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ЯДРА

SPATIAL DEPENDENCE OF PAIRING IN DEFORMED NUCLEI

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The solution of time-dependent Hartree–Fock–Bogoliubov equations by the Wigner-function-moments method leads to the appearance of refined low-lying modes whose description requires the accurate knowledge of the anomalous density matrix. It is shown that calculations with Woods–Saxon potential satisfy this requirement, producing an anomalous density matrix of the same quality as more complicated calculations with realistic forces.

1. INTRODUCTION

The problem of the spatial dependence of the pairing field is at the moment the object of strong interest of nuclear theorists [1–6] because of the necessity to explain the properties of nuclei disposed far from the beta-stability line. We met this problem when studying the nuclear scissors mode. It is known [7, 8] that one must take into account pair correlations to describe correctly nuclear scissors motion, therefore Time-Dependent Hartree–Fock–Bogoliubov (TDHFB) equations should be the natural instrument to work with.

Being the isovector mode the nuclear scissors appear in the frame of our approach (Wigner-function moments) together with isoscalar low-lying excitations (ISLLE), which are generated by quantum corrections to the semiclassical limit of TDHFB equations, implying their subtle structure. In this sense ISLLE modes turn out to be even more subtle and refined modes than the scissors modes. Naturally, one cannot use semiclassical expressions for the anomalous density and pairing field to describe such excitations – one needs quantum-mechanical expressions, which can be found by solving static HFB equations. In the previous paper [9] we performed methodical calculations of the anomalous density, pairing field and coherence length for the spherical nucleus ^{134}Ba using Woods–Saxon single-particle wave functions as a test case. Our results turned out to be in very good agreement with the respective results of [1–5] obtained in self-consistent calculations with various realistic forces (Argonne, Gogny, Skyrme).

The aim of this paper is to repeat the analogous calculations for deformed nuclei (again with the Woods–Saxon potential). It is necessary to note that all known HFB calculations of deformed nuclei [6, 10–12] use the oscillator basis, which allows one to transform the anomalous density to the relative and center-of-mass coordinates with the help of Moshinski coefficients. In this case one is forced to use very large basis sets to get the correct asymptotic behavior of the solution. Our approach is free from this problem because the Woods–Saxon potential ensures the correct asymptotics automatically.

2. ANOMALOUS DENSITY MATRIX

The anomalous density matrix is defined [13] as

$$\kappa(\mathbf{r}_1,s_1;\mathbf{r}_2,s_2) = \tag{1}$$
$$= \sum_{k>0} u_k v_k [\phi_k(\mathbf{r}_1,s_1)\phi_{\bar{k}}(\mathbf{r}_2,s_2) -$$
$$- \phi_{\bar{k}}(\mathbf{r}_1,s_1)\phi_k(\mathbf{r}_2,s_2)],$$

where $k \equiv n,l,j,m$ is the set of shell-model quantum numbers and $\bar{k}$ is that of the time-conjugate state. Bogoliubov coefficients u,v are defined in the usual way:

$$v_k^2 = \frac{1}{2}\left(1 - \frac{\tilde{\epsilon}_k}{\sqrt{\tilde{\epsilon}_k^2 + \Delta_k^2}}\right), \quad u_k^2 = 1 - v_k^2$$

with $\Delta_k = -\sum_{k'>0} \bar{v}_{k\bar{k},k'\bar{k}'} u_{k'} v_{\bar{k}'}$, $\bar{v}_{ij,mn} = v_{ij,mn} - v_{ij,nm}$, and $\tilde{\epsilon}_k = \epsilon_k - \lambda$. ϵ_k's are single-particle energies and λ is the chemical potential. Inserting expressions for u,v into the formula for Δ one finds the set of gap equations

$$\Delta_k = -\frac{1}{2}\sum_{k'>0} \bar{v}_{k\bar{k},k'\bar{k}'} \frac{\Delta_{k'}}{\sqrt{\tilde{\epsilon}_{k'}^2 + \Delta_{k'}^2}}. \tag{2}$$

The solution of these equations for ^{174}Yb is shown in Fig. 1.

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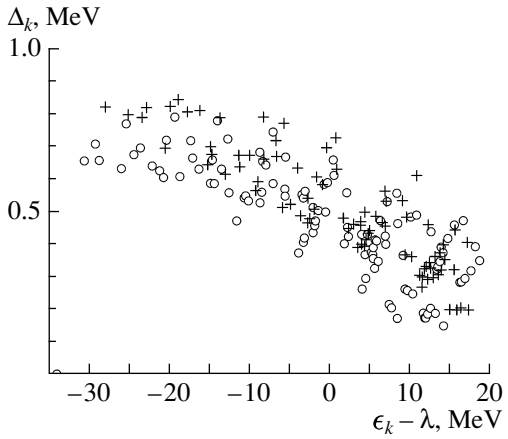

Fig. 1. Neutron (circles) and proton (crosses) pairing gaps Δ_k for ^{174}Yb depending on the state k.

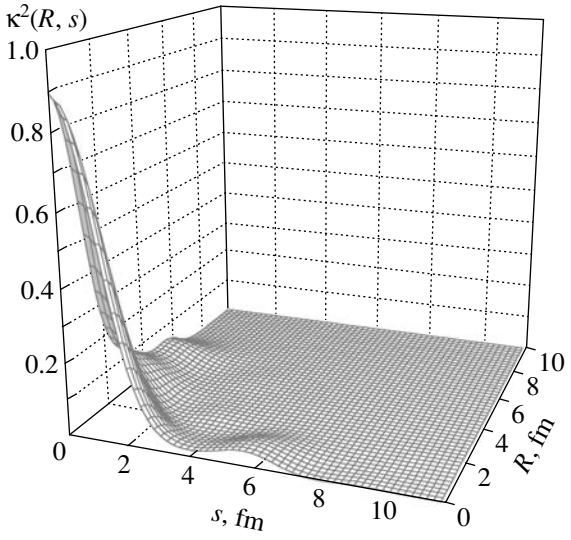

Fig. 2. Function $\kappa^2(R, s)$ for ^{134}Ba with deformation $\delta = 0.14$. Vertical scale has been multiplied by a factor of 10^4.

Matrix elements $v_{k\bar{k},k'\bar{k}'}$ of the Gaussian pair interaction $v(\mathbf{r}_1, \mathbf{r}_2) = -V_0 e^{-|\mathbf{r}_1 - \mathbf{r}_2|^2/r_p^2}$ were calculated with the Woods–Saxon single-particle wave functions [14], the values $V_0 = 25$ MeV and $r_p = 1.9$ fm being used to ensure reasonable gap values.

Having Δ_k one can calculate $\kappa(\mathbf{r}_1, s_1; \mathbf{r}_2, s_2)$. We take $s_1 = -1/2, s_2 = 1/2$ (see [8]) and introduce coordinates $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2, \mathbf{s} = \mathbf{r}_1 - \mathbf{r}_2$. The vector $\mathbf{R}$ will be usually directed along the z-axis, that corresponds to the configuration (a) of paper [6]. In what follows the notation $\kappa(R, s)$ will be used for $\kappa(\mathbf{r}_1, -1/2; \mathbf{r}_2, 1/2)$ averaged over the angle between $\mathbf{R}$ and $\mathbf{s}$. Figure 2 demonstrates $\kappa^2(R, s)$ calculated for ^{134}Ba with the deformation $\delta = 0.14$.

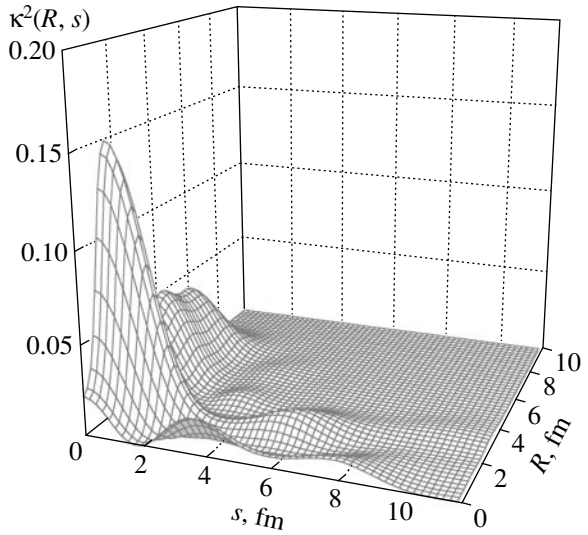

Fig. 3. Function $\kappa^2(R, s)$ for ^{174}Yb with deformation $\delta = 0.26$. Vertical scale has been multiplied by a factor of 10^4.

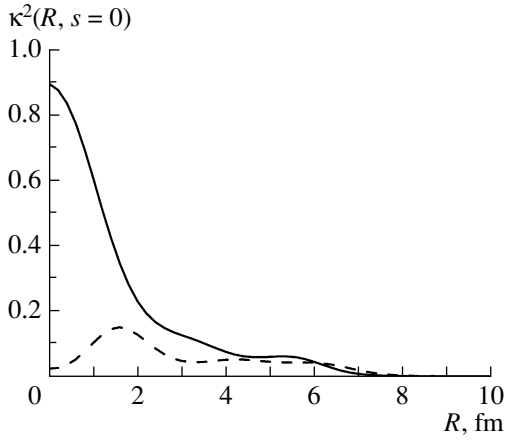

Fig. 4. Sections of the function $\kappa^2(R, s)$ at $s = 0$ for ^{134}Ba (solid curve) and ^{174}Yb (dashed curve). Vertical scale has been multiplied by a factor of 10^4.

The picture is qualitatively quite close to that of [9], calculated with $\delta = 0$. The remarkable difference is only in the height of the spike on the nuclear surface — it became approximately 5 times lower. The anomalous density $\kappa^2(R, s)$ calculated for ^{174}Yb ($\delta = 0.26$) is shown on Fig. 3.

One sees immediately the main difference with ^{134}Ba: in the center of nucleus (at the point $R = s = 0$) one has now a very small value of $\kappa^2(R, s)$ instead of huge pike in the case of ^{134}Ba. For the sake of more convenient comparison we have shown on Fig. 4 the section of $\kappa^2(R, s)$ at $s = 0$ in the same scale for both nuclei.

The reason of such striking difference in the

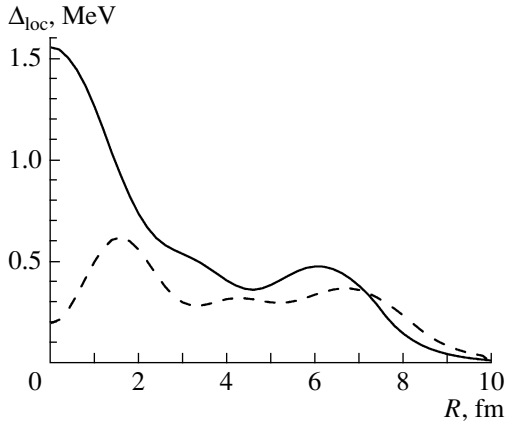

Fig. 5. The pairing field $\Delta_{\rm loc}$ calculated in the local approximation for ^{134}Ba (solid curve) and ^{174}Yb (dashed curve).

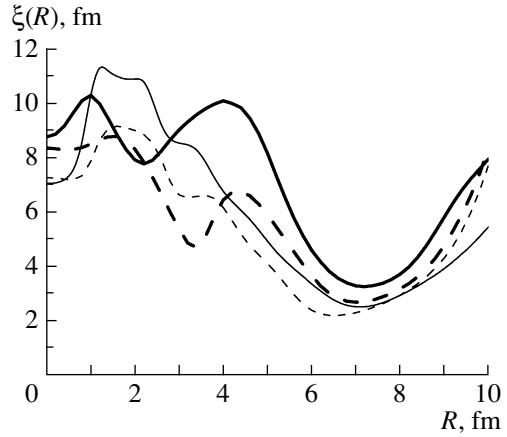

Fig. 6. Coherence length $\xi(R)$ of ^{174}Yb calculated for spherical (s) and deformed (d) states (thick solid curve — neutrons (d), thin solid curve — neutrons (s), thick dashed curve — protons (d), thin dashed curve — protons (s)).

behavior of $\kappa^2(R,s)$ in different nuclei lays naturally in their shell structure. In the case of zero deformation of ^{134}Ba the main contribution into the value of $\kappa^2(R=0,s=0)$ gives the level $3s1/2$, which is disposed practically on the Fermi surface. Exactly the same situation was noticed by the authors of [1] for ^{120}Sn. In the case of deformed ^{134}Ba the big value of $\kappa^2(R=0,s=0)$ is ensured by the level (in the Nilsson notation) $[Nn_z\Lambda\Sigma]=[400\uparrow]$ which has the energy -9.001 MeV, the chemical potential being -8.84 MeV. Expanding its wave function in the spherical basis one finds that the main ($\sim$60%) constituent of this state is $3s1/2$ state. On the other hand, in the vicinity of the Fermi level of ^{174}Yb there are no levels having the big s-state component. As a result, one has a "hole" in the anomalous density at the nucleus center. Exactly as in the spherical case [9] the three-spike structure, observed in the $\kappa^2(R,s)$ behavior is repeated in the behavior of the pairing field calculated in the local approximation (Fig. 5)

$$\Delta_{\rm loc}(R)\equiv\Delta(R,p_{\rm F}(R)),\quad p_{\rm F}^2=2m[\lambda-V(R)].$$

This correlation is especially well seen when one compares Figs. 4 and 5. The value of $\Delta(R,p_{\rm F}(R))$ is obtained by averaging the function $\Delta(R,\mathbf{p})$ over angle between $\mathbf{R}$ and $\mathbf{p}$. The formula for this function reads:

$$\Delta(R,\mathbf{p})=\int\exp(-i\mathbf{ps}/\hbar)\Delta(R,\mathbf{s})d^3s= \qquad (3)$$

$$=|V_0|\int\exp(-i\mathbf{ps}/\hbar)\exp(-s^2/r_p^2)\kappa(R,\mathbf{s})d^3s.$$

Finally we calculated the coherence length

$$\xi(R)=\frac{(\int s^2|\kappa(R,\mathbf{s})|^2d^3s)^{1/2}}{(\int|\kappa(R,\mathbf{s})|^2d^3s)^{1/2}}, \qquad (4)$$

which is shown in Fig. 6.

In general, the picture for deformed nuclei is similar to that of spherical ones. There are, however, some new details. First, in deformed cases the function $\xi(R)$ has more oscillations inside of nucleus. Second, the minimal value of the coherence length on the nuclear surface $\xi(R_{\rm surf})$ in deformed nuclei is 0.3–0.7 fm bigger than in spherical ones. It is necessary also to note that our results for $\xi(R_{\rm surf})$ are systematically higher than that of the paper [6]. We suppose that it is connected with essentially different asymptotics of Woods–Saxon and harmonic-oscillator wave functions.

3. CONCLUSION

We have demonstrated that calculations of the abnormal density and the pair field (gap) with the Woods–Saxon mean-field potential are able to reproduce very well the results obtained in the self-consistent calculations with realistic interactions, the statement being true for spherical nuclei and deformed ones as well. As we understand, the lack of self-consistency is completely compensated by the proper choice of Woods–Saxon parameters, which are fitted to reproduce the nuclear single-particle levels near the Fermi surface. These results shall be used in the calculation of scissors mode.

REFERENCES

1. N. Pillet, N. Sandulescu, and P. Schuck, Phys. Rev. C **76**, 024310 (2007).
2. N. Pastore, F. Barranco, R. A. Broglia, *et al.*, Phys. Rev. C **78**, 024315 (2008).
3. S. S. Pankratov, E. E. Saperstein, M. V. Zverev, and E. Vigezzi, Phys. Rev. C **79**, 024309 (2009).

4. N. Pastore, T. Duguet, and T. Lesinski, in *Proceedings of the International Conference on Nuclear Structure and Related Topics, Dubna, 2009*, Ed. by A. I. Vdovin *et al.* (JINR, Dubna, 2009), Vol. 2, p. 181.
5. S. S. Pankratov, M. Baldo, U. Lombardo, *et al.*, see [4], p. 276.
6. N. Pillet, N. Sandulescu, P. Schuck, and J.-F. Berger, Phys. Rev. C **81**, 034307 (2010).
7. D. Zawischa, J. Phys. G **24**, 683 (1998).
8. E. B. Balbutsev, L. A. Malov, P. Schuck, *et al.*, Phys. Atom. Nucl. **72**, 1305 (2009).
9. E. B. Balbutsev, L. A. Malov, Izv. RAN. Ser. Fiz. **74**, 898 (2010).
10. S. Péru and H. Goutte, Phys. Rev. C **77**, 044313 (2008).
11. J. Dobaczewski, W. Satula, B. G. Carlsson, *et al.*, Comput. Phys. Commun. **180**, 2361 (2009).
12. R. Rodriguez-Guzman, P. Sarriguren, L. M. Robledo, and J. E. Garsia-Ramos, Phys. Rev. C **81**, 024310 (2010).
13. P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer-Verlag, Berlin, 1980).
14. L. A. Malov, V. G. Soloviev, Fiz. Elem. Chastits At. Yadra **11**, 301 (1980).

ПРОСТРАНСТВЕННАЯ ЗАВИСИМОСТЬ ПАРНЫХ КОРРЕЛЯЦИЙ В ДЕФОРМИРОВАННЫХ ЯДРАХ

Е. Б. Бальбуцев, Л. А. Малов, П. Шук

Решение зависящих от времени уравнений Хартри–Фока–Боголюбова методом моментов функции Вигнера приводит к появлению низколежащих мод сложной структуры. Для описания этих возбуждений необходимо точное знание аномальной матрицы плотности. Показано, что расчеты с потенциалом Вудса–Саксона удовлетворяют таким требованиям — аномальную матрицу плотности в них получают того же качества, что и в более сложных самосогласованных расчетах с реалистическими силами.