FINAL STATE INTERACTION OF PIONS IN $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$ DECAY AND $\pi\pi$ SCATTERING LENGTHS

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In 2003-2004 the collaboration NA48/2 at the CERN SPS collected a large amount of data ($\sim 6 \times 10^{7}$) on the decay $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$. The dependence of the distribution on the invariant mass of two neutral pions M_{00} (Fig. 1) shows a cusp-like anomaly in the vicinity of charged pions threshold $M_{00} = 2m_c$ [1, 2].

As was pointed out by N.Cabibbo [3] this anomaly is a result of charge exchange process $\pi^{+}\pi^{-} \to \pi^{0}\pi^{0}$ in the decay $K^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$. The amplitude of the decay $K^{+} \to \pi^{+}\pi^{0}\pi^{0}$ reads:

$$T = T_0 + 2ia_x kT_+; \quad a_x(\pi^{+}\pi^{-} \to \pi^{0}\pi^{0}) = \frac{a_2 - a_0}{3}; \tag{1}$$

$T_0(K^{+} \to \pi^{+}\pi^{0}\pi^{0})$; $T_+(K^{+} \to \pi^{+}\pi^{-}\pi^{+})$ – unperturbed (without final state interaction) amplitudes; a_0, a_2 – s-wave $\pi\pi$ scattering lengths in the isospin $I = 0$ and $I = 2$ states; $k = \frac{1}{2}\sqrt{M_{00}^2 - 4m_c^2}$ - the relative momentum of charged pion in the reaction $\pi^{+} + \pi^{-} \to \pi^{0} + \pi^{0}$.

Under charged pions threshold this momentum becomes imaginary $k = i\kappa$ thus

$$|T|^2 = T_0^2 + 4\frac{(a_0 - a_2)^2 k^2}{9}T_+^2; M_{00}^2 > 4m_c^2,$$
$$|T|^2 = T_0^2 + 4\frac{(a_0 - a_2)^2 \kappa^2}{9}T_+^2 - 4\frac{(a_0 - a_2)\kappa}{3}T_0T_+; M_{00}^2 < 4m_c^2. \tag{2}$$

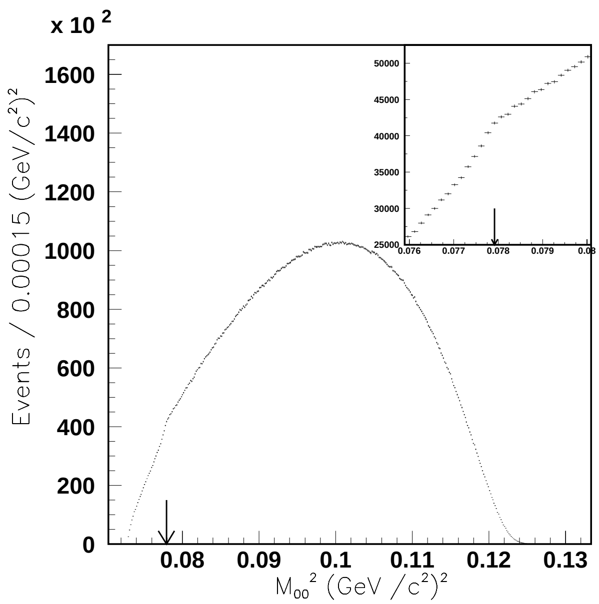

Fig. 1: Decay rate vs M_{00}^2

Above the threshold $M_{00}^2 > 4m_c^2$ the decay rate is proportional to the square of scattering lengths difference $(a_0 - a_2)^2$, the fact known for many years [4] while under threshold $M_{00}^2 < 4m_c^2$ the decay rate acquires interference term linear in $(a_0 - a_2)$. Such irregular behavior of the decay rate at threshold allows one to extract the value of the difference $(a_0 - a_2)$ fitting the experimental data [1,2] on the rate of $K^\pm \to \pi^\pm\pi^0\pi^0$ decay.

For many years the semileptonic decay $K^\pm \to \pi^+\pi^-e^\pm\nu$ (K_{e4} decay) was seen as a cleanest method for measurement of $\pi\pi$ scattering lengths due to only two pions in the final state and well known connection between the pions phases difference in s and p- wave states with scattering lengths [5]. In addition it is desirable to know scattering lengths values with highest possible accuracy as at present the Chiral Perturbation Theory (ChPT) predicts their values with unusual for strong interaction precision ($\sim 2\%$).

The discovery of cusp effect by NA48/2 collaboration open a new challenges for precise determination of scattering lengths. The fit

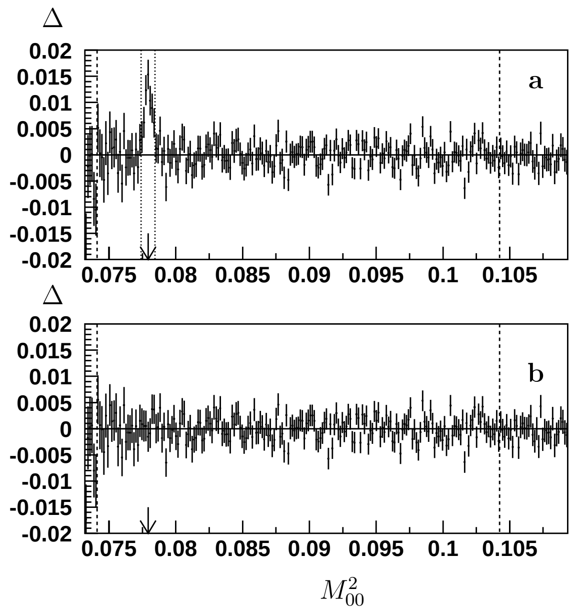

Fig. 2: Δ=(data-fit)/data versus M_{00}^2: a – fit without pionium formation; b – fit including the electromagnetic effects

of the experimental distribution making use of two theoretical models [6, 7] being the extension of Cabibbo approach to higher order in scattering lengths ($\sim a^2$) allows to determine the them with accuracy comparable to theoretical predictions [1, 2].

Nevertheless there are two issues unsolved in the above models. At charged pions threshold one has to account for electromagnetic interaction of pions, leading to bound states (pionium atoms) just under threshold. In Fig. 2(a) the result of the fit without electromagnetic effects is shown [1]. The discrepancy at charged pions threshold is a result of neglect of this effect in the theoretical approaches. The better fit can be obtained (Fig. 2(b)), when the authors add a free parameter relevant to a term describing the expected formation of pionium atoms decaying to $\pi^0\pi^0$ at threshold. However the extracted in such a way probability of pionium atom creation $K^\pm \to \pi^\pm + A_{\pi^+\pi^-}$ normalized to the $K^\pm \to \pi^\pm\pi^+\pi^-$ decay rate turn out to be twice larger than the theoretical prediction [8].

The second problem closely connected to the bound state formation is the absence of a reliable way in upgraded approaches [6, 7] to estimate the contribution of higher order terms in strong interaction. To account for electromagnetic interaction leading to unstable bound states one needs expressions for decay amplitudes including the strong interaction between pions in all orders,the task which can't be obtained in the framework of two mentioned above [6, 7] theoretical models.

To resolve these challenges, we make use the well known methods of non-relativistic quantum mechanics. The amplitudes of kaon decay to two pions can be presented as the convolution of the unperturbed amplitude $M_0(r)$ with the two pions wave functions describing strong interactions in the $\pi^+\pi^-$; $\pi^0\pi^0$ states and appear to be a solution of couple Shrödinger equations [9]:

$$M_c(K \to \pi^+\pi^-) = \int \Psi_c^+(r)M_0(r)d^3r;$$
$$M_n(K \to \pi^0\pi^0) = \int \Psi_n^+(r)M_0(r)d^3r; \tag{3}$$

$$-\Delta\Psi_c(r) + U_{cc}\Psi_c(r) + U_{cn}\Psi_n(r) = k_c^2\Psi_c(r);$$
$$-\Delta\Psi_n(r) + U_{nn}\Psi_n(r) + U_{nc}\Psi_c(r) = k_n^2\Psi_n(r). \tag{4}$$

Here k_c, k_n are pions relative momenta in charge $\pi^+\pi^-$ and neutral $\pi^0\pi^0$ pairs, while U_{ik} are relative potentials in $\pi\pi$ elastic and charge exchange interaction. Assuming that only s-wave scattering $\pi\pi$ scattering take place and strong potentials with sharp boundary $U_{ik} \gg k_{c(n)}^2$, making use the known asymptotic behavior of wave functions and unitarity constrains we obtain a set of relations expressing the decay amplitudes through unperturbed one M_{0c}, M_{0n} and $\pi\pi$ scattering amplitudes $f_x(\pi^+\pi^- \to \pi^0\pi^0)$, $f_c(\pi^+\pi^- \to \pi^+\pi^-)$, $f_n(\pi^0\pi^0 \to \pi^0\pi^0)$

$$M_c = M_{0c}(1 + ik_cf_{cc}) + ik_nM_{0n}f_x; \quad f_{cc} = \frac{a_{cc}(1 - ik_na_{nn}) + ik_na_x^2}{D};$$
$$M_n = M_{0n}(1 + ik_nf_{nn}) + ik_cM_{0c}f_x; \quad f_{nn} = \frac{a_{nn}(1 - ik_ca_{cc}) + ik_ca_x^2}{D};$$
$$f_x = \frac{a_x}{D}; \quad D = (1 - ik_ca_{cc})(1 - ik_na_{nn}) + k_nk_ca_x^2. \tag{5}$$

In the case of exact isospin symmetry in the vicinity of threshold these amplitudes manifest itself through the scattering lengths:

$$a_x = \frac{a_2 - a_0}{3}; \quad a_{nn} = \frac{a_0 + 2a_2}{3}; \quad a_{cc} = \frac{2a_0 + a_2}{6}. \tag{6}$$

To account for electromagnetic interaction among the pions we took advantage of well known receipt [11]. It turn out that to include the electromagnetic interaction in the considered approach it is sufficient to do a substitution: $ik_c \to \tau = \frac{d \log[G_0(kr) + iF_0(kr)]}{dr}\Big|_{r=r_0}$ where F_0, G_0 are the regular and irregular solutions of the Coulomb problem (Hypergeometric functions). In relevant region $kr_0 \ll 1$ (r_0 – strong potential radius):

$$\tau = ik - \alpha m \left[\log(-2ikr_0) + 2\gamma + \psi(1 - i\xi)\right];$$

$$\mathrm{Re}\ \tau = -\alpha m \left[\log(2kr_0) + 2\gamma + \mathrm{Re}\,\psi(1 - i\xi)\right], \xi = \frac{\alpha m}{2k};$$

$$\mathrm{Im}\ \tau = kA^2, \quad A = \exp\left(\frac{\pi\xi}{2}\right)|\Gamma(1 + i\xi)|. \tag{7}$$

Here $\gamma = 0.5772$ is the Euler constant while $\psi(\xi) = \frac{d \log \Gamma(\xi)}{d\xi}$-digamma function.

Inclusion of the electromagnetic interactions in a such way leads to the bound state in the very narrow region under charged pions threshold and correctly accounts for the electromagnetic interaction in all kinematical region [10]. It seems that the difference between predicted rate for creation of pionium atoms and fit result at threshold is due to the fact that in the vicinity of threshold besides the bound states there are contribution from unbound pairs, whose electromagnetic interaction gives almost the same size contribution as the pionium decay [12].

Another place where the developed approach can be applied is the K_{e4} decay. As was mentioned above it is ideally suited for determination of $\pi\pi$ scattering lengths. From experimental data on K_{e4} decay the phases difference between s and p pions states $\delta = \delta_s - \delta_p$, can be extracted.

The preliminary analysis of the experimental data obtained by NA48/2 collaboration leads to large discrepancy between the prediction for the value of scattering length a_0 from ChPT and its value obtained from experimental data [13] on K_{e4} decay.

As was shown in works [14, 15] this discrepancy is due to the neglect of isospin breaking effects in the experimental data processing. Account [14] for the electromagnetic interaction between the pions in the decay $K^{\pm} \to \pi^{+}\pi^{-}e\nu$ and isospin breaking effects due to the possibility of charge exchange reaction among the pions in the final state [15] allows to adjust the data for scattering lengths from NA48/2 and theoretical predictions.

The considered approach can be applied to the wide class of the decays with two or more hadrons in the final state, leading to better understanding of strong interaction and giving the unique possibility to check different theoretical models predictions for meson-meson interactions at low energies.

1. J. Batley *et al.*, Phys. Lett. B **633**, 173 (2006).
2. J. Batley *et al.*, Eur. Phys. J. C **64**, 589 (2009).
3. N. Cabibbo, Phys. Rev. Lett. **93**, 121801 (2004).
4. V. Gribov, Nucl. Phys. **5**, 653 (1958).
5. B.Ananthanarayan *et al.*, Phys. Rept. **353**, 207 (2001).
6. N. Cabibbo, G. Isidori, JHEP **0503**, 021 (2005).
7. G. Colangelo, J. Gasser, B. Kubis, A. Rusetsky, Phys. Lett. B **638**, 187 (2006).
8. Z. Silagadze, JETP Lett. **60**, 689 (1994).
9. S. Gevorkyan, A. Tarasov, O. Voskresenskaya, Eur. Phys. J. C **67**, 143 (2010).
10. S. Gevorkyan, A. Tarasov, O. Voskresenskaya, Phys. Lett. B **649**, 159 (2007).
11. E. Wigner, Phys. Rev. **73**, 1002 (1948).
12. S. Gevorkyan, D. Madigozhin, A. Tarasov, O. Voskresenskaya, Phys. Part. Nucl. Lett. **5**, 85 (2008).
13. B. Bloch-Devaux, *Proceedings of the Kaon International Conference, Frascati, May* (2007).
14. S. Gevorkyan, A. Sissakian, A. Tarasov, H. Torosyan, O. Voskresenskaya, Phys. Atom. Nucl. **73**, 937 (2010).
15. S. Gevorkyan, A. Sissakian, A. Tarasov, H. Torosyan, O. Voskresenskaya, Phys. Atom. Nucl. **73**, 961 (2010).