1

A Critique of the Mathematical Abilities of CA Systems

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Abstract: Computer algebra systems (CASs) have become an essential computational tool in the last decade. General purpose CASs, which are designed to solve a wide variety of problems, have gained special prominence. In this chapter, the capabilities of seven major general purpose CASs (Axiom, Derive, Macsyma, Maple, Mathematica, MuPAD and Reduce) are reviewed on 542 short problems covering a broad range of (primarily) symbolic mathematics.[1]

1.1 Introduction

It is not so easy to write a review. One desires to be complete and accurate as well as balanced and informative; however, the process of reviewing requires attending to a seemingly unending set of details. "Is this action *really* correct in *all* situations? If I read this manual entry from a sideways perspective, does it make *more* sense, or *less*? Are these people really idiots or am I just getting a tiny bit carried away?" It is always an internal struggle to decide when enough is really enough and it is now time to just turn loose of the baby (sob!).

Yes, writing a review can be an emotional experience if one cares at all about the subject matter. And once the review is completed is when the fun really begins, of course. "What do you mean by *this*? What about *that*? Are you sure?" Some readers will flow with praise,[2] while others will attack. (Another segment will be indifferent.) It is the mark of a good review, though, if it makes people *think*.[3]

Presented below is a tabular summary of 542 mathematical problems, primarily symbolic, that were given to the seven general purpose computer algebra systems

[1] Earlier versions of this chapter have appeared in [Wester94, Wester95].
[2] At least reviewers sometimes have a passing fancy this may happen!
[3] Failing that, it can at least try to entertain!

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Table 1.1 CASs used.

Ax	Axiom 2.1	1997
De	Derive for DOS Version 4.11	1998
Mc	Macsyma 422.0	1998
Mp	Maple V Release 5.1	1998
Mm	Mathematica 3.0	1996
Mu	MuPAD 1.4.0	1998
Re	REDUCE 3.6	15–Apr–96

(CASs) listed in Table 1.1 [Jenks92, NAG95, Rich94, Kutzler97, Macsyma95, Bogen96, Heal98, Monagan98, Redfern96, Wolfram96, Martin96, Fuchssteiner94, Fuchssteiner96, Hearn95]. The CAS versions tested were those that were available to the author, and were typically the newest and most comprehensive versions that were generally available at the time of this evaluation.

1.2 The Summary Table

The summary is divided into 30 sections, with each group of problems sorted loosely in order of increasing logical difficulty. Problem descriptions are abbreviated due to space limitations with this format and may not always be complete. However, full descriptions and references can be found as comments in the input and output files.[4]

The notations used in the summary are explained in Table 1.3. For example, if the font of the problem number is not Roman type, this indicates that the problem is either particularly easy (*italics*) or particularly difficult (**boldface**).

In previous reviews, the results of a particular system on a given problem was indicated by one of a small number of symbols. It became clear on doing this new review that additional codes were needed in order to give a more precise indication on how well a system really performed, especially as there seem to be quite a number of full and partial failure modes that can occur. For example, a system can give up with an error message which might simply say a particular algorithm has not yet been implemented, or it might abort issuing some totally incomprehensible message spewed from deep within the bowels of the program (ε versus $\mathcal{E}$).

As another example, a simple solution producing a correct answer is awarded a $\bullet$; however, sometimes one needs to be just a little clever to arrive at a nice result or maybe the answer is just not quite as simple as it could be. In these situations, a $\star$ is given. If even more effort is needed to generate a nice solution or the final result is somehow unsimplified or incomplete, then a $\circ$ is bestowed on the effort. Especially tricky solutions, very inelegant output and minimal success result in a $\natural$.

For a specific instance, consider solving the cubic equation

$$3x^3 - 18x^2 + 33x - 19 = 0 \tag{1.1}$$

for x (problem M2). It is well known ([CRC73]p. 103) that if the coefficients of a cubic polynomial are real and satisfy a specific inequality, then the roots of that polynomial are guaranteed to be real and unequal. These roots can thus be expressed

[4] The output files are available from `http://math.unm.edu/~wester/cas_review.html`.

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using trigonometric functions in a manner that makes it trivially obvious that they are indeed real (no explicit i's) instead of in terms of the usual cubic formula where expressions containing i's to $\frac{1}{3}$ powers make such an observation very unobvious. Compare

$$x = \frac{\frac{\sqrt{3}\,i}{2} - \frac{1}{2}}{3\,\sqrt[3]{\frac{i}{6\sqrt{3}} + \frac{1}{6}}} + \sqrt[3]{\frac{i}{6\sqrt{3}} + \frac{1}{6}}\left(-\frac{\sqrt{3}\,i}{2} - \frac{1}{2}\right) + 2$$

with

$$x = \frac{\sqrt{3}\,\sin\left(\frac{\pi}{18}\right) - \cos\left(\frac{\pi}{18}\right) + 2\sqrt{3}}{\sqrt{3}}\,,$$

which is the same root of (1.1) in the two different forms as produced by Macsyma.

When given this problem, Derive and Reduce immediately produce the explicitly real solution in a simple form, so they are ranked completely successful. MuPAD also uses the trig formula, but its results even after simplification are unnecessarily complicated (it does not recognize that $\frac{1}{3}\tan^{-1}\frac{\sqrt{3}}{3} = \frac{\pi}{18}$), so it is considered good but not perfect. The other systems need some help from the user to transform their solutions to an explicitly real form. With Macsyma and Maple, it is not so difficult:

Macsyma	`ratsimp(rectform(%));`
Maple	`simplify(evalc({%}));`

Hence, they are also deemed to be good here (it is a little more difficult than with MuPAD, but they also produce complete simplification of the results). Axiom and Mathematica require a little more ingenuity, since the application of their versions of the 'map' command are not as clear cut for this problem as was the case with Macsyma and Maple:

Axiom	`map(e +-> lhs(e) = simplify(complexForm(rhs(e))), %)`
Mathematica	`Map[#[[1]] -> ComplexExpand[#[[2]]] &, Flatten[%]]`

These systems are thus considered to be OK on this example.

Other considerations in evaluating a problem are:

- \# The CAS attempted a solution, but only managed to go part way. (Typically, a simplification was left undone that the system should have been able to do.)
- (blank) The system tried, but it could not do the problem. (Often, the output is simply the original input command.)
- — There is no simple way to even state the problem in the CAS or a necessary capability is lacking to do such an example.
- τ No solution after one hour of CPU time (quite possibly an unending calculation).
- $\boxed{\star}$ There was some small error in the solution. (Perhaps an absolute value was omitted, etc.)
- $\otimes$ Some success, but also some mistakes. (For example, when solving $\sqrt{x} < 2$ for x (N12), both Mathematica and MuPAD discover that $x < 4$, but do not realize that $x \geq 0$ must also be true [Mathematica does warn that it may produce an incorrect result].)

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⊎ The answer is mostly wrong, but there are still a few positive elements.
× Totally wrong.
See Table 1.3 for additional details.

Some entries in the summary are divided in two. This may indicate that a problem had two parts and the system fared unequally on them, or it may signify that the problem could be attempted in more than one way and these are the results from two major approaches. Often, a footnote will accompany the problem description or section title, explicitly defining what the two ratings mean.

1.3 Selection of Problems

The problems in this review have come from a variety of sources. Various references [CRC73, Gradshteyn94, Zwillinger92], textbooks [Bradley75, Caviness93, Cohen78, Cullen90, Davis75, Fitzgerald75, Gantmacher77, Geddes92, Johnson81, Knopp90, Konopinski81, Koopmans87, Levinson70, Lovelock75, Olver93, Perron50, Roxin72, Sanchez83, Smythe66, Stark84, Symon71, Taylor72, Venables94, Wilkinson65] and theses [Farhat93, Gruntz96, Wester92] provided many interesting problems from an assortment of subject areas. Some amusing examples were described or suggested by random articles [Coutsias97, Hong97, van Hulsen94, Keady, Li95, Liska95, Moses71, Pinch93, Robidoux93, Stoutemyer91, Wilf9x, Xu9x], messages posted to the Usenet newsgroup `sci.math.symbolic`, as well as by discussions in person or via email with various people (see the actual input/output files for complete credits).

Some of the most intriguing problems came from trying to compare the output of a system with a known solution. In some cases, it was extremely difficult (or even impossible) to coerce an answer to a recognizable form without resorting to hand calculation. For example, the expression in C22:

$$\frac{(6-4\sqrt{2})\log(3-2\sqrt{2})+(3-2\sqrt{2})\log(17-12\sqrt{2})+32-24\sqrt{2}}{48\sqrt{2}-72} = \frac{\log(\sqrt{2}+1)+\sqrt{2}}{3}$$

is Axiom's solution to W24, $\int_0^1\int_0^1\sqrt{x^2+y^2}\,dx\,dy$. Only Derive was able to simplify this expression in any really useful fashion. This was because it recognized that

$$\begin{aligned}\log(3-2\sqrt{2}) &= \log\left((\sqrt{2}-1)^2\right) &= 2\log(\sqrt{2}-1),\\ \log(17-12\sqrt{2}) &= \log\left((\sqrt{2}-1)^4\right) &= 4\log(\sqrt{2}-1).\end{aligned}$$

Another example is the simplification requested in F5:

$$\frac{\Gamma(n+\frac{1}{2})}{\sqrt{\pi}\,n!} \Rightarrow \frac{(2n)!}{2^{2n}n!^2} \quad\text{or}\quad \frac{(2n-1)!!}{2^n n!}\,.$$

None of the CASs reviewed can do either of these transformations. This expression was produced in some form by every system but Axiom when computing $\prod_{k=1}^n \frac{2k-1}{2k}$ (S5). All these CASs know that $\Gamma(n+\frac{1}{2}) = \frac{(2n-1)!!}{2^n}\sqrt{\pi}$, n a positive integer ([Wilf62]p. 179), for specific values of n, but they cannot apply this identity for generic n even when n is declared explicitly to be an integer > 0.

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Yet one more example that was brought up by another problem is what should in theory be a very easy simplification of $2 \cdot 2^n$ into 2^{n+1} (H1) and the related transformation of $4 \cdot 2^n$ to 2^{n+2} (H2). Derive and Mathematica have no trouble doing these, but the other systems surprisingly flounder on one or the other. One tricky solution ($\natural$) is to substitute 2 by a, simplify $a \cdot a^n$ to a^{n+1} or $a^a \cdot a^n$ to a^{n+a} (here, 4 must first be factored as 2^2), and then substitute 2 back for a!

The last category of problems are those that were made up to see just how clever the CASs are. An especially amusing example is trying various operations on the expression

$$\frac{\tan^2 x + 1 - \sec^2 x}{\sin^2 x + \cos^2 x - 1}\,, \tag{1.2}$$

which, of course, is $\frac{0}{0}$ and hence undefined (I10–I12). Derive and Reduce always realize that there is something funny happening (MuPAD usually does as well, although one has to simplify the derivative expression before it will complain there); however, the other systems are spotty. Some of them simplify (1.2) to $1/\cos^2 x$, for instance, entirely removing the singularity!

The very last set of problems in the summary, entitled "Mathematics versus Computer Science", are another made-up collection. They test two things:

1. Do CASs consider k to be an implicit local variable in expressions like $\sum_{k=1}^{4} k$, $\lim_{k \to 0} k$, etc. so that assignments to k outside the expression do not affect the answer because k is, at least mathematically, simply a dummy name for the variable in the sum, limit, etc.?
2. Do CASs recognize the equivalence of the different forms that can be used to specify e^x, $\sqrt{x}$ and i when performing pattern matching?[5]

The results seem to be (1) usually no, (2) usually yes, but there are exceptions for both questions.

1.4 Point of View

The philosophy that I followed in making these comparisons included several facets. The choice of problems was broad, as the primary goal was to provide a useful indication of the breadth of coverage of each general purpose system. A secondary goal was to give a feeling of the depth of coverage provided for the various classes of problems.

I approached this review with an applied mathematician's bias—I wanted to produce an absolutely correct answer with a minimum of fuss and as quickly as possible. I was less interested in the exact manner needed to produce a result, although programming style was judged for elegance and power of expression, since this is an important ingredient of the CAS environment. I tended to emphasize exact, symbolic calculations, since this is what originally made CASs special in relation to numeric packages; however, a few approximate, numerical problems are also included.

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[5] This problem is significant in other situations as well. For example, in Maple Vr4, $\int_0^2 \frac{x^2}{\sqrt{(1+x^3)^3}}dx \Rightarrow \frac{4}{9}$ while $\int_0^2 \frac{x^2}{(1+x^3)^{3/2}}$ produced a very complicated expression (this has since been fixed in Maple Vr5).

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Issues of graphics, language design, user interface and computational speed are mostly ignored, all of which are important items of concern as well. The seven general purpose CASs examined can, with varying amounts of user assistance, solve a wide variety of problems. I have tried to emphasize those problems that should involve minimal user intervention.

Packages not provided with the standard distribution of a CAS were excluded from consideration in order to make the scope of the review manageable. Many systems do, however, have some quite nice user packages that are available on the Internet or which must be purchased separately. Other people have also made comparisons of multiple CASs, emphasizing different aspects than was done here. For example, see [Aslaksen96, Bernardin96, Fateman96, Faugère95, Gräbe95, Gräbe96, Gruntz96, Harper91, Hereman94, Hereman95, Hereman96, Koepf95, Pinch93, Postel96, Rua98, Simon92, Simon95, Simon97] as well as the other chapters in this volume. For an overview of the general issues associated with computer algebra, please see [MacCallum98]. In addition, nice comprehensive discussions about computer algebra systems and algorithms can be found in [Barton72, Caviness86, Geddes92, van Hulzen83, de Souza93].

1.5 Observations

Playing with seven different CASs, each with a somewhat different philosophy, on such a large number of problems has provided some useful insights. In general, these systems are simply amazing. They can do an incredible variety of problems and they typically do them well. They embed a great deal of knowledge that in total transcends the expertise of a majority of their users. They are flexible and can be taught new ideas. Each system has its own personality and is almost like a living creature.

Of course, each CAS has annoying quirks, spotty intelligence, poor communication at times, and is not as flexible as one might desire. How can these systems improve? Here are some observations.

1.5.1 Documentation

Certainly the worst problem with these systems is how they communicate with the user. The most basic methods are through the user/reference manual(s) and the CAS's online help system. Most of the CAS manuals are not so user-friendly. The most important element they lack typically is a good comprehensive topical index. Most indices today seem to be basically an alphabetical list of commands, and even if the commands are well named, the user must often still hunt around looking for the correct synonym for the operation he is interested in. This is why cross-referencing was invented—to obviate this very problem! The index in *The Mathematica Book*[Wolfram96] is actually pretty good, although it commits the sin of omitting most references to functionality in the '*Standard* Add-On Packages' (if they are standard, why not cover them too?). Hence, one may need to search the indices of *two* manuals when checking on the availability of some particular functionality.

Manuals should be reasonably comprehensive and certainly written clearly (not always the case now). Well-organized tabular summaries can help the user to focus on important aspects of the systems. Examples can be extremely helpful in clarifying

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details. All of this should be obvious, but it does not hurt to repeat it here.

It is certainly critical to make definitions clear. As can be seen in the footnotes to the summary table, different systems have different conventions about what $\sum_k$ (indefinite summation) actually means and what the constant is in front of the integral in the definition of the Fourier transform.[6] The difference between using a biased rather than an unbiased estimator (so that division is by n rather than by $n-1$) in the statistical problems is yet another point of subtlety.

Of course, all the above comments carry over directly to the online documentation, which in addition, should be easily searchable, well cross referenced (for example, by hyperlinks) and have plenty of examples that can be easily attempted. Searching should be case insensitive (usually) with wildcards allowed, as in MuPAD's implementation.

Finally, error conditions should produce useful and comprehensible messages, not simply saying little more than that something bad happened, which the user probably already guessed.

1.5.2 Language

A second major obstacle to effectively using CASs is their languages. There are many items, but I will touch on just a few of them here. One user interface issue is sets versus lists. In some systems, such as Maple and MuPAD, commands sometimes require a set of objects and sometimes a list of objects (a set is like a list except that duplicate entries have been removed and element order is not guaranteed). For example, the solve commands in Maple and MuPAD insist on sets of equations and variables. Although it is easy to convert between lists and sets, it seems that the CASs could do this on their own without having to burden the user with conforming exactly to one specific syntax (it sometimes can be quite convenient to work with lists of equations). Similar remarks can be said about matrices versus lists of lists or one-element lists versus isolated expressions. The idea, of course, is to free the user from having to worry so much about syntax, rather allowing him (or her) to spend additional time concentrating on more substantive matters (like mathematics!).

A second syntax issue is the ease of selecting parts of expressions. It can be quite cumbersome in some systems to extract a particular portion of a given expression (here, I mean both operators such $+$, $-$, log, sin, etc. as well as their operands). Picking expressions apart (and putting them back together in new ways) is such a common activity that it is surprising that more attention has not been paid to this matter. Consider one example.

All major CASs have some ability to deal with matrices. At least one major numerical package, MATLAB [MathWorks92b, MathWorks92a], has a prime emphasis on matrix operations and a well-designed interface so that these operations can be applied in a reasonably "natural" manner. In MATLAB, it is quite easy to extract rows, columns and subblocks of matrices in various combinations, and rearrange the results into new matrices. (This is encouraged by allowing vector indices to select a sequence of objects, such as matrix elements.)

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[6] In most systems, the only way to discover these conventions was to deduce them from the output.

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For example, suppose A is a $4{\times}4$ matrix. Then the MATLAB statement

```
B = [A(1:3,2:4), A([1,2,4],[3,1,4]); A, [A(1:2,3:4); A([4,1],[3,2])]]
```

will define B to be the $7{\times}6$ matrix

$$B = \left(\begin{array}{cccccc}
a_{12} & a_{13} & \multicolumn{1}{c|}{a_{14}} & a_{13} & a_{11} & a_{14} \\
a_{22} & a_{23} & \multicolumn{1}{c|}{a_{24}} & a_{23} & a_{21} & a_{24} \\
a_{32} & a_{33} & \multicolumn{1}{c|}{a_{34}} & a_{43} & a_{41} & a_{44} \\ \hline
a_{11} & a_{12} & a_{13} & \multicolumn{1}{c|}{a_{14}} & a_{13} & a_{14} \\
a_{21} & a_{22} & a_{23} & \multicolumn{1}{c|}{a_{24}} & a_{23} & a_{24} \\ \cline{5-6}
a_{31} & a_{32} & a_{33} & \multicolumn{1}{c|}{a_{34}} & a_{43} & a_{42} \\
a_{41} & a_{42} & a_{43} & \multicolumn{1}{c|}{a_{44}} & a_{13} & a_{12}
\end{array}\right).$$

Square brackets (`[ ]`) delimit matrices and vectors in MATLAB, semicolons (`;`) separate rows, and colons (`:`) are used to make integer sequences (e.g., `1:3` is equivalent to `[1,2,3]`). Such notation can be used on the left side of assignment statements as well. For instance, `B([2,5],:) = B([5,2],:)` will swap rows 2 and 5 of B (the colon here expands to all relevant columns). This flexibility in manipulating matrices (along with the apparent ease in doing so) should be a vital ingredient in any system that deals with matrices, and in fact, a variation of this notation has been adopted by Macsyma (P3, P4).[7]

The last issue that will be considered in this subsection is how does one keep track of the ever increasing numbers of commands in CASs and their corresponding functionalities? Besides better documentation, one idea is to have fewer but more capable commands that fit in naturally with the mathematical ideas expressed. This is the extended object-oriented approach in which the command detects the domains of its arguments and acts appropriately for objects in those domains. Moreover, if the arguments are not formally in an appropriate domain, but with a little effort could be coerced to be (see above), then do so if the results will be unambiguous.

Some of the operators in Derive provide a good model for the first two-thirds of this idea. For example, $A \bmod n$, $\frac{dA}{dx}$, $|A|$ and A^{-1} (P5, P6, P9, P11) act appropriately for A either a scalar or a matrix ($|A|$ computes the Frobenius norm when A is a matrix). It is true that all CASs do domain detection and coercion at times (consider adding an integer to a rational number), but more of this would be better.

1.5.3 Simplification

"Simplify, simplify."
— Henry David Thoreau in *Walden*

"Everything should be as simple as possible, but not simpler."
— Albert Einstein

"If complexity is perplexity, then is simplicity duplicity?"
— The author

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[7] Derive has recently adopted some of these ideas as well.

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Once a user has started playing with a CAS, one of the first difficulties he encounters is the problem of simplification. Many times expressions will be generated that must be massaged considerably in order to produce a simplified form. A representative example is the following expression resulting from MuPAD's solution to

$$\frac{dx}{dt} = x\left(1 + \frac{\cos t}{2 + \sin t}\right)$$

(Z23):

$$\sqrt{16\sin t - 2\cos 2t + 18} = 2(\sin t + 2)\ .$$

MuPAD had to be greatly assisted in order to perform this simplification.

In the systems under review, the major strategies presented to the user for performing simplification are

1. a few, hopefully powerful, simplification functions that the user should normally apply before trying anything else (typical names are simplify, factor, expand, etc.);
2. many different transformation functions that operate in various circumstances (Macsyma and Maple[8] especially);
3. simplification depending on properties of the unknowns contained in the expression, the properties typically declared with a command like assume or declare[9] (Derive, Macsyma, Maple and to a lesser extent Mathematica[10] and MuPAD).

Many of the examples in this review ultimately involved nested radicals, exponential, trigonometric and hyperbolic functions and their inverses, and complex results (see below), so that those systems that are facile manipulating such quantities tended to do well. Moreover, a number of problems involved restrictions on parameters, a common circumstance in physical problems, and therefore those CASs that do not easily handle such assumptions were at a major disadvantage for these examples.

It should be noted that there are situations where assumptions are implicit in the statement of the problem. For instance, consider the behavior of Macsyma when confronted with $\int_{-1}^{1}\int_{0}^{2}|y - x^2|\,dy\,dx$ (W26):

```
(c1) integrate(integrate(abs(y - x^2), y, 0, 2), x, -1, 1);
Is   x   zero or nonzero?
nonzero;
      2
Is   x  - 2   positive, negative, or zero?
negative;
                                       46
(d1)                                   --
                                       15
```

[8] Primarily categorized under simplify and convert. MuPAD has the beginnings of such a strategy categorized mainly under simplify and rewrite.

[9] It is not uncommon for various functions to make implicit assumptions about their arguments—e.g., Mathematica's ComplexExpand assumes that all variables it operates on are real.

[10] Explicitly only with Integrate.

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Here, Macsyma fails to infer that $-1 \le x \le 1$ as implied by the limits on the outer integral. Other systems exhibit such behavior for other examples.

See also [Moses71] for a very nice discussion on how CASs handle simplification, and also [van Hulzen83].

1.5.4 The Complex Domain

This is an area in which CASs sometimes have great difficulty. Many of the problems in this section are derived from the theorems stated in [Aslaksen96] (see his updated article in this volume: Chapter **??**) and are evaluated strictly for consistency with the domains assumed by the variables involved. Note that by default Axiom, Derive and Macsyma assume that variables are real, while the other CASs defaultly take variables to be complex.

One interesting pair of problems is K23 and K24. Most CASs can compute exactly the principal value of $\tan^{-1}(\tan(11+30i)) = 11-4\pi+30i$, but have great trouble with the floating point variant $\tan^{-1}(\tan(11.0+30.0i)) \approx -1.56637+30.0i$. This is because $\tan(11.0+30.0i) \approx -1.5 \cdot 10^{-28}+i$ and the low-precision calculations these systems do normally do not carry enough digits. Two possible solutions are to carry more digits or to use a hybrid symbolic/numeric approach like that used by Macsyma which produced $\frac{\pi}{2} - 3.137169 + 30.0i$.

One other interesting problem pair is M34 and M35. Here, the systems are asked to solve an equation involving a complex variable z and its conjugate for z. They are not happy. Even explicitly rewriting the equation in terms of x and y where $z = x+iy$ and declaring x and y to be real if possible does not help. The CASs fail to understand that the real and imaginary components are linearly independent and so really form two separate equations.

1.5.5 General Remarks

What are some other important considerations that authors of CASs should concern themselves with? One is that CASs need to be able to intercommunicate freely: with each other, with numerical and statistical packages, with user code (e.g., scripts, programs and subroutines via dynamic linking, code generation, etc.), with text and word processors (TEX, HTML, etc.) and with files in various formats. Some of this is already available and some is being worked on. For example, the OpenMath project is trying to develop a platform-independent standard for the representation of mathematical objects so that they can be exchanged in a meaningful way between various software tools—clearly, not an easy task.

Mathematical problems often involve objects from multiple domains (e.g., polynomials, trig functions, matrices, etc.), and so a CAS needs to be able to smoothly deal with interactions between various types of objects. Here, 'domain' is being used loosely and so could even refer to different operators. For example, most systems know that the derivative of a (finite) sum is the sum of the derivatives, but how many know a similar rule for the integral or the Laplace transform of a sum? See Table 1.2 and also Y7 (an example involving the Laplace transform of an infinite square wave).

CASs need to be robust. Fernando J. Corbató, a designer of CTSS—the first timesharing system—in his 1990 Turing Award lecture "On Building Systems That

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Table 1.2 Applying linearity ($\mathcal{L}$ denotes the Laplace transform).

	Ax	De	Mc	Mp	Mm	Mu	Re
$\frac{d}{dx}\sum^{n} \Rightarrow \sum^{n}\frac{d}{dx}$	•	•	•	•	•	•	
$\int\sum^{n} \Rightarrow \sum^{n}\int$			•			•	
$\mathcal{L}\sum^{n} \Rightarrow \sum^{n}\mathcal{L}$			•	•			

Will Fail" [Corbató91] discusses 'ambitious systems'. These systems never quite work as expected. The question is not if, but when will something go wrong? Therefore, such systems should be designed from the ground up to anticipate failure, since if the system is sufficiently complex, failure will be inevitable.

In Mathematica 2.2, if one tried to find the eigenvalues of a matrix whose characteristic polynomial contained a general quintic factor, the system would complain that it could not find all the eigenvalues and return nothing, even if the other roots of the characteristic polynomial were easy to deduce! (Mathematica 3.0 has since fixed this problem.)

A CAS is certainly an "ambitious system". There will be times when a problem will not be completely solvable, so the CAS should try to do the best it can and present to the user what facts it has discovered. A message like "Fail" does not convey any insight.

In 1996, Cherri M. Pancake of Oregon State University[11] gave a talk in Albuquerque, New Mexico on the topic "Can We Bridge the Gap between User Needs and Parallel Tool Support?" [Pancake96]. Cherri Pancake is an anthropologist by training who works now in the High Performance Computing (HPC) community trying to act as a bridge between developers and users. Many of her remarks are quite relevant to the computer algebra community as well.

She notes that many software tools lack robustness for real-world *size* problems (see above) as well as usefulness for many tasks that users desire to perform. Why is this?

Developers and users speak different languages. Developers are comfortable with computer science and enjoy experimenting with new tools, while users are comfortable with their application area and typically are just interested in completing the task at hand. Many users are only "occasional programmers", and their attention spans for learning/relearning tool use are notoriously short.

Each side makes assumptions about the other, justified or not. Software designs are based on assumptions about user needs and habits. A common occurrence, though, is that the applicability of tools is not as obvious as their makers think. Here is an actual scenario. A developer commissioned "real" users to test a new tool. The developer's comments were: "The interface is fine. They found some bugs in the software, but didn't have any trouble with the interface." One of the users' comments was: "Sure, we tried to break it—and did. The interface? It was so awful we figured they just

[11] See `http://www.nero.net/~pancake/`.

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hadn't gotten around to developing it yet."

So what is the solution? Designers must actually involve users throughout the design process. Users must articulate their needs and provide constructive criticisms. Both sides must explain, not just state, their point of view. I think that the computer algebra community does try to engender conversations among developers and users to some extent, but Ms. Pancake's observations are illuminating.

For instance, it would be extremely useful to perform usability studies on the current crop of mathematical software and in particular, the general purpose CASs, and see how easy it really is for people to solve problems using these packages. The only such study that I am aware of is [Rua98], in which the help systems of Derive, Macsyma and Maple were compared in order to assess their effectiveness in assisting users previously unacquainted with CASs (approximately 40 high school students) solve mathematical problems. The study was somewhat flawed as the authors note, but nonetheless, this is a direction that should definitely be pursued further.

It certainly would be wonderful if CASs were adjustable to the mind-sets of their users (mathematicians, physicists, engineers, etc.).[12] One could invoke mindset(elementary_math_student) to initially declare all variables to be real, make $\sqrt{-1}$ undefined, etc., for example.

I conclude with a comment on quality control. I ran my tests on various versions of the CASs as I tried to get this review into final form. I would compare the output from a new version of a CAS with that from an earlier version using the Unix diff command. There were times when some example that ran successfully before would now fail (this happens with "ambitious systems"). I did try to inform the vendors when such an occurrence happened.

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I wish to thank Paul Abbott (Mathematica), Ed Bedrick (statistics), Vageli Coutsias (helpful references), Richard J. Fateman (Macsyma), Jeffrey P. Golden (Macsyma), Tony Hearn (for making Reduce 3.5 available), Tom Issaevitch (Mathematica), Richard Liska (Reduce and for several examples), Jack McIver (for giving me access to a nice machine with lots of software), Rolf Mertig (Mathematica), Michael Monagan (Maple), Winfried Neun (Reduce and making Reduce 3.6 available), Albert D. Rich (for making Derive available), Nicolas Robidoux (Axiom, triangular system of two ODEs example [Robidoux93]), Margaret Sanders (statistics examples), Stanly Steinberg (several examples), David Wood (readability), Paul Zimmermann (MuPAD), Ecole Polytechnique, France for making Axiom 2.1 available, and finally the Department of Mathematics and Statistics and the Department of Physics and Astronomy at the University of New Mexico for their support and excellent computing environments.[13]

[12] Of course, exposing users to new ideas can be good too!

[13] I would also like to note that much excellent advice in designing my tables was provided by [Tufte83, Tufte90, Tufte97] which I encourage all people interested in presenting data to read.

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Table 1.3 Notations used.

A1	a particularly easy problem
A1	an unrated problem
A1	a particularly hard problem
$\bullet$	success! (hurrah!)
$\star$	success, but a little fudging or subtlety required, or the answer could be just a little nicer or more complete (yes!)
$\circ$	success, but indirectly, incomplete or unsimplified (ok)
$\natural$	tricky, very inelegant or minimal success (so-so)
$\#$	incompletely simplified, but some useful transformations were performed (groan)
	could not do the problem (boo!)
—	lacks the capability to do or state the problem (sigh)
ε	an error message was generated (erk!)
$\mathcal{E}$	a *surprising* error occurred (ack!)
$\textcircled{\varepsilon}$	a fatal program error occurred (yuck!)
τ	very slow (yawn)
$\boxed{\star}$	almost correct (oops)
$\otimes$	partial success, but also partially incorrect (hmmm)
$\uplus$	mostly, but not completely wrong (well ...)
$\times$	produced the wrong answer (hiss!)
$\Rightarrow$, $\rightarrow$	yields/then
"	previous result
?	cannot determine an answer
$\bar{z}$	complex conjugate of z
$\binom{n}{m}$	binomial coefficient
$[A, B]$	closed interval *or* commutator of A and B $(= AB - BA)$
C, **R**, **Z**	complex numbers, real numbers, integers
$\mathsf{CDF}(\ldots, x)$, $\mathsf{PDF}(\ldots, x)$	cumulative/probability density function at x
$\mathsf{cf}(x)$	continued fraction of x
$\mathsf{charpoly}(A)$, $\mathsf{minpoly}(A)$	characteristic/minimum polynomial of the matrix A
$\operatorname{Ci} x$, $\operatorname{Si} x$	cosine/sine integrals
$\operatorname{cn} u$, $\operatorname{dn} u$, $\operatorname{sn} u$	cosine/delta/sine amplitudes (Jacobian elliptic functions)
D, diff	derivative operator/differentiate
δ_{ij}, $\delta(x)$	Kronecker delta/Dirac delta

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f_n	n^{th} Fibonacci number
$F(\phi, k)$, $E(\phi, k)$	elliptic integral of the $1^{\text{st}}/2^{\text{nd}}$ kind
$F(a, b; c; z)$	hypergeometric function
$\mathsf{factor}(\ldots,\ \alpha)$	factor over the algebraic field extension α
$\mathsf{flops}(\ldots)$	count (floating point) operations
$\mathsf{fourier}(f(x),\ p)$	Fourier series (over the interval $[-p, p]$) or transform of $f(x)$
γ	Euler's constant
Γ_{ijk}, $\Gamma_{j\ k}^{\ i}$	Christoffel symbol of the $1^{\text{st}}/2^{\text{nd}}$ kind
$H(t)$	Heaviside (unit step) function
$I_{m\times n}$	$m \times n$ identity matrix
$J_\mu(z)$	Bessel function of the 1^{st} kind of order μ
$K_{i\ jk\mid\ell}^{\ h}$	covariant derivative of the Riemann curvature tensor
$\mathsf{laplace}(f(t),\ t \to s)$	Laplace transform of $f(t)$ under $t \to s$
$\mathrm{Li}_n(x)$	Jonquière's polylogarithm function
$\mathsf{match}(y,\ x)$	match the instance y with the pattern x
$\mathsf{maximize/minimize}(\ldots\{, I\})$	maximize/minimize (over the interval I)
$\mathsf{N}(\ldots\{, k\})$	numerically evaluate (to k-digit precision)
(operator)	define as an operator
$P_n(x)$	n^{th} Legendre polynomial
$P_\nu^\mu(z)$	associated Legendre (spherical) function of the 1^{st} kind
$\psi(x)$	psi function $[= \frac{d}{dx} \log \Gamma(x)]$
$\mathsf{pade}(f(x),\ x = a)$	Pade rational approximation of $f(x)$ about $x = a$
$\mathsf{power\_series}(f(x),\ x = a)$	general power series formula of $f(x)$ about $x = a$
$\mathsf{rectform}(f(z))$	rectangular form of the complex function $f(z)$
(rewrite rules)	using user-supplied rewrite rules
$\mathsf{solve}(\ldots,\ \mathbf{R})$	produce an explicitly real solution
$\mathsf{solve}(\ldots,\ \mathbf{Z})$	produce only integer solutions
$\mathsf{solve}(f(x, y) = 0,\ \frac{dy}{dx})$	find the derivative using implicit differentiation
$\mathsf{solve}(x = f(y),\ y = a,\ \text{series})$	solve for $y(x)$ about $y = a$ using series reversion
$\mathsf{stdev}(\ldots)$	unbiased sample standard deviation
$\mathsf{SVD}(A)$	singular value decomposition of the matrix A
$T_n(x)$	Chebyshev polynomial of the 1^{st} kind
$\mathsf{taylor}(f(x),\ x = a)$	truncated Taylor/Laurent/Puiseux series of $f(x)$ about $x = a$
$W_n(x)$	n^{th} branch of Lambert's W function
$\zeta(n)$	Riemann zeta function

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Footnotes to the Summary Table of Problems

[a] ability was **a**dded after the current manual came out
[d] need to supply the **d**efinition of the operation
[g] **g**eneral (usually complex) case
[m] does not indicate some or all solutions may have a **m**ultiplicity > 1
[n] found a **n**umerical solution
[p] defined via a procedure | via a special **p**iecewise function
[r] only provides solutions within a **r**estricted interval
[s] one or more **s**purious solutions were produced

---

[1] Maple: not using | using bsimp
[2] $x < y$ is apparently some sort of lexographic order relation
[3] Derive: translation involved subscripted variables a_i | function calls $a(i)$
Macsyma/Reduce: fortran/on fort | gentran
[4] incorrectly computes the x^2 coefficient for $([-4,2]x + [1,3])\,([-4,2]x + [1,3])$
[5] the values for the two quantities are both reasonable, but inconsistent with each other
[6] STUDENT is incorrectly identified as a cumulative probability distribution (it is really a central area distribution)
[7] $b_0 + b_1 2^{-d/\theta}$ produces an error; however, $b_0 + b_1 2.0^{-d/\theta}$ works just fine
[8] e^x | $\exp(x)$
[9] FACTOR(_, Complex) determines the field extension automatically!
[10] simplify produced an incorrect answer
[11] Derive: BESSEL_J | JN or CHEBYCHEV_T | TN or GAUSS | F21
[12] $E(\phi, k) = \int_0^A \frac{\sqrt{1-Bt^2}}{\sqrt{1-t^2}}\,dt$. Here, I use (like Gradshteyn and Ryzhik) $A = \sin\phi, B = k^2$ while Derive, Macsyma, Mathematica, Reduce use (like Abramowitz and Stegun) $A = \sin\phi, B = k$ and Maple uses $A = \phi, B = k^2$
[13] it is necessary to simplify the expression found at the end of a long series of if statements to obtain the correct answer
[14] ComplexExpand (which assumes all variables are real) produces a very complicated expression
[15] this usage is only documented in the program help
[16] completely general answer!
[17] online help warns that radcan performs transformations that are not always valid for all values of the variables involved
[18] unsuccessful using E^x, but successful using exp(x)
[19] same output when no solutions are possible as when no solutions were discovered
[20] Maple and Mathematica: x is by default a complex variable
[21] Maple: used an undocumented function to compute matrix square roots
[22] ratmx:false and sparse:true. All other settings cause the calculation to be very slow or die a horrible death
[23] MuPAD: eigenValues(float(*rosser*), All) | numeric::eigenvalues(float(*rosser*))
[24] produced one solution
[25] found the exact singular values and numerical U and V such that $A = U\Sigma V^T$
[26] demonstrated for dimension = 2
[27] demonstrated componentwise rather than indicially

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28 Macsyma and Reduce: use the convention that $\sum_k = \lim_{K \to k} \sum_{k=0}^{K}$ rather than $\sum_k = \lim_{K \to k} \sum_{k=0}^{K-1}$

29 Reduce: sum | gosper

30 performed an invalid transformation

31 Mathematica: normal behavior | after loading `Calculus‘Limit‘`

32 Derive: GAUSS | F21

33 cannot do one variation of this problem

34 Mathematica: Abs[x] | Sqrt[x^2]

35 produces solutions for both $a < b$ and $a > b$

36 Axiom: without | with `"noPole"` option—only examples in which the answer improved are shown

37 Macsyma: intanalysis:true | intanalysis:false

38 on some runs, the integral was left unevaluated and on others it produced zero

39 an unexpected error occurs if too few terms are asked for

40 Reduce: laplace | laplace_transform. fourier results are for the cos and sin transform where applicable

41 fourier(f, $x \to z$) $= A \int_{-\infty}^{\infty} f e^{izx}\, dx$. Here, I use (like Gradshteyn and Ryzhik) $A = \frac{1}{\sqrt{2\pi}}$ while $A = \frac{1}{\pi}$ (Macsyma), 1 (Maple, Mathematica, MuPAD), $\frac{1}{2}$ (Reduce)

42 Derive: need to extract coefficients by hand to fit into the pattern specified in the manual | ODE (except discontinuous ODE)
Mathematica (2×2 system and verification): using regular functions | pure functions

43 more difficult in Macsyma to obtain the answer produced by the other systems, but it is the only system able to show that its result is equivalent to f_{n+1}

44 requires some user sophistication to solve

45 for systems that claim that this is a PDE(!), solve as an ODE | as a PDE

46 no system produces any of the nontrivial solutions $y(x) = A \sin([\frac{\pi}{2} + n\pi]x)$, $n \in \mathbf{Z}$

47 Reduce: using a regular operator | a generic_function

48 Several queries about temporary variables unseen by the user need to be answered

49 Derive: DSOLVE2 | ODE (in a fresh invocation)

50 rule cannot be generalized successfully to admit other variables or functions

51 need to be careful about the statement order

52 direct definition | recursive definition

53 simplify($p(\begin{pmatrix} 1 & -2 \\ 2 & 3 \end{pmatrix})$) produces $b^2 - 4b + 7!$, while evalm($p(\begin{pmatrix} 1 & -2 \\ 2 & 3 \end{pmatrix})$) is correct

54 Reduce: packages RLFI | TRI

55 Axiom: match(__, $(-1)^{1/2}x$) produces a syntax error! | match(__, $x(-1)^{1/2}$)

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#	PROBLEM	**Ax**	**De**	**Mc**	**Mp**	**Mm**	**Mu**	**Re**
	A. Boolean Logic and Quantifier Elimination							
A1	true and false $\Rightarrow$ false	•	•	•	•	•	•	•
A2	x or $\neg x \Rightarrow$ true		•	•	•	•	•	•
A3	x or y or $(x$ and $y) \Rightarrow x$ or y		•	•	•	•	•	•
A4	x xor y xor $y \Rightarrow x$		•		•	•	—	—
A5	$(w$ and $x)$ implies $(y$ and $z)$		•	—	•	•	—	•
A6	x iff $y \Rightarrow (x$ and $y)$ or $\neg(x$ or $y)$	—	—	—	•	—	—	—
A7	x and $1 > 2 \Rightarrow$ false		•	ε	•	•	•	∘
A8	$\forall \lambda \in C\,\{a\lambda^2 + b\lambda + c = 0$ implies $\ldots\}$	—	—	—	—	—	—	—
A9	$\exists w \in \mathbf{R} \ni \{v > 0$ and $w > 0 \ldots\} \Rightarrow v > 1$	—	—	—	—	—	—	—
A10	$\forall c \in \mathbf{R}\,\{-1 \leq c \leq 1$ implies $\ldots \leq 4\}$	—	—	—	—	—	—	—
A11	$\forall y \in \mathbf{C}\,\{v > 0 \ldots$implies $\mathrm{Re}\, y < 0\}$	—	—	—	—	—	—	—
A12	$4wv > 0$ and $4w(\ldots) > 0 \ldots \Rightarrow w > \lvert W\rvert$ [1]	—[2]		ε	| $\mathcal{E}$			
A13	$\exists y, n \in \mathbf{C}, \exists e \in \mathbf{R} \ni \{\mathrm{Re}\, y > 0$ and $\ldots\}$	—	—	—	—	—	—	—
	B. Set Theory							
B1	$\{a,b,b,c,c,c\} \bigcup \{d,c,b\} \bigcup \{b,e,b\}$	⋆	•	•	•	•	•	•
B2	$\{a,b,b,c,c,c\} \bigcap \{d,c,b\} \bigcap \{b,e,b\} \Rightarrow \{b\}$	⋆	•	•	•	•	•	•
B3	$\{a,b,c,d,e\} - \{b\} \Rightarrow \{a,c,d,e\}$	•	•	•	•	⋆	•	•
B4	$\{a,b\} \times \{c,d\} \Rightarrow \{ac,ad,bc,bd\}$	$\circ^{\mathbf{d}}$	$\circ^{\mathbf{d}}$	⋆	$\circ^{\mathbf{d}}$	⋆	•	$\circ^{\mathbf{d}}$
	C. Numbers							
C1	$50! \Rightarrow 3.0414093201\ldots0000000000 \cdot 10^{64}$	•	•	•	•	•	•	•
C2	factor$(50!) \Rightarrow 2^{47}3^{22}5^{12}7^{8}11^{4}13^{3}\cdots 47$	•	•	•	•	•	•	⋆
C3	$10!! \Rightarrow 3840; \quad 9!! \Rightarrow 945$	—	—	•	—	•	—	—
C4	$\mathrm{ABC}_{16} \Rightarrow 2748_{10}$	—	•		$\natural$	•	—	—
C5	$123_{10} \Rightarrow 234_{7}$	•	•	•	∘	•	—	—
C6	$677_8 \Rightarrow 1\mathrm{BF}_{16} \quad (= 447_{10})$	#	•	•	$\natural$	•	—	—
C7	$\log_8 32768 \Rightarrow 5$		•	•	•	•		
C8	$5^{-1} \bmod 7 \Rightarrow 3; \quad 5^{-1} \bmod 6 \Rightarrow 5$	• | ⋆	•	⋆	•	•	•	• | ε
C9	$\gcd(1776, 1554, 5698) \Rightarrow 74$	⋆	•	•	•	•	•	⋆
C10	$\frac{1}{2} + \cdots + \frac{1}{10} \Rightarrow \frac{4861}{2520}$	•	•	•	•	•	•	•

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#	PROBLEM	**Ax**	**De**	**Mc**	**Mp**	**Mm**	**Mu**	**Re**
C11	$\mathsf{N}(\frac{1}{7}) \Rightarrow 0.\overline{142857}$	$\bullet$	—	—	—	$\bullet$	$\bullet$	$\bullet$[a]
C12	$\mathsf{N}(\frac{7}{11})\mathsf{N}(\frac{22}{7}) = 6.\overline{36} \cdot 3.\overline{142857} \Rightarrow 2$	$\bullet$	—	—	—		—	[a]
C13	$\frac{10}{7}\sqrt[3]{1+\frac{29}{1000}} \Rightarrow \sqrt[3]{3}$	$\bullet$	$\bullet$	$\bullet$	$\star$	$\bullet$	$\star$	$\bullet$
C14	$\sqrt{2\sqrt{3}+4} \Rightarrow 1+\sqrt{3}$		$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
C15	$\sqrt{14+3\sqrt{3+\cdots}} \Rightarrow 3+\sqrt{2}$		$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
C16	$\sqrt{10+2\sqrt{6}+2\sqrt{10}+2\sqrt{15}} \Rightarrow \sqrt{2}+\cdots$		$\bullet$					
C17	$\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \Rightarrow 5+2\sqrt{6}$	$\bullet$	$\bullet$	$\star$	$\circ$	$\bullet$	$\bullet$	
C18	$\sqrt{-2+\sqrt{-5}}\,\sqrt{-2-\sqrt{-5}} \Rightarrow 3$		$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	
C19	$\sqrt[3]{90+34\sqrt{7}} \Rightarrow 3+\sqrt{7}$		$\bullet$		$\bullet$	$\bullet$	$\bullet$	
C20	$\frac{[(135+78\sqrt{3})^{2/3}+3]\sqrt{3}}{(135+78\sqrt{3})^{1/3}} \Rightarrow 12$	#	$\bullet$	#	$\bullet$	$\bullet$	$\bullet$	
C21	$\sqrt[5]{41+29\sqrt{2}} \Rightarrow 1+\sqrt{2}$							
C22	$\frac{(6-4\sqrt{2})\log(3-2\sqrt{2})+(3-2\sqrt{2})\log(17-12\sqrt{2})+\cdots}{48\sqrt{2}-72}$		$\bullet$	#	#	#		
C23	$2\infty - 3 \Rightarrow \infty$	$\bullet$	$\bullet$	$\star$	$\bullet$	$\bullet$	$\bullet$	—
C24	$2^{\aleph_0} \Rightarrow \aleph_1$	$\mathcal{E}$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	—
	D. Numerical Analysis[3]							
D1	$\frac{0.0}{\sqrt{2}} \Rightarrow 0.0$ (immediate)	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$		$\bullet$
D2	$\mathsf{N}(e^{-1000000}) \approx 3.29683 \cdot 10^{-434295}$	$\bullet$	τ	$\star$	$\bullet$	$\bullet$	$\star$	$\star$
D3	$\mathsf{N}(e^{\pi\sqrt{163}}, 50) \approx 262537412640768744.0$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
D4	$\lfloor -\frac{5}{3} \rfloor \Rightarrow -2; \quad \lceil -\frac{5}{3} \rceil \Rightarrow -1$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
D5	compute cubic spline s; $s(3) \Rightarrow \frac{27}{8}$	—	$\circ$	$\circ$	$\bullet$	$\circ$	—	—
D6	translate $p = \sum_{i=1}^{n} a_i x^i$ to Fortran	$\mathcal{E}$	$\times$ | $\natural$	$\times$ | $\uplus$	ε	$\uplus$	$\uplus$	$\uplus$ | $\uplus$
D7	translate $p = \sum_{i=1}^{n} a_i x^i$ to C	—	$\uplus$ | $\natural$	$\uplus$	$\uplus$	$\times$	$\uplus$	$\uplus$
D8	Horner's rule applied to $p = \sum_{i=1}^{5} a_i x^i$	—	—	$\bullet$	$\bullet$	$\bullet$	$\circ$	—
D9	translate above to Fortran	$\star$	$\times$ | $\bullet$	$\bullet$ | $\bullet$	$\star$	$\circ$	$\bullet$	$\bullet$ | $\star$
D10	translate above to C	—	$\otimes$ | $\bullet$	$\bullet$	$\star$	$\circ$	$\bullet$	$\star$
D11	$\mathsf{flops}(\sum_{k=1}^{n} \prod_{i=1}^{k} f_{ik})$	—	—	$\bullet$	$\times$	—	—	—
D12	$([-4,2]x+[1,3])^2$ (interval analysis)	—	—	—	—	$\bullet$[4]	$\bullet$	—
D13	$\frac{df}{dt} = \frac{d^2f}{dx^2} \Rightarrow \frac{f_{i,j+1}-f_{ij}}{\Delta t} = \frac{f_{i+1,j}-2f_{ij}+f_{i-1,j}}{(\Delta x)^2}$	—	—	$\bullet$	—	—	—	—

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#	PROBLEM	**Ax**	**De**	**Mc**	**Mp**	**Mm**	**Mu**	**Re**
	E. Statistics							
E1	mean([3, 7, 11, 5, 19]) $\Rightarrow 9$	$\circ^{\mathbf{d}}$	•	•	•	•	•	$\circ^{\mathbf{d}}$
E2	median([3, 7, 11, 5, 19]) $\Rightarrow 7$	—	—	•	•	•	•	—
E3	quartile(1, [1, ..., 8]) = quantile($\frac{1}{4}$, ") $\Rightarrow 2$	—	—	—	•	$\star^{\mathbf{5}}$	—	—
E4	mode([3, 7, 11, 7, 3, 5, 7]) $\Rightarrow 7$	—	—	—	•	•	—	#
E5	stdev([1, 2, 3, 4, 5]) $\Rightarrow \sqrt{\frac{5}{2}}$	—	•	•	•	•	•	—
E6	PDF(discrete binomial distribution)	—	•	•	•	•	—	—
E7	CDF(discrete binomial distribution)	—	•	•	•	•	—	—
E8	CDF(continuous normal distribution)	—	•	•	•	•	—	—
E9	hypothesis testing: t distribution	—	$\natural^{\mathbf{6}}$	◦	◦	•	•	—
E10	hypothesis testing: normal distribution	—	◦	◦	◦	⋆	•	—
E11	compute χ^2 statistic by hand $\Rightarrow \frac{1153}{252}$	•	•	•	•	•	•	•
E12	χ^2 test $\Rightarrow 0.46986$	—	⋆	—	⋆	•	•	—
E13	linear regression $\Rightarrow y' \approx 0.7365x + 6.964$	—	•	•	•	•	—	—
E14	multiple linear regression (2 variables)	—	•	⋆	•	•	—	—
E15	multiple linear regression using L_1 norm	—	—	—	—	—	—	—
E16	nonlinear regression: $w = b_0 + b_1 2^{-d/\theta}$	—	—	$\star^{\mathbf{7}}$	—	•	—	—
	F. Combinatorial Theory							
F1	$(a)_3 \Rightarrow a(a+1)(a+2)$ (Pochhammer)	—	•	•	•	•	—	•
F2	$\binom{n}{3} \Rightarrow \frac{n(n-1)(n-2)}{6}$	⋆	•	•	⋆	•		⋆
F3	$2^n n!\,(2n-1)!! \Rightarrow (2n)!$ or $\Gamma(2n+1)$	—	—		—		—	—
F4	$2^n n! \prod_{k=1}^{n} 2k-1 \Rightarrow (2n)!$ or $\Gamma(2n+1)$		#	#	•	•	#	#
F5	$\frac{\Gamma(n+1/2)}{\sqrt{\pi}\,n!} \Rightarrow \frac{(2n)!}{2^{2n}\,n!^2}$ or $\frac{(2n-1)!!}{2^n\,n!}$							
F6	partitions of 4 $\Rightarrow \{4, 2+2, 1+3, \ldots\}$	—	—	—	•	—	—	—
F7	number of partitions of 4 $\Rightarrow 5$	—	—	•	•	•	—	—
F8	$S_1(5,2) \Rightarrow -50$ (Stirling numbers)	•	—	—	•	•	—	•
F9	$\phi(1776) \Rightarrow 576$ (Euler's totient fun.)	•	—	•	•	•	•	—
	G. Number Theory							
G1	discover primes 999983 and 1000003	•	⋆	◦	•	⋆	•	—
G2	primitive root of 191 $\Rightarrow 19$	—	—	—	•	•	•	—

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#	PROBLEM	**Ax**	**De**	**Mc**	**Mp**	**Mm**	**Mu**	**Re**
G3	$[(a+b)^p \Rightarrow a^p + b^p] \bmod p$ (p prime)	—	—	ε				—
G4	solve$(9x \equiv 15 \bmod 21) \Rightarrow x \equiv 4 \bmod 7$	ε	$\bullet$	ε	$\bullet$	$\bullet$		$\bullet$
G5	solve$(7x \equiv 22 \bmod 39) \Rightarrow x \equiv 31 \bmod 39$	ε	$\bullet$	$\bullet$	$\bullet$	$\bullet$		$\bullet$
G6	solve$(x^2 + x + 4 \equiv 0 \bmod 8)$	ε	—	ε	$\bullet$	$\bullet$		$\bullet$
G7	solve$(x^3 + 2x^2 + 5x + 6 \equiv 0 \bmod 11)$	$\bullet^{\mathbf{m}}$	—	$\bullet$	$\bullet$	$\bullet$		$\bullet^{\mathbf{m}}$
G8	solve$(x \equiv 7 \bmod 9 \equiv 13 \bmod 23 \equiv \cdots)$	$\star$	—	—	$\star$	—	$\star$	—
G9	solve$([5x + 4y \equiv 6,\, 3x - 2y \equiv 6] \bmod 7)$	$\bullet$	—	$\bullet$	ε	$\bullet$		$\bullet$
G10	solve$(2x + 3y \equiv 1 \bmod 5,\, [x,\, y])$	ε	—	$\circ$	ε	$\circ$		$\bullet$
G11	solve$(2x + 3y \equiv 1 \bmod 6,\, [x,\, y])$	ε	—	ε	ε	$\otimes$		$\bullet$
G12	solve$(x^4 + 9 = y^2,\, \mathbf{Z}) \Rightarrow x,\, y = 2, 5$	ε		—		—	—	
G13	solve$(x^2 + 4 = y^3,\, \mathbf{Z}) \Rightarrow x,\, y = 11, 5$	ε		—		—	—	
G14	solve$([x^2 + y^2 = t^2,\, t^2 + z^2 = w^2],\, \mathbf{Z})$	$\mathcal{E}$		—		—	—	—
G15	rational approximation of $\sqrt{3} \Rightarrow \frac{26}{15}$ (e.g.)	$\bullet$	$\star$	$\bullet$	$\star$	$\bullet$	$\star$	$\circ$
G16	cf$(3.1415926535) \Rightarrow <3, 7, 15, 1, 292, \ldots>$	$\bullet$	—	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
G17	cf$(\sqrt{23}) \Rightarrow <4, \overline{1, 3, 1, 8}>$	$\star$	—	$\bullet$	$\bullet$	$\star$	$\star$	$\star$
G18	cf$(\frac{1+\sqrt{5}}{2}) \Rightarrow <\overline{1}>$	$\star$	—	$\boxed{\star}$	$\bullet$	$\star$	$\star$	$\star$
G19	cf$(\frac{e^{1/x}-1}{e^{1/x}+1}) \Rightarrow <0, 2x, 6x, 10x, 14x, \ldots>$	—	—	ε	$\mathcal{E}$		$\mathcal{E}$	$\varepsilon^{\mathbf{a}}$
G20	cf$(\sqrt{x^2+1} - x) \Rightarrow <\overline{2x}>$	—	—	ε	$\star$		$\star$	$\star^{\mathbf{a}}$
	H. Algebra							
H1	$2 \cdot 2^n \Rightarrow 2^{n+1}$		$\bullet$	$\natural$	$\bullet$	$\bullet$	$\star$	
H2	$4 \cdot 2^n \Rightarrow 2^{n+2}$		$\bullet$	$\circ$	$\natural$	$\bullet$		
H3	$(-1)^{n(n+1)} \Rightarrow 1$ ($n \in \mathbf{Z}$)	—		$\bullet$	$\bullet$	—		—
H4	factor$(6x - 10) \Rightarrow 2(3x - 5)$	$\bullet$	$\bullet$	$\bullet$	#	$\bullet$	#	$\bullet$
H5	univariate gcd $\Rightarrow 1$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
H6	univariate gcd $\not\Rightarrow 1$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
H7	multivariate gcd $\Rightarrow 1$ (3 variables)	$\bullet$	τ	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\mathcal{E}$
H8	multivariate gcd $\not\Rightarrow 1$ (3 variables)	$\bullet$	τ	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\mathcal{E}$
H9	gcd$(2x^{n+4} - x^{n+2},\, 4x^{n+1} + 3x^n) \Rightarrow x^n$	$\times$	$\bullet$	$\bullet$	$\times$	$\bullet$	$\times$	$\bullet$
H10	resultant$(3x^4 + \cdots - 2,\, x^3 - \cdots + 5) \Rightarrow 0$	$\bullet$	—	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$

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#	PROBLEM	**Ax**	**De**	**Mc**	**Mp**	**Mm**	**Mu**	**Re**
H11	resultant(expand($p_1 q$), expand($p_2 q$)) $\Rightarrow 0$	•	—	•	•	•	•	$\mathcal{E}$
H12	$\frac{x^2-4}{x^2+4x+4} \Rightarrow \frac{x-2}{x+2}$	•	•	•	•	•	•	•
H13	$\frac{e^x-1}{e^{x/2}+1} \Rightarrow e^{x/2}-1$ [8]	•	•	•	•	•	•	
H14	expand($(x+1)^{20}$) $\to$ diff $\to$ factor	•	•	•	•	•	•	•
H15	factor(x^3+x^2-7) $\to$ expand	•	#	•	•	•	#	#
H16	factor($x^{100}-1$)	•	•	•	•	•	•	•
H17	factor(expand($(64x^{34}-\cdots)(72x^{60}-\cdots)$))			•	•	•	•	•
H18	factor($4x^4+8x^3+77x^2+18x+53$)	⋆	•	•	•	•	⋆	•
H19	let($a^2=2$); $\quad \frac{1}{a-1} \Rightarrow a+1$	•	∘	⋆	•	⋆		•
H20	let("); $\quad \frac{x^3+(a-2)x^2-(2a+3)x-3a}{x^2-2} \Rightarrow \frac{x^2-2x-3}{x-a}$	•	∘	∘	•	⋆	#	⋆
H21	let($b^3=2,\ c^2=3$); $\quad (b+c)^4$	•	—	⋆	•	⋆		—
H22	factor($x^4-3x^2+1 \bmod 5$)	•	—	•	•	•	⋆	•
H23	factor($x^{11}+x+1 \bmod 65537$)	•	—	•	•	ε	⋆	•
H24	factor(x^4-3x^2+1, RootOf($\phi^2-\phi-1$))	•	•[9]	•	•	⋆	⋆	—
H25	expand($(x-2y^2+3z^3)^{20}$) $\to$ factor	•	τ	•	•	•	•	•
H26	expand($(\sin x - 2\cos^2 y + \cdots)^{20}$) $\to$ factor		⋆	•	•	•	⋆	•
H27	factor(expand($(24xy^{19}z^8 - \cdots + 5)(\ldots)$))	•	#	•	•	•	•	•
H28	expand($\cdots(c^2+s^2)^{10}$) with $c^2+s^2=1$	∘	—	∘	•	∘	∘	∘
H29	factor($4x^2-21xy+20y^2 \bmod 3$)	ε	—	•	•	#	⋆	•
H30	factor($x^3+y^3,\ i\sqrt{3}$) $\Rightarrow \frac{x+y}{4}\cdots$	⋆	•	•	•	•	⋆	#
H31	$\frac{x^2+2x+3}{x^3+4x^2+5x+2} \Rightarrow \frac{3}{x+2} - \frac{2}{x+1} + \frac{2}{(x+1)^2}$	⋆	•	•	•	•	•	⋆
H32	$[ABC-(ABC)^{-1}]ACB$ (noncommuting)	—	⋆	•			—	×
H33	$[A,B,C]+[B,C,A]+[C,A,B]=0$	—	•	•	•	$\natural$	—	•
	I. Trigonometry							
I1	$\tan\frac{7\pi}{10} \Rightarrow -\frac{\sqrt{5}+1}{\sqrt{5}-1}\frac{\sqrt{2}}{(\sqrt{5}+5)^{1/2}} = -\sqrt{1+\frac{2}{\sqrt{5}}}$		•	⋆	•	•	#	
I2	$\sqrt{\frac{1+\cos 6}{2}} \Rightarrow -\cos 3$	#	•			•		
I3	$\cos n\pi + \sin\frac{4n-1}{2}\pi \Rightarrow (-1)^n - 1 \quad (n \in \mathbf{Z})$	—	•	•	•	#	#	—
I4	$\cos(\pi\cos n\pi) + \sin(\frac{\pi}{2}\cos n\pi) \quad (n \in \mathbf{Z})$	—	•	•	∘	—		—
I5	$\sin([\frac{n^5}{5}+\frac{n^4}{2}+\frac{n^3}{3}-\frac{n}{30}]\pi) \Rightarrow 0 \quad (n \in \mathbf{Z})$	—			•	—		—

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#	PROBLEM	**Ax**	**De**	**Mc**	**Mp**	**Mm**	**Mu**	**Re**
I6	$\lvert\cos x\rvert, \lvert\sin x\rvert \quad (-3\pi < x < -\frac{5\pi}{2})$	—	○		•	—		—
I7	$\frac{\cos 3x}{\cos x} \Rightarrow \cos^2 x - 3\sin^2 x$ (or similar)	○	•	•	•	•	•	•
I8	$\frac{\cos 3x}{\cos x} \Rightarrow 2\cos 2x - 1$	$\natural$	•	○	$\star$	$\star$	$\star$	$\star$
I9	$\frac{\cos 3x}{\cos x} \Rightarrow \cos^2 x - 3\sin^2 x$ (rewrite rules)	•	—	•	•	•	$\natural$	•
I10	$\mathsf{simplify}(\frac{\tan^2 x+1-\sec^2 x}{\sin^2 x+\cos^2 x-1}) \Rightarrow \frac{0}{0}$, error or ?	$\otimes$[10]	•	×	×	•	•	•
I11	$\lim_{x\to 0} \frac{\tan^2 x+1-\sec^2 x}{\sin^2 x+\cos^2 x-1} \Rightarrow$ error or ?	×	•	•	•	•	•	•
I12	$\frac{d}{dx}\left(\frac{\tan^2 x+1-\sec^2 x}{\sin^2 x+\cos^2 x-1}\right) \Rightarrow$ error or ?	$\otimes$[10]	•	×	×	×	$\boxed{\star}$	•
	J. Special Functions[11]							
J1	$B_{16} \Rightarrow -\frac{3617}{510}$ (Bernoulli number)	•	•	•	•	•	•	•
J2	$\frac{dE(\phi,k)}{dk} \Rightarrow \frac{E(\phi,k)-F(\phi,k)}{k}$ [12]	—	○	•	•	•	—	×
J3	$\frac{d}{du}\,\mathrm{dn}\,u \Rightarrow -k^2\,\mathrm{sn}\,u\,\mathrm{cn}\,u$	—	—	•	•		—	
J4	$\Gamma(-\frac{1}{2}) \Rightarrow -2\sqrt{\pi}$	○[n]	•	•	•	•	•	•
J5	$\psi(\frac{1}{3}) \Rightarrow -\gamma - \frac{\pi}{2}\sqrt{\frac{1}{3}} - \frac{3}{2}\log 3$	○[n]	#	•	•			
J6	$\mathsf{N}(J_2(1+i)) \approx 0.04158 + 0.24740i$	•	•	•	•	•	•	•
J7	$J_{-5/2}(\frac{\pi}{2}) \Rightarrow \frac{12}{\pi^2}$	○[n]	× | •	•	•	•		
J8	$J_{3/2}(z) \Rightarrow \sqrt{\frac{2}{\pi z}}(\frac{\sin z}{z} - \cos z)$		○[13] | •	•	•	•		
J9	$\frac{d}{dz}J_0(z) \Rightarrow -J_1(z)$	$\star$	$\mathcal{E}$ | #	•	•	•	•	•
J10	$P_\nu^\mu(0) \Rightarrow \frac{2^\mu\sqrt{\pi}}{\Gamma(\frac{\nu-\mu}{2}+1)\Gamma(\frac{-\nu-\mu+1}{2})}$	—		•	$\boxed{\star}$		—	•
J11	$P_3^1(x) \Rightarrow -\frac{3}{2}\sqrt{1-x^2}(5x^2-1)$	—	•	•		•	—	•
J12	$T_{1008}(x) - 2xT_{1007}(x) + T_{1006}(x) \Rightarrow 0$	•	$\mathcal{E}$ | τ	•	•	•	$\natural$	•
J13	$T_n(-1) \Rightarrow (-1)^n \quad (n \in \mathbf{Z} \geq 0)$	ε		•		○	$\mathcal{E}$	•
J14	$F(\frac{1}{2},\frac{1}{2};\frac{3}{2};z^2) \Rightarrow \frac{\sin^{-1} z}{z}$	—	$\uplus$ | •	•	$\star$	$\star$	—	$\star$
J15	$F(\frac{n+2}{2}, -\frac{n-2}{2}; \frac{3}{2}; \sin^2 z) \Rightarrow \frac{\sin nz}{n\sin z\cos z}$	—	| $\star$	○	$\star$	$\star$	—	
J16	$\zeta'(0) \Rightarrow -\frac{1}{2}\log 2\pi$	—		•	•	•		
J17	$\int_0^3 f(\frac{x+2}{5})\delta(\frac{x-2}{3}) - g(x)\delta'(x-1)\,dx$	—	—	○	$\star$	•		—
J18	define an antisymmetric function f	—	—	•	$\star$	○	—	•
	K. The Complex Domain							
K1	$[\mathrm{Re}(x+iy), \mathrm{Im}(x+iy)] \quad (x, y \in \mathbf{C})$[g]	×	$\natural$	•		•	•	•
K2	$\lvert 3-\sqrt{7}+i(6\sqrt{7}-15)^{1/2}\rvert \Rightarrow 1$	•	•	•	•	•	•	

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K3	$\left\lvert\frac{1}{a+i/a+ib}\right\rvert \Rightarrow \frac{1}{\sqrt{a^2+(1/a+b)^2}} \quad (a, b \in \mathbf{R})$	$\bullet$	$\star$	$\bullet$	$\bullet$		$\bullet$	—
K4	$\mathsf{rectform}(\log(3+4i)) \Rightarrow \log 5 + i\tan^{-1}\frac{4}{3}$	$\star$	$\star$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	
K5	$\mathsf{rectform}(\tan(x+iy))$	$\circ$	$\star$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	
K6	$\frac{\sqrt{xy\lvert z\rvert^2}}{\sqrt{x}\lvert z\rvert} \Rightarrow \frac{\sqrt{xy}}{\sqrt{x}} \not\Rightarrow \sqrt{y}\ ^{\mathbf{g}}$		$\bullet$	$\bullet$	$\bullet$	$\star^{\mathbf{14}}$	$\bullet$	$\bullet$
K7	$\frac{\sqrt{xy\lvert z\rvert^2}}{\sqrt{x}\lvert z\rvert} \Rightarrow \sqrt{y} \quad (y \geq 0)$	—	$\bullet$	$\bullet$		$\bullet$	$\bullet$	—
K8	$\sqrt{\frac{1}{z}} - \frac{1}{\sqrt{z}} \Rightarrow 0 \quad (z \text{ is not real negative})^{\mathbf{g}}$	$\times$	$\bullet^{\mathbf{15}}$	$\bullet$	$\bullet$	$\bullet$	$\boxed{\star}$	$\times$
K9	$\sqrt{\frac{1}{z}} - \frac{1}{\sqrt{z}} \Rightarrow 0 \quad (z > 0)$	—	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	—
K10	$\sqrt{\frac{1}{z}} + \frac{1}{\sqrt{z}} \Rightarrow 0 \quad (z < 0)$	—	$\bullet$	$\bullet$	$\bullet$	—		—
K11	$\sqrt{e^z} - e^{z/2} \Rightarrow 0 \quad (-\pi < \operatorname{Im} z \leq \pi, \text{ etc.})^{\mathbf{g}}$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\times$	$\times$
K12	$\sqrt{e^z} - e^{z/2} \Rightarrow 0 \quad (z \in \mathbf{R})$		$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
K13	$\sqrt{e^{6i}} \Rightarrow -e^{3i} \quad \text{(principal value)}$	$\circ$	$\bullet$	$\bullet$	$\bullet$	$\star$	$\bullet$	$\times$
K14	$\log e^z \Rightarrow z \quad (-\pi < \operatorname{Im} z \leq \pi)^{\mathbf{g}}$	$\times$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\otimes$	$\times$
K15	$\log e^z \Rightarrow z \quad (z \in \mathbf{R})$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
K16	$\log e^{10i} \Rightarrow (10 - 4\pi)i \quad \text{(principal value)}$	$\times$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\circ$	
K17	$(xy)^{1/n} - x^{1/n}y^{1/n} \Rightarrow 0 \quad (\operatorname{Re} x, y > 0)^{\mathbf{g}}$	$\uplus$	$\bullet$	$\bullet$	$\bullet$	$\star^{\mathbf{14}}$	$\bullet$	$\bullet$
K18	$(xy)^{1/n} - x^{1/n}y^{1/n} \Rightarrow 0 \quad (y > 0)$	—	$\bullet$	$\bullet$	$\bullet$	$\star^{\mathbf{16}}$	$\otimes$	—
K19	$(xy)^n - x^n y^n \Rightarrow 0 \quad (n \in \mathbf{Z})$	—		$\bullet$	$\bullet$	—		—
K20	$\tan^{-1}(\tan z) \Rightarrow z \quad (-\frac{\pi}{2} < z \leq \frac{\pi}{2})^{\mathbf{g}}$	$\otimes$	$\bullet^{\mathbf{15}}$	$\bullet$	$\bullet$	$\star^{\mathbf{14}}$	$\bullet$	$\bullet$
K21	$\tan^{-1}(\tan z) \Rightarrow z \quad (-\frac{\pi}{2} < z < \frac{\pi}{2})$	—		$\bullet$		—		—
K22	$\tan^{-1}(\tan 10) \Rightarrow 10 - 3\pi \quad \text{(princ. value)}$		$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	#
K23	$\tan^{-1}[\tan(11 + 30i)] \Rightarrow 11 - 4\pi + 30i$		$\natural$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	
K24	$\tan^{-1}[\tan(11.0 + 30.0i)] \approx -1.56637 + 30i$	$\mathcal{E}$		$\star$	$\times$	$\times$	$\mathcal{E}$	
K25	$w = \frac{z + \frac{1}{z}}{2}; \quad w + \sqrt{w+1}\sqrt{w-1} \Rightarrow z \text{ or } \frac{1}{z}$			$\circ$	$\circ$			
	L. Determining Zero Equivalence							
L1	$\sqrt{997} - (997^3)^{1/6} \Rightarrow 0$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\star$
L2	$\sqrt{999983} - (999983^3)^{1/6} \Rightarrow 0$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\star$
L3	$(2^{1/3} + 4^{1/3})^3 - 6(2^{1/3} + 4^{1/3}) - 6 \Rightarrow 0$	$\star$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
L4	$\cos^3 x + \cos x \sin^2 x - \cos x \Rightarrow 0$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\star$	#
L5	$\log(\tan(\frac{1}{2}x + \frac{\pi}{4})) - \sinh^{-1}(\tan x) \Rightarrow 0$		#	$\natural^{\mathbf{17}}$	$\natural$			

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L6	derivative of above is 0 & above at 0 is 0	#	#	$\star^{\mathbf{17}}$	$\star$	$\circ$	#	$\bullet$
L7	$\log \frac{2\sqrt{r}+1}{\sqrt{4r+4\sqrt{r}+1}} \Rightarrow 0$		$\circ$	$\star$	$\bullet$	$\circ$	$\bullet$	$\bullet$
L8	$(4r+4\sqrt{r}+1)^{\frac{\sqrt{r}}{2\sqrt{r}+1}}(2\sqrt{r}+1)^{\frac{1}{2\sqrt{r}+1}}\cdots$	$\bullet$	$\circ$			$\circ$	$\bullet$	
L9	$2^{1-z}\Gamma(z)\zeta(z)\cos\frac{z\pi}{2} - \pi^z\zeta(1-z) \Rightarrow 0$	—				$\bullet$		
	M. Equations							
M1	$\frac{x=2}{2} + (1=1) \Rightarrow \frac{x}{2}+1=2$	$\bullet$		$\bullet$	$\bullet$	$\circ$	$\bullet$	$\bullet$
M2	$\mathsf{solve}(3x^3-18x^2+33x-19=0,\,\mathbf{R})$	$\circ$	$\bullet$	$\star$	$\star$	$\circ$	$\star$	$\bullet$
M3	$\mathsf{solve}(x^4+x^3+x^2+x+1=0)$	$\natural$	$\bullet$	$\bullet$	$\bullet$	$\circ$	$\bullet$	$\bullet$
M4	verify a solution of the above	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\star$	$\bullet$
M5	$\mathsf{solve}(x^6-9x^4-4x^3+27x^2-36x-23)$							
M6	$\mathsf{solve}(x^7-1=0) \Rightarrow x=\{1,\{e^{\pm 2k\pi i/7}\}_{k=1}^{3}\}$	$\natural$	$\star$	$\bullet$	$\bullet$	$\star$	$\natural$	$\natural$
M7	$\mathsf{solve}(x^8-8x^7+\cdots-140x+46=0)$			$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
M8	$\mathsf{solve}(e^{2x}+2e^x+1=z,\,x)$	$\circ^{\mathbf{r}}$	$\circ^{\mathbf{r}}$	$\bullet$	$\circ^{\mathbf{r}}$	$\circ^{\mathbf{r}}$	$\star^{\mathbf{18}}$	$\bullet$
M9	$\mathsf{solve}(e^{2-x^2}=e^{-x}) \Rightarrow x=\{-1,2\}[+\mathbf{C}]$		$\circ^{\mathbf{r}}$	$\bullet$	$\circ^{\mathbf{r}}$	$\circ^{\mathbf{r}}$	$\star^{\mathbf{18}}$	
M10	$\mathsf{solve}(e^x=x) \Rightarrow x=-W_n(-1) \quad (n\in\mathbf{Z})$			$\circ^{\mathbf{r}}$	$\star^{\mathbf{r}}$	$\circ^{\mathbf{r}}$		$\circ^{\mathbf{r}}$
M11	$\mathsf{solve}(x^x=x) \Rightarrow x=\{-1,1\}$				$\circ$	$\circ$		
M12	$\mathsf{solve}((x+1)(\sin^2 x+1)^2\cos^3 3x=0)$		$\circ^{\mathbf{m,r}}$	$\star$	$\circ^{\mathbf{m}}$	$\circ^{\mathbf{m,r}}$	$\circ^{\mathbf{m}}$	$\star$
M13	$\mathsf{solve}(\sin x=\cos x) \Rightarrow x=\frac{\pi}{4}\,[+n\pi]$	$\circ^{\mathbf{r}}$	$\circ^{\mathbf{r}}$	$\star$	$\bullet$	$\circ^{\mathbf{r}}$	$\star$	$\star$
M14	$\mathsf{solve}(\tan x=1) \Rightarrow x=\frac{\pi}{4}\,[+n\pi]$	$\circ^{\mathbf{r}}$	$\circ^{\mathbf{r}}$	$\bullet$	$\bullet$	$\circ^{\mathbf{r}}$	$\bullet$	$\bullet$
M15	$\mathsf{solve}(\sin x=\frac{1}{2}) \Rightarrow \frac{\pi}{6},\frac{5\pi}{6}\,[+n2\pi,\,+n2\pi]$	$\natural^{\mathbf{r}}$	$\circ^{\mathbf{r}}$	$\star$	$\bullet$	$\circ^{\mathbf{r}}$	$\bullet$	$\bullet$
M16	$\mathsf{solve}(\sin x=\tan x) \Rightarrow 0,0\,[+n\pi,\,+n2\pi]$	$\circ^{\mathbf{m,r}}$	$\circ^{\mathbf{r}}$	$\bullet$	$\circ^{\mathbf{m,r}}$	$\circ^{\mathbf{m}}$	$\bullet$	$\otimes^{\mathbf{s}}$
M17	$\mathsf{solve}(\sin^{-1}x=\tan^{-1}x) \Rightarrow x=\{0,0,0\}$	$\circ^{\mathbf{m}}$	$\circ^{\mathbf{m}}$	$\boxed{\star}^{\mathbf{s}}$	$\circ^{\mathbf{m}}$	$\circ^{\mathbf{m}}$		$\bullet$
M18	$\mathsf{solve}(\cos^{-1}x=\tan^{-1}x) \Rightarrow x=\sqrt{\frac{\sqrt{5}-1}{2}}$	$\uplus^{\mathbf{s}}$	$\bullet$	$\bullet$	$\bullet$	$\circ$		$\boxed{\star}^{\mathbf{s}}$
M19	$\mathsf{solve}(\frac{x-2}{x^{1/3}}=0) \Rightarrow x=2$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
M20	$\mathsf{solve}(\sqrt{x^2+1}=x-2) \Rightarrow x=\{\}$	$\times^{\mathbf{s}}$	$\circ^{\mathbf{19}}$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
M21	$\mathsf{solve}(x+\sqrt{x}=2) \Rightarrow x=1$	$\otimes^{\mathbf{s}}$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
M22	$\mathsf{solve}(2\sqrt{x}+3x^{1/4}-2=0) \Rightarrow x=\frac{1}{16}$	$\uplus^{\mathbf{s}}$	$\bullet$		$\bullet$	$\bullet$		$\bullet$
M23	$\mathsf{solve}(x=\frac{1}{\sqrt{1+x^2}}) \Rightarrow \{\sqrt{\frac{\sqrt{5}-1}{2}},-i\sqrt{\frac{\sqrt{5}+1}{2}}\}$	$\otimes^{\mathbf{s}}$		$\bullet$	$\star$	$\bullet$	$\bullet$	$\circ$
M24	$\mathsf{solve}(\binom{m}{2}2^k=1,\,k) \Rightarrow \log_2\frac{2}{m(m-1)}\,(k\in\mathbf{R})$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$	$\star$

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#	PROBLEM	**Ax**	**De**	**Mc**	**Mp**	**Mm**	**Mu**	**Re**
M25	$\mathsf{solve}(ab^x = cd^x,\, x) \Rightarrow \frac{\log(c/a)}{\log(b/d)}$		$\star$	$\bullet$	$\bullet$	$\boxed{\star}$		
M26	$\mathsf{solve}(\sqrt{\log x} = \log\sqrt{x}) \Rightarrow x = \{1, e^4\}$	$\otimes^{\mathbf{s}}$	$\bullet$	$\mathcal{E}$	$\bullet$	$\bullet$	$\bullet$	
M27	$\mathsf{solve}(\log(\cos^{-1}(\sin^{-1}(x^{2/3} - b) - 1)) + 2)$	$\bullet$		$\circ$	$\bullet$	$\bullet$		$\bullet$
M28	$\mathsf{N}(\mathsf{solve}(5x + e^{(x-5)/2} = 8x^3))$	—	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$
M29	$\mathsf{solve}(\|x - 1\| = 2) \Rightarrow x = \{-1, 3\}$		$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
M30	$\mathsf{solve}(\|2x + 5\| = \|x - 2\|) \Rightarrow x = \{-1, -7\}$		$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\natural$	$\natural$
M31	$\mathsf{solve}(1 - \|x\| = \max(-x - 2, x - 2)) \Rightarrow \pm\frac{3}{2}$		$\bullet$		$\bullet$	$\bullet$		
M32	$\mathsf{solve}(\max(2 - x^2, x) = \max(-x, \frac{x^3}{9}))$	$\uplus^{\mathbf{s}}$	$\natural$		$\bullet$	$\bullet$		
M33	$\mathsf{solve}(\max(2 - x^2, x) = \frac{x^3}{9}) \Rightarrow x = \{\pm 3, \}$	$\otimes^{\mathbf{s}}$	$\natural$		$\star$	$\star$		
M34	$\mathsf{solve}((1 + i)z + (2 - i)\bar{z} = -3i, z)$		$\uplus$					
M35	$\mathsf{solve}(3x - 2y - iy = -3i,\, [x, y]) \Rightarrow 2 + 3i$			$\uplus$				$\uplus$
M36	$\mathsf{solve}(f^2(x) + f(x) - 2 = 0, x)$		$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
M37	solve a 3×3 dependent linear system	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
M38	solve a 189×49 simple linear system	$\bullet$	$\bullet$	$\bullet$	$\star$	$\star$	$\star$	$\bullet$
M39	solve a 3×3 nonlinear system	$\bullet$		$\bullet$	$\bullet$	$\otimes$	$\bullet$	$\bullet$
	N. Inequalities							
N1	$\mathsf{is}(e^\pi > \pi^e) \Rightarrow$ true	$\times$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	—
N2	$[\mathsf{is}(x^4 - x + 1 > 0), \mathsf{is}(x^4 - x + 1 > 1)]$	—		$\circ$	$\bullet$			—
N3	$\mathsf{assume}(\|x\| < 1);\quad \mathsf{is}(-1 < x < 1)$	—	—		$\times$	—		—
N4	$\mathsf{assume}(x > y > 0);\quad \mathsf{is}(2x^2 > 2y^2)$	—	—	$\bullet$	$\bullet$	—	$\bullet$	—
N5	$\mathsf{assume}('',\, k > 0);\quad \mathsf{is}(kx^2 > ky^2)$	—	—	$\bullet$	$\bullet$	—		—
N6	$\mathsf{assume}('',\, n > 0);\quad \mathsf{is}(kx^n > ky^n)$	—	—			—		—
N7	$\mathsf{assume}(x > 1,\, y \geq x - 1);\quad \mathsf{is}(y > 0)$	—	—		$\bullet$	—	$\bullet$	—
N8	$\mathsf{assume}(x \geq y \geq z \geq x);\quad \mathsf{is}(x = y = z)$	—	—	$\bullet$	$\bullet$	—	$\uplus$	—
N9	$\mathsf{solve}(\|x - 1\| > 2) \Rightarrow x < -1$ or $x > 3$	—	$\bullet$	$\circ$	$\bullet$	$\bullet$	$\bullet$	$\mathcal{E}$
N10	$\mathsf{solve}(\mathsf{expand}((x - 1)\cdots(x - 5)) < 0)$	—	$\bullet$	$\uplus$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
N11	$\mathsf{solve}(\frac{6}{x-3} \leq 3) \Rightarrow x < 3$ or $x \geq 5$	—	$\star$	$\natural$	$\bullet$	$\bullet$	$\bullet$	
N12	$\mathsf{solve}(\sqrt{x} < 2) \Rightarrow 0 \leq x < 4$	—	$\star$	—	$\bullet$	$\otimes$	$\otimes$	ε
N13	$\mathsf{solve}(\sin x < 2) \Rightarrow x \in \mathbf{R}$	—	$\bullet$	—	$\uplus^{\mathbf{20}}$	$\uplus^{\mathbf{20}}$	$\mathcal{E}$	ε

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#	PROBLEM	**Ax**	**De**	**Mc**	**Mp**	**Mm**	**Mu**	**Re**
N14	$\mathsf{solve}(\sin x < 1) \Rightarrow x \neq \frac{\pi}{2}\,[+n2\pi]\ (n \in \mathbf{Z})$	—		—		$\circ^{\mathbf{r}}$	$\mathcal{E}$	ε
N15	$\mathsf{solve}(\lvert 2A(\cos t - 1) + 1\rvert \leq 1,\ A)$	—		—			$\mathcal{E}$	
N16	$\mathsf{solve}(A^2(\cos t - 4)^2 \sin^2 t < 9,\ A)$	—		$\times$			$\mathcal{E}$	
N17	$\mathsf{solve}([x + y > 0,\ x - y < 0]) \Rightarrow \lvert x\rvert < y$	—		#	#	—		
	O. Vector Analysis							
O1	$\Vert(1+i, -2, 3i)\Vert \Rightarrow \sqrt{15}$	$\circ^{\mathbf{d}}$	$\bullet$	$\bullet$	$\star$	$\circ^{\mathbf{d}}$	$\star$	$\times$
O2	(2, 2, -3) $\times$ (1, 3, 1) $\Rightarrow$ (11, -5, 4)	—	$\bullet$	$\star$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
O3	$(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) \Rightarrow (\mathbf{a} \cdot \mathbf{c})(\mathbf{b} \cdot \mathbf{d}) - \cdots$	—		$\bullet$	—		—	—
O4	$\nabla \times (xyz,\ x^2y^2z^2,\ y^2z^3)$	—	$\bullet$	$\circ$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
O5	$\nabla \cdot (\mathbf{f} \times \mathbf{g}) \Rightarrow \mathbf{g} \cdot (\nabla \times \mathbf{f}) - \mathbf{f} \cdot (\nabla \times \mathbf{g})$	—			—		—	—
O6	$\nabla \cdot (a_r, a_\theta, a_\phi) \Rightarrow \frac{da_r}{dr} + \frac{2a_r}{r} + \frac{1}{r}\frac{da_\theta}{d\theta} + \cdots$	—	$\bullet$	$\star$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
O7	$\frac{d\mathbf{r}}{dt} \Rightarrow (\frac{dr}{dt}, r\frac{d\theta}{dt}, r\sin\theta\frac{d\phi}{dt})$	—	—	—	—	—	—	—
O8	$\mathsf{solve}(\nabla\phi = (2xy,\ x^2 + 1,\ 6z^2),\ \phi)$	—	$\bullet$	$\bullet$	$\bullet$	—	—	—
O9	$\mathsf{solve}(\nabla \times \mathbf{A} = (2yz^3 - 2x^2y^2z,\ xy,\ \ldots))$	—	$\bullet$	$\bullet$	$\bullet$	—	$\bullet$	—
O10	orthogonalize a set of 4 vectors	$\star$	$\star$	$\bullet$	$\bullet$	$\star$	—	$\bullet$
	P. Matrix Theory[21]							
P1	extract superdiagonal of a 3×3 matrix	$\circ^{\mathbf{d}}$	$\circ^{\mathbf{d}}$	$\bullet$	$\circ^{\mathbf{d}}$	$\circ^{\mathbf{d}}$	$\circ^{\mathbf{d}}$	$\circ^{\mathbf{d}}$
P2	(2, 3)-minor of 3×3 matrix $A \Rightarrow \begin{pmatrix} 1 & 2 \\ 7 & 8 \end{pmatrix}$	—	$\bullet$	$\bullet$	$\bullet$	—	$\circ^{\mathbf{d}}$	$\bullet$
P3	ease of MATLAB style matrix creation	$\natural$	$\circ$	$\bullet$	$\star$	$\star$	$\star$	$\circ$
P4	$\mathsf{diag}(\begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix}, b, \ldots) \Rightarrow$ block diagonal mat.		—	$\bullet$	$\bullet$	ε	—	$\natural$
P5	$\begin{pmatrix} 7 & 11 \\ 3 & 8 \end{pmatrix}$ mod 2 $\Rightarrow \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$	$\bullet$	$\bullet$	$\star$	ε	$\bullet$	ε	$\bullet$
P6	$\frac{d^2}{d\theta^2}\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \Rightarrow -\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$	$\bullet$	$\bullet$	$\bullet$	$\times$	$\bullet$	$\bullet$	
P7	$\begin{pmatrix} x & y \end{pmatrix} \cdot [a\begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix} + \begin{pmatrix} 7 & -9 & 11 \\ -8 & 10 & -12 \end{pmatrix}]$	$\bullet$	$\bullet$	$\bullet$	$\star$	$\bullet$	$\bullet$	$\bullet$
P8	$\left\Vert \begin{matrix} 1 & -2i \\ -3i & 4 \end{matrix} \right\Vert_\infty \Rightarrow 7$	—	—	$\bullet$	$\bullet$	—	$\bullet$	—
P9	$\Vert A\Vert_F \Rightarrow \frac{a^2+b^2+c^2}{\lvert a\rvert\ \lvert b\rvert\ \lvert c\rvert} \quad (a, b, c \in \mathbf{R})$	—	$\bullet$	$\star$	$\star$	—	$\circ$	—
P10	$\begin{pmatrix} 1 & 2+3i \\ f(4-5i) & 6 \end{pmatrix}^H \Rightarrow \begin{pmatrix} 1 & \bar{f}(4+5i) \\ 2-3i & 6 \end{pmatrix}$	$\varepsilon^{\mathbf{d}}$	$\mathcal{E}^{\mathbf{d}}$	$\bullet$	$\bullet$	$\circ^{\mathbf{d}}$	$\circ^{\mathbf{d}}$	$\boxed{\star}$
P11	$\begin{pmatrix} a & b \\ 1 & ab \end{pmatrix}^{-1} \Rightarrow \frac{1}{a^2-1}\begin{pmatrix} a & -1 \\ -1/b & a/b \end{pmatrix}$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
P12	$\begin{pmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{pmatrix}^{-1} \Rightarrow \begin{pmatrix} A_{11}^{-1} & -A_{11}^{-1}A_{12}A_{22}^{-1} \\ 0 & A_{22}^{-1} \end{pmatrix}$ (blk)	—	$\otimes$	$\otimes$	$\otimes$	—	—	$\uplus$
P13	LU decomposition of a 3×3 poly. mat.	—	—	$\bullet$	$\bullet$	$\times$	—	—
P14	reduced row echelon form of a 4×5 mat.	$\bullet$	$\bullet$	$\circ$	$\bullet$	$\bullet$	$\star$	—

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#	PROBLEM	**Ax**	**De**	**Mc**	**Mp**	**Mm**	**Mu**	**Re**
P15	rank(3×4 integer matrix) $\Rightarrow 2$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\circ^{\mathbf{d}}$	$\bullet$	$\bullet$
P16	rank$\begin{pmatrix} 2\sqrt{2} & 8 \\ 6\sqrt{6} & 24\sqrt{3} \end{pmatrix} \Rightarrow 1$	$\bullet$	$\bullet$	$\times$	ε	$\circ^{\mathbf{d}}$	$\times$	$\bullet$
P17	rank$\begin{pmatrix} \sin 2\theta & \cos 2\theta \\ 2(1-\cos^2\theta)\cos\theta & (1-2\sin^2\theta)\sin\theta \end{pmatrix} \Rightarrow 1$	$\times$	$\bullet$	$\times$	ε	$\circ^{\mathbf{d}}$	$\times$	$\times$
P18	null_space(3×4 matrix) $\Rightarrow 4 \times 2$ matrix	$\bullet$	$\circ$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
P19	det(4×4 Vandermonde matrix)	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
P20	minpoly(4×4 matrix) $\Rightarrow (\lambda-1)^2(\lambda+1)$	ε	—	—	$\bullet$	—	—	—
P21	charpoly(3×3 matrix) $\Rightarrow \lambda = \{1, -2, 3\}$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
P22	eigenvalues($(2-a)I_{100\times100}$) $\Rightarrow \{2-a\}_{k=1}^{100}$	$\circ^{\mathbf{m}}$	$\circ^{\mathbf{m}}$	$\natural^{\mathbf{22}}$	$\circ$	$\star$	τ	$\star$
P23	eigenvalues(5×5 Wilkinson matrix)	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\star$
P24	eigenvalues(8×8 Rosser matrix)	$\bullet$	$\boxed{\star}^{\mathbf{m}}$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\star$
P25	N(eigenvalues(8×8 Rosser matrix))[23]	$\boxed{\star}^{\mathbf{m}}$	$\natural$	$\boxed{\star}$	$\bullet$	$\bullet$	$\star \mid \mathcal{E}$	$\star$
P26	eigenvalues(9×9 hypercompanion mat.)	$\circ$	$\boxed{\star}^{\mathbf{m}}$	$\bullet$	$\bullet$	$\star$	$\star$	$\star$
P27	eigenvalues/vectors(5×5 simple matrix)	$\bullet$		$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\boxed{\star}$
P28	generalized eigenvectors of a 3×3 matrix	$\circ$	$\natural^{\mathbf{m}}$	$\circ$	$\circ$	$\otimes$	$\circ$	$\circ$
P29	generalized eigenvectors of a 6×6 matrix	$\circ$		$\circ$	$\circ$	$\otimes$	$\circ$	$\circ$
P30	jordan(5×5) $\Rightarrow$ diag($\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, -1$)	—	—	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
P31	smith_form$\begin{pmatrix} x^2 & x-1 \\ x+1 & x^2 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & x^4-x^2+1 \end{pmatrix}$	—	—	—	$\bullet$	—	—	$\bullet$
P32	$\exp\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} e\cos 2 & -e\sin 2 \\ e\sin 2 & e\cos 2 \end{pmatrix}$	—	—	$\star$	$\bullet$	$\star$	$\circ$	—
P33	exp(4×4 matrix)	—	—	$\star$	$\bullet$	$\star$	$\circ$	—
P34	sin(6×6 Jordan matrix)	—	—	$\bullet$	$\circ$	$\circ$		—
P35	$\sin(\frac{\pi}{2}[3 \times 3 \text{ integer matrix}]) \Rightarrow I$	—	—	$\bullet$	$\times$	$\star$		—
P36	$\begin{pmatrix} 10 & 7 \\ 7 & 17 \end{pmatrix}^{1/2} \Rightarrow \{\pm\begin{pmatrix} 3 & 1 \\ 1 & 4 \end{pmatrix}, \pm\frac{1}{\sqrt{5}}\begin{pmatrix} -1 & 7 \\ 7 & 6 \end{pmatrix}\}$	—	—	$\circ^{\mathbf{24}}$	$\natural^{\mathbf{24}}$	$\circ^{\mathbf{24}}$	ε	—
P37	$(3 \times 3 \text{ non-singular integer matrix})^{1/2}$	—	—	$\uplus$	$\natural^{\mathbf{24}}$	ε	ε	—
P38	$(3 \times 3 \text{ singular integer matrix})^{1/2}$	—	—	$\mathcal{E}$	ε	ε	ε	—
P39	SVD($\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}^T$) $\Rightarrow (3^2)\begin{pmatrix} 2\sqrt{7} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}^T(2^2)$	—	—	$\bullet$	$\circ^{\mathbf{25}}$	$\natural^{\mathbf{n}}$	—	$\natural^{\mathbf{n}}$
P40	jacobian($r\cos\theta, r\sin\theta$) $\Rightarrow \begin{pmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{pmatrix}$	—	$\bullet$	$\bullet$	$\bullet$	$\circ^{\mathbf{d}}$	$\bullet$	$\bullet$
P41	hessian($r^2\sin\theta$) $\Rightarrow \begin{pmatrix} 2\sin\theta & 2r\cos\theta \\ 2r\cos\theta & -r^2\sin\theta \end{pmatrix}$	—	—	—	$\bullet$	—	$\bullet$	$\bullet$
P42	wronskian($\cos\theta, \sin\theta$) $\Rightarrow \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$	—	—	$\bullet$	$\bullet$	—	—	—
P43	compute jacobian($r\cos\theta, r\sin\theta$) by hand	$\bullet$	$\bullet$	$\bullet$	$\star$	$\bullet$	$\bullet$	$\star$

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P44	compute $\mathsf{hessian}(r^2 \sin\theta)$ by hand	•	•	•	⋆	•	•	⋆
P45	compute $\mathsf{wronskian}(\cos\theta, \sin\theta)$ by hand	⋆	•	•	♮	•	•	⋆
	Q. Tensor Analysis							
Q1	$\delta_{ik}^{jh} \Rightarrow \delta_i^j \delta_k^h - \delta_k^j \delta_i^h$	—	∘[26]	$\boxed{\star}$	—	—	—	—
Q2	$[\varepsilon_{213}, \varepsilon_{131}] \Rightarrow [-1, 0]$ (Levi-Civita symbol)	⋆	•	•	∘	•	—	∘
Q3	$\begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix} \otimes \begin{pmatrix} 5 & 6 \\ 7 & -8 \end{pmatrix}$ (outer product)	⋆	•	•	•	•	∘[d]	⋆
Q4	$\Gamma_{\ell hk} \Rightarrow \frac{1}{2}(\frac{\partial a_{kh}}{\partial x^\ell} + \frac{\partial a_{h\ell}}{\partial x^k} - \frac{\partial a_{\ell k}}{\partial x^h})$	—	—	•	∘[27]	—	—	—
Q5	$T^i_{j\vert k} \Rightarrow T^i_{j,k} + \Gamma_m{}^i{}_k T^m_j - \Gamma_j{}^m{}_k T^i_m$	—	—	•	—	—	—	—
Q6	$K_i{}^h{}_{jk\vert\ell} + K_i{}^h{}_{k\ell\vert j} + K_i{}^h{}_{\ell j\vert k} \Rightarrow 0$ [Bianchi]	—	—	•	#[27]	—	—	—
	R. Sums[28]							
R1	$\sum_{i=1}^n (x_i - \frac{1}{n}\sum_{j=1}^n x_j)^2 \Rightarrow \sum_{i=1}^n x_i^2 - \cdots$		⋆	∘				
R2	least squares derivation		•		•			•
R3	$\sum_k (-1)^k \binom{2n}{k}^2 \Rightarrow (-1)^n \binom{2n}{n}$					—		
R4	$\sum_k \frac{1}{2^n}\binom{n}{k} - \frac{1}{2^{n+1}}\binom{n+1}{k} \Rightarrow \frac{1}{2^{n+1}}\binom{n}{k-1}$ [29]	⋆		⋆	⋆	—		⋆
R5	$\sum_k (-1)^k \binom{a+b}{a+k}\binom{b+c}{b+k}\binom{c+a}{c+k} \Rightarrow \frac{(a+b+c)!}{a!\,b!\,c!}$					—		
R6	$\sum_{k=0}^n g_{k+1} - g_k \Rightarrow g_{n+1} - g_0$		•					
R7	$\sum_{k=1}^n k^3 \Rightarrow \frac{n^2(n+1)^2}{4}$	•	•	•	•	•	•	•
R8	$\sum_{k=1}^n k^2 \binom{n}{k} \Rightarrow n(n+1)2^{n-2}$			#	•	•	⋆	
R9	$\sum_{k=0}^n \frac{1}{k+1}\binom{n}{k} \Rightarrow \frac{2^{n+1}-1}{n+1}$				•	•		
R10	$\sum_{k=0}^r \binom{n}{k}\binom{m}{r-k} \Rightarrow \binom{n+m}{r}$ [Vandermonde]				♮	•	×	
R11	$\sum_{k=0}^n \binom{n}{k} f_k \Rightarrow f_{2n}$	ε						—
R12	$\sum_{k=1}^n f_k^2 \Rightarrow f_n f_{n+1}$	ε		∘				—
R13	$\sum_{k=1}^n \sin kx \Rightarrow \frac{1}{2}\cot\frac{x}{2} - \frac{\cos([2n+1]x/2)}{2\sin(x/2)}$		•	⋆	⋆	•	∘	•
R14	$\sum_{k=1}^n \sin([2k-1]x) \Rightarrow \frac{\sin^2 nx}{\sin x}$		•		⋆	•	∘	•
R15	$\sum_{k=0}^{\lfloor n/2 \rfloor} \binom{n-k}{k} \Rightarrow f_{n+1}$	ε			⊎	♮		
R16	$\sum_{k=1}^\infty (\frac{1}{k^2} + \frac{1}{k^3}) \Rightarrow \frac{\pi^2}{6} + \zeta(3)$		•	•	•	•	•	•
R17	$\mathsf{N}(\sum_{k=1}^\infty (\frac{1}{k^2} + \frac{1}{k^3})) \approx 2.84699$		•	•	•	•	•	•
R18	$\sum_{k=1}^\infty \frac{1}{2^k k^2} \Rightarrow \frac{\pi^2}{12} - \frac{1}{2}\log^2 2$				⋆	•	•	
R19	$\sum_{k=0}^\infty \frac{1}{(3k+1)(3k+2)(3k+3)} \Rightarrow \frac{\pi}{12}\sqrt{3} - \frac{1}{4}\log 3$		•	•	•	•	#	
R20	$\sum_{k=0}^\infty \binom{n}{4k} \Rightarrow \frac{1}{2}(2^{n-1} + 2^{n/2}\cos\frac{n\pi}{4})$				∘	∘		

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R21	$\sum_{k=1}^{\infty} \frac{1}{\sqrt{k(k+1)}(\sqrt{k}+\sqrt{k+1})} \Rightarrow 1$							
R22	$\sum_{n=0}^{\infty} \sum_{k=0}^{\lfloor n/2 \rfloor} \binom{n}{k}\binom{n-k}{n-2k} x^n y^{n-2k}$ $(\lvert xy \rvert < 1)$	ε			#		$\mathcal{E}$	
R23	$\sum_{k=0}^{\infty} \sum_{n=2k}^{\infty} \frac{n!}{k!^2 (n-2k)!} (\frac{x}{y})^k (xy)^{n-k}$ (")				$\star$	$\star$		
R24	$\sum_{m=2}^{\infty} \prod_{k=1}^{m} \frac{k}{2k-1} \Rightarrow \frac{\pi}{2}$		#	#	•	•	#	#
	S. Products							
S1	$\prod_{k=1}^{8} \Gamma(\frac{k}{3}) \Rightarrow \frac{640\pi^3}{2187\sqrt{3}}$		•	#	•	•	$\star$	
S2	$\prod_{k=1}^{n} k \Rightarrow n!$ or $\Gamma(n+1)$		•	•	•	•	•	•
S3	$\prod_{k=1}^{n} x^k \Rightarrow x^{n(n+1)/2}$		•	•		•		•
S4	$\prod_{k=1}^{n-1} \frac{1}{1+1/k} \Rightarrow n$		•	•	•	•		•
S5	$\prod_{k=1}^{n} \frac{2k-1}{2k} \Rightarrow \frac{1}{2^{2n}}\binom{2n}{n}$		$\star$	$\star$	$\star$	$\star$	$\star$	$\star$
S6	$\prod_{k=1}^{n-1} x^2 - 2x\cos\frac{k\pi}{n} + 1 \Rightarrow \frac{x^{2n}-1}{x^2-1}$							
S7	$\prod_{k=2}^{\infty} \frac{k^3-1}{k^3+1} \Rightarrow \frac{2}{3}$		[30]	•	•			$\star$
S8	$\prod_{k=1}^{\infty} 1 - \frac{1}{(2k)^2} \Rightarrow \frac{2}{\pi}$		•	•	•	•		ε
S9	$\prod_{k=1}^{\infty} 1 + \frac{(-1)^{k+1}}{2k-1} \Rightarrow \sqrt{2}$				ε	•		$\mathcal{E}$
S10	$\prod_{k=0}^{\infty} \frac{k(k+1)+1+i}{k(k+1)+1-i} \Rightarrow -1$			•	ε			$\mathcal{E}$
	T. Limits[31]							
T1	$\lim_{n\to\infty}(1+\frac{1}{n})^n \Rightarrow e$; $\lim_{x\to 0} \frac{1-\cos x}{x^2} \Rightarrow \frac{1}{2}$	•	•	•	•	• / •	•	•
T2	$\lim_{x\to\infty}(3^x + 5^x)^{1/x} \Rightarrow 5$		$\mathcal{E}$	•	•	/ ×		×
T3	$\lim_{x\to\infty} \frac{\log x}{\log x + \sin x} \Rightarrow 1$			•	×	/ •		
T4	$\lim_{x\to\infty} \frac{1}{x}[\exp(\frac{xe^{-x}}{e^{-x}+e^{-2x^2/(x+1)}}) - e^x] \Rightarrow -e^2$		τ		•	/ ×	•	
T5	$\lim_{x\to\infty} \frac{x\log x \log^2(xe^x - x^2)}{\log(\log(x^2+2\exp(e^{3x^3\log x})))} \Rightarrow \frac{1}{3}$		τ		•	/ ×	•	ε
T6	$\lim_{n\to\infty} \frac{1}{n}\sqrt[n]{n!} \Rightarrow \frac{1}{e}$		$\mathcal{E}$	•	•	/ •	$\mathcal{E}$	×
T7	$\lim_{n\to\infty} \frac{1}{n}\sqrt[n]{\Gamma(n+1)} \Rightarrow \frac{1}{e}$		$\mathcal{E}$	•		/ ×	×	ε
T8	$\lim_{z\to\infty} \frac{\Gamma(z+a)}{\Gamma(z)} e^{-a\log z} \Rightarrow 1$		•	$\star$		/ •	$\mathcal{E}$	ε
T9	$\lim_{k\to\infty} F(1,k;1;\frac{z}{k}) \Rightarrow e^z$ [32]	—	/ •	•	$\star$	• / •	—	
T10	$\lim_{x\to 0} \zeta(x) - \frac{1}{x-1} \Rightarrow \gamma$	—		×	•	• / •		×
T11	$\lim_{n\to\infty} \frac{n^x}{x\prod_{k=1}^{n}(1+\frac{x}{k})} \Rightarrow \Gamma(x)$		τ	×	#	/ •		ε
T12	$\lim_{x\to 0} \frac{x\int_0^x e^{-t^2}\,dt}{1-e^{-x^2}} \Rightarrow 1$		•	•	•	• / ×	•	•
T13	$\lim_{x\to 0-} \frac{x}{\lvert x \rvert} \Rightarrow -1$; $\lim_{x\to 0+} \frac{x}{\lvert x \rvert} \Rightarrow 1$	•	•	•	•	• / •	•	•

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#	PROBLEM	**Ax**	**De**	**Mc**	**Mp**	**Mm**	**Mu**	**Re**
T14	$\lim_{x\to 0+} \tan^{-1}(-\log x) \Rightarrow \frac{\pi}{2}$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$ / $\bullet$	$\mathcal{E}$	
	U. Calculus							
U1	$\frac{d}{dx}\lvert x\rvert \Rightarrow \frac{x}{\lvert x\rvert}$ or $\mathrm{sign}(x)$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\circ$	$\bullet$	$\bullet$
U2	$\frac{d}{dx}\lvert x\rvert$ (piecewise defined)[P]	$\times$	$\bullet$ / $\bullet$	$\bullet$ / $\bullet$	$\bullet$ / $\bullet$	$\star$ / $\natural$	$\bullet$ / $\natural$	$\mathcal{E}$
U3	$f(x) = \begin{cases} x^2 - 1 & (x = 1) \\ x^3 & \text{otherwise} \end{cases}$; $f'(1) \Rightarrow 3$	$\bullet$	$\times$	$\bullet$	$\star$[33]	$\times$	$\bullet$	$\bullet$
U4	$\frac{d^n}{dx^n} x^n \Rightarrow n!$	ε		$\star$			$\mathcal{E}$	$\times$
U5	$\frac{d^2}{dt^2} y(x(t)) \Rightarrow \frac{d^2y}{dx^2}\left(\frac{dx}{dt}\right)^2 + \frac{dy}{dx}\frac{d^2x}{dt^2}$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\star$
U6	$\frac{d}{dx}\int_{g(x)}^{h(x)} f(y)\,dy \Rightarrow f(h(x))\frac{dh}{dx} - f(g(x))\frac{dg}{dx}$	ε	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	
U7	$d(V(P,T)) \Rightarrow \frac{\partial V}{\partial P}dP + \frac{\partial V}{\partial T}dT$	—		$\bullet$	—	$\bullet$	—	—
U8	solve$(y = \cos xy + x,\ \frac{dy}{dx}) \Rightarrow \frac{1 - y\sin xy}{1 + x\sin xy}$	—	$\star$	$\circ$	$\bullet$	$\natural$	—	—
U9	substitute$(f = g(x^2+y^2),\ \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y})$		$\bullet$	$\circ$	$\bullet$	$\star$	$\bullet$	
U10	residue$(\frac{z^3+5}{(z^4-1)(z+1)}, z = -1) \Rightarrow -\frac{9}{4}$	—	—	$\bullet$	$\bullet$	$\bullet$	—	$\bullet$
U11	$(2\,dx + dz) \wedge (3\,dx + dy + dz) \wedge \cdots$	$\star$	—	$\bullet$	$\bullet$	—	—	$\bullet$
U12	$d(3x^5\,dy \wedge dz + 5xy^2\,dz \wedge dx + 8z\,dx \wedge dy)$	$\mathcal{E}$	—	$\bullet$	$\mathcal{E}$	—	—	$\mathcal{E}$
U13	minimize$(x^4 - x + 1) \Rightarrow 1 - \frac{3}{8}\sqrt[3]{2}$	—	—	ε	$\bullet$	$\circ$[n]	—	—
U14	minimize/maximize$(\frac{1}{x^2+y^2+1}) \Rightarrow [0,1]$	—	—	—	$\times$	ε	—	—
U15	minimize$(a + bx + cy + dxy,\ [-1,1]^2)$	—	—	—	$\otimes$	—	—	—
U16	minimize/maximize$(x^2y^3,\ [-1,1]^2)$	—	—	—	$\mathcal{E}$	—	—	—
U17	minimize(linear function + constraints)	—	—	$\star$	$\star$	$\bullet$	$\star$	$\bullet$
	V. Indefinite Integrals[34]							
V1	$\int \lvert x\rvert\,dx \Rightarrow \frac{x\lvert x\rvert}{2}$ $(x \in \mathbf{R})$		$\otimes$	$\bullet$	$\otimes$	/ $\star$	$\otimes$	$\bullet$
V2	$\int \lvert x\rvert\,dx$ (piecewise defined)[P]	$\otimes$	/ $\bullet$	$\bullet$ / $\bullet$	$\mathcal{E}$ / $\bullet$	/ $\circ$	$\mathcal{E}$ /	
V3	$\int \frac{1}{x^3+2}\,dx \to$ diff $\to$ simplify	$\bullet$	$\bullet$	$\bullet$	$\star$	$\bullet$	$\bullet$	$\bullet$
V4	$\int \frac{2^x}{\sqrt{1+4^x}}\,dx \Rightarrow \frac{\sinh^{-1} 2^x}{\log 2} = \frac{2^x + \sqrt{4^x+1}}{\log 2}$	$\mathcal{E}$	$\bullet$	$\circ$	$\times$			
V5	$\int \frac{(3x-5)^2}{(2x-1)^{7/2}}\,dx \Rightarrow \frac{-9x^2+16x-41/5}{(2x-1)^{5/2}}$	$\star$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\star$	$\star$
V6	$\int \frac{1}{2e^{mx} - 5e^{-mx}}\,dx \Rightarrow \frac{1}{2m\sqrt{10}} \log \frac{-5 + e^{mx}\sqrt{10}}{-5 - e^{mx}\sqrt{10}}$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\circ$	$\bullet$
V7	$\int \frac{\sinh^4 x}{\cosh^2 x}\,dx \Rightarrow -\frac{3x}{2} + \frac{1}{4}\sinh 2x + \tanh x$	$\bullet$	$\circ$	$\star$	$\bullet$	$\bullet$	$\bullet$	$\circ$
V8	$\int \frac{1}{a + b\cos x}\,dx$ $(a < b)$ [real solution]	$\bullet$[35]		$\bullet$	$\bullet$	$\bullet$	$\natural$	
V9	$\int \frac{1}{a + b\cos x}\,dx \to$ diff $\to$ simplify	$\bullet$	$\bullet$	$\star$	$\star$	$\bullet$	$\circ$	$\star$

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V10	$\int \frac{1}{3+3\cos x+4\sin x}\,dx \Rightarrow \frac{1}{4}\log(3+4\tan\frac{x}{2})$	$\bullet$	$\boxed{\star}$	$\bullet$	$\bullet$	$\star$	$\mathcal{E}$	$\bullet$
V11	$\int \frac{1}{4+3\cos x+4\sin x}\,dx \Rightarrow \frac{1}{3}\log\frac{\tan(x/2)+1}{\tan(x/2)+7}$	$\bullet$	$\star$	$\bullet$	$\star$	$\star$	$\star$	$\bullet$
V12	$\int \frac{1}{5+3\cos x+4\sin x}\,dx \Rightarrow -\frac{1}{2+\tan\frac{x}{2}}$	$\bullet$	$\star$	$\star$	$\bullet$	$\bullet$	$\mathcal{E}$	$\star$
V13	$\int \frac{1}{6+3\cos x+4\sin x}\,dx \Rightarrow \frac{2}{\sqrt{11}}\tan^{-1}\frac{3\tan(x/2)+4}{\sqrt{11}}$	$\star$	$\uplus$	$\boxed{\star}$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
V14	$\int \log\lvert x^2-a^2\rvert\,dx \Rightarrow x\log\lvert x^2-a^2\rvert - \cdots$	$\boxed{\star}$	$\otimes$	$\boxed{\star}$	$\boxed{\star}$	$\mathcal{E}\ \vert\ \boxed{\star}$		$\boxed{\star}$
V15	$\int x\cot^{-1}\frac{x}{a}\,dx \Rightarrow \frac{ax}{2}+\frac{1}{2}(x^2+a^2)\cot^{-1}\frac{x}{a}$	$\otimes$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\boxed{\star}$	$\bullet$
V16	$\int \cos 5x\,\mathrm{Ci}\,2x\,dx \Rightarrow \frac{\sin 5x\,\mathrm{Ci}\,2x}{5} - \frac{\mathrm{Si}\,7x+\mathrm{Si}\,3x}{10}$	$\mathcal{E}$	#			$\bullet$		
V17	$\int \frac{\frac{df(x)}{dx}g(x)-f(x)\frac{dg(x)}{dx}}{f^2(x)-g^2(x)}\,dx \Rightarrow \frac{1}{2}\frac{f(x)-g(x)}{f(x)+g(x)}$			$\circ$		$\mathcal{E}$		
	W. Definite Integrals[36]							
W1	$\int_{a-1}^{a+1}\frac{1}{x-a}\,dx \Rightarrow 0$ (principal value)	ε	$\times$	$\uplus$	$\bullet$	$\circ$	$\bullet$	$\times$
W2	$\int_{a-1}^{a+1}\frac{1}{(x-a)^2}\,dx \Rightarrow$ divergent		$\times$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\times$
W3	$\int_0^1\sqrt{x+\frac{1}{x}-2}\,dx \Rightarrow \frac{4}{3}$		$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\uplus$
W4	$\int_1^2\sqrt{x+\frac{1}{x}-2}\,dx \Rightarrow \frac{4-\sqrt{8}}{3}$	$\vert\ \star$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
W5	$\int_0^2\sqrt{x+\frac{1}{x}-2}\,dx \Rightarrow \frac{8-\sqrt{8}}{3}$		$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\uplus$
W6	$\int_{-3\pi/4}^{-\pi/4}\frac{\sqrt{2-2\cos 2x}}{2}\,dx \Rightarrow \sqrt{2}$		$\bullet$	$\times$	$\bullet$	$\bullet$	$\bullet$	
W7	$\int_{-\infty}^{\infty}\frac{\cos x}{x^2+a^2}\,dx \Rightarrow \frac{\pi}{a}e^{-a}\quad (a>0)$			$\bullet$	$\bullet$	$\bullet$	$\bullet$	
W8	$\int_0^{\infty}\frac{t^{a-1}}{1+t}\,dt \Rightarrow \frac{\pi}{\sin\pi a}\quad (0<a<1)$			$\bullet$		$\bullet$	$\circ$	$\mathcal{E}$
W9	$\int_{-\infty}^{\infty}\frac{5x^3}{1+x+x^2+x^3+x^4} \Rightarrow -2\pi(\sin\frac{\pi}{5}+\cdots)$ [37]			$\vert\ \times$		$\times$		$\times$
W10	$\int_{-\infty}^{\infty}\frac{1}{1+x+x^2+x^4} \Rightarrow \frac{2\pi}{5}(1+\cos\frac{\pi}{5})\csc\frac{2\pi}{5}$	τ		τ	$\times$			
W11	$\int_{-1}^{1}\frac{\sqrt{1-x^2}}{1+x^2} \Rightarrow \pi(\sqrt{2}-1)$	$\vert\ \circ$	$\bullet$	$\bullet$	$\bullet$	$\bullet$		$\times$
W12	$\int_{-\infty}^{\infty}xe^{-px^2+2qx}\,dx \Rightarrow \frac{q}{p}\sqrt{\frac{\pi}{p}}e^{q^2/p}\quad (p>0)$		$\bullet$	$\bullet$	$\bullet$	$\boxed{\star}$	#	$\times$
W13	$\int_0^1\frac{1}{\log t}+\frac{1}{1-t}-\log(\log\frac{1}{t})\,dt \Rightarrow 2\gamma$							$\mathcal{E}$
W14	$\int_{-\infty}^{\infty}\frac{\sin t}{t}e^{2it}\,dt \Rightarrow 0$	ε		$\bullet$	$\natural$[38]	$\bullet$		
W15	$\int_0^1\log\Gamma(x)\cos 6\pi x\,dx \Rightarrow \frac{1}{12}$							τ
W16	$\int_{-1}^{1}(1+x)^3P_1(x)P_2(x)\,dx \Rightarrow \frac{36}{35}$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\mathcal{E}$	$\times$
W17	$\int_0^{\infty}e^{-ax}J_0(bx)\,dx \Rightarrow \frac{1}{\sqrt{a^2+b^2}}\quad (a>0)$		$\mathcal{E}$	$\bullet$	$\boxed{\star}$	$\bullet$	$\bullet$	$\bullet$
W18	$\int_0^{\infty}\left(\frac{J_1(x)}{x}\right)^2\,dx \Rightarrow \frac{4}{3\pi}$		$\mathcal{E}$		$\bullet$	$\bullet$	$\mathcal{E}$	
W19	$\int_0^{\infty}\mathrm{Ci}\,x\,J_0(2\sqrt{7x})\,dx \Rightarrow \frac{\cos 7-1}{7}$	$\mathcal{E}$	$\mathcal{E}$				—	
W20	$\int_0^1 x^2\mathrm{Li}_3(\frac{1}{x+1})\,dx \Rightarrow \frac{\frac{17}{3}+\pi^2}{36}+\cdots+\frac{\zeta(3)}{4}$	—						—

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W21	$\mathsf{N}(\int_0^1 x^2 \mathrm{Li}_3(\frac{1}{x+1})\, dx) = \mathsf{N}(") \approx 0.210883$	—	$\star$	$\bullet$	$\bullet$	$\bullet$	$\circ$	$\natural$
W22	$s(t) = \begin{cases} 1 & (1 \le t \le 2) \\ 0 & \text{otherwise} \end{cases}$; $\int_0^u s(t) \cos t\, dt$ **P**	$\times$	$\mid \bullet$	$\mid \uplus$	$\mathcal{E} \mid \bullet$	$\mid \star$	$\mathcal{E} \mid$	$\mathcal{E}$
W23	$\int_a^b \int_{-\infty}^{\infty} \frac{x}{x^2+y^2}\, dy\, dx = \int_{-\infty}^{\infty} \int_a^b \frac{x}{x^2+y^2}\, dx\, dy$	$\varepsilon \mid \circ$	$\bullet$	$\bullet$	$\otimes$	$\natural$	$\circ$	$\natural$
W24	$\int_0^1 \int_0^1 \sqrt{x^2+y^2}\, dx\, dy \Rightarrow \frac{\log(\sqrt{2}+1)+\sqrt{2}}{3}$	$\mathcal{E} \mid \natural$	$\bullet$	$\star$		$\bullet$		$\star$
W25	$\int_0^{\pi/2} \int_0^{\pi/2} \frac{\sin\alpha \sin y}{\sqrt{1-\sin^2\alpha \sin^2 x \sin^2 y}}\, dx\, dy \Rightarrow \frac{\pi\alpha}{2}$	ε	$\mathcal{E}$		$\star$		#	$\mathcal{E}$
W26	$\int_{-1}^1 \int_0^2 \lvert y - x^2 \rvert\, dy\, dx \Rightarrow \frac{46}{15}$	ε	$\bullet$	$\bullet$	$\bullet$	$\bullet$		#
W27	$\int_0^a \int_0^{b(1-x/a)} \int_0^{c(1-x/a-y/b)} 1\, dz\, dy\, dx \Rightarrow \frac{abc}{6}$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
	X. Series							
X1	$\mathsf{taylor}(\frac{1}{\sqrt{1-(v/c)^2}},\, v = 0) \Rightarrow 1 + \frac{1}{2}\frac{v^2}{c^2} + \cdots$	$\bullet$	$\star$	$\star$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
X2	$\frac{1}{\text{above}^2} \Rightarrow 1 - \frac{v^2}{c^2} + \cdots$	$\bullet$	$\circ$	$\bullet$	$\star$	$\bullet$	$\bullet$	$\star$
X3	$\frac{\mathsf{taylor}(\sin x,\, x=0)}{\mathsf{taylor}(\cos x,\, x=0)} = \mathsf{taylor}(\tan x,\, x = 0)$	$\bullet$	$\circ$	$\star$	$\star$	$\bullet$	$\bullet$	$\star$
X4	$\log[\mathsf{taylor}(\frac{\sin x}{x},\, x = 0)] \Rightarrow -\frac{x^2}{6} - \frac{x^4}{180} - \cdots$	$\bullet$	$\circ$	$\circ$	$\star$	$\star$	$\circ$	$\star$
X5	$\mathsf{taylor}(f^{(1)}(ax) + g(bx) + \int_0^x h(cy)\, dy, \ldots)$	ε	$\bullet$	$\circ$	$\bullet$	$\bullet$	#	$\circ$
X6	$\mathsf{taylor}(e^{(A+B)t} - e^{At}e^{Bt}, \ldots) \quad (AB \ne BA)$	—	$\times$	$\times$	$\times$	—	—	$\times$
X7	$\mathsf{taylor}(\frac{1}{x(e^x-1)},\, x = 0) \Rightarrow \frac{1}{x^2} - \frac{1}{2x} + \frac{1}{12} - \cdots$	$\bullet$	$\natural$	$\star$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
X8	$\mathsf{taylor}(\sqrt{\sec x},\, x = \frac{3\pi}{2}) \Rightarrow \frac{1}{\sqrt{x-3\pi/2}} + \cdots$	$\bullet$		$\star$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
X9	$\mathsf{taylor}(x^x,\, x = 0) \Rightarrow \sum_{k=0}^{\infty} \frac{(x \log x)^k}{k!}$	$\bullet$		$\star$	$\bullet$	$\bullet$	$\bullet$	
X10	$\mathsf{taylor}(\log(\sinh z) + \log(\cosh(z + w)))$	$\star$		$\star$	$\star$	$\bullet$	$\bullet$[39]	$\bullet$
X11	$\mathsf{taylor}(\log(\sinh z \cosh(z + w)),\, z = 0)$	$\star$		$\star$	$\star$[39]	$\bullet$	$\bullet$[39]	
X12	$\mathsf{taylor}((\log x)^a e^{-bx},\, x = 1) \Rightarrow \frac{(x-1)^a}{e^b} + \cdots$	$\mathcal{E}$		$\star$	$\mathcal{E}$		[39]	$\bullet$
X13	$\mathsf{taylor}(\sqrt{2x^2+1},\, x = \infty) \Rightarrow \sqrt{2}x + O(\frac{1}{x})$	ε		$\star$	$\bullet$	$\bullet$	$\bullet$[39]	$\bullet$
X14	$\mathsf{taylor}(\frac{1}{2^{2n}}\binom{2n}{n},\, n = \infty) \Rightarrow \frac{1}{\sqrt{\pi n}} + \cdots$	ε		$\mathcal{E}$	$\mathcal{E}$	ε	$\mathcal{E}$	
X15	$\mathsf{taylor}(e^x \int_x^{\infty} \frac{e^{-t}}{t}\, dx),\, x = \infty) \Rightarrow \frac{1}{x} - \cdots$	ε	$\mathcal{E}$	$\mathcal{E}$	$\bullet$			$\mathcal{E}$
X16	$\mathsf{taylor}(\cos(x + y),\, x = y = 0) \Rightarrow 1 - \cdots$	ε	$\circ$	$\star$	$\times$	$\star$	$\natural$	$\bullet$
X17	$\mathsf{power\_series}(\log \frac{\sin x}{x},\, x = 0)$	—	—	$\bullet$	$\mathcal{E}$	$\circ$		
X18	$\mathsf{power\_series}(e^{-x} \sin x,\, x = 0)$	—	—	$\uplus$	$\bullet$	$\star$	$\bullet$	$\otimes$
X19	$\mathsf{solve}(x = \sin y + \cos y,\, y = 0,\, \text{series})$	$\mathcal{E}$	$\star$	$\bullet$	$\star$	$\star$	$\circ$	$\star$
X20	$\mathsf{pade}(e^{-x},\, x = 0) \Rightarrow \frac{2-x}{2+x}$	$\star$	$\bullet$	$\star$	$\bullet$	$\bullet$	$\star$	$\bullet$[a]
X21	$\mathsf{fourier}(x,\, p) \Rightarrow -\frac{2p}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin \frac{n\pi x}{p}$	—	$\star$	$\bullet$	—	$\star$	—	—

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X22	fourier$(\lvert x\rvert,)\Rightarrow \frac{p}{2}-\frac{2p}{\pi^2}\sum_{n=1}^{\infty}\frac{1-(-1)^n}{n^2}\cos\frac{n\pi x}{p}$	—	$\star$	$\bullet$	—	$\circ$	—	—
	Y. Transforms[40]							
Y1	laplace$(\cos((\omega-1)t),\, t\to s)\Rightarrow \frac{s}{s^2+(\omega-1)^2}$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
Y2	inverse Laplace transform of above	$\star$	—	$\bullet$	$\bullet$	$\bullet$	$\star$	$\circ$
Y3	laplace$(\sinh\omega t\cosh\omega t,\, t\to s)\Rightarrow \frac{\omega}{s^2-4\omega^2}$	$\bullet$	$\bullet$	$\bullet$		$\bullet$	$\star$	$\bullet$
Y4	laplace$\left(\operatorname{erf}\left(\frac{3}{\sqrt{t}}\right),\, t\to s\right)\Rightarrow \frac{e^{-6\sqrt{s}}}{s}\quad (s>0)$	ε			$\otimes$			$\uplus$
Y5	laplace$(\frac{d^2y}{dt^2}+y=4[H(t-1)-H(t-2)])$	#	#	$\bullet$	$\bullet$	$\bullet$	$\bullet$	
Y6	solve " using inverse Laplace transforms		—	$\bullet$	$\bullet$	$\bullet$	#	—
Y7	laplace$(1+2\sum_{n=1}^{\infty}(-1)^n H(t-na),\, t\to s)$	ε		#	#			ε
Y8	fourier$(1,\, x\to z)\Rightarrow\sqrt{2\pi}\delta(z)$ [41]	d	d		$\bullet$	$\bullet$	$\bullet$	—
Y9	fourier$(e^{-9x^2},\, x\to z)\Rightarrow \frac{e^{-z^2/36}}{3\sqrt{2}}$ [41]	d	d	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
Y10	fourier$(\lvert x\rvert e^{-3\lvert x\rvert},\, x\to z)\Rightarrow\sqrt{\frac{2}{\pi}}\frac{9-z^2}{(9+z^2)^2}$ [41]	$\mathcal{E}^{\mathbf{d}}$	$\star^{\mathbf{d}}$	$\bullet$		$\bullet$		
Y11	Mellin_transform$(\frac{1}{1-x},\, x\to s)\Rightarrow\pi\cot\pi s$	d	d	d	$\bullet$	$\bullet^{\mathbf{d}}$		$\mathcal{E}^{\mathbf{d}}$
Y12	Mellin_transform$(\frac{J_3(x)}{x^3},\, x\to s)\Rightarrow\frac{2^{s-4}\Gamma(s/2)}{\Gamma(4-s/2)}$	$\mathcal{E}^{\mathbf{d}}$	$\mathcal{E}^{\mathbf{d}}$	d	$\star$	$\bullet^{\mathbf{d}}$		d
Y13	Z_transform$(H(t-3T),\, t\to z)\Rightarrow\frac{z}{z^3(z-1)}$	—	—	—	$\bullet$	—	—	—
Y14	Z_transform$(H(t-mT),\, t\to z)\Rightarrow\frac{z}{z^m(z-1)}$	—	—	—		—	—	—
	Z. Ordinary Difference and Differential Equations[42]							
Z1	solve$([r_{n+2}-2r_{n+1}+r_n=2,\,\ldots],\, r_n)$	—	$\star$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	—
Z2	solve$([r_n=5r_{n-1}-6r_{n-2},\,\ldots],\, r_n)$	—	$\star$	$\star$	$\bullet$	$\bullet$	$\bullet$	—
Z3	solve$([r_n=r_{n-1}+r_{n-2},\, r_1=1,\, r_2=2])$	—	$\circ$	$\circ$[43]	$\circ$	$\circ$	$\circ$	—
Z4	solve$([r_n=\frac{1+c-c^{n-1}-c^{n+1}}{1-c^n}r_{n-1}-\cdots],\, r_n)$	—	—	ε	ε	ε		—
Z5	solve$(\frac{d^2f}{dt^2}+4f=\sin 2t,\, f(0)=f'(0)=0)$	$\bullet$	$\circ$ | $\circ$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\natural$
Z6	above solution using Laplace transforms	$\natural$	#	$\bullet$	$\bullet$	$\star$	$\star$	
Z7	solve$(\frac{dy}{dx}=\frac{x^2}{y(1+x)^3},\, y(x))$	$\bullet$	$\star$ | $\star$	$\bullet$	$\star$	$\bullet$	$\star$	
Z8	solve$(\frac{dy}{dx}=\frac{y}{x}+\frac{x}{y})\Rightarrow y^2(x)=2x^2\log\lvert Ax\rvert$	$\boxed{\star}$	$\boxed{\star}$ | $\star$	$\bullet$	$\boxed{\star}$	$\bullet$	$\otimes$	$\mathcal{E}$
Z9	solve$(x^2\frac{dy}{dx}+3xy=\frac{\sin x}{x})\Rightarrow y=\frac{A-\cos x}{x^3}$	$\bullet$	$\star$ | $\star$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
Z10	solve$(\frac{dy}{dx}=-\frac{1+2x\sin y}{1+x^2\cos y},\, y(x))$	$\bullet$	$\natural$ | $\star$	$\bullet$	$\bullet$	τ		
Z11	solve$(\frac{d^2y}{dx^2}+y(\frac{dy}{dx})^3=0,\, y(x))\Rightarrow\frac{y^3(x)}{6}\cdots$	$\mathcal{E}$	$\natural$[44] | $\times$	$\bullet$	$\bullet$	$\star$	$\bullet$	
Z12	solve$([",\, y(0)=0,\, y'(0)=2],\, y(x))$	$\mathcal{E}$	$\times$	$\mathcal{E}$	$\uplus^{\mathbf{s}}$	$\uplus^{\mathbf{s}}$		—

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#	PROBLEM	**Ax**	**De**	**Mc**	**Mp**	**Mm**	**Mu**	**Re**
Z13	$\mathsf{solve}(\frac{d}{dx}y(x,a) = ay(x,a),\, y(x,a))$ [45]	$\mathcal{E}$		$\bullet$	$\bullet$	ε | $\bullet$	$\bullet$	$\bullet$
Z14	$\mathsf{solve}([\frac{d^2y}{dx^2} + k^2y = 0,\, y(0)\cdots = 0],\, [y,k])$ [46]		#					
Z15	$\mathsf{solve}(\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + \frac{1}{4x}(1 - \frac{\nu^2}{x})y = 0,\, y(x))$		$\star$	$\bullet$	$\bullet$	$\bullet$		
Z16	$\mathsf{solve}(\frac{dy}{dt} + ay(t-1) = 0,\, y(t))$		—		$\mathcal{E}$	ε	ε	
Z17	discontinuous ODE [P]	$\times$ | ε	$\uplus$	$\mathcal{E}$ | $\times$	$\mathcal{E}$ | $\times$	$\times$ | $\times$	$\mathcal{E}$	$\mathcal{E}$
Z18	$\mathsf{solve}(\frac{di}{dt} + 2i + 5\int_0^t i(\tau)\,d\tau = 10e^{-4t},\, i(t))$	ε	—		$\mathcal{E}$			
Z19	above solution using Laplace transforms	ε	#	$\bullet$	$\bullet$	$\star$	#	
Z20	$\mathsf{solve}([\frac{dx}{dt} = x - y,\, \frac{dy}{dt} = x + y],\, [x(t),\, y(t)])$	—	—	$\bullet$	$\star$	$\bullet$ | $\circ$	$\star$	—
Z21	verify the solution of the above	$\circ$	$\bullet$	$\circ$	$\circ$	$\circ$ | $\star$	$\boxed{\star}$	$\star$
Z22	$\mathsf{solve}([\frac{dx}{dt} = x\left(1 + \frac{\cos t}{2+\sin t}\right),\, \frac{dy}{dt} = x - y])$	—	—	$\bullet$	$\star$	$\bullet$	$\mathcal{E}$	—
Z23	as above, but one equation at a time	$\star$	$\circ$ | $\circ$	$\star$	$\star$	$\star$	$\circ$	$\natural$
Z24	3×3 linear system $(\lambda = 2,1,3)$	—	—	$\bullet$	$\bullet$	$\bullet$	$\bullet$	—
Z25	3×3 linear system $(\lambda = 0, -1 \pm i\sqrt{2})$	—	—	$\bullet$	$\bullet$	$\circ$	$\circ$	—
Z26	3×3 linear system $(\lambda = 2,2,2)$	—	—	$\bullet$	$\bullet$	$\bullet$	$\uplus$	—
Z27	$\mathsf{solve}([\frac{d^2x}{dt^2} = 2\omega\frac{dy}{dt},\, \frac{d^2y}{dt^2} = -2\omega\frac{dx}{dt} + 3\omega^2y])$	—	—	$\bullet$	$\bullet$	$\bullet$	$\uplus$	—
	a. Partial Differential Equations[47]							
a1	$\mathsf{solve}(\frac{\partial^2 f}{\partial x\,\partial y} = 0,\, f(x,y)) \Rightarrow g(x) + h(y)$	—	—		$\bullet$	$\boxed{\star}$	—	$\bullet$
a2	$\mathsf{solve}(\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},\, u(x,t))$ [Heat equation]	—	—	$\circ$[48]	$\natural$		—	
a3	$\mathsf{substitute}(u = \frac{1}{\sqrt{t}}f(\frac{x}{\sqrt{t}}),\, \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2})$	$\circ$	$\bullet$	$\star$	$\star$	$\circ$		$\circ$ | $\star$
a4	$\mathsf{substitute}(x = z\sqrt{t},$ above)	$\star$	$\otimes$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\otimes$ | $\star$
a5	$\mathsf{solve}(2\frac{d^2f}{dz^2} + z\frac{df}{dz} + f = 0,\, f(z))$ [49]	$\mathcal{E}$	$\circ$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	
a6	$\mathsf{solve}(\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial v}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 v}{\partial\theta^2} = 0,\, v(r,\theta))$	—	—		$\circ$		—	
	b. Operators							
b1	$f = e^x,\, g = x^2;\quad (f+2g)(y) \Rightarrow e^y + 2y^2$	ε	—	$\bullet$	$\bullet$	#	$\bullet$	
b2	$(f\circ g)(y) \Rightarrow e^{y^2}$	ε	—	$\bullet$	$\bullet$	$\otimes$	$\bullet$	
b3	$L = (D-1)(D+2)$ (operator)	$\star$	—	$\bullet$	$\star$	$\bullet$	$\bullet$	—
b4	$L(f) \Rightarrow D^2f + Df - 2f$ (pure function)	$\star$	—	$\star$	$\bullet$	$\natural$	$\bullet$	—
b5	$L_y(g(y)) \Rightarrow \frac{d^2g}{dy^2} + \frac{dg}{dy} - 2g(y)$	$\circ$	—	$\star$	$\bullet$	$\natural$	$\bullet$	—
b6	$L_z(A\sin z^2) \Rightarrow 2A[(1+z)\cos z^2 - \cdots]$	$\circ$	—	$\star$	$\bullet$	$\natural$	$\bullet$	—
b7	$T = \sum_{k=0}^{2}\frac{(D^kf)(a)}{k!}(x-a)^k$ (operator)	$\circ$	—	$\star$	$\star$	$\star$	$\bullet$	$\star$

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#	PROBLEM	**Ax**	**De**	**Mc**	**Mp**	**Mm**	**Mu**	**Re**
b8	$T(f) \Rightarrow f(a) + (Df)(a)(x-a) + \cdots$	$\star$	—	$\star$	$\bullet$	$\circ$	$\star$	$\circ$
b9	$T_{y,b}(g(y)) \Rightarrow g(b) + \frac{dg}{dy}\vert_{y=b}(y-b) + \cdots$	$\bullet$	—	$\bullet$	$\bullet$	$\star$	$\bullet$	$\star$
b10	$T_{z,c}(\sin z) \Rightarrow \sin c + \cos c\,(z-c) - \cdots$	$\bullet$	—	$\bullet$	$\bullet$	$\star$	$\bullet$	$\star$
b11	define $\star$ so that $x \star y = \sqrt{x^2+y^2}$	—	—	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
b12	$3 \star 4 \star 12 \Rightarrow 13$ (make $\star$ associative)	—	—	$\bullet$	$\bullet$	$\circ$	$\times$	$\bullet$
b13	define $\vert\ \vert$ so that $\vert x\vert = \mathrm{abs}(x)$	—	—	$\bullet$	—	—	—	—
	c. Programming and Miscellaneous							
c1	substitute$(a+b=x,\ (a+b+c)^2+\cdots)$	$\circ$	$\circ$	$\star$	$\star$	$\star$	$\circ$	$\circ$
c2	substitute$(\sqrt{x^2+y^2}=r,\ \frac{x}{\sqrt{x^2+y^2}}) \Rightarrow \frac{x}{r}$	$\bullet$	$\circ$	$\bullet$	$\star$	$\star$	$\star$	$\bullet$
c3	change variables in a messy expression	$\natural$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$
c4	$f'g''+f'g' \Rightarrow (fg')'$ (rewrite rule)		—	$\natural$[50]		$\bullet$		$\mathcal{E}$
c5	$2(f'g''+f'g')+fg \Rightarrow 2(fg')'+fg$ (")	$\mathcal{E}$	—	$\natural$[50]	$\natural$[50]	$\bullet$		$\mathcal{E}$
c6	multiply two infinite lists together	$\bullet$	$\circ$	$\star$	$\circ$	$\star$	$\circ$	$\circ$
c7	compute Legendre polys directly	$\bullet$	$\bullet$	$\bullet$	$\star$	$\bullet$	$\star$	$\bullet$
c8	compute Legendre polys recursively	$\star$[51]	$\bullet$	$\bullet$	$\star$[51]	$\bullet$	$\star$[51]	$\bullet$
c9	evaluate the 4^{th} Legendre poly at 1 [52]	$\star \mid \bullet$	$\star$	$\bullet$	$\star$	$\star \mid \bullet$	$\star$	$\star \mid \bullet$
c10	define iterative Fibonacci number func.	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
c11	translate to Fortran syntax	ε	$\times$	ε	$\star$	$\times$	ε	
c12	create $[f_0,\ldots,f_{10}] \Rightarrow [0,1,1,2,\ldots,55]$	$\bullet$	$\bullet$	$\bullet$	$\star$	$\bullet$	$\bullet$	$\bullet$
c13	define a simple derivative operator	$\star$		$\circ$	$\circ$	$\bullet$	$\circ$	$\bullet$
c14	define p so $p(2+i\sqrt{3})=0$, $p(\begin{pmatrix} 1 & -2 \\ 2 & 3 \end{pmatrix}) = \mathbf{0}$	$\bullet$	$\bullet$	$\otimes$	$\otimes$[53]	$\otimes$	$\otimes$	$\otimes$
c15	define a function as a calculation result	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
c16	display an expression's top-level structure	—	—	$\bullet$	$\natural$	$\natural$	$\natural$	$\circ$
c17	translate $y = \sqrt{\frac{e^{x^2}+e^{-x^2}}{\sqrt{3}\,x-\sqrt{2}}}$ to TeX/LaTeX[54]	$\star$	—	$\bullet$	$\bullet$	$\circ$	$\natural$	$\star \mid \star$
	d. Mathematics versus Computer Science							
d1	(3^{rd} k local?) $k \leftarrow 1;$ $\sum_{k=1}^{4} k \Rightarrow 10$	ε	$\bullet$	$\bullet$	ε	$\bullet$	ε	ε
d2	(3^{rd} k local?) $k \leftarrow 1;$ $\prod_{k=1}^{3} k \Rightarrow 6$	ε	$\bullet$	$\bullet$	ε	$\bullet$	ε	ε
d3	(3^{rd} k local?) $k \leftarrow 1;$ $\lim_{k\to 0} k \Rightarrow 0$	ε	$\bullet$	ε	ε	ε	ε	$\times$
d4	(3^{rd} k local?) $k \leftarrow 1;$ $\int_{k=0}^{1} k\,dk \Rightarrow \frac{1}{2}$	ε	$\bullet$	ε	ε	ε	ε	ε

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#	PROBLEM	**Ax**	**De**	**Mc**	**Mp**	**Mm**	**Mu**	**Re**
d5	match$(\exp(y),\, \exp(x))$	•	—	•	•	•	•	•
d6	match$(e^y,\, \exp(x))$	•	—	•	⋆	•		
d7	match$(\exp(y),\, e^x)$	•	—	•	⋆	•		•
d8	match$(e^y,\, e^x)$	•	—	•	⋆	•	•	•
d9	match$(\sqrt{y},\, \sqrt{x})$	•	—	•	⋆	•	•	•
d10	match$(y^{1/2},\, \sqrt{x})$	•	—	•	⋆	•	•	
d11	match$(\sqrt{y},\, x^{1/2})$	•	—	•	⋆	•	•	•
d12	match$(y^{1/2},\, x^{1/2})$	•	—	•	⋆	•	•	•
d13	match$(iy,\, ix)$		—	•	•	•	•	•
d14	match$(\sqrt{-1}\,y,\, ix)$	ε	—	•	•	•	•	•
d15	match$((-1)^{1/2}y,\, ix)$	ε	—	•	•	•	•	•
d16	match$(iy,\, \sqrt{-1}\,x)$	ε	—	•	•	•	•	•
d17	match$(\sqrt{-1}\,y,\, \sqrt{-1}\,x)$	•	—	•	•	•	•	•
d18	match$((-1)^{1/2}y,\, \sqrt{-1}\,x)$	•	—	•	•	•	•	•
d19	match$(iy,\, (-1)^{1/2}x)$ [55]	$\mathcal{E}$ | ε	—	•	•	•	•	•
d20	match$(\sqrt{-1}\,y,\, (-1)^{1/2}x)$ [55]	$\mathcal{E}$ | •	—	•	•	•	•	•
d21	match$((-1)^{1/2}y,\, (-1)^{1/2}x)$ [55]	$\mathcal{E}$ | •	—	•	•	•	•	•

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