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Beyond the Standard Model

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I. Introduction

I would like to invite you to go beyond the Standard Model of the three fundamental interactions of matter. How far we go depends on your patience and intellectual curiosity. We do not know what is waiting us there in terra incognita. We hope to find it out with the help of future experiments. However modern theories give us several guiding ideas, some of which are very exciting. In the present lectures we will discuss some of these ideas looking more attractive today.

1. The Standard Model: Achievements and Drawbacks

The Standard Model (SM) describes the *strong*, *weak* and *electromagnetic* interactions and is based on a *gauge principle*. According to this principle all the forces of nature are mediated by an exchange of the gauge fields of the corresponding gauge group. The group of the SM is

$$SU_{colour}(3) \otimes SU_{left}(2) \otimes U_{hypercharge}(1) \qquad (1.1)$$

whereas the field content is the following:

Gauge sector

The gauge bosons are spin 1 vector particles belonging to the adjoint representation of the group (1.1). Their quantum numbers under $SU(3) \otimes SU(2) \otimes U(1)$ are respectively:

$$\begin{array}{lllll} \textit{gluons} & G^a_\mu: & (\underline{8},\underline{1},0) & SU_{colour}(3) & g_s \\ \textit{intermediate weak bosons} & A^i_\mu: & (\underline{1},\underline{3},0) & SU_{left}(2) & g \\ \textit{abelian boson} & B_\mu: & (\underline{1},\underline{1},0) & U_Y(1) & g'. \end{array}$$

The coupling constants are usually denoted by g_s, g and g' respectively.

Fermion sector

The matter fields are fermions belonging to the fundamental representation of the gauge group. These are believed to be quarks and leptons of at least of three generations. The SM is left-right asymmetric. Left-handed and right-handed fermions have different quantum numbers:

quarks

$$\begin{array}{llllllll} Q^i_{\alpha L} = & \begin{pmatrix} U^i_\alpha \\ D^i_\alpha \end{pmatrix}_L & = & \begin{pmatrix} u^i \\ d^i \end{pmatrix}_L, & \begin{pmatrix} c^i \\ s^i \end{pmatrix}_L, & \begin{pmatrix} t^i \\ b^i \end{pmatrix}_L, & \ldots & (\underline{3},\underline{2},1/3) \\ & U^i_{\alpha R} & = & u_{iR}, & c_{iR}, & t_{iR}, & \ldots & (\underline{3},\underline{1},4/3) \\ & D^i_{\alpha R} & = & d_{iR}, & s_{iR}, & b_{iR}, & \ldots & (\underline{3},\underline{1},-2/3) \\ \textit{leptons} & L_{\alpha L} & = & \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, & \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, & \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, & \ldots & (\underline{1},\underline{2},-1) \\ & l_{\alpha R} & = & e_R, & \mu_R, & \tau_R, & \ldots & (\underline{1},\underline{1},-2) \end{array}$$

$i = 1,2,3$ - colour, $\alpha = 1,2,3,\ldots$ - generation.

Higgs sector

In the minimal version of the SM there is only one doublet of Higgs scalar fields

$$H = \begin{pmatrix} H^+ \\ H_0 \end{pmatrix} \quad (1, 2, -1),$$

which is introduced in order to give masses to quarks, leptons and intermediate weak bosons via spontaneous symmetry breaking.

The Lagrangian of the SM is

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{Yukawa} + \mathcal{L}_{Higgs}, \tag{1.2}$$

$$\begin{aligned} \mathcal{L}_{gauge} = & -\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} - \frac{1}{4}A^i_{\mu\nu}A^i_{\mu\nu} - \frac{1}{4}B_{\mu\nu}B_{\mu\nu} \\ & + i\overline{L}_\alpha\gamma^\mu D_\mu L_\alpha + i\overline{Q}_\alpha\gamma^\mu D_\mu Q_\alpha + i\overline{l}_\alpha\gamma^\mu D_\mu l_\alpha \\ & + i\overline{U}_\alpha\gamma^\mu D_\mu U_\alpha + i\overline{D}_\alpha\gamma^\mu D_\mu D_\alpha + (D_\mu H)^\dagger(D_\mu H), \end{aligned}$$

where

$$\begin{aligned} G^a_{\mu\nu} &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu, \\ A^i_{\mu\nu} &= \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g\epsilon^{ijk} A^j_\mu A^k_\nu, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \\ D_\mu L_\alpha &= (\partial_\mu - i\frac{g}{2}\tau^i A^i_\mu + i\frac{g'}{2}B_\mu)L_\alpha, \\ D_\mu l_\alpha &= (\partial_\mu + ig'B_\mu)l_\alpha, \\ D_\mu Q_\alpha &= (\partial_\mu - i\frac{g}{2}\tau^i A^i_\mu - i\frac{g'}{6}B_\mu - i\frac{g_s}{2}\lambda^a G^a_\mu)Q_\alpha, \\ D_\mu U_\alpha &= (\partial_\mu - i\frac{2}{3}g'B_\mu - i\frac{g_s}{2}\lambda^a G^a_\mu)U_\alpha, \\ D_\mu D_\alpha &= (\partial_\mu + i\frac{1}{3}g'B_\mu - i\frac{g_s}{2}\lambda^a G^a_\mu)D_\alpha. \end{aligned}$$

$$\mathcal{L}_{Yukawa} = f^l_{\alpha\beta}\overline{L}_\alpha l_\beta H + f^d_{\alpha\beta}\overline{Q}_\alpha D_\beta H + f^u_{\alpha\beta}\overline{Q}_\alpha U_\beta \tilde{H} + h.c.,$$

where $\tilde{H} = i\tau_2 H^\dagger$.

$$\mathcal{L}_{Higgs} = -V = m^2 H^\dagger H - \lambda(H^\dagger H)^2.$$

Here $\{f\}$ are the Yukawa and λ is the Higgs coupling constants respectively, both dimensionless and m is the only dimensional mass parameter.

The Lagrangian of the SM contains the following set of free parameters:

- 3 gauge couplings,

- Yukawa couplings,
- Higgs coupling,
- mass parameter,
- quark mixing matrix,
- number of matter fields (generations).

All the particles obtain their masses due to spontaneous symmetry breaking of $SU_{left}(2)$ group via a non-zero vacuum expectation value of the Higgs field

$$< H >= 1/\sqrt{2}\begin{pmatrix} 0 \\ v \end{pmatrix}, \quad v = m/\sqrt{\lambda}.$$

As a result the gauge group of the SM is spontaneously broken down to

$$SU_c(3) \otimes SU_L(2) \otimes U_Y(1) \Rightarrow SU_c(3) \otimes U_{EM}(1).$$

The weak bosons now are

$$W^\pm_\mu = \frac{A^1_\mu \pm iA^2_\mu}{\sqrt{2}}, \quad Z_\mu = \sin\theta_W B_\mu - \cos\theta_W A^3_\mu$$

with masses

$$m_W = 1/2gv, \quad m_Z = m_W/\cos\theta_W, \quad \tan\theta_W = g'/g,$$

while the photon field

$$\gamma_\mu = \cos\theta_W B_\mu + \sin\theta_W A^3_\mu$$

remains massless.

The matter fields acquire masses proportional to the corresponding Yukawa couplings:

$$M^u_{\alpha\beta} = f^u_{\alpha\beta}v/\sqrt{2}, \; M^d_{\alpha\beta} = f^d_{\alpha\beta}v/\sqrt{2}, \; M^l_{\alpha\beta} = f^l_{\alpha\beta}v/\sqrt{2}, \; m_H = \sqrt{2}m.$$

Explicit mass terms in the Lagrangian are forbidden because they are not $SU_{left}(2)$ symmetrical and would destroy the renormalizability of the Standard model.

The SM has been constructed as a result of numerous efforts both theoretical and experimental. Achievements of the SM are obvious. At present the SM is extraordinary successful, there is no any experiment that contradicts the SM. Moreover there is nothing observed beyond the SM.

However the SM has its natural drawbacks. Among them are:

- large number of free parameters,
- formal unification of strong and electroweak interactions,
- the Higgs particles have not yet been observed and it is not clear whether they are fundamental or composite,
- the problem of CP-violation is not well understood including CP-violation in strong interaction,
- One of the main problems of the SM is the origin of the mass spectrum.

There are also some problems of more fundamental nature:

- why the gauge group is $SU(3) \otimes SU(2) \otimes U(1)$?
- what is the number of generations?
- how to incorporate gravity in unified theory? etc.

The answer to these problems definitely lies beyond the SM.

II. Just beyond the Standard Model

2. Compositeness and Technicolour

One of the main problems of the SM still not truly understood is the symmetry breaking mechanism. Unbroken symmetry means that *all* fundamental particles are *massless*. This is because both the fermion mass term $f_L \cdot f_R$ and that of the gauge bosons W_μ^2 and Z_μ^2 are not $SU_{left}(2)$ invariant.

If the symmetry is broken on some scale, these particles can acquire masses ~ the breaking scale. Its typical value is ~ 250 GeV (v.e.v. of the Higgs field).

2.1. Symmetry Breaking without Higgs Fields (A Simple Model)

If the Higgs particles will not be found, we will be faced with the problem of replacing the Higgs mechanism of spontaneous symmetry breaking by some alternative. Such an alternative already exists. This is the so-called dynamical symmetry breaking like the chiral symmetry breaking in QCD.

To understand how this mechanism works, we consider QCD with u and d quarks. If they are massless, then the QCD Lagrangian is invariant under chiral group $SU_L(2) \otimes SU_R(2)$. However quarks can create a vacuum condensate

$$< \bar{u}u + \bar{d}d > \neq 0,$$

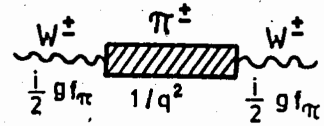

Figure 1: Pion intermediate state contribution to the W polarization operator

which breaks the chiral group down to that of the Isospin $SU_I(2)$. The symmetry breaking is spontaneous as far as Lagrangian is still invariant but this is no longer true for the vacuum state. Hence due to the *Goldstone* theorem one should get massless spin 0 goldstone bosons. Indeed they exist. These are pions π. (Later they obtain small masses becoming the pseudo-goldstone bosons. However the smallness of pion mass is protected by chiral invariance.)

Due to a non-zero matrix element of the axial isospin current

$$< 0|J^\mu_{5a}|\pi_b(\vec{q}) >= f_\pi q^\mu \delta_{ab}, \qquad (2.1)$$

where q^μ is the momentum of the pion and $f_\pi \approx 93Mev$ is the pion decay constant, the pions appear to be the longitudinal components of *massive* vector bosons W and Z. To see this, consider the propagator of W. Because of radiative corrections the total expression has the form

$$\frac{g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}}{q^2} \quad \rightarrow \quad \frac{g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}}{q^2(1+\Pi(q^2))},$$

where $\Pi(q^2)$ is the polarization operator.

To obtain a non-zero mass of W we need a pole term in $\Pi(q^2)$ at $q^2 = 0$. The contribution to $\Pi(q^2)$ due to the interaction with the axial current $\frac{g}{2} W^\pm_\mu J^\mu_{\pm 5}$ is given by

$$(g_{\mu\nu} - q_\mu q_\nu)\Pi(q^2) = \frac{g^2}{4} \int dx e^{iqx} < 0|T J^\pm_{\mu 5}(x) J^\pm_{\nu 5}(0)|0 > . \qquad (2.2)$$

The pole term we are interested in comes from the pion intermediate state (see Fig.1) According to eqs.(2.1,2.2) we get

$$\Pi(q^2) = -\frac{g^2 f_\pi}{4q^2},$$

which leads to a shift of the pole thus giving the desired mass

$$m_W = \frac{1}{2} g f_\pi .$$

The same mechanism works for the neutral bosons. However one should take into account the mixing between B_μ and A^3_μ. The mass matrix here has the form

$$m^2 = \begin{pmatrix} g^2 & g'g \\ g'g & g'^2 \end{pmatrix} \frac{f_\pi}{4}.$$

After diagonalization this leads to

$$\begin{aligned} m_\gamma^2 &= 0 \\ m_Z^2 &= \frac{1}{2}(g^2 + g'^2) f_\pi^2 \end{aligned}$$

with a standard ratio

$$\frac{m_W}{m_Z} = \cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}}.$$

Thus we get what we wanted but ... the masses of W and Z are of the order of 100 MeV rather than 100 GeV, i.e. 10^3 times smaller than they actually are.

Hence, the conclusion is the following : the mechanism works, however the scales are wrong. In order to obtain appropriate masses, one needs some new kind of interaction with a different scale. This is just the idea of *technicolour*.

2.2. Technicolour

The *technicolour* idea is the following: one introduces *new* strong interactions called *Quantum Technicolour Dynamics* (QTD) and a *new* set of fields called technifermions. In a simplest case one has a doublet of technifermions $T = \begin{pmatrix} A \\ B \end{pmatrix}$. QTD is constructed in full analogy with QCD. The only difference is the scale of interaction

$$\begin{aligned} f_\pi \rightarrow \quad & F_\pi \sim \quad 250 Gev \\ \frac{\Lambda_{TC}}{\Lambda_{QCD}} \sim \quad & \frac{F_\pi}{f_\pi} \sim \quad 2600 \end{aligned}$$

Hence if $\Lambda_{QCD} \sim 200 Mev, \Lambda_{TC} \sim 500 Gev$. This gives $m_{W,Z} \sim 100$GeV as it should be.

Therefore we come to the following picture:

The Higgs fields are replaced by the strong interacting technifermions. Instead of a scalar field with a contact self-interaction we introduce a set of

fermion fields interacting through a new gauge force. Instead of a condensate of scalar fields we have a condensate of technifermion pairs which spontaneously breaks the symmetry thus giving masses to the gauge bosons. The appeared goldstone bosons (the technipions) are absorbed by W and Z becoming their longitudinal components exactly like it happens with fundamental scalars in the Higgs mechanism.

The described picture is slightly modified because of the presence of both QCD and QTD, i.e. of ordinary pions as well as technipions. As a result we have two orthogonal combinations

$$|pion\ absorbed> = \frac{F_\pi|technipion> + f_\pi|pion>}{\sqrt{F_\pi^2 + f_\pi^2}}$$

and

$$|pion\ physical> = \frac{F_\pi|pion> - f_\pi|technipion>}{\sqrt{F_\pi^2 + f_\pi^2}},$$

with the matrix elements of the full axial current being

$$\left.\begin{aligned} &<0|J^5_\mu|pion> = f_\pi q_\mu \\ &<0|J^5_\mu|technipion> = F_\pi q_\mu \end{aligned}\right\} \begin{aligned} &<0|J^5_\mu|pion\ absorbed> = \sqrt{F_\pi^2 + f_\pi^2}\, q_\mu \\ &<0|J^5_\mu|pion\ physical> = 0 . \end{aligned}$$

Due to a relative smallness of f_π the physical pion is mostly the ordinary one. However its decay modes now are described in a more complicated non-contact way.

2.3. Technispectroscopy

In analogy with QCD one can expect that QTD will predict a spectrum of bound states of techniquarks. It should be very much like the QCD spectrum but rescaled by a factor of $\Lambda_{TC}/\Lambda_{QCD} \sim 2600$. There should exist techni-ρ, techni-ω, etc. mesons with masses of the order of 1 TeV and with spacings and widths also $\sim 10^3$ larger than in QCD.

These techni-resonances will manifest themselves in e^+e^- annihilation process as the new resonance peaks in the cross-section ratio (see Fig.2) The main difference from the SM here is that the longitudinal parts of W and Z bosons become strongly interacting particles. In e^+e^- annihilation into a pair of W^+W^- this means that the transverse parts of Ws will behave like in the SM while the longitudinal ones will produce resonances, which is a specific prediction of technicolour.

Another manifestation of technicolour is high p_T jets in e^+e^- or $p\bar{p}$ collisions. The p_T will be also rescaled by a factor of 10^3 with respect to QCD.

The spectrum of technistates appears to be strongly model-dependent. It depends on the group of technicolour, representation , etc. For the simplest

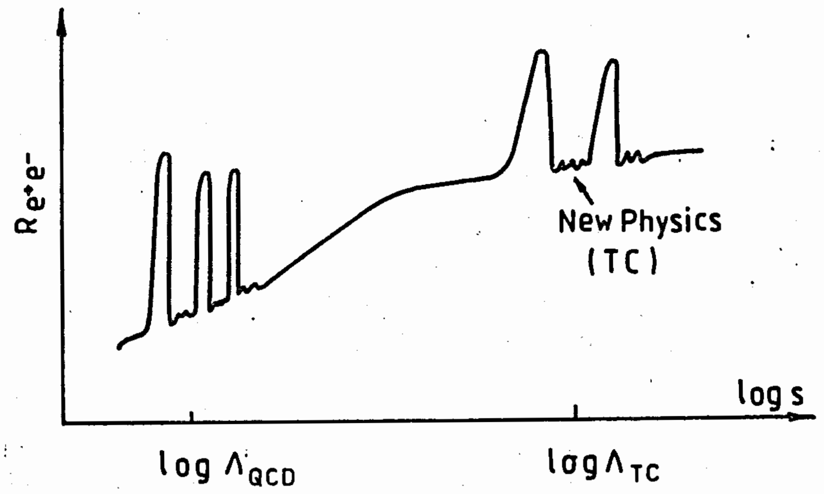

Figure 2: e^+e^- annihilation cross-section ratio with techni- resonances

one-doublet model

$$T = \begin{pmatrix} A \\ B \end{pmatrix}_L, \quad A_R, \quad B_R$$

the spectrum is trivial and contains no (pseudo)goldstone bosons. However already in the one-family model, which was constructed in analogy with the family content of the SM in order to suppress FCNC by GIM mechanism, the situation is more complicated. In this case the techniquarks and technileptons are organised like quarks and leptons in the SM

$$\begin{pmatrix} U_c^\alpha \\ D_c^\alpha \end{pmatrix}_L \quad \begin{matrix} U_{cR}^\alpha \\ D_{cR}^\alpha \end{matrix} \quad \begin{pmatrix} N^\alpha \\ E^\alpha \end{pmatrix}_L \quad \begin{matrix} N_R^\alpha \\ E_R^\alpha \end{matrix} \quad \begin{matrix} \alpha = 1,2,3,\ldots - \textit{technicolour} \\ c = 1,2,3 - \textit{colour} \end{matrix}$$

The symmetry breaking now is achieved by the condensate of all techniparticles (for simplicity we take them to be equal)

$$< \bar{U}U >=< \bar{D}D >=< \bar{N}N >=< \bar{E}E >\neq 0.$$

In this case

$$\Lambda_{TC} \sim 1300\sqrt{\frac{3}{N}}Gev \quad \text{for} \quad SU(N) \quad \text{TC} \quad \text{group}.$$

This model predicts a rich spectrum of technihadrons with masses $\sim$ 1TeV having an integer (technimesons) or a half-integer (technibaryons) spin. The latter is possible for $N = 2k + 1$.

The lightest states of spin 0 may have masses $\leq$ 1 TeV or even $\sim$ 100 GeV. These are the pseudo-goldstone bosons with unusual production and decay properties. To find them out, let us first remind QCD. There the $SU_L(2)\otimes SU_R(2)$ chiral symmetry is broken down to $SU_{Isospin}(2)$ by the quark condensate $< \bar{u}u + \bar{d}d >\neq 0$ thus producing a triplet of goldstone bosons - pions $\vec{\pi}$. They become the pseudo-goldstones acquiring small masses $m_\pi^2 \sim m_q^2$ due to quark mass terms in the QCD Lagrangian.

For the TC one-family model the initial chiral group is $SU_L(8)\otimes SU_R(8)$ which is broken down to $SU_{vector}(8)$. Hence the number of goldstone bosons equals : $2\times 63 - 63 = 63$, i.e. the number of broken generators.These goldstone bosons become pseudo-goldstones due to $SU(3)\otimes SU(2)\otimes U(1)$ corrections. They can be constructed out of techniquark and technilepton doublets

$$Q^c = \begin{pmatrix} U \\ D \end{pmatrix}^c, \quad L = \begin{pmatrix} N \\ E \end{pmatrix}$$

and can be colour as well as technicolour singlets or multiplets:

$$\begin{array}{lll} \Theta_a^i & \sim & \bar{Q}\gamma^5\lambda_a\tau^i Q, \\ \Theta_a & \sim & \bar{Q}\gamma^5\lambda_a Q, \\ T_c^i & \sim & \bar{Q}_c\gamma^5\tau^i L, \qquad \bar{T}_c^i \sim \bar{L}\gamma^5\tau_i Q_c, \\ T_c & \sim & \bar{Q}_c\gamma^5 L, \qquad \bar{T}_c \sim \bar{L}\gamma^5 Q_c, \\ \Pi^i & \sim & \bar{Q}\gamma^5\tau^i Q + \bar{L}\gamma^5\tau^i L, \\ P^\pm & \sim & \bar{Q}\gamma^5\tau^\pm Q - 3\bar{L}\gamma^5\tau^\pm L, \\ P^3 & \sim & \bar{Q}\gamma^5\tau^3 Q - 3\bar{L}\gamma^5\tau^3 L, \\ P^0 & \sim & \bar{Q}\gamma^5 Q - 3\bar{L}\gamma^5 L, \end{array}$$

where a = 1,2,... 8; i = 1,2,3.

The Θ -mesons have typical masses $m_\Theta \sim \sqrt{\frac{4}{N}}$ 200 GeV and the decay modes are:

$$\begin{array}{lllll} \Theta_a & \rightarrow & \bar{q}q, & gg, & g\gamma, \quad etc, \\ \Theta_a^3 & \rightarrow & \bar{q}q, & g\gamma, & gZ, \quad etc, \\ \Theta_a^\pm & \rightarrow & \bar{q}q, & gW^\pm, & etc. \end{array}$$

For the T-mesons one has $m_T \sim 2/3 m_\Theta$ and the decay modes are:

$$\begin{array}{llll} T_c^+ & \rightarrow & \bar{b}_c + \nu, & \\ T_c^3 & \rightarrow & \bar{t}_c + \nu, & \bar{b}_c + \tau, \\ T_c & \rightarrow & \bar{t}_c + \nu, & \bar{b}_c + \tau, \\ T_c^- & \rightarrow & T_c^3 + \mu_- + \nu. & \end{array}$$

The Π^i mesons are "eaten" by Ws and Z and become their longitudinal components.

$P^\pm$ are the so-called axions. Their masses are of electromagnetic origin

$$(m_{P^\pm}^{em})^2 \sim \frac{3\alpha}{4\pi}m_Z^2\ln\frac{M_{TC}^2}{m_Z^2} \sim (5 - 10Gev)^2.$$

The decay modes are

$$P^+ \to c + \bar{b},$$

$$(\bar{t}t) \to \quad P^+ + \bar{t} + b\,, \quad if \quad m_{P^+} < m_t - m_b\,,$$
$$\hookrightarrow P^- + b$$

P^3 and P^0 have masses $0 \le m^2_{P^{3,0}} \le (100Gev)^2$. They can be produced in e^+e^- annihilation

$$e^+e^- \to \bar{t}t + P^0, \quad P^3 + \gamma,$$

other modes being suppressed.

2.4. Fermion Masses

We have shown how TC can provide us with gauge boson masses. The gauge group of the TC-extended SM now is

$$SU_c(3) \otimes SU_L(2) \otimes U_Y(1) \otimes TC.$$

However this is not enough to give masses to fermions. They are still massless. The way out of this puzzle is found along the same lines. One introduces a *new* type of interactions with a *new* scale $\Lambda_E > \Lambda_{TC}$. These new interactions will produce on Λ_E some new effective (nonrenormalizable) operators (four-fermion, etc) that will give us finally the mass terms.

To find out what type of operators plays the main role, let us consider the coupling constant corresponding to an n-fermion operator. On dimensional grounds it behaves like

$$g^{(n)} \sim \frac{1}{M^{\frac{3n}{2}-4}},$$

where M is some scale of dimension of the mass. Therefore the larger M is and the larger n is the less is the coupling. Hence the main role is played by the operator with minimal n. Because of Lorentz invariance n is even,so we have

$n = 2:$ $\bar{\psi}\psi$. This is a mass term. It is forbidden due to $SU_L(2)$ symmetry.

$n = 4$: To construct the four-fermion operator in a one-doublet model $T = \begin{pmatrix} A \\ B \end{pmatrix}$, we choose the 2×2 matrix

$$M_T = 1 \cdot \bar{T}T + i\vec{\tau}\bar{T}\gamma^5\vec{\tau}T.$$

Then the effective four-fermion $SU_L(2)$-invariant interaction is

$$\frac{a}{\Lambda_E^2}\bar{q}_L M_T q_R + \frac{b}{\Lambda_E^2}\bar{q}_L M_T \tau^3 q_R + h.c. \qquad (2.3)$$

If techniquark condensate exists

$$< \bar{T}T > = < \bar{A}A + \bar{B}B > \neq 0,$$

Figure 3: The diagrams contributing to quark masses due to TC condensate

then eq.(2.3) will lead to quark masses

$$\begin{aligned} m_u &\sim \frac{< \bar{A}A + \bar{B}B >}{\Lambda_E^2}(a+b) \\ m_d &\sim \frac{< \bar{A}A + \bar{B}B >}{\Lambda_E^2}(a-b) \end{aligned} \qquad (2.4)$$

The diagrams contributing to eq.(2.4) are shown in Fig.3.

Estimating the value of techniquark condensate for the $SU(N)$ TC group to be

$$\frac{< \bar{T}T >}{2} \sim \sqrt{\frac{3}{N}} < \bar{q}q > (\frac{F_\pi}{f_\pi})^3 \sim \begin{matrix} (600Gev)^3 & 1 \text{ doublet model} \\ (300Gev)^3 & 1 \text{ family model} \end{matrix}$$

we get an estimate for the new scale Λ_E respectively

$$\Lambda_E \sim \begin{matrix} 20Tev \\ 7Tev \end{matrix}.$$

The same is true for the one family model, however here we have more possibilities. With quark and lepton doublets

$$q = \begin{pmatrix} u \\ d \end{pmatrix}, l = \begin{pmatrix} \nu \\ e \end{pmatrix}, Q = \begin{pmatrix} U \\ D \end{pmatrix}, L = \begin{pmatrix} N \\ E \end{pmatrix}$$

we have

$$\begin{aligned} M_Q &= 1 \cdot \bar{Q}Q + i\vec{\tau}\bar{Q}\gamma^5\vec{\tau}Q \\ M_L &= 1 \cdot \bar{L}L + i\vec{\tau}\bar{L}\gamma^5\vec{\tau}L\,. \end{aligned}$$

And $SU_{Left}(2)$ singlets now are

$$\begin{matrix} \bar{q}_L M_Q q_R, & \bar{q}_L M_Q \tau^3 q_R, & \bar{q}_L M_L q_R, & \bar{q}_L M_L \tau^3 q_R \\ \bar{l}_L M_Q l_R, & \bar{l}_L M_Q \tau^3 l_R, & \bar{l}_L M_L l_R, & \bar{l}_L M_L \tau^3 l_R \end{matrix}$$

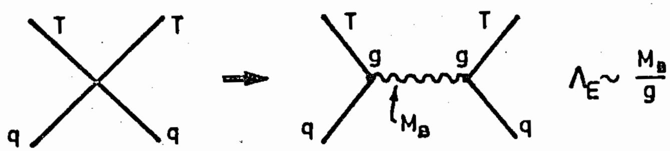

Figure 4: Effective four-fermion interaction due to gauge boson exchange

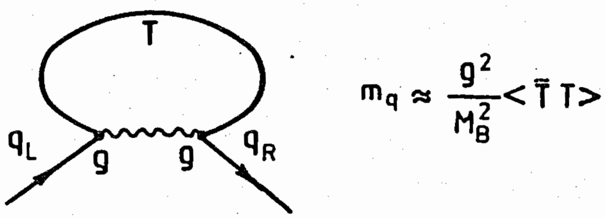

Figure 5: The diagram contributing to the quark mass due to a gauge boson exchange in TC theories

Introducing a generation mixing matrix we also get the mixing angles.

Thus, to obtain fermion masses one needs an effective four-fermion interaction. The latter can be also obtained from the gauge boson exchange, just like effective four-fermion operators in weak interactions appear due to W and Z exchange (see Fig.4). Then the mass terms arise from the diagrams shown in Fig.5. This is an extended Technicolour idea.

2.5. Extended Technicolour

The guiding idea: Techniparticles and ordinary particles belong to the same representation of a large gauge group that is dynamically broken on some scale

$$G_{ETC} \supset G_{TC}.$$

For example

$$SU(N+1) \supset SU(N)$$

and the fundamental representation

$$\underline{N+1} = \underline{N} + \underline{1} \quad ETC \left\{ \begin{pmatrix} T_1 \\ \vdots \\ T_N \\ q \end{pmatrix} \right. \begin{array}{l} \left.\vphantom{\begin{matrix} T_1 \\ \vdots \\ T_N \end{matrix}}\right\} \; TC \\ TC \quad singlet\,. \end{array}$$

The other possibilities are

$$SU(N+m) \supset SU(N), \quad SU(N+3) \supset SU_{TC}(N) \otimes SU_c(3)$$
$$\underline{N+m} = \underline{N} + m \cdot \underline{1}, \qquad \underline{N+3} = \underline{N} + \underline{3} \cdot \underline{1}$$

On the scale Λ_E the ETC group should be dynamically broken

$$G_{ETC} \overset{\Lambda_{ETC}}{\Rightarrow} U_Y(1) \otimes SU_L(2) \otimes SU_c(3) \otimes G_{TC}\,.$$

The broken generators of the ETC group will lead to massive gauge bosons which will mediate the fermion-technifermion transformations. On Λ_E the ordinary fermions will mix with technifermions and with themselves thus producing rare decays like

$$K \to \mu e, \quad \pi^+ \to e^+ e^+ e^-, \quad etc.$$

This is an indication of Extended Technicolour.

2.6. GUTs and Technicolour

As you probably noticed, ETC idea is closely related to that of Grand Unified Theories. In the latter case the group of the SM is embedded into a larger group of GUT.

$$GUT: SU(3) \otimes SU(2) \otimes U(1) \subset G_{GUT} = SU(5).$$

The same can be done with TC.

$$GUT + TC: G_{TC} \otimes SU(3) \otimes SU(2) \otimes U(1) \subset G_{GUT}.$$

For instance, if G_{TC} is $SU(N)$, the natural embedding is to $SU(N+5)$:

$$\left(\begin{array}{c|c|c} SU_{TC}(N) & & \\ \hline & SU_c(3) & \\ \hline & & SU_L(2) \end{array} \right)$$

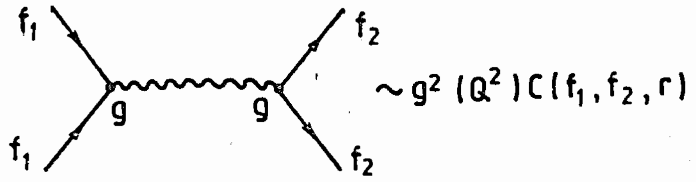

Figure 6: Creation of new fermions in the most attractive channel

The pattern of symmetry breaking will then have two stages

$$G_{GUT} = SU(N+5) \stackrel{\Lambda_B}{\rightarrow} G_{TC} \otimes SU(3) \otimes SU(2) \otimes U(1) \stackrel{\Lambda_{TC}}{\rightarrow} SU(3) \otimes U(1)$$

To conclude, we mention the successes and problems of Technicolour.

Successes:

i) Low energy Higgs bosons are replaced by fermions. These fermions are unified with ordinary fermions;

ii) More than one family of light fermions can be naturally incorporated in a large representation of a unifying group.

Problems:

i) It is difficult to implement the required symmetry breaking in many stages;

ii) Many technifermions may change the β-functions and ruin successful predictions for the proton decay and low-energy value of θ_W;

iii)Spectra of light fermions is unrealistic.

2.7. Tumbling Theory (Groups Breaking Themselves)

An interesting possibility of dynamical breaking of gauge symmetry, which can be useful to obtain several stages of breaking, is the so-called *tumbling* phenomenon.

Let the gauge group G be asymptotically free. Consider the scattering process shown in Fig.6.

The amplitude of this process is proportional to the square of the running coupling $g^2(Q^2)$ which increases as we come down with energy due to the asymptotical freedom. The coefficient $C(f_1, f_2, r)$ depends on the group and representation of fermion fields.

The final fermions may condense in a singlet channel. The condensation will take place if the force between final fermions is attractive and the amplitude is large enough (in AF theory it becomes large as we go down with energy). It will take place first in the most attractive channel. If a condensate is not a singlet under some group, the latter will be dynamically broken.

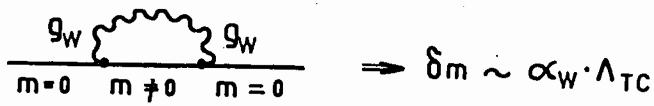

Figure 7: Weak radiative correction to the quark mass

Thus the chain looks as follows: One starts at a high energy with an unbroken group G. All of the particles are massless. As the energy goes down, some fermions condense. This leads to a symmetry breaking. Some particles acquire masses of the order of the symmetry breaking scale, while the others remain massless. Then the procedure is repeated several times.

G	$\stackrel{Condensate}{\rightarrow}$	G_1	$\stackrel{Condensate}{\rightarrow}$	G_2	$\rightarrow$	$\cdots$	$\rightarrow$	G_N
massless	E_1	*some*	E_2	*some*	E_3		E_N	
particles		$m \sim E_1$		$m \sim E_2$				

We get a chain of symmetry breakings due to condensation. The described picture is called a *tumbling gauge theory*. This phenomenon may naturally produce a sequence of scales thus solving the problem equally important in TC as well in any other theories.

Another possibility to reach the same goal is the radiative corrections. Suppose that as a result of some symmetry breaking (like in TC theory) some quarks become massive while the others remain massless. Then they will get mass corrections due to the weak interactions(see Fig.7). Hence ordinary fermions will have masses of the order of α and/or $\alpha^2 \times \Lambda_{TC}$. This mechanism will give masses to the light fermions without introducing a new scale.

However in any case the origin of the fermion spectrum remains unclear. Note that there is no solution of this problem in the Higgs model as well. Understanding of the origin of fermion masses is one of the greatest challenges facing particle physicists today.

3. Supersymmetry

3.1. Motivations of SUSY

Supersymmetry or fermion-boson symmetry have not yet been observed in Nature. This is a purely theoretical invention. Its validity in particle physics follows from the common belief in unification. The *general idea* is a unification of all forces of Nature. It defines the *strategy* : increasing unification towards smaller distances up to $l_{Pl} \sim 10^{-33}$ cm including quantum gravity. However the graviton has spin 2, while the other gauge bosons (photon, gluons, W and Z weak bosons) have spin 1. Unification of spin 2 and spin 1 gauge forces within unique algebra is forbidden due to the no-go theorems for any symmetry but SUSY.

If Q is a generator of SUSY algebra, then

$$Q|boson> = |fermion> \quad and \quad Q|fermion> = |boson> .$$

Hence starting with the graviton spin 2 state and acting by SUSY generators we get the following chain of states

$$spin\ 2 \quad \rightarrow \quad spin\ 3/2 \quad \rightarrow \quad spin\ 1 \quad \rightarrow \quad spin\ 1/2 \quad \rightarrow \quad spin\ 0$$

Thus a partial unification of matter (fermions) with forces (bosons) naturally arises out of an attempt to unify gravity with other interactions.

This very promising unification pattern gives us a justification of research in spite of the absence of even a shred of experimental evidence of SUSY.

The *uniqueness* of SUSY is due to a strict mathematical statement that algebra of SUSY is the *only* graded (i.e. containing anticommutators as well as commutators) Lie algebra possible within relativistic field theory.

Theoretical attractiveness of SUSY field theories is explained by remarkable properties of SUSY models. This is first of all a cancellation of ultraviolet divergences in rigid SUSY theories which is the origin of

- possible solution of the hierarchy problem in GUTs;
- construction of a finite field theory;
- possible construction of quantum (super)gravity;
- solution of the problem of vanishing cosmological constant, etc.

What is essential, the standard concepts of QFT allow SUSY without any further assumptions.

3.2. Global SUSY. Algebra and Representations

As can be easily seen, supersymmetry transformations differ from ordinary global transformations as far as they convert bosons into fermions and vice versa. Indeed if we symbolically write SUSY transformation as

$$\delta B = \varepsilon \cdot f,$$

where B and f are boson and fermion fields respectively and ε is an infinitesimal transformation parameter, then from the usual (anti)commutation relations for (fermions) bosons

$$[B, B] = 0, \qquad \{f, f\} = 0$$

we immediately find

$$\{\varepsilon, \varepsilon\} = 0.$$

This means that all the generators of SUSY must be *fermionic* , i.e. they must change the spin by a half-odd amount and change the statistics.

Combined with the usual Poincaré and internal symmetry algebra the superPoincaré Lie algebra is

$$\begin{aligned}
&[P_\mu, P_\nu] = 0,\\
&[P_\mu, M_{\rho\sigma}] = i(g_{\mu\rho}P_\sigma - g_{\mu\sigma}P_\rho),\\
&[M_{\mu\nu}, M_{\rho\sigma}] = i(g_{\nu\rho}M_{\mu\sigma} - g_{\nu\sigma}M_{\mu\rho} - g_{\mu\rho}M_{\nu\sigma} + g_{\mu\sigma}M_{\nu\rho}),\\
&[B_r, B_s] = iC^t_{rs}B_t,\\
&[B_r, P_\mu] = [B_r, M_{\mu\sigma}] = 0,\\
&[Q^i_\alpha, P_\mu] = [\bar{Q}^i_{\dot\alpha}, P_\mu] = 0,\\
&[Q^i_\alpha, M_{\mu\nu}] = \tfrac{1}{2}(\sigma_{\mu\nu})^\beta_\alpha Q^i_\beta, \quad [\bar{Q}^i_{\dot\alpha}, M_{\mu\nu}] = -\tfrac{1}{2}\bar{Q}^i_{\dot\beta}(\bar\sigma_{\mu\nu})^{\dot\beta}_{\dot\alpha},\\
&[Q^i_\alpha, B_r] = (b_r)^i_j Q^j_\alpha, \quad [\bar{Q}^i_{\dot\alpha}, B_r] = -\bar{Q}^j_{\dot\alpha}(b_r)^i_j,\\
&\{Q^i_\alpha, \bar{Q}^j_{\dot\beta}\} = 2\delta^{ij}(\sigma^\mu)_{\alpha\dot\beta}P_\mu,\\
&\{Q^i_\alpha, Q^j_\beta\} = 2\epsilon_{\alpha\beta}Z^{ij}, \quad Z_{ij} = a^r_{ij}b_r, \quad Z^{ij} = Z^+_{ij},\\
&\{\bar{Q}^i_{\dot\alpha}, \bar{Q}^j_{\dot\beta}\} = -2\epsilon_{\dot\alpha\dot\beta}Z^{ij}, \quad [Z_{ij}, anything] = 0,\\
&\alpha, \dot\alpha = 1,2 \quad i,j = 1,2,\ldots,N.
\end{aligned} \tag{3.1}$$

Here P_μ and $M_{\mu\nu}$ are four-momentum and angular-momentum operators respectively, B_r are internal symmetry generators, Q^i and $\bar{Q}^i$ are spinorial SUSY generators and Z_{ij} are the so-called central charges.

A natural question arises: how many SUSY generators are possible, i.e. what is the value of N ? To answer this question, let us consider the massless states in SUSY theory. The ground state defined by $Q_i|E, \lambda>= 0$ is labelled by two quantum numbers: energy E and helicity λ. Then a one-particle state is produced by acting of a creation operator

$$1\ particle\ state \qquad \bar{Q}_i|E, \lambda> = |E, \lambda + 1>_i$$

It has the same energy E since a SUSY generator commutes with four-momentum (see eq.(3.1)) and helicity $\lambda + 1/2$. The number of one-particle states equals that of SUSY generators, i.e. $\binom{N}{1} = N$.

Two-particle state is constructed by acting of two creation operators

$$2\ particle\ state \qquad \bar{Q}_i\bar{Q}_j|E, \lambda> = |E, \lambda + 1>_{ij}.$$

The number of such states is $\binom{N}{2} = \frac{N(N+1)}{2}$. Continuing the procedure we end up with an N-particle state

$$N\ particle\ state \qquad \bar{Q}_1 \cdots \bar{Q}_N|E, \lambda + \frac{N}{2}> .$$

The number of these states is $\binom{N}{N} = 1$. Thus the total number of states is

$$\sum_{k=0}^{N} \binom{N}{k} = 2^N = 2^{N-1} bosons + 2^{N-1} fermions.$$

We see that the number of bosonic states equals that of fermionic. This is a general feature of SUSY. If SUSY is realized linearly,bosonic and fermionic states are always equal in number.

Remind now the so-called PCT-theorem valid for any local Lorentz- invariant QFT.

PCT-Theorem: In any Lorentz-invariant local field theory for every state with helicity λ there should be a parity reflected state with helicity $-\lambda$.

Consider $N=1$ case with $\lambda = 0$. According to the previous discussion we have

$N=1$	*helicity*	0	1/2		*helicity*	$-1/2$	0
$\lambda = 0$	*No of states*	1	1	$\overset{PCT}{\Rightarrow}$	*No of states*	1	1

Thus a complete $N=1$ multiplet consists of the following set of states:

$N=1$	*helicity*	$-1/2$	0	1/2
multiplet	*No of states*	1	2	1

This is an example of a self-conjugated multiplet. Some other self-conjugated multiplets are also important

$N=4$	*helicity*	-1	$-1/2$	0	1/2	1
SUSY YM	*No of states*	1	4	6	4	1

$N=8$	*helicity*	-2	$-3/2$	-1	$-1/2$	0	1/2	1	3/2	2
SUGRA	*No of states*	1	8	28	56	70	56	28	8	1

As can be seen, the maximal helicity (or spin) is related to the number of SUSY generators N by equation

$$4s \leq N.$$

This constraint becomes very essential since any theory containing particles with spin > 1 is non-renormalizable, and any theory containing a finite number of particles with spin $\geq 5/2$ has no consistent coupling to gravity. Since we are not able to proceed with non-renormalizable interactions, we get the following constraint on the number of SUSY generators

$N \leq 4$ for renormalizable theories (YM),
$N \leq 8$ for (super)gravity.

3.3. Component Fields and Superfields

Return now to an $N=1$ supermultiplet. We denote it by (A, ψ, F), where A and F are bosonic and ψ is a fermionic field. Supersymmetry transformations are (we use the two-component spinor notation):

$$\begin{aligned} \delta_\epsilon A &= \sqrt{2}\varepsilon\psi, \\ \delta_\epsilon \psi &= i\sqrt{2}\sigma^\mu \bar{\varepsilon}\partial_\mu A + \sqrt{2}\varepsilon F, \\ \delta_\epsilon F &= i\sqrt{2}\bar{\varepsilon}\sigma^\mu \partial_\mu \psi, \end{aligned} \qquad (3.2)$$

where $\sigma^\mu = (1, \vec{\sigma})$ are Pauli matrices. Transformations (3.2) form a closed algebra. However not all of the fields are the usual ones. To clear it up, let us look at their dimensions. If for A and ψ we have, as usual,

$$[A] = m^1, \quad [\psi] = m^{3/2},$$

then from eq.(3.2) it follows that

$$[\varepsilon] = [\bar{\varepsilon}] = m^{-1/2}, \quad [F] = m^2.$$

One can see that the field F has a "wrong" dimension. This field is called an *auxiliary* field, it has no physical meaning and is needed to close the algebra (3.2). As we will see, one can get rid of the auxiliary fields with the help of equations of motion.

The Lagrangian which is invariant under transformations (3.2) (up to a total derivative) is

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_0 + m\mathcal{L}_m, \\ \mathcal{L}_0 &= i\bar{\psi}\sigma^\mu\partial_\mu\psi + A^*\Box A + F^*F, \\ \mathcal{L}_m &= AF + A^*F^* - \frac{1}{2}\psi\psi - \frac{1}{2}\bar{\psi}\bar{\psi}. \end{aligned} \qquad (3.3)$$

Equations of motion that follows from eq.(3.3) are

$$\begin{aligned} i\sigma^\mu\partial_\mu\psi + m\bar{\psi} &= 0, \\ \Box A + mF^* &= 0, \\ F + mA^* &= 0. \end{aligned}$$

One can see that the last equation contains no derivatives, i.e. this is a constraint rather than a dynamical equation. Solving this constraint with respect to an auxiliary field F we get the so-called mass-shell formulation of SUSY. Lagrangian (3.3) becomes

$$\mathcal{L} = i\bar{\psi}\sigma^\mu\partial_\mu\psi - \frac{1}{2}m(\psi\psi + \bar{\psi}\bar{\psi}) + A^*\Box A - m^2A^*A,$$

which is a usual free field Lagrangian for spinor and scalar fields.

Superspace

More elegant formulation of supersymmetry transformations and invariants can be achieved in the framework of superspace. Superspace differs from the ordinary Euclidean (Minkowski) space by addition of two new coordinates, which are grassmanian, i.e. anticommuting, variables $\{\theta_\alpha, \theta_\beta\} = 0, \quad \theta^2 = 0.$

Space	*Superspace*
x_μ	$x_\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}}$

A SUSY group element can be constructed in superspace in the same way as an ordinary translation in the usual space

$$G(x, \theta, \bar{\theta}) = e^{i(-x^\mu P_\mu + \theta Q + \bar{\theta}\bar{Q})}.$$

It leads to the supertranslation in superspace

$$\begin{array}{lcl} x_\mu & \to & x_\mu + i\theta\sigma_\mu\bar{\xi} - i\xi\sigma_\mu\bar{\theta}, \\ \theta & \to & \theta + \xi, \\ \bar{\theta} & \to & \bar{\theta} + \bar{\xi}, \end{array}$$

where ξ and $\bar{\xi}$ are grassmanian transformation parameters.

Now we are in a position to introduce a superfield as a function on a superspace which is a representation of a superPoincaré group (3.1). The simplest one is a scalar superfield $F(x, \theta, \bar{\theta})$ which is SUSY invariant. Its Taylor expansion in θ and $\bar{\theta}$ has only several terms due to the nilpotent character of grassmanian parameters.

$$\begin{aligned} F(x, \theta, \bar{\theta}) &= f(x) + \theta\varphi(x) + \bar{\theta}\bar{\chi}(x) \\ &+ \theta\theta m(x) + \bar{\theta}\bar{\theta} n(x) + \theta\sigma^\mu\bar{\theta} v_\mu(x) \\ &+ \theta\theta\bar{\theta}\lambda(x) + \bar{\theta}\bar{\theta}\theta\psi(x) + \theta\theta\bar{\theta}\bar{\theta} d(x). \end{aligned} \quad (3.4)$$

The coefficients are ordinary functions of x , being the usual fields. Superfield (3.4) is a reducible representation of SUSY. To get an irreducible one, we consider a function which depends only on θ but not on $\bar{\theta}$, i.e. obeys the equation

$$\frac{\partial}{\partial\bar{\theta}} F(x, \theta) = 0.$$

Unfortunately this equation is not SUSY invariant. To improve it, one has to use the covariant derivative instead of the ordinary one

$$\bar{D}F = 0, \quad \text{where } \bar{D} = \frac{\partial}{\partial\bar{\theta}} - i\theta\sigma^\mu\partial_\mu. \quad (3.5)$$

The superfield obeying eq.(3.5) is called a *chiral* superfield

$$\bar{D}\Phi = 0 \quad \Rightarrow \Phi = \Phi(y, \theta), \quad y = x + i\theta\sigma\bar{\theta}.$$

Its Taylor expansion looks like

$$\begin{aligned} \Phi(y, \theta) &= A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \\ &= A(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu A(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box A(x) \\ &+ \sqrt{2}\theta\psi(x) - \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\psi(x)\sigma^\mu\bar{\theta} + \theta\theta F(x) \end{aligned}$$

and manifests the $N = 1$ supermultiplet considered above.

The product of chiral superfields Φ^2, Φ^3, etc. is also a chiral superfield, while the product of chiral and antichiral ones $\Phi^+\Phi$ is a general superfield.

We are now ready to construct SUSY invariant Lagrangians.

3.4. SUSY Lagrangians

In the superfield notation SUSY invariant Lagrangians are simply polynomials of superfields. Having in mind that for component fields we should have the ordinary terms, the general SUSY invariant Lagrangian has the form

$$\begin{aligned} \mathcal{L} &= \Phi_i^+\Phi_i|_{\theta\theta\bar{\theta}\bar{\theta}} + [(\lambda_i\Phi_i + \frac{1}{2}m_{ij}\Phi_i\Phi_j \\ &+ \frac{1}{3}g_{ijk}\Phi_i\Phi_j\Phi_k)|_{\theta\theta} + h.c.]. \end{aligned} \quad (3.6)$$

Hereafter the vertical line means the corresponding term of a Taylor expansion. Performing this expansion we get in components

$$\begin{aligned} \mathcal{L} &= i\bar{\psi}_i\sigma^\mu\partial_\mu\psi_i + A_i^*\Box A_i + F_i^*F_i \\ &+ [\lambda_i F_i + m_{ij}(A_i F_j - \frac{1}{2}\psi_i\psi_j) + g_{ijk}(A_i A_j F_k - \psi_i\psi_j A_k) + h.c.] \end{aligned}$$

The last two terms which are additional to that of eq.(3.3) are the interaction ones. To obtain a familiar form of the Lagrangian, we have to solve the constraints:

$$\begin{aligned} \frac{\partial\mathcal{L}}{\partial F_k^*} &= F_k + \lambda_k^* + m_{ik}^* A_i^* + g_{ijk}^* A_i^* A_j^* = 0, \\ \frac{\partial\mathcal{L}}{\partial F_k} &= F_k^* + \lambda_k + m_{ik} A_i + g_{ijk} A_i A_j = 0. \end{aligned}$$

After that we finally get

$$\begin{aligned} \mathcal{L} &= i\bar{\psi}_i\sigma^\mu\partial_\mu\psi_i + A_i^*\Box A_i - \frac{1}{2}m_{ij}\psi_i\psi_j - \frac{1}{2}m_{ij}^*\bar{\psi}_i\bar{\psi}_j \\ &- g_{ijk}\psi_i\psi_j A_k - g_{ijk}^*\bar{\psi}_i\bar{\psi}_j A_k^* - V(A_i, A_j), \end{aligned} \quad (3.7)$$

where $V = F_k^* F_k$. Note that because of the renormalizability constraint $V \leq A^4$ the superpotential should be limited by $W \leq \Phi^3$ as in eq.(3.6).

To construct the gauge invariant interactions, we will need a real vector superfield $V = V^+$. It is not chiral but rather a general superfield

$$\begin{aligned} V(x,\theta,\bar\theta) &= C(x) + i\theta\chi(x) - i\bar\theta\bar\chi(x) \\ &+ \frac{i}{2}\theta\theta[M(x)+iN(x)] - \frac{i}{2}\bar\theta\bar\theta[M(x)-iN(x)] \\ &- \theta\sigma^\mu\bar\theta v_\mu(x) + i\theta\theta\bar\theta[\lambda(x) + \frac{i}{2}\bar\sigma^\mu\partial_\mu\chi(x)] \\ &- i\bar\theta\bar\theta\theta[\lambda + \frac{i}{2}\sigma^\mu\partial_\mu\bar\chi(x)] + \frac{1}{2}\theta\theta\bar\theta\bar\theta[D(x) + \frac{1}{2}\Box C(x)]. \end{aligned} \quad (3.8)$$

The physical degrees of freedom corresponding to a real vector superfield are the vector gauge field v_μ and majorana spinor field λ. All other components are unphysical and can be eliminated.

Under the abelian (super)gauge transformation the superfield V is transformed as

$$V \rightarrow V + \Phi + \Phi^+,$$

where Φ and Φ^+ are some chiral superfields. In components it looks like

$$\begin{aligned} C &\rightarrow C + A + A^*, \\ \chi &\rightarrow \chi - i\sqrt{2}\psi, \\ M+iN &\rightarrow M + iN - 2iF, \\ v_\mu &\rightarrow v_\mu - i\partial_\mu(A - A^*), \\ \lambda &\rightarrow \lambda, \\ D &\rightarrow D \end{aligned} \quad (3.9)$$

and corresponds to ordinary gauge transformations for physical components. According to eq.(3.9) one can choose a gauge (Wess-Zumino gauge) where $C = \chi = M = N = 0$, leaving us with the physical degrees of freedom except for the auxiliary field D. In this gauge

$$\begin{aligned} V &= -\theta\sigma^\mu\bar\theta v_\mu(x) + i\theta\theta\bar\theta\bar\lambda(x) - i\bar\theta\bar\theta\theta\lambda(x) + \frac{1}{2}\theta\theta\bar\theta\bar\theta D(x), \\ V^2 &= -\frac{1}{2}\theta\theta\bar\theta\bar\theta v_\mu(x)v^\mu(x), \\ V^3 &= 0, \quad etc. \end{aligned}$$

One can define also a field strength tensor (as analog of $F_{\mu\nu}$ in gauge theories)

$$\begin{aligned} W_\alpha &= -\frac{1}{4}\bar D^2 e^V D_\alpha e^{-V}, \\ \bar W_{\dot\alpha} &= -\frac{1}{4}D^2 e^V \bar D_\alpha e^{-V}, \end{aligned}$$

which is a polynomial in the Wess-Zumino gauge. (Here Ds are the covariant derivatives.) The strength tensor is a chiral superfield

$$\bar D_{\dot\beta} W_\alpha = 0, \quad D_\beta \bar W_{\dot\alpha} = 0.$$

The gauge invariant Lagrangian now is

$$\begin{aligned} \mathcal{L} &= \frac{1}{4}(W^\alpha W_\alpha|_{\theta\theta} + \bar W^{\dot\alpha}\bar W_{\dot\alpha}|_{\bar\theta\bar\theta}) \\ &= \frac{1}{2}D^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - i\bar\lambda\sigma^\mu D_\mu\lambda. \end{aligned}$$

To obtain a gauge-invariant interaction with matter chiral superfields, consider their gauge transformation (abelian)

$$\Phi \rightarrow e^{-ig\Lambda}\Phi, \quad \Phi^+ \rightarrow \Phi^+ e^{ig\Lambda^+}, \quad V \rightarrow V + i(\Lambda - \Lambda^+),$$

where Λ is a gauge parameter (chiral superfield).

It is clear now how to construct both the SUSY and gauge invariant interaction

$$\begin{aligned} \mathcal{L}_{inv} &= \frac{1}{4}(W^\alpha W_\alpha|_{\theta\theta} + \bar W^{\dot\alpha}\bar W_{\dot\alpha}|_{\bar\theta\bar\theta}) \\ &+ \Phi_i^+ e^{gV}\Phi_i|_{\theta\theta\bar\theta\bar\theta} \\ &+ (\frac{1}{2}m_{ij}\Phi_i\Phi_j + \frac{1}{3}g_{ijk}\Phi_i\Phi_j\Phi_k)|_{\theta\theta} + h.c. \end{aligned}$$

In particular the SUSY generalization of QED looks as follows

$$\begin{aligned} \mathcal{L}_{SUSY\ QED} &= \frac{1}{4}(W^\alpha W_\alpha|_{\theta\theta} + \bar W^{\dot\alpha}\bar W_{\dot\alpha}|_{\bar\theta\bar\theta}) \\ &+ (\Phi_+^+ e^{gV}\Phi_+ + \Phi_-^+ e^{-gV}\Phi_-)|_{\theta\theta\bar\theta\bar\theta} \\ &+ m\Phi_+\Phi_-|_{\theta\theta} + m\Phi_+^+\Phi_-^+|_{\bar\theta\bar\theta}. \end{aligned}$$

The non-abelian generalization is straightforward

$$\begin{aligned} \mathcal{L}_{SUSY\ YM} &= \frac{1}{4}\int Tr(W^\alpha W_\alpha)\, d^2\theta + \int \bar\Phi_{ia}(e^{gV})^a_b \Phi^b_i \, d^2\theta d^2\bar\theta \\ &+ \int V(\Phi_i)\, d^2\theta \quad + h.c., \end{aligned}$$

where instead of taking the proper components we use an integration over the superspace according to the rules of grassmanian integration

$$\int d\theta = 0, \quad \int \theta\, d\theta = 1.$$

This trick allows one to write down the action as an integral over the whole superspace in exact analogy with an ordinary QFT.

Note that the form of the Lagrangian is practically fixed by symmetry requirements. The only freedom is the field content, the value of the gauge coupling g, Yukawa couplings g_{ijk} and the masses. All members of a supermultiplet have the same masses, i.e. bosons and fermions are degenerate in masses. This property of SUSY theories contradicts the phenomenology and requires supersymmetry breaking.

3.5. Spontaneous Symmetry Breaking

Apart from non-supersymmetric theories in SUSY models the energy is always nonnegative definite. Indeed, according to quantum mechanics

$$E = <0|\ H\ |0>$$

and due to SUSY algebra eq.(3.1)

$$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2(\sigma^\mu)_{\alpha\dot\beta} P_\mu,$$

taking into account that $tr(\sigma^\mu P_\mu) = 2P_0$, we get

$$E = \frac{1}{4}\sum_{\alpha=1,2} <0|\{Q_\alpha, \bar{Q}_\alpha\}|0> = \frac{1}{4}\sum_\alpha |Q_\alpha|0>|^2 \geq 0.$$

Hence

$$E = <0|\ H\ |0> \neq 0 \quad if\ and\ only\ if \quad Q_\alpha \neq 0.$$

Therefore *a supersymmetry is spontaneously broken, i.e. vacuum is not invariant* $(Q_\alpha|0>\neq 0)$, *if and only if the minimum of the potential is positive* $(i.e. E > 0)$.
The situation is illustrated in Fig.8 .

Mechanisms for SSB

1) Fayet-Illiopoulos (D-term) mechanism

Let the Lagrangian be SUSY and $U(1)$ gauge invariant and contain two matter superfields Φ_1 and Φ_2. We add to the Lagrangian a linear term

$$\Delta\mathcal{L} = \xi\ V|_{\theta\theta\bar\theta\bar\theta}, \tag{3.10}$$

which is a D-term of a vector superfield (cf. eq.(3.8)). Eq.(3.10) is SUSY and gauge invariant (up to a total derivative).
Solving now equations of motion for the auxiliary fields we obtain the potential

$$\begin{aligned} V_{pot} &= (m^2 + g\xi)A_1^*A_1 + (m^2 - g\xi)A_2^*A_2 \\ &+ \frac{1}{8}g^2(A_1^*A_1 - A_2^*A_2)^2 + 1/2\xi^2. \end{aligned}$$

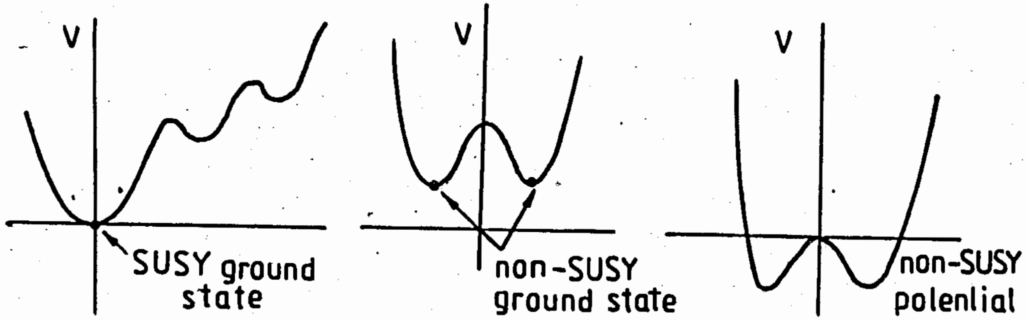

Figure 8: Spontaneous symmetry breaking in SUSY theories

There are two possibilities:
i) $g\xi < m^2$. In this case SUSY is spontaneously broken while the gauge invariance is preserved. The masses are

$$m^2_{A_{1,2}} = m^2 \pm 1/2g\xi, \quad m_{\psi_{1,2}} = m \quad m_{A_\mu,\lambda} = 0.$$

ii) $g\xi > m^2$. In this case both SUSY and gauge invariance are spontaneously broken. The minimum of the potential is at a finite value for the matter fields. One choice is

$$<A_1> = 0, \quad <A_2> = \mu/g \quad \text{with} \quad \mu^2 = 2(\xi g - m^2).$$

A Higgs mechanism will eliminate the goldstone boson, give mass to the vector A_μ and mix spinors into mass eigenstates $\tilde\psi$ and $\tilde\lambda$. The masses become

$$\begin{aligned} m_{\tilde\psi_{1,2}} &= m^2 + \mu^2, \quad m_{\tilde\lambda} = 0, \\ m_{A_\mu} &= m_{A_1} = \mu, \quad m_{A_2} = \sqrt{2}m. \end{aligned}$$

The potential is shown in Fig.9. The spectrum of particles is illustrated in Fig.10.Note that in both cases the following sum rule is always valid

$$\sum_{boson\ states} m^2 = \sum_{fermion\ states} m^2.$$

The drawback of this mechanism is the necessity of $U(1)$ gauge invariance. It can be used in SUSY generalizations of the SM but not in GUTs.

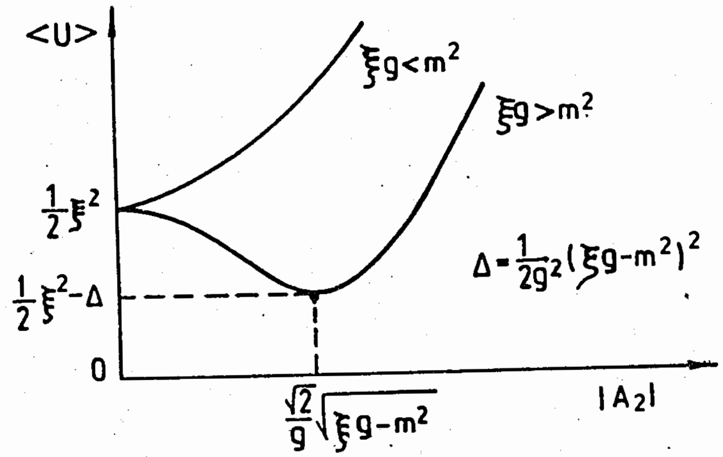

Figure 9: The potential for Fayet-Illiopoulos model

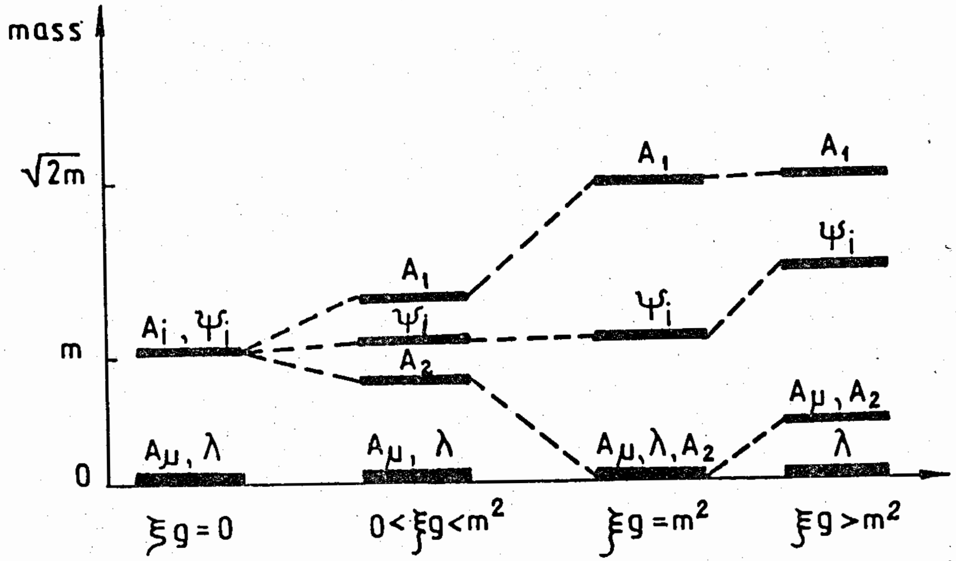

Figure 10: Mass spectrum of Fayet-illiopoulos model

2) O'Raifeartaigh (F-term) mechanism

This mechanism works when we have several chiral fields. Let the superpotential be

$$W(\Phi) = \lambda\Phi_3 + m\Phi_1\Phi_2 + g\Phi_3\Phi_1^2.$$

Equations of motion for the auxiliary fields in this case are

$$\begin{aligned} F_1^* &= mA_2 + 2gA_1A_3, \\ F_2^* &= mA_1, \\ F_3^* &= \lambda + gA_1^2. \end{aligned}$$

Looking for a vacuum solution $< F_i^* >= 0$ we find that it is absent. This means that SUSY is spontaneously broken since $V = F_i^* F_i \neq 0$. The potential takes the form

$$\begin{aligned} V(A) &= \frac{1}{2}|mA_2 + 2gA_1A_3|^2 + \frac{m^2}{2}|A_1|^2 \\ &+ \frac{1}{2}|\lambda + gA_1^2|^2. \end{aligned}$$

The minimum is achieved by the choice

$$< A_1 > = < A_2 > = 0, \quad < A_3 > = \frac{\mu}{2g}, \quad \mu \text{ is arbitrary}$$

Again there are two possibilities

i) $|2g\lambda| < m^2$, then

$$\begin{aligned} m_{\psi_{1,2}} &= \sqrt{\mu^2/4 + m^2} \pm \mu/2, \quad m_{\psi_3} = 0, \\ m^2_{\tilde{A}_{1,2}} &= \mu^2/2 + m^2 + g\lambda \pm \sqrt{(\mu^2 + 2g\lambda)^2/4 + \mu^2 m^2}, \quad m_{A_3} = 0, \\ m^2_{\tilde{B}_{1,2}} &= \mu^2/2 + m^2 - g\lambda \pm \sqrt{(\mu^2 - 2g\lambda)^2/4 + \mu^2 m^2}, \quad A = \tilde{A} + i\tilde{B}. \end{aligned}$$

ii) $|2g\lambda| > m^2$, then

$$\begin{aligned} m^2_{\psi_{1,2}} &= 4g\lambda - m^2, \quad m_{\psi_3} = 0, \quad m_{A_3} = 0, \\ m^2_{A_1} &= 4g\lambda - m^2, \quad m^2_{\tilde{A}_2} = 4g\lambda - 2m^2, \quad m^2_{\tilde{B}_2} = 4g\lambda. \end{aligned}$$

In both cases SUSY is spontaneously broken but $\sum m^2_{boson} = \sum m^2_{fermion}$ as before. The potential and the spectrum are shown in Figs.11,12. The drawback of this mechanism is a lot of arbitrariness in the choice of potential. The mass spectrum also causes some troubles for a phenomenology.

3) A supergravity induced mechanism

This mechanism is based on effective non-renormalizable interactions arising as the low energy limit of supergravity theories. In spite of attractiveness of this mechanism,in general, it is not truly substantiated due to the lack of a consistent theory of quantum (super)gravity.

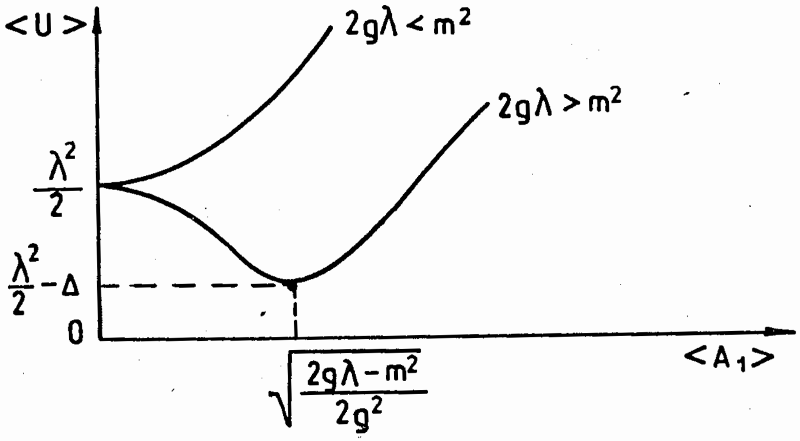

Figure 11: The potential for O'Raifeartaigh model

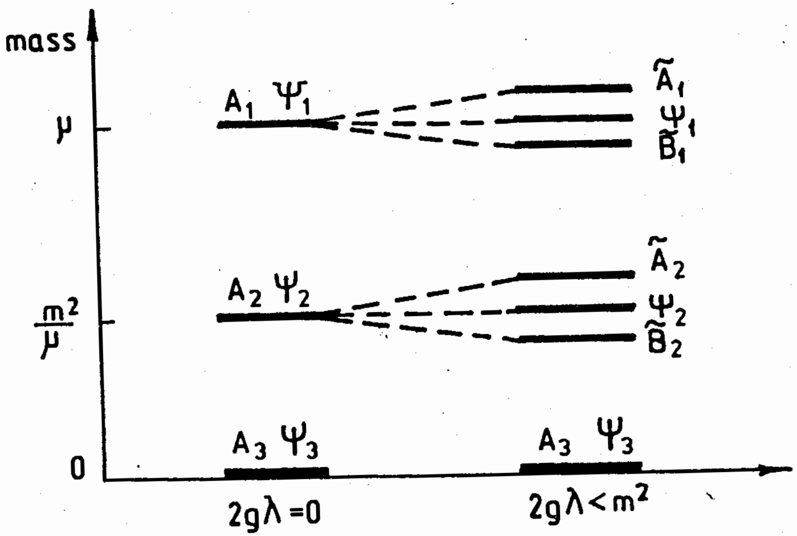

Figure 12: Mass spectrum of O'Raifeartaigh model

3.6. Phenomenology of Global SUSY Theories

As was already mentioned, in SUSY theories the number of bosonic degrees of freedom equals that of fermionic. Let us count these numbers in the SM. In the minimal version the number of bosonic degrees of freedom equals 28, while that of fermionic equals 90. So the SM is in great deal non-supersymmetric. Trying to add some new particles to supersymmetrize the SM, one should take into account the following observations:

i) There is no fermions with quantum numbers of the gauge bosons;

ii) Higgs fields have a non-zero v.e.v., hence they cannot be superpartners of quarks and leptons since this would induce a spontaneous violation of baryon and lepton numbers;

iii) One needs at least two complex chiral multiplets to give masses to Up and Down quarks. This is due to the form of superpotential and chirality of matter superfields. Therefore the Higgs sector is inevitably enlarged.

$$SM: \quad 4 \stackrel{SSB}{\rightarrow} 1 \qquad SUSY: \quad 8 \stackrel{SSB}{\rightarrow} 5$$

Conclusion: In SUSY models supersymmetry associates *known* bosons with *new* fermions and *known* fermions with *new* bosons.

Content of SUSY Particle Theory

Superfield	Spin 1	Spin 1/2	Spin 0
Gauge	Photon Gluons $W^\pm, Z$	Photino Gluinos "Gauginos"	
Higgs		Spin 1/2 partners of Higgs bosons	Higgs bosons
Lepton and Quark		Leptons and Quarks	Spin 0 partners of leptons and quarks

Content of Spontaneously Broken SUSY

Superfield	Spin 1	Spin 1/2	Spin 0
Massless Gauge	Photon Gluons	Photino Gluinos	
Massive Gauge	$W^\pm$ $Wino^\pm, Zino$	Heavy fermions $w^\pm, z$	Higgs bosons
Lepton and Quark		Leptons and Quarks	Spin 0 sleptons and squarks

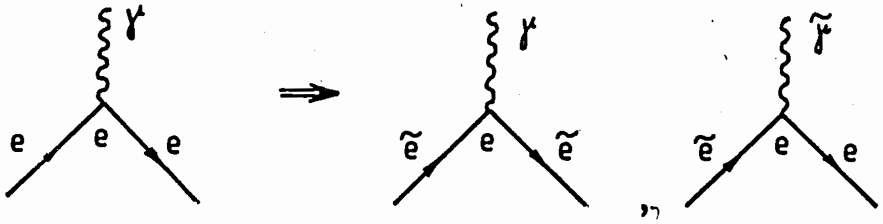

Figure 13: Typical vertices involving superpartners

Characteristic feature of any supersymmetric generalization of the SM is the presence of superpartners of ordinary particles. The absence of them at modern energies is believed to be explained by their masses being very heavy. This means that if the energy will be high enough, the superpartners will be created.

The interactions of superpartners are essentially the same as in the SM but two of three particles involved into an interaction at any vertex are replaced by superpartners. The reason for this is the so-called *R-parity*, defined by

$$R - parity: \quad (-1)^R = (-)^{2s}(-)^{3(B-L)}.$$

For all known particles $R = 0$, whence for all superpartners $R = \pm 1$. A conservation of R-parity has two consequences: superpartners are created in pairs and the lightest one is stable. Usually it is supposed that it is a photino $\tilde{\gamma}$, the superpartner of a photon. A typical vertex is shown in Fig.13. The tilde above a letter denotes a corresponding superpartner. Note that the coupling is the same at all the vertices. The above-mentioned rule together with the Feynman rules for the SM enables us to draw diagrams describing creation of superpartners. One of the most promising processes is e^+e^- annihilation (see Fig.14). Creation of superpartners can be accompanied by creation of the ordinary particles as well.

The decay properties of superpartners depend on their masses. For the quark and lepton superpartners the main processes are shown in Fig.15

Since R-parity is conserved, new particles will eventually end up giving photinos (lightest superparticle) whose interactions are comparable to those of neutrinos and they leave undetected. Therefore the way to notice them is to look for the missing transverse momentum. This is a signature of SUSY in the nearest future experiments.

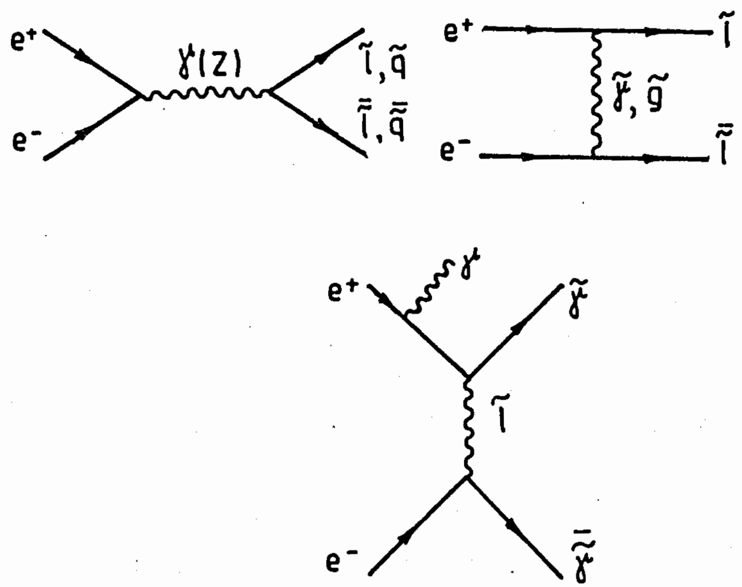

Figure 14: Creation of superpartners in e^+e^- annihilation

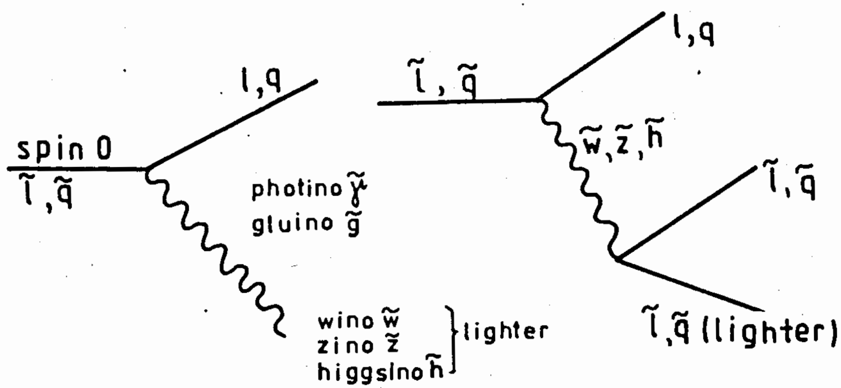

Figure 15: Decay modes of superpartners

Examples. We end up with some explicit examples of superpartners decays.

squarks :	$\tilde{q}$	$\to$	$q+\tilde{\gamma}$	(*quark + photino*)
	$\tilde{q}$	$\to$	$q+\tilde{g}$	(*quark + gluino*)
sleptons :	$\tilde{l}$	$\to$	$l+\tilde{\gamma}$	(*lepton + photino*)
gluino :	$\tilde{g}$	$\to$	$q+\bar{q}+\tilde{\gamma}$	(*quark + antiquark + photino*)
	$\tilde{g}$	$\to$	$g+\tilde{\gamma}$	(*gluon + photino*)
wino :	$\tilde{w}$	$\to$	$e+\nu_e+\tilde{\gamma}$	(*electron + neutrino + photino*)

If $M_W > M_{\tilde{w}} + M_{\tilde{\gamma}}$, then the following processes are also possible

$$W \to \tilde{w}+\tilde{\gamma}, \quad \hookrightarrow \tilde{\gamma}+\nu_e+e \qquad W \to \tilde{e} + \tilde{\nu}_e, \quad \tilde{e}\hookrightarrow l+\tilde{\gamma}, \ \tilde{\nu}_e \hookrightarrow \nu_e+\tilde{\gamma}.$$

Thus if supersymmetry exists in Nature and if it is broken somewhere below 1 TeV, then it will be possible to detect it in the nearest future.

III. Far beyond the Standard Model

4. Grand Unified Theories

4.1. The Idea of GUTs

The philosophy of Grand Unification is based on a *hypothesis*: Gauge symmetry increases with energy. Having in mind unification of all forces of Nature on a common basis and neglecting gravity for the time being due to its weakness the idea of GUTs is the following:

All known interactions are different branches of unique interaction associated with a simple gauge group. The unification (or splitting) occurs at high energy

	Low energy			High energy
$SU_c(3)\otimes$	$SU_L(2)\otimes$	$U_Y(1)$	$\Rightarrow$	G_{GUT} (*or* G^n + discrete symmetry)
gluons	W, Z	photon	$\Rightarrow$	gauge bosons
quarks	leptons		$\Rightarrow$	fermions
g_3	g_2	g_1	$\Rightarrow$	g_{GUT}

At first sight this is impossible due to a big difference in the values of the couplings of strong, weak and electromagnetic interactions. The crucial point here is the running coupling constants. According to the renormalization group equations all the couplings depend on the energy scale ($\alpha_i \equiv g_i^2/4\pi$)

$$RG: \quad \alpha_i = \alpha_i(\frac{Q^2}{\Lambda^2}) = \alpha_i(\text{distance}).$$

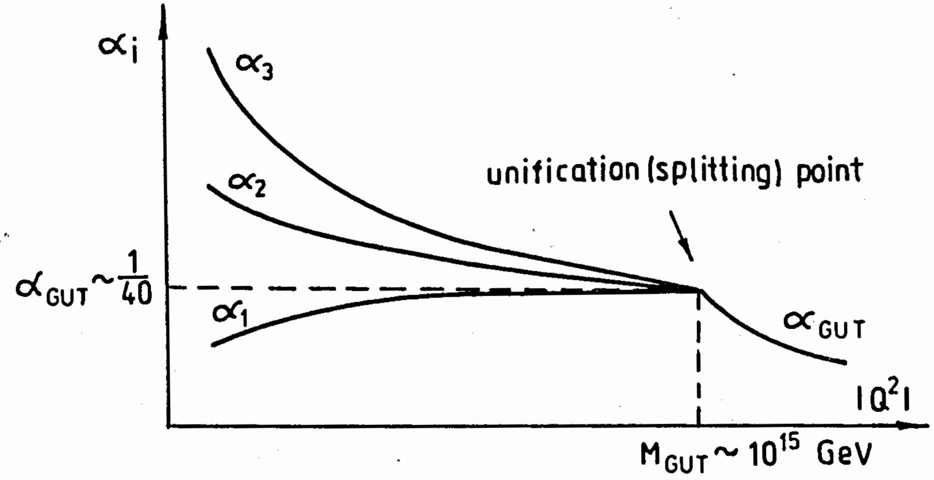

Figure 16: The running coupling constants in GUT scenario

In the SM the strong and weak couplings associated with non-abelian gauge groups decrease with energy, while the electromagnetic one associated with the abelian group on the contrary increases. Thus it becomes possible that on some energy scale they become equal (see Fig.16). According to the GUT idea this equality is not occasional but is a manifestation of unique origin of these three interactions. As a result of spontaneous symmetry breaking, the unifying group is broken and unique interaction is splitted into three branches which we call strong, weak and electromagnetic interactions. This happens at a very high energy of the order of 10^{15} GeV. Of course, this energy is out of the range of accelerators, however, some crucial predictions follow from the very fact of unification.

4.2. General Features of GUTs

While most of the GUT predictions are model dependent, some general features reflect the very idea of a simple gauge group.
i)*Prediction for* $\sin^2\theta_W$. As far as $\sin^2\theta_W$ in the SM is expressed through the ratio of $SU(2)$ and $U(1)$ couplings and at the unification point they are equal, the value of $\sin^2\theta_W$ can be calculated. Indeed, consider a part of the SM Lagrangian

$$g\bar{\psi}\gamma^\mu\tau_3\psi A^3_\mu + g'\bar{\psi}\gamma^\mu\frac{Y}{2}\psi B_\mu$$

In GUT it looks like

$$g_{GUT}(\bar{\psi}\gamma^\mu T_3\psi A^3_\mu + \bar{\psi}\gamma^\mu T_0\psi B_\mu)$$

Comparing these equations we find

$$g = g_{GUT}, \quad \frac{1}{2} g' Y = g_{GUT} T_0.$$

Evaluating now the electric charge operator $Q = T_3 + Y/2$ and having in mind the normalization condition $Tr\ T_i T_j = 1/2\delta_{ij}$ we finally get

$$\frac{Tr\ T_3^2}{Tr\ Q^2} = \frac{g'^2}{g^2 + g'^2} = \sin^2\theta_W.$$

Hence calculating the traces of operators for any representation one can calculate the value of $\sin^2\theta_W$ .We will do it for some particular models below.
ii)*Quantization of electric charge.* If Q belongs to the generators of G_{GUT} which is a compact group, then $Tr\ Q = 0$ for any representation. This means that all the charges are comparable, i.e. are the multiplets of electron charge.
iii)*Baryon No non-conservation.* If quarks and leptons belong to the same irreducible representation of a GUT group, then it would be possible to achieve quark-lepton transitions due to the gauge interactions. They will cause a proton decay, neutron-antineutron oscillations and the other processes with baryon No violation. Fortunately, the rate of proton decay happens to be very small so that the life-time does not contradict the reality.
iv)*Grand desert.* The renormalization group plot shown in Fig.16 is based on an extrapolation of the SM from 100 GeV up to 10^{15} GeV. It is assumed that there is nothing new in this huge energy region, which is usually called the *Grand desert.* However, it could be that the Grand desert is not empty but inhabited by some animals like superpartners of ordinary particles, additional heavy Higgses, etc. This would influence the rate of the running couplings, changing some low energy predictions of GUTs. One should take care of this in model building.

To construct some GUT we have to fulfil the following requirements:

1. Grand unifying group should include the group of the SM , i.e.

$$G_{GUT} \supset SU(3) \otimes SU(2) \otimes U(1)$$

This means that $rank\ G_{GUT} \geq 4$ (recall that $rank\ SU(N) = N - 1$).

2. Multiplets of G_{GUT} should include known quarks and leptons.

3. The theory should be renormalizable and contain no anomalies. Recall that this requirement is fulfilled in the SM by cancellation of anomalies between quarks and leptons of each generation.

4. The scalar multiplets (Higgs fields) should provide spontaneous symmetry breaking in several stages

$$G_{GUT} \underset{M_{GUT}}{\Rightarrow} \cdots \Rightarrow SU(3) \otimes SU(2) \otimes U(1) \underset{M_W}{\Rightarrow} SU(3) \otimes U(1)$$

Alternatively G_{GUT} should include G_{TC} or else.

4.3. SU(5) GUT - Minimal GUT

$SU(5)$ is a minimal group (rank 4) into which $SU(3) \otimes SU(2) \otimes U(1)$ can be embedded and which has complex representations needed for chiral fermions. This group satisfies all the requirements mentioned above. The natural embedding of $SU(3) \otimes SU(2)$ to $SU(5)$ is

$$\left(\begin{array}{ccc|cc} & SU_c(3) & & & \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ & & & SU_L(2) & \end{array}\right)$$

The only generator of $SU(5)$ commuting with $SU(3)$ and $SU(2)$ is the hypercharge

$$Y = \sqrt{\frac{3}{20}} \begin{pmatrix} -2/3 & & & & \\ & -2/3 & & 0 & \\ & & -2/3 & & \\ & 0 & & 1 & \\ & & & & 1 \end{pmatrix},$$

which coincides with that of $U_Y(1)$ of the SM.

Particle content of the $SU(5)$ GUT is the following:

Gauge sector. $W_\mu = W_\mu^A T^A$, $A = 1, 2, \ldots 24$, T^A are the generators of $SU(5)$. It is a $24 -$ *plet* which can be represented as a traceless 5×5 matrix

$$W_\mu = \left(\begin{array}{ccc|cc} & & & X_\mu^1 & Y_\mu^1 \\ G_\mu^a \frac{\lambda^a}{2} & - & \frac{1}{\sqrt{15}} B_\mu 1_3 & X_\mu^2 & Y_\mu^2 \\ & & & X_\mu^3 & Y_\mu^3 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ X_\mu^{*1} & X_\mu^{*2} & X_\mu^{*3} & \frac{1}{2} A_\mu^3 + \sqrt{\frac{3}{20}} B_\mu & W_\mu^+ \\ Y_\mu^{*1} & Y_\mu^{*2} & Y_\mu^{*3} & W_\mu^- & -\frac{1}{2} A_\mu^3 + \sqrt{\frac{3}{20}} B_\mu \end{array}\right)$$

Among 24 gauge bosons there are 8 gluons G_μ^a, 3 weak bosons $W_\mu^\pm$ and A_μ^3 and 1 $U(1)$ boson B_μ. There are also 12 *new* fields X_μ and Y_μ. They are usually called lepto-quarks because they mediate lepto-quark transition leading to baryon No violation. The gauge multiplet has the following $SU(3)\otimes SU(2)$ decomposition

$$\underline{24} = \underset{gluons}{(\underline{8},\underline{1})} \quad \underset{W\ and\ Z}{+(\underline{1},\underline{3})} \quad \underset{leptoquarks}{+(\underline{3},\underline{2})+(\underline{3},\underline{2})}$$

Fermion sector. All fermions are taken to be left-handed. Right-handed particles are replaced by the corresponding left-handed conjugated. The minimal fundamental representation of $SU(5)$ is $\underline{5}$. However it is more convenient to use the conjugated one which has appropriate $SU(3)\otimes SU(2)\otimes U(1)$ quantum numbers

$$\underline{5}^* = (\underline{3},\underline{1},-2/3)+(\underline{1},\underline{2},1)$$

It is naturally identified with d-quark and electron-neutrino doublet

$$\underline{5}^* = (d_1^c, d_2^c, d_3^c, e^-, \nu_e)_{Left}$$

To find place for the other members of the same family, we have to go beyond the fundamental representation. Surprisingly, the next (after $\underline{5}$) representation, $\underline{10} = (5\times 5)_{asym}$ has precisely correct quantum numbers

$$\underline{10} = (\underline{3},\underline{2},1/3)+(\underline{3}^*,\underline{1},-4/3)+(\underline{1},\underline{1},-2)$$

It is a 5×5 antisymmetric matrix and its fermion assignment is

$$\underline{10} = \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ & 0 & u_1^c & u_2 & d_2 \\ & & 0 & u_3 & d_3 \\ & & & 0 & e^+ \\ & & & & 0 \end{pmatrix}_{Left}, \qquad \begin{matrix} u_L^c \to u_R \\ e_L^+ \to e_R \end{matrix}.$$

Thus, all known fermions exactly fit to $(\underline{5}^*+\underline{10})$ representations of $SU(5)$. Now new fermions appear. Note that there is no room for the right-handed neutrino ν_R. Hence either neutrino is massless in the $SU(5)$ model or it could be a singlet that does not take part in gauge interaction. In spite of the left-right asymmetry of the model there is no anomalies in the gauge currents. They automatically cancel between contributions of $\underline{5}^*$ and $\underline{10}$.

We can now follow all the general features of GUTs with an example of the minimal $SU(5)$ model. Looking at the electromagnetic current

$$\bar{e}_L\gamma^\mu[-\frac{g_{GUT}}{2}A_\mu^3 - g_{GUT}\sqrt{\frac{3}{20}}B_\mu]e_L$$

and comparing it with the SM , we find

$$g = g_{GUT},\quad g' = \sqrt{\frac{3}{5}}g_{GUT},\quad \tan\theta_W = \frac{g'}{g} = \sqrt{\frac{3}{5}} \quad \text{or} \quad \sin^2\theta_W = \frac{3}{8}.$$

Note, however, that this is true only at $E = E_{GUT} \sim 10^{15}$ *Gev*. Charge quantization is also straightforward. From the requirement $Tr\ Q = 0$ under the assumption that $SU_{Colour}(3)$ is exact we obtain

$$Q_d = \frac{1}{3}Q_e, \quad Q_u = -\frac{2}{3}Q_e.$$

It is interesting to estimate the value of the meeting point E_{GUT}. For this purpose consider the RG equations for the running couplings in the SM assuming that no new particles exist. To the one-loop order we have

$$\begin{aligned} \mu\frac{d}{d\mu}g_3 &= -\frac{g_3^3}{16\pi^2}(11-\frac{2}{3}f), \\ \mu\frac{d}{d\mu}g_3 &= -\frac{g_2^3}{16\pi^2}(\frac{22}{3}-\frac{2}{3}f), \\ \mu\frac{d}{d\mu}g_1 &= \frac{g_1^3}{16\pi^2}\frac{2}{3}f, \end{aligned} \tag{4.1}$$

where the couplings g_3, g_2 and g_1 belong to the gauge groups $SU(3), SU(2)$ and $U(1)$,respectively, and f is the number of flavours. Solving these equations and imposing the boundary conditions $g_1 = g_2 = g_3 = g_{GUT}$ at $\mu = M_{GUT}$, we get the values of M_{GUT}, g_{GUT} and $\sin^2\theta_W$. Taking the values of α_s and α_{EM} as input we find (note that $e_{EM}^2 = \frac{g'^2}{g^2+g'^2}$)

$$\begin{aligned} \ln\frac{M_{GUT}}{M_W} &= \frac{\pi}{11}(\frac{1}{\alpha_{EM}} - \frac{8}{3}\frac{1}{\alpha_s(M_W)}), \\ \sin^2\theta_W &= \frac{1}{6}+\frac{5}{9}\frac{\alpha_{EM}}{\alpha_s(M_W)}. \end{aligned}$$

This gives

$$M_{GUT} \simeq 10^{15}\ Gev, \quad \sin^2\theta_W(M_W) \simeq 0,214, \quad \alpha_{GUT} \simeq \frac{1}{40}.$$

It should be stressed once more that the value of $\sin^2\theta_W$ strongly depends on energy scale. The variation of $\sin^2\theta_W$ with the energy is shown in Figs.17,18.

One of the successes of the minimal $SU(5)$ model is the prediction of $\sin^2\theta_W$, which is in a very good agreement with experimental data.

Spontaneous symmetry breaking in the $SU(5)$ model occurs in two stages. Within the Higgs mechanism of SSB one introduces two Higgs multiplets: $\underline{24}$ and $\underline{5}$. The v.e.v are chosen to be

$$<\Phi_{24}> = \begin{pmatrix} V & & & & \\ & V & & & \\ & & V & & \\ & & & -3/2\ V & \\ & & & & -3/2\ V \end{pmatrix}, \qquad V \sim M_{GUT} \sim 10^{15}\ Gev,$$

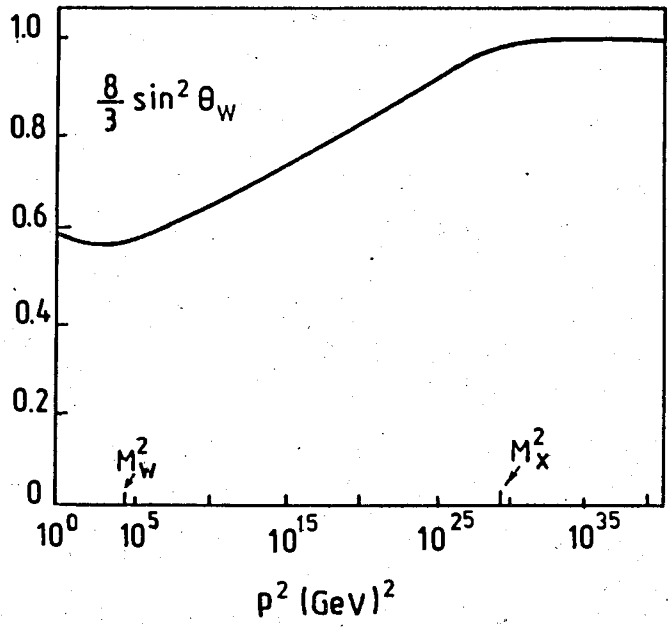

Figure 17: The variation of $\sin^2\theta_W$ with energy scale

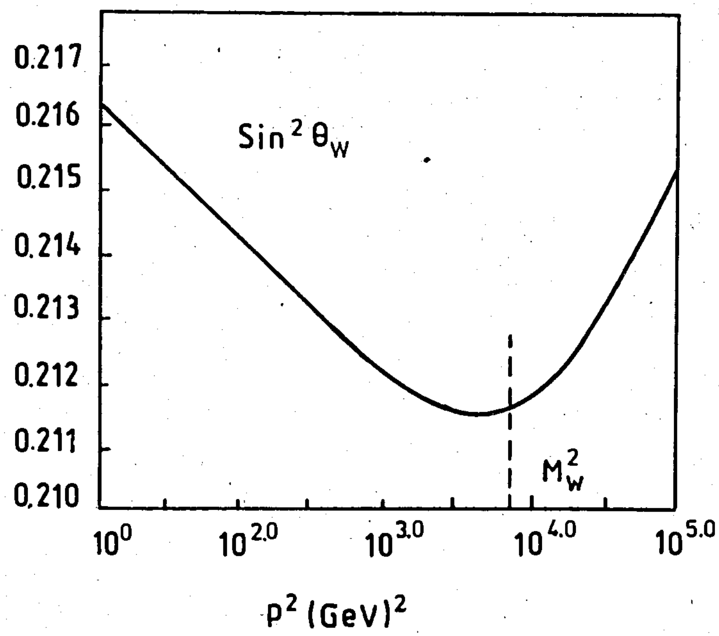

Figure 18: Expanded view near M_W

which breaks $SU(5)$ down to $SU(3)\otimes SU(2)\otimes U(1)$ and

$$< H_5 >= \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v/\sqrt{2} \end{pmatrix}, \qquad v \sim 250 \text{ GeV},$$

which breaks $SU(3)\otimes SU(2)\otimes U(1)$ down to $SU(3)\otimes U(1)$.

4.4. Hierarchy Problem

The appearance of two different scales $V \gg v$ leads to a very serious problem for GUTs, which is called the *hierarchy problem.* There are two aspects of this problem.

The first one is the very existence of hierarchy. To get the desired spontaneous symmetry breaking pattern, we need

$$\begin{matrix} m_H & \sim & v & \sim & 10^2 \text{ GeV} \\ m_\Phi & \sim & V & \sim & 10^{15} \text{ GeV} \end{matrix} \qquad \frac{m_H}{m_\Phi} \sim 10^{-13} \ll 1. \tag{4.2}$$

The question arises how to get a so small number in a natural way. One needs some kind of fine tuning in a theory, and we don't know is there anything behind it.

The second aspect of the hierarchy problem is connected with the preservation of a given hierarchy. Even if we choose the hierarchy like eq.(4.2) the radiative corrections will destroy it! To see how this happens, we consider the interaction of Higgs fields. Using the notation $\Phi_{24} \equiv \Phi, H_5 \equiv H$ we write down the interaction terms. They are

$$\alpha(H^\dagger H)^2, \quad \beta Tr\ \Phi^4, \quad \gamma(Tr\ \Phi^2)^2.$$

There is no direct interaction between heavy (Φ) and light (H) Higgs particles. However, because of the ultraviolet divergences in the diagrams like that shown in Fig.19 and the necessity of renormalization, the corresponding counterterms appear:

$$\Delta\mathcal{L} = \delta H^\dagger \Phi^2 H.$$

As a result, the radiative corrections to the light Higgs field mass become inevitable. Corresponding diagrams are shown in Fig. 20. Another source of mass corrections is the interaction with heavy gauge bosons (see Fig.21). We see that the mass corrections happen to be much larger than the masses themselves thus violating the chosen hierarchy. The way out of this trouble is the cancellation of radiative corrections. This very accurate cancellation with a precision $\sim 10^{-13}$ also needs a fine tuning of the coupling constants.

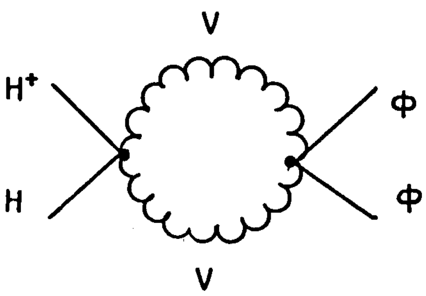

Figure 19: Divergent graph leading to the interaction between light and heavy Higgs fields

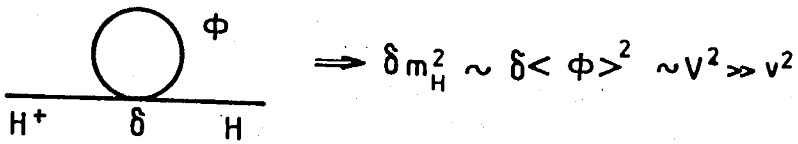

Figure 20: The diagram contributing to the light Higgs field mass due to the interaction with heavy Higgs particles

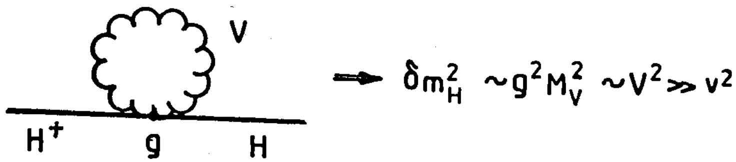

Figure 21: The diagram contributing to the light Higgs field mass due to the interaction with heavy gauge bosons

It can be naturally achieved in SUSY models. That is why it is usually said that supersymmetry solves the hierarchy problem. However, we see that it solves only the second aspect of the problem. The origin of the hierarchy remains unclear.

4.5. Non-Minimal Models

Before considering the other Grand Unified Theories let us focus our attention on the problems of the minimal $SU(5)$ GUT. They are:

1. Fermions of the same generation belong to different representations $(\underline{5}^* + \underline{10})$;
2. The mass matrix for quarks and leptons is unsatisfactory $(m_e/m_\mu = m_d/m_s = \cdots)$;
3. $B - L$ conservation depends on a Higgs sector and looks occasional;
4. Neutrino is massless;
5. Proton life-time is too small;
6. There exists a Grand desert for $E = 10^2 \div 10^{15}$ GeV.

Trying to solve some of these problems, we briefly consider some more complicated models.

SO(10) *GUT*

For the $SO(10)$ group, which is a next possible group of rank 5 , all the fermions of the same generation belong to a single irreducible representation $\underline{16}$

$$\underline{16} = (u_1\ u_2\ u_3\ d_1\ d_2\ d_3\ \nu_e\ e^-\ u_1^c\ u_2^c\ u_3^c\ d_1^c\ d_2^c\ d_3^c\ \nu_e^c\ e^+)_{Left}$$

Note that contrary to the $SU(5)$ model the right-handed neutrino (left-handed antineutrino) is present now. This means that the neutrino in the $SO(10)$ model is massive.

The symmetry breaking in the $SO(10)$ model can be achieved in two different ways and needs at least three different scales $M_1 \gg M_2 \gg \cdots M_W$

$$SO(10) \overset{M_1}{\begin{array}{l}\nearrow\ SU(5) \overset{M_2}{\longrightarrow} SU(3)\otimes SU(2)\otimes U(1) \overset{M_W}{\longrightarrow} SU(3)\otimes U(1)\\ \searrow\ SO(6)\otimes SO(4) \sim SU(4)\otimes SU_L(2)\otimes SU_R(2)\end{array}}$$

If one chooses an $SU(5)$ chain, then $SO(10)$ multiplets find their natural decomposition in terms of that of $SU(5)$

$$\underline{16} = \underline{5}^* + \underline{10} + \underline{1} \qquad fermions,$$
$$\underline{45} = \underline{24} + \underline{10} + \underline{10}^* + \underline{1} \qquad gauge\ bosons.$$

In $SO(10)$ GUT we solve the problems *No* 1,3,4,5 and 6 of the above mentioned list. However, the price for this is a more complicated model with a more freedom but a less predictive power.

E(6) GUT

This model is based on the exceptional group $E(6)$ of rank 6. It is left-right symmetric

$$E(6) \supset SU_C(3) \otimes SU_L(3) \otimes SU_R(3).$$

Fermions belong to a single fundamental representation $\underline{27}$ which has the following decomposition under $SO(10)$

$$\underline{27} = \underline{16} + \underline{10} + \underline{1},$$

while the gauge bosons form an adjoint representation $\underline{78}$.

The model contains a lot of new particles. Its attractiveness is mainly due to appearance of $E(6)$ GUT in superstring inspired models (see below).

SUSY SU(5) GUT

Another extension of the minimal $SU(5)$ model is SUSY $SU(5)$. In the minimal version the matter content of the theory is the following:

$$\begin{array}{rll} \text{Matter superfields}: & \underline{5}^* & \psi^i \quad i,j = 1,2,\dots 5 \\ & \underline{10} & \psi_{ij} \\ \text{Higgs superfields}: & \underline{5} & H_i \\ & \underline{5}^* & \bar{H}^i \\ & \underline{24} & \Phi_{ij} \end{array}$$

The superpotential contains several terms:

$$\left.\begin{array}{c} \psi_{ij}\psi_{kl}H_m\epsilon^{ijklm} \\ \psi^i\psi_{ij}\bar{H}^j \end{array}\right\} \text{ needed for } SU(2) \text{ breaking and to give masses to quarks and leptons}$$

$$Tr\ \Phi^2 \quad Tr\ \Phi^3 \Big\} \text{ needed for } SU(5) \text{ breaking and to give masses to leptoquarks.}$$

As was already mentioned, the number of fields here is larger than in the minimal $SU(5)$ model. However due to supersymmetry an interaction between light $(H, \bar{H})$ and heavy Φ Higgs fields is absent, and as a result, the hierarchy is preserved.

In conclusion we list some properties of various GUT models.

Model	$SU(5)$	$SO(10)$	$E(6)$
Automatic absence of anomalies	-	+	+
One irreducible representation	-	+	+
Left-Right Symmetry	-	+	+
Ambiguities in $\sin^2\theta_W$ due to different patterns of SSB	-	+	+
m_ν	0	$\sim \frac{M_W^2}{M_X}$	$\sim \frac{M_W^2}{M_X}$
Possibility of dynamical SSB	-	-	+

4.6. Phenomenological Consequences of GUTs

Looking for phenomenological consequences of GUTs at low energies, one should take into account the existence of a large variety of models. The GUT group is not a priori known. This puts the severe problem of choice. Moreover, even if the group is chosen,in all cases except $SU(5)$ there exist several patterns of spontaneous symmetry breaking. This also contributes to the arbitrariness of GUT predictions. However, some phenomenological properties like proton decay, the presence of monopoles, the problem of fermion mass spectrum are common to all GUT models.

Proton Decay

The leptoquarks X_μ and Y_μ present in any GUT model mediate quark-lepton transitions thus leading to baryon No non-conservation. In $SU(5)$ we have the following interaction vertices

$$\underline{5}^*: \quad g\frac{1}{\sqrt{2}}X^i_\mu \bar{d}_R \gamma^\mu e^+_R$$

$$\underline{10}: \quad g\frac{1}{\sqrt{2}}\epsilon_{ijk}X^i_\mu \bar{u}^{cj}_L \gamma^\mu u^k_L \, .$$

The corresponding diagrams are shown in Fig.22. A combination of these two vertices leads to proton decay according to leptoquark exchange process shown in Fig.23. As suggested by Fig.23, the dominant mode is

$$p^+ \quad \rightarrow \quad e^+\pi^0. \tag{4.3}$$

The amplitude is proportional to g^2_{GUT}/M^2_X, where $M_X \sim M_{GUT}$. Hence the proton life-time behaves like M^4_{GUT}/g^4_{GUT}. Then from dimensional considerations one has

$$\tau_p = C(\frac{M_{GUT}}{M_p})^4 \frac{1}{\alpha^2_{GUT}} \frac{1}{M_p},$$

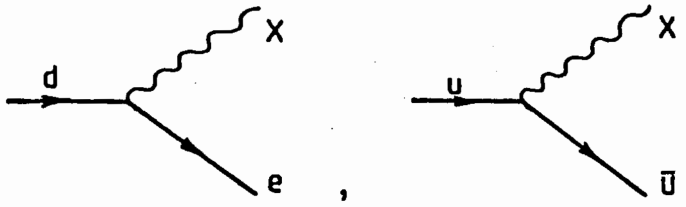

Figure 22: Leptoquark interaction vertices

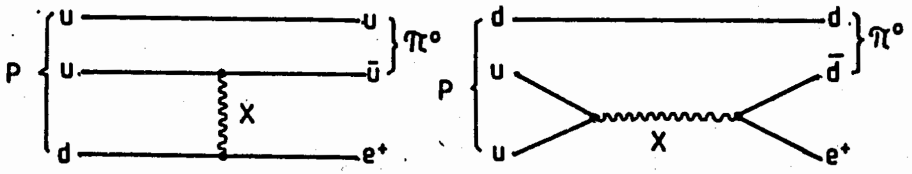

Figure 23: Proton decay diagrams due to X boson exchange

where M_p is the proton mass. A rough estimate is

$$M_{GUT} \sim 10^{14}\ Gev, \quad \alpha_{GUT} \sim 10^{-2}, \quad \tau_p \sim 10^{36} \text{ sec} \sim 10^{29} \text{ years}.$$

A more accurate calculation taking into account the structure of the proton gives, with some uncertainties

$$\tau_p = (0.2 \div 10) \cdot 10^{28} \left(\frac{M_{GUT}}{10^{14} Gev}\right)^4 \quad \text{years}.$$

With account of some ambiguities in determining of M_{GUT} the predicted life-time is

$$\tau_p \ < \ 2 \cdot 10^{31} \quad \text{years},$$

while the modern experimental limit on the decay mode (4.3) is

$$\tau_p \ > \ 3 \cdot 10^{32} \quad \text{years}.$$

This seems to rule out the minimal $SU(5)$ GUT. Predictions for other models strongly depend on the particle content, mainly, on the number of light Higgs bosons. Some results are listed below.

Minimal $SU(5)$	$\tau_p \sim 10^{27} \div 10^{31}$ *years*
	$(N \to e^+\pi) : (N \to \bar{\nu}\pi) : (N \to \mu^+ K) = 1 : 0.2 : 0.1$
Modified $SU(5)$	$\tau_p \sim 10^{31} \div 10^{34}$ *years*
SUSY $SU(5)$	$\tau_p \sim 10^{31} \div 10^{32}$ *years*
SUSY $SU(5)$	$\tau_p \sim 10^{30} \div 10^{34}$ *years*
with Higgs mediated proton decay	$(N \to \bar{\nu}K, \mu^+ K) : (N \to \mu^+\pi, e^+K) : (N \to e^+\pi, \bar{\nu}\pi) = 1 : 0.1 : 0.01$

Monopoles

One of the model-independent predictions of GUTs is the presence of monopoles. The reason is that for any semisimple gauge group G which is broken to a subgroup $H \subset G$ containing an $U(1)$ factor $H = h \otimes U(1)$, there exist topologically non-trivial stable configurations of gauge and/or Higgs fields - monopoles.

In our case the GUT group is broken in exactly the needed way

$$G_{GUT} \overset{M_{GUT}}{\Rightarrow} \quad SU(3) \otimes SU(2) \otimes U(1).$$

At large distances the magnetic field of such configurations behaves like that of the Dirac monopole with magnetic charge $g^* = \frac{2\pi}{e}$. The mass of the monopole is

$$M_{Mon} \sim \frac{M_X}{\alpha} \sim 10^{16} \div 10^{17}\,\text{GeV}.$$

Being so heavy the monopoles cannot be produced at accelerators, however, they could have been created at the early stage of the Universe. Hence as far as they are stable, one may hope to detect them now. One of the characteristic processes to look for monopoles is the monopole-catalysed proton decay

$$\begin{aligned} p + Mon &\rightarrow e^+ + Mon\ (+pions), \\ p + Mon &\rightarrow e^+ + \mu^+\mu^- + Mon\ (+pions). \end{aligned}$$

The cross-section of the process is of the order of $(1\text{GeV})^{-2}$ while the probability of proton decay in the presence of a monopole depends on the monopole flux. However, magnetic monopoles are not yet found in Nature.

One of the crucial problems of the SM which is not solved in GUTs is the origin of the fermion spectrum. Practically, in all GUTs the situation with fermion masses is the same

$$\begin{array}{ll} E \sim M_{GUT} & E \sim M_W \\ m_d = m_e & \\ m_s = m_\mu & \dfrac{m_e}{m_\mu} = \dfrac{m_d}{m_s}, \cdots \\ m_b = m_\tau & \end{array}$$

The last relation evidently contradicts the experiment

$$\frac{m_e}{m_\mu} \sim \frac{1}{200}, \qquad \frac{m_d}{m_s} \sim \frac{1}{20}.$$

Solution of this problem requires a new insight in the symmetry breaking mechanism.

In conclusion, we mention some of the problems which have found their solution within GUTs or on the contrary have been put forward by GUTs:

⋆ Group and representation	?
⋆ Number of generations	?
⋆ Unique gauge coupling	+
⋆ Prediction of $\sin^2\theta_W$	+
⋆ Quantization of charge	+
⋆ Quark and lepton masses	?
⋆ Mixing matrix	?
⋆ Hierarchy problem	?

5. Supergravity, Superstrings, etc.

Before going further towards higher energy, let us first have a look at the high energy physics panorama from the present point of view (see Fig.24).

What is shown here is a simplified picture of what we understand today about high energy physics and how we imagine the microworld structure at very small distances.

In this section we are going to discuss possible physics beyond the Planck scale (10^{19} *Gev* or 10^{-33} *cm*). This is the region where gravity becomes essential and comparable in strength with other interactions. So, one cannot ignore it any more. Because of lack of the quantum theory of gravity due to ultraviolet divergences and much softer ultraviolet behaviour of supersymmetric theories, the hopes are associated with a supersymmetrical generalization of gravity - the supergravity.

5.1. Supergravity

Supergravity (SUGRA) is the theory of *local* supersymmetry. The motivations for SUGRA are purely theoretical:

i) All fundamental symmetries in Nature are local except supersymmetry. It is natural to gauge supersymmetry as well.

ii) Superalgebra contains translations

$$[\bar{\varepsilon}Q, \bar{Q}\varepsilon] = 2\bar{\varepsilon}\sigma^\mu\varepsilon P_\mu.$$

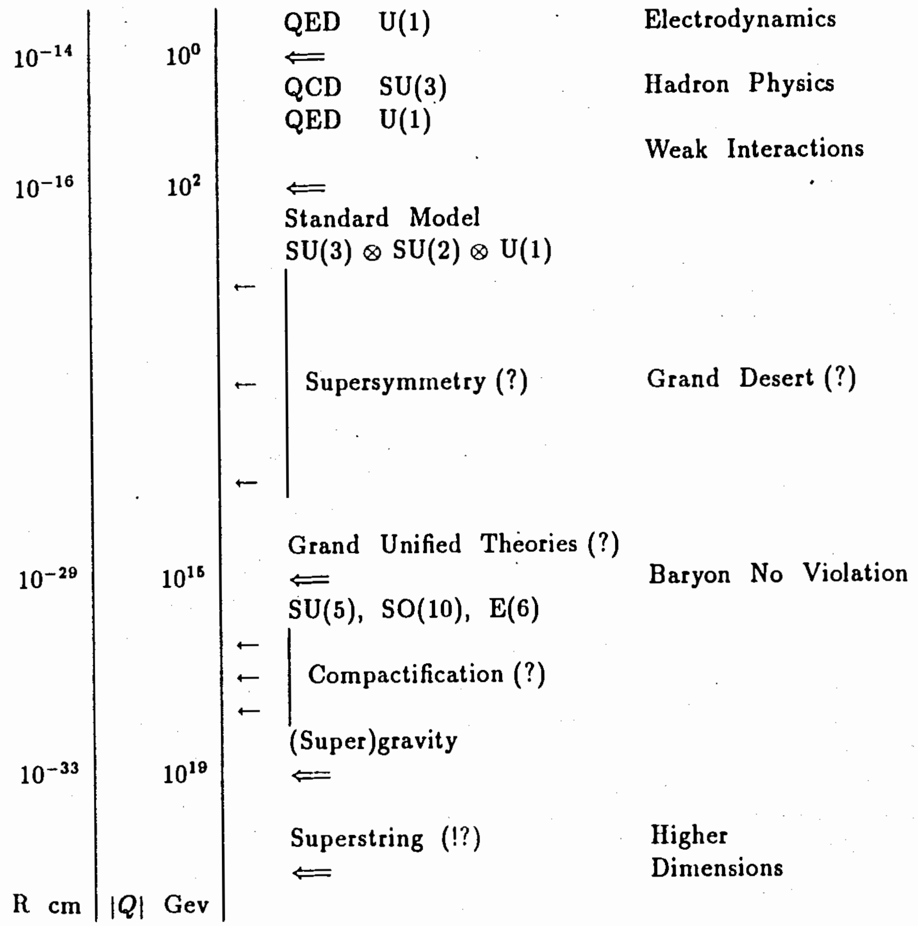

Figure 24: High energy physics panorama

If they are local, i.e. $\varepsilon = \varepsilon(x)$, we get local translational invariance with a parameter

$$a_\mu = \bar{\varepsilon}\sigma^\mu \varepsilon P_\mu.$$

But the theory invariant under the general coordinate transformation is General Relativity. Hence, local SUSY is the theory of (super)gravity.
iii) In SUSY GUTs the unification scale approaches the Planck scale where gravity cannot be neglected.
iv) Ultraviolet behaviour of SUGRA is much better than in ordinary gravity.

Anyhow much hopes in construction of quantum gravity have been connected with the development of SUGRA.

As is well known, gauging any symmetry leads to the appearance of appropriate gauge fields. They are proportional to the derivatives of local gauge parameters and have the corresponding quantum numbers:

gauge parameter		gauge field
$\theta \to \theta(x)$	$\Rightarrow$	$v_\mu(x) \sim \partial_\mu \theta(x)$
scalar (spin 0)		vector (spin 1)
$a_\nu \to a_\nu(x)$	$\Rightarrow$	$g_{\mu\nu}(x) \sim \partial_\mu a_\nu(x)$
vector (spin 1)		tensor (spin 2)
$\varepsilon^i_\alpha \to \varepsilon^i_\alpha(x)$	$\Rightarrow$	$\psi^i_{\mu\alpha} \sim \partial_\mu \varepsilon^i_\alpha(x)$
spinor (spin 1/2)		spin-tensor (spin 3/2)

Thus, gauging supersymmetry leads to appearance of a new particle of spin 3/2 called the *gravitino*, which is a superpartner of the graviton. This is a crucial prediction of the SUGRA theory. Gravitinos are always present in any SUGRA model playing an essential role in anomaly and UV divergences cancellation.

N = 1 Supergravity

This is the simplest version of SUGRA. It has only one supersymmetry generator, and hence, only one gravitino field. The particle content of the model is the following:
- One spin 2 state $g_{\mu\nu}$ - graviton,
- $N = 1$ spin 3/2 state $\psi_{\mu\alpha}$ - gravitino,
- Matter supermultiplets.

The Lagrangian of pure $N = 1$ SUGRA consists of two parts. The first one is the General Relativity Lagrangian

$$\mathcal{L}_G = -\frac{1}{2K^2}\sqrt{-g}R = \frac{1}{2K^2}eR, \qquad (5.1)$$

where $g_{\mu\nu}$ is the metric tensor, $g = det\, g_{\mu\nu}$, R is a scalar curvature, $K^2 = G$ is the Newton gravitational constant and e^m_μ is a vierbein defined by

$$g_{\mu\nu} = e^m_\mu e^n_\nu \eta_{mn},$$

η_{mn} being the Minkowski metric tensor.

The second part is the Rarita-Shwinger Lagrangian for the spin-tensor gravitino field

$$\mathcal{L}_{RS} = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu \gamma_5 \gamma_\nu D_\rho \psi_\sigma, \qquad (5.2)$$

where the covariant derivative

$$D_\rho = \partial_\rho + \frac{1}{2}\omega^{mn}_\rho \gamma_{mn}, \qquad \gamma_{mn} = \frac{1}{2}[\gamma_m, \gamma_n]$$

and ω^{mn}_ρ is the spin connection expressed in terms of the vierbein e^m_μ.

The total Lagrangian

$$\mathcal{L}_{SUGRA} = \mathcal{L}_G + \mathcal{L}_{RS} \qquad (5.3)$$

is invariant under *local* SUSY transformations

$$\begin{aligned} \delta e^m_\mu &= \frac{K}{2}\bar{\varepsilon}(x)\gamma^m \psi_\mu, \\ \delta\omega^{mn}_\mu &= 0, \\ \delta\psi_\mu &= \frac{1}{K}D_\mu \varepsilon(x) = \frac{1}{K}(\partial_\mu + \frac{1}{2}\omega^{mn}_\mu \gamma_{mn})\varepsilon(x). \end{aligned} \qquad (5.4)$$

SUSY algebra (5.4) is closed only "on-shell". For the "off-shell" formulation like in global SUSY theories, a set of auxiliary fields is needed.

To construct a realistic model, one should add to Lagrangian (5.1 - 5.3) the terms describing a supersymmetric matter. We will not do it here because of complexity of formulae.

As was mentioned above, the quantum theory of gravity is non-renormalizable. In pure gravity the divergences are cancelled only in the one-loop order on mass shell. If matter fields are added, these cancellations take place only in supergravity. However, even there cancellation fails in higher loops. Possible improvement of the UV behaviour is associated with extended supergravities. Maximally extended $N = 8$ SUGRA is believed to be finite up to eight loops. Unfortunately, we have no complete formulation of extended supergravities. Presumably they require an infinite number of auxiliary fields.

From the modern point of view supergravity as well as super-gauge models are only effective theories being the low energy limits of superstrings.

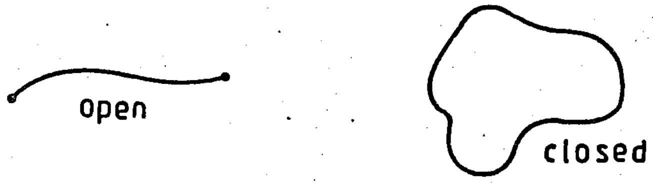

Figure 25: Examples of open and closed strings

5.2. Superstrings

The superstring theory is the most ambitious theory in particle physics. Two years ago it was sometimes called the *Theory of Everything*. It pretends to describe the origin of particle physics being the ultimate fundamental theory.

What is it, superstring? The basis of a superstring is the classical relativistic one-dimensional object - a string. Like an ordinary violin string, a superstring has a sequence of vibrational modes. A single string possesses an infinite series of such normal modes, i.e. an infinite series of massive states in local quantum field theory. These normal modes are identified with the ordinary particles. The mass spectrum is quantized with

$$\Delta m^2 \sim T,$$

where T is the string tension, the only dimensional parameter of the string theory. In superstring models it is supposed that $T \sim (10^{19}\ \mathrm{GeV})^2$.

The strings could be *open* or *closed* as shown in Fig. 25. Examining the spectrum of string states, we find that for an open string the massless states contain spin 1 bosons which are identified with the gauge bosons of a corresponding local gauge group. For a closed string such bosons have spin 2 and, therefore, are identified with gravitons. As far as due to the string interactions these two types of strings are transformed into one another, the gravitational and gauge interactions become intimately connected having the same origin. This way, in a string theory a unification of all forces takes place.

There exist now several string theory models. Only few of them are mathematically self-consistent. One of the problems is the appearance of tachyons. To avoid unphysical tachyon states, supersymmetry is needed. We present here, for illustration, the spectrum of one of the most popular models, the heterotic string model (see Fig.26).

For $p \ll 10^{19}$ GeV or $x \ll 10^{-33}$ cm all the massive states "decouple" leaving us with effective point-like field theory of (super)gravity and

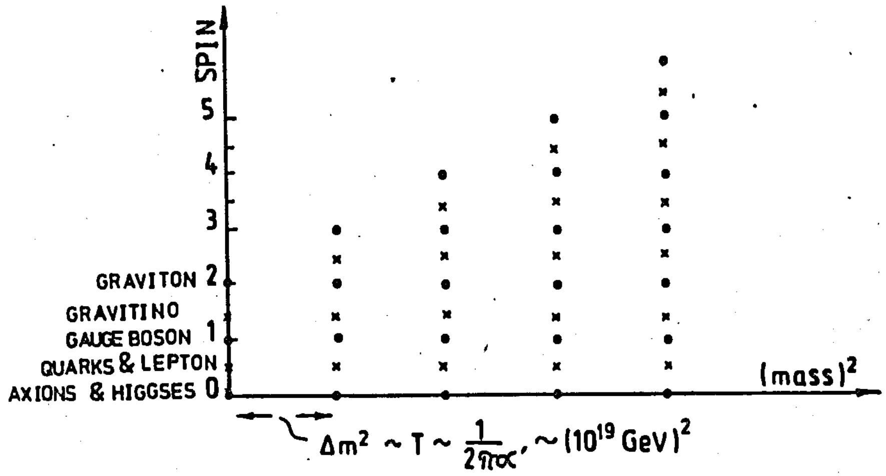

Figure 26: Spectrum of the Heterotic string model

(super)Yang-Mills with fixed parameters and particle content. Observed particles (quarks, leptons, gauge bosons, etc.) should be among massless excitations ($m \ll 10^{19}$ GeV).

(Super)String Dynamics

The dynamics of a string theory is described by the *least action principle.* In analogy with a point-like particle, where the action S is the length of a world line, for the string the action is the square of the world sheet (see Fig. 27). The action for a string is essentially that of the two-dimensional σ-model

$$S = -\frac{1}{4\pi\alpha'}\int d\sigma d\tau \eta_{\mu\nu}\sqrt{-g}g^{\alpha\beta}\partial_\alpha x^\mu \partial_\beta x^\nu \ + \ \theta - terms,$$

where $\eta_{\mu\nu}$ is the metric of D-dimensional "target space", $g_{\alpha\beta}$ is the metric of the world sheet, $\alpha, \beta = \sigma, \tau$. This is the so-called first quantized approach to the string theory.

To define a viable quantum field theory, some conditions should be imposed:

i) Modular invariance (reparametrization invariance of the world sheet);
ii) Conformal invariance (the underlying theory is a 2-D conformally invariant field theory);
iii) Absence of anomalies.

The absence of conformal anomalies puts a strong constraint on a target space:

$$D \ = \ 26 \quad \textit{for bosonic string},$$

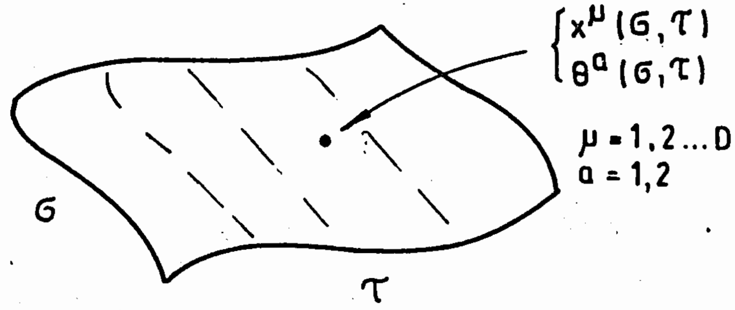

Figure 27: Superstring world sheet

$$D = 10 \quad for\ fermionic\ or\ superstring.$$

Now it is realized that the additional dimensions may correspond to internal degrees of freedom which eventually give rise to the gauge and other symmetries observed in the $D = 4$ world (remind the Kaluza - Klein idea).
iv) Supersymmetry (provides the absence of tachyons and finiteness(?) of a superstring model).

The Kaluza -Klein Idea

The necessity of considering higher space-time dimensions in the superstring approach renewed the interest in the Kaluza-Klein idea of 50 year ago. To begin with, we consider the simplest case.

Imagine the General Relativity but in 5 dimensions. The interval now is

$$ds^2 = \hat{g}_{MN}\, dx^M dx^N,$$

where $x^M = (x^\mu, y)$, $\mu = 1,2,3,4$. The $D = 4$ interpretation of 5-D metric is the following:

$$\begin{array}{lllll} \hat{g}_{\mu\nu} = g_{\mu\nu}(x) & tensor & spin\ 2 & graviton & \\ \hat{g}_{\mu 5} = A_\mu(x) & vector & spin\ 1 & photon & (5.5) \\ \hat{g}_{55} = \phi(x) & scalar & spin\ 0 & dilaton & \end{array}$$

If so, then Einstein equations in $D = 5$ give Einstein + Maxwell + Klein-Gordon equations in $D = 4$. Thus, 5-D gravity theory describes gravity plus the $U(1)$ gauge theory (electrodynamics) plus the scalar field theory in $D = 4$. In the framework of this approach the gauge invariance (charge conservation) is a consequence of general coordinate invariance (energy conservation), i.e. one has a remarkable unification of gravity with electromagnetism.

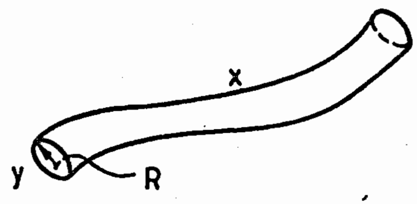

Figure 28: A hose-pipe. Example of the object with one compact dimension

A natural question arises: what is the meaning of the fifth dimension? Why it is not observed yet? To answer these questions, let us suppose that the topology of space is

$$R^4 \times S^1,$$

where R^4 is the Euclidean (Minkowski) 4-D space and S^1 is one-sphere or a circle. This means that the fifth dimension is *compact*

$$-\infty < x^\mu < \infty, \quad 0 \le my \le 2\pi,$$

where $m \sim 1/R$ and R is the radius of compactification (see Fig.28). Performing the Fourier expansion in the fifth coordinate we have

$$\hat{g}_{MN}(x,y) = \sum_{n=-\infty}^{\infty} g_{MN}^{(n)}(x) e^{inmy}.$$

According to eq.(5.5), $n = 0$ states correspond to massless chargeless particles (graviton, photon, dilaton), while $n \neq 0$ states are massive with $m_n = n \cdot m$ and charges $q_n = n \cdot K \cdot m$, where $K = \sqrt{G}$, G being the Newton gravitational constant. If the fundamental charge is that of electron

$$e = K \cdot m,$$

then $m \sim 10^{19}$ GeV.

To incorporate the other interactions in the Kaluza-Klein scenario, i.e. to get strong, weak, and electromagnetic interactions of the SM from higher-dimensional gravity, we have to substitute

$$U(1) \Rightarrow G \supset SU(3) \otimes SU(2) \otimes U(1).$$

This needs the dimension D higher than 5, i.e. the circle should be replaced by a compact space of R dimensions (R depends on the rank of the group G) with G- symmetry

$$x^M = (x^\mu, y^m), \quad m = 1,2,\ldots,R.$$

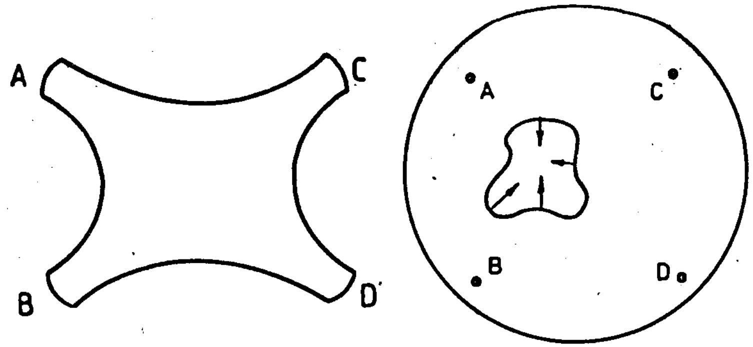

Figure 29: The world sheet of the scattering process having the topology of a sphere

For example: R-sphere S^R with $SO(R+1)$ symmetry.

Now gravity in $D = 4 + R$ dimensions leads to gravity plus non-abelian gauge interactions in $D = 4$:

$$\hat{g}_{MN} = \begin{cases} g_{\mu\nu} & \textit{graviton} \\ g_{\mu n} & \textit{Y-M gauge bosons} \\ g_{mn} & \textit{scalar particles} \\ & \textit{(Higgs bosons)} \end{cases} \quad + \textit{heavy } (> M_{Pl}) \textit{ states}$$

These ideas find their realization in superstring models giving rise to the universal description of gravitational ($N = 1$ supergravity) and gauge ($N = 1, 2$ super Yang-Mills) interactions.

String Interactions

In the superstring theory, the string is considered as a fundamental object at very small distances. The string interactions are then the origin of all point-lke particle interactions at low energy. As for the string interactions themselves, they have a topological origin, being a result of the complicated topology of the world sheet.

Consider the scattering of two open strings. The world sheet corresponding to this process is shown in Fig.29. Topologically it is equivalent to a sphere with four points A,B,C and D associated with emission of strings. A sphere has a trivial topology. Any contour on the surface of a sphere can be continuously deformed into a point (see Fig.29). It is said that a sphere has a genus (it is roughly speaking the number of handles) equal zero. However, to describe the scattering of strings, one should also take into account more complicated world sheet topologies with higher genus. For example, a torus has genus one. A torus is a sphere with one handle. There are two contours on the surface of a torus that cannot be continuously deformed into a point (see Fig.30). (Of course, what is shown is a three dimensional torus. The author is apologizing for not being able to draw a ten-dimensional one.)

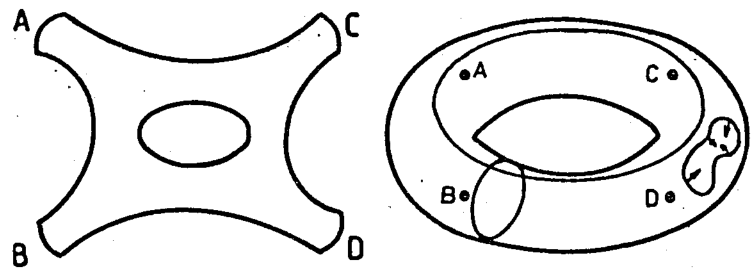

Figure 30: The world sheet of the scattering process having the topology of a torus

If one attentively looks at Fig.30, one finds that the world sheet corresponding to the interacting strings can be divided into several parts consisting of two elements: the triple vertices (the trousers) and the pipes. There is a rigorous mathematical statement that any closed oriented surface can be constructed of these two elements. This means that the string interactions are always cubic. What is essential is that, in spite of the non-local nature of strings, their interactions are local. However, the interaction point is not well defined. It depends on the position of an observable (see Fig.31). This explains, in particular, why superstring models have no severe ultraviolet divergences. This local cubic interaction gives rise to the local particle interactions of the corresponding low energy effective theory. Higher order contact terms appear there as effective ones, just like in the SM we get an effective four-fermion interaction.

5.3. Superstring Models

At the beginning of the "string revolution" the number of string models was very limited. Mathematical self-consistency happened to be a very restrictive requirement that was considered in favour of the string approach. Later on it has been realized that there are much more possibilities, and investigation is still continuing. Our aim now is to mention some of the most popular models. To begin with, we consider a block-scheme of fundamental interactions which is associated mainly with the heterotic 10-dimensional string theory , however, its main features remain true in general (see Fig.32).

The fundamental object of the theory is a superstring. It lives in a D-dimensional space-time ($D = 10$ for the model at hand). Extra dimensions are compactified on some scale giving rise to a tower of states with a quantized *mass*^{2}. Another result of compactification is the appearance of a local gauge group. Going down with energy after decoupling of heavy massive modes,

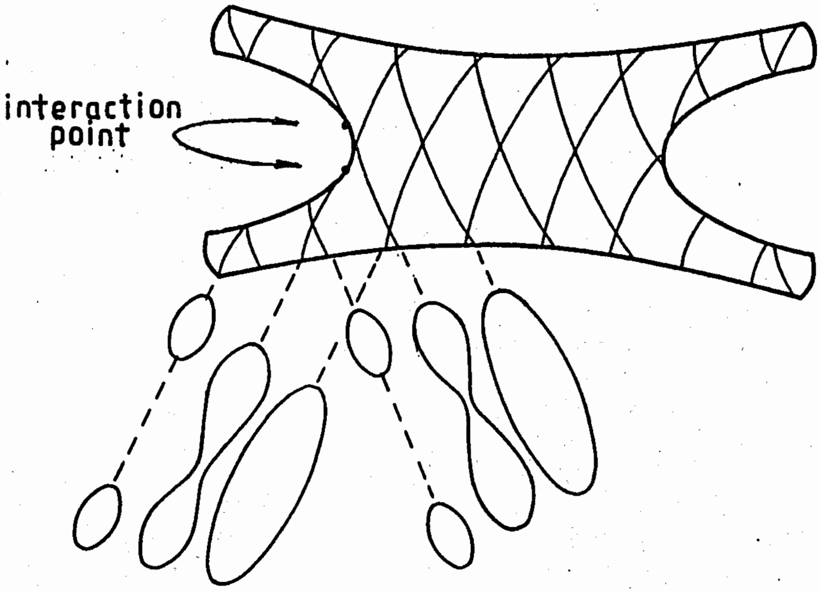

Figure 31: String interaction point seen by different observables

we end up with a point-like quantum field theory which is supergravity and super GUT. As a result of symmetry breaking on lower energy scales, they lead to Einstein gravity and a supersymmetrical generalization of the SM, respectively. Some extra gauge symmetries may also appear. Further on the situation is more or less standard.

We see that the advocated scheme contains some new particles and forces (shown in italic) which are absent in the SM. This is a usual prediction of superstring theories. We consider now some examples of string models in more detail.

10-D Heterotic String

This is one of the first and still reliable models based on a heterotic construction. Left and right moving states in this approach are considered separately. The left movers are those of a 26-dimensional bosonic string theory and right movers belong to a 10-dimensional fermionic string. Compactification occurs in two stages as shown in Fig.33. At first stage the 26 dimensions of the left movers are compactified to ten with the remaining 16 dimensions being internal ones, thus giving rise to $E_8 \otimes E_8$ gauge group. The resulting theory is the anomaly-free 10-dimensional supersymmetric string model. At the subsequent stage of compactification to 4 space-time dimensions the residual gauge group becomes $E_6 \otimes E_8$ (for the Calabi-Yau compact spaces)

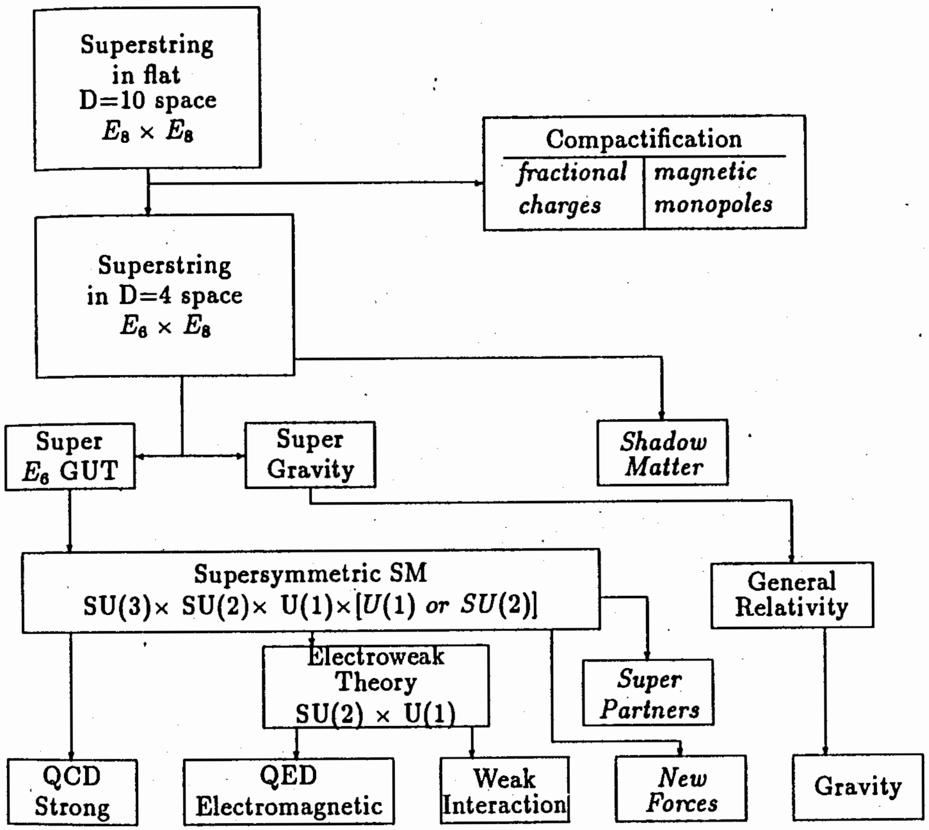

Figure 32: Fundamental interactions in Superstring approach

$$
\begin{array}{lcl}
(D & = & 26\ Bosonic\ String)_{Left\ Movers} \times (D = 10\ Fermionic\ String)_{Right\ Movers} \\
 & \Downarrow & \\
 & \Downarrow & Compactification \\
 & \Downarrow & \\
(D & = & 16\ "Internal" + D = 10\ Bosonic)_{LM} \times (D = 10\ Fermionic)_{RM} \\
 & \Downarrow & \\
 & \Downarrow & Anomaly\ Cancellation\ \Rightarrow\ Modular\ Invariance \\
 & \Downarrow & \\
(E_8 & \otimes & E_8\ Internal\ Symmetry) \times (D = 10\ Superstring\ Theory) \\
 & \Downarrow & \\
 & \Downarrow & Compactification\ (Calabi - Yau, Orbifold) \\
 & \Downarrow & \\
(E_6 & \otimes & E_8\ Internal\ Symmetry) \times (N = 1\ Supersymmetry\ in\ D = 4)
\end{array}
$$

Figure 33: Calabi-Yau type compactification

and $N = 1$ supersymmetry is preserved. The "predicted" gauge group of associated GUT is E_6, while another E_8 factor is supposed to be confining on a very high energy scale and to describe the shadow matter.

4-D Superstrings

Recently alternative schemes of compactification have been proposed which do not require two steps of compactification. They are known as 4D strings (see Fig.34).

In this case the left moving states and right moving states are compactified simultaneously to the required 4 flat dimensions. The gauge symmetry which has appeared as a result of this procedure is much larger than in the previous case. Groups up to $SO(44)$ with four supersymmetries are possible. Thus, the uniqueness of the gauge group prediction which was treated as an achievement of the string theory is lost now. It is hoped that some other criteria for making a definite choice will be found in future.

Although the investigations towards the building of realistic models are in progress, the modern phenomenology is essentially based on E_6 symmetry. However, in view of the above discussion, this should be considered as a representative rather than an exhaustive example.

5.4. Superstring Inspired GUTs

Now at least three viable three-generation models have been developed based on different compactification schemes. Remarkably, these schemes give rise to very similar phenomenological signals, basically the supersymmetric SM with a few additional neutrino or lepton-like states and possibly a richer

$$
\begin{array}{lcl}
(D & = & 26\ Bosonic\ String)_{Left\ Movers} \times (D = 10\ Fermionic\ String)_{Right\ Movers} \\
 & \Downarrow & \\
 & \Downarrow & Compactification \\
 & \Downarrow & \\
(D & = & 22\ "Internal" + D = 4\ Bosonic)_{LM} \times (D = 6\ "Internal" + D = 4\ Fermionic) \\
 & \Downarrow & \\
 & \Downarrow & Anomaly\ Cancellation\ \Rightarrow\ Modular\ Invariance \\
 & \Downarrow & \\
 & & \begin{array}{ccc} SO(44) & \times & (N = 4\ Supersymmetry\ in\ D = 4 \\ or\ smaller & & or\ smaller) \end{array}
\end{array}
$$

Figure 34: 4 - D superstring construction

Higgs structure. The problems of inhibiting nucleon decay and generating an acceptable mass pattern have been solved in these models either via new discrete symmetries or by additional gauge symmetries which are expected to be broken on a high scale.

E_6 Based Models

In the models, where after the first stage of compactification the gauge symmetry is $E_8 \otimes E_8$, the low energy group of the visible sector will be a subgroup of E_6.

$$E_8 \otimes E_8 \overset{Calabi-Yau\ compactification}{\Longrightarrow} E_6 \otimes E_8$$

The subgroup of E_6 is $[SU(3)]^3$, so the group of the SM must be embedded in this group structure

$$
\begin{array}{lcl}
E_6 & \supset & SU_C(3) \otimes SU_L(3) \otimes SU_R(3) \\
 & \supset & SU_C(3) \otimes SU_L(2) \otimes U_Y(1) .
\end{array}
$$

We show below the $[SU(3)]^3$ decomposition of E_6 supermultiplets $\underline{78}$ and $\underline{27}$ which should contain the known gauge bosons and quarks and leptons, respectively. (The underlined states are not in the supersymmetric SM.)

<u>Gauge boson - Gaugino supermultiplets</u>

$$
\begin{array}{cccc}
\underline{78} = & (\underline{8}, \underline{1}, \underline{1}) + & (\underline{1}, \underline{8}, \underline{1}) + & (\underline{1}, \underline{1}, \underline{8}) \\
 & Gluons & W^{\pm}, Z, \gamma & \underline{8\ new\ states} \\
 & & (+\ Gauginos) & \\
 & (\underline{3}, \underline{3}, \underline{3}) + & (\underline{3^*}, \underline{3^*}, \underline{3^*}) & \\
 & \underline{J = 1} & \underline{Leptoquarks} & \cdots\ Must\ be\ heavy \\
 & & (+\ Inos) &
\end{array}
$$

$$\begin{array}{lllll}
E_8\otimes E_8 & \overset{M_c\sim 10^{18}\,\mathrm{GeV}}{\Longrightarrow} & E_6\otimes SU(3)\otimes SU(2) & \otimes & U(1)\otimes U'(1)\times[N=1\ SUSY] \\
 & \textit{Breaking via} & \Downarrow & \Downarrow & \\
 & \textit{Wilson loops} & \Downarrow & \Downarrow & \\
 & & \textit{Confining} & \Downarrow & \\
 & & m\sim 10^{18}\mathrm{GeV?} & \Downarrow & \\
 & & & \Downarrow & \textit{Standard Model} + \\
 & & & \Downarrow & \textit{A new neutral current} \\
 & & \textit{Radiative SUSY} & \Downarrow & <H_1^0>,<H_2^0> = O(10^2\mathrm{GeV}) \\
 & & \textit{Breaking } O(10^3\mathrm{GeV})? & \Downarrow & \textit{Electroweak Breaking} \\
 & & SU_c(3) & \otimes & U_{EM}(1)
\end{array}$$

Figure 35: Minimal rank 5 superstring inspired model

Fermion - Sfermion supermultiplets

$$\begin{array}{lcl}
\underline{27} & = & (\underline{1},\underline{3},\underline{3}^*) \quad + (\underline{3}^*,\underline{3}^*,\underline{1}) \quad + (\underline{3},\underline{1},\underline{3}) \\
\textit{Higgs} + \textit{Higgsinos } H_{1,2} & & \\
\textit{Leptons} + \textit{Sleptons } e,\nu,N & : & \begin{pmatrix} H_1^0 & H_2^+ & e^+ \\ H_1^- & H_2^0 & \bar{\nu}_e \\ e^- & \nu_e & \underline{N} \end{pmatrix}_R + \begin{pmatrix} u \\ d \\ \underline{D} \end{pmatrix}_L + \begin{pmatrix} u \\ d \\ \underline{D} \end{pmatrix}_R \\
\textit{Quarks} + \textit{Squarks } u,d,D & &
\end{array}$$

A specific realization of the compactification procedure will give more details about the low energy structure of the model: the number of families, the low energy gauge group, the masses and couplings.

Minimal Rank 5 Model

This is the model where E_6 is broken on the compactification scale by Wilson loops, i.e. by topologically non-trivial configurations of the gauge fields. In this case one finds that only $\underline{27}$ representation survive and the E_6 group breaks , but not further than to the rank 5 group $SU_C(3)\otimes SU_L(2)\otimes U(1)\otimes U(1)'$ with one additional Z' boson whose mass should be of the order of the electroweak breaking scale. The breaking pattern is shown in Fig.35.

The light states ($m \leq O(M_W)$) are

$$3\times\left[\begin{pmatrix} u \\ d \\ \underline{D} \end{pmatrix}_L + \begin{pmatrix} u \\ d \\ \underline{D} \end{pmatrix}_R + \begin{pmatrix} H_1^0 & H_2^+ & e^+ \\ H_1^- & H_2^0 & \bar{\nu}_e \\ e^- & \nu_e & \underline{N} \end{pmatrix}_R\right]$$

The new states are underlined.

Three-Generation Calabi-Yau Model

Another possibility is to allow Higgs fields to acquire very large intermediate scale vacuum expectation values, thus breaking the gauge group.

$$\begin{array}{lllll}
E_8\otimes E_8 & \overset{M_c\sim 10^{18}\,\mathrm{GeV}}{\Longrightarrow} & E_6\otimes SU(3)\otimes SU(3) & \otimes & SU(3)\times[N=1\ SUSY] \\
 & \textit{Wilson loops} & \Downarrow & \Downarrow & \\
 & & \Downarrow & \Downarrow & <\nu_R>\sim 10^{17}\mathrm{GeV} \\
 & & \textit{Confining} & \Downarrow & \langle N\rangle\sim 10^{15}\mathrm{GeV} \\
 & & M\sim 10^{18}\,\mathrm{GeV} & \Downarrow & \textit{Intermediate} \\
 & & & \Downarrow & \textit{Breaking} \\
 & & SU(3)\otimes SU(2)\otimes U(1) & \Downarrow & \textit{Standard Model} \\
 & & & \Downarrow & \\
 & & \textit{Radiative SUSY} & \Downarrow & <H_1^0>,<H_2^0>\neq 0 \\
 & & \textit{Breaking } O(1\mathrm{GeV})? & \Downarrow & \\
 & & SU_c(3) & \otimes & U_{EM}(1)
\end{array}$$

Figure 36: Three-generation Calabi-Yau superstring model

It happens to be energetically favourable in the three-generation Calabi-Yau model. Because this model has the known Calabi-Yau manifold, many details of its properties are known, including the multiplet structure, the number of generations, etc. The pattern of symmetry breaking leads now to the standard SUSY model without new Z' boson (see Fig.36).

The light states ($m \leq O(M_W)$) here are

$$3\times\left[\begin{pmatrix} u \\ d \end{pmatrix}_L, u_R, d_R, \begin{pmatrix} \nu \\ l \end{pmatrix}_L, l_R\right] + \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix}_L + \begin{pmatrix} H_2^0 \\ H_2^- \end{pmatrix}_L + \begin{array}{l}\textit{Three neutral} \\ \textit{One charged} \\ \textit{Leptons}\end{array}$$

The Yukawa couplings in this case can be calculated as integrals over the Calabi-Yau manifold. It is argued that the pattern of symmetry breaking is possible leading to quark and lepton masses consistent with experiment.

We list below some general properties of the superstring inspired models:

1. $N \geq 1$ supersymmetry;
2. The number of generations N_F (of E_6) is defined by the topology of the compact space

$$N_F = \frac{1}{2}|\textit{Euler characteristics}|$$

The "realistic" Calabi-Yau spaces have $N_F \sim 3,4$;

3. Symmetry breaking pattern may lead to some additional symmetries (new neutral currents);

4. All Yukawa couplings of low energy theory are defined by topology and can be calculated;

5. There should exist stable heavy particles with $M \sim M_{Pl}$ and rational electric charges e/R and magnetic monopoles with $g = \frac{2\pi}{e} R$.

It would be ridiculous, at this stage, to claim that any model is the effective low energy theory following from the superstring. But its existence does show that the low energy structure emerging from the underlying compactified theory may closely resemble the observed world.

It was believed some time ago that with the superstring theory we have already found a unique fundamental theory of Nature. Now it is realized that this is not a single theory, rather we have opened a window in a new wonderful and exciting world of superstrings. You are welcome!

IV. References

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Казаков Д.И. E2-89-711
Физика вне Стандартной Модели

Настоящие лекции содержат краткое описание идей и методов физики вне Стандартной Модели. Они могут быть разделены на две части: физика сразу за Стандартной Моделью и физика далеко от Стандартной Модели. В первой части обсуждаются модель Техницвета, являющаяся альтернативой механизму Хиггса спонтанного нарушения симметрии, и Суперсимметрия – новая симметрия в природе. Вторая часть посвящена теориям Великого объединения, Супергравитации и теории Суперструны.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Kazakov D.I. E2-89-711
Beyond the Standard Model

The present lectures contain a brief guide to the ideas and methods of physics beyond the Standard Model. It can be divided into two parts: physics just beyond the Standard Model and physics far beyond it. The first part contains a discussion of Technicolour, an alternative to the Higgs mechanism of spontaneous symmetry breaking, and of Supersymmetry, the new type of symmetry in Nature. The second part is devoted to Grand unified theories, Supergravity, and Superstring theory.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.