Precision Measurement of ππ Scattering Lengths in Ke4 Decays

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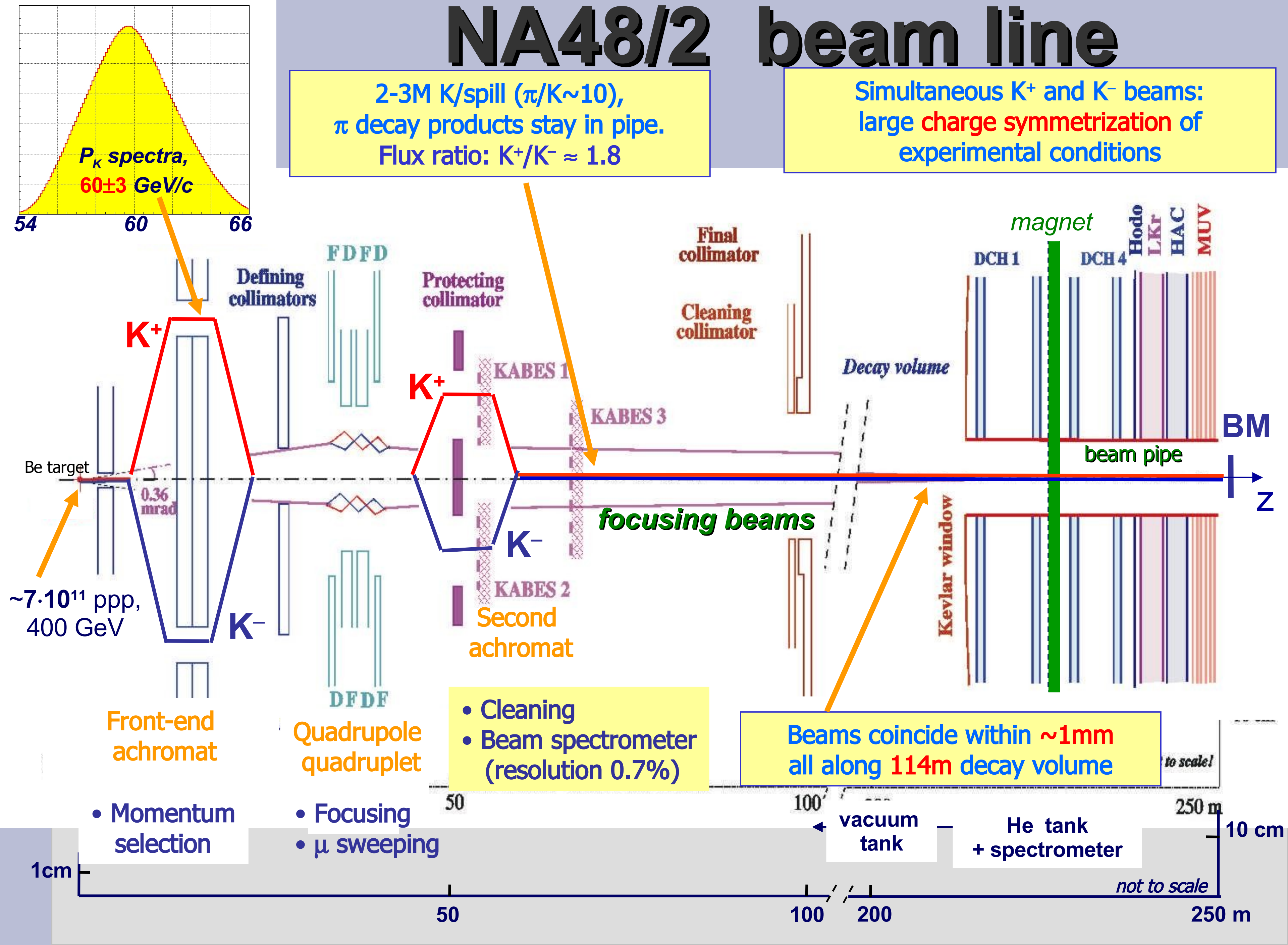
on behalf of the NA48/2 Collaboration:

Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Vienna

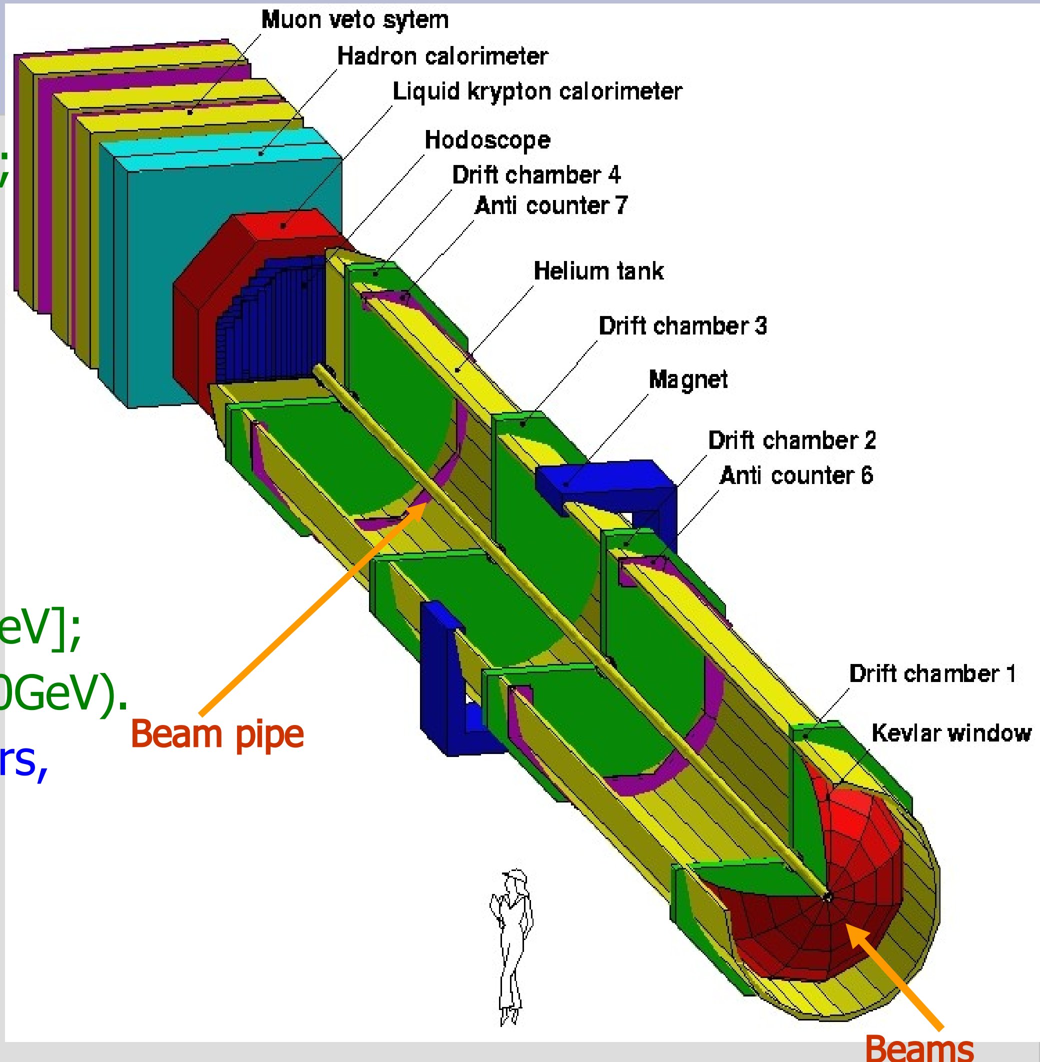
NA48/2 beam line



The NA48 detector

Main detector components:

- Magnetic spectrometer (4 DCHs):
  4 views/DCH: redundancy ⇒efficiency;
  used in trigger logic;
  $\Delta p/p = 1.0\% + 0.044\%{*}p$ [GeV/c].
- Hodoscope
  fast trigger;
  precise time measurement (150ps).
- Liquid Krypton EM calorimeter (LKr)
  High granularity, quasi-homogenious;
  $\sigma_E/E = 3.2\%/E^{1/2} + 9\%/E + 0.42\%$ [GeV];
  $\sigma_x=\sigma_y=0.42/E^{1/2} + 0.6$mm (1.5mm@10GeV).
- Hadron calorimeter, muon veto counters, photon vetoes.



NA48/2 data:

2003 run: ~ 50 days

2004 run: ~ 60 days

A view of the NA48/2 beam line

Total statistics in 2 years:

$K^{\pm} \rightarrow \pi^{-}\pi^{+}\pi^{\pm}$: ~$4 \cdot 10^{9}$

$K^{\pm} \rightarrow \pi^{0}\pi^{0}\pi^{\pm}$: ~$1 \cdot 10^{8}$

Rare $K^{\pm}$ decays:
BR's down to 10^{-9}
can be measured

>200 TB of data recorded

Pion scattering lengths

The important free parameter of ChPT is the quark condensate <qq>, it determines the relative size of mass and momentum terms in the power expansion.

a_0 and a_2 are S-wave $\pi\pi$ scattering lengths in isospin states I=0 and I=2, correspondingly. They enter into all $\pi\pi$ scattering amplitudes.

The relation between <qq> and the scattering lengths a_0 and a_2 is known from this theory, so the experimental measurement of a_0 and a_2 provides an important constraints for ChPT Lagrangian parameters.

Pion scattering lengths are measured also in the

study of the cusp-effect in $K^{\pm}\rightarrow\pi^{\pm}\pi^{0}\pi^{0}$ decays

[NA48/2 final result: EPJ C64(2009)589]

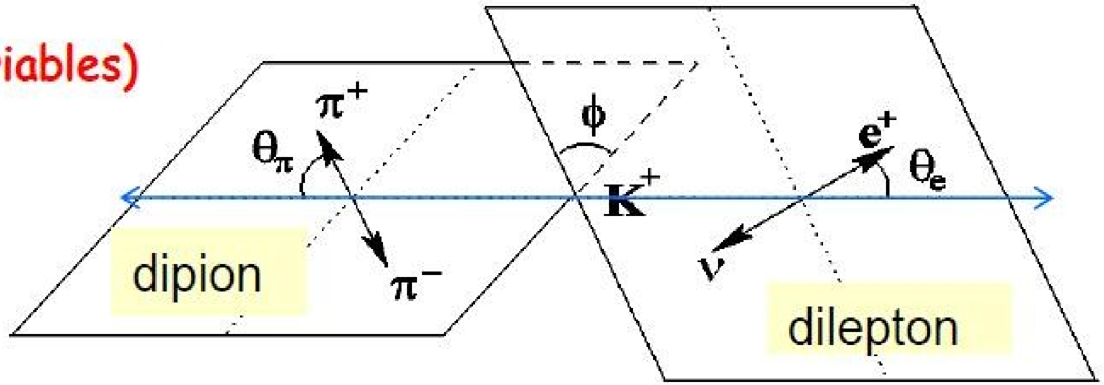
Ke4 decays : formalism

Five kinematic variables (Ca.Ma. variables)

(Cabibbo-Maksymowicz 1965)

S_π ($M^2_{\pi\pi}$), S_e ($M^2_{e\nu}$),

$\cos\theta_\pi$, $\cos\theta_e$ and ϕ.



Partial Wave expansion of the amplitude

into s and p waves (Pais-Treiman 1968)

+ Watson theorem (T-invariance) for δ_ℓ^I

$\delta_0^0 \equiv \delta_s$ and $\delta_1^1 \equiv \delta_p$

F, G = 2 Axial Form Factors

$F = F_s e^{i\delta s} + F_p e^{i\delta p} \cos\theta_\pi$

$G = G_p e^{i\delta g}$

H = 1 Vector Form Factor

$H = H_p e^{i\delta h}$

F, G, H are complex and dimensionless

Map the distributions of the Ca.Ma. variables in the **five-dimensional space** with 4 Form factors and one phase shift , assuming identical phases for the p-wave Form Factors F_p, G_p, H_p:

The fit parameters are :

F_s F_p G_p H_p and $\delta = \delta_s - \delta_p$

(F_s , F_p , G_p , H_p are real)

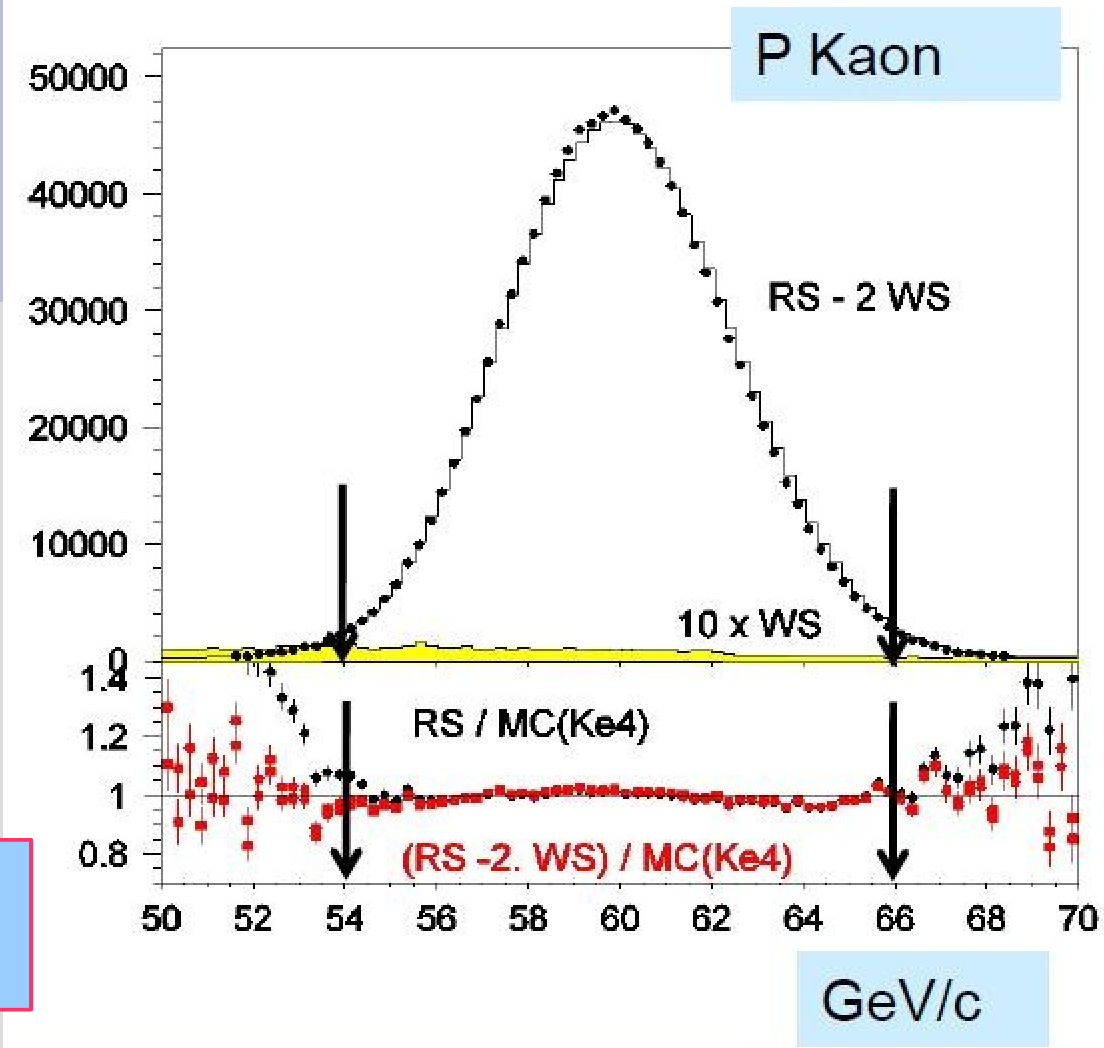
Signal (π+ π- e+/- ν) topology:

- 3 charged tracks, good vertex
- Opposite sign 2 pions
- 1 electron (Elkr /P ~ 1)

Main background sources:

- a) $K^+ \rightarrow [\pi^- \rightarrow e^- \nu]\pi^+\pi^+$
- b) $K^+ \rightarrow [\pi^0 \rightarrow e^+e^- (\gamma)]\ (\pi^0)\ \pi^+$

Background has a «wrong sign» (WS) component (same sign pion charges): 1/3 for a) and 1/2 for b).



Total background is at the level of 2 x 0.3 % (estimated from WS events and checked by MC)

Ke4 decays : fitting procedure

Total (2003+2004) 1.13 Million Ke4 decays

Using iso-populated boxes in the 5-dimension space of the Ca.Ma. variables, ($M_{\pi\pi}$, $M_{e\nu}$, $\cos\theta_\pi$, $\cos\theta_e$ and ϕ) one defines a grid of

10x5x5x5x12=15000 variable size boxes.

In each $M_{\pi\pi}$ "slice" (1500 boxes), a set of 4 fit parameters is found which minimizes the difference between the data and predicted populations

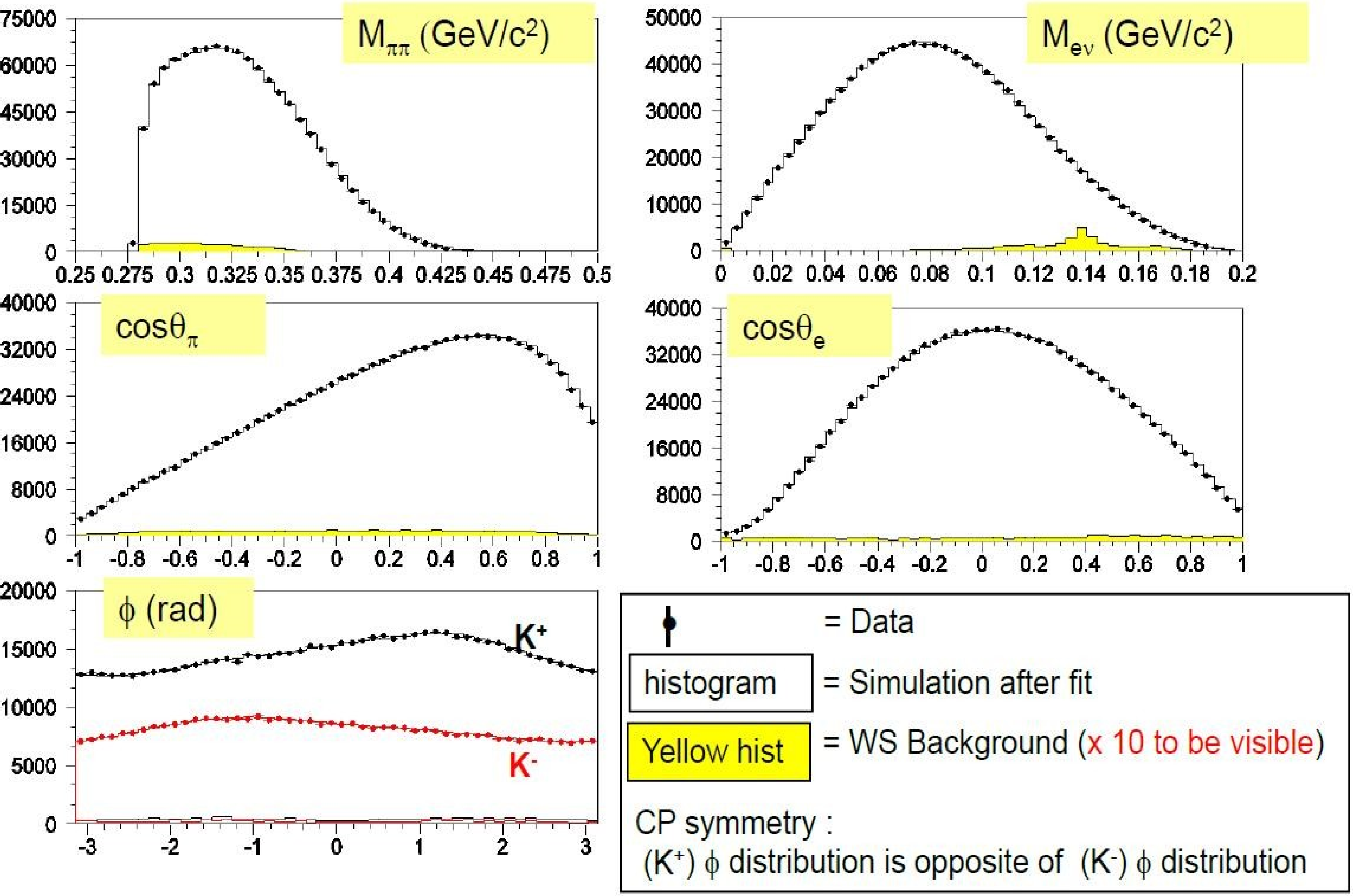
The normalisation F_s^2 is obtained in each bin/slice by the ratio $x_{slice} = \Sigma_{j\ in\ slice} Nj / \Sigma_{j\ in\ slice} MCj$

Data sample	MC sample
K^+ sample (726 400 events) 48 events/box	K^+ MC (17.4 Million events) 1160 events/box
K^- sample (404 400 events) 27 events/box	K^- MC (9.7 Million events) 650 events/box

K^+ and K^- samples fitted separately in 10 independent $M\pi\pi$ bins/slices, then combined in each slice according to their statistical error.

No assumption is made on the shape of the variation of the phase δ (and FF) from one $M_{\pi\pi}$ slice to the next (i.e. "model independent" analysis)

Ke4 decays : Data/MC comparison after fit



Ke4 formfactors: fit results

Series expansion with:

- $q^2 = S\pi/4m_\pi^2 - 1$
- $Se/4m_\pi^2$

$F_s^2 = f_s^2(1+f'_s/f_s q^2+f''_s/f_s q^4+f'_e/f_s Se/4m_\pi^2)^2$

$G_p/f_s = g'_p/f_s + g'_p/f_s q^2$

Preliminary (2003+2004)

	value	stat	syst
f_s'/f_s	0.152	± 0.007	± 0.005
f_s''/f_s	-0.073	± 0.007	± 0.006
f_e'/f_s	0.068	± 0.006	± 0.007
f_p/f_s constant	-0.048	± 0.003	± 0.004
g_p/f_s	0.868	± 0.010	± 0.010
g_p'/f_s	0.089	± 0.017	± 0.013
h_p/f_s constant	-0.398	±0.015	± 0.008

Comparison of Ke4 phase shift experimental measurements

Apply Isospin corrections (10-15 mrad) to all published points :

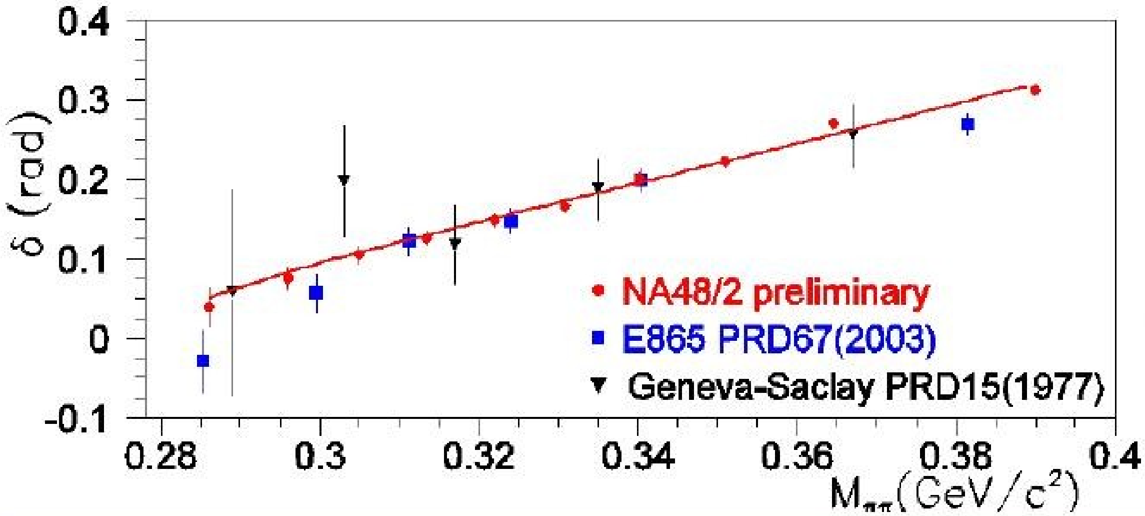
S118 (Geneva-Saclay): typical error 40-50 mrad

E865: typical error 15-20 mrad

Correction small wrt experimental error but coherent shift downwards for all data points

NA48/2 typical error 7-8 mrad

improved precision due to both
- larger statistics ~3 x E865
- larger acceptance at high $\pi\pi$ mass



- All Phase points corrected for isospin mass effects
- Independent experiments
- Errors = stat + syst

Line from a 2p fit to NA48 data alone

Systematic errors are mailny bin to bin uncorrelated:
- Binning choise
- Trigger efficiency
- Acceptance control
- Background level (the only correlated source)
- Background shape
- Electron identification
- Radiative correction

From phase shifts to pion-pion scattering lengths

$\pi\pi$ phases at threshold can be predicted from data above 0.8 GeV using Roy equations (unitarity, analyticity and crossing symmetries) and 2 subtraction constants a_0 and a_2

Numerical solutions have been developed (ACGL, DFGS) valid in the Isospin symmetry limit **(Universal Band in the [a_2,a_0] plane)**, but broken in the experimental world.

factorization of electromagnetic and mass effects :

Gamow factor x PHOTOS generator × Isospin corrections

Radiative effects (except mass effects) included in the simulation,

Gamow factor : "classical" Coulomb attraction between the 2 charged pions

PHOTOS generator: real photon(s) are emitted and tracked in the simulation

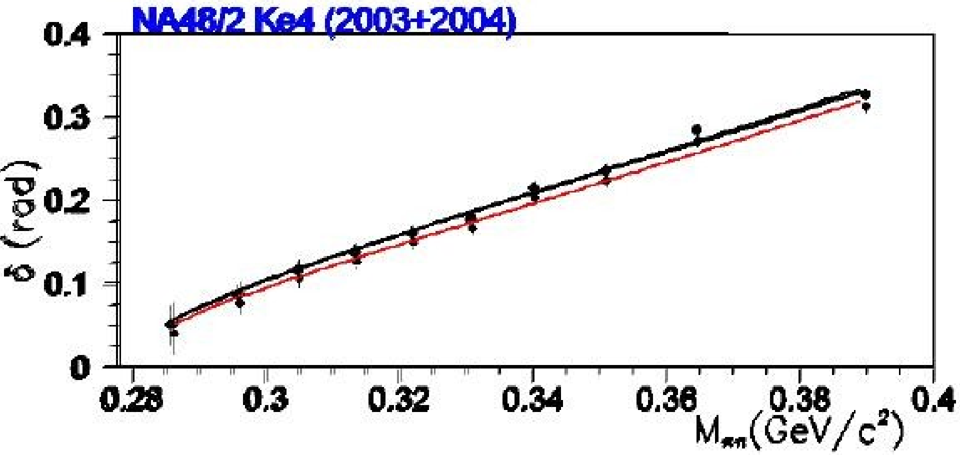
(-> effect on event selection + possible bias on reconstructed quantities)

Mass effects:

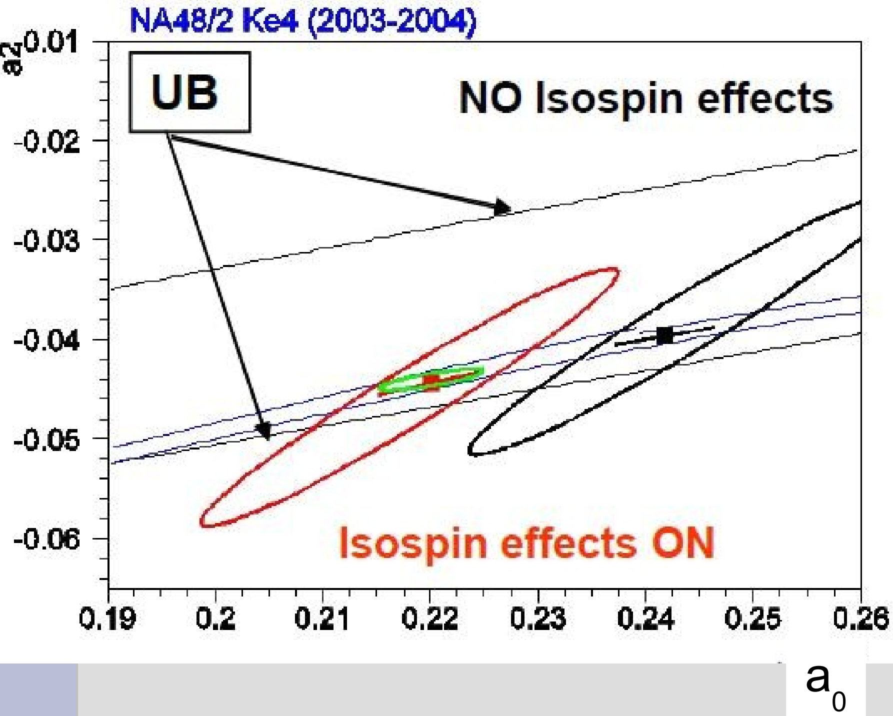
- recently computed as a correction to the measurements
- even larger than current experimental precision !

(CGR EPJ C59 (2009) 777,
DK preliminary June 2008 in progress)

Ke4 decays: from phase shifts to scattering lengths (a_0,a_2)



a tiny effect from theory…. a big change in now precise experimental measurement !



This induces a large change on (a_0,a_2) values

from a 2p fit			from a 1p fit	
Δa_0 = -0.025, Δa_2 = -0.007			Δa_0 = -0.022	
error	stat	syst	stat	syst
$\sigma(a_0)$:	± 0.0128	± 0.0050	± 0.005	± 0.002
$\sigma(a_2)$:	± 0.0084	± 0.0034		

Ellipses are 68% CL contours in 2p fits

Ke4 decays: comparison with theoretical predictions

Preliminary (2003+2004)

THEORY prediction

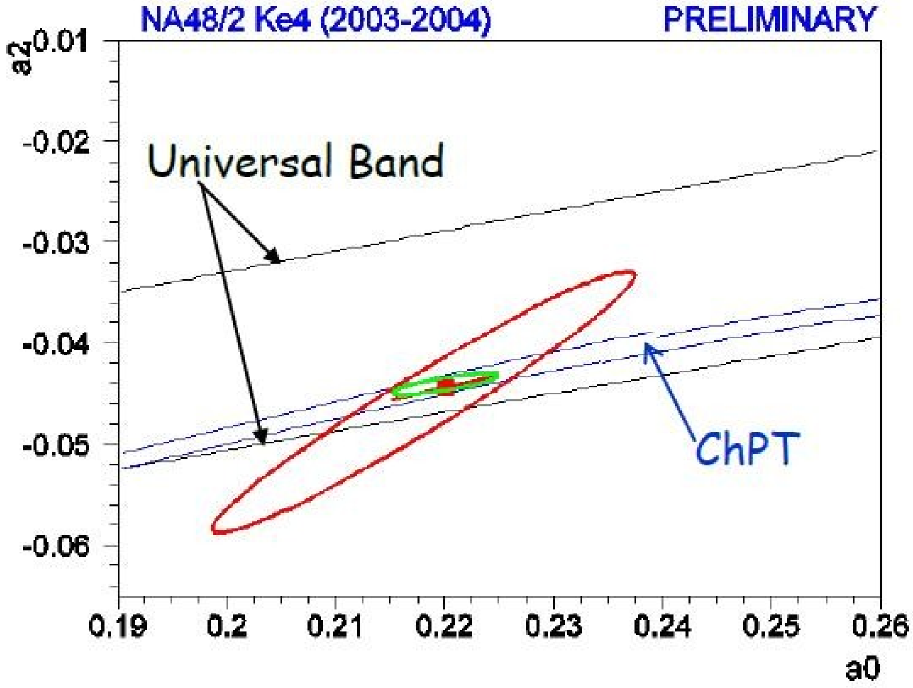
Using more inputs from ChPT and low energy constants, the prediction is better constrained (CGL NPB603(2001),PRL86(2001)):

a_0 = 0.220 ± 0.005

a_2 = -0.0444 ± 0.0008

Experimental measurement

a_0 ChPT 1p fit	0.2206 ± 0.0049 stat ± 0.0018 syst ± 0.0064 theo *
a_0 free a_2 free 2p fit	0.2220 ± 0.0128 stat ± 0.0050 syst ± 0.0037 theo* -0.0432 ± 0.0086 stat ± 0.0034 syst ± 0.0028 theo* Correlation 96.7%



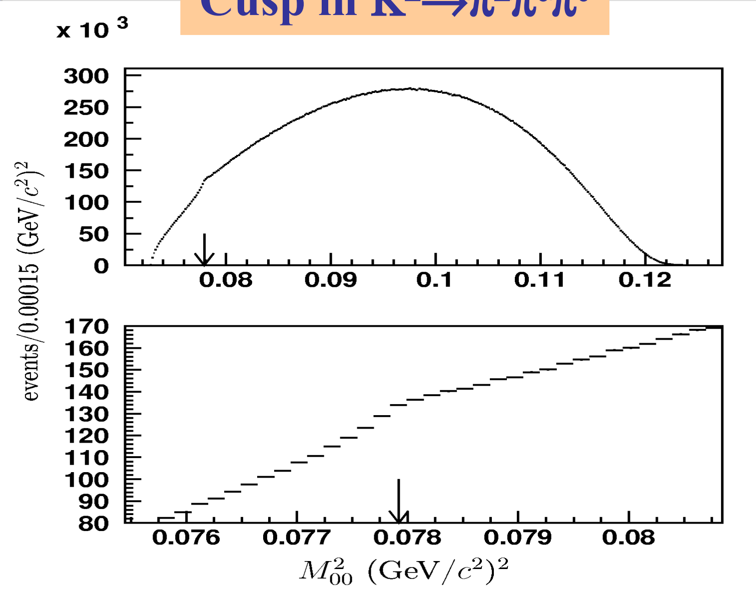
*Theory error evaluated from control of the isospin corrections & inputs to Roy equation numerical solutions (CGR EPJ C59 (2009)777)

Final results of pion scattering lengths measurement from the Ke4 and the cusp results

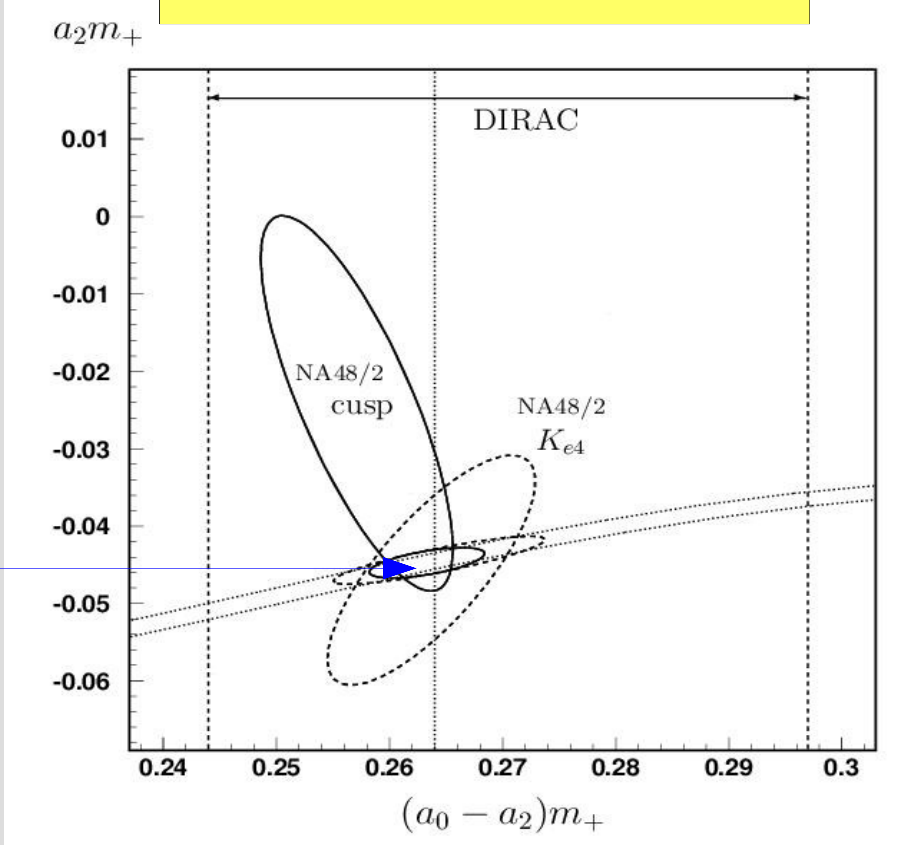
68% - probability ellipses, full uncertainties:

combined
- Statistical,
- Systematical,
- External.

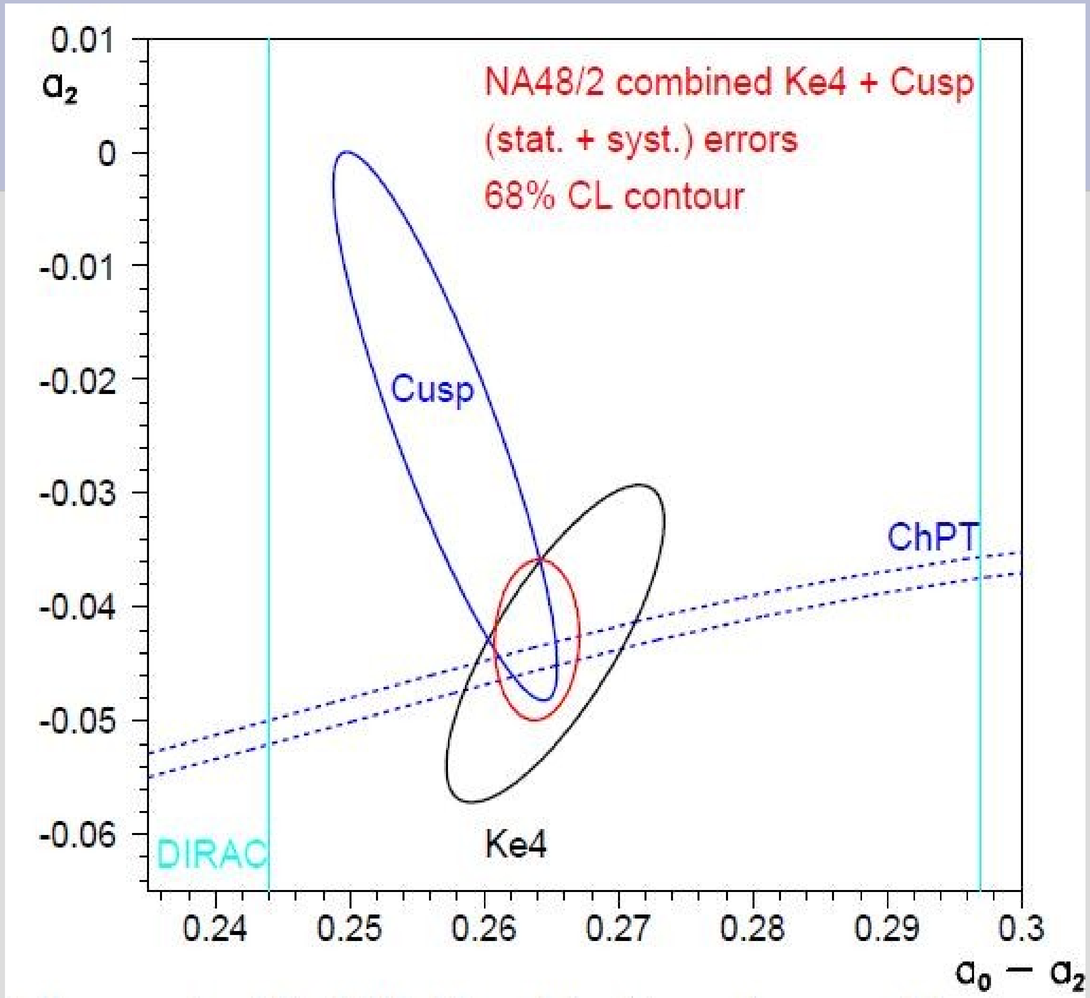
Cusp in $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$



Cusp and Ke4 results using Chiral constraint (shown with a Gaussian error)



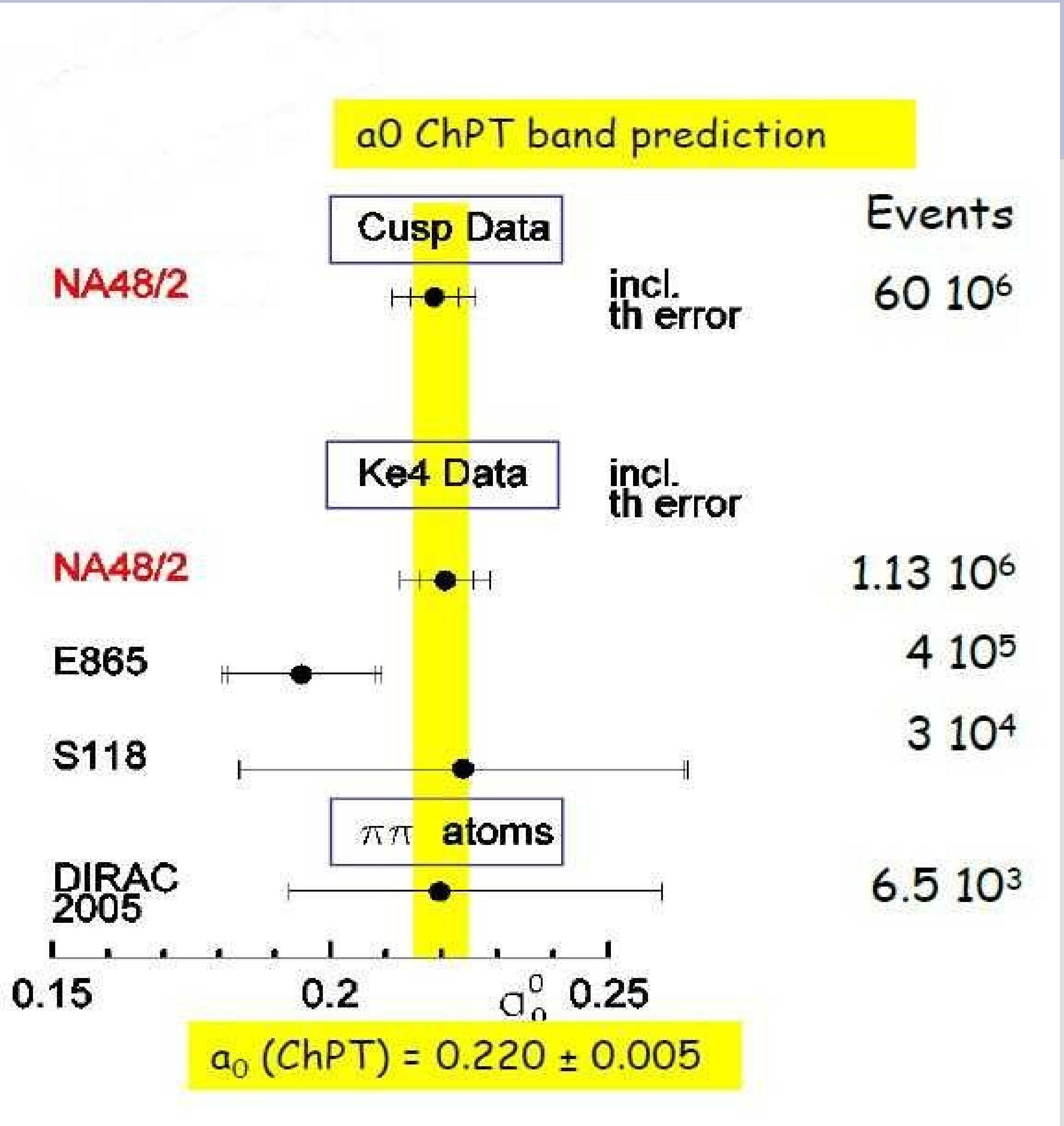
NA48/2 Ke4 + cusp combined



$$a_0 = 0.2210 \pm 0.0047_{\mathrm{stat}} \pm 0.0015_{\mathrm{syst}}, \quad a_2 = -0.0429 \pm 0.0044_{\mathrm{stat}} \pm 0.0016_{\mathrm{syst}}$$

$$a_0 - a_2 = 0.2639 \pm 0.0020_{\mathrm{stat}} \pm 0.0004_{\mathrm{syst}}.$$

Comparison with another measurements



Summary

- 1.13 millons of fully reconstructed $K^{\pm} \to \pi^{+}\pi^{-}\nu\, e^{\pm}$ (Ke4) decays (2003+2004 data).
- Ke4 Form Factors measured with an improved precision.
- Final result with ChPT link between a_0 and a_2 :

$$a_0 = 0.2206 \pm 0.0049(\text{stat}) \pm 0.0018(\text{syst}) \pm 0.0064(\text{theor})$$

  is in good agreement with the cusp measurement and with the prediction of ChPT.
- Experimental uncertainty is at the level of theoretical ChPT prediction 0.220 ± 0.005 (Colangelo, Gasser, Leutwyler 2000).