Statistical Model Analysis of (n,t) Cross Sections for 14–15 MeV Neutrons

G. Khuukhenkhuu, J. Munkhsaikhan, M. Odsuren, Ch. Saikhanbayar, and B. Batchimeg

Nuclear Research Center, National University of Mongolia, Ulaanbaatar, Mongolia

Yu.M. Gledenov, E. Sansarbayar, and M.V. Sedysheva

Frank Laboratory of Neutron Physics, JINR, Dubna, Russia

1. Introduction

Fast neutron induced (n,t) reaction data analysis is of important for nuclear reaction study and fusion reactor technology, particularly to evaluate three nucleons clustering effect and enhancing tritium in the structural materials. However, experimental data base of the (n,t) reaction is very scarce due to small cross sections and their available values for the same isotopes are varied in the wide range. So, theoretical model calculations and systematical analysis of available data are useful to estimate unmeasured (n,t) cross sections and to conclude which value is a probably correct.

A few attempts of compilation, systematical analysis and evaluation for the (n,t) cross sections were carried out using different approaches [1–10] and some systematical regularities were observed. But, the results of these studies are not consistent and up to now a common explanation of the observed regularities are not available.

In past years we have been analyzing the (n,α) and (n,p) cross sections for fast neutrons using a unified method, namely the statistical model, and some regular behaviours in known experimental data were observed [11–14] for wide energy range of 2 to 20 MeV. Moreover, in the framework of the statistical model, alpha clustering factors (or probabilities) for slow and fast neutron induced (n,α) reactions were obtained [13,14].

In this work we carried out a systematical analysis of known (n,t) cross sections around 14–15 MeV neutrons using the statistical model and determined the triton clustering factor.

2. Theoretical background

In the framework of the statistical model based upon Bohr's assumption of compound mechanism the (n,t) cross section can be expressed as following:

$$\sigma(n,t) = \sigma_c(n)G(t). \quad (1)$$

Here:

$$\sigma_c(n) = \pi(R + \lambda\!\!\!^{-}_n)^2 \quad (2)$$

is the compound nucleus formation cross section, where $R = r_0 A^{1/3}$ is the target nucleus radius, $r_0 = 1.3\cdot 10^{-13}$ cm and A is target nucleus mass number;

$$\lambda\!\!\!^{-}_n = \frac{4.55\cdot 10^{-13}}{\sqrt{E_n(\mathrm{MeV})}}(\mathrm{cm}) \quad (3)$$

is the wavelength of the incident neutrons divided by 2π, where E_n is the neutron energy.

The triton decay probability of the compound nucleus is given by

$$G(t)=\frac{\Gamma_t}{\Gamma}=\frac{\Gamma_t}{\sum_i \Gamma_i}, \quad (4)$$

where Γ_t and Γ are the triton and total level widths.
Using the detailed balancing principle and Weisskopf-Ewing's evaporation model [15,16] the triton width of level, Γ_t, is determined as following:

$$\Gamma_t=\frac{2S_t+1}{\pi^2\hbar^2\rho_c(E_c)}M_t\int_{V_t}^{E_t^{\max}}E_t\sigma_c(E_t)\rho_y(U_t)dE_t. \quad (5)$$

Here: S_t, M_t, E_t, and V_t are the spin, mass, energy and the Coulomb potential energy for the outgoing triton, respectively; $\rho_c(E_c)$ and $\rho_y(U_t)$ are the level densities of the compound and residual nuclei, respectively; U_t is the excitation energy of the residual nuclei; $\sigma_c(E_t)$ is the inverse reaction cross section which is determined in the semiclassical approximation [16] as follows:

$$\sigma_c(E_t)=\begin{cases}\pi R^2\left(1-\dfrac{V_t}{E_t}\right) & \text{for} \quad E_t>V_t\\ 0 & \text{for} \quad E_t<V_t\end{cases}. \quad (6)$$

Using the nuclear entropy [17] and constant temperature approximation [16] we can get from (1), (2) and (4)–(6) the following simple formula for fast neutron induced (n,t) reaction cross section:

$$\sigma_{(n,t)}=\pi(R+\lambda\!\!\!^{-}_n)^2\frac{2S_t+1}{2S_n+1}\cdot\frac{M_t}{M_n}e^{\frac{Q_{nt}-V_t}{\theta}}. \quad (7)$$

The (n,t) reaction energy, Q_{nt}, can be obtained using Weizsäcker's formula [18] for binding energy as:

$$Q_{nt}=-\alpha A+\alpha(A-2)+\beta\left[A^{2/3}-(A-2)^{2/3}\right]+\gamma\left[\frac{Z^2}{A^{1/3}}-\frac{(Z-1)^2}{(A-2)^{1/3}}\right]+$$
$$\xi\left\{\frac{(A-2Z)^2}{A}-\frac{[(A-2)-2(Z-1)]^2}{A-2}\right\}\pm\left[\frac{\delta_f}{(A-2)^{3/4}}-\frac{\delta_i}{A^{3/4}}\right]+\varepsilon_t. \quad (8)$$

Here: $\varepsilon_t=8.48$ MeV is the internal binding energy of triton; $\alpha=15.7$ MeV, $\beta=17.8$ MeV, $\gamma=0.71$ MeV, $\xi=23.7$ MeV and $|\delta|=34$ MeV or 0 are the Weizsäcker's constants; Z is the proton number. For medium mass and heavy nuclei approximation A>>2 can be used. Then, the reaction energy is found from (8) as follows:

$$Q_{nt}=\gamma\frac{2Z-1}{A^{1/3}}-4\xi\frac{N-Z+1}{A}+\varepsilon_t. \quad (9)$$

The Coulomb potential energy of triton can be written [19] in the form:

$$V_t = 1.029\frac{(Z-1)}{A^{1/3}+3^{1/3}}\,. \tag{10}$$

If we use Fermi gas model for level density parameter [20], the nuclear thermodynamic temperature [13] is expressed as:

$$\Theta = \sqrt{\frac{13.5(E_n + Q_{nt})}{A}}\,. \tag{11}$$

So, from (7), (9) and (10) the (n,t) cross section can be obtained as follows:

$$\sigma(n,t) = C\pi\left(R + \lambda\!\!\!^{-}_n\right)^2 e^{-K\frac{N-Z+1}{A}}\,, \tag{12}$$

where: *N* is the neutron number of the target nucleus; The parameters *C* and *K* are expressed as follows:

$$C = 3\exp\left\{\frac{\gamma\frac{2Z-1}{A^{1/3}} + \varepsilon_t - 1.029\frac{(Z-1)}{A^{1/3}+3^{1/3}}}{\Theta}\right\} \tag{13}$$

and

$$K = \frac{4\xi}{\Theta}\,. \tag{14}$$

Here the thermodynamic temperature, θ, is determined by formula (11) and reaction energy, Q_{nt}, is obtained by expression (9). The parameters *C* and *K* can be, also, obtained by fitting of theoretical (n,t) cross sections to experimental data as constants for all isotopes. Formulas (7) and (11)–(14) are used in the systematical analysis of the fast neutron induced (n,t) cross sections.

3. Results and Discussion

From formula (12) can be found a reduced (n,t) cross section as:

$$\frac{\sigma(n,t)}{\pi\left(R + \lambda\!\!\!^{-}_n\right)^2} = Ce^{-K\frac{N-Z+1}{A}}\,. \tag{15}$$

The dependence of the reduced (n,t) cross sections on the relative neutron excess parameter (N–Z+1)/A in the target nuclei, for which experimental data are available, is shown in Fig.1.

Some regular dependence of the reduced (n,t) cross sections on the parameter (N–Z+1)/A is seen form Fig.1 except light nuclei with mass number A=6–18 which perhaps have a cluster structure and strong resonances in the cross sections. Systematical behaviour of the reduced (n,t) cross sections for odd-even and even-even nuclei are separately and satisfactorily described by formula (15) with fitting parameters C and K which are given in Fig.1, also.

Using the approximation A>>2, the absolute value of the (n,t) cross section can be obtained by formulae (11)–(14). Results of such calculations in comparison with experimental data for odd-even and even-even nuclei are shown in Fig.2.

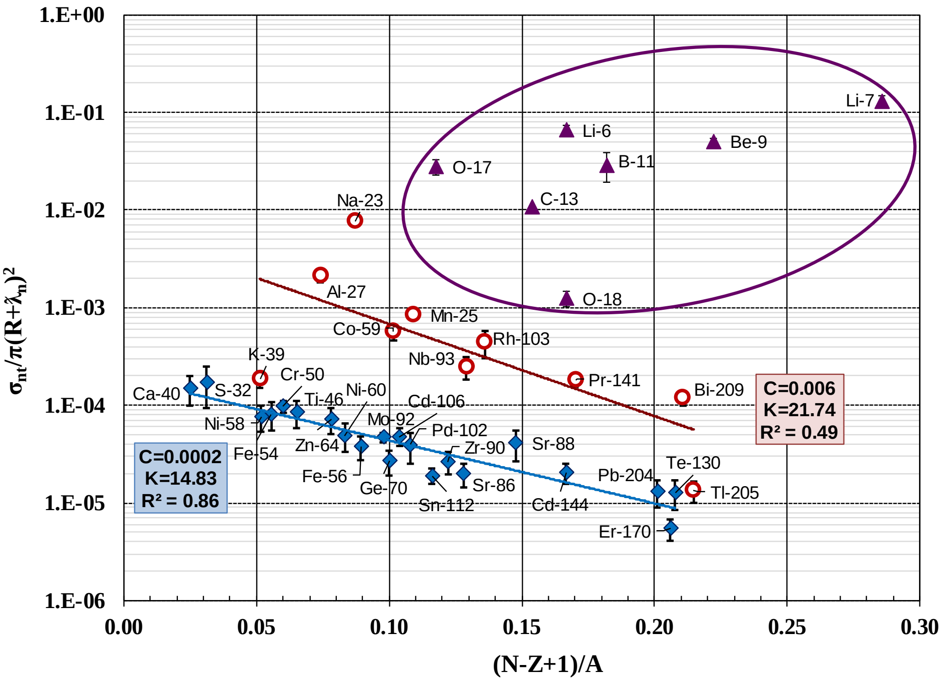

Fig.1. Experimental data for the reduced (n,t) cross sections:
▲ -light nuclei; ○ -odd-even nuclei; ◆-even-even nuclei.

Theoretical lines were drawn by formula (15) with fitted parameters C and K.

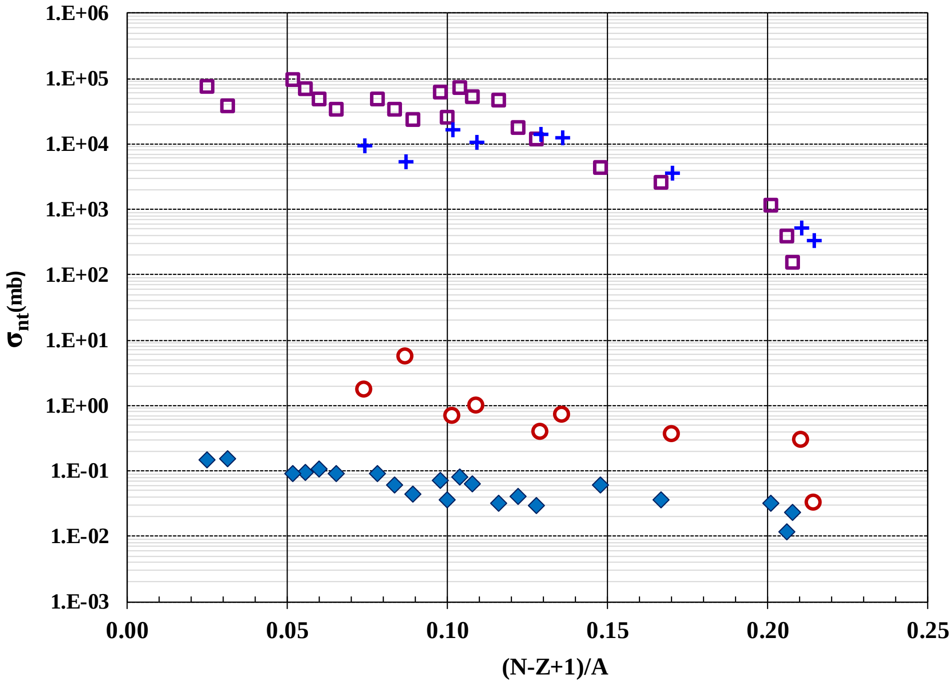

Fig.2. Experimental data and calculated by formulae (11)–(14) (n,t) cross sections:
□ and + -theoretical cross sections for even-even and odd-even nuclei, respectively;
◆ and ○-experimental data for even-even and odd-even nuclei without ^{39}K, respectively;

It is seen from Fig.2 that the statistical model formulas give overestimated values of the (n,t) cross sections for fast neutrons. We assume that this fact is probably caused by three nucleons (or triton) clustering effect that is not considered in the above deduced formulae.
So, the triton clustering factor (or probability), by analogy with the spectroscopic factor [21] and alpha-clustering probability [13,22], can be obtained as a ratio of experimental (n,t) cross sections to theoretical ones as:

$$\phi_t = \frac{\sigma_{(n,t)}^{\exp}}{\sigma_{(n,t)}^{theor}}. \qquad (16)$$

Results of such calculations are given in Fig.3.

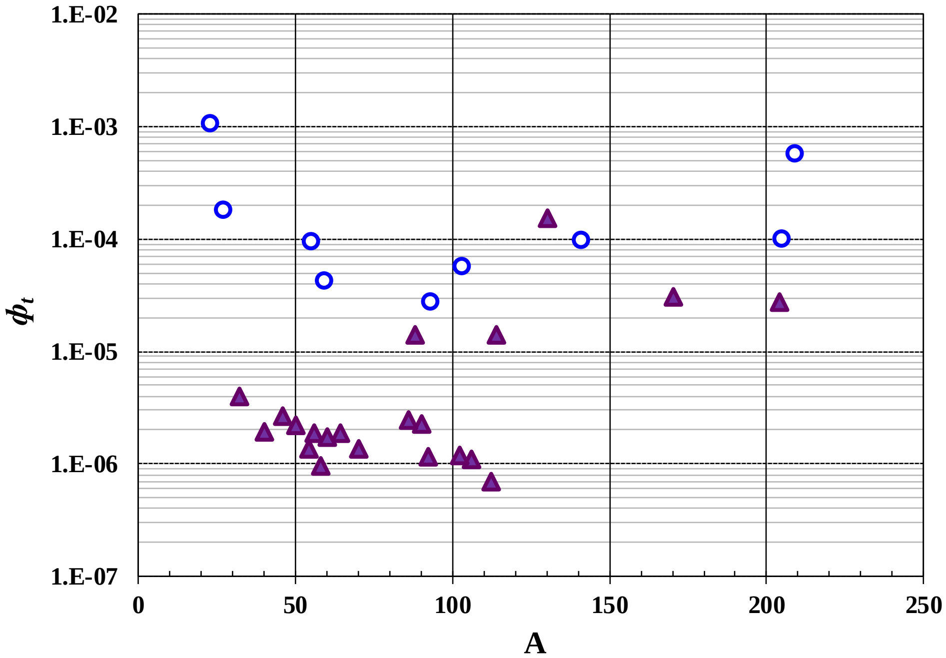

Fig.3. The triton clustering factor versus mass number of target nuclei:
▲-even-even nuclei and ○-odd-even nuclei.

It is seen from Fig.3 that the triton clustering factors for even-even nuclei are on average lower by factor around 10–100 than ones for odd-even nuclei.
The values of the triton clustering factor ($\phi_t \approx 10^{-3}$–10^{-6}) obtained in this work are much lower than α-clustering (four nucleons) probability ($\phi_t \approx 0.28$) for 14.5 MeV neutrons determined by the same method in our previous work [10].

4. Conclusions

1. Simple formulae for systematical analysis of the fast neutron induced (n,t) reaction cross sections were deduced using the statistical model.
2. The systematical analysis of known (n,t) cross sections for 14–15 MeV neutrons shows that general regularity of experimental data for medium mass and heavy nuclei are satisfactorily described by the statistical model which, however, gives overestimated values for absolute (n,t) cross sections. We assume that this fact is, probably, caused by three nucleons (triton) clustering effect which was not considered in the above mentioned approach.

3. The triton clustering factor (or probability) was obtained, by analogy with the spectroscopic factor, as a ratio of experimental (n,t) cross sections to theoretical ones.
4. The values of the triton clustering factor ($\phi_t \approx 10^{-3}$–10^{-6}) obtained in this work are much lower than α-clustering probability ($\phi_t \approx 0.28$) for 14.5 MeV neutrons determined by the same method in our previous work.

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