nu+3}{4}, \frac{3}{4}; \frac{7}{4}, \frac{3}{2}; \frac{(-1)^\gamma b^4}{c^2}\right) - \\ - \begin{cases} 0 \\ 1 \end{cases} \frac{2^{\alpha-1}}{\pi c^\alpha} \Gamma\left(\frac{\alpha+\nu}{2}\right) \Gamma\left(\frac{\alpha-\nu}{2}\right) \cos \gamma \frac{\alpha-\nu}{2} \pi$$

$$3. \int_0^\infty x^{\alpha-1} e^{\mp b^2 x^2} \begin{Bmatrix} \operatorname{erf}(ibx) \\ \operatorname{erfc}(bx) \end{Bmatrix} Y_\nu(cx) dx = W(1)$$

$$[c > 0; |\operatorname{Re} \nu| - (1 \pm 1)/2 < \operatorname{Re} \alpha < 5/2; |\arg b| < (2 \mp 1)\pi/4] \\ W(\gamma) = - \frac{c^\nu \cos \gamma \nu \pi}{2^{\nu+(1 \pm 1)/2} b^{\alpha+\nu}} \Gamma\left[\frac{-\nu}{1-(\alpha+\nu)/2}\right] \begin{Bmatrix} i \sec[(\alpha+\nu)\pi/2] \\ \operatorname{cosec}(\alpha+\nu)\pi \end{Bmatrix} \times \\ \times {}_1F_1\left(\frac{\alpha+\nu}{2}; 1+\nu; \pm \frac{(-1)^\gamma c^2}{4b^2}\right) - \frac{2^{\nu-(1 \pm 1)/2}}{b^{\alpha-\nu} c^\nu} \times \\ \times \Gamma\left[\frac{\nu}{1-(\alpha-\nu)/2}\right] \begin{Bmatrix} i \sec[(\alpha-\nu)\pi/2] \\ \operatorname{cosec}(\alpha-\nu)\pi \end{Bmatrix} {}_1F_1\left(\frac{\alpha-\nu}{2}; 1-\nu; \pm \frac{(-1)^\gamma c^2}{4b^2}\right) - \\ - \frac{2^{\alpha-2}}{\pi^{3/2} b^{\alpha-1}} \begin{Bmatrix} i \\ 1 \end{Bmatrix} \sin \gamma \frac{\alpha-\nu}{2} \pi \Gamma\left(\frac{\alpha+\nu-1}{2}\right) \Gamma\left(\frac{\alpha-\nu-1}{2}\right) \times \\ \times {}_2F_2\left(\frac{1}{2}, 1; \frac{3-\alpha-\nu}{2}, \frac{3-\alpha+\nu}{2}; \pm \frac{(-1)^\gamma c^2}{4b^2}\right).$$

$$4. \int_0^\infty x^{\nu+1} e^{-b^2 x^2} \operatorname{erf}(ibx) Y_\nu(cx) dx = \frac{ic^\nu}{2^{\nu+1} b^{2\nu+2}} \exp\left(-\frac{c^2}{4b^2}\right) \\ [c > 0; -3/2 < \operatorname{Re} \nu < 1/2; |\arg b| < \pi/4]$$

$$5. \int_0^\infty x^{\nu+1} e^{b^2 x^2} \operatorname{erfc}(bx) Y_\nu(cx) dx = \frac{c^\nu}{2^{\nu+1} \pi b^{2\nu+2}} \Gamma(\nu+1) \exp\left(\frac{c^2}{4b^2}\right) \Gamma\left(-\nu; \frac{c^2}{4b^2}\right) \\ [c > 0; |\arg b| < 3\pi/4; -1 < \operatorname{Re} \nu < 1/2]$$

$$6. \int_0^\infty x^{\nu+3} e^{b^2 x^2} \operatorname{erfc}(bx) Y_\nu(cx) dx = - \frac{\Gamma(\nu+2)}{\pi b^{\nu+2}} \exp\left(\frac{c^2}{8b^2}\right) W_{-(\nu+3)/2, \nu/2}\left(\frac{c^2}{4b^2}\right) \\ [c > 0; |\arg b| < 3\pi/4; -2 < \operatorname{Re} \nu < -3/2]$$

2.13.31. Интегралы от $x^\alpha \begin{Bmatrix} S(bx) \\ C(bx) \end{Bmatrix} Y_\nu(cx)$.

$$1. \int_0^\infty x^{\alpha-1} \begin{Bmatrix} S(bx) \\ C(bx) \end{Bmatrix} Y_\nu(cx) dx = U(1)$$

$$[b, c > 0; |\operatorname{Re} \nu| - (2 \pm 1)/2 < \operatorname{Re} \alpha < 3/2, \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}],$$

$$U(\gamma) = - \frac{2^{\alpha+\delta} b^{1/2+\delta}}{3^\delta \pi^{3/2} c^{\alpha+\delta+1/2}} \times \\ \times \cos \gamma \frac{2\alpha-2\nu+2\delta+1}{4} \pi \Gamma\left(\frac{\alpha+\nu+\delta}{2} + \frac{1}{4}\right) \Gamma\left(\frac{\alpha-\nu+\delta}{2} + \frac{1}{4}\right) \times \\ \times {}_3F_2\left(\frac{1+2\delta}{4}, \frac{\alpha+\nu+\delta}{2} + \frac{1}{4}, \frac{\alpha-\nu+\delta}{2} + \frac{1}{4}; \frac{5+2\delta}{4}, \frac{1}{2} + \delta; \frac{(-1)^{\gamma+1} b^2}{c}\right).$$

2.13.32. Интегралы, содержащие $\left\{ \frac{\gamma(\mu, bx^n)}{\Gamma(\mu, bx^n)} \right\} Y_\nu(cx)$.

$$1. \int_0^\infty x^{\alpha-1} \left\{ \frac{\gamma(\mu, bx)}{\Gamma(\mu, bx)} \right\} Y_\nu(cx) dx = V(1)$$

$$\left[a, \operatorname{Re} b > 0; \operatorname{Re}(\alpha + \mu) > |\operatorname{Re} \nu|, \left\{ \begin{array}{l} \operatorname{Re} \mu > 0; \operatorname{Re} \alpha < 3/2 \\ \operatorname{Re} \alpha > |\operatorname{Re} \nu| \end{array} \right\} \right],$$

$$V(\gamma) = \mp \frac{2^{\alpha+\mu-1} b^\mu}{\mu \pi c^{\alpha+\mu}} \Gamma\left(\frac{\alpha+\mu+\nu}{2}\right) \Gamma\left(\frac{\alpha+\mu-\nu}{2}\right) \cos^\gamma \frac{\alpha+\mu-\nu}{2} \pi \times$$

$$\times {}_2F_2\left(\frac{\mu}{2}, \frac{\alpha+\mu+\nu}{2}, \frac{\alpha+\mu-\nu}{2}; \frac{\mu}{2} + 1, \frac{1}{2}; \frac{(-1)^\gamma b^2}{c^2}\right) \pm$$

$$\pm (-1)^\gamma \frac{2^{\alpha+\mu} b^{\mu+1}}{(\mu+1) \pi c^{\alpha+\mu+1}} \Gamma\left(\frac{\alpha+\mu+\nu+1}{2}\right) \Gamma\left(\frac{\alpha+\mu-\nu+1}{2}\right) \sin^\gamma \frac{\alpha+\mu-\nu}{2} \pi \times$$

$$\times {}_2F_2\left(\frac{\mu+1}{2}, \frac{\alpha+\mu+\nu+1}{2}, \frac{\alpha+\mu-\nu+1}{2}; \frac{\mu+3}{2}, \frac{3}{2}; \frac{(-1)^\gamma b^2}{c^2}\right) -$$

$$- \left\{ \begin{array}{l} 0 \\ 1 \end{array} \right\} \frac{2^{\alpha-1}}{\pi c^\alpha} \Gamma\left[\mu, \frac{\alpha+\nu}{2}, \frac{\alpha-\nu}{2}\right] \cos^\gamma \frac{\alpha-\nu}{2} \pi$$

$$2. \int_0^\infty x^{\alpha-1} \left\{ \frac{\gamma(\mu, bx^2)}{\Gamma(\mu, bx^2)} \right\} Y_\nu(cx) dx = W(1)$$

$$\left[a, \operatorname{Re} b > 0; \operatorname{Re}(\alpha + 2\mu) > |\operatorname{Re} \nu|, \left\{ \begin{array}{l} \operatorname{Re} \mu < 0, \operatorname{Re} \alpha < 3/2 \\ \operatorname{Re} \alpha > |\operatorname{Re} \nu| \end{array} \right\} \right],$$

$$W(\gamma) = \mp \frac{2^\nu b^{(\nu-\alpha)/2} \Gamma(\nu)}{(\nu-\alpha) \pi c^\nu} \Gamma\left(\mu + \frac{\alpha-\nu}{2}\right) \times$$

$$\times {}_2F_2\left(\frac{\alpha-\nu}{2}, \mu + \frac{\alpha-\nu}{2}; \frac{\alpha-\nu}{2} + 1, 1-\nu; \frac{(-1)^\gamma c^2}{4b}\right) \pm \frac{c^\nu \cos^\gamma \nu \pi \Gamma(-\nu)}{2^\nu (\alpha+\nu) \pi b^{(\alpha+\nu)/2}} \times$$

$$+ \Gamma\left(\mu + \frac{\alpha+\nu}{2}\right) {}_2F_2\left(\frac{\alpha+\nu}{2}, \mu + \frac{\alpha+\nu}{2}; \frac{\alpha+\nu}{2} + 1, 1+\nu; \frac{(-1)^\gamma c^2}{4b}\right) -$$

$$- \left\{ \begin{array}{l} 1 \\ 0 \end{array} \right\} \frac{2^{\alpha-1}}{\pi c^\alpha} \Gamma\left[\mu, \frac{\alpha+\nu}{2}, \frac{\alpha-\nu}{2}\right] \cos^\gamma \frac{\alpha-\nu}{2} \pi$$

$$3. \int_0^\infty x^{\nu+2n-2\mu+2} \Gamma(\mu, bx^2) Y_\nu(cx) dx =$$

$$= (-1)^{n-1} \frac{b^{\mu-n-\nu/2-1}}{c} \Gamma\left[\frac{3}{2} + n, \frac{(3+\nu)}{2} + n\right] \exp\left(\frac{c^2}{8b}\right) W_{-\nu/2-n, \nu/2}\left(\frac{c^2}{4b}\right)$$

$$[a, \operatorname{Re} b > 0; \operatorname{Re} \mu < n + 3/2; \operatorname{Re} \nu > -n - 3/2; \operatorname{Re}(\nu - \mu) > -n - 3/2]$$

$$4. \int_0^\infty x^{\alpha-1} e^{bx^2} \Gamma(\mu, bx^2) Y_\nu(cx) dx =$$

$$= - \frac{2^{\nu-1} b^{(\nu-\alpha)/2}}{c^\nu} \Gamma\left[\nu, \frac{(\alpha-\nu)}{2}\right] \operatorname{cosec} \frac{\alpha-\nu+2\mu}{2} \pi {}_1F_1\left(\frac{\alpha-\nu}{2}; 1-\nu; \frac{c^2}{4b}\right) -$$

$$- \frac{c^\nu}{2^{\nu+1} b^{(\alpha+\nu)/2}} \Gamma\left[-\nu, \frac{(\alpha+\nu)}{2}\right] \cos \nu \pi \operatorname{cosec} \frac{\alpha+\nu+2\mu}{2} \pi \times$$

$$\times {}_1F_1\left(\frac{\alpha+\nu}{2}; 1+\nu; \frac{c^2}{4b}\right) + \frac{2^{\alpha+2\mu-3} b^{\mu-1}}{c^{\alpha+2\mu-2\pi}} \Gamma\left(\frac{\alpha+\nu}{2} + \mu - 1\right) \times$$

$$\times \Gamma\left(\frac{\alpha-\nu}{2} + \mu - 1\right) \cos \frac{\alpha-\nu+2\mu}{2} \pi {}_2F_2\left(1-\mu, 1, 2-\mu + \frac{\nu-\alpha}{2}, 2-\mu - \frac{\nu+\alpha}{2}; \frac{c^2}{4b}\right)$$

$$[c > 0; \arg b < \pi; \operatorname{Re} \alpha > |\operatorname{Re} \nu|; |\operatorname{Re} \nu| < \operatorname{Re}(\alpha + 2\mu) < \dots]$$

2.13.33. Интегралы, содержащие $D_\mu(bx^\gamma) Y_\nu(cx)$.

$$1. \int_0^\infty x^{\alpha-1} e^{\pm b^2 x^2/4} D_\mu(bx) Y_\nu(cx) dx = I \quad (1)$$

$$\left[\operatorname{Re} \alpha > |\operatorname{Re} \nu|; |\arg b| < (2 \pm 1) \pi/4, \begin{cases} c > 0; & \operatorname{Re}(\alpha + \mu) < 3/2 \\ c > 0 \end{cases} \right],$$

$$\begin{aligned} I(\gamma) = & -\frac{2^{(3\nu-\alpha-1 \mp \mu \mp 1)/2}}{\pi b^{\alpha-\nu} c^\nu} \Gamma(\nu) \Gamma(\alpha-\nu) \Gamma^{\pm 1} \left(\frac{1 \mp 1}{4} \mp \frac{\alpha-\nu \pm \mu}{2} \right) \times \\ & \times \left\{ \frac{\Gamma^{-1}(-\mu)}{\sqrt{\pi}} \right\} {}_2F_2 \left(\frac{\alpha-\nu}{2}, \frac{\alpha-\nu+1}{2}; 1-\nu, \frac{3 \pm 1}{4} \mp \frac{\alpha-\nu \pm \mu}{2}; \mp \frac{(-1)^\nu c^2}{2b^2} \right) - \\ & - \frac{2^{-(\alpha+3\nu \pm \mu + 1 \pm 1)/2} c^\nu}{\pi b^{\alpha+\nu}} \Gamma(-\nu) \Gamma(\alpha+\nu) \cos^\nu \nu \pi \Gamma^{\pm 1} \left(\frac{1 \mp 1}{4} \mp \frac{\alpha+\nu \pm \mu}{2} \right) \times \\ & \times \left\{ \frac{\Gamma^{-1}(-\mu)}{\sqrt{\pi}} \right\} {}_2F_2 \left(\frac{\alpha+\nu}{2}, \frac{\alpha+\nu+1}{2}; \nu+1, \frac{3 \pm 1}{4} \mp \frac{\alpha+\nu \pm \mu}{2}; \mp \frac{(-1)^\nu c^2}{2b^2} \right) - \\ & - \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{2^{\alpha+\mu-1} b^\mu}{\pi c^{\alpha+\mu}} \Gamma \left(\frac{\alpha+\mu+\nu}{2} \right) \Gamma \left(\frac{\alpha+\mu-\nu}{2} \right) \cos^\nu \frac{\alpha+\mu-\nu}{2} \pi \times \\ & \times {}_2F_2 \left(-\frac{\mu}{2}, \frac{1-\mu}{2}; 1-\frac{\alpha+\mu+\nu}{2}, 1-\frac{\alpha+\mu-\nu}{2}; \frac{(-1)^{\nu-1} c^2}{2b^2} \right). \end{aligned}$$

$$2. \int_0^\infty x^\nu e^{b^2 x^2/4} D_{(\nu-1)/2}(bx) Y_\nu(cx) dx =$$

$$= -\frac{2^{3(\nu+1)/4}}{\pi b^\nu c} \Gamma(\nu+1) \exp\left(\frac{c^2}{4b^2}\right) W_{(\nu-1)/2, \nu/2}\left(\frac{c^2}{2b^2}\right) \\ [c > 0; |\arg b| < 3\pi/4 \quad -1/2 < \operatorname{Re} \nu < 2/3]$$

$$3. \int_0^\infty x^{\alpha-1} e^{\pm b^2 x/4} D_\mu(b\sqrt{x}) Y_\nu(cx) dx = V \quad (1)$$

$$\left[\operatorname{Re} \alpha > |\operatorname{Re} \nu|; |\arg b| < (2 \pm 1) \pi/4, \begin{cases} c > 0; & \operatorname{Re}(2\alpha + \mu) < 3 \\ c > 0 \end{cases} \right],$$

$$\begin{aligned} V(\gamma) = & -\frac{2^{2\nu \mp (\mu \mp 1 + 1)/2 - \alpha}}{\pi b^{2\alpha-2\nu} c^\nu} \left\{ \frac{\Gamma^{-1}(-\mu)}{\sqrt{\pi}} \right\} \Gamma(\nu) \Gamma(2\alpha-2\nu) \times \\ & \times \Gamma^{\pm 1} \left(\frac{1 \mp 1 - 2\mu}{4} \mp \alpha \pm \nu \right) {}_4F_3 \left(\frac{\alpha-\nu}{2}, \frac{2\alpha-2\nu+1}{4}, \frac{\alpha-\nu+1}{2}, \frac{2\alpha-2\nu+3}{4}; \right. \\ & \left. 1-\nu, \frac{7 \pm 1 \pm 2\mu}{8} \mp \frac{\alpha-\nu}{2}, \frac{3 \pm 1 \pm 2\mu}{8} \mp \frac{\alpha-\nu}{2}; \frac{4(-1)^\nu c^2}{b^4} \right) - \\ & - \frac{2^{-2\nu \mp (\mu \mp 1 + 1)/2 - \alpha} c^\nu \cos^\nu \nu \pi}{\pi b^{2\alpha+2\nu}} \left\{ \frac{\Gamma^{-1}(-\mu)}{\sqrt{\pi}} \right\} \Gamma(-\nu) \Gamma(2\alpha+2\nu) \times \\ & \times \Gamma^{\pm 1} \left(\frac{1 \mp 1 - 2\mu}{4} \mp \alpha \pm \nu \right) {}_4F_3 \left(\frac{\alpha+\nu}{2}, \frac{2\alpha+2\nu+1}{4}, \frac{\alpha+\nu+1}{2}, \frac{2\alpha+2\nu+3}{4}; \right. \\ & \left. 1+\nu, \frac{7 \pm 1 \pm 2\mu}{8} \mp \frac{\alpha+\nu}{2}, \frac{3 \pm 1 \pm 2\mu}{8} \mp \frac{\alpha+\nu}{2}; \frac{4(-1)^\nu c^2}{b^4} \right) - \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{2^{\alpha-\mu/2-1} b^\mu}{\pi c^{\alpha+\mu/2}} \times \\ & \times \Gamma \left(\frac{2\alpha+2\nu+\mu}{4} \right) \Gamma \left(\frac{2\alpha-2\nu+\mu}{4} \right) \cos^\nu \frac{2\alpha-2\nu+\mu}{4} \pi {}_4F_3 \left(-\frac{\mu}{4}, \frac{1-\mu}{4}, \frac{2-\mu}{4}, \right. \\ & \left. \frac{3-\mu}{4}; 1-\frac{2\alpha+2\nu+\mu}{4}, 1-\frac{2\alpha-2\nu+\mu}{4}, \frac{1}{2}; \frac{4(-1)^\nu c^2}{b^4} \right) - \\ & - \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{2^{\alpha-\mu/2-3} b^{\mu-2}}{\pi c^{\alpha+\mu/2-1}} \mu(1-\mu) \Gamma \left(\frac{2\alpha+2\nu+\mu-2}{4} \right) \times \end{aligned}$$

$$\times \Gamma\left(\frac{2\alpha-2\nu+\mu-2}{4}\right) \sin \gamma \frac{2\alpha-2\nu+\mu}{4} {}_4F_3\left(\frac{2-\mu}{4}, \frac{3-\mu}{4}, 1-\frac{\mu}{4}, \frac{5-\mu}{4}; \frac{6-2\alpha+2\nu-\mu}{4}, \frac{6-2\alpha-2\nu-\mu}{4}, \frac{3}{2}; \frac{4(-1)^\gamma c^2}{b^4}\right).$$

2.14. Функции Ганкеля $H_\nu^{(1)}(x)$ и $H_\nu^{(2)}(x)$

Функции $H_\nu^{(j)}(x)$, $j=1, 2$, связаны с функциями $J_\nu(x)$, $Y_\nu(x)$ формулой

$$H_\nu^{(j)}(cx) = J_\nu(cx) - (-1)^j i Y_\nu(cx), \quad j=1, 2.$$

2.14.1. Интегралы, содержащие элементарные функции и $H_\nu^{(j)}(cx)$.

Обозначение: $j=1, 2$.

$$1. \int_a^\infty x^{1-\nu} (x^2 - a^2)^{\beta-1} H_\nu^{(j)}(cx) dx = 2^{\beta-1} \Gamma(\beta) a^{\beta-\nu} c^{-\beta} H_{\nu-\beta}^{(j)}(ac)$$

$$[a, \operatorname{Re} \beta > 0; (-1)^j \operatorname{Im} c < 0]$$

$$2. \int_0^\infty e^{-px} H_\nu^{(j)}(cx) dx = \frac{(-1)^j i}{\sqrt{p^2 + c^2} \sin \nu \pi} \left[\frac{(p + \sqrt{p^2 + c^2})^\nu}{c^\nu} - \frac{e^{(-1)^j i \nu \pi} c^\nu}{(p + \sqrt{p^2 + c^2})^\nu} \right]$$

$$[\operatorname{Re} p > |\operatorname{Im} c|; |\operatorname{Re} \nu| < 1; \nu \neq 0]$$

$$3. \int_0^\infty e^{-px} H_0^{(j)}(cx) dx = \frac{1}{\sqrt{p^2 + c^2}} \left[1 + (-1)^j \frac{2i}{\pi} \ln \frac{p + \sqrt{p^2 + c^2}}{c} \right]$$

$$[\operatorname{Re} p > |\operatorname{Im} c|]$$

$$4. \int_0^\infty x e^{-px} H_0^{(j)}(cx) dx = \frac{p}{(p^2 + c^2)^{3/2}} \left[1 + \frac{2(-1)^j i}{\pi} \ln \frac{p + \sqrt{p^2 + c^2}}{c} \right] - \frac{2(-1)^j i}{\pi (p^2 + c^2)}$$

$$[\operatorname{Re} p > |\operatorname{Im} c|]$$

$$5. \int_0^\infty x e^{-px} H_1^{(j)}(cx) dx =$$

$$= \frac{1}{p^2 + c^2} \left[\frac{c}{\sqrt{p^2 + c^2}} \left(1 + \frac{2(-1)^j i}{\pi} \ln \frac{p + \sqrt{p^2 + c^2}}{c} \right) + \frac{2i(-1)^j p}{\pi c} \right]$$

$$[\operatorname{Re} p > |\operatorname{Im} c|]$$

$$6. \int_0^\infty e^{ipx} H_0^{(j)}(cx) dx = \frac{1 + (-1)^j}{\sqrt{c^2 - p^2}} - \frac{2(-1)^j}{\pi \sqrt{c^2 - p^2}} \arccos \frac{p}{c}$$

$$[0 \leq p < c]$$

$$7. = 2i \frac{1 + (-1)^j}{\sqrt{p^2 - c^2}} + \frac{(-1)^j}{\pi \sqrt{p^2 - c^2}} \ln \frac{p - \sqrt{p^2 - c^2}}{c}$$

$$[0 < c < p]$$

$$8. \int_0^\infty e^{icx} H_0^{(1)}(cx) dx = \frac{2}{\pi c}$$

$$[c > 0]$$

$$9. \int_0^\infty e^{-px^2} H_\nu^{(j)}(cx) dx =$$

$$= \frac{\sqrt{\pi}}{2 \sqrt{p}} \sec \frac{\nu \pi}{2} \exp\left(-\frac{c^2}{8p}\right) \left[e^{(-1)^j i \nu \pi / 2} I_{\nu/2}\left(\frac{c^2}{8p}\right) + \frac{(-1)^j i}{\pi} K_{\nu/2}\left(\frac{c^2}{8p}\right) \right]$$

$$[\operatorname{Re} p > 0; |\operatorname{Re} \nu| < 1]$$

$$10. \int_0^{\infty} x^{\alpha-1} e^{-px^2} H_{\nu}^{(j)}(cx) dx =$$

$$= \frac{p^{(1-\alpha)/2}}{\pi c} \Gamma\left(\frac{\alpha+\nu}{2}\right) \Gamma\left(\frac{\alpha-\nu}{2}\right) \exp\left[(-1)^j i \frac{\nu\pi}{2} - \frac{c^2}{8p}\right] W_{(1-\alpha)/2, \nu/2}\left(\frac{c^2}{4p} e^{(-1)^j i\pi}\right)$$

[Re p > 0; Re α > |Re ν|]

$$11. \int_0^{\infty} x^{\alpha-1} e^{-px^2} H_1^{(j)}(cx) dx =$$

$$= \frac{(-1)^{j-1} ic}{p^{(\alpha-1)/2}} \sec \frac{\alpha\pi}{2} \Gamma\left(\frac{\alpha+1}{2}\right) \exp\left(-\frac{c^2}{8p}\right) k_{1-\alpha}\left(\frac{c^2}{8p} e^{(-1)^j i\pi}\right)$$

[Re p > 0; Re α > 1]

$$12. \int_0^{\infty} x^{1-\nu} e^{-px^2} H_{\nu}^{(j)}(cx) dx = (-1)^j \frac{(2p)^{\nu-1} i}{\pi c^{\nu}} \Gamma(1-\nu) \Gamma\left(\nu, e^{(-1)^j i\pi} \frac{c^2}{4p}\right)$$

[Re p > 0; Re ν < 1]

$$13. \int_0^{\infty} \frac{1}{x} e^{-p/x} H_{\nu}^{(j)}(cx) dx = 2 H_{\nu}^{(j)}(\sqrt{2pc}) K_{\nu}(\sqrt{2pc})$$

[Re p > 0; (-1)^j Im c < 0]

2.14.2. Интегралы, содержащие элементарные функции и $H_{\nu}^{(j)}(\varphi(x))$.

Обозначение $j=1, 2$.

$$1. \int_0^{\infty} e^{-px} H_0^{(j)}(c \sqrt{x^2 - b^2}) dx =$$

$$= \frac{\exp(-b \sqrt{p^2 + c^2})}{\sqrt{p^2 + c^2}} \left[1 + \frac{2(-1)^j i}{\pi} \ln \frac{p + \sqrt{p^2 + c^2}}{c} \right]$$

[b > 0; Re p > |Im c|]

$$2. \int_0^{\infty} x e^{-px} H_0^{(j)}(c \sqrt{x^2 - b^2}) dx = \frac{\exp(-b \sqrt{p^2 + c^2})}{p^2 + c^2} \times$$

$$\times \left[p \left(b + \frac{1}{\sqrt{p^2 + c^2}} \right) \left(1 + \frac{2(-1)^j i}{\pi} \ln \frac{p + \sqrt{p^2 + c^2}}{c} \right) - \frac{2(-1)^j i}{\pi} \right]$$

[b > 0; Re p > |Im c|]

$$3. \int_0^{\infty} \sqrt{x^2 - b^2} e^{-px} H_0^{(j)}(c \sqrt{x^2 - b^2}) dx = \frac{\exp(-b \sqrt{p^2 + c^2})}{p^2 + c^2} \times$$

$$\times \left[c \left(b + \frac{1}{\sqrt{p^2 + c^2}} \right) \left(1 + \frac{2(-1)^j i}{\pi} \ln \frac{p + \sqrt{p^2 + c^2}}{c} \right) + \frac{2i(-1)^j p}{\pi c} \right]$$

[b > 0; Re p > |Im c|]

$$4. \int_{-\infty}^{\infty} e^{ipx} H_0^{(j)}(c \sqrt{b^2 - x^2}) dx = \frac{2(-1)^{j+1}}{\sqrt{c^2 + p^2}} \exp[(-1)^{j+1} ib \sqrt{c^2 + p^2}]$$

[Im c = Im p = 0; $0 \leq (-1)^{j+1} \arg \sqrt{b^2 - x^2} < \pi$; $0 \leq (-1)^{j+1} \arg b < \pi$]

$$5. \int_{-\infty}^{\infty} e^{\nu\varphi - \mu x} H_{\nu}^{(j)}(\sqrt{a^2 + b^2 + 2ab \operatorname{ch} x}) dx = (-1)^{j+1} \pi i H_{\nu-\mu}^{(j)}(a) H_{\mu}^{(j)}(b)$$

[a, b > 0; $\operatorname{th} \varphi = b \operatorname{sh} x / (a + b \operatorname{ch} x)$]

$$6. \int_0^{\pi} \frac{\cos 2nx}{\sqrt{a^2 - b^2 \sin^2 x}} H_1^{(j)}(x) dx =$$

$$= \frac{(-1)^n \pi}{2a \sqrt{a^2 - b^2}} \left[(a + \sqrt{a^2 - b^2}) J_n \left(\frac{a - \sqrt{a^2 - b^2}}{2} \right) H_{n+1}^{(j)} \left(\frac{a + \sqrt{a^2 - b^2}}{2} \right) - \right.$$

$$\left. - (a - \sqrt{a^2 - b^2}) J_{n+1} \left(\frac{a - \sqrt{a^2 - b^2}}{2} \right) H_n^{(j)} \left(\frac{a + \sqrt{a^2 - b^2}}{2} \right) \right]$$

[0 < b ≤ a]

$$7. \int_0^{2\pi} \frac{\sin x \sin (n+1)x}{a^2 + b^2 + 2ab \cos x} H_2^{(j)}(x) dx = \frac{(-1)^n \pi}{b^2 - a^2} [J_n(a) H_{n+2}^{(j)}(b) - J_{n+2}(a) H_n^{(j)}(b)]$$

[0 < a < b]

$$8. \int_0^{2\pi} \frac{1}{\sqrt{a^2 + b^2 + 2ab \cos x}} \left\{ \frac{\sin(x/2) \sin[(2n+1)x/2]}{\cos(x/2) \cos[(2n+1)x/2]} \right\} H_1^{(j)}(x) dx =$$

$$= \frac{(-1)^n \pi}{b \mp a} [J_n(a) H_{n+1}^{(j)}(b) \mp J_{n+1}(a) H_n^{(j)}(b)]$$

[0 < a < b]

$$9. \int_0^{\infty} \cos ax H_0^{(j)}(c \sqrt{b^2 - x^2}) dx = \frac{(-1)^j i}{\sqrt{a^2 + c^2}} \exp [(-1)^{j+1} ib \sqrt{a^2 + c^2}]$$

[a, c > 0; 0 ≤ (-1)^{j+1} arctg(b^2 - x^2) < π]

$$10. \int_0^{\pi} \cos 2nx H_0^{(j)}(\sqrt{a^2 - b^2 \sin^2 x}) dx = (-1)^n \pi J_n \left(\frac{a - \sqrt{a^2 - b^2}}{2} \right) \times$$

$$\times H_n^{(j)} \left(\frac{a + \sqrt{a^2 - b^2}}{2} \right)$$

[0 < b ≤ a]

$$11. \int_0^{2\pi} \cos nx H_0^{(j)}(\sqrt{a^2 + b^2 + 2ab \cos x}) dx = (-1)^n 2\pi J_n(a) H_n^{(j)}(b)$$

[0 < a < b].

2.14.3. Интегралы, содержащие $J_\nu(bx)$ и $H_\nu^{(j)}(cx)$.

Обозначение: $j=1, 2$.

$$1. \int_0^{\infty} x^{\alpha-1} J_\mu(bx) H_\nu^{(j)}(cx) dx =$$

$$= \frac{2^{\alpha-1} b^\mu}{\pi c^{\alpha+\mu}} e^{(-1)^{j+1}(\alpha+\mu-\nu-1)/2} \Gamma \left[\frac{(\alpha+\mu+\nu)/2, (\alpha+\mu-\nu)/2}{\mu+1} \right] \times$$

$$\times {}_2F_1 \left(\frac{\alpha+\mu+\nu}{2}, \frac{\alpha+\mu-\nu}{2}; \mu+1; \frac{b^2}{c^2} \right)$$

[|b| < |c|; Re(tc + (-1)^{j+1}ib) > 0
при Re(α + μ ± ν) > 0; Re(tc + (-1)^{j+1}ib) = 0
при Re(α + μ ± ν) > 0, Re α < 2].

$$2. \int_0^{\infty} \frac{x^{\mu+1}}{(x^2 + z^2)^{\nu/2}} J_\mu(bx) H_\nu^{(j)}(c \sqrt{x^2 + z^2}) dx =$$

$$= \frac{b^\mu (c^2 - b^2)^{(\nu-\mu-1)/2}}{c^{\nu/2} \nu^{\mu-1}} H_{\nu-\mu-1}^{(j)}(z \sqrt{c^2 - b^2})$$

[0 < b < c; Re z, Re(μ - ν) > 0; Re μ > -1]

$$3. \int_0^{\infty} \frac{1}{x^2} J_0\left(\frac{b}{x}\right) H_{\nu}^{(1)}(cx) H_{\nu}^{(2)}(cx) dx =$$

$$= \frac{16}{\pi^2 b} \cos \nu \pi K_{2\nu}(2e^{\pi i/4} \sqrt{bc}) K_{2\nu}(2e^{-\pi i/4} \sqrt{bc})$$

$$[b, \operatorname{Re} c > 0; |\operatorname{Re} \nu| < 1/4]$$

$$2. \int_0^{\infty} x^{\alpha-1} e^{-px^r} I_{\nu}(cx) dx =$$

$$= \frac{c^{\nu}}{2^{\nu} r p^{(\alpha+\nu)/r} \Gamma(\nu+1)} \sum_{k=0}^{\infty} \frac{1}{k! (\nu+1)_k} \Gamma\left(\frac{2k+\alpha+\nu}{r}\right) \left(\frac{c}{2p^{1/r}}\right)^{2k}$$

[Re p, Re(\alpha+\nu) > 0; r > 1 (r=1 см. в 2.15.3.2)]

$$3. \int_0^a x^{\alpha-1} (a^r - x^r)^{\beta-1} e^{-cx} I_{\nu}(cx) dx =$$

$$= \frac{a^{\alpha+\nu+r\beta-r} c^{\nu}}{2^{\nu} r \Gamma(\nu+1)} \sum_{k=0}^{\infty} \frac{(\nu+1/2)_k (-2ac)^k}{k! (2\nu+1)_k} \Gamma\left[\frac{(k+\alpha+\nu)/r}{\beta+(k+\alpha+\nu)/r}\right]$$

[a, r, Re \beta, Re(\alpha+\nu) > 0]

$$4. \int_a^{\infty} x^{\alpha-1} (x^r - a^r)^{\beta-1} e^{-cx} I_{\nu}(cx) dx =$$

$$= \frac{a^{\alpha+\nu+r\beta-r} c^{\nu}}{2^{\nu} r} \Gamma\left[\frac{\beta}{\nu+1}\right] \sum_{k=0}^{\infty} \frac{(\nu+1/2)_k (-2ac)^k}{k! (2\nu+1)_k} \Gamma\left[\frac{1-\beta-(\alpha+\nu+k)/r}{1-(\alpha+\nu+k)/r}\right] +$$

$$+ \frac{(2c)^{r-r\beta-\alpha}}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(1-\beta)_k}{k!} \Gamma\left[\frac{\nu+\alpha-r(1-\beta+k), 1/2-\alpha+r(1-\beta+k)}{1+\nu-\alpha+r(1-\beta+k)}\right] (2ac)^{rk}$$

[a, r, Re c, Re \beta > 0; Re(\alpha+r\beta) < r+1/2]

$$5. \int_0^{\infty} \frac{x^{\alpha-1}}{(x^r + z^r)^{\rho}} e^{-cx} I_{\nu}(cx) dx =$$

$$= \frac{c^{\nu} z^{\alpha+\nu-r\rho}}{2^{\nu} r \Gamma(\nu+1) \Gamma(\rho)} \sum_{k=0}^{\infty} \frac{(\nu+1/2)_k (-2cz)^k}{k! (2\nu+1)_k} \Gamma\left(\frac{k+\alpha+\nu}{r}\right) \Gamma\left(\rho - \frac{k+\alpha+\nu}{r}\right) +$$

$$+ \frac{(2c)^{r\rho-\alpha}}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k (\rho)_k}{k!} \Gamma\left[\frac{\alpha+\nu-r\rho-rk, 1/2-\alpha+r\rho+rk}{1+\nu-\alpha+r\rho+rk}\right] (2cz)^{rk}$$

[r, Re c, Re(\alpha+\nu) > 0; Re(\alpha-r\rho) < 1/2]

$$6. \int_0^{\infty} \frac{x^{\alpha-1}}{x^r - y^r} e^{-cx} I_{\nu}(cx) dx =$$

$$= -\frac{c^{\nu} y^{\alpha+\nu-r}}{2^{\nu} r \Gamma(\nu+1)} \sum_{k=0}^{\infty} \frac{(\nu+1/2)_k (-2cy)^k}{k! (2\nu+1)_k} \operatorname{ctg} \frac{\alpha+\nu+k}{r} \pi +$$

$$+ \frac{(2c)^{r-\alpha}}{\sqrt{\pi}} \sum_{k=0}^{\infty} \Gamma\left[\frac{\alpha+\nu-r-rk, 1/2-\alpha+r+rk}{1+\nu-\alpha+r+rk}\right] (2cy)^{rk}$$

[r, y, Re c, Re(\alpha+\nu) > 0; Re \alpha < r+1/2]

$$7. \int_0^{\infty} x^{\alpha-1} e^{-px^r - cx} I_{\nu}(cx) dx = U$$

[r, Re p, Re(\alpha+\nu), Re c > 0]

$$U = \left(\frac{c}{2}\right)^{\nu} \frac{p^{-(\alpha+\nu)r}}{r \Gamma(\nu+1)} \sum_{k=0}^{\infty} \frac{(\nu+1/2)_k}{k! (2\nu+1)_k} \Gamma\left(\frac{\alpha+\nu+k}{r}\right) \left(-\frac{2c}{p^{1/r}}\right)^{2k}$$

[r > 1]

$$U = \frac{p^{(1-2\alpha)/(2r)}}{r \sqrt{2\pi c}} \sum_{k=0}^{\infty} \frac{(1/2+v)_k (1/2-v)_k}{k!} \Gamma\left(\frac{2\alpha-2k-1}{2r}\right) \left(\frac{p^{1/r}}{2c}\right)^k +$$

$$+ \frac{1}{\sqrt{\pi} (2c)^\alpha} \sum_{k=0}^{\infty} \frac{1}{k!} \Gamma\left[\frac{\alpha+v+kr, 1/2-\alpha-kr}{1+v-\alpha-kr}\right] \left(-\frac{p}{2rc^r}\right)^k \quad |r < 1|$$

U при $r=1$ см. в 2.15.3.2.

$$8. \int_0^{\infty} x^{\alpha-1} e^{-px^{-r}-cx} I_\nu(cx) dx =$$

$$= \left(\frac{c}{2}\right)^\nu \frac{p^{(\alpha+v)/r}}{r \Gamma(v+1)} \sum_{k=0}^{\infty} \frac{(v+1/2)_k}{k! (2v+1)_k} \Gamma\left(-\frac{k+\alpha+v}{r}\right) (-2cp^{1/r})^k +$$

$$+ \frac{1}{\sqrt{\pi} (2c)^\alpha} \sum_{k=0}^{\infty} \frac{1}{k!} \Gamma\left[\frac{\alpha+v-kr, 1/2-\alpha+kr}{1+v-\alpha+kr}\right] (-2^r c^r p)^k$$

$|r, \operatorname{Re} \alpha, \operatorname{Re} p > 0; \operatorname{Re} x < 1/2|$

$$9. \int_0^{\infty} x^{\alpha-1} e^{-px^r} \left\{ \frac{\sin bx^r}{\cos bx^r} \right\} I_\nu(cx) dx =$$

$$= \frac{2^{-\nu} c^\nu}{r (\sqrt{b^2+p^2})^{(\alpha+v)/r} \Gamma(v+1)} \sum_{k=0}^{\infty} \frac{1}{k! (v+1)_k} \Gamma\left(\frac{\alpha+v+2k}{r}\right) \times$$

$$\times \cos\left(\frac{\delta\pi}{2} - \frac{\alpha+v+2k}{r} \arccos \frac{p}{\sqrt{b^2+p^2}}\right) \frac{c^{2k}}{2^{2k} (b^2+p^2)^{k/r}}$$

$[b > 0; \operatorname{Re}(\alpha+v) > -\delta r; \operatorname{Re} p > 0]$

при $r > 1; \operatorname{Re} p > |\operatorname{Re} \alpha|, |\alpha|^2 < |b^2+p^2|$ при $r=1; \delta = \left\{ \begin{smallmatrix} 1 \\ 0 \end{smallmatrix} \right\}$.

$$10. \int_0^{\infty} x^{\alpha-1} e^{-cx} \left\{ \frac{\sin bx^r}{\cos bx^r} \right\} I_\nu(cx) dx = V$$

$[b, r, \operatorname{Re} c > 0; \operatorname{Re}(\alpha+v) > -\delta r; \operatorname{Re} \alpha < r+1/2; \delta = \left\{ \begin{smallmatrix} 1 \\ 0 \end{smallmatrix} \right\}]$,

$$V = \frac{b^{-(\alpha+v)/r}}{r \Gamma(v+1)} \left(\frac{c}{2}\right)^\nu \sum_{k=0}^{\infty} \frac{(v+1/2)_k}{k! (2v+1)_k} \Gamma\left(\frac{k+\alpha+v}{r}\right) \times$$

$$\times \left\{ \frac{\sin [(k+\alpha+v) \pi/(2r)]}{\cos [(k+\alpha+v) \pi/(2r)]} \right\} \left(-\frac{2c}{b^{1/r}}\right)^k \quad |r > 1|$$

$$V = \frac{b^{(1-2\alpha)/(2r)}}{r \sqrt{2\pi c}} \sum_{k=0}^{\infty} \frac{(1/2+v)_k (1/2-v)_k}{k!} \Gamma\left(\frac{2\alpha-2k-1}{2r}\right) \times$$

$$\times \left\{ \frac{\sin [(2\alpha-2k-1) \pi/(4r)]}{\cos [(2\alpha-2k-1) \pi/(4r)]} \right\} \left(\frac{b^{1/r}}{2c}\right)^k +$$

$$+ \frac{b^\delta}{\sqrt{\pi} (2c)^{\alpha+r\delta}} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (\delta+1/2)_k} \Gamma\left[\frac{\alpha+v+r\delta+2rk, 1/2-\alpha-r\delta-2rk}{1+v-\alpha-r\delta-2rk}\right] \left(\frac{b}{2^{r+1}c^r}\right)^k$$

$|r < 1|$

U при $r=1$ см в 2.15.11.8.

$$11. \int_0^{\infty} x^{\alpha-1} e^{-px^r - cx} \begin{Bmatrix} \sin bx^r \\ \cos bx^r \end{Bmatrix} I_{\nu}(cx) dx = W$$

$$[b, r, \operatorname{Re} c, \operatorname{Re} p > 0; \operatorname{Re}(\alpha + \nu) > -\delta r; \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}],$$

$$W = \left(\frac{c}{2}\right)^{\nu} \frac{(\sqrt{b^2 + p^2})^{-(\alpha + \nu)/r}}{r \Gamma(\nu + 1)} \sum_{k=0}^{\infty} \frac{(\nu + 1/2)_k}{k! (2\nu + 1)_k} \times \\ \times \Gamma\left(\frac{\alpha + \nu + k}{r}\right) \cos\left(\frac{\delta\pi}{2} - \frac{\alpha + \nu + k}{r} \arccos \frac{p}{\sqrt{b^2 + p^2}}\right) \frac{(-2c)^k}{(\sqrt{b^2 + p^2})^{k/r}}$$

$$[r > 1] \text{ или } [r = 1, 4|c|^2 < |b^2 + p^2|]$$

$$W = \frac{(b^2 + p^2)^{(1-2\alpha)/(4r)}}{r \sqrt{2\pi c}} \sum_{k=0}^{\infty} \frac{(1/2 + \nu)_k (1/2 - \nu)_k}{k!} \Gamma\left(\frac{2\alpha - 2k - 1}{2r}\right) \times \\ \times \cos\left(\frac{\delta\pi}{2} + \frac{1 - 2\alpha + 2k}{2r} \arccos \frac{p}{\sqrt{b^2 + p^2}}\right) \frac{(\sqrt{b^2 + p^2})^{k/r}}{(2c)^k} + \\ + \frac{(2c)^{-\alpha}}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{1}{k!} \Gamma\left[\frac{\alpha + \nu + rk, 1/2 - \alpha - rk}{1 + \nu - \alpha - rk}\right] \left(-\frac{\sqrt{b^2 + p^2}}{2rc^r}\right)^k$$

$$[r < 1] \text{ или } [r = 1; 4|c|^2 > |b^2 + p^2|]$$

$$12. \int_0^{\infty} x^{\alpha-1} e^{-cx} \begin{Bmatrix} \ln(x^r + z^r) \\ \ln|x^r - z^r| \end{Bmatrix} I_{\nu}(cx) dx =$$

$$= \left(\frac{c}{2}\right)^{\nu} \frac{\pi z^{\alpha}}{\Gamma(\nu + 1)} \sum_{k=0}^{\infty} \frac{(\nu + 1/2)_k}{k! (2\nu + 1)_k} \begin{Bmatrix} \operatorname{cosec}[(\alpha + \nu + k)\pi/r] \\ \operatorname{ctg}[(\alpha + \nu + k)\pi/r] \end{Bmatrix} (-2cz)^k - \\ - \frac{(2c)^{-\alpha}}{\sqrt{\pi}} \sum_{k=1}^{\infty} \frac{(-1)^k}{k} \Gamma\left[\frac{\nu + \alpha - rk, 1/2 - \alpha + rk}{1 + \nu - \alpha + rk}\right] (2cz)^{rk} + \\ + \frac{(2c)^{-\alpha r}}{\sqrt{\pi}} \Gamma\left[\frac{\nu + \alpha, 1/2 - \alpha}{1 + \nu - \alpha}\right] [\psi(\nu + \alpha) + \psi(1 + \nu - \alpha) - \psi(1/2 - \alpha) - \ln(2c)]$$

$$[r, \operatorname{Re} c, \operatorname{Re}(\alpha + \nu) > 0; \operatorname{Re} \alpha < 1/2, \begin{Bmatrix} r|\arg z| < \pi \\ z > 0 \end{Bmatrix}]$$

$$13. \int_0^{\infty} x^{\alpha-1} e^{-cx} J_{\mu}(bx^r) I_{\nu}(cx) dx = X$$

$$[b, r, \operatorname{Re} c, \operatorname{Re}(\alpha + \nu + r\mu) > 0; \operatorname{Re} \alpha < (3r + 1)/2]$$

$$X = \frac{2^{(\alpha + \nu)/r - \nu - 1} c^{\nu}}{b^{(\alpha + \nu)/r} r \Gamma(\nu + 1)} \sum_{k=0}^{\infty} \frac{(\nu + 1/2)_k}{k! (2\nu + 1)_k} \Gamma\left[\frac{(r\mu + \alpha + \nu + k)/(2r)}{1 + (r\mu - \alpha - \nu - k)/(2r)}\right] \left(-\frac{2^{1+1/r} c}{b^{1/r}}\right)^k$$

$$[r > 1],$$

$$X = \frac{2^{(2\alpha - 3r - 1)/(2r)}}{r \sqrt{\pi c} b^{(2\alpha - 1)/(2r)}} \sum_{k=0}^{\infty} \frac{(1/2 + \nu)_k (1/2 - \nu)_k}{k!} \times \\ \times \Gamma\left[\frac{(2\mu r + 2\alpha - 2k - 1)/(4r)}{1 + (2\mu r - 2\alpha + 2k + 1)/(4r)}\right] \left(\frac{b^{1/r}}{2^{1+1/r} c}\right)^k + \\ + \frac{2^{-\alpha - \mu - r\mu} b^{\mu}}{c^{\alpha + \mu r} \sqrt{\pi} \Gamma(\mu + 1)} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (\mu + 1)_k} \times \\ \times \Gamma\left[\frac{\alpha + \nu + \mu r + 2rk, 1/2 - \alpha - \mu r - 2rk}{1 + \nu - \alpha - \mu r - 2rk}\right] \left(\frac{b}{2^{r+1} c^r}\right)^{2k}$$

$$[r < 1]$$

X при $r = 1$ см в 2.15.16.3

$$14. \int_0^{\infty} x^{\alpha-1} e^{-bx^r-cx} I_{\mu}(bx^r) I_{\nu}(cx) dx = Z$$

$$\{r \geq 1; \operatorname{Re} b, \operatorname{Re} c > 0; -\operatorname{Re}(\mu r + \nu) < \operatorname{Re} \alpha < (r+1)/2\}$$

$$Z = \frac{2^{-\nu-(\alpha+\nu)/r} c^{\nu}}{r \sqrt{\pi} b^{(\alpha+\nu)/r} \Gamma(\nu+1)} \sum_{k=0}^{\infty} \frac{(1/2+\nu)_k}{k! (2\nu+1)_k} \times \\ \times \Gamma \left[\frac{\mu + (\alpha + \nu + k)/r, 1/2 - (\alpha + \nu + k)/r}{1 + \mu - (\alpha + \nu + k)/r} \right] \left(-\frac{2^{1-1/r} c}{b^{1/r}} \right)^k + \\ + \frac{c^{r/2-\alpha}}{2^{\alpha+(1-r)/2} \pi \sqrt{b}} \sum_{k=0}^{\infty} \frac{(1/2+\mu)_k (1/2-\mu)_k}{k!} \times \\ \times \Gamma \left[\frac{\alpha + \nu - r/2 - rk, (1+r)/2 + rk - \alpha}{1 + \nu - \alpha + r/2 + kr} \right] \left(\frac{2^{r-1} c^r}{b} \right)^k \quad [r > 1]$$

Z при $r=1$ см в 2.15.20.6

$$15. \int_0^{\infty} x^{\alpha-1} e^{-cx} \left\{ \begin{matrix} Y_{\mu}(bx^r) \\ K_{\mu}(bx^r) \end{matrix} \right\} I_{\nu}(cx) dx = U_{\mu}$$

$$\left[r, \operatorname{Re} c > 0; \operatorname{Re}(\alpha + \nu) > r + |\operatorname{Re} \mu|, \left\{ \begin{matrix} b > 0; \operatorname{Re} \alpha < (3r+1)/2 \\ \operatorname{Re} b > 0 \end{matrix} \right\} \right],$$

$$U_{\mu} = \mp \frac{2^{\nu+(\alpha+\nu)/r-2} c^{\nu}}{\pi^{3/2} b^{(\alpha+\nu)/r} r \Gamma(\nu+1)} \left\{ \begin{matrix} 2 \\ \pi \end{matrix} \right\} \sum_{k=0}^{\infty} \frac{(\nu+1/2)_k}{k! (2\nu+1)_k} \times \\ \times \Gamma \left(\frac{\alpha + \nu + k + \mu r}{2r} \right) \Gamma \left(\frac{\alpha + \nu + k - \mu r}{2r} \right) \cos^{(1 \pm 1)/2} \frac{\alpha + \nu + k - \mu r}{2} \pi \left(-\frac{2^{1/r+1} c}{b^{1/r}} \right)^k \quad [r > 1],$$

$$U_{\mu} = \mp \frac{(2c)^{-\alpha}}{2\pi^{3/2}} \left\{ \begin{matrix} 2 \\ \pi \end{matrix} \right\} \sum_{k=0}^{\infty} \frac{(\mp 1)^k}{k!} \left[A_k(\mu) + \left\{ \begin{matrix} \cos \mu \pi \\ 1 \end{matrix} \right\} A_k(-\mu) \right] \left(\frac{b}{2^{r+1} c^r} \right)^{2k} + V_{\mu},$$

$$A_k(\mu) = \left(\frac{2^{r+1} c^r}{b} \right)^{\mu} \frac{\Gamma(\mu)}{(1-\mu)_k} \Gamma \left[\frac{\alpha + \nu - \mu r + 2rk, 1/2 - \alpha + \mu r - 2rk}{1 + \nu - \alpha + \mu r - 2rk} \right],$$

$$V_{\mu} = \mp \frac{2^{(2\alpha-5r-1)/(2r)}}{\pi^{3/2} b^{(2\alpha-1)/(2r)} \sqrt{c} r} \left\{ \begin{matrix} 2 \\ \pi \end{matrix} \right\} \sum_{k=0}^{\infty} \frac{(1/2+\nu)_k (1/2-\nu)_k}{k!} \times \\ \times \Gamma \left(\frac{r\mu + \alpha - k - 1/2}{2r} \right) \Gamma \left(\frac{\alpha - r\mu - k - 1/2}{2r} \right) \cos^{(1 \pm 1)/2} \frac{\mu r - \alpha + k + 1/2}{2r} \pi \left(\frac{b^{1/r}}{2^{1/r+1} c} \right)^k$$

$$[r < 1, \mu \neq \dots, -2, -1, 0, 1, 2, \dots, l - 2rk \neq \alpha + \mu r - 1/2, \alpha - \mu r - 1/2; l, k = 0, 1, 2, \dots]$$

$$U_n = \mp \frac{(2c)^{-\alpha}}{2\pi^{3/2}} \left\{ \begin{matrix} 2 \\ \pi \end{matrix} \right\} \left\{ \sum_{k=0}^{n-1} \frac{(\mp 1)^k}{k!} A_k(n) \left(\frac{b}{2^{r+1} c^r} \right)^{2k} + \right. \\ \left. + \left(\pm \frac{b}{2^{r+1} c^r} \right)^n \sum_{k=0}^{\infty} \frac{(\mp 1)^k}{k! (n+k)!} \Gamma \left[\frac{\alpha + \nu + nr + 2rk, 1/2 - \alpha - nr - 2rk}{1 + \nu - \alpha - nr - 2rk} \right] \times \right. \\ \left. \times \left[\psi(k+1) + \psi(n+k+1) - 2r\psi(\alpha + \nu + nr + 2rk) + 2r\psi \left(\frac{1}{2} - \alpha - nr - 2rk \right) - \right. \right. \\ \left. \left. - 2r\psi(1 + \nu - \alpha - nr - 2rk) - 2 \ln \frac{b}{2^{r+1} c^r} \right] \left(\frac{b}{2^{r+1} c^r} \right)^{2k} + V_n \right.$$

$$[r < 1, l - 2rk \neq \alpha + \mu r - 1/2, \alpha - \mu r - 1/2, l, k, n = 0, 1, 2,$$

U_{μ} при $r=1$ см в 2.15 18.1, 2.16.29.2.

$$16. \int_0^{\infty} x^{\alpha-1} \text{Ei}(-bx^r) I_{\nu}(cx) dx =$$

$$= -\frac{c^{\nu}}{2^{\nu} b^{(\alpha+\nu)/r} \Gamma(\nu+1)} \sum_{k=0}^{\infty} \frac{1}{k! (\nu+1)_k (2k+\alpha+\nu)} \Gamma\left(\frac{\alpha+\nu+2k}{r}\right) \left(\frac{c}{2b^{1/r}}\right)^{2k}$$

$$[r \geq 1; \operatorname{Re}(\alpha+\nu) > 0 \quad \operatorname{Re} b > 0 \text{ при } r > 1; \operatorname{Re} b > |\operatorname{Re} c|, |c| < |b| \text{ при } r = 1]$$

$$17. \int_0^{\infty} x^{\alpha-1} e^{-cx} \text{Ei}(-bx^r) I_{\nu}(cx) dx = \hat{X},$$

$$[r, \operatorname{Re} b, \operatorname{Re} c, \operatorname{Re}(\alpha+\nu) > 0]$$

$$\hat{X} = -\frac{b^{-(\alpha+\nu)/r}}{\Gamma(\nu+1)} \left(\frac{c}{2}\right)^{\nu} \sum_{k=0}^{\infty} \frac{(1/2+\nu)_k}{k! (2\nu+1)_k (k+\alpha+\nu)} \Gamma\left(\frac{k+\alpha+\nu}{r}\right) \left(-\frac{2c}{b^{1/r}}\right)^k$$

$$[r > 1]$$

$$\hat{X} = \frac{b^{(1-2\alpha)/(2r)}}{\sqrt{2\pi c}} \sum_{k=0}^{\infty} \frac{(1/2+\nu)_k (1/2-\nu)_k}{k! (1/2-\alpha+k)} \Gamma\left(\frac{2\alpha-2k-1}{2r}\right) \left(\frac{b^{1/r}}{2c}\right)^k +$$

$$+ \frac{(2c)^{-\alpha}}{\sqrt{\pi}} \sum_{k=1}^{\infty} \frac{(-1)^k}{k! k} \Gamma\left[\frac{\alpha+\nu+rk, 1/2-\alpha-rk}{1+\nu-\alpha-rk}\right] \left(\frac{b}{2c^r}\right)^k +$$

$$+ \Gamma\left[\frac{\alpha+\nu, 1/2-\alpha}{1+\nu-\alpha}\right] [C + r\psi(\alpha+\nu) + r\psi(1+\nu-\alpha) - r\psi(1/2-\alpha) - r \ln(2c) + \ln b,$$

$$[r < 1]$$

$\hat{X}$ при $r=1$ см. в 2.15.24.8.

$$18. \int_0^{\infty} x^{\alpha-1} e^{\pm bx^r - cx} \text{Ei}(\mp bx^r) I_{\nu}(cx) dx = \hat{Y}$$

$$[r, \operatorname{Re} b, \operatorname{Re} c, \operatorname{Re}(\alpha+\nu) > 0; \operatorname{Re} \alpha < r + 1/2]$$

$$\hat{Y} = -\frac{b^{-(\alpha+\nu)/r}}{r \Gamma(\nu+1)} \left(\frac{c}{2}\right)^{\nu} \sum_{k=0}^{\infty} \frac{(1/2+\nu)_k}{k! (1+2\nu)_k} \times$$

$$\times \left\{ \begin{array}{l} \operatorname{cosec}[(\alpha+\nu+k)\pi/r] \\ \operatorname{ctg}[(\alpha+\nu+k)\pi/r] \end{array} \right\} \Gamma\left(\frac{\alpha+\nu+k}{r}\right) \left(-\frac{2c}{b^{1/r}}\right)^k \mp$$

$$\mp \frac{(2c)^{r-\alpha}}{b \sqrt{\pi}} \sum_{k=0}^{\infty} \frac{1}{k!} \Gamma\left[\frac{\alpha+\nu-r-rk, 1/2-\alpha+r+rk}{1+\nu-\alpha+r+rk}\right] \left(\mp \frac{2c^r}{b}\right)^k$$

$$[r > 1]$$

$$\hat{Y} = -\frac{\sqrt{\pi} b^{(1-2\alpha)/(2r)}}{r \sqrt{2c}} \sum_{k=0}^{\infty} \frac{(1/2+\nu)_k (1/2-\nu)_k}{k!} \times$$

$$\times \Gamma\left(\frac{2\alpha-2k-1}{2r}\right) \left\{ \begin{array}{l} \operatorname{cosec}[(2\alpha-2k-1)\pi/(2r)] \\ \operatorname{ctg}[(2\alpha-2k-1)\pi/(2r)] \end{array} \right\} \left(\frac{b^{1/r}}{2c}\right)^k -$$

$$- \frac{(2c)^{-\alpha}}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{1}{k!} \Gamma\left[\frac{\alpha+\nu+rk, 1/2-\alpha-rk}{1+\nu-\alpha-rk}\right] [\psi(k+1) -$$

$$- r\psi(\alpha+\nu+rk) - r\psi(1+\nu-\alpha-rk) + r\psi(1/2-\alpha-rk) +$$

$$+ r \ln(2c) - \ln b] \left(\pm \frac{b}{2c^r}\right)^k$$

$$[r < 1],$$

$\hat{Y}$ при $r=1$ см. в 2.15.24.9.

$$1. \int_0^{\infty} x^{\alpha-1} e^{-cx} \left\{ \begin{matrix} \operatorname{si} (bx^r) \\ \operatorname{ci} (bx^r) \end{matrix} \right\} I_{\nu}(cx) dx = \hat{Z}$$

$$\left[b, r, \operatorname{Re} c, \operatorname{Re}(\alpha + \nu) > 0; \operatorname{Re} \alpha < 2r + 1/2, \delta = \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \right],$$

$$Z = - \frac{b^{-(\alpha+\nu)/r}}{\Gamma(\nu+1)} \left(\frac{c}{2} \right)^{\nu} \sum_{k=0}^{\infty} \frac{(\nu+1/2)_k}{k! (2\nu+1)_k (\alpha+\nu+k)} \times \\ \times \Gamma \left(\frac{\alpha+\nu+k}{r} \right) \left\{ \begin{matrix} \sin [(\alpha+\nu+k) \pi/(2r)] \\ \cos [(\alpha+\nu+k) \pi/(2r)] \end{matrix} \right\} \left(- \frac{2c}{b^{1/r}} \right)^k \quad [r > 1],$$

$$\begin{aligned} Z = & \sqrt{\frac{2}{\pi c}} b^{(1-2\alpha)/(2r)} \sum_{k=0}^{\infty} \frac{(1/2+\nu)_k (1/2-\nu)_k}{k! (1-2\alpha+2k)} \Gamma \left(\frac{2\alpha-2k-1}{2r} \right) \times \\ & \times \left\{ \begin{matrix} \sin [(2\alpha-2k-1)\pi/(4r)] \\ \cos [(2\alpha-2k-1)\pi/(4r)] \end{matrix} \right\} \left(\frac{b^{1/r}}{2c} \right)^k + \\ & + \frac{b^{\delta}}{(2c)^{\alpha+r\delta} \sqrt{\pi}} \sum_{k=1}^{\infty} \frac{(-1)^k}{(\delta+1/2)_k k! (2k+\delta)} \times \\ & \times \Gamma \left[\begin{matrix} \alpha+\nu+r\delta+2rk, 1/2-\alpha-r\delta-2rk \\ 1+\nu-\alpha-r\delta-2rk \end{matrix} \right] \left(\frac{b}{2^{r+1}c^r} \right)^{2k} + \\ & + \frac{b^{\delta}}{(2c)^{\alpha+r\delta} \sqrt{\pi}} \Gamma \left[\begin{matrix} \alpha+\nu+r\delta, 1/2-\alpha-r\delta \\ 1+\nu-\alpha-r\delta \end{matrix} \right] \left\{ \begin{matrix} 1 \\ R \end{matrix} \right\} - \\ & - \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{(2c)^{-\alpha} \sqrt{\pi}}{2} \Gamma \left[\begin{matrix} \alpha+\nu, 1/2-\alpha \\ 1+\nu-\alpha \end{matrix} \right] \\ & \left[r < 1; R = C + r\psi(\alpha+\nu) - r\psi\left(\frac{1}{2}-\alpha\right) + r\psi(1+\nu-\alpha) - \ln \frac{(2c)^r}{b} \right], \end{aligned}$$

$\hat{Z}$ при $r=1$ см. в 2.15.25.1.

$$u. \int_0^{\infty} x^{\alpha-1} \operatorname{erfc}(bx^r) I_{\nu}(cx) dx =$$

$$= \frac{b^{-(\alpha+\nu)/r}}{\sqrt{\pi} \Gamma(\nu+1)} \left(\frac{c}{2} \right)^{\nu} \sum_{k=0}^{\infty} \frac{1}{k! (\nu+1)_k (2k+\alpha+\nu)} \Gamma \left(\frac{2k+\alpha+\nu+r}{2r} \right) \left(\frac{c}{2b^{1/r}} \right)^{2k}$$

$[r \geq 1/2, \operatorname{Re}(\alpha + \nu) > 0, |\arg b| < \pi/4$ при $r > 1/2$,
 $\operatorname{Re} b^2 > |\operatorname{Re} c|, |c| < |b|^2$ при $r = 1/2$; также см
 2.15.26 3].

$$21. \int_0^{\infty} x^{\alpha-1} e^{-cx} \left\{ \begin{matrix} \operatorname{erf}(bx^r) \\ \operatorname{erfc}(bx^r) \end{matrix} \right\} I_{\nu}(cx) dx = \hat{U}$$

$$\left[r, \operatorname{Re} c > 0, \left\{ \begin{matrix} \operatorname{Re} \alpha < 1/2; \operatorname{Re}(\alpha + \nu) > -r \\ \operatorname{Re}(\alpha + \nu) > 0 \end{matrix} \right\}, |\arg b| < \pi/4 \right],$$

$$\hat{U} = \mp \frac{c^{\nu}}{2^{\nu} \sqrt{\pi} b^{(\alpha+\nu)/r} \Gamma(\nu+1)} \sum_{k=0}^{\infty} \frac{(\nu+1/2)_k}{k! (2\nu+1)_k (k+\alpha+\nu)} \times \\ \times \Gamma \left(\frac{k+r+\alpha+\nu}{2r} \right) \left(- \frac{2c}{b^{1/r}} \right)^k + \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{(2c)^{-\alpha}}{\sqrt{\pi}} \Gamma \left[\begin{matrix} \alpha+\nu, 1/2-\alpha \\ 1+\nu-\alpha \end{matrix} \right] \\ [r > 1/2]$$

$$\hat{U} = \mp \frac{b^{(1-2\alpha)/(2r)}}{\pi \sqrt{2c}} \sum_{k=0}^{\infty} \frac{(1/2+v)_k (1/2-v)_k}{k! (\alpha - 1/2 - k)} \Gamma\left(\frac{2r+2\alpha-2k-1}{4r}\right) \times$$

$$\times \left(\frac{b^{1/r}}{2c}\right)^k \pm \frac{2b}{\pi (2c)^{\alpha+r}} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (2k+1)} \Gamma\left[\frac{\alpha+v+r+2rk, 1/2-\alpha-r-2rk}{1+v-\alpha-r-2rk}\right] \times$$

$$\times \left(\frac{b}{2rc^r}\right)^{2k} + \begin{cases} 0 \\ 1 \end{cases} \frac{(2c)^{-\alpha}}{\sqrt{\pi}} \Gamma\left[\frac{\alpha+v, 1/2-\alpha}{1+v-\alpha}\right] \quad [r < 1/2],$$

$\hat{U}$ при $r=1/2$ см. в 2.15.26.5.

$$22. \int_0^{\infty} x^{\alpha-1} e^{-cx} \left\{ \begin{matrix} S(bx^r) \\ C(bx^r) \end{matrix} \right\} I_v(cx) dx = \hat{V}$$

$$\left[b, r, \operatorname{Re} c > 0; \operatorname{Re}(\alpha+v) > -(2 \pm 1)r/2; \operatorname{Re} \alpha < (3r+1)/2; \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right],$$

$$\hat{V} = - \frac{c^v}{2^{v+1/2} \sqrt{\pi} b^{(\alpha+v)/r} \Gamma(v+1)} \sum_{k=0}^{\infty} \frac{(1/2+v)_k}{k! (2v+1)_k (\alpha+v+k)} \times$$

$$\times \Gamma\left(\frac{r+2v+2\alpha+2k}{2r}\right) \left\{ \begin{matrix} \cos[(r-2v-2\alpha-2k)\pi/(4r)] \\ \sin[(r-2v-2\alpha-2k)\pi/(4r)] \end{matrix} \right\} \times$$

$$\times \left(-\frac{2c}{b^{1/r}}\right)^k + \frac{(2c)^{-\alpha}}{2\sqrt{\pi}} \Gamma\left[\frac{\alpha+v, 1/2-\alpha}{1+v-\alpha}\right] \quad [r > 1],$$

$$\hat{V} = \frac{b^{(1-2\alpha)/(2r)}}{2\pi \sqrt{c}} \sum_{k=0}^{\infty} \frac{(1/2+v)_k (1/2-v)_k}{k! (1/2-\alpha+k)} \Gamma\left(\frac{r+2\alpha-2k-1}{2r}\right) \times$$

$$\times \left\{ \begin{matrix} \cos[(r+1-2\alpha+2k)\pi/(4r)] \\ \sin[(r+1-2\alpha+2k)\pi/(4r)] \end{matrix} \right\} \left(\frac{b^{1/r}}{2c}\right)^k +$$

$$+ \frac{b^{\delta+1/2} \sqrt{2}}{(2c)^{\alpha+r\delta+r/2} \pi} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (\delta+1/2)_k (1+2\delta+4k)} \times$$

$$\times \Gamma\left[\frac{\alpha+v+r\delta+r/2+2rk, 1/2-\alpha-r/2-r\delta-2rk}{1+v-\alpha-r/2-r\delta-2rk}\right] \left(\frac{b}{2^{r+1}c^r}\right)^{2k} \quad [r < 1],$$

$\hat{V}$ при $r=1$ см. в 2.15.27.1.

$$23. \int_0^{\infty} x^{\alpha-1} \Gamma(\mu, bx^r) I_v(cx) dx = \frac{b^{-(\alpha+v)/r}}{\Gamma(v+1)} \left(\frac{c}{2}\right)^v \times$$

$$\times \sum_{k=0}^{\infty} \frac{1}{k! (v+1)_k (2k+\alpha+v)} \Gamma\left(\mu + \frac{2k+\alpha+v}{r}\right) \left(\frac{c}{2b^{1/r}}\right)^{2k}$$

$$[r \geq 1; \operatorname{Re}(\alpha+v+\mu r), \operatorname{Re}(\alpha+v) > 0; \operatorname{Re} b > 0 \text{ при } r > 1;$$

$$\operatorname{Re} b > |\operatorname{Re} c|, |c| < |b| \text{ при } r=1; \text{ также см. 2.15.28.1}].$$

$$24. \int_0^{\infty} x^{\alpha-1} e^{-cx} \left\{ \begin{matrix} \gamma(\mu, bx^r) \\ \Gamma(\mu, bx^r) \end{matrix} \right\} I_v(cx) dx = \hat{W}$$

$$\left[r, \operatorname{Re} b, \operatorname{Re} c, \operatorname{Re}(\alpha+v+ru) > 0, \left\{ \begin{matrix} \operatorname{Re} \mu > 0; \operatorname{Re} \alpha < 1/2 \\ \operatorname{Re}(\alpha+v) > 0 \end{matrix} \right\} \right],$$

$$\hat{W} = \mp \frac{b^{-(\alpha+v)/r}}{\Gamma(v+1)} \left(\frac{c}{2}\right)^v \sum_{k=0}^{\infty} \frac{(1/2+v)_k}{k! (2v+1)_k (\alpha+v+k)} \times$$

$$\times \Gamma\left(\mu + \frac{\alpha+v+k}{r}\right) \left(-\frac{2c}{b^{1/r}}\right)^k + \begin{cases} 1 \\ 0 \end{cases} \frac{(2c)^{-\alpha}}{\sqrt{\pi}} \Gamma\left[\frac{\mu, \alpha+v, 1/2-\alpha}{1+v-\alpha}\right] \quad [r > 1],$$

$$\hat{W} = \pm \frac{b^{(1-2\alpha)/(2r)}}{\sqrt{2\pi c}} \sum_{k=0}^{\infty} \frac{(1/2+v)_k (1/2-v)_k}{k! (1/2-\alpha+k)} \Gamma\left(\mu + \frac{2\alpha-2k-1}{2r}\right) \times$$

$$\times \left(\frac{b^{1/r}}{2c}\right)^k \pm \frac{b^\mu}{\sqrt{\pi} (2c)^{\alpha+\mu r}} \sum_{k=0}^{\infty} \frac{1}{k! (\mu+k)} \times$$

$$\times \Gamma\left[\frac{\alpha+v+\mu r+rk}{1+v-\alpha-\mu r-rk}, \frac{1/2-\alpha-\mu r-rk}{1}\right] \left(-\frac{b}{2rc}\right)^k +$$

$$+ \begin{cases} 0 \\ 1 \end{cases} \frac{(2c)^{-\alpha}}{\sqrt{\pi}} \Gamma\left[\mu, \alpha+v, \frac{1/2-\alpha}{1+v-\alpha}\right] \quad [r < 1].$$

$\hat{W}$ при $r=1$ см. в 2.15.28.3.

$$25. \int_0^\infty x^{\alpha-1} e^{-b^2 x^{2r}/4} D_\mu(bx^r) I_\nu(cx) dx =$$

$$= \frac{2^{-(\alpha+v)/(2r)-v+\mu/2} \sqrt{\pi} c^v}{b^{(\alpha+v)/r} r \Gamma(v+1)} \sum_{k=0}^{\infty} \frac{1}{k! (v+1)_k} \times$$

$$\times \Gamma\left[\frac{(2k+\alpha+v)/r}{(2k+r+\alpha+v-r\mu)/(2r)}\right] \left(\frac{c^2}{2^{2+1/r} b^{2/r}}\right)^k$$

$[r \geq 1/2; \operatorname{Re}(\alpha+v) > 0; |\arg b| < \pi/4 \text{ при } r > 1/2;$
 $\operatorname{Re} b^2 > 2 |\operatorname{Re} c|, 2|c| < |b|^2 \text{ при } r = 1/2]$

$$26. \int_0^\infty x^{\alpha-1} e^{\pm b^2 x^{2r}/4 - cx} D_\mu(bx^r) I_\nu(cx) dx = \tilde{X}$$

$$\left[r, \operatorname{Re} c > 0; |\arg b| < (2 \pm 1) \pi/4, \left\{ \begin{array}{l} -\operatorname{Re} v < \operatorname{Re} \alpha < -r \operatorname{Re} \mu + 1/2 \\ \operatorname{Re}(\alpha+v) > 0 \end{array} \right\}\right],$$

$$\tilde{X} = \frac{2^{-(v+\alpha)/(2r)-v \mp (\mu+1 \pm 1)/2} c^v}{b^{(\alpha+v)/r} r \Gamma(v+1)} \left\{ \frac{\Gamma^{-1}(-\mu)}{\sqrt{\pi}} \right\} \times$$

$$\times \sum_{k=0}^{\infty} \frac{(v+1/2)_k}{k! (2v+1)_k} \Gamma\left(\frac{\alpha+v+k}{r}\right) \Gamma^{\pm 1}\left(\frac{1 \mp 1}{4} \mp \frac{v+\alpha+k \pm r\mu}{2r}\right) \times$$

$$\times \left(-\frac{2^{1-1/(2r)} c}{b^{1/r}}\right)^k + \begin{cases} 1 \\ 0 \end{cases} \frac{b^\mu}{\sqrt{\pi} (2c)^{\alpha+\mu r}} \sum_{k=0}^{\infty} \frac{(-\mu)_{2k}}{k!} \times$$

$$\times \Gamma\left[\frac{\alpha+v+\mu r-2rk}{1+v-\alpha-\mu r+2rk}, \frac{1/2-\alpha-\mu r+2rk}{1}\right] \left(-\frac{2^{2r-1} c^{2r}}{b^2}\right)^k \quad [r > 1/2].$$

$$\tilde{X} = \frac{2^{(1-2\alpha)/(4r) \mp (\mu+1 \pm 2)/2}}{\sqrt{\pi c} b^{(2\alpha-1)/(2r)} r} \left\{ \frac{\Gamma^{-1}(-\mu)}{\sqrt{\pi}} \right\} \times$$

$$\times \sum_{k=0}^{\infty} \frac{(1/2+v)_k (1/2-v)_k}{k!} \Gamma\left(\frac{2\alpha-2k-1}{2r}\right) \Gamma^{\pm 1}\left(\frac{1 \mp 1}{4} \mp \frac{2\alpha-2k-1 \pm 2r\mu}{4r}\right) \times$$

$$\times \left(\frac{b^{1/r}}{2^{1-1/(2r)} c}\right)^k + \frac{2^{\mp (\mu+1 \pm 1)/2 - \alpha}}{c^\alpha \sqrt{\pi}} \left\{ \frac{\Gamma^{-1}(-\mu)}{\sqrt{\pi}} \right\} \sum_{k=0}^{\infty} \frac{1}{k!} \times$$

$$\times \Gamma\left[\frac{\alpha+v+rk}{1+v-\alpha-rk}, \frac{1/2-\alpha-rk}{1}\right] \Gamma^{\pm 1}\left(\frac{1 \mp 1}{4} \pm \frac{k \mp \mu}{2}\right) \left(-\frac{b}{2^{r-1/2} c^r}\right)^k$$

$[r < 1/2]$

$\tilde{X}$ при $r=1/2$ см. в 2.15.29.4.

2.15.2. Интегралы от $A(x) I_\nu(\varphi(x))$.

1.
$$\int_0^a x^{\alpha-1} (a-x)^{\beta-1} I_\nu(cx) dx = a^{\alpha+\beta+\nu-1} \left(\frac{c}{2}\right)^\nu \Gamma\left[\frac{\alpha+\nu}{2}, \frac{\beta}{2}\right] \times$$

$$\times {}_2F_2\left(\frac{\alpha+\nu}{2}, \frac{\alpha+\nu+1}{2}; \nu+1, \frac{\alpha+\nu+\beta}{2}, \frac{\alpha+\nu+\beta+1}{2}; \frac{a^2 c^2}{4}\right)$$

$$[a, \operatorname{Re} \beta, \operatorname{Re}(\alpha+\nu) > 0].$$
2.
$$\int_0^a \frac{x^n}{\sqrt{a-x}} I_\nu(cx) dx =$$

$$= \pi \left(\frac{a}{2}\right)^{n+1/2} \sum_{k=0}^n \binom{n}{k} I_{(2\nu+2n+1)/4-k} \left(\frac{ac}{2}\right) I_{(2\nu-2n-1)/4+k} \left(\frac{ac}{2}\right)$$

$$[a > 0, \operatorname{Re} \nu > -n-1].$$
3.
$$\int_0^a \frac{1}{\sqrt{a^2-x^2}} I_\nu(cx) dx = \frac{\pi}{2} I_{\nu/2}^2 \left(\frac{ac}{2}\right)$$

$$[a > 0; \operatorname{Re} \nu > -1]$$
4.
$$\int_0^a \frac{1}{\sqrt{a^2-x^2}} I_1(cx) dx = \frac{\operatorname{ch} ac - 1}{ac}$$

$$[a > 0]$$
5.
$$\int_0^a x^{\alpha-1} (a^2-x^2)^{\beta-1} I_\nu(cx) dx =$$

$$= 2^{-\nu-1} a^{2\beta+\alpha+\nu-2} c^\nu \Gamma\left[\frac{\beta}{2}, \frac{(\alpha+\nu)/2}{2}\right] {}_1F_2\left(\frac{\alpha+\nu}{2}; \beta + \frac{\alpha+\nu}{2}, \nu+1; \frac{a^2 c^2}{4}\right)$$

$$[a, \operatorname{Re} \beta, \operatorname{Re}(\alpha+\nu) > 0]$$
6.
$$\int_0^a x^{\nu+1} (a^2-x^2)^{\beta-1} I_\nu(cx) dx = 2^{\beta-1} a^{\nu+\beta} c^{-\beta} \Gamma(\beta) I_{\nu+\beta}(ac)$$

$$[a, \operatorname{Re} \beta > 0; \operatorname{Re} \nu > -1]$$
7.
$$\int_0^a x^\nu (a^2-x^2)^{\nu-1/2} I_\nu(cx) dx = 2^{\nu-1} \sqrt{\pi} a^{2\nu} c^{-\nu} \Gamma\left(\nu + \frac{1}{2}\right) I_\nu^2\left(\frac{ac}{2}\right)$$

$$[a > 0; \operatorname{Re} \nu > -1/2]$$
8.
$$\int_0^a x^\nu (a^2-x^2)^{\nu-3/2} I_\nu(cx) dx =$$

$$= \frac{a^{2\nu-1} c^{1-\nu} \sqrt{\pi}}{2^{2-\nu}} \Gamma\left(\nu - \frac{1}{2}\right) I_{\nu-1}\left(\frac{ac}{2}\right) I_\nu\left(\frac{ac}{2}\right)$$

$$[a > 0; \operatorname{Re} \nu > 1/2]$$
9.
$$\int_0^a x^{-\nu} (a^2-x^2)^{-\nu-1/2} I_\nu(cx) dx = \frac{c^\nu \sqrt{\pi}}{2^{\nu+1} a^{2\nu}} \Gamma\left(\frac{1}{2} - \nu\right) I_\nu\left(\frac{ac}{2}\right) I_{-\nu}\left(\frac{ac}{2}\right)$$

$$[a > 0; \operatorname{Re} \nu < 1/2]$$
10.
$$\int_0^a \frac{x^{1-\nu}}{\sqrt{a^2-x^2}} I_\nu(cx) dx = a^{1/2-\nu} \sqrt{\frac{\pi}{2c}} L_{\nu-1/2}(ac)$$

$$[a > 0]$$
11.
$$\int_0^a \frac{x}{\sqrt{a^2-x^2}} I_\nu(cx) dx = \frac{\pi a}{2} I_{(\nu+1)/2}\left(\frac{ac}{2}\right) I_{(\nu-1)/2}\left(\frac{ac}{2}\right)$$

$$[a > 0, \operatorname{Re} \nu > -2]$$

$$12. \int_0^a \frac{x}{\sqrt{a^2 - x^2}} I_0(cx) dx = \frac{\text{sh} ac}{c} \quad [a > 0].$$

$$13. \int_a^b x^{1-\nu} (b^2 - x^2)^{\nu-1/2} (x^2 - a^2)^{\nu-1/2} I_\nu(cx) dx = \\ = 2^{\nu-1} \sqrt{\pi} \Gamma\left(\nu + \frac{1}{2}\right) (b^2 - a^2)^\nu c^{-\nu} I_\nu\left(\frac{cb + ca}{2}\right) I_\nu\left(\frac{cb - ca}{2}\right) \\ [a, b > 0; \text{Re } \nu > -1/2].$$

$$14. \int_0^a x^{-\mu} \frac{(a + \sqrt{a^2 - x^2})^\mu + (a - \sqrt{a^2 - x^2})^\mu}{\sqrt{a^2 - x^2}} I_\nu(cx) dx = \\ = \frac{1}{ac} \Gamma\left[\frac{(1 + \nu + \mu)/2, (1 + \nu - \mu)/2}{1 + \nu, 1 + \nu}\right] M_{\mu/2, \nu/2}(ac) M_{\mu/2, \nu/2}(-ac) \\ [a > 0, \text{Re } (\mu - \nu) < 1]$$

$$15. \int_0^a x^\mu \frac{(x + i\sqrt{a^2 - x^2})^\mu + (x - i\sqrt{a^2 - x^2})^\mu}{\sqrt{a^2 - x^2}} I_\nu(cx) dx = \\ = \pi a^{2\mu} I_{(\nu+\mu)/2}\left(\frac{ac}{2}\right) I_{(\nu-\mu)/2}\left(\frac{ac}{2}\right) \quad [a > 0; \text{Re } (\mu + \nu) > -1].$$

$$16. \int_0^\infty \frac{x^{\alpha-1}}{x^2 + z^2} I_\nu\left(\frac{cx}{x^2 + z^2}\right) dx = \\ = \frac{z^{\alpha-1}}{2^{\nu+1}c} \Gamma\left[\frac{(\nu + \alpha)/2, 1 + (\nu - \alpha)/2}{\nu + 1, \nu + 1}\right] M_{(\alpha-1)/2, \nu/2}\left(\frac{c}{z}\right) M_{(1-\alpha)/2, \nu/2}\left(\frac{c}{z}\right) \\ [c, \text{Re } z > 0; -\text{Re } \nu < \text{Re } \alpha < \text{Re } \nu + 2].$$

2.15.3. Интегралы от $x^\alpha e^{-px} I_\nu(cx)$.

$$1. \int_0^\infty e^{-px} I_\nu(cx) dx = \frac{c^\nu}{\sqrt{p^2 - c^2} (p + \sqrt{p^2 - c^2})^\nu} \quad [\text{Re } p > |\text{Re } c|; \text{Re } \nu > -1].$$

$$2. \int_0^\infty x^{\alpha-1} e^{-px} I_\nu(cx) dx = I_\nu^\alpha \\ [\text{Re } (\alpha + \nu) > 0; \text{Re } p > |\text{Re } c|; k = \sqrt{2c}/\sqrt{p+c}], \\ I_\nu^\alpha = p^{-\alpha-\nu} \left(\frac{c}{2}\right)^\nu \Gamma\left[\frac{\nu + \alpha}{\nu + 1}\right] {}_2F_1\left(\frac{\alpha + \nu}{2}, \frac{\alpha + \nu + 1}{2}; \nu + 1; \frac{c^2}{p^2}\right) = \\ = e^{-\pi\nu/2} \Gamma(\alpha + \nu) (p^2 - c^2)^{-\alpha/2} p_{\alpha-1}^{-\nu} \left(\frac{p}{\sqrt{p^2 - c^2}}\right),$$

$$I_\nu^{\nu+1} = \frac{(2c)^\nu \Gamma(\nu + 1/2)}{\sqrt{\pi} (p^2 - c^2)^{\nu+1/2}} \quad [\text{Re } \nu > -1/2],$$

$$I_\nu^{\nu+2} = \frac{2p (2c)^\nu \Gamma(\nu + 3/2)}{\sqrt{\pi} (p^2 - c^2)^{\nu+3/2}} \quad [\text{Re } \nu > -1],$$

$$I_\nu^{-1} = \frac{c^{-\nu}}{\nu(\nu^2 - 1)} (p + \nu \sqrt{p^2 - c^2}) (p - \sqrt{p^2 - c^2})^\nu \quad [\text{Re } \nu > 1],$$

$$I_\nu^0 = \frac{c^\nu}{\nu} (p + \sqrt{p^2 - c^2})^{-\nu} \quad [\text{Re } \nu > 0],$$

$$I_\nu^n = c^\nu (-1)^{n-1} \frac{\partial^{n-1}}{\partial p^{n-1}} \left[\frac{(p + \sqrt{p^2 - c^2})^{-\nu}}{\sqrt{p^2 - c^2}} \right] \quad [\text{Re } \nu > -n].$$

$$I_{\nu}^2 = \frac{c^{\nu} (p + \nu \sqrt{p^2 - c^2})}{(p + \sqrt{p^2 - c^2})^{\nu} (p^2 - c^2)^{3/2}} \quad [\operatorname{Re} \nu > -2],$$

$$I_{\nu}^{1/2} = \sqrt{\frac{2}{\pi c}} Q_{\nu-1/2} \left(\frac{p}{c} \right) \quad [\operatorname{Re} \nu > -1/2],$$

$$I_0^{1/2} = k \sqrt{\frac{2}{\pi c}} K(k), \quad I_0^{3/2} = \frac{k^3}{2(1-k^2) \sqrt{2\pi c^3}} E(k)$$

$$I_0^{5/2} = \frac{k^5}{8(1-k^2)^2 \sqrt{2\pi c^5}} [2(2-k^2) E(k) - (1-k^2) K(k)],$$

$$I_0^{7/2} = \frac{k^7}{32(1-k^2)^3 \sqrt{2\pi c^7}} [(23-23k^2+8k^4) E(k) - 4(1-k^2)(2-k^2) K(k)],$$

$$I_1^{-1/2} = \frac{4}{3} \sqrt{\frac{2c}{\pi k^3}} [(2-k^2) E(k) - 2(1-k^2) K(k)],$$

$$I_1^{1/2} = \frac{1}{k} \sqrt{\frac{2}{\pi c}} [(2-k^2) K(k) - 2E(k)],$$

$$I_1^{3/2} = \frac{k}{2(1-k^2) \sqrt{2\pi c^3}} [(2-k^2) E(k) - 2(1-k^2) K(k)],$$

$$I_1^{5/2} = \frac{k^3}{8(1-k^2)^2 \sqrt{2\pi c^5}} [2(1-k^2+k^4) E(k) - (1-k^2)(2-k^2) K(k)],$$

$$I_2^{-1/2} = \frac{4}{15} \sqrt{\frac{2c}{\pi k^5}} [(16-16k^2+k^4) E(k) - 8(1-k^2)(2-k^2) K(k)],$$

$$I_2^{1/2} = \frac{1}{3k^3} \sqrt{\frac{2}{\pi c}} [(16-16k^2+3k^4) K(k) - 8(1-k^2)(2-k^2) E(k)],$$

$$I_2^{3/2} = \frac{1}{2k(1-k^2) \sqrt{2\pi c^3}} [(16-16k^2+k^4) E(k) - 8(1-k^2)(2-k^2) K(k)],$$

$$I_2^{5/2} = \frac{k}{8(1-k^2)^2 \sqrt{2\pi c^5}} [(16-16k^2-k^4)(1-k^2) K(k) - 2(2-k^2)(4-4k^2-k^4) E(k)],$$

$$I_2^{7/2} = \frac{k^3}{32(1-k^2)^3 \sqrt{2\pi c^7}} [4(2-2k^2-k^4)(1-k^2)(2-k^2) K(k) - \\ - (16-32k^2+9k^4+7k^6-8k^8) E(k)],$$

$$I_3^{-5/2} = \frac{64}{10395} \sqrt{\frac{2c^5}{\pi k^{11}}} [(128-256k^2+99k^4+29k^6+8k^8)(2-k^2) E(k) - \\ - 2(1-k^2)(128-256k^2+123k^4+5k^6+2k^8) K(k)],$$

$$I_3^{-3/2} = \frac{16}{945} \sqrt{\frac{2c^3}{\pi k^9}} [(128-128k^2-k^4)(1-k^2)(2-k^2) K(k) - \\ - 2(128-256k^2+135k^4-7k^6-k^8) E(k)],$$

$$I_3^{-1/2} = \frac{4}{105} \sqrt{\frac{2c}{\pi k^7}} [(128-128k^2+3k^4)(2-k^2) E(k) - \\ - 2(1-k^2)(128-128k^2+27k^4) K(k)],$$

$$I_3^{1/2} = \frac{2}{15k^5 \sqrt{2\pi c}} [(128-128k^2+15k^4)(2-k^2) K(k) - 2(128-128k^2+23k^4) E(k)],$$

$$I_3^{3/2} = \frac{1}{6k^3(1-k^2) \sqrt{2\pi c^3}} [(128-128k^2+3k^4)(2-k^2) E(k) - \\ - 2(1-k^2)(128-128k^2+27k^4) K(k)],$$

$$I_8^{5/2} = \frac{1}{8k(1-k^2)^2 \sqrt{2\pi c^5}} [(128 - 128k^2 - k^4)(1-k^2)(2-k^2)K(k) - \\ - 2(128 - 256k^2 + 135k^4 - 7k^6 - k^8)E(k)]$$

$$I_8^{7/2} = \frac{k}{32(1-k^2)^3 \sqrt{2\pi c^7}} [(128 - 256k^2 + 99k^4 + 29k^6 + 8k^8)(2-k^2)E(k) - \\ - 2(1-k^2)(128 - 256k^2 + 123k^4 + 5k^6 + 2k^8)K(k)]$$

$$3. \int_0^\infty x^{\alpha-1} e^{-cx} I_\nu(cx) dx = \frac{(2c)^{-\alpha}}{\sqrt{\pi}} \Gamma \left[\begin{matrix} \alpha + \nu, 1/2 - \alpha \\ 1 + \nu - \alpha \end{matrix} \right] \\ [\operatorname{Re} c > 0; -\operatorname{Re} \nu < \operatorname{Re} \alpha < 1/2].$$

2.15.4. Интегралы от $A(x, e^{px}) I_\nu(cx)$

$$1. \int_0^a x^{\alpha-1} (a-x)^{\beta-1} e^{\pm cx} I_\nu(cx) dx = a^{\alpha+\beta+\nu-1} \left(\frac{c}{2}\right)^\nu \times \\ \times \Gamma \left[\begin{matrix} \alpha + \nu, \beta \\ \alpha + \beta + \nu, \nu + 1 \end{matrix} \right] {}_2F_2 \left(\nu + \frac{1}{2}, \alpha + \nu; 2\nu + 1, \alpha + \beta + \nu; \pm 2ac \right) \\ [a, \operatorname{Re} \beta, \operatorname{Re}(\alpha + \nu) > 0]$$

$$2. \int_a^\infty x^{\alpha-1} (x-a)^{\beta-1} e^{-cx} I_\nu(cx) dx = \frac{(2c)^{1-\alpha-\beta}}{\sqrt{\pi}} \Gamma \left[\begin{matrix} \alpha + \beta + \nu - 1, 3/2 - \alpha - \beta \\ 2 - \alpha - \beta + \nu \end{matrix} \right] \times \\ \times {}_2F_2 \left(1 - \beta, \frac{3}{2} - \alpha - \beta; 2 - \alpha - \beta - \nu, 2 + \nu - \alpha - \beta; -2ac \right) + \\ + a^{\alpha+\beta+\nu-1} \left(\frac{c}{2}\right)^\nu \Gamma \left[\begin{matrix} \beta, 1 - \alpha - \beta - \nu \\ 1 - \alpha - \nu, \nu + 1 \end{matrix} \right] \times \\ \times {}_2F_2 \left(\alpha + \nu, \nu + \frac{1}{2}; \alpha + \beta + \nu, 2\nu + 1; -2ac \right) \\ [a, \operatorname{Re} c, \operatorname{Re} \beta > 0; \operatorname{Re}(\alpha + \beta) < 3/2].$$

$$3. \int_0^\infty \frac{e^{-cx}}{x+z} I_0(cx) dx = e^{cz} K_0(cz) \quad [\operatorname{Re} c > 0; |\arg z| < \pi].$$

$$4. \int_0^\infty x^{\alpha-1} \frac{e^{-cx}}{(x+z)^\rho} I_\nu(cx) dx = \frac{(2c)^{\rho-\alpha}}{\sqrt{\pi}} \Gamma \left[\begin{matrix} \nu + \alpha - \rho, 1/2 + \rho - \alpha \\ \nu + \rho - \alpha + 1 \end{matrix} \right] \times \\ \times {}_2F_2 \left(\frac{1}{2} + \rho - \alpha, \rho; 1 - \nu - \alpha + \rho, 1 + \nu + \rho - \alpha; 2cz \right) + \\ + \left(\frac{c}{2}\right)^\nu z^{\alpha+\nu-\rho} \Gamma \left[\begin{matrix} \rho - \alpha - \nu, \alpha + \nu \\ \rho, \nu + 1 \end{matrix} \right] {}_2F_2 \left(\nu + \frac{1}{2}, \alpha + \nu, 1 + \alpha + \nu - \rho, 2\nu + 1; 2cz \right) \\ [-\operatorname{Re} \nu < \operatorname{Re} \alpha < \operatorname{Re} \rho + 1/2; \operatorname{Re} c > 0; |\arg z| < \pi]$$

$$5. \int_0^\infty \frac{x^\nu}{(x+z)^\rho} e^{-cx} I_\nu(cx) dx = \\ = \frac{z^{\nu-\rho/2}}{\sqrt{\pi}(2c)^{1-\rho/2}} B \left(\nu + \frac{1}{2}, \rho - \nu - \frac{1}{2} \right) e^{cz} W_{(1-\rho)/2, \nu+(1-\rho)/2}(2cz) \\ [\operatorname{Re} c > 0, \operatorname{Re} \nu > -1/2; \operatorname{Re}(\rho - \nu) > 1/2, |\arg z| < \pi]$$

$$6. \int_0^\infty \frac{x^\nu}{e^{ax}-1} I_\nu(cx) dx = \frac{(2c)^\nu}{\sqrt{\pi}} \Gamma \left(\nu + \frac{1}{2} \right) \sum_{k=1}^\infty \frac{1}{(a^2 k^2 - c^2)^{\nu+1/2}} \\ [\operatorname{Re} \nu > 0, \operatorname{Re} a > |\operatorname{Re} c|]$$

2.15.5. Интегралы от $A(x) e^{-px^2} I_\nu(cx)$.

$$1. \int_0^a x^\nu e^{-px^2} I_\nu(cx) dx = a^{\nu+1} e^{-ap^2} \sum_{k=0}^{\infty} \left(\frac{2a^2}{c}\right)^k I_{k+\nu+1}(ac)$$

$$[a > 0; \operatorname{Re} \nu > -1/2].$$

$$2. \int_0^{\infty} e^{-px^2} I_\nu(cx) dx = \frac{1}{2} \sqrt{\frac{\pi}{p}} \exp\left(\frac{c^2}{8p}\right) I_{\nu/2}\left(\frac{c^2}{8p}\right)$$

$$[\operatorname{Re} p > 0; \operatorname{Re} \nu > -1; |\arg c| < \pi].$$

$$3. \int_0^{\infty} e^{-px^2} I_1(cx) dx = \frac{1}{c} \left[\exp\left(\frac{c^2}{4p}\right) - 1 \right]$$

$$[\operatorname{Re} p > 0; |\arg c| < \pi]$$

$$4. \int_0^{\infty} x^{\alpha-1} e^{-px^2} I_\nu(cx) dx = A_\nu^\alpha$$

$$[\operatorname{Re} p, \operatorname{Re}(\alpha+\nu) > 0; |\arg c| < \pi].$$

$$A_\nu^\alpha = 2^{-\nu-1} c^\nu p^{-(\alpha+\nu)/2} \Gamma\left[\frac{(\alpha+\nu)/2}{\nu+1}\right] {}_1F_1\left(\frac{\alpha+\nu}{2}; \nu+1; \frac{c^2}{4p}\right),$$

$$A_\nu^{2-\nu} = \frac{(2p)^{\nu-1}}{c^\nu \Gamma(\nu)} \exp\left(\frac{c^2}{4p}\right) \gamma\left(\nu, \frac{c^2}{4p}\right),$$

$$A_\nu^\nu = \frac{2^{\nu-1} e^{-\pi\nu i}}{c^\nu} \gamma\left(\nu, \frac{c^2 e^{\pi i}}{4p}\right)$$

$$[\operatorname{Re} \nu > 0].$$

$$A_\nu^{\nu+2} = \frac{c^\nu}{(2p)^{\nu+1}} \exp\left(\frac{c^2}{4p}\right)$$

$$[\operatorname{Re} \nu > -1].$$

$$A_\nu^{\nu+2n+2} = \frac{n! c^\nu}{2^{\nu+1} p^{n+\nu+1}} \exp\left(\frac{c^2}{4p}\right) L_n^\nu\left(-\frac{c^2}{4p}\right)$$

$$[\operatorname{Re} \nu > -n-1].$$

$$A_\nu^2 = \frac{c}{8p} \sqrt{\frac{\pi}{p}} \exp\left(\frac{c^2}{8p}\right) \left[J_{(\nu-1)/2}\left(\frac{c^2}{8p}\right) + J_{(\nu+1)/2}\left(\frac{c^2}{8p}\right) \right]$$

$$[\operatorname{Re} \nu > -2]$$

2.15.6. Интегралы от $A(x) e^{f(x)} I_\nu(cx)$.

$$1. \int_0^{\infty} x^{\alpha-1} e^{-cx-b\sqrt{x}} I_\nu(cx) dx =$$

$$= b^{1-2\alpha} \sqrt{\frac{2}{\pi c}} \Gamma(2\alpha-1) {}_2F_2\left(\frac{1}{2}-\nu, \frac{1}{2}+\nu; 1-\alpha, \frac{3}{2}-\alpha; \frac{b^2}{8c}\right) +$$

$$+ \frac{(2c)^{-\alpha}}{\sqrt{\pi}} \Gamma\left[\frac{1/2-\alpha, \nu+\alpha}{\nu+1-\alpha}\right] {}_2F_2\left(\alpha-\nu, \alpha+\nu; \frac{1}{2}, \frac{1}{2}+\alpha; \frac{b^2}{8c}\right) -$$

$$- \frac{b}{\sqrt{\pi} (2c)^{\alpha+1/2}} \Gamma\left[\frac{-\alpha, \alpha+\nu-1/2}{\nu-\alpha+1/2}\right] {}_2F_2\left(\frac{1}{2}+\alpha-\nu, \frac{1}{2}+\alpha+\nu; \frac{3}{2}; 1+\alpha; \frac{b^2}{8c}\right)$$

$$[\operatorname{Re} b, \operatorname{Re} c, \operatorname{Re}(\alpha+\nu) > 0]$$

$$2. \int_0^{\infty} x^{\nu-1/2} e^{-cx-b\sqrt{x}} I_\nu(cx) dx =$$

$$= \frac{2^{\nu/2+1} b^{-\nu-1}}{\pi c^{\nu/2}} \Gamma\left(2\nu+\frac{1}{2}\right) \Gamma\left(\nu+\frac{1}{2}\right) \exp\left(\frac{b^2}{16c}\right) W_{-\nu/2, \nu/2}\left(\frac{b^2}{8c}\right)$$

$$[\operatorname{Re} b, \operatorname{Re} c > 0, \operatorname{Re} \nu > -1/4]$$

$$3. \int_0^{\infty} x^{-1/2} e^{-cx-b\sqrt{x}} I_0(cx) dx = \frac{1}{\sqrt{2\pi c}} \exp\left(\frac{b^2}{16c}\right) K_0\left(\frac{b^2}{16c}\right)$$

$$[\operatorname{Re} b, \operatorname{Re} c > 0]$$

4. $\int_0^\infty \frac{1}{x} e^{-px-b/x} I_\nu(cx) dx = 2I_\nu(z_-) K_\nu(z_+)$
 $[z_\pm = \sqrt{b} (\sqrt{p+c} \pm \sqrt{p-c}); \operatorname{Re} b > 0; \operatorname{Re} p > |\operatorname{Re} c|].$
5. $\int_0^\infty \frac{1}{x^2} e^{-px-b/x} I_0(cx) dx = 2c [z_-^{-1} I_0(z_-) K_1(z_+) - z_+^{-1} I_1(z_-) K_0(z_+)]$
 $[z_\pm = \sqrt{b} (\sqrt{p+c} \pm \sqrt{p-c}); \operatorname{Re} b > 0; \operatorname{Re} p > |\operatorname{Re} c|].$
6. $\int_0^\infty x^{\alpha-1} e^{-cx-b/x} I_\nu(cx) dx =$
 $= \frac{(2c)^{-\alpha}}{\sqrt{\pi}} \Gamma \left[\begin{matrix} \alpha + \nu, 1/2 - \alpha \\ 1 + \nu - \alpha \end{matrix} \right] {}_1F_2 \left(\frac{1}{2} - \alpha; 1 - \nu - \alpha, 1 + \nu - \alpha; 2bc \right) +$
 $+ b^{\alpha+\nu} \left(\frac{c}{2} \right)^\nu \Gamma \left[\begin{matrix} -\nu - \alpha \\ 1 + \nu \end{matrix} \right] {}_1F_2 \left(\frac{1}{2} + \nu; 1 + \nu + \alpha, 1 + 2\nu; 2bc \right)$
 $[\operatorname{Re} b, \operatorname{Re} c > 0; \operatorname{Re} \alpha < 1/2].$
7. $\int_0^\infty x^{-3/2} e^{-cx-b/x} I_0(cx) dx = \sqrt{\frac{\pi}{c}} [I_0(\sqrt{8bc}) - L_0(\sqrt{8bc})]$
 $[\operatorname{Re} b, \operatorname{Re} c > 0].$
8. $\int_0^\infty \exp(-p\sqrt{x^2+z^2}) I_1(cx) dx = \frac{p}{c\sqrt{p^2-c^2}} \exp(-z\sqrt{p^2-c^2}) - \frac{1}{c} e^{-pz}$
 $[\operatorname{Re} p > |\operatorname{Re} c|; \operatorname{Re} z > 0].$
9. $\int_0^\infty x^{\nu+1} \exp(-p\sqrt{x^2+z^2}) I_\nu(cx) dx =$
 $= \sqrt{\frac{2}{\pi}} p z^{\nu+3/2} c^\nu (p^2 - c^2)^{-(2\nu+3)/4} K_{\nu+3/2}(z\sqrt{p^2-c^2})$
 $[\operatorname{Re} p > |\operatorname{Re} c|; \operatorname{Re} z > 0, \operatorname{Re} \nu > -1].$
10. $\int_0^\infty x \exp(-p\sqrt{x^2+z^2}) I_0(cx) dx = p(p^2 - c^2)^{-3/2} (1 + z\sqrt{p^2 - c^2}) \times$
 $\times \exp(-z\sqrt{p^2 - c^2})$
 $[\operatorname{Re} p > |\operatorname{Re} c|; \operatorname{Re} z > 0]$
11. $\int_0^\infty \frac{\exp(-p\sqrt{x^2+z^2})}{\sqrt{x^2+z^2}} I_\nu(cx) dx = A_\nu$
 $\left[\operatorname{Re} p > |\operatorname{Re} c|; \operatorname{Re} z > 0; \operatorname{Re} \nu > -1; z_\pm = \frac{z}{2} (p \pm \sqrt{p^2 - c^2}) \right],$
 $A_\nu = I_{\nu/2}(z_-) K_{\nu/2}(z_+), \quad A_1 = \frac{1}{cz} [\exp(-z\sqrt{p^2 - c^2}) - e^{-pz}].$
12. $\int_0^\infty x^{\alpha-1} \frac{\exp(-p\sqrt{x^2+z^2})}{\sqrt{x^2+z^2}} I_\nu(cx) dx =$
 $= \frac{c^\nu z^{\alpha+\nu-1}}{2^{(1+\nu-\alpha)/2} \sqrt{\pi} p^{(\alpha+\nu-1)/2}} \sum_{k=0}^\infty \frac{(cz)^{2k}}{(2p)^k k!} \Gamma \left[\begin{matrix} (\alpha + \nu)/2 + k \\ \nu + k + 1 \end{matrix} \right] K_{(\alpha+\nu-1)/2+k}(pz)$
 $[\operatorname{Re} p > |\operatorname{Re} c|; \operatorname{Re} z, \operatorname{Re}(\alpha + \nu) > 0]$

$$13. \int_0^{\infty} x^{\nu+1} \frac{\exp(-p\sqrt{x^2+z^2})}{\sqrt{x^2+z^2}} I_{\nu}(cx) dx = B_{\nu}$$

$$[\operatorname{Re} p > |\operatorname{Re} c|; \operatorname{Re} z > 0; \operatorname{Re} \nu > -1]$$

$$B_{\nu} = (cz)^{\nu} \sqrt{\frac{2z}{\pi}} (p^2 - c^2)^{-(2\nu+1)/4} K_{\nu+1/2}(z\sqrt{p^2 - c^2}),$$

$$B_0 = (p^2 - c^2)^{-1/2} \exp(-z\sqrt{p^2 - c^2}),$$

$$B_1 = c \exp(-z\sqrt{p^2 - c^2}) \left[\frac{z}{p^2 - c^2} + \frac{1}{(p^2 - c^2)^{3/2}} \right].$$

$$14. \int_0^{\infty} \frac{x^{-\mu}}{\sqrt{x^2+z^2}} (\sqrt{x^2+z^2} \pm z)^{\mu} \exp(-p\sqrt{x^2+z^2}) I_{\nu}(cx) dx =$$

$$= \frac{1}{cz} \Gamma \left[\frac{(1+\nu \mp \mu)/2}{1+\nu} \right] W_{\pm \mu/2, \nu/2}(z_{\pm}) M_{\pm \mu/2, \nu/2}(z_{\pm})$$

$$[z_{\pm} = z(p \pm \sqrt{p^2 - c^2}); \operatorname{Re} p > |\operatorname{Re} c|; \operatorname{Re} z > 0; \operatorname{Re}(\nu \mp \mu) > -1].$$

$$15. \int_0^{\infty} x^{1 \pm \nu} \frac{(\sqrt{x^2+z^2} \pm z)^{\mp \nu}}{\sqrt{x^2+z^2}} \exp(-p\sqrt{x^2+z^2}) I_{\nu}(cx) dx =$$

$$= \frac{c^{\nu}}{\sqrt{p^2 - c^2}} \frac{\exp(-z\sqrt{p^2 - c^2})}{(p + \sqrt{p^2 - c^2})^{\nu}}$$

$$[\operatorname{Re} p > |\operatorname{Re} c|; \operatorname{Re} z > 0; \operatorname{Re} \nu > -1]$$

2.15.7. Интегралы от $A(x)e^{-px}I_{\nu}(\varphi(x))$

$$1. \int_0^a x^{\alpha-1} (a-x)^{-\alpha} e^{-px} I_{\nu}(\sqrt{ax-x^2}) dx = \frac{2}{a} \Gamma \left[\frac{\nu/2 + \alpha, 1 + \nu/2 - \alpha}{\nu+1, \nu+1} \right] \times$$

$$\times e^{-pz/2} M_{\alpha-1/2, \nu/2} \left[\frac{z}{2} (\sqrt{p^2+1} + p) \right] M_{1/2-\alpha, \nu/2} \left[\frac{z}{2} (\sqrt{p^2+1} - p) \right]$$

$$[\alpha > 0; -\operatorname{Re} \nu < 2 \operatorname{Re} \alpha < 2 + \operatorname{Re} \nu]$$

$$2. \int_0^{\infty} e^{-px} I_0(c\sqrt{x^2+xz}) dx = \frac{1}{\sqrt{p^2 - c^2}} \exp \left[\frac{z}{2} (p - \sqrt{p^2 - c^2}) \right]$$

$$[\operatorname{Re} p > |\operatorname{Re} c|; |\arg z| < \pi]$$

$$3. \int_0^{\infty} \frac{x^{\rho-1}}{(x+z)^{\rho}} e^{-px} I_{\nu}(c\sqrt{x^2+xz}) dx =$$

$$= \frac{2}{cz} \Gamma \left[\frac{\rho + \nu/2}{\nu+1} \right] e^{pz/2} M_{1/2-\rho, \nu/2}(z_{-}) W_{1/2-\rho, \nu/2}(z_{+})$$

$$[z_{\pm} = z(p \pm \sqrt{p^2 - c^2})/2; \operatorname{Re} p > |\operatorname{Re} c|; \operatorname{Re}(2\rho + \nu) > 0; |\arg z| < \pi].$$

$$4. \int_0^{\infty} \frac{x^{\rho}}{(x+z)^{\rho}} e^{-px} I_{\nu}(c\sqrt{x^2+xz}) dx =$$

$$= \left(\frac{c}{2} \right)^{\nu} \frac{z^{(\nu+1)/2-\rho}}{\sqrt{\pi}} \Gamma \left(\frac{\nu}{2}, \rho+1 \right) (p^2 - c^2)^{(2\rho-\nu-1)/4} e^{pz/2} \times$$

$$\times \sum_{k=0}^{\infty} \frac{(-1)^k (\nu/2 + \rho)_k}{k! \Gamma(\nu + k + 1)} \left(\frac{c}{2\sqrt{p^2 - c^2}} \right)^k K_{(\nu+1)/2+k-\rho} \left(\frac{z}{2} \sqrt{p^2 - c^2} \right)$$

$$[\operatorname{Re} p > |\operatorname{Re} c|; \operatorname{Re}(\nu - 2\rho) > -2; |\arg z| < \pi]$$

$$5. \int_0^{\infty} \frac{x^{v/2}}{(x+z)^{v/2}} e^{-px} I_v(c \sqrt{x^2+xz}) dx =$$

$$= \frac{c^v}{\sqrt{p^2-c^2}} (p + \sqrt{p^2-c^2})^{-v} \exp\left(\frac{z}{2} (p - \sqrt{p^2-c^2})\right)$$

$$[\operatorname{Re} p > |\operatorname{Re} c|; \operatorname{Re} v > -1, |\arg z| < \pi].$$

$$6. \int_0^{\infty} x^{v/2} (x+z)^{v/2} e^{-px} I_v(c \sqrt{x^2+xz}) dx =$$

$$= \frac{c^v}{2^v \sqrt{\pi}} \left(\frac{z}{\sqrt{p^2-c^2}}\right)^{v+1/2} e^{pz/2} K_{v+1/2}\left(\frac{z}{2} \sqrt{p^2-c^2}\right)$$

$$[\operatorname{Re} p > |\operatorname{Re} c|; \operatorname{Re} v > -1; |\arg z| < \pi].$$

$$7. \int_0^{\infty} \frac{x^{-1/2}}{\sqrt{x+z}} e^{-px} I_v(c \sqrt{x^2+xz}) dx = e^{pz/2} I_{v/2}(z_-) K_{v/2}(z_+)$$

$$\left[z_{\pm} = \frac{z}{4} (p \pm \sqrt{p^2-c^2}); \operatorname{Re} p > |\operatorname{Re} c|; \operatorname{Re} v > -1; |\arg z| < \pi\right].$$

$$8. \int_0^{\infty} \frac{x^{-1/2}}{\sqrt{x+z}} e^{-px} I_1(c \sqrt{x^2+xz}) dx = \frac{2}{cz} \left\{ \exp\left[\frac{z}{2} (p - \sqrt{p^2-c^2})\right] - 1 \right\}$$

$$[\operatorname{Re} p > |\operatorname{Re} c|; |\arg z| < \pi].$$

$$9. \int_0^{\infty} x^{1/2} (x+z)^{1/2} e^{-px} I_1(c \sqrt{x^2+xz}) dx =$$

$$= \frac{c}{(p^2-c^2)^{3/2}} \left(1 + \frac{z}{2} \sqrt{p^2-c^2}\right) \exp\left[\frac{z}{2} (p - \sqrt{p^2-c^2})\right]$$

$$[\operatorname{Re} p > |\operatorname{Re} c|; |\arg z| < \pi].$$

2.15.8. Интегралы от $A(x) e^{f(x)} I_v(\varphi(x))$

$$1. \int_0^a \frac{x^{\alpha-1}}{a^2-x^2} \exp\left(-b \frac{a^2+x^2}{a^2-x^2}\right) I_v\left(\frac{2cx}{a^2-x^2}\right) dx =$$

$$= -\frac{a^{\alpha-1}}{2c} \Gamma\left[\frac{(\alpha+v)/2}{v+1}\right] M_{(1-\alpha)/2, v/2}(z_-) W_{(1-\alpha)/2, v/2}(z_+)$$

$$[z_{\pm} = a^{-1} (ab \pm \sqrt{a^2 b^2 - c^2}); ab > c > 0; a, \operatorname{Re}(\alpha+v) > 0].$$

$$2. \int_a^{\infty} \frac{x^{\alpha-1}}{x^2-a^2} \exp\left(-b \frac{x^2+a^2}{x^2-a^2}\right) I_v\left(\frac{2cx}{x^2-a^2}\right) dx =$$

$$= \frac{a^{\alpha-1}}{2c} \Gamma\left[\frac{1+(v-\alpha)/2}{v+1}\right] M_{(\alpha-1)/2, v/2}(z_-) W_{(\alpha-1)/2, v/2}(z_+)$$

$$[z_{\pm} = a^{-1} (ab \pm \sqrt{a^2 b^2 - c^2}); ab > c > 0; a > 0; \operatorname{Re}(\alpha-v) < 2].$$

$$3. \int_0^{\infty} \frac{x^{\alpha-1}}{x^2+z^2} \exp\left(\pm b \frac{x^2-z^2}{x^2+z^2}\right) I_v\left(\frac{2cx}{x^2+z^2}\right) dx =$$

$$= \frac{z^{\alpha-1}}{2c} \Gamma\left[\frac{1+(v-\alpha)/2, (v+\alpha)/2}{1+v, 1+v}\right] M_{\mp(1-\alpha)/2, v/2}(z_-) M_{\mp(\alpha-1)/2, v/2}(z_+)$$

$$[z_{\pm} = z^{-1} (\sqrt{b^2 z^2 + c^2} \pm bz); b, \operatorname{Re} z > 0; -\operatorname{Re} v < \operatorname{Re} \alpha < \operatorname{Re} v + 2]$$

2.15.9. Интегралы, содержащие гиперболические функции и $I_\nu(\varphi(x))$.

$$1. \int_0^\infty x^{\nu+1} \frac{\operatorname{sh} ax}{\operatorname{sh} bx} I_\nu(cx) dx = \left(\frac{\pi}{b}\right)^{\nu+2} e^{-(\nu+1/2)\pi} \sum_{k=1}^\infty (-1)^{k-1} k^{\nu+1} \sin \frac{ak\pi}{b} H_{\nu}^{(2)}\left(\frac{ck\pi}{b}\right)$$

[$\operatorname{Re} \nu > -1$; $\operatorname{Re} b > |\operatorname{Re} a| + |\operatorname{Re} c|$]

$$2. \int_0^a \frac{\operatorname{ch}(b\sqrt{a^2-x^2})}{\sqrt{a^2-x^2}} I_\nu(cx) dx = \frac{\pi}{2} I_{\nu/2}(a\sqrt{b^2+c^2}+ab) I_{\nu/2}(a\sqrt{b^2+c^2}-ab)$$

[$a > 0$; $\operatorname{Re} \nu > -1$]

$$3. \int_0^\infty \frac{\operatorname{ch} ax}{\operatorname{ch} x} I_\nu\left(\frac{c}{\operatorname{ch} x}\right) dx =$$

$$= \frac{1}{2c} \Gamma\left[\frac{(1+\nu+a)/2}{\nu+1}, \frac{(1+\nu-a)/2}{\nu+1}\right] M_{a/2, \nu/2}(c) M_{-a/2, \nu/2}(c)$$

[$c > 0$; $|\operatorname{Re} a| < 1 + \operatorname{Re} \nu$]

$$4. \int_{-\infty}^\infty e^{-px-b \operatorname{th} x} I_\nu\left(\frac{c}{\operatorname{ch} x}\right) \frac{dx}{\operatorname{ch} x} = \frac{1}{c} \Gamma\left[\frac{(1+\nu+p)/2}{1+\nu}, \frac{(1+\nu-p)/2}{1+\nu}\right] \times$$

$$\times M_{-p/2, \nu/2}(\sqrt{b^2+c^2}+b) M_{p/2, \nu/2}(\sqrt{b^2+c^2}-b)$$

[$c > 0$; $|\operatorname{Re} p| < \operatorname{Re} \nu + 1$]

2.15.10. Интегралы, содержащие тригонометрические функции и $I_\nu(\varphi(x))$.

$$1. \int_0^a \frac{1}{\sqrt{a^2-x^2}} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} I_\nu(cx) dx = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \frac{\pi}{2} J_0(ab) I_{\nu/2}^2\left(\frac{ac}{2}\right) +$$

$$+ \pi \sum_{k=1}^\infty (-1)^k J_{2k+\delta}(ab) I_{(\nu+\delta)/2+k}\left(\frac{ac}{2}\right) I_{(\nu-\delta)/2-k}\left(\frac{ac}{2}\right)$$

[$a > 0$; $\operatorname{Re} \nu > -(3 \pm 1)/2$; $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$]

$$2. \int_0^a x^{\nu+1} \begin{Bmatrix} \sin [cx^2/(2a)] \\ \cos [cx^2/(2a)] \end{Bmatrix} I_\nu(cx) dx =$$

$$= \frac{a^{\nu+1}}{2c} \left[\begin{Bmatrix} \sin (ac/2) \\ \cos (ac/2) \end{Bmatrix} \int_0^{ac} I_\nu(x) dx \mp \begin{Bmatrix} \cos (ac/2) \\ \sin (ac/2) \end{Bmatrix} \int_0^{ac} I_{\nu+1}(x) dx \right]$$

[$a > 0$; см. 1.11.1]

$$3. \int_0^a x^{\nu+1} \sin(b\sqrt{a^2-x^2}) I_\nu(cx) dx =$$

$$= \sqrt{\frac{\pi}{2}} a^{\nu+3/2} b c^\nu (b^2-c^2)^{-(2\nu+3)/4} J_{\nu+3/2}(a\sqrt{b^2-c^2})$$

[$a > 0$; $\operatorname{Re} \nu > -1$]

$$4. \int_0^a x \sin(b\sqrt{a^2-x^2}) I_0(cx) dx = \frac{b \sin(a\sqrt{b^2-c^2})}{(b^2-c^2)^{3/2}} - \frac{ab \cos(a\sqrt{b^2-c^2})}{b^2-c^2}$$

[$a > 0$]

$$5. \int_0^a \frac{\cos(b\sqrt{a^2-x^2})}{\sqrt{a^2-x^2}} I_\nu(cx) dx = \frac{\pi}{2} J_{\nu/2}(z_+) J_{\nu/2}(z_-)$$

[$2z_\pm = a(b \pm \sqrt{b^2-c^2})$; $a > 0$; $\operatorname{Re} \nu > -1$]

$$6. \int_0^a \frac{\cos(b \sqrt{a^2 - x^2})}{\sqrt{a^2 - x^2}} I_1(cx) dx = \frac{1}{ac} [\cos(a \sqrt{b^2 - c^2}) - \cos ab] \quad [a > 0]$$

$$7. \int_0^a x^{v+1} \frac{\cos(b \sqrt{a^2 - x^2})}{\sqrt{a^2 - x^2}} I_v(cx) dx = \\ = \sqrt{\frac{\pi}{2}} a^{v+1/2} c^v (b^2 - c^2)^{-(2v+1)/4} J_{v+1/2}(a \sqrt{b^2 - c^2}) \\ [a > 0; \operatorname{Re} v > -1]$$

$$8. \int_0^a x \frac{\cos(b \sqrt{a^2 - x^2})}{\sqrt{a^2 - x^2}} I_0(cx) dx = \frac{\sin(a \sqrt{b^2 - c^2})}{\sqrt{b^2 - c^2}} \quad [a > 0]$$

$$9. \int_0^a x^{n+1} \sin(c \sqrt{a^2 - x^2}) I_n(cx) dx = \frac{a^{2n+3} c^{n+1}}{(2n+3)!!} \quad [a > 0]$$

$$10. \int_0^a x^{v+1} \frac{\cos(c \sqrt{a^2 - x^2})}{\sqrt{a^2 - x^2}} I_v(cx) dx = \frac{\sqrt{\pi} a^{2v+1} c^v}{2^{v+1} \Gamma(v+3/2)} \quad [a > 0; \operatorname{Re} v > -1]$$

$$11. \int_0^\pi \left\{ \begin{matrix} \sin bx \\ \cos bx \end{matrix} \right\} I_v(c \sin x) \quad x = \pi \left\{ \begin{matrix} \sin(b\pi/2) \\ \cos(b\pi/2) \end{matrix} \right\} I_{(v-b)/2}\left(\frac{c}{2}\right) I_{(v+b)/2}\left(\frac{c}{2}\right) \\ [b > 0; \operatorname{Re} v > -(3 \pm 1)/2]$$

$$12. \int_0^\pi \cos 2nx I_0\left(c \left\{ \begin{matrix} \sin x \\ \cos x \end{matrix} \right\}\right) dx = (-1)^n \pi I_n^2\left(\frac{c}{2}\right).$$

$$13. \int_0^{2\pi} \cos nx I_0(\sqrt{a^2 + b^2 - 2ab \cos x}) dx = (-1)^n 2\pi I_n(a) I_n(b) \quad [a, b > 0]$$

$$14. \int_0^{2\pi} \frac{1}{\sqrt{a^2 + b^2 - 2ab \cos x}} \left\{ \begin{matrix} \sin(x/2) \sin[(2n+1)x/2] \\ \cos(x/2) \cos[(2n+1)x/2] \end{matrix} \right\} \times \\ \times I_1(\sqrt{a^2 + b^2 - 2ab \cos x}) dx = \frac{(-1)^n \pi}{b \pm a} [I_n(a) I_{n+1}(b) \pm I_{n+1}(a) I_n(b)] \\ [a, b > 0; a \neq b]$$

$$15. \int_0^{2\pi} \frac{\sin x \sin(n+1)x}{\sqrt{a^2 + b^2 - 2ab \cos x}} I_2(\sqrt{a^2 + b^2 - 2ab \cos x}) dx = \\ = \frac{(-1)^n \pi}{b^2 - a^2} [I_n(a) I_{n+2}(b) - I_{n+2}(a) I_n(b)] \quad [a, b > 0; a \neq b]$$

2.15.11. Интегралы от $x^\alpha e^{-px} \left\{ \begin{matrix} \sin bx \\ \cos bx \end{matrix} \right\} I_v(cx)$.

Обозначение: $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$.

$$1. \int_0^\infty e^{-px} \cos bx I_0(cx) \quad x = \sqrt{\frac{R^2 + p^2 - b^2 - c^2}{2R^4}}$$

$$[R^4 = (p^2 - b^2 - c^2)^2 + 4p^2 b^2; \operatorname{Re} p > |\operatorname{Re} c| + |\operatorname{Im} b|]$$

$$2. \int_0^{\infty} x^{\alpha-1} e^{-px} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} I_{\nu}(cx) dx = A_{\alpha}$$

[Re $p > |\operatorname{Re} c| + |\operatorname{Im} b|$; Re $(\alpha + \nu) > -(1 \pm 1)/2$]

$$A_{\nu+1} = \frac{2^{2\nu} c^{\nu} r^{3\nu+2}}{\sqrt{\pi}} \Gamma\left(\nu + \frac{1}{2}\right) (r^4 + 16b^2 c^2)^{-\nu-1/2} \begin{Bmatrix} \sin \varphi \\ \cos \varphi \end{Bmatrix}$$

$$A_{\nu+2} = \frac{2^{2\nu+1}}{\sqrt{\pi}} c^{\nu} r^{3\nu+6} \Gamma\left(\nu + \frac{3}{2}\right) (r^4 + 16b^2 c^2)^{-\nu-3/2} \begin{Bmatrix} \sin \psi \\ \cos \psi \end{Bmatrix}$$

[Re $\nu > -(3 \pm 1)/4$]

$$A_0 = \frac{(2c)^{\nu}}{\nu} (r + \sqrt{r^2 + 4c^2})^{-\nu} \begin{Bmatrix} \sin \theta \\ \cos \theta \end{Bmatrix}$$

[Re $\nu > -(1 \pm 1)/2$]

$$\left[r = \sqrt{2} \{b^2 + p^2 - c^2 + \sqrt{[b^2 + (p+c)^2][b^2 + (p-c)^2]}\}^{1/2}, \right.$$

$$\varphi = (2\nu + 2\delta + 1) \operatorname{arctg} \frac{2b}{r} \sqrt{\frac{r^2 + 4c^2}{r^2 - 4b^2}},$$

$$\psi = (2\nu + 3) \operatorname{arctg} \frac{2b}{r} \sqrt{\frac{r^2 + 4c^2}{r^2 - 4b^2}}, \quad \theta = \nu \arcsin \frac{2b}{r} \Big].$$

$$3. \int_0^{\infty} \frac{1}{x} e^{-px} \sin bx I_0(cx) dx = \arcsin \frac{2b}{r}$$

[Re $p > |\operatorname{Re} c| + |\operatorname{Im} b|$; r см. в 2.15.11.2]

$$4. \int_0^{\infty} \frac{1}{x} e^{-px} \sin bx I_1(cx) dx = \frac{1}{c} (R \sin \varphi - b)$$

[$R^4 = (p^2 - b^2 - c^2)^2 + 4p^2 b^2$, $2pb \operatorname{ctg} 2\varphi = p^2 - b^2 - c^2$; Re $p > |\operatorname{Re} c| + |\operatorname{Im} b|$]

$$5. \int_0^{\infty} \frac{1}{x} e^{-px} \cos bx I_1(cx) dx = \frac{1}{c} (p - R \cos \varphi)$$

[R , φ см. в 2.15.11.4; Re $p > |\operatorname{Re} c| + |\operatorname{Im} b|$]

$$6. \int_0^{\infty} \frac{1}{x^2} e^{-px} \sin bx I_1(cx) dx = \frac{1}{2b^2 c} [(p^2 + b^2) \sin 2\theta - R \sqrt{p^2 + b^2} \sin(\varphi + \theta)] +$$

$$+ \frac{c}{2b^2} \operatorname{arctg} \left(\frac{R \sin \varphi + \sqrt{p^2 + b^2} \sin \theta}{R \cos \varphi + \sqrt{p^2 + b^2} \cos \theta} \right)$$

[$R^4 = (p^2 - b^2 - c^2)^2 + 4b^2 p^2$, $\operatorname{tg} \theta = b/p$, $2pb \operatorname{ctg} 2\varphi = p^2 - b^2 - c^2$; Re $p > |\operatorname{Re} c| + |\operatorname{Im} b|$]

$$7. \int_0^{\infty} e^{-cx} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} I_0(cx) dx = \frac{(2c)^{1-\delta}}{\sqrt{2b(b^2 + 4c^2)}} (b + \sqrt{b^2 + 4c^2})^{\pm 1/2}$$

[b , Re $c > 0$]

$$8. \int_0^{\infty} x^{\alpha-1} e^{-\alpha x} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} I_{\nu}(cx) dx = \frac{b^{-\nu-\alpha}}{\sqrt{\pi}} \left(\frac{c}{2}\right)^{\nu} \Gamma\left[\frac{\alpha+\nu}{2}\right] \begin{Bmatrix} \sin [(\alpha+\nu)\pi/2] \\ \cos [(\alpha+\nu)\pi/2] \end{Bmatrix} \times$$

$$\times {}_4F_3\left(\frac{\alpha+\nu}{2}, \frac{\alpha+\nu+1}{2}, \frac{2\nu+1}{4}, \frac{2\nu+3}{4}; \frac{1}{2}, \nu+\frac{1}{2}, \nu+1; -\frac{4c^2}{b^2}\right) +$$

$$+ (-1)^{\delta} \frac{b^{-\alpha-\nu-1} c^{\nu+1}}{2^{\nu} \sqrt{\pi}} \begin{Bmatrix} \cos [(\alpha+\nu)\pi/2] \\ \sin [(\alpha+\nu)\pi/2] \end{Bmatrix} \Gamma\left[\frac{\alpha+\nu+1}{2}\right] \times$$

$$\times {}_4F_3\left(\frac{\alpha+\nu+1}{2}, \frac{\alpha+\nu+2}{2}, \frac{2\nu+3}{4}, \frac{2\nu+5}{4}; \frac{3}{2}, \nu+1, \nu+\frac{3}{2}; -\frac{4c^2}{b^2}\right)$$

[$0 < 2c < b$; $-\delta - \operatorname{Re} \nu < \operatorname{Re} \alpha < 3/2$]

$$\begin{aligned}
9. \quad &= \frac{b^0}{(2c)^{\alpha+\delta} \sqrt{\pi}} \Gamma \left[\frac{\alpha+\nu+\delta}{\nu-\alpha-\delta+1}, \frac{1/2-\alpha-\delta}{2} \right] {}_4F_3 \left(\frac{\alpha-\nu+1}{2}, \frac{\alpha-\nu}{2}+\delta, \right. \\
&\quad \left. \frac{\alpha+\nu+1}{2}, \frac{\alpha+\nu}{2}+\delta; \frac{3+2\alpha}{4}, \frac{1+2\alpha}{4}+\delta, \delta+\frac{1}{2}; -\frac{b^2}{4c^2} \right) + \\
&\quad + \frac{b^{1/2-\alpha}}{\sqrt{2\pi c}} \left\{ \frac{\sin [(2\alpha-1)\pi/4]}{\cos [(2\alpha-1)\pi/4]} \right\} \Gamma \left(\alpha - \frac{1}{2} \right) \times \\
&\times {}_4F_3 \left(\frac{1+2\nu}{4}, \frac{1-2\nu}{4}, \frac{3+2\nu}{4}, \frac{3-2\nu}{4}; \frac{1}{2}, \frac{5-2\alpha}{4}, \frac{3-2\alpha}{4}; -\frac{b^2}{4c^2} \right) + \\
&\quad + \frac{b^{3/2-\alpha}}{(2c)^{3/2} \sqrt{\pi}} \left\{ \frac{\cos [(1-2\alpha)\pi/4]}{\sin [(1-2\alpha)\pi/4]} \right\} \Gamma \left(\alpha - \frac{3}{4} \right) \times \\
&\times {}_4F_3 \left(\frac{3+2\nu}{4}, \frac{3-2\nu}{4}, \frac{5+2\nu}{4}, \frac{5-2\nu}{4}; \frac{3}{2}, \frac{7-2\alpha}{4}, \frac{5-2\alpha}{4}; -\frac{b^2}{4c^2} \right) \\
&\quad [0 < b < 2c; -\delta - \operatorname{Re} \nu < \operatorname{Re} \alpha < 3/2]
\end{aligned}$$

2.15.12. Интегралы от $x^\alpha e^{-px} \left\{ \frac{\sin b \sqrt{x}}{\cos b \sqrt{x}} \right\} I_\nu(cx)$.

Обозначение $\delta = \begin{cases} 1 \\ 0 \end{cases}$.

$$\begin{aligned}
1. \quad &\int_0^\infty x^{\alpha-1} e^{-cx} \left\{ \frac{\sin b \sqrt{x}}{\cos b \sqrt{x}} \right\} I_\nu(cx) dx = (-1)^\delta \sqrt{\frac{2}{\pi c}} b^{1-2\alpha} \left\{ \frac{\cos \alpha\pi}{\sin \alpha\pi} \right\} \Gamma(2\alpha-1) \times \\
&\quad \times {}_2F_2 \left(\frac{1}{2}-\nu, \frac{1}{2}+\nu, \frac{3}{2}-\alpha, 1-\alpha; -\frac{b^2}{8c} \right) + \\
&\quad + \frac{b^\delta}{(2c)^{\alpha+\delta/2} \sqrt{\pi}} \Gamma \left[\frac{(1-\delta)/2-\alpha}{1+\nu-\alpha-\delta/2}, \nu+\alpha+\delta/2 \right] \times \\
&\quad \times {}_2F_2 \left(\frac{\delta}{2}+\alpha-\nu, \frac{\delta}{2}+\alpha+\nu; \frac{1+\delta}{2}+\alpha, \frac{1}{2}+\delta; -\frac{b^2}{8c} \right) \\
&\quad [b, \operatorname{Re} c > 0; \operatorname{Re} \alpha < 1; \operatorname{Re}(\alpha+\nu) > -\delta/2]
\end{aligned}$$

$$\begin{aligned}
2. \quad &\int_0^\infty x^{-\nu-(1\mp 1)/4} e^{-cx} \left\{ \frac{\sin b \sqrt{x}}{\cos b \sqrt{x}} \right\} I_\nu(cx) dx = \\
&\quad = 2^{1-(\delta+\nu)/2} b^{\nu-1} c^{(\nu-\delta)/2} \exp \left(-\frac{b^2}{16c} \right) W_{(\delta-3\nu)/2, (\nu-\delta)/2} \left(\frac{b^2}{8c} \right) \\
&\quad [b, \operatorname{Re} c > 0; \operatorname{Re} \nu > (\delta-1)/2]
\end{aligned}$$

$$\begin{aligned}
3. \quad &\int_0^\infty x^{-1/2} e^{-cx} \left\{ \frac{\sin b \sqrt{x}}{\cos b \sqrt{x}} \right\} I_0(cx) dx = \sqrt{\frac{\pi \pm 1}{2c}} \exp \left(-\frac{b^2}{16c} \right) \left\{ \begin{matrix} I_0(b^2/(16c)) \\ K_0(b/(16c)) \end{matrix} \right\} \\
&\quad [b, \operatorname{Re} c > 0]
\end{aligned}$$

2.15.13. Интегралы, содержащие логарифмическую функцию и $I_\nu(\varphi(x))$

$$1. \quad \int_a^b x \ln \frac{b}{x} I_0(cx) dx = \frac{1}{c^2} I_0(bc) - \frac{1}{c^2} I_0(ac) - \frac{a}{c} I_1(ac) \ln \frac{b}{a}$$

$[0 < a < b]$

$$2. \quad \int_0^1 x \ln x I_0(cx) dx = \frac{1}{c^2} - \frac{1}{c^2} I_0(c).$$

$$3. \int_0^{\infty} e^{-px} \ln x I_0(cx) dx = (p^2 - c^2)^{-1/2} [\ln(p + \sqrt{p^2 - c^2}) - \ln(p^2 - c^2) - \ln 2 - C] \\ [\operatorname{Re} p > |\operatorname{Re} c|]$$

$$4. \int_0^{\infty} e^{-px^2} \ln x I_0(cx) dx = \frac{2}{p} \exp\left(\frac{c^2}{4p}\right) \left[\ln \frac{c}{2p} - \frac{1}{2} \operatorname{Ei}\left(-\frac{c^2}{4p}\right) \right] \quad [\operatorname{Re} p > 0]$$

$$5. \int_0^{\infty} x e^{-px^2} \ln x I_0(cx) dx = -\frac{1}{4p} \exp\left(\frac{c^2}{4p}\right) \left[2 \ln^2 p + \operatorname{Ei}\left(-\frac{c^2}{4p}\right) \right] \\ [\operatorname{Re} p > 0]$$

$$6. \int_0^{\infty} x \frac{\exp(-p \sqrt{x^2 + z^2})}{\sqrt{x^2 + z^2}} \ln(\sqrt{x^2 + z^2} \pm z) I_0(cx) dx = \\ = \frac{1}{r} \left[e^{-rz} \ln \frac{z(p \pm r)}{r} - e^{rz} \operatorname{Ei}(-2rz) \right] + \left\{ \begin{matrix} 0 \\ A \end{matrix} \right\} \\ \left[r = \sqrt{p^2 - c^2}, A = \frac{1}{r} [e^{rz} \operatorname{Ei}(-rz - pz) - e^{-rz} \operatorname{Ei}(rz - pz)]; \operatorname{Re} z > 0; \operatorname{Re} p > |\operatorname{Re} c| \right].$$

$$7. \int_0^{\infty} e^{-px} \ln x I_0(c \sqrt{x^2 + xz}) dx = \\ = \frac{1}{r} e^{pz/2} \left\{ e^{-rz/2} \left[\ln \frac{z(p-r)}{2r} - \operatorname{Ei}\left(\frac{rz-pz}{2}\right) \right] + e^{rz/2} \left[\operatorname{Ei}\left(-\frac{rz+pz}{2}\right) - \operatorname{Ei}(-rz) \right] \right\} \\ [r = \sqrt{p^2 - c^2}; \operatorname{Re} p > |\operatorname{Re} c|; |\arg z| < \pi]$$

$$8. \int_0^{\infty} e^{-px} \ln(x+z) I_0(c \sqrt{x^2 + xz}) dx = \\ = \frac{1}{r} e^{pz/2} \left[e^{-rz/2} \ln \frac{z(p+r)}{2r} - e^{rz/2} \operatorname{Ei}(-rz) \right] \\ [r = \sqrt{p^2 - c^2}; \operatorname{Re} p > |\operatorname{Re} c|; |\arg z| < \pi]$$

$$9. \int_0^{\infty} \frac{1}{x^2} \ln I_0(x) dx = \frac{\pi}{2}.$$

2.15.14. Интегралы, содержащие обратные тригонометрические функции и $I_\nu(cx)$

$$1. \int_0^1 \frac{\cos(b \arccos x)}{\sqrt{1-x^2}} I_\nu(cx) dx = \frac{\pi}{2} I_{(\nu+b)/2}\left(\frac{c}{2}\right) I_{(\nu-b)/2}\left(\frac{c}{2}\right) \\ [\operatorname{Re} \nu > -1]$$

2.15.15. Интегралы от $A(x) J_\mu(\varphi(x)) I_\nu(cx)$.

$$1. \int_0^a x^{\nu+1} (a^2 - x^2)^{\mu/2} J_\mu(b \sqrt{a^2 - x^2}) I_\nu(cx) dx = \\ = a^{\mu+\nu+1} b^\mu c^\nu (b^2 - c^2)^{-(\mu+\nu+1)/2} J_{\mu+\nu+1}(a \sqrt{b^2 - c^2}) \\ [a > 0; \operatorname{Re} \mu, \operatorname{Re} \nu > -1]$$

$$2. \int_0^a x^{\nu+1} (a^2 - x^2)^{-\nu/2-1} J_\mu(c \sqrt{a^2 - x^2}) I_\nu(cx) dx = \Gamma\left[\frac{(\mu-\nu)/2}{(\mu+\nu)/2+1}\right] \left(\frac{ac}{2}\right)^\nu I_\mu(ac) \\ [a > 0; -1 < \operatorname{Re} \nu < \operatorname{Re} \mu]$$

$$3. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\mu/2} J_{\mu}(c \sqrt{a^2 - x^2}) I_{\nu}(cx) dx = \frac{1}{2^{\mu+\nu+1}} \Gamma \left[\frac{(\alpha + \nu)/2}{\nu + 1, 1 + \mu + (\alpha + \nu)/2} \right] \times \\ \times a^{\alpha+2\mu+\nu} c^{\mu+\nu} {}_1F_2 \left(\frac{\nu - \alpha}{2} + 1; \nu + 1, \frac{\alpha + \nu}{2} + \mu + 1; -\frac{a^2 c^2}{4} \right) \\ [a, \operatorname{Re}(\alpha + \nu) > 0; \operatorname{Re} \mu > -1]$$

$$4. \int_0^a x^{-\mu-1} (a^2 - x^2)^{\mu/2} J_{\mu}(c \sqrt{a^2 - x^2}) I_{\nu}(cx) dx = \\ = 2^{-\mu-1} \Gamma \left[\frac{(\nu - \mu)/2}{(\mu + \nu)/2 + 1} \right] a^{\mu} c^{\mu} J_{\nu}(ac) \quad [a > 0; -1 < \operatorname{Re} \mu < \operatorname{Re} \nu]$$

$$5. \int_0^a x^{1-\nu} (a^2 - x^2)^{\mu/2} J_{\mu}(c \sqrt{a^2 - x^2}) I_{\nu}(cx) dx = \frac{a^{2\mu-2\nu+2} c^{\mu-\nu}}{2^{\nu-1} \Gamma(\nu)} \int_0^{ac} x^{2\nu-\mu-2} J_{\mu+1}(x) dx \\ [a, \operatorname{Re} \nu > 0; \operatorname{Re} \mu > -1]$$

$$6. \int_0^a x (a^2 - x^2)^{\mu/2} J_{\mu}(c \sqrt{a^2 - x^2}) I_0(cx) dx = \frac{a^{2\mu+2} c^{\mu}}{2^{\mu+1} \Gamma(\mu + 2)} \\ [a > 0; \operatorname{Re} \mu > -1]$$

$$7. \int_0^a x^{\mp \nu} (a^2 - x^2)^{\pm \nu - 1/4} J_{\pm 2\nu - 1/2}(c \sqrt{a^2 - x^2}) I_{\nu}(cx) dx = \\ = 2^{\mp \nu - 1/2} \pi^{(1 \pm 1)/2} \Gamma^{\mp 1} \left(\nu + \frac{1}{2} \right) a^{\pm 2\nu} c^{\pm \nu - 1/2} J_{\nu} \left(\frac{ac}{2} \right) J_{\pm \nu} \left(\frac{ac}{2} \right) \\ [a > 0; \pm \operatorname{Re} \nu > -1/4]$$

2.15.16. Интегралы от $x^{\alpha} e^{-px^n} J_{\mu}(bx) I_{\nu}(cx)$.

$$1. \int_0^{\infty} e^{-px} J_{\mu}(bx) I_{\nu}(cx) dx = A_{\mu, \nu}$$

$$\left[\operatorname{Re} p > |\operatorname{Re} c| + |\operatorname{Im} b|; \operatorname{Re}(\mu + \nu) > -1; k = \frac{1}{\sqrt{2}} \left(1 - \frac{p^2 + b^2 - c^2}{\sqrt{(p^2 + b^2 - c^2)^2 + 4b^2 c^2}} \right)^{1/2} \right],$$

$$A_{0,0} = \frac{2}{\pi} \sqrt{\frac{k}{bc}} (1 - k^2)^{1/4} K(k),$$

$$A_{1,1} = \frac{2}{\pi \sqrt{bck}} (1 - k^2)^{-1/4} [E(k) - (1 - k^2) K(k)],$$

$$A_{2,2} = \frac{2}{3\pi \sqrt{bck^3}} (1 - k^2)^{-3/4} [(1 - k^2)(2 - 3k^2) K(k) - 2(1 - 2k^2) E(k)],$$

$$A_{3,3} = \frac{2}{15\pi \sqrt{bck^5}} (1 - k^2)^{-5/4} [(8 - 23k^2 + 23k^4) E(k) - (1 - k^2)(8 - 19k^2 + \\ + 15k^4) K(k)].$$

$$2. \int_0^{\infty} x^{\alpha-1} e^{-px} J_{\mu}(bx) I_{\nu}(cx) dx = B_{\mu, \nu}^{\alpha}$$

$$[\operatorname{Re} p > |\operatorname{Re} c| + |\operatorname{Im} b|; \operatorname{Re}(\alpha + \mu + \nu) > 0; \\ r = 2^{-1/2} \{p^2 + c^2 - b^2 + \sqrt{[c^2 + (p + b)^2][c^2 + (p - b)^2]}\}^{1/2}, \\ k = \frac{1}{\sqrt{2}} \left(1 - \frac{p^2 + b^2 - c^2}{\sqrt{(p^2 + r^2 - c^2)^2 + 4b^2 c^2}} \right)^{1/2}],$$

$$B_{\mu, \nu}^{\alpha} = \frac{b^{\mu} c^{\nu}}{\Gamma(\mu+1)} 2^{-\mu-\nu} p^{-\alpha-\mu-\nu} \sum_{k=0}^{\infty} \frac{1}{k!} \Gamma \left[\begin{matrix} \alpha + \mu + \nu + 2k \\ \nu + k + 1 \end{matrix} \right] \times \\ \times {}_2F_1 \left(-k, -\nu - k; \mu + 1; -\frac{b^2}{c^2} \right) \left(\frac{c}{2p} \right)^{2k},$$

$$B_{\mu, \nu}^{1/2} = r^{-1/2} \Gamma \left(\mu + \nu + \frac{1}{2} \right) P_{\mu-1/2}^{-\nu} \left(\frac{\sqrt{r^2 + c^2}}{r} \right) P_{\nu-1/2}^{-\mu} \left(\frac{\sqrt{r^2 - b^2}}{r} \right) e^{-i\nu\pi/2} \\ [\operatorname{Re}(\mu + \nu) > -1/2].$$

$$B_{0,0}^2 = \frac{2p}{\pi} \left(\frac{k}{bc} \right)^{3/2} (1 - k^2)^{3/4} [2E(k) - K(k)],$$

$$B_{1,1}^2 = \frac{2p \sqrt{k}}{\pi (ac)^{3/2}} (1 - k^2)^{-1/4} [(1 - k^2) K(k) - (1 - 2k^2) E(k)],$$

$$B_{2,2}^2 = \frac{2p}{\pi (ac)^{3/2} \sqrt{k}} (1 - k^2)^{-1/4} [2(1 - k^2 + k^4) E(k) - (1 - k^2)(2 - k^2) K(k)],$$

$$B_{3,3}^2 = \frac{2p}{3\pi (ack)^{3/2}} (1 - k^2)^{-3/4} [(8 - 15k^2 + 3k^4)(1 - k^2) K(k) - \\ - (8 - 19k^2 + 9k^4 - 6k^6) E(k)]$$

$$3. \int_0^{\infty} x^{\alpha-1} e^{-cx} J_{\mu}(bx) I_{\nu}(cx) dx = \frac{2^{\alpha-1} c^{\nu}}{b^{\alpha+\nu}} \Gamma \left[\begin{matrix} (\alpha + \mu + \nu)/2 \\ \nu + 1, (\mu - \alpha - \nu)/2 + 1 \end{matrix} \right] \times \\ \times {}_4F_3 \left(\frac{\alpha + \nu - \mu}{2}, \frac{\alpha + \mu + \nu}{2}, \frac{2\nu + 1}{4}, \frac{2\nu + 3}{4}; \frac{1}{2}, \nu + \frac{1}{2}, \nu + 1; -\frac{4c^2}{b^2} \right) - \\ - \frac{2^{\alpha} c^{\nu+1}}{b^{\alpha+\nu+1}} \Gamma \left[\begin{matrix} (\alpha + \mu + \nu + 1)/2 \\ \nu + 1, (1 + \mu - \alpha - \nu)/2 \end{matrix} \right] \times \\ \times {}_4F_3 \left(\frac{1 + \alpha - \mu + \nu}{2}, \frac{1 + \alpha + \mu + \nu}{2}, \frac{2\nu + 3}{4}, \frac{2\nu + 5}{4}; \frac{3}{2}, \nu + 1, \nu + \frac{3}{2}; -\frac{4c^2}{b^2} \right) \\ [0 < 2c < b; \operatorname{Re}(\alpha + \mu + \nu) > 0; \operatorname{Re} \alpha < 2]$$

$$4. = \frac{2^{\alpha-2} b^{1/2-\alpha}}{\sqrt{\pi} c} \Gamma \left[\begin{matrix} (\mu + \alpha)/2 - 1/4 \\ (\mu - \alpha)/2 + 5/4 \end{matrix} \right] \times \\ \times {}_4F_3 \left(\frac{1+2\nu}{4}, \frac{1-2\nu}{4}, \frac{3+2\nu}{4}, \frac{3-2\nu}{4}; \frac{1}{2}, \frac{5}{4} - \frac{\mu + \alpha}{2}, \frac{5}{4} + \frac{\mu - \alpha}{2}; -\frac{b^2}{4c^2} \right) - \\ - \frac{2^{\alpha-4} b^{3/2-\alpha}}{c^{3/2} \sqrt{\pi}} \left(\nu^2 - \frac{1}{4} \right) \Gamma \left[\begin{matrix} (\mu + \alpha)/2 - 3/4 \\ (\mu - \alpha)/2 + 7/4 \end{matrix} \right] \times \\ \times {}_4F_3 \left(\frac{3+2\nu}{4}, \frac{3-2\nu}{4}, \frac{5+2\nu}{4}, \frac{5-2\nu}{4}; \frac{3}{2}, \frac{7}{4} - \frac{\mu + \alpha}{2}, \frac{7}{4} + \frac{\mu - \alpha}{2}; -\frac{b^2}{4c^2} \right) + \\ + \frac{b^{\mu} 2^{-2\mu-\alpha}}{c^{\mu+\alpha} \sqrt{\pi}} \Gamma \left[\begin{matrix} 1/2 - \alpha - \mu, \mu + \nu + \alpha \\ 1 + \mu, \nu - \mu - \alpha + 1 \end{matrix} \right] {}_4F_3 \left(\frac{1 + \mu + \alpha - \nu}{2}, \frac{\mu + \alpha - \nu}{2}, \right. \\ \left. \frac{1 + \mu + \nu + \alpha}{2}, \frac{\mu + \nu + \alpha}{2}; \frac{3}{4} + \frac{\mu + \alpha}{2}, \frac{1}{4} + \frac{\mu + \alpha}{2}, 1 + \mu; -\frac{b^2}{4c^2} \right) \\ [0 < b < 2c; \operatorname{Re}(\alpha + \mu + \nu) > 0; \operatorname{Re} \alpha < 2]$$

$$5. \int_0^{\infty} x^{1/2} e^{-cx} J_{\nu+1/2}(bx) I_{\nu}(cx) dx = \frac{(2c)^{\nu}}{b \sqrt{\pi}} (b^2 + 4c^2)^{-1/2} (\sqrt{b^2 + 4c^2} + b)^{1/2-\nu} \\ [b, \operatorname{Re} c > 0; \operatorname{Re} \nu > -1]$$

$$6. \int_0^{\infty} x^{\alpha-1} e^{-px^2} J_{\mu}(bx) I_{\nu}(cx) dx =$$

$$= \frac{b^{\mu} c^{\nu}}{2^{\mu+\nu+1} \Gamma(\mu+1)} p^{-(\alpha+\mu+\nu)/2} \sum_{k=0}^{\infty} \frac{1}{k!} \Gamma \left[\frac{k+(\alpha+\mu+\nu)/2}{\nu+k+1} \right] \left(\frac{c}{2p} \right)^{2k} \times \\ \times {}_2F_1 \left(-k, -\nu-k; \mu+1; -\frac{b^2}{c^2} \right) \quad [\operatorname{Re} p, \operatorname{Re}(\alpha+\mu+\nu) > 0]$$

$$7. \int_0^{\infty} x e^{-px^2} J_{\nu}(bx) I_{\nu}(cx) dx = \frac{1}{2p} \exp \left(\frac{c^2-b^2}{4p} \right) J_{\nu} \left(\frac{bc}{2p} \right)$$

$$[\operatorname{Re} p > 0; \operatorname{Re} \nu > -1]$$

$$8. \int_0^{\infty} x^3 e^{-px^2} J_{n+2}(bx) I_n(cx) dx = \frac{1}{8p^3 b^2} \exp \left(\frac{c^2-b^2}{4p} \right) \left\{ bc(b^2-4p) J_{n+1} \left(\frac{bc}{2p} \right) - \right. \\ \left. - bc(4pn+8p+b^2) J_{n-1} \left(\frac{bc}{2p} \right) + [16p^2 n(n+2) + (4pn-c^2)b^2 + b^4] J_n \left(\frac{bc}{2p} \right) \right\} \\ [\operatorname{Re} p > 0]$$

$$9. \int_0^{\infty} x^3 e^{-px^2} J_n(bx) I_{n+2}(cx) dx = -\frac{1}{8p^3 c^2} \exp \left(\frac{c^2-b^2}{4p} \right) \times \\ \times \left\{ bc(c^2+4p) J_{n+1} \left(\frac{bc}{2p} \right) + bc(4pn+8p-c^2) J_{n-1} \left(\frac{bc}{2p} \right) - \right. \\ \left. - [16p^2 n(n+2) - (4pn+b^2)c^2 + c^4] J_n \left(\frac{bc}{2p} \right) \right\} \quad [\operatorname{Re} p > 0]$$

$$10. \int_0^{\infty} x^3 e^{-px^2} J_1(cx) I_1(cx) dx = \frac{c^2}{4p^3} J_0 \left(\frac{c^2}{2p} \right) \quad [\operatorname{Re} p > 0]$$

2.15.17. Интегралы от $x^{\alpha} e^{-px} J_{\mu}(b\sqrt{x}) I_{\nu}(cx)$.

$$1. \int_0^{\infty} e^{-px} J_{2\nu}(b\sqrt{x}) I_{\nu}(cx) dx = \frac{1}{\sqrt{p^2-c^2}} \exp \left(-\frac{b^2 p}{4p^2-4c^2} \right) I_{\nu} \left(\frac{b^2 c}{4p^2-4c^2} \right) \\ [b > 0; \operatorname{Re} p > |\operatorname{Re} c|; \operatorname{Re} \nu > -1]$$

$$2. \int_0^{\infty} e^{-cx} J_{2\nu}(b\sqrt{x}) I_{\nu}(cx) dx = \frac{1}{b\sqrt{2\pi c}} \exp \left(-\frac{b^2}{8c} \right) \\ [b, \operatorname{Re} c > 0; \operatorname{Re} \nu > -1].$$

$$3. \int_0^{\infty} x^{\alpha-1} e^{-cx} J_{\mu}(b\sqrt{x}) I_{\nu}(cx) dx = \frac{b^{\mu} c^{-\alpha-\mu/2}}{2^{\alpha+3\mu/2} \sqrt{\pi}} \Gamma \left[\frac{(1-\mu)/2-\alpha, \nu+\alpha+\mu/2}{\mu+1, 1+\nu-\alpha+\mu/2} \right] \times \\ \times {}_2F_2 \left(\alpha+\frac{\mu}{2}-\nu, \alpha+\frac{\mu}{2}+\nu; \frac{1+\mu}{2}+\alpha, \mu+1; -\frac{b^2}{8c} \right) + \\ + \frac{2^{2\alpha-3/2} b^{1-2\alpha}}{\sqrt{\pi c}} \Gamma \left[\frac{\alpha+(\mu-1)/2}{(\mu+3)/2-\alpha} \right] \times \\ \times {}_2F_2 \left(\frac{1}{2}-\nu, \frac{1}{2}+\nu; \frac{3-\mu}{2}-\alpha, \frac{3+\mu}{2}-\alpha; -\frac{b^2}{8c} \right) \\ [b, \operatorname{Re} c, \operatorname{Re}(\alpha+\nu+\mu/2) > 0; \operatorname{Re} \alpha < 5/2]$$

$$4. \int_0^{\infty} x^{\pm \nu \mp \mu/2} e^{-cx} J_{\mu}(b\sqrt{x}) I_{\nu}(cx) dx =$$

$$= \frac{2^{(5 \pm 2\mu \pm 2\nu)/4}}{\sqrt{\pi} b^{3/2 \pm \nu}} c^{(\pm 2\mu \mp 2\nu - 1)/4} \Gamma(1 \pm 1/2) \left[\frac{\nu + 1/2}{\mu - \nu + 1/2} \right] e^{-z/2} \left\{ \begin{matrix} M_{\rho, \sigma}(z) \\ W_{\rho, \sigma}(z) \end{matrix} \right\}$$

$$[b, \operatorname{Re} c > 0; z = \frac{b^2}{8c}, \rho = (1 \pm 6\nu \mp 2\mu)/4, \sigma = (\pm 2\mu \mp 2\nu - 1)/4, \left\{ \begin{matrix} -1 \leq 2\operatorname{Re} \nu < \operatorname{Re} \mu - 1/2 \\ -1 \leq \operatorname{Re} \mu < 2\operatorname{Re} \nu + 1/2 \end{matrix} \right\}].$$

$$5. \int_0^{\infty} x^{(\pm 1 - 1 \mp \nu)/2} e^{-cx} J_{\nu - (1 \mp 1)/2}(b\sqrt{x}) I_{\nu}(cx) dx =$$

$$= \frac{b^{\nu - 1/2 \mp 1/2}}{\sqrt{\pi} c^{(1 \pm 1)/4}} \exp\left(-\frac{b^2}{16c}\right) D_{-2\nu - (1 \mp 1)/2}\left(\frac{b}{2\sqrt{c}}\right)$$

$$[b, \operatorname{Re} c > 0; \operatorname{Re} \nu > \mp 1/2]$$

$$6. \int_0^{\infty} x^{(2\nu - 1)/4} e^{-cx} J_{3\nu - 1/2}(b\sqrt{x}) I_{\nu}(cx) dx =$$

$$= \frac{2^{2\nu + 2} b^{\nu - 1/2} c^{\nu + 1/2}}{\pi} \exp\left(-\frac{b^2}{16c}\right) K_{\nu}\left(\frac{b^2}{16c}\right)$$

$$[b, \operatorname{Re} c > 0, -1/6 < \operatorname{Re} \nu < 1]$$

$$7. \int_0^{\infty} x^{-(2\nu + 1)/4} e^{-cx} J_{3\nu + 1/2}(b\sqrt{x}) I_{\nu}(cx) dx =$$

$$= 2^{2\nu} b^{-\nu - 1/2} c^{\nu - 1/2} \exp\left(-\frac{b^2}{16c}\right) K_{\nu}\left(\frac{b^2}{16c}\right)$$

$$[b, \operatorname{Re} c > 0; \operatorname{Re} \nu > -1/2]$$

2.15.18. Интегралы от $x^{\alpha} e^{-px^2} Y_{\mu}(bx) I_{\nu}(cx)$

$$1. \int_0^{\infty} x^{\alpha - 1} e^{-cx} Y_{\mu}(bx) I_{\nu}(cx) dx =$$

$$= -\frac{2^{\alpha + 2\nu - 1} c^{\nu}}{\pi^{3/2} b^{\alpha + \nu} \Gamma(\nu + 1)} \cos \frac{\alpha - \mu + \nu}{2} \pi \Gamma\left(\frac{\alpha + \mu + \nu}{2}\right) \Gamma\left(\frac{\alpha - \mu + \nu}{2}\right) \times$$

$$\times {}_4F_3\left(\frac{\alpha + \mu + \nu}{2}, \frac{\alpha - \mu + \nu}{2}, \frac{2\nu + 1}{4}, \frac{2\nu + 3}{4}; \nu + \frac{1}{2}, \nu + 1, \frac{1}{2}; -\frac{4c^2}{b^2}\right) -$$

$$- \frac{2^{\alpha + 2\nu} c^{\nu + 1}}{\pi^{3/2} b^{\alpha + \nu + 1}} \sin \frac{\alpha - \mu + \nu}{2} \pi \Gamma\left[\frac{(\alpha + \mu + \nu + 1)/2, (\alpha - \mu + \nu + 1)/2}{\nu + 1}\right] \times$$

$$\times {}_4F_3\left(\frac{\alpha + \mu + \nu + 1}{2}, \frac{\alpha - \mu + \nu + 1}{2}, \frac{2\nu + 3}{4}, \frac{2\nu + 5}{4}; \nu + 1, \nu + \frac{3}{2}, \frac{3}{2}; -\frac{4c^2}{b^2}\right)$$

$$[b, \operatorname{Re} c > 0; \operatorname{Re}(\alpha + \nu) > |\operatorname{Re} \mu|, \operatorname{Re} \alpha < 2]$$

$$2. \int_0^{\infty} x e^{-px^2} Y_0(cx) I_0(cx) dx = \frac{1}{4p} H_0\left(\frac{c^2}{2p}\right) + \frac{1}{4p} Y_0\left(\frac{c^2}{2p}\right)$$

$$[\operatorname{Re} p > 0]$$

2.15.19. Интегралы от $A(x) I_{\mu}(\varphi(x)) I_{\nu}(cx)$.

$$1. \int_0^a x^{\alpha - 1} (a^2 - x^2)^{\beta - 1} I_{\mu}(cx) I_{\nu}(cx) dx =$$

$$= 2^{-\mu - \nu - 1} a^{\alpha + \mu + \nu + 2\beta - 2} c^{\mu + \nu} \Gamma\left[\frac{\beta, (\alpha + \mu + \nu)/2}{\beta + (\alpha + \mu + \nu)/2, 1 + \mu, 1 + \nu}\right] \times$$

$$\times {}_3F_4\left(\frac{\alpha + \mu + \nu}{2}, \frac{\mu + \nu + 1}{2}, \frac{\mu + \nu}{2} + 1; \beta + \frac{\alpha + \mu + \nu}{2}, \mu + \nu + 1, \mu + 1, \nu + 1; a^2 c^2\right)$$

$$[a, \operatorname{Re} \beta, \operatorname{Re}(\alpha + \mu + \nu) > 0]$$

$$2. \int_0^a x^{\mu+\nu+1} (a^2-x^2)^{-(\mu+\nu)/2-1} I_\mu(cx) I_\nu(cx) dx =$$

$$= \frac{1}{2\sqrt{\pi}} \Gamma \left[\begin{matrix} (\mu+\nu+1)/2, -(\mu+\nu)/2 \\ \mu+1, \nu+1 \end{matrix} \right] (ac)^{\mu+\nu} {}_1F_2 \left(\frac{\mu+\nu+1}{2}; \mu+1, \nu+1; a^2c^2 \right)$$

[a > 0; -1 < Re(\mu + \nu) < 0]

$$3. \int_0^a x^{2\nu} (a^2-x^2)^{-\nu-1/2} I_{\nu-1}(cx) I_\nu(cx) dx = \frac{1}{2} \Gamma \left(\frac{1}{2} - \nu \right) (ac)^{\nu-1} I_\nu(2ac)$$

[a > 0, 0 < Re \nu < 1/2]

$$4. \int_0^a \frac{1}{x \sqrt{a^2-x^2}} I_1^2(cx) dx = \frac{1}{2a^2c} I_1(2ac) - \frac{1}{2a}$$

[a > 0]

$$5. \int_0^a \frac{1}{x^3 \sqrt{a^2-x^2}} I_{\frac{3}{2}}^2(cx) dx = \frac{1}{4a^3c^2} \left[\frac{a^2c^2}{2} - \frac{a^4c^4}{6} - J_2(2ac) + 2ac J_3(2ac) \right]$$

[a > 0]

$$6. \int_0^a I_{-\nu}(ac-cx) I_\nu(cx) dx = \frac{\text{sh } ac}{c}$$

[a > 0; |Re \nu| < 1]

$$7. \int_0^a I_{1-\nu}(ac-cx) I_\nu(cx) dx = \frac{1}{c} \text{ch } ac - \frac{1}{c} I_0(ac)$$

[a > 0, -1 < Re \nu < 2]

$$8. \int_0^a x^{\alpha-1} I_\mu(ac-cx) I_\nu(cx) dx =$$

$$= \left(\frac{2}{c} \right)^\alpha \sum_{k=0}^{\infty} \frac{(-1)^k (\alpha)_k}{k!} \Gamma \left[\begin{matrix} \alpha + \nu + k \\ \nu + k + 1 \end{matrix} \right] I_{\alpha+\mu+\nu+2k}(ac)$$

[a, Re(\alpha + \nu) > 0; Re \mu > -1]

$$9. \int_0^a \frac{1}{x} I_\mu(ac-cx) I_\nu(cx) dx = \frac{1}{\nu} I_{\mu+\nu}(ac)$$

[a, Re \nu > 0; Re \mu > -1]

$$10. \int_0^a \frac{x^{\alpha-1}}{a-x} I_\mu(ac-cx) I_\nu(cx) dx =$$

$$= \frac{1}{a\mu} \left(\frac{2}{c} \right)^\alpha \sum_{k=0}^{\infty} \frac{(\alpha)_k}{k!} \Gamma \left[\begin{matrix} \alpha + \nu + k \\ \nu + k + 1 \end{matrix} \right] (\alpha + \mu + \nu + 2k) I_{\alpha+\mu+\nu+2k}(ac)$$

[a, Re \mu, Re(\alpha + \nu) > 0]

$$11. \int_0^a \frac{1}{x(a-x)} I_\mu(ac-cx) I_\nu(cx) dx = \frac{\mu+\nu}{a\mu\nu} I_{\mu+\nu}(ac)$$

[a, Re \mu, Re \nu > 0]

$$12. \int_0^a x^\lambda (a-x)^{\mu+(1\pm 1)/2} I_\mu(ac-cx) I_\nu(cx) dx =$$

$$= \frac{a^{\mu+\nu+1\pm 1/2}}{\sqrt{2\pi c}} \Gamma \left[\begin{matrix} \mu+1\pm 1/2, \nu+1/2 \\ \mu+\nu+(3\pm 1)/2 \end{matrix} \right] I_{\mu+\nu+1/2}(ac)$$

[a > 0; Re \mu > -(3\pm 1)/4, Re \nu > -1/2]

$$13. \int_0^a x^v (a-x)^{-v-1} I_\mu(ac-cx) I_\nu(cx) dx = \frac{(2ac)^v}{\sqrt{\pi}} \Gamma \left[\begin{matrix} v+1/2, \mu-v \\ \mu+v+1 \end{matrix} \right] I_\mu(ac)$$

[$a > 0$; $-1/2 < \operatorname{Re} v < \operatorname{Re} \mu$]

$$14. \int_0^a x^{v+1} (a^2-x^2)^{\mu/2} I_\mu(b\sqrt{a^2-x^2}) I_\nu(cx) dx =$$

$$= a^{\mu+v+1} b^\mu c^v (b^2+c^2)^{-(\mu+v+1)/2} I_{\mu+v+1}(a\sqrt{b^2+c^2})$$

[$a, \operatorname{Re} b, \operatorname{Re} c > 0$; $\operatorname{Re} \mu, \operatorname{Re} v > -1$]

2.15.20. Интегралы от $x^\alpha e^{-px^n} I_\mu(bx) I_\nu(cx)$.

$$1. \int_0^\infty e^{-px} I_\nu(bx) I_\nu(cx) dx = A_\nu$$

$$\left[\operatorname{Re} p > |\operatorname{Re} b| + |\operatorname{Re} c|; \operatorname{Re} v > -1/2; k = \sqrt{\frac{4bc}{p^2 - (b-c)^2}} \right],$$

$$A_\nu = \frac{1}{\pi \sqrt{bc}} Q_{\nu-1/2} \left(\frac{p^2 - b^2 - c^2}{2bc} \right),$$

$$A_0 = \frac{k}{\pi \sqrt{bc}} K(k),$$

$$A_1 = \frac{1}{\pi k \sqrt{bc}} [(2-k^2) K(k) - 2E(k)].$$

$$2. \int_0^\infty x^{\alpha-1} e^{-px} I_\mu(bx) I_\nu(cx) dx = \frac{b^\mu c^\nu}{2^{\mu+\nu} p^{\alpha+\mu+\nu}} \Gamma \left[\begin{matrix} \alpha+\mu+\nu \\ \mu+1, \nu+1 \end{matrix} \right] \times$$

$$\times F_4 \left(\frac{\alpha+\mu+\nu}{2}, \frac{\alpha+\mu+\nu+1}{2}; \mu+1, \nu+1; \frac{b^2}{p^2}, \frac{c^2}{p^2} \right) =$$

$$= \frac{b^\mu c^\nu p^{-\alpha-\mu-\nu}}{2^{\mu+\nu} \Gamma(\nu+1)} \sum_{k=0}^\infty \frac{1}{k!} \Gamma \left[\begin{matrix} \alpha+\mu+\nu+2k \\ \mu+k+1 \end{matrix} \right] {}_2F_1 \left(-k, -\mu-k, \nu+1; \frac{c^2}{b^2} \right) \left(\frac{b}{2p} \right)^{2k}$$

[$\operatorname{Re} p > |\operatorname{Re} b| + |\operatorname{Re} c|$; $\operatorname{Re}(\alpha+\mu+\nu) > 0$]

$$3. \int_0^\infty x^{-1/2} e^{-px} I_\mu(bx) I_\nu(cx) dx =$$

$$= r^{-1/2} \Gamma \left(\mu + \nu + \frac{1}{2} \right) P_{\nu-1/2}^{\mu} \left(\frac{\sqrt{r^2+b^2}}{r} \right) P_{\mu-1/2}^{\nu} \left(\frac{\sqrt{r^2+c^2}}{r} \right)$$

[$2r = \sqrt{p^2 - (b+c)^2} + \sqrt{p^2 - (b-c)^2}$; $\operatorname{Re}(\mu+\nu) > -1/2$ $\operatorname{Re} p > |\operatorname{Re} b| + |\operatorname{Re} c|$]

$$4. \int_0^\infty e^{-px} I_\mu(cx) I_\nu(cx) dx = B_{\mu,\nu}$$

[$\operatorname{Re} p > 2|\operatorname{Re} c|$, $\operatorname{Re}(\mu+\nu) > -1$]

$$B_{0,0} = \frac{2}{\pi p} E \left(\frac{2c}{p} \right), \quad B_{0,1} = \frac{1}{\pi c} K \left(\frac{2c}{p} \right) - \frac{1}{2c},$$

$$B_{1,1} = \frac{p}{\pi c^2} \left[\left(1 - \frac{2c^2}{p^2} \right) K \left(\frac{2c}{p} \right) - E \left(\frac{2c}{p} \right) \right].$$

$$5. \int_0^\infty x^{\alpha-1} e^{-px} I_\mu(cx) I_\nu(cx) dx = C_{\mu,\nu}^\alpha$$

[$\operatorname{Re}(\alpha+\mu+\nu) > 0$; $\operatorname{Re} p > 2|\operatorname{Re} c|$; $z = \frac{\sqrt{p^2-4c^2}}{p}$]

$$C_{\mu, \nu}^{\alpha} = \left(\frac{c}{2}\right)^{\mu+\nu} p^{-\alpha-\mu-\nu} \Gamma \left[\begin{matrix} \alpha + \mu + \nu \\ \mu + 1, \nu + 1 \end{matrix} \right] {}_4F_3 \left(\begin{matrix} \frac{\mu + \nu + 1}{2}, \frac{\mu + \nu}{2} + 1, \\ \frac{\alpha + \mu + \nu}{2}, \frac{\alpha + \mu + \nu + 1}{2} \end{matrix} ; \mu + \nu + 1, \nu + 1, \mu + 1; \frac{4c^2}{p^2} \right),$$

$$C_{\nu, \nu}^{2\nu+1} = \frac{2^{4\nu} c^{2\nu}}{\pi p^{4\nu+1}} \Gamma \left[\begin{matrix} \nu + 1/2, 2\nu + 1/2 \\ \nu + 1 \end{matrix} \right] {}_2F_1 \left(\nu + \frac{1}{2}, 2\nu + \frac{1}{2} ; \nu + 1; \frac{4c^2}{p^2} \right) \quad [\operatorname{Re} \nu > -1/4],$$

$$C_{\nu, \nu}^{1/2} = 2^{-2\nu} p^{-1/2} \Gamma \left(2\nu + \frac{1}{2} \right) [P_{-1/4}^{-\nu}(z)]^2 \quad [\operatorname{Re} \nu > -1/4],$$

$$C_{-\nu, \nu}^{1/2} = \sqrt{\frac{\pi}{p}} P_{-1/4}^{\nu}(z) P_{-1/4}^{-\nu}(z),$$

$$C_{0,1}^0 = \frac{p}{2c} - \frac{p}{\pi c} E \left(\frac{2c}{p} \right),$$

$$C_{1,1}^0 = \frac{p^2}{2\pi c^2} \left[E \left(\frac{2c}{p} \right) - \left(1 - \frac{4c^2}{p^2} \right) K \left(\frac{2c}{p} \right) \right] - \frac{1}{2},$$

$$C_{0,0}^2 = \frac{2}{\pi (p^2 - 4c^2)} E \left(\frac{2c}{p} \right),$$

$$C_{0,1}^2 = \frac{1}{2\pi c} \left[\frac{p}{p^2 - 4c^2} E \left(\frac{2c}{p} \right) - \frac{2}{p} K \left(\frac{2c}{p} \right) \right].$$

$$\begin{aligned} 6. \int_0^{\infty} x^{\alpha-1} e^{-(b+c)x} I_{\mu}(bx) I_{\nu}(cx) dx &= \frac{b^{\mu} c^{-\alpha-\mu}}{2^{\alpha+2\mu} \sqrt{\pi}} \Gamma \left[\begin{matrix} 1/2 - \mu - \alpha, \alpha + \mu + \nu \\ 1 + \nu - \mu - \alpha, \mu + 1 \end{matrix} \right] \times \\ &\times {}_3F_2 \left(\alpha + \mu - \nu, \alpha + \mu + \nu, \mu + \frac{1}{2} ; \alpha + \mu + \frac{1}{2}, 2\mu + 1; -\frac{b}{c} \right) + \\ &+ \frac{b^{1/2-\alpha}}{2^{\alpha} \pi \sqrt{c}} \Gamma \left[\begin{matrix} \mu + \alpha - 1/2, 1 - \alpha \\ \mu + 3/2 - \alpha \end{matrix} \right] \times \\ &\times {}_3F_2 \left(\frac{1}{2} - \nu, \frac{1}{2} + \nu, 1 - \alpha; \frac{3}{2} - \alpha + \mu, \frac{3}{2} - \alpha - \mu; -\frac{b}{c} \right) \\ &[\operatorname{Re}(b+c), \operatorname{Re}(\alpha + \mu + \nu) > 0; \operatorname{Re} \alpha < 1]. \end{aligned}$$

$$\begin{aligned} 7. \int_0^{\infty} x^{\alpha-1} e^{-px^2} I_{\mu}(bx) I_{\nu}(cx) dx &= \frac{b^{\mu} c^{\nu} p^{-(\alpha+\mu+\nu)/2}}{2^{\mu+\nu+1} \Gamma(\nu+1)} \times \\ &\times \sum_{k=0}^{\infty} \Gamma \left[\begin{matrix} k + (\alpha + \mu + \nu)/2 \\ \mu + k + 1 \end{matrix} \right] \frac{1}{k!} \left(\frac{b}{2p} \right)^{2k} {}_2F_1 \left(-k, -\mu - k; \nu + 1; \frac{c^2}{b^2} \right) \\ &[\operatorname{Re} p, \operatorname{Re}(\alpha + \mu + \nu) > 0]. \end{aligned}$$

$$8. \int_0^{\infty} x e^{-px^2} I_{\nu}(bx) I_{\nu}(cx) dx = \frac{1}{2p} \exp \left(\frac{b^2 + c^2}{4p} \right) I_{\nu} \left(\frac{bc}{2p} \right) \quad [\operatorname{Re} p > 0; \operatorname{Re} \nu > -1].$$

$$\begin{aligned} 9. \int_0^{\infty} x^3 e^{-px^2} I_{n+2}(bx) I_n(cx) dx &= \\ &= \frac{1}{8p^3 b^2} \exp \left(\frac{b^2 + c^2}{4p} \right) \left\{ bc(4p + b^2) I_{n+1} \left(\frac{bc}{2p} \right) - bc(4pn + 8p - b^2) I_{n-1} \left(\frac{bc}{2p} \right) + \right. \\ &\quad \left. + [16p^2 n(n+2) - (4pn - c^2)b^2 + b^4] I_n \left(\frac{bc}{2p} \right) \right\} \quad [\operatorname{Re} p > 0]. \end{aligned}$$

$$10. \int_0^{\infty} x^{\alpha-1} e^{-px^2} I_{\mu}(cx) I_{\nu}(cx) dx = 2^{-\mu-\nu-1} c^{\mu+\nu} p^{-(\alpha+\mu+\nu)/2} \Gamma \left[\frac{(\alpha+\mu+\nu)/2}{\mu+1, \nu+1} \right] \times \\ \times {}_3F_3 \left(\frac{\mu+\nu+1}{2}, \frac{\mu+\nu}{2}+1, \frac{\alpha+\mu+\nu}{2}; \mu+1, \nu+1, \mu+\nu+1; \frac{c^2}{p} \right) \\ [\operatorname{Re} p, \operatorname{Re}(\alpha+\mu+\nu) > 0].$$

$$11. \int_0^{\infty} \frac{1}{x} e^{-px^2} I_{\nu}^2(cx) dx = \frac{1}{2\nu} \exp \left(\frac{c^2}{2p} \right) \left[I_{\nu} \left(\frac{c^2}{2p} \right) + 2 \sum_{k=1}^{\infty} (-1)^k I_{\nu+k} \left(\frac{c^2}{2p} \right) \right] \\ [\operatorname{Re} p, \operatorname{Re} \nu > 0].$$

$$12. \int_0^{\infty} x e^{-px^2} I_{\nu}^2(cx) dx = \frac{1}{2p} \exp \left(\frac{c^2}{2p} \right) I_{\nu} \left(\frac{c^2}{2p} \right) \quad [\operatorname{Re} p > 0; \operatorname{Re} \nu > -1].$$

2.15.21. Интегралы, содержащие произведения трех и более функций $I_{\nu}(\varphi(x))$ и $J_{\nu}(\chi(x))$.

$$1. \int_0^{\infty} e^{-(2b+c)x} I_0^2(bx) I_0(cx) dx = \frac{4\sqrt{2}}{\pi^2 c \sqrt{b}} (\sqrt{2b+2c} - \sqrt{2b+c}) K(k) K(\sqrt{1-k^2}) \\ [k = c^{-1}(\sqrt{2b+c} - \sqrt{2b}) (\sqrt{2b+2c} - \sqrt{2b+c}); \operatorname{Re} b, \operatorname{Re} c > 0].$$

$$2. \int_0^{\infty} e^{-3cx} I_0^3(cx) dx = \frac{4}{c\pi^2} (18 + 12\sqrt{2} - 10\sqrt{3} - 7\sqrt{6}) K^2[(2 - \sqrt{3})(\sqrt{3} - \sqrt{2})] \\ [\operatorname{Re} c > 0].$$

$$3. \int_0^a x J_{\mu}(cx) J_{\nu}(c\sqrt{a^2-x^2}) I_{\mu}(cx) I_{\nu}(c\sqrt{a^2-x^2}) dx = \\ = \left(\frac{c}{2} \right)^{2\mu+2\nu} \frac{a^{2\mu+2\nu+2}}{2\Gamma[\mu+1, \nu+1, \mu+\nu+2]} \times \\ \times {}_1F_4 \left(\frac{\mu+\nu+1}{2}; \mu+1, \nu+1, \mu+\nu+1, \frac{\mu+\nu+3}{2}; -\frac{a^4 c^4}{16} \right) \\ [a > 0, \operatorname{Re} \mu, \operatorname{Re} \nu > -1]$$

2.15.22. Интегралы по индексу, содержащие $I_{\nu \pm x}(c)$.

$$1. \int_0^{\infty} I_x(c) dx = \frac{c}{2} - \int_0^{\infty} \frac{e^{-c \operatorname{ch} x}}{x^2 + \pi^2} dx \quad [\operatorname{Re} c > 0].$$

$$2. \int_0^{\infty} \cos bx I_{\nu+x}(c) I_{\nu-x}(c) dx = \frac{1}{2} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} I_{2\nu} \left(2c \cos \frac{b}{2} \right) \quad \left[\begin{Bmatrix} 0 < b < \pi \\ b > \pi \end{Bmatrix} \right].$$

2.15.23. Интегралы от $I_{\nu}(x)/I_{\nu}(x)$.

$$1. \int_0^{\infty} \frac{x^{\nu}}{I_0(x)} dx = \pi \operatorname{cosec} \frac{\pi \nu}{2} \sum_{k=1}^{\infty} \frac{\gamma_k^{\nu-1}}{J_1(\gamma_k)} \\ [\gamma_k - k\text{-й положительный нуль } J_0(x)].$$

2.15.24. Интегралы, содержащие $\operatorname{Ei}(\varphi(x)) I_{\nu}(cx)$.

$$1. \int_0^{\infty} \operatorname{Ei}(-bx) I_0(cx) dx = -\frac{1}{c} \arcsin \frac{c}{b} \quad [\operatorname{Re}(b-c) > 0].$$

$$2. \int_0^{\infty} x^{\alpha-1} \operatorname{Ei}(-bx) I_{\nu}(cx) dx =$$

$$= -\frac{c^{\nu}}{2^{\nu} b^{\alpha+\nu} (\alpha+\nu)} \Gamma\left[\frac{\alpha+\nu}{\nu+1}\right] {}_3F_2\left(\frac{\alpha+\nu}{2}, \frac{\alpha+\nu}{2}, \frac{\alpha+\nu+1}{2}; \frac{\alpha+\nu}{2}+1, \nu+1; \frac{c^2}{b^2}\right)$$

[Re(b-c), Re(α+ν) > 0].

$$3. \int_0^{\infty} x \operatorname{Ei}(-bx) I_0(cx) dx = \frac{1}{c^2} \left(1 - \frac{b}{\sqrt{b^2 - c^2}}\right)$$

[Re(b-c) > 0].

$$4. \int_0^{\infty} x^{\alpha-1} \operatorname{Ei}(-bx^2) I_{\nu}(cx) dx = -\frac{c^{\nu}}{2^{\nu} (\alpha+\nu) b^{(\alpha+\nu)/2}} \Gamma\left[\frac{(\alpha+\nu)/2}{\nu+1}\right] \times$$

$$\times {}_3F_2\left(\frac{\alpha+\nu}{2}, \frac{\alpha+\nu}{2}; \nu+1, \frac{\alpha+\nu}{2}+1; \frac{c^2}{4b}\right)$$

[Re b, Re(α+ν) > 0].

$$5. \int_0^{\infty} x^{n+1} \operatorname{Ei}(-bx^2) I_n(cx) dx = (-1)^n n! \frac{2^{n+1}}{c^{n+2}} \left[1 - \exp \frac{c^2}{4b} \sum_{k=0}^n \frac{1}{k!} \left(-\frac{c^2}{4b}\right)^k\right]$$

[Re b > 0].

$$6. \int_0^{\infty} x \operatorname{Ei}[-b(z + \sqrt{x^2 + z^2})] I_0(cx) dx = -\frac{\exp[-z(b + \sqrt{b^2 - c^2})]}{(b + \sqrt{b^2 - c^2}) \sqrt{b^2 - c^2}}$$

[Re(b-c), Rez > 0].

$$7. \int_0^{\infty} e^{\pm ax} \operatorname{Ei}(-bx) I_0(cx) dx = -\frac{2}{\sqrt{b}(\sqrt{b} + \sqrt{b \mp 2c})}$$

[Re(b-c ± c) > 0].

$$8. \int_0^{\infty} x^{\alpha-1} e^{-cx} \operatorname{Ei}(-bx) I_{\nu}(cx) dx = -\frac{b^{-\alpha-\nu}}{\alpha+\nu} \left(\frac{c}{2}\right)^{\nu} \Gamma\left[\frac{\alpha+\nu}{\nu+1}\right] \times$$

$$\times {}_3F_2\left(\alpha+\nu, \alpha+\nu, \nu+\frac{1}{2}; \alpha+\nu+1, 2\nu+1; -\frac{2c}{b}\right)$$

[Re b, Re c, Re(α+ν) > 0].

$$9. \int_0^{\infty} x^{\alpha-1} e^{(\pm b-c)x} \operatorname{Ei}(\mp bx) I_{\nu}(cx) dx =$$

$$= -b^{-\alpha-\nu} \left(\frac{c}{2}\right)^{\nu} \left\{ \frac{\operatorname{cosec}(\alpha+\nu)\pi}{\operatorname{ctg}(\alpha+\nu)\pi} \right\} \Gamma\left[\frac{\alpha+\nu}{\nu+1}\right] {}_2F_1\left(\nu+\frac{1}{2}, \alpha+\nu; 2\nu+1; \pm \frac{2c}{b}\right) \mp$$

$$\mp \frac{(2c)^{1-\alpha}}{b \sqrt{\pi}} \Gamma\left[\frac{\alpha+\nu-1}{2+\nu-\alpha}, \frac{3/2-\alpha}{2}\right] {}_3F_2\left(1, 1, \frac{3}{2}-\alpha; 2-\alpha-\nu, 2-\alpha+\nu; \pm \frac{2c}{b}\right)$$

[Re b, Re c, Re(α+ν) > 0; Re α < 3/2].

$$10. \int_0^{\infty} x^{\nu} e^{(b-c)x} \operatorname{Ei}(-bx) I_{\nu}(cx) dx =$$

$$= -\frac{2^{\nu+1} c^{\nu} \Gamma(2\nu+1)}{(2\nu+1) \sqrt{\pi} b^{2\nu+1}} \Gamma\left(\frac{1}{2}-\nu\right) {}_2F_1\left(\nu+\frac{1}{2}, 2\nu+1; \nu+\frac{3}{2}; 1-\frac{2c}{b}\right)$$

[Re b, Re c > 0; |Re ν| < 1/2].

$$11. \int_0^{\infty} x^{\nu} e^{cx} \operatorname{Ei}(-2cx) I_{\nu}(cx) dx = -\frac{2^{\nu} \sec \nu \pi \Gamma(\nu+1)}{(2\nu+1) c^{\nu+1}}$$

[Re c > 0; |Re ν| < 1/2].

$$12. \int_0^{\infty} x^{\alpha-1} \operatorname{Ei}(-bx^2) I_{\mu}(cx) I_{\nu}(cx) dx =$$

$$= -\frac{b^{-(\alpha+\mu+\nu)/2}}{\alpha+\mu+\nu} \left(\frac{c}{2}\right)^{\mu+\nu} \Gamma\left[\frac{(\alpha+\mu+\nu)/2}\right] {}_4F_4\left(\frac{\mu+\nu+1}{2}, \frac{\mu+\nu}{2}+1, \frac{\alpha+\mu+\nu}{2}, \frac{\alpha+\mu+\nu}{2}; \mu+1, \nu+1, \mu+\nu+1, \frac{\alpha+\mu+\nu}{2}+1; \frac{c^2}{b^2}\right)$$

[Re b , Re $(\alpha+\mu+\nu) > 0$; $|\arg c| < \pi$].

2.15.25. Интегралы от $x^{\alpha} e^{-cx} \begin{Bmatrix} \operatorname{si}(bx) \\ \operatorname{ci}(bx) \end{Bmatrix} I_{\nu}(cx)$.

$$1. \int_0^{\infty} x^{\alpha-1} e^{-cx} \begin{Bmatrix} \operatorname{si}(bx) \\ \operatorname{ci}(bx) \end{Bmatrix} I_{\nu}(cx) dx =$$

$$= -\frac{b^{-\alpha-\nu}}{\alpha+\nu} \left(\frac{c}{2}\right)^{\nu} \Gamma\left[\frac{\alpha+\nu}{2}\right] \begin{Bmatrix} \sin[(\alpha+\nu)\pi/2] \\ \cos[(\alpha+\nu)\pi/2] \end{Bmatrix} {}_5F_4\left(\frac{2\nu+1}{4}, \frac{2\nu+3}{4}, \frac{\alpha+\nu}{2}, \frac{\alpha+\nu+1}{2}; \frac{\alpha+\nu}{2}+1, \nu+\frac{1}{2}, \nu+1, \frac{1}{2}; \frac{4c^2}{b^2}\right) \pm$$

$$\pm \frac{c^{\nu+1}}{2^{\nu} b^{\alpha+\nu+1}} \Gamma\left[\frac{\alpha+\nu+1}{2}\right] \begin{Bmatrix} \cos[(\alpha+\nu)\pi/2] \\ \sin[(\alpha+\nu)\pi/2] \end{Bmatrix} {}_5F_4\left(\frac{2\nu+3}{4}, \frac{2\nu+5}{4}, \frac{\alpha+\nu+1}{2}, \frac{\alpha+\nu+3}{2}; \nu+1, \nu+\frac{3}{2}, \frac{3}{2}; \frac{4c^2}{b^2}\right)$$

[b , Re c , Re $(\alpha+\nu) > 0$; Re $\alpha < 5/2$].

2.15.26. Интегралы, содержащие $\begin{Bmatrix} \operatorname{erf}(bx^r) \\ \operatorname{erfc}(bx^r) \end{Bmatrix} I_{\nu}(cx)$.

$$1. \int_0^{\infty} \operatorname{erfc}(bx) I_0(cx) dx = -\frac{i}{c} \operatorname{erf}\left(i\frac{c}{2b}\right) \quad [|\arg b| < \pi/4].$$

$$2. \int_0^{\infty} x^{\alpha-1} \operatorname{erfc}(bx) I_{\nu}(cx) dx = \frac{c^{\nu}}{2^{\nu}(\alpha+\nu)\sqrt{\pi} b^{\alpha+\nu}} \Gamma\left[\frac{(\alpha+\nu+1)/2}\right] \times$$

$$\times {}_2F_2\left(\frac{\alpha+\nu}{2}, \frac{\alpha+\nu+1}{2}; \frac{\alpha+\nu}{2}+1, \nu+1; \frac{c^2}{4b^2}\right)$$

[Re $(\alpha+\nu) > 0$; $|\arg b| < \pi/4$; $|\arg c| < \pi$].

$$3. \int_0^{\infty} x^{\alpha-1} \operatorname{erfc}(b\sqrt{x}) I_{\nu}(cx) dx = \frac{b^{-2\alpha-2\nu}}{(\alpha+\nu)\sqrt{\pi}} \left(\frac{c}{2}\right)^{\nu} \Gamma\left[\frac{\alpha+\nu+1/2}{2}\right] \times$$

$$\times {}_3F_3\left(\frac{\alpha+\nu}{2}, \frac{2\alpha+2\nu+1}{4}, \frac{2\alpha+2\nu+3}{4}; \nu+1, \frac{\alpha+\nu}{2}+1; \frac{c^2}{b^4}\right)$$

[$|\arg b| < \pi/4$; Re $(\alpha+\nu) > 0$; Re $(b^2-c) > 0$].

$$4. \int_0^{\infty} x^{\alpha-1} e^{-cx} \begin{Bmatrix} \operatorname{erf}(bx) \\ \operatorname{erfc}(bx) \end{Bmatrix} I_{\nu}(cx) dx =$$

$$= \mp \frac{c^{\nu}}{2^{\nu}(\alpha+\nu)\sqrt{\pi} b^{\alpha+\nu}} \Gamma\left[\frac{(\alpha+\nu+1)/2}\right] \times$$

$$\times {}_4F_4\left(\frac{2\nu+1}{4}, \frac{2\nu+3}{4}, \frac{\alpha+\nu}{2}, \frac{\alpha+\nu+1}{2}; \nu+\frac{1}{2}, \nu+1, \frac{\alpha+\nu}{2}+1, \frac{1}{2}; \frac{c^2}{b^2}\right) \pm$$

$$\pm \frac{c^{\nu+1}}{2^{\nu}(\alpha+\nu+1)\sqrt{\pi} b^{\alpha+\nu+1}} \Gamma\left[\frac{(\alpha+\nu)/2+1}{2}\right] {}_4F_4\left(\frac{2\nu+3}{4}, \frac{2\nu+5}{4}, \frac{\alpha+\nu+1}{2}, \frac{\alpha+\nu+3}{2}; \nu+1, \nu+\frac{3}{2}, \frac{3}{2}; \frac{c^2}{b^2}\right)$$

$$\frac{\alpha+v+1}{2}, \frac{\alpha+v}{2}+1; v+1, v+\frac{3}{2}, \frac{\alpha+v+3}{2}, \frac{3}{2}; \frac{c^2}{b^2} \Big) + \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{(2c)^{-\alpha}}{\sqrt{\pi}} \times \\ \times \Gamma \left[\begin{matrix} \alpha+v, 1/2-\alpha \\ 1+v-\alpha \end{matrix} \right] \quad \left[\operatorname{Re} c > 0; |\arg b| < \pi/4, \begin{Bmatrix} -\operatorname{Re} v-1 < \operatorname{Re} \alpha < 1/2 \\ \operatorname{Re}(\alpha+v) > 0 \end{Bmatrix} \right].$$

$$5. \int_0^\infty x^{\alpha-1} e^{-cx} \begin{Bmatrix} \operatorname{erf}(b\sqrt{x}) \\ \operatorname{erfc}(b\sqrt{x}) \end{Bmatrix} I_v(cx) dx = \\ = \mp \frac{c^v}{2^v(\alpha+v)\sqrt{\pi} b^{2\alpha+2v}} \Gamma \left[\begin{matrix} \alpha+v+1/2 \\ v+1 \end{matrix} \right] \times \\ \times {}_3F_2 \left(v+\frac{1}{2}, \alpha+v+\frac{1}{2}, \alpha+v; \alpha+v+1, 2v+1; -\frac{2c}{b^2} \right) + \\ + \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{(2c)^{-\alpha}}{\sqrt{\pi}} \Gamma \left[\begin{matrix} \alpha+v, 1/2-\alpha \\ 1+v-\alpha \end{matrix} \right] \\ \left[\operatorname{Re} c > 0; |\arg b| < \pi/4, \begin{Bmatrix} -\operatorname{Re} v-1/2 < \operatorname{Re} \alpha < 1/2 \\ \operatorname{Re}(\alpha+v) > 0 \end{Bmatrix} \right].$$

$$6. \int_0^\infty x^{-1/2} e^{-cx} \operatorname{erfc}(b\sqrt{x}) I_0(cx) dx = \\ = \frac{1}{\sqrt{2\pi c}} \ln \left(1 + \frac{2c}{b^2} + \frac{2\sqrt{c}}{b^2} \sqrt{b^2+c} \right) \quad [\operatorname{Re} c > 0; |\arg b| < \pi/4].$$

$$7. \int_0^\infty x^v e^{(b^2-c)x} \operatorname{erfc}(b\sqrt{x}) I_v(cx) dx = \\ = \frac{2^{v+1} c^v \Gamma(-v) \Gamma(2v+3/2)}{(2v+1) \pi^{3/2} b^{4v+2}} {}_2F_1 \left(v+\frac{1}{2}, 2v+\frac{3}{2}; v+\frac{3}{2}; 1-\frac{2c}{b^2} \right) \\ [\operatorname{Re} c > 0; |\arg b| < \pi/4; -1/2 < \operatorname{Re} v < 0].$$

$$8. \int_0^\infty x^{v-1/2} e^{cx} \operatorname{erfc}(\sqrt{2cx}) I_v(cx) dx = \frac{(2c)^{-v-1/2}}{\cos v\pi} \Gamma \left[\begin{matrix} 2v+1/2 \\ v+1 \end{matrix} \right] \\ [\operatorname{Re} c > 0; -1/4 < \operatorname{Re} v < 1/2].$$

$$9. \int_0^\infty x^{v \pm v} \operatorname{erfc}(bx) I_v^2(cx) dx = \frac{b^{-3v \mp v-1} c^{2v}}{2^{v \mp v} (3v \pm v+1) \pi} \Gamma \left[\begin{matrix} (v \pm v+1)/2 \\ v+1 \end{matrix} \right] \times \\ \times {}_2F_2 \left(v+\frac{1}{2}, \frac{3v \pm v+1}{2}; \frac{3v \mp v}{2}+1, \frac{3v \pm v+3}{2}; \frac{c^2}{b^2} \right) \\ [\operatorname{Re} v > -(2 \mp 1)/4; |\arg b| < \pi/4].$$

$$10. \int_0^\infty x^{\alpha-1} \operatorname{erfc}(bx) I_\mu(cx) I_v(cx) dx = \\ = \frac{1}{(\alpha+\mu+v) \sqrt{\pi} b^{\alpha+\mu+v}} \left(\frac{c}{2} \right)^{\mu+v} \Gamma \left[\begin{matrix} (\alpha+\mu+v+1)/2 \\ \mu+1, v+1 \end{matrix} \right] \times \\ \times {}_4F_4 \left(\frac{\mu+v}{2}+1, \frac{\mu+v+1}{2}, \frac{\alpha+\mu+v}{2}, \frac{\alpha+\mu+v+1}{2}; \mu+1, \right. \\ \left. v+1, \mu+v+1, \frac{\alpha+\mu+v}{2}+1; \frac{c^2}{b^2} \right) \quad [\operatorname{Re}(\alpha+\mu+v) > 0; |\arg b| < \pi/4]$$

$$11. \int_0^{\infty} x^{\alpha-1} \operatorname{erfc}(bx^2) J_\nu(cx) I_\nu(cx) dx =$$

$$= \frac{c^{2\nu}}{2^{2\nu} (\alpha+2\nu) \sqrt{\pi} b^{\alpha/2+\nu}} \Gamma \left[\frac{(\alpha+2\nu+2)/4}{\nu+1, \nu+1} \right] \times$$

$$\times {}_2F_4 \left(\frac{\alpha+2\nu}{4}, \frac{\alpha+2\nu+2}{4}; \frac{\alpha+2\nu}{4}+1, \frac{\nu+1}{2}, \frac{\nu}{2}+1, \nu+1; -\frac{c^4}{64b^2} \right)$$

[Re($\alpha+2\nu$) > 0; |arg b | < $\pi/4$]

2.15.27. Интегралы от $x^\alpha e^{-cx} \left\{ \begin{matrix} S(bx) \\ C(bx) \end{matrix} \right\} I_\nu(cx)$.

$$1. \int_0^{\infty} x^{\alpha-1} e^{-cx} \left\{ \begin{matrix} S(bx) \\ C(bx) \end{matrix} \right\} I_\nu(cx) dx =$$

$$= -\frac{c^\nu}{2^{\nu+1/2} (\alpha+\nu) \sqrt{\pi} b^{\alpha+\nu}} \Gamma \left[\frac{\alpha+\nu+1/2}{\nu+1} \right] \left\{ \begin{matrix} \cos[(1-2\alpha-2\nu)\pi/4] \\ \sin[(1-2\alpha-2\nu)\pi/4] \end{matrix} \right\} \times$$

$$\times {}_5F_4 \left(\frac{2\nu+1}{4}, \frac{2\nu+3}{4}, \frac{2\alpha+2\nu+1}{4}, \frac{2\alpha+2\nu+3}{4}, \frac{\alpha+\nu}{2}; \frac{\alpha+\nu}{2}+1, \nu+\frac{1}{2}, \right.$$

$$\left. \nu+1, \frac{1}{2}; -\frac{4c^2}{b^2} \right) \pm \frac{c^{\nu+1}}{2^{\nu+1/2} (\alpha+\nu+1) \sqrt{\pi} b^{\alpha+\nu+1}} \Gamma \left[\frac{\alpha+\nu+3/2}{\nu+1} \right] \times$$

$$\times \left\{ \begin{matrix} \sin[(1-2\alpha-2\nu)\pi/4] \\ \cos[(1-2\alpha-2\nu)\pi/4] \end{matrix} \right\} {}_5F_4 \left(\frac{2\nu+3}{4}, \frac{2\nu+5}{4}, \frac{2\alpha+2\nu+3}{4}, \frac{2\alpha+2\nu+5}{4}, \right.$$

$$\left. \frac{\alpha+\nu+1}{2}; \frac{\alpha+\nu+3}{2}, \nu+1, \nu+\frac{3}{2}, \frac{3}{2}; -\frac{4c^2}{b^2} \right) + \frac{(2c)^{-\alpha}}{2\sqrt{\pi}} \Gamma \left[\frac{\alpha+\nu, 1/2-\alpha}{1+\nu-\alpha} \right]$$

[$b, \operatorname{Re} c > 0, -(2 \pm 1)/2 - \operatorname{Re} \nu < \operatorname{Re} \alpha < 2$].

2.15.28. Интегралы, содержащие $\left\{ \begin{matrix} \gamma(\mu, bx^n) \\ \Gamma(\mu, bx^n) \end{matrix} \right\} I_\nu(cx)$.

$$1. \int_0^{\infty} x^{\alpha-1} \Gamma(\mu, bx) I_\nu(cx) dx = \frac{b^{-\alpha-\nu}}{\alpha+\nu} \left(\frac{c}{2} \right)^\nu \Gamma \left[\frac{\alpha+\mu+\nu}{\nu+1} \right] \times$$

$$\times {}_2F_2 \left(\frac{\alpha+\nu}{2}, \frac{\alpha+\mu+\nu}{2}, \frac{\alpha+\mu+\nu+1}{2}; \frac{\alpha+\nu}{2}+1, \nu+1; \frac{c^2}{b^2} \right)$$

[Re($\alpha+\mu+\nu$), Re($\alpha+\nu$) > 0; Re b > |Re c |].

$$2. \int_0^{\infty} x^{\alpha-1} \Gamma(\mu, bx^2) I_\nu(cx) dx = \frac{b^{-(\alpha+\nu)/2}}{\alpha+\nu} \left(\frac{c}{2} \right)^\nu \times$$

$$\times \Gamma \left[\frac{\mu+(\alpha+\nu)/2}{\nu+1} \right] {}_2F_2 \left(\frac{\alpha+\nu}{2}, \mu+\frac{\alpha+\nu}{2}; \nu+1, \frac{\alpha+\nu}{2}+1; \frac{c^2}{4b} \right)$$

[Re b , Re($\alpha+\nu+2\mu$), Re($\alpha+\nu$) > 0]

$$3. \int_0^{\infty} x^{\alpha-1} e^{-cx} \left\{ \begin{matrix} \gamma(\mu, bx) \\ \Gamma(\mu, bx) \end{matrix} \right\} I_\nu(cx) dx = \mp \frac{b^{-\alpha-\nu}}{\alpha+\nu} \left(\frac{c}{2} \right)^\nu \Gamma \left[\frac{\alpha+\mu+\nu}{\nu+1} \right] \times$$

$$\times {}_2F_2 \left(\nu+\frac{1}{2}, \alpha+\mu+\nu, \alpha+\nu; \alpha+\nu+1, 2\nu+1; -\frac{2c}{b} \right) +$$

$$+ \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{(2c)^{-\alpha}}{\sqrt{\pi}} \Gamma \left[\frac{\mu, \alpha+\nu, 1/2-\alpha}{1+\nu-\alpha} \right]$$

[Re b , Re c , Re($\alpha+\mu+\nu$) > 0, $\left\{ \begin{matrix} \operatorname{Re} \mu > 0; \operatorname{Re} \alpha < 1/2 \\ \operatorname{Re}(\alpha+\nu) > 0 \end{matrix} \right\}$].

2.15.29. Интегралы, содержащие $D_\mu(bx) I_\nu(cx)$

1.
$$\int_0^\infty x^{\alpha-1} e^{-b^2 x^2/4} D_\mu(bx) I_\nu(cx) dx =$$
$$= \frac{2^{-(\alpha+3\nu-\mu)/2} \sqrt{\pi} c^\nu}{b^{\alpha+\nu}} \Gamma \left[\begin{matrix} \alpha+\nu \\ \nu+1, (1+\alpha+\nu-\mu)/2 \end{matrix} \right] \times$$
$$\times {}_2F_2 \left(\frac{\alpha+\nu}{2}, \frac{\alpha+\nu+1}{2}; \frac{1+\alpha+\nu-\mu}{2}, \nu+1; \frac{c^2}{2b^2} \right)$$

[$\operatorname{Re}(\alpha+\nu) > 0; |\arg b| < \pi/4$]

2.
$$\int_0^\infty x^{\alpha-1} e^{-b^2 x^2/4 - cx} D_\mu(bx) I_\nu(cx) dx =$$
$$= \frac{2^{-(\alpha+3\nu-\mu)/2} \sqrt{\pi} c^\nu}{b^{\alpha+\nu}} \Gamma \left[\begin{matrix} \alpha+\nu \\ \nu+1, (1+\alpha+\nu-\mu)/2 \end{matrix} \right] \times$$
$$\times {}_4F_4 \left(\frac{\alpha+\nu}{2}, \frac{\alpha+\nu+1}{2}, \frac{2\nu+1}{4}, \frac{2\nu+3}{4}; \frac{1}{2}, \nu+\frac{1}{2}, \nu+1, \frac{1+\alpha+\nu-\mu}{2}; \frac{2c^2}{b^2} \right) -$$
$$- \frac{2^{-(\alpha-\mu+3\nu+1)/2} \sqrt{\pi} c^{\nu+1}}{b^{\alpha+\nu+1}} \Gamma \left[\begin{matrix} \alpha+\nu+1 \\ \nu+1, (\alpha+\nu-\mu)/2+1 \end{matrix} \right] \times$$
$$\times {}_4F_4 \left(\frac{\alpha+\nu+1}{2}, \frac{\alpha+\nu}{2}+1, \frac{2\nu+3}{4}, \frac{2\nu+5}{4}; \frac{3}{2}, \nu+1, \nu+\frac{3}{2}, \frac{\alpha+\nu-\mu}{2}+1; \frac{2c^2}{b^2} \right)$$

[$\operatorname{Re} c, \operatorname{Re}(\alpha+\nu) > 0; |\arg b| < \pi/4$].

3.
$$\int_0^\infty x^\nu e^{-b^2 x^2/4 - cx} D_{\nu+(2\mp 1)/2}(bx) I_\nu(cx) dx =$$
$$= \frac{(2c)^\nu}{\sqrt{\pi} b^{2\nu+1}} \Gamma \left(\nu + \frac{2\mp 1}{2} \right) e^{c^2 b^{-2}} D_{-\nu-(2\mp 1)/2} \left(\frac{2c}{b} \right)$$

[$\operatorname{Re} \nu > -1/2; |\arg b| < \pi/4$].

4.
$$\int_0^\infty x^{\alpha-1} e^{(\pm b^2/4 - c)x} D_\mu(b\sqrt{x}) I_\nu(cx) dx =$$
$$= \frac{2^{-\alpha-2\nu\mp(\mu+1\mp 1)/2} c^\nu}{b^{2\alpha+2\nu}} \Gamma \left[\frac{2\alpha+2\nu}{\nu+1} \right] \left\{ \frac{\Gamma^{-1}(-\mu)}{\sqrt{\pi}} \right\} \Gamma^{\pm 1} \left(\frac{1\mp 1-2\mu}{4} \mp \nu \mp \alpha \right) \times$$
$$\times {}_3F_2 \left(\alpha+\nu, \alpha+\nu+\frac{1}{2}, \nu+\frac{1}{2}; 2\nu+1, \frac{3\pm 1\pm 2\mu}{4} + \alpha+\nu; \pm \frac{4c}{b^2} \right) +$$
$$+ \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{b^\mu}{\sqrt{\pi} (2c)^{\alpha+\mu/2}} \Gamma \left[\begin{matrix} \alpha+\nu+\mu/2, (1-\mu)/2-\alpha \\ 1+\nu-\alpha-\mu/2 \end{matrix} \right] \times$$
$$\times {}_3F_2 \left(-\frac{\mu}{2}, \frac{1-\mu}{2}, \frac{1-\mu}{2}-\alpha; 1-\alpha-\nu-\frac{\mu}{2}, 1+\nu-\alpha-\frac{\mu}{2}; \frac{4c}{b^2} \right)$$

[$\operatorname{Re} c > 0; |\arg b| < (2\pm 1)\pi/4, \left\{ \begin{matrix} -\operatorname{Re} \nu < \operatorname{Re} \alpha < (1-\operatorname{Re} \mu)/2 \\ \operatorname{Re}(\alpha+\nu) > 0 \end{matrix} \right\}$].

5.
$$\int_0^\infty x^{\alpha-1} D_{-\mu-1}(b\sqrt{x}) D_\mu(b\sqrt{x}) I_\nu(cx) dx =$$
$$= \frac{2^{1/2-2\alpha-3\nu} \pi c^\nu}{b^{2\alpha+2\nu}} \Gamma \left[\begin{matrix} 2\alpha+2\nu \\ \nu+1, (\alpha+\mu+\nu)/2+1, (\alpha+\nu-\mu+1)/2 \end{matrix} \right] \times$$
$$\times {}_4F_3 \left(\frac{\alpha+\nu}{2}, \frac{2\alpha+2\nu+1}{4}, \frac{\alpha+\nu+1}{2}, \frac{2\alpha+2\nu+3}{4}; \right.$$
$$\left. \nu+1, \frac{\alpha+\mu+\nu}{2}+1, \frac{\alpha-\mu+\nu+1}{2}; \frac{4c^2}{b^4} \right)$$

[$\operatorname{Re}(\alpha+\nu) > 0; \operatorname{Re}(b^2-2c) > 0$]

$$6. \int_0^{\infty} x^{-1/2} D_{-\nu-1}(b\sqrt{x}) D_{\nu}(b\sqrt{x}) I_0(cx) dx =$$

$$= \frac{\pi}{2^{3/2}b} P_{-1/4}^{(2\nu+1)/4} \left(\frac{\sqrt{b^4-4c^2}}{b^2} \right) P_{-1/4}^{-(2\nu+1)/4} \left(\frac{\sqrt{b^4-4c^2}}{b^2} \right) \quad [\operatorname{Re}(b^2-2c) > 0].$$

2.16. Функция Макдональда $K_{\nu}(x)$

При $\nu = \pm \frac{1}{2}, \pm \frac{3}{2}, \dots$ функция $K_{\nu}(x)$ сводится к элементарным, в частности,

$$K_{\pm 1/2}(cx) = \sqrt{\frac{\pi}{2cx}} e^{-cx}.$$

При вычислении интегралов от $K_{\nu}(x)$ можно использовать формулу

$$K_{\nu}(cx) = \frac{\pi}{2 \sin \nu \pi} [I_{-\nu}(cx) - I_{\nu}(cx)] \quad [\nu \neq 0, \pm 1, \pm 2, \dots].$$

2.16.1. Интегралы общего вида

(Интегралы, содержащие $K_{\nu}(cx)$, см. также в 2.13.1).

$$1. \int_0^a x^{\alpha-1} (a^r - x^r)^{\beta-1} e^{\pm cx} K_{\nu}(cx) dx = U_{\nu}^{\pm} \quad [a, r, \operatorname{Re} \beta > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu|].$$

$$U_{\nu}^{\pm} = \frac{a^{\alpha+r\beta-r}}{2r} \Gamma(\beta) \sum_{k=0}^{\infty} \frac{1}{k!} [A_k(\nu) + A_k(-\nu)] (\pm 2ac)^k,$$

$$A_k(\nu) = \left(\frac{2}{ac} \right)^{\nu} \Gamma(\nu) \frac{(1/2 - \nu)_k}{k! (1 - 2\nu)_k} \Gamma \left[\frac{(\alpha - \nu + k)/r}{\beta + (\alpha - \nu + k)/r} \right] \quad [2\nu \neq \dots; -2, -1, 0, 1, 2, \dots].$$

$$U_n^{\pm} = \frac{a^{\alpha+r\beta-r}}{2r} \Gamma(\beta) \left\{ \sum_{k=0}^{2n-1} \frac{A_k(n)}{k!} (\pm 2ac)^k + \right. \\ \left. + \frac{2(2n)!}{n!} \left(-\frac{ac}{2} \right)^n \sum_{k=0}^{\infty} \frac{(1/2 + n)_k}{k! (2n + k)!} \Gamma \left[\frac{(\alpha + n + k)/r}{\beta + (\alpha + n + k)/r} \right] \times \right. \\ \left. \times \left[\psi(1 + k) + \psi(1 + 2n + k) - \psi \left(\frac{1}{2} \pm n \pm k \right) - \frac{1}{r} \psi \left(\frac{\alpha + n + k}{r} \right) + \right. \right. \\ \left. \left. + \frac{1}{r} \psi \left(\beta + \frac{\alpha + n + k}{r} \right) - \ln(2ac) \right] (\pm 2ac)^k \right\},$$

$$U_{n+1/2}^{-} = \frac{a^{\alpha+r\beta-r}}{2r} \Gamma(\beta) \left\{ \sum_{k=0}^n \frac{1}{k!} A_k \left(n + \frac{1}{2} \right) (-2ac)^k - \right. \\ \left. - (-1)^n 2 \sqrt{\pi} (2ac)^{n+1/2} \sum_{k=0}^{\infty} \frac{(n+k)!}{k! (2n+k+1)!} \times \right. \\ \left. \times \Gamma \left[\frac{(\alpha + n + k + 1/2)/r}{\beta + (\alpha + n + k + 1/2)/r} \right] (-2ac)^k \right\}.$$

$$2. \int_a^\infty x^{\alpha-1} (x^r - a^r)^{\beta-1} e^{\pm cx} K_\nu(cx) dx = V_\nu^\pm$$

$$\left[a, r, \operatorname{Re} \beta > 0; rl - k - \alpha - r\beta + r \neq \nu, -\nu; \right. \\ \left. k, l = 0, 1, 2, \dots, \left\{ \operatorname{Re}(\alpha + r\beta) < r + 1/2; |\arg c| < \pi \right\} \right],$$

$$V_\nu^\pm = \frac{a^{\alpha+r\beta-r}}{2r} \Gamma(\beta) \sum_{k=0}^\infty \frac{1}{k!} [B_k(\nu) + B_k(-\nu)] (\pm 2ac)^k + \sum_{l=0}^\infty C_l(\nu),$$

$$B_k(\nu) = \left(\frac{2}{ac}\right)^\nu \Gamma(\nu) \frac{(1/2 - \nu)_k}{k! (1 - 2\nu)_k} \Gamma\left[\frac{1 - \beta - (\alpha - \nu + k)/r}{1 - (\alpha - \nu + k)/r}\right],$$

$$C_l(\nu) = (2c)^{r-r\beta-\alpha} \sqrt{\pi} \frac{(1-\beta)_l}{l!} \Gamma\left[\frac{\alpha + \nu - (1 - \beta + l)r}{1/2 + \alpha - (1 - \beta + l)r}\right] \times \\ \times [\sec(\alpha - (1 - \beta + l)r) \pi \cos \nu\pi]^{(1 \pm 1)/2} (2ac)^{lr} \\ [2\nu \neq \dots, -2, -1, 0, 1, 2, \dots],$$

$$V_n^\pm = \frac{a^{\alpha+r\beta-r}}{2r} \Gamma(\beta) \left\{ \sum_{k=0}^{2n-1} \frac{B_k(n)}{k!} (\pm 2ac)^k + \sum_{m=0}^\infty D_m \right\} + \sum_{l=0}^\infty C_l(n),$$

$$D_m = \frac{2(2n)!}{n!} \left(-\frac{ac}{2}\right)^n \frac{(1/2 + n)_m}{m! (m + 2n)!} \Gamma\left[\frac{1 - \beta - (m + n + \alpha)/r}{1 - (m + n + \alpha)/r}\right] \times \\ \times \left[\psi(m+1) + \psi(m+2n+1) - \psi\left(\frac{1}{2} \pm n \pm m\right) - \frac{1}{r} \psi\left(1 - \frac{m+n+\alpha}{r}\right) + \right. \\ \left. + \frac{1}{r} \psi\left(1 - \beta - \frac{m+n+\alpha}{r}\right) - \ln(2ac) \right] (\pm 2ac)^m,$$

$$V_{n+1/2}^- = \frac{a^{\alpha+r\beta-r} \Gamma(\beta)}{2r} \left\{ \sum_{k=0}^n \frac{1}{k!} B_k\left(n + \frac{1}{2}\right) (-2ac)^k - \right. \\ \left. - (-1)^n 2 \sqrt{\pi} (2ac)^{n+1/2} \sum_{k=0}^\infty \frac{(n+k)!}{k! (2n+k+1)!} \times \right. \\ \left. \times \Gamma\left[\frac{1 - \beta - (\alpha + n + k + 1/2)/r}{1 - (\alpha + n + k + 1/2)/r}\right] (-2ac)^k \right\} + \sum_{l=0}^\infty C_l\left(n + \frac{1}{2}\right).$$

$$3. \int_0^\infty \frac{x^{\alpha-1}}{(x^r + z^r)^\rho} e^{\pm cx} K_\nu(cx) dx = W_\nu^\pm$$

$$\left[r > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu|; r |\arg z| < \pi; rl - k - \alpha + r\rho \neq \nu, -\nu; \right. \\ \left. k, l = 0, 1, 2, \dots, \left\{ \operatorname{Re}(\alpha - r\rho) < 1/2; |\arg c| < \pi \right\} \right],$$

$$W_\nu^\pm = \frac{z^{\alpha-r\rho}}{2r\Gamma(\rho)} \sum_{k=0}^\infty \frac{1}{k!} [E_k(\nu) + E_k(-\nu)] (\pm 2cz)^k + \sum_{l=0}^\infty F_l(\nu),$$

$$E_k(\nu) = \left(\frac{2}{cz}\right)^\nu \frac{\Gamma(\nu) (1/2 - \nu)_k}{k! (1 - 2\nu)_k} \Gamma\left(\frac{\alpha - \nu + k}{r}\right) \Gamma\left(\rho - \frac{\alpha - \nu + k}{r}\right),$$

$$F_l(\nu) = (2c)^{r\rho-\alpha} \sqrt{\pi} \frac{(-1)^l (\rho)_l}{l!} \Gamma\left[\frac{\alpha + \nu - r\rho - lr}{1/2 + \alpha - r\rho - lr}\right] \times \\ \times [\sec(\alpha - r\rho - lr) \pi \cos \nu\pi]^{(1 \pm 1)/2} (2cz)^{lr} \\ [2\nu \neq \dots, -2, -1, 0, 1, 2, \dots],$$

$$W_n^{\pm} = \frac{z^{\alpha-r\rho}}{2r\Gamma(\rho)} \left\{ \sum_{k=0}^{2n-1} \frac{E_k(n)}{k!} (\pm 2cz)^k + \sum_{m=0}^{\infty} G_m \right\} + \sum_{l=0}^{\infty} F_l(n),$$

$$G_m = \frac{2(2n)!}{n!} \left(-\frac{cz}{2}\right)^n \frac{(1/2+n)_m}{m!(m+2n)!} \Gamma\left(\frac{\alpha+m+n}{r}\right) \times$$

$$\times \Gamma\left(\rho - \frac{\alpha+m+n}{r}\right) \left[\psi(1+m) + \psi(1+m+2n) - \psi\left(\frac{1}{2} \pm m \pm n\right) - \right.$$

$$\left. - \frac{1}{r} \psi\left(\frac{\alpha+m+n}{r}\right) + \frac{1}{r} \psi\left(\rho - \frac{\alpha+m+n}{r}\right) - \ln(2cz) \right] (\pm 2cz)^m,$$

$$W_{n+1/2}^{-} = \frac{z^{\alpha-r\rho}}{2r\Gamma(\rho)} \left\{ \sum_{k=0}^n \frac{1}{k!} E_k\left(n + \frac{1}{2}\right) (-2cz)^k - \right.$$

$$\left. - (-1)^n 2\sqrt{\pi} (2cz)^{n+1/2} \sum_{k=0}^{\infty} \frac{(n+k)!}{k!(2n+k+1)!} \times \right.$$

$$\left. \times \Gamma\left(\frac{\alpha+n+k+1/2}{r}\right) \Gamma\left(\rho - \frac{\alpha+n+k+1/2}{r}\right) (-2cz)^k \right\} + \sum_{l=0}^{\infty} F_l\left(n + \frac{1}{2}\right).$$

$$4. \int_0^{\infty} \frac{x^{\alpha-1}}{x^r - y^r} e^{\pm cx} K_{\nu}(cx) dx = X_{\nu}^{\pm}$$

$$\left[r, y > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu|; rl - k - \alpha + r \neq \nu, -\nu; k, l = 0, 1, 2, \dots, \right.$$

$$\left. \left\{ \operatorname{Re} \alpha < r + 1/2; |\arg c| < \pi \right\} \right],$$

$$X_{\nu}^{\pm} = -\frac{y^{\alpha-r\pi}}{2r} \sum_{k=0}^{\infty} \frac{1}{k!} [M_k(\nu) + M_k(-\nu)] (\pm 2cy)^k + \sum_{l=0}^{\infty} N_l(\nu),$$

$$M_k(\nu) = \left(\frac{2}{cy}\right)^{\nu} \frac{\Gamma(\nu) (1/2 - \nu)_k}{k! (1 - 2\nu)_k} \operatorname{ctg} \frac{\alpha - \nu + k}{r} \pi,$$

$$N_l(\nu) = (2c)^{r-\alpha} \sqrt{\pi} \Gamma\left[\frac{\alpha + \nu - r - lr}{1/2 + \alpha - r - lr}\right] \times$$

$$\times [\sec(\alpha - r - lr)\pi \cos \nu\pi]^{(1 \pm 1)/2} (2cy)^r$$

$$[2\nu \neq \dots, -2, -1, 0, 1, 2, \dots],$$

$$X_n^{\pm} = -\frac{y^{\alpha-r\pi}}{2r} \left\{ \sum_{k=0}^{2n-1} \frac{M_k(n)}{k!} (\pm 2cy)^k + \sum_{m=0}^{\infty} P_m \right\} + \sum_{l=0}^{\infty} N_l(n),$$

$$P_m = \frac{2(2n)!}{n!} \left(-\frac{cy}{2}\right)^n \frac{(1/2+n)_m}{m!(m+2n)!} \operatorname{ctg} \frac{\alpha+m+n}{r} \pi \times$$

$$\times \left[\psi(1+m) + \psi(1+m+2n) - \psi\left(\frac{1}{2} \pm m \pm n\right) + \right.$$

$$\left. + \frac{2\pi}{r} \operatorname{cosec} 2 \frac{\alpha+m+n}{r} \pi - \ln(2cy) \right] (\pm 2cy)^m,$$

$$X_{n+1/2}^{-} = -\frac{y^{\alpha-r\pi}}{2r} \left\{ \sum_{k=0}^n \frac{1}{k!} M_k\left(n + \frac{1}{2}\right) (-2cy)^k - \right.$$

$$\left. - (-1)^n 2\sqrt{\pi} (2cy)^{n+1/2} \sum_{k=0}^{\infty} \frac{(n+k)!}{k!(2n+k+1)!} \times \right.$$

$$\left. \times \operatorname{ctg} \frac{\alpha+n+k+1/2}{r} \pi (-2cy)^k \right\} + \sum_{l=0}^{\infty} N_l\left(n + \frac{1}{2}\right).$$

$$5. \int_0^{\infty} x^{\alpha-1} e^{-px^r} \pm cx K_{\nu}(cx) dx = Y_{\nu}^{\pm}$$

$$\left[r, \operatorname{Re} p > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu|, \left\{ \begin{array}{l} |\arg c| < \pi \\ \operatorname{Re} c > 0 \end{array} \right\} \right],$$

$$Y_{\nu}^{\pm} = \frac{p^{-\alpha/r}}{2r} \sum_{k=0}^{\infty} \frac{1}{k!} [R_k(\nu) + R_k(-\nu)] \left(\pm \frac{2c}{p^{1/r}} \right)^k,$$

$$R_k(\nu) = \left(\frac{2p^{1/r}}{c} \right)^{\nu} \frac{\Gamma(\nu) (1/2 - \nu)_k}{k! (1 - 2\nu)_k} \Gamma\left(\frac{\alpha - \nu + k}{r}\right)$$

[r > 1; 2ν ≠ ..., -2, -1, 0, 1, 2, ...],

$$Y_{\frac{n}{2}}^{\pm} = \frac{p^{-\alpha/r}}{2r} \left\{ \sum_{k=0}^{2n-1} \frac{R_k(n)}{k!} \left(\pm \frac{2c}{p^{1/r}} \right)^k + \frac{2(2n)!}{n!} \left(-\frac{c}{2p^{1/r}} \right)^n \times \right.$$

$$\times \sum_{k=0}^{\infty} \frac{(1/2 + n)_k}{k! (2n + k)!} \Gamma\left(\frac{\alpha + k + n}{r}\right) \left[\psi(1 + k) + \psi(1 + 2n + k) - \right.$$

$$\left. - \psi\left(\frac{1}{2} \pm n \pm k\right) - \frac{1}{r} \psi\left(\frac{\alpha + n + k}{r}\right) - \ln \frac{2c}{p^{1/r}} \right] \left(\pm \frac{2c}{p^{1/r}} \right)^k \Big\} \quad [r > 1],$$

$$Y_{n+1/2}^{\pm} = \frac{p^{-\alpha/r}}{2r} \left\{ \sum_{k=0}^n \frac{1}{k!} R_k\left(n + \frac{1}{2}\right) \left(-\frac{2c}{p^{1/r}} \right)^k - \right.$$

$$\left. - (-1)^n 2\sqrt{\pi} \left(\frac{2c}{p^{1/r}} \right)^{n+1/2} \sum_{k=0}^{\infty} \frac{(n+k)!}{k! (2n+k+1)!} \Gamma\left(\frac{\alpha + n + k + 1/2}{r}\right) \left(-\frac{2c}{p^{1/r}} \right)^k \right\}$$

[r > 1].

$$Y_{\nu}^{\pm} = (2c)^{-\alpha} \sqrt{\pi} \sum_{k=0}^{\infty} \frac{1}{k!} \Gamma\left[\frac{\alpha + \nu + rk}{1/2 + \alpha + rk}\right] \times$$

$$\times [\sec(\alpha + rk)\pi \cos \nu\pi]^{(1 \pm 1)/2} \left(-\frac{p}{2rc^r} \right)^k + \left\{ \begin{array}{l} 1 \\ 0 \end{array} \right\} \frac{\sqrt{\pi} p^{(1-2\alpha)/(2r)}}{r\sqrt{2c}} \times$$

$$\times \sum_{k=0}^{\infty} \frac{(1/2 + \nu)_k (1/2 - \nu)_k}{k!} \Gamma\left(\frac{2\alpha - 2k - 1}{2r}\right) \left(-\frac{p^{1/r}}{2c} \right)^k \quad [r < 1],$$

$Y_{\nu}^{\pm}$ при $r=1$ см. в 2.16.6.3.

$$6. \int_0^{\infty} x^{\alpha-1} e^{-px^{-r}} \pm cx K_{\nu}(cx) dx = Z_{\nu}^{\pm}$$

$$\left[r, \operatorname{Re} p > 0; rl - k - \alpha \neq \nu, -\nu; k, l = 0, 1, 2, \dots, \left\{ \begin{array}{l} \operatorname{Re} \alpha < 1/2; |\arg c| < \pi \\ \operatorname{Re} c > 0 \end{array} \right\} \right],$$

$$Z_{\nu}^{\pm} = \frac{p^{\alpha/r}}{2r} \sum_{k=0}^{\infty} \frac{1}{k!} [S_k(\nu) + S_k(-\nu)] (\pm 2cp^{1/r})^k + \sum_{l=0}^{\infty} T_l(\nu),$$

$$S_k(\nu) = \left(\frac{cp^{1/r}}{2} \right)^{-\nu} \frac{\Gamma(\nu) (1/2 - \nu)_k}{k! (1 - 2\nu)_k} \Gamma\left(\frac{\nu - \alpha - k}{r}\right),$$

$$T_l(\nu) = (2c)^{-\alpha} \sqrt{\pi} \frac{1}{l!} \Gamma\left[\frac{\alpha + \nu - rl}{1/2 + \alpha - rl}\right] \times$$

$$\times [\sec(\alpha - rl)\pi \cos \nu\pi]^{(1 \pm 1)/2} (-2rc^r p)^l \quad [2\nu \neq \dots, -2, -1, 0, 1, 2, \dots],$$

$$Z_n^{\pm} = \frac{p^{\alpha/r}}{2r} \left\{ \sum_{k=0}^{2n-1} \frac{S_k(n)}{k!} (\pm 2cp^{1/r})^k + \sum_{m=0}^{\infty} \Delta_m \right\} + \sum_{l=0}^{\infty} T_l(n),$$

$$\Delta_m = \frac{2(2n)!}{n!} \left(-\frac{cp^{1/r}}{2} \right)^n \frac{(1/2+n)_m}{m!(m+2n)!} \Gamma\left(-\frac{\alpha+m+n}{r}\right) \times \\ \times \left[\psi(1+m) + \psi(1+m+n) + \frac{1}{r} \psi\left(-\frac{\alpha+m+n}{r}\right) - \psi\left(\frac{1}{2} \pm m \pm n\right) - \right. \\ \left. - \ln(2cp^{1/r}) \right] (\pm 2cp^{1/r})^m,$$

$$Z_{n+1/2}^- = \frac{p^{\alpha/r}}{2r} \left\{ \sum_{k=0}^n \frac{1}{k!} S_k\left(n + \frac{1}{2}\right) (-2cp^{1/r})^k - \right. \\ \left. - (-1)^n 2\sqrt{\pi} (2cp^{1/r})^{n+1/2} \sum_{k=0}^{\infty} \frac{(n+k)!}{k!(2n+k+1)!} \times \right. \\ \left. \times \Gamma\left(-\frac{\alpha+n+k+1/2}{r}\right) (-2cp^{1/r})^k \right\} + \sum_{l=0}^{\infty} T_l\left(n + \frac{1}{2}\right).$$

$$7. \int_0^{\infty} x^{\alpha-1} \left\{ \frac{\sin bx^r}{\cos bx^r} \right\} K_{\nu}(cx) dx = -\frac{\pi}{2} U_{\nu}(0, 0)$$

$$[b, r, \operatorname{Re} c > 0, |\operatorname{Re} \nu| - (r \pm r)/2 < \operatorname{Re} \alpha; U_{\nu}(\gamma, \varepsilon) \text{ см в 2.13.1.7}].$$

$$8. \int_0^{\infty} x^{\alpha-1} e^{-px^r} \left\{ \frac{\sin bx^r}{\cos bx^r} \right\} K_{\nu}(cx) dx = -\frac{\pi}{2} V_{\nu}(0, 0)$$

$$[b, r, \operatorname{Re} c, \operatorname{Re} p > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu| - r(1 \pm 1)/2; V_{\nu}(\gamma, \varepsilon) \text{ см в 2.13.1.8}]$$

$$9. \int_0^{\infty} x^{\alpha-1} e^{-cx} \left\{ \frac{\sin bx^r}{\cos bx^r} \right\} K_{\nu}(cx) dx = \hat{V}_{\nu}(1)$$

$$[b, r, \operatorname{Re} c > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu| - r\delta; \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}],$$

$$\hat{V}_{\nu}(\gamma) = \frac{b^{-\alpha/r}}{2r} \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma}}{k!} [A_k(\nu) + A_k(-\nu)] \left(\frac{2c}{b^{1/r}} \right)^k,$$

$$A_k(\nu) = \left(\frac{2b^{1/r}}{c} \right)^{\nu} \frac{\Gamma(\nu) (1/2 - \nu)_k}{(1 - 2\nu)_k} \Gamma\left(\frac{\alpha - \nu + k}{r}\right) \left\{ \frac{\sin[(\alpha + k - \nu)\pi/(2r)]}{\cos[(\alpha + k - \nu)\pi/(2r)]} \right\} \\ [r > 1; 2\nu \neq \dots, -2, -1, 0, 1, 2, \dots].$$

$$\hat{V}_n(\gamma) = \frac{b^{-\alpha/r}}{2r} \left[\sum_{k=0}^{2n-1} \frac{(-1)^{k\gamma} A_k(n)}{k!} \left(\frac{2c}{b^{1/r}} \right)^k + \right. \\ \left. + \frac{2(2n)!}{n!} \left(-\frac{c}{2b^{1/r}} \right)^n \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma} (1/2 + n)_k}{k!(2n+k)!} \Gamma\left(\frac{\alpha + n + k}{r}\right) \times \right. \\ \left. \times \left\{ \frac{\sin[(\alpha + n + k)\pi/(2r)]}{\cos[(\alpha + n + k)\pi/(2r)]} \right\} \left[\psi(1+k) + \psi(1+2n+k) - \right. \right. \\ \left. \left. - \psi\left(\frac{1}{2} + (-1)^{\gamma}(n+k)\right) - \frac{1}{r} \psi\left(\frac{\alpha + n + k}{r}\right) \mp \frac{\pi}{2r} \operatorname{tg}^{\mp 1} \left(\frac{\alpha + n + k}{2r} \pi \right) - \right. \right. \\ \left. \left. - \ln \frac{2c}{b^{1/r}} \right] \left(\frac{2c}{b^{1/r}} \right)^k \right] \quad [r > 1],$$

$$\hat{V}_{n+1/2}(1) = \frac{b^{-\alpha/r}}{2r} \sum_{k=0}^n \frac{1}{k!} A_k \left(n + \frac{1}{2} \right) \left(-\frac{2c}{b^{1/r}} \right)^k -$$

$$- \frac{(-1)^n \sqrt{\pi}}{b^{\alpha/r} r} \left(\frac{2c}{b^{1/r}} \right)^{n+1/2} \sum_{k=0}^{\infty} \frac{(n+k)!}{k! (2n+k+1)!} \times$$

$$\times \Gamma \left(\frac{\alpha+n+k+1/2}{r} \right) \left\{ \begin{array}{l} \sin [(\alpha+n+k+1/2) \pi/(2r)] \\ \cos [(\alpha+n+k+1/2) \pi/(2r)] \end{array} \right\} \left(-\frac{2c}{b^{1/r}} \right)^k \quad [r > 1],$$

$$\hat{V}_v(\gamma) = \frac{b^\delta \sqrt{\pi} \cos^{1-\gamma} v \pi}{(2c)^{\alpha+r\delta}} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (1/2+\delta)_k} \times$$

$$\times \Gamma \left[\begin{array}{l} \alpha+v+r\delta+2rk, \alpha-v+r\delta+2rk \\ 1/2+\alpha+r\delta+2rk \end{array} \right] \cos^{v-1} (\alpha+r\delta+2rk) \pi \times$$

$$\times \left(\frac{b}{2^{r+1} c^r} \right)^{2k} + (1-\gamma) \frac{\sqrt{\pi} b^{(1-2\alpha)/(2r)}}{r \sqrt{2c}} \sum_{k=0}^{\infty} \frac{(1/2+v)_k (1/2-v)_k}{k!} \Gamma \left(\frac{2\alpha-2k-1}{2r} \right) \times$$

$$\times \left\{ \begin{array}{l} \sin [(2\alpha-2k-1) \pi/(4r)] \\ \cos [(2\alpha-2k-1) \pi/(4r)] \end{array} \right\} \left(-\frac{b^{1/r}}{2c} \right)^k \quad [r < 1],$$

$\hat{V}_v(\gamma)$ при $r=1$ см. в 2.16.18.1.

$$10. \int_0^\infty x^{\alpha-1} e^{cx} \left\{ \begin{array}{l} \sin bx^r \\ \cos bx^r \end{array} \right\} K_v(cx) dx = \hat{V}_v(0)$$

$[b, r > 0; |\operatorname{Re} v| - r(1 \pm 1)/2 < \operatorname{Re} \alpha < r + 1/2; |\arg c| < \pi, \hat{V}_v(\gamma)$ см. в 2.16.1.9].

$$11. \int_0^\infty x^{\alpha-1} \left\{ \begin{array}{l} \ln(x^r + z^r) \\ \ln|x^r - z^r| \end{array} \right\} K_v(cx) dx = -\frac{\pi}{2} W_v(0, 0)$$

$[r, \operatorname{Re} c > 0; |\operatorname{Re} v| < \operatorname{Re} \alpha; rl - 2k - \alpha \neq v, -v, k = 0, 1, 2, \dots, \{r | \arg z| < \pi\}, W_v(\gamma, s)$ см. в 2.13.1.9].

$$12. \int_0^\infty x^{\alpha-1} J_\mu(bx^r) K_v(cx) dx = -\frac{\pi}{2} \tilde{U}_v(0, 0)$$

$[b, r, \operatorname{Re} c > 0; \operatorname{Re}(\alpha + \mu r) > |\operatorname{Re} v|; \tilde{U}_v(\gamma, s)$ см. в 2.13.1.10].

$$13. \int_0^\infty x^{\alpha-1} e^{\pm cx} J_\mu(bx^r) K_v(cx) dx = \tilde{Y}_v^\pm$$

$[b, r > 0; \operatorname{Re}(\alpha + \mu r) > |\operatorname{Re} v|, \left\{ \begin{array}{l} \operatorname{Re} \alpha < (3r+1)/2; |\arg c| < \pi \\ \operatorname{Re} c > 0 \end{array} \right\}],$

$$\tilde{Y}_v^\pm = \frac{2^{\alpha/r-2}}{b^{\alpha/r} r} \sum_{k=0}^{\infty} \frac{1}{k!} [\tilde{A}_k(v) + \tilde{A}_k(-v)] \left(\pm \frac{2^{1/r+1} c}{b^{1/r}} \right)^k,$$

$$\tilde{A}_k(v) = \left(\frac{b^{1/r}}{2^{1/r-1} c} \right)^v \frac{\Gamma(v) (1/2-v)_k}{(1-2v)_k} \Gamma \left[\begin{array}{l} (\mu r + \alpha + k - v)/(2r) \\ (\mu r - \alpha - k + v)/(2r) + 1 \end{array} \right] +$$

$[r > 1; 2v \neq \dots, -2, -1, 0, 1, 2, \dots],$

$$\begin{aligned} \tilde{Y}_n^\pm = & \frac{2^{\alpha/r-2}}{b^{\alpha/r}} \left\{ \sum_{k=0}^{2n-1} \frac{\tilde{A}_k(n)}{k!} \left(\pm \frac{2^{1/r+1}c}{b^{1/r}} \right)^k + 2 \left(-\frac{2^{1/r-1}c}{b^{1/r}} \right)^n \times \right. \\ & \times \frac{(2n)!}{n!} \sum_{k=0}^{\infty} \frac{(1/2+n)_k}{k! (2n+k)!} \Gamma \left[\frac{(\mu r + \alpha + k + n)/(2r)}{[(\mu r - \alpha - k - n)/(2r) + 1]} \right] \times \\ & \times \left[\psi(k+1) + \psi(2n+k+1) - \psi\left(\frac{1}{2} \pm n \pm k\right) - \frac{1}{2r} \psi\left(\frac{\mu r + \alpha + k + n}{2r}\right) - \right. \\ & \left. \left. - \frac{1}{2r} \psi\left(\frac{\mu r - \alpha - k - n}{2r} + 1\right) - \ln \frac{2^{1/r+1}c}{b^{1/r}} \right] \left(\pm \frac{2^{1/r+1}c}{b^{1/r}} \right)^k \right\} \\ & [r > 1; n = 0, 1, 2, \dots], \end{aligned}$$

$$\begin{aligned} \tilde{Y}_{n+1/2}^- = & \frac{2^{\alpha/r-2}}{b^{\alpha/r}} \left[\sum_{k=0}^n \frac{1}{k!} \tilde{A}_k\left(n + \frac{1}{2}\right) \left(-\frac{2^{1/r+1}c}{b^{1/r}} \right)^k - \right. \\ & - 2\sqrt{\pi} (-1)^n \left(\frac{2^{1/r+1}c}{b^{1/r}} \right)^{n+1/2} \sum_{k=0}^{\infty} \frac{(n+k)!}{k! (2n+k+1)!} \times \\ & \times \Gamma \left[\frac{(\mu r + \alpha + k + n + 1/2)/(2r)}{[(\mu r - \alpha - k - n - 1/2)/(2r) + 1]} \right] \left(-\frac{2^{1/r+1}c}{b^{1/r}} \right)^k \quad [r > 1], \end{aligned}$$

$$\begin{aligned} \tilde{Y}_v^\pm = & \frac{\sqrt{\pi} b^\mu}{2^{\alpha+\mu r+\mu} c^{\alpha+\mu r} \Gamma(1+\mu)} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (1+\mu)_k} \times \\ & \times \Gamma \left[\frac{\alpha + v + \mu r + 2rk}{1/2 + \alpha + \mu r + 2rk} \right] \times \\ & \times [\sec(\alpha + \mu r + 2rk) \pi \cos v \pi]^{(1 \pm 1)/2} \left(\frac{b}{2^{r+1}c^r} \right)^{2k} + \\ & + \begin{cases} 1 \\ 0 \end{cases} \frac{2^{(2\alpha-3r-1)/(2r)} \sqrt{\pi}}{r \sqrt{c} b^{(2\alpha-1)/(2r)}} \sum_{k=0}^{\infty} \frac{(1/2+v)_k (1/2-v)_k}{k!} \times \\ & \times \Gamma \left[\frac{(\mu r + \alpha - k - 1/2)/(2r)}{[(\mu r - \alpha + k + 1/2)/(2r) + 1]} \right] \left(-\frac{b^{1/r}}{2^{1/r+1}c} \right)^k \quad [r < 1]. \end{aligned}$$

$$\begin{aligned} 14. \int_0^\infty x^{\alpha-1} \begin{Bmatrix} Y_\mu(bx^r) \\ K_\mu(bx^r) \end{Bmatrix} K_\nu(cx) dx = -\frac{\pi}{2} U_{\mu,\nu}(0, 0) \\ [r, \operatorname{Re} c > 0; \operatorname{Re} \alpha > r; \operatorname{Re} \mu + |\operatorname{Re} v|, \begin{cases} b > 0 \\ \operatorname{Re} b > 0 \end{cases}, \\ U_{\mu,\nu}(\gamma, \varepsilon) \text{ см. в 2.13.1.11}]. \end{aligned}$$

$$\begin{aligned} 15. \int_0^\infty x^{\alpha-1} I_\mu(bx^r) K_\nu(cx) dx = -\frac{\pi}{2} U_\nu^+(0, 0) \\ [r \leq 1; |\arg b| < \pi; \operatorname{Re} c > 0 \text{ при } r < 1; \operatorname{Re}(c-b) > 0 \\ \text{при } r = 1; \operatorname{Re}(\alpha + \mu r) > |\operatorname{Re} v|; U_\nu^+(\gamma, \varepsilon) \text{ см. в 2.13.1.10}]. \end{aligned}$$

$$\begin{aligned} 16. \int_0^\infty x^{\alpha-1} e^{-cx} I_\mu(bx^r) K_\nu(cx) dx = \frac{\sqrt{\pi} b^\mu}{2^{\alpha+\mu r+\mu} c^{\alpha+\mu r} \Gamma(\mu+1)} \sum_{k=0}^{\infty} \frac{1}{k! (\mu+1)_k} \times \\ \times \Gamma \left[\frac{\alpha + v + \mu r + 2rk}{1/2 + \alpha + \mu r + 2rk} \right] \left(\frac{b}{2^{r+1}c^r} \right)^{2k} \\ [r \leq 1, |\arg b| < \pi; \operatorname{Re} c > 0 \text{ при } r < 1; \operatorname{Re}(2c-b) > 0 \text{ при } r = 1; \\ \operatorname{Re}(\alpha + \mu r) > |\operatorname{Re} v|]. \end{aligned}$$

$$17. \int_0^{\infty} x^{\alpha-1} e^{-bx^r \pm cx} I_{\mu}(bx^r) K_{\nu}(cx) dx = \tilde{Z}_{\frac{\alpha}{r}}^{\pm}$$

$$\left[r, \operatorname{Re} b > 0; \operatorname{Re}(\alpha + r\mu) > |\operatorname{Re} \nu|, \begin{cases} |\arg c| < \pi; \operatorname{Re} \alpha < (r+1)/2 \\ \operatorname{Re} c > 0 \end{cases} \right],$$

$$\tilde{Z}_{\nu}^{\pm} = \frac{(2b)^{-\alpha/r}}{2r\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{1}{k!} [\tilde{B}_k(\nu) + \tilde{B}_k(-\nu)] \left(\pm \frac{2^{1-1/r} c}{b^{1/r}} \right)^k + V_{\nu}^{\pm},$$

$$\tilde{B}_k(\nu) = \left(\frac{2^{1+1/r} b^{1/r}}{c} \right)^{\nu} \frac{\Gamma(\nu) (1/2 - \nu)_k}{(1 - 2\nu)_k} \Gamma \left[\mu + (\alpha + k - \nu)/r, 1/2 + (\nu - \alpha - k)/r \right],$$

$$V_{\nu}^{\pm} = \frac{2^{(r-1)/2-\alpha}}{c^{\alpha-r/2} \sqrt{b}} \sum_{k=0}^{\infty} \frac{(1/2 + \mu)_k (1/2 - \mu)_k}{k!} \times \\ \times \Gamma \left[\frac{\alpha + \nu - r/2 - rk}{(1-r)/2 + \alpha - rk}, \frac{\alpha - \nu - r/2 - rk}{(1-r)/2 + \alpha - rk} \right] \left[\sec \left(\alpha - \frac{r}{2} - rk \right) \pi \cos \nu \pi \right]^{(1 \pm 1)/2} \times \\ \times \left(\frac{2^{r-1} c^r}{b} \right)^k \quad \left[r > 1; 2\nu \neq \dots -2, -1, 0, 1, 2, \dots; \right. \\ \left. l - kr - r/2 - \alpha \neq \nu, -\nu; k, l = 0, 1, 2, \dots \right],$$

$$\tilde{Z}_{\frac{\alpha}{r}}^{\pm} = \frac{(2b)^{-\alpha/r}}{2r\sqrt{\pi}} \left\{ \sum_{k=0}^{2n-1} \frac{\tilde{B}_k(n)}{k!} \left(\pm \frac{2^{1-1/r} c}{b^{1/r}} \right)^k + \right. \\ \left. + 2 \left(-\frac{c}{2^{1/r+1} b^{1/r}} \right)^n \frac{(2n)!}{n!} \sum_{k=0}^{\infty} \frac{(1/2 + n)_k}{k! (2n+k)!} \Gamma \left[\mu + (\alpha + k + n)/r, 1/2 - (\alpha + k + n)/r \right] \right. \\ \times \left[\psi(k+1) + \psi(2n+k+1) - \psi \left(\frac{1}{2} \pm n \pm k \right) - \frac{1}{r} \psi \left(\mu + \frac{\alpha + k + n}{r} \right) + \right. \\ \left. + \frac{1}{r} \psi \left(\frac{1}{2} - \frac{\alpha + k + n}{r} \right) - \frac{1}{r} \psi \left(1 + \mu - \frac{\alpha + k + n}{r} \right) - \right. \\ \left. - \ln \frac{2^{1-1/r} c}{b^{1/r}} \right] \left(\pm \frac{2^{1-1/r} c}{b^{1/r}} \right)^k + V_{\frac{\alpha}{r}}^{\pm} \quad [r > 1].$$

$$\tilde{Z}_{\frac{\alpha}{r}+1/2}^{\pm} = \frac{(2b)^{-\alpha/r}}{2r\sqrt{\pi}} \left\{ \sum_{k=0}^n \frac{1}{k!} \tilde{B}_k \left(n + \frac{1}{2} \right) \left(-\frac{2^{1-1/r} c}{b^{1/r}} \right)^k - \right. \\ \left. - 2(-1)^n \sqrt{\pi} \left(\frac{2^{1-1/r} c}{b^{1/r}} \right)^{n+1/2} \sum_{k=0}^{\infty} \frac{(n+k)!}{k! (2n+k+1)!} \times \right. \\ \left. \times \Gamma \left[\mu + (\alpha + k + n + 1/2)/r, 1/2 - (\alpha + k + n + 1/2)/r \right] \left(-\frac{2^{1-1/r} c}{b^{1/r}} \right)^k \right\} \quad [r > 1].$$

$$\tilde{Z}_{\nu}^{\pm} = \begin{cases} 1 \\ 0 \end{cases} \frac{2^{(1-2\alpha-r)/(2r)}}{r\sqrt{c} b^{(2\alpha-1)/(2r)}} \sum_{k=0}^{\infty} \frac{(1/2 + \nu)_k (1/2 - \nu)_k}{k!} \times \\ \times \Gamma \left[\mu + (\alpha - k - 1/2)/r, (1+r-2\alpha+2k)/(2r) \right] \left(-\frac{b^{1/r}}{2^{1-1/r} c} \right)^k + \\ + \frac{b\mu \sqrt{\pi}}{2\alpha + \mu + r\mu c^{\alpha+r\mu} \Gamma(\mu+1)} \sum_{k=0}^{\infty} \frac{(1/2 + \mu)_k}{k! (2\mu+1)_k} \Gamma \left[\alpha + \nu + r\mu + rk, \alpha - \nu + r\mu + rk \right] \times \\ \times [\sec(\alpha + r\mu + rk) \pi \cos \nu \pi]^{(1 \pm 1)/2} \left(-\frac{b}{2^{r-1} c^r} \right)^k \quad [r < 1].$$

$$\tilde{Z}_{\nu}^{\pm} \text{ при } r=1 \text{ см. в 2.16.29.4.}$$

$$18. \int_0^{\infty} x^{\alpha-1} e^{\pm bx^r} \operatorname{Ei}(\mp bx^r) K_{\nu}(cx) dx = -\frac{\pi}{2} \hat{U}_{\nu}(0, 0)$$

$[r, \operatorname{Re} b, \operatorname{Re} c > 0; |\operatorname{Re} \nu| < \operatorname{Re} \alpha; \hat{U}_{\nu}(\nu, c) \text{ см. в 2.13.1.13}].$

$$19. \int_0^{\infty} x^{\alpha-1} e^{\pm cx} \operatorname{Ei}(-bx^r) K_{\nu}(cx) dx = X_{\nu}^{\pm}$$

$[r, \operatorname{Re} c, \operatorname{Re} \beta > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu|].$

$$X_{\nu}^{\pm} = \frac{b^{-\alpha/r}}{2} \sum_{k=0}^{\infty} \frac{1}{k!} [B_k(\nu) + B_k(-\nu)] \left(\pm \frac{2c}{b^{1/r}} \right)^k,$$

$$B_k(\nu) = \left(\frac{2b^{1/r}}{c} \right)^{\nu} \frac{\Gamma(\nu) (1/2 - \nu)_k}{(\nu - \alpha - k) (1 - 2\nu)_k} \Gamma\left(\frac{\alpha + k - \nu}{r}\right)$$

$[r > 1; 2\nu \neq \dots, -2, -1, 0, 1, 2, \dots].$

$$X_n^{\pm} = \frac{b^{-\alpha/r}}{2} \left\{ \sum_{k=0}^{2n-1} \frac{B_k(n)}{k!} \left(\pm \frac{2c}{b^{1/r}} \right)^k - 2 \left(-\frac{c}{2b^{1/r}} \right)^n \times \right. \\ \times \frac{(2n)!}{n!} \sum_{k=0}^{\infty} \frac{(1/2 + n)_k}{k! (2n + k)! (\alpha + n + k)} \Gamma\left(\frac{\alpha + n + k}{r}\right) \times \\ \times \left[\psi(k+1) + \psi(2n+k+1) - \psi\left(\frac{1}{2} \pm n \pm k\right) + \frac{1}{\alpha + n + k} - \right. \\ \left. - \frac{1}{r} \psi\left(\frac{\alpha + n + k}{r}\right) - \ln \frac{2c}{b^{1/r}} \right] \left(\pm \frac{2c}{b^{1/r}} \right)^k \Big\} \quad [r > 1].$$

$$X_{n+1/2}^{\pm} = \frac{b^{-\alpha/r}}{2} \sum_{k=0}^n \frac{1}{k!} B_k\left(n + \frac{1}{2}\right) \left(-\frac{2c}{b^{1/r}} \right)^k - \frac{(-1)^n \sqrt{\pi}}{b^{\alpha/r}} \left(\frac{2c}{b^{1/r}} \right)^{n+1/2} \times \\ \times \sum_{k=0}^{\infty} \frac{(n+k)!}{k! (2n+k+1)! (\alpha + n + k + 1/2)} \Gamma\left(\frac{\alpha + n + k + 1/2}{r}\right) \left(-\frac{2c}{b^{1/r}} \right)^k \\ [r > 1; n = 0, 1, 2, \dots].$$

$$X_{\nu}^{\pm} = \frac{\sqrt{\pi}}{(2c)^{\alpha}} \sum_{k=1}^{\infty} \frac{1}{k! k} \Gamma\left[\begin{matrix} \alpha + \nu + rk, \alpha - \nu + rk \\ \alpha + 1/2 + rk \end{matrix}\right] \times \\ \times [\sec(\alpha + rk)\pi \cos \nu\pi]^{(1 \pm 1)/2} \left(-\frac{b}{2rc^r} \right)^k + \frac{\sqrt{\pi}}{(2c)^{\alpha}} \Gamma\left[\begin{matrix} \alpha + \nu, \alpha - \nu \\ \alpha + 1/2 \end{matrix}\right] \times \\ \times (\sec \alpha\pi \cos \nu\pi)^{(1 \pm 1)/2} \left[C - r\psi(\alpha + \nu) - r\psi(\alpha - \nu) - r\psi\left(\frac{1}{2} \mp \alpha\right) - \ln \frac{2rc^r}{b} \right] + \\ + \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \sqrt{\frac{\pi}{2c}} b^{(1-2\alpha)/(2r)} \sum_{k=0}^{\infty} \frac{(1/2 + \nu)_k (1/2 - \nu)_k}{k! (1/2 - \alpha + k)} \Gamma\left(\frac{2\alpha - 2k - 1}{2r}\right) \left(-\frac{b^{1/r}}{2c} \right)^k \\ [r < 1; rl - k \neq 1/2 - \alpha; k, l = 0, 1, 2, \dots].$$

$X_{\nu}^{\pm}$ при $r=1$ см. в 2.16.57.5.

$$20. \int_0^{\infty} x^{\alpha-1} e^{bx^r} \pm cx \operatorname{Ei}(-bx^r) K_{\nu}(cx) dx = Y_{\nu}^{\pm}(1)$$

$[r, \operatorname{Re} b > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu|, \left\{ \begin{matrix} \operatorname{Re} \alpha < r + 1/2; |\arg c| < \pi \\ \operatorname{Re} c > 0 \end{matrix} \right\}],$

$$Y_{\nu}^{\pm}(\gamma) = -\frac{b^{-\alpha/r} \pi}{2r} \sum_{k=0}^{\infty} \frac{1}{k!} [C_k(\nu) + C_k(-\nu)] \left(\pm \frac{2c}{b^{1/r}} \right)^k + V_{\nu}^{\pm}(\gamma).$$

$$C_k(v) = \left(\frac{2b^{1/r}}{c}\right)^v \frac{\Gamma(v) (1/2-v)_k}{(1-2v)_k} \Gamma\left(\frac{\alpha+k-v}{r}\right) \frac{\cos^{1-v}[(\alpha+k-v)\pi/r]}{\sin[(\alpha+k-v)\pi/r]},$$

$$V_{\frac{v}{r}}^{\pm}(\gamma) = \frac{(-1)^v \sqrt{\pi}}{b(2c)^{\alpha-r}} \sum_{k=0}^{\infty} (-1)^{kv} k! \Gamma\left[\frac{\alpha+v-r-rk}{1/2+\alpha-r-rk}, \alpha-v-r-rk\right] \times$$

$$\times [\sec(\alpha-r-rk)\pi \cos v\pi]^{(1\pm 1)/2} \left(\frac{2^r c^r}{b}\right)^k$$

$[r > 1; 2v \neq \dots, -2, -1, 0, 1, 2, \dots]$

$$Y_n^{\pm}(\gamma) = -\frac{b^{-\alpha/r} \pi}{2r} \left\{ \sum_{k=0}^{2n-1} \frac{C_k(n)}{k!} \left(\pm \frac{2c}{b^{1/r}}\right)^k + 2 \left(-\frac{c}{2b^{1/r}}\right)^n \frac{(2n)!}{n!} \times \right.$$

$$\times \sum_{k=0}^{\infty} \frac{(1/2+n)_k}{k! (2n+k)!} \Gamma\left(\frac{\alpha+n+k}{r}\right) \frac{\cos^{1-v}[(\alpha+n+k)\pi/r]}{\sin[(\alpha+n+k)\pi/r]} \left[\psi(k+1) + \psi(2n+k+1) - \right.$$

$$- \psi\left(\frac{1}{2} \pm n \pm k\right) - \frac{1}{r} \psi\left(\frac{\alpha+n+k}{r}\right) + \frac{\pi}{r} \operatorname{ctg} \frac{\alpha+n+k}{r} \pi +$$

$$\left. + (1-v) \frac{\pi}{r} \operatorname{tg} \frac{\alpha+n+k}{r} \pi - \ln \frac{2c}{b^{1/r}} \right] \left(\pm \frac{2c}{b^{1/r}}\right)^k \Big\} + V_n^{\pm}(\gamma)$$

$[r > 1; rl - k - \alpha + r \neq v, -v; l, n = 0, 1, 2, \dots]$

$$Y_{n+1/2}^{-}(\gamma) = -\frac{b^{-\alpha/r} \pi}{2r} \left\{ \sum_{k=0}^n \frac{1}{k!} C_k\left(n+\frac{1}{2}\right) \left(-\frac{2c}{b^{1/r}}\right)^k - \right.$$

$$- (-1)^n 2 \sqrt{\pi} \left(\frac{2c}{b^{1/r}}\right)^{n+1/2} \sum_{k=0}^{\infty} \frac{(n+k)!}{k! (2n+k+1)!} \Gamma\left(\frac{\alpha+n+k+1/2}{r}\right) \times$$

$$\times \frac{\cos^{1-v}[(\alpha+n+k+1/2)\pi/r]}{\sin[(\alpha+n+k+1/2)\pi/r]} \left(-\frac{2c}{b^{1/r}}\right)^k \Big\} + V_{n+1/2}^{-}(\gamma)$$

$[r > 1; rl - k - \alpha + r \neq n+1/2, -n-1/2; l, n = 0, 1, 2, \dots]$

$$Y_{\frac{v}{r}}^{\pm}(\gamma) = -\frac{\sqrt{\pi}}{(2c)^{\alpha}} \sum_{k=0}^{\infty} \frac{(-1)^{(1-v)k}}{k!} \Gamma\left[\frac{\alpha+v+rk}{1/2+\alpha+rk}, \alpha-v+rk\right] \times$$

$$\times [\sec(\alpha+rk)\pi \cos v\pi]^{(1\pm 1)/2} \left[\psi(k+1) - r\psi(\alpha+v+rk) - \right.$$

$$- r\psi(\alpha-v+rk) + r\psi\left(\frac{1}{2} \mp \alpha \mp rk\right) + \ln \frac{2^r c^r}{b} \Big] \left(\frac{b}{2^r c^r}\right)^k -$$

$$- \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{b^{(1-2\alpha)/(2r)} \pi^{3/2}}{r \sqrt{2c}} \sum_{k=0}^{\infty} \frac{(1/2+v)_k (1/2-v)_k}{k!} \times$$

$$\times \Gamma\left(\frac{2\alpha-2k-1}{2r}\right) \frac{\cos^{1-v}[(2\alpha-2k-1)\pi/(2r)]}{\sin[(2\alpha-2k-1)\pi/(2r)]} \left(-\frac{b^{1/r}}{2c}\right)^k$$

$[r < 1; rl - k \neq 1/2 - \alpha; k, l = 0, 1, 2, \dots]$

$Y_{\frac{v}{r}}^{\pm}(\gamma)$ при $r=1$ см. в 2.16.57.6.

$$21. \int_0^{\infty} x^{\alpha-1} e^{-bx^r} \pm cx \operatorname{Ei}(bx^r) K_v(cx) dx = Y_{\frac{v}{r}}^{\pm}(0)$$

$$\left[r, \operatorname{Re} b > 0; \operatorname{Re} \alpha > |\operatorname{Re} v|, \begin{cases} \operatorname{Re} \alpha < r + 1/2; |\arg c| < \pi \\ \operatorname{Re} c > 0 \end{cases} \right],$$

$Y_v(0)$ см. в 2.16.1.20].

$$22. \int_0^{\infty} x^{\alpha-1} \left\{ \begin{matrix} \operatorname{si}(bx^r) \\ \operatorname{ci}(bx^r) \end{matrix} \right\} K_v(cx) dx = -\frac{\pi}{2} \hat{V}_v(0, 0)$$

$$[b, r, \operatorname{Re} c > 0; \operatorname{Re} \alpha > |\operatorname{Re} v|; \hat{V}_v(\gamma, c) \text{ см. в 2.13.1.14}].$$

$$23. \int_0^{\infty} x^{\alpha-1} e^{-cx} \left\{ \begin{matrix} \operatorname{si} (bx^r) \\ \operatorname{ci} (bx^r) \end{matrix} \right\} K_{\nu}(cx) dx = Z_{\nu}(1)$$

$$\left[b, r, \operatorname{Re} c > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu|; \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right],$$

$$Z_{\nu}(\gamma) = \frac{b^{-\alpha/r}}{2} \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma}}{k!} [D_k(\nu) + D_k(-\nu)] \left(\frac{2c}{b^{1/r}} \right)^k,$$

$$D_k(\nu) = \left(\frac{2b^{1/r}}{c} \right)^{\nu} \frac{\Gamma(\nu) (1/2 - \nu)_k}{(1 - 2\nu)_k (\nu - \alpha - k)} \Gamma \left(\frac{\alpha + k - \nu}{r} \right) \left\{ \begin{matrix} \sin [(\alpha + k - \nu) \pi / (2r)] \\ \cos [(\alpha + k - \nu) \pi / (2r)] \end{matrix} \right\}$$

$[r > 1; 2\nu \neq \dots, -2, -1, 0, 1, 2, \dots]$

$$Z_n(\gamma) = \frac{b^{-\alpha/r}}{2} \left\{ \sum_{k=0}^{2n-1} \frac{(-1)^{k\gamma} D_k(n)}{k!} \left(\frac{2c}{b^{1/r}} \right)^k - \right. \\ \left. - 2 \left(-\frac{c}{2b^{1/r}} \right)^n \frac{(2n)!}{n!} \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma} (1/2 + n)_k}{k! (2n + k)! (\alpha + n + k)} \Gamma \left(\frac{\alpha + k + n}{r} \right) \times \right. \\ \times \left\{ \begin{matrix} \sin [(\alpha + k + n) \pi / (2r)] \\ \cos [(\alpha + k + n) \pi / (2r)] \end{matrix} \right\} \left[\psi(k+1) + \psi(2n+k+1) - \right. \\ \left. - \psi \left(\frac{1}{2} + (-1)^{\gamma} (n+k) \right) + \frac{1}{\alpha + n + k} - \frac{1}{r} \psi \left(\frac{\alpha + k + n}{r} \right) \mp \right. \\ \left. \mp \frac{\pi}{2r} \operatorname{tg}^{\mp 1} \left(\frac{\alpha + k + n}{2r} \pi \right) - \ln \frac{2c}{b^{1/r}} \right] \left(\frac{2c}{b^{1/r}} \right)^k \Big\} \quad [r > 1].$$

$$Z_{n+1/2}(1) = \frac{b^{-\alpha/r}}{2} \sum_{k=0}^n \frac{1}{k!} D_k \left(n + \frac{1}{2} \right) \left(-\frac{2c}{b^{1/r}} \right)^k - \\ - \frac{(-1)^n \sqrt{\pi}}{b^{\alpha/r}} \left(\frac{2c}{b^{1/r}} \right)^{n+1/2} \sum_{k=0}^{\infty} \frac{(n+k)!}{k! (2n+k+1)! (\alpha + k + n + 1/2)} \times \\ \times \Gamma \left(\frac{\alpha + k + n + 1/2}{r} \right) \left\{ \begin{matrix} \sin [(\alpha + k + n + 1/2) \pi / (2r)] \\ \cos [(\alpha + k + n + 1/2) \pi / (2r)] \end{matrix} \right\} \left(-\frac{2c}{b^{1/r}} \right)^k \quad [r > 1].$$

$$Z_{\nu}(\gamma) = \frac{\sqrt{\pi} b^{\delta} \cos^{1-\gamma} \nu \pi}{2 (2c)^{\alpha+\delta r}} \sum_{k=1}^{\infty} \frac{(-1)^k}{k! (1/2 + \delta), (k + \delta/2)} \times \\ \times \Gamma \left[\begin{matrix} \alpha + \nu + \delta r + 2rk, \alpha - \nu + \delta r + 2rk \\ 1/2 + \alpha + \delta r + 2rk \end{matrix} \right] \cos^{\gamma-1} (\alpha + \delta r + 2rk) \pi \times \\ \times \left(\frac{b}{2^{r+1} c^r} \right)^{2k} + \frac{\sqrt{\pi} b^{\delta}}{(2c)^{\alpha+\delta r}} \left(\frac{\cos \nu \pi}{\cos (\alpha + \delta r) \pi} \right)^{1-\gamma} \Gamma \left[\begin{matrix} \alpha + \nu + \delta r, \alpha - \nu + \delta r \\ 1/2 + \alpha + \delta r \end{matrix} \right] \left\{ \begin{matrix} 1 \\ R \end{matrix} \right\} + \\ + \frac{\delta \pi^{3/2}}{2^{\alpha+1} c^{\alpha}} \Gamma \left[\begin{matrix} \alpha + \nu, \alpha - \nu \\ \alpha + 1/2 \end{matrix} \right] \left(\frac{\cos \nu \pi}{\cos \alpha \pi} \right)^{1-\gamma} + (1 - \gamma) \sqrt{\frac{\pi}{2c}} b^{(1-2\alpha)/(2r)} \times \\ \times \sum_{k=0}^{\infty} \frac{(1/2 + \nu)_k (1/2 - \nu)_k}{k! (1/2 - \alpha + k)} \Gamma \left(\frac{2\alpha - 2k - 1}{2r} \right) \times \\ \times \left\{ \begin{matrix} \sin [(2\alpha - 2k - 1) \pi / (4r)] \\ \cos [(2\alpha - 2k - 1) \pi / (4r)] \end{matrix} \right\} \left(-\frac{b^{1/r}}{2c} \right)^k,$$

$$\left[r < 1; R = c + r\psi(\alpha + \nu) + r\psi(\alpha - \nu) - r\psi \left(\alpha + \frac{1}{2} \right) - (1 - \gamma) \pi r \operatorname{tg} \alpha \pi - \right. \\ \left. - \ln \frac{2^r c^r}{b} \right],$$

$Z_{\nu}(\gamma)$ при $r=1$ см. в 2.16.58.2.

$$24. \int_0^{\infty} x^{\alpha-1} e^{cx} \left\{ \begin{matrix} \operatorname{si} (bx^r) \\ \operatorname{ci} (bx^r) \end{matrix} \right\} K_{\nu}(cx) dx = Z_{\nu}(0)$$

[$b, r, \operatorname{Re} c > 0; |\operatorname{Re} \nu| < \operatorname{Re} \alpha < r + 1/2; Z_{\nu}(\gamma)$ см в 2.16.1 23].

$$25. \int_0^{\infty} x^{\alpha-1} \left\{ \begin{matrix} \operatorname{erf} (bx^r) \\ \operatorname{erfc} (bx^r) \end{matrix} \right\} K_{\nu}(cx) dx = -\frac{\pi}{2} \hat{W}_{\nu}(0, 0)$$

[$r, \operatorname{Re} c > 0; |\arg b| < \pi/4; |\operatorname{Re} \nu| - (1 \pm 1)r/2 < \operatorname{Re} \alpha; \hat{W}_{\nu}(\gamma, s)$ см в 2.13.1.15]

$$26. \int_0^{\infty} x^{\alpha-1} e^{\pm cx} \operatorname{erf} (bx^r) K_{\nu}(cx) dx = \tilde{U}_{\nu}^{\pm}(1)$$

[$r, \operatorname{Re} c > 0; |\arg b| < \pi/4, \left\{ \begin{matrix} |\operatorname{Re} \nu| - r < \operatorname{Re} \alpha < 1/2 \\ \operatorname{Re} \alpha > |\operatorname{Re} \nu| - r \end{matrix} \right\}$],

$$\tilde{U}_{\nu}^{\pm}(\gamma) = (-1)^{\gamma} \frac{b^{-\alpha/r}}{2\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{1}{k!} [E_k(\nu) + E_k(-\nu)] \left(\pm \frac{2c}{b^{1/r}} \right)^k + \gamma V_{\nu}^{\pm},$$

$$E_k(\nu) = \left(\frac{2b^{1/r}}{c} \right)^{\nu} \frac{\Gamma(\nu) (1/2 - \nu)_k}{(1 - 2\nu)_k (\alpha - \nu + k)} \Gamma\left(\frac{r + \alpha - \nu + k}{2r}\right),$$

$$V_{\nu}^{\pm} = \frac{\sqrt{\pi}}{(2c)^{\alpha}} \Gamma\left[\begin{matrix} \alpha + \nu, \alpha - \nu \\ \alpha + 1/2 \end{matrix}\right] (\sec \alpha\pi \cos \nu\pi)^{(1 \pm 1)/2}$$

[$r > 1/2; 2\nu \neq \dots, -2, -1, 0, 1, 2, \dots$].

$$\begin{aligned} \tilde{U}_n^{\pm}(\gamma) = & (-1)^{\gamma} \frac{b^{-\alpha/r}}{2\sqrt{\pi}} \left\{ \sum_{k=0}^{2n-1} \frac{E_k(n)}{k!} \left(\pm \frac{2c}{b^{1/r}} \right)^k + \right. \\ & + 2 \left(-\frac{c}{2b^{1/r}} \right)^n \frac{(2n)!}{n!} \sum_{k=0}^{\infty} \frac{(1/2 + n)_k}{k! (2n + k)! (\alpha + n + k)} \times \\ & \times \Gamma\left(\frac{r + \alpha + n + k}{2r}\right) \left[\psi(k+1) + \psi(2n + k + 1) - \psi\left(\frac{1}{2} \pm n \pm k\right) + \right. \\ & \left. + \frac{1}{\alpha + n + k} - \frac{1}{2r} \psi\left(\frac{r + \alpha + n + k}{2r}\right) - \ln \frac{2c}{b^{1/r}} \right] \left(\pm \frac{2c}{b^{1/r}} \right)^k \left. \right\} + \gamma V_n^{\pm} \quad [r > 1/2]. \end{aligned}$$

$$\begin{aligned} \tilde{U}_{n+1/2}^{-}(\gamma) = & (-1)^{\gamma} \frac{b^{-\alpha/r}}{2\sqrt{\pi}} \sum_{k=0}^n \frac{1}{k!} E_k\left(n + \frac{1}{2}\right) \left(-\frac{2c}{b^{1/r}} \right)^k - \\ & - \frac{(-1)^{n+\gamma}}{b^{\alpha/r}} \left(\frac{2c}{b^{1/r}} \right)^{n+1/2} \sum_{k=0}^{\infty} \frac{(n+k)!}{k! (2n+k+1)! (n+1/2+\alpha+k)} \times \\ & \times \Gamma\left(\frac{r + \alpha + n + k + 1/2}{2r}\right) \left(-\frac{2c}{b^{1/r}} \right)^k + \frac{\gamma \sqrt{\pi}}{(2c)^{\alpha}} \Gamma\left[\begin{matrix} \alpha + n + 1/2, \alpha - n - 1/2 \\ \alpha + 1/2 \end{matrix}\right] \quad [r > 1/2]. \end{aligned}$$

$$\begin{aligned} U_{\nu}^{\pm}(\gamma) = & \frac{(-1)^{\gamma-1} b}{(2c)^{\alpha+r}} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (k+1/2)} \Gamma\left[\begin{matrix} \alpha + \nu + r + 2rk, \alpha - \nu + r + 2rk \\ 1/2 + \alpha + r + 2rk \end{matrix}\right] \times \\ & \times [\sec(\alpha + r + 2rk)\pi \cos \nu\pi]^{(1 \pm 1)/2} \left(\frac{b}{2rc^r} \right)^{2k} - \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{(-1)^{\gamma} b^{(1-2\alpha)/(2r)}}{\sqrt{2c}} \times \\ & \times \sum_{k=0}^{\infty} \frac{(1/2 + \nu)_k (1/2 - \nu)_k}{k! (1/2 - \alpha + k)} \Gamma\left(\frac{r + \alpha - k - 1/2}{2r}\right) \left(-\frac{b^{1/r}}{2c} \right)^k + (1 - \gamma) V_{\nu}^{\pm} \quad [r < 1/2]. \end{aligned}$$

$\tilde{U}_{\nu}^{\pm}(\gamma)$ при $r = 1/2$ см. в 2.16.59.6.

$$27. \int_0^{\infty} x^{\alpha-1} e^{\pm cx} \operatorname{erfc}(bx^r) K_\nu(cx) dx = \tilde{U}_\nu^\pm(0)$$

$$[r, \operatorname{Re} c > 0; |\arg b| < \pi/4; \operatorname{Re} \alpha > |\operatorname{Re} \nu|; \tilde{U}_\nu^\pm(\gamma) \text{ см. в 2.16.1.26}].$$

$$28. \int_0^{\infty} x^{\alpha-1} \left\{ \begin{matrix} S(bx^r) \\ C(bx^r) \end{matrix} \right\} K_\nu(cx) dx = -\frac{\pi}{2} X_\nu(0, 0)$$

$$[b, r, \operatorname{Re} c > 0; |\operatorname{Re} \nu| - (2 \pm 1)r/2 < \operatorname{Re} \alpha, X_\nu(\gamma, s) \text{ см. в 2.13.1.16}].$$

$$29. \int_0^{\infty} x^{\alpha-1} e^{-cx} \left\{ \begin{matrix} S(bx^r) \\ C(bx^r) \end{matrix} \right\} K_\nu(cx) dx = \tilde{U}_\nu(1)$$

$$[b, r, \operatorname{Re} c > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu| - (2 \pm 1)r/2; \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}],$$

$$\tilde{U}_\nu(\gamma) = \frac{b^{-\alpha/r}}{2\sqrt{2\pi}} \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma}}{k!} [F_k(\nu) + F_k(-\nu)] \left(\frac{2c}{b^{1/r}}\right)^k + V_\nu(\gamma),$$

$$F_k(\nu) = \left(\frac{2b^{1/r}}{c}\right)^\nu \frac{\Gamma(\nu)(1/2-\nu)_k}{(1-2\nu)_k(\nu-\alpha-k)} \Gamma\left(\frac{1}{2} + \frac{\alpha+k-\nu}{r}\right) \times \\ \times \begin{cases} \sin[(r+2\alpha+2k-2\nu)\pi/(4r)] \\ \cos[(r+2\alpha+2k-2\nu)\pi/(4r)] \end{cases},$$

$$V_\nu(\gamma) = \frac{\sqrt{\pi}}{2(2c)^\alpha} \left(\frac{\cos \nu\pi}{\cos \alpha\pi}\right)^{1-\gamma} \Gamma\left[\begin{matrix} \alpha+\nu, \alpha-\nu \\ \alpha+1/2 \end{matrix}\right] \\ [r > 1; 2\nu \neq \dots -2, -1, 0, 1, 2, \dots],$$

$$\tilde{U}_n(\gamma) = \frac{b^{-\alpha/r}}{2\sqrt{2\pi}} \left\{ \sum_{k=0}^{2n-1} \frac{(-1)^{k\gamma}}{k!} F_k(n) \left(\frac{2c}{b^{1/r}}\right)^k - \right. \\ \left. - 2 \left(-\frac{c}{2b^{1/r}}\right)^n \frac{(2n)!}{n!} \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma}(1/2+n)_k}{k! (2n+k)! (\alpha+n+k)} \Gamma\left(\frac{1}{2} + \frac{\alpha+k+n}{r}\right) \times \right. \\ \times \begin{cases} \sin[(r+2\alpha+2k+2n)\pi/(4r)] \\ \cos[(r+2\alpha+2k+2n)\pi/(4r)] \end{cases} \left[\psi(k+1) + \psi(2n+k+1) - \right. \\ \left. - \psi\left(\frac{1}{2} + (-1)^\gamma(n+k)\right) + \frac{1}{\alpha+n+k} - \frac{1}{r} \psi\left(\frac{1}{2} + \frac{\alpha+k+n}{r}\right) \right] \mp \\ \mp \frac{\pi}{2r} \operatorname{tg} \mp \frac{r+2\alpha+2k+2n}{4r} \pi - \ln \frac{2c}{b^{1/r}} \left(\frac{2c}{b^{1/r}}\right)^k \left. \right\} + V_n(\gamma) \quad [r > 1].$$

$$\tilde{U}_{n+1/2}(1) = \frac{b^{-\alpha/r}}{2\sqrt{2\pi}} \sum_{k=0}^n \frac{1}{k!} F_k\left(n+\frac{1}{2}\right) \left(-\frac{2c}{b^{1/r}}\right)^k - \\ - \frac{(-1)^n}{\sqrt{2} b^{\alpha/r}} \left(\frac{2c}{b^{1/r}}\right)^{n+1/2} \sum_{k=0}^{\infty} \frac{(n+k)!}{k! (2n+k+1)! (\alpha+n+k+1/2)} \times \\ \times \Gamma\left(\frac{1}{2} + \frac{\alpha+k+n+1/2}{r}\right) \begin{cases} \sin[(1+r+2\alpha+2k+2n)\pi/(4r)] \\ \cos[(1+r+2\alpha+2k+2n)\pi/(4r)] \end{cases} \times \\ \times \left(-\frac{2c}{b^{1/r}}\right)^k + V_{n+1/2}(1) \quad [r > 1].$$

$$\begin{aligned} \tilde{U}_v(\gamma) = & \frac{\sqrt{2} b^{\delta+1/2} \cos^{1-\gamma} v \pi}{(2c)^\alpha + \delta r + r/2} \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (1/2 + \delta)_k (4k + 2\delta + 1)} \times \\ & \times \Gamma \left[\frac{\alpha + v + \delta r + 2rk + r/2}{(1+r)/2 + \delta r + 2rk + \alpha}, \frac{\alpha - v + \delta r + 2rk + r/2}{(1+r)/2 + \delta r + 2rk + \alpha} \right] \times \\ & \times \cos^{\gamma-1} \left(\frac{r}{2} + \delta r + 2rk + \alpha \right) \pi \left(\frac{b}{2^{r+1} c^r} \right)^{2k} + \\ & + (1-\gamma) \frac{b^{(1-2\alpha)/(2r)}}{2 \sqrt{c}} \sum_{k=0}^{\infty} \frac{(1/2 + v)_k (1/2 - v)_k}{k! (1/2 - \alpha + k)} \times \\ & \times \Gamma \left(\frac{r + 2\alpha - 2k - 1}{2r} \right) \left\{ \frac{\sin [(r + 2\alpha - 2k - 1) \pi / (4r)]}{\cos [(r + 2\alpha - 2k - 1) \pi / (4r)]} \right\} \left(-\frac{b^{1/r}}{2c} \right)^k \end{aligned} \quad [r < 1],$$

$\tilde{U}_v(\gamma)$ при $r=1$ см. в 2.16.60.3.

$$30. \int_0^\infty x^{\alpha-1} e^{cx} \left\{ \frac{S(bx^r)}{C(bx^r)} \right\} K_v(cx) dx = \tilde{U}_v(0)$$

$[b, r > 0; |\operatorname{Re} v| - (2 \pm 1) r/2 < \operatorname{Re} \alpha < 1/2; |\arg c| < \pi;$
 $\tilde{U}_v(\gamma)$ см в 2.16.1.29].

$$31. \int_0^\infty x^{\alpha-1} \left\{ \frac{\gamma(\mu, bx^r)}{\Gamma(\mu, bx^r)} \right\} K_v(cx) dx = -\frac{\pi}{2} Y_v(0, 0)$$

$[r, \operatorname{Re} b, \operatorname{Re} c > 0; \operatorname{Re}(\alpha + \mu r) > |\operatorname{Re} v|, \left\{ \begin{array}{l} \operatorname{Re} \mu > 0 \\ \operatorname{Re} \alpha > |\operatorname{Re} v| \end{array} \right\}, Y_v(\gamma) \text{ см. в 2.13.1.17}]$.

$$32. \int_0^\infty x^{\alpha-1} e^{-cx} \left\{ \frac{\gamma(\mu, bx^r)}{\Gamma(\mu, bx^r)} \right\} K_v(cx) dx = \tilde{W}_v(1)$$

$[r, \operatorname{Re} b, \operatorname{Re} c > 0; \operatorname{Re}(\alpha + \mu r) > |\operatorname{Re} v|, \left\{ \begin{array}{l} \operatorname{Re} \mu > 0 \\ \operatorname{Re} \alpha > |\operatorname{Re} v| \end{array} \right\}]$,

$$\tilde{W}_v(\gamma) = \pm \frac{b^{-\alpha/r}}{2} \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma}}{k!} [G_k(v) + G_k(-v)] \left(\frac{2c}{b^{1/r}} \right)^k + \left\{ \begin{array}{l} 1 \\ 0 \end{array} \right\} V_v(\gamma),$$

$$G_k(v) = \left(\frac{2b^{1/r}}{c} \right)^v \frac{\Gamma(v) (1/2 - v)_k}{(1 - 2v)_k (v - \alpha - k)} \Gamma \left(\mu + \frac{\alpha + k - v}{r} \right),$$

$$V_v(\gamma) = \frac{\sqrt{\pi}}{(2c)^{\alpha}} \left(\frac{\cos v\pi}{\cos \alpha\pi} \right)^{1-\gamma} \Gamma \left[\begin{array}{l} \mu, \alpha + v, \alpha - v \\ \alpha + 1/2 \end{array} \right]$$

$[r > 1; 2v \neq \dots, -2, -1, 0, 1, 2, \dots]$.

$$\begin{aligned} \tilde{W}_n(\gamma) = & \pm \frac{b^{-\alpha/r}}{2} \left\{ \sum_{k=0}^{2n-1} \frac{(-1)^{k\gamma} G_k(n)}{k!} \left(\frac{2c}{b^{1/r}} \right)^k - \right. \\ & - 2 \left(-\frac{c}{2b^{1/r}} \right)^n \frac{(2n)!}{n!} \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma} (1/2 + n)_k}{k! (2n + k)! (\alpha + n + k)} \Gamma \left(\mu + \frac{\alpha + k + n}{r} \right) \times \\ & \times \left[\psi(k+1) + \psi(2n + k + 1) - \psi \left(\frac{1}{2} + (-1)^\gamma (n + k) \right) + \frac{1}{\alpha + n + k} - \right. \\ & \left. \left. - \frac{1}{r} \psi \left(\mu + \frac{\alpha + k + n}{r} \right) - \ln \frac{2c}{b^{1/r}} \right] \left(\frac{2c}{b^{1/r}} \right)^k \right\} + \left\{ \begin{array}{l} 1 \\ 0 \end{array} \right\} V_n(\gamma) \end{aligned} \quad [r > 1],$$

$$\begin{aligned} \tilde{W}_{n+1/2}(1) = & \pm \frac{b^{-\alpha/r}}{2} \sum_{k=0}^n \frac{1}{k!} G_k \left(n + \frac{1}{2} \right) \left(-\frac{2c}{b^{1/r}} \right)^k \mp \\ & \mp \frac{(-1)^n \sqrt{\pi}}{b^{\alpha/r}} \left(\frac{2c}{b^{1/r}} \right)^{n+1/2} \sum_{k=0}^{\infty} \frac{(n+k)!}{k! (2n+k+1)! (\alpha+k+n+1/2)} \times \\ & \times \Gamma \left(\mu + \frac{\alpha+k+n+1/2}{r} \right) \left(-\frac{2c}{b^{1/r}} \right)^k + \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} V_{n+1/2}(1) \quad [r > 1]. \end{aligned}$$

$$\begin{aligned} \tilde{W}_v(\gamma) = & \pm \frac{b^\mu \sqrt{\pi} \cos^{1-\gamma} v \pi}{(2c)^{\gamma+\mu r}} \sum_{k=0}^{\infty} \frac{1}{k! (\mu+k)} \times \\ & \times \Gamma \left[\begin{matrix} \alpha+v+\mu r+rk, \alpha-v+\mu r+rk \\ \alpha+1/2+\mu r+rk \end{matrix} \right] \cos^{\gamma-1} (\alpha+\mu r+rk) \pi \times \\ & \times \left(-\frac{b}{2^r c^r} \right)^k \pm (1-\gamma) \frac{b^{(1-2\alpha)/(2r)} \sqrt{\pi}}{\sqrt{2c}} \sum_{k=0}^{\infty} \frac{(1/2+v)_k (1/2-v)_k}{k! (1/2-\alpha+k)} \times \\ & \times \Gamma \left(\mu + \frac{2\alpha-2k-1}{2r} \right) \left(-\frac{b^{1/r}}{2c} \right)^k + \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} V_v(\gamma) \quad [r < 1]. \end{aligned}$$

$\tilde{W}_v(\gamma)$ при $r=1$ см в 2.16.61.3.

$$\begin{aligned} 33. \int_0^\infty x^{\alpha-1} e^{cx} \left\{ \begin{matrix} \gamma(\mu, bx^r) \\ \Gamma(\mu, bx^r) \end{matrix} \right\} K_v(cx) dx = & \tilde{W}_v(0) \\ & [r, \operatorname{Re} b, \operatorname{Re} c > 0; \operatorname{Re}(\alpha + \mu r) > |\operatorname{Re} v|, \left\{ \begin{matrix} \operatorname{Re} \mu > 0; \operatorname{Re} \alpha < 1/2 \\ \operatorname{Re} \alpha > |\operatorname{Re} v| \end{matrix} \right\}, \\ & \tilde{W}_v(\gamma) \text{ см в 2.16.1.32}]. \end{aligned}$$

$$\begin{aligned} 34. \int_0^\infty x^{\alpha-1} e^{\pm b^2 x^{2r}/4} D_\mu(bx^r) K_v(cx) dx = & -\frac{\pi}{2} Z_v(0, 0) \\ & [r, \operatorname{Re} c > 0; \operatorname{Re} \alpha > |\operatorname{Re} v|; |\arg b| < (2 \pm 1)\pi/4; Z_v(\gamma, s) \text{ см в 2.13 1.18}]. \end{aligned}$$

$$\begin{aligned} 35. \int_0^\infty x^{\alpha-1} e^{\pm b^2 x^{2r}/4 - cx} D_\mu(bx^r) K_v(cx) dx = & \tilde{X}_v^\pm(1) \\ & [r, \operatorname{Re} c > 0; \operatorname{Re} \alpha > |\operatorname{Re} v|; |\arg b| < (2 \pm 1)\pi/4]. \end{aligned}$$

$$\begin{aligned} \tilde{X}_v^\pm(\gamma) = & \frac{2^{\alpha/(2r)} \mp (\pm 7 + 1 + \mu)/2}{b^{\alpha/r r}} \left\{ \frac{\Gamma^{-1}(-\mu)}{\sqrt{\pi}} \right\} \times \\ & \times \sum_{k=0}^{\infty} \frac{(-1)^{k\gamma}}{k!} [A_k^\pm(v) + A_k^\pm(-v)] \left(\frac{2^{1/(2r)+1} c}{b^{1/r}} \right)^k + \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} V_v^\pm(\gamma), \end{aligned}$$

$$A_k^\pm(v) = \left(\frac{b^{1/r}}{2^{1/(2r)-1} c} \right)^v \frac{\Gamma(v) (1/2-v)_k}{(1-2v)_k} \Gamma \left(\frac{\alpha+k-v}{r} \right) \Gamma^{\pm 1} \left(\frac{1 \mp 1 - 2\mu}{4} \mp \frac{\alpha+k-v}{2r} \right),$$

$$\begin{aligned} V_v^\pm(\gamma) = & \frac{\sqrt{\pi} b^\mu \cos^{1-\gamma} v \pi}{2^{\alpha+\mu+\mu r} c^{\alpha+\mu r}} \sum_{k=0}^{\infty} \frac{(-\mu)_{2k}}{k!} \times \\ & \times \Gamma \left[\begin{matrix} \alpha+v+\mu r-2rk, \alpha-v+\mu r-2rk \\ 1/2+\alpha+\mu r-2rk \end{matrix} \right] \cos^{\gamma-1} (\alpha+\mu r-2rk) \pi \left(-\frac{2^{2r+1} c^{2r}}{b^2} \right)^k \\ & [r > 1/2, 2v \neq \dots, -2, -1, 0, 1, 2, \dots]. \end{aligned}$$

$$\begin{aligned}\tilde{X}_n^\pm(\gamma) = & \frac{2^{\alpha/(2r) \mp (\pm 7 + 1 + \mu)/2}}{b^{\alpha/r}} \left\{ \frac{\Gamma^{-1}(-\mu)}{\sqrt{\pi}} \right\} \left\{ \sum_{k=0}^{2n-1} \frac{(-1)^{kv} A_k^\pm(n)}{k!} \left(\frac{2^{1/(2r)+1} c}{b^{1/r}} \right)^k + \right. \\ & + 2 \left(-\frac{2^{1/(2r)-1} c}{b^{1/r}} \right)^n \frac{(2n)!}{n!} \sum_{k=0}^{\infty} \frac{(-1)^{kv} (1/2+n)_k}{k! (2n+k)!} \Gamma\left(\frac{\alpha+k+n}{r}\right) \times \\ & \times \Gamma^{\pm 1} \left(\frac{1 \mp 1 - 2\mu}{4} \mp \frac{\alpha+k+n}{2r} \right) \left[\psi(k+1) + \psi(2n+k+1) - \psi\left(\frac{1}{2} + (-1)^v(n+k)\right) - \right. \\ & - \frac{1}{r} \psi\left(\frac{\alpha+k+n}{r}\right) + \frac{1}{2r} \psi\left(\frac{1 \mp 1 - 2\mu}{4} \mp \frac{\alpha+k+n}{2r}\right) - \\ & \left. \left. - \ln \frac{2^{1/(2r)+1} c}{b^{1/r}} \right] \left(\frac{2^{1/(2r)+1} c}{b^{1/r}} \right)^k \right\} + V_n^\pm(\gamma) \quad [r > 1/2].\end{aligned}$$

$$\begin{aligned}\tilde{X}_{n+1/2}^\pm(1) = & \frac{2^{\alpha/(2r) \mp (\mu+1 \pm 7)/2}}{b^{\alpha/r}} \left\{ \frac{\Gamma^{-1}(-\mu)}{\sqrt{\pi}} \right\} \times \\ & \times \left[\sum_{k=0}^n \frac{1}{k!} A_k^\pm\left(n+\frac{1}{2}\right) \left(-\frac{2^{1/(2r)+1} c}{b^{1/r}} \right)^k - (-1)^n 2\sqrt{\pi} \left(\frac{2^{1/(2r)+1} c}{b^{1/r}} \right)^{n+1/2} \times \right. \\ & \times \sum_{k=0}^{\infty} \frac{(n+k)!}{k! (2n+k+1)!} \Gamma\left(\frac{\alpha+k+n+1/2}{r}\right) \times \\ & \left. \times \Gamma^{\pm 1} \left(\frac{1 \mp 1 - 2\mu}{4} \mp \frac{\alpha+k+n+1/2}{2r} \right) \left(-\frac{2^{1/(2r)+1} c}{b^{1/r}} \right)^k \right] + V_{n+1/2}^\pm(\gamma) \quad [r > 1/2].\end{aligned}$$

$$\begin{aligned}\tilde{X}_v^\pm(\gamma) = & \frac{2^{\mp (\mu+1 \pm 1)/2 - \alpha} \sqrt{\pi}}{c^\alpha \cos^{v-1} v\pi} \left\{ \frac{\Gamma^{-1}(-\mu)}{\sqrt{\pi}} \right\} \sum_{k=0}^{\infty} \frac{1}{k!} \Gamma\left[\frac{\alpha+v+rk}{1/2+\alpha+rk}\right] \times \\ & \times \Gamma^{\pm 1} \left(\frac{1 \mp 1 - 2\mu \pm 2k}{4} \right) \cos^{v-1}(\alpha+rk) \pi \left(-\frac{b}{2^{r+1/2} c^r} \right)^k + \\ & + (1-\gamma) \frac{2^{(2\alpha-1)/(4r) \mp (\mu+1 \pm 3)/2} \sqrt{\pi}}{r \sqrt{c} b^{(2\alpha-1)/(2r)}} \left\{ \frac{\Gamma^{-1}(-\mu)}{\sqrt{\pi}} \right\} \sum_{k=0}^{\infty} \frac{(1/2+v)_k (1/2-v)_k}{k!} \times \\ & \times \Gamma\left(\frac{2\alpha-2k-1}{2r}\right) \Gamma^{\pm 1} \left(\frac{1 \mp 1 - 2\mu}{4} \mp \frac{2\alpha-2k-1}{4r} \right) \left(-\frac{b^{1/r}}{2^{1+1/(2r)} c} \right)^k \quad [r < 1/2].\end{aligned}$$

$\tilde{X}_v^\pm(\gamma)$ при $r=1/2$ см в 2.16.62.4

$$36. \int_0^\infty x^{\alpha-1} e^{\pm b^2 x^{2r}/4 + cx} D_\mu(bx^r) K_v(cx) dx = \tilde{X}_v^\pm(0)$$

$$\left[r, \operatorname{Re} c > 0; |\arg b| < (2 \pm 1) \pi/4, \left\{ \begin{array}{l} |\operatorname{Re} v| < \operatorname{Re} \alpha \\ |\operatorname{Re} v| < \operatorname{Re} \alpha < 1/2 - r \operatorname{Re} \mu \end{array} \right\}, \right. \\ \left. \tilde{X}_v^\pm(\gamma) \text{ см в 2.16.1.35} \right].$$

2.16.2. Интегралы от $x^\alpha (z \pm x)^\beta K_v(cx)$.

$$1. \int_0^\infty K_v(cx) dx = \frac{\pi}{2c} \sec \frac{v\pi}{2} \quad [|\operatorname{Re} v| < 1; \operatorname{Re} c > 0]$$

$$2. \int_0^\infty x^{\alpha-1} K_v(cx) dx = 2^{\alpha-2} c^{-\alpha} \Gamma\left(\frac{\alpha+v}{2}\right) \Gamma\left(\frac{\alpha-v}{2}\right) \quad [\operatorname{Re} \alpha > |\operatorname{Re} v|; \operatorname{Re} c > 0]$$

$$\begin{aligned}
3. \int_0^a x^{\alpha-1} (a-x)^{\beta-1} K_\nu(cx) dx &= 2^{\nu-1} a^{\alpha+\beta-\nu-1} c^{-\nu} \Gamma(\nu) B(\beta, \alpha-\nu) \times \\
&\times {}_2F_3\left(\frac{\alpha-\nu}{2}, \frac{1+\alpha-\nu}{2}; 1-\nu, \frac{\alpha+\beta-\nu}{2}, \frac{1+\alpha+\beta-\nu}{2}; \frac{a^2 c^2}{4}\right) + \\
&+ 2^{-\nu-1} a^{\alpha+\beta+\nu-1} c^\nu \Gamma(-\nu) B(\beta, \alpha+\nu) \times \\
&\times {}_2F_3\left(\frac{\alpha+\nu}{2}, \frac{1+\alpha+\nu}{2}; 1+\nu, \frac{\alpha+\beta+\nu}{2}, \frac{1+\alpha+\beta+\nu}{2}; \frac{a^2 c^2}{4}\right) \\
&\quad [a, \operatorname{Re} \beta > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu|]
\end{aligned}$$

$$\begin{aligned}
4. \int_a^\infty x^{\alpha-1} (x-a)^{\beta-1} K_\nu(cx) dx &= 2^{\alpha+\beta-3} c^{1-\alpha-\beta} \Gamma\left(\frac{\alpha+\beta+\nu-1}{2}\right) \Gamma\left(\frac{\alpha+\beta-\nu-1}{2}\right) \times \\
&\times {}_2F_3\left(1-\frac{\beta}{2}, 1-\frac{\beta}{2}; \frac{1}{2}, \frac{3-\alpha-\beta-\nu}{2}, \frac{3+\nu-\alpha-\beta}{2}; \frac{a^2 c^2}{4}\right) - \\
&- 2^{\alpha+\beta-4} a c^{2-\alpha-\beta} (\beta-1) \Gamma\left(\frac{\alpha+\beta+\nu}{2}-1\right) \Gamma\left(\frac{\alpha+\beta-\nu}{2}-1\right) \times \\
&\times {}_2F_3\left(1-\frac{\beta}{2}, \frac{3-\beta}{2}; \frac{3}{2}, 2+\frac{\nu-\alpha-\beta}{2}, 2-\frac{\nu+\alpha+\beta}{2}; \frac{a^2 c^2}{4}\right) + \\
&+ 2^\nu a^{\alpha+\beta-\nu-1} c^{-\nu} \Gamma(\nu) B(\beta, 1-\alpha-\beta+\nu) \times \\
&\times {}_2F_3\left(\frac{\alpha-\nu}{2}, \frac{1+\alpha-\nu}{2}; 1-\nu, \frac{\alpha+\beta-\nu}{2}, \frac{1+\alpha+\beta-\nu}{2}; \frac{a^2 c^2}{4}\right) + \\
&+ 2^{-\nu} a^{\alpha+\beta+\nu-1} c^\nu \Gamma(-\nu) B(\beta, 1-\alpha-\beta-\nu) \times \\
&\times {}_2F_3\left(\frac{\alpha+\nu}{2}, \frac{1+\alpha+\nu}{2}; 1+\nu, \frac{\alpha+\beta+\nu}{2}, \frac{1+\alpha+\beta+\nu}{2}; \frac{a^2 c^2}{4}\right) \\
&\quad [a, \operatorname{Re} c, \operatorname{Re} \beta > 0]
\end{aligned}$$

$$\begin{aligned}
5. \int_0^\infty \frac{1}{x+z} K_\nu(cx) dx &= \\
&= \frac{\pi^2}{2} \operatorname{cosec}^2 \nu \pi [I_\nu(cz) + I_{-\nu}(cz) - e^{-i\pi\nu/2} J_\nu(icz) - e^{i\pi\nu/2} J_{-\nu}(icz)] \\
&\quad [\operatorname{Re} c > 0; |\operatorname{Re} \nu| < 1; |\arg z| < \pi]
\end{aligned}$$

$$\begin{aligned}
6. \int_0^\infty \frac{x^{\alpha-1}}{(x+z)^\rho} K_\nu(cx) dx &= 2^{\alpha-\rho-2} c^{\rho-\alpha} \Gamma\left(\frac{\alpha+\nu-\rho}{2}\right) \Gamma\left(\frac{\alpha-\nu-\rho}{2}\right) \times \\
&\times {}_2F_3\left(\frac{\rho}{2}, \frac{1+\rho}{2}; \frac{1}{2}, 1+\frac{\rho-\nu-\alpha}{2}, 1+\frac{\rho+\nu-\alpha}{2}; \frac{c^2 z^2}{4}\right) - \\
&- 2^{\alpha-\rho-3} c^{\rho-\alpha+1} \rho z \Gamma\left(\frac{\alpha+\nu-\rho-1}{2}\right) \Gamma\left(\frac{\alpha-\nu-\rho-1}{2}\right) \times \\
&\times {}_2F_3\left(\frac{\rho+1}{2}, 1+\frac{\rho}{2}; \frac{3}{2}, \frac{3+\rho-\nu-\alpha}{2}, \frac{3+\rho+\nu-\alpha}{2}; \frac{c^2 z^2}{4}\right) + \\
&+ 2^{\nu-1} c^{-\nu} z^{\alpha-\rho-\nu} \Gamma(\nu) B(\alpha-\nu, \rho-\alpha+\nu) \times \\
&\times {}_2F_3\left(\frac{\alpha-\nu}{2}, \frac{1+\alpha-\nu}{2}; 1-\nu, 1+\frac{\alpha-\nu-\rho}{2}, \frac{1+\alpha-\nu-\rho}{2}; \frac{c^2 z^2}{4}\right) + \\
&+ 2^{-\nu-1} c^\nu z^{\alpha+\nu-\rho} \Gamma(-\nu) B(\alpha+\nu, \rho-\alpha-\nu) \times \\
&\times {}_2F_3\left(\frac{\alpha+\nu}{2}, \frac{\alpha+\nu+1}{2}; 1+\nu, 1+\frac{\alpha+\nu-\rho}{2}, \frac{1+\alpha+\nu-\rho}{2}; \frac{c^2 z^2}{4}\right) \\
&\quad [\operatorname{Re} c > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu|; |\arg z| < \pi]
\end{aligned}$$

$$\begin{aligned}
7. \int_0^{\infty} \frac{x^{\alpha-1}}{x-y} K_{\nu}(cx) dx = & \\
= 2^{\alpha-3} c^{1-\alpha} \Gamma\left(\frac{\alpha+\nu-1}{2}\right) \Gamma\left(\frac{\alpha-\nu-1}{2}\right) {}_1F_2\left(1; \frac{3-\nu-\alpha}{2}, \frac{3+\nu-\alpha}{2}; \frac{c^2 y^2}{4}\right) + & \\
+ 2^{\alpha-4} c^{2-\alpha} y \Gamma\left(\frac{\alpha+\nu}{2}-1\right) \Gamma\left(\frac{\alpha-\nu}{2}-1\right) {}_1F_2\left(1; 2-\frac{\nu+\alpha}{2}, 2+\frac{\nu-\alpha}{2}; \frac{c^2 y^2}{4}\right) + & \\
+ \frac{\pi^2 y^{\alpha-1}}{2 \sin \nu \pi} [\operatorname{ctg}(\alpha+\nu) \pi I_{\nu}(cy) - \operatorname{ctg}(\alpha-\nu) \pi I_{-\nu}(cy)] & \\
[y, \operatorname{Re} c > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu|] &
\end{aligned}$$

2.16.3. Интегралы от $x^{\alpha} (z^2 \pm x^2)^{\beta} K_{\nu}(cx)$

$$1. \int_0^a \frac{1}{\sqrt{a^2-x^2}} K_{\nu}(cx) dx = \frac{\pi}{2} \sec \frac{\nu \pi}{2} \left[I_{\nu/2}\left(\frac{ac}{2}\right) + I_{-\nu/2}\left(\frac{ac}{2}\right) \right] K_{\nu/2}\left(\frac{ac}{2}\right)$$

[$a > 0; |\operatorname{Re} \nu| < 1$].

$$\begin{aligned}
2. \int_0^a x^{\alpha-1} (a^2-x^2)^{\beta-1} K_{\nu}(cx) dx = & \\
= 2^{\nu-2} a^{\alpha+2\beta-\nu-2} c^{-\nu} \Gamma(\nu) B\left(\beta, \frac{\alpha-\nu}{2}\right) {}_1F_2\left(\frac{\alpha-\nu}{2}; 1-\nu, \beta+\frac{\alpha-\nu}{2}; \frac{a^2 c^2}{4}\right) + & \\
+ 2^{-\nu-2} a^{\alpha+2\beta+\nu-2} c^{\nu} \Gamma(-\nu) B\left(\beta, \frac{\alpha+\nu}{2}\right) {}_1F_2\left(\frac{\alpha+\nu}{2}; 1+\nu, \beta+\frac{\alpha+\nu}{2}; \frac{a^2 c^2}{4}\right) & \\
[a, \operatorname{Re} \beta > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu|] &
\end{aligned}$$

$$\begin{aligned}
3. \int_0^a x^{1-\nu} (a^2-x^2)^{\beta-1} K_{\nu}(cx) dx = \frac{2^{\beta-2} \pi a^{\beta-\nu} c^{-\beta}}{\sin \nu \pi} \Gamma(\beta) I_{\beta-\nu}(ac) + & \\
+ \frac{2^{-\nu-2} a^{2\beta} c^{\nu}}{\beta} \Gamma(-\nu) {}_1F_2\left(1; \nu+1, \beta+1; \frac{a^2 c^2}{4}\right) & \\
[a, \operatorname{Re} \beta > 0; \operatorname{Re} \nu < 1] &
\end{aligned}$$

$$4. \int_0^a x^{\nu} (a^2-x^2)^{\nu-1/2} K_{\nu}(cx) dx = 2^{\nu-1} \sqrt{\pi} a^{2\nu} c^{-\nu} \Gamma\left(\nu+\frac{1}{2}\right) I_{\nu}\left(\frac{ac}{2}\right) K_{\nu}\left(\frac{ac}{2}\right)$$

[$a > 0; \operatorname{Re} \nu > -1/2$].

$$5. \int_a^{\infty} \sqrt{x^2-a^2} K_{\nu}(cx) dx = \frac{a}{2c} K_{(1+\nu)/2}\left(\frac{ac}{2}\right) K_{\nu/2}\left(\frac{ac}{2}\right)$$

[$a, \operatorname{Re} c > 0$].

$$6. \int_a^{\infty} \frac{1}{\sqrt{x^2-a^2}} K_{\nu}(cx) dx = \frac{1}{2} K_{\nu/2}^2\left(\frac{ac}{2}\right)$$

[$a, \operatorname{Re} c > 0$].

$$\begin{aligned}
7. \int_a^{\infty} x^{\alpha-1} (x^2-a^2)^{\beta-1} K_{\nu}(cx) dx = 2^{\nu-2} a^{\alpha+2\beta-\nu-2} c^{-\nu} \times & \\
\times \Gamma(\nu) B\left(\beta, 1-\beta+\frac{\nu-\alpha}{2}\right) {}_1F_2\left(\frac{\alpha-\nu}{2}; 1-\nu, \beta+\frac{\alpha-\nu}{2}; \frac{a^2 c^2}{4}\right) + & \\
+ 2^{-\nu-2} a^{\alpha+2\beta+\nu-2} c^{\nu} \Gamma(-\nu) B\left(\beta, 1-\beta-\frac{\alpha+\nu}{2}\right) \times & \\
\times {}_1F_2\left(\frac{\alpha+\nu}{2}; 1+\nu, \beta+\frac{\alpha+\nu}{2}; \frac{a^2 c^2}{4}\right) + 2^{\alpha+2\beta-4} c^{2-\alpha-2\beta} \Gamma\left(\beta+\frac{\alpha+\nu}{2}-1\right) \times & \\
\times \Gamma\left(\beta+\frac{\alpha-\nu}{2}-1\right) {}_1F_2\left(1-\beta; 2-\beta-\frac{\alpha+\nu}{2}, 2-\beta-\frac{\alpha-\nu}{2}; \frac{a^2 c^2}{4}\right) & \\
[a, \operatorname{Re} c, \operatorname{Re} \beta > 0] &
\end{aligned}$$

$$8. \int_a^\infty x^{1 \pm \nu} (x^2 - a^2)^{\beta-1} K_\nu(cx) dx = 2^{\beta-1} a^\beta \pm^\nu c^{-\beta} \Gamma(\beta) K_{\nu \pm \beta}(ac)$$

[a, Re c, Re β > 0].

$$9. \int_a^\infty x^\nu (x^2 - a^2)^{\nu-1/2} K_\nu(cx) dx = \frac{2^{\nu-1}}{\sqrt{\pi}} a^{2\nu} c^{-\nu} \Gamma\left(\nu + \frac{1}{2}\right) K_\nu^2\left(\frac{ac}{2}\right)$$

[a, Re c > 0; Re ν > -1/2].

$$10. \int_a^\infty \frac{x}{\sqrt{x^2 - a^2}} K_0(cx) dx = \frac{\pi}{2c} e^{-ac}$$

[a, Re c > 0].

$$11. \int_0^\infty \frac{1}{x^2 + z^2} K_\nu(cx) dx =$$

$$= \frac{\pi^2}{4z} \sec \frac{\nu\pi}{2} \left\{ \operatorname{tg} \frac{\nu\pi}{2} [J_\nu(cz) - Y_\nu(cz)] - E_\nu(cz) - Y_\nu(cz) \right\}$$

[Re c, Re z > 0; |Re ν| < 1].

$$12. \int_0^\infty \frac{1}{\sqrt{x^2 + z^2}} K_\nu(cx) dx = \frac{\pi^2}{8} \sec \frac{\nu\pi}{2} \left[J_{\nu/2}^2\left(\frac{cz}{2}\right) + Y_{\nu/2}^2\left(\frac{cz}{2}\right) \right]$$

[Re c, Re z > 0; |Re ν| < 1].

$$13. \int_0^\infty \frac{x^{\alpha-1}}{(x^2 + z^2)^\rho} K_\nu(cx) dx = 2^{\nu-2} c^{-\nu} z^{\alpha-2\rho-\nu} \Gamma(\nu) B\left(\rho + \frac{\nu-\alpha}{2}, \frac{\alpha-\nu}{2}\right) \times \\ \times {}_1F_2\left(\frac{\alpha-\nu}{2}; 1-\nu, 1-\rho + \frac{\alpha-\nu}{2}; -\frac{c^2 z^2}{4}\right) + \\ + 2^{-\nu-2} c^\nu z^{\alpha+\nu-2\rho} \Gamma(-\nu) B\left(\rho - \frac{\alpha+\nu}{2}, \frac{\alpha+\nu}{2}\right) \times \\ \times {}_1F_2\left(\frac{\alpha+\nu}{2}; 1+\nu, 1-\rho + \frac{\alpha+\nu}{2}; -\frac{c^2 z^2}{4}\right) + 2^{\alpha-2\rho-2} c^{2\rho-\alpha} \Gamma\left(\frac{\alpha+\nu}{2} - \rho\right) \times \\ \times \Gamma\left(\frac{\alpha-\nu}{2} - \rho\right) {}_1F_2\left(\rho; 1+\rho - \frac{\alpha+\nu}{2}, 1+\rho - \frac{\alpha-\nu}{2}; -\frac{c^2 z^2}{4}\right)$$

[Re α, Re z > 0; |Re ν| < Re α].

$$14. \int_0^\infty \frac{x^{\nu+1}}{(x^2 + z^2)^\rho} K_\nu(cx) dx = (2z)^\nu \left(\frac{z}{c}\right)^{1-\rho} \Gamma(\nu+1) S_{-\nu-\rho, 1+\nu-\rho}(cz)$$

[Re c, Re z > 0; Re ν > -1].

$$15. \int_0^\infty \frac{x^{\alpha-1}}{x^2 + z^2} K_\nu(cx) dx = (2z)^{\alpha-2} \Gamma\left(\frac{\alpha-\nu}{2}\right) \Gamma\left(\frac{\alpha+\nu}{2}\right) S_{1-\alpha, \nu}(cz)$$

[Re c, Re z > 0; Re α > |Re ν|]

$$16. \int_0^\infty \frac{x^{\pm \nu}}{x^2 + z^2} K_\nu(cx) dx = \frac{\pi^2 z^{\pm \nu - 1}}{4 \cos \nu\pi} [H_{\mp \nu}(cz) - Y_{\mp \nu}(cz)]$$

[Re c, Re z > 0; ± Re ν > -1/2].

$$17. \int_0^{\infty} \frac{x}{\sqrt{x^2+z^2}} K_0(cx) dx = \frac{2}{c} [\sin cz \operatorname{ci}(cz) - \cos cz \operatorname{si}(cz)]$$

[Re c, Re z > 0].

$$18. \int_0^{\infty} \frac{x^{\alpha-1}}{x^2-y^2} K_\nu(cx) dx =$$

$$= 2^{\alpha-1} c^{2-\alpha} \Gamma\left(\frac{\alpha+\nu}{2}-1\right) \Gamma\left(\frac{\alpha-\nu}{2}-1\right) {}_1F_2\left(1; 2-\frac{\nu+\alpha}{2}, 2+\frac{\nu-\alpha}{2}; \frac{c^2 y^2}{4}\right) +$$

$$+ \frac{\pi^2 y^{\alpha-2}}{4 \sin \nu \pi} \left[\operatorname{ctg} \frac{\alpha+\nu}{2} \pi I_\nu(cy) - \operatorname{ctg} \frac{\alpha-\nu}{2} \pi I_{-\nu}(cy) \right]$$

[y, Re c > 0; Re α > |Re ν|].

$$19. \int_0^{\infty} \frac{x^{\pm \nu}}{x^2-y^2} K_\nu(cx) dx = \frac{\pi^2 y^{\pm \nu-1}}{4 \cos \nu \pi} [L_{\mp \nu}(cy) - I_{\pm \nu}(cy)]$$

[y, Re c > 0; ± Re ν > -1/2]

2.16.4. Интегралы от $A(x) K_\nu(cx)$.

$$1. \int_a^b \frac{x}{\sqrt{(b^2-x^2)(x^2-a^2)}} K_0(cx) dx = \frac{\pi}{2} I_0\left(\frac{bc-|a|c}{2}\right) K_0\left(\frac{bc+|a|c}{2}\right)$$

[|a| < b].

$$2. \int_0^{\infty} x^{\alpha-1} \frac{(\sqrt{x^2+z^2} \pm z)^\mu}{(x^2+z^2)^r} K_\nu(cx) dx =$$

$$= 2^{\alpha+\mu-3} (\pm \mu z)^{1-2r} c^{1-\mu-\alpha} \Gamma\left(\frac{\alpha+\mu+\nu-1}{2}\right) \Gamma\left(\frac{\alpha+\mu-\nu-1}{2}\right) \times$$

$$\times {}_2F_3\left(\frac{1+\mu}{2}, \frac{1-\mu}{2}; \frac{3}{2}-2r, \frac{3-\nu-\alpha-\mu}{2}, \frac{3+\nu-\alpha-\mu}{2}; -\frac{c^2 z^2}{4}\right) +$$

$$+ 2^{\alpha+\mu-2r-2} (\pm \mu z)^{2r} c^{r-\alpha-\mu} \Gamma\left(\frac{\alpha+\mu+\nu}{2}-2r\right) \Gamma\left(\frac{\alpha+\mu-\nu}{2}-2r\right) \times$$

$$\times {}_2F_3\left(2r+\frac{\mu}{2}, 2r-\frac{\mu}{2}; \frac{1}{2}+2r, 1+2r-\frac{\alpha+\mu+\nu}{2}, 1+2r-\frac{\alpha+\mu-\nu}{2}; -\frac{c^2 z^2}{4}\right) +$$

$$+ 2^{\alpha+\mu-2} (\mp \mu)^{1-2r} c^{-\nu} z^{\alpha+\mu-\nu-2r} \Gamma\left[\nu, \frac{(\alpha+\mu \mp \mu-\nu)/2, \nu-\alpha-\mu+2r}{1-(\mu \pm \mu-\nu+\alpha)/2}\right] \times$$

$$\times {}_2F_3\left(\mu+\frac{\alpha-\nu}{2}, \frac{\alpha-\nu}{2}; 1-\nu, 1-2r+\frac{\alpha+\mu-\nu}{2}, \frac{1+\alpha+\mu-\nu}{2}; -\frac{c^2 z^2}{4}\right) +$$

$$+ 2^{\alpha+\mu-2} (\mp \mu)^{1-2r} c^{\nu} z^{\alpha+\mu+\nu-2r} \Gamma\left[-\nu, \frac{(\alpha+\nu+\mu \mp \mu)/2, 2r-\alpha-\mu-\nu}{1-(\mu \pm \mu+\nu+\alpha)/2}\right] \times$$

$$\times {}_2F_3\left(\mu+\frac{\alpha+\nu}{2}, \frac{\alpha+\nu}{2}; 1+\nu, 1-2r+\frac{\alpha+\mu+\nu}{2}, \frac{1+\alpha+\mu+\nu}{2}; -\frac{c^2 z^2}{4}\right)$$

[r = 0 или 1/2; Re z, Re c > 0; Re(α + μ ∓ μ) > |Re ν|].

$$3. \int_0^{\infty} x^{-\mu} \frac{(\sqrt{x^2+z^2} \pm z)^\mu}{\sqrt{x^2+z^2}} K_\nu(cx) dx =$$

$$= \frac{1}{2cz} \Gamma\left(\frac{1+\nu \mp \mu}{2}\right) \Gamma\left(\frac{1-\nu \mp \mu}{2}\right) W_{\pm \mu/2, \nu/2}(icz) W'_{\pm \mu/2, \nu/2}(-icz)$$

[Re z, Re c > 0; |Re ν| ± Re μ < 1].

$$\begin{aligned}
4. \int_0^\infty x^{\alpha-1} \frac{(V x^2 + z^2 \pm x)^\mu}{(x^2 + z^2)^r} K_\nu(cx) dx = \\
= 2^{\alpha \pm 2\mu - 2r - 2} c^{2r - \alpha \mp \mu} z^{\mu \mp \mu} \Gamma\left(\frac{\alpha + \nu \pm \mu}{2} - r\right) \Gamma\left(\frac{\alpha - \nu \pm \mu}{2} - r\right) \times \\
\times {}_2F_3\left(2r \mp \frac{\mu}{2}, \frac{1 \mp \mu}{2}; 1 \mp \mu, 1 + r - \frac{\alpha + \nu \pm \mu}{2}, 1 + r - \frac{\alpha - \nu \pm \mu}{2}; -\frac{c^2 z^2}{4}\right) + \\
+ 2^{2r + 2\nu - \alpha - 2} (\mp \mu)^{1-2r} c^{-\nu} z^{\alpha + \mu - 2r - \nu} \Gamma\left[\frac{\nu, r + (\nu - \alpha \mp \mu)/2, \alpha - \nu}{1 - r + (\alpha - \nu \mp \mu)/2}\right] \times \\
\times {}_2F_3\left(\frac{\alpha - \nu}{2}, \frac{1 + \alpha - \nu}{2}; 1 - r + \frac{\alpha - \nu \mp \mu}{2}, 1 - \nu, 1 - r + \frac{\alpha - \nu \pm \mu}{2}; -\frac{c^2 z^2}{4}\right) + \\
+ 2^{2r - 2\nu - \alpha - 2} (\mp \mu)^{1-2r} c^{\nu} z^{\alpha + \mu + \nu - 2r} \Gamma\left[\frac{-\nu, r - (\alpha + \nu \pm \mu)/2, \alpha + \nu}{1 - r + (\alpha + \nu \mp \mu)/2}\right] \times \\
\times {}_2F_3\left(\frac{\alpha + \nu}{2}, \frac{\alpha + \nu + 1}{2}; 1 + \nu, 1 - r + \frac{\alpha + \nu - \mu}{2}, 1 - r + \frac{\alpha + \mu + \nu}{2}; -\frac{c^2 z^2}{4}\right) \\
[r = 0 \text{ или } 1/2, \operatorname{Re} z, \operatorname{Re} c > 0; |\operatorname{Re} \nu| < \operatorname{Re} \alpha].
\end{aligned}$$

$$\begin{aligned}
5. \int_0^\infty \frac{(V x^2 + z^2 \pm x)^\mu}{V x^2 + z^2} K_\nu(cx) dx = \\
= \frac{\pi^{2\mu}}{4 \sin \nu \pi} \left[J_{(\nu \mp \mu)/2}\left(\frac{cz}{2}\right) Y_{(\mp \mu - \nu)/2}\left(\frac{cz}{2}\right) - J_{(\mp \mu - \nu)/2}\left(\frac{cz}{2}\right) Y_{(\nu \mp \mu)/2}\left(\frac{cz}{2}\right) \right] \\
[\operatorname{Re} z, \operatorname{Re} c > 0; |\operatorname{Re} \nu| < 1].
\end{aligned}$$

$$\begin{aligned}
6. \int_a^\infty \frac{(x + V x^2 - a^2)^\mu + (x - V x^2 - a^2)^\mu}{V x^2 - a^2} K_\nu(cx) dx = \\
= a^\mu K_{(\nu + \mu)/2}\left(\frac{ac}{2}\right) K_{(\nu - \mu)/2}\left(\frac{ac}{2}\right) \quad [a, \operatorname{Re} c > 0].
\end{aligned}$$

$$\begin{aligned}
7. \int_a^\infty x^{-\mu} \frac{(a + i V x^2 - a^2)^\mu + (a - i V x^2 - a^2)^\mu}{V x^2 - a^2} K_\nu(cx) dx = \\
= \frac{\pi}{ac} W_{\mu/2, \nu/2}(ac) W_{-\mu/2, \nu/2}(ac) \quad [a, \operatorname{Re} c > 0].
\end{aligned}$$

2.16.5. Интегралы от $A(x) K_\nu(\varphi(x))$.

$$\begin{aligned}
1. \int_0^1 x^{\alpha-1} K_\nu\left(\frac{c}{x} - cx\right) dx = \\
= \frac{\pi^2}{4 \sin \nu \pi} [J_{(\alpha + \nu)/2}(c) Y_{(\alpha - \nu)/2}(c) - J_{(\alpha - \nu)/2}(c) Y_{(\alpha + \nu)/2}(c)] \\
[\operatorname{Re} c > 0; |\operatorname{Re} \nu| < 1].
\end{aligned}$$

$$\begin{aligned}
2. \int_1^\infty x^{\alpha-1} K_\nu\left(cx - \frac{c}{x}\right) dx = \\
= \frac{\pi^2}{4 \sin \nu \pi} [J_{(\nu - \alpha)/2}(c) Y_{-(\nu + \alpha)/2}(c) - J_{-(\nu + \alpha)/2}(c) Y_{(\nu - \alpha)/2}(c)] \\
[\operatorname{Re} c > 0; |\operatorname{Re} \nu| < 1].
\end{aligned}$$

$$3. \int_0^\infty x^{\alpha-1} K_0\left(c \left|x - \frac{i}{x}\right|\right) dx = \frac{\pi^2}{4} [J_{\alpha/2}^2(c) + Y_{\alpha/2}^2(c)] \quad [\operatorname{Re} c > 0].$$

$$4. \int_0^{\infty} x^{\alpha-1} K_{\nu} \left(cx + \frac{c}{x} \right) dx = K_{(\nu+\alpha)/2}(c) K_{(\nu-\alpha)/2}(c) \quad [\operatorname{Re} c > 0].$$

$$5. \int_0^{\infty} x^{\alpha-1} \left(\frac{a+bx}{b+ax} \right)^{\nu/2} K_{\nu} (\sqrt{a^2+b^2+ab(x+x^{-1})}) dx = 2K_{\alpha+\nu/2}(a) K_{\alpha-\nu/2}(b) \\ [a, b > 0].$$

2.16.6. Интегралы от $x^{\alpha} e^{-px} K_{\nu}(cx)$.

$$1. \int_0^{\infty} e^{-px} K_{\nu}(cx) dx = \frac{\pi c^{-\nu}}{2 \sin \nu \pi \sqrt{p^2 - c^2}} [(p + \sqrt{p^2 - c^2})^{\nu} - (p - \sqrt{p^2 - c^2})^{\nu}] = \\ = \frac{\pi \operatorname{cosec} \nu \pi}{\sqrt{c^2 - p^2}} \sin \left(\nu \arccos \frac{p}{c} \right) \quad [\operatorname{Re}(c+p) > 0; |\operatorname{Re} \nu| < 1, \nu \neq 0].$$

$$2. \int_0^{\infty} e^{-px} K_0(cx) dx = \frac{1}{\sqrt{p^2 - c^2}} \ln \frac{p + \sqrt{p^2 - c^2}}{c} = \frac{\arccos(p/c)}{\sqrt{c^2 - p^2}} \\ [\operatorname{Re}(c+p) > 0].$$

$$3. \int_0^{\infty} x^{\alpha-1} e^{-px} K_{\nu}(cx) dx = I_{\nu}^{\alpha} \quad [\operatorname{Re}(c+p) > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu|;$$

$$I_{\nu}^{\alpha} = \frac{p^{\nu-\alpha} c^{-\nu}}{2^{\alpha}} \sqrt{\pi} \Gamma \left[\frac{\alpha-\nu}{2}, \frac{\alpha+\nu}{2} \right] {}_2F_1 \left(\frac{\alpha-\nu}{2}, \frac{\alpha-\nu+1}{2}; \alpha + \frac{1}{2}; 1 - \frac{c^2}{p^2} \right) = \\ = \frac{(2c)^{\nu} \sqrt{\pi}}{(p+c)^{\alpha+\nu}} \Gamma \left[\frac{\alpha-\nu}{2}, \frac{\alpha+\nu}{2} \right] {}_2F_1 \left(\alpha+\nu, \nu + \frac{1}{2}; \alpha + \frac{1}{2}; \frac{p-c}{p+c} \right) = \\ = e^{-\pi i \nu} \Gamma(\alpha-\nu) (p^2 - c^2)^{-\alpha/2} Q_{\alpha-1}^{\nu} \left(\frac{p}{\sqrt{p^2 - c^2}} \right) =$$

$$= \sqrt{\frac{\pi}{2c}} \Gamma(\alpha-\nu) \Gamma(\alpha+\nu) (c^2 - p^2)^{(1-2\alpha)/4} p_{\nu-1/2}^{1/2-\alpha} \left(\frac{p}{c} \right);$$

$$I_0^{\alpha} = \frac{1}{p^2 - c^2} \left(\frac{p}{\sqrt{p^2 - c^2}} \ln \frac{p + \sqrt{p^2 - c^2}}{c} - 1 \right);$$

$$I_1^{\alpha} = \frac{p}{c(p^2 - c^2)} - \frac{c}{(p^2 - c^2)^{3/2}} \ln \frac{p + \sqrt{p^2 - c^2}}{c};$$

$$I_0^{1/2} = 2 \sqrt{\frac{\pi}{p+c}} K(k_1) \quad [0 < c < p],$$

$$I_0^{1/2} = \sqrt{\frac{2\pi}{c}} K(k_2) \quad [0 < p < c];$$

$$I_0^{3/2} = \frac{\sqrt{\pi(p+c)}}{p^2 - c^2} [K(k_2) - E(k_1)] \quad [0 < c < p],$$

$$I_0^{3/2} = \frac{\sqrt{\pi}}{(c^2 - p^2) \sqrt{2c}} [2cE(k_2) - (p+c)K(k_2)] \quad [0 < p < c];$$

$$I_0^{5/2} = \frac{\sqrt{\pi(p+c)}}{2(p^2 - c^2)^2} [(3p+c)K(k_2) - 4pE(k_1)] \quad [0 < c < p];$$

$$I_0^{5/2} = \frac{\sqrt{\pi}}{2(c^2 - p^2)^2 \sqrt{2c}} [(p+c)(3p+c)K(k_2) - 8cpE(k_2)] \\ [0 < p < c];$$

$$I_0^{7/2} = \frac{\sqrt{\pi(p+c)}}{4(p^2-c^2)^3} [(15p^2+8cp+9c^2) K(k_1) - (23p^2+9c^2) E(k_1)]$$

[0 < c < p].

$$I_0^{7/2} = \frac{\sqrt{\pi}}{4(c^2-p^2)^3 \sqrt{2c}} [2c(23p^2+9c^2) E(k_2) - (p+c)(15p^2+8cp+9c^2) K(k_2)]$$

[0 < p < c];

$$I_1^{3/2} = \frac{\sqrt{\pi(p+c)}}{c(p^2-c^2)} [pE(k_1) - cK(k_1)]$$

[0 < c < p].

$$I_1^{3/2} = \frac{\sqrt{\pi}}{(c^2-p^2) \sqrt{2c}} [(p+c) K(k_2) - 2pE(k_2)]$$

[0 < p < c];

$$I_1^{5/2} = \frac{\sqrt{\pi(p+c)}}{2c(p^2-c^2)^2} [(p^2+3c^2) E(k_1) - c(p+3c) K(k_1)]$$

[0 < c < p].

$$I_1^{5/2} = \frac{\sqrt{\pi}}{2(c^2-p^2)^2 \sqrt{2c}} [2(p^2+3c^2) E(k_2) - (p+c)(p+3c) K(k_2)]$$

[0 < p < c];

$$I_1^{7/2} = \frac{\sqrt{\pi(p+c)}}{4c(p^2-c^2)^3} [p(3p^2+29c^2) E(k_1) - c(3p^2+24cp+5c^2) K(k_1)]$$

[0 < c < p].

$$I_1^{7/2} = \frac{\sqrt{\pi}}{4(c^2-p^2)^3 \sqrt{2c}} [(p+c)(3p^2+24cp+5c^2) K(k_2) - 2p(3p^2+29c^2) E(k_2)]$$

[0 < p < c];

$$I_2^{5/2} = \frac{\sqrt{\pi(p+c)}}{2c^2(p^2-c^2)^2} [c(5c^2+3cp-4p^2) K(k_1) - 4p(2c^2-p^2) E(k_1)]$$

[0 < c < p].

$$I_2^{5/2} = \frac{\sqrt{\pi}}{2c(c^2-p^2)^2 \sqrt{2c}} [(p+c)(5c^2+3cp-4p^2) K(k_2) - 8p(2c^2-p^2) E(k_2)]$$

[0 < p < c].

$$4. \int_0^\infty x^{\alpha-1} e^{-cx} K_\nu(cx) dx = \sqrt{\pi} (2c)^{-\alpha} \Gamma \left[\begin{matrix} \alpha+\nu, \alpha-\nu \\ \alpha+1/2 \end{matrix} \right]$$

[Re c > 0; Re α > |Re ν|].

$$5. \int_0^\infty x^{\alpha-1} e^{cx} K_\nu(cx) dx = \frac{\cos \nu\pi}{\sqrt{\pi}} (2c)^{-\alpha} \Gamma [\alpha+\nu, \alpha-\nu, 1/2-\alpha]$$

[Re c > 0; |Re ν| < Re α < 1/2]

$$6. \int_{-\infty}^\infty |x|^\nu e^{-px} K_\nu(c|x|) dx = \frac{\sqrt{\pi} \Gamma(\nu+1/2) (2c)^\nu}{(c^2-p^2)^{\nu+1/2}}$$

[Re p < |Re c|; Re ν < 1/2]

2.16.7. Интегралы от $x^\alpha (z \pm x)^\beta e^{px} K_\nu(cx)$.

$$1. \int_0^a x^{\alpha-1} (a-x)^{\beta-1} e^{\pm cx} K_\nu(cx) dx = 2^{\nu-1} a^{\alpha+\beta-\nu-1} c^{-\nu} \Gamma \left[\begin{matrix} \nu, \beta, \alpha-\nu \\ \alpha+\beta-\nu \end{matrix} \right] \times$$

$$\times {}_2F_2 \left(\begin{matrix} \frac{1}{2}-\nu, \alpha-\nu, 1-2\nu, \alpha+\beta-\nu; \pm 2ac \end{matrix} \right) + 2^{\nu-1} a^{\alpha+\beta+\nu-1} c^\nu \times$$

$$\times \Gamma \left[\begin{matrix} -\nu, \beta, \alpha+\nu \\ \alpha+\beta+\nu \end{matrix} \right] {}_2F_2 \left(\begin{matrix} \frac{1}{2}+\nu, \alpha+\nu, 1+2\nu, \alpha+\beta+\nu; \pm 2ac \end{matrix} \right)$$

[a, Re β > 0; Re α > |Re ν|]

$$\begin{aligned}
2. \int_a^\infty x^{\alpha-1} (x-a)^{\beta-1} e^{\mp cx} K_\nu(cx) dx &= 2^{\nu-1} a^{\alpha+\beta-\nu-1} c^{-\nu} \Gamma \left[\begin{matrix} \nu, \beta, 1-\beta-\alpha+\nu \\ 1-\alpha+\nu \end{matrix} \right] \times \\
&\times {}_2F_2 \left(\alpha-\nu, \frac{1}{2}-\nu; 1-2\nu, \alpha+\beta-\nu; \mp 2ac \right) + 2^{\nu-1} a^{\alpha+\beta+\nu-1} c^\nu \times \\
&\times \Gamma \left[\begin{matrix} -\nu, \beta, 1-\beta-\alpha-\nu \\ 1-\alpha-\nu \end{matrix} \right] {}_2F_2 \left(\alpha+\nu, \frac{1}{2}+\nu; 1+2\nu, \alpha+\beta+\nu; \mp 2ac \right) \pm \\
&\pm \left\{ \cos \nu\pi \sec(\alpha+\beta)\pi \right\} \sqrt{\pi} (2c)^{1-\alpha-\beta} \Gamma \left[\begin{matrix} \alpha+\beta+\nu-1, \alpha+\beta-\nu-1 \\ \alpha+\beta-1/2 \end{matrix} \right] \times \\
&\times {}_2F_2 \left(1-\beta, \frac{3}{2}-\alpha-\beta; 2-\alpha-\beta-\nu, 2-\alpha-\beta+\nu; \mp 2ac \right) \\
&\left[a, \operatorname{Re} \beta > 0, \left\{ \begin{matrix} \operatorname{Re} c > 0 \\ \operatorname{Re} c > 0, \operatorname{Re}(\alpha+\beta) < 3/2 \end{matrix} \right\} \right].
\end{aligned}$$

$$\begin{aligned}
3. \int_a^\infty x^{-\beta-1/2} (x-a)^{\beta-1} e^{-cx} K_\nu(cx) dx &= \sqrt{\frac{\pi}{2a^2c}} \Gamma(\beta) e^{-ac} W_{-\beta, \nu}(2ac) \\
&[a, \operatorname{Re} c, \operatorname{Re} \beta > 0].
\end{aligned}$$

$$4. \int_a^\infty \frac{x^{-1}}{\sqrt{x-a}} e^{-cx} K_{i\tau}(cx) dx = \frac{\sqrt{2\pi c}}{\tau} e^{-ac} \operatorname{Im} K_{1/2+i\tau}(ac) \quad [a, c, \tau > 0].$$

$$\begin{aligned}
5. \int_a^\infty x^{-\nu} (x-a)^{\beta-1} e^{\pm cx} K_\nu(cx) dx &= \\
&= \frac{\sqrt{\pi} a^{(\beta-1)/2-\nu}}{(2c)^{(\beta+1)/2}} \left\{ \frac{B(\beta, 1/2+\nu-\beta)}{\Gamma(\beta)} \right\} e^{\pm ac} W_{\pm\beta/2, \nu-\beta/2}(2ac) \\
&\left[a, \operatorname{Re} c > 0, \left\{ \begin{matrix} 0 < \operatorname{Re} \beta < \operatorname{Re} \nu + 1/2 \\ \operatorname{Re} \beta > 0 \end{matrix} \right\} \right].
\end{aligned}$$

$$\begin{aligned}
6. \int_0^\infty \frac{x^{\alpha-1}}{(x+z)^\rho} e^{\pm cx} K_\nu(cx) dx &= \\
&= \sqrt{\pi} (2c)^{\rho-\alpha} (\cos \nu\pi \sec(\rho-\alpha)\pi)^{(1\pm 1)/2} \Gamma \left[\begin{matrix} \alpha+\nu-\rho, \alpha-\nu-\rho \\ \alpha+1/2-\rho \end{matrix} \right] \times \\
&\times {}_2F_2 \left(\frac{1}{2}+\rho-\alpha, \rho; 1+\rho-\alpha-\nu, 1+\rho-\alpha+\nu; \mp 2cz \right) + 2^{\nu-1} c^{-\nu} z^{\alpha-\nu-\rho} \times \\
&\times \Gamma \left[\begin{matrix} \nu, \rho+\nu-\alpha, \alpha-\nu \\ \rho \end{matrix} \right] {}_2F_2 \left(\frac{1}{2}-\nu, \alpha-\nu; 1-2\nu, 1+\alpha-\rho-\nu; \mp 2cz \right) + \\
&+ 2^{\nu-1} c^\nu z^{\alpha+\nu-\rho} \Gamma \left[\begin{matrix} -\nu, \rho-\nu-\alpha, \alpha+\nu \\ \rho \end{matrix} \right] \times \\
&\times {}_2F_2 \left(\frac{1}{2}+\nu, \alpha+\nu; 1+2\nu, 1+\alpha+\nu-\rho; \mp 2cz \right) \\
&\left[\operatorname{Re} \alpha > |\operatorname{Re} \nu|; |\arg z| < \pi, \left\{ \begin{matrix} \operatorname{Re} c > 0; \operatorname{Re}(\alpha-\rho) < 1/2 \\ \operatorname{Re} c > 0 \end{matrix} \right\} \right].
\end{aligned}$$

$$\begin{aligned}
7. \int_0^\infty \frac{x^{\rho-3/2}}{(x+z)^\rho} e^{-cx} K_0(cx) dx &= \sqrt{\frac{\pi}{2c}} \Gamma \left[\begin{matrix} \rho-1/2, \rho-1/2 \\ \rho \end{matrix} \right] \frac{1}{z} e^{cz} W_{1-\rho, 0}(2cz) \\
&[\operatorname{Re} c > 0; \operatorname{Re} \rho > 1/2; |\arg z| \leq \pi].
\end{aligned}$$

$$\begin{aligned}
8. \int_0^\infty \frac{x^{-1/2}}{x+z} e^{-cx} K_\nu(cx) dx &= \frac{\pi \sec \nu\pi e^{cz}}{\sqrt{z}} K_\nu(cz) \\
&[\operatorname{Re} c > 0; |\operatorname{Re} \nu| < 1/2; |\arg z| < \pi]
\end{aligned}$$

$$9. \int_0^{\infty} \frac{x^{\alpha-1}}{x-y} e^{\mp cx} K_{\nu}(cx) dx = \pm (2c)^{1-\alpha} \sqrt{\pi} \left\{ \frac{1}{\cos \nu\pi \sec \alpha\pi} \right\} \times$$

$$\times \Gamma \left[\frac{\alpha+\nu-1}{\alpha-1/2}, \frac{\alpha-\nu-1}{\alpha-1/2} \right] {}_2F_2 \left(1, \frac{3}{2}-\alpha; 2-\alpha-\nu, 2-\alpha+\nu; \mp 2cy \right) -$$

$$- \frac{\pi c^{\nu} y^{\alpha+\nu-1}}{2^{\nu+1}} \Gamma(-\nu) \operatorname{ctg}(\alpha+\nu) {}_1F_1 \left(\nu + \frac{1}{2}; 2\nu+1; \mp 2cy \right) -$$

$$- \frac{\pi y^{\alpha-\nu-1}}{2^{1-\nu} c^{\nu}} \Gamma(\nu) \operatorname{ctg}(\alpha-\nu) {}_1F_1 \left(\frac{1}{2}-\nu; 1-2\nu; \mp 2cy \right)$$

$$\left[y > 0, |\operatorname{Re} \nu| < \operatorname{Re} \alpha, \begin{cases} \operatorname{Re} c > 0 \\ \operatorname{Re} c > 0; \operatorname{Re} \alpha < 3/2 \end{cases} \right].$$

2.16.8. Интегралы от $x^{\alpha} e^{-px^{\pm 1}} K_{\nu}(cx)$.

$$1. \int_a^{\infty} x^{\nu+1} e^{\pm cx^2/(2a)} K_{\nu}(cx) dx = \frac{a^{\nu+1}}{2c} e^{\mp ac/2} \int_{ac}^{\infty} e^{\pm x} [K_{\nu}(x) \mp K_{\nu+1}(x)] dx$$

[a, c > 0].

$$2. \int_a^{\infty} x^{\nu+1} e^{icx^2/(2a)} K_{\nu}(cx) dx = \frac{a^{\nu+1}}{2c} e^{iac/2} \int_{ac}^{\infty} [K_{\nu}(x) + iK_{\nu+1}(x)] dx$$

[a, c > 0].

$$3. \int_0^{\infty} e^{-px^2} K_{\nu}(cx) dx = \frac{1}{4} \sqrt{\frac{\pi}{p}} \sec \frac{\nu\pi}{2} \exp \left(\frac{c^2}{8p} \right) K_{\nu/2} \left(\frac{c^2}{8p} \right)$$

[Re p > 0; |Re ν| < 1].

$$4. \int_0^{\infty} x^{\alpha-1} e^{-px^2} K_{\nu}(cx) dx =$$

$$= \frac{p^{(1-\alpha)/2}}{2c} \Gamma \left(\frac{\alpha+\nu}{2} \right) \Gamma \left(\frac{\alpha-\nu}{2} \right) \exp \left(\frac{c^2}{8p} \right) W_{(1-\alpha)/2, \nu/2} \left(\frac{c^2}{4p} \right)$$

[Re α > |Re ν|; Re p > 0].

$$5. \int_0^{\infty} x^{\nu+1} e^{-px^2} K_{\nu}(cx) dx = \frac{c^{\nu} \Gamma(\nu+1)}{2^{\nu+2} p^{\nu+1}} \exp \left(\frac{c^2}{4p} \right) \Gamma \left(-\nu, \frac{c^2}{4p} \right)$$

[Re p > 0; Re ν > -1].

$$6. \int_0^{\infty} x e^{-px^2} K_0(cx) dx = -\frac{1}{4p} \exp \left(\frac{c^2}{4p} \right) \operatorname{Ei} \left(-\frac{c^2}{4p} \right)$$

[Re p > 0].

$$7. \int_0^{\infty} x e^{-px^2} K_1(cx) dx = \frac{c \sqrt{\pi}}{16p^{3/2}} \exp \left(\frac{c^2}{8p} \right) \left[K_1 \left(\frac{c^2}{8p} \right) - K_0 \left(\frac{c^2}{8p} \right) \right]$$

[Re p > 0].

$$8. \int_0^{\infty} x^{\alpha-1} e^{-p\sqrt{x}} K_{\nu}(cx) dx = \frac{2^{\alpha-2}}{c^{\alpha}} \Gamma \left(\frac{\alpha+\nu}{2} \right) \Gamma \left(\frac{\alpha-\nu}{2} \right) \times$$

$$\times {}_2F_3 \left(\frac{\alpha+\nu}{2}, \frac{\alpha-\nu}{2}; \frac{1}{4}, \frac{1}{2}, \frac{3}{4}; \frac{p^4}{64c^2} \right) - \frac{2^{\alpha-3/2} p}{c^{\alpha+1/2}} \Gamma \left(\frac{1+2\alpha+2\nu}{4} \right) \times$$

$$\times \Gamma \left(\frac{1+2\alpha-2\nu}{4} \right) {}_2F_3 \left(\frac{1+2\alpha+2\nu}{4}, \frac{1+2\alpha-2\nu}{4}; \frac{1}{2}, \frac{3}{4}, \frac{5}{4}; \frac{p^4}{64c^2} \right) +$$

$$\begin{aligned}
& + \frac{2^{\alpha-2} p^2}{c^{\alpha+1}} \Gamma\left(\frac{1+\alpha+\nu}{2}\right) \Gamma\left(\frac{1+\alpha-\nu}{2}\right) {}_2F_3\left(\frac{1+\alpha+\nu}{2}, \frac{1+\alpha-\nu}{2}; \frac{3}{4}, \frac{5}{4}, \frac{3}{2}; \frac{p^4}{64c^2}\right) - \\
& - \frac{2^{\alpha-3/2} p^2}{3c^{\alpha+3/2}} \Gamma\left(\frac{3+2\alpha+2\nu}{4}\right) \Gamma\left(\frac{3+2\alpha-2\nu}{4}\right) \times \\
& \times {}_2F_3\left(\frac{3+2\alpha+2\nu}{4}, \frac{3+2\alpha-2\nu}{4}; \frac{5}{4}, \frac{3}{2}, \frac{7}{4}; \frac{p^4}{4c^2}\right) \\
& \quad [\operatorname{Re} c > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu|]
\end{aligned}$$

$$\begin{aligned}
9. \quad \int_0^\infty x^{\alpha-1} e^{-p/x} K_\nu(cx) dx = \\
= \frac{2^{\alpha-2}}{c^\alpha} \Gamma\left(\frac{\alpha+\nu}{2}\right) \Gamma\left(\frac{\alpha-\nu}{2}\right) {}_0F_3\left(\frac{1}{2}, 1-\frac{\alpha+\nu}{2}, 1-\frac{\alpha-\nu}{2}; \frac{c^2 p^2}{16}\right) - \\
- \frac{2^{\alpha-3} p}{c^{\alpha-1}} \Gamma\left(\frac{\alpha+\nu-1}{2}\right) \Gamma\left(\frac{\alpha-\nu-1}{2}\right) {}_0F_3\left(\frac{3}{2}, \frac{3-\alpha-\nu}{2}, \frac{3+\nu-\alpha}{2}; \frac{c^2 p^2}{16}\right) + \\
+ 2^{-\nu-1} p^{\alpha+\nu} c^\nu \Gamma(-\nu) \Gamma(-\nu-\alpha) {}_0F_3\left(1+\nu, 1+\frac{\alpha+\nu}{2}, \frac{1+\alpha+\nu}{2}; \frac{c^2 p^2}{16}\right) + \\
+ 2^{\nu-1} p^{\alpha-\nu} c^{-\nu} \Gamma(\nu) \Gamma(\nu-\alpha) {}_0F_3\left(1-\nu, 1+\frac{\alpha-\nu}{2}, \frac{1+\alpha-\nu}{2}; \frac{c^2 p^2}{16}\right) \\
\quad [\operatorname{Re} c, \operatorname{Re} p > 0].
\end{aligned}$$

$$10. \quad \int_0^\infty x e^{-p/x} K_0(cx) dx = \frac{2p}{c} K_1(\sqrt{2icp}) K_1(\sqrt{-2icp}) \quad [\operatorname{Re} c, \operatorname{Re} p > 0].$$

$$11. \quad \int_0^\infty \frac{1}{x} e^{-p/x} K_0(cx) dx = 2 K_0(\sqrt{2icp}) K_0(\sqrt{-2icp}) \quad [\operatorname{Re} c, \operatorname{Re} p > 0].$$

$$\begin{aligned}
12. \quad \int_0^\infty \frac{1}{x^2} e^{-p/x} K_0(cx) dx = 2 \sqrt{\frac{2c}{\pi}} \{ \ker(\sqrt{2cp}) [\ker_1(\sqrt{2cp}) - \operatorname{kei}_1(\sqrt{2cp})] + \\
+ \operatorname{kei}(\sqrt{2cp}) [\ker_1(\sqrt{2cp}) + \operatorname{kei}_1(\sqrt{2cp})] \} \\
\quad [\operatorname{Re} c, \operatorname{Re} p > 0].
\end{aligned}$$

$$\begin{aligned}
13. \quad \int_0^\infty x^{\alpha-1} e^{-p/x^2} K_\nu(cx) dx = \\
= \frac{2^{\alpha-2}}{c^\alpha} \Gamma\left(\frac{\alpha+\nu}{2}\right) \Gamma\left(\frac{\alpha-\nu}{2}\right) {}_0F_2\left(1-\frac{\alpha+\nu}{2}, 1-\frac{\alpha-\nu}{2}; -\frac{c^2 p}{4}\right) + \\
+ \frac{c^\nu p^{(\alpha+\nu)/2}}{2^{\nu+2}} \Gamma(-\nu) \Gamma\left(-\frac{\alpha+\nu}{2}\right) {}_0F_2\left(1+\frac{\alpha+\nu}{2}, 1+\nu; -\frac{c^2 p}{4}\right) + \\
+ \frac{c^{-\nu} p^{(\alpha-\nu)/2}}{2^{2-\nu}} \Gamma(\nu) \Gamma\left(\frac{\nu-\alpha}{2}\right) {}_0F_2\left(1+\frac{\alpha-\nu}{2}, 1-\nu; -\frac{c^2 p}{4}\right) \\
\quad [\operatorname{Re} c, \operatorname{Re} p > 0].
\end{aligned}$$

$$14. \quad \int_0^\infty e^{-p/x^2} K_{1/3}(cx) dx = \frac{2\pi}{\sqrt{3}} \exp\left[-3\left(\frac{c^2 p}{4}\right)^{1/3}\right] \quad [\operatorname{Re} c, \operatorname{Re} p > 0].$$

$$15. \quad \int_0^\infty \frac{1}{x^2} e^{-p/x^2} K_{1/3}(cx) dx = \frac{\pi}{\sqrt{3}} \left(\frac{2}{cp^2}\right)^{1/3} \exp\left[-3\left(\frac{c^2 p}{4}\right)^{1/3}\right] \\
\quad [\operatorname{Re} c, \operatorname{Re} p > 0]$$

$$16. \int_0^{\infty} \frac{1}{x} e^{-p/x^2} K_{2/3}(cx) dx = \pi \left(\frac{4}{3c^2 p} \right)^{1/3} \exp \left[-3 \left(\frac{c^2 p}{4} \right)^{1/3} \right]$$

[Re c, Re p > 0].

2.16.9. Интегралы от $A(x) e^{f(x)} K_\nu(cx)$.

$$1. \int_0^{\infty} \frac{1}{x} e^{-px-b/x} K_\nu(cx) dx = 2K_\nu(z_+) K_\nu(z_-)$$

$[z_{\pm} = \sqrt{b} (\sqrt{p+c} \pm \sqrt{p-c}); \operatorname{Re}(p+c), \operatorname{Re} b > 0]$.

$$2. \int_0^{\infty} \frac{1}{x^2} e^{-px-b/x} K_0(cx) dx = 2c [z_-^{-1} K_0(z_+) K_1(z_-) + z_+^{-1} K_1(z_+) K_0(z_-)]$$

$[z_{\pm} = \sqrt{b} (\sqrt{p+c} \pm \sqrt{p-c}); \operatorname{Re}(p+c), \operatorname{Re} b > 0]$.

$$3. \int_0^{\infty} x^{\alpha-1} e^{-cx-b/x} K_\nu(cx) dx =$$

$$= \frac{b^{\alpha+\nu} c^\nu}{2^{\nu+1}} \Gamma(-\nu) \Gamma(-\alpha-\nu) {}_1F_2 \left(\frac{1}{2} + \nu; 1+2\nu, 1+\nu+\alpha; \pm 2bc \right) +$$

$$+ \frac{b^{\alpha-\nu} c^{-\nu}}{2^{1-\nu}} \Gamma(\nu) \Gamma(\nu-\alpha) {}_1F_2 \left(\frac{1}{2} - \nu; 1-2\nu, 1+\alpha-\nu; \pm 2bc \right) +$$

$$+ \left\{ \frac{1}{\cos \nu\pi \sec \alpha\pi} \right\} \frac{\sqrt{\pi}}{(2c)^\alpha} \Gamma \left[\begin{matrix} \alpha+\nu, \alpha-\nu \\ \alpha+1/2 \end{matrix} \right] {}_1F_2 \left(\frac{1}{2} - \alpha; 1-\alpha+\nu, 1-\alpha-\nu; \pm 2bc \right)$$

[Re b, Re c > 0].

$$4. \int_0^{\infty} x^{-3/2} e^{-cx-b/x} K_\nu(cx) dx = 2 \sqrt{\frac{\pi}{b}} K_{2\nu}(\sqrt{8bc})$$

[Re b, Re c > 0]

$$5. \int_0^{\infty} \frac{1}{x} e^{cx-b/x} K_\nu(cx) dx = \frac{\pi^2}{2} [J_\nu^2(\sqrt{2bc}) + Y_\nu^2(\sqrt{2bc})]$$

[Re b, Re c > 0]

$$6. \int_0^{\infty} x^{-3/2} e^{cx-b/x} K_0(cx) dx = \sqrt{\frac{\pi^3}{b}} [H_0(\sqrt{8bc}) - Y_0(\sqrt{8bc})]$$

[Re b, Re c > 0].

$$7. \int_0^{\infty} x^{-1/2} e^{cx-b/x} [K_1(cx) - K_0(cx)] dx = -\sqrt{\frac{2\pi^3}{c}} [Y_1(\sqrt{8bc}) - H_{-1}(\sqrt{8bc})]$$

[Re b, Re c > 0].

$$8. \int_0^{\infty} \frac{\exp(-p\sqrt{x^2+z^2})}{\sqrt{x^2+z^2}} K_\nu(cx) dx = \frac{1}{2} \sec \frac{\nu\pi}{2} K_{\nu/2} \left(\frac{z_+}{2} \right) K_{\nu/2} \left(\frac{z_-}{2} \right)$$

$[\operatorname{Re} z, \operatorname{Re}(p+c) > 0; |\operatorname{Re} \nu| < 1; z_{\pm} = z(p \pm \sqrt{p^2 - c^2})]$.

$$9. \int_0^{\infty} \frac{x}{\sqrt{x^2+z^2}} \exp(-p\sqrt{x^2+z^2}) K_0(cx) dx = \frac{1}{2R} [e^{zR} \operatorname{Ei}(-z_+) - e^{-zR} \operatorname{Ei}(-z_-)]$$

$[\operatorname{Re} z, \operatorname{Re}(p+c) > 0, R = \sqrt{p^2 - c^2}; z_{\pm} = z(p \pm \sqrt{p^2 - c^2})]$.

$$10. \int_0^{\infty} \frac{x^{1-\nu}}{\sqrt{x^2+z^2}} (\sqrt{x^2+z^2} \pm z)^{\nu} \exp(-p \sqrt{x^2+z^2}) K_{\nu}(cx) dx =$$

$$= \frac{(cz)^{\pm \nu} \Gamma(1 \mp \nu)}{2R} \left\{ z_{-}^{\mp \nu} e^{-Rz/2} \Gamma(\pm \nu, z_{-}) - z_{+}^{\mp \nu} e^{Rz/2} \Gamma(\pm \nu, z_{+}) \right\}$$

$$[\operatorname{Re} z, \operatorname{Re}(p+c) > 0; \operatorname{Re} \nu < 1; R = \sqrt{p^2 - c^2}; z_{\pm} = z(p \pm \sqrt{p^2 - c^2})].$$

$$11. \int_0^{\infty} \frac{x^{-\mu}}{\sqrt{x^2+z^2}} (\sqrt{x^2+z^2} \pm z)^{\mu} \exp(-p \sqrt{x^2+z^2}) K_{\nu}(cx) dx =$$

$$= \frac{1}{2cz} \Gamma\left(\frac{1+\nu \mp \mu}{2}\right) \Gamma\left(\frac{1-\nu \mp \mu}{2}\right) W_{\pm \mu/2, \nu/2}(z_{+}) W_{\pm \mu/2, \nu/2}(z_{-})$$

$$[\operatorname{Re} z, \operatorname{Re}(p+c) > 0; |\operatorname{Re} \nu| < 1 \mp \operatorname{Re} \mu; z_{\pm} = z(p \pm \sqrt{p^2 - c^2})].$$

$$12. \int_0^{\infty} x^{-1/2} \frac{\exp(-c \sqrt{(x-a)^2 + b^2})}{\sqrt{(x-a)^2 + b^2}} K_{\nu}(cx) dx =$$

$$= \frac{\pi \sec \nu \pi}{c(a^2 + b^2)^{1/4}} P_{\nu-1/2}\left(-\frac{a}{\sqrt{a^2 + b^2}}\right) K_{\nu}(c \sqrt{a^2 + b^2})$$

$$[a, b, c > 0; |\operatorname{Re} \nu| < 1/2]$$

2.16.10. Интегралы от $A(x) e^{f(x)} K_{\nu}(\varphi(x))$.

$$1. \int_0^{\infty} e^{-px} K_0(c \sqrt{x^2 + xz}) dx = -\frac{1}{2R} e^{pz/2} \left[e^{-Rz/2} \operatorname{Ei}\left(-\frac{z_{-}}{2}\right) - e^{Rz/2} \operatorname{Ei}\left(-\frac{z_{+}}{2}\right) \right]$$

$$[\operatorname{Re}(p+c) > 0; |\arg z| < \pi; R = \sqrt{p^2 - c^2}; z_{\pm} = z(p \pm \sqrt{p^2 - c^2})].$$

$$2. \int_0^{\infty} \frac{x^{\rho-1}}{(x+z)^{\rho}} e^{-px} K_{\nu}(c \sqrt{x^2 + xz}) dx =$$

$$= \frac{1}{cz} \Gamma\left(\rho + \frac{\nu}{2}\right) \Gamma\left(\rho - \frac{\nu}{2}\right) e^{pz/2} W_{1/2-\rho, \nu/2}\left(\frac{z_{+}}{2}\right) W_{1/2-\rho, \nu/2}\left(\frac{z_{-}}{2}\right)$$

$$[\operatorname{Re}(p+c) > 0; |\arg z| < \pi; 2 \operatorname{Re} \rho > |\operatorname{Re} \nu|; z_{\pm} = z(p \pm \sqrt{p^2 - c^2})].$$

$$3. \int_0^{\infty} \frac{x^{-1/2}}{\sqrt{x+z}} e^{-px} K_{\nu}(c \sqrt{x^2 + xz}) dx = \frac{e^{pz/2}}{2 \cos(\nu \pi/2)} K_{\nu/2}\left(\frac{z_{+}}{4}\right) K_{\nu/2}\left(\frac{z_{-}}{4}\right)$$

$$[\operatorname{Re}(p+c) > 0; |\arg z| < \pi; |\operatorname{Re} \nu| < 1; z_{\pm} = z(p \pm \sqrt{p^2 - c^2})].$$

$$4. \int_0^{\infty} \frac{x^{\nu/2}}{(x+z)^{\nu/2}} e^{-px} K_{\nu}(c \sqrt{x^2 + xz}) dx =$$

$$= \frac{\Gamma(1+\nu)}{2R (cz)^{\nu}} e^{pz/2} \left[z_{-}^{\nu} e^{-Rz/2} \Gamma\left(-\nu, \frac{z_{-}}{2}\right) - z_{+}^{\nu} e^{Rz/2} \Gamma\left(-\nu, \frac{z_{+}}{2}\right) \right]$$

$$[\operatorname{Re}(p+c) > 0; |\arg z| < \pi; \operatorname{Re} \nu > -1; R = \sqrt{p^2 - c^2}; z_{\pm} = z(p \pm \sqrt{p^2 - c^2})].$$

$$5. \int_0^{\frac{1}{b}} \frac{x^{\alpha-1}}{x^2 - y^2} \exp\left(-b \frac{y^2 + x^2}{y^2 - x^2}\right) K_{\nu}\left(\frac{2cx}{y^2 - x^2}\right) dx =$$

$$= -\frac{y^{\alpha-1}}{4c} \Gamma\left(\frac{\alpha-\nu}{2}\right) \Gamma\left(\frac{\alpha+\nu}{2}\right) W_{(1-\alpha)/2, \nu/2}(y_{+}) W_{(1-\alpha)/2, \nu/2}(y_{-})$$

$$[b, y, \operatorname{Re} c > 0, \operatorname{Re} \alpha > |\operatorname{Re} \nu|; y_{\pm} = b \pm \sqrt{b^2 - c^2 y^{-2}}].$$

$$6. \int_y^\infty \frac{x^{\alpha-1}}{x^2-y^2} \exp\left(-b \frac{x^2+y^2}{x^2-y^2}\right) K_\nu\left(\frac{2bx}{x^2-y^2}\right) dx =$$

$$= \frac{y^{\alpha-1}}{4c} \Gamma\left(1-\frac{\alpha+\nu}{2}\right) \Gamma\left(1+\frac{\nu-\alpha}{2}\right) W_{(\alpha-1)/2, \nu/2}(y_+) W_{(\alpha-1)/2, \nu/2}(y_-)$$

$$[b, y, \operatorname{Re} c > 0; \operatorname{Re} \alpha < |\operatorname{Re} \nu| + 2; y_{\pm} = b \pm \sqrt{b^2 - c^2 y^2}].$$

2.16.11. Интегралы от $x^\alpha \begin{Bmatrix} \operatorname{sh} bx \\ \operatorname{ch} bx \end{Bmatrix} K_\nu(cx)$.

$$1. \int_0^\infty \begin{Bmatrix} \operatorname{sh} bx \\ \operatorname{ch} bx \end{Bmatrix} K_\nu(cx) dx = \frac{\pi}{2\sqrt{c^2-b^2}} \begin{Bmatrix} \operatorname{cosec}(\nu\pi/2) \sin \varphi \\ \sec(\nu\pi/2) \cos \varphi \end{Bmatrix}$$

$$[\varphi = \nu \arcsin(b/c), \operatorname{Re} c > |\operatorname{Re} b|; |\operatorname{Re} \nu| < (3 \pm 1)/2].$$

$$2. \int_0^\infty \operatorname{sh} bx K_0(cx) dx = \frac{\arcsin(b/c)}{\sqrt{c^2-b^2}} \quad [\operatorname{Re} c > |\operatorname{Re} b|]$$

$$3. \int_0^\infty x^{\alpha-1} \begin{Bmatrix} \operatorname{sh} bx \\ \operatorname{ch} x \end{Bmatrix} K_\nu(cx) dx =$$

$$= \frac{2^{\alpha+\delta-2} b^\delta}{c^{\alpha+\delta}} \Gamma\left(\frac{\alpha+\delta+\nu}{2}\right) \Gamma\left(\frac{\alpha+\delta-\nu}{2}\right) {}_2F_1\left(\frac{\alpha+\delta+\nu}{2}, \frac{\alpha+\delta-\nu}{2}; \delta + \frac{1}{2}; \frac{b^2}{c^2}\right)$$

$$[\operatorname{Re} c > |\operatorname{Re} b|; |\operatorname{Re} \nu| < \operatorname{Re} \alpha + \delta, \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}].$$

$$4. \int_0^\infty x^{m-1/2} \begin{Bmatrix} \operatorname{sh} bx \\ \operatorname{ch} bx \end{Bmatrix} K_n(cx) dx = J_n^m$$

$$[\operatorname{Re} c > |\operatorname{Re} b|; m-n > -1 \mp 1/2; k = \sqrt{b^2+c^2}/\sqrt{2c}]$$

$$J_0^0 = \sqrt{\frac{\pi}{2c}} [K(k) \mp K(\sqrt{1-k^2})],$$

$$J_0^1 = \frac{\sqrt{\pi}}{2(c^2-b^2)\sqrt{2c}} [2cE(k) - (c-b)K(k) \pm (c+b)K(\sqrt{1-k^2}) \mp 2cE(\sqrt{1-k^2})],$$

$$J_0^2 = \frac{\sqrt{\pi}}{4(c^2-b^2)^2\sqrt{2c}} [(c-b)(c-3b)K(k) + 8bcE(k) \pm$$

$$\pm 8bcE(\sqrt{1-k^2}) \mp (b+c)(3b+c)K(\sqrt{1-k^2})],$$

$$J_0^3 = \frac{\sqrt{\pi}}{8(c^2-b^2)^3\sqrt{2c}} [2c(9c^2+23b^2)E(k) - (c-b)(9c^2-8bc+15b^2)K(k) \pm$$

$$\pm (c+b)(9c^2+8bc+15b^2)K(\sqrt{1-k^2}) \mp 2c(9c^2+23b^2)E(\sqrt{1-k^2})],$$

$$J_1^1 = \frac{\sqrt{\pi}}{2(c^2-b^2)\sqrt{2c}} [(c-b)K(k) + 2E(k) \pm 2bE(\sqrt{1-k^2}) \mp (c+b)K(\sqrt{1-k^2})],$$

$$J_1^2 = \frac{\sqrt{\pi}}{4(c^2-b^2)^2\sqrt{2c}} [2(3c^2+b^2)E(k) - (c-b)(3c-b)K(k) \pm$$

$$\pm (c+b)(3c+b)K(\sqrt{1-k^2}) \mp 2(3c^2+b^2)E(\sqrt{1-k^2})],$$

$$J_1^3 = \frac{\sqrt{\pi}}{8(c^2-b^2)^3\sqrt{2c}} [(5c^2-24bc+3b^2)(c-b)K(k) +$$

$$+ 2b(29c^2+3b^2)E(k) \pm 2b(29c^2+3b^2)E(\sqrt{1-k^2}) \mp$$

$$\mp (5c^2+24bc+3b^2)(b+c)K(\sqrt{1-k^2})],$$

$$J_2^2 = \frac{\sqrt{\pi}}{4(c^2 - b^2)^2 \sqrt{2c^3}} [(5c^2 - 3bc - 4b^2)(c - b) K(k) + \\ + 8b(2c^2 - b^2) E(k) \pm 8b(2c^2 - b^2) E(\sqrt{1 - k^2}) \mp \\ \mp (b + c)(5c^2 + 3bc - 4b^2) K(\sqrt{1 - k^2})].$$

$$5. \int_0^\infty \frac{1}{x} \operatorname{sh} bx K_\nu(cx) dx = \frac{\pi}{2\nu} \sec \frac{\nu\pi}{2} \sin \left(\nu \arcsin \frac{b}{c} \right) \\ [\operatorname{Re} c > |\operatorname{Re} b|; |\operatorname{Re} \nu| < 1].$$

$$6. \int_0^\infty \frac{1}{x} \operatorname{sh} bx K_0(cx) dx = \frac{\pi}{2} \arcsin \frac{b}{c} \quad [\operatorname{Re} c > |\operatorname{Re} b|].$$

2.16.12. Интегралы от $A(x)e^{-px} \begin{Bmatrix} \operatorname{sh} \varphi(x) \\ \operatorname{ch} \varphi(x) \end{Bmatrix} K_\nu(cx)$.

Условие: $|\arg b|, |\arg c| < \pi$.

$$1. \int_0^a \frac{\operatorname{ch}(b\sqrt{a^2 - x^2})}{\sqrt{a^2 - x^2}} K_\nu(cx) dx = \\ = \frac{\pi^2}{4} \operatorname{cosec} \frac{\nu\pi}{2} [I_{-\nu/2}(z_+) I_{-\nu/2}(z_-) - I_{\nu/2}(z_+) I_{\nu/2}(z_-)] \\ [a > 0; |\operatorname{Re} \nu| < 1, \nu \neq 0; 2z_\pm = a(\sqrt{b^2 + c^2} \pm b)].$$

$$2. \int_0^a \frac{\operatorname{ch}(b\sqrt{a^2 - x^2})}{\sqrt{a^2 - x^2}} K_0(cx) dx = \frac{\pi}{2} [I_0(z_-) K_0(z_+) + I_0(z_+) K_0(z_-)] \\ [a > 0; 2z_\pm = a(\sqrt{b^2 + c^2} \pm b)].$$

$$3. \int_a^\infty x^{1 \pm \nu} \operatorname{sh}(b\sqrt{x^2 - a^2}) K_\nu(cx) dx = \\ = \sqrt{\frac{\pi}{2}} a^{3/2 \pm \nu} b c^{\pm \nu} (c^2 - b^2)^{(\mp 2\nu - 3)/4} K_{\mp \nu - 3/2}(a\sqrt{c^2 - b^2}) \\ [a > 0; \operatorname{Re} c > |\operatorname{Re} b|].$$

$$4. \int_a^\infty x^{1 \pm \nu} \frac{\operatorname{ch}(b\sqrt{x^2 - a^2})}{\sqrt{x^2 - a^2}} K_\nu(cx) dx = \\ = \sqrt{\frac{\pi}{2}} c^{\pm \nu} a^{1/2 \pm \nu} (c^2 - b^2)^{(\mp 2\nu - 1)/4} K_{\mp \nu - 1/2}(a\sqrt{c^2 - b^2}) \\ [a > 0; \operatorname{Re} c > |\operatorname{Re} b|].$$

$$5. \int_a^\infty x \frac{\operatorname{ch}(b\sqrt{x^2 - a^2})}{\sqrt{x^2 - a^2}} K_0(cx) dx = \frac{\pi}{2\sqrt{c^2 - b^2}} e^{-a\sqrt{c^2 - b^2}} \\ [a > 0; \operatorname{Re} c > |\operatorname{Re} b|].$$

$$6. \int_0^\infty \frac{1}{x} e^{-px} \operatorname{sh} bx K_0(cx) dx = \\ = \frac{1}{4} \ln^2 \frac{p + b + \sqrt{(p + b)^2 - c^2}}{c} - \frac{1}{4} \ln^2 \frac{p - b + \sqrt{(p - b)^2 - c^2}}{c} \\ [\operatorname{Re}(p + c) > |\operatorname{Re} b|].$$

2.16.13. Интегралы, содержащие гиперболические функции и $K_\nu(\varphi(\operatorname{sh} x, \operatorname{ch} x))$.

$$1. \int_0^\infty \operatorname{ch} bx K_0(c \operatorname{sh} x) dx = \frac{\pi^2}{8} \left[J_{b/2}^2\left(\frac{c}{2}\right) + Y_{b/2}^2\left(\frac{c}{2}\right) \right] \quad [\operatorname{Re} c > 0]$$

$$2. \int_0^\infty \operatorname{ch} bx K_\nu(c \operatorname{ch} x) dx = \frac{1}{2} K_{(\nu+b)/2}\left(\frac{c}{2}\right) K_{(\nu-b)/2}\left(\frac{c}{2}\right) \quad [\operatorname{Re} c > 0]$$

$$3. \int_0^\infty \operatorname{cth}^\mu \frac{x}{2} K_\nu(c \operatorname{sh} x) dx = \\ = \frac{1}{2c} \Gamma\left(\frac{1+\nu-\mu}{2}\right) \Gamma\left(\frac{1-\nu-\mu}{2}\right) W_{\mu/2, \nu/2}(ic) W_{\mu/2, \nu/2}(-ic) \\ [\operatorname{Re} c > 0; |\operatorname{Re} \nu| + \operatorname{Re} \mu < 1]$$

$$4. \int_{-\infty}^\infty e^{cx} \left(\frac{a + be^x}{ae^x + b}\right)^\nu K_{2\nu}(\sqrt{a^2 + b^2 + 2ab \operatorname{ch} x}) dx = 2K_{\nu+c}(a) K_{\nu-c}(b) \\ [\operatorname{Re} a, \operatorname{Re} b > 0]$$

2.16.14. Интегралы о $x^\alpha \left\{ \begin{smallmatrix} \sin bx \\ \cos bx \end{smallmatrix} \right\} K_\nu(cx)$.

$$1. \int_0^\infty \left\{ \begin{smallmatrix} \sin bx \\ \cos bx \end{smallmatrix} \right\} K_\nu(cx) dx = \\ = \frac{\pi}{4c^\nu} \left\{ \begin{smallmatrix} \operatorname{cosec}(\nu\pi/2) \\ \sec(\nu\pi/2) \end{smallmatrix} \right\} \frac{1}{\sqrt{b^2 + c^2}} [(\sqrt{b^2 + c^2} + b)^\nu \mp (\sqrt{b^2 + c^2} - b)^\nu] \\ [\operatorname{Re} c > |\operatorname{Im} b|; |\operatorname{Re} \nu| < (3 \pm 1)/2, \nu \neq 0]$$

$$2. \int_0^\infty \sin bx K_0(cx) dx = \frac{1}{\sqrt{b^2 + c^2}} \ln \frac{b + \sqrt{b^2 + c^2}}{c} \quad [\operatorname{Re} c > |\operatorname{Im} b|]$$

$$3. \int_0^\infty x^{\alpha-1} \left\{ \begin{smallmatrix} \sin bx \\ \cos bx \end{smallmatrix} \right\} K_\nu(cx) dx = \frac{2^{\alpha+\delta-2} b^\delta}{c^{\alpha+\delta}} \Gamma\left(\frac{\alpha+\delta+\nu}{2}\right) \Gamma\left(\frac{\alpha+\delta-\nu}{2}\right) \times \\ \times {}_2F_1\left(\frac{\alpha+\delta+\nu}{2}, \frac{\alpha+\delta-\nu}{2}; \delta + \frac{1}{2}; -\frac{b^2}{c^2}\right) = \\ = \mp \frac{\pi}{4} \Gamma(\alpha+\nu) (b^2 + c^2)^{-\alpha/2} \left\{ \begin{smallmatrix} \sec[(\alpha-\nu)\pi/2] \\ \operatorname{cosec}[(\alpha-\nu)\pi/2] \end{smallmatrix} \right\} \times \\ \times \left[P_{\alpha-1}^{-\nu}\left(\frac{b}{\sqrt{b^2 + c^2}}\right) \mp P_{\alpha-1}^{-\nu}\left(-\frac{b}{\sqrt{b^2 + c^2}}\right) \right] \\ [\operatorname{Re} c > |\operatorname{Im} b|; |\operatorname{Re} \nu| < \operatorname{Re} \alpha + \delta, \delta = \left\{ \begin{smallmatrix} 1 \\ 0 \end{smallmatrix} \right\}]$$

$$4. \int_0^\infty x^{\nu+(1 \pm 1)/2} \left\{ \begin{smallmatrix} \sin bx \\ \cos bx \end{smallmatrix} \right\} K_\nu(x) dx = \\ = 2^{\nu+\delta-1} \sqrt{\pi} b^\delta c^\nu \Gamma\left(\nu + \delta + \frac{1}{2}\right) (b^2 + c^2)^{-\nu-\delta-1/2} \\ [\operatorname{Re} c > |\operatorname{Im} b|; \operatorname{Re} \nu > -1/2 - \delta, \delta = \left\{ \begin{smallmatrix} 1 \\ 0 \end{smallmatrix} \right\}]$$

$$5. \int_0^{\infty} x^{m-1/2} \begin{cases} \sin bx \\ \cos bx \end{cases} K_n(cx) dx = I_n^m$$

$$\left[\operatorname{Re} c > |\operatorname{Im} b|; m-n > -1 \mp 1/2; k = \frac{b + \sqrt{b^2 + c^2}}{2\sqrt{b^2 + c^2}} \right],$$

$$I_0^0 = \sqrt{\frac{\pi k}{c}} (1-k^2)^{1/4} [K(k) \mp K(\sqrt{1-k^2})],$$

$$I_0^1 = \sqrt{\frac{\pi k^3}{c^3}} (1-k^2)^{3/4} [2E(\sqrt{1-k^2}) - K(\sqrt{1-k^2}) \mp 2E(k) \pm K(k)],$$

$$I_0^2 = -\sqrt{\frac{\pi k^5}{c^5}} (1-k^2)^{5/4} [8(1-2k^2)(E(k) \pm E(\sqrt{1-k^2})) - (5-8k^2)K(k) \mp (3-8k^2)K(\sqrt{1-k^2})],$$

$$I_0^3 = \sqrt{\frac{\pi k^7}{c^7}} (1-k^2)^{7/4} [(15-112k^2+128k^4)K(\sqrt{1-k^2}) \mp (31-144k^2+128k^4)K(k) - 2(23-128k^2+128k^4)(E(\sqrt{1-k^2}) \mp E(k))],$$

$$I_1^1 = \sqrt{\frac{\pi k}{c^3}} (1-k^2)^{1/4} [(1-k^2)K(k) \mp k^2K(\sqrt{1-k^2}) - (1-2k^2)(E(k) \pm E(\sqrt{1-k^2}))],$$

$$I_1^2 = -\sqrt{\frac{\pi k^3}{c^5}} (1-k^2)^{3/4} [k^2(7-8k^2)K(\sqrt{1-k^2}) \pm (1-k^2)(1-8k^2)K(k) + (1-16k^2+16k^4)(E(\sqrt{1-k^2}) \mp E(k))],$$

$$I_1^3 = -\sqrt{\frac{\pi k^5}{c^7}} (1-k^2)^{5/4} [(1-k^2)(3-80k^2+128k^4)K(k) \mp k^2(51-176k^2+128k^4)K(\sqrt{1-k^2}) - (3-134k^2+384k^4-256k^6)(E(k) \pm E(\sqrt{1-k^2}))],$$

$$I_2^2 = \sqrt{\frac{\pi k}{c^5}} (1-k^2)^{1/4} [(1-k^2)(2+5k^2-8k^4)K(k) \pm k^2(1-11k^2+8k^4)K(\sqrt{1-k^2}) - 2(1-2k^2)(1+4k^2-4k^4)(E(k) \pm E(\sqrt{1-k^2}))],$$

$$I_2^3 = \sqrt{\frac{\pi k^3}{c^7}} (1-k^2)^{3/4} [k^2(1-103k^2-520k^4-128k^6)K(\sqrt{1-k^2}) \mp (1-k^2)(2+15k^2-144k^4+128k^6)K(k) - 2(1+7k^2-135k^4+256k^6-128k^8)(E(\sqrt{1-k^2}) \mp E(k))].$$

$$6. \int_0^{\infty} x^{n-3/2} \sin bx K_n(cx) dx = J_n \quad \left[\operatorname{Re} c > |\operatorname{Im} b|; k = \frac{b + \sqrt{b^2 + c^2}}{2\sqrt{b^2 + c^2}} \right],$$

$$J_0 = -\sqrt{\frac{\pi c}{k}} (1-k^2)^{-1/4} [2E(k) - K(k) + K(\sqrt{1-k^2}) - 2E(\sqrt{1-k^2})],$$

$$J_1 = \sqrt{\frac{\pi}{ck}} (1-k^2)^{-1/4} [E(k) - (1-k^2)K(k) + k^2K(\sqrt{1-k^2}) - E(\sqrt{1-k^2})],$$

$$J_2 = \sqrt{\frac{\pi}{kc^3}} (1-k^2)^{-1/4} [2(1-k^2+k^4)(E(k) - E(\sqrt{1-k^2})) + k^2(1+k^2)K(\sqrt{1-k^2}) - (1-k^2)(2-k^2)K(k)].$$

$$7. \int_0^{\infty} x \cos bx K_0(cx) dx = \frac{1}{b^2 + c^2} - \frac{b}{(b^2 + c^2)^{3/2}} \ln \frac{b + \sqrt{b^2 + c^2}}{c}$$

$$[\operatorname{Re} c > |\operatorname{Im} b|].$$

$$8. \int_0^{\infty} x^{-\nu} \sin ax \sin bx K_{\nu}(cx) dx =$$

$$= \frac{\sqrt{\pi} ab \Gamma(3/2 - \nu)}{(2c)^{\nu} (a^2 + b^2 + c^2)^{3/2 - \nu}} {}_2F_1 \left(\frac{5-2\nu}{4}, \frac{3-2\nu}{4}; \frac{3}{2}; \frac{4a^2b^2}{(a^2 + b^2 + c^2)^2} \right) \\ [Re c > |Im a| + |Im b|, Re \nu < 3/2]$$

$$9. \int_0^{\infty} x^{\alpha-1} (1 - \cos bx) K_{\nu}(cx) dx = \frac{2^{\alpha-1} b^2}{c^{\alpha+2}} \Gamma \left(1 + \frac{\alpha + \nu}{2} \right) \Gamma \left(1 + \frac{\alpha - \nu}{2} \right) \times \\ \times {}_2F_2 \left(1, 1 + \frac{\alpha + \nu}{2}, 1 + \frac{\alpha - \nu}{2}; \frac{3}{2}, 2, -\frac{b^2}{c^2} \right) \\ [Re c > |Im b|, Re \alpha > |Re \nu - 2|]$$

2.16.15. Интегралы от $x^{\alpha} \left\{ \begin{matrix} \sin bx^{\pm r} \\ \cos bx^{\pm r} \end{matrix} \right\} K_{\nu}(cx)$

$$1. \int_a^{\infty} x^{\nu+1} \left\{ \begin{matrix} \sin [cx^2/(2a)] \\ \cos [cx^2/(2a)] \end{matrix} \right\} K_{\nu}(cx) dx = \\ = \frac{a^{\nu+1}}{2c} \left[\left\{ \begin{matrix} \sin(ac/2) \\ \cos(ac/2) \end{matrix} \right\} \int_{ac}^{\infty} K_{\nu}(x) dx - \left\{ \begin{matrix} \cos(ac/2) \\ \sin(ac/2) \end{matrix} \right\} \int_{ac}^{\infty} K_{\nu+1}(x) dx \right] \quad [a, c > 0].$$

$$2. \int_0^{\infty} \left\{ \begin{matrix} \sin bx^2 \\ \cos bx^2 \end{matrix} \right\} K_{\nu}(cx) dx = \mp \frac{\pi^3}{\sqrt{8b}} \sec \frac{\nu\pi}{2} \left[\left\{ \begin{matrix} \sin \varphi \\ \cos \varphi \end{matrix} \right\} J_{\nu/2} \left(\frac{c^2}{8b} \right) \pm \left\{ \begin{matrix} \cos \varphi \\ \sin \varphi \end{matrix} \right\} Y_{\nu/2} \left(\frac{c^2}{8b} \right) \right] \\ [b, Re c > 0, |Re \nu| < 2 \pm 1, \varphi = -c/(8b) + (\nu + 1)/4].$$

$$3. \int_0^{\infty} x \cos bx^2 K_0(cx) dx = \frac{1}{4b} \left[\sin \frac{c^2}{4b} ci \left(\frac{c^2}{4b} \right) - \cos \frac{c^2}{4b} si \left(\frac{c^2}{4b} \right) \right] \\ [b, c > 0].$$

$$4. \int_0^{\infty} x^{\alpha-1} \left\{ \begin{matrix} \sin bx^2 \\ \cos bx^2 \end{matrix} \right\} K_{\nu}(cx) dx = \\ = 2^{-\nu/4-2} b^{-(2\alpha+\nu)/4} c^{\nu/2} \left\{ \begin{matrix} \sin [(\alpha + \nu) \pi/4] \\ \cos [(\alpha + \nu) \pi/4] \end{matrix} \right\} \Gamma(-\nu) \Gamma \left(\frac{\alpha + \nu}{2} \right) \times \\ \times {}_2F_3 \left(\frac{\alpha + \nu}{4}, \frac{\alpha + \nu + 2}{4}; \frac{1}{2}, 1 + \frac{\nu}{2}, \frac{1 + \nu}{2}; -\frac{c^2}{8b} \right) + \\ + 2^{\nu/4-2} b^{(\nu-2\alpha)/4} c^{-\nu/2} \left\{ \begin{matrix} \sin [(\alpha - \nu) \pi/4] \\ \cos [(\alpha - \nu) \pi/4] \end{matrix} \right\} \Gamma(\nu) \Gamma \left(\frac{\alpha - \nu}{2} \right) \times \\ \times {}_2F_3 \left(\frac{\alpha - \nu}{4}, \frac{\alpha - \nu + 2}{4}; \frac{1}{2}, 1 - \frac{\nu}{2}, \frac{1 - \nu}{2}; -\frac{c^2}{8b} \right) \mp \\ \mp 2^{-(\nu+10)/4} b^{-(2\alpha+\nu+2)/4} c^{\nu/2+1} \left\{ \begin{matrix} \cos [(\alpha + \nu) \pi/4] \\ \sin [(\alpha + \nu) \pi/4] \end{matrix} \right\} \Gamma(-\nu-1) \Gamma \left(1 + \frac{\alpha + \nu}{2} \right) \times \\ \times {}_2F_3 \left(\frac{\alpha + \nu + 2}{4}, \frac{\alpha + \nu}{4} + 1; \frac{3}{2}, \frac{3 + \nu}{2}, 1 + \frac{\nu}{2}; -\frac{c^2}{8b} \right) \mp \\ \mp 2^{(\nu-10)/4} b^{(\nu-2\alpha-2)/4} c^{1-\nu/2} \left\{ \begin{matrix} \cos [(\alpha - \nu) \pi/4] \\ \sin [(\alpha - \nu) \pi/4] \end{matrix} \right\} \Gamma(\nu-1) \Gamma \left(1 + \frac{\alpha - \nu}{2} \right) \times \\ \times {}_2F_3 \left(\frac{\alpha - \nu + 2}{4}, \frac{\alpha - \nu}{4} + 1; \frac{3}{2}, \frac{3 - \nu}{2}, 1 - \frac{\nu}{2}; -\frac{c^2}{8b} \right) \\ [b, Re c > 0; |Re \nu| < Re \alpha + 1 \pm 1].$$

$$\begin{aligned}
5. \int_0^\infty x^{\alpha-1} \left\{ \frac{\sin b \sqrt{x}}{\cos b \sqrt{x}} \right\} K_\nu(cx) dx = \\
= 2^{\alpha+\delta/4-2} b^{\delta/2} c^{-\alpha-\delta/4} \Gamma\left(\frac{\alpha+\nu}{2} + \frac{\delta}{4}\right) \Gamma\left(\frac{\alpha-\nu}{2} + \frac{\delta}{4}\right) \times \\
\times {}_2F_3\left(\frac{\alpha+\nu}{2} + \frac{\delta}{4}, \frac{\alpha-\nu}{2} + \frac{\delta}{4}; \frac{1}{2}, \frac{1}{4} + \delta, \frac{3}{4}; \frac{b^2}{8c}\right) - \\
- 2^{\alpha+(\delta-2)/4} b^{1+\delta/2} c^{-\alpha-(\delta+2)/4} \Gamma\left(\frac{\alpha+\nu+1}{2} + \frac{\delta}{4}\right) \Gamma\left(\frac{\alpha-\nu+1}{2} + \frac{\delta}{4}\right) \times \\
\times {}_2F_3\left(\frac{\alpha+\nu+1}{2} + \frac{\delta}{4}, \frac{\alpha-\nu+1}{2} + \frac{\delta}{4}; \frac{3}{2}, \frac{3}{4} + \delta, \frac{5}{4}; \frac{b^2}{8c}\right) - \\
\left[b, \operatorname{Re} c > 0, |\operatorname{Re} \nu| < \operatorname{Re} \alpha + \delta/2, \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}\right].
\end{aligned}$$

$$\begin{aligned}
6. \int_0^\infty x^{-1/4} \left\{ \frac{\sin b \sqrt{x}}{\cos b \sqrt{x}} \right\} K_{1/4}(cx) dx = \sqrt{\frac{\pi^3 b}{8c}} \left[I_{\pm 1/4}\left(\frac{b^2}{4c}\right) - L_{\pm 1/4}\left(\frac{b^2}{4c}\right) \right] \\
[b, \operatorname{Re} c > 0].
\end{aligned}$$

$$\begin{aligned}
7. \int_0^\infty x^{-1/2} \cos b \sqrt{x} K_\nu(cx) dx = \\
= \frac{\pi \sec \nu \pi}{2 \sqrt{c}} \left[D_{\nu-1/2}\left(\frac{b}{\sqrt{2c}}\right) D_{-\nu-1/2}\left(-\frac{b}{\sqrt{2c}}\right) + D_{\nu-1/2}\left(-\frac{b}{\sqrt{2c}}\right) D_{-\nu-1/2}\left(\frac{b}{\sqrt{2c}}\right) \right] \\
[b, \operatorname{Re} c > 0, |\operatorname{Re} \nu| < 1/2]
\end{aligned}$$

$$\begin{aligned}
8. \int_0^\infty x^{\alpha-1} \left\{ \frac{\sin(b/x)}{\cos(b/x)} \right\} K_\nu(cx) dx = \\
= 2^{\alpha-\delta-2} b^\delta c^{\delta-\alpha} \Gamma\left(\frac{\alpha+\nu-\delta}{2}\right) \Gamma\left(\frac{\alpha-\nu-\delta}{2}\right) \times \\
\times {}_0F_3\left(1 + \frac{\delta-\alpha-\nu}{2}, 1 + \frac{\delta+\nu-\alpha}{2}, \frac{1}{2} + \delta; -\frac{b^2 c^2}{16}\right) \mp \\
\mp 2^{-1-\nu} b^{\alpha+\nu} c^\nu \left\{ \frac{\sin[(\alpha+\nu)\pi/2]}{\cos[(\alpha+\nu)\pi/2]} \right\} \Gamma(-\nu) \Gamma(-\alpha-\nu) \times \\
\times {}_0F_3\left(1+\nu, 1 + \frac{\alpha+\nu}{2}, \frac{1+\alpha+\nu}{2}; -\frac{b^2 c^2}{16}\right) \mp \\
\mp 2^{\nu-1} b^{\alpha-\nu} c^{-\nu} \left\{ \frac{\sin[(\alpha-\nu)\pi/2]}{\cos[(\alpha-\nu)\pi/2]} \right\} \Gamma(\nu) \Gamma(\nu-\alpha) \times \\
\times {}_0F_3\left(1-\nu, 1 + \frac{\alpha-\nu}{2}, \frac{1+\alpha-\nu}{2}; -\frac{b^2 c^2}{16}\right) \\
\left[b, \operatorname{Re} c > 0, |\operatorname{Re} \nu| < \operatorname{Re} \alpha + 1, \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}\right].
\end{aligned}$$

$$9. \int_0^\infty \frac{1}{x} \left\{ \frac{\sin(b/x)}{\cos(b/x)} \right\} K_0(cx) dx = \pm \pi \left\{ \frac{J_0(\sqrt{2bc})}{Y_0(\sqrt{2bc})} \right\} K_0(\sqrt{2bc}) \quad [b, \operatorname{Re} c > 0]$$

$$10. \int_0^\infty x \left\{ \frac{\sin(b/x)}{\cos(b/x)} \right\} K_0(cx) dx = \pm \frac{\pi b}{c} \left\{ \frac{J_1(\sqrt{2bc})}{Y_1(\sqrt{2bc})} \right\} K_1(\sqrt{2bc}) \quad [b, \operatorname{Re} c > 0]$$

2.16.16. Интегралы от $A(x) \begin{cases} \sin(b\sqrt{z^2-x^2}) \\ \cos(b\sqrt{z^2-x^2}) \end{cases} K_\nu(cx)$.

$$1. \int_0^a \frac{\cos(b\sqrt{a^2-x^2})}{\sqrt{a^2-x^2}} K_\nu(cx) dx =$$

$$= \frac{\pi^2}{4 \sin(\nu\pi/2)} [J_{-\nu/2}(z_+) J_{-\nu/2}(z_-) - J_{\nu/2}(z_+) J_{\nu/2}(z_-)]$$

$$[0 < c < b; a > 0; |\operatorname{Re} \nu| < 1, \nu \neq 0; 2z_{\pm} = a(b \pm \sqrt{b^2 - c^2})].$$

$$2. \int_0^a \frac{\cos(b\sqrt{a^2-x^2})}{\sqrt{a^2-x^2}} K_0(cx) dx = -\frac{\pi^2}{8} [J_0(z_+) Y_0(z_-) + J_0(z_-) Y_0(z_+)]$$

$$[0 < c < b; a > 0; 2z_{\pm} = a(b \pm \sqrt{b^2 - c^2})].$$

$$3. \int_0^a \frac{x}{\sqrt{a^2-x^2}} \cos(b\sqrt{a^2-x^2}) K_0(cx) dx =$$

$$= \frac{1}{2} (b^2 - c^2)^{-1/2} \{ \cos u [\operatorname{si}(2z_+) - \operatorname{si}(2z_-)] - \sin u [\operatorname{ci}(2z_+) + \operatorname{ci}(2z_-)] \}$$

$$[0 < c < b; a > 0; u = a\sqrt{b^2 - c^2}; 2z_{\pm} = a(b \pm \sqrt{b^2 - c^2})].$$

$$4. \int_a^\infty x^{1 \pm \nu} \sin(b\sqrt{x^2 - a^2}) K_\nu(cx) dx =$$

$$= \sqrt{\frac{\pi}{2}} a^{3/2 \pm \nu} b c^{\pm \nu} (b^2 + c^2)^{(\mp 2\nu - 3)/4} K_{\mp \nu - 3/2}(a\sqrt{b^2 + c^2})$$

$$[a > 0; \operatorname{Re} c > |\operatorname{Im} b|].$$

$$5. \int_a^\infty \frac{\sin(b\sqrt{x^2 - a^2})}{\sqrt{x^2 - a^2}} K_0(cx) dx - \int_0^a \frac{\exp(-b\sqrt{a^2 - x^2})}{\sqrt{a^2 - x^2}} K_0(cx) dx =$$

$$= -\frac{\pi}{2} I_0(z_-) K_0(z_+)$$

$$[a > 0; \operatorname{Re} c > |\operatorname{Im} b|; 2z_{\pm} = a(\sqrt{b^2 + c^2} \pm b)].$$

$$6. \int_a^\infty \frac{\cos(b\sqrt{x^2 - a^2})}{\sqrt{x^2 - a^2}} K_\nu(cx) dx = \frac{1}{2} K_{\nu/2}(z_+) K_{\nu/2}(z_-)$$

$$[a > 0; \operatorname{Re} c > |\operatorname{Im} b|; 2z_{\pm} = a(\sqrt{b^2 + c^2} \pm b)].$$

$$7. \int_a^\infty x^{1 \pm \nu} \frac{\cos(b\sqrt{x^2 - a^2})}{\sqrt{x^2 - a^2}} K_\nu(cx) dx =$$

$$= \sqrt{\frac{\pi}{2}} a^{1/2 \pm \nu} c^{\pm \nu} (b^2 + c^2)^{(\mp 2\nu - 1)/4} K_{\mp \nu - 1/2}(a\sqrt{b^2 + c^2})$$

$$[a > 0; \operatorname{Re} c > |\operatorname{Im} b|].$$

$$8. \int_a^\infty x \frac{\cos(b\sqrt{x^2 - a^2})}{\sqrt{x^2 - a^2}} K_0(cx) dx = \frac{\pi}{2\sqrt{b^2 + c^2}} \exp(-a\sqrt{b^2 + c^2})$$

$$[a > 0; \operatorname{Re} c > |\operatorname{Im} b|].$$

$$9. \int_a^\infty \frac{\cos(b\sqrt{x^2-a^2})}{x\sqrt{x^2-a^2}} K_0(cx) dx = \frac{1}{4a} \frac{\partial}{\partial \lambda} [K_{\lambda+1/2}(z_+) K_{\lambda+1/2}(z_-)]_{\lambda=0}$$

$$[a > 0; \operatorname{Re} c > |\operatorname{Im} b|; 2z_{\pm} = a(\sqrt{b^2+c^2} \pm b)].$$

$$10. \int_0^\infty \frac{1}{\sqrt{x^2+z^2}} \left\{ \frac{\sin(b\sqrt{x^2+z^2})}{\cos(b\sqrt{x^2+z^2})} \right\} K_\nu(cx) dx =$$

$$= \frac{\pi^2}{8} \sec \frac{\nu\pi}{2} \left[J_{\nu/2}(z_-) \left\{ \frac{Y_{\nu/2}(z_+)}{J_{\nu/2}(z_+)} \right\} \mp Y_{\nu/2}(z_-) \left\{ \frac{J_{\nu/2}(z_+)}{Y_{\nu/2}(z_+)} \right\} \right]$$

$$[\operatorname{Re} z > 0; \operatorname{Re} c > |\operatorname{Im} b|; |\operatorname{Re} \nu| < 1; 2z_{\pm} = z(\sqrt{b^2+c^2} \pm b)].$$

$$11. \int_0^\infty \frac{x}{\sqrt{x^2+z^2}} \left\{ \frac{\sin(b\sqrt{x^2+z^2})}{\cos(b\sqrt{x^2+z^2})} \right\} K_0(cx) dx =$$

$$= \frac{1}{2} (b^2+c^2)^{-1/2} \left[\left\{ \frac{\cos \varphi}{\sin \varphi} \right\} [\operatorname{ci}(2z_+) \mp \operatorname{ci}(2z_-)] \pm \left\{ \frac{\sin \varphi}{\cos \varphi} \right\} [\operatorname{si}(2z_+) \mp \operatorname{si}(2z_-)] \right]$$

$$[\operatorname{Re} z > 0; \operatorname{Re} c > |\operatorname{Im} b|; \varphi = z\sqrt{b^2+c^2}; 2z_{\pm} = z(\sqrt{b^2+c^2} \pm b)].$$

2.16.17. Интегралы от $\left\{ \frac{\sin bx}{\cos bx} \right\} K_\nu(\varphi(x))$.

$$1. \int_{-\infty}^\infty \left\{ \frac{\sin bx}{\cos bx} \right\} K_0(c\sqrt{(x-y)^2+z^2}) dx = \frac{\pi}{\sqrt{b^2+c^2}} \exp(-z\sqrt{b^2+c^2}) \left\{ \frac{\sin by}{\cos by} \right\}$$

$$[b, y, \operatorname{Re} c, \operatorname{Re} z > 0].$$

2.16.18. Интегралы, содержащие e^{px} , тригонометрические функции и $K_\nu(cx)$.

Обозначение $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$.

$$1. \int_0^\infty x^{\alpha-1} e^{-cx} \left\{ \frac{\sin bx}{\cos bx} \right\} K_\nu(cx) dx =$$

$$= \sqrt{\pi} (2c)^{-\alpha-\delta} b^\delta \Gamma \left[\frac{\alpha+\delta+\nu}{\alpha+\delta+1/2} \right] \times$$

$$\times {}_4F_3 \left(\frac{\alpha+\nu}{2} + \delta, \frac{\alpha+\nu+1}{2}, \frac{\alpha-\nu}{2} + \delta, \frac{\alpha-\nu+1}{2}; \frac{1}{2} + \delta, \right.$$

$$\left. \frac{2\alpha+3}{4}, \frac{2\alpha+1}{4} + \delta; -\frac{b^2}{4c^2} \right) \quad [b, \operatorname{Re} c > 0; |\operatorname{Re} \nu| < \operatorname{Re} \alpha + \delta].$$

$$2. \int_0^\infty x^{\alpha-1} e^{-cx} \left\{ \frac{\sin b\sqrt{x}}{\cos b\sqrt{x}} \right\} K_\nu(cx) dx =$$

$$= \sqrt{\pi} (2c)^{-\alpha-\delta/2} b^\delta \Gamma \left[\frac{\alpha+\nu+\delta/2}{\alpha+(1+\delta)/2} \right] \times$$

$$\times {}_2F_2 \left(\alpha+\nu+\frac{\delta}{2}, \alpha-\nu+\frac{\delta}{2}; \delta+\frac{1}{2}, \alpha+\frac{1+\delta}{2}; -\frac{b^2}{8c} \right)$$

$$[b, \operatorname{Re} c > 0; |\operatorname{Re} \nu| < \operatorname{Re} \alpha + \delta/2].$$

$$3. \int_0^\infty x^{-1/2} e^{-cx} \cos b\sqrt{x} K_0(cx) dx = \sqrt{\frac{\pi^3}{2c}} \exp\left(-\frac{b^2}{16c}\right) I_0\left(\frac{b^2}{16c}\right) \quad [b, \operatorname{Re} c > 0]$$

$$4. \int_0^{\infty} \frac{1}{x} e^{-b\sqrt{x}} \cos\left(b\sqrt{x} + \frac{1-2\nu}{4}\pi\right) K_{\nu}(cx) dx = \\ = \sqrt{\frac{\pi}{2}} \Gamma\left(\frac{1}{2}-\nu\right) D_{\nu-1/2}\left(\frac{b}{\sqrt{-ic}}\right) D_{\nu-1/2}\left(\frac{b}{\sqrt{ic}}\right) \\ [b, \operatorname{Re} c > 0; |\operatorname{Im} \nu| < 1/2].$$

$$5. \int_0^{\infty} x^{\alpha-1} e^{cx} \left\{ \begin{matrix} \sin b\sqrt{x} \\ \cos b\sqrt{x} \end{matrix} \right\} K_{\nu}(cx) dx = \\ = (2c)^{-\alpha-\delta/2} \frac{b^{\delta}}{\sqrt{\pi}} \cos \nu\pi \Gamma\left[\frac{1-\delta}{2}-\alpha, \alpha+\nu+\frac{\delta}{2}, \alpha-\nu+\frac{\delta}{2}\right] \times \\ \times {}_2F_2\left(\alpha+\nu+\frac{\delta}{2}, \alpha-\nu+\frac{\delta}{2}; \alpha+\frac{1+\delta}{2}, \delta+\frac{1}{2}; \frac{b^2}{8c}\right) \pm \\ \pm \sqrt{\frac{\pi}{2c}} b^{1-2\alpha} \cos \nu\pi \left\{ \begin{matrix} \operatorname{cosec} \alpha\pi \\ \sec \alpha\pi \end{matrix} \right\} \Gamma\left[\frac{1}{2}+\nu, \frac{1}{2}-\nu\right] \times \\ \times {}_2F_2\left(\frac{1}{2}+\nu, \frac{1}{2}-\nu; \frac{3}{2}-\alpha, 1-\alpha; \frac{b^2}{8c}\right) \\ [b, \operatorname{Re} c > 0; |\operatorname{Re} \nu| - \delta/2 < \operatorname{Re} \alpha < 1]$$

$$6. \int_0^{\infty} x^{-(1\mp 1)/4-\nu} e^{cx} \left\{ \begin{matrix} \sin b\sqrt{x} \\ \cos b\sqrt{x} \end{matrix} \right\} K_{\nu}(cx) dx = \\ = 2^{1-(\nu+\delta)/2} \pi b^{\nu-1} c^{(\nu-\delta)/2} \Gamma\left[\frac{\delta+1/2-2\nu}{\nu+1/2}\right] \exp\left(\frac{b^2}{16c}\right) W_{(3\nu-\delta)/2, (\delta-\nu)/2}\left(\frac{b^2}{8c}\right) \\ [b, \operatorname{Re} c > 0; -(1\mp 1)/4 < \operatorname{Re} \nu < (2\pm 1)/4].$$

$$7. \int_0^{\infty} x^{-1/2} e^{cx} \cos b\sqrt{x} K_0(cx) dx = \sqrt{\frac{\pi}{2c}} \exp\left(\frac{b^2}{16c}\right) K_0\left(\frac{b^2}{16c}\right) \\ [b, \operatorname{Re} c > 0].$$

2.16.19. Интегралы, содержащие тригонометрические функции и $K_{\nu}(c \operatorname{sh} x)$ или $K_{\nu}(c \cos^{\pm 1} x)$.

$$1. \int_0^{\infty} \cos bx K_0(c \operatorname{sh} x) dx = \frac{\pi^2}{8} \left[J_{ib/2}^2\left(\frac{c}{2}\right) + Y_{ib/2}^2\left(\frac{c}{2}\right) \right] \\ [b, \operatorname{Re} c > 0].$$

$$2. \int_0^{\pi/2} \cos bx K_{\nu}(c \cos x) dx = \\ = \frac{\pi}{2 \sin \nu\pi} \left[I_{-(\nu+b)/2}\left(\frac{c}{2}\right) I_{(b-\nu)/2}\left(\frac{c}{2}\right) - I_{(\nu+b)/2}\left(\frac{c}{2}\right) I_{(\nu-b)/2}\left(\frac{c}{2}\right) \right] \\ [|\operatorname{Re} \nu| < 1]$$

$$3. \int_0^{\pi/2} \cos(\nu+2n)x K_{\nu}(c \cos x) dx = \frac{(-1)^n \pi^2}{4} I_n\left(\frac{c}{2}\right) K_{\nu+n}\left(\frac{c}{2}\right) \\ [|\operatorname{Re} \nu| < 1]$$

$$4. \int_0^{\pi/2} \frac{\cos bx}{\cos x} K_{\nu}\left(\frac{c}{\cos x}\right) dx = \frac{\pi}{2c} W_{b/2, \nu/2}(c) W_{-b/2, \nu/2}(c) \\ [b, \operatorname{Re} c > 0].$$

2.16.20. Интегралы, содержащие логарифмическую функцию и $K_\nu(cx)$

$$1. \int_0^\infty x^{\alpha-1} \ln x K_\nu(cx) dx = \\ = \frac{2^{\alpha-3}}{c^\alpha} \Gamma\left(\frac{\alpha+\nu}{2}\right) \Gamma\left(\frac{\alpha-\nu}{2}\right) \left[\psi\left(\frac{\alpha+\nu}{2}\right) + \psi\left(\frac{\alpha-\nu}{2}\right) - 2 \ln \frac{c}{2} \right] \\ [c > 0; |\operatorname{Re} \alpha| > \operatorname{Re} \nu]$$

$$2. \int_0^\infty \cos bx \ln(ax) K_0(cx) dx = \frac{\pi}{2} (b^2 + c^2)^{-1/2} \left[\ln \frac{ac}{2} - C - \ln(b^2 + c^2) \right] \\ [a, b, \operatorname{Re} c > 0]$$

$$3. \int_1^\infty \frac{1}{\sqrt{x^2-1}} \cos[b \ln(x + \sqrt{x^2-1})] K_0(cx) dx = \frac{1}{2} K_{ib/2}^2\left(\frac{c}{2}\right) \\ [b, \operatorname{Re} c > 0].$$

2.16.21. Интегралы от $x^\alpha J_\mu(bx) K_\nu(cx)$.

$$1. \int_0^\infty x^{\alpha-1} J_\mu(bx) K_\nu(cx) dx = A_{\mu, \nu}^\alpha \\ [\operatorname{Re} c > |\operatorname{Im} b|, \operatorname{Re}(\alpha + \mu) > |\operatorname{Re} \nu|; k = b/\sqrt{b^2 + c^2}; l = b/(\sqrt{b^2 + c^2} + c)], \\ A_{\mu, \nu}^\alpha = 2^{\alpha-2} b^\mu c^{-\alpha-\mu} \Gamma\left[\begin{matrix} (\alpha + \mu + \nu)/2, (\alpha + \mu - \nu)/2 \\ \mu + 1 \end{matrix}\right] \times \\ \times {}_2F_1\left(\frac{\alpha + \mu + \nu}{2}, \frac{\alpha + \mu - \nu}{2}; \mu + 1; -\frac{b^2}{c^2}\right),$$

$$A_{\mu, \nu}^{\mu \pm \nu + 2} = 2^{\mu \pm \nu} b^\mu c^{\pm \nu} \Gamma(\mu \pm \nu + 1) (b^2 + c^2)^{\mp \nu - \mu - 1},$$

$$A_{\nu, \nu}^\alpha = 2^{\alpha-2} \Gamma\left(\frac{\alpha}{2}\right) \Gamma\left(\nu + \frac{\alpha}{2}\right) (b^2 + c^2)^{-\alpha/2} P_{-\alpha/2}^{-\nu}\left(\frac{c^2 - b^2}{c^2 + b^2}\right),$$

$$A_{0,0}^1 = \frac{k}{b} K(k), \quad A_{0,0}^3 = \frac{1}{b^2 + c^2}, \quad A_{0,0}^5 = \frac{k^3}{b^3} [2E(k) - K(k)],$$

$$A_{0,0}^7 = \frac{3k^5}{b^5} [8(1 - 2k^2)E(k) - (5 - 8k^2)K(k)],$$

$$A_{0,0}^9 = \frac{15k^7}{b^7} [2(23 - 128k^2 + 128k^4)E(k) - (31 - 144k^2 + 128k^4)K(k)],$$

$$A_{1,0}^0 = k^{-1} [K(k) - E(k)],$$

$$A_{1,0}^1 = \frac{1}{2b} \ln \frac{b^2 + c^2}{c^2}, \quad A_{1,0}^3 = \frac{k}{b^2} [K(k) - E(k)],$$

$$A_{1,0}^5 = \frac{2b}{(b^2 + c^2)^2}, \quad A_{1,0}^7 = \frac{k^3}{b^4} [(1 - 4k^2)K(k) - (1 - 8k^2)E(k)],$$

$$A_{1,0}^9 = \frac{3k^5}{b^6} [(3 - 4k^2)(1 - 16k^2)K(k) - (3 - 88k^2 + 128k^4)E(k)],$$

$$A_{2,0}^1 = \frac{b}{9k^3} [(2 + k^2)K(k) - 2(1 + k^2)E(k)],$$

$$A_{2,0}^3 = \frac{1}{bk} [(2 - k^2)K(k) - 2E(k)],$$

$$A_{2,0}^5 = \frac{k}{b^3} [(2 + k^2)K(k) - 2(1 + k^2)E(k)],$$

$$\begin{aligned}
A_{2,0}^5 &= \frac{k^3}{b^5} [(2 + 7k^2 - 24k^4) K(k) - 2(1 + 4k^2 - 24k^4) E(k)], \\
A_{2,0}^{-3} &= \frac{b^2}{225k^5} [(8 + 3k^2 + 4k^4) K(k) - (8 + 7k^2 + 8k^4) E(k)], \\
A_{2,0}^0 &= \frac{1}{9k^3} [(8 - 5k^2) K(k) - (8 - k^2) E(k)], \\
A_{2,0}^2 &= \frac{1}{b^2k} [(8 - 5k^2) K(k) - (8 - k^2) E(k)], \\
A_{2,0}^4 &= \frac{k}{b^4} [(8 + 3k^2 + 4k^4) K(k) - (8 + 7k^2 + 8k^4) E(k)], \\
A_{2,0}^6 &= \frac{k^3}{b^6} [(8 + 19k^2 + 60k^4 - 192k^6) K(k) - (8 + 23k^2 + 72k^4 - 384k^6) E(k)], \\
A_{0,1}^2 &= \frac{k}{bc} E(k), \quad A_{0,1}^3 = \frac{2c}{(b^2 + c^2)^2}, \\
A_{0,1}^4 &= \frac{k^3}{b^3c} [(7 - 8k^2) E(k) - 4(1 - k^2) K(k)], \\
A_{0,1}^6 &= \frac{3k^5}{b^5c} [(43 - 168k^2 + 128k^4) E(k) - 4(1 - k^2)(7 - 16k^2) K(k)], \\
A_{0,1}^8 &= \frac{15k^7}{b^7c} [(337 - 2784k^2 + 5504k^4 - 3072k^6) E(k) - \\
&\quad - 8(1 - k^2)(29 - 176k^2 + 192k^4) K(k)], \\
A_{1,1}^1 &= \frac{1}{ck} [E(k) - (1 - k^2) K(k)], \quad A_{1,1}^{12} = \frac{b}{c(b^2 + c^2)}, \\
A_{1,1}^2 &= \frac{k}{b^2c} [(1 - k^2) K(k) - (1 - 2k^2) E(k)], \\
A_{1,1}^3 &= \frac{3k^3}{b^4c} [(1 - 8k^2)(1 - k^2) K(k) - (1 - 16k^2 + 16k^4) E(k)], \\
A_{1,1}^5 &= \frac{15k^5}{b^6c} [(1 - k^2)(3 - 80k^2 + 128k^4) K(k) - (3 - 134k^2 + 384k^4 - 256k^6) E(k)], \\
A_{2,1}^0 &= \frac{b}{3ck^3} [(2 - k^2) E(k) - 2(1 - k^2) K(k)], \\
A_{2,1}^2 &= \frac{1}{bck} [(2 - k^2) E(k) - 2(1 - k^2) K(k)], \\
A_{2,1}^4 &= \frac{k}{b^3c} [2(1 - k^2)(1 + 2k^2) K(k) - (2 + 3k^2 - 8k^4) E(k)], \\
A_{2,1}^6 &= \frac{3k^3}{b^5c} [2(1 - k^2)(1 + 6k^2 - 32k^4) K(k) - (2 + 11k^2 - 136k^4 + 128k^6) E(k)], \\
A_{2,1}^{-1} &= \frac{b^2}{45ck^5} [(8 - 3k^2 - 2k^4) E(k) - (1 - k^2)(8 + k^2) K(k)], \\
A_{2,1}^1 &= \frac{1}{3ck^3} [(8 - 7k^2) E(k) - (8 - 3k^2)(1 - k^2) K(k)], \\
A_{2,1}^3 &= \frac{1}{b^2ck} [(8 - 3k^2 - 2k^4) E(k) - (1 - k^2)(8 + k^2) K(k)], \\
A_{2,1}^5 &= \frac{k}{b^4c} [(1 - k^2)(8 + 13k^2 + 24k^4) K(k) - (8 + 9k^2 + 16k^4 - 48k^6) E(k)], \\
A_{0,2}^2 &= \frac{k}{bc^2} [2(2 - k^2) E(k) - (1 - k^2) K(k)], \\
A_{0,2}^3 &= \frac{k^3}{b^3c^2} [2(19 - 44k^2 + 24k^4) E(k) - (1 - k^2)(23 - 24k^2) K(k)],
\end{aligned}$$

$$\begin{aligned}
A_{0,2}^7 &= \frac{3k^5}{b^5c^2} [2(158 - 923k^2 + 1408k^4 - 640k^6) E(k) - \\
&\quad - (1 - k^2)(211 - 848k^2 + 640k^4) K(k)], \\
A_{1,2}^2 &= \frac{1}{c^2k} [(1 + k^2) E(k) - (1 - k^2) K(k)], \\
A_{1,2}^4 &= \frac{k}{b^2c^2} [(1 - k^2)(3 - 4k^2) K(k) - (3 - 13k^2 + 8k^4) E(k)], \\
A_{1,2}^6 &= \frac{3k^3}{b^4c^2} [(1 - k^2)(5 - 68k^2 + 64k^4) K(k) - (5 - 123k^2 + 248k^4 - 128k^6) E(k)], \\
A_{2,2}^1 &= \frac{b}{3c^2k^3} [(1 - k^2)(2 - 3k^2) K(k) - 2(1 - 2k^2) E(k)], \\
A_{2,2}^3 &= \frac{1}{bc^2k} [2(1 - k^2 + k^4) E(k) - (1 - k^2)(2 - k^2) K(k)], \\
A_{2,2}^5 &= \frac{3k}{b^3c^2} [(1 - k^2)(2 + 5k^2 - 8k^4) K(k) - 2(1 - 2k^2)(1 + 4k^2 - 4k^4) E(k)], \\
A_{2,2}^7 &= \frac{15k^3}{b^5c^2} [(2 + 15k^2 - 144k^4 + 128k^6)(1 - k^2) K(k) - \\
&\quad - 2(1 + 7k^2 - 135k^4 + 256k^6 - 128k^8) E(k)], \\
A_{3,2}^0 &= \frac{b^2}{15c^2k^5} [(1 - k^2)(8 - 9k^2) K(k) - (8 - 13k^2 + 3k^4) E(k)], \\
A_{3,2}^2 &= \frac{1}{3c^2k^3} [(1 - k^2)(8 - 9k^2) K(k) - (8 - 13k^2 + 3k^4) E(k)], \\
A_{4,2}^1 &= \frac{1}{b^2c^2k} [(1 + k^2)(8 - 13k^2 + 8k^4) E(k) - (1 - k^2)(8 - k^2 - 4k^4) K(k)], \\
A_{4,2}^3 &= \frac{k}{bc^3} [(23 - 23k^2 + 8k^4) E(k) - 4(1 - k^2)(2 - k^2) K(k)], \\
A_{4,2}^5 &= \frac{k^3}{b^3c^3} [(281 - 985k^2 + 1080k^4 - 384k^6) E(k) - \\
&\quad - 4(1 - k^2)(44 - 93k^2 + 48k^4) K(k)], \\
A_{1,3}^3 &= \frac{1}{c^3k} [(3 + 7k^2 - 2k^4) E(k) - (1 - k^2)(3 + k^2) K(k)], \\
A_{1,3}^5 &= \frac{k}{b^2c^3} [(15 - 43k^2 + 24k^4)(1 - k^2) K(k) - (15 - 103k^2 + 128k^4 - 48k^6) E(k)], \\
A_{2,3}^1 &= \frac{b}{3c^3k^3} [2(1 - k^2)(1 - 3k^2) K(k) - (2 - 7k^2 - 3k^4) E(k)], \\
A_{2,3}^3 &= \frac{1}{bc^3k} [(6 - 9k^2 + 19k^4 - 8k^6) E(k) - 2(1 - k^2)(3 - 3k^2 + 2k^4) K(k)], \\
A_{3,3}^1 &= \frac{b^2}{15c^3k^5} [(8 - 23k^2 + 23k^4) E(k) - (1 - k^2)(8 - 19k^2 + 15k^4) K(k)], \\
A_{3,3}^3 &= \frac{1}{3c^3k^3} [(8 - 15k^2 + 3k^4)(1 - k^2) K(k) - (8 - 19k^2 + 9k^4 - 6k^6) E(k)], \\
A_{4,3}^5 &= \frac{k}{bc^4} [8(11 - 11k^2 + 6k^4)(2 - k^2) E(k) - (1 - k^2)(71 - 71k^2 + 24k^4) K(k)], \\
A_{1,4}^4 &= \frac{1}{b^4k} [(15 + 58k^2 - 33k^4 + 8k^6) E(k) - (1 - k^2)(15 + 13k^2 - 4k^4) K(k)], \\
A_{2,4}^3 &= \frac{b}{c^4k^3} [(1 - k^2)(2 - 9k^2 - k^4) K(k) - 2(1 + k^2)(1 - 6k^2 + k^4) E(k)], \\
A_{3,4}^2 &= \frac{b^2}{15c^4k^5} [(8 - 33k^2 + 58k^4 + 15k^6) E(k) - (1 - k^2)(8 - 29k^2 + 45k^4) K(k)],
\end{aligned}$$

$$A_{0,0}^{1/2} = \sqrt{\frac{l}{b}} \frac{\Gamma^2(1/4)}{\pi} K(l), \quad A_{0,0}^{3/2} = \frac{2\sqrt{l}\Gamma^2(3/4)}{\pi\sqrt{b(b^2+c^2)}} K(l),$$

$$A_{1,0}^{1/2} = \frac{4\Gamma^2(3/4)}{\pi\sqrt{bl}} [K(l) - E(l)],$$

$$A_{2,0}^{1/2} = \frac{\Gamma^2(1/4)}{9\pi\sqrt{bl^3}} [(2+l^2)K(l) - 2(1+l^2)E(l)],$$

$$A_{1,1}^{1/2} = \frac{\sqrt{b}\Gamma^2(1/4)}{6\pi cl^{3/2}} [(1+l^2)E(l) - (1-l^2)K(l)],$$

$$A_{1,1}^{3/2} = \frac{\sqrt{b}\Gamma^2(3/4)}{\pi c\sqrt{l^3(b^2+c^2)}} [(1+l^2)E(l) - (1-l^2)K(l)],$$

$$A_{2,1}^{1/2} = \frac{2\sqrt{b}\Gamma^2(3/4)}{5\pi cl^{5/2}} [2(1-l^2+l^4)E(l) - (1-l^2)(2-l^2)K(l)],$$

$$A_{2,2}^{1/2} = \frac{b^{3/2}\Gamma^2(1/4)}{84\pi c^2 l^{7/2}} [(2-9l^2-l^4)(1-l^2)K(l) - 2(1+l^2)(1-6l^2+l^4)E(l)].$$

$$2. \int_0^\infty x^{\alpha-1} J_\nu(cx) K_\nu(cx) dx = 2^\alpha c^{-\alpha} \Gamma \left[\begin{matrix} \alpha/2, (\alpha+2\nu)/4 \\ 1+(2\nu-\alpha)/4 \end{matrix} \right]$$

$$[\operatorname{Re} \alpha, \operatorname{Re}(\alpha+2\nu) > 0; |\arg c| < \pi/4]$$

2.16.22. Интегралы от $A(x) J_\mu(\varphi(x)) K_\nu(\chi(x))$.

$$1. \int_0^\infty \frac{x^{\alpha-1}}{(x^4+z^4)^\rho} J_\nu(cx) K_\nu(cx) dx =$$

$$\begin{aligned} &= \frac{z^{\alpha-4\rho}}{8\nu} B\left(\rho - \frac{\alpha}{4}, \frac{\alpha}{4}\right) {}_1F_4\left(\frac{\alpha}{4}; \frac{1}{2}, 1 - \frac{\nu}{2}, 1 + \frac{\nu}{2}, 1 + \frac{\alpha}{4} - \rho; \frac{c^4 z^4}{64}\right) - \\ &- \frac{c^2 z^{\alpha+2-4\rho}}{16(\nu^2-1)} B\left(\rho - \frac{\alpha+2}{4}, \frac{\alpha+2}{4}\right) {}_1F_4\left(\frac{\alpha+2}{4}; \frac{3}{2}, \frac{3-\nu}{2}, \frac{3+\nu}{2}, \frac{\alpha+6}{4} - \rho; \frac{c^4 z^4}{64}\right) + \\ &+ \frac{c^{2\nu} z^{\alpha+2\nu-4\rho}}{2^{2\nu+3}} \Gamma \left[\begin{matrix} -\nu, \rho - (\alpha+2\nu)/4, (\alpha+2\nu)/4 \\ \nu+1, \rho \end{matrix} \right] \times \\ &\times {}_1F_4\left(\frac{\alpha+2\nu}{4}; 1 + \frac{\nu}{2}, \frac{1+\nu}{2}, \nu+1, \rho - \frac{\alpha+2\nu}{4}; \frac{c^4 z^4}{64}\right) + \\ &+ \frac{2^{\alpha-4\rho-3}}{c^{\alpha-4\rho}} \Gamma \left[\begin{matrix} \alpha/2 - 2\rho, (2\nu+\alpha)/4 - \rho \\ (2\nu-\alpha)/4 + \rho + 1 \end{matrix} \right] \times \\ &\times {}_1F_4\left(\rho, 1 + \rho - \frac{\alpha}{4}, \rho + \frac{2-\alpha}{4}, 1 + \rho + \frac{2\nu-\alpha}{4}, 1 - \frac{2\nu+\alpha}{4} + \rho; \frac{c^4 z^4}{64}\right) \end{aligned}$$

$$[\operatorname{Re} \alpha, \operatorname{Re}(\alpha-2\nu) > 0; |\arg c|, |\arg z| < \pi/4]$$

$$2. \int_0^\infty x^{\alpha-1} J_\mu(bx^2) K_\nu(cx) dx =$$

$$\begin{aligned} &= 2^{(\alpha-\nu)/2-4} b^{-(\alpha+\nu)/2} c^\nu \Gamma \left[\begin{matrix} -\nu, (\alpha+2\mu+\nu)/4 \\ 1+(\alpha+2\mu-\nu)/4 \end{matrix} \right] \times \\ &\times {}_2F_3\left(\frac{\nu-\alpha-2\mu}{4}, \frac{\nu+\alpha+2\mu}{4}; \frac{1}{2}, 1 + \frac{\nu}{2}, \frac{1+\nu}{2}; -\frac{c^4}{64b^2}\right) - \\ &- 2^{(\alpha-\nu)/2-5} b^{-(\alpha+\nu)/2-1} c^{\nu+1} \Gamma \left[\begin{matrix} -\nu-1, (2+2\mu+\alpha+\nu)/4 \\ (2+2\mu+\alpha-\nu)/4 \end{matrix} \right] \times \\ &\times {}_2F_3\left(\frac{2+\nu-\alpha-2\mu}{4}, \frac{2+\nu+\alpha+2\mu}{4}; \frac{3}{2}, \frac{3+\nu}{2}, 1 + \frac{\nu}{2}; -\frac{c^4}{64b^2}\right) + \end{aligned}$$

$$\begin{aligned}
& + 2^{(\alpha+\nu)/2-4} b^{(\nu-\alpha)/2} c^{-\nu} \Gamma \left[\begin{matrix} \nu, (2\mu+\alpha-\nu)/4 \\ 1+(2\mu+\alpha+\nu)/4 \end{matrix} \right] \times \\
& \times {}_2F_3 \left(-\frac{2\mu+\alpha+\nu}{4}, \frac{2\mu+\alpha-\nu}{4}; \frac{1}{2}, 1-\frac{\nu}{2}, \frac{1-\nu}{2}; -\frac{c^4}{64b^2} \right) - \\
& - 2^{(\alpha+\nu)/2-5} b^{(\nu-\alpha)/2-1} c^{2-\nu} \Gamma \left[\begin{matrix} 1+\nu, (2+2\mu+\alpha-\nu)/4 \\ (2+2\mu+\alpha+\nu)/4 \end{matrix} \right] \times \\
& \times {}_2F_3 \left(\frac{2-\alpha-2\mu-\nu}{4}, \frac{2+\alpha+2\mu-\nu}{4}; \frac{3}{2}, \frac{3-\nu}{2}, 1-\frac{\nu}{2}; -\frac{c^4}{64b^2} \right) \\
& [b, \operatorname{Re} c > 0, \operatorname{Re}(\alpha+2\mu) > |\operatorname{Re} \nu|]
\end{aligned}$$

$$\begin{aligned}
3. \int_0^\infty x J_{\nu/2}(bx^2) K_\nu(cx) dx &= \frac{\pi \sec(\nu\pi/2)}{8b} \left[H_{-\nu/2} \left(\frac{c^2}{4b} \right) - Y_{-\nu/2} \left(\frac{c^2}{4b} \right) \right] \\
& [b, \operatorname{Re} c > 0; \operatorname{Re} \nu > -1].
\end{aligned}$$

$$\begin{aligned}
4. \int_0^\infty x^2 J_{(\nu-1)/2}(bx^2) K_\nu(cx) dx &= \\
& = -\frac{\pi c \operatorname{cosec}(\nu\pi/2)}{16b^2} \left[H_{-(\nu+1)/2} \left(\frac{c^2}{4b} \right) - Y_{-(\nu+1)/2} \left(\frac{c^2}{4b} \right) \right] \\
& [b, \operatorname{Re} c > 0; \operatorname{Re} \nu > -1]
\end{aligned}$$

$$\begin{aligned}
5. \int_0^\infty J_{2\nu}(b\sqrt{x}) K_\nu(cx) dx &= \frac{\pi}{2c} I_\nu \left(\frac{b^2}{4c} \right) - \frac{\pi}{2c} L_\nu \left(\frac{b^2}{4c} \right) \\
& [\operatorname{Re} c > 0; \operatorname{Re} \nu > -1/2]
\end{aligned}$$

$$\begin{aligned}
6. \int_0^\infty x^\alpha {}_1J_\mu(b\sqrt{x}) K_\nu(cx) dx &= \\
& = 2^{\alpha-\mu/2-2} b^\mu c^{-\mu/2-\alpha} \Gamma \left[\begin{matrix} (\alpha+\nu)/2+\mu/4, (\alpha-\nu)/2+\mu/4 \\ \mu+1 \end{matrix} \right] \times \\
& \times {}_2F_3 \left(\frac{\alpha+\nu}{2} + \frac{\mu}{4}, \frac{\alpha-\nu}{2} + \frac{\mu}{4}; \frac{1}{2}, \frac{\mu+1}{2}, \frac{\mu}{2}+1; \frac{b^4}{64c^2} \right) - \\
& - 2^{\alpha-\mu/2-3} b^{\mu+2} c^{-\mu/2-\alpha-1} \Gamma \left[\begin{matrix} (\alpha+\nu+1)/2+\mu/4, (\alpha-\nu+1)/2+\mu/4 \\ \mu+2 \end{matrix} \right] \times \\
& \times {}_2F_3 \left(\frac{\alpha+\nu+1}{2} + \frac{\mu}{4}, \frac{\alpha-\nu+1}{2} + \frac{\mu}{4}; \frac{3}{2}, \frac{\mu}{2}+1, \frac{\mu+3}{2}; \frac{b^4}{64c^2} \right) \\
& [\operatorname{Re} c > 0; \operatorname{Re} \mu > 2|\operatorname{Re} \nu| - 2\operatorname{Re} \alpha]
\end{aligned}$$

$$\begin{aligned}
7. \int_0^\infty x^{1/2} J_{2\nu-1}(b\sqrt{x}) K_\nu(cx) dx &= \frac{\pi b}{4c^2} \left[I_{\nu-1} \left(\frac{b^2}{4c} \right) - L_{\nu-1} \left(\frac{b^2}{4c} \right) \right] \\
& [\operatorname{Re} c > 0; \operatorname{Re} \nu > -1/2].
\end{aligned}$$

$$\begin{aligned}
8. \int_0^\infty x^{\alpha-1} J_\mu \left(\frac{b}{x} \right) K_\nu(cx) dx &= 2^{-\alpha-2} b^{\alpha+\nu} c^\nu \Gamma \left[\begin{matrix} -\nu, (\mu-\alpha-\nu)/2 \\ 1+(\alpha+\mu+\nu)/2 \end{matrix} \right] \times \\
& \times {}_0F_3 \left(1+\nu, 1+\frac{\alpha-\mu+\nu}{2}, 1+\frac{\alpha+\mu+\nu}{2}; -\frac{b^2c^2}{16} \right) + \\
& + 2^{2\nu-\alpha-2} b^{2-\nu} c^{-\nu} \Gamma \left[\begin{matrix} (\mu+\nu-\alpha)/2 \\ 1+(\mu-\nu+\alpha)/2 \end{matrix} \right] {}_0F_3 \left(1-\nu, 1-\frac{\mu+\nu-\alpha}{2}, \right. \\
& \left. 1+\frac{\mu-\nu+\alpha}{2}; -\frac{b^2c^2}{16} \right) + 2^{2\nu-\alpha-2} b^\nu c^{-\alpha} \Gamma \left[\begin{matrix} (\alpha+\nu-\mu)/2, (\alpha-\mu-\nu)/2 \\ 1+\mu \end{matrix} \right] \times \\
& \times {}_0F_3 \left(1+\frac{\mu-\nu-\alpha}{2}, 1+\frac{\mu+\nu-\alpha}{2}, 1+\mu; -\frac{b^2c^2}{16} \right) \\
& [b, \operatorname{Re} c > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu| - 3/2]
\end{aligned}$$

$$9. \int_0^{\infty} x^{\pm 1-1} J_{\nu} \left(\frac{b}{x} \right) K_{\nu}(cx) dx =$$

$$= i \left\{ \frac{c^{-1}}{b^{-1}} \right\} [e^{\pi \nu i/2} K_{2\nu}(e^{\pi i/4} 2 \sqrt{bc}) - e^{-\pi \nu i/2} K_{2\nu}(e^{-\pi i/4} 2 \sqrt{bc})]$$

[$b, \operatorname{Re} c > 0; |\operatorname{Re} \nu| < 3/2 \pm 1$].

$$10. \int_0^{\infty} x^{2\nu-3/2} J_{\nu+1/2} \left(\frac{b}{x} \right) K_{\nu}(cx) dx = \frac{\sqrt{2\pi}}{c^{\nu}} b^{\nu-1/2} J_{2\nu}(\sqrt{2bc}) K_{2\nu}(\sqrt{2bc})$$

[$b, \operatorname{Re} c > 0; \operatorname{Re} \nu > -1/3$].

$$11. \int_0^{\infty} x^{2\nu+1/2} J_{\nu+1/2} \left(\frac{b}{x} \right) K_{\nu}(cx) dx =$$

$$= \sqrt{2\pi} b^{\nu+1/2} c^{-\nu-1} J_{2\nu+1}(\sqrt{2bc}) K_{2\nu+1}(\sqrt{2bc})$$

[$b, \operatorname{Re} c > 0; \operatorname{Re} \nu > -1$].

$$12. \int_0^{\infty} x^{-2\nu+1/2} J_{\nu-1/2} \left(\frac{b}{x} \right) K_{\nu}(cx) dx =$$

$$= \sqrt{2\pi} b^{1/2-\nu} c^{\nu-1} K_{2\nu-1}(\sqrt{2bc}) [\sin \nu \pi J_{2\nu-1}(\sqrt{2bc}) + \cos \nu \pi Y_{2\nu-1}(\sqrt{2bc})]$$

[$b, \operatorname{Re} c > 0; \operatorname{Re} \nu < 1$].

$$13. \int_a^{\infty} x^{-\nu} (x^2 - a^2)^{\mu/2} J_{\mu}(b \sqrt{x^2 - a^2}) K_{\nu}(cx) dx =$$

$$= a^{\mu-\nu+1} b^{\mu} c^{-\nu} (b^2 + c^2)^{(\nu-\mu-1)/2} K_{\nu-\mu-1}(a \sqrt{b^2 + c^2})$$

[$a > 0; \operatorname{Re} c > |\operatorname{Im} b|; \operatorname{Re} \mu > -1$].

$$14. \int_0^{\infty} x^{\mu+1} (x^2 - y^2)^{-\nu/2} J_{\mu}(bx) K_{\nu}(c \sqrt{x^2 - y^2}) dx =$$

$$= \frac{\pi b^{\mu}}{2c^{\nu}} y^{1+\mu-\nu} e^{\pi i(\mu-\nu+1/2)} (b^2 + c^2)^{(\nu-\mu-1)/2} H_{\nu-\mu-1}^{(2)}(y \sqrt{b^2 + c^2})$$

[$\operatorname{Re} c > |\operatorname{Im} b|; \operatorname{Re} \mu > -1; \operatorname{Re} \nu < 1, \operatorname{Im} y = 0,$
 $\arg \sqrt{x^2 - y^2} = 0$ при $x > y, \arg (x^2 - y^2)^{\nu} = \pi \nu$ при $x < y$].

$$15. \int_0^{\infty} x^{\alpha-1} J_{\nu}(c \sqrt{x^2 + z^2} - cz) K_{\nu}(c \sqrt{x^2 + z^2} + cz) dx =$$

$$= 2^{\alpha-2} \left(\frac{z}{c} \right)^{\alpha/2} \Gamma \left[\frac{(\alpha + 2\nu)/4}{1 + (2\nu - \alpha)/4} \right] K_{\alpha/2}(2cz)$$

[$\operatorname{Re} z > 0; |\arg c| < \pi/4; \operatorname{Re}(\alpha + 2\nu) > 0$]

2.16.23. Интегралы от $x^{\alpha} e^{-px^2} J_{\mu}(bx) K_{\nu}(cx)$.

$$1. \int_0^{\infty} x^{-1/2} e^{-px} J_{\mu}(bx) K_{\nu}(cx) dx = A_{\mu, \nu}$$

$$\left[\operatorname{Re}(p + c) > |\operatorname{Im} b|; k = \left(\frac{2}{1 + \sqrt{1 + c^2 \gamma^2}} \right)^{1/2}, l = \left(\frac{1 - \sqrt{1 - b^2 \gamma^2}}{2} \right)^{1/2}, \right.$$

$$\left. \gamma = \frac{2}{p^2 + b^2 - c^2 + \sqrt{(p^2 + b^2 - c^2)^2 + 4b^2 c^2}} \right],$$

$$A_{0,0} = 2k \sqrt{\frac{\gamma}{\pi}} K(k) K(l),$$

$$A_{1,0} = \frac{2}{kl} \sqrt{\frac{\gamma}{\pi(1-l^2)}} [(2-k^2) K(k) - 2E(k)] [E(l) - (1-l^2) K(l)],$$

$$A_{2,0} = \frac{2\sqrt{\gamma}}{9\sqrt{\pi} k^{3/2}(1-l^2)} [(16-16k^2+3k^4) K(k) - 8(2-k^2) E(k)] \times \\ \times [(1-l^2)(2-3l^2) K(l) - 2(1-2l^2) E(l)],$$

$$A_{1,1} = \frac{2\sqrt{\gamma}}{3kl\sqrt{\pi(1-k^2)(1-l^2)}} [(2-k^2) E(k) - 2(1-k^2) K(k)] \times \\ \times [(1-l^2) K(l) - (1-2l^2) E(l)],$$

$$A_{2,1} = \frac{2\sqrt{\gamma}}{15\sqrt{\pi} k^{3/2}(1-l^2)\sqrt{1-k^2}} [(16-16k^2+k^4) E(k) - \\ - 8(1-k^2)(2-k^2) K(k)] [2(1-l^2+l^4) E(l) - (1-l^2)(2-l^2) K(l)],$$

$$A_{2,2} = \frac{2\sqrt{\gamma}}{105\sqrt{\pi} k^{3/2}(1-k^2)(1-l^2)} [(1-k^2)(16-16k^2-k^4) K(k) - \\ - 2(2-k^2)(4-4k^2-k^4) E(k)] [(1-l^2)(2+5l^2-8l^4) K(l) - \\ - 2(1-2l^2)(1+4l^2-4l^4) E(l)].$$

$$2. \int_0^\infty x e^{-px^2} J_0(cx) K_0(cx) dx = \frac{\pi}{8p} H_0\left(\frac{c^2}{2p}\right) - \frac{\pi}{8p} Y_0\left(\frac{c^2}{2p}\right) \quad [\operatorname{Re} p > 0].$$

$$3. \int_0^\infty e^{cx} J_0(b\sqrt{x}) K_0(cx) dx = \frac{1}{b\sqrt{\pi^3 c}} \exp\left(\frac{b^2}{8c}\right) \operatorname{erfc}\left(\frac{b}{\sqrt{8c}}\right) \\ [b, \operatorname{Re} c > 0]$$

$$4. \int_0^\infty x^{\alpha-1} e^{\mp cx} J_\mu(b\sqrt{x}) K_\nu(cx) dx = \\ = \left\{ \frac{1}{\cos \nu\pi \sec(\alpha + \mu/2)\pi} \right\} 2^{-\alpha-3/2} \sqrt{\pi} b^\mu c^{-\alpha-\mu/2} \Gamma\left[\frac{\mu/2+\alpha+\nu}{\mu+1}, \frac{\mu/2+\alpha-\nu}{\alpha+(\mu+1)/2}\right] \times \\ \times {}_2F_2\left(\frac{\mu}{2}+\alpha+\nu, \frac{\mu}{2}+\alpha-\nu; \mu+1, \alpha+\frac{\mu+1}{2}; \mp \frac{b^2}{8c}\right) + \\ + \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \sqrt{\frac{\pi}{c}} 2^{2\alpha-3/2} b^{1-2\alpha} \Gamma\left[\frac{\alpha+(\mu-1)/2}{(3+\mu)/2-\alpha}\right] \times \\ \times {}_2F_2\left(\frac{1}{2}+\nu, \frac{1}{2}-\nu; \frac{3-\mu}{2}-\alpha, \frac{3+\mu}{2}-\alpha; \frac{b^2}{8c}\right) \\ \left[\operatorname{Re}(2\alpha+\mu) > 2 + |\operatorname{Re} \nu|, \left\{ \begin{matrix} \operatorname{Re} c > 0 \\ b, \operatorname{Re} c > 0; \operatorname{Re} \alpha < 5/4 \end{matrix} \right\} \right].$$

2.16.24. Интегралы от $x^\alpha \left\{ \begin{matrix} \operatorname{sh} ax \\ \operatorname{ch} ax \end{matrix} \right\} J_\mu(bx) K_\nu(cx)$.

$$\int_0^\infty \operatorname{sh} ax J_0(bx) K_1(cx) dx = \frac{1}{c} \left[u E(k) - K(k) E(u) + \frac{\operatorname{sn}(u) \operatorname{dn}(u)}{\operatorname{cn}(u)} K(k) \right]$$

$$[\operatorname{Re} c > |\operatorname{Re} a| + |\operatorname{Im} b|;$$

$$\operatorname{cn} u = \frac{b\sqrt{2}}{[\sqrt{(c^2-a^2-b^2)^2+4b^2c^2}-(c^2-a^2-b^2)]^{1/2}},$$

$$k = \frac{1}{\sqrt{2}} \left(1 - \frac{c^2-a^2-b^2}{\sqrt{(c^2-a^2-b^2)^2+4b^2c^2}} \right)^{1/2} \Big].$$

$$2. \int_0^{\infty} x \operatorname{sh} ax J_n(bx) K_n(cx) dx = A_n$$

[Re $c > |\operatorname{Re} a| + |\operatorname{Im} b|$; k см. в 2.16.24.1].

$$A_0 = a \left(\frac{k}{bc} \right)^{3/2} (1 - k^2)^{3/4} [2E(k) - K(k)],$$

$$A_1 = \frac{a \sqrt{k}}{(bc)^{3/2}} (1 - k^2)^{1/4} [(1 - k^2) K(k) - (1 - 2k^2) E(k)],$$

$$A_2 = \frac{a}{\sqrt{k} (bc)^{3/2}} (1 - k^2)^{-1/4} [2(1 - k^2 + k^4) E(k) - (1 - k^2)(2 - k^2) K(k)],$$

$$A_3 = \frac{a}{3 (bck)^{3/2}} (1 - k^2)^{-3/4} [(8 - 15k^2 + 3k^4) (1 - k^2) K(k) - (8 - 19k^2 + 9k^4 - 6k^6) E(k)].$$

$$3. \int_0^{\infty} \operatorname{ch} ax J_n(bx) K_n(cx) dx = B_n \quad [\operatorname{Re} c > |\operatorname{Re} a| + |\operatorname{Im} b|; k \text{ см. в 2.16.24.1}],$$

$$B_0 = \sqrt{\frac{k}{bc}} (1 - k^2)^{1/4} K(k), \quad B_1 = \frac{1}{\sqrt{bck}} (1 - k^2)^{-1/4} [E(k) - (1 - k^2) K(k)],$$

$$B_2 = \frac{1}{3 \sqrt{bck^3}} (1 - k^2)^{-3/4} [(1 - k^2) (2 - 3k^2) K(k) - 2(1 - 2k^2) E(k)],$$

$$B_3 = \frac{1}{15 \sqrt{bck^5}} (1 - k^2)^{-5/4} [(8 - 23k^2 + 23k^4) E(k) - (1 - k^2) (8 - 19k^2 + 15k^4) K(k)].$$

$$4. \int_0^{\infty} x^{1/2} \operatorname{sh} cx J_{\nu+1/2}(bx) K_{\nu}(cx) dx = \frac{\sqrt{\pi} c^{1-\nu}}{2^{\nu} b} (b^2 + 4c^2)^{-1/2} (\sqrt{b^2 + 4c^2} + b)^{\nu-1/2} \quad [b, \operatorname{Re} c > 0; \operatorname{Re} \nu > -3/2].$$

2.16.25. Интегралы от $x^{\alpha} \begin{cases} \sin ax^n \\ \cos ax^n \end{cases} J_{\nu}(bx) K_{\nu}(cx)$.

$$1. \int_0^{\infty} x \sin ax J_n(bx) K_n(cx) dx = A_n$$

$$\left[\operatorname{Re} c > |\operatorname{Im} a| + |\operatorname{Im} b|; k = \frac{1}{\sqrt{2}} \left(1 - \frac{a^2 + c^2 - b^2}{\sqrt{(a^2 + c^2 - b^2)^2 + 4b^2 c^2}} \right)^{1/2}; A_n \text{ см. в 2.16.24.2} \right].$$

$$2. \int_0^{\infty} \cos ax J_n(bx) K_n(cx) dx = B_n$$

[Re $c > |\operatorname{Im} a| + |\operatorname{Im} b|$; k см. в 2.16.25.1; B_n см. в 2.16.24.3].

$$3. \int_0^{\infty} x^{\alpha-1} \begin{cases} \sin ax^2 \\ \cos ax^2 \end{cases} J_{\nu}(cx) K_{\nu}(cx) dx = \frac{a^{-\alpha/2}}{4^{\nu}} \Gamma\left(\frac{\alpha}{2}\right) \begin{cases} \sin(\alpha\pi/4) \\ \cos(\alpha\pi/4) \end{cases} \times \\ \times {}_2F_3\left(\frac{\alpha+2}{4}, \frac{\alpha}{4}; \frac{1}{2}, 1-\frac{\nu}{2}, 1+\frac{\nu}{2}; \frac{c^4}{16a^2}\right) \mp$$

$$\mp \frac{a^{-\alpha/2-1}c^2}{8(v^2-1)} \Gamma\left(\frac{\alpha}{2}+1\right) \left\{ \frac{\cos(\alpha\pi/4)}{\sin(\alpha\pi/4)} \right\} \times \\ \times {}_2F_3\left(1+\frac{\alpha}{4}, \frac{2+\alpha}{4}; \frac{3}{2}, \frac{3-v}{2}, \frac{3+v}{2}; \frac{c^4}{16a^2}\right) + \\ + 2^{2v-2}a^{-\alpha/2-v}c^{2v}\Gamma\left[\frac{-v}{v+1}, \frac{v+\alpha/2}{v+1}\right] \left\{ \frac{\sin[(2v+\alpha)\pi/4]}{\cos[(2v+\alpha)\pi/4]} \right\} \times \\ \times {}_2F_3\left(\frac{2v+\alpha}{4}, \frac{1+v}{2}+\frac{\alpha}{4}; 1+\frac{v}{2}, \frac{1+v}{2}, 1+v; \frac{c^4}{16a^2}\right) \\ [a > 0; |\arg c| < \pi/4; \operatorname{Re} \alpha, \operatorname{Re}(\alpha+2v) > -1 \mp 1].$$

$$4. \int_0^\infty x \sin ax^2 J_0(cx) K_0(cx) dx = \frac{1}{4a} K_0\left(\frac{c^2}{2a}\right) \quad [a > 0; |\arg c| < \pi/4].$$

$$5. \int_0^\infty x \cos ax^2 J_0(cx) K_0(cx) dx = \frac{\pi}{8a} \left[I_0\left(\frac{c^2}{2a}\right) - L_0\left(\frac{c^2}{2a}\right) \right] \quad [a > 0; |\arg c| < \pi/4].$$

2.16.26. Интегралы содержащие $Y_0(cx) - \frac{2}{\pi} K_0(cx)$.

$$1. \int_a^\infty x^3 (x^4 - a^4)^{\beta-1} \left[Y_0(cx) - \frac{2}{\pi} K_0(cx) \right] dx = \\ = \frac{a^{4\beta} \operatorname{cosec} \beta\pi}{4\beta} {}_0F_3\left(\frac{1}{2}, \frac{1}{2}, \beta+1; \frac{a^4 c^4}{256}\right) - 2^{8\beta-2} c^{-4\beta} \Gamma\left[\frac{1}{2}-\beta, \frac{1}{2}-\beta\right] \times \\ \times {}_0F_3\left(\frac{1}{2}-\beta, \frac{1}{2}-\beta, 1-\beta; \frac{a^4 c^4}{256}\right) \quad [a, c > 0, 0 < \operatorname{Re} \beta < 3/8].$$

$$2. \int_0^\infty [(z^2 + ix^2)^{\rho-1} + (z^2 - ix^2)^{\rho-1}] \left[Y_0(cx) - \frac{2}{\pi} K_0(cx) \right] dx = \\ = -\frac{2^\rho}{\Gamma(1-\rho)} c^{-\rho} z^\rho \left[e^{-i\rho\pi/4} K_\rho(e^{i\pi/4} cz) + e^{i\rho\pi/4} K_\rho(e^{-i\pi/4} cz) \right] \\ [c, \operatorname{Re} z > 0, \operatorname{Re} \rho < 3/4].$$

$$3. \int_0^\infty \frac{x}{x^4 + z^4} \left[Y_0(cx) - \frac{2}{\pi} K_0(cx) \right] dx = -\frac{1}{z^2} \ker(cz) \\ [c > 0; |\arg z| < \pi/4]$$

$$4. \int_0^\infty \frac{x}{\sqrt{x^4 + z^4}} \left[Y_0(cx) - \frac{2}{\pi} K_0(cx) \right] dx = Y_0\left(\frac{cz}{\sqrt{2}}\right) K_0\left(\frac{cz}{\sqrt{2}}\right) \\ [c > 0; |\arg z| < \pi/4]$$

$$5. \int_0^\infty x \frac{(\sqrt{x^4 + z^4} + x^2)^{1/2}}{\sqrt{x^4 + z^4}} \left[Y_0(cx) - \frac{2}{\pi} K_0(cx) \right] dx = \\ = \frac{1}{c} \exp\left(-\frac{cz}{\sqrt{2}}\right) \left(\sin \frac{cz}{\sqrt{2}} - \cos \frac{cz}{\sqrt{2}} \right) \quad [c > 0; |\arg z| < \pi/4]$$

$$6. \int_0^\infty x \frac{(\sqrt{x^4 + z^4} + x^2)^v + (\sqrt{x^4 + z^4} - x^2)^v}{\sqrt{x^4 + z^4}} \left[Y_0(cx) - \frac{2}{\pi} K_0(cx) \right] dx = \\ = 2z^{2v} \cos \frac{v\pi}{2} K_v\left(\frac{cz}{\sqrt{2}}\right) \left[\sin \frac{v\pi}{2} J_v\left(\frac{cz}{\sqrt{2}}\right) + \cos \frac{v\pi}{2} Y_v\left(\frac{cz}{\sqrt{2}}\right) \right] \\ [c > 0, |\arg z| < \pi/4; |\operatorname{Re} v| < 3/4]$$

$$7. \int_0^{\infty} \frac{1}{x} \sin bx^2 \left[Y_0(cx) - \frac{2}{\pi} K_0(cx) \right] dx = \frac{1}{2} \operatorname{ci} \left(\frac{c^2}{4b} \right) \quad [b, c > 0]$$

$$8. \int_0^{\infty} \frac{1}{x^3} \sin^2 bx^2 \left[Y_0(cx) - \frac{2}{\pi} K_0(cx) \right] dx = \frac{b}{2} \operatorname{ci} \left(\frac{c^2}{8b} \right) + \frac{b}{2} \cos \frac{c^2}{8b} + \frac{c^2}{16} \operatorname{si} \left(\frac{c^2}{8b} \right) \quad [b, c > 0]$$

$$9. \int_0^{\infty} x e^{-px} \cos px \left[Y_0(cx) - \frac{2}{\pi} K_0(cx) \right] dx = \\ = -\sqrt{2} p (4p^4 + c^4)^{-3/4} \cos \left(\frac{3}{2} \operatorname{arctg} \frac{c^2}{2p^2} \right) \quad [c > 0, \quad |\arg p| < \pi/4]$$

$$10. \int_0^{\infty} e^{-px} \cos \left(px + \frac{\pi}{4} \right) \left[Y_0(cx) - \frac{2}{\pi} K_0(cx) \right] dx = \\ = -\frac{1}{\sqrt{2}} (4p^4 + c^4)^{-1/2} (\sqrt{4p^4 + c^4} + 2p^2)^{1/2} \quad [c > 0; \quad |\arg p| < \pi/4]$$

$$11. \int_0^1 x \ln x \left[Y_0(cx) - \frac{2}{\pi} K_0(cx) \right] dx = \frac{1}{c^2} Y_0(c) + \frac{2}{\pi c^2} K_0(c) \quad [|\arg c| < \pi]$$

$$12. \int_0^{\infty} x \ln \frac{x^4 + z^4}{x^4} \left[Y_0(cx) - \frac{2}{\pi} K_0(cx) \right] dx = \\ = \frac{2z}{c} [e^{-i\pi/4} K_1(e^{i\pi/4} cz) + e^{i\pi/4} K_1(e^{-i\pi/4} cz)] \quad [c > 0; \quad |\arg z| < \pi/4]$$

$$13. \int_0^{\infty} x \ln \frac{|x^4 - a^4|}{x^4} \left[Y_0(cx) - \frac{2}{\pi} K_0(cx) \right] dx = -\frac{\pi a}{c} J_1(ac) \quad [a, c > 0]$$

$$14. \int_0^{\infty} \frac{x}{\sqrt{x^4 + z^4}} \ln \frac{\sqrt{x^4 + z^4} + z^2}{x^2} \left[Y_0(cx) - \frac{2}{\pi} K_0(cx) \right] dx = -\frac{\pi^2}{2} J_0(cz) K_0(cz) \quad [c > 0; \quad |\arg z| < \pi/4]$$

2.16.27. Интегралы, содержащие $Y_\mu(bx^{\pm r}) K_\nu(cx)$.

$$1. \int_0^{\infty} Y_0(bx) K_0(cx) dx = -\frac{1}{\sqrt{b^2 + c^2}} K \left(\frac{c}{\sqrt{b^2 + c^2}} \right) \quad [\operatorname{Re} c > |\operatorname{Im} b|]$$

$$2. \int_0^{\infty} x^{\alpha-1} Y_\mu(bx) K_\nu(cx) dx = -\frac{2^{\alpha-2}}{\pi} b^{-\mu} c^\mu \alpha \times \\ \times \Gamma \left[\mu, \frac{\alpha + \nu - \mu}{2}, \frac{\alpha - \nu - \mu}{2} \right] {}_2F_1 \left(\frac{\alpha + \nu - \mu}{2}, \frac{\alpha - \nu - \mu}{2}; 1 - \mu; -\frac{b^2}{c^2} \right) - \\ - \frac{2^{\alpha-2} b^\mu}{\pi c^{\alpha+\mu}} \cos \mu \pi \Gamma \left[-\mu, \frac{\alpha + \mu + \nu}{2}, \frac{\alpha + \mu - \nu}{2} \right] {}_2F_1 \left(\frac{\alpha + \mu + \nu}{2}, \right. \\ \left. \frac{\alpha + \mu - \nu}{2}; 1 + \mu; -\frac{b^2}{c^2} \right) \quad [\operatorname{Re} c > |\operatorname{Im} b|, \operatorname{Re} \alpha > |\operatorname{Re} \mu| + |\operatorname{Re} \nu|]$$

$$3. \int_0^{\infty} x^{\alpha-1} Y_{\nu}(cx) K_{\nu}(cx) dx = -\frac{2^{\alpha-3}}{\pi c^{\alpha}} \cos \frac{\alpha-2\nu}{4} \pi \Gamma \left[\frac{\alpha}{2}, \frac{\alpha+2\nu}{4}, \frac{\alpha-2\nu}{4} \right]$$

[Re $\alpha > 2$; |Re ν |; |arg c | < $\pi/4$].

$$4. \int_0^{\infty} x Y_{\nu/2}(bx^2) K_{\nu}(cx) dx =$$

$$= \frac{\pi}{8b} \left[\operatorname{cosec} \frac{\nu\pi}{2} H_{-\nu/2} \left(\frac{c^2}{4b} \right) - \sec \frac{\nu\pi}{2} J_{-\nu/2} \left(\frac{c^2}{4b} \right) - 2 \operatorname{cosec} \nu\pi H_{\nu/2} \left(\frac{c^2}{4b} \right) \right]$$

[$b, \operatorname{Re} c > 0$; |Re ν | < 1].

$$5. \int_0^{\infty} Y_{2\nu}(b\sqrt{x}) K_{\nu}(cx) dx =$$

$$= \frac{\pi}{2c} \left[\operatorname{cosec} 2\nu\pi L_{-\nu} \left(\frac{b^2}{4c} \right) - \operatorname{ctg} 2\nu\pi L_{\nu} \left(\frac{b^2}{4c} \right) - \operatorname{tg} \nu\pi I_{\nu} \left(\frac{b^2}{4c} \right) - \frac{\sec \nu\pi}{\pi} K_{\nu} \left(\frac{b^2}{4c} \right) \right]$$

[Re $c > 0$; |Re ν | < 1/2]

$$6. \int_0^{\infty} x^{\alpha-1} Y_{\mu} \left(\frac{b}{x} \right) K_{\nu}(cx) dx =$$

$$= -\frac{2^{\alpha-2\mu-2b\mu}}{\pi c^{\alpha-\mu}} \cos \mu\pi \Gamma \left[-\mu, \frac{\alpha+\nu-\mu}{2}, \frac{\alpha-\nu-\mu}{2} \right] \times$$

$$\times {}_0F_3 \left(1+\mu, 1+\frac{\mu-\nu-\alpha}{2}, 1+\frac{\mu+\nu-\alpha}{2}; -\frac{b^2c^2}{16} \right) -$$

$$- \frac{2^{\alpha+2\mu-2}}{\pi b^{\mu} c^{\alpha+\mu}} \Gamma \left[\mu, \frac{\alpha+\mu+\nu}{2}, \frac{\alpha+\mu-\nu}{2} \right] \times$$

$$\times {}_0F_3 \left(1-\mu, 1-\frac{\alpha+\mu+\nu}{2}, 1-\frac{\alpha+\mu-\nu}{2}; -\frac{b^2c^2}{16} \right) -$$

$$- \frac{b^{\alpha+\nu} c^{\nu}}{2^{\alpha+2\nu+2\pi}} \cos \frac{\alpha+\mu+\nu}{2} \pi \Gamma \left[-\nu, \frac{\mu-\nu-\alpha}{2}, -\frac{\mu+\nu+\alpha}{2} \right] \times$$

$$\times {}_0F_3 \left(1+\nu, 1+\frac{\alpha-\mu+\nu}{2}, 1+\frac{\alpha+\mu+\nu}{2}; -\frac{b^2c^2}{16} \right) -$$

$$- \frac{b^{\alpha-\nu} c^{-\nu}}{2^{\alpha-2\nu+2\pi}} \cos \frac{\alpha+\mu-\nu}{2} \pi \Gamma \left[\nu, \frac{\mu+\nu-\alpha}{2}, \frac{\nu-\mu-\alpha}{2} \right] \times$$

$$\times {}_0F_3 \left(1-\nu, 1+\frac{\alpha-\mu-\nu}{2}, 1+\frac{\alpha+\mu-\nu}{2}; -\frac{b^2c^2}{16} \right)$$

[$b, \operatorname{Re} c > 0$; Re $\alpha > |\operatorname{Re} \nu| - 3/2$]

$$7. \int_0^{\infty} x^{-1 \pm 1} Y_{\nu} \left(\frac{b}{x} \right) K_{\nu}(cx) dx =$$

$$= 2 \left\{ \frac{c^{-1}}{b^{-1}} \right\} \left[\pm \left\{ \frac{\sin(3\nu\pi/2)}{\cos(3\nu\pi/2)} \right\} \operatorname{kei}_{2\nu}(2\sqrt{bc}) - \left\{ \frac{\cos(3\nu\pi/2)}{\sin(3\nu\pi/2)} \right\} \operatorname{ker}_{2\nu}(2\sqrt{bc}) \right] =$$

$$= \left\{ \frac{c^{-1}}{-ib^{-1}} \right\} \left[\mp e^{-\pi\nu i/2} K_{2\nu}(2\sqrt{-ibc}) - e^{\pi\nu i/2} K_{2\nu}(2\sqrt{ibc}) \right]$$

[$b, \operatorname{Re} c > 0$; |Re ν | < 3/2 $\pm$ 1].

$$8. \int_0^{\infty} x^{2\nu \pm 1 - 1/2} Y_{\nu+1/2} \left(\frac{b}{x} \right) K_{\nu}(cx) dx =$$

$$= \sqrt{2\pi} b^{\nu \pm 1/2} c^{-\nu - (1 \pm 1)/2} Y_{2\nu + (1 \pm 1)/2}(\sqrt{2bc}) K_{2\nu + (1 \pm 1)/2}(\sqrt{2bc})$$

[$b, \operatorname{Re} c > 0, \operatorname{Re} \nu > (-2 \mp 1)/3$]

$$9. \int_0^{\infty} x^{-2\nu+1/2} Y_{\nu-1/2} \left(\frac{b}{x} \right) K_{\nu}(cx) dx = \\ = \sqrt{\frac{\pi}{2}} \frac{b^{1/2-\nu} c^{\nu-1}}{\cos \nu\pi} K_{2\nu-1}(\sqrt{2bc}) [J_{1-2\nu}(\sqrt{2bc}) - J_{2\nu-1}(\sqrt{2bc})] \\ [b, \operatorname{Re} c > 0; \operatorname{Re} \nu < 1].$$

$$10. \int_0^{\infty} x^{\nu-1/2} e^{cx} Y_{\nu}(b\sqrt{x}) K_{\nu}(cx) dx = \\ = -\frac{2\sqrt{\pi} (2c)^{-\nu}}{b} \Gamma \left[\begin{matrix} 1/2+2\nu, & 1/2+2\nu \\ 1+2\nu \end{matrix} \right] \exp \left(\frac{b^2}{16} \right) W_{2\nu, 0} \left(\frac{b^2}{8c} \right) \\ [b, \operatorname{Re} c > 0, -1/4 < \operatorname{Re} \nu < 3/4]$$

$$11. \int_0^{\infty} x^{-1/2} e^{cx} Y_0(b\sqrt{x}) K_{\nu}(cx) dx = -\frac{\pi}{\sqrt{2c}} \exp \left(\frac{b^2}{16c} \right) K_0 \left(\frac{b^2}{16c} \right) \\ [b, \operatorname{Re} c > 0].$$

$$12. \int_0^{\infty} x^{\alpha-1} \left\{ \begin{matrix} \sin \alpha x^2 \\ \cos \alpha x^2 \end{matrix} \right\} Y_{\nu}(cx) K_{\nu}(cx) dx = \\ = \frac{a^{-\alpha/2}}{4\pi\nu} \operatorname{ctg} \frac{\nu\pi}{2} \left\{ \begin{matrix} \sin(\alpha\pi/4) \\ \cos(\alpha\pi/4) \end{matrix} \right\} \Gamma \left(\frac{\alpha}{2} \right) {}_2F_3 \left(\frac{\alpha}{4}, \frac{\alpha+2}{4}; \frac{1}{2}, 1-\frac{\nu}{2}, 1+\frac{\nu}{2}; \frac{c^4}{16a^2} \right) \mp \\ \mp \frac{c^2}{8\sqrt{\pi} a^{\alpha/2+1} (1-\nu^2)} \operatorname{tg} \frac{\nu\pi}{2} \left\{ \begin{matrix} \cos(\alpha\pi/4) \\ \sin \alpha\pi/4 \end{matrix} \right\} \Gamma \left(1+\frac{\alpha}{2} \right) \times \\ \times {}_2F_3 \left(\frac{2+\alpha}{4}, 1+\frac{\alpha}{4}; \frac{3}{2}, \frac{3-\nu}{2}, \frac{3+\nu}{2}; \frac{c^4}{16a^2} \right) - \\ - \frac{2^{-2\nu-2}}{\pi} a^{-\nu-\alpha/2} c^{2\nu} \cos \nu\pi \Gamma \left[-\nu, -\nu, \nu+\frac{\alpha}{2} \right] \times \\ \times {}_2F_3 \left(\frac{1+\nu}{2} + \frac{\alpha}{4}, \frac{2\nu+\alpha}{4}; 1+\frac{\nu}{2}, \frac{1+\nu}{2}, 1+\nu, \frac{c^4}{16a^2} \right) - \\ - \frac{2^{\nu-2}}{\pi} a^{\nu-\alpha/2} c^{-2\nu} \left\{ \begin{matrix} \sin [(\alpha-2\nu)\pi/4] \\ \cos [(\alpha-2\nu)\pi/4] \end{matrix} \right\} \Gamma \left[\nu, \nu, \frac{\alpha}{2}-\nu \right] \times \\ \times {}_2F_3 \left(\frac{1-\nu}{2} + \frac{\alpha}{4}, \frac{\alpha-2\nu}{4}; 1-\frac{\nu}{2}, \frac{1-\nu}{2}, 1-\nu; \frac{c^4}{16a^2} \right) \\ [a > 0, |\arg c| < \pi/4, \operatorname{Re} \alpha > 2, |\operatorname{Re} \nu| - 1 \mp 1]$$

$$13. \int_0^{\infty} x^{1+2\sigma} \cos \alpha x^2 Y_{\sigma}(cx) K_{\sigma}(cx) dx = -\frac{c^{2\sigma}}{2^{2+\sigma} a^{1+2\sigma}} K_0 \left(\frac{c^2}{2a} \right) \\ [\sigma = 0 \text{ или } 1, a > 0, |\arg c| < \pi/4]$$

2.16.28. Интегралы от $A(x) I_{\mu}(\rho(x)) K_{\nu}(\chi(x))$

$$1. \int_0^{\infty} x^{\alpha-1} I_{\mu}(bx) K_{\nu}(cx) dx = B_{\mu, \nu}^{\alpha} \quad [\operatorname{Re} c > \operatorname{Re} b; \operatorname{Re}(\alpha + \mu) > |\operatorname{Re} \nu|,$$

$B_{\mu, \nu}^{\alpha} = i^{-\mu} A_{\mu, \nu}^{\alpha}(ib), A_{\mu, \nu}^{\alpha}(b) = A_{\mu, \nu}^{\alpha}$ см. в 2.16.21.1, $B_{m, n}^{\alpha}$ получаются из $A_{m, n}^{\alpha}$ с помощью формул

$$K(z) = \frac{1}{\sqrt{1-z^2}} K \left(\frac{iz}{\sqrt{1-z^2}} \right), \quad E(z) = \sqrt{1-z^2} E \left(\frac{iz}{\sqrt{1-z^2}} \right),$$

$$B_{\mu, \nu}^{\alpha} = 2^{\alpha-2} b^{\mu} c^{-\alpha-\mu} \Gamma \left[\frac{(\alpha+\mu+\nu)/2, (\alpha+\mu-\nu)/2}{\mu+1} \right] \times \\ \times {}_2F_1 \left(\frac{\alpha+\mu+\nu}{2}, \frac{\alpha+\mu-\nu}{2}; \mu+1; \frac{b^2}{c^2} \right),$$

$$B_{\nu, \nu}^{\alpha} = 2^{\alpha-2} (c^2 - b^2)^{-\alpha/2} \Gamma \left(\frac{\alpha}{2} \right) \Gamma \left(\nu + \frac{\alpha}{2} \right) P_{-\alpha/2}^{-\nu} \left(\frac{c^2 + b^2}{c^2 - b^2} \right) = \\ = \frac{2^{\alpha-2}}{\sqrt{\pi b c}} (c^2 - b^2)^{(1-\alpha)/2} \Gamma \left(\frac{\alpha}{2} \right) \exp \left(-i \frac{\alpha-1}{2} \pi \right) Q_{\nu-1/2}^{(\alpha-1)/2} \left(\frac{c^2 + b^2}{2cb} \right),$$

$$B_{0, 1}^1 = \frac{1}{c} K \left(\frac{b}{c} \right), \quad B_0^1 = \frac{1}{c(c^2 - b^2)} \left[\frac{2c^2}{c^2 - b^2} E \left(\frac{b}{c} \right) - K \left(\frac{b}{c} \right) \right],$$

$$B_{1, 0}^0 = \frac{c}{b} E \left(\frac{b}{c} \right) - \frac{c^2 - b^2}{bc} K \left(\frac{b}{c} \right), \quad B_{1, 0}^1 = -\frac{1}{2b} \ln \left(1 - \frac{b^2}{c^2} \right),$$

$$B_{1, 1}^1 = \frac{2b}{(c^2 - b^2)}, \quad B_{0, 1}^2 = \frac{1}{c^2 - b^2} E \left(\frac{b}{c} \right),$$

$$B_{0, 1}^3 = \frac{2c}{(c^2 - b^2)^2}, \quad B_{1, 1}^1 = \frac{1}{b^2} \left[K \left(\frac{b}{c} \right) - E \left(\frac{b}{c} \right) \right], \quad B_{1, 1}^2 = \frac{b}{c(c^2 - b^2)},$$

$$B_{1, 1}^3 = \frac{b}{(c^2 - b^2)^2} \left\{ E \left(\frac{b}{c} \right) + K \left(\frac{b}{c} \right) - \frac{c^2}{b^2} \left[K \left(\frac{b}{c} \right) - E \left(\frac{b}{c} \right) \right] \right\}.$$

$$2. \int_0^{\infty} \frac{1}{x} [I_0(bx) - 1] K_1(cx) dx = \frac{\pi}{2} - E \left(\frac{b}{c} \right) \quad [\operatorname{Re} c > |\operatorname{Re} b|]$$

$$3. \int_0^{\infty} x^{\alpha-1} I_{\mu}(cx) K_{\nu}(cx) dx = \frac{2^{\alpha-2}}{c^{\alpha}} \Gamma \left[\frac{(\alpha+\mu+\nu)/2, (\alpha+\mu-\nu)/2, 1-\alpha}{1+(\mu+\nu-\alpha)/2, 1+(\mu-\nu-\alpha)/2} \right] \\ [\operatorname{Re} c > 0, |\operatorname{Re} \nu| - \operatorname{Re} \mu < \operatorname{Re} \alpha < 1]$$

$$4. \int_0^{\infty} \frac{1}{x} \left[I_{\nu}(cx) - \left(\frac{cx}{2} \right)^{\nu} \frac{1}{\Gamma(\nu+1)} \right] K_{\nu}(cx) dx = \frac{1}{4\nu} \left[\psi(\nu) + \gamma + \frac{1}{\nu} \right] \\ [\operatorname{Re} c > 0, \operatorname{Re} \nu > -1]$$

$$5. \int_0^{\infty} I_{2\nu}(b\sqrt{x}) K_{\nu}(cx) dx = \frac{\pi}{2c} I_{\nu} \left(\frac{b^2}{4c} \right) + \frac{\pi}{2c} L_{\nu} \left(\frac{b^2}{4c} \right) \\ [\operatorname{Re} c > 0; \operatorname{Re} \nu > -1/2]$$

$$6. \int_0^{\infty} x^{\alpha-1} I_{\mu}(b\sqrt{x}) K_{\nu}(cx) dx = \\ = 2^{\alpha-\mu/2-2} b^{\mu} c^{-\mu/2-\alpha} \Gamma \left[\frac{(\alpha+\nu)/2 + \mu/4, (\alpha-\nu)/2 + \mu/4}{\mu+1} \right] \times \\ \times {}_2F_3 \left(\frac{\alpha+\nu}{2} + \frac{\mu}{4}, \frac{\alpha-\nu}{2} + \frac{\mu}{4}; \frac{1}{2}, \frac{\mu+1}{2}, \frac{\mu}{2} + 1; \frac{b^4}{64c^2} \right) + \\ + 2^{\alpha-\mu/2-3} b^{\mu+2} c^{-\mu/2-1} \Gamma \left[\frac{(\alpha+\nu+1)/2 + \mu/4, (\alpha-\nu+1)/2 + \mu/4}{\mu+2} \right] \times \\ \times {}_2F_3 \left(\frac{\alpha+\nu+1}{2} + \frac{\mu}{4}, \frac{\alpha-\nu+1}{2} + \frac{\mu}{4}; \frac{3}{2}, \frac{\mu}{2} + 1, \frac{\mu+3}{2}; \frac{b^4}{64c^2} \right) \\ [\operatorname{Re} c > 0, \operatorname{Re} \mu > 2, |\operatorname{Re} \nu| - 2 \operatorname{Re} \alpha]$$

$$7. \int_0^{\infty} x^{1/2} I_{2\nu-1}(b\sqrt{x}) K_{\nu}(cx) dx = \frac{\pi b}{4c^2} \left[I_{\nu-1}\left(\frac{b^2}{4c}\right) + L_{\nu-1}\left(\frac{b^2}{4c}\right) \right]$$

[Re $c > 0$; Re $\nu > -1/2$].

$$8. \int_a^{\infty} x^{-\nu} (x^2 - a^2)^{\mu/2} I_{\mu}(b\sqrt{x^2 - a^2}) K_{\nu}(cx) dx =$$

$$= a^{\mu-\nu+1} b^{\mu} c^{-\nu} (c^2 - b^2)^{(\nu-\mu-1)/2} K_{\nu-\mu-1}(a\sqrt{c^2 - b^2})$$

[$a > 0$; Re $c > |\text{Re } b|$; Re $\mu > -1$].

$$9. \int_a^{\infty} x^{1-\nu} (x^2 - a^2)^{\mu/2} I_{\mu}(c\sqrt{x^2 - a^2}) K_{\nu}(cx) dx =$$

$$= 2^{2\nu-2\mu-3} \Gamma(\nu - \mu - 1) a^{3(\mu-\nu)/2+1} c^{3(\mu-\nu+1)/2}$$

[$a, \text{Re } c > 0$; $-1 < \text{Re } \mu < \text{Re } \nu - 1$]

$$10. \int_0^{\infty} \frac{x^{\mu+1}}{(x^2 - y^2)^{\nu/2}} I_{\mu}(bx) K_{\nu}(c\sqrt{x^2 - y^2}) dx =$$

$$= \frac{\pi b^{\mu}}{2c^{\nu}} y^{1+\mu-\nu} e^{-\pi i(\nu-\mu-1/2)} (c^2 - b^2)^{(\nu-\mu-1)/2} H_{\nu-\mu-1}^{(2)}(y\sqrt{c^2 - b^2})$$

[$y > 0$; Re $c > |\text{Re } b|$; Re $\mu > -1$; Re $\nu < 1$; arg $\sqrt{x^2 - y^2} = 0$ при $x > y$,
arg $(x^2 - y^2)^{\varphi} = \pi\varphi$ при $x < y$].

$$11. \int_0^{\infty} x^{\alpha-1} I_{\nu}(c\sqrt{x^2 + z^2} - cz) K_{\nu}(c\sqrt{x^2 + z^2} + cz) dx =$$

$$= \frac{1}{2\sqrt{\pi}} \left(\frac{z}{c}\right)^{\alpha/2} \Gamma\left[\frac{\nu+\alpha/2, (1-\alpha)/2}{1+\nu-\alpha/2}\right] K_{\alpha/2}(2cz)$$

[Re $c > 0$; $-2 \text{Re } \nu < \text{Re } \alpha < 1$].

2.16.29. Интегралы от $x^{\alpha} e^{-px^n} I_{\mu}(bx^r) K_{\nu}(cx)$.

$$1. \int_0^{\infty} x^{-1/2} e^{-px} I_{\mu}(bx) K_{\nu}(cx) dx =$$

$$= \frac{\sqrt{d} \cos \mu\pi}{\cos(\mu+\nu)\pi} \Gamma\left(\mu - \nu + \frac{1}{2}\right) p_{\nu-1/2}^{-\mu} (\sqrt{1+b^2d^2}) Q_{\mu-1/2}^{-\nu} (\sqrt{1+c^2d^2})$$

[($1+b^2d^2$)($1+c^2d^2$) = p^2d^2 ; Re $\mu > |\text{Re } \nu| - 1/2$; Re $(p+c-b) > 0$]

$$2. \int_0^{\infty} x^{\alpha-1} e^{-bx} I_{\mu}(bx) K_{\nu}(cx) dx =$$

$$= \frac{2^{\alpha-\mu} b^{\mu}}{c^{\alpha+\mu}} \Gamma\left[\frac{(\alpha+\mu+\nu)/2, (\alpha+\mu-\nu)/2}{\mu+1}\right] \times$$

$$\times {}_4F_3\left(\frac{\alpha+\mu+\nu}{2}, \frac{\alpha+\mu-\nu}{2}, \frac{2\mu+1}{4}, \frac{2\mu+3}{4}; \mu+\frac{1}{2}, \mu+1, \frac{1}{2}; \frac{4b^2}{c^2}\right) -$$

$$- \frac{2^{\alpha-1} b^{\mu+1}}{c^{\alpha+\mu+1}} \Gamma\left[\frac{(\alpha+\mu+\nu+1)/2, (\alpha+\mu-\nu+1)/2}{\mu+1}\right] \times$$

$$\times {}_4F_3\left(\frac{\alpha+\mu+\nu+1}{2}, \frac{\alpha+\mu-\nu+1}{2}, \frac{2\mu+3}{4}, \frac{2\mu+5}{4}; \mu+1, \mu+\frac{3}{2}, \frac{3}{2}; \frac{4b^2}{c^2}\right)$$

[Re $b, \text{Re } c > 0$; Re $(\alpha+\mu) > |\text{Re } \nu|$].

$$3. \int_0^{\infty} e^{-px} [I_0(bx) K_0(cx) + I_0(cx) K_0(bx)] dx = \\ = \frac{2}{\sqrt{p^2 - (b-c)^2}} K \left(\sqrt{\frac{p^2 - (b+c)^2}{p^2 - (b-c)^2}} \right) \quad [\operatorname{Re} p > |\operatorname{Re}(b-c)|].$$

$$4. \int_0^{\infty} x^{\alpha-1} e^{-bx \pm cx} I_{\mu}(bx) K_{\nu}(cx) dx = \\ = \frac{\sqrt{\pi} b^{\mu}}{2^{\alpha+2\mu} \Gamma(\alpha+\mu)} \Gamma \left[\begin{matrix} \alpha+\mu+\nu, \alpha+\mu-\nu \\ \alpha+\mu+1/2 \end{matrix} \right] \left\{ \frac{\cos \nu\pi \sec(\alpha+\mu)\pi}{1} \right\} \times \\ \times {}_3F_2 \left(\begin{matrix} 1/2+\mu, \alpha+\mu+\nu, \alpha+\mu-\nu \\ 1/2+\alpha+\mu, 2\mu+1 \end{matrix}; \pm \frac{b}{c} \right) + \\ + \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{b^{1/2-\alpha}}{2^{\alpha} \sqrt{c}} \Gamma \left[\begin{matrix} \mu+\alpha-1/2, 1-\alpha \\ 3/2+\mu-\alpha \end{matrix} \right] {}_3F_2 \left(\begin{matrix} 1/2+\nu, 1/2-\nu, 1-\alpha \\ 3/2-\alpha-\mu, 3/2+\mu-\alpha \end{matrix}; \pm \frac{b}{c} \right) \\ \left[\operatorname{Re} b > 0; \operatorname{Re}(\alpha+\mu) > |\operatorname{Re} \nu|, \left\{ \begin{matrix} |\arg c| < \pi; \operatorname{Re} \alpha < 1 \\ \operatorname{Re} c > 0 \end{matrix} \right\} \right].$$

$$5. \int_0^{\infty} x^{-3/2} e^{-(b+c)x} I_{\mu}(bx) K_0(cx) dx = \sqrt{\frac{\pi c}{2}} \left[\Gamma \left(\mu + \frac{1}{2} \right) P_{-1/2}^{-\mu} \left(\frac{b+c}{c} \right) \right]^2 \\ [\operatorname{Re} \mu > 1/2, \operatorname{Re} c > 0]$$

$$6. \int_0^{\infty} x^{\alpha-1} e^{-px} I_{\mu}(cx) K_{\nu}(cx) dx = \\ = 2^{-\mu-\nu-1} c^{\mu+\nu} p^{-\alpha-\mu-\nu} \Gamma \left[\begin{matrix} -\nu, \alpha+\mu+\nu \\ 1+\mu \end{matrix} \right] \times \\ \times {}_4F_3 \left(\begin{matrix} \frac{\mu+\nu+1}{2}, \frac{\mu+\nu}{2}+1, \frac{\alpha+\mu+\nu}{2}, \frac{\alpha+\mu+\nu+1}{2} \\ 1+\nu, 1+\mu, 1+\mu+\nu \end{matrix}; \frac{4c^2}{p^2} \right) + 2^{\nu-\mu-1} c^{\mu-\nu} p^{\nu-\mu-\alpha} \Gamma \left[\begin{matrix} \nu, \alpha+\mu-\nu \\ 1+\mu \end{matrix} \right] \times \\ \times {}_4F_3 \left(\begin{matrix} \frac{1+\mu-\nu}{2}, 1+\frac{\mu-\nu}{2}, \frac{\alpha+\mu-\nu}{2}, \frac{\alpha+\mu-\nu+1}{2} \\ 1-\nu, 1+\mu, 1+\mu-\nu \end{matrix}; \frac{4c^2}{p^2} \right) \\ [\operatorname{Re} c, \operatorname{Re} p > 0; \operatorname{Re}(\alpha+\mu) > |\operatorname{Re} \nu|].$$

$$7. \int_0^{\infty} e^{-px} [I_{\nu}(cx) + I_{-\nu}(cx)] K_{\nu}(cx) dx = \frac{\pi \sec \nu\pi}{2c} P_{\nu-1/2} \left(\frac{p^2 - 2c^2}{2c^2} \right) \\ [\operatorname{Re} p, \operatorname{Re} c > 0; |\operatorname{Re} \nu| < 1/2]$$

$$8. \int_0^{\infty} x^{\alpha-1} e^{-px} I_{\nu}(cx) K_{\nu}(cx) dx = A_{\nu}^{\alpha} \quad [\operatorname{Re} c, \operatorname{Re} p, \operatorname{Re} \alpha, \operatorname{Re}(\alpha+2\nu) > 0].$$

$$A_{\nu}^{2\nu+1} = \frac{\pi c^{-2\nu-1}}{4 \cos^2 \nu\pi} P_{\nu-1/2} \left(\frac{p^2 - 2c^2}{2c^2} \right) \quad [\operatorname{Re} \nu > -1/4].$$

$$A_{\nu}^{1/2} = -\frac{1}{2c} \sqrt{\frac{\pi p}{2}} \left[P_{\nu-1/2}^{-1/4}(z) Q_{\nu-1/2}^{1/4}(z) - P_{\nu-1/2}^{1/4}(z) Q_{\nu-1/2}^{-1/4}(z) \right] \\ [z = \sqrt{4c^2 - p^2}/(2c); \operatorname{Re} \nu > -1/4],$$

$$A_0^1 = \frac{2}{p+2c} K(k),$$

$$A_0^2 = \frac{1}{p(4c^2 - p^2)} [(p + 2c) E(k) - 2pK(k)],$$

$$A_0^3 = \frac{1}{p^2(2c + p)(2c - p)^2} [(4c^2 - 3p^2) E(k) + 4p(p - c) K(k)]$$

[k = (p - 2c)/(p + 2c)]

$$9. \int_0^\infty x e^{-px^2} [I_0(bx) K_0(cx) + I_0(cx) K_0(bx)] dx = \frac{1}{2p} \exp\left(\frac{b^2 + c^2}{4p}\right) K_0\left(\frac{bc}{2p}\right)$$

[Re p > 0]

$$10. \int_0^\infty x^{\alpha-1} e^{-px^2} I_\mu(cx) K_\nu(cx) dx =$$

$$= 2^{-\mu-\nu-2} c^{\mu+\nu} p^{-(\alpha+\mu+\nu)/2} \Gamma\left[-\nu, \frac{(\alpha+\mu+\nu)/2}{1+\mu}\right] \times$$

$$\times {}_3F_3\left(\frac{1+\mu+\nu}{2}, 1+\frac{\mu+\nu}{2}, \frac{\alpha+\mu+\nu}{2}; 1+\nu, 1+\mu, 1+\mu+\nu; \frac{c^2}{p}\right) +$$

$$+ 2^{\nu-\mu-2} c^{\mu-\nu} p^{(\nu-\mu-\alpha)/2} \Gamma\left[\nu, \frac{(\alpha+\mu-\nu)/2}{1+\mu}\right] \times$$

$$\times {}_3F_3\left(\frac{1+\mu-\nu}{2}, 1+\frac{\mu-\nu}{2}, \frac{\alpha+\mu-\nu}{2}; 1-\nu, 1+\mu-\nu, 1+\mu; \frac{c^2}{p}\right)$$

[Re p > 0; Re(\alpha + \mu) > |Re \nu|]

$$11. \int_0^\infty x^{2\nu+1} e^{-px^2} I_\nu(cx) K_\nu(cx) dx =$$

$$= \frac{c^{\nu-1}}{4p(3\nu+1)/2} \Gamma\left(\nu + \frac{1}{2}\right) \exp\left(\frac{c^2}{2p}\right) W_{-\nu/2, \nu/2}\left(\frac{c^2}{p}\right)$$

[Re p > 0; Re \nu > -1/2]

$$12. \int_0^\infty x^{\alpha-1} e^{-bx^2} I_\mu(bx^2) K_\nu(cx) dx =$$

$$= \frac{2^{-(\alpha+3\nu)/2-2}}{\sqrt{\pi}} c^\nu p^{-(\alpha+\nu)/2} \Gamma\left[-\nu, \frac{(1-\alpha-\nu)/2, \mu+(\alpha+\nu)/2}{1+\mu-(\alpha+\nu)/2}\right] \times$$

$$\times {}_3F_3\left(\frac{\alpha+\nu}{2}-\mu, \frac{\alpha+\nu}{2}+\mu; 1+\nu, \frac{1+\alpha+\nu}{2}; \frac{c^2}{8b}\right) +$$

$$+ \frac{2^{(3\nu-\alpha)/2-2}}{\sqrt{\pi}} b^{(\nu-\alpha)/2} c^{-\nu} \Gamma\left[\nu, \frac{(1+\nu-\alpha)/2, \mu+(\alpha-\nu)/2}{1+\mu+(\nu-\alpha)/2}\right] \times$$

$$\times {}_3F_3\left(\frac{\alpha-\nu}{2}-\mu, \frac{\alpha-\nu}{2}+\mu; 1-\nu, \frac{1+\alpha-\nu}{2}; \frac{c^2}{8b}\right) +$$

$$+ 2^{\alpha-7/2} \frac{c^{1-\alpha}}{\sqrt{\pi p}} \Gamma\left(\frac{\alpha+\nu-1}{2}\right) \Gamma\left(\frac{\alpha-\nu-1}{2}\right) \times$$

$$\times {}_2F_2\left(\frac{1}{2}-\mu, \frac{1}{2}+\mu; \frac{3-\nu-\alpha}{2}, \frac{3+\nu-\alpha}{2}; \frac{c^2}{8b}\right)$$

[Re b, Re c > 0; Re(\alpha + 2\mu) > |Re \nu|]

$$13. \int_0^\infty x^{2\mu+\nu+1} e^{-bx^2} I_\mu(bx^2) K_\nu(cx) dx =$$

$$= \frac{2^{(2\mu-2\nu-3)/4}}{\sqrt{\pi}} b^{-(2\mu+2\nu+1)/4} c^{-\mu-3/2} \Gamma(2\mu+\nu+1) \Gamma\left(\mu + \frac{1}{2}\right) \exp\left(\frac{c^2}{16b}\right) W_{k,m}\left(\frac{c^2}{8b}\right)$$

[2k = -3\mu - \nu - 1/2, 2m = \mu + \nu + 1/2, Re c, Re b > 0; Re \mu > -1/2; Re(2\mu + \nu) > -1]

$$14. \int_0^{\infty} x^{\alpha-1} e^{-cx} I_{\mu}(b\sqrt{x}) K_{\nu}(cx) dx =$$

$$= 2^{-\alpha-3\mu/2} \sqrt{\pi} b^{\mu} c^{-\alpha-\mu/2} \Gamma \left[\begin{matrix} \alpha+\nu+\mu/2, \alpha-\nu+\mu/2 \\ \mu+1, \alpha+(\mu+1)/2 \end{matrix} \right] \times \\ \times {}_2F_2 \left(\alpha+\nu+\frac{\mu}{2}, \alpha-\nu+\frac{\mu}{2}; \mu+1, \alpha+\frac{\mu+1}{2}; \frac{b^2}{8c} \right) \\ [\operatorname{Re} c > 0; \operatorname{Re} (\alpha+\mu/2) > |\operatorname{Re} \nu|].$$

2.16.30. Интегралы от $x^{\alpha} \begin{Bmatrix} \operatorname{sh} ax \\ \operatorname{ch} ax \end{Bmatrix} I_{\mu}(bx) K_{\nu}(cx)$

$$1. \int_0^{\infty} \operatorname{sh} ax I_0(bx) K_1(cx) dx = \frac{1}{c} \left[u E(k) - K(k) E(u) + \frac{\operatorname{sn} u \operatorname{dn} u}{\operatorname{cn} u} K(k) \right]$$

$$[\operatorname{cn}^2 u = -2c^2 (\sqrt{(a^2 - b^2 + c^2)^2 - 4a^2 c^2} + a^2 - b^2 - c^2)^{-1}, \\ 2k^2 = 1 + (a^2 - b^2 - c^2) ((a^2 - b^2 + c^2)^2 - 4a^2 c^2)^{-1/2}, \operatorname{Re} c > |\operatorname{Re} a| + |\operatorname{Re} b|].$$

$$2. \int_0^{\infty} x \operatorname{sh} ax I_n(bx) K_n(cx) dx = B_n$$

$$[\operatorname{Re} c > |\operatorname{Re} a| + |\operatorname{Re} b|, k = 2\sqrt{bc}/\sqrt{(b+c)^2 - a^2}].$$

$$B_0 = \frac{ak^3}{8(1-k^2)(bc)^{3/2}} E(k),$$

$$B_1 = \frac{ak}{8(1-k^2)(bc)^{3/2}} [(2-k^2) E(k) - 2(1-k^2) K(k)],$$

$$B_2 = \frac{a}{8k(1-k^2)(bc)^{3/2}} [(16-16k^2+k^4) E(k) - 8(1-k^2)(2-k^2) K(k)],$$

$$B_3 = \frac{a}{24k^3(1-k^2)(bc)^{3/2}} [(2-k^2)(128-128k^2+3k^4) E(k) - \\ - 2(1-k^2)(128-128k^2+27k^4) K(k)].$$

$$3. \int_0^{\infty} \operatorname{ch} ax I_n(bx) K_n(cx) dx = C_n$$

$$[\operatorname{Re} c > |\operatorname{Re} a| + |\operatorname{Re} b|, k \text{ см. в 2.16.30.2}].$$

$$C_0 = \frac{k}{2\sqrt{bc}} K(k), \quad C_1 = \frac{1}{2k\sqrt{bc}} [(2-k^2) K(k) - 2E(k)],$$

$$C_2 = \frac{1}{6k^3\sqrt{bc}} [(4-k^2)(4-3k^2) K(k) - 8(2-k^2) E(k)],$$

$$C_3 = \frac{1}{30k^5\sqrt{bc}} [(128-128k^2+15k^4)(2-k^2) K(k) - 2(128-128k^2+23k^4) E(k)],$$

$$C_4 = \frac{1}{210k^7\sqrt{bc}} [(6144-12288k^2+8000k^4-1856k^6 + \\ + 105k^8) K(k) - 32(2-k^2)(96-96k^2+11k^4) E(k)].$$

2.16.31. Интегралы от $x^{\alpha} \begin{Bmatrix} \sin ax^r \\ \cos ax^r \end{Bmatrix} I_{\mu}(bx) K_{\nu}(cx)$.

$$1. \int_0^{\infty} \sin ax I_{n+1}(bx) K_n(cx) dx = P_n$$

$$[\operatorname{Re} c > |\operatorname{Re} b| + |\operatorname{Im} a|; k = \sqrt{bc}/\sqrt{(b+c)^2 + a^2} = \sin \alpha, \\ \sin \beta = a/\sqrt{(b-c)^2 + a^2}],$$

$$P_0 = -\frac{ak}{4\sqrt{b^3c}} K(k) + \frac{\pi}{4b} \Lambda_0(\alpha, \beta),$$

$$P_1 = \frac{a}{k\sqrt{b^3c}} E(k) - \frac{ak(a^2 + b^2 + 2c^2)}{4b(bc)^{3/2}} K(k) + \frac{\pi c}{4b^2} \Lambda_0(\alpha, \beta) + \frac{\pi}{2c}.$$

$$2. \int_0^\infty x \sin ax I_\nu(bx) K_\nu(cx) dx = A_\nu$$

$$[\operatorname{Re} c > |\operatorname{Re} b| + |\operatorname{Im} a|; \operatorname{Re} \nu > -3/2; k = 2\sqrt{bc}/\sqrt{a^2 + (b+c)^2}],$$

$$A_\nu = -\frac{a(u^2 - 1)^{-1/2}}{2(bc)^{3/2}} Q_{\nu-1/2}^1(u) \quad \left[u = \frac{a^2 + b^2 + c^2}{2bc} \right],$$

$$A_n = B_n \text{ при } k = 2\sqrt{bc}/\sqrt{a^2 + (b+c)^2} \text{ [см. 2.16.30.2].}$$

$$3. \int_0^\infty \cos ax I_\nu(bx) K_\nu(cx) dx = D_\nu$$

$$[\operatorname{Re} c > |\operatorname{Re} b| + |\operatorname{Im} a|; \operatorname{Re} \nu > -1/2; k = 2\sqrt{bc}/\sqrt{a^2 + (b+c)^2}],$$

$$D_\nu = \frac{1}{2\sqrt{bc}} Q_{\nu-1/2} \left(\frac{a^2 + b^2 + c^2}{2bc} \right),$$

$$D_n = C_n \text{ при } k = 2\sqrt{bc}/\sqrt{a^2 + (b+c)^2} \text{ [см. 2.16.30.3].}$$

$$4. \int_0^\infty x \cos ax I_0(bx) K_1(cx) dx =$$

$$= \frac{k}{16(1-k^2)c(bc)^{3/2}} [k^2(c^2 - a^2 - b^2) E(k) + 4bc(1-k^2) K(k)]$$

$$[\operatorname{Re} c > |\operatorname{Re} b| + |\operatorname{Im} a|; k = 2\sqrt{bc}/\sqrt{a^2 + (b+c)^2}].$$

$$5. \int_0^\infty \frac{1}{x} \cos ax I_1(bx) K_0(cx) dx = \frac{\sqrt{c}}{k\sqrt{b}} E(k) + \frac{k(b^2 - c^2)}{4\sqrt{b^3c}} K(k) - \frac{\pi a}{4b} \Lambda_0(\alpha, \beta)$$

$$[\operatorname{Re} c > |\operatorname{Re} b| + |\operatorname{Im} a|; k = 2\sqrt{bc}/\sqrt{a^2 + (b+c)^2} = \sin \alpha, \sin \beta = a/\sqrt{a^2 + (b-c)^2}].$$

$$6. \int_0^\infty x^{\alpha-1} \begin{Bmatrix} \sin ax \\ \cos ax \end{Bmatrix} I_\mu(cx) K_\nu(cx) dx =$$

$$= 2^{-\mu-\nu-1} a^{-\alpha-\mu-\nu} c^{\mu+\nu} \begin{Bmatrix} \sin [(\alpha+\mu+\nu)\pi/2] \\ \cos [(\alpha+\mu+\nu)\pi/2] \end{Bmatrix} \Gamma \left[\begin{matrix} -\nu, \alpha+\mu+\nu \\ 1+\mu \end{matrix} \right] \times$$

$$\times {}_4F_3 \left(\frac{\alpha+\mu+\nu}{2}, \frac{1+\alpha+\mu+\nu}{2}, \frac{1+\mu+\nu}{2}, 1 + \frac{\mu+\nu}{2}; \right.$$

$$\left. 1+\nu, 1+\mu, 1+\mu+\nu; -\frac{4c^2}{a^2} \right) +$$

$$+ 2^{\nu-\mu-1} a^{\nu-\mu-\alpha} c^{\mu-\nu} \begin{Bmatrix} \sin [(\alpha+\mu-\nu)\pi/2] \\ \cos [(\alpha+\mu-\nu)\pi/2] \end{Bmatrix} \Gamma \left[\begin{matrix} \nu, \alpha+\mu-\nu \\ 1+\mu \end{matrix} \right] \times$$

$$\times {}_4F_3 \left(\frac{\alpha+\mu-\nu}{2}, \frac{1+\alpha+\mu-\nu}{2}, \frac{1+\mu-\nu}{2}, \right.$$

$$\left. 1 + \frac{\mu-\nu}{2}; 1-\nu, 1+\mu, 1+\mu-\nu; -\frac{4c^2}{a^2} \right)$$

$$[a, \operatorname{Re} c > 0; |\operatorname{Re} \nu| - \operatorname{Re} \mu - (1 \pm 1)/2 < \operatorname{Re} \alpha < 2].$$

$$7. \int_0^{\infty} x^{1/2} \begin{Bmatrix} \sin ax \\ \cos ax \end{Bmatrix} I_{\nu \pm 1/2}(cx) K_{\nu}(cx) dx =$$

$$= 2^{2\nu - (1 \mp 1)/2} \sqrt{\frac{\pi}{a}} c^{2\nu \pm 1/2} \frac{(a + \sqrt{a^2 + 4c^2})^{-2\nu \mp 1/2}}{\sqrt{a^2 + 4c^2}}$$

$$[a, \operatorname{Re} c > 0; \operatorname{Re} \nu > -(3 \pm 3)/4].$$

$$8. \int_0^{\infty} x^{-1/2} \begin{Bmatrix} \sin ax \\ \cos ax \end{Bmatrix} I_0(cx) K_0(cx) dx = \frac{1 - 2k^2}{\sqrt{\pi a}} \sqrt{2} K(k) [K(\sqrt{1 - k^2}) \mp K(k)]$$

$$[a, \operatorname{Re} c > 0; 2k^2 = 1 - \sqrt{2a}/(a + \sqrt{a^2 + 4c^2})].$$

$$9. \int_0^{\infty} \frac{1}{x} \cos ax I_{n+1}(cx) K_n(cx) dx = C_n$$

$$[a, \operatorname{Re} c > 0; k = 2c/\sqrt{a^2 + 4c^2}],$$

$$C_1 = \frac{1}{3k^3} [4(1 - k^2) K(k) - (4 - 5k^2) E(k)] - \frac{\pi a}{4c},$$

$$C_2 = \frac{1}{15k^5} [4(1 - k^2)(8 + k^2) K(k) - (32 - 12k^2 - 23k^4) E(k)] - \frac{\pi a}{4c}.$$

$$10. \int_0^{\infty} x^{\alpha-1} \begin{Bmatrix} \sin a \sqrt{x} \\ \cos a \sqrt{x} \end{Bmatrix} I_{\mu}(cx) K_{\nu}(cx) dx =$$

$$= \frac{a^{\delta}}{4} \left(\frac{2}{c} \right)^{\alpha+\delta/2} \Gamma \left[\begin{matrix} 1 - \alpha - \delta/2, (\alpha + \mu + \nu)/2 + \delta/4, (\alpha + \mu - \nu)/2 + \delta/4 \\ 1 + (\mu - \nu - \alpha)/2 - \delta/4, 1 + (\mu + \nu - \alpha)/2 - \delta/4 \end{matrix} \right] \times$$

$$\times {}_4F_5 \left(\begin{matrix} \frac{\delta}{4} - \frac{\mu - \nu - \alpha}{2}, \frac{\delta}{4} - \frac{\mu + \nu - \alpha}{2}, \frac{\delta}{4} + \frac{\alpha + \mu + \nu}{2}, \\ \frac{\delta}{4} + \frac{\alpha + \mu - \nu}{2}; \frac{1}{2}, \frac{1 + \alpha}{2} + \frac{\delta}{4}, \frac{2\alpha + \delta}{4}, \frac{2\delta + 1}{4}, \\ \frac{2\delta + 3}{4}; \frac{a^4}{256c^2} \right) - 2^{\alpha+\delta/2-2} \frac{a^{\delta+2}}{3^{\delta}} c^{-\alpha-\delta/2-1} \times$$

$$\times \Gamma \left[\begin{matrix} -\alpha - \delta/2, \delta/4 + (\alpha + \mu + \nu + 1)/2, \delta/4 + (\alpha + \mu - \nu + 1)/2 \\ (1 + \mu - \nu - \alpha)/2 - \delta/4, (1 + \mu + \nu - \alpha)/2 - \delta/4 \end{matrix} \right] \times$$

$$\times {}_4F_5 \left(\begin{matrix} \frac{\delta}{4} + \frac{1 + \alpha + \nu - \mu}{2}, \frac{\delta}{4} + \frac{1 + \alpha - \mu - \nu}{2}, \frac{\delta}{4} + \frac{\alpha + \mu + \nu + 1}{2}, \\ \frac{\delta}{4} + \frac{\alpha + \mu - \nu + 1}{2}; \frac{3}{2}, 1 + \frac{2\alpha + \delta}{4}, \frac{1 + \alpha}{2} + \frac{\delta}{4}, \frac{2\delta + 3}{4}, \\ \frac{2\delta + 5}{4}; \frac{a^4}{256c^2} \right) - \frac{a^{2-2\alpha}}{c} \begin{Bmatrix} \sin \alpha\pi \\ \cos \alpha\pi \end{Bmatrix} \Gamma(2\alpha - 2) \times$$

$$\times {}_4F_5 \left(\begin{matrix} \frac{1 + \nu - \mu}{2}, \frac{1 - \mu - \nu}{2}, \frac{1 + \mu + \nu}{2}, \frac{1 + \mu - \nu}{2}; \frac{1}{2}, \frac{3 - \alpha}{2}, 1 - \frac{\alpha}{2}, \frac{3 - 2\alpha}{4}, \\ \frac{5 - 2\alpha}{4}; \frac{a^4}{256c^2} \right) - \frac{a^{4-2\alpha}}{8c^2} (\mu^2 - \nu^2) \begin{Bmatrix} \sin \alpha\pi \\ \cos \alpha\pi \end{Bmatrix} \Gamma(2\alpha - 4) \times$$

$$\times {}_4F_5 \left(\begin{matrix} 1 + \frac{\nu - \mu}{2}, 1 - \frac{\mu + \nu}{2}, 1 + \frac{\mu + \nu}{2}, 1 + \frac{\mu - \nu}{2}; \frac{3}{2}, \\ 2 - \frac{\alpha}{2}, \frac{3 - \alpha}{2}, \frac{5 - 2\alpha}{4}, \frac{7 - 2\alpha}{4}; \frac{a^4}{256c^2} \right)$$

$$[a, \operatorname{Re} c > 0; |\operatorname{Re} \nu| - \operatorname{Re} \mu - \delta/2 < \operatorname{Re} \alpha < 3/2, \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}].$$

$$11. \int_0^{\infty} x^{1/4} \left\{ \begin{matrix} \sin a \sqrt{x} \\ \cos a \sqrt{x} \end{matrix} \right\} I_{\pm 1/4 - \nu}(cx) K_{\nu}(cx) dx =$$

$$= 2 \sqrt{2\pi} a^{-3/2} \Gamma \left[\begin{matrix} (2 \pm 1)/4 - \nu \\ 1 \pm 1/4 \end{matrix} \right] W_{\nu \mp 1/8, \pm 1/8} \left(\frac{a^2}{8c} \right) M_{-\nu \pm 1/8, \pm 1/8} \left(\frac{a^2}{8c} \right)$$

[a, Re c > 0; Re v < (2 ± 1)/4].

2.16.32. Интегралы, содержащие $I_1(x)/K_1(x)$.

$$1. \int_0^{\infty} \frac{1}{x} \left[4x - \ln \left(1 + \frac{\pi^2 I_1^2(x)}{K_1^2(x)} \right) \right] dx = \frac{\pi^2}{2}.$$

$$2. \int_0^{\infty} \frac{x}{x^2 + z^2} \left[2x - \ln \left(1 + \frac{\pi I_1(x)}{K_1(x)} \right) \right] dx = -\pi i \left[i \left(\frac{3\pi}{4} - z \right) + \ln \left(\sqrt{\frac{\pi z}{2}} H_1^{(1)}(z) \right) \right].$$

$$3. \int_0^{\infty} x \operatorname{arctg} \frac{K_1(x)}{\pi I_1(x)} dx = \frac{3\pi}{16}.$$

2.16.33. Интегралы от $A(x) K_{\mu}(\varphi(x)) K_{\nu}(\chi(x))$.

$$1. \int_0^{\infty} x^{\alpha-1} K_{\mu}(bx) K_{\nu}(cx) dx = A_{\mu, \nu}^{\alpha}$$

$$\left[\operatorname{Re}(b+c) > 0; \operatorname{Re} \alpha > |\operatorname{Re} \mu| + |\operatorname{Re} \nu|; k = \frac{b-c}{b+c}, k_b = (b - \sqrt{b^2 - c^2})^{1/2} / (2b)^{1/2}, \right.$$

$$\left. k_c = (c - \sqrt{c^2 - b^2})^{1/2} / (2c)^{1/2} \right],$$

$$A_{\mu, \nu}^{\alpha} = 2^{\alpha-3} c^{-\alpha-\mu} \frac{b^{\mu}}{\Gamma(\alpha)} \Gamma \left[\frac{\alpha+\mu+\nu}{2}, \frac{\alpha+\mu-\nu}{2}, \frac{\alpha-\mu+\nu}{2}, \frac{\alpha-\mu-\nu}{2} \right] \times$$

$$\times {}_2F_1 \left(\frac{\alpha+\mu+\nu}{2}, \frac{\alpha-\mu+\nu}{2}; \alpha; 1 - \frac{b^2}{c^2} \right),$$

$$A_{\nu, \nu}^{\alpha} = 2^{\alpha-3} \sqrt{\frac{\pi}{bc}} \Gamma \left[\frac{\alpha}{2}, \frac{\alpha}{2} + \nu, \frac{\alpha}{2} - \nu \right] |b^2 - c^2|^{(1-\alpha)/2} P_{\nu-1/2}^{(1-\alpha)/2} \left(\frac{b^2 + c^2}{2bc} \right) =$$

$$= 2^{\alpha-3} \Gamma \left(\frac{\alpha}{2} \right) \Gamma \left(\nu + \frac{\alpha}{2} \right) e^{i\nu\pi} |b^2 - c^2|^{-\alpha/2} Q_{\alpha/2-1}^{-\nu} \left(\frac{b^2 + c^2}{|b^2 - c^2|} \right)$$

[Re α > 2 | Re ν |],

$$A_{\nu, \nu}^3 = \frac{\pi (bc)^{-\nu} (c^{2\nu} - b^{2\nu})}{2 \sin \nu\pi (c^2 - b^2)} \quad [b \neq c; |\operatorname{Re} \nu| < 1],$$

$$A_{0, \nu}^{1-\nu} = \frac{\pi}{2^{\nu+2} b} (c^2 - b^2)^{\nu/2} \Gamma^2 \left(\frac{1}{2} - \nu \right) P_{-1/2}^{\nu} \left(\frac{2c^2 - b^2}{b^2} \right)$$

[Re ν < 1/2],

$$A_{0, 0}^1 = \frac{\pi}{b+c} K(k); \quad A_{0, 0}^2 = \frac{\ln c - \ln b}{c^2 - b^2} \quad [b \neq c],$$

$$A_{0, 0}^3 = \frac{1}{2a^2} \quad [c = a],$$

$$A_{0, 0}^3 = \frac{\pi}{(b+c)(b-c)^2} [K(k) - E(k)];$$

$$A_{1, 0}^3 = \frac{\pi}{2b(b^2 - c^2)} [2bK(k) - (b+c)E(k)];$$

$$A_{1, 0}^3 = \frac{1}{b(c^2 - b^2)} \left(1 + \frac{2b^2}{c^2 - b^2} \ln \frac{b}{c} \right) \quad [b \neq c]$$

$$A_{1,1}^1 = \frac{\pi}{2b(b+c)^2(b-c)^2} [2b(3b+c)K(k) - (7b^2+c^2)E(k)];$$

$$A_{2,0}^1 = \frac{\pi}{b^2(b+c)(b-c)^2} [b(3b-2c)K(k) - (2b^2-c^2)E(k)];$$

$$A_{1,1}^1 = \frac{\pi}{2bc(b+c)(b-c)^2} [(b^2+c^2)E(k) - 2bcK(k)];$$

$$A_{2,1}^1 = \frac{\pi}{2b^2c(b+c)^2(b-c)^2} [(3b^4+7b^2c^2-2c^4)E(k) - 2bc(3b^2+3bc-2c^2)K(k)];$$

$$A_{0,0}^{1/2} = \frac{\Gamma^2(1/4)}{2\sqrt{2b}} [K(k_b) + K(\sqrt{1-k_b^2})];$$

$$A_{0,0}^{3/2} = \frac{\Gamma^2(3/4)}{\sqrt{2b^3}(1-2k_b^2)} [K(\sqrt{1-k_b^2}) - K(k_b)];$$

$$A_{1,0}^{3/2} = \frac{\Gamma^2(1/4)}{4\sqrt{2b^3}(1-2k_b^2)} [2E(k_b) - K(k_b) + K(\sqrt{1-k_b^2}) - 2E(\sqrt{1-k_b^2})]$$

$$[0 < c < b],$$

$$A_{1,0}^{3/2} = \frac{b\Gamma^2(1/4)}{8\sqrt{2c^5}(1-2k_c^2)k_c^2(1-k_c^2)} [(1-k_c^2)K(k_c) -$$

$$-E(k_c) + E(\sqrt{1-k_c^2}) - k_c^2K(\sqrt{1-k_c^2})]$$

$$[0 < b < c].$$

$$2. \int_0^\infty x^{\alpha-1} K_\mu(cx) K_\nu(cx) dx =$$

$$= \frac{2^{\alpha-3}}{c^\alpha \Gamma(\alpha)} \Gamma \left[\frac{\alpha+\mu+\nu}{2}, \frac{\alpha+\mu-\nu}{2}, \frac{\alpha+\nu-\mu}{2}, \frac{\alpha-\mu-\nu}{2} \right]$$

$$[\operatorname{Re} c > 0; \operatorname{Re} \alpha > |\operatorname{Re} \mu| + |\operatorname{Re} \nu|].$$

$$3. \int_0^\infty K_{2\nu}(b\sqrt{x}) K_\nu(cx) dx = \frac{\pi}{4c \cos \nu\pi} \left\{ K_\nu\left(\frac{b^2}{4c}\right) + \frac{\pi}{2 \sin \nu\pi} \left[L_{-\nu}\left(\frac{b^2}{4c}\right) - L_\nu\left(\frac{b^2}{4c}\right) \right] \right\}$$

$$[\operatorname{Re} c > 0; |\operatorname{Re} \nu| < 1/2].$$

$$4. \int_0^\infty x^{\alpha-1} K_\mu(b\sqrt{x}) K_\nu(cx) dx =$$

$$= 2^{\alpha-\mu/2-3} b^\mu c^{-\alpha-\mu/2} \Gamma \left[-\mu, \frac{\alpha+\nu}{2} + \frac{\mu}{4}, \frac{\alpha-\nu}{2} + \frac{\mu}{4} \right] \times$$

$$\times {}_2F_3 \left(\frac{\alpha+\nu}{2} + \frac{\mu}{4}, \frac{\alpha-\nu}{2} + \frac{\mu}{4}; \frac{1}{2}, 1 + \frac{\mu}{2}, \frac{1+\mu}{2}; \frac{b^4}{64c^2} \right) +$$

$$+ 2^{\alpha+\mu/2-3} b^{-\mu} c^{\mu/2-\alpha} \Gamma \left[\mu, \frac{\alpha+\nu}{2} - \frac{\mu}{4}, \frac{\alpha-\nu}{2} - \frac{\mu}{4} \right] \times$$

$$\times {}_2F_3 \left(\frac{\alpha+\nu}{2} - \frac{\mu}{4}, \frac{\alpha-\nu}{2} - \frac{\mu}{4}; \frac{1}{2}, 1 - \frac{\mu}{2}, \frac{1-\mu}{2}; \frac{b^4}{64c^2} \right) -$$

$$- 2^{\alpha-\mu/2-4} b^{\mu+2} c^{-\alpha-\mu/2-1} \Gamma \left[-\mu-1, \frac{\alpha+\nu+1}{2} + \frac{\mu}{4}, \frac{\alpha-\nu+1}{2} + \frac{\mu}{4} \right] \times$$

$$\times {}_2F_3 \left(\frac{\alpha+\nu+1}{2} + \frac{\mu}{4}, \frac{\alpha-\nu+1}{2} + \frac{\mu}{4}; \frac{3}{2}, \frac{3+\mu}{2}, 1 + \frac{\mu}{2}; \frac{b^4}{64c^2} \right) -$$

$$- 2^{\alpha+\mu/2-4} b^{2-\mu} c^{\mu/2-\alpha-1} \Gamma \left[\mu-1, \frac{\alpha+\nu+1}{2} - \frac{\mu}{4}, \frac{\alpha-\nu+1}{2} - \frac{\mu}{4} \right] \times$$

$$\times {}_2F_3 \left(\frac{\alpha+\nu+1}{2} - \frac{\mu}{4}, \frac{\alpha-\nu+1}{2} - \frac{\mu}{4}; \frac{3}{2}, \frac{3-\mu}{2}, 1 - \frac{\mu}{2}; \frac{b^4}{64c^2} \right)$$

$$[\operatorname{Re} c > 0; 2 \operatorname{Re} \alpha > |\operatorname{Re} \mu| + 2|\operatorname{Re} \nu|].$$

$$\begin{aligned}
5. \int_0^\infty x^{\alpha-1} K_\mu\left(\frac{b}{x}\right) K_\nu(cx) dx &= 2^{\alpha-2\mu} b^\mu c^{\mu-\alpha} \Gamma\left[-\mu, \frac{\alpha+\nu-\mu}{2}, \frac{\alpha-\nu-\mu}{2}\right] \times \\
&\times {}_0F_3\left(1+\mu, 1+\frac{\mu-\nu-\alpha}{2}, 1+\frac{\mu+\nu-\alpha}{2}; \frac{b^2 c^2}{16}\right) + 2^{\alpha+2\mu-3} b^{-\mu} c^{-\mu-\alpha} \times \\
&\times \Gamma\left[\mu, \frac{\alpha+\mu+\nu}{2}, \frac{\alpha+\mu-\nu}{2}\right] {}_0F_3\left(1-\mu, 1-\frac{\alpha+\mu+\nu}{2}, 1-\frac{\alpha+\mu-\nu}{2}; \frac{b^2 c^2}{16}\right) + \\
&+ 2^{-\alpha-2\nu-3} b^{\alpha+\nu} c^\nu \Gamma\left[-\nu, \frac{\mu-\nu-\alpha}{2}, -\frac{\mu+\nu+\alpha}{2}\right] {}_0F_3\left(1+\nu, 1+\frac{\alpha+\nu-\mu}{2}, \right. \\
&\quad \left. 1+\frac{\alpha+\mu+\nu}{2}; \frac{b^2 c^2}{16}\right) + 2^{2\nu-\alpha-3} b^{\alpha-\nu} c^{-\nu} \Gamma\left[\nu, \frac{\mu+\nu-\alpha}{2}, \frac{\nu-\mu-\alpha}{2}\right] \times \\
&\times {}_0F_3\left(1-\nu, 1+\frac{\alpha-\mu-\nu}{2}, 1+\frac{\alpha+\mu-\nu}{2}; \frac{b^2 c^2}{16}\right) \quad [\operatorname{Re} b, \operatorname{Re} c > 0].
\end{aligned}$$

$$6. \int_0^\infty x^{-1 \pm 1} K_\nu\left(\frac{b}{x}\right) K_\nu(cx) dx = \pi \left\{ \frac{c^{-1}}{b^{-1}} \right\} K_{2\nu}(2\sqrt{bc}) \quad [\operatorname{Re} b, \operatorname{Re} c > 0].$$

$$\begin{aligned}
7. \int_0^\infty x^{2\nu-3/2+2\sigma} K_{\nu+1/2}\left(\frac{b}{x}\right) K_\nu(cx) dx &= \\
&= \sqrt{2\pi} \frac{b^{\nu-1/2+\sigma}}{c^{\nu+\sigma}} K_{2\nu+\sigma}(e^{\pi i/4} \sqrt{2bc}) K_{2\nu+\sigma}(e^{-\pi i/4} \sqrt{2bc}) \\
&\quad [\sigma = 0 \text{ или } 1; \operatorname{Re} b, \operatorname{Re} c > 0].
\end{aligned}$$

$$\begin{aligned}
8. \int_0^\infty x^{\mu-1} |x-y|^{-\mu} K_\mu(|x-y|) K_\nu(x) dx &= \\
&= \frac{(2y)^{-\mu}}{\sqrt{\pi}} \Gamma\left[\frac{1}{2}-\mu, \mu+\nu, \mu-\nu\right] K_\nu(y) \quad [y > 0; |\operatorname{Re} \nu| < \operatorname{Re} \mu < 1/2].
\end{aligned}$$

$$\begin{aligned}
9. \int_0^\infty x^{\mu-1} (x+z)^{-\mu} K_\mu(x+z) K_\nu(x) dx &= \frac{\sqrt{\pi}}{(2z)^\mu} \Gamma\left[\frac{\mu+\nu}{2}, \frac{\mu-\nu}{2}\right] K_\nu(z) \\
&\quad [|\arg z| < \pi; \operatorname{Re} \mu > |\operatorname{Re} \nu|].
\end{aligned}$$

$$\begin{aligned}
10. \int_0^\infty x^{\alpha-1} K_\nu(c\sqrt{x^2+z^2}-cz) K_\nu(c\sqrt{x^2+z^2}+cz) dx &= \\
&= \frac{\sqrt{\pi}}{2} \left(\frac{z}{c}\right)^{\alpha/2} \Gamma\left[\frac{\alpha/2-\nu}{2}, \frac{\alpha/2+\nu}{2}\right] K_{\alpha/2}(2cz) \\
&\quad [\operatorname{Re} c, \operatorname{Re} z > 0; \operatorname{Re} \alpha > 2|\operatorname{Re} \nu|].
\end{aligned}$$

$$\begin{aligned}
11. \int_0^\infty \frac{K_1(c\sqrt{x^2+z^2+2xz\cos b})}{\sqrt{x^2+z^2+2xz\cos b}} K_0(cx) dx &= \frac{bK_0(cz)}{cz \sin b} \\
&\quad [|b| < \pi; \operatorname{Re} c, \operatorname{Re} z > 0].
\end{aligned}$$

2.16.34. Интегралы от $x^\alpha e^{i(x)} K_\mu(bx^{\pm n}) K_\nu(cx)$.

$$\begin{aligned}
1. \int_0^\infty x^{-1/2} e^{-px} K_\mu(bx) K_\nu(cx) dx &= \\
&= \frac{\sqrt{d} \cos \mu\pi \cos \nu\pi}{\cos(\mu+\nu)\pi \cos(\mu-\nu)\pi} \Gamma\left(\frac{1}{2}-\mu-\nu\right) Q_{\nu+1/2}^{-\mu}(\sqrt{1+b^2 d^2}) Q_{\mu-1/2}^{-\nu}(\sqrt{1+c^2 d^2}) \\
&\quad [(1+b^2 d^2)(1+c^2 d^2) = p^2 d^2; \operatorname{Re}(p+b+c) > 0; |\operatorname{Re} \mu| + |\operatorname{Re} \nu| < 1/2].
\end{aligned}$$

$$\begin{aligned}
2. \int_0^\infty x^{\alpha-1} e^{(b \mp c)x} K_\mu(bx) K_\nu(cx) dx = \\
= \frac{\sqrt{\pi} b^\mu}{2^{2\mu+\alpha-1} c^{\alpha+\mu}} [\cos \nu\pi \sec(\alpha+\mu)\pi]^{(1 \mp 1)/2} \Gamma \left[\begin{matrix} -\mu, \alpha+\mu+\nu, \alpha+\mu-\nu \\ \alpha+\mu+1/2 \end{matrix} \right] \times \\
\times {}_3F_2 \left(\frac{1}{2} + \mu, \alpha+\mu+\nu, \alpha+\mu-\nu; 2\mu+1, \alpha+\mu+\frac{1}{2}; \pm \frac{b}{c} \right) + \\
+ \frac{\sqrt{\pi} b^{-\mu}}{2^{2\mu-1} c^{\alpha-\mu}} [\cos \nu\pi \sec(\alpha-\mu)\pi]^{(1 \mp 1)/2} \Gamma \left[\begin{matrix} \mu, \alpha+\nu-\mu, \alpha-\mu-\nu \\ \alpha-\mu+1/2 \end{matrix} \right] \times \\
\times {}_3F_2 \left(\frac{1}{2} - \mu, \alpha+\nu-\mu, \alpha-\nu-\mu; 1-2\mu, \alpha-\mu+\frac{1}{2}; \pm \frac{b}{c} \right) + \\
+ \begin{cases} 0 \\ 1 \end{cases} \frac{b^{1/2-\alpha} \cos \mu\pi}{2^\alpha \sqrt{c}} \Gamma \left[\alpha+\mu-\frac{1}{2}, \alpha-\mu-\frac{1}{2}, 1-\alpha \right] \times \\
\times {}_3F_2 \left(1-\alpha, \frac{1}{2}+\nu, \frac{1}{2}-\nu; \frac{3}{2}+\mu-\alpha, \frac{3}{2}-\mu-\alpha; -\frac{b}{c} \right) \\
\left[\operatorname{Re} \alpha > |\operatorname{Re} \mu| + |\operatorname{Re} \nu|, \begin{cases} \operatorname{Re} c > 0 \\ \operatorname{Re} b, \operatorname{Re} c > 0; \operatorname{Re} \alpha < 1 \end{cases} \right].
\end{aligned}$$

$$\begin{aligned}
3. \int_0^\infty x^{\alpha-1} e^{-(b+c)x} K_\mu(bx) K_\nu(cx) dx = 2^{-\alpha-2\nu-1} \sqrt{\pi} b^{-\alpha-\nu} c^\nu \times \\
\times \Gamma \left[\begin{matrix} -\nu, \alpha+\mu+\nu, \alpha+\nu-\mu \\ \alpha+\nu+1/2 \end{matrix} \right] {}_3F_2 \left(\frac{1}{2} + \nu, \alpha+\mu+\nu, \alpha+\nu-\mu; 1+2\nu, \right. \\
\left. \frac{1}{2} + \nu + \alpha; -\frac{c}{b} \right) + 2^{2\nu-\alpha-1} \sqrt{\pi} b^{\nu-\alpha} c^{-\nu} \Gamma \left[\begin{matrix} \nu, \alpha+\mu-\nu, \alpha-\mu-\nu \\ \alpha-\nu+1/2 \end{matrix} \right] \times \\
\times {}_3F_2 \left(\frac{1}{2} - \nu, \alpha+\mu-\nu, \alpha-\mu-\nu; 1-2\nu, \frac{1}{2} + \alpha - \nu; -\frac{c}{b} \right) \\
\left[\operatorname{Re}(b+c) > 0; \operatorname{Re} \alpha > |\operatorname{Re} \mu| + |\operatorname{Re} \nu| \right].
\end{aligned}$$

$$\begin{aligned}
4. \int_0^\infty x^{\alpha-1} e^{b/x \mp cx} K_\mu \left(\frac{b}{x} \right) K_\nu(cx) dx = \\
= \frac{2^{\alpha-1}}{\sqrt{\pi}} b^{\alpha+\nu} c^\nu \cos \mu\pi \Gamma \left[-\nu, \mu-\alpha-\nu, -\mu-\alpha-\nu, \frac{1}{2} + \alpha + \nu \right] \times \\
\times {}_2F_3 \left(\frac{1}{2} + \nu, \frac{1}{2} + \alpha + \nu; 1+2\nu, 1+\alpha+\nu-\mu, 1+\alpha+\mu+\nu; \mp 4bc \right) + \\
+ \frac{2^{\alpha-1}}{\sqrt{\pi}} b^{\alpha-\nu} c^{-\nu} \cos \mu\pi \Gamma \left[\nu, \mu+\nu-\alpha, \nu-\mu-\alpha, \frac{1}{2} + \alpha - \nu \right] \times \\
\times {}_2F_3 \left(\frac{1}{2} - \nu, \frac{1}{2} + \alpha - \nu; 1-2\nu, 1+\alpha+\mu-\nu, 1+\alpha-\mu-\nu; \mp 4bc \right) + \\
+ 2^{-\alpha-1} \sqrt{\pi} b^\mu c^{\mu-\alpha} [\cos \nu\pi \sec(\alpha-\mu)\pi]^{(1 \mp 1)/2} \Gamma \left[\begin{matrix} -\mu, \alpha+\nu-\mu, \alpha-\nu-\mu \\ 1/2 + \alpha - \mu \end{matrix} \right] \times \\
\times {}_2F_3 \left(\frac{1}{2} + \mu - \alpha, \frac{1}{2} + \mu; 1+2\mu, 1+\mu-\nu-\alpha, 1+\mu+\nu-\alpha; \mp 4bc \right) + \\
+ 2^{-\alpha-1} \sqrt{\pi} b^{-\mu} c^{\mu-\alpha} [\cos \nu\pi \sec(\alpha+\mu)\pi]^{(1 \mp 1)/2} \Gamma \left[\begin{matrix} \mu, \alpha+\mu+\nu, \alpha+\mu-\nu \\ 1/2 + \alpha + \mu \end{matrix} \right] \times \\
\times {}_2F_3 \left(\frac{1}{2} - \alpha - \mu, \frac{1}{2} - \mu; 1-2\mu, 1-\nu-\mu-\alpha, 1+\nu-\mu-\alpha; \mp 4bc \right) \\
\left[\operatorname{Re} \alpha > |\operatorname{Re} \nu| - 1/2, \begin{cases} \operatorname{Re} c > 0 \\ \operatorname{Re} b, \operatorname{Re} c > 0; \operatorname{Re} \alpha + |\operatorname{Re} \mu| < 1/2 \end{cases} \right].
\end{aligned}$$

$$\begin{aligned}
5. \int_0^\infty x^{\alpha-1} e^{-b/x-cx} K_\mu\left(\frac{b}{x}\right) K_\nu(cx) dx = & \\
& = 2^{-\alpha-1} \pi^{3/2} b^{-\mu} c^{-\alpha-\mu} \Gamma\left[\begin{matrix} \mu, \alpha+\nu-\mu, \alpha-\nu-\mu \\ 1/2+\alpha+\mu \end{matrix}\right] \times \\
& \times {}_2F_3\left(\frac{1}{2}-\mu, \frac{1}{2}-\alpha-\mu; 1-2\mu, 1+\mu-\nu-\alpha, 1+\mu+\nu-\alpha; 4bc\right) + \\
& + 2^{-\alpha-1} \pi^{3/2} b^\mu c^{\alpha-\mu} \Gamma\left[\begin{matrix} -\mu, \alpha+\mu+\nu, \alpha+\mu-\nu \\ 1/2+\alpha-\mu \end{matrix}\right] \times \\
& \times {}_2F_3\left(\frac{1}{2}+\mu, \frac{1}{2}+\mu-\alpha; 1+2\mu, 1-\alpha-\mu-\nu, 1+\nu-\mu-\alpha; 4bc\right) + \\
& + 2^\alpha \pi^{3/2} b^{\alpha+\nu} c^\nu \Gamma\left[\begin{matrix} -\nu, \mu-\alpha-\nu, -\mu-\alpha-\nu \\ 1/2-\alpha-\nu \end{matrix}\right] \times \\
& \times {}_2F_3\left(\frac{1}{2}+\nu, \frac{1}{2}+\alpha+\nu; 1+2\nu, 1+\alpha+\nu-\mu, 1+\alpha+\nu+\mu; 4bc\right) + \\
& + 2^{\alpha-1} \pi^{3/2} b^{\alpha-\nu} c^{-\nu} \Gamma\left[\begin{matrix} \nu, \mu+\nu-\alpha, \nu-\mu-\alpha \\ 1/2+\nu-\alpha \end{matrix}\right] \times \\
& \times {}_2F_3\left(\frac{1}{2}-\nu, \frac{1}{2}+\alpha-\nu; 1-2\nu, 1+\alpha-\mu-\nu, 1+\alpha+\mu-\nu; 4bc\right) \\
& \quad [\operatorname{Re} b, \operatorname{Re} c > 0]
\end{aligned}$$

$$\begin{aligned}
6. \int_0^\infty x^\alpha e^{-bx^2} K_\mu(bx^2) K_\nu(cx) dx = & \frac{\sqrt{\pi} c^\nu}{2^{(\alpha+3\nu)/2+2} b^{(\alpha+\nu)/2}} \times \\
& \times \left(\cos \mu\pi \sec \frac{\alpha+\nu}{2} \pi\right)^{(1\mp 1)/2} \Gamma\left[\begin{matrix} -\nu, (\alpha+\nu)/2+\mu, (\alpha+\nu)/2-\mu \\ (1+\alpha+\nu)/2 \end{matrix}\right] \times \\
& \times {}_2F_2\left(\frac{\alpha+\nu}{2}+\mu, \frac{\alpha+\nu}{2}-\mu; \nu+1, \frac{1+\alpha+\nu}{2}; \pm \frac{c^2}{8b}\right) + \\
& + \frac{\sqrt{\pi} b^{(\nu-\alpha)/2} c^{-\nu}}{2^{(\alpha-3\nu)/2+2}} \left(\cos \mu\pi \sec \frac{\alpha-\nu}{2} \pi\right)^{(1\mp 1)/2} \times \\
& \times \Gamma\left[\begin{matrix} \nu, (\alpha-\nu)/2+\mu, (\alpha-\nu)/2-\mu \\ (1+\alpha-\nu)/2 \end{matrix}\right] {}_2F_2\left(\frac{\alpha-\nu}{2}+\mu, \frac{\alpha-\nu}{2}-\mu; 1-\nu, \right. \\
& \left. \frac{1+\alpha-\nu}{2}; \pm \frac{c^2}{8b}\right) + \begin{cases} 0 \\ 1 \end{cases} 2^{\alpha-7/2} c^{1-\alpha} \sqrt{\frac{\pi}{b}} \Gamma\left(\frac{\alpha+\nu-1}{2}\right) \Gamma\left(\frac{\alpha-\nu-1}{2}\right) \times \\
& \times {}_2F_2\left(\frac{1}{2}+\mu, \frac{1}{2}-\mu; \frac{3-\nu-\alpha}{2}, \frac{3+\nu-\alpha}{2}; -\frac{c^2}{8b}\right) \\
& \quad [\operatorname{Re} \alpha > 2, |\operatorname{Re} \mu| + |\operatorname{Re} \nu|, \begin{cases} \operatorname{Re} b > 0 \\ \operatorname{Re} c > 0 \end{cases}].
\end{aligned}$$

$$\begin{aligned}
7. \int_0^\infty x^{\alpha-1} e^{-bx^2} K_\mu\left(\frac{b}{x^2}\right) K_\nu(cx) dx = & 2^{\alpha-3\mu-3} b^\mu c^{2\mu-\alpha} \Gamma\left[-\mu, \frac{\alpha+\nu}{2}-\mu, \frac{\alpha-\nu}{2}-\mu\right] \times \\
& \times {}_1F_3\left(\frac{1}{2}+\mu; 1+2\mu, 1+\mu-\frac{\alpha+\nu}{2}, 1+\mu-\frac{\alpha-\nu}{2}; \mp \frac{bc^2}{2}\right) + \\
& + 2^{\alpha+3\mu-3} b^{-\mu} c^{-2\mu-\alpha} \Gamma\left[\mu, \frac{\alpha+\nu}{2}+\mu, \frac{\alpha-\nu}{2}+\mu\right] \times \\
& \times {}_1F_3\left(\frac{1}{2}-\mu; 1-2\mu, 1-\mu-\frac{\alpha+\nu}{2}, 1-\mu-\frac{\alpha-\nu}{2}; \mp \frac{bc^2}{2}\right) + \\
& + 2^{(\alpha-\nu)/2-2} \sqrt{\pi} b^{(\alpha+\nu)/2} c^\nu \left(\cos \mu\pi \sec \frac{\alpha+\nu}{2} \pi\right)^{(1\mp 1)/2} \times
\end{aligned}$$

$$\begin{aligned}
& \times \Gamma \left[\begin{matrix} -\nu, \mu - (\alpha + \nu)/2, -\mu - (\alpha + \nu)/2 \\ (1 - \alpha - \nu)/2 \end{matrix} \right] \times \\
& \times {}_1F_3 \left(\frac{1 + \alpha + \nu}{2}; 1 + \nu, 1 - \mu + \frac{\alpha + \nu}{2}, 1 + \mu + \frac{\alpha + \nu}{2}; -\frac{bc^2}{2} \right) + \\
& + 2^{(\alpha + \nu)/2 - 2} \sqrt{\pi} b^{(\alpha - \nu)/2} c^{-\nu} \left(\cos \mu \pi \sec \frac{\alpha - \nu}{2} \pi \right)^{(1 \mp 1)/2} \times \\
& \times \Gamma \left[\begin{matrix} \nu, \mu + (\nu - \alpha)/2, (\nu - \alpha)/2 - \mu \\ (1 + \nu - \alpha)/2 \end{matrix} \right] \times \\
& \times {}_1F_3 \left(\frac{1 + \alpha - \nu}{2}; 1 - \nu, 1 - \mu + \frac{\alpha - \nu}{2}, 1 + \mu + \frac{\alpha - \nu}{2}; -\frac{bc^2}{2} \right) \\
& \left[\operatorname{Re} c > 0, \begin{cases} \operatorname{Re} b > 0 \\ \operatorname{Re} \alpha > |\operatorname{Re} \nu| - 1 \end{cases} \right].
\end{aligned}$$

2.16.35. Интегралы от $x^\alpha \begin{Bmatrix} \operatorname{sh} ax \\ \operatorname{ch} ax \end{Bmatrix} K_\mu(bx) K_\nu(cx)$.

$$1. \int_0^\infty x \operatorname{sh} ax K_n(bx) K_n(cx) dx = I_n \quad [\operatorname{Re}(b+c) > |\operatorname{Re} a|; k = (a^2 - (b-c)^2)/(4bc)]$$

$$I_0 = \frac{\pi a (1 - k^2)}{8k^2 (bc)^{3/2}} [E(k) - (1 - k^2) K(k)],$$

$$I_1 = \frac{\pi a}{8k^2 (1 - k^2) (bc)^{3/2}} [(1 - k^2) K(k) - (1 - 2k^2) E(k)],$$

$$2. \int_0^\infty \operatorname{ch} ax K_0(bx) K_0(cx) dx = \frac{\pi}{2 \sqrt{bc}} K \left(\frac{a^2 - (b-c)^2}{4bc} \right) \quad [\operatorname{Re}(b+c) > |\operatorname{Re} a|]$$

$$3. \int_0^\infty x^{2n+2} \operatorname{sh} ax K_{n+1}(cx) K_n(cx) dx = J_n \quad [2 \operatorname{Re} c > |\operatorname{Re} a|; k = a/(2c)],$$

$$J_0 = \frac{\pi a}{16c^4 k^2 (1 - k^2)^2} [(1 + k^2) E(k) - (1 - k^2) K(k)],$$

$$J_1 = \frac{3\pi a}{64c^6 k^4 (1 - k^2)^4} [(1 - k^2) (2 - 9k^2 - k^4) K(k) - 2(1 + k^2) (1 - 6k^2 + k^4) E(k)].$$

$$4. \int_0^\infty x^{2n+1} \operatorname{ch} ax K_{n+1}(cx) K_n(cx) dx = R_n \quad [2 \operatorname{Re} c > |\operatorname{Re} a|; k = a/(2c)],$$

$$R_0 = \frac{\pi}{4c^2 (1 - k^2)} E(k),$$

$$R_1 = \frac{\pi}{16c^4 k^2 (1 - k^2)^3} [(3 + 7k^2 - 2k^4) E(k) - (1 - k^2) (3 + k^2) K(k)].$$

2.16.36. Интегралы от $x^\alpha \begin{Bmatrix} \sin ax^n \\ \cos ax^n \end{Bmatrix} K_\mu(bx) K_\nu(cx)$.

$$1. \int_0^\infty x \sin ax K_\nu(bx) K_\nu(cx) dx = I_\nu$$

$$[\operatorname{Re}(b+c) > |\operatorname{Im} a|; |\operatorname{Re} \nu| < 3/2; k = \sqrt{a^2 + (b-c)^2} / \sqrt{a^2 + (b+c)^2}, l = (a^2 + b^2 + c^2)/(2bc)],$$

$$I_\nu = \frac{(1 - 4\nu^2) \pi^2 a}{16 (bc)^{3/2} \cos \nu \pi} (l^2 - 1)^{-1/2} P_{\nu-1/2}^{-1}(l),$$

$$I_0 = \frac{\pi a}{8k^2} \left(\frac{1-k^2}{bc} \right)^{3/2} [K(k) - E(k)],$$

$$I_1 = \frac{\pi a \sqrt{1-k^2}}{8k^2 (bc)^{3/2}} [(1+k^2) E(k) - (1-k^2) K(k)].$$

$$2. \int_0^\infty \cos ax K_\nu(bx) K_\nu(cx) dx = J_\nu \quad [2 \operatorname{Re} c > |\operatorname{Im} a|; |\operatorname{Re} \nu| < 1/2]$$

$$J_\nu = \frac{\pi^2}{4 \sqrt{bc} \cos \nu \pi} P_{\nu-1/2} \left(\frac{a^2 + b^2 + c^2}{2bc} \right),$$

$$J_0 = \frac{\pi}{2} \sqrt{\frac{1-k^2}{bc}} K(k) = \frac{\pi}{2} \sqrt{\frac{u}{bc}} K(\sqrt{1-u^2})$$

$$[k = \sqrt{a^2 + (b-c)^2} / \sqrt{a^2 + (b+c)^2}, u = 1 - \sqrt{1-k^2}, l = (a^2 + b^2 + c^2)/(2bc)].$$

$$3. \int_0^\infty x^{\alpha-1} \left\{ \frac{\sin ax}{\cos ax} \right\} K_\mu(cx) K_\nu(cx) dx = \frac{2^{\alpha+\delta-3} a^\delta}{c^{\alpha+\delta}} \times$$

$$\times \Gamma \left[\frac{(\alpha+\delta+\mu+\nu)/2, (\alpha+\delta+\mu-\nu)/2, (\alpha+\delta-\mu+\nu)/2, (\alpha+\delta-\mu-\nu)/2}{\alpha+\delta} \right] \times$$

$$\times {}_4F_3 \left(\frac{\alpha+\delta+\mu+\nu}{2}, \frac{\alpha+\delta+\mu-\nu}{2}, \frac{\alpha+\delta-\mu+\nu}{2}, \frac{\alpha+\delta-\mu-\nu}{2}; \right.$$

$$\left. \delta + \frac{1}{2}, \frac{\alpha+1}{2}, \frac{\alpha}{2} + \delta; -\frac{a^2}{4c^2} \right)$$

$$\left[2 \operatorname{Re} c > |\operatorname{Im} a|; \operatorname{Re} \alpha > |\operatorname{Re} \mu| + |\operatorname{Re} \nu| - \delta, \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right].$$

$$4. \int_0^\infty x^{-1/2} \left\{ \frac{\sin ax}{\cos ax} \right\} K_0^2(cx) dx = \sqrt{\frac{\pi}{2c}} (1-2k^2) [K(k) \mp K(\sqrt{1-k^2})]^2$$

$$\left[2 \operatorname{Re} c > |\operatorname{Im} a|; k = \frac{1}{\sqrt{2}} \left(1 - \sqrt{\frac{2a}{a + \sqrt{a^2 + 4c^2}}} \right)^{1/2} \right].$$

$$5. \int_0^\infty x^{1/2} \left\{ \frac{\sin ax}{\cos ax} \right\} K_0(cx) K_1(cx) dx =$$

$$= \frac{(1-2k^2) \sqrt{2\pi}}{4k \sqrt{(1-k^2) a (a^2 + 4c^2)}} [K(\sqrt{1-k^2}) \mp K(k)] [k^2 K(\sqrt{1-k^2}) -$$

$$-(1-2k^2) E(\sqrt{1-k^2}) \mp (1-k^2) K(k) \pm (1-2k^2) E(k)]$$

$$[2 \operatorname{Re} c > |\operatorname{Im} a|; k \text{ см. в 2.16.36.4}]$$

$$6. \int_0^\infty x^{2n+2} \sin ax K_{n+1}(cx) K_n(cx) dx = J_n \quad [2 \operatorname{Re} c > |\operatorname{Im} a|; k = a/\sqrt{a^2 + 4c^2}]$$

$$J_0 = \frac{\pi k}{2a^2 c} [(1-k^2) K(k) - (1-2k^2) E(k)],$$

$$J_1 = \frac{3\pi k}{2a^4 c} [(1-k^2) (2+5k^2-8k^4) K(k) - 2(1-2k^2) (1+4k^2-4k^4) E(k)].$$

$$7. \int_0^\infty x^{2n+1} \cos ax K_{n+1}(cx) K_n(cx) dx = I_n \quad [2 \operatorname{Re} c > |\operatorname{Im} a|; k = a/\sqrt{a^2 + 4c^2}]$$

$$I_0 = \frac{\pi k}{2ac} E(k),$$

$$I_1 = \frac{\pi k}{2a^3 c} [(1-k^2) (3-4k^2) K(k) - (3-13k^2+8k^4) E(k)].$$

$$8. \int_0^{\infty} \cos ax^2 K_\nu (ce^{\pi i/4} x) K_\nu (ce^{-\pi i/4} x) dx = \\ = \frac{\sqrt{2a\pi}}{8c^2} \Gamma\left(\frac{1}{4} + \nu\right) \Gamma\left(\frac{1}{4} - \nu\right) W_{1/4, \nu}\left(\frac{c^2}{2a} e^{\pi i/2}\right) W_{1/4, \nu}\left(\frac{c^2}{2a} e^{-\pi i/2}\right) \\ [a > 0; |\arg c| < \pi/4; |\operatorname{Re} \nu| < 1/2].$$

$$9. \int_0^{\infty} x \cos ax^2 K_0 (ce^{\pi i/4} x) K_0 (ce^{-\pi i/4} x) dx = \frac{\pi^2}{16a} \left[H_0\left(\frac{c^2}{2a}\right) - Y_0\left(\frac{c^2}{2a}\right) \right] \\ [a > 0; |\arg c| < \pi/4].$$

2.16.37. Интегралы от $x^\alpha J_\lambda(ax) J_\mu(bx) K_\nu(cx)$.

$$1. \int_0^{\infty} J_\lambda(ax) J_\mu(bx) K_\nu(cx) dx = \\ = \frac{a^\lambda b^\mu}{2c^{\lambda+\mu+1}} \Gamma\left[\frac{(\lambda+\mu+\nu+1)/2, (\lambda+\mu-\nu+1)/2}{\lambda+1, \mu+1}\right] A_{\mu, \lambda}(u) A_{\lambda, \mu}(v) \\ \left[A_{\mu, \lambda}(u) = {}_2F_1\left(\frac{\lambda+\mu+\nu}{2}, \frac{\lambda+\mu-\nu}{2}; \mu+1; u\right), \right. \\ u = \frac{c^2 - a^2 + b^2 - \sqrt{(c^2 - a^2 + b^2)^2 + 4a^2c^2}}{2c^2}, \\ \left. v = \frac{c^2 + a^2 - b^2 - \sqrt{(c^2 + a^2 - b^2)^2 + 4b^2c^2}}{2c^2}; \operatorname{Re}(\lambda+\mu) > |\operatorname{Re} \nu| - 1 \right].$$

$$2. \int_0^{\infty} x^{\alpha-1} J_\lambda(ax) J_\mu(bx) K_\nu(cx) dx = B^\alpha(\lambda, \mu, \nu) \\ [\operatorname{Re}(\alpha + \lambda + \mu) > |\operatorname{Re} \nu|; |\alpha| > |\alpha| + |\beta|; \operatorname{Re} c > |\operatorname{Im} a| + |\operatorname{Im} b|; \\ u = (a^2 + b^2 + c^2)/(2ab)],$$

$$B^\alpha(\lambda, \mu, \nu) = \frac{2^{\alpha-2} a^\lambda b^\mu}{c^{\alpha+\lambda+\mu}} \Gamma\left[\frac{(\alpha+\lambda+\mu-\nu)/2, (\alpha+\lambda+\mu+\nu)/2}{\lambda+1, \mu+1}\right] \times \\ \times F_4\left(\frac{\alpha+\lambda+\mu-\nu}{2}, \frac{\alpha+\lambda+\mu+\nu}{2}; \lambda+1, \mu+1; -\frac{a^2}{c^2}, -\frac{b^2}{c^2}\right),$$

$$B^{\lambda+\nu-\mu+2}(\lambda, \mu, \nu) = 2^{\lambda+\nu-\mu-1} b^{\mu-\nu-\lambda-2} \Gamma\left[\frac{\lambda+1, \lambda+\nu+1, \nu+1}{\mu+1}\right] \times \\ \times (\operatorname{ch} \varphi - \cos \psi) P_{\lambda+\nu-\mu}^{-\nu}(\cos \psi) P_{\lambda+\nu-\mu}^{-\lambda}(\operatorname{ch} \varphi) \\ \left[\operatorname{Re} \lambda, \operatorname{Re} \mu, \operatorname{Re} \nu, \operatorname{Re}(\lambda+\nu) > -1; a+ic = ib \operatorname{ctg} \frac{\psi+i\varphi}{2}, \text{ т. е.} \right.$$

$$\frac{c}{b} = \frac{\operatorname{ch} \varphi \sin \psi}{\operatorname{ch}^2 \varphi - \cos^2 \psi} (1 + \cos \psi \operatorname{ch} \varphi - \operatorname{sh} \varphi \sin \psi), \\ \frac{a}{b} = \frac{\cos \psi \operatorname{sh} \varphi}{\operatorname{ch}^2 \varphi - \cos^2 \psi} (1 + \cos \psi \operatorname{ch} \varphi - \operatorname{sh} \varphi \sin \psi) \Big],$$

$$B^{\pm \nu}(\mu, \mu, \nu) = \frac{c^{\pm \nu} (ab)^{\mp \nu-1}}{\sqrt{2\pi}} \exp\left[\left(\mp \nu - \frac{1}{2}\right) \pi i\right] (u^2 - 1)^{-(1 \mp 2\nu)/4} Q_{\mu-1/2}^{1/2 \pm \nu}(u) \\ [\operatorname{Re} \mu, \operatorname{Re}(\mu + \nu) > -1]$$

$$B^{\nu+2}(\nu, \nu, \nu) = \frac{2^{\nu-1} c^\nu \Gamma(\nu+1/2)}{\sqrt{\pi} (ab)^{\nu+1} (u^2 - 1)^{\nu+1/2}} \quad [\operatorname{Re} \nu > -1/2]$$

$$B^{\mu+2}(\nu, \mu, \nu) = \frac{(2a)^\mu r^{2\mu+3} \Gamma(\mu+\nu+1) (r^4 - a^2 b^2)^{-\mu-1/2}}{(r^2 - b^2)^{3/2}} P_\mu^{-\nu}\left(\frac{r^4 - 2b^2 r^2 + a^2 b^2}{r^4 - a^2 b^2}\right) = \\ = \frac{a^\mu}{\sqrt{2\pi} (bc)^{\mu+1}} \exp\left[-\frac{i\pi(4\mu-2\nu+1)}{4}\right] (z^2 - 1)^{-(2\mu+1)/4} Q_{\nu-1/2}^{\mu+1/2}(iz) \\ [2bcz = a^2 + c^2 - b^2, r = \sqrt{ab(u + \sqrt{u^2 - 1})}; \operatorname{Re} \mu, \operatorname{Re}(\mu + \nu) > -1]$$

$$B^2(\mu + \nu, \mu, \nu) = \frac{a^\mu b^{\mu+\nu}}{r^{2\mu+\nu-2} (r^4 - a^2 b^2)} \left(\frac{r^2 - a^2}{r^2 - b^2} \right)^{\nu/2} \\ [r = \sqrt{ab(u + \sqrt{u^2 - 1})}; \operatorname{Re} \mu, \operatorname{Re}(\mu + \nu) > -1],$$

$$B^2(\mu, \mu, 0) = r_1^{-1} r_2^{-1} (r_2 - r_1)^\mu (r_2 + r_1)^{-\mu} \\ [r_1 = \sqrt{(a-b)^2 + c^2} \quad r_2 = \sqrt{(a+b)^2 + c^2}; \operatorname{Re} \mu > -1],$$

$$B^2(1, 1, 0) = \frac{1}{2ab} \left(\frac{u}{\sqrt{u^2 - 1}} - 1 \right), \quad B^3(0, 1, 0) = \frac{au - b}{2a^2 b^2 (u^2 - 1)^{3/2}},$$

$$B^3(0, 0, 1) = \frac{cu}{2a^2 b^2 (u^2 - 1)^{3/2}}, \quad B^3(1, 1, 1) = \frac{c}{2a^2 b^2 (u^2 - 1)^{3/2}}.$$

$$3. \int_0^\infty x^{\alpha-1} J_\lambda(bx) J_\mu(bx) K_\nu(cx) dx = A^\alpha(\lambda, \mu, \nu)$$

$$[\operatorname{Re}(\alpha + \lambda + \mu) > |\operatorname{Re} \nu|; |c| > |2b|; \operatorname{Re} c > 2|\operatorname{Im} b|; z = \sqrt{4b^2 + c^2}/c, \\ k = 2b/(\sqrt{4b^2 + c^2} + c)],$$

$$A^\alpha(\lambda, \mu, \nu) = \frac{2^{\alpha-2} b^{\lambda+\mu}}{c^{\alpha+\lambda+\mu}} \Gamma \left[\frac{(\alpha + \lambda + \mu + \nu)/2, (\alpha + \lambda + \mu - \nu)/2}{\mu + 1, \lambda + 1} \right] \times \\ \times {}_4F_3 \left(\frac{1 + \lambda + \mu}{2}, 1 + \frac{\lambda + \mu}{2}, \frac{\alpha + \lambda + \mu + \nu}{2}, \frac{\alpha + \lambda + \mu - \nu}{2}; \right. \\ \left. \lambda + \mu + 1, \mu + 1, \lambda + 1; -\frac{4b^2}{c^2} \right),$$

$$A^{2-\nu}(\mu, \mu, \nu) = \frac{\Gamma(1 + \mu - \nu)}{2^\nu c} (4b^2 + c^2)^{(\nu-1)/2} P_{\nu-1}^{-\mu} \left(\frac{2b^2 + c^2}{c \sqrt{4b^2 + c^2}} \right) \\ [\operatorname{Re} \mu > -1; -1/2 < \operatorname{Re} \nu < \operatorname{Re} \mu + 1],$$

$$A^{2+\nu}(\nu, \nu, \nu) = \frac{2^{3\nu} b^{2\nu}}{\sqrt{\pi} c^{\nu+1}} \Gamma \left(\nu + \frac{1}{2} \right) (4b^2 + c^2)^{-\nu-1/2} \\ [\operatorname{Re} \nu > -1/2],$$

$$A^1(\mu, \mu, \nu) = \frac{1}{2c} \Gamma \left(\mu + \frac{\nu+1}{2} \right) \Gamma \left(\mu + \frac{1-\nu}{2} \right) [P_{(\nu-1)/2}^{-\mu}(z)]^2 \\ [2 \operatorname{Re} \mu > |\operatorname{Re} \nu| - 1]$$

$$A^2(\mu, \mu, \nu) = \Gamma \left(\mu + \frac{\nu}{2} + 1 \right) \Gamma \left(\mu - \frac{\nu}{2} + 1 \right) \frac{1}{c^2 z} P_{\nu/2}^{-\mu}(z) P_{-\nu/2}^{-\mu}(z) \\ [2 \operatorname{Re} \mu > |\operatorname{Re} \nu| - 2],$$

$$A^1(-\mu, \mu, \nu) = \frac{\pi}{2c} \sec \frac{\nu\pi}{2} P_{(\nu-1)/2}^{\mu}(z) P_{(\nu-1)/2}^{-\mu}(z) \\ [|\operatorname{Re} \nu| < 1],$$

$$A^2(-\mu, \mu, \nu) = -\frac{\pi}{2c^2 z} \operatorname{cosec} \frac{\nu\pi}{2} \left[\left(\mu - \frac{\nu}{2} \right) P_{\nu/2}^{\mu}(z) P_{-\nu/2}^{-\mu}(z) - \right. \\ \left. - \left(\mu + \frac{\nu}{2} \right) P_{\nu/2}^{-\mu}(z) P_{-\nu/2}^{\mu}(z) \right] \\ [|\operatorname{Re} \nu| < 2],$$

$$A^2(-\mu-1, \mu, \nu) = \frac{\pi}{2c^2 z} \sec \frac{\nu\pi}{2} \left[P_{(\nu-1)/2}^{-\mu}(z) P_{(\nu-1)/2}^{\mu+1}(z) + \right. \\ \left. + \frac{\nu^2 - (\mu+1)^2}{4} P_{(\nu-1)/2}^{-\mu-1}(z) P_{\nu-1/2}^{\mu}(z) \right] - \frac{\sin \mu\pi}{4\pi} \sec \frac{\nu\pi}{2} \\ [|\operatorname{Re} \nu| < 1],$$

$$A^2(\mu+1, \mu, \nu) = \Gamma \left(\mu + \frac{3+\nu}{2} \right) \Gamma \left(\mu + \frac{3-\nu}{2} \right) c^{-3} \sqrt{c^2 + 4b^2} P_{(\nu-1)/2}^{-\mu}(z) P_{(\nu-1)/2}^{-\mu-1}(z) \\ [2 \operatorname{Re} \mu > |\operatorname{Re} \nu| - 3],$$

$$A^1(0, 0, 0) = \frac{2k}{\pi b} K^2(k), \quad A^1(1, 1, 0) = \frac{2}{\pi b R} [K(k) - E(k)]^2.$$

$$A^1(2, 2, 0) = \frac{2}{9\pi b k^3} [(2 + k^2) K(k) - 2(1 + k^2) E(k)]^2,$$

$$A^1(3, 3, 0) = \frac{2}{225\pi b k^5} [(8 + 3k^2 + 4k^4) K(k) - (8 + 7k^2 + 8k^4) E(k)]^2,$$

$$A^1(1, 1, 1) = \frac{k^2}{2c},$$

$$A^1(1, 1, 2) = \frac{2b}{3\pi c^2 k^3} [(1 + k^2) E(k) - (1 - k^2) K(k)]^2,$$

$$A^1(2, 2, 2) = \frac{2b}{15\pi c^2 k^5} [2(1 - k^2 + k^4) E(k) - (1 - k^2)(2 - k^2) K(k)]^2$$

$$A^1(3, 3, 2) = \frac{2b}{315\pi c^2 k^7} [(8 - 13k^2 + 8k^4)(1 + k^2) E(k) - (1 - k^2)(8 - k^2 - 4k^4) K(k)]^2,$$

$$A^2(0, 0, 0) = \frac{1}{c \sqrt{4b^2 + c^2}},$$

$$A^2(1, 0, 0) = \frac{2 \sqrt{4b^2 + c^2}}{\pi b c^2} K(k) [K(k) - E(k)], \quad A^2(1, 1, 0) = \frac{1}{c^2} \frac{z-1}{z+1},$$

$$A^2(2, 1, 0) = \frac{2 \sqrt{4b^2 + c^2}}{\pi b c^2 k^2} [K(k) - E(k)] [(2 + k^2) K(k) - 2(1 + k^2) E(k)],$$

$$A^2(3, 2, 0) = \frac{2 \sqrt{4b^2 + c^2}}{9\pi b c^2 k^4} [(2 + k^2) K(k) - 2(1 + k^2) E(k)] \times \\ \times [(8 + 3k^2 + 4k^4) K(k) - (8 + 7k^2 + 8k^4) E(k)],$$

$$A^2(0, 0, 1) = \frac{2}{\pi c \sqrt{4b^2 + c^2}} K(k) [2E(k) - (1 - k^2) K(k)],$$

$$A^2(0, 1, 1) = \frac{z-1}{2bcz},$$

$$A^2(1, 1, 1) = \frac{2}{\pi c k^2 \sqrt{4b^2 + c^2}} [K(k) - E(k)] [(1 + k^2) E(k) - (1 - k^2) K(k)],$$

$$A^2(2, 2, 1) = \frac{2}{3\pi c k^4 \sqrt{4b^2 + c^2}} [2(1 - k^2 + k^4) E(k) - (1 - k^2)(2 - k^2) K(k)] \times \\ \times [(2 + k^2) K(k) - 2(1 + k^2) E(k)],$$

$$A^2(1, 0, 2) = \frac{2b \sqrt{4b^2 + c^2}}{\pi c^4 k^2} [(1 + k^2) E(k) - (1 - k^2) K(k)] [2E(k) - (1 - k^2) K(k)],$$

$$A^1(2, 1, 2) = \frac{2b \sqrt{4b^2 + c^2}}{3\pi c^4 k^4} [(1 + k^2) E(k) - (1 - k^2) K(k)] \times \\ \times [2(1 - k^2 + k^4) E(k) - (1 - k^2)(2 - k^2) K(k)],$$

$$A^2(3, 2, 2) = \frac{2b \sqrt{4b^2 + c^2}}{15\pi c^4 k^6} [2(1 - k^2 + k^4) E(k) - (1 - k^2)(2 - k^2) K(k)] \times \\ \times [(1 + k^2)(8 - 13k^2 + 8k^4) E(k) - (1 - k^2)(8 - k^2 - 4k^4) K(k)],$$

$$A^2(1, 1, 3) = \frac{2b^2}{3\pi c^3 k^4 \sqrt{4b^2 + c^2}} [(1 + 14k^2 + k^4) E(k) - (1 - k^2)(1 + 7k^2) K(k)] \times \\ \times [(1 + k^2) E(k) - (1 - k^2) K(k)].$$

2.16.38. Интегралы от $x^\alpha J_\lambda(ax^m) J_\mu(bx^n) K_\nu(cx)$.

$$1. \int_0^\infty x^{\alpha-1} J_\mu(ax^2) J_\nu(cx) K_\nu(cx) dx = \frac{1}{8\nu} \left(\frac{2}{a}\right)^{\alpha/2} \Gamma\left[\frac{(2\mu+\alpha)/4}{1+(2\mu-\alpha)/4}\right] \times$$

$$\times {}_2F_3\left(\frac{\alpha-2\mu}{4}, \frac{\alpha+2\mu}{4}; \frac{1}{2}, 1-\frac{\nu}{2}, 1+\frac{\nu}{2}; \frac{c^4}{16a^2}\right) - \frac{2^{\alpha/2-3} c^2}{(\nu^2-1) a^{\alpha/2+1}} \times$$

$$\times \Gamma\left[\frac{(2+2\mu+\alpha)/4}{(2+2\mu-\alpha)/4}\right] {}_2F_3\left(\frac{1-\mu}{2} + \frac{\alpha}{4}, \frac{1+\mu}{2} + \frac{\alpha}{4}; \frac{3}{2}, \frac{3-\nu}{2}, \frac{3+\nu}{2}; \frac{c^4}{16a^2}\right) +$$

$$+ \frac{2^{\alpha/2-\nu-3} c^{2\nu}}{a^{\alpha/2+\nu}} \Gamma\left[\frac{-\nu, (2\mu+2\nu+\alpha)/4}{\nu+1, 1+(2\mu-2\nu-\alpha)/4}\right] \times$$

$$\times {}_2F_3\left(\frac{\alpha}{4} + \frac{\nu-\mu}{2}, \frac{\alpha}{4} + \frac{\nu+\mu}{2}; 1+\frac{\nu}{2}, \frac{1+\nu}{2}, 1+\nu; \frac{c^4}{16a^2}\right)$$

[$a, \operatorname{Re} c, \operatorname{Re}(\alpha+2\mu), \operatorname{Re}(\alpha+2\mu+2\nu) > 0$].

$$2. \int_0^\infty x J_\mu(ax^2) J_\nu(cx) K_\nu(cx) dx =$$

$$= \frac{1}{2c^2} \Gamma\left[\frac{(\mu+\nu+1)/2}{\nu+1}\right] M_{-\mu/2, \nu/2}\left(\frac{c^2}{2a}\right) W_{-\mu/2, \nu/2}\left(\frac{c^2}{2a}\right)$$

[$a > 0; |\arg c| < \pi/4; \operatorname{Re} \mu, \operatorname{Re}(\mu+\nu) > -1$].

$$3. \int_0^\infty x J_0(ax^2) J_\nu(cx) K_\nu(cx) dx = \frac{1}{4a} I_{\nu/2}\left(\frac{c^2}{4a}\right) K_{\nu/2}\left(\frac{c^2}{4a}\right)$$

[$a > 0; |\arg c| < \pi/4; \operatorname{Re} \nu > -1$].

$$4. \int_0^\infty x^{\nu+2} J_{(\nu+1)/2}(ax^2) J_\nu(cx) K_\nu(cx) dx = \frac{c^\nu}{2^{(\nu+3)/2} \sqrt{\pi} a^{\nu+3/2}} K_{\nu/2}\left(\frac{c^2}{2a}\right)$$

[$a > 0; |\arg c| < \pi/4; \operatorname{Re} \nu > -1$].

$$5. \int_0^\infty x^{\nu+3} J_{\nu/2}(ax^2) J_\nu(cx) K_\nu(cx) dx = \frac{c^{\nu+1}}{2^{\nu/2+2} \sqrt{\pi} a^{\nu+5/2}} K_{(\nu-1)/2}\left(\frac{c^2}{2a}\right)$$

[$a > 0; |\arg c| < \pi/4; \operatorname{Re} \nu > -1$].

$$6. \int_0^\infty x^{2-\nu} J_{(\nu-1)/2}(ax^2) J_\nu(cx) K_\nu(cx) dx =$$

$$= 2^{(\nu-5)/2} \sqrt{\pi} a^{\nu-3/2} c^{-\nu} \left[I_{\nu/2}\left(\frac{c^2}{2a}\right) - L_{\nu/2}\left(\frac{c^2}{2a}\right) \right]$$

[$a > 0; |\arg c| < \pi/4; \operatorname{Re} \nu > -1$].

$$7. \int_0^\infty x^3 J_\nu(ax^2) J_\nu(cx) K_\nu(cx) dx = \frac{1}{4a^2} \exp\left(-\frac{c^2}{2a}\right)$$

[$a, \operatorname{Re} c > 0; \operatorname{Re} \nu > -1$].

$$8. \int_0^\infty x^5 J_1(ax^2) J_2(cx) K_2(cx) dx = \frac{c^2}{8a^4} \exp\left(-\frac{c^2}{2a}\right)$$

[$a > 0; |\arg c| < \pi/4$].

$$9. \int_0^\infty x J_{\mu+\nu/4}(bx^2) J_{\mu-\nu/4}(bx^2) K_\nu(cx) dx =$$

$$= \frac{1}{\pi c^2} \Gamma\left(\mu + \frac{2+\nu}{4}\right) \Gamma\left(\mu + \frac{2-\nu}{4}\right) W_{-\mu, \nu/4}\left(\frac{ic^2}{8b}\right) W_{-\mu, \nu/4}\left(-\frac{ic^2}{8b}\right)$$

[$b, \operatorname{Re} c > 0; 4 \operatorname{Re} \mu > |\operatorname{Re} \nu| - 2$].

$$10. \int_0^{\infty} x J_{\nu/4}(bx^2) J_{-\nu/4}(bx^2) K_{\nu}(cx) dx = \frac{\pi}{2^5 b} \sec \frac{\nu\pi}{4} \left[J_{\nu/2}^2 \left(\frac{c^2}{16b} \right) + Y_{\nu/2}^2 \left(\frac{c^2}{16b} \right) \right]$$

[b, Re c > 0; |Re ν| < 2]

2.16.39. Интегралы от $x^{\alpha} J_{\lambda}(ax^{\pm m}) Y_{\mu}(bx^{\pm n}) K_{\nu}(cx)$.

$$1. \int_0^{\infty} x^{\nu+1} [J_{\mu}(ax) Y_{-\mu}(bx) + J_{-\mu}(bx) Y_{\mu}(ax)] K_{\nu}(cx) dx =$$

$$= - \frac{c^{\nu}}{\sqrt{2\pi} (ab)^{\nu+1}} \Gamma(\mu + \nu + 1) \Gamma(\nu - \mu + 1) (z^2 - 1)^{-(2\nu+1)/4} P_{\mu-1/2}^{-\nu-1/2}(z)$$

[2abz = a^2 + b^2 + c^2; Re c > |Im a| + |Im b|; Re μ, Re ν, Re(μ + ν) > -1].

$$2. \int_0^{\infty} x J_0(ax) Y_0(cx) K_0(cx) dx = - \frac{2(a^4 + 4c^4)^{-1/2}}{\pi} \ln \frac{a^2 + \sqrt{a^4 + 4c^4}}{2c^2}$$

[a > 0; |arg c| < π/4].

$$3. \int_0^{\infty} x J_{\nu}(ax^2) Y_{\nu}(cx) K_{\nu}(cx) dx = - \frac{1}{2c^2} W_{\nu/2, \nu/2} \left(\frac{c^2}{2a} \right) W_{-\nu/2, \nu/2} \left(\frac{c^2}{2a} \right)$$

[a > 0; |arg c| < π/4; Re ν > -1/2].

$$4. \int_0^{\infty} x J_0(ax^2) Y_0(cx) K_0(cx) dx = - \frac{1}{4\pi a} K_0^2 \left(\frac{c^2}{4a} \right)$$

[a > 0; |arg c| < π/4].

$$5. \int_0^{\infty} x J_0(ax^2) Y_{\nu}(cx) K_{\nu}(cx) dx =$$

$$= - \frac{1}{4a} \sec \frac{\nu\pi}{2} K_{\nu/2} \left(\frac{c^2}{4a} \right) \left[\frac{1}{\pi} K_{\nu/2} \left(\frac{c^2}{4a} \right) + \sin \frac{\nu\pi}{2} I_{\nu/2} \left(\frac{c^2}{4a} \right) \right]$$

[a > 0; |arg c| < π/4; |Re ν| < 1].

$$6. \int_0^{\infty} x^{\nu+2} J_{\nu}(cx) Y_{(\nu+1)/2}(bx^2) K_{\nu}(cx) \cdot x =$$

$$= \frac{\sqrt{\pi} c^{\nu}}{2^{(\nu+5)/2} b^{\nu+3/2}} \sec \frac{\nu\pi}{2} \left[L_{-\nu/2} \left(\frac{c^2}{2b} \right) - I_{\nu/2} \left(\frac{c^2}{2b} \right) \right]$$

[b > 0; |arg c| < π/4; Re ν > -1].

$$7. \int_0^{\infty} J_{\mu} \left(\frac{b}{x} \right) Y_{\mu} \left(\frac{b}{x} \right) K_0(cx) dx = - \frac{2}{c} J_{2\mu}(2\sqrt{bc}) K_{2\mu}(2\sqrt{bc})$$

[b, Re c > 0].

$$8. \int_0^{\infty} \left[J_{\mu}^2 \left(\frac{b}{x} \right) - Y_{\mu}^2 \left(\frac{b}{x} \right) \right] K_0(cx) dx = \frac{4}{c} Y_{2\mu}(2\sqrt{bc}) K_{2\mu}(2\sqrt{bc})$$

[b, Re c > 0].

2.16.40. Интегралы от $x^{\alpha} J_{\lambda}(ax^{\pm r}) I_{\mu}(\varphi(x)) K_{\nu}(\chi(x))$.

$$1. \int_0^{\infty} x^{\alpha-1} J_{\lambda}(ax) I_{\mu}(bx) K_{\nu}(cx) dx = B^{\alpha}(\lambda, \mu, \nu)$$

[Re(α + λ + μ) > |Re ν|; |c| > |a| + |b|; Re c > |Re b| + |Im a|;
u = (a^2 + b^2 + c^2)/(2bc)],

$$B^{\alpha}(\lambda, \mu, \nu) = 2^{\alpha-2} a^{\lambda} b^{\mu} c^{-\alpha-\lambda-\mu} \Gamma \left[\frac{(\alpha+\lambda+\mu-\nu)/2, (\alpha+\lambda+\mu+\nu)/2}{\lambda+1, \mu+1} \right] \times \\ \times F_4 \left(\frac{\alpha+\lambda+\mu-\nu}{2}, \frac{\alpha+\lambda+\mu+\nu}{2}; \lambda+1, \mu+1; -\frac{a^2}{c^2}, \frac{b^2}{c^2} \right),$$

$$B^{\nu+2}(\mu, \mu, \nu) = \frac{c^{\nu}}{\sqrt{2\pi} (ab)^{\nu+1}} \exp \left[-\frac{(4\nu-2\mu+1)\pi i}{4} \right] (v^2+1)^{-(2\nu+1)/4} Q_{\mu-1/2}^{\nu+1/2}(iv)$$

$$[2abv = c^2 + a^2 - b^2; \operatorname{Re} \mu, \operatorname{Re}(\mu+\nu) > -1],$$

$$B^{\nu+2}(\nu, \nu, \nu) = \frac{2^{\nu-1} a^{\nu} \Gamma(\nu+1/2)}{\sqrt{\pi} (bc)^{\nu+1} (u^2-1)^{\nu+1/2}} \quad [\operatorname{Re} \nu > -1/2],$$

$$B^{\mu+2}(\mu, \nu, \nu) = \frac{(bc)^{-\mu-1} a^{\mu} e^{-(2\mu+1)\pi i/2}}{\sqrt{2\pi} (u^2-1)^{(2\mu+1)/4}} Q_{\nu-1/2}^{\mu+1/2}(u) \\ [\operatorname{Re} \mu, \operatorname{Re}(\mu+\nu) > -1],$$

$$B^0(0, \nu, \nu) = \frac{1}{2bc} (u^2-1)^{-1/2} (u + \sqrt{u^2-1})^{-\nu} \\ [\operatorname{Re} \nu > -1],$$

$$B^2(0, 0, 0) = \frac{1}{2bc \sqrt{u^2-1}}, \quad B^2(0, 1, 1) = \frac{1}{2bc} \left(\frac{u}{\sqrt{u^2-1}} - 1 \right),$$

$$B^3(1, 0, 0) = \frac{au}{2b^2c^2 (u^2-1)^{3/2}}, \quad B^3(0, 0, 1) = \frac{cu-b}{2b^2c^2 (u^2-1)^{3/2}},$$

$$B^3(1, 1, 1) = \frac{a}{2b^2c^2 (u^2-1)^{3/2}}.$$

$$2. \int_0^{\infty} x^{\alpha-1} J_{\lambda}(ax) I_{\mu}(cx) K_{\nu}(cx) dx = A^{\alpha}(\lambda, \mu, \nu)$$

$$[a, \operatorname{Re} c > 0; \operatorname{Re}(\alpha+\lambda+\mu) > |\operatorname{Re} \nu|; \operatorname{Re} \alpha < 5/2; k = a/(\sqrt{a^2+4c^2}+2c)],$$

$$A^{\alpha}(\lambda, \mu, \nu) = 2^{\alpha-2} a^{-\mu-\nu-\alpha} c^{\mu+\nu} \Gamma \left[\frac{-\nu, (\lambda+\mu+\nu+\alpha)/2}{1+(\lambda-\mu-\nu-\alpha)/2, 1+\mu} \right] \times \\ \times {}_4F_3 \left(\frac{\alpha+\mu+\nu-\lambda}{2}, \frac{\lambda+\mu+\nu+\alpha}{2}, \frac{1+\mu+\nu}{2}, 1+\frac{\mu+\nu}{2}; 1+\nu, \right. \\ \left. 1+\mu, 1+\mu+\nu; -\frac{4c^2}{a^2} \right) + 2^{\alpha-2} a^{\nu-\alpha-\mu} c^{\mu-\nu} \Gamma \left[\frac{\nu, (\alpha+\mu+\lambda-\nu)/2}{1+(\lambda+\nu-\mu-\alpha)/2, 1+\mu} \right] \times \\ \times {}_4F_3 \left(\frac{\alpha+\mu-\lambda-\nu}{2}, \frac{\alpha+\lambda+\mu-\nu}{2}, \frac{1+\mu-\nu}{2}, 1+\frac{\mu-\nu}{2}; \right. \\ \left. 1-\nu, 1+\mu, 1+\mu-\nu; -\frac{4c^2}{a^2} \right),$$

$$A^{\nu+2}(\mu, \mu, \nu) = \frac{2^{\nu} c^{\nu-1/2}}{\sqrt{\pi} a^{\nu+1}} \exp \left[-\frac{(4\nu-2\mu+1)\pi i}{4} \right] (a^2+4c^2)^{-(2\nu+1)/4} Q_{\mu-1/2}^{\nu+1/2} \left(\frac{ia}{2c} \right) \\ [\operatorname{Re} \mu > -1; -\operatorname{Re} \mu < \operatorname{Re} \nu < 1/2],$$

$$A^{\nu+3/2} \left(\nu + \frac{1}{2}, \nu, \nu \right) = a^{-\nu-1/2} (4c)^{\nu-1/2} \Gamma \left(\nu + \frac{1}{2} \right) P_{-\nu-1/2} \left(\frac{a^2+8c^2}{4c \sqrt{a^2+4c^2}} \right) \\ [-1/2 < \operatorname{Re} \nu < 1],$$

$$A^1(\mu, \nu, \nu) = \frac{e^{-i\pi\nu}}{a} P_{(\mu-1)/2}^{-\nu} \left(\frac{\sqrt{a^2+4c^2}}{a} \right) Q_{(\mu-1)/2}^{\nu} \left(\frac{\sqrt{a^2+4c^2}}{a} \right) \\ [\operatorname{Re} \mu, \operatorname{Re}(\mu+2\nu) > -1],$$

$$A^2(\mu+\nu, \mu, \nu) = \frac{(2c)^{\mu-\nu}}{a \sqrt{a^2+4c^2}} (a + \sqrt{a^2+4c^2})^{\nu-\mu} \\ [\operatorname{Re} \mu, \operatorname{Re}(\mu+\nu) > -1],$$

$$A^1(0, 0, 0) = \frac{2}{\pi a} (1 - k^2) K(k) K(\sqrt{1 - k^2}),$$

$$A^1(1, 0, 0) = \frac{1}{a} \ln \frac{a + \sqrt{a^2 + 4c^2}}{2c},$$

$$A^1(0, 1, 1) = \frac{2(1 - k^2)}{\pi ck^2} E(\sqrt{1 - k^2}) [K(k) - E(k)], \quad A^1(1, 1, 1) = 0,$$

$$A^1(2, 0, 0) = \frac{2}{\pi a(1 - k^2)} [2E(k) - (1 - k^2)K(k)] \times \\ \times [(1 + k^2)K(\sqrt{1 - k^2}) - 2E(\sqrt{1 - k^2})],$$

$$A^1(2, 1, 1) = \frac{2}{3\pi ck^2(1 - k^2)} [(1 + k^2)E(k) - (1 - k^2)K(k)] \times \\ \times [(1 + k^2)E(\sqrt{1 - k^2}) - 2k^2K(\sqrt{1 - k^2})],$$

$$A^1(0, 2, 2) = \frac{2(1 - k^2)}{9\pi ck^4} [(2 + k^2)K(k) - 2(1 + k^2)E(k)] \times \\ \times [2(1 + k^2)E(\sqrt{1 - k^2}) - k^2K(\sqrt{1 - k^2})].$$

$$3. \int_0^\infty J_{2\nu+2\mu}(a\sqrt{x}) I_\mu(cx) K_\nu(cx) dx = \\ = \frac{4}{a^2} \Gamma\left[\frac{\mu+1/2}{\nu+\mu+1}\right] M_{(\mu-\nu)/2, (\mu+\nu)/2}\left(\frac{a^2}{8c}\right) W_{(\nu-\mu)/2, (\nu+\mu)/2}\left(\frac{a^2}{8c}\right) \\ [a, \operatorname{Re} c > 0; \operatorname{Re} \mu, \operatorname{Re}(\mu + \nu) > -1/2].$$

$$4. \int_0^\infty J_{4\nu}(a\sqrt{x}) I_\nu(cx) K_\nu(cx) dx = \frac{1}{2c} I_\nu\left(\frac{a^2}{16c}\right) K_\nu\left(\frac{a^2}{16c}\right) \\ [a, \operatorname{Re} c > 0 \quad \operatorname{Re} \nu > -1/4].$$

$$5. \int_0^\infty x^{\nu+1} J_{2\nu}(b\sqrt{x}) I_{2\nu}(b\sqrt{x}) K_\nu(cx) dx = \frac{\sqrt{\pi}}{2^{\nu+1}} b^{2\nu+1} c^{-2\nu-5/2} J_{\nu-1/2}\left(\frac{b^2}{2c}\right) \\ [\operatorname{Re} c > 0, \operatorname{Re} \nu > -1/2].$$

$$6. \int_0^\infty x^{1-\nu} [J_{-2\nu}(b\sqrt{x}) I_{2\nu}(b\sqrt{x}) - J_{2\nu}(b\sqrt{x}) I_{-2\nu}(b\sqrt{x})] K_\nu(cx) dx = \\ = -2^\nu \sqrt{\pi} b^{1-2\nu} c^{2\nu-5/2} \sin \nu\pi J_{\nu+1/2}\left(\frac{b^2}{2c}\right) \\ [\operatorname{Re} c > 0, \operatorname{Re} \nu < 3/2].$$

$$7. \int_0^\infty x^\nu J_{2\nu-1}(b\sqrt{x}) I_{2\nu-1}(b\sqrt{x}) K_\nu(cx) dx = 2^{-\nu} \sqrt{\pi} b^{2\nu-1} c^{-2\nu-1/2} J_{\nu-1/2}\left(\frac{b^2}{2c}\right) \\ [\operatorname{Re} c, \operatorname{Re} \nu > 0].$$

$$8. \int_0^\infty x^{-\nu} J_{\pm(2\nu+1)}(b\sqrt{x}) I_{\mp(2\nu+1)}(b\sqrt{x}) K_\nu(cx) dx = \\ = \pm 2^\nu \sqrt{\pi} b^{-2\nu-1} c^{2\nu-1/2} \left[\cos \nu\pi H_{\nu+1/2}\left(\frac{b^2}{2c}\right) \mp \sin \nu\pi J_{\nu+1/2}\left(\frac{b^2}{2c}\right) \right] \\ [\operatorname{Re} c > 0, \operatorname{Re} \nu < 1/2].$$

$$9. \int_0^\infty x J_0(b\sqrt{x}) I_0(b\sqrt{x}) K_0(cx) dx = \frac{1}{c^2} \cos \frac{b^2}{2c} \\ [\operatorname{Re} c > 0].$$

$$10. \int_0^{\infty} \frac{1}{x^2} J_0\left(\frac{a}{x}\right) I_\nu(cx) K_\nu(cx) dx = \frac{2}{a} J_{2\nu}(2\sqrt{ac}) K_{2\nu}(2\sqrt{ac})$$

$$[a, \operatorname{Re} c > 0; \operatorname{Re} \nu > -1/4].$$

$$11. \int_0^{\infty} \frac{1}{x^2} J_0\left(\frac{a}{x}\right) I_{\pm 1/4}(cx) K_{1/4}(cx) dx = \frac{1}{\sqrt{a^2 c}} \exp(-2\sqrt{ac}) \begin{cases} \sin 2\sqrt{ac} \\ \cos 2\sqrt{ac} \end{cases}$$

$$[a, \operatorname{Re} c > 0].$$

$$12. \int_0^{\infty} x J_\nu(ax) I_{\nu/2}(c\sqrt{x^2+z^2}-cz) K_{\nu/2}(c\sqrt{x^2+z^2}+cz) dx =$$

$$= \frac{1}{a\sqrt{a^2+4c^2}} \exp(-z\sqrt{a^2+4c^2})$$

$$[a, \operatorname{Re} a, \operatorname{Re} z > 0; \operatorname{Re} \nu > -1].$$

2.16.41. Интегралы от $x^\alpha Y_\mu(b\sqrt{x}) I_\mu(b\sqrt{x}) K_\nu(cx)$.

$$1. \int_0^{\infty} x^\nu Y_{2\nu-1}(b\sqrt{x}) I_{2\nu-1}(b\sqrt{x}) K_\nu(cx) dx = 2^{-\nu-1} \sqrt{\pi} b^{2\nu-1} c^{-2\nu-1/2} \times$$

$$\times \cos \nu \pi \left[H_{1/2-\nu}\left(\frac{b^2}{2c}\right) + \cos \nu \pi J_{\nu-1/2}\left(\frac{b^2}{2c}\right) + \sin \nu \pi Y_{\nu-1/2}\left(\frac{b^2}{2c}\right) \right]$$

$$[b, \operatorname{Re} c, \operatorname{Re} \nu > 0].$$

2.16.42. Интегралы, содержащие $x^\alpha I_\lambda(ax) I_\mu(bx) K_\nu(cx)$.

$$1. \int_0^{\infty} x^{\alpha-1} I_\lambda(ax) I_\mu(bx) K_\nu(cx) dx = A^\alpha(\lambda, \mu, \nu)$$

$$[\operatorname{Re}(\alpha + \lambda + \mu) > |\operatorname{Re} \nu|; |c| > |a| + |b|; \operatorname{Re} c > |\operatorname{Re} a| + |\operatorname{Re} b|; z = (b^2 + c^2 - a^2)/(2ab)],$$

$$A^\alpha(\lambda, \mu, \nu) = \frac{2^{\alpha-1} a^\lambda b^\mu}{c^{\alpha+\lambda+\mu}} \Gamma\left[\frac{(\alpha + \lambda + \mu - \nu)/2, (\alpha + \lambda + \mu + \nu)/2}{\lambda + 1, \mu + 1}\right] \times$$

$$\times F_4\left(\frac{\alpha + \lambda + \mu - \nu}{2}, \frac{\alpha + \lambda + \mu + \nu}{2}; \lambda + 1, \mu + 1; \frac{a^2}{c^2}, \frac{b^2}{c^2}\right),$$

$$A^{\mu+2}(\mu, \nu, \nu) = \frac{a^\nu}{\sqrt{2\pi} (bc)^{\nu+1}} (z^2 - 1)^{-(2\mu+1)/4} \exp\left(-\frac{2\mu+1}{4} \pi i\right) Q_{\nu-1/2}^{\mu+1/2}(z)$$

$$[\operatorname{Re} \mu, \operatorname{Re}(\mu + \nu) > -1].$$

$$A^{\nu+2}(\nu, \mu, \nu) = \frac{a^\mu}{\sqrt{2\pi} (bc)^{\mu+1}} (z^2 - 1)^{-(2\nu+1)/4} \exp\left[-\frac{2\nu+1}{4} \pi i\right] Q_{\mu-1/2}^{\nu+1/2}(z)$$

$$[\operatorname{Re} \nu, \operatorname{Re}(\mu + \nu) > -1].$$

$$2. \int_0^{\infty} x^{\alpha-1} I_\lambda(bx) I_\mu(bx) K_\nu(cx) dx = B^\alpha(\lambda, \mu, \nu)$$

$$[\operatorname{Re}(\alpha + \lambda + \mu) > |\operatorname{Re} \nu|; |c| > 2|b|; \operatorname{Re} c > 2|\operatorname{Re} b|; z = \sqrt{c^2 - 4b^2}/c],$$

$$B^\alpha(\lambda, \mu, \nu) = \frac{2^{\alpha-2} b^{\lambda+\mu}}{c^{\alpha+\lambda+\mu}} \Gamma\left[\frac{(\alpha + \lambda + \mu + \nu)/2, (\alpha + \lambda + \mu - \nu)/2}{\mu + 1, \lambda + 1}\right] \times$$

$$\times {}_4F_3\left(\frac{1 + \lambda + \mu}{2}, 1 + \frac{\lambda + \mu}{2}, \frac{\alpha + \lambda + \mu + \nu}{2}, \frac{\alpha + \lambda + \mu - \nu}{2};$$

$$\lambda + \mu + 1, \mu + 1, \lambda + 1; \frac{4b^2}{c^2}\right),$$

$$B^1(\mu, \mu, \nu) = \frac{1}{2c} \Gamma\left(\mu + \frac{\nu+1}{2}\right) \Gamma\left(\mu + \frac{1-\nu}{2}\right) [P_{(\nu-1)/2}^{-\mu}(z)]^2$$

$$[|\operatorname{Re} \nu| < 2\operatorname{Re} \mu + 1].$$

$$B^1(-\mu, \mu, \nu) = \frac{\pi}{2c} \sec \frac{\nu\pi}{2} P_{(\nu-1)/2}^{\mu}(2) P_{(\nu-1)/2}^{-\mu}(2)$$

$$[|\operatorname{Re} \nu| < 1]$$

$$B^1(0, 0, 0) = \frac{2}{\pi c} K^2 \left(\sqrt{\frac{1-z}{2}} \right).$$

$$3. \int_0^{\infty} x^{\nu+1} \left[I_{-\mu}^2 \left(\frac{cx}{2} \right) - I_{\mu}^2 \left(\frac{cx}{2} \right) \right] K_{\nu}(cx) dx = \frac{2^{\nu+1} \sin \mu\pi}{\sqrt{\pi} c^{\nu+2}} \Gamma \left[\frac{\nu+\mu+1}{2}, \frac{\nu-\mu+1}{2} \right]$$

$$[\operatorname{Re} c > 0; \operatorname{Re} \nu < -1/2; |\operatorname{Re}(\nu+2\mu)| < 2 + \operatorname{Re} \nu]$$

2.16.43. Интегралы от $x^{\alpha} J_{\lambda}(ax^{\pm q}) K_{\mu}(bx^r) K_{\nu}(cx)$.

$$1. \int_0^{\infty} x^{\alpha-1} J_{\lambda}(ax) K_{\mu}(bx) K_{\nu}(cx) dx = \frac{2^{\alpha-3} a^{\lambda}}{c^{\alpha+\lambda} \Gamma(\lambda+1)} [A(\mu) + A(-\mu)],$$

$$A(\mu) = \left(\frac{b}{c} \right)^{\mu} \Gamma \left[-\mu, \frac{\alpha+\lambda+\mu-\nu}{2}, \frac{\alpha+\lambda+\mu+\nu}{2} \right] \times \\ \times F_4 \left(\frac{\alpha+\lambda+\mu-\nu}{2}, \frac{\alpha+\lambda+\mu+\nu}{2}; \lambda+1, \mu+1; -\frac{a^2}{c^2}, \frac{b^2}{c^2} \right)$$

$$[\operatorname{Re}(\alpha+\lambda) > |\operatorname{Re} \mu| + |\operatorname{Re} \nu|; \operatorname{Re}(b+c) > |\operatorname{Im} a|]$$

$$2. \int_0^{\infty} x^{\mu+1} J_{\mu}(ax) K_{\nu}(bx) K_{\nu}(cx) dx = A(\mu, \nu)$$

$$[\operatorname{Re} \mu > |\operatorname{Re} \nu| - 1; \operatorname{Re}(b+c) > |\operatorname{Im} a|; a = (a^2 + b^2 + c^2)/(2bc)],$$

$$A(\mu, \nu) = \frac{\sqrt{\pi} a^{\mu}}{2^{3/2} (bc)^{\mu+1}} (u^2 - 1)^{-(2\mu+1)/4} \Gamma(\mu+\nu+1) \Gamma(\mu-\nu+1) P_{\nu-1/2}^{-\mu-1/2}(u),$$

$$A(0, \nu) = \frac{\pi}{4bc \sin \nu\pi} (u^2 - 1)^{-1/2} [(u + \sqrt{u^2 - 1})^{\nu} - (u + \sqrt{u^2 - 1})^{-\nu}]$$

$$[|\operatorname{Re} \nu| < 1]$$

$$A(0, 0) = \frac{1}{2bc \sqrt{u^2 - 1}} \ln(u + \sqrt{u^2 - 1}),$$

$$A(1, 0) = \frac{1}{2b^2c(u^2 - 1)} \left[\frac{u}{\sqrt{u^2 - 1}} \ln(u + \sqrt{u^2 - 1}) - 1 \right],$$

$$A(1, 1) = \frac{a}{2b^2c^2(u^2 - 1)} \left[u - \frac{1}{\sqrt{u^2 - 1}} \ln(u + \sqrt{u^2 - 1}) \right].$$

$$3. \int_0^{\infty} x^{\alpha-1} J_{\lambda}(ax) K_{\mu}(cx) K_{\nu}(cx) dx = A^{\alpha}(\lambda, \mu, \nu)$$

$$[\operatorname{Re}(\alpha+\lambda) > |\operatorname{Re} \mu| + |\operatorname{Re} \nu|; 2\operatorname{Re} c > |\operatorname{Im} a|; k = a/(\sqrt{a^2 + 4c^2} + 2c), \\ z = \sqrt{a^2 + 4c^2}/a],$$

$$A^{\alpha}(\lambda, \mu, \nu) = \frac{2^{\alpha-3} a^{\lambda}}{c^{\alpha+\lambda}} \times$$

$$\times \Gamma \left[\frac{(\alpha+\lambda+\mu+\nu)/2, (\alpha+\lambda+\mu-\nu)/2, (\alpha+\lambda+\nu-\mu)/2, (\alpha+\lambda-\mu-\nu)/2}{\alpha+\lambda, \lambda+1} \right] \times$$

$$\times {}_4F_3 \left(\frac{\alpha+\lambda+\mu+\nu}{2}, \frac{\alpha+\lambda+\mu-\nu}{2}, \frac{\alpha+\lambda+\nu-\mu}{2}, \frac{\alpha+\lambda-\mu-\nu}{2}; \right.$$

$$\left. \frac{\alpha+\lambda}{2}, \frac{\alpha+\lambda+1}{2}, \lambda+1; -\frac{a^2}{4c^2} \right),$$

$$A^1(\mu, \nu, \nu) = \frac{e^{2\nu\pi i}}{a} \left[\frac{(\mu+1)/2 + \nu}{(\mu+1)/2 - \nu} \right] \left[Q_{(\mu-1)/2}^{-\nu}(z) \right]^2$$

[Re $\mu > 2$ | Re ν | -1],

$$A^2(\mu, \nu, \nu) = \frac{e^{2\nu\pi i}}{a^2 z} \Gamma \left[\frac{1 + \nu + \mu/2}{\mu/2 - \nu} \right] Q_{\mu/2}^{-\nu}(z) Q_{\mu/2-1}^{-\nu}(z)$$

[Re $\mu > 2$ | Re ν | -2],

$$A^2(\mu, \nu+1, \nu) = \frac{2e^{2\nu\pi i}}{a^2 z} \Gamma \left[\frac{(\mu+3)/2 + \nu}{(\mu-1)/2 - \nu} \right] Q_{(\mu-1)/2}^{-\nu}(z) Q_{(\nu-1)/2}^{-\nu-1}(z)$$

[Re $\mu, \text{Re}(\mu-2\nu) > -1$; Re $(\mu+2\nu) > -3$],

$$A^{3/2-\nu} \left(\nu - \frac{1}{2}, \nu, \nu \right) = \frac{\pi a^{2\nu-1} z^{\nu-1/2}}{2^{2\nu-1} \Gamma(\nu+1/2)} \Gamma \left(\frac{1}{2} - \nu \right) P_{\nu-1/2} \left(\frac{a^2 + 8c^2}{4c \sqrt{a^2 + 4c^2}} \right)$$

[Re $\nu < 1/2$],

$$A^1(0, 0, 0) = \frac{4k}{a} K^2(k) \quad A^1(1, 0, 0) = \frac{1}{a} \ln^3 \frac{a + \sqrt{a^2 + 4c^2}}{2c},$$

$$A^1(2, 0, 0) = \frac{4}{ak} [K(k) - E(k)]^2$$

$$A^1(4, 0, 0) = \frac{4}{9ak^3} [(2+k^2)K(k) - 2(1+k^2)E(k)]^2,$$

$$A^1(2, 1, 1) = \frac{a}{12c^2 k^3} [(1+k^2)E(k) - (1-k^2)K(k)]^2,$$

$$A^2(0, 0, 0) = \frac{1}{a^2 z} \ln \frac{z+1}{z-1},$$

$$A^2(1, 0, 0) = \frac{4}{a \sqrt{a^2 + 4c^2}} K(k) [K(k) - E(k)],$$

$$A^2(3, 0, 0) = \frac{4}{ak^3 \sqrt{a^2 + 4c^2}} [K(k) - E(k)] [(2+k^2)K(k) - 2(1+k^2)E(k)],$$

$$A^2(0, 0, 1) = \frac{1}{c \sqrt{a^2 + 4c^2}} K(k) [2E(k) - (1-k^2)K(k)],$$

$$A^2(2, 0, 1) = \frac{1}{ck^2 \sqrt{a^2 + 4c^2}} [K(k) - E(k)] [(1+k^2)E(k) - (1-k^2)K(k)],$$

$$A^2(4, 0, 1) = \frac{1}{32k^4 c \sqrt{a^2 + 4c^2}} [(2+k^2)K(k) - 2(1+k^2)E(k)] \times$$

$$\times [2(1-k^2+k^4)E(k) - (1-k^2)(2-k^2)K(k)],$$

$$A^2(1, 1, 1) = \frac{a}{4k^2 c^2 \sqrt{a^2 + 4c^2}} [(1+k^2)E(k) - (1-k^2)K(k)] [2E(k) - (1-k^2)K(k)],$$

$$A^2(2, 1, 2) = \frac{a}{48k^4 c^3 \sqrt{a^2 + 4c^2}} [(1+k^2)E(k) - (1-k^2)K(k)] \times$$

$$\times [(1+14k^2+k^4)E(k) - (1-k^2)(1+7k^2)K(k)],$$

$$A^2(4, 2, 3) = \frac{a^4}{26880k^8 c^6 \sqrt{a^2 + 4c^2}} [(1-k^2)(2-9k^2-k^4)K(k) -$$

$$- 2(1+k^2)(1-6k^2+k^4)E(k)] [(1-k^2)(2-21k^2-108k^4-6k^6) \times$$

$$\times K(k) - 2(1-11k^2-108k^4-11k^6+k^8)E(k)].$$

$$4. \int_0^\infty \frac{1}{x^2} J_0 \left(\frac{a}{x} \right) K_\nu(cx) dx =$$

$$= -\frac{2\pi}{a} K_{2\nu}(2\sqrt{ac}) [\sin \nu\pi J_{2\nu}(2\sqrt{ac}) + \cos \nu\pi Y_{2\nu}(2\sqrt{ac})]$$

[a, Re $c > 0$; |Re ν | < 1/4]

$$5. \int_0^{\infty} \frac{1}{x^2} J_0\left(\frac{a}{x}\right) K_{1/4}^2(cx) dx = \frac{\pi}{a \sqrt{2ac}} \exp(-2\sqrt{ac}) [\cos 2\sqrt{ac} - \sin 2\sqrt{ac}]$$

[a, Re c > 0]

$$6. \int_0^{\infty} x J_{\mu}(ax^2) K_{\nu}(ce^{\pi i/4}x) K_{\nu}(ce^{-\pi i/4}x) dx = \\ = \frac{1}{4c^2} \Gamma\left(\frac{\mu+\nu+1}{2}\right) \Gamma\left(\frac{\mu-\nu+1}{2}\right) W_{-\mu/2, \nu/2}\left(\frac{ic^2}{2a}\right) W_{-\mu/2, \nu/2}\left(-\frac{ic^2}{2a}\right) \\ [a > 0; |\arg c| < \pi/4; \operatorname{Re} \mu > |\operatorname{Re} \nu| - 1]$$

$$7. \int_0^{\infty} x J_0(ax^2) K_{\nu}(ce^{\pi i/4}x) K_{\nu}(ce^{-\pi i/4}x) dx = \frac{\pi^2}{32a} \sec \frac{\nu\pi}{2} H_{\nu/2}^{(1)}\left(\frac{c^2}{4a}\right) H_{\nu/2}^{(2)}\left(\frac{c^2}{4a}\right)$$

[a > 0; |\arg c| < \pi/4; |\operatorname{Re} \nu| < 1]

$$8. \int_0^{\infty} x^{2-\nu} J_{(\nu-1)/2}(ax^2) K_{\nu}(ce^{\pi i/4}x) K_{\nu}(ce^{-\pi i/4}x) dx = \\ = \frac{2^{(\nu-5)/2} \pi^{3/2} a^{\nu-3/2}}{c^{\nu}} \sec \frac{\nu\pi}{2} \left[H_{\nu/2}\left(\frac{c^2}{2a}\right) - Y_{\nu/2}\left(\frac{c^2}{2a}\right) \right] \\ [a > 0; |\arg c| < \pi/4; |\operatorname{Re} \nu| < 1]$$

$$9. \int_0^{\infty} x^{\alpha-1} J_{\mu}(b\sqrt{x}) K_{\mu}(b\sqrt{x}) K_{\nu}(cx) dx = \\ = \frac{2^{\alpha-3}}{\mu c^{\alpha}} \Gamma\left(\frac{\alpha+\nu}{2}\right) \Gamma\left(\frac{\alpha-\nu}{2}\right) {}_2F_3\left(\frac{\alpha+\nu}{2}, \frac{\alpha-\nu}{2}; \frac{1}{2}, 1-\frac{\mu}{2}, 1+\frac{\mu}{2}; -\frac{b^4}{16c^2}\right) - \\ - \frac{2^{\alpha-2}b^3}{c^{\alpha+1}(\mu^2-1)} \Gamma\left(\frac{\alpha+\nu+1}{2}\right) \Gamma\left(\frac{\alpha-\nu+1}{2}\right) \times \\ \times {}_2F_3\left(\frac{\alpha+\nu+1}{2}, \frac{\alpha-\nu+1}{2}; \frac{3}{2}, \frac{3-\mu}{2}, \frac{3+\mu}{2}; -\frac{b^4}{16c^2}\right) + \\ + \frac{2^{\alpha-\mu-3}b^{2\mu}}{c^{\alpha+\mu}} \Gamma\left[\frac{-\mu, (\alpha+\mu+\nu)/2, (\alpha+\mu-\nu)/2}{1+\mu}\right] \times \\ \times {}_2F_3\left(\frac{\alpha+\mu+\nu}{2}, \frac{\alpha+\mu-\nu}{2}; 1+\frac{\mu}{2}, \frac{1+\mu}{2}, 1+\mu; -\frac{b^4}{16c^2}\right) \\ [\operatorname{Re}(\alpha+\nu), \operatorname{Re}(\alpha+\mu+\nu), \operatorname{Re} c > 0]$$

$$10. \int_0^{\infty} x^{\nu} J_{2\nu-1}(b\sqrt{x}) K_{2\nu-1}(b\sqrt{x}) K_{\nu}(cx) dx = \\ = 2^{-\nu-2} \pi^{3/2} b^{2\nu-1} c^{-2\nu-1/2} \operatorname{cosec} \nu\pi \left[H_{1/2-\nu}\left(\frac{b^2}{2c}\right) - Y_{1/2-\nu}\left(\frac{b^2}{2c}\right) \right] \\ [\operatorname{Re} c, \operatorname{Re} \nu > 0]$$

2.16.44. Интегралы, содержащие $x^{\alpha} Y_{\lambda}(ax^{\pm q}) K_{\mu}(bx^r) K_{\nu}(cx)$.

$$1. \int_0^{\infty} \frac{1}{x^2} Y_0\left(\frac{a}{x}\right) K_{\nu}^2(cx) dx = \\ = \frac{2\pi}{c} K_{2\nu}(2\sqrt{ac}) [\cos \nu\pi J_{2\nu}(2\sqrt{ac}) - \sin \nu\pi Y_{2\nu}(2\sqrt{ac})] \\ [a, \operatorname{Re} c > 0; |\operatorname{Re} \nu| < 1/4]$$

$$2. \int_0^{\infty} \left[\sin \frac{\mu - \nu}{2} \pi J_{\nu}(b \sqrt{x}) + \cos \frac{\mu - \nu}{2} \pi Y_{\mu}(b \sqrt{x}) \right] K_{\mu}(b \sqrt{x}) K_{\nu}(cx) dx = I_{\nu}$$

[Re c > 0; |Re (μ + ν)| < 1]

$$I_{\nu} = -\frac{1}{2b^2} \Gamma\left(\frac{1+\mu-\nu}{2}\right) \Gamma\left(\frac{1-\mu-\nu}{2}\right) W_{\nu/2, \mu/2}\left(\frac{ib^2}{2c}\right) W_{\nu/2, \mu/2}\left(-\frac{ib^2}{2c}\right),$$

$$I_0 = -\frac{\pi^2}{16c} \sec \frac{\mu\pi}{2} H_{\mu/2}^{(1)}\left(\frac{b^2}{4c}\right) H_{\mu/2}^{(2)}\left(\frac{b^2}{4c}\right).$$

2.16.45. Интегралы от $x^{\alpha} I_{\lambda}(ax) K_{\mu}(bx) K_{\nu}(cx)$.

$$1. \int_0^{\infty} I_{\mu}(bx) K_{\mu}(bx) K_{\nu}(cx) dx =$$

$$= -\frac{\pi}{4b \sin \nu\pi} \left[P_{\mu-1/2}^{-\nu/2}(z) Q_{\mu-1/2}^{\nu/2}(z) - P_{\mu-1/2}^{\nu/2}(z) Q_{\mu-1/2}^{-\nu/2}(z) \right]$$

[0 < c < 2b; z = √(4b² - c²)/(2b); |Re ν| < 1; 2 Re μ > |Re ν| - 1]

$$2. = \frac{\pi}{4c} \sec \mu\pi \sec \frac{\nu\pi}{2} P_{-(\nu+1)/2}^{-\mu}(\zeta) \left[Q_{-(\nu+1)/2}^{\mu}(\zeta) + Q_{(\nu-1)/2}^{\mu}(\zeta) \right]$$

[0 < 2b < c; ζ = √(c² - 4b²)/c; |Re ν| < 1; 2 Re μ > |Re ν| - 1]

$$3. \int_0^{\infty} I_0(bx) K_0(bx) K_0(cx) dx = \frac{1}{c} K\left(\sqrt{\frac{c - \sqrt{c^2 - 4b^2}}{2c}}\right) K\left(\sqrt{\frac{c + \sqrt{c^2 - 4b^2}}{2c}}\right)$$

[0 < 2b < c]

$$4. \int_0^{\infty} x^{\alpha-1} I_{\lambda}(bx) K_{\mu}(bx) K_{\nu}(cx) dx =$$

$$= 2^{\alpha-3} b^{\lambda+\mu} c^{-\alpha-\lambda-\mu} \Gamma\left[\begin{matrix} -\mu, (\alpha+\lambda+\mu+\nu)/2, (\alpha+\lambda+\mu-\nu)/2 \\ 1+\lambda \end{matrix}\right] {}_4F_3\left(\frac{1+\lambda+\mu}{2}, \right.$$

$$\left. 1+\frac{\lambda+\mu}{2}, \frac{\alpha+\lambda+\mu+\nu}{2}, \frac{\alpha+\lambda+\mu-\nu}{2}; 1+\lambda, 1+\mu, 1+\lambda+\mu; \frac{4b^2}{c^2}\right) +$$

$$+ 2^{\alpha-3} b^{\lambda-\mu} c^{\alpha-\lambda-\alpha} \Gamma\left[\begin{matrix} \mu, (\alpha+\lambda+\nu-\mu)/2, (\alpha+\lambda-\mu-\nu)/2 \\ 1+\lambda \end{matrix}\right] {}_4F_3\left(\frac{1+\lambda-\mu}{2}, \right.$$

$$\left. 1+\frac{\lambda-\mu}{2}, \frac{\alpha+\lambda+\nu-\mu}{2}, \frac{\alpha+\lambda-\mu-\nu}{2}; 1-\mu, 1+\lambda-\mu, 1+\lambda; \frac{4b^2}{c^2}\right)$$

[Re c > 0; Re (α + λ) > |Re μ| + |Re ν|]

$$5. \int_0^{\infty} x^{-1/2} I_{\mu}(bx) K_{\mu}(bx) K_{\nu}(2bx) dx =$$

$$= \frac{1}{4\sqrt{b}} \Gamma\left[\begin{matrix} (1+2\nu)/4, (1-2\nu)/4, (1+2\nu)/4+\mu, (1-2\nu)/4+\mu \\ (3+2\nu)/4+\mu, (3-2\nu)/4+\mu \end{matrix}\right]$$

[Re b > 0; |Re ν| < 1/2, 2 Re μ + 1/2]

$$6. \int_0^{\infty} x^{\alpha-1} I_{\lambda}(ax) K_{\mu}(cx) K_{\nu}(cx) dx = \frac{2^{\alpha-3} a^{\lambda}}{c^{\alpha+\lambda}} \times$$

$$\times \Gamma\left[\begin{matrix} (\alpha+\lambda+\mu+\nu)/2, (\alpha+\lambda+\mu-\nu)/2, (\alpha+\lambda+\nu-\mu)/2, (\alpha+\lambda-\mu-\nu)/2 \\ \alpha+\lambda, \lambda+1 \end{matrix}\right] \times$$

$$\times {}_4F_3\left(\frac{\alpha+\lambda+\mu+\nu}{2}, \frac{\alpha+\lambda+\mu-\nu}{2}, \frac{\alpha+\lambda+\nu-\mu}{2}, \right.$$

$$\left. \frac{\alpha+\lambda-\mu-\nu}{2}; \frac{\alpha+\lambda}{2}, \frac{\alpha+\lambda+1}{2}, \lambda+1; \frac{a^2}{4c^2}\right)$$

[Re (α + λ) > |Re μ| + |Re ν|; 2 Re c > |Re a|]

2.16.46. Интегралы от $x^\alpha K_\lambda(ax^\pm 1) K_\mu(bx^\pm 1) K_\nu(cx)$.

$$1. \int_0^\infty x^{\alpha-1} K_\lambda(ax) K_\mu(bx) K_\nu(cx) dx =$$

$$= \frac{2^{\alpha-1}}{c^\alpha} [A(\lambda, \mu) + A(\lambda, -\mu) + A(-\lambda, \mu) + A(-\lambda, -\mu)],$$

$$A(\lambda, \mu) = \left(\frac{a}{c}\right)^\lambda \left(\frac{b}{c}\right)^\mu \Gamma\left[-\lambda, -\mu, \frac{\alpha+\lambda+\mu-\nu}{2}, \frac{\alpha+\lambda+\mu+\nu}{2}\right] \times \\ \times F_4\left(\frac{\alpha+\lambda+\mu-\nu}{2}, \frac{\alpha+\lambda+\mu+\nu}{2}; \lambda+1, \mu+1; \frac{a^2}{c^2}, \frac{b^2}{c^2}\right) \\ [\operatorname{Re} \alpha > |\operatorname{Re} \lambda| + |\operatorname{Re} \mu| + |\operatorname{Re} \nu|; \operatorname{Re}(a+b+c) > 0].$$

$$2. \int_0^\infty K_\mu^2(bx) K_\nu(cx) dx = \frac{\pi^3}{16b} \sec \mu\pi \sec^2 \frac{\nu\pi}{2} \times$$

$$\times \{P_{\mu-1/2}^{\nu/2}(z) [Q_{\mu-1/2}^{-\nu/2}(z) + Q_{-\mu-1/2}^{-\nu/2}(z)] + P_{\mu-1/2}^{-\nu/2}(z) [Q_{\mu-1/2}^{\nu/2}(z) + Q_{-\mu-1/2}^{\nu/2}(z)]\} \\ [0 < c < 2b; z = \sqrt{4b^2 - c^2}/(2b); |\operatorname{Re} \nu| < 1; |\operatorname{Re} \mu| < 1/2].$$

$$3. = \frac{\pi^2}{4c} \sec \frac{\nu\pi}{2} \operatorname{cosec} 2\mu\pi \{P_{-(\nu+1)/2}^\mu(\xi) [Q_{-(\nu+1)/2}^{-\mu}(\xi) + Q_{(\nu-1)/2}^{-\mu}(\xi)] + \\ + P_{-(\nu+1)/2}^{-\mu}(\xi) [Q_{-(\nu+1)/2}^\mu(\xi) + Q_{(\nu-1)/2}^\mu(\xi)]\} \\ [0 < 2b < c; \xi = \sqrt{c^2 - 4b^2}/c; |\operatorname{Re} \nu| < 1; |\operatorname{Re} \mu| < 1/2].$$

$$4. \int_0^\infty K_0^2(bx) K_0(cx) dx =$$

$$= \frac{\pi}{2b} K\left(\sqrt{\frac{2b - \sqrt{4b^2 - c^2}}{4b}}\right) K\left(\sqrt{\frac{2b + \sqrt{4b^2 - c^2}}{4b}}\right) \quad [0 < c < 2b].$$

$$5. = \frac{\pi}{2c} \left[K^2\left(\sqrt{\frac{c - \sqrt{c^2 - 4b^2}}{2c}}\right) + K^2\left(\sqrt{\frac{c + \sqrt{c^2 - 4b^2}}{2c}}\right) \right] \quad [0 < 2b < c].$$

$$6. \int_0^\infty x^{\alpha-1} K_\lambda(bx) K_\mu(bx) K_\nu(cx) dx = 2^{\alpha-1} b^{\nu-\alpha} c^{-\nu} \times$$

$$\times \Gamma\left[\nu, \frac{(\alpha+\lambda+\mu-\nu)/2, (\alpha+\mu-\lambda-\nu)/2, (\alpha+\lambda-\mu-\nu)/2, (\alpha-\lambda-\mu-\nu)/2}{\alpha-\nu}\right] \times$$

$$\times {}_4F_3\left(\frac{\alpha+\lambda+\mu-\nu}{2}, \frac{\alpha+\mu-\lambda-\nu}{2}, \frac{\alpha+\lambda-\mu-\nu}{2}, \frac{\alpha-\lambda-\mu-\nu}{2};\right.$$

$$1-\nu, \frac{\alpha-\nu}{2}, \frac{1+\alpha-\nu}{2}; \frac{c^2}{4b^2}\bigg) + 2^{\alpha-1} b^{\nu-\alpha} c^\nu \times$$

$$\times \Gamma\left[\frac{-\nu, (\alpha+\lambda+\mu+\nu)/2, (\alpha+\mu+\nu-\lambda)/2, (\alpha+\lambda+\nu-\mu)/2, (\alpha+\nu-\lambda-\mu)/2}{\alpha+\nu}\right] \times$$

$$\times {}_4F_3\left(\frac{\alpha+\lambda+\mu+\nu}{2}, \frac{\alpha+\mu+\nu-\lambda}{2}, \frac{\alpha+\lambda+\nu-\mu}{2}, \frac{\alpha+\nu-\lambda-\mu}{2};\right.$$

$$1+\nu, \frac{\alpha+\nu}{2}, \frac{1+\alpha+\nu}{2}; \frac{c^2}{4b^2}\bigg)$$

$$[\operatorname{Re} \alpha > |\operatorname{Re} \lambda| + |\operatorname{Re} \mu| + |\operatorname{Re} \nu|; \operatorname{Re}(2b+c) > 0]$$

$$7. \int_0^\infty K_\mu^2\left(\frac{b}{x}\right) K_0(cx) dx = \frac{2\pi}{c} K_{\frac{\mu}{2}}(2e^{\pi i/4} \sqrt{bc}) K_{\frac{\mu}{2}}(2e^{-\pi i/4} \sqrt{bc})$$

$$[\operatorname{Re} b, \operatorname{Re} c > 0].$$

2.16.47 Интегралы, содержащие четыре и более функции $J_\mu(bx)$ и $K_\nu(cx)$

$$1. \int_0^\infty x^{2\nu+1} J_\nu(bx) J_\nu(cx) K_\nu(bx) K_\nu(cx) dx = \\ = \frac{2^{\nu-3} b^{2\nu}}{\sqrt{\pi} c^{4\nu+2}} \Gamma \left[\frac{(\nu+1)/2, \nu+1/2, (3\nu+1)/2}{\nu+1} \right] {}_2F_1 \left(\nu+\frac{1}{2}, \frac{3\nu+1}{2}; 2\nu+1; 1-\frac{b}{c^2} \right) \\ [0 < b < c, \operatorname{Re} \nu > -1/3]$$

$$2. \int_0^\infty x J_0(bx) J_0(cx) K_0(bx) K_0(cx) dx = \\ = \frac{\pi}{4c^2} K \left(\frac{\sqrt{c^4 - b^4}}{c^2} \right) = \frac{1}{2(b^2 + c^2)} K \left(\frac{c^2 - b^2}{c^2 + b^2} \right) \quad [0 < b < c]$$

$$3. \int_0^\infty x J_1^2(bx) K_0^2(cx) dx = \frac{1}{2bc} \operatorname{arctg} \frac{b}{c} \quad [b, \operatorname{Re} c > 0]$$

$$4. \int_0^\infty x^{\alpha-1} \prod_{j=1}^n J_{\mu_j}(b_j x) K_\nu(cx) dx = \\ = 2^{\alpha-2} c^{-\alpha-\lambda} \Gamma \left(\frac{\alpha+\lambda-\nu}{2} \right) \Gamma \left(\frac{\alpha+\lambda+\nu}{2} \right) \prod_{j=1}^n \frac{b_j^{\mu_j}}{\Gamma(\mu_j+1)} \times \\ \times F_C^{(n)} \left(\frac{\alpha+\lambda-\nu}{2}, \frac{\alpha+\lambda+\nu}{2}; \mu_1+1, \dots, \mu_n+1; -\frac{b_1^2}{c^2}, \dots, -\frac{b_n^2}{c^2} \right) \\ \left[\lambda = \sum_{j=1}^n \mu_j; \operatorname{Re}(\alpha+\lambda) > |\operatorname{Re} \nu|; \operatorname{Re} c > \sum_{j=1}^n |\operatorname{Im} b_j| \right].$$

2.16.48. Интегралы по индексу от произведений $K_{ix}(c)$ на элементарные функции.

Условие. $c > 0$.

$$1. \int_0^\infty K_{ix}(c) dx = \frac{\pi}{2} e^{-c}.$$

$$2. \int_0^\infty x^{2n} K_{ix}(c) dx = (-1)^n \frac{\pi}{2} \left[\frac{\partial^{2n}}{\partial z^{2n}} e^{-c \operatorname{ch} z} \right]_{z=0}.$$

$$3. \int_0^\infty \frac{K_{ix}(c)}{x^2 + z^2} dx = \frac{\pi}{2z} \int_0^\infty e^{-c \operatorname{ch} t - zt} dt.$$

$$4. \int_0^\infty \frac{K_{ix}(c)}{\sqrt{x^2 + z^2}} dx = \int_0^\infty e^{-c \operatorname{ch} t} K_0(zt) dt.$$

$$5. \int_0^\infty e^{-px} K_{ix}(c) dx = p \int_0^\infty \frac{e^{-c \operatorname{ch} t}}{p^2 + t^2} dt \quad [\operatorname{Re} p > -\pi/2].$$

$$6. \int_0^\infty \operatorname{ch} \frac{\pi x}{2} K_{ix}(c) dx = \frac{\pi}{2}.$$

7. $\int_0^{\infty} \frac{1}{\operatorname{cn}(\pi x/2)} K_{ix}(c) dx = -\frac{\pi c}{2} [K_0(c) L_{-1}(c) + K_1(c) L_0(c)].$
8. $\int_0^{\infty} \frac{x^2}{\operatorname{ch} \pi x} K_{ix}(c) dx = -\frac{e^{-c}}{2} \left[\sqrt{\frac{\pi c}{2}} - \pi c e^{2c} \operatorname{erfc}(\sqrt{2c}) \right].$
9. $\int_0^{\infty} \left\{ \frac{\operatorname{sh} ax \operatorname{sh} bx}{\operatorname{ch} ax \operatorname{ch} bx} \right\} K_{ix}(c) d\lambda = \frac{\pi}{2} e^{-c \cos a \cos b} \left\{ \frac{\operatorname{sh}(c \sin a \sin b)}{\operatorname{ch}(c \sin a \sin b)} \right\}$
 $\quad \quad \quad [\operatorname{Re} a + \operatorname{Re} b \leq \pi/2].$
10. $\int_0^{\infty} \frac{\operatorname{sh} ax}{\operatorname{sh} \pi x} K_{ix}(c) dx = \frac{\sin a}{2} \int_0^{\infty} e^{-t \cos a} K_0(c+t) dt$
 $\quad \quad \quad [\operatorname{Re} a < \pi/2]$
11. $\int_0^{\infty} \frac{\operatorname{sh} ax}{\operatorname{sh}(\pi x/2)} K_{ix}(c) dx = \sin a \int_0^{\infty} e^{-t \cos a} K_0(\sqrt{c^2+t^2}) dt$
 $\quad \quad \quad [\operatorname{Re} a \leq \pi]$
12. $\int_0^{\infty} \frac{\operatorname{ch} ax}{\operatorname{ch} \pi x} K_{ix}(c) dx = \frac{\pi}{2} e^{c \cos a} \operatorname{erfc} \left(\sqrt{2c} \left| \cos \frac{a}{2} \right| \right)$
 $\quad \quad \quad [\operatorname{Re} a \leq 3\pi/2].$
13. $\int_0^{\infty} \frac{\operatorname{ch} ax}{\operatorname{ch}(\pi x/2)} K_{ix}(c) dx = c \int_0^{\infty} (c^2+t^2)^{-1/2} e^{-t \cos a} K_1(\sqrt{c^2+t^2}) dt$
 $\quad \quad \quad [\operatorname{Re} a \leq \pi].$
14. $\int_0^{\infty} x \operatorname{th} \frac{\pi x}{2} K_{ix}(c) dx = c K_0(c).$
15. $\int_0^{\infty} \operatorname{sh} ax \operatorname{th} \pi x K_{ix}(c) dx = \frac{\pi}{2} e^{-c \cos a} \operatorname{erf} \left(\sqrt{2c} \sin \frac{a}{2} \right)$
 $\quad \quad \quad [\operatorname{Re} a \leq \pi/2].$
16. $\int_0^{\infty} \frac{1}{\operatorname{ch} \pi x} \left\{ \frac{\operatorname{sh} ax \operatorname{sh} bx}{\operatorname{ch} ax \operatorname{ch} bx} \right\} K_{ix}(c) dx =$
 $= \frac{\pi}{4} \left[e^{c \cos(a+b)} \operatorname{erfc} \left(\sqrt{2c} \left| \cos \frac{a+b}{2} \right| \right) \mp e^{c \cos(a-b)} \operatorname{erfc} \left(\sqrt{2c} \left| \cos \frac{a-b}{2} \right| \right) \right]$
 $\quad \quad \quad [|\operatorname{Re} a| + |\operatorname{Re} b| \leq 3\pi/2].$
17. $\int_0^{\infty} \frac{\operatorname{sh} \pi x \operatorname{ch} ax}{\operatorname{sh} bx} K_{ix}(c) dx = \frac{\pi^2}{2b} I_0(c) + \frac{\pi^2}{b} \sum_{k=1}^{\infty} (-1)^k I_{k\pi/b}(c) \cos \frac{k a \pi}{b}$
 $\quad \quad \quad [|\operatorname{Re} b| - |\operatorname{Re} a| \geq \pi/2].$
18. $\int_0^{\infty} \frac{\operatorname{sh} ax \operatorname{th} \pi x}{\operatorname{sh} bx} K_{ix}(c) dx =$
 $= \frac{\pi}{4} \left[e^{-c \cos(a+b)} \operatorname{erf} \left(\sqrt{2c} \sin \frac{a+b}{2} \right) + e^{-c \cos(a-b)} \operatorname{erf} \left(\sqrt{2c} \sin \frac{a-b}{2} \right) \right]$
 $\quad \quad \quad [|\operatorname{Re} a| - |\operatorname{Re} b| \leq \pi/2].$
19. $\int_0^{\infty} \left\{ \frac{x \sin bx}{\cos bx} \right\} K_{ix}(c) dx = \frac{\pi}{2} \left\{ \frac{c \operatorname{sh} b}{1} \right\} e^{-c \operatorname{ch} b}$
 $\quad \quad \quad [|\operatorname{Im} b| < \pi/2]$

$$20. \int_0^{\infty} \left\{ \begin{matrix} \operatorname{sh} ax \sin bx \\ \operatorname{ch} ax \cos bx \end{matrix} \right\} K_{ix}(c) dx = \frac{\pi}{2} e^{-c \cos a \operatorname{ch} b} \left\{ \begin{matrix} \sin (c \sin a \operatorname{sh} b) \\ \cos (c \sin a \operatorname{sh} b) \end{matrix} \right\}$$

[|Re a| + |Im b| < π/2].

$$21. \int_0^{\infty} \operatorname{th} \pi x \sin bx K_{ix}(c) dx = \frac{\pi}{2} e^{-c \operatorname{ch} b} \operatorname{erfi} \left(\sqrt{2c} \operatorname{sh} \frac{b}{2} \right) \quad [|\operatorname{Im} b| \leq \pi/2].$$

2.16.49. Интегралы по индексу от произведений $K_{ix}(c)$ на специальные функции.

Условие: $b, c > 0$.

$$1. \int_0^{\infty} x \operatorname{sh} \pi x \left| \Gamma \left(\nu + \frac{ix}{2} \right) \right|^2 K_{ix}(c) dx = 2^{1-2\nu} \pi^2 c^{2\nu} \quad [0 \leq \operatorname{Re} \nu \leq 1/4].$$

$$2. \int_0^{\infty} x \operatorname{sh} \pi x \left| \Gamma \left(\nu + \frac{ix}{2} \right) \Gamma \left(\frac{1}{2} - \nu + \frac{ix}{2} \right) \right|^2 K_{ix}(c) dx = 2^{3/2} \pi^{5/2} \sqrt{c} K_{1/2-2\nu}(c).$$

$$3. \int_0^{\infty} [\operatorname{Re} J_{ix}(b)] K_{ix}(c) dx = \frac{\pi}{2} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} J_0(\sqrt{b^2 - c^2}) \quad \left[\begin{Bmatrix} 0 < a < b \\ 0 < b < c \end{Bmatrix} \right].$$

$$4. \int_0^{\infty} \left\{ \begin{matrix} \sin ax \operatorname{Im} J_{ix}(b) \\ \cos ax \operatorname{Re} J_{ix}(b) \end{matrix} \right\} K_{ix}(c) dx = I$$

$$[b \neq c; z_{\pm} = \sqrt{b^2 - c^2 \pm 2bc \operatorname{sh} a}],$$

$$I = \frac{\pi}{4} [J_0(z_-) \mp J_0(z_+)] \quad [c < b; 2bc \operatorname{sh} a < b^2 - c^2].$$

$$I = 0 \quad [c > b; 2bc \operatorname{sh} a < c^2 - b^2].$$

$$I = \mp \frac{\pi}{4} J_0(z_+) \quad [2bc \operatorname{sh} a > |b^2 - c^2|].$$

$$5. \int_0^{\infty} x \operatorname{Im} [J_{\nu+ix/2}(b) Y_{\nu-ix/2}(b)] K_{ix}(c) dx =$$

$$= 2^{2\nu} b^{2\nu} (4b^2 + c^2)^{-1/2} (c + \sqrt{4b^2 + c^2})^{-2\nu}.$$

$$6. \int_0^{\infty} x \operatorname{sh} \frac{\pi x}{2} \operatorname{Re} [J_{ix/2}(b) Y_{-ix/2}(a)] K_{ix}(c) dx =$$

$$= - \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{c}{\sqrt{4ab - c^2}} \cos \left(\frac{a+b}{2\sqrt{ab}} \sqrt{4ab - c^2} \right) \quad \left[\begin{Bmatrix} 0 < c < 2\sqrt{ab} \\ c > 2\sqrt{ab} \end{Bmatrix} \right].$$

$$7. \int_0^{\infty} x \operatorname{sh} \frac{\pi x}{2} \left[J_{ix/2}(a) \begin{Bmatrix} J_{ix/2}(b) \\ Y_{ix/2}(b) \end{Bmatrix} \pm Y_{ix/2}(a) \begin{Bmatrix} Y_{ix/2}(b) \\ J_{ix/2}(b) \end{Bmatrix} \right] K_{ix}(c) dx =$$

$$= \frac{2c}{\sqrt{c^2 + 4ab}} \begin{Bmatrix} \cos \varphi \\ \sin \varphi \end{Bmatrix} \quad [\varphi = (b-a) \sqrt{c^2 + 4ab} / (2\sqrt{ab})].$$

$$8. \int_0^{\infty} \frac{1}{\operatorname{sh}(\pi x/2)} \operatorname{Im} [J_{ix/2}(b) Y_{-ix/2}(b)] K_{ix}(c) dx =$$

$$= \frac{1}{\pi} K_0 \left(\frac{c + \sqrt{c^2 - 4b^2}}{2} \right) K_0 \left(\frac{c - \sqrt{c^2 - 4b^2}}{2} \right).$$

$$\begin{aligned}
9. \quad & \int_0^\infty \frac{x}{\operatorname{ch} \pi x} \operatorname{Im} [J_{ix}(b) Y_{-ix}(b)] K_{ix}(c) dx = \frac{1}{2} \exp\left(\frac{b^2}{2c} - c\right) \operatorname{erfc}\left(\frac{b}{\sqrt{2c}}\right). \\
10. \quad & \int_0^\infty [J_{ix/2}^2(b) + Y_{ix/2}^2(b)] K_{ix}(c) dx = \int_0^\infty H_{ix/2}^{(1)}(b) H_{ix/2}^{(2)}(b) K_{ix}(c) dx = \\
& = \frac{2}{\pi} K_0\left(\frac{c + \sqrt{c^2 - 4b^2}}{2}\right) K_0\left(\frac{c - \sqrt{c^2 - 4b^2}}{2}\right). \\
11. \quad & \int_0^\infty x \operatorname{sh} \frac{\pi x}{2} [J_{ix/2}^2(b) + Y_{ix/2}^2(b)] K_{ix}(c) dx = \\
& = \int_0^\infty x \operatorname{sh} \frac{\pi x}{2} H_{ix/2}^{(1)}(b) H_{ix/2}^{(2)}(b) K_{ix}(c) dx = \frac{2c}{\sqrt{4b^2 + c^2}}. \\
12. \quad & \int_0^\infty x \operatorname{th} \pi x [J_{ix}^2(b) + Y_{ix}^2(b)] K_{ix}(c) dx = \\
& = \int_0^\infty x \operatorname{th} \pi x H_{ix}^{(1)}(b) H_{ix}^{(2)}(b) K_{ix}(c) dx = \exp\left(\frac{b^2}{2c} - c\right) \operatorname{erfc}\left(\frac{b}{\sqrt{2c}}\right). \\
13. \quad & \int_0^\infty x \operatorname{Im} [I_{ix/2}(a) I_{ix/2}(b)] K_{ix}(c) dx = - \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{c}{\sqrt{4ab - c^2}} \operatorname{ch} \left(\frac{a-b}{2\sqrt{ab}} \sqrt{4ab - c^2} \right) \\
& \quad \left[\begin{Bmatrix} 0 < c < 2\sqrt{ab} \\ c > 2\sqrt{ab} \end{Bmatrix} \right]. \\
14. \quad & \int_0^\infty x \operatorname{th} \pi x \operatorname{Re} [D_{ix-1/2}(b) D_{-ix-1/2}(-b)] K_{ix}(c) dx = \frac{\pi \sqrt{c}}{2} \cos(b\sqrt{2c}).
\end{aligned}$$

2.16.50. Интегралы по индексу от произведений $\begin{Bmatrix} \operatorname{Re} K_{\nu+ix}(c) \\ \operatorname{Im} K_{\nu+ix}(c) \end{Bmatrix}$ на элементарные функции.

Условие $c > 0$

$$\begin{aligned}
1. \quad & \int_0^\infty \begin{Bmatrix} \operatorname{Re} K_{1/2+ix}(c) \\ x \operatorname{Im} K_{1/2+ix}(c) \end{Bmatrix} dx = \frac{\pi}{2(3 \mp 1)^{1/2}} e^{-c}. \\
2. \quad & \int_0^\infty \frac{1}{x^2 + 1/4} \begin{Bmatrix} \operatorname{Re} K_{1/2+ix}(c) \\ x \operatorname{Im} K_{1/2+ix}(c) \end{Bmatrix} dx = \frac{\pi}{2(3 \mp 1)^{1/2}} \left[K_0(c) \pm K_1(c) \mp \frac{e^{-c}}{c} \right]. \\
3. \quad & \int_0^\infty \frac{x}{\operatorname{sh} \pi x} \operatorname{Re} K_{1/2+ix}(c) dx = \frac{\pi}{4} e^c \operatorname{erfc}(\sqrt{2c}). \\
4. \quad & \int_0^\infty \begin{Bmatrix} \operatorname{ch} bx \operatorname{Re} K_{1/2+ix}(c) \\ \operatorname{sh} bx \operatorname{Im} K_{1/2+ix}(c) \end{Bmatrix} dx = \frac{\pi}{2} \begin{Bmatrix} \cos(b/2) \\ \sin(b/2) \end{Bmatrix} e^{-c \cos b} \quad [0 < b < \pi/2]. \\
5. \quad & \int_0^\infty \begin{Bmatrix} \operatorname{sech} \pi x \operatorname{Re} K_{1/2+ix}(c) \\ \operatorname{th} \pi x \operatorname{Im} K_{1/2+ix}(c) \end{Bmatrix} dx = \frac{1}{2} K_0(c).
\end{aligned}$$

$$6. \int_0^{\infty} \frac{1}{\operatorname{ch} (\pi x/2)} \operatorname{Re} K_{1/2+ix}(c) dx = \frac{\pi}{\sqrt{2}} \operatorname{erfc}(\sqrt{c}).$$

$$7. \int_0^{\infty} \frac{\operatorname{sh} bx}{\operatorname{sh} \pi x} \operatorname{Re} K_{1/2+ix}(c) dx = \frac{\pi}{2} \sin \frac{\nu}{2} e^{c \cos b} \operatorname{erfc} \left(\sqrt{2c} \cos \frac{b}{2} \right)$$

$$[0 \leq b \leq \pi]$$

$$8. \int_0^{\infty} \frac{1}{\operatorname{ch} 2\pi x + \cos 2\pi \nu} \left\{ \operatorname{ch} \pi x \operatorname{Re} K_{1/2+ix}(c) \right. \\ \left. \operatorname{sh} 2\pi x \operatorname{Im} K_{1/2+ix}(c) \right\} dx = \frac{1}{4} \left\{ \frac{\sec \nu \pi}{2} \right\} K_{\nu}(c)$$

$$[|\operatorname{Re} \nu| < 1/2.]$$

$$9. \int_0^{\infty} \operatorname{cth} \pi x \left\{ \frac{x \operatorname{Re} K_{1/2+ix}(c)}{x^2 \operatorname{Im} K_{1/2+ix}(c)} \right\} dx = \sqrt{\frac{\pi c}{2}} e^{-c}.$$

$$10. \int_0^{\infty} \left\{ \frac{\cos bx \operatorname{Re} K_{1/2+ix}(c)}{\sin bx \operatorname{Im} K_{1/2+ix}(c)} \right\} dx = \frac{\pi}{2} \left\{ \frac{\operatorname{ch}(b/2)}{\operatorname{sh}(b/2)} \right\} e^{-c \operatorname{ch} b}$$

$$[|\operatorname{Im} b| < \pi/2]$$

$$11. \int_0^{\infty} \cos bx \operatorname{Re} K_{\nu-ix}(c) dx = \frac{\pi}{2} e^{-c \operatorname{ch} b} \operatorname{ch} \nu b$$

$$[b > 0]$$

2.16.51. Интегралы по индексу от произведений $\left\{ \begin{matrix} \operatorname{Re} K_{1/2+ix}(c) \\ \operatorname{Im} K_{1/2+ix}(c) \end{matrix} \right\}$ на специальные функции.

Условие: $c > 0$.

$$1. \int_0^{\infty} \left| \Gamma \left(\frac{\nu + ix}{2} \right) \right|^2 \left\{ \begin{matrix} \operatorname{Re} K_{1/2+ix}(c) \\ x \operatorname{Im} K_{1/2+ix}(c) \end{matrix} \right\} dx = \\ = 2^{\pm 1/2} \pi^{3/2} \Gamma \left(\nu + \frac{1 \mp 1}{2} \right) c^{(2\nu-3)/4} e^{-c/2} W_{-(2\nu+1 \mp 2)/4, -(2\nu-1)/4}(c)$$

$$[\operatorname{Re} \nu > -(1 \mp 1)/2]$$

$$2. \int_0^{\infty} \left| \Gamma \left(\frac{1}{4} + \frac{ix}{2} \right) \right|^2 \operatorname{Re} K_{1/2+ix}(c) dx = \pi^{3/2} e^{-c/2} K_0 \left(\frac{c}{2} \right).$$

$$3. \int_0^{\infty} \operatorname{ch} \pi x \left| \Gamma(\nu + ix) \right|^2 \left\{ \begin{matrix} \operatorname{Re} K_{1/2+ix}(c) \\ x \operatorname{Im} K_{1/2+ix}(c) \end{matrix} \right\} dx = 2^{\nu-3/2} \pi^{3/2} \Gamma \left(\nu + \frac{1 \mp 1}{2} \right) c^{\nu-1/2} e^{-c}$$

$$[\operatorname{Re} \nu > (-1 \pm 1)/2].$$

$$4. \int_0^{\infty} H_{ix}^{(1)}(b) H_{ix}^{(2)}(b) \operatorname{Re} K_{1/2+ix}(c) dx = \frac{1}{\sqrt{2\pi c}} \exp \left(\frac{b^2}{4c} - c \right) K_0 \left(\frac{b^2}{4c} \right) \quad [b > 0].$$

2.16.52. Интегралы по индексу от произведений $K_{\mu+ix}(b) K_{\nu \pm ix}(c)$ на элементарные функции.

$$1. \int_{-\infty}^{\infty} e^{ipx} K_{\nu+ix}(b) K_{\nu-ix}(c) dx = \pi \left(\frac{b+ce^p}{be^p+c} \right)^{\nu} K_{2\nu}(\sqrt{b^2+c^2+2bc \operatorname{ch} p})$$

$$[|\arg b| + |\arg c| + |\operatorname{Im} p| < \pi].$$

2. $\int_{-\infty}^{\infty} e^{(\pi-A)x} K_{i\nu+ix}(b) K_{i\mu+ix}(c) dx = \pi K_{i(\nu-\mu)}(a) e^{-\mu B - \nu C}$
 [A, B и C — углы треугольника, стороны которого равны a, b и c].
3. $= \pi e^{-\mu\pi} K_{i(\nu-\mu)}(b-c)$ [A = 0; 0 < c < b].
4. $= \pi K_{i(\nu-\mu)}(b+c)$ [A = π ; |arg b| + |arg c| < π].
5. $\int_0^{\infty} \operatorname{ch} ax K_{\nu+ix}(c) K_{\nu-ix}(c) ix = \frac{\pi}{2} K_{2\nu} \left(2c \cos \frac{a}{2} \right)$
 [2 |arg c| + |Re a| < π].
6. $\int_0^{\infty} \operatorname{ch} ax K_{ix}(b) K_{ix}(c) dx = \frac{\pi}{2} K_0 (\sqrt{b^2 + c^2 + 2bc \cos a})$
 [|Re a| + |arg b| + |arg c| < π].
7. $\int_0^{\infty} \frac{1}{\operatorname{ch} \pi x} K_{ix}^2(c) dx = -\frac{\pi}{2} E_1(-2c)$ [c > 0].
8. $\int_0^{\infty} x \operatorname{sh} ax K_{ix}(b) K_{ix}(c) dx = \frac{\pi bc}{2z} \sin a K_1(z)$
 [z = $\sqrt{b^2 + c^2 + 2bc \cos a}$; 0 ≤ a < π].
9. $\int_0^{\infty} x \operatorname{sh} \pi x K_{2ix}(b) K_{ix}(c) dx = \frac{b}{4} \sqrt{\frac{\pi^3}{2c}} \exp \left(-\frac{b^2}{8c} - c \right)$
 [|arg b| < $\pi/4$; c > 0].
10. $\int_0^{\infty} x \operatorname{th} \pi x K_{ix}(b) K_{ix}(c) dx = \frac{\pi}{2} \frac{\sqrt{bc}}{b+c} e^{-b-c}$ [|arg b|, |arg c| < π].
11. $\int_0^{\infty} \frac{x \operatorname{sh} \pi x}{x^2 + z^2} K_{ix}(b) K_{ix}(c) dx = \frac{\pi^2}{2} I_z(c) K_z(b)$ [0 < c < b].
12. $= \frac{\pi^2}{2} I_z(b) K_z(c)$ [0 < b < c].
13. $\int_0^{\infty} \cos ax K_{ix}(b) K_{ix}(c) dx = \frac{\pi}{2} K_0 (\sqrt{b^2 + c^2 + 2bc \operatorname{ch} a})$
 [|Im a| + |arg b| + |arg c| < π].
14. $\int_0^{\infty} x \sin \frac{\pi x}{2} K_{ix/2}(b) K_{ix}(c) dx = c \sqrt{\frac{\pi^3}{2b}} \exp \left(-\frac{c^2}{8b} - b \right)$
 [|arg c| < $\pi/2$; b > 0].
15. $\int_{-\infty}^{\infty} \frac{\sin(\nu+ix)\pi}{n+\nu+ix} K_{\nu+ix}(b) K_{\nu-ix}(c) dx = \pi^2 I_n(b) K_{n+2\nu}(c)$ [0 < b < c].
16. $= \pi^2 I_n(c) K_{n+2\nu}(b)$ [0 < c < b].
17. $\int_0^{\infty} \left\{ \frac{\operatorname{sh} \pi x \sin bx}{\operatorname{ch} \pi x \cos bx} \right\} K_{ix}^2(c) dx = \pm \frac{\pi^2}{4} \begin{Bmatrix} J_0(z) \\ Y_0(z) \end{Bmatrix}$ [b, c > 0; z = 2c sh (b/2)].

2.16.53. Интегралы по индексу от произведений $K_{\mu+ix}(b) K_{\nu+ix}(c)$ на специальные функции.
Условие: $a, b, c > 0$.

$$1. \int_0^{\infty} x \operatorname{sh}(\pi x) |\Gamma(\nu+ix)|^2 K_{ix}(b) K_{ix}(c) dx = I_{\nu,1},$$

$$I_{1,1} = 2^{\nu-1} \pi^{3/2} (bc)^{\nu} (b+c)^{-\nu} \Gamma(\nu+1/2) K_{\nu}(b+c) \quad [\operatorname{Re} \nu \geq 0],$$

$$I_{2,1} = \frac{2^{\nu} \pi^{5/2}}{\Gamma(1/2-\nu)} (bc)^{\nu} |b-c|^{-\nu} K_{\nu}(|b-c|) \quad [0 \leq \operatorname{Re} \nu \leq 1/2],$$

$$I_{1,1/2} = 2^{1-2\nu} \pi^2 (bc)^{2\nu} (b^2+c^2)^{-\nu} K_{2\nu}(\sqrt{b^2+c^2}) \quad [\operatorname{Re} \nu \geq 0].$$

$$2. \int_0^{\infty} \frac{x \operatorname{th} \pi x}{|\Gamma(3/4+ix/2)|^2} K_{ix}(b) K_{ix}(c) dx = \frac{\sqrt{\pi bc}}{2} (b^2+c^2)^{-1/4} \exp(-\sqrt{b^2+c^2}).$$

$$3. \int_0^{\infty} x \left\{ \begin{array}{l} \operatorname{sh}(\pi x/2) \operatorname{Re} J_{ix}(b) \\ \operatorname{ch}(\pi x/2) \operatorname{Im} J_{ix}(b) \end{array} \right\} K_{ix}(b) K_{ix}(c) dx = \pm \frac{\pi}{4} \left\{ \begin{array}{l} \sin z \\ \cos z \end{array} \right\} \quad [z = b^2/(2c)].$$

$$4. \int_0^{\infty} x \left\{ \begin{array}{l} \operatorname{sh}(\pi x/2) \operatorname{Re} Y_{ix}(b) \\ \operatorname{ch}(\pi x/2) \operatorname{Im} Y_{ix}(b) \end{array} \right\} K_{ix}(b) K_{ix}(c) dx = -\frac{\pi}{4} \left\{ \begin{array}{l} \cos z \\ \sin z \end{array} \right\} \quad [z = b^2/(2c)].$$

$$5. \int_0^{\infty} x \operatorname{sh} \frac{\pi x}{2} \operatorname{Re} [I_{ix/2}(a)] K_{ix/2}(b) K_{ix}(c) dx = \frac{\pi c}{2 |c^2-4ab|^{1/2}} \left\{ \begin{array}{l} \exp z \\ \sin z \end{array} \right\} \\ \left[z = \frac{a-b}{2\sqrt{ab}} |c^2-4ab|^{1/2}, \left\{ \begin{array}{l} c < 2\sqrt{ab} \\ c > 2\sqrt{ab} \end{array} \right\} \right].$$

$$6. \int_0^{\infty} x \operatorname{th} \pi x \operatorname{Re} [I_{ix}(b)] K_{ix}(b) K_{ix}(c) dx = -\frac{\pi i}{4} \exp\left(-\frac{b^2}{2c}-c\right) \operatorname{erf}\left(\frac{ib}{\sqrt{2c}}\right).$$

$$7. \int_0^{\infty} \operatorname{Re} [I_{\nu+ix/2}(b) K_{\nu-ix/2}(b)] K_{ix}(c) dx = \\ = \frac{\pi}{2} I_{\nu}\left(\frac{\sqrt{4b^2+c^2}-c}{2}\right) K_{\nu}\left(\frac{\sqrt{4b^2+c^2}+c}{2}\right).$$

2.16.54. Интегралы по индексу, содержащие $K_{ix}(b) \times$
 $\times \left\{ \begin{array}{l} \operatorname{Re} K_{1/2+ix}(c) \\ \operatorname{Im} K_{1/2+ix}(c) \end{array} \right\}$.

Условия: $b, c > 0$.

$$1. \int_0^{\infty} K_{ix}(b) \left\{ \begin{array}{l} \operatorname{Re} K_{1/2+ix}(c) \\ x \operatorname{Im} K_{1/2+ix}(c) \end{array} \right\} dx = \frac{\pi^{3/2} b^{(1 \mp 1)/2} e^{-b-c}}{2^{(4 \mp 1)/2} (b+c)^{(2 \mp 1)/2}}.$$

$$2. \int_0^{\infty} x^{(1 \mp 1)/2} K_{ix}(b) \left\{ \begin{array}{l} \operatorname{ch} ax \operatorname{Re} K_{1/2+ix}(c) \\ \operatorname{sh} ax \operatorname{Im} K_{1/2+ix}(c) \end{array} \right\} dx = \\ = \left(\frac{\pi}{2}\right)^{3/2} \left\{ \begin{array}{l} \operatorname{Re} (c+be^{-ia})^{-1/2} \\ \operatorname{Im} (c+be^{-ia})^{-1/2} \end{array} \right\} \exp(-\sqrt{b^2+c^2+2bc \cos a}) \quad [0 \leq a \leq \pi].$$

$$3. \int_0^{\infty} \operatorname{ch} \pi x K_{ix}(b) \operatorname{Re} K_{1/2+ix}(c) dx = \left(\frac{\pi}{2}\right)^{3/2} \frac{e^{b-c}}{\sqrt{c-b}} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \quad \left[\begin{Bmatrix} 0 < b < c \\ 0 < c < b \end{Bmatrix}\right],$$

$$4. \int_0^{\infty} \operatorname{sh} \pi x K_{ix}(b) \operatorname{Im} K_{1/2+ix}(c) dx = \left(\frac{\pi}{2}\right)^{3/2} \frac{e^{c-b}}{\sqrt{b-c}} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \quad \left[\begin{Bmatrix} 0 < c < b \\ 0 < b < c \end{Bmatrix}\right].$$

$$5. \int_0^{\infty} \operatorname{ch} \pi x K_{ix}^2(b) \operatorname{Re} K_{1/2+ix}(c) dx = \frac{\pi^{3/2}}{2^{5/2} \sqrt{c}} \exp\left(\frac{b^2}{4c} - c\right) K_0\left(\frac{b^2}{4c}\right).$$

2.16.55. Интегралы по индексу, содержащие

$$\begin{Bmatrix} \operatorname{Re} K_{\nu \pm ix}(b) \operatorname{Re} K_{1/2+ix}(c) \\ \operatorname{Im} K_{\nu \pm ix}(b) \operatorname{Im} K_{1/2+ix}(c) \end{Bmatrix}.$$

Условия: $a, b, c > 0$.

$$1. \int_0^{\infty} \begin{Bmatrix} \operatorname{Re} K_{1/2+ix}(b) \operatorname{Re} K_{1/2+ix}(c) \\ \operatorname{Im} K_{1/2+ix}(b) \operatorname{Im} K_{1/2+ix}(c) \end{Bmatrix} dx = \frac{\pi}{4} K_1(b+c) \pm \frac{\pi}{4} K_0(b+c).$$

$$2. \int_0^{\infty} \cos ax \begin{Bmatrix} \operatorname{Re} K_{1/2+ix}(b) \operatorname{Re} K_{1/2+ix}(c) \\ \operatorname{Im} K_{1/2+ix}(b) \operatorname{Im} K_{1/2+ix}(c) \end{Bmatrix} dx = \frac{\pi}{4} \operatorname{ch} \frac{a}{2} \left[\pm K_0(z) + \frac{b+c}{z} K_1(z) \right] \\ [z = \sqrt{b^2 + c^2 + 2bc \operatorname{ch} a}].$$

$$3. \int_0^{\infty} \cos ax \operatorname{Re} [K_{1/2+ix}(b) K_{1/2+ix}(c)] dx = \frac{\pi}{2} \operatorname{ch} \frac{a}{2} K_0(z) \\ [z = \sqrt{b^2 + c^2 + 2bc \operatorname{ch} a}].$$

$$4. \int_0^{\infty} \operatorname{Re} [I_{\nu+ix}(b) K_{\nu-ix}(b)] \operatorname{Re} K_{1/2+ix}(c) dx = \frac{\pi^{3/2}}{2^{5/2} \sqrt{c}} \exp\left(-c - \frac{b^2}{4c}\right) I_{\nu}\left(\frac{b^2}{4c}\right) \\ [\operatorname{Re} \nu > -1/2].$$

$$5. \int_0^{\infty} \operatorname{cth} \pi x \operatorname{Im} [I_{ix}(b) K_{ix}(b)] \operatorname{Re} K_{1/2+ix}(c) dx = \frac{1}{8} \sqrt{\frac{\pi}{2c}} \exp\left(\frac{b^2}{4c} - c\right) K_0\left(\frac{b^2}{4c}\right).$$

2.16.56. Интегралы по индексу, содержащие

$$K_{irx}(a) K_{irx}(b) K_{ix}(c).$$

$$1. \int_0^{\infty} x \operatorname{sh} \pi x K_{irx}(a) K_{irx}(b) K_{ix}(c) dx = I_r,$$

$$I_1 = \frac{\pi^2}{4} \exp\left(-\frac{a^2 c^2 + b^2 c^2 + a^2 b^2}{2abc}\right) \\ [|\arg a| + |\arg b| + |\arg c| < \pi/2],$$

$$I_{1/2} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{\pi^2 c}{\sqrt{c^2 - 4ab}} \cos\left(\frac{a-b}{2\sqrt{ab}} \sqrt{c^2 - 4ab}\right) \quad \left[\begin{Bmatrix} c > 2\sqrt{ab} \\ c < 2\sqrt{ab} \end{Bmatrix}\right].$$

$$2. \int_0^{\infty} x \operatorname{sh} \frac{\pi x}{2} K_{ix/2}(a) K_{ix/2}(b) K_{ix}(c) dx = \frac{\pi^2 c}{2 \sqrt{c^2 + 4ab}} \exp \left(-\frac{a+b}{2 \sqrt{ab}} \sqrt{c^2 + 4ab} \right) \\ \left[|\arg a| + |\arg b| < \pi; c > 0 \right].$$

$$3. \int_0^{\infty} x \operatorname{sh} \frac{\pi x}{2} K_{ix/2}(ai) K_{ix/2}(-ai) K_{ix}(c) dx = \frac{\pi^2 c}{4 \sqrt{4a^2 + c^2}} \quad [a, b, c > 0].$$

$$4. \int_0^{\infty} \operatorname{ch} \frac{\pi x}{2} K_{ix/2}^2(b) K_{ix}(c) dx = \frac{\pi^2}{2} K_0 \left(\frac{\sqrt{c^2 + 4b^2} + c}{2} \right) K_0 \left(\frac{\sqrt{c^2 + 4b^2} - c}{2} \right) \\ [b, c > 0].$$

$$5. \int_0^{\infty} x \operatorname{sh} \pi x K_{\nu+ix/2}(b) K_{\nu-ix/2}(b) K_{ix}(c) dx = \\ = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} 2^{-2\nu-1} \frac{\pi^2 c}{b^{2\nu} \sqrt{c^2 - 4b^2}} [(c + \sqrt{c^2 - 4b^2})^{2\nu} + (c - \sqrt{c^2 - 4b^2})^{2\nu}] \\ \left[\begin{Bmatrix} 0 < 2b < c \\ 0 < c < 2b \end{Bmatrix} \right].$$

$$6. \int_0^{\infty} x \operatorname{th} \pi x K_{ix}(a) K_{ix}(b) K_{ix}(c) dx = \\ = \frac{\pi^2}{4} \exp \left(\frac{a^2 b^2 + b^2 c^2 + a^2 c^2}{2abc} \right) \operatorname{erfc} \left(\frac{ab + bc + ac}{\sqrt{2abc}} \right) \quad [a, b, c > 0].$$

$$7. \int_0^{\infty} \operatorname{sh} \pi x \operatorname{Im} [K_{(1+ix)/2}(a) K_{(1+ix)/2}(b)] K_{ix}(c) dx = \\ = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{\pi^2}{\sqrt{c^2 - 4ab}} \cos \left(\frac{b-a}{2 \sqrt{ab}} \sqrt{c^2 - 4ab} \right) \quad \left[\begin{Bmatrix} c > 2 \sqrt{ab} \\ c < 2 \sqrt{ab} \end{Bmatrix} \right].$$

$$8. \int_0^{\infty} x \operatorname{sh} \pi x K_{ix}^2(a) K_{ix}^2(b) dx = \frac{\pi^2}{4} K_0(2 \sqrt{a^2 + b^2}) \quad [a, b > 0].$$

$$9. \int_0^{\infty} x \operatorname{th} \pi x K_{ix}^2(a) K_{ix}^2(b) dx = -\frac{\pi^2}{4} \operatorname{Ei}(-2a - 2b) \quad [a, b > 0].$$

2.16.57. Интегралы, содержащие $\operatorname{Ei}(bx^{\pm n}) K_{\nu}(cx)$.

$$1. \int_0^{\infty} x^{\alpha-1} \operatorname{Ei}(-bx) K_{\nu}(cx) dx = \\ = -\frac{2^{\alpha-1} b}{c^{\alpha+1}} \Gamma \left(\frac{\alpha+\nu+1}{2} \right) \Gamma \left(\frac{\alpha-\nu+1}{2} \right) {}_3F_2 \left(\frac{\alpha+\nu+1}{2}, \frac{\alpha-\nu+1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; \frac{b^2}{c^2} \right) + \\ + \frac{2^{\alpha-2} b^2}{c^{\alpha+1}} \Gamma \left(\frac{\alpha+\nu}{2} + 1 \right) \Gamma \left(\frac{\alpha-\nu}{2} + 1 \right) {}_4F_3 \left(\frac{\alpha+\nu}{2} + 1, \frac{\alpha-\nu}{2} + 1, 1, 1; 2, 2, \frac{3}{2}; \frac{b^2}{c^2} \right) + \\ + \frac{2^{\alpha-3}}{c^{\alpha}} \Gamma \left(\frac{\alpha+\nu}{2} \right) \Gamma \left(\frac{\alpha-\nu}{2} \right) \left[\psi \left(\frac{\alpha+\nu}{2} \right) + \psi \left(\frac{\alpha-\nu}{2} \right) + 2\zeta + 2 \ln \frac{2b}{c} \right] \\ [\operatorname{Re} b, \operatorname{Re} c > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu|].$$

$$2. \int_0^{\infty} x^{\alpha-1} \operatorname{Ei}(-bx^2) K_{\nu}(cx) dx =$$

$$= \frac{2^{\nu-1} b^{(\nu-\alpha)/2} \Gamma(\nu) \Gamma\left(\frac{\alpha-\nu}{2}\right) {}_2F_2\left(\frac{\alpha-\nu}{2}, \frac{\alpha-\nu}{2}; \frac{\alpha-\nu}{2}+1, 1-\nu; \frac{c^2}{4b}\right) -$$

$$- \frac{c^{\nu} \Gamma(-\nu)}{2^{\nu+1} (\alpha+\nu) b^{(\alpha+\nu)/2}} \Gamma\left(\frac{\alpha+\nu}{2}\right) {}_2F_2\left(\frac{\alpha+\nu}{2}, \frac{\alpha+\nu}{2}; \frac{\alpha+\nu}{2}+1, 1+\nu; \frac{c^2}{4b}\right)$$

[Re $b, \operatorname{Re} c > 0$; Re $\alpha > |\operatorname{Re} \nu|$]

$$3. \int_0^{\infty} \operatorname{Ei}\left(-\frac{b}{x}\right) K_1(cx) dx = -\frac{2}{c} K_0(e^{\pi i/4} \sqrt{2bc}) K_0(e^{-\pi i/4} \sqrt{2bc})$$

[Re $b, \operatorname{Re} c > 0$]

$$4. \int_0^{\infty} x^{\alpha-1} e^{\pm bx} \operatorname{Ei}(\mp bx) K_{\nu}(cx) dx = -\frac{\pi}{2} U(0)$$

[Re $b, \operatorname{Re} c > 0$; Re $\alpha > |\operatorname{Re} \nu|$; $U(\gamma)$ см. в 2.13.28.6]

$$5. \int_0^{\infty} x^{\alpha-1} e^{\pm cx} \operatorname{Ei}(-bx) K_{\nu}(cx) dx =$$

$$= \frac{2^{\nu-1} b^{\nu-\alpha} \Gamma(\nu) \Gamma(\alpha-\nu)}{(\nu-\alpha) c^{\nu}} {}_2F_2\left(\frac{1}{2}-\nu, \alpha-\nu, \alpha-\nu; \alpha-\nu+1, 1-2\nu; \pm \frac{2c}{b}\right) -$$

$$- \frac{c^{\nu} \Gamma(-\nu) \Gamma(\alpha+\nu)}{2^{\nu+1} (\alpha+\nu) b^{\alpha+\nu}} {}_2F_2\left(\frac{1}{2}+\nu, \alpha+\nu, \alpha+\nu; \alpha+\nu+1, 1+2\nu; \pm \frac{2c}{b}\right)$$

[Re $b, \operatorname{Re} c > 0$; Re $\alpha > |\operatorname{Re} \nu|$]

$$6. \int_0^{\infty} x^{\alpha-1} e^{(b \mp c)x} \operatorname{Ei}(-bx) K_{\nu}(cx) dx = V(1)$$

[Re $b > 0$; Re $\alpha > |\operatorname{Re} \nu|$, $\left\{ \begin{array}{l} \operatorname{Re} c > 0 \\ \operatorname{Re} \alpha < 3/2; \quad |\arg c| < \pi \end{array} \right\}$],

$$V(\gamma) = -\frac{2^{\nu-1} \pi b^{\nu-\alpha}}{c^{\nu}} \Gamma(\nu) \Gamma(\alpha-\nu) \frac{\cos^{1-\gamma}(\alpha-\nu) \pi}{\sin(\alpha-\nu) \pi} {}_2F_1\left(\frac{1}{2}-\nu, \alpha-\nu; 1-2\nu; \right.$$

$$\left. \mp \frac{(-1)^{\gamma} 2c}{b}\right) - \frac{\pi c^{\nu}}{2^{\nu+1} b^{\alpha+\nu}} \Gamma(-\nu) \Gamma(\alpha+\nu) \frac{\cos^{1-\gamma}(\alpha+\nu) \pi}{\sin(\alpha+\nu) \pi} {}_2F_1\left(\frac{1}{2}+\nu, \alpha+\nu; \right.$$

$$\left. 1+2\nu; \mp \frac{(-1)^{\gamma} 2c}{b}\right) \pm \frac{(-1)^{\gamma} \sqrt{\pi}}{(2c)^{\alpha-1} b} \left\{ \frac{1}{\cos \nu \pi \sec \alpha \pi} \right\} \Gamma\left[\begin{matrix} \alpha+\nu-1, \alpha-\nu-1 \\ \alpha-1/2 \end{matrix}\right] \times$$

$$\times {}_2F_2\left(1, 1, \frac{3}{2}-\alpha; 2-\nu-\alpha, 2+\nu-\alpha; \mp \frac{(-1)^{\gamma} 2c}{b}\right)$$

[Re $b > 0$; Re $\alpha > |\operatorname{Re} \nu|$, $\left\{ \begin{array}{l} \operatorname{Re} c > 0 \\ \operatorname{Re} \alpha < 3/2; \quad |\arg c| < \pi \end{array} \right\}$].

$$7. \int_0^{\infty} x^{\alpha-1} e^{-(b \pm c)x} \operatorname{Ei}(bx) K_{\nu}(cx) dx = V(0)$$

[Re $b > 0$; Re $\alpha > |\operatorname{Re} \nu|$, $\left\{ \begin{array}{l} \operatorname{Re} c > 0 \\ \operatorname{Re} \alpha < 3/2; \quad |\arg c| < \pi \end{array} \right\}$; $V(\gamma)$ см. в 2.16.57.6]

$$8. \int_0^{\infty} x^{-1/2} e^{(b-c)x} \operatorname{Ei}(-bx) K_0(cx) dx =$$

$$= -\frac{\pi^{5/2}}{\sqrt{2c}} {}_2F_1\left(\frac{1}{2}, \frac{1}{2}; 1; 1-\frac{b}{2c}\right) = -\frac{\pi^{5/2}}{\sqrt{2c}} P_{-1,2}\left(\frac{b}{c}-1\right)$$

[Re $b, \operatorname{Re} c > 0$]

$$9. \int_0^{\infty} x^{-3/2} e^{b/x-cx} \operatorname{Ei}\left(-\frac{b}{x}\right) K_0(cx) dx = -\frac{2\pi^{3/2}}{\sqrt{b}} s_{0,0}(\sqrt{8bc})$$

[Re b , Re $c > 0$].

2.16.58. Интегралы, содержащие $\begin{Bmatrix} \operatorname{si}(bx) \\ \operatorname{ci}(bx) \end{Bmatrix} K_\nu(cx)$.

$$1. \int_0^{\infty} x^{\alpha-1} \begin{Bmatrix} \operatorname{si}(bx) \\ \operatorname{ci}(bx) \end{Bmatrix} K_\nu(cx) dx = -\frac{\pi}{2} V(0)$$

[b , Re $c > 0$; Re $\alpha > |\operatorname{Re} \nu|$; $V(\gamma)$ см. в 2.13.29.14.

$$2. \int_0^{\infty} x^{\alpha-1} e^{-cx} \begin{Bmatrix} \operatorname{si}(bx) \\ \operatorname{ci}(bx) \end{Bmatrix} K_\nu(cx) dx = V(1) + \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \hat{V}$$

[b , Re $c > 0$; Re $\alpha > |\operatorname{Re} \nu|$, $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$],

$$V(\gamma) = -\frac{\sqrt{\pi} b^{\delta+2}}{2(2+\delta)(1+2\delta)(2c)^{2+\alpha+\delta}} \Gamma\left[\begin{matrix} \alpha+\nu+\delta+2, \alpha-\nu+\delta+2 \\ 5/2+\alpha+\delta \end{matrix}\right] \times \\ \times {}_3F_3\left(\begin{matrix} \frac{\alpha+\nu+2\delta}{2}+1, \frac{\alpha+\nu+3}{2}, \frac{\alpha-\nu+2\delta}{2}+1, \frac{\alpha-\nu+3}{2}, 1+\frac{\delta}{2}, \\ 1; \frac{5+2\alpha}{4}+\delta, \frac{7+2\alpha}{4}, \frac{\delta}{2}+2, 2, \frac{3}{2}+\delta; -\frac{b^2}{4c^2} \end{matrix}\right) + \\ + \frac{\sqrt{\pi} b^\delta}{(2c)^{\alpha+\delta}} \Gamma\left[\begin{matrix} \alpha+\nu+\delta, \alpha-\nu+\delta \\ 1/2+\alpha+\delta \end{matrix}\right] \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \left\{ R(\gamma) \right\},$$

$$R(\gamma) = C + \psi(\alpha+\nu) + \psi(\alpha-\nu) - \psi\left(\alpha + \frac{1}{2}\right) - \ln \frac{2c}{b} - (1-\gamma)\pi \operatorname{tg} \alpha\pi,$$

$$\hat{V} = \frac{\pi^{3/2}}{2^{\alpha+1} c^\alpha} \Gamma\left[\begin{matrix} \alpha+\nu, \alpha-\nu \\ \alpha+1/2 \end{matrix}\right].$$

$$3. \int_0^{\infty} x^{\alpha-1} e^{cx} \begin{Bmatrix} \operatorname{si}(bx) \\ \operatorname{ci}(bx) \end{Bmatrix} K_\nu(cx) dx = \mp \frac{\cos \nu\pi}{\cos \alpha\pi} \left[V(0) + \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \hat{V} \right] + \\ + \frac{b^{1/2-\alpha}}{1-2\alpha} \sqrt{\frac{2\pi}{-}} \Gamma\left(\alpha - \frac{1}{2}\right) \begin{Bmatrix} \sin[(2\alpha-1)\pi/4] \\ \cos[(2\alpha-1)\pi/4] \end{Bmatrix} \times \\ \times {}_4F_4\left(\begin{matrix} \frac{1+2\nu}{4}, \frac{3+2\nu}{4}, \frac{1-2\nu}{4}, \frac{3-2\nu}{4}, \frac{1-2\alpha}{4}, \frac{1}{2}, \frac{5-2\alpha}{4}, \frac{5-2\alpha}{4}, \frac{3-2\alpha}{4}; -\frac{b^2}{4c^2} \end{matrix}\right) \pm \\ \pm \frac{\sqrt{\pi} b^{3/2-\alpha} (1-4\nu^2)}{2(2c)^{3/2} (3-2\alpha)} \Gamma\left(\alpha - \frac{3}{2}\right) \begin{Bmatrix} \cos[(2\alpha-1)\pi/4] \\ \sin[(2\alpha-1)\pi/4] \end{Bmatrix} \times \\ \times {}_5F_4\left(\begin{matrix} \frac{3+2\nu}{4}, \frac{5+2\nu}{4}, \frac{3-2\nu}{4}, \frac{5-2\nu}{4}, \frac{3-2\alpha}{4}; \frac{3}{2}, \\ \frac{5-2\alpha}{4}, \frac{7-2\alpha}{4}, \frac{7-2\alpha}{4}; -\frac{b^2}{4c^2} \end{matrix}\right)$$

[b , Re $c > 0$; $|\operatorname{Re} \nu| < \operatorname{Re} \alpha < 3/2$; $V(\gamma)$; $\hat{V}$ см. в 2.16.58.2].

2.16.59. Интегралы, содержащие $\begin{Bmatrix} \operatorname{erf}(\varphi(x)) \\ \operatorname{erfc}(\varphi(x)) \end{Bmatrix} K_\nu(cx)$.

$$1. \int_0^{\infty} x^{\alpha-1} \begin{Bmatrix} \operatorname{erf}(bx) \\ \operatorname{erfc}(bx) \end{Bmatrix} K_\nu(cx) dx = -\frac{\pi}{2} U(0)$$

[Re $c > 0$; $|\arg b| < \pi/4$; Re $\alpha > |\operatorname{Re} \nu| - (1 \pm 1)/2$; $U(\gamma)$ см. в 2.13.30.1].

$$2. \int_0^{\infty} x \operatorname{erf}(bx) K_0(cx) dx = \frac{1}{8b^2} \exp\left(\frac{c^2}{8b^2}\right) \left[K_1\left(\frac{c^2}{8b^2}\right) - K_0\left(\frac{c^2}{8b^2}\right) \right]$$

[Re c > 0; |arg b| < π/4].

$$3. \int_0^{\infty} x^{\alpha-1} \left\{ \begin{matrix} \operatorname{erf}(b\sqrt{x}) \\ \operatorname{erfc}(b\sqrt{x}) \end{matrix} \right\} K_\nu(cx) dx = -\frac{\pi}{2} V(0)$$

[Re c > 0; |arg b| < π/4; Re α > |Re ν| - (1 ± 1)/4; V(ν) см. в 2.13.30.2].

$$4. \int_0^{\infty} x^{\alpha-1} e^{\mp b^2 x^2} \left\{ \begin{matrix} \operatorname{erf}(ibx) \\ \operatorname{erfc}(bx) \end{matrix} \right\} K_\nu(cx) dx = -\frac{\pi}{2} W(0)$$

[Re c > 0; |arg b| < (2 ∓ 1)π/4; Re α > |Re ν| - (1 ± 1)/2; W(ν) см. в 2.13.30.3].

$$5. \int_0^{\infty} x^{\nu+1} e^{-b^2 x^2} \operatorname{erf}(ibx) K_\nu(cx) dx =$$

$$= \frac{ic^\nu}{2^{\nu+2} b^{2\nu+2}} \Gamma\left(\nu + \frac{3}{2}\right) \exp\left(\frac{c^2}{4b^2}\right) \Gamma\left(-\nu - \frac{1}{2}, \frac{c^2}{4b^2}\right)$$

[Re c > 0; |arg b| < π/4; Re ν > -3/2].

$$6. \int_0^{\infty} x^{\alpha-1} e^{\pm cx} \operatorname{erf}(b\sqrt{x}) K_\nu(cx) dx = X(1)$$

[Re c > 0; |arg b| < π/4, { |Re ν| - 1/2 < Re α < 1/2 }
|Re ν| - 1/2 < Re α]],

$$X(\gamma) = \pm (-1)^\gamma \frac{2b}{(2c)^{\alpha+1/2}} \Gamma\left[\begin{matrix} \alpha + \nu + 1/2, \alpha - \nu + 1/2 \\ \alpha + 1 \end{matrix}\right] \left\{ \begin{matrix} \cos \nu\pi \operatorname{cosec} \alpha\pi \\ 1 \end{matrix} \right\} \times$$

$$\times {}_2F_2\left(\frac{1}{2}, \alpha + \nu + \frac{1}{2}, \alpha - \nu + \frac{1}{2}; \alpha + 1, \frac{3}{2}; \pm \frac{b^2}{2c}\right) -$$

$$- \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{(-1)^\gamma 2b^{1-2\alpha} \Gamma(\alpha)}{\sqrt{2c} (1-2\alpha)} {}_3F_2\left(\frac{1}{2} + \nu, \frac{1}{2} - \nu, \frac{1}{2} - \alpha; \frac{3}{2} - \alpha, 1 - \alpha; \frac{b^2}{2c}\right) +$$

$$+ (1 - \gamma) \frac{\sqrt{\pi}}{(2c)^\alpha} \Gamma\left[\begin{matrix} \alpha + \nu, \alpha - \nu \\ \alpha + 1/2 \end{matrix}\right] \left\{ \begin{matrix} \cos \nu\pi \sec \alpha\pi \\ 1 \end{matrix} \right\}.$$

$$7. \int_0^{\infty} x^{\alpha-1} e^{\pm cx} \operatorname{erfc}(b\sqrt{x}) K_\nu(cx) dx = X(0)$$

[Re c > 0; |arg b| < π/4; Re α > |Re ν|; X(γ) см. в 2.16.59.6].

$$8. \int_0^{\infty} x^{\alpha-1} e^{(\mp b^2 - c)x} \left\{ \begin{matrix} \operatorname{erf}(ib\sqrt{x}) \\ \operatorname{erfc}(b\sqrt{x}) \end{matrix} \right\} K_\nu(cx) dx = Y(1)$$

[Re c > 0; |arg b| < (2 ∓ 1)π/4; Re α > |Re ν| - (1 ± 1)/4].

$$Y(\gamma) = \frac{(2c)^{1/2-\alpha}}{b} \left\{ \begin{matrix} i \\ 1 \end{matrix} \right\} \Gamma\left[\begin{matrix} \alpha + \nu - 1/2, \alpha - \nu - 1/2 \\ \alpha \end{matrix}\right] \left(\frac{\cos \nu\pi}{\sin \alpha\pi}\right)^{1-\gamma} \times$$

$$\times {}_3F_2\left(1, 1 - \alpha, \frac{1}{2}; \frac{3}{2} - \alpha - \nu, \frac{3}{2} - \alpha + \nu; \frac{(-1)^\gamma 2c}{b^2}\right) +$$

$$+ \frac{\pi c^\nu}{2^{\nu+(1\pm 1)/2} b^{2\alpha+2\nu}} \Gamma\left[\begin{matrix} -\nu \\ 1 - \alpha - \nu \end{matrix}\right] \left\{ \begin{matrix} i \sec(\alpha + \nu)\pi \\ \operatorname{cosec} 2(\alpha + \nu)\pi \end{matrix} \right\} \times$$

$$\times {}_2F_1\left(\frac{1}{2} + \nu, \alpha + \nu; 1 + 2\nu; \frac{(-1)^\gamma 2c}{b^2}\right) + \frac{2^{\nu-(1\pm 1)/2} \pi}{b^{2\alpha-2\nu} c^\nu} \times$$

$$\times \Gamma\left[\begin{matrix} \nu \\ 1 - \alpha + \nu \end{matrix}\right] \left\{ \begin{matrix} i \sec(\alpha - \nu)\pi \\ \operatorname{cosec} 2(\alpha - \nu)\pi \end{matrix} \right\} {}_2F_1\left(\frac{1}{2} - \nu, \alpha - \nu; 1 - 2\nu; \frac{(-1)^\gamma 2c}{b^2}\right).$$

$$9. \int_0^{\infty} x^{\alpha-1} e^{(\mp b^2 + c)x} \begin{Bmatrix} \operatorname{erf}(ib\sqrt{x}) \\ \operatorname{erfc}(b/\sqrt{x}) \end{Bmatrix} K_{\nu}(cx) dx = Y(0)$$

[Re $c > 0$; $|\arg b| < (2 \mp 1)\pi/4$; $|\operatorname{Re} \nu| - (1 \pm 1)/4 < \operatorname{Re} \alpha < 1$; $Y(\nu)$ см. в 2.16.59.8].

$$10. \int_0^{\infty} \frac{1}{x} e^{(b^2 - c)x} \operatorname{erf}(b\sqrt{x}) K_0(cx) dx = \frac{2\pi\sqrt{2c}}{b} \arcsin \frac{b^2}{2c}$$

[Re $(2c - b^2) > 0$; $|b|^2 < 2c$].

$$11. \int_0^{\infty} x^{\alpha-1} e^{\mp b^2/x - cx} \begin{Bmatrix} \operatorname{erf}(ib/\sqrt{x}) \\ \operatorname{erfc}(b/\sqrt{x}) \end{Bmatrix} K_{\nu}(cx) dx = Z(1)$$

[Re $c > 0$; $|\arg b| < (2 \mp 1)\pi/4$; $\operatorname{Re} \alpha > |\operatorname{Re} \nu| - 1/2$].

$$\begin{aligned} Z(\nu) = & 2^{3/2-\alpha} e^{(3 \mp 1)\pi i/4} b c^{1/2-\alpha} \Gamma \left[\begin{matrix} \alpha + \nu - 1/2, \alpha - \nu - 1/2 \\ \alpha \end{matrix} \right] \times \\ & \times \left(\frac{\cos \nu \pi}{\sin \alpha \pi} \right)^{1-\nu} {}_2F_3 \left(1, 1-\alpha; \frac{3}{2} - \nu - \alpha, \frac{3}{2} + \nu - \alpha, \frac{3}{2}; \mp (-1)^{\nu} 2b^2 c \right) + \\ & + \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \frac{\sqrt{\pi}}{(2c)^{\alpha}} \Gamma \left[\begin{matrix} \alpha + \nu, \alpha - \nu \\ \alpha + 1/2 \end{matrix} \right] \left(\frac{\cos \nu \pi}{\cos \alpha \pi} \right)^{1-\nu} \times \\ & \times {}_1F_2 \left(\frac{1}{2} - \alpha; 1 - \nu - \alpha, 1 + \nu - \alpha; (-1)^{\nu} 2b^2 c \right) \pm \\ & \pm \frac{\pi b^{2\alpha+2\nu} c^{\nu}}{2^{\nu+(1 \pm 1)/2}} \Gamma \left[\begin{matrix} -\nu \\ 1 + \alpha + \nu \end{matrix} \right] \left\{ \frac{i \sec(\alpha + \nu)\pi}{\operatorname{cosec} 2(\alpha + \nu)\pi} \right\} \times \\ & \times {}_1F_2 \left(\frac{1}{2} + \nu; 1 + 2\nu, 1 + \alpha + \nu; \mp (-1)^{\nu} 2b^2 c \right) + \\ & + \frac{2^{\nu-(1 \pm 1)/2} \pi}{b^{2\nu-2\alpha} c^{\nu}} \Gamma \left[\begin{matrix} \nu \\ 1 + \alpha - \nu \end{matrix} \right] \left\{ \frac{i \sec(\alpha - \nu)\pi}{\operatorname{cosec} 2(\alpha - \nu)\pi} \right\} \times \\ & \times {}_1F_2 \left(\frac{1}{2} - \nu; 1 - 2\nu, 1 + \alpha - \nu; \mp (-1)^{\nu} 2b^2 c \right). \end{aligned}$$

$$12. \int_0^{\infty} x^{\alpha-1} e^{\mp b^2/x + cx} \begin{Bmatrix} \operatorname{erf}(ib/\sqrt{x}) \\ \operatorname{erfc}(b/\sqrt{x}) \end{Bmatrix} K_{\nu}(cx) dx = Z(0)$$

[Re $c > 0$; $|\arg b| < (2 \mp 1)\pi/4$; $|\operatorname{Re} \nu| - 1/2 < \operatorname{Re} \alpha < (3 \pm 1)/4$; $Z(\nu)$ см. в 2.16.59.11].

$$13. \int_0^{\infty} \frac{1}{x} e^{-c(x+1/x)} \operatorname{erf}\left(i\sqrt{\frac{c}{x}}\right) K_0(cx) dx = 2\pi i I_0(c\sqrt{2}) K_0(c\sqrt{2})$$

[Re $c > 0$].

$$14. \int_0^{\infty} \frac{1}{x} e^{b^2/x - cx} \operatorname{erfc}\left(\frac{b}{\sqrt{x}}\right) K_{\nu}(cx) dx = \frac{\pi^{5/2}}{4 \cos \nu \pi} [J_{\nu}^2(b\sqrt{2c}) + Y_{\nu}^2(b\sqrt{2c})]$$

[Re $b, \operatorname{Re} c > 0$; $|\operatorname{Re} \nu| < 1/2$].

$$15. \int_0^{\infty} \frac{1}{x} e^{(ax+b/x)^2} \operatorname{erfc}\left(ax + \frac{b}{x}\right) K_0(cx) dx = 2K_0(z_+) K_0(z_-)$$

[$z_{\pm} = \sqrt{b/2} (a \pm \sqrt{a^2 - 4c})^{1/2}$; $\operatorname{Re} a, \operatorname{Re} b, \operatorname{Re} c > 0$].

2.16.60. Интегралы, содержащие $\left\{ \begin{matrix} S(bx^r) \\ C(bx^r) \end{matrix} \right\} K_\nu(cx)$.

$$1. \int_0^\infty x^{\alpha-1} \left\{ \begin{matrix} S(bx) \\ C(bx) \end{matrix} \right\} K_\nu(cx) dx = -\frac{\pi}{2} U(0)$$

[$b, \operatorname{Re} c > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu| - (2 \pm 1)/2; U(\gamma)$ см. в 2.13.31.1].

$$2. \int_0^\infty x^{-1/2} C(b\sqrt{x}) K_0(cx) dx = \frac{1}{2} \sqrt{\frac{\pi}{2c}} F\left(\arccos \frac{c-b^2}{c+b^2}; \frac{1}{\sqrt{2}}\right)$$

[$c > b^2 > 0; b > 0$].

$$3. \int_0^\infty x^{\alpha-1} e^{-cx} \left\{ \begin{matrix} S(bx) \\ C(bx) \end{matrix} \right\} K_\nu(cx) dx = V(1)$$

[$b, \operatorname{Re} c > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu| - (2 \pm 1)/2, \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$],

$$V(\gamma) = (\pm 1)^{\gamma-1} \frac{\sqrt{2} b^{\delta+1/2}}{(2\delta+1)(2c)^{\alpha+\delta+1/2}} \left(\frac{\cos \nu\pi}{\sin \alpha\pi} \right)^{1-\gamma} \times$$

$$\times \Gamma\left[\begin{matrix} \alpha+\delta+\nu+1/2, \alpha+\delta-\nu+1/2 \\ \alpha+\delta+1 \end{matrix}\right] {}_5F_4\left(\begin{matrix} 2\alpha+4\delta+2\nu+1, 2\alpha+2\nu+3 \\ 4, 4 \end{matrix}, \right.$$

$$\left. \frac{2\alpha+4\delta-2\nu+1}{4}, \frac{2\alpha-2\nu+3}{4}, \frac{2\delta+1}{4}, \frac{2\delta+5}{4}, \delta+\frac{1}{2}, \frac{1+\alpha+2\delta}{2}, \right.$$

$$\left. 1+\frac{\alpha}{2}; -\frac{b^2}{4c^2} \right) + (1-\gamma) \frac{b^{1/2-\alpha} \Gamma(\alpha)}{(1-2\alpha)\sqrt{c}} \left\{ \begin{matrix} \sin(\alpha\pi/2) \\ \cos(\alpha\pi/2) \end{matrix} \right\} {}_5F_4\left(\begin{matrix} 2\nu+1, 2\nu+3 \\ 4, 4 \end{matrix}, \right.$$

$$\left. \frac{1-2\nu}{4}, \frac{3-2\nu}{4}, \frac{1-2\alpha}{4}, \frac{1}{2}, \frac{5-2\alpha}{4}, 1-\frac{\alpha}{2}, \frac{1-\alpha}{2}; -\frac{b^2}{4c^2} \right) -$$

$$-(1-\gamma) \frac{(1-4\nu^2) b^{3/2-\alpha}}{8(3-2\alpha)c^{3/2}} {}_5F_4\left(\begin{matrix} 3+2\nu, 5+2\nu, 3-2\nu, 5-2\nu \\ 4, 4, 4, 4 \end{matrix}, \frac{3-2\alpha}{4}; \right.$$

$$\left. \frac{3}{2}, \frac{7-2\alpha}{4}, \frac{3-\alpha}{2}, 1-\frac{\alpha}{2}; -\frac{b^2}{4c^2} \right).$$

$$4. \int_0^\infty x^{\alpha-1} e^{cx} \left\{ \begin{matrix} S(bx) \\ C(bx) \end{matrix} \right\} K_\nu(cx) dx = V(0)$$

[$b > 0; |\operatorname{Re} \nu| - (2 \pm 1)/2 < \operatorname{Re} \alpha < 1/2; |\arg c| < \pi; V(\gamma)$ см. в 2.16.60.3].

2.16.61. Интегралы, содержащие $\left\{ \begin{matrix} \gamma(\mu, bx^{\pm n}) \\ \Gamma(\mu, bx^{\pm n}) \end{matrix} \right\} K_\nu(cx)$.

$$1. \int_0^\infty x^{\alpha-1} \left\{ \begin{matrix} \gamma(\mu, bx) \\ \Gamma(\mu, bx) \end{matrix} \right\} K_\nu(cx) dx = -\frac{\pi}{2} V(0)$$

[$\operatorname{Re} b, \operatorname{Re} c > 0; \operatorname{Re}(\alpha+\mu) > |\operatorname{Re} \nu|, \begin{Bmatrix} \operatorname{Re} \mu > 0 \\ \operatorname{Re} \alpha > |\operatorname{Re} \nu| \end{Bmatrix}; V(\gamma)$ см. в 2.13.32.1].

$$2. \int_0^\infty x^{\alpha-1} \left\{ \begin{matrix} \gamma(\mu, bx^2) \\ \Gamma(\mu, bx^2) \end{matrix} \right\} K_\nu(cx) dx = -\frac{\pi}{2} W(0)$$

[$\operatorname{Re} b, \operatorname{Re} c > 0; \operatorname{Re}(\alpha+2\mu) > |\operatorname{Re} \nu|, \begin{Bmatrix} \operatorname{Re} \mu > 0 \\ \operatorname{Re} \alpha > |\operatorname{Re} \nu| \end{Bmatrix}; W(\gamma)$ см. в 2.13.32.2].

$$3. \int_0^\infty x^{\alpha-1} e^{-cx} \left\{ \begin{matrix} \gamma(\mu, bx) \\ \Gamma(\mu, bx) \end{matrix} \right\} K_\nu(cx) dx = W(1)$$

[$\operatorname{Re} b, \operatorname{Re} c > 0; \operatorname{Re}(\alpha+\mu) > |\operatorname{Re} \nu|, \begin{Bmatrix} \operatorname{Re} \mu > 0 \\ \operatorname{Re} \alpha > |\operatorname{Re} \nu| \end{Bmatrix}$].

$$\begin{aligned}
W(\gamma) = & \pm \frac{\sqrt{\pi} b^\mu}{\mu (2c)^{\alpha+\mu}} \left(\frac{\cos v\pi}{\cos(\alpha+\mu)\pi} \right)^{1-\gamma} \Gamma \left[\begin{matrix} \alpha+\mu+v, \alpha+\mu-v \\ \alpha+\mu+1/2 \end{matrix} \right] \times \\
& \times {}_3F_2 \left(\mu, \alpha+\mu+v, \alpha+\mu-v; \alpha+\mu+\frac{1}{2}, \mu+1; \frac{(-1)^\gamma b}{2c} \right) \pm \\
& \pm (1-\gamma) \frac{\sqrt{2\pi} b^{1/2-\alpha}}{(1-2\alpha)\sqrt{c}} \Gamma \left(\alpha+\mu-\frac{1}{2} \right) {}_3F_2 \left(\frac{1}{2}-\alpha, \frac{1}{2}+v, \frac{1}{2}-v; \right. \\
& \left. \frac{3}{2}-\alpha, \frac{3}{2}-\alpha-\mu; \frac{b}{2c} \right) \mp (1-\gamma) \frac{(1-4v^2)\sqrt{\pi} b^{3/2-\alpha}}{4\sqrt{2}(3-2\alpha)c^{3/2}} \Gamma \left(\alpha+\mu-\frac{3}{2} \right) \times \\
& \times {}_3F_2 \left(\frac{3}{2}-\alpha, \frac{3}{2}+v, \frac{3}{2}-v; \frac{5}{2}-\alpha, \frac{5}{2}-\alpha-\mu; \frac{b}{2c} \right) + \\
& + \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{\sqrt{\pi}}{(2c)^\alpha} \left(\frac{\cos v\pi}{\cos \alpha\pi} \right)^{1-\gamma} \Gamma \left[\begin{matrix} \mu, \alpha+v, \alpha-v \\ \alpha+1/2 \end{matrix} \right].
\end{aligned}$$

$$4. \int_0^\infty x^{\alpha-1} e^{cx} \left\{ \begin{matrix} \gamma(\mu, bx) \\ \Gamma(\mu, bx) \end{matrix} \right\} K_\nu(cx) dx = W(0)$$

$$\left[\operatorname{Re} b, \operatorname{Re} c > 0; \operatorname{Re}(\alpha+\mu) > |\operatorname{Re} v|, \left\{ \begin{matrix} \operatorname{Re} \mu > 0; \operatorname{Re} \alpha < 1/2 \\ \operatorname{Re} \alpha > |\operatorname{Re} v| \end{matrix} \right\}; \right. \\
\left. W(\gamma) \text{ см. в 2.16.61.3} \right].$$

$$\begin{aligned}
5. \int_0^\infty x^{\alpha-1} e^{(b \pm c)x} \Gamma(\mu, bx) K_\nu(cx) dx = & \frac{\sqrt{\pi}}{(2c)^\alpha} \left\{ \begin{matrix} \cos v\pi \sec \alpha\pi \\ 1 \end{matrix} \right\} \times \\
& \times \Gamma \left[\begin{matrix} \alpha+v, \alpha-v, \mu \\ \alpha+1/2 \end{matrix} \right] {}_2F_1 \left(\alpha+v, \alpha-v; \alpha+\frac{1}{2}; \mp \frac{b}{2c} \right) - \\
& - \frac{\sqrt{\pi} b^\mu}{\mu (2c)^{\alpha+\mu}} \left\{ \begin{matrix} \cos v\pi \sec(\alpha+\mu)\pi \\ 1 \end{matrix} \right\} \Gamma \left[\begin{matrix} \alpha+\mu+v, \alpha+\mu-v \\ 1/2+\alpha+\mu \end{matrix} \right] \times \\
& \times {}_3F_2 \left(1, \alpha+\mu+v, \alpha+\mu-v; \frac{1}{2}+\alpha+\mu, \mu+1; \mp \frac{b}{2c} \right) - \\
& - \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{\pi^{3/2} b^{1/2-\alpha}}{\sqrt{2c} \cos(\alpha+\mu)\pi} \Gamma \left[\begin{matrix} \alpha-1/2 \\ 1-\mu \end{matrix} \right] {}_2F_1 \left(\frac{1}{2}+v, \frac{1}{2}-v; \frac{3}{2}-\alpha; -\frac{b}{2c} \right) \\
& \left[\operatorname{Re} b > 0; \operatorname{Re} \alpha > |\operatorname{Re} v|; \operatorname{Re}(\alpha+\mu) > |\operatorname{Re} v|, \right. \\
& \left. \left\{ \begin{matrix} |\arg c| < \pi; \operatorname{Re}(\alpha+\mu) < 3/2 \\ \operatorname{Re} c > 0 \end{matrix} \right\} \right].
\end{aligned}$$

$$\begin{aligned}
6. \int_0^\infty x^{\alpha-1} e^{b/x \pm cx} \Gamma \left(\mu, \frac{b}{x} \right) K_\nu(cx) dx = \\
= \frac{\sqrt{\pi}}{(2c)^\alpha} \left\{ \begin{matrix} \cos v\pi \sec \alpha\pi \\ 1 \end{matrix} \right\} \Gamma \left[\begin{matrix} \mu, \alpha+v, \alpha-v \\ \alpha+1/2 \end{matrix} \right] \times \\
\times {}_1F_2 \left(\frac{1}{2}-\alpha; 1-\alpha-v, 1-\alpha+v; \mp 2bc \right) - \frac{\sqrt{\pi} b^\mu}{\mu (2c)^{\alpha-\mu}} \times \\
\times \Gamma \left[\begin{matrix} \alpha+v-\mu, \alpha-v-\mu \\ 1/2+\alpha-\mu \end{matrix} \right] \left\{ \begin{matrix} \cos v\pi \sec(\alpha-\mu)\pi \\ 1 \end{matrix} \right\} \times \\
\times {}_2F_3 \left(\frac{1}{2}+\mu-\alpha, 1; \mu+1, 1+\mu-v-\alpha, 1+\mu+v-\alpha; \mp 2bc \right) +
\end{aligned}$$

$$+ \frac{\pi b^{\alpha+\nu} c^\nu}{2^{\nu+1} \sin(\mu-\nu-\alpha) \pi} \Gamma \left[\begin{matrix} -\nu, -\nu-\alpha \\ 1-\mu \end{matrix} \right] {}_1F_2 \left(\frac{1}{2} + \nu; 1+2\nu, 1+\alpha+\nu; \mp 2bc \right) +$$

$$+ \frac{\pi b^{\alpha-\nu} c^{-\nu}}{2^{1-\nu} \sin(\mu+\nu-\alpha) \pi} \Gamma \left[\begin{matrix} \nu, \nu-\alpha \\ 1-\mu \end{matrix} \right] {}_1F_2 \left(\frac{1}{2} - \nu; 1-2\nu, 1+\alpha-\nu; \mp 2bc \right)$$

$$\left[\begin{matrix} \operatorname{Re} b > 0; \operatorname{Re}(\alpha-\mu) > |\operatorname{Re} \nu| - 1, \\ \{ |\arg c| < \pi; \operatorname{Re} \alpha, \operatorname{Re}(\alpha-\mu) < 1/2 \} \\ \operatorname{Re} c > 0 \end{matrix} \right].$$

$$7. \int_0^\infty x^{-3/2} e^{b/x-cx} \Gamma \left(\mu, \frac{b}{x} \right) K_0(cx) dx =$$

$$= \frac{2^{1-\mu} \sqrt{\pi}}{\sqrt{b}} \Gamma \left[\begin{matrix} 1/2 - \mu, 1/2 - \mu \\ 1 - \mu \end{matrix} \right] S_{2\mu,0}(2\sqrt{2bc})$$

$$[\operatorname{Re} b, \operatorname{Re} c > 0; \operatorname{Re} \mu < 1/2]$$

2.16.62. Интегралы, содержащие $D_\mu(bx^{\pm r}) K_\nu(cx)$.

$$1. \int_0^\infty x^{\alpha-1} e^{\pm b^2 x^2/4} D_\mu(bx) K_\nu(cx) dx = -\frac{\pi}{2} U(0)$$

$$[\operatorname{Re} c > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu|; |\arg b| < (2 \pm 1)\pi/4; U(\gamma) \text{ см. в 2.13.33.1}].$$

$$2. \int_0^\infty x^{\nu+1-\sigma} e^{-b^2 x^2/4} D_{2\nu+\sigma}(bx) K_\nu(cx) dx =$$

$$= \sqrt{\frac{\pi}{2}} \frac{c^{\nu-1+\sigma}}{b^{2\nu+1}} e^{(2\nu-1+2\sigma)\pi i/2} \Gamma(2\nu+2-\sigma) \exp\left(\frac{c^2}{4b^2}\right) D_{-2\nu-2+\sigma}\left(\frac{c}{b}\right)$$

$$[\sigma=0 \text{ или } 1; \operatorname{Re} c > 0; \operatorname{Re} \nu > -(2-\sigma)/2; |\arg b| < \pi/4].$$

$$3. \int_0^\infty x^{\alpha-1} e^{\pm b^2 x^2/4} D_\mu(b\sqrt{x}) K_\nu(cx) dx = -\frac{\pi}{2} V(0)$$

$$[\operatorname{Re} c > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu|; |\arg b| < (2 \pm 1)\pi/4; V(\gamma) \text{ см. в 2.13.33.3}].$$

$$4. \int_0^\infty x^{\alpha-1} e^{(\pm b^2/4 - c)x} D_\mu(b\sqrt{x}) K_\nu(cx) dx = W(1)$$

$$[\operatorname{Re} c > 0; \operatorname{Re} \alpha > |\operatorname{Re} \nu|; |\arg b| < (2 \pm 1)\pi/4]$$

$$W(\gamma) = \frac{2^{\mp(\mu+1 \pm 1)/2 - \alpha} \sqrt{\pi}}{c^\alpha} \left(\frac{\cos \nu\pi}{\cos \alpha\pi} \right)^{1-\gamma} \left\{ \frac{\Gamma^{-1}(-\mu)}{\sqrt{\pi}} \right\} \Gamma \left[\begin{matrix} \alpha+\nu, \alpha-\nu \\ 1/2+\alpha \end{matrix} \right] \times$$

$$\times \Gamma^{\pm 1} \left(\frac{1 \mp 1-2\mu}{4} \right) {}_3F_2 \left(\alpha+\nu, \alpha-\nu, \frac{1 \mp 1 \mp 2\mu}{4}; \frac{1}{2} + \alpha, \frac{1}{2}; \mp \frac{(-1)^\nu b^2}{16c} \right) +$$

$$+ \frac{(-1)^\nu 2^{\mp(\mu+1 \pm 1)/2 - \alpha} \sqrt{\pi} b}{c^{\alpha+1/2}} \left(\frac{\cos \nu\pi}{\sin \alpha\pi} \right)^{1-\gamma} \left\{ \frac{\Gamma^{-1}(-\mu)}{\sqrt{\pi}} \right\} \times$$

$$\times \Gamma \left[\begin{matrix} \alpha+\nu+1/2, \alpha-\nu+1/2 \\ \alpha+1 \end{matrix} \right] \Gamma^{\pm 1} \left(\frac{1 \pm 1-2\mu}{4} \right) \times$$

$$\times {}_3F_2 \left(\alpha+\nu+\frac{1}{2}, \alpha-\nu+\frac{1}{2}, \frac{3 \mp 1 \mp 2\mu}{4}; \frac{3}{2}, \alpha+1; \mp \frac{(-1)^\nu b^2}{16c} \right) +$$

$$+ (1-\gamma) \frac{2^{\alpha \mp (\mu+1 \pm 1)/2} \sqrt{\pi}}{b^{2\alpha-1} \sqrt{c}} \left\{ \frac{\Gamma^{-1}(-\mu)}{\sqrt{\pi}} \right\} \Gamma(2\alpha-1) \Gamma^{\pm 1} \left(\frac{1 \pm 1-2\mu}{4} \mp \alpha \right) \times$$

$$\times {}_3F_2 \left(\frac{1}{2} + \nu, \frac{1}{2} - \nu, \frac{3 \mp 1 \mp 2\mu}{4} - \alpha; \frac{3}{2} - \alpha, 1 - \alpha; \mp \frac{b^2}{16c} \right).$$

$$5. \int_0^{\infty} x^{\alpha-1} e^{(\pm b^2/4 + c)x} D_{\mu}(b\sqrt{x}) K_{\nu}(cx) dx = W(0) \\ \left[\operatorname{Re} c > 0; |\arg b| < (2 \pm 1)\pi/4, \begin{cases} |\operatorname{Re} \nu| < \operatorname{Re} \alpha < (1 - \operatorname{Re} \mu)/2 \\ |\operatorname{Re} \nu| < \operatorname{Re} \alpha \end{cases}; \right. \\ \left. W(\gamma) \text{ см. в 2.16.62.4} \right].$$

$$6. \int_0^{\infty} x^{-(\mu+3)/2} e^{(b^2/4 - c)x} D_{\mu}(b\sqrt{x}) K_0(cx) dx = \\ = -\frac{\pi \sqrt{2} \Gamma(-\mu-1)}{(\mu+1) c^{1/4}} (b^2 - 4c)^{(2\mu+3)/4} p_{-1/2}^{\mu+3/2} \left(\frac{b}{2\sqrt{c}} \right) \\ [\operatorname{Re} c > 0; \operatorname{Re} \mu < -1; |\arg b| < 3\pi/4].$$

$$7. \int_0^{\infty} x^{\alpha-1} e^{\pm b^2/(4x^2)} D_{\mu}\left(\frac{b}{x}\right) K_{\nu}(cx) dx = \\ = \frac{2^{(\alpha+\nu-3 \mp 1 \mp \mu)/2}}{b^{\nu-\alpha} c^{\nu}} \Gamma(\nu) \Gamma(\nu-\alpha) \Gamma^{\pm 1} \left(\frac{1 \mp 1}{4} \mp \frac{\nu-\alpha \pm \mu}{2} \right) \left\{ \frac{\Gamma^{-1}(-\mu)}{\sqrt{\pi}} \right\} \times \\ \times {}_1F_3 \left(\frac{1 \mp 1}{4} + \frac{\alpha-\nu \mp \mu}{2}; 1-\nu, 1+\frac{\alpha-\nu}{2}, \frac{1+\alpha-\nu}{2}; \pm \frac{b^2 c^2}{8} \right) + \\ + 2^{(\alpha-\nu-3 \mp 1 \mp \mu)/2} b^{\alpha+\nu} c^{\nu} \Gamma(-\nu) \Gamma(-\nu-\alpha) \Gamma^{\pm 1} \left(\frac{1 \mp 1}{4} \pm \frac{\alpha+\nu \mp \mu}{2} \right) \times \\ \times \left\{ \frac{\Gamma^{-1}(-\mu)}{\sqrt{\pi}} \right\} {}_1F_3 \left(\frac{1 \mp 1}{4} + \frac{\alpha+\nu \mp \mu}{2}; 1+\nu, 1+\frac{\alpha+\nu}{2}, \frac{1+\alpha+\nu}{2}; \pm \frac{b^2 c^2}{8} \right) + \\ + \frac{2^{\alpha+\mu/2-2} \sqrt{\pi}}{c^{\alpha}} \Gamma \left[\frac{(\alpha+\nu)/2, (\alpha-\nu)/2}{(1-\mu)/2} \right] {}_1F_3 \left(\frac{1 \mp 1 \mp 2\mu}{4}; \frac{1}{2}, 1-\frac{\alpha-\nu}{2}, \right. \\ \left. 1-\frac{\alpha+\nu}{2}; \pm \frac{b^2 c^2}{8} \right) - \frac{2^{\alpha+(\mu-5)/2} \sqrt{\pi} b}{c^{\alpha-1}} \Gamma \left[\frac{(\alpha+\nu-1)/2, (\alpha-\nu-1)/2}{-\mu/2} \right] \times \\ \times {}_1F_3 \left(\frac{3 \mp 1 \mp 2\mu}{4}; \frac{3}{2}, \frac{3-\nu-\alpha}{2}, \frac{3+\nu-\alpha}{2}; \pm \frac{b^2 c^2}{8} \right) \\ \left[\operatorname{Re} c > 0, \begin{cases} |\arg b| < 3\pi/4; \operatorname{Re}(\alpha-\mu) > |\operatorname{Re} \nu| \\ |\arg b| < \pi/4 \end{cases} \right].$$

$$8. \int_0^{\infty} x^{1/2} D_{\nu-1/2}\left(\frac{b}{\sqrt{x}}\right) D_{-\nu-1/2}\left(\frac{b}{\sqrt{x}}\right) K_{\nu}(cx) dx = \frac{\pi}{2c^{3/2}} \exp(-b\sqrt{2c}) \\ [\operatorname{Re} c > 0; |\arg b| < \pi/4].$$

2.17. МНОГОЧЛЕНЫ ЛЕЖАНДРА $P_n(x)$

Многочлены $P_n(x)$ являются частными случаями многочленов Гегенбауэра и Якоби:

$$P_n(x) = C_n^{1/2}(x) = P_n^{(0,0)}(x),$$

поэтому интегралы общего вида, содержащие $P_n(x)$, можно получить из п. 2.21.1 и п. 2.22.1. При вычислении интегралов от $P_n(x)$ может оказаться полезной формула

$$P_n(-x) = (-1)^n P_n(x).$$

2.17.1. Интегралы от $(z^m \pm x^m)^{\beta} P_n(cx)$.

$$1. \int_0^a x^{\alpha-1} P_{2n+\varepsilon}\left(\frac{x}{a}\right) dx = \frac{(-1)^n ((1-\alpha+\varepsilon)/2)_n a^{\alpha}}{2 ((\alpha+\varepsilon)/2)_{n+1}}$$

$$[\varepsilon=0 \text{ или } 1; a>0; \operatorname{Re} \alpha > -\varepsilon].$$

$$2. \int_0^a x^m P_n \left(\frac{x}{a} \right) dx = 0 \quad [a > 0; n-m=2, 4, 6, \dots]$$

$$3. \int_{-a}^a x^m P_n \left(\frac{x}{a} \right) dx = I_{m,n} \quad [a > 0]$$

$$I_{m,n} = 0 \quad [m+n=1, 3, 5, \dots, \text{или } m+n=2, 4, 6, \dots \text{ и } n-m=2, 4, 6, \dots]$$

$$I_{m,n} = \frac{\sqrt{\pi} m! a^{m+1}}{2^m \Gamma((m+n+3)/2) \Gamma((m-n)/2+1)} \quad [m+n=0, 2, 4, \dots; n-m \neq 2, 4, 6, \dots]$$

$$4. \int_a^\infty x^{\alpha-1} P_n \left(\frac{x}{a} \right) dx = \frac{a^\alpha}{2^{\alpha+1} \sqrt{\pi}} \Gamma \left[\frac{-(n+\alpha)/2, (n-\alpha+1)/2}{1-\alpha} \right] \quad [a > 0; \operatorname{Re} \alpha < -n]$$

$$5. \int_a^b (x-a)^{\alpha-1} P_n \left(\frac{x}{b} \right) dx = \frac{n! (b-a)^\alpha}{(\alpha)_{n+1}} P_n^{(\alpha, -\alpha)} \left(\frac{a}{b} \right) \quad [-b < a < b; \operatorname{Re} \alpha > 0]$$

$$6. \int_a^b (x-a)^{n-1} P_n \left(\frac{x}{b} \right) dx = \frac{(b-a)^n}{n} \left(\frac{a+b}{2b} \right)^n \quad [-b < a < b]$$

$$7. \int_a^b \frac{1}{\sqrt{x-a}} P_n \left(\frac{x}{b} \right) dx = \frac{2\sqrt{2b}}{2n+1} \sin \left[(2n+1) \arcsin \sqrt{\frac{b-a}{2b}} \right] =$$

$$= \frac{2a}{(2n+1) \sqrt{b-a}} \left[T_n \left(\frac{a}{b} \right) - T_{n+1} \left(\frac{a}{b} \right) \right] \quad [-b < a < b]$$

$$8. \int_{-a}^b (b-x)^{\beta-1} P_n \left(\frac{x}{a} \right) dx = \frac{n!}{(\beta)_{n+1}} (b \mp a)^\beta P_n^{(\pm \beta, \mp \beta)} \left(\frac{b}{a} \right) \quad [-a < b < a; \operatorname{Re} \beta > 0]$$

$$9. \int_{-a}^b \frac{1}{\sqrt{b-x}} P_n \left(\frac{x}{a} \right) dx = \frac{2a}{(2n+1) \sqrt{b+a}} \left[T_n \left(\frac{b}{a} \right) + T_{n+1} \left(\frac{b}{a} \right) \right] \quad [-a < b < a]$$

$$10. \int_{-a}^a (x+a)^{\alpha-1} P_n \left(\frac{x}{a} \right) dx = \frac{(-1)^n (1-\alpha)_n}{(\alpha)_{n+1}} (2a)^\alpha \quad [a, \operatorname{Re} \alpha > 0]$$

$$11. \int_{-a}^a \frac{1}{\sqrt{x+a}} P_n \left(\frac{x}{a} \right) dx = (-1)^n \frac{2\sqrt{2a}}{2n+1} \quad [a > 0]$$

$$12. \int_{-a}^a \frac{1}{(x+z)^\rho} P_n \left(\frac{x}{a} \right) dx = I_\rho \quad [a > 0; |\arg(z^2 - a^2)| < \pi]$$

$$I_\rho = \frac{(-1)^n (\rho)_n (2a)^{n+1}}{(n+1)_{n+1} (z-a)^{n+\rho}} {}_2F_1 \left(\rho+n, n+1; 2n+2; \frac{2a}{a-z} \right),$$

$$I_{3/2} = \frac{(-1)^n 2^{3/2} (z - \sqrt{z^2 - a^2})^{n+1/2}}{a^n \sqrt{z^2 - a^2}},$$

$$I_{1/2} = \frac{(-1)^n 2\sqrt{2}}{(2n+1) a^n} (z - \sqrt{z^2 - a^2})^{n+1/2}.$$

$$13. \int_{-a}^a \frac{1}{x-y} P_n \left(\frac{x}{a} \right) dx = -2Q_n \left(\frac{y}{a} \right) \quad [a > 0; y \notin [-a, a]]$$

$$14. \int_{-a}^a \frac{1}{x-y} \left[P_n \left(\frac{y}{a} \right) P_{n-1} \left(\frac{x}{a} \right) - P_{n-1} \left(\frac{y}{a} \right) P_n \left(\frac{x}{a} \right) \right] dx = -\frac{2}{n} \quad [a > 0; -a < y < a]$$

$$15. \int_a^\infty \frac{1}{(x+a)^\rho} P_n \left(\frac{x}{a} \right) dx = \frac{(-1)^{n+1} (\rho)_n}{(1-\rho)_{n+1}} (2a)^{1-\rho} \quad [a > 0; \operatorname{Re} \rho > n+1]$$

$$16. \int_a^\infty \frac{1}{(x+z)^\rho} P_n \left(\frac{x}{a} \right) dx = \frac{(-1)^{n+1} n!}{(1-\rho)_{n+1}} (a+z)^{1-\rho} P_n^{(\rho-1, 1-\rho)} \left(\frac{z}{a} \right) \quad [a > 0; \operatorname{Re} \rho > n+1; |\arg(z-a)| < \pi]$$

$$17. \int_0^a \frac{1}{(x^2+z^2)^\rho} P_{2n} \left(\frac{x}{a} \right) dx = \frac{(-1)^n (\rho)_n a^{2n+1}}{2 (n+1/2)_{n+1} z^{2\rho+2n}} \times \\ \times {}_2F_1 \left(\rho+n, n+\frac{1}{2}; 2n+\frac{3}{2}; -\frac{a^2}{z^2} \right) \quad [a, \operatorname{Re} z > 0]$$

$$18. \int_{-a}^a \frac{1}{(x^2+z^2)^{n+3/2}} P_{2n} \left(\frac{x}{a} \right) dx = \frac{2 (-1)^n a^{2n+1}}{(2n+1) z^{2n+2} (a^2+z^2)^{n+1/2}} \quad [a, \operatorname{Re} z > 0]$$

$$19. \int_{-a}^a (a^2-x^2)^{\beta-1} P_n \left(\frac{x}{a} \right) dx = \\ = (-1)^n \sqrt{\pi} \Gamma \left[\frac{\beta}{2} \right] a^{2\beta-1} {}_3F_2 \left(-n, n+1, \beta; 2\beta, 1; 1 \right) \quad [a, \operatorname{Re} \beta > 0]$$

$$20. \int_{-a}^a (a^2-x^2)^m P_{2n} \left(\frac{x}{a} \right) dx = \frac{2m^2 a^2}{(m-n)(2m+2n+1)} \int_{-a}^a (a^2-x^2)^{m-1} P_{2n} \left(\frac{x}{a} \right) dx \quad [a > 0; n < m]$$

$$21. \int_{-a}^a (a^2-x^2)^{-3/4} P_{2n}(cx) dx = \frac{1}{\sqrt{2\pi a}} \Gamma^2 \left(\frac{1}{4} \right) C_n^{1/4} (2a^2 c^2 - 1) \quad [a > 0]$$

2.17.2. Интегралы от $x^\alpha (z^2 \pm x^2)^\beta P_n(cx)$.

$$1. \int_0^a x^{\alpha-1} (a^2-x^2)^{\beta-1} P_{2n+\varepsilon}(cx) dx = \\ = \frac{(-1)^n (1/2)_{n+\varepsilon}}{2^{1-\varepsilon} n!} a^{\alpha+2\beta+\varepsilon-2} c^\varepsilon B \left(\frac{\alpha+\varepsilon}{2}, \beta \right) \times \\ \times {}_3F_2 \left(-n, \frac{1}{2}+n+\varepsilon, \frac{\alpha+\varepsilon}{2}; \varepsilon+\frac{1}{2}, \frac{\alpha+\varepsilon}{2}+\beta; a^2 c^2 \right) \quad [\varepsilon=0 \text{ или } 1; a, \operatorname{Re} \beta > 0; \operatorname{Re} \alpha > -\varepsilon]$$

$$2. \int_0^a x^{n-2\beta} (a^2-x^2)^{\beta-1} P_n(cx) dx = \frac{1}{2} B \left(\frac{1}{2}-\beta, \beta \right) a^{n-1} C_n^{1/2-\beta}(ac) \quad [a > 0; 0 < \operatorname{Re} \beta < (n+1)/2]$$

$$3. \int_0^a x^\varepsilon (a^2 - x^2)^{\beta-1} P_{2n+\varepsilon} \left(\frac{x}{a} \right) dx =$$

$$= \frac{\sqrt{\pi} (1/2)_{n+\varepsilon}}{2 (n!)} (1-\beta)_n a^{2\beta+\varepsilon-1} \Gamma \left[\begin{matrix} \beta \\ \beta + \varepsilon + n + 1/2 \end{matrix} \right]$$

[$\varepsilon=0$ или 1; $a, \operatorname{Re} \beta > 0$].

$$4. \int_0^a x^{2n+\varepsilon-2\beta} (a^2 - x^2)^{\beta-1} P_{2n+\varepsilon} \left(\frac{x}{a} \right) dx = \frac{(1-\beta)_n a^{2n+\varepsilon-1}}{2 (n!)} B \left(\beta, n - \beta + \varepsilon + \frac{1}{2} \right)$$

[$\varepsilon=0$ или 1; $a > 0$; $0 < \operatorname{Re} \beta < n + \varepsilon + 1/2$].

$$5. \int_0^a \frac{x^{n-1}}{\sqrt{a^2 - x^2}} P_n \left(\frac{x}{a} \right) dx = \frac{a^{n-1}}{n}$$

[$a, n > 0$].

$$6. \int_0^a \frac{x^{\alpha-1}}{(z^2 \pm x^2)^\rho} P_{2n+\varepsilon} \left(\frac{x}{a} \right) dx = \frac{(-1)^n ((1-\alpha+\varepsilon)/2)_n a^\alpha}{2 ((\alpha+\varepsilon)/2)_{n+1} z^{2\rho}} \times$$

$$\times {}_3F_2 \left(\rho, \frac{\alpha}{2}, \frac{\alpha+1}{2}; \frac{\alpha+\varepsilon}{2} + n + 1, \frac{1+\alpha-\varepsilon}{2} - n; \mp \frac{a^2}{z^2} \right)$$

[$\varepsilon=0$ или 1; $\operatorname{Re} \alpha > -\varepsilon$, $\left\{ \begin{matrix} a, \operatorname{Re} z > 0 \\ z > a > 0 \end{matrix} \right\}$].

$$7. \int_0^a \frac{x^{\alpha-1}}{x^2 - y^2} P_{2n+\varepsilon} \left(\frac{x}{a} \right) dx = I$$

[$\varepsilon=0$ или 1; $a, y > 0$; $\operatorname{Re} \alpha > -\varepsilon$].

$$I = \frac{(-1)^{n+1} a^\alpha ((1-\alpha+\varepsilon)/2)_n}{2 y^2 ((\alpha+\varepsilon)/2)_{n+1}} {}_3F_2 \left(1, \frac{\alpha}{2}, \frac{\alpha+1}{2}; \right.$$

$$\left. \frac{\alpha+\varepsilon}{2} + n + 1, \frac{1+\alpha-\varepsilon}{2} - n; \frac{a^2}{y^2} \right)$$

[$a < y$].

$$I = \frac{(-1)^{\varepsilon+1} \pi}{2} y^{\alpha-2} \operatorname{tg}^{2\varepsilon-1} \frac{\alpha \pi}{2} P_{2n+\varepsilon} \left(\frac{y}{a} \right) + \frac{(-1)^n a^{\alpha-2} ((3-\alpha+\varepsilon)/2)_n}{2 ((\alpha+\varepsilon)/2 - 1)_{n+1}} \times$$

$$\times {}_3F_2 \left(1, 1 - \frac{\alpha+\varepsilon}{2} - n, \frac{3-\alpha+\varepsilon}{2} + n; \frac{3-\alpha}{2}, 2 - \frac{\alpha}{2}; \frac{y^2}{a^2} \right)$$

[$y < a$].

$$8. \int_a^b x^{\alpha-1} (b^2 - x^2)^{\beta-1} P_{2n+\varepsilon} \left(\frac{x}{a} \right) dx = \frac{(-1)^{n+1} ((1-\alpha+\varepsilon)/2)_n a^\alpha b^{2\beta-2}}{2 ((\alpha+\varepsilon)/2)_{n+1}} \times$$

$$\times {}_3F_2 \left(1-\beta, \frac{\alpha}{2}, \frac{\alpha+1}{2}; \frac{1+\alpha-\varepsilon}{2} - n, \frac{\alpha+\varepsilon}{2} + n + 1; \frac{a^2}{b^2} \right) +$$

$$+ \frac{2^{2n+\varepsilon-1} (1/2)_{2n+\varepsilon}}{(2n+\varepsilon)! a^{2n+\varepsilon}} b^{\alpha+2\beta+\varepsilon+2n-2} B \left(\beta, \frac{\alpha+\varepsilon}{2} + n \right) \times$$

$$\times {}_3F_2 \left(-n, 1 - \frac{\alpha+\varepsilon}{2} - \beta - n, \frac{1}{2} - \varepsilon - n; \frac{1}{2} - \varepsilon - 2n, 1 - \frac{\alpha+\varepsilon}{2} - n; \frac{a^2}{b^2} \right)$$

[$\varepsilon=0$ или 1; $b > a > 0$; $\operatorname{Re} \beta > 0$].

$$9. \int_a^b x^{\alpha-1} (x^2 - a^2)^{\beta-1} P_{2n+\varepsilon} \left(\frac{x}{b} \right) dx =$$

$$= \frac{(-1)^n ((3-\alpha+\varepsilon)/2 - \beta)_n}{2 ((\alpha+\varepsilon)/2 + \beta - 1)_{n+1}} b^{\alpha+2\beta-2} \times$$

$$\times {}_3F_2 \left(1-\beta, 1 - \frac{\alpha+\varepsilon}{2} - \beta - n, \frac{3-\alpha+\varepsilon}{2} - \beta + n; \frac{3-\alpha}{2} - \beta, 2 - \beta - \frac{\alpha}{2}; \frac{a^2}{b^2} \right) +$$

$$+ \frac{(-1)^n (1/2)_{n+\varepsilon} a^{\alpha+2\beta+\varepsilon-2}}{2^{1-\varepsilon} n! b^\varepsilon} B\left(\beta, 1-\beta-\frac{\alpha+\varepsilon}{2}\right) \times \\ \times {}_3F_2\left(-n, \frac{\alpha+\varepsilon}{2}, \frac{1}{2}+\varepsilon+n; \frac{\alpha+\varepsilon}{2}+\beta, \varepsilon+\frac{1}{2}; \frac{a^2}{b^2}\right) \\ [\varepsilon=0 \text{ или } 1; b>a>0; \operatorname{Re} \beta>0].$$

$$10. \int_a^b x^{-2\beta-n-1} (x^2-a^2)^{\beta-1} P_n\left(\frac{x}{b}\right) dx = \frac{n! (b^2-a^2)^\beta}{(2\beta)_{n+1} a^{n+2} b^{2\beta}} C_n^{\beta+1/2}\left(\frac{a}{b}\right) \\ [b>a>0; \operatorname{Re} \beta>0].$$

$$11. \int_{-a}^a \frac{x^\varepsilon}{\sqrt{a^2-x^2}} P_{2n+\varepsilon}\left(\frac{x}{a}\right) dx = \pi \left[\frac{(1/2)_n}{n!}\right]^2 \left(\frac{2n+1}{2n+2} a\right)^\varepsilon \\ [\varepsilon=0 \text{ или } 1; a>0].$$

$$12. \int_0^\infty \frac{x^\varepsilon}{(x^2+a^2)^\rho} P_{2n+\varepsilon}\left(\frac{ix}{a}\right) dx = \frac{(-1)^{n+\varepsilon} \sqrt{\pi}}{2 (n!)} a^{1+\varepsilon-2\rho} \left(\frac{1}{2}\right)_{n+\varepsilon} (\rho)_n \Gamma\left[\frac{\rho-\varepsilon-n-1/2}{\rho}\right] \\ [\varepsilon=0 \text{ или } 1; \operatorname{Re} a>0; \operatorname{Re} \rho>n+\varepsilon+1/2].$$

$$13. \int_a^\infty x^{\alpha-1} (x^2-a^2)^{\beta-1} P_n\left(\frac{x}{b}\right) dx = \frac{2^{n-1} a^{\alpha+2\beta+n-2} (1/2)_n}{n! b^n} B\left(\beta, 1-\frac{\alpha+n}{2}-\beta\right) \times \\ \times {}_3F_2\left(-\left[\frac{n}{2}\right], 1-\frac{\alpha+n}{2}-\beta, \frac{1}{2}+\left[\frac{n}{2}\right]-n; 1-\frac{\alpha+n}{2}, \frac{1}{2}-n; \frac{b^2}{a^2}\right) \\ [a>b>0; \operatorname{Re} \beta>0; \operatorname{Re}(\alpha+2\beta)<2-n].$$

$$14. \int_a^\infty \frac{x^{\alpha-1}}{(x^2 \pm z^2)^\rho} P_{2n+\varepsilon}\left(\frac{x}{a}\right) dx = \frac{(-1)^{n+1} a^{\alpha-2\rho} ((1-\alpha+\varepsilon)/2+\rho)_n}{2 ((\alpha+\varepsilon)/2-\rho)_{n+1}} \times \\ \times {}_3F_2\left(\rho, \frac{1-\alpha+\varepsilon}{2}+\rho+n, \rho-n-\frac{\alpha+\varepsilon}{2}; 1+\rho-\frac{\alpha}{2}, \frac{1-\alpha}{2}+\rho; \mp \frac{z^2}{a^2}\right) \\ [\varepsilon=0 \text{ или } 1; \operatorname{Re}(\alpha-2\rho)<-2n-\varepsilon, \left\{\begin{matrix} a, \pm z > 0 \\ a>z>0 \end{matrix}\right\}].$$

$$15. \int_a^\infty \frac{x^{\alpha-1}}{x^2-y^2} P_{2n+\varepsilon}\left(\frac{x}{a}\right) dx = I \\ [\varepsilon=0 \text{ или } 1; a, y>0; \operatorname{Re} \alpha<2-2n-\varepsilon].$$

$$I = (-1)^n \frac{a^\alpha ((1-\alpha+\varepsilon)/2)_n}{2y^2 ((\alpha+\varepsilon)/2)_{n+1}} {}_3F_2\left(1, \frac{\alpha}{2}, \frac{\alpha+1}{2}; \frac{1+\alpha-\varepsilon}{2}-n, 1+\frac{\alpha+\varepsilon}{2}+n; \frac{a^2}{y^2}\right) + \frac{(-1)^{\varepsilon-1} \pi}{2} y^{\alpha-2} \operatorname{tg} 2\varepsilon-1 \frac{\alpha\pi}{2} P_{2n+\varepsilon}\left(\frac{y}{a}\right) \\ [a<y].$$

$$I = (-1)^{n+1} \frac{a^{\alpha-2} ((3-\alpha+\varepsilon)/2)_n}{2 ((\alpha+\varepsilon)/2-1)_{n+1}} \times \\ \times {}_3F_2\left(1, \frac{3-\alpha+\varepsilon}{2}+n, 1-\frac{\alpha+\varepsilon}{2}-n; 2-\frac{\alpha}{2}, \frac{3-\alpha}{2}; \frac{y^2}{a^2}\right) \\ [y<a].$$

2.17.3. Интегралы от $(x \pm a)^\alpha (b \pm x)^\beta P_n(\varphi(x))$.

$$1. \int_a^b \frac{1}{\sqrt{(x-a)(b-x)}} P_n(cx) dx = \pi P_n(z_+) P_n(z_-) \\ [b>a; \sqrt{2} z_\pm = (1+abc \pm \sqrt{(1-a^2c^2)(1-b^2c^2)})^{1/2}].$$

$$2. \int_a^b (x-a)^{\alpha-1} (b-x)^{\beta-1} P_n \left(\frac{x}{a} \right) dx = B(\alpha, \beta) (b-a)^{\alpha+\beta-1} \times \\ \times {}_3F_2 \left(-n, n+1, \alpha; \alpha+\beta, 1; \frac{a-b}{2a} \right) \quad [b > a; \operatorname{Re} \alpha, \operatorname{Re} \beta > 0].$$

$$3. \int_a^b (x-a)^{n-\beta} (b-x)^{\beta-1} P_n \left(\frac{x}{a} \right) dx = \frac{\pi (1-\beta)_n}{n! \sin \beta \pi} (b-a)^n P_n^{(0, -\beta)} \left(\frac{b}{a} \right) \\ [b > a; 0 < \operatorname{Re} \beta < n+1].$$

$$4. \int_{-a}^b (b-x)^{\beta-1} (a-x)^{-\tau} P_n \left(\frac{x}{a} \right) dx = B(\beta, \tau-\beta) (a-b)^{\beta-\tau} \times \\ \times {}_3F_2 \left(-n, n+1, 1-\tau; 1, 1+\beta-\tau; \frac{a-b}{2a} \right) + \frac{(1+\tau-\beta)_n}{(\beta-\tau)_{n+1}} (2a)^{\beta-\tau} \times \\ \times {}_3F_2 \left(1-\beta, 1+\tau+n-\beta, \tau-\beta-n; 1+\tau-\beta, 1+\tau-\beta; \frac{a-b}{2a} \right) \\ [-a < b < a; \operatorname{Re} \beta > 0].$$

$$5. \int_a^b (b-x)^{\beta-1} (x+a)^{-\tau} P_n \left(\frac{x}{a} \right) dx = \frac{2^n (1/2)_n}{n! a^n} B(1+n-\tau, \beta) (a+b)^{\beta+n-\tau} \times \\ \times {}_3F_2 \left(-n, -n, \tau-\beta-n; -2n, \tau-n; \frac{2a}{a+b} \right) + \frac{(-1)^{n+1} (\tau)_n}{(1-\tau)_{n+1}} (2a)^{1-\tau} \times \\ \times (a+b)^{\beta-1} {}_3F_2 \left(1-\beta, 1-\tau, 1-\tau; 2+n-\tau, 1-\tau-n; \frac{2a}{a+b} \right) \\ [b > a > 0; \operatorname{Re} \beta > 0].$$

$$6. \int_a^b (x-a)^{\alpha-1} (b-x)^{\beta-1} P_n \left(\frac{x}{b} \right) dx = B(\alpha, \beta) (b-a)^{\alpha+\beta-1} \times \\ \times {}_3F_2 \left(-n, n+1, \beta; 1, \alpha+\beta; \frac{b-a}{2b} \right) \quad [-b < a < b; \operatorname{Re} \alpha, \operatorname{Re} \beta > 0].$$

$$7. \int_a^b (x-a)^{n-\beta} (b-x)^{\beta-1} P_n \left(\frac{x}{b} \right) dx = \frac{\pi (1-\beta)_n}{n! \sin \beta \pi} (b-a)^n P_n^{(0, \beta-n-1)} \left(\frac{a}{b} \right) \\ [b > a; 0 < \operatorname{Re} \beta < n+1].$$

$$8. \int_a^b (x-a)^{\alpha-1} (x+b)^{-\tau} P_n \left(\frac{x}{b} \right) dx = (-1)^n B(\alpha, \tau-\alpha) (a+b)^{\alpha-\tau} \times \\ \times {}_3F_2 \left(-n, n+1, 1-\tau; 1, 1+\alpha-\tau; \frac{a+b}{2b} \right) + \frac{(-1)^n (1+\tau-\alpha)_n}{(\alpha-\tau)_{n+1}} (2b)^{\alpha-\tau} \times \\ \times {}_3F_2 \left(1-\alpha, 1+\tau+n-\alpha, \tau-\alpha-n; 1+\tau-\alpha, 1+\tau-\alpha; \frac{a+b}{2b} \right) \\ [b > a > 0; \operatorname{Re} \alpha > 0].$$

$$9. \int_a^b (x-a)^{\tau-n-2} (x+b)^{-\tau} P_n \left(\frac{x}{b} \right) dx = \\ = \frac{(-1)^{n+1} n! (2b)^{1+n-\tau}}{(1-\tau)_{n+1}} (b-a)^{\tau-n-1} (b+a)^{-n-1} P_n^{(\tau-n-1, 0)} \left(\frac{a}{b} \right) \\ [b > a > 0; \operatorname{Re} \tau > n+1].$$

$$10. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\beta-1} P_n\left(\frac{x}{a}\right) dx = \\ = (2a)^{\alpha+\beta-1} B(\alpha, \beta) {}_3F_2(-n, n+1, \beta; 1, \alpha+\beta; 1) \\ [a, \operatorname{Re} \alpha, \operatorname{Re} \beta > 0].$$

$$11. \int_{-a}^a (a+x)^{n-\beta} (a-x)^{\beta-1} P_n\left(\frac{x}{a}\right) dx = \left[\frac{(1-\beta)_n}{n!}\right]^2 \frac{(2a)^n \pi}{\sin \beta \pi} \\ [a > 0; 0 < \operatorname{Re} \beta < n+1].$$

$$12. \int_{-a}^a (a-x)^{\beta-1} (\pm x \mp z)^{-\theta} P_n\left(\frac{x}{a}\right) dx = \frac{(1-\beta)_n}{(\beta)_{n+1}} (2a)^\beta (\pm a \mp z)^{-\theta} \times \\ \times {}_3F_2\left(\theta, \beta, \beta; \beta-n, \beta+n+1; \frac{2a}{a-z}\right) \\ \left[a, \operatorname{Re} \beta > 0, \left\{\begin{array}{l} z < -a \\ |\arg(z^2 - a^2)| < \pi \end{array}\right\}\right].$$

$$13. \int_{-a}^a (x+a)^{\alpha-1} (z \pm x)^{-\theta} P_n\left(\frac{x}{a}\right) dx = \frac{(-1)^n (1-\alpha)_n}{(\alpha)_{n+1}} (2a)^\alpha (z \mp a)^{-\theta} \times \\ \times {}_3F_2\left(\alpha, \alpha, \theta; \alpha-n, \alpha+n+1; \frac{2a}{a \mp z}\right) \\ [a, \operatorname{Re} \alpha > 0; |\arg(z^2 - a^2)| < \pi].$$

$$14. \int_{-a}^a \frac{x^m}{x-y} P_n\left(\frac{x}{a}\right) dx = -2y^m Q_n\left(\frac{y}{a}\right) - \left\{\begin{array}{l} 0 \\ 1 \end{array}\right\} \frac{n! a^{n+1}}{2^{n-1} (3/2)_n} \\ \left[\left\{\begin{array}{l} m \leq n \\ m = n+1 \end{array}\right\}, a > 0, y \notin [-a, a]\right].$$

$$15. \int_{-a}^a \frac{(a-x)^{\beta-1}}{x-y} P_n\left(\frac{x}{a}\right) dx = I \quad [a, \operatorname{Re} \beta > 0].$$

$$I = \pi \operatorname{ctg} \beta \pi (a-y)^{\beta-1} P_n\left(\frac{y}{a}\right) - \frac{(2-\beta)_n}{(\beta-1)_{n+1}} (2a)^{\beta-1} \times \\ \times {}_3F_2\left(1, 2+n-\beta, 1-\beta-n; 2-\beta, 2-\beta; \frac{a-y}{2a}\right) \quad [-a < y < a].$$

$$I = \frac{(1-\beta)_n}{(a-y)(\beta)_{n+1}} (2a)^\beta {}_3F_2\left(1, \beta, \beta; \beta-n, \beta+n+1; \frac{2a}{a-y}\right) \quad [y < -a].$$

$$16. \int_a^\infty \frac{(x+a)^{-\tau}}{x-y} P_n\left(\frac{x}{a}\right) dx = I \quad [a > 0; \operatorname{Re} \tau > n].$$

$$I = (-1)^{n+1} \frac{(\tau+1)_n}{(-\tau)_{n+1}} (2a)^{-\tau} {}_3F_2\left(1, \tau-n, \tau+n+1; \tau+1, \tau+1; \frac{a+y}{2a}\right) \\ [-a < y < a].$$

$$I = \pi \operatorname{ctg} \tau \pi (a+y)^{-\tau} P_n\left(\frac{y}{a}\right) + (-1)^n \frac{(\tau)_n (2a)^{1-\tau}}{(a+y)(1-\tau)_{n+1}} \times \\ \times {}_3F_2\left(1, 1-\tau, 1-\tau; 2+n-\tau, 1-\tau-n; \frac{2a}{a+y}\right) \quad [y > a].$$

$$17. \int_a^\infty (x-a)^{\alpha-1} (x+a)^{-\tau} P_n\left(\frac{x}{a}\right) dx = \\ = B(\alpha, \tau-\alpha-n) (2a)^{\alpha-\tau} {}_3F_2(-n, -n, \alpha; 1, \tau-n; 1) \\ [a > 0; 0 < \operatorname{Re} \alpha < \operatorname{Re} \tau - n].$$

$$18. \int_a^\infty (x+a)^{-\tau} (x+z)^{-\theta} P_n \left(\frac{x}{a} \right) dx = \frac{(-1)^{n+1} (\theta + \tau)_n (2a)^{1-\theta-\tau}}{(1-\theta-\tau)_{n+1}} \times \\ \times {}_3F_2 \left(\theta, \theta + \tau - n - 1, \theta + \tau + n; \theta + \tau, \theta + \tau; \frac{a-z}{2a} \right) \\ [a > 0; \operatorname{Re}(\theta + \tau) > n + 1; |\arg(z-a)| < \pi]$$

$$19. \int_a^\infty (x-a)^{\alpha-1} (x+z)^{-\theta} P_n \left(\frac{x}{a} \right) dx = \\ = B(\alpha, \theta - \alpha) (a+z)^{\alpha-\theta} {}_3F_2 \left(-n, n+1, \alpha; 1, 1+\alpha-\theta; \frac{a+z}{2a} \right) \\ [a > 0; 0 < \operatorname{Re} \alpha < \operatorname{Re} \theta - n; |\arg(a+z)| < \pi]$$

$$20. \int_a^\infty (x-a)^{\alpha-1} (x+b)^{-\tau} P_n \left(\frac{x}{b} \right) dx = \frac{2^n (1/2)_n}{n! b^n} B(\alpha, \tau - \alpha - n) (a+b)^{\alpha+n-\tau} \times \\ \times {}_3F_2 \left(-n, -n, \tau - \alpha - n; \tau - n, -2n; \frac{2b}{a+b} \right) \\ [a > b > 0; 0 < \operatorname{Re} \alpha < \operatorname{Re} \tau - n]$$

$$21. \int_a^\infty (x-a)^{\alpha-1} (x-b)^{-\tau} P_n \left(\frac{x}{b} \right) dx = \\ = B(\alpha, \tau - \alpha) (a-b)^{\alpha-\tau} {}_3F_2 \left(-n, n+1, 1-\tau; 1, 1+\alpha-\tau; \frac{b-a}{2b} \right) \\ [a > b > 0; 0 < \operatorname{Re} \alpha < \operatorname{Re} \tau - n]$$

$$22. \int_a^\infty \frac{(x + \sqrt{x^2 - a^2})^{-\rho}}{\sqrt{x^2 - a^2}} P_{2n+\varepsilon} \left(\frac{x}{a} \right) dx = \frac{a^{-\rho}}{\rho} \left(\frac{\rho^2}{\rho^2 - 1} \right)^n \prod_{k=1}^n \frac{\rho^2 - (2k + \varepsilon - 1)^2}{\rho^2 - (2k + \varepsilon)^2} \\ [\varepsilon = 0 \text{ или } 1; a > 0; \operatorname{Re} \rho > 2n + \varepsilon]$$

$$23. \int_0^a x^{\alpha-1} P_n \left(\frac{x+a}{2\sqrt{ax}} \right) dx = 2^{2n+1} (2\alpha - n)_{n+1} a^\alpha \\ [a > 0; \operatorname{Re} \alpha > n/2]$$

$$24. \int_0^\infty x^{\alpha-1} (x+a)^{-(n+1)/2} P_n \left(\frac{2a+x}{2\sqrt{a(a+x)}} \right) dx = \\ = \frac{a^{\alpha-(n+1)/2}}{n! \sqrt{\pi}} (1-\alpha)_n \Gamma(\alpha) \Gamma\left(\frac{1}{2} - \alpha\right) \\ [0 < \operatorname{Re} \alpha < 1/2, |\arg a| < \pi]$$

2.17.4. Интегралы от $(x \pm a)^\alpha (b \pm x)^\beta (d \pm x)^\gamma P_n(cx)$.

$$1. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\beta-1} (z-x)^{n-\alpha-\beta+1} P_n \left(\frac{x}{a} \right) dx = \\ = B(\beta, \alpha - n - 1) (2a)^{\alpha+\beta-1} (z+a)^{-\beta} (z-a)^{n-\alpha+1} \times \\ \times {}_3F_2 \left(n+1, n+1, \beta; 1, 2+n-\alpha; \frac{z-a}{z+a} \right) + \\ + (-1)^n \left[\frac{(1-\alpha)_n}{n!} \right]^2 \frac{\pi}{\sin \alpha \pi} (2a)^{\alpha+\beta-1} (a+z)^{n-\alpha-\beta+1} \times \\ \times {}_3F_2 \left(\alpha, \alpha, \alpha+\beta-n-1; \alpha-n, \alpha-n; \frac{z-a}{z+a} \right) \\ [a, \operatorname{Re} \alpha, \operatorname{Re} \beta > 0; |\arg(z^2 - a^2)| < \pi]$$

$$2. \int_{-a}^a \frac{(x+a)^{n-\beta+1}}{x-y} (a-x)^{\beta-1} P_n \left(\frac{x}{a} \right) dx = \left[\frac{(2-\beta)_n}{n!} \right]^2 \frac{2^{n+1}\pi}{\sin \beta\pi} \times \\ \times a^{\beta+n-1} (a+y)^{1-\beta} {}_3F_2 \left(1, 2+n-\beta, 2+n-\beta; 2-\beta, 2-\beta; \frac{y-a}{y+a} \right) + \\ + \pi \operatorname{ctg} \beta\pi (a+y)^{n-\beta+1} (a-y)^{\beta-1} P_n \left(\frac{y}{a} \right) \quad [-a < y < a; 0 < \operatorname{Re} \beta < n+2].$$

$$3. \int_{\pm a}^b (x \mp a)^{\alpha-1} (b-x)^{\beta-1} (a \pm x)^{n-\alpha-\beta+1} P_n \left(\frac{x}{a} \right) dx = \\ = (\pm 1)^n B(\alpha, \beta) (2a)^{n-\beta+1} (a \pm b)^{-\alpha} (b \mp a)^{\alpha+\beta-1} \times \\ \times {}_3F_2 \left(n+1, n+1, \alpha; 1, \alpha+\beta; \frac{b \mp a}{b \pm a} \right) \quad [\operatorname{Re} \alpha, \operatorname{Re} \beta > 0, \left\{ \begin{matrix} b > a \\ -a < b < a \end{matrix} \right\}].$$

$$4. \int_a^b (x-a)^{\alpha-1} (b-x)^{\beta-1} (x+b)^{n-\alpha-\beta+1} P_n \left(\frac{x}{b} \right) dx = \\ = B(\alpha, \beta) (2b)^{n-\alpha+1} (a+b)^{-\beta} (b-a)^{\alpha+\beta-1} {}_3F_2 \left(n+1, n+1, \beta; 1, \alpha+\beta; \frac{a-b}{a+b} \right) \\ [b > a > 0; \operatorname{Re} \alpha, \operatorname{Re} \beta > 0].$$

2.17.5. Интегралы от $A(x) e^{-px \pm m} P_n(cx)$.

$$1. \int_{-a}^a e^{px} P_n \left(\frac{x}{a} \right) dx = \sqrt{\frac{2\pi a}{p}} e^{n\pi i/2} J_{n+1/2}(ap) \quad [a > 0].$$

$$2. \int_{-a}^a e^{-px} P_n \left(\frac{x}{a} \right) dx = (-1)^n \sqrt{\frac{2\pi a}{p}} I_{n+1/2}(ap) \quad [a > 0].$$

$$3. \int_{-a}^{\infty} e^{-px} P_n \left(\frac{x}{a} \right) dx = (-1)^n \left(\frac{p}{a} \right)^n \left(\frac{1}{p} \frac{d}{dp} \right)^n \left(\frac{e^{ap}}{p} \right) = \\ = \frac{(-1)^n}{p} e^{ap} {}_2F_0 \left(-n, n+1; \frac{1}{2ap} \right) \quad [a, \operatorname{Re} p > 0].$$

$$4. \int_a^{\infty} e^{-px} P_n \left(\frac{x}{a} \right) dx = \sqrt{\frac{2a}{\pi p}} K_{n+1/2}(ap) \quad [a, \operatorname{Re} p > 0].$$

$$5. \int_0^{\infty} x^{\alpha-1} e^{-px} P_{n+s}(cx) dx = \frac{(-1)^n (2c)^s (1/2)_{n+s} \Gamma(\alpha+s)}{n! p^{\alpha+s}} \times \\ \times {}_4F_1 \left(-n, n+s+\frac{1}{2}, \frac{\alpha+1}{2}, \frac{\alpha}{2}+s; s+\frac{1}{2}; \frac{4c^2}{p^2} \right) \\ [s=0 \text{ или } 1; \operatorname{Re} p > 0; \operatorname{Re} \alpha > -s].$$

$$6. \int_{-a}^a (x+a)^{\alpha-1} e^{-px} P_n \left(\frac{x}{a} \right) dx = \\ = (-1)^n \frac{(1-\alpha)_n}{(\alpha)_{n+1}} (2a)^{\alpha} e^{ap} {}_2F_2(\alpha, \alpha; \alpha+n+1, \alpha-n; -2ap) \\ [a, \operatorname{Re} \alpha > 0].$$

$$7. \int_a^{\infty} (x-a)^{\alpha-1} e^{-px} P_n \left(\frac{x}{a} \right) dx =$$

$$= \frac{2^n (1/2)_n \Gamma(\alpha+n)}{n! a^n p^{\alpha+n}} e^{-ap} {}_2F_2(-n, -n; 1-\alpha-n, -2n; 2ap)$$

[a, Re p, Re α > 0]

$$8. \int_a^{\infty} (x+a)^{-\tau} e^{-px} P_n \left(\frac{x}{a} \right) dx = \frac{(-1)^{n+1} (\tau)_n (2a)^{1-\tau}}{(1-\tau)_{n+1}} e^{ap} \times$$

$$\times {}_2F_2(1-\tau, 1-\tau; 2+n-\tau, 1-\tau-n; -2ap) +$$

$$+ \frac{2^{2n} \Gamma(1+n-\tau) (1/2)_n}{n! (2a)^n} p^{\tau-n-1} e^{ap} {}_2F_2(-n, -n; -2n, \tau-n; -2ap)$$

[a, Re p > 0]

$$9. \int_0^a x^{\alpha-1} e^{-px^2} P_{2n+\varepsilon} \left(\frac{x}{a} \right) dx = \frac{(-1)^n a^{\alpha} ((1-\alpha+\varepsilon)/2)_n}{2 ((\alpha+\varepsilon)/2)_n} \times$$

$$\times {}_2F_2 \left(\frac{\alpha}{2}, \frac{\alpha+1}{2}; \frac{1+\alpha-\varepsilon}{2}-n, \frac{\alpha+\varepsilon}{2}+n+1; -a^2 p \right)$$

[ε = 0 или 1; a > 0; Re α > -ε]

$$10. \int_0^{\infty} x^{\alpha-1} e^{-px^2} P_{2n+\varepsilon}(cx) dx = \frac{(-1)^n 2^{\varepsilon-1} c^{\varepsilon} (1/2)_{n+\varepsilon} \Gamma \left(\frac{\alpha+\varepsilon}{2} \right)}{p^{(\alpha+\varepsilon)/2} n!} \times$$

$$\times {}_2F_1 \left(-n, \frac{1}{2}+n+\varepsilon, \frac{\alpha+\varepsilon}{2}; \varepsilon+\frac{1}{2}; \frac{c^2}{p} \right)$$

[ε = 0 или 1; Re p > 0; Re α > -ε]

$$11. \int_0^{\infty} x^{\varepsilon} e^{-px^2} P_{2n+\varepsilon}(cx) dx = \frac{(-1)^n \sqrt{\pi} c^{\varepsilon} (1/2)_{n+\varepsilon}}{2 p^{\varepsilon+1/2} n!} {}_2F_0 \left(-n, n+\varepsilon+\frac{1}{2}; \frac{c^2}{p} \right)$$

[ε = 0 или 1; Re p > 0]

$$12. \int_0^{\infty} x^{\alpha-1} e^{-px^2} P_{2n+\varepsilon} \left(\frac{x}{a} \right) dx = \frac{(-1)^{n+1} a^{\alpha} ((1-\alpha+\varepsilon)/2)_n}{2 ((\alpha+\varepsilon)/2)_{n+1}} +$$

$$\times {}_2F_2 \left(\frac{\alpha}{2}, \frac{\alpha+1}{2}; \frac{1+\alpha-\varepsilon}{2}-n, \frac{\alpha+\varepsilon}{2}+n+1; -a^2 p \right) +$$

$$+ \frac{2^{2n+\varepsilon-1} (1/2)_{2n+\varepsilon}}{(2n+\varepsilon)! a^{2n+\varepsilon} p^{(\alpha+\varepsilon)/2+n}} \Gamma \left(\frac{\alpha+\varepsilon}{2}+n \right) \times$$

$$\times {}_2F_2 \left(-n, \frac{1}{2}-\varepsilon-n; \frac{1}{2}-\varepsilon-2n, 1-n-\frac{\alpha+\varepsilon}{2}; -a^2 p \right)$$

[ε = 0 или 1; a, Re p > 0]

$$13. \int_0^a x^{\alpha-1} e^{-px^{-2}} P_{2n+\varepsilon} \left(\frac{x}{a} \right) dx = \frac{(-1)^n a^{\alpha} ((1-\alpha+\varepsilon)/2)_n}{2 ((\alpha+\varepsilon)/2)_{n+1}} \times$$

$$\times {}_2F_2 \left(-\frac{\alpha+\varepsilon}{2}-n, \frac{1-\alpha+\varepsilon}{2}+n; \frac{1-\alpha}{2}, 1-\frac{\alpha}{2}; -\frac{p}{a^2} \right) +$$

$$+ \frac{(-1)^n 2^{\varepsilon-1}}{n! a^{\varepsilon}} p^{(\alpha+\varepsilon)/2} \left(\frac{1}{2} \right)_{n+\varepsilon} \Gamma \left(-\frac{\alpha+\varepsilon}{2} \right) \times$$

$$\times {}_2F_2 \left(-n, \frac{1}{2}+\varepsilon+n; \frac{1}{2}+\varepsilon, 1+\frac{\alpha+\varepsilon}{2}; -\frac{p}{a^2} \right)$$

[ε = 0 или 1, a, Re p > 0]

$$14. \int_0^a x^{-n-2} e^{-px^{-2}} P_n \left(\frac{x}{a} \right) dx = \frac{e^{-pa^{-2}}}{2^{n+1} p^{n/2+1}} H_n \left(\sqrt{p} \frac{a}{a} \right) \quad [a, \operatorname{Re} p > 0].$$

$$15. \int_a^\infty x^{\alpha-1} e^{-px^{-2}} P_{2n+\varepsilon} \left(\frac{x}{a} \right) dx = \frac{(-1)^{n+1} a^\alpha ((1-\alpha+\varepsilon)/2)_n}{2 ((\alpha+\varepsilon)/2)_{n+1}} \times \\ \times {}_2F_2 \left(-\frac{\alpha+\varepsilon}{2} - n, \frac{1-\alpha+\varepsilon}{2} + n; \frac{1-\alpha}{2}, 1 - \frac{\alpha}{2}; -\frac{p}{a^2} \right) \\ [\varepsilon = 0 \text{ или } 1; a > 0; \operatorname{Re} \alpha < -2n - \varepsilon].$$

2.17.6. Интегралы от $A(x) e^{\Phi(x)} P_n(cx)$.

$$1. \int_{-a}^a (x+a)^{\alpha-1} \exp \left(-p \frac{(a-x)^\varepsilon}{a+x} \right) P_n \left(\frac{x}{a} \right) dx = \\ = \frac{(-1)^n (1-\alpha)_n}{(\alpha)_{n+1}} (2a)^\alpha e^{\varepsilon p} {}_2F_2 \left(-\alpha - n, n - \alpha + 1; 1 - \alpha, 1 - \alpha; -\frac{p}{(2a)^{1-\varepsilon}} \right) + \\ + (-1)^n \Gamma(-\alpha) (2a)^{\alpha\varepsilon} p^\alpha e^{\varepsilon p} {}_2F_2 \left(-n, n + 1; 1, \alpha + 1; -\frac{p}{(2a)^{1-\varepsilon}} \right) \\ [\varepsilon = 0 \text{ или } 1; a, \operatorname{Re} p > 0].$$

$$2. \int_{-a}^a (x+a)^{-n-2} \exp \left(-p \frac{(a-x)^\varepsilon}{a+x} \right) P_n \left(\frac{x}{a} \right) dx = \\ = \frac{(-1)^n n!}{(2a)^{(n+1)\varepsilon} p^{n+1}} \exp \left(\frac{\varepsilon p - p}{2a} \right) L_n \left(\frac{p}{(2a)^{1-\varepsilon}} \right) \quad [\varepsilon = 0 \text{ или } 1; a, \operatorname{Re} p > 0].$$

$$3. \int_{-a}^a (x+a)^{-n-\beta-1} (a-x)^{\beta-1} \exp \left(-p \frac{(a-x)^\varepsilon}{a+x} \right) P_n \left(\frac{x}{a} \right) dx = \\ = (-1)^n \Gamma(\beta + n) (2a)^{\beta-\beta\varepsilon-n\varepsilon-1} p^{-\beta-n} \exp \left(\frac{\varepsilon p - p}{2a} \right) \times \\ \times {}_2F_2 \left(-n, -n; 1 - \beta - n, 1; \frac{p}{(2a)^{1-\varepsilon}} \right) \quad [\varepsilon = 0 \text{ или } 1; a, \operatorname{Re} p, \operatorname{Re} \beta > 0].$$

$$4. \int_{-a}^a (x+a)^{n-\beta} (a-x)^{\beta-1} \exp \left(-p \frac{(a-x)^\varepsilon}{a+x} \right) P_n \left(\frac{x}{a} \right) dx = \\ = \left[\frac{(1-\beta)_n}{n!} \right]^2 \frac{\pi (2a)^n}{\sin \beta \pi} \exp \left(\frac{\varepsilon p - p}{2a} \right) {}_2F_2 \left(\beta, \beta; \beta - n, \beta - n; \frac{p}{(2a)^{1-\varepsilon}} \right) + \\ + (-1)^n \Gamma(\beta - n - 1) p^{n-\beta+1} (2a)^{\beta-\beta\varepsilon+n\varepsilon-1} \exp \left(\frac{\varepsilon p - p}{2a} \right) \times \\ \times {}_2F_2 \left(n+1, n+1; 1, n - \beta + 2; \frac{p}{(2a)^{1-\varepsilon}} \right) \quad [\varepsilon = 0 \text{ или } 1; a, \operatorname{Re} p, \operatorname{Re} \beta > 0].$$

$$5. \int_{-a}^a (x+a)^{n-\beta} (a-x)^{\beta-1} \exp \left(-p \frac{(a+x)^\varepsilon}{a-x} \right) P_n \left(\frac{x}{a} \right) dx = \left[\frac{(1-\beta)_n}{n!} \right]^2 \times \\ \times \frac{\pi}{\sin \beta \pi} (2a)^n \exp \left(\frac{\varepsilon p - p}{2a} \right) {}_2F_2 \left(n - \beta + 1, n - \beta + 1; 1 - \beta, 1 - \beta; \frac{p}{(2a)^{1-\varepsilon}} \right) + \\ + \Gamma(-\beta) (2a)^{n-\beta+\beta\varepsilon} p^\beta \exp \left(\frac{\varepsilon p - p}{2a} \right) {}_2F_2 \left(n+1, n+1; 1, \beta + 1; \frac{p}{(2a)^{1-\varepsilon}} \right) \\ [\varepsilon = 0 \text{ или } 1; a, \operatorname{Re} p > 0; \operatorname{Re} \beta < n + 1].$$

$$6. \int_{-a}^a (x+a)^{-\beta-n-1} (a-x)^{\beta-1} \exp \left(-p \frac{(a+x)^s}{a-x} \right) P_n \left(\frac{x}{a} \right) dx =$$

$$= \frac{\Gamma(-\beta) p^\beta}{(2a)^{\beta-\beta s+n+1}} \exp \left(\frac{ep-p}{2a} \right) {}_2F_2 \left(-n, -n; 1, \beta+1; \frac{\rho}{(2a)^{1-s}} \right)$$

[$s=0$ или 1 ; n , $\operatorname{Re} p > 0$; $\operatorname{Re} \beta < -n$]

$$7. \int_a^\infty (x-a)^{\alpha-1} \exp \left(-p \frac{(x+a)^s}{x-a} \right) P_n \left(\frac{x}{a} \right) dx =$$

$$= \Gamma(-\alpha) (2a)^{\alpha s} p^\alpha e^{-ep} {}_2F_2 \left(-n, n+1; 1, \alpha+1; \frac{\rho}{(2a)^{1-s}} \right)$$

[$s=0$ или 1 ; a , $\operatorname{Re} p > 0$; $\operatorname{Re} \alpha < -n$]

$$8. \int_a^\infty (x+a)^{-\tau} \exp \left(-p \frac{(a-x)^s}{a+x} \right) P_n \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^{n+1} (\tau)_n}{(1-\tau)_{n+1}} (2a)^{1-\tau} e^{ep} {}_2F_2 \left(\tau-n-1, \tau+n; \tau, \tau; -\frac{\rho}{(2a)^{1-s}} \right)$$

[$s=0$ или 1 ; $a > 0$; $\operatorname{Re} \tau > n+1$]

$$9. \int_{-a}^a (x+a)^{\alpha-1} e^{-p\sqrt{x+a}} P_n \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^n (1-\alpha)_n}{(\alpha)_{n+1}} (2a)^\alpha {}_2F_3 \left(\alpha, \alpha; \alpha+n+1, \alpha-n, \frac{1}{2}; \frac{ap^2}{2} \right) -$$

$$- \frac{(-1)^n (1/2-\alpha)_n}{(\alpha+1/2)_{n+1}} (2a)^{\alpha+1/2} {}_2F_3 \left(\alpha+\frac{1}{2}, \alpha+\frac{1}{2}; \frac{3}{2}, \alpha+n+\frac{3}{2}, \alpha-n+\frac{1}{2}; \frac{ap^2}{2} \right)$$

[a , $\operatorname{Re} \alpha > 0$]

$$10. \int_{-a}^a \frac{1}{\sqrt{x+a}} e^{-p\sqrt{x+a}} P_n \left(\frac{x}{a} \right) dx =$$

$$= (-1)^n 2\sqrt{2a} I_{n+1/2} \left(p \sqrt{\frac{a}{2}} \right) K_{n+1/2} \left(p \sqrt{\frac{a}{2}} \right)$$

[$a > 0$]

$$11. \int_{-a}^a (x+a)^{\alpha-1} \exp \left(-\frac{p}{\sqrt{x+a}} \right) P_n \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^n (1-\alpha)_n}{(\alpha)_{n+1}} (2a)^\alpha {}_2F_3 \left(-\alpha-n, 1+n-\alpha; 1-\alpha, 1-\alpha, \frac{1}{2}; \frac{p^2}{8a} \right) +$$

$$+ (-1)^n 2\Gamma(-2\alpha) p^{2\alpha} {}_2F_3 \left(-n, 1+n; 1, \alpha+1, \alpha+\frac{1}{2}; \frac{p^2}{8a} \right) -$$

$$- \frac{(-1)^n (2-\alpha)_n}{(\alpha-1/2)_{n+1}} (2a)^{\alpha-1/2} {}_2F_3 \left(\frac{1}{2}-\alpha-n, \frac{3}{2}+n-\alpha; \frac{3}{2}-\alpha, \frac{3}{2}-\alpha, \frac{3}{2}; \frac{p^2}{8a} \right)$$

[a , $\operatorname{Re} p > 0$]

$$12. \int_{-a}^a (x+a)^{-\beta-n-1} (a-x)^{\beta-1} \exp \left(-p \sqrt{\frac{a-x}{a+x}} \right) P_n \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^n \Gamma(2\beta+2n)}{2^n a^{n+1} p^{2\beta+2n}} {}_2F_2 \left(-n, -n; 1, 1-\beta-n; \frac{1}{2}-\beta-n; -\frac{p^2}{4} \right)$$

[a , $\operatorname{Re} p$, $\operatorname{Re} \beta > 0$]

$$\begin{aligned}
13. \int_{-a}^a (x+a)^{n-\beta} (a-x)^{\beta-1} \exp\left(-p \sqrt{\frac{a-x}{a+x}}\right) P_n\left(\frac{x}{a}\right) dx = \\
= (-1)^n 2^{n+1} \Gamma(2\beta - 2n - 2) a^n p^{2(n-\beta+1)} \times \\
\times {}_2F_3\left(n+1, n+1; 1, n-\beta+2, \frac{3}{2} + n - \beta; -\frac{p^2}{4}\right) + \\
+ \left[\frac{(1-\beta)_n}{n!}\right]^2 \frac{\pi (2a)^n}{\sin \beta\pi} {}_2F_3\left(\beta, \beta; \beta-n, \beta-n, \frac{1}{2}; -\frac{p^2}{4}\right) - \\
- \left[\frac{(1/2-\beta)_n}{n!}\right]^2 \frac{\pi (2a)^n p}{\cos \beta\pi} {}_2F_3\left(\beta + \frac{1}{2}, \beta + \frac{1}{2}; \frac{3}{2}, \frac{1}{2} + \beta - n, \frac{1}{2} + \beta - n; -\frac{p^2}{4}\right) \\
[a, \operatorname{Re} p, \operatorname{Re} \beta > 0].
\end{aligned}$$

$$\begin{aligned}
14. \int_{-a}^a (x+a)^{-\beta-n-1} (a-x)^{\beta-1} \exp\left(-p \sqrt{\frac{a+x}{a-x}}\right) P_n\left(\frac{x}{a}\right) dx = \\
= \frac{p^{2\beta} \Gamma(-2\beta)}{2n a^{n+1}} {}_2F_3\left(-n, -n; 1, \beta + \frac{1}{2}, \beta + 1; -\frac{p^2}{4}\right) \\
[a, \operatorname{Re} p > 0; \operatorname{Re} \beta < -n].
\end{aligned}$$

$$\begin{aligned}
15. \int_{-a}^a (x+a)^{n-\beta} (a-x)^{\beta-1} \exp\left(-p \sqrt{\frac{a+x}{a-x}}\right) P_n\left(\frac{x}{a}\right) dx = \\
= 2^{n+1} \Gamma(-\beta) a^n p^{2\beta} {}_2F_3\left(n+1, n+1; \beta+1, \beta + \frac{1}{2}, 1; -\frac{p^2}{4}\right) + \\
+ \left[\frac{(1-\beta)_n}{n!}\right]^2 \frac{\pi (2a)^n}{\sin \beta\pi} {}_2F_3\left(1+n-\beta, 1+n-\beta; 1-\beta, 1-\beta, \frac{1}{2}; -\frac{p^2}{4}\right) + \\
+ \left[\frac{(3/2-\beta)_n}{n!}\right]^2 \frac{\pi (2a)^n p}{\cos \beta\pi} {}_2F_3\left(\frac{3}{2} + n - \beta, \frac{3}{2} + n - \beta; \frac{3}{2}, \frac{1}{2} - \beta, \frac{3}{2} - \beta; -\frac{p^2}{4}\right) \\
[a, \operatorname{Re} p > 0; \operatorname{Re} \beta < n+1].
\end{aligned}$$

$$\begin{aligned}
16. \int_a^\infty (x-a)^{\alpha-1} e^{-p \sqrt{x-a}} P_n\left(\frac{x}{a}\right) dx = \\
= \frac{2^{n+1} \Gamma(2\alpha+2n) (1/2)_n}{n! a^n p^{2\alpha+2n}} {}_2F_3\left(-n, -n; -2n, 1-\alpha-n, \frac{1}{2} - \alpha - n; -\frac{ap^2}{2}\right) \\
[a, \operatorname{Re} p, \operatorname{Re} \alpha > 0].
\end{aligned}$$

$$\begin{aligned}
17. \int_a^\infty (x-a)^{\alpha-1} \exp\left(-\frac{p}{\sqrt{x-a}}\right) P_n\left(\frac{x}{a}\right) dx = \\
= 2 \Gamma(-2\alpha) p^{2\alpha} {}_2F_3\left(-n, n+1; 1, \alpha+1, \alpha + \frac{1}{2}; -\frac{p^2}{8a}\right) \\
[a, \operatorname{Re} p > 0; \operatorname{Re} \alpha < -n].
\end{aligned}$$

$$\begin{aligned}
18. \int_a^\infty (x+a)^{-\tau} e^{-p \sqrt{x+a}} P_n\left(\frac{x}{a}\right) dx = \\
= \frac{(-1)^{n+1} (\tau)_n}{(1-\tau)_{n+1}} (2a)^{1-\tau} {}_2F_3\left(1-\tau, 1-\tau; 1-\tau-n, 2+n-\tau, \frac{1}{2}; \frac{ap^2}{2}\right) + \\
+ \frac{2^{n+1} (1/2)_n (2+2n-2\tau)}{n! a^n} p^{2\tau-2n-2} {}_2F_3\left(-n, -n; -2n, \tau-n, \tau-n-\frac{1}{2}; \frac{ap^2}{2}\right) + \\
+ \frac{(-1)^n (3/2-\tau)_n}{(3/2-\tau)_{n+1}} (2a)^{3/2-\tau} p {}_2F_3\left(\frac{3}{2}-\tau, \frac{3}{2}-\tau; \frac{3}{2}, \frac{5}{2}+n-\tau, \frac{3}{2}-\tau-n; \frac{ap^2}{2}\right) \\
[a, \operatorname{Re} p > 0].
\end{aligned}$$

$$\begin{aligned}
 19. \int_a^\infty (x+a)^{-\tau} \exp\left(-\frac{p}{\sqrt{x+a}}\right) P_n\left(\frac{x}{a}\right) dx = \\
 = \frac{(-1)^{n+1} (\tau)_n}{(1-\tau)_{n+1}} (2a)^{1-\tau} {}_2F_3\left(\tau-n-1, \tau+n; \frac{1}{2}, \tau, \tau; \frac{p^2}{8a}\right) + \\
 + \frac{(-1)^n (\tau+1/2)_n}{(1/2-\tau)_{n+1}} (2a)^{1/2-\tau} {}_2F_3\left(\tau-n-\frac{1}{2}, \tau+n+\frac{1}{2}; \frac{3}{2}, \tau+\frac{1}{2}, \tau+\frac{1}{2}; \frac{p^2}{8a}\right) \\
 (a > 0; \operatorname{Re} \tau > n+1).
 \end{aligned}$$

2.17.7. Интегралы, содержащие $\begin{Bmatrix} \sin \varphi(x) \\ \cos \varphi(x) \end{Bmatrix} P_n(\chi(x))$.

Обозначение: $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$.

$$1. \int_0^1 \begin{Bmatrix} \sin bx P_{2n+1}(x/a) \\ \cos bx P_{2n}(x/a) \end{Bmatrix} dx = (-1)^n \sqrt{\frac{\pi a}{2b}} J_{2n+\delta+1/2}(ab) \quad (a > 0).$$

$$\begin{aligned}
 2. \int_0^1 x^{\alpha-1} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} P_{2n+\delta}\left(\frac{x}{a}\right) dx = \frac{(-1)^n ((1-\alpha-\delta+\varepsilon)/2)_n}{2 ((\alpha+\delta+\varepsilon)/2)_{n+1}} a^{\alpha+\delta} b^\delta \times \\
 \times {}_2F_3\left(\frac{\alpha}{2}+\delta, \frac{\alpha+1}{2}; \frac{1+\alpha+\delta-\varepsilon}{2}-n, \delta+\frac{1}{2}, \frac{\alpha+\delta+\varepsilon}{2}+n+1; -\frac{a^2 b^2}{4}\right) \\
 (\varepsilon=0 \text{ или } 1; a > 0; \operatorname{Re} \alpha > -\delta-\varepsilon).
 \end{aligned}$$

$$\begin{aligned}
 3. \int_0^\infty x^{\alpha-1} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} P_{2n+\delta}\left(\frac{x}{a}\right) dx = \frac{(-1)^{n+1} ((1-\alpha-\delta+\varepsilon)/2)_n}{2 ((\alpha+\delta+\varepsilon)/2)_n} a^{\alpha+\delta} b^\delta \times \\
 \times {}_2F_3\left(\frac{\alpha}{2}+\delta, \frac{\alpha+1}{2}; \frac{1+\alpha+\delta-\varepsilon}{2}-n, \frac{\alpha+\delta+\varepsilon}{2}+n+1, \delta+\frac{1}{2}; -\frac{a^2 b^2}{4}\right) \pm \\
 \pm \frac{(-1)^n 2^{2n+\varepsilon} (1/2)_{2n+\varepsilon}! (\alpha+\varepsilon+2n)! (\cos((\alpha+\varepsilon)\pi/2)}{((2n+\varepsilon)!) x^{2n+\varepsilon} b^{\alpha+\varepsilon+2n}} \begin{Bmatrix} \cos((\alpha+\varepsilon)\pi/2) \\ \sin((\alpha+\varepsilon)\pi/2) \end{Bmatrix} \times \\
 \times {}_2F_3\left(-n, \frac{1}{2}-\varepsilon-n; \frac{1}{2}-\varepsilon-2n, 1-\frac{\alpha}{2}-\varepsilon-n, \frac{1-\alpha}{2}-n; -\frac{a^2 b^2}{4}\right) \\
 (\varepsilon=0 \text{ или } 1; a, b > 0; \operatorname{Re} \alpha < 1-2n-\varepsilon).
 \end{aligned}$$

$$\begin{aligned}
 4. \int_{-a}^a (x+a)^{\alpha-1} \begin{Bmatrix} \sin(b\sqrt{x+a}) \\ \cos(b\sqrt{x+a}) \end{Bmatrix} P_n\left(\frac{x}{a}\right) dx = \frac{(-1)^n (1-\alpha-\delta/2)_n}{(\alpha+\delta/2)_{n+1}} (2a)^{\alpha+\delta/2} b^\delta \times \\
 \times {}_2F_3\left(\alpha+\frac{\delta}{2}, \alpha+\frac{\delta}{2}; 1+\alpha+n+\frac{\delta}{2}, \frac{\delta}{2}+\alpha-n, \delta+\frac{1}{2}; -\frac{ab^2}{2}\right) \\
 (a > 0; \operatorname{Re} \alpha > -\delta/2).
 \end{aligned}$$

$$\begin{aligned}
 5. \int_{-a}^a \frac{1}{\sqrt{x+a}} \begin{Bmatrix} \sin(b\sqrt{x+a}) \\ \cos(b\sqrt{x+a}) \end{Bmatrix} P_n\left(\frac{x}{a}\right) dx = \\
 = (-1)^n \pi \sqrt{2a} J_{n+1/2}\left(b\sqrt{\frac{a}{2}}\right) J_{\pm n \pm 1/2}\left(b\sqrt{\frac{a}{2}}\right) \quad (a > 0).
 \end{aligned}$$

$$\begin{aligned}
 6. \int_{-a}^a \frac{1}{\sqrt{x+z}} \begin{Bmatrix} \sin(b\sqrt{x+z}) \\ \cos(b\sqrt{x+z}) \end{Bmatrix} P_n\left(\frac{x}{a}\right) dx = \\
 = (-1)^n \pi \sqrt{2a} J_{n+1/2}(by_-) \begin{Bmatrix} J_{n+1/2}(by_+) \\ y_{n+1/2}(by_+) \end{Bmatrix} \\
 [\sqrt{2} y_{\pm} = (z \pm \sqrt{z^2 - a^2})^{1/2}, z > a > 0].
 \end{aligned}$$

$$7. \int_{-a}^a (x+a)^{\alpha-1} \left\{ \frac{\sin(b/\sqrt{x+a})}{\cos(b/\sqrt{x+a})} \right\} P_n \left(\frac{x}{a} \right) dx = \frac{(-1)^n (1+\delta/2-\alpha)_n}{(\alpha-\delta/2)_{n+1}} (2a)^{\alpha-\delta/2} b^\delta \times \\ \times {}_2F_3 \left(\frac{\delta}{2}-\alpha-n, 1+n+\frac{\delta}{2}-\alpha; 1+\frac{\delta}{2}-\alpha, 1+\frac{\delta}{2}-\alpha, \delta+\frac{1}{2}; -\frac{b^2}{8a} \right) + \\ + (-1)^{n+\delta} 2^{2\alpha} \left\{ \frac{\sin \alpha\pi}{\cos \alpha\pi} \right\} {}_2F_3 \left(-n, n+1; 1, \alpha+1, \alpha+\frac{1}{2}; -\frac{b^2}{8a} \right) \\ [a, b > 0; \operatorname{Re} \alpha > -1/2]$$

$$8. \int_{-a}^a (x+a)^{n-\beta} (a-x)^{\beta-1} \left\{ \frac{\sin(b\sqrt{a-x}/\sqrt{a+x})}{\cos(b\sqrt{a-x}/\sqrt{a+x})} \right\} P_n \left(\frac{x}{a} \right) dx = \\ = (-2)^{n+1} \Gamma(2\beta-2n-2) a^n b^{2+2n-2\beta} \left\{ \frac{\sin \beta\pi}{\cos \beta\pi} \right\} {}_2F_3 \left(n+1, n+1; \right. \\ \left. 1, 2+n-\beta, \frac{3}{2}+n-\beta; \frac{b^2}{4} \right) + \pi \left[\frac{(1-\beta-\delta/2)_n}{n!} \right]^2 \left\{ \frac{\sec \beta\pi}{\operatorname{cosec} \beta\pi} \right\} (2a)^n b^\delta \times \\ \times {}_2F_3 \left(\beta+\frac{\delta}{2}, \beta+\frac{\delta}{2}; \frac{\delta}{2}+\beta-n, \frac{\delta}{2}+\beta-n, \delta+\frac{1}{2}; \frac{b^2}{4} \right) \\ [a, b > 0; -\delta/2 < \operatorname{Re} \beta < n+3/2]$$

$$9. \int_a^\infty (x+a)^{-\tau} \left\{ \frac{\sin(b\sqrt{x+a})}{\cos(b\sqrt{x+a})} \right\} P_n \left(\frac{x}{a} \right) dx = \frac{(-1)^{n+1} (\tau-\delta/2)_n}{(1+\delta/2-\tau)_{n+1}} (2a)^{1+\delta/2-\tau} b^\delta \times \\ \times {}_2F_3 \left(1+\frac{\delta}{2}-\tau, 1+\frac{\delta}{2}-\tau; 2+\frac{\delta}{2}+n-\tau, 1+\frac{\delta}{2}-\tau-n, \delta+\frac{1}{2}; -\frac{ab^2}{2} \right) \pm \\ \pm \frac{(-1)^n 2^{n+1} (1/2)_n}{n! a^n} \left\{ \frac{\sin \tau\pi}{\cos \tau\pi} \right\} b^{2\tau-2n-2} \Gamma(2n-2\tau+1) \times \\ \times {}_2F_3 \left(-n, -n; -2n, \tau-n, \tau-n-\frac{1}{2}; -\frac{ab^2}{2} \right) \\ [b > 0; \operatorname{Re} \tau > n+1/2]$$

$$10. \int_a^\infty (x+a)^{-\tau} \left\{ \frac{\sin(b/\sqrt{x+a})}{\cos(b/\sqrt{x+a})} \right\} P_n \left(\frac{x}{a} \right) dx = \frac{(-1)^{n+1} (\tau+\delta/2)_n}{(1-\tau-\delta/2)_{n+1}} (2a)^{1-\tau-\delta/2} b^\delta \times \\ \times {}_2F_3 \left(\tau-n+\frac{\delta}{2}-1, \tau+n+\frac{1}{2}; \tau+\frac{\delta}{2}, \tau+\frac{\delta}{2}, \delta+\frac{1}{2}; -\frac{b^2}{8a} \right) \\ [\operatorname{Re} \tau > 1+n-\delta/2]$$

$$11. \int_0^{2\pi} P_{2n}(\cos x) dx = \frac{\pi}{2^{2n-1}} \binom{2n}{n}.$$

$$12. \int_0^{2\pi} \cos x P_{2n+1}(\cos x) dx = \frac{\pi}{2^{2n+1}} \binom{2n}{n} \binom{2n+2}{n+1}.$$

$$13. \int_0^n \sin mx P_n(\cos x) dx = \frac{2(m-n+1)(m-n+3)\dots(m+n-1)}{(m-n)(m-n+2)\dots(m+n)}$$

$$= 0$$

$$[m > n, m+n=1, 3, 5, \dots], \\ [m \leq n \text{ или } m+n=2, 4, 6, \dots]$$

$$14. \int_0^\pi \frac{1}{\sin(x/2)} \sin\left(\frac{2n+1}{2}x\right) P_n(\cos x) dx = \pi \quad [a > 0]$$

$$15. \int_{-\pi/2}^{\pi/2} x \cos x P_n(\sin x) dx = \frac{1 - (-1)^n}{2^{n+2}} \pi \left[\frac{(n-2)!!}{[(n+1)/2]!} \right]^2 \quad [n=1, 2, 3, \dots]$$

$$16. \int_0^\infty e^{-px} P_{2n+\varepsilon} \left(\begin{Bmatrix} \cos bx \\ \operatorname{ch} bx \end{Bmatrix} \right) dx = I_{2n+\varepsilon} \quad \left[\begin{Bmatrix} \operatorname{Re} p > (2n+\varepsilon) |\operatorname{Im} b| \\ \operatorname{Re} p > (2n+\varepsilon) |\operatorname{Re} b| \end{Bmatrix} \right],$$

$$I_{2n+\varepsilon} = \frac{p^{2\varepsilon-1}}{(p^2 \pm b^2)^\varepsilon} \prod_{k=1}^n \frac{p^2 \pm (2k+\varepsilon-1)^2 b^2}{p^2 \pm (2k+\varepsilon)^2 b^2},$$

$$I_0 = \frac{1}{p}, \quad I_1 = \frac{p}{p^2 \pm b^2}, \quad I_2 = \frac{p^2 \pm b^2}{p(p^2 \pm 4b^2)}.$$

2.17.8. Интегралы от $A(x) \ln \varphi(x) P_n(x)$.

$$1. \int_{-a}^a (x+a)^{\alpha-1} \ln \frac{x+a}{2a} P_n \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^{n-1} (1-\alpha)_n (2a)^\alpha}{(\alpha)_{n+1}} \sum_{k=0}^n \frac{1}{k+\alpha} + \sum_{k=0}^{n-1} \frac{1}{k-\alpha+1}$$

[$a, \operatorname{Re} \alpha > 0; \alpha \neq 1$]

$$2. \int_{-a}^a \ln \frac{x+a}{2a} P_n \left(\frac{x}{a} \right) dx = \frac{(-1)^{n-1} 2a}{n(n+1)} \quad [a, n > 0].$$

$$3. \int_{-a}^a (x+a)^{\alpha-1} \left\{ \begin{matrix} \ln(x+z) \\ \ln|x-z| \end{matrix} \right\} P_n \left(\frac{x}{a} \right) dx = \pm \frac{(-1)^n (-\alpha)_n}{(z-a)(\alpha-1)_{n+1}} (2a)^{\alpha+1} \times$$

$$\times {}_4F_3 \left(1, 1, 1+\alpha, 1-\alpha-n; 2, 1+\alpha-n, 2-\alpha; \frac{2a}{a \pm z} \right) +$$

$$+ \frac{(-1)^n (1-\alpha)_n}{(\alpha)_{n+1}} (2a)^\alpha \ln(z \mp a) \quad \left[a, \operatorname{Re} \alpha > 0, \left\{ \begin{matrix} |\arg(z-a)| < \pi \\ z > -a \end{matrix} \right\} \right].$$

$$4. \int_a^\infty (x+a)^{-\tau} \left\{ \begin{matrix} \ln(x+z) \\ \ln|x-z| \end{matrix} \right\} P_n \left(\frac{x}{a} \right) dx = \pm \frac{(-1)^{n+1} (\tau-1)_n (2a)^{\tau-1}}{(z \mp a)(2-\tau)_{n+1}} \times$$

$$\times {}_4F_3 \left(1, 1, 2-\tau, 2-\tau; 2, 2-\tau-n, 3+n-\tau; \frac{2a}{a \pm z} \right) +$$

$$+ \frac{(\mp 1)^{n+1} 2^n \pi (1/2)_n (z \mp a)^{1-\tau+n}}{n! (\tau-n-1) a^n} \left\{ \begin{matrix} \operatorname{cosec} \tau \pi \\ \operatorname{ctg} \tau \pi \end{matrix} \right\} \times$$

$$\times {}_3F_2 \left(-n, -n, \tau-n-1; \tau-n, -2n; \frac{2a}{a \pm z} \right) - \frac{(-1)^n (\tau)_n}{(1-\tau)_{n+1}} (2a)^{1-\tau} \ln(z \mp a)$$

[$a > 0; \operatorname{Re} \tau > n+1, \left\{ \begin{matrix} |\arg(z-a)| < \pi \\ z > -a \end{matrix} \right\}$].

$$5. \int_0^a x^{\alpha-1} \left\{ \begin{matrix} \ln(x^2+z^2) \\ \ln|x^2-z^2| \end{matrix} \right\} P_{2n+\varepsilon} \left(\frac{x}{a} \right) dx = \pm \frac{(-1)^n a^{\alpha+2} ((\varepsilon-\alpha-1)/2)_n}{2z^2 ((\alpha+\varepsilon)/2+1)_{n+1}} \times$$

$$\times {}_4F_3 \left(1, 1, \frac{\alpha}{2}+1, \frac{\alpha+3}{2}; 2, \frac{3+\alpha-\varepsilon}{2}-n, 2+n+\frac{\alpha+\varepsilon}{2}; \mp \frac{a^2}{z^2} \right) +$$

$$+ \frac{(-1)^n a^\alpha ((1+\varepsilon-\alpha)/2)_n}{((\alpha+\varepsilon)/2)_{n+1}} \ln z \quad \left[\varepsilon=0 \text{ или } 1, a>0; \operatorname{Re} \alpha > -\varepsilon, \left\{ \begin{matrix} \operatorname{Re} z > 0 \\ z > 0 \end{matrix} \right\} \right].$$

$$6. \int_a^\infty x^{\alpha-1} \left\{ \frac{\ln(x^2+z^2)}{\ln x^2-z^2} \right\} P_{2n+\varepsilon} \left(\frac{x}{a} \right) dx = \mp \frac{(-1)^n a^{\alpha-2z^2} ((3+\varepsilon-\alpha)/2)_n}{2(1+(\alpha+\varepsilon)/2)_{n+1}} \times$$

$$\times {}_4F_3 \left(1, 1, 1 - \frac{\alpha+\varepsilon}{2} - n, \frac{3+\varepsilon-\alpha}{2} + n; 2, 2 - \frac{\alpha}{2}, \frac{3-\alpha}{2}; \mp \frac{z^2}{a^2} \right) +$$

$$+ \frac{(-1)^n a^\alpha ((1+\varepsilon-\alpha)/2)_n}{((\alpha+\varepsilon)/2)_{n+1}} \left[\sum_{k=0}^n \frac{1}{\alpha+\varepsilon+2k} + \sum_{k=0}^{n-1} \frac{1}{2k-\alpha+\varepsilon+1} - \ln a \right]$$

$$\left[\varepsilon=0 \text{ или } 1; a>0, \operatorname{Re} \alpha < -2n-\varepsilon, \begin{cases} \operatorname{Re} z > 0 \\ z > 0 \end{cases} \right].$$

2.17.9. Интегралы от $A(x) \operatorname{Ei}(a+bx^m) P_n(cx)$.

$$1. \int_{-a}^a (x+a)^{\alpha-1} \operatorname{Ei}(-ab-bx) P_n \left(\frac{x}{a} \right) dx = \frac{(-1)^{n-1} (-\alpha)_n}{(\alpha+1)_{n+1}} (2a)^{\alpha+1} b \times$$

$$\times {}_4F_4(1, 1, \alpha+1, \alpha+1; 2, 2, \alpha-n+1, \alpha+n+2; -2ab) -$$

$$- \frac{(-1)^n (1-\alpha)_n}{(\alpha)_{n+1}} (2a)^\alpha \left[C - \sum_{k=0}^n \frac{1}{k+\alpha} - \sum_{k=0}^{n-1} \frac{1}{k-\alpha+1} + \ln(2ab) \right]$$

$$[a, \operatorname{Re} \alpha > 0].$$

$$2. \int_0^\infty (x+a)^{-\tau} \operatorname{Ei}(-ab-bx) P_n \left(\frac{x}{a} \right) dx = (-1)^n \frac{(\tau-1)_n}{(2-\tau)_{n+1}} (2a)^{2-\tau} b \times$$

$$\times {}_4F_4(1, 1, 2-\tau, 2-\tau; 2, 2, 2-\tau-n, 3+n-\tau; -2ab) +$$

$$+ \frac{n(1/2)_n b^{\tau-n-1} \Gamma(1+n-\tau)}{n! (\tau-n-1) a^n} {}_3F_3(-n, -n, \tau-n-1; \tau-n, \tau-n, -2n; -2ab) +$$

$$+ \frac{(-1)^{n+1} (\tau)_n}{(1-\tau)_{n+1}} (2a)^{1-\tau} \left[C - \sum_{k=0}^n \frac{1}{k-\tau+1} - \sum_{k=0}^{n-1} \frac{1}{k+\tau} + \ln(2ab) \right]$$

$$[a, \operatorname{Re} b > 0].$$

$$3. \int_1^a x^{\alpha-1} \operatorname{Ei}(-bx^2) P_{2n+\varepsilon} \left(\frac{x}{a} \right) dx = \frac{(-1)^{n+1} a^{\alpha+2b} ((\varepsilon-\alpha-1)/2)_n}{2((\alpha+\varepsilon)/2+1)_{n+1}} \times$$

$$\times {}_4F_4 \left(1, 1, \frac{\alpha+3}{2}, \frac{\alpha}{2}+1; 2, 2, 2+\frac{\alpha+\varepsilon}{2}+n, \frac{3+\alpha-\varepsilon}{2}-n; -a^2b \right) +$$

$$+ \frac{(-1)^n a^\alpha ((1-\alpha+\varepsilon)/2)_n}{2((\alpha+\varepsilon)/2)_{n+1}} \left[C - \sum_{k=0}^n \frac{2}{2k+\alpha+\varepsilon} - \sum_{k=0}^{n-1} \frac{2}{2k-\alpha+\varepsilon+1} + \ln(a^2b) \right]$$

$$[\varepsilon=0 \text{ или } 1, a>0, \operatorname{Re} \alpha > -\varepsilon, \arg b_1 < \pi].$$

$$4. \int_a^\infty x^{\alpha-1} \operatorname{Ei}(-bx^2) P_{2n+\varepsilon} \left(\frac{x}{a} \right) dx = \frac{(-1)^n a^{\alpha+2b} ((\varepsilon-\alpha-1)/2)_n}{2((\alpha+\varepsilon)/2+1)_{n+1}} \times$$

$$\times {}_4F_4 \left(1, 1, \frac{\alpha}{2}+1, \frac{\alpha+3}{2}; 2, 2, \frac{3+\alpha-\varepsilon}{2}-n, 2+\frac{\alpha+\varepsilon}{2}+n; -a^2b \right) +$$

$$+ \frac{(-1)^{n+1} a^\alpha ((1-\alpha+\varepsilon)/2)_n}{2((\alpha+\varepsilon)/2)_{n+1}} \left[C - \sum_{k=0}^n \frac{2}{2k+\alpha+\varepsilon} - \sum_{k=0}^{n-1} \frac{2}{2k-\alpha+\varepsilon+1} + \ln(a^2b) \right] -$$

$$- \frac{2^{2n+\varepsilon} (1/2)_{2n+\varepsilon}}{(2n+\varepsilon)! a^{2n+\varepsilon} (\alpha+\varepsilon)/2+n} \Gamma \left(\frac{\alpha+\varepsilon}{2} + n \right) \times$$

$$\times {}_3F_3 \left(-n, \frac{1}{2}-\varepsilon-n, -\frac{\alpha+\varepsilon}{2}-n; \frac{1}{2}-\varepsilon-2n, 1-\frac{\alpha+\varepsilon}{2}-n, 1-\frac{\alpha+\varepsilon}{2}-n; -a^2b \right)$$

$$[\varepsilon=0 \text{ или } 1, a, \operatorname{Re} b > 0].$$

2.17.10. Интегралы от $A(x) \begin{Bmatrix} \text{si}(\varphi(x)) \\ \text{ci}(\varphi(x)) \end{Bmatrix} P_n(cx)$.

$$1. \int_0^a x^{\alpha-1} \begin{Bmatrix} \text{si}(bx) \\ \text{ci}(bx) \end{Bmatrix} P_{2n+\varepsilon}\left(\frac{x}{a}\right) dx = \frac{(-1)^{n+1} a^{\alpha+0+2\delta+2} ((\varepsilon-\alpha-\delta-1)/2)_n}{2^{\varepsilon-\delta} 3^{2\delta} ((\alpha+\delta+\varepsilon)/2+1)_{n+1}} \times$$

$$\times {}_4F_3\left(1, 1+\frac{\delta}{2}, \frac{\alpha}{2}+\delta+1, \frac{3+\alpha}{2}; 2, \frac{\alpha+\delta+\varepsilon}{2}+n+2, 2+\frac{\delta}{2}, \frac{3}{2}+\delta, \frac{3+\alpha+\delta-\varepsilon}{2}-n; -\frac{a^2 b^2}{4}\right) +$$

$$+ \frac{(-1)^n a^{\alpha+\delta} b^\delta ((1-\alpha-\delta+\varepsilon)/2)_n}{2 ((\alpha+\delta+\varepsilon)/2)_n} \{R\} - \frac{\delta (-1)^n \pi a^\alpha ((1-\alpha+\varepsilon)/2)_n}{4 ((\alpha+\varepsilon)/2)_{n+1}},$$

$$R = C - \sum_{k=0}^n \frac{1}{2k+\alpha+\varepsilon} - \sum_{k=0}^{n-1} \frac{1}{2k-\alpha+\varepsilon+1} + \ln(ab)$$

[$\varepsilon=0$ или 1 ; $a>0$; $\text{Re } \alpha > -\varepsilon$].

$$2. \int_a^\infty x^{\alpha-1} \begin{Bmatrix} \text{si}(bx) \\ \text{ci}(bx) \end{Bmatrix} P_{2n+\varepsilon}\left(\frac{x}{a}\right) dx = \frac{(-1)^n a^{\alpha+\delta+2\delta+2} ((\varepsilon-\alpha-\delta-1)/2)_n}{2^{\varepsilon-\delta} 3^{2\delta} ((\alpha+\varepsilon+\delta)/2+1)_{n+1}} \times$$

$$\times {}_4F_3\left(1, 1+\frac{\delta}{2}, \frac{\alpha+\delta}{2}, \frac{\alpha}{2}+\delta+1; 2, \frac{\alpha+\delta+\varepsilon}{2}+n+2, \delta+\frac{3}{2}, 2+\frac{\delta}{2}, \frac{3+\alpha+\delta-\varepsilon}{2}-n; -\frac{a^2 b^2}{4}\right) +$$

$$+ \frac{(-1)^{n-1} 2^{2n+\varepsilon} (1/2)_{2n+\varepsilon} \Gamma(\alpha+\varepsilon+2n)}{(2n+\varepsilon)! (\alpha+\varepsilon+2n) a^{2n+\varepsilon} b^{\alpha+\varepsilon+2n}} \times$$

$$\times \begin{Bmatrix} \sin[(\alpha+\varepsilon)\pi/2] \\ \cos[(\alpha+\varepsilon)\pi/2] \end{Bmatrix} {}_3F_4\left(-n, \frac{1}{2}-\varepsilon-n, -\frac{\alpha+\varepsilon}{2}-n; 1-\frac{\alpha+\varepsilon}{2}-n; \frac{1}{2}-\varepsilon-2n, \frac{1-\alpha}{2}-n, 1-\frac{\alpha}{2}-\varepsilon-n; -\frac{a^2 b^2}{4}\right) +$$

$$+ \frac{(-1)^{n+1} a^{\alpha+\delta} b^\delta ((1-\alpha+\varepsilon-\delta)/2)_n}{2 ((\alpha+\varepsilon+\delta)/2)_{n+1}} \{R\} + \frac{(-1)^n \delta a^\alpha \pi ((1-\alpha+\varepsilon)/2)_n}{4 ((\alpha+\varepsilon)/2)_{n+1}},$$

$$R = C - \sum_{k=0}^n \frac{1}{2k+\alpha+\varepsilon} - \sum_{k=0}^{n-1} \frac{1}{2k-\alpha+\varepsilon+1} + \ln(ab)$$

[$\varepsilon=0$ или 1 ; $a, b>0$; $\text{Re } \alpha < 2-2n-\varepsilon$].

$$3. \int_{-a}^a (x+a)^{\alpha-1} \begin{Bmatrix} \text{si}(b\sqrt{x+a}) \\ \text{ci}(b\sqrt{x+a}) \end{Bmatrix} P_n\left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^{n+1} (-\alpha-\delta/2)_n b^{\delta+2} 2^{\alpha+\delta/2-1} a^{\alpha+\delta/2+1}}{3^{2\delta} (\alpha+\delta/2+1)_{n+1}} {}_4F_3\left(1, 1+\frac{\delta}{2}+\alpha, \frac{\delta}{2}+1, 1+\alpha+\frac{\delta}{2}; 2, 2+\frac{\delta}{2}+\alpha+n, \frac{3}{2}+\delta, 2+\frac{\delta}{2}, 1+\frac{\delta}{2}+\alpha-n; -\frac{ab^2}{2}\right) +$$

$$+ \frac{(-1)^n (1-\alpha-\delta/2)_n (2a)^{\alpha+\delta/2} b^\delta}{(\alpha+\delta/2)_{n+1}} \{R\} - \frac{\delta (-1)^n 2^{\alpha-1} \pi a^\alpha (1-\alpha)_n}{(\alpha)_{n+1}},$$

$$R = C - \frac{1}{2} \sum_{k=0}^n \frac{1}{\alpha+k} - \frac{1}{2} \sum_{k=0}^{n-1} \frac{1}{k+1-\alpha} + \frac{1}{2} \ln(2ab^2)$$

[$a, \text{Re } \alpha > 0$].

$$\begin{aligned}
4. \int_a^\infty (x+a)^{-\tau} \left\{ \begin{matrix} \text{si} (b \sqrt{x+a}) \\ \text{ci} (b \sqrt{x+a}) \end{matrix} \right\} P_n \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^n 2^{3\delta/2-\tau} (\tau-\delta/2-1)_n a^{2+\delta/2-\tau} b^{\delta+2}}{3^{2\delta} (2+\delta/2-\tau)_{n+1}} {}_4F_5 \left(1, 1+\frac{\delta}{2}, 2+\frac{\delta}{2}-\tau, \right. \\
\left. 2+\frac{\delta}{2}-\tau, 2, \frac{3}{2}+\delta, 2+\frac{\delta}{2}, 2+\frac{\delta}{2}-\tau-n, 3+\frac{\delta}{2}+n-\tau; -\frac{ab^2}{2} \right) \pm \\
\pm (-1)^n \frac{(1/2)_n 2^n \Gamma(2n-2\tau+2) b^{2\tau-2n-2}}{n! (\tau-n-1) a^n} \left\{ \begin{matrix} \sin \tau\pi \\ \cos \tau\pi \end{matrix} \right\} \times \\
\times {}_2F_4 \left(-n, -n, \tau-n-1; -2n, \tau-n, \tau-n, \tau-n-\frac{1}{2}; -\frac{ab^2}{2} \right) + \\
+ \frac{(-1)^{n+1} (\tau-\delta/2)_n}{(1+\delta/2-\tau)_{n+1}} (2a)^{\delta/2-\tau+1} b^\delta \left\{ \begin{matrix} 1 \\ R \end{matrix} \right\} + \frac{\delta (-1)^n (\tau)_n \pi a^{1-\tau}}{2^\tau (1-\tau)_{n+1}}, \\
R = C - \frac{1}{2} \sum_{k=0}^n \frac{1}{k-\tau+1} - \frac{1}{2} \sum_{k=0}^{n-1} \frac{1}{k+\tau} + \frac{1}{2} \ln(2ab^2)
\end{aligned}$$

[a, b > 0; Re τ > n].

2.17.11. Интегралы, содержащие $\left\{ \begin{matrix} \text{erf}(\varphi(x)) \\ \text{erfc}(\varphi(x)) \end{matrix} \right\} P_n(cx)$.

$$\begin{aligned}
1. \int_0^\infty x^{\alpha-1} \text{erfc}(bx) P_{2n+\varepsilon}(cx) dx = \frac{(-1)^n (2c)^\varepsilon (1/2)_{n+\varepsilon}}{\sqrt{\pi} (\alpha+\varepsilon) n! b^{\alpha+\varepsilon}} \Gamma\left(\frac{\alpha+\varepsilon+1}{2}\right) \times \\
\times {}_4F_2 \left(-n, \frac{1}{2}+\varepsilon+n, \frac{\alpha+1}{2}, \frac{\alpha}{2}+\varepsilon, \frac{1}{2}+\varepsilon, \frac{\alpha+\varepsilon}{2}+1; \frac{c^2}{b^2} \right) \\
[\varepsilon=0 \text{ или } 1; \text{Re } \alpha > -\varepsilon; |\arg b| < \pi/4].
\end{aligned}$$

$$\begin{aligned}
2. \int_0^a \text{erf}(bx) P_{2n+1}\left(\frac{x}{a}\right) dx = \frac{(-1)^n}{2 \sqrt{\pi ab^3} (n+1/2)_{n+1}} e^{-a^2 b^2/2} M_{-3/4, n+3/4}(a^2 b^2) \\
[a > 0].
\end{aligned}$$

$$\begin{aligned}
3. \int_0^a x^{\alpha-1} \left\{ \begin{matrix} \text{erf}(bx) \\ \text{erfc}(bx) \end{matrix} \right\} P_{2n+\varepsilon}\left(\frac{x}{a}\right) dx = \pm \frac{(-1)^n a^{\alpha+1} b ((\varepsilon-\alpha)/2)_n}{\sqrt{\pi} ((\alpha+\varepsilon+1)/2)_{n+1}} \times \\
\times {}_2F_3 \left(\frac{1}{2}, \frac{\alpha+1}{2}, \frac{\alpha}{2}+1; \frac{3}{2}; 1+\frac{\alpha-\varepsilon}{2}-n, \frac{\alpha+\varepsilon+3}{2}+n; -a^2 b^2 \right) + \\
+ \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{(-1)^n a^\alpha ((1-\alpha+\varepsilon)/2)_n}{2 ((\alpha+\varepsilon)/2)_{n+1}} \\
[\varepsilon=0 \text{ или } 1; a > 0; \text{Re } \alpha > (\mp 1-1)/2-\varepsilon].
\end{aligned}$$

$$\begin{aligned}
4. \int_0^a x^{2m+\varepsilon-3} \text{erf}(bx) P_{2n+\varepsilon}\left(\frac{x}{a}\right) dx = \\
= \frac{(-1)^{m+n-1} a^{2n+\varepsilon+1} b^{2n-2m+3}}{\sqrt{\pi} (n+\varepsilon+1/2)_{n+1} (3+2n-2m) (n-m+1)!} {}_2F_3 \left(\frac{3}{2}+n-m, n+\frac{\varepsilon+1}{2}, \right. \\
\left. 1+n+\frac{\varepsilon}{2}; \frac{5}{2}+n-m, 2n+\varepsilon+\frac{3}{2}, n-m+2; -a^2 b^2 \right) \\
[\varepsilon=0 \text{ или } 1; a > 0; 1-\varepsilon \leq m \leq n].
\end{aligned}$$

$$\begin{aligned}
5. \int_a^\infty x^{\alpha-1} \left\{ \begin{matrix} \operatorname{erf}(bx) \\ \operatorname{erfc}(bx) \end{matrix} \right\} P_{2n+\varepsilon} \left(\frac{x}{a} \right) dx = & \mp \frac{(-1)^n a^{\alpha+1} b ((\varepsilon-\alpha)/2)_n}{\sqrt{\pi} ((\alpha+\varepsilon+1)/2)_{n+1}} \times \\
& \times {}_3F_2 \left(\frac{1}{2}, \frac{\alpha+1}{2}, \frac{\alpha}{2}+1; \frac{\alpha+\varepsilon+3}{2}+n, 1+\frac{\alpha-\varepsilon}{2}-n, \frac{3}{2}; -a^2 b^2 \right) \mp \\
& \mp \frac{2^{2n+\varepsilon} (1/2)_{2n+\varepsilon}}{\sqrt{\pi} (2n+\varepsilon)! (\alpha+\varepsilon+2n) a^{2n+\varepsilon} b^{\alpha+\varepsilon+2n}} \Gamma \left(\frac{\alpha+\varepsilon+1}{2}+n \right) \times \\
& \times {}_3F_2 \left(-n, \frac{1}{2}-\varepsilon-n, -\frac{\alpha+\varepsilon}{2}-n; 1-\frac{\alpha}{2}-\varepsilon-n, \frac{1}{2}-\varepsilon-2n, \frac{1-\alpha}{2}-n; -a^2 b^2 \right) + \\
& + \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \frac{(-1)^{n+1} a^\alpha ((1-\alpha+\varepsilon)/2)_n}{2 ((\alpha+\varepsilon)/2)_{n+1}} \\
& \left[\varepsilon=0 \text{ или } 1; a>0, \begin{cases} \operatorname{Re} b > 0; \operatorname{Re} \alpha < -2n-\varepsilon \\ \operatorname{Re} b > 0 \end{cases} \right].
\end{aligned}$$

$$\begin{aligned}
6. \int_a^\infty (x-a)^{\alpha-1} \operatorname{erfc}(bx-ab) P_n \left(\frac{x}{a} \right) dx = \\
= \frac{1}{\alpha \sqrt{\pi} b a^\alpha} \Gamma \left(\frac{\alpha+1}{2} \right) {}_4F_2 \left(-n, n+1, \frac{\alpha}{2}, \frac{\alpha+1}{2}; 1, \frac{\alpha}{2}+1; \frac{1}{4a^2 b^2} \right) \\
[a, \operatorname{Re} \alpha > 0; |\arg b| < \pi/4].
\end{aligned}$$

$$\begin{aligned}
7. \int_{-a}^a (x+a)^{\alpha-1} \left\{ \begin{matrix} \operatorname{erf}(ab+bx) \\ \operatorname{erfc}(ab+bx) \end{matrix} \right\} P_n \left(\frac{x}{a} \right) dx = & \pm \frac{(-1)^n (-\alpha)_n 2^{\alpha+2} a^{\alpha+1} b}{\sqrt{\pi} (\alpha+1)_{n+1}} \times \\
& \times {}_3F_2 \left(\frac{1}{2}, \frac{\alpha+1}{2}, \frac{\alpha}{2}+1, \frac{\alpha+1}{2}, \frac{\alpha}{2}+1; \frac{3}{2}, \frac{\alpha-n+1}{2}, \frac{\alpha-n}{2}+1, \right. \\
& \left. \frac{\alpha+n}{2}+1, \frac{\alpha+n+3}{2}; -4a^2 b^2 \right) + \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} (-1)^n (2a)^\alpha \frac{(1-\alpha)_n}{(\alpha)_{n+1}} \\
& [a>0; \operatorname{Re} \alpha > -(1 \pm 1)/2].
\end{aligned}$$

$$\begin{aligned}
8. \int_{-a}^a (x+a)^{\alpha-1} \left\{ \begin{matrix} \operatorname{erf}(b\sqrt{x+a}) \\ \operatorname{erfc}(b\sqrt{x+a}) \end{matrix} \right\} P_n \left(\frac{x}{a} \right) dx = \\
= \pm \frac{(-1)^n (1/2-\alpha)_n 2^{\alpha+3/2} a^{\alpha+1/2} b}{\sqrt{\pi} (\alpha+1/2)_{n+1}} {}_3F_2 \left(\frac{1}{2}, \alpha+\frac{1}{2}, \alpha+\frac{1}{2}; \frac{3}{2}, \right. \\
\left. \frac{3}{2}+\alpha+n, \frac{1}{2}+\alpha-n; -2ab^2 \right) + \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \frac{(-1)^n (1-\alpha)_n}{(\alpha)_{n+1}} (2a)^\alpha \\
[a>0; \operatorname{Re} \beta > -(1 \pm 1)/4].
\end{aligned}$$

$$\begin{aligned}
9. \int_{-a}^a (x+a)^{m-1/2} \operatorname{erf}(b\sqrt{x+a}) P_n \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^{m+n} 2^{n+2} a^{n+1} b^{2n-2m+1} (n-m+1)_m}{\sqrt{\pi} (n+1)_{n+1} (1+2n-2m)} \times \\
\times {}_3F_2 \left(\frac{1}{2}+n-m, n+1, n+1; \frac{3}{2}+n-m, 2n+2, 1+n-m; -2ab^2 \right) \\
[a>0; m < n].
\end{aligned}$$

$$\begin{aligned}
10. \int_a^\infty (x-a)^{\alpha-1} \operatorname{erfc}(b\sqrt{x-a}) P_n \left(\frac{x}{a} \right) dx = \\
= \frac{1}{\alpha \sqrt{\pi} b a^\alpha} \Gamma \left(\alpha + \frac{1}{2} \right) {}_4F_2 \left(-n, n+1, \alpha, \alpha + \frac{1}{2}; 1, \alpha+1; \frac{1}{2ab^2} \right) \\
[a, \operatorname{Re} \alpha > 0; |\arg b| < \pi/4].
\end{aligned}$$

$$\begin{aligned}
 11. \int_a^\infty (x+a)^{-\tau} \left\{ \begin{matrix} \operatorname{erf}(b\sqrt{x+a}) \\ \operatorname{erfc}(b\sqrt{x+a}) \end{matrix} \right\} P_n\left(\frac{x}{a}\right) dx = \pm \frac{(-1)^{n+1} (\tau-1/2)_n 2^{5/2-\tau}}{\sqrt{\pi} (3/2-\tau)_{n+1}} \times \\
 \times a^{3/2-\tau} {}_3F_3\left(\frac{1}{2}, \frac{3}{2}-\tau, \frac{3}{2}-\tau; \frac{3}{2}, \frac{5}{2}+n-\tau, \frac{3}{2}-\tau-n; -2ab^2\right) \pm \\
 \pm \frac{2^n (1/2)_n \Gamma(3/2+n-\tau) b^{2\tau-2n-2}}{n! \sqrt{\pi} (\tau-n-1) a^n} \times \\
 \times {}_3F_3\left(-n, -n, \tau-n-1; -2n, \tau-n-\frac{1}{2}, \tau-n; -2ab^2\right) + \\
 + \begin{cases} 0 \\ 1 \end{cases} \frac{(-1)^{n-1} (\tau)_n}{(1-\tau)_{n+1}} (2a)^{1-\tau} \quad \left[a > 0, \begin{cases} |\arg b| < \pi/4, \operatorname{Re} \tau > n+1 \\ |\arg b| < \pi/4 \end{cases} \right].
 \end{aligned}$$

$$\begin{aligned}
 12. \int_0^a x^{\alpha-1} e^{o^2 x^2} \operatorname{erf}(bx) P_n\left(\frac{x}{a}\right) dx = \frac{a^{\alpha+1} b}{2^\alpha} \Gamma\left[\frac{\alpha+1}{2}, \frac{(\alpha-n)/2+1, (\alpha+n+3)/2}{2}\right] \times \\
 \times {}_3F_3\left(1, \frac{\alpha+1}{2}, \frac{\alpha}{2}+1; \frac{3}{2}, \frac{\alpha-n}{2}+1, \frac{\alpha+n+3}{2}; a^2 b^2\right) \\
 [a > 0; \operatorname{Re} \alpha > 2[n/2] - n - 1].
 \end{aligned}$$

$$\begin{aligned}
 13. \int_{-a}^a (x+a)^{\alpha-1} e^{b^2 x} \operatorname{erf}(b\sqrt{x+a}) P_n\left(\frac{x}{a}\right) dx = \\
 = \frac{(-1)^n (1/2-\alpha)_n 2^{\alpha+3/2} a^{\alpha+1/2} b}{\sqrt{\pi} (\alpha+1/2)_{n+1}} e^{-ab^2} \times \\
 \times {}_3F_3\left(1, \alpha+\frac{1}{2}, \alpha+\frac{1}{2}; \frac{3}{2}, \frac{1}{2}+\alpha-n, \frac{3}{2}+\alpha+n; 2ab^2\right) \\
 [a > 0; \operatorname{Re} \alpha > -1/2].
 \end{aligned}$$

2.17.12. Интегралы, содержащие $\left\{ \begin{matrix} J_\nu(\varphi(\lambda)) \\ I_\nu(\varphi(\lambda)) \end{matrix} \right\} P_n(cx)$.

$$\begin{aligned}
 1. \int_0^a x^{\alpha-1} \left\{ \begin{matrix} J_\nu(bx) \\ I_\nu(bx) \end{matrix} \right\} P_{2n+\varepsilon}\left(\frac{x}{a}\right) dx = \frac{(-1)^n a^{\alpha+\nu} b^\nu ((1-\alpha+\varepsilon-\nu)/2)_n}{2^{\nu+1} ((\alpha+\varepsilon+\nu)/2)_{n+1} \Gamma(\nu+1)} \times \\
 \times {}_3F_3\left(\frac{\alpha+\nu+1}{2}, \frac{\alpha+\nu}{2}; \nu+1, \frac{\alpha+\varepsilon+\nu}{2}+n+1, \frac{1+\alpha-\varepsilon+\nu}{2}-n; \mp \frac{a^2 b^2}{4}\right) \\
 [\varepsilon=0 \text{ или } 1; a > 0, \operatorname{Re}(\alpha+\nu) > -\varepsilon].
 \end{aligned}$$

$$\begin{aligned}
 2. \int_a^\infty x^{\alpha-1} J_\nu(bx) P_{2n+\varepsilon}\left(\frac{x}{a}\right) dx = \frac{(-1)^{n+1} a^{\alpha+\nu} b^\nu ((1-\alpha+\varepsilon-\nu)/2)_n}{2^{\nu+1} ((\alpha+\varepsilon+\nu)/2)_{n+1} \Gamma(\nu+1)} \times \\
 \times {}_3F_3\left(\frac{\alpha+\nu+1}{2}, \frac{\alpha+\nu}{2}; \nu+1, \frac{\alpha+\varepsilon+\nu}{2}+n+1, \frac{1+\alpha-\varepsilon+\nu}{2}-n; -\frac{a^2 b^2}{4}\right) + \\
 + \frac{2^{4n+2\varepsilon+\alpha-1} (1/2)_{2n+3}}{(2n+\varepsilon)! a^{2n+\varepsilon} b^{\alpha+\varepsilon+2n}} \Gamma\left[\frac{(\alpha+\varepsilon+\nu)/2+n}{1+(v-\alpha-\varepsilon)/2-n}\right] \times \\
 \times {}_3F_3\left(-n, \frac{1}{2}-\varepsilon-n; \frac{1}{2}-\varepsilon-2n, 1-\frac{\nu+\alpha+\varepsilon}{2}-n, 1+\frac{\nu-\alpha-\varepsilon}{2}-n; -\frac{a \cdot b^2}{4}\right) \\
 [\varepsilon=0 \text{ или } 1; a, b > 0; \operatorname{Re} \alpha < 3/2 - 2n - \varepsilon].
 \end{aligned}$$

$$\begin{aligned}
 3. \int_0^a x^{\alpha-1} J_\nu\left(\frac{b}{x}\right) P_{2n+\varepsilon}\left(\frac{x}{a}\right) dx = \frac{(-1)^n a^{\alpha-\nu} b^\nu ((1-\alpha+\varepsilon+\nu)/2)_n}{2^{\nu+1} ((\alpha+\varepsilon-\nu)/2)_{n+1} \Gamma(\nu+1)} \times \\
 \times {}_3F_3\left(\frac{\nu-\alpha-\varepsilon}{2}-n, \frac{1-\alpha+\varepsilon+\nu}{2}+n; \nu+1, 1+\frac{\nu-\alpha}{2}, \frac{1+\nu-\alpha}{2}; -\frac{b^2}{4a^2}\right) +
 \end{aligned}$$

$$+ \frac{(-1)^n b^{\alpha+s}}{2^{\alpha+s+1} n! a^s} \Gamma \left[\frac{(\nu - \alpha - s)/2}{(\nu + \alpha + s)/2 + 1} \right] \left(s + \frac{1}{2} \right)_n \times \\ \times {}_2F_3 \left(-n, \frac{1}{2} + s + n; s + \frac{1}{2}, 1 + \frac{\alpha + s - \nu}{2}, 1 + \frac{\alpha + s + \nu}{2}; -\frac{b^2}{4a^2} \right) \\ [s=0 \text{ или } 1; a, b > 0; \operatorname{Re} \alpha > -3/2 - s].$$

$$4. \int_0^a x^{-5/2} J_{2n+3/2} \left(\frac{b}{x} \right) P_{2n+1} \left(\frac{x}{a} \right) dx = (-1)^n \sqrt{\frac{2}{\pi b^3}} \sin \frac{b}{a} \quad [a, b > 0]$$

$$5. \int_{-a}^a (x+a)^{\alpha-1} \left\{ \begin{matrix} J_\nu(b\sqrt{x+a}) \\ J_\nu(b\sqrt{x+a}) \end{matrix} \right\} P_n \left(\frac{x}{a} \right) dx = (-1)^n A_{\alpha, \nu} \\ [a, \operatorname{Re}(2\alpha + \nu) > 0]$$

$$A_{\alpha, \nu} = \frac{2^{\alpha-\nu/2} a^{\alpha+\nu/2} b^\nu (1-\alpha-\nu/2)_n}{(\alpha+\nu/2)_{n+1} \Gamma(\nu+1)} \times \\ \times {}_2F_3 \left(\alpha + \frac{\nu}{2}, \alpha + \frac{\nu}{2}; \nu+1, \alpha + \frac{\nu}{2} + n + 1, \alpha + \frac{\nu}{2} - n; \mp \frac{ab^2}{2} \right),$$

$$A_{0,1} = \frac{(\pm 1)^n \sqrt{2}}{b \sqrt{a} (2n+1)} [(n+1) J_{2n+2}(b\sqrt{2a}) \mp n J_{2n}(b\sqrt{2a})],$$

$$A_{3/2,1} = \frac{(\pm 1)^n 4a}{b (2n+1)} [(n+1) J_{2n+2}(b\sqrt{2a}) \mp n J_{2n}(b\sqrt{2a})],$$

$$A_{1,0} = \frac{(\pm 1)^n 2 \sqrt{2a}}{b} J_{2n+1}(b\sqrt{2a}),$$

$$A_{2,0} = \frac{(\pm 1)^n 2 (2a)^{3/2}}{b (2n+1)} [(n+1) J'_{2n+2}(b\sqrt{2a}) \mp n J'_{2n}(b\sqrt{2a})].$$

$$6. \int_a^\infty (x-a)^{\alpha-1} J_\nu(b\sqrt{x-a}) P_n \left(\frac{x}{a} \right) dx = \frac{2^{2n+2\alpha} (1/2)_n}{n! a^n b^{2\alpha+2n}} \Gamma \left[\frac{\nu/2 + \alpha + n}{1 + \nu/2 - \alpha - n} \right] \times \\ \times {}_2F_3 \left(-n, -n; -2n, 1 - \frac{\nu}{2} - \alpha - n, 1 + \frac{\nu}{2} - \alpha - n; \frac{ab^2}{2} \right) \\ [a, b, \operatorname{Re}(2\alpha + \nu) > 0; \operatorname{Re} \alpha < 3/4 - n]$$

$$7. \int_a^\infty (x+a)^{-\tau} J_\nu(b\sqrt{x+a}) P_n \left(\frac{x}{a} \right) dx = \frac{(-1)^{n+1} (\tau - \nu/2)_n 2^{2-\tau-\nu/2}}{(1 + \nu/2 - \tau)_{n+1} \Gamma(\nu+1)} a^{\nu/2+1-\tau} b^\nu \times \\ \times {}_2F_3 \left(1 + \frac{\nu}{2} - \tau, 1 + \frac{\nu}{2} - \tau; 1 + \frac{\nu}{2} - \tau - n, 2 + \frac{\nu}{2} + n - \tau, \nu + 1; -\frac{ab^2}{2} \right) + \\ + \frac{2^{2\tau-2n+2} (1/2)_n}{n! a^n} b^{2\tau-2n-2} \Gamma \left[\frac{1+n-\tau+\nu/2}{\nu/2+\tau-n} \right] \times \\ \times {}_2F_3 \left(-n, -n; -2n, \frac{\nu}{2} + \tau - n, \tau - \frac{\nu}{2} - n; -\frac{ab^2}{2} \right) \\ [a, b > 0; \operatorname{Re} \tau > n + 1/4]$$

$$8. \int_{-a}^a (x+a)^{\alpha-1} J_\nu \left(\frac{b}{\sqrt{x+a}} \right) P_n \left(\frac{x}{a} \right) dx = (-1)^n \left(\frac{b}{2} \right)^{2\alpha} \Gamma \left[\frac{\nu/2 - \alpha}{1 + \alpha + \nu/2} \right] \times \\ \times {}_2F_3 \left(-n, n+1; 1, 1 + \alpha - \frac{\nu}{2}, 1 + \alpha + \frac{\nu}{2}; -\frac{b^2}{8a} \right) +$$

$$+ (-1)^n \frac{(1 + \nu/2 - \alpha)_n 2^{\alpha-3\nu/2} a^{\alpha-\nu/2} b^\nu}{(\alpha - \nu/2)_{n+1} \Gamma(\nu+1)} \times \\ \times {}_2F_3\left(1 + \frac{\nu}{2} + n - \alpha, \frac{\nu}{2} - \alpha - n; \nu+1, 1 + \frac{\nu}{2} - \alpha, 1 + \frac{\nu}{2} - \alpha; -\frac{b^2}{8a}\right) \\ [a, b > 0; \operatorname{Re} \alpha > -3/4].$$

$$9. \int_a^\infty (x-a)^{\alpha-1} J_\nu\left(\frac{b}{\sqrt{x-a}}\right) P_n\left(\frac{x}{a}\right) dx = \\ = \left(\frac{b}{2}\right)^{2\alpha} \Gamma\left[\frac{\nu/2 - \alpha}{1 + \alpha + \nu/2}\right] {}_2F_3\left(-n, n+1; 1, 1 + \alpha + \frac{\nu}{2}, 1 + \alpha - \frac{\nu}{2}; \frac{b^2}{8a}\right) \\ [a, b > 0; -3/4 < \operatorname{Re} \alpha < \operatorname{Re} \nu/2 - n].$$

$$10. \int_a^\infty (x+a)^{-\tau} \left\{ \begin{matrix} J_\nu(b/\sqrt{x+a}) \\ I_\nu(b/\sqrt{x+a}) \end{matrix} \right\} P_n\left(\frac{x}{a}\right) dx = \\ = \frac{(-1)^{n+1} (\tau + \nu/2)_n 2^{1-3\nu/2-\tau}}{(1 - \tau - \nu/2)_{n+1} \Gamma(\nu+1)} a^{1-\nu/2-\tau} b^\nu \times \\ \times {}_2F_3\left(\frac{\nu}{2} + \tau - n - 1, \frac{\nu}{2} + \tau + n; \tau + \frac{\nu}{2}, \tau + \frac{\nu}{2}, \nu+1; \mp \frac{b^2}{8a}\right) \\ [a > 0; \operatorname{Re}(\nu + 2\tau) > 2n + 2].$$

$$11. \int_{-a}^a (x+a)^{-\beta-n-1} (a-x)^{\beta-1} J_\nu\left(b \sqrt{\frac{a-x}{a+x}}\right) P_n\left(\frac{x}{a}\right) dx = \frac{(-1)^n 2^{2\beta+n-1}}{a^{n+1} b^{2\beta+2n}} \times \\ \times \Gamma\left[\frac{\nu/2 + \beta + n}{1 + \nu/2 - \beta - n}\right] {}_2F_3\left(-n, -n; 1, 1 - \beta - n - \frac{\nu}{2}, 1 + \frac{\nu}{2} - \beta - n; \frac{b^2}{4}\right) \\ [a, b, \operatorname{Re}(2\beta + \nu) > 0; \operatorname{Re} \beta < 3/4 - n].$$

$$12. \int_{-a}^a (x+a)^{n-\beta} (a-x)^{\beta-1} J_\nu\left(b \sqrt{\frac{a-x}{a+x}}\right) P_n\left(\frac{x}{a}\right) dx = \\ = \left[\frac{(1 - \beta - \nu/2)_n}{n!}\right]^2 \frac{2^{n-\nu} \pi a^n b^\nu}{\Gamma(\nu+1) \sin(\beta + \nu/2)\pi} \times \\ \times {}_2F_3\left(\frac{\nu}{2} + \beta, \frac{\nu}{2} + \beta; \frac{\nu}{2} + \beta - n, \frac{\nu}{2} + \beta - n, \nu+1; \frac{b^2}{4}\right) + \\ + (-1)^n 2^{2\beta-n-2} a^n b^{2n-2\beta+2} \Gamma\left[\frac{\nu/2 + \beta - n - 1}{2 + \nu/2 + n - \beta}\right] \times \\ \times {}_2F_3\left(n+1, n+1; 1, 2 + n - \beta - \frac{\nu}{2}, 2 + n - \beta + \frac{\nu}{2}; \frac{b^2}{4}\right) \\ [a, b, \operatorname{Re}(2\beta + \nu) > 0; \operatorname{Re} \beta < 7/4 + n].$$

$$13. \int_{-a}^a (x+a)^{n-1} J_0\left(b \sqrt{\frac{a-x}{a+x}}\right) P_n\left(\frac{x}{a}\right) dx = \frac{a^n b^{2n}}{2^{n-1} (n!)^2} K_0(b) \quad [a > 0]$$

$$14. \int_0^a x^{n-1} J_0\left(\frac{c}{x} \sqrt{a^2 - x^2}\right) P_n\left(\frac{x}{a}\right) dx = \frac{a^n c^{n-1}}{n!} e^{-c} \quad [a > 0].$$

$$15. \int_{-a}^a e^{bx} I_0(ab - bx) P_n\left(\frac{x}{a}\right) dx = \frac{2a}{2n+1} e^{-ab} [I_n(2ab) + I_{n+1}(2ab)] \\ [a > 0].$$

$$16. \int_0^a \left\{ \begin{matrix} \sin bx \\ \cos bx \end{matrix} \right\} J_0(c \sqrt{a^2 - x^2}) P_n\left(\frac{x}{a}\right) dx =$$

$$= (-1)^{[n/2]} \sqrt{\frac{\pi a}{2}} (b^2 + c^2)^{-1/4} J_{n+1/2}(a \sqrt{b^2 + c^2}) P_n\left(\frac{b}{\sqrt{b^2 + c^2}}\right)$$

$$[a > 0, n = \{0, 2, 4, \dots\}].$$

$$17. \int_0^a x^{-5/2} J_{2n+3/2}\left(\frac{b}{x}\right) J_0\left(\frac{c}{x} \sqrt{a^2 - x^2}\right) P_n\left(\frac{x}{a}\right) dx =$$

$$= \sqrt{\frac{2}{\pi b}} (b^2 - a^2 c^2)^{-1/2} \sin\left(\frac{\sqrt{b^2 - a^2 c^2}}{a} - \frac{\pi n}{2}\right) P_n\left(\frac{\sqrt{b^2 - a^2 c^2}}{b}\right)$$

$$[a, b, c > 0; b \neq ac].$$

$$18. \int_0^a x^{-3/2} J_{n+1/2}\left(\frac{b}{x}\right) J_0\left(\frac{c}{x} \sqrt{a^2 - x^2}\right) P_n\left(\frac{x}{a}\right) dx =$$

$$= (-1)^{[n/2]} \sqrt{\frac{2b}{\pi}} (b^2 - a^2 c^2)^{-1/2} \left\{ \begin{matrix} \sin(\sqrt{b^2 - a^2 c^2}/a) \\ \cos(\sqrt{b^2 - a^2 c^2}/a) \end{matrix} \right\} P_n\left(\sqrt{\frac{b^2 - a^2 c^2}{b^2}}\right)$$

$$[a, b, c > 0; b \neq ac; n = \{0, 2, 4, \dots\}].$$

$$19. \int_{-a}^a J_0^2(b \sqrt{x+a}) P_n\left(\frac{x}{a}\right) dx = (-1)^n \frac{2a}{2n+1} [J_n^2(b \sqrt{2a}) + J_{n+1}^2(b \sqrt{2a})]$$

$$[a > 0].$$

$$20. \int_{-a}^a (x+a)^{\alpha-1} \left\{ \begin{matrix} J_\mu(b \sqrt{x+a}) J_\nu(b \sqrt{x+a}) \\ I_\mu(b \sqrt{x+a}) I_\nu(b \sqrt{x+a}) \end{matrix} \right\} P_n\left(\frac{x}{a}\right) dx =$$

$$= (-1)^n 2^{\alpha-(\mu+\nu)/2} \frac{(1-(\mu+\nu)/2-\alpha)_n a^{\alpha+(\mu+\nu)/2} b^{\mu+\nu}}{(\alpha+(\mu+\nu)/2)_{n+1} \Gamma(\mu+1) \Gamma(\nu+1)} \times$$

$$\times {}_4F_5\left(\frac{\mu+\nu}{2}+\alpha, \frac{\mu+\nu}{2}+\alpha, \frac{\mu+\nu+1}{2}, \frac{\mu+\nu}{2}+1; \frac{\mu+\nu}{2}+\alpha+n+1, \frac{\mu+\nu}{2}+\alpha-n, \mu+1, \nu+1, \mu+\nu+1; \mp 2ab^2\right)$$

$$[a, \operatorname{Re}(2\alpha + \mu + \nu) > 0].$$

2.17.13. Интегралы, содержащие $\left\{ \begin{matrix} Y_\nu(\varphi(x)) \\ K_\nu(\varphi(x)) \end{matrix} \right\} P_n(cx)$

$$1. \int_{-a}^a Y_0(b \sqrt{x+a}) P_n\left(\frac{x}{a}\right) dx = (-1)^n \frac{2\sqrt{2a}}{b\pi} [S_{2n+1}(b \sqrt{2a}) + \pi Y_{2n+1}(b \sqrt{2a})]$$

$$[a > 0].$$

$$2. \int_{-a}^a J_0(b \sqrt{x+a}) Y_0(b \sqrt{x+a}) P_n\left(\frac{x}{a}\right) dx =$$

$$= (-1)^n \frac{2a}{2n+1} [J_n(b \sqrt{2a}) Y_n(b \sqrt{2a}) + J_{n+1}(b \sqrt{2a}) Y_{n+1}(b \sqrt{2a})]$$

$$[a > 0].$$

$$\begin{aligned}
3. \int_{-a}^a (x+a)^{\alpha-1} K_\nu(b\sqrt{x+a}) P_n\left(\frac{x}{a}\right) dx = \\
= \frac{(-1)^n 2^{\alpha+\nu/2-1} (1+\nu/2-\alpha)_n a^{\alpha-\nu/2} \Gamma(\nu)}{(\alpha-\nu/2)_{n+1} b^\nu} \times \\
\times {}_2F_3\left(\alpha-\frac{\nu}{2}, \alpha-\frac{\nu}{2}; 1+\alpha+n-\frac{\nu}{2}, \alpha-n-\frac{\nu}{2}, 1-\nu; \frac{ab^2}{2}\right) + \\
+ \frac{(-1)^n 2^{\alpha-\nu/2-1} (1-\nu/2-\alpha)_n a^{\alpha+\nu/2} b^\nu \Gamma(-\nu)}{(\alpha+\nu/2)_{n+1}} \times \\
\times {}_2F_3\left(\alpha+\frac{\nu}{2}, \alpha+\frac{\nu}{2}; 1+\alpha+\frac{\nu}{2}+n, \alpha-n+\frac{\nu}{2}, \nu+1; \frac{ab^2}{2}\right) \\
[a > 0; 2 \operatorname{Re} \alpha > |\operatorname{Re} \nu|].
\end{aligned}$$

$$\begin{aligned}
4. \int_{-a}^a K_0(b\sqrt{x+a}) P_n\left(\frac{x}{a}\right) dx = \\
= -\frac{2\sqrt{2a}}{b} \left[K_{2n+1}(b\sqrt{2a}) + (-1)^{n+1} \frac{i}{2} S_{2n+1}(ib\sqrt{2a}) \right] \quad [a > 0].
\end{aligned}$$

$$\begin{aligned}
5. \int_a^\infty (x-a)^{\alpha-1} K_\nu(b\sqrt{x-a}) P_n\left(\frac{x}{a}\right) dx = \\
= \frac{2^{\alpha+n-1} (1/2)_n}{n! a^n b^{2\alpha+2n}} \Gamma\left(\alpha+n+\frac{\nu}{2}\right) \Gamma\left(\alpha+n-\frac{\nu}{2}\right) \times \\
\times {}_2F_3\left(-n, -n; -2n, 1-n-\alpha-\frac{\nu}{2}, 1+\frac{\nu}{2}-\alpha-n; -\frac{ab^2}{2}\right) \\
[a, \operatorname{Re} b > 0; 2 \operatorname{Re} \alpha > |\operatorname{Re} \nu|].
\end{aligned}$$

$$\begin{aligned}
6. \int_a^\infty (x+a)^{-\tau} K_\nu(b\sqrt{x+a}) P_n\left(\frac{x}{a}\right) dx = \\
= \frac{2^{\alpha-\tau+1} (1/2)_n}{n! a^n b^{2\alpha+2n-\tau}} \Gamma\left(1+n+\frac{\nu}{2}-\tau\right) \Gamma\left(1+n-\frac{\nu}{2}-\tau\right) \times \\
\times {}_2F_3\left(-n, -n; -2n, \frac{\nu}{2}+\tau-n, \tau-\frac{\nu}{2}-n; \frac{ab^2}{2}\right) + \\
+ \frac{(-1)^{n+1} (\nu/2+\tau)_n 2^{\nu/2-\tau} a^{1-\nu/2-\tau} \Gamma(\nu)}{(1-\nu/2-\tau)_{n+1} b^\nu} \times \\
\times {}_2F_3\left(1-\tau-\frac{\nu}{2}, 1-\tau-\frac{\nu}{2}; 1-\nu, 2+n-\frac{\nu}{2}-\tau, 1-n-\frac{\nu}{2}-\tau; \frac{ab^2}{2}\right) + \\
+ \frac{(-1)^{n+1} (\tau-\nu/2)_n b^{1+\nu/2-\tau} b^\nu \Gamma(-\nu)}{(1+\nu/2-\tau)_{n+1} 2^{\nu/2+\nu}} \times \\
\times {}_2F_3\left(1+\frac{\nu}{2}-\tau, 1+\frac{\nu}{2}-\tau; 1+\nu, 2+n+\frac{\nu}{2}-\tau, 1+\frac{\nu}{2}-n-\tau; \frac{ab^2}{2}\right) \\
[a, \operatorname{Re} b > 0]
\end{aligned}$$

$$\begin{aligned}
7. \int_{-a}^a (x+a)^{\alpha-1} K_\nu\left(\frac{b}{\sqrt{x+a}}\right) P_n\left(\frac{x}{a}\right) dx = \frac{(-1)^n a^\alpha}{2^{\alpha+1}} \Gamma\left(\frac{\nu}{2}-\alpha\right) \Gamma\left(-\frac{\nu}{2}-\alpha\right) \times \\
\times {}_2F_3\left(-n, n+1; 1, 1+\alpha+\frac{\nu}{2}, 1+\alpha-\frac{\nu}{2}; \frac{b^2}{8a}\right) +
\end{aligned}$$

$$\begin{aligned}
& + \frac{(-1)^n 2^{\alpha+3\nu/2-1} (1-\nu/2-\alpha)_n}{(\alpha+\nu/2)_{n+1} b^\nu} a^{\alpha+\nu/2} \Gamma(\nu) \times \\
& \times {}_2F_3 \left(-\frac{\nu}{2}-\alpha-n, 1-\frac{\nu}{2}+n-\alpha; 1-\nu, 1-\frac{\nu}{2}-\alpha, 1-\frac{\nu}{2}-\alpha; \frac{b^2}{8a} \right) + \\
& + \frac{(-1)^n 2^{\alpha-3\nu/2-1} (1+\nu/2-\alpha)_n}{(\alpha-\nu/2)_{n+1}} a^{\alpha-\nu/2} b^\nu \Gamma(-\nu) \times \\
& \times {}_2F_3 \left(\frac{\nu}{2}-\alpha-n, 1+\frac{\nu}{2}+n-\alpha; 1+\nu, 1+\frac{\nu}{2}-\alpha, 1+\frac{\nu}{2}-\alpha; \frac{b^2}{8a} \right) \\
& [a, \operatorname{Re} b > 0].
\end{aligned}$$

$$\begin{aligned}
8. \int_a^\infty (x-a)^{\alpha-1} K_\nu \left(\frac{b}{\sqrt{x-a}} \right) P_n \left(\frac{x}{a} \right) dx &= \frac{b^{2\alpha}}{2^{2\alpha+1}} \Gamma \left(-\frac{\nu}{2}-\alpha \right) \Gamma \left(\frac{\nu}{2}-\alpha \right) \times \\
& \times {}_2F_3 \left(-n, n+1; 1, 1+\alpha-\frac{\nu}{2}, 1+\alpha+\frac{\nu}{2}; -\frac{b^2}{4a} \right) \\
& [a, \operatorname{Re} b > 0, 2 \operatorname{Re} \alpha < -|\operatorname{Re} \nu - 2n|.
\end{aligned}$$

$$\begin{aligned}
9. \int_a^\infty (x+a)^{-\tau} K_\nu \left(\frac{b}{\sqrt{x+a}} \right) P_n \left(\frac{x}{a} \right) dx &= \frac{(-1)^{n+1} 2^{3\nu/2-\tau} (\tau-\nu/2)_n a^{1+\nu/2-\tau}}{(1+\nu/2-\tau)_{n+1} b^\nu} \times \\
& \times \Gamma(\nu) {}_2F_3 \left(\tau-n-\frac{\nu}{2}-1, \tau+n-\frac{\nu}{2}, \tau-\frac{\nu}{2}, \tau+\frac{\nu}{2}, 1-\nu; \frac{b^2}{8a} \right) + \\
& + \frac{(-1)^{n+1} (\tau+\nu/2)_n a^{1-\nu/2-\tau} b^\nu}{(1-\nu/2-\tau)_{n+1} 2^{3\nu/2-\tau}} \Gamma(-\nu) \times \\
& \times {}_2F_3 \left(\tau+\frac{\nu}{2}-n-1, \tau+n+\frac{\nu}{2}; \tau+\frac{\nu}{2}, \tau+\frac{\nu}{2}, \nu+1; \frac{b^2}{8a} \right) \\
& [a > 0, 2 \operatorname{Re} \tau > |\operatorname{Re} \nu| + 2n + 2].
\end{aligned}$$

$$\begin{aligned}
10. \int_{-a}^a (x+a)^{-\beta-n-1} (a-x)^{\beta-1} K_\nu \left(b \sqrt{\frac{a-x}{a+x}} \right) P_n \left(\frac{x}{a} \right) dx &= \\
& = \frac{(-1)^n 2^{2\beta+n-2}}{a^{n+1} b^{2\beta+2n}} \Gamma \left(\beta+n+\frac{\nu}{2} \right) \Gamma \left(\beta+n-\frac{\nu}{2} \right) \times \\
& \times {}_2F_3 \left(-n, -n; 1, 1-n-\beta-\frac{\nu}{2}, 1+\frac{\nu}{2}-n-\beta; -\frac{b^2}{4} \right) \\
& [a, \operatorname{Re} b > 0, 2 \operatorname{Re} \beta > |\operatorname{Re} \nu|].
\end{aligned}$$

$$\begin{aligned}
11. \int_{-a}^a (x+a)^{n-\beta} (a-x)^{\beta-1} K_\nu \left(b \sqrt{\frac{a-x}{a+x}} \right) P_n \left(\frac{x}{a} \right) dx &= \\
& = (-1)^n 2^{2\beta-n-3} a^n b^{2n-2\beta+2} \Gamma \left(\frac{\nu}{2}+\beta-n-1 \right) \Gamma \left(\beta-\frac{\nu}{2}-n-1 \right) \times \\
& \times {}_2F_3 \left(n+1, n+1; 1, 2+n-\beta-\frac{\nu}{2}, 2+n-\beta+\frac{\nu}{2}; -\frac{b^2}{4} \right) + \\
& + \left[\frac{(1-\nu/2-\beta)_n}{n!} \right]^2 \frac{2^{n-\nu-1} \pi a^n b^\nu}{\sin(\beta+\nu/2)\pi} {}_2F_3 \left(\beta+\frac{\nu}{2}, \beta+\frac{\nu}{2}; \beta+\frac{\nu}{2}-n, \right. \\
& \left. \beta+\frac{\nu}{2}-n, \nu+1; -\frac{b^2}{4} \right) + \left[\frac{(1+\nu/2-\beta)_n}{n!} \right]^2 \frac{2^{\nu+n-1} \pi a^n b^{-\nu}}{\sin(\beta-\nu/2)\pi} \times \\
& \times {}_2F_3 \left(\beta-\frac{\nu}{2}, \beta-\frac{\nu}{2}; \beta-\frac{\nu}{2}-n, \beta-\frac{\nu}{2}-n, 1-\nu; -\frac{b^2}{4} \right) \\
& [a, \operatorname{Re} b > 0; 2 \operatorname{Re} \beta > |\operatorname{Re} \nu|]
\end{aligned}$$

$$12. \int_{-a}^a \frac{1}{\sqrt{x+a}} I_0(b\sqrt{x+a}) K_0(b\sqrt{x+a}) P_n\left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^n \sqrt{2a}}{2n+1} [I_n(b\sqrt{2a}) K_n(b\sqrt{2a}) + I_{n+1}(b\sqrt{2a}) K_{n+1}(b\sqrt{2a})]$$

[a > 0].

2.17.14. Интегралы от $A(x) P_m(bx) P_n(cx)$.

$$1. \int_0^a P_m\left(\frac{x}{a}\right) P_n\left(\frac{x}{a}\right) dx = 0 \quad [a > 0; m - n = \pm 2, \pm 4, \pm 6, \dots]$$

$$2. \int_0^a \left[P_n\left(\frac{x}{a}\right)\right]^2 dx = \frac{a}{2n+1} \quad [a > 0]$$

$$3. \int_0^a P_{2m+\varepsilon}\left(\frac{x}{a}\right) P_{2n}\left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^{m+n} (2n+\varepsilon+1) (2n-\varepsilon) (1/2+\varepsilon)_m (-\varepsilon/2)_n a}{2 (2n-2m-\varepsilon) (2n+2m+\varepsilon+1) ((\varepsilon+1)/2)_{n+1} m!} \quad [\varepsilon = 0 \text{ или } 1; a > 0].$$

$$4. \int_0^a x^{\alpha-1} P_{2m+\varepsilon}\left(\frac{x}{a}\right) \begin{Bmatrix} P_{2n+1}(x/a) \\ P_{2n}(x/a) \end{Bmatrix} dx =$$

$$= \frac{(-1)^{m+n} 2^{\varepsilon-1} a^{\alpha} (1/2)_{m+\varepsilon} ((1+\delta-\varepsilon-\alpha)/2)_n}{m! ((\alpha+\delta+\varepsilon)/2)_{n+1}} \times$$

$$\times {}_4F_3\left(-m, \frac{1}{2}+\varepsilon+m, \frac{\alpha+1}{2}, \frac{\alpha}{2}+\varepsilon; \varepsilon+\frac{1}{2}, \frac{1+\alpha+\varepsilon-\delta}{2}-n, \right.$$

$$\left. 1+\frac{\alpha+\delta+\varepsilon}{2}+n; 1\right) \quad \left[\varepsilon=0 \text{ или } 1; a > 0; \operatorname{Re} \alpha > -\varepsilon-\delta, \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}\right].$$

$$5. \int_0^a x^{\alpha-1} P_m\left(\frac{x}{a}\right) P_n\left(\frac{x}{a}\right) dx =$$

$$= \frac{\sqrt{\pi} (m+n)! a^{\alpha}}{2^{\alpha} m! n!} \Gamma\left[\frac{\alpha}{2}\right] \left[\frac{(\alpha-m-n+1)/2, (\alpha+m+n)/2+1}{(\alpha-m-n+1)/2, (\alpha+m+n)/2+1}\right] \times$$

$$\times {}_3F_2\left(-m, -n, \frac{\alpha-m-n}{2}; -m-n, \frac{\alpha-m-n+1}{2}; 1\right)$$

[a > 0; Re α > 2[m/2] + 2[n/2] - m - n]

$$6. \int_0^a x^l P_m\left(\frac{x}{a}\right) P_n\left(\frac{x}{a}\right) dx = \frac{2^{m+n} ((l-m-n)/2+1)_{m+n} a^{l+1}}{(l+1)_{m+n+1}} \times$$

$$\times {}_3F_2\left(-m, -n, \frac{1}{2}; 1, \frac{l-m-n}{2}+1; 1\right) \quad [a > 0]$$

$$7. \int_0^a x^{m+n-1} P_m\left(\frac{x}{a}\right) P_n\left(\frac{x}{a}\right) dx = \frac{(m+n-1)!}{2^{m+n} m! n!} a^{m+n} \quad [a > 0; m+n \geq 1]$$

$$8. \int_0^a x^2 P_{n+2}\left(\frac{x}{a}\right) P_n\left(\frac{x}{a}\right) dx = \frac{a^3 (n+1) (n+2)}{(2n+1) (2n+3) (2n+5)} \quad [a > 0]$$

$$9. \int_0^a x^{\alpha-1} P_{2m+\varepsilon} \left(\frac{x}{b} \right) \left\{ \begin{matrix} P_{2n+1}(x/a) \\ P_{2n}(x/a) \end{matrix} \right\} dx =$$

$$= \frac{(-1)^{m+n} 2^{\varepsilon-1} a^{\alpha+\varepsilon} (1/2)_{m+\varepsilon} ((1+\delta-\varepsilon-\alpha)/2)_n}{m! ((\alpha+\delta+\varepsilon)/2)_{n+1} b^{\varepsilon}} \times$$

$$\times {}_4F_3 \left(-m, \frac{1}{2} + \varepsilon + m, \frac{\alpha+1}{2}, \frac{\alpha}{2} + \varepsilon; \frac{1}{2} + \varepsilon, \frac{1+\alpha+\varepsilon-\delta}{2} - n, \right.$$

$$\left. 1 + \frac{\alpha+\delta+\varepsilon}{2} + n; \frac{a^2}{b^2} \right) \quad \left[\varepsilon=0 \text{ или } 1; b>a>0; \operatorname{Re} \alpha > -\varepsilon-\delta, \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right].$$

$$10. \int_{-a}^a P_m \left(\frac{x}{a} \right) P_n \left(\frac{x}{a} \right) dx = \frac{2a}{2n+1} \delta_{m,n} \quad [a>0].$$

$$11. \int_{-a}^a P_{n-2m} \left(\frac{x}{a} \right) P_n(cx) dx = \frac{(-1)^m 2a^{n+1} c^n}{(1+2n-2m)} P_m^{(-n-1/2, -1)} \left(1 - \frac{2}{a^2 c^2} \right)$$

$$[a>0].$$

$$12. \int_{-a}^a (a \mp x)^{\beta-1} P_m \left(\frac{x}{a} \right) P_n \left(\frac{x}{a} \right) dx = \frac{(\pm 1)^{m+n} (1-\beta)_n}{(\beta)_{n+1}} (2a)^{\beta} \times$$

$$\times {}_2F_3(-m, m+1, \beta, \beta; 1, \beta+n+1, \beta-n; 1) \quad [a, \operatorname{Re} \beta > 0].$$

$$13. \int_{-a}^a (x+a)^{m+n} P_m \left(\frac{x}{a} \right) P_n \left(\frac{x}{a} \right) dx = \frac{[(n+1)_m]^2 (m+1)_n}{2^{m+n-1} m! (3/2)_{m+n}} a^{m+n+1} \quad [a>0].$$

$$14. \int_{-a}^a (x+a)^{2n} \left[P_n \left(\frac{x}{a} \right) \right]^2 dx = \frac{[(1/2)_n]^2 2^{4n+1}}{n! (3/2)_{2n}} a^{2n+1} \quad [a>0].$$

$$15. \int_{-a}^a (x+a)^{m-n-1} P_m \left(\frac{x}{a} \right) P_n \left(\frac{x}{a} \right) dx = 0 \quad [a>0; m>n].$$

$$16. \int_{-a}^a \frac{1}{x-y} P_m \left(\frac{x}{a} \right) P_n \left(\frac{x}{a} \right) dx = -2P_m \left(\frac{y}{a} \right) Q_n \left(\frac{y}{a} \right) + \frac{2}{n+1} \delta_{m,n+1}$$

$$[m \leq n+1; a>0].$$

$$17. \int_{-a}^a \frac{x}{x-y} P_m \left(\frac{x}{a} \right) P_n \left(\frac{x}{a} \right) dx = -2yP_m \left(\frac{y}{a} \right) Q_n \left(\frac{y}{a} \right) + \frac{2a}{2n+1} \delta_{m,n}$$

$$[m \leq n; a>0].$$

$$18. \int_{-a}^a \frac{x^m}{x-y} \left[P_n \left(\frac{x}{a} \right) \right]^2 dx = I_m \quad [a>0].$$

$$I_2 = -2y^2 P_n \left(\frac{y}{a} \right) Q_n \left(\frac{y}{a} \right) + \frac{2ay}{2n+1},$$

$$I_3 = -2y^3 P_n \left(\frac{y}{a} \right) Q_n \left(\frac{y}{a} \right) + \frac{2a}{2n+1} \left[y^3 + \frac{(n+1)(n+2)}{2(2n+3)} a^2 - \frac{n(n-1)}{2(2n-1)} a^2 \right].$$

$$19. \int_{-a}^a \left(\frac{x-y}{x-a} \right)^2 \left[P_m \left(\frac{x}{a} \right) - P_{m-1} \left(\frac{x}{a} \right) \right] \left[P_n \left(\frac{x}{a} \right) - P_{n-1} \left(\frac{x}{a} \right) \right] dx = \frac{2m}{na} (y-a)^2$$

$$[a, y>0; m \leq n].$$

$$20. \int_a^b P_m \left(\frac{x}{b} \right) P_n \left(\frac{x}{a} \right) dx = \frac{b^{n-1} (b^2 - a^2)}{2a^n} {}_2F_1 \left(1 + \frac{m-n}{2}, \frac{1-m-n}{2}; 2; 1 - \frac{a^2}{b^2} \right) \\ [b > a > 0].$$

$$21. \int_a^b x^{\alpha-1} P_{2m+\varepsilon} \left(\frac{x}{b} \right) \left\{ \begin{matrix} P_{2n+1}(x/a) \\ P_{2n}(x/a) \end{matrix} \right\} dx = \\ = \frac{(-1)^{m+n-1} 2^{\varepsilon-1} a^{\alpha+\varepsilon} (1/2)_{m+\varepsilon} ((1+\delta-\varepsilon-\alpha)/2)_n}{m! b^{\varepsilon} ((\alpha+\delta+\varepsilon)/2)_{n+1}} \times \\ \times {}_4F_3 \left(-m, m+\varepsilon+\frac{1}{2}, \frac{\alpha+1}{2}, \frac{\alpha}{2}+\varepsilon; \varepsilon+\frac{1}{2}, \frac{1+\alpha+\varepsilon-\delta}{2}-n, \frac{\alpha+\delta+\varepsilon}{2}+n+1; \right. \\ \left. \frac{a^2}{b^2} \right) + \frac{(-1)^{m+2n+\delta-1} b^{\alpha+2n-\delta} (1/2)_{2n+\delta} ((1+\varepsilon-\alpha-\delta)/2-n)_m}{(2n+\delta)! a^{2n+\delta} ((\alpha+\delta+\varepsilon)/2+n)_{m+1}} \times \\ \times {}_4F_3 \left(-n, \frac{1}{2}-\delta-n, \frac{1+\varepsilon-\alpha-\delta}{2}+m-n, -m-n-\frac{\alpha+\delta+\varepsilon}{2}; \right. \\ \left. \frac{1}{2}-2n-\delta, \frac{1-\alpha}{2}-n, 1-\frac{\alpha}{2}-\delta-n; \frac{a^2}{b^2} \right) \\ [\varepsilon=0 \text{ или } 1; b > a > 0, \delta=\begin{pmatrix} 1 \\ 0 \end{pmatrix}].$$

$$22. \int_a^{\infty} (x+a)^{-\tau} P_m \left(\frac{x}{a} \right) P_n \left(\frac{x}{a} \right) dx = \frac{(-1)^{n+1} 2^{2m-\tau+1} (1/2)_m (\tau-m)_n a^{1-\tau}}{m! (1-\tau+m)_{n+1}} \times \\ \times {}_4F_3 (-m, -m, \tau+n-m, \tau-m-n-1; -2m, \tau-m, \tau-m; 1) \\ [a > 0; \operatorname{Re} \tau > m+n+1].$$

$$23. \int_a^{\infty} x^{\alpha-1} P_{2m+\varepsilon} \left(\frac{x}{b} \right) \left\{ \begin{matrix} P_{2n+1}(x/a) \\ P_{2n}(x/a) \end{matrix} \right\} dx = \\ = \frac{(-1)^{n+1} 2^{2m+\varepsilon-1} a^{\alpha+2m+\varepsilon} (1/2)_{2m+\varepsilon} ((1+\delta-\alpha-\varepsilon)/2-m)_n}{(2m+\varepsilon)! b^{2m+\varepsilon} ((\alpha+\delta+\varepsilon)/2+m)_{n+1}} \times \\ \times {}_4F_3 \left(-m, \frac{1}{2}-\varepsilon-m, \frac{1+\delta-\alpha-\varepsilon}{2}-m+n, -m-n-\frac{\alpha+\varepsilon+\delta}{2}; \right. \\ \left. 1-\frac{\alpha}{2}-\varepsilon-m, \frac{1}{2}-\varepsilon-2m, \frac{1-\alpha}{2}-m; \frac{b^2}{a^2} \right) \\ [\varepsilon=0 \text{ или } 1; a > b > 0; \operatorname{Re} \alpha < -2m-2n-\varepsilon-\delta].$$

$$24. \int_{-1}^1 \frac{d^p P_m(x)}{dx^p} \frac{d^q P_n(x)}{dx^q} dx = 0 \\ [m-n-p+q=1, 3, 5, \dots] \\ = 2^{2-p-q} \sum_{k=1}^p (-1)^{k-1} (p-k)! (q+k-1)! \binom{m+p-k}{p-k} \times \\ \times \binom{m}{p-k} \binom{n+q+k-1}{q+k-1} \binom{n}{q+k-1} \\ [m-p-n+q=0, 2, 4, \dots].$$

2.17.15. Интегралы, содержащие $P_m(\varphi(x)) P_n(cx)$.

$$1. \int_{-a}^a (x+a)^{\alpha-1} P_m(2bx+2ab-1) P_n \left(\frac{x}{a} \right) dx = (-1)^{m+n} \frac{(1-\alpha)_n}{(\alpha)_{n+1}} (2a)^{\alpha} \times \\ \times {}_4F_3 (-m, m+1, \alpha, \alpha; \alpha+n+1, 1, \alpha-n; 2ab) \\ [a, b, \operatorname{Re} \alpha > 0; 2ab < 1].$$

$$2. \int_a^b P_m \left(\frac{2x+a-b}{a+b} \right) P_n \left(\frac{x}{a} \right) dx = \frac{(-1)^m 2^n (1/2)_n (a+b)^{n+1}}{a^n (m+n+1)!} \times \\ \times {}_2F_1 \left(m-n, -m-n-1; -2n; \frac{2a}{a+b} \right) \quad [b > a > 0].$$

$$3. \int_a^b P_m \left(\frac{2x+b-a}{a+b} \right) P_n \left(\frac{x}{b} \right) dx = \frac{(-1)^n 2^{3m+1} (1/2)_m (-m)_n b^{m+1}}{(a+b)^m (m+n+1)!} \times \\ \times {}_2F_1 \left(n-m, -m-n-1; -2m; \frac{a+b}{2b} \right) \quad [b > a > 0].$$

$$4. \int_{\pm a}^b (x \mp a)^{\alpha-1} P_m \left(\frac{2x \mp a-b}{b \mp a} \right) P_n \left(\frac{x}{a} \right) dx = \\ = \frac{(-1)^m (\pm 1)^n (1-\alpha)_m (b \mp a)^\alpha}{(\alpha)_{m+1}} {}_4F_3 \left(-n, n+1, \alpha, \alpha; 1, \alpha+m+1, \alpha-m; \frac{a \mp b}{2a} \right) \\ \left[\operatorname{Re} \alpha > 0, \left\{ \begin{matrix} b > a > 0 \\ a > b > 0 \end{matrix} \right\} \right].$$

$$5. \int_a^b (x+a)^{-\tau} P_m \left(\frac{2x+a-b}{a+b} \right) P_n \left(\frac{x}{a} \right) dx = \\ = \frac{(-1)^{m+n-1} (\tau)_n}{(1-\tau)_{n+1}} (2a)^{1-\tau} {}_4F_3 \left(-m, m+1, 1-\tau, 1-\tau; 2+n-\tau, 1-\tau-n, 1; \frac{2a}{a+b} \right) + \\ + \frac{(-1)^m 2^n (1/2)_n (\tau-n)_m (a+b)^{n-\tau+1}}{n! a^n (1+n-\tau)_{m+1}} \times \\ \times {}_4F_3 \left(-n, -n, \tau+m-n, \tau-m-n-1; -2n, \tau-n, \tau-n; \frac{2a}{a+b} \right) \\ [b > a > 0].$$

$$6. \int_a^b (b+x)^{-\tau} P_m \left(\frac{2x+b-a}{a+b} \right) P_n \left(\frac{x}{b} \right) dx = \frac{(-1)^{m+n+1} (\tau)_m}{(1-\tau)_{m+1}} (a+b)^{1-\tau} \times \\ \times {}_4F_3 \left(-n, n+1, 1-\tau, 1-\tau; 2+m-\tau, 1-m-\tau, 1; \frac{a+b}{2b} \right) + \\ + \frac{(-1)^n 2^{3m-\tau+1} (1/2)_m (\tau-m)_n b^{m-\tau+1}}{m! (1-\tau+m)_{n+1} (a+b)^m} \times \\ \times {}_4F_3 \left(-m, -m, \tau+n-m, \tau-m-n-1; -2m, \tau-m, \tau-m; \frac{a+b}{2b} \right) \\ [b > a > 0].$$

$$7. \int_a^\infty (x+b)^{-\tau} P_m \left(\frac{2x+b-a}{a+b} \right) P_n \left(\frac{x}{b} \right) dx = \frac{(-1)^{m+1} 2^n (1/2)_n (\tau-n)_m}{n! b^n (1+n-\tau)_{m+1}} (a+b)^{n-\tau+1} \times \\ \times {}_4F_3 \left(-n, -n, \tau+m-n, \tau-m-n-1; -2n, \tau-n, \tau-n; \frac{2b}{a+b} \right) \\ [a > b > 0; \operatorname{Re} \tau > m+n+1].$$

$$8. \int_a^\infty (x+a)^{-\tau} P_n (2bx+2ab-1) P_n \left(\frac{x}{a} \right) dx = \frac{(-1)^{n-1} 2^{3m-\tau+1} (1/2)_m (\tau-m)_n}{m! (1+m-\tau)_{n+1}} \times \\ \times a^{m-\tau+1} b^m {}_4F_3 \left(-m, -m, \tau+n-m, \tau-m-n-1; -2m, \tau-m, \tau-m; \frac{1}{2ab} \right) \\ [a > 0; 2ab > 1; \operatorname{Re} \tau > m+n+1].$$

$$9. \int_{-a}^a (x+a)^{\alpha-1} P_{2m+\varepsilon}(b\sqrt{x+a}) P_n\left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^{m+n} 2^{\alpha+\varepsilon/2} (1/2)_{m+\varepsilon} (1-\varepsilon/2-\alpha)_n}{m! (\alpha+\varepsilon/2)_{n+1}} a^{\alpha+\varepsilon/2} b^\varepsilon \times$$

$$\times {}_4F_3\left(-m, \frac{1}{2}+\varepsilon+m, \alpha+\frac{\varepsilon}{2}, \alpha+\frac{\varepsilon}{2}; \alpha+\frac{\varepsilon}{2}+n+1, \varepsilon+\frac{1}{2}, \alpha+\frac{\varepsilon}{2}-n; 2ab^2\right)$$

[$\varepsilon = 0$ или 1 ; $a, b > 0$; $2ab^2 < 1$; $\operatorname{Re} \alpha > -\varepsilon/2$].

$$10. \int_{-a}^a (x+a)^{\alpha-1} P_{2m+\varepsilon}\left(\sqrt{\frac{x+a}{2a}}\right) P_n\left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^{m+n} 2^{\alpha+\varepsilon} (1/2)_{m+\varepsilon} (1-\varepsilon/2-\alpha)_n}{m! (\alpha+\varepsilon/2)_{n+1}} \times$$

$$\times a^\alpha {}_4F_3\left(-m, \frac{1}{2}+\varepsilon+m, \alpha+\frac{\varepsilon}{2}, \alpha+\frac{1+\varepsilon}{2}; 1+\alpha+\frac{\varepsilon}{2}+n, \varepsilon+\frac{1}{2}, \alpha+\frac{\varepsilon}{2}-n; 1\right)$$

[$\varepsilon = 0$ или 1 ; $a > 0$; $\operatorname{Re} \alpha > -\varepsilon/2$].

$$11. \int_a^b P_{2m}\left(\sqrt{\frac{x+a}{b+a}}\right) P_n\left(\frac{x}{a}\right) dx = \frac{(-1)^m 2^n (1/2)_n (-1/2-n)_m (a+b)^{n+1}}{a^n (m+n+1)!} \times$$

$$\times {}_3F_2\left(-n, m-n-\frac{1}{2}, -m-n-1; -2n, -\frac{1}{2}-n; \frac{2a}{a+b}\right) \quad [b > a > 0].$$

$$12. \int_a^b P_{2m}\left(\sqrt{\frac{x+b}{a+b}}\right) P_n\left(\frac{x}{b}\right) dx = \frac{(-1)^n 2^{m+1} (1/2+m)_m (-m)_n b^{m+1}}{m! (m+n+1)! (a+b)^m} \times$$

$$\times {}_3F_2\left(\frac{1}{2}-m, n-m, -m-n-1; \frac{1}{2}-2m, -m; \frac{a+b}{2b}\right) \quad [b > a > 0].$$

$$13. \int_{\pm a}^b (x \mp a)^{\alpha-1} P_{2m+\varepsilon}\left(\sqrt{\frac{x \mp a}{b \mp a}}\right) P_n\left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^m (\pm 1)^n ((1+\varepsilon)/2-\alpha)_m (b \mp a)^\alpha}{(\alpha+\varepsilon/2)_{m+1}} \times$$

$$\times {}_4F_3\left(-n, n+1, \alpha, \alpha+\frac{1}{2}; 1, \frac{1-\varepsilon}{2}-\alpha-m, \alpha+\frac{\varepsilon}{2}+m+1; \frac{a \mp b}{2a}\right)$$

[$\varepsilon = 0$ или 1 ; $\operatorname{Re} \alpha > -\varepsilon/2$, $\left\{\begin{smallmatrix} b > a > 0 \\ a > b > 0 \end{smallmatrix}\right\}$].

$$14. \int_a^b (x+a)^{-\tau} P_{2m+\varepsilon}\left(\sqrt{\frac{x+a}{b+a}}\right) P_n\left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^{m+n-1} 2^{\varepsilon/2-\tau+1} (1/2)_{m+\varepsilon} (\tau-\varepsilon/2)_n a^{\varepsilon/2-\tau+1}}{m! (1+\varepsilon/2-\tau)_{n+1} (a+b)^{\varepsilon/2}} \times$$

$$\times {}_4F_3\left(-m, \frac{1}{2}+\varepsilon+m, 1+\frac{\varepsilon}{2}-\tau, 1+\frac{\varepsilon}{2}-\tau; 2+n+\frac{\varepsilon}{2}-\tau, \frac{1}{2}+\varepsilon, \right.$$

$$\left. 1+\frac{\varepsilon}{2}-\tau-n; \frac{2a}{a+b}\right) + \frac{(-1)^m 2^n (1/2)_n ((\varepsilon-1)/2+\tau-n)_m (a+b)^{n-\tau+1}}{n! a^n (1+n-\tau+\varepsilon/2)_{m+1}} \times$$

$$+ {}_4F_3\left(-n, -n, \frac{\varepsilon-1}{2}+\tau-n+m, \tau-m-n-1-\frac{\varepsilon}{2}; -2n, \tau-n, \tau-n-\frac{1}{2}; \frac{2a}{a+b}\right)$$

[$\varepsilon = 0$ или 1 ; $b > a > 0$].

$$\begin{aligned}
15. \int_a^b (x+b)^{-\tau} P_{2m+\varepsilon} \left(\sqrt{\frac{x+b}{a+b}} \right) P_n \left(\frac{x}{b} \right) dx = & \frac{(-1)^{m+n-1} ((\varepsilon-1)/2 + \tau)_m (a+b)^{1-\tau}}{(1 + \varepsilon/2 - \tau)_{m+1}} \times \\
& \times {}_4F_3 \left(-n, n+1, \frac{3}{2} - \tau, 1 - \tau; 2 + \frac{\varepsilon}{2} + m - \tau, \frac{3-\varepsilon}{2} - \tau - m, 1; \frac{a+b}{2b} \right) + \\
& + \frac{(-1)^n 2^{3m-\tau+3\varepsilon/2+1} (1/2)_{2m+\varepsilon} (\tau-m-\varepsilon/2)_n b^{m-\tau+\varepsilon/2+1}}{(2m+\varepsilon)! (1 + \varepsilon/2 + m - \tau)_{n+1} (a+b)^{m+\varepsilon/2}} \times \\
& \times {}_4F_3 \left(-m, \frac{1}{2} - \varepsilon - m, \tau - \frac{\varepsilon}{2} - m + n, \tau - \frac{\varepsilon}{2} - m - n - 1; \tau - \frac{\varepsilon}{2} - m, \right. \\
& \left. \frac{1}{2} - \varepsilon - 2m, \tau - \frac{\varepsilon}{2} - m; \frac{a+b}{2b} \right) \quad [\varepsilon = 0 \text{ или } 1; b > a > 0].
\end{aligned}$$

$$\begin{aligned}
16. \int_a^\infty (x+a)^{-\tau} P_{2m+\varepsilon} (b \sqrt{x+a}) P_n \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^{n+1} 2^{3m+3\varepsilon/2-\tau+1} (1/2)_{2m+\varepsilon} (\tau-m-\varepsilon/2)_n a^{m+\varepsilon/2-\tau+1} b^{2m+\varepsilon}}{(2m+\varepsilon)! (m + \varepsilon/2 - \tau + 1)_{n+1}} \times \\
\times {}_4F_3 \left(-m, \frac{1}{2} - \varepsilon - m, \tau - \frac{3}{2} - m + n, \tau - \frac{\varepsilon}{2} - m - n - 1; \right. \\
\left. \frac{1}{2} - \varepsilon - 2m, \tau - m - \frac{\varepsilon}{2}, \tau - m - \frac{\varepsilon}{2}; \frac{1}{2ab^2} \right) \\
[\varepsilon = 0 \text{ или } 1; a > 0; 2ab^2 > 1; \operatorname{Re} \tau > m + n + \varepsilon/2 + 1].
\end{aligned}$$

$$\begin{aligned}
17. \int_0^\infty (x+b)^{-\tau} P_{2m+\varepsilon} \left(\sqrt{\frac{x+b}{a+b}} \right) P_n \left(\frac{x}{b} \right) dx = \\
= \frac{(-1)^{m+1} 2^n (1/2)_n ((\varepsilon-1)/2 + \tau - n)_m (a+b)^{n-\tau+1}}{n! b^n (1 + n + \varepsilon/2 - \tau)_{m+1}} \times \\
\times {}_4F_3 \left(-n, -n, \frac{\varepsilon-1}{2} + \tau + m - n, \tau - m - n - \frac{\varepsilon}{2} - 1; -2n, \tau - n - \frac{1}{2}, \tau - n; \frac{2b}{a+b} \right) \\
[\varepsilon = 0 \text{ или } 1; a > b > 0; \operatorname{Re} \tau > m + n + \varepsilon/2 + 1].
\end{aligned}$$

$$\begin{aligned}
18. \int_a^\infty (x+a)^{-\tau} P_{2m+\varepsilon} \left(\sqrt{\frac{x+a}{2a}} \right) P_n \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^{m+1} 2^{2n-\tau+1} (1/2)_n ((\varepsilon-1)/2 + \tau - n)_m a^{1-\tau}}{n! (n + \varepsilon/2 - \tau + 1)_{m+1}} \times \\
\times {}_4F_3 \left(-n, -n, \frac{\varepsilon-1}{2} + \tau + m - n, \tau - m - n - \frac{\varepsilon}{2} - 1; -2n, \tau - n - \frac{1}{2}, \tau - n; 1 \right) \\
[\varepsilon = 0 \text{ или } 1; a > 0; \operatorname{Re} \tau > m + n + \varepsilon/2 + 1].
\end{aligned}$$

$$\begin{aligned}
19. \int_{-a}^a P_{n_1} \left(\frac{x}{a} \right) P_{n_2} \left(\frac{x}{a} \right) \dots P_{n_k} \left(\frac{x}{a} \right) P_{n_1+n_2+\dots+n_k} \left(\frac{x}{a} \right) dx = \\
= \frac{(n_1+n_2+\dots+n_k)!}{(2(n_1+n_2+\dots+n_k)+1)!!} 2a \prod_{j=1}^k \frac{(2n_j-1)!!}{n_j!} \quad (a > 0).
\end{aligned}$$

$$20. \int_{-a}^a P_m\left(\frac{x}{a}\right) P_n\left(\frac{x}{a}\right) P_p\left(\frac{x}{a}\right) P_q\left(\frac{x}{a}\right) dx =$$

$$= 2a \sum_{k=0}^n \frac{2m+2n-4k+1}{2m+2n-2k+1} \frac{A_{m-k} A_k A_{n-k} A_{(p+q-m-n)/2+k}}{p+q+m+n-2k+1} \times$$

$$\times \frac{A_{(p-q+m+n)/2-k} A_{(q-p+m+n)/2-k}}{A_{m+n-k} A_{(p+q+m+n)/2-k}},$$

$$A_j = \frac{(2j-1)!!}{j!}$$

$$[a > 0; p - q \leq m \pm n \leq p + q; p + q + m + n = 2, 4, 6, \dots].$$

$$21. \int_{-a}^a (a-x)^{\beta-1} \operatorname{erf}(ab-bx) \left[P_n\left(\frac{x}{a}\right) \right]^2 dx = \frac{2^{\beta+2n!} a^{\beta+1} b}{\sqrt{\pi} (\beta+1)_{n+1}} \times$$

$$\times {}_3F_3\left(\frac{1}{2}, \frac{\beta+1}{2}, \frac{\beta}{2}+1; \frac{3}{2}, \frac{\beta+n}{2}+1, \frac{\beta+n+3}{2}; -a^2 b^2\right)$$

$$[a > 0; \operatorname{Re} \beta > -1].$$

$$22. \int_{-a}^a (a-x)^{\beta-1} \operatorname{erf}(b\sqrt{a-x}) \left[P_n\left(\frac{x}{a}\right) \right]^2 dx =$$

$$= \frac{2^{\beta+3/2} n! a^{\beta+1/2} b}{\sqrt{\pi} (\beta+1/2)_{n+1}} {}_2F_2\left(\frac{1}{2}, \beta+\frac{1}{2}; \frac{3}{2}, \beta+n+\frac{3}{2}; -ab^2\right)$$

$$[a > 0; \operatorname{Re} \beta > -1/2].$$

2.18. МНОГОЧЛЕНЫ ЧЕБЫШЕВА $T_n(x)$ и $U_n(x)$

Многочлены $T_n(x)$, $U_n(x)$ являются частными случаями многочленов Гегенбауэра и Якоби:

$$T_n(x) = \frac{n}{2} \lim_{\lambda \rightarrow 0} [\lambda^{-1} C_n^\lambda(x)] = \frac{n!}{(1/2)_n} P_n^{(-1/2, -1/2)}(x),$$

$$U_n(x) = C_n^1(x) = \frac{(n+1)!}{(3/2)_n} P_n^{(1/2, 1/2)}(x),$$

поэтому для вычисления интегралов, содержащих эти многочлены, можно воспользоваться формулами из разделов 2.21, 2.22.

2.18.1. Интегралы, содержащие $T_n(\varphi(x))$.

$$1. \int_0^1 \frac{x^{\alpha-1}}{\sqrt{a^2-x^2}} T_n\left(\frac{x}{a}\right) dx = \frac{\pi a^{\alpha-1}}{2^\alpha} \Gamma\left[\frac{\alpha}{(\alpha+n+1)/2, (\alpha-n+1)/2}\right]$$

$$[a, \operatorname{Re} \alpha > 0].$$

$$2. \int_a^b \frac{1}{\sqrt{(x-a)(b-x)}} T_n\left(\frac{x}{b}\right) dx = \frac{\pi}{2} \left[P_n\left(\frac{a}{b}\right) + P_{n-1}\left(\frac{a}{b}\right) \right]$$

$$[b > a; n = 1, 2, 3, \dots].$$

$$3. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\beta-1} T_n\left(\frac{x}{a}\right) dx =$$

$$= (2a)^{\alpha+\beta-1} B(\alpha, \beta) {}_3F_2\left(-n, n, \beta; \frac{1}{2}, \alpha+\beta; 1\right)$$

$$[a, \operatorname{Re} \alpha, \operatorname{Re} \beta > 0].$$

$$4. \int_{-a}^a \frac{(x+a)^{\alpha-1}}{\sqrt{a-x}} T_n \left(\frac{x}{a} \right) dx = 2^{1/2-\alpha} \pi a^{\alpha-1/2} \Gamma \left[\begin{matrix} 2\alpha \\ \alpha+n+1/2, \alpha-n+1/2 \end{matrix} \right]$$

[a, Re $\alpha > 0$].

$$5. \int_{-a}^a \frac{1}{\sqrt{a^2-x^2}(x-y)} T_n \left(\frac{x}{a} \right) dx = \frac{\pi}{a} U_{n-1} \left(\frac{y}{a} \right)$$

[-a < y < a; n = 1, 2, 3, ...].

$$6. \int_a^\infty \frac{x^{\alpha-1}}{\sqrt{x^2-a^2}} T_n \left(\frac{x}{a} \right) dx = \frac{a^{\alpha-1}}{2^{\alpha+1}} \Gamma \left[\begin{matrix} (1-\alpha+n)/2, (1-\alpha-n)/2 \\ 1-\alpha \end{matrix} \right]$$

[a > 0; Re $\alpha < 1-n$].

$$7. \int_0^\infty e^{-px} T_n \left(\frac{x}{a} \right) dx = i^{n+1} a O_n (iap)$$

[Re p > 0].

$$8. \int_0^a \frac{x^{-1/2}}{\sqrt{a^2-x^2}} e^{-px} T_n \left(\frac{x}{a} \right) dx =$$

$$= \frac{\pi}{2} \sqrt{\frac{\pi p}{2}} \left[I_{-(2n+1)/4} \left(\frac{ap}{2} \right) I_{(2n-1)/4} \left(\frac{ap}{2} \right) - I_{(2n+1)/4} \left(\frac{ap}{2} \right) I_{(1-2n)/4} \left(\frac{ap}{2} \right) \right]$$

[a > 0].

$$9. \int_{-a}^a \frac{1}{\sqrt{a^2-x^2}} e^{ipx} T_n \left(\frac{x}{a} \right) dx = i^n \pi J_n (ap)$$

[a > 0].

$$10. \int_{-a}^a \frac{1}{\sqrt{a^2-x^2}} e^{-px} T_n \left(\frac{x}{a} \right) dx = (-1)^n \pi I_n (ap)$$

[a > 0].

$$11. \int_a^\infty \frac{1}{\sqrt{x-a}} e^{-px} T_n \left(\frac{x}{a} \right) dx = \sqrt{8\pi a} e^{-ap} O_{2n} (\sqrt{8ap})$$

[a, Re p > 0].

$$12. \int_a^\infty \frac{1}{\sqrt{x^2-a^2}} e^{-px} T_n \left(\frac{x}{a} \right) dx = K_n (ap)$$

[a, Re p > 0].

$$13. \int_0^a \frac{x^{-1/2}}{\sqrt{a^2-x^2}} e^{-p/x} T_n \left(\frac{x}{a} \right) dx = \sqrt{\frac{\pi}{a}} D_{n-1/2} \left(\sqrt{\frac{2p}{a}} \right) D_{-n-1/2} \left(\sqrt{\frac{2p}{a}} \right)$$

[a, Re p > 0].

$$14. \int_{-a}^a (x+a)^{-n-3/2} (a-x)^{-1/2} \exp \left(-p \frac{a-x}{a+x} \right) T_n \left(\frac{x}{a} \right) dx = \frac{\sqrt{\pi} 2^{-3n-1}}{a^{n+1} p^{n+1/2}} H_{2n} (\sqrt{p})$$

[a, Re p > 0].

$$15. \int_a^\infty \frac{1}{\sqrt{x-a}} e^{-p\sqrt{x-a}} T_n \left(\frac{x}{a} \right) dx = 2 \sqrt{2a} O_{2n} (p \sqrt{2a})$$

[a, Re p > 0].

$$16. \int_0^a \frac{x^{n-2}}{\sqrt{a^2-x^2} \operatorname{ch}^m(b\sqrt{a^2-x^2}/x)} T_n\left(\frac{x}{a}\right) dx = I_m \quad [a, \operatorname{Re} b > 0].$$

$$I_1 = \left(\frac{b}{\pi}\right)^{n-1} \frac{a^{n-2}}{2^n} \left[\zeta\left(n, \frac{2b+\pi}{4\pi}\right) - \zeta\left(n, \frac{2b+3\pi}{4\pi}\right) \right] = \\ = \left(\frac{b}{\pi}\right)^{n-1} a^{n-2} \Phi\left(-1, n, \frac{2b+\pi}{2\pi}\right), \\ I_2 = \frac{na^{n-2}b^{n-1}}{\pi^n} \zeta\left(n+1, \frac{2b+\pi}{2\pi}\right).$$

$$17. \int_0^a \frac{1}{\sqrt{a^2-x^2}} \left\{ \frac{\sin bx T_{2n+1}(x/a)}{\cos bx T_{2n}(x/a)} \right\} dx = (-1)^n \frac{\pi}{2} J_{2n+(1\pm 1)/2}(ab) \quad [a > 0].$$

$$18. \int_{-a}^a \cos(b\sqrt{a^2-x^2}) \left\{ \frac{\sin cx T_{2n+1}(x/a)}{\cos cx T_{2n}(x/a)} \right\} dx = \\ = (-1)^n \pi a J_{2n+(1\pm 1)/2}(a\sqrt{b^2+c^2}) T_{2n+(1\pm 1)/2}\left(\frac{c}{\sqrt{b^2+c^2}}\right) \quad [a > 0].$$

$$19. \int_0^a \frac{x^{\alpha-1}}{\sqrt{a-x}} \ln \frac{x}{a} T_n\left(\frac{x}{a}\right) dx = \\ = \sqrt{\pi} a^{\alpha-1/2} \Gamma\left[\frac{\alpha}{\alpha+1/2}\right] \sum_{k=0}^n \frac{(-n)_k (n)_k}{(\alpha+1/2)_k k! 2^k} \left[\psi(\alpha) - \psi\left(\alpha+k+\frac{1}{2}\right) \right] \\ [a, \operatorname{Re} \alpha > 0].$$

$$20. \int_0^a \frac{1}{\sqrt{a^2-x^2}} \left\{ \frac{\operatorname{erf}(bx)}{\operatorname{erfi}(bx)} \right\} T_{2n+1}\left(\frac{x}{a}\right) dx = \\ = \frac{(\mp 1)^n \sqrt{\pi} (ab)^{2n+1}}{2^{2n+1} (2n+1) n!} {}_1F_1\left(n+\frac{1}{2}; 2n+2; \mp a^2 b^2\right) \quad [a > 0].$$

$$21. \int_0^a \frac{x^{\alpha-1}}{\sqrt{a^2-x^2}} \operatorname{erf}(bx) T_n\left(\frac{x}{a}\right) dx = \frac{\sqrt{\pi} a^{\alpha} b}{2^{\alpha}} \Gamma\left[\frac{\alpha+1}{(\alpha+n)/2+1, (\alpha-n)/2+1}\right] \times \\ \times {}_3F_3\left(\frac{1}{2}, \frac{\alpha+1}{2}, \frac{\alpha}{2}+1; \frac{3}{2}, \frac{\alpha+n}{2}+1, \frac{\alpha-n}{2}+1; -a^2 b^2\right) \\ [a > 0; \operatorname{Re} \alpha > 2[n/2]-n-1]$$

$$22. \int_a^{\infty} (x-a)^{\alpha-1} \operatorname{erfc}(b\sqrt{x-a}) T_n\left(\frac{x}{a}\right) dx = \\ = \frac{1}{\sqrt{\pi} a b^{2\alpha}} \Gamma\left(\alpha+\frac{1}{2}\right) {}_4F_3\left(-n, n, \alpha, \alpha+\frac{1}{2}; \frac{1}{2}, \alpha+1; -\frac{1}{2ab^2}\right) \\ [a, \operatorname{Re} \alpha > 0; |\arg b| < \pi/4].$$

$$23. \int_0^a \frac{1}{\sqrt{a^2-x^2}} e^{\pm b^2 x^2} \left\{ \frac{\operatorname{erf}(bx)}{\operatorname{erfi}(bx)} \right\} T_{2n+1}\left(\frac{x}{a}\right) dx = (\pm 1)^n \frac{\pi}{2} e^{\pm a^2 b^2/2} I_{n+1/2}\left(\frac{a^2 b^2}{2}\right) \\ [a > 0]$$

$$24. \int_0^a \frac{x^{\alpha-1}}{\sqrt{a^2-x^2}} e^{b^2 x^2} \operatorname{erf}(bx) T_n\left(\frac{x}{a}\right) dx = \frac{\sqrt{\pi} a^\alpha b}{2^\alpha} \Gamma\left[\frac{\alpha+1}{(\alpha+n)/2+1}, \frac{(\alpha-n)/2+1}\right] \times \\ \times {}_3F_3\left(1, \frac{\alpha+1}{2}, \frac{\alpha}{2}+1; \frac{3}{2}, \frac{\alpha+n}{2}+1, \frac{\alpha-n}{2}+1; a^2 b^2\right) \\ [a > 0; \operatorname{Re} \alpha > 2[n/2] - n - 1].$$

$$25. \int_0^a \frac{1}{\sqrt{a^2-x^2}} J_\nu(bx) T_n\left(\frac{x}{a}\right) dx = \frac{\pi}{2} J_{(\nu+n)/2}\left(\frac{ab}{2}\right) J_{(\nu-n)/2}\left(\frac{ab}{2}\right) \\ [a > 0; \operatorname{Re} \nu > 2[n/2] - n - 1].$$

$$26. \int_0^a \frac{x^{-1}}{\sqrt{a^2-x^2}} K_\nu\left(\frac{b}{x}\right) T_n\left(\frac{x}{a}\right) dx = \frac{\pi}{2b} W_{n/2, \nu/2}\left(\frac{b}{a}\right) W_{-n/2, \nu/2}\left(\frac{b}{a}\right) \\ [a, \operatorname{Re} b > 0].$$

$$27. \int_{-a}^a T_n^2\left(\frac{x}{a}\right) dx = \frac{4n^2-2}{4n^2-1} a \quad [a > 0].$$

$$28. \int_{-a}^a \frac{1}{\sqrt{a^2-x^2}} T_m\left(\frac{x}{a}\right) T_n\left(\frac{x}{a}\right) dx = \left\{\frac{\pi/2}{\pi}\right\} \delta_{m,n} \quad [a > 0, \left\{\begin{matrix} n > 0 \\ n = 0 \end{matrix}\right\}].$$

$$29. \int_{-a}^a (a \pm x)^{-1/2} (a \mp x)^{\beta-1} T_m\left(\frac{x}{a}\right) T_n\left(\frac{x}{a}\right) dx = \\ = \frac{(\pm 1)^{m+n} \sqrt{\pi} (1/2-\beta)_n}{(2a)^{1/2-\beta}} \Gamma\left[\frac{\beta}{\beta+n+1/2}\right] \times \\ \times {}_4F_3\left(-m, m, \beta, \beta+\frac{1}{2}; \frac{1}{2}, \beta+n+\frac{1}{2}, \beta-n+\frac{1}{2}; 1\right) \quad [a, \operatorname{Re} \beta > 0].$$

$$30. \int_{-a}^a \frac{(x+a)^{m \pm n - 3/2}}{\sqrt{a-x}} T_m\left(\frac{x}{a}\right) T_n\left(\frac{x}{a}\right) dx = \begin{cases} 1 \\ 0 \end{cases} \frac{\pi (2m+2n-2)! a^{m+n-1}}{2^{m+n} (2m-1)! (2n-1)!} \\ [a > 0, \left\{\begin{matrix} m+n \neq 0 \\ m > n \end{matrix}\right\}].$$

$$31. \int_a^b \frac{1}{\sqrt{(b^2-x^2)(x^2-a^2)}} T_m\left(\frac{x}{b}\right) T_n\left(\frac{x}{a}\right) dx = \\ = \frac{\pi b^{n-1}}{2a^n} {}_2F_1\left(\frac{m-n+1}{2}, \frac{1-m-n}{2}; 1; 1-\frac{a^2}{b^2}\right) \quad [b > a > 0].$$

$$32. \int_a^b \frac{1}{\sqrt{(b^2-x^2)(x^2-a^2)}} T_{n-1}\left(\frac{x}{b}\right) T_n\left(\frac{x}{a}\right) dx = \frac{\pi b^{n-1}}{2a^n} \\ [b > a > 0; n = 1, 2, 3, \dots].$$

2.18.2. Интегралы, содержащие $U_n(cx)$.

$$1. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\beta-1} U_n\left(\frac{x}{a}\right) dx = \\ = (2a)^{\alpha+\beta-1} (n+1) B(\alpha, \beta) {}_3F_2\left(-n, n+2, \beta; \alpha+\beta, \frac{3}{2}; 1\right) \\ [a, \operatorname{Re} \alpha, \operatorname{Re} \beta > 0].$$

$$2. \int_{-a}^a (x+a)^{\alpha-1} \sqrt{a-x} U_n \left(\frac{x}{a} \right) dx =$$

$$= (-1)^n (n+1) \sqrt{\pi} 2^{\alpha-1/2} \left(\frac{3}{2} - \alpha \right)_n \Gamma \left[\frac{\alpha}{\alpha+n+3/2} \right] a^{\alpha+1/2} \quad [a, \operatorname{Re} \alpha > 0].$$

$$3. \int_{-a}^a \frac{1}{\sqrt{a^2-x^2}} U_{2n}(cx) dx = \pi P_n(2a^2c^2-1) \quad [a > 0].$$

$$4. \int_{-a}^a \frac{\sqrt{a^2-x^2}}{x-y} U_n \left(\frac{x}{a} \right) dx = -a\pi T_{n+1} \left(\frac{y}{a} \right)$$

$$[-a < y < a; n = 1, 2, 3, \dots].$$

$$5. \int_{-a}^a e^{ipx} U_n \left(\frac{x}{a} \right) dx = \frac{i^{n-1} \pi n}{p} J_n(ap) \quad [a > 0].$$

$$6. \int_a^\infty \sqrt{x^2-a^2} e^{-px} U_n \left(\frac{x}{a} \right) dx = \frac{(n+1)a}{p} K_{n+1}(ap) \quad [a, \operatorname{Re} p > 0].$$

$$7. \int_0^\infty \frac{e^{-px}}{\sqrt{x^2+a^2}} U_n \left(i \frac{x}{a} \right) dx = \frac{e^{\pi(n-1)i/2}}{2} S_n(ap) \quad [\operatorname{Re} p > 0].$$

$$8. \int_{-a}^a (x+a)^{-n-5/2} \sqrt{a-x} \exp \left(-p \frac{a-x}{a+x} \right) U_n \left(\frac{x}{a} \right) dx =$$

$$= \frac{\sqrt{\pi} (n+1)}{2^{3n+3} a^{n+1} p^{n+2}} H_{2n+1}(\sqrt{p}) \quad [a, \operatorname{Re} p > 0].$$

$$9. \int_a^\infty e^{-p\sqrt{x-a}} U_n \left(\frac{x}{a} \right) dx = \frac{4(n+1)a}{2n+1} O_{2n+1}(p\sqrt{2a}) \quad [a, \operatorname{Re} p > 0].$$

$$10. \int_0^a \frac{x^{n-1}}{\exp(p\sqrt{a^2-x^2}/x) \pm 1} U_n \left(\frac{x}{a} \right) dx =$$

$$= \pm \frac{a^n}{2n} \mp \frac{1}{2} \left(\frac{ap}{\pm \pi} \right)^n \zeta \left(n+1, \frac{p}{2\pi} + \frac{1 \pm 1}{4} \right) - \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{\pi}{2p} \quad [a, \operatorname{Re} p > 0].$$

$$11. \int_{-a}^a \sin(b\sqrt{a^2-x^2}) \left\{ \begin{matrix} \sin cx U_{2n+1}(x/a) \\ \cos cx U_{2n}(x/a) \end{matrix} \right\} dx =$$

$$= (-1)^n \frac{\pi ab}{\sqrt{b^2+c^2}} J_{2n+1+\delta}(a\sqrt{b^2+c^2}) U_{2n+\delta} \left(\frac{c}{\sqrt{b^2+c^2}} \right) \quad [a > 0, \delta = \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\}]$$

$$12. \int_{-a}^a (x+a)^{\alpha-1} \sqrt{a-x} \operatorname{erf}(b\sqrt{x+a}) U_n \left(\frac{x}{a} \right) dx =$$

$$= (-1)^n (n+1) (1-\alpha)_n (2a)^{\alpha+1} b \Gamma \left[\frac{\alpha+1/2}{\alpha+n+2} \right] \times$$

$$\times {}_3F_2 \left(\frac{1}{2}, \alpha, \alpha + \frac{1}{2}; \frac{3}{2}, \alpha+n+2, \alpha-n; -2ab^2 \right) \quad [a > 0; \operatorname{Re} \alpha > -1/2].$$

$$\begin{aligned}
13. \int_{-a}^a \sqrt{x+a} (a-x)^{\beta-1} U_m\left(\frac{x}{a}\right) U_n\left(\frac{x}{a}\right) dx = \\
= 2^{\beta-1/2} \sqrt{\pi} (m+1) (n+1) (3/2-\beta)_n \Gamma\left[\frac{\beta}{3/2+\beta+n}\right] a^{\beta+1/2} \times \\
\times {}_4F_3\left(-m, m+2, \beta, \beta-\frac{1}{2}; \frac{3}{2}, \beta+n+\frac{3}{2}, \beta-n-\frac{1}{2}; 1\right) \\
[a, \operatorname{Re} \beta > 0].
\end{aligned}$$

$$\begin{aligned}
14. \int_{-a}^a \sqrt{x+a} (a-x) U_m\left(\frac{x}{a}\right) U_n\left(\frac{x}{a}\right) dx = \\
= \frac{(-1)^{m+n} 16 \sqrt{2} (m+1) (n+1) a^{5/2}}{(2m+2n+3) (2m+2n+5) [1-4(m-n)^2]} \quad [a > 0].
\end{aligned}$$

$$\begin{aligned}
15. \int_{-a}^a (x+a)^{m+n+3/2} \sqrt{a-x} U_m\left(\frac{x}{a}\right) U_n\left(\frac{x}{a}\right) dx = \\
= \frac{\pi (2m+2n+2)!}{2^{m+n+2} (2m+1)! (2n+1)!} a^{m+n+3} \quad [a > 0].
\end{aligned}$$

$$16. \int_{-a}^a (x+a)^{m-n-1/2} \sqrt{a-x} U_m\left(\frac{x}{a}\right) U_n\left(\frac{x}{a}\right) dx = 0 \quad [m > n, a > 0].$$

$$17. \int_{-a}^a \sqrt{a^2-x^2} U_m\left(\frac{x}{a}\right) U_n\left(\frac{x}{a}\right) dx = \frac{\pi}{2} a^2 \delta_{m,n} \quad [a > 0]$$

2.19. МНОГОЧЛЕНЫ ЛАГЕРРА $L_n^\lambda(x)$

2.19.1. Интегралы общего вида.

$$\begin{aligned}
1. \int_0^a x^{\alpha-1} (a^r-x^r)^{\beta-1} e^{-cx} L_n^\lambda(cx) dx = \\
= \frac{a^{\alpha+r\beta-r}}{n!r} \Gamma(\beta) \sum_{k=0}^{\infty} \frac{(1+\lambda+k)_n}{k!} \Gamma\left[\frac{(\alpha+k)/r}{\beta+(\alpha+k)/r}\right] (-ac)^k \\
[a, r, \operatorname{Re} \alpha, \operatorname{Re} \beta > 0].
\end{aligned}$$

$$\begin{aligned}
2. \int_a^\infty x^{\alpha-1} (x^r-a^r)^{\beta-1} e^{-cx} L_n^\lambda(cx) dx = \\
= \frac{a^{\alpha+r\beta-r}}{n!r} \Gamma(\beta) \sum_{k=0}^{\infty} \frac{(1+\lambda+k)_n}{k!} \Gamma\left[\frac{1-\beta-(\alpha+k)/r}{1-(\alpha+k)/r}\right] (-ac)^k + \\
+ \frac{a^{\alpha-r\beta-\alpha}}{n!} \sum_{k=0}^{\infty} \frac{(1-\beta)_k (1-\alpha+\lambda+r-r\beta+rk)_n}{k!} \Gamma(\alpha-r+r\beta-rk) (ac)^{rk} \\
[a, r, \operatorname{Re} c, \operatorname{Re} \beta > 0].
\end{aligned}$$

$$3. \int_0^1 \frac{x^{\alpha-1}}{(x^r + z^r)^\rho} e^{-cx} L_n^\lambda(cx) dx =$$

$$= \frac{z^{\alpha-r\rho}}{n! r \Gamma(\rho)} \sum_{k=0}^{\infty} \frac{(1+\lambda+k)_n}{k!} \Gamma\left(\frac{\alpha+k}{r}\right) \Gamma\left(\rho - \frac{\alpha+k}{r}\right) (-cz)^k +$$

$$+ \frac{c^{\rho-\alpha}}{n!} \sum_{k=0}^{\infty} \frac{(\rho)_k (1-\alpha+\lambda+r\rho+rk)_n}{k!} \Gamma(\alpha-r\rho-rk) (-c^r z^r)^k$$

[$r, \operatorname{Re} c, \operatorname{Re} \alpha > 0; r \mid \arg z < \pi$].

$$4. \int_0^\infty \frac{x^{\alpha-1}}{x^r - y^r} e^{-cx} L_n^\lambda(cx) dx = -\frac{\pi y^{\alpha-r}}{n! r} \sum_{k=0}^{\infty} \frac{(1+\lambda+k)_n}{k!} \operatorname{ctg} \frac{\alpha+k}{r} \pi (-cy)^k +$$

$$+ \frac{c^{\rho-\alpha}}{n!} \sum_{k=0}^{\infty} \frac{(1-\alpha+\lambda+r+rk)_n}{k!} \Gamma(\alpha-r-rk) (cy)^{rk}$$

[$r, y, \operatorname{Re} c, \operatorname{Re} \alpha > 0$].

$$5. \int_0^\infty x^{\alpha-1} e^{-\rho x^r - cx} L_n^\lambda(cx) dx = U$$

[$r, \operatorname{Re} c, \operatorname{Re} \rho, \operatorname{Re} \alpha > 0$].

$$U = \frac{\rho^{-\alpha/r}}{n! r} \sum_{k=0}^{\infty} \frac{(1+\lambda+k)_n}{k!} \Gamma\left(\frac{\alpha+k}{r}\right) \left(-\frac{c}{\rho^{1/r}}\right)^k \quad [r > 1],$$

$$U = \frac{c^{-\alpha}}{n!} \sum_{k=0}^{\infty} \frac{1}{k!} (1-\alpha+\lambda-rk)_n \Gamma(\alpha+rk) \left(-\frac{\rho}{c^r}\right)^k \quad [r < 1],$$

U при $r=1$ см в 2.19.3.2.

$$6. \int_0^\infty x^{\alpha-1} e^{-\rho x^{-r} - cx} L_n^\lambda(cx) dx = \frac{\rho^{\alpha/r}}{n! r} \sum_{k=0}^{\infty} \frac{(1+\lambda+k)_n}{k!} \Gamma\left(-\frac{\alpha+k}{r}\right) (-c\rho^{1/r})^k +$$

$$+ \frac{c^{-\alpha}}{n!} \sum_{k=0}^{\infty} \frac{(1-\alpha+\lambda+rk)_n}{k!} \Gamma(\alpha-rk) (-c^r \rho)^k$$

[$r, \operatorname{Re} c, \operatorname{Re} \rho > 0$].

$$7. \int_0^\infty x^{\alpha-1} e^{-cx} \begin{Bmatrix} \sin bx^r \\ \cos bx^r \end{Bmatrix} L_n^\lambda(cx) dx = V$$

[$b, r, \operatorname{Re} c > 0; \operatorname{Re} \alpha > -\delta r, \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$],

$$V = \frac{b^{-\alpha/r}}{n! r} \sum_{k=0}^{\infty} \frac{(1+\lambda+k)_n}{k!} \Gamma\left(\frac{\alpha+k}{r}\right) \begin{Bmatrix} \sin [(\alpha+k)\pi/(2r)] \\ \cos [(\alpha+k)\pi/(2r)] \end{Bmatrix} \left(-\frac{c}{b^{1/r}}\right)^k \quad [r > 1],$$

$$V = \frac{b^0}{n! c^{\alpha+\delta r}} \sum_{k=0}^{\infty} \frac{(-1)^k (1-\alpha+\lambda-\delta r-2rk)_n}{k! (\delta+1/2)_k} \Gamma(\alpha+\delta r+2rk) \left(\frac{b}{2c^r}\right)^{2k} \quad [r < 1].$$

V при $r=1$ см. в 2.19.5.1.

$$\begin{aligned}
8. \int_0^\infty x^{\alpha-1} e^{-cx} \left\{ \frac{\ln(x^r + z^r)}{\ln|x^r - z^r|} \right\} L_n^\lambda(cx) dx = \\
= \frac{\pi z^\alpha}{n!} \sum_{k=0}^\infty \frac{(1+\lambda+k)_n}{k! (\alpha+k)} \left\{ \frac{\operatorname{cosec}[(\alpha+k)\pi/r]}{\operatorname{ctg}[(\alpha+k)\pi/r]} \right\} (-z)^k \pm \\
\pm \frac{c^{r-\alpha} z^r}{n!} \sum_{k=0}^\infty \frac{(1-\alpha+\lambda+r+rk)_n}{k!} \Gamma(\alpha-r-rk) (\mp c^r z^r)^k + \\
+ \frac{r(1-\alpha+\lambda)_n}{n! c^\alpha} \Gamma(\alpha) \left[\psi(\alpha) - \sum_{k=1}^n \frac{1}{k-\alpha+\lambda} - \ln c \right] \\
\left[r, \operatorname{Re} c, \operatorname{Re} \alpha > 0, \left\{ \begin{array}{l} r | \arg z | < \pi \\ z > 0 \end{array} \right\} \right].
\end{aligned}$$

$$9. \int_0^\infty x^{\alpha-1} e^{-bx^r - cx} L_m^\gamma(bx^r) L_n^\lambda(cx) dx = W \quad [r, \operatorname{Re} b, \operatorname{Re} c, \operatorname{Re} \alpha > 0],$$

$$W = \frac{b^{-\alpha/r}}{m! n! r} \sum_{k=0}^\infty \frac{(1+\lambda+k)_n}{k!} \left(1 + \gamma - \frac{\alpha+k}{r} \right)_m \Gamma\left(\frac{\alpha+k}{r}\right) \left(-\frac{c}{b^{1/r}}\right)^k \quad [r > 1],$$

W при $r=1$ см. в 2.19.14.9.

$$10. \int_0^\infty x^{\alpha-1} e^{-cx} \operatorname{Ei}(\mp bx^r) L_n^\lambda(cx) dx = X \quad [r, \operatorname{Re} b, \operatorname{Re} c, \operatorname{Re} \alpha > 0],$$

$$X = -\frac{b^{-\alpha/r}}{n!} \sum_{k=0}^\infty \frac{(1+\lambda+k)_n}{k! (\alpha+k)} \Gamma\left(\frac{\alpha+k}{r}\right) \left(-\frac{c}{b^{1/r}}\right)^k \quad [r > 1],$$

$$\begin{aligned}
X = -\frac{b}{n! c^{\alpha+r}} \sum_{k=0}^\infty \frac{(1-\alpha+\lambda-r-rk)_n}{k! (k+1)^2} \Gamma(\alpha+r+rk) \left(-\frac{b}{c^r}\right)^k + \\
+ \frac{(1-\alpha+\lambda)_n}{n! c^\alpha} \Gamma(\alpha) \left[C + r\psi(\alpha) - r \sum_{k=1}^n \frac{1}{k-\alpha+\lambda} - \ln \frac{c^r}{b} \right] \quad [r < 1].
\end{aligned}$$

X при $r=1$ см. в 2.19.7.2.

$$11. \int_0^\infty x^{\alpha-1} e^{\pm bx^r - cx} \operatorname{Ei}(\mp bx^r) L_n^\lambda(cx) dx = Y \quad \left[r, \operatorname{Re} c, \operatorname{Re} \alpha > 0, \left\{ \begin{array}{l} | \arg b | < \pi \\ b > 0 \end{array} \right\} \right],$$

$$\begin{aligned}
Y = -\frac{b^{-\alpha/r} \pi}{n! r} \sum_{k=0}^\infty \frac{(1+\lambda+k)_n}{k!} \Gamma\left(\frac{\alpha+k}{r}\right) \left\{ \frac{\operatorname{cosec}[(\alpha+k)\pi/r]}{\operatorname{ctg}[(\alpha+k)\pi/r]} \right\} \left(-\frac{c}{b^{1/r}}\right)^k \mp \\
\mp \frac{c^{r-\alpha}}{n! b} \sum_{k=0}^\infty k! (1-\alpha+\lambda+r+rk)_n \Gamma(\alpha-r-rk) \left(\mp \frac{c^r}{b}\right)^k \quad [r > 1],
\end{aligned}$$

$$\begin{aligned}
Y = -\frac{c^{-\alpha}}{n!} \sum_{k=0}^\infty \frac{(1-\alpha+\lambda-rk)_n}{k!} \Gamma(\alpha+rk) \left[\psi(k+1) - r\psi(\alpha+rk) + \right. \\
\left. + r \sum_{l=1}^n \frac{1}{l-\alpha+\lambda-rk} - \ln \frac{b}{c^r} \right] \left(\pm \frac{b}{c^r}\right)^k \quad [r < 1].
\end{aligned}$$

Y при $r=1$ см. в 2.19.7.3.

$$12. \int_0^{\infty} x^{\alpha-1} e^{-cx} \begin{Bmatrix} \text{si}(bx^r) \\ \text{ci}(bx^r) \end{Bmatrix} L_n^{\lambda}(cx) dx = Z \quad [b, r, \text{Re} c, \text{Re} \alpha > 0],$$

$$Z = -\frac{b^{-\alpha/r}}{n!} \sum_{k=0}^{\infty} \frac{(1+\lambda+k)_n}{k! (\alpha+k)} \Gamma\left(\frac{\alpha+k}{r}\right) \begin{Bmatrix} \sin [(\alpha+k) \pi/(2r)] \\ \cos [(\alpha+k) \pi/(2r)] \end{Bmatrix} \left(-\frac{c}{b^{1/r}}\right)^k \quad [r > 1],$$

$$Z = -\frac{b^{\delta+r}}{2 \cdot 3^{\delta} n! c^{\alpha+\delta r+2r}} \sum_{k=0}^{\infty} \frac{(-1)^k}{(k+1)! (\delta+3/2)_k} \times \\ \times \frac{(1-\alpha+\lambda-\delta r-2r-2rk)_n}{(\delta+2+2k)} \Gamma(\alpha+\delta r+2r+2rk) \left(\frac{b}{2c^r}\right)^{2k} + \\ + \frac{b^{\delta} (1-\alpha+\lambda-\delta r)_n}{n! c^{\alpha+\delta r}} \Gamma(\alpha+\delta r) \begin{Bmatrix} 1 \\ R \end{Bmatrix} - \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{\pi (1-\alpha+\lambda)_n}{2c^{\alpha} n!} \Gamma(\alpha), \\ R = C + r\psi(\alpha) - r \sum_{k=1}^n \frac{1}{k-\alpha+\lambda} - \ln \frac{c^r}{b} \quad [r < 1],$$

Z при $r=1$ см. в 2.19.8.1.

$$13. \int_0^{\infty} x^{\alpha-1} e^{-cx} \begin{Bmatrix} \text{erf}(bx^r) \\ \text{erfc}(bx^r) \end{Bmatrix} L_n^{\lambda}(cx) dx = U$$

$[r, \text{Re} c > 0; |\arg b| < \pi/4; \text{Re} \alpha > -r(1 \pm 1)/2],$

$$U = \mp \frac{b^{-\alpha/r}}{\sqrt{\pi} n!} \sum_{k=0}^{\infty} \frac{(1+\lambda+k)_n}{k! (\alpha+k)} \Gamma\left(\frac{\alpha+k+r}{2r}\right) \left(-\frac{c}{b^{1/r}}\right)^k + \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{(1-\alpha+\lambda)_n}{n! c^{\alpha}} \Gamma(\alpha) \quad [r > 1/2],$$

$$U = \pm \frac{b}{\sqrt{\pi} n! c^{\alpha+r}} \sum_{k=0}^{\infty} \frac{(-1)^k (1-\alpha+\lambda-r-2rk)_n}{k! (k+1/2)} \Gamma(\alpha+r+2rk) \times \\ \times \left(\frac{b}{c^r}\right)^{2k} + \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \frac{(1-\alpha+\lambda)_n}{n! c^{\alpha}} \Gamma(\alpha) \quad [r < 1/2],$$

U при $r=1/2$ см. в 2.19.9.4.

$$14. \int_0^{\infty} x^{\alpha-1} e^{-cx} \begin{Bmatrix} S(bx^r) \\ C(bx^r) \end{Bmatrix} L_n^{\lambda}(cx) dx = V$$

$[b, r, \text{Re} c > 0; \text{Re} \alpha > -r(\delta+1/2), \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}],$

$$V = -\frac{b^{-\alpha/r}}{\sqrt{2\pi} n!} \sum_{k=0}^{\infty} \frac{(1+\lambda+k)_n}{k! (\alpha+k)} \Gamma\left(\frac{\alpha+k}{r} + \frac{1}{2}\right) \times \\ \times \begin{Bmatrix} \sin [(2\alpha+r+2k) \pi/(4r)] \\ \cos [(2\alpha+r+2k) \pi/(4r)] \end{Bmatrix} \left(-\frac{c}{b^{1/r}}\right)^k + \frac{(1-\alpha+\lambda)_n}{2 (n!) c^{\alpha}} \Gamma(\alpha) \quad [r > 1],$$

$$V = \frac{b^{\delta+1/2}}{\sqrt{2\pi} n! c^{\alpha+r\delta+r/2}} \sum_{k=0}^{\infty} \frac{(-1)^k (1-\alpha+\lambda-r\delta-r/2-2rk)_n}{k! (\delta+1/2)_k (\delta+1/2+2k)} \times \\ \times \Gamma(\alpha+r\delta+r/2+2rk) \left(\frac{b}{2c^r}\right)^{2k} \quad [r < 1],$$

V при $r=1$ см. в 2.19.10.1.

$$15. \int_0^{\infty} x^{\alpha-1} e^{-cx} \left\{ \frac{\gamma(\mu, bx^r)}{\Gamma(\mu, bx^r)} \right\} L_n^{\lambda}(cx) dx = W$$

$$\left[r, \operatorname{Re} b, \operatorname{Re} c, \operatorname{Re}(\alpha + \mu r) > 0, \begin{cases} \operatorname{Re} \mu > 0 \\ \operatorname{Re} \alpha > 0 \end{cases} \right],$$

$$W = \mp \frac{b^{-\alpha/r}}{n!} \sum_{k=0}^{\infty} \frac{(1+\lambda+k)_n}{k! (\alpha+k)} \Gamma\left(\frac{\alpha+k}{r} + \mu\right) \left(-\frac{c}{b^{1/r}}\right)^k +$$

$$+ \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{(1-\alpha+\lambda)_n}{n! c^{\alpha}} \Gamma(\alpha) \Gamma(\mu) \quad [r > 1],$$

$$W = \pm \frac{b^{\mu}}{n! c^{\alpha+\mu r}} \sum_{k=0}^{\infty} \frac{(1-\alpha+\lambda-\mu r-rk)_n}{k! (\mu+k)} \Gamma(\alpha+\mu r+rk) \times$$

$$\times \left(-\frac{b}{c^r}\right)^k + \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \frac{(1-\alpha+\lambda)_n}{n! c^{\alpha}} \Gamma(\alpha) \Gamma(\mu) \quad [r < 1],$$

W при $r=1$ см. в 2.19.11.1.

$$16. \int_0^{\infty} x^{\alpha-1} e^{-cx} J_{\mu}(bx^r) L_n^{\lambda}(cx) dx = X \quad [b, r, \operatorname{Re} c, \operatorname{Re}(\alpha + \mu r) > 0],$$

$$X = \frac{2^{\alpha/r-1}}{b^{\alpha/r} n! r} \sum_{k=0}^{\infty} \frac{(-1)^k (1+\lambda+k)_n}{k!} \Gamma\left[\frac{(\mu r + \alpha + k)/(2r)}{(\mu r - \alpha - k)/(2r) + 1}\right] \left(\frac{2c^r}{b}\right)^{k/r}$$

$$X = X_{-}, \quad [r > 1],$$

$$X_{\mp} = \frac{b^{\mu}}{2^{\mu} n! c^{\alpha+\mu r} \Gamma(\mu+1)} \sum_{k=0}^{\infty} \frac{(\mp 1)^k (1-\alpha+\lambda-\mu r-2rk)_n}{k! (\mu+1)_k} \times$$

$$\times \Gamma(\alpha+\mu r+2rk) \left(\frac{b}{2c^r}\right)^{2k} \quad [r < 1],$$

X при $r=1$ см. в 2.19.12.3.

$$17. \int_0^{\infty} x^{\alpha-1} e^{-cx} I_{\mu}(bx^r) L_n^{\lambda}(cx) dx = X_{+}$$

$$[r, \operatorname{Re} c, \operatorname{Re}(\alpha + \mu r) > 0; |\arg b| < \pi; X_{+} \text{ см. в 2.19.1.16 и 2.19.12.3}].$$

$$18. \int_0^{\infty} x^{\alpha-1} e^{-cx} \left\{ \frac{Y_{\mu}(bx^r)}{K_{\mu}(bx^r)} \right\} L_n^{\lambda}(cx) dx = U_{\mu}$$

$$\left[\operatorname{Re} c > 0; \operatorname{Re} \alpha > r |\operatorname{Re} \mu|, \begin{cases} b > 0 \\ \operatorname{Re} b > 0 \end{cases} \right],$$

$$U_{\mu} = \mp \frac{2^{\alpha/r-2}}{\pi b^{\alpha/r} n! r} \sum_{k=0}^{\infty} \frac{(1+\lambda+k)_n}{k!} \Gamma\left(\frac{\alpha+\mu r+k}{2r}\right) \times$$

$$\times \Gamma\left(\frac{\alpha+k-\mu r}{2r}\right) \pi^{(1 \mp 1)/2} \left(2 \cos \frac{\mu r - \alpha - k}{2r} \pi\right)^{(1 \pm 1)/2} \left(-\frac{2^{1/r} c}{b^{1/r}}\right)^k$$

$$[r > 1],$$

$$U_{\mu} = \mp \frac{c^{-\alpha}}{2\pi n!} \begin{Bmatrix} 2 \\ \pi \end{Bmatrix} \sum_{k=0}^{\infty} \frac{1}{k!} \left[A_k(\mu) + \begin{Bmatrix} \cos \mu \pi \\ 1 \end{Bmatrix} A_k(-\mu) \right] \left(\mp \frac{b^2}{4c^{2r}}\right)^k,$$

$$A_k(\mu) = \left(\frac{2c^r}{b}\right)^{\mu} \frac{\Gamma(\mu)}{(1-\mu)_k} \Gamma(\alpha-\mu r+2rk) (1-\alpha+\lambda+\mu r-2rk)_n$$

$$[r < 1; \mu \neq \dots, -2, -1, 0, 1, 2, \dots],$$

$$U_m = \mp \frac{c^{-\alpha}}{2\pi n!} \left\{ \frac{2}{\pi} \right\} \left\{ \sum_{k=0}^{\infty} \frac{(-1)^k (m)!}{k!} \left(\mp \frac{b^2}{4c^{2r}} \right)^k + \right. \\ \left. + \left(\pm \frac{b}{2c^r} \right)^m \sum_{k=0}^{\infty} \frac{1}{k! (m+k)!} \Gamma(\alpha + mr + 2rk) \times \right. \\ \left. \times (1 - \alpha + \lambda - mr - 2rk)_n \left[\psi(k+1) + \psi(k+m+1) - 2r\psi(\alpha + mr + 2rk) + \right. \right. \\ \left. \left. + 2r \sum_{l=0}^{n-1} \frac{1}{1 - \alpha + \lambda - mr - 2rk + l} + 2 \ln \frac{2c^r}{b} \right] \left(\mp \frac{b^2}{4c^{2r}} \right)^k \right\} \\ [r < 1; m = 0, 1, 2, \dots]$$

U_μ при $r=1$ см. в 2.19.13.3.

2.19.2. Интегралы от $A(x) L_n^\lambda(cx)$.

$$1. \int_0^a x^{\alpha-1} (a-x)^{\beta-1} L_n^\lambda(cx) dx = \\ = \frac{a^{\alpha+\beta-1} (\lambda+1)_n}{n!} B(\alpha, \beta) {}_2F_2(-n, \alpha; \alpha+\beta, \lambda+1; ac) \\ [a, \operatorname{Re} \alpha, \operatorname{Re} \beta > 0].$$

$$2. \int_0^a x^\lambda (a-x)^{\beta-1} L_n^\lambda(cx) dx = a^{\beta+\lambda} B(\beta, \lambda+n+1) L_n^{\beta+\lambda}(ac) \\ [a, \operatorname{Re} \beta > 0; \operatorname{Re} \lambda > -1].$$

$$3. \int_a^\infty x^{\alpha-1} (x-a)^{\beta-1} L_n^\lambda(cx) dx = \\ = \frac{a^{\alpha+\beta-1} (\lambda+1)_n}{n!} B(\beta, 1-\alpha-\beta) {}_2F_2(-n, \alpha; \alpha+\beta, \lambda+1; ac) \\ [a, \operatorname{Re} \beta > 0; \operatorname{Re}(\alpha+\beta) < 1-n].$$

$$4. \int_0^\infty \frac{x^{\alpha-1}}{(x+z)^\rho} L_n^\lambda(cx) dx = \frac{z^{\alpha-\rho} (\lambda+1)_n}{n!} B(\alpha, \rho-\alpha) {}_2F_2(-n, \alpha; \lambda+1, 1+\alpha-\rho; -cz) \\ [\operatorname{Re} \alpha > 0; \operatorname{Re}(\alpha-\rho) < -n; |\arg z| < \pi].$$

2.19.3. Интегралы от $A(x) e^{-px} L_n^\lambda(cx)$.

$$1. \int_0^\infty e^{-px} L_n^\lambda(cx) dx = \sum_{k=0}^n \frac{(\lambda)_k (p-c)^{n-k}}{p^{n-k+1}} = \\ = \frac{(\lambda+1)_n}{n! (p-c)} {}_2F_1\left(1, \lambda+n+1; \lambda+1; \frac{c}{c-p}\right) \\ [\operatorname{Re} p > 0].$$

$$2. \int_0^\infty x^{\alpha-1} e^{-px} L_n^\lambda(cx) dx = \frac{\Gamma(\alpha)}{p^\alpha} P_n^{(\lambda, \alpha-\lambda-n-1)}\left(1 - \frac{2c}{p}\right) \\ [\operatorname{Re} p, \operatorname{Re} \alpha > 0].$$

$$3. \int_0^\infty x^\lambda e^{-px} L_n^\lambda(cx) dx = \frac{\Gamma(\lambda+n+1) (p-c)^n}{n! p^{\lambda+n+1}} \\ [\operatorname{Re} p > 0; \operatorname{Re} \lambda > -1].$$

$$4. \int_0^{\infty} x^{n+2\lambda} e^{-px} L_n^{\lambda}(cx) dx = \frac{2^{2\lambda} \Gamma(\lambda + 1/2)}{\sqrt{\pi} p^{\lambda+n+1}} \Gamma(\lambda + n + 1) C_n^{\lambda+1/2} \left(1 - \frac{2c}{p}\right)$$

[Re p > 0; Re λ > -(n+1)/2].

$$5. \int_0^{\infty} x^{\alpha-1} e^{-cx} L_n^{\lambda}(cx) dx = \frac{(1-\alpha+\lambda)_n}{n! c^{\alpha}} \Gamma(\alpha)$$

[Re c, Re α > 0].

$$6. \int_0^a x^{\alpha-1} (a-x)^{\beta-1} e^{-cx} L_n^{\lambda}(cx) dx =$$

$$= \frac{a^{\alpha+\beta-1} (1+\lambda)_n}{n!} B(\alpha, \beta) {}_2F_2(\alpha, 1+\lambda+n; \alpha+\beta, 1+\lambda; -ac)$$

[a, Re α, Re β > 0].

$$7. \int_a^{\infty} (x-a)^{\beta-1} e^{-cx} L_n^{\lambda}(cx) dx = \frac{\Gamma(\beta)}{c^{\beta}} e^{-ac} L_n^{\lambda-\beta}(ac)$$

[a, Re c, Re β > 0].

$$8. \int_a^{\infty} x^{\alpha-1} (x-a)^{\beta-1} e^{-cx} L_n^{\lambda}(cx) dx =$$

$$= \frac{a^{\alpha+\beta-1} (1+\lambda)_n}{n!} B(\beta, 1-\alpha-\beta) {}_2F_2(\alpha, 1+\lambda+n; \alpha+\beta, 1+\lambda; -ac) +$$

$$+ \frac{c^{1-\alpha-\beta}}{n!} (2-\alpha-\beta+\lambda)_n \Gamma(\alpha+\beta-1) \times$$

$$\times {}_2F_2(1-\beta, 2-\alpha-\beta+\lambda+n; 2-\alpha-\beta, 2-\alpha-\beta+\lambda; -ac)$$

[a, Re c, Re β > 0].

$$9. \int_0^{\infty} \frac{x^{\alpha-1}}{(x+z)^{\rho}} e^{-cx} L_n^{\lambda}(cx) dx =$$

$$= \frac{z^{\alpha-\rho} (1+\lambda)_n}{n!} B(\alpha, \rho-\alpha) {}_2F_2(\alpha, 1+\lambda+n; 1+\alpha-\rho, 1+\lambda; cz) + \frac{c^{\rho-\alpha}}{n!} \times$$

$$\times (1-\alpha+\lambda+\rho)_n \Gamma(\alpha-\rho) {}_2F_2(\rho, 1-\alpha+\lambda+\rho+n; 1-\alpha+\lambda+\rho, 1-\alpha+\rho; cz)$$

[Re c, Re α > 0; |arg z| < π].

$$10. \int_0^{\infty} \frac{x^{\alpha-1}}{x-y} e^{-cx} L_n^{\lambda}(cx) dx =$$

$$= \frac{c^{1-\alpha}}{n!} (2-\alpha+\lambda)_n \Gamma(\alpha-1) {}_2F_2(1, 2-\alpha+\lambda+n; 2-\alpha, 2-\alpha+\lambda; -cy) -$$

$$- (-1)^n \pi y^{\alpha-1} \operatorname{ctg} \alpha \pi (\lambda+1)_n e^{-cy} L_n^{\lambda}(cy)$$

[y, Re c, Re α > 0].

$$11. \int_0^a x^{\alpha-1} (a^2-x^2)^{\beta-1} e^{-cx} L_n^{\lambda}(cx) dx = \frac{a^{\alpha+2\beta-2} (1+\lambda)_n}{2(n!)} B\left(\frac{\alpha}{2}, \beta\right) \times$$

$$\times {}_2F_4\left(\frac{\alpha}{2}, \frac{1+\lambda+n}{2}, 1+\frac{\lambda+n}{2}; \frac{1}{2}, \frac{\alpha}{2}+\beta, \frac{1+\lambda}{2}, 1+\frac{\lambda}{2}; \frac{a^2 c^2}{4}\right) -$$

$$- \frac{a^{\alpha+2\beta-1} c (2+\lambda)_n}{2(n!)} B\left(\frac{\alpha+1}{2}, \beta\right) \times$$

$$\times {}_2F_4\left(\frac{\alpha+1}{2}, 1+\frac{\lambda+n}{2}, \frac{3+\lambda+n}{2}; \frac{3}{2}, \frac{\alpha+1}{2}+\beta, 1+\frac{\lambda}{2}, \frac{3+\lambda}{2}; \frac{a^2 c^2}{4}\right)$$

[a, Re α, Re β > 0].

$$\begin{aligned}
12. \int_1^{\infty} x^{\alpha-1} (x^2 - a^2)^{\beta-1} e^{-cx} L_n^{\lambda}(cx) dx &= \frac{a^{\alpha+2\beta-2} (1+\lambda)_n}{2(n!)} B\left(1 - \frac{\alpha}{2} - \beta, \beta\right) \times \\
&\times {}_3F_4\left(\frac{\alpha}{2}, \frac{1+\lambda+n}{2}, 1 + \frac{\lambda+n}{2}; \frac{1}{2}, \frac{\alpha}{2} + 3, \frac{1+\lambda}{2}, 1 + \frac{\lambda}{2}; \frac{a^2 c^2}{4}\right) - \\
&- \frac{a^{\alpha+2\beta-1} c (2+\lambda)_n}{2(n!)} B\left(\frac{1-\alpha}{2} - \beta, \beta\right) \times \\
&\times {}_3F_4\left(\frac{\alpha+1}{2}, 1 + \frac{\lambda+n}{2}, \frac{3+\lambda+n}{2}; \frac{3}{2}, \frac{\alpha+1}{2} + \beta, 1 + \frac{\lambda}{2}, \frac{3+\lambda}{2}; \frac{a^2 c^2}{4}\right) + \\
&+ \frac{c^{2-\alpha-2\beta}}{n!} (3-\alpha-2\beta+\lambda)_n \Gamma(\alpha+2\beta-2) {}_3F_4\left(1-\beta, \frac{3+n-\alpha+\lambda}{2}, \right. \\
&\left. -\beta, 2-\beta + \frac{n-\alpha+\lambda}{2}; \frac{3-\alpha+\lambda}{2} - \beta, 2-\beta + \frac{\lambda-\alpha}{2}, 2-\frac{\alpha}{2} - \beta, \frac{3-\alpha}{2} - \beta; \frac{a^2 c^2}{4}\right) \\
&\quad [a, \operatorname{Re} c, \operatorname{Re} \beta > 0]
\end{aligned}$$

$$\begin{aligned}
13. \int_0^{\infty} \frac{x^{\alpha-1}}{(x^2 + z^2)^{\rho}} e^{-cx} L_n^{\lambda}(cx) dx &= \frac{z^{\alpha-2\rho}}{2(n!)} B\left(\frac{\alpha}{2}, \rho - \frac{\alpha}{2}\right) (1+\lambda)_n \times \\
&\times {}_3F_4\left(\frac{\alpha}{2}, \frac{1+\lambda+n}{2}, 1 + \frac{\lambda+n}{2}; \frac{1}{2}, 1 + \frac{\alpha}{2} - \rho, \frac{1+\lambda}{2}, 1 + \frac{\lambda}{2}; -\frac{c^2 z^2}{4}\right) - \\
&- \frac{c z^{\alpha-2\rho+1}}{2(n!)} B\left(\frac{\alpha+1}{2}, \rho - \frac{\alpha+1}{2}\right) (2+\lambda)_n \times \\
&\times {}_3F_4\left(\frac{\alpha+1}{2}, 1 + \frac{\lambda+n}{2}, \frac{3+\lambda+n}{2}; \frac{3}{2}, \frac{3+\alpha}{2} - \rho, 1 + \frac{\lambda}{2}, \frac{3+\lambda}{2}; -\frac{c^2 z^2}{4}\right) + \\
&+ \frac{c^{2\rho-\alpha}}{n!} (1-\alpha+\lambda+2\rho)_n \Gamma(\alpha-2\rho) {}_3F_4\left(\rho, \frac{1-\alpha+\lambda+n}{2} + \rho, \right. \\
&\left. 1 - \frac{\alpha-\lambda-n}{2} + \rho; \frac{1-\alpha+\lambda}{2} + \rho, 1 + \frac{\lambda-\alpha}{2} + \rho, 1 + \rho - \frac{\alpha}{2}, \frac{1-\alpha}{2} + \rho; -\frac{c^2 z^2}{4}\right) \\
&\quad [\operatorname{Re} c, \operatorname{Re} z, \operatorname{Re} \alpha > 0].
\end{aligned}$$

$$\begin{aligned}
14. \int_0^{\infty} \frac{x^{\alpha-1}}{x^2 - y^2} e^{-cx} L_n^{\lambda}(cx) dx &= -\frac{\pi y^{\alpha-2}}{2(n!)} (1+\lambda)_n \operatorname{ctg} \frac{\alpha\pi}{2} \times \\
&\times {}_2F_3\left(\frac{1+\lambda+n}{2}, 1 + \frac{\lambda+n}{2}; \frac{1}{2}, \frac{1+\lambda}{2}, 1 + \frac{\lambda}{2}; \frac{c^2 y^2}{4}\right) - \\
&- \frac{\pi y^{\alpha-1} c (2+\lambda)_n}{2(n!)} \operatorname{tg} \frac{\alpha\pi}{2} {}_2F_3\left(1 + \frac{\lambda+n}{2}, \frac{3+\lambda+n}{2}; \frac{3}{2}, 1 + \frac{\lambda}{2}, \frac{3+\lambda}{2}; \frac{c^2 y^2}{4}\right) + \\
&+ \frac{c^{2-\alpha}}{n!} \Gamma(\alpha-2) (3-\alpha+\lambda)_n {}_3F_4\left(\frac{3-\alpha+\lambda+n}{2}, 2 - \frac{\alpha-\lambda-n}{2}, \right. \\
&\left. 1, \frac{3-\alpha}{2}, 2 - \frac{\alpha}{2}, \frac{3-\alpha+\lambda}{2}, 2 + \frac{\lambda-\alpha}{2}; \frac{c^2 y^2}{4}\right) \\
&\quad [y, \operatorname{Re} c, \operatorname{Re} \alpha > 0].
\end{aligned}$$

$$\begin{aligned}
15. \int_0^a x^{\alpha-1} (\sqrt{a} - \sqrt{x})^{\beta-1} e^{-cx} L_n^{\lambda}(cx) dx &= \frac{2a^{\alpha+(\beta-1)/2} (1+\lambda)_n}{n!} B(2\alpha, \beta) \times \\
&\times {}_3F_3\left(\alpha, \alpha + \frac{1}{2}, 1 + \lambda + n; \alpha + \frac{\beta}{2}, \alpha + \frac{\beta+1}{2}, 1 + \lambda; -ac\right) \\
&\quad [a, \operatorname{Re} \alpha, \operatorname{Re} \beta > 0].
\end{aligned}$$

$$\begin{aligned}
16. \int_a^\infty x^{\alpha-1} (\sqrt{x}-\sqrt{a})^{\beta-1} e^{-cx} L_n^\lambda(cx) dx &= \frac{2a^{\alpha+(\beta-1)/2} (1+\lambda)_n}{n!} B(1-2\alpha-\beta, \beta) \times \\
&\times {}_3F_2\left(\alpha, \frac{1}{2}+\alpha, 1+\lambda+n; \frac{1+\beta}{2}+\alpha, \frac{\beta}{2}+\alpha, 1+\lambda; -ac\right) + \\
&+ \frac{c^{(1-\beta)/2-\alpha}}{n!} \left(\frac{3-\beta}{2}-\alpha+\lambda\right)_n \Gamma\left(\alpha+\frac{\beta-1}{2}\right) \times \\
&\times {}_3F_3\left(\frac{1-\beta}{2}, 1-\frac{\beta}{2}, \frac{3-\beta}{2}-\alpha+\lambda+n; \frac{1}{2}, \frac{3-\beta}{2}-\alpha, \frac{3-\beta}{2}-\alpha+\lambda; -ac\right) + \\
&+ \frac{c^{1-\alpha-\beta/2}\sqrt{a}(1-\beta)}{n!} \left(2-\frac{\beta}{2}-\alpha+\lambda\right)_n \Gamma\left(\alpha+\frac{\beta}{2}-1\right) \times \\
&\times {}_3F_3\left(1-\frac{\beta}{2}, \frac{3-\beta}{2}, 2-\alpha-\frac{\beta}{2}+\lambda+n; \frac{3}{2}, 2-\alpha-\frac{\beta}{2}+\lambda, 2-\alpha-\frac{\beta}{2}; -ac\right) \\
&\quad [a, \operatorname{Re} \alpha, \operatorname{Re} \beta > 0].
\end{aligned}$$

$$\begin{aligned}
17. \int_0^\infty \frac{x^{\alpha-1}}{(\sqrt{x}+\sqrt{z})^\rho} e^{-cx} L_n^\lambda(cx) dx &= \frac{2z^{\alpha-\rho/2}}{n!} B(2\alpha, \rho-2\alpha) (1+\lambda)_n \times \\
&\times {}_3F_3\left(\alpha, \alpha+\frac{1}{2}, 1+\lambda+n; 1+\alpha-\frac{\rho}{2}, \frac{1-\rho}{2}+\alpha, 1+\lambda; -cz\right) + \\
&+ \frac{c^{\rho/2-\alpha}}{n!} \left(1-\alpha+\lambda+\frac{\rho}{2}\right)_n \Gamma\left(\alpha-\frac{\rho}{2}\right) \times \\
&\times {}_3F_3\left(\frac{\rho}{2}, \frac{\rho+1}{2}, 1-\alpha+\lambda+\frac{\rho}{2}+n; \frac{1}{2}, 1+\frac{\rho}{2}-\alpha, 1-\alpha+\lambda+\frac{\rho}{2}; -cz\right) - \\
&- \frac{c^{(\rho+1)/2-\alpha}\sqrt{z}}{n!} \rho \left(\frac{3+\rho}{2}-\alpha+\lambda\right)_n \Gamma\left(\alpha-\frac{\rho+1}{2}\right) \times \\
&\times {}_3F_3\left(\frac{\rho+1}{2}, \frac{\rho}{2}+1, \frac{3+\rho}{2}-\alpha+\lambda+n; \frac{3}{2}, \frac{3+\rho}{2}-\alpha, \frac{3+\rho}{2}-\alpha+\lambda; -cz\right) \\
&\quad [\operatorname{Re} \alpha, \operatorname{Re} z > 0; |\arg z| < 2\pi].
\end{aligned}$$

$$\begin{aligned}
18. \int_0^\infty \frac{x^{\alpha-1}}{\sqrt{x}-\sqrt{y}} e^{-cx} L_n^\lambda(cx) dx &= -2\pi \frac{y^{\alpha-1/2}}{n!} (1+\lambda)_n \operatorname{ctg}(2\alpha\pi) \times \\
&\times {}_1F_1(1+\lambda+n; 1+\lambda; -cy) + \frac{c^{1/2-\alpha}}{n!} \left(\frac{3}{2}-\alpha+\lambda\right)_n \Gamma\left(\alpha-\frac{1}{2}\right) \times \\
&\times {}_2F_2\left(\frac{3}{2}-\alpha+\lambda+n, 1; \frac{3}{2}-\alpha+\lambda, \frac{3}{2}-\alpha; -cy\right) + \\
&+ \frac{c^{1-\alpha}\sqrt{y}}{n!} (2-\alpha+\lambda)_n \Gamma(\alpha-1) {}_2F_2(2-\alpha+\lambda+n, 1; 2-\alpha+\lambda, 2-\alpha; -cy) \\
&\quad [y, \operatorname{Re} \alpha, \operatorname{Re} \alpha > 0].
\end{aligned}$$

2.19.4. Интегралы от $x^\alpha e^{\varphi(x)} L_n^\lambda(cx)$.

$$\begin{aligned}
1. \int_0^\infty x^{\alpha-1} e^{-\rho\sqrt{x}} L_n^\lambda(cx) dx &= \frac{(-1)^n 2c^n}{n! \rho^{2\alpha+2n}} \Gamma(2\alpha+2n) \times \\
&\times {}_2F_2\left(-n, -\lambda-n; \frac{1}{2}-\alpha-n, 1-\alpha-n; -\frac{\rho^2}{4c}\right) \\
&\quad [\operatorname{Re} \rho, \operatorname{Re} \alpha > 0]
\end{aligned}$$

$$2. \int_0^{\infty} x^{\alpha-1} e^{-p\sqrt{x}} L_n^{\lambda}(cx) dx =$$

$$= \frac{(-1)^n 3c^n}{n! p^{3\alpha+3n}} \Gamma(3\alpha+3n) {}_2F_3\left(-n, -\lambda-n; 1-\alpha-n, \frac{3}{2}-\alpha-n, \frac{1}{2}-\alpha-n; \frac{p^3}{27c}\right) \quad [\operatorname{Re} p, \operatorname{Re} \alpha > 0].$$

$$3. \int_0^{\infty} x^{\alpha-1} e^{-p/x} L_n^{\lambda}(cx) dx = \frac{(\lambda+1)_n}{n!} \Gamma(-\alpha) p^{\alpha} {}_1F_2(-n; \alpha+1, \lambda+1; -cp) \quad [\operatorname{Re} p > 0; \operatorname{Re} \alpha < -n].$$

$$4. \int_0^{\infty} x^{-n-1} e^{-p/x} L_n^{-2n}(cx) dx = \frac{(-1)^n 2c^{n+1/2}}{n! \sqrt{p}} O_{2n}(2\sqrt{cp}) \quad [\operatorname{Re} p > 0].$$

$$5. \int_0^{\infty} x^{-n-1} e^{-p/x} L_n^{-2n-1}(cx) dx = \frac{(-1)^n 4c^{n+1}}{n! (2n+1)} O_{2n+1}(2\sqrt{cp}) \quad [\operatorname{Re} p > 0].$$

$$6. \int_0^{\infty} x^{-n-1} e^{-p/x} L_n^{-n-1/2}(cx) dx = \frac{(-c)^n}{p^{n+1}} y_n\left(\frac{1}{2i\sqrt{cp}}\right) y_n\left(-\frac{1}{2i\sqrt{cp}}\right) \quad [\operatorname{Re} p > 0].$$

$$7. \int_0^{\infty} x^{\alpha-1} e^{-p\sqrt{x}} L_n^{\lambda}(cx) dx =$$

$$= \frac{2(\lambda+1)_n p^{2\alpha}}{n!} \Gamma(-2\alpha) {}_1F_3\left(-n; \alpha+1, \alpha+\frac{1}{2}, \lambda+1; \frac{cp^3}{4}\right) \quad [\operatorname{Re} p > 0; \operatorname{Re} \alpha < -n].$$

$$8. \int_0^{\infty} x^{\alpha-1} e^{-px^2-cx} L_n^{\lambda}(cx) dx = \frac{(1+\lambda)_n}{2(n!) p^{\alpha/2}} \Gamma\left(\frac{\alpha}{2}\right) \times$$

$$\times {}_3F_3\left(\frac{1+\lambda+n}{2}, 1+\frac{\lambda+n}{2}, \frac{\alpha}{2}; \frac{1+\lambda}{2}, 1+\frac{\lambda}{2}, \frac{1}{2}; \frac{c^2}{4p}\right) -$$

$$- \frac{c(2+\lambda)_n}{2(n!) p^{(\alpha+1)/2}} \Gamma\left(\frac{\alpha+1}{2}\right) {}_3F_3\left(\frac{\alpha+1}{2}, 1+\frac{\lambda+n}{2}, \frac{3+\lambda+n}{2}; \frac{3}{2}, 1+\frac{\lambda}{2}, \frac{3+\lambda}{2}; \frac{c^2}{4p}\right) \quad [\operatorname{Re} p, \operatorname{Re} \alpha > 0].$$

$$9. \int_0^{\infty} x^{\alpha-1} e^{-p\sqrt{x}-cx} L_n^{\lambda}(cx) dx =$$

$$= \frac{\Gamma(\alpha) (1-\alpha+\lambda)_n}{n! c^{\alpha}} {}_2F_2\left(\alpha, \alpha-\lambda; \frac{1}{2}, \alpha-\lambda-n; \frac{p^3}{4c}\right) -$$

$$- \frac{p\Gamma(\alpha+1/2)}{n! c^{\alpha+1/2}} \left(\frac{1}{2}-\alpha+\lambda\right)_n {}_2F_2\left(\alpha+\frac{1}{2}, \frac{1}{2}+\alpha-\lambda; \frac{3}{2}, \frac{1}{2}+\alpha-\lambda-n; \frac{p^3}{4c}\right) \quad [\operatorname{Re} c, \operatorname{Re} \alpha > 0].$$

$$10. \int_0^{\infty} x^{(m-1)/2} e^{p\sqrt{x}-cx/2} L_n^m(cx) dx =$$

$$= \frac{\sqrt{\pi}}{n!} 2^{(3-m)/2-n} c^{-(m+1)/2} \exp\left(\frac{p^2}{2c} - \frac{m\pi i}{2}\right) H_{m+n}\left(\frac{ip}{\sqrt{2c}}\right) H_n\left(\frac{ip}{\sqrt{2c}}\right)$$

[Re c > 0].

$$11. \int_0^{\infty} x^{\alpha-1} e^{-p\sqrt[3]{x}-cx} L_n^{\lambda}(cx) dx =$$

$$= \frac{(1-\alpha+\lambda)_n}{n! c^{\alpha}} \Gamma(\alpha) {}_2F_3\left(\alpha, \alpha-\lambda; \frac{1}{3}, \frac{2}{3}, \alpha-\lambda-n; -\frac{p^3}{27c}\right) +$$

$$+ \frac{p^3}{2(n!) c^{\alpha+2/3}} \Gamma\left(\frac{2}{3}+\alpha\right) \left(\frac{1}{3}-\alpha+\lambda\right)_n \times$$

$$\times {}_2F_3\left(\frac{2}{3}+\alpha, \frac{2}{3}+\alpha-\lambda; \frac{4}{3}, \frac{5}{3}, \frac{2}{3}+\alpha-\lambda-n; -\frac{p^3}{27c}\right) -$$

$$\left(-\frac{p}{n! c^{\alpha+1/3}} \Gamma\left(\alpha+\frac{1}{3}\right) \left(\frac{2}{3}-\alpha+\lambda\right)_n \times\right.$$

$$\left. \times {}_2F_3\left(\frac{1}{3}+\alpha, \frac{1}{3}+\alpha-\lambda; \frac{2}{3}, \frac{4}{3}, \frac{1}{3}+\alpha-\lambda-n; -\frac{p^3}{27c}\right)\right)$$

[Re c, Re α > 0].

$$12. \int_0^{\infty} x^{\alpha-1} e^{-p/x-cx} L_n^{\lambda}(cx) dx =$$

$$= \frac{p^{\alpha} (1+\lambda)_n}{n!} \Gamma(-\alpha) {}_1F_2(1+\lambda+n; 1+\alpha, 1+\lambda; cp) +$$

$$+ \frac{(1-\alpha+\lambda)_n}{n! c^{\alpha}} \Gamma(\alpha) {}_1F_2(1-\alpha+\lambda+n; 1-\alpha, 1-\alpha+\lambda; cp)$$

[Re c, Re p > 0].

$$13. \int_0^{\infty} x^{\lambda+n-1} e^{-p/x-cx} L_n^{\lambda}(cx) dx = \frac{(-1)^n 2p^{\lambda/2+n}}{n! c^{\lambda/2}} K_{\lambda}(2\sqrt{cp})$$

[Re c, Re p > 0].

$$14. \int_0^{\infty} x^{\alpha-1} e^{-p/x^2-cx} L_n^{\lambda}(cx) dx = \frac{p^{\alpha/2} (1+\lambda)_n}{2(n!)} \Gamma\left(-\frac{\alpha}{2}\right) \times$$

$$\times {}_2F_4\left(\frac{1+\lambda+n}{2}, 1+\frac{\lambda+n}{2}; \frac{1}{2}, 1+\frac{\alpha}{2}, \frac{1+\lambda}{2}, 1+\frac{\lambda}{2}; -\frac{c^2 p}{4}\right) -$$

$$- \frac{cp^{(\alpha+1)/2} (2+\lambda)_n}{2(n!)} \Gamma\left(-\frac{\alpha+1}{2}\right) \times$$

$$\times {}_2F_4\left(1+\frac{\lambda+n}{2}, \frac{3+\lambda+n}{2}; \frac{3}{2}, \frac{3+\alpha}{2}, 1+\frac{\lambda}{2}, \frac{3+\lambda}{2}; -\frac{c^2 p}{4}\right) +$$

$$+ \frac{c^{-\alpha}}{n!} (1-\alpha+\lambda)_n \Gamma(\alpha) \times {}_2F_4\left(\frac{1-\alpha+\lambda+n}{2}, 1-\frac{\alpha-\lambda-n}{2}; 1-\frac{\alpha}{2},\right.$$

$$\left.\frac{1-\alpha}{2}, \frac{1-\alpha+\lambda}{2}, 1+\frac{\lambda-\alpha}{2}; -\frac{c^2 p}{4}\right)$$

[Re c, Re p > 0].

$$\begin{aligned}
15. \int_0^\infty x^{\alpha-1} e^{-p/\sqrt{\lambda-cx}} L_n^\lambda(cx) dx = \\
= \frac{2p^{2\alpha}}{n!} (1+\lambda)_n \Gamma(-2\alpha) {}_1F_3\left(1+\lambda+n; \alpha+1, \frac{1}{2}+\alpha, 1+\lambda; -\frac{cp^2}{4}\right) + \\
+ \frac{(1-\alpha+\lambda)_n}{n! c^\alpha} \Gamma(\alpha) {}_1F_3\left(1-\alpha+\lambda+n; \frac{1}{2}, 1-\alpha, 1-\alpha+\lambda; -\frac{cp^2}{4}\right) - \\
- \frac{p\Gamma(\alpha-1/2)}{n! c^{\alpha-1/2}} \left(\frac{3}{2}-\alpha+\lambda\right)_n {}_1F_3\left(\frac{3}{2}-\alpha+\lambda+n; \frac{3}{2}, \frac{3}{2}-\alpha, \frac{3}{2}-\alpha+\lambda; -\frac{cp^2}{4}\right) \\
\quad [Re c, Re p > 0].
\end{aligned}$$

2.19.5. Интегралы от $x^\alpha e^{-cx} \left\{ \begin{matrix} \sin bx^{\pm r} \\ \cos bx^{\pm r} \end{matrix} \right\} L_n^\lambda(cx)$.

Обозначение: $\delta = \begin{cases} 1 \\ 0 \end{cases}$.

$$\begin{aligned}
1. \int_0^\infty x^{\alpha-1} e^{-cx} \left\{ \begin{matrix} \sin bx \\ \cos bx \end{matrix} \right\} L_n^\lambda(cx) dx = \frac{b^\delta (1-\alpha-\delta+\lambda)_n}{n! c^{\alpha+\delta}} \Gamma(\alpha+\delta) \times \\
\times {}_4F_3\left(\frac{\alpha+1}{2}, \frac{\alpha}{2}+\delta, \frac{1+\alpha-\lambda}{2}, \frac{\alpha-\lambda}{2}+\delta; \delta+\frac{1}{2}, \frac{1+\alpha-\lambda-n}{2}, \frac{\alpha-\lambda-n}{2}+\delta; -\frac{b^2}{c^2}\right) \\
\quad [b, Re c > 0; Re \alpha > -\delta].
\end{aligned}$$

$$\begin{aligned}
2. \int_0^\infty x^\lambda e^{-cx} \left\{ \begin{matrix} \sin bx \\ \cos bx \end{matrix} \right\} L_n^\lambda(cx) dx = \\
= e^{\pi i (n+\delta)/2} \frac{\Gamma(\lambda+n+1)}{2(n!)} b^n [(c-ib)^{-\lambda-n-1} + (-1)^{n+\delta} (c+ib)^{-\lambda-n-1}] \\
\quad [b, Re c > 0; Re \lambda > -1-\delta].
\end{aligned}$$

$$\begin{aligned}
3. \int_0^\infty x^{\alpha-1} e^{-cx} \left\{ \begin{matrix} \sin b \sqrt{x} \\ \cos b \sqrt{x} \end{matrix} \right\} L_n^\lambda(cx) dx = \frac{b^\delta}{n! c^{\alpha+\delta/2}} \left(1-\alpha-\frac{\delta}{2}+\lambda\right)_n \Gamma\left(\alpha+\frac{\delta}{2}\right) \times \\
\times {}_2F_2\left(\alpha+\frac{\delta}{2}, \alpha+\frac{\delta}{2}-\lambda; \delta+\frac{1}{2}, \alpha+\frac{\delta}{2}-\lambda-n; -\frac{b^2}{4c}\right) \\
\quad [b, Re c > 0; Re \alpha > -\delta/2].
\end{aligned}$$

$$\begin{aligned}
4. \int_0^\infty x^{n-(1\mp 1)/4} e^{-cx} \left\{ \begin{matrix} \sin b \sqrt{x} \\ \cos b \sqrt{x} \end{matrix} \right\} L_n^{\lambda \pm 1/2}(cx) dx = \\
= \sqrt{\pi} \left(\frac{b}{2}\right)^{2n+\delta} c^{-2n-\delta-1/2} \exp\left(-\frac{b^2}{4c}\right) L_n^{\lambda \pm 1/2}\left(\frac{b^2}{4c}\right) \\
\quad [b, Re c > 0].
\end{aligned}$$

$$\begin{aligned}
5. \int_0^\infty x^{\alpha-1} e^{-cx} \left\{ \begin{matrix} \sin(b/\sqrt{x}) \\ \cos(b/\sqrt{x}) \end{matrix} \right\} L_n^\lambda(cx) dx = b^\delta c^{\delta/2-\alpha} \Gamma\left(\alpha-\frac{\delta}{2}\right) \frac{(1-\alpha+\delta/2+\lambda)_n}{n!} \times \\
\times {}_1F_3\left(1-\alpha+\frac{\delta}{2}+\lambda+n; \frac{1}{2}+\delta, 1-\alpha+\frac{\delta}{2}, 1-\alpha+\frac{\delta}{2}+\lambda; \frac{b^2 c}{4}\right) \mp \\
\mp \frac{2(1+\lambda)_n b^{2\alpha}}{n!} \Gamma(-2\alpha) \left\{ \begin{matrix} \sin \alpha\pi \\ \cos \alpha\pi \end{matrix} \right\} {}_1F_3\left(1+\lambda+n; 1+\alpha, \frac{1}{2}+\alpha, 1+\lambda; \frac{b^2 c}{4}\right) \\
\quad [b, Re c > 0; Re \alpha > -1/2].
\end{aligned}$$

$$6. \int_0^{\infty} x^{-1/2} e^{-cx} \sin b \sqrt{x} L_n(cx) dx = \\ = \frac{(-1)^n n!}{2 \sqrt{\pi c}} \left[D_{-n-1}^2 \left(\frac{ib}{\sqrt{2c}} \right) - D_{-n-1}^2 \left(-\frac{ib}{\sqrt{2c}} \right) \right] \quad [b, \operatorname{Re} c > 0].$$

$$7. \int_0^{\infty} x^{-1/2} e^{-cx} \cos b \sqrt{x} L_n(cx) dx = (-1)^n \sqrt{\frac{\pi}{c}} \exp \left(-\frac{b^2}{4c} \right) L_n^{-n-1/2} \left(\frac{b^2}{4c} \right) \\ [b, \operatorname{Re} c > 0].$$

2.19.6. Интегралы от $x^\alpha e^{-cx} \ln^m \varphi(x) L_n^\lambda(cx)$.

$$1. \int_0^{\infty} x^{\alpha-1} e^{-cx} \ln(cx) L_n^\lambda(cx) dx = \frac{(1-\alpha+\lambda)_n}{n! c^\alpha} \Gamma(\alpha) \left[\psi(\alpha) - \sum_{k=1}^n \frac{1}{k-\alpha+\lambda} \right] \\ [\operatorname{Re} c, \operatorname{Re} \alpha > 0].$$

$$2. \int_0^{\infty} x^{\lambda+m} e^{-cx} \ln(cx) L_n^\lambda(cx) dx = \frac{(-1)^{m-1} m! (n-m-1)!}{n!} \Gamma(\lambda+m+1) c^{-\lambda-m-1} \\ [m=0, 1, 2, \dots, n-1; n \geq 1; \operatorname{Re} c > 0].$$

$$3. \int_0^{\infty} x^{\alpha-1} e^{-cx} \ln^2(cx) L_n^\lambda(cx) dx = \\ = \frac{(1-\alpha+\lambda)_n}{n! c^\alpha} \Gamma(\alpha) \left[\left(\psi(\alpha) - \sum_{k=1}^n \frac{1}{k-\alpha+\lambda} \right)^2 + \psi'(\alpha) - \sum_{k=1}^n \frac{1}{(k-\alpha+\lambda)^2} \right] \\ [\operatorname{Re} c, \operatorname{Re} \alpha > 0].$$

$$4. \int_0^{\infty} x^{\lambda+m} e^{-cx} \ln^2(cx) L_n^\lambda(cx) dx = \\ = (-1)^m \frac{2(m!)(n-m-1)!}{n!} \Gamma(\lambda+m+1) [\psi(n-m) - \psi(m+1) - \psi(\lambda+m+1)] c^{-\lambda-m-1} \\ [m=0, 1, 2, \dots, n-1; n \geq 1; \operatorname{Re} c > 0].$$

$$5. \int_0^{\infty} x^{\alpha-1} e^{-cx} \left\{ \frac{\ln(x+z)}{1+x-z} \right\} L_n^\lambda(cx) dx = \\ = \frac{\pi z^\alpha (1+\lambda)_n}{n! \alpha} \left\{ \operatorname{cosec} \alpha \pi \right\} {}_2F_2(1+\lambda+n, \alpha; \alpha+1, 1+\lambda; \pm cz) \pm \\ \pm \frac{c^{1-\alpha} z}{n!} (2-\alpha+\lambda)_n \Gamma(\alpha-1) {}_3F_3(1, 1, 2-\alpha+\lambda+n; 2, 2-\alpha, 2-\alpha+\lambda; \pm cz) + \\ + \frac{(1-\alpha+\lambda)_n}{n! c^\alpha} \Gamma(\alpha) \left[\psi(\alpha) - \sum_{k=1}^n \frac{1}{k-\alpha+\lambda} - \ln c \right] \\ [\operatorname{Re} c, \operatorname{Re} \alpha > 0, \left\{ \begin{array}{l} |\arg z| < \pi \\ z > 0 \end{array} \right\}].$$

$$\begin{aligned}
6. \int_0^{\infty} x^{\alpha-1} e^{-cx} \left\{ \frac{\ln(x^2+z^2)}{\ln|x^2-z^2|} \right\} L_n^{\lambda}(cx) dx &= \frac{\pi z^{\alpha}}{n! \alpha} (1+\lambda)_n \left\{ \frac{\operatorname{cosec}(\alpha\pi/2)}{\operatorname{ctg}(\alpha\pi/2)} \right\} \times \\
&\times {}_3F_4 \left(\frac{\alpha}{2}, \frac{1+\lambda+n}{2}, 1+\frac{\lambda+n}{2}; \frac{1}{2}, \frac{\alpha}{2}+1, \frac{1+\lambda}{2}, 1+\frac{\lambda}{2}; \mp \frac{c^2 z^2}{4} \right) \mp \\
&\mp \frac{\pi c z^{\alpha+1} (2+\lambda)_n \{\sec(\alpha\pi/2)\}}{n! (\alpha+1) \{\operatorname{tg}(\alpha\pi/2)\}} {}_3F_4 \left(\frac{\alpha+1}{2}, 1+\frac{\lambda+n}{2}, \frac{3+\lambda+n}{2}; \right. \\
&\left. \frac{3}{2}, \frac{\alpha+3}{2}, 1+\frac{\lambda}{2}, \frac{3+\lambda}{2}; \mp \frac{c^2 z^2}{4} \right) \pm \frac{c^{\alpha-2} z^2}{n!} (3-\alpha+\lambda)_n \Gamma(\alpha-2) \times \\
&\times {}_4F_5 \left(1, 1, \frac{3-\alpha+\lambda+n}{2}, 2+\frac{\lambda-\alpha+n}{2}; 2, 2-\frac{\alpha}{2}, \frac{3-\alpha}{2}, \frac{3-\alpha+\lambda}{2}, \right. \\
&\left. 2+\frac{\lambda-\alpha}{2}; \mp \frac{c^2 z^2}{4} \right) + \frac{2(1-\alpha+\lambda)_n \Gamma(\alpha)}{c^{\alpha}} \left[\psi(\alpha) - \sum_{k=1}^n \frac{1}{k-\alpha+\lambda} - \ln c \right] \\
&\quad \left[\operatorname{Re} c, \operatorname{Re} \alpha > 0, \begin{cases} \operatorname{Re} z > 0 \\ z > 0 \end{cases} \right].
\end{aligned}$$

2.19.7. Интегралы, содержащие $\operatorname{Ei}(bx^m) L_n^{\lambda}(cx)$.

$$1. \int_0^{\infty} \operatorname{Ei}(-bx) L_n^{\lambda}(cx) dx = \frac{1}{(n+1)c} \left[\left(1 - \frac{c}{b} \right)^{n+1} - 1 \right] \quad [\operatorname{Re} b > 0].$$

$$\begin{aligned}
2. \int_0^{\infty} x^{\alpha-1} e^{-cx} \operatorname{Ei}(-bx) L_n^{\lambda}(cx) dx &= \\
&= -\frac{\Gamma(\alpha) (1+\lambda)_n}{n! b^{\alpha} c} {}_3F_2 \left(1+\lambda+n, \alpha, \alpha; 1+\alpha, 1+\lambda; -\frac{c}{b} \right) \\
&\quad [\operatorname{Re} b, \operatorname{Re} c, \operatorname{Re} \alpha > 0].
\end{aligned}$$

$$\begin{aligned}
3. \int_0^{\infty} x^{\alpha-1} e^{(\pm b-c)x} \operatorname{Ei}(\mp bx) L_n^{\lambda}(cx) dx &= \\
&= -\frac{\pi (1+\lambda)_n \Gamma(\alpha)}{n! b^{\alpha}} \left\{ \frac{\operatorname{cosec} \alpha\pi}{\operatorname{ctg} \alpha\pi} \right\} {}_2F_1 \left(1+\lambda+n, \alpha; 1+\lambda; \pm \frac{c}{b} \right) \mp \\
&\mp \frac{c^{1-\alpha}}{n! b} (2-\alpha+\lambda)_n \Gamma(\alpha-1) {}_3F_2 \left(1, 1, 2-\alpha+\lambda+n; 2-\alpha, 2-\alpha+\lambda; \pm \frac{c}{b} \right) \\
&\quad \left[\operatorname{Re} c, \operatorname{Re} \alpha > 0, \begin{cases} |\arg b| < \pi \\ b > 0 \end{cases} \right].
\end{aligned}$$

$$\begin{aligned}
4. \int_0^{\infty} e^{(b-c)x} \operatorname{Ei}(-bx) L_n^{\lambda}(cx) dx &= -\frac{1}{b(n+1)} {}_2F_1 \left(1, n+1; n+2; 1-\frac{c}{b} \right) \\
&\quad [\operatorname{Re} c > 0; |\arg b| < \pi].
\end{aligned}$$

$$\begin{aligned}
5. \int_0^{\infty} x^{\alpha-1} e^{-cx} \operatorname{Ei}(-bx^2) L_n^{\lambda}(cx) dx &= -\frac{(1+\lambda)_n \Gamma(\frac{\alpha}{2})}{\alpha n! b^{\alpha/2}} \times \\
&\times {}_4F_4 \left(\frac{\alpha}{2}, \frac{\alpha}{2}, \frac{1+\lambda+n}{2}, 1+\frac{\lambda+n}{2}; \frac{1}{2}, \frac{\alpha}{2}+1, \frac{1+\lambda}{2}, 1+\frac{\lambda}{2}; \frac{c^2}{4b} \right) + \\
&+ \frac{(2+\lambda)_n c}{(\alpha+1) n! b^{(\alpha+1)/2}} \Gamma\left(\frac{\alpha+1}{2}\right) \times \\
&\times {}_4F_4 \left(\frac{\alpha+1}{2}, \frac{\alpha+1}{2}, 1+\frac{\lambda+n}{2}, \frac{3+\lambda+n}{2}; \frac{3}{2}, \frac{\alpha+3}{2}, 1+\frac{\lambda}{2}, \frac{3+\lambda}{2}; \frac{c^2}{4b} \right) \\
&\quad [\operatorname{Re} b, \operatorname{Re} \alpha > 0].
\end{aligned}$$

$$\begin{aligned}
6. \int_0^{\infty} x^{\alpha-1} e^{\pm bx^2 - cx} \operatorname{Ei}(\mp bx^2) L_n^{\lambda}(cx) dx = \\
= -\frac{\pi(1+\lambda)_n}{2(n!) b^{\alpha/2}} \Gamma\left(\frac{\alpha}{2}\right) \left\{ \operatorname{cosec}(\alpha\pi/2) \right\} \times \\
\times {}_3F_3\left(\frac{\alpha}{2}, \frac{1+\lambda+n}{2}, 1+\frac{\lambda+n}{2}; \frac{1}{2}, \frac{1+\lambda}{2}, 1+\frac{\lambda}{2}; \mp \frac{c^2}{4b}\right) \pm \\
\pm \frac{\pi c(2+\lambda)_n}{2(n!) b^{(\alpha+1)/2}} \Gamma\left(\frac{\alpha+1}{2}\right) \left\{ \sec(\alpha\pi/2) \right\} \times \\
\times {}_3F_3\left(\frac{\alpha+1}{2}, 1+\frac{\lambda+n}{2}, \frac{3+\lambda+n}{2}; \frac{3}{2}, 1+\frac{\lambda}{2}, \frac{3+\lambda}{2}; \mp \frac{c^2}{4b}\right) \mp \\
\mp \frac{c^{\alpha-\alpha}}{n! b} (3-\alpha+\lambda)_n \Gamma(\alpha-2) {}_4F_4\left(1, 1, \frac{3-\alpha+\lambda+n}{2}, 2-\frac{\alpha-\lambda-n}{2}; 2-\frac{\alpha}{2}, \right. \\
\left. \frac{3-\alpha}{2}, \frac{3-\alpha+\lambda}{2}, 2+\frac{\lambda-\alpha}{2}; \mp \frac{c^2}{4b}\right) \quad \left[\operatorname{Re} c, \operatorname{Re} \alpha > 0, \left\{ \begin{array}{l} |\arg b| < \pi \\ b > 0 \end{array} \right\} \right].
\end{aligned}$$

2.19.8. Интегралы от $x^{\alpha} e^{-cx} \left\{ \begin{array}{l} \operatorname{si}(bx^r) \\ \operatorname{ci}(bx^r) \end{array} \right\} L_n^{\lambda}(cx)$.

$$\begin{aligned}
1. \int_0^{\infty} x^{\alpha-1} e^{-cx} \left\{ \begin{array}{l} \operatorname{si}(bx) \\ \operatorname{ci}(bx) \end{array} \right\} L_n^{\lambda}(cx) dx = -\frac{(1+\lambda)_n \Gamma(\alpha)}{n! b^{\alpha}} \left\{ \begin{array}{l} \sin(\alpha\pi/2) \\ \cos(\alpha\pi/2) \end{array} \right\} \times \\
\times {}_4F_4\left(\frac{\alpha}{2}, \frac{\alpha}{2}, \frac{\alpha+1}{2}, \frac{1+\lambda+n}{2}, 1+\frac{\lambda+n}{2}; \frac{1}{2}, \frac{\alpha}{2}+1, 1+\frac{\lambda}{2}, \frac{\lambda+1}{2}; -\frac{c^2}{b^2}\right) \pm \\
\pm \frac{c \Gamma(\alpha+1) (\lambda+2)_n}{n! b^{\alpha+1} (\alpha+1)} \left\{ \begin{array}{l} \cos(\alpha\pi/2) \\ \sin(\alpha\pi/2) \end{array} \right\} {}_4F_4\left(\frac{\alpha+1}{2}, \frac{\alpha+1}{2}, \frac{\alpha}{2}+1, \right. \\
\left. 1+\frac{\lambda+n}{2}, \frac{3+\lambda+n}{2}; \frac{3}{2}, \frac{\alpha+3}{2}, 1+\frac{\lambda}{2}, \frac{3+\lambda}{2}; -\frac{c^2}{b^2}\right) \\
[b, \operatorname{Re} c, \operatorname{Re} \alpha > 0].
\end{aligned}$$

$$\begin{aligned}
2. \int_0^{\infty} x^{\alpha-1} e^{-cx} \left\{ \begin{array}{l} \operatorname{si}(b\sqrt{x}) \\ \operatorname{ci}(b\sqrt{x}) \end{array} \right\} L_n^{\lambda}(cx) dx = \\
= -\frac{b^{\delta+2} \Gamma(\alpha+\delta/2+1)}{2^{\delta+1} 3^{\delta} n! c^{\alpha+\delta/2+1}} \left(\lambda - \alpha - \frac{\delta}{2} \right)_n {}_4F_4\left(1, \alpha+\frac{\delta}{2}+1, 1+\frac{\delta}{2}, \right. \\
\left. 1+\alpha+\frac{\delta}{2}-\lambda, 2, \delta+\frac{3}{2}, 2+\frac{\delta}{2}, 1+\alpha+\frac{\delta}{2}-\lambda-n; -\frac{b^2}{4c}\right) + \\
+ \frac{b^{\delta} (1-\alpha-\delta/2+\lambda)_n}{n! c^{\alpha+\delta/2}} \Gamma\left(\alpha+\frac{\delta}{2}\right) \left\{ \begin{array}{l} 1 \\ R \end{array} \right\} - \left\{ \begin{array}{l} 1 \\ 0 \end{array} \right\} \frac{\pi(1-\alpha+\lambda)_n \Gamma(\alpha)}{2(n!) c^{\alpha}}, \\
R = C + \frac{\psi(\alpha)}{2} - \frac{1}{2} \sum_{k=1}^n \frac{1}{k-\alpha+\lambda} - \ln \frac{\sqrt{c}}{b} \\
[b, \operatorname{Re} c, \operatorname{Re} \alpha > 0; \delta = \left\{ \begin{array}{l} 1 \\ 0 \end{array} \right\}].
\end{aligned}$$

2.19.9. Интегралы, содержащие $\left\{ \begin{array}{l} \operatorname{erf}(bx^r) \\ \operatorname{erfc}(bx^r) \end{array} \right\} L_n^{\lambda}(cx)$.

$$\begin{aligned}
1. \int_0^{\infty} x^{\alpha-1} \operatorname{erfc}(b\sqrt{x}) L_n^{\lambda}(cx) dx = \\
= \frac{(1+\lambda)_n \Gamma(1/2+\alpha)}{\sqrt{\pi} n! b^{\alpha}} {}_3F_3\left(-n, \alpha, \frac{1}{2}+\alpha; 1+\alpha, 1+\lambda; \frac{c}{b^2}\right) \\
[\operatorname{Re} \alpha > 0; |\arg b| < \pi/4]
\end{aligned}$$

$$2. \int_0^{\infty} x^{\lambda} \operatorname{erfc}(\sqrt{cx}) L_n^{\lambda}(cx) dx = \frac{\Gamma(3/2 + \lambda) \Gamma(1/2 + n)}{\pi n! (1 + \lambda + n) c^{\lambda+1}}$$

[Re $c > 0$; Re $\lambda > -1$].

$$3. \int_0^{\infty} x^{\alpha-1} e^{-cx} \left\{ \begin{matrix} \operatorname{erf}(bx) \\ \operatorname{erfc}(bx) \end{matrix} \right\} L_n^{\lambda}(cx) dx = \mp \frac{(1+\lambda)_n}{\sqrt{\pi} n! b^{\alpha} c^{\alpha}} \Gamma\left(\frac{\alpha+1}{2}\right) \times$$

$$\times {}_4F_4\left(\frac{\alpha}{2}, \frac{\alpha+1}{2}, \frac{1+\lambda+n}{2}, 1+\frac{\lambda+n}{2}; \frac{1}{2}, \frac{\alpha}{2}+1, \frac{1+\lambda}{2}, 1+\frac{\lambda}{2}; \frac{c^2}{4b^2}\right) \pm$$

$$\pm \frac{c(2+\lambda)_n}{\sqrt{\pi} n! b^{\alpha+1} (\alpha+1)} \Gamma\left(\frac{\alpha}{2}+1\right) {}_4F_4\left(\frac{\alpha+1}{2}, \frac{\alpha}{2}+1, \frac{3+\lambda+n}{2}, 1+\frac{\lambda+n}{2}; \frac{3}{2}, \frac{\alpha+3}{2}, \frac{3+\lambda}{2}, 1+\frac{\lambda}{2}; \frac{c^2}{4b^2}\right) + \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{(1-\alpha+\lambda)_n}{n! c^{\alpha}} \Gamma(\alpha)$$

[Re $c > 0$; Re $\alpha > -(1 \pm 1)/2$; $|\arg b| < \pi/4$].

$$4. \int_0^{\infty} x^{\alpha-1} e^{-cx} \left\{ \begin{matrix} \operatorname{erf}(b\sqrt{x}) \\ \operatorname{erfc}(b\sqrt{x}) \end{matrix} \right\} L_n^{\lambda}(cx) dx = \mp \frac{(1+\lambda)_n}{\sqrt{\pi} n! b^{2\alpha} c^{\alpha}} \Gamma\left(\alpha + \frac{1}{2}\right) \times$$

$$\times {}_3F_2\left(\alpha, \alpha + \frac{1}{2}, 1+\lambda+n; \alpha+1, 1+\lambda; -\frac{c}{b^2}\right) + \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{(1-\alpha+\lambda)_n}{n! c^{\alpha}} \Gamma(\alpha)$$

[Re $c > 0$; Re $\alpha > -(1 \pm 1)/4$; $|\arg b| < \pi/4$].

$$5. \int_0^{\infty} x^{\alpha-1} e^{(b^2-c)x} \operatorname{erfc}(b\sqrt{x}) L_n^{\lambda}(cx) dx =$$

$$= \frac{(\lambda+1)_n \Gamma(\alpha)}{n! b^{2\alpha} \cos \alpha\pi} {}_2F_1\left(\lambda+n+1, \alpha; \lambda+1; \frac{c}{b^2}\right) +$$

$$+ \frac{(3/2-\alpha+\lambda)_n \Gamma(\alpha-1/2)}{\sqrt{\pi} n! b c^{\alpha-1/2}} {}_3F_2\left(1, \frac{1}{2}, \frac{3}{2}-\alpha+\lambda+n; \frac{3}{2}-\alpha, \frac{3}{2}-\alpha+\lambda; \frac{c}{b^2}\right)$$

[Re $c, \operatorname{Re} \alpha > 0$; $|\arg b| < \pi/4$].

$$6. \int_0^{\infty} x^{\lambda} e^{(b^2-c)x} \operatorname{erfc}(b\sqrt{x}) L_n^{\lambda}(cx) dx = \frac{1}{\pi n! b^{2\lambda+2} (\lambda+n+1)} \Gamma\left(\lambda + \frac{3}{2}\right) \times$$

$$\times \Gamma\left(n + \frac{1}{2}\right) {}_2F_1\left(\lambda + \frac{3}{2}, \lambda+n+1; \lambda+n+2; 1 - \frac{c}{b^2}\right)$$

[Re $c > 0$; Re $\lambda > -1$; $|\arg b| < \pi/4$].

2.19.10. Интегралы от $x^{\alpha} e^{-cx} \left\{ \begin{matrix} S(bx^r) \\ C(bx^r) \end{matrix} \right\} L_n^{\lambda}(cx)$.

Обозначение: $\delta = \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\}$.

$$1. \int_0^{\infty} x^{\alpha-1} e^{-cx} \left\{ \begin{matrix} S(bx) \\ C(bx) \end{matrix} \right\} L_n^{\lambda}(cx) dx =$$

$$= \frac{\sqrt{2} b^{\delta+1/2} (1/2-\alpha-\delta+\lambda)_n}{3^{\delta} \sqrt{\pi} n! c^{\alpha+\delta+1/2}} \Gamma\left(\alpha+\delta+\frac{1}{2}\right) {}_5F_4\left(\frac{2\alpha+3}{4}, \frac{2\alpha+1}{4}+\delta, \frac{2\delta+1}{4}, \frac{3+2\alpha-2\lambda}{4}, \frac{1+2\alpha-2\lambda}{4}+\delta; \delta+\frac{1}{2}, \frac{2\delta+5}{4}, \frac{3+2\alpha-2\lambda-2n}{4}, \frac{1+2\alpha-2\lambda-2n}{4}+\delta; -\frac{b^2}{c^2}\right)$$

[$b, \operatorname{Re} c > 0$; Re $\alpha > -\delta - 1/2$].

$$\begin{aligned}
2. \int_0^{\infty} x^{\alpha-1} e^{-cx} \left\{ \begin{matrix} S(b\sqrt{x}) \\ C(b\sqrt{x}) \end{matrix} \right\} L_n^{\lambda}(cx) dx = \\
= \frac{\sqrt{2} b^{\delta+1/2}}{3^{\delta} \sqrt{\pi} n! c^{\alpha+(2\delta+1)/4}} \left(\frac{3-2\delta}{4} - \alpha + \lambda \right)_n \Gamma\left(\frac{1+2\delta}{4} + \alpha\right) \times \\
\times {}_3F_3\left(\frac{2\delta+1}{4}, \frac{1+2\delta}{4} + \alpha, \frac{1+2\delta}{4} + \alpha - \lambda; \delta + \frac{1}{2}, \right. \\
\left. \frac{2\delta+5}{4}, \frac{1+2\delta}{4} + \alpha - \lambda - n; -\frac{b^2}{4c}\right) \quad [b, \operatorname{Re} c > 0; \operatorname{Re} \alpha > -(2\delta+1)/4].
\end{aligned}$$

2.19.11. Интегралы от $x^{\alpha} e^{-cx} \left\{ \begin{matrix} \gamma(\mu, bx^r) \\ \Gamma(\mu, bx^r) \end{matrix} \right\} L_n^{\lambda}(cx)$.

$$\begin{aligned}
1. \int_0^{\infty} x^{\alpha-1} e^{-cx} \left\{ \begin{matrix} \gamma(\mu, bx) \\ \Gamma(\mu, bx) \end{matrix} \right\} L_n^{\lambda}(cx) dx = \\
= \mp \frac{(1+\lambda)_n}{n! b^{\alpha} c^{\alpha}} \Gamma(\alpha + \mu) {}_3F_3\left(\alpha, \alpha + \mu, 1 + \lambda + n; 1 + \alpha, 1 + \lambda; -\frac{c}{b}\right) + \\
+ \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{(1-\alpha+\lambda)_n}{n! c^{\alpha}} \Gamma(\alpha) \Gamma(\mu) \quad \left[\operatorname{Re} b, \operatorname{Re} c, \operatorname{Re}(\alpha + \mu) > 0, \left\{ \begin{matrix} \operatorname{Re} \mu > 0 \\ \operatorname{Re} \alpha > 0 \end{matrix} \right\} \right].
\end{aligned}$$

$$2. \int_0^{\infty} e^{-cx} \gamma(\lambda, bx) L_n^{\lambda}(cx) dx = \frac{\Gamma(\lambda + n)}{n! c} \left(\frac{b}{b+c} \right)^{n+\lambda} \quad [\operatorname{Re} b, \operatorname{Re} c, \operatorname{Re} \lambda > 0].$$

$$\begin{aligned}
3. \int_0^{\infty} x^{\alpha-1} e^{-cx} \left\{ \begin{matrix} \gamma(\mu, bx^2) \\ \Gamma(\mu, bx^2) \end{matrix} \right\} L_n^{\lambda}(cx) dx = \mp \frac{(1+\lambda)_n}{n! b^{\alpha/2} c^{\alpha}} \Gamma\left(\frac{\alpha}{2} + \mu\right) \times \\
\times {}_4F_4\left(\frac{\alpha}{2}, \frac{\alpha}{2} + \mu, \frac{1+\lambda+n}{2}, 1 + \frac{\lambda+n}{2}; \frac{1}{2}, 1 + \frac{\alpha}{2}, \frac{1+\lambda}{2}, 1 + \frac{\lambda}{2}; \frac{c^2}{4b}\right) \pm \\
\pm \frac{c(2+\lambda)_n}{b^{(\alpha+1)/2}(\alpha+1)} \Gamma\left(\frac{1+\alpha}{2} + \mu\right) {}_4F_4\left(\frac{1+\alpha}{2}, \frac{\alpha+1}{2} + \mu, 1 + \frac{\lambda+n}{2}, \frac{3+\lambda+n}{2}; \right. \\
\left. \frac{3}{2}, \frac{3+\alpha}{2}, 1 + \frac{\lambda}{2}, \frac{3+\lambda}{2}; \frac{c^2}{4b}\right) + \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{(1-\alpha+\lambda)_n}{n! c^{\alpha}} \Gamma(\alpha) \Gamma(\mu) \\
\left[\operatorname{Re} b, \operatorname{Re} c, \operatorname{Re}(\alpha + 2\mu) > 0, \left\{ \begin{matrix} \operatorname{Re} \mu > 0 \\ \operatorname{Re} \alpha > 0 \end{matrix} \right\} \right].
\end{aligned}$$

$$\begin{aligned}
4. \int_0^{\infty} x^{\alpha-1} e^{-cx} \left\{ \begin{matrix} \gamma(\mu, b\sqrt{x}) \\ \Gamma(\mu, b\sqrt{x}) \end{matrix} \right\} L_n^{\lambda}(cx) dx = \pm \frac{b^{\mu} (1-\alpha-\mu/2+\lambda)_n}{n! c^{\alpha+\mu/2} \mu} \Gamma\left(\alpha + \frac{\mu}{2}\right) \times \\
\times {}_3F_3\left(\frac{\mu}{2}, \alpha + \frac{\mu}{2}, \alpha + \frac{\mu}{2} - \lambda; 1 + \frac{\mu}{2}, \frac{1}{2}, \alpha + \frac{\mu}{2} - \lambda - n; \frac{b^2}{4c}\right) \mp \\
\mp \frac{b^{\mu+1}}{n! c^{\alpha+(\mu+1)/2} (\mu+1)} \Gamma\left(\alpha + \frac{\mu+1}{2}\right) \left(\frac{1-\mu}{2} + \lambda - \alpha\right)_n \times \\
\times {}_3F_3\left(\frac{\mu+1}{2}, \alpha + \frac{\mu+1}{2}, \alpha + \frac{1+\mu}{2} - \lambda; \alpha + \frac{1+\mu}{2} - \lambda - n, \frac{\mu+3}{2}, \frac{3}{2}; \frac{b^2}{4c}\right) + \\
+ \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{(1-\alpha+\lambda)_n}{n! c^{\alpha}} \Gamma(\alpha) \Gamma(\mu) \quad \left[\operatorname{Re} b, \operatorname{Re} c, \operatorname{Re}(2\alpha + \mu) > 0, \left\{ \begin{matrix} \operatorname{Re} \mu > 0 \\ \operatorname{Re} \alpha > 0 \end{matrix} \right\} \right].
\end{aligned}$$

2.19.12. Интегралы, содержащие $\left\{ \frac{J_\mu(\varphi(x))}{I_\mu(\varphi(x))} \right\} L_n^\lambda(cx)$.

$$1. \int_0^\infty x^{\alpha-1} J_\mu(b\sqrt{x}) L_n^\lambda(cx) dx = \frac{(-c)^n}{n!} \left(\frac{2}{b}\right)^{2\alpha+2n} \Gamma\left[\frac{\alpha+\mu/2+n}{1-\alpha+\mu/2-n}\right] \times \\ \times {}_2F_2\left(-n, -\lambda-n; 1-\alpha-\frac{\mu}{2}-n, 1-\alpha+\frac{\mu}{2}-n; \frac{b^2}{4c}\right) \\ [b > 0; -\operatorname{Re} \mu < 2 \operatorname{Re} \alpha < 3/2-2n].$$

$$2. \int_0^\infty x^{\alpha-1} J_\mu\left(\frac{b}{\sqrt{x}}\right) L_n^\lambda(cx) dx = \frac{(\lambda+1)_n}{n!} \left(\frac{b}{2}\right)^{2\alpha} \Gamma\left[\frac{\mu/2-\alpha}{1+\mu/2+\alpha}\right] \times \\ \times {}_1F_3\left(-n; 1+\lambda, 1+\alpha+\frac{\mu}{2}, 1+\alpha-\frac{\mu}{2}; -\frac{b^2c}{4}\right) \\ [b > 0; -3/2 < 2 \operatorname{Re} \alpha < \operatorname{Re} \mu - 2n].$$

$$3. \int_0^\infty x^{\alpha-1} e^{-cx} \left\{ \frac{J_\mu(bx)}{I_\mu(bx)} \right\} L_n^\lambda(cx) dx = \\ = \frac{b^\mu (1-\alpha-\mu+\lambda)_n}{2^\mu n! c^{\alpha+\mu}} \Gamma\left[\frac{\alpha+\mu}{1+\mu}\right] {}_4F_3\left(\frac{\alpha+\mu}{2}, \frac{\alpha+\mu+1}{2}, \frac{1+\alpha+\mu-\lambda}{2}, \frac{\alpha+\mu-\lambda}{2}; \mu+1, \frac{1+\alpha+\mu-\lambda-n}{2}, \frac{\alpha+\mu-\lambda-n}{2}; \mp \frac{b^2}{c^2}\right) \\ [\operatorname{Re} \alpha, \operatorname{Re}(\alpha+\mu) > 0, \left\{ \begin{matrix} b > 0 \\ \operatorname{Re}(c-b) > 0 \end{matrix} \right\}].$$

$$4. \int_0^\infty e^{-cx/2} J_0(bx) L_n(cx) dx = \frac{2}{\sqrt{4b^2+c^2}} P_n\left(\frac{c^2-4b^2}{c^2+4b^2}\right) \quad [b, \operatorname{Re} c > 0].$$

$$5. \int_0^a (a-x)^{n+1/2} e^{-cx/2} I_{n+1/2}\left(c\frac{a-x}{2}\right) L_n(cx) dx = \\ = \frac{n!}{\sqrt{\pi} c^{n+3/2}} e^{-ac/2} \left[1 + 2^{-2n-2} \sum_{k=1}^{2n+2} k \binom{2n+2}{k} \operatorname{Ki}_{2k}(ac) \right] \quad [a > 0].$$

$$6. \int_0^\infty x^{\lambda/2} e^{-px} \left\{ \frac{J_\lambda(b\sqrt{x})}{I_\lambda(b\sqrt{x})} \right\} L_n^\lambda(cx) dx = J(p, c) \\ [\operatorname{Re} p > 0; \operatorname{Re} \lambda > -1; |\arg b| < \pi], \\ J(p, c) = \left(\frac{b}{2}\right)^\lambda \frac{(p-c)^n}{p^{\lambda+n+1}} \exp\left(\mp \frac{b^2}{4p}\right) L_n^\lambda\left(\pm \frac{b^2c}{4pc-4p^2}\right), \\ J(c, c) = \frac{(\pm 1)^n}{n! c^{\lambda+n+1}} \left(\frac{b}{2}\right)^{\lambda+n+1} \exp\left(\mp \frac{b^2}{4c}\right).$$

$$7. \int_0^\infty x^{\alpha-1} e^{-cx} \left\{ \frac{J_\mu(b\sqrt{x})}{I_\mu(b\sqrt{x})} \right\} L_n^\lambda(cx) dx = \\ = \frac{b^\mu (1-\alpha-\mu/2+\lambda)_n}{2^\mu n! c^{\alpha+\mu/2}} \Gamma\left[\frac{\alpha+\mu/2}{\mu+1}\right] {}_2F_2\left(\alpha+\frac{\mu}{2}, \alpha+\frac{\mu}{2}-\lambda; \alpha+\frac{\mu}{2}-\lambda-n, \mu+1; \mp \frac{b^2}{4c}\right) \\ [\operatorname{Re} c, \operatorname{Re}(2\alpha+\mu) > 0; |\arg b| < \pi].$$

$$8. \int_0^{\infty} x^{(\lambda+n)/2} e^{-cx} \left\{ \begin{matrix} J_{\lambda-n}(b\sqrt{x}) \\ I_{\lambda-n}(b\sqrt{x}) \end{matrix} \right\} L_n^{\lambda}(cx) dx =$$

$$= (\pm 1)^n \left(\frac{b}{2} \right)^{\lambda+n} c^{-\lambda-n-1} \exp\left(\mp \frac{b^2}{4c}\right) L_n^{\lambda}\left(\pm \frac{b^2}{4c}\right)$$

[Re $c > 0$; Re $\lambda > -1$; $|\arg b| < \pi$].

$$9. \int_0^{\infty} x^{\lambda/2} e^{-cx/2} \left\{ \begin{matrix} J_{\lambda}(b\sqrt{x}) \\ I_{\lambda}(b\sqrt{x}) \end{matrix} \right\} L_n^{\lambda}(cx) dx = \frac{(-1)^n 2b^{\lambda}}{c^{\lambda+1}} \exp\left(\mp \frac{b^2}{2c}\right) L_n^{\lambda}\left(\pm \frac{b^2}{c}\right)$$

[Re $c > 0$; Re $\lambda > -1$; $|\arg b| < \pi$].

$$10. \int_0^{\infty} x^{\alpha-1} e^{-cx} J_{\mu}\left(\frac{b}{\sqrt{x}}\right) L_n^{\lambda}(cx) dx =$$

$$= \frac{(\lambda+1)_n}{n!} \left(\frac{b}{2}\right)^{2\alpha} \Gamma\left[\frac{\mu/2-\alpha}{1+\alpha-\mu/2}\right] {}_1F_3\left(1+\lambda+n; 1+\alpha+\frac{\mu}{2}, 1+\alpha-\frac{\mu}{2}, 1+\lambda; \frac{b^2 c}{4}\right) +$$

$$+ \frac{c^{\mu/2-\alpha}}{n!} \left(\frac{b}{2}\right)^{\mu} \left(1-\alpha+\frac{\mu}{2}+\lambda\right)_n \Gamma\left[\frac{\alpha-\mu/2}{\mu+1}\right] \times$$

$$\times {}_1F_3\left(1-\alpha+\frac{\mu}{2}+\lambda+n; \mu+1, 1-\alpha+\frac{\mu}{2}, 1-\alpha+\frac{\mu}{2}+\lambda; \frac{b^2 c}{4}\right)$$

[$b, \operatorname{Re} c > 0, \operatorname{Re} \alpha > -3/4$].

$$11. \int_{-a}^a (a^2-x^2)^{(2\lambda-1)/4} e^{-cx/2} J_{\lambda-1/2}\left(\frac{c}{2} \sqrt{a^2-x^2}\right) L_n^{\lambda}(cx+d) dx =$$

$$= \frac{\sqrt{\pi} n! a^{2\lambda} c^{\lambda-1/2}}{2^{2\lambda-1} \Gamma(\lambda+n+1)} L_n^{\lambda}(z_+) L_n^{\lambda}(z_-)$$

[$2z_{\pm} = \sqrt{d+ac} \pm \sqrt{d-ac}; a > 0; \operatorname{Re} \lambda > -1/2$].

$$12. \int_0^{\infty} x^{\lambda} e^{-px} \left\{ \begin{matrix} J_{2\lambda}(b\sqrt{x}) \\ I_{2\lambda}(b\sqrt{x}) \end{matrix} \right\} [L_n^{\lambda}(cx)]^2 dx =$$

$$= \left(\frac{b}{2}\right)^{2\lambda} \frac{\Gamma(\lambda+n+1)}{\pi n! p^{2\lambda+1}} \exp\left(\mp \frac{b^2}{4p}\right) \times$$

$$\times \sum_{k=0}^n \frac{(-1)^k}{(n-k)!} \Gamma\left[\frac{n-k+1/2}{1+\lambda+k}, k+1/2\right] \left(1-\frac{2c}{p}\right)^{2k} L_{2k}^{2\lambda}\left(\pm \frac{b^2 c}{4pc-2p^2}\right)$$

[Re $p > 0$; Re $\lambda > -1/2$; $|\arg b| < \pi$].

$$13. \int_0^{\infty} x^{\lambda} e^{-cx} \left\{ \begin{matrix} J_{2\lambda}(b\sqrt{x}) \\ I_{2\lambda}(b\sqrt{x}) \end{matrix} \right\} [L_n^{\lambda}(cx)]^2 dx = \left(\frac{b}{2}\right)^{2\lambda} c^{-2\lambda-1} \exp\left(\mp \frac{b^2}{4c}\right) \left[L_n^{\lambda}\left(\pm \frac{b^2}{4c}\right)\right]^2$$

[Re $c > 0$; Re $\lambda > -1/2$; $|\arg b| < \pi$].

$$14. \int_0^{\infty} x^{(\gamma+\lambda)/2} e^{-cx} J_{\gamma+\lambda}(b\sqrt{x}) L_m^{\gamma}(cx) L_n^{\lambda}(cx) dx =$$

$$= \frac{(-1)^{m+n}}{c^{\gamma+\lambda+1}} \left(\frac{b}{2}\right)^{\gamma+\lambda} \exp\left(\mp \frac{b^2}{4c}\right) L_m^{\gamma+m-n}\left(\pm \frac{b^2}{4c}\right) L_n^{\lambda-m+n}\left(\pm \frac{b^2}{4c}\right)$$

[Re $c > 0$; Re $(\gamma+\lambda) > -1$; $|\arg b| < \pi$].

2.19.13. Интегралы, содержащие $\left\{ \begin{matrix} Y_\mu(bx^{\pm 1/2}) \\ K_\mu(bx^{\pm 1/2}) \end{matrix} \right\} L_n^\lambda(cx)$.

$$1. \int_0^\infty x^{\alpha-1} K_\mu(b\sqrt{x}) L_n^\lambda(cx) dx = \frac{(-c)^n 2^{2\alpha+2n-1}}{n! b^{2\alpha+2n}} \Gamma\left(\alpha + \frac{\mu}{2} + n\right) \times \\ \times \Gamma\left(\alpha - \frac{\mu}{2} + n\right) {}_2F_2\left(-n, -\lambda - n; 1 - \alpha - \frac{\mu}{2} - n, 1 - \alpha + \frac{\mu}{2} - n; -\frac{b^2}{4c}\right) \\ [\operatorname{Re} b > 0; 2 \operatorname{Re} \alpha > |\operatorname{Re} \mu|].$$

$$2. \int_0^\infty x^{\alpha-1} K_\mu\left(\frac{b}{\sqrt{x}}\right) L_n^\lambda(cx) dx = \frac{(\lambda+1)_n b^{2\alpha}}{2^{2\alpha+1} n!} \Gamma\left(\frac{\mu}{2} - \alpha\right) \Gamma\left(-\alpha - \frac{\mu}{2}\right) \times \\ \times {}_1F_3\left(-n; \lambda+1, 1 + \alpha + \frac{\mu}{2}, 1 + \alpha - \frac{\mu}{2}; \frac{b^2 c}{4}\right) \\ [\operatorname{Re} b > 0; 2 \operatorname{Re} \alpha + |\operatorname{Re} \mu| < -2n].$$

$$3. \int_0^\infty x^{\alpha-1} e^{-cx} \left\{ \begin{matrix} Y_\mu(bx) \\ K_\mu(bx) \end{matrix} \right\} L_n^\lambda(cx) dx = \\ = \mp \frac{2^{\alpha-2} (1+\lambda)_n}{\pi n! b^\alpha} \Gamma\left(\frac{\alpha+\mu}{2}\right) \Gamma\left(\frac{\alpha-\mu}{2}\right) \left\{ \frac{2 \cos[(\alpha-\mu)\pi/2]}{\pi} \right\} \times \\ \times {}_4F_3\left(\frac{\alpha+\mu}{2}, \frac{\alpha-\mu}{2}, \frac{1+\lambda+n}{2}, 1 + \frac{\lambda+n}{2}; \frac{1}{2}, \frac{1+\lambda}{2}, 1 + \frac{\lambda}{2}; \mp \frac{c^2}{b^2}\right) \mp \\ \mp \frac{2^{\alpha-1} c (2+\lambda)_n}{\pi n! b^{\alpha+1}} \Gamma\left(\frac{\alpha+\mu+1}{2}\right) \Gamma\left(\frac{\alpha-\mu+1}{2}\right) \left\{ \frac{2 \sin[(\alpha-\mu)\pi/2]}{\pi} \right\} \times \\ \times {}_4F_3\left(\frac{\alpha-\mu+1}{2}, \frac{\alpha+\mu+1}{2}, 1 + \frac{\lambda+n}{2}, \frac{3+\lambda+n}{2}; \frac{3}{2}, 1 + \frac{\lambda}{2}, \frac{3+\lambda}{2}; \mp \frac{c^2}{b^2}\right) \\ \left[\operatorname{Re} c > 0; \operatorname{Re} \alpha > |\operatorname{Re} \mu|, \left\{ \begin{matrix} b > 0 \\ \operatorname{Re}(b+c) > 0 \end{matrix} \right\} \right].$$

$$4. \int_0^\infty x^{\alpha-1} e^{-(b+c)x} K_\mu(bx) L_n^\lambda(cx) dx = \frac{\sqrt{\pi} (\lambda+1)_n}{n! (2b)^\alpha} \Gamma\left[\begin{matrix} \alpha+\mu, \alpha-\mu \\ \alpha+1/2 \end{matrix}\right] \times \\ \times {}_3F_2\left(\alpha+\mu, \alpha-\mu, 1+\lambda+n; 1+\lambda, \frac{1}{2}+\alpha; -\frac{c}{2b}\right) \\ [\operatorname{Re}(2b+c) > 0; \operatorname{Re} \alpha > |\operatorname{Re} \mu|].$$

$$5. \int_0^\infty x^{\alpha-1} e^{-cx} \left\{ \begin{matrix} Y_\mu(b\sqrt{x}) \\ K_\mu(b\sqrt{x}) \end{matrix} \right\} L_n^\lambda(cx) dx = \\ = \mp \left\{ \frac{2}{\pi} \right\} \frac{2^{\mu-1} (1-\alpha+\mu/2+\lambda)_n}{\pi n! b^\mu c^{\alpha-\mu/2}} \Gamma(\mu) \Gamma\left(\alpha - \frac{\mu}{2}\right) \times \\ \times {}_2F_2\left(\alpha - \frac{\mu}{2}, \alpha - \frac{\mu}{2} - \lambda; 1-\mu, \alpha - \frac{\mu}{2} - \lambda - n; \mp \frac{b^2}{4c}\right) \mp \\ \mp \left\{ \frac{2 \cos \mu\pi}{\pi} \right\} \frac{b^\mu \Gamma(-\mu)}{2^{\mu+1} \pi n! c^{\alpha+\mu/2}} \Gamma\left(\alpha + \frac{\mu}{2}\right) \left(1 - \alpha - \frac{\mu}{2} + \lambda\right)_n \times \\ \times {}_2F_2\left(\alpha + \frac{\mu}{2}, \alpha + \frac{\mu}{2} - \lambda; 1+\mu, \alpha + \frac{\mu}{2} - \lambda - n; \mp \frac{b^2}{4c}\right) \\ [\operatorname{Re} c > 0; 2 \operatorname{Re} \alpha > |\operatorname{Re} \mu|; |\arg b| < \pi; \text{при } \mu = 0, 1, 2, \dots \text{ см. 2.19.1.18}].$$

$$\begin{aligned}
6. \int_0^{\infty} x^{\alpha-1} e^{-cx} K_{\mu} \left(\frac{b}{\sqrt{x}} \right) L_n^{\lambda}(cx) dx = & \frac{(1-\alpha+\mu/2+\lambda)_n b^{\mu}}{2^{\mu+1} n! c^{\alpha-\mu/2}} \Gamma \left(\alpha - \frac{\mu}{2} \right) \times \\
& \times {}_1F_3 \left(1-\alpha+\frac{\mu}{2}+\lambda+n; 1+\mu, 1-\alpha+\frac{\mu}{2}, 1-\alpha+\frac{\mu}{2}+\lambda; -\frac{b^2 c}{4} \right) + \\
& + \frac{2^{\mu-1} (1-\alpha-\mu/2+\lambda)_n}{n! b^{\mu} c^{\alpha+\mu/2}} \Gamma \left(\alpha + \frac{\mu}{2} \right) \times \\
& \times {}_1F_3 \left(1-\alpha-\frac{\mu}{2}+\lambda+n; 1-\mu, 1-\alpha-\frac{\mu}{2}, 1-\alpha-\frac{\mu}{2}+\lambda; -\frac{b^2 c}{4} \right) \\
& [\operatorname{Re} b, \operatorname{Re} c > 0].
\end{aligned}$$

2.19.14. Интегралы, содержащие $L_m^{\gamma}(a+bx^k) L_n^{\lambda}(cx)$.

$$\begin{aligned}
1. \int_0^a x^{\lambda} (a-x)^{\gamma} L_m^{\gamma}(ca-cx) L_n^{\lambda}(cx) dx = \\
= a^{\gamma+\lambda+1} \frac{(m+n)!}{m! n!} B(\gamma+m+1, \lambda+n+1) L_{m+n}^{\gamma+\lambda+1}(ac) \\
[a > 0; \operatorname{Re} \gamma, \operatorname{Re} \lambda > -1].
\end{aligned}$$

$$2. \int_0^a L_m(ca-cx) L_n(cx) dx = \frac{1}{c} [L_{m+n}(ac) - L_{m+n+1}(ac)] \quad [a > 0].$$

$$\begin{aligned}
3. \int_0^a x^{\lambda} (a-x)^{\gamma+1} L_m^{\gamma}(ca-cx) L_n^{\lambda}(cx) dx = \\
= a^{\gamma+\lambda+2} (\gamma+1)(\gamma+\lambda+1) \frac{(m+n)!}{m! n!} B(\gamma+m+1, \lambda+n+1) L_{m+n}^{\gamma+\lambda+1}(ac) \\
[a > 0; \operatorname{Re} \gamma > -2; \operatorname{Re} \lambda > -1].
\end{aligned}$$

$$\begin{aligned}
4. \int_0^{\infty} e^{-px} L_n(bx) L_n(cx) dx = \frac{(p-b-c)^n}{p^{n+1}} P_n \left(\frac{p^2 - (b+c)p + 2bc}{p(p-b-c)} \right) \\
[\operatorname{Re} p > 0].
\end{aligned}$$

$$\begin{aligned}
5. \int_0^{\infty} x^{\alpha-1} e^{-px} L_n^{\lambda}(bx) L_n^{\lambda}(cx) dx = \\
= \frac{(\lambda+1)_n}{(n!)^2} \Gamma(\alpha) \left\{ \frac{\partial^n}{\partial h^n} \left[(1-h)^{-\lambda-1} B^{-\alpha} {}_2F_1 \left(\frac{\alpha}{2}, \frac{\alpha+1}{2}; \lambda+1; \frac{A^2}{B^2} \right) \right] \right\}_{h=0} \\
\left[A^2 = \frac{4bch}{(1-h)^2}, B = p + \frac{(b+c)h}{1-h}; \operatorname{Re} p, \operatorname{Re} \alpha > 0 \right].
\end{aligned}$$

$$\begin{aligned}
6. \int_0^{\infty} x^{\lambda} e^{-px} L_m^{\lambda}(bx) L_n^{\lambda}(cx) dx = \\
= \frac{(\lambda+1)_m (\lambda+1)_n}{m! n! p^{m+n+\lambda+1}} \Gamma(\lambda+1) (p-b)^m (p-c)^n {}_2F_1 \left(-m, -n; \lambda+1; \frac{bc}{(p-b)(p-c)} \right) = \\
= \frac{\Gamma(m+n+\lambda+1) (p-b)^m (p-c)^n}{m! n! p^{m+n+\lambda+1}} {}_2F_1 \left(-m, -n; -m-n-\lambda; \frac{p(p-b-c)}{(p-b)(p-c)} \right) \\
[\operatorname{Re} p > 0; \operatorname{Re} \lambda > -1].
\end{aligned}$$

7. $\int_0^\infty x^{2\lambda} e^{-px} L_n^\lambda(bx) L_n^\lambda(cx) dx = \frac{\Gamma(2\lambda+1) \Gamma(n+\lambda+1)}{n! p^{2\lambda+1}} \times$
 $\times \sum_{k=0}^\infty \frac{(-1)^k}{k! \Gamma(\lambda-k+1)} \left(1 - \frac{b+c}{2p}\right)^{n-k} C_{n+k}^{\lambda+1/2} \left(\frac{p^2 + (b+c)p + 2bc}{p(p-b-c)}\right)$
 $[\operatorname{Re} p > 0; \operatorname{Re} \lambda > -1/2].$
8. $\int_0^\infty x^{\alpha-1} e^{-cx} L_m^\gamma(bx) L_n^\lambda(cx) dx =$
 $= \frac{(1+\gamma)_m (1-\alpha+\lambda)_n \Gamma(\alpha)}{m! n! b^\alpha} {}_3F_2\left(-m, \alpha, \alpha-\lambda; 1+\gamma, \alpha-\lambda-n; \frac{b}{c}\right)$
 $[\operatorname{Re} c, \operatorname{Re} \alpha > 0].$
9. $\int_0^\infty x^{\alpha-1} e^{-(b+c)x} L_m^\gamma(bx) L_n^\lambda(cx) dx =$
 $= c^{-\alpha} \frac{(1+\lambda)_m}{m! n!} (1-\alpha+\lambda)_n \Gamma(\alpha) {}_3F_2\left(\alpha, \alpha-\lambda, 1+\gamma+m; \alpha-\lambda-n, 1+\lambda; -\frac{b}{c}\right)$
 $[\operatorname{Re}(b+c), \operatorname{Re} \alpha > 0].$
10. $\int_0^\infty x^\lambda e^{-cx} L_m^\lambda(bx) L_n^\lambda(cx) dx = \frac{(-1)^n \Gamma(\lambda+m+1) (-m)_n (c-b)^{m-n}}{m! n! b^{\lambda-n+1} c^m}$
 $[\operatorname{Re} c > 0; \operatorname{Re} \lambda > -1].$
11. $\int_0^\infty e^{-bx/2} L_m(bx) L_n(cx) dx = \frac{(-1)^m 2(b-2c)^n}{b^{n+1}} \sum_{k=0}^{\min(m, n)} \left(\frac{4c}{2c-b}\right)^k \binom{m}{k} \binom{n}{k}$
 $[\operatorname{Re} b > 0; b \neq 2c].$
 $= \frac{(-1)^{m+n} 2^n}{c} \binom{m}{n}$
 $[\operatorname{Re} c > 0; b = 2c; n \leq m].$
12. $\int_0^\infty e^{-2cx} L_m(bx) L_n(cx) dx = \frac{(2c-b)^m}{2^{m+n+1} c^{m+1}} \sum_{k=0}^{\min(m, n)} \left(\frac{b}{2c-b}\right)^k \binom{m}{k} \binom{n}{k}$
 $[\operatorname{Re} c > 0; b \neq 2c].$
13. $= \frac{2^{-n-1}}{c} \binom{n}{m}$
 $[\operatorname{Re} c > 0; b = 2c; m \leq n].$
14. $\int_0^\infty e^{-px} L_m(cx) L_n(cx) dx = \frac{(p-c)^{m+n}}{p^{m+n+1}} \sum_{k=0}^{\min(m, n)} \left(\frac{c}{p-c}\right)^{2k} \binom{m}{k} \binom{n}{k}$
 $[\operatorname{Re} p > 0].$
15. $\int_0^\infty x^{\alpha-1} e^{-cx} L_m^\gamma(cx) L_n^\lambda(cx) dx =$
 $= \frac{(1+\gamma)_m (\lambda-\alpha+1)_n \Gamma(\alpha)}{m! n! c^\alpha} {}_3F_2(-m, \alpha, \alpha-\lambda; \gamma+1, \alpha-\lambda-n; 1)$
 $[\operatorname{Re} \alpha, \operatorname{Re} c > 0].$
16. $\int_0^\infty x^{\lambda-m-n-1} e^{-cx} L_m^\gamma(cx) L_n^\lambda(cx) dx =$
 $= \frac{c^{m+n-\lambda} (1+m+n)_n (1+\gamma-\lambda+m+n)_{m+n} \Gamma(\lambda-m-n)}{n! m! (\gamma+m+1)_n}$
 $[\operatorname{Re} c > 0; \operatorname{Re} \lambda > m+n].$

$$17. \int_0^{\infty} x^{\lambda-1} e^{-cx} L_m^{\gamma}(cx) L_n^{\lambda}(cx) dx = (-1)^n \frac{(1+\gamma)_m}{m! c^{\lambda}} \Gamma(\lambda) \quad [\operatorname{Re} c, \operatorname{Re} \lambda > 0].$$

$$18. \int_0^{\infty} x^{\gamma+\lambda} e^{-cx} L_m^{\gamma}(cx) L_n^{\lambda}(cx) dx = \frac{(-1)^{m+n}}{c^{\gamma+\lambda+1}} \binom{\gamma+m}{n} \binom{\lambda+n}{m} \Gamma(\gamma+\lambda+1) \\ [\operatorname{Re} c > 0; \operatorname{Re}(\gamma+\lambda) > -1].$$

$$19. \int_0^{\infty} x^{\lambda} e^{-cx} L_m^{\lambda}(cx) L_n^{\lambda}(cx) dx = \frac{\Gamma(\lambda+n+1)}{n! c^{\lambda+1}} \delta_{m,n} \quad [\operatorname{Re} c > 0; \operatorname{Re} \lambda > -1].$$

$$20. \int_0^{\infty} x^{\alpha-1} e^{-bx^2-cx} L_m^{\gamma}(bx^2) L_n^{\lambda}(cx) dx = \frac{(1+\lambda)_n}{m! n! 2b^{\alpha/2}} \Gamma\left(\frac{\alpha}{2}\right) \left(1+\gamma-\frac{\alpha}{2}\right)_m \times \\ \times {}_4F_4\left(\frac{1+\lambda+n}{2}, 1+\frac{\lambda+n}{2}, \frac{\alpha}{2}, \frac{\alpha}{2}-\gamma; \frac{\alpha}{2}-\gamma-m, \frac{1}{2}, \frac{1+\lambda}{2}, 1+\frac{\lambda}{2}; \frac{c^2}{4b}\right) - \\ - \frac{c(2+\lambda)_n}{m! n! 2b^{(\alpha+1)/2}} \left(\frac{1-\alpha}{2}+\gamma\right)_m \Gamma\left(\frac{\alpha+1}{2}\right) {}_4F_4\left(\frac{1+\alpha}{2}, \frac{1+\alpha}{2}-\gamma, 1+\frac{\lambda+n}{2}, \right. \\ \left. \frac{3+\lambda+n}{2}; \frac{3}{2}, \frac{1+\alpha}{2}-\gamma-m, 1+\frac{\lambda}{2}, \frac{3+\lambda}{2}; \frac{c^2}{4b}\right) \\ [\operatorname{Re} b, \operatorname{Re} c, \operatorname{Re} \alpha > 0].$$

$$21. \int_0^{\infty} e^{-cx} \sin b \sqrt{x} L_n^{1/2-\lambda}(cx) L_n^{\lambda}(cx) dx = \\ = \sqrt{\frac{\pi}{c^3}} \frac{b}{2} \exp\left(-\frac{b^2}{4c}\right) L_n^{1/2-\lambda}\left(\frac{b^2}{4c}\right) L_n^{\lambda}\left(\frac{b^2}{4c}\right) \quad [b, \operatorname{Re} c > 0].$$

$$22. \int_0^{\infty} x^{-1/2} e^{-cx} \cos b \sqrt{x} L_n^{-\lambda-1/2}(cx) L_n^{\lambda}(cx) dx = \\ = \sqrt{\frac{\pi}{c}} \exp\left(-\frac{b^2}{4c}\right) L_n^{-1/2-\lambda}\left(\frac{b^2}{4c}\right) L_n^{\lambda}\left(\frac{b^2}{4c}\right) \quad [b, \operatorname{Re} c > 0].$$

$$23. \int_0^{\infty} x^{\lambda_1+\dots+\lambda_m} e^{-cx} L_{n_1}^{\lambda_1}(cx) \dots L_{n_m}^{\lambda_m}(cx) dx = \\ = \frac{(-1)^{n_1+\dots+n_m}}{c^{\lambda_1+\dots+\lambda_m+1}} (1+\lambda_1)_{n_1} \dots (1+\lambda_m)_{n_m} (\lambda_1+\dots+\lambda_m)! \times \\ \times F_A^n(\lambda_1+\dots+\lambda_m; -n_1, \dots, -n_m; 1+\lambda_1, \dots, 1+\lambda_m; 1, \dots, 1) \\ [\lambda_1, \dots, \lambda_m = 0, 1, 2, \dots; \operatorname{Re} c > 0].$$

2.20. МНОГОЧЛЕНЫ ЭРМИТА $H_n(x)$

При вычислении некоторых интегралов, содержащих многочлены Эрмита $H_n(x)$, можно воспользоваться соотношением

$$H_{2n+\varepsilon}(x) = (-1)^n 2^{2n+\varepsilon} n! x^{\varepsilon} L_n^{\varepsilon-1/2}(x^2) \quad [\varepsilon=0 \text{ или } 1]$$

и формулами из раздела 2.19.

2.20.1. Интегралы общего вида.

Обозначение: $\delta = \begin{cases} 1 \\ 0 \end{cases}$.

$$1. \int_0^a x^{\alpha-1} (a^r - x^r)^{\beta-1} e^{-c^2 x^2} \left\{ \begin{matrix} H_{2n+1}(cx) \\ H_{2n}(cx) \end{matrix} \right\} dx = (-1)^n \frac{2^{2n+\delta}}{r} a^{\alpha+r\beta-r+\delta} c^\delta \Gamma(\beta) \times \\ \times \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \left(k + \delta + \frac{1}{2} \right)_n \Gamma \left[\frac{(2k + \alpha + \delta)/r}{\beta + (2k + \alpha + \delta)/r} \right] (ac)^{2k} \\ [a, r, \operatorname{Re} \beta > 0; \operatorname{Re} \alpha > -\delta].$$

$$2. \int_a^\infty x^{\alpha-1} (x^r - a^r)^{\beta-1} e^{-c^2 x^2} \left\{ \begin{matrix} H_{2n+1}(cx) \\ H_{2n}(cx) \end{matrix} \right\} dx = \\ = (-1)^n \frac{2^{2n+\delta}}{r} a^{\alpha+r\beta-r+\delta} c^\delta \Gamma(\beta) \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \left(k + \delta + \frac{1}{2} \right)_n \times \\ \times \Gamma \left[\frac{1 - \beta - (2k + \alpha + \delta)/r}{1 - (2k + \alpha + \delta)/r} \right] (ac)^{2k} + (-1)^n 2^{2n+\delta-1} c^{r-\beta-\alpha} \times \\ \times \sum_{k=0}^{\infty} \frac{(1-\beta)_k}{k!} \left(\frac{r(k-\beta+1) - \alpha + \delta + 1}{2} \right)_n \Gamma \left(\frac{\alpha + \delta - r(k-\beta+1)}{2} \right) (ac)^{rk} \\ [a, r, \operatorname{Re} \beta > 0; |\arg c| < \pi/4].$$

$$3. \int_0^\infty \frac{x^{\alpha-1}}{(x^r + z^r)^\rho} e^{-c^2 x^2} \left\{ \begin{matrix} H_{2n+1}(cx) \\ H_{2n}(cx) \end{matrix} \right\} dx = \\ = (-1)^n \frac{2^{2n+\delta}}{r \Gamma(\rho)} c^\delta z^{\alpha-r\rho+\delta} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \left(k + \delta + \frac{1}{2} \right)_n \Gamma \left(\frac{2k + \alpha + \delta}{r} \right) \times \\ \times \Gamma \left(\rho - \frac{2k + \alpha + \delta}{r} \right) (cz)^{2k} + (-1)^n 2^{2n+\delta-1} c^{r\rho-\alpha} \times \\ \times \sum_{k=0}^{\infty} \frac{(-1)^k (\rho)_k}{k!} \left(\frac{r\rho + rk - \alpha + \delta + 1}{2} \right)_n \Gamma \left(\frac{\alpha + \delta - r\rho - rk}{2} \right) (cz)^{rk} \\ [r > 0; r |\arg z| < \pi; \operatorname{Re} \alpha > -\delta; |\arg c| < \pi/4].$$

$$4. \int_0^\infty \frac{x^{\alpha-1}}{x^r - y^r} e^{-c^2 x^2} \left\{ \begin{matrix} H_{2n+1}(cx) \\ H_{2n}(cx) \end{matrix} \right\} dx = \\ = (-1)^{n+1} \frac{2^{2n+\delta}}{r} \pi c^0 y^{\alpha+\delta-r} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \left(k + \delta + \frac{1}{2} \right)_n \times \\ \times \operatorname{ctg} \left(\frac{2k + \alpha + \delta}{r} \pi \right) (cy)^{2k} + (-1)^n 2^{2n+\delta-1} c^{r-\alpha} \times \\ \times \sum_{k=0}^{\infty} \left(\frac{r + rk - \alpha + \delta + 1}{2} \right)_n \Gamma \left(\frac{\alpha + \delta - r - rk}{2} \right) (cy)^{rk} \\ [r, y > 0; \operatorname{Re} \alpha > -\delta; |\arg c| < \pi/4].$$

$$5. \int_0^\infty x^{\alpha-1} e^{-px^r - c^2 x^2} \left\{ \begin{matrix} H_{2n+1}(cx) \\ H_{2n}(cx) \end{matrix} \right\} dx = U \\ [r, \operatorname{Re} p > 0; \operatorname{Re} \alpha > -\delta; |\arg c| < \pi/4].$$

$$U = (-1)^n \frac{2^{2n+\delta} c^\delta}{p^{(\alpha+\delta)/r}} \sum_{k=0}^{\infty} \frac{1}{k!} \left(k + \delta + \frac{1}{2} \right)_n \Gamma \left(\frac{2k + \alpha + \delta}{r} \right) \left(-\frac{c^2}{p^{2/r}} \right)^k \quad [r > 2]$$

$$U = (-1)^n \frac{2^{2n+\delta-1}}{c^\alpha} \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{\delta - \alpha - rk + 1}{2} \right)_n \Gamma \left(\frac{\delta + \alpha + rk}{2} \right) \left(-\frac{p}{c^r} \right)^k \quad [r < 2]$$

U при $r=2$ см. в 2.20.3.4.

$$6. \int_0^\infty x^{\alpha-1} e^{-px-r-c^2x^2} \left\{ \begin{matrix} H_{2n+1}(cx) \\ H_{2n}(cx) \end{matrix} \right\} dx$$

$$= (-1)^n \frac{2^{2n+\delta}}{r} c^\delta p^{(\alpha+\delta)/r} \sum_{k=0}^{\infty} \frac{1}{k!} \left(k + \delta + \frac{1}{2} \right)_n \Gamma \left(-\frac{2k + \alpha + \delta}{r} \right) (-c^2 p^{2/r})^k +$$

$$+ (-1)^n \frac{2^{2n+\delta-1}}{c^\alpha} \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{rk - \alpha + \delta + 1}{2} \right)_n \Gamma \left(\frac{\alpha + \delta - rk}{2} \right) (-pc^r)^k$$

[$r, \operatorname{Re} p > 0; |\arg c| < \pi/4$]

$$7. \int_0^\infty x^{\alpha-1} e^{-c^2x^2} \left\{ \begin{matrix} \sin bx^r \\ \cos bx^r \end{matrix} \right\} H_{2n+\varepsilon}(cx) dx = V$$

[$\varepsilon = 0$ или $1; b, r > 0; \operatorname{Re} \alpha > -\varepsilon - \delta r; |\arg c| < \pi/4$]

$$V = (-1)^n \frac{2^{2n+\varepsilon} c^\varepsilon}{b^{(\alpha+\varepsilon)/r}} \sum_{k=0}^{\infty} \frac{1}{k!} \left(k + \varepsilon + \frac{1}{2} \right)_n \Gamma \left(\frac{2k + \alpha + \varepsilon}{r} \right) \times$$

$$\times \left\{ \begin{matrix} \sin [(2k + \alpha + \varepsilon) \pi / (2r)] \\ \cos [(2k + \alpha + \varepsilon) \pi / (2r)] \end{matrix} \right\} \left(-\frac{c^2}{b^{2/r}} \right)^k \quad [r > 2]$$

$$V = (-1)^n \frac{2^{2n+\varepsilon-1} b^\delta}{c^{\alpha+\delta r}} \sum_{k=0}^{\infty} \frac{1}{k! (\delta + 1/2)_k} \left(\frac{1 + \varepsilon - \alpha - \delta r}{2} - rk \right)_n \times$$

$$\times \Gamma \left(\frac{\alpha + \varepsilon + \delta r}{2} + rk \right) \left(-\frac{b^2}{4c^{2r}} \right)^k \quad [r < 2]$$

V при $r=2$ см. в 2.20.5.4.

$$8. \int_0^\infty x^{\alpha-1} e^{-c^2x^2} \left\{ \begin{matrix} \ln(x^r + z^r) \\ \ln|x^r - z^r| \end{matrix} \right\} H_{2n+\varepsilon}(cx) dx =$$

$$= (-1)^n 2^{2n+\varepsilon} \pi c^{\varepsilon} z^{\alpha+\varepsilon} \sum_{k=0}^{\infty} \frac{(-c^2 z^2)^k}{k! (2k + \alpha + \varepsilon)} \left(k + \varepsilon + \frac{1}{2} \right)_n \left\{ \begin{matrix} \operatorname{cosec} [(2k + \alpha + \varepsilon) \pi / r] \\ \operatorname{ctg} [(2k + \alpha + \varepsilon) \pi / r] \end{matrix} \right\} \pm$$

$$\pm (-1)^n 2^{2n+\varepsilon-1} \frac{z^r}{c^{\alpha-r}} \sum_{k=0}^{\infty} \frac{(\mp c^r z^r)^k}{k+1} \left(\frac{r + rk - \alpha + \varepsilon + 1}{2} \right)_n \Gamma \left(\frac{\alpha + \varepsilon - r - rk}{2} \right) +$$

$$+ (-1)^n \frac{2^{2n+\varepsilon-2} r}{c^\alpha} \left(\frac{\varepsilon - \alpha + 1}{2} \right)_n \Gamma \left(\frac{\alpha + \varepsilon}{2} \right) \left[\psi \left(\frac{\alpha + \varepsilon}{2} \right) - \sum_{k=0}^{n-1} \frac{2}{2k - \alpha + \varepsilon + 1} - 2 \ln c \right]$$

[$\varepsilon = 0$ или $1; r > 0; \operatorname{Re} \alpha > -\varepsilon; |\arg c| < \pi/4; r \mid \arg z| < \pi$]

$$9. \int_0^\infty x^{\alpha-1} e^{-c^2x^2} \operatorname{Ei}(-bx^r) \left\{ \begin{matrix} H_{2n+1}(cx) \\ H_{2n}(cx) \end{matrix} \right\} dx = W$$

[$r, \operatorname{Re} b > 0; \operatorname{Re} \alpha > -\delta; |\arg c| < \pi/4$]

$$W = (-1)^{n+1} \frac{2^{2n+\delta} c^\delta}{b^{(\alpha+\delta)/r}} \sum_{k=0}^{\infty} \frac{1}{k! (2k+\alpha+\delta)} \left(k+\delta+\frac{1}{2}\right)_n \Gamma\left(\frac{2k+\alpha+\delta}{r}\right) \left(-\frac{c^2}{b^{2/r}}\right)^k$$

$[r > 2]$

$$W = (-1)^{n+1} \frac{2^{2n+\delta-1} b}{c^{\alpha+r}} \sum_{k=0}^{\infty} \frac{1}{k! (k+1)^2} \left(\frac{\delta-\alpha-r-rk+1}{2}\right)_n \times$$

$$\times \Gamma\left(\frac{\alpha+\delta+r+rk}{2}\right) \left(-\frac{b}{c^r}\right)^k + (-1)^n \frac{2^{2n+\delta-1}}{c^\alpha} \left(\frac{\delta-\alpha+1}{2}\right)_n \times$$

$$\times \Gamma\left(\frac{\alpha+\delta}{2}\right) \left[C + \frac{r}{2} \psi\left(\frac{\alpha+\delta}{2}\right) - r \sum_{k=0}^{n-1} \frac{1}{2k-\alpha+\delta+1} - \ln \frac{c^r}{b}\right]$$

$[r < 2]$

W при $r=2$ см. в 2.20.7.2.

$$10. \int_0^\infty x^{\alpha-1} e^{\pm bx^r - c^2 x^2} \operatorname{Ei}(\mp bx^r) H_{2n+\varepsilon}(cx) dx = U$$

$$[\varepsilon=0 \text{ или } 1; r>0; \operatorname{Re} \alpha > -\varepsilon; |\arg c| < \pi/4, \left\{ \begin{array}{l} |\arg b| < \pi \\ b > 0 \end{array} \right\}],$$

$$U = (-1)^{n+1} \frac{2^{2n+\varepsilon} \pi c^\varepsilon}{b^{(\alpha+\varepsilon)/r}} \sum_{k=0}^{\infty} \frac{1}{k!} \left(k+\varepsilon+\frac{1}{2}\right)_n \Gamma\left(\frac{2k+\alpha+\varepsilon}{r}\right) \times$$

$$\times \left\{ \operatorname{cosec} [(2k+\alpha+\varepsilon) \pi/r] \right\} \left(-\frac{c^2}{b^{2/r}}\right)^k \mp (-1)^n \frac{2^{2n+\varepsilon-1}}{b c^{\alpha-r}} \times$$

$$\times \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{r+rk-\alpha+\varepsilon+1}{2}\right)_n \Gamma\left(\frac{\alpha+\varepsilon-r-rk}{2}\right) \left(\mp \frac{c^r}{b}\right)^k$$

$[r > 2]$

$$U = (-1)^{n+1} \frac{2^{2n+\varepsilon-1}}{c^\alpha} \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{\varepsilon-\alpha-rk+1}{2}\right)_n \Gamma\left(\frac{\alpha+\varepsilon+rk}{2}\right) \times$$

$$\times \left[\psi(k+1) - \frac{r}{2} \psi\left(\frac{\alpha+\varepsilon+rk}{2}\right) + r \sum_{m=0}^n \frac{1}{2m-rk-\alpha+\varepsilon+1} + \ln \frac{c^r}{b} \right] \left(\pm \frac{b}{c^r}\right)^k$$

$[r < 2]$

U при $r=2$ см. в 2.20.7.4.

$$11. \int_0^\infty x^{\alpha-1} e^{-c^2 x^2} \left\{ \begin{array}{l} \operatorname{si}(bx^r) \\ \operatorname{ci}(bx^r) \end{array} \right\} H_{2n+\varepsilon}(cx) dx = V$$

$$[\varepsilon=0 \text{ или } 1; b, r > 0; \operatorname{Re} \alpha > -\varepsilon; |\arg c| < \pi/4]$$

$$V = (-1)^{n+1} \frac{2^{2n+\varepsilon} c^\varepsilon}{b^{(\alpha+\varepsilon)/r}} \sum_{k=0}^{\infty} \frac{1}{k! (2k+\alpha+\varepsilon)} \left(k+\varepsilon+\frac{1}{2}\right)_n \Gamma\left(\frac{2k+\alpha+\varepsilon}{r}\right) \times$$

$$\times \left\{ \begin{array}{l} \sin [(2k+\alpha+\varepsilon) \pi/(2r)] \\ \cos [(2k+\alpha+\varepsilon) \pi/(2r)] \end{array} \right\} \left(-\frac{c^2}{b^{2/r}}\right)^k$$

$[r > 2]$

$$V = (-1)^{n+1} \frac{2^{2n+\varepsilon-2} b^{\delta+2}}{3\delta c^{\alpha+2r+\delta r}} \sum_{k=0}^{\infty} \frac{1}{(k+1)! (\delta+3/2)_k (2k+\delta+2)} \times$$

$$\times \left(\frac{1-\alpha+\varepsilon-\delta r}{2} - r - rk \right)_n \Gamma \left(\frac{\alpha+\varepsilon+\delta r}{2} + r + rk \right) \left(-\frac{b^2}{4c^{2r}} \right)^k +$$

$$+ (-1)^n \frac{2^{2n+\varepsilon-1} b^{\delta}}{c^{\alpha+\delta r}} \left(\frac{\varepsilon-\alpha-\delta r+1}{2} \right)_n \Gamma \left(\frac{\alpha+\varepsilon+\delta r}{2} \right) \left\{ R \right\} -$$

$$- \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} (-1)^n \frac{2^{2n+\varepsilon-2} \pi}{c^{\alpha}} \left(\frac{\varepsilon-\alpha+1}{2} \right)_n \Gamma \left(\frac{\alpha+\varepsilon}{2} \right),$$

$$R = C + \frac{r}{2} \Psi \left(\frac{\alpha+\varepsilon}{2} \right) - r \sum_{k=0}^{n-1} \frac{1}{2k-\alpha+\varepsilon-\delta r+1} - \ln \frac{c^r}{b}$$

$[r < 2]$.

V при $r=2$ см. в 2.20.8.2.

$$12. \int_0^{\infty} x^{\alpha-1} e^{-c^2 x^2} \left\{ \begin{matrix} \operatorname{erf}(bx^r) \\ \operatorname{erfc}(bx^r) \end{matrix} \right\} H_{2n+\varepsilon}(cx) dx = W$$

$\{\varepsilon=0 \text{ или } 1; r>0; \operatorname{Re} \alpha > -\varepsilon - r(1 \pm 1)/2; |\arg b|, |\arg c| < \pi/4\}$

$$W = \mp \frac{(-1)^n 2^{2n+\varepsilon} c^{\varepsilon}}{\sqrt{\pi} b^{(\alpha+\varepsilon)/r}} \sum_{k=0}^{\infty} \frac{1}{k! (2k+\alpha+\varepsilon)} \left(k + \varepsilon + \frac{1}{2} \right)_n \times$$

$$\times \Gamma \left(\frac{2k+\alpha+\varepsilon+r}{2r} \right) \left(-\frac{c^2}{b^{2/r}} \right)^k + \left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \frac{(-1)^n 2^{2n+\varepsilon-1}}{c^{\alpha}} \left(\frac{\varepsilon-\alpha+1}{2} \right)_n \Gamma \left(\frac{\alpha+\varepsilon}{2} \right)$$

$[r > 1]$.

$$W = \pm \frac{(-1)^n 2^{2n+\varepsilon} b}{\sqrt{\pi} c^{\alpha+r}} \sum_{k=0}^{\infty} \frac{1}{k! (2k+1)} \left(\frac{\varepsilon-\alpha-r-2rk+1}{2} \right)_n \times$$

$$\times \Gamma \left(\frac{\alpha+\varepsilon+r+2rk}{2} \right) \left(-\frac{b^2}{c^{2r}} \right)^k + \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{(-1)^n 2^{2n+\varepsilon-1}}{c^{\alpha}} \left(\frac{\varepsilon-\alpha+1}{2} \right)_n \Gamma \left(\frac{\alpha+\varepsilon}{2} \right)$$

$[r < 1]$.

W при $r=1$ см. в 2.20.10.2.

$$13. \int_0^{\infty} x^{\alpha-1} e^{-c^2 x^2} \left\{ \begin{matrix} S(bx^r) \\ C(bx^r) \end{matrix} \right\} H_{2n+\varepsilon}(cx) dx = J$$

$\{\varepsilon=0 \text{ или } 1; b, r>0; \operatorname{Re} \alpha > -\varepsilon - r(2 \pm 1)/2; |\arg c| < \pi/4\}$

$$U = (-1)^{n-1} \frac{2^{2n+\varepsilon-1/2} c^{\varepsilon}}{\sqrt{\pi} b^{(\alpha+\varepsilon)/r}} \sum_{k=0}^{\infty} \frac{1}{k! (2k+\alpha+\varepsilon)} \left(k + \varepsilon + \frac{1}{2} \right)_n \times$$

$$\times \Gamma \left(\frac{2k+\alpha+\varepsilon}{r} + \frac{1}{2} \right) \left\{ \begin{matrix} \sin[(4k+2\alpha+2\varepsilon+r)\pi/(4r)] \\ \cos[(4k+2\alpha+2\varepsilon+r)\pi/(4r)] \end{matrix} \right\} \left(-\frac{c^2}{b^{2/r}} \right)^k +$$

$$+ (-1)^n \frac{2^{2n+\varepsilon-2}}{c^{\alpha}} \left(\frac{\varepsilon-\alpha+1}{2} \right)_n \Gamma \left(\frac{\alpha+\varepsilon}{2} \right)$$

$[r > 2]$.

$$U = (-1)^n \frac{2^{2n+\varepsilon-1/2} b^{\delta+1/2}}{\sqrt{\pi} c^{\alpha+(2\delta+1)r/2}} \sum_{k=0}^{\infty} \frac{1}{k! (\delta+1/2)_k (4k+2\delta+1)} \times$$

$$\times \left(\frac{2\varepsilon-2\alpha-r-2\delta r-4rk+2}{4} \right)_n \Gamma \left(\frac{2\alpha+2\varepsilon+r+2\delta r+4rk}{4} \right) \left(-\frac{b^2}{4c^{2r}} \right)^k$$

$[r < 2]$.

U при $r=2$ см в 2.20.11.2.

$$14. \int_0^{\infty} x^{\alpha-1} e^{-c^2 x^2} \left\{ \frac{\gamma(\mu, bx^r)}{\Gamma(\mu, bx^r)} \right\} H_{2n+\varepsilon}(cx) dx = V$$

$$\left[\varepsilon = 0 \text{ или } 1; r, \operatorname{Re} b > 0; \operatorname{Re}(\alpha + \mu r) > -\varepsilon, \begin{cases} \operatorname{Re} \mu > 0 \\ \operatorname{Re} \alpha > -\varepsilon \end{cases}, |\arg c| < \pi/4 \right],$$

$$V = \mp (-1)^n \frac{2^{2n+\varepsilon} c^\varepsilon}{b^{(\alpha+\varepsilon)/r}} \sum_{k=0}^{\infty} \frac{1}{k! (2k+\alpha+\varepsilon)} \left(k + \varepsilon + \frac{1}{2} \right)_n \times \\ \times \Gamma\left(\frac{2k+\alpha+\varepsilon+\mu r}{r}\right) \left(-\frac{c^2}{b^{2/r}}\right)^k + \begin{cases} 1 \\ 0 \end{cases} (-1)^n \frac{2^{2n+\varepsilon-1}}{c^\alpha} \Gamma(\mu) \Gamma\left(\frac{\alpha+\varepsilon}{2}\right) \left(\frac{\varepsilon-\alpha+1}{2}\right)_n$$

$[r > 2],$

$$V = \pm \frac{(-1)^n 2^{2n+\varepsilon-1} b^\mu}{c^{\alpha+\mu r}} \sum_{k=0}^{\infty} \frac{1}{k! (k+\mu)} \left(\frac{\varepsilon-\alpha-\mu r-rk+1}{2} \right)_n \times \\ \times \Gamma\left(\frac{\alpha+\varepsilon+\mu r+rk}{2}\right) \left(-\frac{b}{c^r}\right)^k + \begin{cases} 0 \\ 1 \end{cases} (-1)^n \frac{2^{2n+\varepsilon-1}}{c^\alpha} \left(\frac{\varepsilon-\alpha+1}{2}\right)_n \Gamma(\mu) \Gamma\left(\frac{\alpha+\varepsilon}{2}\right)$$

$[r < 2],$

V при $r=2$ см. в 2.20.12.2.

$$15. \int_0^{\infty} x^{\alpha-1} e^{-c^2 x^2} J_\mu(bx^r) \left\{ \frac{H_{2n+1}(cx)}{H_{2n}(cx)} \right\} dx = W$$

$$[b, r > 0; \operatorname{Re}(\alpha + \mu r) > -\delta; |\arg c| < \pi/4],$$

$$W = (-1)^n \frac{2^{2n+\delta+(\alpha+\delta)/r-1} c^\delta}{b^{(\alpha+\delta)/r}} \sum_{k=0}^{\infty} \frac{1}{k!} \left(k + \delta + \frac{1}{2} \right)_n \times \\ \times \Gamma\left[\frac{(\mu r + 2k + \alpha + \delta)/(2r)}{(\mu r - 2k - \alpha - \delta)/(2r) + 1}\right] \left(-\frac{2^{2/r} c^2}{b^{2/r}}\right)^k$$

$[r > 2],$

$$W = W_-$$

$$[r < 2],$$

$$W_{\pm} = (-1)^n \frac{2^{2n+\delta-\mu-1} b^\mu}{c^{\alpha+\mu r} \Gamma(\mu+1)} \sum_{k=0}^{\infty} \frac{1}{k! (\mu+1)_k} \times \\ \times \left(\frac{\delta-\alpha-\mu r-2rk+1}{2} \right)_n \Gamma\left(\frac{\alpha+\delta+\mu r+2rk}{2}\right) \left(\pm \frac{b^2}{4c^{2r}}\right)^k,$$

W при $r=2$ см. в 2.20.13.5.

$$16. \int_0^{\infty} x^{\alpha-1} e^{-c^2 x^2} I_\mu(bx^r) \left\{ \frac{H_{2n+1}(cx)}{H_{2n}(cx)} \right\} dx = W_+$$

$$[0 < r < 2; \operatorname{Re}(\alpha + \mu r) > -\delta; |\arg b| < \pi; |\arg c| < \pi/4; W_+ \text{ см. в 2.20.1.15, } W_+ \text{ при } r=2 \text{ см. в 2.20.13.5}]$$

$$17. \int_0^{\infty} x^{\alpha-1} e^{-c^2 x^2} \left\{ \frac{Y_\mu(bx^r)}{K_\mu(bx^r)} \right\} H_{2n+\varepsilon}(cx) dx = U_\mu$$

$$\left[\varepsilon = 0 \text{ или } 1; r > 0; \operatorname{Re} \alpha > r |\operatorname{Re} \mu| - \varepsilon; |\arg c| < \pi/4, \begin{cases} \operatorname{Re} b > 0 \\ \operatorname{Re} \mu > 0 \end{cases} \right],$$

$$U_\mu = \mp \frac{(-1)^n 2^{2n+\varepsilon+(\alpha+\varepsilon)/r-2} c^\varepsilon}{\pi b^{(\alpha+\varepsilon)/r}} \sum_{k=0}^{\infty} \frac{1}{k!} \left(k + \varepsilon + \frac{1}{2} \right)_n \Gamma\left(\frac{2k+\alpha+\varepsilon+\mu r}{2r}\right) \times \\ \times \Gamma\left(\frac{2k+\alpha+\varepsilon-\mu r}{2r}\right) \pi^{(1 \mp 1)/2} \left(2 \cos \frac{2k+\alpha+\varepsilon-\mu r}{2r} \pi \right)^{(1 \pm 1)/2} \left(-\frac{2^{2/r} c^2}{b^{2/r}}\right)^k$$

$[r > 2],$

$$U_{\mu} = \mp (-1)^n \frac{2^{2n+\delta-2}}{\pi c^{\alpha}} \left\{ \frac{2}{\pi} \right\} \sum_{k=0}^{\infty} \frac{1}{k!} \left[A_k(\mu) + \left\{ \frac{\cos \mu \pi}{1} \right\} A_k(-\mu) \right] \left(\mp \frac{b^2}{4c^{2r}} \right)^k$$

[$r < 2$; $\mu \neq -2, -1, 0, 1, 2, \dots$].

$$A_k(\mu) = \left(\frac{2c^r}{b} \right)^{\mu} \frac{\Gamma(\mu)}{(1-\mu)_k} \left(\frac{\varepsilon - \alpha + \mu r - 2rk + 1}{2} \right)_n \Gamma \left(\frac{\alpha + \varepsilon - \mu r + 2rk}{2} \right),$$

$$U_m = \mp (-1)^n \frac{2^{2n+\delta-2}}{\pi c^{\alpha}} \left\{ \frac{2}{\pi} \right\} \left[\sum_{k=0}^{m-1} \frac{A_k(m)}{k!} \left(\mp \frac{b^2}{4c^{2r}} \right)^k + \right. \\ \left. + \left(\pm \frac{2c^r}{b} \right)^{-m} \sum_{k=0}^{\infty} \frac{1}{k! (k+m)!} \left(\frac{\varepsilon - \alpha - rm - 2rk + 1}{2} \right)_n \times \right. \\ \left. \times \Gamma \left(\frac{\alpha + \varepsilon + rm + 2rk}{2} \right) \left(\psi(k+1) + \psi(k+m+1) - r\psi \left(\frac{\alpha + \varepsilon + rm}{2} + rk \right) + \right. \right. \\ \left. \left. + 2r \sum_{l=0}^{n-1} \frac{1}{2l - 2rk - rm - \alpha + \varepsilon + 1} + 2 \ln \frac{2c^r}{b} \right) \left(\mp \frac{b^2}{4c^{2r}} \right)^k \right]$$

[$r < 2$; $m = 0, 1, 2, \dots$].

U_{μ} при $r=2$ см. в 2.20.14.4.

$$18. \int_0^{\infty} x^{\alpha-1} e^{-bx^r - c^2 x^2} L_m^{\gamma}(bx^r) \begin{Bmatrix} H_{2n+1}(cx) \\ H_{2n}(cx) \end{Bmatrix} dx = V$$

[$r, \operatorname{Re} b > 0$; $\operatorname{Re} \alpha > -\delta$; $|\arg c| < \pi/4$].

$$V = \frac{(-1)^n 2^{2n+\delta} c^{\delta}}{m! b^{(\alpha+\delta)/r}} \sum_{k=0}^{\infty} \frac{1}{k!} \left(k + \delta + \frac{1}{2} \right)_n \left(1 + \gamma - \frac{2k + \alpha + \delta}{r} \right)_m \times \\ \times \Gamma \left(\frac{2k + \alpha + \delta}{r} \right) \left(-\frac{c^2}{b^{2/r}} \right)^k$$

[$r > 2$].

$$V = \frac{(-1)^n 2^{2n+\delta-1}}{m! c^{\alpha}} \sum_{k=0}^{\infty} \frac{(k + \gamma + 1)_m}{k!} \left(\frac{\delta - \alpha - rk + 1}{2} \right)_n \Gamma \left(\frac{\alpha + \delta + rk}{2} \right) \left(-\frac{b}{c^r} \right)^k$$

[$r < 2$].

V при $r=2$ см. в 2.20.15.2.

$$19. \int_0^{\infty} x^{\alpha-1} e^{-b^2 x^{2r} - c^2 x^2} H_m(bx^r) \begin{Bmatrix} H_{2n+1}(cx) \\ H_{2n}(cx) \end{Bmatrix} dx = W$$

[$r > 0$; $\operatorname{Re} \alpha > -\delta - rm/2 + r[m/2]$; $|\arg b|, |\arg c| < \pi/4$].

$$W = \frac{(-1)^n + [m/2] 2m + 2n + \delta - 1}{b^{(\alpha+\delta)/r}} c^{\delta} \sum_{k=0}^{\infty} \frac{1}{k!} \left(k + \delta + \frac{1}{2} \right)_n \times \\ \times \left(\frac{m+1}{2} - \left[\frac{m}{2} \right] - \frac{2k + \alpha + \delta}{2r} \right)_{[m/2]} \Gamma \left(\frac{m}{2} - \left[\frac{m}{2} \right] + \frac{2k + \alpha + \delta}{2r} \right) \left(-\frac{c^2}{b^{2/r}} \right)^k$$

[$r > 1$].

W при $r=1$ см. в 2.20.16.13.

2.20.2. Интегралы от $A(x)H_n(cx)$.

Обозначение: $\delta = \begin{cases} 1 \\ 0 \end{cases}$.

$$1. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\beta-1} \left\{ \frac{H_{2n+1}(cx)}{H_{2n}(cx)} \right\} dx = (-1)^n 2^{2n+\delta-1} a^{\alpha+2\beta+\delta-2} c^\delta \times \\ \times B\left(\frac{\alpha+\delta}{2}, \beta\right) \left(\delta + \frac{1}{2}\right)_n {}_2F_2\left(-n, \frac{\alpha+\delta}{2}; \delta + \frac{1}{2}, \frac{\alpha+2\beta+\delta}{2}; a^2 c^2\right) \\ [a, \operatorname{Re} \beta > 0; \operatorname{Re} \alpha > -\delta].$$

$$2. \int_0^a x^{(1 \pm 1)/2} (a^2 - x^2)^{\beta-1} \left\{ \frac{H_{2n+1}(cx)}{H_{2n}(cx)} \right\} dx = \\ = (-1)^n 2^{2n-1} n! \sqrt{\pi} a^{2\beta+2\delta-1} c^\delta \Gamma\left[\frac{\beta}{n+\beta+\delta+1/2}\right] L_n^{\beta+\delta-1/2}(a^2 c^2) \\ [a, \operatorname{Re} \beta > 0].$$

$$3. \int_a^\infty x^{\alpha-1} (x^2 - a^2)^{\beta-1} \left\{ \frac{H_{2n+1}(cx)}{H_{2n}(cx)} \right\} dx = \\ = (-1)^n 2^{2n+\delta-1} a^{\alpha+2\beta+\delta-2} c^\delta \left(\delta + \frac{1}{2}\right)_n B\left(\beta, 1-\beta-\frac{\alpha+\delta}{2}\right) \times \\ \times {}_2F_2\left(-n, \frac{\alpha+\delta}{2}; \delta + \frac{1}{2}, \frac{\alpha+2\beta+\delta}{2}; a^2 c^2\right) \\ [a, \operatorname{Re} \beta > 0; \operatorname{Re}(\alpha+2\beta) < 2-2n-\delta].$$

$$4. \int_0^\infty \frac{x^{\alpha-1}}{(x^2 + z^2)^\rho} \left\{ \frac{H_{2n+1}(cx)}{H_{2n}(cx)} \right\} dx = \\ = (-1)^n 2^{2n+\delta-1} c^\delta z^{\alpha+\delta-2\rho} \left(\delta + \frac{1}{2}\right)_n B\left(\frac{\alpha+\delta}{2}, \rho - \frac{\alpha+\delta}{2}\right) \times \\ \times {}_2F_2\left(-n, \frac{\alpha+\delta}{2}; \delta + \frac{1}{2}, 1-\rho + \frac{\alpha+\delta}{2}; -c^2 z^2\right) \\ [\operatorname{Re} z > 0; -\delta < \operatorname{Re} \alpha < 2\operatorname{Re} \rho - 2n - \delta].$$

2.20.3. Интегралы от $x^\alpha e^{\varphi(x)} H_n(b+cx)$.

Обозначение: $\delta = \begin{cases} 1 \\ 0 \end{cases}$.

$$1. \int_0^\infty e^{-px} \left\{ \frac{H_{2n+1}(cx)}{H_{2n}(cx)} \right\} dx = \frac{(-1)^n (2n+\delta)! (2c)^\delta}{n! p^{\delta+1}} {}_2F_0\left(-n, 1; \frac{4c^2}{p^2}\right) \quad [\operatorname{Re} p > 0].$$

$$2. \int_0^\infty x^{\alpha-1} e^{-px} H_n(cx) dx = \frac{n! (2c)^n}{p^{\alpha+n}} \sum_{k=0}^{[n/2]} \frac{\Gamma(\alpha+n-2k)}{k! (n-2k)!} \left(-\frac{p^2}{4c^2}\right)^k = \\ = \frac{(2c)^n}{p^{\alpha+n}} \Gamma(\alpha+n) {}_2F_2\left(-\frac{n}{2}, \frac{1-n}{2}; \frac{1-n-\alpha}{2}, 1-\frac{n+\alpha}{2}; -\frac{p^2}{4c^2}\right) \\ [\operatorname{Re} p > 0; \operatorname{Re} \alpha > 2[n/2] - n].$$

$$3. \int_0^\infty e^{-px^2} H_{2n}(cx) dx = \frac{\sqrt{\pi} (2n)! (c^2 - p)^n}{2 (n!) p^{n+1/2}} \quad [\operatorname{Re} p > 0].$$

$$4. \int_0^{\infty} x^{\alpha-1} e^{-px^2} \left\{ \begin{matrix} H_{2n+1}(cx) \\ H_{2n}(cx) \end{matrix} \right\} dx =$$

$$= (-1)^n \frac{2^{2n+\delta-1}}{c^\alpha} \left(\frac{\delta-\alpha+1}{2} \right)_n \Gamma\left(\frac{\alpha+\delta}{2}\right) {}_2F_1\left(\frac{\alpha}{2}, \frac{\alpha+1}{2}; \frac{\alpha-\delta+1}{2}-n; \frac{c^2-p}{c^2}\right)$$

[Re $\alpha > -\delta$; Re $p > 0$].

$$5. \int_0^{\infty} x^n e^{-px^2} H_{2n}(cx) dx = \frac{2^{n/2-1} n! \sqrt{\pi}}{p^{(n+1)/2}} P_n\left(\frac{c}{\sqrt{p}}\right)$$

[Re $p > 0$].

$$6. \int_0^{\infty} x^{2n-1} e^{-px^2} H_{2n}(cx) dx = \frac{(-1)^n 2^{n-1} (2n-1)!}{p^n} T_n\left(1 - \frac{2c^2}{p}\right)$$

[Re $p > 0$; $n = 1, 2, 3, \dots$].

$$7. \int_0^{\infty} x e^{-px^2} H_{2n+1}(cx) dx = \frac{\sqrt{\pi} (2n+1)! c (c^2-p)^n}{2 (n!) p^{n+3/2}}$$

[Re $p > 0$].

$$8. \int_0^{\infty} x^{2n+2} e^{-px^2} H_{2n+1}(cx) dx = \frac{(2n+1)! c}{p^{n+2}} U_n\left(\frac{2c^2}{p} - 1\right)$$

[Re $p > 0$].

$$9. \int_{-\infty}^{\infty} e^{-2c^2 x^2} H_{2n}(cx) dx = \frac{(-1)^n (2n)! \sqrt{\pi}}{2^{n+1/2} n! c}$$

[|arg c | < $\pi/4$].

$$10. \int_{-\infty}^{\infty} x^n e^{-px^2} H_n(cx) dx = \frac{n! \sqrt{\pi}}{p^{(n+1)/2}} P_n\left(\frac{c}{\sqrt{p}}\right)$$

[Re $p > 0$].

$$11. \int_{-\infty}^{\infty} x^{(1 \pm 1)/2} e^{-px^2} \left\{ \begin{matrix} H_{2n+1}(cx) \\ H_{2n}(cx) \end{matrix} \right\} dx = \frac{(2n+\delta)! \sqrt{\pi} c^\delta (c^2-p)^n}{n! p^{n+\delta+1/2}}$$

[Re $p > 0$].

$$12. \int_0^{\infty} e^{-px^2} [H_n(b-cx) + H_n(b+cx)] dx = \frac{\sqrt{\pi} (p-c^2)^{n/2}}{2^{n+1/2} p^{(n+1)/2}} H_n\left(b \sqrt{\frac{p}{p-c^2}}\right)$$

[Re $p > 0$].

$$13. \int_0^{\infty} e^{-px-c^2 x^2} H_n(cx) dx = \frac{\sqrt{\pi} (-p)^n}{c^{n+1}} \exp\left(-\frac{p^2}{2c^2}\right)$$

[|arg c | < $\pi/4$].

$$14. \int_0^{\infty} x^{\alpha-1} e^{-px-c^2 x^2} \left\{ \begin{matrix} H_{2n+1}(cx) \\ H_{2n}(cx) \end{matrix} \right\} dx = (-1)^n \frac{2^{2n+\delta-1}}{c^\alpha} \left(\frac{\delta-\alpha+1}{2} \right)_n \Gamma\left(\frac{\alpha+\delta}{2}\right) \times$$

$$\times {}_2F_2\left(\frac{\alpha}{2}, \frac{\alpha+1}{2}; \frac{\alpha-n+1}{2}, \frac{1}{2}; \frac{p^2}{4c^2}\right) - (-1)^n \frac{2^{2n+\delta-1} p}{c^{\alpha+1}} \left(\frac{\delta-\alpha}{2} \right)_n \times$$

$$\times \Gamma\left(\frac{\alpha+\delta+1}{2}\right) {}_2F_2\left(\frac{\alpha+1}{2}, \frac{\alpha+2}{2}; \frac{\alpha-n+2}{2}, \frac{3}{2}; \frac{p^2}{4c^2}\right)$$

[Re $\alpha > -\delta$; |arg c | < $\pi/4$].

$$15. \int_{-\infty}^{\infty} e^{ibx - c^2 x^2 / 2} H_n(cx) dx = \frac{\sqrt{2\pi}}{c} i^n \exp\left(-\frac{b^2}{2c^2}\right) H_n\left(\frac{b}{c}\right)$$

[$b > 0$; $|\arg c| < \pi/4$].

$$16. \int_{-\infty}^{\infty} e^{-p(x-y)^2} H_n(cx) dx = \frac{\sqrt{\pi} (p - c^2)^{n/2}}{p^{(n+1)/2}} H_n\left(cy \sqrt{\frac{p}{p - c^2}}\right)$$

[y , $\operatorname{Re} p > 0$].

$$17. \int_{-\infty}^{\infty} e^{-c^2(x-y)^2} H_n(cx) dx = \sqrt{\pi} c^{n-1} (2y)^n$$

[$y > 0$; $|\arg c| < \pi/4$].

$$18. \int_0^{\infty} x^{\alpha-1} e^{-p/x} \left\{ \frac{H_{2n+1}(cx)}{H_{2n}(cx)} \right\} dx = (-1)^n 2^{2n+\delta} c^{\delta} p^{\alpha+\delta} \left(\delta + \frac{1}{2}\right)_n \times$$

$$\times \Gamma(-\alpha - \delta) {}_1F_3\left(-n; \delta + \frac{1}{2}, \frac{\alpha}{2} + 1, \frac{\alpha + 2\delta + 1}{2}; \frac{p^2 c^2}{4}\right)$$

[$\operatorname{Re} p > 0$; $\operatorname{Re} \alpha < -2n - \delta$].

$$19. \int_0^{\infty} x^{\alpha-1} e^{-p/x^2} \left\{ \frac{H_{2n+1}(cx)}{H_{2n}(cx)} \right\} dx = (-1)^n 2^{2n+\delta-1} c^{\delta} p^{(\alpha+\delta)/2} \left(\delta + \frac{1}{2}\right)_n \times$$

$$\times \Gamma\left(-\frac{\alpha + \delta}{2}\right) {}_1F_2\left(-n; \delta + \frac{1}{2}, \frac{\alpha + \delta}{2} + 1; -pc^2\right)$$

[$\operatorname{Re} p > 0$; $\operatorname{Re} \alpha < -2n - \delta$].

$$20. \int_0^{\infty} x^{\alpha-1} e^{-p/x - c^2 x^2} \left\{ \frac{H_{2n+1}(cx)}{H_{2n}(cx)} \right\} dx = (-1)^n 2^{2n+\delta} c^{\delta} p^{\alpha+\delta} \left(\delta + \frac{1}{2}\right)_n \times$$

$$\times \Gamma(-\alpha - \delta) {}_1F_3\left(n + \delta + \frac{1}{2}; \delta + \frac{1}{2}, \frac{\alpha}{2} + 1, \frac{\alpha + 1}{2} + \delta; -\frac{c^2 p^2}{4}\right) +$$

$$+ (-1)^n \frac{2^{2n+\delta-1}}{c^{\alpha}} \left(\frac{\delta - \alpha + 1}{2}\right)_n \Gamma\left(\frac{\alpha + \delta}{2}\right) {}_1F_3\left(\frac{2n - \alpha + \delta + 1}{2}; \frac{1}{2},$$

$$1 - \frac{\alpha}{2}, \frac{1 - \alpha}{2}; -\frac{c^2 p^2}{4}\right) - (-1)^n \frac{2^{2n+\delta-1} p}{c^{\alpha-1}} \left(\frac{\delta - \alpha}{2} + 1\right)_n \times$$

$$\times \Gamma\left(\frac{\alpha + \delta - 1}{2}\right) {}_1F_3\left(1 + n + \frac{\delta - \alpha}{2}; \frac{3}{2}, 1 - \frac{\alpha}{2}, \frac{3 - \alpha}{2}; -\frac{c^2 p^2}{4}\right)$$

[$\operatorname{Re} p > 0$; $|\arg c| < \pi/4$].

$$21. \int_0^{\infty} x^{n-2} e^{-p/x^2 - c^2 x^2} H_n(cx) dx = 2^{n-1} \sqrt{\pi} p^{(n-1)/2} e^{-2c\sqrt{p}}$$

[$\operatorname{Re} p > 0$; $|\arg c| < \pi/4$].

$$22. \int_0^{\infty} x^{\alpha-1} e^{-p/x^2 - c^2 x^2} \left\{ \frac{H_{2n+1}(cx)}{H_{2n}(cx)} \right\} dx = (-1)^n 2^{2n+\delta-1} c^{\delta} p^{(\alpha+\delta)/2} \left(\delta + \frac{1}{2}\right)_n \times$$

$$\times \Gamma\left(-\frac{\alpha + \delta}{2}\right) {}_1F_2\left(n + \delta + \frac{1}{2}; \delta + \frac{1}{2}, \frac{\alpha + \delta}{2} + 1; c^2 p\right) +$$

$$+ (-1)^n \frac{2^{2n+\delta-1}}{c^{\alpha}} \left(\frac{\delta - \alpha + 1}{2}\right)_n \Gamma\left(\frac{\alpha + \delta}{2}\right) {}_1F_2\left(\frac{2n - \alpha + \delta + 1}{2}; \frac{1 - \alpha}{2}, 1 - \frac{\alpha}{2}; c^2 p\right)$$

[$\operatorname{Re} p > 0$; $|\arg c| < \pi/4$].

$$\begin{aligned}
23. \int_0^\infty x^{\alpha-1} e^{-px^4 - c^2 x^2} \frac{H_{2n+1}(cx)}{H_{2n}(cx)} dx &= (-1)^n \frac{2^{2n+\delta-2} c^\delta}{p^{(\alpha+\delta)/4}} \left(\delta + \frac{1}{2}\right)_n \Gamma\left(\frac{\alpha+\delta}{4}\right) \times \\
&\times {}_3F_3\left(\frac{\alpha+\delta}{4}, \frac{2n+3}{4}, \frac{2n+1}{4} + \delta; \delta + \frac{1}{4}, \frac{3}{4}, \frac{1}{2}; \frac{c^4}{4p}\right) - \\
&- (-1)^n \frac{2^{2n+\delta-2} c^{\delta+2}}{p^{(\alpha+\delta+2)/4}} \left(\delta + \frac{3}{2}\right)_n \Gamma\left(\frac{\alpha+\delta+2}{4}\right) \times \\
&\times {}_3F_3\left(\frac{\alpha+\delta+2}{4}, \frac{2n+5}{4}, \frac{2n+3}{4} + \delta; \delta + \frac{3}{4}, \frac{5}{4}, \frac{3}{2}; \frac{c^4}{4p}\right) \\
&\quad [\operatorname{Re} p > 0; \operatorname{Re} \alpha > -\delta].
\end{aligned}$$

$$\begin{aligned}
24. \int_0^\infty x^{\alpha-1} e^{-p/x^4 - c^2 x^2} \frac{H_{2n+1}(cx)}{H_{2n}(cx)} dx &= \\
&= (-1)^n 2^{2n+\delta-2} c^\delta p^{(\alpha+\delta)/4} \left(\delta + \frac{1}{2}\right)_n \Gamma\left(-\frac{\alpha+\delta}{4}\right) \times \\
&\times {}_2F_4\left(\frac{2n+3}{4}, \frac{2n+1}{4} + \delta; \delta + \frac{1}{4}, \frac{3}{4}, \frac{\alpha+\delta}{4} + 1, \frac{1}{2}; -\frac{c^4 p}{4}\right) - \\
&- (-1)^n 2^{2n+\delta-2} c^{\delta+2} p^{(\alpha+\delta+2)/4} \left(\delta + \frac{3}{2}\right)_n \Gamma\left(-\frac{\alpha+\delta+2}{4}\right) \times \\
&\times {}_2F_4\left(\frac{2n+5}{4}, \frac{2n+3}{4} + \delta; \delta + \frac{3}{4}, \frac{5}{4}, \frac{\alpha+\delta+6}{4}, \frac{3}{2}; -\frac{c^4 p}{4}\right) + \\
&+ (-1)^n \frac{2^{2n+\delta-1}}{c^\alpha} \left(\frac{\delta-\alpha+1}{2}\right)_n \Gamma\left(\frac{\alpha+\delta}{2}\right) \times \\
&\times {}_2F_4\left(\frac{2n-\alpha+\delta+1}{4}, \frac{2n-\alpha+\delta+3}{4}; \frac{1-\alpha}{4}, \frac{2-\alpha}{4}, \frac{3-\alpha}{4}, \frac{4-\alpha}{4}; -\frac{c^4 p}{4}\right) \\
&\quad [\operatorname{Re} p > 0; |\arg c| < \pi/4].
\end{aligned}$$

2.20.4. Интегралы от $A(x) e^{-c^2 x^2} H_n(cx)$.

Обозначение: $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$.

$$\begin{aligned}
1. \int_0^a x^{\alpha-1} (a-x)^{\beta-1} e^{-c^2 x^2} \frac{H_{2n+1}(cx)}{H_{2n}(cx)} dx &= \\
&= (-1)^n 2^{2n+\delta} a^{\alpha+\beta+\delta-1} c^\delta B(\beta, \alpha+\delta) \left(\delta + \frac{1}{2}\right)_n \times \\
&\times {}_3F_3\left(n+\delta+\frac{1}{2}, \frac{\alpha+1}{2}, \frac{\alpha}{2} + \delta; \delta + \frac{1}{2}, \frac{\alpha+\beta+1}{2}, \frac{\alpha+\beta}{2} + \delta; -a^2 c^2\right) \\
&\quad [a, \operatorname{Re} \beta > 0; \operatorname{Re} \alpha > -\delta].
\end{aligned}$$

$$\begin{aligned}
2. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\beta-1} e^{-c^2 x^2} \frac{H_{2n+1}(cx)}{H_{2n}(cx)} dx &= \\
&= (-1)^n 2^{2n+\delta-1} a^{\alpha+2\beta+\delta-1} c^\delta B\left(\beta, \frac{\alpha+\delta}{2}\right) \left(\delta + \frac{1}{2}\right)_n \times \\
&\times {}_2F_2\left(n+\delta+\frac{1}{2}, \frac{\alpha+\delta}{2}; \delta + \frac{1}{2}, \frac{\alpha+2\beta+\delta}{2}; -a^2 c^2\right) \\
&\quad [a, \operatorname{Re} \beta > 0; \operatorname{Re} \alpha > -\delta].
\end{aligned}$$

$$\begin{aligned}
3. \int_0^a x^{\alpha-1} (a^4 - x^4)^{\beta-1} e^{-c^2 x^2} \frac{H_{2n+1}(cx)}{H_{2n}(cx)} dx = \\
= (-1)^n 2^{2n+\delta-2} a^{\alpha+4\beta+\delta-4} c^\delta B\left(\beta, \frac{\alpha+\delta}{4}\right) \left(\delta + \frac{1}{2}\right)_n \times \\
\times {}_3F_4\left(\frac{2n+3}{4}, \frac{2n+1}{4} + \delta, \frac{\alpha+\delta}{4}; \frac{3}{4}, \frac{1}{4} + \delta, \beta + \frac{\alpha+\delta}{4}, \frac{1}{2}; \frac{a^4 c^4}{4}\right) - \\
- (-1)^n 2^{2n+\delta-2} a^{\alpha+4\beta+\delta-4} c^{\delta+2} B\left(\beta, \frac{\alpha+\delta+2}{4}\right) \left(\delta + \frac{3}{2}\right)_n \times \\
\times {}_3F_4\left(\frac{2n+5}{4}, \frac{2n+3}{4} + \delta, \frac{\alpha+\delta+2}{4}; \frac{5}{4}, \frac{3}{4} + \delta, \beta + \frac{\alpha+\delta+2}{4}, \frac{3}{2}; \frac{a^4 c^4}{4}\right) \\
[a, \operatorname{Re} \beta > 0; \operatorname{Re} \alpha > -\delta].
\end{aligned}$$

$$\begin{aligned}
4. \int_a^\infty x^{\alpha-1} (x-a)^{\beta-1} e^{-c^2 x^2} \frac{H_{2n+1}(cx)}{H_{2n}(cx)} dx = \\
= (-1)^n 2^{2n+\delta} a^{\alpha+\beta+\delta-1} c^\delta B\left(\beta, 1-\alpha-\beta-\delta\right) \left(\delta + \frac{1}{2}\right)_n \times \\
\times {}_3F_3\left(n+\delta+\frac{1}{2}, \frac{\alpha+1}{2}, \frac{\alpha}{2} + \delta; \frac{\alpha+\delta}{2} + \delta, \frac{\alpha+\beta+1}{2}, \delta + \frac{1}{2}; -a^2 c^2\right) + \\
+ (-1)^n 2^{2n+\delta-1} c^{1-\alpha-\beta} \left(\frac{\delta-\alpha-\beta}{2} + 1\right)_n \Gamma\left(\frac{\alpha+\beta+\delta-1}{2}\right) \times \\
\times {}_3F_3\left(\frac{1-\beta}{2}, 1-\frac{\beta}{2}, \frac{\delta-\alpha-\beta}{2} + n+1; 1-\frac{\alpha+\beta}{2}, \frac{3-\alpha-\beta}{2}, \frac{1}{2}; -a^2 c^2\right) + \\
+ (-1)^n 2^{2n+\delta-1} a c^{2-\alpha-\beta} (1-\beta) \left(\frac{\delta-\alpha-\beta+3}{2}\right)_n \Gamma\left(\frac{\alpha+\beta+\delta-1}{2}\right) \times \\
\times {}_3F_3\left(1-\frac{\beta}{2}, \frac{3-\beta}{2}, \frac{\delta-\alpha-\beta+3}{2} + n; \frac{3-\alpha-\beta}{2}, 2-\frac{\alpha+\beta}{2}, \frac{3}{2}; -a^2 c^2\right) \\
[a, \operatorname{Re} \beta > 0; |\arg c| < \pi/4].
\end{aligned}$$

$$\begin{aligned}
5. \int_a^\infty x^{\alpha-1} (x^2 - a^2)^{\beta-1} e^{-c^2 x^2} \frac{H_{2n+1}(cx)}{H_{2n}(cx)} dx = \\
= (-1)^n 2^{2n+\delta-1} a^{\alpha+2\beta+\delta-2} c^\delta B\left(\beta, 1-\beta-\frac{\alpha+\delta}{2}\right) \left(\delta + \frac{1}{2}\right)_n \times \\
\times {}_2F_2\left(n+\delta+\frac{1}{2}, \frac{\alpha+\delta}{2}; \beta + \frac{\alpha+\delta}{2}, \delta + \frac{1}{2}; -a^2 c^2\right) + \\
+ (-1)^n 2^{2n+\delta-1} c^{2-\alpha-2\beta} \left(\frac{3+\delta-\alpha}{2} - \beta\right)_n \Gamma\left(\frac{\alpha+\delta}{2} + \beta - 1\right) \times \\
\times {}_2F_2\left(\frac{3+\delta-\alpha}{2} + n - \beta, 1-\beta; \frac{3-\alpha}{2} - \beta, 2-\beta-\frac{\alpha}{2}; -a^2 c^2\right) \\
[a, \operatorname{Re} \beta > 0; |\arg a| < \pi/4].
\end{aligned}$$

$$\begin{aligned}
6. \int_a^\infty x^{\alpha-1} (x^4 - a^4)^{\beta-1} e^{-c^2 x^2} \frac{H_{2n+1}(cx)}{H_{2n}(cx)} dx = \\
= (-1)^n 2^{2n+\delta-2} a^{\alpha+4\beta+\delta-4} c^\delta B\left(\beta, 1-\beta-\frac{\alpha+\delta}{4}\right) \left(\delta + \frac{1}{2}\right)_n \times \\
\times {}_3F_4\left(\frac{2n+3}{4}, \frac{2n+1}{4} + \delta, \frac{\alpha+\delta}{4}; \beta + \frac{\alpha+\delta}{4}, \frac{3}{4}, \frac{1}{4} + \delta, \frac{1}{2}; \frac{a^4 c^4}{4}\right) -
\end{aligned}$$

$$\begin{aligned}
& -(-1)^n 2^{2n+\delta-2} a^{\alpha+\delta-2} c^{\delta+2} B\left(\beta, \frac{2-\alpha-\delta}{4}-\beta\right)\left(\delta+\frac{3}{2}\right)_n \times \\
& \times {}_3F_4\left(\frac{2n+5}{4}, \frac{2n+3}{4}+\delta, \frac{\alpha+\delta+2}{4}; \frac{\alpha+\delta+2}{4}+\beta, \frac{5}{4}, \frac{3}{4}+\delta, \frac{3}{2}; \frac{a^4 c^4}{4}\right) + \\
& + (-1)^n 2^{2n+\delta-1} c^{4-\alpha-4\beta} \left(\frac{5+\delta-\alpha}{2}-2\beta\right)_n \Gamma\left(\frac{\alpha+\delta}{2}+2\beta-2\right) \times \\
& \times {}_3F_4\left(1-\beta, \frac{2n-\alpha+\delta+5}{4}-\beta, \frac{2n-\alpha+\delta+7}{4}-\beta; \frac{5-\alpha}{4}-\beta, \right. \\
& \left. \frac{6-\alpha}{4}-\beta, \frac{7-\alpha}{4}-\beta, 2-\frac{\alpha}{4}-\beta; \frac{a^4 c^4}{4}\right)
\end{aligned}$$

[a, Re $\beta > 0$; |arg c | < $\pi/4$].

$$\begin{aligned}
7. \int_0^\infty \frac{x^{\alpha-1}}{(x+z)^\rho} e^{-c^2 x^2} \frac{H_{2n+1}(cx)}{H_{2n}(cx)} dx = \\
= (-1)^n 2^{2n+\delta} c^{\delta} z^{\alpha+\delta-\rho} B\left(\alpha+\delta, \rho-\alpha-\delta\right)\left(\delta+\frac{1}{2}\right)_n \times \\
\times {}_3F_3\left(n+\delta+\frac{1}{2}, \frac{\alpha+1}{2}, \frac{\alpha}{2}+\delta; \frac{\alpha-\rho}{2}+1, \frac{\alpha-\rho+1}{2}+\delta, \delta+\frac{1}{2}; -c^2 z^2\right) + \\
+ (-1)^n 2^{2n+\delta-1} c^{\rho-\alpha} \left(\frac{1+\delta+\rho-\alpha}{2}\right)_n \Gamma\left(\frac{\alpha+\delta-\rho}{2}\right) \times \\
\times {}_3F_3\left(\frac{\rho}{2}, \frac{\rho+1}{2}, n+\frac{1+\delta+\rho-\alpha}{2}; 1-\frac{\alpha-\rho}{2}, \frac{1-\alpha+\rho}{2}, \frac{1}{2}; -c^2 z^2\right) - \\
- (-1)^n 2^{2n+\delta-1} c^{1-\alpha+\rho} z^\rho \left(1+\frac{\delta+\rho-\alpha}{2}\right)_n \Gamma\left(\frac{\alpha+\delta-\rho-1}{2}\right) \times \\
\times {}_3F_3\left(\frac{\rho+1}{2}, \frac{\rho}{2}+1, 1+n+\frac{\delta+\rho-\alpha}{2}; 1-\frac{\alpha-\rho}{2}, \frac{3-\alpha+\rho}{2}, \frac{3}{2}; -c^2 z^2\right)
\end{aligned}$$

[Re $\alpha > -\delta$; |arg c | < $\pi/4$; |arg z | < π].

$$\begin{aligned}
8. \int_0^\infty \frac{x^{\alpha-1}}{(x^2+z^2)^\rho} e^{-c^2 x^2} \frac{H_{2n+1}(cx)}{H_{2n}(cx)} dx = \\
= (-1)^n 2^{2n+\delta-1} c^{\delta} z^{\alpha+\delta-2\rho} B\left(\frac{\alpha+\delta}{2}, \rho-\frac{\alpha+\delta}{2}\right)\left(\delta+\frac{1}{2}\right)_n \times \\
\times {}_2F_2\left(\frac{\alpha+\delta}{2}, n+\delta+\frac{1}{2}; \delta+\frac{1}{2}, \frac{\alpha+\delta}{2}-\rho+1; c^2 z^2\right) + \\
+ (-1)^n 2^{2n+\delta-1} c^{2\rho-\alpha} \left(\frac{\delta-\alpha+1}{2}+\rho\right)_n \Gamma\left(\frac{\alpha+\delta}{2}-\rho\right) \times \\
\times {}_2F_2\left(\frac{1-\alpha+\delta}{2}+n+\rho, \rho; \frac{1-\alpha}{2}+\rho, \rho+1-\frac{\alpha}{2}; c^2 z^2\right)
\end{aligned}$$

[Re $z > 0$; Re $\alpha > -\delta$; |arg c | < $\pi/4$].

$$\begin{aligned}
9. \int_0^\infty \frac{x^{-1}}{x^2+z^2} e^{-c^2 x^2} H_{2n+1}(cx) dx = \\
= (-2)^n \frac{\sqrt{\pi}}{z^2} [2^n n! - (2n+1)! e^{c^2 z^2/2} D_{-2n-2}(cz\sqrt{2})]
\end{aligned}$$

[Re $z > 0$; |arg c | < $\pi/4$].

$$\begin{aligned}
10. \int_0^{\infty} \frac{x^{\alpha-1}}{(x^4+z^4)^{\rho}} e^{-c^2 x^2} \left\{ \begin{matrix} H_{2n+1}(cx) \\ H_{2n}(cx) \end{matrix} \right\} dx = \\
= (-1)^n 2^{2n+\delta-2} c^{\delta} z^{\alpha+\delta-4\rho} B\left(\frac{\alpha+\delta}{4}, \rho - \frac{\alpha+\delta}{4}\right) \left(\delta + \frac{1}{2}\right)_n \times \\
\times {}_3F_4\left(\frac{2n+3}{4}, \frac{2n+1}{4} + \delta, \frac{\alpha+\delta}{4}; \frac{1}{2}, \frac{3}{4}, \frac{1}{4} + \delta, \frac{\alpha+\delta}{4} - \rho + 1; -\frac{c^4 z^4}{4}\right) - \\
- (-1)^n 2^{2n+\delta-2} c^{\delta+2} z^{\alpha+\delta-4\rho+2} B\left(\frac{\alpha+\delta+2}{4}, \rho - \frac{\alpha+\delta+2}{4}\right) \left(\delta + \frac{3}{2}\right)_n \times \\
\times {}_3F_4\left(\frac{2n+5}{4}, \frac{2n+3}{4} + \delta, \frac{\alpha+\delta+2}{4}; \frac{3}{2}, \frac{5}{4}, \frac{3}{4} + \delta, \frac{2-\alpha-\delta}{4} - \rho; -\frac{c^4 z^4}{4}\right) + \\
+ (-1)^n 2^{2n+\delta-1} c^{4\rho-\alpha} \left(2\rho + \frac{\delta-\alpha+1}{2}\right)_n \Gamma\left(\frac{\alpha+\delta}{2} - 2\rho\right) \times \\
\times {}_3F_4\left(\rho, \frac{1-\alpha+\delta+2n}{4} + \rho, \frac{3-\alpha+\delta+2n}{4} + \rho; \frac{1-\alpha}{4} + \rho, \right. \\
\left. \frac{2-\alpha}{4} + \rho, \frac{3-\alpha}{4} + \rho, 1 - \frac{\alpha}{4} + \rho; -\frac{c^4 z^4}{4}\right) \quad [\operatorname{Re} \alpha > -\delta, |\arg c|, |\arg z| < \pi/4].
\end{aligned}$$

$$\begin{aligned}
11. \int_0^{\infty} \frac{x^{\alpha-1}}{x-y} e^{-c^2 x^2} \left\{ \begin{matrix} H_{2n+1}(cx) \\ H_{2n}(cx) \end{matrix} \right\} dx = \\
= (-1)^{n+1} 2^{2n+\delta} \pi c^{\delta} y^{\alpha+\delta-1} \operatorname{ctg} \alpha \pi \left(\delta + \frac{1}{2}\right)_n {}_1F_1\left(n + \delta + \frac{1}{2}; \delta + \frac{1}{2}; -c^2 y^2\right) + \\
+ (-1)^n 2^{2n+\delta-1} c^{1-\alpha} \left(\frac{\delta-\alpha}{2} + 1\right)_n \Gamma\left(\frac{\alpha+\delta-1}{2}\right) \times \\
\times {}_2F_3\left(1, \frac{\delta-\alpha}{2} + n + 1; 1 - \frac{\alpha}{2}, \frac{3-\alpha}{2}; -c^2 y^2\right) + \\
+ (-1)^n 2^{2n+\delta-1} c^{2-\alpha} y \left(\frac{\delta-\alpha+3}{2}\right)_n \Gamma\left(\frac{\alpha+\delta}{2} - 1\right) \times \\
\times {}_2F_3\left(1, \frac{3-\alpha+\delta}{2} + n; \frac{3-\alpha}{2}, 2 - \frac{\alpha}{2}; -c^2 y^2\right) \quad [y > 0; \operatorname{Re} \alpha > -\delta; |\arg c| < \pi/4].
\end{aligned}$$

$$\begin{aligned}
12. \int_0^{\infty} \frac{x^{\alpha-1}}{x^2-y^2} e^{-c^2 x^2} \left\{ \begin{matrix} H_{2n+1}(cx) \\ H_{2n}(cx) \end{matrix} \right\} dx = \\
= (-1)^{n+1} 2^{2n+\delta-1} \pi c^{\delta} y^{\alpha+\delta-2} \operatorname{ctg} \frac{\alpha+\delta}{2} \pi \left(\delta + \frac{1}{2}\right)_n \times \\
\times {}_1F_1\left(n + \delta + \frac{1}{2}; \delta + \frac{1}{2}; -c^2 y^2\right) + (-1)^n 2^{2n+\delta-1} c^{2-\alpha} \times \\
\times \left(\frac{\delta-\alpha+3}{2}\right)_n \Gamma\left(\frac{\alpha+\delta}{2} - 1\right) {}_2F_3\left(\frac{3-\alpha+\delta}{2} + n, 1; 2 - \frac{\alpha}{2}, \frac{3-\alpha}{2}; -c^2 y^2\right) \quad [y > 0; \operatorname{Re} \alpha > -\delta; |\arg c| < \pi/4].
\end{aligned}$$

$$\begin{aligned}
13. \int_0^{\infty} \frac{x^{\alpha-1}}{x^4-y^4} e^{-c^2 x^2} \left\{ \begin{matrix} H_{2n+1}(cx) \\ H_{2n}(cx) \end{matrix} \right\} dx = \\
= (-1)^{n+1} 2^{2n+\delta-2} \pi c^{\delta} y^{\alpha+\delta-3} \operatorname{ctg} \frac{\alpha+\delta}{4} \pi \left(\delta + \frac{1}{2}\right)_n \times \\
\times {}_2F_3\left(\frac{2n+3}{4}, \frac{2n+1}{4} + \delta; \frac{1}{2}, \frac{3}{4}, \frac{1}{4} + \delta; \frac{c^4 y^4}{4}\right) -
\end{aligned}$$

$$\begin{aligned}
& -(-1)^n 2^{2n+\delta-2} \pi c^\delta y^{\alpha+\delta-4} \operatorname{tg} \frac{\alpha+\delta}{4} \pi \left(\delta + \frac{3}{2} \right)_n \times \\
& \times {}_2F_3 \left(\frac{2n+5}{4}, \frac{2n+3}{4} + \delta; \frac{5}{4}, \frac{3}{4} + \delta, \frac{3}{2}; \frac{c^4 y^4}{4} \right) + \\
& + (-1)^n 2^{2n+\delta-1} c^{4-\alpha} \left(\frac{\delta-\alpha+5}{2} \right)_n \Gamma \left(\frac{\alpha+\delta}{2} - 2 \right) \times \\
& \times {}_3F_4 \left(\frac{5-\alpha+\delta+2n}{4}, \frac{2n-\alpha+\delta+7}{4}, 1; \frac{5-\alpha}{4}, \frac{6-\alpha}{4}, \frac{7-\alpha}{4}, 2 - \frac{\alpha}{4}; \frac{c^4 y^4}{4} \right) \\
& [y > 0; \operatorname{Re} \alpha > -\delta; |\arg c| < \pi/4].
\end{aligned}$$

2.20.5. Интегралы от $x^\alpha e^{-px^2} \begin{Bmatrix} \sin bx^m \\ \cos bx^m \end{Bmatrix} H_n(cx)$.

Обозначение: $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$.

$$1. \int_0^\infty e^{-c^2 x^2} \begin{Bmatrix} \sin bx H_{2n+1}(cx) \\ \cos bx H_{2n}(cx) \end{Bmatrix} dx = \frac{(-1)^n \sqrt{\pi}}{2c} \left(\frac{b}{c} \right)^{2n + (1 \pm 1)/2} \exp \left(-\frac{b^2}{4c^2} \right)$$

[$|\arg b| < \pi/4$].

$$\begin{aligned}
2. \int_0^\infty e^{-px^2} \begin{Bmatrix} \sin bx H_{2n+1}(cx) \\ \cos bx H_{2n}(cx) \end{Bmatrix} dx = \\
= \frac{i^\delta \sqrt{\pi} (p-c^2)^{n/2}}{2p^{(n+1)/2}} \exp \left(-\frac{b^2}{4p} \right) H_n \left(\frac{bc}{2\sqrt{p(c^2-p)}} \right)
\end{aligned}$$

[$\operatorname{Re} p > 0$].

$$\begin{aligned}
3. \int_0^\infty x^{\alpha-1} e^{-c^2 x^2} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} H_{2n+\varepsilon}(cx) dx = \\
= (-1)^n \frac{2^{2n+\varepsilon-1} b^\delta}{c^{\alpha+\delta}} \left(\frac{1-\alpha-\delta+\varepsilon}{2} \right)_n \Gamma \left(\frac{\alpha+\delta+\varepsilon}{2} \right) \times \\
\times {}_2F_2 \left(\frac{\alpha}{2} + \delta, \frac{\alpha+1}{2}; \delta + \frac{1}{2}, \frac{\alpha+\delta-\varepsilon+1}{2} - n; -\frac{b^2}{4c^2} \right) \\
[\varepsilon = 0 \text{ или } 1; b > 0; \operatorname{Re} \alpha > -\delta - \varepsilon; |\arg c| < \pi/4].
\end{aligned}$$

$$\begin{aligned}
4. \int_0^\infty x^{\alpha-1} e^{-c^2 x^2} \begin{Bmatrix} \sin bx^2 \\ \cos bx^2 \end{Bmatrix} H_{2n+\varepsilon}(cx) dx = \\
= (-1)^n \frac{2^{2n+\varepsilon-1} b^\delta}{c^{\alpha+2\delta}} \left(\frac{1-\alpha+\varepsilon-\delta}{2} \right)_n \Gamma \left(\frac{\alpha+\varepsilon}{2} + \delta \right) {}_4F_3 \left(\frac{\alpha+3}{4}, \frac{\alpha+1}{4} + \delta, \right. \\
\left. \frac{\alpha+2}{4}, \frac{\alpha}{4} + \delta; \frac{\alpha-\varepsilon-2n+3}{4}, \frac{\alpha-\varepsilon-2n+1}{4} + \delta, \delta + \frac{1}{2}; -\frac{b^2}{c^4} \right) \\
[\varepsilon = 0 \text{ или } 1; b > 0; \operatorname{Re} \alpha > -2\delta - \varepsilon; |\arg c| < \pi/4].
\end{aligned}$$

$$5. \int_{-\infty}^\infty e^{-c^2 x^2} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} H_n(cx) dx = \frac{\sqrt{\pi} b^n}{c^{n+1}} \exp \left(-\frac{b^2}{4c^2} \right) \begin{Bmatrix} \sin (\pi n/2) \\ \cos (\pi n/2) \end{Bmatrix}$$

[$|\arg c| < \pi/4$].

2.20.6. Интегралы от $x^\alpha e^{-c^2 x^2} \ln^m \varphi(x) H_n(cx)$.

$$1. \int_0^\infty e^{-x^2} \ln x H_{2n}(x) dx = -\frac{\sqrt{\pi}}{4} (C + 2 \ln 2)$$

[$n = 0$].

$$2. = (-1)^{n-1} (n-1)! 2^{2n-2} \sqrt{\pi}$$

[$n = 1, 2, 3, \dots$].

$$3. \int_0^{\infty} e^{-x^2} \ln x H_{2n+1}(x) dx = (-1)^{n-1} 2^{n-1} \left(C + 2 \sum_{k=1}^n \frac{1}{2k-1} \right) (2n-1)!!.$$

$$4. \int_0^{\infty} e^{-x^2} \ln^2 x H_{2n}(x) dx = \frac{\sqrt{\pi}}{8} \left[\Psi^2\left(\frac{1}{2}\right) - \Psi\left(\frac{1}{2}\right) \right] \quad [n=0],$$

$$5. = (-1)^n (n-1)! 2^{2n-1} \sqrt{\pi} \left(C + 2 \ln 2 + \sum_{k=1}^{n-1} \frac{1}{k} \right) \quad [n=1, 2, 3, \dots].$$

$$6. \int_0^{\infty} e^{-x^2} \ln^2 x H_{2n+1}(x) dx =$$

$$= (-1)^n 2^{n-1} \left[\frac{\pi^2}{6} + \left(C + 2 \sum_{k=1}^n \frac{1}{2k-1} \right)^2 - 4 \sum_{k=1}^n \frac{1}{(2k-1)^2} \right] (2n-1)!!.$$

$$7. \int_0^{\infty} x^{\alpha-1} e^{-c^2 x^2} \left\{ \frac{\ln(x+z)}{\ln|x-z|} \right\} H_{2n+\varepsilon}(cx) dx =$$

$$= (-1)^n \frac{2^{2n+\varepsilon} \pi}{\alpha+\varepsilon} c^{\varepsilon} z^{\alpha+\varepsilon} \left(\varepsilon + \frac{1}{2} \right)_n \left\{ \operatorname{cosec}(\alpha+\varepsilon)\pi \right\} \times$$

$$\times {}_2F_2 \left(\frac{\alpha+\varepsilon}{2}, n+\varepsilon+\frac{1}{2}; \varepsilon+\frac{1}{2}, \frac{\alpha+\varepsilon}{2}+1; -c^2 z^2 \right) \pm$$

$$\pm (-1)^n \frac{2^{2n+\varepsilon-1} z}{c^{\alpha-1}} \left(\frac{\varepsilon-\alpha}{2} + 1 \right)_n \Gamma\left(\frac{\alpha+\varepsilon-1}{2}\right) \times$$

$$\times {}_3F_3 \left(\frac{1}{2}, 1, \frac{\varepsilon-\alpha}{2} + n + 1; \frac{3}{2}, 1 - \frac{\alpha}{2}, \frac{3-\alpha}{2}; -c^2 z^2 \right) -$$

$$- (-1)^n \frac{2^{2n+\varepsilon-2} z^2}{c^{\alpha-2}} \left(\frac{3-\alpha+\varepsilon}{2} \right)_n \Gamma\left(\frac{\alpha+\varepsilon-1}{2}\right) \times$$

$$\times {}_3F_3 \left(1, 1, n - \frac{\alpha-\varepsilon-3}{2}; 2, \frac{3-\alpha}{2}, 2 - \frac{\alpha}{2}; -c^2 z^2 \right) +$$

$$+ (-1)^n \frac{2^{2n+\varepsilon-2}}{c^{\alpha}} \left(\frac{1-\alpha+\varepsilon}{2} \right)_n \Gamma\left(\frac{\alpha+\varepsilon}{2}\right) \left[\Psi\left(\frac{\alpha+\varepsilon}{2}\right) - \sum_{k=0}^{n-1} \frac{2}{2k-\alpha+\varepsilon+1} - 2 \ln c \right]$$

[$\varepsilon = 0$ или 1 ; $\operatorname{Re} \alpha > -\varepsilon$; $|\arg c| < \pi/4$; $|\arg z| < \pi$].

$$3. \int_0^{\infty} x^{\alpha-1} e^{-c^2 x^2} \left\{ \frac{\ln(x^2+z^2)}{\ln|x^2-z^2|} \right\} H_{2n+\varepsilon}(cx) dx =$$

$$= (-1)^n \frac{2^{2n+\varepsilon} \pi}{\alpha+\varepsilon} c^{\varepsilon} z^{\alpha+\varepsilon} \left(\varepsilon + \frac{1}{2} \right)_n \left\{ \operatorname{cosec}[(\alpha+\varepsilon)\pi/2] \right\} \times$$

$$\times {}_2F_2 \left(\frac{\alpha+\varepsilon}{2}, n+\varepsilon+\frac{1}{2}; \frac{\alpha+\varepsilon}{2}+1, \varepsilon+\frac{1}{2}; \pm c^2 z^2 \right) \pm$$

$$\pm (-1)^n \frac{2^{2n+\varepsilon-1} z^2}{c^{\alpha-2}} \left(\frac{3-\alpha+\varepsilon}{2} \right)_n \Gamma\left(\frac{\alpha+\varepsilon-1}{2}\right) \times$$

$$\times {}_3F_3 \left(1, 1, \frac{3-\alpha+\varepsilon}{2} + n; 2, 2 - \frac{\alpha}{2}, \frac{3-\alpha}{2}; \pm c^2 z^2 \right) +$$

$$+ (-1)^n \frac{2^{2n+\varepsilon-1}}{c^{\alpha}} \left(\frac{1-\alpha+\varepsilon}{2} \right)_n \Gamma\left(\frac{\alpha+\varepsilon}{2}\right) \left[\Psi\left(\frac{\alpha+\varepsilon}{2}\right) - \sum_{k=0}^{n-1} \frac{2}{2k-\alpha+\varepsilon+1} - 2 \ln c \right]$$

[$\varepsilon = 0$ или 1 ; $\operatorname{Re} z > 0$; $\operatorname{Re} \alpha > -\varepsilon$; $|\arg c| < \pi/4$].

2.20.7. Интегралы от $x^\alpha e^{\varphi(x)} \operatorname{Ei}(bx^m) H_n(cx)$.

Обозначение: $\delta = \begin{cases} 1 \\ 0 \end{cases}$.

$$\begin{aligned}
 1. \int_0^\infty x^{\alpha-1} e^{-c^2 x^2} \operatorname{Ei}(\mp bx) \left\{ \begin{matrix} H_{2n+1}(cx) \\ H_{2n}(cx) \end{matrix} \right\} dx = \\
 = (-1)^{n+1} \frac{2^{2n+\delta-1} b}{c^{\alpha+1}} \left(\frac{\delta-\alpha}{2} \right)_n \Gamma\left(\frac{\alpha+\delta+1}{2}\right) \times \\
 \times {}_3F_3\left(\frac{1}{2}, \frac{\alpha+1}{2}, \frac{\alpha}{2}+1; \frac{3}{2}, \frac{3}{2}, \frac{\alpha-\delta}{2}-n+1; \frac{b^2}{4c^2}\right) + \\
 + (-1)^n \frac{2^{2n+\delta-1} b^3}{c^{\alpha+3}} \left(\frac{\delta-\alpha-1}{2} \right)_n \Gamma\left(\frac{\alpha+\delta}{2}+1\right) \times \\
 \times {}_4F_4\left(1, 1, \frac{\alpha}{2}+1, \frac{\alpha+3}{2}; 2, 2, \frac{3}{2}, \frac{\alpha-\delta+3}{2}-n; \frac{b^2}{4c^2}\right) + \\
 + (-1)^n \frac{2^{2n+\delta-1}}{c^\alpha} \Gamma\left(\frac{\alpha+\delta}{2}\right) \left(\frac{1-\alpha+\delta}{2}\right)_n \left[c + \frac{1}{2} \psi\left(\frac{\alpha+\delta}{2}\right) - \sum_{k=0}^{n-1} \frac{1}{2k-\alpha+\delta+1} - \ln \frac{c}{b} \right] \\
 [\operatorname{Re} \alpha > -\delta; |\arg b| < \pi; |\arg c| < \pi/4].
 \end{aligned}$$

$$\begin{aligned}
 2. \int_0^\infty x^{\alpha-1} e^{-c^2 x^2} \operatorname{Ei}(-bx^2) \left\{ \begin{matrix} H_{2n+1}(cx) \\ H_{2n}(cx) \end{matrix} \right\} dx = \\
 = (-1)^{n+1} \frac{2^{2n+\delta} c^\delta}{b^{(\alpha+\delta)/2} (\alpha+\delta)} \left(\delta + \frac{1}{2} \right)_n \Gamma\left(\frac{\alpha+\delta}{2}\right) \times \\
 \times {}_3F_3\left(\frac{\alpha+\delta}{2}, \frac{\alpha+\delta}{2}, n+\delta+\frac{1}{2}; \frac{\alpha+\delta}{2}+1, \delta+\frac{1}{2}; -\frac{c^2}{b}\right) \\
 [\operatorname{Re}(b+c^2) > 0; \operatorname{Re} \alpha > -\delta].
 \end{aligned}$$

$$\begin{aligned}
 3. \int_0^\infty x^{\alpha-1} e^{-c^2 x^2} \operatorname{Ei}(-bx^4) \left\{ \begin{matrix} H_{2n+1}(cx) \\ H_{2n}(cx) \end{matrix} \right\} dx = \\
 = (-1)^{n+1} \frac{2^{2n+\delta} c^\delta}{b^{(\alpha+\delta)/4} (\alpha+\delta)} \left(\delta + \frac{1}{2} \right)_n \Gamma\left(\frac{\alpha+\delta}{4}\right) \times \\
 \times {}_4F_4\left(\frac{\alpha+\delta}{4}, \frac{\alpha+\delta}{4}, \frac{2n+3}{4}, \frac{2n+1}{4}+\delta; \frac{1}{2}, \frac{3}{4}, \frac{1}{4}+\delta, \frac{\alpha+\delta}{4}+1; \frac{c^4}{4b}\right) + \\
 + (-1)^n \frac{2^{2n+\delta} c^{\delta+2}}{b^{(\alpha+\delta+2)/4} (\alpha+\delta+2)} \left(\delta + \frac{3}{2} \right)_n \Gamma\left(\frac{\alpha+\delta+2}{4}\right) \times \\
 \times {}_4F_4\left(\frac{\alpha+\delta+2}{4}, \frac{\alpha+\delta+2}{4}, \frac{2n+5}{4}, \frac{2n+3}{4}+\delta; \frac{3}{2}, \frac{5}{4}, \frac{3}{4}+\delta, \frac{\alpha+\delta+6}{4}; \frac{c^4}{4b}\right) \\
 [\operatorname{Re} b > 0, \operatorname{Re} \alpha > -\delta].
 \end{aligned}$$

$$\begin{aligned}
 4. \int_0^\infty x^{\alpha-1} e^{(\pm b - c^2) x^2} \operatorname{Ei}(\mp bx^2) H_{2n+\varepsilon}(cx) dx = \\
 = (-1)^{n+1} \frac{2^{2n+\varepsilon-1} \pi c^\varepsilon}{b^{(\alpha+\varepsilon)/2}} \left(\varepsilon + \frac{1}{2} \right)_n \Gamma\left(\frac{\alpha+\varepsilon}{2}\right) \left\{ \operatorname{cosec}[(\alpha+\varepsilon)\pi/2] \right\} \times \\
 \times {}_2F_1\left(n+\varepsilon+\frac{1}{2}, \frac{\alpha+\varepsilon}{2}; \varepsilon+\frac{1}{2}; \pm \frac{c^2}{b}\right) \mp (-1)^n \frac{2^{2n+\varepsilon-1}}{bc^{\alpha-2}} \left(\frac{3-\alpha+\varepsilon}{2} \right)_n \times \\
 \times \Gamma\left(\frac{\alpha+\varepsilon}{2}-1\right) {}_3F_3\left(1, 1, \frac{3-\alpha+\varepsilon}{2}+n; \frac{3-\alpha}{2}, 2-\frac{\alpha}{2}; \pm \frac{c^2}{b}\right) \\
 \left[\varepsilon = 0 \text{ или } 1; \operatorname{Re} \alpha > -\varepsilon; |\arg c| < \pi/4, \left\{ \begin{matrix} |\arg b| < \pi \\ b > 0 \end{matrix} \right\} \right].
 \end{aligned}$$

$$\begin{aligned}
5. \int_0^{\infty} x^{\alpha-1} e^{\pm bx^2 - c^2 x^2} \operatorname{Ei}(\mp bx^2) H_{2n+\varepsilon}(cx) dx = \\
= (-1)^{n+1} \frac{2^{2n+\varepsilon-2} \pi c^{\varepsilon}}{b^{(\alpha+\varepsilon)/4}} \left(\varepsilon + \frac{1}{2}\right)_n \Gamma\left(\frac{\alpha+\varepsilon}{4}\right) \left\{ \operatorname{cosec} [(\alpha+\varepsilon)\pi/4] \right\} \times \\
\times {}_3F_3\left(\frac{\alpha+\varepsilon}{4}, \frac{2n+3}{4}, \frac{2n+1}{4} + \varepsilon; \frac{1}{2}, \frac{3}{4}, \frac{1}{4} + \varepsilon; \mp \frac{c^4}{4b}\right) \pm \\
\pm (-1)^n \frac{2^{2n+\varepsilon-2} \pi c^{\varepsilon+2}}{b^{(\alpha+\varepsilon+2)/4}} \left(\varepsilon + \frac{3}{2}\right)_n \Gamma\left(\frac{\alpha+\varepsilon+2}{4}\right) \left\{ \operatorname{tg} [(\alpha+\varepsilon)\pi/4] \right\} \times \\
\times {}_3F_3\left(\frac{\alpha+\varepsilon+2}{4}, \frac{2n+5}{4}, \frac{2n+3}{4} + \varepsilon; \frac{3}{2}, \frac{5}{4}, \frac{3}{4} + \varepsilon; \mp \frac{c^4}{4b}\right) \mp \\
\mp (-1)^n \frac{2^{2n+\varepsilon-1}}{bc^{\alpha-1}} \left(\frac{5-\alpha+\varepsilon}{2}\right)_n \Gamma\left(\frac{\alpha+\varepsilon}{2}-2\right) \times \\
\times {}_4F_4\left(1, 1, \frac{7-\alpha}{4} + n, \frac{5-\alpha}{4} + \varepsilon + n; \frac{5-\alpha}{4}, \frac{6-\alpha}{4}, \frac{7-\alpha}{4}, 2 - \frac{\alpha}{4}; \mp \frac{c^4}{4b}\right) \\
\left[\varepsilon = 0 \text{ или } 1; \operatorname{Re} \alpha > -\varepsilon; |\arg c| < \pi/4, \left\{ \begin{array}{l} |\arg b| < \pi \\ b > 0 \end{array} \right\}\right].
\end{aligned}$$

2.20.8. Интегралы от $x^{\alpha} e^{-c^2 x^2} \left\{ \begin{array}{l} \operatorname{si} (bx^m) \\ \operatorname{ci} (bx^m) \end{array} \right\} H_n(cx)$.

$$\begin{aligned}
1. \int_0^{\infty} x^{\alpha-1} e^{-c^2 x^2} \left\{ \begin{array}{l} \operatorname{si} (bx) \\ \operatorname{ci} (bx) \end{array} \right\} H_{2n+\varepsilon}(cx) dx = \\
= (-1)^{n+1} \frac{2^{2n+\varepsilon-2} b^{\delta+2}}{3\delta c^{\alpha+\delta+2} (\delta+2)} \left(\frac{\varepsilon-\alpha-\delta-1}{2}\right)_n \Gamma\left(\frac{\alpha+\delta+\varepsilon}{2}+1\right) \times \\
\times {}_4F_4\left(1, \frac{\delta}{2}+1, \frac{\alpha+3}{2}, \frac{\alpha}{2}+\delta+1; 2, \frac{\delta}{2}+2, \delta+\frac{3}{2}, \frac{\alpha+\delta-\varepsilon+\varepsilon}{2}-n; -\frac{b^2}{4c^2}\right) + \\
+ (-1)^n \frac{2^{2n+\varepsilon-1} b^{\delta}}{c^{\alpha+\delta}} \left(\frac{1-\alpha-\delta+\varepsilon}{2}\right)_n \Gamma\left(\frac{\alpha+\delta+\varepsilon}{2}\right) \left\{ \begin{array}{l} 1 \\ R \end{array} \right\} - \\
- \left\{ \begin{array}{l} 1 \\ 0 \end{array} \right\} (-1)^n \frac{2^{2n+\varepsilon-2} \pi}{c^{\alpha}} \left(\frac{1-\alpha+\varepsilon}{2}\right)_n \Gamma\left(\frac{\alpha+\varepsilon}{2}\right), \\
R = C + \frac{1}{2} \psi\left(\frac{\alpha+\varepsilon}{2}\right) - \sum_{k=0}^{n-1} \frac{1}{2k-\alpha-\delta+\varepsilon+1} - \ln \frac{c}{b} \\
\left[\varepsilon = 0 \text{ или } 1; b > 0; \operatorname{Re} \alpha > -\varepsilon; |\arg c| < \pi/4; \delta = \left\{ \begin{array}{l} 1 \\ 0 \end{array} \right\}\right].
\end{aligned}$$

$$\begin{aligned}
2. \int_0^{\infty} x^{\alpha-1} e^{-c^2 x^2} \left\{ \begin{array}{l} \operatorname{si} (bx^2) \\ \operatorname{ci} (bx^2) \end{array} \right\} H_{2n+\varepsilon}(cx) dx = \\
= (-1)^{n+1} \frac{2^{2n+\varepsilon} c^{\varepsilon}}{b^{(\alpha+\varepsilon)/2} (\alpha+\varepsilon)} \left(\varepsilon + \frac{1}{2}\right)_n \Gamma\left(\frac{\alpha+\varepsilon}{2}\right) \left\{ \begin{array}{l} \sin [(\alpha+\varepsilon)\pi/4] \\ \cos [(\alpha+\varepsilon)\pi/4] \end{array} \right\} \times \\
\times {}_5F_4\left(\frac{\alpha+\varepsilon}{4}, \frac{\alpha+\varepsilon}{4}, \frac{\alpha+\varepsilon+2}{4}, \frac{2n+3}{4}, \frac{2n+1}{4} + \varepsilon; \frac{1}{2}, \frac{3}{4}, \frac{1}{4} + \varepsilon, \frac{\alpha+\varepsilon}{4} + 1; -\frac{c^4}{b^2}\right) \pm \\
\pm (-1)^n \frac{2^{2n+\varepsilon} c^{\varepsilon+2}}{b^{(\alpha+\varepsilon)/2+1} (\alpha+\varepsilon+2)} \left(\varepsilon + \frac{3}{2}\right)_n \Gamma\left(\frac{\alpha+\varepsilon}{2}+1\right) \left\{ \begin{array}{l} \cos [(\alpha+\varepsilon)\pi/4] \\ \sin [(\alpha+\varepsilon)\pi/4] \end{array} \right\} \times \\
\times {}_5F_4\left(\frac{\alpha+\varepsilon+2}{4}, \frac{\alpha+\varepsilon+2}{4}, \frac{\alpha+\varepsilon}{4} + 1, \frac{2n+5}{4}, \frac{2n+3}{4} + \varepsilon; \right. \\
\left. \frac{3}{2}, \frac{5}{4}, \frac{3}{4} + \varepsilon, \frac{\alpha+\varepsilon+6}{4}; -\frac{c^4}{b^2}\right) \\
\left[\varepsilon = 0 \text{ или } 1; b > 0; \operatorname{Re} \alpha > -\varepsilon; |\arg c| < \pi/4\right]
\end{aligned}$$

2.20.9. Интегралы от $x^\alpha \begin{Bmatrix} \operatorname{erf}(a+bx) \\ \operatorname{erfc}(a+bx) \end{Bmatrix} H_n(cx)$.

$$1. \int_0^\infty [\operatorname{erf}(cx+d) - \operatorname{erf}(cx-d)] H_{2n}(cx) dx = \frac{(2d)^{2n+1}}{(2n+1)c} \quad [|\arg c| < \pi/4].$$

$$2. \int_{-\infty}^\infty [\operatorname{erf}(cx+d) - \operatorname{erf}(cx)] H_n(cx) dx = \frac{(-1)^n (2d)^{n+1}}{(n+1)c} \quad [|\arg c| < \pi/4].$$

$$3. \int_0^\infty x^{\alpha-1} \operatorname{erfc}(bx) \begin{Bmatrix} H_{2n+1}(cx) \\ H_{2n}(cx) \end{Bmatrix} dx = (-1)^n \frac{(2n+\delta)! (2c)^\delta}{n! \sqrt{\pi} (\alpha+\delta) b^{\alpha+\delta}} \times \\ \times \Gamma\left(\frac{\alpha+\delta+1}{2}\right) {}_3F_2\left(-2n-\delta, \frac{\alpha+1}{2}, \frac{\alpha}{2}+\delta; \delta+\frac{1}{2}, \frac{\alpha+\delta}{2}+1; \frac{c^2}{b^2}\right) \\ \left[\operatorname{Re} \alpha > -\delta; |\arg b| < \pi/4; \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right].$$

2.20.10. Интегралы от $x^\alpha e^{\varphi(x)} \begin{Bmatrix} \operatorname{erf}(\chi(x)) \\ \operatorname{erfc}(\chi(x)) \end{Bmatrix} H_n(cx)$.

$$1. \int_0^\infty e^{-c^2 x^2} \operatorname{erf}(bx) H_{2n+1}(cx) dx = \frac{(-1)^n 2^{2n} b^{2n+1} \Gamma(n+1/2)}{\sqrt{\pi} c (b^2 + c^2)^{n+1/2}} \\ [|\arg b|, |\arg c| < \pi/4].$$

$$2. \int_0^\infty x^{\alpha-1} e^{-c^2 x^2} \begin{Bmatrix} \operatorname{erf}(bx) \\ \operatorname{erfc}(bx) \end{Bmatrix} H_{2n+\varepsilon}(cx) dx = \\ = \mp (-1)^n \frac{2^{2n+\varepsilon} c^\varepsilon}{\sqrt{\pi} b^{\alpha+\varepsilon} (\alpha+\varepsilon)} \left(\varepsilon + \frac{1}{2}\right)_n \Gamma\left(\frac{\alpha+\varepsilon+1}{2}\right) \times \\ \times {}_3F_2\left(\frac{\alpha+1}{2}, \frac{\alpha}{2}+\varepsilon, n+\varepsilon+\frac{1}{2}; \varepsilon+\frac{1}{2}, \frac{\alpha+\varepsilon}{2}+1; -\frac{c^2}{b^2}\right) + \\ + \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{(-1)^n 2^{2n+\varepsilon-1}}{c^\alpha} \left(\frac{1-\alpha+\varepsilon}{2}\right)_n \Gamma\left(\frac{\alpha+\varepsilon}{2}\right) \\ [\varepsilon=0 \text{ или } 1; \operatorname{Re} \alpha > -\varepsilon - (1 \pm 1)/2; |\arg b|, |\arg c| < \pi/4].$$

$$3. \int_0^\infty x^{\alpha-1} e^{-c^2 x^2} \begin{Bmatrix} \operatorname{erf}(bx^2) \\ \operatorname{erfc}(bx^2) \end{Bmatrix} H_{2n+\varepsilon}(cx) dx = \\ = \mp \frac{(-1)^n 2^{2n+\varepsilon} c^\varepsilon}{\sqrt{\pi} b^{(\alpha+\varepsilon)/2} (\alpha+\varepsilon)} \left(\varepsilon + \frac{1}{2}\right)_n \Gamma\left(\frac{\alpha+\varepsilon+2}{4}\right) \times \\ \times {}_4F_4\left(\frac{\alpha+\varepsilon}{4}, \frac{\alpha+\varepsilon+1}{4}, \frac{2n+3}{4}, \frac{2n+1}{4}+\varepsilon; \frac{1}{2}, \frac{3}{4}, \frac{1}{4}+\varepsilon, \frac{\alpha+\varepsilon}{4}+1; \frac{c^2}{4b^2}\right) \pm \\ \pm \frac{(-1)^n 2^{2n+\varepsilon} c^{\varepsilon+2}}{\sqrt{\pi} b^{(\alpha+\varepsilon)/2+1} (\alpha+\varepsilon+2)} \left(\varepsilon + \frac{3}{2}\right)_n \Gamma\left(\frac{\alpha+\varepsilon}{4}+1\right) \times \\ \times {}_4F_4\left(\frac{\alpha+\varepsilon+2}{4}, \frac{\alpha+\varepsilon}{4}+1, \frac{2n+5}{4}, \frac{2n+3}{4}+\varepsilon; \frac{3}{2}, \frac{5}{4}, \frac{3}{4}+\varepsilon, \frac{\alpha+\varepsilon+6}{4}; \frac{c^2}{4b^2}\right) + \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{(-1)^n 2^{2n+\varepsilon-1}}{c^\alpha} \left(\frac{1-\alpha+\varepsilon}{2}\right)_n \Gamma\left(\frac{\alpha+\varepsilon}{2}\right) \\ [\varepsilon=0 \text{ или } 1; \operatorname{Re} \alpha > -\varepsilon - 1 \mp 1; |\arg b|, |\arg c| < \pi/4].$$

$$\begin{aligned}
4. \int_0^{\infty} x^{\alpha-1} e^{-c^2 x^2} \left\{ \begin{array}{l} \operatorname{erf}(b \sqrt{x}) \\ \operatorname{erfc}(b \sqrt{x}) \end{array} \right\} H_{2n+\varepsilon}(cx) dx = \\
= \pm \frac{(-1)^n 2^{2n+\varepsilon} b}{\sqrt{\pi} c^{\alpha+1/2}} \left(\frac{1-2\alpha+2\varepsilon}{4} \right)_n \Gamma \left(\frac{2\alpha+2\varepsilon+1}{4} \right) \times \\
\times {}_3F_3 \left(\frac{1}{4}, \frac{2\alpha+1}{4}, \frac{2\alpha+3}{4}; \frac{1}{2}, \frac{5}{4}, \frac{2\alpha-2\varepsilon+3}{4} - n; \frac{b^4}{4c^2} \right) \mp \\
\mp \frac{(-1)^n 2^{2n+\varepsilon} b^3}{3 \sqrt{\pi} c^{\alpha+3/2}} \left(\frac{2\varepsilon-2\alpha-1}{4} \right)_n \Gamma \left(\frac{2\alpha+2\varepsilon+3}{4} \right) \times \\
\times {}_3F_3 \left(\frac{3}{4}, \frac{2\alpha+3}{4}, \frac{2\alpha+5}{4}; \frac{3}{2}, \frac{7}{4}, \frac{2\alpha-2\varepsilon+5}{4} - n; \frac{b^4}{4c^2} \right) + \\
+ \begin{cases} 0 \\ 1 \end{cases} \frac{(-1)^n 2^{2n+\varepsilon-1}}{c^{\alpha}} \left(\frac{1-\alpha+\varepsilon}{2} \right)_n \Gamma \left(\frac{\alpha+\varepsilon}{2} \right) \\
[\varepsilon=0 \text{ или } 1; \operatorname{Re} \alpha > -\varepsilon - (1 \pm 1)/4; |\arg b|, |\arg c| < \pi/4]
\end{aligned}$$

$$\begin{aligned}
5. \int_0^{\infty} e^{-c^2 x^2} [\operatorname{erf}(cx+d) - \operatorname{erf}(cx-d)] H_{2n}(cx) dx = -\frac{2^{2-n}}{c} e^{-d^2/2} H_{2n-1} \left(\frac{d}{\sqrt{2}} \right) \\
[|\arg c| < \pi/4; n=1, 2, 3, \dots]
\end{aligned}$$

$$\begin{aligned}
6. \int_0^{\infty} e^{-c^2 x^2} [1 + \operatorname{erf}(cx)] \operatorname{erfc}(cx) H_{2n}(cx) dx = \\
= \frac{(-1)^n 2^n}{\pi c} \Gamma \left(n + \frac{1}{2} \right) {}_2F_1 \left(\frac{1}{2}, n + \frac{1}{2}; \frac{3}{2}; \frac{1}{4} \right) \quad [|\arg c| < \pi/4].
\end{aligned}$$

2.20.11. Интегралы от $x^{\alpha} e^{-c^2 x^2} \left\{ \begin{array}{l} S(bx^m) \\ C(bx^m) \end{array} \right\} H_n(cx)$.

Обозначение: $\delta = \begin{cases} 1 \\ 0 \end{cases}$.

$$\begin{aligned}
1. \int_0^{\infty} x^{\alpha-1} e^{-c^2 x^2} \left\{ \begin{array}{l} S(bx) \\ C(bx) \end{array} \right\} H_{2n+\varepsilon}(cx) dx = \\
= (-1)^n \frac{2^{2n+\varepsilon-1/2} b^{\delta+1/2}}{3^{\delta} \sqrt{\pi} c^{\alpha+\delta+1/2}} \left(\frac{1-2\alpha-2\delta+2\varepsilon}{4} \right)_n \Gamma \left(\frac{2\alpha+2\delta+2\varepsilon+1}{4} \right) \times \\
\times {}_3F_3 \left(\frac{2\delta+1}{4}, \frac{2\alpha+3}{4}, \frac{2\alpha+1}{4} + \delta; \delta + \frac{1}{2}, \frac{2\delta+5}{4}, \frac{2\alpha+2\delta-2\varepsilon+3}{4} - n; -\frac{b^2}{4c^2} \right) \\
[\varepsilon=0 \text{ или } 1; b>0; \operatorname{Re} \alpha > -\varepsilon - (2 \pm 1)/2; |\arg c| < \pi/4].
\end{aligned}$$

$$\begin{aligned}
2. \int_0^{\infty} x^{\alpha-1} e^{-c^2 x^2} \left\{ \begin{array}{l} S(bx^2) \\ C(bx^2) \end{array} \right\} H_{2n+\varepsilon}(cx) dx = \\
= (-1)^n \frac{2^{2n+\varepsilon-1/2} b^{\delta+1/2}}{3^{\delta} \sqrt{\pi} c^{\alpha+2\delta+1}} \left(\frac{\varepsilon-\alpha-2\delta}{2} \right)_n \Gamma \left(\frac{\alpha+2\delta+\varepsilon+1}{2} \right) \times \\
\times {}_4F_4 \left(\frac{2\delta+1}{4}, \frac{\alpha+3}{4}, \frac{\alpha}{4} + 1, \frac{\alpha+1}{4} + \delta, \frac{\alpha+2}{4} + \delta; \delta + \frac{1}{2}, \frac{2\delta+5}{4}, \right. \\
\left. \frac{\alpha-2n-\varepsilon+2}{4} + \delta, \frac{\alpha-2n-\varepsilon}{4} + 1; -\frac{b^2}{c^4} \right) \\
[\varepsilon=0 \text{ или } 1; b>0; \operatorname{Re} \alpha > -\varepsilon - 2 \mp 1; |\arg c| < \pi/4].
\end{aligned}$$

2.20.12. Интегралы от $x^\alpha e^{-cx^2} \left\{ \frac{\gamma(\mu, bx^m)}{\Gamma(\mu, bx^m)} \right\} H_n(cx)$.

$$\begin{aligned}
 1. \int_0^\infty x^{\alpha-1} e^{-cx^2} \left\{ \frac{\gamma(\mu, bx)}{\Gamma(\mu, bx)} \right\} H_{2n+\varepsilon}(cx) dx = \\
 = \pm (-1)^n \frac{2^{2n+\varepsilon-1} b^\mu}{c^{\alpha+\mu}} \left(\frac{1-\alpha+\varepsilon-\mu}{2} \right)_n \Gamma\left(\frac{\alpha+\varepsilon+\mu}{2}\right) \times \\
 \times {}_3F_3\left(\frac{\mu}{2}, \frac{\alpha+\mu}{2}, \frac{\alpha+\mu+1}{2}; \frac{1}{2}, \frac{\mu}{2}+1, \frac{\alpha+\mu-\varepsilon+1}{2}-n; \frac{b^2}{4c^2}\right) \mp \\
 \mp (-1)^n \frac{2^{2n+\varepsilon-1} b^{\mu+1}}{c^{\alpha+\mu+1}(\mu+1)} \left(\frac{\varepsilon-\alpha-\mu}{2} \right)_n \Gamma\left(\frac{\alpha+\varepsilon+\mu+1}{2}\right) \times \\
 \times {}_3F_3\left(\frac{\mu+1}{2}, \frac{\alpha+\mu+1}{2}, \frac{\alpha+\mu}{2}+1; \frac{3}{2}, \frac{\mu+3}{2}, \frac{\alpha-\varepsilon+\mu}{2}-n+1; \frac{b^2}{4c^2}\right) + \\
 + \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} (-1)^n \frac{2^{2n+\varepsilon-1}}{c^\alpha} \left(\frac{1-\alpha+\varepsilon}{2} \right)_n \Gamma(\mu) \Gamma\left(\frac{\alpha+\varepsilon}{2}\right) \\
 \left[\varepsilon=0 \text{ или } 1; \operatorname{Re} b > 0; \operatorname{Re}(\alpha+\mu) > -\varepsilon, \begin{Bmatrix} \operatorname{Re} \mu > 0 \\ \operatorname{Re} \alpha > -\varepsilon \end{Bmatrix}, |\arg c| < \pi/4 \right].
 \end{aligned}$$

$$\begin{aligned}
 2. \int_0^\infty x^{\alpha-1} e^{-cx^2} \left\{ \frac{\gamma(\mu, bx^2)}{\Gamma(\mu, bx^2)} \right\} H_{2n+\varepsilon}(cx) dx = \\
 = \mp (-1)^n \frac{2^{2n+\varepsilon} c^\varepsilon}{b^{(\alpha+\varepsilon)/2} (\alpha+\varepsilon)} \left(\varepsilon + \frac{1}{2} \right)_n \Gamma\left(\frac{\alpha+\varepsilon}{2} + \mu\right) \times \\
 \times {}_3F_3\left(\frac{\alpha+\varepsilon}{2}, \frac{\alpha+\varepsilon}{2} + \mu, n + \varepsilon + \frac{1}{2}; \varepsilon + \frac{1}{2}, \frac{\alpha+\varepsilon}{2} + 1; -\frac{c^2}{b}\right) + \\
 + \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} (-1)^n \frac{2^{2n+\varepsilon-1}}{2c^\alpha} \left(\frac{1-\alpha+\varepsilon}{2} \right)_n \Gamma(\mu) \Gamma\left(\frac{\alpha+\varepsilon}{2}\right) \\
 \left[\varepsilon=0 \text{ или } 1; \operatorname{Re} b > 0; \operatorname{Re}(\alpha+2\mu) > -\varepsilon, \begin{Bmatrix} \operatorname{Re} \mu > 0 \\ \operatorname{Re} \alpha > -\varepsilon \end{Bmatrix}, |\arg c| < \pi/4 \right].
 \end{aligned}$$

2.20.13. Интегралы, содержащие $J_\mu(bx^{\pm m}) H_n(cx)$.

$$\begin{aligned}
 1. \int_0^\infty x^{\alpha-1} J_\mu(bx) H_n(cx) dx = \frac{2^{2n+\alpha-1} c^n}{b^{n+\alpha}} \Gamma\left[\frac{(\alpha+\mu+n)/2}{(\mu-\alpha-n)/2+1}\right] \times \\
 \times {}_2F_2\left(-\frac{n}{2}, \frac{1-n}{2}; 1-\frac{\alpha+\mu+n}{2}, \frac{\alpha+n-\mu}{2}; \frac{b^2}{4c^2}\right) \\
 [b > 0; 2[n/2]-n-\operatorname{Re} \mu < \operatorname{Re} \alpha < 3/2-n].
 \end{aligned}$$

$$\begin{aligned}
 2. \int_0^\infty x^{\alpha-1} J_\mu\left(\frac{b}{x}\right) H_{2n+\varepsilon}(cx) dx = (-1)^n 2^{2n-\alpha-1} b^{\alpha+\varepsilon} c^\varepsilon \left(\varepsilon + \frac{1}{2} \right)_n \times \\
 \times \Gamma\left[\frac{(\mu-\alpha-\varepsilon)/2}{(\alpha+\mu+\varepsilon)/2+1}\right] {}_1F_3\left(-n; \varepsilon + \frac{1}{2}, \frac{\alpha+\varepsilon+\mu}{2}+1, \frac{\alpha+\varepsilon-\mu}{2}+1; -\frac{b^2 c^2}{4}\right) \\
 [\varepsilon=0 \text{ или } 1; b > 0; -\varepsilon-3/2 < \operatorname{Re} \alpha < \operatorname{Re} \mu-2n-\varepsilon].
 \end{aligned}$$

$$\begin{aligned}
 3. \int_0^\infty x^{\alpha-1} e^{-cx^2} \left\{ \frac{J_\mu(bx)}{I_\mu(bx)} \right\} H_{2n+\varepsilon}(cx) dx = \\
 = (-1)^n \frac{2^{2n+\varepsilon-\mu-1} b^\mu}{c^{\alpha+\mu}} \left(\frac{1-\alpha+\varepsilon-\mu}{2} \right)_n \Gamma\left[\frac{(\alpha+\varepsilon+\mu)/2}{\mu+1}\right] \times \\
 \times {}_2F_2\left(\frac{\alpha+\mu}{2}, \frac{\alpha+\mu+1}{2}; \mu+1, \frac{\alpha-\varepsilon+\mu+1}{2}-n; \mp \frac{b^2}{4c^2}\right) \\
 [\varepsilon=0 \text{ или } 1; \operatorname{Re}(\alpha+\mu) > -\varepsilon; |\arg b| < \pi; |\arg c| < \pi/4].
 \end{aligned}$$

$$4. \int_0^{\infty} x^{\mu+\sigma} e^{-c^2 x^2} J_{\mu}(bx) H_n(cx) dx = \frac{2^{\sigma/2} b^{n/2-1}}{c^{\mu+(n+\sigma)/2}} \Gamma \left[\frac{\mu+\sigma+1/2}{\mu+(2+\sigma-n)/2} \right] \times \\ \times \exp \left(-\frac{b^2}{8c^2} \right) M_{(2\mu+n+3\sigma)/4, (2\mu-n+\sigma)/4} \left(\frac{b^2}{4c^2} \right) \\ [\sigma=0 \text{ или } 1; \operatorname{Re} \mu > -(1+\sigma)/2; |\arg c| < \pi/4]$$

$$5. \int_0^{\infty} x^{\alpha-1} e^{-c^2 x^2} \left\{ \begin{matrix} J_{\mu}(bx^2) \\ I_{\mu}(bx^2) \end{matrix} \right\} H_{2n+\varepsilon}(cx) dx = \\ = (-1)^n \frac{2^{2n+\varepsilon-\mu-1} b^{\mu}}{c^{\alpha+2\mu}} \left(\frac{1-\alpha+\varepsilon}{2} - \mu \right)_n \Gamma \left[\frac{(\alpha+\varepsilon)/2 + \mu}{\mu+1} \right] \times \\ \times {}_4F_3 \left(\frac{\alpha+2\mu}{4}, \frac{\alpha+2\mu+1}{4}, \frac{\alpha+2\mu+2}{4}, \frac{\alpha+2\mu+3}{4}; \mu+1, \right. \\ \left. \frac{\alpha-\varepsilon+2\mu-2n+1}{4}, \frac{\alpha-\varepsilon+2\mu-2n+3}{4}; \mp \frac{b^2}{c^4} \right) \\ \left[\varepsilon=0 \text{ или } 1; \operatorname{Re}(\alpha+2\mu) > -\varepsilon; |\arg c| < \pi/4, \left\{ \begin{matrix} b>0 \\ |\arg b| < \pi; \operatorname{Re}(c^2-b) > 0 \end{matrix} \right\} \right]$$

2.20.14. Интегралы, содержащие $\left\{ \begin{matrix} Y_{\mu}(bx^{\pm m}) \\ K_{\mu}(bx^{\pm m}) \end{matrix} \right\} H_n(cx)$.

$$1. \int_0^{\infty} x^{\alpha-1} K_{\mu}(bx) H_n(cx) dx = \frac{2^{2n+\alpha-2} c^n}{b^{n+\alpha}} \Gamma \left(\frac{\alpha+\mu+n}{2} \right) \Gamma \left(\frac{\alpha-\mu+n}{2} \right) \times \\ \times {}_2F_2 \left(-\frac{n}{2}, \frac{1-n}{2}; 1-\frac{\alpha+\mu+n}{2}, 1-\frac{\alpha-\mu+n}{2}; -\frac{b^2}{4c^2} \right) \\ [\operatorname{Re} b > 0; \operatorname{Re} \alpha > -n+2[n/2]+|\operatorname{Re} \mu|]$$

$$2. \int_0^{\infty} x^{\alpha-1} K_{\mu} \left(\frac{b}{x} \right) H_{2n+\varepsilon}(cx) dx = \\ = (-1)^n 2^{2n-\alpha-2} b^{\alpha+\varepsilon} c^{\varepsilon} \left(\varepsilon + \frac{1}{2} \right)_n \Gamma \left(\frac{\mu-\alpha-\varepsilon}{2} \right) \Gamma \left(-\frac{\alpha+\varepsilon+\mu}{2} \right) \times \\ \times {}_1F_3 \left(-n; \varepsilon + \frac{1}{2}, \frac{\alpha+\varepsilon+\mu}{2} + 1, \frac{\alpha+\varepsilon-\mu}{2} + 1; \frac{b^2 c^2}{4} \right) \\ [\varepsilon=0 \text{ или } 1; \operatorname{Re} b > 0; \operatorname{Re} \alpha < -2n-\varepsilon-|\operatorname{Re} \mu|]$$

$$3. \int_0^{\infty} x^{\alpha-1} e^{-c^2 x^2} \left\{ \begin{matrix} Y_{\mu}(bx) \\ K_{\mu}(bx) \end{matrix} \right\} H_{2n+\varepsilon}(cx) dx = \\ = \mp (-1)^n \frac{2^{2n+\varepsilon+\mu-2}}{\pi b^{\mu} c^{\alpha-\mu}} \left(\frac{1-\alpha+\varepsilon+\mu}{2} \right)_n \Gamma(\mu) \Gamma \left(\frac{\alpha+\varepsilon-\mu}{2} \right) \times \\ \times \left\{ \begin{matrix} 2 \\ \pi \end{matrix} \right\} {}_2F_2 \left(\frac{\alpha-\mu}{2}, \frac{\alpha-\mu+1}{2}; 1-\mu, \frac{\alpha-\varepsilon-\mu+1}{2}-n; \mp \frac{b^2}{4c^2} \right) \mp \\ \mp (-1)^n \frac{2^{2n+\varepsilon-\mu-2} b^{\mu}}{\pi c^{\alpha+\mu}} \left(\frac{1-\alpha+\varepsilon-\mu}{2} \right)_n \Gamma(-\mu) \Gamma \left(\frac{\alpha+\varepsilon+\mu}{2} \right) \times \\ \times \left\{ \begin{matrix} 2 \cos \mu\pi \\ \pi \end{matrix} \right\} {}_2F_2 \left(\frac{\alpha+\mu}{2}, \frac{\alpha+\mu+1}{2}; \mu+1, \frac{\alpha-\varepsilon+\mu+1}{2}-n; \mp \frac{b^2}{4c^2} \right) \\ [\varepsilon=0 \text{ или } 1; \operatorname{Re} \alpha > |\operatorname{Re} \mu| - \varepsilon; |\arg b| < \pi; \\ |\arg c| < \pi/4; \text{ при } \mu=0, 1, 2, \dots \text{ см } 2.20.1.17]$$

$$\begin{aligned}
4. \int_0^\infty x^{\alpha-1} e^{-cx^2} \left\{ \begin{matrix} Y_\mu(bx^2) \\ K_\mu(bx^2) \end{matrix} \right\} H_{2n+2}(cx) dx = \mp (-1)^n \frac{2^{2n+(3\varepsilon+\alpha)/2-3} c^\varepsilon}{\pi b^{(\alpha+\varepsilon)/2}} \times \\
\times \left(\varepsilon + \frac{1}{2} \right)_n \Gamma\left(\frac{\alpha+\varepsilon+2\mu}{4}\right) \Gamma\left(\frac{\alpha+\varepsilon-2\mu}{4}\right) \pi^{(1\mp 1)/2} \left[2 \cos \frac{\alpha+\varepsilon-2\mu}{4} \pi \right]^{(1\pm 1)/2} \times \\
\times {}_4F_3\left(\frac{2n+3}{4}, \frac{2n+1}{4} + \varepsilon, \frac{\alpha+\varepsilon+2\mu}{4}, \frac{\alpha+\varepsilon-2\mu}{4}; \frac{1}{2}, \frac{3}{4}, \frac{1}{4} + \varepsilon; \mp \frac{c^2}{b^2}\right) - \\
- (-1)^n \frac{2^{2n+(3\varepsilon+\alpha)/2-3} c^{\varepsilon+2}}{\pi b^{(\alpha+\varepsilon)/2+1}} \left(\varepsilon + \frac{3}{2} \right)_n \Gamma\left(\frac{\alpha+\varepsilon+2\mu+2}{4}\right) \times \\
\times \Gamma\left(\frac{\alpha+\varepsilon-2\mu+2}{4}\right) \pi^{(1\mp 1)/2} \left[2 \sin \frac{\alpha+\varepsilon-2\mu}{4} \pi \right]^{(1\pm 1)/2} \times \\
\times {}_4F_3\left(\frac{2n+5}{4}, \frac{2n+3}{4} + \varepsilon, \frac{\alpha+\varepsilon+2\mu+2}{4}, \frac{\alpha+\varepsilon-2\mu+2}{4}; \right. \\
\left. \frac{3}{2}, \frac{5}{4}, \frac{3}{4} + \varepsilon; \mp \frac{c^2}{b^2}\right) \\
\left[\varepsilon=0 \text{ или } 1; \operatorname{Re} \alpha > 2|\operatorname{Re} \mu| - \varepsilon; |\arg b| < \pi; |\arg c| < \pi/4, \left\{ \begin{matrix} \operatorname{Re} c^2 > |\operatorname{Im} b| \\ \operatorname{Re}(b+c^2) > 0 \end{matrix} \right\} \right].
\end{aligned}$$

2.20.15. Интегралы, содержащие $L_m^\gamma(bx^k) H_n(cx)$.

Обозначение: $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$.

$$\begin{aligned}
1. \int_0^\infty x^{\alpha-1} e^{-bx-c^2x^2} L_m^\gamma(bx) \left\{ \begin{matrix} H_{2n+1}(cx) \\ H_{2n}(cx) \end{matrix} \right\} dx = \\
= \frac{(-1)^n 2^{2n+\delta-1} (1+\gamma)_m}{m! c^\alpha} \Gamma\left(\frac{\alpha+\delta}{2}\right) \left(\frac{1-\alpha+\delta}{2}\right)_n \times \\
\times {}_4F_4\left(\frac{1+\gamma+m}{2}, 1+\frac{\gamma+m}{2}, \frac{\alpha}{2}, \frac{1+\alpha}{2}; \frac{1}{2}, \frac{1+\alpha-\delta}{2} - n, \right. \\
\left. \frac{1+\gamma}{2}, 1+\frac{\gamma}{2}; \frac{b^2}{4c^2}\right) - \frac{(-1)^n 2^{2n+\delta} b (2+\gamma)_m}{m! c^{\alpha+1}} \Gamma\left(\frac{\alpha+\delta+1}{2}\right) \left(\frac{\delta-\alpha}{2}\right)_n \times \\
\times {}_4F_4\left(1+\frac{\gamma+m}{2}, \frac{3+\gamma+m}{2}, \frac{\alpha+1}{2}, 1+\frac{\alpha}{2}; \frac{3}{2}, \right. \\
\left. 1+\frac{\alpha-\delta}{2} - n, 1+\frac{\gamma}{2}, \frac{3+\gamma}{2}; \frac{b^2}{4c^2}\right) \\
[\operatorname{Re} \alpha > -\delta; |\arg c| < \pi/4].
\end{aligned}$$

$$\begin{aligned}
2. \int_0^\infty x^{\alpha-1} e^{-(b+c^2)x^2} L_m^\gamma(bx^2) \left\{ \begin{matrix} H_{2n+1}(cx) \\ H_{2n}(cx) \end{matrix} \right\} dx = \\
= \frac{(-1)^n 2^{2n+\delta-1} (1+\gamma)_m}{m! c^\alpha} \Gamma\left(\frac{\alpha+\delta}{2}\right) \left(\frac{1-\alpha+\delta}{2}\right)_n \times \\
\times {}_3F_2\left(1+\gamma, \frac{\alpha}{2}, \frac{1+\alpha}{2}; \frac{1+\alpha-\delta}{2} - n, 1+\gamma; -\frac{b}{c^2}\right) \\
[\operatorname{Re}(b+c^2) > 0; \operatorname{Re} \alpha > -\delta].
\end{aligned}$$

$$\begin{aligned}
3. \int_0^\infty x^{\alpha-1} e^{-bx^4-c^2x^2} L_m^\gamma(bx^4) \left\{ \begin{matrix} H_{2n+1}(cx) \\ H_{2n}(cx) \end{matrix} \right\} dx = \\
= \frac{(-1)^n 2^{2n+\delta-2} c^\delta}{b^{(\alpha+\delta)/4} m!} \left(\frac{1}{2} + \delta\right)_n \left(1+\gamma - \frac{\alpha+\delta}{4}\right)_m \Gamma\left(\frac{\alpha+\delta}{4}\right) \times
\end{aligned}$$

$$\times {}_4F_4 \left(\frac{2n+3}{4}, \frac{2n+1}{4} + \delta, \frac{\alpha+\delta}{4}, \frac{\alpha+\delta}{4} - \gamma; \frac{1}{2}, \frac{3}{4}, \frac{1}{4} + \delta, \frac{\alpha+\delta}{4} - \gamma - m; \frac{c^4}{4b} \right) - \frac{(-1)^n 2^{2n+\delta-2} c^{\delta+2}}{b^{(\alpha+\delta+2)/4} m!} \left(\frac{3}{2} + \delta \right)_n \left(\gamma + \frac{2-\alpha-\delta}{4} \right)_m \Gamma \left(\frac{\alpha+\delta+2}{4} \right) \times$$

$$\times {}_4F_4 \left(\frac{2n+5}{4}, \frac{2n+3}{4} + \delta, \frac{2+\alpha+\delta}{4}, \frac{2+\alpha+\delta}{4} - \gamma; \frac{3}{2}, \frac{5}{4}, \frac{3}{4} + \delta, \frac{2+\alpha+\delta}{4} - \gamma - m; \frac{c^4}{4b} \right) \quad [\operatorname{Re} b > 0; \operatorname{Re} \alpha > -\delta].$$

$$4. \int_0^\infty e^{-4c^2 x^2} \left\{ \frac{\sin bx}{\cos bx} \right\} L_n(4c^2 x^2) \left\{ \frac{H_{2n+1}(cx)}{H_{2n}(cx)} \right\} dx =$$

$$= \frac{\sqrt{\pi}}{4c} \exp \left(-\frac{b^2}{16c^2} \right) L_n \left(\frac{b^2}{16c^2} \right) H_{2n+\delta} \left(\frac{b}{8c} \right) \quad \left[b, \operatorname{Re} c > 0; \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right].$$

2.20.16. Интегралы, содержащие $H_m(ax^k+b)H_n(cx+d)$.

Обозначение: $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$.

$$1. \int_0^\infty e^{-px^2} \left\{ \frac{H_{2m+1}(bx) H_{2n+1}(cx)}{H_{2m}(bx) H_{2n}(cx)} \right\} dx =$$

$$= (\pm 1)^{m+n+\delta} \sqrt{\pi} (bc)^\delta \frac{(2m+2n+2\delta)! (p-b^2)^m (p-c^2)^n}{2(m+n+\delta)! p^{m+n+\delta+1/2}} \times$$

$$\times {}_2F_1 \left(-m, -n; -m-n-\delta-\frac{1}{2}; \frac{p(p-b^2-c^2)}{(p-b^2)(p-c^2)} \right) \quad [\operatorname{Re} p > 0].$$

$$2. \int_0^\infty e^{-px^2} H_n(bx) H_n(cx) dx = \frac{2^{n-1} n! \sqrt{\pi}}{p^{(n+1)/2}} (b^2+c^2-p)^{n/2} P_n \left(\frac{bc}{\sqrt{p(b^2+c^2-p)}} \right)$$

$$[\operatorname{Re} p > 0].$$

$$3. \int_0^\infty e^{-px^2} H_n(-cx) H_n(cx) dx = 2^{n-1} n! e^{n\pi i/2} \sqrt{\frac{\pi}{p}} P_n \left(\frac{c^2}{p} \right)$$

$$[\operatorname{Re} p > 0].$$

$$4. \int_{-\infty}^\infty e^{-px^2} \left\{ \frac{H_{2m+1}(bx) H_{2n+1}(cx)}{H_{2m}(bx) H_{2n}(cx)} \right\} dx = (2m+\delta)! (2n+\delta)! \sqrt{\frac{\pi}{p}} \times$$

$$\times \sum_{k=0}^{\min(m,n)} \frac{1}{(m-k)! (n-k)! (2k+\delta)!} \left(\frac{b^2-p}{p} \right)^{m-k} \left(\frac{c^2-p}{p} \right)^{n-k} \left(\frac{2bc}{p} \right)^{2k+\delta}$$

$$[\operatorname{Re} p > 0].$$

$$5. \int_{-\infty}^\infty e^{-px^2} H_{2m}(bx) H_{2n+1}(cx) dx = 0 \quad [\operatorname{Re} p > 0].$$

$$6. \int_{-\infty}^\infty e^{-px^2} H_m(cx) H_n(cx) dx =$$

$$= \frac{2^{m+n} (c^2-p)^{(m+n)/2}}{p^{(m+n+1)/2}} \Gamma \left(\frac{m+n+1}{2} \right) {}_2F_1 \left(-m, -n; \frac{1-m-n}{2}; \frac{p}{2(p-c^2)} \right)$$

$$[\operatorname{Re} p > 0; m+n=2, 4, 6, \dots].$$

$$7. \int_{-\infty}^{\infty} e^{-c^2 x^2} H_m(bx) H_n(cx) dx = \begin{cases} 1 \\ 0 \end{cases} \frac{\sqrt{\pi} m! n b^n (b^2 - c^2)^{(m-n)/2}}{((m-n)/2)! c^{m+1}} \left[|\arg c| < \pi/4, \begin{cases} m-n=2, 4, 6, \dots \\ m < n \end{cases} \right].$$

$$8. \int_{-\infty}^{\infty} e^{-c^2 x^2} H_m(cx) H_n(cx) dx = \frac{2^n n! \sqrt{\pi}}{c} \delta_{m,n} \quad [|\arg c| < \pi/4].$$

$$9. \int_{-\infty}^{\infty} e^{-2c^2 x^2} H_m(cx) H_n(cx) dx = (-1)^{(m+n)/2} 2^{(m+n-1)/2} c^{-1} \Gamma\left(\frac{m+n+1}{2}\right) \quad [|\arg c| < \pi/4; m+n=2, 4, 6, \dots].$$

$$10. \int_{-\infty}^{\infty} e^{-c^2 x^2} H_m(a+cx) H_n(b+cx) dx = \frac{2^n \sqrt{\pi} m! b^{n-m}}{c} L_m^{n-m}(-2ab) \quad [|\arg c| < \pi/4; m \leq n].$$

$$11. \int_0^{\infty} e^{-px^2} [H_m(b+ax) H_n(d+cx) + H_m(b-ax) H_n(d-cx)] dx = \\ = \sqrt{\frac{\pi}{2}} p^{-(m+n+1)/2} \sum_{k=0}^{\min(m,n)} \binom{m}{k} \binom{n}{k} k! (p-a^2)^{(m+k)/2} \times \\ \times (p-c^2)^{(n+k)/2} \left(\frac{2}{ac}\right)^k H_{m-k}\left(b \sqrt{\frac{2p}{p-a^2}}\right) H_{n-k}\left(d \sqrt{\frac{2p}{p-c^2}}\right) \quad [\operatorname{Re} p > 0].$$

$$12. \int_0^{\infty} x^{\alpha-1} e^{-px^2} \left\{ \begin{matrix} H_{2m+1}(bx) H_{2n+1}(cx) \\ H_{2m}(bx) H_{2n}(cx) \end{matrix} \right\} dx = \\ = \frac{(-1)^{m+n} (2m+\delta)! (2n+\delta)! (bc)^{\delta}}{m! n! 2p^{\alpha/2+\delta}} \Gamma\left(\frac{\alpha}{2} + \delta\right) \times \\ \times F_2\left(-m, -n, \frac{\alpha}{2} + \delta; \frac{1}{2} + \delta, \frac{1}{2} + \delta; \frac{b^2}{p}, \frac{c^2}{p}\right) \quad [\operatorname{Re} p > 0; \operatorname{Re} \alpha > -2\delta].$$

$$13. \int_0^{\infty} x^{\alpha-1} e^{-(b^2+c^2)x^2} H_{2m+\varepsilon}(bx) \left\{ \begin{matrix} H_{2n+1}(cx) \\ H_{2n}(cx) \end{matrix} \right\} dx = \\ = \frac{(-1)^{m+n} 2^{2m+2n+\delta+\varepsilon-1} c^{\delta}}{b^{\alpha+\delta}} \left(\frac{1}{2} + \delta\right)_n \left(\frac{1+\varepsilon-\alpha-\delta}{2}\right)_m \Gamma\left(\frac{\alpha+\varepsilon+\delta}{2}\right) \times \\ \times {}_3F_2\left(\frac{\alpha+\delta}{2}, \frac{1+\alpha+\delta}{2}, \frac{1}{2} + n + \delta; \frac{1+\alpha+\delta-\varepsilon}{2} - m, \frac{1}{2} + \delta; -\frac{c^2}{b^2}\right) \quad [\varepsilon=0 \text{ или } 1; \operatorname{Re}(b^2+c^2) > 0; \operatorname{Re} \alpha > -\delta-\varepsilon].$$

$$14. \int_0^{\infty} x e^{-px^2} H_{n+1}(cx) H_n(cx) dx = \\ = \frac{2^{n-1} n! (n+1)! \sqrt{\pi} c}{p^{3/2}} \sum_{k=0}^n \frac{(2k+1)!}{k! (k+1)! (n-k)!} \left(\frac{p-c^2}{p}\right)^k \quad [\operatorname{Re} p > 0].$$

$$15. \int_0^{\infty} x^{\alpha-1} e^{-c^2 x^2} H_m(cx) H_n(cx) dx =$$

$$= \frac{\sqrt{\pi}}{2^{\alpha-m-n} c^{\alpha}} \Gamma \left[\frac{\alpha}{(\alpha-m-n+1)/2} \right] {}_2F_1 \left(-m, -n; \frac{\alpha-m-n+1}{2}; \frac{1}{2} \right)$$

[Re c > 0].

$$16. \int_0^{\infty} x^{l+m+n} e^{-c^2 x^2} H_m(cx) H_n(cx) dx =$$

$$= \frac{\sqrt{\pi} (l+m+n)!}{2^{l+1} \Gamma(l/2+1) c^{l+m+n+1}} {}_2F_1 \left(-m, -n; 1 + \frac{l}{2}; \frac{1}{2} \right)$$

[|arg c| < π/4].

$$17. \int_{-\infty}^{\infty} e^{-(x-y)^2} H_m(cx) H_n(cx) dx =$$

$$= \sqrt{\pi} \sum_{k=0}^{\min(m, n)} 2^k k! \binom{m}{k} \binom{n}{k} (1-c^2)^{(m+n)/2-k} H_{m+n-2k} \left(\frac{cy}{\sqrt{1-c^2}} \right).$$

$$18. \int_{-\infty}^{\infty} e^{-(x-c)^2} H_m(cx) H_n(cx) dx = 2^n \sqrt{\pi} m! c^{n-m} L_n^{n-m}(-2c^2)$$

[m ≤ n].

$$19. \int_{-\infty}^{\infty} e^{-3c^2 x^2/2 - px} H_m(a+cx) H_n(cx) dx =$$

$$= \frac{(-1)^{m+n} \sqrt{2\pi}}{3^{(m+n+1)/2} c} \exp \left(\frac{p^2}{6c^2} \right) \sum_{k=0}^{\min(m, n)} k! 4^k \binom{m}{k} \binom{n}{k} H_{m-k} \left(\frac{p}{c\sqrt{3}} - a\sqrt{3} \right) H_{n-k} \left(\frac{p}{c\sqrt{3}} \right)$$

[|arg c| < π/4].

$$20. \int_0^{\infty} x^{\alpha-1} e^{-b^2 x^4 - c^2 x^2} H_{2m+\varepsilon}(bx^2) \begin{Bmatrix} H_{2n+1}(cx) \\ H_{2n}(cx) \end{Bmatrix} dx =$$

$$= \frac{(-1)^{m+n} 2^{2m+2n+\delta+\varepsilon-2} c^{\delta}}{b^{(\alpha+\delta)/2}} \left(\frac{1}{2} + \delta \right)_n \left(\frac{2+2\varepsilon-\alpha-\delta}{4} \right)_m \Gamma \left(\frac{2\varepsilon+\alpha+\delta}{4} \right) \times$$

$$\times {}_4F_4 \left(\frac{\alpha+\delta}{4}, \frac{2+\alpha+\delta}{4}, \frac{2n+3}{4}, \frac{2n+1}{4} + \delta; \frac{1}{2}, \frac{3}{4}, \frac{1}{4} + \delta, \right.$$

$$\left. \frac{2+\alpha+\delta-2\varepsilon}{4} - m; \frac{c^4}{4b^2} \right) - \frac{(-1)^{m+n} 2^{2m+2n+\delta+\varepsilon-2} c^{\delta+2}}{b^{(\alpha+\delta)/2+1}} \left(\frac{3}{2} + \delta \right)_n \left(\frac{2\varepsilon-\delta-\alpha}{4} \right)_m \times$$

$$\times \Gamma \left(\frac{2+\alpha+\delta+2\varepsilon}{4} \right) {}_4F_4 \left(\frac{2+\alpha+\delta}{4}, 1 + \frac{\alpha+\delta}{4}, \frac{2n+5}{2}, \right.$$

$$\left. \frac{2n+3}{4} + \delta; \frac{3}{2}, \frac{5}{4}, \frac{3}{4} + \delta, 1 - m + \frac{\alpha+\delta-2\varepsilon}{4}; \frac{c^4}{4b^2} \right)$$

[ε = 0 или 1; Re α > -δ - 2ε; |arg b| < π/4].

$$21. \int_0^{\infty} \frac{x^{-1}}{x^2+z^2} e^{-c^2 x^2} H_{n+1}(cx) H_n(cx) dx =$$

$$= \frac{2^n n! (n+1)! \sqrt{\pi}}{z^2} \sum_{k=0}^n \frac{(-2)^k}{(k+1)! (n-k)!} [1 - (2k+1)! e^{c^2 z^2/2} D_{-2k-2}(cz\sqrt{2})]$$

[Re z > 0; |arg c| < π/4].

$$22. \int_0^{\infty} e^{-c^2 x^2} \left\{ \begin{matrix} \sin bx H_{n+2m+1}(cx) \\ \cos bx H_{n+2m}(cx) \end{matrix} \right\} H_n(cx) dx =$$

$$= (-1)^m 2^{n-1} \sqrt{\pi} n! \frac{b^{2m+(1\pm 1)/2}}{c^{2m+(3\pm 1)/2}} \exp\left(-\frac{b^2}{4c^2}\right) L_n^{2m+(1\pm 1)/2}\left(\frac{b^2}{2c^2}\right)$$

[$b > 0$; $|\arg c| < \pi/4$].

$$23. \int_0^{\infty} \operatorname{erfc}(cx) H_{n+1}(cx) H_n(cx) dx = \frac{2^{n+1} n!}{c} - \frac{1}{4(n+1)c} H_{n+1}^2(0)$$

[$|\arg c| < \pi/4$].

$$24. \int_0^{\infty} \operatorname{erfc}(\sqrt{2} cx) H_{n+1}(cx) H_n(cx) dx =$$

$$= \frac{(-1)^{n+1} 2^{n-1}}{\sqrt{\pi} (n+1)c} \Gamma\left(n + \frac{3}{2}\right) - \frac{1}{4(n+1)c} H_{n+1}^2(0)$$

[$|\arg c| < \pi/4$].

$$25. \int_0^{\infty} e^{-2c^2 x^2} \operatorname{erf}(bx) H_{n+1}(cx) H_n(cx) dx =$$

$$= \frac{(-1)^n 2^{2n-1} \Gamma(n+1/2) b (b^2+c^2)^n}{\sqrt{\pi} c (b^2+2c^2)^{n+1/2}} {}_2F_1\left(-n, -n; \frac{1}{2}-n; \frac{b^2+2c^2}{2(b^2+c^2)}\right)$$

[$|\arg c| < \pi/4$].

$$26. \int_0^{\infty} e^{-c^2 x^2} \operatorname{erf}(cx) H_{n+2m+1}(cx) H_n(cx) dx = \frac{(-1)^m (2m+2n+2)!}{2^{m+n+3/2} (2m+1)(m+n+1)! c}$$

[$|\arg c| < \pi/4$].

$$27. \int_{-\infty}^{\infty} x^{\alpha-1} e^{-c^2 x^2} \operatorname{erfc}(bx) H_m(cx) H_n(cx) dx =$$

$$= \frac{2^{m+n-1}}{\alpha \pi b^{\alpha}} \Gamma\left(\frac{\alpha+1}{2}\right) \cos \frac{m+n}{2} \pi \sum_{k=0}^{\min(m,n)} (-1)^k k! \Gamma\left(\frac{m+n+1}{2} - k\right) \times$$

$$\times 2^{-k} \binom{m}{k} \binom{n}{k} {}_2F_2\left(\frac{\alpha}{2}, \frac{\alpha+1}{2}, \frac{m+n+1}{2} - k; \frac{1}{2}, \frac{\alpha}{2} + 1; -\frac{c^2}{b^2}\right)$$

[$\operatorname{Re} \alpha > 0$; $|\arg b|, |\arg c| < \pi/4$].

2.20.17. Интегралы, содержащие $\prod_{j=1}^m H_{n_j}(cx)$, $m=3, 4$.

$$1. \int_{-\infty}^{\infty} e^{-2c^2 x^2} H_l(cx) H_m(cx) H_n(cx) dx = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{2^{s-1}}{\pi c} \Gamma[s-l, s-m, s-n]$$

[$|\arg c| < \pi/4, \left\{ \begin{matrix} l+m+n=2s-1=1, 3, 5, \dots \\ l+m+n=2s-1=2, 4, 6, \dots \end{matrix} \right\} \right]$.

$$2. \int_{-\infty}^{\infty} e^{-c^2 x^2} H_l(cx) H_m(cx) H_n(cx) dx =$$

$$= \frac{2^{(l+m+n)/2} \sqrt{\pi} l! m! n!}{c [(l+m-n)/2]! [(m+n-l)/2]! [(l+m-n)/2]!}$$

[$l+m+n=2, 4, 6, \dots$; стороны с длинами l, m, n составляют треугольник; $|\arg c| < \pi/4$].

= 0 [в остальных случаях; $|\arg c| < \pi/4$].

$$3. \int_{-\infty}^{\infty} e^{-c^2 x^2} H_{l+m+n}(cx) H_m(cx) H_n(cx) dx = \frac{2^{l+m+n} \sqrt{\pi}}{c} (l+m+n)! \\ [|\arg c| < \pi/4; l=1, 3, 5, \dots]$$

$$4. \int_0^{\infty} e^{-3c^2 x^2} \operatorname{erf}(icx) H_{2n+1}(cx) H_{2n}^2(cx) dx = \frac{2^{3n+3/2} i}{3\pi^{3/2} c} \Gamma^3\left(n + \frac{1}{2}\right) \\ [|\arg c| < \pi/4].$$

$$5. \int_0^{\infty} e^{-3c^2 x^2} \operatorname{erf}(ic\sqrt{2}x) H_{2n+1}(cx) H_{2n}^2(cx) dx = \frac{2^{3n+1/2} i}{3c} \left[\frac{(2n)!}{n!}\right]^3 \\ [|\arg c| < \pi/4].$$

$$6. \int_{-\infty}^{\infty} e^{-c^2 x^2} H_m^2(cx) H_n^2(cx) dx = \frac{2^{m+n} \sqrt{\pi}}{c} m! n! \sum_{k=0}^{\min(m, n)} \binom{m}{k} \binom{n}{k} \binom{2k}{k} \\ [|\arg c| < \pi/4].$$

2.21. МНОГОЧЛЕНЫ ГЕГЕНБАУЭРА $C_n^\lambda(x)$

При вычислении интегралов, содержащих многочлены Гегенбауэра $C_n^\lambda(x)$, могут оказаться полезными формулы

$$C_n^\lambda(x) = \frac{(2\lambda)_n}{(\lambda + 1/2)_n} P_n^{(\lambda-1/2, \lambda-1/2)}(x), \\ C_n^\lambda(-x) = (-1)^n C_n^\lambda(x).$$

2.21.1. Интегралы общего вида.

(Также см. 2.22.1 при $\rho = \sigma = \lambda - 1/2$.)

Обозначение: $\delta = \begin{cases} 1 \\ 0 \end{cases}$.

$$1. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\lambda-1/2} (b^r - x^r)^{\beta-1} C_{2n+\varepsilon}^\lambda\left(\frac{x}{a}\right) dx = \\ = \frac{(-1)^n}{2(2n+\varepsilon)!} a^{\alpha+2\lambda-1} b^{\beta r-r} (2\lambda)_{2n+\varepsilon} \Gamma\left(\lambda + \frac{1}{2}\right) \times \\ \times \sum_{k=0}^{\infty} \frac{(1-\beta)_k}{k!} \Gamma\left[\frac{(\alpha+\varepsilon+rk)/2}{(\alpha+\varepsilon+rk+1)/2 + \lambda + n}\right] \left(\frac{1-\alpha+\varepsilon-rk}{2}\right)_n \left(\frac{a}{b}\right)^{rk} \\ [\varepsilon = 0 \text{ или } 1; a, r > 0, b > a; \operatorname{Re} \lambda > -1/2; \operatorname{Re} \alpha > -\varepsilon].$$

$$2. \int_0^a x^{\alpha-1} (a^r - x^r)^{\beta-1} (b^2 - x^2)^{\lambda-1/2} C_{2n+\varepsilon}^\lambda\left(\frac{x}{b}\right) dx = \\ = \frac{(-1)^n 2^{2n+\varepsilon} a^{\alpha+\varepsilon+\beta r-r} b^{2\lambda-\varepsilon-1}}{(2n+\varepsilon)! r} (\lambda)_{n+\varepsilon} \Gamma(\beta) \times \\ \times \sum_{k=0}^{\infty} \frac{(1/2-\lambda-n)_k}{k!} \Gamma\left[\frac{(\alpha+\varepsilon+2k)/r}{\beta + (\alpha+\varepsilon+2k)/r}\right] \left(\varepsilon + k + \frac{1}{2}\right)_n \left(\frac{a}{b}\right)^{2k} \\ [\varepsilon = 0 \text{ или } 1; a, r, \operatorname{Re} \beta > 0; b > a; \operatorname{Re} \alpha > -\varepsilon].$$

$$\begin{aligned}
3. \int_a^b x^{\alpha-1} (x^2 - a^2)^{\lambda-1/2} (b^r - x^r)^{\beta-1} C_{2n+\varepsilon}^\lambda \left(\frac{x}{a}\right) dx = \\
= \frac{a^{\alpha+2\lambda-1} b^{r\beta-r}}{2(2n+\varepsilon)!} \Gamma\left(\lambda + \frac{1}{2}\right) (2\lambda)_{2n+\varepsilon} \sum_{k=0}^{\infty} \frac{(1-\beta)_k}{k!} \times \\
\times \Gamma\left[\frac{(1-\alpha-\varepsilon-rk)/2-\lambda-n}{1-(\alpha+\varepsilon+rk)/2}\right] \left(\frac{1-\alpha+\varepsilon-rk}{2}\right)_n \left(\frac{a}{b}\right)^{rk} + \\
+ \frac{2^{2n+\varepsilon} b^{\alpha+\varepsilon+2\lambda+r\beta-r+2n-1} (\lambda)_{n+\varepsilon}}{(2n+\varepsilon)! a^{2n+\varepsilon} r} \Gamma(\beta) \sum_{k=0}^{\infty} \frac{(1/2-\lambda-n)_k}{k!} \times \\
\times \Gamma\left[\frac{(\alpha+\varepsilon+2\lambda-2k+2n-1)/r}{\beta+(\alpha+\varepsilon+2\lambda-2k+2n-1)/r}\right] (\varepsilon+\lambda-k+n)_n \left(\frac{a}{b}\right)^{2k} \\
[\varepsilon = 0 \text{ или } 1; b > a > 0; r, \operatorname{Re} \beta > 0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
4. \int_a^b x^{\alpha-1} (x^r - a^r)^{\beta-1} (b^2 - x^2)^{\lambda-1/2} C_{2n+\varepsilon}^\lambda \left(\frac{x}{b}\right) dx = \\
= \frac{(-1)^n}{2(2n+\varepsilon)!} b^{\alpha+2\lambda+r\beta-r-1} (2\lambda)_{2n+\varepsilon} \Gamma\left(\lambda + \frac{1}{2}\right) \times \\
\times \sum_{k=0}^{\infty} \frac{(1-\beta)_k}{k!} \Gamma\left[\frac{(\alpha+\varepsilon+r\beta-rk-r)/2}{(\alpha+\varepsilon+r\beta-rk-r+1)/2+\lambda+n}\right] \times \\
\times \left(\frac{\varepsilon+r(k-\beta+1)-\alpha+1}{2}\right)_n \left(\frac{a}{b}\right)^{rk} + \frac{(-1)^n 2^{2n+\varepsilon}}{(2n+\varepsilon)! r} a^{\alpha+\varepsilon+\beta r-r} \times \\
\times b^{2\lambda-\varepsilon-1} (\lambda)_{n+\varepsilon} \Gamma(\beta) \sum_{k=0}^{\infty} \frac{(1/2-n-\lambda)_k}{k!} \times \\
\times \Gamma\left[\frac{1-\beta-(\alpha+\varepsilon+2k)/r}{1-(\alpha+\varepsilon+2k)/r}\right] \left(\frac{1}{2}+\varepsilon+k\right)_n \left(\frac{a}{b}\right)^{2k} \\
[\varepsilon = 0 \text{ или } 1; b > a > 0; r, \operatorname{Re} \beta > 0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
5. \int_a^\infty x^{\alpha-1} (x^r - a^r)^{\beta-1} (x^2 - b^2)^{\lambda-1/2} C_{2n+\varepsilon}^\lambda \left(\frac{x}{b}\right) dx = \\
= \frac{2^{2n+\varepsilon} a^{\alpha+\varepsilon+2\lambda+r\beta-r+2n-1}}{(2n+\varepsilon)! b^{2n+\varepsilon} r} (\lambda)_{n+\varepsilon} \Gamma(\beta) \times \\
\times \sum_{k=0}^{\infty} \frac{(1/2-n-\lambda)_k}{k!} \Gamma\left[\frac{1-\beta-(\alpha+\varepsilon+2\lambda+2n-2k-1)/r}{1-(\alpha+\varepsilon+2\lambda+2n-2k-1)/r}\right] (\varepsilon+\lambda+n-k)_n \left(\frac{b}{a}\right)^{2k} \\
[\varepsilon = 0 \text{ или } 1; a > b > 0; r, \operatorname{Re} \beta > 0; \operatorname{Re}(\alpha + 2\lambda + r\beta) < r - \varepsilon - 2n + 1].
\end{aligned}$$

$$\begin{aligned}
6. \int_a^\infty x^{\alpha-1} (x^2 - a^2)^{\lambda-1/2} (x^r - b^r)^{\beta-1} C_{2n+\varepsilon}^\lambda \left(\frac{x}{a}\right) dx = \\
= \frac{a^{\alpha+2\lambda+r\beta-r-1}}{2(2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \Gamma\left(\lambda + \frac{1}{2}\right) \sum_{k=0}^{\infty} \frac{(1-\beta)_k}{k!} \times \\
\times \Gamma\left[\frac{(1-\alpha-\varepsilon-r\beta+r+rk)/2-\lambda-n}{1+(r+rk-\alpha-\varepsilon-r\beta)/2}\right] \left(\frac{1-\alpha+\varepsilon+r+rk-r\beta}{2}\right)_n \left(\frac{b}{a}\right)^{rk} \\
[\varepsilon = 0 \text{ или } 1; a > b > 0, r > 0; \operatorname{Re} r > -1/2; \operatorname{Re}(\alpha + 2\lambda + r\beta) < r - \varepsilon - 2n + 1]
\end{aligned}$$

$$7. \int_0^a \frac{x^{\alpha-1}}{(x^r+z^r)^\rho} (a^2-x^2)^{\lambda-1/2} C_{2n+\varepsilon}^\lambda \left(\frac{x}{a}\right) dx = I$$

$$[\varepsilon = 0 \text{ или } 1; a, r > 0; \operatorname{Re} \alpha > -\varepsilon; \operatorname{Re} \lambda > -1/2; r | \arg z| < \pi]$$

$$I = \frac{(-1)^n}{2(2n+\varepsilon)!} a^{\alpha+2\lambda-1} z^{-r\rho} (2\lambda)_{2n+\varepsilon} \Gamma\left(\lambda + \frac{1}{2}\right) \times \\ \times \sum_{k=0}^{\infty} \frac{(-1)^k (\rho)_k}{k!} \Gamma\left[\frac{(\alpha+\varepsilon+rk)/2}{(\alpha+\varepsilon+rk+1)/2+\lambda+n}\right] \left(\frac{\varepsilon-\alpha-rk+1}{2}\right)_n \left(\frac{a}{z}\right) \\ [|z| > a],$$

$$I = \frac{(-1)^n 2^{2n+\varepsilon} a^{2\lambda-\varepsilon-1} z^{\alpha+\varepsilon-r\rho}}{(2n+\varepsilon)! r \Gamma(\rho)} (\lambda)_{n+\varepsilon} \sum_{k=0}^{\infty} \frac{(1/2-\lambda-n)_k}{k!} \times \\ \times \Gamma\left(\frac{\alpha+\varepsilon+2k}{r}\right) \Gamma\left(\rho - \frac{\alpha+\varepsilon+2k}{r}\right) \left(\varepsilon+k+\frac{1}{2}\right)_n \left(\frac{z}{a}\right)^{2k} + \\ + \frac{(-1)^n}{2(2n+\varepsilon)!} a^{\alpha+2\lambda-r\rho-1} (2\lambda)_{2n+\varepsilon} \Gamma\left(\lambda + \frac{1}{2}\right) \times \\ \times \sum_{k=0}^{\infty} \frac{(-1)^k (\rho)_k}{k!} \Gamma\left[\frac{(\alpha+\varepsilon-rk-r\rho)/2}{(\alpha+\varepsilon-rk-r\rho+1)/2+\lambda+n}\right] \left(\frac{\varepsilon-\alpha+rk+r\rho+1}{2}\right)_n \left(\frac{z}{a}\right)^r \\ [|z| < a].$$

$$8. \int_a^\infty \frac{x^{\alpha-1}}{(x^r+z^r)^\rho} (x^2-a^2)^{\lambda-1/2} C_{2n+\varepsilon}^\lambda \left(\frac{x}{a}\right) dx = I$$

$$[\varepsilon = 0 \text{ или } 1; a, r > 0; \operatorname{Re} \lambda > -1/2; \operatorname{Re}(\alpha+2\lambda-r\rho) < 1-2n-\varepsilon; r | \arg z| < \pi],$$

$$I = \frac{a^{\alpha+2\lambda-1}}{2(2n+\varepsilon)! z^{r\rho}} (2\lambda)_{2n+\varepsilon} \Gamma\left(\lambda + \frac{1}{2}\right) \sum_{k=0}^{\infty} \frac{(-1)^k (\rho)_k}{k!} \times \\ \times \Gamma\left[\frac{(1-\alpha-\varepsilon-rk)/2-\lambda-n}{1-(\alpha+\varepsilon+rk)/2}\right] \left(\frac{1-\alpha+\varepsilon-rk}{2}\right)_n \left(\frac{a}{z}\right)^{rk} + \\ + \frac{2^{2n+\varepsilon} z^{\alpha+\varepsilon+2\lambda-r\rho+2n-1}}{(2n+\varepsilon)! a^{2n+\varepsilon} r \Gamma(\rho)} (\lambda)_{n+\varepsilon} \sum_{k=0}^{\infty} \frac{(1/2-n-\lambda)_k}{k!} \Gamma\left(\frac{\alpha+\varepsilon+2\lambda+2n-2k-1}{r}\right) \times \\ \times \Gamma\left(\rho - \frac{\alpha+\varepsilon+2\lambda+2n-2k-1}{r}\right) (\lambda+\varepsilon+n-k)_n \left(\frac{a}{z}\right)^{2k} \\ [|z| > a],$$

$$I = \frac{a^{\alpha+2\lambda-r\rho-1}}{2(2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \Gamma\left(\lambda + \frac{1}{2}\right) \sum_{k=0}^{\infty} \frac{(-1)^k (\rho)_k}{k!} \times \\ \times \Gamma\left[\frac{(1-\alpha-\varepsilon+r\rho+rk)/2-\lambda-n}{1+(rk+r\rho-\alpha-\varepsilon)/2}\right] \left(\frac{1-\alpha+\varepsilon+rk+r\rho}{2}\right)_n \left(\frac{z}{a}\right)^r \\ [|z| < a].$$

$$9. \int_0^a \frac{x^{\alpha-1}}{x^r-y^r} (a^2-x^2)^{\lambda-1/2} C_{2n+\varepsilon}^\lambda \left(\frac{x}{a}\right) dx = I$$

$$[\varepsilon = 0 \text{ или } 1; a, r, y > 0; \operatorname{Re} \alpha > -\varepsilon; \operatorname{Re} \lambda > -1/2],$$

$$I = \frac{(-1)^{n+1}}{2(2n+\varepsilon)!} a^{\alpha+2\lambda-1} y^{-r} (2\lambda)_{2n+\varepsilon} \Gamma\left(\lambda + \frac{1}{2}\right) \times \\ \times \sum_{k=0}^{\infty} \Gamma\left[\frac{(\alpha+\varepsilon+rk)/2}{(\alpha+\varepsilon+rk+1)/2+\lambda+n}\right] \left(\frac{\varepsilon-\alpha-rk+1}{2}\right)_n \left(\frac{a}{y}\right)^{rk} \\ [y > a].$$

$$I = \frac{(-1)^{n+1} 2^{2n+\varepsilon} \pi}{(2n+\varepsilon)! r} a^{2\lambda-\varepsilon-1} y^{\alpha+\varepsilon-r} (\lambda)_{n+\varepsilon} \sum_{k=0}^{\infty} \frac{(1/2-\lambda-n)_k}{k!} \operatorname{ctg} \left(\frac{\alpha+\varepsilon+2k}{r} \pi \right) \times$$

$$\times \left(\frac{1}{2} + \varepsilon + k \right)_n \left(\frac{y}{a} \right)^{2k} + \frac{(-1)^n}{2 (2n+\varepsilon)!} a^{\alpha+2\lambda-r-1} (2\lambda)_{2n+\varepsilon} \Gamma \left(\lambda + \frac{1}{2} \right) \times$$

$$\times \sum_{k=0}^{\infty} \Gamma \left[\frac{(\alpha+\varepsilon-rk-r)/2}{(\alpha+\varepsilon-rk-r+1)/2 + \lambda + n} \right] \left(\frac{\varepsilon-\alpha+rk+r+1}{2} \right)_n \left(\frac{y}{a} \right)^{rk}$$

[y < a].

$$10. \int_a^{\infty} \frac{x^{\alpha-1}}{x^r - y^r} (x^2 - a^2)^{\lambda-1/2} C_{2n+\varepsilon}^{\lambda} \left(\frac{x}{a} \right) dx = I$$

[ε = 0 или 1; a, r, y > 0; Re λ > -1/2; Re (α + 2λ) < r - ε - 2n + 1].

$$I = -\frac{a^{\alpha+2\lambda-1}}{2 (2n+\varepsilon)! y^r} (2\lambda)_{2n+\varepsilon} \Gamma \left(\lambda + \frac{1}{2} \right) \times$$

$$\times \sum_{k=0}^{\infty} \Gamma \left[\frac{(1-\alpha-\varepsilon-rk)/2 - \lambda - n}{1 - (\alpha+\varepsilon+rk)/2} \right] \left(\frac{\varepsilon-\alpha-rk+1}{2} \right)_n \left(\frac{a}{y} \right)^{rk} -$$

$$- \frac{2^{2n+\varepsilon} \pi y^{\alpha+\varepsilon+2\lambda-r+2n-1}}{(2n+\varepsilon)! a^{2n+\varepsilon} r} (\lambda)_{n+\varepsilon} \sum_{k=0}^{\infty} \frac{(1/2-\lambda-n)_k}{k!} \times$$

$$\times (\varepsilon + \lambda + n - k)_n \operatorname{ctg} \left(\frac{\alpha+\varepsilon+2\lambda+2n-2k-1}{r} \pi \right) \left(\frac{a}{y} \right)^{2k}$$

[y > a].

$$I = \frac{a^{\alpha+2\lambda-r-1}}{2 (2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \Gamma \left(\lambda + \frac{1}{2} \right) \times$$

$$\times \sum_{k=0}^{\infty} \Gamma \left[\frac{(1-\alpha-\varepsilon+rk+r)/2 - \lambda - n}{1 + (rk+r-\alpha-\varepsilon)/2} \right] \left(\frac{\varepsilon-\alpha+rk+r+1}{2} \right)_n \left(\frac{y}{a} \right)^{rk}$$

[y < a].

$$11. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\lambda-1/2} e^{-px^r} C_{2n+\varepsilon}^{\lambda} \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^n}{2 (2n+\varepsilon)!} a^{\alpha+2\lambda-1} (2\lambda)_{2n+\varepsilon} \Gamma \left(\lambda + \frac{1}{2} \right) \sum_{k=0}^{\infty} \frac{1}{k!} \times$$

$$\times \Gamma \left[\frac{(\alpha+\varepsilon+rk)/2}{(\alpha+\varepsilon+rk+1)/2 + \lambda + n} \right] \left(\frac{\varepsilon-\alpha-rk+1}{2} \right)_n (-a^r p)^k$$

[ε = 0 или 1; a, r > 0; Re α > -ε; Re λ > -1/2].

$$12. \int_a^{\infty} x^{\alpha-1} (x^2 - a^2)^{\lambda-1/2} e^{-px^r} C_{2n+\varepsilon}^{\lambda} \left(\frac{x}{a} \right) dx =$$

$$= \frac{a^{\alpha+2\lambda-1}}{2 (2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \Gamma \left(\lambda + \frac{1}{2} \right) \sum_{k=0}^{\infty} \frac{1}{k!} \times$$

$$\times \Gamma \left[\frac{(1-\alpha-\varepsilon-rk)/2 - \lambda - n}{1 - (\alpha+\varepsilon+rk)/2} \right] \left(\frac{1-\alpha+\varepsilon-rk}{2} \right)_n (-a^r p)^k +$$

$$+ \frac{2^{2n+\varepsilon} p^{(1-\alpha-\varepsilon-2\lambda-2n)/r}}{(2n+\varepsilon)! a^{2n+\varepsilon} r} (\lambda)_{n+\varepsilon} \times$$

$$\times \sum_{k=0}^{\infty} \frac{(1/2-\lambda-n)_k}{k!} (\lambda + \varepsilon + n - k)_n \Gamma \left(\frac{\alpha+\varepsilon+2\lambda+2n-2k-1}{r} \right) (a p^{1/r})^{2k}$$

[ε = 0 или 1; a, r, Re p > 0; Re λ > -1/2].

$$\begin{aligned}
13. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\lambda-1/2} e^{-px^{-r}} C_{2n+\varepsilon}^{\lambda} \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^n}{2(2n+\varepsilon)!} a^{\alpha+2\lambda-1} (2\lambda)_{2n+\varepsilon} \Gamma\left(\lambda + \frac{1}{2}\right) \sum_{k=0}^{\infty} \frac{1}{k!} \times \\
\times \Gamma\left[\frac{(\alpha+\varepsilon-rk)/2}{(\alpha+\varepsilon-rk+1)/2 + \lambda + n}\right] \left(\frac{\varepsilon-\alpha+rk+1}{2}\right)_n \left(-\frac{p}{a^r}\right)^k + \\
+ \frac{(-1)^n 2^{2n+\varepsilon}}{(2n+\varepsilon)! r} a^{2\lambda-\varepsilon-1} p^{(\alpha+\varepsilon)/r} (\lambda)_{n+\varepsilon} \times \\
\times \sum_{k=0}^{\infty} \frac{(1/2-\lambda-n)_k}{k!} \Gamma\left(-\frac{\alpha+\varepsilon+2k}{r}\right) \left(\frac{1}{2} + \varepsilon + k\right)_n \left(\frac{p^{1/r}}{a}\right)^{2k} \\
[\varepsilon = 0 \text{ или } 1; a, r, \operatorname{Re} p > 0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
14. \int_a^{\infty} x^{\alpha-1} (x^2 - a^2)^{\lambda-1/2} e^{-px^{-r}} C_{2n+\varepsilon}^{\lambda} \left(\frac{x}{a} \right) dx = \\
= \frac{a^{\alpha+2\lambda-1}}{2(2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \Gamma\left(\lambda + \frac{1}{2}\right) \sum_{k=0}^{\infty} \frac{1}{k!} \times \\
\times \Gamma\left[\frac{(1-\alpha-\varepsilon+rk)/2 - \lambda - n}{1 - (\alpha+\varepsilon-rk)/2}\right] \left(\frac{\varepsilon-\alpha+rk+1}{2}\right)_n \left(-\frac{p}{a^r}\right)^k \\
[\varepsilon = 0 \text{ или } 1; a, r > 0; -1/2 < \operatorname{Re} \lambda < (1 - \operatorname{Re} \alpha - \varepsilon - 2n)/2].
\end{aligned}$$

$$\begin{aligned}
15. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\lambda-1/2} \begin{Bmatrix} \sin bx^r \\ \cos bx^r \end{Bmatrix} C_{2n+\varepsilon}^{\lambda} \left(\frac{a}{x} \right) dx = \\
= \frac{(-1)^n}{2(2n+\varepsilon)!} a^{\alpha+2\lambda+0r-1} b^{\delta} (2\lambda)_{2n+\varepsilon} \Gamma\left(\lambda + \frac{1}{2}\right) \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (\delta+1/2)_k} \times \\
\times \Gamma\left[\frac{rk + (\alpha+\varepsilon+r\delta)/2}{\lambda + rk + n + (\alpha+\varepsilon+\delta r+1)/2}\right] \left(\frac{\varepsilon-\alpha-\delta r+1}{2} - rk\right)_n \left(\frac{a^r b}{2}\right)^{2k} \\
[\varepsilon = 0 \text{ или } 1; a, r > 0; \operatorname{Re} \alpha > -\varepsilon - \delta r; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
16. \int_a^{\infty} x^{\alpha-1} (x^2 - a^2)^{\lambda-1/2} \begin{Bmatrix} \sin bx^r \\ \cos bx^r \end{Bmatrix} C_{2n+\varepsilon}^{\lambda} \left(\frac{x}{a} \right) dx = \\
= \frac{a^{\alpha+2\lambda+\delta r-1}}{2(2n+\varepsilon)!} b^{\delta} (2\lambda)_{2n+\varepsilon} \Gamma\left(\lambda + \frac{1}{2}\right) \times \\
\times \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (1/2+\delta)_k} \Gamma\left[\frac{(1-\alpha-\varepsilon-\delta r)/2 - \lambda - rk - n}{1 - (\alpha+\varepsilon+\delta r)/2 - rk}\right] \times \\
\times \left(\frac{\varepsilon-\alpha-\delta r+1}{2} - rk\right)_n \left(\frac{a^r b}{2}\right)^{2k} + \frac{2^{2n+\varepsilon}}{(2n+\varepsilon)! a^{2n+\varepsilon r}} \times \\
\times b^{(1-\alpha-\varepsilon-2\lambda-2n)/r} (\lambda)_{n+\varepsilon} \sum_{k=0}^{\infty} \frac{(1/2-\lambda-n)_k}{k!} (\varepsilon + \lambda + n - k)_n \times \\
\times \Gamma\left(\frac{\alpha+\varepsilon+2\lambda+2n-2k-1}{r}\right) \begin{Bmatrix} \sin [(\alpha+\varepsilon+2\lambda+2n-2k-1)\pi/(2r)] \\ \cos [(\alpha+\varepsilon+2\lambda+2n-2k-1)\pi/(2r)] \end{Bmatrix} (ab^{1/r})^{2k} \\
[\varepsilon = 0 \text{ или } 1; a, b, r > 0; -1/2 < \operatorname{Re} \lambda < (r - \operatorname{Re} \alpha - \varepsilon - 2n + 1)/2].
\end{aligned}$$

$$17. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\lambda-1/2} \left\{ \frac{\ln(x^r + z^r)}{\ln|x^r - z^r|} \right\} C_{2n+\varepsilon}^{\lambda} \left(\frac{x}{a} \right) dx = I$$

$$\left[\varepsilon = 0 \text{ или } 1; a, r > 0; \operatorname{Re} \alpha > -\varepsilon; \operatorname{Re} \lambda > -1/2, \left\{ \begin{array}{l} r |\arg z| < \pi \\ z > 0 \end{array} \right\} \right],$$

$$I = \pm \frac{(-1)^n}{2(2n+\varepsilon)!} a^{\alpha+2\lambda+r-1} z^{-r} (2\lambda)_{2n+\varepsilon} \Gamma\left(\lambda + \frac{1}{2}\right) \times \\ \times \sum_{k=0}^{\infty} \frac{(\mp 1)^k}{k+1} \Gamma\left[\frac{(\alpha+\varepsilon+r+rk)/2}{(\alpha+\varepsilon+r+rk+1)/2+\lambda+n}\right] \left(\frac{\varepsilon-\alpha-r-rk+1}{2}\right)_n \left(\frac{a}{z}\right)^{rk} + \\ + \frac{(-1)^n a^{\alpha+2\lambda-1} r}{2(2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{\varepsilon-\alpha+1}{2}\right)_n \Gamma\left[\frac{(\alpha+\varepsilon)/2, \lambda+1/2}{(\alpha+\varepsilon+1)/2+\lambda+n}\right] \ln z \\ [|z| > a],$$

$$I = \pm \frac{(-1)^n}{2(2n+\varepsilon)!} a^{\alpha+2\lambda-r-1} z^r (2\lambda)_{2n+\varepsilon} \Gamma\left(\lambda + \frac{1}{2}\right) \times \\ \times \sum_{k=0}^{\infty} \frac{(\mp 1)^k}{k+1} \Gamma\left[\frac{(\alpha+\varepsilon-r-rk)/2}{(\alpha+\varepsilon-r-rk+1)/2+\lambda+n}\right] \left(\frac{\varepsilon-\alpha+r+rk+1}{2}\right)_n \left(\frac{z}{a}\right)^{rk} + \\ + \frac{(-1)^n a^{\alpha+2\lambda-1} r}{4(2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \Gamma\left[\frac{(\alpha+\varepsilon)/2, \lambda+1/2}{(\alpha+\varepsilon+1)/2+\lambda+n}\right] \left(\frac{\varepsilon-\alpha+1}{2}\right)_n \times \\ \times \left[\psi\left(\frac{\alpha+\varepsilon}{2}\right) - \psi\left(\frac{\alpha+\varepsilon+1}{2} + \lambda + n\right) - \sum_{j=0}^{n-1} \frac{2}{2j-\alpha+\varepsilon+1} + 2 \ln a \right] + \\ + \frac{(-1)^n 2^{2n+\varepsilon} \pi a^{2\lambda-\varepsilon-1} z^{\alpha+\varepsilon}}{(2n+\varepsilon)!} (\lambda)_{n+\varepsilon} \sum_{k=0}^{\infty} \frac{(1/2-\lambda-n)_k}{k! (\alpha+\varepsilon+2k)} \times \\ \times \left(\frac{1}{2} + \varepsilon + k\right)_n \left\{ \operatorname{cosec}[(\alpha+\varepsilon+2k)\pi/r] \right\} \left(\frac{z}{a}\right)^{2k} \\ [|z| < a].$$

$$18. \int_a^{\infty} x^{\alpha-1} (x^2 - a^2)^{\lambda-1/2} \left\{ \frac{\ln(x^r + z^r)}{\ln|x^r - z^r|} \right\} C_{2n+\varepsilon}^{\lambda} \left(\frac{x}{a} \right) dx = I$$

$$\left[\varepsilon = 0 \text{ или } 1; a, r > 0; -1/2 < \operatorname{Re} \lambda < (1 - \operatorname{Re} \alpha - \varepsilon - 2n)/2, \left\{ \begin{array}{l} r |\arg z| < \pi \\ z > 0 \end{array} \right\} \right],$$

$$I = \pm \frac{a^{\alpha+2\lambda+r-1}}{2(2n+\varepsilon)! z^r} (2\lambda)_{2n+\varepsilon} \Gamma\left(\lambda + \frac{1}{2}\right) \sum_{k=0}^{\infty} \frac{(\mp 1)^k}{k+1} \times \\ \times \Gamma\left[\frac{(1-\alpha-\varepsilon-r-rk)/2-\lambda-n}{1-(\alpha+\varepsilon+rk+r)/2}\right] \left(\frac{\varepsilon-\alpha-r-rk+1}{2}\right)_n \left(\frac{a}{z}\right)^{rk} + \\ + \frac{2^{2n+\varepsilon} \pi}{(2n+\varepsilon)! a^{2n+\varepsilon}} z^{\alpha+\varepsilon+2\lambda+2n-1} (\lambda)_{n+\varepsilon} \sum_{k=0}^{\infty} \frac{(1/2-\lambda-n)_k (\varepsilon+\lambda+n-k)_n}{k! (2k-\alpha-\varepsilon-2\lambda-2n+1)} \times \\ \times \left\{ \operatorname{cosec}[(1-\alpha-\varepsilon-2\lambda-2n+2k)\pi/r] \right\} \left(\frac{a}{z}\right)^{2k} + \frac{a^{\alpha+2\lambda-1} r}{2(2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \times \\ \times \left(\frac{\varepsilon-\alpha+1}{2}\right)_n \Gamma\left[\frac{\lambda+1/2, (1-\alpha-\varepsilon)/2-\lambda-n}{1-(\alpha+\varepsilon)/2}\right] \ln z \\ [|z| > a].$$

$$I = \pm \frac{a^{2+2\lambda-r-1} z^r}{2(2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \Gamma\left(\lambda + \frac{1}{2}\right) \sum_{k=0}^{\infty} \frac{(\mp 1)^k}{k+1} \times$$

$$\times \Gamma\left[\frac{(1-\alpha-\varepsilon+r+rk)/2-\lambda-n}{1-(\alpha+\varepsilon-r-rk)/2}\right] \left(\frac{1-\alpha+\varepsilon+r+rk}{2}\right)_n \left(\frac{z}{a}\right)^{rk} -$$

$$- \frac{a^{\alpha+2\lambda-1} r}{4(2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{1-\alpha+\varepsilon}{2}\right)_n \Gamma\left[\lambda+1/2, \frac{(1-\alpha-\varepsilon)/2-\lambda-n}{1-(\alpha+\varepsilon)/2}\right] \times$$

$$\times \left[\psi\left(\frac{1-\alpha-\varepsilon}{2}-\lambda-n\right) - \psi\left(1-\frac{\alpha+\varepsilon}{2}\right) + \sum_{j=0}^{n-1} \frac{2}{2j-\alpha+\varepsilon+1} - 2 \ln a \right]$$

[|z| < a].

$$19. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\lambda-1/2} \text{Ei}(-bx^r) C_{2n+\varepsilon}^{\lambda} \left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^{n+1}}{2(2n+\varepsilon)!} a^{\alpha+2\lambda+r-1} b (2\lambda)_{2n+\varepsilon} \Gamma\left(\lambda + \frac{1}{2}\right) \times$$

$$\times \sum_{k=0}^{\infty} \frac{1}{k!(k+1)^2} \Gamma\left[\frac{(\alpha+\varepsilon+r+rk)/2}{(\alpha+\varepsilon+r+rk+1)/2+\lambda+n}\right] \times$$

$$\times \left(\frac{\varepsilon-\alpha-r-rk+1}{2}\right)_n (-a^r b)^k + \frac{(-1)^n}{2(2n+\varepsilon)!} a^{\alpha+2\lambda-1} \times$$

$$\times (2\lambda)_{2n+\varepsilon} \Gamma\left[\frac{(\alpha+\varepsilon)/2, \lambda+1/2}{(\alpha+\varepsilon+1)/2+\lambda+n}\right] \left(\frac{\varepsilon-\alpha+1}{2}\right)_n \left[C + \frac{r}{2} \psi\left(\frac{\alpha+\varepsilon}{2}\right) - \right.$$

$$\left. - \frac{r}{2} \psi\left(\frac{\alpha+\varepsilon+1}{2}+\lambda+n\right) - \sum_{j=0}^{n-1} \frac{r}{2j-\alpha+\varepsilon+1} + \ln(a^r b) \right]$$

[$\varepsilon=0$ или 1 ; $a, r > 0$; $\text{Re } \alpha > -\varepsilon$; $\text{Re } \lambda > -1/2$; $|\arg b| < \pi$].

$$20. \int_a^{\infty} x^{\alpha-1} (x^2 - a^2)^{\lambda-1/2} \text{Ei}(-bx^r) C_{2n+\varepsilon}^{\lambda} \left(\frac{x}{a}\right) dx =$$

$$= - \frac{a^{\alpha+2\lambda+r-1} b}{2(2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \Gamma\left(\lambda + \frac{1}{2}\right) \sum_{k=0}^{\infty} \frac{1}{k!(k+1)^2} \times$$

$$\times \Gamma\left[\frac{(1-\alpha-\varepsilon-r-rk)/2-\lambda-n}{1-(\alpha+\varepsilon+r+rk)/2}\right] \left(\frac{\varepsilon-\alpha-r-rk+1}{2}\right)_n (-a^r b)^k +$$

$$+ \frac{a^{\alpha+2\lambda-1}}{2(2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \Gamma\left[\lambda+1/2, \frac{(1-\alpha-\varepsilon)/2-\lambda-n}{1-(\alpha+\varepsilon)/2}\right] \left(\frac{\varepsilon-\alpha+1}{2}\right)_n \times$$

$$\times \left[C - \frac{r}{2} \psi\left(\frac{1-\alpha-\varepsilon}{2}-\lambda-n\right) + \frac{r}{2} \psi\left(1-\frac{\alpha+\varepsilon}{2}\right) - \sum_{j=0}^{n-1} \frac{r}{2j-\alpha+\varepsilon+1} + \ln(a^r b) \right] +$$

$$+ \frac{2^{2n+\varepsilon} b^{(1-\alpha-\varepsilon-2\lambda-2n)/r}}{(2n+\varepsilon)! a^{2n+\varepsilon}} (\lambda)_{n+\varepsilon} \times$$

$$\times \sum_{k=0}^{\infty} \frac{(1/2-\lambda-n)_k (\varepsilon+\lambda+n-k)_n}{k! (2k-\alpha-\varepsilon-2\lambda-2n+1)} \Gamma\left(\frac{\alpha+\varepsilon+2\lambda+2n-2k-1}{r}\right) (ab^{1/r})^{2k}$$

[$\varepsilon=0$ или 1 ; $a, r, \text{Re } b > 0$; $\text{Re } \lambda > -1/2$].

$$\begin{aligned}
21. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\lambda-1/2} \begin{Bmatrix} \text{si}(bx^r) \\ \text{ci}(bx^r) \end{Bmatrix} C_{2n+\varepsilon}^{\lambda} \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^{n+1}}{4 \cdot 3^{\delta} (2n+\varepsilon)!} a^{\alpha+2\lambda+\delta r+2r-1} b^{\delta+2} (2\lambda)_{2n+\varepsilon} \Gamma\left(\lambda + \frac{1}{2}\right) \times \\
\times \sum_{k=0}^{\infty} \frac{(-1)^k}{(k+1)! (\delta+3/2)_k (2k+\delta+2)} \Gamma\left[\frac{r+rk+(\alpha+\varepsilon+\delta r)/2}{\lambda+r+rk+n+(\alpha+\varepsilon+\delta r+1)/2}\right] \times \\
\times \left(\frac{\varepsilon-\alpha-\delta r+1}{2} - r - rk\right)_n \left(\frac{a^r b}{2}\right)^{2k} + \frac{(-1)^n}{2 (2n+\varepsilon)!} a^{\alpha+2\lambda+\delta r-1} b^{\delta} \times \\
\times (2\lambda)_{2n+\varepsilon} \Gamma\left[\frac{(\alpha+\varepsilon+\delta r)/2, \lambda+1/2}{(\alpha+\varepsilon+\delta r+1)/2+\lambda+n}\right] \left(\frac{\varepsilon-\alpha-\delta r+1}{2}\right)_n \left\{R\right\} - \\
- \delta \frac{(-1)^n \pi}{4 (2n+\varepsilon)!} a^{\alpha+2\lambda-1} (2\lambda)_{2n+\varepsilon} \Gamma\left[\frac{(\alpha+\varepsilon)/2, \lambda+1/2}{(\alpha+\varepsilon+1)/2+\lambda+n}\right] \left(\frac{\varepsilon-\alpha+1}{2}\right)_n, \\
R = C + \frac{r}{2} \psi\left(\frac{\alpha+\varepsilon}{2}\right) - \frac{r}{2} \psi\left(\frac{\alpha+\varepsilon+1}{2} + \lambda + n\right) - \sum_{j=0}^{n-1} \frac{r}{2j-\alpha+\varepsilon+1} + \ln(a^r b) \\
[\varepsilon = 0 \text{ или } 1; a, r > 0; \text{Re } \alpha > -\varepsilon; \text{Re } \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
22. \int_a^{\infty} x^{\alpha-1} (x^2 - a^2)^{\lambda-1/2} \begin{Bmatrix} \text{si}(bx^r) \\ \text{ci}(bx^r) \end{Bmatrix} C_{2n+\varepsilon}^{\lambda} \left(\frac{x}{a} \right) dx = \\
= - \frac{a^{\alpha+2\lambda+\delta r+2r-1} b^{\delta+2}}{3^{\delta} \cdot 4 (2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \Gamma\left(\lambda + \frac{1}{2}\right) \times \\
\times \sum_{k=0}^{\infty} \frac{(-1)^k}{(k+1)! (\delta+3/2)_k (2k+\delta+2)} \Gamma\left[\frac{(1-\alpha-\varepsilon-\delta r)/2 - \lambda - r - rk - n}{1 - (\alpha+\varepsilon+\delta r)/2 - r - rk}\right] \times \\
\times \left(\frac{\varepsilon-\alpha-\delta r+1}{2} - r - rk\right)_n \left(\frac{a^r b}{2}\right)^{2k} + \frac{2^{2n+\varepsilon} b^{(1-\alpha-\varepsilon-2\lambda-2n)/r}}{(2n+\varepsilon)! a^{2n+\varepsilon}} (\lambda)_{n+\varepsilon} \times \\
\times \sum_{k=0}^{\infty} \frac{(1/2-\lambda-n)_k (\varepsilon+\lambda+n-k)_n}{k! (2k-\alpha-\varepsilon-2\lambda-2n+1)} \Gamma\left(\frac{\alpha+\varepsilon+2\lambda+2n-2k-1}{r}\right) \times \\
\times \begin{Bmatrix} \sin[(\alpha+\varepsilon+2\lambda+2n-2k-1)\pi/(2r)] \\ \cos[(\alpha+\varepsilon+2\lambda+2n-2k-1)\pi/(2r)] \end{Bmatrix} (ab^{1/r})^{2k} + \\
+ \frac{a^{\alpha+2\lambda+\delta r-1} b^{\delta}}{2 (2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \Gamma\left[\frac{\lambda+1/2, (1-\alpha-\varepsilon-\delta r)/2 - \lambda - n}{1 - (\alpha+\varepsilon+\delta r)/2}\right] \times \\
\times \left(\frac{1-\alpha+\varepsilon-\delta r}{2}\right)_n \left\{R\right\} - \delta \frac{a^{\alpha+2\lambda-1} \pi}{4 (2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \times \\
\times \Gamma\left[\frac{\lambda+1/2, (1-\alpha-\varepsilon)/2 - \lambda - n}{1 - (\alpha+\varepsilon)/2}\right] \left(\frac{1-\alpha+\varepsilon}{2}\right)_n, \\
R = C - \frac{r}{2} \psi\left(\frac{1-\alpha-\varepsilon}{2} - \lambda - n\right) + \frac{r}{2} \psi\left(1 - \frac{\alpha+\varepsilon}{2}\right) - \sum_{j=0}^{n-1} \frac{r}{2j-\alpha+\varepsilon+1} + \ln(a^r b) \\
[\varepsilon = 0 \text{ или } 1; a, b, r > 0; -1/2 < \text{Re } \lambda < (2r - \text{Re } \alpha - \varepsilon - 2n + 1)/2].
\end{aligned}$$

$$\begin{aligned}
23. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\lambda-1/2} \begin{Bmatrix} \text{erfi}(bx^r) \\ \text{erfc}(bx^r) \end{Bmatrix} C_{2n+\varepsilon}^{\lambda} \left(\frac{x}{a} \right) dx = \\
= \pm \frac{(-1)^n}{(2n+\varepsilon)! \sqrt{\pi}} a^{\alpha+2\lambda+r-1} b (2\lambda)_{2n+\varepsilon} \Gamma\left(\lambda + \frac{1}{2}\right) \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (2k+1)} \times
\end{aligned}$$

$$\times \Gamma \left[\frac{rk + (\alpha + \varepsilon + r)/2}{\lambda + rk + n + (\alpha + \varepsilon + r + 1)/2} \right] \left(\frac{\varepsilon - \alpha - r + 1}{2} - rk \right)_n (a^r b)^{2k} +$$

$$+ \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \frac{(-1)^n}{2(2n + \varepsilon)!} a^{\alpha + 2\lambda - 1} (2\lambda)_{2n + \varepsilon} \Gamma \left[\frac{(\alpha + \varepsilon)/2, \lambda + 1/2}{(\alpha + \varepsilon + 1)/2 + \lambda + n} \right] \left(\frac{\varepsilon - \alpha + 1}{2} \right)_n$$

[$\varepsilon = 0$ или 1 ; $a, r > 0$; $\operatorname{Re} \lambda > -1/2$, $\operatorname{Re} \alpha > -r(1 \pm 1)/2 - \varepsilon$].

$$24. \int_a^\infty x^{\alpha-1} (x^2 - a^2)^{\lambda-1/2} \begin{Bmatrix} \operatorname{erf}(bx^r) \\ \operatorname{erfc}(bx^r) \end{Bmatrix} C_{2n+\varepsilon}^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \pm \frac{a^{\alpha+2\lambda+r-1} b}{\sqrt{\pi} (2n + \varepsilon)!} (2\lambda)_{2n + \varepsilon} \Gamma \left(\lambda + \frac{1}{2} \right) \sum_{k=0}^\infty \frac{(-1)^k}{k! (2k + 1)} \times$$

$$\times \Gamma \left[\frac{(1 - \alpha - \varepsilon - r)/2 - rk - \lambda - n}{1 - (\alpha + \varepsilon + r)/2 - rk} \right] \left(\frac{\varepsilon - \alpha - r + 1}{2} - rk \right)_n (a^r b)^{2k} \pm$$

$$\pm \frac{2^{2n+\varepsilon} b^{(1-\varepsilon-\alpha-2\lambda-2n)/r} (\lambda)_{n+\varepsilon}}{\sqrt{\pi} (2n + \varepsilon)! a^{2n+\varepsilon}} \sum_{k=0}^\infty \frac{(1/2 - \lambda - n)_k (\varepsilon + \lambda + n - k)_n}{k! (2k - \alpha - \varepsilon - 2\lambda - 2n + 1)} \times$$

$$\times \Gamma \left(\frac{\alpha + \varepsilon + 2\lambda - 2k + 2n + r - 1}{2r} \right) (a^r b)^{2k/r} +$$

$$+ \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \frac{a^{\alpha+2\lambda-1}}{2(2n + \varepsilon)!} (2\lambda)_{2n + \varepsilon} \Gamma \left[\frac{\lambda + 1/2, (1 - \alpha - \varepsilon)/2 - \lambda - n}{1 - (\alpha + \varepsilon)/2} \right] \left(\frac{\varepsilon - \alpha + 1}{2} \right)_n$$

[$\varepsilon = 0$ или 1 ; $a, r > 0$, $\operatorname{Re} \lambda > -1/2$, $\begin{cases} \operatorname{Re} b > 0; \operatorname{Re} (\alpha + 2\lambda) < 1 - 2n - \varepsilon \\ \operatorname{Re} b > 0 \end{cases}$].

$$25. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\lambda-1/2} \begin{Bmatrix} J_\nu(bx^r) \\ I_\nu(bx^r) \end{Bmatrix} C_{2n+\varepsilon}^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^n}{2^{\nu+1} (2n + \varepsilon)!} a^{\alpha+2\lambda+\nu r-1} b^\nu (2\lambda)_{2n + \varepsilon} \Gamma \left[\frac{\lambda + 1/2}{\nu + 1} \right] \times$$

$$\times \sum_{k=0}^\infty \frac{(\mp 1)^k}{k! (\nu + 1)_k} \Gamma \left[\frac{(\alpha + \varepsilon + \nu r)/2 + rk}{(\alpha + \varepsilon + \nu r + 1)/2 + rk + \lambda + n} \right] \times$$

$$\times \left(\frac{\varepsilon - \alpha - \nu r + 1}{2} - rk \right)_n \left(\frac{a^r b}{2} \right)^{2k}$$

[$\varepsilon = 0$ или 1 ; $a, r > 0$; $\operatorname{Re} (\alpha + \nu r) > -\varepsilon$, $\operatorname{Re} \lambda > -1/2$].

$$26. \int_a^\infty x^{\alpha-1} (x^2 - a^2)^{\lambda-1/2} J_\nu(bx^r) C_{2n+\varepsilon}^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{a^{\alpha+2\lambda+\nu r-1} b^\nu}{2^{\nu+1} (2n + \varepsilon)!} (2\lambda)_{2n + \varepsilon} \Gamma \left[\frac{\lambda + 1/2}{\nu + 1} \right] \sum_{k=0}^\infty \frac{(-1)^k}{k! (\nu + 1)_k} \times$$

$$\times \Gamma \left[\frac{(1 - \alpha - \varepsilon - \nu r)/2 - \lambda - rk - n}{1 - (\alpha + \varepsilon + \nu r)/2 - rk} \right] \left(\frac{\varepsilon - \alpha - \nu r + 1}{2} - rk \right)_n \left(\frac{a^r b}{2} \right)^{2k} \times$$

$$+ \frac{2^{2n+\varepsilon-1} (\lambda)_{n+\varepsilon}}{(2n + \varepsilon)! a^{2n+\varepsilon} r} \left(\frac{b}{2} \right)^{(1-\alpha-\varepsilon-2\lambda-2n)/r} \sum_{k=0}^\infty \frac{(1/2 - \lambda - n)_k (\varepsilon + \lambda + n - k)_n}{k!} \times$$

$$\times \Gamma \left[\frac{(\alpha + \varepsilon + 2\lambda + \nu r + 2n - 2k - 1)/(2r)}{1 - (\alpha + \varepsilon + 2\lambda - \nu r + 2n - 2k - 1)/(2r)} \right] \left(\frac{a^r b}{2} \right)^{2k/r}$$

[$\varepsilon = 0$ или 1 ; $a, b, r > 0$; $-1/2 < \operatorname{Re} \lambda < (3r/2 - \operatorname{Re} \alpha - \varepsilon - 2n)/2$].

$$\begin{aligned}
27. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\lambda-1/2} L_m^\gamma (bx^r) C_{2n+\varepsilon}^\lambda \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^n}{m! (2n+\varepsilon)! 2} a^{\alpha+2\lambda-1} (2\lambda)_{2n+\varepsilon} (\gamma+1)_m \Gamma \left(\lambda + \frac{1}{2} \right) \times \\
\times \sum_{k=0}^m \frac{(-m)_k}{k! (\gamma+1)_k} \Gamma \left[\frac{(\alpha+\varepsilon+rk)/2}{(\alpha+\varepsilon+rk+1)/2 + \lambda + n} \right] \left(\frac{\varepsilon-\alpha-rk+1}{2} \right)_n (a^r b)^k \\
[\varepsilon = 0 \text{ или } 1; a, r > 0; \operatorname{Re} \alpha > -\varepsilon; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
28. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\lambda-1/2} e^{-bx^r} L_m^\gamma (bx^r) C_{2n+\varepsilon}^\lambda \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^n}{m! (2n+\varepsilon)! 2} a^{\alpha+2\lambda-1} (2\lambda)_{2n+\varepsilon} \Gamma \left(\lambda + \frac{1}{2} \right) \times \\
\times \sum_{k=0}^{\infty} \frac{1}{k!} (k+\gamma+1)_m \Gamma \left[\frac{(\alpha+\varepsilon+rk)/2}{(\alpha+\varepsilon+rk+1)/2 + \lambda + n} \right] \left(\frac{\varepsilon-\alpha-rk+1}{2} \right)_n (-a^r b)^k \\
[\varepsilon = 0 \text{ или } 1; a, r > 0; \operatorname{Re} \alpha > -\varepsilon; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
29. \int_a^\infty x^{\alpha-1} (x^2 - a^2)^{\lambda-1/2} e^{-bx^r} L_m^\gamma (bx^r) C_{2n+\varepsilon}^\lambda \left(\frac{x}{a} \right) dx = \\
= \frac{2^{2n+\varepsilon} b^{(1-\alpha-\varepsilon-2\lambda-2n)/r}}{m! (2n+\varepsilon)! a^{2n+\varepsilon}} (\lambda)_{n+\varepsilon} \times \\
\times \sum_{k=0}^{\infty} \frac{(1/2-\lambda-n)_k}{k!} (\varepsilon+\lambda+n-k)_n \left(1+\gamma+\frac{1-\alpha-\varepsilon-2\lambda-2n+2k}{r} \right)_m \times \\
\times \Gamma \left(\frac{\alpha+\varepsilon+2\lambda+2n-2k-1}{r} \right) (a^r b)^{2k/r} + \frac{a^{\alpha+2\lambda-1} (2\lambda)_{2n+\varepsilon}}{m! (2n+\varepsilon)! 2} \times \\
\times \Gamma \left(\lambda + \frac{1}{2} \right) \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{1+\varepsilon-\alpha+rk}{2} \right)_n (k+\gamma+1)_m \times \\
\times \Gamma \left[\frac{(1-\alpha-\varepsilon-2n-rk)/2 - \lambda}{1 - (\alpha+\varepsilon+rk)/2} \right] (-a^r b)^k \\
[\varepsilon = 0 \text{ или } 1; a, r, \operatorname{Re} b > 0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
30. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\lambda-1/2} H_{2m+\varepsilon} (bx^r) \left\{ \begin{matrix} C_{2n+1}^\lambda (x/a) \\ C_{2n}^\lambda (x/a) \end{matrix} \right\} dx = \\
\times \frac{(-1)^{m+n} (2m+\varepsilon)! 2^{\varepsilon-1}}{m! (2n+\delta)!} a^{\alpha+2\lambda+r\varepsilon-1} b^\varepsilon (2\lambda)_{2n+\delta} \Gamma \left(\lambda + \frac{1}{2} \right) \times \\
\times \sum_{k=0}^m \frac{(-m)_k}{(1/2+\varepsilon)_k k!} \left(\frac{\delta-\varepsilon-\alpha+1}{2} - rk \right)_n \times \\
\times \Gamma \left[\frac{rk + (\alpha+\delta+\varepsilon)/2}{\lambda+n+rk+(\alpha+\delta+r\varepsilon+1)/2} \right] (a^r b)^{2k} \\
[\varepsilon = 0 \text{ или } 1; a, r > 0; \operatorname{Re} \alpha > -\varepsilon - \delta; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
31. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\lambda-1/2} e^{-b^2 x^{2r}} H_{2m+\varepsilon}(bx^r) \begin{Bmatrix} C_{2n+1}^\lambda(x/a) \\ C_{2n}^\lambda(x/a) \end{Bmatrix} dx = \\
= \frac{(-1)^{m+n} 2^{2m+\varepsilon-1}}{(2n+\delta)!} a^{\alpha+2\lambda+\varepsilon r-1} b^\varepsilon (2\lambda)_{2n+\delta} \Gamma\left(\lambda + \frac{1}{2}\right) \times \\
\times \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \left(\frac{1}{2} + \varepsilon + k\right)_m \left(\frac{1+\delta-\varepsilon r-\alpha}{2} - rk\right)_n \times \\
\times \Gamma\left[\frac{rk + (\delta + \varepsilon r + \alpha)/2}{\lambda + n + rk + (\varepsilon r + \alpha + \delta + 1)/2}\right] (a^r b)^{2k} \\
[\varepsilon = 0 \text{ или } 1; a, r > 0; \operatorname{Re} \alpha > -\varepsilon r - \delta; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
32. \int_a^\infty x^{\alpha-1} (x^2 - a^2)^{\lambda-1/2} e^{-b^2 x^{2r}} H_{2m+\varepsilon}(bx^r) \begin{Bmatrix} C_{2n+1}^\lambda(x/a) \\ C_{2n}^\lambda(x/a) \end{Bmatrix} dx = \\
= \frac{(-1)^m 2^{2m+\varepsilon-1} a^{\alpha+2\lambda+\varepsilon r-1} b^\varepsilon}{(2n+\delta)!} (2\lambda)_{2n+\delta} \Gamma\left(\lambda + \frac{1}{2}\right) \times \\
\times \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \left(\frac{1}{2} + \varepsilon + k\right)_m \left(\frac{1+\delta-\varepsilon r-\alpha}{2} - rk\right)_n \times \\
\times \Gamma\left[\frac{(1-\alpha-\delta-\varepsilon r)/2 - \lambda - rk - n}{1 - rk - (\alpha + \delta + \varepsilon r)/2}\right] (a^r b)^{2k} + \\
+ \frac{(-1)^m b^{(1-\alpha-2\lambda-2\varepsilon-\delta)/r} 2^{2m+2n+\delta+\varepsilon-1}}{(2n+\delta)! a^{2n+\delta} r} (\lambda_{n+\delta}) \times \\
\times \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{1}{2} - \lambda - n\right)_k (\delta + \lambda + n - k)_n \left(\frac{1+r-\alpha-\delta-2\lambda+\varepsilon r-2n+2k}{2r}\right)_m \times \\
\times \Gamma\left(\frac{\alpha + \delta + \varepsilon r + 2\lambda + 2n - 2k - 1}{2r}\right) (a^r b)^{2k/r} \\
[\varepsilon = 0 \text{ или } 1; a, r, \operatorname{Re} b > 0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
33. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\lambda-1/2} (b^{2r} - x^{2r})^{\gamma-1/2} C_{2m+\varepsilon}^\gamma\left(\frac{x^r}{b^r}\right) \begin{Bmatrix} C_{2n+1}^\lambda(x/a) \\ C_{2n}^\lambda(x/a) \end{Bmatrix} dx = \\
= \frac{(-1)^{m+n} 2^{2m+\varepsilon-1} a^{\alpha+2\lambda+\varepsilon r-1} b^{(2\gamma-\varepsilon-1)r} (\gamma)_{m+\varepsilon} (2\lambda)_{2n+\delta}}{(2m+\varepsilon)! (2n+\delta)!} \times \\
\times \Gamma\left(\lambda + \frac{1}{2}\right) \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{1}{2} - \gamma - m\right)_k \left(\frac{1}{2} + \varepsilon + k\right)_m \left(\frac{1+\delta-\alpha-\varepsilon r}{2} - rk\right)_n \times \\
\times \Gamma\left[\frac{(\alpha + \delta + \varepsilon r)/2 + rk}{\lambda + n + rk + (\alpha + \delta + \varepsilon r + 1)/2}\right] \left(\frac{a}{b}\right)^{2rk} \\
[\varepsilon = 0 \text{ или } 1; r > 0; b > a > 0; \operatorname{Re} \alpha > -\delta - \varepsilon r; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
34. \int_a^b x^{\alpha-1} (x^2 - a^2)^{\lambda-1/2} (b^{2r} - x^{2r})^{\gamma-1/2} C_{2m+\varepsilon}^\gamma\left(\frac{x^r}{b^r}\right) \begin{Bmatrix} C_{2n+1}^\lambda(x/a) \\ C_{2n}^\lambda(x/a) \end{Bmatrix} dx = \\
= \frac{(-1)^m 2^{2m+\varepsilon-1} a^{\alpha+2\lambda+\varepsilon r-1}}{(2m+\varepsilon)! (2n+\delta)!} b^{(2\gamma-\varepsilon-1)r} (2\lambda)_{2n+\delta} (\gamma)_{m+\varepsilon} \times \\
\times \Gamma\left(\lambda + \frac{1}{2}\right) \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{1}{2} - \gamma - m\right)_k \left(\frac{1}{2} + \varepsilon + k\right)_m \left(\frac{1+\delta-\varepsilon r-\alpha}{2} - rk\right)_n \times
\end{aligned}$$

$$\times \Gamma \left[\frac{(1-\alpha-\delta-\varepsilon r)^r - 1 - r\lambda - n}{1-rk-(\alpha+\delta+\varepsilon r)/2} \right] \left(\frac{a}{b} \right)^{2rk} +$$

$$+ \frac{(-1)^m 2^{2n+\delta-1} b^{\alpha+\delta+2\lambda+2n+2r\gamma-1}}{(2m+\varepsilon)! (2n+\delta)! a^{2n+\delta r}} (\lambda)_{n+\delta} (2\gamma)_{2m+\varepsilon} \times$$

$$\times \Gamma \left(\gamma + \frac{1}{2} \right) \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{1}{2} - \lambda - n \right)_k (\lambda + \delta + n - k)_n \left(\frac{1+r+2k+\varepsilon r - \alpha - 2\lambda - 2n - \delta}{2r} \right)_m \times$$

$$\times \Gamma \left[\frac{(\alpha + \delta + 2\lambda + 2n + \varepsilon r - 2k - 1)/(2r)}{(\alpha + \delta + 2\lambda + r + 2rm + re + 2n - 2k - 1)/(2r) + \gamma} \right] \left(\frac{a}{b} \right)^{2k}$$

[$\varepsilon = 0$ или 1 ; $b > a > 0$; $r > 0$; $\operatorname{Re} \gamma, \operatorname{Re} \lambda > -1/2$].

$$35. \int_a^{\infty} x^{\alpha-1} (x^2 - a^2)^{\lambda-1/2} (x^{2r} - b^{2r})^{\gamma-1/2} C_{2m+\varepsilon}^{\gamma} \left(\frac{x^r}{b^r} \right) \left\{ \begin{matrix} C_{2n+1}^{\lambda} (x/a) \\ C_{2n}^{\lambda} (x/a) \end{matrix} \right\} dx =$$

$$= \frac{2^{2m+\varepsilon-1} (\gamma)_{m+\varepsilon} (2\lambda)_{2n+\delta} a^{\alpha+2\lambda+2\gamma r+2rm+\varepsilon r-r-1}}{(2m+\varepsilon)! (2n+\delta)! b^{2rm+\varepsilon r}} \times$$

$$\times \Gamma \left(\lambda + \frac{1}{2} \right) \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{1}{2} - \gamma - m \right)_k (\gamma + \varepsilon + m - k)_m \left(\frac{1-\alpha+\delta-\varepsilon r+r}{2} - \gamma r - rm + rk \right)_n \times$$

$$\times \Gamma \left[\frac{(1+r-\delta-\alpha-\varepsilon r)/2 + rk - \gamma r - \lambda - n - rm}{1+rk-rm-\gamma r - (\alpha + \varepsilon r + \delta - r)/2} \right] \left(\frac{b}{a} \right)^{2rk}$$

[$\varepsilon = 0$ или 1 ; $r > 0$; $a > b > 0$;
 $-1/2 < \operatorname{Re} \lambda < (1+r-2rm-2\gamma r - \operatorname{Re} \alpha - \delta - \varepsilon r - 2n)/2$].

2.21.2. Интегралы от $(x \pm a)^{\alpha} (x^m \pm b^m)^{\beta} C_n^{\lambda}(\varphi(x))$.

$$1. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\beta-1} C_{2n+\varepsilon}^{\lambda}(cx) dx =$$

$$= \frac{(-1)^n (\lambda)_{n+\varepsilon} c^{\varepsilon}}{2^{1-\varepsilon} n!} a^{\alpha+2\beta+\varepsilon-2} B \left(\frac{\alpha+\varepsilon}{2}, \beta \right) \times$$

$$\times {}_3F_2 \left(-n, \lambda+n+\varepsilon, \frac{\alpha+\varepsilon}{2}; \varepsilon+\frac{1}{2}, \frac{\alpha+\varepsilon}{2}+\beta; a^2 c^2 \right)$$

[$\varepsilon = 0$ или 1 ; $a, \operatorname{Re} \beta > 0$; $\operatorname{Re} \alpha > -\varepsilon$].

$$2. \int_0^a x^{2\lambda-2\beta+2n+\varepsilon-1} (a^2 - x^2)^{\beta-1} C_{2n+\varepsilon}^{\lambda}(cx) dx =$$

$$= \frac{1}{2} B(\lambda - \beta, \beta) a^{2\lambda+2n+\varepsilon-2} C_{2n+\varepsilon}^{\lambda-\beta}(ac)$$

[$\varepsilon = 0$ или 1 ; $a, \operatorname{Re} \beta > 0$; $\operatorname{Re}(\lambda - \beta) > -\varepsilon/2 - n$].

$$3. \int_0^a x^{\varepsilon} (a^2 - x^2)^{\beta-1} C_{2n+\varepsilon}^{\lambda} \left(\frac{x}{a} \right) dx =$$

$$= \frac{\sqrt{\pi}}{2(n!)} a^{2\beta+\varepsilon-1} (\lambda)_{n+\varepsilon} \left(\lambda - \beta + \frac{1}{2} \right)_n \Gamma \left[\begin{matrix} \beta \\ \beta + \varepsilon + n + 1/2 \end{matrix} \right]$$

[$\varepsilon = 0$ или 1 ; $a, \operatorname{Re} \beta > 0$].

$$4. \int_0^a x^{2\lambda-2\beta+2n+\varepsilon-1} (a^2 - x^2)^{\beta-1} C_{2n+\varepsilon}^{\lambda} \left(\frac{x}{a} \right) dx =$$

$$= \frac{2^{2n+\varepsilon-1}}{(2n+\varepsilon)!} a^{2\lambda+2n+\varepsilon-2} \left(\lambda - \beta + \frac{1}{2} \right)_n \Gamma \left[\begin{matrix} \beta, \lambda + n - \beta + \varepsilon \\ \lambda \end{matrix} \right]$$

[$\varepsilon = 0$ или 1 ; $a > 0$; $0 < \operatorname{Re} \beta < \operatorname{Re} \lambda + n + \varepsilon$]

5. $\int_0^a x^{\alpha-1} (a^2 - x^2)^{\lambda-1/2} C_{2n+\varepsilon}^\lambda \left(\frac{x}{a} \right) dx =$

$$= \frac{(-1)^n 2^{2n+\varepsilon-1}}{(2n+\varepsilon)!} (\lambda)_{n+\varepsilon} a^{\alpha+2\lambda-1} \left(\frac{\varepsilon-\alpha+1}{2} \right)_n B \left(\lambda+n+\frac{1}{2}, \frac{\alpha+\varepsilon}{2} \right)$$

$$[\varepsilon = 0 \text{ или } 1; a > 0; \operatorname{Re} \alpha > -\varepsilon; \operatorname{Re} \lambda > -1/2].$$
6. $\int_0^a x^m (a^2 - x^2)^{\lambda-1/2} C_n^\lambda \left(\frac{x}{a} \right) dx = 0$

$$[n-m=2, 4, 6, \dots; a > 0; \operatorname{Re} \lambda > -1/2].$$
7. $\int_a^b (x-a)^{\lambda-1} (b-x)^{\lambda-1} C_n^\lambda(cx) dx = n! \Gamma \left[\begin{matrix} \lambda, \lambda \\ 2\lambda+n \end{matrix} \right] (b-a)^{2\lambda-1} C_n^\lambda(z_+) C_n^\lambda(z_-)$

$$[V^2 z_\pm = (1 + abc^2 \pm V(1-a^2c^2)(1-b^2c^2))^{1/2}; \operatorname{Re} \lambda > 0].$$
8. $\int_{\pm a}^b (x \mp a)^{\alpha-1} (b-x)^{\beta-1} C_n^\lambda \left(\frac{x}{a} \right) dx =$

$$= \frac{(\pm 1)^n}{n!} (2\lambda)_n B(\alpha, \beta) (b \mp a)^{\alpha+\beta-1} {}_3F_2 \left(-n, 2\lambda+n, \alpha; \alpha+\beta, \lambda+\frac{1}{2}; \frac{a \mp b}{2a} \right)$$

$$[b > \pm a; a, \operatorname{Re} \alpha, \operatorname{Re} \beta > 0].$$
9. $\int_{-a}^b (x+a)^{\lambda-1/2} (b-x)^{\beta-1} C_n^\lambda \left(\frac{x}{a} \right) dx =$

$$= (2\lambda)_n (a+b)^{\beta+\lambda-1/2} \Gamma \left[\begin{matrix} \beta, \lambda+1/2 \\ \beta+\lambda+n+1/2 \end{matrix} \right] P_n^{(\rho, \sigma)} \left(\frac{b}{a} \right)$$

$$[\rho = \lambda - \beta - 1/2, \sigma = \beta + \lambda - 1/2; -a < b < a; \operatorname{Re} \beta > 0; \operatorname{Re} \lambda > -1/2].$$
10. $\int_{-a}^b (x+a)^{2\lambda-\beta+n-1} (b-x)^{\beta-1} C_n^\lambda \left(\frac{x}{a} \right) dx =$

$$= \frac{1}{(\lambda+1/2)_n} \Gamma \left[\begin{matrix} \beta, 2\lambda-\beta+n \\ 2\lambda \end{matrix} \right] (a+b)^{2\lambda+n-1} P_n^{(\lambda-\beta-1/2, \lambda-1/2)} \left(\frac{b}{a} \right)$$

$$[-a < b < a; \operatorname{Re} \beta > 0; \operatorname{Re} (2\lambda - \beta) > -n].$$
11. $\int_a^b (x-a)^{\lambda-1/2} (b-x)^{\beta-1} C_n^\lambda \left(\frac{x}{a} \right) dx = 2^{2n} (\lambda)_n B \left(\lambda+n+\frac{1}{2}, \beta \right) \times$

$$\times (a+b)^{\beta+\lambda+n-1/2} {}_2F_1 \left(1-2\lambda-n, \frac{1}{2}-\beta-\lambda-n; 1-2\lambda-2n; \frac{2a}{a+b} \right)$$

$$[b > a > 0; \operatorname{Re} \lambda > -1/2; \operatorname{Re} \beta > 0].$$
12. $\int_a^b (x-a)^{\alpha-1} (b-x)^{\beta-1} C_n^\lambda \left(\frac{x}{b} \right) dx = \frac{(2\lambda)_n}{n!} B(\alpha, \beta) (b-a)^{\alpha+\beta-1} \times$

$$\times {}_3F_2 \left(-n, 2\lambda+n, \beta; \lambda+\frac{1}{2}, \alpha+\beta; \frac{b-a}{2b} \right)$$

$$[-b < a < b; \operatorname{Re} \alpha, \operatorname{Re} \beta > 0].$$
13. $\int_a^b (x-a)^{2\lambda-\beta+n-1} (b-x)^{\beta-1} C_n^\lambda \left(\frac{x}{b} \right) dx =$

$$= \frac{1}{(\lambda+1/2)_n} \Gamma \left[\begin{matrix} \beta, 2\lambda-\beta+n \\ 2\lambda \end{matrix} \right] (b-a)^{2\lambda+n-1} P_n^{(\lambda-1/2, \beta-\lambda-n-1/2)} \left(\frac{a}{b} \right)$$

$$[-b < a < b; \operatorname{Re} \beta > 0; \operatorname{Re} (2\lambda - \beta) > -n].$$

$$14. \int_a^b (x-a)^{\alpha-1} (b-x)^{\lambda-1/2} C_n^\lambda \left(\frac{x}{b} \right) dx =$$

$$= (2\lambda)_n \Gamma \left[\begin{matrix} \alpha, \lambda + 1/2 \\ \alpha + \lambda + n + 1/2 \end{matrix} \right] (b-a)^{\alpha+\lambda-1/2} p_n^{(\rho, \sigma)} \left(\frac{a}{b} \right)$$

[$\rho = \lambda + \alpha - 1/2, \sigma = \lambda - \alpha - 1/2; -b < a < b; \operatorname{Re} \lambda > -1/2; \operatorname{Re} \alpha > 0$].

$$15. \int_{-a}^a (a^2 - x^2)^{\beta-1} C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^n (2\lambda)_n \sqrt{\pi}}{n!} \Gamma \left[\begin{matrix} \beta \\ \beta + 1/2 \end{matrix} \right] a^{2\beta-1} {}_3F_2 \left(-n, n+2\lambda, \beta; 2\beta, \lambda + \frac{1}{2}; 1 \right)$$

[$a, \operatorname{Re} \beta > 0$].

$$16. \int_{-a}^a (a^2 - x^2)^{\lambda/2-1} C_{2n}^\lambda (cx) dx = \sqrt{\pi} a^{\lambda-1} \Gamma \left[\begin{matrix} \lambda/2 \\ (\lambda+1)/2 \end{matrix} \right] C_n^{\lambda/2} (2a^2 c^2 - 1)$$

[$a, \operatorname{Re} \lambda > 0$].

$$17. \int_{-a}^a (a^2 - x^2)^{\lambda-1/2} C_n^\lambda \left(\frac{x}{a} \right) dx = \delta_{n,0} \sqrt{\pi} a^{2\lambda} \Gamma \left[\begin{matrix} \lambda + 1/2 \\ \lambda + 1 \end{matrix} \right]$$

[$a > 0; \operatorname{Re} \lambda > -1/2$].

$$18. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\beta-1} C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= (2\lambda)_n \frac{(2a)^{\alpha+\beta-1}}{n!} B(\alpha, \beta) {}_3F_2 \left(-n, n+2\lambda, \beta; \alpha+\beta, \lambda + \frac{1}{2}; 1 \right)$$

[$a, \operatorname{Re} \alpha, \operatorname{Re} \beta > 0$].

$$19. \int_{-a}^a (x+a)^{\lambda-1/2} (a-x)^{\beta-1} C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(2\lambda)_n}{n!} \left(\frac{1}{2} + \lambda - \beta \right)_n \Gamma \left[\begin{matrix} \beta, \lambda + 1/2 \\ \lambda + \beta + n + 1/2 \end{matrix} \right] (2a)^{\lambda+\beta-1/2}$$

[$a, \operatorname{Re} \beta > 0, \operatorname{Re} \lambda > -1/2$].

$$20. \int_{-a}^a (x+a)^{2\lambda-\beta+n-1} (a-x)^{\beta-1} C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(2a)^{2\lambda+n-1}}{n! (\lambda + 1/2)_n} \left(\lambda - \beta + \frac{1}{2} \right)_n \Gamma \left[\begin{matrix} \beta, 2\lambda - \beta + n \\ 2\lambda \end{matrix} \right]$$

[$a > 0; 0 < \operatorname{Re} \beta < 2 \operatorname{Re} \lambda + n$].

$$21. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\lambda-1/2} C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= (-1)^n \frac{(2\lambda)_n}{n!} \left(\lambda - \alpha + \frac{1}{2} \right)_n \Gamma \left[\begin{matrix} \alpha, \lambda + 1/2 \\ \alpha + \lambda + n + 1/2 \end{matrix} \right] (2a)^{\alpha+\lambda-1/2}$$

[$a, \operatorname{Re} \alpha > 0; \operatorname{Re} \lambda > -1/2$].

$$22. \int_{-a}^a x^m (a^2 - x^2)^{\lambda-1/2} C_n^\lambda \left(\frac{x}{a} \right) dx = I_{m,n}$$

[$a > 0; \operatorname{Re} \lambda > -1/2$].

$$I_{m,n} = 0$$

[$m+n=1, 3, 5, \dots$, или $m+n=2, 4, 6, \dots$ и $n-m=2, 4, 6, \dots$].

$$I_{m,n} = \frac{(2\lambda)_n \sqrt{\pi} m!}{n! 2^m} \Gamma \left[\begin{matrix} \lambda + 1/2 \\ (m+n)/2 + \lambda + 1, (m-n)/2 + 1 \end{matrix} \right] a^{m+2\lambda}$$

[$m+n=0, 2, 4, \dots$; $n-m \neq 2, 4, 6, \dots$].

$$23. \int_0^\infty x^\rho (x^2 + a^2)^{-\rho} C_{2n+\varepsilon}^\lambda \left(\frac{ix}{a} \right) dx =$$

$$= \frac{(-1)^n \sqrt{\pi} i^\varepsilon}{2 (n!)} a^{1+\varepsilon-2\rho} (\lambda)_{n+\varepsilon} \left(\lambda + \rho - \frac{1}{2} \right)_n \Gamma \left[\begin{matrix} \rho - \varepsilon - n - 1/2 \\ \rho \end{matrix} \right]$$

[$\varepsilon = 0$ или 1 ; $\operatorname{Re} a > 0$; $\operatorname{Re} \rho > n + \varepsilon + 1/2$].

$$24. \int_a^\infty x^{2\lambda-2\beta+2n+\varepsilon-1} (x^2 - a^2)^{\beta-1} C_{2n+\varepsilon}^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^{n+\varepsilon} 2^{2n+\varepsilon-1}}{(2n+\varepsilon)!} a^{2\lambda+2n+\varepsilon-2} \left(\lambda - \beta + \frac{1}{2} \right)_n \Gamma \left[\begin{matrix} 1-\lambda, \beta \\ \beta - \lambda - \varepsilon - n + 1 \end{matrix} \right]$$

[$\varepsilon = 0$ или 1 ; $a, \operatorname{Re} \beta > 0$; $\operatorname{Re} \lambda < 1 - 2n - \varepsilon$].

$$25. \int_a^\infty x^{\alpha-1} (x^2 - a^2)^{\lambda-1/2} C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(2\lambda)_n}{2^{\alpha+1} n! \sqrt{\pi}} a^{\alpha+2\lambda-1} \Gamma \left[\begin{matrix} (1-n-\alpha)/2 - \lambda, (1+n-\alpha)/2, \lambda + 1/2 \\ 1-\alpha \end{matrix} \right]$$

[$a > 0$; $\operatorname{Re} (\alpha + 2\lambda) < 1 - n$; $\operatorname{Re} \lambda > -1/2$].

$$26. \int_a^\infty (x-a)^{\alpha-1} (x+a)^{-\tau} C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(2\lambda)_n}{n!} B(\alpha, \tau - \alpha - n) (2a)^{\alpha-\tau} {}_3F_2 \left(-n, \alpha, \frac{1}{2} - \lambda - n; \lambda + \frac{1}{2}, \tau - n; 1 \right)$$

[$a, \operatorname{Re} \alpha > 0$; $\operatorname{Re} (\alpha - \tau) < -n$].

$$27. \int_a^\infty (x-a)^{\lambda-1/2} (x+a)^{-\tau} C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(\lambda + \tau - 1/2)_n}{n!} (2\lambda)_n \Gamma \left[\begin{matrix} \lambda + 1/2, \tau - \lambda - n - 1/2 \\ \tau \end{matrix} \right] (2a)^{\lambda-\tau+1/2}$$

[$a > 0$; $\operatorname{Re} \lambda > -1/2$; $\operatorname{Re} (\tau - \lambda) > n + 1/2$].

$$28. \int_a^\infty (x-a)^{2\lambda+\tau+n-2} (x+a)^{-\tau} C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(\lambda + \tau - 1/2)_n}{n! (\lambda + 1/2)_n} \Gamma \left[\begin{matrix} 1-2\lambda, 2\lambda + \tau + n - 1 \\ \tau \end{matrix} \right] (2a)^{2\lambda+n-1}$$

[$a > 0$; $1 - n - \operatorname{Re} \tau < 2 \operatorname{Re} \lambda < 1 - 2n$].

$$29. \int_a^\infty (x-a)^{\alpha-1} (x+a)^{\lambda-1/2} C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^n (\lambda - \alpha + 1/2)_n (2\lambda)_n}{n!} \Gamma \left[\begin{matrix} \alpha, 1/2 - \alpha - \lambda - n \\ 1/2 - \lambda \end{matrix} \right] (2a)^{\alpha+\lambda-1/2}$$

[$a > 0$; $0 < \operatorname{Re} \alpha < 1/2 - \operatorname{Re} \lambda - n$].

$$30. \int_0^a x^{\alpha-1} (a-x)^{2\lambda-1} C_n^\lambda \left(\frac{x+a}{2\sqrt{ax}} \right) dx = \frac{a^{\alpha+2\lambda-1}}{n!} \left(\alpha + \lambda - \frac{n}{2} \right)_n B \left(2\lambda + n, \alpha - \frac{n}{2} \right)$$

[$a, \operatorname{Re} \lambda > 0$; $\operatorname{Re} \alpha > n/2$].

$$31. \int_0^a x^{\alpha-1} (a-x)^{-\lambda-n/2} C_n^\lambda \left(\frac{2a-x}{2\sqrt{a(a-x)}} \right) dx = \frac{a^{\alpha-\lambda-n/2}}{n!} (2\lambda-\alpha)_n B(1-\lambda, \alpha)$$

[a, Re $\alpha > 0$; Re $\lambda < 1-n$].

$$32. \int_0^\infty x^{\alpha-1} (x+a)^{-\lambda-n/2} C_n^\lambda \left(\frac{2a+x}{2\sqrt{a(a+x)}} \right) dx = \frac{a^{\alpha-\lambda-n/2}}{n!} (2\lambda-\alpha)_n B(\alpha, \lambda-\alpha)$$

[0 < Re $\alpha < \text{Re } \lambda$; |arg a | < π].

2.21.3. Интегралы от $\frac{A(x)}{x^m - y^m} C_n^\lambda(cx)$.

$$1. \int_0^a \frac{x^{\alpha-1}}{x^2 - y^2} (a^2 - x^2)^{\lambda-1/2} C_{2n+\varepsilon}^\lambda \left(\frac{x}{a} \right) dx = I$$

[$\varepsilon = 0$ или 1; $a, y > 0$; Re $\alpha > -\varepsilon$; Re $\lambda > -1/2$].

$$I = \frac{(-1)^{n+1} a^{\alpha+2\lambda-1}}{2(2n+\varepsilon)! y^2} (2\lambda)_{2n+\varepsilon} \left(\frac{1+\varepsilon-\alpha}{2} \right)_n \Gamma \left[\frac{\lambda+1/2, (\alpha+\varepsilon)/2}{(\alpha+\varepsilon+1)/2 + \lambda + n} \right] \times$$

$$\times {}_3F_2 \left(1, \frac{\alpha}{2}, \frac{1+\alpha}{2}; \frac{1+\alpha+\varepsilon}{2} + \lambda + n, \frac{1+\alpha-\varepsilon}{2} - n; \frac{a^2}{y^2} \right) \quad [a < y].$$

$$I = \frac{(-1)^{n+\varepsilon+1} 2^{\varepsilon-1} \pi}{n!} a^{2\lambda-\varepsilon-1} y^{\alpha+\varepsilon-2} (\lambda)_{n+\varepsilon} \operatorname{tg}^{2\varepsilon-1} \frac{\alpha\pi}{2} \times$$

$$\times {}_2F_1 \left(\frac{1}{2} - \lambda - n, \frac{1}{2} + \varepsilon + n; \frac{1}{2} + \varepsilon; \frac{y^2}{a^2} \right) +$$

$$+ \frac{(-1)^n a^{\alpha+2\lambda-3}}{2(2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{\varepsilon-\alpha+3}{2} \right)_n \Gamma \left[\frac{\lambda+1/2, (\alpha+\varepsilon)/2-1}{(\alpha+\varepsilon-1)/2 + \lambda + n} \right] \times$$

$$\times {}_3F_2 \left(1, \frac{3-\alpha-\varepsilon}{2} - \lambda - n, \frac{3+\varepsilon-\alpha}{2} + n; \frac{3-\alpha}{2}, 2 - \frac{\alpha}{2}; \frac{y^2}{a^2} \right) \quad [y < a].$$

$$2. \int_{-a}^a \frac{(x+a)^{\lambda-1/2}}{x-y} (a-x)^{\beta-1} C_n^\lambda \left(\frac{x}{a} \right) dx = I$$

[a, Re $\beta > 0$; Re $\lambda > -1/2$].

$$I = \pi \operatorname{ctg} \beta\pi (a-y)^{\beta-1} (a+y)^{\lambda-1/2} C_n^\lambda \left(\frac{y}{a} \right) -$$

$$- \frac{(2\lambda)_n}{n!} \left(\lambda - \beta + \frac{3}{2} \right)_n \Gamma \left[\frac{\beta-1, \lambda+1/2}{\beta+\lambda+n-1/2} \right] (2a)^{\beta+\lambda-3/2} \times$$

$$\times {}_3F_2 \left(1, \frac{3}{2} + n + \lambda - \beta, \frac{3}{2} - \beta - \lambda - n; 2 - \beta, \frac{3}{2} + \lambda - \beta; \frac{a-y}{2a} \right)$$

[-a < y < a].

$$I = \frac{(2\lambda)_n}{n! (a-y)} \left(\lambda - \beta + \frac{1}{2} \right)_n \Gamma \left[\frac{\beta, \lambda+1/2}{\beta+\lambda+n+1/2} \right] (2a)^{\lambda+\beta-1/2} \times$$

$$\times {}_3F_2 \left(1, \beta, \frac{1}{2} + \beta - \lambda; \frac{1}{2} + \beta - \lambda - n, \frac{1}{2} + \beta + \lambda + n; \frac{2a}{a-y} \right) \quad [y < -a].$$

$$3. \int_{-a}^a \frac{(x+a)^{2\lambda-\beta+n}}{x-y} (a-x)^{\beta-1} C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= - \frac{2^{2\lambda+n}}{n! (\lambda+1/2)_n} \left(\lambda - \beta + \frac{3}{2} \right)_n \Gamma \left[\frac{\beta-1, 2\lambda-\beta+n+1}{2\lambda} \right] a^{\beta+\lambda+n-3/2} (a+y)^{\lambda-\beta+1/2} \times$$

$$\times {}_3F_2 \left(1, 2\lambda-\beta+n+1, \lambda-\beta+n+\frac{3}{2}; \lambda-\beta+\frac{3}{2}, 2-\beta; \frac{y-a}{y+a} \right) +$$

$$+ \pi \operatorname{ctg} \beta\pi (a+y)^{2\lambda-\beta+n} (a-y)^{\beta-1} C_n^\lambda \left(\frac{y}{a} \right)$$

[-a < y < a; Re $\beta > 0$; Re $(\beta - 2\lambda) < n+1$].

$$4. \int_{-a}^a \frac{x^m}{x-y} (a^2-x^2)^{\lambda-1/2} C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= - \frac{\sqrt{\pi} 2^{3/2-\lambda}}{\Gamma(\lambda)} e^{(1/2-\lambda)\pi i} a^{\lambda-1/2} y^m (y^2-a^2)^{(2\lambda-1)/4} \times$$

$$\times Q_{n+\lambda-1/2}^{\lambda-1/2} \left(\frac{y}{a} \right) - \begin{cases} 0 \\ 1 \end{cases} \frac{n! 2^{1-2\lambda-n\pi}}{\Gamma(\lambda) \Gamma(\lambda+n+1)} a^{2\lambda+n}$$

$$\left[\begin{cases} m \leq n \\ m = n+1 \end{cases}, a > 0; y \notin [-a, a]; \operatorname{Re} \lambda > -1/2 \right].$$

$$5. \int_{-a}^a \frac{1}{x-y} \left[C_n^\lambda \left(\frac{y}{a} \right) C_{n+1}^\lambda \left(\frac{x}{a} \right) - C_{n+1}^\lambda \left(\frac{y}{a} \right) C_n^\lambda \left(\frac{x}{a} \right) \right] dx =$$

$$= \frac{4(2\lambda-1)_{n+1}}{(n+1)!} \sum_{k=0}^{[n/2]} \frac{2k+\lambda}{(2k+1)(2k+2\lambda-1)} C_{2k}^\lambda \left(\frac{y}{a} \right) \quad [a > 0; -a < y < a].$$

$$6. \int_a^\infty \frac{x^{\alpha-1}}{x^2-y^2} (x^2-a^2)^{\lambda-1/2} C_{2n+\varepsilon}^\lambda \left(\frac{x}{a} \right) dx = I$$

$$[\varepsilon = 0 \text{ или } 1; a, y > 0; \operatorname{Re} \lambda > -1/2; \operatorname{Re}(\alpha+2\lambda) < 3-2n-\varepsilon].$$

$$I = - \frac{a^{\alpha+2\lambda-1}}{2(2n+\varepsilon)! y^2} (2\lambda)_{2n+\varepsilon} \left(\frac{1+\varepsilon-\alpha}{2} \right)_n \Gamma \left[\lambda+1/2, (1-\alpha-\varepsilon)/2-\lambda-n \right] \times$$

$$\times {}_3F_2 \left(1, \frac{\alpha}{2}, \frac{1+\alpha}{2}; \frac{1+\alpha-\varepsilon}{2}-n, \frac{1+\alpha+\varepsilon}{2}+\lambda+n; \frac{a^2}{y^2} \right) +$$

$$+ (-1)^\varepsilon \frac{2^{\varepsilon+2n-1} \pi y^{\alpha+2\lambda+\varepsilon+2n-3}}{(2n+\varepsilon)! a^{2n+\varepsilon}} (\lambda)_{2n+\varepsilon} \operatorname{ctg}^{2\varepsilon-1} \left(\frac{\alpha}{2} + \lambda \right) \pi \times$$

$$\times {}_2F_1 \left(\frac{1}{2}-\lambda-n, 1-\lambda-\varepsilon-n; 1-\lambda-\varepsilon-2n; \frac{a^2}{y^2} \right) \quad [a < y].$$

$$I = \frac{a^{\alpha+2\lambda-3}}{2(2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{3+\varepsilon-\alpha}{2} \right)_n \Gamma \left[\lambda+1/2, (3-\alpha-\varepsilon)/2-\lambda-n \right] \times$$

$$\times {}_3F_2 \left(1, \frac{3+\varepsilon-\alpha}{2}+n, \frac{3-\alpha-\varepsilon}{2}-\lambda-n; 2-\frac{\alpha}{2}, \frac{3-\alpha}{2}; \frac{y^2}{a^2} \right) \quad [y < a].$$

$$7. \int_a^\infty \frac{(x-a)^{\lambda-1/2}}{x-y} (x+a)^{-\tau} C_n^\lambda \left(\frac{x}{a} \right) dx = I$$

$$[a > 0; \operatorname{Re} \lambda > -1/2; \operatorname{Re}(\lambda-\tau) < 1/2-n].$$

$$I = \frac{(\lambda+\tau+1/2)_n}{n!} (2\lambda)_n \Gamma \left[\lambda+1/2, \tau-\lambda-n+1/2 \right] (2a)^{\lambda-\tau-1/2} \times$$

$$\times {}_3F_2 \left(1, \tau-\lambda-n+\frac{1}{2}, \tau+\lambda+n+\frac{1}{2}; \tau+1, \tau+\lambda+\frac{1}{2}; \frac{a+y}{2a} \right)$$

$$[-a < y < a].$$

$$I = \pi \operatorname{tg}(\lambda-\tau) \pi (a+y)^{-\tau} (y-a)^{\lambda-1/2} C_n^\lambda \left(\frac{y}{a} \right) -$$

$$- \frac{(2a)^{\lambda-\tau+1/2} (\tau+\lambda-1/2)_n (2\lambda)_n}{n! (a+y)} \Gamma \left[\lambda+1/2, \tau-\lambda-n-1/2 \right] \times$$

$$\times {}_3F_2 \left(1, 1-\tau, \frac{3}{2}-\tau-\lambda; \frac{3}{2}+\lambda+n-\tau, \frac{3}{2}-\tau-\lambda-n; \frac{2a}{a+y} \right) \quad [y > a].$$

$$8. \int_a^\infty \frac{(x-a)^{\alpha-1}}{x-y} (x+a)^{\lambda-1/2} C_n^\lambda \left(\frac{x}{a} \right) dx = -\pi e^{i\pi} g_{\alpha\pi} (y-a)^{\alpha-1} (y+a)^{\lambda-1/2} C_n^\lambda \left(\frac{y}{a} \right) +$$

$$+ (-1)^n \frac{(3/2 + \lambda - \alpha)_n}{n!} (2\lambda)_n \Gamma \left[\frac{\alpha-1}{1/2-\lambda}, \frac{3/2-\alpha-\lambda-n}{1/2-\lambda} \right] \times$$

$$\times {}_3F_2 \left(1, \frac{3}{2} + \lambda + n - \alpha, \frac{3}{2} - \lambda - \alpha - n; \frac{3}{2} + \lambda - \alpha, 2 - \alpha; \frac{a-y}{2a} \right)$$

[$y > a$; $\operatorname{Re} \alpha > 0$; $\operatorname{Re} (\alpha + \lambda) < 3/2 - n$].

2.21.4. Интегралы от $(x^m \pm a^m)^\alpha (b^m \pm x^m)^\beta (d^m \pm x^m)^\gamma C_n^\lambda(cx)$.

$$1. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\beta-1} (b^2 - x^2)^{\lambda-1/2} C_{2n+\varepsilon}^\lambda \left(\frac{x}{b} \right) dx =$$

$$= \frac{(-1)^n a^{\alpha+2\beta+\varepsilon-2} b^{2\lambda-\varepsilon-1} (\lambda)_{n+\varepsilon}}{2^{1-\varepsilon} n!} B \left(\beta, \frac{\alpha+\varepsilon}{2} \right) \times$$

$$\times {}_3F_2 \left(\frac{1}{2} - \lambda - n, \frac{\alpha+\varepsilon}{2}, \frac{1}{2} + \varepsilon + n; \frac{1}{2} + \varepsilon, \frac{\alpha+\varepsilon}{2} + \beta; \frac{a^2}{b^2} \right)$$

[$\varepsilon = 0$ или 1 ; $b > a > 0$; $\operatorname{Re} \beta > 0$; $\operatorname{Re} \alpha > -\varepsilon$].

$$2. \int_0^a \frac{x^{\alpha-1}}{(z^2 \pm x^2)^\rho} (a^2 - x^2)^{\lambda-1/2} C_{2n+\varepsilon}^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^n a^{\alpha+2\lambda-1}}{2 (2n+\varepsilon)! z^{2\rho}} (2\lambda)_{2n+\varepsilon} \left(\frac{\varepsilon-\alpha+1}{2} \right)_n \Gamma \left[\frac{\lambda+1/2}{(\alpha+\varepsilon+1)/2+\lambda+n}, \frac{(\alpha+\varepsilon)/2}{(\alpha+\varepsilon+1)/2+\lambda+n} \right] \times$$

$$\times {}_3F_2 \left(\rho, \frac{\alpha}{2}, \frac{\alpha+1}{2}; \frac{\alpha+\varepsilon+1}{2} + \lambda + n, \frac{\alpha-\varepsilon+1}{2} - n; \mp \frac{a^2}{z^2} \right)$$

[$\varepsilon = 0$ или 1 ; $\operatorname{Re} \alpha > -\varepsilon$; $\operatorname{Re} \lambda > -1/2$, $\left\{ \begin{smallmatrix} a, \operatorname{Re} z > 0 \\ z > a > 0 \end{smallmatrix} \right\}$].

$$3. \int_a^b x^{\alpha-1} (x^2 - a^2)^{\beta-1} (b^2 - x^2)^{\lambda-1/2} C_{2n+\varepsilon}^\lambda \left(\frac{x}{b} \right) dx =$$

$$= \frac{(-1)^n b^{\alpha+2\beta+2\lambda-3}}{2 (2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{\varepsilon-\alpha+3}{2} - \beta \right)_n \Gamma \left[\frac{\lambda+1/2}{(\alpha+\varepsilon-1)/2+\beta+\lambda+n}, \frac{(\alpha+\varepsilon)/2+\beta-1}{(\alpha+\varepsilon-1)/2+\beta+\lambda+n} \right] \times$$

$$\times {}_3F_2 \left(1-\beta, \frac{3-\alpha-\varepsilon}{2} - \beta - \lambda - n, \frac{3+\varepsilon-\alpha}{2} - \beta + n; \frac{3-\alpha}{2} - \beta, 2-\beta - \frac{\alpha}{2}; \frac{a^2}{b^2} \right) +$$

$$+ \frac{(-1)^n a^{\alpha+2\beta+\varepsilon-2} b^{2\lambda-\varepsilon-1} (\lambda)_{n+\varepsilon}}{2^{1-\varepsilon} n!} B \left(\beta, 1-\beta - \frac{\alpha+\varepsilon}{2} \right) \times$$

$$\times {}_3F_2 \left(\frac{\alpha+\varepsilon}{2}, \frac{1}{2} + \varepsilon + n, \frac{1}{2} - \lambda - n; \frac{\alpha+\varepsilon}{2} + \beta, \frac{1}{2} + \varepsilon; \frac{a^2}{b^2} \right)$$

[$\varepsilon = 0$ или 1 ; $b > a > 0$; $\operatorname{Re} \beta > 0$; $\operatorname{Re} \lambda > -1/2$].

$$4. \int_a^b x^{-2\lambda-2\beta-n} (x^2 - a^2)^{\beta-1} (b^2 - x^2)^{\lambda-1/2} C_n^\lambda \left(\frac{x}{b} \right) dx =$$

$$= \frac{(2\lambda)_n}{2 (2\beta+2\lambda)_n} B \left(\lambda + \frac{1}{2}, \beta \right) \frac{(b^2 - a^2)^{\beta+\lambda-1/2}}{a^{2\lambda+n+1} b^{2\beta}} C_n^{\lambda+\beta} \left(\frac{a}{b} \right)$$

[$b > a > 0$; $\operatorname{Re} \lambda > -1/2$; $\operatorname{Re} \beta > 0$].

$$\begin{aligned}
5. \int_a^b x^{\alpha-1} (x^2 - a^2)^{\lambda-1/2} (b^2 - x^2)^{\beta-1} C_{2n}^{\lambda} + \varepsilon \left(\frac{x}{a} \right) dx = \\
= \frac{a^{\alpha+2\lambda-1} b^{2\beta-2}}{2(2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{1+\varepsilon-\alpha}{2} \right)_n \Gamma \left[\lambda + 1/2, (1-\alpha-\varepsilon)/2 - \lambda - n \right] \times \\
\times {}_3F_2 \left(1-\beta, \frac{\alpha}{2}, \frac{1+\alpha}{2}; \frac{1+\alpha-\varepsilon}{2} - n, \frac{1+\alpha+\varepsilon}{2} + \lambda + n; \frac{a^2}{b^2} \right) + \\
+ \frac{2^{2n+\varepsilon-1} b^{\alpha+2\beta+2\lambda+\varepsilon+2n-3}}{(2n+\varepsilon)! a^{2n+\varepsilon}} (\lambda)_{2n+\varepsilon} B \left(\beta, \frac{\alpha+\varepsilon-1}{2} + \lambda + n \right) \times \\
\times {}_3F_2 \left(\frac{1}{2} - \lambda - n, \frac{3-\alpha-\varepsilon}{2} - \lambda - \beta - n, 1-\varepsilon - \lambda - n; 1-\varepsilon - \lambda - 2n, \right. \\
\left. \frac{3-\alpha-\varepsilon}{2} - \lambda - n; \frac{a^2}{b^2} \right) \quad [\varepsilon = 0 \text{ или } 1; b > a > 0; \operatorname{Re} \beta > 0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
6. \int_{\pm a}^b (x \mp a)^{\alpha-1} (b-x)^{\beta-1} (a \pm x)^{\lambda-1/2} C_n^{\lambda} \left(\frac{x}{a} \right) dx = \\
= \frac{(\pm 1)^n (2\lambda)_n}{n!} B(\alpha, \beta) (2a)^{\lambda-1/2} (b \mp a)^{\alpha+\beta-1} \times \\
\times {}_3F_2 \left(\alpha, \frac{1}{2} - \lambda - n, \frac{1}{2} + \lambda + n; \alpha + \beta, \lambda + \frac{1}{2}; \frac{a \mp b}{2a} \right) \\
\cdot \left[\operatorname{Re} \alpha, \operatorname{Re} \beta > 0, \left\{ \begin{matrix} b > a \\ -a < b < a \end{matrix} \right\} \right].
\end{aligned}$$

$$\begin{aligned}
7. \int_{\pm a}^b (x \mp a)^{\alpha-1} (b-x)^{\beta-1} (a \pm x)^{2\lambda+n-\alpha-\beta} C_n^{\lambda} \left(\frac{x}{a} \right) dx = \\
= \frac{(\pm 1)^n (2\lambda)_n}{n!} B(\alpha, \beta) (2a)^{2\lambda+n-\beta} (a \pm b)^{-\alpha} (b \mp a)^{\alpha+\beta-1} \times \\
\times {}_3F_2 \left(\alpha, \lambda + n + \frac{1}{2}, 2\lambda + n; \alpha + \beta, \lambda + \frac{1}{2}; \frac{b \mp a}{b \pm a} \right) \\
[-a < b < a; \operatorname{Re} \alpha, \operatorname{Re} \beta > 0].
\end{aligned}$$

$$\begin{aligned}
8. \int_{-a}^b (x+a)^{\lambda-1/2} (b-x)^{\beta-1} (a-x)^{-\tau} C_n^{\lambda} \left(\frac{x}{a} \right) dx = \\
= \frac{(2\lambda)_n}{n!} B(\beta, \tau-\beta) (2a)^{\lambda-1/2} (a-b)^{\beta-\tau} \times \\
\times {}_3F_2 \left(1-\tau, \frac{1}{2} - \lambda - n, \lambda + n + \frac{1}{2}; \lambda + \frac{1}{2}, \beta - \tau + 1; \frac{a-b}{2a} \right) + \\
+ \frac{(\lambda + \tau - \beta + 1/2)_n}{n!} (2\lambda)_n \Gamma \left[\lambda + 1/2, \beta - \tau \right. \\
\left. \beta + \lambda - \tau + n + 1/2 \right] (2a)^{\beta+\lambda-\tau-1/2} \times \\
\times {}_3F_2 \left(1-\beta, \frac{1}{2} + \lambda + \tau + n - \beta, \frac{1}{2} + \tau - \beta - \lambda - n; 1 + \tau - \beta, \frac{1}{2} + \lambda + \tau - \beta; \frac{a-b}{2a} \right) \\
[-a < b < a; \operatorname{Re} \beta > 0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
9. \int_a^b (x-a)^{\lambda-1/2} (b-x)^{\beta-1} (x+a)^{-\tau} C_n^{\lambda} \left(\frac{x}{a} \right) dx = \\
= \frac{2^n (\lambda)_n}{n! a^n} B \left(\frac{1}{2} + \lambda + n - \tau, \beta \right) (a+b)^{\beta+\lambda-\tau+n-1/2} \times
\end{aligned}$$

$$\times {}_3F_2\left(\frac{1}{2}-\lambda-n, 1-2\lambda-n, \frac{1}{2}+\tau-\beta-\lambda-n; 1-2\lambda-2n, \frac{1}{2}+\tau-\lambda-n; \frac{2a}{a+b}\right) +$$

$$+ \frac{(\tau+\lambda-1/2)_n}{n!} (2\lambda)_n \Gamma\left[\frac{\lambda+1/2}{\tau}, \tau-n-\lambda-1/2\right] (2a)^{\lambda-\tau+1/2} (a+b)^{\beta-1} \times$$

$$\times {}_3F_2\left(1-\beta, 1-\tau, \frac{3}{2}-\tau-\lambda; \frac{3}{2}+\lambda+n-\tau, \frac{3}{2}-\tau-\lambda-n; \frac{2a}{a+b}\right)$$

$$[b > a > 0; \operatorname{Re} \lambda > -1/2; \operatorname{Re} \beta > 0].$$

$$10. \int_a^b (x-a)^{\alpha-1} (b-x)^{\beta-1} (x+b)^{\lambda-1/2} C_n^\lambda\left(\frac{x}{b}\right) dx =$$

$$= \frac{(2\lambda)_n}{n!} B(\alpha, \beta) (2b)^{\lambda-1/2} (b-a)^{\alpha+\beta-1} \times$$

$$\times {}_3F_2\left(\beta, \frac{1}{2}-\lambda-n, \frac{1}{2}+\lambda+n; \alpha+\beta, \lambda+\frac{1}{2}; \frac{b-a}{2b}\right)$$

$$[b > a > 0; a, \operatorname{Re} \alpha, \operatorname{Re} \beta > 0].$$

$$11. \int_a^b (x-a)^{\alpha-1} (b-x)^{\beta-1} (x+b)^{2\lambda+n-\alpha-\beta} C_n^\lambda\left(\frac{x}{b}\right) dx =$$

$$= \frac{(2\lambda)_n}{n!} B(\alpha, \beta) (2b)^{2\lambda+n-\alpha} (b-a)^{\alpha+\beta-1} (a+b)^{-\beta} \times$$

$$\times {}_3F_2\left(\beta, 2\lambda+n, \frac{1}{2}+\lambda+n; \alpha+\beta, \lambda+\frac{1}{2}; \frac{a-b}{a+b}\right)$$

$$[b > a > 0; \operatorname{Re} \alpha, \operatorname{Re} \beta > 0].$$

$$12. \int_a^b (x-a)^{\alpha-1} (b-x)^{\lambda-1/2} (x+b)^{-\tau} C_n^\lambda\left(\frac{x}{b}\right) dx =$$

$$= \frac{(-1)^n (2\lambda)_n}{n!} B(\alpha, \tau-\alpha) (2b)^{\lambda-1/2} (a+b)^{\alpha-\tau} \times$$

$$\times {}_3F_2\left(1-\tau, \frac{1}{2}-\lambda-n, \frac{1}{2}+\lambda+n; \lambda+\frac{1}{2}, 1+\alpha-\tau; \frac{a+b}{2b}\right) +$$

$$+ \frac{(-1)^n}{n!} \left(\frac{1}{2}+\lambda+\tau-\alpha\right)_n (2\lambda)_n \Gamma\left[\frac{\lambda+1/2}{1/2+\lambda+\alpha+n-\tau}, \alpha-\tau\right] (2b)^{\alpha+\lambda-\tau-1/2} \times$$

$$\times {}_3F_2\left(1-\alpha, \frac{1}{2}+\lambda+\tau+n-\alpha, \frac{1}{2}+\tau-\lambda-\alpha-n; \frac{1}{2}+\lambda+\tau-\alpha, 1+\tau-\alpha; \frac{a+b}{2b}\right)$$

$$[b > a > 0; \operatorname{Re} \alpha > 0; \operatorname{Re} \lambda > -1/2].$$

$$13. \int_a^b (x-a)^{\tau-\lambda-n-3/2} (b-x)^{\lambda-1/2} (x+b)^{-\tau} C_n^\lambda\left(\frac{x}{b}\right) dx =$$

$$= (2\lambda)_n \Gamma\left[\frac{\lambda+1/2}{\tau}, \tau-\lambda-n-1/2\right] (2b)^{\lambda-\tau+n+1/2} (b-a)^{\tau-n-1} \times$$

$$\times (b+a)^{-\lambda-n-1/2} P_n^{(\tau-n-1, \lambda-1/2)}\left(\frac{a}{b}\right)$$

$$[b > a > 0; \operatorname{Re}(\tau-\lambda) > n+1/2; \operatorname{Re} \lambda > -1/2].$$

$$14. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\beta-1} (z-x)^{2\lambda+n-\alpha-\beta} C_n^\lambda\left(\frac{x}{a}\right) dx =$$

$$= \frac{(2\lambda)_n}{n!} B(\beta, \alpha-2\lambda-n) (2a)^{\alpha+\beta-1} (z+a)^{-\beta} (z-a)^{2\lambda+n-\alpha} \times$$

$$\times {}_3F_2\left(\beta, 2\lambda+n, \lambda+n+\frac{1}{2}; \lambda+\frac{1}{2}, 2\lambda+n+1-\alpha; \frac{z-a}{z+a}\right) +$$

$$+ \frac{(-1)^n (\lambda-\alpha+1/2)_n}{n! (\lambda+1/2)_n} \Gamma\left[\frac{\alpha, 2\lambda+n-\alpha}{2\lambda}\right] (2a)^{\alpha+\beta-1} (z+a)^{2\lambda+n-\alpha-\beta} \times$$

$$\times {}_3F_2\left(\alpha, \frac{1}{2}+\alpha-\lambda, \alpha+\beta-2\lambda-n; 1+\alpha-2\lambda-n, \frac{1}{2}+\alpha-\lambda-n; \frac{z-a}{z+a}\right)$$

[a, Re α, Re β > 0; |arg(z² - a²)| < π].

$$15. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\lambda-1/2} (z \pm x)^{-\theta} C_n^\lambda\left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^n}{n!} \left(\frac{1}{2} + \lambda - \alpha\right)_n (2\lambda)_n \Gamma\left[\frac{\alpha, \lambda+1/2}{\alpha+\lambda+n+1/2}\right] (2a)^{\alpha+\lambda-1/2} (z \mp a)^{-\theta} \times$$

$$\times {}_3F_2\left(\alpha, \theta, \frac{1}{2} + \alpha - \lambda; \frac{1}{2} + \alpha - \lambda - n, \frac{1}{2} + \alpha + \lambda + n; \frac{2a}{a \mp z}\right)$$

[a, Re α > 0; Re λ > -1/2; |arg(z² - a²)| < π].

$$16. \int_a^\infty x^{\alpha-1} (x^2-a^2)^{\beta-1} (x^2-b^2)^{\lambda-1/2} C_{2n+\varepsilon}^\lambda\left(\frac{x}{b}\right) dx =$$

$$= \frac{2^{2n+\varepsilon-1} a^{\alpha+2\beta+2\lambda+\varepsilon+2n-3}}{(2n+\varepsilon)! b^{2n+\varepsilon}} (\lambda)_{2n+\varepsilon} \times$$

$$\times B\left(\beta, \frac{3-\alpha-\varepsilon}{2} - \beta - \lambda - n\right) {}_3F_2\left(\frac{1}{2} - \lambda - n, \frac{3-\alpha-\varepsilon}{2} - \beta - \lambda - n, \right.$$

$$\left. 1 - \lambda - \varepsilon - n; \frac{3-\alpha-\varepsilon}{2} - \lambda - n, 1 - \lambda - \varepsilon - 2n; \frac{b^2}{a^2}\right)$$

[ε = 0 или 1; a > b > 0; Re β > 0; Re(α + 2λ + 2β) < 3 - 2n - ε].

$$17. \int_a^\infty x^{\alpha-1} (x^2-a^2)^{\lambda-1/2} (x^2-b^2)^{\beta-1} C_{2n+\varepsilon}^\lambda\left(\frac{x}{a}\right) dx =$$

$$= \frac{a^{2+2\beta+2\lambda-\varepsilon}}{2(2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{3+\varepsilon-\alpha}{2} - \beta\right)_n \Gamma\left[\frac{\lambda+1/2, (3-\alpha-\varepsilon)/2 - \beta - \lambda - n}{2 - \beta - (\alpha+\varepsilon)/2}\right] \times$$

$$\times {}_3F_2\left(1 - \beta, \frac{3-\alpha-\varepsilon}{2} - \beta - \lambda - n, \frac{3+\varepsilon-\alpha}{2} - \beta + n; 2 - \beta - \frac{\alpha}{2}, \frac{3-\alpha}{2} - \beta; \frac{b^2}{a^2}\right)$$

[ε = 0 или 1; a > b > 0; Re λ > -1/2; Re(α + 2β + 2λ) < 3 - 2n - ε].

$$18. \int_a^\infty \frac{x^{\alpha-1}}{(x^2+z^2)^\rho} (x^2-a^2)^{\lambda-1/2} C_{2n+\varepsilon}^\lambda\left(\frac{x}{a}\right) dx =$$

$$= \frac{a^{\alpha+2\lambda-2\rho-1}}{2(2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{1+\varepsilon-\alpha}{2} + \rho\right)_n \Gamma\left[\frac{\lambda+1/2, (1-\alpha-\varepsilon)/2 + \rho - \lambda - n}{1 + \rho - (\alpha+\varepsilon)/2}\right] \times$$

$$\times {}_3F_2\left(\rho, \frac{1+\varepsilon-\alpha}{2} + \rho + n, \frac{1-\alpha-\varepsilon}{2} + \rho - \lambda - n; 1 + \rho - \frac{\alpha}{2}, \frac{1-\alpha}{2} + \rho; -\frac{z^2}{a^2}\right)$$

[ε = 0 или 1; a, Re z > 0; Re λ > -1/2; Re(α + 2λ - 2ρ) < 1 - 2n - ε].

$$19. \int_a^\infty (x-a)^{\lambda-1/2} (x+a)^{-\tau} (x+z)^{-\theta} C_n^\lambda\left(\frac{x}{a}\right) dx =$$

$$= \frac{(\lambda+\theta+\tau-1/2)_n}{n!} (2\lambda)_n \Gamma\left[\frac{\lambda+1/2, \theta+\tau-\lambda-n-1/2}{\theta+\tau}\right] (2a)^{1/2+\lambda-\theta-\tau} \times$$

$$\times {}_3F_2\left(\theta, \theta+\tau-\lambda-n-\frac{1}{2}, \theta+\tau+\lambda+n-\frac{1}{2}; \theta+\tau, \theta+\tau+\lambda-\frac{1}{2}; \frac{a-z}{2a}\right)$$

[a > 0; Re λ > -1/2; Re(λ - θ - τ) < -n - 1/2; |arg(z - a)| < π].

$$\begin{aligned}
20. \int_a^\infty (x-a)^{\alpha-1} (x+a)^{\lambda-1/2} (x+z)^{-\theta} G_n^\lambda \left(\frac{x}{a} \right) dx = \\
= \frac{(2\lambda)_n}{n!} B(\alpha, \theta-\alpha) (2a)^{\lambda-1/2} (a+z)^{\alpha-\theta} \times \\
\times {}_3F_2 \left(\alpha, \frac{1}{2} + \lambda + n, \frac{1}{2} - \lambda - n; \frac{1}{2} + \lambda, 1 + \alpha - \theta; \frac{a+z}{2a} \right) + \\
+ (-1)^n \frac{(1/2 + \lambda + \theta - \alpha)_n}{n!} (2\lambda)_n \Gamma \left[\frac{\alpha - \theta, 1/2 + \theta - \alpha - \lambda - n}{1/2 - \lambda} \right] (2a)^{\alpha + \lambda - \theta - 1/2} \times \\
\times {}_3F_2 \left(\frac{1}{2} + \lambda + \theta + n - \alpha, \frac{1}{2} + \theta - \lambda - \alpha - n, \theta; 1 + \theta - \alpha, \frac{1}{2} + \lambda + \theta - \alpha; \frac{a+z}{2a} \right) \\
[a, \operatorname{Re} \alpha > 0; \operatorname{Re} (\alpha + \lambda - \theta) < 1/2 - n; |\arg (a+z)| < \pi].
\end{aligned}$$

$$\begin{aligned}
21. \int_a^\infty (x-a)^{\alpha-1} (x-b)^{\lambda-1/2} (x+b)^{-\tau} C_n^\lambda \left(\frac{x}{b} \right) dx = \\
= \frac{2^n (\lambda)_n}{n! b^n} B \left(\alpha, \frac{1}{2} + \tau - \alpha - \lambda - n \right) (a+b)^{\alpha + \lambda + n - \tau - 1/2} \times \\
\times {}_3F_2 \left(\frac{1}{2} - \lambda - n, 1 - 2\lambda - n, \frac{1}{2} + \tau - \lambda - \alpha - n; \frac{1}{2} + \tau - \lambda - n, 1 - 2\lambda - 2n; \frac{2b}{a+b} \right) \\
[a > b > 0; \operatorname{Re} \alpha > 0; \operatorname{Re} (\tau - \lambda - \alpha) > n - 1/2].
\end{aligned}$$

$$\begin{aligned}
22. \int_a^\infty (x-a)^{\alpha-1} (x-b)^{-\tau} (x+b)^{\lambda-1/2} C_n^\lambda \left(\frac{x}{b} \right) dx = \\
= \frac{(2\lambda)_n}{n!} B(\alpha, \tau - \alpha) (2b)^{\lambda-1/2} (a-b)^{\alpha-\tau} \times \\
\times {}_3F_2 \left(1 - \tau, \frac{1}{2} + \lambda + n, \frac{1}{2} - \lambda - n; \lambda + \frac{1}{2}, 1 + \alpha - \tau; \frac{b-a}{2b} \right) + \\
+ (-1)^n \frac{(1/2 + \tau + \lambda - \alpha)_n}{n!} (2\lambda)_n \Gamma \left[\frac{\alpha - \tau, 1/2 + \tau - \alpha - \lambda - n}{1/2 - \lambda} \right] (2b)^{\alpha + \lambda - \tau - 1/2} \times \\
\times {}_3F_2 \left(\beta, \frac{1}{2} + \tau + \lambda + n - \alpha, \frac{1}{2} + \tau - \alpha - \lambda - n; 1 + \tau - \alpha, \frac{1}{2} + \lambda + \tau - \alpha; \frac{b-a}{2b} \right) \\
[a > b > 0; \operatorname{Re} \alpha > 0; \operatorname{Re} (\tau - \alpha - \lambda) > n - 1/2].
\end{aligned}$$

2.21.5. Интегралы от $A(x) e^{-px} {}^m C_n^\lambda(cx)$.

$$1. \int_{-a}^a (a^2 - x^2)^{\lambda-1/2} e^{-px} C_n^\lambda \left(\frac{x}{a} \right) dx = \frac{(-1)^n 2\pi}{n!} \Gamma \left[\frac{2\lambda + n}{\lambda} \right] \left(\frac{a}{2p} \right)^\lambda I_{\lambda+n}(ap) \\
[a > 0; \operatorname{Re} \lambda > -1/2].$$

$$\begin{aligned}
2. \int_{-a}^a (x+a)^{\lambda-1/2} (a-x)^{\beta-1} e^{-px} C_n^\lambda \left(\frac{x}{a} \right) dx = \\
= \frac{(\lambda - \beta + 1/2)_n}{n!} (2\lambda)_n \Gamma \left[\frac{\beta, \lambda + 1/2}{\beta + \lambda + n + 1/2} \right] (2a)^{\beta + \lambda - 1/2} e^{-ap} \times \\
\times {}_3F_2 \left(\beta, \beta - \lambda + \frac{1}{2}; \beta + \lambda + n + \frac{1}{2}, \beta - \lambda - n + \frac{1}{2}; 2ap \right) \\
[a, \operatorname{Re} \beta > 0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$3. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\lambda-1/2} e^{-px} C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^n}{n!} (2\lambda)_n \left(\lambda - \alpha + \frac{1}{2} \right)_n \Gamma \left[\alpha, \lambda + 1/2 \right]_{\alpha + \lambda + n + 1/2} (2a)^{\alpha + \lambda - 1/2} e^{ap} \times$$

$$\times {}_2F_2 \left(\alpha, \alpha - \lambda + \frac{1}{2}; \alpha + \lambda + n + \frac{1}{2}, \alpha - \lambda - n + \frac{1}{2}; -2ap \right)$$

[a, Re $\alpha > 0$; Re $\lambda > -1/2$]

$$4. \int_0^\infty x^{\alpha-1} e^{-px} C_{2n+\varepsilon}^\lambda(cx) dx = \frac{(-1)^n (2c)^\varepsilon}{n! p^{\alpha+\varepsilon}} (\lambda)_{n+\varepsilon} \Gamma(\alpha + \varepsilon) \times$$

$$\times {}_4F_1 \left(-n, \frac{\alpha+1}{2}, \frac{\alpha}{2} + \varepsilon, n + \lambda + \varepsilon; \varepsilon + \frac{1}{2}; \frac{4c^2}{p^2} \right)$$

[$\varepsilon = 0$ или 1; Re $p > 0$; Re $\alpha > -\varepsilon$]

$$5. \int_{\pm a}^\infty (x \mp a)^{\alpha-1} e^{-px} C_n^\lambda \left(\frac{x}{a} \right) dx = \frac{2^n (\lambda)_n \Gamma(\alpha + n)}{n! a^n p^{\alpha+n}} e^{\mp ap} \times$$

$$\times {}_2F_2 \left(-n, \frac{1}{2} - \lambda - n; 1 - \alpha - n, 1 - 2\lambda - 2n; \pm 2ap \right)$$

[a, Re p , Re $\alpha > 0$]

$$6. \int_{-a}^\infty (x+a)^{\lambda-1/2} e^{-px} C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^n 2^{1-2\lambda} \sqrt{\pi}}{n! p^{\lambda+1/2}} \Gamma \left[\begin{matrix} 2\lambda + n \\ \lambda \end{matrix} \right] e^{ap} {}_2F_0 \left(-n, 2\lambda + n; \frac{1}{2ap} \right)$$

[a, Re $p > 0$; Re $\lambda > -1/2$]

$$7. \int_a^\infty (x-a)^{\lambda-1/2} (x+a)^{-\tau} e^{-px} C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(\tau + \lambda - 1/2)_n (2\lambda)_n}{n!} \Gamma \left[\begin{matrix} \lambda + 1/2, \tau - \lambda - n - 1/2 \\ \tau \end{matrix} \right] (2a)^{\lambda - \tau + 1/2} e^{ap} \times$$

$$\times {}_2F_2 \left(1 - \tau, \frac{3}{2} - \tau - \lambda; \frac{3}{2} + \lambda + n - \tau, \frac{3}{2} - \tau - \lambda - n; -2ap \right) +$$

$$+ \frac{2^n (\lambda)_n \Gamma(\lambda - \tau + n + 1/2)}{n! (2a)^n} p^{\tau - \lambda - n - 1/2} e^{ap} \times$$

$$\times {}_2F_2 \left(1 - 2\lambda - n, \frac{1}{2} - \lambda - n; 1 - 2\lambda - 2n, \frac{1}{2} + \tau - \lambda - n; -2ap \right)$$

[a, Re $p > 0$; Re $\lambda > -1/2$]

$$8. \int_a^\infty (x-a)^{\alpha-1} (x+a)^{\lambda-1/2} e^{-px} C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^n (\lambda - \alpha + 1/2)_n (2\lambda)_n}{n!} \Gamma \left[\begin{matrix} \alpha, 1/2 - \alpha - \lambda - n \\ 1/2 - \lambda \end{matrix} \right] (2a)^{\alpha + \lambda - 1/2} e^{-ap} \times$$

$$\times {}_2F_2 \left(\alpha, \frac{1}{2} + \alpha - \lambda; \frac{1}{2} + \alpha - \lambda - n, \frac{1}{2} + \alpha + \lambda + n; 2ap \right) +$$

$$+ \frac{2^n (\lambda)_n \Gamma(\alpha + \lambda + n - 1/2)}{n! a^n} p^{1/2 - \alpha - \lambda - n} e^{-ap} \times$$

$$\times {}_2F_2 \left(\frac{1}{2} - \lambda - n, 1 - 2\lambda - n; \frac{3}{2} - \alpha - \lambda - n, 1 - 2\lambda - 2n; 2ap \right)$$

[a, Re p , Re $\alpha > 0$]

$$9. \int_0^{\infty} x^{\alpha-1} e^{-px^2} C_{2n+\varepsilon}^{\lambda}(ax) dx =$$

$$= \frac{(-1)^n 2^{\varepsilon-1} c^{\varepsilon} (\lambda)_{n+\varepsilon}}{p^{(\alpha+\varepsilon)/2} n!} \Gamma\left(\frac{\alpha+\varepsilon}{2}\right) {}_2F_1\left(-n, \frac{\alpha+\varepsilon}{2}, \lambda+n+\varepsilon; \varepsilon+\frac{1}{2}; \frac{c^2}{p}\right)$$

[$\varepsilon=0$ или 1 ; $\operatorname{Re} p > 0$; $\operatorname{Re} \alpha > -\varepsilon$].

$$10. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\lambda-1/2} e^{-px^2} C_{2n+\varepsilon}^{\lambda}\left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^n a^{\alpha+2\lambda-1}}{2 (2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{1+\varepsilon-\alpha}{2}\right)_n \Gamma\left[\lambda+1/2, \frac{(\alpha+\varepsilon)/2}{(1+\alpha+\varepsilon)/2+\lambda+n}\right] \times$$

$$\times {}_2F_2\left(\frac{\alpha}{2}, \frac{1+\alpha}{2}; \frac{1+\alpha-\varepsilon}{2}-n, \frac{1+\alpha+\varepsilon}{2}+\lambda+n; -a^2 p\right)$$

[$\varepsilon=0$ или 1 ; $a > 0$; $\operatorname{Re} \alpha > -\varepsilon$; $\operatorname{Re} \lambda > -1/2$].

$$11. \int_a^{\infty} x^{\alpha-1} (x^2 - a^2)^{\lambda-1/2} e^{-px^2} C_{2n+\varepsilon}^{\lambda}\left(\frac{x}{a}\right) dx =$$

$$= \frac{a^{\alpha+2\lambda-1}}{2 (2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{1+\varepsilon-\alpha}{2}\right)_n \Gamma\left[\lambda+1/2, \frac{(1-\alpha-\varepsilon)/2-\lambda-n}{1-(\alpha+\varepsilon)/2}\right] \times$$

$$\times {}_2F_2\left(\frac{\alpha}{2}, \frac{1+\alpha}{2}; \frac{1+\alpha-\varepsilon}{2}-n, \frac{1+\alpha+\varepsilon}{2}+\lambda+n; -a^2 p\right) +$$

$$+ \frac{2^{2n+\varepsilon-1} p^{(1-\alpha-\varepsilon)/2-\lambda-n}}{(2n+\varepsilon)! a^{2n+\varepsilon}} (\lambda)_{2n+\varepsilon} \Gamma\left(\frac{\alpha+\varepsilon-1}{2}+\lambda+n\right) \times$$

$$\times {}_2F_2\left(\frac{1}{2}-\lambda-n, 1-\lambda-\varepsilon-n; 1-\lambda-\varepsilon-2n, \frac{3-\alpha-\varepsilon}{2}-\lambda-n; -a^2 p\right)$$

[$\varepsilon=0$ или 1 ; $a, \operatorname{Re} p > 0$; $\operatorname{Re} \lambda > -1/2$].

$$12. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\lambda-1/2} e^{-px^{-2}} C_{2n+\varepsilon}^{\lambda}\left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^n a^{\alpha+2\lambda-1}}{2 (2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{1+\varepsilon-\alpha}{2}\right)_n \Gamma\left[\lambda+1/2, \frac{(\alpha+\varepsilon)/2}{(1+\alpha+\varepsilon)/2+\lambda+n}\right] \times$$

$$\times {}_2F_2\left(\frac{1-\alpha-\varepsilon}{2}-\lambda-n, \frac{1+\varepsilon-\alpha}{2}+n; \frac{1-\alpha}{2}, 1-\frac{\alpha}{2}; -\frac{p}{a^2}\right) +$$

$$+ \frac{(-1)^n 2^{\varepsilon-1}}{n!} a^{2\lambda-\varepsilon-1} p^{(\alpha+\varepsilon)/2} (\lambda)_{n+\varepsilon} \Gamma\left(-\frac{\alpha+\varepsilon}{2}\right) \times$$

$$\times {}_2F_2\left(\frac{1}{2}-\lambda-n, \frac{1}{2}+\varepsilon+n; \frac{1}{2}+\varepsilon, 1+\frac{\alpha+\varepsilon}{2}; -\frac{p}{a^2}\right)$$

[$\varepsilon=0$ или 1 ; $a, \operatorname{Re} p > 0$; $\operatorname{Re} \lambda > -1/2$].

$$13. \int_a^{\infty} x^{\alpha-1} (x^2 - a^2)^{\lambda-1/2} e^{-px^{-2}} C_{2n+\varepsilon}^{\lambda}\left(\frac{x}{a}\right) dx =$$

$$= \frac{a^{\alpha+2\lambda-1}}{2 (2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{1+\varepsilon-\alpha}{2}\right)_n \Gamma\left[\lambda+1/2, \frac{(1-\alpha-\varepsilon)/2-\lambda-n}{1-(\alpha+\varepsilon)/2}\right] \times$$

$$\times {}_2F_2\left(\frac{1-\alpha-\varepsilon}{2}-\lambda-n, \frac{1+\varepsilon-\alpha}{2}+n; \frac{1-\alpha}{2}, 1-\frac{\alpha}{2}; -\frac{p}{a^2}\right)$$

[$\varepsilon=0$ или 1 ; $a > 0$; $\operatorname{Re} \lambda > -1/2$; $\operatorname{Re}(\alpha+2\lambda) < 1-2n-\varepsilon$].

2.21.6. Интегралы от $A(x) e^{\psi(x)} C_n^\lambda(cx)$.

$$\begin{aligned}
 1. \int_{-a}^a (x+a)^{-\beta-n-1} (a-x)^{\beta-1} \exp\left(-p \frac{(a-x)^\varepsilon}{a+x}\right) C_n^\lambda\left(\frac{x}{a}\right) dx = \\
 = \frac{(-1)^n}{n!} (2\lambda)_n \Gamma(\beta+n) (2a)^{\beta-\beta\varepsilon-n\varepsilon-1} p^{-\beta-n} \exp\left(\frac{\varepsilon p - p}{2a}\right) \times \\
 \times {}_2F_2\left(-n, \frac{1}{2}-\lambda-n; 1-\beta-n, \frac{1}{2}+\lambda; \frac{p}{(2a)^{1-\varepsilon}}\right) \\
 [\varepsilon=0 \text{ или } 1; a, \operatorname{Re} p, \operatorname{Re} \beta > 0].
 \end{aligned}$$

$$\begin{aligned}
 2. \int_{-a}^a (x+a)^{2\lambda-\beta+n-1} (a-x)^{\beta-1} \exp\left(-p \frac{(a-x)^\varepsilon}{a+x}\right) C_n^\lambda\left(\frac{x}{a}\right) dx = \\
 = \frac{(1/2+\lambda-\beta)_n}{n! (\lambda+1/2)_n} \Gamma\left[\begin{matrix} \beta, 2\lambda+n-\beta \\ 2\lambda \end{matrix}\right] (2a)^{2\lambda+n-1} \exp\left(\frac{\varepsilon p - p}{2a}\right) \times \\
 \times {}_2F_2\left(\beta, \frac{1}{2}+\beta-\lambda; 1+\beta-2\lambda-n, \frac{1}{2}+\beta-\lambda-n; \frac{p}{(2a)^{1-\varepsilon}}\right) + \\
 + \frac{(-1)^n (2\lambda)_n}{n!} \Gamma(\beta-2\lambda-n) (2a)^{(2\lambda-\beta+n)\varepsilon+\beta-1} p^{2\lambda-\beta+n} \exp\left(\frac{\varepsilon p - p}{2a}\right) \times \\
 \times {}_2F_2\left(\frac{1}{2}+\lambda+n, 2\lambda+n; \frac{1}{2}+\lambda, 1+2\lambda+n-\beta; \frac{p}{(2a)^{1-\varepsilon}}\right) \\
 [\varepsilon=0 \text{ или } 1; a, \operatorname{Re} p, \operatorname{Re} \beta > 0].
 \end{aligned}$$

$$\begin{aligned}
 3. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\lambda-1/2} \exp\left(-p \frac{(a-x)^\varepsilon}{a+x}\right) C_n^\lambda\left(\frac{x}{a}\right) dx = \\
 = \frac{(-1)^n}{n!} \left(\frac{1}{2}+\lambda-\alpha\right)_n (2\lambda)_n \Gamma\left[\begin{matrix} \alpha, \lambda+1/2 \\ \alpha+\lambda+n+1/2 \end{matrix}\right] (2a)^{\alpha+\lambda-1/2} e^{\varepsilon p} \times \\
 \times {}_2F_2\left(\frac{1}{2}-\alpha-n-\lambda, \frac{1}{2}+\lambda+n-\alpha; 1-\alpha, \frac{1}{2}+\lambda-\alpha; -\frac{p}{(2a)^{1-\varepsilon}}\right) + \\
 + \frac{(-1)^n}{n!} (2\lambda)_n \Gamma(-\alpha) (2a)^{\alpha\varepsilon+\lambda-1/2} p^\alpha e^{\varepsilon p} \times \\
 \times {}_2F_2\left(\frac{1}{2}+\lambda+n, \frac{1}{2}-\lambda-n; \lambda+\frac{1}{2}, \alpha+1; -\frac{p}{(2a)^{1-\varepsilon}}\right) \\
 [\varepsilon=0 \text{ или } 1; a, \operatorname{Re} p > 0; \operatorname{Re} \lambda > -1/2].
 \end{aligned}$$

$$\begin{aligned}
 4. \int_{-a}^a (x+a)^{n+2\lambda-\beta-1} (a-x)^{\beta-1} \exp\left(-p \frac{(a+x)^\varepsilon}{a-x}\right) C_n^\lambda\left(\frac{x}{a}\right) dx = \\
 = \frac{(1/2+\lambda-\beta)_n}{n! (\lambda+1/2)_n} \Gamma\left[\begin{matrix} \beta, 2\lambda+n-\beta \\ 2\lambda \end{matrix}\right] (2a)^{2\lambda+n-1} \exp\left(\frac{\varepsilon p - p}{2a}\right) \times \\
 \times {}_2F_2\left(\frac{1}{2}+\lambda+n-\beta, 2\lambda+n-\beta; 1-\beta, \frac{1}{2}+\lambda-\beta; \frac{p}{(2a)^{1-\varepsilon}}\right) + \\
 + \frac{(2\lambda)_n}{n!} \Gamma(-\beta) (2a)^{2\lambda+n-\beta+\beta\varepsilon-1} p^\beta \exp\left(\frac{\varepsilon p - p}{2a}\right) \times \\
 \times {}_2F_2\left(\frac{1}{2}+\lambda+n, 2\lambda+n; \beta+1, \frac{1}{2}+\lambda; \frac{p}{(2a)^{1-\varepsilon}}\right) \\
 [\varepsilon=0 \text{ или } 1; a, \operatorname{Re} p > 0; \operatorname{Re}(\beta-2\lambda) < n].
 \end{aligned}$$

$$5. \int_{-a}^a (x+a)^{-\beta-n-1} (a-x)^{\beta-1} \exp\left(-p \frac{(a+x)^{\varepsilon}}{a-x}\right) C_n^{\lambda}\left(\frac{x}{a}\right) dx =$$

$$= \frac{(2\lambda)_n \Gamma(-\beta)}{n! (2a)^{\beta-\beta\varepsilon+n+1}} p^{\beta} \exp\left(\frac{\varepsilon p - p}{2a}\right) {}_2F_2\left(-n, \frac{1}{2}-\lambda-n; \frac{1}{2}+\lambda, 1+\beta; \frac{p}{(2a)^{1-\varepsilon}}\right)$$

[$\varepsilon=0$ или 1 ; $a, \operatorname{Re} p > 0$; $\operatorname{Re} \beta < -n$].

$$6. \int_{-a}^a (x+a)^{\lambda-1/2} (a-x)^{-\lambda-n-3/2} \exp\left(-p \frac{(a+x)^{\varepsilon}}{a-x}\right) C_n^{\lambda}\left(\frac{x}{a}\right) dx =$$

$$= \frac{(2\lambda)_n \Gamma(\lambda+1/2)}{(2a)^{(\lambda+n+1/2)\varepsilon-\lambda+1/2} p^{\lambda+n+1/2}} \exp\left(\frac{\varepsilon p - p}{2a}\right) L_n^{\lambda-1/2}\left(\frac{p}{(2a)^{1-\varepsilon}}\right)$$

[$\varepsilon=0$ или 1 ; $a, \operatorname{Re} p > 0$; $\operatorname{Re} \lambda > -1/2$].

$$7. \int_a^{\infty} (x-a)^{\lambda-1/2} (x+a)^{-\tau} \exp\left(-p \frac{(a-x)^{\varepsilon}}{a+x}\right) C_n^{\lambda}\left(\frac{x}{a}\right) dx =$$

$$= \frac{(\tau+\lambda-1/2)_n}{n!} (2\lambda)_n \Gamma\left[\begin{matrix} \lambda+1/2, \tau-\lambda-n-1/2 \\ \tau \end{matrix}\right] (2a)^{\lambda-\tau+1/2} \times$$

$$\times e^{\varepsilon p} {}_2F_2\left(\tau-\lambda-n-\frac{1}{2}, \tau+\lambda+n-\frac{1}{2}; \tau, \tau+\lambda-\frac{1}{2}; -\frac{p}{(2a)^{1-\varepsilon}}\right)$$

[$\varepsilon=0$ или 1 ; $a > 0$; $\operatorname{Re} \lambda > -1/2$; $\operatorname{Re}(\tau-\lambda) > n+1/2$].

$$8. \int_a^{\infty} (x-a)^{\alpha-1} (x+a)^{\lambda-1/2} \exp\left(-p \frac{(x+a)^{\varepsilon}}{x-a}\right) C_n^{\lambda}\left(\frac{x}{a}\right) dx =$$

$$= (-1)^n \frac{(1/2+\lambda-\alpha)_n}{n!} (2\lambda)_n \Gamma\left[\begin{matrix} \alpha, 1/2-\alpha-\lambda-n \\ 1/2-\lambda \end{matrix}\right] (2a)^{\alpha+\lambda-1/2} \times$$

$$\times e^{-\varepsilon p} {}_2F_2\left(\frac{1}{2}-\alpha-\lambda-n, \frac{1}{2}+n+\lambda-\alpha; 1-\alpha, \frac{1}{2}+\lambda-\alpha; \frac{p}{(2a)^{1-\varepsilon}}\right) +$$

$$+ \frac{(2\lambda)_n}{n!} \Gamma(-\alpha) (2a)^{\alpha+\lambda-1/2} p^{\alpha} e^{-\varepsilon p} \times$$

$$\times {}_2F_2\left(\frac{1}{2}-\lambda-n, \frac{1}{2}+\lambda+n; \frac{1}{2}+\lambda, \alpha+1; \frac{p}{(2a)^{1-\varepsilon}}\right)$$

[$\varepsilon=0$ или 1 ; $a, \operatorname{Re} p > 0$; $\operatorname{Re}(\alpha+\lambda) < 1/2-n$].

$$9. \int_{-a}^a (x+a)^{\lambda-1/2} (a-x)^{\beta-1} e^{-p\sqrt{a-x}} C_n^{\lambda}\left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^n (\lambda-\beta+1/2)_n}{n!} (2\lambda)_n \Gamma\left[\begin{matrix} \beta, \lambda+1/2 \\ \beta+\lambda+n+1/2 \end{matrix}\right] (2a)^{\beta+\lambda-1/2} \times$$

$$\times {}_2F_2\left(\beta, \beta-\lambda+\frac{1}{2}; \beta+\lambda+n+\frac{1}{2}, \beta-\lambda-n+\frac{1}{2}, \frac{1}{2}; \frac{ap^2}{2}\right) -$$

$$- \frac{(-1)^n (\lambda-\beta)_n}{n!} (2\lambda)_n \Gamma\left[\begin{matrix} \beta+1/2, \lambda+1/2 \\ \beta+\lambda+n+1 \end{matrix}\right] (2a)^{\beta+\lambda} p \times$$

$$\times {}_2F_2\left(\beta+\frac{1}{2}, \beta-\lambda+1; \frac{3}{2}, \beta+\lambda+n+1, \beta-\lambda-n+1; \frac{ap^2}{2}\right)$$

[$a, \operatorname{Re} \beta > 0$; $\operatorname{Re} \lambda > -1/2$].

$$\begin{aligned}
10. \quad & \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\lambda-1/2} \exp\left(-\frac{p}{\sqrt{x+a}}\right) C_n^\lambda\left(\frac{x}{a}\right) dx = \\
& = \frac{(-1)^n}{n!} \left(\lambda - \alpha + \frac{1}{2}\right)_n (2\lambda)_n \Gamma\left[\alpha, \lambda + 1/2\right] (2a)^{\alpha+\lambda-1/2} \times \\
& \times {}_2F_3\left(\frac{1}{2} - \lambda - \alpha - n, \frac{1}{2} + \lambda + n - \alpha; 1 - \alpha, \frac{1}{2} + \lambda - \alpha, \frac{1}{2}; \frac{p^2}{8a}\right) + \\
& + \frac{(-1)^n (2\lambda)_n}{n!} 2^{\lambda+1/2} \Gamma(-2\alpha) a^{\lambda-1/2} p^{2\alpha} \times \\
& \times {}_2F_3\left(\frac{1}{2} + \lambda + n, \frac{1}{2} - \lambda - n; \frac{1}{2} + \lambda, 1 + \alpha, \frac{1}{2} + \alpha; \frac{p^2}{8a}\right) - \\
& - \frac{(-1)^n}{n!} (\lambda - \alpha + 1)_n (2\lambda)_n \Gamma\left[\alpha - 1/2, \lambda + 1/2\right] (2a)^{\alpha+\lambda-1} p \times \\
& \times {}_2F_3\left(1 - \alpha - \lambda - n, 1 + \lambda + n - \alpha; \frac{3}{2} - \alpha, 1 + \lambda - \alpha, \frac{3}{2}; \frac{p^2}{8a}\right) \\
& [a, \operatorname{Re} p > 0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
11. \quad & \int_{-a}^a (x+a)^{-\beta-n-1} (a-x)^{\beta-1} \exp\left(-p \sqrt{\frac{a-x}{a+x}}\right) C_n^\lambda\left(\frac{x}{a}\right) dx = \\
& = \frac{(-1)^n (2\lambda)_n \Gamma(2\beta + 2n)}{n! 2^n a^{n+1} p^{2\beta+2n}} {}_2F_3\left(-n, \frac{1}{2} - \lambda - n; \lambda + \frac{1}{2}, 1 - \beta - n, \frac{1}{2} - \beta - n; -\frac{p^2}{4}\right) \\
& [a, \operatorname{Re} p, \operatorname{Re} \beta > 0].
\end{aligned}$$

$$\begin{aligned}
12. \quad & \int_{-a}^a (x+a)^{n+2\lambda-\beta-1} (a-x)^{\beta-1} \exp\left(-p \sqrt{\frac{a-x}{a+x}}\right) C_n^\lambda\left(\frac{x}{a}\right) dx = \\
& = \frac{(-1)^n (2\lambda)_n}{n!} 2^{2\lambda+n} \Gamma(2\beta - 4\lambda - 2n) a^{2\lambda+n-1} p^{2(2\lambda+n-\beta)} \times \\
& \times {}_2F_3\left(2\lambda + n, \lambda + n + \frac{1}{2}; \lambda + \frac{1}{2}, 1 + n + 2\lambda - \beta, \frac{1}{2} + n + 2\lambda - \beta; -\frac{p^2}{4}\right) + \\
& + \frac{(1/2 + \lambda - \beta)_n}{n! (\lambda + 1/2)_n} \Gamma\left[\beta, 2\lambda + n - \beta\right] (2a)^{2\lambda+n-1} \times \\
& \times {}_2F_3\left(\beta, \frac{1}{2} + \beta - \lambda; 1 + \beta - 2\lambda - n, \frac{1}{2} + \beta - \lambda - n, \frac{1}{2}; -\frac{p^2}{4}\right) - \\
& - \frac{(\lambda - \beta)_n}{n! (\lambda + 1/2)_n} \Gamma\left[\beta + 1/2, 2\lambda - \beta + n - 1/2\right] (2a)^{2\lambda+n-1} p \times \\
& \times {}_2F_3\left(\beta + \frac{1}{2}, 1 + \beta - \lambda; \frac{3}{2}, 1 + \beta - \lambda - n, \frac{3}{2} + \beta - 2\lambda - n; -\frac{p^2}{4}\right) \\
& [a, \operatorname{Re} p, \operatorname{Re} \beta > 0].
\end{aligned}$$

$$\begin{aligned}
13. \quad & \int_{-a}^a (x+a)^{-\beta-n-1} (a-x)^{\beta-1} \exp\left(-p \sqrt{\frac{a+x}{a-x}}\right) C_n^\lambda\left(\frac{x}{a}\right) dx = \\
& = \frac{(2\lambda)_n}{n! 2^n a^{n+1}} p^{2\beta} \Gamma(-2\beta) {}_2F_3\left(-n, \frac{1}{2} - n - \lambda; \lambda + \frac{1}{2}, \beta + 1, \beta + \frac{1}{2}; -\frac{p^2}{4}\right) \\
& [a, \operatorname{Re} p > 0; \operatorname{Re} \beta < -n].
\end{aligned}$$

$$\begin{aligned}
14. \int_{-a}^a (x+a)^{n+2\lambda-\beta-1} (a-x)^{\beta-1} \exp\left(-p \sqrt{\frac{a+x}{a-x}}\right) C_n^\lambda\left(\frac{x}{a}\right) dx = \\
= \frac{(1/2+\lambda-\beta)_n}{n! (\lambda+1/2)_n} \Gamma\left[\begin{matrix} \beta, 2\lambda+n-\beta \\ 2\lambda \end{matrix}\right] (2a)^{2\lambda+n-1} \times \\
\times {}_2F_3\left(\frac{1}{2}+n+\lambda-\beta, 2\lambda+n-\beta; \frac{1}{2}, 1-\beta, \frac{1}{2}+\lambda-\beta; -\frac{p^2}{4}\right) + \\
+ \frac{(2\lambda)_n}{n!} 2^{2\lambda+n} \Gamma(-2\beta) a^{2\lambda+n-1} p^{2\beta} \times \\
\times {}_2F_3\left(\lambda+n+\frac{1}{2}, 2\lambda+n; 1+\beta, \frac{1}{2}+\beta, \frac{1}{2}+\lambda; -\frac{p^2}{4}\right) - \\
- \frac{(1+\lambda-\beta)_n (2a)^{2\lambda+n-1} p}{n! (\lambda+1/2)_n} \Gamma\left[\begin{matrix} \beta-1/2, 2\lambda-\beta+n+1/2 \\ 2\lambda \end{matrix}\right] \times \\
\times {}_2F_3\left(\lambda-\beta+n+1, 2\lambda-\beta+n+\frac{1}{2}; \frac{3}{2}-\beta, 1+\lambda-\beta, \frac{3}{2}; -\frac{p^2}{4}\right) \\
[a, \operatorname{Re} p > 0; \operatorname{Re}(\beta-2\lambda) < n].
\end{aligned}$$

$$\begin{aligned}
15. \int_{\pm a}^{\infty} (x \mp a)^{\alpha-1} e^{-p \sqrt{x \mp a}} C_n^\lambda\left(\frac{x}{a}\right) dx = \frac{2^{n+1} (\lambda)_n \Gamma(2\alpha+2n)}{n! a^n p^{2\alpha+2n}} \times \\
\times {}_2F_3\left(-n, \frac{1}{2}-n-\lambda; 1-2\lambda-2n, 1-\alpha-n, \frac{1}{2}-\alpha-n; \mp \frac{ap^2}{2}\right) \\
[a, \operatorname{Re} p, \operatorname{Re} \alpha > 0].
\end{aligned}$$

$$\begin{aligned}
16. \int_{\pm a}^{\infty} (x \mp a)^{\alpha-1} \exp\left(-\frac{p}{\sqrt{x \mp a}}\right) C_n^\lambda\left(\frac{x}{a}\right) dx = 2 \frac{(\pm 1)^n (2\lambda)_n}{n!} \Gamma(-2\alpha) \times \\
\times p^{2\alpha} {}_2F_3\left(-n, 2\lambda+n; \alpha+1, \alpha+\frac{1}{2}, \lambda+\frac{1}{2}; \mp \frac{p^2}{8a}\right) \\
[a, \operatorname{Re} p > 0; \operatorname{Re} \alpha < -n].
\end{aligned}$$

$$\begin{aligned}
17. \int_{-a}^{\infty} (x-a)^{\lambda-1/2} (x+a)^{-\tau} e^{-p \sqrt{x+a}} C_n^\lambda\left(\frac{x}{a}\right) dx = \\
= \frac{(\lambda+\tau-1/2)_n}{n!} (2\lambda)_n \Gamma\left[\begin{matrix} \lambda+1/2, \tau-\lambda-n-1/2 \\ \tau \end{matrix}\right] (2a)^{\lambda-\tau+1/2} \times \\
\times {}_2F_3\left(1-\tau, \frac{3}{2}-\tau-\lambda; \frac{3}{2}-\tau-\lambda-n, \frac{3}{2}+\lambda+n-\tau, \frac{1}{2}; \frac{ap^2}{2}\right) + \\
+ \frac{2^{n+1} (\lambda)_n \Gamma(1+2n+2\lambda-2\tau)}{n! a^n} p^{2\tau-2\lambda-2n-1} \times \\
\times {}_2F_3\left(1-2\lambda-n, \frac{1}{2}-\lambda-n; 1-2\lambda-2n, \frac{1}{2}+\tau-\lambda-n, \tau-\lambda-n; \frac{ap^2}{2}\right) - \\
- \frac{(\tau+\lambda-1)_n}{n!} (2\lambda)_n \Gamma\left[\begin{matrix} \lambda+1/2, \tau-\lambda-n-1 \\ \tau-1/2 \end{matrix}\right] (2a)^{1+\lambda-\tau} p \times \\
\times {}_2F_3\left(\frac{3}{2}-\tau, 2-\tau-\lambda; \frac{3}{2}, 2+n+\lambda-\tau, 2-\lambda-\tau-n; \frac{ap^2}{2}\right) \\
[a, \operatorname{Re} p > 0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
18. \int_a^\infty (x-a)^{\alpha-1} (x+a)^{\lambda-1/2} e^{-p\sqrt{x-a}} C_n^\lambda \left(\frac{x}{a} \right) dx = \\
= \frac{2^{n+1} (\lambda)_n \Gamma(2\alpha+2\lambda+2n-1)}{n! a^n p^{2\alpha+2\lambda+2n-1}} \times \\
\times {}_2F_3 \left(\frac{1}{2} - \lambda - n, 1 - 2\lambda - n; 1 - 2\lambda - 2n, \frac{3}{2} - \alpha - \lambda - n, 1 - \alpha - \lambda - n; -\frac{ap^2}{2} \right) + \\
+ \frac{(-1)^n (1/2 + \lambda - \alpha)_n (2\lambda)_n}{n!} \Gamma \left[\frac{\alpha}{1/2 - \lambda}, \frac{1/2 - \alpha - \lambda - n}{1/2 - \lambda} \right] (2a)^{\alpha+\lambda-1/2} \times \\
\times {}_2F_3 \left(\alpha, \frac{1}{2} + \alpha - \lambda; \frac{1}{2}, \frac{1}{2} + \alpha - \lambda - n, \frac{1}{2} + \alpha + \lambda + n; -\frac{ap^2}{2} \right) - \\
- \frac{(-1)^n (\lambda - \alpha)_n (2\lambda)_n}{n!} \Gamma \left[\frac{\alpha + 1/2}{1/2 - \lambda}, -\alpha - \lambda - n \right] (2a)^{\alpha+\lambda} p \times \\
\times {}_2F_3 \left(\alpha + \frac{1}{2}, 1 + \alpha - \lambda; \frac{3}{2}, 1 + \alpha - \lambda - n, 1 + \alpha + \lambda + n; -\frac{ap^2}{2} \right) \\
[a, \operatorname{Re} p, \operatorname{Re} \alpha > 0].
\end{aligned}$$

$$\begin{aligned}
19. \int_a^\infty (x-a)^{\alpha-1} (x+a)^{\lambda-1/2} \exp \left(-\frac{p}{\sqrt{x-a}} \right) C_n^\lambda \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^n (\lambda - \alpha + 1/2)_n (2\lambda)_n}{n!} \Gamma \left[\frac{\alpha}{1/2 - \lambda}, \frac{1/2 - \alpha - \lambda - n}{1/2 - \lambda} \right] (2a)^{\alpha+\lambda-1/2} \times \\
\times {}_2F_3 \left(\frac{1}{2} - \alpha - \lambda - n, \frac{1}{2} + \lambda + n - \alpha; 1 - \alpha, \frac{1}{2} + \lambda - \alpha, \frac{1}{2}; -\frac{p^2}{8a} \right) + \\
+ \frac{2^{\lambda+1/2} (2\lambda)_n}{n!} \Gamma(-2\alpha) a^{\lambda-1/2} p^{2\alpha} \times \\
\times {}_2F_3 \left(\frac{1}{2} - \lambda - n, \frac{1}{2} + \lambda + n; \alpha + 1, \alpha + \frac{1}{2}, \lambda + \frac{1}{2}; -\frac{p^2}{8a} \right) \\
[a, \operatorname{Re} p > 0; \operatorname{Re}(\alpha + \lambda) < 1/2 - n].
\end{aligned}$$

$$\begin{aligned}
20. \int_a^\infty (x-a)^{\lambda-1/2} (x+a)^{-\tau} \exp \left(-\frac{p}{\sqrt{x+a}} \right) C_n^\lambda \left(\frac{x}{a} \right) dx = \\
= \frac{(\tau + \lambda - 1/2)_n (2\lambda)_n}{n!} \Gamma \left[\frac{\lambda + 1/2}{\tau}, \frac{\tau - \lambda - n - 1/2}{\tau} \right] (2a)^{\lambda-\tau+1/2} \times \\
\times {}_2F_3 \left(\tau - \lambda - n - \frac{1}{2}, \tau + \lambda + n - \frac{1}{2}; \frac{1}{2}, \tau, \tau + \lambda - \frac{1}{2}; \frac{p^2}{8a} \right) - \\
- \frac{(\lambda + \tau)_n (2\lambda)_n}{n!} \Gamma \left[\frac{\lambda + 1/2}{\tau + 1/2}, \frac{\tau - \lambda - n}{\tau + 1/2} \right] (2a)^{\lambda-\tau} p \times \\
\times {}_2F_3 \left(\tau - \lambda - n, \tau + \lambda + n; \tau + \frac{1}{2}, \tau + \lambda, \frac{3}{2}; \frac{p^2}{8a} \right) \\
[a > 0; \operatorname{Re} \lambda > -1/2; \operatorname{Re}(\tau - \lambda) > \tau + 1/2].
\end{aligned}$$

2.21.7. Интегралы от $A(x) \left\{ \frac{\sin \varphi(x)}{\cos \varphi(x)} \right\} C_n^\lambda(cx)$.

Обозначение: $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$.

$$\begin{aligned}
1. \int_0^a (a^2 - x^2)^{\lambda-1/2} \left\{ \frac{\sin bx C_{2n+1}^\lambda(x/a)}{\cos bx C_{2n}^\lambda(x/a)} \right\} dx = \\
= \frac{(-1)^n \pi}{(2n + \delta)!} \left(\frac{a}{2b} \right)^\lambda \Gamma \left[\frac{2\lambda + 2n + \delta}{\lambda} \right] J_{2n+\lambda+\delta}(ab) \\
[a > 0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$2. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\lambda-1/2} \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} C_{2n+\varepsilon}^{\lambda} \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^n}{2(2n+\varepsilon)!} a^{\alpha+2\lambda+\varepsilon-1/2} (2\lambda)_{2n+\varepsilon} \left(\frac{1+\varepsilon-\delta-\alpha}{2} \right)_n \Gamma \left[\frac{\lambda+1/2, (\alpha+\delta+\varepsilon)/2}{(1+\alpha+\delta+\varepsilon)/2+\lambda+n} \right] \times$$

$$\times {}_2F_3 \left(\frac{\alpha}{2} + \delta, \frac{1+\alpha}{2}; \frac{1+\alpha+\delta-\varepsilon}{2} - n, \delta + \frac{1}{2}, \frac{1+\alpha+\delta+\varepsilon}{2} + \lambda + n; -\frac{a^2 b^2}{4} \right)$$

$$[\varepsilon=0 \text{ или } 1; a>0; \operatorname{Re} \alpha > -\varepsilon-\delta; \operatorname{Re} \lambda > -1/2].$$

$$\begin{aligned} 3. \quad & \int_a^{\infty} x^{\alpha-1} (x^2 - a^2)^{\lambda-1/2} \left\{ \begin{matrix} \sin bx \\ \cos bx \end{matrix} \right\} C_{2n+\varepsilon}^{\lambda} \left(\frac{x}{a} \right) dx = \\ & = \frac{a^{\alpha+2\lambda+\delta-1} b^{\delta}}{2 (2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{1+\varepsilon-\alpha-\delta}{2} \right)_n \Gamma \left[\begin{matrix} \lambda+1/2, (1-\alpha-\delta-\varepsilon)/2 - \lambda - n \\ 1 - (\alpha+\delta+\varepsilon)/2 \end{matrix} \right] \times \\ & \times {}_2F_3 \left(\frac{\alpha}{2} + \delta, \frac{\alpha+1}{2}; \frac{1+\alpha+\delta-\varepsilon}{2} - n, \frac{1+\alpha+\delta+\varepsilon}{2} + \lambda + n, \delta + \frac{1}{2}; -\frac{a^2 b^2}{4} \right) \mp \\ & \mp \frac{(-1)^n 2^{2n+\varepsilon} b^{1-\alpha-2\lambda-\varepsilon-2n}}{(2n+\varepsilon)! a^{2n+\varepsilon}} (\lambda)_{2n+\varepsilon} \Gamma(\alpha+\varepsilon+2\lambda+2n-1) \left\{ \begin{matrix} \cos [(\alpha+\varepsilon+2\lambda) \pi/2] \\ \sin [(\alpha+\varepsilon+2\lambda) \pi/2] \end{matrix} \right\} \times \\ & \times {}_2F_3 \left(\frac{1}{2} - \lambda - n, 1 - \lambda - \varepsilon - n; 1 - \lambda - \varepsilon - 2n, \frac{3-\alpha}{2} - \varepsilon - \lambda - n, \right. \\ & \left. 1 - \frac{\alpha}{2} - \lambda - n; -\frac{a^2 b^2}{4} \right) \end{aligned}$$

$$[\varepsilon=0 \text{ или } 1; a, b>0; \operatorname{Re} \lambda > -1/2; \operatorname{Re}(\alpha + 2\lambda) < 2 - 2n - \varepsilon].$$

$$\begin{aligned}
 4. \quad & \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\lambda-1/2} \left\{ \begin{matrix} \sin(b\sqrt{x+a}) \\ \cos(b\sqrt{x+a}) \end{matrix} \right\} C_n^\lambda\left(\frac{x}{a}\right) dx = \\
 & = \frac{(-1)^n (2\lambda)_n}{n!} \left(\frac{1-\delta}{2} - \alpha + \lambda\right)_n \Gamma\left[\frac{\alpha+\delta/2, \lambda+1/2}{\alpha+n+\lambda+(\delta+1)/2}\right] (2a)^{\alpha+\lambda+(\delta-1)/2} b^\delta \times \\
 & \quad \times {}_2F_3\left(\alpha+\frac{\delta}{2}, \frac{\delta+1}{2}+\alpha-\lambda; \frac{1+\delta}{2}+\alpha+\lambda+n, \right. \\
 & \quad \left. \frac{1+\delta}{2}+\alpha-\lambda-n, \delta+\frac{1}{2}; -\frac{ab^2}{2}\right) \quad [a>0; \operatorname{Re}\alpha>-\delta/2; \operatorname{Re}\lambda>-1/2].
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\lambda-1/2} \left\{ \begin{matrix} \sin(b/\sqrt{x+a}) \\ \cos(b/\sqrt{x+a}) \end{matrix} \right\} C_n^\lambda\left(\frac{x}{a}\right) dx = \\
 & = \frac{(-1)^n}{n!} (2\lambda)_n \left(\frac{1+\delta}{2} + \lambda - \alpha\right)_n \Gamma\left[\frac{\lambda+1/2, \alpha-\delta/2}{\alpha+\lambda+n+(1-\delta)/2}\right] (2a)^{\alpha+\lambda-(\delta+1)/2} b^\delta \times \\
 & \times {}_2F_3\left(\frac{1+\delta}{2} - \alpha - \lambda - n, \frac{1+\delta}{2} + \lambda + n - \alpha; 1 + \frac{\delta}{2} - \alpha, \frac{1+\delta}{2} + \lambda - \alpha, \right. \\
 & \quad \left. \delta + \frac{1}{2}; -\frac{b^2}{8a}\right) + \frac{(-1)^{n+\delta} 2^{\lambda+1/2} (2\lambda)_n}{n!} a^{\lambda-1/2} b^{2\alpha} \left\{ \begin{matrix} \sin \alpha\pi \\ \cos \alpha\pi \end{matrix} \right\} \times \\
 & \times {}_2F_3\left(\lambda + n + \frac{1}{2}, \lambda - n - \frac{1}{2}; \alpha + \frac{1}{2}, \alpha + 1, \lambda + \frac{1}{2}; -\frac{b^2}{8a}\right)
 \end{aligned}$$

$$\{a, b \geq 0; \operatorname{Re} \alpha, \operatorname{Re} \lambda \geq -1/2\}.$$

$$\begin{aligned}
6. \quad & \int_{-a}^a (x+a)^{2\lambda+n-\beta-1} (a-x)^{\beta-1} \left\{ \frac{\sin(b\sqrt{a-x}/\sqrt{a+x})}{\cos(b\sqrt{a-x}/\sqrt{a+x})} \right\} C_n^\lambda\left(\frac{x}{a}\right) dx = \\
& = \frac{(-1)^n (2\lambda)_n}{n!} 2^{2\lambda+n} \Gamma(2\beta-4\lambda-2n) \left\{ \frac{\sin(\beta-2\lambda)\pi}{\cos(\beta-2\lambda)\pi} \right\} a^{2\lambda+n-1} b^{4\lambda+2n-2\beta} \times \\
& \times {}_2F_3\left(2\lambda+n, \lambda+n+\frac{1}{2}; \lambda+\frac{1}{2}, 2\lambda-\beta+n+1, 2\lambda-\beta+n+\frac{1}{2}; \frac{b^2}{4}\right) + \\
& + \frac{1}{n! (\lambda+1/2)_n} \left(\frac{1-\delta}{2} + \lambda - \beta\right)_n \Gamma\left[\frac{\beta+\delta/2}{2\lambda}, 2\lambda+n-\beta-\delta/2\right] (2a)^{2\lambda+n-1} \times \\
& \times b^\delta {}_2F_3\left(\beta+\frac{\delta}{2}, \frac{1+\delta}{2} + \beta - \lambda; \frac{1+\delta}{2} + \beta - \lambda - n, 1 + \frac{\delta}{2} + \beta - 2\lambda - n, \delta + \frac{1}{2}; \frac{b^2}{4}\right) \\
& [a, b > 0; \operatorname{Re} \beta > -\delta/2; \operatorname{Re}(\beta-2\lambda) < n+1/2].
\end{aligned}$$

$$\begin{aligned}
7. \quad & \int_a^\infty (x-a)^{\lambda-1/2} (x+a)^{-\tau} \left\{ \frac{\sin(b\sqrt{x+a})}{\cos(b\sqrt{x+a})} \right\} C_n^\lambda\left(\frac{x}{a}\right) dx = \\
& = \frac{(2\lambda)_n}{n!} \left(\tau + \lambda - \frac{\delta+1}{2}\right)_n \Gamma\left[\frac{\lambda+1/2}{\tau-\delta/2}, \tau-\lambda-n-(\delta+1)/2\right] (2a)^{(1+\delta)/2+\lambda-\tau} b^\delta \times \\
& \times {}_2F_3\left(1+\frac{\delta}{2}-\tau, \frac{3+\delta}{2}-\tau-\lambda; \frac{3+\delta}{2}+\lambda+n-\tau, \frac{3+\delta}{2}-\lambda-\tau-n, \delta+\frac{1}{2}; -\frac{ab^2}{2}\right) + \\
& + \frac{(-1)^n 2^{n+1} (\lambda)_n}{n! a^n} \left\{ \frac{\cos(\tau-\lambda)\pi}{\sin(\tau-\lambda)\pi} \right\} b^{2\tau-2\lambda-2n-1} \Gamma(2n+2\lambda-2\tau+1) \times \\
& \times {}_2F_3\left(\frac{1}{2}-\lambda-n, 1-2\lambda-n; 1-2\lambda-2n, \frac{1}{2}+\tau-\lambda-n, \tau-\lambda-n; -\frac{ab^2}{2}\right) \\
& [b > 0; \operatorname{Re} \lambda > -1/2; \operatorname{Re}(\tau-\lambda) > n].
\end{aligned}$$

$$\begin{aligned}
8. \quad & \int_a^\infty (x-a)^{\alpha-1} (x+a)^{\lambda-1/2} \left\{ \frac{\sin(b\sqrt{x-a})}{\cos(b\sqrt{x-a})} \right\} C_n^\lambda\left(\frac{x}{a}\right) dx = \\
& = \mp \frac{(-1)^n 2^{n+1} (\lambda)_n}{n! a^n b^{2\alpha+2\lambda+2n-1}} \Gamma(2\alpha+2\lambda+2n-1) \left\{ \frac{\cos(\alpha+\lambda)\pi}{\sin(\alpha+\lambda)\pi} \right\} \times \\
& \times {}_2F_3\left(\frac{1}{2}-\lambda-n, 1-2\lambda-n; 1-2\lambda-2n, \frac{3}{2}-\alpha-\lambda-n, 1-\alpha-\lambda-n; \frac{ab^2}{2}\right) + \\
& + \frac{(-1)^n}{n!} \left(\frac{1-\delta}{2} + \lambda - \alpha\right)_n (2\lambda)_n \times \\
& \times \Gamma\left[\frac{\alpha+\delta/2}{1/2-\lambda}, (1-\delta)/2-\lambda-\alpha-n\right] 2^{\alpha+\lambda-1/2} a^{\alpha+\lambda+(\delta-1)/2} b^\delta \times \\
& \times {}_2F_3\left(\alpha+\frac{\delta}{2}, \frac{1+\delta}{2} + \alpha - \lambda; \frac{1+\delta}{2} + \alpha - \lambda - n, \frac{1+\delta}{2} + \alpha + \lambda + n, \delta + \frac{1}{2}; \frac{ab^2}{2}\right) \\
& [b > 0; \operatorname{Re} \alpha > -\delta/2; \operatorname{Re}(\alpha+\lambda) < 1-n].
\end{aligned}$$

$$\begin{aligned}
9. \quad & \int_a^\infty (x-a)^{\lambda-1/2} (x+a)^{-\tau} \left\{ \frac{\sin(b/\sqrt{x+a})}{\cos(b/\sqrt{x+a})} \right\} C_n^\lambda\left(\frac{x}{a}\right) dx = \\
& = \frac{(2\lambda)_n}{n!} \left(\tau + \lambda + \frac{\delta-1}{2}\right)_n \Gamma\left[\frac{\lambda+1/2}{\tau+\delta/2}, \tau-\lambda-n+(\delta-1)/2\right] (2a)^{\lambda-\tau+(1-\delta)/2} b^\delta \times \\
& \times {}_2F_3\left(\tau-\lambda-n+\frac{\delta-1}{2}, \tau+\lambda+n+\frac{\delta-1}{2}; \tau+\frac{\delta}{2}, \right. \\
& \left. \tau+\lambda+\frac{\delta-1}{2}, \delta+\frac{1}{2}; -\frac{b^2}{8a}\right) \\
& [\operatorname{Re} \lambda > -1/2; \operatorname{Re}(\lambda-\tau) < (\delta-1)/2-n].
\end{aligned}$$

$$\begin{aligned}
10. \int_a^\infty (x-a)^{\alpha-1} (x+a)^{\lambda-1/2} \left\{ \begin{matrix} \sin(b/\sqrt{x-a}) \\ \cos(b/\sqrt{x-a}) \end{matrix} \right\} C_n^\lambda\left(\frac{x}{a}\right) dx = \\
= \frac{(-1)^n (2\lambda)_n}{n!} \Gamma\left[\alpha-\delta/2, (1+\delta)/2-\alpha-\lambda-n\right] \left(\frac{1+\delta}{2}+\lambda-\alpha\right)_n \times \\
\times (2a)^{\alpha+\lambda-(\delta+1)/2} {}_2F_3\left(\frac{1+\delta}{2}+n+\lambda-\alpha, \frac{1+\delta}{2}-\alpha-\lambda-n; \right. \\
\left. \frac{1+\delta}{2}+\lambda-\alpha, 1+\frac{\delta}{2}-\alpha, \delta+\frac{1}{2}; \frac{b^2}{8a}\right) \mp \frac{(2\lambda)_n}{n!} \Gamma(-2\alpha) \left\{ \begin{matrix} \sin \alpha\pi \\ \cos \alpha\pi \end{matrix} \right\} 2^{\lambda+1/2} a^{\lambda-1/2} b^{2\alpha} \times \\
\times {}_2F_3\left(\frac{1}{2}-\lambda-n, \frac{1}{2}+\lambda+n; \frac{1}{2}+\lambda, \frac{1}{2}+\alpha, 1+\alpha; \frac{b^2}{8a}\right) \\
[b>0; \operatorname{Re}(\alpha+\lambda)<(1+\delta)/2-n; \operatorname{Re}\alpha>-1/2].
\end{aligned}$$

2.21.8. Интегралы от $A(x) \ln \varphi(x) C_n^\lambda(cx)$.

$$\begin{aligned}
1. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\lambda-1/2} \ln \frac{x+a}{2a} C_n^\lambda\left(\frac{x}{a}\right) dx = \\
= \frac{(-1)^n (2\lambda)_n}{n!} \left(\frac{1}{2}+\lambda-\alpha\right)_n \Gamma\left[\lambda+1/2, \alpha\right] (2a)^{\alpha+\lambda-1/2} \times \\
\times \left[\psi(\alpha) - \psi\left(\frac{1}{2}+\alpha+\lambda+n\right) - \sum_{j=0}^{n-1} \frac{2}{2j-2\alpha+2\lambda+1} \right] \\
[a, \operatorname{Re}\alpha>0; \operatorname{Re}\lambda>-1/2].
\end{aligned}$$

$$\begin{aligned}
2. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\lambda-1/2} \left\{ \begin{matrix} \ln(x+z) \\ \ln|x-z| \end{matrix} \right\} C_n^\lambda\left(\frac{x}{a}\right) dx = \\
= \pm \frac{(-1)^n (2\lambda)_n}{n! (z-a)} \left(\lambda-\alpha-\frac{1}{2}\right)_n \Gamma\left[\lambda+1/2, \alpha-1\right] (2a)^{\alpha+\lambda+1/2} \times \\
\times {}_4F_3\left(1, 1, \frac{3}{2}+\alpha-\lambda, \frac{3}{2}-\alpha-\lambda-n; 2, \frac{3}{2}+\alpha-\lambda-n, 2-\alpha; \mp \frac{2a}{z-a}\right) + \\
+ \frac{(-1)^n}{n!} (2a)^{\alpha+\lambda-1/2} (2\lambda)_n \left(\lambda-\alpha+\frac{1}{2}\right)_n \Gamma\left[\alpha, \lambda+1/2\right] \ln z \\
[a, \operatorname{Re}\alpha>0; \operatorname{Re}\lambda>-1/2, \left\{ \begin{matrix} |\arg(z-a)|<\pi \\ z>-a \end{matrix} \right\}].
\end{aligned}$$

$$\begin{aligned}
3. \int_a^\infty (x-a)^{\lambda-1/2} (x+a)^{-\tau} \left\{ \begin{matrix} \ln(x+z) \\ \ln|x-z| \end{matrix} \right\} C_n^\lambda\left(\frac{x}{a}\right) dx = \\
= \pm \frac{(2\lambda)_n}{n! (z-a)} \left(\tau+\lambda-\frac{3}{2}\right)_n \Gamma\left[\lambda+1/2, \tau-\lambda-n-3/2\right] (2a)^{3/2+\lambda-\tau} \times \\
\times {}_4F_3\left(1, 1, \frac{5}{2}-\tau-\lambda, 2-\tau; 2, \frac{5}{2}-\tau-\lambda-n, \frac{5}{2}+\lambda+n-\tau; \mp \frac{2a}{z-a}\right) - \\
- \frac{(\mp 1)^n 2^{n+1} \pi (\lambda)_n (z-a)^{\lambda-\tau+n+1/2}}{n! (2\tau-2\lambda-2n-1) a^n} \left\{ \begin{matrix} \sec(\tau-\lambda)\pi \\ \operatorname{tg}(\tau-\lambda)\pi \end{matrix} \right\} {}_3F_2\left(\tau-\lambda-n-\frac{1}{2}, \right. \\
\left. \frac{1}{2}-\lambda-n, 1-2\lambda-n; \frac{1}{2}+\tau-\lambda-n, 1-2\lambda-2n; \mp \frac{2a}{z-a}\right) + \\
+ \frac{(2a)^{\lambda-\tau+1/2}}{n!} (2\lambda)_n (\lambda+\tau-1/2)_n \Gamma\left[\lambda+1/2, \tau-\lambda-n-1/2\right] \ln z \\
[a>0; \operatorname{Re}\lambda>-1/2; \operatorname{Re}(\tau-\lambda)>n+1/2, \left\{ \begin{matrix} |\arg(z-a)|<\pi \\ z>-a \end{matrix} \right\}].
\end{aligned}$$

$$\begin{aligned}
4. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\lambda-1/2} \left\{ \frac{\ln(x^2 + z^2)}{\ln|x^2 - z^2|} \right\} C_{2n+\varepsilon}^{\lambda} \left(\frac{x}{a} \right) dx = \\
= \pm \frac{(-1)^n a^{\alpha+2\lambda+1}}{2(2n+\varepsilon)! z^2} (2\lambda)_{2n+\varepsilon} \left(\frac{\varepsilon - \alpha - 1}{2} \right)_n \Gamma \left[\frac{\lambda + 1/2, (\alpha + \varepsilon)/2 + 1}{(3 + \alpha + \varepsilon)/2 + \lambda + n} \right] \times \\
\times {}_4F_3 \left(1, 1, \frac{\alpha}{2} + 1, \frac{3 + \alpha}{2}; 2, \frac{3 + \alpha - \varepsilon}{2} - n, \frac{3 + \alpha + \varepsilon}{2} + \lambda + n; \mp \frac{a^2}{z^2} \right) + \\
+ \frac{(-1)^n a^{\alpha+2\lambda-1}}{(2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{1 + \varepsilon - \alpha}{2} \right)_n \Gamma \left[\frac{\lambda + 1/2, (\alpha + \varepsilon)/2}{(1 + \alpha + \varepsilon)/2 + \lambda + n} \right] \ln z \\
\left[\varepsilon = 0 \text{ или } 1; a > 0; \operatorname{Re} \alpha > -\varepsilon; \operatorname{Re} \lambda > -1/2, \left\{ \begin{array}{l} \operatorname{Re} z > 0 \\ z > 0 \end{array} \right\} \right].
\end{aligned}$$

$$\begin{aligned}
5. \int_a^{\infty} x^{\alpha-1} (x^2 - a^2)^{\lambda-1/2} \left\{ \frac{\ln(x^2 + z^2)}{\ln|x^2 - z^2|} \right\} C_{2n+\varepsilon}^{\lambda} \left(\frac{x}{a} \right) dx = \\
= \pm \frac{a^{\alpha+2\lambda-3/2}}{2(2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{3 + \varepsilon - \alpha}{2} \right)_n \Gamma \left[\frac{\lambda + 1/2, (3 - \alpha - \varepsilon)/2 - \lambda - n}{2 - (\alpha + \varepsilon)/2} \right] \times \\
\times {}_4F_3 \left(1, 1, \frac{3 - \alpha - \varepsilon}{2} - \lambda - n, \frac{3 + \varepsilon - \alpha}{2} + n; 2, 2 - \frac{\alpha}{2}, \frac{3 - \alpha}{2}; \mp \frac{z^2}{a^2} \right) - \\
- \frac{a^{\alpha+2\lambda-1}}{2(2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{1 + \varepsilon - \alpha}{2} \right)_n \Gamma \left[\frac{\lambda + 1/2, (1 - \alpha - \varepsilon)/2 - \lambda - n}{1 - (\alpha + \varepsilon)/2} \right] \times \\
\times \left[\psi \left(\frac{1 - \alpha - \varepsilon}{2} - \lambda - n \right) - \psi \left(1 - \frac{\alpha + \varepsilon}{2} \right) + \sum_{j=0}^{n-1} \frac{2}{2j - \alpha + \varepsilon + 1} - 2 \ln a \right] \\
\left[\varepsilon = 0 \text{ или } 1; a > 0; \operatorname{Re} \lambda > -1/2; \operatorname{Re}(\alpha + 2\lambda) < 1 - 2n - \varepsilon, \left\{ \begin{array}{l} \operatorname{Re} z > 0 \\ z > 0 \end{array} \right\} \right].
\end{aligned}$$

2.21.9. Интегралы от $A(x) \operatorname{Ei}(a + bx^m) C_n^{\lambda}(cx)$.

$$\begin{aligned}
1. \int_{-\frac{1}{2}}^a (x+a)^{\alpha-1} (a-x)^{\lambda-1/2} \operatorname{Ei}(-ab - bx) C_n^{\lambda} \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^{n+1} (2\lambda)_n}{n!} \left(\lambda - \alpha - \frac{1}{2} \right)_n \Gamma \left[\frac{\alpha + 1, \lambda + 1/2}{\alpha + \lambda + n + 3/2} \right] (2a)^{\alpha+\lambda+1/2} b \times \\
\times {}_4F_4 \left(1, 1, \alpha + 1, \alpha - \lambda + \frac{3}{2}; 2, 2, \alpha - \lambda - n + \frac{3}{2}, \alpha + \lambda + n + \frac{3}{2}; -2ab \right) - \\
- \frac{(-1)^n}{n!} (2\lambda)_n \left(\lambda - \alpha + \frac{1}{2} \right)_n \Gamma \left[\frac{\alpha, \lambda + 1/2}{\alpha + \lambda + n + 1/2} \right] (2a)^{\alpha+\lambda-1/2} \times \\
\times \left[C + \psi(\alpha) - \psi \left(\alpha + \lambda + n + \frac{1}{2} \right) - \sum_{j=0}^{n-1} \frac{2}{2j - 2\alpha + 2\lambda + 1} + \ln(2ab) \right] \\
[a, \operatorname{Re} \alpha > 0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
2. \int_a^{\infty} (x-a)^{\lambda-1/2} (x+a)^{-\tau} \operatorname{Ei}(-ab - bx) C_n^{\lambda} \left(\frac{x}{a} \right) dx = \\
= - \frac{(2\lambda)_n}{n!} \left(\lambda + \tau - \frac{3}{2} \right)_n \Gamma \left[\frac{\lambda + 1/2, \tau - \lambda - n - 3/2}{\tau - 1} \right] (2a)^{\lambda-\tau+3/2} b \times \\
\times {}_4F_4 \left(1, 1, \frac{5}{2} - \lambda - \tau, 2 - \tau; 2, 2, \frac{5}{2} - \lambda - \tau - n, \frac{5}{2} + \lambda + n - \tau; -2ab \right) + \\
+ \frac{(\lambda)_n 2^{n+1} b^{\tau-\lambda-n-1/2}}{n! (2\tau - 2\lambda - 2n - 1) a^n} \Gamma \left(\frac{1}{2} + n + \lambda - \tau \right) {}_2F_2 \left(\tau - \lambda - n - \frac{1}{2}, \frac{1}{2} - \lambda - n, \right.
\end{aligned}$$

$$1+2\lambda-n, \frac{1}{2}+\tau-\lambda-n, \frac{1}{2}+\tau-\lambda-n, 1-2\lambda-2n; -2ab) +$$

$$+ \frac{(2\lambda)_n}{n!} \left(\tau + \lambda - \frac{1}{2} \right)_n \Gamma \left[\frac{\lambda+1/2, \tau-\lambda-n-1/2}{\tau} \right] \times$$

$$\times (2a)^{\lambda-\tau+1/2} \left[\zeta + \psi(\tau) - \psi \left(\tau - \lambda - n - \frac{1}{2} \right) - \sum_{j=0}^{n-1} \frac{2}{2j+2\lambda+2\tau-1} + \ln(2ab) \right]$$

[a, Re b > 0; Re λ > -1/2]

$$3. \int_0^a x^{\alpha-1} (a^2-x^2)^{\lambda-1/2} \operatorname{Ei}(-bx^2) C_{2n+\varepsilon}^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^{n+1} a^{\alpha+2\lambda+1} b}{2(2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{\varepsilon-\alpha-1}{2} \right)_n \Gamma \left[\frac{\lambda+1/2, (\alpha+\varepsilon)/2+1}{(3+\alpha+\varepsilon)/2+\lambda+n} \right] \times$$

$$\times {}_4F_4 \left(1, 1, \frac{3+\alpha}{2}, \frac{\alpha}{2}+1; 2, 2, \frac{3+\alpha+\varepsilon}{2}+\lambda+n, \frac{3+\alpha-\varepsilon}{2}-n; -a^2b \right) +$$

$$+ \frac{(-1)^n a^{\alpha+2\lambda-1}}{2(2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{1+\varepsilon-\alpha}{2} \right)_n \Gamma \left[\frac{\lambda+1/2, (\alpha+\varepsilon)/2}{(1+\alpha+\varepsilon)/2+\lambda+n} \right] \times$$

$$\times \left[\zeta + \psi \left(\frac{\alpha+\varepsilon}{2} \right) - \psi \left(\frac{\alpha+\varepsilon+1}{2} + \lambda + n \right) - \sum_{j=0}^{n-1} \frac{2}{2j-\alpha+\varepsilon+1} + \ln(a^2b) \right]$$

[ε=0 или 1; α>0; Re α > -ε; Re λ > -1/2; |arg b| < π].

$$4. \int_a^\infty x^{\alpha-1} (x^2-a^2)^{\lambda-1/2} \operatorname{Ei}(-bx^2) C_{2n+\varepsilon}^\lambda \left(\frac{x}{a} \right) dx =$$

$$= -\frac{a^{\alpha+2\lambda+1} b}{2(2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{\varepsilon-\alpha-1}{2} \right)_n \Gamma \left[\frac{\lambda+1/2, -(\alpha+\varepsilon+1)/2-\lambda-n}{-(\alpha+\varepsilon)/2} \right] \times$$

$$\times {}_4F_4 \left(1, 1, 1+\frac{\alpha}{2}, \frac{3+\alpha}{2}; 2, 2, \frac{3+\alpha-\varepsilon}{2}-n, \frac{3+\alpha+\varepsilon}{2}+\lambda+n; -a^2b \right) +$$

$$+ \frac{a^{\alpha+2\lambda-1}}{2(2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{1+\varepsilon-\alpha}{2} \right)_n \Gamma \left[\frac{\lambda+1/2, (1-\alpha-\varepsilon)/2-\lambda-n}{1-(\alpha+\varepsilon)/2} \right] \times$$

$$\times \left[\zeta - \psi \left(\frac{1-\alpha-\varepsilon}{2} - \lambda - n \right) + \psi \left(1 - \frac{\alpha+\varepsilon}{2} \right) - \sum_{j=0}^{n-1} \frac{2}{1-\alpha+\varepsilon+2j} + \ln(a^2b) \right] +$$

$$+ \frac{2^{2n+\varepsilon} b^{(1-\alpha-\varepsilon)/2-\lambda-n} (\lambda)_{2n+\varepsilon}}{(2n+\varepsilon)! (1-\alpha-\varepsilon-2\lambda-2n) a^{2n+\varepsilon}} \Gamma \left(\frac{\alpha+\varepsilon-1}{2} + \lambda + n \right) \times$$

$$\times {}_3F_3 \left(\frac{1}{2} - \lambda - n, 1 - \lambda - \varepsilon - n, \frac{1-\alpha-\varepsilon}{2} - \lambda - n; 1 - \lambda - \varepsilon - 2n, \right.$$

$$\left. \frac{3-\alpha-\varepsilon}{2} - \lambda - n, \frac{3-\alpha-\varepsilon}{2} - \lambda - n; -a^2b \right)$$

[ε=0 или 1; a, Re b > 0; Re λ > -1/2].

2.21.10. Интегралы от $A(x) \begin{Bmatrix} \operatorname{si}(\varphi(x)) \\ \operatorname{ci}(\varphi(x)) \end{Bmatrix} C_n^\lambda(cx)$.

Обозначение: $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$.

$$1. \int_0^a x^{\alpha-1} (a^2-x^2)^{\lambda-1/2} \begin{Bmatrix} \operatorname{si}(bx) \\ \operatorname{ci}(bx) \end{Bmatrix} C_{2n+\varepsilon}^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^{n+1} a^{\alpha+2\lambda+0+1}}{2^{1-\delta} 3^{2\delta} (2n+\varepsilon)!} b^{\delta+2} (2\lambda)_{2n+\varepsilon} \left(\frac{\varepsilon-\alpha-\delta-1}{2} \right)_n \times$$

$$\begin{aligned}
& \times \Gamma \left[\lambda + 1/2, (\alpha + \varepsilon + \delta)/2 + 1 \right] {}_4F_5 \left(1, 1 + \frac{\delta}{2}, \frac{\alpha}{2} + \delta + 1, \frac{3 + \alpha}{2}; \right. \\
& 2, \frac{\alpha + \varepsilon + \delta + 3}{2} + \lambda + n, 2 + \frac{\delta}{2}, \frac{3}{2} + \delta, \frac{3 + \alpha + \delta - \varepsilon}{2} - n; \left. -\frac{a^2 b^2}{4} \right) + \\
& + \frac{(-1)^n a^{\alpha + 2\lambda + \delta - 1}}{2(2n + \varepsilon)!} b^\delta (2\lambda)_{2n + \varepsilon} \left(\frac{1 + \varepsilon - \delta - \alpha}{2} \right)_n \times \\
& \times \Gamma \left[\lambda + 1/2, (\alpha + \delta + \varepsilon)/2 \right] \left\{ \begin{matrix} 1 \\ R \end{matrix} \right\} - \delta \frac{(-1)^n \pi}{4(2n + \varepsilon)!} a^{\alpha + 2\lambda - 1} \times \\
& \times (2\lambda)_{2n + \varepsilon} \left(\frac{1 + \varepsilon - \alpha}{2} \right)_n \Gamma \left[\lambda + 1/2, (\alpha + \varepsilon)/2 \right] \\
& R = C + \frac{1}{2} \psi \left(\frac{\alpha + \varepsilon}{2} \right) - \frac{1}{2} \psi \left(\frac{1 + \alpha + \varepsilon}{2} + \lambda + n \right) - \sum_{j=0}^{n-1} \frac{1}{2j - \alpha + \varepsilon + 1} + \ln(ab) \\
& [\varepsilon = 0 \text{ или } 1; a > 0; \operatorname{Re} \alpha > -\varepsilon, \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
2. \int_a^\infty x^{\alpha-1} (x^2 - a^2)^{\lambda-1/2} \left\{ \begin{matrix} \operatorname{si}(bx) \\ \operatorname{ci}(bx) \end{matrix} \right\} C_{2n+\varepsilon}^\lambda \left(\frac{x}{a} \right) dx = \\
= - \frac{a^{\alpha+2\lambda+\delta+1} b^{\delta+2}}{2^{3-\delta} 3^{2\delta} (2+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{\varepsilon - \alpha - \delta - 1}{2} \right)_n \times \\
\times \Gamma \left[\lambda + 1/2, -(1 + \alpha + \delta + \varepsilon)/2 - \lambda - n \right] {}_4F_5 \left(1, 1 + \frac{\delta}{2}, \frac{3 + \alpha}{2}, \frac{\alpha}{2} + \delta + 1; \right. \\
2, \frac{3 + \alpha + \delta + \varepsilon}{2} + \lambda + n, \frac{3}{2} + \delta, 2 + \frac{\delta}{2}, \frac{3 + \alpha + \delta - \varepsilon}{2} - n; \left. -\frac{a^2 b^2}{4} \right) + \\
+ \frac{(-1)^n 2^{2n+\varepsilon} b^{1-\alpha-\varepsilon-2\lambda-2n} (\lambda)_{2n+\varepsilon}}{(2n + \varepsilon)! (1 - \alpha - \varepsilon - 2\lambda - 2n) a^{2n+\varepsilon}} \times \\
\times \Gamma(\alpha + \varepsilon + 2\lambda + 2n - 1) \left\{ \begin{matrix} \cos [(\alpha + \varepsilon + 2\lambda) \pi/2] \\ \sin [(\alpha + \varepsilon + 2\lambda) \pi/2] \end{matrix} \right\} \times \\
\times {}_3F_4 \left(\frac{1}{2} - \lambda - n, \frac{1 - \alpha - \varepsilon}{2} - \lambda - n, 1 - \lambda - \varepsilon - n; \frac{3 - \alpha - \varepsilon}{2} - \lambda - n, \right. \\
1 - \lambda - \varepsilon - 2n, \frac{3 - \alpha}{2} - \varepsilon - \lambda - n, 1 - \frac{\alpha}{2} - \lambda - n; \left. -\frac{a^2 b^2}{4} \right) + \\
+ \frac{a^{\alpha+2\lambda+\delta-1} b^\delta}{2(2n + \varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{1 + \varepsilon - \alpha - \delta}{2} \right)_n \Gamma \left[\lambda + 1/2, (1 - \alpha - \varepsilon - \delta)/2 - \lambda - n \right] \times \\
\times \left\{ \begin{matrix} 1 \\ R \end{matrix} \right\} - \delta \frac{a^{\alpha+2\lambda-1} \pi}{4(2n + \varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{1 + \varepsilon - \alpha}{2} \right)_n \Gamma \left[\lambda + 1/2, (1 - \alpha - \varepsilon)/2 - \lambda - n \right], \\
R = C - \frac{1}{2} \psi \left(\frac{1 - \alpha - \varepsilon}{2} - \lambda - n \right) + \frac{1}{2} \psi \left(1 - \frac{\alpha + \varepsilon}{2} \right) - \\
- \sum_{j=0}^{n-1} \frac{1}{2j - \alpha + \varepsilon + 1} + \ln(ab) \\
[\varepsilon = 0 \text{ или } 1; a, b > 0; \operatorname{Re} \lambda > -1/2, \operatorname{Re}(\alpha + 2\lambda) < 3 - 2n - \varepsilon].
\end{aligned}$$

$$\begin{aligned}
3. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\lambda-1/2} \left\{ \begin{matrix} \operatorname{si}(b\sqrt{x+a}) \\ \operatorname{ci}(b\sqrt{x+a}) \end{matrix} \right\} C_n^\lambda \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^{n+1} (2\lambda)_n}{3^{2\delta} n!} \left(\lambda - \alpha - \frac{\delta + 1}{2} \right)_n 2^{\alpha+\lambda+3(\delta-1)/2} a^{\alpha+\lambda+(\delta+1)/2} b^{\delta+2} \times \\
\times \Gamma \left[\lambda + 1/2, \alpha + \delta/2 + 1 \right] {}_4F_5 \left(1, \frac{3 + \delta}{2} + \alpha - \lambda, 1 + \frac{\delta}{2}, 1 + \alpha + \frac{\delta}{2}; \right.
\end{aligned}$$

$$2, \frac{3+\delta}{2} + \alpha + \lambda + n, \frac{3}{2} + \delta, 2 + \frac{\delta}{2}, \frac{3+\delta}{2} + \alpha - \lambda - n; -\frac{ab^2}{2} \Big) +$$

$$+ \frac{(-1)^n (2\lambda)_n}{n!} \left(\frac{1-\delta}{2} + \lambda - \alpha \right)_n \Gamma \left[\frac{\alpha + \delta/2, \lambda + 1/2}{\alpha + \lambda + n + (\delta + 1)/2} \right] (2a)^{\alpha + \lambda + (\delta - 1)/2} b^\delta \times$$

$$\times \left\{ \begin{matrix} 1 \\ R \end{matrix} \right\} - \delta \frac{(-1)^n}{2(n!)} (2a)^{\alpha + \lambda - 1/2} \pi (2\lambda)_n \left(\lambda - \alpha + \frac{1}{2} \right)_n \Gamma \left[\frac{\alpha, \lambda + 1/2}{\alpha + \lambda + n + 1/2} \right],$$

$$R = C + \frac{1}{2} \psi(\alpha) - \frac{1}{2} \psi \left(\alpha + \lambda + n + \frac{1}{2} \right) - \sum_{j=0}^{n-1} \frac{1}{2j - 2\alpha + 2\lambda + 1} + \frac{1}{2} \ln(2ab^2)$$

[a, Re α > 0, Re λ > -1/2].

$$4. \int_a^\infty (x-a)^{\lambda-1/2} (x+a)^{-\tau} \left\{ \begin{matrix} \text{si}(b\sqrt{x+a}) \\ \text{ci}(b\sqrt{x+a}) \end{matrix} \right\} C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= -\frac{(2\lambda)_n}{3^{\delta} n!} \left(\tau + \lambda - \frac{\delta+3}{2} \right)_n 2^{\lambda-\tau+(3\delta-1)/2} \times$$

$$\times \Gamma \left[\frac{\lambda + 1/2, \tau - \lambda - n - (\delta+3)/2}{\tau - \delta/2 - 1} \right] a^{\lambda-\tau+(\delta+3)/2} b^{\delta+2} \times$$

$$\times {}_4F_5 \left(1, \frac{5+\delta}{2} - \tau - \lambda, 1 + \frac{\delta}{2}, 2 - \tau + \frac{\delta}{2}; 2, \frac{3}{2} + \delta, 2 + \frac{\delta}{2}, \right.$$

$$\left. \frac{5+\delta}{2} - \tau - \lambda - n, \frac{5+\delta}{2} + \lambda + n - \tau; -\frac{ab^2}{2} \right) +$$

$$+ (-1)^n \frac{(\lambda)_n 2^{n+1} \Gamma(1+2n+2\lambda-2\tau) b^{2\tau-2\lambda-2n-1}}{n! (\tau-2\lambda-2n-1) a^n} \left\{ \begin{matrix} \cos(\tau-\lambda)\pi \\ \sin(\tau-\lambda)\pi \end{matrix} \right\} \times$$

$$\times {}_3F_4 \left(\frac{1}{2} - \lambda - n, \tau - \lambda - n - \frac{1}{2}, 1 - 2\lambda - n; 1 - 2\lambda - 2n, \frac{1}{2} + \tau - \lambda - n, \right.$$

$$\left. \frac{1}{2} + \tau - \lambda - n, \tau - \lambda - n; -\frac{ab^2}{2} \right) + \frac{(2\lambda)_n}{n!} \left(\tau + \lambda - \frac{\delta+1}{2} \right)_n \times$$

$$\times \Gamma \left[\frac{\lambda + 1/2, \tau - \lambda - n - (\delta+1)/2}{\tau - \delta/2} \right] (2a)^{\lambda-\tau+(\delta+1)/2} b^\delta \left\{ \begin{matrix} 1 \\ R \end{matrix} \right\} -$$

$$- \delta \frac{(2\lambda)_n}{n!} \left(\tau + \lambda - \frac{1}{2} \right)_n 2^{\lambda-\tau-1/2} \pi \Gamma \left[\frac{\lambda + 1/2, \tau - \lambda - n - 1/2}{\tau} \right] a^{\lambda-\tau+1/2},$$

$$R = C + \frac{1}{2} \psi(\tau) - \frac{1}{2} \psi \left(\tau - \lambda - n - \frac{1}{2} \right) - \sum_{j=0}^{n-1} \frac{1}{2j + 2\lambda + 2\tau - 1} + \frac{1}{2} \ln(2ab^2)$$

[a, b > 0, Re λ > -1/2; Re(λ - τ) < 1/2 - n].

2.21.11. Интегралы, содержащие $\left\{ \begin{matrix} \text{erf}(\varphi(x)) \\ \text{erfc}(\varphi(x)) \end{matrix} \right\} C_n^\lambda(cx)$.

$$1. \int_0^\infty x^{\alpha-1} \text{erfc}(bx) C_{2n+\varepsilon}^\lambda(cx) dx = \frac{(-1)^n 2^\varepsilon (\lambda)_{n+\varepsilon} c^\varepsilon}{\sqrt{\pi} n! (\alpha + \varepsilon) b^{\alpha+\varepsilon}} \Gamma \left(\frac{\alpha + \varepsilon + 1}{2} \right) \times$$

$$\times {}_4F_2 \left(-n, n + \lambda + \varepsilon, \frac{\alpha+1}{2}, \frac{\alpha}{2} + \varepsilon; \frac{1}{2} + \varepsilon, \frac{\alpha + \varepsilon + 1}{2} + 1; \frac{c^2}{b^2} \right)$$

[ε = 0 или 1; Re α > -ε, |arg b| < π/4].

$$2. \int_0^a (a^2 - x^2)^{\lambda-1/2} \text{erf}(bx) C_{2n+1}^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^n (2\lambda)_{2n+1}}{2^{2n+1} (2n+1) n!} \Gamma \left[\frac{\lambda + 1/2}{2n + \lambda + 2} \right] \frac{a^{\lambda-1}}{b^{\lambda+1}} e^{-a^2 b^2/2} M_{-(\lambda+1)/2, n+(\lambda+1)/2}(a^2 b^2)$$

[a > 0; Re λ > -1/2].

$$\begin{aligned}
3. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\lambda-1/2} \left\{ \begin{array}{l} \operatorname{erf}(bx) \\ \operatorname{erfc}(bx) \end{array} \right\} C_{2n+\varepsilon}^\lambda \left(\frac{x}{a} \right) dx = \\
= \pm \frac{(-1)^n a^{\alpha+2\lambda} b}{\sqrt{\pi} (2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{\varepsilon-\alpha}{2} \right)_n \Gamma \left[\frac{\lambda+1/2, (1+\alpha+\varepsilon)/2}{(\alpha+\varepsilon)/2+\lambda+n+1} \right] \times \\
\times {}_3F_3 \left(\frac{1}{2}, \frac{1+\alpha}{2}, 1+\frac{\alpha}{2}; \frac{3}{2}, 1+\frac{\alpha-\varepsilon}{2}-n, \frac{\alpha+\varepsilon}{2}+\lambda+n+1; -a^2 b^2 \right) + \\
+ \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \frac{(-1)^n}{2(2n+\varepsilon)!} a^{\alpha+2\lambda-1} (2\lambda)_{2n+\varepsilon} \left(\frac{1+\varepsilon-\alpha}{2} \right)_n \Gamma \left[\frac{\lambda+1/2, (\alpha+\varepsilon)/2}{(1+\alpha+\varepsilon)/2+\lambda+n} \right] \\
[\varepsilon=0 \text{ или } 1; a>0; \operatorname{Re} \lambda > -1/2; \operatorname{Re} \alpha > (\mp 1-1)/2-\varepsilon].
\end{aligned}$$

$$\begin{aligned}
4. \int_a^\infty x^{\alpha-1} (x^2 - a^2)^{\lambda-1/2} \left\{ \begin{array}{l} \operatorname{erf}(bx) \\ \operatorname{erfc}(bx) \end{array} \right\} C_{2n+\varepsilon}^\lambda \left(\frac{x}{a} \right) dx = \\
= \pm \frac{a^{\alpha+2\lambda} b}{\sqrt{\pi} (2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{\varepsilon-\alpha}{2} \right)_n \Gamma \left[\frac{\lambda+1/2, -(\alpha+\varepsilon)/2-\lambda-n}{(1-\alpha-\varepsilon)/2} \right] \times \\
\times {}_3F_3 \left(\frac{1}{2}, \frac{1+\alpha}{2}, 1+\frac{\alpha}{2}; \frac{3}{2}, \frac{\alpha+\varepsilon}{2}+\lambda+n+1, 1+\frac{\alpha-\varepsilon}{2}-n; -a^2 b^2 \right) \pm \\
\pm \frac{2^{2n+\varepsilon} b^{1-\alpha-\varepsilon-2\lambda-2n} (\lambda)_{2n+\varepsilon}}{\sqrt{\pi} (2n+\varepsilon)! (1-\alpha-\varepsilon-2\lambda-2n) a^{2n+\varepsilon}} \Gamma \left(\frac{\alpha+\varepsilon}{2} + \lambda + n \right) \times \\
\times {}_3F_3 \left(\frac{1}{2}-\lambda-n, 1-\lambda-\varepsilon-n, \frac{1-\alpha-\varepsilon}{2}-\lambda-n; \right. \\
\left. \frac{3-\alpha}{2}-\varepsilon-\lambda-n, 1-\lambda-\varepsilon-2n, 1-\lambda-n-\frac{\alpha}{2}; -a^2 b^2 \right) + \\
+ \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \frac{a^{\alpha+2\lambda-1}}{2(2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{1+\varepsilon-\alpha}{2} \right)_n \Gamma \left[\frac{\lambda+1/2, (1-\alpha-\varepsilon)/2-\lambda-n}{1-(\alpha+\varepsilon)/2} \right] \\
[\varepsilon=0 \text{ или } 1; a>0, \operatorname{Re} \lambda > -1/2, \left\{ \begin{array}{l} \operatorname{Re} b > 0, \operatorname{Re}(\alpha+2\lambda) < 1-2n-\varepsilon \\ \operatorname{Re} b > 0 \end{array} \right\}].
\end{aligned}$$

$$\begin{aligned}
5. \int_{-a}^\infty (x+a)^{\alpha-1} \operatorname{erfc}(ab+bx) C_n^\lambda \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^n (2\lambda)_n}{\alpha \sqrt{\pi} n! b^\alpha} \Gamma \left(\frac{\alpha+1}{2} \right) {}_4F_2 \left(-n, n+2\lambda, \frac{\alpha}{2}, \frac{\alpha+1}{2}; \lambda+\frac{1}{2}, \frac{\alpha}{2}+1; \frac{1}{4a^2 b^2} \right) \\
[a, \operatorname{Re} \alpha > 0; |\arg b| < \pi/4].
\end{aligned}$$

$$\begin{aligned}
6. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\lambda-1/2} \left\{ \begin{array}{l} \operatorname{erf}(a^2+bx) \\ \operatorname{erfc}(ab+bx) \end{array} \right\} C_n^\lambda \left(\frac{x}{a} \right) dx = \\
= \pm \frac{(-1)^n (2\lambda)_n}{\sqrt{\pi} n!} \left(\lambda-\alpha-\frac{1}{2} \right)_n 2^{\alpha+\lambda+3/2} a^{\alpha+\lambda+1/2} b \Gamma \left[\frac{\alpha+1, \lambda+1/2}{\alpha+\lambda+n+3/2} \right] \times \\
\times {}_5F_5 \left(\frac{1}{2}, \frac{\alpha+1}{2}, \frac{\alpha}{2}+1, \frac{2\alpha-2\lambda+3}{4}, \frac{2\alpha-2\lambda+5}{4}; \frac{3}{2}, \frac{2\alpha-2\lambda-2n+3}{4}, \right. \\
\left. \frac{2\alpha-2\lambda-2n+5}{4}, \frac{2\alpha+2n+2\lambda+3}{4}, \frac{2\alpha+2\lambda+2n+5}{4}; -4a^2 b^2 \right) + \\
+ \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \frac{(-1)^n (2\lambda)_n}{n!} (2a)^{\alpha+\lambda-1/2} \left(\lambda-\alpha+\frac{1}{2} \right)_n \Gamma \left[\frac{\alpha, \lambda+1/2}{\alpha+\lambda+n+1/2} \right] \\
[a>0; \operatorname{Re} \lambda > -1/2; \operatorname{Re} \alpha > -(1\pm 1)/2].
\end{aligned}$$

$$\begin{aligned}
7. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\lambda-1/2} \left\{ \begin{matrix} \operatorname{erf} (b \sqrt{x+a}) \\ \operatorname{erfc} (b \sqrt{x+a}) \end{matrix} \right\} C_n^\lambda \left(\frac{x}{a} \right) dx = \\
= \pm \frac{(-1)^n (2\lambda)_n}{\sqrt{\pi} n!} (\lambda - \alpha)_n 2^{\alpha+\lambda+1} a^{\alpha+\lambda} b \Gamma \left[\begin{matrix} \alpha+1/2, \lambda+1/2 \\ \alpha+\lambda+n+1 \end{matrix} \right] \times \\
\times {}_3F_3 \left(\frac{1}{2}, \alpha + \frac{1}{2}, 1 + \alpha - \lambda; \frac{3}{2}, 1 + \alpha - \lambda - n, 1 + n + \alpha + \lambda; -2ab^2 \right) + \\
+ \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{(-1)^n}{n!} (2a)^{\alpha+\lambda-1/2} \left(\lambda - \alpha + \frac{1}{2} \right)_n (2\lambda)_n \Gamma \left[\begin{matrix} \alpha, \lambda+1/2 \\ \alpha+\lambda+n+1/2 \end{matrix} \right] \\
[a > 0; \operatorname{Re} \lambda > -1/2; \operatorname{Re} \alpha > -(1 \pm 1)/4].
\end{aligned}$$

$$\begin{aligned}
8. \int_{\pm a}^{\infty} (x \mp a)^{\alpha-1} \operatorname{erfc} (b \sqrt{x \mp a}) C_n^\lambda \left(\frac{x}{a} \right) dx = \\
= \frac{(\pm 1)^n (2\lambda)_n}{a \sqrt{\pi} n! b^{2\alpha}} \Gamma \left(\alpha + \frac{1}{2} \right) {}_4F_3 \left(-n, 2\lambda + n, \alpha, \alpha + \frac{1}{2}; \alpha + 1, \lambda + \frac{1}{2}; \frac{1}{2ab^2} \right) \\
[a, \operatorname{Re} \alpha > 0; |\arg b| < \pi/4].
\end{aligned}$$

$$\begin{aligned}
9. \int_a^\infty (x-a)^{\lambda-1/2} (x+a)^{-\tau} \left\{ \begin{matrix} \operatorname{erf} (b \sqrt{x+a}) \\ \operatorname{erfc} (b \sqrt{x+a}) \end{matrix} \right\} C_n^\lambda \left(\frac{x}{a} \right) dx = \\
= \pm \frac{(2\lambda)_n}{\sqrt{\pi} n!} (\tau + \lambda - 1)_n 2^{\lambda-\tau+2} \Gamma \left[\begin{matrix} \lambda+1/2, \tau-\lambda-n-1 \\ \tau-1/2 \end{matrix} \right] a^{\lambda+\tau-1} \times \\
\times {}_3F_3 \left(\frac{1}{2}, 2-\tau-\lambda, \frac{3}{2}-\tau; \frac{3}{2}, 2+\lambda+n-\tau, 2-\tau-\lambda-n; -2ab^2 \right) \pm \\
\pm \frac{(\lambda)_n 2^{n+1} \Gamma(1+\lambda+\lambda-\tau) b^{2\tau-2\lambda-2n-1}}{n! \sqrt{\pi} (2\tau-2\lambda-2n-1) a^n} {}_3F_3 \left(\frac{1}{2}-\lambda-n, 1-2\lambda-n, \right. \\
\left. \tau-\lambda-n-\frac{1}{2}; 1-2\lambda-2n, \tau-\lambda-n, \frac{1}{2}+\tau-\lambda-n; -2ab^2 \right) + \\
+ \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{(2\lambda)_n}{n!} \left(\tau + \lambda - \frac{1}{2} \right)_n \Gamma \left[\begin{matrix} \lambda+1/2, \tau-\lambda-n-1/2 \\ \tau \end{matrix} \right] (2a)^{\lambda-\tau+1/2} \\
[a > 0; \operatorname{Re} \lambda > -1/2, \left\{ \begin{matrix} |\arg b| < \pi/4, \operatorname{Re}(\tau-\lambda) > n+1/2 \\ |\arg b| < \pi/4 \end{matrix} \right\}].
\end{aligned}$$

$$\begin{aligned}
10. \int_0^a x^{\alpha-1} (a^2-x^2)^{\lambda-1/2} e^{b^2 x^2} \operatorname{erf} (bx) C_n^\lambda \left(\frac{x}{a} \right) dx = \\
= \frac{a^{\alpha+2\lambda} b (2\lambda)_n}{2^{\alpha} n!} \Gamma \left[\begin{matrix} \alpha+1, \lambda+1/2 \\ (\alpha-n)/2+1, (\alpha+n)/2+\lambda+1 \end{matrix} \right] \times \\
\times {}_3F_3 \left(1, \frac{\alpha+1}{2}, \frac{\alpha}{2}+1; \frac{3}{2}, \frac{\alpha-n}{2}+1, \frac{\alpha+n}{2}+\lambda+1; a^2 b^2 \right) \\
[a > 0; \operatorname{Re} \alpha > 2[n/2] - n - 1; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
11. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\lambda-1/2} e^{b^2 x} \operatorname{erf} (b \sqrt{x+a}) C_n^\lambda \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^n (2\lambda)_n}{\sqrt{\pi} n!} (\lambda - \alpha)_n 2^{\alpha+\lambda+1} a^{\alpha+\lambda} b \Gamma \left[\begin{matrix} \alpha+1/2, \lambda+1/2 \\ \alpha+\lambda+n+1 \end{matrix} \right] e^{-ab^2} \times \\
\times {}_3F_3 \left(1, \alpha + \frac{1}{2}, \alpha - \lambda + 1; \frac{3}{2}, \alpha - \lambda - n + 1, \alpha + \lambda + n + 1; 2ab^2 \right) \\
[a > 0, \operatorname{Re} \lambda, \operatorname{Re} \alpha > -1/2].
\end{aligned}$$

2.21.12. Интегралы, содержащие $\left\{ \begin{matrix} J_\nu(\varphi(x)) \\ I_\nu(\varphi(x)) \end{matrix} \right\} C_n^\lambda(cx)$.

$$1. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\lambda-1/2} \left\{ \begin{matrix} J_\nu(bx) \\ I_\nu(bx) \end{matrix} \right\} C_{2n+\varepsilon}^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^n a^{\alpha+2\lambda+\nu-1} b^\nu}{2^{\nu+1} (2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{1+\varepsilon-\alpha-\nu}{2} \right)_n \times$$

$$\times \Gamma \left[\lambda + 1/2, (\alpha + \varepsilon + \nu)/2 \right] {}_2F_3 \left(\frac{1+\alpha+\nu}{2}, \frac{\alpha+\nu}{2}; \right.$$

$$\left. \nu+1, \frac{1+\alpha+\nu+\varepsilon}{2} + \lambda + n, \frac{1+\alpha+\nu-\varepsilon}{2} - n; -\frac{a^2 b^2}{4} \right)$$

[$\varepsilon = 0$ или 1 ; $a > 0$; $\operatorname{Re}(\alpha + \nu) > -\varepsilon$; $\operatorname{Re} \lambda > -1/2$].

$$2. \int_a^\infty x^{\alpha-1} (x^2 - a^2)^{\lambda-1/2} J_\nu(bx) C_{2n+\varepsilon}^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{a^{\alpha+2\lambda+\nu-1} b^\nu}{2^{\nu+1} (2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{1+\varepsilon-\alpha-\nu}{2} \right)_n \Gamma \left[\lambda + 1/2, (1-\alpha-\varepsilon-\nu)/2 - \lambda - n \right] \times$$

$$\times {}_2F_3 \left(\frac{1+\alpha+\nu}{2}, \frac{\alpha+\nu}{2}; \nu+1, \frac{1+\alpha+\varepsilon+\nu}{2} + \lambda + n, \frac{1+\alpha+\nu-\varepsilon}{2} - n; -\frac{a^2 b^2}{4} \right) +$$

$$+ \frac{(-1)^n (2\lambda)_{2n+\varepsilon} a^{2\lambda+\nu-1} b^\nu}{(2n+\varepsilon)! a^{2n+\varepsilon}} \Gamma \left[(\alpha + \nu + \varepsilon - 1)/2 + \lambda + n \right] \times$$

$$\times {}_2F_3 \left(\frac{1}{2} - \lambda - n, 1 - \lambda - \varepsilon - n; 1 - \lambda - \varepsilon - 2n, \right.$$

$$\left. \frac{3-\alpha-\nu-\varepsilon}{2} - \lambda - n, \frac{3+\nu-\alpha-\varepsilon}{2} - \lambda - n; -\frac{a^2 b^2}{4} \right)$$

[$\varepsilon = 0$ или 1 ; $a, b > 0$; $\operatorname{Re} \lambda > -1/2$; $\operatorname{Re}(\alpha + 2\lambda) < 5/2 - 2n - \varepsilon$].

$$3. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\lambda-1/2} J_\nu \left(\frac{b}{x} \right) C_{2n+\varepsilon}^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^n a^{\alpha+2\lambda-\nu-1} b^\nu}{2^{\nu+1} (2n+\varepsilon)!} (2\lambda)_{2n+\varepsilon} \left(\frac{1+\nu+\varepsilon-\alpha}{2} \right)_n \Gamma \left[(\alpha + \varepsilon - \nu)/2, \lambda + 1/2 \right] \times$$

$$\times {}_2F_3 \left(\frac{1+\nu-\alpha-\varepsilon}{2} - \lambda - n, \frac{1+\varepsilon+\nu-\alpha}{2} + n; \nu+1, 1 + \frac{\nu-\alpha}{2}, \frac{1+\nu-\alpha}{2}; -\frac{b^2}{4a^2} \right) +$$

$$+ \frac{(-1)^n (\lambda)_{n+\varepsilon} a^{2\lambda-\varepsilon-1} b^{\alpha+\varepsilon}}{2^{\alpha+1} n!} \Gamma \left[(\nu - \alpha - \varepsilon)/2 \right] \times$$

$$\times {}_2F_3 \left(\frac{1}{2} + \varepsilon + n, \frac{1}{2} - \lambda - n; \frac{1}{2} + \varepsilon, 1 + \frac{\alpha + \varepsilon - \nu}{2}, 1 + \frac{\alpha + \nu + \varepsilon}{2}; -\frac{b^2}{4a^2} \right)$$

[$\varepsilon = 0$ или 1 ; $a, b > 0$; $\operatorname{Re} \lambda > -1/2$; $\operatorname{Re} \alpha > -3/2 - \varepsilon$].

$$4. \int_0^a x^{-\lambda-2} (a^2 - x^2)^{\lambda-1/2} J_{\lambda+n} \left(\frac{b}{x} \right) C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^{[n/2]} (2\lambda)_n 2^\lambda a^{2\lambda-1} \Gamma(\lambda + 1/2)}{\sqrt{\pi} n! b^{\lambda+1}} \left\{ \begin{matrix} \sin(b/a) \\ \cos(b/a) \end{matrix} \right\}$$

[$a, b > 0$; $-1/2 < \operatorname{Re} \lambda < 1 \pm 1/2$, $n = \begin{cases} 1, 3, 5, \dots \\ 0, 2, 4, \dots \end{cases}$].

$$\begin{aligned}
5. \int_{\pm a}^{\infty} (x \mp a)^{\alpha-1} J_{\nu}(b \sqrt{x \mp a}) C_n^{\lambda}\left(\frac{x}{a}\right) dx = \\
= \frac{(\pm 1)^n (\lambda)_n 2^{2\alpha+3n}}{n! a^n b^{2\alpha+2n}} \Gamma\left[\frac{\nu/2 + \alpha + n}{\nu/2 - \alpha - n + 1}\right] \times \\
\times {}_2F_3\left(-n, \frac{1}{2} - \lambda - n; 1 - 2\lambda - 2n, 1 - n - \alpha - \frac{\nu}{2}, 1 + \frac{\nu}{2} - \alpha - n; \pm \frac{ab^2}{2}\right) \\
[a, b, \operatorname{Re}(2\alpha + \nu) > 0; \operatorname{Re} \alpha < 3/4 - n].
\end{aligned}$$

$$\begin{aligned}
6. \int_{\pm a}^{\infty} (x \mp a)^{\alpha-1} J_{\nu}\left(\frac{b}{\sqrt{x \mp a}}\right) C_n^{\lambda}\left(\frac{x}{a}\right) dx = \\
= \frac{(\pm 1)^n (2\lambda)_n}{n!} \left(\frac{b}{2}\right)^{2\alpha} \Gamma\left[\frac{\nu/2 - \alpha}{1 + \alpha + \nu/2}\right] \times \\
\times {}_2F_3\left(-n, 2\lambda + n; \frac{1}{2} + \lambda, 1 + \alpha + \frac{\nu}{2}, 1 + \alpha - \frac{\nu}{2}; \pm \frac{b^2}{8a}\right) \\
[a, b > 0; \operatorname{Re} \alpha > -3/4; \operatorname{Re}(\nu - 2\alpha) > 2n].
\end{aligned}$$

$$\begin{aligned}
7. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\lambda-1/2} \left\{ \frac{J_{\nu}(b \sqrt{x+a})}{I_{\nu}(b \sqrt{x+a})} \right\} C_n^{\lambda}\left(\frac{x}{a}\right) dx = \\
= \frac{(-1)^n (2\lambda)_n}{n!} \left(\frac{1-\nu}{2} + \lambda - \alpha\right)_n 2^{\alpha+\lambda-(\nu+1)/2} \times \\
\times \Gamma\left[\frac{\alpha + \nu/2, \lambda + 1/2}{\nu + 1, \alpha + \lambda + n + (\nu + 1)/2}\right] a^{\alpha+\lambda+(\nu-1)/2} b^{\nu} \times \\
\times {}_2F_3\left(\frac{\nu}{2} + \alpha, \frac{1+\nu}{2} + \alpha - \lambda; \frac{1+\nu}{2} + \alpha + \lambda + n, \frac{1+\nu}{2} + \alpha - \lambda - n, \nu + 1; \mp \frac{ab^2}{2}\right) \\
[a, \operatorname{Re}(2\alpha + \nu) > 0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
8. \int_{-a}^a (x+a)^{\lambda-1/2} (a-x)^{\beta-1} \left\{ \frac{J_{\nu}(b \sqrt{a-x})}{I_{\nu}(b \sqrt{a-x})} \right\} C_n^{\lambda}\left(\frac{x}{a}\right) dx = \\
= \frac{(2\lambda)_n}{n!} \left(\frac{1-\nu}{2} + \lambda - \beta\right)_n 2^{\beta+\lambda-(\nu+1)/2} \Gamma\left[\frac{\beta + \nu/2, \lambda + 1/2}{\nu + 1, \beta + \lambda + n + (\nu + 1)/2}\right] \times \\
\times a^{\beta+\lambda+(\nu-1)/2} b^{\nu} {}_2F_3\left(\beta - \lambda + \frac{\nu+1}{2}, \beta + \frac{\nu}{2}; \right. \\
\left. \frac{1+\nu}{2} + \beta - \lambda - n, \frac{1+\nu}{2} + \lambda + \beta + n, \nu + 1; \mp \frac{ab^2}{2}\right) \\
[a, \operatorname{Re}(2\beta + \nu) > 0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
9. \int_a^{\infty} (x-a)^{\lambda-1/2} (x+a)^{-\tau} J_{\nu}(b \sqrt{x+a}) C_n^{\lambda}\left(\frac{x}{a}\right) dx = \\
= \frac{(2\lambda)_n}{n!} \left(\tau + \lambda - \frac{\nu+1}{2}\right)_n 2^{\lambda-\tau+(3-\nu)/2} \Gamma\left[\frac{\lambda + 1/2, \tau - \lambda - n - (\nu + 1)/2}{\nu + 1, \tau - \nu/2}\right] \times \\
\times a^{\lambda-\tau+(\nu+1)/2} b^{\nu} {}_2F_3\left(1 + \frac{\nu}{2} - \tau, \frac{3+\nu}{2} - \tau - \lambda; \frac{3+\nu}{2} - \tau - \lambda - n, \frac{3+\nu}{2} + \lambda + n - \tau, \right. \\
\left. \nu + 1; -\frac{ab^2}{2}\right) + \frac{(\lambda)_n 2^{2\lambda-2\tau+3n+1}}{n! a^{\tau}} \Gamma\left[\frac{n + \lambda - \tau + (\nu + 1)/2}{(1+\nu)/2 + \tau - \lambda - n}\right] b^{2\tau-2\lambda-2n-1} \times \\
\times {}_2F_3\left(\frac{1}{2} - \lambda - n, 1 - 2\lambda - n; 1 - 2\lambda - 2n, \frac{1+\nu}{2} + \tau - \lambda - n, \frac{1-\nu}{2} + \tau - \lambda - n; -\frac{ab^2}{2}\right) \\
[a, b > 0; \operatorname{Re} \lambda > -1/2; \operatorname{Re}(\lambda - \tau) < 1/4 - n].
\end{aligned}$$

$$\begin{aligned}
10. \int_a^\infty (x-a)^{\alpha-1} (x+a)^{\lambda-1/2} J_\nu(b\sqrt{x-a}) C_n^\lambda\left(\frac{x}{a}\right) dx = \\
= \frac{(-1)^n (2\lambda)_n}{n!} \left(\frac{1-\nu}{2} + \lambda - \alpha\right)_n 2^{\alpha+\lambda-(\nu+1)/2} \Gamma\left[\begin{matrix} \alpha+\nu/2, (1-\nu)/2-\alpha-\lambda-n \\ \nu+1, 1/2-\lambda \end{matrix}\right] \times \\
\times a^{\alpha+\lambda+(\nu-1)/2} b^\nu {}_2F_3\left(\frac{1+\nu}{2} + \alpha - \lambda, \alpha + \frac{\nu}{2}; \right. \\
\left. \frac{1+\nu}{2} + \alpha - \lambda - n, \frac{1+\nu}{2} + \alpha + \lambda + n, \nu+1; \frac{ab^2}{2}\right) + \\
+ \frac{(\lambda)_n 2^{2\alpha+2\lambda+2n-1}}{n!} \Gamma\left[\begin{matrix} (\nu-1)/2 + \alpha + \lambda + n \\ (3+\nu)/2 - \lambda - \alpha - n \end{matrix}\right] a^{-n} b^{1-2\alpha-2\lambda-2n} \times \\
\times {}_2F_3\left(1-2\lambda-n, \frac{1}{2} - \lambda - n; 1-2\lambda-2n, \frac{3-\nu}{2} - \lambda - \alpha - n, \frac{3+\nu}{2} - \alpha - \lambda - n; \frac{ab^2}{2}\right) \\
\{a, b, \operatorname{Re}(2\alpha + \nu) > 0; \operatorname{Re}(\alpha + \lambda) < 5/4 - a\}.
\end{aligned}$$

$$\begin{aligned}
11. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\lambda-1/2} J_\nu\left(\frac{b}{\sqrt{x+a}}\right) C_n^\lambda\left(\frac{x}{a}\right) dx = \\
= \frac{(-1)^n (2\lambda)_n}{n!} 2^{\lambda-2\alpha-1/2} \Gamma\left[\begin{matrix} \nu/2 - \alpha \\ 1 + \alpha + \nu/2 \end{matrix}\right] a^{\lambda-1/2} b^{2\alpha} \times \\
\times {}_2F_3\left(\frac{1}{2} + \lambda + n, \frac{1}{2} - \lambda - n; \frac{1}{2} + \lambda, 1 + \alpha - \frac{\nu}{2}, 1 + \alpha + \frac{\nu}{2}; -\frac{b^2}{8a}\right) + \\
+ \frac{(-1)^n (2\lambda)_n}{n!} \left(\frac{1+\nu}{2} + \lambda - \alpha\right)_n \Gamma\left[\begin{matrix} \lambda + 1/2, \alpha - \nu/2 \\ \nu+1, \alpha + \lambda + n + (1-\nu)/2 \end{matrix}\right] \times \\
\times 2^{\alpha+\lambda-(3\nu+1)/2} a^{\alpha+\lambda-(\nu+1)/2} b^\nu {}_2F_3\left(\frac{1+\nu}{2} + \lambda + n - \alpha, \right. \\
\left. \frac{1+\nu}{2} - \alpha - \lambda - n; \nu+1, 1 + \frac{\nu}{2} - \alpha, \frac{1+\nu}{2} + \lambda - \alpha; -\frac{b^2}{8a}\right) \\
\{a, b > 0; \operatorname{Re} \lambda > -1/2; \operatorname{Re} \alpha > -3/4\}.
\end{aligned}$$

$$\begin{aligned}
12. \int_a^\infty (x-a)^{\lambda-1/2} (x+a)^{-\tau} \left\{ \begin{matrix} J_\nu(b/\sqrt{x+a}) \\ I_\nu(b/\sqrt{x+a}) \end{matrix} \right\} C_n^\lambda\left(\frac{x}{a}\right) dx = \\
= \frac{(2\lambda)_n}{n!} \left(\tau + \lambda + \frac{\nu-1}{2}\right)_n 2^{(1-3\nu)/2+\lambda-\tau} \Gamma\left[\begin{matrix} \lambda + 1/2, (\nu-1)/2 + \tau - \lambda - n \\ \nu+1, \tau + \nu/2 \end{matrix}\right] a^{(1-\nu)/2+\lambda-\tau} b^\nu \times \\
\times {}_2F_3\left(\frac{\nu-1}{2} + \tau - \lambda - n, \frac{\nu-1}{2} + \tau + \lambda + n; \tau + \frac{\nu}{2}, \frac{\nu-1}{2} + \tau + \lambda, \nu+1; \mp \frac{b^2}{8a}\right) \\
\{a > 0; \operatorname{Re} \lambda > -1/2; \operatorname{Re}(\nu + 2\tau - 2\lambda) > 2n + 1\}.
\end{aligned}$$

$$\begin{aligned}
13. \int_a^\infty (x-a)^{\alpha-1} (x+a)^{\lambda-1/2} J_\nu\left(\frac{b}{\sqrt{x-a}}\right) C_n^\lambda\left(\frac{x}{a}\right) dx = \\
= \frac{(2\lambda)_n}{n!} 2^{\lambda-2\alpha-1/2} \Gamma\left[\begin{matrix} \nu/2 - \alpha \\ 1 + \alpha + \nu/2 \end{matrix}\right] a^{\lambda-1/2} b^{2\alpha} {}_2F_3\left(\frac{1}{2} + \lambda + n, \frac{1}{2} - \lambda - n; \frac{1}{2} + \lambda, \right. \\
\left. 1 + \alpha + \frac{\nu}{2}, 1 + \alpha - \frac{\nu}{2}; \frac{b^2}{8a}\right) + \frac{(-1)^n (2\lambda)_n}{n!} \left(\frac{1+\nu}{2} + \lambda - \alpha\right)_n 2^{\alpha+\lambda-(3\nu+1)/2} \times \\
\times \Gamma\left[\begin{matrix} \alpha - \nu/2, (1+\nu)/2 - \alpha - \lambda - n \\ \nu+1, 1/2 - \lambda \end{matrix}\right] a^{\alpha+\lambda-(\nu+1)/2} b^\nu \times \\
\times {}_2F_3\left(\frac{1+\nu}{2} + \lambda + n - \alpha, \frac{1+\nu}{2} - \alpha - \lambda - n; \frac{1+\nu}{2} + \lambda - \alpha, 1 + \frac{\nu}{2} - \alpha, \nu+1; \frac{b^2}{8a}\right) \\
\{a, b > 0; \operatorname{Re} \alpha > -3/4; \operatorname{Re}(2\alpha + 2\lambda - \nu) < 1 - 2n\}.
\end{aligned}$$

$$\begin{aligned}
14. \int_{-a}^a (x+a)^{-\beta-n-1} (a-x)^{\beta-1} J_\nu \left(b \sqrt{\frac{a-x}{a+x}} \right) C_n^\lambda \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^n (2\lambda)_n 2^{2\beta+n-1}}{n! a^{n+1} b^{2\beta+2n}} \Gamma \left[\frac{\nu}{2} + \beta + n \right] \times \\
\times {}_2F_3 \left(-n, \frac{1}{2} - \lambda - n; \frac{1}{2} + \lambda, 1 - \beta - n - \frac{\nu}{2}, 1 + \frac{\nu}{2} - \beta - n; \frac{b^2}{4} \right) \\
[a, b, \operatorname{Re} (2\beta + \nu) > 0; \operatorname{Re} \beta < 3/4 - n].
\end{aligned}$$

$$\begin{aligned}
15. \int_{-a}^a (x+a)^{n+2\lambda-\beta-1} (a-x)^{\beta-1} J_\nu \left(b \sqrt{\frac{a-x}{a+x}} \right) C_n^\lambda \left(\frac{x}{a} \right) dx = \\
= \frac{2^{n+2\lambda-\nu-1}}{n! (\lambda + 1/2)_n} \left(\frac{1-\nu}{2} + \lambda - \beta \right)_n \Gamma \left[\frac{\beta + \nu/2, 2\lambda + n - \beta - \nu/2}{\nu + 1, 2\lambda} \right] a^{n+2\lambda-1} b^\nu \times \\
\times {}_2F_3 \left(\frac{1+\nu}{2} + \beta - \lambda, \beta + \frac{\nu}{2}; \frac{1+\nu}{2} + \beta - \lambda - n, 1 + \frac{\nu}{2} + \beta - 2\lambda - n, \nu + 1; \frac{b^2}{4} \right) + \\
+ \frac{(-1)^n (2\lambda)_n 2^{2\beta-2\lambda-n-1}}{n!} \Gamma \left[\frac{\nu/2 + \beta - 2\lambda - n}{1 + \nu/2 + 2\lambda + n - \beta} \right] a^{2\lambda+n-1} b^{2(2\lambda+n-\beta)} \times \\
\times {}_2F_3 \left(2\lambda + n, \frac{1}{2} + \lambda + n; \frac{1}{2} + \lambda, 1 + 2\lambda + n - \beta - \frac{\nu}{2}, 1 + 2\lambda + n + \frac{\nu}{2} - \beta; \frac{b^2}{4} \right) \\
[a, b, \operatorname{Re} (2\beta + \nu) > 0; \operatorname{Re} (\beta - 2\lambda) < 3/4 + n].
\end{aligned}$$

$$\begin{aligned}
16. \int_{-a}^a (a^2 - x^2)^{\lambda-1/2} (b-x)^{-\lambda/2} \left\{ \begin{matrix} J_\lambda(c\sqrt{b-x}) \\ Y_\lambda(c\sqrt{b-x}) \end{matrix} \right\} C_n^\lambda \left(\frac{x}{a} \right) dx = \\
= \frac{2\pi}{n!} \Gamma \left[\frac{2\lambda + n}{\lambda} \right] \left(\frac{a}{c} \right)^\lambda J_{\lambda+n}(z_-) \left\{ \begin{matrix} J_{\lambda+n}(z_+) \\ Y_{\lambda+n}(z_+) \end{matrix} \right\} \\
[\sqrt{2} z_\pm = c(b \pm \sqrt{b^2 - a^2})^{1/2}; b > a > 0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
17. \int_0^a (a^2 - x^2)^{(2\lambda-1)/4} \left\{ \begin{matrix} \sin bx \\ \cos bx \end{matrix} \right\} J_{\lambda-1/2}(c\sqrt{a^2 - x^2}) C_n^\lambda \left(\frac{x}{a} \right) dx = \\
= (-1)^{[n/2]} \sqrt{\frac{\pi}{2}} \frac{a^\lambda c^{\lambda-1/2}}{(b^2 + c^2)^{\lambda/2}} J_{\lambda+n}(a\sqrt{b^2 + c^2}) C_n^\lambda \left(\frac{b}{\sqrt{b^2 + c^2}} \right) \\
[a > 0; \operatorname{Re} \lambda > -1/2, n = \{1, 3, 5, \dots\}].
\end{aligned}$$

$$\begin{aligned}
18. \int_0^a x^{-5/2} (a^2 - x^2)^{(2\lambda-1)/4} J_{\lambda+2n+1} \left(\frac{b}{x} \right) J_{\lambda-1/2} \left(\frac{c}{x} \sqrt{a^2 - x^2} \right) C_n^\lambda \left(\frac{x}{a} \right) dx = \\
= \sqrt{\frac{2}{\pi}} (b^2 - a^2 c^2)_+^{-1/2} a^{2\lambda-1} b^{-\lambda} c^{\lambda-1/2} \sin \left(\frac{\sqrt{b^2 - a^2 c^2}}{a} - \frac{\pi n}{2} \right) C_n^\lambda \left(\frac{\sqrt{b^2 - a^2 c^2}}{b} \right) \\
[a, b, c > 0; b \neq ac; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
19. \int_0^a x^{-3/2} (a^2 - x^2)^{(2\lambda-1)/4} J_{\lambda+n} \left(\frac{b}{x} \right) J_{\lambda-1/2} \left(\frac{c}{x} \sqrt{a^2 - x^2} \right) C_n^\lambda \left(\frac{x}{a} \right) dx = \\
= (-1)^{[n/2]} \sqrt{\frac{2}{\pi}} (b^2 - a^2 c^2)_+^{-1/2} a^{2\lambda-1} b^{1-\lambda} c^{\lambda-1/2} \times \\
\times \left\{ \begin{matrix} \sin(\sqrt{b^2 - a^2 c^2}/a) \\ \cos(\sqrt{b^2 - a^2 c^2}/a) \end{matrix} \right\} C_n^\lambda \left(\frac{\sqrt{b^2 - a^2 c^2}}{b} \right) \\
[a, b, c > 0; b \neq ac; \operatorname{Re} \lambda > -1/2, n = \{1, 3, 5, \dots\}].
\end{aligned}$$

$$\begin{aligned}
20. \int_{-a}^a (x+a)^{\lambda-1/2} (a-x)^{\beta-1} \left\{ \frac{J_\mu(b\sqrt{a-x}) J_\nu(b\sqrt{a-x})}{I_\mu(b\sqrt{a-x}) I_\nu(b\sqrt{a-x})} \right\} C_n^\lambda\left(\frac{x}{a}\right) dx = \\
= \frac{2^{\beta+\lambda-(\mu+\nu+1)/2} (2\lambda)_n}{n!} \left(\frac{1-\mu-\nu}{2} + \lambda - \beta \right)_n a^{\beta+\lambda+(\mu+\nu-1)/2} b^{\mu+\nu} \times \\
\times \Gamma \left[\begin{matrix} (\mu+\nu)/2 + \beta, \lambda + 1/2 \\ \mu+1, \nu+1, \beta+n+\lambda+(\mu+\nu+1)/2 \end{matrix} \right] {}_4F_3 \left(\frac{\mu+\nu}{2} + \beta, \right. \\
\left. \frac{1+\mu+\nu}{2} + \beta - \lambda, \frac{\mu+\nu+1}{2}, \frac{\mu+\nu}{2} + 1; \frac{1+\mu+\nu}{2} + \beta + \lambda + n, \right. \\
\left. \frac{1+\mu+\nu}{2} + \beta - \lambda - n, \mu+1, \nu+1, \mu+\nu+1; \mp 2ab^2 \right) \\
[a, \operatorname{Re}(2\beta + \mu + \nu) > 0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

2.21.13. Интегралы от $A(x) K_\nu(\varphi(x)) C_n^\lambda(cx)$.

$$\begin{aligned}
1. \int_{\pm a}^{\infty} (x \mp a)^{\alpha-1} K_\nu(b\sqrt{x \mp a}) C_n^\lambda\left(\frac{x}{a}\right) dx = \\
= \frac{(\pm 1)^n (\lambda)_n 2^{2\alpha+3n-1}}{n! a^n b^{2\alpha+2n}} \Gamma\left(\alpha+n+\frac{\nu}{2}\right) \Gamma\left(\alpha+n-\frac{\nu}{2}\right) \times \\
\times {}_2F_3\left(-n, \frac{1}{2} - \lambda - n; 1-2\lambda-2n, 1-n-\alpha-\frac{\nu}{2}, 1+\frac{\nu}{2}-\alpha-n; \mp \frac{ab^2}{2}\right) \\
[a, \operatorname{Re} b > 0; 2 \operatorname{Re} \alpha > |\operatorname{Re} \nu|].
\end{aligned}$$

$$\begin{aligned}
2. \int_{\pm a}^{\infty} (x \mp a)^{\alpha-1} K_\nu\left(\frac{b}{\sqrt{x \mp a}}\right) C_n^\lambda\left(\frac{x}{a}\right) dx = \\
= \frac{(\pm 1)^n (2\lambda)_n}{n! 2^{2\alpha+1}} \Gamma\left(\frac{\nu}{2} - \alpha\right) \Gamma\left(-\frac{\nu}{2} - \alpha\right) b^{2\alpha} \times \\
\times {}_2F_3\left(-n, 2\lambda+n; \frac{1}{2} + \lambda, 1+\alpha-\frac{\nu}{2}, 1+\alpha+\frac{\nu}{2}; \mp \frac{b^2}{8a}\right) \\
[a, \operatorname{Re} b > 0; 2 \operatorname{Re} \alpha < -2n - |\operatorname{Re} \nu|].
\end{aligned}$$

$$\begin{aligned}
3. \int_{-a}^a (x+a)^{\lambda-1/2} (a-x)^{\beta-1} K_\nu(b\sqrt{a-x}) C_n^\lambda\left(\frac{x}{a}\right) dx = \\
= \frac{(2\lambda)_n}{n!} \left(\frac{1-\nu}{2} + \lambda - \beta \right)_n 2^{\beta+\lambda-(\nu+3)/2} a^{\beta+\lambda+(\nu-1)/2} b^\nu \times \\
\times \Gamma \left[\begin{matrix} -\nu, \beta+\nu/2, \lambda+1/2 \\ \beta+\lambda+n+(\nu+1)/2 \end{matrix} \right] {}_2F_3 \left(\beta+\frac{\nu}{2}, \frac{1+\nu}{2} + \beta - \lambda, \right. \\
\left. \nu+1, \frac{1+\nu}{2} + \beta - \lambda - n, \frac{1+\nu}{2} + \beta + \lambda + n; \frac{ab^2}{2} \right) + \\
+ \frac{(2\lambda)_n}{n!} \left(\frac{1+\nu}{2} + \lambda - \beta \right)_n 2^{\beta+\lambda+(\nu-3)/2} \Gamma \left[\begin{matrix} \nu, \beta-\nu/2, \lambda+1/2 \\ \beta+\lambda+n+(1-\nu)/2 \end{matrix} \right] a^{\beta+\lambda-(\nu+1)/2} b^{-\nu} \times \\
\times {}_2F_3 \left(\beta-\frac{\nu}{2}, \frac{1-\nu}{2} + \beta - \lambda; 1-\nu, \frac{1-\nu}{2} + \beta - \lambda - n, \frac{1-\nu}{2} + \beta + \lambda + n; \frac{ab^2}{2} \right) \\
[a > 0; \operatorname{Re} \lambda > -1/2; 2 \operatorname{Re} \beta > |\operatorname{Re} \nu|].
\end{aligned}$$

$$\begin{aligned}
4. \int_a^\infty (x-a)^{\lambda-1/2} (x+a)^{-\tau} K_\nu(b\sqrt{x+a}) C_n^\lambda\left(\frac{x}{a}\right) dx = \\
= \frac{(\lambda)_n 2^{2n+2\lambda-2\tau}}{n! a^n} \Gamma\left(\frac{1+\nu}{2} + \lambda + n - \tau\right) \Gamma\left(\frac{1-\nu}{2} + \lambda + n - \tau\right) b^{2\tau-2\lambda-2n-1} \times \\
\times {}_2F_3\left(\frac{1}{2} - \lambda - n, 1 - 2\lambda - n; 1 - 2\lambda - 2n, \frac{1+\nu}{2} + \tau - \lambda - n, \frac{1-\nu}{2} + \tau - \lambda - n; \frac{ab^2}{2}\right) + \\
+ \frac{(2\lambda)_n}{n!} \left(\frac{\nu-1}{2} + \tau + \lambda\right)_n 2^{\lambda+(\nu-1)/2-\tau} \times \\
\times \Gamma\left[\begin{matrix} \nu, \lambda+1/2, (\nu-1)/2 + \tau - \lambda - n \\ \tau + \nu/2 \end{matrix}\right] a^{(1-\nu)/2+\lambda-\tau} b^{-\nu} \times \\
\times {}_2F_3\left(1 - \tau - \frac{\nu}{2}, \frac{3-\nu}{2} - \tau - \lambda; 1 - \nu, \frac{3-\nu}{2} + \lambda + n - \tau, \frac{3-\nu}{2} - \lambda - \tau - n; \frac{ab^2}{2}\right) + \\
+ \frac{(2\lambda)_n}{n!} \left(\tau + \lambda - \frac{\nu+1}{2}\right)_n 2^{\lambda-\tau-(\nu+1)/2} \times \\
\times \Gamma\left[\begin{matrix} -\nu, \lambda+1/2, \tau - \lambda - n - (\nu+1)/2 \\ \tau - \nu/2 \end{matrix}\right] a^{(1+\nu)/2+\lambda-\tau} b^\nu \times \\
\times {}_2F_3\left(1 + \frac{\nu}{2} - \tau, \frac{3+\nu}{2} - \tau - \lambda; 1 + \nu, \frac{3+\nu}{2} + \lambda - \tau + n, \frac{3+\nu}{2} - \lambda - \tau - n; \frac{ab^2}{2}\right) \\
[a, \operatorname{Re} b > 0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
5. \int_a^\infty (x-a)^{\alpha-1} (x+a)^{\lambda-1/2} K_\nu(b\sqrt{x-a}) C_n^\lambda\left(\frac{x}{a}\right) dx = \\
= \frac{(\lambda)_n 2^{2\alpha+2\lambda+2n-2}}{n! a^n} \Gamma\left(\alpha + \lambda + n + \frac{\nu-1}{2}\right) \Gamma\left(\alpha + \lambda + n - \frac{\nu+1}{2}\right) \times \\
\times b^{1-2\alpha-2\lambda-2n} {}_2F_3\left(\frac{1}{2} - \lambda - n, 1 - 2\lambda - n; 1 - 2\lambda - 2n, \frac{3-\nu}{2} - \lambda - \alpha - n, \right. \\
\left. \frac{3+\nu}{2} - \lambda - \alpha - n; -\frac{ab^2}{2}\right) + \frac{(-1)^n (2\lambda)_n}{n!} \left(\frac{1-\nu}{2} + \lambda - \alpha\right)_n 2^{\alpha+\lambda-(\nu+3)/2} \times \\
\times \Gamma\left[\begin{matrix} \alpha + \nu/2, (1-\nu)/2 - \alpha - \lambda - n \\ 1/2 - \lambda \end{matrix}\right] a^{\alpha+\lambda+(\nu-1)/2} b^\nu \times \\
\times {}_2F_3\left(\frac{1+\nu}{2} + \alpha - \lambda, \alpha + \frac{\nu}{2}; \nu + 1, \frac{1+\nu}{2} + \alpha - \lambda - n, \frac{1+\nu}{2} + \alpha + \lambda + n; -\frac{ab^2}{2}\right) + \\
+ \frac{(-1)^n (2\lambda)_n}{n!} \left(\frac{1+\nu}{2} + \lambda - \alpha\right)_n 2^{\alpha+\lambda+(\nu-3)/2} \times \\
\times \Gamma\left[\begin{matrix} \alpha - \nu/2, (1+\nu)/2 - \alpha - \lambda - n \\ 1/2 - \lambda \end{matrix}\right] a^{\alpha+\lambda-(\nu+1)/2} b^{-\nu} \times \\
\times {}_2F_3\left(\frac{1-\nu}{2} + \alpha - \lambda, \alpha - \frac{\nu}{2}; 1 - \nu, \frac{1-\nu}{2} + \alpha - \lambda - n, \frac{1-\nu}{2} + \alpha + \lambda + n; -\frac{ab^2}{2}\right) \\
[a, \operatorname{Re} b > 0; 2 \operatorname{Re} \alpha > |\operatorname{Re} \nu|].
\end{aligned}$$

$$\begin{aligned}
6. \int_a^\infty (x-a)^{(\lambda-1)/2} (x+a)^{\lambda-1/2} K_\lambda(b\sqrt{x-a}) C_n^\lambda\left(\frac{x}{a}\right) dx = \\
= \frac{\pi^2}{2 (n!)} \Gamma[2\lambda + n] \left(\frac{a}{b}\right)^\lambda \left[J_{\lambda+n}^2\left(b\sqrt{\frac{a}{2}}\right) + Y_{\lambda+n}^2\left(b\sqrt{\frac{a}{2}}\right)\right] \\
[a, \operatorname{Re} b > 0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$7. \int_a^\infty (x^2 - a^2)^{\lambda-1/2} (x+z)^{-\lambda/2} K_\lambda(b\sqrt{x+z}) C_n^\lambda\left(\frac{x}{a}\right) dx =$$

$$= \frac{1}{n!} \left(\frac{a}{\sqrt{b}}\right)^\lambda \Gamma\left[\begin{matrix} 2\lambda+n \\ \lambda \end{matrix}\right] K_{\lambda+n}(z_+) K_{\lambda+n}(z_-)$$

$$[\sqrt{2} z_\pm = b^{1/2} (z \pm \sqrt{z^2 - a^2})^{1/2}; a, \operatorname{Re} b > 0; \operatorname{Re} \lambda > -1/2; |\arg z| < \pi].$$

$$8. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\lambda-1/2} K_\nu\left(\frac{b}{\sqrt{x+a}}\right) C_n^\lambda\left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^n (2\lambda)_n}{n!} 2^{\lambda-2\alpha-3/2} \Gamma\left(\frac{\nu}{2} - \alpha\right) \Gamma\left(-\frac{\nu}{2} - \alpha\right) a^{\alpha+\lambda-1/2} \times$$

$$\times {}_2F_3\left(\frac{1}{2} - \lambda - n, \frac{1}{2} + \lambda + n; \frac{1}{2} + \lambda, 1 + \alpha + \frac{\nu}{2}, 1 + \alpha - \frac{\nu}{2}; \frac{b^2}{8a}\right) +$$

$$+ \frac{(-1)^n (2\lambda)_n}{n!} \left(\frac{1-\nu}{2} + \lambda - \alpha\right)_n 2^{\alpha+\lambda+3(\nu-1)/2} \times$$

$$\times \Gamma\left[\begin{matrix} \nu, \lambda+1/2, \alpha+\nu/2 \\ \alpha+\lambda+n+(\nu+1)/2 \end{matrix}\right] a^{\alpha+\lambda+(\nu-1)/2} b^{-\nu} {}_2F_3\left(\frac{1-\nu}{2} - \alpha - \lambda - n, \right.$$

$$\left. \frac{1-\nu}{2} + \lambda - \alpha + n; 1-\nu, 1-\alpha - \frac{\nu}{2}, \frac{1-\nu}{2} + \lambda - \alpha; \frac{b^2}{8a}\right) + \frac{(-1)^n (2\lambda)_n}{n!} \times$$

$$\times \left(\frac{1+\nu}{2} + \lambda - \alpha\right)_n 2^{\alpha+\lambda-3(\nu+1)/2} \Gamma\left[\begin{matrix} -\nu, \lambda+1/2, \alpha-\nu/2 \\ \alpha+\lambda+n+(1-\nu)/2 \end{matrix}\right] a^{\alpha+\lambda-(\nu+1)/2} b^\nu \times$$

$$\times {}_2F_3\left(\frac{1+\nu}{2} - \alpha - \lambda - n, \frac{1+\nu}{2} + \lambda + n - \alpha; 1+\nu, 1+\frac{\nu}{2} - \alpha, \frac{1+\nu}{2} + \lambda - \alpha; \frac{b^2}{8a}\right)$$

$$[a, \operatorname{Re} b > 0; \operatorname{Re} \lambda > -1/2].$$

$$9. \int_a^\infty (x-a)^{\lambda-1/2} (x+a)^{-\tau} K_\nu\left(\frac{b}{\sqrt{x+a}}\right) C_n^\lambda\left(\frac{x}{a}\right) dx =$$

$$= \frac{(2\lambda)_n}{n!} \left(\tau + \lambda - \frac{\nu+1}{2}\right)_n 2^{\lambda-\tau+3(\nu-1)/2} \times$$

$$\times \Gamma\left[\begin{matrix} \nu, \lambda+1/2, \tau-\lambda-n-(\nu+1)/2 \\ \tau-\nu/2 \end{matrix}\right] a^{(1+\nu)/2+\lambda-\tau} b^{-\nu} \times$$

$$\times {}_2F_3\left(\tau-\lambda-n-\frac{\nu+1}{2}, \lambda+\tau+n-\frac{\nu+1}{2}; \tau-\frac{\nu}{2}, \lambda+\tau-\frac{\nu+1}{2}, 1-\nu; \frac{b^2}{8a}\right) +$$

$$+ \frac{(2\lambda)_n}{n!} \left(\tau + \lambda + \frac{\nu-1}{2}\right)_n 2^{\lambda-\tau-3(\nu+1)/2} \times$$

$$\times \Gamma\left[\begin{matrix} -\nu, \lambda+1/2, \tau-\lambda-n+(\nu-1)/2 \\ \tau+\nu/2 \end{matrix}\right] a^{(1-\nu)/2+\lambda-\tau} b^\nu \times$$

$$\times {}_2F_3\left(\tau-\lambda-n+\frac{\nu-1}{2}, \lambda+\tau+n+\frac{\nu-1}{2}; \tau+\frac{\nu}{2}, \lambda+\tau+\frac{\nu-1}{2}, 1+\nu; \frac{b^2}{8a}\right)$$

$$[a > 0; \operatorname{Re} \lambda > -1/2; 2 \operatorname{Re}(\tau - \lambda) > |\operatorname{Re} \nu| + 2n + 1].$$

$$10. \int_a^\infty (x-a)^{\alpha-1} (x+a)^{\lambda-1/2} K_\nu\left(\frac{b}{\sqrt{x-a}}\right) C_n^\lambda\left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^n (2\lambda)_n}{n!} \left(\frac{1+\nu}{2} + \lambda - \alpha\right)_n 2^{\alpha+\lambda-3(\nu+1)/2} \times$$

$$\times \Gamma\left[\begin{matrix} -\nu, \alpha-\nu/2, (1+\nu)/2 - \alpha - \lambda - n \\ 1/2 - \lambda \end{matrix}\right] a^{\alpha+\lambda-(\nu+1)/2} b^\nu \times$$

$$\begin{aligned}
& \times {}_2F_3 \left(\frac{1+v}{2} + \lambda - \alpha + n, \frac{1+v}{2} - \alpha - \lambda - n; v+1, 1 + \frac{v}{2} - \alpha, \frac{1+v}{2} + \lambda - \alpha; -\frac{b^2}{8a} \right) + \\
& + \frac{(-1)^n (2\lambda)_n}{n!} \left(\frac{1-v}{2} + \lambda - \alpha \right)_n 2^{\alpha+\lambda+3(v-1)/2} \times \\
& \times \Gamma \left[\begin{matrix} v, \alpha + v/2, (1-v)/2 - \alpha - \lambda - n \\ 1/2 - \lambda \end{matrix} \right] a^{\alpha+\lambda+(v-1)/2} b^{-v} \times \\
& \times {}_2F_3 \left(\frac{1-v}{2} + \lambda + n - \alpha, \frac{1-v}{2} - \alpha - \lambda - n; 1-v, 1 - \frac{v}{2} - \alpha, \frac{1-v}{2} + \lambda - \alpha; -\frac{b^2}{8a} \right) + \\
& + \frac{(2\lambda)_n}{n!} 2^{\lambda-2\alpha-3/2} \Gamma \left(\frac{v}{2} - \alpha \right) \Gamma \left(-\frac{v}{2} - \alpha \right) a^{\lambda-1/2} b^{2\alpha} \times \\
& \times {}_2F_3 \left(\frac{1}{2} - \lambda - n, \frac{1}{2} + \lambda + n; \frac{1}{2} + \lambda, 1 + \alpha - \frac{v}{2}, 1 + \alpha + \frac{v}{2}; -\frac{b^2}{8a} \right) \\
& [a, \operatorname{Re} b > 0; 2 \operatorname{Re} (\alpha + \lambda) < 1 - |\operatorname{Re} v| - 2n].
\end{aligned}$$

$$\begin{aligned}
11. \int_{-a}^a (x+a)^{-\beta-n-1} (a-x)^{\beta-1} K_v \left(b \sqrt{\frac{a-x}{a+x}} \right) C_n^\lambda \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^n (2\lambda)_n 2^{2\beta+n-2}}{n! a^{n+1} b^{2\beta+2n}} \Gamma \left(\beta + n + \frac{v}{2} \right) \Gamma \left(\beta + n - \frac{v}{2} \right) \times \\
\times {}_2F_3 \left(-n, \frac{1}{2} - \lambda - n; \frac{1}{2} + \lambda, 1 - n - \beta - \frac{v}{2}, 1 + \frac{v}{2} - n - \beta; -\frac{b^2}{4} \right) \\
[a, \operatorname{Re} b > 0; 2 \operatorname{Re} \beta > |\operatorname{Re} v|].
\end{aligned}$$

$$\begin{aligned}
12. \int_{-a}^a (x+a)^{n+2\lambda-\beta-1} (a-x)^{\beta-1} K_v \left(b \sqrt{\frac{a-x}{a+x}} \right) C_n^\lambda \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^n (2\lambda)_n 2^{2\beta-2\lambda-n-2}}{n!} \Gamma \left(\frac{v}{2} + \beta - 2\lambda - n \right) \Gamma \left(\beta - 2\lambda - n - \frac{v}{2} \right) \times \\
\times a^{2\lambda+n-1} b^{2(2\lambda+n-\beta)} {}_2F_3 \left(\frac{1}{2} + \lambda + n, 2\lambda + n; \frac{1}{2} + \lambda, \right. \\
\left. 1 + 2\lambda + n - \beta - \frac{v}{2}, 1 + 2\lambda + n - \beta + \frac{v}{2}; -\frac{b^2}{4} \right) + \\
+ \frac{1}{n! (\lambda + 1/2)_n} \left(\frac{1-v}{2} + \lambda - \beta \right)_n 2^{2\lambda+n-v-2} \Gamma \left[\begin{matrix} \beta - v/2, n + 2\lambda - \beta - v/2 \\ 2\lambda \end{matrix} \right] a^{2\lambda+n-1} b^v \times \\
\times {}_2F_3 \left(\beta + \frac{v}{2}, \frac{1+v}{2} + \beta - \lambda; \frac{1+v}{2} + \beta - \lambda - n, 1 + \frac{v}{2} + \beta - 2\lambda - n, 1 + v; -\frac{b^2}{4} \right) + \\
+ \frac{1}{n! (\lambda + 1/2)_n} \left(\frac{1+v}{2} + \lambda - \beta \right)_n 2^{2\lambda+n+v-2} \Gamma \left[\begin{matrix} \beta + v/2, n + 2\lambda - \beta + v/2 \\ 2\lambda \end{matrix} \right] a^{2\lambda+n-1} b^{-v} \times \\
\times {}_2F_3 \left(\beta - \frac{v}{2}, \frac{1-v}{2} + \beta - \lambda; \frac{1-v}{2} + \beta - \lambda - n, 1 - \frac{v}{2} + \beta - 2\lambda - n, 1 - v; -\frac{b^2}{4} \right) \\
[a, \operatorname{Re} b > 0; 2 \operatorname{Re} \beta > |\operatorname{Re} v|].
\end{aligned}$$

2.21.14. Интегралы от $A(x) P_m(bx) C_n^\lambda(cx)$.

$$\begin{aligned}
1. \int_0^a (a^2 - x^2)^{\lambda-3/2} P_{2m} \left(\frac{x}{a} \right) C_n^\lambda \left(\frac{x}{a} \right) dx = \\
= \frac{a^{2\lambda-2} \pi (1/2)_{m+n-2[n/2]} \Gamma \left[\begin{matrix} \lambda - 1/2 \\ \lambda, m + n - 2[n/2] + 1/2 \end{matrix} \right]}{2} \quad [a > 0; \operatorname{Re} \lambda > 1/2].
\end{aligned}$$

$$\begin{aligned}
2. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\lambda-1/2} P_{2m+\varepsilon} \left(\frac{x}{b} \right) \left\{ \frac{C_{2n+1}^\lambda(x/a)}{C_{2n}^\lambda(x/a)} \right\} dx = \\
= \frac{(-1)^{m+n} 2^{\varepsilon-1} a^{\alpha+2\lambda+\varepsilon-1} (1/2)_{m+\varepsilon} (2\lambda)_{2n+\delta}}{m! (2n+\delta)! b^\varepsilon} \times \\
\times \left(\frac{1-\alpha+\delta-\varepsilon}{2} \right)_n \Gamma \left[\frac{\lambda+1/2, (\alpha+\delta+\varepsilon)/2}{(1+\alpha+\delta+\varepsilon)/2+\lambda+n} \right] {}_4F_3 \left(-m, \frac{1}{2}+\varepsilon+m, \frac{\alpha+1}{2}, \right. \\
\left. \frac{\alpha}{2}+\varepsilon, \frac{1}{2}+\varepsilon, \frac{1+\alpha-\delta+\varepsilon}{2}-n, \frac{1+\alpha+\delta+\varepsilon}{2}+\lambda+n; \frac{a^2}{b^2} \right) \\
\left[\varepsilon=0 \text{ или } 1; b>a>0; \operatorname{Re} \lambda > -1/2; \operatorname{Re} \alpha > -\varepsilon-\delta, \delta=\begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right].
\end{aligned}$$

$$\begin{aligned}
3. \int_0^a x^{\alpha-1} (b^2 - x^2)^{\lambda-1/2} P_{2n+\varepsilon} \left(\frac{x}{a} \right) \left\{ \frac{C_{2n+1}^\lambda(x/b)}{C_{2n}^\lambda(x/b)} \right\} dx = \\
= \frac{(-1)^{m+n} 2^{\delta-1} a^{\alpha+\delta} b^{2\lambda} \delta^{-1} (\lambda)_{n+\delta} ((1-\alpha-\delta+\varepsilon)/2)_m}{n! ((\alpha+\delta+\varepsilon)/2)_{m+1}} \times \\
\times {}_4F_3 \left(\frac{1}{2}-\lambda-n, \frac{1}{2}+\delta+n, \frac{\alpha+1}{2}, \frac{\alpha}{2}+\delta; \right. \\
\left. \delta+\frac{1}{2}, \frac{1+\alpha+\delta-\varepsilon}{2}-m, \frac{\alpha+\delta+\varepsilon}{2}+m+1; \frac{a^2}{b^2} \right) \\
\left[\varepsilon=0 \text{ или } 1; b>a>0; \operatorname{Re} \alpha > -\delta-\varepsilon, \delta=\begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right].
\end{aligned}$$

$$\begin{aligned}
4. \int_{-a}^a P_{n-2m} \left(\frac{x}{a} \right) C_n^\lambda(cx) dx = \\
= \frac{(\lambda)_n a^{n+1} c^n}{(1/2)_{n-m+1} (1-\lambda-n)_m} P_m^{(-\lambda-n, \lambda-3/2)} \left(1 - \frac{2}{a^2 c^2} \right) \quad [a>0].
\end{aligned}$$

$$\begin{aligned}
5. \int_{-a}^a (x+a)^{\alpha-1} P_m \left(\frac{x}{a} \right) C_n^\lambda \left(\frac{x}{a} \right) dx = \frac{(-1)^{m+n} (2a)^\alpha (2\lambda)_n (1-\alpha)_m}{n! (\alpha)_{m+1}} \times \\
\times {}_4F_3 \left(-n, 2\lambda+n, \alpha, \alpha; \lambda+\frac{1}{2}, \alpha+m+1, \alpha-m; 1 \right) \quad [a, \operatorname{Re} \alpha > 0].
\end{aligned}$$

$$\begin{aligned}
6. \int_{-a}^a (a^2 - x^2)^{\lambda-1/2} P_m \left(\frac{x}{a} \right) C_n^\lambda \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^n (n+1)_m (-m)_n \sqrt{\pi} (2\lambda)_n}{m! n! 2^{2n}} \Gamma \left[\frac{\lambda+1/2}{\lambda+n+1} \right] a^{2\lambda} \times \\
\times {}_3F_2 \left(n-m, 1+m+n, \lambda+n+\frac{1}{2}; 2\lambda+n+1, n+1; 1 \right) \\
[a, n>0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
7. \int_{-a}^a (a^2 - x^2)^{\lambda-1/2} P_{n+2m} \left(\frac{x}{b} \right) C_n^\lambda \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^m (1/2)_{m+n} (2\lambda)_n \sqrt{\pi} a^{2\lambda+n+2m}}{(\lambda)_{m+n+1} n! b^{2m+n}} \Gamma \left[\frac{\lambda+1/2}{\lambda} \right] \times \\
\times P_m^{(-1/2-n-2m, -\lambda-1/2)} \left(1 - \frac{2b^2}{a^2} \right) \quad [a>0, \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
8. \int_{-a}^a (x+a)^{\lambda-1/2} (a-x)^{\beta-1} P_m \left(\frac{x}{a} \right) C_n^{\lambda} \left(\frac{x}{a} \right) dx = \\
= \frac{(2\lambda)_n (1/2 + \lambda - \beta)_n}{n!} \Gamma \left[\begin{matrix} \beta, \lambda + 1/2 \\ \beta + \lambda + n + 1/2 \end{matrix} \right] (2a)^{\beta + \lambda - 1/2} \times \\
\times {}_4F_3 \left(-m, m+1, \frac{1}{2} + \beta - \lambda, \beta; 1, \frac{1}{2} + \beta + \lambda + n, \frac{1}{2} + \beta - \lambda - n; 1 \right) \\
[a, \operatorname{Re} \beta > 0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
9. \int_{-a}^a (x+a)^{\lambda-1/2} (a-x)^{\lambda-3/2} P_m \left(\frac{x}{a} \right) C_n^{\lambda} \left(\frac{x}{a} \right) dx = \\
= \frac{\sqrt{\pi}}{2^{2\lambda-1}} \Gamma \left[\begin{matrix} \lambda - 1/2 \\ \lambda \end{matrix} \right] (2a)^{2\lambda-1} \left[1 + (-1)^n \frac{(-m)_{n+1} (m+1)_{n+1} (\lambda - 1/2)_{n+1}}{(n+2\lambda)_{n+1} [(n+1)!]^2} \times \right. \\
\left. \times {}_4F_3 \left(1+n-m, 2+m+n, \frac{1}{2} + \lambda + n, n+1; n+2, n+2, 2\lambda + 2n + 1; 1 \right) \right] \\
[a > 0; \operatorname{Re} \lambda > 1/2].
\end{aligned}$$

$$\begin{aligned}
10. \int_a^b (x^2 - a^2)^{\lambda-1/2} P_m \left(\frac{x}{b} \right) C_n^{\lambda} \left(\frac{x}{a} \right) dx = \\
= \frac{(2\lambda)_n b^{n-1}}{(2\lambda+1) n! a^n} (b^2 - a^2)^{\lambda+1/2} {}_2F_1 \left(1 + \frac{m-n}{2}, 1 - \frac{m+n}{2}; \lambda + \frac{3}{2}; 1 - \frac{a^2}{b^2} \right) \\
[b > a > 0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
11. \int_a^b (b^2 - x^2)^{\lambda-1/2} P_m \left(\frac{x}{a} \right) C_n^{\lambda} \left(\frac{x}{b} \right) dx = \\
= \frac{(2\lambda)_n b^{m-1}}{(2\lambda+1) n! a^m} (b^2 - a^2)^{\lambda+1/2} {}_2F_1 \left(\lambda + \frac{n-m+1}{2}, \frac{1-m-n}{2}; \lambda + \frac{3}{2}; 1 - \frac{a^2}{b^2} \right) \\
[b > a > 0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
12. \int_a^b x^{\alpha-1} (x^2 - a^2)^{\lambda-1/2} P_{2m+\varepsilon} \left(\frac{x}{b} \right) \left\{ \begin{matrix} C_{2n+\delta}^{\lambda} (x/a) \\ C_{2n}^{\lambda} (x/a) \end{matrix} \right\} dx = \\
= \frac{(-1)^m 2^{\varepsilon-1} a^{\alpha+2\lambda+\varepsilon-1} (2\lambda)_{2n+\delta} (1/2)_{m+\varepsilon}}{m! (2n+\delta)! b^{\varepsilon}} \left(\frac{1+\delta-\varepsilon-\alpha}{2} \right)_n \times \\
\times \Gamma \left[\begin{matrix} \lambda + 1/2, (1-\alpha-\delta-\varepsilon)/2 - \lambda - n \\ 1 - (\alpha + \delta + \varepsilon)/2 \end{matrix} \right] {}_4F_3 \left(-m, \frac{1}{2} + \varepsilon + m, \frac{\alpha+1}{2}, \right. \\
\left. \frac{\alpha}{2} + \varepsilon; \varepsilon + \frac{1}{2}, \frac{1+\alpha+\varepsilon-\delta}{2} - n, \frac{1+\alpha+\delta+\varepsilon}{2} + \lambda + n; \frac{a^2}{b^2} \right) + \\
+ \frac{(-1)^m 2^{2n+\delta-1} b^{\alpha+2\lambda+2n+\delta-1} (\lambda)_{2n+\delta} ((\varepsilon-\alpha-\delta)/2 + 1 - \lambda - n)_m}{(2n+\delta)! a^{2n+\delta} ((\alpha+\delta+\varepsilon-1)/2 + \lambda + n)_{m+1}} \times \\
\times {}_4F_3 \left(\frac{1}{2} - \lambda - n, 1 - \lambda - \delta - n, \frac{\varepsilon-\alpha-\delta}{2} + 1 + m - \lambda - n, \right. \\
\left. \frac{1-\alpha-\delta-\varepsilon}{2} - \lambda - m - n; 1 - \lambda - \delta - 2n, 1 - \lambda - n - \frac{\alpha}{2}, \frac{3-\alpha-\delta-\lambda-n}{2}; \frac{a^2}{b^2} \right) \\
[\varepsilon = 0 \text{ или } 1; b > a > 0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
13. \int_a^b x^{\alpha-1} (b^2 - x^2)^{\lambda-1/2} P_{2m+\varepsilon} \left(\frac{x}{a} \right) \left\{ \frac{C_{2n+1}^\lambda(x/b)}{C_{2n}^\lambda(x/b)} \right\} dx = \\
= \frac{(-1)^{n+m-1} 2^{2\delta-1} a^{\alpha+\varepsilon} b^{2\lambda-\varepsilon-1} (\lambda)_{n+\delta} ((1+\varepsilon-\delta-\alpha)/2)_m}{n! ((\alpha+\delta+\varepsilon)/2)_{m+1}} \times \\
\times {}_4F_3 \left(\frac{1}{2} - \lambda - n, \frac{1}{2} + \delta + n, \frac{\alpha+1}{2}, \frac{\alpha}{2} + \delta; \frac{1}{2} + \delta, \frac{1+\alpha+\delta-\varepsilon}{2} - m, \right. \\
\left. \frac{\alpha+\delta+\varepsilon}{2} + m + 1; \frac{a^2}{b^2} \right) + \frac{(-1)^n 2^{2m+\varepsilon-1} b^{\alpha+2\lambda+2m+\varepsilon-1} (1/2)_{2m+\varepsilon} (2\lambda)_{2n+\delta}}{(2m+\varepsilon)! (2n+\delta)! a^{2m+\varepsilon}} \times \\
\times \left(\frac{1+\delta-\alpha-\varepsilon}{2} - m \right)_n \Gamma \left[\lambda + 1/2, (\alpha+\delta+\varepsilon)/2 + m \right. \\
\left. (1+\alpha+\delta+\varepsilon)/2 + m + n + \lambda \right] \times \\
\times {}_4F_3 \left(-m, \frac{1}{2} - \varepsilon - m, \frac{1+\delta-\alpha-\varepsilon}{2} + n - m, \frac{1-\alpha-\delta-\varepsilon}{2} - \lambda - m - n; \right. \\
\left. \frac{1}{2} - \varepsilon - 2m, \frac{1-\alpha}{2} - m, 1 - \frac{\alpha}{2} - \varepsilon - m; \frac{a^2}{b^2} \right) \\
[\varepsilon = 0 \text{ или } 1; b > a > 0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
14. \int_a^\infty x^{\alpha-1} (x^2 - a^2)^{\lambda-1/2} P_{2m+\varepsilon} \left(\frac{x}{b} \right) \left\{ \frac{C_{2n+1}^\lambda(x/a)}{C_{2n}^\lambda(x/a)} \right\} dx = \\
= \frac{2^{2m+\varepsilon-1} a^{\alpha+2\lambda+2m+\varepsilon-1} (2\lambda)_{2n+\delta} (1/2)_{2m+\varepsilon}}{(2m+\varepsilon)! (2n+\delta)! b^{2m+\varepsilon}} \times \\
\times \left(\frac{1+\delta-\alpha-\varepsilon}{2} - m \right)_n \Gamma \left[\lambda + 1/2, (1-\alpha-\varepsilon-\delta)/2 - \lambda - m - n \right] \times \\
\times {}_4F_3 \left(-m, \frac{1}{2} - \varepsilon - m, \frac{1+\delta-\alpha-\varepsilon}{2} + n - m, \frac{1-\alpha-\delta-\varepsilon}{2} - \lambda - m - n; \right. \\
\left. 1 - \frac{\alpha}{2} - \varepsilon - m, \frac{1}{2} - \varepsilon - 2m, \frac{1-\alpha}{2} - m; \frac{b^2}{a^2} \right) \\
[\varepsilon = 0 \text{ или } 1; a > b > 0; \operatorname{Re} \lambda > -1/2; \operatorname{Re} (\alpha + 2\lambda) < 1 - 2m - 2n - \varepsilon - \delta].
\end{aligned}$$

$$\begin{aligned}
15. \int_a^\infty x^{\alpha-1} (x^2 - b^2)^{\lambda-1/2} P_{2m+\varepsilon} \left(\frac{x}{a} \right) \left\{ \frac{C_{2n+1}^\lambda(x/b)}{C_{2n}^\lambda(x/b)} \right\} dx = \\
= \frac{(-1)^{m-1} 2^{2n+\delta-1} a^{\alpha+2\lambda+2n+\delta-1} (\lambda)_{2n+\delta} ((\varepsilon-\alpha-\delta)/2 + 1 - \lambda - n)_m}{(2n+\delta)! b^{2n+\delta} ((\alpha+\delta+\varepsilon-1)/2 + \lambda + n)_{m+1}} \times \\
\times {}_4F_3 \left(\frac{1}{2} - \lambda - n, 1 - \lambda - \delta - n, \frac{\varepsilon-\alpha-\delta}{2} + 1 - \lambda + m - n, \right. \\
\left. \frac{1-\alpha-\varepsilon-\delta}{2} - \lambda - m - n; \frac{3-\alpha}{2} - \delta - \lambda - n, 1 - \lambda - \delta - 2n, 1 - \lambda - n - \frac{\alpha}{2}; \frac{b^2}{a^2} \right) \\
[\varepsilon = 0 \text{ или } 1; a > b > 0; \operatorname{Re} (\alpha + 2\lambda) < 1 - 2m - 2n - \varepsilon - \delta].
\end{aligned}$$

$$\begin{aligned}
16. \int_a^\infty (x-a)^{\lambda-1/2} (x+a)^{-\tau} P_m \left(\frac{x}{a} \right) C_n^\lambda \left(\frac{x}{a} \right) dx = \\
= \frac{2^{2m+\lambda-\tau+1/2} (1/2)_m (2\lambda)_n (\tau+\lambda-m-1/2)_n a^{\lambda-\tau+1/2}}{m! n!} \times \\
\times \Gamma \left[\lambda + 1/2, \tau - \lambda - m - n - 1/2 \right] {}_4F_3 \left(-m, -m, \tau + \lambda + n - m - \frac{1}{2}, \right. \\
\left. \tau - \lambda - m - n - \frac{1}{2}; -2m, \tau + \lambda - m - \frac{1}{2}, \tau - m; 1 \right) \\
[a > 0; -1/2 < \operatorname{Re} \lambda < \operatorname{Re} \tau - m - n - 1/2].
\end{aligned}$$

2.21.15. Интегралы от $A(x) P_m(\varphi(x)) C_n^\lambda(cx)$

$$1. \int_{-a}^a (a \pm x)^{\alpha-1} (a \mp x)^{\lambda-1/2} P_m(2bx \pm 2ab \mp 1) C_n^\lambda\left(\frac{x}{a}\right) dx =$$

$$= \frac{(\mp 1)^{m+n} (2\lambda)_n (1/2 + \lambda - \alpha)_n}{n!} \Gamma\left[\alpha, \lambda + 1/2\right] (2a)^{\alpha+\lambda-1/2} \times$$

$$\times {}_4F_3\left(-m, m+1, \frac{1}{2} + \alpha - \lambda, \alpha; 1, \alpha + \lambda + n + \frac{1}{2}, \frac{1}{2} + \alpha - \lambda - n; 2ab\right)$$

[a, b, Re α > 0; 2ab < 1; Re λ > -1/2]

$$2. \int_a^b (x-a)^{\lambda-1/2} P_m\left(\frac{2x+a-b}{a+b}\right) C_n^\lambda\left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^m (2\lambda)_{2n} (1/2 - \lambda - n)_m}{n! (2a)^n (\lambda + 1/2)_{m+n+1}} (a+b)^{\lambda+n+1/2} {}_3F_2\left(1-2\lambda-n, \frac{1}{2} + m - \lambda - n, \right.$$

$$\left. -\lambda - m - n - \frac{1}{2}; 1-2\lambda-2n, \frac{1}{2} - \lambda - n; \frac{2a}{a+b}\right)$$

[b > a > 0; Re λ > -1/2]

$$3. \int_a^b (b-x)^{\lambda-1/2} P_m\left(\frac{2x+b-a}{a+b}\right) C_n^\lambda\left(\frac{x}{b}\right) dx =$$

$$= \frac{(-1)^n 2^{3m+\lambda+1/2} (1/2)_m (\lambda - m - 1/2)_n}{(a+b)^m (\lambda + 1/2)_{m+n+1}} b^{\lambda+m+1/2} \times$$

$$\times {}_3F_2\left(-m, \lambda + n - m - \frac{1}{2}, -m - n - \lambda - \frac{1}{2}; -2m, \lambda - m - \frac{1}{2}; \frac{a+b}{2b}\right)$$

[b > a > 0; Re λ > -1/2]

$$4. \int_{\pm a}^b (x \mp a)^{\alpha-1} (a \pm x)^{\lambda-1/2} P_m\left(\frac{2x \mp a - b}{b \mp a}\right) C_n^\lambda\left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^m (\pm 1)^n (2\lambda)_n (1-\alpha)_m (2a)^{\lambda-1/2} (b \mp a)^\alpha}{n! (\alpha)_{m+1}} \times$$

$$\times {}_4F_3\left(\frac{1}{2} - \lambda - n, \frac{1}{2} + \lambda + n, \alpha, \alpha; \alpha + m + 1, \alpha - m, 1; \frac{a \mp b}{2a}\right)$$

[Re α > 0, {b > a > 0; a > b > 0}]

$$5. \int_a^b (x-a)^{\lambda-1/2} (x+a)^{-\tau} P_m\left(\frac{2x+a-b}{a+b}\right) C_n^\lambda\left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^m (2\lambda)_n (\tau + \lambda - 1/2)_n}{n!} \Gamma\left[\tau, \lambda + 1/2, \tau - \lambda - n - 1/2\right] (2a)^{1/2+\lambda-\tau} \times$$

$$\times {}_4F_3\left(-m, m+1, \frac{3}{2} - \tau - \lambda, 1 - \tau; \frac{3}{2} + n + \lambda - \tau, \frac{3}{2} - \lambda - \tau - n, 1; \frac{2a}{a+b}\right) +$$

$$+ \frac{(-1)^m 2^n (\lambda)_n (\tau - \lambda - n + 1/2)_m}{n! a^n (1/2 + n + \lambda - \tau)_{m+1}} (a+b)^{1/2+\lambda+n-\tau} \times$$

$$\times {}_4F_3\left(\frac{1}{2} - \lambda - n, 1 - 2\lambda - n, \frac{1}{2} + \tau + m - \lambda - n, \tau - \lambda - m - n - \frac{1}{2}; \right.$$

$$\left. 1 - 2\lambda - 2n, \frac{1}{2} + \tau - \lambda - n, \frac{1}{2} + \tau - \lambda - n; \frac{2a}{a+b}\right)$$

[b > a > 0; Re λ > -1/2]

$$\begin{aligned}
6. \int_a^b (b-x)^{\lambda-1/2} (b+x)^{-\tau} P_m \left(\frac{2x+b-a}{a+b} \right) C_n^\lambda \left(\frac{x}{b} \right) dx = \\
= \frac{(-1)^{m+n-1} (2\lambda)_n (\tau)_m (2b)^{\lambda-1/2}}{n! (1-\tau)_{m+1}} (a+b)^{1-\tau} \times \\
\times {}_4F_3 \left(\frac{1}{2} - \lambda - n, \frac{1}{2} + \lambda + n, 1 - \tau, 1 - \tau; 2 + m - \tau, 1 - m - \tau, \lambda + \frac{1}{2}; \frac{a+b}{2b} \right) + \\
+ \frac{(-1)^n 2^{3m+\lambda-\tau+1/2} (1/2)_m (2\lambda)_n (\lambda + \tau - m - 1/2)_n}{m! n! (a+b)^m} \times \\
\times \Gamma \left[\frac{\lambda + 1/2, 1 + m - \tau}{3/2 + \lambda + m + n - \tau} \right] b^{1/2+\lambda+m-\tau} {}_4F_3 \left(-m, -m, \lambda + \tau + n - m - \frac{1}{2}, \right. \\
\left. \tau - \lambda - m - n - \frac{1}{2}; -2m, \lambda + \tau - m - \frac{1}{2}, \tau - m; \frac{a+b}{2b} \right) \\
[b > a > 0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
7. \int_a^\infty (x-b)^{\lambda-1/2} P_m \left(\frac{2x+b-a}{a+b} \right) C_n^\lambda \left(\frac{x}{b} \right) dx = \\
= \frac{(-1)^{m-1} 2^n (\lambda)_n (1/2 - \lambda - n)_m (a+b)^{\lambda+n+1/2}}{n! b^n (1/2 + \lambda + n)_{m+1}} \times \\
\times {}_3F_2 \left(1 - 2\lambda - n, \frac{1}{2} + m - \lambda - n, -\lambda - m - n - \frac{1}{2}; 1 - 2\lambda - 2n, \right. \\
\left. \frac{1}{2} - \lambda - n; \frac{2b}{a+b} \right) \quad [a > b > 0; \operatorname{Re} \lambda < -m - n - 1/2].
\end{aligned}$$

$$\begin{aligned}
8. \int_a^\infty (x-b)^{\lambda-1/2} (x+b)^{-\tau} P_m \left(\frac{2x+b-a}{a+b} \right) C_n^\lambda \left(\frac{x}{b} \right) dx = \\
= \frac{(-1)^{m-1} 2^n (\lambda)_n (1/2 + \tau - \lambda - n)_m (a+b)^{\lambda+n-\tau+1/2}}{n! b^n (1/2 - \tau + \lambda + n)_{m+1}} \times \\
\times {}_4F_3 \left(\frac{1}{2} - \lambda - n, 1 - 2\lambda - n, \frac{1}{2} + \tau + m - \lambda - n, \tau - \lambda - m - n - \frac{1}{2}; \right. \\
\left. 1 - 2\lambda - 2n, \frac{1}{2} + \tau - \lambda - n, \frac{1}{2} + \tau - \lambda - n; \frac{2b}{a+b} \right) \\
[a > b > 0; \operatorname{Re} (\tau - \lambda) > m + n + 1/2].
\end{aligned}$$

$$\begin{aligned}
9. \int_a^\infty (x-a)^{\lambda-1/2} (x+a)^{-\tau} P_m (2bx + 2ab - 1) C_n^\lambda \left(\frac{x}{a} \right) dx = \\
= \frac{2^{3m+\lambda-\tau+1/2} (1/2)_m (2\lambda)_n (\tau + \lambda - m - 1/2)_n}{m! n!} \times \\
\times \Gamma \left[\frac{\lambda + 1/2, \tau - \lambda - m - n - 1/2}{\tau - m} \right] a^{m+\lambda-\tau+1/2} b^m \times \\
\times {}_4F_3 \left(-m, -m, \tau + \lambda + n - m - \frac{1}{2}, \tau - \lambda - m - n - \frac{1}{2}; \right. \\
\left. -2m, \tau + \lambda - m - \frac{1}{2}, \tau - m; \frac{1}{2ab} \right) \\
[a > 0; 2ab > 1; \operatorname{Re} \lambda > -1/2; \operatorname{Re} (\tau - \lambda) > m + n + 1/2]
\end{aligned}$$

$$\begin{aligned}
10. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\lambda-1/2} P_{2m+\varepsilon}(b\sqrt{x+a}) C_n^\lambda\left(\frac{x}{a}\right) dx = \\
= \frac{(-1)^{m+n} (1/2)_{m+\varepsilon} (2\lambda)_n ((1-\varepsilon)/2 + \lambda - \alpha)_n}{m! n!} \times \\
\times \Gamma\left[\lambda + 1/2, \alpha + \varepsilon/2\right] 2^{\alpha+\lambda+(3\varepsilon-1)/2} a^{\alpha+\lambda+(\varepsilon-1)/2} b^\varepsilon \times \\
\times {}_4F_3\left(-m, \frac{1}{2} + \varepsilon + m, \alpha + \frac{\varepsilon}{2}, \frac{1+\varepsilon}{2} + \alpha - \lambda; \right. \\
\left. \frac{1+\varepsilon}{2} + \alpha + \lambda + n, \frac{1}{2} + \varepsilon, \frac{1+\varepsilon}{2} + \alpha - \lambda - n; 2ab^2\right) \\
[\varepsilon = 0 \text{ или } 1; a, b > 0; 2ab^2 \leq 1; \operatorname{Re} \alpha > -\varepsilon/2; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
11. \int_a^b (x-a)^{\lambda-1/2} P_{2m}\left(\sqrt{\frac{x+a}{b+a}}\right) C_n^\lambda\left(\frac{x}{a}\right) dx = \\
= \frac{(-1)^m (2\lambda)_{2n} (-\lambda-n)_m (a+b)^{\lambda+n+1/2}}{n! (2a)^n (\lambda+1/2)_{m+n+1}} \times \\
\times {}_3F_2\left(1-2\lambda-n, m-n-\lambda, -m-n-\lambda-\frac{1}{2}; 1-2\lambda-2n, -\lambda-n; \frac{2a}{a+b}\right) \\
[b > a > 0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
12. \int_a^b (b-x)^{\lambda-1/2} P_{2m}\left(\sqrt{\frac{x+b}{a+b}}\right) C_n^\lambda\left(\frac{x}{b}\right) dx = \\
= \frac{(-1)^n (2b)^{m+\lambda+1/2} (1/2+m)_m (2\lambda)_n (\lambda-m-1/2)_n}{n! (\lambda+1/2)_{m+n+1} (a+b)^m} \times \\
\times {}_3F_2\left(\frac{1}{2}-m, \lambda+n-m-\frac{1}{2}, -\lambda-m-n-\frac{1}{2}; \frac{1}{2}-2m, \lambda-m-\frac{1}{2}; \frac{a+b}{2b}\right) \\
[b > a > 0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
13. \int_{\pm a}^b (x \mp a)^{\alpha-1} (a \pm x)^{\lambda-1/2} P_{2m+\varepsilon}\left(\sqrt{\frac{x \mp a}{b \mp a}}\right) C_n^\lambda\left(\frac{x}{a}\right) dx = \\
= \frac{(-1)^m (\pm 1)^n (2\lambda)_n ((1+\varepsilon)/2 - \alpha)_m (b \mp a)^\alpha (2a)^{\lambda-1/2}}{n! (\alpha + \varepsilon/2)_{m+1}} \times \\
\times {}_4F_3\left(\frac{1}{2}-\lambda-n, \frac{1}{2} + \lambda + n, \alpha, \alpha + \frac{1}{2}; \frac{1}{2} + \lambda, \frac{1-\varepsilon}{2} - \alpha - m, \alpha + m + \frac{\varepsilon}{2} \mp 1; \frac{a \mp b}{2a}\right) \\
[\varepsilon = 0 \text{ или } 1; \operatorname{Re} \alpha > -\varepsilon/2, \left\{\begin{matrix} b > a > 0 \\ a > b > 0 \end{matrix}\right\}].
\end{aligned}$$

$$\begin{aligned}
14. \int_a^b (x-a)^{\lambda-1/2} (x+a)^{-\tau} P_{2m+\varepsilon}\left(\sqrt{\frac{x+a}{b+a}}\right) C_n^\lambda\left(\frac{x}{a}\right) dx = \\
= \frac{(-1)^m (1/2)_{m+\varepsilon} (2\lambda)_n}{m! n! (a+b)^{\varepsilon/2}} \left(\tau + \lambda - \frac{\varepsilon+1}{2}\right)_n 2^{\lambda-\tau+(3\varepsilon+1)/2} \times \\
\times a^{\lambda-\tau+(\varepsilon+1)/2} \Gamma\left[\lambda + 1/2, \tau - \lambda - n - (\varepsilon+1)/2\right] {}_4F_3\left(-m, \frac{1}{2} + \varepsilon + m, \right. \\
\left. \frac{3+\varepsilon}{2} - \tau - \lambda, 1 + \frac{\varepsilon}{2} - \tau; \frac{3+\varepsilon}{2} + \lambda + n - \tau, \frac{1}{2} + \varepsilon, \frac{3+\varepsilon}{2} - \tau - \lambda - n; \frac{2a}{a+b}\right) +
\end{aligned}$$

$$+ \frac{(-1)^m 2^n (\lambda)_n (\varepsilon/2 + \tau - \lambda - n)_m (a+b)^{\lambda+n-\tau+1/2}}{n! a^n ((\varepsilon+1)/2 + \lambda + n - \tau)_{m+1}} \times$$

$$\times {}_4F_3 \left(\frac{1}{2} - \lambda - n, 1 - 2\lambda - n, \frac{\varepsilon}{2} + \tau - \lambda + m - n, \tau - m - n - \lambda - \frac{\varepsilon+1}{2}; \right.$$

$$\left. 1 - 2\lambda - 2n, \frac{1}{2} + \tau - \lambda - n, \tau - \lambda - n; \frac{2a}{a+b} \right)$$

[$\varepsilon = 0$ или 1 ; $b > a > 0$; $\operatorname{Re} \lambda > -1/2$]

$$15. \int_a^b (b-x)^{\lambda-1/2} (b+x)^{-\tau} P_{2m+\varepsilon} \left(\sqrt{\frac{x+b}{a+b}} \right) C_n^\lambda \left(\frac{x}{b} \right) dx =$$

$$= \frac{(-1)^{m+n-1} (2\lambda)_n ((\varepsilon-1)/2 + \tau)_m (2b)^{\lambda-1/2} (a+b)^{1-\tau}}{n! (1 + \varepsilon/2 - \tau)_{m+1}} \times$$

$$\times {}_4F_3 \left(\frac{1}{2} - \lambda - n, \frac{1}{2} + \lambda + n, \frac{3}{2} - \tau, 1 - \tau; 2 + \frac{\varepsilon}{2} + m - \tau, \right.$$

$$\left. \frac{3-\varepsilon}{2} - \tau - m, \lambda + \frac{1}{2}; \frac{a+b}{2b} \right) + \frac{(-1)^n (2\lambda)_n (1/2)_{2m+\varepsilon}}{n! (2m+\varepsilon)!} \times$$

$$\times \left(\lambda + \tau - m - \frac{\varepsilon+1}{2} \right)_n \Gamma \left[\frac{\lambda+1/2, \varepsilon/2+1+m-\tau}{(3+\varepsilon)/2 + \lambda + m + n - \tau} \right] \times$$

$$\times 2^{\lambda+2m-\tau+(2\varepsilon+1)/2} b^{\lambda+m-\tau+(2\varepsilon+1)/2} (a+b)^{-m-\varepsilon/2} \times$$

$$\times {}_4F_3 \left(-m, \frac{1}{2} - \varepsilon - m, \lambda + \tau - m + n - \frac{\varepsilon+1}{2}, \tau - \lambda - m - n - \frac{\varepsilon+1}{2}; \right.$$

$$\left. \tau - m - \frac{\varepsilon}{2}, \frac{1}{2} - \varepsilon - 2m, \lambda + \tau - m - \frac{\varepsilon+1}{2}; \frac{a+b}{2b} \right)$$

[$\varepsilon = 0$ или 1 ; $b > a > 0$; $\operatorname{Re} \lambda > -1/2$]

$$16. \int_a^\infty (x-a)^{\lambda-1/2} (x+a)^{-\tau} P_{2m+\varepsilon} (b \sqrt{x+a}) C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(1/2)_{2m+\varepsilon} (2\lambda)_n}{(2m+\varepsilon)! n!} \left(\tau + \lambda - m - \frac{\varepsilon+1}{2} \right)_n 2^{\lambda+2m-\tau+(2\varepsilon+1)/2} \times$$

$$\times a^{\lambda+m-\tau+(2\varepsilon+1)/2} b^{2m+\varepsilon} \Gamma \left[\frac{\lambda+1/2, \tau-\lambda-m-n-(\varepsilon+1)/2}{\tau-m-\varepsilon/2} \right] \times$$

$$\times {}_4F_3 \left(-m, \frac{1}{2} - \varepsilon - m, \tau + \lambda + n - m - \frac{\varepsilon+1}{2}, \tau - \lambda - m - n - \frac{\varepsilon+1}{2}; \right.$$

$$\left. \frac{1}{2} - \varepsilon - 2m, \tau + \lambda - m - \frac{\varepsilon+1}{2}, \tau - m - \frac{\varepsilon}{2}; \frac{1}{2ab^2} \right)$$

[$\varepsilon = 0$ или 1 ; $a > 0$; $2ab^2 > 1$; $\operatorname{Re} \lambda > -1/2$; $\operatorname{Re} (\tau - \lambda) > m + n + (\varepsilon+1)/2$]

$$17. \int_a^\infty (x-b)^{\lambda-1/2} (x+b)^{-\tau} P_{2m+\varepsilon} \left(\sqrt{\frac{x+b}{a+b}} \right) C_n^\lambda \left(\frac{x}{b} \right) dx =$$

$$= \frac{(-1)^{m-1} 2^n (\lambda)_n (\varepsilon/2 + \tau - \lambda - n)_m (a+b)^{1/2+\lambda+n-\tau}}{n! b^n ((1+\varepsilon)/2 + \lambda + n - \tau)_{m+1}} \times$$

$$\times {}_4F_3 \left(\frac{1}{2} - \lambda - n, 1 - 2\lambda - n, \frac{\varepsilon}{2} + \tau + m - \lambda - n, \tau - \lambda - m - n - \frac{\varepsilon+1}{2}; \right.$$

$$\left. 1 - 2\lambda - 2n, \tau - \lambda - n, \frac{1}{2} + \tau - \lambda - n; \frac{2b}{a+b} \right)$$

[$\varepsilon = 0$ или 1 ; $a \geq b > 0$; $\operatorname{Re} (\tau - \lambda) > m + n + (\varepsilon+1)$
 $\operatorname{Re} \lambda > -1/2$ при $a = b$]

2.21.16. Интегралы, содержащие $L_m^\gamma(a+bx^2) C_n^\lambda(cx)$.

$$1. \int_0^a x^{\alpha-1} (a^2-x^2)^{\lambda-1/2} L_m^\gamma(bx^2) C_{2n+\varepsilon}^\lambda\left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^n a^{\alpha+2\lambda-1}}{m! (2n+\varepsilon)! 2} (2\lambda)_{2n+\varepsilon} (\gamma+1)_m \left(\frac{1+\varepsilon-\alpha}{2}\right)_n \Gamma\left[\frac{\lambda+1/2, (\alpha+\varepsilon)/2}{(\alpha+\varepsilon+1)/2+\lambda+n}\right] \times$$

$$\times {}_3F_3\left(-m, \frac{\alpha}{2}, \frac{1+\alpha}{2}; \frac{1+\alpha-\varepsilon}{2}-n, \gamma+1, \frac{\alpha+\varepsilon+1}{2}+\lambda+n; a^2b\right)$$

[$\varepsilon=0$ или 1 ; $a>0$; $\operatorname{Re} \lambda > -1/2$; $\operatorname{Re} \alpha > -\varepsilon$].

$$2. \int_0^a x^{\alpha-1} (a^2-x^2)^{\lambda-1/2} e^{-bx^2} L_m^\gamma(bx^2) C_{2n+\varepsilon}^\lambda\left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^n a^{\alpha+2\lambda-1}}{m! (2n+\varepsilon)! 2} (2\lambda)_{2n+\varepsilon} (1+\gamma)_m \left(\frac{1+\varepsilon-\alpha}{2}\right)_n \Gamma\left[\frac{\lambda+1/2, (\alpha+\varepsilon)/2}{(\alpha+\varepsilon+1)/2+\lambda+n}\right] \times$$

$$\times {}_3F_3\left(1+\gamma+m, \frac{\alpha}{2}, \frac{1+\alpha}{2}; \frac{1+\alpha-\varepsilon}{2}-n, 1+\gamma, \frac{\alpha+\varepsilon+1}{2}+\lambda+n; -a^2b\right)$$

[$\varepsilon=0$ или 1 ; $a>0$; $\operatorname{Re} \lambda > -1/2$; $\operatorname{Re} \alpha > -\varepsilon$].

$$3. \int_a^\infty x^{\alpha-1} (x^2-a^2)^{\lambda-1/2} e^{-bx^2} L_m^\gamma(bx^2) C_{2n+\varepsilon}^\lambda\left(\frac{x}{a}\right) dx =$$

$$= \frac{2^{2n+\varepsilon-1} b^{(1-\alpha-\varepsilon)/2-\lambda-n}}{m! (2n+\varepsilon)! a^{2n+\varepsilon}} (\lambda)_{2n+\varepsilon} \left(\frac{3-\alpha-\varepsilon}{2}+\gamma-\lambda-n\right)_m \times$$

$$\times \Gamma\left(\frac{\alpha+\varepsilon-1}{2}+\lambda+n\right) {}_3F_3\left(\frac{1}{2}-\lambda-n, 1-\lambda-\varepsilon-n, \frac{3-\alpha-\varepsilon}{2}+\gamma-\lambda-n+m; \right.$$

$$\left. \frac{3-\alpha-\varepsilon}{2}+\gamma-\lambda-n, 1-\lambda-\varepsilon-2n, \frac{3-\alpha-\varepsilon}{2}-\lambda-n; -a^2b\right) +$$

$$+ \frac{a^{\alpha+2\lambda-1} (2\lambda)_{2n+\varepsilon}}{m! (2n+\varepsilon)! 2} \left(\frac{1+\varepsilon-\alpha}{2}\right)_n (1+\gamma)_m \Gamma\left[\frac{\lambda+1/2, (1-\alpha-\varepsilon)/2-\lambda-n}{1-(\alpha+\varepsilon)/2}\right] \times$$

$$\times {}_3F_3\left(1+\gamma+m, \frac{1+\varepsilon-\alpha}{2}+n, \frac{\alpha+\varepsilon}{2}; \frac{1+\alpha+\varepsilon}{2}+\lambda+n, 1+\gamma, \frac{1+\varepsilon-\alpha}{2}; -a^2b\right)$$

[$\varepsilon=0$ или 1 ; $a, \operatorname{Re} b > 0$; $\operatorname{Re} \lambda > -1/2$].

$$4. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\lambda-1/2} e^{-bx} L_m^\gamma(ab+bx) C_n^\lambda\left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^n (1+\gamma)_m (2\lambda)_n}{m! n!} \left(\frac{1}{2}+\lambda-\alpha\right)_n \Gamma\left[\frac{\lambda+1/2, \alpha}{\alpha+\lambda+n+1/2}\right] (2a)^{\alpha+\lambda-1/2} \times$$

$$\times e^{ab} {}_3F_3\left(1+\gamma+m, \frac{1}{2}+\alpha-\lambda, \alpha; 1+\gamma, \frac{1}{2}+\alpha-\lambda-n, \frac{1}{2}+\alpha+\lambda+n; -2ab\right)$$

[$a, \operatorname{Re} \alpha > 0$; $\operatorname{Re} \lambda > -1/2$].

$$5. \int_a^\infty (x-a)^{\lambda-1/2} e^{-bx} L_m^\gamma(ab+bx) C_n^\lambda\left(\frac{x}{a}\right) dx =$$

$$= \frac{2^n (\lambda)_n (1/2+\gamma-\lambda-n)_m}{m! n! a^n b^{\lambda+n+1/2}} \Gamma\left(\frac{1}{2}+\lambda+n\right) e^{ab} \times$$

$$\times {}_2F_2\left(1-2\lambda-n, \frac{1}{2}+\gamma-\lambda-n+m; 1-2\lambda-2n, \frac{1}{2}+\gamma-\lambda-n; -2ab\right)$$

[$a, \operatorname{Re} b > 0$; $\operatorname{Re} \lambda > -1/2$].

$$\begin{aligned}
6. \int_a^\infty (x-a)^{\lambda-1/2} (x+a)^{-\tau} e^{-bx} L_m^\gamma(ab+bx) C_n^\lambda\left(\frac{x}{a}\right) dx = \\
= \frac{(1+\gamma)_m (2\lambda)_n}{m! n!} \left(\lambda+\tau-\frac{1}{2}\right)_n \Gamma\left[\frac{\lambda+1/2, \tau-\lambda-n-1/2}{\tau}\right] (2a)^{1/2+\lambda-\tau} e^{ab} \times \\
\times {}_3F_3\left(\frac{3}{2}-\tau-\lambda, 1+\gamma+m, 1-\tau; \frac{3}{2}-\tau-\lambda-n, 1+\gamma, \frac{3}{2}+\lambda+n-\tau; -2ab\right) + \\
+ \frac{2^n (\lambda)_n (1/2+\gamma+\tau-\lambda-n)_m}{m! n! a^n} \Gamma\left(\frac{1}{2}+n+\lambda-\tau\right) b^{\tau-\lambda-n-1/2} e^{ab} \times \\
\times {}_3F_3\left(\frac{1}{2}-\lambda-n, 1-2\lambda-n, \frac{1}{2}+\gamma+\tau-\lambda-n+m; \right. \\
\left. 1-2\lambda-2n, \frac{1}{2}+\gamma+\tau-\lambda-n, \frac{1}{2}+\tau-\lambda-n; -2ab\right) \\
[a, \operatorname{Re} b > 0; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

2.21.17. Интегралы, содержащие $H_m(\varphi(x)) C_n^\lambda(cx)$.

Обозначение: $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$.

$$\begin{aligned}
1. \int_0^a x^{\alpha-1} (a^2-x^2)^{\lambda-1/2} H_{2m+\varepsilon}(bx) \left\{ \begin{matrix} C_{2n+1}^\lambda(x/a) \\ C_{2n}^\lambda(x/a) \end{matrix} \right\} dx = \\
= \frac{(-1)^{m+n} 2^{2m+2\varepsilon-1}}{(2n+\delta)!} a^{\alpha+2\lambda+\varepsilon-1} b^\varepsilon (2\lambda)_{2n+\delta} \left(\frac{1}{2}\right)_{m+\varepsilon} \left(\frac{1+\delta-\varepsilon-\alpha}{2}\right)_n \times \\
\times \Gamma\left[\frac{(\alpha+\varepsilon+\delta)/2}{(1+\alpha+\varepsilon+\delta)/2+\lambda+n}\right] {}_3F_3\left(-m, \frac{1+\alpha}{2}, \frac{\alpha}{2}+\varepsilon; \right. \\
\left. \frac{1}{2}+\varepsilon, \frac{1+\alpha+\varepsilon-\delta}{2}-n, \frac{1+\alpha+\varepsilon+\delta}{2}+\lambda+n; a^2 b^2\right) \\
[\varepsilon=0 \text{ или } 1; a>0; \operatorname{Re} \lambda > -1/2; \operatorname{Re} \alpha > -\varepsilon-\delta].
\end{aligned}$$

$$\begin{aligned}
2. \int_0^a x^{\alpha-1} (a^2-x^2)^{\lambda-1/2} e^{-b^2 x^2} H_{2m+\varepsilon}(bx) \left\{ \begin{matrix} C_{2n+1}^\lambda(x/a) \\ C_{2n}^\lambda(x/a) \end{matrix} \right\} dx = \\
= \frac{(-1)^{m+n} 2^{2m+2\varepsilon-1}}{(2n+\delta)!} a^{\alpha+2\lambda+\varepsilon-1} b^\varepsilon (2\lambda)_{2n+\delta} \left(\frac{1}{2}\right)_{m+\varepsilon} \left(\frac{1+\delta-\varepsilon-\alpha}{2}\right)_n \times \\
\times \Gamma\left[\frac{\lambda+1/2, (\alpha+\delta+\varepsilon)/2}{(1+\alpha+\delta+\varepsilon)/2+\lambda+n}\right] {}_3F_3\left(\frac{1}{2}+\varepsilon+m, \frac{1+\alpha+\varepsilon}{2}, \frac{\alpha+\varepsilon}{2}; \right. \\
\left. \frac{1+\alpha+\varepsilon-\delta}{2}-n, \frac{1}{2}+\varepsilon, \frac{1+\alpha+\delta+\varepsilon}{2}+\lambda+n; -a^2 b^2\right) \\
[\varepsilon=0 \text{ или } 1; a>0; \operatorname{Re} \lambda > -1/2; \operatorname{Re} \alpha > -\varepsilon-\delta].
\end{aligned}$$

$$\begin{aligned}
3. \int_a^\infty x^{\alpha-1} (x^2-a^2)^{\lambda-1/2} e^{-b^2 x^2} H_{2m+\varepsilon}(bx) \left\{ \begin{matrix} C_{2n+1}^\lambda(x/a) \\ C_{2n}^\lambda(x/a) \end{matrix} \right\} dx = \\
= \frac{(-1)^m 2^{2m+2\varepsilon-1} a^{\alpha+2\lambda+\varepsilon-1} b^\varepsilon}{(2n+\delta)!} (2\lambda)_{2n+\delta} \left(\frac{1}{2}\right)_{m+\varepsilon} \left(\frac{1+\delta-\varepsilon-\alpha}{2}\right)_n \times \\
\times \Gamma\left[\frac{\lambda+1/2, (1-\alpha-\varepsilon-\delta)/2-\lambda-n}{1-(\alpha+\delta+\varepsilon)/2}\right] {}_3F_3\left(\frac{1}{2}+\varepsilon+m, \frac{\alpha}{2}+\varepsilon, \frac{\alpha+1}{2}; \right. \\
\left. \frac{1+\alpha+\varepsilon+\delta}{2}+\lambda+n, \frac{1+\alpha+\varepsilon-\delta}{2}-n, \frac{1}{2}+\varepsilon; -a^2 b^2\right) +
\end{aligned}$$

$$+ \frac{(-1)^m 2^{m+2n+\delta+1} b^{1-\alpha-2\lambda-\delta-2n}}{(2n+\delta)! a^{2n+\delta}} (\lambda)_{2n+\delta} \left(\frac{\varepsilon-\alpha-\delta}{2} + 1 - \lambda - n \right)_m \times$$

$$\times \Gamma \left(\frac{\alpha+\varepsilon+\delta-1}{2} + \lambda + n \right) {}_3F_3 \left(\frac{1}{2} - \lambda - n, 1 - \lambda - \delta - n, \frac{\varepsilon-\alpha-\delta}{2} + 1 - \lambda - n + m; \right.$$

$$\left. 1 - \lambda - \delta - 2n, 1 - \lambda - n - \frac{\alpha}{2}, \frac{3-\alpha}{2} - \delta - \lambda - n; -a^2 b^2 \right)$$

[$\varepsilon = 0$ или 1 ; $a, \operatorname{Re} b > 0$; $\operatorname{Re} \lambda > -1/2$].

4. $\int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\lambda-1/2} e^{-b^2 x} H_{2m+\varepsilon}(b\sqrt{x+a}) C_n^\lambda \left(\frac{x}{a} \right) dx =$

$$= \frac{(-1)^{m+n} (2\lambda)_n}{n!} \left(\frac{1-\varepsilon}{2} + \lambda - \alpha \right)_n 2^{\alpha+\lambda+2m+(3\varepsilon-1)/2} \left(\frac{1}{2} + \varepsilon \right)_m \times$$

$$\times \Gamma \left[\frac{\alpha+\varepsilon/2, \lambda+1/2}{\alpha+\lambda+n+(\varepsilon+1)/2} \right] a^{\alpha+\lambda+(\varepsilon-1)/2} b^\varepsilon e^{ab^2} {}_3F_3 \left(\frac{1}{2} + \varepsilon + m, \right.$$

$$\left. \frac{1+\varepsilon}{2} + \alpha - \lambda, \alpha + \frac{\varepsilon}{2}; \frac{1+\varepsilon}{2} + \alpha + \lambda + n, \frac{1+\varepsilon}{2} + \alpha - \lambda - n, \frac{1}{2} + \varepsilon; -2ab^2 \right)$$

[$\varepsilon = 0$ или 1 ; $a > 0$; $\operatorname{Re} \lambda > -1/2$; $\operatorname{Re} \alpha > -\varepsilon/2$].

5. $\int_a^\infty (x-a)^{\lambda-1/2} (x+a)^{-\tau} e^{-b^2 x} H_{2m+\varepsilon}(b\sqrt{x+a}) C_n^\lambda \left(\frac{x}{a} \right) dx =$

$$= \frac{(-1)^m (2\lambda)_n}{n!} \left(\tau + \lambda - \frac{\varepsilon+1}{2} \right)_n \left(\frac{1}{2} + \varepsilon \right)_m 2^{2m+\lambda+(3\varepsilon+1)/2-\tau} \times$$

$$\times \Gamma \left[\frac{\lambda+1/2, \tau-\lambda-n-(\varepsilon+1)/2}{\tau-\varepsilon/2} \right] a^{\lambda-\tau+(\varepsilon+1)/2} b^\varepsilon e^{ab^2} \times$$

$$\times {}_3F_3 \left(\frac{1}{2} + \varepsilon + m, \frac{3+\varepsilon}{2} - \tau - \lambda, 1 + \frac{\varepsilon}{2} - \tau; \frac{1}{2} + \varepsilon, \frac{3+\varepsilon}{2} + \lambda + n - \tau, \right.$$

$$\left. \frac{3+\varepsilon}{2} - \tau - \lambda - n; -2ab^2 \right) + \frac{(-1)^m (\lambda)_n}{n! a^n} \left(\tau - \lambda - n + \frac{\varepsilon}{2} \right)_m 2^{2m+\varepsilon+n} b^{2\tau-2\lambda-2n-1} \times$$

$$\times \Gamma \left(\frac{1+\varepsilon}{2} + \lambda + n - \tau \right) e^{ab^2} {}_3F_3 \left(\frac{1}{2} - \lambda - n, 1 - 2\lambda - n, \frac{\varepsilon}{2} + \tau - \lambda + m - n; \right.$$

$$\left. 1 - 2\lambda - 2n, \tau - \lambda - n, \frac{1}{2} + \tau - \lambda - n; -2ab^2 \right)$$

[$\varepsilon = 0$ или 1 ; $a > 0$; $\operatorname{Re} \lambda > -1/2$; $|\arg b| < \pi/4$].

2.21.18. Интегралы от $A(x) C_m^\gamma(bx) C_n^\lambda(cx)$.

1. $\int_0^1 (a^2 - x^2)^{\lambda-3/2} C_{2m}^\gamma \left(\frac{x}{a} \right) C_n^\lambda \left(\frac{x}{a} \right) dx =$

$$= \frac{a^{2\lambda-2n} (\gamma+1/2)_m (\gamma)_{m+n-2[n/2]} \Gamma \left[\frac{\lambda-1/2}{\lambda, m+n-2[n/2]+1/2} \right]}{2(m!)}$$

[$a > 0$; $\operatorname{Re} \lambda > 1/2$].

2. $\int_1^a x^l (a^2 - x^2)^{\lambda-1/2} C_m^\lambda \left(\frac{x}{a} \right) C_n^\lambda \left(\frac{x}{a} \right) dx =$

$$= 2^{m+n-1} \frac{(2\lambda)_m (2\lambda)_n l!}{m! n! (l+m+n)!} \left(\frac{l-m-n}{2} + 1 \right)_{m+n} a^{2\lambda+l} {}_2F_1 \left(\lambda + \frac{1}{2}, \frac{l+m+n+1}{2} \right) \times$$

$$\times {}_3F_2 \left(-m, -n, \lambda; 2\lambda, \frac{l-m-n}{2} + 1; 1 \right)$$

[$a > 0$; $\operatorname{Re} \lambda > -1/2$].

$$3. \int_0^a x^{m+n+2\lambda-2} (a^2-x^2)^{\lambda-1/2} C_m^\lambda \left(\frac{x}{a} \right) C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(2\lambda-1)_{m+n} \sqrt{\pi}}{2^{m+n+2\lambda} m! n!} \Gamma \left[\lambda - 1/2 \right] a^{m+n+4\lambda-2} \quad [a > 0; \operatorname{Re} \lambda > -1/2].$$

$$4. \int_{-a}^a (x+a)^{\gamma+\lambda-1} (a-x)^{\beta-1} C_m^\gamma \left(\frac{x}{a} \right) C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(2\gamma)_m (2\lambda)_n}{m! n!} \left(\frac{1}{2} + \lambda - \beta \right)_n \Gamma \left[\beta, \lambda + 1/2 \right]_{\beta + \lambda + n + 1/2} (2a)^{\beta+\gamma+\lambda-1} \times$$

$$\times {}_4F_3 \left(\frac{1}{2} - \gamma - m, \frac{1}{2} + \gamma + m, \frac{1}{2} + \beta - \lambda, \beta; \beta + \lambda + n + \frac{1}{2}, \gamma + \frac{1}{2}, \frac{1}{2} + \beta - \lambda - n; 1 \right)$$

$$[a, \operatorname{Re} \beta, \operatorname{Re} (\gamma + \lambda) > 0].$$

$$5. \int_{-a}^a (x+a)^{\lambda-1/2} (a-x)^{\beta-1} C_m^\gamma \left(\frac{x}{a} \right) C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(2a)^{\beta+\lambda-1/2} (2\gamma)_m (2\lambda)_n (1/2 + \lambda - \beta)_n}{m! n!} \Gamma \left[\beta, \lambda + 1/2 \right]_{\beta + \lambda + n + 1/2} \times$$

$$\times {}_4F_3 \left(-m, 2\gamma + m, \beta, \beta - \lambda + \frac{1}{2}; \gamma + \frac{1}{2}, \beta + \lambda + n + \frac{1}{2}, \beta - \lambda - n + \frac{1}{2}; 1 \right)$$

$$[a, \operatorname{Re} \beta > 0; \operatorname{Re} \lambda > -1/2].$$

$$6. \int_{-a}^a (x+a)^{3\lambda+m+n-3/2} (a-x)^{\lambda-1/2} C_m^\lambda \left(\frac{x}{a} \right) C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{\sqrt{\pi} (2\lambda)_{m+n}}{2^{m+n} (\lambda + 1/2)_m (\lambda + 1/2)_n m! n!} \times$$

$$\times \Gamma \left[\begin{matrix} \lambda + m + n + 1/2 & 3\lambda + m + n - 1/2 \\ 2\lambda & 2\lambda + m + n + 1/2 \end{matrix} \right] a^{4\lambda+m+n-1}$$

$$[a > 0; \operatorname{Re} \lambda > -1/2, (1-2m-2n)/6].$$

$$7. \int_{-a}^a (x+a)^{\lambda+m-n-3/2} (a-x)^{\lambda-1/2} C_m^\lambda \left(\frac{x}{a} \right) C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^m 2^{1+n-m} \pi (2\lambda)_n (1/2 - \lambda - n)_m}{\sqrt{\pi} m! (n-m)! (1/2 + \lambda)_m} \Gamma \left[\begin{matrix} \lambda + m - n - 1/2 \\ \lambda, 1/2 + m - n \end{matrix} \right] a^{2\lambda+m-n-1}$$

$$[a > 0; n \geq m; \operatorname{Re} \lambda > 1/2 + n - m].$$

$$8. \int_{-a}^a (x+a)^{\lambda-1/2} (a-x)^{2\lambda-1} C_m^\lambda \left(\frac{x}{a} \right) C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(2a)^{3\lambda-1/2} (2\lambda)_m}{m! n!} \Gamma \left[\begin{matrix} \lambda + 1/2, 2\lambda + n, 1/2 + \lambda + m + n, 1/2 - \lambda + n - m \\ 1/2 - \lambda, 1/2 + \lambda + n - m, 1/2 + 3\lambda + m + n \end{matrix} \right]$$

$$[a, \operatorname{Re} \lambda > 0].$$

$$9. \int_{-a}^a \frac{(a^2-x^2)^{\lambda-1/2}}{x-y} C_m^\lambda \left(\frac{x}{a} \right) C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= -\frac{2\sqrt{\pi}}{\Gamma(\lambda)} \left(\frac{a}{2} \right)^{\lambda-1/2} e^{(1/2-\lambda)\pi i} (y^2-a^2)^{(2\lambda-1)/4} C_m^\lambda \left(\frac{y}{a} \right) Q_{n+\lambda-1/2}^{\lambda-1/2} \left(\frac{y}{a} \right)$$

$$[a > 0; n \geq m; \operatorname{Re} \lambda > -1/2].$$

$$10. \int_{-a}^a (a^2 - x^2)^{\lambda-1/2} C_m^\lambda \left(\frac{x}{a} \right) C_n^\lambda \left(\frac{x}{a} \right) dx = \frac{\sqrt{\pi} (2\lambda)_n}{n! (n+\lambda)} \Gamma \left[\lambda + 1/2 \right] a^{2\lambda} \delta_{m,n}$$

[$a > 0$; $\operatorname{Re} \lambda > -1/2$].

$$11. \int_{-a}^a (x+a)^{2\lambda-1} (a-x)^{\lambda-1/2} \left[C_n^\lambda \left(\frac{x}{a} \right) \right]^2 dx =$$

$$= \left[\frac{(2\lambda)_n}{n!} \right]^2 B \left(\lambda + 2n + \frac{1}{2}, 2\lambda \right) (2a)^{3\lambda-1/2}$$

[$a, \operatorname{Re} \lambda > 0$].

$$12. \int_{-a}^a (x+a)^{\lambda-1/2} (a-x)^{\lambda-3/2} \left[C_n^\lambda \left(\frac{x}{a} \right) \right]^2 dx = \frac{\sqrt{\pi} (2\lambda)_n}{n!} \Gamma \left[\lambda - 1/2 \right] a^{2\lambda-1}$$

[$a > 0$; $\operatorname{Re} \lambda > 1/2$].

$$13. \int_{-a}^a (x+a)^{\lambda-1/2} (a-x)^{2\lambda+2n-3/2} \left[C_n^\lambda \left(\frac{x}{a} \right) \right]^2 dx =$$

$$= \frac{\pi (\lambda)_n (\lambda+n+1/2)_n}{a^{2\lambda-1} (n!)^2} \Gamma \left[\frac{3\lambda+2n-1/2}{2\lambda+2n+1/2, \lambda} \right] a^{4\lambda+2n-1}$$

[$a > 0$; $\operatorname{Re} \lambda > -1/2, (1-4n)/6$].

$$14. \int_a^\infty (x-a)^{\gamma+\lambda-1} (x+a)^{-\tau} C_m^\gamma \left(\frac{x}{a} \right) C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{2^{2m+\lambda+\gamma-\tau} (\gamma)_m (2\lambda)_n}{m! n!} (\tau+\lambda-\gamma-m)_n \Gamma \left[\frac{\lambda+1/2, \tau-\lambda-\gamma-m-n}{1/2+\tau-\gamma-m} \right] \times$$

$$\times a^{\lambda+\gamma-\tau} {}_4F_3 \left(\frac{1}{2}-\gamma-m, 1-2\gamma-m, \tau+\lambda-\gamma+n-m, \tau-\lambda-\gamma-m-n; \right.$$

$$\left. 1-2\gamma-2m, \tau+\lambda-\gamma-m, \frac{1}{2}+\tau-\gamma-m; 1 \right)$$

[$a > 0$; $0 < \operatorname{Re} (\gamma + \lambda) < \operatorname{Re} \tau - m - n$].

$$15. \int_{-a}^a (a^2 - x^2)^{\gamma-1/2} C_{n-2m}^\gamma \left(\frac{x}{a} \right) C_n^\lambda (cx) dx =$$

$$= \frac{(\lambda)_n (2\gamma)_{n-2m} \sqrt{\pi} a^{2\gamma+n} c^n}{(\gamma)_{n-m+1} (1-\lambda-n)_m (n-2m)!} \Gamma \left[\gamma + 1/2 \right] P_m^{(-\lambda-n, \lambda-\gamma-1)} \left(1 - \frac{2}{a^2 c^2} \right)$$

[$a > 0$; $\operatorname{Re} \gamma > -1/2$].

$$16. \int_a^b (x^2 - a^2)^{\lambda-1/2} (b^2 - x^2)^{\gamma-1/2} C_m^\gamma \left(\frac{x}{b} \right) C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{(2\gamma)_m (2\lambda)_n}{2a^n m! n!} B \left(\gamma + \frac{1}{2}, \lambda + \frac{1}{2} \right) (b^2 - a^2)^{\gamma+\lambda} \times$$

$$\times {}_2F_1 \left(\gamma + \frac{m-n+1}{2}, \frac{1-m-n}{2}; \gamma + \lambda + 1; 1 - \frac{a^2}{b^2} \right)$$

[$b > a > 0$; $\operatorname{Re} \gamma, \operatorname{Re} \lambda > -1/2$].

$$\begin{aligned}
17. \int_0^a x^{\alpha-1} (a^2 - x^2)^{\lambda-1/2} (b^2 - x^2)^{\gamma-1/2} C_{2m+\varepsilon}^{\gamma} \left(\frac{x}{b} \right) \left\{ \frac{C_{2n+1}^{\lambda}(x/a)}{C_{2n}^{\lambda}(x/a)} \right\} dx = \\
= \frac{(-1)^{m+n} 2^{\varepsilon-1} a^{\alpha+2\lambda+\varepsilon-1}}{m! (2n+\delta)!} b^{2\lambda-\varepsilon-1} (\gamma)_{m+\varepsilon} (2\lambda)_{2n+\delta} \left(\frac{1+\delta-\varepsilon-\alpha}{2} \right)_n \times \\
\times \Gamma \left[\lambda+1/2, (\alpha+\delta+\varepsilon)/2 \right] {}_4F_3 \left(\frac{1}{2}-\gamma-m, \frac{1}{2}+\varepsilon+m, \frac{1+\alpha}{2}, \frac{\alpha}{2}+\varepsilon; \right. \\
\left. \frac{1}{2}+\varepsilon, \frac{1+\alpha+\varepsilon-\delta}{2}-n, \frac{1+\alpha+\delta+\varepsilon}{2}+\lambda+n; \frac{a^2}{b^2} \right) \\
\left[\varepsilon=0 \text{ или } 1; b>a>0; \operatorname{Re} \lambda > -1/2; \operatorname{Re} \alpha > -\varepsilon-\delta, \delta=\begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right].
\end{aligned}$$

$$\begin{aligned}
18. \int_a^b x^{\alpha-1} (x^2 - a^2)^{\lambda-1/2} (b^2 - x^2)^{\gamma-1/2} C_{2m+\varepsilon}^{\gamma} \left(\frac{x}{b} \right) \left\{ \frac{C_{2n+1}^{\lambda}(x/a)}{C_{2n}^{\lambda}(x/a)} \right\} dx = \\
= \frac{(-1)^m 2^{\varepsilon-1} a^{\alpha+2\lambda+\varepsilon-1} b^{2\gamma-\varepsilon-1}}{m! (2n+\delta)!} (2\lambda)_{2n+\delta} (\gamma)_{m+\varepsilon} \left(\frac{1+\delta-\varepsilon-\alpha}{2} \right)_n \times \\
\times \Gamma \left[\lambda+1/2, (1-\alpha-\delta-\varepsilon)/2 - \lambda - n \right] {}_4F_3 \left(\frac{1}{2}-\gamma-m, \frac{1}{2}+\varepsilon+m, \frac{1+\alpha}{2}, \frac{\alpha}{2}+\varepsilon; \right. \\
\left. \frac{1}{2}+\varepsilon, \frac{1+\alpha+\varepsilon-\delta}{2}-n, \frac{1+\alpha+\delta+\varepsilon}{2}+\lambda+n; \frac{a^2}{b^2} \right) + \\
+ \frac{(-1)^m 2^{2n+\delta-1} b^{\alpha+2\gamma+2\lambda+2n+\delta-2}}{(2m+\varepsilon)! (2n+\delta)! a^{2n+\delta}} (\lambda)_{2n+\delta} (2\gamma)_{2m+\varepsilon} \left(\frac{\varepsilon-\alpha-\delta}{2} + 1 - \lambda - n \right)_m \times \\
\times \Gamma \left[\gamma+1/2, (\alpha+\delta+\varepsilon-1)/2 + \lambda + n \right] {}_4F_3 \left(\frac{1}{2}-\lambda-n, 1-\lambda-\delta-n, \right. \\
\left. \frac{\varepsilon-\alpha-\delta}{2} + 1 + m - \lambda - n, 1-\lambda-\gamma-m-n - \frac{\alpha+\delta+\varepsilon}{2}; 1-\lambda-\delta-2n, \right. \\
\left. 1-\lambda-n - \frac{\alpha}{2}, \frac{3-\alpha}{2} - \delta - \lambda - n; \frac{a^2}{b^2} \right) \\
\left[\varepsilon=0 \text{ или } 1; b>a>0; \operatorname{Re} \gamma, \operatorname{Re} \lambda > -1/2, \delta=\begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right].
\end{aligned}$$

$$\begin{aligned}
19. \int_a^{\infty} x^{\alpha-1} (x^2 - a^2)^{\lambda-1/2} (x^2 - b^2)^{\gamma-1/2} C_{2m+\varepsilon}^{\gamma} \left(\frac{x}{b} \right) \left\{ \frac{C_{2n+1}^{\lambda}(x/a)}{C_{2n}^{\lambda}(x/a)} \right\} dx = \\
= \frac{2^{2m+\varepsilon-1} a^{\alpha+2\lambda+2\gamma+2m+\varepsilon-2}}{(2m+\varepsilon)! (2n+\delta)! b^{2m+\varepsilon}} (2\lambda)_{2n+\delta} (\gamma)_{2m+\varepsilon} \times \\
\times \left(\frac{\delta-\alpha-\varepsilon}{2} + 1 - \gamma - m \right)_n \Gamma \left[\lambda+1/2, 1-\gamma-\lambda-m-n - (\alpha+\varepsilon+\delta)/2 \right] \times \\
\times {}_4F_3 \left(\frac{1}{2}-\gamma-m, 1-\gamma-\varepsilon-m, \frac{\delta-\alpha-\varepsilon}{2} + 1 + n - \gamma - m, \right. \\
\left. 1-\gamma-\lambda-m-n - \frac{\alpha+\varepsilon+\delta}{2}; \frac{3-\alpha}{2} - \varepsilon - \gamma - m, 1-\gamma-\varepsilon-2m, 1-\gamma-m - \frac{\alpha}{2}; \frac{b^2}{a^2} \right) \\
\left[\varepsilon=0 \text{ или } 1; a>b>0; \operatorname{Re} \lambda > -1/2; \right. \\
\left. \operatorname{Re} (\alpha+2\gamma+2\lambda) < 2-2m-2n-\varepsilon-\delta, \delta=\begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right].
\end{aligned}$$

2.21.19. Интегралы, содержащие $C_m^\gamma \varphi(x) C_n^\lambda(cx)$.

$$1. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\lambda-1/2} (1-ab-bx)^{\gamma-1/2} C_m^\gamma(2bx+2ab-1) C_n^\lambda\left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^{m+n} (2\gamma)_m (2\lambda)_n}{m! n!} \left(\frac{1}{2} + \lambda - \alpha\right)_n \Gamma\left[\alpha, \lambda + 1/2\right]_{\alpha + \lambda + n + 1/2} (2a)^{\alpha + \lambda - 1/2} \times$$

$$\times {}_4F_3\left(\frac{1}{2} + \gamma + m, \frac{1}{2} - \gamma - m, \frac{1}{2} + \alpha - \lambda, \alpha; \alpha + \lambda + n + \frac{1}{2}, \frac{1}{2} + \gamma, \frac{1}{2} + \alpha - \lambda - n; 2ab\right)$$

[a, b, Re α > 0; 2ab < 1; Re λ > -1/2].

$$2. \int_a^b (x-a)^{\lambda-1/2} (b-x)^{\gamma-1/2} C_m^\lambda\left(\frac{2x+a-b}{a+b}\right) C_n^\lambda\left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^m (2\lambda)_{2n} (\gamma - \lambda - n)_m (2\gamma)_m}{m! n! (2a)^n} \Gamma\left[\gamma + 1/2, \lambda + 1/2\right]_{\lambda + \gamma + m + n + 1} (a+b)^{\lambda + \gamma + n} \times$$

$$\times {}_3F_2\left(1 - 2\lambda - n, \gamma + m - \lambda - n, -\lambda - \gamma - m - n; 1 - 2\lambda - 2n, \gamma - \lambda - n; \frac{2a}{a+b}\right)$$

[b > a > 0; Re λ, Re γ > -1/2].

$$3. \int_a^b (x-a)^{\gamma-1/2} (b-x)^{\lambda-1/2} C_m^\gamma\left(\frac{2x+b-a}{a+b}\right) C_n^\lambda\left(\frac{x}{b}\right) dx =$$

$$= \frac{(-1)^n (2\gamma)_{2m} (\lambda - \gamma - m)_n (2\lambda)_n}{m! n! (a+b)^m} \Gamma\left[\lambda + 1/2, \gamma + 1/2\right]_{\lambda + \gamma + m + n + 1} (2b)^{\lambda + \gamma + m} \times$$

$$\times {}_3F_2\left(1 - 2\gamma - m, \lambda + n - \gamma - m, -\lambda - \gamma - m - n; 1 - 2\gamma - 2m, \lambda - \gamma - m; \frac{a+b}{2b}\right)$$

[b > a > 0; Re λ, Re γ > -1/2].

$$4. \int_{\pm a}^b (x \mp a)^{\alpha-1} (b-x)^{\gamma-1/2} (a \pm x)^{\lambda-1/2} C_m^\gamma\left(\frac{2x \mp a - b}{\mp a + b}\right) C_n^\lambda\left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^m (\pm 1)^n (2\lambda)_n}{m! n!} \left(\frac{1}{2} + \gamma - \alpha\right)_m (2\gamma)_m \Gamma\left[\alpha, \gamma + 1/2\right]_{\alpha + \gamma + m + 1/2} \times$$

$$\times (2a)^{\lambda-1/2} (b \mp a)^{\alpha + \gamma - 1/2} {}_4F_3\left(\frac{1}{2} - \lambda - n, \lambda + n + \frac{1}{2}, \alpha, \alpha - \gamma + \frac{1}{2};\right.$$

$$\left. \alpha + \gamma + m + \frac{1}{2}, \frac{1}{2} + \alpha - \gamma - m, \lambda + \frac{1}{2}; \frac{a \mp b}{2a}\right)$$

[Re α > 0; Re γ > -1/2, {b > a > 0}, {a > b > 0}].

$$5. \int_a^b (x-a)^{\lambda-1/2} (b-x)^{\gamma-1/2} (x+a)^{-\tau} C_m^\gamma\left(\frac{2x+a-b}{a+b}\right) C_n^\lambda\left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^m (2\gamma)_m (2\lambda)_n}{m! n!} \left(\tau + \lambda - \frac{1}{2}\right)_n \Gamma\left[\lambda + 1/2, \tau - \lambda - n - 1/2\right]_{\tau} \times$$

$$\times (2a)^{1/2 + \lambda - \tau} (a+b)^{\gamma-1/2} {}_4F_3\left(\frac{1}{2} - \gamma - m, \frac{1}{2} + \gamma + m, \frac{3}{2} - \tau - \lambda, 1 - \tau;\right.$$

$$\left. \frac{3}{2} + n + \lambda - \tau, \frac{3}{2} - \lambda - \tau - n, \gamma + \frac{1}{2}; \frac{2a}{a+b}\right) + \frac{(-1)^m 2^n (\lambda)_n (2\gamma)_m}{m! n! a^n} \times$$

$$\times (\tau + \gamma - \lambda - n)_m \Gamma\left[\gamma + 1/2, 1/2 + n + \lambda - \tau\right]_{1 + \gamma + \lambda + m + n - \tau} (a+b)^{\gamma + \lambda + n - \tau} \times$$

$$\times {}_4F_3\left(\frac{1}{2}-\lambda-n, 1-2\lambda-n, \tau+\gamma+m-\lambda-n, \tau-\lambda-\gamma-m-n;\right.$$

$$1-2\lambda-2n, \tau+\gamma-\lambda-n, \frac{1}{2}+\tau-\lambda-n; \frac{2a}{a+b}\Big)$$

$$[b>a>0; \operatorname{Re} \lambda, \operatorname{Re} \gamma > -1/2].$$

$$6. \int_a^b (x-a)^{\gamma-1/2} (b-x)^{\lambda-1/2} (b+x)^{-\tau} C_m^\gamma \left(\frac{2x+b-a}{a+b} \right) C_n^\lambda \left(\frac{x}{b} \right) dx =$$

$$= \frac{(-1)^n (2\gamma)_m (2\lambda)_n}{m! n!} \left(\tau + \gamma - \frac{1}{2} \right)_m \Gamma \left[\begin{matrix} \gamma + 1/2, \tau - \gamma - m - 1/2 \\ \tau \end{matrix} \right] \times$$

$$\times (2b)^{\lambda-1/2} (a+b)^{1/2+\gamma-\tau} {}_4F_3 \left(\frac{1}{2}-\lambda-n, \lambda+n+\frac{1}{2}, \frac{3}{2}-\tau-\gamma, 1-\tau; \right.$$

$$\frac{3}{2}+\gamma+m-\tau, \frac{3}{2}-m-\tau-\gamma, \lambda+\frac{1}{2}; \frac{a+b}{2b} \Big) + \frac{(-1)^n 2^{\lambda+\gamma+m-n} (\gamma)_m (2\lambda)_n}{m! n!} \times$$

$$\times (\lambda+\tau-\gamma-m)_n \Gamma \left[\begin{matrix} \lambda+1/2, 1/2+m+\gamma-\tau \\ 1+\lambda+\gamma+m+n-\tau \end{matrix} \right] b^{\lambda+\gamma+m-\tau} (a+b)^{-m} \times$$

$$\times {}_4F_3 \left(\frac{1}{2}-\gamma-m, 1-2\gamma-m, \lambda+\tau+n-\gamma-m, \tau-\gamma-\lambda-m-n; \right.$$

$$1-2\gamma-2m, \lambda+\tau-\gamma-m, \frac{1}{2}+\tau-\gamma-m; \frac{a+b}{2b} \Big)$$

$$[a, b > 0; \operatorname{Re} \gamma, \operatorname{Re} \lambda > -1/2].$$

$$7. \int_a^\infty (x-a)^{\gamma-1/2} (x-b)^{\lambda-1/2} C_m^\gamma \left(\frac{2x+b-a}{a+b} \right) C_n^\lambda \left(\frac{x}{b} \right) dx =$$

$$= \frac{2^n (\lambda)_n (2\gamma)_m}{m! n! b^n} (\gamma-\lambda-n)_m \Gamma \left[\begin{matrix} \gamma+1/2, -\lambda-\gamma-m-n \\ 1/2-\lambda-n \end{matrix} \right] (a+b)^{\gamma+\lambda+n} \times$$

$$\times {}_3F_2 \left(1-2\lambda-n, \gamma+m-\lambda-n, -\lambda-\gamma-m-n; 1-2\lambda-2n, \gamma-\lambda-n; \frac{2b}{a+b} \right)$$

$$[a>b>0; \operatorname{Re} \gamma > -1/2, \operatorname{Re} (\lambda+\gamma) < -m-n].$$

$$8. \int_a^\infty (x-a)^{\gamma-1/2} (x-b)^{\lambda-1/2} (x+b)^{-\tau} C_m^\gamma \left(\frac{2x+b-a}{a+b} \right) C_n^\lambda \left(\frac{x}{b} \right) dx =$$

$$= \frac{2^n (\lambda)_n (2\gamma)_m}{m! n! b^n} (\gamma+\tau-\lambda-n)_m \Gamma \left[\begin{matrix} \gamma+1/2, \tau-\lambda-\gamma-m-n \\ 1/2+\tau-\lambda-n \end{matrix} \right] \times$$

$$\times (a+b)^{\gamma+\lambda+n-\tau} {}_4F_3 \left(\frac{1}{2}-\lambda-n, 1-2\lambda-n, \gamma+\tau+m-\lambda-n, \right.$$

$$\tau-\lambda-\gamma-m-n; 1-2\lambda-2n, \gamma+\tau-\lambda-n, \frac{1}{2}+\tau-\lambda-n; \frac{2b}{a+b} \Big)$$

$$[a>b>0; \operatorname{Re} \gamma > -1/2; \operatorname{Re} (\tau-\gamma-\lambda) > m+n].$$

$$9. \int_a^\infty (x-a)^{\lambda-1/2} (x+a)^{-\tau} (bx+ab-1)^{\gamma-1/2} C_m^\gamma (2bx+2ab-1) C_n^\lambda \left(\frac{x}{a} \right) dx =$$

$$= \frac{2^{\lambda+\gamma+m-n} (\gamma)_m (2\lambda)_n}{m! n!} (\tau+\lambda-\gamma-m)_n \Gamma \left[\begin{matrix} \lambda+1/2, \tau-\lambda-\gamma-m-n \\ 1/2+\tau-\gamma-m \end{matrix} \right] \times$$

$$\times a^{m+\lambda+\gamma-\tau} b^{\gamma+m-1/2} {}_4F_3 \left(\frac{1}{2}-\gamma-m, 1-2\gamma-m, \tau+\lambda-\gamma+n-m, \right.$$

$$\tau-\lambda-\gamma-m-n; 1-2\gamma-2m, \tau+\lambda-\gamma-m, \frac{1}{2}+\tau-\gamma-m; \frac{1}{2ab} \Big)$$

$$[a>0, 2ab>1; \operatorname{Re} \lambda > -1/2, \operatorname{Re} (\tau-\gamma-\lambda) > m+n].$$

$$10. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\lambda-1/2} (1-ab^2-b^2x)^{\gamma-1/2} C_{2m+\varepsilon}^{\gamma} (b\sqrt{x+a}) C_n^{\lambda} \left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^{m+n} (\gamma)_{m+\varepsilon} (2\lambda)_n}{m! n!} \left(\frac{1-\varepsilon}{2} + \lambda - \alpha\right)_n \Gamma \left[\lambda + 1/2, \alpha + \varepsilon/2 \right] \times$$

$$\times 2^{\alpha+\lambda+(3\varepsilon-1)/2} a^{\alpha+\lambda+(\varepsilon-1)/2} b^{\varepsilon} {}_3F_2 \left(\frac{1}{2} - \gamma - m, \frac{1}{2} + \varepsilon + m, \alpha + \frac{\varepsilon}{2}, \frac{1+\varepsilon}{2} + \alpha - \lambda; \right.$$

$$\left. \frac{1+\varepsilon}{2} + \alpha + \lambda + n, \frac{1}{2} + \varepsilon, \frac{1+\varepsilon}{2} + \alpha - \lambda - n; 2ab^2 \right)$$

[$\varepsilon=0$ или 1 ; $a, b > 0$; $2ab^2 < 1$; $\operatorname{Re} \alpha > -\varepsilon/2$; $\operatorname{Re} \lambda > -1/2$].

$$11. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\gamma+\lambda-1} C_{2m+\varepsilon}^{\gamma} \left(\sqrt{\frac{x+a}{2a}} \right) C_n^{\lambda} \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^{m+n} (\gamma)_{m+\varepsilon} (2\lambda)_n}{m! n!} \left(\frac{1-\varepsilon}{2} + \lambda - \alpha\right)_n \Gamma \left[\lambda + 1/2, \alpha + \varepsilon/2 \right] \times$$

$$\times 2^{\alpha+\lambda+\gamma+\varepsilon-1} a^{\alpha+\lambda+\gamma-1} {}_3F_2 \left(\frac{1}{2} - \gamma - m, \frac{1}{2} + \varepsilon + m, \alpha + \frac{\varepsilon}{2}, 1 + \frac{\varepsilon}{2} + \alpha - \lambda; \right.$$

$$\left. \frac{1+\varepsilon}{2} + \alpha + \lambda + n, \varepsilon + \frac{1}{2}, \frac{1+\varepsilon}{2} + \alpha - \lambda - n; 1 \right)$$

[$\varepsilon=0$ или 1 ; $a, \operatorname{Re}(\lambda+\gamma) > 0$; $\operatorname{Re} \alpha > -\varepsilon/2$].

$$12. \int_{\pm a}^b (x \mp a)^{\alpha-1} (b-x)^{\gamma-1/2} (a \pm x)^{\lambda-1/2} C_{2m+\varepsilon}^{\gamma} \left(\sqrt{\frac{x \mp a}{b \mp a}} \right) C_n^{\lambda} \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^m (\pm 1)^n (2\gamma)_{2m+\varepsilon} (2\lambda)_n}{n! (2m+\varepsilon)!} \left(\frac{1+\varepsilon}{2} - \alpha\right)_m \Gamma \left[\gamma + 1/2, \alpha + \varepsilon/2 \right] \times$$

$$\times (b \mp a)^{\alpha+\gamma-1/2} (2a)^{\lambda-1/2} {}_3F_2 \left(\frac{1}{2} - \lambda - n, \frac{1}{2} + \lambda + n, \alpha, \alpha + \frac{1}{2}; \right.$$

$$\left. \frac{1}{2} + \lambda, \frac{1-\varepsilon}{2} - \alpha - m, \alpha + \gamma + m + \frac{\varepsilon+1}{2}; \frac{a \mp b}{2a} \right)$$

[$\varepsilon=0$ или 1 ; $\operatorname{Re} \alpha > -\varepsilon/2$, $\operatorname{Re} \gamma > -1/2$, $\left\{ \begin{smallmatrix} b > a > 0 \\ a > b > 0 \end{smallmatrix} \right\}$].

$$13. \int_a^b (x-a)^{\lambda-1/2} (b-x)^{\gamma-1/2} C_{2m}^{\gamma} \left(\sqrt{\frac{x+a}{b+a}} \right) C_n^{\lambda} \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^m (2\gamma)_{2m} (2\lambda)_{2n}}{(2m)! n! (2a)^n} (-\lambda - n)_m \Gamma \left[\gamma + 1/2, \lambda + 1/2 \right] (a+b)^{\gamma+\lambda+n} \times$$

$$\times {}_3F_2 \left(1 - 2\lambda - n, m - n - \lambda, -m - n - \lambda - \gamma; 1 - 2\lambda - 2n, -\lambda - n; \frac{2a}{a+b} \right)$$

[$b > a > 0$; $\operatorname{Re} \gamma, \operatorname{Re} \lambda > -1/2$].

$$14. \int_a^b (x-a)^{\gamma-1/2} (b-x)^{\lambda-1/2} C_{2m}^{\gamma} \left(\sqrt{\frac{x+b}{a+b}} \right) C_n^{\lambda} \left(\frac{x}{b} \right) dx =$$

$$= \frac{(-1)^n (\gamma)_{2m} (2\lambda)_n}{(2m)! n!} (\lambda - \gamma - m)_n \Gamma \left[\lambda + 1/2, 1/2 + \gamma + m \right] \times$$

$$\times 2^{\lambda+\gamma+2m} b^{\lambda+\gamma+m} (a+b)^{-m} {}_3F_2 \left(1 - \gamma - m, n + \lambda - \gamma - m, -\lambda - \gamma - m - n; \right.$$

$$\left. 1 - \gamma - 2m, \lambda - \gamma - m; \frac{a+b}{2b} \right)$$

[$b > a > 0$, $\operatorname{Re} \gamma, \operatorname{Re} \lambda > -1/2$].

$$\begin{aligned}
15. \int_a^b (x-a)^{\lambda-1/2} (b-x)^{\gamma-1/2} (x+a)^{-\tau} C_{2m+\varepsilon}^{\gamma} \left(\sqrt{\frac{x+a}{b+a}} \right) C_n^{\lambda} \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^m (\gamma)_{m+\varepsilon} (2\lambda)_n}{m! n!} \left(\tau + \lambda - \frac{\varepsilon+1}{2} \right)_n \Gamma \left[\begin{matrix} \lambda+1/2, \tau-\lambda-n-(\varepsilon+1)/2 \\ \tau-\varepsilon/2 \end{matrix} \right] \times \\
\times 2^{\lambda-\tau+(3\varepsilon+1)/2} a^{\lambda-\tau+(\varepsilon+1)/2} (a+b)^{\gamma-(\varepsilon+1)/2} {}_4F_3 \left(\begin{matrix} \frac{1}{2}-\gamma-m, \frac{1}{2}+\varepsilon+m, \\ \frac{3+\varepsilon}{2}-\tau-\lambda, 1+\frac{\varepsilon}{2}-\tau, \frac{3+\varepsilon}{2}+\lambda+n-\tau, \frac{1}{2}+\varepsilon, \frac{3+\varepsilon}{2}-\tau-\lambda-n; \frac{2a}{a+b} \end{matrix} \right) + \\
+ \frac{(-1)^n 2^n (\gamma)_{2m+\varepsilon} (\lambda)_n}{(2m+\varepsilon)! n! a^n} \left(\frac{\varepsilon}{2} + \tau - \lambda - n \right)_m \times \\
\times \Gamma \left[\begin{matrix} \gamma+1/2, (\varepsilon+1)/2+\lambda+n-\tau \\ 1+\varepsilon/2+\lambda+\gamma+m+n-\tau \end{matrix} \right] (a+b)^{\gamma+\lambda+n-\tau} \times \\
\times {}_4F_3 \left(\begin{matrix} \frac{1}{2}-\lambda-n, 1-2\lambda-n, \frac{\varepsilon}{2}+\tau+m-\lambda-n, \tau-m-n-\lambda-\gamma-\frac{\varepsilon}{2}; \\ 1-2\lambda-2n, \frac{1}{2}+\tau-\lambda-n, \tau-\lambda-n; \frac{2a}{a+b} \end{matrix} \right) \\
[\varepsilon=0 \text{ или } 1; b>a>0; \operatorname{Re} \gamma, \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
16. \int_a^b (x-a)^{\gamma-1/2} (b-x)^{\lambda-1/2} (x+b)^{-\tau} C_{2m+\varepsilon}^{\gamma} \left(\sqrt{\frac{x+b}{a+b}} \right) C_n^{\lambda} \left(\frac{x}{b} \right) dx = \\
= \frac{(-1)^n (2\gamma)_{2m+\varepsilon} (2\lambda)_n}{n! (2m+\varepsilon)!} \left(\frac{\varepsilon-1}{2} + \tau \right)_m \Gamma \left[\begin{matrix} \gamma+1/2, \tau-\gamma-m-(\varepsilon+1)/2 \\ \tau-\varepsilon/2 \end{matrix} \right] \times \\
\times (2b)^{\lambda-1/2} (a+b)^{1/2+\gamma-\tau} {}_4F_3 \left(\begin{matrix} \frac{1}{2}-\lambda-n, \frac{1}{2}+\lambda+n, \frac{3}{2}-\tau, 1-\tau; \\ \frac{3+\varepsilon}{2}+\gamma+m-\tau, \frac{3-\varepsilon}{2}-\tau-m, \frac{1}{2}+\lambda; \frac{a+b}{2b} \end{matrix} \right) + \frac{(-1)^n (2\lambda)_n (\gamma)_{2m+\varepsilon}}{n! (2m+\varepsilon)!} \times \\
\times \left(\lambda + \tau - \gamma - m - \frac{\varepsilon}{2} \right)_n \Gamma \left[\begin{matrix} \lambda+1/2, (1+\varepsilon)/2+\gamma+m-\tau \\ 1+\lambda+\gamma+m+n+\varepsilon/2-\tau \end{matrix} \right] 2^{\lambda+\gamma+3m-\tau+3\varepsilon/2} \times \\
\times b^{\gamma+\lambda+m-\tau+\varepsilon/2} (a+b)^{-m-\varepsilon/2} {}_4F_3 \left(\begin{matrix} \frac{1}{2}-\gamma-m, 1-\varepsilon-\gamma-m, \\ \lambda+\tau-\gamma-m+n-\frac{\varepsilon}{2}, \tau-\lambda-\gamma-m-n-\frac{\varepsilon}{2}; \frac{1-\varepsilon}{2}-\gamma-m+\tau, \\ 1-\varepsilon-\gamma-2m, \lambda+\tau-\gamma-m-\frac{\varepsilon}{2}; \frac{a+b}{2b} \end{matrix} \right) \\
[\varepsilon=0 \text{ или } 1; b>a>0; \operatorname{Re} \gamma, \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
17. \int_a^{\infty} (x-a)^{\lambda-1/2} (x+a)^{-\tau} (b^2x+ab^2-1)^{\gamma-1/2} C_{2m+\varepsilon}^{\gamma} (b\sqrt{x+a}) C_n^{\lambda} \left(\frac{x}{a} \right) dx = \\
= \frac{(\gamma)_{2m+\varepsilon} (2\lambda)_n}{(2m+\varepsilon)! n!} \left(\tau + \lambda - \gamma - m - \frac{\varepsilon}{2} \right)_n 2^{\lambda+\gamma+3m-\tau+3\varepsilon/2} \times \\
\times a^{\lambda+\gamma+m-\tau+\varepsilon/2} b^{2\gamma+\varepsilon+2m-1} \Gamma \left[\begin{matrix} \lambda+1/2, \tau-\gamma-\lambda-m-n-\varepsilon/2 \\ \tau-\gamma-m+(1-\varepsilon)/2 \end{matrix} \right] \times \\
\times {}_4F_3 \left(\begin{matrix} \frac{1}{2}-m-\gamma, 1-\varepsilon-\gamma-m, \tau+\lambda-\gamma-m+n-\frac{\varepsilon}{2}, \tau-\gamma-\lambda-m-n-\frac{\varepsilon}{2}; \\ 1-\varepsilon-\gamma-2m, \tau+\lambda-\gamma-m-\frac{\varepsilon}{2}, \tau-\gamma-m+\frac{1-\varepsilon}{2}; \frac{1}{2ab^2} \end{matrix} \right) \\
[\varepsilon=0 \text{ или } 1; a>0; 2ab^2>1; \operatorname{Re} \lambda > -1/2; \operatorname{Re} (\tau-\gamma-\lambda) > m+n+\varepsilon/2].
\end{aligned}$$

$$\begin{aligned}
18. \int_a^\infty (x-a)^{\gamma-1/2} (x-b)^{\lambda-1/2} (x+b)^{-\tau} C_{2m+\varepsilon}^\gamma \left(\sqrt{\frac{x+b}{a+b}} \right) C_n^\lambda \left(\frac{x}{b} \right) dx = \\
= \frac{2^n (\lambda)_n (2\gamma)_{2m+\varepsilon}}{(2m+\varepsilon)! n! b^n} \left(\frac{\varepsilon}{2} + \tau - \lambda - n \right)_m \times \\
\times \Gamma \left[\begin{matrix} \gamma+1/2, \tau-\lambda-\gamma-m-n-\varepsilon/2 \\ \tau-\lambda-n+(1-\varepsilon)/2 \end{matrix} \right] (a+b)^{\gamma+\lambda+n-\tau} \times \\
\times {}_4F_3 \left(\frac{1}{2} - \lambda - n, 1-2\lambda-n, \frac{\varepsilon}{2} + \tau + m - \lambda - n, \tau - \lambda - \gamma - m - n - \frac{\varepsilon}{2}; \right. \\
\left. 1-2\lambda-2n, \tau-\lambda-n, \frac{1}{2} + \tau - \lambda - n; \frac{2b}{a+b} \right) \\
[\varepsilon=0 \text{ или } 1; a>b>0; \operatorname{Re} \gamma > -1/2; \operatorname{Re}(\tau-\gamma-\lambda) > m+n+\varepsilon/2].
\end{aligned}$$

$$\begin{aligned}
19. \int_a^\infty (x-a)^{\gamma+\lambda-1} (x+a)^{-\tau} C_{2m+\varepsilon}^\gamma \left(\sqrt{\frac{x+a}{2a}} \right) C_n^\lambda \left(\frac{x}{a} \right) dx = \\
= \frac{2^{\lambda+\gamma+2n-\tau} (\lambda)_n (2\gamma)_{2m+\varepsilon}}{(2m+\varepsilon)! n!} \left(\frac{\varepsilon}{2} + \tau - \lambda - n \right)_m \Gamma \left[\begin{matrix} \gamma+1/2, \tau-\lambda-\gamma-m-n-\varepsilon/2 \\ \tau-\lambda-n+(1-\varepsilon)/2 \end{matrix} \right] \times \\
\times a^{\gamma+\lambda-\tau} {}_4F_3 \left(\frac{1}{2} - \lambda - n, 1-2\lambda-n, \frac{\varepsilon}{2} + \tau + m - \lambda - n, \right. \\
\left. \tau - \lambda - \gamma - m - n - \frac{\varepsilon}{2}; 1-2\lambda-2n, \tau-\lambda-n, \frac{1}{2} + \tau - \lambda - n; 1 \right) \\
[\varepsilon=0 \text{ или } 1; a>0; 0 < \operatorname{Re}(\gamma+\lambda) < \operatorname{Re} \tau - m - n - \varepsilon/2].
\end{aligned}$$

$$\begin{aligned}
20. \int_{-a}^a (x+a)^{\lambda-1/2} (a-x)^{\beta-1} \operatorname{erf}(ab-bx) \left[C_n^\lambda \left(\frac{x}{a} \right) \right]^2 dx = \\
= \frac{[(2\lambda)_n]^2}{\sqrt{\pi} n!} 2^{\beta+\lambda+3/2} a^{\beta+\lambda+1/2} b \Gamma \left[\begin{matrix} \beta+1, \lambda+1/2 \\ \beta+\lambda+n+3/2 \end{matrix} \right] \times \\
\times {}_3F_3 \left(\frac{1}{2}, \frac{\beta+1}{2}, \frac{\beta}{2} + 1, \frac{3}{2}, \frac{\beta+\lambda+n}{2} + \frac{3}{4}, \frac{\beta+\lambda+n}{2} + \frac{5}{4}; -a^2 b^2 \right) \\
[a>0; \operatorname{Re} \beta > -1; \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

$$\begin{aligned}
21. \int_{-a}^a (x+a)^{\lambda-1/2} (a-x)^{\beta-1} \operatorname{erf}(b \sqrt{a-x}) \left[C_n^\lambda \left(\frac{x}{a} \right) \right]^2 dx = \\
= \frac{[(2\lambda)_n]^2}{\sqrt{\pi} n!} 2^{\beta+\lambda+1} a^{\beta+\lambda} b \Gamma \left[\begin{matrix} \beta+1/2, \lambda+1/2 \\ \beta+\lambda+n+1 \end{matrix} \right] \times \\
\times {}_2F_3 \left(\frac{1}{2}, \beta + \frac{1}{2}; \frac{3}{2}, \beta+\lambda+n+1; -ab^2 \right) \\
[a>0; \operatorname{Re} \beta, \operatorname{Re} \lambda > -1/2].
\end{aligned}$$

2.22. МНОГОЧЛЕНЫ ЯКОБИ $P_n^{(\rho, \sigma)}(x)$

При вычислении интегралов, содержащих многочлены Якоби $P_n^{(\rho, \sigma)}(x)$, может оказаться полезной формула

$$P_n^{(\rho, \sigma)}(-x) = (-1)^n P_n^{(\sigma, \rho)}(x).$$

2.22.1. Интегралы общего вида.

Обозначение: $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$.

$$\begin{aligned}
 1. \quad & \int_{-a}^b (x+a)^{\alpha-1} (a-x)^{\rho} [(a+b)^r - (a+x)^r]^{\beta-1} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
 & = \frac{(-1)^n}{n! r} (2a)^{\rho} (a+b)^{\alpha+r\beta-r} \Gamma(\beta) \sum_{k=0}^{\infty} \frac{(-\rho-n)_k}{k!} (\sigma+k+1)_n \times \\
 & \quad \times \Gamma \left[\frac{(\alpha+k)/r}{\beta + (\alpha+k)/r} \right] \left(\frac{a+b}{2a} \right)^k \quad [-a < b < a; r, \operatorname{Re} \alpha, \operatorname{Re} \beta > 0].
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\rho} [z^r - (a+x)^r]^{-\theta} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
 & = \frac{(-1)^n}{n! z^{\theta}} (2a)^{\alpha+\rho} \Gamma(\rho+n+1) \sum_{k=0}^{\infty} \frac{(\theta)_k}{k!} (\sigma-\alpha-rk+1)_n \Gamma \left[\frac{\alpha+rk}{\alpha+\rho+rk+n+1} \right] \left(\frac{2a}{z} \right)^{rk} \\
 & \quad [a, r, \operatorname{Re} \alpha > 0; \operatorname{Re} \rho > -1; |\arg z(z-2a)| < \pi].
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & \int_a^b (x-a)^{\rho} (x+a)^{-\tau} [(a+b)^r - (a+x)^r]^{\beta-1} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
 & = \frac{(a+b)^{\rho+r\beta-r-\tau+n+1}}{n! (2a)^n r} \Gamma(\beta) \sum_{k=0}^{\infty} \frac{(-\rho-n)_k}{k!} (\sigma+\rho-k+n+1)_n \times \\
 & \quad \times \Gamma \left[\frac{(\rho-\tau-k+n+1)/r}{\beta + (\rho-\tau-k+n+1)/r} \right] \left(\frac{2a}{a+b} \right)^k + \frac{(2a)^{\rho-\tau+1}}{n!} (a+b)^{r\beta-r} \times \\
 & \quad \times \Gamma(\rho+n+1) \sum_{k=0}^{\infty} \frac{(1-\beta)_k}{k!} (\sigma+\tau-rk)_n \Gamma \left[\frac{\tau-\rho-n-rk-1}{\tau-rk} \right] \left(\frac{2a}{a+b} \right)^{rk} \\
 & \quad [b > a > 0; r, \operatorname{Re} \beta > 0; \operatorname{Re} \rho > -1].
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & \int_a^b (b-x)^{\rho} (x+b)^{-\tau} [(b+x)^r - (a+b)^r]^{\alpha-1} P_n^{(\rho, \sigma)} \left(\frac{x}{b} \right) dx = \\
 & = \frac{(-1)^n}{n! r} (2b)^{\rho} (a+b)^{r\alpha-r-\tau+1} \Gamma(\alpha) \sum_{k=0}^{\infty} \frac{(-\rho-n)_k}{k!} (1+\sigma+k)_n \times \\
 & \quad \times \Gamma \left[\frac{1-\alpha+(\tau-k-1)/r}{1+(\tau-k-1)/r} \right] \left(\frac{a+b}{2b} \right)^k + \frac{(-1)^n}{n!} (2b)^{\rho+r\alpha-r-\tau+1} \times \\
 & \quad \times \Gamma(\rho+n+1) \sum_{k=0}^{\infty} \frac{(1-\alpha)_k}{k!} (r+rk+\sigma+\tau-\alpha r)_n \times \\
 & \quad \times \Gamma \left[\frac{\alpha r-r-rk-\tau+1}{\rho+\alpha r-r-rk-\tau+n+2} \right] \left(\frac{a+b}{2b} \right)^{rk} \\
 & \quad [b > a > 0, r, \operatorname{Re} \alpha > 0; \operatorname{Re} \rho > -1].
 \end{aligned}$$

$$5. \int_a^\infty (x-b)^\rho (x+b)^{-\tau} [(b+x)^r - (a+b)^r]^{\alpha-1} P_n^{(\rho, \sigma)} \left(\frac{x}{b} \right) dx =$$

$$= \frac{(a+b)^{\rho+\tau\alpha-r-\tau+n+1}}{n! (2b)^n r} \Gamma(\alpha) \sum_{k=0}^{\infty} \frac{(-\rho-n)_k}{k!} (\rho+\sigma-k+n+1)_n \times \\ \times \Gamma \left[\frac{1-\alpha-(\rho-\tau-k+n+1)/r}{1-(\rho-\tau-k+n+1)/r} \right] \left(\frac{2b}{a+b} \right)^k \\ [a > b > 0; r, \operatorname{Re} \alpha > 0; \operatorname{Re}(\rho+\tau\alpha-\tau) < r-n-1].$$

$$6. \int_{-a}^a (a-x)^\rho (x+a)^{\alpha-1} [(a+x)^r + z^r]^{-\theta} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = I$$

$$[a, r, \operatorname{Re} \alpha > 0; \operatorname{Re} \rho > -1; r|\arg z| < \pi].$$

$$I = \frac{(-1)^n (2a)^\rho z^{\alpha-r\theta}}{n! r \Gamma(\theta)} \sum_{k=0}^{\infty} \frac{(-\rho-n)_k}{k!} (\sigma+k+1)_n \Gamma \left(\frac{\alpha+k}{r} \right) \Gamma \left(\theta - \frac{\alpha+k}{r} \right) \left(\frac{z}{2a} \right)^k + \\ + \frac{(-1)^n}{n!} (2a)^{\alpha+\rho-\theta r} \Gamma(\rho+n+1) \sum_{k=0}^{\infty} \frac{(-1)^k (\theta)_k}{k!} (\sigma+\theta r+rk-\alpha+1)_n \times \\ \times \Gamma \left[\frac{\alpha-\theta r-kr}{\alpha+\rho-\theta r-kr+n+1} \right] \left(\frac{z}{2a} \right)^{rk} \quad [|z| < 2a].$$

$$I = \frac{(-1)^n (2a)^{\alpha+\rho} z^{-\theta r}}{n!} \Gamma(\rho+n+1) \sum_{k=0}^{\infty} \frac{(-1)^k (\theta)_k}{k!} (\sigma-\alpha-rk+1)_n \times \\ \times \Gamma \left[\frac{\alpha+rk}{\alpha+\rho+rk+n+1} \right] \left(\frac{2a}{z} \right)^{rk} \quad [|z| > 2a].$$

$$7. \int_a^\infty (x-a)^\rho (x+a)^{-\tau} [(x+a)^r + z^r]^{-\theta} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = I$$

$$[a, r > 0; \operatorname{Re} \rho > -1; \operatorname{Re}(\rho-\tau-r\theta) < -n-1; r|\arg z| < \pi].$$

$$I = \frac{(2a)^{1+\rho-\tau-r\theta}}{n!} \Gamma(\rho+n+1) \sum_{k=0}^{\infty} \frac{(-1)^k (\theta)_k}{k!} (\sigma+\tau+r\theta+rk)_n \times \\ \times \Gamma \left[\frac{\tau+rk+r\theta-\rho-n-1}{\tau+rk+r\theta} \right] \left(\frac{z}{2a} \right)^{rk} \quad [|z| < 2a].$$

$$I = \frac{(2a)^{1+\rho-\tau}}{n! z^{r\theta}} \Gamma(\rho+n+1) \times \\ \times \sum_{k=0}^{\infty} \frac{(-1)^k (\theta)_k}{k!} (\sigma+\tau-rk)_n \Gamma \left[\frac{\tau-\rho-n-rk-1}{\tau-rk} \right] \left(\frac{2a}{z} \right)^{rk} + \\ + \frac{z^{\rho-\tau-r\theta+n+1}}{n! r (2a)^n \Gamma(\theta)} \sum_{k=0}^{\infty} \frac{(-\rho-n)_k}{k!} (\sigma+\rho-k+n+1)_n \Gamma \left(\frac{\rho-\tau-k+n+1}{r} \right) \times \\ \times \Gamma \left(\theta - \frac{\rho-\tau-k+n+1}{r} \right) \left(\frac{2a}{z} \right)^k \quad [|z| > 2a].$$

$$8. \int_{-a}^a \frac{(x+a)^{\alpha-1} (a-x)^\rho}{(a+x)^r - (a+y)^r} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = I$$

$$[a, r, \operatorname{Re} \alpha > 0; \operatorname{Re} \rho > -1].$$

$$I = \frac{(-1)^{n+1} \pi (2a)^{\rho} (a+y)^{\alpha-r}}{n! r} \sum_{k=0}^{\infty} \frac{(-\rho-n)_k}{k!} (\sigma+k+1)_n \times \\ \times \operatorname{ctg} \frac{\alpha+k}{r} \pi \left(\frac{a+y}{2a} \right)^k + \frac{(-1)^n}{n!} (2a)^{\alpha+\rho-r} \Gamma(\rho+n+1) \times \\ \times \sum_{k=0}^{\infty} (\sigma+r+rk-\alpha+1)_n \Gamma \left[\frac{\alpha-r-rk}{\alpha+\rho-r-rk+n+1} \right] \left(\frac{a+y}{2a} \right)^{rk} \\ [-a < y < a],$$

$$I = \frac{(-1)^{n-1} (2a)^{\alpha+\rho}}{n! (a+y)^r} \Gamma(\rho+n+1) \sum_{k=0}^{\infty} (\sigma-\alpha-rk+1)_n \times \\ \times \Gamma \left[\frac{\alpha+rk}{\alpha+\rho+rk+n+1} \right] \left(\frac{2a}{a+y} \right)^{rk} \quad [y > a].$$

$$9. \int_a^{\infty} \frac{(x-a)^{\rho} (x+a)^{-\tau}}{(x+a)^r - (y+a)^r} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = I \\ [a, r > 0; \operatorname{Re} \rho > -1; \operatorname{Re}(\rho - \tau) < r - n - 1],$$

$$I = \frac{(2a)^{\rho-\tau-r+1}}{n!} \Gamma(\rho+n+1) \sum_{k=0}^{\infty} (\sigma+\tau+r+rk)_n \times \\ \times \Gamma \left[\frac{\tau+rk+r-\rho-n-1}{\tau+r+rk} \right] \left(\frac{a+y}{2a} \right)^{rk} \quad [-a < y < a],$$

$$I = -\frac{(2a)^{\rho-\tau+1}}{n! (a+y)^r} \Gamma(\rho+n+1) \sum_{k=0}^{\infty} (\sigma+\tau-rk)_n \Gamma \left[\frac{\tau-\rho-rk-n-1}{\tau-rk} \right] \left(\frac{2a}{a+y} \right)^{rk} - \\ - \frac{(a+y)^{\rho-\tau-r+n+1}}{n! r (2a)^n} \pi \sum_{k=0}^{\infty} \frac{(-\rho-n)_k}{k!} (\sigma+\rho-k+n+1)_n \times \\ \times \operatorname{ctg} \frac{\rho-\tau-k+n+1}{r} \pi \left(\frac{2a}{a+y} \right)^k \quad [y > a].$$

$$10. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\rho} e^{-p(x+a)^r} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\ = \frac{(-1)^n}{n!} (2a)^{\alpha+\rho} \Gamma(\rho+n+1) \sum_{k=0}^{\infty} \frac{1}{k!} (\sigma-\alpha-rk+1)_n \times \\ \times \Gamma \left[\frac{\alpha+rk}{\alpha+\rho+rk+n+1} \right] (-2^r a^r p)^k \quad [a, r, \operatorname{Re} \alpha > 0; \operatorname{Re} \rho > -1].$$

$$11. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\rho} e^{-p(x+a)^r} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\ = \frac{(-1)^n}{n!} (2a)^{\alpha+\rho} \Gamma(\rho+n+1) \sum_{k=0}^{\infty} \frac{1}{k!} (\sigma+kr-\alpha+1)_n \Gamma \left[\frac{\alpha-rk}{\alpha+\rho-rk+n+1} \right] \times \\ \times \left(-\frac{p}{2^r a^r} \right)^k + \frac{(-1)^n}{n! r} (2a)^{\rho} p^{\alpha/r} \sum_{k=0}^{\infty} \frac{(-\rho-n)_k}{k!} (\sigma+k+1)_n \Gamma \left(-\frac{\alpha+k}{r} \right) \left(\frac{p}{2^r a^r} \right)^{k/r} \\ [a, r, \operatorname{Re} p > 0; \operatorname{Re} \rho > -1].$$

$$\begin{aligned}
 12. \int_a^\infty (x-a)^\rho (x+a)^{-\tau} e^{-p(x+a)^r} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
 = \frac{(2a)^{\rho-\tau+1}}{n!} \Gamma(\rho+n+1) \sum_{k=0}^\infty \frac{1}{k!} (\sigma+\tau-rk)_n \times \\
 \times \Gamma \left[\begin{matrix} \tau-\rho-rk-n-1 \\ \tau-rk \end{matrix} \right] (-2^r a^r p)^k + \frac{p^{(\tau-\rho-n-1)/r}}{n! r (2a)^n} \times \\
 \times \sum_{k=0}^\infty \frac{(-\rho-n)_k}{k!} (\sigma+\rho-k+n+1)_n \Gamma \left(\frac{\rho-\tau-k+n+1}{r} \right) (2^r a^r p)^{k/r} \\
 [a, r, \operatorname{Re} p > 0; \operatorname{Re} \rho > -1].
 \end{aligned}$$

$$\begin{aligned}
 13. \int_a^\infty (x-a)^\rho (x+a)^{-\tau} e^{-p(x+a)^r} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
 = \frac{(2a)^{\rho-\tau+1}}{n!} \Gamma(\rho+n+1) \sum_{k=0}^\infty \frac{(-1)^k}{k!} (\sigma+\tau+rk)_n \times \\
 \times \Gamma \left[\begin{matrix} \tau+rk-\rho-n-1 \\ \tau+rk \end{matrix} \right] \left(\frac{p}{2^r a^r} \right)^k \\
 [a, r > 0; \operatorname{Re} \rho > -1; \operatorname{Re}(\rho-\tau) < -n-1].
 \end{aligned}$$

$$\begin{aligned}
 14. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^\rho \left\{ \frac{\sin b(x+a)^r}{\cos b(x+a)^r} \right\} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
 = \frac{(-1)^n}{n!} (2a)^{\alpha+\rho+r\delta} b^\delta \Gamma(\rho+n+1) \sum_{k=0}^\infty \frac{(-1)^k}{k! (\delta+1/2)_k} \times \\
 \times (1+\sigma-\alpha-r\delta-2rk)_n \Gamma \left[\begin{matrix} \alpha+r\delta+2rk \\ \alpha+\rho+r\delta+2rk+n+1 \end{matrix} \right] \left(\frac{a^r b}{2^{1-r}} \right)^{2k} \\
 [a, r > 0; \operatorname{Re} \rho > -1; \operatorname{Re} \alpha > -\delta r].
 \end{aligned}$$

$$\begin{aligned}
 15. \int_a^\infty (x-a)^\rho (x+a)^{-\tau} \left\{ \frac{\sin b(x+a)^r}{\cos b(x+a)^r} \right\} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
 = \frac{(2a)^{\rho+\delta r-\tau+1} b^\delta}{n!} \Gamma(\rho+n+1) \sum_{k=0}^\infty \frac{(-1)^k}{(1/2+\delta)_k k!} \times \\
 \times (\sigma+\tau-\delta r-2rk)_n \Gamma \left[\begin{matrix} \tau-\rho-n-\delta r-2rk-1 \\ \tau-\delta r-2rk \end{matrix} \right] \left(\frac{a^r b}{2^{1-r}} \right)^{2k} + \\
 + \frac{b^{(\tau-\rho-n-1)/r}}{n! r (2a)^n} \sum_{k=0}^\infty \frac{(-\rho-n)_k}{k!} (\sigma+\rho-k+n+1)_n \times \\
 \times \Gamma \left(\frac{\rho-\tau-k+n+1}{r} \right) \left\{ \frac{\sin [(\rho-\tau-k+n+1)\pi/(2r)]}{\cos [(\rho-\tau-k+n+1)\pi/(2r)]} \right\} (2^r a^r b)^{k/r} \\
 [a, b, r > 0; \operatorname{Re} \rho > -1; \operatorname{Re}(\rho-\tau) < r-n-1].
 \end{aligned}$$

$$\begin{aligned}
 16. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^\rho \left\{ \frac{\ln [(a+x)^r + z^r]}{\ln |(a+x)^r - z^r|} \right\} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = I \\
 [a, r, \operatorname{Re} \alpha > 0; \operatorname{Re} \rho > -1, \left\{ \begin{matrix} r | \arg z | < \pi \\ z > 0 \end{matrix} \right\}],
 \end{aligned}$$

$$I = \frac{(-1)^n \pi}{n!} (2a)^\rho z^\alpha \sum_{k=0}^{\infty} \frac{(-\rho-n)_k}{k! (\alpha+k)} (\sigma+k+1)_n \times$$

$$\times \left\{ \operatorname{cosec} [(\alpha+k) \pi/r] \right\} \left(\frac{z}{2a} \right)^k \pm \frac{(-1)^n}{n!} (2a)^{\alpha+\rho-r} z^r \Gamma(\rho+n+1) \times$$

$$\times \sum_{k=0}^{\infty} \frac{(\mp 1)^k}{k+1} (\sigma-\alpha+r+rk+1)_n \Gamma \left[\frac{\alpha-r-rk}{\alpha+\rho-r-rk+n+1} \right] \left(\frac{z}{2a} \right)^{rk} +$$

$$+ \frac{(-1)^n r}{n!} (2a)^{\alpha+\rho} B(\alpha, \rho+n+1) (\sigma-\alpha+1)_n \times$$

$$\times \left[\psi(\alpha) - \psi(\alpha+\rho+n+1) - \sum_{j=0}^{r-1} \frac{1}{\sigma-\alpha+j+1} + \ln(2a) \right]$$

$$[|z| < 2a],$$

$$I = \pm \frac{(-1)^n}{n!} (2a)^{\alpha+\rho+r} z^{-r} \Gamma(\rho+n+1) \times$$

$$\times \sum_{k=0}^{\infty} \frac{(\mp 1)^k}{k+1} (\sigma-\alpha-r-rk+1)_n \Gamma \left[\frac{\alpha-r-rk}{\rho+\alpha-r-rk+n+1} \right] \left(\frac{2a}{z} \right)^{rk} +$$

$$+ \frac{(-1)^n r}{n!} (2a)^{\alpha+\rho} (1-\alpha+\sigma)_n B(\alpha, \rho+n+1) \ln z$$

$$[|z| > 2a].$$

$$17. \int_a^\infty (x-a)^\rho (x+a)^{-\tau} \left\{ \frac{\ln [(x+a)^r + z^r]}{\ln [(x+a)^r - z^r]} \right\} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = I$$

$$\left[a, r > 0; \operatorname{Re} \rho > -1; \operatorname{Re} (\rho - \tau) < -n-1, \left\{ \begin{array}{l} r | \arg z | < \pi \\ z > 0 \end{array} \right\} \right],$$

$$I = \pm \frac{(2a)^{\rho-\tau-r+1} z^r}{n!} \Gamma(\rho+n+1) \times$$

$$\times \sum_{k=0}^{\infty} \frac{(\mp 1)^k}{k+1} (\sigma+\tau+r+rk)_n \Gamma \left[\frac{\tau+r+rk-\rho-n-1}{\tau+r+rk} \right] \left(\frac{z}{2a} \right)^{rk} -$$

$$- \frac{(2a)^{\rho-\tau+1} r (\tau+\sigma)_n}{n!} B(\rho+n+1, \tau-\rho-n-1) \times$$

$$\times \left[\psi(\tau-\rho-n-1) - \psi(\tau) + \sum_{j=0}^{n-1} \frac{1}{\sigma+\tau+j} - \ln(2a) \right]$$

$$[|z| < 2a],$$

$$I = \pm \frac{(2a)^{\rho+\tau-r+1}}{n! z^r} \Gamma(\rho+n+1) \sum_{k=0}^{\infty} \frac{(\mp 1)^k}{k+1} (\sigma+\tau-r-rk)_n \times$$

$$\times \Gamma \left[\frac{\tau-\rho-r-rk-n-1}{\tau-r-rk} \right] \left(\frac{2a}{z} \right)^{rk} + \frac{\pi z^{\rho-\tau+n+1}}{n! (2a)^n} \times$$

$$\times \sum_{k=0}^{\infty} \frac{(-\rho-n)_k (\sigma+\rho-k+n+1)_n}{k! (\tau-\rho+k-n-1)} \left\{ \frac{\operatorname{cosec} [(\tau-\rho+k-n-1) \pi/r]}{\operatorname{ctg} [(\tau-\rho+k-n-1) \pi/r]} \right\} \left(\frac{2a}{z} \right)^k +$$

$$+ \frac{(2a)^{\rho-\tau+1} r}{n!} (\sigma+\tau)_n B(\rho+n+1, \tau-\rho-n-1) \ln z$$

$$[|z| > 2a].$$

$$\begin{aligned}
18. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\rho} \operatorname{Ei}(-b(x+a)^r) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
= \frac{(-1)^{n+1}}{n!} (2a)^{\alpha+\rho+r} b \Gamma(\rho+n+1) \times \\
\times \sum_{k=0}^{\infty} \frac{(\sigma-\alpha-r-rk+1)_n}{k! (k+1)^2} \Gamma\left[\frac{\alpha+r+rk}{\alpha+\rho+r+rk+n+1}\right] (-2^r a^r b)^k - \\
- \frac{(-1)^n}{n!} (\sigma-\alpha+1)_n B(\rho+n+1, \alpha) (2a)^{\alpha+\rho} \left[C+r\psi(\alpha)-r\psi(\alpha+\rho+n+1)- \right. \\
\left. - \sum_{j=0}^{n-1} \frac{r}{\sigma-\alpha+j+1} + \ln(2^r a^r b) \right] \quad [a, r, \operatorname{Re} \alpha > 0; \operatorname{Re} \rho > -1].
\end{aligned}$$

$$\begin{aligned}
19. \int_a^{\infty} (x-a)^{\rho} (x+a)^{-\tau} \operatorname{Ei}(-b(x+a)^r) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
= - \frac{(2a)^{\rho-\tau+r+1} b}{n!} \Gamma(\rho+n+1) \sum_{k=0}^{\infty} \frac{(\sigma+\tau-r-rk)_n}{k! (k+1)^2} \times \\
\times \Gamma\left[\frac{\tau-\rho-r-rk-n-1}{\tau-r-rk}\right] (-2^r a^r b)^k + \frac{b^{(\tau-\rho-n-1)/r}}{n! (2a)^n} \times \\
\times \sum_{k=0}^{\infty} \frac{(-\rho-n)_k (\sigma+\rho-k+n+1)_n}{k! (\tau-\rho+k-n-1)} \Gamma\left(\frac{\rho-\tau-k+n+1}{r}\right) (2^r a^r b)^{k/r} + \\
+ \frac{(2a)^{\rho-\tau+1}}{n!} B(\rho+n+1, \tau-\rho-n-1) (\sigma+\tau)_n \times \\
\times \left[C-r\psi(\tau-\rho-n-1)+r\psi(\tau)-\sum_{j=0}^{n-1} \frac{r}{\sigma+\tau+j} + \ln(2^r a^r b) \right] \\
[a, r, \operatorname{Re} b > 0; \operatorname{Re} \rho > -1].
\end{aligned}$$

$$\begin{aligned}
20. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\rho} \begin{Bmatrix} \operatorname{si}(b(x+a)^r) \\ \operatorname{ci}(b(x+a)^r) \end{Bmatrix} P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
= \frac{(-1)^{n+1} (2a)^{\alpha+\rho+r\delta+2r}}{2 \cdot 3^{\delta} n!} b^{\delta+2} \Gamma(\rho+n+1) \times \\
\times \sum_{k=0}^{\infty} \frac{(-1)^k (\sigma-\alpha-r\delta-2r-2rk+1)_n}{(k+1)! (3/2+\delta)_k (2k+\delta+2)} \Gamma\left[\frac{\alpha+r\delta+2r+2rk}{\alpha+\rho+r\delta+2r+2rk+n+1}\right] \left(\frac{a^r b}{2^{1-r}}\right)^{2k} + \\
+ \frac{(-1)^n}{n!} B(\rho+n+1, \alpha+r\delta) (\sigma-\alpha-r\delta+1)_n (2a)^{\alpha+\rho+r\delta} b^{\delta} \left\{ \frac{1}{R} \right\} - \\
- \delta \frac{(-1)^n}{2(n!)} (2a)^{\alpha+\rho} \pi B(\rho+n+1, \alpha) (\sigma-\alpha+1)_n, \\
R = C+r\psi(\alpha)-r\psi(\alpha+\rho+n+1)-\sum_{j=0}^{n-1} \frac{r}{\sigma-\alpha+j+1} + \ln(2^r a^r b) \\
[a, r, \operatorname{Re} \alpha > 0; \operatorname{Re} \rho > -1].
\end{aligned}$$

$$\begin{aligned}
21. \int_a^\infty (x-a)^\rho (x+a)^{-\tau} \left\{ \begin{matrix} \text{si} (b(x+a)^r) \\ \text{ci} (b(x+a)^r) \end{matrix} \right\} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= - \frac{(2a)^{\rho+\delta r+2r-\tau+1}}{2 \cdot 3^\delta n!} b^{\delta+2} \Gamma(\rho+n+1) \times \\
\times \sum_{k=0}^\infty \frac{(-1)^k (\tau+\sigma-\delta r-2r-2rk)_n}{(k+1)! (3/2+\delta)_k (2k+\delta+2)} \Gamma \left[\begin{matrix} \tau-\rho-\delta r-2r-2rk-n-1 \\ \tau-\delta r-2r-2rk \end{matrix} \right] \left(\frac{a^r b}{2^{1-r}} \right)^{2k} + \\
+ \frac{b^{(\tau-\rho-n-1)/r}}{n! (2a)^n} \sum_{k=0}^\infty \frac{(-\rho-n)_k}{k! (\tau-\rho+k-n-1)} (\sigma+\rho-k+n+1)_n \times \\
\times \Gamma \left(\frac{\rho-\tau-k+n+1}{r} \right) \left\{ \begin{matrix} \sin [(\rho-\tau-k+n+1) \pi/(2r)] \\ \cos [(\rho-\tau-k+n+1) \pi/(2r)] \end{matrix} \right\} (2^r a^r b)^{k/r} + \\
+ \frac{(2a)^{\rho+\delta r-\tau+1} b^\delta}{n!} (\sigma+\tau-\delta r)_n B(\rho+n+1, \tau-\rho-\delta r-n-1) \left\{ \begin{matrix} 1 \\ R \end{matrix} \right\} - \\
- \delta \frac{(2a)^{\rho-\tau+1} \pi}{2(n!)} (\sigma+\tau)_n B(\rho+n+1, \tau-\rho-n-1), \\
R = C - r\psi(\tau-\rho-n-1) + r\psi(\tau) - \sum_{j=0}^{n-1} \frac{r}{\sigma+\tau+j} + \ln(2^r a^r b) \\
[a, b, r > 0; \operatorname{Re} \rho > -1; \operatorname{Re}(\rho-\tau) < 2r-n-1].
\end{aligned}$$

$$\begin{aligned}
22. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^\rho \left\{ \begin{matrix} \operatorname{erf} (b(x+a)^r) \\ \operatorname{erfc} (b(x+a)^r) \end{matrix} \right\} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \pm \frac{(-1)^n 2 (2a)^{\alpha+\rho+r} b}{n! \sqrt{\pi}} \Gamma(\rho+n+1) \times \\
\times \sum_{k=0}^\infty \frac{(-1)^k}{k! (2k+1)} (\sigma-\alpha-r-2rk+1)_n \Gamma \left[\begin{matrix} \alpha+r+2rk \\ \alpha+\rho+r+2rk+n+1 \end{matrix} \right] (2^r a^r b)^{2k} + \\
+ \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{(-1)^n}{n!} (2a)^{\alpha+\rho} B(\alpha, \rho+n+1) (\sigma-\alpha+1)_n \\
[a, r > 0; \operatorname{Re} \rho > -1; \operatorname{Re} \alpha > -(1 \pm 1)r/2].
\end{aligned}$$

$$\begin{aligned}
23. \int_a^\infty (x-a)^\rho (x+a)^{-\tau} \left\{ \begin{matrix} \operatorname{erf} (b(x+a)^r) \\ \operatorname{erfc} (b(x+a)^r) \end{matrix} \right\} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \pm \frac{2 (2a)^{\rho+r-\tau+1} b}{n! \sqrt{\pi}} \Gamma(\rho+n+1) \times \\
\times \sum_{k=0}^\infty \frac{(-1)^k}{k! (2k+1)} (\sigma+\tau-r-2rk)_n \Gamma \left[\begin{matrix} \tau-\rho-r-2rk-n-1 \\ \tau-r-2rk \end{matrix} \right] (2^r a^r b)^{2k} \pm \\
\pm \frac{b^{(\tau-\rho-n-1)/r}}{n! (2a)^n \sqrt{\pi}} \sum_{k=0}^\infty \frac{(-\rho-n)_k (\sigma+\rho-k+n+1)_n}{k! (\tau-\rho+k-n-1)} \times \\
\times \Gamma \left(\frac{r+\rho-\tau-k+n+1}{2r} \right) (2^r a^r b)^{k/r} + \\
+ \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{(2a)^{\rho-\tau+1}}{n!} B(\rho+n+1, \tau-\rho-n-1) (\sigma+\tau)_n \\
[a, r > 0; \operatorname{Re} \rho > -1, \left\{ \begin{matrix} |\arg b| < \pi/4, \operatorname{Re}(\rho-\tau) < -n-1 \\ |\arg b| < \pi/4 \end{matrix} \right\}].
\end{aligned}$$

$$\begin{aligned}
24. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\rho} \left\{ \frac{J_{\nu}(b(x+a)^r)}{I_{\nu}(b(x+a)^r)} \right\} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^n 2^{\alpha+\rho+\nu r-\nu}}{n!} a^{\alpha+\rho+\nu r} b^{\nu} \Gamma \left[\frac{\rho+n+1}{\nu+1} \right] \times \\
\times \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (\nu+1)_k} (\sigma-\alpha-\nu r-2rk+1)_n \Gamma \left[\frac{\alpha+\nu r+2rk}{\alpha+\rho+\nu r+2rk+n+1} \right] \left(\frac{a^r b}{2^{1-r}} \right)^{2k} \\
[a, r, \operatorname{Re}(\alpha+\nu r) > 0; \operatorname{Re} \rho > -1].
\end{aligned}$$

$$\begin{aligned}
25. \int_a^{\infty} (x-a)^{\rho} (x+a)^{-\tau} J_{\nu}(b(x+a)^r) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \frac{2^{\rho-\tau+\nu r-\nu+1}}{n!} a^{\rho-\tau+\nu r+1} b^{\nu} \Gamma \left[\frac{\rho+n+1}{\nu+1} \right] \times \\
\times \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (\nu+1)_k} (\sigma+\tau-\nu r-2rk)_n \Gamma \left[\frac{\tau-\rho-\nu r-2rk-n-1}{\tau-\nu r-2rk} \right] \left(\frac{a^r b}{2^{1-r}} \right)^{2k} + \\
+ \frac{2^{(\rho-\tau+n+1)/r-n-1} b^{(\tau-\rho-n-1)/r}}{n! r a^n} \times \\
\times \sum_{k=0}^{\infty} \frac{(-\rho-n)_k}{k!} (\sigma+\rho-k+n+1)_n \Gamma \left[\frac{(\nu r+\rho-\tau-k+n+1)/(2r)}{1+(\nu r-\rho+\tau+k-n-1)/(2r)} \right] \left(\frac{a^r b}{2^{1-r}} \right)^{k/r} \\
[a, b, r > 0; \operatorname{Re} \rho > -1; \operatorname{Re}(\rho-\tau) < 3r/2 - n - 1].
\end{aligned}$$

$$\begin{aligned}
26. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\rho} e^{-b(x+a)^r} L_m^{\nu}(b(x+a)^r) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^n}{m! n!} (2a)^{\alpha+\rho} \Gamma(\rho+n+1) \times \\
\times \sum_{k=0}^{\infty} \frac{1}{k!} (\gamma+k+1)_m (\sigma-\alpha-rk+1)_n \Gamma \left[\frac{\alpha+rk}{\alpha+\rho+rk+n+1} \right] (-2^r a^r b)^k \\
[a, r, \operatorname{Re} \alpha > 0; \operatorname{Re} \rho > -1].
\end{aligned}$$

$$\begin{aligned}
27. \int_a^{\infty} (x-a)^{\rho} (x+a)^{-\tau} e^{-b(x+a)^r} L_m^{\nu}(b(x+a)^r) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \frac{(2a)^{\rho-\tau+1}}{m! n!} \Gamma(\rho+n+1) \times \\
\times \sum_{k=0}^{\infty} \frac{1}{k!} (\sigma+\tau-kr)_n (\gamma+k+1)_m \Gamma \left[\frac{\tau-\rho-rk-n-1}{\tau-rk} \right] (-2^r a^r b)^k + \\
+ \frac{b^{(\tau-\rho-n-1)/r}}{m! n! r (2a)^n} \sum_{k=0}^{\infty} \frac{(-\rho-n)_k}{k!} (\sigma+\rho-k+n+1)_n \times \\
\times \left(\gamma+1+\frac{\tau-\rho+k-n-1}{r} \right)_m \Gamma \left(\frac{\rho-\tau-k+n+1}{r} \right) (2^r a^r b)^{k/r} \\
[a, r, \operatorname{Re} b > 0; \operatorname{Re} \rho > -1].
\end{aligned}$$

$$\begin{aligned}
28. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\rho} e^{-b^2(x+a)^{2r}} H_{2m+\varepsilon}(b(x+a)^r) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
= \frac{(-1)^{m+n} 2^{2m+\varepsilon}}{n!} (2a)^{\alpha+\rho+r\varepsilon} b^{\varepsilon} \Gamma(\rho+n+1) \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \left(\varepsilon+k+\frac{1}{2}\right)_m \times \\
\times (\sigma-\alpha-re-2rk+1)_n \Gamma\left[\frac{\alpha+re+2rk}{\alpha+\rho+re+2rk+n+1}\right] (2^r a^r b)^{2k} \\
[\varepsilon=0 \text{ или } 1; a, r>0; \operatorname{Re} \rho>-1; \operatorname{Re} \alpha>-re]
\end{aligned}$$

$$\begin{aligned}
29. \int_a^{\infty} (x-a)^{\rho} (x+a)^{-\tau} e^{-b^2(x+a)^{2r}} H_{2m+\varepsilon}(b(x+a)^r) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
= \frac{(-1)^m 2^{\rho+re+2m+\varepsilon-\tau+1}}{n!} a^{\rho+re-\tau+1} b^{\varepsilon} \Gamma(\rho+n+1) \times \\
\times \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} (\sigma+\tau-re-2rk)_n \left(\varepsilon+k+\frac{1}{2}\right)_m \Gamma\left[\frac{\tau-\rho-re-2rk-n-1}{\tau-re-2rk}\right] \times \\
\times (2^r a^r b)^{2k} + \frac{(-1)^m b^{(\tau-\rho-n-1)/r} 2^{2m+\varepsilon-n-1}}{n! r a^n} \sum_{k=0}^{\infty} \frac{(-\rho-n)_k}{k!} (\sigma+\rho-k+n+1)_n \times \\
\times \left(\frac{r+re-\rho+\tau+k-n-1}{2r}\right)_m \Gamma\left(\frac{re+\rho-\tau-k+n+1}{2r}\right) (2^r a^r b)^{k/r} \\
[\varepsilon=0 \text{ или } 1; a, r>0; \operatorname{Re} \rho>-1; |\arg b|<\pi/4].
\end{aligned}$$

$$\begin{aligned}
30. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\rho} [1-b^2(x+a)^{2r}]^{\gamma-1/2} C_{2m+\varepsilon}^{\gamma}(b(x+a)^r) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
= \frac{(-1)^{m+n} 2^{\alpha+\rho+r\varepsilon+2m+\varepsilon} a^{\alpha+\rho+r\varepsilon}}{n! (2m+\varepsilon)!} (\gamma)_{m+\varepsilon} \Gamma(\rho+n+1) b^{\varepsilon} \times \\
\times \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{1}{2}-\gamma-m\right)_k \left(\frac{1}{2}+\varepsilon+k\right)_m (\sigma-\alpha-re-2rk+1)_n \times \\
\times \Gamma\left[\frac{\alpha+re+2rk}{\alpha+\rho+re+2rk+n+1}\right] (2^r a^r b)^{2k} \\
[\varepsilon=0 \text{ или } 1; a, b, r>0; 2ab^{1/r}<1; \operatorname{Re} \alpha>-re \\
\text{и } \operatorname{Re} \rho>-1 \text{ (или } \operatorname{Re}(\rho+\gamma)>-1/2 \text{ при } 2ab^{1/r}=1)].
\end{aligned}$$

$$\begin{aligned}
31. \int_{\pm a}^b (x \mp a)^{\alpha-1} [(b \mp a)^{2r} - (x \mp a)^{2r}]^{\gamma-1/2} \left\{ \frac{(x+a)^{\sigma}}{(a-x)^{\rho}} \right\} \times \\
\times C_{2m+\varepsilon}^{\gamma}\left(\frac{(x \mp a)^r}{(b \mp a)^r}\right) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
= \frac{(-1)^m (\pm 1)^n (2a)^{\psi} (b \mp a)^{\alpha+2\gamma r-r}}{2(n!) r (2m+\varepsilon)!} \Gamma\left(\gamma+\frac{1}{2}\right) (2\gamma)_{2m+\varepsilon} \sum_{k=0}^{\infty} \frac{(-\psi-n)_k}{k!} \times \\
\times (\psi+k+1)_n \left(\frac{r+re-\alpha-k}{2r}\right)_m \Gamma\left[\frac{(re+\alpha+k)/(2r)}{m+\gamma+(r+re+\alpha+k)/(2r)}\right] \left(\frac{a \mp b}{2a}\right)^k \\
[\varepsilon=0 \text{ или } 1; r>0; \operatorname{Re} \alpha>-re; \operatorname{Re} \gamma>-1/2, \\
\{b>a>0\}, \varphi=\{\rho\}, \psi=\{\sigma\}].
\end{aligned}$$

$$\begin{aligned}
32. \int_a^b (x-a)^\rho [(a+b)^{2r} - (a+x)^{2r}]^{\gamma-1/2} (a+x)^{-\tau} C_{2m+\varepsilon}^\gamma \left(\left(\frac{a+x}{a+b} \right)^r \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^m 2^{2m+\varepsilon} (2a)^{\rho+r\varepsilon-\tau+1} (a+b)^{2r\gamma-r\varepsilon-r}}{n! (2m+\varepsilon)!} (\gamma)_{m+\varepsilon} \Gamma(\rho+n+1) \times \\
\times \sum_{k=0}^{\infty} \frac{(1/2-\gamma-m)_k}{k!} \left(\varepsilon+k+\frac{1}{2} \right)_m (\sigma+\tau-r\varepsilon-2rk)_n \times \\
\times \Gamma \left[\frac{\tau-\rho-r\varepsilon-2rk-n-1}{\tau-r\varepsilon-2rk} \right] \left(\frac{2a}{a+b} \right)^{2rk} + \\
+ \frac{(-1)^m (a+b)^{\rho+2r\gamma-\tau-r+n+1}}{n! 2r (2m+\varepsilon)! (2a)^n} (2\gamma)_{2m+\varepsilon} \Gamma \left(\gamma+\frac{1}{2} \right) \times \\
\times \sum_{k=0}^{\infty} \frac{(-\rho-n)_k}{k!} (\sigma+\rho-k+n+1)_n \left(\frac{r+r\varepsilon+\tau+k-\rho-n-1}{2r} \right)_m \times \\
\times \Gamma \left[\frac{(r\varepsilon+\rho-\tau-k+n+1)/(2r)}{\gamma+m+(r+r\varepsilon+\rho-\tau-k+n+1)/(2r)} \right] \left(\frac{2a}{a+b} \right)^k \\
[\varepsilon=0 \text{ или } 1; r>0; b>a>0; \operatorname{Re} \rho > -1; \operatorname{Re} \gamma > -1/2].
\end{aligned}$$

$$\begin{aligned}
33. \int_a^b [(x+b)^{2r} - (a+b)^{2r}]^{\gamma-1/2} (b-x)^\rho (x+b)^{-\tau} C_{2m+\varepsilon}^\gamma \left(\left(\frac{x+b}{a+b} \right)^r \right) P_n^{(\rho, \sigma)} \left(\frac{x}{b} \right) dx = \\
= \frac{(-1)^n (2b)^\rho (a+b)^{2r\gamma-\tau-r+1}}{n! 2r (2m+\varepsilon)!} \Gamma \left(\gamma+\frac{1}{2} \right) (2\gamma)_{2m+\varepsilon} \times \\
\times \sum_{k=0}^{\infty} \frac{(-\rho-n)_k}{k!} (\sigma+k+1)_n \left(\frac{r+r\varepsilon+\tau-k-1}{2r} \right)_m \times \\
\times \Gamma \left[\frac{(r+\tau-r\varepsilon-k-1)/(2r)-m-\gamma}{1-(r\varepsilon-\tau+k+1)/(2r)} \right] \left(\frac{a+b}{2b} \right)^k + \\
+ \frac{(-1)^n 2^{\rho+(2\gamma+\varepsilon+2m-1)r+2m+\varepsilon-\tau+1} b^{\rho+(2\gamma+\varepsilon+2m-1)r-\tau+1}}{n! (2m+\varepsilon)! (a+b)^{2mr+r\varepsilon}} \Gamma(\rho+n+1) \times \\
\times \sum_{k=0}^{\infty} \frac{(1/2-\gamma-m)_k}{k!} (\varepsilon+\gamma+m-k)_m (\sigma+\tau+r+2kr-2r\gamma-2rm-r\varepsilon)_n \times \\
\times \Gamma \left[\frac{2r\gamma+2rm+r\varepsilon-r-2rk-\tau+1}{\rho+2r\gamma+2rm+r\varepsilon-r-2rk-\tau+n+2} \right] \left(\frac{a+b}{2b} \right)^{2rk} \\
[\varepsilon=0 \text{ или } 1; a, b, r>0; \operatorname{Re} \rho > -1; \operatorname{Re} \gamma > -1/2].
\end{aligned}$$

$$\begin{aligned}
34. \int_a^\infty (x-a)^\rho (x+a)^{-\tau} [b^2 (x+a)^{2r} - 1]^{\gamma-1/2} C_{2m+\varepsilon}^\gamma (b (x+a)^r) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \frac{2^{2m+\varepsilon} (2a)^{\rho+r\varepsilon+2r\gamma+2mr-r-\tau+1} b^{\varepsilon+2\gamma+2m-1}}{n! (2m+\varepsilon)!} (\gamma)_{m+\varepsilon} \Gamma(\rho+n+1) \times \\
\times \sum_{k=0}^{\infty} \frac{(1/2-\gamma-m)_k}{k!} (\varepsilon+\gamma+m-k)_m (\tau+\sigma+r+2rk-r\varepsilon-2r\gamma-2rm)_n \times \\
\times \Gamma \left[\frac{\tau+r+2rk-r\varepsilon-2r\gamma-2rm-\rho-n-1}{\tau+r+2rk-r\varepsilon-2r\gamma-2rm} \right] (2^r a^r b)^{-2k} \\
[\varepsilon=0 \text{ или } 1; a, r>0; 2ab^{1/r}>1; \operatorname{Re} \rho > -1; \\
\operatorname{Re}(\rho-\tau+2r\gamma) < r-n-2rm-r\varepsilon-1].
\end{aligned}$$

$$\begin{aligned}
35. \int_a^\infty [(x+b)^{2r} - (a+b)^{2r}]^{\gamma-1/2} (x-b)^\rho (x+b)^{-\tau} C_{2m+\varepsilon}^\gamma \left(\left(\frac{x+b}{a+b} \right)^r \right) \times \\
\times P_n^{(\rho, \sigma)} \left(\frac{x}{b} \right) dx = \frac{(a+b)^{\rho+2r\gamma-\tau-r+n+1}}{2^{n+1} n! r (2m+\varepsilon)! b^n} (2\gamma)_{2m+\varepsilon} \Gamma \left(\gamma + \frac{1}{2} \right) \times \\
\times \sum_{k=0}^\infty \frac{(-\rho-n)_k}{k!} (\rho+\sigma-k+n+1)_n \left(\frac{r+r\varepsilon+\tau+k-\rho-n-1}{2r} \right)_m \times \\
\times \Gamma \left[\frac{(r-r\varepsilon+\tau+k-\rho-n-1)/(2r) - \gamma - m}{1 + (\tau-\rho+k-r\varepsilon-n-1)/(2r)} \right] \left(\frac{2b}{a+b} \right)^k \\
[\varepsilon=0 \text{ или } 1; r>0; a>b>0; \operatorname{Re} \gamma > -1/2; \\
\operatorname{Re}(\rho-\tau+2r\gamma) < r-2rm-r\varepsilon-n-1].
\end{aligned}$$

$$\begin{aligned}
36. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^\rho [1-b(x+a)^r]^\lambda P_m^{(\lambda, \nu)} (2b(x+a)^r-1) \times \\
\times P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \frac{(-1)^{m+n}}{m! n!} (2a)^{\alpha+\rho} \Gamma(\rho+n+1) \sum_{k=0}^\infty \frac{(-m-\lambda)_k}{k!} (\nu+k+1)_m \times \\
\times (\sigma-\alpha-rk+1)_n \Gamma \left[\frac{\alpha+rk}{\rho+\alpha+n+rk+1} \right] (2^r a^r b)^k \\
[a, b, r, \operatorname{Re} \alpha > 0; 2ab^{1/r} < 1; \operatorname{Re} \rho > -1].
\end{aligned}$$

$$\begin{aligned}
37. \int_{\pm a}^b (x \mp a)^{\alpha-1} [(b \mp a)^r - (x \mp a)^r]^\lambda \left\{ \frac{(x+a)^\sigma}{(a-x)^\rho} \right\} P_m^{(\lambda, \nu)} \left(2 \left(\frac{x \mp a}{b \mp a} \right)^r - 1 \right) \times \\
\times P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \frac{(-1)^m (\pm 1)^n (2a)^\psi (b \mp a)^{\alpha+\lambda r}}{m! n! r} \Gamma(\lambda+m+1) \times \\
\times \sum_{k=0}^\infty \frac{(-\psi-n)_k}{k!} (\psi+k+1)_n \left(\nu - \frac{\alpha+k}{r} + 1 \right)_m \Gamma \left[\frac{(\alpha+k)/r}{\lambda + (\alpha+k)/r + m + 1} \right] \left(\frac{a \mp b}{2a} \right)^k \\
[r, \operatorname{Re} \alpha > 0; \operatorname{Re} \lambda > -1, \left\{ \begin{smallmatrix} b > a > 0 \\ a > b > 0 \end{smallmatrix} \right\}, \varphi = \left\{ \begin{smallmatrix} \rho \\ \sigma \end{smallmatrix} \right\}, \psi = \left\{ \begin{smallmatrix} \sigma \\ \rho \end{smallmatrix} \right\}].
\end{aligned}$$

$$\begin{aligned}
38. \int_a^b (x-a)^\rho [(a+b)^r - (a+x)^r]^\lambda (a+x)^{-\tau} P_m^{(\lambda, \nu)} \left(2 \left(\frac{a+x}{a+b} \right)^r - 1 \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^m (2a)^{\rho-\tau+1} (a+b)^{r\lambda}}{m! n!} \Gamma(\rho+n+1) \sum_{k=0}^\infty \frac{(-\lambda-m)_k}{k!} \times \\
\times (\nu+k+1)_m (\sigma+\tau-rk)_n \Gamma \left[\frac{\tau-\rho-rk-n-1}{\tau-rk} \right] \left(\frac{2a}{a+b} \right)^{rk} + \\
+ \frac{(-1)^m (a+b)^{r\lambda+\rho-\tau+n+1}}{n! m! r (2a)^n} \Gamma(\lambda+m+1) \sum_{k=0}^\infty \frac{(-\rho-n)_k}{k!} (\sigma+\rho-k+n+1)_n \times \\
\times \left(\frac{\tau+k-\rho-n-1}{r} + \nu + 1 \right)_m \Gamma \left[\frac{(\rho-\tau-k+n+1)/r}{1+\lambda+m+(\rho-\tau-k+n+1)/r} \right] \left(\frac{2a}{a+b} \right)^k \\
[b > a > 0; r > 0; \operatorname{Re} \rho, \operatorname{Re} \lambda > -1].
\end{aligned}$$

$$\begin{aligned}
39. \int_a^b [(x+b)^r - (a+b)^r]^\lambda (b-x)^\rho (b+x)^{-\tau} P_m^{(\lambda, \nu)} \left(2 \left(\frac{x+b}{a+b} \right)^r - 1 \right) \times \\
\times P_n^{(\rho, \sigma)} \left(\frac{x}{b} \right) dx = \frac{(-1)^n}{m! n! r} (2b)^\rho (a+b)^{\lambda r - \tau + 1} \Gamma(\lambda + m + 1) \times \\
\times \sum_{k=0}^{\infty} \frac{(-\rho - n)_k}{k!} (\sigma + k + 1)_n \left(\frac{\tau - k - 1}{r} + \nu + 1 \right)_m \Gamma \left[\frac{(\tau - k - 1)/r - \lambda - m}{(\tau - k - 1)/r + 1} \right] \times \\
\times \left(\frac{a+b}{2b} \right)^k + \frac{(-1)^n}{m! n!} (2b)^{\rho + r\lambda + rm - \tau + 1} (a+b)^{-mr} \Gamma(\rho + n + 1) \times \\
\times \sum_{k=0}^{\infty} \frac{(-\lambda - m)_k}{k!} (\lambda + \nu - k + m + 1)_m (\sigma + \tau + rk - r\lambda - rm)_n \times \\
\times \Gamma \left[\frac{rm + r\lambda - \tau - rk + 1}{\rho + rm + r\lambda - \tau - rk + n + 2} \right] \left(\frac{a+b}{2b} \right)^{kr} \\
[b > a > 0; r > 0; \operatorname{Re} \rho, \operatorname{Re} \lambda > -1].
\end{aligned}$$

$$\begin{aligned}
40. \int_a^\infty (x-a)^\rho (x+a)^{-\tau} [b(x+a)^r - 1]^\lambda P_m^{(\lambda, \nu)} (2b(x+a)^r - 1) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \frac{(2a)^{\rho + \lambda r + mr - \tau + 1} b^{\lambda + m}}{m! n!} \Gamma(\rho + n + 1) \times \\
\times \sum_{k=0}^{\infty} \frac{(-\lambda - m)_k}{k!} (\nu + \lambda + m - k + 1)_m (\sigma + \tau - \lambda r - mr + rk)_n \times \\
\times \Gamma \left[\frac{\tau + rk - \rho - \lambda r - mr - n - 1}{\tau + rk - \lambda r - mr} \right] (2^r a^r b)^{-k} \\
[a, r > 0; 2ab^{1/r} > 1; \operatorname{Re} \rho > -1; \operatorname{Re}(\rho - \tau + \lambda r) < -mr - n - 1].
\end{aligned}$$

$$\begin{aligned}
41. \int_a^\infty [(x+b)^r - (a+b)^r]^\lambda (x-b)^\rho (x+b)^{-\tau} P_m^{(\lambda, \nu)} \left(2 \left(\frac{x+b}{a+b} \right)^r - 1 \right) \times \\
\times P_n^{(\rho, \sigma)} \left(\frac{x}{b} \right) dx = \frac{(a+b)^{\lambda r + \rho - \tau + n + 1}}{m! n! r (2b)^n} \Gamma(\lambda + m + 1) \times \\
\times \sum_{k=0}^{\infty} \frac{(-\rho - n)_k}{k!} (\sigma + \rho - k + n + 1)_n \left(\frac{\tau + k - \rho - n - 1}{r} + \nu + 1 \right)_m \times \\
\times \Gamma \left[\frac{(\tau - \rho + k - n - 1)/r - \lambda - m}{1 + (\tau - \rho + k - n - 1)/r} \right] \left(\frac{2b}{a+b} \right)^k \\
[r > 0; a > b > 0; \operatorname{Re} \lambda > -1; \operatorname{Re}(\rho - \tau + \lambda r) < -mr - n - 1].
\end{aligned}$$

2.22.2. Интегралы от $(x \pm a)^\alpha (b \pm x)^\beta P_n^{(\rho, \sigma)}(cx)$.

$$\begin{aligned}
1. \int_{\pm a}^b (x \mp a)^{\alpha-1} (b-x)^{\beta-1} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \frac{(\pm 1)^n}{n!} (\varphi + 1)_n B(\alpha, \beta) (b \mp a)^{\alpha + \beta - 1} \times \\
\times {}_2F_2 \left(-n, \rho + \sigma + n + 1, \alpha; \alpha + \beta, \varphi + 1; \frac{a \mp b}{2a} \right) \\
[a > 0; b > \pm a; \operatorname{Re} \alpha, \operatorname{Re} \beta > 0, \varphi = \left\{ \begin{smallmatrix} \rho \\ \sigma \end{smallmatrix} \right\}].
\end{aligned}$$

$$2. \int_{-a}^b (x+a)^\sigma (b-x)^{\beta-1} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx =$$

$$= B(\beta, \sigma+n+1) (a+b)^{\beta+\sigma} P_n^{(\rho-\beta, \sigma+\beta)} \left(\frac{b}{a} \right)$$

$$[-a < b < a; \operatorname{Re} \beta > 0; \operatorname{Re} \sigma > -1].$$

$$3. \int_{-a}^b (x+a)^{n+\rho+\sigma-\beta} (b-x)^{\beta-1} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx =$$

$$= B(\beta, \rho+\sigma-\beta+n+1) (a+b)^{n+\rho+\sigma} P_n^{(\rho-\beta, \sigma)} \left(\frac{b}{a} \right)$$

$$[-a < b < a; \operatorname{Re} \beta > 0; \operatorname{Re}(\rho+\sigma-\beta) > -n-1].$$

$$4. \int_a^b (x-a)^\rho (b-x)^{\beta-1} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx =$$

$$= \frac{(\rho+\sigma+n+1)_n}{n! (2a)^n} B(\rho+n+1, \beta) (a+b)^{\beta+\rho+n} \times$$

$$\times {}_2F_1 \left(-\rho-\sigma-n, -\beta-\rho-n; -\rho-\sigma-2n; \frac{2a}{a+b} \right)$$

$$[b > a > 0; \operatorname{Re} \rho > -1; \operatorname{Re} \beta > 0].$$

$$5. \int_a^b (x-a)^{\alpha-1} (b-x)^{\beta-1} P_n^{(\rho, \sigma)} \left(\frac{x}{b} \right) dx = \frac{(\rho+1)_n}{n!} B(\alpha, \beta) (b-a)^{\alpha+\beta-1} \times$$

$$\times {}_3F_2 \left(-n, \rho+\sigma+n+1, \beta; \rho+1, \alpha+\beta; \frac{b-a}{2b} \right)$$

$$[-b < a < b; \operatorname{Re} \alpha, \operatorname{Re} \beta > 0].$$

$$6. \int_a^b (x-a)^{\rho+\sigma-\beta+n} (b-x)^{\beta-1} P_n^{(\rho, \sigma)} \left(\frac{x}{b} \right) dx =$$

$$= B(\beta, \rho+\sigma-\beta+n+1) (b-a)^{\rho+\sigma+n} P_n^{(\rho, \beta-\rho-n-1)} \left(\frac{a}{b} \right)$$

$$[-b < a < b; \operatorname{Re} \beta > 0; \operatorname{Re}(\rho+\sigma-\beta) > -n-1].$$

$$7. \int_a^b (x-a)^{\alpha-1} (b-x)^\rho P_n^{(\rho, \sigma)} \left(\frac{x}{b} \right) dx =$$

$$= B(\alpha, \rho+n+1) (b-a)^{\alpha+\rho} P_n^{(\rho+\alpha, \sigma-\alpha)} \left(\frac{a}{b} \right)$$

$$[-b < a < b; \operatorname{Re} \alpha > 0; \operatorname{Re} \rho > -1].$$

$$8. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\beta-1} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \frac{(\rho+1)_n}{n!} B(\alpha, \beta) (2a)^{\alpha+\beta-1} \times$$

$$\times {}_3F_2(-n, \rho+\sigma+n+1, \beta; \rho+1, \alpha+\beta; 1) \quad [a, \operatorname{Re} \alpha, \operatorname{Re} \beta > 0].$$

$$9. \int_{-a}^a (x+a)^\sigma (a-x)^{\beta-1} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \frac{(\rho-\beta+1)_n}{n!} B(\beta, \sigma+n+1) (2a)^{\beta+\sigma}$$

$$[a, \operatorname{Re} \beta > 0; \operatorname{Re} \sigma > -1].$$

$$10. \int_{-a}^a (x+a)^{\rho+\sigma-\beta+n} (a-x)^{\beta-1} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx =$$

$$= \frac{(\rho-\beta+1)_n}{n!} B(\beta, \rho+\sigma-\beta+n+1) (2a)^{\rho+\sigma+n}$$

[$a > 0$; $0 < \operatorname{Re} \beta < \operatorname{Re}(\rho+\sigma)+n+1$].

$$11. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\rho} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \frac{(-1)^n}{n!} (\sigma-\alpha+1)_n B(\alpha, \rho+n+1) (2a)^{\alpha+\rho}$$

[$a, \operatorname{Re} \alpha > 0$; $\operatorname{Re} \rho > -1$].

$$12. \int_{-a}^a (x+a)^{m/2} (a-x)^{l/2} P_n^{(l, m)} \left(\frac{x}{a} \right) dx = 0$$

[$n = l+1, l+2, \dots$; $a > 0$].

$$13. \int_a^\infty (x-a)^{\alpha-1} (x+a)^{-\tau} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx =$$

$$= \frac{(\rho+1)_n}{n!} B(\alpha, \tau-\alpha-n) (2a)^{\alpha-\tau} {}_3F_2(-n, \alpha, -n-\sigma; \rho+1, \tau-n; 1)$$

[$a, \operatorname{Re} \alpha > 0$; $\operatorname{Re}(\alpha-\tau) < -n$].

$$14. \int_a^\infty (x-a)^{\rho} (x+a)^{-\tau} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx =$$

$$= \frac{(\tau+\sigma)_n}{n!} B(\rho+n+1, \tau-\rho-n-1) (2a)^{\rho-\tau+1}$$

[$a > 0$; $\operatorname{Re} \rho > -1$; $\operatorname{Re}(\tau-\rho) > n+1$].

$$15. \int_a^\infty (x-a)^{\tau+\rho+\sigma+n-1} (x+a)^{-\tau} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^n (2a)^{\rho+\sigma+n}}{n!} (\sigma+\tau)_n B(\rho+\sigma+\tau+n, -\rho-\sigma-n)$$

[$a > 0$; $\operatorname{Re}(\rho+\sigma+\tau) > -n$; $\operatorname{Re}(\rho+\sigma) < -2n$].

$$16. \int_a^\infty (x-a)^{\alpha-1} (x+a)^{\sigma} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \frac{(\rho-\alpha+1)_n}{n!} B(\alpha, -\alpha-\sigma-n) (2a)^{\alpha+\sigma}$$

[$a > 0$; $0 < \operatorname{Re} \alpha < -\operatorname{Re} \sigma - n$].

2.22.3. Интегралы от $\frac{x^k}{x-y} (x \pm a)^\alpha (b \pm x)^\beta P_n^{(\rho, \sigma)}(cx)$.

$$1. \int_{-a}^a \frac{(x+a)^\sigma}{x-y} (a-x)^{\beta-1} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = I$$

[$a, \operatorname{Re} \beta > 0$; $\operatorname{Re} \sigma > -1$].

$$I = \pi \operatorname{ctg} \beta \pi (a-y)^{\beta-1} (a+y)^\sigma P_n^{(\rho, \sigma)} \left(\frac{y}{a} \right) -$$

$$- \frac{(\rho-\beta+2)_n}{n!} B(\beta-1, \sigma+n+1) (2a)^{\beta+\sigma-1} \times$$

$$\times {}_3F_2 \left(1, \rho-\beta+n+2, 1-\beta-\sigma-n; 2-\beta, \rho-\beta+2; \frac{a-y}{2a} \right)$$

[$-a < y < a$].

$$I = \frac{(\rho - \beta + 1)_n}{n!} B(\beta, \sigma + n + 1) \frac{(2a)^{\beta + \sigma}}{a - y} \times \\ \times {}_2F_2\left(1, \beta, \beta - \rho; \beta - \rho - n, \beta + \sigma + n + 1; \frac{2a}{a - y}\right) \quad [y < -a].$$

$$2. \int_{-a}^a \frac{(x + a)^{n + \rho + \sigma - \beta + 1}}{x - y} (a - x)^{\beta - 1} P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\ = -\frac{2^{\rho + \sigma + n + 1}}{n!} (\rho - \beta + 2)_n B(\beta - 1, \rho + \sigma - \beta + n + 2) a^{n + \rho + \beta - 1} (a + y)^{\sigma - \beta + 1} \times \\ \times {}_2F_2\left(1, \rho + \sigma - \beta + n + 2, \rho - \beta + n + 2; \rho - \beta + 2, 2 - \beta; \frac{y - a}{y + a}\right) + \\ + \pi \operatorname{ctg} \beta \pi (a + y)^{\rho + \sigma - \beta + n + 1} (a - y)^{\beta - 1} P_n^{(\rho, \sigma)}\left(\frac{y}{a}\right) \\ [-a < y < a; \operatorname{Re} \beta > 0; \operatorname{Re}(\beta - \rho - \sigma) < n + 2].$$

$$3. \int_{-a}^a \frac{(x + a)^{\alpha - 1}}{x - y} (a - x)^{\rho} P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = I \quad [a, \operatorname{Re} \alpha > 0; \operatorname{Re} \rho > -1].$$

$$I = -\pi \operatorname{ctg} \alpha \pi (a + y)^{\alpha - 1} (a - y)^{\rho} P_n^{(\rho, \sigma)}\left(\frac{y}{a}\right) + \\ + \frac{(-1)^n}{n!} (\sigma - \alpha + 2)_n B(\alpha - 1, \rho + n + 1) (2a)^{\alpha + \rho - 1} \times \\ \times {}_2F_2\left(1, 1 - \alpha - \rho - n; \sigma - \alpha + n + 2; \sigma - \alpha + 2, 2 - \alpha; \frac{a + y}{2a}\right) \\ [-a < y < a].$$

$$I = \frac{(-1)^{n+1}}{n!} (\sigma - \alpha + 1)_n B(\alpha, \rho + n + 1) \frac{(2a)^{\alpha + \rho}}{a + y} \times \\ \times {}_2F_2\left(1, \alpha, \alpha - \sigma; \alpha + \rho + n + 1, \alpha - \sigma - n; \frac{2a}{a + y}\right) \quad [y > a].$$

$$4. \int_{-a}^a \frac{(x + a)^{m/2}}{x - y} (a - x)^{l/2} P_n^{(l, m)}\left(\frac{x}{a}\right) dx = \\ = -\frac{2^{n - (l - m)/2 + 1} a^{n+1}}{(y - a)^{n - (l + m)/2 + 1} (2n + 1)!} \Gamma\left(n + \frac{l - m}{2} + 1\right) \Gamma\left(n + \frac{l + m}{2} + 1\right) \times \\ \times {}_2F_1\left(n - \frac{l + m}{2} + 1, n + \frac{l - m}{2} + 1; 2n + 2; \frac{2a}{a - y}\right) \quad [-a < y < a].$$

$$5. \int_{-a}^a \frac{x^m (x + a)^{\sigma}}{x - y} (a - x)^{\rho} P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = -2y^m (y - a)^{\rho} (y + a)^{\sigma} Q_n^{(\rho, \sigma)}\left(\frac{y}{a}\right) \\ [m = 0, 1, 2, \dots, n; a > 0; |\arg(y^2 - a^2)| < \pi; \operatorname{Re} \sigma, \operatorname{Re} \rho > -1].$$

$$6. \int_a^{\infty} \frac{(x - a)^{\rho}}{x - y} (x + a)^{-\tau} P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = I \\ [a > 0; \operatorname{Re} \rho > -1; \operatorname{Re}(\rho - \tau) < -n].$$

$$I = \frac{(\sigma + \tau + 1)_n}{n!} B(\rho + n + 1, \tau - \rho - n) (2a)^{\rho - \tau} \times \\ \times {}_2F_2 \left(1, \tau - \rho - n, \sigma + \tau + n + 1; \tau + 1, \sigma + \tau + 1; \frac{a + y}{2a} \right) \\ [-a < y < a].$$

$$I = \pi \operatorname{ctg}(\tau - \rho) \pi (a + y)^{-\tau} (y - a)^{\rho} P_n^{(\rho, \sigma)} \left(\frac{y}{a} \right) - \\ - \frac{(2a)^{\rho - \tau + 1} (\tau + \sigma)_n}{n! (a + y)} B(\rho + n + 1, \tau - \rho - n - 1) \times \\ \times {}_2F_2 \left(1, 1 - \tau, 1 - \sigma - \tau; \rho - \tau + n + 2, 1 - \sigma - \tau - n; \frac{2a}{a + y} \right) \\ [y > a]$$

$$7. \int_a^\infty \frac{(x - a)^{\alpha - 1}}{x - y} (x + a)^{\sigma} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\ = -\pi \operatorname{ctg} \alpha \pi (y - a)^{\alpha - 1} (y + a)^{\sigma} P_n^{(\rho, \sigma)} \left(\frac{y}{a} \right) + \\ + \frac{(\rho - \alpha + 2)_n}{n!} B(\alpha - 1, 1 - \alpha - \sigma - n) (2a)^{\alpha + \sigma - 1} \times \\ \times {}_2F_2 \left(1, \rho - \alpha + n + 2, 1 - n - \sigma - \alpha; \rho - \alpha + 2, 2 - \alpha; \frac{a - y}{2a} \right) \\ [y > a; \operatorname{Re} \alpha > 0; \operatorname{Re}(\alpha + \sigma) < 1 - n].$$

2.22.4. Интегралы от $(x \pm a)^{\alpha} (b \pm x)^{\beta} (d \pm x)^{\gamma} P_n^{(\rho, \sigma)}(cx)$.

$$1. \int_{\pm a}^b (x \mp a)^{\alpha - 1} (b - x)^{\beta - 1} \left\{ \frac{(a + x)^{\sigma}}{(a - x)^{\rho}} \right\} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\ = \frac{(\pm 1)^n (\varphi + 1)_n}{n!} B(\alpha, \beta) (2a)^{\psi} (b \mp a)^{\alpha + \beta - 1} \times \\ \times {}_2F_2 \left(\alpha, -\psi - n, \varphi + n + 1; \varphi + 1, \alpha + \beta; \frac{a \mp b}{2a} \right) \\ \left[\operatorname{Re} \alpha, \operatorname{Re} \beta > 0, \left\{ \begin{matrix} b > a \\ -a < b < a \end{matrix} \right\}, \varphi = \left\{ \begin{matrix} \rho \\ \sigma \end{matrix} \right\}, \psi = \left\{ \begin{matrix} \rho \\ \sigma \end{matrix} \right\} \right].$$

$$2. \int_{\pm a}^b (x \mp a)^{\alpha - 1} (b - x)^{\beta - 1} (a \pm x)^{\rho + \sigma - \alpha - \beta + n + 1} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\ = \frac{(\pm 1)^n (\varphi + 1)_n}{n!} B(\alpha, \beta) (2a)^{\rho + \sigma - \beta + n + 1} (a \pm b)^{-\alpha} (b \mp a)^{\alpha + \beta - 1} \times \\ \times {}_2F_2 \left(\varphi + n + 1, \rho + \sigma + n + 1, \alpha; \varphi + 1, \alpha + \beta; \frac{b \mp a}{b \pm a} \right) \\ [-a < b < a; \operatorname{Re} \alpha, \operatorname{Re} \beta > 0; \varphi = \left\{ \begin{matrix} \rho \\ \sigma \end{matrix} \right\}].$$

$$\begin{aligned}
3. \quad & \int_{-a}^b (x+a)^\sigma (b-x)^{\beta-1} (a-x)^{-\tau} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
& = \frac{(\rho+1)_n}{n!} B(\beta, \tau-\beta) (2a)^\sigma (a-b)^{\beta-\tau} \times \\
& \times {}_3F_2 \left(1-\tau, -\sigma-n, \rho+n+1; \rho+1, \beta-\tau+1; \frac{a-b}{2a} \right) + \\
& + \frac{(\rho+\tau-\beta+1)_n}{n!} B(\sigma+n+1, \beta-\tau) (2a)^{\beta+\sigma-\tau} \times \\
& \times {}_3F_2 \left(1-\beta, \rho+\tau-\beta+n+1, \tau-\beta-\sigma-n; \tau-\beta+1, \rho+\tau-\beta+1; \frac{a-b}{2a} \right) \\
& [-a < b < a; \operatorname{Re} \beta > 0; \operatorname{Re} \sigma > -1].
\end{aligned}$$

$$\begin{aligned}
4. \quad & \int_a^b (x-a)^\rho (b-x)^{\beta-1} (x+a)^{-\tau} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
& = \frac{(\rho+\sigma+n+1)_n}{n! (2a)^n} B(\rho-\tau+n+1, \beta) (a+b)^{\beta+\rho-\tau+n} \times \\
& \times {}_3F_2 \left(-\rho-n, -\rho-\sigma-n, \tau-\beta-\rho-n; -\rho-\sigma-2n, \tau-\rho-n; \frac{2a}{a+b} \right) + \\
& + \frac{(2a)^{\rho-\tau+1}}{n!} (\sigma+\tau)_n B(\rho+n+1, \tau-\rho-n-1) (a+b)^{\beta-1} \times \\
& \times {}_3F_2 \left(1-\beta, 1-\tau, 1-\tau-\sigma; \rho-\tau+n+2, 1-\tau-\sigma-n, \frac{2a}{a+b} \right) \\
& [b > a > 0; \operatorname{Re} \rho > -1; \operatorname{Re} \beta > 0].
\end{aligned}$$

$$\begin{aligned}
5. \quad & \int_a^b (x-a)^{\alpha-1} (b-x)^{\beta-1} (x+b)^\sigma P_n^{(\rho, \sigma)} \left(\frac{x}{b} \right) dx = \\
& = \frac{(\rho+1)_n}{n!} B(\alpha, \beta) (2b)^\sigma (b-a)^{\alpha+\beta-1} {}_3F_2 \left(-\sigma-n, \beta, \rho+n+1; \rho+1, \alpha+\beta; \frac{b-a}{2b} \right) \\
& [b > a > 0; a, \operatorname{Re} \alpha, \operatorname{Re} \beta > 0].
\end{aligned}$$

$$\begin{aligned}
6. \quad & \int_a^b (x-a)^{\alpha-1} (b-x)^{\beta-1} (x+b)^{\rho+\sigma-\alpha-\beta+n+1} P_n^{(\rho, \sigma)} \left(\frac{x}{b} \right) dx = \\
& = \frac{(\rho+1)_n}{n!} B(\alpha, \beta) (2b)^{\rho+\sigma-\alpha+n+1} (b-a)^{\alpha+\beta-1} (b+a)^{-\beta} \times \\
& \times {}_3F_2 \left(\beta, \rho+\sigma+n+1, \rho+n+1; \alpha+\beta, \rho+1; \frac{a-b}{a+b} \right) \\
& [b > a > 0; \operatorname{Re} \alpha, \operatorname{Re} \beta > 0].
\end{aligned}$$

$$\begin{aligned}
7. \quad & \int_a^b (x-a)^{\alpha-1} (b-x)^\rho (x+b)^{-\tau} P_n^{(\rho, \sigma)} \left(\frac{x}{b} \right) dx = \\
& = \frac{(-1)^n}{n!} (1+\sigma)_n B(\alpha, \tau-\alpha) (2b)^\rho (a+b)^{\alpha-\tau} \times \\
& \times {}_3F_2 \left(1-\tau, -\rho-n, \sigma+n+1; \sigma+1, \alpha-\tau+1; \frac{a+b}{2b} \right) + \\
& + \frac{(-1)^n}{n!} (\sigma+\tau-\alpha+1)_n B(\rho+n+1, \alpha-\tau) (2b)^{\alpha+\rho-\tau} \times \\
& \times {}_3F_2 \left(1-\alpha, \sigma+\tau-\alpha+n+1, \tau-\alpha-\rho-n; \sigma+\tau-\alpha+1, \tau-\alpha+1; \frac{a+b}{2b} \right) \\
& [b > a > 0; \operatorname{Re} \alpha > 0; \operatorname{Re} \rho > -1]
\end{aligned}$$

$$8. \int_a^b (x-a)^{\tau-\rho-n-2} (b-x)^{\rho} (x+b)^{-\tau} P_n^{(\rho, \sigma)} \left(\frac{x}{b} \right) dx =$$

$$= B(\tau-\rho-n-1, \rho+n+1) (2b)^{\rho-\tau+n+1} (b-a)^{\tau-n-1} \times$$

$$\times (b+a)^{-\rho-n-1} P_n^{(\tau-n-1, \sigma)} \left(\frac{a}{b} \right)$$

$$[b > a > 0; \operatorname{Re}(\tau-\rho) > n+1; \operatorname{Re} \rho > -1].$$

$$9. \int_{-a}^a x^m (x+a)^{\sigma} (a-x)^{\rho} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = I_m \quad [a > 0; \operatorname{Re} \rho, \operatorname{Re} \sigma > -1].$$

$$I_m = 0 \quad [m = 0, 1, 2, \dots, n-1].$$

$$I_n = B(\rho+n+1, \sigma+n+1) (2a)^{\rho+\sigma+n+1},$$

$$I_m = \frac{m!}{n! (m-n)!} B(\rho+n+1, \sigma+n+1) 2^{\rho+\sigma+n+1} a^{\rho+\sigma+m+1} \times$$

$$\times {}_2F_1(n-m, \rho+n+1; \rho+\sigma+2n+2; 2)$$

$$[m = n+1, n+2, \dots].$$

$$10. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\beta-1} (z-x)^{\rho+\sigma-\alpha-\beta+n+1} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx =$$

$$= \frac{(\rho+1)_n}{n!} B(\beta, \alpha-\rho-\sigma-n-1) (2a)^{\alpha+\beta-1} (z+a)^{-\beta} (z-a)^{\rho+\sigma-\alpha+n+1} \times$$

$$\times {}_3F_2 \left(\beta, \rho+\sigma+n+1, \rho+n+1; \rho+1, \rho+\sigma-\alpha+n+2; \frac{z-a}{z+a} \right) +$$

$$+ \frac{(-1)^n (\sigma-\alpha+1)_n}{n!} B(\rho+\sigma-\alpha+n+1, \alpha) (2a)^{\alpha+\beta-1} (a+z)^{\rho+\sigma-\alpha-\beta+n+1} \times$$

$$\times {}_3F_2 \left(\alpha, \alpha-\sigma, \alpha+\beta-\rho-\sigma-n-1; \alpha-\rho-\sigma-n, \alpha-\sigma-n; \frac{z-a}{z+a} \right)$$

$$[a, \operatorname{Re} \alpha, \operatorname{Re} \beta > 0; |\arg(z^2 - a^2)| < \pi].$$

$$11. \int_{-a}^a (x+a)^{\sigma} (a-x)^{\beta-1} (\pm x \mp z)^{-\theta} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx =$$

$$= \frac{(\rho-\beta+1)_n}{n!} B(\beta, \sigma+n+1) (2a)^{\beta+\sigma} (\pm a \mp z)^{-\theta} \times$$

$$\times {}_3F_2 \left(\theta, \beta, \beta-\rho; \beta-\rho-n, \beta+\sigma+n+1; \frac{2a}{a-z} \right)$$

$$\left[a, \operatorname{Re} \beta > 0; \operatorname{Re} \sigma > -1, \left\{ \begin{array}{l} z < -a \\ |\arg(z^2 - a^2)| < \pi \end{array} \right\} \right].$$

$$12. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\rho} (z \pm x)^{-\theta} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^n (\sigma-\alpha+1)_n}{n!} B(\alpha, \rho+n+1) (2a)^{\alpha+\rho} (z \mp a)^{-\theta} \times$$

$$\times {}_3F_2 \left(\alpha, \theta, \alpha-\sigma; \alpha-\sigma-n, \alpha+\rho+n+1; \frac{2a}{a \mp z} \right)$$

$$[a, \operatorname{Re} \alpha > 0; \operatorname{Re} \rho > -1; |\arg(z^2 - a^2)| < \pi].$$

$$\begin{aligned}
 13. \int_a^\infty (x-a)^\rho (x+a)^{-\tau} (x+z)^{-\theta} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
 = \frac{(\theta + \tau + \sigma)_n}{n!} B(\rho + n + 1, \theta + \tau - \rho - n - 1) (2a)^{\rho - \theta - \tau + 1} \times \\
 \times {}_3F_2 \left(\theta, \theta + \tau - \rho - n - 1, \theta + \tau + \sigma + n; \theta + \tau, \theta + \tau + \sigma; \frac{a+z}{2a} \right) \\
 [a > 0; \operatorname{Re} \rho > -1; \operatorname{Re}(\rho - \theta - \tau) < -n - 1; |\operatorname{Re}(z-a)| < \pi].
 \end{aligned}$$

$$\begin{aligned}
 14. \int_a^\infty (x-a)^{\alpha-1} (x+a)^\sigma (x+z)^{-\theta} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
 = \frac{(\rho + 1)_n}{n!} B(\alpha, \theta - \alpha) (2a)^\sigma (a+z)^{\alpha-\theta} \times \\
 \times {}_3F_2 \left(\alpha, \rho + n + 1, -\sigma - n; \rho + 1, \alpha - \theta + 1; \frac{a+z}{2a} \right) + \\
 + \frac{(\rho + \theta - \alpha + 1)_n}{n!} B(\alpha - \theta, \theta - \sigma - \alpha - n) (2a)^{\alpha + \sigma - \theta} \times \\
 \times {}_3F_2 \left(\rho + \theta - \alpha + n + 1, \theta - \sigma - \alpha - n, \theta; \theta - \alpha + 1, \rho + \theta - \alpha + 1; \frac{a+z}{2a} \right) \\
 [a, \operatorname{Re} \alpha > 0; \operatorname{Re}(\alpha + \sigma - \theta) < -n; |\arg(a+z)| < \pi].
 \end{aligned}$$

$$\begin{aligned}
 15. \int_a^\infty (x-a)^{\alpha-1} (x-b)^\rho (x+b)^{-\tau} P_n^{(\rho, \sigma)} \left(\frac{x}{b} \right) dx = \\
 = \frac{(\rho + \sigma + n + 1)_n}{n! (2b)^n} B(\alpha, \tau - \alpha - \rho - n) (a+b)^{\alpha + \rho - \tau + n} \times \\
 \times {}_3F_2 \left(-\rho - n, -\rho - \sigma - n, \tau - \rho - \alpha - n; \tau - \rho - n, -\rho - \sigma - 2n; \frac{2b}{a+b} \right) \\
 [a > b > 0; \operatorname{Re} \alpha > 0; \operatorname{Re}(\tau - \rho - \alpha) > n].
 \end{aligned}$$

$$\begin{aligned}
 16. \int_a^\infty (x-a)^{\alpha-1} (x-b)^{-\tau} (x+b)^\sigma P_n^{(\rho, \sigma)} \left(\frac{x}{b} \right) dx = \\
 = \frac{(\rho + 1)_n}{n!} B(\alpha, \tau - \alpha) (2b)^\sigma (a-b)^{\alpha-\tau} \times \\
 \times {}_3F_2 \left(1-\tau, \rho + n + 1, -\sigma - n; \rho + 1, \alpha - \tau + 1; \frac{b-a}{2b} \right) + \\
 + \frac{(\rho - \alpha + \tau + 1)_n}{n!} B(\alpha - \tau, \tau - \alpha - \sigma - n) (2b)^{\alpha + \sigma - \tau} \times \\
 \times {}_3F_2 \left(\beta, \tau + \rho - \alpha + n + 1, \tau - \alpha - \sigma - n; \tau - \alpha + 1, \rho + \tau - \alpha + 1; \frac{b-a}{2b} \right) \\
 [a > b > 0; \operatorname{Re} \alpha > 0; \operatorname{Re}(\tau - \alpha - \sigma) > n].
 \end{aligned}$$

2.22.5. Интегралы от $A(x) e^{-px} P_n^{(\rho, \sigma)}(cx)$.

$$\begin{aligned}
 1. \int_{-a}^a (x+a)^\sigma (a-x)^{\beta-1} e^{-px} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
 = \frac{(\rho - \beta + 1)_n}{n!} B(\beta, \sigma + n + 1) (2a)^{\beta + \sigma} e^{-ap} \times \\
 \times {}_3F_2(\beta, \beta - \rho; \beta + \sigma + n + 1, \beta - \rho - n; 2ap) \\
 [a, \operatorname{Re} \beta > 0; \operatorname{Re} \sigma > -1].
 \end{aligned}$$

$$2. \int_{\pm a}^{\infty} (x \mp a)^{\alpha-1} e^{-px} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \frac{(\rho + \sigma + n + 1)_n}{n! (2a)^n} \Gamma(\alpha + n) \times \\ \times p^{-\alpha-n} e^{\mp ap} {}_2F_2(-n, -\varphi - n; 1 - \alpha - n, -\rho - \sigma - 2n; \pm 2ap) \\ \left[a, \operatorname{Re} p, \operatorname{Re} \alpha > 0; \varphi = \begin{cases} \rho \\ \sigma \end{cases} \right].$$

$$3. \int_a^{\infty} (x-a)^{\rho} (x+a)^{-\tau} e^{-px} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\ = \frac{(\sigma + \tau)_n}{n!} B(\rho + n + 1, \tau - \rho - n - 1) (2a)^{\rho-\tau+1} e^{ap} \times \\ \times {}_2F_2(1-\tau, 1-\sigma-\tau; \rho-\tau+n+2, 1-\sigma-\tau-n; -2ap) + \\ + \frac{(\rho + \sigma + n + 1)_n}{n! (2a)^n} \Gamma(\rho - \tau + n + 1) p^{\tau-\rho-n-1} e^{ap} \times \\ \times {}_2F_2(-\rho - \sigma - n, -\rho - n; -\rho - \sigma - 2n, \tau - \rho - n; -2ap) \\ [a, \operatorname{Re} p > 0; \operatorname{Re} \rho > -1].$$

$$4. \int_a^{\infty} (x-a)^{\alpha-1} (x+a)^{\sigma} e^{-px} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\ = \frac{(\rho - \alpha + 1)_n}{n!} B(\alpha, -\alpha - \sigma - n) (2a)^{\alpha+\sigma} e^{-ap} {}_2F_2(\alpha, \alpha - \rho; \alpha - \rho - n, \\ \alpha + \sigma + n + 1; 2ap) + \frac{(\rho + \sigma + n + 1)_n}{n! (2a)^n} \Gamma(\alpha + \sigma + n) p^{-\alpha-\sigma-n} e^{-ap} \times \\ \times {}_2F_2(-\sigma - n, -\rho - \sigma - n; 1 - \alpha - \sigma - n, -\rho - \sigma - 2n; 2ap) \\ [a, \operatorname{Re} p, \operatorname{Re} \alpha > 0].$$

2.22.6. Интегралы от $A(x) e^{\varphi(x)} P_n^{(\rho, \sigma)}(cx)$.

$$1. \int_{-a}^a (x+a)^{-\beta-n-1} (a-x)^{\beta-1} \exp\left(-p \frac{(a-x)^{\varepsilon}}{a+x}\right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\ = \frac{(-1)^n}{n!} (\sigma + 1)_n \Gamma(\beta + n) (2a)^{\beta-\beta\varepsilon-n\varepsilon-1} p^{-\beta-n} \exp\left(\frac{\varepsilon p - p}{2a}\right) \times \\ \times {}_2F_2\left(-n, -\rho - n; 1 - \beta - n, \sigma + 1; \frac{p}{(2a)^{1-\varepsilon}}\right) \\ [\varepsilon = 0 \text{ или } 1; a, \operatorname{Re} p, \operatorname{Re} \beta > 0].$$

$$2. \int_{-a}^a (x+a)^{\rho+\sigma-\beta+n} (a-x)^{\beta-1} \exp\left(-p \frac{(a-x)^{\varepsilon}}{a+x}\right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\ = \frac{(\rho - \beta + 1)_n}{n!} B(\beta, \rho + \sigma - \beta + n + 1) (2a)^{\rho+\sigma+n} \exp\left(\frac{\varepsilon p - p}{2a}\right) \times \\ \times {}_2F_2\left(\beta, \beta - \rho; \beta - \rho - \sigma - n, \beta - \rho - n; \frac{p}{(2a)^{1-\varepsilon}}\right) + \\ + \frac{(-1)^n}{n!} (\sigma + 1)_n \Gamma(\beta - \rho - \sigma - n - 1) \times \\ \times (2a)^{\beta+\varepsilon(\rho+\sigma-\beta+n+1)-1} p^{\rho+\sigma-\beta+n+1} \exp\left(\frac{\varepsilon p - p}{2a}\right) \times \\ \times {}_2F_2\left(\sigma + n + 1, \rho + \sigma + n + 1; \sigma + 1, \rho + \sigma - \beta + n + 2; \frac{p}{(2a)^{1-\varepsilon}}\right) \\ [\varepsilon = 0 \text{ или } 1; a, \operatorname{Re} p, \operatorname{Re} \rho > 0].$$

$$\begin{aligned}
3. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\rho} \exp\left(-p \frac{(a-x)^{\varepsilon}}{a+x}\right) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
= \frac{(-1)^n}{n!} (\sigma - \alpha + 1)_n B(\alpha, \rho + n + 1) (2a)^{\alpha+\rho} e^{\varepsilon p} \times \\
\times {}_2F_2\left(-\alpha - \rho - n, \sigma - \alpha + n + 1; 1 - \alpha, \sigma - \alpha + 1; -\frac{\rho}{(2a)^{1-\varepsilon}}\right) + \\
+ \frac{(-1)^n}{n!} (\sigma + 1)_n \Gamma(-\alpha) (2a)^{\rho+\alpha\varepsilon} \rho^{\alpha} e^{\varepsilon p} \times \\
\times {}_2F_2\left(\sigma + n + 1, -\rho - n; \sigma + 1, \alpha + 1; -\frac{\rho}{(2a)^{1-\varepsilon}}\right) \\
[\varepsilon = 0 \text{ или } 1; a, \operatorname{Re} p > 0; \operatorname{Re} \rho > -1].
\end{aligned}$$

$$\begin{aligned}
4. \int_{-a}^a (x+a)^{\rho+\sigma-\beta+n} (a-x)^{\beta-1} \exp\left(-p \frac{(a+x)^{\varepsilon}}{a-x}\right) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
= \frac{(\rho - \beta + 1)_n}{n!} B(\beta, \rho + \sigma - \beta + n + 1) (2a)^{\rho+\sigma+n} \exp\left(\frac{\varepsilon p - \rho}{2a}\right) \times \\
\times {}_2F_2\left(\rho - \beta + n + 1, \rho + \sigma - \beta + n + 1; 1 - \beta, \rho - \beta + 1; \frac{\rho}{(2a)^{1-\varepsilon}}\right) + \\
+ \frac{(\rho + 1)_n}{n!} \Gamma(-\beta) (2a)^{\rho+\sigma-\beta+\varepsilon n} \rho^{\beta} \exp\left(\frac{\varepsilon p - \rho}{2a}\right) \times \\
\times {}_2F_2\left(\rho + n + 1, \rho + \sigma + n + 1; \beta + 1, \rho + 1; \frac{\rho}{(2a)^{1-\varepsilon}}\right) \\
[\varepsilon = 0 \text{ или } 1; a, \operatorname{Re} p > 0; \operatorname{Re}(\beta - \rho - \sigma) < n + 1].
\end{aligned}$$

$$\begin{aligned}
5. \int_{-a}^a (x+a)^{-\beta-n-1} (a-x)^{\beta-1} \exp\left(-p \frac{(a+x)^{\varepsilon}}{a-x}\right) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
= \frac{(\rho + 1)_n \Gamma(-\beta)}{n! (2a)^{\beta-\beta\varepsilon+n+1}} \rho^{\beta} \exp\left(\frac{\varepsilon p - \rho}{2a}\right) {}_2F_2\left(-n, -n - \sigma; \rho + 1, \beta + 1; \frac{\rho}{(2a)^{1-\varepsilon}}\right) \\
[\varepsilon = 0 \text{ или } 1; a, \operatorname{Re} p > 0; \operatorname{Re} \beta < -n].
\end{aligned}$$

$$\begin{aligned}
6. \int_{-a}^a (x+a)^{\sigma} (a-x)^{-\sigma-n-1} \exp\left(-p \frac{(a+x)^{\varepsilon}}{a-x}\right) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
= \frac{\Gamma(\sigma + n + 1) (2a)^{\sigma-\varepsilon(\sigma+n+1)}}{\rho^{\sigma+n+1}} \exp\left(\frac{\varepsilon p - \rho}{2a}\right) L_n^{\rho}\left(\frac{\rho}{(2a)^{1-\varepsilon}}\right) \\
[\varepsilon = 0 \text{ или } 1; a, \operatorname{Re} p > 0; \operatorname{Re} \sigma > -1].
\end{aligned}$$

$$\begin{aligned}
7. \int_{\pm a}^{\infty} (x \mp a)^{\alpha-1} \exp\left(\mp p \frac{(x \pm a)^{\varepsilon}}{x \mp a}\right) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
= \frac{(\pm 1)^n}{n!} (\varphi + 1)_n \Gamma(-\alpha) (2a)^{\alpha\varepsilon} \rho^{\alpha} e^{\mp \varepsilon p} \times \\
\times {}_2F_2\left(-n, \rho + \sigma + n + 1; \alpha + 1, \varphi + 1; \pm \frac{\rho}{(2a)^{1-\varepsilon}}\right) \\
[\varepsilon = 0 \text{ или } 1; a, \operatorname{Re} p > 0; \operatorname{Re} \alpha < -n; \varphi = \left\{\frac{\rho}{\sigma}\right\}].
\end{aligned}$$

$$\begin{aligned}
8. \int_a^\infty (x-a)^\rho (x+a)^{-\tau} \exp \left(-p \frac{(a-x)^\varepsilon}{a+x} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \frac{(\tau+\sigma)_n}{n!} B(\rho+n+1, \tau-\rho-n-1) (2a)^{\rho-\tau+1} e^{\varepsilon p} \times \\
\times {}_2F_2 \left(\tau-\rho-n-1, \sigma+\tau+n; \tau, \sigma+\tau; -\frac{p}{(2a)^{1-\varepsilon}} \right) \\
[\varepsilon=0 \text{ или } 1; a>0; \operatorname{Re} \rho > -1; \operatorname{Re} (\tau-\rho) > n+1].
\end{aligned}$$

$$\begin{aligned}
9. \int_a^\infty (x-a)^{\alpha-1} (x+a)^\sigma \exp \left(-p \frac{(x+a)^\varepsilon}{x-a} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \frac{(\rho-\alpha+1)_n}{n!} B(\alpha, -\alpha-\sigma-n) (2a)^{\alpha+\sigma} e^{-\varepsilon p} \times \\
\times {}_2F_2 \left(-\alpha-\sigma-n, \rho-\alpha+n+1; 1-\alpha, \rho-\alpha+1; \frac{p}{(2a)^{1-\varepsilon}} \right) + \\
+ \frac{(\rho+1)_n}{n!} \Gamma(-\alpha) (2a)^{\sigma+\alpha\varepsilon} p^\alpha e^{-\varepsilon p} {}_2F_2 \left(-\sigma-n, \rho+n+1; \rho+1, \alpha+1; \frac{p}{(2a)^{1-\varepsilon}} \right) \\
[\varepsilon=0 \text{ или } 1; a, \operatorname{Re} \rho > 0; \operatorname{Re} (\alpha+\sigma) < -n].
\end{aligned}$$

$$\begin{aligned}
10. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^\rho e^{-p\sqrt{x+a}} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^n}{n!} (\sigma-\alpha+1)_n B(\alpha, \rho+n+1) (2a)^{\alpha+\rho} \times \\
\times {}_2F_3 \left(\alpha, \alpha-\sigma; \alpha+\rho+n+1, \alpha-\sigma-n, \frac{1}{2}; \frac{ap^2}{2} \right) - \\
- \frac{(-1)^n}{n!} \left(\sigma-\alpha+\frac{1}{2} \right)_n B \left(\alpha+\frac{1}{2}, \rho+n+1 \right) (2a)^{\alpha+\rho+1/2} p \times \\
\times {}_2F_3 \left(\alpha+\frac{1}{2}, \alpha-\sigma+\frac{1}{2}; \frac{3}{2}, \alpha+\rho+n+\frac{3}{2}, \alpha-\sigma-n+\frac{1}{2}; \frac{ap^2}{2} \right) \\
[a, \operatorname{Re} \alpha > 0; \operatorname{Re} \rho > -1].
\end{aligned}$$

$$\begin{aligned}
11. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^\rho \exp \left(-\frac{p}{\sqrt{x+a}} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^n}{n!} (\sigma-\alpha+1)_n B(\alpha, \rho+n+1) (2a)^{\alpha+\rho} \times \\
\times {}_2F_3 \left(-\alpha-\rho-n, \sigma-\alpha+n+1; \frac{1}{2}, 1-\alpha, \sigma-\alpha+1; \frac{p^2}{8a} \right) + \\
+ \frac{(-1)^n (\sigma+1)_n}{n!} 2^{\rho+1} \Gamma(-2\alpha) a^\rho p^{2\alpha} \times \\
\times {}_2F_3 \left(\sigma+n+1, -\rho-n; \alpha+\frac{1}{2}, \alpha+1, \sigma+1; \frac{p^2}{8a} \right) - \\
- \frac{(-1)^n}{n!} \left(\sigma-\alpha+\frac{3}{2} \right)_n B \left(\alpha-\frac{1}{2}, \rho+n+1 \right) (2a)^{\alpha+\rho-1/2} p \times \\
\times {}_2F_3 \left(\frac{1}{2}-\alpha-\rho-n, \sigma-\alpha+n+\frac{3}{2}; \frac{3}{2}, \frac{3}{2}-\alpha, \sigma-\alpha+\frac{3}{2}; \frac{p^2}{8a} \right) \\
[a, \operatorname{Re} \rho > 0; \operatorname{Re} \rho > -1].
\end{aligned}$$

$$\begin{aligned}
12. \int_{-a}^a (x+a)^{-\beta-n-1} (a-x)^{\beta-1} \exp\left(-p \sqrt{\frac{a-x}{a+x}}\right) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
= \frac{(-1)^n (\sigma+1)_n}{n! 2^n a^{n+1}} \Gamma(2\beta+2n) p^{-2\beta-2n} \times \\
\times {}_2F_3\left(-n, -n-\rho; \sigma+1, 1-\beta-n, \frac{1}{2}-\beta-n; -\frac{p^2}{4}\right) \\
[a, \operatorname{Re} p, \operatorname{Re} \beta > 0].
\end{aligned}$$

$$\begin{aligned}
13. \int_{-a}^a (x+a)^{\rho+\sigma-\beta+n} (a-x)^{\beta-1} \exp\left(-p \sqrt{\frac{a-x}{a+x}}\right) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
= \frac{(-1)^n}{n!} (\sigma+1)_n 2^{\rho+\sigma+n+1} \Gamma(2\beta-2\rho-2\sigma-2n-2) a^{\rho+\sigma+n} p^{2(\rho+\sigma-\beta+n+1)} \times \\
\times {}_2F_3\left(\rho+\sigma+n+1, \sigma+n+1; \sigma+1, \rho+\sigma-\beta+n+2, \rho+\sigma-\beta+n+\frac{3}{2}; -\frac{p^2}{4}\right) + \\
+ \frac{(\rho-\beta+1)_n}{n!} B(\beta, \rho+\sigma-\beta+n+1) (2a)^{\rho+\sigma+n} \times \\
\times {}_2F_3\left(\beta, \beta-\rho; \beta-\rho-\sigma-n, \beta-\rho-n, \frac{1}{2}; -\frac{p^2}{4}\right) - \\
- \frac{(\rho-\beta+1/2)_n}{n!} B\left(\beta+\frac{1}{2}, \rho+\sigma-\beta+n+\frac{1}{2}\right) (2a)^{\rho+\sigma+n} p \times \\
\times {}_2F_3\left(\beta+\frac{1}{2}, \beta-\rho+\frac{1}{2}; \frac{3}{2}, \beta-\rho-n+\frac{1}{2}, \beta-\rho-\sigma-n+\frac{1}{2}; -\frac{p^2}{4}\right) \\
[a, \operatorname{Re} p, \operatorname{Re} \beta > 0].
\end{aligned}$$

$$\begin{aligned}
14. \int_{-a}^a (x+a)^{-\beta-n-1} (a-x)^{\beta-1} \exp\left(-p \sqrt{\frac{a+x}{a-x}}\right) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
= \frac{(\rho+1)_n}{n! 2^n} \Gamma(-2\beta) a^{-n-1} p^{2\beta} {}_2F_3\left(-n, -n-\sigma; \beta+1, \beta+\frac{1}{2}, \rho+1; -\frac{p^2}{4}\right) \\
[a, \operatorname{Re} p > 0; \operatorname{Re} \beta < -n].
\end{aligned}$$

$$\begin{aligned}
15. \int_{-a}^a (x+a)^{\rho+\sigma-\beta+n} (a-x)^{\beta-1} \exp\left(-p \sqrt{\frac{a+x}{a-x}}\right) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
= \frac{(\rho-\beta+1)_n}{n!} B(\beta, \rho+\sigma-\beta+n+1) (2a)^{\rho+\sigma+n} \times \\
\times {}_2F_3\left(\rho-\beta+n+1, \rho+\sigma-\beta+n+1; \frac{1}{2}, 1-\beta, \rho-\beta+1; -\frac{p^2}{4}\right) + \\
+ \frac{(\rho+1)_n}{n!} 2^{\rho+\sigma+n+1} \Gamma(-2\beta) a^{\rho+\sigma+n} p^{2\beta} \times \\
\times {}_2F_3\left(\rho+n+1, \rho+\sigma+n+1; \beta+1, \beta+\frac{1}{2}, \rho+1; -\frac{p^2}{4}\right) - \\
- \frac{(\rho-\beta+3/2)_n}{n!} B\left(\beta-\frac{1}{2}, \rho+\sigma-\beta+n+\frac{3}{2}\right) (2a)^{\rho+\sigma+n} p \times \\
\times {}_2F_3\left(\rho-\beta+n+\frac{3}{2}, \rho+\sigma-\beta+n+\frac{3}{2}; \frac{3}{2}, \frac{3}{2}-\beta, \rho-\beta+\frac{3}{2}; -\frac{p^2}{4}\right) \\
[a, \operatorname{Re} p > 0; \operatorname{Re}(\beta-\rho-\sigma) < n+1].
\end{aligned}$$

$$16. \int_{-a}^{\infty} (x \mp a)^{\alpha-1} e^{-p\sqrt{x \mp a}} p_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \frac{(\rho + \sigma + n + 1)_n}{2^{n-1} n! a^n} \Gamma(2\alpha + 2n) p^{-2\alpha-2n} \times \\ \times {}_2F_3 \left(-n, -n-\varphi; -\rho-\sigma-2n, 1-\alpha-n, \frac{1}{2}-\alpha-n; \mp \frac{ap^2}{2} \right) \\ \left[a, \operatorname{Re} p, \operatorname{Re} \alpha > 0, \varphi = \left\{ \frac{\rho}{\sigma} \right\} \right].$$

$$17. \int_{-a}^{\infty} (x \mp a)^{\alpha-1} \exp \left(-\frac{\rho}{\sqrt{x \mp a}} \right) p_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = 2 \frac{(\pm 1)^n}{n!} (\varphi + 1)_n \Gamma(-2\alpha) \times \\ \times p^{2\alpha} {}_2F_3 \left(-n, \rho + \sigma + n + 1; \alpha + 1, \alpha + \frac{1}{2}, \varphi + 1; \mp \frac{\rho^2}{8a} \right) \\ \left[a, \operatorname{Re} p > 0; \operatorname{Re} \alpha < -n, \varphi = \left\{ \frac{\rho}{\sigma} \right\} \right].$$

$$18. \int_a^{\infty} (x-a)^{\rho} (x+a)^{-\tau} e^{-p\sqrt{x+a}} p_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\ = \frac{(\tau + \sigma)_n}{n!} B(\rho + n + 1, \tau - \rho - n - 1) (2a)^{\rho-\tau+1} \times \\ \times {}_2F_3 \left(1-\tau, 1-\sigma-\tau; 1-\sigma-\tau-n, \rho-\tau+n+2, \frac{1}{2}; \frac{ap^2}{2} \right) + \\ + \frac{(\rho + \sigma + n + 1)_n}{2^{n-1} n! a^n} \Gamma(2\rho - 2\tau + 2n + 2) p^{2\tau-2\rho-2n-2} \times \\ \times {}_2F_3 \left(-\rho-\sigma-n, -\rho-n; -\rho-\sigma-2n, \tau-\rho-n, \tau-\rho-n-\frac{1}{2}; \frac{ap^2}{2} \right) - \\ - \frac{(\sigma + \tau - 1/2)_n}{n!} B \left(\rho + n + 1, \tau - \rho - n - \frac{3}{2} \right) (2a)^{\rho-\tau+3/2} p \times \\ \times {}_2F_3 \left(\frac{3}{2}-\tau, \frac{3}{2}-\sigma-\tau; \frac{3}{2}, \rho-\tau+n+\frac{5}{2}, \frac{3}{2}-\sigma-\tau-n; \frac{ap^2}{2} \right) \\ [a, \operatorname{Re} p > 0; \operatorname{Re} \rho > -1].$$

$$19. \int_a^{\infty} (x-a)^{\alpha-1} (x+a)^{\sigma} e^{-p\sqrt{x+a}} p_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\ = \frac{(\rho + \sigma + n + 1)_n}{2^{n-1} n! a^n} \Gamma(2\alpha + 2\sigma + 2n) p^{-2\alpha-2\sigma-2n} \times \\ \times {}_2F_3 \left(-\sigma-n, -\rho-\sigma-n; -\rho-\sigma-2n, 1-\alpha-\sigma-n, \frac{1}{2}-\alpha-\sigma-n; -\frac{ap^2}{2} \right) + \\ + \frac{(\rho - \alpha + 1)_n}{n!} B(\alpha, -\alpha - \sigma - n) (2a)^{\alpha+\sigma} \times \\ \times {}_2F_3 \left(\alpha, \alpha - \rho; \alpha - \rho - n, \alpha + \sigma + n + 1, \frac{1}{2}; -\frac{ap^2}{2} \right) - \\ - \frac{(\rho - \alpha + 1/2)_n}{n!} B \left(\alpha + \frac{1}{2}, -\alpha - \sigma - n - \frac{1}{2} \right) (2a)^{\alpha+\sigma+1/2} p \times \\ \times {}_2F_3 \left(\alpha + \frac{1}{2}, \alpha - \rho + \frac{1}{2}; \alpha - \rho - n + \frac{1}{2}, \alpha + \sigma + n + \frac{3}{2}, \frac{3}{2}; -\frac{ap^2}{2} \right) \\ [a, \operatorname{Re} p, \operatorname{Re} \alpha > 0].$$

$$\begin{aligned}
20. \int_a^\infty (x-a)^{\alpha-1} (x+a)^\sigma \exp\left(-\frac{p}{\sqrt{x-a}}\right) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
= \frac{(\rho-\alpha+1)_n}{n!} B(\alpha, -\alpha-\sigma-n) (2a)^{\alpha+\sigma} {}_2F_3\left(-\alpha-\sigma-n, \rho-\alpha+n+1; \right. \\
\left. \frac{1}{2}, 1-\alpha, \rho-\alpha+1; -\frac{p^2}{8a}\right) + 2^{\sigma+1} \frac{(\rho+1)_n}{n!} \Gamma(-2\alpha) a^\sigma p^{2\alpha} \times \\
\times {}_2F_3\left(-\sigma-n, \rho+n+1; \alpha+1, \alpha+\frac{1}{2}, \rho+1; -\frac{p^2}{8a}\right) \\
[a, \operatorname{Re} p > 0; \operatorname{Re}(\alpha+\sigma) < -n].
\end{aligned}$$

$$\begin{aligned}
21. \int_a^\infty (x-a)^\rho (x+a)^{-\tau} \exp\left(-\frac{p}{\sqrt{x+a}}\right) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
= \frac{(\sigma+\tau)_n}{n!} B(\rho+n+1, \tau-\rho-n-1) (2a)^{\rho-\tau+1} \times \\
\times {}_2F_3\left(\tau-\rho-n-1, \sigma+\tau+n; \frac{1}{2}, \tau, \sigma+\tau; \frac{p^2}{8a}\right) - \\
- \frac{(\sigma+\tau+1/2)_n}{n!} B\left(\rho+n+1, \tau-\rho-n-\frac{1}{2}\right) (2a)^{\rho-\tau+1/2} p \times \\
\times {}_2F_3\left(\tau-\rho-n-\frac{1}{2}, \sigma+\tau+n+\frac{1}{2}; \frac{3}{2}, \tau+\frac{1}{2}, \sigma+\tau+\frac{1}{2}; \frac{p^2}{8a}\right) \\
[a > 0; \operatorname{Re} \rho > -1; \operatorname{Re}(\tau-\rho) > n+1].
\end{aligned}$$

2.22.7. Интегралы от $A(x) \begin{Bmatrix} \sin \varphi(x) \\ \cos \varphi(x) \end{Bmatrix} P_n^{(\rho, \sigma)}(cx)$.

Обозначение: $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$.

$$\begin{aligned}
1. \int_0^a (a^2-x^2)^\sigma \begin{Bmatrix} \sin bx \\ \cos bx \end{Bmatrix} P_n^{(\sigma, \sigma)}\left(\frac{x}{a}\right) dx = \\
= \frac{(-1)^m \sqrt{\pi} 2^{\sigma-1/2}}{n!} \left(\frac{a}{b}\right)^{\sigma+1/2} \Gamma(\sigma+n+1) J_{\sigma+n+1/2}(ab) \\
[a > 0; \operatorname{Re} \sigma > -1; n = \begin{Bmatrix} 2m+1 \\ 2m \end{Bmatrix}].
\end{aligned}$$

$$\begin{aligned}
2. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^\rho \begin{Bmatrix} \sin b \sqrt{x+a} \\ \cos b \sqrt{x+a} \end{Bmatrix} P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
= \frac{(-1)^n}{n!} (1-\alpha-\delta/2+\sigma)_n B\left(\alpha+\frac{\delta}{2}, \rho+n+1\right) (2a)^{\alpha+\delta/2+\rho} b^\delta \times \\
\times {}_2F_3\left(\alpha+\frac{\delta}{2}, \alpha+\frac{\delta}{2}-\sigma; \alpha+\frac{\delta}{2}+\rho+n+1, \alpha+\frac{\delta}{2}-\sigma-n, \delta+\frac{1}{2}; -\frac{ab^2}{2}\right) \\
[a > 0; \operatorname{Re} \alpha > -\delta/2; \operatorname{Re} \rho > -1].
\end{aligned}$$

$$\begin{aligned}
3. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^\rho \begin{Bmatrix} \sin(b/\sqrt{x+a}) \\ \cos(b/\sqrt{x+a}) \end{Bmatrix} P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
= \frac{(-1)^n}{n!} \left(\frac{\delta}{2}-\alpha+\sigma+1\right)_n B\left(\rho+n+1, \alpha-\frac{\delta}{2}\right) (2a)^{\alpha+\rho-\delta/2} b^\delta \times
\end{aligned}$$

$$\times {}_2F_3\left(\frac{\delta}{2}-\alpha-\rho-n, \frac{\delta}{2}-\alpha+\sigma+n+1; \frac{\delta}{2}-\alpha+1, \frac{\delta}{2}-\alpha+\sigma+1, \delta+\frac{1}{2}; -\frac{b^2}{8a}\right) + \frac{(-1)^{n+\delta} 2^{\rho+1}}{n!} (\sigma+1)_n a^\rho b^{2\alpha} \left\{ \frac{\sin \alpha \pi}{\cos \alpha \pi} \right\} \times$$

$$\times {}_2F_3\left(\sigma+n+1, \rho-n; \alpha+1, \alpha+\frac{1}{2}, \sigma+1; -\frac{b^2}{8a}\right)$$

[a, b > 0; Re α > -1/2; Re ρ > -1].

4. $\int_{-a}^a (x+a)^{\rho+\sigma-\beta+n} (a-x)^{\beta-1} \left\{ \frac{\sin [b(a-x)^{1/2}(a+x)^{-1/2}]}{\cos [b(a-x)^{1/2}(a+x)^{-1/2}]} \right\} P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx =$

$$= -\frac{(\sigma+1)_n}{n!} 2^{\rho+\sigma+n+1} \Gamma(2\beta-2\rho-2\sigma-2n-2) \times$$

$$\times \left\{ \frac{\sin(\beta-\rho-\sigma)\pi}{\cos(\beta-\rho-\sigma)\pi} \right\} a^{\rho+\sigma+n} b^{2(\rho+\sigma-\beta+n+1)} \times$$

$$\times {}_2F_3\left(\rho+\sigma+n+1, \sigma+\frac{1}{2}+1; \sigma+1, \rho+\sigma-\beta+n+2, \rho+\sigma-\beta+n+\frac{3}{2}; \frac{b^2}{4}\right) +$$

$$+ \frac{(\rho-\beta-\delta/2+1)_n}{n!} B\left(\beta+\frac{\delta}{2}, \rho+\sigma-\beta-\frac{\delta}{2}+n+1\right) (2a)^{\rho+\sigma+n} b^\delta \times$$

$$\times {}_2F_3\left(\beta+\frac{\delta}{2}, \beta+\frac{\delta}{2}-\rho; \beta+\frac{\delta}{2}-\rho-n, \beta+\frac{\delta}{2}-\rho-\sigma-n, \delta+\frac{1}{2}; \frac{b^2}{4}\right)$$

[a, b > 0; Re β > -δ/2; Re (β - ρ - σ) < 3/2 + n].

5. $\int_a^\infty (x-a)^\rho (x+a)^{-\tau} \left\{ \frac{\sin(b\sqrt{x+a})}{\cos(b\sqrt{x+a})} \right\} P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx =$

$$= \frac{(\tau+\sigma-\delta/2)_n}{n!} B\left(\rho+n+1, \tau-\rho-\frac{\delta}{2}-n-1\right) (2a)^{\rho+\delta/2-\tau+1} b^\delta \times$$

$$\times {}_2F_3\left(\frac{\delta}{2}-\tau+1, \frac{\delta}{2}-\sigma-\tau+1; \rho+\frac{\delta}{2}-\tau+n+2, \frac{\delta}{2}-\sigma-\tau-n+1, \delta+\frac{1}{2}; -\frac{ab^2}{2}\right) + \frac{(-1)^{n-1}}{2^{n-1} n! a^n} (\rho+\sigma+n+1)_n \times$$

$$\times \left\{ \frac{\sin(\rho-\tau)\pi}{\cos(\rho-\tau)\pi} \right\} b^{2(\tau-\rho-n-1)} \Gamma(2\rho-2\tau+2n+2) \times$$

$$\times {}_2F_3\left(-\rho-n, -\rho-\sigma-n; -\rho-\sigma-2n, \tau-\rho-n, \tau-\rho-n-\frac{1}{2}; -\frac{ab^2}{2}\right)$$

[b > 0; Re ρ > -1; Re (ρ - τ) < -n - 1/2].

6. $\int_a^\infty (x-a)^{\alpha-1} (x+a)^\sigma \left\{ \frac{\sin(b\sqrt{x-a})}{\cos(b\sqrt{x-a})} \right\} P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx =$

$$= \frac{(-1)^n}{2^{n-1} n! a^n} (\rho+\sigma+n+1)_n \Gamma(2\alpha+2\sigma+2n) b^{-2\alpha-2\sigma-2n} \left\{ \frac{\sin(\alpha+\sigma)\pi}{\cos(\alpha+\sigma)\pi} \right\} \times$$

$$\times {}_2F_3\left(-\sigma-n, -\rho-\sigma-n; -\rho-\sigma-2n, 1-\alpha-\sigma-n, \frac{1}{2}-\alpha-\sigma-n; \frac{ab^2}{2}\right) +$$

$$+ \frac{(\rho-\alpha-\delta/2+1)_n}{n!} B\left(-\alpha-\sigma-\frac{\delta}{2}-n, \alpha+\frac{\delta}{2}\right) 2^{\alpha+\sigma} a^{\alpha+\delta/2+\sigma} b^\delta \times$$

$$\times {}_2F_3\left(\alpha+\frac{\delta}{2}, \alpha+\frac{\delta}{2}-\rho; \alpha+\frac{\delta}{2}-\rho-n, \alpha+\sigma+\frac{\delta}{2}+n+1, \delta+\frac{1}{2}; \frac{ab^2}{2}\right)$$

[b > 0; Re α > -δ/2; Re (α + σ) < 1/2 - n].

$$\begin{aligned}
7. \int_a^\infty (x-a)^\rho (x+a)^{-\tau} \left\{ \frac{\sin(b/\sqrt{x+a})}{\cos(b/\sqrt{x+a})} \right\} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \frac{(\sigma + \tau + \delta/2)_n}{n!} B \left(\rho + n + 1, \tau + \frac{\delta}{2} - \rho - n - 1 \right) (2a)^{\rho - \delta/2 - \tau + 1} \times \\
\times b^\delta {}_2F_3 \left(\tau + \frac{\delta}{2} - \rho - n - 1, \sigma + \tau + \frac{\delta}{2} + n; \tau + \frac{\delta}{2}, \sigma + \tau + \frac{\delta}{2}, \delta + \frac{1}{2}; -\frac{b^2}{8a} \right) \\
[\operatorname{Re} \rho > -1; \operatorname{Re} (\rho - \tau) < \delta/2 - n - 1]
\end{aligned}$$

$$\begin{aligned}
8. \int_a^\infty (x-a)^{\alpha-1} (x+a)^\sigma \left\{ \frac{\sin(b/\sqrt{x-a})}{\cos(b/\sqrt{x-a})} \right\} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \frac{(\rho + \delta/2 - \alpha + 1)_n}{n!} B \left(\alpha - \frac{\delta}{2}, \frac{\delta}{2} - \alpha - \sigma - n \right) (2a)^{\alpha + \sigma - \delta/2} b^\delta \times \\
\times {}_2F_3 \left(\frac{\delta}{2} - \alpha + \rho + n + 1, \frac{\delta}{2} - \alpha - \sigma - n; \rho + \frac{\delta}{2} - \alpha + 1, \frac{\delta}{2} - \alpha + 1, \delta + \frac{1}{2}; \frac{b^2}{8a} \right) \mp \\
\mp \frac{(\rho + 1)_n}{n!} \Gamma(-2\alpha) \left\{ \frac{\sin \alpha \pi}{\cos \alpha \pi} \right\} 2^{\sigma+1} a^\sigma b^{2\alpha} \times \\
\times {}_2F_3 \left(-\sigma - n, \rho + n + 1; \alpha + 1, \alpha + \frac{1}{2}, \rho + 1; \frac{b^2}{8a} \right) \\
[b > 0; \operatorname{Re} \alpha > -1/2; \operatorname{Re} (\alpha + \sigma) < \delta/2 - n]
\end{aligned}$$

2.22.8. Интегралы от $A(x) \ln \varphi(x) P_n^{(\rho, \sigma)}(cx)$.

$$\begin{aligned}
1. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^\rho \ln \frac{x+a}{2a} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^n}{n!} (\sigma - \alpha + 1)_n B(\alpha, \rho + n + 1) (2a)^{\alpha + \rho} \times \\
\times [\psi(\alpha - \sigma) + \psi(\alpha) - \psi(\alpha - \sigma - n) - \psi(\alpha + \rho + n + 1)] \\
[a, \operatorname{Re} \alpha > 0; \operatorname{Re} \rho > -1]
\end{aligned}$$

$$\begin{aligned}
2. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^\rho \left\{ \frac{\ln(x+z)}{\ln|x-z|} \right\} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \pm \frac{(-1)^n (\sigma - \alpha)_n}{n! (z-a)} B(\rho + n + 1, \alpha - 1) (2a)^{\alpha + \rho + 1} \times \\
\times {}_4F_3 \left(1, 1, \alpha - \sigma + 1, 1 - \alpha - \rho - n; 2, \alpha - \sigma - n + 1, 2 - \alpha; \mp \frac{2a}{z-a} \right) + \\
+ \frac{(-1)^n}{n!} (2a)^{\alpha + \rho} (1 - \alpha + \sigma)_n B(\alpha, \rho + n + 1) \ln z \\
\left[a, \operatorname{Re} \alpha > 0; \operatorname{Re} \rho > -1, \left\{ \left| \arg(z-a) \right| < \pi \right\} \right].
\end{aligned}$$

$$\begin{aligned}
3. \int_a^\infty (x-a)^\rho (x+a)^{-\tau} \left\{ \frac{\ln(x+z)}{\ln|x-z|} \right\} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \pm \frac{(\tau + \sigma - 1)_n}{n! (z-a)} B(\rho + n + 1, \tau - \rho - n - 2) (2a)^{\rho - \tau + 2} \times \\
\times {}_4F_3 \left(1, 1, 2 - \tau - \sigma, 2 - \tau; 2, 2 - \tau - \sigma - n, \rho - \tau + n + 3; \mp \frac{2a}{z-a} \right) \mp \\
\mp \frac{(\mp 1)^n \pi (\rho + \sigma + n + 1)_n (z-a)^{\rho - \tau + n + 1}}{n! (2a)^n (\tau - \rho - n - 1)} \left\{ \frac{\operatorname{cosec}(\tau - \rho) \pi}{\operatorname{ctg}(\tau - \rho) \pi} \right\} \times
\end{aligned}$$

$$\times {}_2F_2\left(\tau-\rho-n-1, -\rho-n, -\rho-\sigma-n; \tau-\rho-n, -\rho-\sigma-2n; \mp \frac{2a}{z-a}\right) +$$

$$+ \frac{(2a)^{\rho-\tau+1}}{n!} (\sigma+\tau)_n B(\rho+n+1, \tau-\rho-n-1) \ln z$$

$$\left[a > 0; \operatorname{Re} \rho > -1; \operatorname{Re} (\rho-\tau) < -n-1, \left\{ \begin{array}{l} |\arg(z-a)| < \pi \\ z > -a \end{array} \right\}\right].$$

2.22.9. Интегралы от $A(x) \operatorname{Ei}(a+bx) P_n^{(\rho, \sigma)}(cx)$.

$$1. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\rho} \operatorname{Ei}(-ab-bx) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^{n+1}}{n!} (\sigma-\alpha)_n B(\rho+n+1, \alpha+1) (2a)^{\alpha+\rho+1} b \times$$

$$\times {}_4F_4(1, 1, \alpha-\sigma+1, \alpha+1; 2, 2, \alpha-\sigma-n+1, \alpha+\rho+n+2; -2ab) -$$

$$- \frac{(-1)^n}{n!} (\sigma-\alpha+1)_n B(\alpha, \rho+n+1) (2a)^{\alpha+\rho} \times$$

$$\times \left[C + \psi(\alpha) - \psi(\alpha+\rho+n+1) - \sum_{j=0}^{n-1} \frac{1}{\sigma-\alpha+j+1} + \ln(2ab) \right]$$

$$[a, \operatorname{Re} \alpha > 0; \operatorname{Re} \rho > -1].$$

$$2. \int_a^{\infty} (x-a)^{\rho} (x+a)^{-\tau} \operatorname{Ei}(-ab-bx) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx =$$

$$= - \frac{(\sigma+\tau-1)_n}{n!} B(\rho+n+1, \tau-\rho-n-2) (2a)^{\rho-\tau+2} b \times$$

$$\times {}_4F_4(1, 1, 2-\sigma-\tau, 2-\tau; 2, 2, 2-\sigma-\tau-n, \rho-\tau+n+3; -2ab) +$$

$$+ \frac{(\rho+\sigma+n+1)_n b^{\tau-\rho-n-1}}{n! (\tau-\rho-n-1) (2a)^n} \Gamma(\rho-\tau+n+1) \times$$

$$\times {}_2F_3(\tau-\rho-n-1, -\rho-n, -\rho-\sigma-n; \tau-\rho-n, \tau-\rho-n, -\rho-\sigma-2n; -2ab) +$$

$$+ \frac{(\sigma+\tau)_n}{n!} B(\rho+n+1, \tau-\rho-n-1) (2a)^{\rho-\tau+1} \times$$

$$\times \left[C + \psi(\tau) - \psi(\tau-\rho-n-1) - \sum_{j=0}^{n-1} \frac{1}{\sigma+\tau+j} + \ln(2ab) \right]$$

$$[a, \operatorname{Re} b > 0; \operatorname{Re} \rho > -1].$$

2.22.10. Интегралы от $A(x) \left\{ \begin{array}{l} \operatorname{si}(b \sqrt{x+a}) \\ \operatorname{ci}(b \sqrt{x+a}) \end{array} \right\} P_n^{(\rho, \sigma)}(cx)$.

Обозначение: $\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$.

$$1. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\rho} \left\{ \begin{array}{l} \operatorname{si}(b \sqrt{x+a}) \\ \operatorname{ci}(b \sqrt{x+a}) \end{array} \right\} P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^{n+1} (\sigma-\alpha-\delta/2)_n}{3^{2\delta} n!} 2^{\alpha+\rho+3\delta/2-1} B\left(\rho+n+1, \alpha+\frac{\delta}{2}+1\right) \times$$

$$\times a^{\alpha+\rho+\delta/2+1} b^{\delta+2} {}_4F_5\left(1, \alpha+\frac{\delta}{2}-\sigma+1, \frac{\delta}{2}+1, \alpha+\frac{\delta}{2}+1;$$

$$2, \alpha+\rho+\frac{\delta}{2}+n+2, \delta+\frac{3}{2}, \frac{\delta}{2}+2, \alpha+\frac{\delta}{2}-\sigma-n+1; -\frac{ab^2}{2}\right) +$$

$$+ \frac{(-1)^n (\sigma - \alpha - \delta/2 + 1)_n}{n!} B \left(\rho + n + 1, \alpha + \frac{\delta}{2} \right) (2a)^{\alpha + \rho + \delta/2} b^\delta \left\{ \frac{1}{R} \right\} -$$

$$- \delta \frac{(-1)^n}{2(n!)} (2a)^{\alpha + \rho} \pi (\sigma - \alpha + 1)_n B(\rho + n + 1, \alpha),$$

$$R = C + \frac{1}{2} \psi(\alpha) - \frac{1}{2} \psi(\alpha + \rho + n + 1) - \frac{1}{2} \sum_{j=0}^{n-1} \frac{1}{\sigma - \alpha + j + 1} + \frac{1}{2} \ln(2ab^2)$$

[a, Re $\alpha > 0$; Re $\rho > -1$].

$$2. \int_a^\infty (x-a)^\rho (x+a)^{-\tau} \left\{ \begin{matrix} \text{si} (b \sqrt{x+a}) \\ \text{ci} (b \sqrt{x+a}) \end{matrix} \right\} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx =$$

$$= - \frac{(\tau + \sigma - \delta/2 - 1)_n}{3^{2\delta} n!} 2^{\rho + 3\delta/2 - \tau} a^{\rho + \delta/2 - \tau + 2} b^{\delta + 2} \times$$

$$\times B \left(\rho + n + 1, \tau - \rho - \frac{\delta}{2} - n - 2 \right) {}_4F_5 \left(1, \frac{\delta}{2} - \tau - \sigma + 2, \frac{\delta}{2} + 1, 2 - \tau + \frac{\delta}{2}; \right.$$

$$2, \delta + \frac{3}{2}, \frac{\delta}{2} + 2, \frac{\delta}{2} + 2 - \tau - \sigma - n, \rho - \tau + \frac{\delta}{2} + n + 3; -\frac{ab^2}{2} \Big) -$$

$$- (-1)^n \frac{(\rho + \sigma + n + 1)_n b^{2(\tau - \rho - n - 1)}}{n! (\tau - \rho - n - 1) (2a)^n} \Gamma(2\rho - 2\tau + 2n + 2) \times$$

$$\times \left\{ \begin{matrix} \sin(\rho - \tau) \pi \\ \cos(\rho - \tau) \pi \end{matrix} \right\} {}_3F_4 \left(-\rho - n, \tau - \rho - n - 1, -\rho - \sigma - n; \right.$$

$$-\rho - \sigma - 2n, \tau - \rho - n, \tau - \rho - n, \tau - \rho - n - \frac{1}{2}; -\frac{ab^2}{2} \Big) +$$

$$+ \frac{(\sigma + \tau - \delta/2)_n}{n!} B \left(\rho + n + 1, \tau - \rho - n - \frac{\delta}{2} - 1 \right) (2a)^{\alpha + \delta/2 - \tau + 1} b^\delta \left\{ \frac{1}{R} \right\} -$$

$$- \delta \frac{(\sigma + \tau)_n}{n!} 2^{\rho - \tau} \pi B(\rho + n + 1, \tau - \rho - n - 1) a^{\rho - \tau + 1},$$

$$R = C + \frac{1}{2} \psi(\tau) - \frac{1}{2} \psi(\tau - \rho - n - 1) - \frac{1}{2} \sum_{j=0}^{n-1} \frac{1}{\sigma + \tau + j} + \frac{1}{2} \ln(2ab^2)$$

[a, b > 0; Re $\rho > -1$; Re $(\rho - \tau) < -n$].

2.22.11. Интегралы, содержащие $\left\{ \begin{matrix} \text{erf}(\varphi(x)) \\ \text{erfc}(\varphi(x)) \end{matrix} \right\} P_n^{(\rho, \sigma)}(\chi(x))$.

$$1. \int_0^a (a^2 - x^2)^\sigma \text{erf}(bx) P_{2n+1}^{(\sigma, \sigma)} \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^n}{2 \sqrt{\pi} (2n+1)!} \Gamma \left[\begin{matrix} n+1/2, \sigma+2n+2 \\ \sigma+2n+5/2 \end{matrix} \right] \frac{a^{\sigma-1/2}}{b^{\sigma+3/2}} M_{-(2\sigma+3)/4, n+(2\sigma+3)/4}(a^2 b^2)$$

[a > 0; Re $\sigma > -1$].

$$2. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^\rho \left\{ \begin{matrix} \text{erf}(ab+bx) \\ \text{erfc}(ab+bx) \end{matrix} \right\} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx =$$

$$= \pm \frac{(-1)^n (\sigma - \alpha)_n}{\sqrt{\pi} n!} 2^{\alpha + \rho + 2} a^{\alpha + \rho + 1} b B \left(\alpha + 1, \rho + n + 1 \right) {}_5F_5 \left(\frac{1}{2}, \frac{\alpha+1}{2}, \frac{\alpha}{2} + 1, \right.$$

$$\frac{\alpha - \sigma + 1}{2}, 1 + \frac{\alpha - \sigma}{2}; \frac{3}{2}, \frac{\alpha - \sigma - n + 1}{2}, \frac{\alpha - \sigma - n}{2} + 1, \frac{\alpha + \rho + n}{2} + 1,$$

$$\frac{\alpha + \rho + n + 3}{2}; -4a^2 b^2 \Big) + \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{(-1)^n}{n!} (2a)^{\alpha + \rho} B(\alpha, \rho + n + 1) (\sigma - \alpha + 1)_n$$

[a > 0; Re $\rho > -1$; Re $\alpha > -(1 \pm 1)/2$].

$$\begin{aligned}
3. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\rho} \left\{ \begin{matrix} \operatorname{erf} (b \sqrt{x+a}) \\ \operatorname{erfc} (b \sqrt{x+a}) \end{matrix} \right\} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \pm \frac{(-1)^n}{n! \sqrt{\pi}} 2^{\alpha+\rho+3/2} \left(\sigma - \alpha + \frac{1}{2} \right)_n a^{\alpha+\rho+1/2} b B \left(\alpha + \frac{1}{2}, \rho + n + 1 \right) \times \\
\times {}_3F_3 \left(\frac{1}{2}, \alpha + \frac{1}{2}, \alpha - \sigma + \frac{1}{2}; \frac{3}{2}, \alpha - \sigma - n + \frac{1}{2}, \alpha + \rho + n + \frac{3}{2}; -2ab^2 \right) + \\
+ \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{(-1)^n}{n!} (2a)^{\alpha+\rho} (\sigma - \alpha + 1)_n B(\alpha, \rho + n + 1) \\
[a > 0; \operatorname{Re} \rho > -1; \operatorname{Re} \alpha > -(1 \pm 1)/4].
\end{aligned}$$

$$\begin{aligned}
4. \int_{\pm a}^{\infty} (x \mp a)^{\alpha-1} \operatorname{erfc} (b \sqrt{x \mp a}) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \frac{(\pm 1)^n (\varphi + 1)_n}{n! \alpha \sqrt{\pi} b^{2\alpha}} \Gamma \left(\alpha + \frac{1}{2} \right) {}_4F_3 \left(-n, \rho + \sigma + n + 1, \alpha, \alpha + \frac{1}{2}; \right. \\
\left. \varphi + 1, \alpha + 1; \frac{1}{2ab^2} \right) \left[a, \operatorname{Re} \alpha > 0, |\arg b| < \pi/4, \varphi = \begin{Bmatrix} \rho \\ \sigma \end{Bmatrix} \right].
\end{aligned}$$

$$\begin{aligned}
5. \int_a^{\infty} (x-a)^{\rho} (x+a)^{-\tau} \left\{ \begin{matrix} \operatorname{erf} (b \sqrt{x+a}) \\ \operatorname{erfc} (b \sqrt{x+a}) \end{matrix} \right\} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \pm \frac{(\tau + \sigma - 1/2)_n}{n! \sqrt{\pi}} 2^{\rho-\tau+5/2} B \left(\rho + n + 1, \tau - \rho - n - \frac{3}{2} \right) \times \\
\times a^{\rho-\tau+3/2} b {}_3F_3 \left(\frac{1}{2}, \frac{3}{2} - \sigma - \tau, \frac{3}{2} - \tau; \frac{3}{2}, \rho - \tau + n + \frac{5}{2}, \right. \\
\left. \frac{3}{2} - \sigma - \tau - n; -2ab^2 \right) \pm \frac{(\sigma + \rho + n + 1)_n b^{2(\tau-\rho-n-1)}}{n! \sqrt{\pi} (\tau - \rho - n - 1) (2a)^n} \times \\
\times \Gamma \left(\frac{3}{2} + \rho - \tau + n \right) {}_3F_3 \left(-\rho - n, -\sigma - \rho - n, \tau - \rho - n - 1; \right. \\
\left. -\sigma - \rho - 2n, \tau - \rho - n - \frac{1}{2}, \tau - \rho - n; -2ab^2 \right) + \\
+ \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \frac{(\tau + \sigma)_n}{n!} B(\rho + n + 1, \tau - \rho - n - 1) (2a)^{\rho-\tau+1} \\
[a > 0; \operatorname{Re} \rho > -1, \{ |\arg b| < \pi/4, \operatorname{Re}(\rho - \tau) < -n - 1 \}].
\end{aligned}$$

$$\begin{aligned}
6. \int_0^{\infty} x^{\alpha-1} (a-x)^n \operatorname{erfc} (b \sqrt{x}) P_n^{(\rho, \sigma)} \left(\frac{a+x}{a-x} \right) dx = \\
= \frac{a^n (\rho + 1)_n}{n! \sqrt{\pi} \alpha b^{2\alpha}} \Gamma \left(\alpha + \frac{1}{2} \right) {}_4F_3 \left(-n, -n - \sigma, \alpha, \alpha + \frac{1}{2}; \rho + 1, \alpha + 1; \frac{1}{ab^2} \right) \\
[\operatorname{Re} \alpha > 0; |\arg b| < \pi/4].
\end{aligned}$$

$$\begin{aligned}
7. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\rho} e^{b^2 x} \operatorname{erf} (b \sqrt{x+a}) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^n}{n! \sqrt{\pi}} \left(\sigma - \alpha + \frac{1}{2} \right)_n 2^{\alpha+\rho+3/2} a^{\alpha+\rho+1/2} b B \left(\alpha + \frac{1}{2}, \rho + n + 1 \right) \times \\
\times e^{-ab^2} {}_3F_3 \left(1, \alpha + \frac{1}{2}, \alpha - \sigma + \frac{1}{2}; \frac{3}{2}, \alpha - \sigma - n + \frac{1}{2}, \alpha + \rho + n + \frac{3}{2}; 2ab^2 \right) \\
[a > 0; \operatorname{Re} \rho > -1; \operatorname{Re} \alpha > -1/2].
\end{aligned}$$

2.22.12. Интегралы, содержащие $\left\{ \begin{matrix} J_\nu(\varphi(x)) \\ I_\nu(\varphi(x)) \end{matrix} \right\} P_n^{(\rho, \sigma)}(cx)$.

$$1. \int_{\pm a}^{\infty} (x \mp a)^{\alpha-1} J_\nu(b \sqrt{x \mp a}) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx =$$

$$= (\pm 1)^n \frac{(\rho + \sigma + n + 1)_n 2^{2\alpha+n}}{n! a^n b^{2\alpha+2n}} \Gamma\left[\frac{\nu/2 + \alpha + n}{\nu/2 - \alpha - n + 1}\right] \times$$

$$\times {}_2F_3\left(-n, -\varphi - n; -\rho - \sigma - 2n, 1 - \alpha - \frac{\nu}{2} - n, \frac{\nu}{2} - \alpha - n + 1; \pm \frac{ab^2}{2}\right)$$

$$\left[a, b, \operatorname{Re}(2\alpha + \nu) > 0; \operatorname{Re}\alpha < 3/4 - n, \varphi = \left\{\frac{\rho}{\sigma}\right\}\right].$$

$$2. \int_{\pm a}^{\infty} (x \mp a)^{\alpha-1} J_\nu\left(\frac{b}{\sqrt{x \mp a}}\right) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx =$$

$$= (\pm 1)^n \frac{(\varphi + 1)_n}{n!} \left(\frac{b}{2}\right)^{2\alpha} \Gamma\left[\frac{\nu/2 - \alpha}{\nu/2 + \alpha + 1}\right] \times$$

$$\times {}_2F_3\left(-n, \rho + \sigma + n + 1, \varphi + 1, \alpha + \frac{\nu}{2} + 1, \alpha - \frac{\nu}{2} + 1; \pm \frac{b^2}{8a}\right)$$

$$\left[a, b > 0; \operatorname{Re}\alpha > -3/4; \operatorname{Re}(2\alpha - \nu) < -2n, \varphi = \left\{\frac{\rho}{\sigma}\right\}\right].$$

$$3. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^\rho \left\{ \begin{matrix} J_\nu(b \sqrt{x+a}) \\ I_\nu(b \sqrt{x+a}) \end{matrix} \right\} P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx =$$

$$= \frac{(-1)^n (\sigma - \alpha - \nu/2 + 1)_n 2^{\alpha+\rho-\nu/2}}{n! \Gamma(\nu+1)} B\left(\alpha + \frac{\nu}{2}, \rho + n + 1\right) a^{\alpha+\rho+\nu/2} b^\nu \times$$

$$\times {}_2F_3\left(\alpha + \frac{\nu}{2}, \alpha + \frac{\nu}{2} - \sigma, \alpha + \frac{\nu}{2} + \rho + n + 1, \alpha + \frac{\nu}{2} - \sigma - n, \nu + 1; \mp \frac{ab^2}{2}\right)$$

$$[a, \operatorname{Re}(2\alpha + \nu) > 0, \operatorname{Re}\rho > -1].$$

$$4. \int_a^\infty (x-a)^\rho (x+a)^{-\tau} J_\nu(b \sqrt{x+a}) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx =$$

$$= \frac{(\tau + \sigma - \nu/2)_n 2^{\rho-\tau-\nu/2+1} a^{\rho+\nu/2-\tau+1} b^\nu}{n! \Gamma(\nu+1)} B\left(\rho + n + 1, \tau - \rho - \frac{\nu}{2} - n - 1\right) \times$$

$$\times {}_2F_3\left(\frac{\nu}{2} - \tau + 1, \frac{\nu}{2} - \sigma - \tau + 1; \frac{\nu}{2} - \sigma - \tau - n + 1, \rho + \frac{\nu}{2} - \tau + n + 2, \nu + 1; -\frac{ab^2}{2}\right) +$$

$$+ \frac{(\rho + \sigma + n + 1)_n 2^{2\rho-2\tau+n+2} b^{2(\tau-\rho-n-1)}}{n! a^n} \times$$

$$\times \Gamma\left[\frac{\rho + \nu/2 - \tau + n + 1}{\tau + \nu/2 - \rho - n}\right] {}_2F_3\left(-\rho - n, -\rho - \sigma - n; -\rho - \sigma - 2n, \tau + \frac{\nu}{2} - \rho - n, \tau - \frac{\nu}{2} - \rho - n; -\frac{ab^2}{2}\right)$$

$$[a, b > 0; \operatorname{Re}\rho > -1; \operatorname{Re}(\rho - \tau) < -n - 1/2].$$

$$5. \int_a^\infty (x-a)^{\alpha-1} (x+a)^\sigma J_\nu(b \sqrt{x-a}) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx =$$

$$= \frac{(1 + \rho - \alpha - \nu/2)_n 2^{\alpha+\sigma-\nu/2}}{n! \Gamma(\nu+1)} B\left(\alpha + \frac{\nu}{2}, -\frac{\nu}{2} - \alpha - \sigma - n\right) a^{\alpha+\sigma+\nu/2} b^\nu \times$$

$$\times {}_2F_3\left(\alpha + \frac{\nu}{2} - \rho, \alpha + \frac{\nu}{2}; \alpha + \frac{\nu}{2} - \rho - n, \alpha + \frac{\nu}{2} + \sigma + n + 1, \nu + 1; \frac{ab^2}{2}\right) +$$

$$+ \frac{(\rho + \sigma + n + 1)_n}{n! (2a)^n} \left(\frac{2}{b}\right)^{2(\alpha + \sigma + n)} \Gamma \left[\frac{v/2 + \alpha + \sigma + n}{v/2 - \alpha - \sigma - n + 1} \right] \times$$

$$\times {}_2F_3 \left(-\rho - \sigma - n, -\sigma - n; -\rho - \sigma - 2n, 1 - n - \sigma - \alpha - v/2, \right.$$

$$\left. v/2 - \alpha - \sigma - n + 1; \frac{ab^2}{2} \right) \quad [a, b, \operatorname{Re}(2\alpha + v) > 0; \operatorname{Re}(\alpha + \sigma) < 3/4 - n].$$

$$6. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\rho} J_v \left(\frac{b}{\sqrt{x+a}} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^n}{n!} (\sigma + 1)_n 2^{\rho-2\alpha} \Gamma \left[\frac{v/2 - \alpha}{v/2 + \alpha + 1} \right] a^{\rho} b^{2\alpha} \times$$

$$\times {}_2F_3 \left(\sigma + n + 1, -\rho - n; \sigma + 1, \alpha - \frac{v}{2} + 1, \alpha + \frac{v}{2} + 1; -\frac{b^2}{8a} \right) +$$

$$+ \frac{(-1)^n (\sigma + v/2 - \alpha + 1)_n}{n! \Gamma(v+1)} 2^{\alpha+\rho-3v/2} B \left(\rho + n + 1, \alpha - \frac{v}{2} \right) a^{\alpha+\rho-v/2} b^v \times$$

$$\times {}_2F_3 \left(\sigma + \frac{v}{2} - \alpha + n + 1, \frac{v}{2} - \alpha - \rho - n; v + 1, \frac{v}{2} - \alpha + 1, \right.$$

$$\left. \sigma + \frac{v}{2} - \alpha + 1; -\frac{b^2}{8a} \right) \quad [a, b > 0; \operatorname{Re} \rho > -1; \operatorname{Re} \alpha > -3/4].$$

$$7. \int_a^{\infty} (x-a)^{\rho} (x+a)^{-\tau} \left\{ \begin{matrix} J_v(b/\sqrt{x+a}) \\ I_v(b/\sqrt{x+a}) \end{matrix} \right\} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx =$$

$$= \frac{(\sigma + \tau + v/2)_n}{n! \Gamma(v+1)} 2^{\rho-3v/2-\tau+1} B \left(\rho + n + 1, \frac{v}{2} - \rho + \tau - n - 1 \right) a^{\rho-\tau-v/2+1} b^v \times$$

$$\times {}_2F_3 \left(\frac{v}{2} + \tau - \rho - n - 1, \sigma + \tau + \frac{v}{2} + n; \tau + \frac{v}{2}, \sigma + \tau + \frac{v}{2}, v + 1; \mp \frac{b^2}{8a} \right)$$

$$[a > 0; \operatorname{Re} \rho > -1; \operatorname{Re}(2\rho - 2\tau - v) < -2n - 2].$$

$$8. \int_a^{\infty} (x-a)^{\alpha-1} (x+a)^{\sigma} J_v \left(\frac{b}{\sqrt{x-a}} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx =$$

$$= \frac{(\rho + 1)_n}{n!} 2^{\sigma-2\alpha} a^{\sigma} b^{2\alpha} \Gamma \left[\frac{v/2 - \alpha}{v/2 + \alpha + 1} \right] \times$$

$$\times {}_2F_3 \left(\rho + n + 1, -\sigma - n; \rho + 1, \alpha + \frac{v}{2} + 1, \alpha - \frac{v}{2} + 1; \frac{b^2}{8a} \right) +$$

$$+ \frac{(\rho - \alpha + v/2 + 1)_n}{n! \Gamma(v+1)} 2^{\alpha+\sigma-3v/2} B \left(\alpha - \frac{v}{2}, \frac{v}{2} - \alpha - \sigma - n \right) a^{\alpha+\sigma-v/2} b^v \times$$

$$\times {}_2F_3 \left(1 - \alpha + \frac{v}{2} + \rho + n, \frac{v}{2} - \alpha - \sigma - n; \rho + \frac{v}{2} - \alpha + 1, \frac{v}{2} - \alpha + 1, v + 1; \frac{b^2}{8a} \right)$$

$$[a, b > 0; \operatorname{Re} \alpha > -3/4; \operatorname{Re}(\alpha + \sigma - v/2) < -n].$$

$$9. \int_{-a}^a (x+a)^{-\beta-n-1} (a-x)^{\beta-1} J_v \left(b \sqrt{\frac{a-x}{a+x}} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^n}{n!} (\sigma + 1)_n 2^{2\beta+n-1} \Gamma \left[\frac{\beta + v/2 + n}{v/2 - \beta - n + 1} \right] a^{-n-1} b^{-2\beta-2n} \times$$

$$\times {}_2F_3 \left(-n, -\rho - n; \sigma + 1, 1 - \beta - \frac{v}{2} - n, \frac{v}{2} - \beta - n + 1; \frac{b^2}{4} \right)$$

$$[b, \operatorname{Re}(2\beta + v) > 0; \operatorname{Re} \beta < 3/4 - n].$$

$$\begin{aligned}
10. \quad & \int_{-a}^a (x+a)^{\rho+\sigma-\beta+n} (a-x)^{\beta-1} J_\nu \left(b \sqrt{\frac{a-x}{a+x}} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
& = \frac{(\rho-\beta-\nu/2+1)_n}{n! \Gamma(\nu+1)} 2^{\rho+\sigma-\nu+n} B \left(\beta + \frac{\nu}{2}, \rho+\sigma-\beta-\frac{\nu}{2}+n+1 \right) \times \\
& \quad \times a^{\rho+\sigma+n} b^\nu {}_2F_3 \left(\beta + \frac{\nu}{2} - \rho, \beta + \frac{\nu}{2}; \beta + \frac{\nu}{2} - \rho - n, \right. \\
& \quad \left. \beta + \frac{\nu}{2} - \rho - \sigma - n, \nu+1; \frac{b^2}{4} \right) + \frac{(-1)^n}{n!} (\sigma+1)_n 2^{2\beta-\rho-\sigma-n-2} \times \\
& \quad \times a^{\rho+\sigma+n} b^{2(\rho+\sigma-\beta+n+1)} \Gamma \left[\beta + \nu/2 - \rho - \sigma - n - 1 \right] \times \\
& \quad \times {}_2F_3 \left(\rho + \sigma + n + 1, \sigma + n + 1; \sigma + 1, \rho + \sigma - \beta - \nu/2 + n + 2, \right. \\
& \quad \left. \rho + \sigma + \nu/2 - \beta + n + 2; \frac{b^2}{4} \right) \\
& \quad [a, b, \operatorname{Re}(2\beta + \nu) > 0; \operatorname{Re}(\alpha - \rho - \sigma) < 7/4 + n].
\end{aligned}$$

$$\begin{aligned}
11. \quad & \int_{-a}^a (x+a)^\sigma (a-x)^{\beta-1} \left\{ \begin{matrix} J_\mu(b\sqrt{a-x}) J_\nu(b\sqrt{a-x}) \\ I_\mu(b\sqrt{a-x}) I_\nu(b\sqrt{a-x}) \end{matrix} \right\} P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
& = \frac{2^{\beta+\sigma-(\mu+\nu)/2}}{n!} a^{\beta+\sigma+(\mu+\nu)/2} b^{\mu+\nu} \left(\rho - \beta - \frac{\mu+\nu}{2} + 1 \right)_n \times \\
& \quad \times \Gamma \left[\beta + (\mu+\nu)/2, \sigma + n + 1 \right. \\
& \quad \left. \mu + 1, \nu + 1, \beta + (\mu+\nu)/2 + \sigma + n + 1 \right] \times \\
& \quad \times {}_4F_3 \left(\frac{\mu+\nu}{2} + \beta, \beta + \frac{\mu+\nu}{2} - \rho, \frac{\mu+\nu+1}{2}, \frac{\mu+\nu}{2} + 1; \beta + \frac{\mu+\nu}{2} + \sigma + n + 1, \right. \\
& \quad \left. \beta + \frac{\mu+\nu}{2} - \rho - n, \mu + 1, \nu + 1, \mu + \nu + 1; \mp 2ab^2 \right) \\
& \quad [a, \operatorname{Re}(2\beta + \mu + \nu) > 0; \operatorname{Re} \sigma > -1].
\end{aligned}$$

2.22.13. Интегралы от $A(x) K_\nu(\varphi(x)) P_n^{(\rho, \sigma)}(cx)$.

$$\begin{aligned}
1. \quad & \int_{\pm a}^{\infty} (x \mp a)^{\alpha-1} K_\nu \left(b \sqrt{x \mp a} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
& = \frac{(\pm 1)^n (\rho + \sigma + n + 1)_n}{n! a^n b^{2\alpha+2n}} 2^{2\alpha+n-1} \Gamma \left(\alpha + \frac{\nu}{2} + n \right) \Gamma \left(\alpha - \frac{\nu}{2} + n \right) \times \\
& \quad \times {}_2F_3 \left(-n, -\varphi - n; -\rho - \sigma - 2n, 1 - n - \alpha - \frac{\nu}{2}, \frac{\nu}{2} - \alpha - n + 1; \mp \frac{ab^2}{2} \right) \\
& \quad [a, \operatorname{Re} b > 0; 2 \operatorname{Re} \alpha > |\operatorname{Re} \nu|, \varphi = \left\{ \begin{matrix} \rho \\ \sigma \end{matrix} \right\}].
\end{aligned}$$

$$\begin{aligned}
2. \quad & \int_{\pm a}^{\infty} (x \mp a)^{\alpha-1} K_\nu \left(\frac{b}{\sqrt{x \mp a}} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
& = \frac{(\pm 1)^n (\varphi + 1)_n}{n! 2^{2\alpha+1}} \Gamma \left(\frac{\nu}{2} - \alpha \right) \Gamma \left(-\frac{\nu}{2} - \alpha \right) b^{2\alpha} \times \\
& \quad \times {}_2F_3 \left(-n, \rho + \sigma + n + 1; \varphi + 1, \alpha - \frac{\nu}{2} + 1, \alpha + \frac{\nu}{2} + 1; \mp \frac{b^2}{8a} \right) \\
& \quad [a, \operatorname{Re} b > 0; 2 \operatorname{Re} \alpha < -2n - |\operatorname{Re} \nu|, \varphi = \left\{ \begin{matrix} \rho \\ \sigma \end{matrix} \right\}].
\end{aligned}$$

$$\begin{aligned}
3. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\rho} K_{\nu}(b \sqrt{x+a}) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
= \frac{(-1)^n}{n!} 2^{\alpha+\rho+\nu/2-1} \left(1+\alpha+\frac{\nu}{2}-\alpha\right)_n \Gamma(\nu) B\left(\rho+n+1, \alpha-\frac{\nu}{2}\right) \times \\
\times a^{\alpha+\rho-\nu/2} b^{-\nu} {}_2F_3\left(\alpha-\frac{\nu}{2}, \alpha-\frac{\nu}{2}-\sigma; \alpha-\frac{\nu}{2}+\rho+n+1, \right. \\
\left. \alpha-\frac{\nu}{2}-\sigma-n, 1-\nu; \frac{ab^2}{2}\right) + \frac{(-1)^n}{n!} 2^{\alpha+\rho-\nu/2-1} \left(\sigma-\alpha-\frac{\nu}{2}+1\right)_n \times \\
\times \Gamma(-\nu) B\left(\rho+n+1, \alpha+\frac{\nu}{2}\right) a^{\alpha+\rho+\nu/2} b^{\nu} {}_2F_3\left(\alpha+\frac{\nu}{2}, \alpha+\frac{\nu}{2}-\sigma; \right. \\
\left. \alpha+\frac{\nu}{2}+\rho+n+1, \alpha+\frac{\nu}{2}-\sigma-n, \nu+1; \frac{ab^2}{2}\right) \\
[a > 0; \operatorname{Re} \rho > -1; 2 \operatorname{Re} \alpha > |\operatorname{Re} \nu|].
\end{aligned}$$

$$\begin{aligned}
4. \int_a^{\infty} (x-a)^{\rho} (x+a)^{-\tau} K_{\nu}(b \sqrt{x+a}) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
= \frac{(\rho+\sigma+n+1)_n}{n!} 2^{2\rho-2\tau+n+1} \Gamma\left(\frac{\nu}{2}+\rho-\tau+n+1\right) \times \\
\times \Gamma\left(\rho-\frac{\nu}{2}-\tau+n+1\right) a^{-n} b^{2(\tau-\rho-n-1)} {}_2F_3\left(-\rho-n, -\rho-\sigma-n; \right. \\
\left. -\rho-\sigma-2n, \frac{\nu}{2}-\rho+\tau-n, \tau-\frac{\nu}{2}-\rho-n; \frac{ab^2}{2}\right) + \frac{(\nu/2+\sigma+\tau)_n}{n!} 2^{\rho+\nu/2-\tau} \times \\
\times \Gamma(\nu) B\left(\rho+n+1, \frac{\nu}{2}+\tau-\rho-n-1\right) a^{\rho-\nu/2-\tau+1} b^{-\nu} {}_2F_3\left(1-\tau-\frac{\nu}{2}, 1-\frac{\nu}{2}-\sigma-\tau; \right. \\
\left. 1-\nu, \rho-\tau-\frac{\nu}{2}+n+2, 1-\frac{\nu}{2}-\tau-\sigma-n; \frac{ab^2}{2}\right) + \\
+ \frac{(\sigma+\tau-\nu/2)_n}{n!} 2^{\rho-\nu/2-\tau} \Gamma(-\nu) B\left(\rho+n+1, \tau-\frac{\nu}{2}-\rho-n-1\right) \times \\
\times a^{\rho+\nu/2-\tau+1} b^{\nu} {}_2F_3\left(\frac{\nu}{2}-\tau+1, \frac{\nu}{2}-\sigma-\tau+1; \nu+1, \rho+\frac{\nu}{2}-\tau+n+2, \right. \\
\left. \frac{\nu}{2}-\sigma-\tau-n+1; \frac{ab^2}{2}\right) \\
[a, \operatorname{Re} b > 0; \operatorname{Re} \rho > -1].
\end{aligned}$$

$$\begin{aligned}
5. \int_a^{\infty} (x-a)^{\alpha-1} (x+a)^{\sigma} K_{\nu}(b \sqrt{x-a}) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
= \frac{(\rho+\sigma+n+1)_n}{n!} 2^{2\alpha+2\sigma+n-1} \Gamma\left(\alpha+\frac{\nu}{2}+\sigma+n\right) \Gamma\left(\alpha-\frac{\nu}{2}+\sigma+n\right) \times \\
\times a^{-n} b^{-2(\alpha+\sigma+n)} {}_2F_3\left(-\sigma-n, -\rho-\sigma-n; -\rho-\sigma-2n, 1-\alpha-\frac{\nu}{2}-\sigma-n, \right. \\
\left. 1-\alpha+\frac{\nu}{2}-\sigma-n; -\frac{ab^2}{2}\right) + \frac{(\rho-\alpha-\nu/2+1)_n}{n!} 2^{\alpha+\sigma-\nu/2-1} \times \\
\times B\left(\alpha+\frac{\nu}{2}, -\alpha-\frac{\nu}{2}-\sigma-n\right) a^{\alpha+\sigma+\nu/2} b^{\nu} {}_2F_3\left(\alpha+\frac{\nu}{2}-\rho, \alpha+\frac{\nu}{2}; \right. \\
\left. \nu+1, \alpha+\frac{\nu}{2}-\rho-n, \alpha+\sigma+\frac{\nu}{2}+n+1; -\frac{ab^2}{2}\right) + \\
+ \frac{(\rho+\nu/2-\alpha+1)_n}{n!} 2^{2\alpha+\sigma+\nu/2-1} B\left(\alpha-\frac{\nu}{2}, \frac{\nu}{2}-\alpha-\sigma-n\right) a^{\alpha+\sigma-\nu/2} \times
\end{aligned}$$

$$\times b^{-\nu} {}_2F_3\left(\alpha - \rho - \frac{\nu}{2}, \alpha - \frac{\nu}{2}; 1 - \nu, \alpha - n - \rho - \frac{\nu}{2}, \alpha + \sigma - \frac{\nu}{2} + n + 1; -\frac{ab^2}{2}\right)$$

[a, Re b > 0; 2 Re α > |Re ν|]

$$\begin{aligned} 6. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\rho} K_{\nu}\left(\frac{b}{\sqrt{x+a}}\right) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\ = \frac{(-1)^n}{n!} (\sigma+1)_n 2^{\rho-2\alpha-1} \Gamma\left(\frac{\nu}{2}-\alpha\right) \Gamma\left(-\frac{\nu}{2}-\alpha\right) a^{\alpha+\rho} \times \\ \times {}_2F_3\left(-\rho-n, \sigma+n+1; \sigma+1, \alpha+\frac{\nu}{2}+1, \alpha-\frac{\nu}{2}+1; \frac{b^2}{8a}\right) + \\ + \frac{(-1)^n}{n!} \left(\sigma-\alpha-\frac{\nu}{2}+1\right)_n 2^{\alpha+\rho+3\nu/2-1} \Gamma(\nu) B\left(\rho+n+1, \alpha+\frac{\nu}{2}\right) \times \\ \times a^{\alpha+\rho+\nu/2} b^{-\nu} {}_2F_3\left(-\alpha-\frac{\nu}{2}-\rho-n, \sigma-\alpha-\frac{\nu}{2}+n+1; 1-\nu, 1-\alpha-\frac{\nu}{2}, \right. \\ \left. \sigma-\alpha-\frac{\nu}{2}+1; \frac{b^2}{8a}\right) + \frac{(-1)^n}{n!} \left(\sigma-\alpha+\frac{\nu}{2}+1\right)_n 2^{\alpha+\rho-3\nu/2-1} \Gamma(-\nu) \times \\ \times B\left(\rho+n+1, \alpha-\frac{\nu}{2}\right) a^{\alpha+\rho-\nu/2} b^{\nu} {}_2F_3\left(\frac{\nu}{2}-\alpha-\rho-n, \sigma-\alpha+\frac{\nu}{2}+n+1; \right. \\ \left. \nu+1, \frac{\nu}{2}-\alpha+1, \frac{\nu}{2}-\alpha+\sigma+1; \frac{b^2}{8a}\right) \end{aligned}$$

[a, Re b > 0; Re ρ > -1]

$$\begin{aligned} 7. \int_a^{\infty} (x-a)^{\rho} (x+a)^{-\tau} K_{\nu}\left(\frac{b}{\sqrt{x+a}}\right) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\ = \frac{(\sigma+\tau-\nu/2)_n}{n!} 2^{\rho+3\nu/2-\tau} \Gamma(\nu) B\left(\rho+n+1, \tau-\rho-\frac{\nu}{2}-n-1\right) \times \\ \times a^{\rho-\tau+\nu/2+1} b^{-\nu} {}_2F_3\left(\tau-\rho-\frac{\nu}{2}-n-1, \sigma+\tau-\frac{\nu}{2}+n; \right. \\ \left. \tau-\frac{\nu}{2}, \sigma+\tau-\frac{\nu}{2}, 1-\nu; \frac{b^2}{8a}\right) + \frac{(\sigma+\tau+\nu/2)_n}{n!} 2^{\rho-3\nu/2-\tau} \Gamma(-\nu) \times \\ \times B\left(\rho+n+1, \tau+\frac{\nu}{2}-\rho-n-1\right) a^{\rho-\tau-\nu/2+1} b^{\nu} {}_2F_3\left(\tau-\rho+\frac{\nu}{2}-n-1, \right. \\ \left. \sigma+\tau+\frac{\nu}{2}+n; \tau+\frac{\nu}{2}, \sigma+\tau+\frac{\nu}{2}, \nu+1; \frac{b^2}{8a}\right) \end{aligned}$$

[a > 0; Re ρ > -1; 2 Re (ρ - τ) < -|Re ν| - 2n - 2]

$$\begin{aligned} 8. \int_a^{\infty} (x-a)^{\alpha-1} (x+a)^{\sigma} K_{\nu}\left(\frac{b}{\sqrt{x-a}}\right) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\ = \frac{(\rho-\alpha+\nu/2+1)_n}{n!} 2^{\alpha+\sigma-3\nu/2-1} \Gamma(-\nu) B\left(\alpha-\frac{\nu}{2}, \frac{\nu}{2}-\alpha-\sigma-n\right) \times \\ \times a^{\alpha+\sigma-\nu/2} b^{\nu} {}_2F_3\left(\rho+\frac{\nu}{2}-\alpha+n+1, \frac{\nu}{2}-\alpha-\sigma-n; \right. \\ \left. \nu+1, \frac{\nu}{2}-\alpha+1, \rho+\frac{\nu}{2}-\alpha+1; -\frac{b^2}{8a}\right) + \frac{(\rho-\alpha-\nu/2+1)_n}{n!} 2^{\alpha+\sigma+3\nu/2-1} \Gamma(\nu) \times \\ \times B\left(\alpha+\frac{\nu}{2}, -\alpha-\frac{\nu}{2}-\sigma-n\right) a^{\alpha+\sigma+\nu/2} b^{-\nu} {}_2F_3\left(\rho-\alpha-\frac{\nu}{2}+n+1, -\alpha-\frac{\nu}{2}-\sigma-n; \right. \\ \left. 1-\nu, 1-\alpha-\frac{\nu}{2}, \rho-\alpha-\frac{\nu}{2}+1; -\frac{b^2}{8a}\right) + \end{aligned}$$

$$+ \frac{(\rho+1)_n}{n!} 2^{\sigma-2\alpha-1} \Gamma\left(\frac{\nu}{2}-\alpha\right) \Gamma\left(-\frac{\nu}{2}-\alpha\right) a^{\sigma} b^{2\alpha} \times \\ \times {}_2F_3\left(-\sigma-n, \rho+n+1; \rho+1, \alpha-\frac{\nu}{2}+1, \alpha+\frac{\nu}{2}+1; -\frac{b^2}{8a}\right) \\ [a, \operatorname{Re} b > 0; 2 \operatorname{Re}(\alpha+\sigma) < -|\operatorname{Re} \nu| - 2n].$$

$$9. \int_{-a}^a (x+a)^{-\beta-n-1} (a-x)^{\beta-1} K_{\nu}\left(b \sqrt{\frac{a-x}{a+x}}\right) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\ = \frac{(-1)^n}{n!} (\sigma+1)_n 2^{2\beta+n-2} \Gamma\left(\beta+\frac{\nu}{2}+n\right) \Gamma\left(\beta-\frac{\nu}{2}+n\right) a^{-n-1} \times \\ \times b^{-2\beta-2n} {}_2F_3\left(-n, -\rho-n; \sigma+1, 1-\beta-\frac{\nu}{2}-n, 1-\beta+\frac{\nu}{2}-n; -\frac{b^2}{4}\right) \\ [a, \operatorname{Re} b > 0; 2 \operatorname{Re} \beta > |\operatorname{Re} \nu|].$$

$$10. \int_{-a}^a (x+a)^{\rho+\sigma-\beta+n} (a-x)^{\beta-1} K_{\nu}\left(b \sqrt{\frac{a-x}{a+x}}\right) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\ = \frac{(-1)^n}{n!} (\sigma+1)_n 2^{2\beta-\rho-\sigma-n-3} \Gamma\left(\beta+\frac{\nu}{2}-\rho-\sigma-n-1\right) \times \\ \times \Gamma\left(\beta-\frac{\nu}{2}-\rho-\sigma-n-1\right) a^{\rho+\sigma+n} b^{2(\rho+\sigma-\beta+n+1)} {}_2F_3\left(\sigma+n+1, \rho+\sigma+n+1; \right. \\ \left. \sigma+1, \rho+\sigma-\beta-\frac{\nu}{2}+n+2, \rho+\sigma-\beta+\frac{\nu}{2}+n+2; -\frac{b^2}{4}\right) + \\ + \frac{(\rho-\beta-\nu/2+1)_n}{n!} 2^{\rho+\sigma-\nu+n-1} B\left(\rho+\sigma-\beta-\frac{\nu}{2}+n+1, \beta+\frac{\nu}{2}\right) a^{\rho+\sigma+n} b^{\nu} \times \\ \times {}_2F_3\left(\beta+\frac{\nu}{2}, \beta+\frac{\nu}{2}-\rho; \beta+\frac{\nu}{2}-\rho-n, \beta+\frac{\nu}{2}-\rho-\sigma-n, \nu+1; -\frac{b^2}{4}\right) + \\ + \frac{(\rho-\beta+\nu/2+1)_n}{n!} 2^{\rho+\sigma+\nu+n-1} B\left(\rho+\sigma-\beta+\frac{\nu}{2}+n+1, \beta-\frac{\nu}{2}\right) a^{\rho+\sigma+n} \times \\ \times b^{-\nu} {}_2F_3\left(\beta-\frac{\nu}{2}, \beta-\frac{\nu}{2}-\rho; \beta-\rho-\frac{\nu}{2}-n, \beta-\rho-\sigma-\frac{\nu}{2}-n, 1-\nu; -\frac{b^2}{4}\right) \\ [a, \operatorname{Re} b > 0; 2 \operatorname{Re} \beta > |\operatorname{Re} \nu|].$$

2.22.14. Интегралы от $A(x) e^{-bx} L_m^{\gamma}(a+bx) P_n^{(\rho, \sigma)}(cx)$.

$$1. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\rho} e^{-bx} L_m^{\gamma}(ab+bx) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\ = \frac{(-1)^n (\sigma-\alpha+1)_n (\gamma+1)_m}{m! n!} B(\rho+n+1, \alpha) (2a)^{\sigma+\rho} e^{ab} \times \\ \times {}_3F_3(\gamma+m+1, \alpha-\sigma, \alpha; \gamma+1, \alpha-\sigma-n, \alpha+\rho+n+1; -2ab) \\ [a, \operatorname{Re} \alpha > 0; \operatorname{Re} \rho > -1].$$

$$2. \int_a^{\infty} (x-a)^{\rho} (x+a)^{-\tau} e^{-bx} L_m^{\gamma}(ab+bx) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\ = \frac{(\sigma+\tau)_n (\gamma+1)_m}{m! n!} B(\rho+n+1, \tau-\rho-n-1) (2a)^{\rho-\tau+1} e^{ab} \times \\ \times {}_3F_3(1-\tau-\sigma, \gamma+m+1, 1-\tau; 1-\tau-\sigma-n, \gamma+1, \rho-\tau+n+2; -2ab) + \\ + \frac{(\sigma+\rho+n+1)_n (\gamma+\tau-\rho-n)_m}{m! n! (2a)^n} \Gamma(\rho-\tau+n+1) b^{\tau-\rho-n-1} e^{ab} \times \\ \times {}_3F_3(-\rho-n, -\rho-\sigma-n, \gamma+\tau-\rho+m-n; \\ -\rho-\sigma-2n, \gamma+\tau-\rho-n, \tau-\rho-n; -2ab) \\ [a, \operatorname{Re} b > 0; \operatorname{Re} \rho > -1].$$

2.22.15. Интегралы от $A(x) e^{-b^2 x} H_m(b \sqrt{x+a}) P_n^{(\rho, \sigma)}(cx)$.

$$\begin{aligned}
 1. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\rho} e^{-b^2 x} H_{2m+\varepsilon}(b \sqrt{x+a}) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
 = \frac{(-1)^{m+n}}{n!} \left(\sigma - \alpha - \frac{\varepsilon}{2} + 1\right)_n \left(\varepsilon + \frac{1}{2}\right)_m 2^{\alpha+\varepsilon/2+\rho+2m} \times \\
 \times B\left(\alpha + \frac{\varepsilon}{2}, \rho + n + 1\right) a^{\alpha+\rho+\varepsilon/2} b^{\varepsilon} e^{ab^2} {}_3F_3\left(m + \varepsilon + \frac{1}{2}, \alpha - \sigma + \frac{\varepsilon}{2}, \alpha + \frac{\varepsilon}{2}; \right. \\
 \left. \alpha + \rho + \frac{\varepsilon}{2} + n + 1, \alpha - \sigma + \frac{\varepsilon}{2} - n, \varepsilon + \frac{1}{2}; -2ab^2\right) \\
 [\varepsilon = 0 \text{ или } 1; a > 0; \operatorname{Re} \rho > -1; \operatorname{Re} \alpha > -\varepsilon/2].
 \end{aligned}$$

$$\begin{aligned}
 2. \int_a^{\infty} (x-a)^{\rho} (x+a)^{-\tau} e^{-b^2 x} H_{2m+\varepsilon}(b \sqrt{x+a}) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
 = \frac{(-1)^m}{n!} \left(\sigma + \tau - \frac{\varepsilon}{2}\right)_n \left(\varepsilon + \frac{1}{2}\right)_m 2^{\rho-\tau+2m+\varepsilon/2+1} \times \\
 \times B\left(\rho + n + 1, \tau - \rho - \frac{\varepsilon}{2} - n - 1\right) a^{\rho-\tau+\varepsilon/2+1} b^{\varepsilon} e^{ab^2} \times \\
 \times {}_3F_3\left(\varepsilon + m + \frac{1}{2}, \frac{\varepsilon}{2} - \tau - \sigma + 1, \frac{\varepsilon}{2} - \tau + 1; \varepsilon + \frac{1}{2}, \rho - \tau + \frac{\varepsilon}{2} + n + 2, \right. \\
 \left. \frac{\varepsilon}{2} - \tau - \sigma - n + 1; -2ab^2\right) + \frac{(-1)^m}{n! a^n} (\sigma + \rho + n + 1)_n \left(\tau + \frac{\varepsilon-1}{2} - \rho - n\right)_m 2^{2m+\varepsilon-n} \times \\
 \times b^{2(\tau-\rho-n-1)} \Gamma\left(\frac{\varepsilon}{2} + \rho - \tau + n + 1\right) e^{ab^2} {}_3F_3\left(-\rho - n, -\rho - \sigma - n, \right. \\
 \left. \frac{\varepsilon-1}{2} + \tau - \rho + m - n; -\rho - \sigma - 2n, \tau - \rho - n, \tau - \rho - n - \frac{1}{2}; -2ab^2\right) \\
 [\varepsilon = 0 \text{ или } 1; a > 0; \operatorname{Re} \rho > -1; |\arg b| < \pi/4].
 \end{aligned}$$

2.22.16. Интегралы, содержащие $C_m^{\gamma}(a+bx) P_n^{(\rho, \sigma)}(cx)$.

$$\begin{aligned}
 1. \int_{-a}^a (x+a)^{\gamma+\sigma-1/2} (a-x)^{\beta-1} C_m^{\gamma}\left(\frac{x}{a}\right) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
 = \frac{(2\gamma)_m}{m! n!} (1 + \rho - \beta)_n B(\beta, \sigma + n + 1) (2a)^{\beta+\sigma+\gamma-1/2} \times \\
 \times {}_4F_3\left(\frac{1}{2} - \gamma - m, \frac{1}{2} + \gamma + m, \beta - \rho, \beta; \beta + \sigma + n + 1, \frac{1}{2} + \gamma, \beta - \rho - n; 1\right) \\
 [a, \operatorname{Re} \beta > 0; \operatorname{Re}(\gamma + \sigma) > -1/2].
 \end{aligned}$$

$$\begin{aligned}
 2. \int_{-a}^a (x+a)^{\gamma+\sigma-1/2} (a-x)^{\rho-1} C_m^{\gamma}\left(\frac{x}{a}\right) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
 = \frac{(2\gamma)_m}{m!} B(\rho, \sigma + n + 1) (2a)^{\rho+\sigma+\gamma-1/2} \left[1 + (-1)^n \times \right. \\
 \times \frac{(1/2 - \gamma - m)_{n+1} (1/2 + \gamma + m)_{n+1} (\rho)_{n+1}}{(\rho + \sigma + n + 1)_{n+1} (1/2 + \gamma)_{n+1} (n+1)!} {}_4F_3\left(\frac{3}{2} + n - m - \gamma, \right. \\
 \left. \frac{3}{2} + m + n + \gamma, \rho + n + 1, n + 1; n + 2, \rho + \sigma + 2n + 2, \frac{3}{2} + \gamma + n; 1\right) \Big] \\
 [a, \operatorname{Re} \rho > 0; \operatorname{Re}(\gamma + \sigma) > -1/2].
 \end{aligned}$$

$$\begin{aligned}
3. \quad & \int_{-a}^a (x+a)^{\gamma+\sigma-1/2} (a-x)^{\rho} C_m^{\gamma} \left(\frac{x}{a} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
& = \frac{(-1)^n (1/2 + \gamma + n)_m (1/2 - \gamma - m)_n (2\gamma)_m}{m! n! (\gamma + 1/2)_m} B(\rho + n + 1, \sigma + n + 1) \times \\
& \quad \times (2a)^{\rho+\sigma+\gamma+1/2} {}_3F_2 \left(\frac{1}{2} + n - m - \gamma, \frac{1}{2} + \gamma + m + n, \rho + n + 1; \right. \\
& \quad \left. \rho + \sigma + 2n + 2, \frac{1}{2} + \gamma + n; 1 \right) \quad [a, n > 0; \operatorname{Re} \rho > -1; \operatorname{Re}(\gamma + \sigma) > -1/2].
\end{aligned}$$

$$\begin{aligned}
4. \quad & \int_a^{\infty} (x-a)^{\gamma+\rho-1/2} (x+a)^{-\tau} C_m^{\gamma} \left(\frac{x}{a} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \frac{2^{\gamma+\rho-\tau+2m+1/2} (\gamma)_m}{m! n!} \times \\
& \times \left(\frac{1}{2} + \tau + \sigma - \gamma - m \right)_n B \left(\rho + n + 1, \tau - \rho - \gamma - m - n - \frac{1}{2} \right) a^{\gamma+\rho-\tau+1/2} \times \\
& \times {}_4F_3 \left(\frac{1}{2} - \gamma - m, 1 - 2\gamma - m, \frac{1}{2} + \tau + \sigma - \gamma + n - m, \tau - \rho - \gamma - m - n - \frac{1}{2}; \right. \\
& \quad \left. 1 - 2\gamma - 2m, \frac{1}{2} + \tau + \sigma - \gamma - m, \frac{1}{2} + \tau - \gamma - m; 1 \right) \\
& \quad [a > 0; -1/2 < \operatorname{Re}(\gamma + \rho) < \operatorname{Re} \tau - m - n - 1/2].
\end{aligned}$$

$$\begin{aligned}
5. \quad & \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\rho} (1-ab-bx)^{\gamma-1/2} C_m^{\gamma} (2bx+2ab-1) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
& = \frac{(-1)^{m+n} (2\gamma)_m}{m! n!} (1 + \sigma - \alpha)_n B(\rho + n + 1, \alpha) (2a)^{\alpha+\rho} \times \\
& \times {}_4F_3 \left(\alpha, \frac{1}{2} + \gamma + m, \frac{1}{2} - \gamma - m, \alpha - \sigma; \frac{1}{2} + \gamma, \alpha + \rho + n + 1, \alpha - \sigma - n; 2ab \right) \\
& \quad [a, b, \operatorname{Re} \alpha > 0; 2ab < 1; \operatorname{Re} \rho > -1].
\end{aligned}$$

$$\begin{aligned}
6. \quad & \int_a^b (x-a)^{\rho} (b-x)^{\gamma-1/2} C_m^{\gamma} \left(\frac{2x+a-b}{a+b} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
& = \frac{(-1)^m (\rho + \sigma + n + 1)_n (2\gamma)_m}{m! n! (2a)^n} \left(\gamma - \rho - n - \frac{1}{2} \right)_m \times \\
& \times \Gamma \left[\gamma + 1/2, \rho + n + 1 \right] (a+b)^{\gamma+\rho+n+1/2} {}_2F_2 \left(-\rho - \sigma - n, \right. \\
& \quad \left. \gamma + m - \rho - n - \frac{1}{2}, -\rho - \gamma - m - n - \frac{1}{2}; -\rho - \sigma - 2n, \gamma - \rho - n - \frac{1}{2}; \frac{2a}{a+b} \right) \\
& \quad [b > a > 0; \operatorname{Re} \gamma > -1/2; \operatorname{Re} \rho > -1].
\end{aligned}$$

$$\begin{aligned}
7. \quad & \int_a^b (x-a)^{\gamma-1/2} (b-x)^{\rho} C_m^{\gamma} \left(\frac{2x+b-a}{a+b} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{b} \right) dx = \\
& = \frac{(-1)^n (2\gamma)_{2m}}{m! n!} \left(\frac{1}{2} + \sigma - \gamma - m \right)_n (2b)^{\rho+\gamma+m+1/2} (a+b)^{-m} \times \\
& \times \Gamma \left[\rho + n + 1, \gamma + 1/2 \right] {}_2F_2 \left(1 - 2\gamma - m, \frac{1}{2} + \sigma + n - \gamma - m, -\gamma - \rho - \right. \\
& \quad \left. - m - n - \frac{1}{2}; 1 - 2\gamma - 2m, \frac{1}{2} + \sigma - \gamma - m; \frac{a+b}{2b} \right) \\
& \quad [b > a > 0; \operatorname{Re} \gamma > -1/2; \operatorname{Re} \rho > -1].
\end{aligned}$$

$$\begin{aligned}
8. \int_{\pm a}^b (x \mp a)^{\alpha-1} (b-x)^{\gamma-1/2} \left\{ \frac{(x+a)^\sigma}{(a-x)^\rho} \right\} C_m^\gamma \left(\frac{2x \mp a - b}{\mp a + b} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^m (\pm 1)^n}{m! n!} (\varphi+1)_n \left(\frac{1}{2} + \gamma - \alpha \right)_m (2\gamma)_m \Gamma \left[\gamma + 1/2, \alpha \right. \\
\left. \times (2a)^\psi (b \mp a)^{\alpha+\gamma-1/2} {}_4F_3 \left(-\psi-n, \varphi+n+1, \alpha, \alpha-\gamma+\frac{1}{2}; \right. \right. \\
\left. \left. \alpha+\gamma+m+\frac{1}{2}, \frac{1}{2}+\alpha-\gamma-m, \varphi+1; \frac{a \mp b}{2a} \right) \right] \times \\
\left[\operatorname{Re} \alpha > 0; \operatorname{Re} \gamma > -1/2, \begin{cases} b > a > 0 \\ a > b > 0 \end{cases}, \varphi = \begin{cases} \rho \\ \sigma \end{cases}, \psi = \begin{cases} \sigma \\ \rho \end{cases} \right].
\end{aligned}$$

$$\begin{aligned}
9. \int_a^b (x-a)^\rho (b-x)^{\gamma-1/2} (x+a)^{-\tau} C_m^\gamma \left(\frac{2x+a-b}{a+b} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^m (2\gamma)_m}{m! n!} (\tau+\sigma)_n B(\rho+n+1, \tau-\rho-n-1) (2a)^{1+\rho-\tau} \times \\
\times (a+b)^{\gamma-1/2} {}_4F_3 \left(\frac{1}{2}-\gamma-m, \frac{1}{2}+\gamma+m, 1-\tau-\sigma, 1-\tau; \right. \\
\left. 2+n+\rho-\tau, 1-\sigma-\tau-n, \frac{1}{2}+\gamma; \frac{2a}{a+b} \right) + \\
+ \frac{(-1)^m (\rho+\sigma+n+1)_n (2\gamma)_m}{m! n! (2a)^n} \left(\tau+\gamma-\rho-n-\frac{1}{2} \right)_m \times \\
\times \Gamma \left[\gamma+1/2, 1+n+\rho-\tau \right] (a+b)^{\gamma+\rho+n+1/2-\tau} \times \\
\times {}_4F_3 \left(-\rho-n, -\rho-\sigma-n, \tau+\gamma+m-\rho-n-\frac{1}{2}, \tau-\rho-\gamma-m-n-\frac{1}{2}; \right. \\
\left. -\rho-\sigma-2n, \tau+\gamma-\rho-n-\frac{1}{2}, \tau-\rho-n; \frac{2a}{a+b} \right) \\
[b > a > 0; \operatorname{Re} \gamma > -1/2; \operatorname{Re} \rho > -1].
\end{aligned}$$

$$\begin{aligned}
10. \int_a^b (x-a)^{\gamma-1/2} (b-x)^\rho (b+x)^{-\tau} C_m^\gamma \left(\frac{2x+b-a}{a+b} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{b} \right) dx = \\
= \frac{(-1)^n (2\gamma)_m}{m! n!} (1+\sigma)_n \left(\tau+\gamma-\frac{1}{2} \right)_m \Gamma \left[\gamma+1/2, \tau-\gamma-m-1/2 \right] \times \\
\times (2b)^\rho (a+b)^{1/2+\gamma-\tau} {}_4F_3 \left(-\rho-n, \sigma+n+1, \frac{3}{2}-\tau-\gamma, 1-\tau; \right. \\
\left. \frac{3}{2}+\gamma+m-\tau, \frac{3}{2}-m-\tau-\gamma, \sigma+1; \frac{a+b}{2b} \right) + \\
+ \frac{(-1)^n (\gamma)_m 2^{\gamma+\rho-\tau+3m+1/2}}{m! n!} \left(\frac{1}{2}+\sigma+\tau-\gamma-m \right)_n \times \\
\times B \left(\rho+n+1, \frac{1}{2}+m+\gamma-\tau \right) b^{\rho+\gamma+m-\tau+1/2} (a+b)^{-m} \times \\
\times {}_4F_3 \left(\frac{1}{2}-\gamma-m, 1-2\gamma-m, \frac{1}{2}+\sigma+\tau+n-\gamma-m, \tau-\gamma-\rho-m-n-\frac{1}{2}; \right. \\
\left. 1-2\gamma-2m, \frac{1}{2}+\sigma+\tau-\gamma-m, \frac{1}{2}+\tau-\gamma-m; \frac{a+b}{2b} \right) \\
[b > a > 0; \operatorname{Re} \gamma > -1/2; \operatorname{Re} \rho > -1].
\end{aligned}$$

$$\begin{aligned}
 11. \int_a^\infty (x-a)^{\gamma-1/2} (x-b)^\rho C_m^\gamma \left(\frac{2x+b-a}{a+b} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{b} \right) dx = \\
 = \frac{(\rho+\sigma+n+1)_n (2\gamma)_m}{m! n! (2b)^n} \left(\gamma-\rho-n-\frac{1}{2} \right)_m \Gamma \left[\begin{matrix} \gamma+1/2, -\rho-\gamma-m-n-1/2 \\ -\rho-n \end{matrix} \right] \times \\
 \times (a+b)^{\gamma+\rho+n+1/2} {}_3F_2 \left(-\rho-\sigma-n, \gamma+m-\rho-n-\frac{1}{2}, -\rho-\gamma-m-n-\frac{1}{2}; \right. \\
 \left. -\rho-\sigma-2n, \gamma-\rho-n-\frac{1}{2}; \frac{2b}{a+b} \right) \\
 [a>b>0; \operatorname{Re} \gamma > -1/2; \operatorname{Re}(\gamma+\rho) < -m-n-1/2].
 \end{aligned}$$

$$\begin{aligned}
 12. \int_a^\infty (x-a)^{\gamma-1/2} (x-b)^\rho (x+b)^{-\tau} C_m^\gamma \left(\frac{2x+b-a}{a+b} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{b} \right) dx = \\
 = \frac{(\sigma+\rho+n+1)_n (2\gamma)_m}{m! n! (2b)^n} \left(\gamma+\tau-\rho-n-\frac{1}{2} \right)_m \times \\
 \times \Gamma \left[\begin{matrix} \gamma+1/2, \tau-\rho-\gamma-m-n-1/2 \\ \tau-\rho-n \end{matrix} \right] (a+b)^{\gamma+\rho-\tau+n+1/2} \times \\
 \times {}_4F_3 \left(-\rho-n, -\rho-\sigma-n, \gamma+\tau+m-\rho-n-\frac{1}{2}, \tau-\rho-\gamma-m-n-\frac{1}{2}; \right. \\
 \left. -\rho-\sigma-2n, \gamma+\tau-\rho-n-\frac{1}{2}, \tau-\rho-n; \frac{2b}{a+b} \right) \\
 [a>b>0; \operatorname{Re} \gamma > -1/2; \operatorname{Re}(\tau-\gamma-\rho) > m+n+1/2]
 \end{aligned}$$

$$\begin{aligned}
 13. \int_a^\infty (x-a)^\rho (x+a)^{-\tau} (bx+ab-1)^{\gamma-1/2} C_m^\gamma (2bx+2ab-1) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
 = \frac{(\gamma)_m 2^{\gamma+\rho-\tau+3m+1/2}}{m! n!} \left(\frac{1}{2} + \tau + \sigma - \gamma - m \right)_n B \left(\rho+n+1, \tau-\rho-\gamma-m-n-\frac{1}{2} \right) \times \\
 \times a^{1/2+m+\rho+\gamma-\tau} b^{\gamma+m-1/2} {}_4F_3 \left(\frac{1}{2} - \gamma - m, 1-2\gamma-m, \frac{1}{2} + \tau + \sigma - \gamma + n - m, \right. \\
 \left. \tau-\rho-\gamma-m-n-\frac{1}{2}; 1-2\gamma-2m, \frac{1}{2} + \tau + \sigma - \gamma - m, \frac{1}{2} + \tau - \gamma - m; \frac{1}{2ab} \right) \\
 [a>0; 2ab>1; \operatorname{Re} \rho > -1; \operatorname{Re}(\tau-\gamma-\rho) > m+n+1/2].
 \end{aligned}$$

$$\begin{aligned}
 14. \int_{-a}^a (x+a)^\sigma (a-x)^{\beta-1} \operatorname{erf}(ab-bx) C_n^{\sigma+1/2} \left(\frac{x}{a} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
 = \frac{(\rho+1)_n (2\sigma+1)_n 2^{\beta+\sigma+2} a^{\beta+\sigma+1} b}{n! \sqrt{\pi}} \Gamma \left[\begin{matrix} \beta+1, \sigma+1 \\ \beta+\sigma+n+2 \end{matrix} \right] \times \\
 \times {}_2F_3 \left(\frac{1}{2}, \frac{\beta+1}{2}, \frac{\beta}{2}+1; \frac{3}{2}, \frac{\beta+\sigma+n}{2}+1, \frac{\beta+\sigma+n+3}{2}; -a^2 b^2 \right) \\
 [a>0; \operatorname{Re} \beta, \operatorname{Re} \sigma > -1].
 \end{aligned}$$

$$\begin{aligned}
 15. \int_{-a}^a (x+a)^\sigma (a-x)^{\beta-1} \operatorname{erf}(b\sqrt{a-x}) C_n^{\sigma+1/2} \left(\frac{x}{a} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
 = \frac{(\rho+1)_n (2\sigma+1)_n 2^{\beta+\sigma+3/2} a^{\beta+\sigma+1/2} b}{n! \sqrt{\pi}} \Gamma \left[\begin{matrix} \beta+1/2, \sigma+1 \\ \beta+\sigma+n+3/2 \end{matrix} \right] \times \\
 \times {}_2F_2 \left(\frac{1}{2}, \beta+\frac{1}{2}; \frac{3}{2}, \beta+\sigma+n+\frac{3}{2}; -ab^2 \right) \\
 [a>0; \operatorname{Re} \beta > -1/2, \operatorname{Re} \sigma > -1].
 \end{aligned}$$

2.22.17. Интегралы от $A(x) C_m^\gamma (b \sqrt{x+a}) P_n^{(\rho, \sigma)}(cx)$.

$$1. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^\rho (1-ab^2-b^2x)^{\gamma-1/2} C_{2m+\varepsilon}^\gamma (b \sqrt{x+a}) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^{m+n}}{m! n!} (\gamma)_{m+\varepsilon} \left(1 + \sigma - \alpha - \frac{\varepsilon}{2} \right)_n B \left(\rho + n + 1, \alpha + \frac{\varepsilon}{2} \right) 2^{\alpha+\rho+3\varepsilon/2} \times$$

$$\times a^{\alpha+\rho+\varepsilon/2} b^\varepsilon {}_4F_3 \left(\frac{1}{2} - \gamma - m, \frac{1}{2} + \varepsilon + m, \alpha + \frac{\varepsilon}{2}, \frac{\varepsilon}{2} + \alpha - \sigma; \right.$$

$$\left. \frac{\varepsilon}{2} + \alpha + \rho + n + 1, \frac{1}{2} + \varepsilon, \frac{\varepsilon}{2} + \alpha - \sigma - n; 2ab^2 \right)$$

[$\varepsilon=0$ или 1 ; $a, b > 0$; $2ab^2 < 1$; $\operatorname{Re} \alpha > -\varepsilon/2$; $\operatorname{Re} \rho > -1$].

$$2. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\gamma+\rho-1/2} C_{2m+\varepsilon}^\gamma \left(\sqrt{\frac{x+a}{2a}} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^{m+n}}{m! n!} (\gamma)_{m+\varepsilon} \left(1 + \sigma - \alpha - \frac{\varepsilon}{2} \right)_n B \left(\rho + n + 1, \alpha + \frac{\varepsilon}{2} \right) \times$$

$$\times 2^{\alpha+\rho+\gamma+\varepsilon-1/2} a^{\alpha+\rho+\gamma-1/2} {}_4F_3 \left(\frac{1}{2} - \gamma - m, \frac{1}{2} + \varepsilon + m, \alpha + \frac{\varepsilon}{2}, \right.$$

$$\left. \frac{\varepsilon}{2} + \alpha - \sigma; \frac{\varepsilon}{2} + \alpha + \rho + n + 1, \varepsilon + \frac{1}{2}, \frac{\varepsilon}{2} + \alpha - \sigma - n; 1 \right)$$

[$\varepsilon=0$ или 1 ; $a > 0$; $\operatorname{Re} \alpha > -\varepsilon/2$; $\operatorname{Re}(\gamma + \rho) > -1/2$].

$$3. \int_{\pm a}^b (x \mp a)^{\alpha-1} (b-x)^{\gamma-1/2} \left\{ \begin{matrix} (x+a)^\sigma \\ (a-x)^\rho \end{matrix} \right\} C_{2m+\varepsilon}^\gamma \left(\sqrt{\frac{x \mp a}{b \mp a}} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^m (\pm 1)^n}{n! (2m+\varepsilon)!} (2\gamma)_{2m+\varepsilon} (1+\varphi)_n \left(\frac{1+\varepsilon}{2} - \alpha \right)_m \times$$

$$\times \Gamma \left[\begin{matrix} \gamma + 1/2, \alpha + \varepsilon/2 \\ \alpha + \gamma + m + (\varepsilon + 1)/2 \end{matrix} \right] (b \mp a)^{\alpha+\gamma-1/2} (2a)^\psi {}_4F_3 \left(-\psi - n, \right.$$

$$\left. \varphi + n + 1, \alpha, \alpha + \frac{1}{2}; \varphi + 1, \frac{1-\varepsilon}{2} - \alpha - m, \alpha + \gamma + m + \frac{\varepsilon+1}{2}; \frac{a \mp b}{2a} \right)$$

[$\varepsilon=0$ или 1 ; $\operatorname{Re} \alpha > -\varepsilon/2$; $\operatorname{Re} \gamma > -1/2$, $\left\{ \begin{matrix} b > a > 0 \\ a > b > 0 \end{matrix} \right\}$, $\varphi = \left\{ \begin{matrix} \rho \\ \sigma \end{matrix} \right\}$, $\psi = \left\{ \begin{matrix} \sigma \\ \rho \end{matrix} \right\}$].

$$4. \int_a^b (x-a)^\rho (b-x)^{\gamma-1/2} C_{2m}^\gamma \left(\sqrt{\frac{x+a}{b+a}} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx =$$

$$= \frac{(-1)^m (2\gamma)_{2m}}{(2m)! n! (2a)^n} (\sigma + \rho + n + 1)_n \left(-\rho - n - \frac{1}{2} \right)_m \times$$

$$\times \Gamma \left[\begin{matrix} \gamma + 1/2, \rho + n + 1 \\ 3/2 + \rho + \gamma + m + n \end{matrix} \right] (a+b)^{\gamma+\rho+n+1/2} {}_3F_2 \left(-\rho - \sigma - n, \right.$$

$$\left. m - n - \rho - \frac{1}{2}, -m - n - \rho - \gamma - \frac{1}{2}; -\rho - \sigma - 2n, -\rho - n - \frac{1}{2}; \frac{2a}{a+b} \right)$$

[$b > a > 0$; $\operatorname{Re} \rho > -1$; $\operatorname{Re} \gamma > -1/2$].

$$5. \int_a^b (x-a)^{\gamma-1/2} (b-x)^\rho C_{2m}^\gamma \left(\sqrt{\frac{x+b}{a+b}} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{b} \right) dx =$$

$$= \frac{(-1)^n (\gamma+m)_m}{(2m)! n!} \left(\sigma - \gamma - m + \frac{1}{2} \right)_n B \left(\rho + n + 1, \frac{1}{2} + \gamma + m \right) \times$$

$$\times 2^{\rho+\gamma+3m+1/2} b^{\rho+\gamma+m+1/2} (a+b)^{-m} {}_3F_2 \left(1-\gamma-m, \frac{1}{2}+n+\sigma-\gamma-m, \right. \\ \left. -\rho-\gamma-m-n-\frac{1}{2}; 1-\gamma-2m, \sigma-\gamma-m+\frac{1}{2}; \frac{a+b}{2b} \right) \\ [b>a>0; \operatorname{Re} \rho > -1; \operatorname{Re} \gamma > -1/2].$$

$$6. \int_a^b (x-a)^{\rho} (b-x)^{\gamma-1/2} (x+a)^{-\tau} C_{2m+\varepsilon}^{\gamma} \left(\sqrt{\frac{x+a}{b+a}} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\ = \frac{(-1)^m (\gamma)_{m+\varepsilon}}{m! n!} \left(\tau + \sigma - \frac{\varepsilon}{2} \right)_n B \left(\rho + n + 1, \tau - \rho - n - \frac{\varepsilon}{2} - 1 \right) \times \\ \times 2^{1+\rho+3\varepsilon/2-\tau} a^{1+\rho+\varepsilon/2-\tau} (a+b)^{\gamma-(\varepsilon+1)/2} {}_4F_3 \left(\frac{1}{2}-\gamma-m, \right. \\ \left. \frac{1}{2}+\varepsilon+m, 1+\frac{\varepsilon}{2}-\tau-\sigma, 1+\frac{\varepsilon}{2}-\tau; \right. \\ \left. 2+\frac{\varepsilon}{2}+n+\rho-\tau, \frac{1}{2}+\varepsilon, 1+\frac{\varepsilon}{2}-\tau-\sigma-n; \frac{2a}{a+b} \right) + \\ + \frac{(-1)^m (2\gamma)_{2m+\varepsilon}}{(2m+\varepsilon)! n! (2a)^n} (\rho + \sigma + n + 1)_n \left(\frac{\varepsilon-1}{2} + \tau - \rho - n \right)_m \times \\ \times \Gamma \left[\frac{\gamma+1/2, \varepsilon/2+\rho+n+1-\tau}{(3+\varepsilon)/2+\rho+\gamma+m+n-\tau} \right] (a+b)^{\gamma+\rho+n-\tau+1/2} \times \\ \times {}_4F_3 \left(-\rho-n, -\rho-\sigma-n, \frac{\varepsilon-1}{2} + \tau - \rho - n + m, \tau - m - n - \rho - \gamma - \frac{\varepsilon+1}{2}; \right. \\ \left. -\rho-\sigma-2n, \tau-\rho-n, \tau-\rho-n-\frac{1}{2}; \frac{2a}{a+b} \right) \\ [\varepsilon=0 \text{ или } 1; b>a>0; \operatorname{Re} \rho > -1; \operatorname{Re} \gamma > -1/2].$$

$$7. \int_a^b (x-a)^{\gamma-1/2} (b-x)^{\rho} (x+b)^{-\tau} C_{2m+\varepsilon}^{\gamma} \left(\sqrt{\frac{x+b}{a+b}} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{b} \right) dx = \\ = \frac{(-1)^n (2\gamma)_{2m+\varepsilon}}{n! (2m+\varepsilon)!} (1+\sigma)_n \left(\frac{\varepsilon-1}{2} + \tau \right)_m \Gamma \left[\frac{\gamma+1/2, \tau-\gamma-m-(\varepsilon+1)/2}{\tau-\varepsilon/2} \right] \times \\ \times (2b)^{\rho} (a+b)^{1/2+\gamma-\tau} {}_4F_3 \left(-\rho-n, 1+\sigma+n, \frac{3}{2}-\tau, 1-\tau; \right. \\ \left. \frac{3+\varepsilon}{2}+\gamma+m-\tau, \frac{3-\varepsilon}{2}-\tau-m, 1+\sigma; \frac{a+b}{2b} \right) + \\ + \frac{(-1)^n (\varepsilon+\gamma+m)_m}{n! (2m+\varepsilon)!} \left(\sigma + \tau - \gamma - m + \frac{1-\varepsilon}{2} \right)_n B \left(\rho + n + 1, \frac{1+\varepsilon}{2} + \gamma + m - \tau \right) \times \\ \times 2^{\rho+\gamma+3m-\tau+(3\varepsilon+1)/2} b^{\rho+\gamma+m-\tau+(\varepsilon+1)/2} (a+b)^{-m-\varepsilon/2} \times \\ \times {}_4F_3 \left(\frac{1}{2}-\gamma-m, 1-\varepsilon-\gamma-m, \sigma+\tau-\gamma-m+n+\frac{1-\varepsilon}{2}, \right. \\ \left. \tau-\rho-\gamma-m-n-\frac{\varepsilon+1}{2}; \frac{1-\varepsilon}{2}-\gamma-m+\tau, 1-\varepsilon-\gamma-2m, \right. \\ \left. \sigma+\tau-\gamma-m+\frac{1-\varepsilon}{2}; \frac{a+b}{2b} \right) \\ [\varepsilon=0 \text{ или } 1; b>a>0; \operatorname{Re} \rho > -1; \operatorname{Re} \gamma > -1/2].$$

$$\begin{aligned}
8. \int_a^\infty (x-a)^\rho (x+a)^{-\tau} (b^2x+ab^2-1)^{\gamma-1/2} C_{2m+\varepsilon}^\gamma (b\sqrt{x+a}) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
= \frac{(\gamma)_{2m+\varepsilon}}{(2m+\varepsilon)! n!} \left(\tau+\sigma-\gamma-m+\frac{1-\varepsilon}{2}\right)_n \times \\
\times 2^{\rho+\gamma+3m-\tau+(3\varepsilon+1)/2} a^{\rho+\gamma+m-\tau+(\varepsilon+1)/2} b^{2\gamma+\varepsilon+2m-1} \times \\
\times B\left(\rho+n+1, \tau-\gamma-\rho-m-n-\frac{\varepsilon+1}{2}\right) {}_4F_3\left(\frac{1}{2}-m-\gamma, 1-\varepsilon-\gamma-m, \right. \\
\left. \tau+\sigma-\gamma-m+n+\frac{1-\varepsilon}{2}, \tau-\gamma-\rho-m-n-\frac{\varepsilon+1}{2}; 1-\varepsilon-\gamma-2m, \right. \\
\left. \tau+\sigma-\gamma-m+\frac{1-\varepsilon}{2}, \tau-\gamma-m+\frac{1-\varepsilon}{2}; \frac{1}{2ab^2}\right) \\
[\varepsilon=0 \text{ или } 1; a>0; 2ab^2>1; \operatorname{Re} \rho>-1; \operatorname{Re}(\rho-\tau+\gamma)<-m-n-(\varepsilon+1)/2].
\end{aligned}$$

$$\begin{aligned}
9. \int_a^\infty (x-a)^{\gamma-1/2} (x-b)^\rho (x+b)^{-\tau} C_{2m+\varepsilon}^\gamma \left(\sqrt{\frac{x+b}{a+b}}\right) P_n^{(\rho, \sigma)}\left(\frac{x}{b}\right) dx = \\
= \frac{(\sigma+\rho+n+1)_n}{(2m+\varepsilon)! n! (2b)^n} (2\gamma)_{2m+\varepsilon} \left(\frac{\varepsilon-1}{2}+\tau-\rho-n\right)_m \times \\
\times \Gamma\left[\gamma+1/2, \tau-\rho-\gamma-m-n-(\varepsilon+1)/2\right] (a+b)^{\gamma+\rho+n-\tau+1/2} \times \\
\times {}_4F_3\left(-\rho-n, -\rho-\sigma-n, \frac{\varepsilon-1}{2}+\tau+m-\rho-n, \tau-\rho-\gamma-m-n-\frac{\varepsilon+1}{2}; \right. \\
\left. -\rho-\sigma-2n, \tau-\rho-n-\frac{1}{2}, \tau-\rho-n; \frac{2b}{a+b}\right) \\
[\varepsilon=0 \text{ или } 1; a>b>0; \operatorname{Re} \gamma>-1/2; \operatorname{Re}(\rho+\gamma-\tau)<-m-n-(\varepsilon+1)/2].
\end{aligned}$$

$$\begin{aligned}
10. \int_a^\infty (x-a)^{\gamma+\rho-1/2} (x+a)^{-\tau} C_{2m+\varepsilon}^\gamma \left(\sqrt{\frac{x+a}{2a}}\right) P_n^{(\rho, \sigma)}\left(\frac{x}{a}\right) dx = \\
= \frac{(\sigma+\rho+n+1)_n}{(2m+\varepsilon)! n!} (2\gamma)_{2m+\varepsilon} \left(\frac{\varepsilon-1}{2}+\tau-\rho-n\right)_m \times \\
\times \Gamma\left[\gamma+1/2, \tau-\rho-\gamma-m-n-(\varepsilon+1)/2\right] (2a)^{\gamma+\rho-\tau+1/2} \times \\
\times {}_4F_3\left(-\rho-n, -\rho-\sigma-n, \frac{\varepsilon-1}{2}+\tau+m-\rho-n, \tau-\rho-\gamma-m-n-\frac{\varepsilon+1}{2}; \right. \\
\left. -\rho-\sigma-2n, \tau-\rho-n-\frac{1}{2}, \tau-\rho-n; 1\right) \\
[\varepsilon=0 \text{ или } 1; a>0; -1/2<\operatorname{Re}(\gamma+\rho)<\operatorname{Re} \tau-m-n-(\varepsilon+1)/2].
\end{aligned}$$

2.22.18. Интегралы от $A(x) P_m^{(\lambda, \nu)}(a+bx) P_n^{(\rho, \sigma)}(cx)$.

$$\begin{aligned}
1. \int_{-a}^a (x+a)^\sigma P_m^{(0, \nu)}\left(\frac{x}{a}\right) P_n^{(0, \sigma)}\left(\frac{x}{a}\right) dx = \\
= \frac{(-1)^m (2n)! (-m)_n (\nu+n+1)_m (\sigma+n-\nu-m+1)_n (2a)^{\sigma+1}}{m! n! (-m-\nu)_n (\sigma+n+1)_{2n+1}} \\
a>0; m>n; \operatorname{Re} \sigma>-1]
\end{aligned}$$

$$\begin{aligned}
2. \int_{-a}^a (x+a)^{\nu+\sigma} (a-x)^{\beta-1} P_m^{(\lambda, \nu)} \left(\frac{x}{a} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \frac{(\lambda+1)_m}{m! n!} (1+\rho-\beta)_n B(\beta, \sigma+n+1) (2a)^{\beta+\sigma+\nu} \times \\
\times {}_4F_3(-m-\nu, \lambda+m+1, \beta-\rho, \beta; \beta+\sigma+n+1, \lambda+1, \beta-\rho-n; 1) \\
[a, \operatorname{Re} \beta > 0; \operatorname{Re}(\sigma+\nu) > -1].
\end{aligned}$$

$$\begin{aligned}
3. \int_{-a}^a (x+a)^{\nu+\sigma} (a-x)^\rho P_m^{(\lambda, \nu)} \left(\frac{x}{a} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^n}{m! n!} (1+\lambda+n)_m (-m-\nu)_n B(\rho+n+1, \sigma+n+1) (2a)^{\nu+\rho+\sigma+1} \times \\
\times {}_3F_2(n-m-\nu, \lambda+m+n+1, \rho+n+1; \rho+\sigma+2n+2, \lambda+n+1; 1) \\
[a, n > 0; \operatorname{Re} \rho, \operatorname{Re}(\nu+\sigma) > -1].
\end{aligned}$$

$$\begin{aligned}
4. \int_{-a}^a (x+a)^{2\sigma} (a-x)^\rho \left[P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) \right]^2 dx = \\
= \left[\frac{(\sigma+1)_n}{n!} \right]^2 B(\rho+2n+1, 2\sigma+1) (2a)^{\rho+2\sigma+1} \\
[a > 0; \operatorname{Re} \rho > -1; \operatorname{Re} \sigma > -1/2].
\end{aligned}$$

$$\begin{aligned}
5. \int_{-a}^a (x+a)^{\alpha-1} (a-x)^{\lambda+\rho} P_m^{(\lambda, \nu)} \left(\frac{x}{a} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^{m+n}}{m! n!} (1+\nu)_m (1+\sigma-\alpha)_n B(\alpha, \rho+n+1) (2a)^{\alpha+\rho+\lambda} \times \\
\times {}_4F_3(-m-\lambda, \nu+m+1, \alpha-\sigma, \alpha; \alpha+\rho+n+1, \nu+1, \alpha-\sigma-n; 1) \\
[a, \operatorname{Re} \alpha > 0; \operatorname{Re}(\lambda+\rho) > -1].
\end{aligned}$$

$$\begin{aligned}
6. \int_{-a}^a (x+a)^\sigma (a-x)^{\lambda+\rho} P_m^{(\lambda, \nu)} \left(\frac{x}{a} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \frac{(-1)^m}{m! n!} (1+\nu+n)_m (-m-\lambda)_n B(\rho+n+1, \sigma+n+1) (2a)^{\rho+\sigma+\lambda+1} \times \\
\times {}_3F_2(n-m-\lambda, \nu+m+n+1, \sigma+n+1; \rho+\sigma+2n+2, \nu+n+1; 1) \\
[a, n > 0; \operatorname{Re}(\lambda+\rho), \operatorname{Re} \sigma > -1].
\end{aligned}$$

$$\begin{aligned}
7. \int_{-a}^a (x+a)^\sigma (a-x)^{2\rho} \left[P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) \right]^2 dx = \\
= \left[\frac{(\rho+1)_n}{n!} \right]^2 B(2\rho+1, \sigma+2n+1) (2a)^{2\rho+\sigma+1} \\
[a > 0; \operatorname{Re} \rho > -1/2; \operatorname{Re} \sigma > -1].
\end{aligned}$$

$$\begin{aligned}
8. \int_{-a}^a (x+a)^\nu (a-x)^\rho P_m^{(\rho, \nu)} \left(\frac{x}{a} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
= \frac{(2a)^{\nu+\rho+1}}{m! (n-m)!} \Gamma \left[\begin{matrix} \rho+\sigma+m+n+1, \rho+n+1, \nu+m+1, 1-\sigma+\nu \\ \rho+\sigma+n+1, \rho+\nu+m+n+2, 1+m-\sigma+\nu-n \end{matrix} \right] \\
[a > 0; \operatorname{Re} \nu, \operatorname{Re} \rho > -1]
\end{aligned}$$

$$\begin{aligned}
9. \quad & \int_a^\infty (x-a)^{\rho+\lambda} (x+a)^{-\tau} P_m^{(\lambda, \nu)} \left(\frac{x}{a} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
& = \frac{(\lambda + \nu + m + 1)_m}{m! n!} (\tau + \sigma - \lambda - m)_n B(\rho + n + 1, \tau - \lambda - \rho - m - n - 1) (2a)^{1+\lambda+\rho-\tau} \times \\
& \quad \times {}_4F_3(-\lambda - m, -\nu - \lambda - m, \tau + \sigma - \lambda + n - m, \tau - \lambda - \rho - m - n - 1; \\
& \quad -\nu - \lambda - 2m, \tau + \sigma - \lambda - m, \tau - \lambda - m; 1) \\
& \quad [a > 0; -1 < \operatorname{Re}(\lambda + \rho) < \operatorname{Re} \tau - m - n - 1].
\end{aligned}$$

$$\begin{aligned}
10. \quad & \int_{-a}^a \frac{x}{x-y} (x+a)^\sigma (a-x)^\rho P_m^{(\rho, \sigma)} \left(\frac{x}{a} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
& = \frac{2^{\rho+\sigma+1} a^{\rho+\sigma+3}}{n! (y^2 - a^2)} \Gamma \left[\begin{matrix} \rho + n + 1, \sigma + n + 1 \\ \rho + \sigma + n + 1, \rho + \sigma + 2n + 1 \end{matrix} \right] \delta_{m, n} - \\
& \quad - 2y (y-a)^\rho (y+a)^\sigma P_m^{(\rho, \sigma)} \left(\frac{y}{a} \right) Q_n^{(\rho, \sigma)} \left(\frac{y}{a} \right) \\
& \quad [m \leq n; a, y > 0; \operatorname{Re} \rho, \operatorname{Re} \sigma > -1].
\end{aligned}$$

$$\begin{aligned}
11. \quad & \int_{-a}^a \frac{x^{n-m+1}}{x-y} (x+a)^\sigma (a-x)^\rho P_m^{(\rho, \sigma)} \left(\frac{x}{a} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
& = \frac{1}{m!} \Gamma \left[\begin{matrix} \rho + n + 1, \sigma + n + 1, \rho + \sigma + 2m + 1 \\ \rho + \sigma + 2n + 2, \rho + \sigma + m + 1 \end{matrix} \right] (2a)^{\rho+\sigma+n-m+1} - \\
& \quad - 2 (y-a)^\rho (y+a)^\sigma y^{n-m+1} P_m^{(\rho, \sigma)} \left(\frac{y}{a} \right) Q_n^{(\rho, \sigma)} \left(\frac{y}{a} \right) \\
& \quad [m < n; a, y > 0; \operatorname{Re} \rho, \operatorname{Re} \sigma > -1].
\end{aligned}$$

$$\begin{aligned}
12. \quad & \int_{-a}^a (x+a)^{\alpha-1} (a-x)^\rho P_m^{(\rho, \nu)} (bx+ab-1) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
& = \frac{(-1)^{m+n}}{m! n!} (1+\nu)_m (1+\sigma-\alpha)_n B(\rho+n+1, \alpha) (2a)^{\alpha+\rho} \times \\
& \quad \times {}_4F_3(-m, \nu+m+1, \alpha-\sigma, \alpha; \alpha+\rho+n+1, \nu+1, \alpha-\sigma-n; ab) \\
& \quad [a, b, \operatorname{Re} \alpha > 0; ab < 1; \operatorname{Re} \rho > -1].
\end{aligned}$$

$$\begin{aligned}
13. \quad & \int_{-a}^a (x+a)^{\alpha-1} (a-x)^\rho (1-ab-bx)^\lambda P_m^{(\lambda, \nu)} (2bx+2ab-1) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
& = \frac{(-1)^{m+n}}{m! n!} (1+\nu)_m (1+\sigma-\alpha)_n B(\rho+n+1, \alpha) (2a)^{\alpha+\rho} \times \\
& \quad \times {}_4F_3(-m-\lambda, \nu+m+1, \alpha-\sigma, \alpha; \alpha+\rho+n+1, \nu+1, \alpha-\sigma-n; 2ab) \\
& \quad [a, b, \operatorname{Re} \alpha > 0; 2ab < 1; \operatorname{Re} \rho > -1].
\end{aligned}$$

$$\begin{aligned}
14. \quad & \int_a^b (x-a)^\rho (b-x)^\lambda P_m^{(\lambda, \nu)} \left(\frac{2x+a-b}{a+b} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
& = \frac{(-1)^m (\rho+\sigma+n+1)_n}{m! n! (2a)^n} (\nu-\rho-n)_m B(\lambda+m+1, \rho+n+1) (a+b)^{\lambda+\rho+n+1} \times \\
& \quad \times {}_3F_2(-\rho-\sigma-n, \nu-\rho+m-n, -\lambda-\rho-m-n-1; \\
& \quad -\rho-\sigma-2n, \nu-\rho-n; \frac{2a}{a+b}) \\
& \quad [b > a > 0; \operatorname{Re} \lambda, \operatorname{Re} \rho > -1].
\end{aligned}$$

$$\begin{aligned}
15. \quad & \int_a^b (x-a)^\rho (b-x)^\lambda P_m^{(\lambda, -\sigma)} \left(\frac{2x+a-b}{a+b} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
& = \frac{(-1)^{m+n} (\lambda+1)_m (\rho+1)_n}{m! n!} B(\lambda+1, \rho+1) \left(\frac{a+b}{2a} \right)^{\lambda-\sigma+m+1} (b-a)^{\lambda+\rho+1} \times \\
& \quad \times {}_2F_1 \left(\lambda+\rho+m+n+2, m-n+\lambda-\sigma+1; \lambda+\rho+2; \frac{a-b}{2a} \right) \\
& \quad [b > a > 0; \operatorname{Re} \lambda, \operatorname{Re} \rho > -1].
\end{aligned}$$

$$\begin{aligned}
16. \quad & \int_a^b (x-a)^\lambda (b-x)^\rho P_m^{(\lambda, \nu)} \left(\frac{2x+b-a}{a+b} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{b} \right) dx = \\
& = \frac{(-1)^n}{m! n!} (2b)^{\rho+\lambda+m+1} (a+b)^{-m} (\nu+\lambda+m+1)_m (\sigma-\lambda-m)_n \times \\
& \times B(\rho+n+1, \lambda+m+1) {}_2F_2 \left(-\nu-\lambda-m, \sigma+n-\lambda-m, -\lambda-\rho-m-n-1; \right. \\
& \quad \left. -\nu-\lambda-2m, \sigma-\lambda-m; \frac{a+b}{2b} \right) \\
& \quad [a, b > 0; \operatorname{Re} \lambda, \operatorname{Re} \rho > -1].
\end{aligned}$$

$$\begin{aligned}
17. \quad & \int_{\pm a}^b (x \mp a)^{\alpha-1} (b-x)^\lambda \left\{ \frac{(x+a)^\sigma}{(a-x)^\rho} \right\} P_m^{(\lambda, \nu)} \left(\frac{2x \mp a-b}{\mp a+b} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
& = \frac{(-1)^m (\pm 1)^n}{m! n!} (\varphi+1)_n (1+\nu-\alpha)_m B(\lambda+m+1, \alpha) (2a)^\psi (b \mp a)^{\alpha+\lambda} \times \\
& \times {}_4F_3 \left(-\psi-n, \varphi+n+1, \alpha, \alpha-\nu; \alpha+\lambda+m+1, \alpha-\nu-m, \varphi+1; \frac{a \mp b}{2a} \right) \\
& \quad \left[\operatorname{Re} \alpha > 0; \operatorname{Re} \lambda > -1, \left\{ \begin{matrix} b > a > 0 \\ a > b > 0 \end{matrix} \right\}, \varphi = \left\{ \begin{matrix} \rho \\ \sigma \end{matrix} \right\}, \psi = \left\{ \begin{matrix} \sigma \\ \rho \end{matrix} \right\} \right].
\end{aligned}$$

$$\begin{aligned}
18. \quad & \int_a^b (x-a)^\rho (b-x)^\lambda (x+a)^{-\tau} P_m^{(\lambda, \nu)} \left(\frac{2x+a-b}{a+b} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
& = \frac{(-1)^m}{m! n!} (1+\nu)_m (\tau+\sigma)_n B(\rho+n+1, \tau-\rho-n-1) (2a)^{1+\rho-\tau} (a+b)^\lambda \times \\
& \quad \times {}_4F_3 \left(-\lambda-m, \nu+m+1, 1-\tau-\sigma, 1-\tau; 2+n+\rho-\tau, \right. \\
& \quad \left. 1-\sigma-\tau-n, \nu+1; \frac{2a}{a+b} \right) + \frac{(-1)^m (\rho+\sigma+n+1)_n}{m! n! (2a)^n} \times \\
& \quad \times (\tau+\nu-\rho-n)_m B(\lambda+m+1, 1+n+\rho-\tau) (a+b)^{\lambda+\rho-\tau+n+1} \times \\
& \quad \times {}_4F_3 \left(-\rho-n, -\rho-\sigma-n, \tau+\nu+m-\rho-n, \tau-\rho-\lambda-m-n-1; \right. \\
& \quad \left. -\rho-\sigma-2n, \tau+\nu-\rho-n, \tau-\rho-n; \frac{2a}{a+b} \right) \\
& \quad [b > a > 0; \operatorname{Re} \lambda, \operatorname{Re} \rho > -1].
\end{aligned}$$

$$\begin{aligned}
19. \quad & \int_a^b (x-a)^\lambda (b-x)^\rho (b+x)^{-\tau} P_m^{(\lambda, \nu)} \left(\frac{2x+b-a}{a+b} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{b} \right) dx = \\
& = \frac{(-1)^n}{m! n!} (2b)^\rho (a+b)^{\lambda-\tau+1} (1+\sigma)_n (\tau+\nu)_m B(\lambda+m+1, \tau-\lambda-m-1) \times \\
& \quad \times {}_4F_3 \left(-\rho-n, \sigma+n+1, 1-\tau-\nu, 1-\tau; 2+\lambda+m-\tau, \right.
\end{aligned}$$

$$\begin{aligned}
& 1-m-\tau-v, \sigma+1; \frac{a+b}{2b} \Big) + \frac{(-1)^n}{m! n!} (2b)^{\rho+\lambda+m-\tau+1} (a+b)^{-m} \times \\
& \times (v+\lambda+m+1)_m (\sigma+\tau-\lambda-m)_n B(\rho+n+1, 1+m+\lambda-\tau) \times \\
& \times {}_2F_2 \left(-\lambda-m, -v-\lambda-m, \sigma+\tau+n-\lambda-m, \tau-\lambda-\rho-m-n-1; \right. \\
& \quad \left. -v-\lambda-2m, \sigma+\tau-\lambda-m, \tau-\lambda-m; \frac{a+b}{2b} \right) \\
& \quad [b > a > 0; \operatorname{Re} \lambda, \operatorname{Re} \rho > -1]
\end{aligned}$$

$$\begin{aligned}
20. \quad & \int_a^\infty (x-a)^\lambda (x-b)^\rho P_m^{(\lambda, v)} \left(\frac{2x+b-a}{a+b} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{b} \right) dx = \\
& = \frac{(\rho+\sigma+n+1)_n}{m! n! (2b)^n} (v-\rho-n)_m B(\lambda+m+1, -\rho-\lambda-m-n-1) (a+b)^{\lambda+\rho+n+1} \times \\
& \times {}_2F_2 \left(-\rho-\sigma-n, v+m-\rho-n, -\rho-\lambda-m-n-1; -\rho-\sigma-2n, v-\rho-n; \frac{2b}{a+b} \right) \\
& \quad [a > b > 0; \operatorname{Re} \lambda > -1; \operatorname{Re} (\rho+\lambda) < -m-n-1]
\end{aligned}$$

$$\begin{aligned}
21. \quad & \int_a^\infty (x-a)^\lambda (x-b)^\rho (x+b)^{-\tau} P_m^{(\lambda, v)} \left(\frac{2x+b-a}{a+b} \right) P_n^{(\rho, \sigma)} \left(\frac{x}{b} \right) dx = \\
& = \frac{(\sigma+\rho+n+1)_n}{m! n! (2b)^n} (v+\tau-\rho-n)_m B(\lambda+m+1, \tau-\rho-\lambda-m-n-1) \times \\
& \times (a+b)^{\lambda+\rho+n-\tau+1} {}_2F_2 \left(-\rho-n, -\rho-\sigma-n, v+\tau+m-\rho-n, \right. \\
& \quad \left. \tau-\rho-\lambda-m-n-1; -\rho-\sigma-2n, v+\tau-\rho-n, \tau-\rho-n; \frac{2b}{a+b} \right) \\
& \quad [a > b > 0; \operatorname{Re} \lambda > -1; \operatorname{Re} (\rho+\lambda-\tau) < -m-n-1]
\end{aligned}$$

$$\begin{aligned}
22. \quad & \int_a^\infty (x-a)^\rho (x+a)^{-\tau} (bx+ab-1)^\lambda P_m^{(\lambda, v)} (2bx+2ab-1) P_n^{(\rho, \sigma)} \left(\frac{x}{a} \right) dx = \\
& = \frac{(\lambda+v+m+1)_m}{m! n!} (\tau+\sigma-\lambda-m)_n B(\rho+n+1, \tau-\rho-\lambda-m-n-1) \times \\
& \times (2a)^{\rho+\lambda-\tau+m+1} b^{\lambda+m} {}_2F_2 \left(-\lambda-m, -v-\lambda-m, \tau+\sigma-\lambda+n-m, \right. \\
& \quad \left. \tau-\rho-\lambda-m-n-1; -v-\lambda-2m, \tau+\sigma-\lambda-m, \tau-\lambda-m; \frac{1}{2ab} \right) \\
& \quad [a > 0; 2ab > 1; \operatorname{Re} \rho > -1; \operatorname{Re} (\rho+\lambda-\tau) < -m-n-1]
\end{aligned}$$

Глава 3. КРАТНЫЕ ИНТЕГРАЛЫ

3.1. ВВЕДЕНИЕ

В этой главе помещены двойные, тройные и многочерные интегралы, содержащие некоторые специальные функции. Основным способом вычисления этих интегралов является сведение их к повторным интегралам, которые находят путем последовательного интегрирования по каждой из переменных в отдельности. Кроме того, часто используется метод замены переменных интегрирования. Ряд формул общего характера, с помощью которых кратные интегралы могут быть приведены к одномерным, имеется в книге [12].

3.2. ДВОЙНЫЕ ИНТЕГРАЛЫ

3.2.1. Общие формулы.

Обозначение: $\hat{f}(p) = \int_0^{\infty} e^{-px} f(x) dx$.

$$1. \int_0^{\infty} \int_0^{\infty} e^{-px-xy} f(x+y) dx dy = -\frac{\hat{f}(p) - \hat{f}(q)}{p-q}.$$

$$2. \int_0^{\infty} \int_0^{\infty} e^{-px-xy} f(|x-y|) dx dy = \frac{\hat{f}(p) + \hat{f}(q)}{p+q}.$$

$$3. \int_0^{\infty} \int_0^{\infty} e^{-px-xy} f(xy) dx dy = 2 \int_0^{\infty} K_0(2\sqrt{pqt}) f(t) dt.$$

$$4. \int_0^{\infty} \int_0^{\infty} e^{-px-xy} f(\sqrt{x^2+y^2}) dx dy =$$

$$= - \int_p^R \frac{1}{\sqrt{R^2-t^2}} \frac{d}{dt} \hat{f}(t) dt - \int_q^R \frac{1}{\sqrt{R^2-t^2}} \frac{d}{dt} \hat{f}(t) dt \quad [R = \sqrt{p^2+q^2}]$$

3.2.2. Интегралы, содержащие $\Gamma(x)$, $E_1(x)$ или $erf(x)$.

Обозначение $\hat{f}(p) = \int_0^{\infty} e^{-px} f(x) dx$.

Условия: $a, \operatorname{Re} p, \operatorname{Re} q > 0$.

$$1. \int_0^\infty \int_0^\infty \frac{y^{x-1}}{\Gamma(x)} e^{-qy} f(x) dx dy = \hat{f}(\ln q).$$

$$2. \int_0^\infty \int_0^\infty \frac{y^{ax}}{\Gamma(ax+1)} e^{-px-xy} f(x) dx dy = \frac{1}{q} \hat{f}(p+a \ln q).$$

$$3. \int_0^\infty \int_0^1 \frac{y^{x+n}}{\Gamma(x+n+1)} e^{-px-xy} dx dy = \frac{q-e^{-p}}{q^{n+2}(p+\ln q)}.$$

$$4. \int_0^\infty \int_1^\infty \frac{y^{x-1}}{\Gamma(x)} e^{-px-xy} dx dy = \frac{e^{-p}}{q(p+\ln q)}.$$

$$5. \int_0^\infty \int_0^\infty \frac{y^x}{\Gamma(x)} e^{-px-xy} dx dy = \frac{1}{q(p+\ln q)^2}.$$

$$6. \int_0^\infty \int_0^\infty \frac{x^m y^{x+n}}{\Gamma(x+n+1)} e^{-px-xy} dx dy = \frac{m!}{q^{n+1}(p+\ln q)^{m+1}}.$$

$$7. \int_0^\infty \int_0^\infty e^{-px-xy} \operatorname{Ei}(axy) dx dy = \frac{1}{pq} \left[e^{-pq/a} \operatorname{Ei}\left(\frac{pq}{a}\right) - \ln \frac{pq}{a} - \epsilon \right].$$

$$8. \int_0^\infty \int_0^\infty x^{-3/2} y^{-5/4} e^{-p/x-q/\sqrt{y}} \operatorname{Ei}(-axy) dx dy = \frac{8\pi a}{\sqrt{pq}} \operatorname{Ei}(-\sqrt{8qVap}).$$

$$9. \int_0^\infty \int_0^\infty e^{-p|x-y|} \operatorname{Ei}(-x) \operatorname{Ei}(-y) dx dy = \\ = \frac{4}{p} \ln 2 - \frac{\pi^2}{6p^2} - \frac{1}{p^2} \ln^2(p+1) - \frac{2}{p^2} \operatorname{Li}_2\left(\frac{p-1}{p+1}\right).$$

$$10. \int_0^\infty \operatorname{Ei}(-y) \int_0^y \operatorname{Ei}(-x) \operatorname{Ei}(x-y) dx dy = \\ = \int_0^\infty \operatorname{Ei}(-y) \int_0^\infty \operatorname{Ei}(-x) \operatorname{Ei}(y-x) dx dy = \frac{\pi^2}{12} - 2 \ln 2.$$

$$11. \int_0^\infty \int_0^\infty \operatorname{Ei}(-x) \operatorname{Ei}(-y) \operatorname{Ei}(-|x-y|) dx dy = \frac{\pi^2}{6} - 4 \ln 2.$$

$$12. \int_0^\infty \int_0^\infty \frac{1}{y^3} e^{-px-q/y^2} \operatorname{erf}(axy) f(x) dx dy = \frac{1}{2q} [\hat{f}(p) - \hat{f}(p+2a\sqrt{q})].$$

$$13. \int_0^\infty \int_0^\infty \frac{1}{y^3} e^{-px-q/y^2} \operatorname{erf}(axy) dx dy = \frac{a}{p\sqrt{q}(p+2a\sqrt{q})}.$$

$$14. \int_0^\infty \int_0^\infty \frac{1}{y^3} e^{-px-q/y^2} \operatorname{erfc}(axy) dx dy = \frac{1}{2q(p+2a\sqrt{q})}.$$

3.2.3. Интегралы, содержащие функцию параболического цилиндра $D_\nu(x)$.

Условия: $a, \operatorname{Re} p, \operatorname{Re} q > 0$.

$$1. \int_0^\infty \int_0^\infty x^{-3/2} y^{v-2} e^{-p/x-q/y^2-a^2x^2y^2/4} D_\nu(axy) dx dy = \\ = \pi p^{-1/2} (2q)^{(v-1)/2} \exp(-\sqrt{ap}\sqrt{8q}).$$

$$2. \int_0^\infty \int_0^\infty y^{v-2} e^{-px-q/y^2-a^2x^2y^2/4} D_\nu(axy) f(x) dx dy = \\ = 2^{(v-2)/2} \sqrt{\pi} q^{(v-1)/2} \int_0^\infty \exp[-(\rho+a\sqrt{2q})x] f(x) dx.$$

$$3. \int_0^\infty e^{-qy} \int_0^y \frac{D_m(\sqrt{2x}) D_m(i\sqrt{2x})}{\sqrt{x(y-x)}} D_n(\sqrt{2(y-x)}) D_n(i\sqrt{2(y-x)}) dx dy = \\ = (-1)^{(m+n)/2} \frac{\pi m! n!}{q} P_m\left(\frac{1}{q}\right) P_n\left(\frac{1}{q}\right) \quad [m+n - \text{четное}].$$

3.2.4. Интегралы, содержащие функцию Бесселя $J_\nu(x)$.

Обозначение: $\hat{f}(p) = \int_0^\infty e^{-px} f(x) dx$.

Условия: $a, b, \operatorname{Re} p, \operatorname{Re} q > 0$.

$$1. \int_0^\infty e^{-qy} \int_0^y e^{-px} J_0(a\sqrt{xy-x^2}) f(x) dx dy = \frac{1}{q} \hat{f}\left(p+q+\frac{a^2}{4q}\right).$$

$$2. \int_0^\infty e^{-qy} \int_0^y \left(\frac{y-x}{x}\right)^{v/2} J_\nu(a\sqrt{xy-x^2}) f(x) dx dy = \frac{a^v}{2^v q^{v+1}} \hat{f}\left(q+\frac{a^2}{4q}\right) \quad [\operatorname{Re} v > -1].$$

$$3. \int_0^\infty e^{-qy} \int_0^y \left(\frac{y-x}{y+x}\right)^{v/2} J_\nu(\sqrt{y^2-x^2}) f(x) dx dy = \frac{(\sqrt{q^2+1}+q)^{-v}}{\sqrt{q^2+1}} \hat{f}(\sqrt{q^2+1}) \\ [\operatorname{Re} v > -1].$$

$$4. \int_0^\infty \int_0^\infty \left(\frac{y}{x}\right)^{v/2} e^{-px-qy} J_\nu(a\sqrt{xy}) f(x) dx = \frac{a^v}{2^v q^{v+1}} \hat{f}\left(p+\frac{a^2}{4q}\right).$$

$$5. \int_0^\infty y^{v/2} e^{-qy} \int_0^\infty (y+2x)^{-v/2} J_\nu(a\sqrt{y(y+2x)}) f(x) dx dy = \\ = \frac{a^{v-2} (\sqrt{q^2+a^2}+q)^{1-v} (\sqrt{q^2+a^2}-q)}{\sqrt{q^2+a^2}} \hat{f}(\sqrt{q^2+a^2} \mp q).$$

$$6. \int_0^\infty e^{-qy} \int_0^y J_\nu(ax) dx dy = \frac{a^\nu}{q \sqrt{q^2 + a^2} (q + \sqrt{q^2 + a^2})^\nu} \quad [\operatorname{Re} \nu > -1].$$

$$7. \int_0^\infty e^{-qy} \int_0^y x^\nu J_\nu(ax) dx dy = \frac{(2a)^\nu \Gamma(\nu + 1/2)}{\sqrt{\pi} q (q^2 + 1)^{\nu+1/2}} \quad [\operatorname{Re} \nu > -1/2].$$

$$8. \int_0^\infty e^{-qy} \int_0^y \frac{J_\nu(a\sqrt{x})}{\sqrt{y-x}} dx dy = \sqrt{\frac{\pi}{q^3}} e^{-a^2/(2q)} I_\nu\left(\frac{a^2}{2q}\right) \quad [\operatorname{Re} \nu > -2].$$

$$9. \int_0^\infty e^{-qy} \int_0^y (y-x)^{n/2} e^{-px} J_n(a\sqrt{y-x}) dx dy = \frac{a^n}{2^n q^{n+1} (p+q)} \exp\left(-\frac{a^2}{4q}\right).$$

$$10. \int_0^\infty e^{-qy} \int_0^y \frac{J_1(a\sqrt{x^2-b^2})}{\sqrt{x^2-b^2}} dx dy = \frac{e^{-bq} - e^{-b\sqrt{q^2+a^2}}}{abq}.$$

$$11. \int_0^\infty e^{-qy} \int_0^y \frac{x J_1(a\sqrt{x^2-b^2})}{\sqrt{x^2-b^2}} dx dy = \frac{1}{aq} \left(e^{-bq} - \frac{q}{\sqrt{q^2+a^2}} e^{-b\sqrt{q^2+a^2}} \right).$$

$$12. \int_0^\infty e^{-qy} \int_y^\infty \frac{J_1(a\sqrt{x^2-b^2})}{\sqrt{x^2-b^2}} dx dy = \frac{e^{-b\sqrt{q^2+a^2}} - e^{-b(a+q)}}{abq}.$$

$$13. \int_0^\infty e^{-qy} \int_y^\infty \frac{x J_1(a\sqrt{x^2-b^2})}{\sqrt{x^2-b^2}} dx dy = \frac{1}{a\sqrt{q^2+a^2}} e^{-b\sqrt{q^2+a^2}}.$$

$$14. \int_0^\infty \frac{1}{\sqrt{y}} e^{-qy} \int_0^\infty \frac{e^{-x^2/(4y)}}{\sqrt{x^2-a^2}} J_1(\sqrt{x^2-a^2}) dx dy = \\ = \frac{1}{a} \sqrt{\frac{\pi}{q}} (e^{-a\sqrt{q}} - e^{-a\sqrt{q^2+1}}).$$

$$15. \int_0^\infty \int_0^\infty e^{-px-xy} J_0(x+y) dx dy = \frac{p+q}{\sqrt{(p^2+1)(q^2+1)} (\sqrt{p^2+1} + \sqrt{q^2+1})}.$$

$$16. \int_0^\infty \int_0^\infty e^{-px-xy} J_1(x+y) dx dy = \frac{p+q}{\sqrt{(p^2+q)(q^2+1)} (p\sqrt{q^2+1} + q\sqrt{p^2+1})}.$$

$$17. \int_0^\infty \int_0^\infty e^{-px-xy} J_0(a\sqrt{x+y}) dx dy = \frac{e^{-a^2/(4q)}}{q(p-q)} - \frac{e^{-a^2/(4p)}}{p(p-q)}.$$

$$18. \int_0^\infty \int_0^\infty \frac{1}{\sqrt{x+y}} e^{-px-xy} J_1(a\sqrt{x+y}) dx dy = \frac{2}{a(p-q)} [e^{-a^2/(4p)} - e^{-a^2/(4q)}].$$

$$19. \int_0^\infty \int_0^\infty e^{-px-xy} J_0(a\sqrt{x^2+y^2}) dx dy = \frac{1}{p^2+q^2+a^2} \left(\frac{p}{\sqrt{q^2+a^2}} + \frac{q}{\sqrt{p^2+a^2}} \right).$$

$$20. \int_0^\infty \int_0^\infty e^{-px-xy} J_0(\sqrt{x^2+2xy}) dx dy = \frac{1}{p^2+1+(q-p)\sqrt{p^2+1}}.$$

$$21. \int_0^\infty \int_0^\infty \frac{x}{\sqrt{y^2+2xy}} e^{-px-xy} J_1(\sqrt{y^2+2xy}) dx dy = -\frac{q-\sqrt{q^2+1}}{p(p-q+\sqrt{q^2+1})}.$$

$$22. \int_0^\infty \int_0^\infty ye^{-px-xy^2} J_0(axy) dx dy = \frac{1}{2a} \sqrt{\frac{\pi}{q}} e^{p^2q/a^2} \operatorname{erfc}\left(\frac{p\sqrt{q}}{a}\right).$$

$$23. \int_0^\infty \int_0^\infty e^{-px-xy} J_0(axy) dx dy = \frac{\pi}{2a} \left[H_0\left(\frac{pq}{a}\right) - Y_0\left(\frac{pq}{a}\right) \right].$$

$$24. \int_0^\infty \int_0^\infty x^{v+1} y^{1-v} e^{-px-xy^2} J_v(axy) dx dy = \frac{2^v \Gamma(v+3/2) p q^{v+1/2}}{\sqrt{\pi} a^{v+2}} e^{p^2q/a^2} \Gamma\left(-v-\frac{1}{2}, \frac{p^2q}{a^2}\right) \\ [\operatorname{Re} v > -1].$$

$$25. \int_0^\infty \int_0^\infty x^v y^{1-v} e^{-px-xy^2} J_v(axy) dx dy = \frac{2^{v-1} \Gamma(v+1/2) q^{v-1/2}}{\sqrt{\pi} a^{v+1}} e^{p^2q/a^2} \Gamma\left(\frac{1}{2}-v, \frac{p^2q}{a^2}\right) \\ [\operatorname{Re} v > -1/2].$$

$$26. \int_0^\infty \int_0^\infty x^{m+n-1} y^{m+1} e^{-px-xy^2} J_m(axy) dx dy = \\ = \frac{\Gamma(2m+n)}{a^{m+n}} (2q)^{n/2-1} e^{2p^2q/a^2} D_{-2m-n}\left(\frac{p\sqrt{2q}}{a}\right).$$

$$27. \int_0^\infty \int_0^\infty x^l y^m e^{-px^2-xy^2} J_v(axy) dx dy = A_v^{l, m} \\ [k_1 = a/\sqrt{4pq+a^2}, k_2 = a/(\sqrt{4pq+a^2}+2\sqrt{pq}); \operatorname{Re} v > -\min(l, m)-1] \\ A_v^{v+1, 0} = \frac{\Gamma(v/2+1/2) k_1^{v+1}}{2ap^{(v+1)/2}}, \\ A_v^{v+1, v-2n+1} = \frac{\Gamma(v-n+1) 2^{v-2n} k_1^{v-n+1}}{a^{v-2n-2pn}} \\ [\operatorname{Re} v > n-1].$$

$$A_v^{v+1, 1-n} = \frac{\Gamma(v/2-n/2+1) k_1^{v-n+2} a^{n-2}}{2^n p^{(v+n)/2}}$$

$$[\operatorname{Re} v > n-2 \text{ при } n=1, 2, \dots; -1 < \operatorname{Re} v \leq 0].$$

$$A_n^{2m-n+1, n+1} = 2^{2m-n} (m!) a^{n-2m-2} q^{m-n} k_1^{2m+2},$$

$$A_0^{1, 2v-1} = \Gamma(v) a^{-2v} (4p)^{v-1} k_1^{2v}, \quad A_1^{v+1, 1-v} = a^{v-2} (2p)^{-v} k_1^2,$$

$$A_0^{0, 0} = \frac{k_1}{a} K(k_1), \quad A_0^{0, 2} = \frac{k_1}{2aq} E(k_1),$$

$$A_0^{1, 0} = -\frac{\sqrt{\pi/p}}{2\sqrt{4pq+a^2}}, \quad A_0^{1, 1} = \frac{k_1^2}{a^2}, \quad A_0^{2, 2} = \frac{k_1^2}{a^2} [2E(k_1) - K(k_1)],$$

$$A_0^{\pm 1/2, \pm 1/2} = \frac{\sqrt{2}}{\pi} \Gamma^2 \left(\frac{2 \pm 1}{4} \right) \left\{ \begin{matrix} 2k_1/a \\ 1 \end{matrix} \right\} \frac{1}{k_2 a} K(k_2),$$

$$A_1^{1,0} = \frac{k_1^2}{2ap}, \quad A_1^{1,-1} = \frac{k_1}{a^2} [K(k_1) - E(k_1)],$$

$$A_1^{-1,0} = \frac{1}{2qk} [E(k_1) - (1 - k_1^2) K(k_1)],$$

$$A_1^{-1,-1} = \frac{1}{k_1} [K(k_1) - E(k_1)], \quad A_1^{-1/2, -1/2} = \frac{4\Gamma^2(3/4)}{\pi \sqrt{ak_2}} [K(k_2) - E(k_2)],$$

$$A_2^{0,0} = \frac{1}{ak_1} [(2 - k_1^2) K(k_1) - 2E(k_1)],$$

$$A_2^{2,-1} = \frac{a^2}{4p^2(4pq + a^4)}.$$

$$28. \int_0^\infty \int_0^\infty x^{v+3} y^{v+1} e^{-px^2 - qy^2} J_v(axy) dx dy = \frac{\sqrt{\pi} a^v e^{a^2/(2\sqrt{pq})}}{2^{2v+4} \sqrt{p} (pq)^{(v+1)/2}} \quad [\operatorname{Re} v > -1].$$

$$29. \int_0^\infty \int_0^\infty \frac{1}{y^2} e^{-px^2 - q/y^2} J_1(axy) dx dy = \frac{\pi a}{2p^2} \left[H_1 \left(\frac{aq}{p} \right) - Y_1 \left(\frac{aq}{p} \right) - \frac{2}{\pi} \right].$$

$$30. \int_0^\infty \int_0^\infty \frac{x}{y^3} e^{-px^2 - q/y^2} J_0(axy) dx dy = \frac{a}{4} \sqrt{\frac{1}{p^3 q}} K_1 \left(a \sqrt{\frac{q}{p}} \right).$$

$$31. \int_0^\infty \int_0^\infty xye^{-px^2 - qy^2} \sin axy J_0(bxy) dx dy =$$

$$= \frac{a}{[(4pq + a^2 - b^2)^2 + 16b^2 pq]^{3/4}} [2E(k) - K(k)]$$

$$\left[k = \frac{1}{\sqrt{2}} \left(1 - \frac{4pq + a^2 - b^2}{\sqrt{(4pq + a^2 - b^2)^2 + 16b^2 pq}} \right)^{1/2} \right].$$

$$32. \int_0^\infty \int_0^\infty e^{-px^2 - qy^2} \cos axy J_0(bxy) dx dy = \frac{1}{[(4pq + a^2 - b^2)^2 + 16b^2 pq]^{1/4}} K(k)$$

[k см. в 3.2.4.31].

$$33. \int_0^\infty \int_0^\infty \frac{y}{x} e^{-px^2 - qy^2} \cos axy J_1(axy) dx dy =$$

$$= \frac{4bp}{[(4pq + a^2 - b^2)^2 + 16b^2 pq]^{1/4} [4pq + a^2 - b^2 + \sqrt{(4pq + a^2 - b^2)^2 + 16b^2 pq}] k^2} \times$$

$$\times [E(k) - (1 - k^2) K(k)] \quad [k \text{ см. в 3.2.4.31}].$$

3.2.5. Интегралы, содержащие две функции Бесселя $J_v(x)$.

Условия: $a, b, \operatorname{Re} p, \operatorname{Re} q > 0$.

$$1. \int_0^a \int_0^b J_0[2i\sqrt{i(a-x)(b-y)}] J_0(2i\sqrt{-ixy}) dx dy = \operatorname{bei}(2\sqrt{ab}).$$

$$2. \int_0^\infty e^{-qy} \int_0^y \frac{J_\nu^2(2\sqrt{x})}{x\sqrt{y-x}} dx dy = \frac{1}{\nu} \sqrt{\frac{\pi}{q}} e^{-2/q} \left[I_\nu\left(\frac{2}{q}\right) + 2 \sum_{k=1}^\infty I_{k+\nu}\left(\frac{2}{q}\right) \right] \\ [\operatorname{Re} \nu > 0].$$

$$3. \int_0^\infty y^{\nu/2} e^{-qy} \int_b^\infty \frac{J_\nu(2\sqrt{xy}) J_1(\sqrt{x^2-b^2})}{\sqrt{x^2-b^2}} dx dy = \frac{b^{\nu/2-1}}{q^{\nu+1}} [e^{-b/q} - e^{-\sqrt{q^2+1}/(bq)}] \\ [\operatorname{Re} \nu > -1].$$

$$4. \int_0^\infty e^{-qy} \int_0^y J_0(2\sqrt{(y-x)x}) J_0(2\sqrt{bx}) dx dy = \frac{1}{q^2+1} \exp\left(-\frac{bq}{q^2+1}\right).$$

$$5. \int_0^\infty \int_0^\infty e^{-px^2-qy^2} J_\nu^2(axy) dx dy = I_\nu \quad [k = a/(\sqrt{pq+a^2} + \sqrt{pq}); \operatorname{Re} \nu > -1/2],$$

$$I_\nu = \frac{\Gamma^2(\nu+1/2)}{4\sqrt{pq}} \left[P_{-1/2}^\nu \left(\sqrt{\frac{pq+a^2}{pq}} \right) \right]^2,$$

$$I_0 = \frac{2}{\pi ka} K^2(k), \quad I_1 = \frac{2}{\pi ka} [K(k) - E(k)]^2.$$

$$6. \int_0^\infty \int_0^\infty xye^{-px^2-qy^2} J_\nu^2(axy) dx dy = \frac{1}{4\sqrt{pq(pq+a^2)}} \left(\frac{\sqrt{pq+a^2} - \sqrt{pq}}{a} \right)^{2\nu} \\ [\operatorname{Re} \nu > -1].$$

$$7. \int_0^\infty \int_0^\infty xy^{2\nu+1} e^{-px^2-qy^2} J_\nu^2(axy) dx dy = \frac{\Gamma(\nu+1/2) a^{2\nu} q^{-\nu}}{4\sqrt{\pi pq} (pq+a^2)^{\nu+1/2}} \\ [\operatorname{Re} \nu > -1/2].$$

$$8. \int_0^\infty \int_0^\infty x^{2\nu+1} y^{-2\nu+1} e^{-px^2-qy^2} J_\nu(axy) J_{-\nu}(axy) dx dy = \frac{q^\nu}{4p^\nu \sqrt{pq(pq+a^2)}} \\ [|\operatorname{Re} \nu| < 1].$$

$$9. \int_0^\infty \int_0^\infty xye^{-px^2-qy^2} J_\nu(axy) J_{\nu+1}(axy) dx dy = \\ = \frac{1}{4} \Gamma^2\left(\nu + \frac{3}{2}\right) \sqrt{\frac{pq+a^2}{(pq)^3}} P_{-1/2}^{-\nu} \left(\sqrt{\frac{pq+a^2}{pq}} \right) P_{-1/2}^{-\nu-1} \left(\sqrt{\frac{pq+a^2}{pq}} \right) \\ [\operatorname{Re} \nu > -3/2].$$

$$10. \int_0^\infty \int_0^\infty xye^{-px^2-qy^2} J_0(axy) J_1(axy) dx dy = \frac{\sqrt{pq+a^2}}{\pi a p q} K(k) [K(k) - E(k)] \\ [k = a/(\sqrt{pq+a^2} + \sqrt{pq})].$$

$$11. \int_0^\infty \int_0^\infty xe^{-px^2-qy^2} J_\nu(\sqrt{2i} xy) J_\nu(\sqrt{-2i} xy) dx dy = \sqrt{\frac{1}{8\pi p}} Q_{\nu-1/2}(pq) \\ [\operatorname{Re} \nu > -1/2]$$

$$12. \int_0^\infty \int_0^\infty x e^{-px^2 - qy^2} J_0(\sqrt{2l} xy) J_0(\sqrt{-2l} xy) dx dy = \\ = \frac{1}{2\sqrt{\pi p(pq+1)}} K\left(\sqrt{\frac{2}{pq+1}}\right) \quad [p, q, pq+1 > 0]$$

$$13. \int_0^\infty \int_0^\infty x e^{-px^2 - qy^2} J_1(\sqrt{2l} xy) J_1(\sqrt{-2l} xy) dx dy = \\ = \frac{1}{k\sqrt{2\pi p}} [(2-k^2) K(k) - 2E(k)] \quad [p, q, pq+1 > 0; k = \sqrt{2}/\sqrt{pq+1}]$$

$$14. \int_0^\infty \int_0^\infty xy \sin(x^2 + y^2) J_0(ax) J_0(by) dx dy = \frac{1}{4} \sin \frac{a^2 + b^2}{4} \quad [a, b > 0]$$

3.2.6. Интегралы, содержащие три функции Бесселя $J_\nu(x)$.

Обозначение: $k = 4c\sqrt{ab}/\sqrt{(4pq+4ab+c^2)^2+16(aq-bp)^2}$.

Условия: $a, b, c, \operatorname{Re} p, \operatorname{Re} q > 0$.

$$1. \int_0^\infty \int_0^\infty f(xy) J_0(xy) J_n(ay) J_n(by) dx dy = \frac{1}{2\pi} \int_0^{2\pi} f(\sqrt{a^2+b^2-2ab\cos\varphi}) \cos n\varphi d\varphi.$$

$$2. \int_0^\infty \int_0^\infty e^{-px-qy} J_0(ax) J_0(by) J_0(c\sqrt{xy}) dx dy = \frac{2}{\pi c\sqrt{ab}k} K(k).$$

$$3. \int_0^\infty \int_0^\infty ye^{-px-qy} J_0(ax) J_0(by) J_0(c\sqrt{xy}) dx dy = \\ = \frac{32(4p^2q+4a^2q+c^2p)}{\pi c\sqrt{ab}[(4pq+4ab+c^2)^2+16(aq-bp)^2-16abc^2]k} E(k).$$

$$4. \int_0^\infty \int_0^\infty e^{-px-qy} J_1(ax) J_1(by) J_0(c\sqrt{xy}) dx dy = \frac{1}{2\pi c\sqrt{ab}k} [(2-k^2) K(k) - 2E(k)].$$

3.2.7. Интегралы, содержащие функцию Неймана $Y_\nu(x)$.

Условия: $a, p, q > 0$.

$$1. \int_0^\infty \int_0^\infty e^{-px^2-qy^2} Y_0(axy) dx dy = -\frac{1}{\sqrt{4pq+a^2}} K\left(\sqrt{\frac{4pq}{4pq+a^2}}\right).$$

$$2. \int_0^\infty \int_0^\infty e^{-px-qy} Y_0(a\sqrt{x^2+y^2}) dx dy = \frac{4pq}{p^2+q^2+a^2}.$$

3.2.8. Интегралы, содержащие модифицированную функцию Бесселя $I_\nu(x)$.

Условия: $a, b, \operatorname{Re} p, \operatorname{Re} q > 0$.

$$1. \int_0^\infty e^{-qy} \int_0^y x^\nu I_\nu(ax) dx dy = \frac{(2a)^\nu \Gamma(\nu+1/2)}{\sqrt{\pi} q (q^2-1)^{\nu+1/2}} \quad [\operatorname{Re} \nu > -1/2]$$

$$2. \int_0^{\infty} e^{-qy} \int_b^y \frac{I_1(a \sqrt{x^2 - b^2})}{\sqrt{x^2 - b^2}} dx dy = \frac{e^{-b \sqrt{q^2 - a^2}} - e^{-bq}}{abq}.$$

$$3. \int_0^{\infty} e^{-qy} \int_b^y \frac{x}{\sqrt{x^2 - b^2}} I_1(a \sqrt{x^2 - b^2}) dx dy = \frac{1}{aq} \left(\frac{q}{\sqrt{q^2 - a^2}} e^{-b \sqrt{q^2 - a^2}} - e^{-bq} \right).$$

$$4. \int_0^{\infty} e^{-qy} \int_b^y \frac{e^{-px}}{\sqrt{x^2 - b^2}} I_1(a \sqrt{x^2 - b^2}) dx dy = \\ = \frac{e^{-b \sqrt{(p+q+a)(p+q-a)}} - e^{-b(p+q)}}{abq}.$$

$$5. \int_0^{\infty} \int_0^{\infty} x^l y^m e^{-px^2 - qy^2} I_n(axy) dx dy = A_n^{\frac{l}{2}, \frac{m}{2}} \quad [p, q, 4pq - a^2 > 0; k = a/\sqrt{4pq}],$$

$$A_0^{0,0} = \frac{1}{2\sqrt{pq}} K(k), \quad A_0^{0,2} = \frac{1}{4(1-k^2)\sqrt{pq^3}} E(k),$$

$$A_0^{1,1} = \frac{1}{4pq - a^2}, \quad A_0^{2,2} = \frac{1}{8(1-k^2)^2\sqrt{p^3q^3}} [2E(k) - (1-k^2)K(k)],$$

$$A_0^{\pm 1/2, \pm 1/2} = \frac{2\pi}{(pq)^{1/4}} \Gamma^2\left(\frac{2 \pm 1}{4}\right) \left\{ \frac{2(4pq - a^2)^{-1/2}}{1} \right\} K\left(\sqrt{\frac{1}{2} - \frac{1}{2}\sqrt{1 - \frac{a^2}{4pq}}}\right),$$

$$A_1^{1,1} = \frac{1}{4pqk(1-k^2)} [E(k) - (1-k^2)K(k)],$$

$$A_1^{1,3} = \frac{1}{8pq^2k(1-k^2)^2} [(1+k^2)E(k) - (1-k^2)K(k)],$$

$$A_1^{2,3} = \frac{1}{16p^2q^2k(1-k^2)^3} [(1+7k^2)E(k) - (1-k^2)(1+3k^2)K(k)],$$

$$A_1^{-1,-1} = \frac{1}{k} [E(k) - (1-k^2)K(k)], \quad A_1^{-1,1} = \frac{1}{2qk} [K(k) - E(k)],$$

$$A_1^{-1,3} = \frac{1}{4q^2k(1-k^2)} [(1-k^2)K(k) - (1-2k^2)E(k)],$$

$$A_1^{-1/2, -1/2} = \frac{8\Gamma^2(3/4)(pq)^{1/4}}{\pi a} \left[E\left(\sqrt{\frac{1}{2} - \frac{1}{2}\sqrt{1 - \frac{a^2}{4pq}}}\right) - \right. \\ \left. - \left(\frac{1}{2} + \sqrt{1 - \frac{a^2}{4pq}}\right) K\left(\sqrt{\frac{1}{2} - \frac{1}{2}\sqrt{1 - \frac{a^2}{4pq}}}\right) \right],$$

$$A_2^{0,0} = \frac{1}{2k^2\sqrt{pq}} [(2-k^2)K(k) - 2E(k)],$$

$$A_2^{0,2} = \frac{1}{4k^2(1-k^2)\sqrt{pq^3}} [(2-k^2)E(k) - 2(1-k^2)K(k)],$$

$$A_2^{2,2} = \frac{1}{8k^2(1-k^2)^2\sqrt{p^3q^3}} [(1-k^2)(2-3k^2)K(k) - 2(1-2k^2)E(k)],$$

$$A_2^{-2,0} = \frac{\sqrt{p}}{3k^2\sqrt{q}} [(2-k^2)E(k) - 2(1-k^2)K(k)],$$

$$A_2^{-2, 2} = \frac{\sqrt{p}}{6k^2 \sqrt{q^3}} [(2 + k^2) K(k) - 2(1 + k^2) E(k)],$$

$$A_2^{-2, -2} = \frac{2\sqrt{pq}}{9k^2} [(2 - 3k^2)(1 - k^2) K(k) - 2(1 - 2k^2) E(k)],$$

$$A_3^{1, 1} = \frac{1}{4pqk^3(1 - k^2)} [(8 - 7k^2) E(k) - (1 - k^2)(8 - 3k^2) K(k)],$$

$$A_3^{1, 3} = \frac{1}{8pq^2k^3(1 - k^2)^2} [(8 - 9k^2)(1 - k^2) K(k) - (8 - 13k^2 + 3k^4) E(k)],$$

$$A_3^{3, 3} = \frac{1}{16p^2q^2k^3(1 - k^2)^3} [(8 - 23k^2 + 23k^4) E(k) - (1 - k^2)(8 - 19k^2 + 15k^4) K(k)],$$

$$A_3^{-1, 3} = \frac{1}{12q^2k^3(1 - k^2)} [(8 - 3k^2 - 2k^4) E(k) - (1 - k^2)(8 + k^2) K(k)],$$

$$A_3^{-1, -1} = \frac{1}{9k^3} [(8 - 7k^2) E(k) - (8 - 3k^2)(1 - k^2) K(k)],$$

$$A_3^{-1, -3} = \frac{2q}{45k^3} [(8 - 9k^2)(1 - k^2) K(k) - (8 - 13k^2 + 3k^4) E(k)],$$

$$A_3^{-3, 1} = \frac{p}{15qk^3} [(8 - 3k^2 - 2k^4) E(k) - (8 + k^2)(1 - k^2) K(k)],$$

$$A_3^{-3, 3} = \frac{p}{30q^2k^3} [(8 + 3k^2 + 4k^4) K(k) - (8 + 7k^2 + 8k^4) E(k)],$$

$$A_3^{-3, -3} = \frac{4pq}{225k^3} [(8 - 23k^2 + 23k^4) E(k) - (1 - k^2)(8 - 19k^2 + 15k^4) K(k)].$$

$$6. \int_0^\infty \int_0^\infty \frac{1}{xy(x+y)^{1/2}} e^{-px-xy-2xy/(x+y)} I_n\left(\frac{2xy}{x+y}\right) dx dy =$$

$$= \frac{2\sqrt{\pi}}{n(2n-1)(\sqrt{p} + \sqrt{q})^{2n-1}} {}_2F_1\left(n - \frac{1}{2}, n; 2n+1; -\frac{4}{(\sqrt{p} + \sqrt{q})^2}\right).$$

$$7. \int_0^\infty \int_0^\infty x^l y^m e^{-px^2-xy^2} \operatorname{sh} axy I_n(bxy) dx dy = B_n^{l, m}(a)$$

$$[a, b, p, q > 0; 4pq > (a+b)^2; k^2 = 8b\sqrt{pq}/((2\sqrt{pq} + b)^2 - a^2)],$$

$$B_0^{1, 1}(a) = \frac{ak^3}{16\sqrt{2} b^{3/2} (pq)^{3/4} (1 - k^2)} E(k),$$

$$B_1^{2, 0}(a) = \frac{ak}{16\sqrt{2} b^{3/2} p^{5/4} q^{1/4} (1 - k^2)} [(2 - k^2) E(k) - 2(1 - k^2) K(k)],$$

$$B_2^{-1, 3}(a) = \frac{ap^{1/4}}{16\sqrt{2} b^{3/2} q^{7/4} k (1 - k^2)} [(k^4 - 16k^2 + 16) E(k) - 8(k^2 - 1)(k^2 - 2) K(k)].$$

$$8. \int_0^\infty \int_0^\infty x^l y^m e^{-px^2-xy^2} \operatorname{ch} axy I_n(bxy) dx dy = C_n^{l, m}(a)$$

$$[a, b, p, q > 0; 4pq > (a+b)^2; k^2 = 8b\sqrt{pq}/((2\sqrt{pq} + b)^2 - a^2)],$$

$$C_0^{0, 0}(a) = \frac{k}{2^{3/2} b^{1/2} (pq)^{1/4}} K(k),$$

$$C_1^{-1, 1}(a) = \frac{p^{1/4}}{2^{3/2} b^{1/2} q^{3/4} k} [(2 - k^2) K(k) - 2E(k)],$$

$$C_{\frac{1}{2}, \frac{1}{2}}^{-2, 2}(a) = \frac{p^{3/4}}{6\sqrt{2} b^{1/2} q^{5/4} k^3} [(k^2 - 4)(3k^2 - 4) K(k) + 8(k^2 - 2) E(k)],$$

$$C_{\frac{1}{2}, \frac{1}{2}}^{-2, 2}(a) = \frac{p^{5/4}}{30\sqrt{2} b^{1/2} q^{7/4} k^5} [(2 - k^2)(15k^4 - 128k^2 + 128) K(k) - \\ - 2(23k^4 - 128k^2 + 128) E(k)].$$

$$9. \int_0^\infty \int_0^\infty x^l y^m e^{-px^2 - qy^2} \begin{cases} \sin axy \\ \cos axy \end{cases} I_n(bxy) dx dy = \begin{cases} -i B_n^{l, m}(ia) \\ C_n^{l, m}(ia) \end{cases} \\ [b, p, q, 4pq - b^2 > 0; \text{ см. 3.2.8. 7-8}].$$

3.2.9. Интегралы, содержащие две модифицированные функции Бесселя $I_\nu(x)$.

Условия: $a, b, \operatorname{Re} p, \operatorname{Re} q > 0$.

$$1. \int_0^\infty \int_0^\infty x^k y^l e^{-px^2 - qy^2} I_m(ax) I_n(ay) dx dy = A_{m, n}^{k, l}$$

$$[p, q, pq - a^2 > 0; k^2 = (\sqrt{pq} - \sqrt{pq - a^2}) / (2\sqrt{pq})],$$

$$A_{0, 0}^{0, 0} = \frac{1}{\pi \sqrt{pq}} K^2(k),$$

$$A_{0, 0}^{2, 0} = \frac{1}{2\pi(1 - 2k^2) \sqrt{p^3 q}} K(k) [2E(k) - K(k)],$$

$$A_{0, 1}^{1, 1} = \frac{1 - 2k^2}{2\pi pqk \sqrt{1 - k^2}} K(k) [E(k) - (1 - k^2) K(k)],$$

$$A_{0, 1}^{-1, 1} = \frac{1 - 2k^2}{2\pi q^2 k \sqrt{1 - k^2}} [2E(k) - K(k)] [(1 - k^2) K(k) - (1 - 2k^2) E(k)],$$

$$A_{1, 1}^{0, 0} = \frac{1}{\pi k^2 (1 - k^2) \sqrt{pq}} [E(k) - (1 - k^2) K(k)],$$

$$A_{1, 1}^{2, 0} = \frac{1}{2\pi k^2 (1 - k^2) (1 - 2k^2) \sqrt{p^3 q}} [E(k) - (1 - k^2) K(k)] \times \\ \times [(1 - k^2) K(k) - (1 - 2k^2) E(k)],$$

$$A_{1, 1}^{-2, 1} = \frac{\sqrt{p}}{3\pi k^2 (1 - k^2) \sqrt{q^3}} [(1 - k^2) K(k) - (1 - 2k^2) E(k)]^2,$$

$$A_{1, 1}^{1, 1} = \frac{1 - 2k^2}{2\pi pqk^3 \sqrt{(1 - k^2)^3}} [E(k) - (1 - k^2) K(k)] \times \\ \times [(2 - 3k^2)(1 - k^2) K(k) - 2(1 - 2k^2) E(k)],$$

$$A_{1, 1}^{-1, 1} = \frac{1 - 2k^2}{6\pi q^2 k^3 \sqrt{(1 - k^2)^3}} [(1 - k^2) K(k) - (1 - 2k^2) E(k)] \times \\ \times [2(1 - k^2 + k^4) E(k) - (1 - k^2)(2 - k^2) K(k)],$$

$$A_{2, 2}^{0, 0} = \frac{1}{9\pi k^4 (1 - k^2)^2 \sqrt{pq}} [(2 - 3k^2)(1 - k^2) K(k) - 2(1 - 2k^2) E(k)]^2,$$

$$A_{2, 2}^{2, 0} = \frac{1}{6\pi k^4 (1 - k^2)^2 (1 - 2k^2) \sqrt{p^3 q}} [2(1 - k^2 + k^4) E(k) - (2 - k^2)(1 - k^2) K(k)] \times \\ \times [(2 - 3k^2)(1 - k^2) K(k) - 2(1 - 2k^2) E(k)],$$

$$A_{2, 2}^{-2, 2} = \frac{\sqrt{p}}{15\pi k^4 (1 - k^2)^2 \sqrt{q^3}} [2(1 - k^2 + k^4) E(k) - (1 - k^2)(2 - k^2) K(k)]^2.$$

$$A_{\frac{1}{2}, \frac{1}{2}}^{\frac{1}{2}} = \frac{1}{225\pi k^2 (1-k^2)^3 \sqrt{pq}} [(8-23k^2+23k^4) E(k) - (1-k^2)(8-19k^2+15k^4) K(k)]^2,$$

$$A_{\frac{3}{2}, \frac{3}{2}}^{\frac{3}{2}} = \frac{\sqrt{p}}{315\pi k^2 (1-k^2)^3 \sqrt{q^3}} [(1-2k^2)(8-3k^2+3k^4) E(k) - (1-k^2)(8-15k^2+3k^4) K(k)]^2,$$

$$A_{\frac{1}{2}, \frac{3}{2}}^{\frac{1}{2}} = \frac{1-2k^2}{18\pi pqk^2 \sqrt{(1-k^2)^5}} [(1-k^2)(2-3k^2) K(k) - 2(1-2k^2) E(k)] \times$$

$$\times [(8-23k^2+23k^4) E(k) - (1-k^2)(8-19k^2+15k^4) K(k)],$$

$$A_{\frac{3}{2}, \frac{1}{2}}^{\frac{3}{2}} = \frac{1-2k^2}{30\pi q^2 k^2 \sqrt{(1-k^2)^5}} [2(1-k^2+k^4) E(k) - (1-k^2)(2-k^2) K(k)] \times$$

$$\times [(1-k^2)(8-15k^2+3k^4) K(k) - (1-2k^2)(8-3k^2+3k^4) E(k)].$$

$$2. \int_0^\infty \int_0^\infty x^k y^l e^{-px^2 - qy^2} I_m(ax^2) I_n(by^2) I_r(cxy) dx dy = B_{m, n, r}^{k, l},$$

$$[p > a, q > b, (p-a)(q-b) > c^2/4; k^2 = 16abc^2 [4(p+a)(q-b) - c^2]^{-1} \times$$

$$\times [4(p-a)(q+b) - c^2]^{-1}],$$

$$B_{0, 0, 0}^{\frac{1}{2}, \frac{1}{2}} = \frac{k}{2\pi c \sqrt{ab}} K(k),$$

$$B_{0, 0, 0}^{\frac{3}{2}, \frac{3}{2}} = \frac{k^3 (4p^2q - 4a^2q - c^2p)}{8\pi (1-k^2) (ab)^{3/2} c^3} E(k),$$

$$B_{0, 1, 0}^{\frac{1}{2}, \frac{1}{2}} = \frac{k^3 [(4p^2q - 4a^2q - c^2p)^2 + 16b^2(p^2 - a^2)^2 - a^2c^4]}{64\pi (1-k^2) (p^2 - a^2) (a^3b^3)^{1/2} c^3} E(k) - \frac{k}{4\pi (ab^3)^{1/2} c} K(k),$$

$$B_{1, 1, 0}^{\frac{1}{2}, \frac{1}{2}} = \frac{1}{2\pi kc \sqrt{ab}} [(2-k^2) K(k) - 2E(k)],$$

$$B_{1, 1, 0}^{\frac{3}{2}, \frac{3}{2}} = \frac{k (4p^2q - 4a^2q - c^2p)}{8\pi (1-k^2) (ab)^{3/2} c^3} [(2-k^2) E(k) - 2(1-k^2) K(k)].$$

$$3. \int_0^\infty \int_0^\infty xy e^{-px^2 - qy^2} J_\nu(axy) I_\nu(axy) dx dy = \frac{(\sqrt{16p^2q^2 + a^4} - 4pq)^\nu}{a^{2\nu} \sqrt{16p^2q^2 + a^4}}$$

$$[\operatorname{Re} \nu > -1]$$

$$4. \int_0^\infty \int_0^\infty y e^{-px - qy^2} J_m(axy) I_m(axy) dx dy =$$

$$= \frac{1}{2a \sqrt{\pi q}} \Gamma\left(m + \frac{1}{2}\right) D_{-m-1/2}\left(\frac{p \sqrt{q}}{a} e^{i\pi/4}\right) D_{-m-1/2}\left(\frac{p \sqrt{q}}{a} e^{-i\pi/4}\right).$$

3.2.10. Интегралы, содержащие функцию Макдональда $K_\nu(x)$.

Условия: $a, \operatorname{Re} p, \operatorname{Re} q > 0$.

$$1. \int_0^\infty \int_0^\infty x^m y^m e^{-px^2 - qy^2} K_0(axy) dx dy = A_m \quad [k = (2\sqrt{pq} - a)/(2\sqrt{pq} + a)],$$

$$A_0 = \frac{\pi}{2\sqrt{pq} + a} K(k), \quad A_1 = \frac{1}{4pq - a^2} \ln \sqrt{\frac{4pq}{a^2}},$$

$$A_2 = \frac{\pi}{(2\sqrt{pq} + a)^3 k^2} [K(k) - E(k)].$$

$$2. \int_0^\infty \int_0^\infty \frac{1}{y} e^{-px - q/y} K_1(axy) dx dy = \frac{\pi^2}{2p} \left[H_1 \left(\sqrt{\frac{aq}{p}} \right) - Y_1 \left(\sqrt{\frac{aq}{p}} \right) \right] - \frac{\pi}{p}.$$

$$3. \int_0^\infty \int_0^\infty \frac{e^{-axy}}{x^{3/2} y} e^{-p(x - q/\sqrt{y})} K_m(axy) dx dy =$$

$$= 8 \sqrt{\frac{\pi}{p}} K_{2m} \left(e^{i\pi/4} \sqrt{8apq^2} \right) K_{2m} \left(e^{-i\pi/4} \sqrt{8apq^2} \right).$$

$$4. \int_0^\infty \int_0^\infty x^l y^m e^{-px^2 - qy^2} \operatorname{sh} axy K_n(bxy) dx dy = A_n^{l,m}$$

$$[a, b, \operatorname{Re} p, \operatorname{Re} q > 0; k_1 = \sqrt{(2\sqrt{pq} - b)^2 - a^2} / \sqrt{(2\sqrt{pq} + b)^2 - a^2},$$

$$k_2 = \sqrt{a^2 - (2\sqrt{pq} - b)^2} / \sqrt{8bp^{1/2}q^{1/2}}],$$

$$A_0^{1,1} = \frac{\pi a (1 - k_1^2)^{3/2}}{2^{9/2} b^{3/2} (pq)^{3/4} k_1^2} [K(k_1) - E(k_1)] \quad [a < |2\sqrt{pq} - b|],$$

$$= \frac{\pi a}{2^{9/2} b^{3/2} (pq)^{3/4} k_2^2 (1 - k_2^2)} [E(k_2) + (k_2^2 - 1) K(k_2)]$$

$$[|2\sqrt{pq} - b| < a < 2\sqrt{pq} + b],$$

$$A_1^{2,0} = \frac{\pi a (1 - k_1^2)^{1/2}}{2^{9/2} b^{3/2} p^{5/4} q^{1/4} k_1^2} [(k_1^2 + 1) E(k_1) + (k_1^2 - 1) K(k_1)]$$

$$[a < |2\sqrt{pq} - b|],$$

$$= \frac{\pi a}{2^{9/2} b^{3/2} p^{5/4} q^{1/4} k_2^2 (1 - k_2^2)} [(2k_2^2 - 1) E(k_2) - (k_2^2 - 1) K(k_2)]$$

$$[|2\sqrt{pq} - b| < a < 2\sqrt{pq} + b],$$

$$5. \int_0^\infty \int_0^\infty e^{-px^2 - qy^2} \operatorname{ch} axy K_0(bxy) dx dy = \frac{\pi (1 - k_1^2)^{1/2}}{2^{3/2} a^{1/2} (pq)^{1/4}} K(k_1)$$

$$[a < |2\sqrt{pq} - b|],$$

$$= \frac{\pi}{2^{3/2} a^{1/2} (pq)^{1/4}} K(k_2)$$

$$[|2\sqrt{pq} - b| < a < 2\sqrt{pq} + b]$$

$$[k_1, k_2 \text{ см. в 3.2.10.4}].$$

$$6. \int_0^\infty \int_0^\infty xy e^{-px^2 - qy^2} \sin axy K_0(bxy) dx dy =$$

$$= \frac{\pi a}{[(2\sqrt{pq} + b)^2 + a^2]^{3/2} k^2} [K(k) - E(k)]$$

$$[k = \sqrt{(2\sqrt{pq} - b)^2 + a^2} / \sqrt{(2\sqrt{pq} + b)^2 + a^2}].$$

$$7. \int_0^\infty \int_0^\infty e^{-px^2 - qy^2} \cos(axy) K_0(bxy) dx dy = \frac{\pi}{[(2\sqrt{pq} + b)^2 + a^2]^{1/2}} K(k)$$

$$[k \text{ см. в 3.2.10.6}].$$

$$8. \int_0^\infty \int_0^\infty x^k y^l e^{-(x^2 + y^2)/2} I_m(axy) K_n(xy) dx dy = A_{m,n}^{k,l}$$

$$[a^2 < 4; k = (2 - \sqrt{4 - a^2})/4],$$

$$A_{0,0}^{0,0} = \frac{4k\sqrt{1-k^2}}{a} K^2(k), \quad A_{0,0}^{2,0} = \frac{2k\sqrt{1-k^2}}{a(1-2k^2)} K(k) [2E(k) - K(k)],$$

$$A_{0,1}^{1,1} = \frac{2k\sqrt{1-k^2}}{a(1-2k^2)} K(k) [2E(k) - K(k)],$$

$$A_{1,0}^{1,0} = \frac{2}{a(1-2k^2)} K(k) [E(k) - (1-k^2)K(k)],$$

$$A_{1,1}^{2,0} = \frac{2}{a(1-2k^2)} [2E(k) - K(k)] [(1-k^2)K(k) - (1-2k^2)E(k)],$$

$$A_{2,0}^{0,0} = \frac{4}{ak\sqrt{1-k^2}} [E(k) - (1-k^2)K(k)]^2,$$

$$A_{2,0}^{2,0} = \frac{2}{ak(1-2k^2)\sqrt{1-k^2}} [E(k) - (1-k^2)K(k)] [(1-k^2)K(k) - (1-2k^2)E(k)],$$

$$A_{2,1}^{1,1} = \frac{2}{ak(1-2k^2)\sqrt{1-k^2}} [E(k) - (1-k^2)K(k)] [(1-k^2)K(k) - (1-2k^2)E(k)],$$

$$A_{2,1}^{-1,1} = \frac{4}{3a\sqrt{1-k^2}} [(1-2k^2)E(k) - (1-k^2)K(k)]^2,$$

$$A_{2,1}^{-1,3} = \frac{2}{3ak(1-2k^2)\sqrt{1-k^2}} [(1-k^2)K(k) - (1-2k^2)E(k)] \times \\ \times [(1-k^2)(1-8k^2)E(k) - (1-16k^2+16k^4)E(k)],$$

$$A_{2,2}^{2,0} = \frac{2}{3ak(1-2k^2)\sqrt{1-k^2}} [(1-k^2)K(k) - (1-2k^2)E(k)] \times \\ \times [(1-8k^2)(1-k^2)K(k) - (1-16k^2+16k^4)E(k)].$$

3.2.11. Интегралы, содержащие функции Кельвина $\text{ber}_\nu(x)$ или $\text{bei}_\nu(x)$.

Условия: $a, b, \text{Re } p, \text{Re } q > 0$.

$$1. \int_0^\infty \int_0^b \frac{xy}{\sqrt{b^2-x^2}} \exp\left[-\frac{a^2 y^2}{2^6(b^2-x^2)}\right] \text{ber}(axy) dx dy = \\ = -\frac{\pi}{3a^2} \left(e^{-b^2} + \varepsilon^2 e^{-\varepsilon b^2} + \varepsilon e^{-\varepsilon^2 b^2}\right) \quad [\varepsilon \neq 1, \varepsilon^2 = 1]$$

$$2. \int_0^\infty \int_0^\infty y [\text{ber}_{n+1/2}^2(axy) + \text{bei}_{n+1/2}^2(axy)] e^{-px-ay^2} dx dy = \\ = \frac{(-1)^n n!}{2a\sqrt{\pi q}} D_{-n-1}\left(\frac{p\sqrt{q}}{a}\right) D_{-n-1}\left(-\frac{p\sqrt{q}}{a}\right).$$

$$3. \int_0^\infty \int_0^\infty xy e^{-px^2-ay^2} \text{bei}(axy) dx dy = \frac{a^2}{16p^2q^2+a^4}.$$

$$4. \int_0^\infty \int_0^\infty xy e^{-px^2-ay^2} \text{ber}(axy) dx dy = \frac{4pq}{16p^2q^2+a^4}.$$

$$5. \int_0^\infty \int_0^\infty x^{\nu+1} y^{1-\nu} e^{-px^2-ay^2} \text{bei}_\nu(axy) dx dy = \\ = \left(\frac{ap}{2}\right)^\nu \frac{4pq \sin(3\nu\pi/4) + a^2 \cos(3\nu\pi/4)}{16p^2q^2+a^4} \quad [\text{Re } \nu > -1].$$

$$6. \int_0^\infty \int_0^\infty x^{\nu+1} y^{1-\nu} e^{-px^2 - qy^2} \operatorname{ber}_\nu (axy) dx dy = \left(\frac{ap}{2}\right)^\nu \frac{4pq \cos(3\nu\pi/4) - a^2 \sin(3\nu\pi/4)}{16p^2q^2 + a^4} \\ [\operatorname{Re} \nu > -1].$$

$$7. \int_0^\infty \int_0^\infty x^{2n+1} y^2 e^{-px^2 - qy^2} \operatorname{bei}_{2n} (axy) dx dy = \\ = (-1)^n \sqrt{\frac{\pi}{2}} \frac{a^{2n}}{2^{5n+3} p^{2n+1} q^{n+1}} e^{-a^4/(2^7 p^2 q)} D_{2n+1} \left(\frac{a^2}{4p \sqrt{2q}} \right).$$

$$8. \int_0^\infty \int_0^\infty x^{2n+1} y e^{-px^2 - qy^2} \operatorname{ber}_{2n} (axy) dx dy = \\ = (-1)^n \sqrt{\frac{\pi}{2}} \frac{a^{2n}}{2^{5n+5/2} p^{2n+1} q^{n+1/2}} e^{-a^4/(2^7 p^2 q)} D_{2n} \left(\frac{a^2}{4p \sqrt{2q}} \right).$$

$$9. \int_0^\infty \int_0^\infty xy^2 e^{-px^2 - qy^2} \operatorname{bei} (axy) dx dy = \frac{\sqrt{\pi} a^2}{2^6 p^2 q^{3/2}} e^{-a^4/(2^6 p^2 q)}.$$

$$10. \int_0^\infty \int_0^\infty xy e^{-px^2 - qy^2} \operatorname{ber} (axy) dx dy = \frac{\sqrt{\pi}}{8p \sqrt{q}} e^{-a^4/(2^6 p^2 q)}.$$

$$11. \int_0^\infty \int_0^\infty xy^2 e^{-px^2 - qy^2} [\operatorname{ber}^2 (axy) + \operatorname{bei}^2 (axy)] dx dy = \frac{1}{8pq} e^{a^4/(16p^2 q)}.$$

$$12. \int_0^\infty \int_0^\infty xy e^{-px^2 - qy^2} \operatorname{ber} (axy) dx dy = \frac{\pi}{16 \sqrt{pq}} J_0 \left(\frac{a^2}{8 \sqrt{pq}} \right).$$

$$13. \int_0^\infty \int_0^\infty xy K_0(ax) K_0(by) (\operatorname{ber} x \operatorname{ber} y - \operatorname{bei} x \operatorname{bei} y) dx dy = \frac{ab - 1}{(a^2 + 1)(b^2 + 1)} \\ [\operatorname{Re} a^2, \operatorname{Re} b^2 > 1/2].$$

3.2.12. Интегралы, содержащие ортогональные многочлены.

$$1. \int_0^1 \int_0^\pi \frac{y^{2\beta+1}}{(1-y^2)^{\beta-\alpha+1}} \sin^{2\beta} x P_n^{(\alpha, \beta)} (2 |\cos a \cos b + ye^{ix} \sin a \sin b|^2 - 1) dx dy = \\ = \frac{\sqrt{\pi}}{2} \Gamma \left[\frac{\alpha - \beta}{\alpha + 1}, \beta + 1/2 \right] \frac{P_n^{(\alpha, \beta)}(\cos 2a) P_n^{(\alpha, \beta)}(\cos 2b)}{P_n^{(\alpha, \beta)}(1)} \quad [\operatorname{Re} \alpha > \operatorname{Re} \beta > -1/2].$$

$$2. \int_{-1}^1 \int_{-1}^1 (1-x^2)^{\alpha-1} (1-y^2)^{\beta-1} C_{2n}^{\alpha+\beta} (ax + by) dx dy = \\ = \pi \Gamma \left[\frac{\alpha}{\alpha + 1/2}, \frac{\beta}{\beta + 1/2} \right] \frac{C_{2n}^{\alpha+\beta}(0) P_n^{(\alpha-1/2, \beta-1/2)}(A) P_n^{(\alpha-1/2, \beta-1/2)}(B)}{P_n^{(\alpha-1/2, \beta-1/2)}(-1) P_n^{(\alpha-1/2, \beta-1/2)}(1)} \\ [2a = \sqrt{(1-A)(1-B)}, 2b = \sqrt{(1+A)(1+B)}, \operatorname{Re} \alpha, \operatorname{Re} \beta > 0].$$

$$3. \iint_{x^2+y^2 \leq 1} \frac{P_m(x) P_n(y)}{\sqrt{1-x^2-y^2}} dx dy = \begin{cases} (-1)^{n/2} 2\pi \frac{(n-1)!!}{n!!(2n+1)} & [m=n - \text{четное число}], \\ 0 & [\text{в остальных случаях}]. \end{cases}$$

$$4. \int_0^{\infty} \int_0^{\infty} e^{-px-xy} L_n(x+y) dx dy = \frac{1}{p^{n+1} q^{n+1} (p+q)} [p^{n+1} (q-1)^n - q^{n+1} (p-1)^n],$$

$$5. \int_0^{\infty} \int_0^{\infty} x^{\nu/2} y^{\alpha-\nu/2} e^{-px-xy} J_{\nu}(2\sqrt{xy}) L_n^{\alpha}(y) dx dy = \\ = \frac{\Gamma(n+\alpha+1) (1+pq-p)^n}{n! p^{\nu-\alpha} (pq+1)^{n+\alpha+1}} \quad [\operatorname{Re} \alpha, \operatorname{Re} \nu > -1].$$

$$6. \int_0^{\infty} \int_0^{\infty} x^{\alpha-1} y^{\beta-1} (x+y)^{\mu} e^{-\lambda(x+y)} L_{n_1}^{\alpha_1}[a_1(x+y)] \dots L_{n_m}^{\alpha_m}[a_m(x+y)] dx dy = \\ = \prod_{k=1}^m \frac{(\alpha_k+1)_{n_k}}{n_k!} \Gamma \left[\begin{matrix} \alpha, \beta, \alpha+\beta+\mu \\ \alpha+\beta \end{matrix} \right] \lambda^{-\alpha-\beta-\mu} \times \\ \times F_A^{(m)} \left(\alpha+\beta+\mu, -n_1, \dots, -n_m; \alpha_1+1, \dots, \alpha_m+1; \frac{a_1}{\lambda}, \dots, \frac{a_m}{\lambda} \right) \\ [\operatorname{Re} \alpha, \operatorname{Re} \beta, \operatorname{Re} \lambda, \operatorname{Re} (\alpha+\beta+\mu) > 0].$$

3.3. ТРОЙНЫЕ ИНТЕГРАЛЫ

Обозначения:

$$\hat{f}(p) = \int_0^{\infty} e^{-px} f(x) dx, \quad \hat{f}(p, q) = \int_0^{\infty} \int_0^{\infty} e^{-px-xy} f(x, y) dx dy.$$

Условия: $a, \operatorname{Re} p, \operatorname{Re} q > 0$.

3.3.1. Интегралы, содержащие гамма-функцию $\Gamma(x)$.

$$1. \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \frac{y^{n+x-1} z^{x-1}}{\Gamma(x) \Gamma(x+n)} e^{-qy-rz} dx dy dz = \frac{1}{q^n \ln(qr)}.$$

$$2. \int_0^{\infty} \int_0^{\infty} e^{-qy-rz} \int_0^y \frac{z^x}{\Gamma^2(x+1)} dx dy dz = \frac{1}{qr} \psi \left(\frac{e^{-q}}{r} \right).$$

3.3.2. Интегралы, содержащие функцию Бесселя $J_{\nu}(x)$.

$$1. \int_{x+y+z \leq 1} \int \int \frac{1}{\sqrt{x}} J_0(y) J_0(z) f(y+z) dx dy dz = 2 \int_0^1 \sqrt{1-t} \sin t f(t) dt.$$

$$2. \int_0^{\infty} e^{-rz} \int_0^z e^{-qy} \int_0^y J_0(x) dx dy dz = \frac{1}{r(q+r) \sqrt{(q+r)^2+1}}.$$

$$3. \int_0^{\infty} \int_0^{\infty} e^{-qy-rz} \int_y^{\infty} \frac{1}{x} J_0(x) dx dy dz = \frac{1}{2qr} \ln \frac{4qr+a^2}{a^2}.$$

$$4. \int_0^{\infty} \int_0^{\infty} e^{-qy-rz} \int_0^z J_0(a\sqrt{z^2-x^2}) f(x+y) dx dy dz = \\ = \frac{1}{\sqrt{r^2+a^2} (q-\sqrt{r^2+a^2})} [\vartheta(\sqrt{r^2+a^2}) - f(q)].$$

$$5. \int_0^\infty \int_0^\infty e^{-qy-rz} \int_0^z J_0(a\sqrt{z^2-x^2}) f(x, y) dx dy dz = \frac{1}{\sqrt{r^2+a^2}} \tilde{f}(\sqrt{r^2+a^2}, q).$$

$$6. \int_0^\infty \int_0^\infty e^{-qy-rz} \int_0^z J_0(a\sqrt{x(z-x)}) f(x, y) dx dy dz = \frac{1}{r} \tilde{f}\left(r + \frac{a^2}{4r}, q\right).$$

$$7. \int_0^\infty \int_0^\infty e^{-qy-rz} \int_0^z \sqrt{\frac{x}{z-x}} J_1(a\sqrt{x(z-x)}) f(x, y) dx dy dz = \\ = \frac{2}{a} \tilde{f}(r, q) - \frac{2}{a} \tilde{f}\left(r + \frac{a^2}{4r}, q\right).$$

$$8. \int_0^\infty \int_0^\infty e^{-qy-rz} \int_0^z \frac{x}{\sqrt{z^2-x^2}} J_1(a\sqrt{z^2-x^2}) f(x, y) dx dy dz = \\ = \frac{1}{a} \tilde{f}(r, q) - \frac{1}{a} \tilde{f}(\sqrt{r^2+a^2}, q).$$

$$9. \int_0^\infty \int_0^\infty \int_0^\infty e^{-qy-rz} J_0(a\sqrt{x(y+z)}) f(x) dx dy dz = \frac{a^2 r \hat{f}\left(\frac{a^2}{4q}\right) - b^2 q \hat{f}\left(\frac{b^2}{4r}\right)}{qr(a^2 r - b^2 q)}.$$

$$10. \int_0^\infty \int_0^\infty \int_0^\infty e^{-qy-rz} J_0(a\sqrt{xy}) J_0(b\sqrt{xz}) f(x) dx dy dz = \frac{1}{qr} \hat{f}\left(\frac{a^2}{4q} + \frac{b^2}{4r}\right).$$

3.4. МНОГОМЕРНЫЕ ИНТЕГРАЛЫ

3.4.1. Интегралы, содержащие функции Бесселя $J_\nu(x)$ и Макдональда $K_\nu(x)$.

Обозначение: $dx = dx_1 dx_2 \dots dx_n$.

$$1. \int_0^\infty \dots \int_0^\infty J_\nu\left(\frac{c}{x_1 x_2 \dots x_n}\right) \prod_{k=1}^n x_k^{2k/(n+1)-1} J_\nu(x_k) dx = J_{n\nu+\nu}((n+1)c^{1/(n+1)}) \\ [c > 0; \operatorname{Re} \nu > 1/2 - 2/(n+1)].$$

$$2. \int_0^\infty \dots \int_0^\infty K_\nu\left(\frac{c}{x_1 x_2 \dots x_n}\right) \prod_{k=1}^n x_k^{2k/(n+1)-1} K_\nu(x_k) dx = \pi^n K_{n\nu+\nu}((n+1)c^{1/(n+1)}) \\ [\operatorname{Re} c > 0].$$

3.4.2. Интегралы, содержащие ортогональные многочлены.

Обозначения: $bx = b_1 x_1 + b_2 x_2 + \dots + b_n x_n$, $dx = dx_1 dx_2 \dots dx_n$.

$$1. \int_{\substack{x_1 \geq 0, \dots, x_n \geq 0 \\ x_1 + \dots + x_n \leq 1}} \prod_{i=1}^n x_i^{\alpha_i-1} P_m(x_1 + \dots + x_n) P_{m+2l}(x_1 + \dots + x_n) dx = \\ = \frac{\delta_{0,l}}{2m+1} \prod_{i=1}^n \Gamma(\alpha_i) \quad [\operatorname{Re} \alpha_1, \operatorname{Re} \alpha_2, \dots, \operatorname{Re} \alpha_n > 0; \alpha_1 + \dots + \alpha_n = 1].$$

$$2. \int \dots \int_{\substack{x_1 \geq 0, \dots, x_n \geq 0 \\ x_1 + \dots + x_n \leq 1}} \prod_{i=1}^n x_i^{\alpha_i - 1} P_m(x_1 + \dots + x_n) P_{m+1}(x_1 + \dots + x_n) dx = \\ = \frac{m+1}{(2m+1)(2m+3)} \prod_{i=1}^n \Gamma(\alpha_i) \quad [\operatorname{Re} \alpha_1, \operatorname{Re} \alpha_2, \dots, \operatorname{Re} \alpha_n \geq 0; \alpha_1 + \dots + \alpha_n = 2].$$

$$3. \int \dots \int_{\substack{\frac{x_1^2}{a_1^2} + \dots + \frac{x_n^2}{a_n^2} \leq 1}} \left(1 - \sum_{k=1}^n \frac{x_k^2}{a_k^2}\right)^{\lambda-1} P_m^{(\alpha, \beta)}(bx) P_m^{(\beta, \alpha)}(bx) dx = \\ = \frac{\pi^{n/2} \Gamma(\lambda)}{\Gamma(\lambda + n/2)} \binom{m+\alpha}{m} \binom{m+\beta}{m} \left(\prod_{i=1}^n a_i\right) {}_2F_4\left(-m, \alpha + \beta + m - 1, \right. \\ \left. \frac{\alpha + \beta + 1}{2}, \frac{\alpha + \beta + 3}{2}, \lambda + \frac{n-1}{2}; \alpha + 1, \beta + 1, \alpha + \beta + 1, \lambda + \frac{n}{2}; 1\right) \\ \left[\operatorname{Re} \lambda, \operatorname{Re}(\alpha + 1), \operatorname{Re}(\beta + 1) > 0; \sum_{k=1}^n a_k^2 b_k^2 = 1 \right].$$

$$4. \int \dots \int_{\substack{\frac{x_1^2}{a_1^2} + \dots + \frac{x_n^2}{a_n^2} \leq 1}} \left(1 - \sum_{k=1}^n \frac{x_k^2}{a_k^2}\right)^{\lambda-1} P_l^{(\alpha, \beta)}(bx) P_m^{(\alpha, \alpha)}(bx) dx = \\ = \frac{2^{2\nu+1} \pi^{(n-1)/2}}{m! (l-m)!} \left(\prod_{i=1}^n a_i\right) \Gamma\left[\lambda + 1/2, \alpha + l + 1, \alpha + m + 1, \alpha + \beta + l + m + 1, \alpha - \beta + 1\right] \\ \Gamma\left[\alpha + 1, \alpha + \beta + l + 1, 2\alpha + l + m + 2, \alpha - \beta - l + m + 1\right] \\ [0 \leq m \leq l, \\ [m > l] \\ 5. = 0$$

$$\left[\alpha + 1 = \lambda + n/2; \operatorname{Re}(\alpha + 1), \operatorname{Re}(\lambda + 1/2) > 0; \sum_{k=1}^n a_k^2 b_k^2 = 1 \right].$$

$$6. \int \dots \int_{\substack{\frac{x_1^2}{a_1^2} + \dots + \frac{x_n^2}{a_n^2} \leq 1}} \left(1 - \sum_{k=1}^n \frac{x_k^2}{a_k^2}\right)^{\lambda-1} C_l^\alpha(bx) C_m^\alpha(bx) dx = \\ = \frac{\pi^{n/2} \Gamma(\lambda) (2\alpha)_l}{l! (\alpha + l) \Gamma(\alpha)} \left(\prod_{i=1}^n a_i\right) \delta_{l,m} \\ \left[\alpha + 1 = \lambda + n/2; \operatorname{Re}(\alpha + 1/2), \operatorname{Re} \lambda > 0; \sum_{k=1}^n a_k^2 b_k^2 = 1 \right].$$

Глава 4. КОНЕЧНЫЕ СУММЫ

4.1. ВВЕДЕНИЕ

В этой главе помещены некоторые конечные суммы, содержащие различные специальные функции. При вычислении конечных сумм иногда может оказаться полезной следующая формула замены индекса суммирования:

$$\sum_{k=n_1}^{n_2} a_k = \sum_{k=n-n_2}^{n-n_1} a_{n-k},$$

в частности,

$$\sum_{k=0}^n a_k = \sum_{k=0}^n a_{n-k}.$$

Всюду в этой главе $l, m, n = 0, 1, 2, 3, \dots$, если не указаны другие условия

4.2. ФУНКЦИИ БЕССЕЛЯ $J_\nu(z)$, $I_\nu(z)$, НЕЙМАНА $Y_\nu(z)$ И МАКДОНАЛЬДА $K_\nu(z)$

4.2.1. Суммы, содержащие $J_\nu(z)$.

$$1. \sum_{k=0}^n J_{\nu \pm 2k}(z) = \pm \frac{1}{2} \int_0^z [J_{\nu \mp 1}(t) - J_{\nu \pm 2n \pm 1}(t)] dt \quad [\operatorname{Re} \nu > 0].$$

$$2. \sum_{k=0}^n (-1)^k J_{2k}(z) = \frac{1}{2} \cos z + \frac{1}{2} J_0(z) + (-1)^n U_{2n+2}(z, z).$$

$$3. \sum_{k=0}^n (-1)^k J_{2k+1}(z) = \frac{1}{2} \sin z + (-1)^n U_{2n+3}(z, z).$$

$$4. \sum_{k=1}^n (-1)^k k J_{2k}(z) = \frac{z}{4} [(-1)^n J_{2n+1}(z) - J_1(z)].$$

$$5. \sum_{k=1}^n (2k+1) J_{2k+1}(z) = \frac{z}{2} - (n+1) z \int_0^z \frac{1}{t} J_{2n+2}(t) dt.$$

$$6. \sum_{k=0}^n (-1)^k (2k+1) J_{2k+1}(z) = \frac{z}{2} [J_0(z) + (-1)^n J_{2n+2}(z)].$$

$$7. \sum_{k=1}^n (-1)^k (2k+v) J_{2k+v}(z) = \frac{z}{2} [(-1)^n J_{2n+v+1}(z) - J_{v+1}(z)].$$

$$8. \sum_{k=0}^n (-1)^k (v-2k) J_{v-2k}(z) = \frac{z}{2} [J_{v+1}(z) + (-1)^n J_{v-2n-1}(z)].$$

$$9. \sum_{k=0}^{n-1} \frac{(z/2)^k}{k!(n-k)!} J_k(z) = \frac{2}{n!} \left(\frac{z}{2}\right)^n \left[\frac{\partial J_v(z)}{\partial v} \Big|_{v=n} - \frac{\pi}{2} Y_n(z) \right].$$

$$10. \sum_{k=0}^n (-1)^k \binom{n}{k} J_{v \pm 2k}(z) = (\pm 2)^n \frac{d^n J_{v \pm n}(z)}{dz^n}.$$

$$11. \sum_{k=1}^n \binom{2n}{n-k} J_{2k}(z) = \frac{2^{2n-1}}{z^n} (2n-1)!! J_n(z) - \frac{1}{2} \binom{2n}{n} J_0(z).$$

$$12. \sum_{k=0}^{n-1} J_{\mu \pm k}(z) J_{\nu \pm k}(z) = \frac{z}{2(\mu-\nu)} [J_{\mu \pm n}(z) J_{\nu \pm n \mp 1}(z) - J_{\mu \pm n \mp 1}(z) J_{\nu \pm n}(z) + J_{\mu \mp 1}(z) J_{\nu}(z) - J_{\mu}(z) J_{\nu \mp 1}(z)].$$

$$13. \sum_{k=0}^n \binom{2n}{n-k} J_k^2(z) = \frac{1}{2} \binom{2n}{n} {}_1F_2\left(\frac{1}{2}; 1, n+1; -z^2\right) + \frac{1}{2} \binom{2n}{n} J_0^2(z).$$

$$14. \sum_{k=0}^n \binom{2n}{n-k} J_{k+1/2}(z) J_{k-1/2}(z) = \frac{(-1)^n z^{-n-1/2} (2n)!}{2(n!) \sqrt{\pi}} \times \\ \times \left[J_{1/2-n}(2z) - z^{1/2-n} \sum_{k=0}^{n-1} \frac{(-1)^k z^{2k}}{k! \Gamma(k-n+3/2)} \right] + \frac{1}{2\pi z} \binom{2n}{n} \sin 2z.$$

$$15. \sum_{k=0}^n (-1)^k \frac{2k+1}{(n-k)! (n+k+1)!} J_{k+1/2}(z) J_{-k-1/2}(z) = \frac{z^{-n-1/2}}{n! \sqrt{\pi}} J_{n+1/2}(2z).$$

$$16. \sum_{k=0}^n \frac{(-1)^k}{(n-k)!} \binom{m-n}{k} \left(\frac{z}{2}\right)^{-k} J_{m-k}(z) = \frac{(-1)^n}{n!} J_{m-2n}(z) \quad [m > n]$$

4.2.2. Суммы, содержащие $Y_v(z)$.

$$1. \sum_{k=0}^n Y_{v \pm 2k}(z) = \pm \frac{1}{2} \int_0^z [Y_{v \mp 1}(t) - Y_{v \pm 2n \pm 1}(t)] dt \quad [\operatorname{Re} v > 0].$$

$$2. \sum_{k=0}^n (-1)^k (2k+1) Y_{2k+1}(z) = \frac{z}{2} [Y_0(z) + (-1)^n Y_{2n+2}(z)].$$

$$3. \sum_{k=0}^n (-1)^k \binom{n}{k} Y_{2k+v}(z) = 2^n \frac{d^n Y_{n+v}(z)}{dz^n}.$$

4.2.3. Суммы, содержащие $I_v(z)$.

$$1. \sum_{k=1}^n (\pm 1)^k I_k(z) = \\ = \frac{(\pm 1)^{n+1}}{2n} \left[e^{\pm z} \int_0^z e^{\mp t} I_n(t) dt - z I_n(z) \mp z I_{n+1}(z) \right] - \frac{1}{2} I_0(z) + \frac{1}{2} e^{\pm z}.$$

$$2. \sum_{k=0}^n (-1)^k I_{2k+\nu}(z) = \frac{1}{2} \int_0^z [I_{\nu-1}(t) + (-1)^n I_{2n+\nu+1}(t)] dt \quad [\operatorname{Re} \nu > 0].$$

$$3. \sum_{k=0}^n \binom{n}{k} I_{\nu \pm 2k}(z) = 2^n \frac{d^n I_{\nu \pm n}(z)}{dz^n}.$$

$$4. \sum_{k=0}^{n-1} \frac{(-z/2)^k}{k! (n-k)!} I_k(z) = \frac{2}{n!} \left(\frac{z}{2}\right)^n \left[K_n(z) + (-1)^n \frac{\partial I_\nu(z)}{\partial \nu} \Big|_{\nu=n} \right].$$

4.2.4. Суммы, содержащие $K_\nu(z)$.

$$1. \sum_{k=0}^n (-1)^k K_{2k+\nu}(z) = \frac{1}{2} \int_z^\infty [K_{\nu-1}(t) + (-1)^n K_{2n+\nu+1}(t)] dt.$$

$$2. \sum_{k=0}^n \binom{n}{k} K_{\nu \pm 2k}(z) = (-2)^n K_{\nu \pm n}(z).$$

$$3. \sum_{k=1}^n (-1)^k \binom{2n}{n-k} K_{2k}(z) = \frac{1}{2} \binom{2n}{n} \left[n! \left(-\frac{2}{z}\right)^n K_n(z) - K_0(z) \right].$$

$$4. \sum_{k=0}^{n-1} \frac{(z/2)^k}{k! (n-k)!} K_k(z) = \frac{2}{n!} \left(\frac{z}{2}\right)^n \frac{\partial K_\nu(z)}{\partial \nu} \Big|_{\nu=n}.$$

$$5. \sum_{k=0}^n \binom{n}{k} \frac{(-z/2)^k}{k!} K_k(z) = \frac{1}{2^n n!} \left(\frac{d}{z dz}\right)^n [z^{2n} K_0(z)].$$

4.3. МНОГОЧЛЕНЫ ЛЕЖАНДРА $P_n(x)$

4.3.1. Суммы вида $\sum a_k P_{kl+m}(x)$.

$$1. \sum_{k=1}^n (2k+2m+1) P_{k+m}(x) = \frac{1}{x-1} [(m+n+1)(P_{m+n+1}(x) - P_{m+n}(x)) - (m+1)(P_{m+1}(x) - P_m(x))].$$

$$2. \sum_{k=0}^n (4k+2\sigma+1) P_{2k+\sigma}(x) = \frac{d}{dx} P_{2n+\sigma+1}(x) \quad [\sigma=0 \text{ или } 1].$$

$$3. \sum_{k=0}^n \binom{n}{k} P_k(x) = 2^{n/2} (x+1)^{n/2} P_n\left(\sqrt{\frac{x+1}{2}}\right).$$

$$4. \sum_{k=0}^n \binom{n}{k} (-x)^k P_k(x) = (1-x^2)^{n/2} P_n(\sqrt{1-x^2}).$$

$$5. \sum_{k=0}^n (-1)^k (4k+2\sigma+1) \frac{(a+\sigma+1)_k}{(n+\sigma+3/2)_k (1/2-a)_k (n-k)!} P_{2k+\sigma}(x) = \frac{(2n+2\sigma+1)!}{(n+\sigma)! 4^\sigma (a+1)^\sigma (-2a)_{2n}} C_{2n+\sigma}^{a-n+1}(x) \quad [\sigma=0 \text{ или } 1].$$

$$6. \sum_{k=1}^{n-1} \cos(n-k)x P_k(\cos x) = n P_n(\cos x).$$

4.3.2. Суммы вида $\sum a_k \prod_i P_{k l_i + m_i}(x)$.

$$1. \sum_{k=0}^n P_k(x) P_{n-k}(x) = U_n(x).$$

$$2. \sum_{k=0}^n (2k+1) P_k(x) P'_k(x) = P'_n(x) P'_{n+1}(x).$$

$$3. \sum_{k=0}^n \frac{1}{k+1} P_k(x) P_{n-k}(x) = \frac{1}{2} P_{n+1}(x) \ln \frac{1+x}{1-x} + Q_{n+1}(x) \quad [-1 < x < 1]$$

4.4. МНОГОЧЛЕНЫ ЛАГЕРРА $L_n^\alpha(x)$

4.4.1. Суммы вида $\sum a_k L_{kl+m}^{k\alpha+\beta}(x)$.

$$1. \sum_{k=0}^n L_k^\alpha(x) = L_n^{\alpha+1}(x).$$

$$2. \sum_{k=1}^n k L_k^\alpha(x) = (n+1) L_n^{\alpha+1}(x) - L_n^{\alpha+2}(x).$$

$$3. \sum_{k=1}^n \frac{1}{k} L_k(x) = \int_0^x \frac{1}{x} [L_n(x) - 1] dx + \psi(n+1) + C.$$

$$4. \sum_{k=0}^n \frac{(-1)^k}{(n-k)! (\beta)_k} L_k^\alpha(x) = \frac{(-1)^n}{(\beta)_n} L_n^{\alpha-\beta-n+1}(x).$$

$$5. \sum_{k=0}^n (-1)^k \binom{n}{k} L_{m+k}^\alpha(x) = (-1)^n L_{m+n}^{\alpha-n}(x).$$

$$6. \sum_{k=0}^n \frac{(\beta+1)_k}{(\alpha+1)_k} L_k^\alpha(x) = \frac{(\beta+2)_n}{n!} {}_2F_2(-n, \beta+1; \alpha+1, \beta+2; x).$$

$$7. \sum_{k=0}^n \frac{t^k}{(n-k)! (\alpha+1)_k} L_k^\alpha(x) = \frac{(t+1)^n}{(\alpha+1)_n} L_n^\alpha\left(\frac{tx}{t+1}\right).$$

$$8. \sum_{k=0}^n \frac{(-1)^k}{(n-k)! (\alpha-n+1/2)_k} L_k^\alpha(x) = \frac{(-1)^n}{n! 2^{2n+1} (1/2-\alpha)_n \sqrt{x}} H_{2n+1}(\sqrt{x}).$$

$$9. \sum_{k=0}^n \frac{(-1)^k (1/2)_k}{(n-k)! (\alpha/2+1)_k (1/2-n)_k} L_{2k}^\alpha(x) = \frac{n!}{(\alpha/2+1)_n (1/2)_n} \left[L_n^{\alpha/2}\left(\frac{x}{2}\right) \right]^2.$$

$$10. \sum_{k=0}^n \frac{(-1)^k (ka+1)^{n-1}}{(n-k)! (\alpha+1)_k} L_k^\alpha\left(\frac{x}{ka+1}\right) = \frac{x^n}{n! (\alpha+1)_n (na+1)}.$$

$$11. \sum_{k=0}^n (-1)^k \binom{n}{k} L_{m+k}^{k+\alpha}(x) = \frac{d^n L_m^\alpha(x)}{dx^n}.$$

$$12. \sum_{k=0}^n \frac{(-x)^k}{(\alpha-n+1)_k} \binom{n}{k} L_{m+k}^{k+\alpha}(x) = \frac{(m+n)!}{m! (\alpha-n+1)_n} L_{m+n}^{\alpha-n}(x).$$

$$13. \sum_{k=0}^n \frac{(-x)^k}{(n-k)! (\alpha-\beta+1)_k (\beta+1)_k} L_k^{\alpha+\beta} (x) = \\ = \frac{n!}{(\alpha-\beta+1)_n (\beta+1)_n} L_n^{\alpha-\beta} (x) L_n^{\beta} (x).$$

$$14. \sum_{k=0}^n \frac{1}{(n-k)! (\beta)_k} L_k^{\alpha-k} (x) = \frac{1}{(\beta)_n} L_n^{\alpha+\beta-1} (x).$$

$$15. \sum_{k=0}^n \binom{n}{k} L_{k+m}^{\alpha-k} (x) = L_{m+n}^{\alpha} (x).$$

$$16. \sum_{k=0}^n \frac{2^{2k}}{(n-k)! (\alpha+1)_k} L_{2k}^{\alpha-k} (x) = L_{2n}^{2\alpha} (2x).$$

$$17. \sum_{k=0}^n \frac{(-t)^k}{(n-k)!} L_k^{n-2k} (x) = \frac{t^{n/2}}{n!} H_n \left(\frac{1+tx}{2\sqrt{t}} \right).$$

$$18. \sum_{k=0}^n \frac{(-1)^k (n-\alpha)_k}{(n-k)! (xy)^k} L_k^{\alpha-2k} (x+y) = \frac{n!}{(xy)^n} L_n^{\alpha-2n} (x) L_n^{\alpha-2n} (y).$$

4.4.2. Суммы вида $\sum a_k \prod_i L_{kl_i+m_i}^{k\alpha_i+\beta_i} (x_i)$.

$$1. \sum_{k=0}^n L_k (x) L_k (y) = \frac{n+1}{x-y} [L_n (x) L_{n+1} (y) - L_{n+1} (x) L_n (y)].$$

$$2. \sum_{k=0}^n [L_k (x)]^2 = (n+1) [L_n (x) L_n^1 (x) - L_{n+1} (x) L_{n-1}^1 (x)].$$

$$3. \sum_{k=0}^n L_k^{\alpha} (x) L_{n-k}^{\beta} (y) = L_n^{\alpha+\beta+1} (x+y).$$

$$4. \sum_{k=0}^n \frac{k!}{(\alpha+1)_k} [L_k^{\alpha} (x)]^2 = \frac{(n+1)!}{(\alpha+1)_n} [L_n^{\alpha} (x) L_n^{\alpha+1} (x) - L_{n+1}^{\alpha} (x) L_{n-1}^{\alpha+1} (x)].$$

$$5. \sum_{k=0}^n (-1)^k \binom{n}{k} L_k (x) L_{n-k} (y) = \frac{(-1)^n}{n!} (x+y)^n P_n \left(\frac{y-x}{y+x} \right).$$

$$6. \sum_{k=0}^n \frac{(\beta)_k}{(\alpha+1)_k} L_k^{\alpha} (x) L_{n-k}^{\alpha-2\beta} (-x) = \frac{(\alpha-\beta+1)_n}{(\alpha+1)_n} L_n^{\alpha} (-x).$$

$$7. \sum_{k=0}^n \frac{(-1)^k}{(n-k)! (\alpha+1)_k} L_k^{\alpha} (x) L_{k+m}^{\alpha-m} (x) = \frac{(-1)^n}{(\alpha+1)_n} \binom{m+2n}{m+n} L_{m+2n}^{\alpha-m-n} (x).$$

$$8. \sum_{k=0}^n L_k^{\alpha-k} (x) L_{n-k}^{\beta+k} (y) = L_n^{\alpha+\beta} (x+y).$$

$$9. \sum_{k=0}^n L_k^{\alpha-2k} (x) L_{n-k}^{2k-\alpha-2n-2} (-x) = (-4)^n.$$

$$10. \sum_{k+l=n} \frac{(-1)^k}{(\alpha+1)_k (\beta+1)_l} L_k^\alpha(x) L_l^\beta(y) = \frac{(x+y)^n}{(\alpha+1)_n (\beta+1)_n} P_n^{(\alpha, \beta)}\left(\frac{y-x}{y+x}\right).$$

$$11. \sum_{\substack{k_1, k_2, \dots, k_m \geq 0 \\ k_1 + k_2 + \dots + k_m = n}} L_{k_1}^{\alpha_1}(x_1) L_{k_2}^{\alpha_2}(x_2) \dots L_{k_m}^{\alpha_m}(x_m) =$$

$$= L_n^{\alpha_1 + \alpha_2 + \dots + \alpha_m + m - 1}(x_1 + x_2 + \dots + x_m).$$

4.5. МНОГОЧЛЕНЫ ЭРМИТА $H_n(x)$

4.5.1. Суммы вида $\sum a_k H_{2k+m}(\varphi(k)x)$.

$$1. \sum_{k=1}^n \frac{1}{(n-k)! (2k)!} H_{2k-1}(x) = \frac{\sqrt{\pi}}{2} e^{x^2} \left[\operatorname{erf}(x) - \frac{1}{\Gamma(n+1/2)} \gamma\left(n+\frac{1}{2}, x^2\right) \right].$$

$$2. \sum_{k=0}^n \frac{1}{(n-k)! (2k+1)!} H_{2k}(x) = \frac{2^{2n}}{(2n+1)!} e^{x^2} [n! - \gamma(n+1, x^2)].$$

$$3. \sum_{k=0}^n \frac{1}{(n-k)! (2k+\sigma)!} H_{2k+\sigma}(x) = \frac{2^{2n+\sigma}}{(2n+\sigma)!} x^{2n+\sigma} \quad [\sigma=0 \text{ или } 1].$$

$$4. \sum_{k=0}^n \frac{(-1)^k}{(n-k)! (2k+1)!} H_{2k+1}(x) = \frac{(-1)^n 2^{n+1/2}}{(2n+1)!} H_{2n+1}\left(\frac{x}{\sqrt{2}}\right).$$

$$5. \sum_{k=0}^n \frac{(-1)^k}{k! 2^{2k}} H_{2k+\sigma}(x) = \frac{(-1)^n}{n! 2^{2n+1} x^{\sigma+1}} [H_{2n+1}(x) + \sigma x H_{2n+2}(x)] \quad [\sigma=0 \text{ или } 1].$$

$$6. \sum_{k=0}^{\infty} \frac{t^k}{(n-k)! (2k+\sigma)!} H_{2k+\sigma}(x) = \frac{(t-1)^n}{(2n+\sigma)!} \left(\frac{t-1}{t}\right)^{\sigma/2} H_{2n+\sigma}\left(x \sqrt{\frac{t}{t-1}}\right) \quad [\sigma=0 \text{ или } 1].$$

$$7. \sum_{k=0}^n \binom{n}{k} t^k H_k(x) = t^n H_n\left(x + \frac{1}{2t}\right).$$

$$8. \sum_{k=0}^n \frac{(-1)^k}{k!} \binom{2n-2k}{n-k} H_{2k}(x) = \frac{2^{2n}}{n!} L_n(x^2).$$

$$9. \sum_{k=0}^{2n} (ka+1)^{2n-k-1} \binom{2n}{k} (-2x)^{-k} H_k((ka+1)x) = \frac{(-1)^n (1/2)_n}{2na+1} x^{-2n}.$$

$$10. \sum_{k=0}^{[n/2]} \frac{(na-2ka+1)^{n/2-1}}{k! (n-2k)!} H_{n-2k}\left(\frac{x}{\sqrt{na-2ka+1}}\right) = \frac{(2x)^n}{n! (na+1)}.$$

$$11. \sum_{k=0}^n k! 2^k \binom{m}{k} \binom{n}{k} H_{m+n-2k}(x) = H_m(x) H_n(x) \quad [n \leq m].$$

4.5.2. Суммы вида $\sum a_k \prod H_{k_i \pm m_i}(x_i)$.

$$12. \sum_{k=0}^n \frac{1}{k! 2^k} H_k(x) H_k(y) = \frac{1}{n! 2^{n+1} (x-y)} [H_{n+1}(x) H_n(y) - H_n(x) H_{n+1}(y)].$$

$$\begin{aligned}
2. \quad & \sum_{k=0}^n \frac{1}{k! 2^k} H_k^2(x) = \frac{1}{n! 2^{n+1}} [H_{n+1}^2(x) - H_n(x) H_{n+2}(x)]. \\
3. \quad & \sum_{k=0}^n \binom{n}{k} t^k H_k(x) H_{n-k}(y) = (t^2 + 1)^{n/2} H_n\left(\frac{tx+y}{\sqrt{t^2+1}}\right). \\
4. \quad & \sum_{k=0}^n \binom{n}{k} H_{2k}(x) H_{2n-2k}(y) = (-4)^n L_n(x^2 + y^2). \\
5. \quad & \sum_{k=0}^n (-1)^k \binom{n}{k} H_{2k}(x) H_{2n-2k}(y) = 2^n H_n\left(\frac{y+x}{\sqrt{2}}\right) H_n\left(\frac{y-x}{\sqrt{2}}\right). \\
6. \quad & \sum_{k=0}^{\min(m, n)} (-2)^k k! \binom{m}{k} \binom{n}{k} H_{m-k}(x) H_{n-k}(x) = H_{m+n}(x). \\
7. \quad & \sum_{k=0}^n \binom{2n}{2k} H_{2k}(x) H_{2n-2k}(y) = 2^{n-1} \left[H_{2n}\left(\frac{x+y}{\sqrt{2}}\right) + H_{2n}\left(\frac{x-y}{\sqrt{2}}\right) \right]. \\
8. \quad & \sum_{k=0}^n \binom{n}{k} H_k^2(x) H_{n-k}^2(y) = 2^n n! \sum_{k=0}^n (-1)^k L_k(2x^2 + 2y^2). \\
9. \quad & \sum_{\substack{k_1 + k_2 + \dots + k_m = n \\ k_1, k_2, \dots, k_m \geq 0}} \frac{a_1^{k_1} a_2^{k_2} \dots a_m^{k_m}}{k_1! k_2! \dots k_m!} H_{k_1}(x_1) H_{k_2}(x_2) \dots H_{k_m}(x_m) = \\
& = \frac{(a_1^2 + a_2^2 + \dots + a_m^2)^{n/2}}{n!} H_n\left(\frac{a_1 x_1 + a_2 x_2 + \dots + a_m x_m}{\sqrt{a_1^2 + a_2^2 + \dots + a_m^2}}\right).
\end{aligned}$$

4.5.3. Суммы, содержащие специальные функции $H_n(x)$.

$$\begin{aligned}
1. \quad & \sum_{k=0}^n \binom{n}{k} t^k P_{n-k}(x) H_k(y) = \sum_{k=0}^{[n/2]} \frac{t^{2k} (x^2 - 1)^k}{2^{2k}} \binom{n}{2k} \binom{2k}{k} H_{n-2k}\left(x + \frac{ty}{2}\right). \\
2. \quad & \sum_{k=0}^n (-1)^k \binom{n}{k} \left(\frac{x}{2y}\right)^k P_{n-k}(x) H_k(y) = \\
& = (-1)^{n/2} \frac{1 + (-1)^n}{2} \frac{n!}{(n/2)!} \left(\frac{x}{2y}\right)^n L_{n/2}\left(\frac{(x^2 - 1)y^2}{x^2}\right). \\
3. \quad & \sum_{k=0}^n \frac{(-1)^k}{k! 2^{2k}} L_{n-k}(x^2 - y^2) H_{2k}(y) = \frac{(-1)^n}{n! x} 2^{-2n} {}_1H_{2n+1}(x). \\
4. \quad & \sum_{k=0}^{2n} \frac{(1/2)_k}{(k!)^2} 2^{2k} L_{2n-k}^{2k}(2x^2) H_{2k}(x) = \frac{1}{(n!)^2} H_{2n}^2(x). \\
5. \quad & \sum_{k=0}^n \frac{(-1)^k}{(k!)^2} L_{n-k}^{2k}(2x^2 + 2y^2) H_{2k}(x) H_{2k}(y) = \frac{(-1)^n}{(n!)^2} H_n^2(x) H_n^2(y)
\end{aligned}$$

4.6. МНОГОЧЛЕНЫ ГЕГЕНБАУЭРА $C_n^v(x)$

4.6.1. Суммы вида $\sum a_k C_k^v(\varphi(x))$

$$1. \sum_{k=0}^n (\pm 1)^k (k+v) C_k^v(x) = \frac{(\pm 1)^k}{2(1 \mp x)} [(n+2v) C_n^v(x) \mp (n+1) C_{n+1}^v(x)].$$

$$2. \sum_{k=0}^n \frac{2k+v+\sigma}{(n-k)! (n+v+\sigma+1)_k} C_{2k+\sigma}^v(x) = \frac{(v+\sigma)_{n+1}}{(2n+\sigma)!} (2x)^{2n+\sigma}$$

$[\sigma=0 \text{ или } 1]$

$$3. \sum_{k=0}^n (-1)^k \frac{(\mu+\sigma)_k (2k+v+\sigma)}{(n-k)! (n+v+\sigma+1)_k (v-\mu+1)_k} C_{2k+\sigma}^v(x) =$$

$$= \frac{(v+1)_{n+\sigma} v}{(1-\mu)_k (v-\mu+1)_n \mu^\sigma} C_{2n+\sigma}^{\mu-n}(x)$$

$[\sigma=0 \text{ или } 1]$

$$4. \sum_{k=0}^n \frac{t^k}{(n-k)! (2v)_k} C_k^v(x) = \frac{1}{(2v)_n} (1-2tx+t^2)^{n/2} C_n^v\left(\frac{1-tx}{\sqrt{1-2t+t^2}}\right).$$

$$5. \sum_{k=0}^{n-1} \cos(n-k)x C_k^v(\cos x) = \frac{n}{2v} C_n^v(\cos x)$$

$$6. \sum_{k=0}^n \frac{t^k}{(n-k)! (1-v)_k} C_k^{v-n}(x) = \frac{(-t)^n}{(v-n)_n} C_n^{v-n}\left(x - \frac{1}{2t}\right).$$

$$7. \sum_{k=0}^n \frac{(-1)^k (n-2v+1)_k}{(n-k)! [(1-v)_k]^2} (4 \sin x \sin y)^{-k} C_k^{v-n}(\cos(x+y)) =$$

$$= \frac{n!}{[(v-1)_n]^2} (4 \sin x \sin y)^{-n} C_n^{v-n}(\cos x) C_n^{v-n}(\cos y)$$

$$8. \sum_{k=0}^{2n} (-1)^k \frac{(ka+1)^{2n-k-1} [(ka+1)^2-4x]^{n/2}}{(2n-k)! (2v)_k} C_k^v\left(\frac{ka+1}{\sqrt{(ka+1)^2-4x}}\right) =$$

$$= \frac{2^{2n} (1/2)_n}{(2n)! (v+1/2)_n (1+2na)} x^n$$

4.6.2. Суммы вида $\sum a_k \prod_i C_{kl_i}^{v_i}(x_i)$.

$$1. \sum_{k=0}^{2n} (-1)^k C_k^v(x) C_{2n-k}^v(x) = C_n^v(2x^2-1)$$

$$2. \sum_{k=0}^n C_k^\mu(x) C_{n-k}^v(x) = C_n^{\mu+v}(x).$$

$$3. \sum_{k=0}^n (k+v) C_k^v(x) C_{n-k}^{v-n}(y) = \frac{(v)_{n+1}}{n!} 2^n (x)$$

$$4. \sum_{k=0}^n \frac{(-1)^k (k+v)}{(n-k)! (n+2v+1)_k} C_k^v(x) C_{k+m}^{v-m}(x) =$$

$$= \frac{(2v+1)_n v}{(v+1/2)_n (m-v+1)_n} 2^{-2n} \binom{m+2n}{n} C_{m+2n}^{v-m-n}(x)$$

$$5. \sum_{k=0}^n \frac{k! (k+v)}{(n-k)! (2v)_k (n+2v+1)_k} C_k^v(x) C_k^v(y) = \frac{(n+2v)(x+y)^n}{(v+1/2)_n 2^{n+1}} C_n^v\left(\frac{xy+1}{x+y}\right).$$

$$6. \sum_{k=0}^n \frac{(2k+\sigma)! (2k+v+\sigma)}{(n-k)! (2v+\sigma)_{2k} (n+v+\sigma+1)_k} C_{2k+\sigma}^v(x) C_{2k+\sigma}^v(y) =$$

$$= \frac{(n+v+\sigma)(2v)^\sigma}{(v+1/2)_n} (x^2+y^2-1)^{n+\sigma/2} C_{2n+\sigma}^v\left(\frac{xy}{\sqrt{x^2+y^2-1}}\right) \quad [\sigma=0 \text{ или } 1].$$

4.7. МНОГОЧЛЕНЫ ЯКОБИ $P_n^{(\alpha, \beta)}(x)$

4.7.1. Суммы вида $\sum a_k P_k^{(\alpha \pm kl, \beta \pm km)}(x)$.

$$1. \sum_{k=0}^n P_k^{(\alpha, 0)}(x) = \frac{(\alpha+2)_n}{n!} {}_3F_2\left(-n, n+\alpha+2, \frac{\alpha+1}{2}; \alpha+1, \frac{\alpha+3}{2}; \frac{1-x}{2}\right).$$

$$2. \sum_{k=0}^n (\pm 1)^k (2k+\alpha+\beta+1) \frac{(\alpha+\beta+1)_k}{(\varphi+1)_k} P_k^{(\alpha, \beta)}(x) =$$

$$= (\pm 1)^n \frac{(\alpha+\beta+1)_{n+1}}{(\varphi+1)_n} P_n^{(\alpha+\delta, \beta-\delta+1)}(x)$$

$$[\varphi = \{\beta\}, \delta = \{1\}].$$

$$3. \sum_{k=0}^n \frac{(\alpha+\beta+n+1)_k}{(n-k)! (\alpha+1)_k (\beta+1)_k} t^k P_k^{(\alpha, \beta)}(x) =$$

$$= \frac{(-1)^n n!}{(\alpha+1)_n (\beta+1)_n} P_n^{(\alpha, \beta)}(-t - \sqrt{t^2+2tx+1}) P_n^{(\alpha, \beta)}(-t + \sqrt{t^2+2tx+1}).$$

$$4. \sum_{k=0}^n \frac{t^k}{(n-k)! (\alpha+1)_k} P_k^{(\alpha, \beta-k)}(x) =$$

$$= \frac{1}{(\alpha+1)_n} \left(1 + \frac{tx+t}{2}\right)^n P_n^{(\alpha, -\alpha-\beta-n-1)}\left(\frac{2+3t-tx}{2+t+tx}\right).$$

$$5. \sum_{k=0}^n \frac{(-1)^k}{(n-k)! (\alpha+1)_k} P_k^{(\alpha, \beta-k)}(x) = (-1)^n \frac{(\alpha+\beta+1)_n}{n! (\alpha+1)_n} \left(\frac{x-1}{2}\right)^n.$$

$$6. \sum_{k=0}^n \frac{(-1)^k}{(n-k)! (\gamma)_k} P_k^{(\alpha, \beta-k)}(x) = \frac{1}{(1-n-\gamma)_n} P_n^{(\alpha-\gamma-n+1, \beta+\gamma-1)}(x).$$

$$7. \sum_{k=0}^n \frac{(-1)^k}{(n-k)! (\gamma)_k} \left(\frac{2}{x+1}\right)^k P_k^{(\alpha, \beta-k)}(x) = \frac{(-1)^n}{(\gamma)_n} \left(\frac{2}{x+1}\right)^n P_n^{(\alpha-\gamma-n-1, \beta-n)}(x)$$

$$8. \sum_{k=0}^n \frac{(-1)^k}{(n-k)! (\alpha+1)_k} \left(\frac{2}{x+1}\right)^k P_k^{(\alpha, \beta-k)}(x) = \frac{(-\beta)_n}{n! (\alpha+1)_n} \left(\frac{x-1}{x+1}\right)^n.$$

$$9. \sum_{k=0}^n \frac{1}{(n-k)! (\gamma)_k} \left(\frac{2}{x+1}\right)^k P_k^{(\alpha-k, \beta-k)}(x) =$$

$$= \frac{1}{(\gamma)_n} \left(\frac{2}{x+1}\right)^n P_n^{(\alpha+\gamma-2n+1, \beta-n)}(x).$$

4.7.2. Суммы вида $\sum a_k \prod_i P_{kl_i + m_i}^{(k\alpha_i + \beta_i, k\gamma_i + \delta_i)}(x_i)$.

$$1. \sum_{k=0}^n \frac{1}{(\alpha+k)(\beta+n-k)} P_k^{(\alpha, \alpha)}(x) P_{n-k}^{(\beta, \beta)}(x) = \frac{\alpha+\beta}{\alpha\beta(\alpha+\beta+n)} P_n^{(\alpha+\beta, \alpha+\beta)}(x).$$

$$2. \sum_{k=0}^n \frac{1}{(\alpha+k)(\alpha+k-n)} P_k^{(\alpha, \alpha)}(x) P_{n-k}^{(-\alpha, -\alpha)}(x) = \frac{1}{\alpha^2} \delta_{0,n}.$$

$$3. \sum_{k=0}^n (2k+\alpha+\beta+1) \frac{k! (\alpha+\beta+1)_k}{(n-k)! (\alpha+1)_k (\beta+1)_k (\alpha+\beta+n+2)_k} P_k^{(\alpha, \beta)}(x) P_k^{(\alpha, \beta)}(y) =$$

$$= \frac{(\alpha+\beta+1)_{n+1}}{(\alpha+1)_n (\beta+1)_n} \left(\frac{x+y}{2}\right)^n P_n^{(\alpha, \beta)}\left(\frac{1+xy}{x+y}\right).$$

$$4. \sum_{k=0}^n (2k+\alpha+\beta+1) \frac{k! (\alpha+\beta+1)_k}{(n-k)! (\alpha+1)_k (\beta+1)_k (\alpha+\beta+n+2)_k} P_k^{(\alpha, \beta)}(x) P_k^{(\alpha, \beta)}(-x) =$$

$$= \frac{(\alpha+\beta+n+1)_n (\alpha+\beta+1)_{n+1}}{n! (\alpha+1)_n (\beta+1)_n} (1-x^2)^n.$$

$$5. \sum_{k=0}^n \frac{2k+\alpha+\beta+1}{(k+\alpha+\beta+1)_{n+1}} P_k^{(\alpha, \beta)}(x) P_{n-k}^{(-\alpha-k-1, -\beta-k-1)}(y) = \frac{1}{n!} \left(\frac{x-y}{2}\right)^n.$$

$$6. \sum_{k=0}^n \frac{(-\beta-n)_k}{(\alpha+1)_k} P_k^{(\alpha, \gamma-k)}(x) P_{n-k}^{(\beta, \delta+k)}(x) =$$

$$= \frac{(\alpha+\beta+\gamma+\delta+n+2)_n}{n!} \left(\frac{x-1}{2}\right)^n {}_3F_2\left(-n, \alpha+\gamma+1, \right.$$

$$\left. \alpha+\beta+n+1; \alpha+1, \alpha+\beta+\gamma+\delta+n+2; 1\right).$$

$$7. \sum_{k=0}^n P_k^{(\alpha-k, \beta-k)}(x) P_{n-k}^{(\gamma+k, \delta+k)}(x) = P_n^{(\alpha+\gamma, \beta+\delta)}(x).$$

$$8. \sum_{k=0}^n \frac{(\alpha+\beta+m+2n+1)_k}{k!} \left(\frac{x^2-1}{4}\right)^k P_{n-k}^{(\alpha+k, \beta+k)}(x) P_{m-k}^{(\alpha+n+k, \beta+n+k)}(x) =$$

$$= \binom{m+n}{n} P_{m+n}^{(\alpha, \beta)}(x) \quad [n \leq m]$$

Г л а в а 5. РЯДЫ

5.1. ВВЕДЕНИЕ

В этой главе приведены суммы рядов, содержащих некоторые специальные функции. При преобразовании таких рядов, в частности при приведении их к форме, которая принята в этой книге, можно воспользоваться формулами из приложения II.

Ряд формул общего характера, которые могут оказаться полезными при суммировании рядов, имеется в книге [12].

5.2. НЕПОЛНЫЕ ГАММА-ФУНКЦИИ $\gamma(a, x)$, $\Gamma(a, x)$

5.2.1. Ряды вида $\sum a_k \gamma(\alpha_k, x)$.

$$1. \sum_{k=0}^{\infty} \frac{1}{k!} \gamma(k+\alpha, x) = \frac{x^\alpha}{\alpha};$$

$$2. \sum_{k=0}^{\infty} \frac{1}{k!} [\gamma(k, x) - \gamma(k, y)] = \ln \frac{x}{y} \quad [x, y > 0].$$

$$3. \sum_{k=0}^{\infty} \frac{1}{k! (k+\beta)} \gamma(k+\alpha, x) = \frac{x^\alpha}{\alpha\beta} {}_2F_2(1, \alpha; \alpha+1, \beta+1; -x).$$

$$4. \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (2k+\alpha)} \gamma(k+\alpha, x) = \frac{1}{4} \gamma^2\left(\frac{\alpha}{2}, x\right).$$

$$5. \sum_{k=0}^{\infty} \frac{1}{\Gamma(k+\beta+1)} \gamma(k+\alpha, x) = \frac{x^\alpha}{(\beta-\alpha)\Gamma(\beta)} [x^{-\alpha}\gamma(\alpha, x) - x^{-\beta}\gamma(\beta, x)].$$

$$6. \sum_{k=0}^{\infty} \frac{(\beta)_k}{k! (\gamma)_k} \gamma(k+\alpha, x) = \frac{x^\alpha}{\alpha} {}_2F_2(\alpha, \gamma-\beta; \alpha+1, \gamma; -x)$$

[$\operatorname{Re} \gamma \geq \operatorname{Re} \beta > 0$].

5.2.2. Ряды вида $\sum \frac{t^k}{ka+b} \gamma(\alpha_k, x)$.

$$1. \sum_{k=0}^{\infty} (\pm 1)^k \frac{t^k}{k} \gamma(k+1, x) = \frac{1}{t} e^{-1/t} \left[E_1\left(\frac{1}{t}\right) - E_1\left(\frac{1}{t} - x\right) \right] \quad [|tx| < 1].$$

$$2. \sum_{k=0}^{\infty} t^k \gamma(2k+1, x) = \frac{1}{2\sqrt{t}} \left\{ e^{1/\sqrt{t}} \left[\text{Ei}\left(-x - \frac{1}{\sqrt{t}}\right) - \text{Ei}\left(-\frac{1}{\sqrt{t}}\right) \right] - \right. \\ \left. - e^{-1/\sqrt{t}} \left[\text{Ei}\left(-x + \frac{1}{\sqrt{t}}\right) - \text{Ei}\left(\frac{1}{\sqrt{t}}\right) \right] \right\} \quad [t > 0; |tx| < 1].$$

$$3. \sum_{k=1}^{\infty} \frac{t^k}{k} \gamma(k+1, x) = e^{-1/t} \left[\text{Ei}\left(\frac{1}{t}\right) - \text{Ei}\left(-x + \frac{1}{t}\right) \right] + e^{-x} \ln(1-tx) \quad [|tx| < 1].$$

$$4. \sum_{k=0}^{\infty} \frac{t^{2k+1}}{2k+1} \gamma(2k+2, x) = \frac{1}{2} e^{1/t} \left[\text{Ei}\left(-x - \frac{1}{t}\right) - \text{Ei}\left(-\frac{1}{t}\right) \right] - \\ - \frac{1}{2} e^{-1/t} \left[\text{Ei}\left(-x + \frac{1}{t}\right) - \text{Ei}\left(\frac{1}{t}\right) \right] - e^{-x} \text{Arth}(tx) \quad [|tx| < 1].$$

$$5. \sum_{k=-\infty}^{\infty} \frac{1}{k+b} x^k \gamma(\alpha-k, x) = \frac{\pi \sin(\alpha+b)\pi}{x^b \sin \alpha\pi \sin b\pi} \gamma(\alpha+b, x).$$

5.2.3. Ряды вида $\sum a_k t^k \gamma(\alpha_k, x)$

$$1. \sum_{k=0}^{\infty} \frac{t^k}{k!} \gamma(k+\alpha, x) = (1-t)^{-\alpha} \gamma(\alpha, x-tx).$$

$$2. \sum_{k=0}^{\infty} \frac{t^{2k}}{(2k)!} \gamma(k+1, x) = \\ = \frac{\sqrt{\pi}}{4} t e^{t^2/4} \left[\text{erf}\left(\frac{t}{2}\right) - \text{erf}\left(\frac{t}{2} + \sqrt{x}\right) - \text{erf}\left(\frac{t}{2} - \sqrt{x}\right) - e^{-x} \text{ch } \sqrt{x} + 1 \right].$$

$$3. \sum_{k=0}^{\infty} \frac{t^{2k+1}}{(2k+1)!} \gamma(k+1, x) = \\ = \frac{\sqrt{\pi}}{2} e^{t^2/4} \left[2 \text{erf}\left(\frac{t}{2}\right) - \text{erf}\left(\frac{t}{2} + \sqrt{x}\right) - \text{erf}\left(\frac{t}{2} - \sqrt{x}\right) \right].$$

$$4. \sum_{k=0}^{\infty} (-1)^k \frac{t^{2k+1}}{k!(2k+1)} \gamma(k+1, x) = \sqrt{\pi} \int_0^{\sqrt{x}} \text{erf}(tu) e^{-u^2} du.$$

$$5. \sum_{k=0}^{\infty} \frac{(-1)^k (k+n)!}{k!} t^k \gamma(k+1, x) = \frac{n!}{t} e^{1/t} \left[\Gamma\left(-n, \frac{1}{t}\right) - \Gamma\left(-n, x + \frac{1}{t}\right) \right] \quad [|tx| < 1].$$

$$6. \sum_{k=0}^{\infty} (-1)^k (a)_k t^k \gamma(k+1, x) = t^{-a} e^{1/t} \left[\gamma\left(1-a, x + \frac{1}{t}\right) - \gamma\left(1-a, \frac{1}{t}\right) \right] \quad [|tx| < 1].$$

$$7. \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} t^{2k+1} \gamma(2k+1, x) = \frac{\sqrt{\pi}}{2} e^{1/(4t^2)} \left[\text{erf}\left(tx + \frac{1}{2t}\right) - \text{erf}\left(\frac{1}{2t}\right) \right].$$

$$\sum_{k=0}^{\infty} \frac{t^k}{k!} \gamma\left(k + \frac{1}{2}, x\right) = \sqrt{\frac{\pi}{1-t}} \text{er}(\sqrt{x-tx}).$$

$$\begin{aligned}
9. \quad \sum_{k=0}^{\infty} \frac{t^{2k}}{(2k)!} \gamma\left(k + \frac{1}{2}, x\right) &= \frac{\sqrt{\pi}}{2} e^{t^2/4} \left[\operatorname{erf}\left(\sqrt{x} + \frac{t}{2}\right) + \operatorname{erf}\left(\sqrt{x} - \frac{t}{2}\right) \right]. \\
10. \quad \sum_{k=0}^{\infty} \frac{t^k}{k! \Gamma(k+1/2)} \gamma\left(k + \frac{1}{2}, x\right) &= \frac{1}{2} e^t \left[\operatorname{erf}(\sqrt{x} + \sqrt{t}) + \operatorname{erf}(\sqrt{x} - \sqrt{t}) \right]. \\
11. \quad \sum_{k=0}^{\infty} \frac{(-1)^k x^k}{k! \Gamma(k+1/2)} \gamma\left(k + \frac{1}{2}, x\right) &= e^{-x} [C(2x) + S(2x)]. \\
12. \quad \sum_{k=0}^{\infty} \frac{t^k}{k!} \gamma\left(\frac{k+1}{2}, x\right) &= \sqrt{\pi} e^{t^2/4} \left[\operatorname{erf}\left(\sqrt{x} - \frac{t}{2}\right) + \operatorname{erf}\left(\frac{t}{2}\right) \right]. \\
13. \quad \sum_{k=0}^{\infty} \frac{x^{k/2}}{k!} \gamma\left(\frac{k+\alpha}{2}, x\right) &= \frac{2x^{\alpha/2}}{\alpha} {}_2F_2\left(1, \alpha; \frac{\alpha+1}{2}, \frac{\alpha}{2} + 1; \frac{x}{4}\right). \\
14. \quad \sum_{k=0}^{\infty} \frac{\Gamma(k-\alpha+1)}{k! x^k} \gamma(k+\alpha, x) &= \frac{\pi^{3/2} x^{1/2}}{\sin \alpha \pi} e^{-x/2} I_{\alpha-1/2}\left(\frac{x}{2}\right) \quad [\alpha > 0]. \\
15. \quad \sum_{k=1}^{\infty} (-1)^k \frac{(2k-1)!!}{(2k)!!} t^k \gamma(k+1, x) &= \sqrt{\frac{\pi}{t}} e^{1/t} \times \\
&\quad \times \left[\operatorname{erf}\left(\sqrt{x + \frac{1}{t}}\right) - \operatorname{erf}\left(\sqrt{\frac{1}{t}}\right) \right].
\end{aligned}$$

5.2.4. Разные ряды, содержащие $\gamma(\alpha_k, x)$.

$$\begin{aligned}
1. \quad \sum_{k=0}^{\infty} \frac{t^k}{k!} \cos ka \gamma\left(\frac{k+1}{2}, x\right) &= \\
&= \frac{\sqrt{\pi}}{2} \left\{ \exp\left(\frac{t^2}{4} e^{2ia}\right) \left[\operatorname{erf}\left(\sqrt{x} - \frac{t}{2} e^{ia}\right) + \operatorname{erf}\left(\frac{t}{2} e^{ia}\right) \right] + \right. \\
&\quad \left. + \exp\left(\frac{t^2}{4} e^{-2ia}\right) \left[\operatorname{erf}\left(\sqrt{x} + \frac{t}{2} e^{-ia}\right) + \operatorname{erf}\left(\frac{t}{2} e^{-ia}\right) \right] \right\}. \\
2. \quad \sum_{k=0}^{\infty} \frac{t^k}{k!} \sin ka \gamma\left(\frac{k+1}{2}, x\right) &= \\
&= \frac{\sqrt{\pi}}{2i} \left\{ \exp\left(\frac{t^2}{4} e^{2ia}\right) \left[\operatorname{erf}\left(\sqrt{x} - \frac{t}{2} e^{ia}\right) + \operatorname{erf}\left(\frac{t}{2} e^{ia}\right) \right] - \right. \\
&\quad \left. - \exp\left(\frac{t^2}{4} e^{-2ia}\right) \left[\operatorname{erf}\left(\sqrt{x} - \frac{t}{2} e^{-ia}\right) + \operatorname{erf}\left(\frac{t}{2} e^{-ia}\right) \right] \right\}. \\
3. \quad \sum_{k=0}^{\infty} \frac{1}{k! \Gamma(k+1/2)} \gamma\left(k + \frac{1}{2}, x\right) \gamma\left(k + \frac{1}{2}, y\right) &= \\
&= (\sqrt{x} + \sqrt{y}) \operatorname{erf}(\sqrt{x} + \sqrt{y}) - (\sqrt{x} - \sqrt{y}) \operatorname{erf}(\sqrt{x} - \sqrt{y}) - \frac{2}{\sqrt{\pi}} e^{-x-y} \operatorname{sh}(2\sqrt{xy}).
\end{aligned}$$

5.2.5. Ряды вида $\sum a_k t^k \Gamma(\alpha_k, x)$.

$$\begin{aligned}
1. \quad \sum_{k=0}^{\infty} t^k \Gamma(k, x) &= \left[e^{-1/\sqrt{t}} \operatorname{Ei}\left(-x + \frac{1}{\sqrt{t}}\right) - \operatorname{Ei}(-x) \right] \quad [|tx| < 1]. \\
2. \quad \sum_{k=0}^{\infty} t^k \Gamma(2k, x) &= \frac{1}{2} \left[e^{-1/\sqrt{t}} \operatorname{Ei}\left(-x + \frac{1}{\sqrt{t}}\right) + e^{1/\sqrt{t}} \operatorname{Ei}\left(-x - \frac{1}{\sqrt{t}}\right) - 2 \operatorname{Ei}(-x) \right] \\
&\quad [|tx| < 1].
\end{aligned}$$

$$3. \sum_{k=0}^{\infty} t^k \Gamma(2k+1, x) = \frac{1}{2t} \left[e^{-1/\sqrt{t}} \operatorname{Ei} \left(-x + \frac{1}{\sqrt{t}} \right) - e^{1/\sqrt{t}} \operatorname{Ei} \left(-x - \frac{1}{\sqrt{t}} \right) \right] \\ [|tx| < 1].$$

$$4. \sum_{k=0}^{\infty} \frac{t^k}{k!} \Gamma(k, x) = \operatorname{Ei}(tx - x) \quad [(1-t)x > 0].$$

$$5. \sum_{k=0}^{\infty} (-1)^k \frac{t^{2k+1}}{k! (2k+1)} \Gamma(k+1, x) = -\frac{\sqrt{\pi}}{2} \int_0^{\sqrt{x}} \operatorname{erf}(tu) e^{-u^2} du + \operatorname{arctg} t.$$

$$6. \sum_{k=0}^{\infty} \frac{(-1)^k}{k! (2k+1)} \Gamma(k+1, x) = \frac{\pi}{4} [1 - \operatorname{erf}^2(\sqrt{x})].$$

$$7. \sum_{k=0}^{\infty} \frac{\Gamma(k+1/2)}{k!} t^{2k} \Gamma(-2k-1, x) = \frac{\sqrt{\pi}}{t} \int_{\arccos(t/x)}^{\pi/2} e^{-t/\cos u} du \\ [t \geq 0, 0 < \arccos(t/x) < \pi/2].$$

$$8. \sum_{k=0}^{\infty} t^k \Gamma(k+\alpha+1) \Gamma(-k-\alpha, x) = \Gamma(\alpha+1) e^{-t} \Gamma(-\alpha, x-t).$$

$$9. \sum_{k=1}^{\infty} \frac{1}{k^n} \Gamma(n, kx) = \int_x^{\infty} \frac{u^{n-1}}{e^u - 1} du \quad [x > 0, n = 2, 3, 4, \dots].$$

5.3. ДЗЕТА-ФУНКЦИЯ РИМАНА $\zeta(z)$

5.3.1. Ряды, содержащие $\zeta(z)$.

$$1. \sum_{k=2}^{\infty} (\pm 1)^k [\zeta(k) - 1] = \frac{1}{2} \begin{Bmatrix} 2 \\ 1 \end{Bmatrix}.$$

$$2. \sum_{k=1}^{\infty} \left[\zeta \left(2k + \frac{1 \pm 1}{2} \right) - 1 \right] = \frac{2 \mp 1}{4}.$$

$$3. \sum_{k=2}^{\infty} \frac{(\pm 1)^k}{k} [\zeta(k) - 1] = \pm 1 \mp C + \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \ln 2.$$

$$4. \sum_{k=2}^{\infty} t^k \zeta(k) = -Ct - t\psi(1-t) \quad [|t| < 1]$$

$$5. \sum_{k=0}^{\infty} (\pm 1)^k t^{2k} \zeta(2k) = -\frac{\pi}{2} t \begin{Bmatrix} \operatorname{ctg} t\pi \\ \operatorname{cth} t\pi \end{Bmatrix} \quad [|t| < 1].$$

$$6. \sum_{k=1}^{\infty} t^{2k+1} \zeta(2k+1) = \frac{1}{2} - \frac{\pi}{2} t \operatorname{ctg} t\pi - Ct - t\psi(t+1) \quad [|t| < 1].$$

$$7. \sum_{k=2}^{\infty} \frac{t^k}{k} \zeta(k) = \ln \Gamma(1-t) - Ct \quad [|t| < 1].$$

$$8. \sum_{k=1}^{\infty} \frac{t^{2k}}{k} \zeta(2k) = \ln(\pi t \operatorname{cosec} t\pi) \quad [|t| < 1]$$

$$9. \sum_{k=1}^{\infty} \frac{1}{2k+1} t^{2k+1} \zeta(2k+1) = \frac{1}{2} \ln(\pi t \operatorname{cosec} t\pi) - \ln \Gamma(t+1) - \mathcal{O}$$

$[|t| < 1]$

$$10. \sum_{k=1}^{\infty} (\pm 1)^k \frac{(a)_k}{k!} [\zeta(k+a) - 1] = \pm \left\{ \frac{1}{2^{-a}} \right\}.$$

$$11. \sum_{k=0}^{\infty} \frac{(a)_k}{k!} t^k \zeta(k+a) = \zeta(a, 1-t)$$

$[|t| < 1]$

$$12. \sum_{k=1}^{\infty} \frac{(a)_k}{k! 2^k} \zeta(k+a) = 2^a (1 - 2^{1-a}) \zeta(a).$$

$$13. \sum_{k=0}^{\infty} \binom{n+k-1}{k} t^k \zeta(n+k) = \frac{(-1)^n}{(n-1)!} \psi^{(n-1)}(1-t)$$

$[|t| < 1]$

5.3.2. Ряды, содержащие произведения $\zeta(z)$.

$$1. \sum_{k=1}^{\infty} \frac{2^{2k}}{k} \zeta(2k) [(1 - 2^{-2k}) \zeta(k) - 1] = \ln \left(\frac{2}{\pi} \prod_{k=3}^{\infty} \sec \frac{\pi}{k} \right).$$

5.4. ИНТЕГРАЛЬНЫЕ СИНУС $\operatorname{Si}(x)$ И КОСИНУС $\operatorname{ci}(x)$

5.4.1. Ряды, содержащие $\operatorname{Si}(x)$

$$1. \sum_{k=1}^{\infty} \frac{1}{k^{2n+1}} \operatorname{Si}(kx) = \frac{(-1)^{n+1}}{(2n+1)! 2} \sum_{k=0}^{2n+1} \binom{2n+1}{k} \frac{B_k (2\pi)^k}{2n-k+1} x^{2n-k+1}$$

$[0 \leq x \leq 2\pi; n = 1, 2, 3, \dots]$

$$2. \sum_{k=1}^{\infty} \frac{(-1)^k}{k} \operatorname{Si}(kx) = -\frac{x}{2}$$

$[-\pi < x < \pi]$

$$3. \sum_{k=-\infty}^{\infty} \frac{(-1)^k}{k+a} \operatorname{Si}(kx) = -\pi \operatorname{cosec} a\pi \operatorname{Si}(ax)$$

$[-\pi < x < \pi]$

$$4. \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^2} \operatorname{Si}((2k+1)x) = \frac{\pi x}{4}$$

$[-\pi/2 \leq x \leq \pi/2]$

$$5. \sum_{k=0}^{\infty} \frac{1}{(2k+1)^3} \operatorname{Si}((2k+1)x) = \frac{\pi x}{8} \left(\pi - \frac{|x|}{2} \right)$$

$[-\pi \leq x \leq \pi]$

$$6. \sum_{k=1}^{\infty} \frac{(-1)^{k+1} k}{k^2 \pm a^2} \operatorname{Si}(kx) = \frac{\pi}{2} \begin{cases} \operatorname{cosech} a\pi \operatorname{shi}(ax) \\ \operatorname{cosec} a\pi \operatorname{Si}(ax) \end{cases}$$

$[-\pi < x < \pi]$

$$7. \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^2 \pm a^2} \operatorname{Si}((2k+1)x) = \frac{\pi}{4a} \begin{cases} \operatorname{sech}(a\pi/2) \operatorname{shi}(ax) \\ \sec(a\pi/2) \operatorname{Si}(ax) \end{cases}$$

$[-\pi/2 < x < \pi/2]$

$$8. \sum_{k=1}^{\infty} \frac{1}{k^2} \operatorname{Si}^2(k\pi) = \frac{\pi^2}{2}.$$

5.4.2. Ряды, содержащие $\text{ci}(x)$.

$$1. \sum_{k=1}^{\infty} \text{ci}(2k\pi) = \frac{1}{4} - \frac{C}{2}.$$

$$2. \sum_{k=0}^{\infty} \text{ci}((2k+1)\pi) = \frac{\ln 2}{2} - \frac{1}{4},$$

$$3. \sum_{k=0}^{\infty} \text{ci}\left(\frac{2k+1}{2}\pi\right) = \frac{\ln 2}{2}.$$

5.5. ИНТЕГРАЛЫ ВЕРОЯТНОСТИ $\text{erf}(x)$, $\text{erfi}(x)$

5.5.1. Ряды, содержащие $\text{erf}(x)$.

$$1. \sum_{k=1}^{\infty} \frac{1}{k^2} e^{-k^2 x^2} [\text{erf}(a + ikx) + \text{erf}(a - ikx)] = \\ = \left(\frac{\pi^2}{3} + x^2\right) \text{erf}(a) + \frac{2x}{\sqrt{\pi}} e^{-a^2} (\pi - ax) - 2\sqrt{\pi} x \quad [0 \leq ax \leq \pi].$$

$$2. \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2} e^{-k^2 x^2} [\text{erf}(a + ikx) + \text{erf}(a - ikx)] = \\ = \left(\frac{\pi^2}{6} - x^2\right) \text{erf}(a) + \frac{2ax^2}{\sqrt{\pi}} e^{-a^2} \quad [-\pi/2 < ax < \pi/2]$$

$$3. \sum_{k=1}^{\infty} (-1)^k \frac{1}{k^2 + b^2} e^{-k^2 x^2} [\text{erf}(a + ikx) + \text{erf}(a - ikx)] = \\ = \frac{\pi e^{b^2 x^2}}{2b \operatorname{sh} b\pi} [\text{erf}(a + bx) + \text{erf}(a - bx)] - \frac{1}{b^2} \text{erf}(a) \quad [-\pi/2 < ax < \pi/2]$$

5.5.2. Ряды, содержащие $\text{erfi}(x)$.

$$1. \sum_{k=-\infty}^{\infty} (-1)^k e^{-k^2 b^2} \text{erfi}(kx + a) = 0.$$

$$2. \sum_{k=1}^{\infty} \frac{1}{k} e^{-k^2 x^2} [2 \text{erfi}(kx) - \text{erfi}(kx + ia) - \text{erfi}(kx - ia)] = \\ = \pi \text{erf}(a) + \frac{2}{\sqrt{\pi}} x (e^{-a^2} - 1) \quad [0 < ax < \pi]$$

$$3. \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} e^{-k^2 x^2} [2 \text{erfi}(kx) - \text{erfi}(kx + ia) - \text{erfi}(kx - ia)] = \frac{2x}{\sqrt{\pi}} (1 - e^{-a^2}) \\ [-\pi/4 < ax < \pi/4]$$

$$4. \sum_{k=1}^{\infty} (-1)^{k+1} \frac{k}{k^2 + b^2} e^{-k^2 x^2} [2 \text{erfi}(kx) - \text{erfi}(kx + ia) - \text{erfi}(kx - ia)] = \\ = \frac{\pi}{2 \operatorname{sh} b\pi} e^{b^2 x^2} [2 \text{erf}(bx) - \text{erf}(bx + a) - \text{erf}(bx - a)] \quad [-\pi/4 < ax < \pi/4]$$

$$\begin{aligned}
 5. \quad \sum_{k=1}^{\infty} \frac{1}{k^3} e^{-k^2 x^2} [2 \operatorname{erfi}(kx) - \operatorname{erfi}(kx + ia) - \operatorname{erfi}(kx - ia)] = \\
 = \frac{2x}{3\sqrt{\pi}} (\pi^2 + 2x^2) - \frac{x}{3\sqrt{\pi}} e^{-a^2} (\pi^2 + 2x^2 + 2a^2 x^2 - 3\pi a x) - \pi x^2 \operatorname{erf}(a) \\
 [0 < ax < \pi].
 \end{aligned}$$

5.6. ФУНКЦИЯ ПАРАБОЛИЧЕСКОГО ЦИЛИНДРА $D_\nu(x)$

5.6.1. Ряды вида $\sum a_k D_{\nu \pm k}(x)$.

$$\begin{aligned}
 1. \quad \sum_{k=0}^{\infty} \frac{t^k}{k!} D_{k+\nu}(x) &= e^{t(2x-t)/4} D_\nu(x-t). \\
 2. \quad \sum_{k=0}^{\infty} \frac{(-\nu)_k}{k!} t^k D_{\nu-k}(x) &= e^{t(t-2x)/4} D_\nu(x-t) \quad [\operatorname{Re} \nu < 0].
 \end{aligned}$$

5.6.2. Ряды вида $\sum a_k D_{nk}(x) D_{\nu-nk}(x)$.

$$\begin{aligned}
 1. \quad \sum_{k=0}^{\infty} \frac{(-\nu)_k}{k!} t^k D_k(x) D_{\nu-k}(y) &= (t^2 + 1)^{\nu/2} \exp \left[-\frac{(x+ty)^2}{4(t^2+1)} \right] D_\nu \left(\frac{y-tx}{\sqrt{t^2+1}} \right) \\
 [x, y > 0; -1 < t \leq 0; \operatorname{Re} \nu \geq 0] \\
 2. \quad \sum_{k=0}^{\infty} (-1)^k \frac{(1/2)_k}{k!} D_{2k}(x) D_{-2k-1}(y) &= \frac{1}{\sqrt{2\pi}} K_0 \left(\frac{x^2 + y^2}{4} \right) \\
 [-\infty < x < \infty; 0 \leq y < \infty].
 \end{aligned}$$

5.7. ФУНКЦИЯ БЕССЕЛЯ $J_\nu(z)$

5.7.1. Ряды вида $\sum (\pm 1)^k J_{\nu \pm nk}(z)$.

Обозначения:

$$V(z) = \int_0^z J_0(t) dt = z J_0(z) + \frac{\pi z}{2} [J_1(z) H_0(z) - J_0(z) H_1(z)], \quad \delta = \begin{cases} 1 \\ 0 \end{cases}.$$

$$1. \quad \sum_{k=1}^{\infty} (\pm 1)^k J_k(z) = \frac{1}{2} - \frac{1}{2} J_0(z) \pm \frac{1}{2} V(z).$$

$$2. \quad \sum_{k=-\infty}^{\infty} J_{n \pm k}(z) = 1.$$

$$3. \quad \sum_{k=-\infty}^{\infty} (-1)^k J_{n \pm k}(z) = (-1)^n.$$

$$4. \quad \sum_{k=1}^{\infty} (\pm 1)^k J_{k+\nu}(z) = \frac{1}{2} \int_0^z [J_{\nu+1}(t) \pm J_\nu(t)] dt.$$

$$5. \quad \sum_{k=0}^{\infty} (\pm 1)^k J_{k+1/2}(z) = C(z) \pm S(z).$$

$$6. \quad \sum_{k=0}^{\infty} J_{2k+\nu}(z) = \frac{1}{2} \int_0^z J_{\nu-1}(t) dt \quad [\operatorname{Re} \nu > 0].$$

$$7. \sum_{k=0}^{\infty} (-1)^k J_{2k+\nu}(z) = \frac{1}{2} \int_0^z J_{\mu}(z-t) J_{\nu-\mu-1}(t) dt$$

$$[\operatorname{Re} \mu > -1; \operatorname{Re}(\nu - \mu) > 0]$$

$$8. \sum_{k=0}^{\infty} J_{2k+2n}(z) = \frac{1}{2} - \frac{1}{2} J_0(z) - \sum_{k=1}^{n-1} J_{2k}(z)$$

$$[n \geq 1]$$

$$9. \sum_{k=0}^{\infty} J_{2k+2n+1}(z) = \frac{1}{2} V(z) - \sum_{k=0}^{n-1} J_{2k+1}(z).$$

$$10. \sum_{k=-\infty}^{\infty} J_{2k \pm 2n}(z) = 1.$$

$$11. \sum_{k=-\infty}^{\infty} J_{2k \pm 2n+1}(z) = 0.$$

$$12. \sum_{k=-\infty}^{\infty} (-1)^k J_{2k \pm 2n}(z) = (-1)^n \cos z.$$

$$13. \sum_{k=-\infty}^{\infty} (-1)^k J_{2k \pm 2n+1}(z) = (-1)^n \sin z.$$

$$14. \sum_{k=1}^{\infty} (\pm 1)^k J_{2k}(z) = \frac{1}{2} \left\{ \frac{1}{\cos z} \right\} - \frac{1}{2} J_0(z).$$

$$15. \sum_{k=0}^{\infty} (\pm 1)^k J_{2k+1}(z) = \frac{1}{2} \left\{ \frac{V(z)}{\sin z} \right\}.$$

$$16. \sum_{k=0}^{\infty} (-1)^{[k/2]} J_{2k+1}(z) = \frac{1}{\sqrt{2}} \sin \frac{z}{\sqrt{2}}.$$

$$17. \sum_{k=0}^{\infty} J_{2k + (2 \pm 1)/2}(z) = \begin{Bmatrix} S(z) \\ C(z) \end{Bmatrix}.$$

$$18. \sum_{k=0}^{\infty} (-1)^k J_{2k + (2 \pm 1)/2}(z) = \frac{\sqrt{2}}{2} \sin z \begin{Bmatrix} C(2z) \\ S(2z) \end{Bmatrix} \mp \frac{\sqrt{2}}{2} \cos z \begin{Bmatrix} S(2z) \\ C(2z) \end{Bmatrix}.$$

$$19. \sum_{k=0}^{\infty} (\pm 1)^k J_{4k}(z) = \frac{1}{2} \left\{ \frac{\cos^2(z/2)}{\cos(z/\sqrt{2})} \right\} + \frac{1}{2} J_0(z).$$

$$20. \sum_{k=0}^{\infty} J_{4k+2}(z) = \frac{1}{2} \sin^2 \frac{z}{2}.$$

$$21. \sum_{k=0}^{\infty} J_{8k}(z) = \frac{1}{4} \cos \frac{z}{\sqrt{2}} + \frac{1}{4} \cos^2 \frac{z}{2} + \frac{1}{2} J_0(z).$$

$$22. \sum_{k=0}^{\infty} J_{8k+4}(z) = \frac{1}{4} \cos \frac{z}{2} - \frac{1}{4} \cos \frac{z}{\sqrt{2}}.$$

5.7.2. Ряды вида $\sum (\pm 1)^k (ka+b)^m J_{nk+v}(z)$.

Обозначения:

$$V(z) = \int_0^z J_0(t) dt = zJ_0(z) + \frac{\pi z}{2} [J_1(z)H_0(z) - J_0(z)H_1(z)], \quad \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}.$$

$$1. \sum_{k=0}^{\infty} (\pm 1)^k (k+v) J_{k+v}(z) = \frac{z}{2} J_{v-1}(z) \pm \frac{z}{2} J_v(z) + \frac{z}{2} \int_0^z [J_v(t) \pm J_{v+1}(t)] dt$$

[Re v > -1].

$$2. \sum_{k=-\infty}^{\infty} k J_{n-k}(z) = n - z.$$

$$3. \sum_{k=1}^{\infty} (\pm 1)^k k J_k(z) = -\frac{z}{2} J_1(z) + \frac{z}{2} V(z) \pm \frac{z}{2}.$$

$$4. \sum_{k=1}^{\infty} (\pm 1)^k (z \mp k \mp v) J_{k+v}(z) = \pm \frac{z}{2} J_{v+1}(z) - \frac{z}{2} J_v(z).$$

$$5. \sum_{k=0}^{\infty} (\pm 1)^k \left(k + \frac{1}{2}\right) J_{k+1/2}(z) = \sqrt{\frac{z}{\pi}} \cos\left(z \pm \frac{\pi}{4}\right) + zS(z) \pm zC(z).$$

$$6. \sum_{k=0}^{\infty} (\pm 1)^k (2k+v) J_{2k+v}(z) = \frac{z}{2} J_{v-1}(z) + \frac{z}{2} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \int_0^z J_v(t) dt \quad [\text{Re } v > -1].$$

$$7. \sum_{k=0}^{\infty} (-1)^k \left(2k + \frac{1 \pm 1}{2}\right) J_{2k+v}(z) = \pm \frac{1}{2} \int_0^z \begin{Bmatrix} \cos(z-t) \\ \sin(z-t) \end{Bmatrix} J_{v-(1 \pm 1)/2}(t) dt$$

[Re v > -(1 \mp 1)/2; см. 2.12.17.1]

$$8. \sum_{k=0}^{\infty} (2k+2n+1) J_{2k+2n+1}(z) = \frac{z}{2} - \sum_{k=0}^{n-1} (2k+1) J_{2k+1}(z).$$

$$9. \sum_{k=0}^{\infty} (-1)^k (k+n) J_{2k+2n}(z) = \frac{z}{4} J_{2n-1}(z).$$

$$10. \sum_{k=1}^{\infty} (\pm 1)^k k J_{2k}(z) = -\frac{z}{4} J_1(z) + \frac{z}{4} \begin{Bmatrix} V(z) \\ 0 \end{Bmatrix}.$$

$$11. \sum_{k=0}^{\infty} (\pm 1)^k (2k+1) J_{2k+1}(z) = \frac{z}{2} \begin{Bmatrix} 1 \\ J_0(z) \end{Bmatrix}.$$

$$12. \sum_{k=0}^{\infty} \left(2k + \delta + \frac{1}{2}\right) J_{2k+\delta+1/2}(z) = z \begin{Bmatrix} C(z) \\ S(z) \end{Bmatrix} \mp \sqrt{\frac{z}{2\pi}} \begin{Bmatrix} \sin z \\ \cos z \end{Bmatrix}.$$

$$13. \sum_{k=0}^{\infty} (-1)^k \left(2k + \delta + \frac{1}{2}\right) J_{2k+\delta+1/2}(z) = \sqrt{\frac{z}{2\pi}} \begin{Bmatrix} \sin z \\ \cos z \end{Bmatrix}.$$

$$14. \sum_{k=1}^{\infty} k J_{4k}(z) = -\frac{z}{8} J_1(z) + \frac{z}{16} V(z).$$

15. $\sum_{k=0}^{\infty} (4k+1) J_{4k+1}(z) = \frac{z}{4} + \frac{z}{4} J_0(z).$
16. $\sum_{k=0}^{\infty} (2k+1) J_{4k+2}(z) = \frac{z}{8} V(z).$
17. $\sum_{k=0}^{\infty} (4k+3) J_{4k+3}(z) = \frac{z}{4} - \frac{z}{4} J_0(z).$
18. $\sum_{k=1}^{\infty} (k+\mu)(k+\nu-z) J_{k+\nu}(z) = \frac{\mu+1}{2} z J_{\nu}(z) - \frac{\mu z}{2} J_{\nu+1}(z).$
19. $\sum_{k=1}^{\infty} (-1)^{k+1} k(k+\mu) J_{2k+\nu}(z) = \frac{1}{4} \int_0^z t J_{\nu-\mu}(z-t) J_{\mu}(t) dt$
 $[Re(\nu-\mu) > -1; Re \mu > -2].$
20. $\sum_{k=1}^{\infty} (-1)^k k(2k+2n+\delta) J_{2k+2n+\delta}(z) = (-1)^n \frac{z}{4} \left[\begin{Bmatrix} \cos z \\ \sin z \end{Bmatrix} + \right.$
 $\left. + J_0(z) \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} - 2 \sum_{k=0}^{n+\delta-1} (-1)^k J_{2k+1-\delta}(z) \right] \quad \left[\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right].$
21. $\sum_{k=0}^{\infty} (-1)^k (2k+1)(2k+2n+\delta) J_{2k+2n+\delta}(z) = \pm (-1)^n \frac{z}{2} \left[\begin{Bmatrix} \cos z \\ \sin z \end{Bmatrix} + \right.$
 $\left. + J_0(z) \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} - 2 \sum_{k=0}^{n+\delta-1} (-1)^k J_{2k+1-\delta}(z) \pm (-1)^n J_{2n-1+\delta}(z) \right] \quad \left[\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right].$
22. $\sum_{k=0}^{\infty} (-1)^k (2k+2n+\delta) J_{2k+2n+\delta}(z) = \pm (-1)^n \frac{z}{2} \left[\begin{Bmatrix} \cos z \\ \sin z \end{Bmatrix} + J_0(z) \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} - \right.$
 $\left. - 2 \sum_{k=0}^{n+\delta-1} (-1)^k J_{2k+1-\delta}(z) + \frac{2n+\delta}{2} z J_{2n-1+\delta}(z) \right] \quad \left[\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right].$
23. $\sum_{k=1}^{\infty} (\pm 1)^k k^2 J_k(z) = \frac{z^2}{2} \pm \frac{z}{2} [J_0(z) - zJ_1(z) + zV(z)].$
24. $\sum_{k=0}^{\infty} (2k+\nu)^2 J_{2k+\nu}(z) = \frac{\nu z}{2} J_{\nu-1}(z) + \frac{z^2}{2} J_{\nu}(z) + \frac{z^2}{2} \int_0^z J_{\nu+1}(t) dt$
 $[Re \nu > -2]$
25. $\sum_{k=0}^{\infty} (-1)^k (2k+\nu)^2 J_{2k+\nu}(z) = \frac{\nu z}{2} J_{\nu-1}(z) - \frac{z}{2} \int_0^z J_{\mu}(z-t) J_{\nu-\mu}(t) dt$
 $[Re \mu, Re(\nu-\mu) > -1]$
26. $\sum_{k=1}^{\infty} (\pm 1)^{k+1} k^2 J_{2k}(z) = \frac{z}{8} \begin{Bmatrix} z \\ \sin z \end{Bmatrix}.$
27. $\sum_{k=0}^{\infty} (2k+1)^2 J_{2k+1}(z) = \frac{z}{2} [J_0(z) - zJ_1(z) + zV(z)].$

$$28. \sum_{k=0}^{\infty} (-1)^k (2k+1)^2 J_{2k+1}(z) = \frac{z}{2} \cos z.$$

$$29. \sum_{k=0}^{\infty} \left(2k + \delta + \frac{1}{2}\right)^2 J_{2k+\delta+1/2}(z) = \\ = z^2 \left\{ \begin{matrix} S(z) \\ C(z) \end{matrix} \right\} + \frac{1}{2} \sqrt{\frac{z}{2\pi}} \left\{ \begin{matrix} \sin z \\ \cos z \end{matrix} \right\} \pm \frac{z^{3/2}}{\sqrt{2\pi}} \left\{ \begin{matrix} \cos z \\ \sin z \end{matrix} \right\}.$$

$$30. \sum_{k=0}^{\infty} (-1)^k \left(2k + \delta + \frac{1}{2}\right)^2 J_{2k+\delta+1/2}(z) = \\ = \mp \frac{1}{2} \sqrt{\frac{z}{2\pi}} \left\{ \begin{matrix} \sin z \\ \cos z \end{matrix} \right\} \pm \frac{z}{\sqrt{2}} \left[\left\{ \begin{matrix} \cos z \\ \sin z \end{matrix} \right\} C(2z) \pm \left\{ \begin{matrix} \sin z \\ \cos z \end{matrix} \right\} S(2z) \right].$$

$$31. \sum_{k=1}^{\infty} k^3 J_k(z) = \frac{z}{2} [zJ_0(z) - z^2J_1(z) + z^2 + (1+z^2)V(z) + 1].$$

$$32. \sum_{k=1}^{\infty} (-1)^k k^3 J_k(z) = \frac{z}{2} [zJ_0(z) - z^2J_1(z) + (z^2+1)V(z)] - \frac{z(z^2+1)}{2}.$$

$$33. \sum_{k=0}^{\infty} (2k+\nu)^3 J_{2k+\nu}(z) = \\ = \frac{z}{2} [\nu^2 + z^2] J_{\nu-1}(z) + \frac{1-\nu}{2} z^2 J_{\nu}(z) + \frac{z}{2} (z^2+1) \int_0^z J_{\nu}(t) dt \quad [\operatorname{Re} \nu > -1].$$

$$34. \sum_{k=0}^{\infty} (-1)^k (2k+\nu)^3 J_{2k+\nu}(z) = \frac{\nu^2 z}{2} J_{\nu-1}(z) - z^2 J_{\nu}(z)$$

$$35. \sum_{k=1}^{\infty} k^3 J_{2k}(z) = \frac{z}{16} [zJ_0(z) - z^2J_1(z) + (1+z^2)V(z)].$$

$$36. \sum_{k=1}^{\infty} (-1)^k k^3 J_{2k}(z) = -\frac{z^2}{8} J_0(z).$$

$$37. \sum_{k=0}^{\infty} (2k+1)^3 J_{2k+1}(z) = \frac{z^3+z}{2}.$$

$$38. \sum_{k=0}^{\infty} (-1)^k (2k+1)^3 J_{2k+1}(z) = \frac{z}{2} J_0(z) - z^2 J_1(z).$$

$$39. \sum_{k=0}^{\infty} \left(2k + \delta + \frac{1}{2}\right)^3 J_{2k+\delta+1/2}(z) = \\ = (z^3+z) \left\{ \begin{matrix} C(z) \\ S(z) \end{matrix} \right\} \pm \frac{1}{4} \sqrt{\frac{z}{2\pi}} \left\{ \begin{matrix} \sin z \\ \cos z \end{matrix} \right\} + \frac{z^{3/2}}{2\sqrt{\pi}} \left\{ \begin{matrix} \cos z \\ \sin z \end{matrix} \right\} \mp \frac{z^{5/2}}{\sqrt{2\pi}} \left\{ \begin{matrix} \sin z \\ \cos z \end{matrix} \right\}.$$

$$40. \sum_{k=0}^{\infty} (-1)^k \left(2k + \delta + \frac{1}{2}\right)^3 J_{2k+\delta+1/2}(z) = \sqrt{\frac{z}{2\pi}} \left[\frac{1}{4} \left\{ \begin{matrix} \sin z \\ \cos z \end{matrix} \right\} \pm 2z \left\{ \begin{matrix} \cos z \\ \sin z \end{matrix} \right\} \right].$$

$$41. \sum_{k=1}^{\infty} (-1)^{k+1} k^4 J_{2k}(z) = \frac{z}{32} \left\{ \frac{z(z^2+4)}{\sin z + 3z \cos z} \right\}.$$

$$42. \sum_{k=1-\sigma}^{\infty} (2k+\sigma)^{2n+\sigma} J_{2k+\sigma}(z) = \\ = \sum_{k=0}^{n+\sigma-1} \left(\frac{z}{2}\right)^{2k-\sigma+2} \sum_{m=0}^k \frac{(-1)^m}{m! (2k-m-\sigma+2)!} (2k-2m-\sigma+2)^{2n+\sigma} \\ [\sigma=0 \text{ или } 1; n+\sigma=1, 2, 3, \dots]$$

$$43. \sum_{k=1-\delta}^{\infty} (-1)^k (2k+\delta)^{2n} J_{2k+\delta}(z) = \\ = \begin{cases} \sin z \\ \cos z \end{cases} \sum_{k=1}^{[n/2]} \frac{(-1)^k}{(2k-1)!} \left(\frac{z}{2}\right)^{2k} \sum_{m=1}^{4k} (-1)^m m(m-2k)^{2n} + \\ + \begin{cases} \cos z \\ \sin z \end{cases} \sum_{k=0}^{[(n+2\delta-3)/2]} \frac{(-1)^k}{(2k)!} \left(\frac{z}{2}\right)^{2k+1} \sum_{m=0}^{4k+2} (-1)^m m(m-2k-1)^{2n} \\ \left[n=1, 2, 3, \dots; \delta=\begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right].$$

5.7.3. Ряды вида $\sum \frac{P(k)}{Q(k)} J_{nk+v}(z)$.

$$1. \sum_{k=1}^{\infty} \frac{1}{k} J_{2k}(z) = -\frac{2}{\pi} \int_0^1 \frac{\cos zt}{\sqrt{1-t^2}} \ln 2t dt.$$

$$2. \sum_{k=1}^{\infty} \frac{(-1)^k}{k} J_{2k}(z) = \frac{1}{2} \left(\ln \frac{z}{2} + C \right) J_0(z) - \frac{\pi}{4} Y_0(z).$$

$$3. \sum_{k=0}^{\infty} \frac{1}{2k+1} J_{2k}(z) = \frac{1}{2} - \frac{\pi}{4} H_1(z) + \frac{1}{2z} \int_0^z J_0(t) dt.$$

$$4. \sum_{k=0}^{\infty} \frac{1}{2k+1} J_{2k+1}(z) = \frac{\pi}{4} H_0(z).$$

$$5. \sum_{k=0}^{\infty} \frac{1}{2k+1} J_{2k+v}(z) = \frac{\pi(v-1)}{4} \int_0^z \frac{1}{t} J_{v-1}(t) H_0(z-t) dt \quad [\operatorname{Re} v > 1]$$

$$6. \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} J_{4k+2}(z) = \int_0^{\pi/4} \cos(z \sin t) dt - \frac{\pi}{4} J_0(z).$$

$$7. \sum_{k=1}^{\infty} \frac{1}{k^{2n+2}} J_{2k}(z) = T_n,$$

$$\sum_{m=0}^{n-1} \frac{(-1)^m \pi^{2n-2m-1}}{(2n-2m-1)!} T_m =$$

$$= \frac{(-1)^{n+1} 2^{2n}}{(2n)!} \frac{\partial^{2n}}{\partial v^{2n}} \left[v \sin \frac{v\pi}{2} s_{-1,v}(z) \right] \Big|_{v=0} - \frac{\pi^{2n+1}}{(2n+1)! 2} J_0(z),$$

$$T_0 = \frac{2}{\pi} \int_0^{\pi/2} t^2 \cos(z \cos t) dt - \frac{\pi^2}{12} J_0(z) = 2S_{-1,0}(z) - \frac{\pi^2}{3} J_0(z) - \frac{\partial^2 J_\nu(z)}{\partial \nu^2} \Big|_{\nu=0}.$$

$$8. \sum_{k=1}^{\infty} \frac{(-1)^k}{k^{2n+2}} J_{2k}(z) = U_n,$$

$$\sum_{m=0}^{n-1} \frac{(-1)^m \pi^{2n-2m}}{(2n-2m-1)!} U_m = \frac{1}{(2n)! 2} \int_0^\pi t^{2n} \cos(z \cos t) dt - \frac{\pi^{2n+1}}{(2n+1)! 2} J_0(z).$$

$$9. \sum_{k=1}^{\infty} (-1)^k \frac{2k+1}{k(k+1)} J_{2k+1}(z) = \left(\ln \frac{z}{2} + C \right) J_1(z) - \frac{1}{z} J_0(z) - \frac{\pi}{2} Y_1(z).$$

$$10. \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2 - \nu^2} J_{2k}(z) = \frac{1}{2\nu^2} J_0(z) - \frac{\operatorname{cosec} \nu\pi}{\nu} \int_0^{\pi/2} \cos 2\nu t \cos(z \sin t) dt.$$

$$11. \sum_{k=0}^{\infty} \frac{1}{(k+\nu)^2 - 1} J_{2k+2\nu}(z) = \frac{z^{-2}}{2\nu(\nu-1)} \int_0^z t^2 J_{2\nu-1}(t) dt \quad [\operatorname{Re} \nu > -1].$$

$$12. \sum_{k=1}^{\infty} \frac{(-1)^k}{(2k+\nu)^2 - 1} J_{2k+\nu}(z) = -\frac{1}{2(\nu+1)} J_{\nu+2}(z) - \frac{1}{z} \sum_{k=1}^{\infty} (-1)^k J_{2k+\nu+1}(z)$$

[см. 5.7.1.7]

$$13. \sum_{k=1}^{\infty} \frac{1}{4k^2 - 1} J_{2k}(z) = \frac{1}{2} J_0(z) + \frac{\pi}{4} H_1(z) - \frac{1}{2}.$$

$$14. \sum_{k=1}^{\infty} \frac{(-1)^k}{4k^2 - 1} J_{2k}(z) = \frac{1}{2} J_0(z) - \frac{1}{2z} \sin z.$$

$$15. \sum_{k=1}^{\infty} \frac{1}{9k^2 - 1} J_{2k}(z) = \frac{2}{9} s_{-1, 2/3}(z) + \frac{1}{2} J_0(z).$$

$$16. \sum_{k=1}^{\infty} \frac{2k+\nu}{(2k+\nu)^2 - 1} J_{2k+\nu}(z) = \frac{1}{2(\nu+1)} J_{\nu+2}(z) + \frac{1}{2z} \int_0^z J_{\nu+2}(t) dt$$

[Re $\nu > -3$]

$$17. \sum_{k=0}^{\infty} \frac{2k+1}{(3k+1)(3k+2)} J_{2k+1}(z) = \frac{2}{9} s_{0, 1/3}(z)$$

5.7.4. Разные ряды вида $\sum a_k J_{nk+\nu}(z)$.

$$1. \sum_{k=0}^{\infty} \frac{\Gamma(k+\mu)}{k!} J_{2k+\nu}(z) = 2^{-\mu} \int_0^z t^{\mu-1} J_{\nu-\mu}(z-t) dt$$

[Re $\mu > 0$; Re $(\mu - \nu) < 1$].

$$2. \sum_{k=0}^{\infty} \frac{(1/2)_k}{k!} J_{2k+\nu}(z) = \frac{\sqrt{\pi z}}{2} J_{\nu/2} \left(\frac{z}{2} \right) J_{(\nu-1)/2} \left(\frac{z}{2} \right).$$

$$3. \sum_{k=0}^{\infty} \frac{(1/2)_k}{k!} J_{2k+(1\pm 1)/2}(z) = \begin{cases} \sin(z/2) \\ \cos(z/2) \end{cases} J_0 \left(\frac{z}{2} \right).$$

$$4. \sum_{k=0}^{\infty} \frac{(v)_k}{k!} (2k+v) J_{2k+v}(z) = \frac{1}{\Gamma(v)} \left(\frac{z}{2}\right)^v \quad [v \neq 0, -1, -2, \dots]$$

$$5. \sum_{k=0}^{\infty} \frac{k!}{(3/2)_k} \left(2k + \frac{3}{2}\right) J_{2k+3/2}(z) = \sqrt{\frac{z}{2\pi}} \operatorname{Si}(z).$$

$$6. \sum_{k=0}^{\infty} \frac{(\delta - v + 3/2)_k}{(v)_k} \left(2k + \delta + \frac{1}{2}\right) J_{2k+\delta+1/2}(z) = \\ = \sqrt{\frac{2}{\pi}} (v-1) z^{\delta-2v+5/2} \int_0^z t^{2v-4} \begin{Bmatrix} \sin t \\ t \cos t \end{Bmatrix} dt \quad \left[\operatorname{Re} v > 1, \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right]$$

$$7. \sum_{k=0}^{\infty} \frac{(1-v)_k}{(v)_k} J_{2k}(z) = \frac{1}{2} {}_1F_2\left(v - \frac{1}{2}; \frac{1}{2}, v; -\frac{z^2}{4}\right) + \frac{1}{2} J_0(z).$$

$$8. \sum_{k=1}^{\infty} \frac{(-1)^k (1-v)_k}{(v)_k} J_{2k}(z) = \frac{1}{2} \Gamma(v) \left(\frac{z}{2}\right)^{1-v} J_{v-1}(z) - \frac{1}{2} J_0(z).$$

$$9. \sum_{k=0}^{\infty} \frac{(2-v)_k}{(v)_k} J_{2k+1}(z) = \frac{z}{2} {}_1F_2\left(v - \frac{1}{2}; \frac{3}{2}, v; -\frac{z^2}{4}\right) \quad [\operatorname{Re} v > 1; v \neq 2, 3, 4, \dots]$$

$$10. \sum_{k=0}^{\infty} \frac{((\mu - \lambda + 1)/2)_k \Gamma(k + (\mu + \lambda)/2)}{k! \Gamma(k + \mu + 1) \Gamma(k + v + 1/2)} J_{k+v}(z) = \\ = \frac{4 \sqrt{\pi} (z/2)^{(\lambda - \mu)/2}}{\Gamma((\mu + \lambda + 1)/2) \Gamma(v - (\mu + \lambda - 1)/2)} \int_0^{\pi/2} J_{v - (\lambda + \mu)/2}(z \sin^2 t) \times \\ \times J_{\mu}(z \cos^2 t) \sin^{2v - \lambda - \mu + 1} t \cos^{2\lambda - 1} t dt \quad [\operatorname{Re}(\lambda + \mu) > 0, \operatorname{Re}(2v - \lambda - \mu) > -1]$$

$$11. \sum_{k=0}^{\infty} \frac{(-1)^k (2k+v) \Gamma(k+v) \Gamma(k - \mu + v) \Gamma(2k - \lambda + \mu + 1)}{k! \Gamma(k + \mu + 1) \Gamma(2k + \lambda - \mu + 2v)} J_{2k+v}(z) = \\ = 2^{2\mu} z^{v-2} \sqrt{\frac{\pi z}{2}} \Gamma\left[\begin{matrix} \mu - \lambda + 1 \\ \mu + \lambda \end{matrix}\right] \int_0^{\pi/2} J_{v - \mu - 1/2}(z \sin^2 t) J_{\mu}(z \cos^2 t) \times \\ \times \sin^{2v - 2\mu} t \cos^{2\lambda - 1} t dt \quad [\operatorname{Re}(\lambda + \mu), \operatorname{Re}(v - \mu) > 0]$$

$$12. \sum_{k=0}^{\infty} \frac{(-1)^k (\mu)_k \Gamma(k + \lambda + \mu)}{k! \Gamma(k + \lambda + 1)} J_{2k+v}(z) = \\ = 2^{1-\mu} z^{\mu} \int_0^{\pi/2} J_{\lambda}(z \sin^2 t) J_{v - \lambda - \mu}(z \cos^2 t) \sin^{2\mu - 1} t \cos t dt \quad [\operatorname{Re}(\lambda + \mu), \operatorname{Re}(v - \lambda - \mu + 1) > 0]$$

$$13. \sum_{k=0}^{\infty} (-1)^k (2k+v) \frac{(2v - \delta)_{2k}}{(2k + \delta)!} J_{2k+v}(z) = \frac{2^{-v} z^{v-\delta}}{\Gamma(v)} \begin{Bmatrix} \sin z \\ \cos z \end{Bmatrix} \quad \left[\operatorname{Re} v > 1/2, \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right].$$

$$14. \sum_{k=0}^{\infty} \frac{(2k+v) \Gamma(k+v)}{k! \Gamma(k + \mu) \Gamma(\mu - v - k)} J_{2k+v}(z) = \frac{1}{\Gamma(\mu - v)} \left(\frac{z}{2}\right)^{v - \mu + 1} J_{\mu-1}(z).$$

$$15. \sum_{k=0}^{\infty} \frac{(2k+\nu) \Gamma(k+\nu)}{k! (2k+1) (2k+2\nu-1)} J_{2k+\nu}(z) = \frac{\sqrt{\pi}}{4} \Gamma\left(\nu - \frac{1}{2}\right) H_{\nu-1}(z)$$

$$16. \sum_{k=0}^{\infty} \frac{(2k+\nu) \Gamma(k+\nu)}{k! [(2k+\nu)^2 - \mu^2]} J_{2k+\nu}(z) = 2^{-\nu} S_{\nu-1, \mu}(z)$$

$$[\nu \pm \mu \neq 0, -2, -4, \dots].$$

$$17. \sum_{k=1}^{\infty} \frac{(2k+\nu) \Gamma(k+\nu)}{k! k (k+\nu)} J_{2k+\nu}(z) = \\ = 2^{2-\nu} S_{\nu-1, \nu}(z) + \Gamma(\nu) \left[\pi Y_{\nu}(z) - 2 \frac{\partial J_{\nu}(z)}{\partial \nu} - J_{\nu}(z) (\psi(\nu+1) + C) \right] \\ [\nu \neq 0, -1, -2, \dots].$$

$$18. \sum_{k=0}^{\infty} \frac{(-1)^k (2k+\nu) \Gamma(k+\mu+1) \Gamma(k+\nu) \Gamma(2k-\mu+\nu)}{k! \Gamma(k-\mu+\nu) \Gamma(2k+\mu+\nu+1)} J_{2k+\nu}(z) = \\ = \frac{z \Gamma^2(\mu+1)}{\Gamma(2\mu+1)} \int_0^{\pi/2} J_{\mu}(z \sin^2 t) J_{\nu-\mu-1}(z \cos^2 t) \sin^{2\mu+1} t \cos t dt \\ [\operatorname{Re} \mu > -1/2; \operatorname{Re}(\nu - \mu) > 0].$$

$$19. \sum_{k=0}^{\infty} \frac{(2k+\nu) (\nu)_k}{k! (\lambda)_k (\mu)_k} (\nu - \lambda + 1)_k (\nu - \mu + 1)_k J_{2k+\nu}(z) = \\ = \frac{1}{\Gamma(\nu)} \left(\frac{z}{2}\right)^{\nu} {}_1F_2\left(\lambda + \mu - \nu - 1; \lambda, \mu; -\frac{z^2}{4}\right).$$

$$20. \sum_{k=0}^{\infty} \frac{(2k+\nu) (\mu)_k}{(\nu - \mu + 1)_k} J_{2k+\nu}(z) = (\nu - \mu) z^{2\mu-\nu} \int_0^z t^{\nu-2\mu} J_{\nu-1}(t) dt \\ [\operatorname{Re}(\nu - \mu) > 0].$$

$$21. \sum_{k=0}^{\infty} \frac{(-1)^k (2k+\nu) (\nu)_k (\nu - \mu)_k}{k! (\mu + 1)_k} J_{2k+\nu}(z) = \Gamma\left[\frac{\mu+1}{\nu}\right] \left(\frac{z}{2}\right)^{\nu-\mu} J_{\mu}(z) \\ [\mu - \nu - \nu - \mu \neq -1, -2, -3, \dots].$$

5.7.5. Ряды вида $\sum (ka+b) t^k J_{nk+\nu}(z)$

$$1. \sum_{k=0}^{\infty} t^k J_{k+\nu}(z) = e^{i\nu\pi/2} t^{-\nu} [U_{\nu}(-itz, z) + iU_{\nu+1}(-itz, z)], \\ = z^{-\nu} e^{tz/2} \int_0^z x^{\nu} e^{-tx^2/(2z)} J_{\nu-1}(x) dx \quad [\operatorname{Re} \nu > 1].$$

$$2. \sum_{k=-\infty}^{\infty} t^k J_{k \pm n}(z) = t^{\mp n} \exp\left[\left(t - \frac{1}{t}\right) \frac{z}{2}\right].$$

$$3. \sum_{k=0}^{\infty} t^{2k} J_{2k+\nu}(z) = z^{-\nu} \int_0^z x^{\nu} \operatorname{ch}\left(\frac{tx^2}{2z} - \frac{tz}{2}\right) J_{\nu-1}(x) dx \quad [\operatorname{Re} \nu > 0]$$

$$4. \sum_{k=0}^{\infty} (-1)^k t^{2k} J_{2k+\nu}(z) = t^{-\nu} U_{\nu}(tz, z).$$

5. $\sum_{k=-\infty}^{\infty} t^{2k+(1\pm 1)/2} J_{2k+2n+(1\pm 1)/2}(z) = t^{-2n-(1\pm 1)/2} \begin{cases} \operatorname{sh} [(a^2-1)x/(2a)] \\ \operatorname{ch} [(a^2-1)x/(2a)] \end{cases}$
6. $\sum_{k=0}^{\infty} t^{2k+1} J_{2k+1}(z) = \frac{1}{2} \exp \left[\left(t - \frac{1}{t} \right) \frac{z}{2} \right] - \frac{1}{\pi} \int_0^{\infty} \frac{1}{x^2+1} \cos \frac{(t^2+x^2)z}{2tx} dx$
 $[0 \leq t < 1; z \geq 0].$
7. $\sum_{k=-\infty}^{\infty} (-1)^k t^{2k} J_{2k+n}(z) = t^{-n} \cos \frac{t^2 z + z - n\pi t}{2t}.$
8. $\sum_{k=-\infty}^{\infty} k t^k J_k(z) = \frac{1}{2} \left(t + \frac{1}{t} \right) z \exp \left[\left(t - \frac{1}{t} \right) \frac{z}{2} \right].$
9. $\sum_{k=-\infty}^{\infty} (\pm 1)^k k t^{2k} J_{2k}(z) = \frac{1}{4} \left(\frac{1}{t} \pm t \right) z \begin{cases} \operatorname{sh} [(t^2-1)z/(2t)] \\ \sin [(t^2+1)z/(2t)] \end{cases}.$
10. $\sum_{k=-\infty}^{\infty} (\pm 1)^k (2k+1) t^{2k+1} J_{2k+1}(z) = \frac{1}{2} \left(t \pm \frac{1}{t} \right) \begin{cases} \operatorname{ch} [(t^2-1)z/(2t)] \\ \cos [(t^2+1)z/(2t)] \end{cases}.$

5.7.6. Ряды вида $\sum \frac{t^k}{\Gamma(nk+\mu)} J_{k+\nu}(z).$

1. $\sum_{k=0}^{\infty} \frac{t^k}{k!} J_{k+\nu}(z) = z^{\nu/2} (z-2t)^{-\nu/2} J_{\nu}(\sqrt{z^2-2tz})$ $[|2t| < z]$
2. $\sum_{k=0}^{\infty} \frac{t^k}{k!} J_{\nu-k}(z) = z^{-\nu/2} (z+2t)^{\nu/2} J_{\nu}(\sqrt{z^2+2tz})$ $[|2t| < z]$
3. $\sum_{k=-\infty}^{\infty} \frac{t^k}{\Gamma(k+\nu+\sigma)} J_{k+\nu}(z) = t^{1-\sigma-\nu} \left(1 - \frac{2t}{z} \right)^{(\sigma-1)/2} J_{1-\sigma}(\sqrt{z^2-2tz})$
 $[\sigma = 0 \text{ или } 1].$
4. $\sum_{k=0}^{\infty} \frac{(z/2)^k}{\Gamma(k+\mu)} J_{k+\nu}(z) = \frac{2z^{\nu-2\mu+2}}{\Gamma(\mu-1)} \int_0^z t^{2\mu-\nu-3} J_{\nu}(t) dt$ $[\operatorname{Re} \mu > 1].$
5. $\sum_{k=-\infty}^{\infty} \frac{t^k}{\Gamma(k-\nu+3/2)} J_{\pm(k-\nu)}(z) = \sqrt{\frac{2}{\pi z}} t^{\nu-1/2} \begin{cases} \cos(\sqrt{z^2-2tz}) \\ \sin(\sqrt{z^2-2tz}) \end{cases}.$
6. $\sum_{k=0}^{\infty} \frac{z^k}{k!} J_{k+\nu}(z) = I_{\nu}(z)$
7. $\sum_{k=0}^{\infty} \frac{(z/2)^k}{k!} J_{k+\nu}(z) = \frac{(z/2)^{\nu}}{\Gamma(\nu+1)}.$
8. $\sum_{k=0}^{\infty} \frac{(-z/2)^k}{k!} J_{\nu-k}(z) = 0$ $[\operatorname{Re} \nu > 0 \text{ или } \nu = 0, \pm 1, \pm 2, \dots].$
9. $\sum_{k=0}^{\infty} \frac{(z/2)^k}{(k+1)!} J_{k+1/2}(z) = \sqrt{\frac{2}{\pi z^3}} (1 - \cos z)$
10. $\sum_{k=0}^{\infty} \frac{(z/2)^k}{(k+2)!} J_{k+1/2}(z) = 4 \sqrt{\frac{2}{\pi z^7}} (\cos z + z \sin z - 1).$

$$11. \sum_{k=0}^{\infty} \frac{z^k}{(2k+1)!!} J_k(z) = \frac{1}{z} V(z) \quad [\text{см. стр. 651}]$$

$$12. \sum_{k=1}^{\infty} \frac{z^k}{(2k-1)!!} J_k(z) = zV(z) - zJ_1(z) \quad [\text{см. стр. 651}]$$

$$13. \sum_{k=0}^{\infty} \frac{z^k}{(2k+1)!!} J_{k+1/2}(z) = \sqrt{\frac{2}{\pi z}} \text{Si}(z).$$

$$14. \sum_{k=0}^{\infty} \frac{z^k}{(2k-1)!!} J_{k+1/2}(z) = \sqrt{\frac{z}{2\pi}} \left[z \text{Si}(z) + \cos z - \frac{3 \sin z}{z} \right]$$

5.7.7. Ряды вида $\sum \frac{t^k}{k! (ka+b)^n} J_{k+v}(z).$

$$1. \sum_{k=0}^{\infty} \frac{(z/2)^k}{k! (k+\mu)} J_{k+v}(z) = \frac{2^{2-\mu}}{\Gamma(\nu)} z^{-\nu} S_{\nu+\mu-1, \nu-\mu}(z)$$

$[\mu, \nu \neq 0, -1, -2, \dots].$

$$2. \sum_{k=0}^{\infty} \frac{(z/2)^k}{k! (k-\nu)} J_{k+v}(z) = \frac{\pi}{\Gamma(\nu+1)} \left(\frac{z}{2}\right)^{\nu} [E_{-2\nu}(z) - \text{ctg } \nu\pi J_{-2\nu}(z)].$$

$$3. \sum_{k=0}^{\infty} \frac{(z/2)^k}{k! (k-\nu+1)} J_{k+v}(z) = \frac{\pi}{\Gamma(\nu-1)} \left(\frac{z}{2}\right)^{\nu-1} [E_{2\nu-1}(z) - \text{ctg } \nu\pi J_{2\nu-1}(z)].$$

$$4. \sum_{k=0}^{\infty} \frac{(z/2)^k}{k! (2k+1)} J_{k+v}(z) = \sqrt{\frac{\pi}{2z}} H_{\nu-1/2}(z)$$

$$5. \sum_{k=0}^{\infty} \frac{(z/2)^k}{k! (k+\nu)} J_{k+(1 \pm 1)/4}(z) = \Gamma(\nu) \left(\frac{2}{z}\right)^{\nu} \left\{ \begin{matrix} H_{\nu-1/2}(z) \\ J_{\nu}(z) \end{matrix} \right\}.$$

$$6. \sum_{k=0}^{\infty} \frac{(z/2)^k}{k! (k+\nu)} J_{k-m}(z) = (-1)^m \Gamma(\nu) \left(\frac{2}{z}\right)^{\nu} J_{m+\nu}(z).$$

$$7. \sum_{k=1}^{\infty} \frac{(z/2)^k}{k! (2k-1)} J_k(z) = 1 - \cos z.$$

$$8. \sum_{k=0}^{\infty} \frac{(z/2)^k}{k! (2k-1)} J_{k-1/2}(z) = \sqrt{\frac{2}{\pi z}} [\pi z H_0(z) - 2].$$

$$9. \sum_{k=0}^{\infty} \frac{(z/2)^k}{k! (2k-1)} J_{k+1/2}(z) = \sqrt{\frac{2z}{\pi}} [Y_1(z) - 2].$$

$$10. \sum_{k=1}^{\infty} \frac{(z/2)^k}{k! k} J_k(z) = \frac{\pi}{2} Y_0(z) - \left(C + \ln \frac{z}{2}\right) J_0(z).$$

$$11. \sum_{k=0}^{\infty} \frac{(z/2)^k}{k! (k+a)^2} J_{k+1}(z) = \Gamma(a) \left(\frac{2}{z}\right)^a \left[\frac{\partial J_{a-1}(z)}{\partial a} + \left(\psi(a) - \ln \frac{z}{2}\right) J_{a-1}(z) \right].$$

$$12. \sum_{k=1}^{\infty} \frac{(z/2)^k}{k! k^2} J_k(z) = \\ = \frac{\pi}{2} \left(C + \ln \frac{z}{2} \right) Y_0(z) - \frac{1}{2} \frac{\partial^2 J_\nu(z)}{\partial \nu^2} \Big|_{\nu=0} - \frac{1}{2} \left[\frac{\pi^2}{6} + \left(C + \ln \frac{z}{2} \right)^2 \right] J_0(z).$$

$$13. \sum_{k=0}^{\infty} \frac{(z/2)^k}{k! (4k^2 - 1)} J_{k+1}(z) = -\frac{1}{2} \sin z.$$

5.7.8. Ряды вида $\sum \frac{\Gamma(ka+b)}{\Gamma(kc+d)\Gamma(kc+f)} t^k J_{nk+\nu}(z).$

$$1. \sum_{k=0}^{\infty} k! \left(\frac{z}{2} \right)^{-k} J_{k+\nu}(z) = \frac{(z/2)^\nu}{(\nu-1)\Gamma(\nu)} {}_1F_2 \left(\nu-1; \nu, \nu; -\frac{z^2}{4} \right)$$

[Re $\nu > 1$]

$$2. \sum_{k=0}^{\infty} k! \left(\frac{z}{2} \right)^{-k} J_{k+2}(z) = 1 - J_0(z).$$

$$3. \sum_{k=0}^{\infty} \frac{(\mu)_k}{k! (\lambda)_\nu} \left(\frac{z}{2} \right)^k J_{k+\nu}(z) = \frac{1}{\Gamma(\nu+1)} \left(\frac{z}{2} \right)^\nu {}_1F_2 \left(\lambda - \mu; \lambda, \nu+1; -\frac{z^2}{4} \right).$$

$$4. \sum_{k=0}^{\infty} \frac{(\mu - \nu - 1)_k}{k! (\mu)_k} \left(\frac{z}{2} \right)^k J_{k+\nu}(z) = \Gamma \left[\frac{\mu}{\nu+1} \right] \left(\frac{z}{2} \right)^{\nu-\mu+1} J_{\mu-1}(z)$$

[$\mu \neq \nu+1$; $\nu \neq -1, -2, -3, \dots$]

$$5. \sum_{k=0}^{\infty} \frac{(-1/2)_k}{k! (\nu+3/2)_k} \left(\frac{z}{2} \right)^k J_{k+\nu}(z) = \Gamma \left[\frac{\nu+3/2}{\nu+2} \right] \sqrt{\frac{1}{2z}} [z J_{\nu-1/2}(z) + J_{\nu+1/2}(z)].$$

$$6. \sum_{k=0}^{\infty} \frac{(1/2)_k}{k! (\nu/2+3/2)_k} \left(\frac{z}{2} \right)^k J_{k+\nu}(z) = \\ = \Gamma \left[\frac{(\nu+1)/2, (\nu+3)/2}{\nu+1} \right] J_{(\nu-1)/2} \left(\frac{z}{2} \right) J_{(\nu+1)/2} \left(\frac{z}{2} \right).$$

$$7. \sum_{k=0}^{\infty} \frac{(1/2)_k}{k! (\nu/2+1)_k} \left(\frac{z}{2} \right)^k J_{k+\nu}(z) = \Gamma \left[\frac{\nu/2+1, \nu/2+1}{\nu+1} \right] J_{\nu/2}^2 \left(\frac{z}{2} \right).$$

$$8. \sum_{k=0}^{\infty} \frac{(2k-1)!!}{(k!)^2} \left(\frac{z}{4} \right)^k J_k(z) = J_0^2 \left(\frac{z}{2} \right).$$

$$9. \sum_{k=0}^{\infty} \frac{(2k+1)!!}{(k!)^2} \left(\frac{z}{4} \right)^k J_k(z) = \left(1 + \frac{z^2}{2} \right) J_0^2 \left(\frac{z}{2} \right) + \frac{z^2}{2} J_1^2 \left(\frac{z}{2} \right) - z J_0 \left(\frac{z}{2} \right) J_1 \left(\frac{z}{2} \right).$$

$$10. \sum_{k=0}^{\infty} \frac{(2k-1)!!}{(k!)^2} \left(\frac{z}{4} \right)^k J_{k+1}(z) = \frac{z}{2} J_0^2 \left(\frac{z}{2} \right) + \frac{z}{2} J_1^2 \left(\frac{z}{2} \right).$$

$$11. \sum_{k=0}^{\infty} \frac{(2k-1)!!}{k! (k+1)!} \left(\frac{z}{4} \right)^k J_k(z) = J_0^2 \left(\frac{z}{2} \right) - J_1^2 \left(\frac{z}{2} \right).$$

$$12. \sum_{k=0}^{\infty} \frac{(2k-1)!!}{k! (k+1)!} \left(\frac{z}{4} \right)^k J_{k+1}(z) = J_0 \left(\frac{z}{2} \right) J_1 \left(\frac{z}{2} \right).$$

$$13. \sum_{k=0}^{\infty} \frac{(2k-1)!!}{k! (k+1)!} \left(\frac{z}{4}\right)^k J_{k+2}(z) = \frac{1}{2} J_1^2\left(\frac{z}{2}\right).$$

$$14. \sum_{k=0}^{\infty} \frac{t^{2k+\delta}}{(2k+\delta)!} J_{2k+\nu+\delta}(z) = \\ = \frac{1}{2} z^{\nu/2} (z-2t)^{-\nu/2} J_{\nu}(\sqrt{z^2-2tz}) \mp \frac{1}{2} z^{\nu/2} (z+2t)^{-\nu/2} J_{\nu}(\sqrt{z^2+2tz}) \\ \left[\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}\right].$$

$$15. \sum_{k=0}^{\infty} \frac{t^{2k+\nu}}{k! (k+\nu+1)!} J_{2k+\nu}(z) = J_{\nu}\left(z \sin\left(\frac{1}{2} \arcsin \frac{4t}{z}\right)\right) J_{\nu}\left(z \cos\left(\frac{1}{2} \arcsin \frac{4t}{z}\right)\right) \\ [|z| \leq 4|t|]$$

$$16. \sum_{k=0}^{\infty} \frac{(z/2)^{2k+\nu} \Gamma(k+\alpha+\nu/2) \Gamma(k+\beta+\nu/2)}{k! \Gamma(k+\nu+1) \Gamma(2k+\alpha+\beta+\nu)} J_{2k+\nu}(z) = \\ = 2 \int_0^{\pi/2} \sin^{2\alpha-1} t \cos^{2\beta-1} t J_{\nu}(z \sin t) J_{\nu}(z \cos t) dt \quad [\operatorname{Re} \alpha \quad \operatorname{Re} \beta > -\operatorname{Re} \nu/2]$$

5.7.9. Ряды, содержащие гиперболические функции и $J_{\nu}(z)$

$$1. \sum_{k=0}^{\infty} (\pm 1)^k \operatorname{ch} ka J_{2k}(z) = \frac{1}{2} J_0(z) + \frac{1}{2} \left\{ \operatorname{ch} [z \operatorname{sh}(a/2)] \right\} \left\{ \cos [z \operatorname{ch}(a/2)] \right\}.$$

$$2. \sum_{k=0}^{\infty} (\pm 1)^k \left\{ \frac{\operatorname{sh}(2k+1)a}{\operatorname{ch}(2k+1)a} \right\} J_{2k+1}(z) = \frac{1}{2} \left\{ \frac{\operatorname{sh}(z \operatorname{sh} a)}{\sin(z \operatorname{ch} a)} \right\}.$$

$$3. \sum_{k=1}^{\infty} (\pm 1)^k k \operatorname{sh} ka J_{2k}(z) = \pm \frac{z}{4} \left\{ \frac{\operatorname{ch}(a/2) \operatorname{sh}[z \operatorname{sh}(a/2)]}{\operatorname{sh}(a/2) \sin[z \operatorname{ch}(a/2)]} \right\}.$$

$$4. \sum_{k=0}^{\infty} (\mp 1)^k \left\{ \frac{\operatorname{sh}(2k+1)a}{\operatorname{ch}(2k+1)a} \right\} J_{2k+1}(z) = \frac{z}{2} \left\{ \frac{\operatorname{sh} a \cos(z \operatorname{ch} a)}{\operatorname{ch} a \operatorname{ch}(z \operatorname{sh} a)} \right\}.$$

$$5. \sum_{k=0}^{\infty} (\pm 1)^k \operatorname{ch} ka J_k^2(z) = \frac{1}{2} J_0^2(z) + \frac{1}{2} \left\{ \frac{I_0(2z \operatorname{sh}(a/2))}{J_0(2z \operatorname{ch}(a/2))} \right\}.$$

5.7.10. Ряды вида $\sum a_k \left\{ \frac{\sin(ka+b)}{\cos(ka+b)} \right\} J_{nk+\nu}(z)$

$$1. \sum_{k=1}^{\infty} (\pm 1)^k \sin ka J_{k+\nu}(z) = \pm \frac{1}{2} \int_0^z \left(\frac{\nu}{t} \pm \cos a \right) \sin[(z-t) \sin a] J_{\nu}(t) dt \\ [\operatorname{Re} \nu > 0]$$

$$2. \sum_{k=-\infty}^{\infty} \left\{ \frac{\sin ka}{\cos la} \right\} J_{k+n}(z) = \left\{ \frac{\sin(-na + z \sin a)}{\cos(na - z \sin a)} \right\}.$$

$$3. \sum_{k=1}^{\infty} (\pm 1)^k \cos ka J_{k+\nu}(z) = \\ = \frac{1}{4} \operatorname{cosec} a \int_0^z \sin[(z-t) \sin a] [\cos 2a J_{\nu}(t) \pm 2 \cos a J'_{\nu}(t) - J_{\nu+2}(t)] dt \\ [\operatorname{Re} \nu > 0]$$

4.
$$\sum_{k=0}^{\infty} (\pm 1)^k \cos ka J_{2k}(z) = \frac{1}{2} \left\{ \cos [z \sin (a/2)] \right\} + \frac{1}{2} J_0(z).$$
5.
$$\sum_{k=0}^{\infty} (\pm 1)^k \left\{ \begin{matrix} \sin (2k+1)a \\ \cos (2k+1)a \end{matrix} \right\} J_{2k+1}(z) = \frac{1}{2} \left\{ \begin{matrix} \sin (z \sin a) \\ \sin (z \cos a) \end{matrix} \right\}.$$
6.
$$\sum_{k=0}^{\infty} \cos (2k+1)a J_{2k+1}(z) = \frac{z}{4\sqrt{2}} \int_{2a}^{\pi} \frac{\sin t}{\sqrt{\cos 2a - \cos t}} J_0\left(z \sin \frac{t}{2}\right) dt.$$
7.
$$\begin{aligned} \sum_{k=0}^{\infty} (-1)^k \left\{ \begin{matrix} \sin ka \\ \cos ka \end{matrix} \right\} J_{2k+1/2}(z) &= \mp \frac{\sqrt{2}}{2} \left\{ \begin{matrix} \sin (a/4) \\ \cos (a/4) \end{matrix} \right\} \times \\ &\times \left[\sin \left(z \cos \frac{a}{2} \right) S \left(2z \cos^2 \frac{a}{4} \right) + \cos \left(z \cos \frac{a}{2} \right) C \left(2z \cos^2 \frac{a}{4} \right) \right] - \\ &- \frac{\sqrt{2}}{2} \left\{ \begin{matrix} \cos (a/4) \\ \sin (a/4) \end{matrix} \right\} \left[\sin \left(z \cos \frac{a}{2} \right) S \left(2z \sin^2 \frac{a}{4} \right) - \cos \left(z \cos \frac{a}{2} \right) C \left(2z \sin^2 \frac{a}{4} \right) \right]. \end{aligned}$$
8.
$$\begin{aligned} \sum_{k=0}^{\infty} (-1)^k \left\{ \begin{matrix} \sin ka \\ \cos ka \end{matrix} \right\} J_{2k+3/2}(z) &= \mp \frac{\sqrt{2}}{2} \left\{ \begin{matrix} \sin (3a/4) \\ \cos (3a/4) \end{matrix} \right\} \times \\ &\times \left[\sin \left(z \cos \frac{a}{2} \right) C \left(2z \cos^2 \frac{a}{4} \right) - \cos \left(z \cos \frac{a}{2} \right) S \left(2z \cos^2 \frac{a}{4} \right) \right] + \\ &+ \frac{\sqrt{2}}{2} \left\{ \begin{matrix} \cos (3a/4) \\ \sin (3a/4) \end{matrix} \right\} \left[\cos \left(z \cos \frac{a}{2} \right) S \left(2z \sin^2 \frac{a}{4} \right) + \sin \left(z \cos \frac{a}{2} \right) C \left(2z \sin^2 \frac{a}{4} \right) \right]. \end{aligned}$$
9.
$$\begin{aligned} \sum_{k=0}^{\infty} (-1)^k \cos \left(2k + \frac{2 \pm 1}{2} \right) a J_{2k + (2 \pm 1)/2}(z) &= \\ &= \frac{\sqrt{2}}{2} \left\{ \begin{matrix} \sin (z \cos a) \\ \cos (z \cos a) \end{matrix} \right\} C \left(2z \cos^2 \frac{a}{2} \right) \mp \frac{\sqrt{2}}{2} \left\{ \begin{matrix} \cos (z \cos a) \\ \sin (z \cos a) \end{matrix} \right\} S \left(2z \cos^2 \frac{a}{2} \right). \end{aligned}$$
10.
$$\sum_{k=1}^{\infty} (\pm 1)^k k \sin ka J_{2k}(z) = \pm \frac{z}{4} \left\{ \begin{matrix} \cos (a/2) \sin [z \sin (a/2)] \\ \sin (a/2) \sin [z \cos (a/2)] \end{matrix} \right\}.$$
11.
$$\sum_{k=0}^{\infty} (\mp 1)^k (2k+1) \left\{ \begin{matrix} \sin (2k+1)a \\ \cos (2k+1)a \end{matrix} \right\} J_{2k+1}(z) = \frac{z}{2} \left\{ \begin{matrix} \sin a \cos (z \cos a) \\ \cos a \cos (z \sin a) \end{matrix} \right\}.$$
12.
$$\sum_{k=-\infty}^{\infty} t^k \sin ka J_k(z) = \exp \left[\left(t - \frac{1}{t} \right) \frac{z}{2} \cos a \right] \sin \left[\left(t + \frac{1}{t} \right) \frac{z}{2} \sin a \right].$$
13.
$$\begin{aligned} \sum_{k=1}^{\infty} (\pm 1)^k \cos ka \cos kb J_{2k}(z) &= \\ &= \frac{1}{2} \left\{ \begin{matrix} \cos [z \cos (a/2) \sin (b/2)] \cos [z \sin (a/2) \cos (b/2)] \\ \cos [z \cos (a/2) \cos (b/2)] \cos [z \sin (a/2) \sin (b/2)] \end{matrix} \right\} - \frac{1}{2} J_0(z). \end{aligned}$$
14.
$$\begin{aligned} \sum_{k=0}^{\infty} (\pm 1)^k \sin (2k+1)a \left\{ \begin{matrix} \sin (2k+1)b \\ \cos (2k+1)b \end{matrix} \right\} J_{2k+1}(z) &= \\ &= \frac{1}{2} \left\{ \begin{matrix} \sin (z \sin a \cos b) \cos (z \cos a \sin b) \\ \sin (z \sin a \sin b) \cos (z \cos a \cos b) \end{matrix} \right\} \end{aligned}$$

$[0 \leq a < \pi; b > 0]$

$$15. \sum_{k=1}^{\infty} e^{i\pi nk/2} \cos ka J_{nk}(z) = \frac{1}{2n} \sum_{k=k_-}^{k_+} \alpha_k \exp\left(iz \cos \frac{2k\pi + a}{n}\right) - \frac{1}{2} J_0(z)$$

$[a > 0; n = 1, 2, \dots; k_{\pm} = \pm [(n\pi \mp a)/(2\pi)]; \alpha_k = 1 \text{ при } k \neq k_{\pm}, \alpha_{k_{\pm}} = 1/2 \text{ при } (n\pi \mp a)/(2\pi) = \dots, -2, -1, 0, 1, 2, \dots, \alpha_{k_{\pm}} = 1 \text{ при } (n\pi \mp a)/(2\pi) \neq \dots, -2, -1, 0, 1, 2, \dots]$

5.7.11. Ряды вида $\sum (\pm 1)^k J_{\mu \pm nk}(w) J_{\nu \pm nk}(z)$.

$$1. \sum_{k=1}^{\infty} (\pm 1)^k J_k(w) J_k(z) = \frac{1}{2} J_0(w \mp z) - \frac{1}{2} J_0(w) J_0(z).$$

$$2. \sum_{k=-\infty}^{\infty} (\pm 1)^k J_k(w) J_{\nu+k}(z) = J_{\nu}(z \mp w) \quad [|w| < |z|].$$

$$3. \sum_{k=-\infty}^{\infty} (\pm 1)^k J_k(w) J_{\nu-k}(z) = J_{\nu}(z \pm w) \quad [|w| < |z|].$$

$$4. \sum_{k=1}^{\infty} (-1)^k J_k(z) J_{k+n}(z) = \frac{1}{2} J_n(2z) - \frac{1}{2} \sum_{k=0}^n J_k(z) J_{n-k}(z).$$

$$5. \sum_{k=1}^{\infty} J_{k+\mu}(z) J_{k+\nu}(z) = \frac{z}{2(\mu-\nu)} [J_{\mu}(z) J_{\nu+1}(z) - J_{\mu+1}(z) J_{\nu}(z)].$$

$$6. \sum_{k=1}^{\infty} J_{k+\nu}^2(z) = \nu \int_0^z \frac{1}{t} J_{\nu}^2(t) dt - \frac{1}{2} J_{\nu}^2(z) \quad [\operatorname{Re} z, \operatorname{Re} \nu > 0; \text{ см. 1.8.3}]$$

$$7. \sum_{k=1}^{\infty} (\pm 1)^k J_{\nu+k}(z) J_{\nu-k}(z) = -\frac{1}{2} J_{\nu}^2(z) + \frac{1}{2} J_{2\nu}(2z) \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}.$$

$$8. \sum_{k=1}^{\infty} (\pm 1)^k J_{k+\nu}(z) J_{k-\nu}(z) = \frac{1}{2\pi} \left\{ \frac{\nu^{-1} \sin \nu\pi}{-4\nu \sin \nu\pi s_{-1} \nu(2z)} \right\} - \frac{1}{2} J_{\nu}(z) J_{-\nu}(z)$$

$$9. \sum_{k=1}^{\infty} (\pm 1)^k J_k^2(z) = \frac{1}{2} \left\{ \begin{matrix} 1 \\ J_0(2z) \end{matrix} \right\} - \frac{1}{2} J_0^2(z).$$

$$10. \sum_{k=-\infty}^{\infty} J_{n \pm k}^2(z) = 1.$$

$$11. \sum_{k=0}^{\infty} (\pm 1)^k J_{k/2}^2(z) = \frac{1}{2} + \frac{1}{2} J_0^2(z) \pm \frac{1}{\pi} \operatorname{Si}(2z).$$

$$12. \sum_{k=0}^{\infty} (-1)^k J_{k+\nu}(z) J_{k-\nu+1}(z) = \frac{1}{\pi} \cos \frac{2\nu-1}{2} \pi s_{0, 2\nu-1}(2z).$$

$$13. \sum_{k=0}^{\infty} (\pm 1)^k J_{k+1/2}^2(z) = \frac{1}{2\pi} \left\{ \begin{matrix} 2 \operatorname{Si}(2x) \\ \pi H_0(2x) \end{matrix} \right\}.$$

$$14. \sum_{k=0}^{\infty} (-1)^{[k/2]} J_{k+\nu}(z) J_{k-\nu+1}(z) = \frac{\sqrt{2}}{\pi} \cos \frac{2\nu-1}{2} \pi s_{0, 2\nu-1}(\sqrt{2} z).$$

$$15. \sum_{k=0}^{\infty} (-1)^k J_{k+1/2}(z) J_{k+3/2}(z) = \frac{1}{2\pi z} \sin 2z + \frac{1}{2} H_1(2z) - \frac{1}{\pi}.$$

$$16. \sum_{k=0}^{\infty} (-1)^k J_{k+\nu}(z) J_{k+\nu+1/2}(z) = \frac{1}{\sqrt{z}} \int_0^z \sqrt{t} J_{\mu}(t) J_{\mu-1/2}(t) J_{2\nu-2\mu}(2z-2t) dt$$

[$-1/2 < \operatorname{Re} \mu < \operatorname{Re} \nu + 1/2$]

$$17. \sum_{k=0}^{\infty} J_{\nu+k}(z) J_{\nu-k-1}(z) = \frac{1}{2} J_{2\nu-1}(2z) \quad [\operatorname{Re} \nu > 0].$$

$$18. \sum_{k=1}^{\infty} (\pm 1)^k J_{2k}(w) J_{2k}(z) = \frac{1}{4} \left\{ \frac{J_0(w+z) + J_0(w-z)}{2J_0(\sqrt{w^2+z^2})} \right\} - \frac{1}{2} J_0(w) J_0(z).$$

$$19. \sum_{k=0}^{\infty} J_{2k+1}(w) J_{2k+1}(z) = \frac{1}{4} J_0(w-z) - \frac{1}{4} J_0(w+z).$$

$$20. \sum_{k=0}^{\infty} J_{2k+\nu}(z) J_{2k-\nu}(z) = \frac{1}{4\nu\pi} \sin \nu\pi - \frac{\nu}{\pi} \sin \nu\pi s_{-1, 2\nu}(2z) + \frac{1}{2} J_{\nu}(z) J_{-\nu}(z)$$

$$21. \sum_{k=0}^{\infty} (-1)^k J_{2k+\nu}(z) J_{2k-\nu}(z) = -\frac{2\nu}{\pi} \sin \nu\pi s_{-1, 2\nu}(\sqrt{2} z) + \frac{1}{2} J_{\nu}(z) J_{-\nu}(z).$$

$$22. \sum_{k=0}^{\infty} J_{2k+\nu}(z) J_{2k-\nu+2}(z) =$$

$$= -\frac{1}{4(\nu-1)\pi} \sin \nu\pi - \frac{\nu-1}{\pi} \sin \nu\pi s_{-1, 2\nu-2}(2z) + \frac{1}{2} J_{\nu}(z) J_{2-\nu}(z).$$

$$23. \sum_{k=1}^{\infty} J_{2k+\delta}(z) J_{2k+n+\delta}(z) = \mp \frac{1}{4} J_n(2z) \pm \frac{1}{4} \sum_{k=0}^n J_k(z) J_{n-k}(z) +$$

$$+ \frac{2}{4n} [J_1(z) J_n(z) - J_0(z) J_{n+1}(z)] - \delta J_1(z) J_{n+1}(z) \quad [\delta = \{1\}].$$

$$24. \sum_{k=-\infty}^{\infty} (-1)^k J_{2k+(1\pm 1)/2}(w) J_{2k+\nu+(1\pm 1)/2}(z) =$$

$$= J_{\nu}(\sqrt{w^2+z^2}) \begin{cases} \sin [\nu \operatorname{arctg}(w/z)] \\ \cos [\nu \operatorname{arctg}(w/z)] \end{cases} \quad [|w| < |z|].$$

$$25. \sum_{k=1}^{\infty} (-1)^k J_{\nu+2k}(z) J_{\nu-2k}(z) = \frac{1}{2} J_{2\nu}(\sqrt{2} z) - \frac{1}{2} J_{\nu}^2(z) \quad [\operatorname{Re} \nu > 0].$$

$$26. \sum_{k=0}^{\infty} (-1)^k J_{2k+\nu}(z) J_{2k+\nu+1/2}(z) = \frac{1}{\pi} \sqrt{\frac{2}{z}} \int_0^z \sqrt{z-t} J_{2\nu-1/2}(2t) dt$$

[$\operatorname{Re} \nu > -1/4$].

5.7.12. Ряды вида $\sum (\pm 1)^k (ka+b)^l J_{\mu \pm mk}(w) J_{\nu \pm nk}(z)$.

$$1. \sum_{k=1}^{\infty} (\pm 1)^k k J_k(w) J_k(z) = \frac{\pm wz}{2(w \mp z)} [J_1(w) J_0(z) \mp J_0(w) J_1(z)].$$

$$2. \sum_{k=1}^{\infty} k J_{k+\nu}^2(z) = \frac{z^2}{2} [J_{\nu}^2(z) + J_{\nu+1}^2(z)] - \frac{(2\nu+1)z}{2} J_{\nu}(z) J_{\nu+1}(z) -$$

$$- \frac{\nu z}{2} \left[J_{\nu+1}(z) \frac{\partial J_{\nu}(z)}{\partial \nu} - J_{\nu}(z) \frac{\partial J_{\nu+1}(z)}{\partial \nu} \right].$$

$$3. \sum_{k=1}^{\infty} k J_{k+v}(z) J_{k-v}(z) = -\frac{z}{4} [J_v(z) J_{1-v}(z) + J_{v+1}(z) J_{-v}(z)] - \\ - \frac{z^2}{2(4v^2-1)} J_v(z) J_{-v}(z) - \frac{z^2}{4(2v+1)} J_{v+1}(z) J_{-v-1}(z) + \frac{z^2}{4(2v-1)} J_{v-1}(z) J_{1-v}(z).$$

$$4. \sum_{k=1}^{\infty} (-1)^k k J_{k+v}(z) J_{k-v}(z) = -\frac{z}{4} [J_v(z) J_{1-v}(z) + J_{v+1}(z) J_{-v}(z)].$$

$$5. \sum_{k=1}^{\infty} (\pm 1)^k k J_k^2(z) = \frac{z^2}{2} [J_0^2(z) + J_1^2(z)] \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} - \frac{z}{2} J_0(z) J_1(z).$$

$$6. \sum_{k=1}^{\infty} k J_k(z) J_{k+1}(z) = \frac{z}{4} [1 - J_0^2(z) - J_1^2(z)].$$

$$7. \sum_{k=1}^{\infty} k J_{k+\mu}(\mu z) J_{k+v}(vz) = \frac{\mu v z}{2(\mu-v)} [J_{\mu+1}(\mu z) J_v(vz) - J_{\mu}(\mu z) J_{v+1}(vz)].$$

$$8. \sum_{k=-\infty}^{\infty} k J_{n-k}(nz) J_{m-k}(mz) = n (\delta_{m,n} - \delta_{0,n}).$$

$$9. \sum_{k=0}^{\infty} (\pm 1)^k \left(k + \frac{1}{2}\right) J_{k+1/2}(\omega) J_{k+1/2}(z) = \frac{V_{\omega z}}{\pi(\omega \mp z)} \sin(\omega \mp z).$$

$$10. \sum_{k=0}^{\infty} \left(k + \frac{1}{2}\right) J_{k+v}(z) J_{k-v+1}(z) = \frac{z \sin v\pi}{4\pi v(1-v)}.$$

$$11. \sum_{k=0}^{\infty} (-1)^k \left(k + \frac{1}{2}\right) J_{k+v}(z) J_{k-v+1}(z) = \frac{z}{4} [J_v(z) J_{-v}(z) + J_{v-1}(z) J_{1-v}(z)].$$

$$12. \sum_{k=0}^{\infty} (\pm 1)^k \left(k + \frac{1}{2}\right) J_{k+1/2}^2(z) = \frac{1}{2\pi} \left\{ \frac{2z}{\sin 2z} \right\}.$$

$$13. \sum_{k=0}^{\infty} (-1)^k \left(k + \frac{2\pm 1}{4}\right) J_{k+(2\pm 1)/4}^2(z) = \\ = \frac{V_z}{\pi} (2z)^{\delta} \Gamma\left[\frac{3}{4} + \frac{\delta}{2}\right] {}_1F_2\left(\frac{3}{4} + \frac{\delta}{2}; \frac{1}{2} + \delta, \frac{5}{4} + \frac{\delta}{2}; -z^2\right) \\ \left[\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}\right].$$

$$14. \sum_{k=1}^{\infty} (2k+v) J_{2k+v}(\omega) J_{2k+v}(z) = \frac{\omega z}{2(\omega^2 - z^2)} [\omega J_v(\omega) J_{v-1}(z) - z J_{v-1}(\omega) J_v(z)]$$

$$15. \sum_{k=0}^{\infty} (2k+v) J_{2k+v}^2(z) = \frac{z^2}{4} [J_{v-1}^2(z) - J_v(z) J_{v-2}(z)].$$

$$16. \sum_{k=1}^{\infty} k J_{\frac{1}{2}k}^2(z) = -\frac{z}{4} J_0(z) J_1(z) + \frac{z^2}{8} [J_0^2(z) + J_1^2(z)].$$

- $$\begin{aligned}
17. \quad & \sum_{k=1}^{\infty} \left(2k + \frac{2 \pm 1}{2}\right) J_{2k+v+(2 \pm 1)/2}(z) J_{2k-v+(2 \pm 1)/2}(z) = \\
& = \frac{z \cos v\pi}{2\pi(1-4v^2)} \mp \frac{z}{8} [J_{v+1/2}(z) J_{-v-1/2}(z) + J_{v-1/2}(z) J_{1/2-v}(z)]. \\
18. \quad & \sum_{k=0}^{\infty} \left(2k + \frac{2 \pm 1}{2}\right) J_{2k+(2 \pm 1)/2}^2(z) = \frac{z}{2\pi} \mp \frac{1}{4\pi} \sin 2z. \\
19. \quad & \sum_{k=0}^{\infty} (-1)^k (4k+2 \pm 1) J_{2k+(2 \pm 1)/2}(w) J_{4k+2 \pm 1}(z) = \\
& = \frac{z}{2^{3/2} \sqrt{\pi w}} \left[U_2(4w, z) \begin{Bmatrix} \cos w \\ \sin w \end{Bmatrix} \mp U_1(4w, z) \begin{Bmatrix} \sin w \\ \cos w \end{Bmatrix} \right]. \\
20. \quad & \sum_{k=1}^{\infty} k^2 J_k(w) J_k(z) = \frac{wz}{2(w-z)} J_1(w-z). \\
21. \quad & \sum_{k=1}^{\infty} k^2 J_k^2(z) = \frac{z^2}{4}. \\
22. \quad & \sum_{k=1}^{\infty} (-1)^k k^2 J_k^2(z) = -\frac{z}{4} J_1(z). \\
23. \quad & \sum_{k=1}^{\infty} k^2 J_{k+v}(z) J_{k-v}(z) = \frac{z^2}{4v(1-v^2)\pi} \sin v\pi. \\
24. \quad & \sum_{k=1}^{\infty} (-1)^k k^2 J_{k+v}(z) J_{k-v}(z) = \frac{z}{8} \sin v\pi \left[s_{0, 2v+1}(2z) - s_{0, 2v-1}(2z) \right]. \\
25. \quad & \sum_{k=-\infty}^{\infty} k^2 J_{n \pm k}^2(z) = n^2 + \frac{z^2}{2}. \\
26. \quad & \sum_{k=0}^{\infty} (-1)^k \left(k + \frac{1}{2}\right)^2 J_{k+v}(z) J_{k-v+1}(z) = \\
& = -\frac{z \sin v\pi}{2\pi} [v s_{-1, 2v}(2z) + (1-v) s_{-1, 2v-2}(2z)]. \\
27. \quad & \sum_{k=1}^{\infty} k^2 J_{2k}(w) J_{2k}(z) = \frac{wz}{16(w-z)} J_1(w-z) - \frac{wz}{16(w+z)} J_1(w+z). \\
28. \quad & \sum_{k=0}^{\infty} (-1)^k \left(k + \frac{1}{2}\right)^2 J_{k+1/2}^2(z) = \frac{z}{2\pi} - \frac{z}{4} H_1(2z). \\
29. \quad & \sum_{k=1}^{\infty} k^3 J_k^2(z) = \frac{1}{12} [(4z^4 + 3z^2) J_0^2(z) + (4z^4 + z^2) J_1^2(z) - 4z^3 J_0(z) J_1(z)]. \\
30. \quad & \sum_{k=1}^{\infty} (-1)^k k^3 J_{k+v}(z) J_{k-v}(z) = \\
& = -\frac{z^2}{4} J_v(z) J_{-v}(z) - \frac{z^2}{8} J_{v+1}(z) J_{v-1}(z) - \frac{z^2}{8} J_{v-1}(z) J_{1-v}(z).
\end{aligned}$$

$$31. \sum_{k=1}^{\infty} (-1)^k k^3 J_k^2(z) = -\frac{z^2}{4} J_0^2(z) + \frac{z^2}{4} J_1^2(z).$$

$$32. \sum_{k=1}^{\infty} k^4 J_{k+\nu}(z) J_{k-\nu}(z) = \frac{z^4}{5 \cdot 2^5 B(3+\nu, 3-\nu)} + \frac{z^2}{24 B(2+\nu, 2-\nu)}.$$

$$33. \sum_{k=1}^{\infty} k^4 J_k^2(z) = \frac{z^2}{4} \left(1 + \frac{3}{4} z^2 \right).$$

$$34. \sum_{k=1}^{\infty} (-1)^{k+1} k^4 J_k^2(z) = \frac{z}{16} J_1(2z) + \frac{3z}{16} [J_0(2z) - J_2(2z)].$$

$$35. \sum_{k=1}^{\infty} k^{2n+2} J_k^2(z) = \frac{z^2}{2} \sum_{k=0}^n \frac{(2k+1)!!}{(k+1)!} \left(\frac{z^2}{2} \right)^k \sum_{l=0}^k \frac{(-1)^l}{l! (2k-l+2)!} (k-l+1)^{2n+2}.$$

$$36. \sum_{k=0}^{\infty} (2k+1)^{2n+1} J_{k+1/2}^2(z) = \\ = \frac{2z}{\pi} \sum_{k=0}^n \frac{k! (2z^2)^k}{(2k+1)!!} \sum_{l=0}^k \frac{(-1)^l}{l! (2k-l+1)!} (2k-2l+1)^{2n+1}.$$

5.7.13. Ряды вида $\sum \frac{P(k)}{Q(k)} J_{\mu \pm mk}(z) J_{\nu \pm nk}(z)$.

$$1. \sum_{k=1}^{\infty} \frac{(-1)^k}{k} J_{\nu+k}(z) J_{\nu-k}(z) = -\frac{2}{\pi} \int_0^1 \frac{\ln 2t}{\sqrt{1-t^2}} J_{2\nu}(2zt) dt$$

[Re $\nu > -1/2$].

$$2. \sum_{k=0}^{\infty} \frac{1}{2k+1} J_k(z) J_{k+1}(z) = \frac{1}{2z} \sin^2 z.$$

$$3. \sum_{k=0}^{\infty} \frac{1}{2k+1} J_{k+1/2}^2(z) = \frac{2z}{\pi} {}_2F_3 \left(1, 1; \frac{3}{2}, \frac{3}{2}, \frac{3}{2}; -z^2 \right).$$

$$4. \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} J_{2k+\nu}(z) J_{2k-\nu+2}(z) = \\ = \frac{4(\nu-1)}{\pi} \sin \nu \pi \int_0^{\pi/4} s_{-1, 2\nu-2}(2z \sin t) dt - \frac{\pi}{4} J_{\nu}(z) J_{-\nu}(z).$$

$$5. \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} J_{2k+1}^2(z) = \int_0^{\pi/4} J_0(2z \sin t) dt - \frac{\pi}{4} J_0^2(z).$$

$$6. \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} J_{\mu+(2k+1)\nu}(z) J_{\mu-(2k+1)\nu}(z) = \frac{\pi}{4} J_{\mu}^2(z) \quad [\operatorname{Re} \mu > 1; -1 < \nu < 1]$$

$$7. \sum_{k=0}^{\infty} \frac{(\pm 1)^k}{2k+1} J_{\mu+kv}(z) J_{\mu+kv \pm 1/2}(z) =$$

$$= \frac{1}{2\sqrt{z}} \int_0^z \frac{\sqrt{z-t}}{t} \left\{ \frac{1 - \cos 2t}{\sin 2t} \right\} J_{\mu-1}(z-t) J_{\mu-1/2}(z-t) dt$$

$$\left[\operatorname{Re} \mu > 0, \nu = \begin{Bmatrix} 2 \\ 1 \end{Bmatrix} \right].$$

$$8. \sum_{k=1}^{\infty} \frac{1}{k^2} J_k^2(z) = \frac{2}{\pi} \int_0^{\pi/2} t^2 J_0(2z \cos t) dt - \frac{\pi^2}{12} J_0^2(z).$$

$$9. \sum_{k=1}^{\infty} \frac{1}{k^2 - a^2} J_k^2(z) = \frac{1}{2a^2} J_0^2(z) - \frac{\pi}{2a} \operatorname{cosec} a\pi J_a(z) J_{-a}(z).$$

$$10. \sum_{k=2}^{\infty} \frac{1}{k^2 - 1} J_k^2(z) = \frac{1}{2} J_0^2(z) - \frac{1}{2} J_0(z) J_1(z) + \frac{1}{4} J_1^2(z).$$

$$11. \sum_{k=1}^{\infty} \frac{1}{4k^2 - 1} J_k^2(z) = \frac{1}{2} J_0^2(z) - \frac{\sin 2z}{4z}.$$

$$12. \sum_{k=1}^{\infty} \frac{(-1)^k}{4k^2 - 1} J_k^2(z) = -\frac{1}{2z} \int_0^z J_0(2t) dt + \frac{1}{2} J_0^2(z).$$

$$13. \sum_{k=0}^{\infty} (-1)^k \frac{2k+1}{(2k+1)^2 - a^2} J_{k+\nu+1}(z) J_{\nu-k}(z) = \frac{\pi}{4} J_{\nu+(1+a)/2}(z) J_{\nu+(1-a)/2}(z).$$

$$14. \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k^2 - a^2} J_{\mu-k\nu}(z) J_{\mu+kv}(z) = \frac{\pi}{2a} \operatorname{cosec} a\pi J_{\mu-a\nu}(z) J_{\mu+a\nu}(z) - \frac{1}{2a^2} J_{\mu}^2(z)$$

$$[\operatorname{Re} \mu > -1/2; -1 < \nu < 1]$$

$$15. \sum_{k=0}^{\infty} (-1)^k \frac{2k+1}{(2k+1)^2 - a^2} J_{\mu-(2k+1)\nu}(z) J_{\mu+(2k+1)\nu}(z) = \frac{\pi}{4} J_{\mu-a\nu}(z) J_{\mu+a\nu}(z)$$

$$[\operatorname{Re} \mu > -1/2, -1/2 < \nu < 1/2]$$

$$16. \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)(2k+3)(2k+5)} J_{\mu-(2k+3)\nu}(z) J_{\mu+(2k+3)\nu}(z) =$$

$$= \frac{\pi}{16} J_{\mu}^2(z) + \frac{\pi}{16} J_{\mu-2\nu}(z) J_{\mu+2\nu}(z) - \frac{1}{3} J_{\mu-\nu}(z) J_{\mu+\nu}(z)$$

$$[\operatorname{Re} \mu > -1/2; -1/2 < \nu < 1/2]$$

5.7.14. Разные ряды вида $\sum a_k J_{\mu \pm mk}(z) J_{\nu \pm nk}(z)$.

$$1. \sum_{k=0}^{\infty} \frac{(\lambda)_k}{k!} J_{k+\mu}(z) J_{k+\nu}(z) = \frac{(z/2)^{\mu+\nu}}{\Gamma(\mu+1) \Gamma(\nu+1)} \times$$

$$\times {}_2F_3 \left(\frac{\mu+\nu-\lambda+1}{2}, \frac{\mu+\nu-\lambda+2}{2}; \mu+1, \nu+1, \mu+\nu-\lambda+1; -z^2 \right).$$

$$2. \sum_{k=0}^{\infty} (-1)^k \frac{(\mu)_k}{k!} J_{k+\nu}(z) J_{k+\mu-\nu}(z) =$$

$$= \frac{(z/2)^{\mu}}{\Gamma(\nu+1) \Gamma(\mu-\nu+1)} {}_1F_2 \left(\frac{\mu+1}{2}; \nu+1, \mu-\nu+1; -z^2 \right).$$

$$3. \sum_{k=0}^{\infty} (-1)^k \frac{(\lambda)_k}{k!} J_{\mu+k}(z) J_{\nu-k}(z) = \Gamma \left[\begin{matrix} \mu + \nu - \lambda + 1 \\ \nu + 1, \mu + \nu + 1, \mu - \lambda + 1 \end{matrix} \right] \times \\ \times \left(\frac{z}{2} \right)^{\mu+\nu} {}_2F_3 \left(\frac{\mu + \nu - \lambda + 1}{2}, \frac{\mu + \nu - \lambda + 2}{2}; \nu + 1, \mu + \nu + 1, \mu - \lambda + 1; -z^2 \right).$$

$$4. \sum_{k=0}^{\infty} \frac{(\nu - 1/2)_k}{k!} J_{k+\nu}(z) J_{k-1/2}(z) = \frac{2^{-\nu+1/2}}{\sqrt{\pi z}} J_{\nu}(2z).$$

$$5. \sum_{k=0}^{\infty} \frac{(-1/2)_k}{k!} J_{k+\nu}(z) J_{k+\nu-1/2}(z) = \sqrt{\frac{2}{\pi z}} J_{2\nu}(2z).$$

$$6. \sum_{k=0}^{\infty} (-1)^k \frac{(1/2 - \nu)_k}{(\nu + 3/2)_k} J_{\nu+k}(z) J_{\nu-k-1}(z) = \frac{\Gamma(\nu + 3/2)}{2\Gamma(\nu + 1)\sqrt{z}} J_{2\nu-1/2}(2z)$$

[Re $\nu > 0$].

$$7. \sum_{k=1}^{\infty} \frac{(1-\mu)_k}{(\mu)_k} J_{\nu+k}(z) J_{\nu-k}(z) = \\ = \frac{1}{2\sqrt{\pi}} \Gamma \left[\begin{matrix} \mu, \nu + 1/2 \\ 2\nu + 1, \mu + \nu \end{matrix} \right] z^{2\nu} {}_1F_2 \left(\nu + \frac{1}{2}; \mu + \nu, 2\nu + 1; -z^2 \right) - \frac{1}{2} J_{\nu}^2(z)$$

[Re $\mu > 1/2$].

$$8. \sum_{k=1}^{\infty} (-1)^k \frac{(1-\nu)_k}{(\nu)_k} J_k^2(z) = \frac{1}{2} {}_1F_2 \left(\frac{1}{2}; 1, \nu; -z^2 \right) - \frac{1}{2} J_0^2(z)$$

[Re $\nu > 1/2$].

$$9. \sum_{k=0}^{\infty} (-1)^k \frac{(2\nu - 1)_k}{k!} J_{k+\nu}(z) J_{k+\nu-1}(z) = \frac{2^{1-2\nu} z^{\nu-1}}{\Gamma(\nu)} J_{\nu}(2z).$$

$$10. \sum_{k=0}^{\infty} (-1)^k (k+\nu) \frac{(4\nu)_{2k}}{(2k)!} J_{k+\nu}^2(z) = \frac{1}{\nu \Gamma^2(\nu)} \left(\frac{z}{2} \right)^{2\nu} {}_1F_2 \left(\nu + \frac{1}{2}; \frac{1}{2}, \nu + 1; -z^2 \right)$$

[Re $\nu > 0$].

$$11. \sum_{k=0}^{\infty} (-1)^k (k+\nu) \frac{(4\nu - 1)_{2k}}{(2k+1)!} J_{k+\nu}^2(z) = \\ = \frac{1}{\nu \Gamma^2(\nu)} \left(\frac{z}{2} \right)^{2\nu} {}_1F_2 \left(\nu + \frac{1}{2}; \frac{3}{2}, \nu + 1; -z^2 \right)$$

[Re $\nu > 1/4$].

$$12. \sum_{k=0}^{\infty} (2k + \mu + \nu) \frac{(\mu + \nu)_k}{k!} J_{k+\mu}(z) J_{k+\nu}(z) = \frac{\mu + \nu}{\Gamma(\mu + 1)\Gamma(\nu + 1)} \left(\frac{z}{2} \right)^{\mu+\nu}.$$

$$13. \sum_{k=0}^{\infty} (-1)^k (k+\nu) \frac{(2\nu)_k}{k!} J_{k+\nu}(w) J_{k+\nu}(z) = \frac{1}{\Gamma(\nu)} \left(\frac{wz}{2} \right)^{\nu} (w+z)^{-\nu} J_{\nu}(w+z).$$

$$14. \sum_{k=0}^{\infty} (2k + 2\mu + 1) \frac{(2\mu + 1)_k}{k!} J_{k+\nu}(z) J_{k+\nu+1/2}(z) = \frac{2\mu + 1}{\sqrt{2\pi} \Gamma(2\nu + 2)} \times \\ \times z^{2\nu+1/2} {}_2F_3 \left(\nu - \mu - \frac{1}{4}, \nu - \mu + \frac{1}{4}; 2\nu - 2\mu + \frac{1}{2}, \nu + 1, \nu + \frac{3}{2}; -z^2 \right)$$

[Re $\mu > -5/4$; Re $(\nu - \mu) > 1/4$]

$$15. \sum_{k=0}^{\infty} (\pm 1)^k \frac{(2k+\mu+\nu)(\mu+\nu)_k}{k! [(2k+\mu+\nu)^2-1]} J_{k+\mu}(z) J_{k+\nu}(z) =$$

$$= \frac{\mu+\nu}{[(\mu+\nu)^2-1] \Gamma(\mu+1) \Gamma(\nu+1)} \left(\frac{z}{2}\right)^{\mu+\nu} \times$$

$$\times {}_2F_3\left(\left(\frac{\mu+\nu+1}{2}\right)^{(1\mp 1)/2}, \frac{\mu+\nu+2}{2}; \mu+1, \nu+1, \frac{\mu+\nu+3}{2}; -z^2\right).$$

$$16. \sum_{k=0}^{\infty} (-1)^k (2k+\nu) \frac{(\nu)_k}{k!} J_{2k+\nu}(w) J_{2k+\nu}(z) =$$

$$= \frac{1}{\Gamma(\nu)} \left(\frac{wz}{2}\right)^{\nu} (w^2+z^2)^{-\nu/2} J_{\nu}(\sqrt{w^2+z^2})$$

$$17. \sum_{k=0}^{\infty} \frac{(2k+\mu-\nu)(\mu-\nu)_k}{k! [(2k+\mu-\nu)^2-1]} J_{\mu+k}(z) J_{\nu-k}(z) =$$

$$= \frac{2\sqrt{\pi} (z/4)^{\mu+\nu}}{[\mu^2-(\nu+1)^2] \Gamma\left[\mu+1, \frac{\mu+\nu+1}{2}, \frac{\mu-\nu}{2}\right]} \times$$

$$\times {}_2F_3\left(\frac{\mu+\nu+1}{2}, \frac{\mu+\nu+2}{2}; \mu+1, \mu+\nu+1, \frac{\mu+\nu+3}{2}; -z^2\right).$$

$$18. \sum_{k=0}^{\infty} (-1)^k \frac{(2k+\mu-\nu)(\mu-\nu)_k}{k! [(2k+\mu-\nu)^2-1]} J_{\mu+k}(z) J_{\nu-k}(z) =$$

$$= \Gamma\left[\frac{(\mu-\nu-1)/2}{(\mu-\nu)/2}\right] 2^{\nu-\mu-1} \sqrt{\pi} J_{(\mu+\nu+1)/2}(z) J_{(\mu+\nu-1)/2}(z)$$

$$19. \sum_{k=0}^{\infty} \frac{(\mu+\nu)_k (\mu+\nu-\lambda+1)_k}{k! (\lambda)_k} J_{k+\mu}(z) J_{k+\nu}(z) = \frac{(z/2)^{\mu+\nu}}{\Gamma(\mu+1) \Gamma(\nu+1)} \times$$

$$\times {}_3F_4\left(\frac{\mu+\nu+1}{2}, \frac{\mu+\nu+2}{2}, \frac{2\lambda-\mu-\nu}{2}; \lambda, \mu+1, \nu+1, \frac{\mu+\nu+2}{2}; -z^2\right)$$

$$20. \sum_{k=0}^{\infty} (-1)^k \frac{(\mu-\nu)_k (\mu-\nu-\lambda)_k}{k! (\lambda)_k} J_{\mu+k}(z) J_{\nu-k}(z) =$$

$$= \Gamma\left[\frac{(\mu-\nu+2)/2, (3\nu-\mu+2\lambda)/2, \lambda}{\nu+1, \mu-\nu+1, (\mu+\nu+2)/2, \nu+\lambda, \nu-2\mu+2\lambda}\right] \left(\frac{z}{2}\right)^{\mu+\nu} \times$$

$$\times {}_3F_4\left(\frac{\mu+\nu+1}{2}, \frac{\mu+\nu+2}{2}, \frac{3\nu-\mu+2\lambda}{2}; \nu+1, \mu+\nu+1, \frac{\mu+\nu+2}{2}, \lambda+\nu; -z^2\right).$$

$$21. \sum_{k=0}^{\infty} (2k+\mu-\nu) \frac{(\mu-\nu)_k (\mu-\nu-\lambda+1)_k}{k! (\lambda)_k} J_{\mu+k}(z) J_{\nu-k}(z) =$$

$$= \Gamma\left[\frac{\lambda}{\nu+1, \mu-\nu, \lambda+\nu}\right] \left(\frac{z}{2}\right)^{\mu+\nu} {}_2F_3\left(\frac{\mu+\nu+1}{2}, \frac{\mu+\nu+2}{2};$$

$$\nu+1, \mu+\nu+1, \lambda+\nu; -z^2\right) \quad [\nu=0, 1, 2, \dots \text{ или } \operatorname{Re}(\mu+\nu-2\lambda) > 0]$$

$$22. \sum_{k=0}^{\infty} (-1)^k (2k+\mu+\nu) \frac{(\mu+\nu)_k (\mu+\nu-\lambda+1)_k}{k! (\lambda)_k} J_{k+\mu}(z) J_{k+\nu}(z) =$$

$$= \frac{\mu+\nu}{\Gamma(\mu+1) \Gamma(\nu+1)} \left(\frac{z}{2}\right)^{\mu+\nu} {}_2F_3\left(\frac{\mu+\nu+1}{2}, \frac{\mu+\nu+2}{2}; \mu+1, \nu+1, \lambda; -z^2\right).$$

5.7.15. Ряды вида $\sum a_k t^k J_{\mu \pm k}(w) J_{\nu \pm k}(z)$.

$$1. \sum_{k=-\infty}^{\infty} (-1)^k t^{2k} J_k^2(z) = J_0 \left[\left(t + \frac{1}{t} \right) z \right].$$

$$2. \sum_{k=1}^{\infty} t^k J_{k+\nu}(tz) J_{k+\nu+1}(z) = \frac{t}{t^2-1} [J_{\nu+1}(tz) J_{\nu}(z) - t J_{\nu}(tz) J_{\nu+1}(z)].$$

$$3. \sum_{k=-\infty}^{\infty} e^{ik\alpha} J_k(w) J_{k+\nu}(z) = \left(\frac{z - we^{-i\alpha}}{z - we^{i\alpha}} \right)^{\nu/2} J_{\nu}(\sqrt{w^2 + z^2 - 2wz \cos \alpha})$$

[$|we^{\pm i\alpha}| < |z|$ при $\nu \neq 0, \pm 1, \pm 2, \dots$].

$$4. \sum_{k=-\infty}^{\infty} (\pm 1)^k t^{2k} J_k(z) J_{\nu-k}(z) = t^{\nu} \begin{cases} J_{\nu}(z/t + tz) \\ J_{\nu}(z/t - tz) \end{cases}$$

[$|t| < 1$ при $\nu \neq \dots, -2, -1, 0, 1, 2, \dots$].

$$5. \sum_{k=0}^{\infty} (-1)^k \frac{\Gamma(k+1/2)}{k!} t^k J_{k+1}(tx) J_{\mu-k}(x) =$$

$$= (2x)^{1/2} \int_0^{\infty} (u^2+1)^{-(2\mu+2\nu+1)/4} J_{\nu}(tx\sqrt{u^2+1}) J_{\mu+1/2}(x\sqrt{u^2+1}) du$$

[$t, x > 0$; $\operatorname{Re}(\mu+\nu) > -3/2$]

5.7.16. Ряды, содержащие три и более функций $J_{\nu}(z)$.

$$1. \sum_{k=1}^{\infty} (-1)^k J_{\nu+k}(w) J_{\nu-k}(w) J_{2k}(z) =$$

$$= \frac{1}{\pi} \int_0^{\pi/2} J_{2\nu}(2w \sin t) \cos(z \sin t) dt - \frac{1}{2} J_{\nu}^2(w) J_0(z)$$

[$\operatorname{Re} \nu > -1/2$].

$$2. \sum_{k=1}^{\infty} (-1)^k J_{\nu+k}(w) J_{\nu-k-1}(w) J_{2k+1}(z) = \frac{1}{\pi} \int_0^{\pi/2} J_{2\nu-1}(2w \sin t) \sin(z \sin t) dt$$

[$\operatorname{Re} \nu > -1/2$].

$$3. \sum_{k=0}^{\infty} (2k+\mu+\nu) J_{k+\mu}(w) J_{k+\nu}(w) J_{2k+\mu+\nu}(z) =$$

$$= \frac{1}{\sqrt{\pi}} G_{11}^{\frac{1}{2}} \left(\frac{w^2 z^2}{4} \middle| \begin{matrix} 1-(\nu-\mu)/2, 1-(\mu-\nu)/2 \\ 1, 1/2 \end{matrix} \right).$$

$$4. \sum_{k=0}^{\infty} (2k+\nu) J_{2k+\nu}(w) J_{2k+\nu}^2(z) = \frac{1}{4} e^{-z^2/(2w)}$$

[$\operatorname{Re} z^2, \operatorname{Re} \nu > 0$].

$$5. \sum_{k=0}^{\infty} (2k+\mu+\nu) J_{k+\mu}(w) J_{k+\nu}(w) J_{k+\mu}(z) J_{k+\nu}(z) =$$

$$= \frac{1}{\pi} G_{11}^{\frac{1}{2}} \left(\frac{w^2}{z^2} \middle| \begin{matrix} 0, 1-(\mu-\nu)/2, 1-(\nu-\mu)/2 \\ (\mu-\nu)/2, (\nu-\mu)/2, 1 \end{matrix} \right)$$

5.7.17. Ряды вида $\sum a_k \begin{Bmatrix} \sin(ka+b) \\ \cos(ka+b) \end{Bmatrix} J_{mk+\mu}(w) J_{nk+\nu}(z)$.

$$1. \sum_{k=0}^{\infty} (\pm 1)^k \cos ka J_k(w) J_k(z) = \\ = \frac{1}{2} J_0(w) J_0(z) + \frac{1}{2} J_0(\sqrt{w^2 + z^2 \mp 2wz \cos a}).$$

$$2. \sum_{k=-\infty}^{\infty} (\pm 1)^k \sin ka J_k(w) J_{k+\nu}(z) = \\ = \pm \sin \left(\nu \arcsin \frac{w \sin a}{\sqrt{w^2 + z^2 \mp 2wz \cos a}} \right) J_{\nu}(\sqrt{w^2 + z^2 \mp 2wz \cos a}) \\ [|we^{ia}|, |we^{-ia}| < |z|; \text{условие опускается при } \nu = 0, 1, 2, \dots].$$

$$3. \sum_{k=-\infty}^{\infty} (\pm 1)^k \cos ka J_k(w) J_{k+\nu}(z) = \\ = \cos \left(\nu \arcsin \frac{w \sin a}{\sqrt{w^2 + z^2 \mp 2wz \cos a}} \right) J_{\nu}(\sqrt{w^2 + z^2 \mp 2wz \cos a}) \\ [|we^{ia}|, |we^{-ia}| < |z|; \text{условие опускается при } \nu = 0, 1, 2, \dots].$$

$$4. \sum_{k=1}^{\infty} \cos ka J_k^2(z) = \frac{1}{2} J_0 \left(2z \sin \frac{a}{2} \right) - \frac{1}{2} J_0^2(z).$$

$$5. \sum_{k=0}^{\infty} (\pm 1)^k \begin{Bmatrix} \sin(2k+1)a \\ \cos(2k+1)a \end{Bmatrix} J_k(z) J_{k+1}(z) = \frac{1}{2} J_1 \left(2z \begin{Bmatrix} \sin a \\ \cos a \end{Bmatrix} \right).$$

$$6. \sum_{k=0}^{\infty} (\pm 1)^k \begin{Bmatrix} \sin(2k+1)a \\ \cos(2k+1)a \end{Bmatrix} J_{k+1/2}^2(z) = \mp \frac{1}{2} H_0 \left(2z \begin{Bmatrix} \sin a \\ \cos a \end{Bmatrix} \right).$$

$$7. \sum_{k=1}^{\infty} (-1)^k \cos ka J_{k+\nu}(z) J_{k-\nu}(z) = \\ = \frac{\operatorname{cosec} \nu \pi}{4} \left[E_{2\nu} \left(2z \cos \frac{a}{2} \right) - E_{-2\nu} \left(2z \cos \frac{a}{2} \right) \right] - \frac{1}{2} J_{\nu}(z) J_{-\nu}(z).$$

$$8. \sum_{k=1}^{\infty} \cos ka J_{nk}(w) J_{nk}(z) = \\ = \frac{1}{2n} \sum_{k=k_{\pm}}^{k_{+}} \alpha_k J_0 \left(\sqrt{w^2 + z^2 - 2wz \cos \frac{2k\pi + a}{n}} \right) - \frac{1}{2} J_0(w) J_0(z) \\ [n \geq 1; k_{\pm} = \pm [(n\pi \mp a)/(2\pi)]; \alpha_k = 1 \text{ при } k \neq k_{\pm}, \alpha_{k_{\pm}} = 1/2 \text{ при } \\ (n\pi \mp a)/(2\pi) = \dots, -2, -1, 0, 1, 2, \dots, \alpha_{k_{\pm}} = 1 \text{ при } (n\pi \mp a)/(2\pi) \neq \\ \neq \dots, -2, -1, 0, 1, 2, \dots].$$

$$9. \sum_{k=1}^{\infty} (\pm 1)^k \cos 2ka J_{k-1/4}(z) J_{k+1/4}(z) = \\ = \frac{1}{\sqrt{2\pi z}} \left\{ \sqrt{\operatorname{cosec} a} [\cos(2z \sin a) C(2z \sin a) + \sin(2z \sin a) S(2z \sin a)] \right\} - \\ - \frac{1}{\sqrt{2\pi z}} \left\{ \sqrt{\sec a} [\cos(2z \cos a) C(2z \cos a) + \sin(2z \cos a) S(2z \cos a)] \right\} - \\ - \frac{1}{2} J_{1/4}(z) J_{-1/4}(z).$$

$$10. \sum_{k=0}^{\infty} (\pm 1)^k \left\{ \frac{\sin (2k+1)a}{\cos (2k+1)a} \right\} J_{k+1/4}(z) J_{k+3/4}(z) = \\ = \frac{1}{\sqrt{2\pi z}} \left\{ \sqrt{\operatorname{cosec} a} [\sin (2z \sin a) C(2z \sin a) - \cos (2z \sin a) S(2z \sin a)] \right\} \\ \left\{ \sqrt{\sec a} [\sin (2z \cos a) C(2z \cos a) - \cos (2z \cos a) S(2z \cos a)] \right\}.$$

$$11. \sum_{k=0}^{\infty} \cos (2k+1)a J_{k+1/2}(w) J_{k+1/2}(z) = \frac{1}{\pi} \int_{t_1}^{t_2} \frac{\sin t dt}{\sqrt{t^2 - w^2 - z^2 + 2wz \cos 2a}} \\ [t_1 = \sqrt{w^2 + z^2 - 2wz \cos 2a}, t_2 = w + z].$$

$$12. \sum_{k=1}^{\infty} (\pm 1)^k \cos (n+k)a J_k(z) J_{2n+k}(z) = \frac{(\mp 1)^n}{2} J_{2n} \left(2z \left\{ \frac{\sin (a/2)}{\cos (a/2)} \right\} \right) - \\ - (\mp 1)^n \sum_{k=1}^n (\mp 1)^k \cos ka J_{n+k}(z) J_{n-k}(z) - \frac{(\mp 1)^n}{2} J_n^2(z).$$

$$13. \sum_{k=0}^{\infty} (\pm 1)^k \left\{ \frac{\sin (2n+2k+1)a}{\cos (2n+2k+1)a} \right\} J_k(z) J_{2n+k+1}(z) = \\ = \frac{(\mp 1)^n}{2} J_{2n+1} \left(2z \left\{ \frac{\sin a}{\cos a} \right\} \right) - (\mp 1)^n \sum_{k=0}^{n-1} (\mp 1)^k \left\{ \frac{\sin (2k+1)a}{\cos (2k+1)a} \right\} J_{n-k}(z) J_{n+k+1}(z).$$

$$14. \sum_{k=0}^{\infty} (-1)^k \cos (2k+1)a J_{k+\nu+1/2}(z) J_{k-\nu+1/2}(z) = \\ = -\frac{\sec \nu\pi}{4} [E_{2\nu}(2z \cos a) + E_{-2\nu}(2z \cos a)].$$

5.7.18. Ряды вида $\sum a_k \left\{ \frac{\sin (ka+b)}{\cos (ka+b)} \right\} J_{\mu+k\nu}(z) J_{\mu-k\nu}(z)$.

$$1. \sum_{k=1}^{\infty} (\pm 1)^k \cos ka J_{\nu+k}(z) J_{\nu-k}(z) = \frac{1}{2} J_{2\nu} \left(2z \left\{ \frac{\cos (a/2)}{|\sin (a/2)|} \right\} \right) - \frac{1}{2} J_{\nu}^2(z) \\ [\operatorname{Re} \nu > -1/2].$$

$$2. \sum_{k=0}^{\infty} (\mp 1)^k \left\{ \frac{\sin (2k+1)a}{\cos (2k+1)a} \right\} J_{\nu+k+1/2}(z) J_{\nu-k-1/2}(z) = \frac{1}{2} J_{2\nu} \left(2z \left\{ \frac{|\sin a|}{\cos a} \right\} \right) \\ [\operatorname{Re} \nu > -1/2].$$

$$3. \sum_{k=1}^{\infty} \cos ka J_{\mu+k\nu}(z) J_{\mu-k\nu}(z) = \frac{1}{2\nu} \sum_{k=k_-}^{k_+} \alpha_k J_{2\mu} \left(2z \cos \frac{2k\pi + a}{2\nu} \right) - \frac{1}{2} J_{\mu}^2(z) \\ [\operatorname{Re} \mu > -1/2; \operatorname{Re} \nu > 0 \text{ при } a = 0, \pm \nu\pi, k_{\pm} = \pm [(\nu\pi \mp a)/(2\pi)], \\ \alpha_k = 1 \text{ при } k \neq k_{\pm}, \alpha_{k_{\pm}} = 1/2 \text{ при } (\nu\pi \mp a)/(2\pi) = \dots -2, -1, 0, \\ 1, 2, \dots, \alpha_{k_{\pm}} = 1 \text{ при } (\nu\pi \mp a)/(2\pi) \neq \dots -2, -1, 0, 1, 2, \dots].$$

$$4. \sum_{k=1}^{\infty} \frac{(\pm 1)^k}{k} \sin k\nu\pi J_{\mu+k\nu}(z) J_{\mu-k\nu}(z) = \frac{\pi}{2} \left(\left\{ \frac{1}{0} \right\} \mp \nu \right) J_{\mu}^2(z) \\ [\operatorname{Re} \mu > -(3 \mp 1)/4; 0 < \nu < (3 \pm 1)/4].$$

$$5. \sum_{k=0}^{\infty} \frac{1}{2k+1} \sin (2k+1)\nu\pi J_{\mu-(2k+1)\nu}(z) J_{\mu+(2k+1)\nu}(z) = \frac{\pi}{4} J_{\mu}^2(z) \\ [\operatorname{Re} \mu > -1; 0 < \nu < 1/2].$$

$$6. \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} \cos (2k+1)v\pi J_{\mu-(2k+1)v}(z) J_{\mu+(2k+1)v}(z) = \frac{\pi}{4} J_{\mu}^2(z) \\ [\operatorname{Re} \mu > -1/2; -1/4 < v < 1/4].$$

$$7. \sum_{k=1}^{\infty} \frac{(\pm 1)^k}{k^2} \cos kv\pi J_{\mu-kv}(z) J_{\mu+kv}(z) = \\ = \frac{\pi^2}{12} \left\{ \frac{3v^2-3v+2}{3v^2-1} \right\} J_{\mu}^2(z) - \frac{v^2}{2} J_{\mu}(z) \frac{\partial^2 J_{\mu}(z)}{\partial \mu^2} + \frac{v^2\pi}{2} \left[\frac{\partial J_{\mu}(z)}{\partial \mu} \right]^2 \\ [\operatorname{Re} \mu > -1/2; -(1 \mp 1)/4 < v < (3 \pm 1)/4].$$

$$8. \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^2} \sin (2k+1)v\pi J_{\mu+(2k+1)v}(z) J_{\mu-(2k+1)v}(z) = \frac{\pi^2 v}{4} J_{\mu}^2(z) \\ [\operatorname{Re} \mu > -1/2; -1/4 < v < 1/4].$$

$$9. \sum_{k=1}^{\infty} \frac{(-1)^k}{k(k+1)} \sin (2k+1)v\pi J_{\mu-(2k+1)v}(z) J_{\mu+(2k+1)v}(z) = \\ = \sin v\pi J_{\mu-v}(z) J_{\mu+v}(z) - 2\pi \cos v\pi J_{\mu-v}(z) J_{\mu+v}(z) - \\ - 2 \sin v\pi \frac{\partial}{\partial v} [J_{\mu-v}(z) J_{\mu+v}(z)] \\ [\operatorname{Re} \mu > -1/2; -1/4 < v < 1/4].$$

$$10. \sum_{k=1}^{\infty} \frac{1}{k(k+1)} \cos (2k+1)v\pi J_{\mu-(2k+1)v}(z) J_{\mu+(2k+1)v}(z) = \\ = \cos v\pi J_{\mu-v}(z) J_{\mu+v}(z) - \pi \sin v\pi J_{\mu-v}(z) J_{\mu+v}(z) + \\ + 2\pi \sin v\pi J_{\mu-v}(z) J_{\mu+v}(z) - 2 \cos v\pi \frac{\partial}{\partial v} [J_{\mu-v}(z) J_{\mu+v}(z)] \\ [\operatorname{Re} \mu > -1/2; 0 < v < 1/2].$$

$$11. \sum_{k=0}^{\infty} \frac{(\pm 1)^{k+1}}{(2k-1)(2k+3)} \left\{ \begin{array}{l} \cos (2k+1)v\pi \\ \sin (2k+1)v\pi \end{array} \right\} J_{\mu-(2k+1)v}(z) J_{\mu+(2k+1)v}(z) = \\ = \mp \frac{\pi}{8} \left\{ \begin{array}{l} \cos 2v\pi \\ \sin 2v\pi \end{array} \right\} J_{\mu-2v}(z) J_{\mu+2v}(z) \\ [\operatorname{Re} \mu > -(3 \mp 1)/4; -(1 \mp 1)/8 < v < (3 \pm 1)/8].$$

$$12. \sum_{k=0}^{\infty} (\pm 1)^{k+1} \frac{2k+1}{(2k-1)(2k+3)} \left\{ \begin{array}{l} \sin (2k+1)v\pi \\ \cos (2k+1)v\pi \end{array} \right\} J_{\mu-(2k+1)v}(z) J_{\mu+(2k+1)v}(z) = \\ = \frac{\pi}{4} \cos 2v\pi J_{\mu-2v}(z) J_{\mu+2v}(z) \\ [\operatorname{Re} \mu > -1/2; -(1 \mp 1)/8 < v < (3 \pm 1)/8].$$

$$13. \sum_{k=1}^{\infty} \frac{(\pm 1)^k}{k^2 - a^2} \cos kv\pi J_{\mu+kv}(z) J_{\mu-kv}(z) = \\ = \frac{1}{2a^2} J_{\mu}^2(z) - \frac{\pi}{2a} \operatorname{cosec} a\pi \cos \left[\left(\frac{1 \pm 1}{2} - v \right) a\pi \right] J_{\mu+av}(z) J_{\mu-av}(z) \\ [\operatorname{Re} \mu > -(3 \mp 1)/4; 0 < v < (3 \pm 1)/4].$$

$$14. \sum_{\substack{k=1 \\ k \neq n}}^{\infty} \frac{1}{k^2 - n^2} \cos kv\pi J_{\mu-kv}(z) J_{\mu+kv}(z) = \\ = \frac{1}{2n^2} J_{\mu}^2(z) + \frac{1}{4n^2} \cos nv\pi J_{\mu-nv}(z) J_{\mu+nv}(z) - \\ - \frac{v\pi}{2n} \sin nv\pi J_{\mu-nv}(z) J_{\mu+nv}(z) - \frac{v\pi}{2n^2} \cos nv\pi \frac{\partial}{\partial v} [J_{\mu-nv}(z) J_{\mu+nv}(z)] \\ [\operatorname{Re} \mu > 1/2; 0 < v < 1].$$

$$15. \sum_{k=2}^{\infty} \frac{(-1)^k}{k^2-1} \cos k\nu\pi J_{\mu-k\nu}(z) J_{\mu+k\nu}(z) = \frac{1}{2} J_{\mu}^2(z) - \frac{1}{4} \cos \nu\pi J_{\mu-\nu}(z) J_{\mu+\nu}(z) - \\ - \frac{\nu}{2} \left[\pi \sin \nu\pi J_{\mu-\nu}(z) J_{\mu+\nu}(z) - \cos \nu\pi \frac{\partial}{\partial \nu} (J_{\mu-\nu}(z) J_{\mu+\nu}(z)) \right] \\ [\operatorname{Re} \mu > -1/2; -1/2 < \nu < 1/2].$$

$$16. \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^2-a^2} \sin (2k+1)\nu\pi J_{\mu-(2k+1)\nu}(z) J_{\mu+(2k+1)\nu}(z) = \\ = \frac{\pi}{4a} \sec \frac{a\pi}{2} \sin a\nu\pi J_{\mu-a\nu}(z) J_{\mu+a\nu}(z) \quad [\operatorname{Re} \mu > -1; -1/4 < \nu < 1/4].$$

$$17. \sum_{k=1}^{\infty} \frac{(\pm 1)^k k}{k^2-a^2} \sin k\nu\pi J_{\mu+k\nu}(z) J_{\mu-k\nu}(z) = \\ = \frac{\pi}{2} \operatorname{cosec} a\pi \sin \left[\left(\frac{1 \pm 1}{2} - \nu \right) a\pi \right] J_{\mu+a\nu}(z) J_{\mu-a\nu}(z) \\ [\operatorname{Re} \mu > -(3 \mp 1)/4; 0 < \nu < (3 \pm 1)/4].$$

$$18. \sum_{\substack{k=1 \\ k \neq n}}^{\infty} \frac{k}{k^2-n^2} \sin k\nu\pi J_{\mu-k\nu}(z) J_{\mu+k\nu}(z) = \\ = \frac{(\nu+1)\pi}{2} \cos n\nu\pi J_{\mu-n\nu}(z) J_{\mu+n\nu}(z) - \frac{\nu}{2} \sin n\nu\pi \frac{\partial}{\partial \nu} [J_{\mu-n\nu}(z) J_{\mu+n\nu}(z)] - \\ - \frac{1}{4n} \sin n\nu\pi J_{\mu-n\nu}(z) J_{\mu+n\nu}(z) \\ [\operatorname{Re} \mu > -1/2; 0 < \nu < 1; n = 1, 2, 3, \dots].$$

$$19. \sum_{k=2}^{\infty} \frac{(-1)^k k}{k^2-1} \sin k\nu\pi J_{\mu-k\nu}(z) J_{\mu+k\nu}(z) = \\ = \frac{1}{4} \sin \nu\pi J_{\mu-\nu}(z) J_{\mu+\nu}(z) + \frac{\nu\pi}{2} \cos \nu\pi J_{\mu-\nu}(z) J_{\mu+\nu}(z) + \\ + \frac{\nu}{2} \sin \nu\pi \frac{\partial}{\partial \nu} [J_{\mu-\nu}(z) J_{\mu+\nu}(z)] \quad [\operatorname{Re} \mu > -1/2; -1/2 < \nu < 1/2].$$

$$20. \sum_{k=0}^{\infty} (-1)^k \frac{2k+1}{(2k+1)^2-a^2} \cos (2k+1)\nu\pi J_{\mu-(2k+1)\nu}(z) J_{\mu+(2k+1)\nu}(z) = \\ = \frac{\pi}{4} \sec \frac{a\pi}{2} \cos a\nu\pi J_{\mu-a\nu}(z) J_{\mu+a\nu}(z) \quad [\operatorname{Re} \mu > -1/2; -1/4 < \nu < 1/4].$$

$$21. \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)(2k+3)(2k+5)} \cos (2k+3)\nu\pi J_{\mu+(2k+3)\nu}(z) J_{\mu-(2k+3)\nu}(z) = \\ = \frac{\pi}{16} J_{\mu}^2(z) + \frac{\pi}{16} \cos 2\nu\pi J_{\mu+2\nu}(z) J_{\mu-2\nu}(z) - \frac{1}{3} \cos \nu\pi J_{\mu+\nu}(z) J_{\mu-\nu}(z) \\ [\operatorname{Re} \mu > -1/2; -1/4 < \nu < 1/4].$$

5.7.19. Ряды вида $\sum a_k J_{\nu}(kx)$.

$$1. \sum_{k=1}^{\infty} J_{2n}(kx) = \frac{1}{x} - \frac{1}{2} \delta_{n,0} + 2 \sum_{k=1}^m (x^2 - 4k^2\pi^2)^{-1/2} \cos \left(2n \arcsin \frac{2k\pi}{x} \right) \\ [m - \text{целое число, для которого } 2m\pi < x < 2(m+1)\pi].$$

$$2. \sum_{k=0}^{\infty} (-1)^k J_0(kx) = \frac{1}{2} \quad [0 < x < \pi].$$

$$3. \sum_{k=1}^{\infty} \frac{1}{k} J_1(kx) = 1 - \frac{x}{4} + \frac{2}{x} \sum_{k=1}^m (x^2 - 4k^2\pi^2)^{1/2}$$

[m — целое число, для которого $2m\pi < x < 2(m+1)\pi$].

$$4. \sum_{k=1}^{\infty} \frac{(\pm 1)^k}{k} J_{2n+1}(kx) = \frac{1}{2n+1} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \quad [n=1, 2, 3, \dots; 0 < x < (3 \pm 1)\pi/2]$$

$$5. \sum_{k=1}^{\infty} \frac{(\pm 1)^k}{k^2} J_{2n}(kx) = \frac{x^2}{8} \delta_{0,n} - \frac{x^2}{16} \delta_{1,n} + \frac{x}{4n^2-1} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \pm \frac{\pi^2}{12} \begin{Bmatrix} 2 \\ 1 \end{Bmatrix} \delta_{0,n}$$

[$-(1 \mp 1)\pi/2 < x < (3 \pm 1)\pi/2$]

$$6. \sum_{k=1}^{\infty} \frac{1}{k^3} J_{2n+1}(kx) = \frac{x^3}{32} \left(\delta_{0,n} - \frac{1}{3} \delta_{1,n} \right) + \frac{x^2}{(2n+3)(4n^2-1)} + \frac{\pi^2 x}{12} \delta_{0,n}$$

[$0 \leq x \leq 2\pi$]

$$7. \sum_{k=1}^{\infty} \frac{(-1)^k}{k^3} J_{2n+1}(kx) = \frac{x^3}{32} \left(\delta_{0,n} - \frac{1}{3} \delta_{1,n} \right) - \frac{\pi^2 x}{24} \delta_{0,n} \quad [-\pi \leq x \leq \pi]$$

$$8. \sum_{k=1}^{\infty} \frac{1}{k^m} J_{2n+m}(kx) = \frac{x^{m-1}}{(2n+1)(2n+3)\dots(2n+2m-1)}$$

[$m, n=1, 2, 3, \dots, 0 < x < 2\pi$]

$$9. \sum_{k=1}^{\infty} k^{2n-\nu} J_{\nu}(kx) = \frac{1}{2n+1} \left(\frac{x}{2} \right)^{\nu-2n-1} \Gamma \left[\frac{n+3/2}{\nu-n+1/2} \right]$$

[$n=1, 2, 3, \dots; \operatorname{Re} \nu > 2n-1/2; 0 \leq x < 2\pi$]

$$10. \sum_{k=1}^{\infty} (-1)^k k^{2n-\nu} J_{\nu}(kx) = 0$$

[$n=1, 2, 3, \dots; -1/2 < \operatorname{Re} \nu - 2n \leq 1/2$ при $0 \leq x < \pi$, $\operatorname{Re} \nu - 2n > 1/2$ при $0 \leq x \leq \pi$].

$$11. \sum_{k=1}^{\infty} \frac{(\pm 1)^k}{k^{\nu}} J_{\nu}(kx) = -\frac{1}{\Gamma(\nu+1)} 2^{-\nu-1} x^{\nu} + \frac{\sqrt{\pi}}{\Gamma(\nu+1/2)} 2^{-\nu+1} x^{-\nu} \times$$

$$\times \sum_{k=1}^m \left[x^2 - \left(2k - \frac{1 \mp 1}{2} \right)^2 \pi^2 \right]^{\nu-1/2} + \frac{\sqrt{\pi}}{\Gamma(\nu+1/2)} 2^{-\nu} x^{\nu-1} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$$

[$\operatorname{Re} \nu > -1/2; m$ — целое число, для которого $(2m - (1 \mp 1)/2)\pi < x < (2m+1 + (1 \pm 1)/2)\pi$].

$$12. \sum_{k=1}^{\infty} \frac{(\pm 1)^k}{k^{\nu+2}} J_{\nu}(kx) = \frac{2^{-\nu-3} x^{\nu+2}}{\Gamma(\nu+2)} \pm \frac{2^{-\nu-2} \pi^2 x^{\nu}}{3\Gamma(\nu+1)} \begin{Bmatrix} 2 \\ 1 \end{Bmatrix} - \frac{2^{-\nu-1} \sqrt{\pi} x^{\nu+1}}{\Gamma(\nu+3/2)} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$$

[$\operatorname{Re} \nu > -5/2, 0 < x < (3 \pm 1)\pi/2$]

$$13. \sum_{k=1}^{\infty} \frac{1}{k^{2n+\nu}} J_{\nu}(kx) = \frac{(-1)^{n+1} x^{2n+\nu}}{(2n)! 2^{\nu+1} \sqrt{\pi}} \sum_{k=0}^{2n} \Gamma \left[\frac{n-(k-1)/2}{n+\nu+1-k/2} \right] \binom{2n}{k} \left(\frac{2\pi}{x} \right)^k B_k$$

[$n=1, 2, 3, \dots; \operatorname{Re} \nu > -2n-1/2; 0 < x < 2\pi$]

$$14. \sum_{k=1}^{\infty} \frac{(-1)^k}{k^{2n+\nu}} J_{\nu}(kx) =$$

$$= \frac{(-1)^{n+1} x^{2n+\nu}}{(2n)! 2^{\nu+1} \sqrt{\pi}} \sum_{k=0}^{2n} \binom{2n}{k} B_k \sum_{l=0}^{2n-k} \binom{2n-k}{l} \frac{2^k \pi^{k+l}}{x^{k+l}} \Gamma \left[\frac{n-(k+l-1)/2}{\nu+n+1-(k+l)/2} \right]$$

[$\operatorname{Re} \nu > -2n-1/2; -\pi < x < \pi$]

$$15. \sum_{k=1}^{\infty} \frac{1}{k^2 \pm a^2} J_{2n}(kx) = \mp \frac{1}{2a^2} \delta_{0,n} - \frac{(-1)^n \pi}{2a} \left\{ \begin{array}{l} \operatorname{cth} a\pi J_{2n}(ax) - L_{2n}(ax) \\ \operatorname{ctg} a\pi J_{2n}(ax) - E_{2n}(ax) \end{array} \right\} \\ [0 < x < \pi].$$

$$16. \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k^2 \pm a^2} J_{2n}(kx) = \frac{\pi}{2a} \left\{ \begin{array}{l} (-1)^{n+1} \operatorname{cosech} a\pi J_{2n}(ax) \\ \operatorname{cosec} a\pi J_{2n}(ax) \end{array} \right\} \pm \frac{1}{2a^2} \delta_{0,n} \\ [-\pi \leq x \leq \pi].$$

$$17. \sum_{\substack{k=1 \\ k \neq m}}^{\infty} \frac{1}{k^2 - m^2} J_{2n}(kx) = \frac{1}{4m^2} J_{2n}(mx) + \frac{\pi}{2m} E_{2n}(mx) + \\ + \frac{x}{4m} [J_{2n+1}(mx) - J_{2n-1}(mx)] + \frac{1}{2m^2} \delta_{n,0} \\ [0 < x < 2\pi].$$

$$18. \sum_{k=2}^{\infty} \frac{(-1)^k}{k^2 - 1} J_{2n}(kx) = \frac{1}{2} \delta_{0,n} - \frac{x}{4} [J_{2n+1}(x) - J_{2n-1}(x)] - \frac{1}{4} J_{2n}(x) \\ [-\pi < x < \pi].$$

$$19. \sum_{k=1}^{\infty} \frac{(-1)^{k+1} k}{k^2 - a^2} J_{2n+1}(kx) = \frac{\pi}{2} \operatorname{cosec} a\pi J_{2n+1}(ax) \\ [-\pi < x < \pi].$$

$$20. \sum_{\substack{k=1 \\ k \neq m}}^{\infty} \frac{k}{k^2 - m^2} J_{2n+1}(kx) = \\ = \frac{\pi}{2} E_{2n+1}(mx) - \frac{1}{4m} J_{2n+1}(mx) + \frac{x}{4} [J_{2n+2}(mx) - J_{2n}(mx)] \\ [m \geq 1; 0 < x < 2\pi]$$

$$21. \sum_{k=2}^{\infty} \frac{(-1)^k k}{k^2 - 1} J_{2n+1}(kx) = \frac{1}{4} J_{2n+1}(x) + \frac{x}{4} [J_{2n}(x) - J_{2n+2}(x)] \\ [-\pi < x < \pi].$$

$$22. \sum_{k=1}^{\infty} \frac{k}{k^2 - a^2} J_{2n+1}(kx) = \frac{\pi}{2} [E_{2n+1}(ax) - \operatorname{ctg} a\pi J_{2n+1}(ax)] \\ [0 < x < 2\pi].$$

$$23. \sum_{\substack{k=1 \\ k \neq m}}^{\infty} \frac{k}{k^2 - m^2} J_{2n+1}(kx) = \frac{\pi}{2} E_{2n+1}(mx) - \\ - \frac{x}{4} [J_{2n}(mx) - J_{2n+2}(mx)] - \frac{1}{4m} J_{2n+1}(mx) \\ [0 < x < 2\pi].$$

$$24. \sum_{k=1}^{\infty} (\pm 1)^k \frac{k^{-\nu}}{k^2 - a^2} J_{\nu}(kx) = \\ = \frac{2^{-\nu-1} x^{\nu}}{a^2 \Gamma(\nu+1)} - \frac{\pi}{2} a^{-\nu-1} \left[J_{\nu}(ax) \left\{ \begin{array}{l} \operatorname{ctg} a\pi \\ \operatorname{cosec} a\pi \end{array} \right\} + H_{\nu}(ax) \left\{ \begin{array}{l} 1 \\ 0 \end{array} \right\} \right] \\ \left[\begin{array}{l} 0 < x < 2\pi \\ 0 < x \leq \pi; \operatorname{Re} \nu > -5/2 \end{array} \right].$$

$$25. \sum_{k=1}^{\infty} \frac{k^{2-\nu}}{k^2 - a^2} J_{\nu}(kx) = \frac{\sqrt{\pi}}{\Gamma(\nu+1/2)} 2^{-\nu} x^{\nu-1} - \frac{\pi}{2} a^{1-\nu} [H_{\nu}(ax) + \operatorname{ctg} a\pi J_{\nu}(ax)] \\ [\operatorname{Re} \nu \geq 1/2; 0 < x < \pi].$$

$$26. \sum_{k=1}^{\infty} (-1)^{k+1} \frac{k^{2n-\nu}}{k^2 - a^2} J_{\nu}(kx) = \frac{\pi}{2} a^{2n-\nu-1} \operatorname{cosec} a\pi J_{\nu}(ax)$$

$$[\operatorname{Re} \nu > 2n - 5/2; n = 1, 2, 3, \dots; 0 < x < \pi]$$

5.7.20. Ряды вида $\sum a_k J_{\nu}((2k+1)x)$.

$$1. \sum_{k=0}^{\infty} \frac{1}{2k+1} J_{2n+1}((2k+1)x) = \frac{1}{2(2n+1)} \quad [n = 1, 2, 3, \dots; 0 < x < \pi]$$

$$2. \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} J_{2n}((2k+1)x) = \frac{\pi}{4} \delta_{0,n} \quad [-\pi/2 < x < \pi/2;$$

$$3. \sum_{k=0}^{\infty} \frac{1}{(2k+1)^2} J_{2n}((2k+1)x) = \frac{\pi^2}{8} \delta_{0,n} - \frac{|x|}{2} \quad [-\pi < x < \pi]$$

$$= \frac{\pi^2}{8} - \frac{x}{2} + \sqrt{x^2 - \pi^2} - \pi \arccos \frac{\pi}{x} \quad [\pi \leq x < 2\pi]$$

$$4. \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^2} J_{2n+1}((2k+1)x) = \frac{\pi x}{8} \delta_{0,n} \quad [-\pi/2 \leq x \leq \pi/2]$$

$$5. \sum_{k=0}^{\infty} \frac{1}{(2k+1)^3} J_{2n+1}((2k+1)x) = \frac{\pi^2 x}{16} \delta_{0,n} + \frac{x|x|}{4(2n+3)(4n^2-1)} \quad [-\pi \leq x \leq \pi]$$

$$6. \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^3} J_{2n}((2k+1)x) = \frac{\pi^2}{32} \delta_{0,n} - \frac{\pi x^2}{16} \delta_{0,n} - \frac{\pi x^2}{32} \delta_{1,n} \quad [-\pi/2 \leq x \leq \pi/2]$$

$$7. \sum_{k=0}^{\infty} \frac{1}{(2k-1)(2k+3)} J_{2n}((2k+1)x) = \frac{\pi}{8} E_{2n}(2x) \quad [0 < x < \pi]$$

$$8. \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k-1)(2k+3)} J_{2n+1}((2k+1)x) = -\frac{\pi}{8} J_{2n+1}(2x) \quad [-\pi/2 < x < \pi/2]$$

$$9. \sum_{k=0}^{\infty} \frac{2k+1}{(a-k)(a+k+1)} J_{2n+1}((2k+1)x) = \\ = \pi \operatorname{cosec} a\pi [\cos a\pi J_{2n+1}((2a+1)x) - \sin a\pi E_{2n+1}((2a+1)x)] \quad [0 < x < \pi]$$

$$10. \sum_{k=0}^{\infty} \frac{(-1)^k}{(4k^2-1)(2k+3)} J_{2n}((2k+1)x) = -\frac{\pi}{16} \delta_{0,n} - \frac{\pi}{16} J_{2n}(2x) \quad [-\pi/2 < x < \pi/2]$$

$$11. \sum_{k=0}^{\infty} \frac{1}{(2k+1)^{\nu+2}} J_{\nu}((2k+1)x) = 2^{-\nu-3} \frac{\pi^2 x^{\nu}}{\Gamma(\nu+1)} - 2^{-\nu-2} \frac{\sqrt{\pi} x^{\nu+1}}{\Gamma(\nu+3/2)} \quad [\operatorname{Re} \nu > -3/2; 0 < x < \pi]$$

$$12. \sum_{k=0}^{\infty} (2k+1)^{2n-\nu} J_{\nu}((2k+1)x) = \frac{2^{2n-\nu}}{2n+1} \left[\frac{n+3/2}{\nu-n+1/2} \right] x^{\nu-2n-1} \quad [\operatorname{Re} \nu \geq 2n+1; 0 < x < 2\pi]$$

$$13. \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^{2n+v+1}} J_v((2k+1)x) =$$

$$= \frac{(-1)^n 2^{-v-2} x^{2n+v} \sqrt{\pi}}{(2n)!} \sum_{k=0}^{2n} \frac{1}{2^k} \binom{2n}{k} \left(\frac{\pi}{x}\right)^k E_k \Gamma \left[\frac{n-(k-1)/2}{v+n+1-k/2} \right]$$

$$[-\pi/2 \leq x < \pi/2].$$

$$14. \sum_{k=0}^{\infty} \frac{1}{(2k+1)^2 - a^2} J_{2n}((2k+1)x) = \frac{\pi}{4a} \operatorname{ctg} \frac{a\pi}{2} J_{2n}(ax) + \frac{\pi}{4a} E_{2n}(ax)$$

$$[0 \leq x \leq \pi].$$

$$15. \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^2 - a^2} J_{2n+1}((2k+1)x) = \frac{\pi}{4a} \operatorname{sech} \frac{a\pi}{2} J_{2n+1}(ax)$$

$$[-\pi/2 \leq x \leq \pi/2].$$

$$16. \sum_{k=0}^{\infty} \frac{2k+1}{(2k+1)^2 - a^2} J_{2n+1}((2k+1)x) =$$

$$= \frac{\pi}{4} \operatorname{sech} \frac{a\pi}{2} [\cos a\pi E_{2n+1}(ax) + \sin a\pi J_{2n+1}(ax)]$$

$$[0 < x < \pi].$$

$$17. \sum_{k=0}^{\infty} (-1)^k \frac{2k+1}{(2k+1)^2 \pm a^2} J_{2n}((2k+1)x) = \frac{\pi}{4} \left\{ \begin{array}{l} (-1)^n \operatorname{sech}(a\pi/2) I_{2n}(ax) \\ \operatorname{sech}(a\pi/2) J_{2n}(ax) \end{array} \right\}$$

$$[-\pi/2 < x < \pi/2].$$

5.7.21. Ряды вида $\sum a_k J_{n+k+v}(z \sqrt{ka+b})$.

$$1. \sum_{k=-\infty}^{\infty} [(2k+\sigma)a+1]^{(2n+\sigma)/2} J_{2k+\sigma}(z \sqrt{(2k+\sigma)a+1}) =$$

$$= \sum_{k=0}^n \left(\frac{z}{2}\right)^{2k+\sigma} \sum_{l=-k-\sigma}^k \frac{(-1)^{k+l} [(2l+\sigma)a+1]^{k+n+\sigma}}{(k-l)! (k+l+\sigma)!}$$

$$[\sigma=0 \text{ или } 1].$$

$$2. \sum_{k=0}^{\infty} \frac{(ka+b+1)^k}{k! (ka+1)^{(k+v+2)/2}} \left(\frac{z}{2}\right)^k J_{k+v}(z \sqrt{ka+1}) =$$

$$= \frac{(z/2)^k}{\Gamma(v+1)} {}_1F_2\left(\frac{1}{a}; 1 + \frac{1}{a}, v+1; \frac{bz^2}{4}\right).$$

$$3. \sum_{k=0}^{\infty} \frac{(ka-b+1)^{k-1}}{k! (ka+1)^{(k+v)/2}} \left(\frac{z}{2}\right)^k J_{k+v}(z \sqrt{ka+1}) = \frac{b^{-v/2}}{1-b} J_v(\sqrt{b} z).$$

$$4. \sum_{k=0}^{\infty} \frac{(z/2)^k}{k! (ka+1)^{(v+2)/2-k}} J_{k+v}(z \sqrt{ka+1}) = \frac{(z/2)^v}{\Gamma(v+1)}.$$

$$5. \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{kx^2}{4}\right)^{(k+n)/2} J_{k+n}(x \sqrt{k}) = S_n.$$

$$S_0 = e^{x^2/4}, \quad S_1 = \frac{x^2}{32} e^{x^2/4}, \quad S_2 = \frac{x^6}{128} \left(\frac{1}{3} + \frac{x^2}{16}\right) e^{x^2/4}.$$

5.7.22. Ряды вида $\sum a_k J_v(x \sqrt{(ka+b)^2 \pm c^2})$.

$$1. \sum_{k=1}^{\infty} (\pm 1)^k J_0(x \sqrt{k^2 + a^2}) = \frac{\cos ax}{x} \left\{ \begin{array}{l} 1 \\ 0 \end{array} \right\} - \frac{1}{2} J_0(ax)$$

$$[0 < x < (3 \pm 1)\pi/2].$$

$$2. \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} J_0(x \sqrt{(2k+1)^2 + a^2}) = \frac{\pi}{4} J_0(ax) \quad [0 < x < \pi].$$

$$3. \sum_{k=1}^{\infty} \frac{(\pm 1)^k}{(k^2 + a^2)^{\nu/2}} J_{\nu}(x \sqrt{k^2 + a^2}) = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \sqrt{\frac{\pi}{2x}} a^{1/2-\nu} J_{\nu-1/2}(ax) - \frac{a^{-\nu}}{2} J_{\nu}(ax) \\ [\operatorname{Re} \nu \geq 0; 0 < x < (3 \pm 1) \pi/2].$$

$$4. \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2 (k^2 + a^2)^{\nu/2}} J_{\nu}(x \sqrt{k^2 + a^2}) = \frac{x}{4a^{\nu+1}} J_{\nu+1}(ax) - \frac{\pi^2}{12a^{\nu}} J_{\nu}(ax) \\ [\operatorname{Re} \nu > -5/2; 0 < x < \pi].$$

$$5. \sum_{k=1}^{\infty} \frac{(\pm 1)^k k^{2n-2}}{(k^2 + a^2)^n} J_{2n}(x \sqrt{k^2 + a^2}) = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \frac{2^{2n+1} (2n)!}{(2n-1) a^{2n+1}} J_{2n+1}(ax) \\ [n = 2, 3, 4, \dots; 0 < x < \pi]$$

$$6. \sum_{k=1}^{\infty} \frac{(-1)^k}{(k^2 - b^2) (k^2 - a^2)^{\nu/2}} J_{\nu}(x \sqrt{k^2 - a^2}) = \\ = \frac{1}{2a^{\nu} b^2} I_{\nu}(ax) - \frac{\pi \operatorname{cosec} b\pi}{2b (b^2 - a^2)^{\nu/2}} J_{\nu}(x \sqrt{b^2 - a^2}) \\ [\operatorname{Re} \nu > -5/2; 0 < x < \pi]$$

5.7.23. Ряды вида $\sum a_k \begin{Bmatrix} \sin ka \\ \cos ka \end{Bmatrix} J_{\nu}(kx)$.

$$1. \sum_{k=1}^{\infty} \sin ka J_0(kx) = \frac{1}{2\pi} \sum_{k=1}^n \frac{1}{k} - \frac{1}{2\pi} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \sum_{k=1}^m \frac{1}{k} - \\ - \sum_{k=n+1}^{\infty} \left[\frac{1}{\sqrt{(2k\pi - a)^2 - x^2}} - \frac{1}{2k\pi} \right] + \sum_{k=\sigma+1}^{\infty} \left[\frac{1}{\sqrt{(2k\pi + a)^2 - x^2}} - \frac{1}{2k\pi} \right] \\ \left[\begin{Bmatrix} \sigma = m; 0 \leq a < x \\ \sigma = 0; a > x > 0 \end{Bmatrix}; m+1, n+1 - \text{натуральные числа, для которых} \right. \\ \left. 2m\pi < \pm x \mp a < 2(m+1)\pi \text{ и } 2n\pi < x + a < 2(n+1)\pi \right].$$

$$2. \sum_{k=1}^{\infty} \cos ka J_0(kx) = -\frac{1}{2} + \sum_{k=\sigma+1}^n \frac{1}{\sqrt{x^2 - (2k\pi - a)^2}} + \\ + \left[\frac{1}{\sqrt{x^2 - a^2}} + \sum_{k=1}^m \frac{1}{\sqrt{x^2 - (2k\pi + a)^2}} \right] \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \\ \left[\begin{Bmatrix} \sigma = 0; 0 \leq a < x \\ \sigma = m; a > x > 0 \end{Bmatrix}; m, n \text{ см. в 5.7.23.1} \right].$$

$$3. \sum_{k=1}^{\infty} (-1)^k \begin{Bmatrix} \sin ka \\ \cos ka \end{Bmatrix} J_0(kx) = \sum_{k=1}^{\infty} \begin{Bmatrix} \sin k(a + \pi) \\ \cos k(a + \pi) \end{Bmatrix} J_0(kx).$$

$$4. \sum_{k=1}^{\infty} \frac{(\pm 1)^k}{k^{\nu+1}} \sin kx J_{\nu}(kx) = -\frac{1}{\Gamma(\nu+1)} \left(\frac{x}{2}\right)^{\nu+1} + \frac{\pi}{2\Gamma(\nu+1)} \left(\frac{x}{2}\right)^{\nu} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \\ [\operatorname{Re} \nu > -(2 \mp 1)/2; 0 < x < (3 \pm 1) \pi/4]$$

$$5. \sum_{k=1}^{\infty} \frac{(\pm 1)^k}{k^{\nu+2}} \cos kx J_{\nu}(kx) = \frac{2^{-\nu-3} x^{\nu}}{3\Gamma(\nu+2)} \left[3x^2 + 2(\nu+1) \begin{Bmatrix} 3x^2 - 6\pi x + 2\pi^2 \\ 3x^2 - \pi^2 \end{Bmatrix} \right] \\ [\operatorname{Re} \nu > -(4 \mp 1)/2; 0 < x < (3 \pm 1) \pi/4]$$

$$6. \sum_{k=1}^{\infty} \frac{1}{k^2 - a^2} \left\{ \frac{k \sin kb}{\cos kb} \right\} J_0(kx) = \pm \frac{\pi}{2a} \operatorname{cosec} a\pi \left\{ \frac{a \sin a (\pi - b)}{\cos a (\pi - b)} \right\} J_0(ax) + \frac{1}{2a^2} \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \\ [0 < b+x < 2\pi; 0 < b-x < 2\pi]$$

$$7. \sum_{k=1}^{\infty} \frac{(\pm 1)^k k^{-\nu}}{k^2 - a^2} \cos kx J_{\nu}(kx) = \\ = \frac{2^{-\nu-1} x^{\nu}}{a^2 \Gamma(\nu+1)} - \frac{\pi}{2} a^{-\nu-1} \operatorname{cosec} a\pi \left\{ \frac{\cos a (x-\pi)}{\cos ax} \right\} J_{\nu}(ax) \\ [\operatorname{Re} \nu > -(4 \mp 1)/2; 0 < x < (3 \pm 1) \pi/4].$$

5.7.24. Ряды вида $\sum a_k J_{\mu}(kx+a) J_{\nu}(ky+b)$.

$$1. \sum_{k=1}^{\infty} \frac{1}{k} J_{2m+1}(kx) J_{2n+1}(kx) = (-1)^{m+n+1} \frac{2}{(2m+1)^2 (4n^2-1) \pi} \\ [m=0, 1, 2, \dots; n=1, 2, 3, \dots; 0 < x < \pi].$$

$$2. \sum_{k=1}^{\infty} \frac{1}{k} J_0(kx) J_{2n+1}(kx) = (-1)^n \left[\frac{2}{(2n+1)^2 \pi} - \frac{x}{4} \delta_{n,0} \right] \quad [0 < x < \pi].$$

$$3. \sum_{k=1}^{\infty} \frac{1}{k} J_m(kx) J_{l+2mn}(kx) = (-1)^{m(n+1)+[(m+l-1)/2]} \frac{2}{[(l+2mn)^2 - m^2] \pi} \\ [n=1, 2, 3, \dots; m=2, 3, 4, \dots; l=\pm 1, \pm 2, \pm 3, \dots; \\ m+l - \text{нечетное число}; 0 < x < \pi].$$

$$4. \sum_{k=1}^{\infty} \frac{(\pm 1)^k}{k} J_{n+\nu+1/2}(kx) J_{n-\nu+1/2}(kx) = \begin{cases} 1 \\ 0 \end{cases} \frac{\sin \nu \pi}{(2n+1) \nu \pi} \\ [n=1, 2, 3, \dots; 0 < x < (3 \pm 1) \pi/4].$$

$$5. \sum_{k=1}^{\infty} \frac{(\pm 1)^k}{k} J_{n+1/2}^2(kx) = \begin{cases} 1 \\ 0 \end{cases} \frac{1}{(2n+1)} \\ [n=1, 2, 3, \dots; 0 < x < (3 \pm 1) \pi/4].$$

$$6. \sum_{k=1}^{\infty} \frac{(\pm 1)^k}{\sqrt{k}} J_{\nu}(kx) J_{1/2-\nu}(kx) = \\ = -\frac{\sqrt{x}}{2 \sqrt{2} \Gamma(1+\nu) \Gamma(3/2-\nu)} + \frac{\pi}{\sqrt{2x} \Gamma(1/2+\nu) \Gamma(1-\nu)} \begin{cases} 1 \\ 0 \end{cases} \\ [0 < x < (3 \pm 1) \pi/4].$$

$$7. \sum_{k=1}^{\infty} k^{-\mu-\nu} J_{\mu}(kx) J_{\nu}(ky) = \frac{1}{\Gamma(\mu+1)} 2^{-\mu-\nu} x^{\mu} y^{\nu} \times \\ \times \left[\frac{\sqrt{\pi}}{y \Gamma(\nu+1/2)} {}_2F_1 \left(\frac{1}{2} - \nu, \frac{1}{2}; \mu+1; \frac{x^2}{y^2} \right) - \frac{1}{2 \Gamma(\nu+1)} \right] \\ [\operatorname{Re} \mu, \operatorname{Re} \nu > -1/2; 0 < x < y < \pi].$$

$$8. \sum_{k=1}^{\infty} (-1)^{k+1} k^{-2\nu} J_{\nu}(kx) J_{\nu}(ky) = \frac{2^{-2\nu-1} (xy)^{\nu}}{\Gamma^2(\nu+1)} \\ [\operatorname{Re} \nu > -1/2; x, y > 0; x+y < \pi; \operatorname{Re} \nu > 0 \text{ при } x+y \leq \pi].$$

$$9. \sum_{k=1}^{\infty} \frac{1}{k^{\mu+\nu}} J_{\mu}(kx) J_{\nu}(kx) = -\frac{x^{\mu+\nu}}{2^{\mu+\nu+1} \Gamma(\mu+1) \Gamma(\nu+1)} + \\ + \frac{\sqrt{\pi} x^{\mu+\nu-1}}{2^{\mu+\nu}} \Gamma \left[\begin{matrix} \mu+\nu \\ \mu+1/2, \nu+1/2, \mu+\nu+1/2 \end{matrix} \right] \\ [\operatorname{Re}(\mu+\nu) > 0; 0 < x < \pi].$$

$$10. \sum_{k=1}^{\infty} \frac{1}{k^{\mu+\nu+2}} J_{\mu}(kx) J_{\nu}(kx) =$$

$$= \frac{2^{-\mu-\nu-1} \pi^2 x^{\mu+\nu}}{3\Gamma(\mu+1)\Gamma(\nu+1)} + \frac{2^{-\mu-\nu-3} (\mu+\nu+2) x^{\mu+\nu+2}}{\Gamma(\mu+2)\Gamma(\nu+2)} -$$

$$- \left(\frac{x}{2}\right)^{\mu+\nu+1} \sqrt{\pi} \Gamma \left[\begin{matrix} \mu+\nu+2 \\ \mu+3/2, \nu+3/2, \mu+\nu+3/2 \end{matrix} \right]$$

[Re μ , Re ν > -1; $0 < x < \pi$].

$$11. \sum_{k=1}^{\infty} (-1)^k k^{2n-\mu-\nu} J_{\mu}(kx) J_{\nu}(kx) = 0$$

[$-1/2 < \text{Re}(\mu+\nu) - 2n \leq 1/2$; $0 \leq x < \pi/2$] или [Re $(\mu+\nu) - 2n > 1/2$; $0 \leq x \leq \pi/2$].

$$12. \sum_{k=1}^{\infty} (-1)^k k^{-\mu-\nu-2} J_{\mu}(kx) J_{\nu}(kx) =$$

$$= \frac{1}{\Gamma(\mu+1)\Gamma(\nu+1)} 2^{-\mu-\nu-2} x^{\mu+\nu} \left[\frac{(\mu+\nu+2)x^2}{2(\mu+1)(\nu+1)} - \frac{\pi^2}{3} \right]$$

[Re $(\mu+\nu) > -3$; $0 < x < \pi/2$].

$$13. \sum_{k=1}^{\infty} (-1)^k \frac{k^{-\mu-\nu}}{k^2 - a^2} J_{\mu}(kx) J_{\nu}(kx) =$$

$$= \frac{1}{a^2 \Gamma(\mu+1)\Gamma(\nu+1)} 2^{-\mu-\nu-1} x^{\mu+\nu} - \frac{\pi}{2 \sin a\pi} a^{-\mu-\nu-1} J_{\mu}(ax) J_{\nu}(ax)$$

[Re $(\mu+\nu) > -3$; $0 < x < \pi/2$].

$$14. \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k^2 - a^2} J_{n+\nu}(kx) J_{n-\nu}(kx) =$$

$$= \frac{\pi}{2a} \operatorname{cosec} a\pi J_{n+\nu}(ax) J_{n-\nu}(ax) - \frac{\sin \nu\pi}{2a^2 \nu} \delta_{0,n}$$

[$-\pi/2 \leq x \leq \pi/2$].

$$15. \sum_{k=1}^{\infty} \frac{\sqrt{k}}{k^2 - a^2} J_{\nu}(kx) J_{-\nu-1/2}(kx) =$$

$$= -\frac{\pi}{2\sqrt{a}} [J_{\nu+1/2}(ax) J_{-\nu}(ax) + \operatorname{ctg} a\pi J_{\nu}(ax) J_{-\nu-1/2}(ax)] + \frac{a^{-2} (2x)^{-1/2}}{\Gamma(\nu+1)\Gamma(1/2-\nu)}$$

[$0 < x < \pi$].

$$16. \sum_{k=1}^{\infty} \frac{(-1)^k k}{k^2 - a^2} J_{n+1/2}^2(kx) = -\frac{\pi}{2} \operatorname{cosec} a\pi J_{n+1/2}^2(ax)$$

[$0 < x < \pi/2$]

$$17. \sum_{k=1}^{\infty} (\pm 1)^k \frac{k^{3/2}}{k^2 - a^2} J_{\nu}(kx) J_{-\nu+1/2}(kx) =$$

$$= -\frac{\pi}{2\sqrt{a}} \operatorname{cosec} a\pi J_{\nu}(ax) J_{-\nu+1/2}(ax) \begin{Bmatrix} \cos a\pi \\ 1 \end{Bmatrix} +$$

$$+ \frac{\pi}{2\sqrt{a}} J_{\nu-1/2}(ax) J_{-\nu}(ax) \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$$

[$0 < x < (3 \pm 1)\pi/4$].

$$18. \sum_{k=0}^{\infty} \frac{1}{2k+1} J_{n+\nu+1/2}((2k+1)x) J_{n-\nu+1/2}((2k+1)x) = \frac{\sin \nu\pi}{2(2n+1)\nu\pi}$$

[$n = 1, 2, 3, \dots$; $0 < x < \pi$].

19. $\sum_{k=0}^{\infty} \frac{1}{2k+1} J_{n+1/2}^2((2k+1)x) = \frac{1}{2(2n+1)} \quad [n=1, 2, 3, \dots; 0 < x < \pi].$
20. $\sum_{k=0}^{\infty} \frac{1}{2k+1} J_m((2k+1)x) J_n((2k+1)y) =$
 $= \frac{y^n}{4x^n} \Gamma \left[\frac{(m+n)/2}{n+1, (m-n)/2+1} \right] {}_2F_1 \left(\frac{m+n}{2}, \frac{n-m}{2}; n+1; \frac{y^2}{x^2} \right)$
 $[m+n - \text{четное}; x+y < \pi].$
21. $\sum_{k=0}^{\infty} \frac{1}{(2k+1)^2} J_{n+v}((2k+1)x) J_{n-v}((2k+1)x) =$
 $= \frac{\pi}{8v} \sin v\pi \delta_{0,n} + \frac{|x|}{(4v^2-1)\pi} \cos v\pi \quad [-\pi/2 \leq x \leq \pi/2].$
22. $\sum_{k=0}^{\infty} \frac{1}{(2k+1)^2} J_n^2((2k+1)x) = \frac{\pi^2}{8} \delta_{0,n} - \frac{|x|}{\pi} \quad [-\pi/2 \leq x \leq \pi/2].$
23. $\sum_{k=1}^{\infty} \frac{1}{(2k+1)^3} J_m((2k+1)x) J_n((2k+1)y) =$
 $= \frac{y^n}{16x^{n-2}} \Gamma \left[\frac{(m+n-2)/2}{n+1, (m-n+4)/2} \right] {}_2F_1 \left(\frac{m+n-2}{2}, \frac{n-m-2}{2}; n+1; \frac{y^2}{x^2} \right) +$
 $+ \frac{\pi^2 x}{16} \delta_{1,m} \delta_{0,n} + \frac{\pi^2 y}{16} \delta_{0,m} \delta_{1,n} \quad [m+n - \text{четное}; x+y < \pi].$
24. $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^3} J_n^2((2k+1)x) = \frac{\pi^3}{32} \delta_{0,n} - \frac{\pi x^2}{16} (2\delta_{0,n} + \delta_{1,n})$
 $[-\pi/4 \leq x \leq \pi/4].$
25. $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^3} J_{n+v}((2k+1)x) J_{n-v}((2k+1)x) =$
 $= \frac{\pi^2 \sin v\pi}{32v} \delta_{0,n} - \frac{x^2 \sin v\pi}{16v(1-v^2)} (2\delta_{0,n} + \delta_{1,n}) \quad [-\pi/4 \leq x \leq \pi/4].$
26. $\sum_{k=0}^{\infty} (-1)^k \frac{(2k+1)^{1-\mu-v}}{(2k+1)^2 - a^2} J_{\mu}((2k+1)x) J_{\nu}((2k+1)x) =$
 $= \frac{\pi}{4 \cos(a\pi/2)} a^{-\mu-v} J_{\mu}(ax) J_{\nu}(ax) \quad [\operatorname{Re}(\mu+v) > -2; 0 < x < \pi/4].$
27. $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)(2k+3)(2k+5)} J_{n+v}((2k+3)x) J_{n-v}((2k+3)x) =$
 $= \frac{\sin v\pi}{16v} \delta_{0,n} + \frac{\pi}{16} J_{n+v}(2x) J_{n-v}(2x) - \frac{1}{3} J_{n+v}(x) J_{n-v}(x)$
 $[-\pi/2 < x < \pi/2].$
28. $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)(2k+3)(2k+5)} J_n^2((2k+3)x) = \frac{\pi}{16} \delta_{0,n} + \frac{\pi}{16} J_n^2(2x) - \frac{1}{3} J_n^2(x)$
 $[-\pi/2 < x < \pi/2].$
29. $\sum_{k=0}^{\infty} (-1)^k (2k+1)^{-\mu-v-1} J_{\mu}((2k+1)x) J_{\nu}((2k+1)x) =$
 $= \frac{\pi}{\Gamma(\mu+1) \Gamma(\nu+1)} 2^{-\mu-v-2} x^{\mu+\nu} \quad [\operatorname{Re}(\mu+v) > -2; 0 < x < \pi/2].$

5.7.25. Ряды вида $\sum a_k J_{k+\mu}(z\sqrt{ka+b}) J_{k+\nu}(z\sqrt{ka+b})$.

$$1. \sum_{k=-\infty}^{\infty} J_{k+2m}(z\sqrt{ka+1}) J_{k+2n}(z\sqrt{ka+1}) = \delta_{m,n}.$$

$$2. \sum_{k=-\infty}^{\infty} \frac{1}{\sqrt{ka+1}} J_{k+2m}(z\sqrt{ka+1}) J_{k+2n+1}(z\sqrt{ka+1}) = 0.$$

5.7.26. Ряды вида $\sum a_k J_{\mu}(x\sqrt{(ka+b)^2+c^2}) J_{\nu}(x\sqrt{(ka+b)^2+c^2})$.

$$1. \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2-a^2} J_0^2(x\sqrt{k^2+b^2}) = \frac{1}{2a^2} J_0^2(bx) - \frac{\pi}{2a} \operatorname{cosec} a\pi J_0^2(x\sqrt{a^2+b^2})$$

[0 < x < \pi/2]

$$2. \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(k^2+a^2)^{(\mu+\nu)/2}} J_{\mu}(x\sqrt{k^2+a^2}) J_{\nu}(x\sqrt{k^2+a^2}) =$$

$$= \frac{a^{-\mu-\nu}}{2} J_{\mu}(ax) J_{\nu}(ax) \quad [\operatorname{Re}(\mu+\nu) > -1; 0 < x < \pi/2]$$

$$3. \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} J_{\nu}(x\sqrt{(2k+1)^2+a^2}) J_{-\nu}(x\sqrt{(2k+1)^2+a^2}) = \frac{\pi}{4} J_{\nu}(ax) J_{-\nu}(ax)$$

[0 < x < \pi/2]

5.7.27. Ряды вида $\sum a_k J_{\nu}(\sqrt{k^2x^2+a^2}+kb_1x+c_1) \times$
 $\times J_{\nu}(\sqrt{k^2x^2+a^2}+kb_2x+c_2)$.

$$1. \sum_{k=1}^{\infty} \frac{(\pm 1)^k}{k} J_{n+1/2}(\sqrt{k^2x^2+a^2}+a) J_{n+1/2}(\sqrt{k^2x^2+a^2}-a) =$$

$$= \frac{1}{2n+1} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} J_0^2(2a) \quad [n=1, 2, 3, \dots; 0 < x < (3 \pm 1)\pi/4].$$

$$2. \sum_{k=1}^{\infty} \frac{(-1)^k}{k^{2\nu+2}} J_{\nu}(\sqrt{k^2x^2+a^2}+a) J_{\nu}(\sqrt{k^2x^2+a^2}-a) =$$

$$= \frac{2^{-2\nu-2}}{\Gamma(\nu+1)} \left(\frac{x^2}{a}\right)^{\nu} \left[\frac{x^2}{a} J_{\nu+1}(2a) - \frac{\pi^2}{3} J_{\nu}(2a) \right]$$

[Re \nu > -3/2; -\pi/2 < x < \pi/2].

$$3. \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} J_n(\sqrt{(2k+1)^2x^2+a^2}+a) J_n(\sqrt{(2k+1)^2x^2+a^2}-a) =$$

$$= \frac{\pi}{4} \delta_{0,n} J_n^2(2a) \quad [-\pi/4 < x < \pi/4]$$

$$4. \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^2-b^2} J_{n+1/2}(\sqrt{(2k+1)^2x^2+a^2}+a) J_{n+1/2}(\sqrt{(2k+1)^2x^2+a^2}-a) =$$

$$= \frac{\pi}{4b} \sec \frac{b\pi}{2} J_{n+1/2}(\sqrt{a^2+b^2x^2}+a) J_{n+1/2}(\sqrt{a^2+b^2x^2}-a)$$

[-\pi/4 \leq x \leq \pi/4]

$$5. \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^{2\nu+1}} J_{\nu}(\sqrt{(2k+1)^2x^2+a^2}+a) J_{\nu}(\sqrt{(2k+1)^2x^2+a^2}-a) =$$

$$= \frac{2^{-2\nu-2\pi}}{\Gamma(\nu+1)} \left(\frac{x^2}{a}\right)^{\nu} J_{\nu}(2a) \quad [\operatorname{Re} \nu > -1; 0 < x < \pi/4].$$

$$6. \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} J_{\nu}(\sqrt{(2k+1)^2 x^2 + a^2} + (2k+1)x) \times \\ \times J_{\nu}(\sqrt{(2k+1)^2 x^2 + a^2} - (2k+1)x) = \frac{\pi}{4} J_{\nu}^2(a) \quad [-\pi/4 < x < \pi/4].$$

$$7. \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2 - b^2} J_{\nu}(\sqrt{k^2 x^2 + a^2} + kx) J_{\nu}(\sqrt{k^2 x^2 + a^2} - kx) = \\ = \frac{1}{2b^2} J_{\nu}^2(a) - \frac{\pi}{2b} \operatorname{cosec} b\pi J_{\nu}(\sqrt{a^2 + b^2 x^2} + bx) J_{\nu}(\sqrt{a^2 + b^2 x^2} - bx) \\ [\operatorname{Re} \nu > -5/2; 0 < x < \pi/2].$$

5.7.28. Ряды вида $\sum a_k \left\{ \frac{\sin ka}{\cos ka} \right\} J_{\mu}(kx) J_{\nu}(ky)$.

$$1. \sum_{k=1}^{\infty} \frac{\sin ka}{k} J_m(kx) J_{2n+m}(ky) = (-1)^n \delta_{n,0} \delta_{m,0} \left(\frac{\pi - a}{2} + \pi \left[\frac{a}{2\pi} \right] \right) \\ [|x| + |y| < \pi - |\pi - a + 2\pi[a/(2\pi)]|].$$

$$2. \sum_{k=1}^{\infty} \frac{\cos ka}{k} J_m(kx) J_{2n-m+1}(ky) = (-1)^{n-1} \delta_{n,0} \left(\delta_{m,1} \frac{x}{4} + \delta_{m,0} \frac{y}{4} \right) \\ [|x| + |y| < \pi - |\pi - a + 2\pi[a/(2\pi)]|].$$

5.7.29. Ряды вида $\sum a_k \left\{ \frac{\sin ka}{\cos ka} \right\} J_{\nu}(\varphi(kx)) J_{\nu}(\chi(kx))$.

$$1. \sum_{k=1}^{\infty} \cos ka J_{\nu}(\sqrt{k^2 x^2 + b^2} + kx) J_{\nu}(\sqrt{k^2 x^2 + b^2} - kx) = \\ = \sum_{k=k_{\pm}}^{k_{+}} \alpha_k [x^2 - (2k\pi + a)^2]^{-1/2} J_{2\nu} \left(\frac{b}{x} \sqrt{x^2 - (2k\pi + a)^2} \right) - \frac{1}{2} J_{\nu}^2(b) \\ [\operatorname{Re} \nu > -1/2; \operatorname{Re} \nu > 1/2 \text{ при } x = \pm a; k_{\pm} = \pm [(x \mp a)/(2\pi)]; \alpha_k = 1 \text{ при } \\ k \neq k_{\pm}, \alpha_{k_{\pm}} = 1/2 \text{ при } (x \mp a)/(2\pi) = \dots, -2, -1, 0, 1, 2, \dots, \alpha_{k_{\pm}} = 1 \text{ при } \\ (x \mp a)/(2\pi) \neq \dots, -2, -1, 0, 1, 2, \dots].$$

$$2. \sum_{k=1}^{\infty} \frac{(\pm 1)^k}{k} \sin 2kx J_{\nu}(\sqrt{k^2 x^2 + a^2} + kx) J_{\nu}(\sqrt{k^2 x^2 + a^2} - kx) = \\ = \frac{\pi}{2} J_{\nu}^2(a) \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} - x J_{\nu}^2(a) \quad [\operatorname{Re} \nu > -(2 \mp 1)\pi/2; 0 < x < (3 \pm 1)\pi/8].$$

$$3. \sum_{k=1}^{\infty} \frac{(\pm 1)^k}{k^2 - b^2} \cos 2kx J_{\nu}(\sqrt{k^2 x^2 + a^2} + kx) J_{\nu}(\sqrt{k^2 x^2 + a^2} - kx) = \\ = \frac{1}{2b^2} J_{\nu}^2(a) - \frac{\pi}{2b} \operatorname{cosec} b\pi \begin{Bmatrix} \cos b(2x - \pi) \\ \cos 2bx \end{Bmatrix} \times \\ \times J_{\nu}(\sqrt{a^2 + b^2 x^2} + b) J_{\nu}(\sqrt{a^2 + b^2 x^2} - b) \\ [\operatorname{Re} \nu > -(4 \mp 1)/2; 0 < x < (3 \pm 1)\pi/8].$$

5.7.30. Ряды вида $\sum a_k J_{k\mu+\nu}(kz+b)$.

Обозначение: $\Delta = \frac{z}{1 + \sqrt{1-z^2}} \exp \sqrt{1-z^2}$.

$$1. \sum_{k=1}^{\infty} (\pm 1)^{k-1} J_k(kz) = \frac{z}{2(1 \mp z)} \quad [|\Delta| < 1].$$

2. $\sum_{k=-\infty}^{\infty} J_{k \pm n}(kz) = \frac{1}{1-z}.$
3. $\sum_{k=1}^{\infty} k [J_{k-1}(kx) - J_{k+1}(kx)] = \frac{1}{(1-x)^2} \quad [-1 < x < 1]$
4. $\sum_{k=-\infty}^{\infty} \frac{1}{k} J_{k-n}(kz) = \frac{1}{n} \quad [n \neq 0].$
5. $\sum_{k=1}^{\infty} \frac{1}{k} [J_{k-1}(kx) - J_{k+1}(kx)] = \frac{x}{2} + 1.$
6. $\sum_{k=1}^{\infty} (\pm 1)^k k^{2n} J_k(kz) = \frac{(-1)^n}{2} \left[\frac{d^{2n}}{da^{2n}} (1 - z \cos b)^{-1} \right] \Big|_{a=(\pi \mp \pi)/2}$
 $[a = b - z \sin b].$
7. $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^2} J_k(kz) = \frac{z(4-z)}{8}.$
8. $\sum_{k=1}^{\infty} \frac{1}{k^2 + a^2} J_k(kz) = -\frac{1}{2a^2} + \frac{1}{2a} \int_0^{\infty} \exp[-a(t - z \sin t)] dt$
 $[\operatorname{Re} a > 0; |\Delta| < 1].$
9. $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^2 - a^2} J_k(kz) = \frac{\pi}{2a} \operatorname{cosec} a\pi J_a(az) - \frac{1}{2a^2}.$
10. $\sum_{k=1}^{\infty} k^p J_{2k}(2kz) = S_p \quad [|\Delta| < 1].$

$$S_2 = \frac{z^2(z^2+1)}{2(z^2-1)^4}, \quad S_0 = \frac{z^2}{2(1-z^2)}, \quad S_{-2} = \frac{z^2}{2},$$

$$S_{-4} = \frac{z^2}{2} - \frac{z^4}{8}, \quad S_{-6} = \frac{z^2}{2} - \frac{5z^4}{32} + \frac{z^6}{72}.$$
11. $\sum_{k=1}^{\infty} \frac{1}{k^2 + a^2} J_{2k}(2kz) = \frac{1}{2a^2 z^2} [{}_1F_2(1; ai, -ai; a^2 z^2) - 1 - z^2]$
 $[|\Delta| < 1].$
12. $\sum_{k=0}^{\infty} (2k+1)^p J_{2k+1}((2k+1)z) = T_p,$

$$T_0 = \frac{z}{2(1-z^2)}, \quad T_{-2} = \frac{z}{2}, \quad T_{-4} = \frac{z}{2} - \frac{z^3}{18},$$

$$T_{-6} = \frac{z}{2} - \frac{5z^3}{81} + \frac{z^5}{450}.$$
13. $\sum_{k=0}^{\infty} \frac{1}{(2k+1)^2 + a^2} J_{2k+1}((2k+1)z) =$

$$= \frac{1}{8a^2 z} \left[{}_1F_2\left(1; \frac{1+ai}{2}, \frac{1-ai}{2}; \frac{a^2 z^2}{2}\right) - 1 \right] \quad [|\Delta| < 1].$$
14. $\sum_{k=0}^{\infty} \frac{\Gamma(k+v)}{k!} (2k+v) J_{2k+v}((2k+v)z) = \left(\frac{z}{2}\right)^v.$

$$15. \sum_{k=v}^{\infty} \frac{\Gamma(k+v)}{k! (2k+v)^{v+1}} J_{2k+v}((2k+v)z) = \frac{1}{v^2} \left(\frac{z}{2}\right)^v \quad [v \neq 0, -1, -2, \dots; |\Delta| < 1].$$

$$16. \sum_{k=0}^{\infty} \frac{\Gamma(k+2v)}{k! (k+v)^{2v+1}} J_{2k+2v}(2(k+v)z) = \frac{z^{2v}}{2(1-z^2)}.$$

$$17. \sum_{k=0}^{\infty} \frac{\Gamma(k+2v)}{k! (k+v)^{2v+1}} J_{2k+2v}(2(k+v)z) = \frac{z^{2v}}{2v^2}.$$

$$18. \sum_{k=0}^{\infty} \frac{\Gamma(k+2v)}{k! (k+v)^{2v+3}} J_{2k+2v}(2(k+v)z) = \frac{z^{2v}}{2v^2} \left[\frac{1}{v^2} - \frac{z^2}{(v+1)^2} \right].$$

5.7.31. Ряды вида $\sum a_k J_{k\mu_1+v_1}(kz+b_1) J_{k\mu_2+v_2}(kz+b_2)$.

Обозначение: $\Delta = \frac{z}{1+\sqrt{1-z^2}} \exp \sqrt{1-z^2}$.

$$1. \sum_{k=1}^{\infty} J_k^2(kz) = \frac{1}{2\sqrt{1-z^2}} - \frac{1}{2} \quad [|\Delta| < 1].$$

$$2. \sum_{k=1}^{\infty} k^2 J_k^2(kz) = \frac{z^2(z^2+4)}{16\sqrt{1-z^2}} \quad [|\Delta| < 1].$$

$$3. \sum_{k=-\infty}^{\infty} \frac{1}{k} J_{k-m}(kz) J_{k-n}(kz) = \frac{1}{n} \delta_{m,n} \quad [m+n \neq 0].$$

$$4. \sum_{k=-\infty}^{\infty} \frac{1}{k^2} J_{k-n}^2(kz) = \frac{1}{n^2} \quad [n \neq 0].$$

$$5. \sum_{k=1}^{\infty} \frac{1}{k^2} J_k^2(kz) = \frac{z^2}{4} \quad [|\Delta| < 1].$$

$$6. \sum_{k=1}^{\infty} \frac{1}{k^2 - a^2} J_k^2(kz) = \frac{1}{2a^2} - \frac{\pi}{2a} \operatorname{cosec} a\pi J_a(az) J_{-a}(az) \quad [|\Delta| < 1].$$

$$7. \sum_{k=1}^{\infty} \frac{1}{4k^2 - 1} J_k^2(kz) = \frac{z - \sin z}{2z}.$$

$$8. \sum_{k=0}^{\infty} J_k((2k+1)z) J_{k+1}((2k+1)z) = \frac{1}{\sqrt{1-4z^2}} - 1.$$

$$9. \sum_{k=0}^{\infty} \frac{\Gamma(k+2v)}{k! (k+v)^{2v+1}} J_{k+v}^2((k+v)z) = \frac{\Gamma(2v+1)}{2v^2 \Gamma^2(v+1)} \left(\frac{z}{2}\right)^{2v}.$$

$$10. \sum_{k=0}^{\infty} \frac{(\mu)_k}{k! (2k+\mu)^{\mu+1}} J_{k+v}((2k+\mu)z) J_{k+\mu-v}((2k+1)z) = \frac{(z/2)^{\mu}}{\mu \Gamma(v+1) \Gamma(\mu-v+1)}.$$

5.7.32. Ряды вида $\sum a_k \begin{Bmatrix} \sin ka \\ \cos ka \end{Bmatrix} J_k(kz)$.

Обозначение: $t - z \sin t = a$, $\operatorname{Im} a = 0$.

Условие: $\left| \frac{z}{1+\sqrt{1-z^2}} \exp \sqrt{1-z^2} \right| < 1$.

$$1. \sum_{k=1}^{\infty} \left\{ \frac{k \sin ka}{\cos ka} \right\} J_k(kz) = \frac{z}{2} \left\{ \frac{(1 - z \cos t)^{-3} \sin t}{(1 - z \cos t)^{-1} \cos t} \right\}.$$

$$2. \sum_{k=1}^{\infty} \frac{1}{k} \sin ka J_k(kz) = \frac{t-a}{2}.$$

$$3. \sum_{k=1}^{\infty} \frac{1}{k} \left\{ \frac{\sin ka}{\cos ka} \right\} [J_{k-n}(kz) \pm J_{k+n}(kz)] = \frac{1}{n} \left\{ \frac{\sin nt}{\cos nt} \right\} \quad [n=1, 2, 3, \dots].$$

5.7.33. Ряды, содержащие нули функции Бесселя $J_\nu(z)$.

Обозначение: $\gamma_{\nu, k}$ — положительные нули функции $J_\nu(x)$, причем $\gamma_{\nu, k+1} > \gamma_{\nu, k}$; $\gamma_k = \gamma_{0, k}$.

Условие: $0 \leq x < 1$.

$$1. \sum_{k=1}^{\infty} \frac{J_\nu(\gamma_{\nu, k} x)}{\gamma_{\nu, k} J_{\nu+1}(\gamma_{\nu, k})} = \frac{x^\nu}{2}.$$

$$2. \sum_{k=1}^{\infty} \frac{J_0(\gamma_k x)}{\gamma_k^2 J_1^2(\gamma_k)} = -\frac{1}{2} \ln x.$$

$$3. \sum_{k=1}^{\infty} \frac{J_0(\gamma_k x)}{\gamma_k^2 J_1(\gamma_k)} = \frac{x^2}{8} - \frac{1}{8}.$$

$$4. \sum_{k=1}^{\infty} \frac{\gamma_{\nu, k} J_\nu(\gamma_{\nu, k} x)}{(\gamma_{\nu, k}^2 - a^2) J_{\nu+1}(\gamma_{\nu, k})} = \frac{J_\nu(ax)}{2J_\nu(a)}.$$

$$5. \sum_{k=1}^{\infty} \frac{J_\nu(\gamma_{\nu, k} x)}{\gamma_{\nu, k} (\gamma_{\nu, k}^2 - a^2) J_{\nu+1}(\gamma_{\nu, k})} = \frac{1}{2a^2} \left[\frac{J_\nu(ax)}{J_\nu(a)} - x^\nu \right].$$

$$6. \sum_{k=1}^{\infty} \frac{J_{\nu+1}(\gamma_{\nu, k} x)}{(\gamma_{\nu, k}^2 - a^2) J_{\nu+1}(\gamma_{\nu, k})} = \frac{1}{2a} \frac{J_{\nu+1}(ax)}{J_\nu(a)}.$$

$$7. \sum_{k=1}^{\infty} \frac{\gamma_{\nu, k} J_\nu(\gamma_{\nu, k} x)}{(\gamma_{\nu, k}^2 - a^2) J_{\nu+1}(\gamma_{\nu, k})} = \frac{1}{2} \frac{J_\nu(ax)}{J_\nu(a)}.$$

$$8. \sum_{k=1}^{\infty} \frac{J_0(\gamma_k a) J_0(\gamma_k x)}{\gamma_k^2 J_1^2(\gamma_k)} = -\frac{1}{2} \ln a \quad [0 \leq x \leq a \leq 1].$$

$$9. \sum_{k=1}^{\infty} \frac{J_{\mu+\nu+1}(\gamma_{\nu, k}) J_\nu(\gamma_{\nu, k} x)}{\gamma_{\nu, k}^{\mu+1} J_{\nu+1}^2(\gamma_{\nu, k})} = \frac{(1-x^2)^\mu x^\nu}{2^{\mu+1} \Gamma(\mu+1)} \quad [0 < x < 1; \operatorname{Re} \mu, \operatorname{Re} \nu > -1].$$

$$10. \sum_{k=1}^{\infty} \frac{J_\nu(\gamma_{\nu, k} a) J_\nu(\gamma_{\nu, k} x)}{(\gamma_{\nu, k}^2 - a^2) J_{\nu+1}^2(\gamma_{\nu, k})} = \frac{\pi J_\nu(ax)}{4J_\nu(a)} [J_\nu(ax) Y_\nu(a) - J_\nu(x) Y_\nu(ax)] \quad [0 \leq x \leq a < 1].$$

$$11. \sum_{k=1}^{\infty} \frac{1}{\gamma_{\nu, k}^{2m}} = S_{2m, \nu}$$

$$S_{2m+2, \nu} = \frac{1}{4} \sum_{k=1}^m \left(-\frac{1}{4} \right)^{m-k} \frac{S_{2k, \nu}}{(m-k+1)! (\nu+1)_{m-k+1}} +$$

$$+ (-1)^m \left(\frac{1}{4} \right)^{m+1} \frac{1}{m! (\nu+1)_{m+1}},$$

$$S_{2,v} = \frac{1}{4(v+1)}, \quad S_{4,v} = \frac{1}{16(v+1)^2(v+2)}, \quad S_{6,v} = \frac{1}{32(v+1)^3(v+2)(v+3)},$$

$$S_{8,v} = \frac{5v+11}{256(v+1)^4(v+2)^2(v+3)(v+4)}.$$

$$12. \sum_{k=1}^{\infty} \frac{1}{\gamma_{v,k}^{2m-v+1} J_{v+1}(\gamma_{v,k})} = T_{2m,v},$$

$$T_{2m+2,v} = \frac{1}{4} \sum_{k=1}^m \left(-\frac{1}{4}\right)^{m-k} \frac{T_{2k,v}}{(m-k+1)!(v+1)_{m-k+1}} +$$

$$+ (-1)^m \left(\frac{1}{2}\right)^{2m-v+3} \frac{\Gamma^2(v+1)}{(m+1)!(m+v+2)},$$

$$T_{2,v} = \left(\frac{1}{2}\right)^{3-v} \frac{\Gamma(v+1)}{v+1}, \quad T_{4,v} = \left(\frac{1}{2}\right)^{6-v} \frac{(v+3)\Gamma(v+1)}{(v+1)^2(v+2)},$$

$$T_{6,v} = \left(\frac{1}{2}\right)^{8-v} \frac{(v^2+8v+19)\Gamma(v+1)}{3(v+1)^3(v+2)(v+3)}.$$

$$13. \sum_{k=1}^{\infty} \frac{\sin \gamma_k}{\gamma_k^3 J_1(\gamma_k)} = \frac{1}{2}.$$

$$14. \sum_{k=1}^{\infty} \frac{\sin \gamma_k}{\gamma_k^3 J_1^2(\gamma_k)} = \frac{1}{2} - \frac{1}{2} \ln 2.$$

$$15. \sum_{k=1}^{\infty} \frac{1}{(\gamma_{1,k}^2 - a^2) J_0(\gamma_{1,k})} = \frac{1}{a^2} - \frac{1}{2a} J_1(a).$$

$$16. \sum_{k=1}^{\infty} \left(\frac{1}{\gamma_k} - \frac{\gamma_k}{\gamma_k^3 - a^2}\right) \frac{1}{J_1(\gamma_k)} = \frac{1}{2} - \frac{1}{2J_0(a)}.$$

$$17. \sum_{k=0}^{\infty} \left[\frac{\gamma_k^4}{(\gamma_k^2 - a^2)^2} - \frac{1}{\gamma_k^2 - a^2}\right] \frac{1}{J_1^2(\gamma_k)} = \frac{1}{4J_0^2(a)} - \frac{1}{4}.$$

$$18. \sum_{k=1}^{\infty} \left[\frac{\gamma_{1,k}^2}{(\gamma_{1,k}^2 - a^2)^2} - \frac{1}{\gamma_{1,k}^2 - a^2}\right] \frac{1}{J_0^2(\gamma_{1,k})} = \frac{1}{4J_1^2(a)} - \frac{1}{a^2} - \frac{1}{4}.$$

Ниже λ_k — положительные нули функции $zJ'_\nu(z) + \mu J_\nu(z)$, $\lambda_{k+1} > \lambda_k$.

$$19. \sum_{k=1}^{\infty} \frac{\lambda_k J_{\nu+1}(\lambda_k) J_\nu(\lambda_k x)}{(\lambda_k^2 - \nu^2) J_\nu^2(\lambda_k) + \lambda_k^2 [J'_\nu(\lambda_k)]^2} = \frac{x^\nu}{2}$$

$[0 \leq x \leq 1; \operatorname{Re}(\mu + \nu) > 0].$

$$20. \sum_{k=1}^{\infty} \frac{\lambda_k^2 J_\nu(\lambda_k x) [\lambda_k J_{\nu+1}(\lambda_k) J_\nu(a) - a J_\nu(\lambda_k) J_{\nu+1}(a)]}{(\lambda_k^2 - a^2) \{\lambda_k^2 [J'_\nu(\lambda_k)]^2 + (\lambda_k^2 - \nu^2) J_\nu^2(\lambda_k)\}} = \frac{1}{2} J_\nu(ax)$$

$[0 \leq x \leq 1].$

5.8. МОДИФИЦИРОВАННАЯ ФУНКЦИЯ БЕССЕЛЯ $I_\nu(z)$

5.8.1. Ряды вида $\sum a_k I_{\nu \pm nk}(z)$.

$$1. \sum_{k=0}^{\infty} (\pm 1)^k I_{k+\nu}(z) = \frac{1}{2(1-\nu)} \left[e^{\pm z} \int_0^z e^{\mp t} I_{\nu-1}(t) dt - z I_{\nu-1}(z) \mp z I_\nu(z) \right]$$

$[\operatorname{Re} \nu > 0].$

2. $\sum_{k=1}^{\infty} (\pm 1)^k I_k(z) = \frac{1}{2} e^{\pm z} - \frac{1}{2} I_0(z).$
3. $\sum_{k=0}^{\infty} (\pm 1)^k I_{k+1/2}(z) = \frac{1}{2} e^{\pm z} \begin{Bmatrix} \operatorname{erf}(\sqrt{2z}) \\ \operatorname{erfi}(\sqrt{2z}) \end{Bmatrix}.$
4. $\sum_{k=0}^{\infty} (-1)^{[(k+\delta)/2]} I_{k+1/2}(z) = \frac{1}{\sqrt{2}} \begin{Bmatrix} \operatorname{erf}(\sqrt{z}) \\ \operatorname{erfi}(\sqrt{z}) \end{Bmatrix} \left[\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \right].$
5. $\sum_{k=0}^{\infty} I_{2k+(2\pm 1)/2}(z) = \frac{1}{4} e^z \operatorname{erf}(\sqrt{2z}) \mp \frac{1}{4} e^{-z} \operatorname{erfi}(\sqrt{2z}).$
6. $\sum_{k=0}^{\infty} (\pm 1)^k I_{2k}(z) = \frac{1}{2} I_0(z) + \frac{1}{2} \begin{Bmatrix} \operatorname{ch} z \\ 1 \end{Bmatrix}.$
7. $\sum_{k=0}^{\infty} I_{2k+1}(z) = \frac{1}{2} \operatorname{sh} z.$
8. $\sum_{k=0}^{\infty} (-1)^k I_{2k+\nu}(z) = \frac{1}{2} \int_0^z I_{\nu-1}(t) dt \quad [\operatorname{Re} \nu > 0; \text{см. 1.11.1}].$
9. $\sum_{k=1}^{\infty} (\pm 1)^k (k+\nu) I_{k+\nu}(z) = \frac{z}{2} [I_{\nu+1}(z) \pm I_{\nu}(z)].$
10. $\sum_{k=0}^{\infty} (\pm 1)^k (\nu-k) I_{\nu-k}(z) = -\frac{z}{2} [I_{\nu+1}(z) \pm I_{\nu}(z)].$
11. $\sum_{k=1}^{\infty} (\pm 1)^k k I_{k+\nu}(z) = \pm \frac{1}{2} e^{\pm z} \int_0^z e^{\mp t} I_{\nu}(t) dt \quad [\operatorname{Re} \nu > -1; \text{см. 1.11.2}].$
12. $\sum_{k=1}^{\infty} k^{2n} I_k(z) = e^z \sum_{k=1}^n \frac{1}{(k-1)!} \left(\frac{z}{2}\right)^k \sum_{m=1}^{2k} (-1)^m m (m-k)^{2n}.$
13. $\sum_{k=1}^{\infty} \frac{1}{k} I_{2k}(z) = \frac{1}{2} K_0(z) + \frac{1}{2} \left(\ln \frac{z}{2} + C \right) I_0(z).$
14. $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^2} I_k(z) = \frac{1}{2} H_{-1,0}(z) - \frac{1}{4} \frac{\partial^2 I_{\nu}(z)}{\partial \nu^2} \Big|_{\nu=0} - \frac{\pi^2}{12} I_0(z).$
15. $\sum_{k=1}^{\infty} \frac{(\pm 1)^k}{4k^2-1} I_k(z) = \frac{1}{2} I_0(z) - \frac{1}{2^{5/2}} \sqrt{\frac{\pi}{z}} e^{\mp z} \begin{Bmatrix} \operatorname{erfi}(\sqrt{2z}) \\ \operatorname{erf}(\sqrt{2z}) \end{Bmatrix}.$
16. $\sum_{k=1}^{\infty} (\pm 1)^k \frac{2k+1}{k(k+1)} I_{k+1/2}(z) =$
 $= \pm \frac{1}{\sqrt{2\pi z}} e^{\pm z} [C + \ln(2z) - \operatorname{Ei}(\mp 2z)] - \sqrt{\frac{2}{\pi z}} \operatorname{sh} z.$

$$17. \sum_{k=1}^{\infty} \frac{n+2k}{k(k+n)} I_{2k+n}(z) =$$

$$= (-1)^n K_n(z) + \left[\ln \frac{z}{2} - \psi(n+1) \right] I_n(z) - \frac{n!}{2} \left(-\frac{2}{z} \right)^n \sum_{k=0}^{n-1} \frac{(-z/2)^k}{k!(k-n)} I_k(z).$$

$$18. \sum_{k=1}^{\infty} (-1)^k (2k+1) I_{2k+1}(z) \sum_{l=1}^k \frac{1}{l} = \frac{z}{2} \int_0^z \frac{1-I_0(t)}{t} dt.$$

5.8.2. Ряды вида $\sum a_k \frac{(b)_k}{(c)_k} I_{nk+v}(z)$.

$$1. \sum_{k=0}^{\infty} (\pm 1)^k \frac{k+v}{k!} (2v)_k I_{k+v}(z) = \frac{1}{\Gamma(v)} \left(\frac{z}{2} \right)^v e^{\pm z} \quad [v \neq 0, -1, -2, \dots].$$

$$2. \sum_{k=0}^{\infty} (-1)^k \frac{k+v}{k! [(k+v)^2 - \mu^2]} (2v)_k I_{k+v}(z) = \frac{2^{-v}}{\Gamma(v)} h_{v-1, \mu}(z)$$

$$[v, v \pm \mu \neq 0, -1, -2, \dots].$$

$$3. \sum_{k=1}^{\infty} (-1)^k \frac{k+v}{k! k (k+2v)} (2v)_k I_{k+v}(z) =$$

$$= \frac{2^{-v}}{\Gamma(v)} H_{v-1, v}(z) - \frac{1}{2} \frac{\partial I_v(z)}{\partial v} - \frac{1}{2} \sec v\pi K_v(z) -$$

$$- \frac{1}{2} [\psi(2v+1) + C] I_v(z)$$

$$[v, 1/2 \pm v \neq 0, -1, -2, \dots].$$

$$4. \sum_{k=0}^{\infty} (-1)^k \frac{k+v}{k! [(k+v)^2 - 1/4]} (2v)_k I_{k+v}(z) =$$

$$= \frac{2^{-2v+1/2}}{(2v-1)\Gamma(v)} z^{-1/2} e^{z\gamma} \left(v + \frac{1}{2}, 2z \right) \quad [\operatorname{Re} v > -1/2].$$

$$5. \sum_{k=0}^{\infty} \frac{k+v}{(\mu+2)_k} (2v-\mu-1)_k I_{k+v}(z) =$$

$$= \frac{\mu(\mu+1)}{2} z^{v-\mu-1} e^z \int_0^z t^{\mu-v} e^{-t} I_{v-1}(t) dt + \frac{\mu+1}{2} I_{v-1}(z) \quad ([\operatorname{Re} \mu > 0]).$$

$$6. \sum_{k=0}^{\infty} (-1)^k \frac{k+v}{(\mu)_k} (2v-\mu+1)_k I_{k+v}(z) =$$

$$= \frac{(z/2)^v}{\Gamma(v)} e^{-z} {}_2F_2 \left(\mu-1, v+\frac{1}{2}; \mu, 2v; 2z \right).$$

$$7. \sum_{k=0}^{\infty} (-1)^k \frac{(2v)_k (2v-\mu)_k}{k! (\mu+1)_k} I_{k+v}(z) = \frac{(z/2)^v}{\Gamma(v)} e^{-z} {}_1F_1 \left(\mu-v+\frac{1}{2}; \mu+1; 2z \right).$$

$$8. \sum_{k=0}^{\infty} \frac{k+v}{k! (\mu)_k} (2v)_k (2v-\mu+1)_k I_{k+v}(z) = \frac{(z/2)^v}{\Gamma(v)} e^{-z} {}_1F_1 \left(v+\frac{1}{2}; \mu; 2z \right).$$

$$9. \sum_{k=0}^{\infty} (-1)^k \frac{(k+v)}{k! (\lambda)_k (\mu)_k} (2v)_k (2v-\lambda+1)_k (2v-\mu+1)_k I_{k+v}(z) = \\ = \frac{(z/2)^v}{\Gamma(v)} e^{-z} {}_2F_2 \left(\lambda + \mu - 2v - 1, v + \frac{1}{2}; \lambda, \mu; 2z \right).$$

$$10. \sum_{k=0}^{\infty} (-1)^k \frac{2k+v}{k+\mu+1} \frac{(v-\mu+1)_k}{(\mu)_k} I_{2k+v}(z) = z^{v-\mu+2} \int_0^z t^{2\mu-v-2} I_{v-1}(t) dt \\ [Re \mu > 1].$$

$$11. \sum_{k=0}^{\infty} (-1)^k \frac{2k+v}{k!} (v)_k I_{2k+v}(z) = \frac{1}{\Gamma(v)} \left(\frac{z}{2} \right)^v \\ [v \neq \dots, -1, -2, \dots].$$

5.8.3. Ряды вида $\sum a_k t^k I_{v \pm nk}(z)$.

$$1. \sum_{k=0}^{\infty} t^k I_{k+v}(z) = z^{-v} e^{tz/2} \int_0^z \tau^v \exp \left(-\frac{t\tau^2}{2z} \right) I_{v-1}(\tau) d\tau \\ [Re v > 0].$$

$$2. \sum_{k=-\infty}^{\infty} t^k I_k(z) = \exp \left(\left(t + \frac{1}{t} \right) \frac{z}{2} \right) \\ [t \neq 0].$$

$$3. \sum_{k=0}^{\infty} (\pm 1)^k t^k I_{k+1/2}(z) = \frac{1}{2\sqrt{t}} \exp \left(\pm \left(t + \frac{1}{t} \right) \frac{z}{2} \right) \times \\ \times \begin{cases} \operatorname{erf} [\sqrt{tz/2} + \sqrt{z/(2t)}] + \operatorname{erf} [\sqrt{tz/2} - \sqrt{z/(2t)}] \\ \operatorname{erfi} [\sqrt{tz/2} + \sqrt{z/(2t)}] + \operatorname{erfi} [\sqrt{tz/2} - \sqrt{z/(2t)}] \end{cases}.$$

$$4. \sum_{k=0}^{\infty} \frac{t^k}{k!} I_{v \pm k}(z) = \left(\frac{2t}{z} + 1 \right)^{\mp v/2} I_v(\sqrt{z^2 + 2tz}) \\ \left[\left\{ \begin{matrix} t, z - \text{любые} \\ |2t| < |z| \end{matrix} \right\} \right].$$

$$5. \sum_{k=0}^{\infty} \frac{(-z)^k}{k!} I_{k+v}(z) = J_v(z).$$

$$6. \sum_{k=0}^{\infty} \frac{t^k}{k!} I_{\pm k + 1/2}(z) = \sqrt{\frac{2}{\pi z}} \begin{cases} \operatorname{ch} \sqrt{z^2 + 2tz} \\ \operatorname{sh} \sqrt{z^2 + 2tz} \end{cases} \\ \left[\left\{ \begin{matrix} t, z - \text{любые} \\ |2t| < |z| \end{matrix} \right\} \right].$$

$$7. \sum_{k=0}^{\infty} \frac{(2k-1)!!}{k!} t^k I_k(z) = \frac{e^z}{2\pi} \int_0^{2\pi} e^{-z \cos \tau (1 + t \cos \tau)} d\tau.$$

$$8. \sum_{k=0}^{\infty} \frac{(-z/2)^k}{k! (k+v)} I_k(z) = \Gamma(v) \left(\frac{z}{2} \right)^{-v} I_v(z).$$

$$9. \sum_{k=1}^{\infty} \frac{(-z/2)^k}{k! k} I_k(z) = -K_0(z) - \left(\ln \frac{z}{2} + C \right) I_0(z).$$

$$10. \sum_{k=1}^{\infty} \frac{(-z/2)^k}{k! k^2} I_k(z) = \\ = - \left(\ln \frac{z}{2} + C \right) K_0(z) - \frac{1}{2} \left[\frac{\pi^2}{6} + \left(\ln \frac{z}{2} + C \right)^2 \right] I_0(z) - \frac{1}{2} \frac{\partial^2 I_v(z)}{\partial v^2} \Big|_{v=0}.$$

$$11. \sum_{k=0}^{\infty} \frac{(-z/2)^k}{k! (2k+1)^2} I_k(z) = \frac{1}{4z} [2 \operatorname{sh} z (C + \ln(2z)) + e^{-z} \operatorname{Ei}(2z) - e^z \operatorname{Ei}(-2z)].$$

$$12. \sum_{k=0}^{\infty} \frac{(-z/2)^k}{k! (2k-1)!} I_k(z) = \frac{1}{2} \operatorname{ch} z [2 - C - \ln(2z)] + \frac{1}{4} [e^{-z} \operatorname{Ei}(2z) + e^z \operatorname{Ei}(-2z)].$$

$$13. \sum_{k=-\infty}^{\infty} \frac{(-z/2)^k}{(k+\mu) \Gamma(k-\nu+1/2)} I_{\nu-k}(z) = \frac{\pi (z/2)^{-\mu} \operatorname{cosec} \mu\pi}{\Gamma(1/2-\mu-\nu)} I_{\mu+\nu}(z).$$

$$14. \sum_{k=0}^{\infty} \left(\frac{z}{2}\right)^k I_{k+\nu}(z) \sum_{l=0}^k \frac{(-1)^l}{l!} = \frac{1}{\Gamma(\nu)} \left(\frac{z}{2}\right)^{-\nu} e^{z^2/4} \gamma\left(\nu, \frac{z^2}{4}\right) \quad [\operatorname{Re} \nu > 0].$$

$$15. \sum_{k=0}^{\infty} \left(\frac{z}{2}\right)^k I_{k+1/2}(z) \sum_{l=0}^k \frac{(-1)^l}{l!} = \sqrt{\frac{2}{z}} e^{z^2/4} \operatorname{erf}\left(\frac{z}{2}\right).$$

$$16. \sum_{k=1}^{\infty} \frac{(-z/2)^k}{k!} I_k(z) \sum_{l=1}^k \frac{1}{l} = -2 \int_0^z \frac{1}{t} [I_0(t) - 1] dt.$$

5.8.4. Ряды, содержащие гиперболические функции и $I_{\nu}(z)$.

$$1. \sum_{k=0}^{\infty} (\pm 1)^k \operatorname{ch} ka I_{2k}(z) = \frac{1}{2} I_0(z) + \frac{1}{2} \left\{ \operatorname{ch} [z \operatorname{ch}(a/2)] \right\}.$$

$$2. \sum_{k=0}^{\infty} (\mp 1)^k \left\{ \frac{\operatorname{sh}(2k+1)a}{\operatorname{ch}(2k+1)a} \right\} I_{2k+1}(z) = \frac{1}{2} \left\{ \frac{\sin(z \operatorname{sh} a)}{\operatorname{sh}(z \operatorname{ch} a)} \right\}.$$

$$3. \sum_{k=1}^{\infty} (\pm 1)^k k \operatorname{sh} ka I_{2k}(z) = \pm \frac{z}{4} \left\{ \frac{\operatorname{sh}(a/2) \operatorname{sh}[z \operatorname{ch}(a/2)]}{\operatorname{ch}(a/2) \sin[z \operatorname{sh}(a/2)]} \right\}.$$

$$4. \sum_{k=0}^{\infty} (\pm 1)^k (2k+1) \left\{ \frac{\operatorname{sh}(2k+1)a}{\operatorname{ch}(2k+1)a} \right\} I_{2k+1}(z) = \frac{z}{2} \left\{ \frac{\operatorname{sh} a \operatorname{ch}(z \operatorname{ch} a)}{\operatorname{ch} a \cos(z \operatorname{sh} a)} \right\}.$$

$$5. \sum_{k=0}^{\infty} (\pm 1)^k \operatorname{ch} ka I_k^2(z) = \frac{1}{2} I_0(z) + \frac{1}{2} \left\{ \frac{I_0(2z \operatorname{ch}(a/2))}{J_0(2z \operatorname{sh}(a/2))} \right\}.$$

5.8.5. Ряды вида $\sum a_k \left\{ \frac{\sin(ka+b)}{\cos(ka+b)} \right\} I_{\mu \pm kv}(z)$.

$$1. \sum_{k=1}^{\infty} (\pm 1)^k \sin ka I_{k+\nu}(z) = \pm \frac{\sin a}{2} \int_0^z e^{\pm(z-t)\cos a} I_{\nu}(t) dt \quad [\operatorname{Re} \nu > -1].$$

$$2. \sum_{k=1}^{\infty} \sin ka I_k(z) = \frac{1}{2} \sin a \int_0^z e^{-t \cos a} I_0(t) dt.$$

$$3. \sum_{k=1}^{\infty} (\pm 1)^k \cos ka I_k(z) = \frac{1}{2} e^{\pm z \cos a} - \frac{1}{2} I_0(z).$$

$$4. \sum_{k=0}^{\infty} \cos ka I_{kn}(z) = \frac{1}{2n} \sum_{k=k_{\pm}}^{k_{+}} \alpha_k \exp\left(z \cos \frac{2k\pi+a}{n}\right) + \frac{1}{2} I_0(z)$$

$[k_{\pm} = \pm[(n\pi \mp a)/(2\pi)]; \alpha_k = 1 \text{ при } k \neq k_{\pm}, \alpha_{k_{\pm}} = 1/2 \text{ при } (n\pi \mp a)/(2\pi) = \dots, -2, -1, 0, 1, 2, \dots, \alpha_{k_{\pm}} = 1 \text{ при } (n\pi \mp a)/(2\pi) \neq \dots, -2, -1, 0, 1, 2, \dots].$

$$5. \sum_{k=1}^{\infty} (\pm 1)^k \cos ka I_{\nu k}(z) = \frac{1}{2} \left\{ \frac{\operatorname{ch}[z \cos(a/2)]}{\operatorname{ch}[z \sin(a/2)]} \right\} - \frac{1}{2} I_{\nu}(z).$$

$$6. \sum_{k=1}^{\infty} (\pm 1)^k \cos ka I_{k/2}(z) = \frac{1}{2} e^{z \cos 2a} \left\{ \frac{1 + \operatorname{erf}(\sqrt{2z} \cos a)}{\operatorname{erfc}(\sqrt{2z} \cos a)} \right\} - \frac{1}{2} I_0(z).$$

$$7. \sum_{k=0}^{\infty} (\pm 1)^k \sin(2k+1)a I_{k+1/2}(z) = \frac{1}{2} e^{\pm z \cos 2a} \left\{ \frac{\operatorname{erfi}(\sqrt{2z} \sin a)}{\operatorname{erf}(\sqrt{2z} \sin a)} \right\}.$$

$$8. \sum_{k=0}^{\infty} (\pm 1)^k \cos(2k+1)a I_{k+1/2}(z) = \frac{1}{2} e^{\pm z \cos 2a} \left\{ \frac{\operatorname{erf}(\sqrt{2z} \cos a)}{\operatorname{erfi}(\sqrt{2z} \cos a)} \right\}.$$

5.8.6. Ряды вида $\sum a_k I_{\mu+k\nu}(w) I_{\rho+k\sigma}(z)$.

$$1. \sum_{k=-\infty}^{\infty} I_k(w) I_{n-k}(z) = I_n(w+z).$$

$$2. \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2 - a^2} I_{\mu-k\nu}(z) I_{\mu+k\nu}(z) = \frac{1}{2a^2} I_{\nu}^2(z) - \frac{\pi}{2a} \operatorname{cosec} a\pi I_{\mu-\nu a}(z) I_{\mu+\nu a}(z)$$

[$\operatorname{Re} \mu > -3/2$; $0 < \nu < 1$].

5.8.7. Ряды вида $\sum a_k \left\{ \begin{matrix} \sin(ka+b) \\ \cos(ka+b) \end{matrix} \right\} I_{\mu \pm k\nu}(w) I_{\rho \pm k\sigma}(z)$.

$$1. \sum_{k=-\infty}^{\infty} (\pm 1)^k \sin ka I_k(w) I_{k+\nu}(z) =$$

$$= \mp \sin \left(\nu \arcsin \frac{w \sin a}{\sqrt{w^2 + z^2 \pm 2wz \cos a}} \right) I_{\nu}(\sqrt{w^2 + z^2 \pm 2wz \cos a})$$

[$|w| < |z|$].

$$2. \sum_{k=-\infty}^{\infty} (\pm 1)^k \cos ka I_k(w) I_{k+\nu}(z) =$$

$$= \cos \left(\nu \arcsin \frac{w \sin a}{\sqrt{w^2 + z^2 \pm 2wz \cos a}} \right) I_{\nu}(\sqrt{w^2 + z^2 \pm 2wz \cos a}).$$

$$3. \sum_{k=1}^{\infty} (\pm 1)^k \cos ka I_k(w) I_k(z) = \frac{1}{2} I_0(\sqrt{w^2 + z^2 \pm 2wz \cos a}) - \frac{1}{2} I_0(w) I_0(z).$$

$$4. \sum_{k=1}^{\infty} (-1)^k \cos ka I_{kn}(w) I_{kn}(z) =$$

$$= \frac{1}{2n} \sum_{k=k_-}^{k_+} I_0 \left(\sqrt{w^2 + z^2 - 2wz \cos \frac{2k\pi + a}{n}} \right) - \frac{1}{2} I_0(w) I_0(z)$$

[$k_{\pm}$ см. в 5.8.5.4].

$$5. \sum_{k=1}^{\infty} (\pm 1)^k \cos 2ka I_{\nu-k}(z) I_{\nu+k}(z) = \frac{1}{2} I_{2\nu} \left(2z \left\{ \begin{matrix} \cos a \\ \sin a \end{matrix} \right\} \right) - \frac{1}{2} I_{\nu}^2(z)$$

[$\operatorname{Re} \nu > -1/2$].

$$6. \sum_{k=0}^{\infty} (\mp 1)^k \left\{ \begin{matrix} \sin(2k+1)a \\ \cos(2k+1)a \end{matrix} \right\} I_{\nu-k-1/2}(z) I_{\nu+k+1/2}(z) = \frac{1}{2} I_{2\nu} \left(2z \left\{ \begin{matrix} \sin a \\ \cos a \end{matrix} \right\} \right)$$

[$\operatorname{Re} \nu > -1/2$].

$$7. \sum_{k=1}^{\infty} \cos ka I_{\mu-k\nu}(z) I_{\mu+k\nu}(z) = \frac{1}{2\nu} \sum_{k=k_{\pm}}^{k_{\mp}} \alpha_k I_{2\mu} \left(2z \cos \frac{2k\pi + a}{2\nu} \right) - \frac{1}{2} I_{\mu}^2(z)$$

$[k_{\pm} = \pm [(\nu\pi \mp a)/(2\pi)]; \operatorname{Re} \mu > -1/2, \operatorname{Re} \mu > 0 \text{ при } a = \pm \nu\pi, \alpha_k = 1 \text{ при } k \neq k_{\pm}, \alpha_{k_{\pm}} = 1/2 \text{ при } (\nu\pi \mp a)/(2\pi) = \dots, -2, -1, 0, 1, 2, \dots, \alpha_{k_{\pm}} = 1 \text{ при } (\nu\pi \mp a)/(2\pi) \neq \dots, -2, -1, 0, 1, 2, \dots]$

$$8. \sum_{k=1}^{\infty} \cos ka \cos \frac{k\nu\pi}{2} I_{\mu-k\nu}(z) I_{\mu+k\nu}(z) = \frac{1}{2\nu} \sum_{k=k_{\pm}}^{k_{\mp}} I_{2\mu} \left(2z \sin \frac{2k\pi + a}{\nu} \right) - \frac{1}{2} I_{\mu}^2(z)$$

$[\operatorname{Re} \mu > -1/2; \operatorname{Re} \mu > 0 \text{ при } a = 0, \pm \pi\nu; k_{\pm} \text{ см. в 5.8.7.7}].$

$$9. \sum_{k=1}^{\infty} \frac{(\pm 1)^k}{k} \sin k\nu\pi I_{\mu-k\nu}(z) I_{\mu+k\nu}(z) = \frac{\pi}{2} \left\{ \frac{1-\nu}{\nu} \right\} I_{\mu}^2(z)$$

$[\operatorname{Re} \mu > -(3 \mp 1)/4; 0 < \nu < (3 \pm 1)/4].$

$$10. \sum_{k=1}^{\infty} \frac{(\pm 1)^k}{k^2 - a^2} \cos k\nu\pi I_{\mu-k\nu}(z) I_{\mu+k\nu}(z) =$$

$$= \frac{1}{2a^2} I_{\mu}^2(z) - \frac{\pi}{2a} \operatorname{cosec} a\pi \left\{ \frac{\cos(\nu-1)a\pi}{\cos \nu a\pi} \right\} I_{\mu-a\nu}(z) I_{\mu+a\nu}(z)$$

$[\operatorname{Re} \mu > -(5 \mp 1)/4; 0 < \nu < (3 \pm 1)/4].$

$$11. \sum_{k=1}^{\infty} \frac{k}{k^2 - a^2} \sin k\nu\pi I_{\mu-k\nu}(z) I_{\mu+k\nu}(z) =$$

$$= -\frac{\pi}{2} \operatorname{cosec} a\pi \left\{ \frac{\sin(\nu-1)a\pi}{\sin \nu a\pi} \right\} I_{\mu-a\nu}(z) I_{\mu+a\nu}(z)$$

$[\operatorname{Re} \mu > -(3 \mp 1)/4; 0 < \nu < (3 \pm 1)/4].$

5.9. РАЗНЫЕ РЯДЫ, СОДЕРЖАЩИЕ ФУНКЦИИ БЕССЕЛЯ $J_{\nu}(z)$, $I_{\nu}(z)$, НЕЙМАНА $Y_{\nu}(z)$ И МАКДОНАЛЬДА $K_{\nu}(z)$

5.9.1. Ряды вида $\sum a_k Y_{k\mu+\nu}(kx+y)$ и $\sum a_k K_{k\mu+\nu}(kx+y)$.

$$1. \sum_{k=0}^{\infty} (-1)^k t^k K_{k+1/2}(z) = \frac{\pi}{2\sqrt{t}} \exp \left[\frac{z}{2} \left(t + \frac{1}{t} \right) \right] \operatorname{erfc} \left(\sqrt{\frac{tz}{2}} + \sqrt{\frac{z}{2t}} \right).$$

$$2. \sum_{k=1}^{\infty} \cos ka Y_0(kx) = \frac{1}{\pi} \left(\ln \frac{4\pi}{x} - \mathbf{C} \right) + \frac{1}{2\pi} \sum_{k=1}^n \frac{1}{k} +$$

$$+ \frac{1}{2\pi} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \sum_{k=1}^m \frac{1}{k} - \sum_{k=n+1}^{\infty} \left[\frac{1}{\sqrt{(2k\pi-a)^2 - x^2}} - \frac{1}{2k\pi} \right] -$$

$$- \sum_{k=\sigma+1}^{\infty} \left[\frac{1}{\sqrt{(2k\pi+a)^2 - x^2}} - \frac{1}{2k\pi} \right] - \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \sum_{k=0}^m \frac{1}{\sqrt{(2k\pi-a)^2 - x^2}}$$

$\left[\begin{cases} \sigma = m; & 0 \leq a < x \\ \sigma = 0; & a > x > 0 \end{cases}; m+1, n+1 - \text{натуральные числа, для которых} \right.$

$\left. 2m\pi < a-x < 2(m+1)\pi \text{ и } 2n\pi < a+x < 2(n+1)\pi \right].$

$$3. \sum_{k=1}^{\infty} (-1)^k \cos ka Y_0(kx) = \sum_{k=1}^{\infty} \cos k(a+\pi) Y_0(kx).$$

$$4. \sum_{k=1}^{\infty} \cos ka K_0(kx) = \frac{\pi}{2\sqrt{x^2+a^2}} + \frac{1}{2} \left(C + \ln \frac{x}{4\pi} \right) +$$

$$+ \frac{\pi}{2} \sum_{k=1}^{\infty} \left[\frac{1}{\sqrt{(2k\pi-a)^2+x^2}} - \frac{1}{2k\pi} \right] + \frac{\pi}{2} \sum_{k=1}^{\infty} \left[\frac{1}{\sqrt{(2k\pi+a)^2+x^2}} - \frac{1}{2k\pi} \right]$$

[$x > 0, \operatorname{Im} a = 0$].

$$5. \sum_{k=0}^{\infty} (-1)^k \left\{ \frac{\sin ka}{\cos ka} \right\} K_{k+1/2}(z) = \mp \frac{\pi}{2} \left\{ \frac{\sin(a/2)}{\cos(a/2)} \right\} e^{z \cos a} \operatorname{erfc} \left(\sqrt{2z \cos \frac{a}{2}} \right).$$

5.9.2. Ряды, содержащие произведения функций $J_\nu(z)$, $Y_\nu(z)$, $I_\nu(z)$, $K_\nu(z)$.

$$1. \sum_{k=0}^{\infty} (\pm 1)^k \cos ka J_k(w) Y_k(z) = \frac{1}{2} Y_0(\sqrt{w^2+z^2 \mp 2wz \cos a}) + \frac{1}{2} J_0(w) Y_0(z)$$

[$|we^{ia}|, |we^{-ia}| < |z|$].

$$2. \sum_{k=-\infty}^{\infty} (\pm 1)^k \cos ka J_k(w) Y_{k+\nu}(z) =$$

$$= \cos \left(\nu \arcsin \frac{w \sin a}{\sqrt{w^2+z^2 \mp 2wz \cos a}} \right) Y_\nu(\sqrt{w^2+z^2 \mp 2wz \cos a})$$

[$|we^{ia}|, |we^{-ia}| < |z|$].

$$3. \sum_{k=-\infty}^{\infty} (\pm 1)^k \sin ka J_k(w) Y_{k+\nu}(z) =$$

$$= \sin \left(\nu \arcsin \frac{w \sin a}{\sqrt{w^2+z^2 \mp 2wz \cos a}} \right) Y_\nu(\sqrt{w^2+z^2 \mp 2wz \cos a})$$

[$|we^{ia}|, |we^{-ia}| < |z|$].

$$4. \sum_{k=0}^{\infty} \cos a J_{kn}(w) Y_{kn}(z) =$$

$$= \frac{1}{2n} \sum_{k=k_-}^{k_+} \alpha_k Y_0 \left(\sqrt{w^2+z^2 - 2wz \cos \frac{2k\pi+a}{n}} \right) + \frac{1}{2} J_0(w) Y_0(z)$$

[$k_{\pm} = \pm \{(n\pi \mp a)/(2\pi)\}$; $\alpha_k = 1$ при $k \neq k_{\pm}$, $\alpha_{k_{\pm}} = 1/2$ при $(n\pi \mp a)/(2\pi) = \dots, -2, -1, 0, 1, 2, \dots$, $\alpha_{k_{\pm}} = 1$ при $(n\pi \mp a)/(2\pi) \neq \dots, -2, -1, 0, 1, 2, \dots$].

$$5. \sum_{k=0}^{\infty} (-1)^{k+1} \cos(2k+1)a J_{k+1/2}(bz) Y_{k+1/2}(cz) =$$

$$= \frac{1}{\pi} \int_{t_1}^{t_2} \frac{\cos tz dt}{\sqrt{b^2+c^2+2bc \cos 2a - t^2}}$$

[$t_1 = c-b, t_2 = \sqrt{b^2+c^2+2bc \cos 2a}$; $c > b$].

$$6. \sum_{k=0}^{\infty} (-1)^k J_{2k+(1 \pm 1)/2}(z) I_{2k+(1 \pm 1)/2}(z) = \frac{1}{2} \left\{ \begin{matrix} \operatorname{bei}(z\sqrt{2}) \\ \operatorname{ber}(z\sqrt{2}) + J_0(z) I_0(z) \end{matrix} \right\}.$$

$$7. \sum_{k=-\infty}^{\infty} (-1)^k J_{2k+n+(1 \pm 1)/2}(z) I_{2k+(1 \pm 1)/2}(z) = (-1)^n \left\{ \begin{matrix} \operatorname{bei}_n(z\sqrt{2}) \\ \operatorname{ber}_n(z\sqrt{2}) \end{matrix} \right\}.$$

8.
$$\sum_{k=1}^{\infty} [t^{-k} + (-t)^k] J_k(z) I_k(z) = J_0\left(z \sqrt{t - \frac{1}{t}}\right) - J_0(z) I_0(z)$$
9.
$$\sum_{k=-\infty}^{\infty} \left\{ \frac{\sin ka}{\cos ka} \right\} I_k(w) K_{k+v}(z) = \left\{ \frac{\sin v\psi}{\cos v\psi} \right\} K_v(\sqrt{w^2 + z^2 - 2wz \cos a})$$

$$[\psi = \arcsin(w \sin a / \sqrt{w^2 + z^2 - 2wz \cos a}); |we^{ia}|, |we^{-ia}| < |z|].$$
10.
$$\sum_{k=0}^{\infty} \cos ka I_k(w) K_k(z) = \frac{1}{2} K_0(\sqrt{w^2 + z^2 - 2wz \cos a}) + \frac{1}{2} I_0(w) K_0(z)$$

$$[|we^{\pm ia}| < |z|].$$
11.
$$\sum_{k=1}^{\infty} \cos ka I_{kn}(w) K_{kn}(z) =$$

$$= \frac{1}{2n} \sum_{k=k_-}^{k_+} K_0\left(\sqrt{w^2 + z^2 - 2wz \cos \frac{2k\pi + a}{n}}\right) - \frac{1}{2} I_0(w) K_0(z)$$

$$[k_{\pm} \text{ см. в 5.9.2.4}].$$
12.
$$\sum_{k=1}^{\infty} (\pm 1)^k \cos ka J_{2k}(z) K_{2k}(z) = \frac{1}{4} \left[K_0\left(2^{1/2} z e^{i\pi/4} \left\{ \frac{\sin(a/2)}{\cos(a/2)} \right\}^{1/2}\right) + \right.$$

$$\left. + K_0\left(2^{1/2} z e^{-i\pi/4} \left\{ \frac{\sin(a/2)}{\cos(a/2)} \right\}^{1/2}\right) \right] - \frac{1}{2} J_0(z) K_0(z).$$
13.
$$\sum_{k=0}^{\infty} (\pm 1)^k \left\{ \frac{\sin(2k+1)a}{\cos(2k+1)a} \right\} J_{2k+1}(z) K_{2k+1}(z) =$$

$$= \frac{i}{4} \left[K_0\left(2^{1/2} z e^{i\pi/4} \left\{ \frac{\sin a}{\cos a} \right\}^{1/2}\right) - K_0\left(2^{1/2} z e^{-i\pi/4} \left\{ \frac{\sin a}{\cos a} \right\}^{1/2}\right) \right].$$
14.
$$\sum_{k=0}^{\infty} (\pm 1)^k \left\{ \frac{\sin(2k+1)a}{\cos(2k+1)a} \right\} I_{k+1/2}(z) K_{k+1/2}(z) =$$

$$= \frac{\pi}{4} \left[I_0\left(2z \left\{ \frac{\sin a}{\cos a} \right\}\right) - L_0\left(2z \left\{ \frac{\sin a}{\cos a} \right\}\right) \right].$$
15.
$$\sum_{k=1}^{\infty} (-1)^k \cos ka I_{k-v}(z) K_{k+v}(z) = -\frac{\pi}{2 \sin 2v\pi} \left[J_{-2v}\left(2iz \cos \frac{a}{2}\right) + \right.$$

$$\left. + J_{-2v}\left(-2iz \cos \frac{a}{2}\right) - 2 \cos v\pi I_{-2v}\left(2z \cos \frac{a}{2}\right) - \frac{1}{2} I_{-v}(z) K_v(z) \right]$$

$$[\operatorname{Re} v < 1/2].$$
16.
$$\sum_{k=0}^{\infty} (-1)^k \cos(2k+1)a I_{k+v+1/2}(z) K_{k-v+1/2}(z) =$$

$$= \frac{\pi}{4 \sin 2v\pi} [i J_{2v}(2iz \cos a) - i J_{2v}(-2iz \cos a) + 2 \sin v\pi I_{2v}(2z \cos a)]$$

$$[\operatorname{Re} v > -1/2].$$
17.
$$\sum_{k=1}^{\infty} (-1)^k \cos ka [I_{k+v}(z) K_{k-v}(z) + I_{k-v}(z) K_{k+v}(z)] =$$

$$= 2 \cos v\pi K_{2v}\left(2z \cos \frac{a}{2}\right) - I_v(z) K_{-v}(z) - I_{-v}(z) K_v(z)$$

$$[\operatorname{Re} v > -1/2].$$

$$18. \sum_{k=0}^{\infty} (-1)^k \cos(2k+1)a [I_{k-\nu+1/2}(z) K_{k+\nu+1/2}(z) - I_{k+\nu+1/2}(z) K_{k-\nu+1/2}(z)] = \\ = 2 \sin \nu \pi K_{2\nu}(2z \cos a) \quad [\operatorname{Re} \nu > -1/2].$$

$$19. \sum_{k=0}^{\infty} (2k+\mu+\nu) J_{2k+\mu+\nu}(x) I_{k+\mu}(y) K_{k+\nu}(y) = \\ = \left(\frac{xy}{2}\right)^{\mu-\nu} \Gamma \left[\begin{matrix} 1-(\mu-\nu)/2, (1-\mu+\nu)/2 \\ 1-\mu \end{matrix} \right] {}_2F_1 \left(1-\frac{\mu-\nu}{2}, \frac{1-\mu+\nu}{2}; 1-\mu; -\frac{4}{x^2 y^2} \right).$$

$$20. \sum_{k=0}^{\infty} (2k+\nu) J_{2k+\nu}(x) J_{2k+\nu}(y) K_{2k+\nu}(z) = \\ = \frac{1}{4} \exp \left(-\frac{\sqrt{yz}}{2x} - \frac{x(z-y)}{2\sqrt{yz}} \right) \quad [x, y, z, \operatorname{Re} \nu > 0].$$

$$21. \sum_{k=0}^{\infty} (2k+\mu+\nu) I_{k+\mu}(x) I_{k+\mu}(y) K_{k+\nu}(x) K_{k+\nu}(y) = \\ = \frac{1}{4\pi} G_{33}^{\frac{1}{2}} \left(\frac{x^2}{y^2} \middle| \begin{matrix} 0, 1-(\mu-\nu)/2, 1+(\mu-\nu)/2 \\ (\mu-\nu)/2, (\nu-\mu)/2, 1 \end{matrix} \right).$$

5.10. МНОГОЧЛЕНЫ ЛЕЖАНДРА $P_n(x)$

5.10.1. Ряды вида $\sum a_k P_{nk+m}(x)$.

$$1. \sum_{k=0}^{\infty} (\pm 1)^k P_k(x) = \frac{1}{\sqrt{2(1 \mp x)}} \quad [|x| < 1].$$

$$2. \sum_{k=0}^{\infty} t^k P_k(x) = \frac{1}{\sqrt{1-2tx+t^2}} \quad [|t| < \min |x \pm \sqrt{x^2-1}|].$$

$$3. \sum_{k=1}^{\infty} k t^k P_k(x) = \frac{t(x-t)}{(1-2tx+t^2)^{3/2}} \quad [|t| < \min |x \pm \sqrt{x^2-1}|].$$

$$4. \sum_{k=1}^{\infty} \frac{t^k}{k} P_k(x) = \ln \frac{2}{1-tx+\sqrt{1-2tx+t^2}} \quad [|x| \leq 1].$$

$$5. \sum_{k=0}^{\infty} \frac{1}{k+1} P_k(x) = \ln \left(1 + \sqrt{\frac{2}{1-x}} \right).$$

$$6. \sum_{k=2}^{\infty} \frac{1}{k-1} P_k(\cos x) = -\cos x \ln \left[\sin \frac{x}{2} \left(1 + \sin \frac{x}{2} \right) \right] - 2 \sin \frac{x}{2} \left(1 - \sin \frac{x}{2} \right).$$

$$7. \sum_{k=0}^{\infty} \frac{k!}{(k+n)!} t^{k+n} P_k(x) = S_n \quad [|t| < 1; -1 < x < 1; \rho = \sqrt{1-2tx+t^2}],$$

S_0 см. в 5.10.1.2,

$$S_1 = \ln \left| \frac{\rho+t-x}{1-x} \right|, \quad S_2 = (t-x) \ln \left| \frac{\rho+t-x}{1-x} \right| + 1 - \rho,$$

$$S_n = \frac{2n-1}{n^2} (t-x) S_{n-1} - \frac{\rho^2}{n^2} S_{n-2} + \frac{t^{n-1}}{n! n}.$$

$$8. \sum_{k=0}^{\infty} \frac{(k+n)!}{k!} t^k P_{k+n}(x) = n! \rho^{-n-1} P_n \left(\frac{x-t}{\rho} \right) \\ [-1 \leq x \leq 1; 0 < t < 1; \rho = \sqrt{1-2tx+t^2}].$$

9. $\sum_{k=0}^{\infty} \frac{1}{2k+1} P_k(x) = \frac{\pi}{4} P_{-1/2}(-x).$
10. $\sum_{k=1}^{\infty} \frac{1}{(2k-1)(2k+3)} P_k(x) = \frac{1}{3} - \sqrt{\frac{1-x}{8}} \quad [-1 < x < 1].$
11. $\sum_{k=0}^{\infty} (-1)^k \frac{2k+1}{(k-a)(k+a+1)} P_k(x) = -\frac{\pi}{\sin a\pi} P_a(x)$
 $[-1 < x \leq 1; a \neq 0, \pm 1, \pm 2, \dots].$
12. $\sum_{k=0}^{\infty} \frac{x^{k+1}}{k+1} P_k(x) = \frac{1}{2} \ln \frac{1+x}{1-x} \quad [-1 < x < 1].$
13. $\sum_{k=0}^{\infty} \frac{t^k}{k!} P_k(x) = e^{tx} J_0(t\sqrt{1-x^2}) \quad [-1 \leq x \leq 1].$
14. $\sum_{k=0}^{\infty} \frac{t^{2k+\delta}}{(2k+\delta)!} P_{2k+\delta}(x) = \left\{ \begin{matrix} \text{sh } tx \\ \text{ch } tx \end{matrix} \right\} J_0(t\sqrt{1-x^2})$
 $[-1 \leq x \leq 1; \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}].$
15. $\sum_{k=0}^{\infty} \frac{t^k}{(a)_k} P_k(x) = \Phi_2\left(\frac{1}{2}, \frac{1}{2}; a; tu_-, tu_+\right)$
 $[u_{\pm} = x \pm \sqrt{x^2-1}; |tu_{\pm}| < 1].$
16. $\sum_{k=0}^{\infty} \frac{(a)_k}{k!} t^k P_k(x) = (1-tu_+)^{-a} {}_2F_1\left(\frac{1}{2}, a; 1; \frac{tu_- - tu_+}{1-tu_+}\right)$
 $[u_{\pm} = x \pm \sqrt{x^2-1}; |tu_{\pm}| < 1],$
 $= (1-2tx+t^2)^{-a/2} {}_2F_1\left(a, 1-a; 1; \frac{1}{2} - \frac{1-tx}{2\sqrt{1-2tx+t^2}}\right).$
17. $\sum_{k=0}^{\infty} (2k+1) \frac{(2-a)_k}{(a)_k} P_k(x) = (a-1) \left(\frac{1-x}{2}\right)^{a-2} \quad [-1 < x < 1; a > 5/4].$
18. $\sum_{k=0}^{\infty} (-1)^k (4k+1) \frac{(3/2-a)_k}{(a)_k} P_{2k}(x) = 2(a-1) |x|^{2a-3}$
 $[-1 < x < 1; a > 1].$
19. $\sum_{k=0}^{\infty} \frac{t^k}{(k!)^2} P_k(x) = I_0(2t\sqrt{x-1}) I_0(2t\sqrt{x+1}).$
20. $\sum_{k=0}^{\infty} \frac{(a)_k}{(k!)^2} t^k P_k(x) = \Psi_2\left(a; 1, 1; \frac{tx-t}{2}, \frac{tx+t}{2}\right).$
21. $\sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{(k!)^2} t^k P_k(x) = F_4\left(a, b; 1, 1; \frac{tx-t}{2}, \frac{tx+t}{2}\right).$
22. $\sum_{k=0}^{\infty} \frac{(a)_k (1-a)_k}{(k!)^2} t^k P_k(x) = {}_2F_1\left(a, 1-a; 1; \frac{1-t-\rho}{2}\right) {}_2F_1\left(a, 1-a; 1; \frac{1+t-\rho}{2}\right)$
 $[\rho = \sqrt{1-2tx+t^2}].$

$$23. \sum_{k=0}^{\infty} (4k+1) \frac{(a)_k (-a)_k}{(3/2+a)_k (3/2-a)_k} P_{2k}(\cos x) = \frac{1-4a^2}{\sin 2a\pi} [\sin 2a(\pi-x) + \sin 2ax] \quad [0 \leq x \leq \pi].$$

$$24. \sum_{k=0}^{\infty} (4k+2n-1) \frac{(-1/2)_k (n-1/2)_k}{k! (k+n)!} P_{2k+n-1}(x) = \frac{4 \sin(n \arccos x)}{n \sqrt{\pi} \Gamma(n-1/2)}.$$

$$25. \sum_{k=0}^{\infty} (4k+2\sigma+1) \frac{(\sigma+1/2)_k (\sigma+3/2-a)_k}{k! (a)_k} P_{2k+\sigma}(x) = \\ = \frac{2^{\sigma+1}}{\sqrt{\pi}} \Gamma \left[\begin{matrix} a \\ a-\sigma-1/2 \end{matrix} \right] x^{\sigma} (1-x^2)^{a-\sigma-3/2} \quad [\sigma=0 \text{ или } 1; -1 < x < 1].$$

$$26. \sum_{k=0}^{\infty} (4k+3) \frac{(1/2-a)_k (1/2+a)_k}{(2-a)_k (2+a)_k} P_{2k+1}(\cos x) = \\ = \frac{4(a^2-1)}{\sin 2a\pi} [\sin 2ax - \sin 2a(\pi-x)].$$

$$27. \sum_{k=0}^{\infty} (4k+3) \left[\frac{(1/2)_k}{(k+1)!} \right]^2 P_{2k+1}(x) = \frac{8}{\pi} \arcsin x.$$

$$28. \sum_{k=0}^{\infty} (-1)^k (4k+1) \left[\frac{(1/2)_k}{k!} \right]^3 P_{2k}(\cos x) = \frac{4}{\pi^2} K(\sin x).$$

$$29. \sum_{k=0}^{\infty} (-1)^k (4k+1) \frac{(-1/2)_k}{(k+1)!} \left[\frac{(1/2)_k}{k!} \right]^2 P_{2k}(\cos x) = \frac{8}{\pi^2} E(\sin x).$$

5.10.2. Ряды вида $\sum a_k \prod_i P_k(x_i)$.

$$1. \sum_{k=0}^{\infty} t^k P_k(\cos x) P_k(\cos y) = \frac{4}{\pi(u_+ + u_-)} K \left(\frac{u_+ - u_-}{u_+ + u_-} \right) \\ [|t| < 1; u_{\pm} = \sqrt{1 - 2t \cos(x \pm y) + t^2}].$$

$$2. \sum_{k=1}^{\infty} \frac{2k+1}{k(k+1)} P_k(x) P_k(y) = 2 \ln 2 - 1 - \ln |(1-x)(1+y)| \\ [-1 < x \leq y < 1].$$

$$3. \sum_{k=0}^{\infty} (-1)^k \cos(2k+1)a [P_k(\cos x)]^2 = \frac{1}{\pi} \left\{ \begin{matrix} \operatorname{cosec} x \\ \sec a \end{matrix} \right\} K(\operatorname{cosec} x \cos^{\pm 1} a) \\ \left[0 < x < \pi, \left\{ \begin{matrix} \cos a < \sin x \\ \cos a > \sin x \end{matrix} \right\} \right].$$

$$4. \sum_{k=0}^{\infty} (2k+1) P_k(x) P_k(y) P_k(z) = \frac{2}{\pi} (1 + 2xyz - x^2 - y^2 - z^2)^{-1/2}_+.$$

5.10.3. Ряды, содержащие специальные функции и $P_n(x)$.
Обозначение: $\rho = (w^2 + z^2 - 2wz \cos x)^{1/2}$.

$$1. \sum_{k=0}^{\infty} (2k+1) J_{2k+1}(z) P_k(x) = \frac{z}{2} J_0 \left(z \sqrt{\frac{1-x}{2}} \right) \quad [-1 \leq x \leq 1].$$

$$2. \sum_{k=0}^{\infty} (2k+1) I_{k+1/2}(z) P_k(x) = \sqrt{\frac{2z}{\pi}} e^{xz} \quad [-1 \leq x \leq 1].$$

$$3. \sum_{k=0}^{\infty} (\pm 1)^k J_{k+1/2}(w) J_{\pm k \pm 1/2}(z) P_k(\cos x) = \frac{2\sqrt{wz}}{\pi\rho} \begin{cases} \sin \rho \\ \cos \rho \end{cases} \\ [|we^{ix}|, |we^{-ix}| < |z|].$$

$$4. \sum_{k=0}^{\infty} (2k+1) J_{k+1/2}(w) H_{k+1/2}^{(j)}(z) P_k(\cos x) = \frac{2(-1)^{j+1}\sqrt{wz}}{\pi i \rho} e^{(-1)^{j+1}\rho} \\ [j=1, 2; |we^{\pm ix}| < |z|].$$

$$5. \sum_{k=0}^{\infty} (2k+1) I_{k+1/2}(w) K_{k+1/2}(z) P_k(\cos x) = \frac{\sqrt{wz}}{\rho} e^{-\rho} \quad [|we^{\pm ix}| < |z|].$$

5.11. МНОГОЧЛЕНЫ ЛАГЕРРА $L_n^\alpha(x)$

5.11.1. Ряды вида $\sum a_k L_{k+n}^\alpha(x)$.

$$1. \sum_{k=0}^{\infty} \frac{1}{k+b} L_k^\alpha(x) = \Gamma \left[\begin{matrix} -\alpha, b \\ b-\alpha \end{matrix} \right] {}_1F_1(b; \alpha+1; x) + \Gamma(\alpha) x^{-\alpha} {}_1F_1(b-\alpha; 1-\alpha; x)$$

$$2. \sum_{k=0}^{\infty} \frac{1}{k+\alpha} L_k^\alpha(x) = \Gamma(\alpha) x^{-\alpha} \quad [x > 0].$$

$$3. \sum_{k=1}^{\infty} \frac{1}{k} L_k(x) = -\ln x - C \quad [x > 0].$$

$$4. \sum_{k=0}^{\infty} \frac{1}{k+1} L_k(x) = -e^x \operatorname{Ei}(-x) \quad [x > 0].$$

$$5. \sum_{k=0}^{\infty} \frac{1}{k+1} L_k^\alpha(x) = x^{-\alpha} e^x \Gamma(\alpha, x) \quad [x > 0; \alpha > -1].$$

$$6. \sum_{k=0}^{\infty} \frac{1}{2k+1} L_k(x) = \frac{1}{2} e^{x/2} K_0\left(\frac{x}{2}\right).$$

$$7. \sum_{k=1}^{\infty} \frac{(-1)^k}{(k)_{n+1}} L_{k+n}(x) = \\ = \frac{1}{n!} \left\{ \frac{x^n}{n!} \left[\sum_{k=1}^n \frac{(-1)^k}{k} \binom{n}{k} - \psi(n+1) + \ln x \right] - \sum_{k=1}^n \frac{(-1)^k x^{n-k}}{(n-k)! k} \binom{n}{k} \right\} \quad [x > 0].$$

$$8. \sum_{k=0}^{\infty} \frac{(b)_k}{(c)_k} L_k^\alpha(x) = \Gamma \left[\begin{matrix} c, b-c+\alpha+1 \\ b, c-b \end{matrix} \right] x^{c-b-\alpha-1} {}_1F_1(-\alpha-1; c-b-\alpha; x) + \\ + \Gamma \left[\begin{matrix} c, c-b-\alpha-1 \\ c-b, c-\alpha-1 \end{matrix} \right] {}_1F_1(b; \alpha+b-c+2; x) \\ [-1 < \operatorname{Re} \alpha < 2 \operatorname{Re}(c-b) - 1/2].$$

$$9. \sum_{k=1}^{\infty} \frac{(k-1)!}{(\alpha+1)_k} L_k^\alpha(x) = \psi(\alpha+1) - \ln x \quad [\operatorname{Re} \alpha > -1; x > 0].$$

$$10. \sum_{k=0}^{\infty} \frac{(b)_k}{(\alpha+1)_k} L_k^\alpha(x) = \Gamma \left[\begin{matrix} \alpha+1 \\ \alpha-b+1 \end{matrix} \right] x^{-b} \\ [\operatorname{Re} \alpha > -1; \operatorname{Re} b < \min(\operatorname{Re} \alpha + 1, (2\operatorname{Re} \alpha + 1)/4); x > 0].$$

$$11. \sum_{k=0}^{\infty} \frac{(1/2)_k}{(\alpha+2)_k} L_k^\alpha(x) = \sqrt{\pi} \Gamma\left[\frac{\alpha+2}{\alpha+3/2}\right] e^x \operatorname{erfc}(\sqrt{x}).$$

5.11.2. Ряды вида $\sum a_k t^k L_{k+n}^\alpha(ka+x)$.

$$1. \sum_{k=0}^{\infty} t^k L_k^\alpha(x) = (1-t)^{-\alpha-1} \exp\left(\frac{tx}{t-1}\right) \quad [|t| < 1].$$

$$2. \sum_{k=0}^{\infty} \frac{t^k}{k+\alpha} L_k^\alpha(x) = (tx)^{-\alpha} \psi\left(\alpha, \frac{tx}{1-t}\right) \quad [\operatorname{Re} \alpha > 0; |t| < 1; x > 0].$$

$$3. \sum_{k=0}^{\infty} \frac{t^{k+1}}{k+1} L_k^\alpha(x) = -e^x \operatorname{Ei}(-x) + e^x \operatorname{Ei}\left(\frac{x}{1-t}\right).$$

$$4. \sum_{k=1}^{\infty} \frac{2^{-k}}{k} L_k^\alpha(x) = \operatorname{Ei}(-x) - \ln \frac{x}{2} - C.$$

$$5. \sum_{k=0}^{\infty} \frac{t^k}{(\alpha-n+1)_k} L_k^\alpha(x) = \\ = \Gamma(\alpha-n+1) (tx)^{(n-\alpha)/2} e^t \sum_{k=0}^n (-1)^k \binom{n}{k} \left(\frac{t}{x}\right)^{k/2} J_{\alpha-k}(2\sqrt{tx}).$$

$$6. \sum_{k=0}^{\infty} \frac{(b)_k}{(\alpha+1)_k} t^k L_k^\alpha(x) = (1-t)^{-b} {}_1F_1\left(b; \alpha+1; -\frac{xt}{1-t}\right) \quad [|t| < 1; \alpha > -1; x > 0].$$

$$7. \sum_{k=0}^{\infty} \frac{(a)_k}{(\alpha+\beta)_k} t^k L_k^\alpha(x) = (1-t)^{-a} \Phi_1\left(a, \beta-1; \alpha+\beta; \frac{t}{t-1}, \frac{tx}{t-1}\right) \quad [-1 \leq t < 1; \operatorname{Re} \alpha > -1].$$

$$8. \sum_{k=0}^{\infty} \binom{k+n}{k} t^k L_{k+n}^\alpha(x) = (1-t)^{-\alpha-n-1} e^{-tx/(1-t)} L_n^\alpha\left(\frac{x}{1-t}\right).$$

$$9. \sum_{k=0}^{\infty} \frac{(n+\alpha+1)_k}{(\alpha+1)_k} t^k L_k^\alpha(x) = n! \Gamma\left[\frac{\alpha+1}{n+\alpha+1}\right] (1-t)^{-n-\alpha-1} e^{-tx/(1-t)} L_n^\alpha\left(\frac{tx}{1-t}\right).$$

$$10. \sum_{k=0}^{\infty} \frac{(1/2)_k}{(\alpha+1)_k} t^k L_{2k}^\alpha(x) = (1-t^2)^{-1/2} \left(\frac{tx}{2}\right)^{-\alpha/2} e^{-tx/(1-t)} I_{\alpha/2}\left(\frac{tx}{1-t}\right).$$

$$11. \sum_{k=1}^{\infty} \frac{(-1)^k}{k} 2^{-k/2} \begin{cases} \sin(k\pi/4) \\ \cos(k\pi/4) \end{cases} L_k(x) = \begin{cases} \pi/4 - \operatorname{Si}(x) \\ \ln(\sqrt{2}/x) - C + \operatorname{ci}(x) \end{cases}.$$

$$12. \sum_{k=0}^{\infty} \frac{(ka+b)_k}{(\alpha+1)_k} t^k L_k^\alpha(ka+x) = \frac{\Gamma(\alpha+1) e^{bu}}{1-au} (bxu)^{-\alpha/2} J_\alpha(2\sqrt{bxu})$$

$$[a = te^{au}].$$

$$13. \sum_{k=0}^{\infty} \frac{(ka+1)_k}{(\alpha+1)_k} t^k L_k^\alpha\left(\frac{x}{ka+1}\right) = \frac{\Gamma(\alpha+1) e^z}{(1-az)(xz)^{\alpha/2}} J_\nu(2\sqrt{xz})$$

$$[t = ze^{-az}; |aze^{1-az}| < 1].$$

$$14. \sum_{k=0}^{\infty} \frac{(ka+1)^{k-1}}{(\alpha+1)_k} t^k L_k^{\alpha} \left(\frac{x}{ka+1} \right) = e^{xz} {}_1F_2 \left(\frac{1}{a}; \alpha+1, \frac{1}{a}+1; -xz \right) \\ [z = ze^{-az}; |aze^{1-az}| < 1].$$

$$15. \sum_{k=0}^{\infty} t^k \left\{ \begin{matrix} \sin ka \\ \cos ka \end{matrix} \right\} L_k(x) = \\ = \rho e^{\rho t x (t - \cos a)} \left[\left\{ \frac{-1+t \cos a}{t \sin a} \right\} \sin(\rho t x \sin a) + \left\{ \frac{t \sin a}{1-t \cos a} \right\} \cos(\rho t x \sin a) \right] \\ [\rho = (1 - 2t \cos a + t^2)^{-1/2}].$$

5.11.3. Ряды вида $\sum a_k L_k^{\alpha} L_k^{\beta}(x) L_k^{\beta}(y)$.

$$1. \sum_{k=0}^{\infty} \frac{1}{k+1} L_k(x) L_k(y) = -e^{x+y} \text{Ei}(-x) \quad [0 < y \leq x].$$

$$2. \sum_{k=0}^{\infty} \frac{1}{2k+1} L_k(x) L_k(y) = \frac{1}{2} e^{(x+y)/2} I_0\left(\frac{x}{2}\right) K_0\left(\frac{y}{2}\right) \quad [x \leq y].$$

$$3. \sum_{k=0}^{\infty} \frac{k!}{(\beta+1)_k} L_k^{\alpha}(x) L_k^{\beta}(y) = \Gamma\left[\frac{\beta+1}{\beta-\alpha}\right] y^{-\alpha} (y-x)^{\beta-\alpha-1} e^{xy} \eta(y-x) \\ [\eta(t) = 0, t < 0; \eta(1) = 1, t > 0; \eta(0) = 1/2].$$

$$4. \sum_{k=0}^{\infty} \frac{k!}{(\alpha+1)_k} L_k^{\alpha}(x) L_{k+1}^{\alpha-1}(y) = x^{-\alpha} e^x [\alpha \Gamma(\alpha, y) - \Gamma(\alpha+1) \eta(y-x)] \\ [x, y, \alpha > 0; \eta(t) \text{ см. в 5.11.3.3}].$$

$$5. \sum_{k=0}^{\infty} t^k L_k(x) L_k^n(y) = \\ = (1-t)^{-n-1} \exp\left(\frac{x+y}{t-1}t\right) \sum_{k=0}^n (-1)^k \binom{n}{k} \left(\frac{tx}{y}\right)^{k/2} I_k\left(\frac{2\sqrt{txy}}{1-t}\right).$$

$$6. \sum_{k=0}^{\infty} \frac{k!}{(\alpha+1)_k (k+1)} L_k^{\alpha}(x) L_k^{\alpha}(y) = \alpha (xy)^{-\alpha} e^{x+y} \gamma(c, x) \Gamma(\alpha, y) \\ [0 < x \leq y].$$

$$7. \sum_{k=0}^{\infty} \frac{k!}{(\alpha+1)_k} t^k L_k^{\alpha}(x) L_{k+n}^{\alpha}(y) = \Gamma(\alpha+1) (1-t)^{-n-1} (txy)^{-\alpha/2} \times \\ \times \exp\left(\frac{x+y}{t-1}t\right) \sum_{k=0}^n (-1)^k \binom{n}{k} \left(\frac{tx}{y}\right)^{k/2} I_{k+\alpha}\left(\frac{2\sqrt{txy}}{1-t}\right) \quad [|t| < 1].$$

5.11.4. Ряды вида $\sum a_k L_{nk}^{\alpha} L_m^{\beta}(x)$.

$$1. \sum_{k=1}^{\infty} t^k L_n^{-n-k}(x) = \frac{t}{(1-t)^n} L_n^{-n-1}(x-tx).$$

$$2. \sum_{k=0}^{\infty} \frac{t^k}{k!} L_n^{k-n}(x) = \frac{e^t}{n!} (t-x)^n.$$

$$3. \sum_{k=0}^{\infty} \frac{t^k}{k!} L_n^{k+\alpha}(x) = e^t L_n^{\alpha}(x-t).$$

4. $\sum_{k=0}^{\infty} \frac{t^k}{k!} L_n^{n-k-1}(x) = (-x)^n e^t \sum_{k=0}^n \frac{x^{-k}}{(n-k)!} L_k(-t).$
5. $\sum_{k=0}^{\infty} \frac{(\alpha)_k}{k!} t^k L_n^{k-n}(x) = \frac{(-t)^n}{(1-t)^{-n-\alpha}} L_n^{-n-\alpha} \left(x - \frac{x}{t}\right).$
6. $\sum_{k=0}^{\infty} t^k L_k^{\alpha k + \beta}(x) = \frac{(1+u)^{\beta+1}}{1-\alpha u} e^{-xu} \quad [u = v(t)(1+v(t))^{-\alpha-1}, v(0) = 0].$
7. $\sum_{k=0}^{\infty} t^k L_k^{k+\alpha}(x) = \frac{1}{\sqrt{1-4t}} \left(\frac{2}{1+\sqrt{1-4t}}\right)^{\alpha} \exp\left(\frac{-2tx}{1-2t+\sqrt{1-4t}}\right).$
8. $\sum_{k=0}^{\infty} t^k L_k^{\alpha-k}(x) = (1+t)^{\alpha} e^{-tx} \quad [|t| < 1].$
9. $\sum_{k=0}^{\infty} t^k L_k^{\alpha-2k}(x) = \frac{1}{\sqrt{1+4t}} \left(\frac{1+\sqrt{1+4t}}{2}\right)^{\alpha+1} \exp\left(\frac{1-\sqrt{1+4t}}{2} x\right).$
10. $\sum_{k=0}^{\infty} \frac{t^k}{k+1} L_k^{-2k-1}(x) = \frac{1}{tx} - \frac{1}{tx} \exp\left(\frac{1-\sqrt{1+4t}}{2} x\right).$
11. $\sum_{k=0}^{\infty} \frac{t^k}{(\beta)_k} L_k^{\alpha-k}(x) = \Phi_3(-\alpha; \beta; -t, -tx).$
12. $\sum_{k=0}^{\infty} \frac{t^k}{(\alpha/2+1)_k} L_k^{k+\alpha}(x) = (t^2-tx)^{-\alpha/2} e^{2t} I_{\alpha/2}(2\sqrt{t^2-tx}).$
13. $\sum_{k=0}^{\infty} \binom{n+k}{k} t^k L_{n+k}^{\alpha-k}(x) = (1+t)^{\alpha} e^{-tx} L_n^{\alpha}(tx+x).$
14. $\sum_{k=0}^{\infty} \binom{n+k}{k} t^k L_{n+k}^{-2n-2k-1}(x) =$
 $= (1+4t)^{-n-1/2} \exp\left(\frac{1-\sqrt{1+4t}}{2} x\right) L_n^{-2n-1}(x\sqrt{1+4t}).$
15. $\sum_{k=0}^{\infty} \frac{(\gamma)_k}{(\beta)_k} t^k L_k^{\alpha-k}(x) = \Phi_1(\gamma, -\alpha; \beta; -t, tx) \quad [|t| < 1].$
16. $\sum_{k=0}^{\infty} t^k L_k^{\alpha-k/2}(x) = \frac{2(1+u)^{\alpha+1}}{2+u} e^{-xu} \quad [u = t(t+\sqrt{t^2+4})/2].$
17. $\sum_{k=0}^{\infty} \frac{t^k}{\alpha+k/2} L_k^{\alpha-k/2}(x) = \frac{1}{\alpha} (1+u)^{\alpha} {}_1F_1(2\alpha; 2\alpha+1; -xu)$
 $[u = t(t+\sqrt{t^2+4})/2].$
18. $\sum_{k=0}^{\infty} \frac{(\beta)_k(-\alpha-\beta)_k}{(-\alpha)_k} t^k L_k^{\alpha-2k}(x) =$
 $= \int_0^{\infty} e^{-\tau} {}_2F_1\left(\beta, -\alpha-\beta; 1; \frac{1+tx-u}{2}\right) {}_2F_1\left(\beta, -\alpha-\beta; 1; \frac{1-tx-u}{2}\right) d\tau$
 $[u = (1+2t(x+2\tau)+\tau^2)^{1/2}].$

5.11.5. Ряды вида $\sum a_k L_{n,k}^{\alpha_1 k + \beta_1} (x) L_{n,k}^{\alpha_2 k + \beta_2} (y)$.

$$1. \sum_{k=0}^{\infty} t^k L_k^{\alpha-n} (x) L_n^{k-n} (y) = t^n (1+t)^{\alpha-n} e^{-tx} L_n^{\alpha-n} \left(\frac{1+t}{t} (tx+y) \right) \quad [|t| < 1].$$

$$2. \sum_{k=0}^{\infty} k! t^k L_k^{\alpha-n} (x) L_n^{k-n} (y) = n! t^n (1-ty)^{\alpha-n} e^{txy} L_n^{\alpha-n} \left(-\frac{(1-tx)(1-ty)}{t} \right).$$

$$3. \sum_{k=0}^{\infty} \frac{y^k}{(\alpha+1)_k} L_k^{\alpha} (x) L_n^{k+\beta} (y) = \frac{(\beta+1)_n}{n!} e^y {}_1F_2 (\beta+n+1; \alpha+1, \beta+1; -xy).$$

$$4. \sum_{k=0}^{\infty} \frac{y^k}{(\alpha+1)_k} L_k^{\alpha} (x) L_n^{k-n+\alpha} (y) = \frac{\Gamma(\alpha+1)}{n!} (xy)^{(n-\alpha)/2} J_{\alpha-n} (2\sqrt{xy}).$$

$$5. \sum_{k=0}^{\infty} \frac{k!}{(\alpha+1)_k} t^k L_k^{\alpha} (x) L_n^{k-\beta} (y) = (1+t)^{\beta} e^{-ty} \Phi_3 \left(-\beta; \alpha+1; \frac{tx}{1+t}, txy \right).$$

$$6. \sum_{k=0}^{\infty} \frac{(-1)^k k!}{(\alpha+1)_k} L_k^{\alpha} (x) L_n^{k-\beta} (y) = \Gamma(\alpha+1) x^{(\beta-\alpha)/2} y^{-(\alpha+\beta)/2} e^y J_{\alpha+\beta} (2\sqrt{xy})$$

$[x > 0; \operatorname{Re} \alpha > -1].$

$$7. \sum_{k=0}^{\infty} \frac{k!}{(\alpha+1)_k} t^k L_k^{\alpha} (x) L_n^{k-\alpha} (y) = \Gamma(\alpha+1) (txy)^{-\alpha/2} e^{-ty} I_{\alpha} (2\sqrt{txy}).$$

5.11.6. Ряды, содержащие специальные функции и $L_n^{\alpha} (x)$.

$$1. \sum_{k=0}^{\infty} \frac{1}{\Gamma(k+\alpha+1)} \gamma(k+\alpha+1, x) L_k^{\alpha} (y) = \left(\frac{x}{y} \right)^{(\alpha+1)/2} J_{\alpha+1} (2\sqrt{xy}).$$

$$2. \sum_{k=0}^{\infty} (-1)^k \frac{k! (2k+\alpha+1)}{(\alpha+1)_k} I_{k+(\alpha+1)/2} (z) L_k^{\alpha} (x) L_k^{\alpha} (-x) =$$

$$= \frac{2}{\Gamma((\alpha+1)/2)} \left(\frac{z}{2} \right)^{(\alpha+1)/2} e^{-z} {}_0F_2 \left(\frac{\alpha+1}{2}, \alpha+1; \frac{x^2 z}{2} \right).$$

$$3. \sum_{k=0}^{\infty} t^k P_k (x) L_k (y) = \rho \exp (\rho^2 t y (t-x)) J_0 (\rho^2 t y \sqrt{1-x^2})$$

$[\rho = (1-2tx+t^2)^{-1/2}].$

5.12. МНОГОЧЛЕНЫ ЭРМИТА $H_n (x)$

5.12.1. Ряды вида $\sum a_k H_{nk+m} (\varphi (k, x))$.

$$1. \sum_{k=0}^{\infty} \frac{t^k}{k!} H_k (ka+x) = \frac{e^{2xz-z^2}}{1-2az} \quad [t = ze^{-2az}].$$

$$2. \sum_{k=0}^{\infty} \frac{t^k}{[k/2]!} H_k (x) = (1+4t^2)^{-3/2} (1+2xt+4t^2) e^{4x^2 t^2 / (1+4t^2)}.$$

$$3. \sum_{k=0}^{\infty} \frac{t^k}{k!} H_{k+n} (x) = e^{2xt-t^2} H_n (x-t).$$

4. $\sum_{k=0}^{\infty} \frac{t^k}{k!} H_{2k+n}(x) = (1+4t)^{-(n+1)/2} e^{4tx^2/(1+4t)} H_n\left(\frac{x}{\sqrt{1+4t}}\right) \quad [|t| < 1/2].$
5. $\sum_{k=0}^{\infty} \frac{(\pm 1)^k t^{2k}}{(2k+n)!} H_{2k+n}(x) =$
 $= (\pm 1)^{[n/2]} \frac{1}{t^n} e^{\mp t^2} \left[\frac{1 - (-1)^n}{2} \left\{ \frac{\operatorname{sh} 2xt}{\sin 2xt} \right\} + \frac{1 + (-1)^n}{2} \left\{ \frac{\operatorname{ch} 2xt}{\cos 2xt} \right\} \right].$
6. $\sum_{k=0}^{\infty} \frac{(-1)^k t^{2k+\delta}}{(2k+\delta)!} H_{2k+\delta}(x) = e^{t^2} \begin{Bmatrix} \sin 2xt \\ \cos 2xt \end{Bmatrix} \quad [\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}].$
7. $\sum_{k=0}^{\infty} \frac{t^{2k+1}}{(2k+1)!} H_{2k}(x) = \frac{\sqrt{\pi}}{4} e^{x^2} [\operatorname{erf}(x+t) - \operatorname{erf}(x-t)].$
8. $\sum_{k=0}^{\infty} \frac{t^{2k+2}}{(2k+2)!} H_{2k+1}(x) = \frac{\sqrt{\pi}}{2} e^{x^2} \operatorname{erf}(x) - \frac{\sqrt{\pi}}{4} e^{x^2} [\operatorname{erf}(x+t) + \operatorname{erf}(x-t)].$
9. $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1} H_{2k}(x) = \frac{\sqrt{\pi}}{2} e^{x^2} [C(2x^2) - S(2x^2)].$
10. $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+2)!} x^{2k+2} H_{2k+1}(x) = \frac{\sqrt{\pi}}{2} e^{x^2} [C(2x^2) + S(2x^2)] - \frac{\sqrt{\pi}}{2} e^{x^2} \operatorname{erf}(x).$
11. $\sum_{k=0}^{\infty} \frac{(-1)^k t^{2k+1}}{k! (2k+1)} H_{2k+1}(x) = \frac{\sqrt{\pi}}{2} \operatorname{erf}\left(\frac{2tx}{\sqrt{1-4t^2}}\right).$
12. $\sum_{k=0}^{\infty} \frac{(-1)^k}{k! (2k+1) 2^{2k}} H_{2k}(x) = \frac{\pi}{2} e^{x^2} \operatorname{erfc}(x) \quad [0 \leq x < \infty].$
13. $\sum_{k=0}^{\infty} \frac{(-1)^k}{k! (2k+1) 2^{2k}} H_{2k+1}(x) = \sqrt{\pi} \operatorname{sgn} x.$
14. $\sum_{k=0}^{\infty} \frac{(-1)^k}{k! (k+a) 2^{2k}} H_{2k+a}(x) = 2^{(2a+\sigma)/2} \Gamma(a) e^{x^2/2} D_{\sigma-2a}\left(\frac{x}{\sqrt{2}}\right)$
 $[\sigma = 0 \text{ или } 1].$
15. $\sum_{k=0}^{\infty} \frac{(-1)^k}{k! (2k+1) 2^{2k}} H_{2k}(x) = \frac{\pi}{2\sqrt{2}} e^{x^2} [1 - \operatorname{erf}^2(x)].$
16. $\sum_{k=0}^{\infty} \frac{(-1)^k}{k! (2k+1) 2^{2k}} H_{2k+1}(x) = \sqrt{2\pi} \operatorname{erf}(x).$
17. $\sum_{k=0}^{\infty} \frac{(-1)^k}{k! (2k+1) (2k+3) 2^{2k}} H_{2k+1}(x) = \frac{\pi}{2} x e^{x^2} \operatorname{erfc}(x).$
18. $\sum_{k=0}^{\infty} \frac{(-1)^k}{2^{2k} (3/2)_k} H_{2k+1}(x) = 2\sqrt{\pi} e^{-x^2} \operatorname{erfi}(x).$
19. $\sum_{k=0}^{\infty} \frac{(a)_k}{(2k+\delta)!} t^k H_{2k+\delta}(x) = (1+t)^{-a} \begin{Bmatrix} 2x \\ 1 \end{Bmatrix} {}_1F_1\left(a; \frac{1}{2} + \delta; \frac{x^2 t}{t+1}\right)$
 $[\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}; |t| < 1 \text{ при } \delta = 0].$

$$20. \sum_{k=0}^{\infty} \frac{(-1)^k (a)_k}{(2k+\delta)!} H_{2k+\delta}(x) = \frac{\sqrt{\pi}}{\Gamma(\delta-a+1/2)} |x|^{-2a+\delta} \begin{cases} \operatorname{sgn} x \\ 1 \end{cases} \\ \left[\delta = \begin{cases} 1 \\ 0 \end{cases}, a < (3 \pm 1)/4 \right].$$

$$21. \sum_{k=0}^{\infty} \frac{(a)_{[k/2]} t^k}{[k/2]! (1/2)_{[k/2]}} H_k(x) = (1+4t^2)^{-a} {}_1F_1\left(a; \frac{1}{2}; \frac{4x^2 t^2}{1+4t^2}\right) + \\ + \frac{32at^3 x^3}{3(1+4t^2)^{a+2}} {}_1F_1\left(a+1; \frac{5}{2}; \frac{4x^2 t^2}{1+4t^2}\right) + \\ + \frac{2xt(1+4t^2-8at^2)}{(1+4t^2)^{a+1}} {}_1F_1\left(a; \frac{3}{2}; \frac{4x^2 t^2}{1+4t^2}\right).$$

$$22. \sum_{k=0}^{\infty} \frac{t^{2k}}{k! (3/4)_k} H_{4k}(x) = \frac{\Gamma(3/4)}{(1-64t^2)^{1/2}} (4x^2 t)^{1/4} \exp\left(\frac{64x^2 t^2}{64t^2-1}\right) I_{-1/4}\left(\frac{8x^2 t}{1-64t^2}\right).$$

$$23. \sum_{k=0}^{\infty} \frac{t^{2k}}{k! (5/4)_k} H_{4k+1}(x) = \\ = \frac{x^2 \Gamma(1/4)}{2(1-64t^2)^{1/2}} (-4x^2 t)^{-1/4} \exp\left(\frac{64x^2 t^2}{64t^2-1}\right) I_{1/4}\left(\frac{8x^2 t}{64t^2-1}\right).$$

$$24. \sum_{k=0}^{\infty} \frac{t^k}{k!} H_k(ka+x) = \frac{1}{1-az} e^{xz-z^2/4} \quad [2t = ze^{-az}; |aze^{1-az}| < 1].$$

$$25. \sum_{k=0}^{\infty} \frac{t^k}{k! (ka+x)} H_k(ka+x) = \frac{e^z}{x} {}_1F_1\left(\frac{x}{2a}; \frac{x}{2a}+1; -\frac{z^2}{4}\right) \\ [2t = ze^{-az}; |aze^{1-az}| < 1].$$

$$26. \sum_{k=0}^{\infty} \frac{(ka+1)^{k/2} t^k}{k!} H_k\left(\frac{x}{\sqrt{ka+1}}\right) = \frac{1}{1+2az} e^{2x\sqrt{z}-z} \\ [t = \sqrt{z} e^{az}; |2aze^{1-2az}| < 1].$$

$$27. \sum_{k=0}^{\infty} \frac{(ka+1)^{k/2-1} t^k}{k!} H_k\left(\frac{x}{\sqrt{ka+1}}\right) = e^{-z} {}_1F_1\left(\frac{1}{a}; \frac{1}{a}+1; 2x\sqrt{z}\right) \\ [t = \sqrt{z} e^{az}; |2aze^{1-2az}| < 1].$$

$$28. \sum_{k=0}^{\infty} \frac{(kb+1)^{k/2}}{k!} t^k H_k\left(\frac{ka+x}{\sqrt{kb+1}}\right) = \frac{1}{1+2bz^2+2az} e^{-z^2-2xz} \\ [t = -ze^{bz^2+2az}; |2aze^{bz^2+2az}+1| < 1].$$

$$29. \sum_{k=0}^{\infty} \frac{(kb+1)^{(k-2)/2}}{k!} t^k H_k\left(\frac{ka+x}{\sqrt{kb+1}}\right) = e^{-z^2-2xz} {}_1F_1\left(1; \frac{b+1}{b}; \frac{2z(bx-a)}{b}\right) \\ [\text{условие см. в 5.12.1.28}].$$

$$30. \sum_{k=0}^{\infty} \frac{(ka+1)^{(k-2)/2}}{k!} t^k H_k(x\sqrt{ka+1}) = e^{-z^2-2xz} \\ [\text{условие см. в 5.12.1.28 при } a=b].$$

$$31. \sum_{k=0}^{\infty} \frac{(kb+1)^{k/2}}{k!} t^k H_k\left(\frac{ka+x}{\sqrt{kb+1}}\right) = e^{-z^2-2xz} {}_1F_1\left(1; \frac{x+2a}{2a}; \frac{z^2(a-bx)}{a}\right) \\ [\text{условие см. в 5.12.1.28}].$$

$$32. \sum_{k=0}^{\infty} \frac{t^k}{k!} \cos \left(2tx - \frac{k\pi}{2} \right) H_k(x) = e^{t^2}.$$

$$33. \sum_{k=0}^{\infty} \frac{(-1)^k}{2^k k!} (2t-1)^k \cos \left[(2t-1)x + \frac{k\pi}{2} \right] H_k(x) = e^{(2t-1)^2/4}.$$

$$34. \sum_{k,l=0}^{\infty} \frac{t^k u^l}{k! l!} H_{k+l}(x) = e^{2ux - u^2 + 2t(x-u) - t^2}.$$

5.12.2. Ряды вида $\sum a_k \prod_i H_{n_i k + m_i}(x_i)$.

$$1. \sum_{k=0}^{\infty} \frac{t^k}{k!} H_{k+m}(x) H_{k+n}(y) = (1-4t^2)^{-(m+n+1)/2} \exp \left[\frac{4txy - 4t^2(x^2 + y^2)}{1-4t^2} \right] \times \\ \times \sum_{k=0}^{\min(m,n)} 2^{2k} k! \binom{m}{k} \binom{n}{k} t^k H_{m-k} \left(\frac{x-2ty}{\sqrt{1-4t^2}} \right) H_{n-k} \left(\frac{y-2tx}{\sqrt{1-4t^2}} \right) \\ \|t\| < 1/2.$$

$$2. \sum_{k=0}^{\infty} \frac{1}{(2k+1)! (2k+1) 2^{2k}} H_{2k+1}(x) H_{2k+1}(y) = \pi \operatorname{erfi}(x) \quad [0 \leq x \leq y].$$

$$3. \sum_{k,l=0}^{\infty} \frac{t^k u^l}{k! l!} H_k(x) H_l(y) H_{k+l+m}(z) = (1-4t^2-4u^2)^{-(m+1)/2} \times \\ \times \exp \left[\frac{-4(t^2+u^2)z^2 + 4(tx+uy)z - 4(tx+uy)^2}{1-4t^2-4u^2} \right] H_m \left(\frac{x-2tx-2uy}{\sqrt{1-4t^2-4u^2}} \right).$$

5.12.3. Ряды, содержащие специальные функции и $H_n(x)$.

$$1. \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} \gamma \left(k + \frac{1}{2}, a^2 \right) H_{2k}(x) = \frac{\sin 2ax}{x}.$$

$$2. \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \gamma(k+1, a^2) H_{2k+1}(x) = \frac{1 - \cos 2ax}{x}.$$

$$3. \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} \Gamma(k, a^2) H_{2k}(x) = -2 \operatorname{ci}(2a|x|).$$

$$4. \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \gamma \left(k + \frac{1}{2}, a^2 \right) H_{2k+1}(x) = 2 \operatorname{si}(2ax) + \pi.$$

$$5. \sum_{k=0}^{\infty} \frac{t^k}{k!} \gamma \left(\frac{k+1}{2}, a^2 \right) H_k(x) = \\ = \sqrt{\frac{\pi}{1+t^2}} e^{t^2 x^2 / (1+t^2)} \left[\operatorname{erf} \left(a \sqrt{1+t^2} + \frac{tx}{\sqrt{1+t^2}} \right) - \operatorname{erf} \left(\frac{tx}{\sqrt{1+t^2}} \right) \right].$$

$$6. \sum_{k=0}^{\infty} \frac{t^{2k}}{(2k)!} \gamma \left(k + \frac{1}{2}, a^2 \right) H_{2k}(x) = \\ = \frac{\sqrt{\pi}}{2 \sqrt{1+t^2}} e^{t^2 x^2 / (1+t^2)} \left[\operatorname{erf} \left(a \sqrt{1+t^2} + \frac{tx}{\sqrt{1+t^2}} \right) + \operatorname{erf} \left(a \sqrt{1+t^2} - \frac{tx}{\sqrt{1+t^2}} \right) \right].$$

$$7. \sum_{k=0}^{\infty} \frac{t^k}{k!} L_n^{k+v} (2tx) H_k(x) = \\ = \frac{(v+1)_n}{n!} e^{2tx} {}_2F_2 \left(\frac{n+v+1}{2}, \frac{n+v+2}{2}; \frac{v+1}{2}, \frac{v+2}{2}; -t^2 \right).$$

5.13. МНОГОЧЛЕНЫ ГЕГЕНБАУЭРА $C_n^v(x)$

5.13.1. Ряды вида $\sum a_k C_{nk+m}^v(\varphi(k, x))$.

$$1. \sum_{k=0}^{\infty} t^k C_k^v(x) = (1-2tx+t^2)^{-v} \quad [|t| < 1].$$

$$2. \sum_{k=0}^{\infty} \frac{t^k}{(a)_k} C_k^v(x) = \Phi_2(v, v; a; tu_-, tu_+) \\ [u_{\pm} = x \pm \sqrt{x^2-1}; |tu_{\pm}| < 1].$$

$$3. \sum_{k=0}^{\infty} \frac{t^k}{(2v)_k} C_k^v(x) = \Gamma\left(v + \frac{1}{2}\right) e^{xt} \left(\frac{t}{2} \sqrt{1-x^2}\right)^{1/2-v} J_{v-1/2}(t \sqrt{1-x^2}).$$

$$4. \sum_{k=0}^{\infty} \frac{t^{2k}}{(2v+\delta)_{2k}} C_{2k+\delta}^v(x) = \\ = \Gamma\left(v + \frac{1}{2}\right) \left(\frac{t}{2}\right)^{\mp 1/2-v} (x^2-1)^{(1-2v)/4} \left\{ \begin{matrix} v \operatorname{sh} xt \\ \operatorname{ch} xt \end{matrix} \right\} I_{v-1/2}(t \sqrt{x^2-1}) \\ [\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}].$$

$$5. \sum_{k=0}^{\infty} \frac{(\mu)_k}{(2v)_k} t^k C_k^v(x) = (1-tx)^{-\mu} {}_2P_1\left(\frac{\mu}{2}, \frac{\mu+1}{2}; v + \frac{1}{2}; t^2 \frac{x^2-1}{(1-tx)^2}\right).$$

$$6. \sum_{k=0}^{\infty} (v+k) \frac{(\mu)_k}{(2v-\mu+1)_k} C_k^v(x) = \Gamma\left[\begin{matrix} 2v-\mu+1 \\ v, v-\mu+1/2 \end{matrix}\right] 2^{\mu-2v} \sqrt{\pi} (1-x)^{-\mu} \\ [-1 < x < 1; 2\mu < v+1 \text{ при } v \geq 0, 2\mu < 2v+1 \text{ при } -1/2 < v \leq 0].$$

$$7. \sum_{k=1}^{\infty} (-1)^k (2k+v+1) \frac{(1/2)_k}{(v+3/2)_k} C_{2k+1}^v(x) = \Gamma\left[\begin{matrix} v+3/2 \\ v \end{matrix}\right] \sqrt{\pi} \operatorname{sgn} x \\ [-1 < x < 1; v > -1/2].$$

$$8. \sum_{k=0}^{\infty} \frac{(v+1/2)_k}{(2v)_k} t^k C_k^v(x) = 2^{v-1/2} (1-2tx+t^2)^{-1/2} (1-tx + \sqrt{1-2tx+t^2})^{1/2-v} \\ [|t| < 1].$$

$$9. \sum_{k=0}^{\infty} \binom{n+k}{k} t^k C_{n+k}^v(x) = (1-2tx+t^2)^{-n/2-v} C_n^v\left(\frac{x-t}{\sqrt{1-2tx+t^2}}\right).$$

$$10. \sum_{k=0}^{\infty} \frac{t^k}{(2v)_k (v+1/2)_k} C_k^v(x) = \\ = \Gamma^2\left(v + \frac{1}{2}\right) \left(\frac{t \sqrt{x^2-1}}{2}\right)^{1/2-v} I_{v-1/2}(\sqrt{2t(x-1)}) I_{v-1/2}(\sqrt{2t(x+1)}).$$

$$11. \sum_{k=0}^{\infty} \frac{(\mu)_k (2\nu - \mu)_k}{(2\nu)_k (\nu + 1/2)_k} t^k C_k^\nu(x) =$$

$$= {}_2F_1 \left(\mu, 2\nu - \mu; \nu + \frac{1}{2}; \frac{1-t - \sqrt{1-2tx+t^2}}{2} \right) \times$$

$$\times {}_2F_1 \left(\mu, 2\nu - \mu; \nu + \frac{1}{2}; \frac{1+t - \sqrt{1-2tx+t^2}}{2} \right).$$

$$12. \sum_{k=0}^{\infty} \frac{[(ka+1)^2 + b^2]^{k/2}}{(2\nu)_k} t^k C_k^\nu \left(\frac{ka+1}{\sqrt{(ka+1)^2 + b^2}} \right) = \frac{2^{\nu-1/2} \Gamma(\nu + 1/2) e^z}{(1-az) (bz)^{\nu-1/2}} J_{\nu-1/2}(bz)$$

$$[t = ze^{-az}; |aze^{1-az}| < 1].$$

$$13. \sum_{k=0}^{\infty} \frac{(ka+1)^{k/2-1} (ka+b+1)^{k/2}}{(2\nu)_k} t^k C_k^\nu \left(\frac{2ka+b+2}{2\sqrt{(ka+1)(ka+b+1)}} \right) =$$

$$= e^z {}_2F_2 \left(\nu, \frac{1}{a}; 2\nu, \frac{1}{a} + 1; bz \right) [t = ze^{-az}; |aze^{1-az}| < 1].$$

$$14. \sum_{k=0}^{\infty} \frac{[(ka+1)^2 - 4b]^{k/2}}{(2\nu)_k (ka+1)} t^k C_k^\nu \left(\frac{ka+1}{\sqrt{(ka+1)^2 - 4b}} \right) =$$

$$= e^z {}_1F_2 \left(\frac{1}{2a}; \nu + \frac{1}{2}, \frac{1}{2a} + 1; bz^2 \right) [t = ze^{-az}; |aze^{1-az}| < 1].$$

5.13.2. Ряды вида $\sum a_k t^k C_k^\mu(x) C_k^\nu(y)$.

$$1. \sum_{k=0}^{\infty} \frac{k!}{(2\mu)_k} t^k C_k^\mu(x) C_k^\nu(y) =$$

$$= (y-tx)^{-2\nu} F_4 \left(\nu, \nu + \frac{1}{2}; \mu + \frac{1}{2}, \nu + \frac{1}{2}; \frac{t^2(x^2-1)}{(y-tx)^2}, \frac{y^2-1}{(y-tx)^2} \right).$$

$$2. \sum_{k=0}^{\infty} \frac{k!}{(2\nu)_k} t^k C_k^\nu(\cos x) C_k^\nu(\cos y) =$$

$$= [1 - 2t \cos(x-y) + t^2]^{-\nu} {}_2F_1 \left(\nu, \nu; 2\nu; -\frac{4t \sin x \sin y}{1 - 2t \cos(x-y) + t^2} \right)$$

$$[|t| < 1].$$

5.13.3. Ряды, содержащие специальные функции $C_n^\nu(x)$.

$$1. \sum_{k=0}^{\infty} (-1)^k (2k + \nu + \delta) J_{2k+\nu+\delta}(z) C_{2k+\delta}^\nu(x) = \frac{1}{\Gamma(\nu)} \left(\frac{z}{2} \right)^\nu \begin{Bmatrix} \sin xz \\ \cos xz \end{Bmatrix}$$

$$[\delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}].$$

$$2. \sum_{k=0}^{\infty} (2k + \nu) \frac{(1/2)_k}{(\nu + 1/2)_k} J_{2k+\nu}(z) C_{2k}^\nu(x) =$$

$$= \Gamma \left[\begin{matrix} \nu + 1/2 \\ \nu \end{matrix} \right] \left(\frac{z}{2} \right)^{1/2} (1-x^2)^{(1-2\nu)/4} J_{\nu-1/2}(z\sqrt{1-x^2}).$$

$$3. \sum_{k=0}^{\infty} (\pm 1)^k (k + \nu) I_{k+\nu}(z) C_k^\nu(x) = \frac{1}{\Gamma(\nu)} \left(\frac{z}{2} \right)^\nu e^{\pm xz}.$$

$$4. \sum_{k=0}^{\infty} \frac{k!}{(2\nu)_k} t^k L_k^{2\nu-1}(y) C_k^{\nu}(x) = \rho \Gamma\left(\nu + \frac{1}{2}\right) \left(\frac{tx}{2}\right)^{1/2-\nu} \times \\ \times (y^2 - 1)^{(1-2\nu)/4} \exp(\rho^2 tx(t-y)) I_{\nu-1/2}(\rho^2 tx \sqrt{y^2 - 1}) \\ [\rho = (1 - 2ty + t^2)^{-1/2}].$$

$$5. \sum_{k=0}^{\infty} (k+\nu) J_{k+\nu}(w) Z_{k+\nu}(z) C_k^{\nu}(x) = \frac{1}{\Gamma(\nu)} \left(\frac{wz}{2\rho}\right)^{\nu} Z_{\nu}(\rho) \\ [Z_{\nu}(z) = J_{\nu}(z), Y_{\nu}(z), H_{\nu}^{(1,2)}(z); \rho = \sqrt{w^2 + z^2 - 2xwz}; \nu \neq 0, -1, -2, \dots; \\ |w \exp(\pm i \arccos x)| < |z| \text{ при } Z_{\nu}(z) \neq J_{\nu}(z)].$$

$$6. \sum_{k=0}^{\infty} (-1)^k (k+\nu) I_{k+\nu}(w) M_{k+\nu}(z) C_k^{\nu}(x) = \frac{1}{\Gamma(\nu)} \left(\frac{wz}{2\rho}\right)^{\nu} M_{\nu}(z) \\ [M_{\nu}(z) = I_{\nu}(z), e^{i\pi\nu} K_{\nu}(z); \rho = \sqrt{w^2 + z^2 - 2xwz}; \nu \neq 0, -1, -2, \dots; \\ |w \exp(\pm i \arccos x)| < |z|].$$

$$7. \sum_{k=0}^{\infty} (k+\nu) I_{(2k+2\nu-1)/4}(z) I_{(2k+2\nu+1)/4}(z) C_k^{\nu}(x) = \\ = 2^{2\nu+3/2} \nu z^{\nu} \Gamma\left[\frac{\nu+1}{2\nu+2}\right] {}_1F_1\left(\nu+1; \nu+\frac{3}{2}; 2xz\right).$$

$$8. \sum_{k=0}^{\infty} i^k (k+\nu) \frac{k!}{(2\nu)_k} J_{k+\nu}(z) C_k^{\nu}(\cos x) C_k^{\nu}(\cos y) = \\ = \Gamma\left[\frac{\nu+1/2}{\nu}\right] \sqrt{\frac{z}{2}} (\sin x \sin y)^{(1-2\nu)/2} e^{iz \cos x \cos y} J_{\nu-1/2}(z \sin x \sin y).$$

5.14. МНОГОЧЛЕНЫ ЯКОБИ $P_n^{(\alpha, \beta)}(x)$

5.14.1. Ряды вида $\sum a_k P_{nk+m}^{(\alpha, \beta)}(x)$.

Обозначение: $\rho = \sqrt{t^2 - 2tx + 1}$.

$$1. \sum_{k=0}^{\infty} t^k P_k^{(\alpha, \beta)}(x) = 2^{\alpha+\beta} \rho^{-1} (1-t+\rho)^{-\alpha} (1+t+\rho)^{-\beta} \quad [|t| < 1].$$

$$2. \sum_{k=0}^{\infty} t^k P_{n+k}^{(\alpha, \beta)}(x) = \frac{(\alpha+1)_n}{n!} \left(\frac{1+x}{2}\right)^{-\beta-n} \times \\ \times F_2\left(\alpha+n+1, -\beta-2n, 1; \alpha+1, n+1; \frac{1-x}{2}, \frac{t+tx}{x}\right).$$

$$3. \sum_{k=0}^{\infty} \frac{t^{k+1}}{k+1} P_k^{(\alpha, -\alpha)}(x) = \frac{1}{\alpha} \left(\frac{1-t+\rho}{1+t+\rho}\right)^{-\alpha} - \frac{1}{\alpha}.$$

$$4. \sum_{k=0}^{\infty} \frac{t^k}{k+\alpha} P_k^{(\alpha, \alpha)}(x) = \frac{2^{2\alpha}}{\alpha} (1-t+\rho)^{-\alpha} (1+t+\rho)^{-\alpha}$$

$[\alpha \neq 0, -1, -2, \dots]$

$$5. \sum_{k=0}^{\infty} \frac{t^k}{k+\alpha} P_k^{(\alpha, -1)} = \frac{2^{\alpha}}{\alpha} (1-t+\rho)^{-\alpha}$$

$[\alpha \neq 0, -1, -2, \dots]$

- $$\begin{aligned}
6. \quad & \sum_{k=0}^{\infty} \frac{(\alpha + \beta + 1)_k}{(\beta + 1)_k} t^k P_k^{(\alpha, \beta)}(x) = \\
& = (1+t)^{-\alpha-\beta-1} {}_2F_1 \left(\frac{\alpha + \beta + 1}{2}, \frac{\alpha + \beta + 2}{2}; \beta + 1; \frac{2tx + 2t}{(t+1)^2} \right). \\
7. \quad & \sum_{k=0}^{\infty} \frac{t^k}{(\alpha + 1)_k (\beta + 1)_k} P_k^{(\alpha, \beta)}(x) = \Gamma(\alpha + 1) \Gamma(\beta + 1) \left(\frac{2}{t} \right)^{(\alpha + \beta)/2} \times \\
& \times (x-1)^{-\alpha/2} (x+1)^{-\beta/2} I_{\alpha}(2t \sqrt{x-1}) I_{\beta}(2t \sqrt{x+1}). \\
8. \quad & \sum_{k=0}^{\infty} \frac{(\alpha + \beta + 1)_k (2k + \alpha + \beta + 1)}{(\alpha + 1)_k} t^k P_k^{(\alpha, \beta)}(x) = \\
& = \frac{(\alpha + \beta + 1)(t+1)}{(1-t)^{\alpha+\beta+2}} {}_2F_1 \left(\frac{\alpha + \beta + 2}{2}, \frac{\alpha + \beta + 3}{2}; \alpha + 1; \frac{2t(x-1)}{(t-1)^2} \right). \\
9. \quad & \sum_{k=0}^{\infty} \frac{(\gamma)_k}{(\alpha + 1)_k (\beta + 1)_k} t^k P_k^{(\alpha, \beta)}(x) = \Psi_2 \left(\gamma; \alpha + 1, \beta + 1; \frac{tx-t}{2}, \frac{tx+t}{2} \right). \\
10. \quad & \sum_{k=0}^{\infty} \frac{(\alpha + \beta + n + 1)_k}{(\gamma)_k} \binom{n+k}{k} t^k P_{n+k}^{(\alpha, \beta)}(x) = \frac{(\alpha + 1)_n}{n!} \left(\frac{x+1}{2} \right)^{-\alpha-\beta-n-1} \times \\
& \times F_4 \left(\alpha + n + 1, \alpha + \beta + n + 1; \gamma, \alpha + 1; \frac{2t}{x+1}, \frac{x-1}{x+1} \right). \\
11. \quad & \sum_{k=0}^{\infty} \frac{(\gamma)_k (\alpha + \beta - \gamma + 1)_k}{(\alpha + 1)_k (\beta + 1)_k} t^k P_k^{(\alpha, \beta)}(x) = \\
& = {}_2F_1 \left(\gamma, \alpha + \beta - \gamma + 1; \alpha + 1; \frac{1-t-\rho}{2} \right) {}_2F_1 \left(\gamma, \alpha + \beta - \gamma + 1; \beta + 1; \frac{1+t-\rho}{2} \right). \\
12. \quad & \sum_{k=0}^{\infty} \frac{(1/2)_k (\alpha + \beta + 1/2)_k}{(\alpha + 1)_k (\beta + 1)_k} t^k P_{2k}^{(2\alpha, 2\beta)}(x) = \\
& = (1-t)^{-\alpha-\beta-1/2} F_4 \left(\frac{\alpha + \beta + 1}{2}, \frac{\alpha + \beta + 2}{2}; \beta + 1, \alpha + 1; \right. \\
& \quad \left. -t \left(\frac{1+x}{1-t} \right)^2, -t \left(\frac{1-x}{1-t} \right)^2 \right). \\
13. \quad & \sum_{k=0}^{\infty} \frac{(\gamma)_k (\delta)_k}{(\alpha + 1)_k (\beta + 1)_k} t^k P_k^{(\alpha, \beta)}(x) = F_4 \left(\gamma, \delta; \alpha + 1, \beta + 1; \frac{tx-t}{2}, \frac{tx+t}{2} \right). \\
14. \quad & \sum_{k=0}^{\infty} (2k + \alpha + \beta + 1) \frac{(\alpha + \beta + 1)_k (1-\gamma)_k}{(\alpha + 1)_k (\alpha + \beta + \gamma + 1)_k} P_k^{(\alpha, \beta)}(x) = \\
& = \Gamma \left[\begin{matrix} \alpha + 1, \alpha + \beta + \gamma + 1 \\ \alpha + \beta + 1, \alpha + \gamma \end{matrix} \right] \left(\frac{1-x}{2} \right)^{\gamma-1} \\
& \quad [-1 < x < 1; -\gamma < \min(\alpha, \alpha/2 - 1/4)].
\end{aligned}$$

5.14.2. Ряды вида $\sum a_k P_{nk+m}^{(\alpha, \beta-k\gamma)}(x)$.

$$1. \quad \sum_{k=0}^{\infty} t^k P_k^{(\alpha, \beta-k)}(x) = (1-t)^{\beta} \left(1 - \frac{tx+t}{2} \right)^{-\alpha-\beta-1} \quad [|t| < 1].$$

$$2. \sum_{k=0}^{\infty} t^k P_{n+k}^{(\alpha, \beta-k)}(x) = \frac{(\alpha+1)_n}{n!} \left(\frac{x+1}{2}\right)^{-\beta} \left(\frac{x-1}{2}\right)^n \times \\ \times F_2 \left(n+\alpha+1, -n-\beta, 1; \alpha+1, n+1; \frac{1-x}{2}, \frac{tx+t}{2} \right).$$

$$3. \sum_{k=0}^{\infty} t^k P_k^{(\alpha, \beta-k\gamma)}(x) = \frac{(1-z)^{\alpha+1} [2-(1-x)(1-z)]^{\beta+1}}{(1+x)^{\beta} [2-(1-x)(1-z)(1+z-\gamma z)]} \\ [t = -2z [2-(1-x)(1-z)]^{\gamma-1} (1-z)^{-1} (1+x)^{-\gamma}].$$

$$4. \sum_{k=0}^{\infty} \frac{t^k}{(\alpha+1)_k} P_k^{(\alpha, \beta-k)}(x) = e^t {}_1F_1 \left(\alpha+\beta+1; \alpha+1; \frac{tx-t}{2} \right) \quad [|t| < 1].$$

$$5. \sum_{k=0}^{\infty} \frac{t^k}{(\gamma)_k} P_k^{(\alpha, \beta-k)}(x) = \Phi_2 \left(\alpha+\beta+1, -\beta; \gamma; \frac{tx+t}{2}, t \right).$$

$$6. \sum_{k=0}^{\infty} \frac{t^k}{\beta-k(\gamma-1)} P_k^{(\alpha, \beta-k\gamma)}(x) = \\ = \frac{(1-z)^{\alpha+1} [2-(1-x)(1-z)]^{\beta}}{\beta(1+x)^{\beta}} {}_2F_1 \left(1, \alpha+1; 1-\frac{\beta}{\gamma-1}; z \right) \\ [z \text{ см. в 5.14.2.3}].$$

$$7. \sum_{k=0}^{\infty} \frac{t^k}{k+\alpha} P_k^{(\alpha, -\alpha(\gamma-1)-k\gamma)}(x) = \\ = \frac{1}{\alpha} (1+x)^{\alpha(\gamma-1)} (1-z)^{\alpha} [2-(1-x)(1-z)]^{-\alpha(\gamma-1)} \quad [z \text{ см. в 5.14.2.3}].$$

$$8. \sum_{k=0}^{\infty} \binom{n+k}{k} t^k P_{n+k}^{(\alpha, \beta-k)}(x) = (t+1)^{-\beta} \left(1 - \frac{tx+t}{2} \right)^{\beta-\alpha-n-1} P_n^{(\alpha, \beta)} \left(\frac{2x-tx-t}{2-tx-t} \right).$$

$$9. \sum_{k=0}^{\infty} \frac{(\gamma)_k}{(\alpha+1)_k} t^k P_k^{(\alpha, \beta-k)}(x) = (1-t)^{\gamma} {}_2F_1 \left(\alpha+\beta+1, \gamma; \alpha+1; \frac{tx-t}{2-2t} \right).$$

$$10. \sum_{k=0}^{\infty} \frac{(\delta)_k}{(\gamma)_k} t^k P_k^{(\alpha, \beta-k)}(x) = F_1 \left(\delta, -\beta, \alpha+\beta+1; \gamma; t, \frac{tx+t}{2} \right) \\ [|t| < 1; |t(x+1)| < 2].$$

$$11. \sum_{k=0}^{\infty} \frac{(n+k)! (\gamma)_k}{k! (\delta)_k} t^k P_{n+k}^{(\alpha, \beta-k)}(x) = (\alpha+1)_n \left(\frac{x+1}{2}\right)^{-\beta} \left(\frac{x-1}{2}\right)^n \times \\ \times F_2 \left(n+\alpha+1, -n-\beta, \gamma; \alpha+1, \delta; \frac{1-x}{2}, \frac{tx+t}{2} \right).$$

$$12. \sum_{k=0}^{\infty} \frac{(n+2k)!}{k! (\gamma)_k} t^{2k} P_{n+2k}^{(\alpha, \beta-2k)}(x) = (\alpha+1)_n \left(\frac{2}{1+x}\right)^{\beta} (1+t+tx)^{-n-\alpha-1} \times \\ \times F_2 \left(n+\alpha+1, \gamma-\frac{1}{2}, -\beta-n; 2\gamma-1, \alpha+1; \frac{2tx+2t}{tx+t+1}, \frac{1-x}{2tx+2t+2} \right).$$

$$13. \sum_{k=0}^{\infty} \frac{(ka+b)^k}{(\alpha+1)_k} t^k P_k^{(\alpha, \beta-k)}(ka+x) = \frac{e^{bv}}{1-av} {}_1F_1 \left(\alpha+\beta+1; \alpha+1; \frac{bx-1}{2} v \right) \\ [v = te^{av}].$$

5.14.3. Ряды вида $\sum a_k P_{nk+m}^{(\alpha+k\mu, \beta+k\nu)}(x)$.

$$1. \sum_{k=0}^{\infty} t^k P_k^{(\alpha-k\mu, \beta-k\nu)}(x) = \frac{(u+1)^{-\alpha} (v+1)^{-\beta}}{1 + (1-\mu)u + (1-\nu)v}$$

$$[2u = -t(x+1)(u+1)^\mu (v+1)^{\nu-1}, 2v = -t(x-1)(u+1)^{\mu-1} (v+1)^\nu].$$

$$2. \sum_{k=0}^{\infty} t^k P_k^{(\alpha-k, \alpha-k)}(x) = \left(1 + \frac{tx+t}{2}\right)^\alpha \left(1 + \frac{tx-t}{2}\right)^\alpha.$$

$$3. \sum_{k=0}^{\infty} \frac{t^k}{k+2\alpha} P_k^{(\alpha-k/2, \alpha-k/2)}(x) = \frac{1}{2\alpha} (\rho+1)^\alpha \left(1 + \frac{\rho-\rho x}{2}\right)^{-2\alpha}$$

$$[2\rho = t(t + \sqrt{t^2 + 4})].$$

$$4. \sum_{k=0}^{\infty} \frac{t^k}{k+2\alpha} P_k^{(\alpha-k/2, \beta-k/2)}(x) =$$

$$= \frac{1}{2\alpha} (\rho+1)^\alpha {}_2F_1\left(2\alpha, \alpha+\beta+1; 2\alpha+1; -\frac{\rho-\rho x}{2}\right)$$

$$[2\rho = t(t + \sqrt{t^2 + 4})].$$

$$5. \sum_{k=0}^{\infty} \frac{t^k}{(-\alpha-\beta)_k} P_k^{(\alpha-k, \beta-k)}(x) = e^{-t(x+1)/2} {}_1F_1(-\beta; -\alpha-\beta; t).$$

$$6. \sum_{k=0}^{\infty} \frac{(\gamma)_k}{(-\alpha-\beta)_k} t^k P_k^{(\alpha-k, \beta-k)}(x) =$$

$$= \left(1 + \frac{tx+t}{2}\right)^{-\gamma} {}_2F_1\left(-\beta, \gamma; -\alpha-\beta; \frac{2t}{tx+t+2}\right).$$

$$7. \sum_{k=-n}^{\infty} \frac{(\alpha+1)_k}{(k+n)!} t^k P_n^{(\alpha-k, \beta-k)}(x) = \left(-\frac{2}{t}\right)^n \left(\frac{1-t}{1-x}\right)^n \left(1 + \frac{tx-t}{2}\right)^{-\alpha-1}.$$

$$8. \sum_{k=0}^{\infty} \binom{n+k}{n} t^k P_{n+k}^{(\alpha-k, \beta-k)}(x) =$$

$$= \left(1 + \frac{tx+t}{2}\right)^\alpha \left(1 + \frac{tx-t}{2}\right)^\beta P_n^{(\alpha, \beta)}\left(x + \frac{tx^2-t}{2}\right).$$

$$9. \sum_{k=0}^{\infty} \frac{(-1)^k t^{2k+\delta}}{(-\alpha-\beta)_{2k+\delta}} P_{2k+\delta}^{(\alpha-2k-\delta, \beta-2k-\delta)}(x) =$$

$$= \begin{cases} \sin [t(1-x)/2] \\ \cos [t(1-x)/2] \end{cases} {}_2F_3\left(-\frac{\alpha}{2}, \frac{1-\alpha}{2}; \frac{1}{2}, -\frac{\alpha+\beta}{2}, \frac{1-\alpha-\beta}{2}; -\frac{t^2}{4}\right) \mp$$

$$\mp \frac{2^{1-\delta}\alpha t}{\alpha \mp \beta} \begin{cases} \cos [t(1-x)/2] \\ \sin [t(1-x)/2] \end{cases} \times$$

$$\times {}_2F_3\left(\frac{1-\alpha}{2}, \frac{2-\alpha}{2}; \frac{3}{2}, \frac{1-\alpha-\beta}{2}, \frac{2-\alpha-\beta}{2}; -\frac{t^2}{4}\right) \quad [\delta = \{1\}].$$

$$10. \sum_{k=0}^{\infty} \frac{(ka+b)^k}{(-\alpha-\beta)_k} t^k P_k^{(\alpha-k, \beta-k)}(ka+x) =$$

$$= \frac{e^{bw}}{1-aw} {}_1F_1\left(-\beta; -\alpha-\beta; -\frac{2w}{bx+1}\right) \quad [2w = -t(bx+1)e^{aw}].$$

5.14.4. Ряды вида $\sum a_k P_k^{(\alpha+k\beta, \mu+k\nu)}(x) P_{k+n}^{(\rho+k\sigma, \chi+kn)}(y)$.

$$1. \sum_{k=0}^{\infty} \frac{k! (\alpha+\beta+1)_k}{(\alpha+1)_k (\beta+1)_k} t^k P_k^{(\alpha, \beta)}(x) P_k^{(\alpha, \beta)}(y) = (1+t)^{-\alpha-\beta-1} \times \\ \times F_4 \left(\frac{\alpha+\beta+1}{2}, \frac{\alpha+\beta+2}{2}; \alpha+1, \beta+1; \frac{t(1-x)(1-y)}{(1+t)^2}, \frac{t(1+x)(1+y)}{(1+t)^2} \right).$$

$$2. \sum_{k=0}^{\infty} (2k+\alpha+\beta+1) \frac{k! (\alpha+\beta+1)_k}{(\alpha+1)_k (\beta+1)_k} t^k P_k^{(\alpha, \beta)}(x) P_k^{(\alpha, \beta)}(y) = \\ = \frac{(\alpha+\beta+1)(1-t)}{(1+t)^{\alpha+\beta+2}} F_4 \left(\frac{\alpha+\beta}{2} + 1, \frac{\alpha+\beta+3}{2}; \alpha+1, \beta+1; \right. \\ \left. \frac{t(1-x)(1-y)}{(1+t)^2}, \frac{t(1+x)(1+y)}{(1+t)^2} \right).$$

$$3. \sum_{k=0}^{\infty} (2k+\gamma+1) \frac{k! (\gamma+1)_k}{(\alpha+1)_k (\beta+1)_k} P_k^{(\alpha, \gamma-\alpha)}(x) P_k^{(\gamma-\beta, \beta)}(y) = \\ = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \Gamma \left[\begin{matrix} \alpha+1, \beta+1 \\ \gamma+1, \alpha+\beta-\gamma \end{matrix} \right] \frac{2^{\gamma+1} (y-x)^{\alpha+\beta-\gamma-1}}{(1-x)^{\alpha} (1+y)^{\beta}} \quad \left[\begin{matrix} x < y \leq 1 \\ 1 \leq y < x \end{matrix} \right].$$

$$4. \sum_{k=0}^{\infty} \frac{(n+k)! (\mu+\nu+1)_k}{(\alpha+1)_k (\beta+1)_k} t^k P_k^{(\alpha, \beta)}(x) P_{n+k}^{(\mu, \nu)}(y) = \\ = (\mu+1)_n (\mu+\nu+1)_n \left(\frac{2}{y+1} \right)^{n+\nu+1} \times \\ \times F_C^{(3)} \left(n+\mu+1, n+\mu+\nu+1; \mu+1, \alpha+1, \beta+1; \frac{y-1}{y+1}, t \frac{x-1}{y+1}, t \frac{x+1}{y+1} \right) \\ [|t(x+1)^{1/2}| + |t(x-1)^{1/2}| + |y-1|^{1/2} < |y+1|^{1/2}].$$

$$5. \sum_{k=0}^{\infty} \frac{k!}{(\alpha+1)_k} t^k P_k^{(\alpha, \beta-k)}(x) P_k^{(\alpha, \gamma-k)}(y) = (1-t)^{\alpha+\beta+\gamma+1} u^{-\alpha-\beta-1} v^{-\alpha-\gamma-1} \times \\ \times {}_2F_1 \left(\alpha+\beta+1, \alpha+\gamma+1; \alpha+1; \frac{t(x-1)(y-1)}{4uv} \right) \\ [u=1-t(x+1)/2, v=1-t(y+1)/2].$$

$$6. \sum_{k=0}^{\infty} \frac{k!}{(\sigma+1)_k} t^k P_k^{(\alpha-k, \beta-k)}(x) P_k^{(\rho-k, \sigma)}(y) = \left(1 - \frac{x+1}{2} t \right)^{\alpha} \left(1 - \frac{x-1}{2} t \right)^{\beta} \times \\ \times F_1 \left(\rho+\sigma+1, -\alpha, -\beta; \sigma+1; -\frac{t(x+1)(y+1)}{4-2t(x+1)}, -\frac{t(x-1)(y+1)}{4-2(x-1)t} \right).$$

$$7. \sum_{k=0}^{\infty} \frac{k!}{(-\alpha-\beta)_k} t^k P_k^{(\alpha-k, \beta-k)}(x) P_k^{(\alpha-k, \beta-k)}(y) = \\ = u^{\alpha} v^{\beta} {}_2F_1 \left(-\alpha, -\beta; -\alpha-\beta; \frac{-t}{uv} \right) \\ [u=1-(x+1)(y+1)/4, v=1-(x-1)(y-1)/4].$$

5.14.5. Ряды, содержащие специальные функции и $P_n^{(\alpha, \beta)}(x)$.

$$1. \sum_{k=0}^{\infty} (-1)^k \left(2k+\alpha+\delta+\frac{1}{2} \right) \frac{(2\alpha+\delta+1)_{2k}}{(\alpha+\delta+1)_{2k}} J_{2k+\alpha+\delta+1/2}(z) P_{2k+\delta}^{(\alpha, \alpha)}(x) = \\ = \frac{(2z)^{\alpha+1/2}}{2\sqrt{\pi}} \Gamma \left[\begin{matrix} \alpha+\delta+1 \\ 2\alpha+\delta+1 \end{matrix} \right] \begin{Bmatrix} \sin xz \\ \cos xz \end{Bmatrix} \quad [|x| \leq 1; \delta = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}].$$

$$2. \sum_{k=0}^{\infty} (2k + \alpha + \beta + 1) \frac{(\alpha + \beta + 1)_k}{(\alpha + 1)_k} I_{k+(\alpha+\beta+1)/2}(z) P_k^{(\alpha, \beta)}(x) =$$

$$= \frac{2}{\Gamma((\alpha + \beta + 1)/2)} \left(\frac{z}{2}\right)^{(\alpha+\beta+1)/2} e^z {}_1F_1\left(\frac{\alpha + \beta}{2} + 1; \alpha + 1; xz - z\right).$$

$$3. \sum_{k=0}^{\infty} \frac{k!}{(\alpha + 1)_k} t^k L_k^{\alpha}(x) P_k^{(\alpha, \beta-k)}(y) =$$

$$= (1-t)^{\beta} \left(1 - \frac{1+y}{2} t\right)^{-\alpha-\beta+1} \exp\left(\frac{tx}{t-1}\right) {}_1F_1\left(\alpha + \beta + 1; \alpha + 1; \frac{tx(1-y)}{(1-t)(2-t-ty)}\right).$$

$$4. \sum_{k=0}^{\infty} (2k + \alpha + \beta + 1) \frac{k! (\alpha + \beta + 1)_k}{(\alpha + 1)_k (\beta + 1)_k} I_{k+(\alpha+\beta+1)/2}(z) P_k^{(\alpha, \beta)}(x) P_k^{(\alpha, \beta)}(-x) =$$

$$= \frac{2}{\Gamma((\alpha + \beta + 1)/2)} \left(\frac{z}{2}\right)^{(\alpha+\beta+1)/2} e^{-z} \times$$

$$\times {}_2F_2\left(\frac{\alpha + \beta + 1}{2}, \frac{\alpha + \beta}{2} + 1, \frac{\alpha + \beta}{2} + 1; \alpha + 1, \beta + 1, \alpha + \beta + 1; 4(1-x^2)z^2\right).$$

Приложение I. БИНОМИАЛЬНЫЕ КОЭФФИЦИЕНТЫ $\binom{a}{b}$ И СИМВОЛ ПОХГАММЕРА $(a)_k$

I.1. БИНОМИАЛЬНЫЕ КОЭФФИЦИЕНТЫ $\binom{a}{b}$

$$\binom{a}{k} = (-1)^k \frac{(-a)_k}{k!} = \frac{a(a-1)\dots(a-k+1)}{k!} \quad [k=1, 2, 3, \dots]$$

$$\binom{a}{0} = 1, \quad \binom{a}{-k} = 0 \quad [k=1, 2, 3, \dots; a \neq -1, -2, -3, \dots]$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \quad [k=1, 2, \dots, n]$$

$$\binom{n}{k} = 0 \quad [k=-1, -2, -3, \dots \text{ или } k > n].$$

$$\binom{n}{k} = \binom{n}{n-k} \quad [n \geq k \geq 0]$$

$$\binom{a}{k} = (-1)^k \binom{-a+k-1}{k}, \quad \binom{a}{b} = \frac{\Gamma(a+1)}{\Gamma(b+1)\Gamma(a-b+1)},$$

$$\binom{a}{b} = \binom{a}{a-b} \quad [b \neq -k, k+a, \text{ где } k=-1, -2, -3, \dots]$$

$$\binom{a}{b+1} = \frac{a}{b+1} \binom{a-1}{b} = \frac{a-b}{b+1} \binom{a}{b}, \quad \binom{a}{b} + \binom{a}{b+1} = \binom{a+1}{b+1},$$

$$\binom{n+a}{n} \binom{b}{n+a} = \binom{b}{n} \binom{b-n}{a} = \binom{b}{a} \binom{b-a}{n}.$$

$$\binom{-1/2}{n} = \frac{(-1)^n}{2^{2n}} \binom{2n}{n} = (-1)^n \frac{(2n-1)!!}{(2n)!!},$$

$$\binom{1/2}{n} = \frac{(-1)^{n-1}}{2^{2n-1}n} \binom{2n-2}{n-1} = (-1)^{n-1} \frac{(2n-3)!!}{(2n-2)!!n},$$

$$\binom{n+1/2}{2n+1} = (-1)^n 2^{-2n-1} \binom{2n}{n}, \quad \binom{2n+1/2}{n} = 2^{-2n} \binom{4n+1}{2n},$$

$$\binom{n}{1/2} = \frac{2^{2n+1}}{\pi} \binom{2n}{n}^{-1}, \quad \binom{n}{n/2} = \frac{2^{2n}}{\pi} \binom{n}{(n-1)/2}.$$

I.2. СИМВОЛ ПОХГАММЕРА $(a)_k$

$$(a)_k = a(a+1)\dots(a+k-1) = \frac{\Gamma(a+k)}{\Gamma(a)} = (-1)^k \frac{\Gamma(1-a)}{\Gamma(1-a-k)},$$

$$(a)_0 = 1, \quad (a)_{-k} = \frac{\Gamma(a-k)}{\Gamma(a)} = \frac{(-1)^k}{(1-a)_k} \quad [a \neq 1, 2, \dots, k; k = 1, 2, 3, \dots]$$

$$(n)_k = \frac{(n+k-1)!}{(n-1)!}, \quad (1)_k = k!,$$

$$(1/2)_k = 2^{-2k} \frac{(2k)!}{k!}, \quad (3/2)_k = 2^{-2k} \frac{(2k+1)!}{k!},$$

$$(a)_{n+k} = (a)_n (a+n)_k, \quad (a)_{n-k} = \frac{(-1)^k (a)_n}{(1-a-n)_k},$$

$$(a)_{nk} = \left(\frac{a}{n}\right)_k \left(\frac{a+1}{n}\right)_k \dots \left(\frac{a+n-1}{n}\right)_k n^{nk}, \quad (a)_{2k} = \left(\frac{a}{2}\right)_k \left(\frac{a+1}{2}\right)_k 2^{2k},$$

$$(nk)! = \left(\frac{1}{n}\right)_k \left(\frac{2}{n}\right)_k \dots \left(\frac{n-1}{n}\right)_k k! n^{nk}, \quad (2k)! = \left(\frac{1}{2}\right)_k k! 2^{2k},$$

$$(nk+m)! = m! \left(\frac{m+1}{n}\right)_k \left(\frac{m+2}{n}\right)_k \dots \left(\frac{m+n}{n}\right)_k n^{nk}, \quad (2k+1)! = \left(\frac{3}{2}\right)_k k! 2^{2k},$$

$$(a+mk)_{nk} = \frac{(a)_{mk+nk}}{(a)_{mk}}, \quad (a+k)_k = \frac{(a)_{2k}}{(a)_k}, \quad (a+m)_k = \frac{(a)_k (a+k)_m}{(a)_m},$$

$$(a-mk)_{nk} = (-1)^{mk} (a)_{nk-mk} (1-a)_{mk} \quad [n \geq m],$$

$$= (-1)^{nk} \frac{(1-a)_{mk}}{(1-a)_{mk-nk}} \quad [n < m].$$

$$(a-k)_k = (-1)^k (1-a)_k, \quad (a-m)_k = \frac{(1-a)_m (a)_k}{(1-a-k)_m},$$

$$(a+mk)_{n \pm lk} = \frac{(a)_n (a+n)_{mk \pm lk}}{(a)_{mk}}, \quad (a+k)_{n-k} = \frac{(a)_n}{(a)_k},$$

$$(a-mk)_{n \pm lk} = \frac{(-1)^{lk} (a)_n (1-a)_{mk}}{(1-a-n)_{mk \mp lk}}, \quad (a-k)_{n-k} = \frac{(-1)^k (a)_n (1-a)_k}{(1-a-n)_{2n}}.$$

П р и л о ж е н и е II. НЕКОТОРЫЕ СПЕЦИАЛЬНЫЕ ФУНКЦИИ И ИХ СВОЙСТВА

II.1. ГАММА-ФУНКЦИЯ $\Gamma(z)$

О п р е д е л е н и е:

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt \quad [\operatorname{Re} z > 0].$$

При $\operatorname{Re} z \leq 0$ функция $\Gamma(z)$ определяется как аналитическое продолжение интеграла.

$\Gamma(z)$ — аналитическая функция во всей z -плоскости, за исключением точек $z=0, -1, -2, \dots$, в которых она имеет простые полюсы.

$$\Gamma(z+1) = z\Gamma(z), \quad \Gamma(n+1) = n!, \quad \Gamma(1) = \Gamma(2) = 1.$$

$$\Gamma(z+k) = (z)_k \Gamma(z), \quad \Gamma(z-k) = \frac{(-1)^k}{(1-z)_k} \Gamma(z),$$

$$\Gamma(z+nk) = \Gamma(z) \left(\frac{z}{n}\right)_k \left(\frac{z+1}{n}\right)_k \dots \left(\frac{z+n-1}{n}\right)_k n^{nk},$$

$$\Gamma(z-nk) = \frac{(-1)^{nk} \Gamma(z) n^{-nk}}{\left(\frac{1-z}{n}\right)_k \left(\frac{2-z}{n}\right)_k \dots \left(\frac{n-z}{n}\right)_k},$$

$$\Gamma(z+2k) = \Gamma(z) \left(\frac{z}{2}\right)_k \left(\frac{z+1}{2}\right)_k 2^{2k},$$

$$\Gamma(z-2k) = \frac{\Gamma(z) 2^{-2k}}{\left(\frac{1-z}{2}\right)_k \left(\frac{2-z}{2}\right)_k}.$$

$$\Gamma\left(n + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^n} (2n-1)!!, \quad \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}, \quad \Gamma\left(\frac{3}{2}\right) = \frac{\sqrt{\pi}}{2},$$

$$\Gamma\left(\frac{1}{2} - n\right) = (-1)^n \frac{2^n \sqrt{\pi}}{(2n-1)!!}, \quad \Gamma\left(-\frac{1}{2}\right) = -2\sqrt{\pi}, \quad \Gamma\left(-\frac{3}{2}\right) = \frac{4\sqrt{\pi}}{3}.$$

$$\Gamma\left(n + \frac{1}{3}\right) = \frac{1 \cdot 4 \cdot 7 \dots (3n-2)}{3^n} \Gamma\left(\frac{1}{3}\right), \quad \Gamma\left(\frac{1}{3}\right) = 2,678938 \dots,$$

$$\Gamma\left(n + \frac{2}{3}\right) = \frac{2 \cdot 5 \cdot 8 \dots (3n-1)}{3^n} \Gamma\left(\frac{2}{3}\right), \quad \Gamma\left(\frac{2}{3}\right) = 1,354118 \dots,$$

$$\Gamma\left(n + \frac{1}{4}\right) = \frac{1 \cdot 5 \cdot 9 \dots (4n-3)}{4^n} \Gamma\left(\frac{1}{4}\right), \quad \Gamma\left(\frac{1}{4}\right) = 3,625600 \dots,$$

$$\Gamma\left(n + \frac{3}{4}\right) = \frac{3 \cdot 7 \cdot 11 \dots (4n-1)}{4^n} \Gamma\left(\frac{3}{4}\right), \quad \Gamma\left(\frac{3}{4}\right) = 1,225417 \dots$$

$$\Gamma(z) \Gamma(-z) = -\frac{\pi}{z \sin \pi z}, \quad \Gamma(z) \Gamma(1-z) = \frac{\pi}{\sin \pi z},$$

$$\Gamma\left(\frac{1}{2} + z\right) \Gamma\left(\frac{1}{2} - z\right) = \frac{\pi}{\cos \pi z}, \quad \Gamma(1+z) \Gamma(1-z) = \frac{\pi z}{\sin \pi z}.$$

$$\frac{\Gamma(z+1)}{\Gamma(w+1) \Gamma(z-w+1)} = \binom{z}{w}.$$

$$\Gamma(nz) = (2\pi)^{(1-n)/2} n^{nz-1/2} \prod_{k=0}^{n-1} \Gamma\left(z + \frac{k}{n}\right),$$

$$\Gamma(2z) = \frac{2^{2z-1}}{\sqrt{\pi}} \Gamma(z) \Gamma\left(z + \frac{1}{2}\right),$$

$$\Gamma(3z) = \frac{3^{3z-1/2}}{2\pi} \Gamma(z) \Gamma\left(z + \frac{1}{3}\right) \Gamma\left(z + \frac{2}{3}\right).$$

$$|\Gamma(ix)|^2 = \frac{\pi}{x \operatorname{sh} \pi x}, \quad \left|\Gamma\left(\frac{1}{2} + ix\right)\right|^2 = \frac{\pi}{\operatorname{ch} \pi x},$$

$$|\Gamma(1+ix)|^2 = \frac{\pi x}{\operatorname{sh} \pi x}, \quad \Gamma\left(\frac{1}{4} + ix\right) \Gamma\left(\frac{3}{4} - ix\right) = \frac{\pi \sqrt{2}}{\operatorname{ch} \pi x + i \operatorname{sh} \pi x}.$$

II.2. БЕТА-ФУНКЦИЯ В (a, b)

Определение:

$$B(a, b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt.$$

$$= \int_0^\infty \frac{t^{a-1}}{(t+1)^{a+b}} dt$$

[Re a, Re b > 0].

$$B(a, b) = \frac{\Gamma(a) \Gamma(b)}{\Gamma(a+b)} \equiv \Gamma\left[\begin{matrix} a, b \\ a+b \end{matrix}\right].$$

$$B(m+1, n+1) = \frac{m!n!}{(m+n+1)!}.$$

$$\Psi\left(\frac{1}{2}, \frac{n}{2}\right) = A \frac{(n-2)!!}{(n-1)!!}, \quad \Psi\left(\frac{3}{2}, \frac{n}{2}\right) = A \frac{(n-2)!!}{(n+1)!!},$$

$$A = \begin{cases} 2, & n=2, 4, 6, \dots, \\ \pi, & n=1, 3, 5, \dots \end{cases}$$

II.3. ПСИ-ФУНКЦИЯ $\psi(z)$

О п р е д е л е н и е:

$$\psi(z) = \frac{\Gamma'(z)}{\Gamma(z)} = \frac{d}{dz} \ln \Gamma(z).$$

$\psi(z)$ — аналитическая функция во всей z -плоскости, за исключением точек $z=0, -1, -2, \dots$, в которых она имеет простые полюсы.

$$\psi(z+n) = \psi(z) + \sum_{k=0}^{n-1} \frac{1}{k+z}, \quad \psi(z-n) = \psi(z) - \sum_{k=1}^n \frac{1}{z-k},$$

$$\psi(z+1) = \psi(z) + \frac{1}{z}, \quad \psi(n+1) = -C + \sum_{k=1}^n \frac{1}{k}, \quad \psi(1) = -C,$$

$$C = \lim_{n \rightarrow \infty} \left(\sum_{k=1}^n \frac{1}{k} - \ln n \right) = 0,5772156649\dots$$

$$\psi\left(\frac{1}{2} \pm n\right) = -C - 2 \ln 2 + 2 \sum_{k=0}^{n-1} \frac{1}{2k+1}, \quad \psi\left(\frac{1}{2}\right) = -C - 2 \ln 2.$$

$$\psi\left(\frac{1}{3}\right) = -C - \frac{\pi}{2\sqrt{3}} - \frac{3}{2} \ln 3, \quad \psi\left(\frac{2}{3}\right) = -C + \frac{\pi}{2\sqrt{3}} - \frac{3}{2} \ln 3,$$

$$\psi\left(\frac{1}{4}\right) = -C - \frac{\pi}{2} - 3 \ln 2, \quad \psi\left(\frac{3}{4}\right) = -C + \frac{\pi}{2} - 3 \ln 2.$$

$$\psi\left(\frac{p}{q}\right) = -C - \ln(2q) - \frac{\pi}{2} \operatorname{ctg} \frac{p\pi}{q} + 2 \sum_{k=1}^{[(q-1)/2]} \cos \frac{2pk\pi}{q} \ln \sin \frac{k\pi}{q}$$

[$p=1, 2, \dots, q-1; q=2, 3, 4, \dots$]

$$\psi(z) - \psi(-z) = -\pi \operatorname{ctg} \pi z - \frac{1}{z}, \quad \psi(z) - \psi(1-z) = -\pi \operatorname{ctg} \pi z,$$

$$\psi(1+z) - \psi(1-z) = \frac{1}{z} - \pi \operatorname{ctg} \pi z, \quad \psi\left(\frac{1}{2} + z\right) - \psi\left(\frac{1}{2} - z\right) = \pi \operatorname{tg} \pi z.$$

$$\psi\left(\frac{z}{2}\right) - \psi\left(\frac{z+1}{2}\right) = -2\beta(z).$$

$$\psi(n) - \psi\left(\frac{n+1}{2}\right) = ((-1)^n + 1) \ln 2 + \sum_{k=1}^{n-1} \frac{(-1)^{n+k}}{k}.$$

$$\psi(nz) = \frac{1}{n} \sum_{k=0}^{n-1} \psi\left(z + \frac{k}{n}\right) + \ln n,$$

$$\psi(2z) = \frac{1}{2} \psi(z) + \frac{1}{2} \psi\left(z + \frac{1}{2}\right) + \ln 2.$$

$$\operatorname{Re} \psi(ix) = \operatorname{Re} \psi(-ix) = \operatorname{Re} \psi(1+ix) = \operatorname{Re} \psi(1-ix) = -C + x^2 \sum_{k=1}^{\infty} \frac{1}{k(k^2+x^2)}.$$

$$\operatorname{Im} \psi(ix) = \frac{1}{2x} + \frac{\pi}{2} \operatorname{cth} \pi x.$$

$$\operatorname{Im} \psi(1+ix) = -\frac{1}{2x} + \frac{\pi}{2} \operatorname{cth} \pi x = x \sum_{k=1}^{\infty} \frac{1}{k^2 + x^2}.$$

$$\operatorname{Im} \psi\left(\frac{1}{2} + ix\right) = \frac{\pi}{2} \operatorname{th} \pi x.$$

$$\psi(z) = -C + (z-1) \sum_{k=0}^{\infty} \frac{1}{(k+1)(k+z)}.$$

$$\psi^{(n)}(z) = (-1)^{n+1} n! \sum_{k=0}^{\infty} \frac{1}{(k+z)^{n+1}}.$$

$$\psi^{(n)}(z+1) = \psi^{(n)}(z) + (-1)^n n! z^{-n-1}.$$

$$= (-1)^{n+1} \sum_{k=0}^{\infty} \frac{(-1)^k (n+k)!}{k!} \zeta(n+k+1) z^k \quad [|z| < 1].$$

$$\psi^{(n)}(z+m) = \psi^{(n)}(z) + (-1)^n n! \sum_{k=0}^{m-1} \frac{1}{(k+z)^{n+1}}.$$

$$\psi^{(n)}(1-z) = (-1)^n \psi^{(n)}(z) + (-1)^n \pi \frac{d^n}{dz^n} \operatorname{ctg} \pi z.$$

$$\psi^{(n)}(mz) = \ln m + \frac{1}{m^{n+1}} \sum_{k=0}^{m-1} \psi^{(n)}\left(z + \frac{k}{m}\right) \quad [n=1, 2, 3, \dots].$$

$$\psi^{(n)}(m+1) = (-1)^n n! \left[-\zeta(n+1) + \sum_{k=1}^m \frac{1}{k^{n+1}} \right].$$

$$\psi^{(n)}(1) = (-1)^{n+1} n! \zeta(n+1) \quad [n=1, 2, 3, \dots].$$

$$\psi^{(n)}\left(\frac{1}{2}\right) = (-1)^{n+1} n! (2^{n+1} - 1) \zeta(n+1) \quad [n=1, 2, 3, \dots].$$

$$\psi'(1) = \frac{\pi^2}{6}, \quad \psi'(m) = \frac{\pi^2}{6} - \sum_{k=1}^{m-1} \frac{1}{k^2},$$

$$\psi'\left(\frac{1}{2}\right) = \frac{\pi^2}{2}, \quad \psi'\left(\frac{1}{2} \pm m\right) = \frac{\pi^2}{2} \mp 4 \sum_{k=1}^m \frac{1}{(2k-1)^2}.$$

II.4. ДЗЕТА-ФУНКЦИЯ РИМАНА $\zeta(z)$ И $\zeta(z, v)$

Определения:

$$\zeta(z) = \sum_{k=1}^{\infty} \frac{1}{k^z} = \frac{1}{1-2^{-z}} \sum_{k=1}^{\infty} \frac{1}{(2k-1)^z} \quad [\operatorname{Re} z > 1].$$

$$= \frac{1}{1-2^{1-z}} \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^z} \quad [\operatorname{Re} z > 0; z \neq 1].$$

$$\zeta(z, v) = \sum_{k=0}^{\infty} \frac{1}{(k+v)^z} \quad [\operatorname{Re} z > 1; v \neq 0, -1, -2, \dots].$$

При $\operatorname{Re} z \leq 1$, $z \neq 1$ функции $\zeta(z)$, $\zeta(z, v)$ определяются как аналитические продолжения приведенных рядов.

$\zeta(z)$, $\zeta(z, v)$ — аналитические функции во всей z -плоскости, за исключением точки $z=1$, в которой они имеют простой полюс.

$$\zeta(z) = 2^z \pi^{z-1} \sin \frac{\pi z}{2} \Gamma(1-z) \zeta(1-z).$$

$$\zeta(z, 1) = \zeta(z), \quad \zeta\left(z, \frac{1}{2}\right) = (2^z - 1) \zeta(z).$$

$$\zeta(0, v) = \frac{1}{2} - v, \quad \zeta(-n, v) = -\frac{B_{n+1}(v)}{n+1},$$

$$\zeta(-n) = (-1)^n \frac{B_{n+1}}{n+1}, \quad \zeta(1-2n) = -\frac{B_{2n}}{2n},$$

$$\zeta(-2n) = 0, \quad \zeta(2n) = \frac{(2\pi)^{2n}}{2(2n)!} |B_{2n}| \quad [n=1, 2, 3, \dots].$$

$$\zeta(0) = -\frac{1}{2}, \quad \zeta(2) = \frac{\pi^2}{6}, \quad \zeta(4) = \frac{\pi^4}{90}.$$

**II.5. ИНТЕГРАЛЬНАЯ ПОКАЗАТЕЛЬНАЯ ФУНКЦИЯ $Ei(z)$,
ИНТЕГРАЛЬНЫЕ СИНУСЫ $si(z)$, $Si(z)$, $shi(z)$ И КОСИНУСЫ $ci(z)$,
 $chi(z)$**

О п р е д е л е н и я:

$$Ei(x) = \int_{-\infty}^x \frac{e^t}{t} dt = - \int_{-x}^{\infty} \frac{e^{-t}}{t} dt \quad [x < 0].$$

$$= C + \ln x + \int_0^x \frac{e^t - 1}{t} dt \quad [x > 0]$$

$$si(x) = - \int_x^{\infty} \frac{\sin t}{t} dt.$$

$$Si(x) = \frac{\pi}{2} + si(x) = \int_0^x \frac{\sin t}{t} dt.$$

$$ci(x) = - \int_x^{\infty} \frac{\cos t}{t} dt = C + \ln x + \int_0^x \frac{\cos t - 1}{t} dt.$$

$$shi(x) = \int_0^x \frac{\operatorname{sh} t}{t} dt.$$

$$chi(x) = C + \ln x + \int_0^x \frac{\operatorname{ch} t - 1}{t} dt.$$

На z -плоскости эти функции определяются как аналитические продолжения приведенных интегралов с помощью рядов

$$Ei(z) = -e^z \Psi(1, 1; -z) = C + \ln z + z {}_2F_2(1, 1; 2, 2; z).$$

$$\begin{cases} Si(z) \\ shi(z) \end{cases} = z {}_2F_2\left(\frac{1}{2}; \frac{3}{2}, \frac{3}{2}; \mp \frac{z^2}{4}\right).$$

$$\begin{cases} ci(z) \\ chi(z) \end{cases} = C + \ln z \mp \frac{z^2}{4} {}_2F_2\left(1, 1; 2, 2, \frac{3}{2}; \mp \frac{z^2}{4}\right).$$

$$Ei(\pm iz) = ci(z) \pm i si(z).$$

$$Ei(z) = -\Gamma(0, -z).$$

$$\begin{Bmatrix} \text{si} (z) \\ \text{ci} (z) \end{Bmatrix} = \frac{(-i)^{(1 \pm 1)/2}}{2} [\text{Ei} (iz) \mp \text{Ei} (-iz)].$$

$$\text{si} (-z) = -\pi - \text{si} (z), \quad \text{Si} (-z) = -\text{Si} (z).$$

$$\text{ci} (e^{\pm \pi i} x) = \text{ci} (x) \pm \pi i$$

[$x > 0$].

$$\text{shi} (z) = -i \text{Si} (iz),$$

$$\text{chi} (z) = \text{ci} (iz) - \frac{\pi i}{2}.$$

$$\frac{d}{dz} \text{Ei} (-z) = \frac{e^{-z}}{z}.$$

$$\frac{d}{dz} \begin{Bmatrix} \text{si} (z) \\ \text{ci} (z) \end{Bmatrix} = \frac{1}{z} \begin{Bmatrix} \sin z \\ \cos z \end{Bmatrix}.$$

$$\frac{d}{dz} \begin{Bmatrix} \text{shi} (z) \\ \text{chi} (z) \end{Bmatrix} = \frac{1}{z} \begin{Bmatrix} \text{sh} z \\ \text{ch} z \end{Bmatrix}.$$

II.6. ИНТЕГРАЛЫ ВЕРОЯТНОСТИ $\text{erf} (z)$, $\text{erfc} (z)$, $\text{erfi} (z)$ И ИНТЕГРАЛЫ ФРЕНЕЛЯ $S (z)$, $C (z)$, $S (z, \nu)$, $C (z, \nu)$

О п р е д е л е н и я:

$$\text{erf} (x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$

$$\text{erfc} (x) = 1 - \text{erf} (x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt.$$

$$\text{erfi} (x) = -i \text{erf} (ix) = \frac{2}{\sqrt{\pi}} \int_0^x e^{t^2} dt.$$

$$S (x) = \frac{1}{\sqrt{2\pi}} \int_0^x \frac{\sin t}{\sqrt{t}} dt, \quad \frac{1}{2} - S (x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \frac{\sin t}{\sqrt{t}} dt.$$

$$C (x) = \frac{1}{\sqrt{2\pi}} \int_0^x \frac{\cos t}{\sqrt{t}} dt, \quad \frac{1}{2} - C (x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \frac{\cos t}{\sqrt{t}} dt.$$

$$\begin{Bmatrix} S (x, \nu) \\ C (x, \nu) \end{Bmatrix} = \int_x^\infty t^{\nu-1} \begin{Bmatrix} \sin t \\ \cos t \end{Bmatrix} dt$$

[$\text{Re} \nu < 1$].

На z -плоскости эти функции определяются как аналитические продолжения приведенных интегралов с помощью рядов

$$\begin{Bmatrix} \text{erf} (z) \\ \text{erfi} (z) \end{Bmatrix} = \frac{2}{\sqrt{\pi}} z {}_1F_1 \left(\frac{1}{2}; \frac{3}{2}; \mp z^2 \right).$$

$$\text{erfc} (z) = \frac{1}{\sqrt{\pi}} e^{-z^2} \Psi \left(\frac{1}{2}, \frac{1}{2}; z^2 \right).$$

$$S (z) = \frac{z}{3} \sqrt{\frac{2z}{\pi}} {}_1F_2 \left(\frac{3}{4}; \frac{3}{2}, \frac{7}{4}; -\frac{z^2}{4} \right).$$

$$C (z) = \sqrt{\frac{2z}{\pi}} {}_1F_2 \left(\frac{1}{4}; \frac{1}{2}, \frac{5}{4}; -\frac{z^2}{4} \right).$$

$$\begin{aligned} \left\{ \begin{matrix} S(z, \nu) \\ C(z, \nu) \end{matrix} \right\} &= \left\{ \begin{matrix} \sin(\nu\pi/2) \\ \cos(\nu\pi/2) \end{matrix} \right\} \Gamma(\nu) - \frac{z^{\nu+0}}{\nu+0} {}_1F_2 \left(\frac{\nu+0}{2}; \frac{\nu+0}{2}+1, 0+\frac{1}{2}; -\frac{z^2}{4} \right) \\ &\quad \left[\left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \right]. \end{aligned}$$

$$\operatorname{erf}(z) = \sqrt{2} e^{\mp \pi i/4} [C(\pm iz^2) \pm iS(\pm iz^2)].$$

$$\left\{ \begin{matrix} \operatorname{erf}(z) \\ \operatorname{erfc}(z) \end{matrix} \right\} = \frac{1}{\sqrt{\pi}} \left\{ \begin{matrix} \gamma(1/2, z^2) \\ \Gamma(1/2, z^2) \end{matrix} \right\}.$$

$$\frac{d}{dz} \left\{ \begin{matrix} \operatorname{erf}(z) \\ \operatorname{erfc}(z) \end{matrix} \right\} = \pm \frac{2}{\sqrt{\pi}} e^{-z^2}.$$

$$\left\{ \begin{matrix} S(z) \\ C(z) \end{matrix} \right\} = (-i)^{(1\pm 1)/2} 2^{-3/2} [e^{\pi i/4} \operatorname{erf}(e^{-\pi i/4} \sqrt{z}) \mp e^{-\pi i/4} \operatorname{erf}(e^{\pi i/4} \sqrt{z})].$$

$$\frac{d}{dz} \left\{ \begin{matrix} S(z) \\ C(z) \end{matrix} \right\} = \frac{1}{\sqrt{2\pi z}} \left\{ \begin{matrix} \sin z \\ \cos z \end{matrix} \right\}.$$

$$\left\{ \begin{matrix} S(z) \\ C(z) \end{matrix} \right\} = \frac{1}{\sqrt{2\pi}} \left\{ \begin{matrix} S(z, 1/2) \\ C(z, 1/2) \end{matrix} \right\}.$$

$$\left\{ \begin{matrix} S(z, 0) \\ C(z, 0) \end{matrix} \right\} = - \left\{ \begin{matrix} \operatorname{si}(z) \\ \operatorname{ci}(z) \end{matrix} \right\}.$$

II.7. НЕПОЛНЫЕ ГАММА-ФУНКЦИИ $\gamma(\nu, z)$, $\Gamma(\nu, z)$

О п р е д е л е н и я:

$$\gamma(\nu, x) = \int_0^x t^{\nu-1} e^{-t} dt \quad [\operatorname{Re} \nu > 0].$$

$$\Gamma(\nu, x) = \Gamma(\nu) - \gamma(\nu, x) = \int_x^\infty t^{\nu-1} e^{-t} dt.$$

На z -плоскости функции $\gamma(\nu, z)$, $\Gamma(\nu, z)$ определяются как аналитические продолжения приведенных интегралов с помощью рядов

$$\gamma(\nu, z) = \frac{z^\nu}{\nu} {}_1F_1(\nu; \nu+1; -z).$$

$$\Gamma(\nu, z) = e^{-z} \Psi(1-\nu, 1-\nu; z).$$

$$\gamma(\nu+1, z) = \nu \gamma(\nu, z) - z^\nu e^{-z}.$$

$$\Gamma(\nu+1, z) = \nu \Gamma(\nu, z) + z^\nu e^{-z}.$$

$$\left\{ \begin{matrix} \gamma(n+1, z) \\ \Gamma(n+1, z) \end{matrix} \right\} = n! \left[\left\{ \begin{matrix} 1 \\ 0 \end{matrix} \right\} \mp e^{-z} \sum_{k=0}^n \frac{z^k}{k!} \right].$$

$$\Gamma(\nu, z) = e^{\pi \nu i/2} [C(-iz, \nu) - iS(-iz, \nu)],$$

$$\frac{d}{dz} \left\{ \begin{matrix} \gamma(\nu, z) \\ \Gamma(\nu, z) \end{matrix} \right\} = \pm z^{\nu-1} e^{-z}.$$

$$\left\{ \begin{matrix} \gamma(1/2, z) \\ \Gamma(1/2, z) \end{matrix} \right\} = \sqrt{\pi} \left\{ \begin{matrix} \operatorname{erf}(\sqrt{z}) \\ \operatorname{erfc}(\sqrt{z}) \end{matrix} \right\}.$$

$$\Gamma(0, z) = -\operatorname{Ei}(-z).$$

II.8. ФУНКЦИЯ ПАРАБОЛИЧЕСКОГО ЦИЛИНДРА $D_\nu(z)$

Определение:

$$\begin{aligned} D_\nu(z) &= 2^{\nu/2} e^{-z^2/4} \Psi\left(-\frac{\nu}{2}, \frac{1}{2}; \frac{z^2}{2}\right) = \\ &= 2^{(\nu-1)/2} e^{-z^2/4} {}_2F_1\left(\frac{1-\nu}{2}, \frac{3}{2}; \frac{z^2}{2}\right) = \\ &= 2^{\nu/2} e^{-z^2/4} \left[\frac{\sqrt{\pi}}{\Gamma[(1-\nu)/2]} {}_1F_1\left(-\frac{\nu}{2}; \frac{1}{2}; \frac{z^2}{2}\right) - \right. \\ &\quad \left. - \frac{\sqrt{2\pi}}{\Gamma(-\nu/2)} z {}_1F_1\left(\frac{1-\nu}{2}; \frac{3}{2}; \frac{z^2}{2}\right) \right]. \end{aligned}$$

$$\begin{aligned} D_\nu(z) &= \frac{\Gamma(\nu+1)}{\sqrt{2\pi}} [e^{\nu\pi i/2} D_{-\nu-1}(iz) + e^{-\nu\pi i/2} D_{-\nu-1}(-iz)] = \\ &= e^{\mp \nu\pi i} D_\nu(-z) + \frac{\sqrt{2\pi}}{\Gamma(-\nu)} e^{\mp (\nu+1)\pi i/2} D_{-\nu-1}(\pm iz). \end{aligned}$$

$$D_{\nu+1}(z) - zD_\nu(z) + \nu D_{\nu-1}(z) = 0.$$

$$\frac{d}{dz} D_\nu(z) \pm \frac{z}{2} D_\nu(z) \mp \left\{ \frac{\nu}{1} \right\} D_{\nu \mp 1}(z) = 0.$$

$$\frac{d^n}{dz^n} [e^{\pm z^2/4} D_\nu(z)] = (-1)^n \left\{ \begin{matrix} (-\nu)_n \\ 1 \end{matrix} \right\} e^{\pm z^2/4} D_{\nu \mp n}(z).$$

$$D_{-1}(z) = \sqrt{\frac{\pi}{2}} e^{z^2/4} \operatorname{erfc}\left(\frac{z}{\sqrt{2}}\right).$$

$$D_{-2}(z) = -\sqrt{\frac{\pi}{2}} z e^{z^2/4} \operatorname{erfc}\left(\frac{z}{\sqrt{2}}\right) + e^{-z^2/4}.$$

$$D_{-n-1}(z) = \frac{(-1)^n}{n!} \sqrt{\frac{\pi}{2}} e^{-z^2/4} \frac{d^n}{dz^n} \left[e^{z^2/2} \operatorname{erfc}\left(\frac{z}{\sqrt{2}}\right) \right],$$

$$D_{-1/2}(z) = \sqrt{\frac{z}{2\pi}} K_{1/4}\left(\frac{z^2}{4}\right).$$

$$D_{-3/2}(z) = \frac{z^{3/2}}{\sqrt{2\pi}} \left[K_{3/4}\left(\frac{z^2}{4}\right) - K_{1/4}\left(\frac{z^2}{4}\right) \right].$$

$$D_n(z) = 2^{-n/2} e^{-z^2/4} H_n\left(\frac{z}{\sqrt{2}}\right).$$

II.9. ФУНКЦИИ БЕССЕЛЯ $J_\nu(z)$, НЕЙМАНА $Y_\nu(z)$ И ГАНКЕЛЯ $H_\nu^{(1)}(z)$, $H_\nu^{(2)}(z)$

Определения:

$$J_\nu(z) = \frac{1}{\Gamma(\nu+1)} \left(\frac{z}{2}\right)^\nu {}_0F_1\left(\nu+1; -\frac{z^2}{4}\right) = \sum_{k=0}^{\infty} \frac{(-1)^k (z/2)^{2k+\nu}}{k! \Gamma(k+\nu+1)}.$$

$$Y_\nu(z) = \frac{1}{\sin \nu\pi} [J_\nu(z) \cos \nu\pi - J_{-\nu}(z)] \quad [\nu \neq \pm n].$$

$$\begin{aligned} Y_n(z) &= \lim_{\nu \rightarrow n} Y_\nu(z) = \frac{2}{\pi} J_n(z) \ln \frac{z}{2} - \frac{1}{\pi} \sum_{k=0}^{n-1} \frac{(n-k-1)!}{k!} \left(\frac{z}{2}\right)^{2k-n} - \\ &\quad - \frac{1}{\pi} \sum_{k=0}^{\infty} \frac{(-1)^k}{(n+k)! k!} [\psi(n+k+1) + \psi(k+1)] \left(\frac{z}{2}\right)^{n+2k}, \end{aligned}$$

$$H_{\nu}^{\left(\begin{smallmatrix} 1 \\ 2 \end{smallmatrix}\right)}(z) = J_{\nu}(z) \pm iY_{\nu}(z) = \pm \frac{i}{\sin \nu\pi} [e^{\mp \nu\pi i} J_{\nu}(z) - J_{-\nu}(z)],$$

$$J_{-n}(z) = (-1)^n J_n(z), \quad Y_{-n}(z) = (-1)^n Y_n(z).$$

$$H_{-n}^{(1, 2)}(z) = (-1)^n H_n^{(1, 2)}(z), \quad H_{-\nu}^{\left(\begin{smallmatrix} 1 \\ 2 \end{smallmatrix}\right)}(z) = e^{\pm \nu\pi i} H_{\nu}^{\left(\begin{smallmatrix} 1 \\ 2 \end{smallmatrix}\right)}(z).$$

Ниже $Z_{\nu}(z)$ — любая линейная комбинация функций $J_{\nu}(z)$, $Y_{\nu}(z)$, $H_{\nu}^{(1)}(z)$, $H_{\nu}^{(2)}(z)$, коэффициенты которой не зависят от z и ν .

$$2\nu Z_{\nu}(z) = zZ_{\nu+1}(z) + zZ_{\nu-1}(z).$$

$$z \frac{d}{dz} Z_{\nu}(z) = \pm \nu Z_{\nu}(z) \mp zZ_{\nu \pm 1}(z).$$

$$2 \frac{d}{dz} Z_{\nu}(z) = Z_{\nu-1}(z) - Z_{\nu+1}(z).$$

$$\frac{d}{dz} [z^{\pm \nu} Z_{\nu}(z)] = \pm z^{\pm \nu} Z_{\nu \mp 1}(z).$$

$$\left(\frac{1}{z} \frac{d}{dz}\right)^m [z^{\pm \nu} Z_{\nu}(z)] = (\pm 1)^m z^{\pm \nu - m} Z_{\nu \mp m}(z).$$

$$\frac{d^m}{dz^m} Z_{\nu}(z) = \frac{1}{2^m} \sum_{k=0}^m (-1)^k \binom{m}{k} Z_{\nu - m + 2k}(z).$$

$$\begin{Bmatrix} J_2(z) \\ Y_2(z) \end{Bmatrix} = \frac{2}{z} \begin{Bmatrix} J_1(z) \\ Y_1(z) \end{Bmatrix} - \begin{Bmatrix} J_0(z) \\ Y_0(z) \end{Bmatrix},$$

$$H_2^{(1, 2)}(z) = \frac{2}{z} H_1^{(1, 2)}(z) - H_0^{(1, 2)}(z).$$

$$\frac{d}{dz} \begin{Bmatrix} J_0(z) \\ Y_0(z) \end{Bmatrix} = - \begin{Bmatrix} J_1(z) \\ Y_1(z) \end{Bmatrix}, \quad \frac{d}{dz} H_0^{(1, 2)}(z) = - H_1^{(1, 2)}(z).$$

$$J_{\nu}(ze^{m\pi i}) = e^{m\nu\pi i} J_{\nu}(z).$$

$$J_n(-z) = J_{-n}(z), \quad J_n(iz) = i^n J_n(z).$$

$$J_{\nu}(ze^{(-1 \pm 2)\pi i/2}) = e^{(\pm 2 - 1)\nu\pi i/2} J_{\nu}(z) \quad \left[\begin{Bmatrix} -\pi < \arg z \leq \pi/2 \\ \pi/2 < \arg z \leq \pi \end{Bmatrix} \right].$$

$$Y_{\pm \nu}(ze^{m\pi i}) = e^{-m\nu\pi i} Y_{\pm \nu}(z) + 2i \sin m\nu\pi \begin{Bmatrix} \operatorname{ctg} \nu\pi \\ \operatorname{cosec} \nu\pi \end{Bmatrix} J_{\nu}(z).$$

$$Y_{\nu}(ze^{\pi i/2}) = e^{(\nu+1)\pi i/2} J_{\nu}(z) - \frac{2}{\pi} e^{-\nu\pi i/2} K_{\nu}(z) \quad [-\pi < \arg z \leq \pi/2].$$

$$H_{\nu}^{\left(\begin{smallmatrix} 1 \\ 2 \end{smallmatrix}\right)}(ze^{m\pi i}) = \mp e^{\mp \nu\pi i} \frac{\sin m\nu\pi}{\sin \nu\pi} H_{\nu}^{\left(\begin{smallmatrix} 2 \\ 1 \end{smallmatrix}\right)}(z) \mp \frac{\sin(m \mp 1)\nu\pi}{\sin \nu\pi} H_{\nu}^{\left(\begin{smallmatrix} 1 \\ 2 \end{smallmatrix}\right)}(z) \quad [m = \dots, -2, -1, 0, 1, 2, \dots].$$

$$H_{\nu}^{\left(\begin{smallmatrix} 1 \\ 2 \end{smallmatrix}\right)}(ze^{\pm \pi i}) = - H_{-\nu}^{\left(\begin{smallmatrix} 2 \\ 1 \end{smallmatrix}\right)}(z).$$

$$J_{\nu}(z) = \frac{Y_{-\nu}(z) - Y_{\nu}(z) \cos \nu\pi}{\sin \nu\pi} = H_{\nu}^{\left(\begin{smallmatrix} 1 \\ 2 \end{smallmatrix}\right)}(z) \mp iY_{\nu}(z) = \frac{1}{2} [H_{\nu}^{(1)}(z) + H_{\nu}^{(2)}(z)].$$

$$Y_{\nu}(z) = \pm iJ_{\nu}(z) \mp iH_{\nu}^{\left(\begin{smallmatrix} 1 \\ 2 \end{smallmatrix}\right)}(z) = \frac{1}{2i} [H_{\nu}^{(1)}(z) - H_{\nu}^{(2)}(z)].$$

$$J_{\nu}(z) Y_{\nu+1}(z) - J_{\nu+1}(z) Y_{\nu}(z) = -\frac{2}{\pi z}.$$

$$H_{\pm \nu}^{(1)}(z) = \frac{2}{\pi i} e^{\mp \nu\pi i/2} K_{\nu}(-iz).$$

$$J_{\pm n \pm 1/2}(z) = (\mp 1)^n \sqrt{\frac{2}{\pi}} z^{n+1/2} \left(\frac{1}{z} \frac{d}{dz} \right)^n \left(\frac{1}{z} \begin{Bmatrix} \sin z \\ \cos z \end{Bmatrix} \right) =$$

$$= \sqrt{\frac{2}{\pi z}} \left[\begin{Bmatrix} \sin(z - n\pi/2) \\ \cos(z + n\pi/2) \end{Bmatrix} \sum_{k=0}^{[n/2]} \frac{(-1)^k (n+2k)!}{(2k)! (n-2k)! (2z)^{2k}} \pm \right.$$

$$\left. \pm \begin{Bmatrix} \cos(z - n\pi/2) \\ \sin(z + n\pi/2) \end{Bmatrix} \sum_{k=0}^{[(n-1)/2]} \frac{(-1)^k (n+2k+1)!}{(2k+1)! (n-2k-1)! (2z)^{2k+1}} \right].$$

$$J_{\pm 1/2}(z) = \sqrt{\frac{2}{\pi z}} \begin{Bmatrix} \sin z \\ \cos z \end{Bmatrix},$$

$$J_{\pm 3/2}(z) = \sqrt{\frac{2}{\pi z}} \left[\pm \frac{1}{z} \begin{Bmatrix} \sin z \\ \cos z \end{Bmatrix} - \begin{Bmatrix} \cos z \\ \sin z \end{Bmatrix} \right],$$

$$J_{\pm 5/2}(z) = \sqrt{\frac{2}{\pi z}} \left[\left(\frac{3}{z^2} - 1 \right) \begin{Bmatrix} \sin z \\ \cos z \end{Bmatrix} \mp \frac{3}{z} \begin{Bmatrix} \cos z \\ \sin z \end{Bmatrix} \right].$$

$$Y_{\pm n \pm 1/2}(z) = \mp (-1)^n J_{\mp n \mp 1/2}(z).$$

$$Y_{\pm 1/2}(z) = \mp \sqrt{\frac{2}{\pi z}} \begin{Bmatrix} \cos z \\ \sin z \end{Bmatrix}.$$

$$H_{n-1/2}^{(j)}(z) = \sqrt{\frac{2}{\pi z}} e^{(-1)^j (n\pi/2 - z)i} \sum_{k=0}^{n-1} (-1)^{kj} \frac{(n+k-1)!}{k! (n-k-1)!} \frac{1}{(2iz)^k}$$

$[j=1, 2].$

$$H_{\pm 1/2}^{(j)}(z) = ((-1)^j i)^{(1 \pm 1)/2} \sqrt{\frac{2}{\pi z}} e^{-(-1)^j iz}$$

$[j=1, 2].$

II.10. МОДИФИЦИРОВАННАЯ ФУНКЦИЯ БЕССЕЛЯ $I_\nu(z)$ И ФУНКЦИЯ МАКДОНАЛЬДА $K_\nu(z)$

Определения:

$$I_\nu(z) = \frac{1}{\Gamma(\nu+1)} \left(\frac{z}{2} \right)^\nu {}_0F_1 \left(\nu+1; \frac{z^2}{4} \right) = \sum_{k=0}^{\infty} \frac{(z/2)^{2k+\nu}}{k! \Gamma(k+\nu+1)}.$$

$$K_\nu(z) = \frac{\pi}{2 \sin \nu\pi} [I_{-\nu}(z) - I_\nu(z)] \quad [\nu \neq \pm n].$$

$$K_n(z) = \lim_{\nu \rightarrow n} K_\nu(z) = (-1)^{n+1} I_n(z) \ln \frac{z}{2} + \frac{1}{2} \sum_{k=0}^{n-1} (-1)^k \frac{(n-k-1)!}{k!} \left(\frac{z}{2} \right)^{2k-n} +$$

$$+ \frac{(-1)^n}{2} \sum_{k=0}^{\infty} \frac{\psi(k+n+1) + \psi(k+1)}{(n+k)! k!} \left(\frac{z}{2} \right)^{n+2k}.$$

$$I_{-n}(z) = I_n(z), \quad K_{-n}(z) = K_n(z).$$

$$2\nu \begin{Bmatrix} I_\nu(z) \\ K_\nu(z) \end{Bmatrix} = \begin{Bmatrix} zI_{\nu-1}(z) - zI_{\nu+1}(z) \\ -zK_{\nu-1}(z) + zK_{\nu+1}(z) \end{Bmatrix}.$$

$$z \frac{d}{dz} I_\nu(z) = \pm \nu I_\nu(z) + zI_{\nu \pm 1}(z),$$

$$z \frac{d}{dz} K_\nu(z) = \pm K_\nu(z) - zK_{\nu \pm 1}(z).$$

$$2 \frac{d}{dz} \begin{Bmatrix} I_\nu(z) \\ K_\nu(z) \end{Bmatrix} = \begin{Bmatrix} I_{\nu-1}(z) + I_{\nu+1}(z) \\ -K_{\nu-1}(z) - K_{\nu+1}(z) \end{Bmatrix}.$$

$$\frac{d}{dz} [z^{\pm \nu} I_{\nu}(z)] = z^{\pm \nu} I_{\nu \mp 1}(z),$$

$$\frac{d}{dz} [z^{\pm \nu} K_{\nu}(z)] = -z^{\pm \nu} K_{\nu \mp 1}(z).$$

$$\left(\frac{1}{z} \frac{d}{dz}\right)^m [z^{\pm \nu} I_{\nu}(z)] = z^{\pm \nu - m} I_{\nu \mp m}(z),$$

$$\left(\frac{1}{z} \frac{d}{dz}\right)^m [z^{\pm \nu} K_{\nu}(z)] = (-1)^m z^{\pm \nu - m} K_{\nu \mp m}(z).$$

$$\frac{d^m}{dz^m} \begin{Bmatrix} I_{\nu}(z) \\ K_{\nu}(z) \end{Bmatrix} = \frac{(\pm 1)^m}{2^m} \sum_{k=0}^m \binom{m}{k} \begin{Bmatrix} I_{\nu-m+2k}(z) \\ K_{\nu-m+2k}(z) \end{Bmatrix}.$$

$$I_2(z) = -\frac{2}{z} I_1(z) + I_0(z), \quad K_2(z) = \frac{2}{z} K_1(z) + K_0(z).$$

$$\frac{d}{dz} I_0(z) = I_1(z), \quad \frac{d}{dz} K_0(z) = -K_1(z).$$

$$I_{\nu}(ze^{m\pi i}) = e^{m\nu\pi i} I_{\nu}(z).$$

$$I_n(-z) = (-1)^n I_n(z).$$

$$I_{\nu}(ze^{\pm \pi i/2}) = e^{\pm \nu \pi i/2} J_{\nu}(z)$$

$$\left[\begin{Bmatrix} -\pi < \arg z \leq \pi/2 \\ \pi/2 < \arg z \leq \pi \end{Bmatrix} \right].$$

$$K_{\nu}(ze^{m\pi i}) = e^{-m\nu\pi i} K_{\nu}(z) - i\pi \frac{\sin m\nu\pi}{\sin \nu\pi} I_{\nu}(z) \quad [\nu \neq \pm n].$$

$$K_{\nu}(ze^{\pm \pi i/2}) = \mp \frac{\pi}{2} e^{(\mp \nu + 1)\pi i/2} [J_{\nu}(z) \mp iY_{\nu}(z)] = \mp \frac{\pi}{2} e^{(\mp \nu + 1)\pi i/2} H_{\nu}^{(2)} \left\{ \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\} (z).$$

$$\begin{aligned} I_{\nu}(z) &= e^{-\nu\pi i/2} J_{\nu}(ze^{\pi i/2}) & [-\pi < \arg z \leq \pi/2] \\ &= e^{3\nu\pi i/2} J_{\nu}(ze^{-3\pi i/2}) & [\pi/2 < \arg z \leq \pi] \end{aligned}$$

$$\begin{aligned} K_{\nu}(z) &= \pm \frac{\pi i}{2} e^{\pm \nu \pi i/2} H_{\nu}^{(1)} \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\} (ze^{\pm \pi i/2}) & \left[\begin{Bmatrix} -\pi < \arg z \leq \pi/2 \\ \pi/2 < \arg z \leq \pi \end{Bmatrix} \right] \\ &= \frac{\pi}{2} e^{(\pm \nu + 1)\pi i/2} H_{\pm \nu}^{(1)}(ze^{\pi i/2}). \end{aligned}$$

$$I_{\nu}(z) K_{\nu+1}(z) + I_{\nu+1}(z) K_{\nu}(z) = \frac{1}{z}.$$

$$\begin{aligned} I_{\pm n \pm 1/2}(z) &= z^n \left(\frac{1}{z} \frac{d}{dz} \right)^n \left[\frac{1}{z} \begin{Bmatrix} \text{sh } z \\ \text{ch } z \end{Bmatrix} \right] \\ &= \frac{1}{\sqrt{2\pi z}} \left[e^z \sum_{k=0}^n \frac{(-1)^k (n+k)!}{k! (n-k)! (2z)^k} \pm (-1)^{n+1} e^{-z} \sum_{k=0}^n \frac{(n+k)!}{k! (n-k)! (2z)^k} \right]. \end{aligned}$$

$$I_{\pm 1/2}(z) = \sqrt{\frac{2}{\pi z}} \begin{Bmatrix} \text{sh } z \\ \text{ch } z \end{Bmatrix}, \quad I_{\pm 3/2}(z) = \sqrt{\frac{2}{\pi z}} \left[\begin{Bmatrix} \text{ch } z \\ \text{sh } z \end{Bmatrix} \mp \frac{1}{z} \begin{Bmatrix} \text{sh } z \\ \text{ch } z \end{Bmatrix} \right],$$

$$I_{\pm 5/2}(z) = \sqrt{\frac{2}{\pi z}} \left[\left(\frac{3}{z^2} + 1 \right) \begin{Bmatrix} \text{sh } z \\ \text{ch } z \end{Bmatrix} - \frac{3}{z} \begin{Bmatrix} \text{ch } z \\ \text{sh } z \end{Bmatrix} \right],$$

$$\begin{aligned} K_{n+1/2}(z) &= \sqrt{\frac{\pi}{2z}} e^{-z} \sum_{k=0}^n \frac{(n+k)!}{k! (n-k)! (2z)^k}, \\ &= n! \sqrt{\frac{\pi}{2z}} (-2z)^{-n} e^{-z} L_n^{-2n-1}(2z). \end{aligned}$$

$$K_{\pm 1/2}(z) = \sqrt{\frac{\pi}{2z}} e^{-z}, \quad K_{\pm 3/2}(z) = \frac{\pi}{2z} \left(1 + \frac{1}{z} \right) e^{-z}.$$

**II.11. ОРТОГОНАЛЬНЫЕ МНОГОЧЛЕНЫ ЛЕЖАНДРА $P_n(z)$,
ЧЕБЫШЕВА $T_n(z)$, $U_n(z)$, ЛАГЕРРА $L_n(z)$, $L_n^\lambda(z)$, ЭРМИТА $H_n(z)$,
ГЕГЕНБАУЭРА $C_n^\lambda(z)$ И ЯКОБИ $P_n^{(\rho, \sigma)}(z)$**

Определения:

$$P_n(z) = \frac{1}{2^n} \sum_{k=0}^{[n/2]} (-1)^k \binom{n}{k} \binom{2n-2k}{n} z^{n-2k} \\ = \frac{1}{2^{2n}} \sum_{k=0}^n \binom{2k}{n} \binom{2n-2k}{n-k} \cos [(n-2k) \arccos z]$$

— многочлены Лежандра;

$$T_n(z) = \frac{n}{2} \sum_{k=0}^{[n/2]} (-1)^k \frac{(n-k-1)!}{k! (n-2k)!} (2z)^{n-2k}, \\ = \cos (n \arccos z)$$

— многочлены Чебышева 1-го рода;

$$U_n(z) = \sum_{k=0}^{[n/2]} (-1)^k \frac{(n-k)!}{k! (n-2k)!} (2z)^{n-2k}, \\ = (1-z^2)^{-1/2} \sin [(n+1) \arccos z]$$

— многочлены Чебышева 2-го рода;

$$L_n^\lambda(z) = \sum_{k=0}^n \frac{(-1)^k}{k!} \binom{n+\lambda}{n-k} z^k \quad [\operatorname{Re} \lambda > -1]$$

— обобщенные многочлены Лагерра (при $\lambda=0$ — многочлены Лагерра

$$L_n(z) \equiv L_n^0(z)),$$

$$H_n(z) = n! \sum_{k=0}^{[n/2]} \frac{(-1)^k}{k! (n-2k)!} (2z)^{n-2k}$$

— многочлены Эрмита;

$$C_n^\lambda(z) = \sum_{k=0}^{[n/2]} \frac{(-1)^k (\lambda)_{n-k}}{k! (n-2k)!} (2z)^{n-2k}, \\ = \sum_{k=0}^n \frac{(\lambda)_k (\lambda)_{n-k}}{k! (n-k)!} \cos [(n-2k) \arccos z] \quad [\operatorname{Re} \lambda > -1/2; \lambda \neq 0],$$

$$C_n^0(z) = \lim_{\lambda \rightarrow 0} [\lambda^{-1} C_n^\lambda(z)] = \sum_{k=0}^{[n/2]} (-1)^k \frac{(n-k-1)!}{k! (n-2k)!} (2z)^{n-2k}, \\ = \frac{2}{n} \cos (n \arccos z) \quad [n \neq 0],$$

$$C_0^0(z) = 1$$

— многочлены Гегенбауэра;

$$P_n^{(\rho, \sigma)}(z) = \frac{1}{2^n} \sum_{k=0}^n \binom{n+\rho}{k} \binom{n+\sigma}{n-k} (z+1)^k (z-1)^{n-k}, \\ = \frac{1}{n!} \Gamma \left[\begin{matrix} \rho+n+1 \\ \rho+\sigma+n+1 \end{matrix} \right] \sum_{k=0}^n 2^{-k} \binom{n}{k} \Gamma \left[\begin{matrix} \rho+\sigma+n+k+1 \\ \rho+k+1 \end{matrix} \right] (z-1)^k \quad [\operatorname{Re} \rho, \operatorname{Re} \sigma > -1]$$

— многочлены Якоби.

$$f_n(z) = \frac{1}{b_n \rho(z)} \frac{d^n}{dz^n} \{ \rho(z) [g(z)]^n \}$$

$f_n(z)$	b_n	$\rho(z)$	$g(z)$
$P_n(z)$	$(-1)^n 2^n n!$	1	$1-z^2$
$T_n(z)$	$(-1)^n 2^n (1/2)_n$	$(1-z^2)^{-1/2}$	$1-z^2$
$U_n(z)$	$(-1)^n 2^n \frac{(3/2)_n}{n+1}$	$(1-z^2)^{1/2}$	$1-z^2$
$L_n(z)$	$n!$	e^{-z}	z
$L_n^\lambda(z)$	$n!$	$z^\lambda e^{-z}$	z
$H_n(z)$	$(-1)^n$	e^{-z^2}	1
$C_n^\lambda(z)$	$(-1)^n 2^n n! \frac{(\lambda+1/2)_n}{(2\lambda)_n}$	$(1-z^2)^{\lambda-1/2}$	$1-z^2$
$P_n^{(\rho, \sigma)}(z)$	$(-1)^n 2^n n!$	$(1-z)^\rho (1+z)^\sigma$	$1-z^2$

Четность, частные значения

$f_n(z)$	$f_n(-z)$	$f_n(1)$	$f_n(0)$
$P_n(z)$	$(-1)^n P_n(z)$	1	$\begin{cases} 0, & n=2m+1 \\ \frac{(-1)^m}{4^m} \binom{2m}{m}, & n=2m \end{cases}$
$T_n(z)$	$(-1)^n T_n(z)$	1	$\begin{cases} 0, & n=2m+1 \\ (-1)^m, & n=2m \end{cases}$
$U_n(z)$	$(-1)^n U_n(z)$	$n+1$	$\begin{cases} 0, & n=2m+1 \\ (-1)^m, & n=2m \end{cases}$
$L_n^\lambda(z)$			$\frac{(\lambda+1)_n}{n!}$
$H_n(z)$	$(-1)^n H_n(z)$		$\begin{cases} 0, & n=2m+1 \\ (-1)^m \frac{(2m)!}{m!}, & n=2m \end{cases}$
$C_n^\lambda(z)$ $\lambda \neq 0$	$(-1)^n C_n^\lambda(z)$	$\frac{(2\lambda)_n}{n!}$	$\begin{cases} 0, & n=2m+1 \\ (-1)^m \frac{(\lambda)_m}{m!}, & n=2m \end{cases}$
$C_n^1(z)$ $n \neq 0$	$(-1)^n C_n^0(z)$	$\frac{2}{n}$	$\begin{cases} 0, & n=2m+1 \\ \frac{(-1)^m}{m}, & n=2m \end{cases}$
$P_n^{(\rho, \sigma)}(z)$	$(-1)^n P_n^{(\sigma, \rho)}(z)$	$\frac{(\rho+1)_n}{n!}$	

$$P_0(z) = 1, \quad P_1(z) = z, \quad P_2(z) = \frac{3}{2}z^2 - \frac{1}{2};$$

$$T_0(z) = 1, \quad T_1(z) = z, \quad T_2(z) = 2z^2 - 1;$$

$$U_0(z) = 1, \quad U_1(z) = 2z, \quad U_2(z) = 4z^2 - 1;$$

$$L_0^\lambda(z) = 1, \quad L_1^\lambda(z) = 1 + \lambda - z, \quad L_2^\lambda(z) = \frac{1}{2}[(1 + \lambda - z)(3 + \lambda - z) - 1 - \lambda];$$

$$H_0(z) = 1, \quad H_1(z) = 2z, \quad H_2(z) = 4z^2 - 2;$$

$$C_0^\lambda(z) = 1, \quad C_1^\lambda(z) = 2\lambda z \quad (\lambda \neq 0), \quad C_1^0(z) = 2z,$$

$$C_2^\lambda(z) = 2\lambda(\lambda + 1)z^2 - \lambda \quad (\lambda \neq 0), \quad C_2^0(z) = 2z^2 - 1;$$

$$P_0^{(\rho, \sigma)}(z) = 1, \quad P_1^{(\rho, \sigma)}(z) = \frac{1}{2}[(\rho + \sigma + 2)z + \rho - \sigma],$$

$$P_2^{(\rho, \sigma)}(z) = \frac{1}{8}[(\rho + \sigma + 3)(\rho + \sigma + 4)z^2 + 2(\rho + \sigma + 3)(\rho - \sigma)z + \\ + (\rho + 1)(\rho + 2) + (\sigma + 1)(\sigma + 2) - 2(\rho + 2)(\sigma + 2)].$$

Соотношения между многочленами

$$P_n(z) = C_n^{1/2}(z) = P_n^{(0, 0)}(z)$$

$$T_n(z) = \frac{n}{2} \lim_{\lambda \rightarrow 0} [\lambda^{-1} C_n^\lambda(z)] = \frac{n!}{(1/2)_n} P_n^{(-1/2, -1/2)}(z),$$

$$T_n(z) = U_n(z) - zU_{n-1}(z), \\ = zU_{n-1}(z) - U_{n-2}(z), \\ = 2^{-1}[U_n(z) - U_{n-2}(z)],$$

$$T_{2n+1}(z) = \frac{n!}{(1/2)_n} z P_n^{(-1/2, 1/2)}(2z^2 - 1).$$

$$U_n(z) = C_n^1(z) = \frac{(n+1)!}{(3/2)_n} P_n^{(1/2, 1/2)}(z), \\ = (1 - z^2)^{-1} [zT_{n+1}(z) - T_{n+2}(z)],$$

$$U_{2n}(z) = \frac{n!}{(1/2)_n} P_n^{(1/2, -1/2)}(2z^2 - 1).$$

$$L_n^\lambda(z) = \lim_{\sigma \rightarrow \infty} P_n^{(\lambda, \sigma)}\left(1 - \frac{2z}{\sigma}\right),$$

$$L_n^{-1/2}(z) = \frac{(-1)^n}{n! 2^{2n}} H_{2n}(\sqrt{z}),$$

$$L_n^{1/2}(z) = \frac{(-1)^n}{n! 2^{2n+1} \sqrt{z}} H_{2n+1}(\sqrt{z}).$$

$$H_n(z) = n! \lim_{\lambda \rightarrow \infty} \left[\lambda^{-n/2} C_n^\lambda\left(\frac{z}{\sqrt{\lambda}}\right) \right],$$

$$H_{2n}(z) = (-1)^n 2^{2n} n! L_n^{-1/2}(z^2),$$

$$H_{2n+1}(z) = (-1)^n 2^{2n+1} n! z L_n^{1/2}(z^2).$$

$$C_n^\lambda(z) = \frac{(2\lambda)_n}{(\lambda + 1/2)_n} P_n^{(\lambda - 1/2, \lambda - 1/2)}(z) \quad [\lambda \neq 0],$$

$$= \frac{\Gamma(\lambda + 1/2) (2\lambda)_n}{n!} \left(\frac{z^2 - 1}{4}\right)^{(1-2\lambda)/4} P_{\frac{n}{2} + \frac{\lambda}{2} - 1/2}^{1/2 - \lambda}(z) \quad [\lambda \neq 0],$$

$$C_n^0(z) = \frac{2}{n} T_n(z),$$

$$C_{2n}^\lambda(z) = \frac{(\lambda)_n n! 2^{2n}}{(2n)!} P_n^{(\lambda - 1/2, -1/2)}(2z^2 - 1) \quad [\lambda \neq 0],$$

$$C_{2n+1}^\lambda(z) = \frac{(\lambda)_{n+1} n! 2^{2n+1}}{(2n+1)!} z P_n^{(\lambda - 1/2, 1/2)}(2z^2 - 1) \quad [\lambda \neq 0],$$

$$C_n^\lambda(z) = \frac{(-2)^n (\lambda)_n}{(2\lambda + n)_n} (z^2 - 1)^{n/2} C_n^{1/2 - \lambda - n}\left(\frac{z}{\sqrt{z^2 - 1}}\right).$$

$$P_n^{(\sigma, \sigma)}(z) = \frac{(\sigma + 1)_n}{(2\sigma + 1)_n} C_n^{\sigma + 1/2}(z),$$

$$P_n^{(\rho, -1/2)}(z) = \frac{(1/2)_{n+1}}{(\rho + 1/2)_{n+1}} \sqrt{\frac{2}{z+1}} C_{2n+1}^{\rho + 1/2}\left(\sqrt{\frac{z+1}{2}}\right),$$

$$= \frac{(1/2)_n}{(\rho + 1/2)_n} C_{2n}^{\rho + 1/2}\left(\sqrt{\frac{z+1}{2}}\right),$$

$$P_n^{(1/2, -1/2)}(z) = \frac{(1/2)_n}{n!} U_{2n}\left(\sqrt{\frac{z+1}{2}}\right),$$

$$P_n^{(1/2, 1/2)}(z) = \frac{(3/2)_n}{(n+1)!} U_n(z),$$

$$P_n^{(-1/2, 1/2)}(z) = \frac{(1/2)_n}{n!} \sqrt{\frac{2}{z+1}} T_{2n+1}\left(\sqrt{\frac{z+1}{2}}\right),$$

$$P_n^{(-1/2, -1/2)}(z) = \frac{(1/2)_n}{n!} T_n(z),$$

$$P_n^{(0, 0)}(z) = P_n(z).$$

$$\begin{aligned} P_n^{(\rho, \sigma)}(z) &= \left(\frac{1-z}{2}\right)^n P_n^{(-\rho - \sigma - 2n - 1, \sigma)}\left(\frac{z+3}{z-1}\right), \\ &= \left(-\frac{1+z}{2}\right)^n P_n^{(-\rho - \sigma - 2n - 1, \rho)}\left(\frac{z-3}{z+1}\right), \\ &= \left(\frac{1+z}{2}\right)^n P_n^{(\rho, -\rho - \sigma - 2n - 1)}\left(\frac{3-z}{1+z}\right), \\ &= \left(\frac{z-1}{2}\right)^n P_n^{(\sigma, -\rho - \sigma - 2n - 1)}\left(\frac{3+z}{1-z}\right). \end{aligned}$$

Представления через функцию Гаусса

$$f_n(z) = q {}_2F_1(a, b; c; h(z))$$

$f_n(z)$	q	a	b	c	$h(z)$
$P_n(z)$	$(\pm 1)^n$	$-n$	$n+1$	1	$\frac{1 \mp z}{2}$
$P_n(z)$	$\binom{2n}{n} \left(\frac{z \mp 1}{2}\right)^n$	$-n$	$-n$	$-2n$	$\frac{2}{1 \mp z}$
$P_n(z)$	$\left(\frac{z \pm 1}{2}\right)^n$	$-n$	$-n$	1	$\frac{z \mp 1}{z \pm 1}$
$P_n(z)$	$\binom{2n}{n} \left(\frac{z}{2}\right)^n$	$-\frac{n}{2}$	$\frac{1-n}{2}$	$\frac{1}{2} - n$	$\frac{1}{z^2}$
$P_n(z)$	z^n	$-\frac{n}{2}$	$\frac{1-n}{2}$	1	$1 - \frac{1}{z^2}$
$P_{2n}(z)$	$(-1)^n \frac{(1/2)_n}{n!}$	$-n$	$\frac{1}{2} + n$	$\frac{1}{2}$	z^2
$P_{2n+1}(z)$	$(-1)^n \frac{(3/2)_n}{n!} z$	$-n$	$\frac{3}{2} + n$	$\frac{3}{2}$	z^2
$T_n(z)$	$(\pm 1)^n$	$-n$	n	$\frac{1}{2}$	$\frac{1 \mp z}{2}$
$T_n(z)$	$2^{n-1} (z \mp 1)^n$	$-n$	$\frac{1}{2} - n$	$1 - 2n$	$\frac{2}{1 \mp z}$
$T_n(z)$	$\left(\frac{z \pm 1}{2}\right)^n$	$-n$	$\frac{1}{2} - n$	$\frac{1}{2}$	$\frac{z \mp 1}{z \pm 1}$
$T_n(z)$	$2^n \frac{1}{2} z^n$	$-\frac{n}{2}$	$\frac{1-n}{2}$	$1 - n$	$\frac{1}{z^2}$
$T_{2n}(z)$	$\frac{(-1)^n}{2}$	$-n$	n	$\frac{1}{2}$	z^2
$T_{2n+1}(z)$	$(-1)^n n z$	$-n$	$n+1$	$\frac{3}{2}$	z^2
$U_n(z)$	$(\pm 1)^n (n+1)$	$-n$	$n+2$	$\frac{3}{2}$	$\frac{1 \mp z}{2}$
$U_n(z)$	$2^{2n} (z \mp 1)^n$	$-n$	$-n - \frac{1}{2}$	$-2n - 1$	$\frac{2}{1 \mp z}$
$U_n(z)$	$(n+1) \left(\frac{z \pm 1}{2}\right)^n$	$-n$	$-n - \frac{1}{2}$	$\frac{3}{2}$	$\frac{z \mp 1}{z \pm 1}$
$U_n(z)$	$(2z)^n$	$-\frac{n}{2}$	$\frac{1-n}{2}$	$-n$	$\frac{1}{z^2}$
$U_{2n}(z)$	$(-1)^n$	$-n$	$n+1$	$\frac{1}{2}$	z^2
$U_{2n+1}(z)$	$(-1)^n (n+1) 2z$	$-n$	$n+2$	$\frac{3}{2}$	z^2
$C_n^\lambda(z)$	$(\pm 1)^n \frac{(2\lambda)_n}{n!}$	$-n$	$n+2\lambda$	$\frac{1}{2} + \lambda$	$\frac{1 \mp z}{2}$

$f_n(z)$	a	a	b	c	$h(z)$
$C_n^\lambda(z)$	$\frac{2^n (\lambda)_n}{n!} (z \mp 1)^n$	$-n$	$\frac{1}{2} - \lambda - n$	$1 - 2\lambda - 2n$	$\frac{2}{1 \mp z}$
$C_n^\lambda(z)$	$\frac{(2\lambda)_n}{n!} \left(\frac{z \pm 1}{2}\right)^n$	$-n$	$\frac{1}{2} - \lambda - n$	$\frac{1}{2} + \lambda$	$\frac{z \mp 1}{z \pm 1}$
$C_n^\lambda(z)$	$\frac{(\lambda)_n}{n!} (2z)^n$	$-\frac{n}{2}$	$\frac{1-n}{2}$	$1 - \lambda - n$	$\frac{1}{z^2}$
$C_{2n}^\lambda(z)$	$\frac{(-1)^n (\lambda)_n}{n!}$	$-n$	$n + \lambda$	$\frac{1}{2}$	z^2
$C_{2n+1}^\lambda(z)$	$\frac{(-1)^n (\lambda)_{n+1}}{n!} 2z$	$-n$	$n + \lambda + 1$	$\frac{3}{2}$	z^2
$P_n^{(\rho, \sigma)}(z)$	$\frac{(\rho+1)_n}{n!}$	$-n$	$n + \rho + \sigma + 1$	$\rho + 1$	$\frac{1-z}{2}$
$P_n^{(\rho, \sigma)}(z)$	$\frac{(\rho+\sigma+n+1)_n}{n!} \left(\frac{z-1}{2}\right)^n$	$-n$	$-n - \rho$	$-2n - \rho - \sigma$	$\frac{2}{1-z}$
$P_n^{(\rho, \sigma)}(z)$	$\frac{(\rho+1)_n}{n!} \left(\frac{z+1}{2}\right)^n$	$-n$	$-n - \sigma$	$\rho + 1$	$\frac{z-1}{z+1}$

$$L_n(z) = {}_1F_1(-n; 1; z),$$

$$L_n^\lambda(z) = \frac{(\lambda+1)_n}{n!} {}_1F_1(-n; \lambda+1; z).$$

$$H_n(z) = 2^n \Psi\left(-\frac{n}{2}, \frac{1}{2}; z^2\right) = 2^n \Psi\left(\frac{1-n}{2}, \frac{3}{2}; z^2\right),$$

$$H_{2n+\varepsilon}(z) = (-1)^n \frac{(2n+\varepsilon)!}{n!} (2z)^\varepsilon {}_1F_1\left(-n; \frac{1}{2} + \varepsilon; z^2\right)$$

[$\varepsilon = 0$ или 1].

Рекуррентные формулы

$$\alpha_n f_{n+1}(z) = (\beta_n + \gamma_n z) f_n(z) - \delta_n f_{n-1}(z)$$

$f_n(z)$	α_n	β_n	γ_n	δ_n
$P_n(z)$	$n+1$	0	$2n+1$	n
$T_n(z)$	1	0	2	1
$U_n(z)$	1	0	2	1
$L_n(z)$	$n+1$	$2n+1$	-1	n
$L_n^\lambda(z)$	$n+1$	$2n+\lambda+1$	-1	$n+\lambda$
$H_n(z)$	1	0	2	$2n$
$C_n^\lambda(z)$	$n+1$	0	$2(n+\lambda)$	$n+2\lambda-1$
$P_n^{(\rho, \sigma)}(z)$	$2(n+1) \times$ $\times (n+\rho+\sigma+1) \times$ $\times (2n+\rho+\sigma)$	$(2n+\rho+\sigma+1) \times$ $\times (\rho^2 - \sigma^2)$	$(2n+\rho+\sigma)_3$	$2(n+\rho)(n+\sigma) \times$ $\times (2n+\rho+\sigma+2)$

$$T_n(z) = U_n(z) - zU_{n-1}(z),$$

$$= zU_{n-1}(z) - U_{n-2}(z),$$

$$= \frac{1}{2} [U_n(z) - U_{n-2}(z)].$$

$$(1-z^2)U_{n-1}(z) = zT_n(z) - T_{n+1}(z).$$

$$n! L_n^{m-n}(z) = (-z)^{n-m} m! L_m^{n-m}(z),$$

$$L_n^\lambda(z) = L_n^{\lambda+1}(z) - L_{n-1}^{\lambda+1}(z),$$

$$\begin{aligned} z L_n^{\lambda+1}(z) &= (n+\lambda+1) L_n^\lambda(z) - (n+1) L_{n+1}^\lambda(z), \\ &= (n+\lambda) L_{n-1}^\lambda(z) - (n-z) L_n^\lambda(z), \end{aligned}$$

$$(n+\lambda+1) L_n^\lambda(z) = (n+1) L_{n+1}^{\lambda+1}(z) - (n+1-z) L_n^{\lambda+1}(z).$$

$$nC_n^\lambda(z) = 2\lambda [z C_{n-1}^{\lambda+1}(z) - C_{n-2}^{\lambda+1}(z)],$$

$$= (2\lambda+n-1) z C_{n-1}^\lambda(z) - 2\lambda(1-z^2) C_{n-2}^{\lambda+1}(z),$$

$$(n+\lambda) C_n^\lambda(z) = \lambda [C_n^{\lambda+1}(z) - C_{n-2}^{\lambda+1}(z)],$$

$$(n+2\lambda) C_n^\lambda(z) = 2\lambda [C_n^{\lambda+1}(z) - z C_{n-1}^{\lambda+1}(z)],$$

$$2\lambda(1-z^2) C_{n-1}^{\lambda+1}(z) = (2\lambda+n-1) C_{n-1}^\lambda(z) - n z C_n^\lambda(z),$$

$$= (n+2\lambda) z C_n^\lambda(z) - (n+1) C_{n+1}^\lambda(z).$$

$$\left(n + \frac{\rho+\sigma}{2} + 1\right) (1-z) P_n^{(\rho+1, \sigma)}(z) = (n+\rho+1) P_n^{(\rho, \sigma)}(z) - (n+1) P_{n+1}^{(\rho, \sigma)}(z),$$

$$\left(n + \frac{\rho+\sigma}{2} + 1\right) (1+z) P_n^{(\rho, \sigma+1)}(z) = (n+\sigma+1) P_n^{(\rho, \sigma)}(z) + (n+1) P_{n+1}^{(\rho, \sigma)}(z),$$

$$(2n+\rho+\sigma) P_n^{(\rho-1, \sigma)}(z) = (n+\rho+\sigma) P_n^{(\rho, \sigma)}(z) - (n+\sigma) P_{n-1}^{(\rho, \sigma)}(z),$$

$$(2n+\rho+\sigma) P_n^{(\rho, \sigma-1)}(z) = (n+\rho+\sigma) P_n^{(\rho, \sigma)}(z) + (n+\rho) P_{n-1}^{(\rho, \sigma)}(z),$$

$$(1-z) P_n^{(\rho+1, \sigma)}(z) + (1+z) P_n^{(\rho, \sigma+1)}(z) = 2 P_n^{(\rho, \sigma)}(z),$$

$$P_n^{(\rho, \sigma-1)}(z) - P_n^{(\rho-1, \sigma)}(z) = P_{n-1}^{(\rho, \sigma)}(z).$$

Производные многочленов

$f_n(z)$	$\frac{d^m}{dz^m} f_n(z)$	Условие
$P_n(z)$	$(2m-1)!! C_{n-m}^{m+1/2}(z)$	$m \leq n$
$T_n(z)$	$2^{m-1} (m-1)! n C_{n-m}^m(z)$	$m \leq n$
$U_n(z)$	$2^m m! C_{n-m}^{m+1}(z)$	$m \leq n$
$L_n(z)$	$(-1)^m L_{n-m}^m(z)$	$m \leq n$
$z^\lambda L_n^\lambda(z)$	$(n-m+\lambda+1)_m z^{\lambda-m} L_n^{\lambda-m}(z)$	
$z^\lambda e^{-z} L_n^\lambda(z)$	$\frac{(m+n)!}{n!} z^{\lambda-m} e^{-z} L_{n+m}^{\lambda-m}(z)$	
$z^{-1/2} {}^2H_{2n}(\sqrt{z})$	$(-1)^n 2^{2n} \left(n-m+\frac{1}{2}\right)_m n! z^{-1/2-m} L_n^{-1/2-m}(z)$	
$H_{2n+1}(\sqrt{z})$	$(-1)^n 2^{2n+1} \left(n-m+\frac{3}{2}\right)_m n! z^{1/2-m} L_n^{1/2-m}(z)$	
$z^{-1/2} e^{-z} {}^2H_{2n}(\sqrt{z})$	$(-1)^n 2^{2n} (m+n)! z^{-1/2-m} e^{-z} L_{n+m}^{-1/2-m}(z)$	
$e^{-z} H_{2n+1}(z)$	$(-1)^n 2^{2n+1} (m+n)! z^{1/2-m} e^{-z} L_{n+m}^{1/2-m}(z)$	
$C_n^\lambda(z)$	$2^m (\lambda)_m C_{n-m}^{\lambda+m}(z)$	$m \leq n$
$P_n^{(\rho, \sigma)}(z)$	$2^{-m} (\rho+\sigma+n+1)_m P_{n-m}^{(\rho+m, \sigma+m)}(z)$	$m \leq n$

$$\alpha(z) \frac{d}{dz} f_n(z) = \beta(z) f_n(z) + \gamma(z) f_{n-1}(z)$$

$f_n(z)$	$\alpha(z)$	$\beta(z)$	$\gamma(z)$
$P_n(z)$	$1-z^2$	$-nz$	n
$T_n(z)$	$1-z^2$	$-nz$	n
$U_n(z)$	$1-z^2$	$-nz$	$n+1$
$L_n(z)$	z	n	$-n$
$L_n^\lambda(z)$	z	n	$-n-\lambda$
$H_n(z)$	1	0	$2n$
$C_n^\lambda(z)$	$1-z^2$	$-nz$	$n+2\lambda-1$
$P_n^{(\rho, \sigma)}(z)$	$(2n+\rho+\sigma)(1-z^2)$	$n[\rho-\sigma-(2n+\rho+\sigma)z]$	$2(n+\rho)(n+\sigma)$

$$(1-z^2) \frac{d}{dz} P_n(z) = \frac{n(n+1)}{2n+1} [P_{n-1}(z) - P_{n+1}(z)],$$

$$\frac{d^m}{dz^m} \left[z^{n/2} (1-z)^{m-n-1} P_n \left(\frac{1+z}{2\sqrt{z}} \right) \right] = \frac{n!}{(n-m)!} z^{(n-m)/2} (1-z)^{-n-1} P_{n-m} \left(\frac{1+z}{2\sqrt{z}} \right) \quad [n \geq m],$$

$$\frac{d^m}{dz^m} \left[z^{-(n+1)/2} (z-1)^{m+n} P_n \left(\frac{1+z}{2\sqrt{z}} \right) \right] = \frac{(m+n)!}{n!} z^{-(m+n+1)/2} (z-1)^n P_{m+n} \left(\frac{1+z}{2\sqrt{z}} \right).$$

$$\frac{d}{dz} T_n(z) = nU_{n-1}(z).$$

$$\frac{d}{dz} L_n^\lambda(z) = -L_{n-1}^{\lambda+1}(z),$$

$$z \frac{d}{dz} L_n^\lambda(z) = (n+1) L_{n+1}^\lambda(z) - (n+\lambda+1-z) L_n^\lambda(z),$$

$$\frac{d}{dz} [L_n^\lambda(z) - L_{n+1}^\lambda(z)] = L_n^\lambda(z),$$

$$\frac{d^m}{dz^m} [e^{-z} L_n^\lambda(z)] = (-1)^m e^{-z} L_n^{\lambda+m}(z),$$

$$\frac{d^m}{dz^m} [z^{m-n-1} L_n^\lambda(z)] = (-1)^m (\lambda+n-m+1)_m z^{n-1} L_{n-m}^\lambda(z) \quad [n \geq m],$$

$$\frac{d^m}{dz^m} [z^{m+n+\lambda} e^{-z} L_n^\lambda(z)] = \frac{(m+n)!}{n!} z^{n+\lambda} e^{-z} L_{m+n}^\lambda(z),$$

$$\frac{d^m}{dz^m} \left[z^{m-1} L_n^\lambda \left(\frac{1}{z} \right) \right] = (\lambda+1)_m z^{-m-1} L_{n-m}^{\lambda+m} \left(\frac{1}{z} \right) \quad [n \geq m],$$

$$\frac{d^m}{dz^m} \left[z^n L_n^\lambda \left(\frac{1}{z} \right) \right] = (n-m+\lambda+1)_m z^{n-m} L_{n-m}^\lambda \left(\frac{1}{z} \right) \quad [n \geq m],$$

$$\frac{d^m}{dz^m} \left[z^{m-\lambda-1} L_n^\lambda \left(\frac{1}{z} \right) \right] = (-1)^m (n-m+\lambda+1)_m z^{-\lambda-1} L_{n-m}^{\lambda-m} \left(\frac{1}{z} \right),$$

$$\frac{d^m}{dz^m} \left[z^{m-1} e^{-1/z} L_n^\lambda \left(\frac{1}{z} \right) \right] = z^{-m-1} e^{-1/z} L_n^{\lambda+m} \left(\frac{1}{z} \right),$$

$$\frac{d^m}{dz^m} \left[z^{-m-\lambda-1} e^{-1/z} L_n^\lambda \left(\frac{1}{z} \right) \right] = (-1)^m \frac{(m+n)!}{n!} z^{-2m-\lambda-1} e^{-1/z} L_{m+n}^\lambda \left(\frac{1}{z} \right).$$

$$\frac{d^m}{dz^m} [z^{m-n-1} H_{2n}(\sqrt{z})] = \frac{(2n)!}{(2n-2m)!} z^{n-1} H_{2n-2m}(\sqrt{z}) \quad [n \geq m],$$

$$\frac{d^m}{dz^m} [z^{m-n-3/2} H_{2n+1}(\sqrt{z})] = \frac{(2n+1)!}{(2n-2m+1)!} z^{-n-3/2} H_{2n-2m+1}(\sqrt{z}) \quad [n \geq m],$$

$$\frac{d^m}{dz^m} \left[z^n H_{2n} \left(\frac{1}{\sqrt{z}} \right) \right] = (-1)^m \frac{(2n)!}{(2n-2m)!} z^{n-m} H_{2n-2m} \left(\frac{1}{\sqrt{z}} \right) \quad [n \geq m],$$

$$\frac{d^m}{dz^m} \left[z^{n-1/2} H_{2n+1} \left(\frac{1}{\sqrt{z}} \right) \right] = (-1)^m \frac{(2n+1)!}{(2n-2m+1)!} z^{n-m-1/2} H_{2n-2m+1} \left(\frac{1}{\sqrt{z}} \right) \quad [n \geq m],$$

$$\frac{d^m}{dz^m} [z^{m+n-1/2} e^{-z} H_{2n}(\sqrt{z})] = \frac{(-1)^m}{2^{2n}} z^{n-1/2} e^{-z} H_{2m+2n}(\sqrt{z}),$$

$$\frac{d^m}{dz^m} [z^{m+n} e^{-z} H_{2n+1}(\sqrt{z})] = \frac{(-1)^n}{2^{2n+1}} z^n e^{-z} H_{2m+2n+1}(\sqrt{z}).$$

$$\frac{d}{dz} C_n^\lambda(z) = 2\lambda C_{n-1}^{\lambda+1}(z),$$

$$(1-z^2) \frac{d}{dz} C_n^\lambda(z) = (n+2\lambda) z C_n^\lambda(z) - (n+1) C_{n+1}^\lambda(z),$$

$$\frac{d}{dz} C_{n \pm 1}^\lambda(z) - z \frac{d}{dz} C_n^\lambda(z) = \begin{Bmatrix} n+2\lambda \\ -n \end{Bmatrix} C_n^\lambda(z),$$

$$\frac{d^m}{dz^m} \left[z^{m-1} C_n^\lambda \left(1 - \frac{2}{z} \right) \right] = \frac{(-4)^m}{(1-\lambda-m)_m} z^{-m-1} C_{n-m}^{\lambda+m} \left(1 - \frac{2}{z} \right) \quad [n \geq m],$$

$$\begin{aligned} \frac{d^m}{dz^m} \left[z^{m-2\lambda} (1-z)^{\lambda-1/2} C_n^\lambda \left(1 - \frac{2}{z} \right) \right] = \\ = 2^{-2m} \frac{(m+n)!}{n!} \frac{(1-2\lambda-n)_m}{(1-\lambda)_m} z^{m-2\lambda} (1-z)^{\lambda-m-1/2} C_{m+n}^{\lambda-m} \left(1 - \frac{2}{z} \right), \end{aligned}$$

$$\frac{d^m}{dz^m} [z^{m+n+\lambda-1} C_{2n}^\lambda(\sqrt{z})] = (\lambda)_m z^{\lambda+n-1} C_{2n}^{\lambda+m}(\sqrt{z}),$$

$$\frac{d^m}{dz^m} [z^{m+n+\lambda-1/2} C_{2n+1}^\lambda(\sqrt{z})] = (\lambda)_m z^{\lambda+n-1/2} C_{2n+1}^{\lambda+m}(\sqrt{z}),$$

$$\frac{d^m}{dz^m} [z^{m-n-1} C_{2n}^\lambda(\sqrt{z})] = (-1)^m \frac{(n-m)!}{n!} \frac{z^{-n-1}}{(\lambda)_m} C_{2n-2m}^{\lambda+m}(\sqrt{z}) \quad [n \geq m],$$

$$\frac{d^m}{dz^m} [z^{m-n-3/2} C_{2n+1}^\lambda(\sqrt{z})] = (-1)^m \frac{(n-m)!}{n!} \frac{z^{-n-3/2}}{(\lambda)_m} C_{2n-2m+1}^{\lambda+m}(\sqrt{z}) \quad [n \geq m],$$

$$\begin{aligned} \frac{d^m}{dz^m} [z^{m+n-1/2} (1-z)^{\lambda-1/2} C_{2n}^\lambda(\sqrt{z})] = \\ = \frac{(m+n)!}{n!} \frac{(2m+2n-1)!!}{(2n-1)!!} \frac{z^{n-1/2}}{2^m (1-\lambda)_m} (1-z)^{\lambda-m-1/2} C_{2m+2n}^{\lambda-m}(\sqrt{z}), \end{aligned}$$

$$\begin{aligned} \frac{d^m}{dz^m} [z^{m+n} (1-z)^{\lambda-1/2} C_{2n+1}^\lambda(\sqrt{z})] = \\ = \frac{(m+n)!}{n!} \frac{(2n+2m+1)!!}{(2n+1)!!} \frac{z^n}{2^m (1-\lambda)_m} (1-z)^{\lambda-m-1/2} C_{2m+2n+1}^{\lambda-m}(\sqrt{z}). \end{aligned}$$

$$\frac{d^m}{dz^m} [(1+z)^\sigma P_n^{(\rho, \sigma)}(z)] = (n-m+\sigma+1)_m (1+z)^{\sigma-m} P_n^{(\rho+m, \sigma-m)}(z),$$

$$\frac{d^m}{dz^m} [(1+z)^{m-n-1} P_n^{(\rho, \sigma)}(z)] = (-n-\beta)_m (1+z)^{-n-1} P_{n-m}^{(\rho+m, \sigma)}(z) \quad [n \geq m],$$

$$\frac{d^m}{dz^m} [(1+z)^{m+n+\rho+\sigma} P_n^{(\rho, \sigma)}(z)] = (-m-n-\rho-\sigma)_m (1+z)^{n+\rho+\sigma} P_n^{(\rho+n, \sigma)}(z),$$

$$\begin{aligned}\frac{d^m}{dz^m} \left[(1-z)^\rho (1+z)^\sigma P_n^{(\rho, \sigma)}(z) \right] &= \\ &= (-2)^m \frac{(m+n)!}{n!} (1-z)^{\rho-m} (1+z)^{\sigma-m} P_{m+n}^{(\rho-m, \sigma-m)}(z),\end{aligned}$$

$$\begin{aligned}\frac{d^m}{dz^m} \left[(1-z)^{m+n+\rho} (1+z)^\sigma P_n^{(\rho, \sigma)}(z) \right] &= \\ &= (-2)^m \frac{(m+n)!}{n!} (1-z)^{n+\rho} (1+z)^{\sigma-m} P_{m+n}^{(\rho, \sigma-m)}(z),\end{aligned}$$

$$\begin{aligned}\frac{d^m}{dz^m} \left[z^{m-1} P_n^{(\rho, \sigma)} \left(1 - \frac{2}{z} \right) \right] &= \frac{(n+\rho+\sigma+1)_m}{z^{m+1}} P_{n-m}^{(\rho+m, \sigma+m)} \left(1 - \frac{2}{z} \right) \\ & \quad [n \geq m],\end{aligned}$$

$$\frac{d^m}{dz^m} \left[z^n P_n^{(\rho, \sigma)} \left(1 - \frac{2}{z} \right) \right] = (n-m+\rho+1)_m z^{n-m} P_{n-m}^{(\rho, \sigma+m)} \left(1 - \frac{2}{z} \right) \quad [n \geq m],$$

$$\begin{aligned}\frac{d^m}{dz^m} \left[z^{-n-\rho-\sigma-1} P_n^{(\rho, \sigma)} \left(1 - \frac{2}{z} \right) \right] &= \\ &= (-1)^m (n+\rho+\sigma+1)_m z^{-m-n-\rho-\sigma-1} P_n^{(\rho, \sigma+m)} \left(1 - \frac{2}{z} \right),\end{aligned}$$

$$\begin{aligned}\frac{d^m}{dz^m} \left[z^{-2n-\rho-\sigma-1} P_n^{(\rho, \sigma)} \left(1 - \frac{2}{z} \right) \right] &= \\ &= (-m-n-\rho-\sigma)_m z^{-m-2n-\rho-\sigma-1} P_n^{(\rho, \sigma+m)} \left(1 - \frac{2}{z} \right),\end{aligned}$$

$$\frac{d^m}{dz^m} \left[z^{m-\rho-1} P_n^{(\rho, \sigma)} \left(1 - \frac{2}{z} \right) \right] = (-1)^m (n-m+\rho+1)_m z^{\rho-1} P_n^{(\rho-m, \sigma+m)} \left(1 - \frac{2}{z} \right),$$

$$\frac{d^m}{dz^m} \left[z^n (1-z)^\rho P_n^{(\rho, \sigma)} \left(1 - \frac{2}{z} \right) \right] = (-n-\sigma)_m z^n (1-z)^{\sigma-m} P_n^{(\rho, \sigma-m)} \left(1 - \frac{2}{z} \right),$$

$$\begin{aligned}\frac{d^m}{dz^m} \left[z^n (1-z)^{m-n-1} P_n^{(\rho, \sigma)} \left(1 - \frac{2}{z} \right) \right] &= \\ &= (-n-\sigma)_m z^{n-m} (1-z)^{-n-1} P_{n-m}^{(\rho+m, \sigma)} \left(1 - \frac{2}{z} \right) \quad [n \geq m],\end{aligned}$$

$$\begin{aligned}\frac{d^m}{dz^m} \left[z^n (1-z)^{m-n-\rho-1} P_n^{(\rho, \sigma)} \left(1 - \frac{2}{z} \right) \right] &= \\ &= (-1)^m (-n-\rho)_m z^n (1-z)^{-n-\rho-1} P_n^{(\rho-m, \sigma+m)} \left(1 - \frac{2}{z} \right),\end{aligned}$$

$$\begin{aligned}\frac{d^m}{dz^m} \left[z^{-n-\rho-\sigma-1} (1-z)^\sigma P_n^{(\rho, \sigma)} \left(1 - \frac{2}{z} \right) \right] &= \\ &= \frac{(m+n)!}{n!} z^{-n-\rho-\sigma-1} (1-z)^{\sigma-m} P_{m+n}^{(\rho, \sigma-m)} \left(1 - \frac{2}{z} \right),\end{aligned}$$

$$\begin{aligned}\frac{d^m}{dz^m} \left[z^{m-\rho-\sigma-1} (1-z)^\sigma P_n^{(\rho, \sigma)} \left(1 - \frac{2}{z} \right) \right] &= \\ &= \frac{(m+n)!}{n!} z^{\rho-\sigma-1} (1-z)^{\sigma-m} P_{m+n}^{(\rho-m, \sigma-m)} \left(1 - \frac{2}{z} \right),\end{aligned}$$

$$\begin{aligned}\frac{d^m}{dz^m} \left[z^{-m-\rho-\sigma-1} (1-z)^\sigma P_n^{(\rho, \sigma)} \left(1 - \frac{2}{z} \right) \right] &= \\ &= \frac{(m+n)!}{n!} z^{m-\rho-\sigma-1} (1-z)^{\sigma-m} P_{m+n}^{(\rho-m, \sigma-m)} \left(1 - \frac{2}{z} \right),\end{aligned}$$

$$\begin{aligned}\frac{d^m}{dz^m} \left[z^{-n-\rho-\sigma-1} (1-z)^{m+n+\sigma} P_n^{(\rho, \sigma)} \left(1 - \frac{2}{z} \right) \right] &= \\ &= \frac{(m+n)!}{n!} z^{-n-\rho-\sigma-1} (1-z)^{n+\sigma} P_{m+n}^{(\rho-m, \sigma)} \left(1 - \frac{2}{z} \right).\end{aligned}$$

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УКАЗАТЕЛЬ ОБОЗНАЧЕНИЙ ФУНКЦИЙ И ПОСТОЯННЫХ

$A(x)$ — алгебраическая функция или степенная функция с произвольным показателем

$\operatorname{am} u \equiv \operatorname{am}(u, k)$ — эллиптическая функция Якоби, $u = \int_0^{\operatorname{am} u} \frac{dt}{\sqrt{1-k^2 \sin^2 t}}$

$\operatorname{Ai}(z) = \frac{1}{\pi} \sqrt{\frac{2}{3}} K_{1/3}\left(\frac{2}{3} z^{3/2}\right)$ — функция Эйри

$\arccos z$, $\arcsin z$, $\operatorname{arccotg} z$, $\operatorname{arctg} z$ — обратные тригонометрические функции

$\operatorname{Arch} z$, $\operatorname{Arsh} z$, $\operatorname{Aroth} z$, $\operatorname{Arth} z$ — обратные гиперболические функции

$\arg z$ — аргумент комплексного числа z ($z = |z| e^{i \arg z}$)

B_n — числа Бернулли

$B'_n(z)$ — многочлены Бернулли

$\operatorname{bei}_\nu(z)$, $\operatorname{ber}_\nu(z)$, $\operatorname{bei}(z) = \operatorname{bei}_0(z)$, $\operatorname{ber}(z) = \operatorname{ber}_0(z)$ ($\operatorname{ber}_\nu(z) + i \operatorname{bei}_\nu(z) = J_\nu(e^{3\pi i/4} z)$) — функция Кельвина

$\operatorname{Bi}(z) = \sqrt{\frac{2}{3}} \left[I_{-1/3}\left(\frac{2}{3} z^{3/2}\right) + I_{1/3}\left(\frac{2}{3} z^{3/2}\right) \right]$ — функция Эйри

$C = 0,5772156649 \dots$ — постоянная Эйлера

$C(x) = \frac{1}{\sqrt{2\pi}} \int_0^x \frac{\cos t}{\sqrt{t}} dt$ — косинус-интеграл Френеля

$C(x, \nu) = \int_x^\infty t^{\nu-1} \cos t dt$ [$\operatorname{Re} \nu < 1$] — обобщенный косинус-интеграл Френеля

$C_n^\lambda(z) = \frac{(2\lambda)_n}{n!} {}_2F_1\left(-n, n+2\lambda; \lambda + \frac{1}{2}; \frac{1-z}{2}\right)$ — многочлены Гегенбауэра

$\operatorname{cd} u = \frac{\operatorname{cn} u}{\operatorname{dn} u}$, $\operatorname{cn} u = \cos(\operatorname{am} u)$ — эллиптические функции Якоби

$\operatorname{ch} z = \frac{e^z + e^{-z}}{2}$ — гиперболический косинус

$\operatorname{chl}(x) = C + \ln x + \int_0^x \frac{\operatorname{ch} t - 1}{t} dt$ — интегральный гиперболический косинус

$\operatorname{ci}_1(x) = - \int_x^\infty \frac{\cos t}{t} dt$ — интегральный косинус

$\cos z = \operatorname{ch}(iz) = \frac{e^{iz} + e^{-iz}}{2}$ — тригонометрическая функция

$\operatorname{cosec} z = \frac{1}{\sin z}$ — тригонометрическая функция

$\operatorname{cosech} z = \frac{1}{\operatorname{sh} z}$ — гиперболическая функция

$\operatorname{ctg} z = \frac{\cos z}{\sin z} = i \operatorname{cth} (iz)$ — тригонометрическая функция

$\operatorname{cth} z = \frac{\operatorname{ch} z}{\operatorname{sh} z} = \frac{e^z + e^{-z}}{e^z - e^{-z}}$ — гиперболическая функция

$D(k) = \int_0^{\pi/2} \frac{\sin^2 t \, dt}{\sqrt{1 - k^2 \sin^2 t}} = D\left(\frac{\pi}{2}, k\right)$ — полный эллиптический интеграл

$D(\varphi, k) = \int_0^{\varphi} \frac{\sin^2 t \, dt}{\sqrt{1 - k^2 \sin^2 t}}$ — эллиптический интеграл

$D_\nu(z) = 2^{\nu/2} e^{-z^2/4} \Psi\left(-\frac{\nu}{2}, \frac{1}{2}; \frac{z^2}{2}\right)$ — функция параболического цилиндра

$\operatorname{de} u = \frac{\operatorname{dn} u}{\operatorname{cn} u}$, $\operatorname{dn} u = \sqrt{1 - k^2 \sin^2(\operatorname{am} u)}$, $\operatorname{ds} u = \frac{\operatorname{dn} u}{\operatorname{sn} u}$ — эллиптические функции Якоби

$E(k) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 t} \, dt$ — полный эллиптический интеграл 2-го рода

$E(\varphi, k) = \int_0^{\varphi} \sqrt{1 - k^2 \sin^2 t} \, dt$ — эллиптический интеграл 2-го рода

E_n — числа Эйлера

$E_n(z)$ — многочлены Эйлера

$E_\nu(z) = \frac{1}{\pi} \int_0^{\pi} \sin(\nu t - z \sin t) \, dt$ — функция Вебера

$\operatorname{Ei}(x) = \int_{-\infty}^x \frac{e^t}{t} \, dt$ — интегральная показательная функция

$e = 2,718281828459 \dots$

$e^z = \exp z$ — показательная функция

$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \, dt$ — интеграл вероятности

$\operatorname{erfc}(x) = 1 - \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} \, dt$ — дополнительный интеграл вероятности

$\operatorname{erfi}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{t^2} \, dt$ — интеграл вероятности мнимого аргумента

$F(\varphi, k) = \int_0^{\varphi} \frac{dt}{\sqrt{1 - k^2 \sin^2 t}}$ — эллиптический интеграл 1-го рода

${}_2F_1(a, b; c; z) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{(c)_k} \frac{z^k}{k!} \quad [|z| < 1],$

$$= \Gamma\left[b, \frac{c}{c-b}\right] \int_0^1 t^{b-1} (1-t)^{c-b-1} (1-tz)^{-a} \, dt$$

$[\operatorname{Re} c > \operatorname{Re} b > 0; |\arg(1-z)| < \pi]$
— гипергеометрическая функция Гаусса

$${}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z) \equiv {}_pF_q \left(\begin{matrix} a_1, \dots, a_p \\ b_1, \dots, b_q \end{matrix} ; z \right) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \dots (a_p)_k}{(b_1)_k (b_2)_k \dots (b_q)_k} \frac{z^k}{k!} -$$

обобщенная гипергеометрическая функция

$${}_1F_1(a; b; z) = \sum_{k=0}^{\infty} \frac{(a)_k z^k}{(b)_k k!} - \text{вырожденная гипергеометрическая функция}$$

$$F_A^{(n)}(a, b_1, \dots, b_n; c_1, \dots, c_n; z_1, \dots, z_n) =$$

$$= \sum_{k_1, \dots, k_n=0}^{\infty} \frac{(a)_{k_1+\dots+k_n} (b_1)_{k_1} \dots (b_n)_{k_n}}{(c_1)_{k_1} \dots (c_n)_{k_n}} \frac{z_1^{k_1} \dots z_n^{k_n}}{k_1! \dots k_n!}$$

$$\left[\sum_{j=1}^n |z_j| < 1 \right] - \text{функция Лауричелла}$$

$$F_C^{(n)}(a, b; c_1, \dots, c_n; z_1, \dots, z_n) = \sum_{k_1, \dots, k_n=0}^{\infty} \frac{(a)_{k_1+\dots+k_n} (b)_{k_1+\dots+k_n}}{(c_1)_{k_1} \dots (c_n)_{k_n}} \frac{z_1^{k_1} \dots z_n^{k_n}}{k_1! \dots k_n!}$$

$$\left[\sum_{j=1}^n \sqrt{|z_j|} < 1 \right] - \text{функция Лауричелла}$$

$F_j(\dots; z, \zeta)$ [$j=1, 2, 3, 4$] — функции Аппеля:

$$F_1(a, b, b'; c; z, \zeta) = \sum_{k, l=0}^{\infty} \frac{(a)_{k+l} (b)_k (b')_l}{(c)_{k+l}} \frac{z^k \zeta^l}{k! l!} \quad [|z|, |\zeta| < 1]$$

$$F_2(a, b, b'; c, c'; z, \zeta) = \sum_{k, l=0}^{\infty} \frac{(a)_{k+l} (b)_k (b')_l}{(c)_k (c')_l} \frac{z^k \zeta^l}{k! l!} \quad [|z| + |\zeta| < 1]$$

$$F_3(a, a', b, b'; c; z, \zeta) = \sum_{k, l=0}^{\infty} \frac{(a)_k (a')_l (b)_k (b')_l}{(c)_{k+l}} \frac{z^k \zeta^l}{k! l!} \quad [|z|, |\zeta| < 1]$$

$$F_4(a, b; c, c'; z, \zeta) = \sum_{k, l=0}^{\infty} \frac{(a)_{k+l} (b)_{k+l}}{(c)_k (c')_l} \frac{z^k \zeta^l}{k! l!} \quad [\sqrt{|z|} + \sqrt{|\zeta|} < 1]$$

$$G = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^2} = 0,9159655942 \dots - \text{постоянная Каталана}$$

$$G_{p, q}^{m, n} \left(z \left| \begin{matrix} a_1, \dots, a_p \\ b_1, \dots, b_q \end{matrix} \right. \right) - G\text{-функция Мейера}$$

$$H_\nu(z) = \frac{2}{\sqrt{\pi}} \left(\frac{z}{2} \right)^{\nu+1} \frac{1}{\Gamma(\nu+3/2)} {}_1F_2 \left(1; \frac{3}{2}, \nu + \frac{3}{2}; -\frac{z^2}{4} \right) - \text{функция Струве}$$

$H_\nu^{(1)}(z) = J_\nu(z) + iY_\nu(z)$, $H_\nu^{(2)}(z) = J_\nu(z) - iY_\nu(z)$ — функции Бесселя 3-го рода (функции Ганкеля 1-го и 2-го рода)

$$H_n(z) = (-1)^n e^{z^2} \frac{d^n}{dz^n} e^{-z^2} - \text{многочлены Эрмита}$$

$$h_{\mu, \nu}(z) = \frac{z^{\mu+1} e^{-z}}{(\mu+1)^2 - \nu^2} {}_2F_2 \left(1, \mu + \frac{3}{2}; \mu - \nu + 2, \mu + \nu + 2; 2z \right)$$

$$H_{\mu, \nu}(z) = h_{\mu, \nu}(z) - \frac{\sqrt{\pi} \Gamma(\mu - \nu + 1) \Gamma(\mu + \nu + 1)}{2^{\mu+1} \Gamma(\mu + 3/2)} \left[J_\nu(z) + \frac{\sin(\nu - \mu)\pi}{\pi \cos \mu\pi} K_\nu(z) \right]$$

$I_\nu(z) = \frac{1}{\Gamma(\nu+1)} \left(\frac{z}{2} \right)^\nu {}_0F_1 \left(\nu+1; \frac{z^2}{4} \right) = e^{-\nu\pi i/2} J_\nu(e^{\pi i/2} z)$ — модифицированная функция Бесселя 1-го рода (функция Бесселя мнимого аргумента)

$\text{Im } z$ — мнимая часть комплексного числа $z = x + iy$ ($\text{Im } z = y$)

$J_\nu(z) = \frac{1}{\Gamma(\nu+1)} \left(\frac{z}{2}\right)^\nu {}_0F_1\left(\nu+1, -\frac{z^2}{4}\right)$ — функция Бесселя 1-го рода

$J_\nu(z) = \frac{1}{\pi} \int_0^\pi \cos(\nu t - z \sin t) dt$ — функция Ангера

$J(x, y) = 1 - e^{-y} \int_0^x e^{-t} I_0(2\sqrt{yt}) dt$

$K(k) = \int_0^{\pi/2} \frac{dt}{\sqrt{1-k^2 \sin^2 t}} = F\left(\frac{\pi}{2}, k\right)$ — полный эллиптический интеграл 1-го рода

$K_\nu(z) = \frac{\pi [I_{-\nu}(z) - I_\nu(z)]}{2 \sin \nu\pi}$ [$\nu \neq n$], $K_n(z) = \lim_{\nu \rightarrow n} K_\nu(z)$ [$n = 0, \pm 1, \pm 2, \dots$] —

функция Макдональда (модифицированная функция Бесселя 3-го рода)

$k_\nu(z) = \frac{1}{\Gamma(\nu/2 + 1)} W_{\nu/2, 1/2}(2z)$ — функция Бейтмена

$\text{kei}_\nu(z)$, $\text{ker}_\nu(z)$, $\text{kei}(z) = \text{kei}_0(z)$, $\text{ker}(z) = \text{ker}_0(z)$ ($\text{ker}_\nu(z) + i \text{kei}_\nu(z) = e^{-\nu\pi i/2} K_\nu(e^{\pi i/4} z)$) — функции Кельвина

$L(x) = -\int_0^x \ln \cos t dt$ — функция Лобачевского

$L_\nu(z) = e^{-(\nu+1)\pi i/2} N_\nu(e^{\pi i/2} z)$ — модифицированная функция Струве

$L_n(z) = L_n^0(z)$ — многочлены Лагерра

$L_n^\lambda(z) = \frac{z^{-\lambda} e^z}{n!} \frac{d^n}{dz^n} (z^{n+\lambda} e^{-z})$ — обобщенные многочлены Лагерра

$\text{Li}_\nu(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^\nu}$ [$|z| < 1$],

$= \frac{z}{\Gamma(\nu)} \int_0^\infty \frac{t^{\nu-1} dt}{e^t - z}$ [$\text{Re } \nu > 0$; $|\arg(1-z)| < \pi$]

— полилогарифм порядка ν

$\text{li}(z) = \text{Ei}(i\pi z)$, $\text{li}(x) = \int_0^x \frac{dt}{\ln t}$ — интегральный логарифм

$\ln z = \ln |z| + i \arg z$ — натуральный логарифм [$z = |z| e^{i(\arg z + 2\pi k)}$, $k = 0, \pm 1, \pm 2, \dots$]

$M_{\kappa, \mu}(z) = z^{\mu+1/2} e^{-z/2} {}_1F_1\left(\mu - \kappa + \frac{1}{2}; 2\mu + 1; z\right)$ — вырожденная гипергеометрическая функция Уиттекера

$\text{sn } u$, $\text{nd } u$, $\text{ns } u$, $\text{pc } u = \frac{1}{\text{sn } u}$, $\text{pd } u = \frac{1}{\text{dn } u}$, $\text{ps } u = \frac{1}{\text{sn } u}$ — эллиптические функции Якоби

$O_n(z) = \frac{n}{4} \sum_{k=0}^{[n/2]} \frac{(n-k-1)!}{k!} \left(\frac{z}{2}\right)^{2k-n-1}$ — многочлены Неймана

$P_n(z) = \frac{2^{-n}}{n!} \frac{d^n}{dz^n} (z^2 - 1)^n$ — многочлены Лежандра

$P_\nu(z) = P_\nu^0(z) = {}_2F_1\left(-\nu, 1+\nu; 1; \frac{1-z}{2}\right)$ [$|\arg(1+z)| < \pi$] — функция Лежандра 1-го рода

$P_\nu^\mu(z) = \frac{1}{\Gamma(1-\mu)} \left(\frac{z+1}{z-1}\right)^{\mu/2} {}_2F_1\left(-\nu, 1+\nu; 1-\mu; \frac{1-z}{2}\right)$ [$|\arg(z \pm 1)| < \pi$]

$P_{\nu}^{\mu}(x) = \frac{1}{\Gamma(1-\mu)} \left(\frac{1+x}{1-x}\right)^{\mu/2} {}_2F_1\left(-\nu, 1+\nu; 1-\mu; \frac{1-x}{2}\right) \quad [-1 < x < 1]$ — присоединенная функция Лежандра 1-го рода

$P_n^{(\rho, \sigma)}(z) = \frac{(-1)^n}{2^n n!} (1-z)^{-\rho} (1+z)^{-\sigma} \frac{d^n}{dz^n} [(1-z)^{\rho+n} (1+z)^{\sigma+n}]$ — многочлены Якоби

$Q_{\nu}(z) = Q_{\nu}^0(z)$ — функция Лежандра 2-го рода

$Q_{\nu}^{\mu}(z) = \frac{e^{i\mu\pi} \sqrt{\pi}}{2^{\nu+1}} \Gamma\left[\frac{\mu+\nu+1}{2}\right] z^{-\mu-\nu-1} (z^2-1)^{\mu/2} \times {}_2F_1\left(\frac{\mu+\nu+1}{2}, \frac{\mu+\nu}{2}+1; \nu+\frac{3}{2}; \frac{1}{z^2}\right) \quad [|\arg(z \pm 1)|, |\arg z| < \pi]$

$Q_{\nu}^{\mu}(x) = \frac{1}{2} e^{-i\mu\pi} \left[e^{-i\mu\pi/2} Q_{\nu}^{\mu}(x+i0) + e^{i\mu\pi/2} Q_{\nu}^{\mu}(x-i0) \right] = \frac{\pi}{2 \sin \mu\pi} \left[P_{\nu}^{\mu}(x) \cos \mu\pi - \Gamma\left[\frac{\nu+\mu+1}{2}\right] P_{\nu}^{-\mu}(x) \right] \quad [-1 < x < 1]$ — присоединенная функция Лежандра 2-го рода

$Q_n^{(\alpha, \beta)}(z) = 2^{\alpha+\beta+n} \Gamma\left[\frac{\alpha+n+1}{2}, \frac{\beta+n+1}{2}\right] (z-1)^{-\alpha-n-1} (z+1)^{\beta} {}_2F_1\left(n+1, \alpha+n+1; \alpha+\beta+2n+2; \frac{2}{1-z}\right) \quad [|\arg(z \pm 1)| < \pi]$ — функция Якоби 2-го рода

$\operatorname{Re} z$ — действительная часть комплексного числа $z = x + iy$ ($\operatorname{Re} z = x$)

$S(x) = \frac{1}{\sqrt{2\pi}} \int_0^x \frac{\sin t}{\sqrt{t}} dt$ — синус-интеграл Френеля

$S(x, \nu) = \int_x^{\infty} t^{\nu-1} \sin t dt \quad [\operatorname{Re} \nu < 1]$ — обобщенный синус-интеграл Френеля

$S_{\mu, \nu}(z) = s_{\mu, \nu}(z) + 2^{\mu-1} \Gamma\left[\frac{\nu, (\mu+\nu+1)/2}{(1+\nu-\mu)/2}\right] \left(\frac{z}{2}\right)^{-\nu} {}_0F_1\left(1-\nu; -\frac{z^2}{4}\right) + 2^{\mu-1} \Gamma\left[\frac{-\nu, (1+\mu-\nu)/2}{(1-\mu-\nu)/2}\right] \left(\frac{z}{2}\right)^{\nu} {}_0F_1\left(1+\nu; -\frac{z^2}{4}\right)$ — функция Ломмеля

$s_{\mu, \nu}(z) = \frac{z^{\mu+1}}{(\mu-\nu+1)(\mu+\nu+1)} {}_1F_2\left(1; \frac{\mu-\nu+3}{2}, \frac{\mu+\nu+3}{2}; -\frac{z^2}{4}\right)$ — функция Ломмеля

$\operatorname{Si}(x) = \int_0^x \frac{\sin t}{t} dt$ — интегральный синус

$\operatorname{si}(x) = \operatorname{Si}(x) - \frac{\pi}{2} = - \int_x^{\infty} \frac{\sin t}{t} dt$ — интегральный синус

$\sec z = \frac{1}{\cos z}$ — тригонометрическая функция

$\operatorname{sech} z = \frac{1}{\operatorname{ch} z}$ — гиперболическая функция

$\operatorname{sgn} x = \begin{cases} 1, & x > 0, \\ 0, & x = 0, \\ -1, & x < 0 \end{cases}$

$\operatorname{sh} z = \frac{e^z - e^{-z}}{2}$ — гиперболическая функция

$\operatorname{shi}(x) = \int_0^x \frac{\operatorname{sh} t}{t} dt = -i \operatorname{Si}(ix)$ — интегральный гиперболический синус

$\sin z = -i \operatorname{sh}(iz) = \frac{e^{iz} - e^{-iz}}{2i}$ — тригонометрическая функция

$\operatorname{sn} u = \sin(\operatorname{am} u)$ — эллиптическая функция Якоби

$T_n(z) = \cos(n \arccos z) = \frac{1}{2} [(z + \sqrt{z^2-1})^n + (z - \sqrt{z^2-1})^n]$ — многочлены Чебышева 1-го рода

$$\operatorname{tg} z = \frac{\sin z}{\cos z} = -i \operatorname{th} (iz) \text{ — тангенс}$$

$$\operatorname{th} z = \frac{\operatorname{sh} z}{\operatorname{ch} z} = \frac{e^z - e^{-z}}{e^z + e^{-z}} \text{ — гиперболический тангенс}$$

$$U_n(z) = \frac{\sin [(n+1) \arccos z]}{\sqrt{1-z^2}} \text{ — многочлены Чебышева 2-го рода}$$

$$U_\nu(z, \zeta) = \sum_{k=0}^{\infty} (-1)^k \left(\frac{z}{\zeta}\right)^{2k+\nu} J_{2k+\nu}(\zeta) \text{ — функция Ломмеля двух переменных}$$

$$V_\nu(z, \zeta) = \cos\left(\frac{z}{2} + \frac{\zeta^2}{2z} + \frac{\nu\pi}{2}\right) + U_{2-\nu}(z, \zeta)$$

$$W_{\kappa, \mu}(z) = z^{\mu+1/2} e^{-z/2} \Psi\left(\mu - \kappa + \frac{1}{2}, 2\mu + 1; z\right) \text{ — вырожденная гипергеометрическая функция Уиттекера}$$

$$Y_\nu(z) = \frac{\cos \nu\pi J_\nu(z) - J_{-\nu}(z)}{\sin \nu\pi} \quad [\nu \neq n], \quad Y_n(z) = \lim_{\nu \rightarrow n} Y_\nu(z) \quad [n = 0, \pm 1, \pm 2, \dots] \text{ — функция Неймана (функция Бесселя 2-го рода)}$$

$$y_n(z) = {}_2F_0\left(-n, n+1; -\frac{z}{2}\right) \text{ — многочлены Бесселя}$$

$$B(\alpha, \beta) = \frac{\Gamma(\alpha) \Gamma(\beta)}{\Gamma(\alpha + \beta)} \text{ — бета-функция}$$

$$B_x(\alpha, \beta) = \int_0^x t^{\alpha-1} (1-t)^{\beta-1} dt \text{ — неполная бета-функция}$$

$$\beta(z) = \frac{1}{2} \left[\psi\left(\frac{z+1}{2}\right) - \psi\left(\frac{z}{2}\right) \right]$$

$$\Gamma(z) = \int_0^{\infty} t^{z-1} e^{-t} dt \quad [\operatorname{Re} z > 0] \text{ — гамма-функция}$$

$$\Gamma(\nu, x) = \int_x^{\infty} t^{\nu-1} e^{-t} dt \text{ — дополнительная неполная гамма-функция}$$

$$\Psi(\nu, x) = \Gamma(\nu) - \Gamma(\nu, x) = \int_0^x t^{\nu-1} e^{-t} dt \text{ — неполная гамма-функция}$$

$$\Gamma\left[\begin{matrix} a_1, \dots, a_m \\ b_1, \dots, b_n \end{matrix}\right] = \frac{\prod_{k=1}^m \Gamma(a_k)}{\prod_{l=1}^n \Gamma(b_l)}, \quad \Gamma[a_1, \dots, a_m] = \prod_{k=1}^m \Gamma(a_k)$$

$$\Delta(k, a) = \frac{a}{k}, \frac{a+1}{k}, \dots, \frac{a+k-1}{k}$$

$$\delta_{m,n} = \begin{cases} 0, & m \neq n, \\ 1, & m = n \end{cases} \text{ — символ Кронекера}$$

$$\zeta(z) = \sum_{k=1}^{\infty} \frac{1}{k^z} \quad [\operatorname{Re} z > 1] \text{ — дзета-функция Римана}$$

$$\zeta(z, v) = \sum_{k=0}^{\infty} \frac{1}{(v+k)^z} \quad [\operatorname{Re} z > 1; v \neq 0, -1, -2, \dots] \text{ — обобщенная дзета-функция Римана}$$

Римана

$$\theta_1(x, q) \text{ — эллиптические тэта-функции}$$

$$\theta_0(x, q) = 1 + 2 \sum_{k=1}^{\infty} (-1)^k q^{k^2} \cos 2kx$$

$$\theta_1(x, q) = 2 \sum_{k=0}^{\infty} (-1)^k q^{(k+1/2)^2} \sin(2k+1)x$$

$$\theta_2(x, q) = 2 \sum_{k=0}^{\infty} q^{(k+1/2)^2} \cos(2k+1)x$$

$$\theta_3(x, q) = 1 + 2 \sum_{k=1}^{\infty} q^{k^2} \cos 2kx$$

$$\theta_4(x, q) \equiv \theta_0(x, q)$$

$$\Lambda_0(\varphi, k) = \frac{2}{\pi} [E(k) F(\varphi, \sqrt{1-k^2}) + K(k) E(\varphi, \sqrt{1-k^2}) - K(k) F(\varphi, \sqrt{1-k^2})]$$

$$\lambda(z, \alpha) = \int_0^{\alpha} z^{-t} \Gamma(t+1) dt$$

$$v(z) = \int_0^{\infty} \frac{z^t}{\Gamma(t+1)} dt$$

$$\Pi(\varphi, \alpha^2, k) = \int_0^{\varphi} \frac{dt}{(1 - \alpha^2 \sin^2 t) \sqrt{1 - k^2 \sin^2 t}} \text{ — эллиптический интеграл 3-го рода}$$

$$\Phi(z, s, v) = \sum_{k=0}^{\infty} \frac{z^k}{(v+k)^s} \quad [|z| < 1; v \neq 0, -1, -2, \dots]$$

$$\Phi_1(a, b; c; z, \zeta) = \sum_{k, l=0}^{\infty} \frac{(a)_{k+l} (b)_k}{(c)_{k+l}} \frac{z^k \zeta^l}{k! l!} \quad [|z| < 1]$$

$$\Phi_2(b, b'; c; z, \zeta) = \sum_{k, l=0}^{\infty} \frac{(b)_k (b')_l}{(c)_{k+l}} \frac{z^k \zeta^l}{k! l!}$$

$$\Phi_3(b; c; z, \zeta) = \sum_{k, l=0}^{\infty} \frac{(b)_k}{(c)_{k+l}} \frac{z^k \zeta^l}{k! l!}$$

$$\Psi(a, b; z) = \Gamma\left[\frac{1-c}{1+a-c}\right] {}_1F_1(a; c; z) + \Gamma\left[\frac{c-1}{a}\right] z^{1-c} {}_1F_1(1+a-c; 2-c; z) \text{ — вырожденная гипергеометрическая функция}$$

$$\Psi_1(a, b; c, c'; z, \zeta) = \sum_{k, l=0}^{\infty} \frac{(a)_{k+l} (b)_k}{(c)_k (c')_l} \frac{z^k \zeta^l}{k! l!} \quad [|z| < 1]$$

$$\Psi_2(a; c, c'; z, \zeta) = \sum_{k, l=0}^{\infty} \frac{(a)_{k+l}}{(c)_k (c')_l} \frac{z^k \zeta^l}{k! l!}$$

$$\psi(z) = [\ln \Gamma(z)]' = \frac{\Gamma'(z)}{\Gamma(z)} \text{ — пси-функция}$$

$$\psi^{(n)}(z) = \frac{d^n}{dz^n} \psi(z)$$

УКАЗАТЕЛЬ ОБОЗНАЧЕНИЙ СИМВОЛОВ'

$(a)_k = a(a+1) \dots (a+k-1)$ [$k = 1, 2, 3, \dots$], $(a)_0 = 1$ — символ Похгаммера

$n! = 1 \cdot 2 \cdot 3 \dots (n-1)n$, $0! = 1! = 1$

$(2n)!! = 2 \cdot 4 \cdot 6 \dots (2n-2)2n = 2^n n!$

$(2n+1)!! = 1 \cdot 3 \cdot 5 \dots (2n+1) = \frac{2^{n+1}}{\sqrt{\pi}} \Gamma\left(n + \frac{3}{2}\right) = \left(\frac{3}{2}\right)_n 2^n$

$n!! = \begin{cases} (2k)!!, & n = 2k, \\ (2k+1)!!, & n = 2k+1, \end{cases} \quad 0!! = (-1)!! = 1$

$\binom{n}{k} = \frac{n(n-1) \dots (n-k+1)}{k!} = \frac{n!}{k!(n-k)!} = \frac{(-1)^k (-n)_k}{k!}$ — биномиальные коэффи-

циенты

$\binom{n}{0} = 1$

$(a)_+^\lambda = \begin{cases} a^\lambda, & a > 0, \\ 0, & a < 0 \end{cases}$

$\operatorname{Re} a, \operatorname{Re} b > c$ означает $\operatorname{Re} a > c$ и $\operatorname{Re} b > c$

$[x] = n$ [$n \leq x < n+1$, $n = 0, \pm 1, \pm 2, \dots$] — целая часть числа x

$\bar{z} = x - iy$ [$z = x + iy$]

$|z| = \sqrt{x^2 + y^2}$

$|x| = \begin{cases} x, & x \geq 0, \\ -x, & x < 0 \end{cases}$

$\prod_{k=m}^n a_k = a_m a_{m+1} \dots a_n \quad [n \geq m]$
 $= 1 \quad [n < m]$

$\prod_{k=1}^{\infty} a_k(z) = \lim_{n \rightarrow \infty} \prod_{k=1}^n a_k(z)$

$\sum_{k=m}^n a_k = a_m + a_{m+1} + \dots + a_n \quad [n \geq m]$
 $= 0 \quad [n < m]$

$\sum_{k=1}^{\infty} a_k(z) = \lim_{n \rightarrow \infty} \sum_{k=1}^n a_k(z)$

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