

Resonances of a two-particle system on a 1D lattice

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We consider a system of two spinless quantum particles (possibly with different masses) that hop from one site to another one on the integer lattice $\mathbb{Z}$. Unlike the counterpart in the continuous case, for any $N \geq 2$ the kinetic energy operator of a lattice N -body system does not allow elimination of the motion of the center of mass. However, being taken in quasimomentum representation, the N -body lattice Hamiltonian may be studied separately for any admissible value of the c.m. quasimomentum (see, e.g. [1], [2] and references therein). Such a reduced Schrödinger operator is usually called a fiber Hamiltonian. In the present work, we first prove that for a fixed value of the c.m. quasimomentum, the 1D lattice two-body kinetic energy operator is unitarily equivalent to a two-sided symmetric Jacobi operator (an infinite three-diagonal number matrix) on $\ell_2(\mathbb{Z})$ with the entries that are all real and do not depend on the site number. The resolvent of this operator admits exact analytic computation. As a function of the energy, its kernel (i. e. the Green's function) has a two-sheeted Riemann surface. For a sufficiently rapidly decreasing pairwise interaction potential, we then consider the Fredholm determinant associated with the corresponding Lippmann-Schwinger operator. Zeros of this determinant on the (real axis of the) physical sheet represent binding energies of the fiber Hamiltonian while poles on the unphysical sheet are treated as resonances. Finally, having taken potentials with one- or two-site support we arrive at exactly solvable problems. In this case at least for a part of the binding energies and resonances, we have obtained explicit analytic expressions. We have also plotted trajectories of the binding energies and resonances as the potential strength varies.

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References

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