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Computational Fluid Mechanics and Heat Transfer

Second Edition

John C. Tannehill
Dale A. Anderson
Richard H. Pletcher

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Second Edition

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To our wives and children: Marcia, Michelle, and John Tannehill
Marleen, Greg, and Lisa Anderson
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PREFACE

Almost fifteen years have passed since the first edition of this book was written. During the intervening years the literature in computational fluid dynamics (CFD) has expanded manyfold. Due in part to greatly enhanced computer power, the general understanding of the capabilities and limitations of algorithms has increased. A number of new ideas and methods have appeared. The authors have attempted to include new developments in this second edition while preserving those fundamental ideas covered in the first edition that remain important for mastery of the discipline. Ninety-five new homework problems have been added. The two part, ten chapter format of the book remains the same, although a shift in emphasis is evident in some of the chapters. The book is still intended to serve as an introductory text for advanced undergraduates and/or first-year graduate students. The major emphasis of the text is on finite-difference/finite-volume methods.

The first part, consisting of Chapters 1–4, presents basic concepts and introduces the reader to the fundamentals of finite-difference/finite-volume methods. The second part of the book, Chapters 5–10, is devoted to applications involving the equations of fluid mechanics and heat transfer. Chapter 1 serves as an introduction and gives a historical perspective of the discipline. This chapter has been brought up to date by reflecting the many changes that have occurred since the introduction of the first edition. Chapter 2 presents a brief review of those aspects of partial differential equation theory that have important implications for numerical solution schemes. This chapter has been revised for improved clarity and completeness. Coverage of the basics of discretization methods begins in Chapter 3. The second edition provides a more thorough introduction to the finite-volume method in this chapter. Chapter 4 deals with the application of numerical methods to selected model equations. Several additions have been made to this chapter. Treatment of methods for solving the wave equation now includes a discussion of Runge-Kutta schemes. The Keller box and modified box methods for solving parabolic equations are now included in Chapter 4. The method of approximate factorization is explained and demonstrated. The material on solution strategies for Laplace's equation has been revised and now contains an introduction to the multigrid method for both linear and nonlinear equations. Coloring schemes that can take advantage of vectorization are introduced. The material on discretization methods for the inviscid Burgers equation has been substantially revised in order to reflect the many developments, particularly with regard to upwind methods, that have occurred since the material for the first edition was drafted. Schemes due to Godunov, Roe, and Enquist and Osher are introduced. Higher-order upwind and total variation diminishing (TVD) schemes are also discussed in the revised Chapter 4.

The governing equations of fluid mechanics and heat transfer are presented in Chapter 5. The coverage has been expanded in several ways. The equations necessary to treat chemically reacting flows are discussed. Introductory information on direct and large-eddy simulation of turbulent flows is included. The filtered equations used in large-eddy simulation are presented as well as the Reynolds-averaged equations. The material on turbulence modeling has been augmented and now includes more details on one- and two-equation and Reynolds stress models as well as an introduction to the subgrid-scale modeling required for large-eddy simulation. A section has been added on the finite-volume formulation, a discretization procedure that proceeds from conservation equations in integral form.

Chapter 6 on methods for the inviscid flow equations is probably the most extensively revised chapter in the second edition. The revised chapter contains major new sections on flux splitting schemes, flux difference splitting schemes, the multidimensional case in generalized coordinates, and boundary conditions for the Euler equations. The chapter includes a discussion on implementing the integral form of conservation statements for arbitrarily shaped control volumes, particularly triangular cells, for two-dimensional applications.

Chapter 7 on methods for solving the boundary-layer equations includes new example applications of the inverse method, new material on the use of generalized coordinates, and a useful coordinate transformation for internal flows. In Chapter 8 methods are presented for solving simplified forms of the Navier-Stokes equations including the thin-layer Navier-Stokes (TLNS) equations, the parabolized Navier-Stokes (PNS) equations, the reduced Navier-Stokes (RNS) equations, the partially-parabolized Navier-Stokes (PPNS) equations, the viscous shock layer (VSL) equations, and the conical Navier-Stokes (CNS) equations. New material includes recent developments on pressure relaxation, upwind methods, coupled methods for solving the partially parabolized equations for subsonic flows, and applications.

Chapter 9 on methods for the "complete" Navier-Stokes equations has undergone substantial revision. This is appropriate because much of the research and development in CFD since the first edition appeared has been concentrated on solving these equations. Upwind methods that were first introduced in the context of model and Euler equations are described as they extend to the full Navier-Stokes equations. Methods to efficiently solve the compressible equations at very low Mach numbers through low Mach number preconditioning are described. New developments in methods based on derived variables, such as the dual potential method, are discussed. Modifications to the method of artificial compressibility required to achieve time accuracy are developed. The use of space-marching methods to solve the steady Navier-Stokes equations is described. Recent advances in pressure-correction (segregated) schemes for solving the Navier-Stokes equations such as the use of non-staggered grids and the pressure-implicit with splitting of operators (PISO) method are included in the revised chapter.

Grid generation, addressed in Chapter 10, is another area in which much activity has occurred since the appearance of the first edition. The coverage has been broadened to include introductory material on both structured and unstructured approaches. Coverage now includes algebraic and differential equation methods for constructing structured grids and the point insertion and advancing front methods for obtaining unstructured grids composed of triangles. Concepts employed in constructing hybrid grids composed of both quadrilateral cells (structured) and triangles, solution adaptive grids, and domain decomposition schemes are discussed.

We are grateful for the help received from many colleagues, users of the first edition and others, while this revision was being developed. We especially thank our colleagues Ganesh Rajagopalan, Alric Rothmayer, and Ijaz Parpia. We also continue to be indebted to our students, both past and present, for their contributions. We would like to acknowledge the skillful preparation of several new figures by Lynn Ekblad. Finally, we would like to thank our families for their patience and continued encouragement during the preparation of this second edition.

This text continues to be a collective work by the three of us. There is no junior or senior author. A coin flip determined the order of authors for the first edition, and a new coin flip has determined the order of authors for this edition.

John C. Tannehill
Dale A. Anderson
Richard H. Pletcher

PREFACE TO THE FIRST EDITION

This book is intended to serve as a text for introductory courses in computational fluid mechanics and heat transfer [or, synonymously, computational fluid dynamics (CFD)] for advanced undergraduates and/or first-year graduate students. The text has been developed from notes prepared for a two-course sequence taught at Iowa State University for more than a decade. No pretense is made that every facet of the subject is covered, but it is hoped that this book will serve as an introduction to this field for the novice. The major emphasis of the text is on finite-difference methods.

The material has been divided into two parts. The first part, consisting of Chapters 1–4, presents basic concepts and introduces the reader to the fundamentals of finite-difference methods. The second part of the book, consisting of Chapters 5–10, is devoted to applications involving the equations of fluid mechanics and heat transfer. Chapter 1 serves as an introduction, while a brief review of partial differential equations is given in Chapter 2. Finite-difference methods and the notions of stability, accuracy, and convergence are discussed in the third chapter.

Chapter 4 contains what is perhaps the most important information in the book. Numerous finite-difference methods are applied to linear and nonlinear model partial differential equations. This provides a basis for understanding the results produced when different numerical methods are applied to the same problem with a known analytic solution.

Building on an assumed elementary background in fluid mechanics and heat transfer, Chapter 5 reviews the basic equations of these subjects, emphasizing forms most suitable for numerical formulations of problems. A section on turbulence modeling is included in this chapter. Methods for solving inviscid flows using both conservative and nonconservative forms are presented in Chapter 6. Techniques for solving the boundary-layer equations for both laminar and turbulent flows are discussed in Chapter 7. Chapter 8 deals with equations of a class known as the "parabolized" Navier-Stokes equations which are useful for flows not adequately modeled by the boundary-layer equations, but not requiring the use of the full Navier-Stokes equations. Parabolized schemes for both subsonic and supersonic flows over external surfaces and in confined regions are included in this chapter. Chapter 9 is devoted to methods for the complete Navier-Stokes equations, including the Reynolds averaged form. A brief introduction to methods for grid generation is presented in Chapter 10 to complete the text.

At Iowa State University, this material is taught to classes consisting primarily of aerospace and mechanical engineers, although the classes often include students from other branches of engineering and earth sciences. It is our experience that Part I (Chapters 1–4) can be adequately covered in a one-semester, three-credit-hour course. Part II of the book contains more information than can be covered in great detail in most one-semester, three-credit-hour courses. This permits Part 2 to be used for courses with different objectives. Although we have found that the major thrust of each of Chapters 5 through 10 can be covered in one semester, it would also be possible to use only parts of this material for more specialized courses. Obvious modules would be Chapters 5, 6 and 10 for a course emphasizing inviscid flows or Chapters 5, 7–9, (and perhaps 10) for a course emphasizing viscous flows. Other combinations are clearly possible. If only one course can be offered in the subject, choices also exist. Part I of the text can be covered in detail in the single course or, alternatively, only selected material from Chapters 1–4 could be covered as well as some material on applications of particular interest from Part II. The material in the text is reasonably broad and should be appropriate for courses having a variety of objectives.

For background, students should have at least one basic course in fluid dynamics, one course in ordinary differential equations, and some familiarity with partial differential equations. Of course, some programming experience is also assumed.

The philosophy used throughout the CFD course sequence at Iowa State and embodied in this text is to encourage students to construct their own computer programs. For this reason, "canned" programs for specific problems do not appear in the text. Use of such programs does not enhance basic understanding necessary for algorithm development. At the end of each chapter, numerous problems are listed that necessitate numerical implementation of the text material. It is assumed that students have access to a high-speed digital computer.

We wish to acknowledge the contributions of all of our students, both past and present. We are deeply indebted to F. Blottner, S. Chakravarthy, G. Christoph, J. Daywitt, T. Holst, M. Hussaini, J. Ievalts, D. Jespersen, O. Kwon, M. Malik, J. Rakich, M. Salas, V. Shankar, R. Warming, and many others for helpful suggestions for improving the text. We would like to thank Pat Fox and her associates for skillfully preparing the illustrations. A special thanks to Shirley Riney for typing and editing the manuscript. Her efforts were a constant source of encouragement. To our wives and children, we owe a debt of gratitude for all of the hours stolen from them. Their forbearance is greatly appreciated.

Finally, a few words about the order in which the authors' names appear. This text is a collective work by the three of us. There is no junior or senior author. The final order was determined by a coin flip. Despite the emphasis of finite-difference methods in the text, we resorted to a "Monte Carlo" method for this determination.

Dale A. Anderson
John C. Tannehill
Richard H. Pletcher

PART
ONE

FUNDAMENTALS

PART
TWO

APPLICATION OF NUMERICAL METHODS TO THE EQUATIONS OF FLUID MECHANICS AND HEAT TRANSFER