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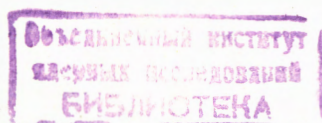
1989

Объединенный институт ядерных исследований
Joint Institute for Nuclear Research

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Дубна 1989

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НЕКОТОРЫЕ ЗАМЕЧАНИЯ ПО ПОВОДУ РЕЗУЛЬТАТОВ ПОИСКА ДИПРОТОННЫХ РЕЗОНАНСОВ

В.Н.Емельяненко, В.А.Никитин, М.Г.Шафранова

Анализируются данные работы Трояна Ю.А. и др.^{/1/}, посвященной поиску дипротонных резонансов в спектре эффективной массы двух протонов из реакции $pp \rightarrow pp \pi^-$ в интервале импульсов нейтронов от 1,25 до 2,23 ГэВ/с. Авторы^{/1/} заявляют об обнаружении ими шестнадцати узких (с шириной ~ 1 МэВ/с²) дипротонных резонансов. Результаты анализа и выводы в определяющей степени зависят от величины и формы фонового распределения. Вместе с тем различные модели фона дают существенно разные результаты. В данной работе сделана попытка описать спектр эффективной массы из работы Трояна Ю.А. и др. с помощью гладкой функции без привлечения узких резонансов. Получены значения $\chi^2/\text{d.f.}$, близкие к единице; они несколько меньше, чем у Трояна Ю.А. и др., при введении резонансов. Вывод работы Трояна Ю.А. и др. о существовании большого числа узких дипротонных резонансов следует признать недостаточно обоснованным.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

Some Remarks on the Results of Diproton Resonance Search

V.N.Emel'yanenko, V.A.Nikitin, M.G.Shafranov

The data from the paper of Troyan et al.^{/1/} on the search for the narrow diproton resonances in the two-proton effective mass spectrum from the reaction $pp \rightarrow pp \pi^-$ are analyzed. These experimental data were obtained over the neutron momentum range from 1.25 to 2.23 GeV/c. The authors^{/1/} declare the existence of sixteen narrow diproton resonances with the width of ~ 1 MeV/c². The results of the analysis and conclusions critically depend on the value and shape of the background distribution. Withal various background models give significantly different results. In the present paper we tried to describe the effective mass spectrum from the paper^{/1/} by the smooth function without using narrow resonances. The values of $\chi^2/\text{d.f.}$ near 1 and somewhat less than those obtained by Troyan et al. using the resonances are obtained. The

conclusion of the paper ^{/1/} on the existence of a large number of narrow diproton resonances should be admitted as not enough justified.

The investigation has been performed at the Laboratory of High Energies, JINR.

Введение

В ряде работ ведется поиск узких дибарионных резонансов. Их обнаружение могло бы свидетельствовать о существовании шестикварковой системы, имеющей уровни возбуждения цветовых степеней свободы. В моделях мешков и струн КХД такие состояния возможны. Существование дибарионов явилось бы фактом фундаментального значения. По этой причине необходим всесторонний критический анализ имеющегося экспериментального материала, чтобы уяснить достоверность выводов о существовании этих экзотических систем.

В данной работе анализируются результаты поиска дипротонных резонансов, полученные при исследовании реакции $p \rightarrow p p \pi^-$ Ю.А.Трояном и др. ^{/1/}. Распределение эффективной массы двух протонов, возникающих в указанной выше реакции, изучалось этими авторами при четырех значениях импульса первичного пучка: 1,257; 1,43; 1,72 и 2,23 ГэВ/с.

На рис. 1 представлен спектр эффективной массы двух протонов, полученный авторами работы ^{/1/} при импульсе нейтрона $p_n = 1,25$ ГэВ/с. Как отмечают сами авторы, вопрос о фоне при исследовании резонансов является одним из главных. На этом же рисунке показаны фоновые кривые, полученные в ^{/1/} тремя методами. Штрихпунктирная фоновая кривая получена методом "перемешивания", в котором протон из одного события комбинируется с протоном из другого. Как видно из рисунка, данный фон даже качественно не отражает экспериментального распределения (авторы ^{/1/} указывают на это). Во втором случае фоновая кривая, которая показана пунктиром на рис. 1, рассчитывается в рамках модели одночастичного обмена. Для описания реакции $p \rightarrow p p \pi^-$ используются матричные элементы трех подпроцессов: с обменом π -мезоном, Δ_{33} -изобарой и обменом протоном. В расчет фона заложен ряд модельных представлений, обоснованность которых не вполне очевидна. Третья фоновая кривая представлена на рис. 1 сплошной линией. В этом случае моделирование ведется на основе экспериментальных распределений в лабораторной системе (МЭЛС). В этом методе точка из графика p_1 vs p_2 в л.с. (p_1 и

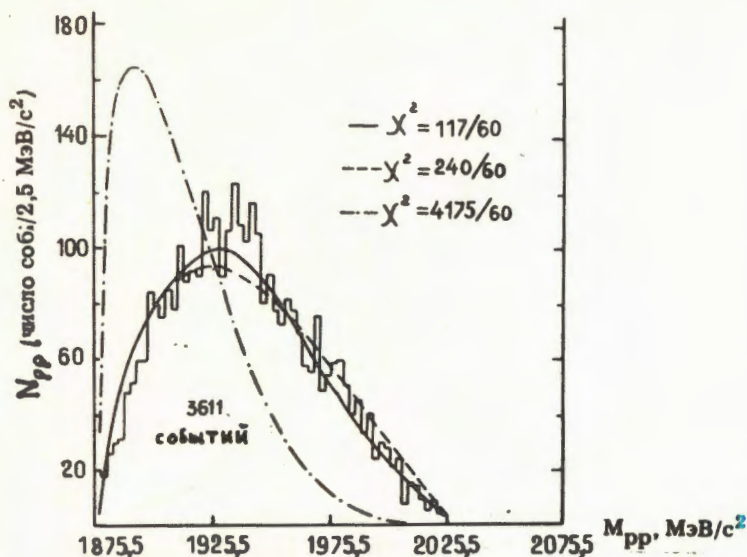


Рис. 1. Спектр эффективных масс двух протонов из реакции $p \rightarrow p p \pi^-$ при импульсе нейтрона $p_n = 1,25$ ГэВ/с из работы Трояна Ю.А. и др.^{/1/}. Штрих-пунктирная кривая получена методом перемешивания протонов из разных событий, пунктирная – по модели одночастичного обмена, сплошная – моделированием (МЭЛС). Рисунок взят из работы Трояна Ю.А. и др.^{/1/}.

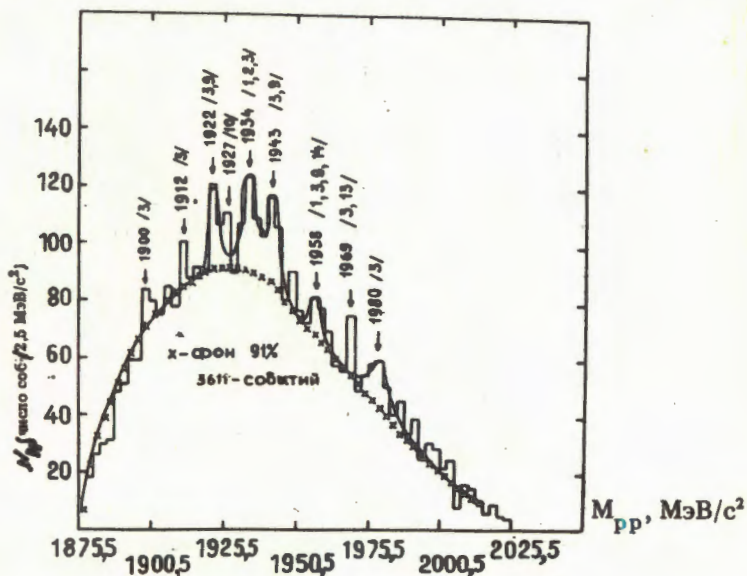


Рис. 2. Тот же спектр эффективных масс, что и на рис. 1. Сплошная кривая – результат описания пятью брейт-вигнеровскими резонансными кривыми и фоновой кривой, полученной методом МЭЛС. Рисунок взят из работы^{/1/}.

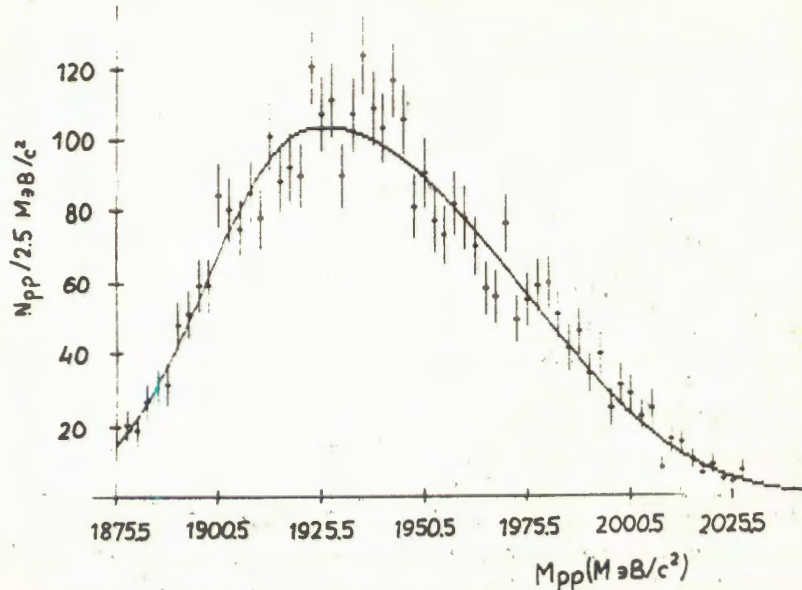


Рис. 3. Тот же спектр эффективных масс, что и на рис. 1 и 2, и результаты нашей аппроксимации его гладкой функцией $F_0(M_{pp})$ без введения резонансов.

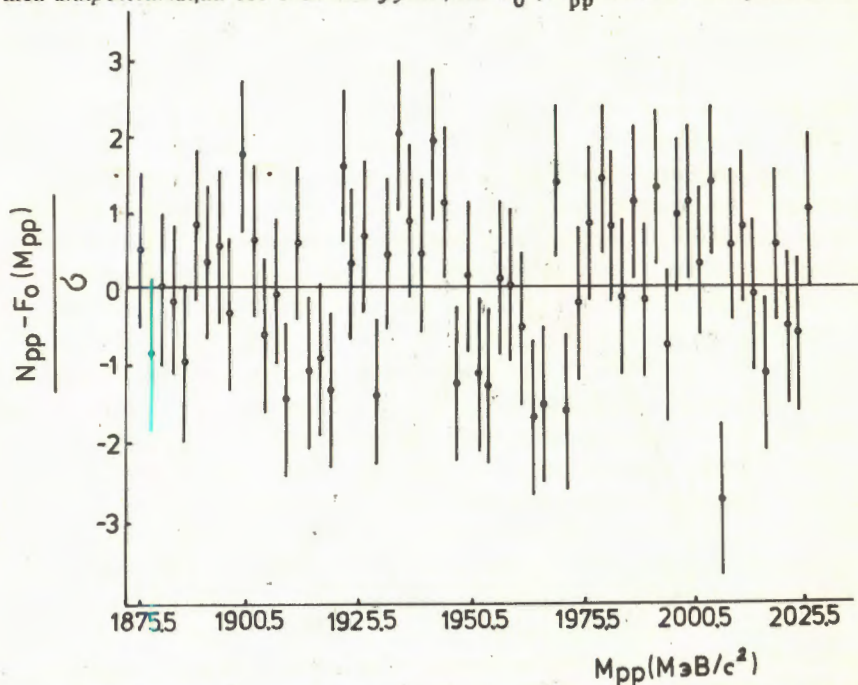


Рис. 4. Значения $(N_{pp} - F_0(M_{pp}))/\sigma$, соответствующие данным на рис. 3. σ — статистическая ошибка в величине N_{pp} .

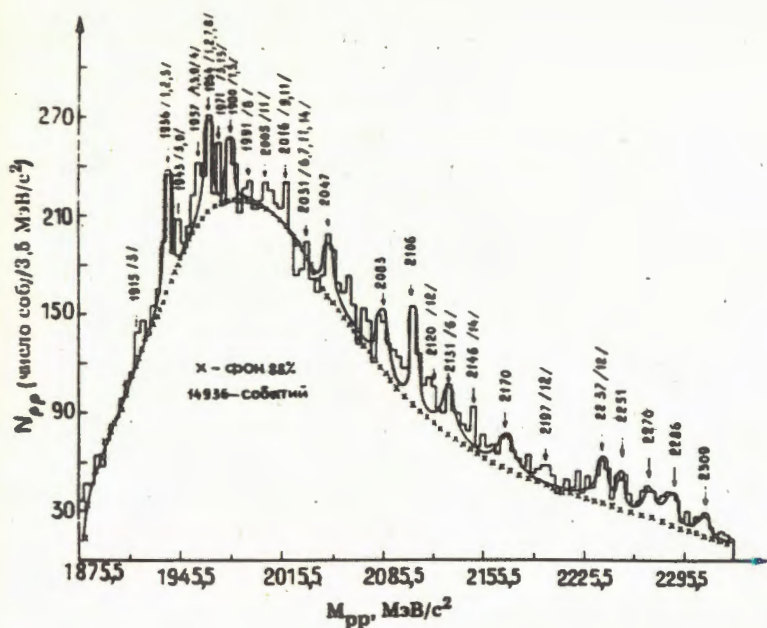


Рис. 5. Суммарный спектр эффективных масс двух протонов из реакции $pp \rightarrow pp \pi^-$ при $p_{\text{п}} = 1,43; 1,72$ и $2,23$ ГэВ/с. Сплошная кривая – результат описания тринадцатью брейт-вигнеровскими резонансными кривыми и фоновой кривой, полученной методом МЭЛС. Рисунок взят из работы Трояна Ю.А. и др. /1/.

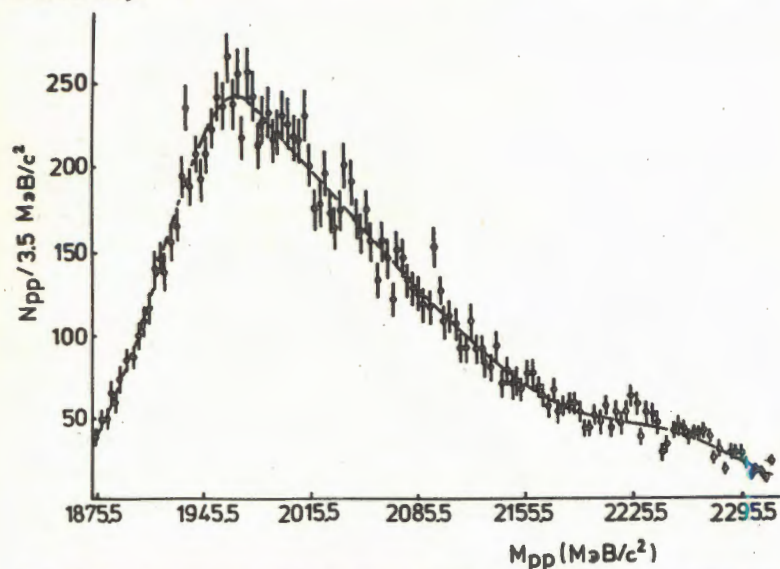


Рис. 6. Тот же спектр эффективных масс, что и на рис. 5, и результаты нашей аппроксимации его гладкой функцией $F(M_{\text{pp}})$ без введения резонансов.

p_2 — импульсы двух протонов в событии) сопоставляется случайным образом со значением угла между двумя протонами в л.с. Вычисленное значение эффективной массы двух протонов используется для получения фонового распределения.

Таким образом, можно констатировать (и это признают авторы работы ^{/1/}), что результаты определения фона в сильной степени зависят от модели, заложенной в его расчет.

В итоге авторы ^{/1/} представляют имеющиеся на рис. 1 данные в виде суммы фоновой кривой и пяти брейт-вигнеровских резонансных кривых (см. рис. 2). Фоновая кривая получена ими методом МЭЛС. Результат обработки дал $\chi^2/\text{d.f.} = 1,3$.

Поскольку неопределенность фона велика, возникает естественный вопрос, нельзя ли экспериментальные данные ^{/1/} описать гладкой кривой без привлечения резонансов.

На рис. 3 показаны тот же спектр эффективной массы двух протонов, что и на рис. 1 и 2, и результаты нашей аппроксимации этого спектра функцией вида

$$F_0(M_{pp}) = A_1 \cdot \exp \{ -|T/(1 + A_5 \cdot S)|^{A_4/(1 + A_6 \cdot S)} \}, \quad (1)$$

$$T = (M_{pp} - A_2) / A_3, \quad S = \text{sign}(T) \cdot [1 - \exp(-|T|)],$$

при значении параметров $A_1 = 103,7$; $A_2 = 1925$; $A_3 = 46,62$; $A_4 = 2,12$; $A_5 = 0,405$; $A_6 = -0,202$. Такая аппроксимация дает $\chi^2/\text{d.f.} = 1,21$.

На рис. 4 приведены значения $(N_{pp} - F_0(M_{pp}))/\sigma$, соответствующие данным на рис. 3. N_{pp} — экспериментальное значение числа двухпротонных событий, σ — статистическая ошибка в величине N_{pp} из ^{/1/}, F_0 — рассчитано по формуле (1). Из рисунка видно, что за пределы трехкратной ошибки из 61 точки не отклоняется ни одна.

Аналогичным образом нами был проанализирован суммарный спектр эффективной массы двух протонов из реакции $p \rightarrow pp \pi^-$ при $p_n = 1,43$; 1,72 и 2,23 ГэВ/с, полученный в той же работе ^{/1/}.

На рис. 5 сплошной кривой показаны результаты аппроксимации этого спектра Трояном Ю.А. и др. ^{/1/} тринадцатью брейт-вигнеровскими резонансными кривыми и фоновой кривой, полученной методом МЭЛС.

На рис. 6 приведены результаты нашей аппроксимации этих данных функцией вида

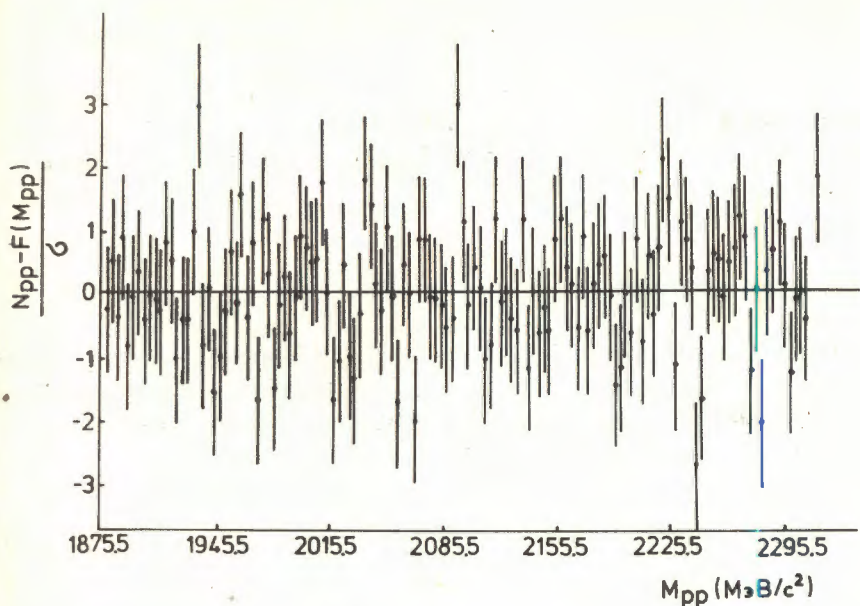


Рис. 7. Значения $(N_{pp} - F(M_{pp})) / \sigma$, соответствующие данным на рис. 6.

$$F(M_{pp}) = F_0(M_{pp}) + A_7 \cdot \exp \left\{ -[(M_{pp} - A_8) / A_9]^2 \right\}, \quad (2)$$

при значении параметров $A_1 = 244,29$; $A_2 = 1970,65$; $A_3 = 104,07$; $A_4 = 1,57$; $A_5 = 0,72$; $A_6 = -0,13$; $A_7 = 21,23$; $A_8 = 2265,89$; $A_9 = 60,80$. Такая аппроксимация дает $\chi^2 / d.f. = 1,00$.

На рис. 7 приведены значения $(N_{pp} - F(M_{pp})) / \sigma$. Ни одна из экспериментальных точек не выпадает за тройную статистическую ошибку.

Вывод

Мы описали спектр эффективной массы двух протонов в реакции $pp \rightarrow pp \pi^-$ /1/ гладкой функцией без введения узких резонансов. Вывод авторов /1/ о наличии большого количества резонансов с $\Gamma \sim 1 \div 3$ МэВ нельзя считать обоснованным, так как выделение из спектра эффективной массы системы (pp) нерезонансного фона модельно-зависимо и имеет большую неопределенность.

З а к л ю ч е н и е

Авторы выражают благодарность А.А.Повторейко за проделанные расчеты по предложенным им формулам (1) и (2), а также Ю.А.Трояну за предоставление таблиц экспериментальных данных и полезные обсуждения.

Л и т е р а т у р а

1. Троян Ю.А., Никитин А.В., Печенов В.Н., Мороз В.И., Иерусалимов А.И., Стельмах А.П., Бешлиу К., Пантеа Д., Которобай Ф., Аракелян С.Г., Равинович И.М., Одинцов В.Г., Абдивалиев А. — ОИЯИ, Д1-88-329, Дубна, 1988.

Рукопись поступила 11 декабря 1989 года.

ON THE POSSIBILITY OF THE DETERMINATION OF THE CONSTITUENT MASSES OF COMPOSITE PARTICLES BY MEASURING FOUR-MOMENTUM TRANSFER

N.G.Fadeev

The method of determining the constituent masses of composite particles by measuring four-momentum transfer is given in this paper. The idea of the new approach proceeds from the quark-parton presentation of deep-inelastic interactions and differs from the traditional methods connected with the phenomena of particle passage through the matter. The method was substantiated and tested using experimental data on the reaction of deuteron breakdown in a hydrogen chamber. The possibility of practical application of the new approach to other inelastic particle interactions is discussed.

The investigation has been performed at the Laboratory of High Energies, JINR.

О возможности определения масс конstituентов
составных частиц на основе измерения
переданного 4-импульса

Н.Г.Фадеев

В работе излагается метод определения масс конstituентов составных частиц на основе измерения переданного 4-импульса. В отличие от традиционных способов, связанных с явлениями прохождения частиц через вещество, идея нового подхода исходит из кварк-партонного представления глубоконеупругих взаимодействий. Метод обоснован и проверен на экспериментальных данных реакции развала дейтрона в водородной камере. Обсуждаются возможности практического применения нового подхода к другим неупругим взаимодействиям частиц.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

Introduction

The present-day methods of particle mass determination are based on a variety of phenomena accompanying the passage of particles through the matter: ionization, multiple scattering and so on. The suggested

approach differs in principle from the traditional methods and its idea lies in the quark-parton picture of inelastic interactions.

A systematic investigation of the deep-inelastic scattering process

$$\ell + N \rightarrow \ell + X, \quad (\ell = e, \mu, \nu, \quad N = n, p) \quad (1)$$

allows one to find an interesting feature of these reactions, namely leptons are elastically scattered on free constituents (partons) of the nucleon^{/1/}. The parton mass xm_N (m_N is the nucleon mass) is completely determined by four-momentum transfer. Thus, the hadron mass M_X can be always presented as an effective mass of two partons xm_N and $(1-x)m_N$.

On the other hand, the presentation of nucleon as a composite particle of unbound partons contradicts the real situation: the nucleon manifests itself as a sufficiently hard system.

In this situation it seems logical to assume the existence of real bound constituents of the nucleon instead of free partons. So, the question on the relation of free partons to real constituents and their masses arises.

From all the variety of properties, which the real constituents may have, the property of constancy of their masses is enough to determine them, at least in the specific cases.

Method

An obvious example of the reaction, in which both assumptions (i.e., constituents exist and their masses are the same in different events) are satisfied,



is the reaction of deuteron breakdown in a hydrogen bubble chamber^{/2/}. In this reaction the proton from the deuteron is simply identified from the target-proton, consequently, four-momentum transfer is known.

To illustrate our approach and also to control its validity, let us restrict the selection of events just to the type (2), i.e., the reactions with pion production will not be under consideration.

It is easy to notice that the kinematics of reaction (2) in the anti-lab. system is identical to that of reaction (1) in the lab. system.

Let us introduce the following notations for reaction (2):

$$K_b + K_a = K'_a + K_x,$$

$$q = K_a - K'_a, \quad Q^2 = -q^2, \quad x = Q^2/2m_b\nu, \quad \nu = E_a - E'_a, \quad (3)$$

$$K_x^2 = M_x^2 = m_b^2 + 2m_b\nu - Q^2,$$

where K_a , K_b and K_x are the four-momenta of the projectile, target and hadron system X, respectively, Q^2 is the 4-momentum transfer squared; x , the Bjorken variable; ν , the difference between the energies of the projectile before and after scattering; and M_x , the effective mass of the hadron system X.

Figure 1 illustrates the x , Q^2 and M_x^2 distributions for reaction (2) at a 3.31 GeV/c deuteron momentum per nucleon.

It should be stressed that the parton representation of inelastic interactions (1) and (2) allows the hadron vector K_x to be expressed as a sum of two four-vectors corresponding to the partons with masses xm_b and $(1-x)m_b$:

$$K_x = K_{x1} + K_{x2},$$

$$K_x^2 = M_x^2 = (xm_b)^2 + (ym_b)^2 + 2xym_b^2 \cosh(\rho_x + \rho_y), \quad (4)$$

where $y = 1 - x$, ρ_x and ρ_y are parton rapidities in the rest system

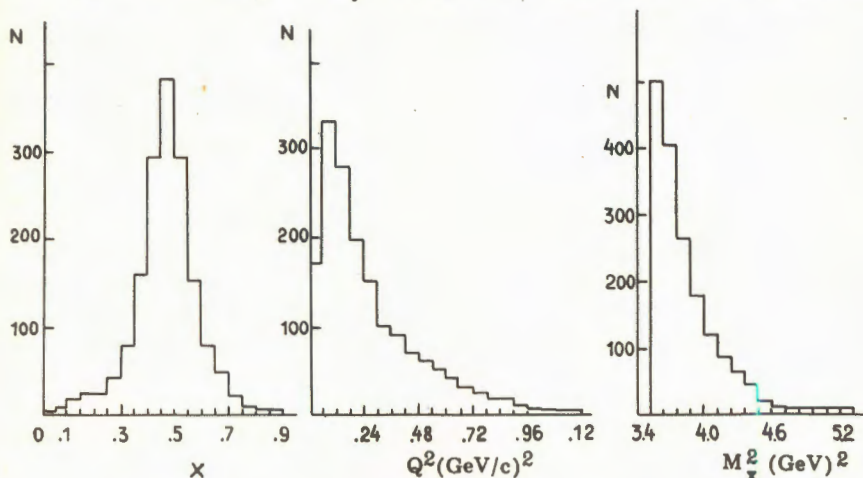


Fig.1. x , Q^2 , M_x^2 distributions for the reaction of deuteron breakdown in a hydrogen bubble chamber at a momentum of 3.31 GeV/c per nucleon.

of the hadron mass M_x (see Fig.2). In Eq. (4) the masses of all particles are known, therefore their velocities (rapidities) are also defined ^{/3/}:

$$\begin{aligned} \text{th } \rho_x &= y m_b \text{sh } \rho_{xy} / (x m_b + y m_b \text{ch } \rho_{xy}), \\ \text{th } \rho_y &= x m_b \text{sh } \rho_{xy} / (y m_b + x m_b \text{ch } \rho_{xy}), \end{aligned} \quad (5)$$

where $\rho_{xy} = \rho_x + \rho_y$.

By analogy with (4), let us introduce the real constituents with masses m_1 and m_2 :

$$\begin{aligned} K_x &= K_1 + K_2, \\ K_x^2 &= M_x^2 = m_1^2 + m_2^2 + 2m_1 m_2 \text{ch}(\rho_1 + \rho_2), \end{aligned} \quad (6)$$

where ρ_1, ρ_2 are the corresponding rapidities of the new particles. From Eq. (6) one can express the masses m_1 and m_2 through their velocities (rapidities):

$$m_1 = M_x \text{sh } \rho_2 / \text{sh } \rho_{12}, \quad m_2 = M_x \text{sh } \rho_1 / \text{sh } \rho_{12}, \quad (7)$$

where $\rho_{12} = \rho_1 + \rho_2$. From Eq. (7) one can see that m_1 and m_2 are defined by the values of ρ_1 and ρ_2 and do not depend on the space orientation of ρ_{12} , i.e. the masses m_1 and m_2 are independent of angle α (Fig.2). Therefore it is more worth-while to change the variables:

$$\rho_1 = \rho_x - \Delta_1, \quad \rho_2 = \rho_y + \Delta_2. \quad (8)$$

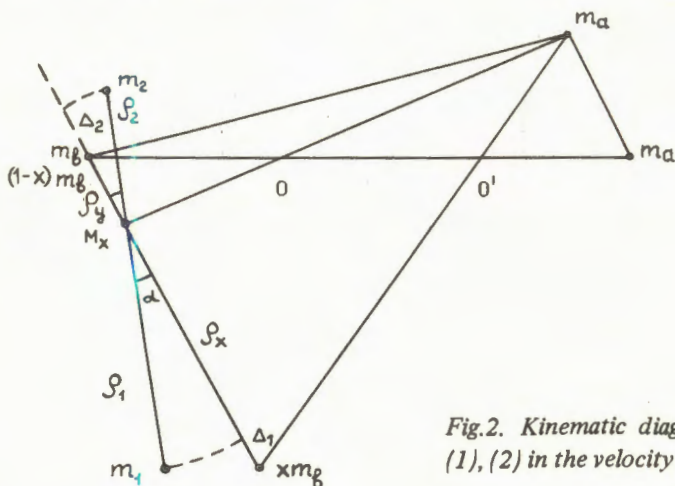


Fig.2. Kinematic diagram of reactions (1), (2) in the velocity space.

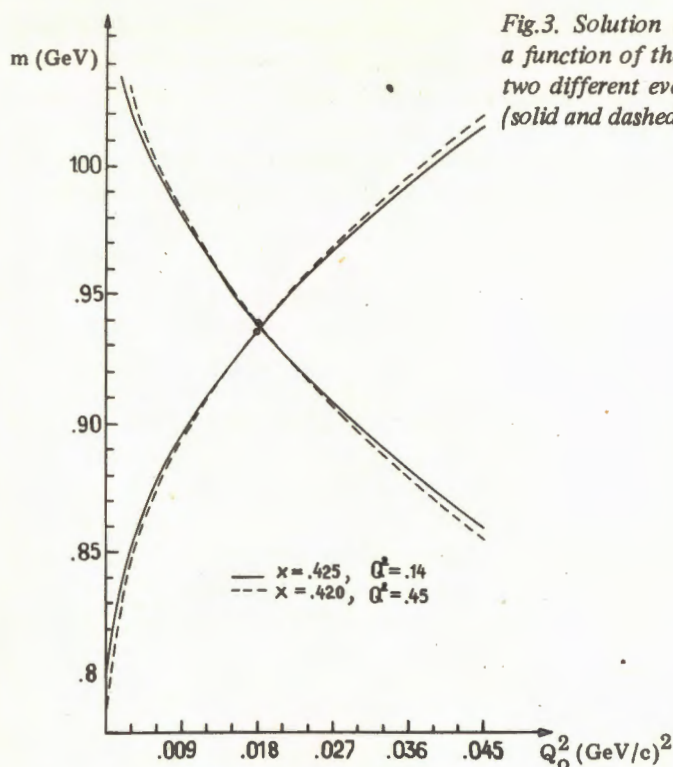


Fig.3. Solution of system (10) as a function of the parameter Q_0^2 for two different events of reaction (3) (solid and dashed lines).

Comparing Eqs.(4) and (6), we have:

$$\begin{aligned} (K_1 - K_{x1})^2 &= (K_2 - K_{x2})^2 = Q_a^2, \\ m_1^2 + (xm_b)^2 - 2xm_b m_1 \text{ch} \Delta_1 &= Q_0^2, \\ m_2^2 + (ym_b)^2 - 2ym_b m_2 \text{ch} \Delta_2 &= Q_0^2, \end{aligned} \quad (9)$$

where Q_0^2 is a new unknown parameter corresponding to Q_a^2 for $\alpha = 0$. The parameter Q_0^2 reflects the above statement that the masses m_1 and m_2 are functions of the values of ρ_1 and ρ_2 and do not depend on the space orientation of ρ_{12} .

Expressing m_1 and m_2 by Eq. (9) and comparing them with (7) taking (8) into account, one can find:

$$\begin{aligned} xm_b \text{ch} \Delta_1 \pm ((xm_b \text{sh} \Delta_1)^2 + Q_0^2)^{1/2} &= M_x \frac{\text{sh}(\rho_y + \Delta_2)}{\text{sh}(\rho_{xy} - \Delta_1 + \Delta_2)} \\ ym_b \text{ch} \Delta_2 \mp ((ym_b \text{sh} \Delta_2)^2 + Q_0^2)^{1/2} &= M_x \frac{\text{sh}(\rho_x - \Delta_1)}{\text{sh}(\rho_{xy} - \Delta_1 + \Delta_2)}, \end{aligned} \quad (10)$$

i.e., the system of two nonlinear equations with three unknown parameters Δ_1 , Δ_2 and Q_0^2 . The experimental data of the reaction (2) show how to select the appropriate sign in Eqs. (10): (+) for the first equation, (−) for the second one if $x \leq 0.5$ and vice versa if $x > 0.5$.

The limits of Q_0^2 can be set for the requirement that m_1 and m_2 are positive (upper limit), and radical expressions are positive too (lower limit):

$$\begin{aligned} - (xm_b \operatorname{sh} \rho_x)^2 &< Q_0^2 < (ym_b)^2, & x \leq 0.5, \\ - (ym_b \operatorname{sh} \rho_y)^2 &< Q_0^2 < (xm_b)^2, & x > 0.5. \end{aligned} \quad (11)$$

For the fixed value of Q_0^2 from system (10) one can find the values of m_1 and m_2 as a function of Q_0^2 . Figure 3 presents the solution of system (10) for two real events. The lower limit of Q_0^2 was chosen only from the positive part of interval (11).

It is necessary to determine the appropriate value of Q_0^2 for a final choice of the values of m_1 and m_2 . For this purpose let us assume that different events of reaction (2) (i.e., different x and Q^2) have the same constituents with the same masses:

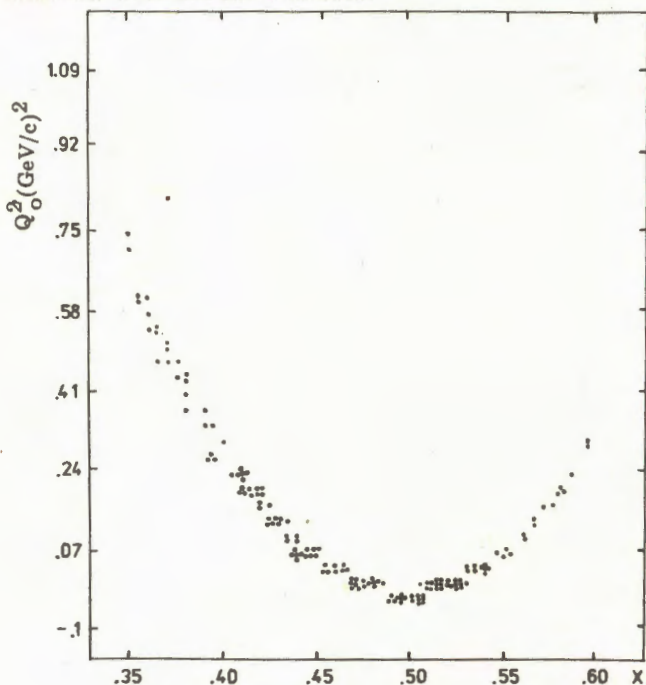


Fig. 4. $x - Q_0^2$ distribution for reaction (2).

$$\begin{aligned}
 m_1(Q_0^2)_i + m_2(Q_0^2)_i &= \text{const}_1, \\
 m_1(Q_0^2)_i \cdot m_2(Q_0^2)_i &= \text{const}_2, \quad i = 1 \div n,
 \end{aligned}
 \tag{12}$$

where n is the number of events.

Figure 4 gives the experimental x and Q_0^2 distributions. The neutron and proton masses for m_1 and m_2 were used to calculate the values of Q_0^2 . From these data one can see that different events have the same value of Q_0^2 over a small interval of x . This experimental fact can be used to choose the appropriate values of Q_0^2 . Namely, from all events one should select those which have crossover points of the curves for $m_1(Q_0^2)$ or $m_2(Q_0^2)$ and find the intersection point of Q_0^2 . Figure 3 illustrates the validity of these statements: two curves, which correspond to different events, intersect at the value of Q_0^2 just for the right masses m_1 and m_2 .

So, the procedure of determination of the real constituent masses reduces to solving the nonlinear system of equations (10) for each event and to finding the groups of events with the same value of Q_0^2 .

Conclusion

For a practical application of the new approach it is useful to stress the main conditions in the framework of which it has been formulated: 1) the components of the four-momentum transfer are measured, 2) real constituents exist, 3) there is a set of events with similar constituents.

Obviously, there is no doubt that this approach can be applicable to reaction (2): just on the basis of this reaction it was substantiated and tested. But its extension to reactions (1) and (2) with secondary pion production still seems unambiguous and requires some further investigation.

On the other hand, poor information on inclusive reactions presents some difficulties in studying details of the dynamics of particle interactions. Thus, the suggested method can be useful in its application to different kinematic regions. This approach seems promising for the investigation of a hadron shower in reaction (1): quark and diquark fragmentation, selection of prompt secondaries and jets. For instance, the selection of prompt particles will make it possible to redetermine the values of four-momentum transfer and to reduce the task to the above version.

Future neutrino experiments at UNK with spectrometers used to measure and to identify all particles of a hadron shower look promising for such kinds of investigations.

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A NEW METHOD OF THE CALCULATION OF THE TWO-NEUTRINO DOUBLE BETA DECAY AMPLITUDES

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A new method is proposed of the calculation of the two-neutrino double-beta decay amplitude, which does not use the spectrum of intermediate nuclear states. The method is based on evaluation of a series of commutators of the nuclear Hamiltonian and weak nuclear hadron current. As a result, two-neutrino double-beta decay amplitude with a nuclear matrix element of a simple form of the two-nucleon operator has been obtained.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Новый метод вычисления амплитуды двухнейтринного двойного бета-распада

Ф.Шимковиц

Предложен новый метод вычисления скорости двухнейтринного двойного бета-распада, основанный на коммутационных соотношениях ядерного гамильтониана и слабых ядерных адронных токов. Метод не требует построения спектра "промежуточного" ядра. Получено замкнутое выражение для амплитуды данного процесса.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Introduction

Recent first experimental observation of the two-neutrino mode of the double beta decay of $^{82}\text{Se}^{1/2}$ has revived interest in this process both among theoreticians and experimentalists.

The two-neutrino emitting mode of the double beta decay ($2\nu 2\beta$):

$$(A, Z) \rightarrow (A, Z+2) + 2e^- + 2\bar{\nu}_e, \quad (1)$$

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occurs as a second-order weak interaction process within the standard model of electroweak interactions.

In the calculations of the $(2\nu 2\beta)$ decay rate the two-nucleon mechanism is considered most frequently ^{/2-4/}.

The modern method of the calculations of the $(2\nu 2\beta)$ amplitude as used recently requires the construction of the spectrum of intermediate nuclear states ^{/2-4/}.

The early calculations by the closure approximation method have systematically overestimated the $(2\nu 2\beta)$ amplitude ^{/2-4/}. The recent progress in calculations of the $(2\nu 2\beta)$ amplitude has been achieved by using quasiparticle random-phase approximation method ^{/5-8/} which leads to a strong suppression of the $(2\nu 2\beta)$ nuclear matrix elements. However, the value of the nuclear matrix element is very sensitive to the particle-particle interaction in the spin-isospin polarisation force.

This substantial sensitivity to the details of the nuclear model used in the construction of intermediate nuclear states showed the importance of the elaboration of alternative methods of the calculations of the $(2\nu 2\beta)$ amplitude.

Recently, C.R.Ching and T.H.Ho^{/9/} have proposed an alternative method for the calculations of the $(2\nu 2\beta)$ amplitude in which the sum over the intermediate nuclear states has been transformed into a series of commutators of two axial vector currents and the nuclear Hamiltonian.

In this article, we present another method which is also based on calculations of commutators of the nuclear Hamiltonian and weak nuclear currents. Performing summation of a series of commutators explicitly, we were able to derive the $(2\nu 2\beta)$ nuclear matrix element in a simple form of the two-nucleon operator for a given nuclear Hamiltonian.

The derivation of the $(2\nu 2\beta)$ amplitude

We assume that the beta decay Hamiltonian has the form

$$H^\beta = \frac{G_F}{\sqrt{2}} 2(\bar{e}_L \gamma_\alpha \nu_{eL}) j_\alpha + \text{h.c.}, \quad (2)$$

where j_α is the strangeness conserving charged hadron current and e_L and ν_{eL} are operators of the left components of fields of the electron and neutrino, respectively.

$$\begin{aligned} \langle f | S^{(2)} | i \rangle = & \frac{(-1)^2}{2} \left(\frac{G_F}{\sqrt{2}} \right)^2 \frac{1}{(2\pi)^6} \frac{1}{\sqrt{16 p_{10} p_{20} k_{10} k_{20}}} \times \\ & \times \bar{u}(p_1) \gamma_\alpha (1 + \gamma_5) u^c(k_1) \cdot \bar{u}(p_2) \gamma_\beta (1 + \gamma_5) u^c(k_2) J_{\alpha\beta} + \\ & - (p_1 \leftrightarrow p_2) - (k_1 \leftrightarrow k_2) + (p_1 \leftrightarrow p_2) (k_1 \leftrightarrow k_2), \end{aligned} \quad (3)$$

where,

$$\begin{aligned} J_{\alpha\beta} = & \int e^{-i(p_1 + k_1) \cdot x_1} e^{-i(p_2 + k_2) \cdot x_2} \times \\ & \times \langle p_f | T(J_\alpha(x_1) J_\beta(x_2)) | p_i \rangle dx_1 dx_2. \end{aligned} \quad (4)$$

Here, p_1 and p_2 (k_1 and k_2) are four-momenta of the electrons (anti-neutrinos), p_i and p_f are four momenta of the initial and final nucleus, and $J_\alpha(x)$ is the weak charged nuclear current in the Heisenberg representation.

If we use the definition of time-order product of two operators in the form

$$T(J_\alpha(x_1) J_\beta(x_2)) = J_\alpha(x_1) J_\beta(x_2) + \theta(x_{20} - x_{10}) [J_\beta(x_2), J_\alpha(x_1)], \quad (5)$$

we obtain

$$\begin{aligned} J_{\alpha\beta} = & \sum_n 2\pi \delta(E_f - E_n + p_{10} + k_{10}) 2\pi \delta(E_n - E_i + p_{20} + k_{20}) \times \\ & \times \int e^{-i(\vec{p}_1 + \vec{k}_1) \cdot \vec{x}_1} e^{-i(\vec{p}_2 + \vec{k}_2) \cdot \vec{x}_2} \times \\ & \times \langle p_f | J_\alpha(0, \vec{x}_1) | p_n \rangle \langle p_n | J_\beta(0, \vec{x}_2) | p_i \rangle d\vec{x}_1 d\vec{x}_2 + \\ & + 2\pi \delta(E_f - E_i + p_{10} + k_{10} + p_{20} + k_{20}) \times \\ & \times \int e^{-i(\vec{p}_1 + \vec{k}_1) \cdot \vec{x}_1} e^{-i(\vec{p}_2 + \vec{k}_2) \cdot \vec{x}_2} \int_0^\infty e^{-i(p_{20} + k_{20})t} \times \\ & \times \langle p_f | [J_\beta(t, \vec{x}_2), J_\alpha(0, \vec{x}_1)] | p_i \rangle \times dt d\vec{x}_1 d\vec{x}_2. \end{aligned} \quad (6)$$

Here, $|p_n\rangle$ is an eigenvector of an intermediate nucleus with energy E_n , and E_i and E_f are energies of the initial and final nucleus, respectively.

We can see that the first term in the r.h.s. of eq. (6) corresponds to two subsequent nuclear beta decay processes. However, in the case of the double beta decay, the beta transition from the parent nucleus (A, Z) to the intermediate nucleus $(A, Z+1)$ is forbidden energetically ($E_n > E_1$) which implies that the first term in the r.h.s. of eq. (6) is equal to zero. So, only the second term in eq. (6) with the non-equal-time commutator of the nuclear hadron currents contributes to the $(2\nu 2\beta)$ amplitude.

Next, if we consider the non-relativistic impulse approximation for the hadronic currents,

$$J_\alpha(0, \vec{x}) = \sum_n r_n^+ (\delta_{\alpha 4} + i g_A \delta_{\alpha k} (\vec{\sigma}_n)_k) \delta(\vec{x} - \vec{x}_n), \quad (7)$$

limit the consideration only to $s_{1/2}$ wave states of the emitted electrons and antineutrinos and the most energetically favoured $0^+ \rightarrow 0^+$ nuclear transition, then for $J_{\alpha\beta}$ we can write,

$$\begin{aligned} J_{\alpha\beta} &= 2\pi \delta(E_f - E_i + p_{10} + k_{10} + p_{20} + k_{20}) \times \\ &\times \int_0^\infty e^{i(p_{20} + k_{20})t} \{ \delta_{\alpha 4} \delta_{\beta 4} \langle p_f(0^+) | [e^{iHt} V(0) e^{-iHt} V(0)] | p_i(0^+) \rangle + \\ &- \frac{1}{3} g_A^2 \delta_{\alpha k} \delta_{\beta k} \langle p_f(0^+) | [e^{iHt} A_\ell(0) e^{-iHt} , A_\ell(0)] | p_i(0^+) \rangle \} dt \end{aligned}$$

with

$$V(0) = \sum_n r_n^+, \quad (9)$$

$$A_\ell(0) = \sum_n r_n^+ (\vec{\sigma}_n)_\ell, \quad (10)$$

where H is the nuclear Hamiltonian. If we use, instead of eq. (5), the fully equivalent formula for the time-ordered product of two operators,

$$T(J_\alpha(x_1) J_\beta(x_2)) = J_\beta(x_2) J_\alpha(x_1) + \theta(x_{10} - x_{20}) [J_\alpha(x_1), J_\beta(x_2)], \quad (11)$$

we obtain the same formula for $J_{\alpha\beta}$ as in eq. (8) but with $p_{20} + k_{20}$ to be replaced by $p_{10} + k_{10}$.

From the equivalence of both the ways of calculation it follows that in the framework of our approximations $J_{\alpha\beta}$ does not depend on the kinematical variables and we can set

$$p_{10} + k_{10} = p_{20} + k_{20} = \frac{E_i - E_f}{2} = \Delta, \quad (12)$$

in eq. (8). The traditional way for the calculation of the $(2\nu 2\beta)$ amplitude is to insert the complete set of intermediate nuclear states into the nuclear matrix elements in eq. (8).

Instead, we proceed as follows. We assume the following form of the nuclear Hamiltonian

$$H = H_0 + V_S, \quad (13)$$

where H_0 is the Hamiltonian of A free nucleons, and V_S is the effective nucleon-nucleon strong interaction

$$V_S = V_W + V_B + V_H + V_M, \quad (14)$$

where

$$V_W = \frac{1}{2} \sum_{i \neq j} g_W(r_{ij}), \quad (15)$$

$$V_B = \frac{1}{2} \sum_{i \neq j} g_B(r_{ij}) P_\sigma, \quad (16)$$

$$V_H = \frac{1}{2} \sum_{i \neq j} g_H(r_{ij}) P_r, \quad (17)$$

$$V_M = \frac{1}{2} \sum_{i \neq j} g_M(r_{ij}) P_\sigma P_r, \quad (18)$$

with

$$P_\sigma = \frac{1}{2} (1 + \vec{\sigma}_i \cdot \vec{\sigma}_j), \quad (19)$$

$$P_r = \frac{1}{2} (1 + \vec{r}_i \cdot \vec{r}_j), \quad (20)$$

where $g_W(r_{ij})$, $g_B(r_{ij})$, $g_H(r_{ij})$ and $g_M(r_{ij})$ are scalar functions of the relative coordinate r_{ij} of two nucleons (i, j) /10/.

The tensor force, spin-orbit interaction and other nonlocal forces as well as the Coulomb interaction are neglected since their contributions are much smaller than those of V_B , V_H and V_M .

The nuclear matrix element with axial current is

$$M_{AA} = \langle p_f(0^+) | [e^{iHt} A_\ell(0) e^{-iHt}, A_\ell(0)] | p_i(0^+) \rangle. \quad (21)$$

If we take into account that

$$[A_\ell(0), H_0 + V_W] = 0, \quad (22)$$

and if we suppose that the main contribution to the nuclear matrix element gives the two-nucleon operators and neglect the contribution of the three- and more-nucleon operators, we obtain

$$M_{AA} = \langle p_f(0^+) | \frac{1}{2} \sum_{n \neq m} [e^{iG_{nm}t} (\vec{A}_{nm})_\ell e^{-iG_{nm}t}, (\vec{A}_{nm})_\ell] | p_i(0^+) \rangle \quad (23)$$

where

$$G_{nm} = g_B(r_{nm}) P_\sigma + g_H(r_{nm}) P_r + g_M(r_{nm}) P_\sigma P_r, \quad (24)$$

$$\vec{A}_{nm} = r_n^+ \vec{\sigma}_n + r_m^+ \vec{\sigma}_m. \quad (25)$$

The two-nucleon operator of the nuclear matrix element in eq. (23) can be simplified in the following way. We have

$$e^{iG_{nm}t} \vec{A}_{nm} e^{-iG_{nm}t} = e^{ig_M P_\sigma P_r t} e^{ig_H P_r t} e^{ig_B P_\sigma t} \vec{A}_{nm} e^{-ig_B P_\sigma t} e^{-ig_H P_r t} e^{-ig_M P_\sigma P_r t}, \quad (26)$$

where, $g_M = g_M(r_{nm})$, $g_H = g_H(r_{nm})$, $g_B = g_B(r_{nm})$, and we can write¹¹ that

$$e^{ig_B P_\sigma t} \vec{A}_{nm} e^{-ig_B P_\sigma t} = \sum_{k=0}^{\infty} \frac{(ig_B t)^k}{k!} [\overbrace{P_\sigma \dots P_\sigma}^k, \vec{A}_{nm}] \dots] \quad (27)$$

It is easy to see that using $P_\sigma^2 = 1$ we can sum up the series of commutators in eq. (27) and obtain

$$e^{ig_B P_\sigma t} \vec{A}_{nm} e^{-ig_B P_\sigma t} = \frac{1}{2} \{ \vec{A}_{nm} + P_\sigma \vec{A}_{nm} P_\sigma + \cos(2g_B t) (\vec{A}_{nm} - P_\sigma \vec{A}_{nm} P_\sigma) + i \sin(2g_B t) [P_\sigma, \vec{A}_{nm}] \}. \quad (28)$$

Proceeding similarly for the Heisenberg force $g_H(r_{nm})$ and for the Majorana force $g_M(r_{nm})$ is eq. (26) and then inserting the result into eq. (23) and calculating the commutator in eq. (23) with the use of the form of $\vec{A}_{nm}$ in eq. (25), we obtain

$$M_{AA} = 6i \langle p_f(0^+) | \sum_{n,m} \tau_n^+ \tau_m^+ \{ \sin(2(g_H - g_B)t) \Pi_s^\sigma + \frac{1}{3} \sin(2(g_H + g_B)t) \Pi_t^\sigma \} | p_i(0^+) \rangle, \quad (29)$$

where

$$\Pi_s^\sigma = \frac{1}{2} (1 - P_\sigma), \quad (30)$$

$$\Pi_t^\sigma = \frac{1}{2} (1 + P_\sigma), \quad (31)$$

are projection operators which project onto the singlet (5) and triplet (t) parts of the nuclear two-body wave function.

Analogously, we get

$$M_{VV} = \langle p_f(0^+) | [e^{iHt} V(0) e^{-iHt}, V(0)] | p_i(0^+) \rangle = 0. \quad (32)$$

By inserting eqs. (29) and (32) into eq. (8) and then into eq. (3) and performing the integration over time variable using the standard procedure of adiabatic switching-off the interaction at $t \rightarrow \infty$,

$$\int_0^\infty e^{iat} \sin(bt) dt \Rightarrow \lim_{\epsilon \rightarrow 0} \int_0^\infty e^{i(a+i\epsilon)t} \sin(bt) dt = \lim_{\epsilon \rightarrow 0} \frac{b}{b^2 - a^2 - i\epsilon}, \quad (33)$$

we obtain the $(2\nu 2\beta)$ amplitude in the form

$$\begin{aligned} \langle f | S^{(2)} | i \rangle &= i \left(\frac{G_F}{\sqrt{2}} \right)^2 \frac{1}{(2\pi)^6} \frac{1}{\sqrt{16 p_{10} p_{20} k_{10} k_{20}}} \times \\ &\times \{ \bar{u}(p_1) \gamma_k (1 + \gamma_5) u^c(k_1) \cdot \bar{u}(p_2) \gamma_k (1 + \gamma_5) u^c(k_2) - (k_1 \leftrightarrow k_2) \} \times \\ &\times g_A^2 M^{2\nu 2\beta} 2\pi \delta(E_f - E_i + p_{10} + k_{10} + p_{20} + k_{20}), \end{aligned} \quad (34)$$

where

$$M^{2\nu 2\beta} = \langle p_f(0^+) | \sum_{n \neq m} \tau_n^+ \tau_m^+ (f(r_{nm}) \Pi_s^\sigma + \frac{1}{3} f^+(r_{nm}) \Pi_t^\sigma) | p_i(0^+) \rangle, \quad (35)$$

with

$$f^\pm(r_{nm}) = \lim_{\epsilon \rightarrow 0} \frac{g_H(r_{nm}) \pm g_B(r_{nm})}{(g_H(r_{nm}) \pm g_B(r_{nm}))^2 - (\frac{\Delta}{2})^2 - i\epsilon}. \quad (36)$$

Discussion and conclusion

We have derived a new formula of the $(2\nu 2\beta)$ amplitude (eq.(34)) with the $(2\nu 2\beta)$ nuclear matrix element in which the summation of a series of commutators of the nuclear Hamiltonian and weak nuclear currents has been performed and a simple form of two-nucleon transition operator has been found.

The two-nucleon operator of the $(2\nu 2\beta)$ nuclear matrix element in eq. (35) in the framework of our approximations depends only on the Bartlett and Heisenberg forces and has a pole structure in their radial dependence (eq. (36)).

We note that if in the L-S coupling we consider only the configuration $L=0, S=0$ in the ground states of even-even nuclei and neglect the $L=1, S=1$ configuration, because of a short range of pairing forces^{/9/}, and choose the Bartlett and Heisenberg forces to be equal, we obtain that the $(2\nu 2\beta)$ amplitude is equal to zero, which is in agreement with the conclusions of C.R.Ching and T.H.Ho^{/9/}.

In conclusion, we add that the same method can be applied to the calculations of other nuclear processes such as the neutrinoless double-beta decay and nuclear pion double charge exchange.

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Note: After the present research has been completed, we have seen the paper by C.R.Ching, T.H.Ho and X.R.Wu^{/12/}. The result obtained by those authors in a completely different way fully supports our result. It is, however, not clear from the text whether the pole structure present in eq. (36) has been noticed in Ref.^{/12/}.

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THE DECAY $K_L \rightarrow \pi^0 \gamma \gamma$

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By using the linear σ -model the estimates of the branching ratio of the decay $K_L \rightarrow \pi^0 \gamma \gamma$ have been obtained. The comparison with the experimental data of the E731 group has been made.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Распад $K_L \rightarrow \pi^0 \gamma \gamma$

М.К.Волков, М.Надь

В рамках линейной σ -модели были получены оценки вероятности распада $K_L \rightarrow \pi^0 \gamma \gamma$. Было проведено сравнение с данными экспериментальной группы E731.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

The authors of recently published paper^{/1/} have performed a search for the rare decay mode $K_L \rightarrow \pi^0 \gamma \gamma$ using a data set from E731 experiment. This decay is of interest for its contribution to the decay $K_L \rightarrow \pi^0 e^+ e^-$, which gives some possibilities of observing a direct CP violation. They have concluded, using world averages for $K_L \rightarrow 2\pi^0$ and for $K_L \rightarrow 3\pi^0$ branching ratios^{/2/} that $B(K_L \rightarrow \pi^0 \gamma \gamma) < 2.7 \times 10^{-6}$ (90% confidence level).

In 1982 one of the authors of the present paper calculated the width of the decay $\eta \rightarrow \pi^0 \gamma \gamma$ ^{/3/} in the framework of the quark model of superconductivity type (QMST)^{/4,5/}, just proposed at that time. This model is a quark analogy of the well-known model of Nambu — Jona-Lasinio^{/6/}. Starting from the effective chirally symmetric four-quark interaction, one can obtain in QMST the well-known chiral phenomenological Lagrangians, describing the linear σ -model and the interactions of scalar, pseudoscalar, vector and axial vector mesons. The number of free parameters in this case is less than that in the old phenomenological approaches.

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The advantages of the linear σ -model compared with the non-linear variant one can realize at the description of the process $\eta \rightarrow \pi^0 \gamma \gamma$. The experimental status of this process was unclear for quite a long time. Before the beginning of eighties the experimental value of the decay rate of this process was considered as equal to the decay rate of the process $\eta \rightarrow \pi^+ \pi^- \gamma$. We have shown already in 1979 the incorrectness of such experimental results from the point of view of phenomenological chiral Lagrangians^{/7/}. Really, the experiments, performed in IHEP (Serpukhov) in 1980-1981, have confirmed, that the width of the decay $\eta \rightarrow \pi^0 \gamma \gamma$ is almost 70-times smaller, than that of the decay $\eta \rightarrow \pi^+ \pi^- \gamma$ ^{/8/}.

From the theoretical results, obtained before the IHEP experiments, the closest to the experimental data were the calculations published by Oppo and Oneda^{/9/}. They have accounted the contributions of the vector mesons in the process $\eta \rightarrow \pi^0 \gamma \gamma$ ($\eta \rightarrow \gamma[(\omega, \rho, \phi) \rightarrow \pi^0 \gamma]$). However, their result, was triple smaller than the next experimental result. In the nonlinear chiral theory the large contributions are also given by the diagrams depicted in Fig.1

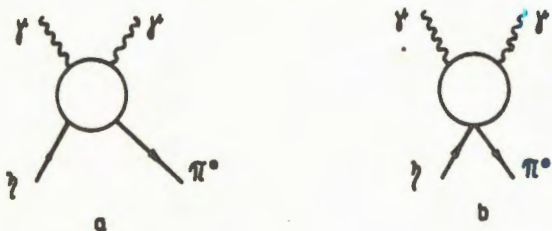


Fig.1

but, these contributions are mutually compensated. Quite a different situation takes place in the linear σ -model with the real intermediate $a_0(980)$ meson (in the old notations $\delta(980)$) in the diagram of the type 1b. In QMST these loop diagrams have the form shown in Fig.2.

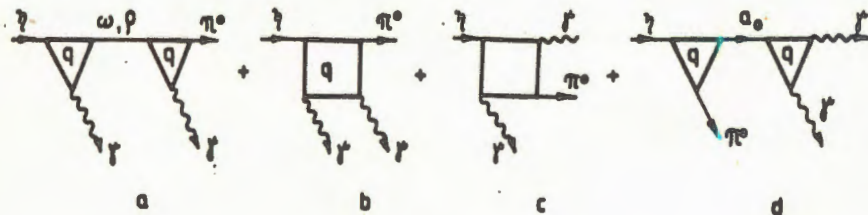


Fig.2

Here the diagram 2d also partially compensates the large contributions of the diagrams 2b and 2c. The remaining part, in contrast to the non-linear model, is nonvanishing and completely commensurate with the contribution of the diagram 2a, having ω and ρ mesons in the intermediate state. As a result we have obtained for the width $\Gamma_{\eta \rightarrow \pi^0 \gamma \gamma}$ the value equal to 1 eV, that is in good agreement with the experimental data^{/8/}.

The contributions of the contact and pole diagrams with an intermediate scalar meson have been accounted in the amplitude of the decay $\eta \rightarrow \pi^0 \gamma \gamma$ for the first time in paper^{/10/}. There the author got the result quite close to the future experimental data^{/8/}. Then, already after the new experimental data appeared, the similar numerical estimates have been obtained in^{/11/}. However, by comparing the results of the paper^{/10/} with those of^{/3,9,11/} one can realize that in^{/10/} the estimates of the contributions of the pole diagrams with vector mesons into the width of the decay $\eta \rightarrow \pi^0 \gamma \gamma$ have been twice higher. Therefore the agreement of the final result with the recent experimental data has only the accidental character. In the articles^{/3/} and^{/11/}, the results of which are completely mutually consistent, the contributions of these diagrams are accounted more correctly.

After these general remarks we overcome to the calculation of the amplitude of the decay $K_L \rightarrow \pi^0 \gamma \gamma$ that can be easily expressed through the amplitude of the decay $\eta \rightarrow \pi^0 \gamma \gamma$. Really, the process $K_L \rightarrow \pi^0 \gamma \gamma$ is completely defined by the three pole diagrams, depicted in Fig.3.

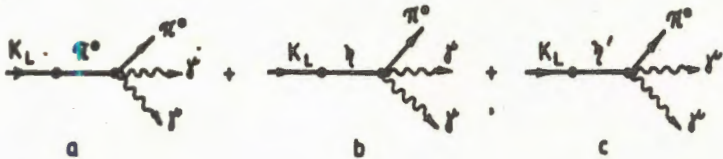


Fig.3

However, the dominant contribution to the amplitude $K_L \rightarrow \pi^0 \gamma \gamma$ will give the diagram 3b, because there is the pole very close to the resonance region. The diagrams 3a and 3c are not only far from the resonance, but even are partially compensated, having opposite signs. That is why we shall neglect their contributions in the following.

The amplitude of the decay $K_L \rightarrow \eta \rightarrow \pi^0 \gamma \gamma$, taking into account the previous arguments, can be expressed in the form

$$T_{K_L \rightarrow \pi^0 \gamma \gamma} = \frac{c}{m_\eta^2 - m_{K_L}^2} T_{\eta \rightarrow \pi^0 \gamma \gamma} = a T_{\eta \rightarrow \pi^0 \gamma \gamma}, \quad (1)$$

where m_η and m_{K_L} are masses of the η and K_L mesons, respectively.

The quantity a , connected with the weak vertex, defining the probability of the transition $K_L \rightarrow \eta$, is considered as a free parameter. We define this parameter from the independent experimental data, namely from the widths of the decays $K_L \rightarrow \gamma\gamma$ and $\eta \rightarrow \gamma\gamma$. The amplitudes of these decays can be expressed analogously to (1)

$$T_{K_L \rightarrow \gamma\gamma} = \frac{c}{m_\eta^2 - m_{K_L}^2} T_{\eta \rightarrow \gamma\gamma} = a T_{\eta \rightarrow \gamma\gamma}. \quad (2)$$

For the amplitude $T_{\eta \rightarrow \gamma\gamma}$ we get from the recent experimental data ^{/2/} the value

$$\Gamma_{\eta \rightarrow \gamma\gamma}^{\text{exp}} = \frac{m_\eta}{\pi} \left(T_{\eta \rightarrow \gamma\gamma}^{\text{exp}} \frac{m_\eta}{8} \right)^2 = 0.42 \text{ keV}, \quad (3)$$

$$T_{\eta \rightarrow \gamma\gamma}^{\text{exp}} = \frac{8}{m_\eta} \left(\frac{\pi \cdot 0.42 \text{ keV}}{m_\eta} \right)^{1/2} = \frac{0.0124}{m_\eta}.$$

Then, by using (2), we have the following formula for the width of the decay $K_L \rightarrow \gamma\gamma$

$$\Gamma_{K_L \rightarrow \gamma\gamma} = \frac{m_{K_L}}{\pi} \left(a T_{\eta \rightarrow \gamma\gamma} \frac{m_{K_L}}{8} \right)^2 = 7.24 \times 10^{-12} \text{ eV}. \quad (4)$$

From here we can easily evaluate the parameter a

$$a = 1.5 \times 10^{-7}. \quad (5)$$

Now we have all necessary aids for the calculation of the width of the decay $K_L \rightarrow \pi^0 \gamma\gamma$

$$\Gamma_{K_L \rightarrow \pi^0 \gamma\gamma} = a^2 \Gamma_{\eta \rightarrow \pi^0 \gamma\gamma} (m_\eta \rightarrow m_{K_L}). \quad (6)$$

The evaluation of the contributions of diagrams in Fig.2 into the width of the decay $K_L \rightarrow \pi^0 \gamma\gamma$, with the change of the mass of η -meson to the mass of K_L -meson, leads to the following results.

The contribution of the diagram with the intermediate vector meson (Fig. 2a) equals

$$\Gamma_{K_L \rightarrow \pi^0 \gamma \gamma}^{(\rho, \omega)} = \frac{\pi}{2} M \left(\frac{M}{8\pi^2 F_\pi} \right)^4 [3a a_\rho a \sin \bar{\theta}]^2 [I_{\rho\rho} + 2I_{\rho\omega} + I_{\omega\omega}] =$$

$$= 4.8 \times 10^{-8}] = 0.5 \times 10^{-14} \text{ eV}, \quad (7)$$

where M denotes the mass of the K_L -meson, $F_\pi = 93 \text{ MeV}$ is the pion decay constant, $a = e^2/4\pi = 1/137$ is the electromagnetic constant, $a_\rho = g_\rho^2/4\pi = 3$ is the constant of the decay $\rho \rightarrow 2\pi$, $\bar{\theta} = \theta_0 - \theta$, $\theta_0 = 35^\circ$ is the ideal mixing angle, $\theta = -18^\circ$, $I_{\rho\rho}$, $I_{\omega\omega}$ and $I_{\rho\omega}$ are the phase integrals, calculated in the paper^{/9/}. We have used the values of these integrals obtained after replacing the mass m_η by the mass m_{K_L} .

The contribution of the diagrams 2b — 2d is given as follows^{/3/}

$$\Gamma_{K_L \rightarrow \pi^0 \gamma \gamma}^{(\square + \delta)} = \frac{M}{9\pi} \left(\frac{m}{2\pi F_\pi} \right)^4 (a a \sin \bar{\theta})^2 \times$$

$$\frac{M^2 + m^2}{2mM} \times \int_1^\infty dx (x^2 - 1)^{1/2} \left(\frac{M^2 + m^2}{2mM} - x \right)^2 \left(1 - \frac{4m_u^2}{m_{a_0}^2 - M^2 - m^2 + 2mMx} \right)^2, \quad (8)$$

where m is the mass of the π^0 -meson, m_{a_0} is the mass of the a_0 -meson and m_u is the mass of the constituent u and d quarks, having in our model the value^{/5/}

$$m_u^2 = \frac{m_{a_1}^2}{12} \left[1 - \left(1 - \left(\frac{2g_\rho F_\pi}{m_{a_1}} \right)^2 \right)^{1/2} \right]. \quad (9)$$

Here m_{a_1} is the mass of the axial-vector a_1 -meson. For the value $m_{a_1} = 1270 \text{ MeV}$ ^{/10/} we get $m_u \approx m_d \approx 280 \text{ MeV}$ and from the formula (8) we get the value

$$\Gamma_{K_L \rightarrow \pi^0 \gamma \gamma}^{(\square + \delta)} = 0.34 \times 10^{-14} \text{ eV}. \quad (10)$$

The last important term, giving the sizeable contribution into the decay $K_L \rightarrow \pi^0 \gamma \gamma$, is coming from the interference of the diagrams of the type 2a and 2b — 2d. This term has the following form

$$\Gamma_{K_L \rightarrow \pi^0 \gamma \gamma}^{\text{interf}} = \frac{\alpha M}{2} \left(\frac{aa \sin \theta}{\pi} \right)^2 \left(\frac{m}{2\pi F_\pi} \right)^4 \left(\frac{m^2}{2mM} I_1 - I_2 \right) = \quad (11)$$

$$= 0.54 \times 10^{-14} \text{ eV},$$

where

$$I_1 = \frac{M^2 + m^2}{2mM} \int_1^{\infty} dx \left(\frac{M^2 + m^2}{2mM} - x \right)^2 \left(1 - \frac{4m_u}{m_{a_0}^2 - M^2 - m^2 + 2mMx} \right) \times \quad (12)$$

$$\times \ln \frac{m^2 - Mm(x - \sqrt{x^2 - 1})}{m^2 - Mm(x + \sqrt{x^2 - 1})},$$

$$I_2 = \frac{M^2 + m^2}{2mM} \int_1^{\infty} dx \sqrt{x^2 - 1} \left(\frac{M^2 + m^2}{2mM} - x \right)^2 \left(1 - \frac{4m_u}{m_{a_0}^2 - M^2 - m^2 + 2mMx} \right). \quad (13)$$

The sum of all these contributions leads to the following value of the width of the decay $K_L \rightarrow \pi^0 \gamma \gamma$

$$\Gamma_{K_L \rightarrow \pi^0 \gamma \gamma} = 1.4 \times 10^{-14} \text{ eV}, \quad (14)$$

or to the value of the branching ratio

$$B(K_L \rightarrow \pi^0 \gamma \gamma) = \frac{\Gamma_{K_L \rightarrow \pi^0 \gamma \gamma}}{\Gamma_{K_L \rightarrow \text{all}}} = 1.1 \times 10^{-6} \quad (15)$$

which does not contradict the experimental data on the upper bound of the branching ratio published in^{/1/}.

The estimates of the width of the decay $K_L \rightarrow \pi^0 \gamma \gamma$ becomes an especially actual meaning at present time, when LEP at CERN is starting to operate. In particular, there are in preparation some new experi-

ments with neutral particles as the final decay products. The test of our theoretical results could undoubtedly provide the interesting information concerning the confirmation of the advantages of using the linear version of σ -model in the elementary particle physics. The result of ^{1/} is close to our estimates and there is the possibility of measuring the upper bound in the framework of LEP activities.

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THE VECTOR AND AXIAL-VECTOR DOMINANCE OF WEAK INTERACTIONS

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The problem of the vector and axial-vector dominance of weak interactions in the framework of the quark model of superconductivity type is discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Векторная и аксиально-векторная доминантность слабых взаимодействий

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Обсуждается проблема векторной и аксиально-векторной доминантности слабых взаимодействий в рамках кварковой модели сверхпроводящего типа.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

The question how to include the weak interactions into the quark model of superconductivity type (QMST)^{/1/} has been discussed in papers^{/2, 3/}. Here we would like to give the final variant of such an inclusion. It will be slightly different from the results, obtained in that part of^{/3/}, where the interaction of axial-vector mesons with W-boson was described. If one uses the approximative mass formula $m_{a_1}^2 = m_\rho^2 + 6m_u^2$ which takes place in our model, the results^{/3/} will coincide with those, obtained in this article. However, for the concrete physical calculations it is better to use the last variant, as a more exact one.

After these general remarks let us overcome to the derivation of formulae, describing the vector and axial-vector dominance in QMST. At the beginning we write the Lagrangian, describing in the framework of our model the interaction of mesons with quarks

$$L_{st} = \bar{q} [i\hat{\partial} - M + g(\bar{\sigma} + i\gamma^5 \bar{\phi}) + \frac{g_\rho}{2} (\hat{V} + \gamma^5 \hat{A})] q -$$

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$$-\frac{\sigma_a^2 + \phi_a^2}{2G_1} + \frac{V_a^2 + A_a^2}{2G_2}, \quad (1)$$

where $\bar{q} = (\bar{u}, \bar{d}, \bar{s})$ are the quark fields with three colours; $i\hat{\partial} = i\gamma^\mu \partial_\mu$; $M = \text{diag}(m_u, m_d, m_s)$ is the constituent quark mass matrix; $\sigma_a, \phi_a, V_a^\mu$ and A_a^μ are the scalar, pseudoscalar, vector and axial-vector mesons, respectively; $\bar{a} = a_a \lambda^a$ ($0 < a < 8$), λ^a are the Gell-Mann matrices; $g_\rho = \sqrt{6}g$ is the constant of decay $\rho \rightarrow 2\pi$, ($g_\rho^2/4\pi = 3$). The constant G_1 and G_2 can be determined from the mass spectrum of pseudoscalar and vector mesons: $G_1 = 4.9 \text{ (GeV)}^{-2}$ and $G_2 = 16 \text{ (GeV)}^{-2/1}$.

From (1) one can obtain the following (necessary for the further investigations) part of "strong" Lagrangian:

$$L'_{st} = i\sqrt{2}g\bar{q}\gamma^5\pi^+r_+q + \frac{g}{\sqrt{2}}\bar{q}(\hat{\rho}^+ + \gamma^5\hat{a}_1^+)r_+q + g_\rho F_\pi Z a_{1\mu}^+ \partial_\mu \pi^- + \text{h.c.} - \frac{1}{2}(\rho_{\mu\nu}^+ \rho_{\mu\nu}^- + a_{1\mu\nu}^+ a_{1\mu\nu}^-) + m_\rho^2 \rho_\mu^+ \rho_\mu^- + m_{a_1}^2 a_{1\mu}^+ a_{1\mu}^-, \quad (2)$$

where $F_\pi = 93 \text{ MeV}$ is the pion decay constant. The factor Z can be expressed in terms of physical quantities as follows

$$Z^{-1} = (1 - \frac{6m_u^2}{m_{a_1}^2}) = \frac{1}{2} [1 + \sqrt{1 - (\frac{2g_\rho F_\pi}{m_{a_1}^2})^2}], \quad (3)$$

where m_{a_1} is the mass of the a_1 -meson. The mass of u -quark is given by the formula

$$m_u^2 = \frac{m_{a_1}^2}{12} [1 - \sqrt{1 - (\frac{2g_\rho F_\pi}{m_{a_1}^2})^2}] \quad (4)$$

and for $m_{a_1} = 1200 \text{ MeV}$ we have $m_u \approx 300 \text{ MeV}$.

Further let us investigate the interaction of the electroweak gauge boson W with quarks and leptons. We write the "weak" Lagrangian in the next form:

$$L_w = -\frac{\kappa}{2\sqrt{2}} \{ \bar{q} \hat{W}^{(+)} (1 - \gamma^5) r_+ q + \ell^{\mu(-)} W_\mu^{(+)} + \text{h.c.} \} -$$

$$-\frac{1}{2} W^{(-)\mu\nu} W_{\mu\nu}^{(+)} + M_W^2 W^{(-)\mu} W_{\mu}^{(+)}, \quad (5)$$

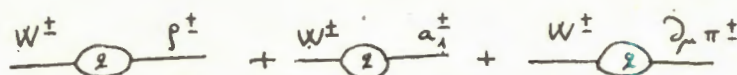
where

$$l^{\mu(-)} = \bar{\nu}_e \gamma^\mu (1 - \gamma^5) e + \bar{\nu}_\mu \gamma^\mu (1 - \gamma^5) \mu + \bar{\nu}_\tau \gamma^\mu (1 - \gamma^5) \tau$$

is the lepton current, W_μ is the vector boson with the mass M_W , $W^{(-)\mu\nu} W_{\mu\nu}^{(+)}$ is the kinetic form of W-boson and κ is a weak constant.

The Lagrangians (2) and (5) lead to an appearance of the diagrams depicted in the Figure, corresponding to the nondiagonal terms in

$$\Delta L = -\frac{\kappa}{2} F_\pi Z W^\mu \partial_\mu \pi^- + \frac{\kappa}{4 g_\rho} [W^{\mu\nu+} (\rho_{\mu\nu}^- + a_{\mu\nu}^-) - 12 m_\pi^2 W^{\mu+} a_{1\mu}^-] + \text{h.c.} \quad (6)$$



Figure

At the evaluation of the divergent diagrams in the Figure, we used the relation ^{1/}

$$g = (4I)^{-1/2}, \quad I = -i \frac{3}{(2\pi)^4} \int d^4 q \frac{\theta(\Lambda^2 - q^2)}{(m^2 - q^2)^2}, \quad (7)$$

where at the regularization of divergent integral, playing an important role at the description of quark loops, we used the cut-off parameter Λ . This parameter bounds the region, where the spontaneous breaking of chiral symmetry takes place ^{1/}.

We perform the diagonalization of kinetic terms by taking the new variables:

$$\rho_\mu^\pm = \rho_\mu^{\prime\pm} + \frac{\kappa}{2 g_\rho} W_\mu^\pm; \quad a_{1\mu}^\pm = a_{1\mu}^{\prime\pm} + \frac{\kappa}{2 g_\rho} W_\mu^\pm. \quad (8)$$

This procedure eliminates the vertices from the Lagrangians L_W and ΔL , describing the interaction of the W-boson with quark fields, as well as the nondiagonal terms $W^{\mu\pm} \partial_\mu \pi^\pm$. The nondiagonal terms with deri-

vations are replaced by the similar terms without derivations and the total Lagrangian will have the form

$$L = L_{st} - \frac{1}{2} W^{(-)\mu\nu} W_{\mu\nu}^{(+)} + M_{\pi}^2 W^{(-)\mu} W_{\mu}^{(+)} + \quad (9)$$

$$+ \frac{\kappa}{2g_{\rho}} [m_{\rho}^2 W_{\mu}^{+} \rho_{\mu}^{-} + \frac{m_{a_1}^2}{Z} W_{\mu}^{+} a_{1\mu}^{-} + \text{h.c.}] - \frac{\kappa}{2\sqrt{2}} [\ell^{\mu(-)} W_{\mu}^{+} + \text{h.c.}].$$

In the limit of large masses M_W the Lagrangian L can be rewritten in the expression

$$L_w = \frac{G_w \cos \theta_c}{g_{\rho}} \ell^{\mu(-)} (m_{\rho}^2 \rho_{\mu}^{+} + \frac{m_{a_1}^2}{Z} a_{1\mu}^{+}) + \text{h.c.} \quad (10)$$

Here θ_c is the Cabbibo angle. The vector and axial-vector dominance of weak interactions in QMST has been firstly discussed in papers ^{/2,3/}. In ^{/3/} the factor $m_{\rho}^2 (\rho_{\mu}^{+} + a_{1\mu}^{+})$ has been obtained instead of that one in (9), i.e. the Lagrangian in ^{/3/} has the form

$$L_w = \frac{G_w \cos \theta_c m_{\rho}^2}{g_{\rho}} \ell^{\mu(-)} (\rho_{\mu}^{+} + a_{1\mu}^{+}) + \text{h.c.} \quad (10a)$$

One can rewrite the Lagrangian (10) into the form (10a) by using the mass formula $m_a^2 = m_{\rho}^2 + 6m_u^2$ and (8), defined in our model

$$m_{\rho}^2 \rho_{\mu}^{+} + (1 - \frac{6m_u^2}{m_{a_1}^2}) m_a^2 a_{1\mu}^{+} = m_{\rho}^2 (\rho_{\mu}^{+} + a_{1\mu}^{+}).$$

However, because the model formula $m_{a_1}^2 = m_{\rho}^2 + 6m_u^2$ leads to the low value of the mass of the a_1 -meson ($m_{a_1} \approx 1100$ MeV) it is better to use the Lagrangian of the form given in (10). We verify it by considering, e.g., the decay $r \rightarrow \nu a_1$. From (10) we get the decay width

$$\Gamma_{r \rightarrow \nu a_1} = \frac{m_r}{8\pi} - [G \cos \theta_c \frac{m_{a_1} m_r}{Z g_{\rho}}]^2 (1 - \frac{m_{a_1}^2}{m_r^2}) (1 + \frac{2m_{\rho}^2}{m_r^2}) =$$

$$= \begin{cases} 3 \times 10^{-10} \text{ MeV} & (m_{a_1} = 1285 \text{ MeV})^{/4/} \\ 2.7 \times 10^{-10} \text{ MeV} & (m_{a_1} = 1200 \text{ MeV})^{/5/} \end{cases} \quad (11)$$

$$\Gamma_{r \rightarrow \nu}^{\text{exp}}(a_1 \rightarrow 3\pi) = (2.85 \pm 0.3 \pm 0.34) \times 10^{-10} \text{ MeV}^{/4/}.$$

By taking the Lagrangian in the form (10a) we obtain

$$\Gamma_{r \rightarrow \nu a_1} = \begin{cases} 0.81 \times 10^{-10} \text{ MeV} & (m_{a_1} = 1285 \text{ MeV}) \\ 1.08 \times 10^{-10} \text{ MeV} & (m_{a_1} = 1200 \text{ MeV}). \end{cases} \quad (12)$$

We write also the theoretical result for the decay $r \rightarrow \nu \rho$:

$$\Gamma_{r \rightarrow \nu \rho} = \frac{m_r}{8\pi} \left[G \cos \theta_c \frac{m_\rho m_r}{g_\rho} \right]^2 \left(1 - \frac{m_\rho^2}{m_r^2}\right)^2 \left(1 + \frac{2m_\rho^2}{m_r^2}\right) = 4.2 \times 10^{-10} \text{ MeV}, \quad (13)$$

$$\Gamma_{r \rightarrow \nu \rho}^{\text{exp}} = (4.35 \pm 0.4 \pm 0.53) \times 10^{-10} \text{ MeV}^{/4/}.$$

There the agreement with the experiment is good. The axial-vector dominance allows to simplify the calculations in any other processes, e.g., the decays $\pi \rightarrow e \nu \gamma^{/6/}$, $\pi \rightarrow \nu e (\ell^+ \ell^-)^{/7/}$, $K \ell_4^{/8/}$, etc.

In conclusion let us discuss how to describe the fundamental decay of pion $\pi^- \rightarrow e \bar{\nu} (\mu \bar{\nu})$ by using the Lagrangian (10). The corresponding part of (10) has the form

$$\Delta L_w = G_w \cos \theta_w \frac{m_{a_1}^2}{g Z} a_{1\mu}^+ \bar{\nu} \gamma^\mu (1 - \gamma^5) e. \quad (10b)$$

To be able to overcome to the physical fields $a_{1\mu}'$, it is necessary to remove the nondiagonal terms in the strong Lagrangian (2). For this purpose it is enough to perform the following transformation of the fields $a_{1\mu}$

$$a_{1\mu}^+ = a_{1\mu}'^+ - \frac{g_\rho F_\pi Z}{m_{a_1}^2} \partial_\mu \pi^+, \quad (14)$$

Then one can easily obtain, using (10b) and (14), the Lagrangian describing the decay $\pi^- \rightarrow e \bar{\nu}$

$$\begin{aligned}
L_{\pi^- \rightarrow e \bar{\nu}} &= -G_w \cos \theta_c \left[\frac{m_{a_1}^2}{g_\rho Z} \frac{g_\rho F_\pi Z}{m_{a_1}^2} = F_\pi \right] \partial_\mu \pi^+ \bar{\nu} \gamma^\mu (1 - \gamma^5) e = \\
&= -G_w \cos \theta_c F_\pi \partial_\mu \pi^+ \bar{\nu} \gamma^\mu (1 - \gamma^5) e.
\end{aligned}
\tag{15}$$

We have demonstrated that the axial vector dominance plays an important role at the calculation of weak processes.

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ЭМИССИЯ ФРАГМЕНТОВ ИЗ ВОЗБУЖДЕННЫХ ЯДЕР, ОБРАЗУЮЩИХСЯ В РЕАКЦИЯХ $^{232}\text{Th} + ^{12}\text{C}$ И $^{181}\text{Ta} + ^{52}\text{Cr}$

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На основе модели каскадного испарения нуклонов и фрагментов проведены расчеты сечений образования фрагментов с $Z = 3 \div 20$ при распаде возбужденных ядер, образующихся в реакциях $^{181}\text{Ta} + ^{52}\text{Cr}$ и $^{232}\text{Th} + ^{12}\text{C}$. Наблюдается качественное различие, для объяснения которого нужно предполагать, что в случае реакции с ионами хрома возрастает деформация остаточного ядра, или лишь $10 \div 20\%$ от выхода равновесных фрагментов определяется каналом распада составного ядра, а остальная доля связана с другими каналами глубоконеупругого взаимодействия ядер.

Работа выполнена в Лаборатории ядерных реакций ОИЯИ.

Emission of Fragments in the Decay of the Excited Nuclei Produced in the Reactions

$^{181}\text{Ta} + ^{52}\text{Cr}$ and $^{232}\text{Th} + ^{12}\text{C}$

Yu.A.Muzychka, B.I.Pustynnik

On the basis of the nucleon evaporation model the cross sections have been calculated for the formation of fragments with $Z = 3 \div 20$ in the decay of the excited nuclei produced in the reactions $^{181}\text{Ta} + ^{52}\text{Cr}$ and $^{232}\text{Th} + ^{12}\text{C}$. It is found that there is a qualitative difference in fitting to the experimental results for these two reactions. This difference can be explained if it is assumed that in the ^{52}Cr -induced reaction the deformation of residual nuclei increases, or only $10\text{-}20\%$ of the equilibrium fragment yield is due to compound nucleus decay, the rest being formed as a result of the deep inelastic interaction of the nuclei.

The investigation has been performed at the Laboratory of Nuclear Reactions, JINR.

В последнее время получены экспериментальные данные, свидетельствующие о том, что заметная доля выхода фрагментов с $Z > 2$, образующихся в ядерных реакциях с тяжелыми ионами, в заднюю полусферу, связана с распадом возбужденных составных ядер, либо долгоживущих промежуточных систем. Эти работы по-

высили интерес к теоретическому рассмотрению процесса испускания равновесных фрагментов^{/1, 2/}. Ранее нами на основе модели каскадного испарения нуклонов и фрагментов были проведены расчеты сечений образования равновесных фрагментов с $Z=3 \div 11$ при распаде возбужденных ядер, образующихся в реакциях $^3\text{He} + \text{Ag}$ и $^{40}\text{Ar} + (\text{Ag}, \text{Sm}, \text{Au})$ ^{/3, 4/}. Окончательная формула для расчетов сечений испарения кластеров из возбужденных ядер с учетом нескольких ступеней нейтронного испарительного каскада была представлена в виде

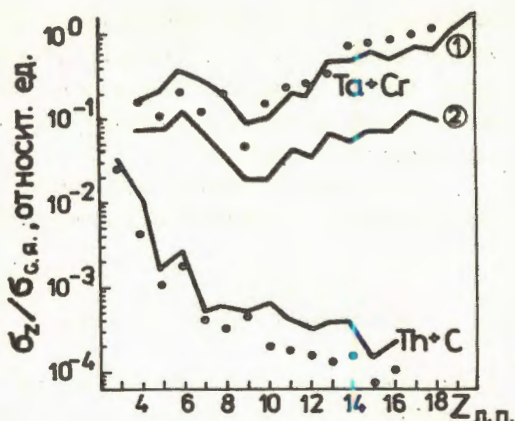
$$\sigma = \sum_{\ell=0}^{\ell_{\text{кр}}} \sigma_{\text{с.я.}}^{\ell} \sum_{i=0}^m \frac{\Gamma_{\nu, \ell}^{(i)}}{\Gamma_{f, \ell}^{(i)} + \sum_{n, p} \Gamma_{n(p), \ell}^{(i)} + \sum_{\nu} \Gamma_{\nu, \ell}^{(i)}} \times$$

$$\times \prod_{k=0}^{i-1} \frac{\Gamma_{n, \ell}^{(k)}}{\Gamma_{f, \ell}^{(k)} + \sum_{n, p} \Gamma_{n(p), \ell}^{(k)} + \sum_{\nu} \Gamma_{\nu, \ell}^{(k)}},$$

где m — число ступеней нейтронного испарительного каскада. Расчеты проведены для каждого ℓ с шагом по энергии возбуждения в 1 МэВ. Испарительные ширины для нуклонов и фрагментов Γ_n , Γ_p и Γ_{ν} рассчитывались в статистической модели с использованием плотности уровней в модели ферми-газа. Использовалось стандартное выражение для делительной ширины Γ_f с учетом эффектов вращения в модели CPS^{/5/}. Анализ показал, что на результаты расчета сильное влияние оказывает учет оболочечных эффектов в параметре плотности уровней остаточных ядер и учет коллективных эффектов, связанных с изменением равновесной деформации и углового момента в остаточных ядрах после вылета нуклонов и фрагментов.

Мы также отмечали, что в случае ионов Ag отождествление долгоживущей промежуточной системы, дающей вклад в выход кластеров в задней полусфере, с составным ядром является более проблематичным, чем в случае реакций с ^3He , однако расчет по испарительной модели и в этом случае приводит к удовлетворительному согласию с экспериментальными данными, если увеличить на $5 \div 7\%$ величину Γ_0 , входящую в формулу для кулоновского взаимодействия кластеров и остаточного ядра. Недавно появились экспериментальные данные по сечениям эмиссии равновесных фрагментов с $Z=3 \div 16$ в реакциях $^{232}\text{Th} + ^{12}\text{C}$ и $^{181}\text{Ta} + ^{52}\text{Cr}$ ^{/6/}, из которых можно сделать более надежное заключение о вкладе

Сравнение экспериментальных сечений ($\bullet$) образования фрагментов с результатами расчета (—). Экспериментальные данные взяты из работы [6], где они представлены в относительных единицах. В реакции Ta+Cr кривая 1 соответствует $r_0 = 1,3$, кривая 2 — $r_0 = 1,23$.



канала эмиссии фрагментов из возбужденных составных ядер $^{244}\text{См}$ и $^{233}\text{Вк}$ в полное сечение образования равновесных фрагментов. Составные ядра имеют близкую энергию возбуждения и предельный угловой момент ($\sim 130 \div 140$ МэВ, $\sim 65\hbar$), вместе с тем сильно отличаются числом нейтронов. Несмотря на то, что $\sigma_{\text{с.я.}}(\text{Th+C})/\sigma_{\text{с.я.}}(\text{Ta+Cr}) \sim 6$, выход фрагментов в реакции с ионами углерода меньше, чем с ионами хрома примерно в $20 \div 500$ раз при Z , меняющемся от 6 до 16. Были проведены расчеты сечений эмиссии фрагментов с $Z=3 \div 20$ из возбужденных составных ядер, образующихся в этих реакциях. Для реакции с ионами ^{12}C удовлетворительное согласие результатов расчета с экспериментом получено при $r_0 = 1,23$ фм, что согласуется с расчетами для ^3He , в то время как в случае ионов хрома также необходимо было использовать $r_0 = 1,3$ фм. При $r_0 = 1,23$ фм в случае ионов хрома расчетное отношение выхода фрагментов $\sigma_{\nu}(\text{Ta+Cr})/\sigma(\text{Th+C})$ меняется всего от 3 до 10 при Z , меняющемся от 6 до 16 (см. рисунок).

Таким образом, наблюдается качественное различие, для объяснения которого нужно либо предполагать, что при реакции с ионами хрома возрастает деформация остаточного ядра, либо в этой реакции лишь $10 \div 20\%$ от выхода равновесных фрагментов определяются каналом распада составного ядра, а остальная доля связана с другими каналами глубоконеупругого взаимодействия ядер [7].

Нам кажется, что дальнейшее исследование процессов с испусканием равновесных фрагментов может явиться хорошим средством изучения характеристик нагретых ядер в широком диапа-

зоне масс и энергий возбуждения. Кроме того, при рассмотрении новых возможных механизмов образования фрагментов необходимо иметь представление о том, какая часть этих фрагментов может быть связана с обычной статистической моделью распада возбужденных ядер.

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