XIII International Conference

Selected Problems of Modern Physics

Proceedings of the Conference

Joint Institute for Nuclear Research
Bogoliubov Laboratory of Theoretical Physics

XIII International Conference on Selected Problems of Modern Physics

Dedicated to the 100th anniversary of the birth
of D. I. Blokhintsev (1908–1979)

Dubna, June 23–27, 2008

Proceedings of the Conference

Edited by B. M. Barbashov and S. M. Eliseev

Dubna • 2009

С 3Т

УДК 530.145(063)
ББК 22.315я431
I69

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International Conference on Selected Problems of Modern Physics

I69 (13; 2008; Dubna).

Proceedings of the XIII International Conference on Selected Problems of Modern Physics, dedicated to the 100th anniversary of the birth of D. I. Blokhintsev (1908–1979), Dubna, June 23–27, 2008 / Ed. by B. M. Barbashov and S. M. Eliseev. — Dubna: JINR, 2009. — 413 p., [1] p. photo.

ISBN 978-5-9530-0207-3

The collection of papers includes the talks given at the XIII International Conference on Selected Problems of Modern Physics, dedicated to the 100th anniversary of the birth of D. I. Blokhintsev (1908–1979). Blokhintsev was the organizer and first director of the Joint Institute for Nuclear Research. This collection contains the papers reflecting Blokhintsev's scientific interests and reminiscences of some theorists about the important events of his many-sided activity.

Международная конференция по избранным проблемам современной физики (13; 2008; Дубна).

Труды XIII Международной конференции по избранным проблемам современной физики, посвященной 100-летию со дня рождения Д. И. Блохинцева (1908–1979), Дубна, 23–27 июня 2008 г. / Под ред. Б. М. Барбашова, С. М. Елисеева. — Дубна: ОИЯИ, 2009. — 413 с., [1] с. фото.

ISBN 978-5-9530-0207-3

В сборник включены избранные доклады, представленные на XIII Международной научной конференции по избранным проблемам современной физики, посвященной 100-летию со дня рождения Д. И. Блохинцева (1908–1979) — организатора и первого директора ОИЯИ. Сборник содержит работы, отражающие научные интересы Д. И. Блохинцева, а также воспоминания сотрудников о важных этапах его многогранной деятельности.

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ISBN 978-5-9530-0207-3

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The Conference is supported by
Russian Foundation for Basic Research,
Heisenberg - Landau Program (BMBF-JINR)
Dynasty Foundation
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Дмитрий Иванович Блохинцев

Opening talk

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The 13th International Conference on Selected Problems of Modern Physics dedicated to the 100th anniversary of the birth of the outstanding scientist Professor Dmitrii Ivanovich Blokhintsev (1908-1979), the first supervisor and director of the first atomic station in the world and the founder of the Joint Institute for Nuclear Research, was held at Dubna on June 23-27, 2008.

This series of conferences was initiated by D.I. Blokhintsev as meetings on fundamental problems of quantum field theory. Forty years passed since the first meeting enriched the methods of field theory and opened many new areas. The present conference was the 13th of this kind organized by the Joint Institute for Nuclear Research. The conference was opened by the memorial session. Further work of the conference proceeded in plenary and six parallel sections.

The topic of the conference reflects the current status of many fundamental problems in modern physics (Quantum Chromodynamics, Electroweak Theory and Its Extensions, Unification, Particle Astrophysics and Cosmology, Modern Mathematical Physics) and those areas of physics to which Blokhintsev has made significant contributions. These Proceedings collect the talks presented at the part "Problems of Quantum Field Theory".

The total number of participants of the conference was the following: thirty five scientists came to Dubna from "remote abroad", one hundred and fifty from "neighboring abroad" (Community of Independent States and Russia). The program compiles the plenary and sectional talks and contains the main part of presented reports. We have twenty three plenary talks and seventy-five sectional talks. We hope this will give an idea of the scientific content of the conference.

The organization of the conference would not be possible without the sponsorship of the Russian Foundation for Basic Research, Heisenberg - Landau Program (BMBF-JINR), Dynasty Foundation, JINR Directorate and on behalf of the Organizing Committee we gratefully acknowledge this support.

Now I would like to give you a short review of D.I. Blokhintsev's research activities. As Dmitry Ivanovich recalled himself, his early interest in mechanical engineering and technical drawing was greatly inspired by scientific genius of captain Nemo. Later the general interest to all kinds of machines was focused at airplanes and rockets. The first serious scientific research dates back to 1925 when Dmitri Ivanovich designed a device measuring the jet thrust. The young researcher was especially oppressed by relatively low temperatures achieved when burning any possible fuel. In his next work, Rocket, which was in fact the first theoretical one, he discusses the application of nuclear energy. When preparing himself for studies in the Air Force Engineering Academy he came across the papers of about splitting of the atom. The amazing results of Rutherford made Blokhintsev change his mind and enter Physics Department of Moscow State University. Since 1926 the life of D.I. Blokhintsev is inseparable from Physics Department of Moscow State University. The first paper on the work function of metals D.I.Blokhintsev published in 1933 in collaboration with I.E.Tamm, his supervisor. In 1936 he was elected the professor of the Theoretical Physics chair. In 1938 D.I. Blohkintsev considered the interaction of an atom with electromagnetic field. Applying the equations of QED and the concept of QED vacuum Blokhintsev predicted the Lamb shift and developed the first theory explaining the effect, which was discovered ten years later. In his calculation Blokhintsev used what now is called high frequency cutt-off. The renormalization of the electron mass was opened in 1947-48. In 1940 he introduced the concept of quasi-probability at which P.Dirac arrived much later. Principles of Quantum Mechanics (1944) is the first university lecture course on quantum mechanics. For the moment, 22 editions of the book were published in 8 languages. During the years of the second world war D.I. Blokhintsev almost completely switched his research to acoustics and its military applications. Soon he became the leading expert in this field. On the basis of the gas-hydrodynamics equations he created the acoustics of inhomogeneous and moving media. He explained and calculated diverse acoustic phenomena, solved many particular problems. These results formed the basis of the theory of aircraft and submarine acoustic location. 1954. Obninsk Nuclear Power Plant became the world's first nuclear power plant to generate electricity for a power grid, and produced around 5 megawatts electric power. D.I. Blokhintsev was the scientific leader of the project. To him belong the physical and design calculations of the reactors of this first Atomic Power Station (APS). In the middle of 1954 the first APS gave current. A long-standing successful operation of the station confirmed the correctness of choosing the reactor type and basic parameters of the first APS. In the subsequent years he calculated and supervised the development of design and construction of a new type of reactors the promising, in industrial sense, fast-neutron reactors with the liquid-metal heat-transfer agent. Now such reactors are exploited at other APS. Reactors attracted Blokhintsev's attention not only as the basis of power plants, but also as an intensive neutron source for diverse scientific studies. He is the author of the remarkable invention (1955) the fast pulsed reactors (IBR-1 and IBR-20). After many years of work this reactor proved to be a remarkable tool for studies in nuclear physics, physics of liquid and solid states and elementary particles physics.

In 1957, based on the "deuteron peaks" in the reactions of quasielastic high-energy proton scattering on nuclei, discovered by the group of M.G. Meshcheryakov, D.I.Blokhintsev proposed and developed the idea of fluctuations of nuclear density, capable as a whole to receive a large momentum transfer. The idea of "Blokhintsev's fluctons" best manifested itself 20 years latter when in reactions with relativistic nuclei the so-called "cumulative" particles were discovered. Later on, D.I. Blokhintsev participated in the development of the multi-quark interpretation of fluctons. These studies grew now in the new promising direction - relativistic nuclear physics. The remarkable confirmation of the flucton idea was obtained in experiments at CERN for deeply inelastic scattering of muons on nuclei and in the production of cumulative protons by a neutrino beam at Serpukhov. In the same years D.I. Blokhintsev investigated (on the basis of the optical "eikonal" model) the structure of nucleons, established its division into the central and peripheral parts and came to the conclusion about the dominant role of peripheral interactions. He showed the contradiction of the hydrodynamic approach to the multi-particle production processes with the basic principles of quantum mechanics (1957). Dmitry Ivanovich investigated the problem of anomalously short time of ultra cold neutron (UCN) storage end explained this effect by heating of the UCN by the hydrogen adsorbed by surface. A large and important cycle of Blokhintsevs works was dedicated to quantum field theory. He was the first to point out the possibility of existence of the so-called "unitary limit" in weak interactions (1957) and the limit of applicability of quantum electrodynamics. Dmitry Ivanovich proposed the idea

of existence of several vacua in quantum field theory and spontaneous transition between them (1960). This idea is intensively used in contemporary unified theories of elementary particles. Investigating substantially nonlinear fields, Blokhintsev came to the conclusion that the concept of point-like coordinates becomes meaningless and requires a change in the geometry of microcosm if the mass spectrum of particles is bounded from above. These questions found their reflection in the book by D.I. Blokhintsev "Space and Time in the Microworld", published in 1970 and in 1982 in our country and repeatedly republished abroad. D.I. Blokhintsev proposed a new approach to nonlocal fields based on the hypothesis of stochastic fluctuations of the space-time metrics. In the last years he repeatedly returned to cosmology. Analyzing Friedman's model he arrived at the conclusion that the visible part of our universe could not be formed within the limits of four-dimensional space-time and proposed his original hypothesis of the existence of extra space dimensions, meta-space, in which meta-bodies and antibodies collide. The conference topics correspond to those areas of theoretical physics to which D.I. Blokhintsev has made significant contribution and which are extensively developed in the Bogoliubov Laboratory of Theoretical Physics at JINR.

Memorial Session

Blokhintsev & Nonlocality & Particles

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I would like to start my talk with enumeration same general ideas which in opinion of D.I. Blokhintsev [1, 2] should be in the background of our understanding of particle physics.

- Particles can not be point-like objects.
- The lowest approximation of a theory should describe the main features of a phenomena under consideration.
- Local QFT – an approximation, when small distances are not essential for physical phenomena under consideration.
- Macroscopic conception of space-time and coordinate x, y, z, t in formulas are different things.
- Locality and microcausality are mathematical but not physical requirements.
- Confinement is defined by a non-trivial QCD vacuum, which is not the Fock vacuum containing plane waves only.
- Quarks and gluons can not be described by plane waves in the confinement region.

One should be stressed that contemporary non-relativistic and relativistic, classical and quantum mechanics are based on the assumptions that time t can be measured ABSOLUTELY PRECISELY. In the classical mechanics it is the Cauchy problem to solve dynamical equation:

$$\ddot{\mathbf{x}}(t) + U(\mathbf{x}(t)) = 0, \qquad \mathbf{x}(t_0) = \mathbf{x}_0, \ \dot{\mathbf{x}}(t_0) = \mathbf{v}_0,$$

In the nonrelativistic quantum mechanics it is the Cauchy problem to solve the Schrödinger equation:

$$i\hbar\frac{d}{dt}\Psi(\mathbf{x},t) = H(\mathbf{x})\Psi(\mathbf{x},t), \qquad \Psi(\mathbf{x},t_0) = \Psi_0(\mathbf{x}).$$

In the relativistic quantum field theory it is the simultaneous canonical commutation relations:

$$[\phi(\mathbf{x},t), \pi(\mathbf{x}',t)] = i\hbar\delta(\mathbf{x}-\mathbf{x}').$$

However in the relativistic quantum field theory the Schrödinger equation is not defined on the Fock space $\mathcal{F}$. Namely according to the Haag theorem (see [3]) the interaction Hamiltonian H_I is not well defined operator on the Fock space.

As consequence we are not able to describe development of a quantum-field system in time and are forced to consider the S-matrix (see [4]), as an operator connecting stable states Ψ_{in} for the time $t = -\infty$ with stable states Ψ_{out} for the time $t = +\infty$:

$$\Psi_{out} = S[g,\phi]\Psi_{in}, \qquad \Psi_{in}, \Psi_{out} \in \mathcal{F}$$

According to correspondence principle formally S-matrix can be represented in the perturbation form

$$S[g,\phi]=\sum_{n=0}^{\infty}\frac{i^n}{n!}\int...\int d^nx\; g(x_1)...g(x_n)S_n(x_1,...,x_n), \qquad (0\le g(x)\le 1),$$

$$S_n(x_1,...,x_n)=T(\mathcal{L}_I(x_1)\cdot...\cdot\mathcal{L}_I(x_n)).$$

The T-exponent is not defined in the coinciding points $x_i = x_j$ $(i,j=1,...,n)$. This uncertainty is used to introduce so-called quasi-local terms

$$K_n(x_1,...,x_n)\sim\prod_{i,j}\delta^{(m_{i,j})}(x_i-x_j)$$

to remove all divergences in the perturbation series. Finally we have the construction

$$S[g,\phi]=\lim_{\Lambda\to\infty}T_\Lambda e^{-i\int dx\; g(x)\mathcal{L}_I(\phi(x),\Lambda)},$$

where an interaction Lagrangian $\mathcal{L}_I(\phi(x),\Lambda)$ contains definite counter-terms. In addition a regularization procedure T_Λ with a parameter Λ should be introduced in order to formulate a rule to calculate terms of perturbation series. Finally we get the formula

$$S[g,\phi]=\lim_{\Lambda\to\infty}T_\Lambda e^{-i\int dx\; g(x)\mathcal{L}_I(\phi(x),\Lambda)}.$$

This S-matrix is not a solution of any quantum field wave equation, so that we have to prove all properties of the S-matrix including unitary and causality:

$$SS^+=1,$$

$$\frac{\delta}{\delta g(x)}\left(\frac{\delta S[g,\phi]}{\delta g(y)}\cdot S^+[g,\phi]\right)=0, \quad \text{for} \quad (x-y)^2<0 \quad \text{and} \quad (x-y)^2>0,\; x_0<y_0.$$

We would like to stress that the unitary and microcausality conditions are direct consequence of correctly formulated Cauchy problem of the quantum field Schrödinger equation if this equation would be mathematically correct formulated.

¿From physical point of view, as it was shown by Bohr and Rosenfeld [5], the quantum electromagnetic field and other quantum fields can be measurable in a small space-time region only

$$\phi(\Gamma)=\int_{\Gamma\in\mathbf{R}^4}dx\;\phi(x), \qquad \Gamma\in\mathbf{R}^4.$$

It means that the microcausality is NOT a physical requirement because it can not be checked in an experiment.

As a result we can introduce into consideration non-local distributions. Idea is very simple. From functional point of view verification of locality requires the space of test functions which contains functions with finite support. Thus if we postulate that the strict locality is not physical requirement then the space of test functions should not contain functions with finite support. Let a space of test functions contains analytical functions only. Then we are not able to check microlocal properties of any functional. On the other

hand in this case one can introduce more wide class distributions defined on the space of analytical functions [6]. As an example let us consider nonlocal distributions of the type

$$G(t-t')=e^{-(l\frac{d}{dt})^2}\cdot\delta(t-t')=\int_{-\infty}^{\infty}\frac{dE}{2\pi}e^{-iE(t-t')+l^2E^2}.$$

This object does not exist as a standard function, but it is defined as a distribution

$$J_{out}(t)=\int_{-\infty}^{\infty}G(t-t')J_{in}(t')=\int_{-\infty}^{\infty}\frac{dE}{2\pi}e^{-iEt}\cdot e^{l^2E^2}\tilde{J}_{in}(E),$$

This integral exists if the test function $\tilde{J}_{in}(E)$ decreases as

$$|\tilde{J}_{in}(E)|\le Ce^{-c|E|^\rho}, \qquad \rho>2.$$

Then the function $J_{in}(t)$ can not be concentrated in a point or any finite region, but

$$|J_{in}(t)|\le Ce^{-b|t|^\gamma}, \quad \gamma=\frac{\rho}{\rho-1}>1.$$

so that

$$e^{l^2E^2}\tilde{J}_{in}(E)\le C_1e^{-c|E|^\rho}, \quad \Rightarrow \quad |J_{out}(t)|\le C_2e^{-b|t|^\gamma}.$$

It means if the test function $J_{in}(t)$ can be concentrated in a vicinity of the point $t=0$ with an accuracy. The function $J_{out}(t)$ is concentrated approximately in the same vicinity.

Using this idea one can describe the confinement phenomenologically in the following way. Let an operator L_x is that the solution of the homogeneous equation $L_x\phi(x)=0$ is zero $\phi(x)=0$, but the solution of the inhomogeneous equation $L_x\phi(x)=J(x)$ is not zero $\phi(x)=\frac{1}{L}J(x)\neq 0$. For example

$$e^{l^2\partial^2}\phi(x)=0 \quad \Longrightarrow \quad \phi(x)=0$$

but

$$e^{l^2\partial^2}\phi(x)=J(x) \quad \Longrightarrow \quad \phi(x)=e^{-\ell^2\partial^2}J(x)=\int\frac{dy}{\pi^2}e^{-y^2}J(x+2\ell y).$$

It means that a particle which is described by the field $\phi(x)$ can exist as a virtual state only. We have an acceptable picture of the analytical confinement. In this picture confinement is vacuum fluctuations but not a statical increasing potential between two non-relativistic quarks. "The Quark Confinement Model of Hadrons" [7] is based on these ideas (see also [8]).

Let us come to QCD as the theory of strong interactions. The QCD Lagrangian is

$$\mathcal{L}=-\frac{1}{8}\mathrm{Tr}\;\breve{G}^2_{\mu\nu}+\left(\bar{q}\left[\hat{p}+g\hat{A}-m\right]q\right)$$

$$\breve{G}_{\mu\nu}(x)=\partial_\nu\breve{A}_\mu-\partial_\mu\breve{A}_\nu+g[\breve{A}_\mu(x),\breve{A}_\nu(x)],$$

One can extract a classical vacuum self-dual field

$$\breve{A}_\mu(x)\;\Rightarrow\;\breve{A}_\mu(x)+\breve{B}_\mu(x),$$

where

$$\breve{B}_\mu(x) = \Lambda^2 \breve{n} b_{\mu\nu} x_\nu, \quad \breve{n} = n^a t^a, \quad n^a n^a = 1$$

$$b_{\mu\nu} = -b_{\nu\mu}, \quad b_{\mu\rho} b_{\rho\nu} = -\delta_{\mu\nu}, \quad \epsilon_{\mu\nu\alpha\beta} b_{\alpha\beta} = \pm b_{\mu\nu}$$

This field satisfies the Yang-Mills equations and provides the analytical confinement because the equation

$$(\gamma_\mu(\partial_\mu - i\breve{B}_{\mu\nu} x_\nu) - m) q(x) = 0$$

has no decreasing solutions and therefore $q(x) = 0$, but the solution of the inhomogeneous equation

$$(\gamma_\mu(\partial_\mu - i\breve{B}_{\mu\nu} x_\nu) - m) S(x) = -\delta(x),$$

is not zero and looks as

$$\tilde{S}_\pm(p) \sim \frac{1}{2\Lambda^2} \int_0^1 du\, e^{-u\frac{p^2}{2\Lambda^2}} \left(\frac{1-u}{1+u}\right)^{\frac{m^2}{4\Lambda^2}} \left\{ i\hat{p} + m \frac{1 \mp \gamma_5 u^2}{1-u^2} \right\}. \qquad (1)$$

The quark propagator $\tilde{S}_\pm(p)$ is an entire analytical function, i.e. it does not describe any real particle state. Thus the self-dual field $\breve{B}_\mu(x)$ can be considered as a candidate to be gluon vacuum field (see [9, 10, 11, 12, 13]).

One can ask why the self-dual vacuum field $\breve{B}_\mu(x)$ exists in QCD and does not exists in QED? One can show [15] that in the quantum electrodynamics of a system of charged fermions and bosons the minimum of vacuum energy is realized with zero self-dual photon field if the number of fermions exceeds the number of bosons and the lightest charged particles are fermions QED

$$E_{vac}(\Lambda) \sim \frac{\Lambda^4}{12\pi^2} \left[\sum_F \ln\left(1 + \frac{2\Lambda^2}{M_F^2}\right) - \sum_B \ln\left(1 + \frac{2\Lambda^2}{m_B^2}\right) \right]$$

$$\Lambda_{min} = 0.$$

Namely, this situation takes place in Nature.

In the QCD the global stability of the quark-gluon system takes place if the number of quarks with different flavors is equal to or more than two. The reason is that gluons in QCD play two-fold role: they are carriers of quark-quark interaction and they are massless vector particles, the existence of which leads to the QCD vacuum which is realized with nonzero self-dual gluon fields.

$$E_{vac}(\Lambda) \sim \frac{\Lambda^4}{12\pi^2} \left[\sum_f \ln\left(1 + \frac{2\Lambda^2}{M_f^2}\right) - \ln\left(\frac{2\Lambda^2}{\Lambda_{QCD}^2}\right) \right].$$

$$\Lambda_{min} > 0$$

Thus our assumption is that the self-dual homogeneous vacuum gluon field $B_\mu(x)$ realizes the true QCD vacuum (details see in [15]).

If these ideas are correct then some general proprties of particles can be explained in the framework of this assumption. Exactly from this point of view let us consider the meson mass spectrum. Mesons are quark-antiquark bound states which are described by currents of the type $(\bar{q}\Gamma_Q q)$ where the vertex Γ_Q defines the corresponding quantum

numbers of a bound states under consideration. Quark-antiquark states are particulary relativistic systems so that the Bethe-Salpeter equation is unique mathematical instrument to calculate masses of relativistic bound states also this equation has its own problems (see [16]). The Bethe-Salpeter equation which determines the mass M of a bound state with a quantum number Q of two constituent quarks with masses m_1 and m_2 can be reduced to diagonalization of the quark-antiquark loop, where the quark propagator is chosen in the form (1) induced by a vacuum self-dual gluon field with constant strength. We get

$$\begin{aligned} -1 &= \frac{\alpha_s}{\pi} \int dy\, V_Q^2(y) \int dx\, e^{i(px)} \mathrm{Tr}\left[\Gamma_Q S_1(x + \mu_2 y) \Gamma_Q S_2(x - \mu_1 y)\right] \qquad (2) \\ &= \frac{\alpha_s}{\pi} \iint_0^1 du_1 du_2 P_Q(u_1,u_2) e^{\frac{(m_1+m_2)^2}{2\Lambda^2} E(\mu,\mu_1,\mu_2,u_1,u_2)}. \end{aligned}$$

where

$$p^2 = -M^2, \quad \mu = \frac{M}{m_1+m_2}, \quad \mu_1 = \frac{m_1}{m_1+m_2}, \quad \mu_2 = \frac{m_2}{m_1+m_2}.$$

The vertex function V_Q is determined by the solution of the Bethe-Salpeter equation, but with acceptable accuracy it can be approximated at large distances $y^2 \sim \frac{1}{\Lambda^2}$ by

$$V_Q(y) \sim D(y) \approx D_0\, e^{-\frac{\Lambda^2}{2} y^2}. \qquad (3)$$

The function $P(u_1,u_2) = P_Q(u_1,u_2,\mu,\mu_1,\mu_2)$ is a polynomial in parameters μ, μ_1, μ_2 and its explicit form is defined by the spin structure of vertices and quark propagators. The quantum numbers $Q = (J^P, n)$ of bound states are defined by these polynomials. In our semi-quantitative approach the explicit form of these polynomials will not be important in our arguments.

In the case $M > m_1 + m_2$ the main contribution to the integral (2) comes from the function

$$\begin{aligned} &E(\mu,\mu_1,\mu_2,u_1,u_2) \qquad (4) \\ &= \mu^2 \cdot \frac{u_1 u_2 + 2(\mu_1^2 u_1 + \mu_2^2 u_2)}{u_1+u_2+2} - \frac{\mu_1^2}{2} \ln\left(\frac{1+u_1}{1-u_1}\right) - \frac{\mu_2^2}{2} \ln\left(\frac{1+u_2}{1-u_2}\right). \end{aligned}$$

It defines the main features of the meson mass spectrum. This function depends on masses of constituent quarks m_1 and m_2 and does not depend on quantum numbers of bound states, so that we can hope that the main features of a set of mesons with the same quantum number $Q = (J^P, n)$ are defined by masses of constituent quarks only.

In the case $M > m_1 + m_2$) the function E is positive and has a positive maximum at a point $0 < u_1^{(0)} < 1$, $0 < u_2^{(0)} < 1$. Thus, one can write approximately for (2)

$$1 = \frac{\alpha_s}{\pi} \iint_0^1 du_1 du_2 P_Q(u_1,u_2) e^{\frac{(m_1+m_2)^2}{2\Lambda^2} E(\mu,\mu_1,\mu_2,u_1,u_2)} \approx C_Q e^{\frac{(m_1+m_2)^2}{2\Lambda^2} \mathcal{E}(M,m_1,m_2)},$$

where

$$\mathcal{E}(M,m_1,m_2) = E(\mu,\mu_1,\mu_2) = \max_{u_1,u_2} E(\mu,\mu_1,\mu_2,u_1,u_2). \qquad (5)$$

The parameter C_Q contains all information about the quantum numbers of states under consideration. We suppose that C_Q depends weakly on the mass parameters μ, μ_1, μ_2.

Calculations of the maximum in (5) give an approximate mass formula

$$M_Q(m_1, m_2) \approx (m_1 + m_2)\left[1 + \frac{A_Q}{(m_1^2 + 1.13 m_1 m_2 + m_2^2)^{0.625}}\right]. \qquad (6)$$

The positive constant A_Q is the same for all mesons with a given fixed quantum number $Q = (J^P, n)$.

This formula was checked for real meson spectrum (see [18]). In the Table Particle Group Data [17] there are only five sets of mesons for $M > m_1 + m_2$ with the fixed quantum number $Q = (J^P, n)$ having all four constituent quarks $u = d$, s, c, b. They are

$$V(1^-, 0),\ S(0^+, 0),\ A(1^+, 0),\ D(2^+, 0),\ D(2^+, 1).$$

The accuracy of the mass formula (6) is around

$$O_Q = \sqrt{\sum_j \left(1 - \frac{M_j}{(M_j)_{exp}}\right)^2} \leq 0.05.$$

All details can be found in [18].

In conclusion one can say

- Description of development of quantum-field systems in time is absent.
- Microcausality is not physical requirement.
- Confinement in the frame of nonlocal approach is not a statical potential between two nonrelativistic quarks but confinement is vacuum fluctuations of quark and gluon fields in space and time.
- Hadronization of quarks takes place in confinement region.
- Self-dual homogeneous vacuum gluon field can realize true QCD vacuum.
- Semi-phenomenological mass formula (6) describes correctly main features of meson mass spectra.

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Development of the Ensemble Approach in Nonclassical Physics (from M. Planck to D.I. Blokhintsev) *

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CONTENTS

1. TWO VERSIONS OF THE NATURE PROBABILITY DESCRIBING IN PYSICS: THE BOLTZMANN ASSEMBLY AND THE GIBBS ENSEMBLE

As well known, in Physics using of probabilities theory mathematical tools received wide spread occurrence in the middle of the 19th century [1,2]. However this fact did not mean that there was the disavowal of the deterministic comprehension of Nature. It continued to be preferable. Of course, its background was confirmed by general admitting of Newton dynamics and Maxwell electrodynamics lows.

In one of his paper such a known specialist in Mathematics as R. fon Mises wrote [3]: "First it must be a collective - only then we may speak about probabilities". With another words, to use probability theory it is necessary to have a set of elementary events called statistical collective.

Initial statistical ideas were introduced in Physics by Boltzmann in his kinetic theory of gases. But he supposed that probability describing was only used due uncertainties of initial data. As a result the primary signification saved the dynamical (deterministic) theories. Now we can surely say the probability theory came in classical Physics as the secondary mathematical describing. In this case the collective called the *Boltzmann assembly* consists of identical objects (for example, of ideal gas atoms) in the same external conditions. As an elementary event it is considered the fact that an object (atom) has a definite coordinate and momentum.

* The paper is done by the support of Fundamental Research Russian Fond (project 07-06-00239)

The origin of non-classical Physics is bounded with papers of Planck, Gibbs and Einstein and related to the beginning of 20-th century [4-6]. Under the notion of nonclassical Physics we understand now any theory considering the influence of stochastic actions of environment. In these situations we suppose physics characteristics of natural objects get random qualities. By the non-classical describing a type of statistical collection is principal changed. An object structure becomes non- essential. The main fact is some object characteristics fluctuate. These cases induce another collection type, non-classical.

There are a lot of situations when we have deal with a single object which intrinsic structure does not play any important role. But it is very important that the object is not isolated, it is always embedded to environment (for example, into thermostat). For these cases J.Gibbs supposed [7] a new notion of statistical collective called the *Gibbs ensemble*. It has a sense of the set of elementary events - soft touches of the system with environment. As a result the probability theory came in nonclassical Physics as the primary mathematical describing.

But it is interesting that Gibbs himself used simultaneously the both models of statistical collectives : the Boltzmann assembly in Statistical mechanics (SM) and the Gibbs ensemble in the prolegomena of Statistical thermodynamics (ST) (see ch.9 of his famous book) [8].

Comparing SM and ST one can say that they are different in principle because they based on the two absolutely different types of statistical collectives.

Real ST including the fluctuations theory of macroparameters was developed by Einstein on the base of the Gibbs ensemble [9].

2. BECOMING OF THE NONCLASSICAL VERSION OF NATURE DESCRIBING: PLANCK - GIBBS- EINSTEIN

The notion of the Gibbs ensemble appeared first in the macrotheory was indirectly used by Planck in his substantiation of the thermal radiation law. His idea about identity of energy quants requires non-standard calculations of states number for obtaining of thermal radiation entropy. As a result he intuitively came to a primary version of Gibbs ensemble [10-12].

In his turn, Einstein developing Planck radiation theory introduced the primary probabilities and thus- the Gibbs ensemble obviously in the microtheory [9]. But at those times (1903-1905) were is not Quantum Mechanics, so the becoming of ensemble non-classical concept in its final shape happened too much later and we can connect this process with the name of Blokhintsev. He made an enormous contribution into correct comprehension of the key role of Gibbs ensemble in non-classical Physics as a whole [13].

Foundations of Quantum Mechanics have been formed more than 75 years ago. Its creation opened up new horizons for the development of thinking. But the process of the new world-outlook familiarization was rather slow and contradictory. In these circumstances a great deal has been done by those scientists, who along with the creators of the new theory contributed to the propagation and adequate interpretation of fundamental ideas of this theory.

Among those who made a significant contribution to this matter a particular position belongs to representatives of the Moscow school, headed by academician L.I. Mandelshtam [14]. One of them was K.V.Nikol'skii [15], in his book "Quantum Processes"(1940) there have been presented for the first time ideas of this school. Starting from the middle of 1940th it was D.I. Blokhintsev who became the leader of this school, and to whose memory this Conference is devoted. The Moscow's interpretation of the Quantum Mechanics, developed by him, was in strong opposition to the initial version of the Copenhagen's interpretation from the Bohr's school.

3. SOME METHODOLOGICAL IDEAS OF D.I. BLOKHINTSEV IN QUANTUM MECHANICS.

Discussions about the probability sense of Nature description have been in progress for a long time. We are in opinion that the most adequate interpretation of Quantum mechanics was reached in the frame of ensemble approach or, better to say, of minimal ensemble approach, according to terminology of D. Home and M. Whitaker [16]. Its foundations were laid by Moscow school of Mandelshtam - Nikol'skii-Blokhintsev [17,13]. In the middle of 20th century the ensemble ideas were also successfully used in classical mechanics. But it is necessary to emphasis that ensemble approach developed by Moscow school and closed to initial Gibbs' point of view does not have any resemblance with the description on the base of an particles assembly used in classical statistical mechanics of Boltzmann.

The ideas of Moscow school were most clearly formulated in the Blokhintsev's books "Some principal problems of Quantum Mechanics"(1966) [18] and "Quantum mechanics. Lectures on selected problems"(1976) [19]. In the last of them there are the next Blokhintsev's sayings: "The principal quarrels are concentrated around the understanding of the wave function. Does the wave function provides one with the objective and complete description of the physical reality or it is just a "notebook"of an observer...? Does the wave function describes the state of the particle, or that of the system of particles?

We restrict ourselves to an explanation of the putted above questions, starting from the concept of quantum ensembles... The concept of quantum ensembles is a very close one to the concept of the Gibbs ensemble.

In the Gibbs ensemble a microsystem is considered in the interaction with a macroscopic thermal bath, having the temperature T. The probability of one or another result of the measurement... is related with the ensemble, formed by unlimited reiterations of the system at one and the same macroscopic conditions, given by the thermostat.

In the full analogy with the Gibbs ensemble a quantum ensemble is formed by unlimited reiterations of situations, made up by one and the same microsystem, which is imbedded into one and the same macroscopic circumstances .

Macrosituation might be artificially formed at the laboratory... , as well as to arise by itself in natural conditions...

The wave function is an objective characteristic of the quantum ensemble and, in principle, might be found by measurement".

In the above expressions there are posed several fundamental problems, which would be commented in what follows. The main problem, which was under discussion, that is whether the probabilistic description of Nature in Quantum Mechanics is the primary, fundamental one or it is a secondary one, as it has been traditionally considered in the Classical (deterministic) Physics.

As it is known, in Classical Physics there is initially presupposed the univalent predeterminance of the course of events. Therefore the impossibility of an univalent prediction of all events, which is encountered in practice, one usually refers to an incompleteness of

the initial data, that is to consider as a secondary effect. However, "... the real collapse of determinism happened along with the development of Quantum Mechanics, starting from the Einstein's work (1916) on the radiation theory, were there have been introduced a priori probabilities for the first time in physics"(V.A. Fock, 1957) [20]. Now it is possible to affirm, that the a priory, a primer character of the probability description in Quantum Mechanics is accepted by majority among investigators.

A substantial progress in this question has been achieved only after elucidation of the meaning of probability, as used in physics as a whole.In relation with that Blokhintsev wrote in his last work (1977) [21] : "Probability is the numerical measure of the potential possibility of that or another event. It happens in a some statistical ensemble of events, which should be defined by clearly formulated material conditions. ... Probability is not a characteristic of an individual mechanical system as itself".

4. CARDINAL SIGNIFICANCE OF BLOKHINTSEV'S IDEAS FOR NONCLASSICAL PHYSICS

Now we can assume that studying of Blokhintsev heritage has for us not only historical significance. His ideas are alive because until now they open new verges of our comprehension of Reality.

He thought

–the deep sense of the Gibbs assembly as a primary description in statistical thermodynamics stipulated by the key role of the thermal stochastic influence on the part of environment on an object;

–the deep resemblance between fundamental ideas of quantum mechanics and thermodynamics. He developed the concept of the Gibbs assembly into quantum mechanics by means his students books and articles.

He defended the idea against attack by the orthodox supporters of Copenhagen school and primitive followings of dialectical materialism. As a result most serious scientists directly or indirectly were agree with his position

Due his efforts:

–the physical sense of wave function became clear as an system objective characteristic under conditions of stochastic influence on the object as a whole. This effect is realized in Nature without observer;

– scientific community started to pay more attention to considering of fluctuations and their correlation in micro- and macro-world;

–particularly, the latest discovery in Astrophysics were done on the base of the fluctuations analysis of the relict radiation.

Now Blokhintsev's approach based on the Gibbs assembly allows us [22, 23]:

– to consider the quantum and thermal stochastic influences together (Thermofield Dynamics by Umezava and another versions of Quantum field theory in the presence of temperature);

–to propagate quantum statements to macro-objects and statistical thermodynamics statements to micro-objects;

– to realize the idea of system entire state and its characteristics independent from characteristics of a system.

It means that we are on the way to the entire description of Nature in the spirit of

Leibnitz -Planck - Einstein. We are of opinion that the global aim of scientific searching must be constructing of the unique state space forming by entire quantum-thermal stochastic (non- force) influence but not the searching of Great Unification Theory (GUT).

More over the conceptual resemblance between quantum and thermal statistical collectives on the base of Gibbs ensemble allows us to speak now about similarity of main ideas on which the two fundamental non-classical theories (Quantum mechanics and Statistical thermodynamics) are building. This fact makes them essentially different from any classical theory. Of course, the formalism of Quantum Mechanics does not like as the formalism of Statistical mechanics. But they are more closely each other than Statistical Mechanics in respect to Statistical thermodynamics.They draw together due the common conception of ensemble and the interpretation of fluctuations

The universality of Gibbs ensemble that Blokhintsev proclaimed together with indestructibility of stochastic influence allowed us now in a different way to have a glance on problem of probability in Nature. Earlier the scientists asked the question whether "did God play dice".Today times changed. Almost nobody doubts about positive reply this question. If, accordingly to Spinose, God is Nature we are sure that there exists chaos (stochastic quantum-thermal influence) in micro-phenomena. And recently the Nobel Prize winner G.t'Hooft has restated the problem [24]: "How does God play dice?".

But His Majesty Chance leads the world not to disorder. It creates "alive sounds". D.I.Blokhintsev well understood this fact. He was not only an outstanding physicist and thinker but he was a very artistic person. That is why he liked to use beautiful metaphors and told about Nature music. Thanks to him we can say that we are able to hear the Nature voice to the accompaniment of Gibbs ensemble.

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On the Contribution of D.I. Blokhintsev to Quantum Physics

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Abstract

A concise survey of the contribution of D.I. Blokhintsev to the quantum physics, including solid state physics, physics of metals, surface physics, statistical physics and optics is given. These achievements have been considered in the context of modern development of these fields of physics.

The name of Corresponding Member of the Academy of Sciences of the USSR D. I. Blokhintsev (January 11, 1908 – January 27, 1979) is widely known in Russia and abroad. His books are being republished; information on his biography and his scientific heritage can be found in multiple papers and collections of papers. However, for many scientists, his name is related mainly to his works in the field of atomic and nuclear physics, applied acoustics, participation in the creation of the first nuclear power station in Obninsk, reactor construction, and multiple studies in high energy and elementary particle physics. It is not so well known that at first he wrote some quite interesting and important works in the field of quantum solid state physics and statistical physics. In the beginning of his distinguished academic career [1, 2], D.I. Blokhintsev has worked in the field of quantum solid state physics and statistical physics, as well as in the field of quantum physics [1]. The aim of my talk is to recall these quite interesting and important works and correlate them with corresponding modern directions in condensed matter physics and quantum physics [3].
D.I. Blokhintsev entered the Physics Faculty of Moscow State University in 1926. At that time L.I. Mandelstam was the head of the Department of theoretical physics and optics and I.E. Tamm was professor of theoretical physics of that Department. Blokhintsev considered L.I. Mandelstam, S.I. Vavilov and I.E. Tamm his teachers. I.E. Tamm become his Ph.D. promotor in postgraduate studies. Thus, Blokhintsev's student years brought him great and fruitful experience in communicating, at lectures and in laboratories, with brilliant and interesting representatives of physical sciences of the time. Blokhintsev was certainly influenced strongly by Mandelstam and learned a lot from him, in particular, his breadth of views on physics as an indivisible science, lecturing skills, understanding the importance of a scientific school, organization of science, etc. As was noted later, "Lectures and seminars given by Mandelstam at the university in 1925-1944 were of special importance. They were devoted to a wide field of the most topical problems in physics in which the lecturer delivered an extremely deep analysis of the modern state of the art without concealing existing difficulties, and he outlined original solutions to very complex problems. These lectures attracted a wide audience of physicists of various ages and ranks from all parts of Moscow." Mandelstam delivered his famous lectures on the principles of quantum mechanics (the theory of indirect measurements) in spring of 1939. He intended to read a

series of lectures on the connection of the mathematical tools of quantum mechanics and its statistical interpretation, causality, etc., as a continuation of these lectures; the basis of this series of lectures was supposed to be the famous book written by J. von Neumann. Later, this program was realized by Blokhintsev.
It was time when quantum mechanics had acquired a certain maturity [4]. In the book by Gurney [5], also referred to in Blokhintsev's works, quantum mechanics is characterized as a new language of physics and chemistry. "The program of quantum mechanics includes no more and no less than the reconsideration of atomic and molecular physics in their entirety on the basis of new laws of behavior of particles following from quantum mechanics". Blokhintsev joined the realization of a program of reconsideration of atomic and solid state physics in their entirety on the basis of new quantum physics with enthusiasm. As he later recollected, "During that period (1927-1929), new quantum mechanics originated and great capabilities in the application of this new physical concept and new methods of calculation of various atomic phenomena were found" [1]. At that time, solid state physics, in particular, the theory of metals, attracted great attention. In 1932, the work "Temperature Dependence of the Photoeffect on Pure Metals" of D.I. Blokhintsev was published. The next paper was "The Work Function of Electrons from Metals" (1933) (jointly with I.E. Tamm). In the monograph [6] this study by Tamm and Blokhintsev was cited together with other basic works on the problem. Thus, from the very beginning, his works were at the highest level of quality. The early works of D.I. Blokhintsev have manifested also his talents of clear vivid presentation of the subject, transparent style, concreteness, the ability to point out most significant things and, most important, emphasis on the *physical meaning*. In a large work by Blokhintsev in 1933 "Theory of Electron Motion in a Crystal Lattice", the F.Bloch theory of motion of tight binding electrons was generalized for the many bands case and for the electron motion in a crystal which is bounded by surface. The next work was the paper "Theory of Anomalous Magnetic and Thermoelectric Effects in Metals"(1933) coauthored with L.W. Nordheim (1899-1985). In this work, the consistent theory of thermoelectric and galvanomagnetic effects in metals was constructed. Unlike earlier works, the case of $s-p$ band metals was considered. The authors studied the behavior of divalent metals in a magnetic field (Thompson and Hall effects). To make their equations compact, Blokhintsev and Nordheim introduced a new notion, the tensor of reciprocal effective masses. In the book of Mott and Jones [6], the priority of Blokhintsev and Nordheim in the creation of this fundamental notion was established. The achievement made by Blokhintsev and Nordheim was that they showed that the concept of effective mass was much more general and workable than had been assumed before and for the first time demonstrated the tensor character of the effective mass by considering the behavior of the electron in external fields. It turned out that the notion of effective mass is extremely useful in the theory of conductivity and other fields of solid state physics, nuclear physics, etc. The concept of effective mass became widely applied, especially in semiconductor physics and the physics of semiconductor devices, the polaron theory, semiconductor superlattices, microelectronics and physics of nanostructures.
A few word should be said about Blokhintsev's coauthor Lotar Wolfgang Nordheim (1899-1985). Nordheim belonged to the Gettingen school of theoretical physics. He was a PhD student with M. Born, and after defending his PhD thesis in 1923, his assistant and colleague till 1933. All his works are marked by bright talent and deep insight into a problem. In Jammer's book [4], the following fact is given: "In autumn of 1926, Hilbert began sys-

tematic studies of the mathematical principles of quantum mechanics. Lotar Wolfgang Nordheim, Born's former student, and the 23-year-old John von Neumann helped him in these studies. Hilbert also gave lectures on the mathematical principles of quantum theory, which were published in shorter form in the spring of 1927." Nordheim worked successfully in the application of quantum mechanics to statistical physics and solid state physics. He gave a successful description of the electron work function in metals, thermoelectron emission, electron kinetics in metals and alloys, etc. Thanks to a grant from the Rockfeller Foundation, Nordheim visited Moscow in 1933 as an invited professor to MSU. His studies were quite close to those performed by the Tamm's group. It was during that visit that he performed his joint work with Blokhintsev.

In 1933, Blokhintsev published "Theory of the Stark Effects in a Time-Dependent Field". In this paper Blokhintsev showed that the atomic levels move under influence of variable electric field (Stark modulation). The picture of light scattering depends nonlinearly on the intensity of the incident light. This work was one of the first in the field of physics, which was latter called *nonlinear optics*.

In 1934, Blokhintsev published paper on the theory of phosphorescence. According to the author, "An attempt was made to explain the phenomenon of phosphorescence in the so called Lenard phosphors on the basis of quantum mechanical ideas of the electron motion in the crystal lattice" [1]. Blokhintsev assumed that duration of the phosphorescence can be related with the capability of formation of *quasilocalized* electronic states in a real crystal as a result of the *local lattice deformation* due to the introduction impurities. Then he estimated the time of reciprocal recombination of these states. Thus, the theory of localized states made it possible to qualitatively (and, partially, quantitatively) interpret the big duration of the phosphorescence. This point of view was included in textbooks on optics. This and subsequent works by Blokhintsev, in which the detailed theory of the kinetics of phosphorescence in heteropolar crystals and the theory of dyed crystals were constructed, contributed considerably to deeper understanding of this problem and showed once more that the quantum mechanical approach is indeed the "new language of physics and chemistry", providing effective description of phenomena considered "mysterious" in classical physics. The same approach was used by Blokhintsev in the work "Quantum Mechanical Theory of Adsorption" (1934) (co-authored with Sh. Shekhter). This work is a very useful and clear survey of the problem as a whole. The paper of the same authors "Lifetime of Particles in Adsorbed State" (1934) was devoted to the calculation of the lifetime of particles in the adsorbed state. In that paper it was demonstrated how the quantum mechanics provides one with the microscopic picture of phenomenon. The authors obtained the correct qualitative behavior of the average lifetime of the adsorbed molecule on the surface, which demonstrated once more the effectiveness of the quantum mechanical approach. In 1934, Blokhintsev presented his Ph.D. thesis to the Institute of Physics of the Moscow State University, entitled *Selected Problems of the Solid State Theory, Especially Metals*. As a result of the high level of the work, he received a degree of Doctor of Science. At the time, Blokhintsev was 26 years old.

In 1935–1936, Blokhintsev continued his work on the theory of light absorption in heteropolar crystals, the theory of phosphorescence, and the theory of dyed crystals. It is interesting to note that in the paper "Theory of Dyed Crystals" (1936), Blokhintsev, in certain sense, anticipated the concept of the *polaron*, which was formulated later by S.I. Pekar (1917-1985). S.I. Pekar wrote this story in his well known monograph [7] in 1951:

"In 1936, Blokhintsev attempted to find out in which crystals autolocalization of electrons pointed out by Landau should be expected on the basis of the approximation of tight-binding electrons...". As is well known, S.I. Pekar coined the very term, *polaron*, in 1946. The main idea was that "excess" electron in ionic crystal polarizes the crystal lattice; this polarization in turn influences the electron, and this action is equivalent to the action of some effective potential well. The depth of this well in some crystals may be sufficiently large for discrete energy levels to exist in it. Local polarization caused by the electron is related to the displacement of ions from their average equilibrium positions. These states of the crystal with the polarization well in which the electron is localized were termed *polarons* by Pekar. The contribution made by Blokhintsev in 1936 to this direction of researches was mentioned later by a few other investigators. The main point was the formulation of the problem of autolocalized electronic states on the basis of approximation of tight-binding electrons. This approximation (LCAO) [3] later become widely used in condensed matter physics, especially for the description of localized states of different nature and disordered systems. The investigation of localized states in the framework of the tight-binding approximation bringed Blokhintsev to the point, namely to the need to describe the interaction of the electron with the lattice vibrations accordingly to the spirit of tight-binding approximation. This was carried out much later (see for details Ref. [3]).

In 1938, Blokhintsev prepared his work "*The Shift of Spectral Lines Caused by the Inverse Action of a Radiation Field*" for publication. He presented it at a seminar of the Physical Institute of the Academy of Sciences of the USSR, where he was employed; he also submitted it to Zhurnal Experimental'noi i Teoreticheskoi Fiziki [Journal of Experimental and Theoretical Physics] (ZhTEF). The work was rejected by the editorial board and published only in 1958 in Dubna in a collection of Blokhintsev's scientific works and papers. This work was mentioned in the survey report delivered by Ya.A. Smorodinskii [8] in 1949. Later on, the following was written [9]: "Already in early works by Blokhintsev, deep understanding of the essence of quantum mechanics, fresh and bold ideas, an original way of thinking that foreshadowed the further development of physics were evident. Typical in this respect was his work on the calculation of the 'shift of spectral lines caused by inverse action of a radiation field,' which in essence contained the theory of the Lamb shift, which was the beginning of quantum electrodynamics. It was reported at the seminar at the Physics Institute of the Academy of Sciences of the USSR and submitted to ZhTEF in 1938. The formula for the Lamb shift obtained by Blokhintsev became famous; it differs from the Bethe formula only by the numerical factor added in 1948 as a result of ultraviolet cutoff. Unfortunately, this important discovery was not published at that time in ZhTEF. There were no other outlets for publication". The genesis of the work "The Shift of Spectral Lines Caused by the Inverse Action of a Radiation Field" was best described by Blokhintsev himself [1]. "I delivered the work that, in essence, contained the theory of the Lamb shift discovered ten years later, at a seminar at the Physics Institute. However, my work was not published, since the editorial board of ZhETF returned the manuscript because the calculations were considered unusual. I kept the manuscript, which was stamped by the journal certifying the date of its receipt (February 25, 1938). I found no support from my colleagues at the Physics Institute. There were no other outlets. Thus, this important work was not published in due time. The main idea of the work followed from my deep belief that a physical vacuum existed in reality; however, I refrained from presenting the affair in this light...". The **Lamb shift** is indeed related to quite remarkable and interesting effects of quantum physics [10]. Lamb and his colleagues performed very precise, thorough, and elegant experimental studies on the determination of the structure of levels with $n = 2$ for hydrogen, deuterium, and singly ionized helium. Since the energy difference for these levels is very small, the probability of spontaneous transitions turns out to be negligible. However, if the atom is placed in a rotating (or oscillating) magnetic field with the corresponding frequency, induced transition can be observed. This frequency can be exactly measured; it is equal to the difference in energies of the two levels divided by $\hbar$. The measurement of the rotation frequency in Lamb's experiments yielded a value of the energy difference of two levels with the same principal quantum number in Rydberg units; it is interesting that this does not require any preliminary data on the Planck constant $\hbar$. The Lamb shift is mainly determined by the variation in the "scale" in wave functions of the atom, which are used upon calculation of the mathematical expectation of corresponding expressions. For fine splitting, Bethe calculated the energy difference of $2P_{1/2}$ and $2P_{3/2}$ states of hydrogen-like atoms. Blokhintsev wrote about his calculations in [1]: "As a result of them, the following expression is obtained for the frequency shift:

$$\delta\omega_0 = k(\frac{e^2}{\hbar c})^3 \frac{Z^4}{n^3} R \lg \left(\frac{\mu c^2}{\Delta E_{av}} \right), \tag{1}$$

where k is the numerical coefficient,ΔE_{av} is the average energy, n is the principal number of the level, and R is the Rydberg constant. Due to the inaccuracy in cutoff, the coefficient k and the values of ΔE_{av} differ somewhat from exact values obtained using the method of electron mass renormalization (note that (1) can be rewritten in the form $\delta\omega_0 \cong |\psi_s(0)|^2$, as is usually done now; here, $\psi_s(0)$ is the value of the wave function at the point $r = 0$). The ratio $\delta\omega_0/\omega = 2.8 \cdot 10^{-8}$ calculated using this formula for the He ion is in good agreement with respect to its absolute value and sign with the value measured by Paschen ($10^{-6}-10^{-7}$). At the time, there were no more precise measurements. This circumstance was of course unfavorable for further improvement of an unpublished work. Only after World War II, in 1948, did the importance of this work for theoretical physics become clear." The Lamb shift in levels in hydrogen, i.e., the energy by which the $2S_{1/2}$ state is higher than the $2P_{1/2}$ state, is obtained by combining different terms contributing to the theoretical expression for the Lamb shift. Experimental investigations of the Lamb shift continue. It was reported not long ago that two-loop corrections to the Lamb shift were first measured in strongly ionized atoms of heavy elements using the ion trap technique [11]. The history of theoretical calculation of the Lamb shift value is quite interesting. It is known from firsthand accounts and has been well described in many papers and books [12, 13]. According to V. Weisskopf [12], "Since 1936, there have been vague data that the position of observed hydrogen levels does not exactly match the predictions following from the Dirac equation, the so-called Pasternak effect. Certain considerations existed on possible ways of calculating this effect using quantum electrodynamics in the presence of deviations. After the war, I decided to investigate this problem together with a very capable *PhD* student, B. French. We wanted to calculate this effect, which was more well known as the Lamb shift, by isolating the infinite self-energy of the electron. These were complicated calculations, since the renormalization technique had not been developed yet. It was necessary to calculate the energy difference of the free and bound electrons when both energies were infinite. We had to be very accurate, since the calculation of the difference of diverging quantities often results in errors. We overcame difficulties slowly, since there were no good experimental results at

that time. Then Lamb and Retherford set up a good experiment, and finally, we obtained a result that agreed well with experimental data. I informed Julian Schwinger and Dick Feynman; they repeated the calculations; however, their results were different from ours, and Schwinger and Feynman obtained the same number. We postponed publication to find the error, spending half a year on it. Meanwhile, Lamb and Kroll published calculation result of the same effect, which more or less agreed with our result. Then Feynman called me from Ithaca, "You were right; I was wrong!" Thus, if we had had courage to publish our results, our paper would have been the first one to explain the experiment performed by Lamb and Retherford. What's the moral of this story? You have to believe in what you do."

In 1939, Blokhintsev published his work "Hydrodynamics of an Elecron gas". In this work, the hydrodynamic description of the system of many particles (electrons), i.e., description in terms of a "reduced" set of variables characterizing the system, the current $I(x)$ and the particle density $\rho(x)$, was considered. Blokhintsev maintained that since a many-particle problem could not be solved exactly, an approximate solution should be sought. It is known that an efficient way for calculating the energy eigenfunctions and eigenvalues is the self-consistent field method. This method was first developed by Hartree without taking into account electron exchange and then by Fock with this exchange taken into account. There exist a large number of works on this method both with and without the exchange account. Blokhintsev wrote in his work that from the very beginning he used the Hartree- Fock approximation, which assigns an individual function $\psi_k(x)$ to each electron n. In this approximation, the system of electrons is described by the density matrix. Considering the dynamic equations (equations of motion) for the current, Blokhintsev derived the "*hydrodynamic*" equation for a system of many particles (electrons) that contained gas density gradients in the stress tensor. To obtain closed expressions, he used approximations characteristic of statistical Fermi-Thomas theory. It is known that the statistical model of the atom describes the electrons of the atom statistically as an electron gas at a temperature of absolute zero. The model yields good approximation only for atoms with a large number of electrons, although it had been used for up to ten electrons. For the statistical approach, the details of the electronic structure had not been described; therefore, the application of a hydrodynamic description was quite relevant. Following the spirit of the statistical model of the atom, the total energy of the atom is obtained from the energy of the electron gas in separate elementary volumes dv by integrating over the whole volume of the atom. Working in this way and using the continuity equation, Blokhintsev derived an expression for the gas energy that (in the statistical case) coincided with the expression obtained earlier by Weizsacker using a different method.

It is appropriate to note here that the work "Hydrodynamics of an Electron Gas" contains one more aspect that does not seem striking at first sight but is nonetheless of great interest. In essence, it was shown in this work that a system in the low-energy limit can be characterized by a small set of "collective" (or hydrodynamic) variables and equations of motion corresponding to these variables. Going beyond the framework of the low-energy region would require the consideration of plasmon excitations, effects of electron shell reconstructing, etc. The existence of two scales, low-energy and high-energy, in the description of physical phenomena is used in physics, explicitly or implicitly. Recently, this topic obtained interesting and deep development, connected with the concept of the "*quantum protectorate*." In a work with a remarkable title, "*The Theory of Everything*" [14], authors

R. Laughlin and D. Pines discussed the most fundamental principles of the description of matter in a wide sense. The authors put forward the question what the "Theory of Everything" should be. In their opinion, "it describes the everyday world of human beings - air, water, rocks, fire, people, and so forth". The answer given by the authors was that "this theory is nonrelativistic quantum mechanics," or, more precisely, the equation of nonrelativistic quantum mechanics, which they wrote in the form

$$H\psi = -\frac{\hbar}{i}\frac{\partial\psi}{\partial t}. \tag{2}$$

That was the only formula in their work; they also gave detailed definition of the Hamiltonian of a system consisting of many interacting particles. The authors agreed, however, that "Less immediate things in the universe, such as the planet Jupiter, nuclear fission, the sun, or isotopic abundances of elements in space are not described by this equation, because important elements such as gravity and nuclear interactions are missing. But except for light, which is easily included, and possibly gravity, these missing parts are irrelevant to people-scale phenomena. Eq.(2) is, for all practical purposes, the Theory of Everything for our everyday world. However, it is obvious glancing through this list that the Theory of Everything is not even remotely a theory of every thing. We know this equation (2) is correct because it has been solved accurately for small numbers of particles (isolated atoms and small molecules) and found to agree in minute detail with experiment. However, it cannot be solved accurately when the number of particles exceeds about 10. No computer existing, or that will ever exist, can break this barrier because it is a catastrophe of dimension. If the amount of computer memory required to represent the quantum wave function of one particle is N then the amount required to represent the wave function of k particles is N^k."
According to R. Laughlin and D. Pines, "The emergent physical phenomena regulated by higher organizing principles have a property, namely their insensitivity to microscopics, that is directly relevant to the broad question of what is knowable in the deepest sense of the term. The low energy excitation spectrum of a conventional superconductor, for example, is completely generic and is characterized by a handful of parameters that may be determined experimentally but cannot, in general, be computed from first principles. An even more trivial example is the low-energy excitation spectrum of a conventional crystalline insulator, which consists of transverse and longitudinal sound and nothing else, regardless of details. It is rather obvious that one does not need to prove the existence of sound in a solid, for it follows from the existence of elastic moduli at long length scales, which in turn follows from the spontaneous breaking of translational and rotational symmetry characteristic of the crystalline state. Conversely, one therefore learns little about the atomic structure of a crystalline solid by measuring its acoustics. The crystalline state is the simplest known example of a quantum protectorate, a stable state of matter whose generic low-energy properties are determined by a higher organizing principle and nothing else. There are many of these, the classic prototype being the Landau fermi liquid, the state of matter represented by conventional metals and normal ^3He... Other important quantum protectorates include superfluidity in Bose liquids such as ^4He and the newly discovered atomic condensates, superconductivity, band insulation, ferromagnetism, antiferromagnetism, and the quantum Hall states. The low-energy excited quantum states of these systems are particles in exactly the same sense that the electron in the vacuum of quantum electrodynamics is a particle: They carry momentum, energy, spin, and charge, scatter off one another according to simple rules, obey Fermi or Bose statistics depending on their nature, and in some cases are

even "relativistic," in the sense of being described quantitatively by Dirac or Klein-Gordon equations at low energy scales. Yet they are not elementary, and, as in the case of sound, simply do not exist outside the context of the stable state of matter in which they live. These quantum protectorates, with their associated emergent behavior, provide us with explicit demonstrations that the underlying microscopic theory can easily have no measurable consequences whatsoever at low energies. The nature of the underlying theory is unknowable until one raises the energy scale sufficiently to escape protection."
The existence of two scales, low-energy and the high-energy, in the description of magnetic phenomena was stressed by Kuzemsky (see Refs. [15, 16, 17]) upon comparative investigation of localized and itinerant quantum models of magnetism. The concept of quantum protectorate was applied to the theory of magnetism in paper [17]. We succeeded in formulating the criterion of applicability of quantum models of magnetism to particular substances on the basis of analyzing their low-energy and high-energy spectra.
In 1940, Blokhintsev's attention was attracted to the problem of statistical description of quantum systems. Interest to these problems stemmed from lectures and works on quantum mechanics by L. I. Mandelstam and K.V. Nikol'skii. Nikol'skii's book *Quantum Processes* [18] is mentioned many times in his papers. In the work "Correlation of a Quantum Ensemble with a Classical Gibbs Ensemble" (1940), the limiting transition from quantum equations of motion for the density matrix to the equations of motion for the classical distribution function was studied. Blokhintsev studied the possibility of correspondence between the classical distribution function $f(q,p)$ and the quantum density matrix ρ from the general point of view. For this purpose, the mixed (q,p) representation for the density matrix was used. Blokhintsev shown in that paper that there does not exist any distribution function depending on (q,p) which could describe the quantum ensemble. In the next work on the topic (1940), the problem of the conditions of approximation of quantum statistics by classical statistics was considered. It was shown that there is no limiting transition $(h \to 0)$ from a quantum ensemble consisting of similar particles to a classical ensemble. The classical description is obtained if the state of the system is characterized by the position in the phase cell $\Omega \gg \hbar$. Thus, in these works, a new direction of physics was initiated: quantum mechanics in the phase space [19].
The title of the next work written by Blokhintsev (jointly with Ya.B. Dashevskii in 1941) is "Partition of a System into Quantum and Classical Parts." According to the authors, "Among physical problems that should be solved using quantum mechanical methods, there are such problems in which the system of interacting particles under study has a property that one of its parts during the processes occurring in the system moves as though it obeys classical laws of motion, i.e., moving along a trajectory." In this work, they studied the possibility of partitioning an interacting system into quantum and classical parts. They demonstrated the type of perturbation when the classical part acts on the quantum part. This field attracted great interest in subsequent years, especially in many problems of physical chemistry. A large number of works are devoted to this topic; some of them are considered in detail in survey [20].
In 1946, after switching to defense problems, Blokhintsev returned to quantum physics. The work performed in 1946 is titled "Calculation of the Natural Width of Spectral Lines Using a Stationary Method". This short work demonstrated high flexibility in handling tools of quantum mechanics when the result was reached in a simple and elegant way. Blokhintsev wrote, "Usually the problem of emission and absorption of light is considered

using the method of quantum transitions. However, this problem, similar to the dispersion problem, can be solved in an extremely simple way using the method of stationary states". Then, the author wrote out the system of equations for state amplitudes of two types: (a) when the emitter is in the state m and light photons are absent, and (b) when the emitter is in the state n and one light photon has been emitted. Taking into account the energy conservation law, the solution for the amplitude was obtained, and on its basis, the approximate expression for the level position of the whole system (emitter and radiation). "This expression resulted in exactly the same shift and smearing of levels as those obtained by Dirac upon calculation of resonance scattering." Then, the spectral distribution within the line width was found. The author noted that upon transformation of the amplitude to the coordinate representation, , "we obtain a divergent wave with an amplitude that slowly increases with increasing distance from the radiation source in the same way as took place for a classical decaying oscillator".

In 1947, Blokhintsev published the work "The Atom under an Electron Microscope". Blokhintsev wrote that "this work, devoted to a very special problem, is worth mentioning due to a somewhat unusual formulation of the problem. The origin is thus. I paid attention to the fact that under the action of a scattered electron, the atom receives recoil and can be knocked out of its position on the surface of the 'object plate.' If it were not knocked out at first scattering, it could be knocked out at subsequent scattering. It should be noted that this experiment is unusual from the point of view of the common formulation of measurements in a quantum ensemble. Indeed, in this case, we consider the repetition of measurements with the same sample of the atom, rather than a set of atoms, as is usually done. After each measurement the state of the atom, generally speaking, changes, and it becomes a sample of another quantum ensemble. Thus, the series of scattering necessary for obtaining an image of the atom consists of a series of scattering related to objects from different quantum ensembles. This seems to be a unique case of such a situation."

Since physicists, chemists, metallurgists, and biologists needed improved microscopes, this problem always stirred interest. It should be noted that remarkable works were performed by Mandelstam on the theory of the microscope. Mandelstam displayed his inherent the strength and depth of thought and his keen understanding of the physical nature in analyzing this problem. Blokhintsev's work continued the development of the theory of the microscope at the new quantum stage. The interest in this problem not only stemmed from the applied value. According to Blokhintsev, "The development of the theory of the microscope is of interest from the theoretical point of view, since when observing a single atom using an electron microscope, the image will emerge as a result of repetition of single scattering acts on the same object, while in quantum mechanics, results are usually formulated with respect to a set of objects in the same initial state. Due to the action on the atom, each new scattering act, generally speaking, will force the atom to be in a new initial state. Therefore, it is important to analyze the influence of electron scattering on the state of the observed atom". Further development in physics proved that Mandelstam and Blokhintsev's interest in problems of the theory of the microscope was justified. This direction was developed in subsequent years greatly and is being extensively developed now. Blokhintsev's name is closely related to the problem of **interpretation of the quantum mechanics** [21]. Blokhintsev recollected [1] that "in the 1930s – 1940s, the interest of many physicists-theoreticians at the Lebedev Physics Institute and MSU was concentrated on the principles of quantum mechanics, which seemed full of paradoxes to many people."

A large number of his books [22, 23, 24] and papers [25, 26] were devoted to this problem. His views of the problem changed and evolved with deepening and perfection of arguments. The most topical problems of interpreting quantum mechanics were the problem of measurement and the role of the observer, and the probabilistic interpretation of the wave function. The variety of opinions concerning the interpretation of quantum mechanics increased with time. Blokhintsev wrote [1]: "Those discussions are reflected in my works; the polemical character of my papers devoted to critical analysis of the ideas of the Copenhagen school and those of Fock gradually brought me to a consistent materialistic concept of quantum ensembles and mathematical measurement theory. Only in the 1960s, after discussions with the Hungarian physicist L. Janosi, did I manage to formulate a reasonable theory of quantum measurements free from inconsistencies in interpreting the role of the observer. In this new concept, the measuring device and its interaction with the microobject were transformed from the subject of philosophical discussions to the subject of theoretical physics".

As a result of longterm research and reflections, Blokhintsev developed his own approach to interpreting quantum mechanics, which included ideas put forward by J. von Neumann, L.I. Mandelstam, and K.V. Nikol'skii. It was called the interpretation of quantum mechanics on the basis of quantum ensembles. He wrote in a summary work [24], "The presentation of quantum mechanics undertaken in these lectures is essentially based on the ideas of von Neumann, which in their time attracted the attention of the Moscow school of theoreticians; in 1930s this school was headed by Academician Mandelstam; also Nikol'skii contributed considerably to our understanding of quantum mechanics." Blokhintsev thought that "this approach to the principles of quantum mechanics had an advantage, as compared to traditional interpretations on the basis of the wave function, since it allowed one to include the theory of quantum measurements as a chapter of quantum mechanics". In Blokhintsev's approach, the statistical operator describing the state of the microsystem in a quantum ensemble of the general type plays the primary role. The wave function describes a special type of quantum ensemble, the coherent ensemble. Blokhintsev's approach to the interpretation of quantum mechanics became widely known. De Witt and Graham [27] in their survey of different approaches to interpreting quantum mechanics wrote about Blokhintsev's books: "...they are both very well written and informative. The departure from orthodoxy occurs, in fact, only in certain attitudes and choice of words, while the general presentation of quantum mechanics is refreshing...; [the second book] contains an excellent account of measurement theory". Blokhintsev's approach to interpreting quantum mechanics is a constituent part of the scope of ideas of various researchers. One of the authoritative historians of quantum mechanics, Hooker [28], noted that "...Einstein and his co-workers Podolsky and Rosen, Blokhintsev, Bopp, de Broglie, Popper, Schrodinger, Lande, and most recently Ballentine constitute a small group of physicists and philosophers, who are determined to treat quantum theory as a species of statistical mechanics, many of them hoping ultimately to reinstate the classical conception of reality." A detailed survey of the interpretation of quantum mechanics on the basis of quantum ensembles can be found in [29]. The interpretation of quantum mechanics on the basis of quantum ensembles is one of many in existence. Thus, the interpretation of quantum mechanics on the basis of quantum ensembles occupies a separate (noticeable) place among other possible approaches to interpretation of quantum mechanics. Interpretation of quantum mechanics on the basis of quantum ensembles was considered in detail by Ya.A. Smorodinskii (1986). The conclusion he made is quite remarkable [30]: "Discussion showed that if the theory of quantum ensembles is used, these ensembles should be assigned unusual properties that could not be consistent with common probability theory; these properties are not manifested for one particle and can be found only in correlated effects; similar to non-Euclidean geometry necessary for the description of the velocity space in special relativity, quantum mechanics has generated the *non-Kolmogorov probability* theory; this is probably the deep meaning of analysis of the properties of a quantum ensemble" (see also recent book [31]).

We conclude this paper with the words by Max Born formulated in his lecture "Experiment and Theory in Physics" delivered in 1943. "Those who want to master the art of scientific prediction should, instead of relying on abstract deduction, try to comprehend the secret language of Nature, which is represented by experimental data." Blokhintsev in his lectures and talks more than once expressed similar thoughts, may be in slightly different words. In these notes and in the extended review [3] we have tried not only to write about Blokhintsev's studies, but build them into appropriate lines of the development of quantum physics and connect them, directly or indirectly, to the modern development of these fields of science. We have tried to show that Blokhintsev's book *Quantum Mechanics* [22], which is justly considered one of the best textbooks in quantum physics, was compiled by a witness to and a participant in the formation and development of quantum mechanics. It organically includes most of his original works in an integrated description of the subject. This, together with the definite literary talent of the author and his gift for presenting the subject clearly and lucidly, is the background on which the book *Quantum Mechanics* stands, and it continues to describes the world using the **language of a quantum**!

In this work due to the lack of space, not all the topics and problems that I wanted to discuss are here. Permit me to refer any reader who wants to reflect on Blokhintsev's works to a collection of selected works in two volumes that will be published in 2008 in Moscow. A more detailed discussion of modern approaches to interpreting quantum mechanics can be found in paper [3]. The full details and precise References are given in paper [3] as well.

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Plenary Session

Light Scalar Mesons Today

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Abstract

Outline: 1. Introduction, 2. Confinement, chiral dynamics and light scalar mesons, 3. Chiral shielding of $\sigma(600)$, chiral constraints, $\sigma(600)$, $f_0(980)$ and their mixing in $\pi\pi \to \pi\pi$, $\pi\pi \to K\bar{K}$, and $\phi \to \gamma\pi^0\pi^0$, 4. The ϕ meson radiative decays on light scalar resonances, 5. Light scalars in $\gamma\gamma$ collisions.

Evidence for four-quark components of light scalars is given. The priority of Quantum Field Theory in revealing the light scalar mystery is emphasized.

1 Introduction

The scalar channels in the region up to 1 GeV became a stumbling block of QCD. The point is that both perturbation theory and sum rules do not work in these channels because there are not solitary resonances in this region.

At the same time the question on the nature of the light scalar mesons is major for understanding the mechanism of the chiral symmetry realization, arising from the confinement, and hence for understanding the confinement itself.

2 Place in QCD

The QCD Lagrangian is given by

$L{=}-\frac{1}{2}Tr\,(G_{\mu\nu}(x)G^{\mu\nu}(x)) + \bar{q}(x)(i\hat{D} - M)q(x)$,

M is a diagonal matrix of quark masses, $\hat{D}{=}\gamma^\mu D_\mu$, $D_\mu{=}\partial_\mu{+}ig_0G_\mu(x)$. M mixes left and right spaces. But in chiral limit, $M_{ff} \to 0$, these spaces separate realize $U_L(3) \times U_R(3)$ flavour symmetry, which, however, is broken by the gluonic anomaly up to $U_{\mathrm{vec}}(1) \times SU_L(3) \times SU_R(3)$. As experiment suggests, confinement forms colourless observable hadronic fields and spontaneous breaking of chiral symmetry with massless pseudoscalar fields. There are two possible scenarios for QCD at low energy. 1. Non-linear σ model. 2. Linear σ model (LSM). The experimental nonet of the light scalar mesons, $f_0(600)$ (or $\sigma(600)$), κ(700-900), $a_0(980)$ and $f_0(980)$ mesons, suggests the $U_L(3) \times U_R(3)$ LSM.

3 History

Hunting the light σ and κ mesons had begun in the sixties already and a preliminary information on the light scalar mesons in PDG Reviews had appeared at that time. But long-standing unsuccessful attempts to prove their existence in a conclusive way entailed general disappointment and an information on these states disappeared from PDG Reviews. One of principal reasons against the σ and κ mesons was the fact that both $\pi\pi$ and πK scattering phase shifts do not pass over 90^0 at putative resonance masses.

4 $SU_L(2) \times SU_R(2)$ LSM, $\pi\pi \to \pi\pi$ [1, 2, 3]

Situation changes when we showed that in LSM there is a negative background phase which hides the σ meson in $\pi\pi \to \pi\pi$. It has been made clear that shielding wide lightest scalar mesons in chiral dynamics is very natural. This idea was picked up and triggered new wave of theoretical and experimental searches for the σ and κ mesons. Our approximation is as follows (see Fig.1):

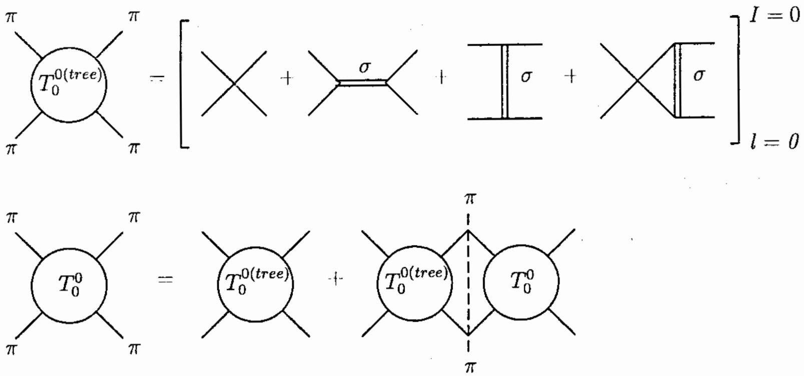


Figure 1. The graphical representation of the S wave $I = 0$ $\pi\pi$ scattering amplitude T_0^0.

$$T_0^0 = \frac{T_0^{0(tree)}}{1 - i\rho_{\pi\pi}T_0^{0(tree)}} = \frac{e^{2i\delta_0^0} - 1}{2i\rho_{\pi\pi}} = \frac{e^{2i(\delta_{bg}+\delta_{res})} - 1}{2i\rho_{\pi\pi}},$$

$$T_0^2 = \frac{T_0^{2(tree)}}{1 - i\rho_{\pi\pi}T_0^{2(tree)}} = \frac{e^{2i\delta_0^2} - 1}{2i\rho_{\pi\pi}}.$$

5 Results in our approximation [3].

$M_{res} = 0.43$ GeV, $\Gamma_{res}(M_{res}^2) = 0.67$ GeV, $m_\sigma = 0.93$ GeV,

$\Gamma_{res}(s) = \frac{g_{res}^2(s)}{16\pi\sqrt{s}}\rho_{\pi\pi}$, $g_{res}(M_{res}^2)/g_{\sigma\pi\pi} = 0.33$,

$a_0^0 = 0.18\, m_\pi^{-1}$, $a_0^2 = -0.04\, m_\pi^{-1}$, $(s_A)_0^0 = 0.45\, m_\pi^2$, $(s_A)_0^2 = 2.02\, m_\pi^2$.

6 Chiral shielding in $\pi\pi \to \pi\pi$ [3]

The chiral shielding of the $\sigma(600)$ meson is illustrated in Fig. 2.

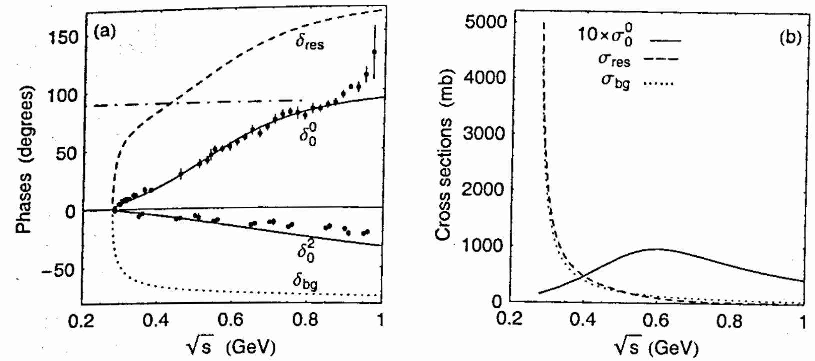


Figure 2. The σ model. Our approximation. $\delta_0^0 = \delta_{res} + \delta_{bg}$. $(\sigma_0^0, \sigma_{res}, \sigma_{bg}) = 32\pi(|T_0^0|^2, |T_{res}|^2, |T_{bg}|^2)/s$.

7 The σ pole in $\pi\pi \to \pi\pi$ [3]

$T_0^0 \to g_\pi^2/(s - s_R)$, $g_\pi^2 = (0.12 + i0.21)$ GeV^2,

$\sqrt{s_R} = M_R - i\Gamma_R/2 = (0.52 - i0.25)$ GeV.

Considering the residue of the σ pole in T_0^0 as the square of its coupling constant to the $\pi\pi$ channel is not a clear guide to understand the σ meson nature for its great obscure imaginary part.

8 The σ propagator [3]

$1/D_\sigma(s) = 1/[M_{res}^2 - s + \mathrm{Re}\Pi_{res}(M_{res}^2) - \Pi_{res}(s)]$.

The σ meson self-energy $\Pi_{res}(s)$ is caused by the intermediate $\pi\pi$ states, that is, by the four-quark intermediate states if we keep in mind that the $SU_L(2) \times SU_R(2)$ LSM could be the low energy realization of the two-flavour QCD. This contribution shifts the Breit-Wigner (BW) mass greatly $m_\sigma - M_{res} = 0.50$ GeV. So, half the BW mass is determined by the four-quark contribution at least. The imaginary part dominates the propagator modulus in the region 300 MeV$< \sqrt{s} <$600 MeV. So, the σ field is described by its four-quark component at least in this energy region.

9 Chiral shielding in $\gamma\gamma \to \pi\pi$ [3]

$T_S(\gamma\gamma \to \pi^+\pi^-) = T_S^{Born}(\gamma\gamma \to \pi^+\pi^-) + 8\alpha I_{\pi^+\pi^-} T_S(\pi^+\pi^- \to \pi^+\pi^-)$,

$T_S(\gamma\gamma \to \pi^0\pi^0) = 8\alpha I_{\pi^+\pi^-}\, T_S(\pi^+\pi^- \to \pi^0\pi^0)$,

$T_S^{Born}(\gamma\gamma \to \pi^+\pi^-) = (8\alpha/\rho_{\pi^+\pi^-})\mathrm{Im}I_{\pi^+\pi^-}$ $\;I_{\pi^+\pi^-} = \frac{m_\pi^2}{s}(\pi + i\ln\frac{1+\rho_{\pi\pi}}{1-\rho_{\pi\pi}})^2 - 1, \quad s \geq 4m_\pi^2$.

Our results are shown in Fig. 3.

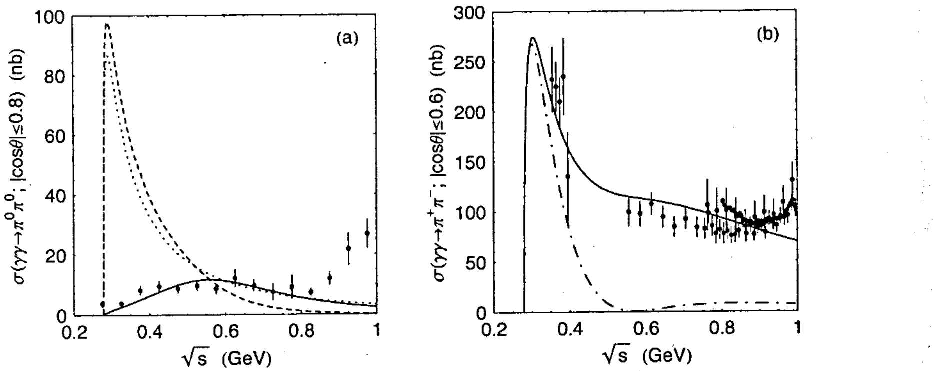


Figure 3. (a) The solid, dashed, and dotted lines are $\sigma_S(\gamma\gamma\to\pi^0\pi^0)$, $\sigma_{res}(\gamma\gamma\to\pi^0\pi^0)$, and $\sigma_{bg}(\gamma\gamma\to\pi^0\pi^0)$. (b) The dashed-dotted line is $\sigma_S(\gamma\gamma\to\pi^+\pi^-)$, the solid line includes the higher waves from $T^{Born}(\gamma\gamma\to\pi^+\pi^-)$.

$$\Gamma(\sigma\to\pi^+\pi^-\to\gamma\gamma, s) = \frac{1}{16\pi\sqrt{s}}|g(\sigma\to\pi^+\pi^-\to\gamma\gamma, s)|^2,$$

where $g(\sigma\to\pi^+\pi^-\to\gamma\gamma, s)=(\alpha/2\pi)\times I_{\pi^+\pi^-}g_{res\,\pi^+\pi^-}(s)$; see Fig. 4. So, the the $\sigma\to\gamma\gamma$ decay is described by the triangle $\pi^+\pi^-$ loop diagram $res\to\pi^+\pi^-\to\gamma\gamma$. Consequently, it is due to the four-quark transition because we imply a low energy realization of the two-flavour QCD by means of the the $SU_L(2)\times SU_R(2)$ LSM.

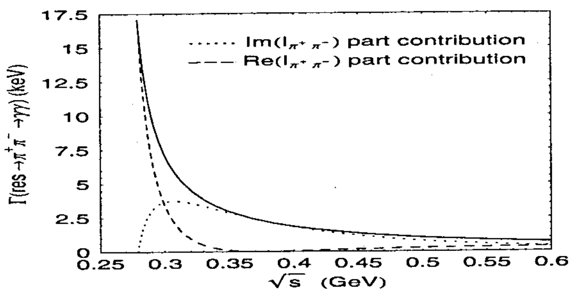


Figure 4. The energy dependent width of the $\sigma\to\pi^+\pi^-\to\gamma\gamma$ decay.

As Fig. 4 suggests, the real intermediate $\pi^+\pi^-$ state dominates in $g(res\to\pi^+\pi^-\to\gamma\gamma)$ in the σ region $\sqrt{s}<0.6$ GeV. Thus the picture in the physical region is clear and informative. But, what about the pole in the complex s plane? Does the pole residue reveal the σ indeed?

10 The σ pole in $\gamma\gamma\to\pi\pi$ [3]

$$\frac{1}{16\pi}\sqrt{\frac{3}{2}}T_S(\gamma\gamma\to\pi^0\pi^0)\to g_\gamma g_\pi/(s-s_R),$$

$g_\gamma g_\pi=(-0.45-i0.19)\cdot 10^{-3}\,\mathrm{GeV}^2$, $g_\gamma/g_\pi=(-1.61+i1.21)\cdot 10^{-3}$, $\Gamma(\sigma\to\gamma\gamma)=\frac{|g_\gamma|^2}{M_R}\approx 2$ keV.

It is hard to believe that anybody could learn the complex but physically clear dynamics of the $\sigma\to\gamma\gamma$ decay from the residues of the σ pole.

11 Discussion [3, 4, 5, 6]

Leutwyler and collaborators obtained $\sqrt{s_R}=M_R-i\Gamma_R/2=\left(441^{+16}_{-8}-i272^{+12.5}_{-9}\right)$ MeV. Our result agrees with the above only qualitatively, $\sqrt{s_R}=M_R-i\Gamma_R/2=(518-i250)$ MeV. It is natural, for our approximation.

Could the above scenario incorporates the primary lightest scalar Jaffe four-quark state? Certainly the direct coupling of this state to $\gamma\gamma$ via neutral vector pairs ($\rho^0\rho^0$ and $\omega\omega$), contained in its wave function, is negligible, $\Gamma(q^2\bar q^2\to\rho^0\rho^0+\omega\omega\to\gamma\gamma)\approx 10^{-3}$ keV, as we showed in 1982 [6]. But its coupling to $\pi\pi$ is strong and leads to $\Gamma(q^2\bar q^2\to\pi^+\pi^-\to\gamma\gamma)$ similar to $\Gamma(res\to\pi^+\pi^-\to\gamma\gamma)$ in the above Fig. 4.

Let us add to $T_S(\gamma\gamma\to\pi^0\pi^0)$ the amplitude for the the direct coupling of σ to $\gamma\gamma$ conserving unitarity $T_{direct}(\gamma\gamma\to\pi^0\pi^0)=sg^{(0)}_{\sigma\gamma\gamma}g_{res}(s)e^{i\delta_{bg}}/D_{res}(s)$. Fitting the $\gamma\gamma\to\pi^0\pi^0$ data gives a negligible value of $g^{(0)}_{\sigma\gamma\gamma}$, $\Gamma^{(0)}_{\sigma\gamma\gamma}=|M^2_{res}g^{(0)}_{\sigma\gamma\gamma}|^2/(16\pi M_{res})\approx 0.0034$ keV, in astonishing agreement with our prediction [6].

12 Phenomenological chiral shielding [7]

$g^2_{\sigma\pi^+\pi^-}/4\pi=0.99\,\mathrm{GeV}^2$, $g^2_{\sigma K^+K^-}/4\pi=2\cdot 10^{-4}\,\mathrm{GeV}^2$, $g^2_{f_0\pi^+\pi^-}/4\pi=0.12\,\mathrm{GeV}^2$, $g^2_{f_0K^+K^-}/4\pi=1.04\,\mathrm{GeV}^2$. The BW masses and width: $m_{f_0}=989$ MeV, $m_\sigma=679$ MeV, $\Gamma_\sigma=498$ MeV. The $l=I=0$ $\pi\pi$ scattering length $a^0_0=0.223\,m^{-1}_{\pi^+}$. Figure 5 illustrates the excellent agreement our phenomenological treatment with the experimental and theoretical data.

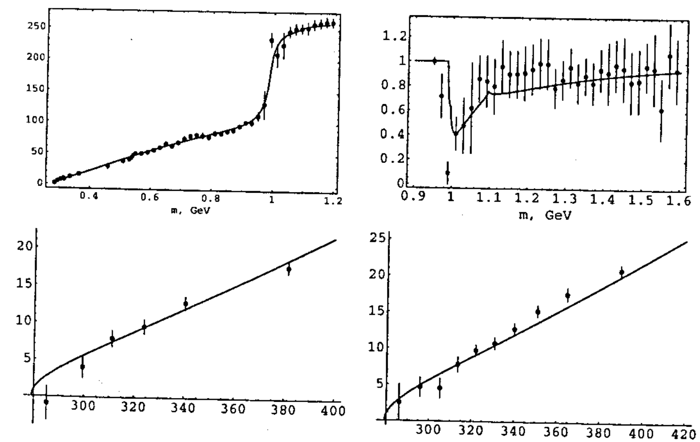


Figure 5. The phenomenological chiral shielding. $\delta^0_0=\delta^{\pi\pi}_B+\delta_{res}$. The comparison with the CERN-Munich data for δ^0_0 and inelasticity η^0_0, with the BNL and NA48 data for δ^0_0, respectively.

13 Four-quark model [5, 8, 9]

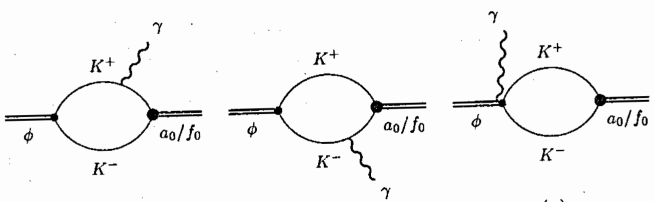
There are numerous evidences in favour of the $q^2\bar{q}^2$ structure of $f_0(980)$ and $a_0(980)$. As for the nonet as a whole, even a dope's look at PDG Review gives an idea of the four-quark structure of the light scalar meson nonet, $\sigma(600)$, $\kappa(700\text{-}900)$, $a_0(980)$, and $f_0(980)$, inverted in comparison with the classical P wave $q\bar{q}$ tensor meson nonet $f_2(1270)$, $a_2(1320)$, $K_2^*(1420)$, and $f_2'(1525)$. Really, it can be easy understood for the $q^2\bar{q}^2$ nonet, where $\sigma(600)$ has no strange quarks, $\kappa(700\text{-}900)$ has the s quark, $a_0(980)$ and $f_0(980)$ have the $s\bar{s}$ pair.

14 Radiative decays of ϕ meson [7, 8, 9, 10, 11, 12]

Twenty years ago we showed [10] that the study of the radiative decays $\phi \to \gamma a_0 \to \gamma\pi\eta$ and $\phi \to \gamma f_0 \to \gamma\pi\pi$ can shed light on the problem of $a_0(980)$ and $f_0(980)$ mesons. Now these decays have been studied not only theoretically but also experimentally. Note that $a_0(980)$ is produced in the radiative ϕ meson decay as intensively as $\eta'(958)$ containing $\approx 66\%$ of $s\bar{s}$, responsible for $\phi \approx s\bar{s} \to \gamma s\bar{s} \to \gamma\eta'(958)$. It is a clear qualitative argument for the presence of the $s\bar{s}$ pair in the isovector $a_0(980)$ state, i.e., for its four-quark nature.

15 K^+K^- loop mechanism [7, 8, 9, 10, 11, 12]

When basing the experimental investigations, we suggested [10] one-loop model $\phi \to K^+K^- \to \gamma\,[a_0(980)/f_0(980)]$; see Fig. 6.



(a) (b) (c)

Figure 6. The K^+K^- loop model.

This model is used in the data treatment and is ratified by experiment, see Figs. 7.

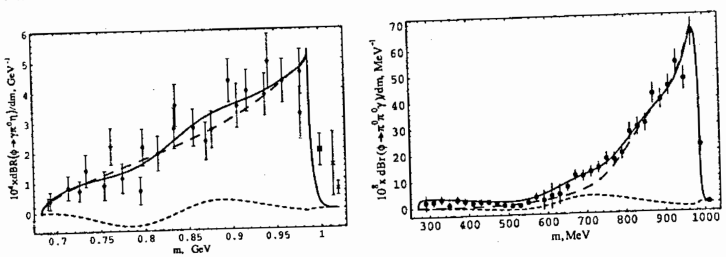


Figure 7. The left (right) plot illustrates the fit to the KLOE data for the $\pi^0\eta$ ($\pi^0\pi^0$) mass spectrum.

For $dBR[\phi \to \gamma(a_0/f_0) \to \gamma(\pi^0\eta/\pi^0\pi^0), m]/dm \sim |g(m)|^2\omega(m)$, the function $|g(m)|^2$ should be smooth at $m \le 0.99$ GeV. But gauge invariance requires that $g(m)$ is proportional to the photon energy $\omega(m)$. Stopping the function $(\omega(m))^3$ at $\omega(990\,\text{MeV})$=29 MeV is the crucial point. The K^+K^- loop model solves this problem in the elegant way, see Fig. 8.

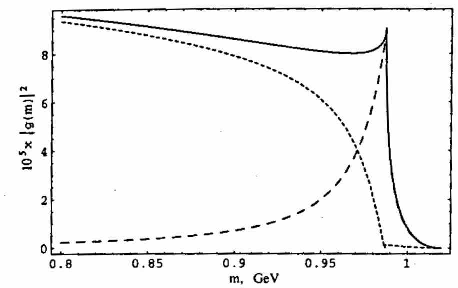


Figure 8. A new threshold phenomenon in $\phi \to K^+K^- \to \gamma R$ decays. The universal in K^+K^- loop model function $|g(m)|^2 = |g_R(m)/g_{RK^+K^-}|^2$ is drawn with the solid line. The contributions of the imaginary and real parts of $g(m)$ are drawn with the dashed and dotted lines, respectively.

16 Four-quark transition and OZI rule [9]

So, we are dealing here with the four-quark transition. A radiative four-quark transition between two $q\bar{q}$ states requires creation and annihilation of an additional $q\bar{q}$ pair, i.e., is forbidden according to OZI rule, while a radiative four-quark transition between $q\bar{q}$ and $q^2\bar{q}^2$ states requires only creation of an additional $q\bar{q}$ pair, i.e., is allowed according to the OZI rule.

17 $a_0(980)/f_0(980) \to \gamma\gamma$ & $q^2\bar{q}^2$ model

Twenty six years ago we predicted [6] the suppression of $a_0(980)/f_0(980) \to \gamma\gamma$ decays basing on $q^2\bar{q}^2$ model. Experiment supported this prediction. The $a_0 \to K^+K^- \to \gamma\gamma$ model [13] describes adequately data and corresponds to the four-quark transition $a_0 \to q^2\bar{q}^2 \to \gamma\gamma$. $\langle\Gamma(a_0 \to K^+K^- \to \gamma\gamma)\rangle \approx 0.3$ keV. $\Gamma^{direct}_{a_0\to\gamma\gamma} \ll 0.1$.

18 $\gamma\gamma \to \pi\pi$ from Belle [14, 15]

Recently, we analyzed the new high statistics Belle data on the reactions $\gamma\gamma \to \pi\pi$ and clarified the current situation around the $\sigma(600)$, $f_0(980)$, and $f_2(1270)$ resonances in $\gamma\gamma$ collisions, see Fig. 9.

$\langle\Gamma(\sigma \to \pi^+\pi^- \to \gamma\gamma)\rangle \approx 0.45\,\text{keV}$, $\langle\Gamma(f_0 \to K^+K^- \to \gamma\gamma)\rangle \approx 0.2\,\text{keV}$, $\Gamma^{direct}_{\sigma\to\gamma\gamma} \ll 0.1\,\text{keV}$, $\Gamma^{direct}_{f_0\to\gamma\gamma} \ll 0.1\,\text{keV}$.

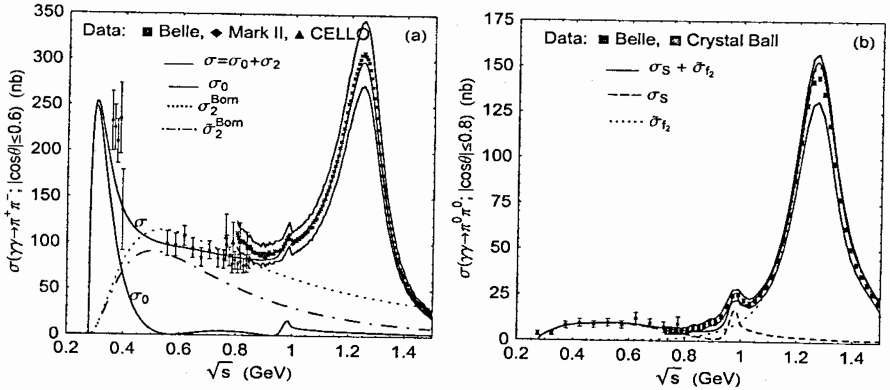


Figure 9. (a) Cross section for $\gamma\gamma \to \pi^+\pi^-$. (b) Cross section for $\gamma\gamma \to \pi^0\pi^0$.

19 The lessons of $\gamma\gamma$ collisions.

The classic P wave $q\bar{q}$ tensor mesons $f_2(1270)$, $a_2(1320)$, and $f_2'(1525)$ are produced by the direct transitions $\gamma\gamma \to q\bar{q}$ in the main, whereas the light scalar mesons $\sigma(600)$, $f_0(980)$, and $a_0(980)$ are produced by the rescattering $\gamma\gamma \to \pi^+\pi^-$, $K^+K^- \to \sigma$, f_0, a_0. The direct transitions $\gamma\gamma \to \sigma$, f_0, a_0 are negligible, as it is expected in four-quark model.

I thank Heiri Leutwyler very much for communications. This work was supported in part by Presidential Grant No. NSh-1027.2008.2 and by RFFI Grant No. 07-02-00093.

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Conformal Gravitation Theory with Initial Data

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Abstract

Supernovae type Ia data and the CMB energy budget is considered in the model inspired by the Dirac conformal approach to General Relativity (GR) with a massless scalar (dilaton) field which scales all masses. A cosmological model obtained from the GR and SM by the average the dilaton and other scalar fields over a large finite volume that does not commute with a variation calculus in general. This approach is in agreement with the dominance of a scalar field kinetic energy density and an intensive cosmological creation of primordial W, Z, and Higgs bosons from vacuum. Arguments are discussed testifying to that two photon processes of the primordial particle annihilations and decays form three peaks in the CMB power spectrum, and their values, and positions $\ell = 220, 546, 800$ are in agreement with the QED coupling constant, the Weinberg angle, and the Higgs particle mass of about 118 GeV.

1 Introduction

Dirac in [1] modified the accepted General Relativity (GR) in spirit of the simplified Weyl's geometry [2], which means that "*a new action principle was set up, much simpler than Weyl's, but requiring a scalar field function*" (called here as dilaton) "*to describe the gravitation field, in additional to* $g_{\mu\nu}$"[1]. In this paper we use this Dirac modification in the Lichnerowicz gauge $|\widetilde{g}^{(3)}| = 1$ [3]. This gauge is compatible with the Wigner representations of the Poincaré group [4] in the tangent space-time distinguished by the Fock simplex (tetrades) [5, 6].

In this case, the Hubble law is explained by the evolution of masses in the constant Universe volume, and the analysis of the supernovae type Ia data [7] is compatible with the dominance of the kinetic energy density of the scalar field zeroth harmonic [8, 9, 10].

This dominance allows us to set in the theory very important free initial data that do not depend on the fundamental parameters of the equations of motion of the type of the Planckian mass and the Higgs potential parameters.

Thus the scale invariance of laws of Nature [2] can be considered as the principle of a choice of variables in GR and the Higgs potential free Standard Model (SM) [11] treated as a theory of the dynamical scale symmetry [1, 12]. In this theory the dilaton Goldstone field compensates all scale transformations of fields including the cosmological scale factor describing expansion of the Universe lengths in the Standard Cosmology [13], so that the

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scale-invariant variables remove non-physical geometrical singularities. Nevertheless, the dilaton explains redshift by the permanent increase of all masses in the Universe and leads to the Conformal Cosmology (CC) [8, 9, 10, 14], where all measurable quantities are identified with the conformal ones (conformal time, coordinate distance, and constant conformal temperature). This CC identification changes the numerical analysis of supernovae type Ia data [7] and shows the dominance of the scalar field *kinetic* energy $\rho = \mathsf{K} + \mathsf{V}, \mathsf{K} \gg \mathsf{V}$ coinciding with pressure $p = \mathsf{K} - \mathsf{V}, \mathsf{K} \gg \mathsf{V}$ in all epochs of the Universe evolution including the chemical evolution, recombination, and the SN explosions. It differs from the accepted approach [13] proclaiming the dominance of the potential term $\mathsf{V}_{\text{Planck}} \gg \mathsf{K}_{\text{Planck}} \sim a^{-6}$ at the Planck epoch, when $a^{-1}_{\text{Planck}} \sim 10^{61}$ is valid, in spite of the kinetic term K has a huge enhancement factor of $a_I^{-6} \sim 10^{366}$ times with respect to the potential one V. Really, the accepted New Inflationary scenario [13] is not cleared up yet, because it proposes "a dynamical inflation" $\mathsf{V}_0 \neq \mathsf{V}_{\text{Planck}}$ and $\mathsf{K} \neq 0$, when the inflation equation $\rho = \mathsf{K} + \mathsf{V} = -p = -\mathsf{K} + \mathsf{V}$ is valid only, if $\mathsf{K} = 0$.

The scale-invariant cosmological model [8, 14, 15] has features different from the ones in both the widely accepted ΛCDM model [13, 16] and the Hoyle – Narlikar model [17]. Recall that the ΛCDM model [13] requires the dominance of the scalar field *potential* energy and explains the Cosmic Microwave Background (CMB) power spectrum [18] by the dynamical scalar metric component that is absent in the Wigner classification of relativistic states with the vacuum postulate [19] and the Dirac condition of the zero value of the local space element velocity [20].

Our construction in cosmological aspects is close to the Hoyle–Narlikar model [17], in which also the redshift mass dependence is considered. But contrary to that model we try to keep a direct relation to the GR and we have quite different phenomenological results. In particular, we avoid the problem of the Hoyle–Narlikar model related to negative energy contributions.

In this paper we construct a model where the dilaton zeroth mode lies in the accepted class of the relativistic state classification [19] in the theory of dynamical scale symmetry compatible with intensive cosmological creation of *primordial* W, Z, and Higgs particles from vacuum in the Early Universe [15].

The content of the paper is as follows. In Sect. 2 we remind the construction of the GR in conformal variables, defined by separation of the dilaton factors. In Sect. 3, a cosmological model obtained from the GR and SM by the average the dilaton and other scalar field over a large finite volume in accord with Einstein's cosmological principle [21], where the initial data Higgs effect is considered. Sect. 4 is devoted to the Early Universe as a factory of primordial particles created from vacuum due to the dilaton interaction.

2 General Relativity in Conformal Variables

2.1 Action and variables

Let us apply the relativistic principles to the theory of General Relativity (GR) supplemented by the Standard Model together with an additional scalar field Q governing the Universe evolution. So we start with the action

$$S_{\mathrm{U}}[g, f] = \int d^4x \sqrt{-g} \left[-\frac{R(g)}{6} + \mathcal{L}_{\mathrm{SM}}(f) + \partial_\mu Q \partial^\mu Q \right], \tag{2.1}$$

and units $\hbar = c = M_{\text{Planck}}\sqrt{3/(8\pi)} = 1$ are used throughout the paper. This action depends on a set of scalar, spinor, vector, and tensor fields $f_{(n)} = \phi, s, V_\mu, g_{\mu\nu}$ with their conformal weights $n = -1, -3/2, 0, 2$, respectively. Let us start with all SM particles being massless and return to the question of their mass generation later on.

In the relativistic theory this action is accompanied by a choice of the frame of reference of the initial data and a certain gauge fixing in order to determine real physical fields [22, 23] compatible with the Wigner classification of fields and their states [19]. So in this article, we use just these variables [22] with gauge-invariant initial data.

The corresponding generating functional is constructed by the Hamiltonian method in [14]. The Wigner classification of massless fields leads unambiguously to radiation variables well known as photons and gravitons with two transverse components [20, 24]. In particular, the transverse metric field components are associated with the tangent Minkowskian space-time and the Dirac-ADM parameterization [20, 25] with 3+1 splitting take the form $\omega_{(0)}$, $\omega_{(b)}$

$$ds^2 = \omega^2_{(0)} - \omega^2_{(b)}, \quad \omega_{(0)} = e^{-2D} N_{\mathrm{d}} dx^0, \quad \omega_{(b)} = e^{-D} \mathrm{e}_{(b)j}(dx^j + N^j dx^0), \tag{2.2}$$

where $\mathrm{e}_{(b)i}$ are the triads with the unit spatial metric determinant $|\mathrm{e}_{(b)i}| = 1$, N_{d} is the Dirac lapse function, and N^j are the shift vector components. Following Dirac [1] we call the spatial determinant logarithm $D[g] = -\log|g^{(3)}|/6$ the dilaton. It compensates scale transformations of fields with the conformal weight (n)

$$\widetilde{f}_{(n)} = e^{nD} f_{(n)}, \qquad \widetilde{g}^{(3)}_{ij} = e^{2D} g^{(3)}_{ij}, \qquad |\widetilde{g}^{(3)}_{ij}| = 1, \tag{2.3}$$

where the scale-invariant Lichnerowicz-type variables [3] appear on the left hand sides. So in this paper we consider GR as a theory of dynamical scale symmetry developed in Refs. [1, 11, 12] in terms of the selected variables (2.3) treated as the observable ones.

In phenomenological applications, one can identify this choice with the CMB co-moving reference frame[2].

These scale-invariant variables (2.3) compatible with relativistic classification of physical states can be considered as physical observables in a relativistic theory. This means that the theory excludes any cosmological expansion of the Lichnerowicz volume and removes any scale factor including the cosmological scale factor from the observable spatial metric components. However, these scale-invariant variables (2.3) do not remove the cosmological evolution. The evolution is described as the evolution of observable masses due to the zeroth dilaton harmonic.

2.2 Zeroth dilaton harmonic as cosmological evolution parameter

The 3+1 splitting (2.2) does not fix the coordinate evolution parameter x^0. The Hamiltonian form of the relativistic theory keeps symmetry with respect to reparameterizations $x^0 \to \widetilde{x}^0 \neq x^0$ [28]. Therefore, the coordinate evolution parameter as an object of reparameterizations can not be considered as a measurable quantity [29]. In such a relativistic theory one can choose two measurable times: an evolution parameter as one of variables in the superspace $D, \phi, Q, ..$ and the proper time-interval in the Minkowskian tangent space

[2] Relativistic invariance means the completeness of a set of reference frames [26, 27].

[30]. In this paper we show that there is a possibility to choose both these times, in the exact theory, so that the initial data of the superspace time-like variable coincide with its integrals of motion. It is just the case of the zeroth dilaton harmonic and so called Misner time [31].

Recall that in accord with Einstein's cosmological principle [21] we can average the dilaton and other scalar field over a large finite volume $V_0 = \int \widetilde{\omega}_{(1)} \wedge \widetilde{\omega}_{(2)} \wedge \widetilde{\omega}_{(3)} = \int d^3x$ [14, 32]. The dilaton zeroth harmonics can be defined as

$$\langle D \rangle = V_0^{-1} \int_{V_0} d^3x D(x^0, \vec{x}). \tag{2.4}$$

After the separation of the zeroth and nonzero harmonics

$$D = \langle D \rangle + \overline{D}, \quad \phi = \langle \phi \rangle + h/\sqrt{2}, \quad Q = \langle Q \rangle + q/\sqrt{2} \tag{2.5}$$

with the strong constraint $\int_{V_0} \overline{D} d^3x = 0$, $\int h d^3x = 0$, action (2.1) takes the form

$$S_{\rm U} \;=\; S_z(\langle D \rangle, \langle \phi \rangle, \langle Q \rangle) + \widetilde{S}_{\rm U}, \tag{2.6}$$

where $\widetilde{S}_{\rm U}$ repeats action (2.1) for nonzero harmonics associated with local excitations and

$$S_z(\langle D \rangle, \langle \phi \rangle, \langle Q \rangle) = \int dx^0 \left[-(\partial_0 \langle D \rangle)^2 + (\partial_0 \langle \phi \rangle)^2 + (\partial_0 \langle Q \rangle)^2\right] \int d^3x N_{\rm d}^{-1}(x^0, \vec{x}) \tag{2.7}$$

is the zeroth mode action, where we can introduce the average of the inverse Dirac lapse function

$$\int d^3x N_{\rm d}^{-1}(x^0, \vec{x}) = V_0 \langle N_{\rm d}^{-1} \rangle = V_0 N_0^{-1}, \tag{2.8}$$

with a finite volume V_0. In this case it is convenient to introduce the diffeo-invariant time interval $d\tau \equiv N_0 dx^0$.

The variation of action (2.6) with respect to the lapse function $N_{\rm d}$ leads to the energy constraint

$$N_{\rm d} \frac{\delta S_{\rm U}}{\delta N_{\rm d}} \;=\; 0 \;\to\; \frac{[\partial_0 \langle D \rangle]^2 - [\partial_0 \langle \phi \rangle]^2 - [\partial_0 \langle Q \rangle]^2}{N_{\rm d}} - N_{\rm d} \widetilde{T}_{\rm d} = 0, \tag{2.9}$$

where

$$\widetilde{T}_{\rm d} = -\frac{\delta \widetilde{S}_{\rm U}}{\delta N_{\rm d}} = -\frac{P_{\overline{D}}^2}{4} + \frac{4}{3} e^{-7D/2} \triangle e^{-D/2} + \sum_{J=0,2,3,4} e^{-JD} \mathcal{T}_J(\widetilde{F}) \tag{2.10}$$

is the local energy density, $\triangle = \partial_i [{\rm e}^i_{(a)} {\rm e}^j_{(a)} \partial_j]$ is the Laplace operator,

$$P_{\overline{D}} = 2\left[(\partial_0 - N^l \partial_l)\overline{D} + \partial_l N^l / 3\right] / N_{\rm d} \tag{2.11}$$

is the local dilaton momentum. We emphasize that the local energy density $\widetilde{T}_{\rm d}$ is a sum of partial energy densities $\mathcal{T}_J = \langle \mathcal{T}_J \rangle + \overline{\mathcal{T}}_J$ in terms of the scale-invariant fields (2.3) and

provides cosmological regimes of the rigid state of matter $J = 0$, radiation $J = 2$, massive matter $J = 3$, and curvature $J = 4$ [14].

Such, the separation of zeroth harmonics in finite space-time does not commutate with variations of the Hilbert action. Therefore, we obtained energy constraint (2.9) that coincided with the accepted Einstein equation $\widetilde{T}_{\rm d} = 0$ only in the infinite volume limit $\langle D \rangle = 0$ [20]. Nevertheless, the additional term solves the energy problem of GR in agreement with the Universe evolution [14].

Averaging the energy constraint (2.9) over the volume V_0 leads to the global constraint

$$[\partial_\tau \langle D \rangle]^2 = [\partial_\tau \langle \phi \rangle]^2 + [\partial_\tau \langle Q \rangle]^2 + \left\langle \sqrt{\widetilde{T}_{\rm d}} \right\rangle^2 \tag{2.12}$$

and determines the diffeo-invariant lapse function

$$\mathcal{N} = \frac{N_{\rm d}}{N_0} = \left\langle \sqrt{\widetilde{T}_{\rm d}} \right\rangle / \sqrt{\widetilde{T}_{\rm d}} \tag{2.13}$$

and the diffeo-invariant interval $d\tau = N_0 dx^0$ through the energy density (2.10).

The dilaton field $D = \langle D \rangle + \overline{D}$ is defined by equation

$$\frac{\delta S_{\rm U}}{\delta D} = 0 \;\to\; 2\partial_\tau^2 \langle D \rangle \;=\; \langle T_D \rangle, \tag{2.14}$$

$$(\partial_\tau - \mathcal{N}_{(b)} \partial_{(b)}) P_{\overline{D}} \;=\; T_D - \langle T_D \rangle, \tag{2.15}$$

where $P_{\overline{D}}$ is the dilaton momentum (2.11), $\mathcal{N}_{(b)} = N^l {\rm e}_{(b)l} / N_0$, $\partial_{(b)} = {\rm e}^l_{(b)} \partial_l$, and

$$T_D = \frac{2}{3} \left\{ 7\mathcal{N} e^{-7D/2} \triangle e^{-D/2} + e^{-D/2} \triangle \left[\mathcal{N} e^{-7D/2}\right] \right\} + \mathcal{N} \sum_{J=0,2,3,4} J e^{-JD} \mathcal{T}_J. \tag{2.16}$$

One can see that Eqs. (2.12), (2.14) can be treated as the exact analogy of the Friedman equations in the tangent space-time defined by simplex components

$$\widetilde{\omega}_{(0)} = e^{-2D} \mathcal{N} d\tau = e^{-2\overline{D}} \mathcal{N} d\eta, \qquad \widetilde{\omega}_{(b)} = {\rm e}_{(b)k} dx^k + \mathcal{N}_{(b)} d\eta. \tag{2.17}$$

After the separation of zero modes $P_{(F)} = 2V_0 \partial_\tau \langle \phi \rangle$, where $F = D, \phi, Q$, the action (2.1) take the form

$$S_{\rm U} \;=\; \widetilde{S}_{\rm U} + S_z \tag{2.18}$$

$$S_z \;=\; \int \left\{ P_{\langle Q \rangle} d\langle Q \rangle + P_{\langle \phi \rangle} d\langle \phi \rangle - P_{\langle D \rangle} d\langle Q \rangle + \frac{P^2_{\langle D \rangle} - P^2_{\langle \phi \rangle} - P^2_{\langle Q \rangle}}{4V_0} N_0(x^0) dx^0 \right\}; \tag{2.19}$$

here $\widetilde{S}_{\rm U}$ repeats Hamiltonian action (??) given in [14] for nonzero harmonics associated with local excitations. In action (2.18), variable $\langle D \rangle$ plays the role of a evolution parameter [14, 32]. In this case, its momentum values determined by the energy constraint

$$P^2_{\langle D \rangle} = {\rm E}^2_{\rm U} \equiv P^2_{\langle \phi \rangle} + P^2_{\langle Q \rangle} + 4\left[\int d^3x \sqrt{\widetilde{T}_{\rm d}}\right]^2 \tag{2.20}$$

are considered as the Universe energy $P_{\langle Q\rangle} = \pm \mathsf{E}_{\mathrm{U}}$ in the Wheeler-DeWitt superspace $[\langle D\rangle|\langle\phi\rangle, \langle Q\rangle, \widetilde{F}]$ [33].

The equation $P_{\langle D\rangle} = 2V_0\partial_\tau\langle D\rangle = \mathsf{E}_{\mathrm{U}}(\langle D\rangle)$ leads to the time-interval as a function of the evolution parameter $\langle D\rangle$ in the superspace, and the initial data

$$\tau[\langle D\rangle_I, \langle D\rangle_0] = 2V_0\int_{\langle D\rangle_I}^{\langle D\rangle_0} d\langle D\rangle/\mathsf{E}_{\mathrm{U}}(\langle D\rangle). \tag{2.21}$$

This relation is treated as the Universe evolution. Thus, in the model we have two measurable times: the zeroth dilaton harmonic $\langle D\rangle$ as evolution parameter in superspace and the proper time-interval $\tau(x^0) = \tau(\widetilde{x}^0)$, where x^0 is the coordinate evolution parameter as an object of reparameterizations $x^0 \to \widetilde{x}^0 \neq x^0$ that can not be considered as a measurable quantity [29].

We emphasize that the reparameterization invariance play the crucial role [28]. It leads to the energy constraint (2.20). Solutions of this constraint with respect to evolution momentum $P_{\langle D\rangle}$ are treated as energies of the relativistic Hamiltonian system in the superspace, and they take positive and negative values. Due to the negative energy this relativistic system is not stable because it has no vacuum. In accord with the QFT experience the stable vacuum is constructed by the primary quantization and secondary one [19, 14]. Quantization of the energy constraint (2.20) leads to a vacuum state with minimal energy and the arrow of time-interval [32].

In [14], where the Hamiltonian cosmological perturbation theory with respect to $\overline{D}$, $(\mathcal{N}-1)$, and $\mathcal{N}_{(b)}$ is constructed, it was shown that the conformal flat approximation $\overline{D} = (\mathcal{N}-1) = 0, \mathcal{N}_{(b)} = 0$ is valid far from heavy bodies.

3 Conformal Cosmology

3.1 Conformal flat space-time

Action (2.18) in the lowest order of interactions, where the metric components $\mathcal{N}_{(b)}, \mathcal{N}$ treated as the Newton like potentials with zero momenta $P_{\mathcal{N}_{(b)},\mathcal{N}} = 0$ are neglected $\mathcal{N}-1 = \mathcal{N}_{(b)} = 0$, takes the form

$$S^0_{\mathrm{U}} = \int d\tau \left\{ \left[\int d^3x \left(\sum_{\widetilde{f}} p_{\widetilde{f}}\partial_\tau\widetilde{f} - \sum_{\widetilde{f}^*} p_{\widetilde{f}^*}\widetilde{f}^*\partial_\tau\langle D\rangle\right)\right] - e^{-2\langle D\rangle}\mathsf{H}_{\mathrm{SM}}[\langle D\rangle]\right\}\Bigg|_{d\tau=N_0dx^0}$$
$$+\int\limits_{\tau=0}^{\tau_0} d\tau\left[P_{\langle Q\rangle}\partial_\tau\langle Q\rangle + P_{\langle\phi\rangle}\partial_\tau\langle\phi\rangle - P_{\langle D\rangle}\partial_\tau\langle D\rangle + \frac{P^2_{\langle D\rangle} - P^2_{\langle\phi\rangle} - P^2_{\langle Q\rangle}}{4V_0}\right]\Bigg|_{d\tau=N_0dx^0}, \tag{3.1}$$

where $\widetilde{f} = h, q, \widetilde{s}, v, \widetilde{h}^{\mathrm{TT}}$ are dynamical degrees of freedom of the initial theory (2.1) treated as the massless and massive particles in the conformal flat space-time $\widetilde{ds}^2 = (d\eta)^2 - (dx^k)^2$; here

$$d\eta = a^2N_0(x^0)dx^0 = a^2d\tau, \qquad a = e^{-\langle D\rangle} \tag{3.2}$$

a is the cosmological scale factor in the homogeneous background of the zeroth mode harmonics (2.7), and H_{SM} is the Standard Model (SM) Hamiltonian, where we distinguish

all fields $\widetilde{f}^* = \widetilde{h}, \widetilde{q}, Z^{||}, W^{||}, \widetilde{h}^{\mathrm{TT}}$ with the direct interaction with the dilaton described the zeroth mode action S_z (2.19)[3]. This interaction leads to both the cosmological particle creation [15, 34, 35, 36, 37], if we quantize the first (particle) action, and the cosmological universe creation, if we quantize the second (universe) action (3.1) [32].

We call an approach as the Universe without particles, if it is filled with only with zeroth harmonics of all the scalar fields $F = D$, ϕ, and Q with equations $\partial^2_\tau\langle F\rangle = 0$ and the initial data [38]:

$$\langle\phi\rangle(\tau\to 0) = \langle\phi\rangle_I, \quad \partial_\tau\langle\phi\rangle(\tau\to 0) = \partial_\tau\langle\phi\rangle_I; \tag{3.3}$$
$$\langle Q\rangle(\tau\to 0) = \langle Q\rangle_I, \quad \partial_\tau\langle Q\rangle(\tau\to 0) = \partial_\tau\langle Q\rangle_I. \tag{3.4}$$

These initial data are chosen as follows

$$\langle\phi\rangle_I = M_{W0}/(g\sqrt{2}), \quad \partial_\tau\langle\phi\rangle_I = 0; \qquad \langle Q\rangle_I = Q_0, \qquad \partial_\tau\langle Q\rangle_I = H_0 \tag{3.5}$$

defined so that the mechanism of the spontaneous electroweak symmetry breaking does not differ from the one accepted in the SM. Quantities M_{W0} and H_0 are the present day values of the W boson mass and the Hubble parameter. Note that in the SM we can start with a completely scale invariant Lagrangian, while all the SM particle masses can be generated by the standard Higgs mechanism. The only difference of our approach from the standard one is the interpretation of the vacuum expectation origin. Namely, in the SM we usually say that $\langle\phi\rangle \neq 0$ appears because of the presence of tachyon-like mass treated as a *fundamental* parameter in the Lagrangian. In our approach we suggest to consider $\langle\phi\rangle \neq 0$ as an external condition or initial data for the equations[4] so that the SM potential in the form $\mathbf{V}(\phi) \sim (\phi^\dagger\phi - \langle\phi\rangle^2)^2$ appeared as an effective potential *after* introduction of the initial data, as discussed below in Sect. 3.3.

There is the approximation used in the accepted cosmological perturbation theory [16], where the "gauge" $N_0 = 1$ is substituted in action (3.1) and reparameterization invariance $x^0 \to \widetilde{x}^0$ is removed, so that one obtains the accepted SM action in flat space without the constraint $\delta S/\delta N_0 = 0$ and all time-intervals become equivalent $d\eta = dx^0$.

If the cosmological dynamics is neglected $\langle D\rangle = \langle Q\rangle = 0$, action (3.1) takes form

$$S_{\mathrm{SM}} = \int d\tau\left\{\left[\int d^3x \sum_{\widetilde{f}} p_{\widetilde{f}}\partial_\tau\widetilde{f}\right] - \mathsf{H}_{\mathrm{SM}}\right\}. \tag{3.6}$$

The flat space-time action (3.6) loses the energy constraint $\delta S/\delta N_0 = 0$, cosmological evolution, and creation of the Universe and its matter content due the direct dilaton interaction $\sum_{\widetilde{f}^*} p_{\widetilde{f}^*}\widetilde{f}^*\partial_\tau\langle D\rangle$.

Thus, the main consequence of the cosmological dynamics $\langle D\rangle = -\log a$ in action (3.1) is the energy constraint

$$P^2_{\log a} = \mathsf{E}^2_{\mathrm{U}}, \tag{3.7}$$
$$\mathsf{E}^2_{\mathrm{U}} \equiv P^2_{\langle\phi\rangle} + P^2_{\langle Q\rangle} + 4V_0a^2\mathsf{H}_{\mathrm{SM}}(a) \tag{3.8}$$

[3] This action does not contain both the dynamical inflation with $\Lambda_I \neq \Lambda_0$ and dynamical acoustic waves with $P_{\overline{D}} \neq 0$ in accord with the Wigner classification of states with positive energy.

[4] Remind analogy of the Higgs mechanism to the classical Ginsburg-Landau potential in the theory of superconductivity where the form of the potential leading to symmetry breaking was also provided by external conditions but not by a *fundamental* term in the Lagrangian.

obtained by the variation of action (3.1) with respect to the lapse function N_0. This energy constraint like its GR analog (2.20) determines the Universe energy as the constraint values of the momentum of evolution parameter $\langle D\rangle = -\log a$ in the WDW superspace of events

$$P_{\log a} = \pm \mathsf{E}_{\mathrm{U}}(a) = \pm 2V_0 \frac{da}{a d\tau} \tag{3.9}$$

together with all attributes of relativistic quantum theory [32]:

1. the proper time relation (2.21)

$$\tau(a_0, a_I) = 2V_0 \int_{a_I}^{a_0} \frac{da}{a \mathsf{E}_{\mathrm{U}}(a)} \simeq H_0^{-1} \int_{\eta_I = r_0}^{\eta_0 = r_0} a^{-2}(\eta) d\eta, \tag{3.10}$$

   treated as the cosmological redshift $(1+z) = a^{-1}$ — coordinate distance $r = \eta_0 - \eta$ relation,

2. the primary quantization $[\hat{P}^2_{\langle \log a\rangle} - \mathsf{E}_{\mathrm{U}}^2]\Psi_{\mathrm{WdW}} = 0$,

3. the secondary quantization $\Psi_{\mathrm{WdW}} = [1/\sqrt{\mathsf{E}_{\mathrm{U}}}][A^+ + A^-]$,

4. the Bogoliubov transformations $A^+ = \alpha B^+ + \beta B^-$ that diagonalize equations of motion $\partial_{(D)} B^{\pm} = \pm i \mathsf{E}_{\mathrm{U}}^B B^{\pm}$, where $\mathsf{E}_{\mathrm{U}}^B$ is the Bogoliubov energy, given in [32],

5. and the vacuum state $B^-|0>= 0$ as the one of minimal energy.

In this case the low-energy decomposition of the cosmological action (3.1)

$$\mathsf{E}_{\mathrm{U}}^B d\log a \simeq \mathsf{E}_{\mathrm{U}} d\log a = 2V_0 \sqrt{H_0^2 + a^2 \mathsf{H}_{\mathrm{SM}}(a)/V_0}\, d\log a \tag{3.11}$$

$$\simeq 2V_0\, d\log a + d\eta \mathsf{H}_{\mathrm{SM}}(a) \tag{3.12}$$

leads to the standard QFT evolution operator

$$< \Phi_I|\hat{U}(\eta_I, \eta_0)|\Phi_0 >=< \Phi_I|T\exp\left\{-i\int_{\eta_I}^{\eta_0} d\eta\, \mathsf{H}_{\mathrm{int}}\right\}|\Phi_0 >= \int \left[\prod_f df\right] \exp\{iS_{\mathrm{SM}}(f|\eta_0)\} \tag{3.13}$$

in the interaction representation in the QFT with action (3.6), where masses depend on the cosmological scale factor and the states defined as the irreducible unitary representations of the Poincaré group, $\mathsf{H}_{\mathrm{int}} = \mathsf{H}_{\mathrm{SM}} - \mathsf{H}_0$.

Eq. (3.8) shows us that the SM particle energy disappears in the Early Universe limit $a \to 0$, where the initial data are defined. Therefore, we can consider the Early Universe neglecting all the SM particle like contribution $\widetilde{S}_{\mathrm{U}} \simeq 0$ in Eq. (2.6).

Thus, the quantum cosmological dynamics $\langle D\rangle = -\log a$ gives us a possibility to describe the creation of Universe at the beginning of the proper time $\tau \geq 0$ and the vacuum creation of primordial particles distinguished by their direct dilaton interactions in action (3.1). In comparison with the Inflationary Model, the Empty Universe in the model under consideration is described by the equation of state $\rho = p$, instead of inflation $\rho = -p$.

3.2 The Universe before particle creation

Let us use as an example the Universe in epoch before particle creation. One can see that the Standard Cosmology observable quantities are connected with the conformal ones by relation (2.3) $F_{(n)\mathrm{SC}} = e^{-n\langle D\rangle}\widetilde{F}_{(n)\mathrm{CC}}$. In the epoch, the dilaton solution of the equation of motion $\partial_\tau^2\langle D\rangle = 0$ takes the form $\langle D\rangle = \langle D\rangle_0 + H_0(\tau - \tau_0)$. In terms of the effective cosmological factor $a = e^{-\langle D\rangle}$ and conformal time $d\eta = a^2 d\tau$ this solution becomes

$$a(\eta) = a_0\sqrt{1 + 2H_0(\eta - \eta_0)}. \tag{3.14}$$

In general case, the dilaton exact equations (2.12) and (2.14) in the approximation $\tilde{\omega}_{(0)} = d\eta$ and $\tilde{\omega}_{(b)} = dx_{(b)}$ coincide the Friedman equations in terms of standard conformal variables

$$(a\,a')^2 = H_0^2\Omega_{(0)}(z), \tag{3.15}$$

$$2[(a\,a')^2 + a^3 a''] = H_0^2\Omega_{(1)}(z), \tag{3.16}$$

where $a' = da/d\eta$ and

$$\Omega_{(n)}(z) = \sum_{J=0,2,3,4,6} (2J)^n (1+z)^{2-J}\Omega_J, \quad \Omega_J = \langle T_J\rangle/H_0^2, \tag{3.17}$$

and $\Omega_{J=0,2,3,4,6}$ are partial density of states: rigid, radiation, matter, curvature, Λ-term, respectively; $\Omega_{(0)}(0) = 1$, and H_0 is Hubble parameter.

The cosmological dynamics of the Conformal Cosmology (CC) differs from the accepted Standard Cosmology (SC) including the ΛCDM model [13] by a constant volume defined by Eqs. (2.2), running masses, conformal time, and the CMB conformal temperature $T_{CC} = T_{SC}a(z) = 2.725$ K during the cosmological evolution process. The dilaton variables (2.3) and (2.2) explain the redshift by a permanent increase of all masses in the Universe [8, 10, 14, 15]. The corresponding luminosity-distance – redshift relation $H_0\widetilde{\ell}(z) = z + z^2/2$ does not contradict the recent SN data [7]. Thus, Conformal Cosmology describes SN Ia data by the dominance of kinetic energy of the scalar field (3.5) and (3.14) instead of unknown source of dynamical inflation $\Lambda_0 \neq \Lambda_I$ used in the ΛCDM model [13]

Calculation of the primordial helium abundance [9, 39] takes into account weak interactions, the Boltzmann factor, (n/p) $e^{\Delta m/T} \sim 1/6$, where Δm is the neutron-proton mass difference, which is the same for both SC and CC, $\Delta m_{SC}/T_{SC} = \Delta m_{CC}/T_{CC} = (1+z)^{-1}m_0/T_0$, and the square root dependence of the z-factor on the measurable time-interval defined in Eq. (3.14) $(1+z)^{-1} \simeq \sqrt{1 + 2H_0(\eta - \eta_0)}$ explained by the dominant rigid state. In the SC, where the measurable time-interval is identified with the Friedman time, this square root dependence of the z-factor is explained by the radiation dominance.

3.3 The Initial Data Higgs Effect in the Early Universe

Recall, that the accepted spontaneous symmetry breaking mechanism is based on the Coleman–Weinberg potential equation $d\mathbf{V}(\langle\phi\rangle_I)/d\langle\phi\rangle_I = 0$ defined by Eq. (3.13),

$$\mathrm{V}_{\mathrm{eff}}(\phi_I) = i\frac{d}{V_0 d\eta_0}\log\left\{\int \left[\prod_f df\right] \exp\{iS_{\mathrm{SM}}(f|\eta_0)\}\right\}\Bigg|_{\phi(\eta,x)=\phi_I + h(\eta,x), \int d^3x h = 0}, \tag{3.18}$$

in the perturbation theory restricted by the constraints $\partial_\tau\langle\phi\rangle = 0$ and $\mathbf{V}(\langle\phi\rangle_I) = 0$. In the perturbation theory (3.5), loop diagrams also lead to an effective potential with the same equation $d\mathbf{V}(\langle\phi\rangle_I)/d\langle\phi\rangle_I = 0$ treated as a constraint that keeps the vacuum equation $\partial^2_\tau\langle\phi\rangle = 0$.

We would like to emphasize that the effective Higgs potential (well known as the Coleman – Weinberg one) is compatible with both mechanisms of the spontaneous scale symmetry breaking the kinetic one define by Eqs (3.3), (3.4), and (3.5) and the accepted Higgs potential with the tachyon term, and the additional potential $\mathbf{V}_q(Q)$ that governs the Universe evolution.

Recall that the accepted Higgs Effect is based on supposition that both the fields (ϕ, Q) are given in the class of the constant fields $(\phi(t) = \phi_{\rm cl}, Q_{\rm cl})$ given by the fundamental parameters of the equations of motion.

4 The Early Universe as Factory of Higgs Particles

4.1 Cosmological creation

The next consequence of the scalar field zeroth modes is the cosmological creation of primordial particles from the stable vacuum [15] as the origin of the Universe and its matter that can be associated with the event known as "Big Bang".

Here we propose that all present-day matter content of the Universe state ${}_{\eta=\eta_0}< U|$ is the final decay product of the primordial particles created from the stable vacuum ${}_{\eta=\eta_I}< 0|$. In this case, one can obtain the expectation value ${}_{\eta=\eta_0}< U^*|\mathsf{H}|U >_{\eta=\eta_0}$ of the energy constraint (3.8) in the interaction representation using the low-energy decomposition (3.11), (3.12) of the cosmological action (3.1)

$$a^2 a'^2 = H_0^2\Omega_{\rm rigid} + a^2{}_{\rm out}< U^*|\mathsf{H}_0|U >_{\rm in} V_0^{-1}, \qquad (4.1)$$

$$|U >_{\rm in} = U|0>_{\rm in} = \hat{T}\exp\left\{-i\int\limits_{\eta_I}^{\eta_0} d\eta\, \mathsf{H}_{\rm int}\right\}|0>_{\rm in}\,. \qquad (4.2)$$

This density reflects the evolution of the matter content of the Universe during its life-time.

The factor $a^2 = e^{-2\langle D\rangle}$ in action (3.1) means that at the initial instance $\eta = 0$, when there were no any particle-like excitations, $< 0|\hat{n}_{\widetilde{f}}|0 >_{(\eta=0)} = 0$, and hence the temperature was equal to zero.

Really, at the beginning, when $\eta_0 \simeq \eta_I$ in accord with the Eq. (4.1), we have the intensive creation of vector bosons $v = W, Z$, Higgs particles h, gravitons $h^{\rm TT}$, and an additional q-field distinguished by the direct interaction with the dilaton $p_{\widetilde{h}}\widetilde{h}\partial_\eta\langle D\rangle$ in action (3.1). Due to this interaction the field equations in terms of creation and annihilation operators

$$\partial_\eta \widetilde{h}^\pm_{\rm k}(\eta) = \pm i\omega_h \widetilde{h}^\pm_{\rm k}(\eta) + i\partial_\eta\left[\langle D\rangle - \log\omega_{{\rm k}h}/2\right]\widetilde{h}^\mp_{\rm k}(\eta) + i[\mathsf{H}_{\rm int}, \widetilde{h}^\pm_{\rm k}(\eta)], \qquad (4.3)$$

are not diagonal; here $h(\eta,\mathbf{x}) = \dfrac{1}{V_0}\sum_{{\rm k},{\rm k}^2\neq 0} e^{i{\rm kx}}h_{\rm k}(\eta)$, $h_{\rm k}(\eta) = \dfrac{1}{\sqrt{2\omega_{hk}}}\left[h^+_{\rm k}(\eta) + h^-_{-{\rm k}}(\eta)\right]$, $\omega_{hk} = \sqrt{k^2_{\rm k} + \widetilde{m}^2_h}$, $\widetilde{m}_h = am_{0h}$, $\mathbf{k} = \dfrac{2\pi}{V_0^{1/3}}\mathbf{l}$.

In order to obtain integrals of motion in the approximation $\mathsf{H}_{\rm int} \sim 0$, these field equations are diagonalized by the Bogoliubov transformation of the operator of particle $\widetilde{h}^+_{\rm l} = \alpha b^+_{\widetilde{h},{\rm l}} + \beta^* b^-_{\widetilde{h},{\rm l}}$, so that the free equations of motion of the Bogoliubov quasiparticle become diagonal $\partial_\eta b^\pm_{\widetilde{h},{\rm l}} = \pm i\omega_b b^\pm_{\widetilde{h},{\rm l}}$, where ω_b is the quasiparticle energy [15, 36, 37]. The stable vacuum is defined by $b^-_{\widetilde{h},{\rm l}}|0>= 0$, where $b^-_{\widetilde{h},{\rm l}}$ is the operator of annihilation of a quasiparticle.

Creation of these primordial particles started at the moment a_{fI} when their wavelengths coincided with the horizon length

$$\widetilde{m}^{-1}_{fI} = [a_{fI}m_{f0}]^{-1} \sim \widetilde{H}^{-1}_{fI} = a^2_{FI}H_0^{-1} \qquad (4.4)$$

as it follows from the uncertainty principle.

In this case, the relativistic gas of primordial particles created at the instance a_{fI} acquires the Boltzmann – Chernikov distribution function [40, 41]

$$B({\rm k}, \widetilde{T}_f) = \left\{\exp\left[\left(\sqrt{{\rm k}^2 + \widetilde{m}^2_f} - \widetilde{m}_f\right)/k_{\rm B}\widetilde{T}_f\right] - 1\right\}^{-1}\theta(\log a - \log a_{fI}). \qquad (4.5)$$

arising due to interactions and collisions as it was shown in Ref. [42] using as an example the scalar field model with $\lambda\phi^4$ interaction [15, 42]. Here the conformal boson temperature $\widetilde{T}_{\rm f} \sim T_0$ is determined by the collision integral kinetic equation

$$\widetilde{n}(\widetilde{T}) = [\widetilde{\sigma}_{\rm f\ scat}r_{\rm f}]^{-1} \qquad (4.6)$$

where $\sigma_{\rm f\ scat}$ is cross section, the free length $r_{\rm f}$ is identified with the horizon $\widetilde{d}(z) = a(z)^2H_0^{-1}$ in CC [8]

$$r_{\rm f} = [\widetilde{n}(\widetilde{T}_{\rm f})\widetilde{\sigma}_{\rm f\ scat}]^{-1} \simeq \widetilde{d}(z) = a(z)^2H_0^{-1}, \qquad (4.7)$$

and $\widetilde{n}(\widetilde{T}_{\rm f})$ is the number density:

$$\widetilde{T}_f \gg m_f \qquad \widetilde{n}_{\rm rel}(\widetilde{T}_f) = N_f\frac{\xi(3)}{\pi^2}\widetilde{T}^3; \qquad \frac{\xi(3)}{\pi^2} \simeq 0.1218 \qquad (4.8)$$

$$\widetilde{T}_f \sim m_f \qquad \widetilde{n}_{\rm Maxwell}(\widetilde{T}_f) = N_f\frac{[\widetilde{m}_f\widetilde{T}_f]^{3/2}}{(2\pi)^{3/2}}. \qquad (4.9)$$

If $|\beta_{f,k}|^2 \simeq |\beta_f|^2$, it is naturally to suppose

$$< 0|\mathsf{H}_0|0 > V_0^{-1} = \sum_{f=h,q,v,h^{\rm TT}} |\beta_f|^2 \int\frac{d^3k}{(2\pi)^3}\omega_{f,k}B({\rm k}, \widetilde{T}_f). \qquad (4.10)$$

In this case, there is the set of arguments [14, 15] that density the primordial particles and their decay products can explain the present day content of the present-day density (4.1) and (4.6) and give us all cosmological parameters, if one supposes initial data and the dominance of kinetic state $\rho_{\rm rigid} = p$ for all epochs including the beginning of the Universe when primordial vector bosons and Higgs particles were created from vacuum. In particular, one can see that there is a remarkable coincidence of the wavelengths of vector bosons at the instance when they equal to the horizon length with the CMB temperature

$$\widetilde{M}^{-1}_{WI} = [a_{WI}M_{0W}]^{-1} \simeq \widetilde{H}^{-1}_{WI} = a^2_{WI}(H_0)^{-1} \rightarrow a_{WI} = \left(\frac{H_0}{M_{W0}}\right)^{1/3} = 2.68\cdot 10^{-15}. \qquad (4.11)$$

with the scale factor $a_{\gamma I}$ at the instance when the initial value of the CMB temperature is equal to the Hubble parameter

$$T_{\gamma I} = H_{\gamma I} \to a_{\gamma I} = (1+z)_{\gamma I}^{-1} = \left(\frac{T_0}{H_0}\right)^{1/2} = 2.56 \cdot 10^{-15} \simeq a_{WI}. \tag{4.12}$$

The CMB temperature estimated from standard collision integral (4.8) is close to the CMB temperature $T_{CC} \simeq (M_W^2 H_0)^{1/3} \sim 3$ K [42, 43, 44], if $\widetilde{n}(\widetilde{T}) \sim \widetilde{T}^3$, $\sigma \sim M_W^{-2}$. Note that the relaxation time is less than the lifetime of the primordial vector bosons by the factor $2\alpha_W$, where α_W is the Weinberg constant.

The lifetime η_L of primordial bosons in the early Universe can be estimated by using the equation of state $a_{WL}^2 = a^2(\eta_L) = a_{WI}^2[1 + 2\widetilde{H}_{WI}(\eta_L - \eta_I)]$ and the W-boson lifetime within the Standard Model. Specifically, we have

$$\frac{a_{WL}^2}{a_{WI}^2} = 1 + 2\widetilde{H}_{WI}(\eta_L - \eta_I) \simeq \frac{\widetilde{H}_{WI}}{\widetilde{M}_{WL}}\frac{2}{\alpha_W} = \frac{a_{WI}}{a_{WL}}\frac{2}{\alpha_W}, \tag{4.13}$$

where α_W is the Weinberg constant and $\widetilde{M}_{WI} \simeq \widetilde{H}_{WI}$. From the solution of Eq. (4.13), $a_{WL}^2/a_{WI}^2 = (2/\alpha_W)^{2/3} \simeq 16$ it follows that the lifetime of primordial bosons is an order of magnitude longer than the Universe relaxation time $\eta_I = (2\widetilde{H}_{WI})^{-1}$:

$$\eta_L - \eta_I \simeq 15(2\widetilde{H}_{WI})^{-1}. \tag{4.14}$$

Also the present-day data of photon density coincides with the primordial vector boson density, $\Omega_{\rm rad} \simeq M_W^2 \cdot a_{\gamma I}^{-2} = 10^{-34}10^{29} \sim 10^{-5}$. The present-day baryon density is calculated by the evolution of the baryon density from the early stage, when it was directly related to the photon density, $\Omega_{\rm b} \sim \alpha_W \simeq 0.03$ [15].

From the ratio of the number of baryons to the number of photons, one can deduce an estimate of the superweak-interaction coupling constant: $X_{\rm CP} \sim 10^{-9}$. Thus, the evolution of the Universe, primary vector bosons, and the aforementioned superweak interaction [45] lead to baryon-antibaryon asymmetry of the Universe

$$\frac{n_{\rm b}(\eta_L)}{n_\gamma(\eta_L)} \simeq X_{\rm CP} = 10^{-9}. \tag{4.15}$$

Thus, the primordial bosons before their decays polarize the Dirac fermion vacuum and give the baryon asymmetry frozen by the CP – violation so that for billion photons there is only one baryon (see [15]).

4.2 Possible CMB data test of two photon decay of Higgs particle

The Bogoliubov transformation of the operator of particle $\widetilde{f}_{\rm k}^+ = \alpha b_{f,{\rm k}}^+ + \beta^* b_{f,{\rm k}}^-$ in the matrix element
${}_{\rm out}\!< U^*|\sum {\sf H}_0|U>_{\rm in} = {}_{\rm out} < U^*|\sum_{f,{\rm k}} \omega_{f,{\rm k}} \widetilde{f}_{\rm k}^+ \widetilde{f}_{\rm -1}^-|U>_{\rm in}$ leads to three terms

$$\begin{aligned}\widetilde{f}_{\rm k}^+ \widetilde{f}_{\rm -k}^- &= (\alpha b_{f,{\rm k}}^+ + \beta^* b_{f,{\rm k}}^-)(\alpha^* b_{f,{\rm k}}^- + \beta b_{f,{\rm -k}}^+) \\ &= |\beta|^2 b_{f,1}^- b_{f,{\rm -k}}^+ + |\alpha|^2 b_{f,{\rm k}}^+ b_{f,{\rm -k}}^- + \left(\alpha\beta b_{f,{\rm k}}^+ b_{f,{\rm -k}}^+ + \alpha^*\beta^* b_{f,{\rm k}}^- b_{f,{\rm k}}^-\right), \end{aligned} \tag{4.16}$$

where U is given by Eq. (4.2). Our hypothesis is that the first term is associated with the formation of the temperature [15] inherited by the CMB, and the second one ${}_{\rm out}\!< U^*|\left((\alpha\beta b_{f,{\rm k}}^+ b_{f,{\rm -k}}^+ + \alpha^*\beta^* b_{f,{\rm k}}^- b_{f,{\rm k}}^-\right)|U>_{\rm in}$, in the next orders of interactions ${\sf H}_{\rm int}$, give additional photons that can explain the CMB power spectrum by the two-gamma processes.

Really, the arguments (4.11) and (4.12) considered before mean that the CMB photons can inform us about the parameters of electroweak interactions and masses, including the Higgs particle mass [46], and a possibility to estimate the magnitude of the CMB anisotropy.

The collision integral equations (4.7) and (4.9) generalized to anisotropic decays $T_0 \to T_0 + \Delta T$, $\sigma \to \sigma + \sigma_{\gamma\gamma}$ gives us the formula $|\Delta T/T_0| \simeq (2/3)|\sigma_{\gamma\gamma}/\sigma|$ and a possibility to estimate the magnitude of the CMB anisotropy.

Its observational value about $\alpha_{\rm QED}^2 \sim 10^{-5}$ [18] testifies to the dominance of the two photon processes. Therefore, the CMB anisotropy revealed in the region of the three peaks $\ell_1 \simeq 220$, $\ell_2 \simeq 546$, and $\ell_3 \simeq 800$ can reflect parameters of the primordial bosons and their decay processes, in particular $h \to \gamma\gamma$, $W^+W^- \to \gamma\gamma$, and $ZZ \to \gamma\gamma$.

In this case, two-photonic decays of primordial Higgs particles and annihilation processes of primordial W, and Z bosons can explain three clear peaks in the CMB power spectrum without any acoustic waves with negative energy [13], if the values for multipole momenta ℓ_P of their processes are proportional to number of emitters at the horizon length $\ell_P = \widetilde{H}^{-1}(z_{\rm P})\widetilde{M}(z_{\rm P}) \sim (1+z_{\rm P})^{-3}$ (in accord with the new CC analysis of supernovae type Ia data) and the energy of any photon is proportional to the mean photon energy in CMB multiplied by the effective scale factor $(1+z_P)^{-1} \sim \ell_P^{1/3}$ in accord with the experience of description of the recombination epoch and the primordial helium abundance [9, 39].

One can be convinced that first three peaks $\ell_1 = 220, \ell_2 = 546, \ell_3 = 800$ reflect the ratio of W and Z masses

$$M_Z/M_W = 1.134 \approx (800/546)^{1/3} = 1.136 \;\to\; [\sin^2\theta_W \approx 0.225] \tag{4.17}$$

and the value of Higgs particle mass as

$$m_h = 2M_W\,(\ell_1/\ell_2)^{1/3} = 2M_W\,(220/546)^{1/3} \simeq 118\,{\rm GeV}. \tag{4.18}$$

The Higgs boson mass is close to the present fit of the LEP experimental data supporting rather low values just above the experimental limit $m_h > 114.4$ GeV. To get a more accurate estimate of the Higgs mass and a better description of the CMB power spectrum within the model under consideration, one has to perform an involved analysis of the kinetic equations for nonequilibrium Universe [44] with primordial particle creation and subsequent decays.

5 Summary

In the paper we tried to describe new observational data in the framework of GR and potential free SM, where all cosmological observables are identified with conformal variables [1, 3, 11, 12] and the expansion of the Universe is replaced by the mass evolution. These conformal variables are consistent with relativistic quantum field theory defined in the tangent space-time [5]. We claim that the model is compatible with a set of cosmological observations: the energy content of the present day Universe, the SN Ia data, the CMB temperature, the baryon-antibaryon asymmetry of matter in the Universe, and n_b/n_γ ratio.

It this framework we discussed a possibility to interpret three peaks in the CMB power spectrum in terms of the parameters of three electroweak decay processes.

The key points of our construction are the choice of variables (2.2) and the definition of the time interval (2.21).

The conformal gravitational theory (2.1) in the CMB reference frame with the initial data (3.5) does not contradict the following scenario of the evolution of the Universe within the conformal cosmology [15]:

$\eta \sim 10^{-12}s$, creation of vector bosons from a "vacuum";

$10^{-12}s < \eta < 10^{-10}s$, formation of baryon-antibaryon asymmetry;

$\eta \sim 10^{-10}s$, decay of vector bosons;

$10^{-10}s < \eta < 10^{11}s$, primordial chemical evolution of matter;

$\eta \sim 10^{11}s$, recombination or separation of cosmic microwave background radiation;

$\eta \sim 10^{15}s$, formation of galaxies;

$\eta > 10^{17}s$, terrestrial experiments and evolution of supernovae.

Our description is not complete, but it gives us a clear consistent statement of the problems in the framework of the well established principles of classification of observational and experimental facts in physics and astrophysics.

Thus, here we propose to consider the conformal gravitation theory [1] with the primary and secondary quantizations of its energy constraint as a basis of quantitative description of the creation of the Universe from vacuum.

Acknowledgements

The authors are grateful to D. Blaschke, A.V. Efremov, Yu.G. Ignatyev, V.B. Priezzhev, and S.A. Smolyansky for interest, criticism and creative discussions. One of us (A.A.) acknowledges the for support the INTAS grant 05-1000008-8328 and the grant of the RF President (Scientific Schools 3312.2008.2). One of us (A.B.) acknowledges the support from the Bogoliubov–Infeld Program 2008.

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ON SOME FEATURES OF COLOR CONFINEMENT

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Abstract

It is argued that a dual symmetry is needed to naturally explain experimental limits on color confinement. Since color is an exact symmetry the only possibility is that this symmetry be a dual symmetry, related to non trivial spatial homotopy. The sphere at infinity of 3-dimensional space being 2-dimensional, the relevant homotopy is Π_2 , the corresponding configurations monopoles, and the mechanism dual superconductivity. The consistency of the order-disorder nature of the deconfining transition is compared with lattice data . It is also shown that the only dual quantum number is magnetic charge and the key quantity is 't Hooft tensor, independent of the gauge group. The general form of the 't Hooft tensor is computed.

1 Introduction

Experimental upper limits to the observation of free quarks in Nature are very stringent [1]. Typically for the abundance of quarks in ordinary matter n_q as compared to that of protons n_p the limit is $n_q/n_p \leq 10^{-27}$ to be compared to the expectation in the Standard Cosmological Model in absence of confinement $n_q/n_p \approx 10^{-12}$[2].

The natural explanation is that n_q is exactly zero due to some symmetry . In this case the deconfining transition is an order disorder transition and can not be a crossover. A crossover indeed means continuity and the theory should explain a factor of 10^{-15} for n_q for a continuous transition between the two phases.

This is similar to what happens in ordinary superconductivity , where a very small upper limit is observed for the resistivity ρ_{SC} in the superconducting phase with respect to the normal one . There the transition is from a Higgs broken phase (superconductor), in which ρ_{SC} is strictly zero for symmetry reasons, to a Coulomb phase (normal) in which electric charge is superselected.

If this argument is correct two main questions raise naturally, namely :

1) What symmetry is responsible for confinement ?

2) Is an order disorder transition compatible with observation?

No clear observation exists yet of deconfinement in heavy ion experiments. There is a clear evidence of it, however, in simulations of the theory on a Lattice from first principles. There the deconfining transition can be observed and its order and universality class can be determined, at least in principle.

2 Symmetry

Color is believed to be an exact (Wigner) symmetry . Perturbative QCD is based on BRST symmetry, which is nothing but the statement that vacuum is a color singlet. Therefore color can not distinguish the confining phase from the deconfined one. What can then be an extra symmetry, besides color, which can do that?

In quenched $SU(N)$ gauge theory (no dynamical quarks) there is a cheap answer : the center of the group , Z_N. The Lagrangean of pure gauge theory is indeed blind to Z_N, since gluons belong to the adjoint representation. Usually, however , the theory is formulated on the lattice in terms of parallel transports in the fundamental representation, to allow the introduction of static quarks as external sources. Z_N is then a symmetry of the particular regularization. Static quarks are described by a parallel transport along the time axis, the Polyakov line $L(\vec{x})$.

$$L(\vec{x}) = P\exp[i\int_0^{\frac{1}{T}} A_0(\vec{x},t)dt] \tag{1}$$

The lattice is supposed to be extended in the time direction from zero to the inverse of the temperature T. The static potential acting between a static $q\bar{q}$ pair, $V(\vec{x})$, is related to Polyakov loop correlators as

$$V(\vec{x}) = -Tln(\langle L^\dagger(\vec{x})L(\vec{0})\rangle) \tag{2}$$

Since, by general arguments, the cluster property holds

$$\langle L^\dagger(\vec{x})L(\vec{0})\rangle \approx |\langle L\rangle|^2 + C\exp(-\frac{\sigma x}{T}) \tag{3}$$

when $\langle L\rangle = 0$ $V(\vec{x}) \approx_{x\to\infty} \sigma x$ (confinement)

when $\langle L\rangle \neq 0$ $V(\vec{x}) \approx_{x\to\infty}$ constant (deconfinement).

$\langle L\rangle$ is the order parameter and the symmetry is Z_N.

However in Nature dynamical quarks do exist and their coupling explicitly breaks Z_N, which then cannot be the symmetry responsible for confinement.

In front of this difficulty there exist in the community two different attitudes:

a) A narrow minded, conservative attitude : the only extra symmetry is a flavor symmetry, namely the chiral symmetry at zero quark mass.

b) A more advanced attitude looking for a dual symmetry related to topologically non trivial spatial boundary conditions. [3] [4][5][6][7][8].

Some comments on the attitude a). If the only relevant degrees of freedom at the deconfining transition at $m_q = 0$ $N_f = 2$ are the chiral ones then a renormalization group argument leads to the conclusion that either the transition is second order and belongs to the universality class of $O(4)$ in 3-d , and in that case the transition at $m_q \neq 0$ around the chiral point is a crossover. Or it is first order and then also at small non zero masses it is first order[9]. The first possibility is very popular in the literature [10], but nobody has found consistency of Lattice data with the $O(4)$ critical indexes. More recent data[11][12], instead, show consistency with a weak first order. Moreover, if chiral degrees of freedom were the only relevant ones, one should expect that also in the analogous system with 2 flavors of quarks in the adjoint representation, they should dominate. Instead that system shows two different transitions [13][14] : a strong first order deconfining transition which is detected, e.g. by the Polyakov line (Z_3 is a symmetry for adjoint quarks) and a very weak chiral transition which is consistent with a cross-over. This demonstrates that there exist other relevant degrees of freedom than light chiral scalars at the deconfining transition.

3 Duality

The key word for the approach b) is **duality**. The prototype example is the 2-d Ising model. It can be viewed as the discretization of a (1+1)-dimensional field theory, the field being the variable $\sigma = \pm 1$ defined on each site of a two dimensional square lattice. The partition function is that of a paramagnetic nearest-neighbours i, j interaction $Z = \Sigma_{i,j} \exp(-\beta\sigma_i\sigma_j)$. The system has a second order phase transition from an ordered phase $\langle\sigma\rangle \neq 0$ to a disordered phase $\langle\sigma\rangle = 0$ at a critical value β_c. The system admits spatial 1-dimensional configurations with non trivial topology, the kinks (anti-kinks), with $\sigma = -1(+1)$ for $x \leq x_0$ and $\sigma = +1(-1)$ for $x > x_0$. The operator μ which creates a kink at a given time reverses the sign of σ for $x \leq x_0$ and time t. If it acts twice the result is the identity, so that $\mu^2 = 1$ or $\mu = \pm 1$. It is a theorem that[4]

$$Z[\sigma, \beta] = Z[\mu, \beta^*] \tag{4}$$

$$sinh(2\beta) = \frac{1}{sinh(2\beta^*)} \tag{5}$$

or $beta \approx 1/over\beta^*$.
It follows that below the critical temperature $\langle\sigma\rangle \neq 0$ and $\langle\mu\rangle = 0$, above it $\langle\sigma\rangle = 0$ and $\langle\mu\rangle \neq 0$. A topological current j_μ can be defined which is identically conserved $\Delta_\mu j_\mu = 0$

$$j_\mu = \epsilon_{\mu,\nu}\Delta_\nu\sigma \tag{6}$$

The corresponding conserved charge is $Q = \int dx j_0(x,t) = \sigma(+\infty) - \sigma(-\infty)$ Is equal to the number of kinks minus the number of anti-kinks. In summary the system admits two equivalent descriptions [eq(4)]: either in terms of the local fields σ, and in this description the topological excitations are non local (direct description) , or in terms of the dual variables μ as local variables, and then the fields σ are non local (dual description). The first one is convenient at low temperature, the second one at high temperature. Duality maps the weak coupling regime of the direct description into the strong coupling of the dual, and viceversa.

In (3+1)dimensional field theories dual configurations have non trivial Π_2 corresponding to a non trivial mapping of the 2-dimensional sphere at infinity on the fields , and are monopoles. In (2+1) dimensions the dual configurations have non trivial Π_1 and are vortices.

4 Monopoles

Monopole configurations in non abelian gauge theories were first studied in a Higgs model with gauge group $SU(2)$ and Higgs field in the adjoint representation [15][16] . Everything is in the adjoint representation so that the theory is blind to Z_2 and is in fact an $SO(3)$ gauge theory. Monopoles of ref's[15][16] are static classical solutions of the equations of motion with finite energy (Solitons). In the "hedgehog" gauge the Higgs field has the form

$$\phi^a(\vec{r}) = f(r)\frac{r^a}{r} \tag{7}$$

The orientation of the field in color space coincides with that of the position vector in physical space. $f(r) \to 1$ as $r \to \infty$ and therefore the solution is a non trivial mapping of the two-dimensional sphere at spatial infinity onto the group $SO(3)/U(1)$, $U(1)$ being the invariance group of $\vec{\phi}$. From the general formula

$$\Pi_2(G/H) = ker[\Pi_1(H) \to \Pi_1(G)] \tag{8}$$

valid for any breaking of a group G to a subgroup H we get [18] $\Pi_2[SU(2)/U(1)] = \Pi_1[U(1)] = Z$ and $\Pi_2[SO(3)/U(1)] = Z/Z_2$. In the model of ref.[15][16] configurations are labeled by an even integer. This integer is nothing but the magnetic charge in units of Dirac units $\frac{1}{2g}$ with g the gauge coupling constant which plays the role of electric charge.

Since a monopole is always an abelian configuration [17] the magnetic charge has to be coupled to the residual $U(1)$ gauge group, i.e. to the abelian field strength

$$F_{\mu\nu} = \partial_\mu A^3_\nu - \partial_\nu A^3_\mu \tag{9}$$

where A^3_μ is the projection of the gauge field along the Higgs field $\vec{\Phi}$ in the unitary gauge in which it is directed along the third axis.

The tensor $F_{\mu\nu}$ can be given a gauge invariant form[15], which is known as 't Hooft tensor. Denoting by $\vec{\phi} = \vec{\Phi}/|\vec{\Phi}/|$ the direction of the Higgs field in color space one can show that [15]

$$F_{\mu\nu} = \vec{\phi}.\vec{G}_{\mu\nu} - \frac{1}{g}\vec{\phi}.(D_\mu\vec{\phi} \wedge D_\nu\vec{\phi}) \tag{10}$$

We define the current

$$j_\nu = \partial_\mu F^*_{\mu\nu} \tag{11}$$

with the usual notation for the dual tensor $F^*_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F_{\rho\sigma}$. Normally, with trivial boundary conditions $\partial_\mu F^*_{\mu\nu} = 0$ an equality known as Bianchi identity. For $j_\nu \neq 0$ one always has

$$\partial_\nu j_\nu = 0 \tag{12}$$

This is a topological symmetry, not related to the action via Noether's theorem and is our dual symmetry. The corresponding conserved charge is the magnetic charge $Q = \int d^3x j_0(\vec{x}, t)$. For the monopole configuration one has

$$E_i = F_{oi} = 0 \tag{13}$$

$$\vec{H} = \frac{1}{g}\frac{\vec{r}}{r^3} + Dirac - string \tag{14}$$

In a compact formulation like lattice the Dirac string is not visible so that $j_0 = \vec{\nabla}\vec{H} = \frac{(4\pi)}{g}\delta^3(\vec{r})$, a violation of Bianchi identity. The monopole is a configuration of charge 2 in agreement with the geometric argument above. The charge Q labels the dual degrees of freedom.

The presence of the Higgs field is only necessary if one wants the monopole as a soliton and a real breaking of the symmetry. In fact the role of $\vec{\Phi}$ can be played by any operator Ψ in the adjoint representation : monopoles will be located at the zeroes of Ψ , and their number and location will depend on the choice of Ψ , but a conserved current will be always defined as in eq(11). However one can think of a theory defined everywhere in space time, except for a discrete but arbitrary number of line like singularities which describe the dual degrees of freedom [Witten's geometric Langland's program[8]]. Creating a new monopole by an operator μ means adding a new singularity and this will be true whatever the choice of Φ . The vacuum expectation value $\langle\mu\rangle$ will be zero if the magnetic charge is super-selected and the vacuum has a definite magnetic charge. If, instead, $\langle\mu\rangle \neq 0$ magnetic gauge symmetry is broken a la Higgs and the vacuum is a dual superconductor. $\langle\mu\rangle$ can be used as an order parameter for confinement.

The above construction can be extended to any gauge group coupled to any matter fields: the basic ingredients are indeed gauge symmetry and the fact that physical space is 3-dimensional. For a generic gauge group there are r independent magnetic currents, with r the rank of the group[18]. The corresponding effective Higgs fields are the fundamental weights of the group. The 't Hooft tensor corresponding to each of them can be explicitly computed[18] and has a more complicated form than that of Eq(10). The residual symmetry can be immediately read from the Dynkin diagram of the Lie algebra, and is the Levy subgroup obtained by eliminating the little circle of the simple root corresponding to the given fundamental weight [18].

5 The order parameter for confinement

An order parameter for monopole condensation has been developed mainly in Pisa in recent years to detect dual superconductivity of QCD vacuum as explained above[19] [20][21][22]. The idea is to translate the component of the gauge field along the residual $U(1)$ direction by a classical monopole field by use of the conjugate momentum. In formulae

$$\mu^a(\vec{x},t) = \exp[i\int d^3y \frac{m}{g}\vec{E}^a(\vec{y},t)\vec{b}_\perp(\vec{x}-\vec{y})] \tag{15}$$

$\frac{m}{g}$ is the magnetic charge of the monopole, $\vec{b}_\perp(\vec{z}) = \frac{\vec{n}\wedge\vec{z}}{z(z-\vec{n}.\vec{z})}$ is the vector potential of the field generated by it in the transverse gauge $\vec{\nabla}\vec{b}=0$ $\vec{\nabla}\wedge\vec{b}_\perp(\vec{z}) = \frac{\vec{z}}{z^3}+$ Dirac - String along the direction $\vec{n}$.

$\vec{E}^a = Tr[\Phi^a\vec{E}]$ with Φ^a the $a-th$ fundamental weight is the component of the Chromo-electric field along the residual $U(1)$ symmetry T_3^a coupled to the magnetic charge. In the convolution with $\vec{b}_\perp$ only the transverse part contributes, which is the conjugate momentum to the transverse vector potential $\vec{A}_\perp^{a3}$ so that

$$\mu^a(\vec{x},t)|\vec{A}_\perp^{a3}(\vec{z},t)\rangle = |\vec{A}_\perp^{a3}(\vec{z},t) + \frac{m}{g}\vec{b}_\perp(\vec{x}-\vec{z})\rangle \tag{16}$$

The operator μ^a simply adds a monopole to the residual abelian projected field. It is easy to show that μ^a depends on $\beta \equiv \frac{2N_c}{g^2}$ as $\mu^a = e^{-\beta\Delta S^a}$ so that

$$\langle\mu^a\rangle = \frac{\int[d\phi]e^{-\beta(S+\Delta S^a)}}{\int[d\phi]e^{-\beta S}} = \frac{Z(S+\Delta S^a)}{Z(S)} \tag{17}$$

the ratio of two partition functions, which is 1 at $\beta = 0$.

If we define[20] [21] $\rho^a \equiv \frac{\partial \ln(\langle\mu^a\rangle)}{\partial\beta}$, we get then

$$\langle\mu^a\rangle = \exp(\int d\beta'\rho^a(\beta')d\beta') \tag{18}$$

If a deconfining transition exists at $T = T_c$, in the thermodynamical limit $V \to \infty$[20][21]

1) $\rho^a \to \bar{\rho}^a$ a finite limit for $T \leq T_c$ so that $\langle\mu^a\rangle \neq 0$ (confinement)

2) $\rho^a \propto -V^{\frac{1}{3}} \to -\infty$ for $T > T_c$ so that $\langle\mu^a\rangle = 0$ (deconfinement)

3) At $T \approx T_c$ $\langle\mu^a\rangle$ drops to zero and therefore ρ^a has a negative peak, which signal the transition. Moreover the finite size scaling dependence on the spatial size of the system L_s ($L_s^3 = V$) is

$$\rho^a = L_s^{\frac{1}{\nu}}\phi^a(\tau L_s^{\frac{1}{\nu}}) \tag{19}$$

where $\tau = 1-\frac{T}{T_c}$ is the reduced temperature, and ν the usual critical index of the correlation length at the transition. Not only ρ^a (or μ^a) detects the transition , but it also provides information on its order and universality class .

The behavior described above has been checked in a number of systems, in particular to study the phase diagram of $N_f = 2$ QCD at small quark masses[23]. In the physical case where the quarks are in the fundamental representation the deconfining transition coincides with the chiral transition: the negative peak of ρ^a seats just at the temperatore where the chiral order parameter $\langle\bar{\psi}\psi\rangle$ drops to zero. The scaling Eq(19) is compatible with a weak first order transition. In the case of the quarks in the adjoint representation of the color group the deconfining transition takes place at lower temperature than the chiral one, it is detected by ρ^a (μ^a) and is consistent with first order[14]. The chiral restoration is instead consistent with a very weak crossover.

6 Order-disorder and Lattice

The deconfining transition is popularly believed to be a crossover in a wide region of the QCD phase diagram[See e.g. ref.[10] for a review]. This only means that no evident jump of any physical quantity has been detected up to presently available volumes. In principle it is not possible to state on the basis of data [numerical or experimental] that a transition is a crossover and not a weak first order: the only correct statement can be " this transition is consistent with a crossover up to the presently available volumes". There are cases , however, in which theoretical arguments allow to state that there is a crossover. For example in $N_f = 2$ QCD if the chiral transition at $m_q = 0$ is second order then by general arguments at small masses in the neighborhood of $m_q = 0$ the transition will be a crossover. [Notice that we are here using a rather improper language by calling transition a crossover.] We have already touched this question in Section 2 above. It is therefore very important to check if the chiral transition is first order or second order in the universality class of $O(4)$. In the first case the transition is first order also at $m_q \neq 0$, a scenario compatible with an order-disorder nature of the deconfinement transition. If, instead the chiral transition proves to be second order then it becomes a crossover at $m_q \neq 0$, and order-disorder is ruled out. In the second case there is no way to define confinement and deconfinement[11][12] . This analysis can be done by use of finite size scaling techniques: the behavior of quantities like e.g. the specific heat or the susceptibility of the order parameter with increasing volume is governed by the critical indexes which are characteristic of the order and universality class of the transition. No consistency has been found of the scaling with second order $O(4)$ [10]. In ref.[11] new tools of investigation were introduced with respect to the previous literature: for example the specific heat, which is independent on any prejudice on the symmetry, was used besides the chiral susceptibility, and a better determination of the reduced temperature including its dependence on the quark mass. The scaling law for the specific heat reads

$$C_V - C_0 = L_s^{\frac{\alpha}{\nu}} \Phi_C(\tau L_s^{\frac{1}{\nu}}, m_q L_s^{y_h}) \tag{20}$$

C_0 is a subtraction corresponding to a quadratic divergence, which is ultraviolet and hence independent on the volume and on the quark mass. The critical index α is equal to 1 for a weak first order transition , whilst for second order $O(4)$ $\alpha = -0.2$, $\nu = \frac{1}{3}$ for weak first order, $\nu = .748(14)$ for second order $O(4)$, $y_h = 3$ for weak first order and $y_h = 2.48$ for second order $O(4)$. In the analysis of Ref[11][12] first C_0 was determined and verified to be independent on L_s and on m_q. Then a number of simulations were made in which one the two variables of the scaling function Φ of eq(20) was kept fixed in turn assuming either $O(4)$ or weak first order. Keeping the second variable fixed while varying m_q and L_s with the appropriate value of y_h, the scaling in the other variable can be tested. In particular at the maximum $C_V - C_0 \propto L_s^{\frac{\alpha}{\nu}}$ The scaling is compatible with first order and definitely excludes $O(4)$: indeed for $O(4)$ α is negative, but the peak strongly increases wit L_s. Keeping the first variable fixed instead, if the transition is first order one expects [12] at large volumes

$$C_V - C_0 = m_q^{-1}\phi^{(1)}(\tau V) + V\phi^{(2)}(\tau V) \tag{21}$$

The first term is non singular in the thermodynamical limit, the second term is singular and produces a latent heat . In the case of second order instead there is no singularity and only the analog of the first term is present[11] namely

$$C_V - C_0 = m_q^{\frac{-\alpha}{\nu y_h}} \phi(\tau L_s^{\frac{1}{\nu}}) \tag{22}$$

The importance of the second term increases with the volume and becomes dominant at large enough volumes. For very weak first order transitions the first term is dominant up to large volumes. The present situation with $N_f = 2$ QCD is that the first term is still big [11][12], and scales with the indexes of weak first order, i.e. as the first term of eq(21): the second term is there but is not dominant at present volumes. More work is needed to give a final clear answer to the question.

7 Conclusions

The only way to have an operative definition of confinement and deconfinement is to have a symmetry to distinguish them . This also appears to be the natural explanation of the strict upper limits on the observation of free quarks in nature. Color is an exact symmetry, and hence the possibility of a symmetry governing confinement relies on duality. This means excitations with non trivial Π_2, i. e. monopoles. Dual superconductivity is then

the candidate mechanism for confinement. This is independent on the gauge group and on the specific matter fields coupled to it. An order parameter can be defined for the dual symmetry, which is the expectation value of an operator which carries magnetic charge. Lattice data support this scenario in $N_f = 2$ QCD and seem to exclude $O(4)$ second order chiral transition, which would imply a crossover at $m_q \neq 0$ which is not compatible with order-disorder transition. More work is needed to definitely clarify the issue.

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On Einstein - Weyl unified model of dark energy and dark matter

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March 10, 2009

Abstract

Here I give a more detailed account of the part of the conference report that was devoted to reinterpreting the Einstein 'unified models of gravity and electromagnetism' (1923) as the unified theory of dark energy (cosmological constant) and dark matter (neutral massive vector particle having only gravitational interactions). After summarizing Einstein's work and related earlier work of Weyl and Eddington, I present an approach to finding spherically symmetric solutions of the simplest variant of the Einstein models that was earlier mentioned in Weyl's work as an example of his generalization of general relativity. The spherically symmetric static solutions and homogeneous isotropic cosmological models are considered in some detail. As the theory is not integrable we study approximate solutions. In the static case, we show that there may exist two horizons and derive solutions near horizons. In cosmology, we show how to find the asymptotic expansions and study in some detail the possible solutions near the origin. These solutions satisfy the Friedmann equations, with the energy density and pressure expressed in terms of the cosmological constant and the vector field. The structure of the solutions seems to hint at a possibility of an inflation mechanism that does not require adding scalar fields.

1 Introduction

In this report I give a new interpretation of the 'unified theory of gravity and electromagnetism' proposed by A.Einstein in 1923 in [1] and briefly summarized in [2]. Einstein gave no details of his derivations, presented no exact or approximate solutions, and did not explain why he completely abandoned his theory (I failed to find any reference to his papers [1] - [2] in his later work). Apparently these papers were soon forgotten by the scientific community and I could not find any reference to these papers in the second half of the 20-th century except for interesting remarks by Schrödinger [3] and a critical discussion by Pauli in addenda to the English translation of his famous book [4]. For these reasons, I first give a brief historical introduction summarizing Einstein's ideas and results as well as earlier related work of Weyl and Eddington.

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Immediately after the general relativity was formulated in its final form (1915 -1916) some attempts to modify it started. Einstein himself added the cosmological constant term Λ to save (unsuccessfully) his static cosmology. After Friedmann's work (1922-1924) this modification was becoming more and more dubious. Weyl, after 1918, developed a much more serious modification aimed at unifying gravity and electromagnetism (most clearly summarized in [5]). Starting from Levy-Civita's ideas on a general (non-Riemannian) connection (1917) he developed the theory of a special space in which the connection depends both on metric tensor and on a vector field which he tried to identify with the electromagnetic potential. To get a consistent theory he introduced a general idea of gauge invariance which survived although the theory itself failed as he admitted later. In paper [6] Einstein discussed Weyl's theory and expressed (like Pauli in [4]) the opinion that the theory is mathematically very interesting but probably not physical, at least, in its original formulation.

In 1919 Eddington proposed a more radical modification of general relativity [7], [8]. His idea was to start with the pure affine formulation of the gravitation, i.e. using first the general symmetric affine connection and only at some later stage introducing a metric tensor. Indeed, the curvature tensor can be defined without metric (here we use Einstein's notation [1] but denote differentiations by commas):

$$r^i_{klm} = -\Gamma^i_{kl,m} + \Gamma^i_{nl}\Gamma^n_{km} + \Gamma^i_{km,l} - \Gamma^i_{nm}\Gamma^n_{kl}\,. \tag{1}$$

Then the Ricci-like (but non-symmetric) curvature tensor can be defined by contracting the indices i, m (or, i, l):

$$r_{kl} = -\Gamma^m_{kl,m} + \Gamma^m_{nl}\Gamma^n_{km} + \Gamma^m_{km,l} - \Gamma^m_{nm}\Gamma^n_{kl} \tag{2}$$

(let us stress once more that $\Gamma^m_{nl} = \Gamma^m_{ln}$ but $r_{kl} \neq r_{lk}$). Using only these tensors and the anti-symmetric tensor density one can build up a rather rich geometric structure. In particular, Eddington discussed different sorts of tensor densities [8]. A notable scalar density is

$$\hat{\mathcal{L}} \equiv \sqrt{-\det(r_{kl})} \equiv \sqrt{-r} \tag{3}$$

which resembles the fundamental scalar density of the Riemannian geometry, $\sqrt{-\det(g_{kl})} \equiv \sqrt{-g}$. For this and some other reasons Eddington suggested to identify the symmetric part of r_{kl} with the metric tensor. The anti-symmetric part,

$$\phi_{kl} \equiv \frac{1}{2}(\Gamma^m_{km,l} - \Gamma^m_{lm,k})\,, \qquad \phi_{kl,m} + \phi_{lm,k} + \phi_{mk,l} \equiv 0\,, \tag{4}$$

strongly resembles the electro-magnetic field tensor and it seems natural to identify it with this tensor. Eddington tried to write consistent equations of the generalized theory but this problem was solved only by Einstein.

The starting point of Einstein in his first paper (72 in [1]) was to write the action principle and to suppose (3) to be the Lagrangian density depending on 40 connection functions Γ^m_{kl}. Varying the action w.r.t. these functions he derived 40 equations that allowed him to find the general expression for Γ^m_{kl} (the derivation is similar to that of the standard general relativity):

$$\Gamma^m_{kl} = \frac{1}{2}[s^{mn}(s_{kn,l} + s_{ln,k} - s_{kl,n}) - s_{kl}\, i^n + \frac{1}{3}(\delta^m_k\, i_l + \delta^m_l\, i_k)]\,. \tag{5}$$

Here s_{kl} is a symmetric tensor (s^{mn} is the inverse matrix to s_{kl}), which Einstein interpreted as the metric tensor (then the first term is the Christoffel symbol for this metric), and i_n is a vector which he tried to connect with the electro-magnetic field. This identification apparently follows from the equations

$$r_{kl} = R_{kl} + \frac{1}{6}[(i_{k,l} - i_{l,k}) + i_k i_l], \tag{6}$$

$$\phi_{kl} = \frac{1}{6}(i_{k,l} - i_{l,k}), \tag{7}$$

which can be obtained by inserting the expression (5) into (2), (4); R_{kl} is the standard Ricci curvature tensor for the metric s_{kl}. Einstein's interpretation of ϕ_{kl} as the Maxwell field is not so natural because of the term $i_k i_l$ in the r.h.s. of Eq.(6) which in fact makes this interpretation impossible. First, this term is not gauge invariant (but the gauge invariance was not yet discovered, the first clear formulation of the gauge principle was given by V.Fock in 1926). For Einstein, the main problem was that the electro-magnetic field in this theory could not exist without charges (i.e. there is no free field). To solve this problem he suggested to make this term 'infinitesimally small' by choosing the corresponding dimensional constant (above, we omit all dimensional constants that can easily be restored). But we, today, cannot be satisfied with this solution because this term violates gauge invariance and makes the photon effectively massive (while it is known that there exist no continuous transition from the massless to massive photon theory)[1]. We return to discussing these facts, on which our interpretation of the Einstein theory is based, after considering the final proposals of Einstein.

In his first paper (*'Zur allgemeinen Relativitätstheorie'*), he considered two limiting cases. He showed that, when the i_n-terms in the connection vanish, the theory is equivalent to the standard general relativity with the cosmological term that emerges naturally and cannot be removed. In the flat space limit he demonstrated that weak fields ϕ_{kl} (linear approximation) satisfy the free Maxwell equations provided that the $i_m i_n$-terms can be neglected. In the second paper (73 in [1]) he gave the following expression for the effective Lagrangian density:

$$\hat{\mathcal{L}} = -2\sqrt{-\det(r_{mn})} + \hat{R} - \frac{1}{6}\hat{s}^{mn} i_m i_n . \tag{8}$$

This should be varied w.r.t. $\hat{s}_{kl}$ and $\hat{f}_{kl}$, which are **the tensor densities defined with the aid of the scalar density** $\sqrt{-\det(s_{kl})}$ and corresponding to the tensors in the decomposition,

$$r_{kl} = s_{kl} + \phi_{kl}; \tag{9}$$

$\hat{R}$ is the scalar curvature density for the metric s_{kl}. The Lagrangian (8) contains a very complex term $\sqrt{-\det r_{mn}}$ which is more general than the so called Born-Infeld Lagrangian proposed ten years later [10] (the first attempts to construct nonlinear electro-dynamics were undertaken in [11]). Apparently, Einstein did not try to find any particular solution of this theory and, instead, in the beautiful third paper '*Zur affinen Feldtheorie*' (74 in [1]) he proposed a significantly simpler effective Lagrangian that is the main subject of this paper.

[1]The present best **experimental** upper bound on the photon mass is $m_\gamma < 10^{-51}$g [9]. Theoretical wisdom says that it must be zero.

As he mentioned in the first two papers the actual form of the Lagrangian is unimportant for getting the connection (5), the only important thing is on which variables it depends.

The main idea of the third paper is to take for the Lagrangian $\hat{\mathcal{L}}$ an arbitrary function of s_{kl} and ϕ_{kl}.[2] Then he introduces the Legendre transformation and the transformed (effective) Lagrangian density $\hat{\mathcal{L}}^*$:

$$\frac{\partial \hat{\mathcal{L}}}{\partial s_{kl}} \equiv \hat{g}^{kl}, \qquad \frac{\partial \hat{\mathcal{L}}}{\partial \phi_{kl}} \equiv \hat{f}^{kl}; \qquad s_{kl} = \frac{\partial \hat{\mathcal{L}}^*}{\partial \hat{g}^{kl}}, \qquad \phi_{kl} = \frac{\partial \hat{\mathcal{L}}^*}{\partial \hat{f}^{kl}} \tag{10}$$

Introducing the Riemann metric tensor g_{kl} and the i^k-vector,

$$g^{kl}\sqrt{-g} = \hat{g}^{kl}, \quad g_{kl}\, g^{lm} = \delta_k^m; \qquad \hat{i}^k = \partial_l \hat{f}^{kl}, \tag{11}$$

he claims (without proof) that Eq.(5) is valid with s_{kl} replaced by g_{kl} and thus the affine geometry is the same for any $\hat{\mathcal{L}}(s_{kl}, \phi_{kl})$. Finally, he uses the freedom in choosing $\hat{\mathcal{L}}^*(\hat{g}^{kl}, \hat{f}^{kl})$ and proposes the following effective Lagrangian density:

$$\hat{\mathcal{L}}^* = 2\alpha\sqrt{-g} - \frac{1}{2}\beta f_{kl}\hat{f}^{kl}, \tag{12}$$

where α and β are some constants not defined by the theory. This Lagrangian incorporates main properties of the theory discussed in previous papers but is easier to deal with.

To further clarify the relation of the new theory to general relativity Einstein rewrite the Lagrangian so that the equations of motion can be obtained by varying it in the metric and the vector field tensors, g_{kl} and f_{kl}. Neglecting dimensions (for example, taking $\hbar = c = \kappa = 1$) and changing Einstein's notation we write it as follows:

$$\hat{\mathcal{L}} = \sqrt{-g}\,[R - 2\Lambda - F_{kl}F^{kl} - m^2 A_k A^k], \qquad F_{kl} \equiv A_{k,l} - A_{k,l}. \tag{13}$$

Now it is absolutely evident that the vector field A_k is not the Maxwell field.[3] Obviously, A_k is a neutral massive vector field with coupling to gravity only. We will call it **vecton**, that is an old fashioned but proper term for this 'geometric' particle. This particle has not been directly observed but it can be considered as one of the possible candidates for **dark matter**. In view of the fact that the affine theory also predicted the cosmological constant term which is one of the best candidates for explaining **dark energy**, Einstein's theory may be considered as the first **unified model of dark energy and dark matter**.

Before we turn to further study of this model let us finish our presentation of its history. If you compare the Einstein model with the concrete models proposed in Weyl's book [5], you will find that Lagrangian similar to Eq.(13) is one of Weyl's examples. Einstein's and Weyl motivations and approaches were quite different, and the Weyl connection does not coincide with Eq.(5) (see Addendum). Weyl's approach was mostly geometrical and he

[2]In his previous work Einstein implied that $\hat{\mathcal{L}}$ depends on r_{kl}. At this point he quoted an unpublished work of 'Droste (from Leiden)' who 'two years ago expressed similar views'. The meaning of this rather cryptic remark is clarified in paper 75 (in [2]), where he confirms that (Johannes) Droste proposed to use a similar effective Lagrangian and, possibly a similar model (maybe, without cosmological constant).

[3]Einstein tried to identify A_k with a 'cosmic current' (this explains his notation i_k). A similar identification reappeared much later in quantum field theory (in the vector dominance model) under the name 'field-current identity'. However, it is meaningless in the classical Maxwell theory.

Here s_{kl} is a symmetric tensor (s^{mn} is the inverse matrix to s_{kl}), which Einstein interpreted as the metric tensor (then the first term is the Christoffel symbol for this metric), and i_n is a vector which he tried to connect with the electro-magnetic field. This identification apparently follows from the equations

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$$\phi_{kl} = \frac{1}{6}(i_{k,l} - i_{l,k}) \,, \tag{7}$$

which can be obtained by inserting the expression (5) into (2), (4); R_{kl} is the standard Ricci curvature tensor for the metric s_{kl}. Einstein's interpretation of ϕ_{kl} as the Maxwell field is not so natural because of the term $i_k i_l$ in the r.h.s. of Eq.(6) which in fact makes this interpretation impossible. First, this term is not gauge invariant (but the gauge invariance was not yet discovered, the first clear formulation of the gauge principle was given by V.Fock in 1926). For Einstein, the main problem was that the electro-magnetic field in this theory could not exist without charges (i.e. there is no free field). To solve this problem he suggested to make this term 'infinitesimally small' by choosing the corresponding dimensional constant (above, we omit all dimensional constants that can easily be restored). But we, today, cannot be satisfied with this solution because this term violates gauge invariance and makes the photon effectively massive (while it is known that there exist no continuous transition from the massless to massive photon theory)[1]. We return to discussing these facts, on which our interpretation of the Einstein theory is based, after considering the final proposals of Einstein.

In his first paper (*'Zur allgemeinen Relativitätstheorie'*), he considered two limiting cases. He showed that, when the i_n-terms in the connection vanish, the theory is equivalent to the standard general relativity with the cosmological term that emerges naturally and cannot be removed. In the flat space limit he demonstrated that weak fields ϕ_{kl} (linear approximation) satisfy the free Maxwell equations provided that the $i_m i_n$-terms can be neglected. In the second paper (73 in [1]) he gave the following expression for the effective Lagrangian density:

$$\hat{\mathcal{L}} = -2\sqrt{-\det(r_{mn})} + \hat{R} - \frac{1}{6}\hat{s}^{mn} i_m i_n \,. \tag{8}$$

This should be varied w.r.t. $\hat{s}_{kl}$ and $\hat{f}_{kl}$, which are **the tensor densities defined with the aid of the scalar density** $\sqrt{-\det(s_{kl})}$ and corresponding to the tensors in the decomposition,

$$r_{kl} = s_{kl} + \phi_{kl} \,; \tag{9}$$

$\hat{R}$ is the scalar curvature density for the metric s_{kl}. The Lagrangian (8) contains a very complex term $\sqrt{-\det r_{mn}}$ which is more general than the so called Born-Infeld Lagrangian proposed ten years later [10] (the first attempts to construct nonlinear electro-dynamics were undertaken in [11]). Apparently, Einstein did not try to find any particular solution of this theory and, instead, in the beautiful third paper *'Zur affinen Feldtheorie'* (74 in [1]) he proposed a significantly simpler effective Lagrangian that is the main subject of this paper.

[1]The present best **experimental** upper bound on the photon mass is $m_\gamma < 10^{-51}$g [9]. Theoretical wisdom says that it must be zero.

As he mentioned in the first two papers the actual form of the Lagrangian is unimportant for getting the connection (5), the only important thing is on which variables it depends.

The main idea of the third paper is to take for the Lagrangian $\hat{\mathcal{L}}$ an arbitrary function of s_{kl} and ϕ_{kl}.[2] Then he introduces the Legendre transformation and the transformed (effective) Lagrangian density $\hat{\mathcal{L}}^*$:

$$\frac{\partial\hat{\mathcal{L}}}{\partial s_{kl}} \equiv \hat{g}^{kl}\,, \qquad \frac{\partial\hat{\mathcal{L}}}{\partial \phi_{kl}} \equiv \hat{f}^{kl}\,; \qquad s_{kl} = \frac{\partial\hat{\mathcal{L}}^*}{\partial \hat{g}^{kl}}\,, \qquad \phi_{kl} = \frac{\partial\hat{\mathcal{L}}^*}{\partial \hat{f}^{kl}} \tag{10}$$

Introducing the Riemann metric tensor g_{kl} and the i^k-vector,

$$g^{kl}\sqrt{-g} = \hat{g}^{kl}\,, \quad g_{kl}\,g^{lm} = \delta_k^m\,; \qquad \hat{i}^k = \partial_l \hat{f}^{kl}\,, \tag{11}$$

he claims (without proof) that Eq.(5) is valid with s_{kl} replaced by g_{kl} and thus the affine geometry is the same for any $\hat{\mathcal{L}}(s_{kl}, \phi_{kl})$. Finally, he uses the freedom in choosing $\hat{\mathcal{L}}^*(\hat{g}^{kl}, \hat{f}^{kl})$ and proposes the following effective Lagrangian density:

$$\hat{\mathcal{L}}^* = 2\alpha\sqrt{-g} - \frac{1}{2}\beta f_{kl}\hat{f}^{kl}\,, \tag{12}$$

where α and β are some constants not defined by the theory. This Lagrangian incorporates main properties of the theory discussed in previous papers but is easier to deal with.

To further clarify the relation of the new theory to general relativity Einstein rewrite the Lagrangian so that the equations of motion can be obtained by varying it in the metric and the vector field tensors, g_{kl} and f_{kl}. Neglecting dimensions (for example, taking $\hbar = c = \kappa = 1$) and changing Einstein's notation we write it as follows:

$$\hat{\mathcal{L}} = \sqrt{-g}\,[R - 2\Lambda - F_{kl}F^{kl} - m^2 A_k A^k], \qquad F_{kl} \equiv A_{k,l} - A_{k,l}\,. \tag{13}$$

Now it is absolutely evident that the vector field A_k is not the Maxwell field.[3] Obviously, A_k is a neutral massive vector field with coupling to gravity only. We will call it **vecton**, that is an old fashioned but proper term for this 'geometric' particle. This particle has not been directly observed but it can be considered as one of the possible candidates for **dark matter**. In view of the fact that the affine theory also predicted the cosmological constant term which is one of the best candidates for explaining **dark energy**, Einstein's theory may be considered as the first **unified model of dark energy and dark matter**.

Before we turn to further study of this model let us finish our presentation of its history. If you compare the Einstein model with the concrete models proposed in Weyl's book [5], you will find that Lagrangian similar to Eq.(13) is one of Weyl's examples. Einstein's and Weyl motivations and approaches were quite different, and the Weyl connection does not coincide with Eq.(5) (see Addendum). Weyl's approach was mostly geometrical and he

[2]In his previous work Einstein implied that $\hat{\mathcal{L}}$ depends on r_{kl}. At this point he quoted an unpublished work of 'Droste (from Leiden)' who 'two years ago expressed similar views'. The meaning of this rather cryptic remark is clarified in paper 75 (in [2]), where he confirms that (Johannes) Droste proposed to use a similar effective Lagrangian and, possibly a similar model (maybe, without cosmological constant).

[3]Einstein tried to identify A_k with a 'cosmic current' (this explains his notation i_k). A similar identification reappeared much later in quantum field theory (in the vector dominance model) under the name 'field-current identity'. However, it is meaningless in the classical Maxwell theory.

wrote the Lagrangian as a simplest illustration of possible physical applications, responding to criticism by Einstein, Pauli and other physicists. Einstein was most interested in physics and, especially, in cosmology. Weyl criticized Einstein for his departure from geometric foundations of physics, in particular, for his derivation of geometry from the variational (action) principle which, probably, was his main achievement in the third paper. Note also that Weyl included the cosmological term only to avoid contradiction to Einstein cosmology of that time ('before Friedmann') while in the original Einstein model (3), (7) it was unavoidable. I think that, conceptually, the model (13) is a step backwards, in comparison with the original theory, (3), (8). There were, probably, two reasons for this step. First, Einstein's deep belief in simplicity of fundamental laws ('...aber *boshaft ist Er nicht*'). Second, his disappointment[4] in static cosmology after accepting Friedmann's results, [12]. Anyway, in his last papers on affine theory [2] he set the cosmological term to zero what is impossible in the original theory and quite unnatural in the framework of the affine approach.

Above, we also mentioned work and ideas of Eddington. The intensive exchange of ideas between Einstein, Weyl and Eddington resulted in interrelations in their work (published in 1918-1923) that are difficult (and, possibly, unnecessary) to disentangle. As the constructive ideas of the affine theory were mostly created by Weyl, Eddington, and Einstein, the resulting model should probably be called **Einstein-Weyl-Eddington unified model of dark energy and dark matter**. However, as far as I am here discussing the concrete Lagrangian (13), I call it Einstein-Weyl model.

Before turning to new results let us briefly summarize the results and thoughts of Weyl, Eddington, and Einstein. 1. **Weyl** had a very clear and original geometric ideas, but: a) his physics was rightly criticized by Einstein, Pauli, and other physicists, b) he considered the theory as a unified theory of gravity and electromagnetism but his vector field was also not electro-magnetic, c) his discussion of dynamics was incomplete and he himself regarded it as preliminary. Nevertheless, it is possible that not all the potential of the Weyl ideas is understood and used. 2. **Eddington** proposed to use, instead of the Weyl's non-Riemannian 'metrical spaces', the most general spaces with symmetric affine connection (without torsion). He discussed possible invariants that can be used in physics, in particular, the square root of the determinant of R_{kl}.[5] He proposed to consider the symmetric part of the curvature matrix as the metric in the general space and the anti-symmetric one as the electro-magnetic field tensor. In later works he discussed a possibility to use this as a Lagrangian (long before the proposal of Born and Infeld). However, he did not find a consistent approach to dynamics. 3. **Einstein** started with formulating dynamics by use of the Hamilton principle similar to one proposed by Palatini in general relativity. The new (and crucial) idea was not to introduce any metric at the beginning and not to fix any special form of the affine connection (apart of the symmetry condition). He soon realized (in paper 74), that he does not need to use a concrete form of the Lagrangian that can be just any function (tensor density) of s_{kl} and ϕ_{kl}-matrices (see (9-11)). For any such Lagrangian

[4]In 1917 de Sitter discovered a non-static solution of the empty - space Einstein equations with the cosmological constant. In 1923 Weyl and Eddington found the effect of recession of test particles in the de Sitter universe. Thus Λ was becoming useless and Einstein finally dismissed it in 1931, after Hubble's discovery (1929).

[5]In addendum to [8], where he gave a clear and detailed account of the Einstein work [1], he also discussed another scalar density that was proposed by R. Weitzenböck

he proved that the affine connection allows one to introduce a symmetric metric and found the expression for connection. Both Einstein's and Weyl's expressions are special cases of the general formula for the symmetric connection (see Appendix).

The most important thing is the following: supposing that the equations of motion follow from an action principle with the general Lagrangian fixes the geometry (connection) and, eventually, allows one to fix some metric compatible with this non-Riemannian connection. Another important thing is that the action can be (and should be) written without metric. Using in paper 74 the Legendre transformation Einstein bypassed difficulties that were met on this way and wrote more tractable effective Lagrangian, but some conceptually beautiful and important features of his new theory were thus hidden (or even lost).

Apparently, Einstein was disappointed in the cosmological constant and also gradually realized that his interpretation of the anti-symmetric field as the electro-magnetic field was not quite satisfactory. Anyway, he completely abandoned this model and left no detailed account of his work. He did not mention any static or cosmological solutions even in the simplified version of the theory, (13). In this paper we try to fill this gap and establish grounds for comparing this model to the present day cosmology.

2 Spherical reduction - static and cosmological solutions

2.1 Vecton-dilaton gravity

At first sight, the theory (13) is very close to the well-understood Einstein-Maxwell theory which can be obtained when $m = 0$. However, we will show that the two theories are qualitatively different and it is hardly possible to construct a reasonable perturbation theory in the parameter m^2. We start our qualitative analysis without assuming that this parameter is small. The natural object for this analysis is the spherically reduced theory. When $m = 0$, the theory automatically reduces to rather simple one-dimensional equations that can be explicitly solved. The solution is the Reissner - Nordström black hole (when the electric charge vanishes it reduces to the Schwarzschild black hole). In general, when gravity couples to other (not electro-magnetic) fields the spherically reduced theory is described by two-dimensional differential equations which are not integrable except very special cases (for many examples and references see, e.g., [13]-[17].).

Following the approach to dimensional reduction and to resulting 1+1 dimensional dilaton gravity (DG) developed in papers [15]-[19] it is not difficult to derive these equations. The general spherically symmetric metric is $(i,j = 0,1;\, x^0 = t, x^1 = r)$:

$$ds^2 = g_{ij}(t,r)\, dx^i dx^j + \varphi(t,r)(\sin^2\theta\, d\theta + d\phi^2)\,. \tag{14}$$

Supposing that all other functions also depend on t, r, inserting the metric (14) into the action with the Lagrangian (13), and integrating out the angle variables θ, ϕ one can derive the following effective two-dimensional Lagrangian[6]:

$$\mathcal{L}^{(2)} = \sqrt{-g}\,[\varphi R^{(2)} + 2 - 2\Lambda\varphi + (\partial\varphi)^2/2\varphi - \varphi F_{ij}F^{ij} - \varphi m^2 A_i A^i], \qquad (15)$$

where $R^{(2)}$ is the two-dimensional Ricci curvature depending on the g_{ij}. It is convenient to remove the fourth term by the Weyl rescaling of the metric, $g_{ij} = \varphi^{-\frac{1}{2}} g_{ij}^{\mathrm{W}}$. Below we use the transformed Lagrangian,

$$\mathcal{L}_W^{(2)} = \sqrt{-g}\,[\varphi R^{(2)} + 2\varphi^{-1/2} - 2\Lambda\varphi^{1/2} - \varphi^{-3/2} F^2 - \varphi m^2 A^2]. \qquad (16)$$

It is easy to derive the equations of motion which in a generic metric g_{ij} are equivalent to the Einstein equations for the spherically symmetric solutions of the four-dimensional theory (13). By varying w.r.t. the diagonal metric functions g_{ii} we fist write the energy and momentum constraints. In the light cone (LC) metric, $ds^2 = -4f(u,v)\,du\,dv$, these constraints are simple:

$$f\partial_i\,(\partial_i\,\varphi/f)] + \varphi m^2 A_i^2 = 0, \qquad i = u, v. \qquad (17)$$

The constraints (17) should be derived using the general metric g_{ij}. The other equations of motion may be obtained directly in the LC-metric:

$$\partial_u\partial_v\,\varphi + f(2\varphi^{-1/2} - 2\Lambda\varphi^{1/2} - \frac{1}{2}\varphi^{3/2} f^{-2} F_{uv}^2) = 0, \qquad F_{uv} \equiv A_{u,v} - A_{v,u}, \qquad (18)$$

$$\partial_j\,(\varphi^{3/2} f^{-1} F_{ij}) = \varphi m^2 A_j \qquad i, j = u, v. \qquad (19)$$

From the last equation immediately follows that $\partial_v(\varphi A_u) + \partial_u(\varphi A_v) = 0$. In the original four-dimensional theory this is the $\partial_\mu(\sqrt{-g}A^\mu) = 0$ condition eliminating spin 0. Weyl, Eddington and Einstein called it the Lorentz condition although we know that its origin and meaning are quite different from the gauge fixing condition in the Maxwell theory first introduced by L.Lorenz and later popularized by H.A.Lorentz.

This dilaton gravity coupled to massive vector field (I suggest to call it **vecton-dilaton gravity, VDG**) is more complex than the well studied models of dilaton gravity coupled to scalar fields and thus it requires a separate study. The natural first question is: are there exact analytical solutions like Schwarzschild or Reissner-Nordström black holes? If the vector field is constant, we return to exactly soluble DG having explicit solutions with horizons. Otherwise, when the vector field is nontrivial, the answer is more difficult to find but it is worked out in some detail below. The second question is: what are the simplest cosmological solutions in this theory? Thus, the first thing to do is to further reduce the theory to static or cosmological configurations. Consider first the static reduction.

2.2 Static states and horizons

The simplest way to derive the corresponding equations is to suppose that all the functions in the equations depend on $r = u + v$. But this is not the most general dimensional

[6]Very similar effective Lagrangians can be obtained from the higher-dimensional analogs of the Lagrangian (13). On the (1+1)-dimensional level it is not difficult to include other sorts of matter that appear in reductions of higher-dimensional supergravity theories (for references see, e.g., [15] - [17]). I hope to discuss some of these generalizations in future publications.

reduction of the two-dimensional theory. There exist more general ones that allow us to simultaneously treat black holes, cosmologies and some waves. These generalized reductions were proposed in papers [20], [21], [17] devoted to dilaton gravity coupled to scalar fields and Abelian gauge fields; here we only discuss in some detail the static and cosmological reductions. In both cases it can be seen that the perturbed theory (with a nonvanishing mass term) is qualitatively different from the non-perturbed one. Indeed, the non-perturbed theory is just dilaton gravity coupled to electromagnetism. This model is equivalent to pure dilaton gravity, which is a topological theory. In particular, it automatically reduces to one-dimensional static or cosmological models that can be analytically solved. Static states are the Reissner-Nordström black holes perturbed by the cosmological constant and having two horizons, while the space between horizons may be considered as an unrealistic cosmology. This object is known from 1916 times; certainly it was familiar to Einstein in 1923 but he did not discuss the static configuration and apparently did not consider black holes or horizons as having any relation to physics.

Let us now write the **static** equations corresponding to the naive reduction to one spatial dimension. To obtain them one can reduce either the equations or the Lagrangian. Following [19], [22], we write the equations of motion in a somewhat unusual form. Let us define two additional functions, χ and B, by the equations (the prime denotes differentiations w.r.t. r)

$$\varphi'(r) = \chi(r), \qquad A'(r) = f(r)\varphi^{-3/2}(r)B(r), \qquad (20)$$

where, as follows from Eq.(19), $A_v(r) = -A_u(r) \equiv -A(r)$. Then the other equations are

$$\chi' = -fU, \qquad B' = -\frac{1}{2}\varphi m^2 A, \qquad f' = (f/\chi)[-fU + \varphi m^2 A^2], \qquad (21)$$

where we defined the potential

$$U \equiv 2(\varphi^{-1/2} - \Lambda\varphi^{1/2} - \varphi^{-3/2}B^2), \qquad (22)$$

These equations are not integrable and cannot be solved analytically. To get numerical solutions we first have to study the analytic and asymptotic properties of their solutions.

Here we only consider **solutions near possible horizons** that are defined as zeroes of the metric, $f \to 0$ for finite values of $\varphi \to \varphi_0$. It is not difficult to understand that we also should require that A is finite near the horizon. To study the behaviour of the solutions for small values of $\tilde{\varphi} \equiv \varphi - \varphi_0$ it is most convenient to **consider the solutions as functions of** φ. Further analysis shows that the solutions can be expanded in power series of $\tilde{\varphi}$ and that the functions $\tilde{F} \equiv f/\chi$ and $\tilde{A} = A/\chi$ should be finite. Thus we have:

$$\tilde{F}'(\varphi) = \varphi\,\tilde{F}(\varphi)\,m^2\tilde{A}^2(\varphi), \qquad \chi'(\varphi) = -\tilde{F}(\varphi)\,U(\varphi), \qquad (23)$$

$$B'(\varphi) = -\frac{1}{2}\varphi\,m^2\tilde{A}(\varphi), \qquad \tilde{A}'(\varphi)\chi(\varphi) = \tilde{F}(\varphi)\,[\varphi^{-3/2}B(\varphi) + U(\varphi)\tilde{A}(\varphi)], \qquad (24)$$

where now the prime denotes differentiation in the new variable φ. It is not very difficult to show that φ_0, $\tilde{A}_0$, B_0, $\tilde{F}_0$ can be taken arbitrary up to one relation that should be satisfied due to the second equation (24):

$$\tilde{A}_0\,U_0 + \varphi_0^{-3/2}B_0 = 0, \qquad U_0 \equiv U(\varphi_0, B_0). \qquad (25)$$

This equation can be solved w.r.t any parameter. It is interesting to see that it has two solutions for φ_0 which means that **there may exist two horizons**[7] as distinct from the Schwarzschild black hole. Note that the solutions with different $\tilde{F}_0$ are equivalent because the equations are invariant under the scale transformation $\tilde{F} \Rightarrow C\tilde{F}$, $\chi \Rightarrow C\chi$.

Now, following the method of [22], one can find several terms in the expansion of the solution. Unfortunately, it is not clear how to construct the complete expansion and therefore our derivations do not allow us to study global properties of the solutions. They say nothing about asymptotic properties and singularities which should be the subject of separate investigations.[8] When the qualitative properties of the black hole type solutions will be understood, the static solutions and their formation can be studied by numerical simulations. As far as I know, the coupling of massive neutral vector particles to gravity did not attract much attention (see, however, numerical simulations of the critical collapse of a massive vector field in [24]).

2.3 Cosmology

Let us turn to cosmological reductions. The simplest cosmology can be obtained by the same naive reductions as was used for static states. However, this cosmology does not coincide with the homogeneous isotropic Friedmann type cosmology. In addition, it can be shown that cosmologies derived by such a naive reduction are closed. If we wish to to get Friedmann type cosmologies from the vecton dilaton gravity corresponding to the spherically symmetric world, we must employ a more complex procedure of dimensional reduction to 1+0 dimension, which was described in [21]. If we only wish to write a Friedmann type cosmology, we can simply use the standard approach and directly obtain the effective (1+0)-dimensional Lagrangian[9]

$$\mathcal{L}_c = 6ke^{\alpha+\gamma} - 6\dot{\alpha}^2 e^{3\alpha-\gamma} - 2\Lambda e^{3\alpha+\gamma} + \dot{A}^2 e^{\alpha-\gamma} - m^2 A^2 e^{\alpha+\gamma}\,, \qquad (26)$$

where the effective (1+1)-dimensional metric is

$$ds^2 = e^{2\alpha}dr^2 - e^{2\gamma}dt^2$$

and α, γ, A depend on t.[10] The equations for this cosmological model are simpler than the static ones. In particular, we immediately see that γ is the Lagrange multiplier, the variation in which gives the Hamiltonian that must be zero. Denoting $f \equiv e^\alpha$ and taking **the gauge fixing condition** $\gamma = 0$ **(the 'standard' gauge)** we have[11]

$$\mathcal{H}_0^c \equiv f[-6\dot{f}^2 - 6k + 2\Lambda f^2 + \dot{A}^2 + m^2 A^2] = 0\,. \qquad (27)$$

[7] This is similar to the charged Reissner-Nordström black hole or to black holes in higher - dimensional theories, [23], although in the present model there is no conserved electric charge.

[8] The asymptotic expansions for $r \to 0$ and $r \to \infty$ can be obtained by following the approach proposed below for the cosmological solutions.

[9] Above we completely neglected the dimensions of all the variables and omitted the gravitational constant. Here we only restore one of the dimensions supposing that $[t^{-2}] = [k] = [\Lambda] = [m^2] = [L^{-2}]$. Recall that $6k$ is the curvature of the three-dimensional 'sphere'; $k > 0$ for the real sphere, $k < 0$ for the pseudo-sphere and $k = 0$ for the flat space. In (15) one should similarly write $2k$ for the curvature of the two-dimensional 'sphere'.

[10] Note that when A_0, A_1 depend only on t, we have $A_0 \equiv 0$, as follows from the (1+1)-dimensional equations of motion; we thus denote $A_1 \equiv A$.

[11] In this paper we treat cosmological solutions independently of the static states and our notation in this section is also independent of the previous one

Another useful **gauge (the LC gauge)** is $\alpha = \gamma$. In this gauge the effective Hamiltonian is:

$$\mathcal{H}_1^c \equiv -6\dot{f}^2 - 6kf^2 + 2\Lambda f^4 + \dot{A}^2 + f^2 m^2 A^2 = 0\,. \qquad (28)$$

Both forms of the constraint tell us that for $\Lambda > 0$ the static cosmology (when $\dot{f} = \dot{A} = 0$) is possible only if $k > 0$, i.e. when the universe is closed; if $k \leq 0$ the constraint can be satisfied only if $\dot{f}^2 \neq 0$.

Let us first write the **equations of motion in the LC gauge** $\alpha = \gamma$. In analogy to the static case we write them in the first order form (the first equation is the definition of F),

$$\dot{f} = fF\,, \qquad \dot{F} + F^2 + k = \frac{2}{3}\Lambda f^2 + \frac{1}{6}m^2 A^2, \qquad \dot{A} = B, \qquad \dot{B} = -m^2 f^2 A\,, \qquad (29)$$

and the Hamiltonian constraint is a simple polynomial function of f, F, A, B:

$$\mathcal{H}_1^c = -6f^2F^2 - 6kf^2 + B^2 + 2\Lambda f^4 + m^2 f^2 A^2 = 0\,. \qquad (30)$$

Similarly to our previous consideration of the static equations, we better change the independent variable to $\alpha \equiv \ln f$. It is convenient to introduce two new functions, $\psi(\alpha)$ and $G(\alpha)$,

$$\psi'(\alpha) \equiv F'(\alpha)/F(\alpha)\,, \qquad G(\alpha) \equiv F^2(\alpha) + k - \frac{1}{3}\Lambda e^{2\alpha}\,, \qquad (31)$$

and use the following equations (the prime denotes differentiation w.r.t. α):

$$G' + 2G = \frac{1}{3}m^2 A^2\,, \qquad A'' + \psi' A' + m^2 e^{2\alpha} F^{-2} A = 0\,. \qquad (32)$$

Of course, instead of the first equation we can use the equivalent equation for F^2 that directly follows from (29):

$$(F^2)' + 2F^2 = -2k + \frac{4}{3}\Lambda e^{2\alpha} + \frac{1}{3}m^2 A^2\,. \qquad (33)$$

Together with the constraint (30), rewritten as

$$\mathcal{H}_1^c = -e^{2\alpha}[F^2(6 - e^{-2\alpha}A'^2) + 6k - 2\Lambda e^{2\alpha} - m^2 A^2] = 0\,, \qquad (34)$$

equations (32) form the complete system describing cosmology in the LC gauge. Note that the constraint (30) is the integral of motion and thus it is sufficient to require that it vanishes just at one point, say, at $t = 0$ or $\alpha = -\infty$. To derive possible asymptotic behaviour of the solutions for $|\alpha| \to \infty$ it is natural to expand A in powers of e^α and to self consistently use the general solution of the first equation,

$$G(\alpha) = \frac{1}{3}m^2 e^{-2\alpha}\int d\alpha A^2(\alpha)e^{2\alpha}\,, \qquad (35)$$

with the relations for the expansion coefficients obtained from the second equation.

In this way we can find, step by step, the asymptotic expansion. In the asymptotic region $\alpha \to -\infty$ we can then find the following possible asymptotic behaviour:

$$A = \sum_{n=0}^{\infty} A_n e^{\alpha n}\,, \qquad F^2 = e^{-2\alpha}\Big[C_\infty + \sum_{n=2}^{\infty} F_n^{(2)} e^{\alpha n}\Big]\,, \qquad \psi' = -1 + \sum_{n=2}^{\infty} n\psi_n e^{n\alpha}\,, \qquad (36)$$

where C_∞, A_0, A_1 are arbitrary constants[12]; A_n, $F_n^{(2)}$ for $n \geq 2$ are derived recursively from (32), (35), and ψ_n from definition (31). The first coefficients are:

$$A_1 = \pm\sqrt{6}, \quad A_2 = 0, \quad A_3 = -\frac{1}{A_1 C_\infty}\left[\frac{1}{6}m^2 A_0^2 - k\right], \quad A_4 = \frac{m^2}{4C_\infty}A_0\,; \tag{37}$$

$$F_2^{(2)} = \frac{1}{6}m^2 A_0^2 - k\,, \quad F_3^{(2)} = \frac{2}{9}m^2 A_0 A_1\,; \qquad \psi_2 = \frac{1}{2C_\infty}F_2^{(2)} \quad \psi_3 = \frac{1}{2C_\infty}F_3^{(2)}\,. \tag{38}$$

Thus we find the differential equation for the metric function $f(t)$ ('scale factor'):

$$\frac{d}{dt}(e^\alpha) \equiv \dot{f} = \sqrt{C_\infty}\,[1 + 2\psi_2 f^2 + 2\psi_3 f^3 + ...]^{\frac{1}{2}}\,, \tag{39}$$

and if we solve it we can find the vector field $A(t)$ by using (36), (37). Neglecting the third term in the r.h.s. it is easy to solve this equation finding the dependence of f on t:

$$f(t) = \sqrt{C_\infty}\left(\frac{1}{6}m^2 A_0^2 - k\right)^{-\frac{1}{2}} \sinh\left[\left(\frac{1}{6}m^2 A_0^2 - k\right)^{\frac{1}{2}}(t - t_0)\right].$$

The exponential growth of $f(t)$ suggests a possibility of an **inflation** character of this solution. However, this is only the first approximation and we should take into account higher order terms to get a more solid conclusion[13]. Moreover, we see that the qualitative character of the solutions essentially depends on the physical parameters A_0, m^2 on which at the moment we have no reliable information[14].

The discussed **solution is not unique**. Using the above equations we can derive another one, for which both A and F are finite for $\alpha \to -\infty$. To get it we take

$$G(\alpha) = \frac{1}{3}m^2 e^{-2\alpha}\int_{-\infty}^{\alpha} d\bar{\alpha} A^2(\bar{\alpha}) e^{2\bar{\alpha}}\,, \tag{40}$$

and then apply the above procedure. Then, using the expansions

$$A = \sum_{n=0}^{\infty} A_n e^{2n\alpha}, \qquad F = \sum_{n=0}^{\infty} F_n e^{2n\alpha}, \qquad F^2 = \sum_{n=0}^{\infty} F_n^{(2)} e^{2n\alpha} \tag{41}$$

we can find that

$$F_0^{(2)} \equiv F_0^2 = \left[\frac{1}{6}m^2 A_0^2 - k\right], \qquad F_1^{(2)} \equiv 2F_0 F_1 = \left[\frac{1}{3}\Lambda + \frac{1}{6}m^2 A_0 A_1\right], \tag{42}$$

$$A_1 = -m^2 A_0/4F_0^2\,, \qquad A_2 = -A_1(m^2 + 6F_1^{(2)})/16F_0^2\,, \tag{43}$$

where now A_0 is the unique arbitrary constant (in the above solution we have one more constant C_∞). Instead of Eq.(39) we now have the equation:

$$\dot{f} \equiv \frac{de^\alpha}{dt} = [F_0^2 + 2F_1 F_0^{-1} f^2(t) + ...]^{\frac{1}{2}}\,, \tag{44}$$

[12] A_1 is defined by the constraint (30): putting the first terms of the expansion (36) into (30) or (34) we get $A_1 = \sqrt{6}$, which is sufficient for satisfying the constraint.

[13] An interesting exercise could be to keep four terms in the r.h.s. of (39) and express the solution in terms of the elliptic functions. The behaviour of $f(t)$ in this approximation essentially depends on all the parameters.

[14] The dependence on Λ only occurs in the omitted fourth-order terms.

which can easily be solved in this approximation:

$$f \equiv e^\alpha = 2e^{F_0 t}(1 - 2F_1 F_0^{-1} e^{2F_0 t})^{-1}\,. \tag{45}$$

The scale factor vanishes if $t \to -\infty$ (LC-gauge!). The parameter $F_1 F_0^{-1}$ strongly depends on A_0, Λ, m^2 and k. It may be positive or negative, small or large; for example, if $k = 0$

$$F_1 F_0^{-1} = \left(\Lambda - \frac{3}{4}m^2\right)\left(m^2 A_0^2\right)^{-1}. \tag{46}$$

We see that for the negative values of $F_1 F_0^{-1}$ the scale factor f can grow with t up to the maximum value $|F_0 F_1^{-1}|$ and then decrease to zero. For positive $F_1 F_0^{-1}$ it blows up when the expression in the brackets vanishes (for a finite value of t). However, we must be very cautious in making definite conclusions basing on this simple result. The approximation (44) can only be reasonable when $|F_1^{(2)}| f^2 \leq F_0^2$. As F_0 and F_1 strongly depend on Λ, on the absolutely unknown mass m and on the arbitrary constant A_0, it is not possible (at the moment) to make conclusive statements on the general properties of this solution though it depends on less parameters than the first one. Note only that both solutions are compatible with existence of a period of fast growing of the scale factor.

To really discuss cosmological applications of the above asymptotic solutions[15] we have first to study possible solutions for $-\infty < \alpha < +\infty$, to glue the asymptotic solutions and, finally, to confront them to known cosmological data. The easiest part of this program is to find the asymptotic solution for $\alpha \to +\infty$. This can be done either in the gauge $\alpha = \gamma$ or, more naturally, in the standard gauge $\gamma = 0$. In the last case, one may use, instead of equations (28), (31) - (33), the constraint (27) and the somewhat different equations for $F^2(\alpha)$ and $A(\alpha)$,

$$(F^2)' + 4F^2 = \frac{4}{3}\Lambda + \left(\frac{1}{3}m^2 A^2 - 2k\right)e^{-2\alpha}\,, \qquad A'' + (1 + \psi')A' + m^2 e^{-2\psi} A = 0\,, \tag{47}$$

where F and ψ are defined as above. It is not very difficult to derive the asymptotic solutions in both gauges but this will be not very useful (even if we forget about the problem of 'ordinary' matter). The asymptotic behaviour at $\alpha \to +\infty$ strongly depends on the unknown mass m and on the other arbitrary parameters. In view of the fact that our model defined by the Hamiltonians (27), (28) is, most probably, not integrable[16] we would expect chaotic behaviour in some domains of the parameters and, correspondingly, strong instabilities in the gluing procedure.

3 Discussion

In this paper we briefly summarized the main ideas of the Einstein - Weyl model and presented its new interpretation, as well as some results obtained investigating its simplest

[15] We should not forget that **it is absolutely necessary to include into consideration 'ordinary' matter** before one can really discuss physical picture of the cosmological evolution.

[16] The Hamiltonian (28) resembles the well known non-integrable Henon - Heiles Hamiltonian [25]. The only difference is that we have the additional condition that $\mathcal{H}_1^c = 0$, which probably does not make the system integrable. Another argument in favor of non-integrability is that our system of four first-order equations (29) is certainly not integrable and one condition (28) reduces it to a third-order system that is also not integrable.

solutions. We only considered the static, spherically symmetric solutions and, in cosmology, only the homogeneous, isotropic model. As we noted in [21], even small deviations from the spherical symmetry may result in a qualitatively different theory. In particular, if we consider axially symmetric configurations infinitesimally deviating from the spherically symmetric ones, we will find additional scalar fields in the vecton gravity, which may be very important in cosmological considerations and in analysing black holes. We did not touch these problems here. Moreover, even in the spherically symmetric case our study is incomplete. In the static case, we have only proven that there may exist two horizons and derived the solutions close to the horizons. In cosmology, we have studied only the asymptotic behavior of the solutions.

As we mentioned above, we expect that the complete solutions should reveal some sort of chaotic behavior. To study these phenomena we must first carefully discuss the physical parameters of the theory. In the original formulation these are: the gravitational constant, the cosmological constant and the vecton mass. In addition, the asymptotic boundary conditions introduce other parameters, the dependence on which is highly nontrivial. This does not allow us to make sound conclusions (or, even guesses) about the global behavior of the solutions derived in our essentially local approach. For example, if we try to glue together the left and the right asymptotic approximations, we will find that the gluing procedure is strongly dependent on the parameters that characterize the influence of the nonlinear terms in the equations, up to producing chaotic effects. This requires a very careful qualitative and numerical study of the equations. Of course, the most important task is taking into account the 'ordinary' matter.

Finally, we must admit that the vecton field is a rather unusual feature of the Einstein-Weyl model. I have found just a few papers in which a massive vector field is introduced (*ad hoc*) as a candidate for dark energy [26], [27]. In this case its mass should be extremely small. Thus the unified model of dark energy and vecton dark matter considered here looks as fresh and new as it was in 1923. In addition, I wish to stress that the original Einstein Lagrangian (3) or (8) is more interesting and exciting than the simplified theory (12) (in particular, one may expect that in the original formulation there exists a relation among dimensional parameters that are arbitrary in the theory (12)). Unfortunately, the original theory is much more difficult to deal with and thus the prime goal must be the study of the simplified theory. In this paper I only give a sketch of how to begin such a study.

A preliminary draft (.ppt file) of this report can be found at the address:
http://atfilippov.googlepages.com/presentations2

4 Appendix I

The connection (5) is a special case of the general expression for the connection in affine spaces. The most **general symmetric affine connection** has the form (see, e.g., [28]):

$$\Gamma^m_{kl} = \frac{1}{2}[s^{mn}(s_{nk,l} + s_{ln,k} - s_{kl,n}) + s^{mn}(s_{nkl} + s_{lnk} - s_{kln})]\,, \tag{48}$$

where s_{kl} is an arbitrary symmetric tensor, s^{mn} is the inverse matrix to s_{kl}, and s_{kln} is an arbitrary tensor that is symmetric in k and l. Both the Weyl and Einstein connections belong to the subclass for which s_{kln} can be presented in the form:

$$s_{kln} = \alpha\, s_{kl}\, i_n + \beta(s_{nk}\, i_l + s_{ln}\, i_k)\,. \tag{49}$$

We may call it the **Weyl-Einstein connection** (defining the Weyl-Einstein spaces).

Inserting (49) into (48), we find:

$$\Gamma^m_{kl} = \frac{1}{2}[s^{mn}(s_{nk,l} + s_{ln,k} - s_{kl,n}) + \alpha(\delta^m_k\, i_l + \delta^m_l\, i_k) - (\alpha - 2\beta)s_{kl}\, i^m]\,. \tag{50}$$

Now it is easy to find that the Einstein connection (5) corresponds to $\alpha = -\beta = \frac{1}{3}$. The Weyl connection introduced in [5] corresponds to $\alpha = 1$, $\beta = 0$. I could not find a discussion of the geometry of spaces with the Einstein connection in accessible literature. The geometry of the Weyl spaces is considered in [8] ($D = 4$, Lorentzian signature), and in [28] ($D = 2, 3, 4$, Euclidean signature).

5 Appendix II

Here we write the **Friedmann equations for the Einstein - Weyl model**. To have the standard equations we use the gauge $\gamma = 0$. Then from the Lagrangian equations of motion and the constraint (27) it is not difficult to find that

$$\dot\alpha^2 + ke^{-2\alpha} = \frac{1}{3}\Lambda + \frac{1}{6}(\dot A^2 + m^2 A^2), \qquad \ddot\alpha - ke^{-2\alpha} = -\frac{1}{3}\dot A^2 - \frac{1}{6}m^2 A^2). \tag{51}$$

This can be compared to the Friedmann equations:

$$\dot\alpha^2 + ke^{-2\alpha} = \frac{1}{3}\rho, \qquad \ddot\alpha - ke^{-2\alpha} = -\frac{1}{2}(P + \rho), \tag{52}$$

where P is the pressure and ρ - the energy density (in a convenient normalization). These parameters in the E - W model are expressed in terms of the dark energy and dark matter:

$$\rho = \Lambda + \frac{1}{2}(\dot A^2 + m^2 A^2), \qquad P = -\Lambda + \frac{1}{6}(\dot A^2 - m^2 A^2), \tag{53}$$

where $A(t)$ satisfies the equation of motion:

$$\ddot A - \dot\alpha \dot A + m^2 A = 0. \tag{54}$$

From these formulas it is not difficult to obtain convenient expressions for the Hubble parameter $H^2 \equiv \dot\alpha^2$ and the deceleration parameter $q \equiv -(1 + \ddot\alpha/\dot\alpha^2)$ as well as to derive them in the asymptotic regions.

Acknowledgment: It is a great pleasure for the author to cordially thank for useful remarks V. de Alfaro, M. Choptuik, P.V. Tretyakov, V.A. Rubakov, A.A. Starobinsky, K. Stelle and E. Witten. The author very much appreciates financial support from the Department of Theoretical Physics of the University of Turin and from INFN (Turin Section), where he began to work on this paper. This work was also supported in part by the Russian Foundation for Basic Research (Grant No. 06-01-00627-a).

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Finiteness and the Higgs Mass Prediction

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Abstract

Finite Unified Theories (FUTs) are $N = 1$ supersymmetric Grand Unified Theories (GUTs) which can be made finite to all-loop orders, leading to a drastic reduction in the number of free parameters. By confronting the predictions of $SU(5)$ FUTs with the top and bottom quark masses we are able to discriminate among different models. Including further low-energy phenomenology constraints, such as B physics observables, the bound on the SM Higgs mass and the cold dark matter density, we derive predictions for the lightest Higgs boson mass and the sparticle spectrum.

1 Introduction

Finite Unified Theories (FUTs) are $N = 1$ supersymmetric Grand Unified Theories (GUTs) which can be made finite to all-loop orders, including the soft supersymmetry breaking sector. FUTs have always attracted interest for their intriguing mathematical properties and their predictive power. To construct GUTs with reduced independent parameters [1,2] one has to search for renormalization group invariant (RGI) relations holding below the Planck scale, which in turn are preserved down to the GUT scale. This programme, called Gauge–Yukawa unification scheme, applied in the dimensionless couplings of supersymmetric GUTs, such as gauge and Yukawa couplings, had already noticeable successes by predicting correctly, among others, the top quark mass in the finite $SU(5)$ GUTs [3,4]. An impressive aspect of the RGI relations is that one can guarantee their validity to all-orders in perturbation theory by studying the uniqueness of the resulting relations at one-loop, as was proven in the early days of the programme of *reduction of couplings* [2]. Even more remarkable is the fact that it is possible to find RGI relations among couplings that guarantee finiteness to all-orders in perturbation theory [5,6].

The search for RGI relations and finiteness has been extended to the soft supersymmetry breaking sector (SSB) of these theories [7–11] which involves parameters of dimension one and two. An interesting observation at the time was that in $N = 1$ Gauge–Yukawa unified theories there exists a RGI sum rule for the soft scalar masses at lower orders; at one-loop for the non-finite case [9] and at two-loops for the finite case [4]. The sum rule, although introducing more parameters in the theory, manages to overcome the problems that the universality condition for the soft scalar masses in FUTs had: that the lightest supersymmetric particle is charged, namely, the stau $\tilde{\tau}$ (although this is not necessarily a problem, as we will see), that it is difficult to comply with the attractive radiative electroweak symmetry breaking, and, worst of all, that the universal soft scalar masses can lead to charge and/or colour breaking minima deeper than the standard vacuum. Moreover, it was proven [10] that the sum rule for the soft scalar massses is RGI to all-orders for both the general as well as for the finite case. Finally the exact β-function for the soft scalar masses in the Novikov-Shifman-Vainstein-Zakharov (NSVZ) scheme [12] for the softly broken supersymmetric QCD has been obtained [10]. Eventually, the full theories can be made all-loop finite and, with use of the sum rule, their predictive power is extended to the Higgs sector and the SUSY spectrum. Thus, we are now in a position to study the spectrum of the full finite $SU(5)$ models in terms of few free parameters with emphasis on the predictions for the masses of the lightest Higgs and LSP and on the constraints imposed by low-energy phenomenology observables.

2 FINITE UNIFIED THEORIES

Finiteness can be understood by considering a chiral, anomaly free, $N = 1$ globally supersymmetric gauge theory based on a group G with gauge coupling constant g. The superpotential of the theory is given by

$$W = \frac{1}{2} m^{ij}\,\Phi_i\,\Phi_j + \frac{1}{6} C^{ijk}\,\Phi_i\,\Phi_j\,\Phi_k\ , \qquad (1)$$

where m^{ij} (the mass terms) and C^{ijk} (the Yukawa couplings) are gauge invariant tensors and the matter field Φ_i transforms according to the irreducible representation R_i of the gauge group G. All the one-loop β-functions of the theory vanish if the β-function of the gauge coupling $\beta_g^{(1)}$, and the anomalous dimensions of the Yukawa couplings $\gamma_i^{j(1)}$, vanish, i.e.

$$\sum_i \ell(R_i) = 3C_2(G)\,,\ \frac{1}{2}C_{ipq}C^{jpq} = 2\delta_i^j g^2 C_2(R_i)\ , \qquad (2)$$

where $\ell(R_i)$ is the Dynkin index of R_i, and $C_2(G)$ is the quadratic Casimir invariant of the adjoint representation of G.

The conditions for finiteness for $N = 1$ field theories with $SU(N)$ gauge symmetry are discussed in [13], and the analysis of the anomaly-freedom and no-charge renormalization requirements for these theories can be found in [14]. A very interesting result is that the conditions (2) are necessary and sufficient for finiteness at the two-loop level [15].

A powerful theorem [5] guarantees the vanishing of the β-functions to all-orders in perturbation theory. This requires that, in addition to the one-loop finiteness conditions (2), the Yukawa couplings are reduced in favour of the gauge coupling. Alternatively, similar results can be obtained [6,16] using an analysis of the all-loop NSVZ gauge beta-function [12].

In the soft breaking sector, it was found that RGI SSB scalar masses in Gauge-Yukawa unified models satisfy a universal sum rule at one-loop [9]. This result was generalized to two-loops for finite theories [4], and then to all-loops for general Gauge-Yukawa and finite unified theories [10]. Then the following soft scalar-mass sum rule is found [4]

$$\frac{(m_i^2 + m_j^2 + m_k^2)}{MM^\dagger} = 1 + \frac{g^2}{16\pi^2}\Delta^{(2)} + O(g^4) \tag{3}$$

for i, j, k with $\rho_{(0)}^{ijk} \neq 0$, where $\Delta^{(2)}$ is the two-loop correction

$$\Delta^{(2)} = -2\sum_l [(m_l^2/MM^\dagger) - (1/3)]\ \ell(R_l), \tag{4}$$

$\Delta^{(2)}$ vanishes for the universal choice, i.e. when all the soft scalar masses are the same at the unification point.

A realistic two-loop finite $SU(5)$ model was presented in [17], and shortly afterwards the conditions for finiteness in the soft susy breaking sector at one-loop [18] were given. Since these finite models have usually an extended Higgs sector, in order to make them viable a rotation of the Higgs sector was proposed [19]. The first all-loop finite theory was studied in [3], without taking into account the soft breaking terms. Naturally, the concept of finiteness was extended to the soft breaking sector, where also one-loop finiteness implies two-loop finiteness [7], and then finiteness to all-loops in the soft sector of realistic models was studied [20,21], although the universality of the soft breaking terms lead to a charged LSP. This fact was also noticed in [22], where the inclusion of an extra parameter in the Higgs sector was introduced to alleviate it. With the derivation of the sum-rule in the soft supersymmetry breaking sector and the proof that it can be made all-loop finite the construction of all-loop phenomenologically viable finite models was made possible [4,10].

Here we will examine such all-loop Finite Unified theories with $SU(5)$ gauge group, where the reduction of couplings has been applied to the third generation of quarks and leptons. An extension to three families, and the generation of quark mixing angles and masses in Finite Unified Theories has been addressed in [23], where several examples are given. These extensions are not considered here. Realistic Finite Unified Theories based on product gauge groups, where the finiteness implies three generations of matter, have also been studied [24].

The particle content of the models we will study consists of the following supermultiplets: three $(\overline{\mathbf{5}} + \mathbf{10})$, needed for each of the three generations of quarks and leptons, four $(\overline{\mathbf{5}} + \mathbf{5})$ and one $\mathbf{24}$ considered as Higgs supermultiplets. When the gauge group of the finite GUT is broken the theory is no longer finite, and we will assume that we are left with the MSSM.

Thus, a predictive Gauge-Yukawa unified $SU(5)$ model which is finite to all orders, in addition to the requirements mentioned already, should also have the following properties:

1. One-loop anomalous dimensions are diagonal, i.e., $\gamma_i^{(1)\,j} \propto \delta_i^j$.

2. Three fermion generations, in the irreducible representations $\overline{\mathbf{5}}_i, \mathbf{10}_i$ $(i = 1,2,3)$, which obviously should not couple to the adjoint $\mathbf{24}$.

3. The two Higgs doublets of the MSSM should mostly be made out of a pair of Higgs quintet and anti-quintet, which couple to the third generation.

In the following we discuss two versions of the all-order finite model. The model of ref. [3], which will be labeled **A**, and a slight variation of this model (labeled **B**), which can

also be obtained from the class of the models suggested in ref. [20] with a modification to suppress non-diagonal anomalous dimensions.

The superpotential which describes the two models takes the form [3,4]

$$\begin{aligned} W &= \sum_{i=1}^{3} [\ \frac{1}{2} g_i^u\, \mathbf{10}_i \mathbf{10}_i H_i + g_i^d\, \mathbf{10}_i \overline{\mathbf{5}}_i\, \overline{H}_i\ ] \\ &+ g_{23}^u\, \mathbf{10}_2 \mathbf{10}_3 H_4 + g_{23}^d\, \mathbf{10}_2 \overline{\mathbf{5}}_3\, \overline{H}_4 + g_{32}^d\, \mathbf{10}_3 \overline{\mathbf{5}}_2\, \overline{H}_4 \\ &+ \sum_{a=1}^{4} g_a^f\, H_a\, \mathbf{24}\, \overline{H}_a + \frac{g^\lambda}{3} (\mathbf{24})^3\ , \end{aligned} \tag{5}$$

where H_a and $\overline{H}_a$ $(a = 1, \ldots, 4)$ stand for the Higgs quintets and anti-quintets.

The non-degenerate and isolated solutions to $\gamma_i^{(1)} = 0$ for the models $\{\mathbf{A}\ ,\ \mathbf{B}\}$ are:

$$\begin{aligned} (g_1^u)^2 &= \{\frac{8}{5}, \frac{8}{5}\} g^2\ ,\ (g_1^d)^2 = \{\frac{6}{5}, \frac{6}{5}\} g^2\ , \\ (g_2^u)^2 &= (g_3^u)^2 = \{\frac{8}{5}, \frac{4}{5}\} g^2\ , \\ (g_2^d)^2 &= (g_3^d)^2 = \{\frac{6}{5}, \frac{3}{5}\} g^2\ , \\ (g_{23}^u)^2 &= \{0, \frac{4}{5}\} g^2\ ,\ (g_{23}^d)^2 = (g_{32}^d)^2 = \{0, \frac{3}{5}\} g^2\ , \\ (g^\lambda)^2 &= \frac{15}{7} g^2\ ,\ (g_2^f)^2 = (g_3^f)^2 = \{0, \frac{1}{2}\} g^2\ , \\ (g_1^f)^2 &= 0\ ,\ (g_4^f)^2 = \{1, 0\} g^2\ . \end{aligned} \tag{6}$$

According to the theorem of ref. [5] these models are finite to all orders. After the reduction of couplings the symmetry of W is enhanced [3,4].

The main difference of the models **A** and **B** is that two pairs of Higgs quintets and anti-quintets couple to the **24** for **B** so that it is not necessary to mix them with H_4 and $\overline{H}_4$ in order to achieve the triplet-doublet splitting after the symmetry breaking of $SU(5)$.

In the dimensionful sector, the sum rule gives us the following boundary conditions at the GUT scale [4]:

$$m_{H_u}^2 + 2m_{\mathbf{10}}^2 = m_{H_d}^2 + m_{\overline{\mathbf{5}}}^2 + m_{\mathbf{10}}^2 = M^2 \ \text{ for }\ \mathbf{A}\ ; \tag{7}$$

$$\begin{aligned} m_{H_u}^2 + 2m_{\mathbf{10}}^2 &= M^2\ ,\ m_{H_d}^2 - 2m_{\mathbf{10}}^2 = -\frac{M^2}{3}\ , \\ m_{\overline{\mathbf{5}}}^2 + 3m_{\mathbf{10}}^2 &= \frac{4M^2}{3} \ \text{ for }\ \mathbf{B}, \end{aligned} \tag{8}$$

where we use as free parameters $m_{\overline{\mathbf{5}}} \equiv m_{\overline{\mathbf{5}}_3}$ and $m_{\mathbf{10}} \equiv m_{\mathbf{10}_3}$ for the model **A**, and $m_{\mathbf{10}} \equiv m_{\mathbf{10}_3}$ for **B**, in addition to M.

3 PREDICTIONS OF LOW ENERGY PARAMETERS

Since the gauge symmetry is spontaneously broken below $M_{\rm GUT}$, the finiteness conditions do not restrict the renormalization properties at low energies, and all it remains are boundary conditions on the gauge and Yukawa couplings (6), the $h = -MC$ relation, and the soft scalar-mass sum rule (3) at $M_{\rm GUT}$, as applied in the two models. Thus we examine the evolution of these parameters according to their RGEs up to two-loops for dimensionless parameters and at one-loop for dimensionful ones with the relevant boundary conditions. Below $M_{\rm GUT}$ their evolution is assumed to be governed by the MSSM. We further assume a unique supersymmetry breaking scale M_s (which we define as the geometric mean of the stop masses) and therefore below that scale the effective theory is just the SM.

We now present the comparison of the predictions of the two models (**FUTA, FUTB**) with the experimental data, starting with the heavy quark masses see ref. [25] for more details. For the top quark pole mass we used the experimental value $M_{\rm top}^{exp} = (170.9 \pm 1.8)$ GeV [26]. For the bottom quark mass we used the running mass evaluated at M_z $m_{\rm bot}(M_Z) = 2.82 \pm 0.07$ [27] to avoid the uncertainties from the running of M_Z to the m_b pole mass, which are not related to the predictions of the FUT models.

In fig.1 we show the **FUTA** and **FUTB** predictions for $M_{\rm top}$ and $m_{\rm bot}(M_Z)$ as a function of the unified gaugino mass M, for the two cases $\mu < 0$ and $\mu > 0$. In the value of the bottom mass $m_{\rm bot}$, we have included the corrections coming from bottom squark-gluino loops and top squark-chargino loops [28], known usually as the Δ_b effects. The bounds on the $m_{\rm bot}(M_Z)$ and the $M_{\rm top}$ mass clearly single out **FUTB** with $\mu < 0$, as the solution most compatible with this experimental constraints. Although $\mu < 0$ is already challenged by present data of the anomalous magnetic moment of the muon a_μ, a heavy SUSY spectrum as the one we have here gives results for a_μ very close to the SM result, and thus cannot be excluded on this fact alone.

In addition the value of $\tan\beta$ is found to be $\tan\beta \sim 54$ and ~ 48 for models **A** and **B**, respectively. Thus the comparison of the model predictions with the experimental data is survived only by **FUTB** with $\mu < 0$.

We now analyze the impact of further low-energy observables on the model **FUTB** with $\mu < 0$. As additional constraints we consider the following observables: the rare b decays BR($b \to s\gamma$) and BR($B_s \to \mu^+\mu^-$), the lightest Higgs boson mass as well as a loose CDM constraint, assuming it consists mainly of neutralinos. More details and a complete set of references can be found in ref. [25].

For the branching ratio BR($b \to s\gamma$), we take the present experimental value estimated by the Heavy Flavour Averaging Group (HFAG) is [29]

$$\mathrm{BR}(b \to s\gamma) = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \times 10^{-4}. \qquad (9)$$

For the branching ratio BR($B_s \to \mu^+\mu^-$), the SM prediction is at the level of 10^{-9}, while the present experimental upper limit from the Tevatron is 5.8×10^{-8} at the 95% C.L. [30], providing the possibility for the MSSM to dominate the SM contribution.

Concerning the lightest Higgs boson mass, M_h, the SM bound of 114.4 GeV [31] can be used. For the prediction we use the code `FeynHiggs` [32].

The lightest supersymmetric particle (LSP) is an excellent candidate for cold dark matter (CDM) [35], with a density that falls naturally within the range

$$0.094 < \Omega_{\rm CDM} h^2 < 0.129 \qquad (10)$$

favoured by a joint analysis of WMAP and other astrophysical and cosmological data [36]. Assuming that the cold dark matter is composed predominantly of LSPs, the determination

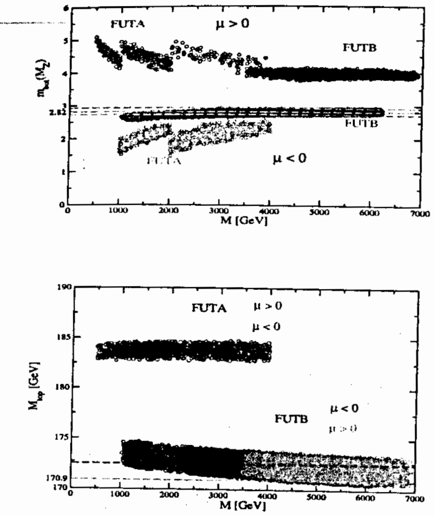

Figure 1: The bottom quark mass at the Z boson scale (upper) and top quark pole mass (lower plot) are shown as function of M for both models.

of $\Omega_{\rm CDM} h^2$ imposes very strong constraints on the MSSM parameter space, and we find that no FUT model points fulfill the strict bound of 10. On the other hand, many model parameters would yield a very large value of $\Omega_{\rm CDM}$. It should be kept in mind that somewhat larger values might be allowed due to possible uncertainties in the determination of the SUSY spectrum (as they might arise at large $\tan\beta$, see below). Therefore, in order to get an impression of the *possible* impact of the CDM abundance on the collider phenomenology in our model, we will analyze the case that the LSP does contribute to the CDM density, and apply a more loose bound of

$$\Omega_{\rm CDM} h^2 < 0.3\ . \qquad (11)$$

Notice that lower values than the ones permitted by (10) are naturally allowed if another particle than the lightest neutralino constitutes CDM. For our evaluation we have used the code MicroMegas [37].

The prediction for M_h of **FUTB** with $\mu < 0$ is shown in Fig. 2. The constraints from the two B physics observables are taken into account. In addition the CDM constraint (evaluated with `Micromegas` [37]) is fulfilled for the darker (red) points in the plot, see ref. [25] for details. The lightest Higgs mass ranges in

$$M_h \sim 121 - 126 \text{ GeV}, \qquad (12)$$

where the uncertainty comes from variations of the soft scalar masses, and from finite (i.e. not logarithmically divergent) corrections in changing renormalization scheme. To this value one has to add ±3 GeV coming from unkonwn higher order corrections [33]. We have

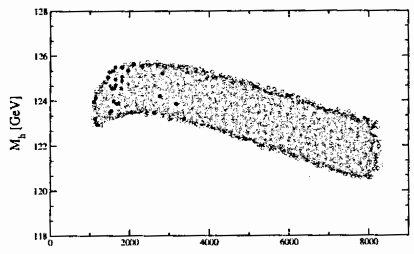

Figure 2: The lightest Higgs mass, M_h, as function of M for the model **FUTB** with $\mu < 0$, see text.

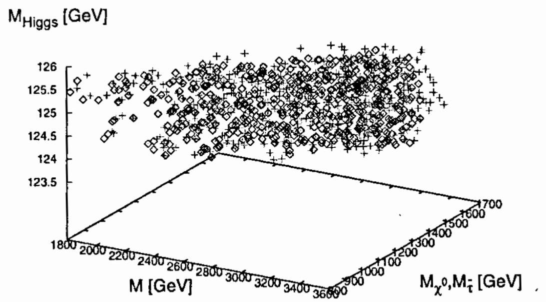

Figure 3: The lightest Higgs mass, M_h, plotted against M and the LSP, which can be the neutralino χ^0 (red crosses) or the stau $\tilde{\tau}$ (blue squares), for the model **FUTB** with $\mu < 0$, see text.

also included a small variation, due to threshold corrections at the GUT scale, of up to 5% of the FUT boundary conditions. Thus, taking into account the B physics constraints (and possibly the CDM constraints) results naturally in a light Higgs boson that fulfills the LEP bounds [31].

In Fig. 3 we present the Higgs mass for **FUTB** for the case when the LSP is the neutralino χ^0 (red crosses) and when it is the stau $\tilde{\tau}$ (blue squares), for the range of values of the gaugino mass M where the loose CDM constraint is fulfilled (left part of Fig. 2). From Fig. 3 it is clear that the prediction for the Higgs mass lies in the same range for both cases. Notice that in case the LSP is the s-tau it can decay by introducing bilinear R-parity violating terms, which respect the finiteness conditions. R-parity violation would have a small impact on the collider phenomenology presented here, but would remove the CDM bound (10) completely and the LSP would not be the CDM candidate.

In the same way the whole SUSY particle spectrum can be derived. The resulting SUSY masses for **FUTB** with $\mu < 0$ are rather large. The lightest SUSY particle starts around 500 GeV, with the rest of the spectrum being very heavy. The observation of SUSY particles

at the LHC or the ILC will only be possible in very favorable parts of the parameter space. For most parameter combinations only a SM-like light Higgs boson in the range of eq. (12) can be observed.

Partially supported by the NTUA programme for basic research "K. Karatheodoris". Work supported in part by EU's Marie-Curie Research Training Network under contract MRTN-CT-2006-035505 'Tools and Precision Calculations for Physics Discoveries at Colliders'. Also supported by the mexican grants PAPIIT-UNAM IN115207 and Conacyt 51554-F.

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Landau Models on Supermanifolds

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Abstract

We report on a recent progress in studying superextensions of the Landau problem of a quantum particle on the two-sphere S^2. These are the superspherical and superflag Landau models describing the motion of non-relativistic particle on the supermanifolds $SU(2|1)/U(1|1)$ and $SU(2|1)/[U(1) \times U(1)]$. For both models the Hilbert space norm can be made positive-definite, so they are unitary. The superflag model is parametrized by an integer $2N'$ and a real number M. At $M = 0$ it is quantum-equivalent to the supersphere Landau model with the $U(1)$ charge $2N = 2N' + 1$. In the generic case the $SU(2|1)$ symmetry is dynamically enhanced to $SU(2|2)$. We also address the planar limit of the superspherical model, in which an analog of the dynamical $SU(2|2)$ is the hidden world-line $\mathcal{N} = 2$ supersymmetry.

Talk at the International Conference "Selected Problems of Modern Theoretical Physics", Dubna, June 23 - 27, 2008

1 Introduction

The Landau model [1] describes a charged particle moving on a plane with a constant uniform magnetic flux passing through the plane. A spherical Landau model [2] describes a charged particle on a 2-sphere $S^2 \sim SU(2)/U(1)$ in the Dirac monopole background.

The aim of the present talk is to sketch the salient features of two minimal superextensions of the S^2 model: **(i)** Landau problem on the $(2+2)$-dimensional supersphere $SU(2|1)/U(1|1)$ [3, 4]; **(ii)** Landau problem on the $(2+4)$-dimensional superflag $SU(2|1)/[U(1) \times U(1)]$ [5, 4]. The large S^2 radius limits of these problems (planar super Landau models) were considered in [6]-[8] (see also [9]). One of these planar models is briefly addressed in Sect. 5.

2 Preliminaries

2.1 Planar bosonic Landau model

- Lagrangian and Hamiltonian:

$$L_b = |\dot z|^2 - i\kappa\,(\dot z\bar z - \dot{\bar z} z) = |\dot z|^2 + (A_z\dot z + A_{\bar z}\dot{\bar z})\ ,\ A_z = -i\kappa\bar z,\ A_{\bar z} = i\kappa z, \quad (2.1)$$

$$H_b = a^\dagger a + \kappa\,, a = i(\partial_{\bar z} + \kappa z),\ a^\dagger = i(\partial_z - \kappa\bar z),\ [a, a^\dagger] = 2\kappa. \quad (2.2)$$

The second term in (2.1) is the simplest example of $d = 1$ Wess-Zumino (WZ) term.

- Invariances: 'magnetic translations' and $2D$ target rotations:

$$P_z = -i(\partial_z + \kappa\bar z), P_{\bar z} = -i(\partial_{\bar z} - \kappa z), F_b = z\partial_z - \bar z\partial_{\bar z},$$
$$[P_z, P_{\bar z}] = 2\kappa, [H, P_z] = [H, P_{\bar z}] = [H, F_b] = 0.$$

2.2 Wave functions

- Lowest Landau level (LLL), $H\Psi_{(0)} = \kappa\Psi_{(0)}$:

$$a\Psi_{(0)}(z,\bar z) = 0 \Leftrightarrow (\partial_{\bar z} + \kappa z)\Psi_{(0)} = 0 \rightarrow \Psi_{(0)} = e^{-\kappa|z|^2}\psi_{(0)}(z)$$

- ℓ-th LL, $H\Psi_{(\ell)} = \kappa(2\ell+1)\Psi_{(\ell)}$:

$$\Psi_{(\ell)}(z,\bar z) = [i(\partial_z - \kappa\bar z)]^\ell e^{-\kappa|z|^2}\psi_{(\ell)}(z)\,.$$

Each LL is infinitely degenerate due to the $(P_z, P_{\bar z})$ invariance.

2.3 Generalization to $S^2 \sim SU(2)/U(1)$

- An S^2 analog of the planar Lagrangian L_b is

$$L_b = \frac{1}{(1+r^2|z|^2)^2}|\dot z|^2 + is\frac{1}{1+r^2|z|^2}\,(\dot z\bar z - \dot{\bar z} z)\,. \quad (2.3)$$

The 2-nd term is the $SU(2)/U(1)$ WZ term, r being the 'inverse' radius of S^2.

- The LL w.f. are finite-dimensional $SU(2)$ irreps, with the "spins" $s, s+1, s+2, \ldots$.

2.4 Toy fermionic "Landau model"

- Lagrangian and Hamiltonian:

$$L_f = \dot\zeta\dot{\bar\zeta} - i\kappa\left(\dot\zeta\bar\zeta + \dot{\bar\zeta}\zeta\right),\ H_f = -\alpha^\dagger\alpha - \kappa,\ \alpha = \partial_{\bar\zeta} - \kappa\zeta,\ \alpha^\dagger = \partial_\zeta - \kappa\bar\zeta. \quad (2.4)$$

Invariances:

$$\Pi_\zeta = \partial_\zeta + \kappa\bar\zeta,\ \Pi_{\bar\zeta} = \partial_{\bar\zeta} + \kappa\zeta,\ F_f = \zeta\partial_\zeta - \bar\zeta\partial_{\bar\zeta},\ [H_f, \Pi_\zeta] = [H_f, \Pi_{\bar\zeta}] = [H_f, F_f] = 0\,.$$

- Ground state and single excited state:

$$\psi^{(0)} = e^{-\kappa\zeta\bar\zeta}\psi_0(\zeta),\ \psi^{(1)} = e^{\kappa\zeta\bar\zeta}\psi_1(\bar\zeta),\ \alpha\psi^{(0)} = \alpha^\dagger\psi^{(1)} = 0,$$
$$\psi_0 = A_0 + \zeta B_0,\ \psi_1 = A_1 + \bar\zeta B_1\ H\psi^{(0)} = -\kappa\psi^{(0)},\ H\psi^{(1)} = \kappa\psi^{(1)}. \quad (2.5)$$

- The natural supertranslation invariant choice of the inner product is

$$<\phi|\psi> = \int d\zeta d\bar\zeta\ \overline{\phi(\zeta,\bar\zeta)}\psi(\zeta,\bar\zeta),\ <\psi^{(0)}|\psi^{(1)}> = 0,$$
$$<\psi^{(0)}|\psi^{(0)}> = 2\kappa\bar A_0 A_0 + \bar B_0 B_0,\ <\psi^{(1)}|\psi^{(1)}> = -2\kappa\bar A_1 A_1 - \bar B_1 B_1\,. \quad (2.6)$$

The unwanted negative norms can be removed by redefining the inner product:

$$<<\phi|\psi>> := <G\phi|\psi>,\ G\left(\psi^{(0)} + \psi^{(1)}\right) = \psi^{(0)} - \psi^{(1)},\ G = -\kappa^{-1}H_f\,.$$

With respect to the new product all norms are *positive.*

- Symmetry generators commute with the metric G, so the new inner product is still invariant. However, the hermitian conjugation properties of the operators which *do not commute* with G, change. Let $\mathcal{O}$ be some operator, such that $[H, \mathcal{O}] = 0$. Then $\mathcal{O}^\ddagger \equiv G\mathcal{O}^\dagger G = \mathcal{O}^\dagger + G\mathcal{O}_G^\dagger$, $\mathcal{O}_G \equiv [G, \mathcal{O}]$, and $[H, \mathcal{O}_G] = 0$. The symmetry generators that do not commute with G thus can generate "hidden" symmetries.

3 Superspherical Landau model

3.1 Definitions

- The supergroup $SU(2|1)$ is the minimal rank 2 semi-simple supergroup extending $U(2) \sim (F, J_{(ik)})$ by the $U(2)$ doublet of fermionic charges $(Q_i, \bar Q^k)$:

$$\{Q_i, \bar Q_k\} = \epsilon_{ik}F + J_{ik}\,,\quad \{Q_i, Q_k\} = \{\bar Q_i, \bar Q_k\} = 0\,. \quad (3.1)$$

- The Riemann supersphere $CP^{(1|1)} \cong SU(2|1)/U(1|1)$ where $U(1|1) \sim (J_3, F, Q_2, \bar Q^2)$ is a complex supermanifold

$$Z^A = (Z^0, Z^1) = (z,\zeta)\,,\qquad \bar Z^{\bar B} = (\bar Z^0, \bar Z^1) = (\bar z, \bar\zeta)\,,$$

where z is a complex coordinate of $CP^1 \sim SU(2)/U(1)$ and ζ is its anti-commuting partner. Their $SU(2|1)$ transformations are analytic

$$\delta z = i\lambda z + \varepsilon + \bar\varepsilon z^2 - (\bar\epsilon_2 + z\bar\epsilon_1)\zeta,\quad \delta\zeta = \frac{i}{2}(\lambda+\mu)\zeta + \epsilon^1 - \epsilon^2 z + \bar\varepsilon z\zeta\,, \quad (3.2)$$

$\lambda, \varepsilon, \bar\varepsilon, \mu$ being infinitesimal $U(2)$ parameters and ϵ^i Grassmann parameters.

- The supersphere is a Kähler supermanifold, with Kähler 2-form

$$\mathcal{F} = 2i\, dZ^A \wedge d\bar Z^{\bar B}\,\partial_{\bar B}\partial_A\mathcal{K} = d\mathcal{A}\,,$$

where $\mathcal{K} = \log(1 + z\bar z + \zeta\bar\zeta)$ is the Kähler potential and

$$\mathcal{A} = -i\left(dZ^A\partial_A - d\bar Z^{\bar B}\partial_{\bar B}\right)\mathcal{K} \equiv dZ^A\mathcal{A}_A + d\bar Z^{\bar B}\mathcal{A}_{\bar B}$$

is the Kähler connection.

3.2 Model

- The invariant Lagrangian reads:

$$L = \dot{Z}^A \dot{\bar{Z}}^{\bar{B}} g_{\bar{B}A} + N\left(\dot{Z}^A \mathcal{A}_A + \dot{\bar{Z}}^{\bar{B}} \mathcal{A}_{\bar{B}}\right), \tag{3.3}$$

$$g_{\bar{z}z} = \frac{1+\zeta\bar{\zeta}}{(1+z\bar{z}+\zeta\bar{\zeta})^2}, \; g_{\bar{z}\zeta} = -\frac{z\bar{\zeta}}{(1+z\bar{z})^2}, \; g_{\bar{\zeta}z} = \frac{\bar{z}\zeta}{(1+z\bar{z})^2}, \; g_{\bar{\zeta}\zeta} = \frac{1}{1+z\bar{z}}.$$

- The quantum Hamiltonian is

$$H = -(-1)^{a(a+b)} g^{A\bar{B}} \nabla_A^{(N)} \nabla_{\bar{B}}^{(N)}, \quad g^{A\bar{C}} g_{\bar{C}B} = \delta_B^A, \tag{3.4}$$

$$\nabla_A^{(N)} = \partial_A - N(\partial_A \mathcal{K}), \qquad \nabla_{\bar{B}}^{(N)} = \partial_{\bar{B}} + N(\partial_{\bar{B}} \mathcal{K}).$$

Here a, b are Grassmann parities associated with the indices A and B.

- The $\ell = 0$ (i.e. LLL) wave function $\Psi_0^{(N)}(Z, \bar{Z})$ is covariantly analytic:

$$\nabla_{\bar{A}}^{(N)} \Psi_0^{(N)} = 0 \;\Rightarrow\; \Psi_0^{(N)} = e^{-N\mathcal{K}}[A_0(z) + \zeta\psi_0(z)], \; H\Psi_0^{(N)} = 0.$$

- For any other $\ell \geq 1$:

$$\Psi_\ell^{(N)} = \Psi_{(+)\ell}^{(N)} + \Psi_{(-)\ell}^{(N)},$$

$$\Psi_{(+)\ell}^{(N)} = \nabla_z^{(N+1)} \cdots \nabla_z^{(N+2\ell-1)} \Phi_\ell^{(+)},$$

$$\Psi_{(-)\ell}^{(N)} = \left[\sum_{p=1}^{\ell} \nabla_z^{(N+1)} \cdots \nabla_\zeta^{(N+2p-1)} \cdots \nabla_z^{(N+2\ell-1)}\right] \Phi_\ell^{(-)},$$

$$\nabla_{\bar{A}}^{(N)} \Phi_\ell^{(\pm)} = 0 \;\Rightarrow\; \Phi_\ell^{(\pm)} = e^{-N\mathcal{K}} \varphi_\ell^{(\pm)}(z,\zeta), \; H\Psi_\ell^{(N)} = \ell(\ell+2N)\Psi_\ell^{(N)}. \tag{3.5}$$

- The natural definition of the $SU(2|1)$ invariant inner product is

$$<\Omega|\Psi> = \int d\mu_0 e^{-\mathcal{K}} \Omega^* \Psi \;\Rightarrow\; ||\Psi||^2 = \int d\mu_0 e^{-\mathcal{K}} \Psi^* \Psi, \; d\mu_0 = dz d\bar{z} \partial_\zeta \partial_{\bar{\zeta}}.$$

$\Psi_\ell^{(N)}$ for different ℓ are orthogonal to each other. The norm at fixed ℓ is expressed through fields in $\varphi_\ell^{(-)} = A_\ell + \zeta\psi_\ell$ and $\varphi_\ell^{(+)} = \chi_\ell + \zeta F_\ell$ as

$$||\Psi_\ell^{(N)}||^2 \sim \int \frac{dz d\bar{z}}{(1+z\bar{z})^{2(N+\ell)+1}} \Bigg[-\ell(2N+\ell)|A_\ell|^2 - \ell\bar{\psi}_\ell\psi_\ell$$

$$-\ell(\bar{\chi}_\ell + \bar{z}\bar{\psi}_\ell)(\chi_\ell + z\psi_\ell) + \frac{2(N+\ell)+1}{1+z\bar{z}} \bar{\chi}_\ell\chi_\ell + |F_\ell|^2 \Bigg]. \tag{3.6}$$

The "natural" norm is *not positive definite* (like in the fermionic Landau model), so the associated quantum theory is *not unitary*. However, there is an alternative $SU(2|1)$ invariant norm that *is positive-definite*. We shall show this in the context of the more general superflag model.

4 Superflag Landau model

4.1 Geometry

- The superflag is the coset superspace $SU(2|1)/[U(1) \times U(1)]$:

$$Z^M = (z, \zeta, \xi), \quad \bar{Z}_M = (\bar{z}, \bar{\zeta}, \bar{\xi}), \quad \delta\xi = -\frac{i}{2}(\lambda - \mu)\xi + \epsilon^2 - \bar{\epsilon}\zeta + (\bar{\epsilon}_1\zeta - \bar{\epsilon}z)\xi.$$

- The superflag is also a Kähler supermanifold:

$$F_1 = 2i dZ \wedge d\bar{Z}\, \bar{\partial}\partial \log K_1 = d\mathcal{B}, \; F_2 = -2i dZ \wedge d\bar{Z}\, \bar{\partial}\partial \log K_2 = d\mathcal{A},$$

$$\mathcal{B} = i\left(dZ^M \partial_M - d\bar{Z}^{\bar{M}} \partial_{\bar{M}}\right) \log K_1 = dZ^M \mathcal{B}_M + d\bar{Z}^{\bar{M}} \mathcal{B}_{\bar{M}},$$

$$\mathcal{A} = -i\left(dZ^M \partial_M - d\bar{Z}^{\bar{M}} \partial_{\bar{M}}\right) \log K_2 = dZ^M \mathcal{A}_M + d\bar{Z}^{\bar{M}} \mathcal{A}_{\bar{M}}.$$

Here $K_1 = 1 + (\bar{\zeta} + \bar{z}\bar{\xi})(\zeta + z\xi) + \bar{\xi}\xi$.

4.2 Superflag model

- The invariant Lagrangian is

$$L = |\omega^+|^2 + \left[\dot{Z}^M (N'\mathcal{A}_M + M\mathcal{B}_M) + c.c.\right], \tag{4.1}$$

$$\omega^+ = K_2^{-1} K_1^{-\frac{1}{2}} \left\{\dot{z}\left[1 - z\xi\bar{\zeta} - K_2\xi\bar{\xi}\right] - \dot{\zeta}\left[z\bar{\zeta} + K_2\bar{\xi}\right]\right\}.$$

Here N' and M are two real numbers. In the quantum theory, M remains arbitrary (it specifies the invariant norm) but $2N'$ must be an integer.

- There are two conjugate pairs of primary constraints in the model:

$$\varphi_\zeta = \mathcal{P}_\zeta + i(\bar{\xi}K_2 + \bar{\zeta}z)\mathcal{P}_z, \qquad \varphi_{\bar{\zeta}} = \mathcal{P}_{\bar{\zeta}} - i(\xi K_2 + \zeta\bar{z})\mathcal{P}_{\bar{z}},$$

$$\varphi_\xi = \pi_\xi - iM\mathcal{B}_\xi, \qquad \varphi_{\bar{\xi}} = \pi_{\bar{\xi}} - iM\mathcal{B}_{\bar{\xi}}, \tag{4.2}$$

where

$$\mathcal{P}_\zeta = \pi_\zeta - iN'\mathcal{A}_\zeta - iM\mathcal{B}_\zeta, \qquad \mathcal{P}_z = p_z - N'\mathcal{A}_z - M\mathcal{B}_z.$$

The quantum Hamiltonian is expressed as

$$H_{N'} = -K_2^2 K_1 \left(\nabla_z^{(N')} - \xi\nabla_\zeta^{(N')}\right)\left(\nabla_{\bar{z}}^{(N')} - \bar{\xi}\nabla_{\bar{\zeta}}^{(N')}\right) = -\mathcal{D}^{--}\mathcal{D}^{++},$$

$$\nabla_A^{(N')} = \partial_A - iN'\mathcal{A}_A, \qquad \nabla_{\bar{A}}^{(N')} = \partial_{\bar{A}} - iN'\mathcal{A}_{\bar{A}}.$$

- The quantization is accomplished by the Gupta-Bleuler method:

$$\varphi_{\bar{\zeta}} \Psi_{ph}^{(N',M)} = \varphi_{\bar{\xi}} \Psi_{ph}^{(N',M)} = 0 \;\Rightarrow\; \Psi_{ph}^{(N',M)} = K_1^M K_2^{-N'} \Phi(z, \bar{z}_{sh}, \zeta, \xi),$$

$$\bar{z}_{sh} = \bar{z} - \xi\bar{\zeta} - \bar{z}(\zeta + z\xi)\bar{\zeta}.$$

These conditions amount to the covariant chirality constraints:

$$\bar{\mathcal{D}}^+ \Psi_{ph}^{(N',M)} = \bar{\mathcal{D}}^- \Psi_{ph}^{(N',M)} = 0,$$

$$\{\mathcal{D}^\pm, \bar{\mathcal{D}}^\pm\} = \pm\mathcal{D}^{\pm\pm}, \; [\mathcal{D}^{--}, \mathcal{D}^{++}] = 2N', \; \{\mathcal{D}^+, \bar{\mathcal{D}}^-\} = \{\mathcal{D}^-, \mathcal{D}^+\} = 0,$$

where $\mathcal{D}^{\pm\pm}, \mathcal{D}^\pm, \bar{\mathcal{D}}^\pm$ are the proper covariant derivatives on $SU(2|1)/[U(1) \times U(1)]$.

- The model spectrum is defined by

$$\Psi_\ell^{(N',M)} = (\mathcal{D}^{--})^\ell \Phi^{(N'+\ell, M-\frac{\ell}{2})}$$
$$\bar{\mathcal{D}}^{\pm}\Phi^{(N'+\ell,M-\frac{\ell}{2})} = \mathcal{D}^{++}\Phi^{(N'+\ell,M-\frac{\ell}{2})} = 0 \Rightarrow \Phi^{(N'+\ell,M-\frac{\ell}{2})} \sim \Phi^{(\ell)}_{an}(z,\zeta,\xi),$$
$$H_{N'}\Psi_\ell^{(N',M)} = -(\mathcal{D}^{--}\mathcal{D}^{++})\Psi_\ell^{(N',M)} = [\ell(\ell+1) + 2N'\ell]\Psi_\ell^{(N',M)} . \qquad (4.3)$$

The spectrum is the same as in the bosonic Landau model on $S^2 \sim SU(2)/U(1)$.

- The inner product is defined as $\langle \Upsilon|\Psi\rangle = \int dz d\bar{z}\, \partial_\zeta \partial_{\bar\zeta}\partial_\xi\partial_{\bar\xi}\, K_2^{-2}\,\Upsilon^*\Psi$. Then, expanding $\Phi^{(\ell)}_{an}$ over (ζ, ξ):

$$\Phi^{(\ell)}_{an} = A^{(\ell)} + \zeta\left[\psi^{(\ell)} + (2N'+2\ell+1)^{-1}\,\partial_z\chi^{(\ell)}\right] + \xi\chi^{(\ell)} + \zeta\xi\, F^{(\ell)} ,$$

where $(A^{(\ell)}, \psi^{(\ell)}, \chi^{(\ell)}, F^{(\ell)})$ are four analytic functions, we obtain

$$||\Psi^{(\ell)}_{N'}||^2 \equiv \langle\Psi|\Psi\rangle \sim \int \frac{dz d\bar{z}}{(1+z\bar{z})^{2(N'+\ell+1)}} \times$$
$$\Big\{ (2M-\ell)\,(2M+2N'+\ell+1)\,\bar{A}^{(\ell)}A^{(\ell)} + \bar{F}^{(\ell)}F^{(\ell)}$$
$$+ \frac{(N'+\ell+1)\,(2N'+2M+\ell+1)}{(2N'+2\ell+1)\,(1+z\bar{z})}\bar{\chi}^{(\ell)}\chi^{(\ell)} + (2M-\ell)\,(1+z\bar{z})\,\bar{\psi}^{(\ell)}\psi^{(\ell)}\Big\} .$$

There are ghosts for any $M = \pm|M|$. How to cure the norm?

- Redefne $<< \Upsilon|\Psi >> = < \Upsilon|G\psi > \Rightarrow |||\Psi|||^2 \equiv \langle\Psi|G\Psi\rangle$, where, for $-2N'-1 < 2M < 0$,

$$G_{an} = -1 + 2\xi\partial_\xi + \frac{2}{2N'+2\ell+1}\,\zeta\partial_z\partial_\xi\,,\ [H_{N'}, G_{an}] = 0\,,\ G^2_{an} = 1\,,$$

and, for $2M < -2N'-1$,

$$\tilde{G}_{an} = 1 - 8\,(F-2M-N') + 8\,(F-2M-N')^2 .$$

All states in these ranges possess non-negative norms [1].

4.3 Quantum equivalence of the SS and SF models

- The covariant conditions singling out SS superfields among the SF ones read:

$$\mathcal{D}^{+}\Psi^{(N',M)} = \bar{\mathcal{D}}^{-}\Psi^{(N',M)} = 0 \Leftrightarrow \partial_{\xi,\bar\xi}\Psi^{(N',M)} = 0 \Rightarrow \Psi^{(N',M)} = \Psi^{(N',M)}(z,\bar{z},\zeta,\bar\zeta).$$

They are consistent only at $M = 0$.

[1]The same can be accomplished for $M > 0$.

- The SS and SF wave superfunctions are related as

$$\Psi^{(N)}_\ell = \mathcal{D}^{+}\tilde{\Psi}_\ell^{(N-\frac{1}{2},0)} = \mathcal{D}^{+}\left[(\mathcal{D}^{--})^\ell\tilde{\Phi}^{(N+\ell-\frac{1}{2},-\frac{\ell}{2})}\right],$$
$$\bar{\mathcal{D}}^{-}\tilde{\Psi}_\ell^{(N-\frac{1}{2},0)} = \bar{\mathcal{D}}^{+}\tilde{\Psi}_\ell^{(N-\frac{1}{2},0)} = 0 \Leftrightarrow \mathcal{D}^{+}\Psi^{(N)}_\ell = \bar{\mathcal{D}}^{-}\Psi^{(N)}_\ell = 0 . \qquad (4.4)$$

The SS model *at* N is thus equivalent to the $M = 0$ SF model *at* $N' = N - \frac{1}{2}$.

- One can define the "master" Hamiltonian which yields both the SS and SF Hamiltonians as its two different limits:

$$H_{mast} = -\frac{1}{2}\left(\mathcal{D}^{++}\mathcal{D}^{--} + \mathcal{D}^{--}\mathcal{D}^{++}\right) + \frac{1}{2}[\mathcal{D}^{-}, \bar{\mathcal{D}}^{+}] + \frac{1}{2}[\mathcal{D}^{+}, \bar{\mathcal{D}}^{-}],$$
$$H_{mast} \Rightarrow -\mathcal{D}^{--}\mathcal{D}^{++} - 2M = H_{SF} - 2M \quad \textit{On chiral SF superfields}$$
$$H_{mast} \Rightarrow H_{SS} = -(-1)^{a(a+b)}\, g^{A\bar{B}}\nabla^{(N)}_A\nabla^{(N)}_{\bar{B}} \quad \textit{On SS superfields}. \qquad (4.5)$$

5 Planar super-Landau: superplane model

Planar limit of the SS and SF models is achieved by making explicit the S^2 radius R, properly rescaling Hamiltonians and taking $R \to \infty$ at fixed $\kappa = N/R^2$. Here we present some salient features of the superplane model following from the SS one.

- It is a hybrid of the bosonic and fermionic Landau models

$$L = L_f + L_b = |\dot{z}|^2 + \dot{\zeta}\dot{\bar\zeta} - i\kappa\left(\dot{z}\bar{z} - \dot{\bar{z}}z + \dot{\zeta}\bar\zeta + \dot{\bar\zeta}\zeta\right), \qquad (5.1)$$
$$H = a^\dagger a - \alpha^\dagger\alpha = \partial_{\bar\zeta}\partial_\zeta - \partial_z\partial_{\bar z} + \kappa\left(\bar{z}\partial_{\bar z} + \bar\zeta\partial_{\bar\zeta} - z\partial_z - \zeta\partial_\zeta\right) + \kappa^2\left(z\bar{z} + \zeta\bar\zeta\right).$$

The invariances are generated by $P_z, P_{\bar z}, \Pi_\zeta, \Pi_{\bar\zeta}$ and the new generators

$$Q = z\partial_\zeta - \bar\zeta\partial_{\bar z}\,,\ Q^\dagger = \bar{z}\partial_{\bar\zeta} + \zeta\partial_z\,, C = z\partial_z + \zeta\partial_\zeta - \bar{z}\partial_{\bar z} - \bar\zeta\partial_{\bar\zeta}\,.$$

They generate $ISU(1|1)$, contraction of $SU(2|1)$:

$$\{Q, Q^\dagger\} = C\,,\ [Q, P_z] = i\Pi_\zeta\,,\ \{Q^\dagger, \Pi_\zeta\} = iP_z\,.$$

- The natural $ISU(1|1)$-invariant inner product given by

$$< \phi|\psi >= \int d\mu\, \overline{\phi\left(z,\bar{z};\zeta,\bar\zeta\right)}\,\psi\left(z,\bar{z};\zeta,\bar\zeta\right),\ d\mu = dzd\bar{z}d\zeta d\bar\zeta,$$

leads to negative norms, like in other examples. All norms become positive after introducing the metric

$$G = -\kappa^{-1}H_f = \frac{1}{\kappa}\left[\partial_\zeta\partial_{\bar\zeta} + \kappa^2\bar\zeta\zeta + \kappa\left(\zeta\partial_\zeta - \bar\zeta\partial_{\bar\zeta}\right)\right].$$

G commutes with all generators, except Q, $Q^\dagger$, whence

$$Q^\ddagger = Q^\dagger - \frac{i}{\kappa}S\,,\ S = i\left(\partial_z\partial_{\bar\zeta} + \kappa^2\bar{z}\zeta - \kappa\bar{z}\partial_{\bar\zeta} - \kappa\zeta\partial_z\right).$$

- According to Sect. 2.4, $S, S^\ddagger$ define a symmetry: $[H,S] = [H,S^\ddagger] = 0$. Also,

$$S = a^\dagger\alpha\,,\; S^\ddagger = a\alpha^\ddagger\,,\; \{S,S^\ddagger\} = 2\kappa H\,, \{S,S\} = 0 = \{S^\ddagger,S^\ddagger\} \tag{5.2}$$

  i.e. $S, S^\ddagger, H$ form $\mathcal{N}=2, d=1$ superalgebra. The LLL ground state is annihilated by $S, S^\ddagger$

$$S\psi^{(0)} = S^\ddagger\psi^{(0)} = 0,$$

  and so it is $\mathcal{N}=2$ SUSY singlet. Hence $\mathcal{N}=2$ SUSY is unbroken and all higher LL form irreps of this SUSY [10]. One can find the realization of this SUSY on the involved $d=1$ fields [7, 8].

- Whether analogous hidden (super)symmetries exist in the SS and SF Landau models where also non-trivial metric operators appear? The answer is YES [4]! After passing to the positive norms one finds the dynamical *enhancement* of the initial $SU(2|1)$ symmetry to $SU(2|2)$ symmetry. The supergroup $SU(2|2)$ involves two independent $SU(2)$ doublets of spinor generators and has $SU(2)\times SU(2)\times U(1)$ as the bosonic subgroup, the $U(1)$ generator being *central.*

6 Summary and outlook

- Self-consistent superextensions of the bosonic Landau models can be constructed and they are WZW type sigma models on the appropriate graded extensions of the "magnetic translation" group and $SU(2)$ group.

- The appearance of negative norms can be evaded by *redefining* the inner products.

- An intriguing common feature of the planar super Landau models is the appearance of the hidden dynamical $\mathcal{N}=2$ SUSY. Its analog in the superflag Landau models is the enhancement of the $SU(2|1)$ symmetry to $SU(2|2)$.

- Possible physical applications: supersymmetric versions of the Quantum Hall Effect [11]? Relations to integrable structures in $\mathcal{N}=4$ SYM and string theory?

Acknowledgements. I thank the Organizers of the Conference for inviting me to give this talk. I also thank my co-authors Andrey Beylin, Thomas Curtright, Luca Mezincescu and Paul K. Townsend. I acknowledge a support from the RFBR grants 06-02-16684-a, 08-02-90490, the grant INTAS-1000005-7928 and a grant of the Heisenberg-Landau program.

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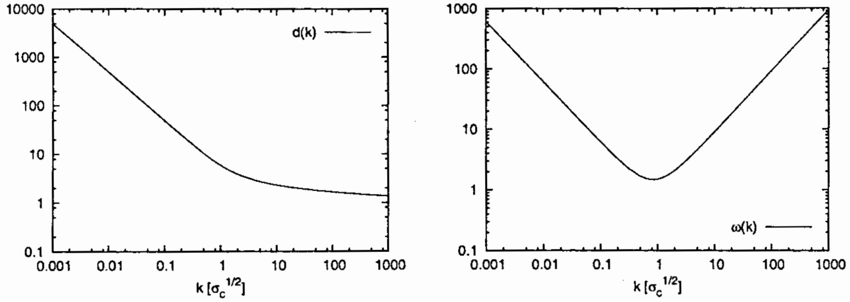

Figure 1: Ghost form factor $d(k)$ (left) and gluon energy $\omega(k)$ from the variational solutions presented in [2].

3 Variational solution

We wish to solve the Schrödinger equation

$$H\psi = E\psi \tag{11}$$

by the variational principle

$$\langle\psi|H|\psi\rangle \rightarrow min \tag{12}$$

with suitable ansätze for the wave functional $\psi(A^{\perp})$. This approach has been studied resently in Refs. [3, 4]. Inspired by the wave functional of a massless particle moving in a spherically symmetric potential in an s-state $\psi = \phi(r)/r$, where $r = J^{1/2}(r)$ is the Jacobian of the transformation from the cartesian to the spherical coordinates for zero angular momentum we choose the following ansatz [4]

$$\psi(A^{\perp}) = \frac{1}{\sqrt{J(A^{\perp})}} \exp\left(-\frac{1}{2}\int d^3x d^3y A_i^{\perp a}(x)\omega(x,y)A_i^{\perp a}(y)\right), \tag{13}$$

where the kernel $\omega(x,y)$ is determined from the variational principle (12). In practice the so resulting equation for $\omega(x,y)$ is converted into a set of Dyson-Schwinger equations for the gluon propagator

$$\langle A_i^{\perp a}(x)A_j^{\perp b}(y)\rangle = \delta^{ab}t_{ij}(x)\frac{1}{2}\omega^{-1}(x,y), \tag{14}$$

with $t_{ij}(x) = \delta_{ij} - \frac{\partial_i\partial_j}{\partial^2}$ being the transverse projector, and the ghost propagator

$$G(x,y) = \left\langle\left(-\hat{\boldsymbol{D}}\cdot\boldsymbol{\partial}\right)^{-1}\right\rangle = \langle x|d(-\Delta)(-\Delta)^{-1}|y\rangle. \tag{15}$$

Here we have introduced the ghost form factor $d(-\Delta)$, which describes the deviation of the QCD ghost propagator from the QED case, where $d(-\Delta) \equiv 1$. The resulting Dyson-Schwinger equations need renormalisation, which is well under control. Fig. 1 shows the solution of the Dyson-Schwinger equation for the gluon energy $\omega(k)$ and the ghost form

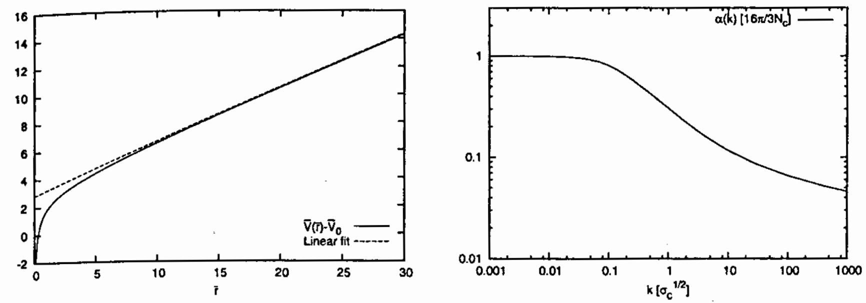

Figure 2: *Left:* Heavy quark potential given by eq. (18). *Right:* Running coupling constant.

factor $d(k)$, as shown in Ref. [2]. An analytic infrared and ultraviolet analysis of the Dyson-Schwinger equation shows the following asymptotic behaviour [4, 5]

$$\begin{aligned}\mathrm{IR}\,(k\rightarrow 0) \;&:\; \omega(k)\sim\frac{1}{k} \qquad d(k)\sim\frac{1}{k}\\ \mathrm{UV}\,(k\rightarrow\infty) \;&:\; \omega(k)\sim k \qquad d(k)\sim k^0\,.\end{aligned} \tag{16}$$

At large momenta the gluon behaves like a photon, which is in agreement with asymptotic freedom, while at small momenta the gluon energy diverges, which implies the absence of gluon states in the physical spectrum. This is nothing but a manifestation of gluon confinement. The infrared divergence of the ghost form factor is a consequence of the horizon condition

$$d^{-1}(k=0) = 0\,, \tag{17}$$

which has been used as input in the renormalisation of the ghost Dyson-Schwinger equation. This is a necessary condition for the Gribov-Zwanziger confinement scenario. In fact, one can show there is a sum rule relating the infrared exponents of the ghost and the gluon propagator and an infrared divergent gluon energy requires also an infrared divergent ghost form factor, i.e. the horizon condition (17), see Ref. [5]. Fig. 2 shows the non-Abelian Coulomb potential

$$V(|x-y|) = g^2\left\langle\langle x|(-\hat{\boldsymbol{D}}\cdot\boldsymbol{\partial})^{-1}(-\boldsymbol{\partial}^2)(-\hat{\boldsymbol{D}}\cdot\boldsymbol{\partial})^{-1}|y\rangle\right\rangle \rightarrow \sigma_C\,|x-y|\,, \tag{18}$$

which for large distance indeed increases linearly [2] as the infrared analysis reveals. The Coulomb string tension σ_C sets the scale of our approach. Also shown in Fig. 2 is the running coupling constant which is infrared finite, for details see Ref. [5]. Fig. 3 shows the continuum results for the gluon energy and the ghost form factor in $D = 2+1$ dimensions [6] together with the corresponding lattice results, Ref. [7]. The agreement is not perfect but, given the approximation involved, quite satisfactory.

In $D = 3+1$ dimensions, previous lattice calculations performed in Coulomb gauge in Ref. [8, 9] showed an anomalous UV behaviour of the gluon propagator — IR : $\omega(k) \sim k^0$, UV : $\omega(k) \sim k^{3/2}$ — which is in strong conflict with the continuum result. However, one should mention that these lattice calculations assumed multiplicative renormalisability of the 4-dimensional gluon propagator, which give rise to scaling violations in the static

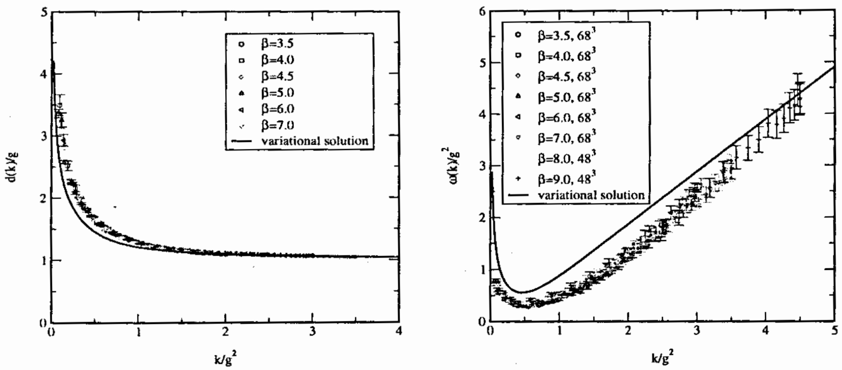

Figure 3: Results for the ghost form factor $d(k)$ (left) and the gluon energy $\omega(k)$ in $2+1$ dimensions, as shown in [6].

propagator. Furthermore, these calculations did not fix the gauge completely, i.e. the residual time-dependent gauge invariance left after Coulomb gauge fixing was left unfixed. Furthermore, the Coulomb gauge fixing was done on a single time-slice, which is sufficient for the calculation of static (time-independent) propagators. However, one should keep in mind that in Coulomb gauge topologically non-trivial gauge configurations, which presumably are responsible for confinement, are discontinuous in time [10] and as a consequence on a small lattice different results are obtained from different time slices.

Recently, we have done improved lattice calculations with a complete gauge fixing [11]. In these studies, the energy dependence of the 4-dimensional gluon propagator could be explicitly extracted and it was found that the static gluon propagator is multiplicatively renormalisable and shows a perfect scaling. Fig. 4 (left panel) shows the results for the gluon propagator of these calculations together with the continuum results. It is assumed here that the Coulomb string tension σ_C is identical to the string tension σ from the Wilson loop. There is a very good agreement, in particular the ultraviolet and infrared behaviour matches perfect for lattice and continuum. What is also remarkable that the lattice result can be very well fitted by Gribov's original formula for the gluon energy

$$\omega(k) = \sqrt{k^2 + \frac{M^4}{k^2}} \tag{19}$$

with $M = 0.88(1)\,\mathrm{GeV}$.

4 The colour dielectric constant

Consider the electric field generated by a charge density ρ in electrodynamics

$$\mathbf{E} = -\boldsymbol{\partial}\phi\,,\ \phi = (-\Delta)^{-1}\rho\,. \tag{20}$$

The longitudinal electric field resulting from the resolution of Gauss' law in the Yang-Mills case is given by a similar expression

$$\mathbf{E} \;=\; \langle \boldsymbol{\Pi} \rangle = -\partial\phi$$

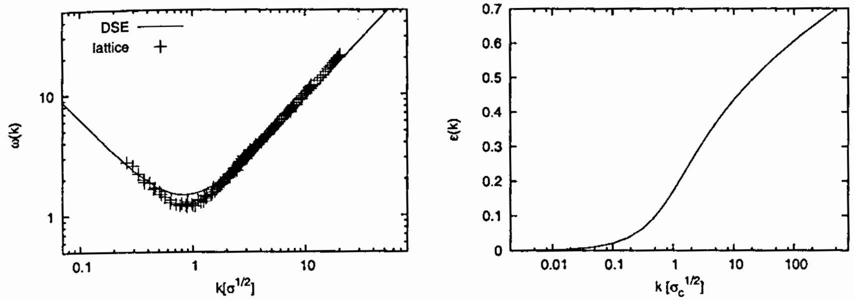

Figure 4: *Left:* Lattice data for $\omega(k)$, compared to the solution of the Dyson-Schwinger equations. *Right:* Dielectric function $\epsilon(k)$.

$$\phi \;=\; \left\langle \left(-\hat{D}\cdot\boldsymbol{\partial}\right)^{-1} \right\rangle \rho = d(-\Delta)(-\Delta)^{-1}\rho \tag{21}$$

except that the Green's function of the Laplacian is replaced by the ghost propagator (15). The last expression has the form of the scalar potential in the presence of a dia-electric medium

$$\phi = \epsilon^{-1}(-\Delta)^{-1}\rho \tag{22}$$

and the inverse of the ghost form factor $d(k)$ can thus be identified as the dielectric function of the Yang-Mills vacuum

$$\epsilon(k) = d^{-1}(k)\,. \tag{23}$$

Fig. 4 (right panel) shows the so defined dielectric function. It satisfies $0 < \epsilon(k) < 1$, which is a manifestation of anti-screening while in QED we have $\epsilon(k) > 1$, which corresponds to ordinary Debye screening. Furthermore, at zero momentum the dielectric function vanishes, showing that in the infrared the Yang-Mills vacuum behaves like a perfect colour dia-electric medium. The vanishing of the dielectric function in the infrared is not an artifact of our solutions of the Dyson-Schwinger equations but is guaranteed by the horizon condition, which is a necessary condition for the Gribov-Zwanziger confinement scenario. A perfect colour dia-electric medium $\epsilon = 0$ is nothing but a dual superconductor. (Here, "dual" refers to an interchange of electric and magnetic fields and charges.) Recall in an ordinary superconductor the magnetic permeability vanishes $\mu = 0$. This shows that the Gribov-Zwanziger confinement scenario implies the dual Meissner effect [12].

5 Topological susceptibility

As first shown by Adler [13] and Bell and Jackiw [14], the $U_A(1)$ symmetry is anomalously broken which gives rise to an extra mass term to the η', which by the Witten-Veneziano formula

$$m^2_{\eta'} + m^2_\eta - 2m^2_K = \frac{2N_f}{F^2_\pi}\chi \tag{24}$$

Coulomb gauge Yang–Mills theory in the Hamiltonian approach[1]

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Abstract

Within the Hamiltonian approach in Coulomb gauge the ghost and gluon propagators are determined from a variational solution of the Yang–Mills Schrödinger equation showing both gluon and heavy quark confinement. The continuum results are in good agreement with lattice data. The ghost form factor is identified as the dielectric function of the Yang–Mills vacuum and a connection between the Gribov–Zwanziger scenario and the dual Meissner effect is established. The topological susceptibility is calculated.

1 Introduction

The aim of the talk is the microscopic description of infrared properties of QCD like confinement. We would like to see, for example, the emergence of the colour flux string between static colour charges. For this purpose, I will use the Hamiltonian approach to Yang-Mills theory in Coulomb gauge. The organisation of my talk is as follows: In section 2, I will briefly summarise the essential ingredients of the Hamiltonian approach to Yang-Mills theory in Coulomb gauge. Then I will present a variational solution of the Yang-Mills Schrödinger equation in section 3, which will result in a set of coupled Dyson-Schwinger equations. I will present the analytic solutions to these equations in the infrared and ultraviolet and the numerical solution for the full momentum range. The resulting propagators will then be compared to the available lattice data. Then I will focus on two non-perturbative properties of the Yang-Mills vacuum: the dielectric constant in section 4 and the topological susceptibility in section 5. Finally, a summary is provided.

2 Canonical quantisation of Yang-Mills theory

In the canonical quantisation approach the gauge fields $A_\mu^a(x)$ are considered as the (cartesian) coordinates and the corresponding conjugate momenta are defined by

$$\Pi_\mu^a(x) = \frac{\delta S}{\delta \partial_0 A_\mu^a(x)} , \tag{1}$$

[1]Invited talk given by H. Reinhardt at the international conference on "Selected Problems in Theoretical Physics, Dubna 23-27 June 2008". Supported in part by the Deutsche Forschungsgemeinschaft (DFG) under contract no. Re856/6-1,2.

where S is the action of the Yang-Mills field. The explicit calculation yields

$$\Pi_i^a(x) = E_i^a(x) \, , \; \Pi_0^a(x) = 0 \, , \tag{2}$$

where $E_i^a(x)$ is the colour electric field. To avoid the problems arising from the vanishing temporal component of the canonical momentum, one imposes Weyl gauge $A_0^a(x) = 0$. The Yang-Mills Hamiltonian is then given by

$$H = \frac{1}{2} \int d^3x \left(\boldsymbol{\Pi}^2(x) + \boldsymbol{B}^2(x) \right) . \tag{3}$$

The canonical quantisation is carried out in the standard fashion by imposing the canonical commutation relation $[A_i^a(x), \Pi_j^b(y)] = \delta_{ij}\delta^{ab}\delta^3(x-y)$, which promotes the canonical momentum to the operator $\Pi_i^a(x) = \delta / i\delta A_i^a(x)$. By imposing Weyl gauge one loses Gauss' law from the Heisenberg equation of motion and Gauss' law has to be imposed as a constraint on the wave functional

$$\hat{\mathbf{D}} \cdot \boldsymbol{\Pi}(x)\psi(x) = -g\rho_m(x)\psi(x) \, , \tag{4}$$

where $\rho_m(x)$ is the colour charge density of the matter fields and $\hat{D}_i^{ab} = \delta^{ab}\partial_i + g\hat{A}_i^{ab}$ ($\hat{A}^{ab} = f^{acb}A^c$) is the covariant derivative in the adjoint representation of the gauge field with f^{abc} being the structure constant of the gauge group. The operator on the left hand side of Gauss' law is nothing but the generator of time-independent gauge transformations and in the absence of external colour charges, $\rho_m(x) = 0$, Gauss' law expresses the invariance of the wave functional under space-dependent but time-independent gauge transformations.

Instead of working with explicit gauge invariant wave functionals it is more convenient to explicitly resolve Gauss' law by fixing the gauge. Coulomb gauge is a particular convenient choice for this purpose. We implement the Coulomb gauge, $\boldsymbol{\partial} \cdot \boldsymbol{A} = 0$, in the standard fashion into the scalar product of the wave functionals by means of the Faddeev-Popov method

$$\langle \psi | \mathcal{O} | \phi \rangle = \int DA^\perp J(A^\perp) \psi^*(A^\perp) \mathcal{O}[A^\perp] \phi(A^\perp) \, , \tag{5}$$

where

$$J = \mathrm{Det}(-\hat{\boldsymbol{D}} \cdot \boldsymbol{\partial}) \tag{6}$$

is the Faddeev-Popov determinant. While in Coulomb gauge the gauge field is transversal the momentum operator $\boldsymbol{\Pi} = \boldsymbol{\Pi}^{||} + \boldsymbol{\Pi}^\perp$ contains both longitudinal $\boldsymbol{\Pi}^{||}$ and transversal $\boldsymbol{\Pi}^\perp$ parts. Resolving Gauss' law for the longitudinal part of the momentum operator yields

$$\boldsymbol{\Pi}^{||}\psi = g\boldsymbol{\partial}(-\hat{\boldsymbol{D}} \cdot \boldsymbol{\partial})^{-1}\rho\psi \, , \; \rho = \rho_g + \rho_m \, , \tag{7}$$

where

$$\rho_g = \hat{\mathbf{A}}^\perp \cdot \boldsymbol{\Pi}^\perp \tag{8}$$

is the colour charge density of the gauge field. With this result the Hamiltonian in Coulomb gauge is found to be

$$H = \frac{1}{2} \int \left(J^{-1} \boldsymbol{\Pi}^\perp J \boldsymbol{\Pi}^\perp + \boldsymbol{B}^2 \right) + H_C \, , \tag{9}$$

where

$$H_C = \frac{1}{2} \int J^{-1} \boldsymbol{\Pi}^{||} J \boldsymbol{\Pi}^{||} = \frac{g^2}{2} \int J^{-1} \rho (-\hat{\boldsymbol{D}} \cdot \boldsymbol{\partial})^{-1} (-\boldsymbol{\partial}^2) (-\hat{\boldsymbol{D}} \cdot \boldsymbol{\partial})^{-1} J \rho \tag{10}$$

is the so-called Coulomb Hamiltonian, which arises from the longitudinal part of the kinetic energy after resolving Gauss' law. The Hamiltonian (9) was first derived in Ref. [1].

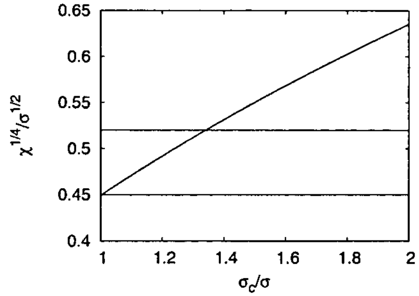

Figure 5: Topological susceptibility χ as a function of the ratio σ_c/σ.

is expressed by the topological susceptibility

$$\chi = -i \int d^4x \langle 0|q(x)q(0)|0\rangle \ , \tag{25}$$

which is the correlation function of the topological charge density

$$q(x) = \frac{g^2}{32\pi^2} F^a_{\mu\nu}(x)\tilde{F}^{a\mu\nu}(x) \ . \tag{26}$$

Furthermore, in eq. (24) N_f denotes the number of flavours and $F_\pi \sim 93\,\mathrm{MeV}$ is the pion decay constant. χ vanishes in all orders of perturbation theory and is thus an ideal observable to test the non-perturbative content of our vacuum wave functional. In the Hamiltonian approach one finds the following expression for the topological susceptibility [15]

$$V\chi = \left(\frac{g^2}{8\pi^2}\right)^2 \left[\langle 0|\int \boldsymbol{B}^2(x)|0\rangle - 2\sum_n \frac{|\langle n|\int \boldsymbol{B}\cdot\boldsymbol{\Pi}|0\rangle|^2}{E_n}\right] \ . \tag{27}$$

Here $|n\rangle$ denotes the exact excited states of the Yang-Mills Hamiltonian with energies E_n. These eigenstates are of course not known. We work out the matrix elements in eq. (27) to two-loop order. In this order only two and three quasi gluon states

$$a_i^{a\dagger}(x)a_j^{b\dagger}(y)|0\rangle \ , \ a_i^{a\dagger}(x)a_j^{b\dagger}(y)a_k^{c\dagger}(z)|0\rangle \tag{28}$$

contribute where our vacuum state is annihilated by the operators $a_i^a(x)$, i.e. $a_i^a(x)|0\rangle = 0$. The resulting expression for the topological susceptibility is ultraviolet divergent and needs renormalisation. For this aim we exploit the fact that χ vanishes to all order perturbation theory and renormalise the expression (27) for χ by subtracting each propagator by its perturbative expression. This renders χ (27) finite. Furthermore, since the momentum integrals in this expression are dominated by the infrared part we replace the coupling constant, which, in principle, should be the running one, by its infrared value. The results obtained in this way for the topological susceptibility are shown in Fig. 5 (right panel) as a function of the ratio σ_C/σ. Choosing $\sigma_C = 1.5\sigma$ which is the value favoured by the lattice calculation [8] we find with $\sqrt{\sigma} = 440\ MeV$

$$\chi = (240\ MeV)^4 \ . \tag{29}$$

This value is somewhat larger than the lattice prediction $\chi = (200 - 230\ MeV)^4$.

6 Summary and Conclusions

I have presented a variational solution of the Yang-Mills Schrödinger equation in Coulomb gauge using a Gaussian type of ansatz for the vacuum wave functional. We find a gluon energy which is infrared divergent, which is a manifestation of gluon confinement. Furthermore, we have found a static colour charge potential which at large distances rises linearly, as one expects for a confining theory. The propagators calculated within this approach are all in satisfactory agreement with the lattice data. I have then shown that the inverse of the ghost form factor can be interpreted as the colour dielectric function of the QCD vacuum. The horizon condition, a necessary condition for the Gribov-Zwanziger confinement scenario to work, implies that in the infrared the QCD vacuum is a perfect colour dia-electric medium, which is nothing but a dual superconductor. In this way the Gribov-Zwanziger confinement scenario implies the dual Meissner effect. Finally I have presented results for the topological susceptibility calculated in the Hamiltonian approach with our vacuum wave functional. For reasonable values of the Coulomb string tension we find results close to but somewhat larger than the lattice data. The results obtained so far in this approach are quite encouraging for further investigations. A natural next step would be the inclusion of dynamical quarks.

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Theoretical Aspects of Spin Program at NICA

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Abstract

The new JINR project on construction of accelerator complex NICA provide the unique possibilities for study of various spin effects in different reactions. Among them the study of Drell-Yan (DY) processes in collisions of transversely polarized protons and deutrons are of extreme importance since they allow to extract the such important transverse-momentum dependent distribuions as transversity, Sivers and Boer-Mulders functions. Here we estimate the size and the feasibility of single-spin asymmetries (SSA) which provide the access to transversity as well as to Boer-Mulders and Sivers PDFs. The feasibility is studied with the new generator of polarized DY events. The performed estimations demonstrate that there exist the such kinematical regions where SSA are presumably measurable.

At present JINR launched the new NICA/MPD project based on the development of the existing Nuclotron accelerator for the new facility creation: the heavy and light nucleus collider NICA [1]. In particular, the possibility is now considered to study the collisions of the polarized proton and deutron beams at the second interaction point (IP) at NICA. That allows to study the DY processes in collisions of transversely polarized protons and deutrons, providing us an access to the very important and still poorly known sea and valence transversity, Boer-Mulders and Sivers PDFs in proton. It is argued [1] that the design of the collider allows to reach the energy of colliding proton beams up to 10 GeV at luminosity up to $10^{30} cm^2 s^{-1}$. At the same time the respective numbers for the deutron collisions are also quite considerable: collision energy per nucleon up to $\sqrt{(s)} \sim 12GeV$ with the average luminosity up to $10^{29} cm^2 s^{-1}$. It is of great importance that both proton and deutron beams can be effectively polarized, with the polarization degree not less than 50% [1].

The such unique possibilities gives the great opportunities to extract from Drell-Yan processes the such important PDFs as transversity, Sivers and Boer-Mulders PDFs. At present the Boer-Mulders PDF is still not measured, while the Sivers [3, 4] and transversity [5] PDFs were preliminarily extracted from the SIDIS data collected by HERMES [6] and COMPASS [7] collaborations.

It is well known that the double transversely polarized DY process allows to directly extract the transversity distributions [2]. However, in the case of pp, pD and DD collisions these asymmetries are much smaller than the single spin asymmetries. Besides, namely unpolarized and single-polarized DY processes give us also an access to Boers-Mulders and Sivers PDFs, which are very intriguing and interesting objects in themselves. On the other hand, in the single-polarized DY processes the access to PDFs we are interesting in is rather difficult since they enter the respective cross-sections [8] in the complex convolution with each other, so that at first sight it is impossible to avoid some models on the k_T dependence of PDFs. To solve this problem the q_T weighting approach [9] was recently applied in Ref. [3] to Sivers effect in the single-polarized DY processes, and in Refs. [10, 11] with respect

to transversity and Boer-Mulders PDF. Here we will estimate both types of single-spin asymmetries (SSA) for proton-proton and proton-deutron collisions.

Transversity and T-odd PDFs via Drell-Yan processes with pp collisions

The procedure proposed in Refs. [10, 11] allows to extract from the single-polarized DY processes the transversity h_1 and the first moment $h_{1q}^{\perp(1)}(x) \equiv \int d^2\mathbf{k}_T \left(\frac{\mathbf{k}_T^2}{2M_p^2}\right) h_{1q}^{\perp}(x_p, \mathbf{k}_T^2)$ of Boer-Mulders $h_1^{\perp(1)}$ PDF directly, without any model assumptions about k_T-dependence of $h_1^{\perp}(x, k_T^2)$. Applied to unpolarized DY process with pp collisions this general procedure gives

$$\hat{k}\Big|_{pp\to l^+l^-X} = 8\frac{\sum_q e_q^2[\bar{h}_{1q}^{\perp(1)}(x_1)h_{1q}^{\perp(1)}(x_2) + (q \to \bar{q})]}{\sum_q e_q^2[\bar{f}_{1q}(x_1)f_{1q}(x_2) + (q \to \bar{q})]}, \tag{1}$$

where $\hat{k}$ is the coefficient at $\cos 2\phi$ dependent part of the properly q_T weighted ratio of unpolarized cross-sections [10, 11]:

$$\hat{R} = \frac{3}{16\pi}(\gamma(1+\cos^2\theta) + \hat{k}\cos 2\phi \sin^2\theta). \tag{2}$$

At the same time, in the case of single-polarized DY process, operating just as in Ref. [10], one gets

$$\hat{A}_h = -\frac{1}{2}\frac{\sum_q e_q^2[\bar{h}_{1q}^{\perp(1)}(x_p)h_{1q}(x_{p\uparrow}) + (q \to \bar{q})]}{\sum_q e_q^2[\bar{f}_{1q}(x_p)f_{1q}(x_{p\uparrow}) + (q \to \bar{q})]}, \tag{3}$$

where the $\sin(\phi + \phi_S)$- and q_T-weighted single spin asymmetry (SSA) $\hat{A}_h$ is defined just as in Refs. [10, 11]. SSA $\hat{A}_h$ is analogous to asymmetry $A_{UT}^{\sin(\phi-\phi_S)\frac{q_T}{M_N}}$ applied in Ref. [3] with respect to the Sivers effect investigation in the single-polarized DY processes. For DY process $pp^\uparrow \to l^+l^-X$ we study here the expressions for $A_{UT}^{\sin(\phi-\phi_S)\frac{q_T}{M_N}}$ look as (see Eqs. (14), (15) in Ref. [3])

$$A_{UT}^{\sin(\phi-\phi_S)\frac{q_T}{M_N}} = 2\frac{\sum_q e_q^2[\bar{f}_{1T}^{\perp(1)q}(x_{p\uparrow})f_{1q}(x_p) + (q \to \bar{q})]}{\sum_q e_q^2[\bar{f}_{1q}(x_{p\uparrow})f_{1q}(x_p) + (q \to \bar{q})]}, \tag{4}$$

where $f_{1T}^{\perp(1)q}(x) \equiv \int d^2\mathbf{k}_T \left(\frac{\mathbf{k}_T^2}{2M_p^2}\right) f_{1T}^{\perp q}(x, \mathbf{k}_T^2)$ is the first moment of the Sivers function $f_{1T}^{\perp q}(x, \mathbf{k}_T^2)$. Notice that factor 2 in Eq. (4) (see also Eq. (7) in Ref. [12]) was introduced in Ref. [3] for consistence with the respective semi-inclusive SSA studied by the HERMES. Since within this paper we also will study SSA given by Eqs. (4) and (4), for comparison purposes it is convenient to introduce, by analogy, SSA $A_{UT}^{\sin(\phi+\phi_S)\frac{q_T}{M_N}} = 2\hat{A}_h$.

Estimations on SSA in pp collisions

Let us first estimate SSA $A_{UT}^{\sin(\phi-\phi_S)\frac{q_T}{M_N}}\Big|_{pp^\uparrow\to l^+l^-X}$ given by Eq. (4) in the NICA kinematical conditions. We perform the calculations for Q^2 values below and above J/Ψ threshold

$Q^2 = 9.5\,GeV^2$. For the estimations we use three different fits for the Sivers function: fits I and II from Ref. [3] and also the latest fit from Ref. [4], which we denote as fit III. For the first moments () of the sea Sivers PDFs entering Eq. (4) we use the model (with the positive sign) proposed in Ref. [12] (see Eqs. (10) and (11) in Ref. [12]). For the unpolarized PDFs entering Eq. (4) we use GRV94 [13] parametrization. The results of estimations for the different Q^2 values are presented in Fig. 1. Looking at Fig. 1 one can

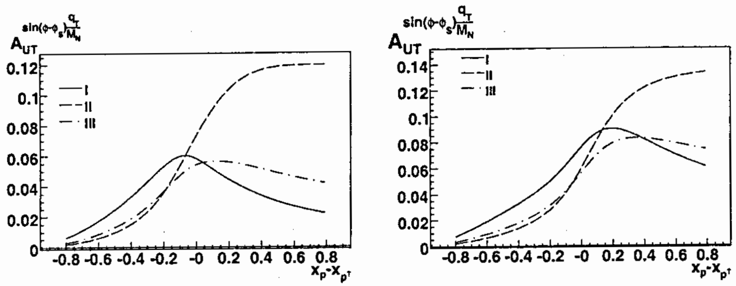

Figure 1: Estimation of SSA $A_{UT}^{\sin(\phi-\phi_S)\frac{q_T}{M_N}}\Big|_{pp^\uparrow}$ for NICA, s=400GeV^2, with $Q^2 = 4\,GeV^2$ (left) and $Q^2 = 15\,GeV^2$ (right). Rome numbers I, II denote respectively fits I and II from Ref. [3] and III denotes the fit from Ref. [4].

see that the asymmetry takes the largest values near zero value of $x_p - x_{p^\uparrow}$ and when this difference becomes positive.

Let us now estimate SSA $A_{UT}^{\sin(\phi+\phi_S)\frac{q_T}{M_N}}$. Since neither the Boer-Mulders function nor its first moment are still not measured, we will use in our calculation the Boer's model (Eq. (50) in Ref. [8]) which produces the good fit for the NA10 [14] and E615 [15] data. We also apply the following assumption for the first moment of the sea Boer-Mulders PDF

$$h_{1\bar{q}}^{\perp(1)}(x)/h_{1q}^{\perp(1)}(x) = f_{1\bar{q}}(x)/f_{1q}(x). \quad (5)$$

Recently, for the fist time, the transversity PDF was extracted [5] from the combined data of HERMES, COMPASS and BELLE collaborations. However, because of the rather poor quality of data the errors surrounding the fit on h_1 is very large, and, besides, the authors of Ref. [5] were compelled to apply the large number of approximations. In particular, the approximation of zero sea transversity PDF was applied. However, as it was stressed before, in the case of proton-proton collisions namely the sea PDFs play the crucial role. That is why here we will apply two versions of evolution model for transversity instead of the fit from Ref. [5]. First is the model where the Soffer inequality is saturated: $h_{1q}(x, Q_0^2) = \frac{1}{2}\left[q(x, Q_0^2) + \Delta q(x, Q_0^2)\right]$, $h_{1\bar{q}}(x, Q_0^2) = \frac{1}{2}\left[\bar{q}(x, Q_0^2) + \Delta\bar{q}(x, Q_0^2)\right]$, at low initial scale ($Q_0^2 = 0.23GeV^2$), and then h_{1q}, $h_{1\bar{q}}$ are evolved with DGLAP. In the second version of evolution model (see [2, 16] and references therein) we use the assumptions $h_{1q(\bar{q})}(x, Q_0^2) = \Delta q(\bar{q})(x, Q_0^2)$ at the same initial scale. This model is much more realistic one because at the model initial scale a lot of models predict [2] that $h_1 = \Delta q$. Besides, we found that the curve corresponding to this version of evolution model is in very good agreement with the fit of Ref. [5].

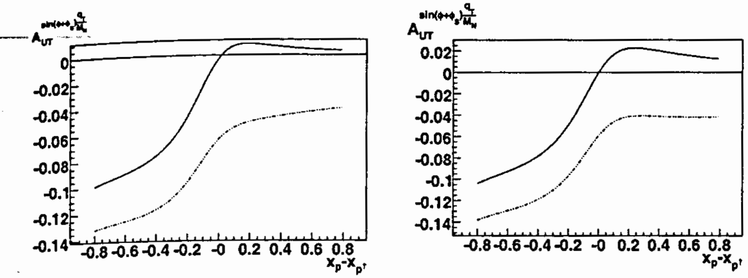

Figure 2: Estimation of SSA $A_{UT}^{\sin(\phi+\phi_S)\frac{q_T}{M_N}}\Big|_{pp^\uparrow}$ for NICA, $s = 400\,GeV^2$, with $Q^2 = 4\,GeV^2$ (left) and $Q^2 = 15\,GeV^2$ (right). The solid and dotted curves correspond to the two different input ansatzes for h_{1u} which are used in evolution model. These are $h_{1q,\bar{q}} = \Delta q, \bar{q}$ and $h_{1q} = (\Delta q + q)/2$ $h_{1\bar{q}} = (\Delta\bar{q} + \bar{q})/2$, respectively. Here GRV94 [13] parametrization for $q(x)$ and GRSV95 [17] parametrization for $\Delta q(x)$ are used.

We present here (Fig. 2) the estimations of SSA $A_{UT}^{\sin(\phi+\phi_S)\frac{q_T}{M_N}}$ below and above J/ψ resonance for NICA kinematical conditions. Looking at Fig. 2 one can see that the asymmetry $A_{UT}^{\sin(\phi+\phi_S)\frac{q_T}{M_N}}$ is negligible at $x_p > x_{p^\uparrow}$ and is quite considerable at $x_p < x_{p^\uparrow}$. In this second case SSA $A_{UT}^{\sin(\phi+\phi_S)\frac{q_T}{M_N}}$ takes its maximal values (about 5-10%) when $x_p - x_{p^\uparrow}$ takes the large negative values.

SSA in pD and DD collisions

As usual, the inclusion of the deutron beam/target can allow us to find PDFs of u and d quark, in separation. Applying $SU_f(2)$ symmetry to the results for SSA in for pp case, one immediately gets the respective results on SSA for Drell-Yan processes in pD and DD collisions. As it was mentioned above there exist the strong theoretical arguments [18] based on $1/N_c$ expansion that the sum of the u and d quark Sivers first moments $f_{1T}^{\perp(1)u}$ and $f_{1T}^{\perp(1)d}$ is very small quantity. Besides, the QCD evolution predicts small values of the sea transversity distributions *even at small x* values [16]. Thus, in the case of polarized deutron in initial state, the respective SSA presumably should be very small quantity (and our calculations confirm it), compatible with zero. It is the great task for NICA to check wether this SSA is zero or not. The only SSA which could take considerable value is SSA containing the sum $h_{1u}(x_{D^\uparrow}) + h_{1d}(x_{D^\uparrow})$. The point is that the analysis [5] of the COMPASS data obtained on the deutron target produced the possibility of nonzero sum $h_{1u} + h_{1d}$. In accordance with this analysis h_{1u} and h_{1d} are of opposite sign but differ in their absolute values (see Fig. 7 in Ref. [5]). However, the uncertainties on h_{1u} and h_{1d} are too large (see the error bands in Fig. 7) to realize is the quantity $h_{1u} + h_{1d}$ zero or not. Thus, the respective measurements of SSA in DY processes with polarized deutron could shed the light on this problem.

On the contrary to the case of polarized deutron, in the case of polarized proton all SSA could take the considerable values. Our calculations show that they are of the same order of magnitude as the respective SSA in the case of $pp^\uparrow$ collisions: ratio $R =$

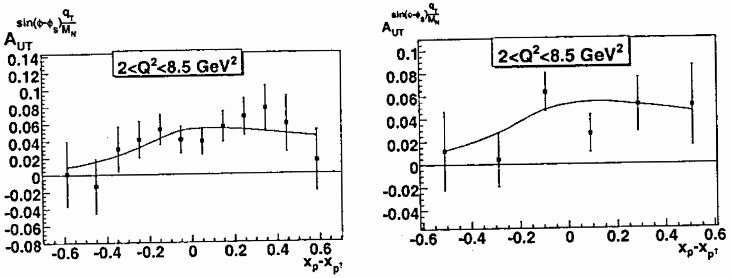

Figure 3: Estimation of asymmetries $A_{UT}^{\sin(\phi-\phi_S)\frac{q_T}{M_N}}\Big|_{pp^\uparrow}$ for NICA, $s = 400 GeV^2$. Here fit from Ref. [4] is used. The points with errors bars are obtained by using simulations with event generator at the applied statistics 100K (left) and 50K (right) pure Drell-Yan events. $\langle Q^2 \rangle \simeq 3.5\, GeV^2$ for both plots.

$A_{UT}^{\sin(\phi-\phi_S)\frac{q_T}{M_N}}\Big|_{Dp^\uparrow} / A_{UT}^{\sin(\phi-\phi_S)\frac{q_T}{M_N}}\Big|_{pp^\uparrow}$ changes from 0.4 to 0.8, while the asymmetries $A_{UT}^{\sin(\phi+\phi_S)\frac{q_T}{M_N}}$ and $A_{UT}^{\sin(\phi+\phi_S)\frac{q_T}{M_N}}\Big|_{pp^\uparrow}$ are almost coincide.

Thus, one can conclude that in the case of $Dp^\uparrow$ collisions both, weighted with $\sin(\phi-\phi_S)$ and weighted with $\sin(\phi+\phi_S)$, SSA are presumably measurable in the same x regions as the respective SSA in the case of $pp^\uparrow$ collisions.

Estimations on the SSA feasibility with the new generator of polarized DY events

Until recently there was no in the free access any generator of Drell-Yan events except for the only PYTHIA generator. However, regretfully, in PYTHIA there are only unpolarized Drell-Yan processes and, besides, they are implemented in PYTHIA without correct q_T and $\cos 2\phi$ dependence, which is absolutely necessary to study Boer-Mulders effect. Thus, we wrote the new generator of polarized DY events (the details will be published elsewhere). The scheme of generator is quite simple and very similar to the event generator GMC_TRANS [19] which was successfully used by HERMES collaboration for simulation of the Sivers effect in semi-inclusive DIS processes [6].

To perform the comprehensive feasibility estimations one needs to take into account the all peculiarities of the concrete experimental setup. This is the subject of our future investigations. Here we present the estimations at the generator level.

We prepared two samples with applied statistics 100K and 50K of pure Drell-Yan events for each of two Q^2 ranges: $2 < Q^2 < 8.5\, GeV^2$ and $Q^2 > 11\, GeV^2$. Cut $2 < Q^2 < 8.5\, GeV^2$ is applied to avoid misidentification of lepton pairs due to numerous background processes (combinatorial background from Dalitz-decays and gamma conversions, etc – see, for instance, section F.4.2 in Ref. [20]) below $Q^2 = 2\, GeV^2$ and to exclude lepton pairs coming from J/ψ region. Cut $Q^2 > 11\, GeV^2$ is also applied to avoid the lepton pairs coming from J/ψ region. The results are presented in Fig. 3. The results in the region $Q^2 > 11\, GeV^2$ are very similar to results in $2 < Q^2 < 8.5\, GeV^2$ region, so that we omit

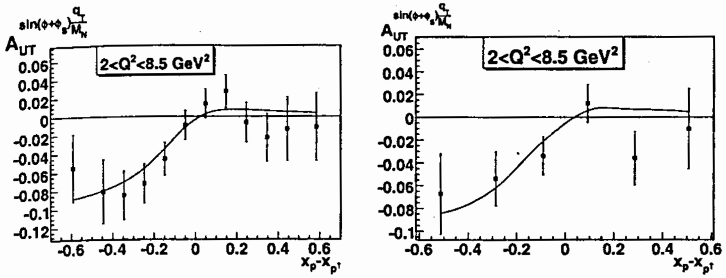

Figure 4: Estimation of asymmetries $A_{UT}^{\sin(\phi+\phi_S)\frac{q_T}{M_N}}\Big|_{pp^\uparrow}$ for NICA, $s = 400 GeV^2$. Here the evolution model with the input ansatz $h_{1q,\bar{q}} = \Delta q, \bar{q}$ at $Q_0^2 = 0.23\, GeV^2$ is used. GRV94 [13] parametrization for $q(x)$ and GRSV95 [17] parametrization for $\Delta q(x)$ are used. The points with errors bars are obtained with the developed generator of polarized DY events at the applied statistics 100K (left) and 50K (right) pure Drell-Yan events. $\langle Q^2 \rangle \simeq 3.5\, GeV^2$ for both plots.

them. For the simulations with the developed generator we use the latest parametrization, fit III (solid line in Fig. 3), from the set [3, 4]. Looking at Fig. 3 one can see that even at relatively low applied statistics 50K pure Drell-Yan events there are three presumably measurable points for $A_{UT}^{\sin(\phi-\phi_S)\frac{q_T}{M_N}}$ in the kinematical region $x_p - x_{p^\uparrow} > 0$, where this SSA is about 4-6%. Moreover, at applied statistics 100K pure Drell-Yan events one can hope to reconstruct the functional form of SSA $A_{UT}^{\sin(\phi-\phi_S)\frac{q_T}{M_N}}$ in the kinematical region $x_p > x_{p^\uparrow}$. In the region $x_p < x_{p^\uparrow}$ SSA $A_{UT}^{\sin(\phi-\phi_S)\frac{q_T}{M_N}}$ is smaller (less than 4%), but still visible within the errors (even at applied statistics 50K events one can see at least one measurable point). Let us now estimate the feasibility of SSA $A_{UT}^{\sin(\phi+\phi_S)\frac{q_T}{M_N}}$. The results are presented in Fig. 4 . For the simulations we again use Boer model for $h_1^\perp$ and the evolution model for h_1 with $h_{1q(\bar{q})} = \Delta q(\Delta\bar{q})$ ansatz at initial scale $Q_0^2 = 0.23\, GeV^2$. Looking at Fig. 4 one can see that in the region $x_p < x_{p^\uparrow}$ even at statistics 50K pure Drell-Yan events one can hope to see within the errors at least three points for $A_{UT}^{\sin(\phi+\phi_S)\frac{q_T}{M_N}}$ in the kinematical region $x_p - x_{p^\uparrow} < 0$. At the same time, at the statistics 100K events one can hope also to reconstruct the functional form of this SSA in the kinematical region $x_p < x_{p^\uparrow}$.

Let us note that due to the close values of $A_{UT}^{\sin(\phi+\phi_S)\frac{q_T}{M_N}}$ in the cases of $pp^\uparrow$ and $Dp^\uparrow$ collisions all conclusions concerning feasibility of $A_{UT}^{\sin(\phi+\phi_S)\frac{q_T}{M_N}}$ in the case of $pp^\uparrow$ collisions are valid in $Dp^\uparrow$ case too.

We estimated also the DY event rates at NICA assuming that the achieved luminosity will be about $10^{30} cm^{-2}s^{-1}$ and using cut $Q > 2\, GeV$ to reduce the background. The results are presented in Table 1 in comparison with the results for PAX experiment. From this table it is seen that the required statistics 50K events could be achieved with only 1 year of data taking while 100K events could be collected within 2 years of data taking. It is of importance that for these estimations we use the strong cut $Q > 2\, GeV$. However, if one

Table 1: DY event rate estimates per month for NICA and PAX

	σ_{DY} total, nb	$L, cm^{-2}s^{-1}$	K events
PAX, $\sqrt{s} = 14.6 GeV$	~ 2	$\sim 10^{30}$	~ 10
NICA, $\sqrt{s} = 20 GeV$	~ 1	$\sim 10^{30}$	~ 5
NICA, $\sqrt{s} = 26 GeV$	~ 1.3	$\sim 10^{30}$	~ 7

Table 2: DY event rate estimates per month for NICA and PAX with different cuts in Q

cut on Q, GeV	1.5	1.6	1.7	1.8	1.9	2.0
NICA, $\sqrt{s} = 20\,GeV$						
σ_{DY} total,nb	2.54	1.94	1.59	1.32	1.1	0.9
N events for a month, K	14.1	10.5	8.8	7.3	6.1	5
NICA, $\sqrt{s} = 26\,GeV$						
σ_{DY} total,nb	3.3	2.7	2.3	1.9	1.6	1.3
N events for a month, K	18	15	13	10	9	7
PAX, $\sqrt{s} = 14.6\,GeV$						
σ_{DY} total,nb	5.1	4.33	3.5	2.9	2.46	2.09
N events for a month, K	24.4	20.7	16.7	13.9	11.8	10

use more soft cuts (see Table 2), the rates per month could be sufficiently increased. As it is stated in PAX proposal [20], this could be done if one carefully study the background in the region $Q < 2\,GeV$ and tune the trigger appropriately.

In summary, the Drell-Yan processes with the colliding protons and deutrons available to NICA were considered. We estimated the single-spin asymmetries $A_{UT}^{\sin(\phi-\phi_S)\frac{q_T}{M_N}}$ and $A_{UT}^{\sin(\phi+\phi_S)\frac{q_T}{M_N}}$, which give us an access to Sivers and to Boer-Mulders and transversity PDFs, respectively. The preliminary estimations demonstrate that SSA $A_{UT}^{\sin(\phi-\phi_S)\frac{q_T}{M_N}}$ can reach quite considerable values (5-10%) in both $x_p > x_{p\uparrow}$ and $x_p < x_{p\uparrow}$ regions. On other hand, the estimations performed for SSA $A_{UT}^{\sin(\phi+\phi_S)\frac{q_T}{M_N}}$ show that this asymmetry is negligible in the region $x_p > x_{p\uparrow}$ and takes considerable values (also about 5-10%) in the region $x_p < x_{p\uparrow}$. The estimations performed for SSA in the case of Dp and DD collisions demonstrate that the asymmetries for DY processes with $pD^\uparrow$ collisions are compatible with zero except for perhaps one, containing sum $h_{1u} + h_{1d}$. On the contrary to DY processes with $pD^\uparrow$ and $DD^\uparrow$ collisions, SSA for $Dp^\uparrow$ collisions are close in their values to the respective SSA for $pp^\uparrow$ collisions and, thus, presumably could be feasible in the same kinematical regions.

The new generator of polarized DY events was developed which allowed us to estimate the feasibility of both weighted with $\sin(\phi - \phi_S)$ and $\sin(\phi + \phi_S)$ single-spin asymmetries. These estimations performed for proton-proton collisions demonstrate that both SSA are presumably measurable even at the applied statistics 50K pure Drell-Yan events. While $A_{UT}^{\sin(\phi-\phi_S)\frac{q_T}{M_N}}$ is presumably measurable in both kinematical regions $x_p > x_{p\uparrow}$ and $x_p < x_{p\uparrow}$, the asymmetry $A_{UT}^{\sin(\phi+\phi_S)\frac{q_T}{M_N}}$ could be measured only in the region $x_p < x_{p\uparrow}$, where it takes quite considerable values.

Acknowledgments

The authors are grateful to R. Bertini, O. Denisov, A. Efremov, Y. Goto, T. Iwata, S. Kazutaka, V. Kekelidze, S. Kumano, A. Maggiora, I. Meshkov, A. Olshevsky, G. Piragino, G. Pontecorvo, S. Sawada, I. Savin, A. Sorin and O. Teryaev, for fruitful discussions.

The work of O. Shevchenko, A. Nagaytsev, W. Kallies, G.V. Mesheryakov and O. Ivanov was supported by the Russian Foundation for Basic Research (Project No. 07-02-01046).

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NUCLOTRON-BASED ION COLLIDER FACILITY (NICA) AT JINR: NEW PROSPECT FOR HEAVY ION COLLISIONS AND SPIN PHYSICS

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The Joint Institute for Nuclear Research (JINR) in Dubna is an international research organization established in accordance with the intergovernmental agreement of 11 countries in 1956. At the present time, eighteen countries are the JINR Member States and five countries, having an Observer status. The JINR basic facility for high-energy physics research is represented by the 6 AGeV Nuclotron. It has replaced the old weak focusing 10 GeV proton accelerator Synchrophasotron, which delivered the first nuclear beams of the relativistic energy of 4.2 AGeV in 1971. Since that time the study of relativistic heavy ion physics became one of the main directions of the JINR research program. The new flagship of the JINR is the NICA/MPD project [1,2]. The main goal of the project is to start in the coming years experimental study of hot and dense strongly interacting matter at the new JINR facility. This goal will be reached by: 1) development of the existing Nuclotron accelerator facility as a basis for generation of intense beams over atomic mass range from protons to uranium and light polarized ions; 2) design and construction of the heavy ion collider having maximum collision energy of $\sqrt{s_{NN}} = 9$ GeV and averaged luminosity of 10^{27} $cm^{-2} \cdot s^{-1}$ and 3) design and construction of Multipurpose Particle Detector (MPD) at colliding beams. Realization of the project will provide unique conditions for the world community research activity. The NICA energy region is of major interest because the highest nuclear (baryonic) density under laboratory conditions can be reached there. Generation of intense polarized light nuclear beams aimed at investigation of polarization phenomena is foreseen as well.

NICA/MPD Goals and Physics Problems

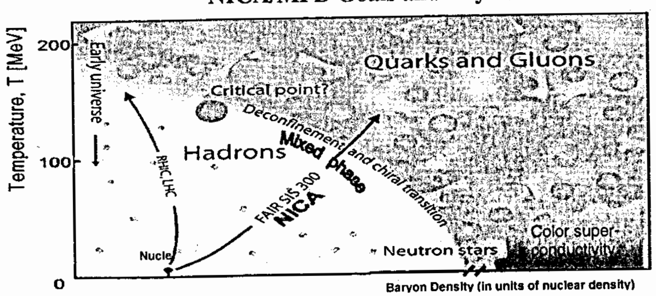

Fig. 1. Phase diagram of nuclear matter (artist's view)

The investigations are
- Relevant to understanding of the evolution of the Early Universe after Big Bang, formation of neutron stars, and the physics of heavy ion collisions. The new JINR facility will make it possible to study in-medium properties of hadrons and nuclear
- matter equation of state, including a search for possible signature of deconfinement and/or chiral symmetry restoration phase transition and critical endpoint in the region of $\sqrt{s_{NN}} = 3-9$ GeV by means of careful scanning in beam energy and centrality of excitation functions The first stage measurements include:

multiplicity and global characteristics of identified hadrons including multi-strange particles; fluctuations in multiplicity and transverse momenta; directed and elliptic flows for various hadrons; HBT and particle correlations. Electromagnetic probes (photons and dileptons) are supposed to be added at the second stage of the project.

The beam energy of the NICA is very much lower than the region of the RHIC (BNL) and the LHC (CERN) but it sits right on the top of the region where the baryon density is expected to be the highest. In this energy range the system occupies a maximal space-time volume in *the mixed quark-hadron phase* (the phase of coexistence of hadron and quark-gluon matter similar to the water-vapor coexistence-phase). The net baryon density at LHC energies is predicted to be lower. The energy region of NICA will allow analyzing the highest baryonic density under laboratory conditions.

The conditions similar to NICA are expected to be reproduced at FAIR facility (GSI) after put the synchrotron SIS300 into operation in 2016. Two different approaches — fixed target experiment CBM at FAIR and collider experiment MPD at NICA will allow a wide variety of methods to be used in these studies. Therefore both facilities, FAIR and NICA/MPD, can be considered as two complementary basic facilities aimed at the study of relevant physics of Hot and Dense Baryonic Matter. GSI and JINR have already a long-term experience of successful cooperation.

NICA General Layout

The NICA (Fig. 2) will consist of a cascade of accelerators. The multicharged ions will be generated in the unique ion source "KRION" developed at JINR, and accelerated in linear accelerator up to 6 MeV per nucleon. Then they are injected in the Booster-Synchrotron – a new machine to be built, accelerated in there, extracted and stripped on a carbon foil into "bare state". Transferred to the Nuclotron they are accelerated up to experiment energy. Before extraction the ion bunch is compressed and becomes of 30 cm length. Such ion bunches are injected, cycle by cycle, into collider rings and provide in collisions the required luminosity. Construction of the new facility is based on the existing buildings and infrastructure of the Synchrophasotron and Nuclotron of JINR.

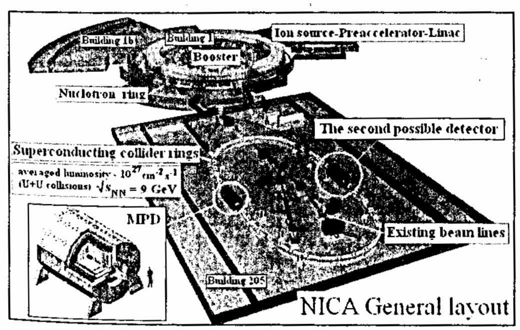

Fig. 2. NICA General Layout. The accelerator chain includes: heavy ion source – RFQ injector – linac – booster ring – Nuclotron – Superconducting collider rings. The peak design kinetic energy of $^{92}U+$ ions in the collider is 3.5 AGeV. Beam cooling and bunching systems are foreseen. The collider magnetic system is fitted to the existing building. The project design presumes realization some of fixed target experiments. Collider operation with polarized deuteron and light ion beams is foreseen as the second stage of the project development.

MPD for Mixed Phase experiments

The proposed MPD (Fig. 3) has to detect the high multiplicity events and perform particle identification. The tracking system includes Inner Tracker (IT) - silicon strip detector, Time Projection Chamber (TPC) – the main tracker, Outer Tracker (straw barrel detector) and End

Cap Tracker (straw wheels). This system is immersed into homogeneous magnetic field of 0,5T of superconducting solenoid with the axis parallel to the beam direction. The detector provides reconstruction and momentum measurement of charged particles in the region $|\eta|<1$. In the extended region of $|\eta|>1$ the accuracy of momenta measurement is lower. For the particles identification Time of Flight (TOF) System based on the RPC is proposed. This system allows pion, kaon and proton identification in the momentum range of 0.2 - 2 GeV/c. The TPC option of the tracker could provide also particle identification by measuring its ionization energy loss. For the electron/positron and gamma detection in the central region the crystal Electromagnetic Calorimeter (ECal) is considered. Two counter systems (Beam-Beam Counters) are located symmetrically at the edges of the detector along the beam axis to provide the trigger information and for precise definition of interaction point.Two Zero Degree Calorimeters (ZDC) provide the energy measurement of spectators and determination of "centrality" in the ion-ion collision.

Some basic parameters are: Interaction rate of U+U events at luminosity of 10^{27} $cm^{-2}\cdot s^{-1}$ is of 10 kHz (interaction rate of central events is of ~ 500 Hz); the accuracy of vertex reconstruction by means of IT is better then 0.2 mm; the TPC produces ~ 50 hits on track and provides momentum measurement accuracy of ~ 1 % in the range of 0.2 – 2 GeV/c; TOFsystem has resolution of ~ 100 ps and provides pion and kaon separation with probability of 5% below 2 GeV/c.

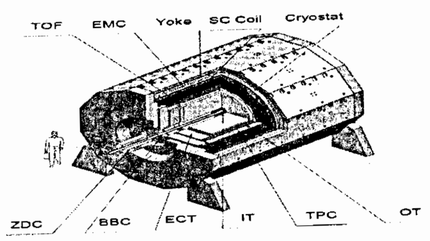

Fig. 3. The MPD schematics. IT – inner tracker (silicon strip detector), TPC –Time Projection Chamber (main tracker), OT (Outer Tracker, straw), ECT (End-Cap Tracker, straw), TOF (Time of Flight, RPC chambers), BBC – beam-beam counters, ZDC – Zero Degree Calorimeters, ECT - End Cap Tracker (straw chambers).

Summary and Outlook

The new facility at JINR in Dubna will allow to study very important unsolved problems of strongly interacting matter. The NICA/MPD commissioning is scheduled in 2014. The design and organization work has been started. The first issue of the NICA/MPD Conceptual Design Report is completed. We suppose a wide world cooperation with many Laboratories both at R&D and construction stages of work. Important innovation aspects of the activity are supposed.

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Landau Gauge QCD: Functional Methods versus Lattice Simulations

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Abstract

The infrared behaviour of QCD Green's functions in Landau gauge has been focus of intense study. Different non-perturbative approaches lead to a prediction in line with the conditions for confinement in local quantum field theory as spelled out in the Kugo-Ojima criterion. Detailed comparisons with lattice studies have revealed small but significant differences, however. But aren't we comparing apples with oranges when contrasting lattice Landau gauge simulations with these continuum results? The answer is yes, and we need to change that. We therefore propose a reformulation of Landau gauge on the lattice which will allow us to perform gauge-fixed Monte-Carlo simulations matching the continuum methods of local field theory which will thereby be elevated to a truly non-perturbative level at the same time.

Introduction

The Green's functions of QCD are the fundamental building blocks of hadron phenomenology [1]. Their infrared behaviour is also known to contain essential information about the realisation of confinement in the covariant formulation of QCD, in terms of local quark and gluon field systems. The Landau gauge Dyson-Schwinger equation (DSE) studies of Refs. [2, 3] established that the gluon propagator alone does not provide long-range interactions of a strength sufficient to confine quarks. This dismissed a widespread conjecture from the 1970's going back to the work of Marciano, Pagels, Mandelstam and others. The idea was revisited that the infrared dominant correlations are instead mediated by the Faddeev-Popov ghosts of this formulation, whose propagator was found to be infrared enhanced. This infrared behaviour is now completely understood in terms of confinement in QCD [1, 4, 5], it is a consequence of the celebrated Kugo-Ojima (KO) confinement criterion.

This criterion is based on the realization of the unfixed global gauge symmetries of the covariant continuum formulation. In short, two conditions are required by the KO criterion to distinguish confinement from Coulomb and Higgs phases: (a) The massless single particle singularity in the transverse gluon correlations of perturbation theory must be screened non-perturbatively to avoid long-range fields and charged superselection sectors as in QED. (b) The global gauge charges must remain well-defined and unbroken to avoid the Higgs mechanism. In Landau gauge, in which the (Euclidean) gluon and ghost propagators,

$$D^{ab}_{\mu\nu}(p) = \delta^{ab}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{Z(p^2)}{p^2}\,, \quad \text{and} \quad D^{ab}_G(p) = -\delta^{ab}\,\frac{G(p^2)}{p^2}\,, \qquad (1)$$

are parametrised by the two invariant functions Z and G, respectively, this criterion requires

$$\text{(a): } \lim_{p^2\to 0} Z(p^2)/p^2 < \infty\,; \quad \text{(b): } \lim_{p^2\to 0} G^{-1}(p^2) = 0\,. \tag{2}$$

The translation of (b) into the infrared enhancement of the ghost propagator (2b) thereby rests on the ghost/anti-ghost symmetry of the Landau gauge or the symmetric Curci-Ferrari gauges. In particular, this equivalence does not hold in linear covariant gauges with non-zero gauge parameter such as the Feynman gauge.

As pointed out in [5], the infrared enhancement of the ghost propagator (2b) represents an additional boundary condition on DSE solutions which then lead to the prediction of a conformal infrared behaviour for the gluonic correlations in Landau gauge QCD consistent with the conditions for confinement in local quantum field theory. In fact, this behaviour is directly tied to the validity and applicability of the framework of local quantum field theory for non-Abelian gauge theories beyond perturbation theory. The subsequent verification of this infrared behaviour with a variety of different functional methods in the continuum meant a remarkable success. These methods which all lead to the same prediction include studies of their Dyson-Schwinger Equations (DSEs) [5], Stochastic Quantisation [6], and of the Functional Renormalisation Group Equations (FRGEs) [7]. This prediction amounts to infrared asymptotic forms

$$Z(p^2) \sim (p^2/\Lambda^2_{\rm QCD})^{2\kappa_Z}\,, \quad \text{and } G(p^2) \sim (p^2/\Lambda^2_{\rm QCD})^{-\kappa_G}\,, \tag{3}$$

for $p^2 \to 0$, which are both determined by a unique critical infrared exponent

$$\kappa_Z = \kappa_G \equiv \kappa\,, \tag{4}$$

with $0.5 < \kappa < 1$. Under a mild regularity assumption on the ghost-gluon vertex [5], the value of this exponent is furthermore obtained as [5, 6]

$$\kappa = (93 - \sqrt{1201})/98 \approx 0.595\,. \tag{5}$$

The conformal nature of this infrared behaviour in the pure Yang-Mills sector of Landau gauge QCD is evident in the generalisation to arbitrary gluonic correlations [8]: a uniform infrared limit of one-particle irreducible vertex functions $\Gamma^{m,n}$ with m external gluon legs and n pairs of ghost/anti-ghost legs of the form

$$\Gamma^{m,n} \sim (p^2/\Lambda^2_{\rm QCD})^{(n-m)\kappa}\,, \tag{6}$$

when all $p_i^2 \propto p^2 \to 0$, $i = 1, \dots 2n+m$. In particular, the ghost-gluon vertex is then infrared finite (with $n = m = 1$) as it must [9], and the non-perturbative running coupling introduced in [2, 3] via the definition

$$\alpha_S(p^2) = \frac{g^2}{4\pi} Z(p^2) G^2(p^2) \tag{7}$$

approaches an infrared fixed-point, $\alpha_S \to \alpha_c$ for $p^2 \to 0$. If the ghost-gluon vertex is regular at $p^2 = 0$, its value is maximised and given by [5]

$$\alpha_c = \frac{8\pi}{N_c} \frac{\Gamma^2(\kappa-1)\Gamma(4-2\kappa)}{\Gamma^2(-\kappa)\Gamma(2\kappa-1)} \approx \frac{9}{N_c} \times 0.99\,. \tag{8}$$

Comparing the infrared scaling behaviour of DSE and FRGE solutions of the form of Eqs. (3), it has in fact been shown that in presence of a single scale, the QCD scale $\Lambda_{\rm QCD}$, the solution with the infrared behaviour (4) and (6), with a positive exponent κ, is unique [10]. Because of its uniqueness, it is nowadays being called the *scaling solution*.

This uniqueness proof does not rule out, however, the possibility of a solution with an infrared finite gluon propagator, as arising from a transverse gluon mass M, which then leads to an essentially free ghost propagator, with the free massless-particle singularity at $p^2 = 0$, *i.e.*,

$$Z(p^2) \sim p^2/M^2\,, \quad \text{and } G(p^2) \sim \text{const.} \tag{9}$$

for $p^2 \to 0$. The constant contribution to the zero-momentum gluon propagator, $D(0) = 3/(4M^2)$, thereby necessarily leads to an infrared constant ghost renormalisation function G. This solution corresponds to $\kappa_Z = 1/2$ and $\kappa_G = 0$. It does not satisfy the scaling relations (4) or (6). This is because in this case the transverse gluons decouple for momenta $p^2 \ll M^2$, below the independent second scale given by their mass M. It is thus not within the class of scaling solutions considered above, and it is termed the *decoupling solution* in contradistinction [11]. The interpretation of the renormalisation group invariant (7) as a running coupling does not make sense in the infrared in this case, in which there is no infrared fixed-point and no conformal infrared behaviour.

Without infrared enhancement of the ghosts in Landau gauge, the global gauge charges of covariant gauge theory are spontaneously broken. Within the language of local quantum field theory the decoupling solution can thus only be realised if and only if it comes along with a Higgs mechanism and massive physical gauge bosons. The Schwinger mechanism can in fact be described in this way, and it can furthermore be shown that a non-vanishing gauge-boson mass, by whatever mechanism it is generated, necessarily implies the spontaneous breakdown of global symmetries [12].

Landau Gauge QCD in the Continuum and on the Lattice

Early lattice studies of the gluon and ghost propagators supported their predicted infrared behaviour qualitatively well. Because of the inevitable finite-volume effects, however, these results could have been consistent with both, the scaling solution as well as the decoupling solution. Recently, the finite-volume effects have been analysed carefully in the Dyson-Schwinger equations to demonstrate how the scaling solution is approached in the infinite volume limit there [13]. Comparing these finite volume DSE results with latest $SU(2)$ lattice data on impressively large lattices [14, 15], corresponding to physical lengths of up to 20 fm in each direction, finite-volume effects appear to be ruled out as the dominant cause of the observed discrepancies with the scaling solution. The lattice results are much more consistent with the decoupling solution which poses the obvious question whether there is something wrong with our general understanding of covariant gauge theory or whether we are perhaps comparing apples with oranges when applying inferences drawn from the infrared behaviour of the lattice Landau gauge correlations on local quantum field theory?

The latter language is based on a cohomology construction of a physical Hilbert space over the indefinite metric spaces of covariant gauge theory from the representations of the Becchi-Rouet-Stora-Tyutin (BRST) symmetry. But do we have a non-perturbative definition of a BRST charge? The obstacle is the existence of the so-called Gribov copies which satisfy the same gauge-fixing condition, *i.e.*, the Lorenz condition in Landau gauge, but are related by gauge transformations, and are thus physically equivalent. In fact, in the direct translation of BRST symmetry on the lattice, there is a perfect cancellation among these gauge copies which gives rise to the famous Neuberger 0/0 problem. It asserts that the expectation value of any gauge invariant (and thus physical) observable in a lattice BRST formulation will always be of the indefinite form 0/0 [16] and therefore prevented such formulations for more than 20 years now.

In present lattice implementations of the Landau gauge this problem is avoided because the numerical procedures are based on minimisations of a gauge fixing potential w.r.t. gauge transformations. To find absolute minima is not feasable on large lattices as this is a non-polynomially hard computational problem. One therefore settles for local minima which in one way or another, depending on the algorithm, samples gauge copies of the first Gribov region among which there is no cancellation. For the same reason, however, this is not a BRST formulation. The emergence of the decoupling solution can thus not be used to dismiss the KO criterion of covariant gauge theory in the continuum.

Strong Coupling Limit of Lattice Landau Gauge

¿From the finite-volume DSE solutions of [13] it follows that a wide separation of scales is necessary before one can even hope to observe the onset of an at least approximate conformal behaviour of the correlation functions in a finite volume of length L. What is needed is a reasonably large number of modes with momenta p sufficiently far below the QCD scale Λ_{QCD} whose corresponding wavelengths are all at the same time much shorter than the finite size L,

$$\pi/L \ll p \ll \Lambda_{\text{QCD}} \,. \tag{10}$$

It was estimated that this requires sizes L of about 15 fm, especially for a power law of the ghost propagator of the form in (3) to emerge in a momentum range with (10). A reliable quantitative determination of the exponents and a verification of their scaling relation (4) on the other hand might even require up to $L = 40$ fm [13].

As an alternative to the brute-force method of using ever larger lattice sizes for the simulations might therefore be to ask what one observes when the formal limit $\Lambda_{\text{QCD}} \to \infty$ is implemented by hand. This should then allow to assess whether the predicted conformal behaviour can be seen for the larger lattice momenta p, after the upper bound in (10) has been removed, in a range where the dynamics due to the gauge action would otherwise dominate and cover it up completely. Therefore, the ghost and gluon propagators of pure $SU(2)$ lattice Landau gauge were studied in the strong coupling limit $\beta \to 0$ in [17, 18].

In this limit, the gluon and ghost dressing functions tend towards the decoupling solution at small momenta and towards the scaling solution at large momenta (in units of the lattice spacing a) as seen in Figure 1. The transition from decoupling to scaling occurs at around $a^2p^2 \approx 1$, independent of the size of the lattice. The observed deviation from scaling at $a^2p^2 < 1$ is thus not a finite-size effect. The high momentum branch can be used to attempt fits of κ_Z and κ_G in (3) and the data is consistent with the scaling relation (4). With some dependence on the model used to fit the data, good global fits are generally obtained for $\kappa = 0.57(3)$, with very little dependence on the lattice size.

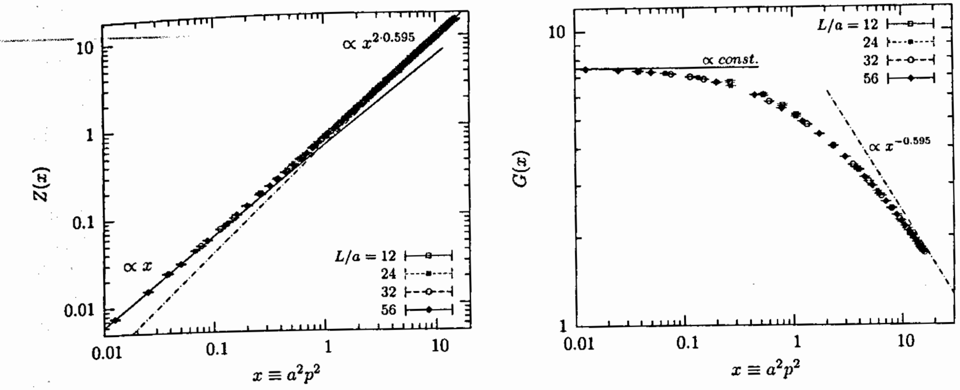

Figure 1: The gluon (left) and ghost (right) dressing functions at $\beta = 0$ compared to the decoupling solution (solid) and the scaling solution (dashed) with κ from (5) (not fitted).

For the scaling solution one would expect the running coupling defined by (7) to approach its constant fixed-point value in the strong-coupling limit, and this is indeed being observed for the scaling branch [17]: The numerical data for the product (7) levels at $\alpha_c \approx 4$ for large a^2p^2. As expected for an exponent κ slightly smaller than the value in (5), see [5], this is just below the upper bound given by (8), $\alpha_c \approx 4.45$ for $SU(2)$.

When comparing various definitions of gauge fields on the lattice, all equivalent in the continuum limit, one furthermore observes that neither the estimate of the critical exponent κ nor the corresponding value of α_c are sensitive to the definition used [17]. This is in contrast to the decoupling branch for $a^2p^2 < 1$, which is very sensitive to that definition. Different definitions, at order a^2 and beyond, lead to different Jacobian factors. This is well known from lattice perturbation theory where, however, the lattice Slavnov-Taylor identities guarantee that the gluon remains massless at every order by cancellation of all quadratically divergent contributions to its self-energy. The strong-coupling limit, where the effective mass in (9) behaves as $M^2 \propto 1/a^2$, therefore shows that such a contribution survives non-perturbatively in minimal lattice Landau gauge. This contribution furthermore depends on the measure for gauge fields whose definition from minimal lattice Landau is therefore ambiguous. One might still hope that this ambiguity will go away at non-zero β, in the scaling limit. While this is true at large momenta, it is not the case in the infrared, at least not for commonly used values of the lattice coupling such as $\beta = 2.5$ or $\beta = 2.3$ in $SU(2)$, as demonstrated in [17].

Lattice BRST and the Neuberger 0/0 Problem

It would obviously be desirable to have a BRST symmetry on the lattice which could then provide lattice Slavnov-Taylor identities beyond perturbation theory. In principle, this could be achieved by inserting the partition function of a topological model with BRST exact action into the gauge invariant lattice measure. Because of its topological nature, this gauge-fixing partition function Z_{GF} will be independent of gauge orbit and gauge parameter. The problem is that in the standard formulation this partition function calculates the Euler characteristic χ of the lattice gauge group which vanishes [19],

$$Z_{GF} = \chi(SU(N)^{\#\text{sites}}) = \chi(SU(N))^{\#\text{sites}} = 0^{\#\text{sites}} . \tag{11}$$

Neuberger's 0/0 problem of lattice BRST arises because we have then inserted zero instead of unity (according to the Faddeev-Popov prescription) into the measure of lattice gauge theory. On a finite lattice, such a topological model is equivalent to a problem of supersymmetric quantum mechanics with Witten index $\mathcal{W} = Z_{GF}$. Unlike the case of primary interest in supersymmetric quantum mechanics, here we need a model with non-vanishing Witten index to avoid the Neuberger 0/0 problem. Then however, just as the supersymmetry of the corresponding quantum mechanical model, such a lattice BRST cannot break.

In Landau gauge, with gauge parameter $\xi = 0$, the Neuberger zero, $Z_{GF} = 0$, arises from the perfect cancellation of Gribov copies via the Poincaré-Hopf theorem. The gauge-fixing potential $V_U[g]$ for a generic link configuration $\{U\}$ thereby plays the role of a Morse potential for gauge transformations g and the Gribov copies are its critical points (the global gauge transformations need to remain unfixed so that there are strictly speaking only (#sites-1) factors of $\chi(SU(N)) = 0$ in (11)). The Morse inequalities then immediately imply that there are at least $2^{(N-1)(\#\text{sites}-1)}$ such copies in $SU(N)$ on the lattice, or $2^{\#\text{sites}-1}$ in compact $U(1)$, and equally many with either sign of the Faddeev-Popov determinant (*i.e.*, that of the Hessian of $V_U[g]$).

The topological origin of the zero originally observed by Neuberger in a certain parameter limit due to uncompensated Grassmann ghost integrations in standard Faddeev-Popov theory [16] becomes particularly evident in the ghost/anti-ghost symmetric Curci-Ferrari gauge with its quartic ghost self-interactions [20]. Due to its Riemannian geometry with symmetric connection and curvature tensor $R_{ijkl} = \frac{1}{4} f^a{}_{ij} f^a{}_{kl}$ for $SU(N)$, in this gauge the same parameter limit leads to computing the zero in (11) from a product of independent Gauss-Bonnet integral expressions,

$$\chi(SU(N)) = \frac{1}{(2\pi)^{(N^2-1)/2}} \int_{SU(N)} dg \int d\bar{c}\, dc\, \exp\left\{\frac{1}{4} R_{abcd}\, \bar{c}^a \bar{c}^b c^c c^d\right\} = 0 , \tag{12}$$

for each site of the lattice. This corresponds to the Gauss-Bonnet limit of the equivalent supersymmetric quantum mechanics model in which only constant paths contribute [21].

The indeterminate form of physical observables as a consequence of (12) is regulated by a Curci-Ferrari mass term. While such a mass m decontracts the double BRST/anti-BRST algebra, which is well-known to result in a loss of unitarity, observables can then be meaningfully defined in the limit $m \to 0$ via l'Hospital's rule [20].

Lattice Landau Gauge from Stereographic Projection

The 0/0 problem due to the vanishing Euler characteristic of $SU(N)$ is avoided when fixing the gauge only up to the maximal Abelian subgroup $U(1)^{N-1}$ because the Euler characteristic of the coset manifold is non-zero. The corresponding lattice BRST has been explicitly constructed for $SU(2)$ [19], where the coset manifold is the 2-sphere and $\chi(SU(2)/U(1)) = \chi(S^2) = 2$. This indicates that the Neuberger problem might be solved

when that of compact $U(1)$ is, where the same cancellation of lattice Gribov copies arises because $\chi(S^1) = 0$. A surprisingly simple solution to this problem is possible, however, by stereographically projecting the circle $S^1 \to \mathbb{R}$ which can be achieved by a simple modification of the minimising potential [22]. The resulting potential is convex to the above and leads to a positive definite Faddeev-Popov operator for compact $U(1)$ where there is thus no cancellation of Gribov copies, but $Z_{GF}^{U(1)} = N_{GC}$, for N_{GC} Gribov copies.

As compared to the standard lattice Landau gauge for compact $U(1)$ their number is furthermore exponentially reduced. This is easily verified explicitly in low dimensional models. While N_{GC} grows exponentially with the number of sites in the standard case as expected, the stereographically projected version has only $N_{GC} = N_x$ copies on a periodic chain of length N_x and $\ln N_{GC} \sim N_t \ln N_x$ on a $2D$ lattice of size $N_t N_x$ in Coulomb gauge, for example, and in both cases their number is verified to be independent of the gauge orbit.

The general proof of $Z_{GF}^{U(1)} = N_{GC}$ with stereographic projection which avoids the Neuberger zero in compact $U(1)$ [22] follows from a simple example of a Nicolai map [21].

Applying the same techniques to the maximal Abelian subgroup $U(1)^{N-1}$, the generalisation to $SU(N)$ lattice gauge theories is possible when the odd-dimensional spheres S^{2n+1}, $n = 1, \dots N-1$, of its parameter space are stereographically projected to $\mathbb{R} \times \mathbb{R}P(2n)$. In absence of the cancellation of the lattice artifact Gribov copies along the $U(1)$ circles, the remaining cancellations between copies of either sign in $SU(N)$, which will persist in the continuum limit, are then necessarily incomplete, however, because $\chi(\mathbb{R}P(2n)) = 1$.

For $SU(2)$ this program is straightforward. One replaces the standard gauge-fixing potential $V_U[g]$ of lattice Landau gauge by $\widetilde{V}_U[g]$, via gauge-transformed links $U^g_{x\mu}$, where

$$V_U[g] = 4\sum_{x,\mu}\left(1 - \frac{1}{2}\,\mathrm{Tr}\, U^g_{x\mu}\right) \quad \text{and} \quad \widetilde{V}_U[g] = -8\sum_{x,\mu} \ln\left(\frac{1}{2} + \frac{1}{4}\,\mathrm{Tr}\, U^g_{x,\hat{\mu}}\right) . \tag{13}$$

The standard and stereographically projected gauge fields on the lattice are defined as

$$A_{x\mu} = \frac{1}{2ia}\left(U_{x\mu} - U^\dagger_{x\mu}\right) \quad \text{and} \quad \widetilde{A}_{x\mu} = \frac{1}{2ia}\left(\widetilde{U}_{x\mu} - \widetilde{U}^\dagger_{x\mu}\right) , \quad \text{with} \quad \widetilde{U}_{x\mu} \equiv \frac{2U_{x\mu}}{1 + \frac{1}{2}\mathrm{Tr}\, U_{x\mu}} .$$

The gauge-fixing conditions $F = 0$ and $\widetilde{F} = 0$ are their respective lattice divergences, in the language of lattice cohomology, $F = \delta A$ and $\widetilde{F} = \delta \widetilde{A}$. A particular advantage of the non-compact $\widetilde{A}$ is that they allow to resolve the modified lattice Landau gauge condition $\widetilde{F} = 0$ by Hodge decomposition. This provides a framework for gauge-fixed Monte-Carlo simulations which is currently being developed for the particularly simple case of $SU(2)$ in 2 dimensions. In the low-dimensional models mentioned above it can furthermore be verified explicitly that the corresponding topological gauge-fixing partition function is indeed given by

$$Z_{GF}^{SU(2)} = Z_{GF}^{U(1)} \neq 0 , \tag{14}$$

as expected from $\chi(\mathbb{R}P(2)) = 1$. The proof of this will be given elsewhere.

Conclusions and Outlook

Comparisons of the infrared behaviour of QCD Green's functions as obtained from lattice Landau gauge implementations based on minimisations of a gauge-fixing potential and from continuum studies based on BRST symmetry have to be taken with a grain of salt. Evidence of the asymptotic conformal behaviour predicted by the latter is seen in the strong coupling limit of lattice Landau gauge where such a behaviour can be observed at large lattice momenta $a^2p^2 \gg 1$. There the strong coupling data is consistent with the predicted critical exponent and coupling from the functional approaches. The deviations from scaling at $a^2p^2 < 1$ are not finite-volume effects, but discretisation dependent and hint at a breakdown of BRST symmetry arguments beyond perturbation theory in this approach. Non-perturbative lattice BRST has been plagued by the Neuberger 0/0 problem, but its improved topological understanding provides ways to overcome this problem. The most promising one at this point rests on stereographic projection to define gauge fields on the lattice together with a modified lattice Landau gauge. This new definition has the appealing feature that it will allow gauge-fixed Monte-Carlo simulations in close analogy to the continuum BRST methods which it will thereby elevate to a non-perturbative level.

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Renormalization-Group Approach to Transverse-Momentum Dependent Parton Distribution Functions in QCD

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Abstract

We discuss the renormalization of gauge-invariant transverse-momentum dependent (TMD), i.e., unintegrated, parton distribution functions (PDFs) and carry out the calculation of their anomalous dimension at one loop. We show that in the light-cone gauge, TMD PDFs contain UV divergences that may be attributed to the renormalization effect on a cusp-like junction point of the gauge contours at infinity. In order to eliminate the anomalous dimension ensuing from this cusp, we propose to use in the definition of the TMD PDFs, a soft counter term in terms of a path-ordered phase factor along a particular cusped contour extending to transverse light-cone infinity and comprising light-like and transverse segments. We argue that this additional factor is analogous to the "intrinsic" Coulomb phase factor found before in QED.

1 Introduction

Parton distribution functions encode the nonperturbative hadronization dynamics at the amplitude level and are, therefore, of fundamental importance in QCD calculations and phenomenological applications (see [1] for a review). While integrated PDFs can be given an unambiguous gauge-invariant definition in terms of Wilson-line operators (gauge links) [2], the analogous definition for unintegrated, i.e., transverse-momentum dependent, PDFs may depend more critically on the details of the gauge contour. As a result, the renormalization of TMD PDFs is a more demanding task to which the present report is devoted.

Indeed, in order to satisfy factorization, one cannot restore gauge invariance in TMD PDFs by inserting a purely light-like Wilson line joining the quark and antiquark field points directly [3]. The reason is that the gluons emitted from the struck quark along the x^- direction have rapidities that cannot match those of the spectator quarks moving along the x^+ direction. Consequently, one is forced to employ a gauge contour that comprises segments going off the light cone and joins the quark field points through infinity. Recall in this context that the gauge link resums the contributions due to collinear and transverse gluons between the struck quark and the spectator remnants (see, e.g., [4]).

Quite recently, Belitsky, Ji, and Yuan [5] (see also [6, 7, 8]) have shown that in the light-cone gauge $A^+ = 0$, one has to include in the definition of TMD PDFs transverse gauge links at light-cone infinity—as illustrated in Fig. 1. These transverse gauge links cancel when the integration over $\mathbf{k}_\perp$ is performed, so that one recovers the correct integrated PDF. Moreover, it was advocated in [5] that adopting the advanced boundary condition (see below), the transverse field $\mathbf{A}_\perp$ vanishes at $\xi^- = \infty$ reducing the transverse gauge link to unity. For that particular boundary condition, the light-cone gauge (one-loop) calculation reproduces the Feynman-gauge PDF.

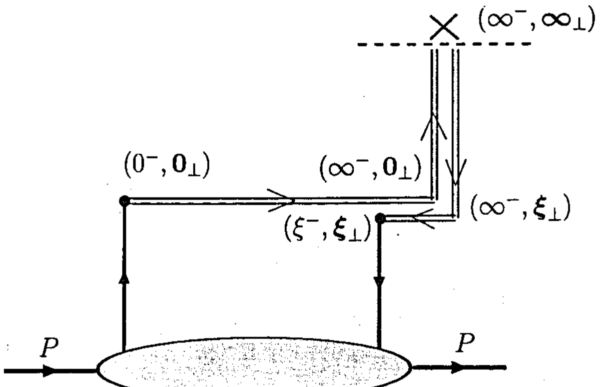

Figure 1: TMD PDF (shaded oval) in coordinate space. Double lines denote lightlike and transverse gauge links, connecting the quark field points $(0^-, \mathbf{0}_\perp)$ and $(\xi^-, \boldsymbol{\xi}_\perp)$, via a composite contour through light-cone infinity $(\infty^-, \infty_\perp)$.

In these investigations it was tacitly assumed that the lightlike-transverse composite contour going through infinity, illustrated in Fig. 1, is everywhere smooth. However, we have shown in [9, 10] by carrying out a one-loop calculation of the gluon radiative corrections to the unpolarized TMD PDF of a quark in a quark in the light-cone gauge that there are UV divergences which are neither related to the quark self energy nor are they caused by the endpoints of the line integral along the gauge contour—as one finds for the direct contour (the "connector" [11, 12]). Instead, the origin of these extra UV divergences can be attributed to a cusp obstruction (denoted by the symbol $\times$ in Fig. 1) in the split gauge contour at transverse light-cone infinity. The concomitant anomalous dimension after renormalization is a local footprint of the cusp and peculiar to the split contour. It turns out to coincide with the leading-order (LO) cusp anomalous dimension [13].[1] The appearance of this extra anomalous dimension necessitates a modification of the definition of the TMD PDF in order to dispense with it. As pointed out in [9], and further outlined in full detail in [10], this can be achieved by including a path-ordered soft factor, in the sense of Collins and Hautmann [14], to be evaluated along a specific gauge contour off-the-light cone (see next section). Having described the cornerstones of our approach, let us now have a closer look to its mathematical details.

[1]It remains to be proved that this coincidence persists at the two-loop order and beyond.

2 One-loop radiative corrections to gauge-invariant TMD PDFs

Taking into account the findings of [5], the strictly gauge-invariant operator definition of the TMD distribution of a quark with momentum $k_\mu = (k^+, k^-, \mathbf{k}_\perp)$ in a quark with momentum $p_\mu = (p^+, p^-, \mathbf{0}_\perp)$, with non-lightlike Wilson lines to light-cone infinity included, reads

$$\begin{aligned} f_{q/q}(x, \boldsymbol{k}_\perp) = \frac{1}{2}\int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{2\pi(2\pi)^2} \exp\left(-ik^+\xi^- + i\boldsymbol{k}_\perp \cdot \boldsymbol{\xi}_\perp\right) \langle q(p)|\bar{\psi}(\xi^-, \boldsymbol{\xi}_\perp)[\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp]^\dagger \\ \times [\infty^-, \boldsymbol{\xi}_\perp; \infty^-, \infty_\perp]^\dagger \gamma^+ [\infty^-, \infty_\perp; \infty^-, \mathbf{0}_\perp][\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp] \\ \times \psi(0^-, \mathbf{0}_\perp)|q(p)\rangle \,|_{\xi^+=0} \,. \end{aligned} \quad (1)$$

Here the gauge links, in the lightlike and the transverse direction, respectively, are defined by the following path-ordered exponentials

$$\begin{aligned} [\infty^-, \boldsymbol{z}_\perp; z^-, \boldsymbol{z}_\perp] &\equiv \mathcal{P} \exp\left[ig \int_0^\infty d\tau \; n_\mu^- \; A_a^\mu t^a(z + n^-\tau)\right] \\ [\infty^-, \infty_\perp; \infty^-, \boldsymbol{\xi}_\perp] &\equiv \mathcal{P} \exp\left[ig \int_0^\infty d\tau \; \boldsymbol{l} \cdot \boldsymbol{A}_a t^a(\boldsymbol{\xi}_\perp + \boldsymbol{l}\tau)\right] , \end{aligned} \quad (2)$$

where the two-dimensional vector $\mathbf{l}$ is arbitrary with no influence on the (local) anomalous dimensions we are interested in.

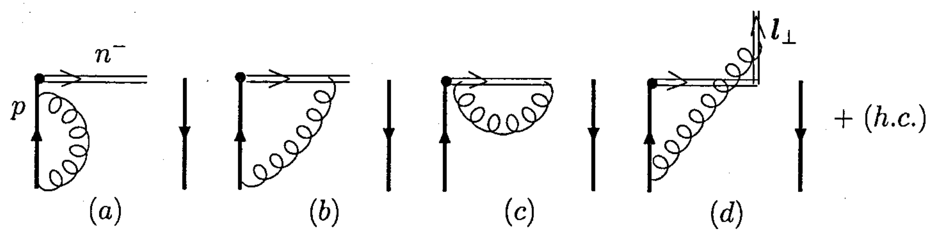

Figure 2: One-loop radiative corrections (curly lines) contributing UV-divergences to $f_{q/q}(x, \boldsymbol{k}_\perp)$ in a general covariant gauge. Double lines denote lightlike and transverse gauge links. Diagrams (b) and (c) are absent in the light-cone gauge, while the Hermitian conjugate ("mirror") diagrams (not shown) are abbreviated by $(h.c)$.

Employing the light-cone gauge $A^+ = (A \cdot n^-) = 0$, $(n^-)^2 = 0$, we calculated in [9, 10] gluon radiative corrections to $f_{q/q}(x, \boldsymbol{k}_\perp)$ at the one-loop level and identified its UV divergences (see Fig. 2). We found that those contributions stemming from the interactions with the gluon field of the transverse gauge link cancel all terms that bear a dependence on the pole prescription applied to regularize the light-cone singularities of the gluon propagator. In the intermediate steps of the calculation the light-cone singularities of the gluon propagator

$$D_{\mu\nu}^{\mathrm{LC}}(q) = \frac{-i}{q^2 - \lambda^2 + i0}\left(g_{\mu\nu} - \frac{q_\mu n_\nu^- + q_\nu n_\mu^-}{[q^+]}\right) \quad (3)$$

are taken into account by means of the term $1/[q^+]$ subject to boundary conditions on the gauge potential. In the present work we apply the following regularization prescriptions to the pole at q^+ [5]:

$$\frac{1}{[q^+]}\Bigg|_{\text{Ret/Adv}} = \frac{1}{q^+ \pm i\eta} \quad , \quad \frac{1}{[q^+]}\Bigg|_{\text{PV}} = \frac{1}{2}\left[\frac{1}{q^+ + i\eta} + \frac{1}{q^+ - i\eta}\right] , \tag{4}$$

where η is a mass-scale parameter kept small but finite. The total UV-divergent contribution is obtained by including also the Hermitian conjugate contributions of diagrams (a) and (d) in Fig. 2. Then, we obtain

$$\begin{aligned} \Sigma_{\text{UV}}^{(a+d)}(p,\mu,\alpha_s;\epsilon) &= -\frac{\alpha_s}{\pi} C_{\text{F}}\frac{1}{\epsilon}\left[\frac{1}{4} - \frac{\gamma^+\hat{p}}{2p^+}\left(1 + \ln\frac{\eta}{p^+} - \frac{i\pi}{2} - i\pi\, C_\infty + i\pi C_\infty\right)\right] \\ &= -\frac{\alpha_s}{\pi} C_{\text{F}}\frac{1}{\epsilon}\left[1 - \frac{\gamma^+\hat{p}}{2p^+}\left(1 + \ln\frac{\eta}{p^+} - \frac{i\pi}{2}\right)\right] , \end{aligned} \tag{5}$$

where $C_{\text{F}} = (N_c^2-1)/(2N_c) = 4/3$ and the parameter C_∞ encodes the adopted pole prescription (cf. Eq. (4)). This expression can be further simplified using

$$\frac{\gamma^+\hat{p}\gamma^+}{2p^+} = \gamma^+$$

and recalling that the mirror counterparts of the evaluated diagrams yield complex-conjugated contributions. As a result, the imaginary terms in Eq. (5) mutually cancel and one is left with

$$\Sigma_{\text{UV}}^{(a+d)}(\alpha_s,\epsilon) = 2\frac{\alpha_s}{\pi}C_{\text{F}}\left[\frac{1}{\epsilon}\left(\frac{3}{4} + \ln\frac{\eta}{p^+}\right) - \gamma_E + \ln 4\pi\right] . \tag{6}$$

The key contribution here is the term $\sim \ln\frac{\eta}{p^+}$ which gives rise to the one-loop anomalous dimension in the light-cone (LC) gauge $\left(\gamma = \frac{\mu}{2}\frac{1}{Z}\frac{\partial\alpha_s}{\partial\mu}\frac{\partial Z}{\partial\alpha_s}\right)$:

$$\gamma_{1-\text{loop}}^{\text{LC}} = \frac{\alpha_s}{\pi}C_{\text{F}}\left(\frac{3}{4} + \ln\frac{\eta}{p^+}\right) = \gamma_{\text{smooth}} - \delta\gamma . \tag{7}$$

Here γ_{smooth} is the anomalous dimension one would obtain in a covariant gauge, or, equivalently, the anomalous dimension associated with a direct smooth contour between the quark fields (i.e., with the connector correction). The term $\delta\gamma$ is the anomalous-dimensions defect entailed by the cusp, we have to compensate in order to recover the same expression as in a covariant gauge according to the factorization proof. Consistent with this finding, one has to modify the multiplication rule for gauge links (or, equivalently, the way of decomposing gauge contours) [10]:

$$\gamma_{\mathcal{C}} = \gamma_{\mathcal{C}_1^\infty \cup \mathcal{C}_2^\infty} + \gamma_{\text{cusp}} \iff [2,1|\mathcal{C}] = [2,\infty|\mathcal{C}_2^\infty]^\dagger[\infty,1|\mathcal{C}_1^\infty]e^{i\Phi_{\text{cusp}}} . \tag{8}$$

The graphics at right of Fig. 3 helps the eye catch the key features of the situation involving two non-lightlike contours $\mathcal{C}_1$ and $\mathcal{C}_2$. For comparison, the smooth decomposition of a purely lightlike contour is shown in the left panel. In that case the junction point 3 creates no anomalous dimension and the standard multiplication rule for gauge links applies.

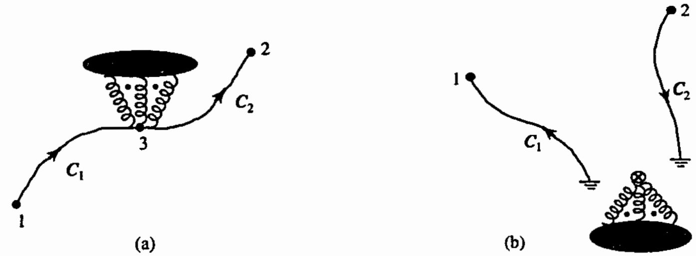

Figure 3: Renormalization effect on the junction point due to gluon corrections (illustrated by a shaded oval with gluon lines attached to it) for (a) two smoothly joined gauge contours $\mathcal{C}_1$ and $\mathcal{C}_2$ at point 3 and (b) the same for two contours joined by a cusp (indicated by the symbol $\otimes$) at infinite transverse distance (marked by the earth symbol) off the light cone. All contours shown are assumed to be arbitrary non-lightlike paths in Minkowski space.

In the above expression, Φ_{cusp} contains a phase entanglement ensuing from the renormalization effect on the cusp-like junction point at infinity. One may associate this phase with final (or initial) state interactions, as proposed by Ji and Yuan in [7], and also by Belitsky, Ji, and Yuan in [5]. However, these authors (and also others) did not recognize that the junction point in the split contour (the latter stretching to light-cone infinity) is no more a simple point, but a cusp obstruction that entails an anomalous dimension $\sim \ln p^+$. More precisely, we have

$$\begin{aligned} \gamma_{\text{cusp}}(\alpha_s,\chi) &= \frac{\alpha_s}{\pi}C_{\text{F}}\,(\chi\coth\chi - 1) , \\ \frac{d}{d\ln p^+}\,\delta\gamma &= \lim_{\chi\to\infty}\frac{d}{d\chi}\gamma_{\text{cusp}}(\alpha_s,\chi) = \frac{\alpha_s}{\pi}C_{\text{F}} , \end{aligned} \tag{9}$$

which makes it apparent that the defect of the anomalous dimension is related to the universal cusp anomalous dimension [13]. To derive this expression, we have used the fact that $p^+ = (p\cdot n^-) \sim \cosh\chi$ defines an angle χ between the direction of the quark momentum p_μ and the lightlike vector n^-. Then, in the large χ limit, one has $\ln p^+ \to \chi$. It is worth recalling in this context that the cusp anomalous dimension of Wilson lines controls the Sudakov factor resulting from gluon resummation and is known to the three-loop order [15]. The Sudakov exponent in next-to-leading logarithmic approximation has been calculated in [16] and expressed as an expansion in inverse powers of the first beta-function coefficient.

3 How to avert the defect of the anomalous dimension

In this section we will show in more depth how to get rid of the cusp anomalous dimension and refurbish the definition of the TMD PDF. The defect of the anomalous dimension, ensuing from the cusp-like junction point of the non-lightlike gauge contours, represents a distortion of the gauge-invariant formulation of the TMD PDF in the light-cone gauge. This is best appreciated by inspecting the composite non-smooth contour $\mathcal{C}_{\text{cusp}}$, visualized

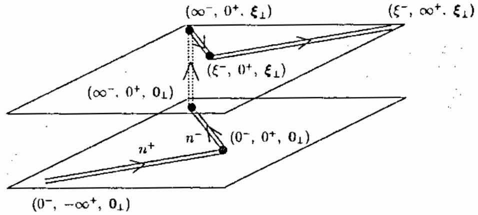

Figure 4: Integration contour associated with the additional soft counter term.

in Fig. 4, and defined by

$$\mathcal{C}_{\rm cusp} : \zeta_\mu = \left\{[p_\mu^+ s, -\infty < s < 0] \cup [n_\mu^- s', 0 < s' < \infty] \cup [\boldsymbol{l}_\perp \tau, 0 < \tau < \infty]\right\} , \tag{10}$$

with n_μ^- being the minus light-cone vector. This contour is obviously cusped: at the origin, the four-velocity p_μ^+, which is parallel to the plus light-cone ray, is replaced—non-smoothly—by the four-velocity n_μ^-, which is parallel to the minus light-cone ray. This means that exactly at this point the contour has a cusp, that is characterized by the angle $\chi \sim \ln p^+ = \ln(p \cdot n^-)$, and will generate an anomalous dimension with the opposite sign relative to $\delta\gamma$—cf. Eq. (7). This contour can be used to define a soft counter term in the sense of Collins and Hautmann [14], namely,

$$R \equiv \Phi(p^+, n^-|0)\Phi^\dagger(p^+, n^-|\xi) , \tag{11}$$

where the eikonal factors are given by

$$\Phi(p^+, n^-|0) = \left\langle 0 \left| \mathcal{P} \exp\left[ig \int_{\mathcal{C}_{\rm cusp}} d\zeta^\mu \ t^a A_\mu^a(\zeta)\right] \right| 0 \right\rangle , \tag{12}$$

$$\Phi^\dagger(p^+, n^-|\xi) = \left\langle 0 \left| \mathcal{P} \exp\left[-ig \int_{\mathcal{C}_{\rm cusp}} d\zeta^\mu \ t^a A_\mu^a(\xi + \zeta)\right] \right| 0 \right\rangle \tag{13}$$

and have to be evaluated along the integration contour $\mathcal{C}_{\rm cusp}$.

Next, we consider the one-loop gluon radiative corrections, contributing to the UV divergences of R and displayed in Fig. 5. Diagrams (a) and (d) give rise to an anomalous dimension that will finally compensate the anomalous-dimensions defect generated by the cusp-like junction point of the contours. On the other hand, by virtue of the light cone gauge $A^+ = 0$, we are employing, diagrams (b) and (c) vanish. The UV parts of diagrams (a) and (d) yield, respectively,

$$\Phi_{\rm UV}^{(a)}(\eta) = -\frac{\alpha_s}{\pi} C_{\rm F} \frac{1}{\epsilon} \left(\ln \frac{\eta}{p^+} - i\frac{\pi}{2} - i\pi C_\infty\right) \tag{14}$$

and

$$\Phi_{\rm UV}^{(d)}(\eta) = -\alpha_s C_{\rm F} i\pi C_\infty \Gamma(\epsilon) \left(-4\pi \frac{\mu^2}{\lambda^2}\right)^\epsilon . \tag{15}$$

Combining these UV terms, we find

$$F_{\rm UV}^{(a+d)}(\eta) = -\frac{\alpha_s}{\pi} C_{\rm F} \frac{1}{\epsilon} \left(\ln \frac{\eta}{p^+} - i\frac{\pi}{2} - i\pi C_\infty + i\pi C_\infty\right) = -\frac{\alpha_s}{\pi} C_{\rm F} \frac{1}{\epsilon} \left(\ln \frac{\eta}{p^+} - i\frac{\pi}{2}\right) . \tag{16}$$

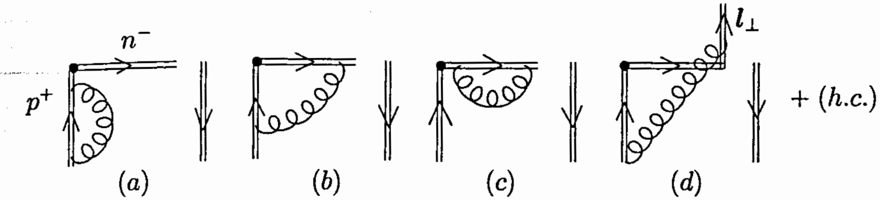

Figure 5: Gluon radiative corrections giving rise to UV-divergences contributing to the soft counter term R. The designations are as in Fig. 2.

Taking into account the Hermitian conjugate ("mirror") terms, we obtain the total UV-divergent part of the soft factor R in one-loop order:

$$\Phi_{\rm UV}^{(1-{\rm loop})}(\eta) = -\frac{\alpha_s}{\pi} C_{\rm F} \frac{2}{\epsilon} \ln \frac{\eta}{p^+} . \tag{17}$$

One notices that this expression bears no dependence on the pole prescription, since all C_∞-dependent terms have mutually canceled. Indeed, only the cusp-dependent term $\sim \ln \frac{\eta}{p^+}$ survives that will ultimately yield $-\gamma_{\rm cusp}$.

The above considerations make it apparent that one may use R and redefine the TMD PDF as follows

$$\begin{aligned} f_{q/q}^{\rm mod}(x, \boldsymbol{k}_\perp; \mu, \eta) = & \frac{1}{2} \int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{2\pi(2\pi)^2} \exp\left(-ik^+\xi^- + i\boldsymbol{k}_\perp \cdot \boldsymbol{\xi}_\perp\right) \Big\langle q(p)|\bar\psi(\xi^-, \boldsymbol{\xi}_\perp) \\ & \times[\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp]^\dagger [\infty^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\infty}_\perp]^\dagger \gamma^+ [\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \mathbf{0}_\perp] \\ & \times[\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp] \psi(0^-, \mathbf{0}_\perp)|q(p)\Big\rangle \\ & \times\Big[\Phi(p^+, n^-|0^-, \mathbf{0}_\perp)\Phi^\dagger(p^+, n^-|\xi^-, \boldsymbol{\xi}_\perp)\Big] . \end{aligned} \tag{18}$$

Before we conclude, let us mention that integrating the above expression over the transverse momenta, we obtain an integrated PDF that coincides with the standard one, containing no artifacts of the cusped contour, and satisfying the DGLAP evolution equation. Moreover, $f_{q/q}^{\rm mod}(x, \boldsymbol{k}_\perp; \mu, \eta)$ satisfies the simple renormalization-group equation

$$\frac{1}{2}\mu\frac{d}{d\mu} \ln f_{q/q}^{\rm mod}(x, \boldsymbol{k}_\perp; \mu, \eta) = \frac{3}{4}\frac{\alpha_s}{\pi} C_{\rm F} + O(\alpha_s^2) . \tag{19}$$

Note that without the soft counter term, R, extra contributions to the anomalous dimension on the right-hand side would appear. In [10] we have outlined the correspondence between the evolution with respect to the scale parameter η in our approach and the Collins-Soper evolution equation with respect to the rapidity parameter ζ, establishing the absence of UV singularities entailed by the light-cone gauge.

4 Conclusions

To summarize the results of this report on the renormalization of gauge-invariant TMD PDFs, the following may be said. First, we have elaborately discussed the one-loop calculation of the UV divergences of a typical TMD PDF which contains lightlike and transverse gauge links in order to fully restore gauge invariance. We found that an extra UV divergence appears, not noticed before in the literature, which is unrelated to the quark self energy and the end-point singularities of the contours. Second, we showed that these divergences give rise to an anomalous dimension, which can be regarded as originating from the renormalization effect on a cusp-like junction point of the integration contours in the gauge links at light-cone infinity. At the considered one-loop order, this anomalous dimension coincides with the universal cusp anomalous dimension of Wilson-line operators and is an ingrained property of the split contours. Third, in order to dispense with this anomalous-dimensions defect and recover the well-known results in a covariant gauge (say, in the Feynman gauge) in which $\mathbf{A}_\perp$ vanishes at infinity, we have proposed a modified definition of the TMD PDF. This definition includes a Collins-Hautmann soft counter term by means of path-ordered eikonal factors that are evaluated along a specific non-smooth contour off the light cone. This cusped contour suffices to neutralize the cusp artifact encountered in the standard definition of the TMD PDF. Finally, as we outlined in [10], the soft counter term can be given an interpretation akin to the "intrinsic" Coulomb phase found by Jakob and Stefanis [17] in QED. In both cases, a phase entanglement appears, ensuing either from the charged "particle behind the moon" (QED) or from the cusp-like junction point at light-cone infinity (QCD). Recently, Collins [18] has considered possible refinements and modifications in the definition of unintegrated parton densities that deserve further examination. An improved definition of TMD PDFs will have tangible consequences in several areas of QCD.

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Constraints on Dark Matter Distribution at the Galactic Center

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Abstract

The existence of dark matter (DM) at scales of few pc down to $\simeq 10^{-5}$ pc around the centers of galaxies and in particular in the Galactic Center region has been considered in the literature. Under the assumption that such a DM clump, principally constituted by non-baryonic matter (like WIMPs) does exist at the center of our galaxy, the study of the γ-ray emission from the Galactic Center region allows us to constrain both the mass and the size of this DM sphere. Further constraints on the DM distribution parameters may be derived by observations of bright infrared stars around the Galactic Center. Here, we discuss the constraints that can be obtained with the orbit analysis of stars (as S2 and S16) moving inside the DM concentration with present and next generations of large telescopes. In particular, consideration of the S2 star apoastron shift may allow improving limits on the DM mass and size.

For the black hole in the Galactic Center, Hall and Gondolo [1] used estimates of the enclosed mass obtained in various ways and tabulated by Ghez et al. [2, 3]. The black hole, stellar cluster and DM could contribute in the mass inside stellar orbits. Moreover, if a DM cusp does exist around the Galactic Center it could modify the trajectories of stars moving around it in a sensible way depending on the DM mass distribution.

In the last years intensive searches for dark matter (DM), especially its non-baryonic component, both in galactic halos and at galaxy centers have been undertaken (see for example [4, 5] for recent results). It is generally accepted that the most promising candidate for the DM non-baryonic component is neutralino. In this case, the γ-flux from galactic halos (and from our Galactic halo in particular) could be explained by neutralino annihilation [6, 7, 8, 9, 10, 11, 12, 13]. Since γ-rays are detected not only from high galactic latitude, but also from the Galactic Center, there is a wide spread hypothesis (see [14] for a discussion) that a DM concentration might be present at the Galactic Center. In this case the Galactic Center could be a strong source of γ-rays and neutrinos [4, 7, 15, 16, 17, 18, 19, 20, 21, 22] due to DM annihilation. Since it is also expected that DM forms spikes at galaxy centers [23, 24, 25] the γ-ray flux from the Galactic Center should increase significantly in that case.

At the same time, progress in monitoring bright stars near the Galactic Center have been reached recently [2, 3, 26]. The astrometric limit for bright stellar sources near the Galactic Center with 10 meter telescopes is today $\delta\theta_{10} \sim 1$ mas and the Next Generation Large

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Telescope (NGLT) will be able to improve this number at least down to $\delta\theta_{30} \sim 0.5$ mas [28, 29] or even to $\delta\theta_{30} \sim 0.1$ mas [27, 28, 29] in the K-band. Therefore, it will be possible to measure the proper motion for about ~ 100 stars with astrometric errors several times smaller than errors in current observations.

Recently it was shown [30, 31] that it is possible to constrain the parameters of the DM distribution possible present around the Galactic Center by considering the induced apoastron shift due to the presence of this DM sphere and either available data obtained with the present generation of telescopes (the so called *conservative* limit) and also expectations from future NGLT observations or with other advanced observational facilities.

Recent advancements in infrared astronomy are allowing to test the scale of the mass profile at the center of our galaxy down to tens of AU. With the Keck 10 m telescope, the proper motion of several stars orbiting the Galactic Center black hole have been monitored and almost entire orbits, as for example that of the S2 star, have been measured allowing an unprecedent description of the Galactic Center region. Measurements of the amount of mass $M(<r)$ contained within a distance r from the Galactic Center are continuously improved as more precise data are collected. Recent observations [2] extend down to the periastron distance ($\simeq 3 \times 10^{-4}$ pc) of the S16 star and they correspond to a value of the enclosed mass within $\simeq 3 \times 10^{-4}$ pc of $\simeq 3.67 \times 10^{6}$ $M_{\odot}$. Several authors have used these observations to model the Galactic Center mass concentration. Here and in the following, we use the three component model for the central region of our galaxy based on estimates of enclosed mass given by Ghez et al [2, 3] recently proposed [1]. This model is constituted by the central black hole, the central stellar cluster and the DM sphere (made of WIMPs), i.e.

$$M(<r) = M_{BH} + M_{*}(<r) + M_{DM}(<r)\ , \tag{1}$$

where M_{BH} is the mass of the central black hole Sagittarius A*. For the central stellar cluster, the empirical mass profile is

$$M_{*}(<r) = \begin{cases} M_{*}\left(\frac{r}{R_{*}}\right)^{1.6}, & r \le R_{*} \\ M_{*}\left(\frac{r}{R_{*}}\right)^{1.0}, & r > R_{*} \end{cases} \tag{2}$$

with a total stellar mass $M_{*} = 0.88 \times 10^{6}$ $M_{\odot}$ and a size $R_{*} = 0.3878$ pc.

As far as the mass profile of the DM concentration is concerned, Hall and Gondolo [1] have assumed a mass distribution of the form

$$M_{DM}(<r) = \begin{cases} M_{DM}\left(\frac{r}{R_{DM}}\right)^{3-\alpha}, & r \le R_{DM} \\ M_{DM}, & r > R_{DM} \end{cases} \tag{3}$$

M_{DM} and R_{DM} being the total amount of DM in the form of WIMPs and the radius of the spherical mass distribution, respectively.

Hall and Gondolo [1] discussed limits on DM mass around the black hole at the Galactic Center. It is clear that present observations of stars around the Galactic Center do not exclude the existence of a DM sphere with mass $\simeq 4 \times 10^{6} M_{\odot}$, well contained within the orbits of the known stars, if its radius R_{DM} is $\lesssim 2 \times 10^{-4}$ pc (the periastron distance of the S16 star in the more recent analysis [3]). However, if one considers a DM sphere with larger radius, the corresponding upper value for M_{DM} decreases (although it tends again to increase for extremely extended DM configurations with $R_{DM} \gg 10$ pc). In the following, we will assume for definiteness a DM mass $M_{DM} \sim 2 \times 10^{5} M_{\odot}$, that is the upper value for the DM sphere in [1] within an acceptable confidence level in the range $10^{-3} - 10^{-2}$ pc for R_{DM}. As it will be clear in the following, we emphasize that even a such small value for the DM mass (that is about only 5% of the standard estimate $3.67 \pm 0.19 \times 10^{6}$ $M_{\odot}$ for the dark mass at the Galactic Center [3]) may give some observational signatures.

Evaluating the S2 apoastron shift [2] as a function of R_{DM}, one can further constrain the DM sphere radius since even now we can say that there is no evidence for negative apoastron shift for the S2 star orbit at the level of about 10 mas. In addition, since at present the precision of the S2 orbit reconstruction is about 1 mas, we can say that even without future upgrades of the observational facilities and simply monitoring the S2 orbit, it will be possible within about 15 years to get much more severe constraints on R_{DM}.

Moreover, observational facilities will allow in the next future to monitor faint infrared objects at the astrometric precision of about 10 μas [32] and, in this case, previous estimates will be sensibly improved since it is naturally expected to monitor eccentric orbits for faint infrared stars closer to the Galactic Center with respect to the S2 star.

In the following section, we study the motion of stars as a consequence of the gravitational potential $\Phi(r)$ due the mass profile given in Eq. (1). As usual, the gravitational potential can be evaluated as

$$\Phi(r) = -G\int_{r}^{\infty} \frac{M(r')}{r'^{2}}\, dr'\ . \tag{4}$$

According to GR, the motion of a test particle can be fully described by solving the geodesic equations. Under the assumption that the matter distribution is static and pressureless, the equations of motion in the PN-approximation become (see, for example, [33])

$$\frac{d\mathbf{v}}{dt} \simeq -\nabla(\Phi_N + 2\Phi_N^2) + 4\mathbf{v}(\mathbf{v}\cdot\nabla)\Phi_N - v^2\nabla\Phi_N\ . \tag{5}$$

We note that the PN-approximation is the first relativistic correction from which the apoastron advance phenomenon arises. In the case of the S2 star, the apoastron shift as seen from Earth (from Eq. (7)) due to the presence of a central black hole is about 1 mas, therefore not directly detectable at present since the available precision in the apoastron shift is about 10 mas (but it will become about 1 mas in 10–15 years even without considering possible technological improvements). It is also evident that higher order relativistic corrections to the S2 apoastron shift are even smaller and therefore may be neglected at present, although they may become important in the future.

As it will be discussed below, the Newtonian effect due to the existence of a sufficiently extended DM sphere around the black hole may cause an apoastron shift in the opposite direction with respect to the relativistic advance due to the black hole. Therefore, we have considered the two effects comparing only the leading terms.

[2] We want to note that the periastron and apoastron shifts $\Delta\Phi$ as seen from the orbit center have the same value whereas they have different values as seen from Earth (see Eq. (7)). When we are comparing our results with orbit reconstruction from observations we refer to the apoastron shift as seen from Earth.

For the DM distribution at the Galactic Center we follow Eq. (3) as done in [1]. Clearly, if in the future faint infrared stars (or spots) closer to the black hole with respect to the S2 star will be monitored [32], this simplified model might well not hold and higher order relativistic corrections may become necessary.

For a spherically symmetric mass distribution (such as that described above) and for a gravitational potential given by Eq. (4), Eq. (5) may be rewritten in the form (see for details [34])

$$\frac{d\mathbf{v}}{dt} \simeq -\frac{GM(r)}{r^3}\left[\left(1+\frac{4\Phi_N}{c^2}+\frac{v^2}{c^2}\right)\mathbf{r}-\frac{4\mathbf{v}(\mathbf{v}\cdot\mathbf{r})}{c^2}\right], \tag{6}$$

r and **v** being the vector radius of the test particle with respect to the center of the stellar cluster and the velocity vector, respectively. Once the initial conditions for the star distance and velocity are given, the rosetta shaped orbit followed by a test particle can be found by numerically solving the set of ordinary differential equations in eq. (6).

We note that the expected apoastron (or, equivalently, periastron) shifts (mas/revolution), $\Delta\Phi$ (as seen from the center) and the corresponding values $\Delta\phi_E^{\pm}$ as seen from Earth (at the distance $R_0 \simeq$ 8 kpc from the GC) are related by

$$\Delta\phi_E^{\pm} = \frac{d(1\pm e)}{R_0}\Delta\Phi, \tag{7}$$

where with the sign $\pm$ are indicated the shift angles of the apoastron (+) and periastron (-), respectively. The S2 star semi-major axis and eccentricity are $d = 919$ AU and $e = 0.87$ [3].

In Fig. 1, the S2 apoastron shift as a function of the DM distribution size R_{DM} is given for $\alpha = 0$ and $M_{DM} \simeq 2\times 10^5$ M$_\odot$. Taking into account that the present day precision for the apoastron shift measurements is of about 10 mas, one can say that the S2 apoastron shift cannot be larger than 10 mas. Therefore, any DM configuration that gives a total S2 apoastron shift larger than 10 mas (in the opposite direction due to the DM sphere) is excluded. The same analysis is done for two different values of the DM mass distribution slope, i.e. $\alpha = 1$ and $\alpha = 2$. In any case, we have calculated the apoastron shift for the S2 star orbit assuming a total DM mass $M_{DM} \simeq 2\times 10^5$ M$_\odot$. As one can see, the upper limit of about 10 mas on the S2 apoastron shift may allow to conclude that DM radii in the range about $10^{-3} - 10^{-2}$ pc are excluded by present observations for DM mass distribution slopes.

We notice that the results of the present analysis allows to further constrain the results of the Hall and Gondolo [1] who have concluded that if the DM sphere radius is in the range $10^{-3} - 1$ pc, configurations with DM mass up to $M_{DM} = 2\times 10^5\ M_\odot$ are acceptable. The present analysis shows that DM configurations of the same mass are acceptable only for R_{DM} out the range between $10^{-3} - 10^{-2}$ pc, almost irrespectively of the α value.

In this paper we have considered the constraints that the upper limit (presently of about 10 mas) of the S2 apoastron shift may put on the DM configurations at the galactic center considered by Hall and Gondolo [1].

When (in about 10–15 years, even without considering improvements in observational facilities) the precision of S2 apoastron shift will be about 1 mas (that is equal to the present accuracy in the S2 orbit reconstruction) our analysis will allow to further constrain the DM distribution parameters. In particular, the asymmetric shape of the curves in Fig.

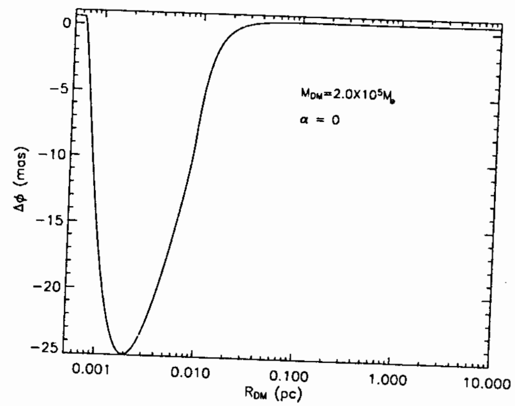

Figure 1: Apoastron shift as a function of the DM radius R_{DM} for $\alpha = 0$ and $M_{DM} \simeq 2\times 10^5$ M$_\odot$. Taking into account present day precision for the apoastron shift measurements (about 10 mas) one can say that DM radii R_{DM} in the range $8\times 10^{-4} - 10^{-2}$ pc are not acceptable.

1 imply that any improvement in the apoastron shift measurements will allow to extend the forbidden region especially for the upper limit for R_{DM}. Quantitatively, we have a similar behavior curves for other choices of slope parameters α for DM concentrations.

In this context, future facilities for astrometric measurements at a level 10 μas of faint infrared stars will be extremely useful [32] and they give an opportunity to put even more severe constraints on DM distribution. In addition, it is also expected to detect faint infrared stars or even hot spots [36] orbiting the Galactic Center. In this case, consideration of higher order relativistic corrections for an adequate analysis of the stellar orbital motion have to be taken into account. Due to a great progress in precision of measurements, one could not exclude a possibility that matter density will be so low that alternative scenarios (to DM annihilation model) will be needed to explain γ-flux from the Galactic Center. Electromagnetic processes in plasma with a presence of a strong gravitational field near the Galactic Center may be important components of such alternative scenarios for the detected γ-flux.

In our considerations we adopted simple analytical expression and reliable values for R_{DM} and M_{DM} parameters following [1] just to illustrate the relevance of the apoastron shift phenomenon in constraining the DM mass distribution at the Galactic Center. If other models for the DM distributions are considered (see, for instance [37] and references therein) the qualitative aspects of the problem are preserved although, of course, quantitative results on apoastron shifts may be different.

AFZ is grateful to Dipartimento di Fisica Universita di Lecce and INFN, Sezione di Lecce where a part of this work was carried out and profs. B.M. Barbashov for his kind attention to our contribution.

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GRAVITY & COSMOLOGY

Initial Data Higgs Effect in Cosmology

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Abstract

The status of the Higgs effect in Cosmology is discussed. A model with simple conditions on the Higgs field dynamics is suggested. Non-trivial cosmological initial data for the Higgs field provide the source of its non-zero vacuum expectation value. Application of the conditions to the first terms in loop expansion of the effective Higgs potential leads to a Gell-Mann–Oakes–Renner type relation between the Higgs vacuum expectation and the condensates of vector bosons and fermions.

1 Introduction

The Standard Model (SM) is a very successful physical theory describing practically all phenomena in high energy physics. But for many reasons we suppose that it is only an *effective* theory appropriate for a certain energy range. Very soon at the Large Hadron Collider (LHC) at CERN we hope to access both the limit of the SM applicability and the mechanism of EW symmetry breaking.

The Higgs-Kibble mechanism [1] gives masses for gauge bosons and certain problems for the theory: scale invariance breaking by a tachyon mass term, the presence of monopole solutions, a non-zero imaginary part of the effective potential, a large Higgs self-coupling, a tremendous contribution to the vacuum energy, the *naturalness* (or *fine-tuning*) problem *etc*, see *e.g.* Ref. [2]. On the other hand, the SM Higgs potential looks as an artificial product of the correspondence principle to the Landau-Ginzburg potential in the theory of superconductivity. Moreover the the status of the Higgs field and of its potential in Cosmology is still under discussion.

Let us remind the treatment of scalar fields in Cosmology [3]. We start with the Lagrangian for the scalar field

$$\mathcal{L} = \partial_\mu \Phi^\dagger \partial^\mu \Phi - V_{eff}(|\Phi|), \qquad |\Phi| = \langle \Phi \rangle + \frac{h}{\sqrt{2}}, \tag{1}$$

where in accord with the Einstein's cosmological principle [4] we separate the zeroth harmonics $\langle \Phi \rangle$ from the particle like excitations h of the Higgs field. The mean value of the field can in this approach depend on time: $\langle \Phi \rangle = \langle \Phi \rangle (t)$. The equation of motion for the zeroth harmonics then reads

$$\partial_0^2 \langle \Phi \rangle + \frac{\partial V_{eff}(\langle \Phi \rangle)}{\partial \langle \Phi \rangle} = 0. \tag{2}$$

It is known that the SM potential of the Higgs field is not appropriate to provide the proper Universe evolution in time. So in Cosmology it was suggested to introduce an additional scalar field (*inflaton*) with a different potential [3]. While the status and possible dynamics of the SM Higgs field in Cosmology is still under discussion.

2 Model

Let us consider a simple model assuming the following conditions on the effective potential of the Higgs field:

$$\frac{\partial V_{eff}(\langle\Phi\rangle)}{\partial\langle\Phi\rangle} = 0 \qquad \Rightarrow \qquad \partial_0^2\langle\Phi\rangle = 0. \qquad (3)$$

In this case we can choose the partial solution

$$\langle\Phi\rangle = \frac{M_W}{g_w\sqrt{2}} = Const, \qquad \partial_0\langle\Phi\rangle = 0,$$

which can be treated as cosmological initial data for the scalar field.

Note that in any case at the beginning of the Universe evolution, the potential term is suppressed by a high power of the cosmological factor: $V_{eff}(\langle\Phi\rangle) \to 0|_{t\to 0}$. We assume also that the effective potential takes into account all loop corrections so that the potential minimum position remains always in the same point $\Phi = \langle\Phi\rangle$. Note that the static value of the Higgs vacuum expectation directly corresponds to the SM case.

It is important that the cosmological principle is applied by averaging of all scalar components over the space:

$$\underline{V_0 < \infty} \quad \Rightarrow \quad \frac{1}{V_0}\int |\Phi| d^3x = \langle\Phi\rangle, \qquad \frac{1}{V_0}\int h\, d^3x = 0,$$

where we assume that the volume V_0 is finite.

The effective potential in a general case can be written as

$$V_{eff}(\langle\Phi\rangle) = i\frac{\partial}{\partial t_0}\left[\ln\int\prod_{F=h,...} dF\; e^{iS_{eff}(F,t_0)}\right], \quad S_{eff} = \int_0^{t_0=1/H_0} dt\, \mathcal{L}_{eff},$$

where Lagrangian $\mathcal{L}_{eff}$ takes into account all the SM interactions.

We suggest to apply assumption (3) for each order in the loop expansion. Let us consider a Lagrangian describing scalar field $|\Phi|$ and its interactions with fermions (s) and vector bosons (V)

$$\mathcal{L}_{\rm Higgs} = \partial_\mu|\Phi|\partial^\mu|\Phi| - |\Phi|\sum_s f_s\bar{s}s + \frac{|\Phi|^2}{4}\sum_{\rm v} g_{\rm v}^2 V^2 - \lambda|\Phi|^4.$$

The idea is to start from a *scale invariant* Lagrangian and to generate masses by initial data for the Bogoliubov condensates

$$\langle\Phi\rangle \equiv \frac{1}{V_0}\int d^3x|\Phi|, \quad \langle{\sf A}\rangle \equiv \frac{1}{V_0}\int d^3x \sum_s f_s\bar{s}s, \quad \langle{\sf B}\rangle \equiv \frac{1}{4V_0}\int d^3x\sum_{\rm v} g_{\rm v}^2 V^2.$$

We again separate the scalar field zeroth harmonics $|\Phi| = \langle\Phi\rangle + h/\sqrt{2}$, and the Lagrangian takes the form

$$\mathcal{L}_{\rm Higgs} = \partial_0\langle\Phi\rangle\partial_0\langle\Phi\rangle - {\sf V}_0(\langle\Phi\rangle) + \frac{1}{2}\partial_\mu h\partial^\mu h - h^2\frac{m_h^2}{2} + h^3\lambda\sqrt{2}\langle\Phi\rangle + \ldots,$$

where

$$\begin{aligned} {\sf V}_0(\langle\Phi\rangle) &= \langle\Phi\rangle\langle{\sf A}\rangle - \langle\Phi\rangle^2\langle{\sf B}\rangle + \lambda\langle\Phi\rangle^4, \\ m_h^2 &= \frac{d^2{\sf V}_0(\langle\Phi\rangle)}{2d\langle\Phi\rangle^2} = 6\lambda\langle\Phi\rangle^2 - \langle{\sf B}\rangle. \end{aligned}$$

Assuming that ${\sf V}_0(\langle\Phi\rangle) = 0$ and $\partial{\sf V}_0(\langle\Phi\rangle)/\partial\langle\Phi\rangle = 0$ we get a set of equations for the condensates giving

$$m_h^2 = \langle{\sf B}\rangle = 3\lambda\langle\phi\rangle^2, \qquad \langle{\sf A}\rangle = 2\lambda\langle\phi\rangle^3.$$

In this way we get a non-trivial relation between the condensates.

3 Outlook

Separation of zeroth modes is not Lorentz-invariant, so for cosmological considerations we choose the *distinguished* CMB reference frame assuming the final size of the Universe volume: $V_0 < \infty$.

¿From the beginning we did not take the classical Higgs potential in the standard form with a tachion mass term. But we can keep the electroweak sector of the SM unchanged as discussed in [5]. The key point of our approach is just the origin of the scale symmetry breaking.

Assuming the same condition for all terms in further loop expansion of V_{eff} we get non-trivial conditions *to be evaluated.* At each order of calculation we can find the Higgs mass as

$$m_h^2 = \frac{d^2V_{eff}(\langle\Phi\rangle)}{2d\langle\Phi\rangle^2}.$$

The difficulties of the SM related to the Higgs sector are not immediately resolved withing the suggested approach, but the latter allows to look at them from other side.

One of us (A.A.) is grateful to the grant of the President RF Scientific Schools 3312.2008.

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Schroedinger Equation in Finite Differences

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Abstract

We consider the spherically symmetric self-gravitating thin dust shell. The direct ("naive") quantization of the proper time Hamiltonian leads to the one-dimensional Schroedinger equation in finite differences. We solved this equation and obtained the discrete mass spectrum for bound states. While using the Lorentzian time, we found the canonical transformation performing the well-known square-root differential operator into the exponential one and, again, obtained the finite differences Schroedinger equation and corresponding discrete mass spectrum which appeared to be the famous Sommerfeld spectrum for the Klein-Gordon equation.

1 The model. Naive quantization.

Our model is the self-gravitating spherically symmetric thin dust shell. Such a shell is a direct generalization of a point particle. The Einstein equations for the shell (Israel equations) are reduced in this case to only one equation

$$\sqrt{\dot\rho^2+1}-\sqrt{\dot\rho^2+1-\frac{2Gm}{\rho}}=\frac{GM}{\rho}, \tag{1}$$

where ρ is the shell radius as a function of the proper time τ, dot denotes the time derivative, G is the Newtonian gravitational constant, m is the total mass of the shell measured by a distant observer, and M is its bare mass (the sum of masses of constituent particles). Squaring Eqn.(1), we easily obtain

$$m=M\sqrt{\dot\rho^2+1}-\frac{GM^2}{2\rho}, \tag{2}$$

where m is the total mass of the system, the first term on the right hand side is the kinetic energy (the square root is nothing more but the famous Lorentz factor written using the proper time derivatives), and the second one is the (negative) gravitational energy. We are interested in the bound motion only. We will consider the right hand side of it as a pre-Hamiltonian E which on the equations of motion takes the value of the total mass $E=m$:

$$E=M\sqrt{\dot\rho^2+1}-\frac{GM^2}{2\rho}. \tag{3}$$

Having the pre-Hamiltonian we obtain rather easy the conjugate momentum p to the radius ρ, the Lagrangian L and the Hamiltonian H, using the following prescriptions:

$$p = \frac{\partial L}{\partial\dot\rho},$$

$$\begin{aligned} E &= p\dot\rho-L=\dot\rho\frac{\partial L}{\partial\dot\rho}-L,\\ L &= \dot\rho\int E\frac{d\dot\rho}{\dot\rho^2}=\dot\rho\int\frac{\partial E}{\partial\dot\rho}\frac{d\dot\rho}{\dot\rho}-E. \end{aligned} \tag{4}$$

The result is

$$\begin{aligned} p &= M\ln(\dot\rho+\sqrt{\dot\rho^2+1})-F(\rho),\\ L &= M(\dot\rho\ln(\dot\rho+\sqrt{\dot\rho^2+1})-\sqrt{\dot\rho^2+1})+\dot\rho F(\rho), \end{aligned} \tag{5}$$

where $F(\rho)$ is an "integration constant" which does not influence the equation of motion, we put it zero, $F(\rho)=0$. Thus, for the proper time velocity $\dot\rho$ and the Hamiltonian H we obtain

$$\begin{aligned} \dot\rho &= \sinh\frac{p}{M},\\ H &= M\cosh\frac{p}{M}-\frac{GM^2}{2\rho}. \end{aligned} \tag{6}$$

Making the canonical transformation to the dimensionless variables $x=M\rho$, $\Pi=\frac{p}{M}$ we get finally

$$H=M\left(\cosh\Pi-\frac{GM^2}{2x}\right). \tag{7}$$

The quantization procedure consists in considering Π and x as operators acting on the wave function Ψ, and and imposing on them the quantum mechanical commutation relation

$$[\Pi,x]=-i. \tag{8}$$

In the coordinate representation x acts as a multiplication, $\Pi=-i\frac{\partial}{\partial x}$, and

$$e^{-i\frac{\partial}{\partial x}}\Psi(x)=\Psi(x-i), \tag{9}$$

and for the stationary Schroedinger equation $H\Psi(x)=E(=m)\Psi(x)$ we obtain ($\varepsilon=\frac{m}{M}$, $\alpha=GM^2$) :

$$\Psi(x+i)+\Psi(x-i)=\left(2\varepsilon+\frac{\alpha}{x}\right)\Psi(x). \tag{10}$$

This is the finite differences equation, unlike a differential equation we are used to. Note, that the shifts in the argument are along the imaginary axes, so the "good" solution has to have special analytical properties in the complex plane. And one more very important property: given some solution we can multiply it by any periodic function with pure imaginary period i and obtain, again, a solution.

At the end of this Section let us examine the non-relativistic limit of our equation. After restoring the dimensional constants $\hbar$ and c the shifted arguments $(x\pm i)$ become

$$x\pm i\longrightarrow M\left(\rho\pm\frac{1}{M}i\right)\longrightarrow\rho\pm\frac{\hbar}{Mc}i, \tag{11}$$

so, the non-relativistic limit corresponds to the inequality $\rho \gg \hbar/Mc$. Expanding Eqn.(10) in series up to the second order in small parameter $\frac{\hbar}{Mc\rho} \ll 1$ we get

$$-\frac{1}{2M\hbar^2}\frac{d^2\Psi}{d\rho^2} - \frac{GM^2}{2\rho}\Psi = (E - Mc^2)\Psi, \tag{12}$$

which is just the non-relativistic Schroedinger equation for the radial (s-wave) function, $(E - Mc^2)$ being the non-relativistic energy of the system. The "good" solutions for the bound states exist only for the discrete energy spectrum (Rydberg's formula):

$$(E - Mc^2)_n = -\hbar^2 \frac{G^2 M^5}{8n^2}, \quad n = 1, 2, \ldots. \tag{13}$$

2 Solution to the Schroedinger equation in finite differences and discrete mass spectrum

It is convenient to introduce new parameters $\varepsilon = \cos\lambda$, $\alpha = GM^2 = 2\beta\sin\lambda$. We choose $\lambda > 0$ and, therefore, $\beta > 0$. Then our equation becomes

$$\Psi(x+i) + \Psi(x-i) = 2\left(\cos\lambda + \frac{\beta\sin\lambda}{x}\right)\Psi(x). \tag{14}$$

We start to construct a solution with transition to the momentum representation. After some simple manipulations we arrive at the following equation

$$i\frac{\partial}{\partial p}\left((\cosh p - \cos\lambda)\Psi_p\right) = \beta\sin\lambda\Psi_p. \tag{15}$$

With a new variable $z = e^p$ it reads as follows:

$$\frac{\partial}{\partial z}\ln\Psi_p = \frac{1}{z} - \frac{\beta+1}{z-z_0} + \frac{\beta-1}{z-\bar{z}_0}, \tag{16}$$

where $z_0 = e^{i\lambda}, \bar{z}_0 = e^{-i\lambda}$. In this form our equation is very easy to solve, the result is

$$\Psi_p = C\frac{z}{(z-z_0)(z-\bar{z}_0)}\left(\frac{z-\bar{z}_0}{z-z_0}\right)^\beta, \quad z = e^p, \tag{17}$$

this solution is periodic with the period $2\pi i$. It means that in the coordinate representation obtaining by the inverse Fourier transform,

$$\Psi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{ipx}\Psi_p dp, \tag{18}$$

the general solution can be written as

$$\Psi_{general}(x) = \left(\sum_{k=-\infty}^{\infty} c_k e^{-2\pi kx}\right)\Psi_0(x). \tag{19}$$

The expression in brackets is nothing more but the Fourier series of some periodic function with the pure imaginary period i, and this shows how the main feature of the equations in finite differences emerges from their Fourier images. Since we have only one solution (up to the multiplicative constant) in the momentum representation there is, essentially, only one fundamental solution $\Psi_0(x)$ in the coordinate representation which deserves the name "super-fundamental".

Our solution Ψ_p, Eqn.(17), has a countable number of branching points in the complex momentum plane which can be combined in pairs $(i\lambda + 2\pi ki, -i\lambda + 2\pi(k+1)i)$, $k = 0, \pm 1, \pm 2, \ldots$. Connecting them we construct the complex plane with countable number of cuts. The specific form of the super-fundamental solution $\Psi_0(x)$ in the coordinate representation depends on the choice of the contour of integration. Using the above-mentioned periodicity of the solution Ψ_p in the momentum representation we are able to build the following closed contour: the integration goes first along the real axes in the positive direction, then back to the left infinity along the line $p = 2\pi i$ for $x > 0$ ($p = -2\pi i$ for $x < 0$), these two straight lines being connected by curves at infinities with zero contributions to the contour integral. Such contour can be distorted to become a contour around a cut. After doing all this we get for $\Psi_0(x)$:

$$\Psi_0(x) = \left(-4\pi\beta e^{-i\lambda}\sin\lambda\right) x e^{-\lambda x} F\left(1 - ix, 1-\beta; 2; 1 - e^{-2i\lambda}\right), \tag{20}$$

where $F(a, b; c; y)$ is the famous Gauss's hypergeometric function, and our primordial equation, Eqn.(14), reduces to the one of its recurrent relations.

Our aim is to find the "good" solutions and obtain the discrete mass spectrum for the quantum shell bound states. For this we need to know the asymptotic behavior of the solutions in two suspicious points, $x = 0$ and $x = \infty$. At $x = 0$ the asymptotics are

$$x^r, \quad r = 1, 2, \ldots, \tag{21}$$

and at $x \to \infty$

$$\begin{aligned} \Psi_0(x) \sim & - 2\pi i\beta e^{-i\lambda\beta} e^{i\pi\beta} \\ & \times \left\{ \frac{(2\sin\lambda)^\beta}{\Gamma(1+\beta)} x^\beta e^{-\lambda x} - \frac{(2\sin\lambda)^\beta}{\Gamma(1-\beta)} x^{-\beta} e^{\lambda x} \right\} \Phi\left(\frac{1}{x}\right), \end{aligned} \tag{22}$$

where $\Gamma(\cdots)$ is the Euler's function. Thus, we see, that the "good" solution at the origin ($x \to 0$) results in the linear combination of the "good" ($x^\beta e^{-\lambda x}$) and "bad" ($x^{-\beta} e^{\lambda x}$) asymptotics at infinity (remember that $x > 0$ and we chose $\beta > 0$). To get rid of the "bad" part we have to put

$$\beta = n, \quad n = 1, 2, \ldots. \tag{23}$$

Remembering now that $\beta = \frac{\alpha}{2\sin\lambda}$, $\cos\lambda = \frac{m}{M}$, $\alpha = GM^2$, we get for the discrete mass spectrum

$$m = M\sqrt{1 - \frac{G^2M^4}{4n^2}}. \tag{24}$$

It is easy to see that for large enough values of quantum number n (namely, for $\frac{GM^2}{2n} \ll 1$) this spectrum is reduced to the non-relativistic Rydberg's formula, Eqn.(13), as it should be. Moreover, it is a part of the Dirac's spectrum (with a suitable change of notations) when the so-called radial quantum number n_r is zero (the latter corresponds to the case of critical angular momentum in classical relativistic Coulomb problem), what is rather unexpected because from the very beginning we have no angular momentum at all. It is interesting to note that the same Schroedinger equation in finite differences was studied by V.Kadyshevsky and R.Mir-Kasimov more than 30 years ago. They have quite a different motivation but proposed the same energy spectrum for bound states!

3 Relativistic Kepler's problem. Square-root quantization.

Till now we used the proper time quantization which is unique in the framework of General Relativity in the sense that the proper time does not depend on the choice of time coordinate both inside and outside the shell. The situation is quite different in Special Relativity where the clocks of any inertial observer can most naturally be used for defining the time units. In this case our pre-Hamiltonian becomes (v is the Lorentz time velocity):

$$E = \frac{M}{\sqrt{1-v^2}} - \frac{GM^2}{2\rho}. \tag{25}$$

Introducing now a conjugate momentum Π corresponding to the velocity v, we arrive at the famous square-root Hamiltonian for the radial motion of our shell:

$$H = \sqrt{\Pi^2 + M^2} - \frac{\alpha}{2\rho}. \tag{26}$$

In quantum mechanics such a Hamiltonian is a nonlocal operator in the coordinate representation. To reveal this non-locality more explicitly we make the following canonical transformation before quantization procedure:

$$\Pi = M \sinh p, \quad \rho = \frac{y}{M \cosh p}. \tag{27}$$

The Hamiltonian, Eqn.(26) now becomes

$$H(p, y) = M \left(1 - \frac{\alpha}{2y}\right) \cosh p. \tag{28}$$

The expression for the quantum counterpart depends on the chosen operator ordering. We write the corresponding Schroedinger equation in the form

$$\left(y - \frac{\alpha}{2}\right) (\Phi(y+i) + \Phi(y-i)) = 2\varepsilon y \Phi(y), \quad \Phi = \left(1 - \frac{\alpha}{2y}\right) \Psi, \quad \varepsilon = \frac{m}{M}. \tag{29}$$

If we were to choose the reverse ordering of momentum and coordinate functions we would get the same equation for $\Psi(y)$ instead of $\Phi(y)$.

The solution to the Eqn.(29) in the momentum representation is

$$\begin{aligned}
\Phi_p &= C \frac{z^{1+i\frac{\alpha}{2}}}{(z-z_0)(z-\bar{z}_0)} \left(\frac{z-\bar{z}_0}{z-z_0}\right)^{\beta}, \\
z &= e^p, \quad z_0 = e^{i\lambda}, \quad \bar{z}_0 = e^{-i\lambda}, \\
\varepsilon &= \cos\lambda, \quad \alpha = GM^2, \quad \beta = \frac{\alpha}{2} \cot\lambda.
\end{aligned} \tag{30}$$

This expression differs from the Eqn.(17) only by factor $z^{i\alpha/2}$. But if we shift the argument in the corresponding solution in coordinate representation $y \to (y - \alpha/2)$, then the Fourier transform of such a shifted function $\tilde{\Phi}_p$ will be exactly the same as Ψ_p in Section III. This means that the discrete spectrum in the case of square-root quantization is determined by $\beta = n, \quad n = 1, 2, \ldots$, and we get

$$m = \frac{M}{\sqrt{1 + \frac{G^2 M^4}{4n^2}}}. \tag{31}$$

This is the so-called Sommerfeld spectrum that can be obtained for the same classical model but using the Klein-Gordon Hamiltonian, local and quadratic in conjugate momenta.

The author is indebted to the Russian Foundation for Fundamental Researches for financial support (Grant N 06-02-16-342 a).

E_6 Unification and Cosmology

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Abstract

In the present paper we have developed a concept of parallel ordinary (O) and mirror (M) worlds, and shown that in the case of the broken mirror parity (MP), the evolutions of fine structure constants in the O- and M-worlds are not identical. It is assumed that E_6-unification inspired by superstring theory restores the broken MP at the scale $\sim 10^{18}$ GeV. Cosmological consequences of this theory is discussed.

Superstring theory is a paramount candidate for the unification of all fundamental gauge interactions with gravity [1]. Superstrings are free of gravitational and Yang-Mills anomalies if a gauge group of symmetry is $SO(32)$ or $E_8 \times E_8$. The 'heterotic' superstring theory $E_8 \times E_8'$ [1, 2] was suggested as a more realistic model for unification. This ten-dimensional Yang-Mills theory can undergo spontaneous compactification: the integration over 6 compactified dimensions of the E_8 superstring theory leads to the effective theory with the E_6-unification in four-dimensional space [1]. In the present investigation [3] we consider the old concept: there exists in Nature a 'mirror' (M) world (hidden sector) [4, 5] parallel to our ordinary (O) world. This M-world is a mirror copy of the O-world and contains the same particles and their interactions as our visible world. Observable elementary particles of our O-world have left-handed (V-A) weak interactions which violate P-parity, and the mirror particles participate in the right-handed (V+A) weak interactions and have an opposite chirality. The idea of the existence of visible and mirror worlds became very attractive in connection with a superstring theory described by $E_8 \times E_8'$. We have discussed cosmological implications of the parallel ordinary and mirror worlds with the broken mirror parity MP [6]. We have considered the parameter characterizing the breaking of MP, which is $\zeta = v'/v$, where v' and v are the VEVs of the Higgs bosons – Electroweak scales – in the M- and O-worlds, respectively. During our numerical calculations, we have used the result [6]: $\zeta \approx 30$. We have assumed in this investigation that at the very small distances there exists E_6-unification predicted by Superstring theory [1]. It was shown that, as a result of the MP-breaking, the evolutions of fine structure constants in O- and M-worlds are not identical, and the extensions of the Standard Model (SM) in the ordinary and mirror worlds are quite different. We have assumed that the E_6-unification, being the same in the O- and M-worlds, restores the broken mirror parity MP. We have considered the following chain of symmetry groups in the ordinary world:

$$SU(3)_C \times SU(2)_L \times U(1)_Y \to SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X \times U(1)_Z$$

$$\to SU(4)_C \times SU(2)_L \times SU(2)_R \times U(1)_Z \to SO(10) \times U(1)_Z \to E_6.$$

A simple logic leads to the following chain in the mirror world:

$$SU(3)'_C \times SU(2)'_L \times SU(2)'_Z \times U(1)'_Y \to SU(3)'_C \times SU(2)'_L \times SU(2)'_Z \times U(1)'_X \times U(1)'_Z$$
$$\to SU(4)'_C \times SU(2)'_L \times SU(2)'_Z \times U(1)'_Z \to SU(6)' \times SU(2)'_Z \to E'_6.$$

The comparison of both evolutions in the ordinary and mirror worlds is given in Figure 1, where we have presented the running of all fine structure constants. Here the SM (SM') is extended by MSSM (MSSM'), and we see different evolutions. Figure 1 corresponds to the SUSY breaking scales

$$M_{SUSY} = 10 \text{ TeV}, \quad M'_{SUSY} \approx 300 \text{ TeV},$$

according to the MP-breaking parameter $\zeta \approx 30$. We have considered the value of seesaw scale in the O-world $M_R \sim 10^{14}$ GeV, and in the M-world: $M'_R \sim 10^{17}$ GeV.

It was shown that the (super)grand unification E'_6 in the mirror world is based on the group

$$E'_6 \supset SU(6)' \times SU(2)'_Z.$$

The existence of a new gauge group $SU(2)'_Z$ in the M-world gives significant consequences for cosmology: it explains the 'quintessence' model of our accelerating Universe.

The reason of our choice of the gauge group $SU(2)'_Z$ was to obtain the correct running of $(\alpha')^{-1}_{2Z}(\mu)$, which: • leads to the new scale $\Lambda_Z \sim 10^{-3}$ eV at extremely low energies [7]; • and is consistent with the running of all inverse gauge coupling constants in the O- and M-worlds with broken mirror parity, considered in [3]. The predicted particle content of $SU(2)'_Z$ is as follows:

1. two doublets of fermions $\psi_i^{(Z)}$ and two doublets of the 'messenger' scalar fields $\phi_i^{(Z)}$ with $i = 1, 2$, and

2. a complex singlet scalar field: $\varphi_Z = (1, 1, 0, 1)$ under the symmetry group

$$G' = [SU(3)'_C \times SU(2)'_L \times U(1)'_Y] \times SU(2)'_Z.$$

The existence of the dark matter DM (non-luminous and non-absorbing matter) in the Universe is now well established in astrophysics. We assume that the best candidate for DM is the mirror M-world (mirror quarks, mirror leptons, mirror bosons and their super-partners). This M-world interacts with the ordinary O-world only by gravity, or by another unknown very weak interaction. Here we want to emphasize that the fermions $\psi_i^{(Z)}$ belonging to the group $SU(2)'_Z$ can be considered as candidates for the Hot Dark Matter (HDM), but the bound states – "hadrons" of the group $SU(2)'_Z$ – can be good candidates for the Cold Dark Matter (CDM).

We have discussed the 'quintessence' model of our Universe [7]: at the scale $\Lambda_Z \sim 10^{-3}$ eV the instantons of the group $SU(2)'_Z$ induce a potential for an axion-like scalar boson a_Z, which can be called "acceleron". The acceleron gives the value $w = -1$ and leads to the acceleration of our Universe. The existence of the scale $\Lambda_Z \sim 10^{-3}$ eV explains the value of cosmological constant CC $\approx (3 \times 10^{-3}\text{ eV})^4$, which is given by astrophysical measurements. Also recent measurements in cosmology fit the equation of state for DE: $w = p/\rho$ (p is the pressure, ρ is the density) with a constant $w \approx -1$. Following [7], we have assumed that at present time our Universe exists in the 'false' vacuum given by the axion potential. The Universe will live there for a long time and its CC (measured in cosmology) is tiny, but nonzero. However, at the end the Universe will jump into the 'true' vacuum and will get a zero CC. But this problem is not trivially solved, and at present time we have only a hypothesis.

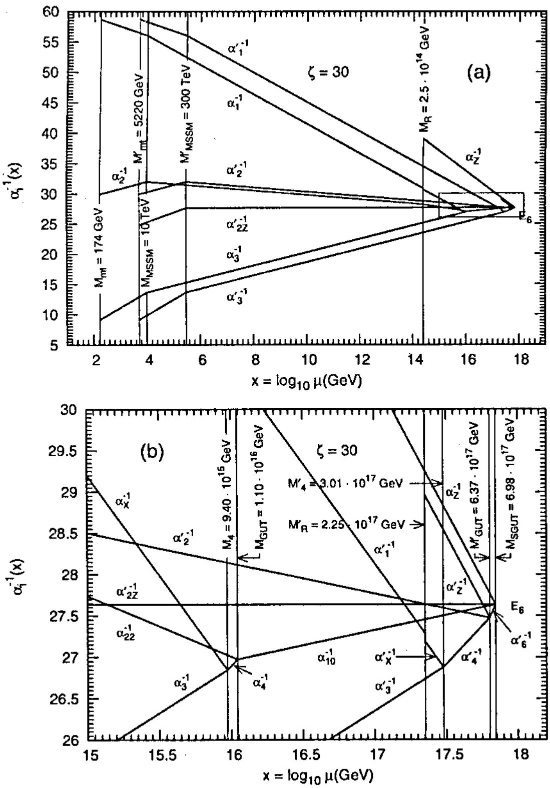

Figure 1: The running of the inverse coupling constants $\alpha_i^{-1}(x)$ in both ordinary and mirror worlds with broken MP from the SM up to the E_6 unification.

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Dynamical symmetry breaking for quark matter in the static Einstein universe

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Abstract

In the framework of an extended Nambu–Jona-Lasinio model we study the formation of quark, diquark and pion condensates in quark matter in a gravitational field of the static Einstein universe at finite temperature, chemical potential and with an asymmetric isospin composition. The phase portraits of the system are constructed. We demonstrate the effect of oscillations of the thermodynamic quantities as functions of the curvature and also refer to a certain similarity between the behavior of these quantities as functions of curvature and temperature.

1 Introduction

Effective field theories with four-fermion interaction of the Nambu – Jona-Lasinio type (NJL) [1] are quite useful in describing the physics of light mesons and diquarks. It was proposed more than twenty years ago [2, 3, 4] that at high baryon densities a colored diquark condensate $\langle qq \rangle$ might appear. In analogy with ordinary superconductivity, this effect was called color superconductivity (CSC). The possibility for the existence of the CSC phase in the region of moderate densities was recently proved (see, e.g., papers [5, 6]). Another interesting phenomenon, the condensation of charged pions, which may appear in dense hadronic matter due to an asymmetry of its isospin composition, has been investigated in the framework of QCD-like effective models, including the NJL model, as well [7]. Note that all these phenomena might be inherent to physics of compact stars, where strong gravitational fields are present.

In several papers, in the framework of the NJL model, the influence of a gravitational field on the dynamical chiral symmetry breaking has been investigated at zero values of temperature and chemical potential (see review [8] and references therein). The dynamical breaking of chiral symmetry at finite temperature and chemical potential in the static Einstein universe was recently considered in [9].

In this talk we discuss the dynamical breaking of chiral, color and isospin symmetries in a constant curvature gravitational field. First, we consider the formation of quark and diquark condensates in dense isotopically symmetric matter and secondly we consider the pion condensation in quark matter with flavor asymmetry. As the simplest model of curved space we have taken the static Einstein universe. This allows us to derive a nonperturbative expression for the thermodynamical potential and to construct the phase portraits of our system (for more details see [10, 11]).

2 The extended NJL model in curved spacetime

The NJL model [1] extended so that it includes up- and down-quarks with $(\bar{q}q)$ and (qq) interactions in the color group $SU_c(N)$ can be used to describe formation of the color superconducting phase. This model can be considered as the low energy limit of QCD. For the color group $SU_c(3)$ its Lagrangian takes the form [10]:

$$\mathcal{L} = \bar{q}\left[i\gamma^\mu\nabla_\mu + \mu\gamma^0\right]q + \frac{G_1}{2N_c}\left[(\bar{q}q)^2 + (\bar{q}i\gamma^5\vec{\tau}q)^2\right] + \frac{G_2}{N_c}\left[i\bar{q}_c\varepsilon\epsilon^b\gamma^5 q\right]\left[i\bar{q}\varepsilon\epsilon^b\gamma^5 q_c\right]. \quad (1)$$

Here, ∇_μ is a covariant derivative with respect to the curved spacetime, $N_c = 3$ is the number of colors, G_1 and G_2 are coupling constants, μ is the quark chemical potential. The quark field $q \equiv q_{i\alpha}$ is a doublet of flavors and triplet of colors, and q_c is a charge-conjugated field. Moreover, $\vec{\tau} \equiv (\tau^1, \tau^2, \tau^3)$ denote Pauli matrices in the flavor space; $(\varepsilon)^{ik} \equiv \varepsilon^{ik}$, $(\epsilon^b)^{\alpha\beta} \equiv \epsilon^{\alpha\beta b}$ are the totally antisymmetric tensors in the flavor and color spaces, respectively.

Next, by applying the usual bosonization procedure, we obtain the linearized version of the Lagrangian (1) with collective boson fields σ, $\vec{\pi}$ and Δ,

$$\begin{aligned}\tilde{\mathcal{L}} &= \bar{q}\left[i\gamma^\mu\nabla_\mu + \mu\gamma^0\right]q - \bar{q}\left(\sigma + i\gamma^5\vec{\tau}\vec{\pi}\right)q - \frac{3}{2G_1}(\sigma^2 + \vec{\pi}^2) \\ &- \frac{3}{G_2}\Delta^{*b}\Delta^b - \Delta^{*b}\left[iq^t C\varepsilon\epsilon^b\gamma^5 q\right] - \Delta^b\left[i\bar{q}\varepsilon\epsilon^b\gamma^5 C\bar{q}^t\right]. \qquad (2)\end{aligned}$$

The Lagrangians (1) and (2) are equivalent, as can be seen by using the Euler-Lagrange equations for bosonic fields, from which it follows that

$$\Delta^b \sim \mathrm{i}q^t C\varepsilon\epsilon^b\gamma^5 q, \quad \sigma \sim \bar{q}q, \quad \vec{\pi} \sim \mathrm{i}\bar{q}\gamma^5\vec{\tau}q. \qquad (3)$$

The fields σ and $\vec{\pi}$ are color singlets, and Δ^b is a color anti-triplet and flavor singlet.

In what follows, it is convenient to consider the effective action for boson fields, which is expressed through the integral over quark fields

$$\exp\left\{ iS_{\text{eff}}(\sigma, \vec{\pi}, \Delta^b, \Delta^{*b})\right\} = N' \int [dq][d\bar{q}] \exp\left\{ i \int d^4x\sqrt{-g}\tilde{\mathcal{L}}\right\}, \qquad (4)$$

where

$$S_{\text{eff}}(\sigma, \vec{\pi}, \Delta^b, \Delta^{*b}) = -\int d^4x\sqrt{-g}\left[\frac{3(\sigma^2 + \vec{\pi}^2)}{2G_1} + \frac{3\Delta^b\Delta^{*b}}{G_2}\right] + S_q, \qquad (5)$$

where S_q is the quark contribution.

In the mean field approximation, the fields σ, $\vec{\pi}$, Δ^b, Δ^{*b} can be replaced by their ground state averages: $\langle\sigma\rangle$, $\langle\vec{\pi}\rangle$, $\langle\Delta^b\rangle$ and $\langle\Delta^{*b}\rangle$, respectively. Let us choose the following ground state of our model:

$$\langle\Delta^1\rangle = \langle\Delta^2\rangle = \langle\vec{\pi}\rangle = 0.$$

If $\langle\sigma\rangle \neq 0$, the chiral symmetry is broken dynamically, and if $\langle\Delta^b\rangle \neq 0$, the color symmetry is broken. Evidently, this choice breaks the color symmetry down to the residual group $SU_c(2)$. Let us denote $\langle\sigma\rangle$, $\langle\Delta^3\rangle \neq 0$, by letters σ, Δ.

The effective potential, the global minimum point of which determines the quantities σ and Δ, follows from the definition: $S_{\text{eff}} = -V_{\text{eff}} \int d^4x\sqrt{-g}$, where

$$V_{\text{eff}} = \frac{3\sigma^2}{2G_1} + \frac{3\Delta\Delta^*}{G_2} + \tilde{V}_{\text{eff}}; \quad \tilde{V}_{\text{eff}} = -\frac{S_q}{v}, \quad v = \int d^4x\sqrt{-g}. \qquad (6)$$

3 Thermodynamic potential

We will use the static D-dimensional Einstein universe as a simple example of curved space. The line element

$$ds^2 = dt^2 - a^2(d\theta^2 + \sin^2\theta d\Omega_{D-2}) \tag{7}$$

gives the global topology $\mathbb{R} \otimes \mathbb{S}^{D-1}$ of the universe, where a is the radius of the universe, related to the scalar curvature by the relation $R = (D-1)(D-2)a^{-2}$. The volume of the universe is $V(a) = 2\pi^{D/2}a^{D-1}/\Gamma(\frac{D}{2})$.

One can introduce the one-particle Hamiltonian $\hat{H} = -i\vec{\alpha}\vec{\nabla} + \sigma\gamma^0$, where $\vec{\alpha} = \gamma^0\vec{\gamma}$. Using this operator the quark contribution to the effective action can be written in the following form:

$$S_q = -\frac{i}{2}\left\{\ln \mathrm{Det}\left[\hat{H}^2 - (\hat{p}_0 + \mu)^2\right] + 2\ln \mathrm{Det}\left[4|\Delta|^2 + (\hat{H}-\mu)^2 - \hat{p}_0^2\right]\right\}, \tag{8}$$

where $\hat{p}_0 = i\partial_0$.

The eigenvalues of the operator $\hat{H}$ may be found exactly for the case of the static D-dimensional Einstein universe. They are expressed through the corresponding eigenvalues of the Dirac operator on the sphere $\mathbb{S}^{D-1}$ [12]:

$$\hat{H}\,\psi_l = \pm E_l\psi_l, \quad E_l = \sqrt{\frac{1}{a^2}\left(l + \frac{D-1}{2}\right)^2 + \sigma^2}, \quad l = 0, 1, 2\dots . \tag{9}$$

The degeneracy of E_l is equal to

$$d_l = \frac{2^{[(D+1)/2]}\Gamma(D+l-1)}{l!\Gamma(D-1)}, \tag{10}$$

where $[x]$ is the integer part of x.

After going over to Euclidean spacetime and summing over Matsubara frequencies we obtain the thermodynamic potential

$$\begin{aligned}\Omega(\sigma,\Delta) &= 3\left(\frac{\sigma^2}{2G_1} + \frac{|\Delta|^2}{G_2}\right) - \frac{N_f}{V}(N_c - 2)\sum_{\xi=\pm}\left\{\sum_{l=0}^{\infty} d_l\left[\frac{E_l}{2} + T\ln\left(1 + e^{-\beta(E_l - \xi\mu)}\right)\right]\right.\\ &\left.-\frac{N_f}{V}\sum_{l=0}^{\infty} d_l\left[\sqrt{(E_l - \xi\mu)^2 + 4|\Delta|^2} + 2T\ln\left(1 + e^{-\beta\sqrt{(E_l-\xi\mu)^2 + 4|\Delta|^2}}\right)\right]\right\}. \end{aligned} \tag{11}$$

In the following Section, we illustrate the numerical calculation of the points of the global minimum of the thermodynamic potential and, with the use of them, consider phase transitions in the Einstein universe.

3.1 Phase transitions

We have fixed the constant G_2, similarly to what has been done in the flat case [6], by using the relation $G_2 = \frac{3}{8}G_1$. For numerical estimates, the constant $G_1 = 10$ has been chosen in such a way that the chiral and/or color symmetries were completely broken. Moreover, we have limited ourselves to the investigation of the case $D = 4$ only.

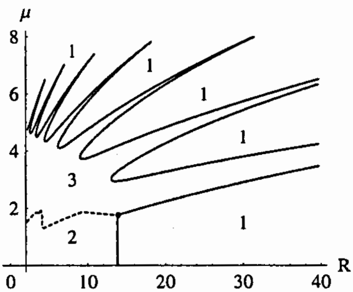

Figure 1: The phase portrait at T=0 for $G_1 = 10$. Dashed (solid) lines denote first (second) order phase transitions. The bold point denotes a tricritical point [10].

In Fig. 1, the $\mu - R$-phase portrait of the system at zero temperature is depicted. For points in the symmetric phase 1, the global minimum of the thermodynamic potential is at $\sigma = 0$, $\Delta = 0$ (chiral and color symmetries are unbroken). In the region of phase 2, only chiral symmetry is broken and $\sigma \neq 0$, $\Delta = 0$. The points in phase 3 correspond to the formation of the diquark condensate (color superconductivity) and the minimum takes place at $\sigma = 0$, $\Delta \neq 0$.

Notice that the oscillation effect, clearly visible in the phase curves of Fig. 1, may be explained by the discreteness of the fermion energy levels (9) in the compact space. This effect may be compared to the similar effect in the magnetic field H, where fermion levels are also discrete (the Landau levels).

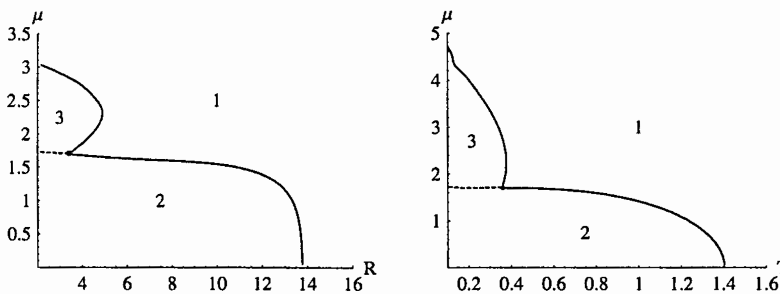

Figure 2: The phase portraits at T=0.35 (left picture) and at R=3 (right picture), $G_1 = 10$.

In addition, we considered also phase transitions at finite temperatures. In Fig. 2, $R - \mu$- and $T - \mu$- phase portraits are depicted. It is clear from Fig.2 that with growing temperature both the chiral and color symmetries are restored. The similarity of plots in $R - \mu$ and $T - \mu$ axes leads one to the conclusion that the parameters of curvature R and temperature T play similar roles in restoring the symmetries of the system.

4 Pion condensation

Suppose that dense, isotopically asymmetric quark matter (in this case the densities of u and d quarks are different) in curved spacetime is described by an extended NJL model with the following Lagrangian [11]:

$$\mathcal{L}_q = \bar{q}\Big[i\gamma^\nu\nabla_\nu + \mu\gamma^0 + \delta\mu\tau_3\gamma^0\Big]q + G\Big[(\bar{q}q)^2 + (\bar{q}i\gamma^5\vec{\tau}q)^2\Big], \tag{12}$$

where μ is the quark chemical potential and $\delta\mu$ is the isospin chemical potential. At $\delta\mu = 0$ the Lagrangian (12) is invariant under chiral $SU_L(2) \times SU_R(2)$ transformations. However, at $\delta\mu \neq 0$ this symmetry is reduced to the subgroup $U_{I_3}(1) \times U_{AI_3}(1)$, where $U_{I_3}(1)$ and $U_{AI_3}(1)$ are the isospin and axial isospin subgroup of the chiral $SU_L(2) \times SU_R(2)$ group.

As in the previous part, we introduce composite meson fields

$$\sigma(x) = -2G(\bar{q}q); \quad \pi_k(x) = -2G(\bar{q}i\gamma^5\tau_k q). \tag{13}$$

In the mean field approximation the fields σ and π_k do not depend on coordinates and may be replaced by their vacuum expectation values.

If $\langle\sigma\rangle \neq 0$, the axial (chiral) isospin symmetry $U_{AI_3}(1)$ is dynamically broken, and if $\langle\pi_3\rangle \neq 0$, the isospin symmetry $U_{I_3}(1)$ is broken. (For simplicity, we chose $\langle\pi_1\rangle = \langle\pi_2\rangle = 0$.) For convenience, we denote $\langle\sigma\rangle, \langle\pi_3\rangle \neq 0$ by letters σ and Δ.

From now on, we will consider only the case of nonzero isospin chemical potential $\delta\mu \neq 0$, whereas the baryon chemical potential is set equal to zero, $\mu = 0$. (Its presence is not of principle importance for us.) So, using the same technic as previously, we obtain the expression for the thermodynamic potential:

$$\begin{aligned}\Omega(\sigma,\Delta) = \frac{\sigma^2+\Delta^2}{4G} - \frac{3}{V}\sum_{l=0}^{\infty}\sum_{\xi=\pm} d_l \Big\{ & \sqrt{(E_l - \xi\delta\mu)^2 + 4|\Delta|^2} \\ & + 2T\ln\Big(1 + e^{-\beta\sqrt{(E_l-\xi\delta\mu)^2+4|\Delta|^2}}\Big)\Big\}.\end{aligned} \tag{14}$$

4.1 Numerical calculations

In the following we consider only the case of zero temperature. The detailed numerical investigation of the global minimum point of thermodynamic potential (14) shows that in the wide range of parameters only the pion condensate Δ is nonzero. The behavior of the pion condensate Δ as function of curvature and isospin chemical potential is shown in Fig.3 [11].

As in the case of quark and diquark condensation, the oscillating behavior of the pion condensate may be explained by the discreteness of fermion energy levels (9) in the compact space.

5 Conclusion

We have considered the chiral, CSC and pion condensate phase transitions of quark matter in the static Einstein universe at finite temperature, curvature, chemical potential and

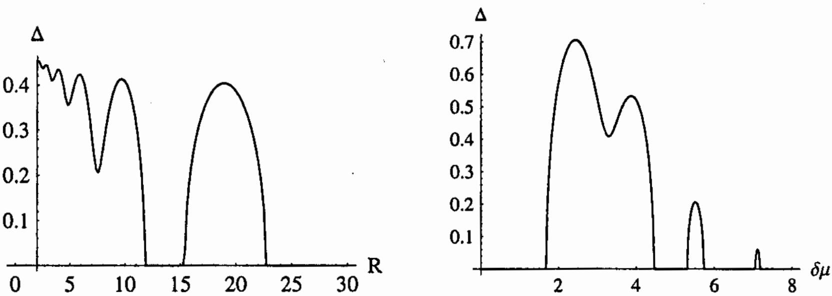

Figure 3: The behavior of the pion condensate Δ. Left picture: $T = 0, \delta\mu = 4.5$. Right picture: $R = 15, T = 0$. $G = 1$ [11].

isospin chemical potential. In spite of the model-theoretical character of this work, we hope that such kind of studies may be applicable to the physics of compact stars or the evolution of the universe.

One of us (D.E.) thanks the colleagues of the Bogoliubov Laboratory for Theoretical Physics of JINR Dubna for kind hospitality and the Bundesministerium für Bildung und Forschung for financial support. This work has also been supported in part by the Deutsche Forschungsgemeinschaft under grant 436 RUS 113/477.

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Classical and quantum equations of motion of spin for particles in nonstatic spacetimes

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Abstract

Relativistic equation describing the motion of classical spin in curved spacetimes is obtained. Classical and quantum equations of motion of spin in the Lense-Thirring (LT) metric are derived.

1 Introduction

The first relativistic equations of motion of spin in curved spacetimes have been obtained by Mathisson [1]. Another approach to the description of dynamics of spin has been developed by Pomeransky and Khriplovich [2]. These two approaches agree in the pole-dipole approximation [3] but differ in the general case [2, 4].

In Ref. [2], the exact relativistic equation defining dynamics of three-component spin in curved spacetimes has been obtained. This equation perfectly describes the dynamics of spin in static spacetimes. However, the Pomeransky-Khriplovich (PK) equation [2] can be used for nonstatic spacetimes only with tetrads satisfying the condition [3] $e^{\hat{0}}_{\hat{i}} = 0$ (hats denote tetrad indices). The symmetric tetrad utilized in Ref. [2] does not satisfy this condition. All calculations of the angular velocity of spin precession carried our in Refs. [2, 4] should be corrected on the Thomas precession [3].

We derive the term to be added to the PK equation in order to make it useful for arbitrary tetrads. We obtain classical and quantum relativistic formulae describing the LT effect [5] which is one of the most exiting predictions of the general relativity.

2 Spin effects in classical and quantum gravity

The correspondence principle formulated by Niels Bohr predicts a similarity between the description of spin effects in classical and quantum gravity. An important property of spin interactions with curved spacetimes is the absence of the anomalous gravitomagnetic moment (AGM) and the gravitoelectric dipole moment [6]. The relations obtained by Kobzarev and Okun [6] lead to equal frequencies of precession of classical and quantum spins in curved spacetimes. As a result, the behavior of classical and quantum spins in curved spacetimes is the same and any quantum effects cannot appear. This is a manifestation of the equivalence principle [7]. In Ref. [8], the earlier experimental data [9] have been reconsidered and the first experimental bound on the AGM have been stated. The fact that dynamics of classical and quantum spins in curved spasetimes is identical has also been proved in Refs. [8, 10]. The full agreement between classical equations of motion of momentum and spin and corresponding quantum equations obtained from solution of the Dirac equation has been established for static gravitational fields and noninertial frames.

Another manifestation of the equivalence principle is the helicity evolution. While the motion of momentum and spin differs in static gravitational fields and uniformly accelerated frames, the helicity evolution is the same [10].

An important problem is dynamics of particles and their spins in nonstatic spacetimes. A similarity of classical and quantum formulae has been shown for rotating frames [8]. A rotation of a central body defining a difference between stationary and static spacetimes leads to an appearance of specific gravitational effects. One of the most important effect has been predicted by Lense and Thirring [5]. This effect consists in frame dragging around rotating bodies and manifests in a precession of satellite orbits and gyroscopes (e.g. classical spins). The nonrelativistic formula for the latter effect has been derived by Schiff [11].

In the present work, we use the weak-field approximation when all components of the metric tensor $g_{\mu\nu}$ are close to the corresponding components of the Minkowski tensor $\eta_{\mu\nu}$ ($|h_{\mu\nu}| \equiv |g_{\mu\nu} - \eta_{\mu\nu}| \ll 1$). The classical equation of motion of spin in the LT metric differs from the previous result [11] by an extension to the relativistic problem. The corresponding quantum equation deduced from the Dirac one is obtained for the first time.

3 Classical equation of motion of spin

The three-component spin is defined in a flat spacetime corresponding to the particle at rest. As a result, the three-component spin $\boldsymbol{S}$ is a tetrad vector. In Ref. [2], an analogy between the motion of spin in electromagnetic and gravitational fields has been used. The derivation has implicitly been based on the admission that the condition $u^{\hat{i}} = 0$ defines the particle rest frame. This admission is not correct in the general case [3]. The corrected equation of motion of spin obtained in the weak field approximation is given by

$$\frac{d\boldsymbol{S}}{dt} = \boldsymbol{\Omega} \times \boldsymbol{S}, \quad \Omega_i = c e_{ikl} \left[\frac{1}{2} \gamma_{klc} \frac{u^c}{u^{\hat{0}}} + \frac{u^{\hat{k}}}{u^{\hat{0}} + 1} \left(\gamma_{0lc} \frac{u^c}{u^{\hat{0}}} + \frac{d e^{0}_{\hat{l}}}{dt} \right) \right], \tag{1}$$

where e^a_μ is the vierbein, e_{ikl} is the antisymmetric tensor, and $\gamma_{abc} = e_{a\mu;\nu} e^\mu_b e^\nu_c = -\gamma_{bac}$ are the Ricci rotation coefficients. We reserve first Latin letters for tetrad indices and denote spatial indices by Latin letters. Spatial and zero tetrad indices are distinguished by hats omitted for the antisymmetric tensor and the Ricci rotation coefficients. When a tetrad satisfies the condition $e^{0}_{\hat{i}} = 0$, Eq. (1) is exact and coincides with the PK equation [2].

4 Classical and quantum description of motion of spin in the Lense-Thirring metric

The motion of spin in the LT metric [5] characterizing a gravitational field of a rotating spherical source has previously been calculated in Ref. [2] with the PK equation. The use of Eq. (1) leads to the different equation for the angular velocity of the spin precession:

$$\boldsymbol{\Omega} = \boldsymbol{\Omega}^{(1)} + \boldsymbol{\Omega}^{(2)}, \tag{2}$$

where $\boldsymbol{\Omega}^{(1)}$ is the angular velocity of spin rotation in the static gravitational field found in Refs. [10] [Eq. (36)]. The newly obtained contribution from the LT effect reads

$$\boldsymbol{\Omega}^{(2)} = \frac{G}{c^2r^3}\left[\frac{3(\boldsymbol{r}\cdot\boldsymbol{J})\boldsymbol{r}}{r^2} - \boldsymbol{J}\right] - \frac{3G}{r^5\epsilon(\epsilon+mc^2)}\left[\boldsymbol{l}(\boldsymbol{l}\cdot\boldsymbol{J}) + (\boldsymbol{r}\cdot\boldsymbol{p})(\boldsymbol{p}\times(\boldsymbol{r}\times\boldsymbol{J}))\right], \qquad (3)$$

where $\boldsymbol{J}$ is the intrinsic angular moment of the source, $\boldsymbol{l} = \boldsymbol{r}\times\boldsymbol{p}$, and $\epsilon = \sqrt{m^2c^4 + c^2p^2}$.

The quantum description is based on the solution of the Dirac equation. We derive the Dirac Hamiltonian and perform the Foldy-Wouthuysen (FW) transformation by the method developed in Ref. [12]. In the weak field approximation

$$\mathcal{H} = \mathcal{H}_{FW}^{(1)} + \mathcal{H}_{FW}^{(2)}, \quad \mathcal{H}_{FW}^{(2)} = \frac{2G}{c^2r^3}\boldsymbol{J}\cdot\boldsymbol{l} + \frac{\hbar G}{2c^2r^3}\left[\frac{3(\boldsymbol{r}\cdot\boldsymbol{J})(\boldsymbol{r}\cdot\boldsymbol{\Sigma})}{r^2} - \boldsymbol{J}\cdot\boldsymbol{\Sigma}\right]$$
$$-\frac{3\hbar G}{8}\left\{\frac{1}{\epsilon(\epsilon+mc^2)}, \left[\frac{2\{(\boldsymbol{J}\cdot\boldsymbol{l}),(\boldsymbol{\Sigma}\cdot\boldsymbol{l})\}}{r^5} + \frac{1}{2}\left\{(\boldsymbol{\Sigma}\cdot(\boldsymbol{p}\times\boldsymbol{l}) - \boldsymbol{\Sigma}\cdot(\boldsymbol{l}\times\boldsymbol{p})), \frac{(\boldsymbol{r}\cdot\boldsymbol{J})}{r^5}\right\}\right.\right. \qquad (4)$$
$$\left.\left. + \left\{\boldsymbol{\Sigma}\cdot(\boldsymbol{p}\times(\boldsymbol{p}\times\boldsymbol{J})), \frac{1}{r^3}\right\}\right]\right\} - \frac{3\hbar^2c^2G}{8}\left\{(5p_r^2 - \boldsymbol{p}^2)\frac{2\epsilon^2 + \epsilon mc^2 + m^2c^4}{\epsilon^4(\epsilon+mc^2)^2}, \frac{(\boldsymbol{J}\cdot\boldsymbol{l})}{r^5}\right\},$$

where $\boldsymbol{p} = -i\hbar\nabla$. The operator $\mathcal{H}_{FW}^{(1)}$ has been calculated in Ref. [10] [Eq. (28)]. The equation of spin rotation obtained via commuting the FW Hamiltonian with the spin operator is in full agreement with Eq. (3) and significantly differs from the corresponding PK equation.

This work was supported in part by the BRFFR, Deutsche Forschungsgemeinschaft, RFBR, and Russian Federation Ministry of Education and Science.

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Born-Infeld Theory as low-energy limit of String Theory: from cosmology to hadron physics

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Abstract

Resent developments in the Random Matrix and Random Lattice Theories give a possibility to find low-energy theorems for many physical models from cosmology to hadron physics in the Born-Infeld form. In our approach that based on the Random Lattice regularization of QCD we try to used the similar ideas in the low-energy baryon physics for finding of the low-energy theory for the chiral fields in the strong-coupling regime.

The aim of this paper is to derive the effective lagrangian from QCD on the lattice at the strong coupling limit. We find that this theory looks like a Born-Infeld theory for the prototype chiral lagrangian. Such form of the effective lagrangian is expected. From the methodological point of view our consideration is very similar with the low-energy theorem in string theory that lead to the Born-Infeld action [1]. Moreover, in [2], it was shown that Chiral Born-Infeld Theory (without logarithmic corrections) has very interesting "bag"-like solution for chiral fields. It was additional motivation of our work.

Why we need in the Random Lattice? The attempts to obtain a chiral effective lagrangian from field theory on the lattice had been performed many times a long ago. Using of the well-known Brezin&Gross trick [4] it could be possible to perform the link's matrix integration in strong coupling regime and obtain the various first order effective theories [5].

In spite of first great success this approach had not been very popular and origin of this stems from the fact that this approach is a very general method for different branches of theoretical physics. For example, modifications of this method is a very popular in cosmology now because it give some theoretical predictions for forms of cosmological actions that result from String theory.

In other hand, there is a some problem in direct realization such ideas in QCD. The lattice regularization of QCD breaks a rotational symmetry of the initial theory from the continues rotation group to a discrete group of rotations on fixed angles. And the lattice regularization approach gives the correct results only such tensors which are invariant by respect to this discrete group. In particular using the ordinary Hyper-Cubical (HC) lattice one can obtains the only first order effective theory and for corrections this method generates non-rotational (non-lorentz) invariant terms. For generating of the high-order effective field theories more symmetrical lattice must be considered. Fortunately this conception is known for a long time and is called the Random Lattice Approach [6].

A general steps of the algorithm of the chiral lagrangian derivation from the lattice QCD that was proposed in [7]. The starting point of our analysis is a standard lattice action with Willson fermions

$$Z = \int [DG][D\bar{\psi}][D\psi] \exp\{-S_{\rm pl}(G) - S_q(G,\bar{\psi},\psi) - S_J\}$$

where $S_{\rm pl}$ - plaquette gauge field term; link fermions term is $S_q = \sum_{x,\mu} Tr(\bar{A}_\mu(x)G_\mu(x) + G^+_\mu(x)A_\mu(x))$ and source term is $S_J = \sum_x J^\alpha_\beta(x)M^\beta_\alpha(x),\ \ M^\beta_\alpha = \frac{1}{N_c}\psi^{a,\beta}(x)\bar{\psi}_{a,\alpha}(x)$.

In order to realize the strong-coupling regime on the lattice let us consider the limit of the large coupling constant g ($g \to \infty$). Main result is that in such limit integral over the gauge field can be performed. Let us consider the leading order contribution in this strong-coupling expansion. The integrals over the gauge degrees of freedom can be calculated into the large N limit by using of the standard procedure [5] and result of these calculations is following

$$Z = \int [D\bar{\psi}][D\psi] \exp\{-N\sum_{x,\nu} \mathrm{Tr}[F(\lambda(x,\nu))] - S_J\} \tag{1}$$

where $\lambda_\nu = -M(x)P^-_\nu M(x+\nu)P^+_\nu$ and $F(\lambda) = \mathrm{Tr}[(1-\sqrt{1-\lambda})] - \mathrm{Tr}[\log(1-\frac{1}{2}\sqrt{1-\lambda})]$.

Now it would be very interesting to point out that the function $F(\lambda)$ has the typical form of the Born-Infeld action with first logarithmic correction.

Our next step is the integration over the fermion degrees of freedom in (1). Using the source technics it was shown [7] that integral (1) can be re-written into the form of the integral over the unitary bosons matrix M_x

$$Z = \int DM \exp S_{\rm eff}(M). \tag{2}$$

As a matter of principle, we already perform the transformation from the color lattice degrees of freedom (G and ψ) to the boson lattice degrees of freedom (M). Now our task is to realize the continuum limit of expression (2).

The nest step of our analysis correspond with the studying of the stationary points of the lattice action $S_{\rm eff}$. Fortunately this is very well studied task [8]. This problem is connected with well-known investigations of the critical behavior of the chiral field on the lattice and with the problem of the phase transformation on the lattice (for references see the issue [9]). In [7], it was shown that for our task the stationary point is $\hat{M}_0 = u_0\hat{I},\ u_0(m_q = 0, r = 1) = 1/4$. Now one can expressed $M(x)$ in terms of the pseudoscalar Goldstone bosons

$$M = u_0 \exp(i\pi_i\tau_i\gamma_5/f_\pi) = u_0[U(x)\frac{1+\gamma_5}{2} + U^+(x)\frac{1-\gamma_5}{2}]$$

and the effective action is given in the form of the Taylor expansion around this stationary point

$$S_{\rm eff}(U) = -N\sum_{k=1}^{\infty}\frac{F^{(k)}(\lambda_0)}{k!}\sum_{x,\nu}\mathrm{Tr}[(\lambda_\nu(x)-\lambda_0)^k] \tag{3}$$

Let us consider the expansion of the chiral field $U = \exp(i\pi_i\tau_i/f_\pi)$ on the lattice around point x (by respect to the small step of the lattice a), $U(x+\nu) = U(x) + a(\partial_\nu U(x)) + \frac{a^2}{2}(\partial^2_\nu U(x)) + \cdots$.

Expressions (3) are very essential because these are a simplest illustration of all aspects of the violation of the rotational symmetry on the lattice. For this moment we specially say nothing about the structure of our lattice.

The basic idea of the RL is the averaging over the big ensemble of various lattices with random distributions of sites and it possible to show that such averaging leads to the restoration of the rotational invariance. The basis of vectors in the CFL method is following $\nu = \nu^{\mu}_{ij} = e^{\mu}_{ij} s_{ij}/l_{ij}$ where s_{ij} is a volume of the corresponding 3-dimensional boundary surface of the Voronoi cells and $l_{ij} = |\vec{r}_i - \vec{r}_j|$ is the length of link. Using the summation formulas from [6] one get that after the averaging only pairs are survive

$$< \prod_a (e^a)_\nu >= \sum_{\text{pairings}} \prod_{\text{pairs}} < (e^a)_i (e^b)_i > \tag{4}$$

At other hand, the result (4) could be obtained by means of the following trick. For beginning let us consider a lattice with fixed position (for simplicity it possible to use the trivial HC lattice) in a flat space. Now let us consider small deformations of the geometry of this space ($\gamma_{ij} \to g_{ij}$). Using of this idea one can rewrite the problem of the random lattice averaging in the terms of the random surface. This is the standard quantum gravity task and using the methods of the Matrix Theory one can show that our result (4) is just the direct consequence of well-known Wick's Theorem about the pairings.

Substituting (4) into the (3) and collecting of all terms which depend on the power of the prototype lagrangian one obtain following expression for the effective chiral lagrangian

$$\mathcal{L}_{\text{eff}} \sim -\text{tr}\left[1 - \sqrt{1 - 1/\beta^2 \partial_\mu U \partial^\mu U^+}\right] - \text{tr}\left[\log(1 - \frac{1}{2}(1 - \sqrt{1 - 1/\beta^2 \partial_\mu U \partial^\mu U^+}))\right] + \cdots \tag{5}$$

where $\cdots$ are all another terms (in particular the Skyrme term) and β is a effective coupling constant that depend on the value of our stationary point u_0.

This work is partially supported by the Russian Federation President's Grant 195.2008.2.

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Hadronic Matter

Effects of mesonic correlations in the PNJL model [1]

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Abstract

The finite temperature chiral and deconfinement phase transitions are studied within a nonlocal chiral quark model coupled to the Polyakov loop. In contrast to previous investigations which were mainly restricted to the mean-field approximation, mesonic correlations are self-consistently included using a strict $1/N_c$ expansion scheme. The quark model parameters are refitted in vacuum taking into account mesonic corrections.

In the present contribution we want to discuss an effective model of low-energy QCD, capable of describing the chiral as well as the deconfinement transitions. In the spirit of the PNJL model [1] which generalizes the well-known Nambu–Jona-Lasinio (NJL) model for the chiral quark dynamics (for an early review see, e.g., Ref. [2]) by coupling it to the Polyakov loop, being an order parameter of the deconfinement transition, we generalize here a nonlocal chiral quark quark model by coupling it to the Polyakov loop [3]. To a large extent, this removes one of the most disturbing features of the NJL-type models, namely the peculiarities due to pressure contributions of unconfined quarks in the hadronic phase. After successfully removing the unphysical quark degrees of freedom from the confined phase, the PNJL model treated in mean-field approximation as in [1] does not contain *any* degree of freedom in this regime. Obviously, this is a rather poor description of the hadronic phase at finite temperature T where mesons are expected to become relevant. We suggest a $1/N_c$ improvement of the PNJL model [3] which is necessary to disentangle hadronic contributions in the vicinity of and below the chiral/deconfinement phase transition.

In our calculations we use a Φ-deriveable approach together with the $1/N_c$ expansion. The central quantity for our analysis is the thermodynamic potential per volume, which is related to the pressure of the system

$$\Omega(T) = \mathrm{Tr}\ln(S^{-1}) - \mathrm{Tr}(\Sigma S) + \Phi(S) + U(\Phi_{\mathrm{PL}}, \bar{\Phi}_{\mathrm{PL}}) - \Omega_0 \,, \qquad (1)$$

where the new element $U(\Phi_{\mathrm{PL}}, \bar{\Phi}_{\mathrm{PL}})$ is the Polyakov loop potential, for which we adopt the logarithmic form of [4]. where $S^{-1} = S_c^{-1} + \Sigma$ represents the full quark propagator, and quark self-energy is $\Sigma = \delta\Phi/\delta(S)$. The symbol Tr denotes a trace over all internal (Dirac-, flavor-, and color-) degrees of freedom as well as momentum and Matsubara frequencies. As usual, we have introduced a constant Ω_0, which is chosen such that $\Omega(0) = 0$.

[1]Work supported by the Heisenberg-Landau programme (M.B., A.E.R., M.K.V), by the grant of Russian President (A.E.R.), by BMBF (A.E.R) and by the Polish Ministry of Science and Higher Education (D.B.).

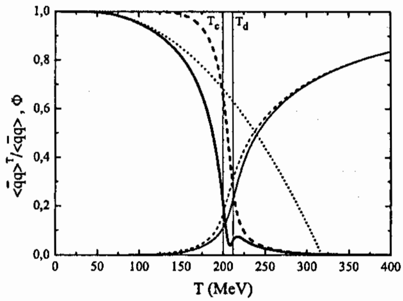

Figure 1: Quark condensate(thick) and Polyakov loop(thin) in the nonlocal model in mean field(dashed) and next order of $1/N_c$ expansion(solid). The chiral perturbation theory (ChPT) prediction for the quark condensate is shown as the dotted line.

The nonlocal four-point interaction is chosen in a separable form motivated by the instanton liquid model (ILM) where it results from the internal nonlocal structure of the nonperturbative QCD vacuum. In the ILM, the nonlocality is represented by the profile function of the quark zero-mode in the instanton field and depends on the gauge.

The mean-field NJL model can be obtained using "glasses" form of the Φ potential which takes the form

$$\Phi_{\text{glasses}} = -\sum_{M=\pi,\sigma} \frac{G}{2} \left[\text{Tr}(\Gamma_M S)\right]^2 , \tag{2}$$

where $\Gamma_\sigma = f(q_1)f(q_2)$ and $\Gamma_\pi = i\gamma_5\tau_a f(q_1)f(q_2)$ and q_1, q_2 are fermion momenta.

We use a Gaussian ansatz for the nonlocal profile functions, $f^2(p^2) = \exp(-p^2/\Lambda^2)$ in Euclidean momentum space, as one of the simplest functional forms of the nonlocality which has a similar behavior as the zero-mode profile obtained in a gauge invariant manner. This choice guarantees convergence at all orders without an additional regularization procedure.

The mean-field potential leads to a dynamical quark self-energy $\Sigma_{mf}(p^2) = -M(p^2)$. The dynamical quark mass function is $M(p^2) = m_c + m_d f^2(p^2)$, where the amplitude $m_d = -\sigma_{\text{mf}}$ is an order parameter for dynamical chiral symmetry breaking.

The next step in the $1/N_c$ expansion is to additionally take into account the "ring sum",

$$\Phi_{\text{ring}} = -\sum_{M=\pi,\sigma} \frac{d_M}{2} \text{Tr}\ln\left[1 - G\Pi_M\right] , \tag{3}$$

where d_M is the mesonic degeneracy factor, Π_M are the meson polarization operators, which in the $1/N_c$ expansion scheme are composed of the mean-field quark propagators.

Beyond the mean-field there are corrections to the dynamical quark mass (B) and the wave function renormalization (A) functions, $\Sigma_{N_c}(p^2) = \not{p}A(p^2) + B(p^2)$.

In Fig. 1 we show the resulting temperature dependence of the quark condensate $\langle\bar{q}q\rangle^T$ (normalized to its vacuum value) and of the Polyakov loop expectation value at mean-field level together with the strict $1/N_c$ scheme results. In the pure quark model the critical temperature for chiral restoration is $T_c = 90$ MeV, whereas the pure gauge sector has a critical

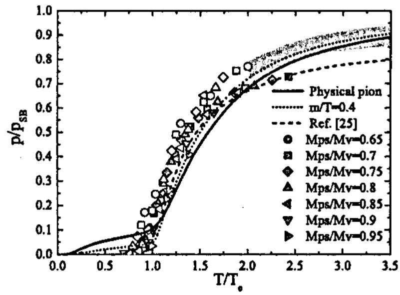

Figure 2: Scaled pressure p/p_{SB} as a function of T/T_c: non-local PNJL model with physical pion mass (solid line) and with $m_c/T = 0.4$ (dotted line). Dashed line: Lattice data for two-flavor QCD with staggered quarks [5]. The shaded region is an estimated continuum extrapolation of these data for massless QCD [5]. Points: Lattice data for two-flavor QCD with Wilson-type quarks [6] for $N_t = 6$ (open symbols) and $N_t = 4$ (filled symbols). The data for the pressure [6] have been divided by the Stefan-Boltzmann limit for $N_t = 6$ and $N_t = 4$, respectively, as given in [6].

temperature for deconfinement T_d = 270 MeV, fixed from lattice data for the Polyakov loop. When the quark and gluon sectors are coupled at mean-field these temperatures get synchronized so that $T_c \approx T_d \approx 200$ MeV. Beyond mean field the temperature of the chiral phase transition is slightly lowered, whereas the deconfinement temperature is increased. At low temperatures $T < 100$ MeV our results coincide with ChPT results. The dip near 200 MeV is connected with a breakdown of the $1/N_c$ expansion near the phase transition.

Our model predictions for the the pressure $p(T) = -\Omega(T)$, divided by the Stefan-Boltzmann limit, are displayed in Fig. 2. For comparison with the full result we also show the mean-field result and the mean-field plus pion contribution as well as the result for an ideal pion and sigma gas with the masses fixed at their vacuum values. We find that at low temperatures the mean-field (i.e., quark) contribution is suppressed and the pressure can be well described by a free pion gas. Near the critical temperature the σ meson gives an additional, visible contribution whereas already for $T > 1.5\ T_c$ the mesonic contributions are negligible and the quark-gluon mean-field dominates the pressure.

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On Finite Width of Quark Gluon Plasma Bags

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Abstract

Within an exactly solvable model I discuss an influence of the medium dependent finite width of QGP bags on their equation of state. It is shown that inclusion of such a width allows one to naturally resolve two conceptual problems of the QGP statistical description. On the basis of the proposed simple kinetic model for a sequential decay of heavy QGP bags formed in high energy elementary particle collisions it is argued that by measuring the energy dependence of life time of these bags it is possible to distinguish the case of critical point existence from the case of tricritical point.

1. Introduction. – A lot of experimental and theoretical efforts is aimed to determine the equation of state (EoS) of the strongly interacting matter. Despite the great achievements of these efforts [1] even the bulk properties of the quark-gluon plasma (QGP) EoS are not well known. Thus, such important characteristics as the mean volume and life time of QGP bags formed in heavy ion collisions have not caught a necessary attention yet. It is clear, however, that right these quantities may put some new bounds on the spacial and temporal properties of the QGP created in high energy collisions. As shown in [2] it is possible to naturally resolve the HBT puzzles at RHIC energies, if one assumes that the QGP consists of droplets of finite (mean) size. On the other hand the short life time of heavy QGP bags found recently within the Hagedorn-Mott resonance model [3] and within the finite width model (FWM) [4, 5] may not only play an important role in all thermodynamic and hydrodynamic phenomena of the strongly coupled QGP matter, but may also explain the absence of strangelets [6] or, more generally, why the finite QGP bags cannot be observed at energy densities typical for hadronic phase [4] (see below). Therefore, an investigation of the mass and volume distributions along with the life time of the QGP bags and the corresponding consequences for both the experimental observables and theoretical studies is vitally necessary for heavy ion phenomenology. The present paper is devoted to a discussion of these problems in the framework of the FWM.

2. The Finite Width Model. – The FWM employs the most convenient way to study the phase structure of any statistical model by analyzing its isobaric partition [7, 8, 9, 10] and to find the rightmost singularities of this partition. Hence, I assume that after the Laplace transform the FWM grand canonical partition $Z(V,T)$ generates the following isobaric partition:

$$\hat{Z}(\lambda,T) \equiv \int_0^\infty dV \exp(-\lambda V)\ Z(V,T) = \frac{1}{[\lambda - F(\lambda,T)]}, \tag{1}$$

where the function $F(\lambda,T)$ is a generalized partition

$$F(\lambda,T) = \int_0^\infty dv \int_0^\infty ds \int_0^\infty dm\ \rho(m,v,s) \exp(-\lambda v) \phi(T,m) \tag{2}$$

of bags of mass m, volume v and surface s defined by their mass-volume-surface spectrum $\rho(m,v,s)$. The partition (2) is a generalization of the statistical ensembles with fluctuating extensive quantities discussed recently in [11]. Note that one could also introduce in (2) the perimeter fluctuations, which may play an important role for small hadronic bubbles [12] or for cosmological phase transition [13], but we neglect it because the curvature term has not been seen in such well established models like the Fisher droplet model (FDM) [14, 15], the statistical multifragmentation model (SMM) [16, 17] and many other cluster systems discussed in [8, 9, 10, 15]. A special analysis of the free energy of 2- and 3-dimensional Ising clusters, using the Complement method [18], did not find any traces of the curvature term (see a detailed discussion in [9]). Such a result is directly related to the QGP bags because quantum chromodynamics (QCD) is expected to be in the same universality class [19] as the 3-dimensional Ising model whose clusters were analyzed in [18].

The thermal density of bags of mass m and a unit degeneracy is given by

$$\phi(T,m) \equiv \frac{1}{2\pi^2} \int\limits_0^\infty p^2 dp \, \exp\left[-\tfrac{(p^2\,+\,m^2)^{1/2}}{T}\right] = \frac{m^2 T}{2\pi^2} \, K_2\left(\tfrac{m}{T}\right). \qquad (3)$$

It is convenient to divide the mass-volume-surface spectrum into the discrete mass-volume spectrum of light hadrons and the continuum contribution of heavy resonances $\rho(m,v)$

$$\rho(m,v,s) = \sum_{j=1}^{J_m} g_j \, \delta(m-m_j)\delta(v-v_j)\,\delta(s) + \Theta(v-V_0)\Theta(m-M_0)\delta(s-a_s v^\kappa)\rho(m,v)\rho_1(s)\,, \quad (4)$$

The first term on the right hand side (r.h.s.) of (4) represents the contribution of a finite number of low-lying hadron states up to mass $M_0 \approx 2$ GeV [4]. This function has no λ-singularities at any temperature T and can generate only a simple pole of the isobaric partition, whereas the mass-volume spectrum of the bags $\rho(m,v)$ on the r.h.s of (4) is chosen to generate an essential singularity $\lambda_Q(T) \equiv p_Q(T)/T$ which defines the QGP pressure $p_Q(T)$. For simplicity here I consider the matter with zero baryonic charge.

The continuous part of the spectrum $\rho(m,v,s)$ introduced in [4] is parameterized as

$$\rho(m,v) = \frac{N_\Gamma}{\Gamma(v)\; m^{a+\frac{3}{2}}} \exp\left[\tfrac{m}{T_H} - \tfrac{(m-Bv)^2}{2\Gamma^2(v)}\right], \quad \text{and} \quad \rho_1\left(\tfrac{v^\kappa}{a_s}\right) = \tfrac{f(T)}{v^b} \, \exp\left[-\tfrac{\sigma(T)}{T}\, v^\kappa\right]. \quad (5)$$

As it is seen from (5) the mass spectrum $\rho(m,v)$ has a Hagedorn like parameterization and the Gaussian attenuation around the bag mass Bv (B is the mass density of a bag of a vanishing width) with the volume dependent Gaussian width $\Gamma(v)$ or width hereafter. I will distinguish it from the true width defined as $\Gamma_R = \alpha\,\Gamma(v)$ ($\alpha \equiv 2\sqrt{2\ln 2}$). It is necessary to stress that the Breit-Wigner attenuation of a resonance mass cannot be used in the spectrum (5) because in case of finite width it would lead to a divergency of the mass integral in (2) above T_H [4, 5].

The normalization factor obeys the condition $N_\Gamma^{-1} = \int\limits_{M_0}^\infty \frac{dm}{\Gamma(v)} \exp\left[-\frac{(m-Bv)^2}{2\Gamma^2(v)}\right]$. The constants $a > 0$ and $b > 0$ define the Fisher exponent $\tau \equiv a + b$ [4, 5] (also see later).

3. Important Features of the FWM Spectrum. – The spectrum in (5) contains the surface free energy ($\kappa = 2/3$) with the T-dependent surface tension which is parameterized as $\sigma(T) = \sigma_0 \cdot \left[\frac{T_c - T}{T_c}\right]^{2l+1}$ ($l = 0, 1, 2, ...$) [9, 20], where $\sigma_0 > 0$ can be a smooth function of temperature. For T not above the tricritical temperature T_c such a parameterization is justified by the usual cluster models like the FDM [14, 15] and SMM [16, 17], whereas the general case for any T can be derived from the surface partitions of the Hills and Dales model [20]. Note that the Hills and Dales model [20] explicitly accounts for all possible surface deformations which correspond to the same cluster volume and, therefore, it is another example of the statistical partition with fluctuating extensive quantity, which in this case is cluster surface.

In Ref. [9] it was rigorously proven that at low baryonic densities the first order deconfinement phase transition degenerates into a cross-over just because of negative surface tension coefficient for $T > T_c$. The other consequences of the present surface tension parameterization and the discussion of the absence of the curvature free energy in (5) can be found in Refs. [9, 18, 21].

The power $\kappa < 1$ which describes the bag's effective surface is a constant that, in principle, can differ from the typical FDM and SMM value $\kappa = \frac{2}{3}$ for which the coefficient a_s is $a_s \equiv (36\,\pi)^{\frac{1}{3}}$. This is so because near the deconfinement phase transition region the QGP has low density and, hence, like in the low density nuclear matter [22], the non-spherical bags (spaghetti-like or lasagna-like [22]) can be favorable (see also [12, 13] for the bubbles of complicated shapes). A similar idea of "polymerization" of gluonic quasiparticles was introduced recently [23].

Note that in contrast to the continuous part of the spectrum (4) its discrete part does not contain the surface free energy because according to the present days status of the statistical model of hadron gas this is not necessary [24].

The spectrum (5) has a simple form, but is rather general since both the width $\Gamma(v)$ and the bag's mass density B can be medium dependent. In [4] it is shown that the FWM has no contradiction, if $\Gamma(v) \equiv \Gamma_1 = \gamma v^{\frac{1}{2}}$ only ($\gamma = const$ of v).

For large bag volumes ($v \gg M_0/B > 0$) the normalization factor N_Γ can be found to be $N_\Gamma \approx 1/\sqrt{2\pi}$. Similarly, one can show that for heavy free bags ($m \gg BV_0$, $V_0 \approx 1$ fm^3 [4], ignoring the hard core repulsion and thermostat)

$$\rho(m) \;\equiv \int\limits_{V_0}^{\infty} dv\, \rho(m, v) \;\approx\; \frac{\rho_1(\frac{m}{B})}{B\;m^{a+\frac{3}{2}}} \exp\left[\frac{m}{T_H}\right]. \tag{6}$$

It originates in the fact that for heavy bags the Gaussian in (5) acts like a Dirac δ-function for Γ_1. Thus, the Hagedorn form of (6) has a clear physical meaning and, hence, it gives an additional argument in favor of the FWM.

Similarly to (6), it is possible to estimate the width of heavy free bags averaged over bag volumes and get $\overline{\Gamma(v)} \approx \Gamma_1(m/B) = \gamma\sqrt{m/B}$. Thus, the mass spectrum of heavy free QGP bags must be the Hagedorn-like one with the property that heavy resonances should have the large mean width because of which they would be hard to be observed.

The FWM allows one to express the pressure of large bags in terms of their most probable mass and width. Comparing the high and low temperature FWM pressures [4, 5] with the lattice QCD data [25, 26, 27], it was possible to estimate the minimal resonance width at zero temperature $\Gamma_R(V_0, T = 0) \approx 600$ MeV and the width at the Hagedorn temperature $\Gamma_R(V_0, T = T_H) = \sqrt{12}\,\Gamma_R(V_0, T = 0) \approx 2000$ MeV. It was also found that these values of the width are almost independent of the number of the lattice QCD elementary degrees of freedom [5]. Clearly, so large widths can naturally explain the huge deficit of the heavy hadronic resonances in the Particle Data Group compared to the exponential mass spectrum used to describe the QGP EoS. Applying the same line of arguments to the strangelets, I conclude that, if their mean volume is a few cubic fermis or larger, they should survive for a very short time. Such a conclusion is similar to the results of Ref. [6] predicting an instability of the strangelets.

Also it is remarkable that at the temperatures below the half of the Hagedorn one the QGP bag pressure of the FWM acquires the linear T dependence, i.e. $p^{-}(T < 0.5T_H) = -T\frac{B^2}{2\gamma^2}$, which is clearly seen in the recent lattice QCD data [27] in the range $T \in [202.5; 419.09]$ MeV [5].

As shown in [5] the relation between the resonance width and the mean mass of the FWM bags at high temperatures obeys the upper bound for the Regge trajectory asymptotic behavior found in [28], whereas a similar relation at low temperatures exactly corresponds to lower bound for the Regge trajectory asymptotic form [28].

4. The Life Time of a Protofireball. - The found FWM width values allow one to get the rough estimates of the life time of the protofireball suggested in [29] to explain the hadron multiplicities measured in the elementary particle collisions at high energies. The microcanoncal analysis of the thermostatic properties of heavy resonances with exponential mass spectrum [30, 21] teaches us two principal facts: first, even a single heavy resonance with the exponential mass spectrum imparts the Hagedorn temperature to any other hadron being in a thermal contact with it, and, second, the splitting of a single heavy resonance into several heavy pieces with mass above M_0 practically does not alter the latter conclusion. These two facts allow us to greatly simplify a treatment of the sequential decay of the heavy QGP bags formed in the elementary particle collisions at high energies. Indeed, the first fact allows one to consider the decay products with the mean kinetic energy which corresponds to the Hagedorn temperature (i.e. $3T_H$ for pions and $3/2\,T_H$ for heavier particles), and the second fact enables us to study the decay of several QGP bags independently of each other. Moreover, these both facts combined with the low particle densities formed in the elementary particle collisions allow one to neglect the treatment of daughter hadrons which may absorb on the decaying bags.

To simplify the problem I study the two particle decays only and consider the evolution of the heaviest QGP bag. The assumption of two particle decay is not too restrictive because it is possible to effectively account for three, four and more particle decays by representing them as the two particle sequential decays with shorter life-time. Therefore, in the rest frame of decaying bag of mass M the mean change of mass of the heaviest of two daughter particles of energies E_1 and E_2 is $\Delta M \approx M - \overline{\max(E_1, E_2)_M} = \overline{\min(E_1, E_2)_M}$, where the bar means the averaging over all possible combinations which obey the energy conservation. Then the mass evolution equation for the heaviest QGP bag can be cast as

$$\frac{\Delta M}{\Delta\, t} \approx -\,q\,\Gamma_M\,\overline{\min(E_1, E_2)_M}\,, \tag{7}$$

where $\Delta\, t$ is the time change, $\Gamma_M = \Gamma_R(V_0, T=0)\sqrt{\frac{M}{M_0}} \equiv \Gamma_0\sqrt{\frac{M}{M_0}} \approx 600 \cdot \sqrt{\frac{M}{M_0}}$ MeV is an average resonance width in the vacuum. Here the constant $q = \frac{\ln N}{\ln 2}$ accounts for the mean number of daughter particles $N > 2$ in a decay.

The mass distribution of the heaviest bag can be constructed from the auxiliary function

$$\Omega_M(E_1) \equiv N_\Omega\, \rho(E_1) \int\limits_{E_{min}}^{M} dE_2\, \rho(E_2)\, \delta\left(1 - \frac{E_1 + E_2}{M}\right), \tag{8}$$

where the density of states of hadrons of energy E is denoted as $\rho(E)$, the Dirac delta function accounts for the energy conservation, and the normalization factor N_Ω is given by

$$1 = \int_{E_{min}}^{M} dE_1 \, \Theta(E_1 - M/2) \, \Omega_M(E_1) \; + \int_{E_{min}}^{M} dE_2 \, \Theta(E_2 - M/2) \, \Omega_M(E_2) \, . \qquad (9)$$

Here the first (second) term corresponds to the fact that the fragment of energy E_1 (E_2) is the heaviest one, and $E_{min} \approx 3\,T_H$ is the minimal energy of the lightest fragment. Using (9), one finds an averaged minimal mass of the daughter fragment as

$$\overline{\min(E_1, E_2)_M} \approx \int_{M/2}^{M-E_{min}} dE_1 \, \rho(E_1) \, [M - E_1] \, \rho(M - E_1) \left[\int_{M/2}^{M-E_{min}} dE_1 \, \rho(E_1) \, \rho(M - E_1) \right]^{-1} . \quad (10)$$

In principle the density of states should be defined from the convolution of the spectrum (4) and Boltzmann density (3) with $T = T_H$. However, to get a simple analytic expression I employ just the Hagedorn mass spectrum and consider all particles nonrelativistically. This can be done because the experimental mass spectrum $\rho_e(m)$ for $m \leq M_0$ is well approximated by $\rho_e(m) \approx C \left[\frac{M_0}{m}\right]^{\tau+3/2} \exp(\frac{m}{T_H})$ [31]. The energy conservation $E = m + \frac{3}{2} T_H + \tilde{\sigma}_0 \, m^{\kappa}$ relates the mean energy of daughter hadron and its mass. Neglecting the surface energy, I get the resulting energy spectrum of the daughter hadron as $\rho(E) \approx C \left[\frac{M_0}{E}\right]^{\tau} \left[\frac{e\, T_H \, M_0}{2\pi}\right]^{\frac{3}{2}}$. As one can see, the leading exponentials cancelled each other and, thus, $\overline{\min(E_1, E_2)_M} \approx E_{min} \left[\frac{M}{E_{min}}\right]^{2-\tau} \left[\ln\left(\frac{M}{E_{min}}\right)\right]^{2\tau-3}$ for $\tau \geq 1$. Hence the life time of the protofireball of mass M_F created in the high energy collision of elementary particles is

$$t_{\tau \neq \frac{3}{2}}(M_F) \approx \frac{\Gamma_0^{-1}}{q\,(\tau - \frac{3}{2})} \sqrt{\frac{M_0}{E_{min}}} \left[\frac{E_{min}^{3/2-\tau}}{M_F^{3/2-\tau}} - \frac{E_{min}^{3/2-\tau}}{M_0^{3/2-\tau}} \right], \text{ and } t_{\tau=\frac{3}{2}}(M_F) \approx \frac{\Gamma_0^{-1}}{q} \ln\left[\frac{M_F}{M_0}\right] . \quad (11)$$

Since for $\frac{3}{2} < \tau \leq 2$ at nonzero baryonic chemical potentials the deconfinement transition is of the first order and for $\frac{4}{3} < \tau \leq \frac{3}{2}$ it degenerates to the second order and in either case there exists the tricritical point [9], whereas there are some arguments that for $\tau > 2$ there should exist the critical point with the first order deconfinement [8]. Therefore, I conclude that the measurements of the energy dependence of the life time (hadronization time) of the protofireballs created in the elementary particle collisions may allow us to distinguish the critical point existence from the tricritical point.

5. *Conclusions.* – Here I discuss some new ideas on the basic properties of the QGP EoS. The main attention is paid to the role of the medium dependent width of heavy QGP bags. Their large width explains a huge deficit of experimental mass spectrum of heavy hadronic resonances and enlightens some important thermodynamic aspects of the color confinement in finite systems. The proposed simple kinetic model for a sequential decay of heavy QGP bags formed in the elementary particle collisions at high energies allows one to distinguish the case of critical point existence from the tricritical one by measuring the energy dependence of the life time of these bags. Of course, the freeze-out process [32] may some what change these conclusions, but the simple kinetic arguments for the nucleus-nucleus collisions [33, 34] teach us that such changes are just a few per cent only and, hence, they cannot affect the result obtained.

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Mesons and diquarks in the CFL phase of dense quark matter

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Abstract

The spectrum of meson and diquark excitations of the color–flavor locked (CFL) phase of dense quark matter is considered in the framework of the Nambu - Jona-Lasinio model. We have found that in this phase all Nambu–Goldstone bosons are realized as scalar and pseudoscalar diquarks. Other diquark excitations are resonances with mass value around 230 MeV. Mesons are stable particles in the CFL phase. Their masses vs chemical potential lie in the interval 300÷500 MeV.

It is well-known that at asymptotically high baryon densities the ground state of massless three-flavor QCD corresponds to the so-called color – flavor locked (CFL) phase [1]. One of the most noticeable differences between color superconductivity phenomena with three and two quark species is that the CFL effect is characterized by a hierarchy of energy scales. Indeed, at the lowest scale of this phase lie NG bosons, which dominate in all physical processes with energies smaller than the superconducting gap Δ. Evident contributors at higher energy scales are quark quasiparticles, which in the CFL phase have an energy greater than Δ. However, up to now we know much less about other excitations, whose energy and mass are of the order of Δ in magnitude. Among these particles are ordinary scalar and pseudoscalar mesons, massive diquarks etc, i.e. particles which might play an essencial role in dynamical processes of the CFL phase. Here we are going to discuss just this type of excitations of the CFL ground state, i.e. mesons and massive diquarks, in the framework of the massless three-flavor NJL model with Lagrangian

$$L=\bar q\Big[\gamma^\nu i\partial_\nu+\mu\gamma^0\Big]q+G_1\sum_{a=0}^{8}\Big[(\bar q\tau_a q)^2+(\bar q i\gamma^5\tau_a q)^2\Big]+$$
$$G_2\sum_{A=2,5,7}\sum_{A'=2,5,7}\Big\{[\bar q^C i\gamma^5\tau_A\lambda_{A'}q][\bar q i\gamma^5\tau_A\lambda_{A'}q^C]+[\bar q^C\tau_A\lambda_{A'}q][\bar q\tau_A\lambda_{A'}q^C]\Big\}. \qquad (1)$$

In (1), $\mu\geq 0$ is the quark number chemical potential which is the same for all quark flavors, $q^C=C\bar q^t$, $\bar q^C=q^tC$ are charge-conjugated spinors, and $C=i\gamma^2\gamma^0$ is the charge conjugation matrix (the symbol t denotes the transposition operation). The quark field q is a flavor and color triplet as well as a four-component Dirac spinor. Furthermore, we use the notations τ_a,λ_a for Gell-Mann matrices in the flavor and color space, respectively ($a=1,...,8$); $\tau_0=\sqrt{\frac{2}{3}}\cdot 1_f$ is proportional to the unit matrix in the flavor space. Clearly, the Lagrangian (1) as a whole is invariant under transformations from the color group $SU(3)_c$. In addition, it is symmetric under the chiral group $SU(3)_L\times SU(3)_R$ as well as under the baryon-number conservation group $U(1)_B$ and the axial group $U(1)_A$. [1] In all numerical calculations below, we used the following values of the model parameters: $\Lambda = 602.3$ MeV, $G_1\Lambda^2 = 2.319$ and $G_2 = 3G_1/4$, where Λ is an ultraviolet cutoff parameter in the three-dimensional momentum space.

Introducing collective scalar $\sigma_a(x), \Delta^s_{AA'}(x)$ and pseudoscalar $\pi_a(x), \Delta^p_{AA'}(x)$ fields,

$$\begin{aligned}\sigma_a(x) &= -2G_1(\bar{q}\tau_a q), \quad \Delta^s_{AA'}(x) = -2G_2(\bar{q}^C i\gamma^5\tau_A\lambda_{A'}q),\\ \pi_a(x) &= -2G_1(\bar{q}i\gamma^5\tau_a q), \quad \Delta^p_{AA'}(x) = -2G_2(\bar{q}^C\tau_A\lambda_{A'}q),\end{aligned} \tag{2}$$

$(a = 0, 1, .., 8; A, A' = 2, 5, 7)$ and then integrating out quark fields from the theory, it is possible to obtain the following generating functional of the two-point one-particle irreducible (1PI) Green functions of the mesons and diquarks in the CFL phase

$$\begin{aligned}\mathcal{S}_{\text{eff}}(\sigma_a, \pi_a, \Delta^{s,p}_{AA'}, \Delta^{s,p*}_{AA'}) = -\int d^4x \left[\frac{\sigma_a^2 + \pi_a^2}{4G_1} + \frac{\Delta^s_{AA'}\Delta^{s*}_{AA'} + \Delta^p_{AA'}\Delta^{p*}_{AA'}}{4G_2}\right] + \\ \frac{i}{4}\text{Tr}\left\{S_0\begin{pmatrix}\Sigma, & K\\ K^*, & \Sigma^t\end{pmatrix}S_0\begin{pmatrix}\Sigma, & K\\ K^*, & \Sigma^t\end{pmatrix}\right\},\end{aligned} \tag{3}$$

where

$$\begin{aligned}\Sigma &= \tau_a\sigma_a + i\gamma^5\pi_a\tau_a, \qquad K = (\Delta^p_{AA'} + i\Delta^s_{AA'}\gamma^5)\tau_A\lambda_{A'},\\ \Sigma^t &= \tau_a^t\sigma_a + i\gamma^5\pi_a\tau_a^t, \qquad K^* = (\Delta^{p*}_{AA'} + i\Delta^{s*}_{AA'}\gamma^5)\tau_A\lambda_{A'},\end{aligned} \tag{4}$$

S_0 is the Nambu – Gorkov representation for the quark propagator in this phase,

$$S_0^{-1} = \begin{pmatrix} i\gamma^\nu\partial_\nu + \mu\gamma^0, & -i\Delta\gamma^5(\tau_2\lambda_2 + \tau_5\lambda_5 + \tau_7\lambda_7)\\ -i\Delta\gamma^5(\tau_2\lambda_2 + \tau_5\lambda_5 + \tau_7\lambda_7), & i\gamma^\nu\partial_\nu - \mu\gamma^0\end{pmatrix}, \tag{5}$$

and Δ is the gap parameter. Using the expression (3), one can find the 1PI Green functions for mesons and diquarks in the CFL phase, namely

$$\Gamma_{XY}(x-y) = -\frac{\delta^2\mathcal{S}^{(2)}_{\text{eff}}}{\delta Y(y)\delta X(x)}, \tag{6}$$

where $X(x), Y(x) = \sigma_a(x), \pi_b(x), \Delta^{s,p}_{AA'}(x), \Delta^{s,p*}_{BB'}(x)$. In total, the Green functions (6) form a 54×54 matrix which is, fortunately, a reducible one. It is well known that in the rest frame of the momentum space representation, i.e. at $p = (p_0, 0, 0, 0)$, the meson and diquark masses are the zeros of the determinant of this matrix. So, after tedious both analytical and numerical calculations (the details are presented in [2]) we have obtained the following results on the mass spectrum of the bosonic excitations of the CFL phase.

In the Figs 1,2 the mass behavior for the scalar and pseudoscalar mesons is presented in the CFL phase. It is easily seen that i) there is a mass splitting between octet and singlet of mesons, ii) the singlet mass is smaller than octet one for scalar mesons, but for pseudoscalar mesons the situation is inverse, iii) in the CFL phase the meson masses are greater than 300 MeV.

[1] In a more realistic case, the additional 't Hooft six-quark interaction term should be taken into account in order to break the axial $U(1)_A$ symmetry.

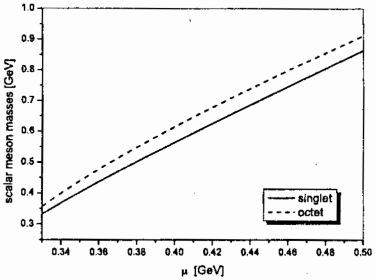

Figure 1: The behavior of the scalar meson masses vs μ in the CFL phase.

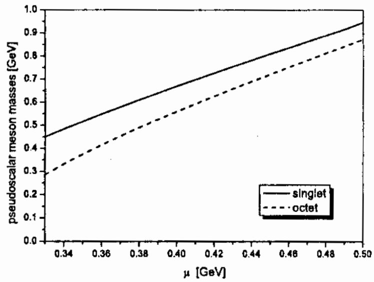

Figure 2: The behavior of the pseudoscalar meson masses in the CFL phase.

In the diquark sector we have found 18 Nambu-Goldstone bosons. Moreover, there are a scalar octet and singlet as well as pseudoscalar octet and singlet of nontrivial diquark excitations of the CFL phase. The diquarks from scalar and pseudoscalar octets are resonances with mass around 230 MeV. The properties of diquarks in the CFL phase were also considered earlier in the papers [2, 3].

For comparison, let us mention the existence of nontrivial excitations of the color superconducting (2SC) phase of quark matter with two quark flavors. In this phase the masses of σ and π mesons lie in the same interval 300÷500 MeV as the meson masses in the CFL phase. But the scalar diquark is a very heavy resonance with mass $\sim$ 1100 MeV [4]. If the electric charge neutrality constraint is imposed, then in the 2SC phase diquark is a stable particle with mass ~200 MeV [5].

We thank V.L. Yudichev for the fruitful cooperation over many years. One of us (D.E.) thanks A.E. Dorokhov and M.K. Volkov for useful discussions and the Bogoliubov Laboratory of Theoretical Physics for kind hospitality.

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Impact of Eight Quark Interactions on Chiral Phase Transitions II: Thermal Effects

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Abstract

In this talk attention is drawn to thermal properties due to the addition of eight quark interactions in the standard $SU(3) \times SU(3)$ chiral Nambu-Jona-Lasinio model (NJL) with 't Hooft interaction (NJLH). The schematic $SU(3)$ flavor limit with massless current quarks as well as the realistic case $m_u = m_d \neq m_s$ are discussed.

The extension of the NJLH model [1]-[4] to include $8q$ (quark) interactions [5] finds its ain motivation in the fact that it stabilizes the scalar effective potential of the theory. his has been discussed at length in [6],[7]. We show here that they play an important role the physics of chiral transitions at finite temperature [8]. Before discussing the thermal roperties, we give a summary of essential features of the model at $T = 0$, relevant for the resent discussion. One of the most attractive features of chiral multiquark interactions the possibility of analyzing patterns of dynamical breakdown of chiral symmetry. In ie original NJL model the symmetry is broken in the massless limit from a critical value the $4q$ interaction strength on, $G > G_{cr}$. The same happens if the $U_A(1)$ breaking $6q$ grangian of 't Hooft is added with strength κ; a realistic fit to the pseudoscalar and scalar ass spectra restricts κ to the role of a perturbative effect on the spontaneously broken hase. This changes radically if the $8q$ interactions are added, for which the most general in 0 non-derivative combination consists of a sum of two terms (with strengths g_1, g_2), one them ($\sim g_1$) violating the OZI-rule. Now, besides the former scenario, it is also possible to ave the Wigner-Weyl phase (i.e. for $G < G_{cr}$ value) in coexistence with another minimum, iduced by the higher multi-quark interactions. Realistic fits to low energy characteristics the mesons show that the second minimum is induced by the strength κ. To these vo possible symmetry breaking patterns one can assign the same meson mass spectra, except for the σ meson), due to an interplay of the $4q$ and $8q$ strengths. Turning now to ie finite temperature case: if symmetry breakdown is induced by the 't Hooft interaction rm, a substantial decrease of the transition temperature is achieved, bringing the model edictions closer to lattice results [9]. This can be easily understood with help of fig. 1. n the left side the effective potential at $T = 0$ shows coexistence of the Wigner-Weyl iase and symmetry broken phase, different curves correspond to different strength of κ. n the right the several curves of the effective potential represent different temperatures, lculated with model parameters fixed at $T = 0$ (dashed curve, $G > G_{cr}$). Increasing the mperature the effective potential runs through configurations of the type shown on the t, changing curvature at origin. Thus starting already at $T = 0$ from a configuration own on the left will obviously reduce the transition temperature of the chiral transition. e discuss now the effect of realistic current quark masses. The gap equations at $T = 0$ in

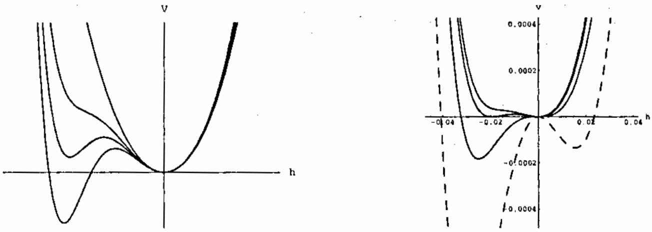

Figure 1: Left: Effective potential at $T = 0$ for different values of κ, axis through origin. Right: Effective potential at different T values, with model parameters fixed at $T = 0$ - dashed line. See also text. Figs. taken from [7],[8] respectively.

the isospin limit $m_u = m_d \neq m_s$ are

$$h_u + \frac{N_c}{6\pi^2} M_u \left(3I_0 - \Delta_{us} I_1\right) = 0, \qquad h_s + \frac{N_c}{6\pi^2} M_s \left(3I_0 + 2\Delta_{us} I_1\right) = 0. \tag{1}$$

This system must be solved selfconsistently with the stationary phase equations

$$\begin{cases} Gh_u + \Delta_u + \frac{\kappa}{16} h_u h_s + \frac{g_1}{4} h_u(2h_u^2 + h_s^2) + \frac{g_2}{2} h_u^3 = 0, \\ Gh_s + \Delta_s + \frac{\kappa}{16} h_u^2 + \frac{g_1}{4} h_s(2h_u^2 + h_s^2) + \frac{g_2}{2} h_s^3 = 0. \end{cases} \tag{2}$$

with $\Delta_{us} = M_u^2 - M_s^2$, $\Delta_l = M_l - m_l$, $h_l \sim$ condensates, $l = u, s$ for constituent quark masses M_l, $I_i = [2J_i(M_u^2) + J_i(M_s^2)]/3$, $i = 0, 1, \ldots$. The one-quark-loop integrals

$$J_i(M^2) = \int\limits_0^\infty \frac{dt}{t^{2-i}} \rho(t\Lambda^2) e^{-tM^2}, \qquad \rho(t\Lambda^2) = 1 - (1 + t\Lambda^2)\exp(-t\Lambda^2), \tag{3}$$

where $\rho(t\Lambda^2)$ denotes the Pauli-Villars regularization kernel with two subtractions and Λ is an ultra-violet cutoff.

To generalize to finite temperatures, the quark loop integrals J_0, J_1 are modified, introducing the Matsubara frequencies

$$J_0(M^2) \to J_0(M^2, T) = 16\pi^2 T \sum_{n=-\infty}^{\infty} \int \frac{d^3p}{(2\pi)^3} \int\limits_0^\infty ds\, \rho(s\Lambda^2) e^{-s[(2n+1)^2\pi^2T^2 + \vec{p}^2 + M^2]}. \tag{4}$$

Using the Poisson formula $\sum_{n=-\infty}^{\infty} F(n) = \sum_{m=-\infty}^{\infty} \int_{-\infty}^{+\infty} dx\, F(x) e^{i2\pi m x}$, where $F(n) = \exp[-s(2n+1)^2\pi^2T^2]$, and after integration over 3-momentum $\vec{p}$ one obtains

$$J_0(M^2, T) = \int\limits_0^\infty \frac{ds}{s^2} \rho(s\Lambda^2) e^{-sM^2} \left[1 + 2\sum_{n=1}^{\infty} (-1)^n \exp\left(\frac{-n^2}{4sT^2}\right)\right], \tag{5}$$

and $J_1(M^2, T) = -\frac{\partial}{\partial M^2} J_0(M^2, T)$. At $T = 0$ the mode $n = 0$ decouples from $T \neq 0$ modes $n > 0$. One recovers the covariant expression at $T = 0$ and $\lim_{T\to\infty} J_{0,1}(M^2, T) = 0$.

In Fig. 2 we display solutions $M_l(T)$, $l = u, s$ of (1)-(2) to the gap equations for the $m_u = m_d \neq m_s$ case, for the parameter sets of [8]. Depending on the strength g_1 of the $8q$ interactions we obtain either a crossover transition (left) or a first order transition (middle); units MeV. The three sets of M_u, M_s are shown as solid line, dotted line, dashed line, and start at $T = 0$ as minima, maxima and saddle points of the effective potential, respectively. On the right side (units GeV) we show the low pseudoscalar and scalar meson spectra for the crossover case obtained with parameters $G, \kappa, g_1, g_2, m_l, \Lambda$ independent on temperature.

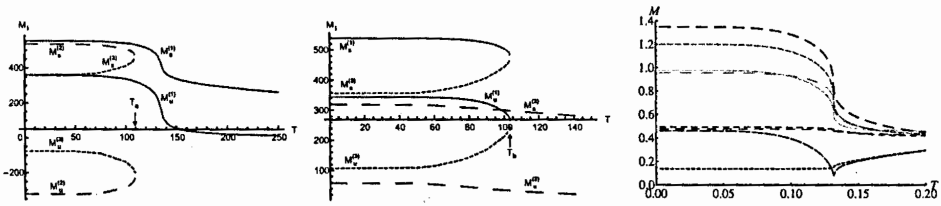

Figure 2: solutions to the gap equations for the light M_u and strange quarks M_s; left: crossover case (only one set of solutions is always positive valued), middle: 1st order transition (all sets of solutions are positive valued); right: pseudoscalar and scalar mass spectrum for the crossover case ($\pi, \sigma, \eta, K, \eta', K_0^*, a_0, f_0(980)$, bottom to top). Details: text and [8].

In conclusion, the chiral eight quark interactions, which cure the global instability of the $4q + 6q$ vacuum, play also a relevant role at finite T, acting as a chiral thermometer for chiral transitions. Temperature, slope and nature of transition are regulated by their strength g_1, which can be adjusted in consonance with G, with all other parameters frozen and leaving meson mass spectra at $T = 0$ unaffected, except for m_σ. Finally, they allow to start from configurations which at $T = 0$ have dynamical chiral symmetry breaking induced by the strength of κ of the 't Hooft $6q$ terms, which turn out to be more favorable to a reduction of the transition temperature, compared to the conventual $4q$ strength G.

This work has been supported in part by grants provided by FCT: POCI/2010, FEDER, POCI/FP/63930/2005, POCI/FP/81926/2007 and SFRH/BD/13528/2003.

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Angular Correlations in the Decays of $\Lambda\Lambda$ and $\Lambda\bar{\Lambda}$ Pairs Produced in Relativistic Heavy Ion Collisions

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Abstract

Spin correlations for the $\Lambda\Lambda$ and $\Lambda\bar{\Lambda}$ pairs, generated in relativistic heavy ion collisions, and related angular correlations at the joint registration of hadronic decays of two hyperons with nonconservation of space parity are analyzed. Within the conventional model of one-particle sources, correlations vanish at enough large relative momenta. However, under these conditions, in the case of two non-identical particles ($\Lambda\bar{\Lambda}$) a noticeable role is played by two-particle annihilation (two-quark, two-gluon) sources, which lead to the difference of the correlation tensor from zero. In particular, such a situation may arise when the system passes through the "mixed phase" .

Spin correlations for $\Lambda\Lambda$ and $\Lambda\bar{\Lambda}$ pairs, generated in relativistic heavy ion collisions, and respective angular correlations at joint registration of hadronic decays of two hyperons, in which space parity is not conserved, give important information on the character of multiple processes .

The spin density matrix of the $\Lambda\Lambda$ and $\Lambda\bar{\Lambda}$ pairs, just as the spin density matrix of two spin-1/2 particles in general, can be presented in the following form [1,2,3]:

$$\hat{\rho}^{(1,2)} = \frac{1}{4}\left[\hat{I}^{(1)}\otimes\hat{I}^{(2)} + (\hat{\boldsymbol{\sigma}}^{(1)}\mathbf{P}_1)\otimes\hat{I}^{(2)} + \hat{I}^{(1)}\otimes(\hat{\boldsymbol{\sigma}}^{(2)}\mathbf{P}_2) + \sum_{i=1}^{3}\sum_{k=1}^{3} T_{ik}\hat{\sigma}_i^{(1)}\otimes\hat{\sigma}_k^{(2)}\right]; \quad (1)$$

in doing so, $tr_{(1,2)}\hat{\rho}^{(1,2)} = 1$.

Here $\hat{I}$ is the two-row unit matrix, $\boldsymbol{\sigma} = (\ \hat{\sigma}_x\ \hat{\sigma}_y,\ \hat{\sigma}_z\ )$ is the vector Pauli operator $(x, y, z \to 1, 2, 3\ )$, $\mathbf{P}_1$ and $\mathbf{P}_2$ are the polarization vectors of first and second particle ($\mathbf{P}_1 = \langle\hat{\boldsymbol{\sigma}}^{(1)}\rangle$, $\mathbf{P}_2 = \langle\hat{\boldsymbol{\sigma}}^{(2)}\rangle$), $T_{ik} = \langle\hat{\sigma}_i^{(1)}\otimes\hat{\sigma}_k^{(2)}\rangle$ are the correlation tensor components . In the general case $T_{ik} \neq P_{1i}P_{2k}$. The tensor with components $C_{ik} = T_{ik} - P_{1i}P_{2k}$ describes the spin correlations of two particles .

It is essential that any decay of an unstable particle may serve as an analyzer of its spin state .

The normalized angular distribution at the decay $\Lambda \to p + \pi^-$ takes the form:

$$\frac{d\,w(\mathbf{n})}{d\,\Omega_{\mathbf{n}}} = \frac{1}{4\,\pi}(1 + \alpha_\Lambda \mathbf{P}_\Lambda\,\mathbf{n}) \quad (2)$$

Here $\mathbf{P}_\Lambda$ is the polarization vector of the Λ particle, $\mathbf{n}$ is the unit vector along the direction of proton momentum in the rest frame of the Λ particle, α_Λ is the coefficient of

P-odd angular asymmetry ($\alpha_\Lambda = 0.642$). The decay $\Lambda \to p + \pi^-$ selects the projections of spin of the Λ particle onto the direction of proton momentum; the analyzing power equals $\boldsymbol{\xi} = \alpha_\Lambda \mathbf{n}$.

Now let us consider the double angular distribution of flight directions for protons formed in the decays of two Λ particles into the channel $\Lambda \to p + \pi^-$, normalized by unity (the analyzing powers are $\boldsymbol{\xi}_1 = \alpha_\Lambda \mathbf{n}_1$, $\boldsymbol{\xi}_2 = \alpha_\Lambda \mathbf{n}_2$). It is described by the following formula [2,3]:

$$\frac{d^2 w(\mathbf{n}_1, \mathbf{n}_2)}{d\Omega_{\mathbf{n}_1}\, d\Omega_{\mathbf{n}_2}} = \frac{1}{16\pi^2}\left[1 + \alpha_\Lambda \mathbf{P}_1 \mathbf{n}_1 + \alpha_\Lambda \mathbf{P}_2 \mathbf{n}_2 + \alpha_\Lambda^2 \sum_{i=1}^{3}\sum_{k=1}^{3} T_{ik} n_{1i} n_{2k}\right], \quad (3)$$

where $\mathbf{P}_1$ and $\mathbf{P}_2$ are polarization vectors of the first and second Λ particle, T_{ik} are the correlation tensor components, $\mathbf{n}_1$ and $\mathbf{n}_2$ are unit vectors in the respective rest frames of the first and second Λ particle, defined in the common (unified) coordinate axes of the c.m. frame of the pair ($i, k = \{1, 2, 3\} = \{x, y, z\}$) .

The polarization parameters can be determined from the angular distribution of decay products by the method of moments [2,3] .

The angular correlation, integrated over all angles except the angle between the vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ and described by the formula [2,3,4,5]

$$d\,w(\cos\theta) = \frac{1}{2}\,(1 + \frac{1}{3}\,\alpha_\Lambda^2 T\,\cos\theta)\sin\theta d\,\theta = \frac{1}{2}\,[1 - \alpha_\Lambda^2\,(W_s - \frac{W_t}{3})\cos\theta]\sin\theta d\,\theta, \quad (4)$$

is determined only by the "trace" of the correlation tensor $T = W_t - 3W_s$ (W_s and W_t are relative fractions of the singlet state and triplet states, respectively), and it does not depend on the polarization vectors (single-particle states may be unpolarized).

Due to CP invariance, the coefficients of P-odd angular asymmetry for the decays $\Lambda \to p + \pi^-$ and $\bar\Lambda \to \bar p + \pi^+$ have equal absolute values and opposite signs: $\alpha_{\bar\Lambda} = -\alpha_\Lambda = -0.642$. The double angular distribution for this case is as follows [2,3]:

$$\frac{d^2 w(\mathbf{n}_1, \mathbf{n}_2)}{d\Omega_{\mathbf{n}_1}\, d\Omega_{\mathbf{n}_2}} = \frac{1}{16\pi^2}\left[1 + \alpha_\Lambda \mathbf{P}_\Lambda \mathbf{n}_1 - \alpha_\Lambda \mathbf{P}_{\bar\Lambda} \mathbf{n}_2 - \alpha_\Lambda^2 \sum_{i=1}^{3}\sum_{k=1}^{3} T_{ik} n_{1i} n_{2k}\right], \quad (5)$$

(here $-\alpha_\Lambda = +\alpha_{\bar\Lambda}$ and $-\alpha_\Lambda^2 = +\alpha_\Lambda \alpha_{\bar\Lambda}$) .

Thus, the angular correlation between the proton and antiproton momenta in the rest frames of the Λ and $\bar\Lambda$ particles is described by the expression:

$$d\,w(\cos\theta) = \frac{1}{2}\,(1 - \frac{1}{3}\,\alpha_\Lambda^2 T\,\cos\theta)\sin\theta d\,\theta = \frac{1}{2}\,[1 + \alpha_\Lambda^2\,(W_s - \frac{W_t}{3})\cos\theta]\sin\theta d\,\theta, \quad (6)$$

where θ is the angle between the proton and antiproton momenta .

Further we will use the model of one-particle sources [6], which is the most adequate one in the case of collisions of relativistic ions .

Two Λ particles are identical particles. Spin and angular correlations at their decays, taking into account Fermi statistics and final-state interaction, were considered previously in the works [2,7].

We will be interested in spin correlations at the decays of $\Lambda\bar{\Lambda}$ pairs . In the framework of the model of independent one-particle sources, spin correlations in the $\Lambda\bar{\Lambda}$ system arise only on account of the difference between the interaction in the final triplet state ($S = 1$) and the interaction in the final singlet state . At small relative momenta, the s-wave interaction plays the dominant role as before, but, contrary to the case of identical particles ($\Lambda\Lambda$), in the case of non-identical particles ($\Lambda\bar{\Lambda}$) the total spin may take both the values $S = 1$ and $S = 0$ at the orbital momentum $L = 0$. In doing so, the interference effect, connected with quantum statistics, is absent .

If the sources emit unpolarized particles, then, in the case under consideration, the correlation function describing momentum-energy correlations has the following structure (in the c.m. frame of the $\Lambda\bar{\Lambda}$ pair):

$$R(\mathbf{k},\mathbf{v}) = 1 + \frac{3}{4}\,B_t^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}) + \frac{1}{4}\,B_s^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}). \tag{7}$$

The spin density matrix of the $\Lambda\bar{\Lambda}$ pair is given by the formula :

$$\hat{\rho}^{(\Lambda\bar{\Lambda})} = \hat{I}^{(1)} \otimes \hat{I}^{(2)} + \frac{B_t^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}) - B_s^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v})}{4\,R(\mathbf{k},\mathbf{v})}\,\hat{\boldsymbol{\sigma}}^{(1)} \otimes \hat{\boldsymbol{\sigma}}^{(2)}, \tag{8}$$

and the components of the correlation tensor are as follows:

$$T_{ik} = \frac{B_t^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}) - B_s^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v})}{4 + 3\,B_t^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}) + B_s^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v})}\,\delta_{ik}; \tag{9}$$

here the contributions of final-state triplet and singlet $\Lambda\bar{\Lambda}$ interaction are determined by the expression obtained in the works [2,7] .

At sufficiently large values of $|k|$, one should expect that [7]:

$$B_s^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}) = 0, \qquad B_t^{(\Lambda\bar{\Lambda})}(\mathbf{k},\mathbf{v}) = 0.$$

In this case the angular correlations in the decays $\Lambda \to p + \pi^-$, $\bar{\Lambda} \to \bar{p} + \pi^+$, connected with the final-state interaction, are absent :

$$T_{ik} = 0, \qquad T = 0.$$

Thus, at sufficiently large relative momenta (for example, $|k| \gtrsim m_\pi$) one should expect that the angular correlations in the decays $\Lambda \to p + \pi^-$ and $\bar{\Lambda} \to \bar{p} + \pi^+$, connected with the interaction of the Λ and $\bar{\Lambda}$ hyperons in the final state (i.e. with one-particle sources) are absent . But, if at the considered energy the dynamical trajectory of the system passes through the so-called "mixed phase", then the two-particle sources, consisting of the free quark and antiquark , start playing a noticeable role . For example, the process $s\,\bar{s} \to \Lambda\bar{\Lambda}$ may be discussed .

In this process, the charge parity of the pairs $s\bar{s}$ and $\Lambda\bar{\Lambda}$ is equal to $C = (-1)^{L+S}$, where L is the orbital momentum and S is the total spin of the fermion and antifermion . Meantime, the CP parity of the fermion-antifermion pair is $CP = (-1)^{S+1}$.

In the case of one-gluon exchange, $CP = 1$, and then $S = 1$, i.e. the $\Lambda\bar{\Lambda}$ pair is generated in the triplet state; in doing so, the "trace" of the correlation tensor $T = 1$.

Even if the frames of one-gluon exchange are overstepped, the quarks s and $\bar{s}$, being ultrarelativistic, interact in the triplet state ($S = 1$) . In so doing, the primary CP parity $CP = 1$, and, due to the CP parity conservation, the $\Lambda\bar{\Lambda}$ pair is also produced in the triplet state. Let us denote the contribution of two-quark sources by x . Then at large relative momenta $T = x > 0$.

Apart from the two-quark sources, there are also two-gluon sources being able to play a comparable role. Analogously with the annihilation process $\gamma\gamma \to \Lambda\bar{\Lambda}$, in this case the trace of the correlation tensor is described by the formula (the process $g\,g \to \Lambda\bar{\Lambda}$ is implied):

$$T = 1 - \frac{4\,(1-\beta^2)}{1 + 2\,\beta^2 \sin^2\theta - \beta^4 - \beta^4 \sin^4\theta}, \tag{10}$$

where β is the velocity of Λ (and $\bar{\Lambda}$ in the c.m. frame of the $\Lambda\bar{\Lambda}$ pair, θ is the angle between the momenta of one of the gluons and Λ in the c.m. frame (see [8]). At small β ($\beta \ll 1$) the $\Lambda\bar{\Lambda}$ pair is produced in the singlet state (total spin $S = 0$, $T = -3$), whereas at $\beta \approx 1$ - in the triplet state ($S = 1$, $T = 1$). Let us remark that at ultrarelativistic velocities β (i.e. at extremely large relative momenta of Λ and $\bar{\Lambda}$) both the two-quark and two-gluon mechanisms lead to the triplet state of the $\Lambda\bar{\Lambda}$ pair ($T = 1$).

In the general case, the appearance of angular correlations in the decays $\Lambda \to p + \pi^-$ and $\bar{\Lambda} \to \bar{p} + \pi^+$ with the nonzero values of the "trace" of the correlation tensor T at large relative momenta of the Λ and $\bar{\Lambda}$ particles may testify to the passage of the system through the "mixed phase" .

So, it is advisable to investigate the spin correlations of $\Lambda\Lambda$ and $\Lambda\bar{\Lambda}$ pairs produced in relativistic heavy ion collisions .

The spin correlations are studied by the method of angular correlations – method of moments .

The spin correlations, as well as the momentum-energy ones, make it possible to determine the space-time characteristics of the generation region and, besides, the parameters of low-energy scattering of Λ on Λ and Λ on $\bar{\Lambda}$. They should be investigated jointly with the momentum-energy correlations .

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Thermal Effects for Quark and Gluon Distributions in Heavy-Ion Collisions

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Abstract

In-medium effects for distributions of quarks and gluons in central A+A collisions are considered. We suggest a duality principle, which means similarity of thermal spectra of hadrons produced in heavy-ion collisions and inclusive spectra which can be obtained within the dynamic quantum scattering theory. Within the suggested approach we show that the mean square of the transverse momentum for these partons grows and then saturates when the initial energy increases. It leads to the energy dependence of hadron transverse mass spectra which is similar to that observed in heavy ion collisions.

1 Introduction

Searching for a new physics in heavy-ion collisions at AGS, SPS and RHIC energies has led to intense theoretical and experimental activities in this field of research [1]. In this respect the search for signals of a possible transition of hadrons into the QCD predicted phase of deconfined quarks and gluons, quark-gluon plasma (QGP), is of particular interest. One of these signals can be the recent experimental observation of the transverse-mass spectra of kaons and pions from central Au+Au and Pb+Pb collisions which revealed "anomalous" dependence on the incident energy. The inverse slope parameter of the transverse mass distribution (the so called effective transverse temperature) at the mid-rapidity rather fast increases with incident energy in the AGS domain [2], then saturates at the SPS [3] and RHIC energies [4].

In this paper we would like to discuss the physical meaning of the so-called thermal spectra of hadrons produced in heavy-ion collisions, see for example [5, 6], and try to understand the dynamic reason of such inclusive spectra. Then we focus on a possible theoretical interpretation of the nontrivial energy dependence for the inverse slope parameter of the transverse mass spectra of mesons produced in central heavy-ion collisions.

2 Duality principle

According to many experimental data, inclusive spectra of hadrons produced in heavy-ion collisions can be fitted by the Fermi-Dirac distribution, corresponding to the thermodynamic equilibrium (TE) for the system of final hadrons, see for example [5, 6]

$$f_h^A = C_T^A \{\exp((\epsilon_h - \mu_h)/T) \pm 1\}^{-1}, \quad (1)$$

where + is for fermions and − is for bosons, ϵ_h and μ_h are the kinetic energy and the chemical potential of the hadron h, T is the temperature, C_T^{HE} is the normalization coefficient depending on T. Actually, the parameter T depends on the incident energy $\sqrt{s}$ in the $N-N$ c.m.s. For mesons simplifying this case we can assume that $\mu_h \simeq 0$, (in fact, it generally cannot be strictly zero [7]); then Eq.(1) is usually presented in the form

$$f_h^A \simeq C_T^A \exp(-\epsilon_h/T) . \tag{2}$$

On the other hand, according to the Regge theory and the $1/N$ expansion in QCD, the inclusive spectrum of hadrons produced, for example in $N-N$ collisions at high energies, has the scaling form, e.g., it depends only on M_X^2/s, where M_X is the missing mass of produced hadrons, s is the initial energy squared in the $N-N$ c.m.s., and $M_X^2/s = 1 - x_r$, where $x_r = 2E_h^*/\sqrt{s}$ is the radial Feynman variable, E_h^* is the energy of the hadron h in the $N-N$ c.m.s. For example, the quantum scattering theory and the fit of the experimental data for inclusive meson spectra at low x_r results in

$$\rho_m^{NN}(x_r) \sim C_N(1-x_r)_N^d \tag{3}$$

If $x_r << 1$, Eq.(3) can be presented in the exponential form

$$\rho_m^{NN} \sim C_N exp(-d_N x_r) \tag{4}$$

Inserting the form for x_r in Eq.(4), we get the inclusive spectrum of mesons in the form similar to that of the thermal spectrum given by Eq.(2)

$$\rho_m^{NN} = C_N \exp(-d_n x_r) \equiv C_N \exp(-\frac{E_h^*}{T_s^N}) , \tag{5}$$

where $T_s^N = \sqrt{s}/2d_N$. However, in contrast to Eq.(2), the form of the inclusive spectrum of mesons produced in $N-N$ collisions given by Eq.(5) does not assume introduction of any temperature of mesons like T. Figure 1 illustrates the approximate equivalence between ρ_m^{NN} given by Eq.(3) and ρ_m^{NN} given by Eq.(5). One can see from Fig.1 that at high energies these two forms for the meson spectrum are very similar to each other to the meson energies about a few GeV. Therefore, ρ_m^{NN} can be presented in the exponential form at low and even moderate energies E_h^*.

Now, let us assume that in heavy-ion reactions at high energies the $q-\bar{q}$ pairs like mesons produced in the first $N-N$ collision mainly in the central rapidity region, have the energy distribution like that in the $N-N$ reaction given by Eq.(5), e.g.,

$$f_m^{AA} = C_A \exp(-\frac{E_h^*}{T_s^A}) , \tag{6}$$

where $T_s^A = \sqrt{s}/2d_A$. In the general case, the parameter d_A is not the same as d_N which can be found from the quantum scattering theory or fitting the experimental data on inclusive spectra of mesons produced in $N-N$ collisions. Let us call the assumption corresponding to Eq.(6) the **Dynamic ansatz (DA)**. One can suggest the **duality principle** which is the similarity of thermal spectra given by Eq.(2) and the dynamical spectra given by Eq.(6).

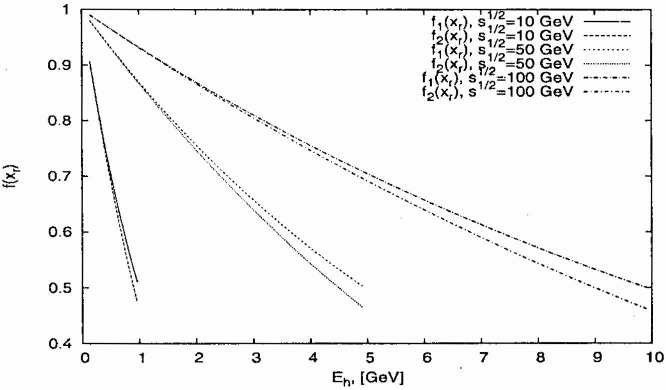

Figure 1: $f_1(x_r) = exp(-dx_r)$ and $f_2(x_r) = (1 - x_r)^d$ as functions of E_h.

3 Parton distribution in medium

Recently the parton distribution in a medium was analyzed on the assumption of the local thermodynamic equilibrium for quark objects like hadrons produced in heavy ion central collisions [8]. It was shown that, for example, the valence quark distribution in the quark object like the hadron h, which is in local thermodynamic equilibrium with surrounding nuclear matter, can be calculated by the following equation:

$$f_{q_v}^A(x, \mathbf{p}_t) = \int_0^1 dx_1 \int_0^1 dx_h \int d^2p_{1t} d^2p_{ht} q_v^h(x, \mathbf{p}_t) q_r^h(x_1, \mathbf{p}_{1t}) \times \tag{7}$$
$$f_h^A(x_h, \mathbf{p}_{ht}) \times \delta(x + x_1 - x_h)\delta^{(2)}(\mathbf{p}_t + \mathbf{p}_{1t} - \mathbf{p}_{ht}) ,$$

where $f_h^A(x_h, \mathbf{p}_{ht})$ is the distribution of quark objects like hadrons locally equilibrated in a medium (**LE**); q_v^h, q_r^h are the probabilities to find the valance quark and other partons (valence, sea quarks (antiquarks) and gluons) in h; x_1, x_h, x are the Feynman variables, $\mathbf{p}_t, \mathbf{p}_{1t}, \mathbf{p}_{ht}$ are the transverse momenta. The thermodynamic distribution like Eq.(2) was assumed in [8] for $f_h^A(x_h, \mathbf{p}_{ht})$. The same form for $f_{q_v}^A$ can be obtained suggesting the dynamic distribution (**DA**) for f_h^A given by Eq.(6) instead of Eq.(2). Assuming the factorized form for $f_q^h(x, p_t) = f_q(x) g_q(p_t)$ we have approximately [8] the following form for the mean transverse momentum squared of the valence quark in a medium:

$$< p_{q,t}^2(x \simeq 0) >_{q,appr.}^A \simeq \frac{< p_t^2 >_q^h + \tilde{T}\sqrt{m_h^2 + s/4}}{1 + \tilde{T}\sqrt{m_h^2 + s\ /\ 4}/(2 < p_t^2 >_q^h)} , \tag{8}$$

where $\tilde{T} = T$ for Eq.(2) (**LE**) and $\tilde{T} = T_s = \sqrt{s}/2d_A$ for Eq.(6) (**DA**). As is seen from Eq.(8) $< p_{q,t}^2 >_{q,appr.}^A$ grows when $\sqrt{s}$ increases and then saturates, its more careful calculation is presented in [8]. Note that in the **LE** case $\sqrt{s}$ is some scale energy which cannot be equal to the initial energy $\sqrt{s_{NN}}$ [8], whereas in the **DA** case it is the same as $\sqrt{s_{NN}}$. For mesons produced in central $A - A$ collisions we have similar broadening for the hadron p_t-spectrum

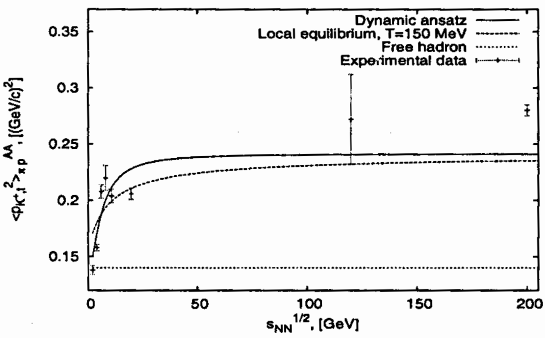

Figure 2: The mean transverse momentum squared of the K^+-meson produced in the central A-A collision.

[8]

$$< p_{h_1 t}^2 >_{appr.}^{AA} \simeq \frac{< p_t^2 >_q^m + \tilde{T}\sqrt{m_m^2 + s/4}}{1 + \tilde{T}\sqrt{m_m^2 + s/4} \; / \; (2 < p_t^2 >_q^m)} + \frac{< p_t^2 >_q^m}{r} , \tag{9}$$

where $< p_{h_1 t}^2 >$ is the mean value for the transverse momentum squared of the meson h_1 produced in the central heavy-ion collision, $< p_t^2 >_q^m$ is the same quantity for a quark in a medium, $r = \gamma_c/\gamma_q$, $\tilde{T} = T$ (**LE**) or $\tilde{T} = \sqrt{s}/2d_A$. Here γ_q and γ_c are the slopes in the Gauss form of the p_t dependence for the quark distribution in the hadron h and its fragmentation function, see details in [8]. As is seen from Eqs.(8,9), the saturation properties for $< p_{q,t}^2 >_q^A$ and $< p_{h_1 t}^2 >^{AA}$ at high $\sqrt{s}$ do not depend on the values of $\tilde{T}$, whereas the growth of these quantities at $\sqrt{s} \leq 20 - 30(GeV)$ is very sensitive to the value of d_A. To describe the experimental data on the transverse momentum squared of K-mesons produced in central $A - A$ collisions at $\sqrt{s} \leq 20 - 30(GeV)$ [2, 3] we took $d_A = (5 - 6) * d_N$.

In Fig.2 we present our estimation for the mean transverse momentum squared of the K^+-meson produced in the A-A collision. One can see from Fig.2 tat this quantity increases when the incident energy increases to the AGS energies and then saturates at higher energies. Note that this calculation is very approximate and we need to improve it including standard nuclear effects like rescattering and others. The suggested approach results in the saturation of the effective slope T_{eff} for the transverse mass spectrum of mesons produced in central heavy-ion collisions that is directly related to the quantity presented in Fig.2. In contrast to this the thermodynamic models predict the increase in T_{eff} when $\sqrt{s_{NN}}$ increases even to very high energies [9]. Therefore, the presented results can be verified by more careful measurements at the SPS energy and future experiments at the LHC.

4 Conclusion

We suggest the duality principle. Thermal spectra of hadrons produced in central A-A collisions can have a dynamical nature. Similar spectra can be obtained within the quantum scattering theory without introducing the temperature. One can assume that hadron jets consisting of colorless quark objects are produced in central A-A collisions. Then we get broadening for the mean transverse momentum squared of quarks in a medium respective to the incident energy. Similar effects can be obtained for transverse momentum spectra of mesons produced in central $A-A$ collisions. The mean transverse momentum squared of these mesons as a function of the incident energy grows and then saturates at high energies.

Acknowledgments
The authors are grateful to A.Andronic, P.Braun-Munzinger, A.V.Efremov, L.L.Frankfurt, M.Gazdzicki, S.B.Gerasimov, J.Cleymans, A.B.Kaidalov, H.Satz, Yu.Sinyukov and V.D.Toneev for very useful discussions. This work was supported in part by RFBR project No 08-02-01003 and by the special program of the Ministry of Education and Science of the Russian Federation (grant RNP.2.1.1.5409).

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Azimuthal Anisotropy and Fundamental Symmetries in QCD Matter at RHIC

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Abstract

A study of collective behavior in heavy ion collisions provides one of the most sensitive and promising probes for investigation of possible formation of new extreme state of strong interacting matter and elucidating its properties. Systematic of experimental results for final state azimuthal anisotropy is presented for heavy ion interactions at RHIC. Experimental data for azimuthal anisotropy indicate that the final state strongly interacting matter under extreme conditions behaves as near-ideal liquid rather, than ideal gas of quarks and gluons. The strong quenching of jets and the dramatic modification of jet-like azimuthal correlations, observed in $Au+Au$ collisions, are evidences of the extreme energy loss of partons traversing matter which contains a large density of color charges. For the first time, dependence of the jet suppression on orientation of a jet with respect to the reaction plane is found at RHIC experimentally. The model of compound collective flow and corresponding analytic approach are discussed. The possible violations of $\mathcal{P}$ and $\mathcal{CP}$ symmetries of strong interactions in heavy ion collisions at different initial energies are considered. Thus, now the fact is established firmly, that extremely hot and dense matter created in relativistic heavy ion collisions at RHIC differs dramatically from everything that was observed and investigated before.

Research a heavy ion interactions at high energies and search of a new state of strongly interacting matter at extremely high density and temperatures has essential interdisciplinary significance. Study of collective and correlation characteristics of interactions allows to obtain new and unique information concerning various stages of space-time evolution of collision process, to establish fundamental relation between geometry of collision and dynamics of final state formation. In heavy ion collisions, metastable vacuum domains may be formed in the QCD vacuum in the vicinity of the deconfinement phase transition in which fundamental symmetries ($\mathcal{P}$ and/or $\mathcal{CP}$) are spontaneously broken. Thus the study of nuclear collisions allows to investigate one of the most important problem of strong interaction theory. Relativistic Heavy Ion Collider (RHIC) of Brookhaven National Laboratory has started to run for physics 2000. RHIC is the world accelerating complex specially designed and intended entirely for researches in the field of the physics of strong interactions. The experimental base of RHIC facility consists of four detectors: small - BRAHMS, PHOBOS and large - PHENIX, STAR for physics programm with heavy ion beams. Now there are huge bases of experimental data with high statistics for Au+Au(Cu+Cu) at $\sqrt{s_{NN}} = 19.6(22.4) - 200$ GeV, for d+Au and p+p at $\sqrt{s_{NN}} = 200$ GeV. Also runs have been executed with small integrated luminosity for Au+Au at $\sqrt{s_{NN}} = 55.8$ GeV, for p+p at $\sqrt{s_{NN}} = 409.8$ GeV, and at low energies: $\sqrt{s_{NN}} = 22/9.2$ GeV - for p+p/Au+Au-interactions. Now experimental data are collected on the large RHIC detectors only.

1 Azimuthal anisotropy

One of the most essential features of non-central AA collisions is the violation of azimuthal symmetry of secondary particle distributions, caused by spatial asymmetry of area of overlapping of colliding heavy ions. The collective behaviour of secondary particles manifests itself both in one-particle p_T-spectra and in asymmetry of azimuthal particle distribution with respect to the reaction plane. The first effect is due to radial azimuthally symmetric expansion, the second effect is characterized by flow parameters $v_n = \langle \cos\left[n\left(\phi - \Psi_{RP}\right)\right]\rangle$.

The directed flow carries the information about very early stages of evolution of collision process. The systematic study of the directed flow v_1 has been executed in experiments at RHIC at various $\sqrt{s_{NN}}$ values [1 - 4]. Dependence $v_1\left(\eta(y) - y_b\right)$ is presented at Fig.1a at initial energy range $\sqrt{s_{NN}} = 8.8 - 200$ GeV for semi-central heavy ion collisions, where y_b - rapidity of beam particles. The results obtained at various RHIC energies agree with SPS data [5] in the region of fragmentation of a beam particle. The most of transport models underestimates of flow v_1 for central rapidity region and agrees with experimental results at the large values of $|\eta|$ more reasonably. The correlation between the first and second harmonics indicates on the development of elliptic flow in plane of event [1]. The recent results show that the directed flow depends on initial energy but not on the type of colliding nuclei [4].

It was observed, that the dependences $v_2\left(p_T\right)$, integrated v_2 and differential $v_2\left(p_T\right)$ parameters on particle mass (type) are described by phenomenological calculations on the basis of hydrodynamics up to $p_T \sim 2$ GeV/c well enough for different particles [6, 7]. The indicated above range of p_T contains $\sim 99\%$ of secondary particles. Thus, the global dynamic feature of the created matter is the collective behaviour described in the framework of relativistic hydrodynamical model at qualitative level. The increasing of v_2 and systematic decreasing of $v_2\left(p_T\right)$ with growth of particle masses is the additional and essential indication on presence of the common velocity fields. Dependence $v_2\left(\sqrt{s_{NN}}\right)$ is presented on the Fig.1b based on [8, 9]. One can see the $v_2\left(\sqrt{s_{NN}}\right)$ increases smoothly at $\sqrt{s_{NN}} > 5$ GeV. The flow energy dependence was fitted by function $a_0 + a_1\left[\sqrt{s_0}/\left(\sqrt{s_{NN}} - 2m_p\right)\right]^{a_2} + a_3\lambda^{a_4} + a_5\lambda^{a_6} + a_7\left[\ln\lambda\right]^{a_8}$ ($\lambda \equiv s_{NN}/s_0$, $s_0 = 1\text{GeV}^2$, m_p - proton mass) which is similar to that for σ_{tot}^{pp} approximation [10]. This function agrees with all available data reasonably, but the approximation with best statistical quality ($\chi^2/ndf = 3.83$) shows the decreasing in TeV energy range (Fig.1b, curve 1). There is some poor quality ($\chi^2/ndf = 4.83$) for curve 2 (Fig.1b) which shows a reasonable behaviour for all energy domain understudy. One can see the experimental data for both ultra-high energy range (LHC) and for intermediate energies $\sqrt{s_{NN}} = 5 - 50$ GeV (FAIR, NICA) are essential for more unambiguous approximation of $v_2\left(\sqrt{s_{NN}}\right)$. As seen, the PHOBOS point at $\sqrt{s_{NN}} = 19.6$ GeV agrees well with the results observed at close SPS energies. It is important to note, that at high RHIC energies the hydrodynamical limit for elliptic collective flow parameter is reached for the first time. The dependence of v_2 on centrality emphasizes additionally of importance of more exact knowledge for initial state in a nuclei-nuclear collisions for the correct description of final state matter evolution. Essential nonzero value and the basic dependences for v_2 are observed for various types of colliding (symmetric) heavy ion beams [11, 12]. Dependence $v_2(\eta)$ has been obtained at RHIC for various initial energies both for Au+Au, and for Cu+Cu collisions. The different phenomenological

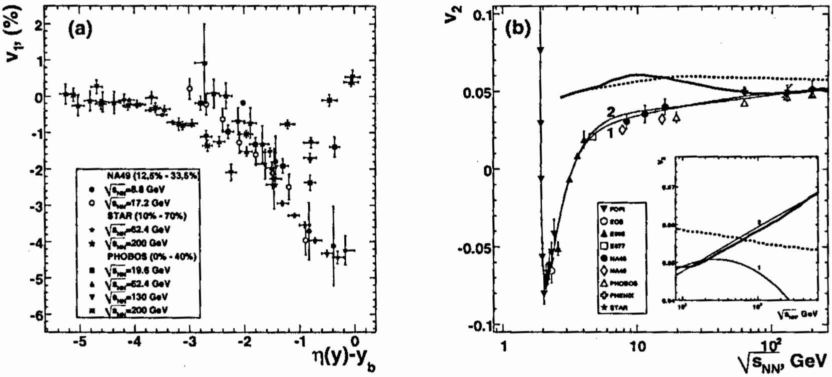

Figure 1: (a) Parameter v_1 for the charged particles at SPS (Pb+Pb) and RHIC (Au+Au) energies. Collision centralities are specified. (b) Energy dependence of v_2 parameter for midrapidity domain in heavy ion collisions. A thick solid curve corresponds to the hydrodynamical calculations for strongly interacting matter EOS with phase transition, a dashed curve – for the EOS of hadron gas without phase transition. Inner picture shows predictions up to the ultra-high energies (see text for more detail). Statistical errors are indicated only.

models describe the $v_2(\eta)$ at qualitative level. Essential excess for partonic cross-sections used in such models above pQCD predictions can indicates on significant non-perturbative effects in the partonic matter formed on the RHIC.

Experimental v_2 results, obtained both for charged hadrons and for the identified particles of various types, allow to make a choice in favour of the equation-of-state (EOS) of strongly interacting matter with presence a quark-gluon phase at early stages of space-time evolution of the formed medium with the subsequent transition in hadronic phase (Fig.1b). Moreover the hybrid model with jet quenching, unlike hydrodynamics, predicts saturation of v_2 in the intermediate p_T domain with the subsequent decreasing at higher transverse momentum and describes dependence $v_2\,(p_T)$ for $p_T > 2$ GeV/c at large density of gluons $dNg/dy \sim 10^3$ qualitatively [6]. This experimental result (together with other ones) is the direct evidence of hot and dense matter formation in which there are partonic hard scattering and finite energy loses of partons and products of their fragmentation at traversing of this medium. The comparative analysis of p+p, d+Au and Au+Au collisions at energy $\sqrt{s_{NN}} = 200$ GeV has demonstrated that there is a significant contribution from anisotropic (elliptic) flow namely in non-central nuclei-nuclear collisions up to, at least, $p_T \simeq 10$ GeV/c [13]. The large elliptic flow signal indicates on the one hand on the fast achieving of thermodynamic equilibrium at RHIC energies, and on the other hand - the medium contituents should interact with each other intensively enough at the early stages of space-time evolution of matter already, that more corresponds to conditions of a liquid, than gas.

At RHIC experimental results have been obtained for parameter of a collective elliptic flow for identified π^0-mesons and inclusive γ in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. The obtained experimental indication on small v_2 values for direct photons makes preferable the naive scenario of direct photon production in processes of the hard scattering, which occurs at the earliest stages of space-time evolution of the created matter [14]. The universal dependence of parameter v_2/n_q on normalized kinematic variables ($p_T/n_q, E_{kT}/n_q, n_q$ – constituent quark number) is observed for a wide set of secondary mesons and baryons [12, 15]. This scale behaviour is the experimental evidence of presence of an essential collective partonic flow and similar character of s-quark flow with elliptic flow of light u, d-quarks.

The experimental RHIC data for elliptic flow v_2 for light flavour particles assume that final state matter is characterized by small free path length of constituents in comparison with the sizes of system and value of $\eta_s/s \sim 0.1$ (η_s - shear viscosity, s - entropy density) is close to the bottom quantum limit for strong coupling systems in the energy domain $\sqrt{s_{NN}} \sim 100$ GeV. The elliptic anisotropy of the particles with heavy flavour quarks has been investigated in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. This result together with results for nuclear modification parameter $R_{AA}(p_T)$ is strong evidence in favour of the conclusion about strong coupling of heavy quarks with the final state matter [16]. Experimental values of v_2^{HF} are larger essentially of model predictions based on pQCD. The Langevin transport model with small relaxation times and / or small diffusion coefficients of heavy quarks K_d^{HQ} allows to obtain a reasonable agreement at a qualitative level between experimental and calculated flow values. Estimations for η_s/s, obtained in such way, are close to the bottom quantum limit also and these estimations agree with results for sector of light quarks. Thus, interpretation of experimental RHIC results for azimuthal anisotropy on the basis of hypothesis about creation at early stage (quasi)ideal partonic liquid is the most proved at present. The alternative approach to explanation of small η_s/s value is based on the presence of anomaly shear viscosity η_s^A, arising due to turbulence of the color magnetic and electric fields generated by expanding quark-gluon system [17].

The higher order even harmonics have been obtained in STAR experiment for charged particles in Au+Au collisions at initial energy $\sqrt{s_{NN}} = 200$ GeV. Values of even harmonics for charged particles, averaged over p_T and η ($|\eta| < 1.2$) for minimum bias events, are equal (%): $v_2 = 5.180 \pm 0.005$; $v_4 = 0.440 \pm 0.099$; $v_6 = 0.043 \pm 0.037$; $v_8 = -0.06 \pm 0.14$ [18].

Identification of jets in relativistic heavy ion interactions was carried out on a statistical basis so far. Two peaks are observed in experimental correlation functions $C_2(\Delta\phi) \propto \int d\Delta\eta N(\Delta\phi, \Delta\eta)$ for AA collisions, which correspond two-jet event structure. The significant suppression of peak at large relative azimuthal angles ($\Delta\phi \simeq \pi$) was observed in azimuthal correlations of two particles with high p_T for central Au+Au in comparison with p+p. Moreover, this effect increases with centrality increasing in nuclei-nuclear interactions. The experimental observations have been interpreted as the critical evidences of large losses of parton energy in dense deconfinement matter, predicted by pQCD as a state of quark-gluon plasma. Control experiments with d+Au collisions have shown that the back-to-back peak is present in this case and its characteristics are close to ones which are observed in p+p interactions. Therefore, experimental RHIC results on two-hadron azimuthal correlations are one of the most obvious and important evidences in favour of creation of new state of strongly interacting matter at final stage of central nuclei-nuclear collisions. This final state matter is characterized by large energy losses and by opacity for partons with high p_T and for products of their fragmentation. The subsequent study at higher p_T both for trigger particles and for associated ones have allowed to find clear two peaks in $C_2(\Delta\phi)$, corresponding two-jet event structure both in semicentral and even central Au+Au collisions as well as in d+Au. As expected, the back-to-back peak is suppressed essentially in central Au+Au events (as well as at smaller p_T) [19]. This is the first direct experimental observation of two-jet event structure in central AA collisions, corresponding to pQCD predictions at qualitative level. Researches of back-to-back peak characteristics have been executed for collisions with various ion types and initial energies [20]. These experimental results indicate on the essential response of medium on energy deposition of hard parton traversed it. At present there are a several scenarios for medium response on hard jet traversing. It seems the experimental results agree with shock wave model at qualitative level only [20, 21]. Thus, additional experimental data and theoretical study are necessary for more unambiguous observation and explanation of cone topology in two dimensions.

Fig.2 shows experimental results for investigations of hadron jet correlations with respect to the reaction plane at SPS energy [22] and RHIC one [13] in Pb+Au and Au+Au collisions, accordingly. The jet was considered as directed in reaction plane, if $|\xi| < \pi/4 \bigcup |\xi| > 3\pi/4$,

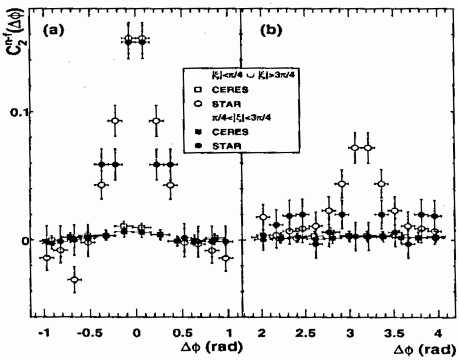

Figure 2: Azimuthal two-particle correlation functions at SPS and RHIC energies in the ranges of small (a) and large (b) relative azimuthal angles. The open symbols correspond to emission of trigger particles in-plane, solid symbols – out of plane direction for AA interactions. Statistical errors are presented only.

and directed out of reaction plane, if $\pi/4 < |\xi| < 3\pi/4$, where $\xi \equiv \phi^{tr} - \Psi_2$ – relative azimuthal angle between a reaction plane estimated by angle of second order event plane (Ψ_2), and a trigger particle. Experimental data obtained at various initial energies, for small and large relative azimuthal angles domain are compared on Fig.2a, 2b, accordingly. As seen, excess over a level of elliptic flow is much weaker at SPS energy $\sqrt{s_{NN}} = 17.3$ GeV, than for RHIC energy in the $\Delta\phi \sim 0$ domain (Fig.2a). At $\Delta\phi \sim \pi$ the values of SPS correlation functions are close to zero level (Fig.2b). One can see correlation functions for various directions of a trigger particle agree with each other well at SPS energy for both cases $\Delta\phi \sim 0$ (Fig.2a) and for $\Delta\phi \sim \pi$ (Fig.2b), that essentially differs from behaviour of jet-like correlations on RHIC in case of hadron jets passed inside medium with various directions with respect to the reaction plane. Therefore, suppression effect at RHIC is stronger significantly for hadron jet traversing of hot and dense matter on direction out of reaction plane, and suppression is weaker in the case of jet passing inside a matter on direction in-plane. This RHIC result is the first experimental evidence of strong correlation of suppression strength of hard hadron jets traversing the volume of hot and dense strongly interacting matter, with path lengths which are passed by jets inside of this matter.

The model of (multi)component collective elliptic flow has been suggested in [23 - 25] based on the experimental RHIC data for azimuthal anisotropy. In the framework of this model the correlations with respect to the reaction plane are supposed both for soft particles, and for hard particles (hadronic jets). The model of compound flow takes into account the number of jets per event, average multiplicity per jet, dependence of jet yield on the orientation with respect to the reaction plane, and independent "soft" particle production. The generalized formulas were derived for two-particle distribution on a relative azimuthal angle and for two-particle distributions in / out with respect to the reaction plane [23 - 25]. These analytic calculations provide the framework for a consistent description of the elliptic flow measured via the single-particle distribution with respect to the reaction plane, jet yield per event, and the amplitude of flow-like modulation in the two-particle distribution in the relative azimuthal angle. It seems, the difference between soft particle flow and parameter of jet correlations with respect to the reaction plane is (very) close to zero at RHIC energies. But jet production will give a more significant contribution at higher (LHC) energies and this difference may be more visible. The model of compound flow agree with expectations, that energy losses of partons are sensitive to energy-momentum tensor $T^{\mu\nu} = (e+p)u^{\mu}u^{\nu} - pg^{\mu\nu}$, described the global properties of created matter. Hence, partons and products of their fragmentation can be sensitive not only to the equation-of-state, but to the common velocity fields of collective flow u^{μ} also, i.e. hard jets appear "enclosed" in collective expansion of surrounding medium at all.

2 Fundamental symmetries in QCD-matter

The possible $\mathcal{P}$ and/or $\mathcal{CP}$ violation in strong interactions requires the deconfinement state of matter with restored chiral symmetry [26, 27]. Thus the positive and reliable experimental results for $\mathcal{P}$ and/or $\mathcal{CP}$ violation in the strong interactions would be prove the clear evidence of deconfinement and chirally symmetric phase creation and establish experimentally the presence of topological configurations of gluon fields and their role in chiral symmetry breaking [27]. The lattice calculations show the non-trivial topological structure of gluon fields which can be characterized by topological charge (winding number) $Q_W = \eta \int d^4x G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a$, $\eta \equiv g_s^2/32\pi^2$. The gauge (color) fields with non-zero Q_W induce difference between number of left-and right-handed fermions. The topological charge changing transitions are exponentially suppressed at zero temperature (instanton modes). But the such transitions unsuppressed at finite temperature (sphaleron modes). In chiral limit the charge difference will be created between two sides of a plane perpendicular to the chromomagnetic field. Thus one can define chiral magnetic effect as a charge separation by gauge field configurations with non-zero Q_W in the presence of background (electric) magnetic field. The clear equation for effect strength was derived in [28]. One needs to emphasize two features which are essential for experimental investigations namely. First of all chiral magnetic effect corresponds the early collision stages because of magnetic field falls off rapidly. Secondly, electrical charge is conserved in hadronization. Thus charge separation for quarks implies charge separation with respect to the reaction plane for hadron states observed experimentally.

The presence of non-zero angular momentum in non-central AA collisions is equal of the external magnetic field. Thus the experimental investigation of electrical charge separation of produced particles in non-central heavy ion collisions makes it possible to study $\mathcal{P}$ and/or $\mathcal{CP}$-odd domains. A correlator, directly sensitive to $\mathcal{P}$-event quantitative parameter is $\langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP}) \rangle$ [29]. The three-particle correlations allow to use a following characteristic: $\langle \cos(\phi_\alpha + \phi_\beta - 2\phi_\gamma) \rangle = \langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP}) \rangle v_2^\gamma$, which is more useful for experimental applications [18]. The preliminary experimental results were obtained by STAR experiment at RHIC for Cu+Cu and for Au+Au collisions at initial energies 62.4 and 200 GeV [30]. The signal of charge separation in Cu+Cu collisions is some larger than that for Au+Au for the same centralities and the experimental signal has a typical hadronic "width" [30]. This feature agrees qualitatively with scenario of stronger suppression of the back-to-back correlations in heavier (Au+Au) collisions. Thus the preliminary STAR experimental results agree at qualitative level with the magnitude and main features of the theoretical predictions for $\mathcal{P}$ violation in nuclear collisions at RHIC [30]. At present there is no indication on the some other effects which could be imitate the experimental signal and the fundamental symmetry violation in heavy ion collisions. But one need more rigorous study of such possible imitation effects for more unambiguous conclusion. The energy dependence of strength of parity violation effect is not trivial. It seems the necessary conditions can be reached in nuclei-nuclear collisions in sufficiently wide energy range. The some important medium properties are very sensitive to the vicinity of phase transition boundary. Therefore, perhaps, the effect of hypothetic fundamental symmetry violation in strong interaction might be more clear at lower energies [27].

3 Summary

The main RHIC results and corresponding phenomenological models are presented for two-particle azimuthal correlations. Investigation of azimuthal anisotropy has allowed to obtain a set of the important experimental results for the first time. Fast space-time evolution is observed for volume of the final state matter. This evolution is described by significant gradients of pressure at early stages. Azimuthal anisotropy agrees well enough with calculations in the framework of hydrodynamical models with phase transition and very small viscosity for the small transverse momentum domain. The manifestation of scale properties predicted by model of quark coalescence, significant values of elliptic flow for multistrange baryons and heavy flavour quarks allow to assume, that the collective behaviour on partonic stages is observed. The final state matter is opaque for partons and products of their fragmentation passed inside of it, and the degree of suppression depends on thickness of a layer of a traversed matter significantly. The generalized formulas have been derived for azimuthal correlation functions with taking into account various components (soft and hard) of elliptic collective flow and for different (in/out) orientations with respect to the reaction plane. These equations are based on experimental RHIC results for azimuthal asymmetry for both soft and hard particles. Thus, the matter created in a final state of nuclei-nuclear collisions at high RHIC energies, differs qualitatively from all matter state created and investigated in laboratory conditions early, and color degrees of freedom namely are adequate ones for the description of the early stages of space-time evolution of the RHIC matter.

The created matter is rather similar on (quasi)ideal liquid of color constituents (partons), than on (quasi)ideal quark-gluon gas, as expected earlier. The preliminary experimental results indicate the possible $\mathcal{P}$-violations in nuclear collisions at RHIC. This observation qualitatively agree with theoretical expectations for fundamental symmetry violations in QCD-matter under extreme conditions. At present the new direction of investigations is formed intensively which can be designated as "relativistic nuclear physics of condensed / continuous matter".

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Dynamics of a phase transition in nuclear matter

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Abstract

A 1+1 dimensional hydrodynamic model is used for describing evolution of a system, produced in heavy-ion central collisions. We consider dynamics of a phase transition taking into account a thermal nonequilibrium effect in the system. Hydrodynamic equations are solved numerically within the Bjorken scaling hypothesis. An effective equation of state is discussed.

Hydrodynamics applied to heavy-ion collisions opens a possibility for extracting information about global properties of compressed and hot nuclear matter. We consider a hydrodynamic approach for describing evolution of a system produced in heavy-ion central collisions, assuming hadron, quark-gluon and mixed phases in nuclear matter. To take into account thermal nonequilibrium, the relaxation time approximation for the first order phase transition is applied here to the system with zero chemical potential.

Hydrodynamic equations can be derived from energy-momentum and charge conservation laws. In our simple model we consider the energy-momentum conservation

$$\frac{\partial T^{\mu\nu}}{\partial x^{\mu}} = 0, \tag{1}$$

but do not take into account baryon, strange and electric charge conservation. For the perfect fluid case the energy-momentum tensor $T^{\mu\nu}$ is given by

$$T^{\mu\nu} = (\varepsilon + P)\, u^{\mu}u^{\nu} - P\, g^{\mu\nu}, \tag{2}$$

where u^{μ}, P and ε are collective velocity, pressure and energy density, respectively. Assuming that both phases have the same collective velocity u^{μ}, we can write expression like (2) for the energy-momentum tensor of each phase, defined by index $i = h, q$:

$$T_i^{\mu\nu} = (\varepsilon_i + P_i)\, u^{\mu}u^{\nu} - P_i\, g^{\mu\nu}, \tag{3}$$

If V_h and V_q are mean volumes containing hadron and quark-gluon phases, the total energy-momentum tensor $T^{\mu\nu}$ may be also expressed through $T_h^{\mu\nu}$ and $T_q^{\mu\nu}$:

$$T^{\mu\nu} = \lambda T_h^{\mu\nu} + (1-\lambda) T_q^{\mu\nu}, \tag{4}$$

where the relative volume $\lambda = \dfrac{V_h}{V_h + V_q}$ is considered as an order parameter.

Assuming mechanical equilibrium for the mixed phase, $P_h = P_q = P$, for our system with zero baryon chemical potential the conservation law (4) may be presented in the following way:

$$T^{\mu\nu} = (\lambda\varepsilon_h + (1-\lambda)\varepsilon_q + P)\, u^{\mu}u^{\nu} - P\, g^{\mu\nu}, \tag{5}$$

where $\lambda\varepsilon_h + (1-\lambda)\varepsilon_q = \varepsilon$ is the total energy density, expressed through energy densities of hadron and quark-gluon phases. Hydrodynamic equations, obtained by substitution of (5) into the conservation law (1), look as follows:

$$u^\nu \partial_\nu(\lambda\varepsilon_h + (1-\lambda)\varepsilon_q) + (\lambda\varepsilon_h + (1-\lambda)\varepsilon_q + P)\ \partial_\nu u^\nu = 0, \tag{6}$$

$$(\lambda\varepsilon_h + (1-\lambda)\varepsilon_q + P)\ u^\mu \partial_\mu u_\nu + u_\nu u^\mu \partial_\mu P - \partial_\nu P = 0. \tag{7}$$

Obviously, in the mixed phase λ changes in the interval from 0 to 1, for pure hadron (quark-gluon) phase one has $\lambda = 1$ ($\lambda = 0$).

To simulate thermal nonequilibrium between two phases ($T_h \neq T_q$) in the system, we define λ in the relaxation time approximation:

$$\Gamma \frac{\partial \lambda}{\partial t} = \lambda_{eq} - \lambda, \tag{8}$$

where Γ is the characteristic time of the equilibration processes. The relative equilibrium volume λ_{eq} is defined for the system satisfying the Gibbs conditions:

$$T_h = T_q, \quad P_h = P_q. \tag{9}$$

In the case of pure phase behavior, the fluid is described by the set of equations (6), (7) supplemented with hadron or quark-gluon Equation of State (EoS). In the mixed phase we should include the relaxation time approximation (8) into the set of equations. It is supposed, that phase transition begins when the system satisfies the Gibbs conditions (9). The phase transition ends if $\lambda = 0$ or $\lambda = 1$.

To find the EoS, the hadron phase is approximated by an ideal massless pion gas model. The quark phase is treated as an ideal massless gas of 2-flavor quarks and gluons confined in a bag with the bag constant $B = 0.41$ GeV/fm^3. For this system we get the critical temperature $T_c \approx$170 MeV and the latent heat about 1.6 GeV/fm^3.

Then the equilibrium value of the order parameter λ_{eq} is defined as:

$$\lambda_{eq} = \frac{(\pi^2/30)(\frac{7}{8}g_q + g_g)\ T_c^4 + B - \varepsilon}{4B}, \tag{10}$$

where g_q and g_g are degeneracy factors for quarks and gluons, respectively.

To investigate the influence of a phase transition on the evolution of expanding fireball, we apply the described relaxation time approximation to the a simple one-dimensional hydrodynamic model with the Bjorken scaling hypothesis [1]. In this case hydrodynamic equations (6), (7) can be reduced to:

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}\tau} + \frac{\varepsilon + P}{\tau} = 0, \tag{11}$$

where $\tau = \sqrt{x_\mu x^\mu}$ is the proper time. It is assumed that main characteristics of the system such as ε and P depend on a single space-time variable τ only. It seems reasonable to suppose, that the order parameter λ is a function of τ as well. For the mixed phase region we use the relaxation equation (8) rewritten in the proper time representation

$$\Gamma \frac{\mathrm{d}\lambda}{\mathrm{d}\tau} = \lambda_{eq} - \lambda. \tag{12}$$

The set of equations (11), (12) should be supplemented by our EoS. It is solved by the Runge-Kutta method of fourth order for following initial conditions:

$$\tau_0 = 1\,\mathrm{fm/c}, \qquad \varepsilon(\tau_0) = 2.75\,\mathrm{GeV/fm^3}, \tag{13}$$

where the value of $\varepsilon(\tau_0)$ corresponds to incident energies $E_{lab} \approx 158\,\mathrm{A\,GeV}$ for central Au-Au collisions [2].

The solution obtained for various values of the relaxation time Γ is shown in Fig.1.

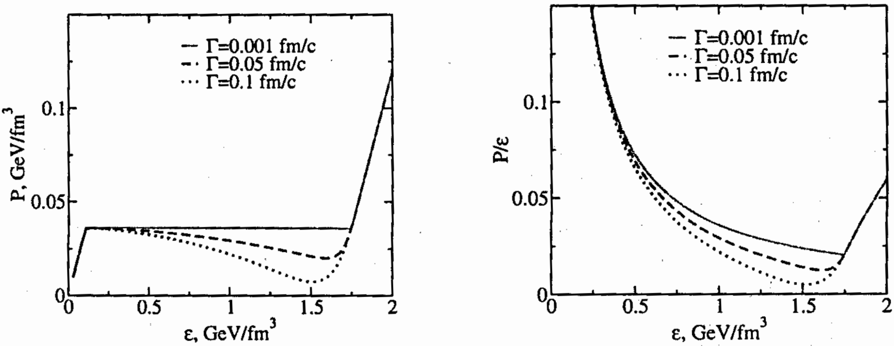

Figure 1: Energy density dependence of pressure and P/ε for various relaxation times Γ.

One can see that in the case of $\Gamma \to 0$ the system evolution started at $\varepsilon(\tau_0)$ really repeats the behavior of EoS with distinct flattening of $P(\varepsilon)$ corresponding to the Gibbs mixed phase. The entrance point into the mixed phase from the high-ε side roughly meets the 'softest point', *i.e.* a minimum in the P/ε ratio. In ultrarelativistic case this ratio is the sound velocity squared. Thus, the system evolves slowly near the softest point $\varepsilon_{s.p.} \approx$ 1.7 GeV/fm^3. Inclusion of the nonequilibrium effect results in somewhat like supercooling, shifts the softest point towards lower energy density and still more decreases the sound velocity of the system at this point. If the Γ parameter increases above $\sim$ 0.1 fm/c, the pressure is getting negative resulting in mechanical instability of the system [3].

We are thankful to D.N. Voskresensky for useful discussions. This work was supported in part by the Russian Foundation for Basic Research (08-02-01003) and special program of the Ministry of Education and Science of the Russian Federation (grant RNP.2.1.1.5409).

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Hydrodynamic View in the NICA Energy Range

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Abstract

Results for penetrating probes treated within a hybrid hydro-kinetic model are projected onto the energy range covered by the NICA and FAIR projects. A new source of dileptons emitted from a mixed quark-hadron phase, quark-hadron bremsstrahlung, is proposed. An estimate for the $\pi\pi \to \sigma \to \gamma\gamma$ process in nuclear collisions is given.

D.I. Blokhintsev, whose centennial anniversary of the birthday this conference is devoted to, has contributed to various fields of physics and its applications, particularly, to hydrodynamics. Sixteen years ago, at the dawn of hydrodynamics he made an important remark [1] concerning possible violation of the uncertainty principle in initial conditions of the Landau hydrodynamic theory. From up-to-date view, this Blokhintsev's estimate looks slightly naive but in principle it is correct until now and should be taken into account in modern development of a relativistic hydrodynamic approach. Here we present a hybrid model which combines kinetic and hydrodynamic descriptions. To certain extent it can be considered as a possible solution of the problem put by D.I. Blokhintsev.

In the hybrid model [2], the initial stage of heavy ion collisions is treated kinetically within the transport Quark Gluon String Model (QGSM) [3] whereas the subsequent stage is considered as an isentropic expansion of a formed dense and hot system (fireball). The transition from one stage to another is solved by considering the entropy evolution.

In Fig.1, the ratio between entropy S and baryon charge Q_B of participants is shown for In+In collisions at the impact parameter $b = 4$ fm and bombarding energy 158 AGeV. Being calculated on a large 3D grid, this ratio is less sensitive to particle fluctuation as compared to the entropy itself. Small values of the baryon charge Q_B at the very beginning of collision result in large values of the S/Q_B ratio. It is clearly seen that for $t_{kin} \gtrsim 1.3$ fm/c this ratio is practically constant and this stage may be considered as isentropic expansion.

To proceed from kinetics to hydrodynamics, we evaluate conserved components of the energy-momentum tensor $T_{00}, T_{01}, T_{02}, T_{03}$ and baryon density n_B (the zero component of the baryon current) within QGSM at the moment $t_{kin} = 1.3$ fm/c in every cell on the 3D grid. This state is treated as an initial state for subsequent hydrodynamic evolution of a fireball. The time dependence of average thermodynamic quantities is presented in the left panel of Fig.1.

The latter stage is evaluated within the relativistic 3D hydrodynamics [2]. The key quantity is the equation of state. In this work, the mixed phase Equation of State (EoS) is applied [5] which allows for coexistence of hadrons and quarks/gluons. This thermodynamically consistent EoS uses the modified Zimanyi mean-field interaction for hadrons and also includes interaction between hadron and quark-gluon phases, which results in a crossover deconfinement phase transition. In addition to [5], the hard thermal loop term was self-consistently added to the interaction of quarks and gluons to get the correct asymptotics

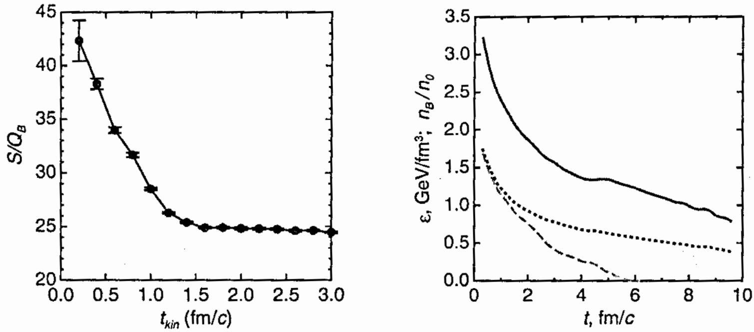

Figure 1: Temporal dependence of entropy S per baryon charge Q_B of participants (left panel) for a semi-central In+In collision at E_{lab} =158 AGeV. In the right panel the average energy (solid line) and baryon (dashed) densities of an expanding fireball formed in this collision. Dotted line shows a contribution of quarks and gluons to the energy density.

at $T >> T_c$ and reasonable agreement of the model results with lattice QCD calculations at finite temperature T and chemical potential μ_B [6]. This agreement is demonstrated in Fig.2 where the reduced pressure $\Delta p/T^4 = (p(\mu_B) - p(\mu_B = 0))/T^4$ is compared with recent lattice QCD data.

The fraction of unbound quarks/gluons defined as $\rho_{pl}/\rho = (n_q + n_{\bar{q}} + n_g)/(n_q + n_{\bar{q}} + n_g + n_B + n_M)$ is presented for the mixed phase EoS in the right panel of Fig.2. It is seen that even at a moderate temperature $T \sim 50 \div 100$ MeV the quark/gluon fraction sharply increases at the baryon density $n_B/n_0 \sim 6$ and dominates thereafter. At $T = 200$ MeV the admixture of hadrons is, naturally, quite small.

Consider now penetrating probes. To find observable dilepton characteristics, one should integrate the emission rate over the whole time-space $x \equiv (t, \mathbf{x})$ evolution, add the contribution from the freeze-out surface ('hadron cocktail'), and take into account the experimental acceptance. To simplify our task, we consider only the main channel $\pi\pi \to \rho \to l^+l^-$. In this case the dilepton emission rate is

$$\frac{d^4R^{l^+l^-}}{dq^4} = -\int d^4x\ \mathcal{L}(M)\ \frac{\alpha^2}{\pi^3 q^2}\ f_B(q_0, T(x))\ \mathrm{Im}\Pi_{em}(q, T(x), \mu_b(x))\ , \tag{1}$$

where the integration is carried out over the whole space grid and time from $t = 0$ till the local freeze-out moment. Here $q^2 = M^2 = q_0^2 - \mathbf{q}^2$, $f_B(q_0, T(x))$ is the Bose distribution function, and $\mathcal{L}(M)$ is the lepton kinematic factor. The imaginary part of the electro-magnetic current correlation function $\mathrm{Im}\Pi_{em}(q, T(x), \mu_b(x)$ includes in-medium effects which may be calculated in different scenarios. The recent precise measurements of muon pairs [7] allowed one to discriminate two main scenarios, in particular those based on the Brown-Rho (BR) scaling hypothesis [8] assuming a dropping ρ mass and on a strong broadening of ρ-meson spectral function as found in the many-body approach by Rapp and Wambach [9]. It was shown [7] that the measured excess of muons is nicely described by the strong broadening of the ρ-meson spectral function. In contrary, the BR scaling hypothesis predicts a large shift of the ρ-meson maximum towards lower invariant mass M in contradiction with

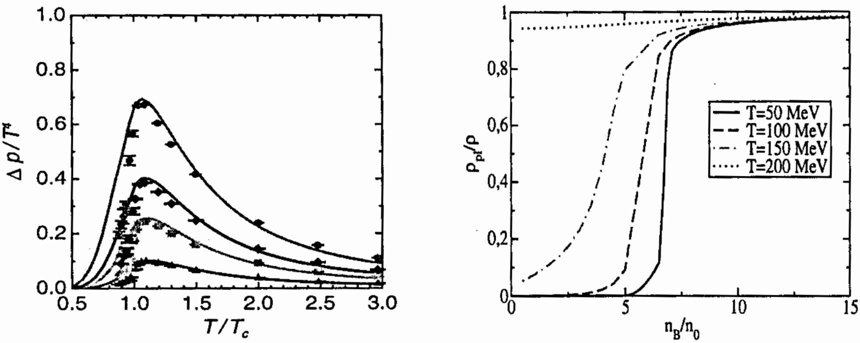

Figure 2: Temperature dependence of the reduced pressure (left panel) at the baryon chemical potential μ_B = 210, 330, 410 and 530 MeV (from the bottom) and fraction of unbound quarks (right panel) within the mixed phase EoS. Points are lattice QCD data for the 2+1 flavor system [6].

experimental data [7].

This result looks quite disappointing. First, one sees no signal of a partial restoration of the chiral symmetry for the sake of which dilepton measurements were originally undertaken. Second, to be consistent with the QCD sum rules *both* collision broadening *and* ρ-mass dropping should be taken into account [10]. Generally, the exact relation between the ρ mass and quark condensate is not fixed by the QCD sum rules in contrast with the BR scaling, so it is questionable whether to prescribe some T dependence to the BR mass shift. This shortcomings of the analysis in [7] are commented by Brown and Rho [11]. In this respect, for the pion annihilation $\pi\pi \to \rho \to \mu^+\mu^-$ we estimated [2] the imaginary part of the ρ-meson self-energy in the one-loop approximation assuming the Hatsuda-Lee relation for the modified ρ mass: $m^*_\rho(x) = m_\rho(1 - 0.15 \cdot n_B(x)/n_0)$. The calculated result is presented in Fig.3 by the solid line [2]. Indeed, the shift of the ρ-meson spectral function is not so drastic as in [7]. In addition, one should note that dileptons carry direct information on the ρ meson spectral function only if the vector dominance is valid [11]. It is not the case in the Harada-Yamawaki vector manifestation of hidden local symmetry [12].

As seen from Fig.3, the muon yield is underestimated at both low and high values of the $\mu^+\mu^-$ invariant mass M. In the broadening scenario a low M component is explained by particle-hole excitations but not a partial chiral symmetry restoration. However, the existence of the mixed quark-hadron phase, those study is the main aim of the Nuclotron-based Ion Collider fAcility (NICA) project [13], may give rise to a new dilepton source, quark(antiquark)-hadron bremsstrahlung. Similarly to the np bremsstrahlung, the process for an antiquark-hadron collision may roughly be estimated in the soft-photon approximation as

$$\frac{dN^{l^+l^-}_{qN}}{dM^2}(s,M) \approx K\,\frac{\alpha^2}{3\pi^2}\,\frac{\bar\sigma(s)}{M^2}\,\ln\left[\frac{s^{1/2}-m_N-m_q}{M}\right]. \qquad (2)$$

Here the averaged cross section $\bar\sigma(s) = \sigma^{qN}_{el}\,[s/(m_N+m_q)^2 - 1]$ and the elastic qN cross

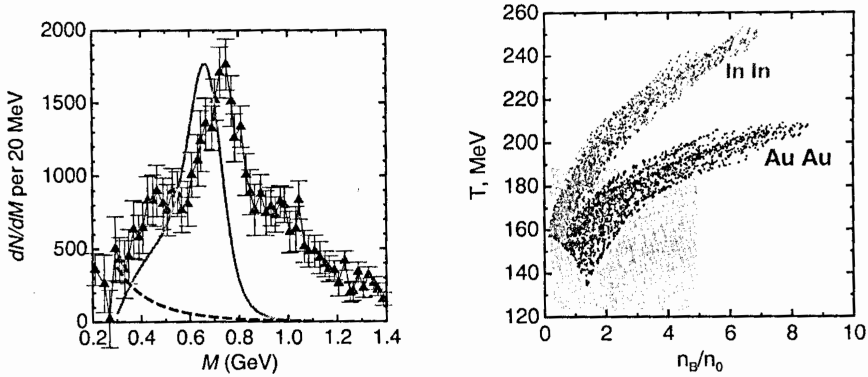

Figure 3: Invariant mass distribution of dimuons (left panel) from semi-central In+In collisions at the beam energy 158 *A*GeV. Experimental points are from [7]. The solid line corresponds to the T-independent dropping mass [2] and the dashed one is the contribution of the quark-hadron bremsstrahlung channel. In the right panel one compares dynamical trajectories projected onto the $T - n_B$ plane for central In+In(158 AGeV) and Au+Au(40 AGeV). The shaded region roughly corresponds to the hadronic phase.

section is approximated by the quark scaled NN cross section [14]

$$\sigma_{el}^{qN} = \left[\frac{18m_N(mb \cdot GeV^2)}{s-(m_N+m_q)^2} - 10\ (mb)\right] \times \frac{1}{3}\ . \tag{3}$$

So the production rate will be

$$\frac{dN_{qN}^{l^+l^-}}{dM^2} = \int d^4x \int \frac{d^3k_q}{(2\pi)^3} f(\mathbf{k}_q, T(x)) \int \frac{d^3N_q}{(2\pi)^3} f(\mathbf{k}_N, T(x))\ \frac{dN_{qN}^{l^+l^-}}{dM^2}(s, M)\ v_{rel}\ , \tag{4}$$

where v_{rel} is the relative velocity of colliding qN particles and the integration in (4) should be carried out over the whole space-time available for the mixed phase. The free mass is used for a nucleon and $m_q = 150$ MeV for an antiquark. In a real case one should also add contributions from all other baryons, as well as that from quark-antibaryon and quark(antiquark)-meson interactions with proper cross sections. All these uncertainities are effectively introduced in (2) by an arbitrary factor K.

As follows from the dashed line in Fig.3, at rather reasonable value of $K = 10$ the qN bremsstrahlung source improves agreement with experiment. It is of interest that the contribution of this new source decreases when the bombarding energy goes down till the NICA energy range (≤ 40 AGeV) while the contribution from particle-hole excitation is expected to grow since the baryon density in this range is higher (see below) allowing, in principle, disentangling of these two sources. The underestimated yield at high M, intermediate mass dileptons, can mainly be described by Drell-Yan process in the quark phase [15]. The mixed quark-hadron phase should also contribute to the intermediate M region. Its contribution can be taken into account in the way as the Drell-Yan process with an additional hadron form factor. Effectively, it will increase the Drell-Yan lepton yield [15].

The phase distribution of all space cells at the early evolution moment $t = 0.3$ fm/c projected on the $T - n_B$ plane is presented in Fig.3. It is seen that at the maximal NICA energy the baryon density in the hadronic phase for central Au+Au collisions is noticeably higher than that at the SPS energy in In+In collisions. It means that the difference between the BR scaling and broadening scenarios is expected to be more pronounced at the NICA energy.

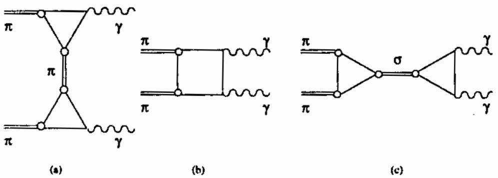

Figure 4: Quark diagrams for the 2γ production in the Born approximation (a), (b) and through the σ resonance (c).

Annihilation of two pions into two photons is of particular interest since its cross section is sensitive to changes of the σ-meson properties which occur in the vicinity of chiral restoration phase transition. With increasing temperature and density the σ meson changes its character from a broad resonance with a large decay width into two pions to a bound state below the two-pion threshold $m_\sigma(T,\mu_B) \lesssim 2m_\pi(T,\mu_B)$. The calculation of the photon pair production rate at the given T as a function of the invariant mass shows a strong enhancement and narrowing of the σ resonance at the threshold due to chiral symmetry restoration [16]. We make the first estimate of this channel for a particular nuclear collision.

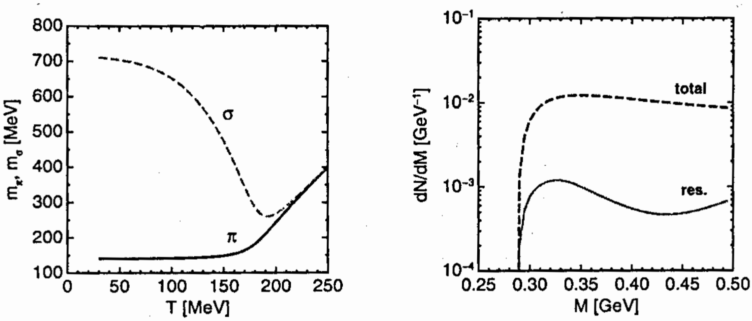

Figure 5: T-dependence of σ and π masses (left panel) and total (dashed line) and resonance (solid line) invariant mass distributions of two photon pairs created in the central Au+Au collision at 40 AGeV (right panel).

In-medium $\pi\pi \to \gamma\gamma$ process is evaluated within the NJL model [16]. Besides the resonance diagram, the dominating Born terms are considered, see Fig.4. So the total rate has the Born term, resonance term and interference between them: $dN^{\gamma\gamma}_{tot}/dM^2 = dN^{\gamma\gamma}_{Born}/dM^2 + dN^{\gamma\gamma}_{res}/dM^2 + dN^{\gamma\gamma}_{interf}/dM^2$ which should be substituted in eq.(4).

The temperature dependence of σ and π masses for this model is shown in Fig.5. The regime $m_\sigma(T, \mu_B) < 2m_\pi(T, \mu_B)$ starts at $T \sim 165$ MeV. The photon yield as a function of photon pair invariant mass is presented in Fig.5 for central Au+Au collisions at 40 AGeV. The total number of photon pairs sharply increases above the threshold $2m_\pi$ and then flattens on the level of $\sim 10^{-2}$. The resonance channel of interest is lower by about the order of magnitude as compared to the total yield and exhibits a spread weak maximum. The maximum predicted in [16] for the fixed T is washed out, as seen from Fig.5. It is not an easy but promising experimental problem to select out this maximum from the total distribution. Note that such an analysis should be carried out on a huge background of γ decays of 'hadron cocktail'.

One of the authors (V.T.) is deeply indebted to D.I. Blokhintsev who many years ago brought him into physics of high-energy interactions. We are thankful to E. Kolomeitsev for valuable remarks. This work was supported in part by the Deutsche Forschungsgemeinschaft (DFG project 436 RUS 113/558/0-3), the Russian Foundation for Basic Research (RFBR grants 06-02-04001 and 08-02-01003), special program of the Ministry of Education and Science of the Russian Federation (grant RNP.2.1.1.5409).

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Modern Mathematical Physics

Basic Properties of Fedosov and Riemannian Supermanifolds

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We discuss some differences in the properties of both even and odd Fedosov and Riemannian supermanifolds.

A *Fedosov supermanifold* (M, ω, Γ) is defined as a symplectic supermanifold (M, ω) equipped with a symmetric connection Γ (or covariant derivative ∇) compatible with a given symplectic structure ω: $\omega\nabla = 0$.[1] In local coordinates $\{x^i\}$, $\epsilon(x^i) = \epsilon_i$, on the supermanifold M the symplectic structure is $\omega = \omega_{ij}dx^j \wedge dx^i$, $\quad \omega_{ij} = -(-1)^{\epsilon_i\epsilon_j}\omega_{ji}$ (Grassmann parity of the symplectic structure, $\epsilon(\omega)$, is equal to 0 for even structure and 1 for odd structure) and the compatibility condition is $\omega_{ij}\nabla_k = \omega_{ij,k} - \Gamma_{ijk} + \Gamma_{jik}(-1)^{\epsilon_i\epsilon_j} = 0$ where $\Gamma_{ijk} = \omega_{in}\Gamma^n_{\ jk}$, $\quad \epsilon(\Gamma_{ijk}) = \epsilon(\omega) + \epsilon_i + \epsilon_j + \epsilon_k$ and $\Gamma^i_{\ jk}$ are components of the connection Γ. Notice that for a given symplectic structure ω there exists a large family of connections satisfying the compatibility condition.

The curvature tensor field $R^i_{\ mjk}$ is defined in terms of the commutator of covariant derivatives, $[\nabla_i, \nabla_j] = \nabla_i\nabla_j - (-1)^{\epsilon_i\epsilon_j}\nabla_j\nabla_i$, whose action on a vector field T^i is $T^i[\nabla_j, \nabla_k] = -(-1)^{\epsilon_m(\epsilon_i+1)}T^m R^i_{\ mjk}$.

It is convenient to describe the basic properties of Fedosov supermanifolds in terms of the symplectic curvature tensor $R_{ijkl} = \omega_{in}R^n_{\ jkl}$, $\quad \epsilon(R_{ijkl}) = \epsilon(\omega) + \epsilon_i + \epsilon_j + \epsilon_k + \epsilon_l$,. This tensor obeys the symmetry properties

$$R_{ijkl} = -(-1)^{\epsilon_k\epsilon_l}R_{ijlk}\,, \quad R_{ijkl} = (-1)^{\epsilon_i\epsilon_j}R_{jikl}\,, \tag{1}$$

and satisfies the Jacobi identity

$$(-1)^{\epsilon_j\epsilon_l}R_{ijkl} + (-1)^{\epsilon_l\epsilon_k}R_{iljk} + (-1)^{\epsilon_k\epsilon_j}R_{iklj} = 0\,, \tag{2}$$

[1] We use conventions and definitions adopted in [1, 2, 3].

the Bianchi identity

$$(-1)^{\epsilon_k\epsilon_m}R_{ijkl;m} + (-1)^{\epsilon_l\epsilon_m}R_{ijmk;l} + (-1)^{\epsilon_k\epsilon_l}R_{ijlm;k} = 0\,, \tag{3}$$

and the special symplectic identity

$$R_{ijkl} + (-1)^{\epsilon_l(\epsilon_i+\epsilon_k+\epsilon_j)}R_{lijk} + (-1)^{(\epsilon_k+\epsilon_l)(\epsilon_i+\epsilon_j)}R_{klij} + (-1)^{\epsilon_i(\epsilon_j+\epsilon_l+\epsilon_k)}R_{jkli} = 0. \tag{4}$$

We see that there are no formal differences in the properties of even and odd Fedosov supermanifolds on the level of symplectic curvature tensor. With the curvature tensor, R_{ijkl}, and the inverse tensor field ω^{ij} ($\omega^{ij} = -(-1)^{\epsilon(\omega)+\epsilon_i\epsilon_j}\omega^{ji}$) of the symplectic structure ω_{ij}, one can construct the only tensor field of type $(0,2)$,

$$K_{ij} = \omega^{kn}R_{nikj}(-1)^{\epsilon_i\epsilon_k+(\epsilon(\omega)+1)(\epsilon_k+\epsilon_n)} \; = \; R^k{}_{ikj}\,(-1)^{\epsilon_k(\epsilon_i+1)}, \quad \epsilon(K_{ij}) = \epsilon_i + \epsilon_j. \tag{5}$$

This tensor satisfies the relations [1]

$$[1 + (-1)^{\epsilon(\omega)}](K_{ij} - (-1)^{\epsilon_i\epsilon_j}K_{ji}) = 0\,, \tag{6}$$

and is called the Ricci tensor. In the even case this tensor is symmetric whereas in the odd case there are not restrictions on its (generalized) symmetry properties. The scalar curvature tensor K is defined by the formula $K = \omega^{ji}K_{ij}(-1)^{\epsilon_i+\epsilon_j}$. From the symmetry properties of R_{ijkl}, it follows that

$$[1 + (-1)^{\epsilon(\omega)}]K = 0. \tag{7}$$

Therefore as in the case of Fedosov manifolds [4], even Fedosov supermanifolds have vanishing scalar curvature K. However, for odd Fedosov supermanifolds this curvature is, in general, not vanishing. This fact was quite recently used in Ref. [5] to generalize the BV formalism [6].

A *Riemannian supermanifold* (M, g, Γ) is defined as a metric supermanifold (M, g) equipped with a symmetric connection Γ (or covariant derivative ∇) compatible with a given metric structure g: $g\nabla = 0$. In local coordinates on the supermanifold M the metric structure is $g = g_{ij}dx^j dx^i$, $\quad g_{ij} = (-1)^{\epsilon_i\epsilon_j}g_{ji}$ (Grassmann parity of the metric structure, $\epsilon(g)$, is equal to 0 for even structure and 1 for odd structure) and the compatibility condition is $g_{ij}\nabla_k = g_{ij,k} - \Gamma_{ijk} - \Gamma_{jik}(-1)^{\epsilon_i\epsilon_j} = 0$ where $\Gamma_{ijk} = g_{in}\Gamma^n_{\ jk}$, $\quad \epsilon(\Gamma_{ijk}) = \epsilon(g) + \epsilon_i + \epsilon_j + \epsilon_k$ and $\Gamma^i{}_{jk}$ are components of the connection Γ. Notice that for a given metric structure g there exists the unique symmetric connection $\Gamma^i{}_{jk}$ which is compatible with a given metric structure,

$$\Gamma^l{}_{ki} = \frac{1}{2}g^{lj}\Big(g_{ij,k}(-1)^{\epsilon_k\epsilon_i} + g_{jk,i}(-1)^{\epsilon_i\epsilon_j} - g_{ki,j}(-1)^{\epsilon_k\epsilon_j}\Big)(-1)^{\epsilon_j\epsilon_i+\epsilon_j+\epsilon(g)(\epsilon_j+\epsilon_l)}, \tag{8}$$

where g^{ij} is the inverse tensor field of the metric g_{ij} ($g^{ij} = (-1)^{\epsilon(g)+\epsilon_i\epsilon_j}g^{ji}$, $\quad \epsilon(g^{ij}) = \epsilon(g) + \epsilon_i + \epsilon_j$).

The curvature tensor $\mathcal{R}_{ijkl} = g_{in}\mathcal{R}^n_{\ jkl}$ ($\epsilon(\mathcal{R}_{ijkl}) = \epsilon(g) + \epsilon_i + \epsilon_j + \epsilon_k + \epsilon_l$), obeys the symmetry properties

$$\mathcal{R}_{ijkl} = -(-1)^{\epsilon_k\epsilon_l}\mathcal{R}_{ijlk}, \quad \mathcal{R}_{ijkl} = -(-1)^{\epsilon_i\epsilon_j}\mathcal{R}_{jikl}, \quad \mathcal{R}_{ijkl} = \mathcal{R}_{klij}(-1)^{(\epsilon_i+\epsilon_j)(\epsilon_k+\epsilon_l)} \tag{9}$$

and satisfies the Jacobi identity (2) and the Bianchi identity (3). Again we find that on the level of the curvature tensor there are no differences in the basic properties of even and odd Riemannian supermanifolds.

¿From the curvature tensor $\mathcal{R}_{ijkl}$ and the inverse tensor field g^{ij} of the metric g_{ij} one can define the only tensor field of type $(0,2)$:

$$\mathcal{R}_{ij} = \mathcal{R}^{k}{}_{ikj}(-1)^{\epsilon_k(\epsilon_i+1)} = g^{kn}\mathcal{R}_{nikj}(-1)^{(\epsilon_k+\epsilon_n)(\epsilon(g)+1)+\epsilon_i\epsilon_k}, \quad \epsilon(\mathcal{R}_{ij}) = \epsilon_i + \epsilon_j. \tag{10}$$

It is the generalized Ricci tensor with the following symmetry properties

$$R_{ij} = (-1)^{\epsilon(g)+\epsilon_i\epsilon_j} R_{ji} \tag{11}$$

depending on Riemannian supermanifolds to be even or odd. A further contraction defines the scalar curvature

$$\mathcal{R} = g^{ji}\mathcal{R}_{ij}\,(-1)^{\epsilon_i+\epsilon_j}, \quad \epsilon(\mathcal{R}) = \epsilon(g) \tag{12}$$

which, in general, is not equal to zero. Notice that for an odd metric structure the scalar curvature tensor squared is identically equal to zero, $\mathcal{R}^2 = 0$.

¿From the Bianchi identity one can deduce the following relation between the scalar curvature and the Ricci tensor

$$\mathcal{R}_{,i} = [1 + (-1)^{\epsilon(g)}]\mathcal{R}^{j}{}_{i;j}(-1)^{\epsilon_j(\epsilon_i+1)}, \tag{13}$$

which in the even case is nothing but $\mathcal{R}_{,i} = 2\mathcal{R}^{j}{}_{i;j}(-1)^{\epsilon_j(\epsilon_i+1)}$, i.e. the supersymmetric generalization of known relation in Riemannian geometry [7]. In the odd case $\mathcal{R}_{,i} = 0$. Therefore odd Riemann supermanifolds have constant scalar curvature, $\mathcal{R} = const$.

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Generalized kinematical symmetries of quantum phase space

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Abstract

Continuous symmetries generated with observables of a quantum theory in the Minkowski spacetime are discussed. An example of an originated in this way algebra of observables is the algebra of observables of the canonical quantum theory, that is contained the Lorentz group algebra and the Heisenberg algebra of phase space operators. In the general case commutation relations between observables depend on c, $\hbar$ and additional fundamental constants. Free field equations are considered, which are invariant with respect to generalized kinematical symmetries of the quantum phase space.

For development of a general theory of fundamental interactions it would be desired to examine in greater detail besides of properties of interactions the properties of a space-time as well [1]. Investigations along these lines have been carried out in the context of both the canonical quantum field theory [2], and various modifications of the canonical theory (e.g. papers submitted to conferences and seminars on nonlocal and nonlinear field theories and selected problems of modern theoretical physics).There are theories with new fundamental constants other than the well known ones, $c$and $\hbar$, among these modifications. Starting with the work [3], a theory with a fundamental length has been elaborated [4, 5]. A possible generalization of the Standard Model has been proposed in the framework of the theory with the minimal length or the maximal mass [6]. Let us consider the problem more generally, when coordinates and momenta are on equal terms and form an operator phase space. In the phase space we investigate admissible symmetries generated with observables of some quantum theory depending on extra fundamental constants other than the well known ones, c and $\hbar$ [3, 7, 8]. In order to restrict a considerable list of such symmetries we require the following natural constraints [9]: a) The generalized algebra (GA) of observables must be a Lie algebra; b) The GA dimension must coincide with the dimension of the algebra of observables for the canonical quantum theory in the Minkowski spacetime; c) The physical dimensions of observables, which are GA generators, should be the same as canonical ones; d) The GA must contain the Lorentz algebra (LA) as its subalgebra and commutation relations of the LA generators with other generators should be identical with canonical ones.

In the papers [3, 7, 8] (see also [9]) the most general algebra under the conditions a) - d) has been found and the new constants with the dimensions of length [3], mass [7] and action [8] have been introduced. The algebra of observables, which satisfy the conditions a)-d), can be presented as

$$[F_{ij}, F_{kl}] = if(g_{jk}F_{il} - g_{ik}F_{jl} + g_{il}F_{jk} - g_{jl}F_{ik}),$$
$$[F_{ij}, p_k] = if(g_{jk}p_i - g_{ik}p_j), [F_{ij}, x_k] = if(g_{jk}x_i - g_{ik}x_j), \quad (1)$$

$[F_{ij}, I] = 0, [p_i, p_j] = (if/L^2)F_{ij}, [x_i, x_j] = (if/M^2)F_{ij},$
$[p_i, x_j] = if(g_{ij}I + F_{ij}/H), [p_i, I] = if(x_i/L^2 - p_i/H), [x_i, I] = if(x_i/H - p_i/M^2)$

Among relations of the system (1) Eq.1 specifies the LA, while Eqs. 2-4 specify the tensor properties for the well-known physical quantities, Eqs. 5-6 lead to the noncommutativity of p and x, Eqs. 7-9 are the generalization of the Heisenberg relation. The system of relations (1) is written in the units with $c = 1$ (c is the velocity of light), it contains four dimensional parameters: f, M, L, and H. But in the limiting case, when $M \to \infty$, $L \to \infty$, $H \to \infty$, the system (1) should transform to the system of relations for observables of the canonical quantum theory, so $f = \hbar$. From mathematical point of view, the generalized algebra (1) contains, as special cases, a great number of algebras of different symmetry groups. If one evaluate the Killing – Cartan form the following condition for the algebra (1) being a semisimple algebra can be written:

$$f^2(M^2L^2 - H^2)/M^2L^2H^2 \neq 0 \quad (2)$$

When the condition (2) is fulfilled the GA(1) is isomorphic to a pseudoorthogonal algebra for one of the $O(3,3)$, $O(4,2)$, $O(5,1)$ groups. In other cases it is isomorphic to some direct or semidirect product of a pseudoorthogonal algebra and an Abelian or an integrable algebra.

Table 1. Domains of H^2, $M^2$and L^2parameters corresponding to the $O(2,4)$, $O(1,5)$ and $O(3,3)$ groups.

Domains of H^2, $M^2$and L^2parameters	Group
$H^2 < M^2L^2, M^2 > 0, L^2 > 0$	$O(2,4)$
$H^2 < M^2L^2, M^2 < 0, L^2 < 0$	$O(2,4)$
$M^2 > 0, L^2 < 0 or M^2 < 0, L^2 > 0$	$O(2,4)$
$H^2 > M^2L^2, M^2 > 0, L^2 > 0$	$O(1,5)$
$H^2 > M^2L^2, M^2 < 0, L^2 < 0$	$O(3,3)$

For the pseudoorthogonal algebras irreducible representations are determined with the help of eigenvalues of the three Casimir operators:

$$K_1 = \epsilon_{ijklmn}F^{ij}F^{kl}F^{mn}, K_2 = F_{ij}F^{ij}, K_3 = (\epsilon_{ijklmn}F^{kl}F^{mn})^2 \quad (3)$$

The second-order invariant operator K_2 in terms of I, p, x and F can be represented in the form:

$$C_2 = \Sigma_{i<j}F_{ij}F^{ij}(1/M^2L^2 - 1/H^2) + I^2 + (x_ip^i + p_ix^i)/H - x_ix^i/L^2 - p_ip^i/M^2 \quad (4)$$

Apart from mathematical properties which have been presented in the Refs. [9, 10] the generalized algebra (1) is the object of interest to the modern physical applications as well. For instance, in paper [11] a suggestion is made to apply the GA(1) in classical physics at the astronomical scales. We consider possible applications of the GA (1) to quantum phenomena at microscales [12, 13]. In this case it is convenient to use the quantum constants $\kappa = \hbar /H$, $\lambda = \hbar /M$, $\mu = \hbar /L$ and to write the algebra (1) in the natural units with $c = \hbar = 1$.

$$\begin{aligned}
&[F_{ij}, F_{kl}] = if(g_{jk}F_{il} - g_{ik}F_{jl} + g_{il}F_{jk} - g_{jl}F_{ik}),\\
&[F_{ij}, p_k] = i(g_{jk}p_i - g_{ik}p_j), [F_{ij}, x_k] = i(g_{jk}x_i - g_{ik}x_j),\\
&[F_{ij}, I] = 0, [p_i, p_j] = i\mu^2F_{ij}, [x_i, x_j] = i\lambda^2F_{ij},\\
&[p_i, x_j] = i(g_{ij}I + \kappa F_{ij}), [p_i, I] = i(\mu^2x_i - \kappa p_i), [x_i, I] = i(\kappa x_i - \lambda^2p_i)
\end{aligned} \quad (5)$$

In the general case one may classify generalized quantum fields (GQF) as the fields which form a space for irreducible representation of GA (5). For the pseudoorthogonal algebra GQF should obey the following equation among others:

$$[\Sigma_{i<j}F_{ij}F^{ij}(\lambda^2\mu^2 - \kappa^2) + I^2 + \kappa(x_ip^i + p_ix^i) - \mu^2x_ix^i - \lambda^2p_ip^i]\Phi = 0 \quad (6)$$

The Eq. (6) is the modification of the Klein-Gordon-Fock equation of the canonical field theory.

Let us apply the GA (5) for description of color particles such as quarks or gluons. Then additional constraints should be required for the form of GA (5). On account of CP-invariance of strong interactions the constraint $\kappa = 0$ holds [12]. Moreover, the presence of a nonzero λ value causes some inconsistencies in the quark descriptions inside hadrons and is superfluous [13]. Thus we put $\kappa = \lambda = 0$. In this case denoting μ as μ_s the following nonzero commutation relations (besides of the standard commutation relations with the Lorentz group generators) take place:

$$[p_i, p_j] = i\mu_s^2 F_{ij}, [p_i, x_j] = ig_{ij}I, [p_i, I] = i\mu_s^2 x_i \qquad (7)$$

¿From these relations it immediately follows nonzero uncertainties for results of simultaneous measurements of quark momentum components. For instance, let $\psi_{1/2}$ is a quark state with a definite value of its spin component along the third axis.Consequently, $[p_1, p_2] = i\mu_s^2/2$, thus $\Delta p_1 \Delta p_2 \gtrsim \mu_s^2/4$ and if $\Delta p_1 \tilde{} \Delta p_2$, one gets $\Delta p_1 \gtrsim \mu_s/2, \Delta p_2 \gtrsim \mu_s/2$, i.e. the transversal quark momentum components are not measurable simultaneously. In the framework of the quark model a rough estimation indicates that the μ_s value lies in the neighborhood of the $0.5GeV$. To find more precise number one can use a quark equation of Dirac-Gursey-Lee type [14, 15, 13].

$$[\gamma_i(p_0^i + dp_0^k L_k^i + i\mu_s\gamma^i/2) + 2i\mu_s S_{ij}(L^{ij} + S^{ij})]\psi = m\psi, \qquad (8)$$

where $p_0 + dp_0 L = p_F$ is the space-time total momentum [16], $d = \mu_s/m_0$, p_0 and L have forms of the usual generators of translations and Lorentz transformations in the Minkowski spacetime respectively, and $p_0^2 = m_0^2$, m_0 is a current quark mass, m is a constituent quark mass.

To estimate a value of μ_s on the basis of $m$and m_0 values we use a ground quark state ψ_0 in a meson so the $L^{ij}\psi_0$ contribution can be neglected. By this means using the Eq. (9) one obtains the approximate relation: $m \cong m_0 + 2i\mu_s$.To account for the well-known inequality $m > m_0$ μ_s should be pure imaginary negative. It follows from the correspondence for ranges of parameters and pseudoorthogonal groups written above (Table 1) that the algebra under consideration is isomorphic to the algebra of the AdS group O(2,3). Now m values can be served, which have been obtained in the independent quark model (IQM) with the hadron spectroscopy data [17]. Moreover the same mass values of constituent quarks have been used for an evaluation of neutrino mixing angles consistent with experimental data [18]. Then if we pick out from the high energy physics data $m_0 \cong 2MeV$ for current u-quark mass and with the help of IQM $m \cong 316MeV$ we obtain $|\mu_s| \cong 157MeV$. Thus we can evaluate mass values of the $d-$, $s-$, $c-$and $b-$current quarks on the scale $\sim 1GeV$, which agree with the values obtained in the QCD framework [19]. In conclusion it may be noted that further investigations of Generalized Algebra (5) and properties of solutions of the equations (6) and (8) are important objectives for an achievement of a mathematical completeness of this approach as well as other physical applications.

The work is supported with the grant # 33 for fundamental research of the Kurchatov Institute in 2008 year.

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Anomalous dimensions in $\mathcal{N} = 4$ SYM

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Abstract

We present results for the universal anomalous dimension $\gamma_{uni}(j)$ of Wilson twist-2 operators in the $\mathcal{N} = 4$ Supersymmetric Yang-Mills theory in the first four orders of perturbation theory.

1 Introduction

The anomalous dimensions (AD) of the twist-2 Wilson operators govern the Bjorken scaling violation for parton distributions in a framework of Quantum Chromodynamics (QCD). In QCD they are known up to the next-to-next-to-leading order (NNLO=N^{2}LO) of the perturbation theory (see [1] and references therein).

The QCD expressions for AD can be transformed to the case of the $\mathcal{N}$-extended Supersymmetric Yang-Mills theories (SYM) if one will use for the Casimir operators C_A, C_F, T_f the following values $C_A = C_F = N_c$, $T_f n_f = \mathcal{N} N_c/2$. For $\mathcal{N}$=2 and $\mathcal{N}$=4-extended SYM the AD get also additional contributions coming from scalar particles.

However, it turns out, that the expressions for eigenvalues of the AD matrix in the $\mathcal{N} = 4$ SYM can be derived directly from the QCD AD without tedious calculations by using a number of plausible arguments. The method elaborated in Ref. [2] for this purpose is based on special properties of the integral kernel for the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation [3]-[5] in this model and a new relation (see [2]) between the BFKL and Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) [6] equations. In the NLO approximation this method gives the correct results for AD eigenvalues, which was checked by *direct calculations* in Ref. [7]. Using the results for the NNLO corrections to QCD AD [1] and the method of Ref. [2] we derive the eigenvalues of the AD matrix for the $\mathcal{N} = 4$ SYM in the N^{2}LO approximation [8]. Moreover, the method of [2] together with long-range asymptotic Bethe-ansatz [9, 10] allows to predict the next-to-next-to-next-to-leading order (NNNLO=N^{3}LO) AD in [11].

2 Leading order AD matrix in $\mathcal{N} = 4$ SYM

In the $\mathcal{N} = 4$ SYM theory [12] one can introduce the following colour and $SU(4)$ singlet local Wilson twist-2 operators [2, 7] (the simbol ˜ is used for spin-dependent case):

$$\mathcal{O}^g_{\mu_1,...,\mu_j} = \hat{S} G^a_{\rho\mu_1} \mathcal{D}_{\mu_2} \mathcal{D}_{\mu_3} ... \mathcal{D}_{\mu_{j-1}} G^a_{\rho\mu_j}, \quad \tilde{\mathcal{O}}^g_{\mu_1,...,\mu_j} = \hat{S} G^a_{\rho\mu_1} \mathcal{D}_{\mu_2} \mathcal{D}_{\mu_3} ... \mathcal{D}_{\mu_{j-1}} \tilde{G}^a_{\rho\mu_j},$$

$$\mathcal{O}^{\lambda}_{\mu_1,...,\mu_j} = \hat{S}\bar{\lambda}^a_i \gamma_{\mu_1} \mathcal{D}_{\mu_2}...\mathcal{D}_{\mu_j} \lambda^{a\,i}, \quad \tilde{\mathcal{O}}^{\lambda}_{\mu_1,...,\mu_j} = \hat{S}\bar{\lambda}^a_i \gamma_5 \gamma_{\mu_1} \mathcal{D}_{\mu_2}...\mathcal{D}_{\mu_j} \lambda^{a\,i},$$
$$\mathcal{O}^{\phi}_{\mu_1,...,\mu_j} = \hat{S}\bar{\phi}^a_r \mathcal{D}_{\mu_1} \mathcal{D}_{\mu_2}...\mathcal{D}_{\mu_j} \phi^a_r, \tag{1}$$

where $\mathcal{D}_\mu$ are covariant derivatives. The spinors λ_i and field tensor $G_{\rho\mu}$ describe gluinos and gluons, respectively, and ϕ_r are the complex scalar fields. For all operators in Eq. (1) the symmetrization of the tensors in the Lorentz indices $\mu_1, ..., \mu_j$ and a subtraction of their traces is assumed. Due to the fact that all twist-2 operators belong to the same supermultiplet the eigenvalues of AD matrix can be expressed through one *universal* AD $\gamma_{uni}(j)$ with a shifted argument.

Indeed, after transform of the above Wilson operators (1) to ones with a multiplicative renormalization (i.e. after *diagonalization*), the *diagonal* leading order AD have the form (see [13, 2]):

$$\gamma^{(0)}_{\pm}(j) = -4S_1(j \mp 2), \quad \gamma^{(0)}_{0}(j) = -4S_1(j), \quad \tilde{\gamma}^{(0)}_{\pm}(j) = -4S_1(j \mp 1),$$

which means, that the evolution equations for the matrix elements of quasi-partonic operators in the multicolour limit $N_c \to \infty$ are equivalent to the Schrödinger equation for an integrable Heisenberg spin model [14, 13]. In QCD the integrability remains only in a small sector of these operators [15] In the case of $\mathcal{N} = 4$ SYM the equations for other sets of operators are also integrable [16]-[19]. Similar results related to the integrability of the multi-colour QCD were obtained earlier in the Regge limit [20].

3 Universal AD for $\mathcal{N} = 4$ SYM

The final three-loop result [1] for the universal AD $\gamma_{uni}(j)$ for $\mathcal{N} = 4$ SYM is [8]

$$\gamma(j) \equiv \gamma_{uni}(j) = \hat{a}\gamma^{(0)}_{uni}(j) + \hat{a}^2\gamma^{(1)}_{uni}(j) + \hat{a}^3\gamma^{(2)}_{uni}(j) + ..., \qquad \hat{a} = \frac{\alpha N_c}{4\pi}, \tag{2}$$

where

$$\frac{1}{4}\gamma^{(0)}_{uni}(j+2) = -S_1, \tag{3}$$
$$\frac{1}{8}\gamma^{(1)}_{uni}(j+2) = \Big(S_3 + \overline{S}_{-3}\Big) - 2\,\overline{S}_{-2,1} + 2\,S_1\Big(S_2 + \overline{S}_{-2}\Big), \tag{4}$$
$$\begin{aligned}\frac{1}{32}\gamma^{(2)}_{uni}(j+2) = {} & 2\,\overline{S}_{-3}\,S_2 - S_5 - 2\,\overline{S}_{-2}\,S_3 - 3\,\overline{S}_{-5} + 24\,\overline{S}_{-2,1,1,1} \\ & +6\Big(\overline{S}_{-4,1} + \overline{S}_{-3,2} + \overline{S}_{-2,3}\Big) - 12\Big(\overline{S}_{-3,1,1} + \overline{S}_{-2,1,2} + \overline{S}_{-2,2,1}\Big) \\ & -\Big(S_2 + 2\,S_1^2\Big)\Big(3\,\overline{S}_{-3} + S_3 - 2\,\overline{S}_{-2,1}\Big) - S_1\Big(8\,\overline{S}_{-4} + \overline{S}^2_{-2} \\ & +4\,S_2\,\overline{S}_{-2} + 2\,S_2^2 + 3\,S_4 - 12\,\overline{S}_{-3,1} - 10\,\overline{S}_{-2,2} + 16\,\overline{S}_{-2,1,1}\Big)\end{aligned} \tag{5}$$

[1] Note, that in an accordance with Ref. [4] our normalization of $\gamma(j)$ contains the extra factor $-1/2$ in comparison with the standard normalization and differs by sign in comparison with one from Ref. [1].

and $S_a \equiv S_a(j)$, $S_{a,b} \equiv S_{a,b}(j)$, $S_{a,b,c} \equiv S_{a,b,c}(j)$ are harmonic sums

$$S_a(j) = \sum_{m=1}^{j} \frac{1}{m^a}, \quad S_{a,b,c,\cdots}(j) = \sum_{m=1}^{j} \frac{1}{m^a} S_{b,c,\cdots}(m), \tag{6}$$

$$S_{-a}(j) = \sum_{m=1}^{j} \frac{(-1)^m}{m^a}, \quad S_{-a,b,c,\cdots}(j) = \sum_{m=1}^{j} \frac{(-1)^m}{m^a} S_{b,c,\cdots}(m),$$

$$\overline{S}_{-a,b,c,\cdots}(j) = (-1)^j S_{-a,b,c,\ldots}(j) + S_{-a,b,c,\cdots}(\infty)\left(1-(-1)^j\right). \tag{7}$$

The expression (7) is defined for all integer values of arguments (see [21, 2, 22]) but can be easily analytically continued to real and complex j by the method of Refs. [21, 23, 22].

4 The N^{3}LO universal AD from Bethe Ansatz

The long-range asymptotic Bethe equations for twist-two operators read [9, 10] ($M = j+2$ hereafter in this Section)

$$\left(\frac{x_k^+}{x_k^-}\right)^2 = \prod_{m=1, m\neq k}^{M} \frac{x_k^- - x_m^+}{x_k^+ - x_m^-} \frac{(1-g^2/x_k^+ x_m^-)}{(1-g^2/x_k^- x_m^+)} \exp\left(2\,i\,\theta(u_k, u_j)\right), \quad \prod_{k=1}^{M} \frac{x_k^+}{x_k^-} = 1\,. \tag{8}$$

These are M equations for $k = 1, \ldots, M$ Bethe roots u_k, with

$$x_k^\pm = x(u_k^\pm)\,, \qquad u^\pm = u \pm \frac{i}{2}, \qquad x(u) = \frac{u}{2}\left(1 + \sqrt{1 - 4\frac{g^2}{u^2}}\right), \tag{9}$$

and where the dressing phase in three loops $\theta \sim \zeta(3)$ is a rather intricate function conjectured in [10].

Once the M Bethe roots are determined from above equations for the state of interest, its asymptotic all-loop AD is given by

$$\gamma^{ABA}(g) = 2\,g^2 \sum_{k=1}^{M} \left(\frac{i}{x_k^+} - \frac{i}{x_k^-}\right). \tag{10}$$

The above equations (8) can be solved recursively order by order in g at arbitrary values of M once the one-loop solution for a given state is known.

Assuming the maximum transcendentality principle [2] at four-loop order one can derive the corresponding expression for the universal AD by making an appropriate ansatz with unknown coefficients multiplying the nested harmonic sums, and subsequently fixing these constants. The latter is done by fitting to the exact anomalous dimension for a sufficiently large list of specific values of M as calculated from the Bethe ansatz.

After much effort the expression was found. It has the form [11]

$$\begin{aligned}
&\frac{1}{256}\,\gamma_{uni}^{ABA}(j+2) = \\
&4\,S_{-7} + 6\,S_7 + 2\,(S_{-3,1,3} + S_{-3,2,2} + S_{-3,3,1} + S_{-2,4,1}) + 3\,(-S_{-2,5} \\
+\;& S_{-2,3,-2}) + 4\,(S_{-2,1,4} + -S_{-2,-2,-2,1} - S_{-2,1,2,-2} - S_{-2,2,1,-2} - S_{1,-2,1,3} \\
-\;& S_{1,-2,2,2} - S_{1,-2,3,1}) + 5\,(-S_{-3,4} + S_{-2,-2,-3}) + 6\,(-S_{5,-2}
\end{aligned}$$

$$
\begin{aligned}
&+ \; S_{1,-2,4} - S_{-2,-2,1,-2} - S_{1,-2,-2,-2}) + 7\,(-S_{-2,-5} + S_{-3,-2,-2} \\
&+ \; S_{-2,-3,-2} + S_{-2,-2,3}) + 8\,(S_{-4,1,2} + S_{-4,2,1} - S_{-5,-2} - S_{-4,3} \\
&- \; S_{-2,1,-2,-2} + S_{1,-2,1,1,-2}) + 9\,S_{3,-2,-2} - 10\,S_{1,-2,2,-2} + 11\,S_{-3,2,-2} \\
&+ \; 12\,(-S_{-6,1} + S_{-2,2,-3} + S_{1,4,-2} + S_{4,-2,1} + S_{4,1,-2} - S_{-3,1,1,-2} - S_{-2,2,-2,1} \\
&- \; S_{1,1,2,3} - S_{1,1,3,-2} - S_{1,1,3,2} - S_{1,2,1,3} - S_{1,2,2,-2} - S_{1,2,2,2} - S_{1,2,3,1} - S_{1,3,1,-2} \\
&- \; S_{1,3,1,2} - S_{1,3,2,1} - S_{2,-2,1,2} - S_{2,-2,2,1} - S_{2,1,1,3} - S_{2,1,2,-2} - S_{2,1,2,2} \\
&- \; S_{2,1,3,1} - S_{2,2,1,-2} - S_{2,2,1,2} - S_{2,2,2,1} - S_{2,3,1,1} - S_{3,1,1,-2} - S_{3,1,1,2} - S_{3,1,2,1} \\
&- \; S_{3,2,1,1}) + 13\,S_{2,-2,3} - 14\,S_{2,-2,1,-2} + 15\,(S_{2,3,-2} + S_{3,2,-2}) \\
&+ \; 16\,(S_{-4,1,-2} + S_{-2,1,-4} - S_{-2,-2,1,2} - S_{-2,-2,2,1} - S_{-2,1,-2,2} - S_{-2,1,1,-3} \\
&- \; S_{1,-3,1,2} - S_{1,-3,2,1} - S_{1,-2,-2,2} - S_{2,-2,-2,1} + S_{-2,1,1,-2,1} + S_{1,1,-2,1,-2} \\
&+ \; S_{1,1,-2,1,2} + S_{1,1,-2,2,1}) - 17\,S_{-5,2} + 18\,(-S_{4,-3} - S_{6,1} + S_{1,-3,3}) \\
&+ \; 20\,(-S_{1,-6} - S_{1,6} - S_{4,3} + S_{-5,1,1} + S_{-4,-2,1} + S_{-3,-2,2} + S_{-2,-4,1} \\
&+ \; S_{-2,-3,2} + S_{1,3,3} + S_{3,1,3} + S_{3,3,1} - S_{1,1,-2,3} - S_{1,2,-2,-2} - S_{2,1,-2,-2}) \\
&- \; 21\,S_{3,4} + 22\,(S_{1,-2,-4} + S_{2,2,3} + S_{2,3,2} + S_{3,-2,2} + S_{3,2,2}) + 23\,(-S_{-3,-4} \\
&- \; S_{5,2} + S_{2,-2,-3}) + 24\,(-S_{-4,-3} + S_{1,-4,-2} - S_{1,-3,1,-2} - S_{1,1,1,4} - S_{1,1,4,1} \\
&- \; S_{1,3,-2,1} - S_{1,4,1,1} - S_{3,-2,1,1} - S_{3,1,-2,1} - S_{4,1,1,1} + S_{-2,-2,1,1,1} + S_{-2,1,-2,1,1} \\
&+ \; S_{1,-2,-2,1,1} + S_{1,-2,1,-2,1} + S_{1,1,-2,-2,1} + S_{1,1,1,-2,-2} + S_{1,1,2,-2,1} + S_{1,2,1,-2,1} \\
&+ \; S_{2,1,1,-2,1}) + 25\,S_{2,-3,-2} + 26\,(-S_{2,5} + S_{1,4,2} + S_{2,4,1} + S_{4,1,2} + S_{4,2,1}) \\
&+ \; 28\,(S_{1,2,4} + S_{2,1,4} - S_{-3,1,-2,1} - S_{-2,1,-3,1} - S_{1,-2,1,-3}) + 30\,S_{-3,1,-3} \\
&+ \; 32\,(S_{1,5,1} + S_{5,1,1} - S_{-3,-2,1,1} - S_{-2,-3,1,1} - S_{1,-3,-2,1} - S_{1,-2,-3,1} \\
&- \; S_{2,2,-2,1} + S_{1,2,-2,1,1} + S_{2,1,-2,1,1} - S_{1,1,1,-2,1,1}) + 36\,(S_{1,1,5} + S_{1,3,-3} \\
&+ \; S_{3,1,-3} - S_{1,1,-3,-2} - S_{1,1,-2,-3} - S_{1,1,2,-3} - S_{1,2,-2,2} - S_{1,2,1,-3} - S_{2,1,-2,2} \\
&- \; S_{2,1,1,-3}) + 38\,S_{-3,-3,1} + 40\,(-S_{1,-4,1,1} - S_{2,-3,1,1} + S_{1,1,1,-2,2}) \\
&- \; 41\,S_{3,-4} + 42\,(-S_{2,-5} + S_{1,-4,2} + S_{1,-3,-3}) + 44\,(S_{1,-5,1} + S_{2,-3,2} + S_{3,-3,1}) \\
&+ \; 46\,S_{2,2,-3} + 48\,S_{1,1,-3,1,1} + 60\,(S_{1,1,-5} - S_{1,1,-3,2}) + 62\,S_{2,-4,1} + 64\,S_{1,1,1,-3,1} \\
&+ \; 68\,(S_{1,2,-4} + S_{2,1,-4} - S_{1,2,-3,1} - S_{2,1,-3,1}) - 72\,S_{1,1,1,-4} - 80\,S_{1,1,-4,1} \\
&- \; \zeta(\mathbf{3}) S_1 (S_3 - S_{-3} + 2\,S_{-2,1}). \qquad (11)
\end{aligned}
$$

The degree of an harmonic sum is defined to be $|a_1| + \ldots |a_n|$. Notice that the total egree of each term is seven in accordance with the maximal transcendentality principle. /e have highlighted the terms in the last line, containing the number $\zeta(3)$ induced by the ressing factor [10].

Now one is ready to analytically continue this expression to the vicinity of the pomeron ɔle at $M = -1 + \omega$. Harmonic sums of degree seven may lead to poles no higher than venth order in ω. In fact, it is known that none of the sums can produce such a high-order ɔle except for the two sums S_7 and S_{-7}, which we have highlighted at the beginning of .e table. Their residues at $1/\omega^7$ are of opposite sign. Thus, one immediately sees that the m of the two residues does *not* cancel. However, the BFKL approach predicts only an istence of the singularity $\sim 1/\omega^4$ (see [11]). So, the results (11) is not complete and, for. ample, the contributions of the wrapping effects should be taken in consideration.

An attempt to phenomenologically improve the erroneous four-loop result (11) was tained in [11] by the dressed asymptotic Bethe ansatz such that all BFKL and double-logarithm constraints are satisfied. Obviously this has to be done in a way which does not ruin the correct features of the expression in (11). In particular, the improvement should not modify the large spin limit nor violate the transcendentality principle. A seemingly natural way to ensure this is to replace the explicit $\zeta(3)$ stemming from the dressing factor by an appropriate linear combination of $\zeta(3)$ and finite harmonic sums of degree three. We found that there is indeed an attractive choice, namely replacing in the last line of the expression in (11) by

$$\zeta(3) \to \frac{47}{24}\,\zeta(3) - \frac{1}{4}S_{-3} + \frac{3}{4}\,S_{-2}\,S_1 + \frac{3}{8}\,S_1\,S_2 + \frac{3}{8}\,S_3 + \frac{1}{6}\,S_{-2,1} - \frac{17}{24}\,S_{2,1}\,. \quad (12)$$

This alteration clearly preserves transcendentality, and it is easy to check that the large spin limit is not modified. In addition, the catastrophic behavior $\sim 1/\omega^7$ is now replaced by the correct one $\sim 1/\omega^4$.

It is nevertheless interesting to work out the four-loop AD of the operator of lowest twist $M = 2$, i.e. the Konishi field, by using the eq. (11) with the replacement (12). One finds

$$\gamma = 12\,g^2 - 48\,g^4 + 336\,g^6 - \left(\frac{5307}{2} + 564\,\zeta(3)\right)\,g^8 + \dots\,. \quad (13)$$

In is necessary to note, however, that (13) is a result based on some reasonable but presumably not unique assumptions and we do not dare calling it a conjecture.

Indeed, very recenly, some investigations [24] have been done for Konishi field and it was shown that the result (13) is not correct. After some checking, now the correct results has the following form [24]

$$\gamma = 12\,g^2 - 48\,g^4 + 336\,g^6 - (2496 - 576\,\zeta(3) + 1440\,\zeta(5))\,g^8 + \dots\,. \quad (14)$$

So, some modification of our predictions (12) for contributions of the wrapping effects is needed.

5 Conclusion

We demonstrated the results for the universal AD at the first four orders of perturbation theory and also the methods which were applied to extract them. The result (11) with the replacement (12) for the N^3LO anomalous dimension is not correct or is not complete and it needs some modifications. We hope to return to study the N^3LO anomalous dimension in our future investigations.

This work was supported by RFBR grant 07-02-00902-a. A.V.K. thanks the Organizing Committee of the XIII International Workshop "Selected Problems of Modern Theoretical Physics" for invitation.

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BRST approach to Lagrangian Construction for Massive Higher Spin Fields

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Abstract

We review the recently developed general gauge invariant approach to Lagrangian construction for massive higher spin fields in Minkowski and AdS spaces of arbitrary dimension. Higher spin Lagrangian, describing the dynamics of the fields with any spin, is formulated with help of BRST-BFV operator in auxiliary Fock space. No off-shell constraints on the fields and gauge parameters are imposed. The construction is also applied to tensor higher spin fields with index symmetry corresponding to a multirow Young tableau.

Higher spin field problem attracts much attention during a long time. At present, there exist the various approaches to this problem although the many aspects are still far to be completely clarified (see e.g. [1] for recent reviews of massless higher spin field theory). This talk is a brief survey of recent state of gauge invariant approach to massive higher spin field theory.

The standard BFV or BRST-BFV construction (see the reviews [2]) arose at operator quantization of dynamical systems with first class constraints. The systems under consideration are characterized by first class constraints in phase space T_a, $[T_a, T_b] = f^c_{ab}T_c$. Then BRST-BFV charge is introduced according to the rule

$$Q \;=\; \eta^a T_a + \frac{1}{2}\eta^b\eta^a f^c_{ab}\mathcal{P}_c, \qquad\qquad Q^2 = 0, \tag{1}$$

where η^a and $\mathcal{P}_a$ are canonically conjugate ghost variables (we consider here the case $gh(T) = 0$, then $gh(\eta^a) = 1$, $gh(\mathcal{P}_a) = -1$) satisfying the relations $\{\eta^a, \mathcal{P}_b\} = \delta^a_b$. After quantization the BRST-BFV charge becomes a Hermitian operator acting in extended space of states including ghost operators, the physical states in the extended space are defined by the equation $Q|\Psi\rangle = 0$. Due to the nilpotency of the BRST-BFV operator, $Q^2 = 0$, the physical states are defined up to transformation $|\Psi'\rangle = |\Psi\rangle + Q|\Lambda\rangle$ which is treated as a gauge transformation.

Application of BRST-BFV construction in the higher spin field theory [3] is inverse to above quantization problem. The initial point are equations, defining the irreducible representations of Poincare or AdS groups with definite spin and mass, the BRST-BFV operator is constructed on the base of these constraints and finally the higher spin Lagrangian is found on the base of BRST-BFV operator. Generic procedure looks as follows. The equations defining the representations are treated as the operators of first class constraints in some auxiliary Fock space. However, in the higher spin field theory a part of these constraints are non-Hermitian operators and in order to construct a Hermitian BRST-BFV perator we have to involve the operators which are Hermitian conjugate to the initial con-raints and which are not the constraints. Then for closing the algebra to the complete set operators we must add some more operators which are not constraints as well. Because presence of such operators the standard BRST-BFV construction can not be applied it s literal form. However this problem can be solved.

This approach to Lagrangian construction was applied for free massive bosonic and rmionic fields in Minkowski space-time in [4] and [5] respecively. Then we generalize e method for constructing Lagrangians in AdS space, where the algebra underlying the RST operator is nonlinear. Construction of Lagrangians for massive bosonic and fermionic elds in AdS space-time is realized in [6] and [7] respectively. Finally it is possible apply e method for constructing Lagrangian for fields with mixed index symmetry [8]. In the ted above papers a reader can find full procedure of Lagrangian construction and detailed xplanation of the BRST method. It is interesting to point out that the Lagrangians btained possess a reducible gauge invariance and for the fermionic fields the order of educibility grows with value of the spin. Recent applications of BRST-BFV approach to teraction higher spin theories are discussed in [9].

Acknowledgements. The work of V.A.K was partially supported by the INTAS grant, roject INTAS-05-7928, the RFBR grant, project No. 06-02-16346 and grant for LRSS, roject No. 2553.2008.2.

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Kac-Moody Algebras & Gauged Supergravity

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Abstract

We review how the surprising relation between two seemingly unrelated subjects, gauged supergravities and Kac-Moody algebras, is demonstrated on a kinematic level. Particularly, focus is placed on the connection between E_{11} and gauged maximal supergravity.

1 Introduction

Maximal supergravity in $D = 11$ dimensions displays a remarkable hidden symmetry upon dimension reduction. Namely, over a torus T^n the resulting scalars transform in cosets $G/K(G)$, with G being an exceptional group (see table 1). For $D = 3$ the global symmetry group, E_8, is still finite, but it has been conjectured and partially proven that for $D = 2$ and below infinite-dimensional Kac-Moody algebras play a role [1, 2, 3].

In this contribution we will review how the correct bosonic field content of all maximal supergravities are embedded within E_{11}. Furthermore, and more strikingly, we will show the same for all the possible gauge deformations of those supergravities [4, 5].

D	11	IIA	9	8	7	6	5	4	3	2
G	1	$\mathbb{R}^+$	$SL(2) \times \mathbb{R}^+$	$SL(3) \times SL(2)$	$SL(5)$	D_5	E_6	E_7	E_8	E_9

Table 1: The hidden symmetries G of $3 \leq D \leq 11$ maximal supergravities upon reduction over a torus.

2 Kac-Moody algebras

We begin with reviewing some basic facts of Kac-Moody algebras [6], and in particular of E_{11}. It is defined in terms of a 11×11 Cartan matrix, which can be read off from the Dynkin diagram (figure 1) as follows:

$$A_{ij} = \begin{cases} 2 & \text{if } i = j, \\ -1 & \text{if there is a line between nodes } i \text{ and } j, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

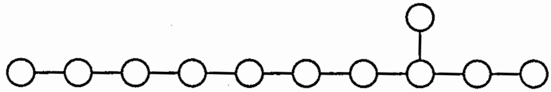

Figure 1: The Dynkin diagram of E_{11}.

The Lie algebra is then generated by multiple commutators of the basic triples of generators $\{h_i, e_i, f_i\}$. The h_i are elements of the Cartan subalgebra, satisfying $[h_i, h_j] = 0$. The e_i and f_i are the positive and negative step operators, respectively. Their commutation relations read

$$[h_i, e_j] = A_{ji} e_j \ , \quad [h_i, f_j] = -A_{ji} f_j \ , \quad [e_i, f_j] = \delta_{ij} h_i \ , \tag{2}$$

with no summation on the repeated indices. The multiple commutators are constrained by the *Serre relations*

$$(\mathrm{ad}_{e_i})^{1-A_{ij}} e_j = 0 \ , \qquad (\mathrm{ad}_{f_i})^{1-A_{ij}} f_j = 0 \ . \tag{3}$$

In order to extract useful information from the thusly generated Lie algebra, one can employ a so-called *level decomposition* [7, 5]. Given the fact that this a rather cumbersome task, we have written a fully automatic computer program to do the job [8]. With the help of this program, the only manual labor required is specifying the Dynkin diagram, and its decomposition into finite-dimensional subgroups corresponding to the symmetries of the supergravity of interest (see table 1). The resulting branching of the E_{11} generators gives then exactly the field content of the that supergravity. And, what's more, it also gives $(D-1)$- and D-form potentials (see table 2). These can, as we will see next, be interpreted as possible gauge deformations.

D	IIA	IIB	9	8	7	6	5	4	3
$p = D-1$	1		2	6	15	144	351	912	1
			3	12	40				3875
$p = D$	2×1	2	2×2	2×3	5	10	27	133	248
		4	4	9	45	126	1728	8645	3875
				15	70	320			147250

Table 2: E_{11} predictions for $(D-1)$–forms and D–forms in all $3 \leq D \leq 10$ maximal supergravities. These are representations of the duality groups G given in table 1.

3 Gauged supergravities

In gauged supergravity, a part of the global symmetry group G is promoted to a local gauge symmetry by means of coupling it to the vector fields of the theory. The gauge group is defined by specifying how its generators X_M are embedded into the generators t_α of the global symmetry group G:

$$X_M = \Theta_M{}^\alpha t_\alpha \,. \tag{4}$$

Iere Θ is known as the *embedding tensor* [9]. In order for Θ to describe a consistent gauging, t must satisfy two constraints: one ensuring the gauge generators X_M close under commu- ation, and another safeguarding unbroken supersymmetry. These two constraints can be nforced by means of Lagrange multipliers [10, 11], which can be written schematically as

$$\mathcal{L} = \mathcal{L}_{\text{sugra}} + A_{(D-1)}\partial\Theta + A_{(D)}\Theta\Theta \,, \tag{5}$$

vhere the Lagrange multipliers A are respectively a $(D-1)$–form and a D–form. Quite urprisingly, it turns out that when calculating the representations of these Lagrange mul- ipliers, they are in exactly the same representation as predicted by E_{11} [12].

4 Summary

Given the fact that E_{11} not only contains the correct bosonic field content of the maximal upergravities, but also somehow knows of all their possible gaugings, seems to suggest a leep connection between the two. This connection does not only hold for the maximal upergravities, as shown here, but can also be extended other cases [10, 13]. However, the nalysis presented here goes only as far as the kinematical level. It will be interesting to ee if the connection can be enlarged as to include dynamics as well.

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Feynman disentangling method and group theory

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Abstract

The subject of this work is to apply the modified Feynman disentangling approach to a problem of transitions in a non-quadratic quantum-mechanical system: a singular oscillator with a time-dependent frequency.

The method of disentangling expressions, containing non-commuting operators (FDM), suggested by Feynman in [1], gave an elegant solution of the harmonic oscillator excitation under the arbitrary time-dependent external force. In further developments the FDM was applied to some other non-stationary quantum mechanical problems, see [2, 3] and [4] for review. And it was shown that the transition matrix elements calculation became much simpler if the FDM was supplied by some considerations from the representation theory of $SU(2)$ or $SU(1,1)$ groups.

In this paper we apply the FDM to a non-quadratic system — singular oscillator with a variable frequency $\omega(t)$. We obtain the self-contained analytic expressions for the transition amplitudes between states with definite quantum numbers (at $t \to \pm\infty$) and calculate the generating functions for transition probabilities. The important role of representation group theory is discussed in this context.

The problems considered in [1, 3] can be generalized to a model of a singular oscillator with variable frequency:

$$\hat{H} = \frac{1}{2}p^2 + \frac{1}{2}\omega(t)^2 x^2 + \frac{g}{8x^2}, \qquad (1)$$
$$0 < x < +\infty, \quad g = \text{const}, \quad g > -1, \quad \hbar = m = 1.$$

The frequency $\omega(t)$ is an arbitrary real time function. As usual, we propose the boundary conditions:

$$\omega(t) \to \omega_\pm \quad \text{at} \quad t \to \pm\infty$$

which allows one to define the final and initial states of the oscillator.

It is well known that at a fixed t the instantaneous spectrum of the Hamiltonian (1) is equidistant (see, e.g., [5]):

$$E_n = 2\omega(n+j), \qquad j = \frac{1}{2} + \frac{1}{4}\sqrt{1+g}, \qquad n = 0, 1, 2, \ldots. \qquad (2)$$

We note further that the operators

$$J_1 = \frac{1}{4\omega_+}\left(-\omega_+^2 x^2 + p^2\right) + \frac{g}{16x^2}, \quad J_2 = \frac{1}{4}(px + xp), \quad J_3 = \frac{1}{4\omega_+}\left(\omega_+^2 x^2 + p^2\right) + \frac{g}{16x^2}, \qquad (3)$$

satisfy the standard commutation relations of the $su(1,1)$ algebra:

$$[J_1, J_2] = -iJ_3, \qquad [J_2, J_3] = +iJ_1, \qquad [J_3, J_1] = +iJ_2, \tag{4}$$

and the Hamiltonian (1) is a linear combination of operators J_1 and J_3:

$$H(t) = \left(\omega_+ + \frac{\omega^2(t)}{\omega_+}\right) \cdot J_3 + \left(\omega_+ - \frac{\omega^2(t)}{\omega_+}\right) \cdot J_1. \tag{5}$$

The instantaneous eigenfunctions of the Hamiltonian (1) realize the irreducible unitary infinite-dimensional representation of the non-compact $su(1,1)$ algebra. The corresponding Casimir operator ("angular momentum" squared) proves to be a constant and can be calculated directly:

$$\mathbf{J}^2 = J_3^2 - J_1^2 - J_2^2 = \frac{g-3}{16} = j(j-1) \tag{6}$$

so the weight of this representation is j.

For the simplest case $\omega_+ = \omega_- = 1$ the initial and the final states are the eigenfunctions of the J_3 operator:

$$J_3\psi_n = \lambda_n\psi_n, \qquad \lambda_n = n + j.$$

According to Ref. [3], the transition probability between initial $|m\rangle$ and final $|n\rangle$ states can be expressed in terms of the generalized Wigner function for the irreducible representation of the $su(1,1)$ algebra with weight j:

$$w_{mn} = \left|f^{(j)}_{n+j,m+j}\right|^2, \qquad n, m = 0, 1, 2, \ldots. \tag{7}$$

The latter can be obtained by an analytic continuation of a standard Wigner function for the compact $su(2)$ algebra; the details of this technique were described in the Appendix A in [6]. In group theory such a method is known as the "Weyl unitary trick" . The probability proves to depend on the only real parameter ρ, $0 \le \rho < 1$, which has the same sense as in the well-known problem of transitions in a regular oscillator (see, e.g., [8, 9]) and can be calculated from the classical oscillator equation of motion.

A generalization of this approach to the case of unequal initial and final frequencies $\omega_+ \neq \omega_-$ is quite obvious, as, according to (5), the transformation from the initial state basis to the final state one is simply a unitary rotation around the axis 2, i.e., an element of the quasi-unitary group $SU(1,1)$. Omitting intermediate calculations (all details can be found in [7]), we derive the following expression for the transition probabilities between states $|m, \omega_-\rangle$ and $|n, \omega_+\rangle$ (see Eq. (13) from Ref. [3], for comparison):

$$w_{mn} = \frac{L!}{((L-S)!)^2 S!}\frac{\Gamma(L+2j)}{\Gamma(S+2j)}\rho^{L-S}(1-\rho)^{2j}\left[{}_2F_1(-S, L+2j; L-S+1; \rho)\right]^2, \tag{8}$$

where j is defined in (2) and

$$L = \max(m,n), \qquad S = \min(m,n), \qquad L - S = |m-n|.$$

The formula (8) furnishes the ultimate answer to the problem given.

The Gauss hypergeometric function ${}_2F_1$ in (8) has its first argument being integer and negative (or zero), so it reduces to the Jacobi polynomial. Making necessary transformations, we obtain from (8):

$$
\begin{aligned}
w_{mn} &= \frac{m!\,\Gamma(n+2j)}{n!\,\Gamma(m+2j)}\rho^{n-m}(1-\rho)^{2j}\left[P_m^{(n-m,2j-1)}(1-2\rho)\right]^2, \quad n \geq m, \\
w_{mn} &= \frac{n!\,\Gamma(m+2j)}{m!\,\Gamma(n+2j)}\rho^{m-n}(1-\rho)^{2j}\left[P_n^{(m-n,2j-1)}(1-2\rho)\right]^2, \quad m \geq n,
\end{aligned}
\tag{9}
$$

which coincides with the standard quantum-mechanical result for the transitions in the time-dependent singular oscillator obtained by means of the Schrödinger equation solution (see [10] and references therein).

The last point to discuss is the generating functions for the transition probabilities of the singular oscillator. Using the expressions (9), one can derive the following relation (see [7] for details):

$$
G(u,v) = \sum_{m,n} w_{mn} u^m v^n = \frac{\nu^{2j}}{1 - uv \cdot \nu^2}, \quad |u|,|v| < 1 \tag{10}
$$

where

$$
\nu = \frac{2(1-\rho)}{1 - \rho(u+v) + uv + \sqrt{[1-\rho(u+v)+uv]^2 - 4uv(1-\rho)^2}} \tag{11}
$$

For the limiting case of a regular oscillator ($g = 0$, i.e. $j = 3/4$) the expression (10) corresponds to the formulas from [11] for generating functions of odd transitions.

The formulas (10) and (11) are quite convenient to compute various operator mean-values over the transition probability distributions. In particular, for the adiabatic invariant $I = \langle H \rangle / 2\omega$ one obtains

$$
\frac{I_+}{I_-} = \frac{\langle n+j \rangle}{m+j} = \frac{1+\rho}{1-\rho} \tag{12}
$$

for transitions from an arbitrary initial level m .

To summarize, we note that in our paper the transition probabilities of the singular oscillator have been calculated for an arbitrary frequency $\omega(t)$. To solve the problem, the modified FDM was applied and the representation theory for non-compact $su(1,1)$ algebra has been used. The final result for the transition amplitudes has been presented in a self-contained form, which is rather convenient for further applications. The expressions for the generating functions for transition probabilities have been derived and the adiabatic invariant variation at the oscillator evolution has been calculated.

Acknowledgements

This work was partially supported by the Russian Foundation for Basic Research (grant No. 07-02-01116) and by the Ministry of Science and Education of the Russian Federation (grant No. RNP 2.1.1. 1972). One of the authors (M.A.T.) also thanks for partial support the President Grant No. NSh-4961.2008.2 and the President Grant No. MK-2130.2008.2 .

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On Lagrangian formulations for mixed-symmetry HS fields on AdS spaces within BFV-BRST approach

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Abstract

The key aspects of a gauge-invariant Lagrangian description of massive and massless half-integer higher-spin fields in AdS spaces subject to two-rows Young tableaux $Y(s_1, s_2)$ are presented both in unconstrained and with off-shell algebraic constraints formulations on a basis of BFV-BRST operators for nonlinear superalgebras.

Problems of an unified description of known interactions and variety of elementary particles are revealed at high energies (partially accessible in LHC), therefore providing the actual development of higher-spin (HS) field theory due to its close relation to superstring theory on constant curvature spaces, which operates with an infinite set of bosonic and fermionic HS fields subject to multi-row Young tableaux (YT) $Y(s_1, ..., s_k), k \geq 1$ (for a review, see [1]). The article considers the last results of constructing Lagrangian formulations (LFs) for free fermionic HS fields on AdS_d-space with $Y(s_1, s_2)$ in metric-like formalism within BFV-BRST approach [4] as a starting tool for an interacting HS field theory in the framework of Quantum Field Theory, and in part based on the results presented in [2, 3].

This method of constructing an LF for HS fields, developed originally to apply to Hamiltonian quantization of gauge theories with a given LF, consists in a solution of the *problem inverse* to that of the method [4] (as in the case of string field theory and first papers on HS fields [5]) in the sense of constructing a gauge LF w.r.t. a nilpotent BFV-BRST operator Q. Q is constructed from a system O_α of 1-class constraints, including in special non-linear operator superalgebra $\{O_I\}$:$\{O_I\} \supset \{O_\alpha\}$, defined on an auxiliary Fock space and encoding the relations extracting the fields with a fixed (m, s) from the AdS group representation spaces.

A massive spin $\mathrm{s} = (s_1, s_2)$, $s_i = n_i + \frac{1}{2}$, $n_1 \geq n_2$, representation of AdS group in AdS$_d$ space are characterized by $Y(s_1, s_2)$ and realized in space of mixed-symmetry spin-tensors

$$\Phi_{(\mu)_{n_1},(\nu)_{n_2}} \equiv \Phi_{\mu_1 \ldots \mu_{n_1}, \nu_1 \ldots \nu_{n_2} A}(x) \longleftrightarrow \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|}\hline \mu_1 & \mu_2 & \cdot\;\;\cdot\;\;\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \mu_{n_1} \\ \hline \nu_1 & \nu_2 & \cdot\;\;\cdot\;\;\cdot & \cdot & \cdot & \cdot & \cdot & \nu_{n_2} & \multicolumn{2}{c}{} \\ \cline{1-8} \end{array}, \qquad (1)$$

subject to the equations, (for $\beta = (2;3) \Longleftrightarrow (n_1 > n_2; n_1 = n_2)$, r being the inverse squared AdS$_d$ radius, with suppressed Dirac's index A and matrices: $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}(x)$):

$$\left([i\gamma^\mu \nabla_\mu - r^{\frac{1}{2}}(n_1 + \tfrac{d}{2} - \beta) - m],\ \gamma^{\mu_1},\ \gamma^{\nu_1}\right) \Phi_{(\mu)_{n_1},\,(\nu)_{n_2}} = \Phi_{\{(\mu)_{n_1},\nu_1\}\nu_2 \ldots \nu_{n_2}} = 0. \qquad (2)$$

To simultaneous describe of all fermionic HS fields, one introduce a Fock space $\mathcal{H} = \mathcal{H}^1 \otimes \mathcal{H}^2$ generated by 2 pairs of creation $a^i_\mu(x)$ and annihilation $a^{j+}_\mu(x)$ operators, $i, j = 1, 2, \mu, \nu = 0, 1..., d-1$: $[a^i_\mu, a^{j+}_\nu] = -g_{\mu\nu}\delta_{ij}$; and a set of constraints for an arbitrary string-like vector $|\Phi\rangle$:

$$\tilde{t}'_0|\Phi\rangle = [-i\tilde{\gamma}^\mu D_\mu + \tilde{\gamma}(m + \sqrt{r}(g^1_0 - \beta))]|\Phi\rangle = 0, \quad (t^i, t)|\Phi\rangle = (\tilde{\gamma}^\mu a^i_\mu, a^{1+}_\mu a^{2\mu})|\Phi\rangle = 0, \quad (3)$$

$$|\Phi\rangle = \textstyle\sum_{n_1=0}^{\infty} \sum_{n_2=0}^{n_1} \Phi_{(\mu)_{n_1},(\nu)_{n_2}}(x)\, a_1^{+\mu_1} \ldots a_1^{+\mu_{n_1}} a_2^{+\nu_1} \ldots a_2^{+\nu_{n_2}}|0\rangle, \ |\Phi\rangle \in \mathcal{H}, \qquad (4)$$

equivalent to Eqs. (2) for all s and given in terms of an operator D_μ being equivalent to ∇_μ, in its action in $\mathcal{H}$. Fermionic operators $\tilde{t}'_0, t^i$ are defined with help of a set of Grassmann-odd gamma-matrix-like objects $\tilde{\gamma}^\mu, \tilde{\gamma}$ ($\{\tilde{\gamma}^\mu, \tilde{\gamma}^\nu\} = 2g^{\mu\nu}$, $\{\tilde{\gamma}^\mu, \tilde{\gamma}\} = 0$, $\tilde{\gamma}^2 = -1$ [2]), related to conventional gamma-matrices by odd non-degenerate transformation: $\gamma^\mu = \tilde{\gamma}^\mu\tilde{\gamma}$.

To derive a Hermitian BFV–BRST charge Q on a total Hilbert space $\mathcal{H}_{\rm tot}=\mathcal{H}\otimes\mathcal{H}'\otimes\mathcal{H}_{\rm gh}$ we need to deduce a set of 1-class quantities O_I: $\{O_\alpha\} \subset \{O_I\}$, closed under the Hermitian conjugation w.r.t. an odd scalar product $\langle\tilde{\Psi}|\Phi\rangle$ [3] with a measure $d^dx\sqrt{-\det g}$ and supercommutator multiplication $[\ ,\ \}$. As a result, the massive half-integer HS symmetry superalgebra in AdS_d space with $Y(s_1, s_2)$: $\mathcal{A}(Y(2), AdS_d)=\{o_I\}=\{\tilde{t}'_0, t_i, t_i^+, t, t^+, l_i, l_i^+, l_{ij}, l_{ij}^+, g_0^i, \tilde{l}'_0\}$,

$$(t^{i+}; g_0^i; t^+; l^i, l^{+i}; l_{ij}) = (\tilde{\gamma}^\mu a_\mu^{i+}; -a_\mu^{i+}a^{\mu i} + \tfrac{d}{2}; a^{\mu 1}a_\mu^{2+}; -i(a^{\mu i}, a^{+\mu i})D_\mu; \tfrac{1}{2}a_i^\mu a_{\mu j}),\ i \le j \quad (5)$$

$$\tilde{l}'_0 = g^{\mu\nu}(D_\nu D_\mu - \Gamma^\sigma_{\mu\nu}D_\sigma) - r\Big(\textstyle\sum_i(g_0^i + t^{i+}t^i) + \frac{d(d-5)}{4}\Big) + \Big(m + \sqrt{r}(g_0^1 - \beta)\Big)^2, \quad (6)$$

contains a central charge $\tilde{m} = (m - \beta\sqrt{r})$, a subset of (4+12) differential $\{l_i, l_i^+\} \subset \{o_a\}$ and algebraic $\{t_i, t_i^+, t, t^+, l_{ij}, l_{ij}^+\} \subset \{o_{\rm a}\}$ 2-class constraints, number particles operators g_0^i, composing, together with $\tilde{m}$, an invertible supermatrix $\|[o_{\rm a}, o_{\rm b}\}\| = \|\Delta_{\rm ab}(g_0^i, \tilde{m})\| + \|\mathcal{O}(o_I)\|$.

From 2 variants of additive conversion for non-linear superalgebras [6] of $\{o_I\}$ into the 1-class system $\{O_\alpha\}$: 1) for $\{o_{\rm a}\}$ resulting to the unconstrained LF; 2) for differential and part of algebraic constraints: l_i, l_i^+, t, t^+ restricting $\mathcal{A}(Y(2), AdS_d)$ to the surface $\{o^r_{\rm a}\} \equiv \{o_{\rm a}\}\backslash\{l_i, l_i^+, t, t^+\}$ on all stages of the construction resulting to the LF with off-shell γ-traceless and traceless conditions, we consider in detail the first case. To find additional parts o'_I: $o_I \to O_I = o_I + o'_I$, $[o_I, o'_I\} = 0$ such that $[O_I, O_J\} \sim O_K$ we need: a) following to [2, 6] pass to another basis of the constraints $o_I \to \tilde{o}_I = u_I^J o_J$, $\mathrm{sdet}\|u_I^J\| \neq 0$ ($\tilde{\gamma} \notin \{\tilde{o}_I\}$), such that only $\tilde{t}'_0, \tilde{l}'_0$ are changed, $t_0 = -i\tilde{\gamma}^\mu D_\mu$, $l_0 = -t_0^2$, having obtained the modified HS symmetry superalgebra $\mathcal{A}_{mod}(Y(2), AdS_d)$; b) construct its auxiliary representation, the Verma module, with use of Cartan-like decomposition, enlarged from one for Lie superalgebra $\{o'_I\} \setminus \{l'_i, l'^+_i, t'_0, l'_0\}$:

$$\mathcal{A}_{mod}(Y(2), AdS_d) = \{\{t'^+_i, l'^{ij+}, t'^+; l'^{i+}\} \oplus \{g'^i_0; t'_0, l'_0\} \oplus \{t'_i, l'^{ij}, t'; l'^i\} \equiv \mathcal{E}^- \oplus H \oplus \mathcal{E}^+; \quad (7)$$

c) realize the Verma module as formal power series, $\sum_{n\ge 0}\sqrt{r}^n\mathcal{P}_n[(a, a^+)_{\rm a}]$, in a Fock space $\mathcal{H}'$ generated by the same numbers of creation and annihilation operators as ones of the converted 2-class constraints $\{o_{\rm a}^{(r)}\}$: $(a, a^+)_{\rm a}^{(r)} = f_i, f_i^+ b_{ij}, b_{ij}^+, (b_i, b_i^+, b, b^+)$ (for constrained LF).

The solution of the item a) follows from the above requirement on $\tilde{O}_I$ to be in involution and compactly written multiplication table for $\{\tilde{o}_i\}$ in $\mathcal{A}_{mod}(Y(2), AdS_d)$ as follows [2, 6]:

$$[o'_I, o'_J\} = f^K_{IJ}o'_K - (-1)^{\varepsilon(o_K)\varepsilon(o_M)}f^{KM}_{IJ}o'_M o'_K \quad \text{if} \quad [\tilde{o}_I, \tilde{o}_J\} = f^K_{IJ}\tilde{o}_K + f^{KM}_{IJ}\tilde{o}_K\tilde{o}_M, \quad (8)$$

with Grassmann parity $\varepsilon(o_I) = 0, 1$ respectively for bosonic and fermionic o_I. In its turn, the solution of items b), c) is more complicated than one made for $\mathcal{A}'(Y(1), AdS_d)$ [2, 6] and $\mathcal{A}'(Y(2), \mathbf{R}^{1,d-1})$ [3], due to nontrivial entanglement of a triple $(l'^+_1 t'^+ l'^+_2)$ being effectively solved iteratively, therefore extending the known results of Verma module construction [7] and its Fock space realization in $\mathcal{H}'$. Note, that within the conversion $\tilde{M} = \tilde{m} + \tilde{m}' = 0$, whereas new constants m_0, h^i and o'_I explicitly in terms of $(a, a^+)_{\rm a}$ are found as in [2, 7].

A nilpotent BFV-BRST charge Q' for open superalgebra $\mathcal{A}_{conv}(Y(2), AdS_d)$ of $\tilde{O}_I$, in case of the Weyl ordering for quadratic combinations of $\tilde{O}_I$ in the r.h.s. of $[\tilde{O}_I, \tilde{O}_J\}=F^K_{IJ}(\tilde{O}, o')\tilde{O}_K$ and for the $(\mathcal{CP})$-ordering for the ghost coordinates and momenta $\mathcal{C}^I$, $\mathcal{P}_I$ [3] has the form,

$$Q' = O_I\mathcal{C}^I + \tfrac{1}{2}\mathcal{C}^{I_1}\mathcal{C}^{I_2}F^J_{I_2I_1}\mathcal{P}_J(-1)^{\varepsilon(O_{I_2}+\varepsilon(O_J))} + \tfrac{1}{6}\mathcal{C}^{I_1}\mathcal{C}^{I_2}\mathcal{C}^{I_3}F^{J_2J_1}_{I_3I_2I_1}\mathcal{P}_{J_2}\mathcal{P}_{J_1} + \mathcal{O}(\mathcal{C}^4) \quad (9)$$

with completely definite functions $F^{J_2 J_1}_{I_3 I_2 I_1}(\tilde{O}, o')$ resolving the Jacobi identity for $\tilde{O}_I$. Note, that Q' is more complicated than one in [2]) and coincides to one in [3] for $r=0$.

A covariant extraction of $G^i_0 = g^i_0 + g'^i_0(h^i)$ from $\{\tilde{O}_I\}$, to pass to BFV-BRST charge Q for 1-class constraints $\{O_\alpha\}$ only, is based on the condition of independence of $\mathcal{H}_{tot}$ of η^i_G and on the elimination from Q' the terms proportional to $\mathcal{P}^i_G, \eta^i_G : \mathcal{K}^i = (\sigma^i + h^i)$ as in [3]:

$$Q' = Q + \eta^i_G \mathcal{K}^i + \mathcal{B}^i \mathcal{P}^i_G;\ \mathcal{K}^i = G^i_0 + (q^+_i p_i + \eta^+_i \mathcal{P}^i + \textstyle\sum_j (1+\delta_{ij})\eta^+_{ij}\mathcal{P}^{ij} + (-1)^i \eta^+ \mathcal{P} + h.c.);\qquad(10)$$

the same applies to the physical vector $|\chi\rangle \in \mathcal{H}_{tot}$, $|\chi\rangle = |\Phi\rangle + |\Phi_A\rangle$, $|\Phi_A\rangle_{\{(a,a^+)_a = \mathcal{C}=\mathcal{P}=0\}} = 0$, with the use of the BFV-BRST equation $Q'|\chi\rangle = 0$ determining the physical states:

$$Q|\chi\rangle = 0, \quad (\sigma^i + h^i)|\chi\rangle = 0, \quad \varepsilon(|\chi\rangle) = 1, \qquad (11)$$

where the 2nd equations determine the spectrum of generalized spin values for $|\chi\rangle$.

The presence of a redundant gauge ambiguity in the definition of an LF permits to expand Q and $|\chi\rangle$ in powers of the zero-mode pairs $q_0, p_0, \eta_0, \mathcal{P}_0$ as follows [2, 3]:

$$(Q;|\chi\rangle) = \Big(q_0\tilde{T}_0 + \eta_0\tilde{L}_0 + (\eta^+_i q_i - \eta_i q^+_i)p_0 + (q_0^2 - \eta^+_i\eta_i)\mathcal{P}_0 + \Delta Q; \textstyle\sum_{k=0}^{\infty} q_0^k(|\chi^k_0\rangle + \eta_0|\chi^k_1\rangle)\Big).\qquad(12)$$

As a result, the 1st equation in (11) is Lagrangian and take the form together with action

$$\Delta Q|\chi^0_0\rangle + \tfrac{1}{2}\{\tilde{T}_0, \eta^+_i\eta_i\}|\chi^1_0\rangle = 0, \qquad \tilde{T}_0|\chi^0_0\rangle + \Delta Q|\chi^1_0\rangle = 0, \qquad (13)$$

$$\mathcal{S} = \langle\tilde{\chi}^0_0|K\tilde{T}_0|\chi^0_0\rangle + \tfrac{1}{2}\langle\tilde{\chi}^1_0|K\{\tilde{T}_0, \eta^+_1\eta_1\}|\chi^1_0\rangle + \langle\tilde{\chi}^0_0|K\Delta Q|\chi^1_0\rangle + \langle\tilde{\chi}^1_0|K\Delta Q|\chi^0_0\rangle, \quad (14)$$

where we have used an operator $K = \hat{1} \otimes K' \otimes \hat{1}_{gh}$ providing the Hermiticity of Q w.r.t. $\langle\ |\ \rangle$ in $\mathcal{H}_{tot}$ and reality of $\mathcal{S}$. The corresponding LF of a HS field with a given value of spin $\mathrm{s} = (n_1 + \frac{1}{2}, n_2 + \frac{1}{2})$ is a reducible gauge theory of $L = (n_1 + n_2)$-th stage of reducibility, whereas the above constrained LF has reduced field spectrum and $L \leq 3$ for any spin.

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Phases of the Dirac determinant, Abelian Chern-Simons terms and Berry's phases in the field theoretic description of graphene

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Abstract

Abstract: This talk presents a study of massless relativistic Dirac fields in three Euclidean dimensions, at finite temperature and density, in the presence of a uniform electromagnetic background. Apart from explaining the behavior of Hall's conductivity for graphene, our results show a direct relationship between the selection of a phase for the Dirac determinant and the generation (or lack thereof) of Berry's phases and Chern-Simons terms.

1 Introduction

Graphene is a bidimensional array of carbon atoms, packed in a honeycomb crystal structure. Even though its theoretical properties were studied decades ago [1], it was only in 2005 that stable monolayer samples of such material were obtained. Among other properties, the Hall conductivity was measured in such samples, independently, by two groups [2]. Later on, a different behavior of the Hall conductivity was reported [3] for bilayer samples. The main difference between the behavior of the Hall conductivity of mono- and bilayer samples lies in the height of the jump around zero carrier density (or, equivalently, around zero chemical potential).

From a theoretical point of view, the most remarkable feature of graphene is that, in a small momentum approximation, the charge carriers or *quasi*–particles behave as two "flavors" (to account for the spin of the elementary constituents) of massless relativistic Dirac particles in the two non–equivalent representations of the Clifford algebra (corresponding to the two non–equivalent vertices in the first Brillouin zone), with an effective "speed of light" about two orders of magnitude smaller than c [1].

In [4], we showed that a field theoretic calculation at finite temperature and density, based upon ζ–function regularization of the Dirac determinant leads, in the zero temperature limit, to a sequence of plateaux in the Hall conductivity consistent with the ones measured each time the chemical potential goes through a nonzero Landau level. Moreover, it was shown in [5] that two of the three possible combinations of phases of the Dirac determinant in both nonequivalent Clifford representations predict a behavior around zero chemical potential consistent with the ones measured in mono- and bilayer graphene.

This paper presents, in section 2, a brief review of our previous results on the subject, with emphasis on the role of the phase of the determinant in giving rise to different behaviors of the Hall conductivity around zero chemical potential. In section 3, we allow for complex chemical potentials, and concentrate on the contribution due to the lowest Landau level, in order to study the invariance of the effective action under large gauge transformations, i.e., under statistics-preserving transformations. We also discuss the connection among phases of the determinant, Berry's phases and Chern-Simons terms.

2 The Hall conductivity and its dependence on the phase of the determinant

As shown in our previous work on the subject [4, 5], the Hall conductivity can be determined by first evaluating the partition function (equivalently, the effective action) for massless Dirac fermions at finite temperature and density, in two spacial dimensions, in the presence of an external magnetic field perpendicular to the plane, and then performing a boost to a reference frame with orthogonal electric and magnetic fields. In this section, we merely list our main results in those references, with emphasis on the role played by the phase of the Dirac determinant, which appears when treating the infinite tower of states associated to the lowest Landau level. We first consider a single flavor, and one of the two nonequivalent representations of the Clifford algebra.

In order to take into account the effects due to finite temperature and density, we study the theory in Euclidean three-dimensional space, with a compact Euclidean "time" $0 \leq x_0 \leq \beta$, where $\beta = \frac{1}{k_B T}$ (here, k_B is the Boltzmann constant and T is the temperature). We introduce the (real) chemical potential and the magnetic field through a minimal coupling of the theory to an electromagnetic potential $A_\mu = (-i\frac{\mu}{e}, 0, Bx_1)$. Natural units ($c = \hbar = 1$) will be used, unless otherwise stated.

In this scenario, the Euclidean effective action is given by $\log \mathcal{Z} = \log\, det(i\partial\!\!\!/ - eA\!\!\!/)_{AP}$, where the subindex AP indicates that antiperiodic boundary conditions must be imposed in the x_0 direction, in order to ensure Fermi statistics. Now, this is a formal expression, which we will define through a zeta-function regularization, i.e.,

$$S_{eff} = \log \mathcal{Z} \equiv -\frac{d}{ds}\bigg]_{s=0} \zeta(s, \frac{(i\partial\!\!\!/ - eA\!\!\!/)_{AP}}{\alpha}) = -\frac{d}{ds}\bigg]_{s=0} \sum_{\omega} \left(\frac{\omega}{\alpha}\right)^{-s}, \tag{1}$$

where ω represents the eigenvalues of the Dirac operator acting on antiperiodic, square-integrable functions, and α is a parameter introduced to render the zeta function dimensionless (as expected on physical grounds, our final predictions will be α-independent).

So, in order order to evaluate the partition function, we first determine the eigenfunctions, and the corresponding eigenvalues, of the Dirac operator. We propose

$$\Psi_{k,l}(x_0, x_1, x_2) = \frac{e^{i\lambda_l x_0} e^{ikx_2}}{\sqrt{2\pi\beta}} \begin{pmatrix} \varphi_{k,l}(x_1) \\ \chi_{k,l}(x_1) \end{pmatrix} \quad \lambda_l = (2l+1)\frac{\pi}{\beta}.$$

Note that, in the last expression, λ_l, $l = -\infty, ..., \infty$ are the Matsubara frequencies adequate to the required antiperiodic conditions, while the continuous index k represents an infinite degeneracy in the x_2 direction.

The resulting spectrum has two pieces: An asymmetric piece, associated to the lowest Landau level of the Hamiltonian:

$$\omega_l = \tilde{\lambda}_l, \quad \text{with} \quad \tilde{\lambda}_l = (2l+1)\frac{\pi}{\beta} + i\mu \quad \text{and} \quad l = -\infty, ..., \infty,$$

and a symmetric piece

$$\omega_{l,n} = \pm\sqrt{\tilde{\lambda}_l^2 + 2neB} \quad \text{with} \quad n = 1, ..., \infty \quad l = -\infty, ..., \infty,$$

corresponding to eigenfunctions with both components different from zero. In all cases, the degeneracy per unit area is given by the well known Landau factor, $\Delta_L = \frac{eB}{2\pi}$.

The asymmetric part of the spectrum is quite particular. In fact, the corresponding eigenfunction is an eigenfunction of the Pauli matrix σ_3, with eigenvalue $+1$. The eigenfunction with the opposite "chirality" was eliminated by the square integrability condition in x_1. As we will discuss in what follows, this part of the spectrum is the one which requires the consideration of a phase of the determinant when evaluating the effective action. Before going to such evaluation, it is interesting to note the invariance of the whole spectrum under $\mu \to \mu + \frac{2ik\pi}{\beta}$. This invariance is a natural one, since such transformations preserve the antiperiodicity of the eigenfunctions and, thus, the Dirac statistics. They are nothing but the so-called large gauge transformations. We will discuss this point in more detail in section 3.

As is clear from (1), in evaluating the effective action, one must perform the analytic extension of the contributions to the zeta function coming from the nonsymmetric piece of the spectrum, $\zeta_1(s,\mu)$, and the one due to the symmetric piece, $\zeta_2(s,\mu,eB)$.

The analytic extension of $\zeta_2(s,\mu,eB)$ is quite standard, and it relies mainly on performing a Mellin transform and making use of the inversion properties of the Jacobi theta functions. A detailed presentation can be found in [4].

As said before, the extension of $\zeta_1(s,\mu,eB)$ requires a careful consideration of the phase of the determinant. In fact, ζ_1 can be written as

$$\zeta_1(s,\mu) = \Delta_L \left(\frac{2\pi}{\alpha\beta}\right)^{-s} \left[\sum_{l=0}^{\infty} \left[(l+\frac{1}{2}) + i\frac{\mu\beta}{2\pi}\right]^{-s} + \sum_{l=0}^{\infty} \left[-\left((l+\frac{1}{2}) - i\frac{\mu\beta}{2\pi}\right)\right]^{-s}\right], \tag{2}$$

and the definition of the overall minus sign in the second sum depends on the selection of the cut in the complex plane of eigenvalues. As discussed in detail in [5], the usual prescription is to choose the cut such that one does not go through vanishing arguments when continuously transforming eigenvalues with positive real part into eigenvalues with negative real part [6]. This prescription then gives rise to what will be called in the following the standard phase of the determinant (characterized from now on by $\kappa = -1$). One could certainly choose the opposite prescription, which we will call the nonstandard phase ($\kappa = +1$). Once one of the phases is selected, the contribution of ζ_1 to the effective action can be evaluated by making use of the well-known properties of the Hurwitz zeta function, to obtain

$$S^I_{eff}(\kappa) = \Delta_L \left\{\log\left[2\cosh(\frac{\mu\beta}{2})\right] + \kappa\frac{|\mu|\beta}{2}\right\}.$$

When this last contribution is added to the one coming from $\zeta_2(s,\mu,eB)$, one gets for the effective action

$$S_{eff}(\kappa) = \Delta_L \Bigg\{\log\left[2\cosh(\frac{\mu\beta}{2})\right] + \kappa\frac{|\mu|\beta}{2} + \beta\sqrt{2eB}\,\zeta_R\left(-\frac{1}{2}\right)$$
$$+ \sum_{n=1}^{\infty} \log\left[\left(1 + e^{-(\sqrt{2neB}-\mu)\beta}\right)\left(1 + e^{-(\sqrt{2neB}+\mu)\beta}\right)\right]\Bigg\}.$$

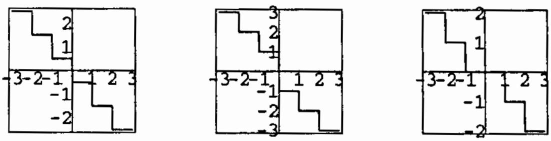

Figure 1: Hall conductivity for different selections of the phase of the determinant. Left to right: $K = 1$, $K = 2$, $K = 0$. In all cases, the horizontal axis represents $\nu_C = \mathrm{sgn}(\mu)\,\mu^2/2eB\hbar c^2$ and the vertical one, $\sigma_{xy}\,h/4e^2$.

From this last expression, the finite-temperature charge density can be obtained as $j^0(\kappa) = \frac{-e}{\beta}\frac{d}{d\mu}S_{eff}(\kappa)$. In the zero-temperature limit ($\beta \to \infty$), and recovering physical units, it reduces to

$$j^0(2ec^2\hbar Bn < \mu^2 < 2eBc^2\hbar(n+1)) = \frac{-(n+\frac{1+\kappa}{2})ce^2B}{h}\,sign(\mu)\,,$$

where $n = [\frac{\mu^2}{2eB\hbar c}]$, and $[x]$ is the integer part of x.

In order to obtain the Hall conductivity, one must perform a boost to a reference frame with crossed electric and magnetic fields. The final contribution to the Hall conductivity from each fermion species and one irreducible representation is given by [5] $\sigma_{xy} = \frac{-(n+\frac{1+\kappa}{2})e^2}{h}\,sign(\mu)$.

Now, the phases of the determinant in both irreducible representations can be selected with the same or with opposite criteria. When this is taken into account, and an overall factor of 2 is included, to take both fermion species into account, on obtains for the total zero-temperature Hall conductivity

$$\sigma_{xy} = \frac{-4(n+\frac{K}{2})e^2}{h}\,sign(\mu)\,,$$

where $K = 0$ corresponds to selecting the standard phase of the determinant in both irreducible representations, $K = 1$ corresponds to choosing opposite criteria for the phases, and $K = 2$, to choosing both phases in the nonstandard way. The dependence of the Hall conductivity on the classical filling factor (ν_C) is presented in figure 1, for the three values of K. From that figure, it is clear that the behavior of monolayer graphene, as reported in [2], corresponds to $K = 1$, i.e., to choosing opposite phases of the determinant in both representations. In fact, in this case the (rescaled) Hall conductivity shows a jump of height 1 for $\nu_C = 0$, and further jumps of the same magnitude for $\nu_C = \pm 1, \pm 2, \ldots$. In turn, the behavior of bilayer graphene, as reported in [3] is exactly reproduced by $K = 2$ (nonstandard selection of the phase in both representations).

3 Phases of the determinant as geometric phases

To analyze the physical meaning of the invariance of the effective action under large gauge transformations in this context, we go back to the zeta function associated to the asymmetric part of the spectrum, for one fermion species and one representation, this time allowing or an imaginary part in the chemical potential, $\tilde{\mu} = \mu + i\gamma$, while always keeping $\mu \neq 0$. n this case, one must be careful when splitting the infinite sum as in (2). In fact, such plitting must be different for different γ-ranges, to make sure that all the eigenvalues in ach infinite sum have a real part with the same sign, which is crucial in defining the phase. 'or example, for $-\frac{1}{2} < \frac{\gamma\beta}{2\pi} < \frac{1}{2}$, one has

$$S^I_{eff}(-\frac{1}{2} < \frac{\gamma\beta}{2\pi} < \frac{1}{2}) = -\Delta_L \frac{d}{ds}\Big]_{s=0} \Big\{ \sum_{l=0}^{\infty} [\,(2l+1)\pi/\beta + i\mu - \gamma\,]^{-s} + \sum_{l=0}^{\infty} e^{-is\theta}\left[(2l+1)\pi/\beta + i(\mu+i\gamma)\,e^{-i\theta}\right]^{-s} \Big\} .$$

Now, the values of θ such that the second term in the RHS does vanish are those ones or which, simultaneously, $(2l+1)\pi/\beta + \mu\sin\theta - \gamma\cos\theta \;=\; 0 \;= \mu\cos\theta + \gamma\sin\theta$.

As before, we consider here two different definitions of the phase of the determinant, vhich correspond to the standard definition for the phase $\kappa = -1$, and to the nonstandard ne $\kappa = +1$. With each one of these prescriptions, the contribution of the asymmetric pectrum to the effective a action in this range is given by

$$S^I_{eff}(-\frac{1}{2} < \frac{\gamma\beta}{2\pi} < \frac{1}{2}) = \Delta_L \left\{ \frac{(\kappa+1)\beta}{2} \operatorname{sgn}\mu\,(\mu+i\gamma) + \log\left(e^{-\frac{\beta}{2}(\mu+i\gamma)(1+\operatorname{sgn}\mu)} + e^{\frac{\beta}{2}(\mu+i\gamma)(1-\operatorname{sgn}\mu)}\right)\right\}. \tag{3}$$

Things are entirely different for $\frac{\gamma\beta}{2\pi} = \pm\frac{1}{2}$. In this case, one mode in the infinite sum lefining the zeta function has a vanishing real part. A careful treatment shows that, at uch points, S^I_{eff} is discontinuous. For instance, $S^I_{eff}(\frac{\gamma\beta}{2\pi} = +\frac{1}{2})$ coincides with $\lim_{\frac{\gamma\beta}{2\pi}\to\frac{1}{2}^-}$ of (3). An equally carefully treatment of the case $\frac{\gamma\beta}{2\pi} = -\frac{1}{2}$ shows that $S^I_{eff}(\frac{\gamma\beta}{2\pi} = -\frac{1}{2}) = S^I_{eff}(\frac{\gamma\beta}{2\pi} = \frac{1}{2})$. This analysis can be extended to other ranges of variation of $\frac{\gamma\beta}{2\pi}$, to obtain

$$S^I_{eff}((k-\frac{1}{2}) < \frac{\gamma\beta}{2\pi} \;\le\; (k+\frac{1}{2})) = \Delta_L \left\{ \frac{(\kappa+1)\beta}{2} \operatorname{sgn}\mu\,[\mu + i(\gamma - \frac{2k\pi}{\beta})] + \log\left(e^{-\frac{\beta}{2}(\mu+i(\gamma-\frac{2k\pi}{\beta}))(1+\operatorname{sgn}\mu)} + e^{\frac{\beta}{2}(\mu+i(\gamma-\frac{2k\pi}{\beta}))(1-\operatorname{sgn}\mu)}\right)\right\}, \tag{4}$$

'or $k = -\infty, ..., \infty$.

This expression shows that the contribution to the effective action of the nonsymmetric art of the spectrum, in this representation of the gamma matrices, is invariant under large ;auge transformations, no matter which phase of the determinant is selected. As already aid, such transformations must constitute an invariance. In fact, an increase of $i\gamma$ in the hemical potential corresponds to the multiplication of the eigenfunctions with a phase, .e., $\psi_{k,l}(x) \to e^{i\gamma x_0}\psi_{k,l}(x)$. So, an increase $i\gamma = \frac{2i\pi}{\beta}$ is a pure gauge transformation which, noreover, preserves the antiperiodicity in x_0.

Due to the fact that these eigenfunctions are eigenfunctions of σ_3, one can equivalently rrite gauge transformations in the form $\psi_{k,l}(x) \to e^{i\frac{\sigma_3}{2}2\gamma x_0}\psi_{k,l}(x)$. This last expression hows that, as x_0 grows from 0 to β, spinors are rotated by $2\gamma\beta$, since $\frac{\sigma_3}{2}$ is the generator f rotations in the plane x_1x_2. In particular, $\gamma = \frac{2\pi}{\beta}$ corresponds to a 4π-rotation around he magnetic field. On the other hand, $\gamma = \frac{\pi}{\beta}$ corresponds to a 2π-rotation. At finite temperature, such transformation changes the statistics to a bosonic one. For $\kappa = +1$, it also gives rise to an overall phase of π per unit degeneracy in the partition function. Such phase is the contribution which survives in the zero temperature limit. Always in the zero temperature limit, $\kappa = +1$ gives rise to a Chern-Simons term in the effective action. Invariance of the partition function under rotations of 2π requires the reduced flux (Δ_L) to be an integer, which fixes the coefficient in front of the Chern-Simons term. Such term is not present for $\kappa = 0$.

To summarize, in each representation, the effective action per unit degeneracy is invariant under large gauge transformations, with any of the two possible selections of phase. As a result, the invariance persists no matter which of the three possible combinations of phases is selected. Moreover, each of the two selections of phase in each representation corresponds to a different geometric phase under the rotation of spinors along a closed path around the magnetic field ($\kappa = -1$: no geometric phase; $\kappa = +1$: geometric phase of π). So, the three possible combinations of phases of the determinant then give a total phase in the partition function of π ($K = 1$, monolayer), 2π ($K = 2$, bilayer), or 0 ($K = 0$), to be compared with the Berry phases studied, for instance, in [7]. Finally, we note that these three values of K also correspond to the three nonequivalent unitary representations of the generator of the cyclic group C_3, which is the relevant symmetry in the case of free graphene.

Acknowledgements: I thank the organizers of SPMTP08 for a very interesting conference and for the nice atmosphere enjoyed during the event. This work was partially supported by Universidad Nacional de La Plata (Proyecto 11/X381) and CONICET (PIP 6160).

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Reconstruction Theorem in Noncommutative Quantum Field Theory

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Abstract

It has been proven that the reconstruction theorem is valid also in NC QFT. Given set of Wightman functions determines quantum field operators and the corresponding vector space.

1 Introduction

The Wightman reconstruction theorem in QFT determines the equivalence between two formulations of QFT in the axiomatic approach. One way is to start with the definition of quantum field operator φ_f, where f is some sufficiently smooth function.

Let us recall that a quantum field operator can not be defined in a point. Only smeared operators, symbolically written down as

$$\varphi_f \equiv \int \varphi(x)\, f(x)\, dx,$$

are well defined. In QFT the standard assumption is that $f(x)$ are test functions of tempered distributions.

The axiom of cyclicity of vacuum states plays a central role in formulation of QFT.

This axiom means that the theory is fully defined by the set of vectors: vacuum vector $\Psi_0, \Phi_1 = \varphi_{f_1}\Psi_0, \ldots \Phi_k = \varphi_{f_k} \ldots \varphi_{f_1}\Psi_0$, that is any vector, which belongs to the space under consideration, can be approximated by some finite linear combination of these vectors with arbitrary accuracy.

The scalar product between vectors $\Phi_k = \varphi_{f_k} \ldots \varphi_{f_1}\Psi_0$ and $\Psi_n = \varphi_{f_{k+1}} \ldots \varphi_{f_n}\Psi_0$ is determined by Wightman functions

$$\langle \Phi_k, \Psi_n \rangle = \int d\,x_1 \ldots d\,x_n$$

$$W(x_1, \ldots, x_n)\, \overline{f_1(x_1)} \cdots \overline{f_k(x_k)}\; f_{k+1}(x_{k+1}) \cdots f_n(x_n), \tag{1}$$

where

$$W(x_1, \ldots, x_n) \equiv \langle \Psi_0, \varphi(x_1) \ldots \varphi(x_n)\, \Psi_0 \rangle.$$

As vacuum vector is a cyclic one, the scalar product between two arbitrary vectors in the space in question is determined by some linear combination of Wightman functions with arbitrary accuracy.

In accordance with the reconstruction theorem, if the set of Wightman functions is given, we can construct the underlying space and quantum field operators [1] - [3].

The aim of this report is to extend this construction on noncommutative quantum field theory (NC QFT).

NC QFT being one of the generalizations of standard QFT is intensively developed during the last years. The idea of such a generalization of QFT ascends still to Heisenberg.

The present development in this direction is connected with the construction of noncommutative geometry and new physical arguments in favour of such a generalization of QFT. Essential interest in NC QFT is also connected with the fact that in some cases it is obtained as a low-energy limit from the string theory (for a review see [4]).

The simplest and at the same time the most studied version of noncommutative theory is based on the following Heisenberg-like commutation relations between coordinates:

$$[\hat{x}_\mu, \hat{x}_\nu] = i\,\theta_{\mu\nu}, \tag{2}$$

where $\theta_{\mu\nu}$ is a constant antisymmetric matrix.

NC QFT can be formulated also in commutative space by replacing the usual product by the star (Moyal-type) product:

$$\varphi(x) \star \varphi(x) = \exp\left(\frac{i}{2}\,\theta^{\mu\nu}\,\frac{\partial}{\partial x^\mu}\,\frac{\partial}{\partial y^\nu}\right)\varphi(x)\varphi(y)|_{x=y}. \tag{3}$$

This product can be extended to the corresponding product in different points:

$$\varphi(x_1) \star \cdots \star \varphi(x_n) = \prod_{a<b\leq n} \exp\left(\frac{i}{2}\,\theta^{\mu\nu}\,\frac{\partial}{\partial x_a^\mu}\,\frac{\partial}{\partial x_b^\nu}\right)\varphi(x_1)\ldots\varphi(x_n). \tag{4}$$

Wightman approach in NC QFT was formulated in [5], [6] and [7].

Formally the Wightman functions can be written down as follows:

$$W(x_1, \ldots, x_n) = \langle \Psi_0, \varphi(x_1) \star \cdots \star \varphi(x_n)\Psi_0 \rangle, \tag{5}$$

where Ψ_0 is a vacuum vector. The formal expression (5) actually means that the scalar product of the vectors $\Phi_k = \varphi_{f_k} \cdots \varphi_{f_1}\Psi_0$ and $\Psi_n = \varphi_{f_{k+1}} \cdots \varphi_{f_n}\Psi_0$ is the following:

$$\langle\, \Phi_k, \Psi_n \,\rangle = \int d\,x_1 \ldots d\,x_n\, W(x_1, \ldots, x_n) \cdot$$

$$\overline{f_1(x_1)} \star \cdots \star \overline{f_k(x_k)} \star f_{k+1}(x_{k+1}) \star \cdots \star f_n(x_n),$$

$$W(x_1, \ldots, x_n) = \langle \Psi_0, \varphi(x_1)\cdots\varphi(x_n)\Psi_0 \rangle. \tag{6}$$

This choice of the Wightman functions reflects the natural physical assumption, that noncommutativity should change the product of operators not only in coinciding points, but also in different ones.

In this report we give a rigorous definition of quantum field operator in NC QFT. We extend the axiomatic construction of field operators on NC QFT and construct the space on the dense domain of which quantum field operator is well defined.

It has been proven that the $\star$-multiplication is well defined for the functions $f_i(x_i)$, if $f_i(x_i) \in S^\beta$, $\beta < 1/2$. S^β is a Gel'fand-Shilov space [8], [9].

Moreover after the $\star$-multiplication we obtain functions which belong to the space S^β with the same β as $f_i(x_i)$ [8].

2 Definition of Quantum Field Operators in NC QFT

et us define rigorously quantum field operator φ_f. To this end we construct a closed and ondegenerate space J such that operators φ_f be well defined on dense domain of J.

The difference of noncommutative case from commutative one is that action of the perator φ_f is defined by the $\star$-product.

Construction of space J we shall begin with introduction of set M of breaking sequences f the following kind

$$g = \{g_0, g_1, \ldots g_k\}, \tag{7}$$

here $\quad g_0 \in \mathbb{C}, \qquad g_1 = g_1^1(x_1), x_1 \in \mathbb{R}^4,$

$$g_i = g_i^1(x_1) \star \cdots \star g_i^i(x_i), \quad x_j \in \mathbb{R}^4, 1 \le j \le i;$$

depends on g.

Addition and multiplication by complex numbers of the above mentioned sequences are efined component by component.

The every possible finite sums of the sequences belonging M form space J_0' on which ction of the operator φ_f, $f = f(x)$, $x \in \mathbb{R}^4$ will be determined.

The operator φ_f is defined as follows

$$\varphi_f\, g = \{f g_0, f \star g_1, \ldots f \star g_k\}, \tag{8}$$

here $f \star g_i = f(x) \star g_i^1(x_1) \star \cdots \star g_i^i(x_i)$.

s $f \star (g_i + \tilde{g}_i) = f \star g_i + f \star \tilde{g}_i$, and any vector of space J_0' is the sum of the vectors belonging o set M, the operator φ_f is determined on any vector of space J_0' and $\varphi_f \Phi \in J_0', \forall\, \Phi \in J_0'$.

Scalar product of vectors in J_0' we shall define with the help of Wightman functions $V(x_1, \ldots, x_n) \equiv \langle \Psi_0, \varphi(x_1) \ldots \varphi(x_n) \Psi_0 \rangle$. We shall consider firstly a chain of vectors: acuum vector $\Psi_0 = \{1, 0, \ldots 0\}, \Phi_1 = \varphi_{f_1} \Psi_0, \ldots \Phi_k = \varphi_{f_k} \cdots \varphi_{f_1} \Psi_0$, $f_i = f_i(x_i)$, $x_i \in \mathbb{R}^4$. vidently, $\Phi_k = \{0, \ldots f_k \star \ldots \star f_1, 0 \ldots 0\}$ and

$$\Psi_n = \varphi_{f_{k+1}} \cdots \varphi_{f_n} \Psi_0 = \{0, \ldots f_{k+1} \star \ldots \star f_n, 0 \ldots 0\}.$$

t is obvious, that J_0' is a span of the vectors of such type. Scalar product of vectors Φ_k nd Ψ_n is

$$\begin{aligned} \langle \Phi_k, \Psi_n \rangle = \langle \Psi_0, \varphi_{\bar{f}_1} \cdots \varphi_{\bar{f}_k} \varphi_{f_{k+1}} \cdots \varphi_{f_n} \Psi_0 \rangle = \\ \int d x_1 \ldots d x_n \, W(x_1, \ldots, x_n) \cdot \\ \overline{f_1(x_1)} \star \cdots \star \overline{f_k(x_k)} \star f_{k+1}(x_{k+1}) \star \cdots \star f_n(x_n). \end{aligned} \tag{9}$$

he adjoined operator φ_f^* is defined by the standard formula. If operator φ_f is Hermitian hen $\varphi_f^* = \varphi_{\bar{f}}$. Here we consider only Hermitian (real) operators, but the construction can e easily extended to complex fields.

Let us point out that a condition

$$\langle \Phi_k, \Psi_n \rangle = \overline{\langle \Psi_n, \Phi_k \rangle} \tag{10}$$

fulfilled, if (as well as in commutative case),

$$W(x_1, \ldots, x_n) = \overline{W(x_n, \ldots, x_1)}. \tag{11}$$

The required condition is satisfied, owing to antisymmetry of $\theta^{\mu\nu}$.

As any vector of space J_0' is a finite sum of the vectors belonging to the set M, we can directly define scalar product of any vectors of space J_0' .

Let us stress that if the $\star$-product acts only in coinciding points and is substituted by usual one in different points then given construction can also be fulfilled, only in the different points we have to put $\theta^{ij} = 0$. But in this case the function $f(x,y) = f(x)f(y), x \neq y, f(x,x) = f(x) \star f(x)$ is not continuous when $x = y$, and we have to consider some regularization.

As well as in commutative case, we need to pass from J_0' to nondegenerate and closed space J.

The space J_0' can contain isotropic, i.e. orthogonal to J_0' vectors which, as is known, form subspace. Designating isotropic space as $\tilde{J}_0$ and passing to factor-space $J_0 = J_0'/\tilde{J}_0$, we obtain nondegenerate space, i.e. a space which does not contain isotropic vectors. For closure of space J_0 we assume, as well as in commutative case, that J_0 is normalized space. If the metrics of J_0 is positive, norm $\Phi \equiv \|\Phi\|$ can be defined by the formula $\|\Phi\| = (\langle\Phi,\Phi\rangle)^{1/2}$.
$\bar{J}_0$ (a closure of J_0) is carried out with the help of standard procedure - closure to the introduced norm. This space, in turn, can contain isotropic subspace $\tilde{J}$.

Factor-space $J = \bar{J}_0/\tilde{J}$, obviously, will be nondegenerate space.

Thus, we constructed closed and nondegenerate space J such that operators φ_f are obviously determined on dense domain J_0. Hence, every vector of J can be approximated with arbitrary accuracy by the vectors of the type

$$\varphi_{f_1} \cdots \varphi_{f_n} \Psi_0, \tag{12}$$

where Ψ_0 is a vacuum vector. In other words the vacuum vector Ψ_0 is cyclic, i.e. the axiom of cyclicity of vacuum is fulfilled.

Let us point out that in commutative case construction of space J begins with introduction of sequences g determined by the formula

$$g = \{g_0, g_1, \dots g_k\}, \tag{13}$$

in which, however, $g_i \equiv g_i(x_1, \dots x_i)$ are smooth functions of variables $x_j \in \mathbb{R}^4$. We shall note that in the commutative case, starting with J_0', we shall come to the same space J. Really, as space of functions of a type $g_i^1(x_1)\, g_i^2(x_2) \dots g_i^i(x_i)$ is dense in space of functions $g_i(x_1, \dots x_i)$, we can complete J_0' up to space of the above mentioned sequences and then carry out the standard construction of space J.

3 Test Functions Space

Let us consider the spaces in which the $\star$-multiplication is well-defined. Gel'fand and Shilov proved [10] that if $f(x) \in S^\beta$ (see ineq. (14)) then the series of derivatives of infinite order can be well-defined in such a space. Thus we assume that $f(x) \in S^\beta$ and prove that the $\star$-product is well-defined only if each f_i belongs to the Gel'fand-Shilov space S^β, $\beta < 1/2$. The $\star$-product can be also well-defined if $\beta = 1/2$, but only for functions which satisfy inequality (14) with sufficiently small B.

Let us recall the definition and basic properties of Gel'fand-Shilov spaces S^{β}. In the case of one variable $f(x)$, $x \in \mathbb{R}^1$ belongs to the space S^{β}, if the following condition is satisfied:

$$\left| x^k \frac{\partial^q f(x)}{\partial x^q} \right| \leq C_k B^q q^{q\beta}, \quad -\infty < x < \infty, \quad k, q \in \mathbb{N}, \tag{14}$$

where the constants C_k and B depend on the function $f(x)$. Below we use the inequality (14) only at $k = 0$, $C_0 \equiv C$. As our results do not depend on constant C, in what follows we put $C = 1$. In the case of a function of several variables, inequality (14) holds for any partial derivative.

In accordance with eq. (3)

$$f(x) \star f(y) = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{i}{2} \theta^{\mu\nu} \frac{\partial}{\partial x^{\mu}} \frac{\partial}{\partial y^{\nu}} \right)^n f(x) f(y) \equiv \sum_{n=0}^{\infty} \frac{D_n}{n!}. \tag{15}$$

We have to find the conditions under which the series (15) converges. After some calculations we come to the inequality [8]:

$$\left| \frac{D_n}{n!} \right| < \tilde{B}^n n^{-2n\gamma}, \quad \tilde{B} = 4 e \theta B^2, \quad \gamma = 1 - 2\beta. \tag{16}$$

For any $\tilde{B}$ the series

$$\sum_{n=0}^{\infty} \tilde{B}^n n^{-2n\gamma} \tag{17}$$

converges if $\gamma > 0$, i.e. $\beta < 1/2$, and diverges if $\beta > 1/2$. If $\beta = 1/2$ the series converges if $\tilde{B} < 1$. Thus we come to the conclusion that the series (15) for arbitrary B and C is a convergent one if $\beta < 1/2$ and divergent if $\beta > 1/2$. If $\beta = 1/2$ the series converges at sufficiently small B.

4 Space-space Noncommutativity

It is known that the construction of NC QFT in a general case ($\theta^{0i} \neq 0$) meets serious difficulties with unitarity and causality. For this reason the version with $\theta^{0i} = 0$ (space-space noncommutativity) draws special attention. Then always there is a system of coordinates, in which only $\theta^{12} = -\theta^{21} \neq 0$. Thus, when $\theta^{0i} = 0$, without loss of generality it is possible to choose coordinates x^0 and x^3 as commutative and coordinates x^1 and x^2 as noncommutative.

Let us point out that in this case the main axiomatic results: CPT and spin-statistics theorems, Haag's theorem remain valid [5] - [7], [11].

Here we consider the condition of local commutativity in NC QFT. We remind that the class of test functions in the ordinary QFT contains functions with compact support. In the case of noncommutative Wightman functions, however, the set of test functions consists of functions only with non-compact support in the NC coordinates.

Note, however, that in the case of space-space noncommutativity, i.e. $\theta^{0i} = 0$, the test functions can still have finite support in the commutative directions x_0, x_3. As a result, the local commutativity condition can be formulated in these directions as

$$\varphi_{f_1} \varphi_{f_2} = \varphi_{f_2} \varphi_{f_1}, \tag{18}$$

where the test functions $f_1(x)$ and $f_2(x')$ are zero everywhere except on space-like separated finite domains O and O' in the commutative coordinates, i.e. for each pair of points $x \in O$ and $x' \in O'$ $(x_0 - x_0')^2 - (x_3 - x_3')^2 < 0$, but without any restriction in the noncommutative directions x_1 and x_2.

This condition is equivalent to the following condition for Wightman functions:

$$W(x_1, \ldots, x_i, x_{i+1}, \ldots, x_n) = W(x_1, \ldots, x_{i+1}, x_i, \ldots, x_n), \tag{19}$$

if supp $f_i \in O_i \times R^2$, supp $f_{i+1} \in O_{i+1} \times R^2$, $O_i \sim O_{i+1}$, which means that the above mentioned condition is fulfilled for any points $x_i \in O_i \times R^2$ $x_{i+1} \in O_{i+1} \times R^2$.

5 Conclusions

We have rigorously constructed field operators in NC QFT and have proven that for the given set of Wihtman functions we can reconstruct QFT.

The carried out construction of the closed and nondegenerate space, such that operators φ_f are determined on its dense domain, is important for any rigorous treatment of the axiomatic approach to NC QFT via NC Wightman functions and the derivation of rigorous results such as CPT and spin-statistics theorems.

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Dirac-type operators on curved spaces

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Abstract

Higher order symmetries corresponding to Killing tensors are investigated. The intimate relation between Killing-Yano tensors and non-standard supersymmetries is pointed out. In the Dirac theory on curved spaces, Killing-Yano tensors generate Dirac type operators involved in interesting algebraic structures.

1 INTRODUCTION

One of the key concepts in physics is that of symmetries. Noether's theorem gives a correspondence between symmetries and conserved quantities. For the geodesic motions on a space-time the usual conserved quantities are correlated with the isometries which correspond to Killing vectors. Sometimes a space-time could admit higher order symmetries corresponding to Killing tensors. These are known as *hidden symmetries* and the typical example is the Runge-Lenz vector in the Kepler-Coulomb problem. The corresponding conserved quantities are quadratic, or, more general, polynomial in momenta.

In the case of the gravitational interactions, a consistent perturbative quantization is not available, even if there are no fermions. The implementation of fermions on curved space-times represents an additional difficulty. Fermions are essentially quantum objects while the space-time backgrounds are classical. Sometimes it can appear anomalies representing discrepancies between the conservation laws at the classical level and the corresponding ones at the quantum level.

The above two problems are correlated. The Killing tensors could be symmetric or antisymmetric. The symmetric ones are used to construct the conserved quantities, polynomials in momenta, while the antisymmetric ones are the appropriate geometrical objects to be coupled with the fermionic degrees of freedom.

2 KILLING TENSORS

Let (M, g) be an $n-$dimensional Riemannian manifold, the covariant derivative in the tensor formalism is defined using the Levi-Civita connection and the indices $\mu, \nu, \ldots$ will be raised and lowered with the metric $g_{\mu\nu}$ or its inverse $g^{\mu\nu}$.

Definition 1 *A symmetric tensor of $K_{\mu_1\ldots\mu_r}$ of rank $r > 1$ satisfying a generalized Killing equation*

$$K_{(\mu_1\ldots\mu_r;\lambda)} = 0\,. \tag{1}$$

is called a Stäckel-Killing (S-K) tensor.

The relevance in physics of the S-K tensors is given by the following proposition which could be easily proved:

Proposition 1 *A symmetric tensor K on M is a S-K tensor if and only if the quantity*

$$K = K_{\mu_1\dots\mu_r}\dot{s}^{\mu_1}\cdots\dot{s}^{\mu_r} \tag{2}$$

is constant along every geodesic s in M.

Definition 2 *A differential p−form f is called a K-Y tensor if its covariant derivative $f_{\mu_1\dots\mu_p;\lambda}$ is totally antisymmetric.*

Equivalently, a tensor is called a K-Y tensor of rank p if it is totally antisymmetric and satisfy the equation

$$f_{\mu_1\dots(\mu_p;\lambda)} = 0\,. \tag{3}$$

These two generalizations (1) and (3) of the Killing vector equation could be related. Let $f_{\mu_1\dots\mu_p}$ be a K-Y tensor, then the tensor field

$$K_{\mu\nu} = f_{\mu\mu_2\dots\mu_p}f^{\mu_2\dots\mu_p}{}_{\nu} \tag{4}$$

is a S-K tensor and it sometimes refers to this S-K tensor as the associated tensor to f. However, the converse statement is not true in general: not all S-K tensors of rank 2 are associated to a K-Y tensor.

3 DIRAC-TYPE OPERATORS

To describe spin-1/2 particles on a curved space-time we use the *standard* Dirac operator D_s

$$D_s = i\gamma^\mu\nabla_\mu \tag{5}$$

where ∇_μ are the spin covariant derivatives including spin-connection, while γ^μ are the standard Dirac matrices carrying natural indices.

Each K-Y tensor $f_{\mu\nu}$ produces a *non-standard* Dirac operator of the form [1]

$$D_f = i\gamma^\mu(f_\mu{}^\nu\nabla_\nu - \frac{1}{6}\gamma^\nu\gamma^\rho f_{\mu\nu;\rho}) \tag{6}$$

which anticommutes with the standard Dirac operator D_s and can be involved in new types of genuine or hidden (super)symmetries.

3.1 COVARIANTLY CONSTANT K-Y TENSORS

Remarkable superalgebras of Dirac-type operators can be produced by special second-order K-Y tensors that represent square roots of the metric tensor.

Definition 3 *The non-singular real or complex-valued K-Y tensor f of rank 2 defined on M which satisfies*

$$f^\mu{}_\alpha f_{\mu\beta} = g_{\alpha\beta}\,, \tag{7}$$

is called an unit root of the metric tensor of M, or simply an unit root of M.

Let us observe that (7) is a particular case of equation (4) with the metric tensor as an ordinary S-K tensor. It was shown that any K-Y tensor that satisfy equation (7) is covariantly constant [2, 3, 4], i. e. $f_{\mu\nu;\sigma} = 0$.

The covariantly constant K-Y tensor gives rise to Dirac-type operators of the form (6) connected with the standard Dirac operators as follows:

Theorem 1 *The Dirac-type operator D_f produced by the K-Y tensor f satisfies the condition*

$$(D_f)^2 = D_s^2 \tag{8}$$

if and only if f is an unit root.

Proof: The arguments of Ref. [2] show that the condition (8) is equivalent with equation (7) f being a covariantly constant K-Y tensor. ■

3.2 COVARIANTLY NON-CONSTANT K-Y TENSORS

The covariantly non-constant K-Y tensors do not represent "square roots" of the metric tensor, but generate non-trivial S-K tensor (4). Consequently, Dirac-type operator (6) associated with a covariantly non-constant K-Y tensor does not close to D_s^2, but to different other conserved operators of the Dirac theory. In general the algebra of the corresponding conserved operators does not close as a finite Lie algebra. For example, in the case of the 4-dimensional Euclidean Taub-NUT space, the dynamical algebra of the conserved operators can be organized as a graded loop superalgebra of the Kac-Moody type [5, 6, 7].

Acknowledgements

This work is supported in part by a joit Romanian-LIT, JINR, Dubna Research Project, theme no. 05-6-1060-2005/2010.

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Quantum Chromodynamics

arity violation in QCD motivated hadronic models at non-zero baryon densities

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Abstract

We investigate how a large baryon density may induce spontaneous parity violation in the meson sector of QCD. The analysis is done for an idealized homogeneous and infinite nuclear matter when the influence of density can be examined with the help of constant chemical potential. We elaborate a novel mechanism of parity breaking based on interplay between lightest and heavy meson condensates which cannot be realized in the pion sector solely. The analysis at intermediate energy scales is done by using an effective Lagrangian that includes two scalar and pseudoscalar multiplets. We scan all possible sectors of meson physics and argue that the parity breaking phenomenon is rather typical than exotic when chemical potentials are large enough.

Introduction

e appearance of P- violation for sufficiently large values of temperature and/or the mical potential has been attracting much interest during last decades to search it both dense nuclear matter (in neutron/quark stars and heavy ion collisions at intermediate ergies) and in strongly interacting quark-gluon matter ("quark-gluon plasma" in heavy collisions at very high energies) . In particular, at finite baryon density it is a possibility jectured by A.B.Migdal in [1] in nuclear physics long ago (and reviewed in [2], see also e recent development in [3]-[6]). One has also to mention a phenomenon of $(C)P$-parity eaking in meta-stable nuclear bubbles created in hot nuclear matter [7] and/or in the esence of a strong background magnetic field [8] which however theoretically is quite ferent in its origin and will not be linked to in the present paper.

In fact, some time ago it was proved quite rigorously in [9] that parity, P, and vector vor symmetry could not undergo spontaneous symmetry breaking in a vector like theory h as QCD. This is thus a well established result in strong interactions at *zero* chemical ential. Finite baryon density however results in a manifest breaking of CP- invariance. e presence of a finite chemical potential leads to the presence of a *constant* imaginary

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zeroth-component of a vector field and the partition function of QCD is not anymore i variant under a CP- transformation. The conditions under which the results of [9] we proven (positivity of the measure) then do not hold anymore.

In this work we shall attempt to explore the interesting issue of P-parity breakin employing effective lagrangian techniques in the range of nuclear densities for which hadro phase persists and quark percolation does not occur yet.

A novel mechanism proposed in our papers [10, 11] is essentially based on interpla between lightest and heavy meson states and cannot be realized solely in the Goldston boson (pion) sector.The analysis is done for an idealized homogeneous and infinite nucle matter when the influence of density can be examined with the help of constant chemic potential.

2 Generalized sigma model

Let us consider a model with two multiplets of scalar/pseudoscalar fields $H_j = \sigma_j \mathbf{I}$ $i\hat{\pi}_j$, $j = 1,2$ with $\hat{\pi}_j \equiv \pi_j^a \tau^a$ with τ^a being a set of Pauli matrices. We shall deal wi a scalar system globally symmetric in respect to $SU(2)_L \times SU(2)_R$ rotations in the exa chiral limit. We should think of these two chiral multiplets as representing the two lowes lying radial states for a given J^{PC}. Let us define the effective potential of this generaliz σ- model. First we write the most general Hermitian potential at zero μ using the chir parameterization

$$H_1(x) = \sigma_1(x)U(x) = \sigma_1(x)\xi^2(x); \quad H_2(x) = \xi(x)\big(\sigma_2(x) + i\hat{\pi}_2(x)\big)\xi(x). \qquad (1)$$

This kind of parameterization preserves the parities of $\sigma_2(x)$ and $\hat{\pi}_2$ to be even and o respectively (in the absence of SPB). It takes the following form

$$\begin{aligned} V_{\text{eff}} &= -\sum_{j,k=1}^{2} \sigma_j \Delta_{jk} \sigma_k - \Delta_{22}(\pi_2^a)^2 + \lambda_2\Big((\pi_2^a)^2\Big)^2 + (\pi_2^a)^2\Big((\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2 \\ &\quad + \lambda_1\sigma_1^4 + \lambda_2\sigma_2^4 + (\lambda_3 + \lambda_4)\sigma_1^2\sigma_2^2 + \lambda_5\sigma_1^3\sigma_2 + \lambda_6\sigma_1\sigma_2^3, \qquad (2) \end{aligned}$$

with 9 real constants Δ_{jk}, λ_A . QCD bosonization rules indicate that $\Delta_{jk} \sim \lambda_A \sim$ The neglected terms will be suppressed by inverse power of the chiral symmetry breaki (CSB) scale $\Lambda \simeq 1.2$GeV. If we assume the v.e.v. of H_j to be of the order of the constitue mass $0.2 \div 0.3$ GeV, it is reasonable to neglect these terms. Let us now investigate t hypothetical appearance of a non-zero v.e.v. of pseudoscalar fields. In order not to bre the charge conservation, we must expect, if at all, only a neutral condensate represent by a solution with $\pi_2^a = \delta^{a0}\rho$. The conditions to have an extremum are derived from t first variation of the effective potential (2),

$$\begin{aligned} 2(\Delta_{11}\sigma_1 + \Delta_{12}\sigma_2) &= 4\lambda_1\sigma_1^3 + 3\lambda_5\sigma_1^2\sigma_2 + 2(\lambda_3 + \lambda_4)\sigma_1\sigma_2^2 + \lambda_6\sigma_2^3 \\ &\quad + \rho^2\Big(2(\lambda_3 - \lambda_4)\sigma_1 + \lambda_6\sigma_2\Big), \\ 2(\Delta_{12}\sigma_1 + \Delta_{22}\sigma_2) &= \lambda_5\sigma_1^3 + 2(\lambda_3 + \lambda_4)\sigma_1^2\sigma_2 + 3\lambda_6\sigma_1\sigma_2^2 + 4\lambda_2\sigma_2^3 + \rho^2\Big(\lambda_6\sigma_1 + 4\lambda_2\sigma_2\Big), \\ 0 &= 2\pi_2^a\Big(- \Delta_{22} + (\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2 + 2\lambda_2\rho^2\Big). \qquad (\end{aligned}$$

To avoid spontaneous parity breaking in normal vacuum phase of QCD, it is *necessary and sufficient* to impose

$$(\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2 > \Delta_{22}. \tag{4}$$

Since QCD in normal conditions does not lead to parity breaking, the low-energy model must necessarily fulfill (4).

The sufficient conditions follow from the positivity of the second variation $\hat{V}^{(2)}$ for a non-trivial solution of the two first equations (3) at $\rho = 0$.

The required conditions are given by $\mathrm{tr}\left\{\hat{V}^{(2)}\right\} > 0$ and $\mathrm{Det}\hat{V}^{(2)} > 0$. For positive matrices it means that $V_{11}^{(2)\sigma} > 0$; $V_{22}^{(2)\sigma} > 0$; $V_{aa}^{(2)\pi} > 0$. The last relation determines the mass squared of π' meson and thereby must be positive according to the inequality (4).

The two set of conditions, namely those presented in eq. (4) represent restrictions that the symmetry breaking pattern of QCD imposes on its low-energy effective realization. At vanishing chemical potential, of course.

3 Finite chemical potential

Finite density is transmitted to the boson sector via $\Delta\mathcal{L} = -(\bar{q}_R H_1 q_L + \bar{q}_L H_1^\dagger q_R)$, where $q_{L,R}$ are assumed to be constituent quarks. Then the one-loop contribution to V_{eff} is

$$\begin{aligned}\Delta V_{\text{eff}}(\mu) &= \frac{\mathcal{N}}{2}\Theta(\mu - |H_1|)\left[\mu|H_1|^2\sqrt{\mu^2 - |H_1|^2} - \frac{2\mu}{3}(\mu^2 - |H_1|^2)^{3/2}\right. \\ &\left. -|H_1|^4 \ln\frac{\mu + \sqrt{\mu^2 - |H_1|^2}}{|H_1|}\right]\left(1 + O\left(\frac{\mu^2}{\Lambda^2}; \frac{|H_1|^2}{\Lambda^2}\right)\right); \qquad \mathcal{N} \equiv \frac{N_c N_f}{4\pi^2}, \end{aligned} \tag{5}$$

where μ is the chemical potential. The higher-order contributions of chiral expansion in $1/\Lambda^2$ are not considered. This effective potential is normalized to reproduce the baryon density for quark matter $\rho_B = -\frac{1}{3}\partial_\mu \Delta V_{\text{eff}}(\mu) = \frac{N_c N_f}{9\pi^2} p_F^3 = \frac{N_c N_f}{9\pi^2}(\mu^2 - |\langle H_1\rangle|^2)^{3/2}$, where the quark Fermi momentum is $p_F = \sqrt{\mu^2 - |\langle H_1\rangle|^2}$. Normal nuclear density is $\rho_B \simeq 0.17$ fm$^{-3} \simeq (1.8\text{ fm})^{-3}$ that corresponds to the average distance 1.8 fm between nucleons in nuclear matter.

The conditions for a minimum of the effective potential are thus modified. For instance (3) is modified to

$$\begin{aligned}&2(\Delta_{11}\sigma_1 + \Delta_{12}\sigma_2) = 4\lambda_1\sigma_1^3 + 3\lambda_5\sigma_1^2\sigma_2 + 2(\lambda_3 + \lambda_4)\sigma_1\sigma_2^2 + \lambda_6\sigma_2^3 \\ &+\rho^2\Big(2(\lambda_3 - \lambda_4)\sigma_1 + \lambda_6\sigma_2\Big) + 2\mathcal{N}\Theta(\mu - \sigma_1)\left[\mu\sigma_1\sqrt{\mu^2 - \sigma_1^2} - \sigma_1^3 \ln\frac{\mu + \sqrt{\mu^2 - \sigma_1^2}}{\sigma_1}\right].\end{aligned} \tag{6}$$

The possibility of SPB is controlled by the inequality (4); in order to approach a SPB phase transition we have to diminish the l.h.s. of inequality (4) and therefore we need to have (assuming that the inequality indeed holds at $\mu = 0$), $\partial_\mu\Big[(\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2\Big] < 0$. This inequality is a *necessary* condition that has to be satisfied by the model at zero chemical potential for it to be potentially capable of yielding SPB. Thus P-violation is not exceptional but rather typical for admissible values of low-energy parameters of our model.

Let us now leave the case $\mu \simeq 0$ and examine the possible existence of a critical point where the strict inequality (4) does not hold and instead for $\mu > \mu_{crit}$

$$(\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\Big(\sigma_2^2 + \rho^2\Big) = \Delta_{22}. \tag{7}$$

After substituting (7) into the second Eq.(3) one finds that

$$\lambda_5\sigma_1^2 + 4\lambda_4\sigma_1\sigma_2 + \lambda_6\Big(\sigma_2^2 + \rho^2\Big) = 2\Delta_{12}, \tag{8}$$

where we have taken into account that $\sigma_1 \neq 0$. Together with (7) it completely fixes the v.e.v.'s of the scalar fields $\sigma_{1,2}$.

Let us now try to determine the critical value of the chemical potential, namely the value where $\rho(\mu_c) = 0$. Combining our equations one gets

$$(4\lambda_2\Delta_{12} - \lambda_6\Delta_{22})r^2 + (2\lambda_6\Delta_{12} - 4\lambda_4\Delta_{22})r + 2(\lambda_3 - \lambda_4)\Delta_{12} - \lambda_5\Delta_{22} = 0; \quad r \equiv \frac{\sigma_2}{\sigma_1}. \tag{9}$$

In order for a SPB phase to exist this equation has to possess real solutions. If $4\lambda_2\Delta_{12} - \lambda_6\Delta_{22} \neq 0$ the SPB phase is bounded by two critical points corresponding to second order transitions. If, on the contrary, $4\lambda_2\Delta_{12} - \lambda_6\Delta_{22} = 0$ there is only one solution corresponding to a second order transition.

4 The physical spectrum in the SPB phase

Once a condensate for π_2^0 appears spontaneously the vector $SU(2)$ symmetry is broken to $U(1)$ and two charged π' mesons are expected to possess zero masses (in the chiral limit). Calculations of the matrix of second variation of the effective potential gives positive masses for two scalar and four pseudoscalar mesons, whereas the doublet of charged of π mesons remain massless. Quantitatively the mass spectrum can be obtained only after kinetic terms are normalized. We take the general kinetic term symmetric under $SU(2)_L \times SU(2)_R$ global rotations to be $\mathcal{L}_{kin} = \frac{1}{4}\sum_{j,k=1}^{2} A_{jk}\mathrm{tr}\left\{\partial_\mu H_j^\dagger \partial^\mu H_k\right\}$. After selecting out the v.e.v. $\langle H_1\rangle = \langle\sigma_1\rangle \equiv \bar\sigma_1$ one can separate the bare Goldstone boson action with the chiral parameterization (1) and expansion around a vacuum configuration $U = 1 + i\hat\pi/F_0 + \cdots$, $\xi = 1 + i\hat\pi/2F_0 + \cdots$ and use the v.e.v.'s $\sigma_j \equiv \bar\sigma_j + \Sigma_j$, $\hat\pi = \tau_3\rho + \hat\Pi$. Then the quadratic part looks as follows

$$\begin{aligned}\mathcal{L}_{kin}^{(2)} &= \frac{1}{2}\sum_{j,k=1}^{2} A_{jk}\left[\partial_\mu\Sigma_j\partial^\mu\Sigma_k + \frac{1}{F_0^2}\bar\sigma_j\bar\sigma_k\partial_\mu\pi^a\partial^\mu\pi^a\right] \\ &+\frac{1}{F_0}\sum_{j=1}^{2} A_{j2}\left[-\rho\partial_\mu\Sigma_j\partial^\mu\pi^0 + \bar\sigma_j\partial_\mu\pi^a\partial^\mu\Pi^a\right] + \frac{1}{2}A_{22}\left[\frac{\rho^2}{F_0^2}\partial_\mu\pi^0\partial^\mu\pi^0 + \partial_\mu\Pi^a\partial^\mu\Pi^a\right],\end{aligned} \tag{10}$$

which shows the mixture between light and heavy pseudoscalar states and, in the SPB phase, also between scalar and pseudoscalar states.

Let us define $F_0^2 = \sum_{j,k=1}^{2} A_{jk}\bar\sigma_j\bar\sigma_k$, $\zeta \equiv \frac{1}{F_0}\sum_{j=1}^{2} A_{j2}\bar\sigma_j$. In the symmetric phase $\rho = 0$ one diagonalizes the Lagrangian by shifting the pion field

$$\mathcal{L}_{kin,\pi}^{(2)} = \frac{1}{2}\partial_\mu\tilde\pi^a\partial^\mu\tilde\pi^a + \frac{1}{2}(A_{22} - \zeta^2)\partial_\mu\Pi^a\partial^\mu\Pi^a, \qquad A_{22} - \zeta^2 = \frac{\bar\sigma_1^2\det A}{F_0^2} > 0, \tag{11}$$

vherefrom, taking into account the second variation of the effective potential , one finds the nasses of the pion triplets. The physical spectrum of the pseudoscalar mesons in the normal ›hase is, $m_\pi^2 = 0$, $m_{\pi'}^2 = \frac{\mathcal{F}_1}{(A_{22}-\zeta^2)}$, where $\mathcal{F}_1 = -\Delta_{22} + (\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2$, .

In the SPB phase the situation is more involved: pseudoscalar states mix with scalar ›nes. In particular, diagonalization is different for neutral and charged pions because the ector isospin symmetry is broken: $SU(2)_V \to U(1)$. Namely $\tilde{\pi}^\pm = \pi^\pm + \zeta\Pi^\pm$, $\tilde{\pi}^0 = \pi^0 + \frac{F_0^2}{F_0^2+A_{22}\rho^2}\left(\zeta\Pi^0 - \frac{\rho}{F_0}\sum_{j=1}^{2} A_{j2}\partial_\mu\Sigma_j\right)$. In this way SPB induces mixing of both massless ınd heavy neutral pions with scalars. The (partially) diagonalized kinetic term has the ollowing form

$$\mathcal{L}_{kin}^{(2)} = \partial_\mu\tilde{\pi}^\pm\partial^\mu\tilde{\pi}^\mp + \frac{1}{2}\left(1+\frac{A_{22}\rho^2}{F_0^2}\right)\partial_\mu\tilde{\pi}^0\partial^\mu\tilde{\pi}^0 + (A_{22}-\zeta^2)\partial_\mu\Pi^\pm\partial^\mu\Pi^\mp \tag{12}$$
$$+\frac{1}{2}(A_{22} - \frac{F_0^2}{F_0^2+A_{22}\rho^2}\zeta^2)\partial_\mu\Pi^0\partial^\mu\Pi^0$$
$$+\frac{1}{2}\sum_{j,k=1}^{2}\frac{A_{jk}F_0^2+\rho^2\det A\delta_{1j}\delta_{1k}}{F_0^2+A_{22}\rho^2}\partial_\mu\Sigma_j\partial^\mu\Sigma_k - \frac{F_0\rho}{F_0^2+A_{22}\rho^2}\zeta\partial_\mu\Pi^0\sum_{j=1}^{2}A_{j2}\partial^\mu\Sigma_j.$$

Ve see that even in the massless pion sector the isospin breaking $SU(2)_V \to U(1)$ occurs: ıeutral pions become less stable with a larger decay constant. Another observation is that n the charged meson sector the relationship between massless π and π' remain the same ıs in the symmetric phase. Further diagonalization $\Pi^0, \Sigma_1, \Sigma_2$ fields leads to mixes neutral ›seudoscalar and scalar states $\tilde{\Pi}^0, \tilde{\Sigma}_1, \tilde{\Sigma}_2$. Therefore genuine mass states do not possess a lefinite parity in decays.

5 Conclusions

Thus our main results. Using an effective quark-meson lagrangian for low-energy QCD that etains the two lowest lying states in the scalar and pseudoscalar sectors parity breaking seems to be quite a realistic possibility in nuclear matter at moderate densities.We have ound the necessary and sufficient conditions for a phase where parity is spontaneously ›roken to exist. Salient characteristics of this phase would be the spontaneous breaking ›f the vector isospin symmetry $SU(2)_V$ down to $U(1)$ and the generation two additional nassless charged pseudoscalar mesons. We also find a strong mixing between scalar and ›seudoscalar states that translate spontaneous parity breaking into meson decays. The nass eigenstates will decay both in odd and even number of pions simultaneously. Isospin ›reaking can also be visible in decay constants.

6 Acknowledgments

This work was supported by research grants FPA2007-66665, RFBR and the Program RNP .1.1.1112. I am very grateful to the Organizing Committee of the XIII Int. Conference SPMTP08" for their hospitality and support and also to Profs. M.K.Volkov, G.V.Efimov nd B.M.Barbashov for many interesting discussions.

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Monopoles in lattice Electroweak theory

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Abstract

There exist several types of monopole - like topological defects in Electroweak theory. We investigate properties of these objects using lattice numerical methods. The intimate connection between them and the dynamics of the theory is established. We find that the density of Nambu monopoles cannot be predicted by the choice of the initial parameters of Electroweak theory and should be considered as the new external parameter of the theory. We also investigate the difference between the versions of Electroweak theory with the gauge groups $SU(2) \otimes U(1)$ and $SU(2) \otimes U(1)/Z_2$. We do not detect any difference at $\alpha \sim \frac{1}{128}$. However, such a difference appears in the unphysical region of large coupling constant $\alpha > 0.1$.

In both cases we use the following lattice variables: 1. The gauge field $\mathcal{U} = (U, \theta)$, where $U \in SU(2)$, $e^{i\theta} \in U(1)$ realized as link variables. 2. A scalar doublet Φ_α, $\alpha = 1,2$. The potential for the scalar field is considered in its simplest form in the London limit, i.e., in the limit of infinite bare Higgs mass. From the very beginning we fix the unitary gauge $\Phi_1 = \sqrt{\gamma}$, $\Phi_2 = 0$. For the case of the $SU(2) \times U(1)/Z_2$ symmetric model we chose the action of the form (A) $S_g = \beta \sum_{\text{plaquettes}} ((1 - \frac{1}{2}\,\mathrm{Tr}\, U_p \cos\theta_p) + \frac{1}{2}(1 - \cos 2\theta_p)) + \gamma \sum_{xy}(1 - Re(U^{11}_{xy} e^{i\theta_{xy}}))$, where the plaquette variables are defined as $U_p = U_{xy}U_{yz}U^*_{wz}U^*_{xw}$, and $\theta_p = \theta_{xy} + \theta_{yz} - \theta_{wz} - \theta_{xw}$ for the plaquette composed of the vertices x, y, z, w. For the case of the conventional $SU(2) \times U(1)$ symmetric model we use the action (B) $S_g = \beta \sum_{\text{plaquettes}} ((1 - \frac{1}{2}\,\mathrm{Tr}\, U_p) + 3(1 - \cos\theta_p)) + \gamma \sum_{xy}(1 - Re(U^{11}_{xy} e^{i\theta_{xy}}))$. The following variables are considered as creating a Z boson and a W boson, respectively: $Z_{xy} = Z^\mu_x = \sin[\mathrm{Arg}U^{11}_{xy} + \theta_{xy}]$, $W_{xy} = W^\mu_x = U^{12}_{xy} e^{-i\theta_{xy}}$. Here, μ represents the direction (xy). In the unitary gauge there is also a $U(1)$ lattice gauge field, which is defined as $A_{xy} = A^\mu_x = [-\mathrm{Arg}U^{11}_{xy} + \theta_{xy}] \bmod 2\pi$. The phase diagrams of the two models under consideration are presented in figure 1. The dashed vertical line represents the phase transition in the $SU(2) \otimes U(1)$-symmetric model. This is the confinement-deconfinement phase transition corresponding to the $U(1)$ constituents of the model. The same transition for the $SU(2) \otimes U(1)/Z_2$-symmetric model is represented by the solid vertical line. The dashed horizontal line corresponds to the transition between the broken and symmetric phases of model A. The continuous horizontal line represents the same transition in model B. Interestingly, in the $SU(2) \otimes U(1)/Z_2$ model both transition lines meet, forming a triple point. Real physics is commonly believed to be achieved within the phases of the two models situated in the right upper corner of Fig. 1. The double-dotted-dashed vertical line on the right-hand side of the diagram represents the line, where the renormalized α is constant and equal to $1/128$. All simulations were performed on lattices of sizes 8^4 and 16^4. Several points were checked using a lattice 24^4. In general we found no significant difference between the mentioned lattice sizes.

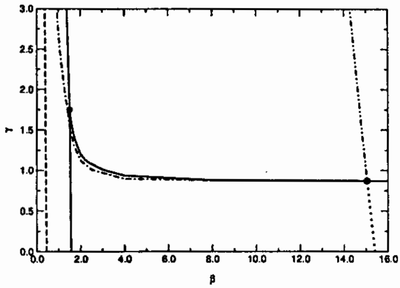

Figure 1: The phase diagrams of the models in the (β, γ)-plane.

We perform the calculation of renormalized fine structure constant α_R using the poten-
ial for infinitely heavy external fermions. We consider Wilson loops for the right-handed
xternal leptons: $\mathcal{W}^{\rm R}_{\rm lept}(l) = \langle {\rm Re}\,\Pi_{(xy)\in l} e^{2i\theta_{xy}} \rangle$. Here l denotes a closed contour on the lat-
ice. We consider the following quantity constructed from the rectangular Wilson loop of
ize $r \times t$: $\mathcal{V}(r) = \lim_{t\to\infty} \log \frac{\mathcal{W}({\rm r\times t})}{\mathcal{W}({\rm r\times(t+1)})}$. Due to exchange by virtual photons at large enough
istances we expect the appearance of the Coulomb interaction $\mathcal{V}(r) = -\frac{\alpha_R}{r} + const.$

The worldlines of the quantum Nambu monopoles could be extracted from the field
onfigurations as follows: $j_Z = \delta \Sigma = \frac{1}{2\pi} {}^*d([dZ']{\rm mod} 2\pi)$ (The notations of differential forms
n the lattice are used here.) The monopole density is defined as $\rho = \left\langle \frac{\sum_{\rm links} |j_{\rm link}|}{4L^4} \right\rangle$, where
L is the lattice size. In order to investigate the condensation of monopoles we use the
ercolation probability $\Pi(A)$. It is the probability that two infinitely distant points are
onnected by a monopole cluster. We show Nambu monopole density and percolation
robability as a function of γ along the line of constant renormalized $\alpha_R = 1/128$. It is
lear that the percolation probability is the order parameter of the transition from the
ymmetric to the broken (both $SU(2)$ and

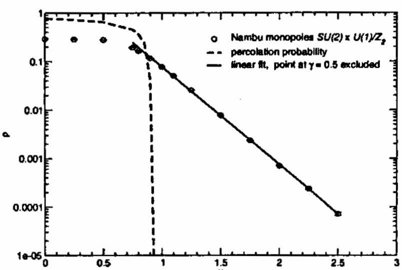

Figure 2: Nambu monopole density and percolation probability as a function of γ along
he line of constant $1/\alpha_R = 128$.

$J(1)$), which is carried by Nambu monopoles. The behavior of ΔS_p shows that a quantum
Nambu monopole may indeed be considered as a physical object.

In order to evaluate the mass of the Z-boson we use the zero - momentum correlator: $\sum_{\bar{x},\bar{y}}\langle\sum_\mu Z^\mu_x Z^\mu_y\rangle \sim e^{-M_Z|x_0-y_0|} + e^{-M_Z(L-|x_0-y_0|)}$. Here the summation $\sum_{\bar{x},\bar{y}}$ is over the three 'space" components of the four - vectors x and y while x_0, y_0 denote their "time" compo-ients. L is the lattice length in the "time" direction. The physical scale is given in our attice theory by the value of the Z-boson mass $M_Z^{phys} \sim 91$ GeV. Therefore the lattice pacing is evaluated to be $a \sim [91\text{GeV}]^{-1} M_Z$, where M_Z is the Z boson mass in lattice inits. The real continuum physics should be approached along the the line of constant $\alpha_R = \frac{1}{128}$, i.e. along the line of constant physics. We investigated the dependence of the iltraviolet cutoff $\Lambda = a^{-1} = (91\text{ GeV})/M_Z$ on γ along the line of constant physics. It occurs that Λ is increasing slowly along this line with decreasing γ and achieves the value 430 ± 40 GeV at the transition point between the physical Higgs phase and the symmetric phase. According to our results this value does not depend on the lattice size. This means hat the largest achievable value of the ultraviolet cutoff is equal to 430 ± 40 GeV if the potential for the Higgs field is considered in the London limit.

Our lattice study also demonstrates another peculiar feature of Electroweak theory. If we are moving along the line of constant $\alpha = 1/128$, then the Nambu-monopole density decreases with increasing γ (for $\gamma > 1$). Its behavior is approximated with a nice accuracy by the simple formula: $\rho \sim e^{2.08-4.6\gamma}$. This means that the density of Nambu monopoles n the continuum theory cannot be predicted by the choice of the usual parameters of the Electroweak theory and should be considered as a new external parameter of the theory.

We found that the two definitions of the theory (with the gauge groups $SU(2)\otimes U(1)/Z_2$ and $SU(2)\otimes U(1)$, respectively) do not lead to different predictions at the values of α around 1/128. However, the corresponding models behave differently at unphysically large values of $\alpha > 0.1$. The main difference is in the behavior of the so-called hypercharge monopoles.

This work was partly supported by RFBR grants 08-02-00661, and 07-02-00237, RFBR-DFG grant 06-02-04010, by Grant for leading scientific schools 679.2008.2, by Federal Program of the Russian Ministry of Industry, Science and Technology No 40.052.1.1.1112.

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Mechanics of Quark Exchange in High-Energy Hadron Reactions at Forward Angles

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Abstract

The 2-quark back-angle scattering mechanism is shown to reproduce main features of high-energy hh flavor-exchange reactions. Prospects for reduction of the reaction matrix element to a form of convolution of hadron wave functions with the hard scattering kernel are discussed. Wave function models suitable for convolution with the kernel, which is singular at small x, and the emerging form-factor types, are discussed.

Reactions of one unit of flavor exchange between two hadrons scattering through small angles at high energy can occur, to LO in Mandelstam s, due to exchange of a single pair of quarks, which enter into a hard collision and completely interchange their large longitudinal momenta (scatter to 180° in c.m.s.). With no rapidity gap between the hadron remnants and their new comoving quarks created, they should with high probability recombine into a new pair of hadrons. In the binary reaction channel the differential cross-section, indeed, exhibits a forward peak of typical $p_\perp \sim 300$ MeV width. The peak decreases with the energy rather slowly, as $s^{-1\div-2}$ – approximately as does the Born-level 2-quark back angle scattering differential cross-section ($\propto s^{-2}$); the rest of the energy dependence must be attributed to reggeization effects.

The Born diagram of the head-on relativistic quark collision is closely similar to that of real photon scattering on a quark, owing to the similarity between hard gluon and hard quark propagators on the light cone. Yet, high energy hadron reactions are much *easier to measure* then hard forward real Compton scattering is, given the coupling constant factor α_s^2 in the cross-section instead of α_{EM}^2, and no radiative or diffractive background involved. On the theory side, it brings complications, since eikonal (gauge link, Wilson line) phases in hadron remnant interactions emerge, which spoil factorization in terms of GPDs. However, that may actually happen in photon-hadron interactions as well, and the $A^+ = 0$ gauge condition does not help [1]. As those links are to be dealt with anyway, it would be better to constrain them using cross-checks from different processes.

Forward Compton scattering is, also, not the best playground for studying hadron spin effects, since quark SSA for it cancel in the sum of two LO diagrams. As for 2-quark scattering in flavor exchange, there is only one LO diagram, so transverse polarization may be open, provided soft contributions supply necessary phase shifts. Generally, soft effects can stem from eikonal links, and also from initial and final state wave functions sandwiching the *complex* hard scattering kernel. Both eikonal and initial/final state contributions are proportional to the interaction strength, but still, it is not excluded that some of them may happen to be more significant. To make a sensible decision, it is instructive first to develop qualitative understanding of the dynamics.

1 Finite Longitudinal Shift + LS-Coupling

Consider the act of the head-on quark collision in the rest frame of one of the hadrons. Instead of backscattering, then, we have braking of a fast quark and knockout of a resting one. The mediating hard gluon, being shed from a fast particle, must propagate along the same direction. Its range (Ioffe time [2]) appears to be collision energy independent and thereby, despite high virtuality of the gluon, finite: $l_z \sim (M_{targ}x_{targ})^{-1} \sim R_H$. So, the stop point of the incident quark on average is shifted with respect to the centre of the well by a distance commensurable with the hadron radius, and with the simultaneous finite transverse momentum transfer in the head-on collision, the captured quark acquires an orbital angular momentum in the normal direction.

Of course, if by definition the final state, as well as the initial one, must have zero orbital angular momentum (s-state), the quark would be forbidden to orbit. But relativistically, the single-quark s-state is the one having upper Dirac bispinor components multiplied by spatial wave functions with $L = 0$, but the lower ones, by parity reasons, must have $L = 1$, composing it with the spin $S = \frac{1}{2}$ to yield the same total momentum $J = \frac{1}{2}$. Now, to flip the spin of the s-state bound quark in a head-on collision, it obviously suffices to change projection of $\mathbf{L}$ for its lower Dirac bispinor components.

¿From the emerging spatial picture it seems clear that the effect is, basically, *bulk*, not due some fine local interference. So, the implementation of real eikonal phases is unlikely to modify the picture drastically. Opacity effects can be of consequence, but their 3d distribution is rather uncertain. Therefore, a reasonable strategy would be to begin with the neglect of eikonal factors, and ultimately look for deriving constraints on them when a nonremovable discrepancy with the data arises[1]. Thus we arrive at the representation of hh-collision amplitudes in terms of wave function overlaps[2].

A popular model of light-front WF with LS-interaction is that of [3]. However, it is of perturbative origin, thence inheriting support $x \in [0, 1]$ and vanishing at the endpoints. Therefore, no imaginary part can result in convolution of such wave functions with the hard kernel $\frac{1}{x_1 x_2 + i0}$. An alternative class of models which tend to give wave functions non-vanishing and continuous at $x = 0$ (with support in x ranging, in principle, from $-\infty$ to $+\infty$, and x normalization being to the single-constituent rest energy, not to that of the whole composite system) assume non-interacting constituents moving in a self-consistent field, static in their c.m.s.[3] The static well is unable to accommodate for recoil effects which must be crucial at $-t > M^2$, but there is no experimental data in that far region anyway. With wave functions continuous at $x = 0$, the imaginary part of the matrix element, in fact, logarithmically diverges in the hard approximation, so it is not purely hard, but that introduces only a weak dependence on the cutoff parameter.

The results obtained with a gaussian WF model were partially presented at the conference, but prior to embarking at model assumptions, it is desirable to exhaust all model-independent means. To this end, one can admit that in general, x-dependence of H and E

[1] Essentially, this is the same attitude as in DGLAP, where, by the way, the role of eikonal factors, to date, still has not become tangible, at typical x.

[2] Note that for elastic scattering that is impossible in principle, because it is *caused* by soft exchanges. The minimum number of gluons exchanged in the t-channel is two, and generally they need not be all attached to the same parton line(though, conditions for the opposite are sought in 'heavy Pomeron' models).

[3] At that, GPDs near $x = 0$ are not to be interpreted as quark densities at $x > 0$ and minus antiquark densities at $x < 0$, at any rate not as those extracted from DIS data using the parton model.

GPDs strongly differ[4], and analyze consequences of factorization in their terms.

2 Factorization With Imaginary Contributions

The factorization of the matrix element proceeds through decomposition of the hard kernel

$$\frac{1}{x_1x_2+i0} = \mathrm{P}\frac{1}{x_1}\mathrm{P}\frac{1}{x_2} - i\pi\frac{1}{|x_1|}\delta(x_2) - i\pi\delta(x_1)\frac{1}{|x_2|},$$

and leads, for a representative reaction $np \to pn$, to the matrix element

$$\begin{aligned} M_{fi} \propto\ & w_n'^{+}\left(H_-(t)+\frac{i\sqrt{-t}}{2M}E_-(t)\sigma^N\right)w_p w_p'^{+}\left(H_-(t)+\frac{i\sqrt{-t}}{2M}E_-(t)\sigma^N\right)w_n \\ -\ & i w_n'^{+}\left(H_+(t)+\frac{i\sqrt{-t}}{2M}E_+(t)\sigma^N\right)w_p w_p'^{+}\left(H_0(t)+\frac{i\sqrt{-t}}{2M}E_0(t)\sigma^N\right)w_n \\ -\ & i w_n'^{+}\left(H_0(t)+\frac{i\sqrt{-t}}{2M}E_0(t)\sigma^N\right)w_p w_p'^{+}\left(H_+(t)+\frac{i\sqrt{-t}}{2M}E_+(t)\sigma^N\right)w_n \\ +\ & \left(\tilde{H}_-^2(t) - 2i\tilde{H}_+(t)\tilde{H}_0(t)\right) w_n'^{+}\sigma^L w_p w_p'^{+}\sigma^L w_n \end{aligned}$$

with

$$H_-(t) = \mathrm{P}\int \frac{dx}{x}H_p^u(x,0,t),\ H_+(t,\lambda_x) = \int_{|x|>\lambda_x}\frac{dx}{|x|}H_p^u(x,0,t) \approx H_+(t),\ H_0(t) = \pi H_p^u(0,0,t),$$

and similarly for E and $\tilde{H}$. Formally, this structure resembles the sum of t-channel pole exchanges, but 2 of them having *imaginary* coupling constants. Polarization emerges due to interference between the "-" exchange and "0+" and "+0" exchanges.

Note that t-dependences of "-" form-factors dominated by typical x and of "0" and "+" form-factors fed by $x \approx 0$ may be of quite different width. Presence of several t-slopes is typically observed in flavor exchange reactions, the pre-QCD interpretation being the difference in masses of the exchanged mesons and onset of central absorption [5]. The quark backscattering theory, however, does not contain correlated $q\bar{q}$ transverse propagation.

Phenomenological consequences of *u-channel* gluon reggeization, and of possible contributions involving extra t-channel color exchanges will be discussed elsewhere.

Acknowledgement. The author is grateful to organizers of the conference for local hospitality during the meeting.

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[4] In contrast to the case of H, x-dependence of E is not constrained by DIS data. Note that in some models [4] those distributions are assumed to be just proportional, but without any motivation beyond the similarity of t-dependence of their x-integrals which are associated with EM form-factors.

Landau gauge propagators and Gribov copy effects in $SU(2)$ lattice gauge theory

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Abstract

For $SU(2)$ lattice gauge theory we study numerically the infrared behavior of the Landau gauge ghost and gluon propagators with the special accent on the Gribov copy dependence. We find that the Gribov copy effect for both propagators is essential in the infrared. In particular, our *best copy* dressing function of the ghost propagator approaches a *plateau* in the infrared, while for the random *first copy* it still grows. Our *best copy* zero-momentum gluon propagator shows a tendency to *decrease* with growing lattice size which excludes singular solutions. The running coupling constant tends to zero.

1 Introduction

In recent years the interest to the lattice results for the field propagators in the IR region has been revived. This interest was stimulated also by the practical progress achieved over the years within the Dyson-Schwinger (DS) approach (for an intermediate review see [1]), and more recently with the help of functional renormalization group (FRG) equations [2, 3]. Infrared QCD has been also investigated using the stochastic quantization method [4, 5], as well as with effective actions [6, 7].

The main motivation for numerical study is triggered by the puzzle posed by the above mentioned continuum approaches. Different kinds of solutions with a quite different IR behavior of the gluon and ghost propagators have been reported by different groups. The power-like solution with relation between gluon κ_D and ghost $\kappa_G > 0$ exponents $\kappa_D = -2\kappa_G$ was called recently a *scaling solution* [8]. This solution [9, 10, 11, 4, 3, 12] allows the gluon propagator to vanish and the ghost dressing function diverge in the IR limit in one-to-one correspondence with both the Gribov-Zwanziger scenario [13, 14] and the Kugo-Ojima criterion [15, 16] for confinement. On the contrary, the so-called *decoupling solutions* [17, 7, 18, 19] provide an IR finite or weakly divergent gluon propagator and a finite ghost dressing function leading to a running coupling vanishing in the infrared. The lattice approach based on the first-principle path integral quantization should be able to resolve the issue.

One of the main goals of lattice studies was to clarify the infrared (IR) asymptotics of the propagators and of the running coupling which can be determined through these propagators. At the same time it has been found that the lattice approach has its own difficulties when applied to such studies. One of them is that to reach small momenta necessary to study the IR limit one has to go to huge lattices which makes the numerical simulations formidable. Another, less apparent, but not less difficult problem is the problem of Gribov copies [1]. Although for many years it was believed that the effect of Gribov copies on both gluon and ghost propagators was weak and could be considered just as a noise in the scaling region (see, e.g., [20]) it has been found first for the ghost propagator [21] and quite recently for the gluon propagator [22] that these effects are in fact quite strong. The presence of these effects makes the task of lattice computations of the field propagators in the IR region even more difficult.

In this paper we continue our lattice study of the influence of Gribov copies on the (minimal) Landau gauge $SU(2)$ gluon and ghost propagators in the IR region by applying global $Z(2)$ flip transformations in combination with an effective optimization algorithm, the so-called *simulated annealing* (SA). The flip transformation was introduced in [22]. Its influence on the gluon propagator was thoroughly studied lateron [23, 24]. We find the Gribov copy dependence to be very strong. Our results look rather as an argument in favor of the decoupling solution with a non-singular gluon propagator. However, we do not yet consider the problem of Gribov copies and, correspondingly, the infrared asymptotics of the gluon propagator to be finally resolved.

2 Gluon and ghost propagators: the definitions

For the Monte Carlo generation of ensembles of non-gauge-fixed gauge field configurations we use the standard Wilson action, which for the case of an $SU(2)$ gauge group is written

$$S = \beta \sum_x \sum_{\mu>\nu} \left[1 - \frac{1}{2}\,\mathrm{Tr}\Big(U_{x\mu}U_{x+\mu;\nu}U^{\dagger}_{x+\nu;\mu}U^{\dagger}_{x\nu}\Big)\right]\,; \qquad \beta = 4/g_0^2\,. \tag{1}$$

Here g_0 is a bare coupling constant and $U_{x\mu} \in SU(2)$ are the link variables. The latter transform as follows under gauge transformations g_x : $U_{x\mu} \stackrel{g}{\mapsto} U^{g}_{x\mu} = g^{\dagger}_x U_{x\mu} g_{x+\mu}$ where $g_x \in SU(2)$. The standard definition of the dimensionless lattice gauge vector potential is $\mathcal{A}_{x+\hat{\mu}/2,\mu} = \Big(U_{x\mu} - U^{\dagger}_{x\mu}\Big)/2i \equiv A^{a}_{x+\hat{\mu}/2,\mu}\sigma_a/2$. In lattice gauge theory the usual choice of the Landau gauge condition is $(\partial\mathcal{A})_x = \sum_{\mu=1}^{4}\Big(\mathcal{A}_{x+\hat{\mu}/2;\mu} - \mathcal{A}_{x-\hat{\mu}/2;\mu}\Big) = 0$, which is equivalent to finding an extremum of the gauge functional

$$F_U(g) = \frac{1}{4V}\sum_{x\mu} \frac{1}{2}\,\mathrm{Tr}\,U^{g}_{x\mu}\,, \tag{2}$$

where $V = L^4$ is the lattice volume, with respect to gauge transformations g_x . The manifold consisting of Gribov copies providing local maxima of the functional (2) and a semi-positive Faddeev-Popov operator (see below) is called *Gribov region* Ω, while that of the global maxima is called the *fundamental modular domain* $\Lambda \subset \Omega$. Our gauge fixing procedure is aimed to approach this domain.

The gluon propagator D and its dressing function Z are then defined (for $p \neq 0$) by

$$D^{ab}_{\mu\nu}(p) = \frac{a^2}{g_0^2}\langle \widetilde{A}^{a}_{\mu}(k)\widetilde{A}^{b}_{\nu}(-k)\rangle \;=\; \left(\delta_{\mu\nu} - \frac{p_\mu\, p_\nu}{p^2}\right)\delta^{ab} D(p)\,; \qquad Z(p) = D(p)\,p^2\,, \tag{3}$$

[1] It is worthwhile to note that the DS approach introduced originally as a method for resummation of the perturbative series is not sensible to different gauge copies.

where $\widetilde{A}(k)$ represents the Fourier transform of the gauge potentials after having fixed the gauge. The momentum p is given by $p_\mu = (2/a)\sin(\pi k_\mu/L),\quad k_\mu \in (-L/2, L/2]$. For $p \neq 0$, one gets from Eq. (3) $D(p) = \frac{1}{9}\sum_{a=1}^{3}\sum_{\mu=1}^{4} D_{\mu\mu}^{aa}(p)$, whereas at $p = 0$ the "zero momentum propagator" $D(0)$ is defined as $D(0) = \frac{1}{12}\sum_{a=1}^{3}\sum_{\mu=1}^{4} D_{\mu\mu}^{aa}(p=0)$. The lattice expression for the Landau gauge Faddeev-Popov operator $M^{ab} = -\partial_\mu D_\mu^{ab}$ (where D_μ^{ab} denotes the covariant derivative in the adjoint representation) for $SU(2)$ is given by

$$M_{xy}^{ab}[U] = \sum_\mu \left\{ \left(\bar{S}_{x\mu}^{ab} + \bar{S}_{x-\hat{\mu};\mu}^{ab}\right)\,\delta_{x;y} - \left(\bar{S}_{x\mu}^{ab} - \bar{A}_{x\mu}^{ab}\right)\,\delta_{y;x+\hat{\mu}} - \left(\bar{S}_{x-\hat{\mu};\mu}^{ab} + \bar{A}_{x-\hat{\mu};\mu}^{ab}\right)\,\delta_{y;x-\hat{\mu}} \right\} \quad (4)$$

where $\bar{S}_{x\mu}^{ab} = \delta^{ab}\,\frac{1}{2}\,\mathrm{Tr}\,U_{x\mu}$ and $\bar{A}_{x\mu}^{ab} = -\frac{1}{2}\,\epsilon^{abc}\,A_{x+\hat{\mu}/2;\mu}^{c}$. From the expression (4) it follows that a trivial zero eigenvalue is always present, such that at the Gribov horizon $\partial\Omega$ the first non-trivial zero-eigenvalue appears. Thus, if the Landau gauge is properly implemented, $M[U]$ is a symmetric and semi-positive matrix.

The ghost propagator $G^{ab}(x,y)$ is defined in a standard way

$$G^{ab}(x,y) = \delta^{ab}\;G(x-y) \equiv \frac{1}{a^2}\Big\langle \left(M^{-1}\right)_{x\,y}^{a\,b}[U] \Big\rangle . \quad (5)$$

The ghost propagator $G(p)$ in momentum space and its dressing function $J(p)$ are

$$G(p) = \frac{a^2}{3V}\sum_{x,\,y,\,a} e^{-\frac{2\pi i}{L}\,k\cdot(x-y)} \Big\langle \left(M^{-1}\right)_{x\,y}^{a\,a}[U] \Big\rangle ; \qquad J(p) = G(p)\;p^2\,, \quad (6)$$

where the coefficient $\frac{1}{3V}$ is taken for a full normalization, including the indicated color average over $a = 1, 2, 3$.

For the gauge fixing procedure we employ the so-called FSA algorithm described in our papers [23, 24].

3 Results

In this section we present the data for the gluon and ghost propagator. In Fig. 1 we show the new data for the gluon propagator $D(p)$ in physical units obtained on the 40^4 lattice at $\beta = 2.20$. We compare the *bc* FSA result with the *fc* SA result (the latter without flips). We clearly see the Gribov copy effect for the lowest accessible momenta moving the data points to lower values for better copies (with the larger gauge functional). The different points at $p \sim 300$MeV belong to different realizations of p^2 and seem to indicate some violation of the hypercubic symmetry.

In Fig. 2 we present these new data together with the ones obtained on smaller lattice sizes always for the FSA *bc* case. We see that the data are nicely consistent with each other and indicate a turnover to decreasing values towards vanishing momentum. A smooth extrapolation to $D(0)$ becomes visible. But still there is no indication for a vanishing gluon propagator at zero momentum for increasing volume. This behavior demonstrates a (slight) tendency to decrease, and looks hardly consistent with $D(0) = 0$ limit. One could consider it rather like an argument in favor of the *decoupling* solution with a finite gluon propagator in the infrared. However, one still cannot exclude that there are even more efficient gauge fixing methods, superior to the one we use, which could make this

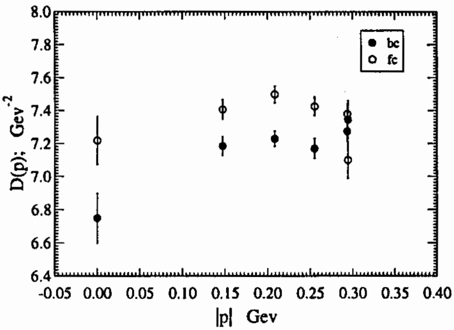

Figure 1: The momentum dependence of the bc and fc gluon propagators in the IR region on the 40^4 lattice.

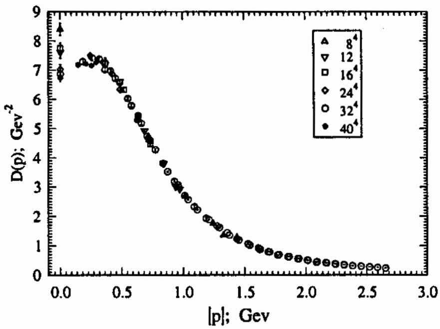

Figure 2: The momentum dependence of the gluon propagator $D(p)$ on various lattice size. bc results are shown throughout.

decreasing more drastic. However, our results for $D(0)$ are in clear disagreement even with a weak divergence advocated in [17, 19].

Analogously to Fig. 1 in Fig. 3 we show the ghost dressing function $J(p)$ obtained on the 40^4 lattice. There is a very clear Gribov copy effect changing $J(p)$ even qualitativly. Whereas the fc SA results seem to support a weakly singular behavior, the bc FSA data provide a *plateau* pointing to a finite IR value of the ghost dressing function, i.e. a tree-level behavior of the ghost propagator. Our data indicate that the plateau starts at $p \lesssim 200$ MeV.

In Fig. 4 the ghost dressing function is shown for lattice sizes from 16^4 to 40^4. We show always bc FSA results, except for 24^4, where we compare also with fc data obtained with the conventional OR algorithm. The latter show an even stronger IR singular behavior than those data obtained with the fc SA algorithm. There is a clear weakening of the singularity visible additionally to a finite-size effect which seems to lead to an IR plateau behavior. Such a plateau would be consistent with the different decoupling solutions and in contradiction with the Kugo-Ojima confinement criterion.

In Fig. 5 for the bc FSA results obtained on lattice sizes from 16^4 up to 40^4 we draw the behavior of the running coupling related to the ghost-ghost-gluon vertex

$$\alpha_s(p) = \frac{g_0^2}{4\pi} J^2(p)\ Z(p) \tag{7}$$

under the assumption that the vertex function is constant as seen in perturbation theory [25] and approximately also in lattice simulations [26, 27]. The decrease towards $p^2 = 0$ is obvious. With the improved gauge fixing the effect is even strengthened, such that an approach to an IR fixed point as expected from the *scaling* DS and FRG solution seems to be excluded.

4 Conclusions

1) For the gluon propagator our new data for the 40^4 lattice agree with data on the smaller lattices (up to 32^4). We confirm our conclusion [23] about the appearance of the local

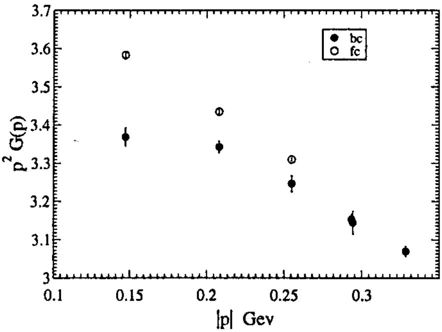

Figure 3: The momentum dependence of the bc and fc ghost dressing functions in the IR region on the 40^4 lattice.

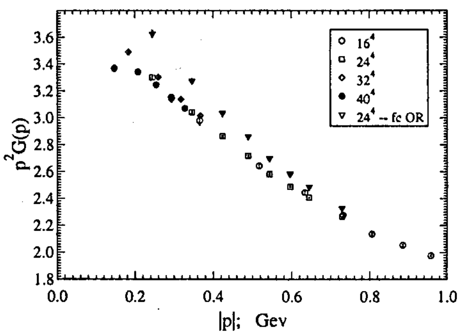

Figure 4: The momentum dependence of the ghost dressing function on the various lattices. For comparison results obtained with OR algorithm on 24^4 are also shown.

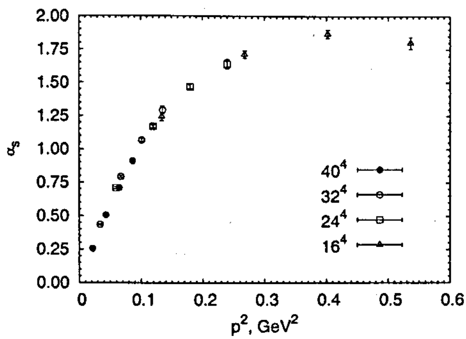

Figure 5: The momentum dependence of the running coupling in the infrared region.

maximum at a non-zero value of the momentum p^2 (this local maximum was absent for lattice sizes $\leq 24^4$). The zero-momentum gluon propagator $D(0)$ has a tendency to decrease with growing lattice size L. This observation is in clear contradiction with the infrared divergent gluon propagator obtained on the basis of Ward-Slavnov-Taylor identities. At the time being, this behavior looks hardly consistent with a $D(0) = 0$ limit at infinite L, and could be considered rather like an argument in favor of the *decoupling* solution with a non-singular gluon propagator. However, we do not yet consider the problem of the infrared asymptotics of the gluon propagator as a finally resolved.

2) We calculated the ghost propagator for lattices up to 40^4. Our bc dressing function $J(p)$ of the ghost propagator demonstrates the approach to a *plateau* in the infrared, while the fc dressing function still grows (as it was in earlier calculations).

This is a first clear indication of the lack of the IR-enhancement of the ghost propagator. This plateau behavior is in a clear contradiction with the Kugo–Ogima confinement criterion. The fate of this confinement criterion still needs a further clarification.

3) We have found that the effect of Gribov copies for both the propagators and in the consequence for the running coupling is essential in the infrared range $p < 1$ GeV. Therefore, the *quality* of the gauge fixing procedure in the study of gauge dependent observables remains important.

We cannot say that we have reached the fundamental modular region when fixing the Landau gauge on larger lattices. One cannot exclude that there is another method superior to our FSA algorithm. We believe that the Gribov problem deserves even more thorough studies.

This investigation has been partly supported by the Heisenberg-Landau program and partly by the joint DFG-RFBR grant 436 RUS 113/866/0-1 and the RFBR-DFG grant 06-02-04014 and by the grant NSh-679.2008.2.

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Solutions of DGLAP evolution equations for the gluon and structure function exponents and determination of the Reduced Cross Section

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Abstract

A set of formulae using the solution of the QCD Dokshitzer-Gribov-Lipatov-Altarelli-parisi (DGLAP) evolution equation to the extract of the exponent λ_g gluon distribution and λ_S structure function from the Regge- like behavior at low x is presented. So, the reduced cross section based on Regge-like behavior of the gluon distribution and structure function is determined. The detailed analysis compared with the HERA experiment H1 data. All results can be consistently described within the framework of perturbative QCD.

Neglecting the quark singlet part, the DGLAP equation for the gluon evolution in the NLO can be written as [1]:

$$Q^2\frac{\partial G}{\partial Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 [P^1_{gg}(z) + \frac{\alpha_s}{2\pi}P^2_{gg}(z)]G(\frac{x}{z},Q^2)dz, \quad (1)$$

where $P^1_{gg}(z)$ and $P^2_{gg}(z)$ are the LO and NLO Altarelli- Parisi splitting kernels and $\alpha_s(Q^2)$ is the running coupling in NLO. To find an analytic solution, we set the splitting kernels at the small x limit. The small- x region of DIS offerees a unique possibility to explore the Regge limit of PQCD [2]. The rapid rise in Q^2 of the structure functions was considered as a sign of departure from the standard Regge behavior. The reason was that the HERA data, when fitted by a single "Regge- pomeron" term $\sim x^{-\lambda_S}$, where λ_S is the pomeron intercept minus one, show that $\lambda_S = \frac{d\ln F_2(x,Q^2)}{d\ln\frac{1}{x}}$ definitely rises with Q^2. The HERA data should determine the small x behavior of gluon and sea quark distribution. Roughly speaking the data on F_2 constrain the sea S and the data on the slope $dF_2/d\ln Q^2$ determine the gluon g. The exponent λ_g is given as the derivative:

$$\lambda_g = \frac{\partial\ln G(x,t)}{\partial\ln\frac{1}{x}}|_{t=cte}. \quad (2)$$

that is

$$G(x,Q^2) = A_g x^{-\lambda_g}, \quad (3)$$

where A_g is a constant and λ_g is the intercept. Using these equations and carrying out the integration, we found [4]:

$$\ln\frac{\lambda_{go}}{\lambda_g - x^{\lambda_g}\int_{t_0}^t x^{-\lambda_g}(\frac{3\alpha}{\pi}-\frac{61\alpha^2}{9\pi^2})dt} = \int_{t_0}^t(\frac{3\alpha}{\pi}-\frac{61\alpha^2}{9\pi^2})\frac{1-x^{\lambda_g}}{\lambda_g}dt. \quad (4)$$

This equation obtain an approximation expression for λ_g. Where $\lambda_{g_0} = \frac{\partial \ln G(x,t_0)}{\partial \ln \frac{1}{x}}$ is the exponent at the starting scale t_0 while $G(x,t_0)$ is the input gluon distribution (t_0=ln($\frac{Q_0^2}{\Lambda^2}$) andΛ is the QCD cut- off parameter). Also with neglecting the quark, the DGLAP evolution equation for the singlet structure function has the form:

$$\frac{dF_2^S}{dt} = \frac{\alpha_s}{2\pi}\int_x^1 dz(2N_f P_{qg}^1(z) + \frac{\alpha_s}{2\pi}P_{qg}^2(z))G(\frac{x}{z},Q^2). \tag{5}$$

The small- x limit of the next- to- leading order splitting function for the evolution of the singlet quark is then [3]:

$$P_{qg}^2(x) \longrightarrow \frac{\alpha_s}{2\pi}\frac{40C_A N_f T_R}{9x}. \tag{6}$$

Now, let us consider the Regge- like behavior of the gluon distribution and carrying out the integration, so the exponent λ_S being directly calculate. Finally we obtain [4]:

$$\begin{aligned}\lambda_S F_2(x,t) - \lambda_{S_0} F_2(x,t_0) &= \frac{0.555}{\pi}\int_{t_0}^t \alpha_s G(x,t)[(\frac{2\lambda_g}{3+\lambda_g}(1-x^{3+\lambda_g}) + \frac{\lambda_g}{1+\lambda_g}(1-x^{1+\lambda_g}) \\ &\quad -\frac{2\lambda_g}{2+\lambda_g}(1-x^{2+\lambda_g})) + (2x^{3+\lambda_g} + x^{1+\lambda_g} - 2x^{2+\lambda_g})]dt \\ &\quad +\frac{1.852}{\pi^2}\int_{t_0}^t \alpha_s^2 G(x,t)dt \end{aligned} \tag{7}$$

which defines the solution for λ_S. To solve this, we use of $\lambda_{S_0} = \frac{\partial \ln F_2(x,t_0)}{\partial \ln \frac{1}{x}}$ at the starting scale t_0 and from $F_2(x,t_0)$ as the input structure function . On the basis of the gluon and structure function exponents we can be evaluated of the reduced cross section as defined [5,6]:

$$\sigma_r \equiv F_2(x,Q^2) - \frac{y^2}{Y_+}.F_L(x,Q^2) \tag{8}$$

and $Y_+ = 1 + (1-y)^2$. Here Q^2 is the squared four-momentum transfer, x denotes the Bjorken scaling variable, $y = Q^2/sx$ is the inelasticity, with s the electron-proton center of mass energy squared, and α is the fine structure constant. Based on the gluon and structure function Regge like behavior, the strong rise of $F_2(x,Q^2)$ and $xg(x,Q^2)$ at small x is corresponding to a rising longitudinal structure function. At low x, using the fact that the nonsinglet contribution F_2^{NS} can be ignored safely. So we can write the longitudinal structure function by an integral over the quark and gluon distributions as:

$$F_L(x,Q^2) = \frac{4\alpha_s}{3\pi}\int_x^1 \frac{dy}{y}(\frac{x}{y})^2 F_2(y,Q^2) + \frac{20\alpha_s}{9\pi}\int_x^1 \frac{dy}{y}(\frac{x}{y})^2(1-\frac{x}{y})G(y,Q^2). \tag{9}$$

With respect to the Regge- like behavior of the gluon and structure function at small x and substitution in Eq.9 and carrying out of the integration, we found:

$$F_L(x,Q^2) = \eta F_2(x,Q^2) + \zeta G(x,Q^2), \tag{10}$$

where

$$\eta = \frac{4\alpha_S}{3\pi}\frac{1 - x^{2+\lambda_S}}{(2+\lambda_S)}, \qquad (11)$$

and

$$\zeta = \frac{20\alpha_s}{9\pi}\frac{(2+\lambda_g)x^{3+\lambda_g} - (3+\lambda_g)x^{2+\lambda_g} + 1}{(2+\lambda_g)(3+\lambda_g)}. \qquad (12)$$

So the reduced cross section σ_r based on and λ_s & λ_g exponents is obtained as:

$$\sigma_r(x,Q^2) = F_2(x,Q_0^2)(\frac{Q_0^2}{Q^2})^{\lambda_S \cdot \frac{lnx}{ln(Q^2/\Lambda^2)}}[1 - \frac{y^2}{Y_+}\eta] - \frac{y^2}{Y_+}\zeta G(x,Q_0^2)(\frac{Q_0^2}{Q^2})^{\lambda_g \cdot \frac{lnx}{ln(Q^2/\Lambda^2)}}, \qquad (13)$$

where $F_2(x,Q_0^2)$ and $G(x,Q_0^2)$ are the input structure function and gluon distribution function. Fig.1 shows the reduced cross section obtained and compared with the experimental results from $H1$ collaboration [7]. In Fig.1 a comparison is made between our obtained values and the existing data, indicating the fact that the reduced cross section σ_r can be determined with reasonable precision. As can be seen, there is some rate of increment as observed in the H1 data, but with a somewhat smaller rate. For Q^2 constant, there is a cross-over point for both of curves whose prediction is numerically equal. To conclude, in

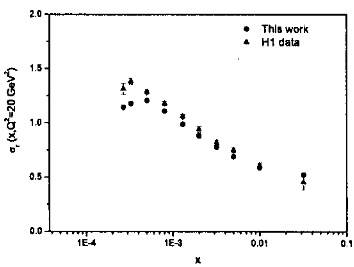

Figure 1: Determination of the reduced DIS scattering cross section (closed points). Triangles represent data from the H1 Collab. [7] with the total errors include the experimental and model uncertainly of the QCD fit.

this paper we have obtained a solution of the DGLAP equation for the exponent $\lambda_S(x,Q^2)$ and $\lambda_g(x,Q^2)$ in the next- to- leading order (NLO) at low x. The behavior of F_2 at low x is consistent with a dependence $F_2(x,Q^2) = A_S x^{-\lambda_S}$ throughout that region. Based upon this behavior an approximate method for the calculation σ_r is presented. In this method the σ_r for low-x values at Q^2 constant value using the DGLAP evolution equation without knowledge of the longitudinal structure function $F_L(x,Q^2)$ determined. There is however a region, a Q^2 interval, where the two regimes, Regge and perturbative QCD, are compatible. We have seen that we can use Regge-like theory with constrain the initial parton densities at $Q^2 = Q_0^2$ and obtain the distributions at higher virtualities with the DGLAP evolution equation. These comparisons indicate that the form of obtained the reduced cross sections are similar to the one predicted from experimental data.

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Radiative Decays of Pseudoscalar (P) and Vector (V) Mesons and the Process $e^+e^- \to \eta'\rho$

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Abstract

Radiative decays of pseudoscalar and vector mesons are calculated in the framework of the chiral Nambu-Jona-Lasinio (NJL) model. We use the amplitude for triangle quark loops of anomalous type. The process of the electron-positron annihilation with production of η' and ρ mesons in the center of mass energy range from 1.6 to 3.5 GeV is considered.

1 Radiative decays of vector and pseudoscalar mesons

For the description of interaction of mesons with quarks we use the NJL model lagrangian [1, 2]:

$$\mathcal{L}_{int} = \bar{q}\left[eQ\hat{A} + (i\gamma_5)\left(g_u\lambda_u\eta_u + g_s\lambda_s\eta_s\right) + \frac{g_\rho}{2}\left(\lambda_3\hat{\rho}_0 + \lambda_u\hat{\omega} + \lambda_s\hat{\phi}\right)\right]q, \qquad (1)$$

where $\bar{q} = \left(\bar{u}, \bar{d}, \bar{s}\right)$ where u, d, s are the quark fields, $Q = \mathrm{diag}\,(2/3, -1/3, -1/3)$ is the quark charge matrix, $\lambda_u = \left(\sqrt{2}\lambda_0 + \lambda_8\right)/\sqrt{3}$, $\lambda_s = \left(-\lambda_0 + \sqrt{2}\lambda_8\right)/\sqrt{3}$ where λ_i are the Gell-Mann matrices and $\lambda_0 = \sqrt{2/3}\,\mathrm{diag}\,(1,1,1)$. $g_u = m_u/f_\pi$, $g_s = m_s/f_s$ are the meson-quark coupling constants which are evaluated by Goldberger-Treiman relation ($m_u =$ 263 MeV, $m_s =$ 407 MeV are quark masses [4], and $f_\pi = 92.4$ MeV is the pion decay constant and $f_s = 1.3 f_\pi$). $g_\rho = 5.94$ is the $\rho \to 2\pi$ coupling constant.

Physical states of η and η' mesons are obtained after taking into account of singlet-octet mixing of η_u and η_s with the angle $\theta = 51.3^o$ [1, 5]:

$$\eta = -\eta_u \sin\theta + \eta_s \cos\theta, \qquad \eta' = \eta_u \cos\theta + \eta_s \sin\theta. \qquad (2)$$

The vector meson decay $V(p_1) \to \gamma(p_2) + P(p_3)$ is described by the amplitude of one loop with quark:

$$M_{V\to P\gamma} = \frac{i}{(2\pi)^2}\, e\; g_P\; g_V\; C_{PV}\; M_q\; J(p_1^2, 0, p_3^2)(e_1 e_2 p_1 p_2), \qquad (3)$$

where $(abcd) \equiv \varepsilon_{\alpha\beta\gamma\delta}a^\alpha b^\beta c^\gamma d^\delta$, $g_V = g_\rho/2$, $g_P = g_u$ if light quarks go through the loop and $g_P = g_s$ if strange quarks are involved; C_{PV} is the flavour-color multiplier corresponding

Decay	Experiment	Approximation I (5)	Approximation II (4)
$\rho \to \eta\gamma$	39.47	65	33.72
$\omega \to \eta\gamma$	4.07	7.83	4.16
$\eta' \to \rho\gamma$	59.68	76.18	41.09
$\eta' \to \omega\gamma$	6.15	7.59	4.04
$\phi \to \eta\gamma$	55.59	71.01	117.9
$\phi \to \eta'\gamma$	0.265	0.497	0.294

Table 1: The table of radiative decays. The values of the widths are in KeV. Approximation I – neglect of the external momentum dependence (5). Approximation II – the real part of exact loop integrals is taken into account (4).

to quark-meson interaction, $C_{\eta\omega} = 2\sin\theta$, $C_{\eta\rho} = 6\sin\theta$, $C_{\eta'\omega} = 2\cos\theta$, $C_{\eta'\rho} = 6\cos\theta$, $C_{\eta\phi} = 4\cos\theta$, $C_{\eta'\phi} = 4\sin\theta$; M_q is the loop quark mass and

$$J(p_1^2, p_2^2, p_3^2) = \mathrm{Re}\left(\int_0^1 dx \int_0^{1-x} dy \frac{1}{M_q^2 - xyp_1^2 - yzp_2^2 - xzp_3^2 - i0}\right), \tag{4}$$

where $z = 1 - x - y$. In the heavy quark approximation (Approximation I) we obtain

$$J(p_1^2, p_2^2, p_3^2) = 1/\left(2M_q^2\right). \tag{5}$$

The matrix element square can be written in the form:

$$|M_{V\to P\gamma}|^2 = \frac{e^2\, g_P^2\, g_V^2\, C_{PV}^2}{(2\pi)^4} \left(M_q\, J(M_V^2, 0, M_P^2)\right)^2 \left[\frac{1}{2}\left(M_V^2 - M_P^2\right)^2\right]. \tag{6}$$

The phase volume of the final state is:

$$d\Phi_{P\gamma} = \frac{d^3p_2}{2E_2}\frac{d^3p_3}{2E_3} = \frac{1}{8\pi}\frac{M_V^2 - M_P^2}{M_V^2}. \tag{7}$$

And then the decay width reads as:

$$\Gamma_{V\to P\gamma} = \frac{1}{3}\frac{\alpha}{2^7\pi^4}\left(\frac{M_V^2 - M_P^2}{M_V}\right)^3 \left[g_P\, g_V\, C_{PV}\, M_q\, J(M_V^2, 0, M_P^2)\right]^2. \tag{8}$$

In Table 1 we present the theoretical results for both the methods – Approximation I (5) and Approximation II (4) – and compare them with the relevant experimental data.

2 Associative production of pseudoscalar and vector mesons in electron-positron annihilation

The matrix elements of the processes of associative production of pseudoscalar and vector mesons $e^+(p_+) + e^-(p_-) \to V(p_1) + P(p_3)$, where $s = (p_+ + p_-)^2$, $p_\pm^2 = m^2$, $p_1^2 = M_V^2$, $p_3^2 = M_P^2$, in the lowest order of the QED coupling constant α have the form :

$$M^{PV} = i\frac{4\pi\alpha}{s} J_\mu^{em} J^{A\mu}, \tag{9}$$

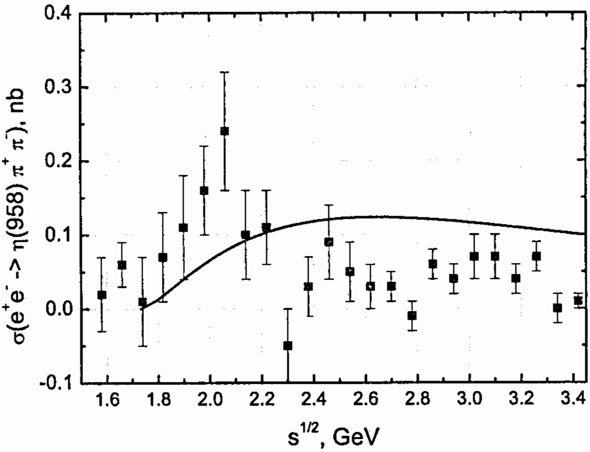

Figure 1: The comparison of our result for the $e^+e^- \rightarrow \eta'\pi^+\pi^-$ with the BABAR-collaboration results for the $e^+e^- \rightarrow \eta'\pi^+\pi^-$ channel [3].

where the QED lepton current is $J^{em}_\mu = \bar{v}(p_+)\gamma_\mu u(p_-)$ and the anomalous current has the form

$$J^{A\mu} = \frac{g_P\, g_V\, C_{PV}}{(2\pi)^2}\,(e_1\mu p_1 p_2)\;\; M_q\;\, J(p_1^2, s, p_3^2), \tag{10}$$

where $p_+ + p_- = p_2 = p_1 + p_3$ and e_1 is the polarization vector of the vector meson (i.e. $(e_1 p_1) = 0$). The total cross section built by general rules is

$$\sigma(s) = \frac{\alpha^2}{96\pi^3 s^3}\lambda^{\frac{3}{2}}\left(s, M_P^2, M_V^2\right)\left|g_V\; g_P\; C_{PV}\; M_q\; J\left(M_P^2, M_V^2, s\right)\right|^2, \tag{11}$$

where $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$ is the well-known triangle function. The cross section of the concrete process $e\bar{e} \rightarrow \eta'(950)\rho$ is drawn on Fig. 1. One can conclude that satisfactory agreement within the experimental errors is observed.

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How to extract information from Green's functions in Landau gauge

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Abstract

The infrared behavior of gluon and ghost propagators offers a crucial test of confinement scenarios in Yang-Mills theories. A nonperturbative study of these propagators from first principles is possible in lattice simulations, but one must consider significantly large lattice sizes in order to approach the infrared limit. We propose constraints based on general properties of the propagators to gain control over the extrapolation of data to the infinite-volume limit. These bounds also provide a way to relate the propagators to simpler, more intuitive quantities. We apply our analysis to the case of pure $SU(2)$ gauge theory in Landau gauge, using the largest lattice sizes to date. Our results seem to contradict commonly accepted confinement scenarios. We argue that it is not so.

1 The Gribov-Zwanziger Confinement Scenario

About thirty years ago, Gribov proposed an interesting confinement mechanism for color charges in Landau (and Coulomb) gauge [1]. His idea was based on the restriction of the physical configuration space to the region Ω of transverse configurations, delimited by the so-called first Gribov horizon, where the smallest (non-trivial) eigenvalue of the Faddeev-Popov (FP) operator $\mathcal{M} = -D_\mu \partial_\mu$ is zero. The limitation of the functional integration to the (first Gribov) region Ω was an attempt to fix the gauge completely, getting rid of spurious gauge copies, known thereafter as Gribov copies. Since the ghost propagator $G(p)$ is given by $\langle p | \mathcal{M}^{-1} | p \rangle$ and $\mathcal{M}$ is semi-positive definite for gluon fields $A \in \Omega$, one cannot have a singularity for $G(p)$ at a finite momentum p. Using perturbation theory up to second order, Gribov wrote (for Landau gauge) the no-pole condition [1, 2]

$$\sigma(0) = \frac{N_c}{4\,(N_c^2 - 1)} \int \frac{d^4 q}{(2\pi)^4} \frac{\langle\, A_\lambda^a(q)\, A_\lambda^a(-q)\,\rangle}{q^2} < 1\,. \tag{1}$$

Here N_c refers to the gauge group $SU(N_c)$, an average over the Lorentz indices λ has been considered and the quantity $\sigma(p)$ enters the ghost propagator as $G(p) \approx p^{-2}\,[1 - \sigma(p)]^{-1}$. The above inequality tells us that, in the infrared (IR) limit, the Landau gluon propagator $D_{\mu\nu}^{ab}(p) = \langle A_\mu^a(p) A_\nu^b(-p) \rangle$ is less singular than $1/p^2$ (in the $4d$ case). Moreover, by using the above no-pole condition as a characterization of the first Gribov region, one can show that the tree-level gluon propagator becomes

$$D(p) = g_0^2\, p^2 / (p^4 + \gamma^4)\,, \tag{2}$$

where the (Gribov) mass parameter γ is fixed by the gap equation $\int d^4q(2\pi)^{-4}(q^4+\gamma^4)^{-1} = 4/(3N_c g^2)$. Then, by using (2) and the gap equation, one can study the IR limit of $\sigma(p)$, obtaining an IR-enhanced ghost propagator $G(p) \propto 1/p^4$. As stressed by Gribov [1], this enhancement is an indication of a long-range effect in the theory that may explain color confinement. In Coulomb gauge, this enhancement of $G(p)$ can be directly related to a color-Coulomb potential linearly rising with distance.

It is interesting that a bound similar to the one above has also been obtained in Ref. [3] by considering a variational method applied to the FP operator $\mathcal{M}$. This bound, known as the ellipsoidal bound, can be written (in the continuum) as

$$\int \frac{d^dq}{(2\pi)^d} \frac{\langle A^b_\mu(q)\, A^b_\mu(-q)\rangle}{q^2} \leq C\,, \tag{3}$$

where C depends on the dimensionality d of the space-time and on the gauge group. One should stress that the ellipsoid $\mathcal{E}$, defined by the ellipsoidal bound, is a region of transverse configurations that includes the first Gribov region Ω. A similar bound can also be obtained on the lattice [4]. Moreover, it is convenient to define on the lattice a region Θ, included in the ellipsoid $\mathcal{E}$ and including the first Gribov region Ω, i.e. $\Omega \subset \Theta \subset \mathcal{E}$. For all configurations belonging to Θ, one can prove [5]

$$\left| \widetilde{A}^b_\mu(0) \right| \leq 2\tan\left(\frac{\pi}{V^{1/d}}\right)\,, \tag{4}$$

where V is the lattice volume and $\widetilde{A}^b_\mu(0) = V^{-1}\sum_x A^b_\mu(x)$ is the gluon field at zero momentum. This quantity may be viewed as a magnetization M. Thus, in the infinite-volume limit one can show that $|M|$ is zero. By adding an external color "magnetic" field H coupled to A in the action and using the above inequality, one also obtains that the free energy per unit volume is null when V goes to infinity [5]. If H is spatially modulated, then the susceptibility χ at zero external field coincides with the usual gluon propagator $D(p)$. The inequalities valid in the region Θ would then suggest that $\lim_{p\to 0}\chi(H=0,p) = \lim_{p\to 0} D(p) = 0$ [4].

In order to restrict the functional integration to the first Gribov region Ω, Zwanziger added to the usual Yang-Mills action a non-local term proportional to $\mathcal{M}^{-1}$ [6]. This term clearly suppresses the probability of configurations near the boundary $\partial\Omega$ of the region Ω. After localizing the action, zeroth-order perturbation theory allows one to obtain again the propagator in Eq. (2). The purely imaginary poles $p^2 = \pm i\gamma^2$ of this propagator make it incompatible with a Kallen-Lehmann representation [6]. These singularities at an unphysical location suggest that gluons are indeed not physical excitations. One should also note [4, 5] that $D(0) = \int d^dx\, D(x) = 0$ means a gluon propagator in position space $D(x)$ that is positive and negative in equal measure. This represents a maximal violation of reflection positivity. In Refs. [4, 5] the violation of reflection positivity for the gluon propagator has been proposed as a confinement mechanism for gluons.

The restriction of the functional integration to Ω has also been discussed in Ref. [7]. The non-local term $-\gamma^4\mathcal{H}$ is added to the Yang-Mills action, where $\mathcal{H}$ is the so-called horizon function, containing an $\mathcal{M}^{-1}$ factor. At the same time, the Gribov mass γ is fixed implicitly by the horizon condition $\langle h\rangle = (N_c^2-1)d$, where h is the horizon function per unit volume. It is interesting that the horizon condition implies a ghost propagator enhanced in the IR limit [8], i.e. $\lim_{p\to 0}[p^2G(p)]^{-1} = 0$. Clearly, the enhancement of the ghost propagator at $p = 0$ should indicate that $G(p)$ feels the singularity of $\mathcal{M}^{-1}$ on $\partial\Omega$. Indeed, since the configuration space has very large dimensionality one expects that in the infinite-volume limit, due to entropy considerations, the Boltzmann weight be concentrated on $\partial\Omega$ [7]. This implies that the smallest nonzero eigenvalue λ_{min} of $\mathcal{M}$ should go to zero in the infinite-volume limit. (This has been verified numerically in Landau gauge [9].)

The IR behavior of propagators and vertices in Landau gauge has also been studied in Ref. [10] by considering the sets of coupled Dyson-Schwinger equations (DSE) for the basic propagators and vertices of Yang-Mills theory (in the $4d$ case). By using a simple power counting for the solutions in the IR limit and constraints obtained using a skeleton expansion, the authors found a consistent solution characterized by an IR-enhanced ghost propagator $G(p) \sim p^{-2(1+\kappa)}$ and by an IR-finite gluon propagator $D(p) \sim p^{2(2\kappa-1)}$ with $\kappa \in [1/2, 3/4]$. Note that $D(0) = 0$ for $\kappa > 1/2$. On the other hand, the analysis carried out in [10] allows also for a solution with a tree-level-like ghost propagator at small momenta $G(p) \sim p^{-2}$ and a finite nonzero gluon propagator $D(0) > 0$. These two consistent solutions have also been obtained by several studies of DSE using specific approximations [11].

Recently it was shown [12, 13] that using the Gribov-Zwanziger approach, i.e. by restricting the functional integration to the Gribov region Ω, one can also obtain in $3d$ and $4d$ a finite nonzero gluon propagator $D(0)$ and a tree-level-like ghost propagator in the IR limit. Here the dynamical mechanism is related to a suitable mass term that may be added to the action while preserving its renormalizability. As a consequence, one can show that the restriction to Ω induces a soft breaking of the BRST symmetry [13]. It is interesting that the same approach cannot be extended to the $2d$ case [14], because the new mass term produces IR singularities that make the restriction to Ω impossible.

An IR-enhanced Landau ghost propagator is also obtained as a consequence of the so-called confinement criterion of Kugo-Ojima [15]. At the same time, this criterion suggests that the perturbative massless pole in the transverse gluon propagator should disappear [16]. In this sense, an IR-suppressed gluon propagator (not necessarily vanishing) can be accommodated in this confinement scenario [17]. Finally, even though the Gribov-Zwanziger and the Kugo-Ojima confinement scenarios seem to predict similar IR behavior for the propagators, it is not clear how to relate the (Euclidean) cutoff at the Gribov horizon to the (Minkowskian) approach of Kugo-Ojima [18].

2 Bounds for the Gluon and the Ghost Propagators

Recently we have introduced [19] rigorous upper and lower bounds for the gluon propagator at zero momentum $D(0)$ by considering the quantity

$$M(0) = \frac{1}{d(N_c^2 - 1)} \sum_{b,\mu} |\widetilde{A}_\mu^b(0)| \, , \tag{5}$$

with $\widetilde{A}_\mu^b(0)$ defined in the previous section. Indeed, by straightforward calculations one finds that

$$V \langle M(0) \rangle^2 \, \leq \, D(0) \, \leq \, V d(N_c^2 - 1) \, \langle M(0)^2 \rangle \, . \tag{6}$$

Thus, if $M(0)$ goes to zero as $V^{-\alpha}$ we obtain that $D(0) \to 0$, $0 < D(0) < +\infty$, or $D(0) \to +\infty$, respectively if α is larger than, equal to or smaller than $1/2$. Recall that

$M(0)$ should go to zero at least as $V^{-1/d}$ in the d-dimensional case [see Eq. (4)]. At the same time, a necessary condition to find $D(0) = 0$ is that $M(0)^2$ goes to zero faster than $1/V$. We note that the above bounds apply to any gauge and that they can be immediately extended to the case $D(p)$ with $p \neq 0$.

We investigated the bounds (6) for pure $SU(2)$ gauge theory in Landau gauge considering several lattice volumes in $2d$, in $3d$ and in $4d$ with the largest lattice corresponding (respectively) to $a^2V \approx (70\,\mathrm{fm})^2$, $a^3V \approx (85\,\mathrm{fm})^3$ and to $a^4V \approx (27\,\mathrm{fm})^4$. By using the Ansatz B_u/L^u for $a^2\langle M(0)^2 \rangle$ we obtain $u = 2.72(1)$ in the $2d$ case, implying $D(0) = 0$. A similar analysis in $3d$ and in $4d$ for the lower and the upper bounds gives $0.4(1)\ \mathrm{GeV}^{-2} \leq a^2D(0) \leq 4(1)\ \mathrm{GeV}^{-2}$ in $3d$ and $2.2(3)\ \mathrm{GeV}^{-2} \leq a^2D(0) \leq 29(5)\ \mathrm{GeV}^{-2}$ in $4d$. Recently, a study for the $4d$ $SU(3)$ case [20] also finds a value for α very close to 1/2. Although the authors conclude that $D(0) = 0$ in the infinite-volume limit, one should observe that in this case the lattice volumes considered are relatively small and the statistics is rather low. Thus, a more detailed analysis in the $SU(3)$ case should be carried out in order to verify if the IR behavior of the gluon propagator agrees [21] or not with the $SU(2)$ case.

One can also obtain lower and upper bounds for the ghost propagator [22]. In Landau gauge, for any nonzero momentum p, one finds

$$\frac{1}{N_c^2-1}\,\frac{1}{\lambda_{min}}\sum_a |\widetilde{\psi}_{min}(a,p)|^2 \leq G(p) \leq \frac{1}{\lambda_{min}}\,, \tag{7}$$

where λ_{min} is the smallest nonzero eigenvalue of the FP operator $\mathcal{M}$ and $\widetilde{\psi}_{min}(a,p)$ is the corresponding eigenvector. If we assume $\lambda_{min} \sim N^{-\delta}$ and $G(p) \sim p^{-2-2\kappa}$ at small p, we should find $2 + 2\kappa \leq \delta$, i.e. $\delta > 2$ is a necessary condition for the IR enhancement of $G(p)$. Note that a similar analysis can be carried out [23] for any generic gauge condition $\mathcal{F}[A] = 0$ imposed on the lattice by minimizing a functional $E[U]$, where U is the (gauge) link variable. Indeed, from the second variation of $E[U]$ one can obtain the corresponding FP matrix $\mathcal{M}$ and the set of local minima of $E[U]$ defines the Gribov region Ω, where all eigenvalues of $\mathcal{M}$ are positive. In the infinite-volume limit, entropy favors configurations near $\partial\Omega$ (where λ_{min} goes to zero). Thus, inequalities of the type (7) can tell us if one should expect an enhancement of the ghost propagator $G(p)$ when the Boltzmann weight gets concentrated on $\partial\Omega$.[1]

A study in the $SU(2)$ Landau case [22] suggests that $\delta > 2$ in 2d, implying IR enhancement of $G(p)$, while $\delta < 2$ in 4d. These results are confirmed if one considers the dressing function $p^2G(p^2)$ for very large lattice volumes [22]. Indeed, the data in the $2d$ case can be fitted by $\sim p^{-2\kappa}$, with κ between 0.1 and 0.2. On the contrary, in $3d$ and in $4d$ the data are well described by $a - b\log(1 + cp^2)$, supporting $\kappa = 0$.

Let us note that our data for the gluon and ghost propagators are in good agreement with results obtained by other groups using very large lattice volumes [25]. Of course, one should also recall that the region Ω is actually not free of Gribov copies [3, 5, 26] and that the configuration space should be identified with the so-called fundamental modular region (FMR) Λ. On the other hand, the restriction of the configuration space to the FMR should not make any difference on the numerical verification of the Gribov-Zwanziger scenario.

[1]For example, in $4d$ Maximally Abelian gauge one sees that λ_{min} goes to zero at large volume but the ghost propagator stays finite at zero momentum [24].

Indeed, as we have seen in the previous section, this scenario is based on the restriction of the configuration space to the region Ω, which includes Λ. Actually, the bounds obtained for the gluon fields (see again Section 1) apply to regions, such as Θ and $\mathcal{E}$, that are even larger than the region Ω. Finally, as explained in [27], the restriction to the FMR can only make the ghost propagator less singular, as confirmed by recent lattice data [28].

3 Conclusions

We have presented simple properties of gluon and ghost propagators that constrain (by upper and lower bounds) their IR behavior. For the gluon case we define a magnetization-like quantity, while for the ghost case we relate the propagator to λ_{min} of the FP matrix. We propose the study of these quantities, as a function of the lattice volume, in order to gain better control of the infinite-volume limit for the propagators in the IR regime.[2]

Our data support a Landau-gauge gluon propagator that is IR finite in $3d$ and $4d$. This result can be interpreted [19] as a consequence of "self-averaging" of a magnetization-like quantity, i.e. $M(0)$ without the absolute value. In particular, one may think of $D(0)$ as a response function (susceptibility) of this magnetization. In this case it is natural to expect $D(0) > 0$ in the infinite-volume limit.[3] In the $2d$ case the magnetization is "over self-averaging" and the susceptibility is zero. These results are in agreement with the suppression of the IR components of the gluon field A due to the limitation of the functional space to the first Gribov region Ω. At the same time the gluon propagator displays a clear violation of reflection positivity in Landau gauge [31], i.e. the confinement mechanism for gluons proposed in [4, 5] is confirmed by lattice data.

For the ghost propagator we find that in $3d$ and $4d$ the behavior at small momenta is essentially tree-level like, while in $2d$ this propagator seems to be clearly enhanced compared to the perturbative behavior p^{-2}. As described in Section 1, these results are not necessarily in contradiction with the Gribov-Zwanziger approach [12, 13, 14].

4 Acknowledgments

We acknowledge partial support from FAPESP and from CNPq. The work of T.M. was supported also by the Alexander von Humboldt Foundation.

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Rare $\pi \to e^+e^-$ decay as a filter for low mass dark matter

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Abstract

Experimental and theoretical progress concerning the rare decay $\pi^0 \to e^+e^-$ is briefly reviewed. It includes the latest data from KTeV and a new model independent estimate of the decay branching which show the deviation between experiment and theory at the level of 3.3σ.

1 Introduction

Astrophysics observables tell us that 95% of the matter in the Universe is not described in terms of the Standard Model (SM) matter. Thus, the search for the traces of New Physics is a fundamental problem of particle physics. There are two strategies to look for the effects of New Physics: experiments at high energy and experiments at low energy. In high-energy experiments it is considered that due to a huge amount of energy the heavy degrees of freedom presumably characteristic of the SM extension sector are possible to excite. In low-energy experiments it is huge statistics that compensates the lack of energy by measuring the rare processes characteristic of such extensions. At present, there is no any evidence for deviation of SM predictions from the results of high-energy experiments and we are waiting for the LHC epoch. On the other hand, in low-energy experiments there are rough edges indicating such deviations. The most famous example is the muon $(g-2)$. Below it will be shown that due to recent experimental and theoretical progress the rare process $\pi^0 \to e^+e^-$ became a good SM test process and that at the moment there is a discrepancy between the SM prediction and experiment at the level of 3.3σ deviation.

2 KTeV data

In 2007, the KTeV collaboration published the result [1] for the branching ratio of the pion decay into an electron-positron pair

$$B^{\rm KTeV}_{\rm no-rad}\left(\pi^0 \to e^+e^-\right) = (7.48 \pm 0.38) \cdot 10^{-8}. \tag{1}$$

The result is based on observation of 794 candidate $\pi^0 \to e^+e^-$ events using $K_L \to 3\pi^0$ as a source of tagged π^0s. Due to a complicated chain of the process and a good technique for final state resolution used by KTeV this is a process with low background.

3 Classical theory of $\pi^0 \to e^+e^-$ decay

The rare decay $\pi^0 \to e^+e^-$ has been studied theoretically over the years, starting with the first prediction of the rate by Drell [2]. Since no spinless current coupling of quarks to leptons exists, the decay is described in the lowest order of QED as a one-loop process via the two-photon intermediate state, as shown in Fig. 1. A factor of $2\,(m_e/m_\pi)^2$ corresponding to the approximate helicity conservation of the interaction and two orders of α suppress the decay with respect to the $\pi^0 \to \gamma\gamma$ decay, leading to an expected branching ratio of about 10^{-7}. In the Standard Model contributions from the weak interaction to this process are many orders of magnitude smaller and can be neglected.

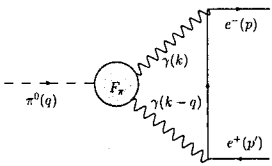

Figure 1: Triangle diagram for the $\pi^0 \to e^+e^-$ process with a pion $\pi^0 \to \gamma^*\gamma^*$ form factor in the vertex.

To the lowest order in QED the normalized branching ratio is given by

$$R\left(\pi^0 \to e^+e^-\right) = \frac{B\left(\pi^0 \to e^+e^-\right)}{B\left(\pi^0 \to \gamma\gamma\right)} = 2\left(\frac{\alpha}{\pi}\frac{m_e}{m_\pi}\right)^2 \beta_e\left(m_\pi^2\right)\left|\mathcal{A}\left(m_\pi^2\right)\right|^2, \tag{2}$$

where $\beta_e\left(q^2\right) = \sqrt{1-4\frac{m_e^2}{q^2}}$, $B\left(\pi^0 \to \gamma\gamma\right) = 0.988$. The amplitude $\mathcal{A}$ can be written as

$$\mathcal{A}\left(q^2\right) = \frac{2i}{q^2}\int \frac{d^4k}{\pi^2} F_{\pi\gamma^*\gamma^*}\left(k^2,(k-q)^2\right)\cdot\frac{q^2k^2-(qk)^2}{\left(k^2+i\varepsilon\right)\left((k-q)^2+i\varepsilon\right)\left((k-p)^2-m_e^2+i\varepsilon\right)}, \tag{3}$$

where $q^2 = m_\pi^2, p^2 = m_e^2$. $F_{\pi\gamma^*\gamma^*}$ is the form factor of the transition $\pi^0 \to \gamma^*\gamma^*$ with off-shell photons.

The imaginary part of $\mathcal{A}$ is defined uniquely as

$$\mathrm{Im}\mathcal{A}\left(q^2\right) = \frac{\pi}{2\beta_e\left(q^2\right)}\ln\left(y_e\left(q^2\right)\right), \tag{4}$$
$$y_e\left(q^2\right) = \frac{1-\beta_e\left(q^2\right)}{1+\beta_e\left(q^2\right)}.$$

It comes from the contribution of real photons in the intermediate state and is model independent since $F_{\pi\gamma^*\gamma^*}(0,0) = 1$. Using inequality $|\mathcal{A}|^2 \geq (\mathrm{Im}\mathcal{A})^2$ one can get the well-known unitary bound for the branching ratio [3]

$$B\left(\pi^0 \to e^+e^-\right) \geq B^{\text{unitary}}\left(\pi^0 \to e^+e^-\right) = 4.69\cdot 10^{-8}. \tag{5}$$

One can attempt to reconstruct the full amplitude by using a once-subtracted dispersion relation [5]

$$\mathcal{A}\left(q^2\right) = \mathcal{A}\left(q^2=0\right) + \frac{q^2}{\pi}\int_0^\infty ds \frac{\mathrm{Im}\mathcal{A}\left(s\right)}{s\left(s-q^2\right)}. \tag{6}$$

If one assumes that Eq. (4) is valid for any q^2, then one arrives for $q^2 \geq 4m_e^2$ at [6, 7, 8]

$$\mathrm{Re}\mathcal{A}\left(q^2\right) = \mathcal{A}\left(q^2=0\right) + \frac{1}{\beta_e\left(q^2\right)}\cdot\left[\frac{1}{4}\ln^2\left(y_e\left(q^2\right)\right) + \frac{\pi^2}{12} + \mathrm{Li}_2\left(-y_e\left(q^2\right)\right)\right], \tag{7}$$

where $\mathrm{Li}_2\left(z\right) = -\int_0^z \left(dt/t\right)\ln\left(1-t\right)$ is the dilogarithm function. The second term in Eq. (7) takes into account a strong q^2 dependence of the amplitude around the point $q^2 = 0$ occurring due to the branch cut coming from the two-photon intermediate state. In the leading order in $\left(m_e/m_\pi\right)^2$, Eq. (7) reduces to

$$\mathrm{Re}\mathcal{A}\left(m_\pi^2\right) = \mathcal{A}\left(q^2=0\right) + \ln^2\left(\frac{m_e}{m_\pi}\right) + \frac{\pi^2}{12}. \tag{8}$$

Thus, the amplitude is fully reconstructed up to a subtraction constant. Usually, this constant containing the nontrivial dynamics of the process is calculated within different models describing the form factor $F_\pi(k^2, q^2)$ [4, 5, 7, 9, 10]. However, it has recently been shown in [10] that this constant may be expressed in terms of the inverse moment of the pion transition form factor given in symmetric kinematics of spacelike photons

$$\mathcal{A}\left(q^2=0\right) = 3\ln\left(\frac{m_e}{\mu}\right) - \frac{5}{4} - \frac{3}{2}\left[\int_0^{\mu^2} dt \frac{F_{\pi\gamma^*\gamma^*}\left(t,t\right)-1}{t} + \int_{\mu^2}^\infty dt \frac{F_{\pi\gamma^*\gamma^*}\left(t,t\right)}{t}\right]. \tag{9}$$

Here, μ is an arbitrary (factorization) scale. One has to note that the logarithmic dependence of the first term on μ is compensated by the scale dependence of the integrals in the brackets. In this way two independent processes becomes related.

4 Importance of CLEO data on $F_{\pi\gamma^*\gamma}$

In order to estimate the integral in Eq. (9), one needs to define the pion transition form factor in symmetric kinematics for spacelike photon momenta. Since it is unknown from the first principles, we will adapt the available experimental data to perform such estimates. Let us first use the fact that $F_{\pi\gamma^*\gamma^*}\left(t,t\right) < F_{\pi\gamma^*\gamma^*}\left(t,0\right)$ for $t>0$ in order to obtain the lower bound of the integral in Eq. (9). For this purpose, we take the experimental results from the CELLO [11] and CLEO [12] Collaborations for the pion transition form factor in asymmetric kinematics for spacelike photon momentum which is well parametrized by the monopole form [12]

$$F_{\pi\gamma^*\gamma^*}^{\mathrm{CLEO}}\left(t,0\right) = \frac{1}{1+t/s_0^{\mathrm{CLEO}}},$$
$$s_0^{\mathrm{CLEO}} = \left(776 \pm 22 \quad \mathrm{MeV}\right)^2. \tag{10}$$

For this type of the form factor one finds from Eq. (9) that

$$\mathcal{A}\left(q^2=0\right) > -\frac{3}{2}\ln\left(\frac{s_0^{\mathrm{CLEO}}}{m_e^2}\right) - \frac{5}{4} = -23.2 \pm 0.1. \tag{11}$$

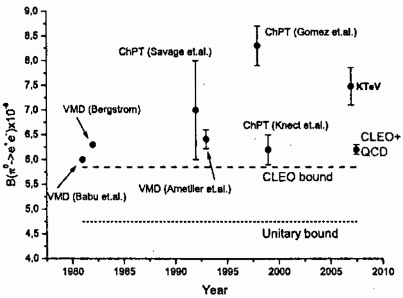

Figure 2: Evolution of model predictions and comparison with the latest KTeV result.

Thus, for the branching ratio we are able to establish the important lower bound which considerably improves the unitary bound given by Eq. (5)

$$B\left(\pi^0 \to e^+e^-\right) > B^{\rm CLEO}\left(\pi^0 \to e^+e^-\right) = (5.84 \pm 0.02)\cdot 10^{-8}. \tag{12}$$

It is natural to assume that the monopole form is also a good parametrization for the form factor in symmetric kinematics

$$F_{\pi\gamma^*\gamma^*}(t,t) = \frac{1}{1+t/s_1}. \tag{13}$$

The scale s_1 can be fixed from the relation for the slopes of the form factors in symmetric and asymmetric kinematics at low t [13],

$$-\frac{\partial F_{\pi\gamma^*\gamma^*}(t,t)}{\partial t}\bigg|_{t=0} = -2\frac{\partial F_{\pi\gamma^*\gamma^*}(t,0)}{\partial t}\bigg|_{t=0}, \tag{14}$$

that gives $s_1 = s_0/2$. Note that a similar reduction of the scale is also predicted by OPE QCD from the large momentum behavior of the form factors: $s_1^{OPE} = s_0^{OPE}/3$ [14]. Thus, the estimate for $\mathcal{A}(0)$ can be obtained from Eq. (11) by shifting the lower bound by a positive number which belongs to the interval $[3\ln(2)/2, 3\ln(3)/2]$

$$\mathcal{A}\left(q^2=0\right) = -\frac{3}{2}\ln\left(\frac{s_1}{m_e^2}\right) - \frac{5}{4} = -21.9 \pm 0.3. \tag{15}$$

With this result the branching ratio becomes

$$B\left(\pi^0 \to e^+e^-\right) = (6.23 \pm 0.09)\cdot 10^{-8}. \tag{16}$$

This is 3.3 standard deviations lower than the KTeV result given by Eq. (1).

5 Possible explanations of the effect

Therefore, it is extremely important to trace possible sources of the discrepancy between the KTeV experiment and theory. There are a few possibilities: (1) problems with (statistic) experiment procession, (2) inclusion of QED radiation corrections by KTeV is wrong, (3) unaccounted mass corrections are important, and (4) effects of new physics. At the moment, the last possibilities were reinvestigated. In [15], the contribution of QED radiative corrections to the $\pi^0 \to e^+e^-$ decay, which must be taken into account when comparing the theoretical prediction (16) with the experimental result (1), was revised. Comparing with earlier calculations [16], the main progress is in the detailed consideration of the $\gamma^*\gamma^* \to e^+e^-$ subprocess and revealing of dynamics of large and small distances. Occasionally, this number agrees well with the earlier prediction based on calculations [16] and, thus, the KTeV analysis of radiative corrections is confirmed. In [17] it was shown that the mass corrections are under control and do not resolve the problem. So our main conclusion is that the inclusion of radiative and mass corrections is unable to reduce the discrepancy between the theoretical prediction for the decay rate (16) and experimental result (1).

6 $\pi^0 \to e^+e^-$ decay as a filtering process for low mass dark matter

If one thinks about an extension of the Standard Model in terms of heavy, of an order of 100 GeV or higher, particles, then the contribution of this sort of particles to the pion decay is negligible. However, there is a class of models for description of Dark Matter with a mass spectrum of particles of an order of 10 MeV [18]. This model postulates a neutral scalar dark matter particle χ which annihilates to produce electron/positron pairs: $\chi\chi \to e^+e^-$. The excess positrons produced in this annihilation reaction could be responsible for the bright 511 keV line emanating from the center of the galaxy [19]. The effects of low mass vector boson U appearing in such model of dark matter (Fig. 3) were considered in [20] where the excess of KTeV data over theory put the constraint on coupling which is consistent with that coming from the muon anomalous magnetic moment and relic radiation [21]. Thus, the pion decay might be a filtering process for light dark matter particles.

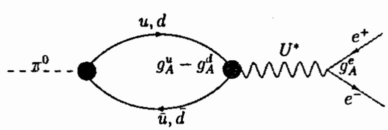

Figure 3: Loop diagram for $\pi^0 \to e^+e^-$ process induced by the low mass exotic U^* boson.

Further independent experiments at KLOE, NA48, WASAatCOSY, BES III and other facilities will be crucial for resolution of the problem. Also important is to get more precise data on the pion transition form factor in asymmetric as well in symmetric kinematics.

7 Acknowledgments

I am grateful to the Organizers for a nice meeting and kind invitation to present our results. I also acknowledge partial support from the Scientific School grant 4476.2006.2.

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Relativistic description of tetraquarks with hidden charm

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Abstract

The masses of the ground and excited heavy tetraquarks with hidden charm are calculated within the relativistic diquark-antidiquark picture. It is argued that recently observed charmonium-like states $X(3872)$, $Y(4260)$, $Y(4360)$, $Z(4248)$, $Z(4433)$ and $Y(4660)$ could be tetraquark states with hidden charm.

Recently, significant experimental progress has been achieved in charmonium spectroscopy. Several new charmonium-like states, such as $X(3872)$, $Y(4260)$, $Y(4360)$, $Y(4660)$, $Z(4248)$, $Z(4430)$, etc., were observed [1] which cannot be simply accommodated in the quark-antiquark ($c\bar{c}$) picture. These states and especially the charged ones can be considered as indications of the possible existence of exotic multiquark states [2, 3]. In our papers [4, 5] we calculated masses of the ground state heavy tetraquarks in the framework of the relativistic quark model based on the quasipotential approach in quantum chromodynamics. Here we extend this analysis to the consideration of the excited tetraquark states with hidden charm. As previously, we use the diquark-antidiquark picture to reduce a complicated relativistic four-body problem to the subsequent two more simple two-body problems. The first step consists in the calculation of the masses, wave functions and form factors of the diquarks, composed from light and heavy quarks. At the second step, a heavy tetraquark is considered to be a bound diquark-antidiquark system. It is important to emphasize that we do not consider the diquark as a point particle but explicitly take into account its structure by calculating the form factor of the diquark-gluon interaction in terms of the diquark wave functions.

In the quasipotential approach the two-particle bound state with the mass M and masses of the constituents $m_{1,2}$ in momentum representation is described by the wave function $\Psi(\mathbf{p})$ satisfying the quasipotential equation of the Schrödinger type [6, 7]

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_{d,T}(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V_{d,T}(\mathbf{p},\mathbf{q};M)\Psi_{d,T}(\mathbf{q}), \tag{1}$$

where the relativistic reduced mass is

$$\mu_R = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3},$$

and the on-mass-shell relative momentum squared

$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}.$$

Table 1: Masses M and form factor parameters of charmed diquarks. S and A denote scalar and axial vector diquarks which are antisymmetric $[\cdots]$ and symmetric $\{\cdots\}$ in flavour, respectively.

Quark content	Diquark type	M (MeV)	ξ (GeV)	ζ (GeV^2)
$[c,q]$	S	1973	2.55	0.63
$\{c,q\}$	A	2036	2.51	0.45
$[c,s]$	S	2091	2.15	1.05
$\{c,s\}$	A	2158	2.12	0.99

The subscript d refers to the diquark and T refers to the tetraquark composed of a diquark and antidiquark. The explicit expressions for the corresponding quasipotentials $V_{d,T}(\mathbf{p},\mathbf{q};M)$ can be found in Ref. [5].

At the first step, we calculate the masses and form factors of the light and heavy diquark. As it is well known, the light quarks are highly relativistic, which makes the v/c expansion inapplicable and thus, a completely relativistic treatment of the light quark dynamics is required. To achieve this goal we closely follow our consideration of the spectra of light mesons [8] and adopt the same procedure to make the relativistic potential local by replacing $\epsilon_{1,2}(p) = \sqrt{m_{1,2}^2 + \mathbf{p}^2} \to E_{1,2} = (M^2 - m_{2,1}^2 + m_{1,2}^2)/2M$. Solving numerically the quasipotential equation (1) with the complete relativistic potential, which depends on the diquark mass in a complicated highly nonlinear way [9], we get the diquark masses and wave functions. In order to determine the diquark interaction with the gluon field, which takes into account the diquark structure, we calculate the corresponding matrix element of the quark current between diquark states. Such calculation leads to the emergence of the form factor $F(r)$ entering the vertex of the diquark-gluon interaction [9]. This form factor is expressed through the overlap integral of the diquark wave functions. Our estimates show that this form factor can be approximated with a high accuracy by the expression

$$F(r) = 1 - e^{-\xi r - \zeta r^2}. \tag{2}$$

The values of the masses and parameters ξ and ζ for heavy-light scalar diquark $[c,q]$ and axial vector diquark $\{c,q\}$ ground states are given in Table 1.

At the second step, we calculate the masses of heavy tetraquarks considered as the bound states of a heavy-light diquark and antidiquark. Masses of both ground and excited states of tetraquarks with hidden charm are obtained. Excitations only of the diquark-antidiquark system are considered.

In the diquark-antidiquark picture of heavy tetraquarks both scalar S (antisymmetric in flavour $(Qq)_{S=0} = [Qq]$) and axial vector A (symmetric in flavour $(Qq)_{S=1} = \{Qq\}$) diquarks are considered. Therefore we get the following structure of the $(Qq)(\bar{Q}\bar{q}')$ ground $(1S)$ states (C is defined only for $q = q'$):

- Two states with $J^{PC} = 0^{++}$:

$$X(0^{++}) = (Qq)_{S=0}(\bar{Q}\bar{q}')_{S=0}$$
$$X(0^{++\prime}) = (Qq)_{S=1}(\bar{Q}\bar{q}')_{S=1}$$

Table 2: Masses of charm diquark-antidiquark ground ($1S$) states (in MeV) calculated in [4]. S and A denote scalar and axial vector diquarks.

State	Diquark	Mass		
J^{PC}	content	$cq\bar{c}\bar{q}$	$cs\bar{c}\bar{s}$	$cq\bar{c}\bar{s}$
0^{++}	$S\bar{S}$	3812	4051	3922
$1^{+\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	3871	4113	3982
0^{++}	$A\bar{A}$	3852	4110	3967
1^{+-}	$A\bar{A}$	3890	4143	4004
2^{++}	$A\bar{A}$	3968	4209	4080

- Three states with $J = 1$:

$$X(1^{++}) = \frac{1}{\sqrt{2}}[(Qq)_{S=1}(\bar{Q}\bar{q}')_{S=0} + (Qq)_{S=0}(\bar{Q}\bar{q}')_{S=1}]$$
$$X(1^{+-}) = \frac{1}{\sqrt{2}}[(Qq)_{S=0}(\bar{Q}\bar{q}')_{S=1} - (Qq)_{S=1}(\bar{Q}\bar{q}')_{S=0}]$$
$$X(1^{+-\prime}) = (Qq)_{S=1}(\bar{Q}\bar{q}')_{S=1}$$

- One state with $J^{PC} = 2^{++}$:

$$X(2^{++}) = (Qq)_{S=1}(\bar{Q}\bar{q}')_{S=1}.$$

The orbitally excited ($1P, 1D \ldots$) states are constructed analogously. As we find, a very rich spectrum of tetraquarks emerges. However the number of states in the considered diquark-antidiquark picture is significantly less than in the genuine four-quark approach.

The diquark-antidiquark model of heavy tetraquarks predicts the existence of a flavour $SU(3)$ nonet of states with hidden charm or beauty ($Q = c, b$): four tetraquarks [$(Qq)(\bar{Q}\bar{q})$, $q = u, d$] with neither open or hidden strangeness, which have electric charges 0 or ± 1 and isospin 0 or 1; four tetraquarks [$(Qs)(\bar{Q}\bar{q})$ and $(Qq)(\bar{Q}\bar{s})$, $q = u, d$] with open strangeness ($S = \pm 1$), which have electric charges 0 or ± 1 and isospin $\frac{1}{2}$; one tetraquark $(Qs)(\bar{Q}\bar{s})$ with hidden strangeness and zero electric charge. Since we neglect in our model the mass difference of u and d quarks and electromagnetic interactions, the corresponding tetraquarks will be degenerate in mass. A more detailed analysis [10] predicts that the tetraquark mass differences can be of a few MeV so that the isospin invariance is broken for the $(Qq)(\bar{Q}\bar{q})$ mass eigenstates and thus in their strong decays. The (non)observation of such states will be a crucial test of the tetraquark model.

The calculated masses of the heavy tetraquark ground ($1S$) states are shown in Table 2. In Table 3 we compare our results (EFG) for the masses of the ground and excited charm diquark-antidiquark bound states with the predictions of Refs. [10, 11, 12, 13] and with the masses of the observed highly-excited charmonium-like states [1, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23]. We assume that the excitations occur only between the bound diquark and antidiquark. Possible excitations of diquarks are not considered. Our calculation of the heavy baryon masses supports such a scheme [9]. In this table we give our predictions only for some of the masses of the orbitally and radially excited states for which possible experimental candidates are observed. The differences in some of the presented theoretical

Table 3: Comparison of theoretical predictions for the masses of the ground and excited charm diquark-antidiquark states (in MeV) and possible experimental candidates.

State J^{PC}	Diquark content	Theory EFG	Theory [10]-[13]	Experiment state	Experiment mass
$1S$					
0^{++}	$S\bar{S}$	3812	3723		
1^{++}	$(S\bar{A}+\bar{S}A)/\sqrt{2}$	3871	$3872^{\dagger}$	$X(3872)$ $X(3876)$	3871.4 ± 0.6 [1] $3875.2 \pm 0.7^{+0.9}_{-1.8}$ [1]
1^{+-}	$(S\bar{A}-\bar{S}A)/\sqrt{2}$	3871	3754		
0^{++}	$A\bar{A}$	3852	3832		
1^{+-}	$A\bar{A}$	3890	3882		
2^{++}	$A\bar{A}$	3968	3952	$Y(3943)$	$3943 \pm 11 \pm 13$ [14] $3914.3^{+4.1}_{-3.8}$ [15]
$1P$					
1^{--}	$S\bar{S}$	4244	4330 ± 70 $(cs\bar{c}\bar{s})$	$Y(4260)$	$4259 \pm 8^{+2}_{-6}$ [16] $4247 \pm 12^{+17}_{-32}$ [17]
1^{-}	$S\bar{S}$	4244		$Z(4248)$	$4248^{+44+180}_{-29-35}$ [18]
0^{-}	$(S\bar{A}\pm\bar{S}A)/\sqrt{2}$	4267			
1^{--}	$(S\bar{A}-\bar{S}A)/\sqrt{2}$	4284		$Y(4260)$	$4284^{+17}_{-16}\pm4$ [19]
1^{--}	$A\bar{A}$	4277			
1^{--}	$A\bar{A}$	4350		$Y(4360)$	$4361 \pm 9 \pm 9$ [20] 4324 ± 24 [21]
$2S$					
1^{+}	$(S\bar{A}\pm\bar{S}A)/\sqrt{2}$	4431		$Z(4430)$	$4433\pm4\pm2$ [22]
0^{+}	$A\bar{A}$	4434			
1^{+}	$A\bar{A}$	4461	~ 4470		
$2P$					
1^{--}	$S\bar{S}$	4666		$Y(4660)$ $X(4630)$	$4664 \pm 11 \pm 5$ [20] 4634^{+8+5}_{-7-8} [23]

† input

mass values can be attributed to the substantial distinctions in the used approaches. We describe the diquarks dynamically as quark-quark bound systems and calculate their masses and form factors, while in Refs.[10, 11, 12, 13] they are treated only phenomenologically. Then we consider the tetraquark as purely the diquark-antidiquark bound system. In distinction, Maini et al. consider a hyperfine interaction between all quarks which, e.g., causes the splitting of 1^{++} and 1^{+-} states arising from the SA diquark-antidiquark compositions. ¿From Table 3 we see that our dynamical calculation supports the assumption [10] that $X(3872)$ can be the axial vector 1^{++} tetraquark state composed from the scalar and axial vector diquark and antidiquark in the relative $1S$ state. Recent Belle and BaBar results indicate the existence of a second $X(3875)$ particle a few MeV above $X(3872)$. This state could be naturally identified with the second neutral particle predicted by the tetraquark model [11]. On the other hand, in our model the lightest scalar 0^{++} tetraquark is predicted to be above the open charm threshold $D\bar{D}$ and thus to be broad, while in the model [10] it lies a few MeV below this threshold, and thus is predicted to be narrow. Our 2^{++} tetraquark also lies higher than the one in Ref.[10], thus making the interpretation of this state as $Y(3943)$ less probable, especially if one averages the original Belle result with the recent BaBar value which is somewhat lower.

The discovery in the initial state radiation at B-factories of the $Y(4260)$, $Y(4360)$ and $Y(4660)$ indicates an overpopulation of the expected charmonium 1^{--} states [1, 16, 17, 19, 20, 21]. Maini et al. [13] argue that $Y(4260)$ is the 1^{--} $1P$ state of the charm-strange diquark-antidiquark tetraquark. We find that $Y(4260)$ cannot be interpreted in this way, since the mass of such $([cs]_{S=0}[\bar{c}\bar{s}]_{S=0})$ tetraquark is found to be ~ 200 MeV higher. A more natural tetraquark interpretation could be the 1^{--} $1P$ state $([cq]_{S=0}[\bar{c}\bar{q}]_{S=0})$ $(S\bar{S})$ which mass is predicted in our model to be close to the mass of $Y(4260)$ (see Table 3). Then the $Y(4260)$ would decay dominantly into $D\bar{D}$ pairs. The other possible interpretations of $Y(4260)$ are the 1^{--} $1P$ states of $(S\bar{A} - \bar{S}A)/\sqrt{2}$ and $A\bar{A}$ tetraquarks which predicted masses have close values. These additional tetraquark states could be responsible for the mass difference of $Y(4260)$ observed in different decay channels. As we see from Table 3, the recently discovered resonances $Y(4360)$ and $Y(4660)$ in the $e^+e^- \to \pi^+\pi^-\psi'$ cross section can be interpreted as the excited 1^{--} $1P$ $(A\bar{A})$ and $2P$ $(S\bar{S})$ tetraquark states, respectively. The peak $X(4630)$ very recently observed by Belle in $e^+e^- \to \Lambda_c^+\Lambda_c^-$ [23] is consistent with a 1^{--} resonance $Y(4660)$ and therefore has the same interpretation in our model.

Recently the Belle Collaboration reported the observation of a relatively narrow enhancement in the $\pi^+\psi'$ invariant mass distribution in the $B \to K\pi^+\psi'$ decay [1, 22]. This new resonance, $Z^+(4430)$, is unique among other exotic meson candidates, since it is the first state which has a non-zero electric charge. Different theoretical interpretations were suggested [1]. Maiani et al. [12] gave qualitative arguments that the $Z^+(4430)$ could be the first radial excitation $(2S)$ of a diquark-antidiquark $X^+_{u\bar{d}}(1^{+-}; 1S)$ state $(A\bar{A})$ with mass 3882 MeV. Our calculations indicate that the $Z^+(4430)$ can indeed be the 1^+ $2S$ $[cu][\bar{c}\bar{d}]$ tetraquark state. It could be the first radial excitation of the ground state $(S\bar{A} - \bar{S}A)/\sqrt{2}$, which has the same mass as $X(3872)$. The other possible interpretation is the 0^+ $2S$ $[cu][\bar{c}\bar{d}]$ tetraquark state $(A\bar{A})$ which has a very close mass. Measurement of the $Z^+(4430)$ spin will discriminate between these possibilities.

Encouraged by this discovery, the Belle Collaboration performed a study of $\bar{B}^0 \to K^-\pi^+\chi_{c1}$ and observed a double peaked structure in the $\pi^+\chi_{c1}$ invariant mass distribution [18]. These two charged hidden charm peaks, $Z(4051)$ and $Z(4248)$, are explicitly exotic. We find no tetraquark candidates for the former, $Z(4051)$, structure. On the other hand, we see from Table 3 that $Z(4248)$ can be interpreted in our model as the charged partner of the 1^- $1P$ state $S\bar{S}$ or as the 0^- $1P$ state of the $(S\bar{A} \pm \bar{S}A)/\sqrt{2}$ tetraquark.

In summary, we calculated the masses of excited heavy tetraquarks with hidden charm in the diquark-antidiquark picture. In contrast to previous phenomenological treatments, we used the dynamical approach based on the relativistic quark model. Both diquark and tetraquark masses were obtained by numerical solution of the quasipotential wave equation with the corresponding relativistic potentials. The diquark structure was taken into account in terms of diquark wave functions. It is important to emphasize that, in our analysis, we did not introduce any free adjustable parameters but used their values fixed from our previous considerations of heavy and light hadron properties. It was found that the $X(3872)$, $Z(4248)$, $Y(4260)$, $Y(4360)$, $Z(4430)$ and $Y(4660)$ exotic meson candidates can be tetraquark states with hidden charm.

We are grateful to organizers of the SPMTP Conference for hospitality. This work was supported in part by the Bundesministerium für Bildung and Forschung (D.E.), the Russian Science Support Foundation (V.O.G.) and the Russian Foundation for Basic Research (RFBR) (grant No.08-02-00582) (R.N.F. and V.O.G.).

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Masses of heavy baryons

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Abstract

The mass spectra of the ground state and excited heavy baryons consisting of two light (u, d, s) and one heavy (c, b) quarks are calculated in the heavy-quark–light-diquark picture within the relativistic quark model.

During last few years a significant experimental progress has been achieved in studying heavy baryons with one heavy quark. At present masses of all ground states of charmed baryons as well as of their excitations are known experimentally with rather good precision [1]. A half of the ground state bottom baryon masses are also known now. Here we briefly review our studies of masses of the ground state and excited heavy baryons containing one heavy quark. All calculations [2, 3] are performed in the framework of the relativistic quark model based on the quasipotential approach in QCD. We use the heavy-quark–light-diquark approximation to reduce a complicated relativistic three-body problem to the subsequent solution of two more simple two-body problems. The first step is the calculation of the masses, wave functions and form factors of the diquarks, composed from two light quarks. Next, at the second step, a heavy baryon is treated as a relativistic bound system of a light diquark and heavy quark. It is important to emphasize that we do not consider a diquark as a point particle but explicitly take into account its structure through the diquark-gluon vertex expressed in terms of the diquark wave functions. The calculated values of the ground state and excited baryon masses are given in Tables 1-4 in comparison with available experimental data [1].

Table 1: Masses of the Λ_Q $(Q = c, b)$ heavy baryons (in MeV).

		$Q=c$		$Q=b$		
$I(J^P)$	Qd state	M	$M^{\rm exp}$ [1]	M	$M^{\rm exp}$ [1]	$M^{\rm exp}$ CDF
$0(\frac{1}{2}^+)$	$1S$	2297	2286.46(14)	5622	5624(9)	5619.7(2.4)
$0(\frac{1}{2}^-)$	$1P$	2598	2595.4(6)	5930		
$0(\frac{3}{2}^-)$	$1P$	2628	2628.1(6)	5947		
$0(\frac{1}{2}^+)$	$2S$	2772	2766.6(2.4)?	6086		
$0(\frac{3}{2}^+)$	$1D$	2874		6189		
$0(\frac{5}{2}^+)$	$1D$	2883	2882.5(2.2)?	6197		

The mass spectra of the Λ_Q and Σ_Q baryons are presented in Tables 1, 2. Masses of the ground states are measured both for charmed and bottom Λ_Q and Σ_Q baryons. For charmed baryons the masses of several excited states are also known. It is important to emphasize

Table 2: Masses of the Σ_Q ($Q = c, b$) heavy baryons (in MeV).

		$Q = c$				$Q = b$	
$I(J^P)$	Qd state	M	$M^{\rm exp}$ [1]	$M^{\rm exp}$ BaBar	$M^{\rm exp}$ Belle	M	$M^{\rm exp}$ CDF
$1(\frac{1}{2}^+)$	$1S$	2439	2453.76(18)			5805	5807.5(2.6)
$1(\frac{3}{2}^+)$	$1S$	2518	2518.0(5)			5834	5829.0(2.4)
$1(\frac{1}{2}^-)$	$1P$	2805				6122	
$1(\frac{1}{2}^-)$	$1P$	2795				6108	
$1(\frac{3}{2}^-)$	$1P$	2799	$2802(^4_7)$			6106	
$1(\frac{3}{2}^-)$	$1P$	2761	2766.6(2.4)?			6076	
$1(\frac{5}{2}^-)$	$1P$	2790				6083	
$1(\frac{1}{2}^+)$	$2S$	2864				6202	
$1(\frac{3}{2}^+)$	$2S$	2912		2939.8(2.3)?	$2938(^3_5)$?	6222	

Table 3: Masses of the Ξ_Q ($Q = c, b$) heavy baryons with scalar diquark (in MeV).

		$Q = c$			$Q = b$	
$I(J^P)$	Qd state	M	$M^{\rm exp}$ [1]	$M^{\rm exp}$ BaBar	M	$M^{\rm exp}$ CDF
$\frac{1}{2}(\frac{1}{2}^+)$	$1S$	2481	2471.0(4)		5812	5792.9(3.0)
$\frac{1}{2}(\frac{1}{2}^-)$	$1P$	2801	2791.9(3.3)		6119	
$\frac{1}{2}(\frac{3}{2}^-)$	$1P$	2820	2818.2(2.1)		6130	
$\frac{1}{2}(\frac{1}{2}^+)$	$2S$	2923			6264	
$\frac{1}{2}(\frac{3}{2}^+)$	$1D$	3030			6359	
$\frac{1}{2}(\frac{5}{2}^+)$	$1D$	3042		3054.2(1.5)	6365	

that the J^P quantum numbers for most excited heavy baryons have not been determined experimentally, but are assigned by PDG on the basis of quark model predictions. For some excited charm baryons such as the $\Lambda_c(2765)$, $\Lambda_c(2880)$ and $\Lambda_c(2940)$ it is even not known if they are excitations of the Λ_c or Σ_c. Our calculations show that the $\Lambda_c(2765)$ can be either the first radial ($2S$) excitation of the Λ_c with $J^P = \frac{1}{2}^+$ containing the light scalar diquark or the first orbital excitation ($1P$) of the Σ_c with $J^P = \frac{3}{2}^-$ containing the light axial vector diquark. The $\Lambda_c(2880)$ baryon in our model is well described by the second orbital ($1D$) excitation of the Λ_c with $J^P = \frac{5}{2}^+$ in agreement with the recent spin assignment by Belle based on the analysis of angular distributions in the decays $\Lambda_c(2880)^+ \to \Sigma_c(2455)^{0,++}\pi^{+,-}$. Our model suggests that the charmed baryon $\Lambda_c(2940)$, recently discovered by BaBar and then also confirmed by Belle, could be the first radial ($2S$) excitation of the Σ_c with $J^P = \frac{3}{2}^+$ which mass is predicted slightly below the experimental value. The $\Sigma_c(2800)$ baryon can be identified in our model with one of the orbital ($1P$) excitations of the Σ_c with $J^P = \frac{1}{2}^-, \frac{3}{2}^-$ or $\frac{5}{2}^-$ which predicted mass differences are less than 15 MeV. Thus masses of all these states are compatible with the experimental values within errors.

Mass spectra of the Ξ_Q baryons with the scalar and axial vector light (qs) diquarks are given in Tables 3, 4. Our model prediction for the M_{Ξ_b} is in a reasonable agreement with recently obtained data. In the excited charmed baryon sector we can identify the $\Xi_c(2790)$ and $\Xi_c(2815)$ with the first orbital ($1P$) excitations of the Ξ_c with $J^P = \frac{1}{2}^-$ and

Table 4: Masses of the Ξ_Q ($Q = c, b$) heavy baryons with axial vector diquark (in MeV).

		$Q=c$				$Q=b$
$I(J^P)$	Qd state	M	$M^{\rm exp}$ [1]	$M^{\rm exp}$ Belle	$M^{\rm exp}$ BaBar	M
$\frac{1}{2}(\frac{1}{2}^+)$	$1S$	2578	2578.0(2.9)			5937
$\frac{1}{2}(\frac{3}{2}^+)$	$1S$	2654	2646.1(1.2)			5963
$\frac{1}{2}(\frac{1}{2}^-)$	$1P$	2934				6249
$\frac{1}{2}(\frac{1}{2}^-)$	$1P$	2928				6238
$\frac{1}{2}(\frac{3}{2}^-)$	$1P$	2931				6237
$\frac{1}{2}(\frac{3}{2}^-)$	$1P$	2900				6212
$\frac{1}{2}(\frac{5}{2}^-)$	$1P$	2921				6218
$\frac{1}{2}(\frac{1}{2}^+)$	$2S$	2984		2978.5(4.1)	2967.1(2.9)	6327
$\frac{1}{2}(\frac{3}{2}^+)$	$2S$	3035				6341
$\frac{1}{2}(\frac{1}{2}^+)$	$1D$	3132				6420
$\frac{1}{2}(\frac{3}{2}^+)$	$1D$	3127				6410
$\frac{1}{2}(\frac{3}{2}^+)$	$1D$	3131				6412
$\frac{1}{2}(\frac{5}{2}^+)$	$1D$	3123			3122.9(1.4)	6403
$\frac{1}{2}(\frac{5}{2}^+)$	$1D$	3087		3082.8(3.3)	3076.4(1.0)	6377
$\frac{1}{2}(\frac{7}{2}^+)$	$1D$	3136				6390

$J^P = \frac{3}{2}^-$, respectively, containing the light scalar diquark, which is in agreement with the PDG [1] assignment. Recently Belle observed two baryons $\Xi_{cx}(2980)$ and $\Xi_{cx}(3077)$, which existence was also confirmed by BaBar. The $\Xi_{cx}(2980)$ can be interpreted in our model as the first radial ($2S$) excitation of the Ξ_c with $J^P = \frac{1}{2}^+$ containing the light axial vector diquark. On the other hand the $\Xi_{cx}(3077)$ corresponds to the second orbital ($1D$) excitation in this system with $J^P = \frac{5}{2}^+$. Very recently BaBar observed two new charmed baryons $\Xi_c(3055)$ and $\Xi_c(3123)$. These states can be interpreted in our model as the second orbital ($1D$) excitations of the Ξ_c with $J^P = \frac{5}{2}^+$ containing scalar and axial vector diquarks, respectively.

We find that all presently available experimental data for the masses of the ground state and excited heavy baryons can be accommodated in the picture treating a heavy baryon as the composite system of the light diquark and heavy quark, experiencing orbital and radial excitations only between these constituents.

We are grateful to organizers of the SPMTP Conference for hospitality. This work was supported in part by the Bundesministerium für Bildung and Forschung (D.E.), the Russian Science Support Foundation (V.O.G.) and the Russian Foundation for Basic Research (RFBR) (grant No.08-02-00582) (R.N.F. and V.O.G.).

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Pion polarizabilities in ChPT at two–loops

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Abstract

We present the two-loop expressions for the $\gamma\gamma \to \pi\pi$ amplitude in the framework of chiral perturbation theory (ChPT). We find for the dipole polarizabilities $(\alpha_1 - \beta_1)_{\pi^\pm} = (5.7 \pm 1.0) \times 10^{-4}\,\mathrm{fm}^3$, which is in conflict with the experimental result recently reported by the MAMI Collaboration.

1 Kinematics

We evaluate the amplitude for $\gamma\gamma \to \pi^+\pi^-$ in the framework of chiral perturbation theory (ChPT) [1, 2, 3] at two-loop order [4], and compare the result with the only previous calculation performed at this accuracy [5].

The amplitude describing the process $\gamma\gamma \to \pi^+\pi^-$ may be extracted from the matrix element

$$\langle\, \pi^+(p_1)\pi^-(p_2)\ \mathrm{out} \mid \gamma(q_1)\gamma(q_2)\ \mathrm{in}\,\rangle \;=i(2\pi)^4\delta^4(P_f - P_i)\, T^{\gamma\gamma\to\pi^+\pi^-}, \tag{1}$$

where

$$\begin{aligned} T^{\gamma\gamma\to\pi^+\pi^-} &= e^2\epsilon_1^\mu\epsilon_2^\nu\, W_{\mu\nu}^{\gamma\gamma\to\pi^+\pi^-}, \\ W_{\mu\nu}^{\gamma\gamma\to\pi^+\pi^-} &= i\int dx\, e^{-i(q_1x+q_2y)}\,\langle\, \pi^+(p_1)\pi^-(p_2)\ \mathrm{out} \mid Tj_\mu(x)j_\nu(y) \mid 0\,\rangle\,. \end{aligned} \tag{2}$$

Here j^μ denotes the electromagnetic current and $\alpha = e^2/4\pi \simeq 1/137$ is the electromagnetic coupling. It is convenient to change the pion coordinates according to $(\pi^\pm, \pi^0) \to (\pi^1, \pi^2, \pi^3)$ and instead of $\pi^+\pi^-$–production, we consider in the following the process $\gamma\gamma \to \pi^1\pi^1$, with

$$W_{\mu\nu}^{\gamma\gamma\to\pi^+\pi^-} = -W_{\mu\nu}^{\gamma\gamma\to\pi^1\pi^1} \doteq -V_{\mu\nu}\,, \tag{3}$$

where the relative minus sign stems from the Condon–Shortly phase convention. [We use the same sign convention as Ref. [5].] The decomposition of the correlator $V_{\mu\nu}$ into Lorentz invariant amplitudes on mass-shell reads

$$\begin{aligned} V_{\mu\nu} &= A(s,t,u)T_{1\,\mu\nu} + B(s,t,u)T_{2\,\mu\nu}, \\ T_{1\,\mu\nu} &= \frac{1}{2}\,s\,g_{\mu\nu} - q_{1\nu}q_{2\mu}\,, \\ T_{2\,\mu\nu} &= 2\,s\,\Delta_\mu\Delta_\nu - \nu^2\,g_{\mu\nu} - 2\,\nu\,(q_{1\,\nu}\Delta_\mu - q_{2\,\mu}\Delta_\nu)\,, \end{aligned} \tag{4}$$

where $\Delta_\mu = (p_1 - p_2)_\mu$ and

$$s \;=\; (q_1+q_2)^2, \quad t = (p_1-q_1)^2, \quad u = (p_2-q_1)^2, \quad \nu = t-u \tag{5}$$

are the standard Mandelstam variables.

It is useful to introduce in addition the helicity amplitudes

$$\bar{H}_{++} = A + 2\,(4\,M_\pi^2 - s)\,B\,, \qquad \bar{H}_{+-} = \frac{8\,(M_\pi^4 - t\,u)}{s}\,B. \tag{6}$$

The helicity components $\bar{H}_{++}$ and $\bar{H}_{+-}$ correspond to photon helicity differences $\lambda = 0, 2$, respectively. With our normalization of states $\langle \mathbf{p}_1|\mathbf{p}_2\rangle = 2\,(2\pi)^3\, p_1^0\, \delta^{(3)}(\mathbf{p}_1 - \mathbf{p}_2)$, the differential cross section for unpolarized photons in the centre-of-mass system is

$$\frac{d\sigma^{\gamma\gamma\to\pi^+\pi^-}}{d\Omega} = \frac{\alpha^2\, s}{32}\beta(s)\, H(s,t)\,, \qquad H(s,t) = |\bar{H}_{++}|^2 + |\bar{H}_{+-}|^2\,, \tag{7}$$

with $\beta(s) = \sqrt{1 - 4\,M_\pi^2/s}$. The relation between the helicity amplitudes $M_{+\pm}$ in Ref. [6] and the amplitudes used here is

$$M_{++}(s,t) = 2\pi\alpha\bar{H}_{++}(s,t)\,, \qquad M_{+-}(s,t) = 16\pi\alpha B(s,t)\,. \tag{8}$$

In the centre-of-mass system, $\vec{q}_1 + \vec{q}_2 = 0$, one has $\vec{q}_1 \cdot \vec{p}_1 = |\vec{q}_1||\vec{p}_1| \cos\theta$, where θ is the scattering angle. Then the Mandelstam variables are given by

$$s = 4\,|\vec{q}|^2, \qquad t = M_\pi^2 - (s/2)\,(1 - \beta(s)\cos\theta)\,. \tag{9}$$

For comparison with experimental data, it is convenient to present also the total cross section for the case having $|\cos\theta|$ less than some fixed value Z,

$$\sigma(s; |\cos\theta| < Z) = \frac{\alpha^2\pi}{8}\int_{t_-}^{t_+} dt H(s,t) \tag{10}$$

with $t_\pm = M_\pi^2 - (s/2)\,(1 \mp \beta(s) Z)$.

2 The effective Lagrangian and its low-energy constants

The effective Lagrangian consists of a string of terms. Here, we consider QCD with two flavours, in the isospin symmetry limit $m_u = m_d = \hat{m}$. At next-to-next-to-leading order (NNLO), one has [2]

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6\,. \tag{11}$$

The subscripts refer to the chiral order. The expression for $\mathcal{L}_2$ is

$$\begin{aligned} \mathcal{L}_2 &= \frac{F^2}{4}\langle D_\mu U\, D^\mu U^\dagger + M^2(U + U^\dagger)\rangle\,, \\ D_\mu U &= \partial_\mu U - i(QU - UQ)A_\mu\,,\ Q = \frac{e}{2}\text{diag}(1,-1)\,, \end{aligned} \tag{12}$$

where e is the electric charge, and A_μ denotes the electromagnetic field. The quantity F denotes the pion decay constant in the chiral limit, and M^2 is the leading term in the quark mass expansion of the pion (mass)^2, $M_\pi^2 = M^2(1 + O(\hat{m}))$. Further, the brackets $\langle \ldots \rangle$ denote a trace in flavour space. In Eq. (12), we have retained only the terms relevant for the present application, i.e., we have dropped additional external fields. We choose the unitary 2×2 matrix U in the form

$$U = \sigma + i\pi/F , \quad \sigma^2 + \frac{\pi^2}{F^2} = 1_{2\times 2} , \quad \pi = \begin{pmatrix} \pi^0 & \sqrt{2}\,\pi^+ \\ \sqrt{2}\,\pi^- & -\pi^0 \end{pmatrix} . \tag{13}$$

The Lagrangian at NLO has the structure [2]

$$\mathcal{L}_4 = \sum_{i=1}^{7} l_i K_i + \sum_{i=1}^{3} h_i \bar{K}_i = \frac{l_1}{4} \langle D_\mu U \, D^\mu U^\dagger \rangle^2 + \cdots , \tag{14}$$

where l_i, h_i denote low-energy couplings, not fixed by chiral symmetry. At NNLO, one has [7, 8, 9]

$$\mathcal{L}_6 = \sum_{i=1}^{57} c_i P_i . \tag{15}$$

For the explicit expressions of the polynomials K_i, $\bar{K}_i$ and P_i, we refer the reader to Refs. [2, 7, 8, 9]. The vertices relevant for $\gamma\gamma \to \pi^+\pi^-$ involve $l_1, \ldots, l_6$ from $\mathcal{L}_4$ and several c_i's from $\mathcal{L}_6$, see below.

The couplings l_i and c_i absorb the divergences at order p^4 and p^6, respectively,

$$\begin{aligned} l_i &= (\mu\, c)^{d-4} \left\{ l_i^r(\mu, d) + \gamma_i \Lambda \right\} , \\ c_i &= \frac{(\mu\, c)^{2(d-4)}}{F^2} \left\{ c_i^r(\mu, d) - \gamma_i^{(2)} \Lambda^2 - (\gamma_i^{(1)} + \gamma_i^{(L)}(\mu, d)) \Lambda \right\} , \\ \Lambda &= \frac{1}{16\,\pi^2(d-4)} , \ \ln c = -\frac{1}{2} \left\{ \ln 4\pi + \Gamma'(1) + 1 \right\} . \end{aligned} \tag{16}$$

The physical couplings are $l_i^r(\mu, 4)$ and $c_i^r(\mu, 4)$, denoted by l_i^r, c_i^r in the following. The coefficients γ_i are given in [2], and $\gamma_i^{(1,2,L)}$ are tabulated in [8]. In order to compare the present calculation with the result of [5], we shall use the scale independent quantities $\bar{l}_i$ introduced in [2],

$$l_i^r = \frac{\gamma_i}{32\pi^2} (\bar{l}_i + l) , \tag{17}$$

where the *chiral logarithm* is $l = \ln(M_\pi^2/\mu^2)$. We shall use [10]

$$\bar{l}_1 = -0.4 \pm 0.6 , \ \bar{l}_2 = 4.3 \pm 0.1 , \ \bar{l}_3 = 2.9 \pm 2.4 , \ \bar{l}_4 = 4.4 \pm 0.2 , \tag{18}$$

and

$$\bar{l}_\Delta \doteq \bar{l}_6 - \bar{l}_5 = 3.0 \pm 0.3 \tag{19}$$

obtained from radiative pion decay to two loop accuracy [11, 12].

The constants c_i^r occur in the combinations

$$\begin{aligned} a_1^r &= -4096\pi^4 \left(6\,c_6^r + c_{29}^r - c_{30}^r - 3\,c_{34}^r + c_{35}^r + 2\,c_{46}^r - 4\,c_{47}^r + c_{50}^r\right), \\ a_2^r &= 256\pi^4 \left(8\,c_{29}^r - 8\,c_{30}^r + c_{31}^r + c_{32}^r - 2\,c_{33}^r + 4\,c_{44}^r + 8\,c_{50}^r - 4\,c_{51}^r\right), \\ b^r &= -128\pi^4 \left(c_{31}^r + c_{32}^r - 2\,c_{33}^r - 4\,c_{44}^r\right). \end{aligned}$$

Their values have been estimated by resonance exchange e.g. in Ref. [5]. We have repeated that analysis. Taking into account ρ, a_1 and b_1 exchange which contribute with a definite sign, we obtain

$$\left(a_1^r, a_2^r, b^r\right) = \left(-3.2, 0.7, 0.4\right) \qquad [\text{ present work }]. \tag{20}$$

Unless stated otherwise, we will use these estimates at the scale $\mu = M_\rho$. In Ref. [13], a large N_C framework and the ENJL model were used to pin down these constants, with the result

$$\left(a_1^r, a_2^r, b^r\right) = \left(-8.7, 5.9, 0.38\right) \qquad \text{Ref. [13]}. \tag{21}$$

Only b^r agrees in the two approaches. So, in the case of b^r, we shall use

$$b^r = 0.4 \pm 0.4\,. \tag{22}$$

As far as the polarizabilities are concerned, $(\alpha_1 - \beta_1)_{\pi^\pm}$ is independent of a_2^r and determined precisely by the chiral expansion to two loops, once a_1^r is fixed. We will then simply display this quantity as a function of a_1^r - the result turns out to be rather independent of its exact value.

The lowest-order contributions to the scattering amplitude are described by tree- and one-loop diagrams. We plot the total cross section in Fig. 1, using the LECs from Eqs. (18) - (20). The data are taken from [17]. It is seen that the two-loop corrections are tiny in this kinematical region.

The *dipole* and *quadrupole* polarizabilities are defined [14, 15] through the expansion of the helicity amplitudes at fixed $t = M_\pi^2$,

$$\frac{\alpha}{M_\pi} H_{+\mp}(s, t = M_\pi^2) = (\alpha_1 \pm \beta_1)_{\pi^+} + \frac{s}{12}(\alpha_2 \pm \beta_2)_{\pi^+} + \mathcal{O}(s^2)\,, \tag{23}$$

where $H_{+\mp}$ denote the helicity amplitudes $\bar{H}_{+\mp}$ with Born-term subtracted. Because we have at our disposal the helicity amplitudes at two-loop order, we can work out the polarizabilities to the same accuracy. It turns out that all relevant integrals can be performed in closed form. Using the same notation as in [5], we find for the *dipole* polarizabilities

$$(\alpha_1 \pm \beta_1)_{\pi^+} = \frac{\alpha}{16\,\pi^2 F_\pi^2 M_\pi} \left\{ c_{1\pm} + \frac{M_\pi^2\, d_{1\pm}}{16\,\pi^2 F_\pi^2} + O(M_\pi^4) \right\}, \tag{24}$$

where

$$c_{1+} = 0, \qquad c_{1-} = \frac{2}{3}\bar{l}_\Delta\,,$$

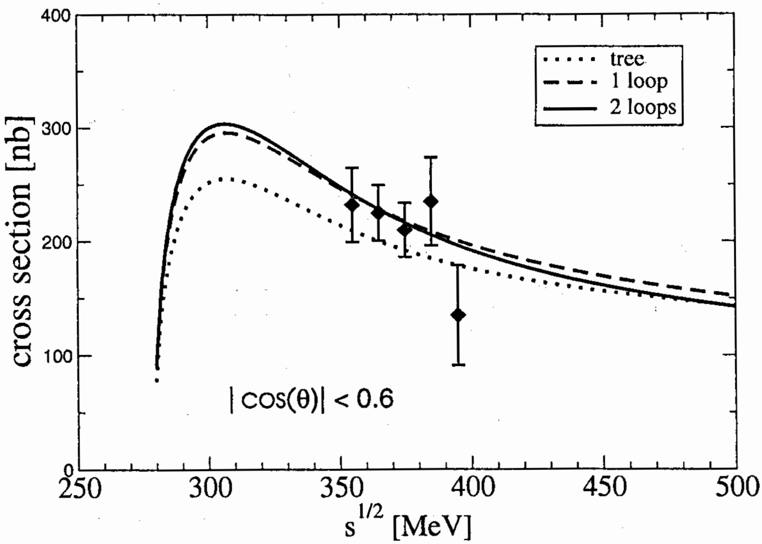

Figure 1: The $\gamma\gamma \to \pi^+\pi^-$ cross section $\sigma(s; |\cos\theta| \le Z = 0.6)$ as a function of $\sqrt{s}$. The experimental data are taken from [17].

$$\begin{aligned} d_{1+} &= 8\,b^r - \frac{4}{9}\left\{l\left(l + \frac{1}{2}\bar{l}_1 + \frac{3}{2}\bar{l}_2\right) - \frac{53}{24}l + \frac{1}{2}\bar{l}_1 + \frac{3}{2}\bar{l}_2 + \frac{91}{72} + \Delta_+\right\}\,, \\ d_{1-} &= a_1^r + 8\,b^r - \frac{4}{3}\left\{l\left(\bar{l}_1 - \bar{l}_2 + \bar{l}_\Delta - \frac{65}{12}\right) - \frac{1}{3}\bar{l}_1 - \frac{1}{3}\bar{l}_2 + \frac{1}{4}\bar{l}_3 - \bar{l}_\Delta\bar{l}_4 \right. \\ &\quad \left. + \frac{187}{108} + \Delta_-\right\}\,, \end{aligned} \tag{25}$$

with

$$\Delta_+ = \frac{8105}{576} - \frac{135}{64}\pi^2\,, \qquad \Delta_- = \frac{41}{432} - \frac{53}{64}\pi^2\,. \tag{26}$$

The uncertainty in the prediction for the polarizability has two sources. First, the low-energy constants are not known precisely. Second, we are dealing here with an expansion in powers of the momenta and of the quark masses.

We now discuss the second source of uncertainties, the truncation of the chiral expansion itself. It is clear that, to have an idea of higher order terms, one needs at least the first two terms in the expansion. This makes it already clear that it is difficult to make reliable predictions for the polarizabilities connected with the helicity flip amplitude, from which we have determined here the leading order contribution only. So, let us concentrate first on the helicity non-flip case H_{++}.

The Born-term subtracted helicity amplitude H_{++} does not have branch points at the Compton threshold. This is why it can be expanded there into an ordinary Taylor series e.g. in the variable s and $\nu = (t - u)$. One then expects that the amplitude at the Compton

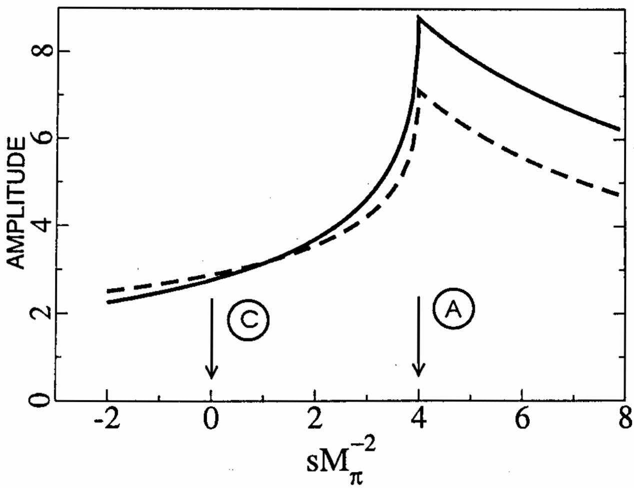

Figure 2: The helicity non-flip amplitude H_{++} in units of M_π^2 as a function of s, at $t = u$, with Born term subtracted. For $s \leq 4M_\pi^2$ the quantity shown is $10^2 M_\pi^2 H_{++}$, and for $s \geq 4M_\pi^2$ we display $10^2 M_\pi^2 |H_{++}|$. The solid (dashed) line is the expression to two loops (to one loop). The Compton threshold in $\gamma\pi^\pm \to \gamma\pi^\pm$ and the threshold in $\gamma\gamma \to \pi^+\pi^-$ are denoted by the encircled letters C and A, respectively. It is clearly seen that two-loop corrections are suppressed at the Compton threshold.

threshold is less affected by chiral logarithms than its counterparts at the threshold for $\gamma\gamma \to \pi\pi$, where unitarity cusps do occur. This is illustrated in Fig. 2, where we display the quantity $10^2 M_\pi^2 H_{++}(s, t = u)$ as a function of s at $t = u$. Above the threshold $s = 4M_\pi^2$, the modulus is shown. The solid (dashed) line is the expression to two loops (to one loop). It is clearly seen that the corrections at the Compton threshold are much smaller than the ones at the threshold for $\gamma\gamma \to \pi\pi$.

The further details of discussion of the uncertainties may be found in [4]. Let us note that the two-loop prediction differs only slightly from the one-loop calculation. This again shows that the value for the dipole polarizability is rather reliable - there is no sign of any large, uncontrolled correction to the two-loop result. We use the maximum deviation 1.0 from the central value 5.7 as the final theoretical uncertainty for the dipole polarizability, and obtain

$$(\alpha_1 - \beta_1)_{\pi^\pm} = (5.7 \pm 1.0) \times 10^{-4}\,\text{fm}^3\,. \tag{27}$$

According to our analysis, the two-loop result Eq. (27) for the dipole polarizability $(\alpha_1 - \beta_1)_{\pi^\pm}$ is particularly reliable. It is in conflict with the recent experimental result obtained at MAMI [16], or with the dispersive analysis performed in [6].

Table 1: Experimental information on $(\alpha_1 - \beta_1)_{\pi^\pm}$, in units of $10^{-4}\,\mathrm{fm}^3$. We indicate the reaction and the data used. In [18] and [19], α_1 was determined, using as a constraint $\alpha_1 = -\beta_1$. To obtain $(\alpha_1 - \beta_1)_{\pi^\pm}$, we multiplied the results by a factor of 2.

Experiments		$(\alpha_1 - \beta_1)_{\pi^\pm}$
$\gamma p \to \gamma\pi^+ n$	Mainz (2005) [16]	$11.6 \pm 1.5_{\text{stat}} \pm 3.0_{\text{syst}} \pm 0.5_{\text{mod}}$
L. Fil'kov, V. Kashevarov (2005) [6]		$13.0^{+2.6}_{-1.9}$
$\gamma\gamma \to \pi^+\pi^-$	MARK II [17],	
TPC/2γ [20], CELLO [21],		
VENUS [22], ALEPH [23], BELLE [24]		
A. Kaloshin, V. Serebryakov (1994) [25]		5.25 ± 0.95
$\gamma\gamma \to \pi^+\pi^-$	MARK II [17]	
	Crystal Ball Coll. [26]	
J.F. Donoghue, B. Holstein (1993) [18]		5.4
$\gamma\gamma \to \pi^+\pi^-$	MARK II [17]	
D. Babusci *et al.* (1992) [19]		
$\gamma\gamma \to \pi^+\pi^-$	PLUTO [27]	$38.2 \pm 9.6 \pm 11.4$
	DM 1 [28]	34.4 ± 9.2
	DM 2 [29]	52.6 ± 14.8
	MARK II [17]	4.4 ± 3.2
$\gamma p \to \gamma\pi^+ n$	Lebedev Inst. (1984) [30]	40 ± 24
$\pi^- Z \to \gamma\pi^- Z$	Serpukhov (1983) [31]	$15.6 \pm 6.4_{\text{stat}} \pm 4.4_{\text{syst}}$

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Hadron multiplicity from the top

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Abstract

The average multiplicity of charged hadrons from the top, as well as average hadron multiplicities in e^+e^- events induced by primary $t\bar{t}$-pair at the collision energy 500 GeV are calculated in perturbative QCD.

As it was revealed by experiments at e^+e^- colliders, the mechanism of multiple production of hadrons depends on a type of the primary quark-antiquark pair. An interesting phenomenon was observed that the difference of hadron multiplicities in light (u, d or s) and heavy (c or b) quark-induced events becomes energy-independent. The calculations in perturbative QCD do describe this phenomenon quite well [1]-[2]. In the present paper we will consider the average multiplicity of charged hadrons in e^+e^- event induced by t-quarks, which is mainly defined by the hadron multiplicity from the top.

It is assumed that the square of the matrix element of the process $e^+e^- \to t^*\bar{t}^* \to X$ has the form [4]:

$$|M(e^+e^- \to t^*\bar{t}^* \to \text{ hadrons})|^2 = |M(e^+e^- \to t^*\bar{t}^* \to t\bar{t} + \text{ hadrons})|^2 \times |M(t \to \text{ hadrons})|^2 \, |M(\bar{t} \to \text{ hadrons})|^2 \, , \qquad (1)$$

where t^* denotes the virtual top quark. Correspondingly, the hadron multiplicity is given by

$$N^h_{t\bar{t}}(W, m_t) = N_t(W, m_t) + 2n_t \, . \qquad (2)$$

Here $N_t(W, m_t)$ describes the average number of hadrons produced in association with the $t\bar{t}$-primary pair, except for the decay products of the top and antitop.

The other term, the hadron multiplicity of the t-quark (antiquark) fragments n_t, is given by

$$n_t = n_W + n_{tb} + n_b \, . \qquad (3)$$

The quantity n_W describes the average number of the W boson decay products (Fig. 1). The hadron multiplicity n_{tb} comes from the emission by t and b quarks (Fig. 2). Finally, the phenomenological quantity $n_b = 5.55 \pm 0.09$ is the average multiplicity of hadrons produced in the decay of the on-shell bottom quark [3].

The multiplicity from the W boson is given by the following expression [4]:

$$n_W = N_{l\bar{l}}(m_W) + \frac{1}{4}\,\delta_{cl} \, , \qquad (4)$$

where $N_{l\bar{l}}(m_W)$ is the average hadron multiplicity in the light quark event taken at the energy $W = m_W$, and $\delta_{cl} = 1.03 \pm 0.34$ is the multiplicity difference in e^+e^- events with

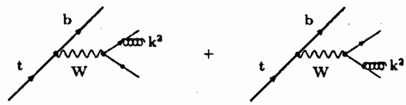

Figure 1: The emission of the massive gluon jets (spiral lines) by the quark pair resulting from the decay of the W^+ boson. The W^+ boson is produced in the weak decay of the top. In their turn, the gluon jets decay into hadrons.

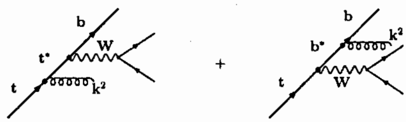

Figure 2: The emission of the massive gluon jets by the on-shell top and off-shell b-quark. The emissions take place before and after the weak decay of the top, respectively.

light and charm primary quarks [3]. The numerical computations of n_W and n_{tb} were made in [4]:

$$n_W = 19.34 \pm 0.10 \,, \tag{5}$$

$$n_{tb} = 16.14 \pm 0.24 \,. \tag{6}$$

As a result, we predict the values of the average multiplicities of charged hadrons from the top:

$$n_t(t \to \text{ hadrons}) = 41.03 \pm 0.54 \,, \tag{7}$$

$$n_t(t \to l\bar{\nu}_l + \text{ hadrons}) = 21.69 \pm 0.53 \,. \tag{8}$$

By estimating quantity N_t in (2) at $W = 500$ GeV and $m_t = 170.9$ GeV,

$$N_t(W = 500 \text{ GeV}) = 4.61 \pm 0.11 \,, \tag{9}$$

we also obtain the average hadron multiplicity in e^+e^- with the primary $t\bar{t}$-pair [4]:

$$N_{t\bar{t}}(e^+e^- \to t\bar{t} \to \text{ hadrons}) = 86.67 \pm 1.11 \,. \tag{10}$$

In the case when W bosons decay into leptons, the average hadron multiplicity is

$$N_{t\bar{t}}(e^+e^- \to t\bar{t} \to W^+W^- + \text{ hadrons}) = 47.99 \pm 0.59 \,. \tag{11}$$

Finally, if b-jets are detected in the final state, we get:

$$N_{t\bar{t}}(e^+e^- \to t\bar{t} \to b\bar{b}\; W^+W^- + \text{ hadrons}) = 36.89 \pm 0.56 \,. \tag{12}$$

All three estimations (10)-(12) correspond to the collision energy 500 GeV.

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Light-cone expansion of heavy-to-light form factors

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Abstract

We present the results of our recent systematic study of the light-cone expansion of heavy-to-light transition form factors in a model with scalar constituents [1]. We show that the higher-twist contributions (represented in this model by off-light-cone effects) all have the same behaviour in the $1/m_Q$ expansion. The suppression parameter of the higher-twist contributions compared to the lower-twist contributions is, in general, the inverse Borel parameter β. The only exception here is the case of the leading and the subleading twists: they are of the same order in $1/\beta$ because of an extra suppression of the leading-twist contribution to the form factor.

Light-cone (LC) sum rules [2] belong to the most widely used approaches for calculating hadron form factors in QCD. The form factor of an individual bound state obtained from a LC sum rule depends on two ingredients: (i) the field-theoretic calculation of the relevant correlator by constructing its LC expansion in terms of hadron distribution amplitudes (DA) of increasing twist, and (ii) the technical "extraction procedure" (cutting the correlator and determining the effective continuum threshold), which introduces a systematic error into the extracted form factor (for a recent study of the serious issue of the systematic errors, see [3]).

In QCD one can calculate only a few terms of the LC expansion of the correlator; it is impossible to study the full series and to estimate the typical size of the omitted higher-twist effects. Therefore, we have systematically analysed the LC expansion in a model with scalar constituents: a heavy "quark" field Q (of mass m_Q) and a light "quark" field φ (of mass m) interacting by exchange of a massless boson in ladder approximation [1]. The basic object here is the heavy-to-light correlator

$$\Gamma(p^2,q^2) \;=\; i\int d^4x \exp(ipx)\langle M(p')|T\left(\varphi(x)Q(x)Q(0)\varphi(0)\right)|0\rangle. \qquad (1)$$

One should (i) obtain the dispersion representation in p^2,

$$\Gamma(p^2,q^2) = \int \frac{ds}{s-p^2-i0}\Delta(s,q^2), \qquad (2)$$

and (ii) perform the Borel transform $1/(s-p^2) \to \exp\left[-s/(2m_Q\beta)\right]$ ($\beta \ll m_Q$) and relate the *cut* Borel image to the form factor of interest:

$$f_{M_Q}\,F_{M_Q\to M}(q^2) = \exp\left(\frac{M_Q^2}{2m_Q\beta}\right)\Gamma(\beta,q^2,s_0) \equiv \int\limits_{(m_Q+m)^2}^{s_0} ds \exp\left(-\frac{s-M_Q^2}{2m_Q\beta}\right)\Delta(s,q^2). \qquad (3)$$

Here, $s_0=(m_Q+z_{\rm eff})^2$ is the effective continuum threshold to be fixed by some criterion. For our analysis it is essential that $z_{\rm eff}$ is a constant which remains finite in the limit $m_Q\to\infty$.

Upon performing the factorization, the spectral density $\Delta(s, q^2)$ may be found via the LC expansion of the soft Bethe–Salpeter (BS) amplitude of the light-quark bound state $M(p')$. The LC expansion of the soft BS amplitude at the factorization scale λ may be written in the form[1]

$$\Psi_{\rm soft}(x, p'|\lambda) \equiv \langle M(p')|T\varphi(x)\varphi(0)|0\rangle_\lambda = \sum_{n=0}^{\infty}(x^2)^n \int_0^1 d\xi \exp(-ip'x\xi)\phi_n(\xi, \lambda). \qquad (4)$$

Here $\phi_n(\xi, \lambda) = C_n(\lambda)(m^2)^n\xi(1-\xi)$, $n = 0, 1, 2, \dots$, are the DAs of increasing twist involving calculable functions $C_n(\lambda)$ of the factorization scale λ. The end-point behaviour of the DAs $\phi_n(\xi, \lambda)$ depends on the inter-"quark" interaction.

The LC expansion of the BS amplitude (4) generates the LC expansion of the correlator. Below, we give the corresponding expressions for $q^2 = 0$. For the uncut correlator ($s_0 \to \infty$), which is *not* related to the contribution of the individual bound state and accordingly *not* to the form factor of interest, neglecting terms of order $O(M^2)$ one obtains

$$\begin{aligned}\Gamma(p^2|\lambda) &= \frac{1}{(2\pi)^4}\int d^4k\, d^4x \frac{\exp[i(p-k)x]}{m_Q^2 - k^2 - i0}\sum_{n=0}^{\infty}(x^2)^n \int_0^1 d\xi \exp(-ip'x\xi)\phi_n(\xi, \lambda) \\ &= \int_0^1 \frac{d\xi\, \phi_0(\xi, \lambda)}{m_Q^2 - p^2(1-\xi)} - 8m_Q^2 \int_0^1 \frac{d\xi\, \phi_1(\xi, \lambda)}{\left[m_Q^2 - p^2(1-\xi)\right]^3} + \dots \equiv \Gamma_0 + \Gamma_1 + \dots. \qquad (5)\end{aligned}$$

The Borelized uncut correlator takes the following form:

$$\Gamma(\beta|\lambda) \simeq \exp\left(-m_Q/2\beta\right)\int_0^1 \frac{d\xi}{1-\xi}\left[\phi_0(\xi, \lambda) - \frac{\phi_1(\xi, \lambda)}{\beta^2(1-\xi)^2} + \dots\right]\exp\left(-\frac{m_Q\xi}{2\beta(1-\xi)}\right).$$

Obviously, the main contribution to the integral arises from the end-point region $\xi \simeq \beta/m_Q$. For this uncut correlator the behaviour of the contributions of increasing twist is found to be

$$\Gamma_0(\beta) \sim \exp\left(-\frac{m_Q}{2\beta}\right)\frac{\beta^2}{m_Q^2}, \qquad \Gamma_1(\beta) \sim \exp\left(-\frac{m_Q}{2\beta}\right)\frac{m^2}{m_Q^2},$$
$$\Gamma_{n+1}(\beta) \sim \Gamma_n(\beta)\left(\frac{m^2}{\beta^2}\right)^n, \qquad n = 0, 1, 2, \dots.$$

Consequently, in the *uncut* correlator contributions of higher twist are suppressed by powers of m^2/β^2 compared to those of lower twist.

For the cut correlator, which *is* related to the form factor of interest, one should take care when applying the cut in the dispersion representation, which leads to surface terms [1]. The surface terms modify the leading behaviour of Γ_0 while leaving the leading behaviour of the higher-twist contributions unchanged:

$$\Gamma_0(\beta, z_{\rm eff}) \sim \exp\left(-\frac{m_Q}{2\beta}\right)\frac{z_{\rm eff}^2}{m_Q^2}, \qquad \Gamma_1(\beta, z_{\rm eff}) \sim \exp\left(-\frac{m_Q}{2\beta}\right)\frac{m^2}{m_Q^2},$$
$$\Gamma_{n+1}(\beta, z_{\rm eff}) \sim \Gamma_n(\beta, z_{\rm eff})\left(\frac{m^2}{\beta^2}\right)^n, \qquad n = 1, 2, 3, \dots.$$

[1] In the ladder-approximation model with scalar constituents, one may obtain the correlator Γ via the BS amplitude without performing the factorization; also the LC contribution to the correlator Γ_0 can be found. However, any proper treatment of the full LC expansion of the correlator still requires factorization.

Note that the parameter $z_{\rm eff}$ here is a fixed parameter (in our model $\sim m$, in QCD $\sim \Lambda_{\rm QCD}$) dictated by dynamics. Thus, the leading and the subleading twists are of the same order of magnitude. For higher twists, the twist hierarchy is preserved in the cut correlator related to the form factor, similar to the case of the uncut correlator.

In [1] we have calculated, without invoking the LC expansion, the full correlator by use of the known solution for the BS amplitude in ladder approximation and, separately, the LC contribution to the correlator. We found that numerically the leading twist provides about 70% of the full correlator. The contributions of the higher twists (mainly, of the subleading twist), however, stay at the order of 30% for all values of the parameter β and of the mass m_Q of the heavy "quark."

In summary, in the LC sum-rule approach to heavy-to-light form factors near $q^2 = 0$ the contributions of all twists exhibit the same behaviour in the $1/m_Q$ expansion. Higher-twist contributions are suppressed by powers of the parameter $\Lambda_{\rm QCD}/\beta$, with the exception of the leading- and the subleading-twist contributions, which exhibit the same behaviour in $1/\beta$. However, since the ground-state contribution is enhanced for small β, one would like to know the correlator for the smallest possible values of β, where the inclusion of higher-twist effects is mandatory. The off-LC and other higher-twist effects in weak decays of heavy mesons in QCD deserve a detailed investigation: for the LC sum-rule method the corresponding DAs are just external objects, which should be provided by other nonperturbative approaches. In particular, the combination of LC sum rules with techniques based on bound-state equations and the constituent quark picture [4] (which successfully describe heavy-meson decays) may prove to be promising.

Acknowledgements. We thank the organizers for the invitation to this highly interesting meeting. D. M. was supported by FWF project P20573 and by RFBR project 07-02-00551.

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Analytic determination of the magnetic dipole moments of the Δ resonance

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Abstract

The pion-nucleon bremsstrahlung $\pi + N \Longrightarrow \gamma' + \pi' + N'$ is studied in a new form of current conservation for the on shell amplitudes [1]. It is shown that the double Δ exchange diagram with the $\Delta - \gamma'\Delta'$ vertex cancel exactly against the appropriate longitudinal part of the external particle radiation diagrams. Consequently, a model independent relation between the magnetic dipole moments of the Δ resonances and the anomalous magnetic moment of the proton μ_p and neutron μ_n is obtained, where $\mu_{\Delta^+} = \frac{M_\Delta}{m_p}\mu_p$, $\mu_{\Delta^o} = \frac{M_\Delta}{m_o}\mu_p$ and $\mu_{\Delta^{++}} = \frac{3}{2}\mu_{\Delta^+}$, $\mu_{\Delta^-} = \frac{3}{2}\mu_{\Delta^o}$. This result is generalized within the field theoretical formulation with the quark degrees of freedom [3], where pions and nucleons are treated as the bound systems of quarks. It is shown that relations generated by current conservation for the on shell πN bremsstrahlung amplitude with composite nucleons and pions have the same form as in the usual quantum field theory without quark degrees of freedom. Consequently, the model independent relations for the magnetic dipole moments of the Δ resonances remain be the same in the quantum field theory with the quark degrees of freedom [2].

The general form of the current conservation for the on shell radiative πN scattering mplitude is obtained based on the modified Ward-Takahashi identity [1]. This identity educes four-divergence of the on mass shell and on energy shell amplitude of the $\pi N \to$ $\gamma'\pi' N'$ reaction $k'_\mu \mathcal{A}^\mu_{\gamma'\pi'N'-\pi N}$ to the sum of the four-divergence of the external particle adiation amplitude in Fig. 1 $k'_\mu \mathcal{E}^\mu_{\gamma'\pi'N'-\pi N}$ and the sum of the off shell elastic πN scattering mplitude $\mathcal{B}_{\pi'N'-\pi N}$, i.e. $k'_\mu \mathcal{A}^\mu_{\gamma'\pi'N'-\pi N} = k'_\mu \mathcal{E}^\mu_{\gamma'\pi'N'-\pi N} + \mathcal{B}_{\pi'N'-\pi N} = 0$, where k'_μ is the four-nomentum of the emitted photon. This form of the current conservation represents the nodel-independent generalization of the low energy photon theorem (Low theorem) for any nergy of an emitted photon.

The present Ward-Takahashi identity allows to connect the internal $\mathcal{I}^\mu_{\gamma'\pi'N'-\pi N}$ and xternal $\mathcal{E}^\mu_{\gamma'\pi'N'-\pi N}$ particle radiation parts of the on shell πN radiation amplitude, beause $k'_\mu \mathcal{E}^\mu_{\gamma'\pi'N'-\pi N} = -k'_\mu \mathcal{I}^\mu_{\gamma'\pi'N'-\pi N} = -\mathcal{B}_{\pi'N'-\pi N}$. Moreover, according to this relation $k'_\mu \mathcal{E}^\mu_{\gamma'\pi'N'-\pi N}$ and $k'_\mu \mathcal{I}^\mu_{\gamma'\pi'N'-\pi N}$ have the opposite sign and thereby they contain the parts vhich must cancel each other. Thus the current conservation generates a screening of the orresponding internal and internal particle radiation parts of the πN radiation amplitudes.

In particular it is shown, that the internal double Δ exchange diagram with the $\Delta -$ $\gamma'\Delta'$ vertex (Fig. 2) cancel exactly against the longitudinal part of the external particle adiation diagrams depicted in Fig. 1. This canceling enables to obtain a model independent stimation of the dipole magnetic moment of the Δ resonances μ_Δ through the anomalous nagnetic moment of the proton μ_p and the anomalous

Model	This work	$SU(6)$ & Bag	Poten. & K-matr.	Skyrme	Low en. pht. theorem	Eff. πN Lagran.	quark
μ_{Δ^+}	3.66	2.79 [10, 11] 2.13[12] 2.20-2.45[13] 3.27[15]		2.0-3.0[19]			3.49[16] 2.85[17] 2.3-2.7[18] 2.79[21]
$\mu_{\Delta^{++}}$	5.49	5.5[10, 11] 4.25[12] 4.41-4.89[13] 6.54[15]	6.9-9.7[14]* 4.52±0.95[6]* 5.6-7.5[5]*	4.2-7.4[19]	3.6±2.0[7]* 5.6±2.1[8]* 4.7-6.9[4]* 3.7-4.9[9]*	6.1±0.5[20]*	6.98[16] 5.33[17] 5.1-5.4[18] 6.17[21]
μ_{Δ^o}	-2.504	0. [10, 11] 0.[12] 0.[13] 0.[15]		-1.33-0.19[19]			0.[16] 0.375[17] -0.3-0.[18]
μ_{Δ^-}	-3.759	-2.79 [10, 11] -2.13[12] -2.20-2.45[13] -3.27[15]		-5.62-2.38[19]			-3.49[16] -2.1[17] -2.72-3.06[18]

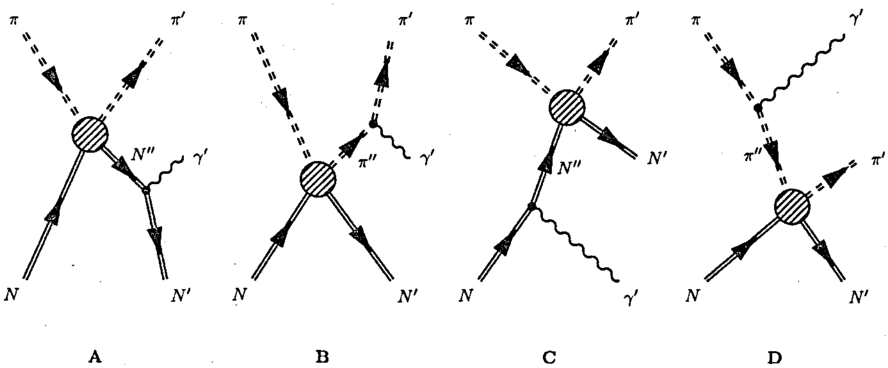

Figure 1: *The external particle radiation diagrams of the πN bremsstrahlung amplitude.*

magnetic moment of the neutron μ_n as

$$\mu_{\Delta^+} = \frac{M_\Delta}{m_p}\mu_p; \quad \mu_{\Delta^{++}} = \frac{3}{2}\mu_{\Delta^+}; \qquad \mu_{\Delta^o} = \frac{M_\Delta}{m_p}\mu_n; \quad \mu_{\Delta^-} = \frac{3}{2}\mu_{\Delta^o} \tag{1}$$

The corresponding numerical values of the magnetic moments of the Δ resonances and the results of other authors [4]-[21] are collected in Table 1. In a number of approaches the magnetic moment of Δ is treated as an adjustable parameter. The corresponding results obtained from the experimental cross sections of the $\pi^+p \to \gamma\pi^+p$ reaction are indicated in Table 1 with the upper index *.

The generalization of the present model independent relation (1) is given in [2].

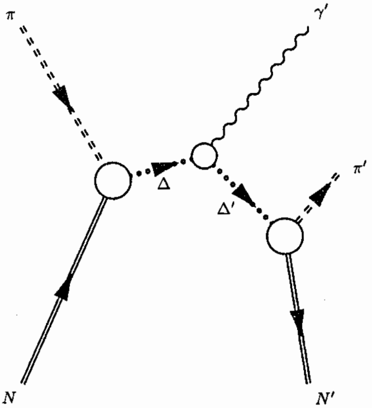

Figure 2: *The double on mass shell Δ exchange diagram with the intermediate Δ radiation vertex. The $\Delta - \gamma\Delta$ vertex contains the dipole magnetic moment of the Δ.*

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The Glueball Spectrum from Constituent Models

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Abstract

We present a model for odd-C (negative charge parity) glueballs with three constituent gluons. The model is an extension of a previous study of two-gluon glueballs. We show that, even if spin-1 gluons seem to reproduce properly the lattice QCD spectrum for $C = +$ states, the extension for $C = -$ cannot match with the lattice results. Resorting to the helicity formalism, we show how transverse gluons fit in better agreement the lattice QCD spectrum.

1 Constituent models for two-gluon glueballs

Quantum Chromodynamics (QCD) allows the self-coupling of the gauge bosons, the gluons. Therefore, states with no valence quarks, the glueballs, are a beautiful consequence and prediction of QCD.

Their observation, however, remains difficult. Probably because the lightest glueball, the scalar 0^{++}, should mix with mesons [1]. Some experimental glueball candidates are currently known, such as the $f_0(980)$, $f_0(1500)$, $f_0(1710)$, ... but no definitive conclusions can be drawn concerning the nature of these states.

On the other hand, pure gauge QCD has been investigated by lattice QCD for many years, leading to a well established glueball spectrum below 4 GeV [2, 3, 4]. Our aim is to reproduce this hierarchy with the most simple models with constituent gluons. Since two gluons can only bind into positive-C, we have to consider three-gluon glueballs for the existence of negative-C states.

In ref. [5], the authors provide a relevant model of two-gluon glueballs. Assuming Casimir scaling for the string tension of the flux tube, the Hamiltonian, endowed with one-gluon exchange (OGE) potentials, reads

$$H_{gg} = 2\sqrt{\boldsymbol{p}^2 + m^2} + \frac{9}{4}\sigma r + V_{oge}(r; \alpha_S, \mu; \boldsymbol{S}, \boldsymbol{L}). \tag{1}$$

Although they use a bare mass $m = 0$ in the kinetic term, their gluons have longitudinal components and are spin-1 particles. Therefore, many states are degenerate and the authors resorted to spin-dependent potentials coming from the OGE to lift these degeneracies. The corrections are of order μ^{-2}, where $\mu = \langle \boldsymbol{p}^2 \rangle$ is an effective constituent mass. The parameters were fitted on the low-lying states and the final spectrum is displayed in Fig. 1 (left).

All states (squares) fall into lattice error bars. However, we noticed some spurious states circles) not found by any lattice study. $J = 1$ states are forbidden by Yang's theorem nd should not be present. The appearance of such states is induced by the longitudinal omponents of gluons and should disappear when considering transverse gluons.

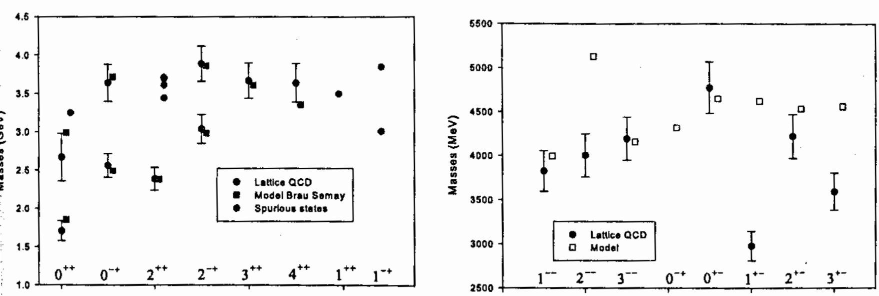
2 Odd-C glueballs

et us forget about the spurious states for the moment and let us generalize the model or three-gluon glueballs. We used a generalisation of the flux tube for the confinement. n heavy baryons, the confinement has a Y-shape, but in our case, we replaced it by a enter-of-mass junction. The Hamiltonian is supplemented by the potential coming from he OGE and reads

$$H_{ggg} = \sum_{i}^{3} \sqrt{\boldsymbol{p}_i^2} + \frac{9}{4} f\sigma |\boldsymbol{r}_i - \boldsymbol{R}_{cm}| + \sum_{i<j} V_{oge}(r_{ij}; \alpha_S, \mu; \boldsymbol{L}_{ij}, \boldsymbol{S}_{ij}). \tag{2}$$

Ve refer the reader to the ref. [6] for further details concerning the Hamiltonian.

We impose the symmetric colour function $d_{abc}A^a_\mu A^b_\nu A^c_\rho$, which ensures a negative C-arity, then the spin symmetry determines the symmetry of the space. Since $\mathbf{1} \otimes \mathbf{1} \otimes \mathbf{1} = \mathbf{3}_s \oplus \mathbf{2}_m \oplus \mathbf{1}_s \oplus \mathbf{0}_a$, 2^{--} has a mixed symmetry and cannot lie in the same mass range as $^{--}$ and 3^{--}, as was already noticed in ref. [7]. Moreover, a positive parity requires an odd ngular momentum. Then, all $(0, 1, 2, 3)^{+-}$ are degenerate with a large component $L = 1$ in he wave function. But the lattice QCD exhibits a gap around 2 GeV between the highest $)^{+-}$ and the lowest 1^{+-}. The spectrum, shown in Fig. 1 (right), is nearly in complete isagreement with lattice QCD. The symmetry arguments are Hamiltonian-independent nd we can therefore conclude that models with longitudinal gluons are not appropriate to eproduce the lattice pure gauge spectrum.



igure 1: Left: Spectrum of Hamiltonian (1) with longitudinal gluons. Right: Spectrum of Iamiltonian (2) with longitudinal gluons.

3 Transverse gluons

In order to solve the problems encountered (spurious states, hierarchy in the $PC = +-$ sector), we implemented a formalism developed by Jacob and Wick [8]. This formalism allows us to handle transverse particles. When applying it to two-gluon glueballs, we remarked that the Bose symmetry (and the parity) implies selection rules. Three families were identified [9]: $(2k)^{++}, (2k+3)^{++}, (2k+2)^{-+}$ with $k \in N$. One easily checks that no spurious $J = 1$ states appear. Moreover, with this special construction, all states are now expressed through a given linear combination of spectroscopic states $|^{2S+1}L_J\rangle$. The degeneracies occurring in Ref. [5] are naturally split by the wave function. One does not need to use complicated spin-dependent potentials.

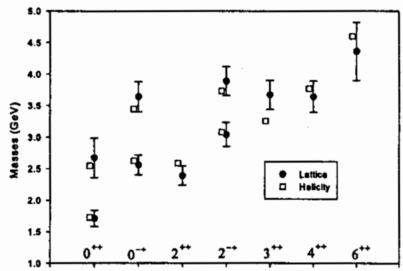


Figure 2: Spectrum of Hamiltonian (3) with transverse gluons.

We tested the wave functions with a simple Hamiltonian:

$$H_{gg} = 2\sqrt{p^2} + \frac{9}{4}\sigma r - 3\frac{\alpha_S}{r}. \tag{3}$$

The resulting spectrum, displayed in Fig. 2 is in good agreement with the lattice QCD data without the inclusion of spin-dependent potentials. But instanton-induced interactions were needed for $J = 0$ states. In addition, all states are present with no spurious state.

The next step is to implement this formalism for three-gluon glueballs. This work is under construction. However, we have some indications that the lowest odd-C are spin 1 and 3 [10]. Symmetry arguments are also in favour of a four-gluon interpretation for 0^{+-}.

Acknowledgement

V. Mathieu thanks the organizers the invitation to the conference and the Dubna Joint Institute for nuclear research for its hospitality.

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Infrared Propagators in MAG and Feynman gauge on the lattice

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Abstract

We propose to investigate infrared properties of gluon and ghost propagators related to the so-called Gribov-Zwanziger confinement scenario, originally formulated for Landau and Coulomb gauges, for other gauges as well. We present results of our investigation of $SU(2)$ lattice gauge theory in the maximally Abelian gauge (MAG), focusing on the behavior of propagators in the off-diagonal (i.e. non-Abelian) sector. We also comment on our preliminary results for general linear covariant gauges, in particular for Feynman gauge.

1 Introduction

Important features of quark and gluon confinement in QCD are believed to be closely related to the behavior of gluon and ghost propagators in the infrared limit. One must notice, however, that the study of infrared properties of these propagators must be performed by nonperturbative methods and at a fixed gauge. The Gribov-Zwanziger confinement scenario [1, 2] — proposed for Landau and Coulomb gauges — provides predictions for gluon and ghost propagators in the infrared limit, which may be tested by lattice simulations and by nonperturbative analytic methods such as Dyson-Schwinger equations. In particular, a suppressed infrared gluon propagator $D(p^2)$ is predicted, with $D(0) = 0$. The latter statement implies maximal violation of reflection positivity for the gluons, a result that may be viewed as an indication of gluon confinement. (Note that it suffices to have violation of reflection positivity, not necessarily maximal.) At the same time, the infinite-volume limit favors gauge configurations on the boundary region known as the first Gribov horizon, where the smallest nonzero eigenvalue λ_{min} of the Faddeev-Popov matrix $\mathcal{M}$ goes to zero. As a consequence, the ghost propagator $G(p^2)$ — which is obtained from $\mathcal{M}^{-1}$ — should be infrared-enhanced, introducing long-range effects in the theory. These, in turn, would be responsible for the color-confinement mechanism.

The nonperturbative study of infrared propagators may be carried out from first principles in lattice simulations, taking into account that the true infrared behavior is however obtained only at large enough lattice volumes. Considerable effort has been dedicated to investigations of the above predictions for the Landau gauge, considering very large lattice sizes (see e.g. [3]). The status of these studies is discussed in [4]. Here we propose to test similar predictions for the propagators as applied to the lattice implementation of other gauges, to try to gain a unified understanding of the mechanism of confinement and its manifestations. We consider the maximally Abelian gauge (MAG) and the linear covariant gauges, in particular Feynman gauge.

In the case of the linear covariant gauges, which include and generalize Landau gauge, some studies suggest that the Gribov-Zwanziger confinement mechanism may apply to the complete class of such gauges [5, 6]. (A recent study of Dyson-Schwinger equations for Feynman gauge has been presented in [7].) On the other hand, for MAG, the usual confinement scenario is based on the concepts of Abelian dominance and of dual superconductivity [8]. Nevertheless, one might argue that a modified Gribov-Zwanziger scenario would likely hold in MAG for the non-Abelian directions in gauge-configuration space. A study of the Yang-Mills Lagrangian restricted to the (MAG) Gribov region by addition of a horizon function with Gribov parameter γ has recently been carried out for $SU(2)$ gauge theory in [9]. As pointed out in that reference and also by other groups, the infrared behavior of propagators in MAG may be modified by the presence of ghost and gluon condensates of mass dimension two. An example of such objects is the ghost condensate v [10, 11], related to the breakdown of a global $SL(2, R)$ symmetry. This quantity is expected to modify the symmetric and anti-symmetric components of the (off-diagonal) ghost propagator. In particular, a nonzero value for v corresponds to nonzero anti-symmetric components of the ghost propagator. In Section 2 we present results of our lattice studies of pure $SU(2)$ theory in MAG. (The implementation of gauge fixing for MAG on the lattice is straightforward.) We consider gluon and ghost propagators, the ghost condensate v mentioned above and the smallest eigenvalue of the Fadeev-Popov matrix. Our preliminary results have also been presented in [12] and [13]. We note that the bounds recently introduced for studying gluon and ghost propagators on large lattices in Landau gauge [14, 15] may be written also for other gauges.

Contrary to the case of MAG, the technical aspect of fixing the linear covariant gauges on the lattice is still not a settled issue. We comment on our recent proposals for gauge-fixing methods for these gauges in Section 3.

2 Infrared propagators in MAG

On the lattice, for the $SU(2)$ case, the MAG is obtained (see e.g. [16]) by minimizing the functional

$$S \;=\; -\frac{1}{2dV}\sum_{x,\mu} Tr\left[\sigma_3 U_\mu(x)\sigma_3 U_\mu^\dagger(x)\right] . \tag{1}$$

At any local minimum one has that the Faddeev-Popov matrix, defined as

$$\sum_{by} M^{ab}(x,y)\gamma^b(y) = \sum_\mu \gamma^a(x)[V_\mu(x) + V_\mu(x-e_\mu)] \;+\; 2\,\{\gamma^a(x-e_\mu)[1 - 2(U_\mu^0(x))^2] \\ - 2\sum_b \gamma^b(x-e_\mu)[\epsilon_{ab}U_\mu^0(x)U_\mu^3(x) + \sum_{cd}\epsilon_{ad}\epsilon_{bc}U_\mu^d(x)U_\mu^c(x)]\}\,, \tag{2}$$

positive-definite. Here the color indices take values 1, 2 and we follow the notation $(x) = U^0_\mu(x)\mathbb{1} + i\,\sigma^a U^a_\mu(x)$ and $V_\mu(x) = (U^0_\mu(x))^2 + (U^3_\mu(x))^2 - (U^1_\mu(x))^2 - (U^2_\mu(x))^2$, here σ^a are the 3 Pauli matrices. Notice that (as in Landau gauge [17]) this matrix is mmetric under the simultaneous exchange of color and space-time indices. Using the ation $U_\mu(x) = \exp[-iag_0 A_\mu(x)]$ one finds (in the formal continuum limit $a \to 0$) the ndard continuum results [18] for the stationary conditions above and for $M^{ab}(x,y)$.

We have considered four values of β (2.2, 2.3, 2.4, 2.512) and lattice volumes up to 40^4. ta for a larger lattice, of volume 56^4, have also been recently produced, but are not fully alyzed yet. We include data for the ghost propagator at this volume in Fig. 2 below, for mparison.

Our results for the gluon propagators are in agreement with the study by Bornyakov et [16]: we see a clear suppression of the off-diagonal propagators compared to the diagonal ansverse) one, supporting Abelian dominance. We have fitted our data for the various ion propagators (at all values of V up to 40^4 and for $\beta = 2.2$), obtaining the following haviors. For $D(p^2)$ (transverse) diagonal, our data favor a Stingl-Gribov form

$$D(p^2) = \frac{1 + d\,p^2}{a + b\,p^2 + c\,p^4}\,, \tag{3}$$

th a mass $m = \sqrt{a/b} \approx 0.72\,GeV$. Note that the above equation corresponds to pair of complex-conjugate poles z and z^*. We can thus write $z = x + iy$ with $x =$ $(2c) \approx 0.32\,GeV^2$ and $y = \sqrt{a/c - x^2} \approx 0.47\,GeV^2$. Let us recall that in the case of Gribov-like propagator these two poles are purely imaginary. For $D(p^2)$ transverse off-agonal our best fit is of Yukawa type, i.e. $D(p^2) = 1/(a + bp^2)$, with a mass $m =$ $a/b \approx 0.97\,GeV$. Finally, the longitudinal off-diagonal gluon propagator is best fitted by $(p^2) = 1/(a + b\,p^2 + c\,p^4)$ (i.e. also of Yukawa type) with a mass $m = \sqrt{a/b} \approx 1.25\,GeV$. s expected from Abelian dominance, the mass is larger in the off-diagonal case.

In Fig. 1 (left) we show our data for the ghost propagator $G(p^2)$, as a function of an proved momentum p (see Ref. [19]). The data show little volume dependence at small (Note that, contrary to Landau gauge, here we can evaluate the ghost propagator at ro momentum.) We see no sign of an enhanced IR propagator. We have fitted our ta (at $\beta = 2.2$), obtaining a behavior of the type (3) above with $a = 0.45(1)\,GeV^2$, $= 1.1(3)$, $c = 0.73(30)\,GeV^{-2}$, $d = 2.1(9)\,GeV^{-2}$. Thus, we see a Stingl-Gribov fit with ass $m \approx 0.6\,GeV$ and complex poles given by $x \approx 0.75$, $y \approx 0.22$.

We next consider (see Fig. 1, right) the smallest eigenvalue λ_{min} of the Faddeev-Popov atrix. We have looked at λ_{min} for several lattice volumes and values of β as a function $1/L$. The data are fitted to $a\,(1/L)^b$ with $b = 1.6(1)$, showing that λ_{min} vanishes more wly than $(1/L)^2$ (Laplacian). This may explain why we do not see a diverging ghost opagator at zero momentum even at rather large lattice volumes [15].

Following the analysis done in Landau gauge [17], we consider the anti-symmetric off-agonal ghost propagator $\langle\,|\,\epsilon_{ab}G^{ab}(p^2)/2\,|\,\rangle$ rescaled by $L^2/\cos(\pi\,\tilde{p}_\mu\,a/L)$, as a function the (unimproved) momentum p for all lattice volumes and β values considered. The ta show nice scaling for all cases considered. The data at $V = 40^4$ and $\beta = 2.2$ can fitted by $\Phi(p) = (a + b\,p/L^2)(p^4 + v^2)$ with $a = 0.0026(7)\,GeV^2$, $b = 32.6(7)\,GeV^{-1}$ d $v^2 = 1.7(1)\,GeV^4$. We thus have a rather large ghost condensate $v \approx 1.3\,GeV^2$, but cannot be sure that it survives in the infinite-volume limit, since the overall constant a

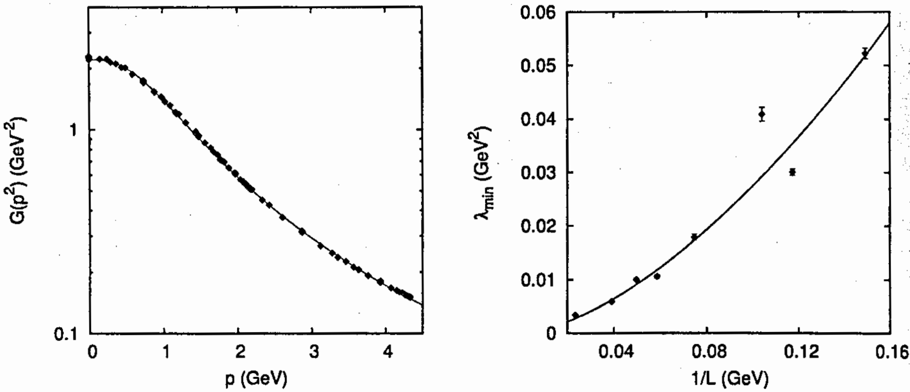

Figure 1: Left: plot of $G(p^2)$ as a function of improved p for lattice volumes $V = 16^4$, 24^4 40^4 and $\beta = 2.2$. Right: plot of the smallest eigenvalue of the Faddeev-Popov operator, as a function of the inverse linear size of the system.

might be null. We can also fit data at several V's and β's for $\Phi(p^2)$ as a function of p and L (see Fig. 2, right). We obtain $\Phi(p) = (a + bp/L^2)(p^4 + v^2)$ with $a = 0.0033(6)\, GeV^2$ $b = 35.8(5)\, GeV^{-1}$ and $v^2 = 1.87(8)\, GeV^4$. We note that the fit parameters change little with the (physical) lattice volume. In fact, data obtained recently for a larger lattice volume 56^4, are seen to fall nicely on top of the fit done for the smaller volumes, as seen in Fig. 2 (right).

We have also investigated possible effects of Gribov copies on our results, by considering the difference between our standard gauge fixing (using the stochastic overrelaxation algorithm [20]) and the so-called smearing method [21]. The effects are found to be of the order of the statistical error.

3 Linear covariant gauges

As mentioned in the Introduction, gauge-fixing to linear covariant gauges (other than Landau gauge) on the lattice is a challenge. More precisely, the gauge condition is given by

$$\partial_\mu A^a_\mu(x) \;=\; \Lambda^a(x) \qquad (4$$

for real-valued functions $\Lambda^a(x)$. As opposed to the case of Landau gauge — for which $\Lambda^a(x) = 0$ and the gauge is fixed by minimizing a simple functional of the gauge-transformed links — in the general case no such functional exists [22]. The solution to this problem presented in [22], based on the consideration of a modified gauge-fixing condition for the minimizing functional, may be affected by spurious minima and it leads to an altered form of the Faddeev-Popov matrix. We propose to consider a class of gauges on the lattice that coincides with the perturbative definition of linear covariant gauges in the formal continuum limit. Our method is based on a three-step process. Instead of minimizing a functional of $\Lambda^a(x)$ directly, we first fix the gauge to Landau gauge, i.e. the transformed gauge field

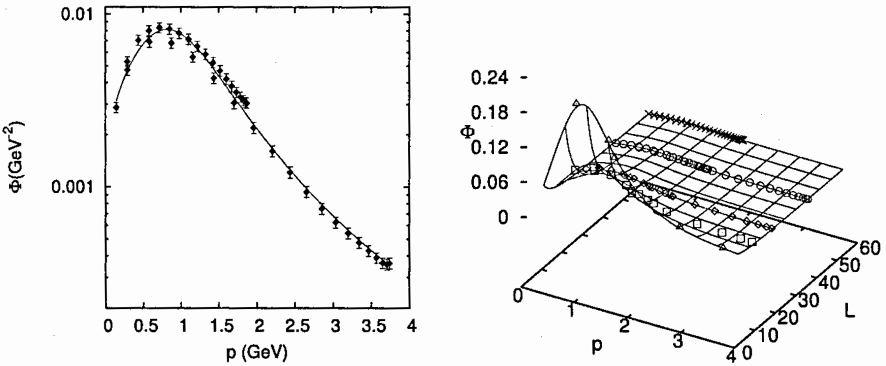

Figure 2: Left: plot of the quantity $\Phi(p^2) = L^2/\cos(\pi\,\tilde{p}_\mu\, a/L)\,\langle\,|\,\epsilon_{ab}G^{ab}(p^2)/2\,|\,\rangle$ as a function of p for lattice volumes $V = 8^4$, 16^4, 24^4, 40^4 and $\beta = 2.2$. Right: plot of $\Phi(p^2)$ as a function of p and L. Data for a larger volume, 56^4, are also included here.

satisfy $\partial_\mu A^a_\mu(x) = 0$. Then we determine a transformation $\phi^b(x)$ such that

$$A'^a_\mu(x) \;\equiv\; A^a_\mu(x) \,+\, D^{ab}_\mu \phi^b(x) \tag{5}$$

satisfies Eq. (4). Finally, we repeat the procedure for several functions $\Lambda^a(x)$ with a Gaussian distribution of width $\sqrt{\xi}$. The case $\xi = 1$ corresponds to Feynman gauge. The resulting distribution of $\partial A'^a_\mu(x)$ is shown for $\xi = 1$ in Fig. 3, in comparison with the original (Gaussian) distribution taken for $\Lambda^a(x)$. We see that the expexted distribution is fairly well reproduced.

Our preliminary results were presented in [23]. We are currently investigating an alternative method for fixing these gauges.

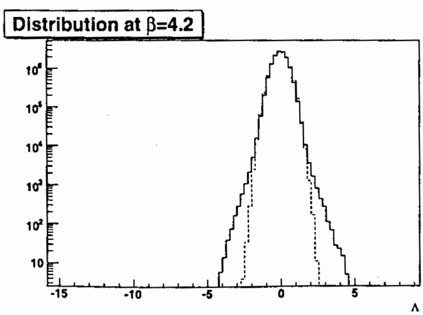

Figure 3: Distribution of $\partial A'^a_\mu(x)$ (solid line) compared with a Gaussian of width $\sqrt{\xi}$ (dashed line).

4 Acknowledgments

We thank S. Sorella for helpful discussions. A. C. and T. M. were partially supported by FAPESP and CNPq. The work of T. M. was also supported by the A. von Humboldt Foundation. A. Maas was supported by the DFG under grants number MA 3935/1-1 and number MA 3935/1-2 and by the FWF under grant number P20330. A. Mihara was partially supported by CNPq.

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New arrangement of common approach to calculating the QCD ground state

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The quark behaviour in the background of intensive stochastic gluon field is studied. An approximate procedure for calculating the effective Hamiltonian is developed and the corresponding ground state within the Hartree-Fock-Bogolyubov approach is found. The comparative analysis of various model Hamiltonian is given and transition to the chiral limit in the Keldysh model is discused in detail.

We study the quark (anti-quark) behaviour while being influenced by intensive stochastic gluon field and work in the context of the Euclidean field theory. The corresponding Lagrangian density is the following

$$\mathcal{L}_E = \bar{q}\ (i\gamma_\mu D_\mu + im)\ q\ , \tag{1}$$

here q ($\bar{q}$) — are the quark (anti-quarks) fields with covariant derivative $D_\mu = \partial_\mu - igA^a_\mu t^a$ where A^a_μ is the gluon field, $t^a = \lambda^a/2$ are the generators of colour gauge group $SU(N_c)$ and m is the current quark mass. As the model of stochastic gluon field we refer to the example of (anti-)instantons considering an ensemble of these quasi-classical configurations. On our way to construct an effective theory (which usually encodes the predictions of a quantum field theory at low energies) the assumptions done are not of special importance. However, what is entirely restrictive to fix the effective action at really low energy (i.e. low cutoff) up to a few coupling constants is an idea to neglect all the contributions coming from gluon fields A_{ex} generated by the (anti-)quarks.

$$A_{ex} \ll A\ .$$

Actually, it means the removal of corresponding cutoff(s) from consideration, but by the definition of an effective theory this operation does not pose itself. Then the corresponding Hamiltonian description results from

$$\mathcal{H} = \pi\dot{q} - \mathcal{L}_E\ ,\quad \pi = \frac{\partial \mathcal{L}_E}{\partial \dot{q}} = iq^+\ , \tag{2}$$

and

$$\mathcal{H}_0 = -\bar{q}\ (i\boldsymbol{\gamma}\boldsymbol{\nabla} + im)\ q\ , \tag{3}$$

for noninteracting quarks. In Schrödinger representation the quark field evolution is determined by the equation for the quark probability amplitude Ψ as

$$\dot{\Psi} = -H\Psi\ , \tag{4}$$

with the density of interaction Hamiltonian

$$\mathcal{V}_S = \bar{q}(\boldsymbol{x})\ t^a \gamma_\mu A^a_\mu(t,\boldsymbol{x})\ q(\boldsymbol{x})\ . \tag{5}$$

The explicit dependence on "time" is present at the gluon field only. The creation and annihilation operators of quarks and anti-quarks $a^+, a,\ b^+, b$ have no "time" dependence and consequently

$$q_{\alpha i}(\boldsymbol{x}) = \int \frac{d\boldsymbol{p}}{(2\pi)^3} \frac{1}{(2|p_4|)^{1/2}} \left[a(\boldsymbol{p},s,c)\ u_{\alpha i}(\boldsymbol{p},s,c)\ e^{i\boldsymbol{px}} + b^+(\boldsymbol{p},s,c)\ v_{\alpha i}(\boldsymbol{p},s,c)\ e^{-i\boldsymbol{px}} \right] . \tag{6}$$

The stochastic character of gluon field (which we supposed) allows us to develop the approximate description of the state Ψ if the following procedure of averaging

$$\Psi \to \langle \Psi \rangle = \int_0^t d\tau\ \Psi(\tau)/\ t$$

is intoduced. With this procedure taken the futher step is to turn to the approach of constructing a density matrix $\langle \overset{*}{\Psi}\Psi \rangle$. However, here we believe that at calculating the ground state (or more generally with quasi-stationary state) it might be sufficiently informative to operate with the averaged amplitude directly. Then in the interaction representation $\Psi = e^{H_0 t}\Phi$ we have the equation for state Φ as

$$\dot{\Phi} = -V\Phi\ ,\quad V = e^{H_0 t} V_S e^{-H_0 t}\ . \tag{7}$$

Now the "time" dependence appears in quark operators as well and after averaging over the short-wavelength component one may obtain the following equation

$$\langle \dot{\Phi}(t) \rangle = + \int_0^\infty d\tau\ \langle V(t) V(t-\tau) \rangle\ \langle \Phi(t) \rangle\ . \tag{8}$$

The limitations to have such a factorization validated are well known in the theory of stochastic differential equations (see, for example, [1]). The integration interval in Eq.(8) may be extended to the infinite "time" because of the rapid decrease (supposed) of the corresponding correlation function. Now we are allowed to deal with amplitude $\langle \Phi(t) \rangle$ in the right hand side of Eq.(8) instead the amplitude with the shifted arguments in order to get an ordinary integro-differential equation. In the quantum field theory applications it is usually difficult to construct the correlation function in the most general form. However, if we are going to limit our interest by describing the long-wavelength quark component only then gluon field correlator $\langle A^a_\mu(x) A^b_\nu(y) \rangle$ may be factorized and as a result we have

$$\langle \dot{\Phi}(t) \rangle = \int d\boldsymbol{x}\ \bar{q}(\boldsymbol{x},t)\ t^a\gamma_\mu\ q(\boldsymbol{x},t) \int_0^\infty d\tau \int d\boldsymbol{y}\ \bar{q}(\boldsymbol{y},t-\tau)\ t^b\gamma_\nu\ q(\boldsymbol{y},t-\tau)\ g^2 \langle A^a_\mu(t,\boldsymbol{x}) A^b_\nu(t-\tau,\boldsymbol{y}) \rangle\ \langle \Phi(t) \rangle$$

Having assumed the correlation function rapidly decreasing in "time" we could ignore all the retarding effects in the quark operators. Turning back to the Schrödinger representation we have for the state amplitude $\chi = e^{-H_0 t}\langle \Phi \rangle$ the following equation

$$\begin{aligned} &\dot{\chi} = -H_{ind}\ \chi\ , \\ &\mathcal{H}_{ind} = -\bar{q}\ (i\gamma\boldsymbol{\nabla} + im)\ q - \bar{q}\ t^a\gamma_\mu\ q \int d\boldsymbol{y}\ \bar{q}'\ t^b\gamma_\nu\ q' \int_0^\infty d\tau\ g^2 \langle A^a_\mu A'^b_\nu \rangle\ , \end{aligned} \tag{9}$$

with $q = q(\boldsymbol{x})$, $\bar{q} = \bar{q}(\boldsymbol{x})$, $q' = q(\boldsymbol{y})$, $\bar{q}' = \bar{q}(\boldsymbol{y})$ and $A^a_\mu = A^a_\mu(t, \boldsymbol{x})$, $A'^b_\nu = A^b_\nu(t-\tau, \boldsymbol{y})$. Now the correlation function might be presented as

$$\int_0^\infty d\tau\; g^2 \langle A^a_\mu A'^b_\nu \rangle = \delta^{ab}\; F_{\mu\nu}(\boldsymbol{x}-\boldsymbol{y}) \;,$$

with the corresponding formfactors $F_{\mu\nu}(\boldsymbol{x}-\boldsymbol{y}) = \delta_{\mu\nu}\; I(\boldsymbol{x}-\boldsymbol{y}) + J_{\mu\nu}(\boldsymbol{x}-\boldsymbol{y})$. In our consideration we ignore the contribution of the second formfactor spanning on the components of the vector $\boldsymbol{x}-\boldsymbol{y}$. Thus, on output we receive the Hamiltonian of four-fermion interaction with the formfactor rooted in the presence of two quark currents in the points $\boldsymbol{x}$ and $\boldsymbol{y}$. With this form of the effective Hamiltonian we could apply the Hartree–Fock–Bogolyubov method [2] to find its ground state as one constructed by the quark–anti-quark pairs with the oppositely directed momenta

$$\begin{aligned} &|\sigma\rangle = T\;|0\rangle\;, \\ &T = \Pi_{p,s,c}\; \exp\left\{ \frac{\theta}{2}\; \left[\, a^+(\boldsymbol{p},s,c)\; b^+(-\boldsymbol{p},s,c) + a(\boldsymbol{p},s,c)\; b(-\boldsymbol{p},s,c)\,\right] \right\}\;, \end{aligned} \tag{10}$$

where the parameter $\theta(\boldsymbol{p})$ characterizes the pairing strength. Introducing the creation and annihilation operators of quasi-particles $A = T\; a\; T^{-1}$, $A^+ = T\; a^+ T^{-1}$, $B = T\; b\; T^{-1}$, $B^+ = T\; b^+ T^{-1}$, we can rewrite the quark (anti-quark) operators as

$$\begin{aligned} q(\boldsymbol{x}) &= \int \frac{d\boldsymbol{p}}{(2\pi)^3} \frac{1}{(2|p_4|)^{1/2}}\; \left[\, A(\boldsymbol{p},s,c)\; U(\boldsymbol{p},s,c)\; e^{i\boldsymbol{px}} + B^+(\boldsymbol{p},s,c)\; V(\boldsymbol{p},s,c)\; e^{-i\boldsymbol{px}} \right]\;, \\ \bar{q}(\boldsymbol{x}) &= \int \frac{d\boldsymbol{p}}{(2\pi)^3} \frac{1}{(2|p_4|)^{1/2}}\; \left[\, A^+(\boldsymbol{p},s,c)\; \overline{U}(\boldsymbol{p},s,c)\; e^{-i\boldsymbol{px}} + B(\boldsymbol{p},s,c)\; \overline{V}(\boldsymbol{p},s,c)\; e^{i\boldsymbol{px}} \right]\;, \end{aligned}$$

with the quasi-particle spinors

$$\begin{aligned} U(\boldsymbol{p},s,c) &= \cos\left(\frac{\theta}{2}\right)\; u(\boldsymbol{p},s,c) - \sin\left(\frac{\theta}{2}\right)\; v(-\boldsymbol{p},s,c)\;, \\ V(\boldsymbol{p},s,c) &= \sin\left(\frac{\theta}{2}\right)\; u(-\boldsymbol{p},s,c) + \cos\left(\frac{\theta}{2}\right)\; v(\boldsymbol{p},s,c)\;, \end{aligned} \tag{11}$$

where $\overline{U}(\boldsymbol{p},s,c) = U^+(\boldsymbol{p},s,c)\;\gamma_4$, $\overline{V}(\boldsymbol{p},s,c) = V^+(\boldsymbol{p},s,c)\;\gamma_4$. Minimizing the mean energy functional one is able to determine the angle θ magnitude

$$\frac{d\langle\sigma|H_{ind}|\sigma\rangle}{d\theta} = 0\;. \tag{12}$$

Dropping the calculation details out we present here the following result for the mean energy as a function of the θ angle

$$\begin{aligned} \langle\sigma|H_{ind}|\sigma\rangle = &-\int \frac{d\boldsymbol{p}}{(2\pi)^3}\; \frac{2N_c\, p_4^2}{|p_4|}\,(1-\cos\theta) - \\ &-\widetilde{G}\int \frac{d\boldsymbol{p}d\boldsymbol{q}}{(2\pi)^6} \left\{ -(3\widetilde{I}-\widetilde{J}) \frac{p_4\; q_4}{|p_4||q_4|} + (4\widetilde{I}-\widetilde{J}) \frac{p\; q}{|p_4||q_4|} \left(\sin\theta - \frac{m}{p}\cos\theta\right) \left(\sin\theta' - \frac{m}{q}\cos\theta'\right) + \right. \\ &\left. +(-2\widetilde{I}\delta_{ij} - 2\widetilde{J}_{ij} + \widetilde{J}\delta_{ij})\; \frac{p_i\; q_j}{|p_4||q_4|}\; \left(\cos\theta + \frac{m}{p}\sin\theta\right) \left(\cos\theta' + \frac{m}{q}\sin\theta'\right)\; \right\}\;, \end{aligned} \tag{13}$$

here the following designations are used $p = |\boldsymbol{p}|$, $q = |\boldsymbol{q}|$, $\widetilde{I} = \widetilde{I}(\boldsymbol{p}+\boldsymbol{q})$, $\widetilde{J}_{ij} = \widetilde{J}_{ij}(\boldsymbol{p}+\boldsymbol{q})$, $\widetilde{J} = \sum_{i=1}^{3} \widetilde{J}_{ii}$, $p^2 = q^2 = -m^2$, $\theta' = \theta(q)$ where $\widetilde{G}$ is the constant of corresponding four-fermion interaction (the relevant details can be found in [3]). The first integral in Eq. (13) comes from free Hamiltonian, and we make a natural subtraction (adding the unit) in order to have zero mean free energy when the angle of pairing is trivial.

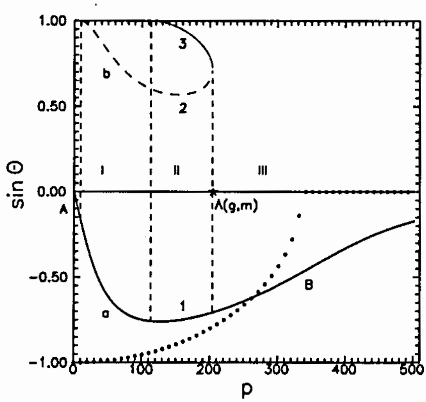

Figure 1: Phase portrait of the Keldysh model, $\sin\theta$ as a function of momentum p(MeV). The dotted curve corresponds to the solution with the negative values of angle in the chiral limit $m = 0$.

Nambu–Jona-Lasinio model

In order to get an idea of the parameter scales we continue with handling the model in which the formfactor behaves in the coordinate space as $I(\boldsymbol{x}-\boldsymbol{y}) = \delta(\boldsymbol{x}-\boldsymbol{y})$, $J_{\mu\nu} = 0$, dropping contribution spanned on the $p_i q_j$ tensor also. Actually, it corresponds to the Nambu–Jona-Lasinio model [4]. As well known the model with such a formfactor requires the regularization and, hence, the cutoff parameter Λ comes to the play

$$W = \int^{\Lambda} \frac{d\boldsymbol{p}}{(2\pi)^3} \left[|p_4|\,(1-\cos\theta) - G\frac{p}{|p_4|}\left(\sin\theta - \frac{m}{p}\cos\theta\right) \int^{\Lambda} \frac{d\boldsymbol{q}}{(2\pi)^3}\frac{q}{|q_4|}\left(\sin\theta' - \frac{m}{q}\cos\theta'\right)\right] . \tag{14}$$

We adjust the NJL model with the parameter set given by Hatsuda and Kunihiro [4] in which $\Lambda = 631$MeV, $m = 5.5$MeV. One curious point of this model is that the solution for optimal angle θ in the whole interval $p \in [0, \Lambda]$ can be found by solving the simple trigonometrical equation

$$(p^2 + m^2)\ \sin\theta - M_q\,(p\cos\theta + m\sin\theta) = 0\ , \tag{15}$$

with the dynamical quark mass

$$M_q = 2G \int^{\Lambda} \frac{d\boldsymbol{p}}{(2\pi)^3}\frac{p}{|p_4|}\left(\sin\theta - \frac{m}{p}\cos\theta\right)\ . \tag{16}$$

Eventually the results obtained look like $M_q = -335$ MeV for dynamical quark mass and $\langle\sigma|\bar{q}q|\sigma\rangle = -i$ (245 MeV)^3 for the quark condensate with the following definition of the quark condensate

$$\langle\sigma|\bar{q}q|\sigma\rangle = \frac{i\,N_c}{\pi^2} \int_0^\infty dp\, \frac{p^2}{|p_4|}\, (p\sin\theta - m\cos\theta)\ . \tag{17}$$

The Keldysh model

Now we are going to analyse the limit, in some extent, opposite to the NJL model, i.e. we are dealing with the formfactor behaving as a delta function but in the momentum space (analogously the Keldysh model, well known in the physics of condensed matter [5]), $I(\boldsymbol{p}) = (2\pi)^3\, \delta(\boldsymbol{p})$. Here the mean energy functional has the following form

$$W(m) = \int \frac{d\boldsymbol{p}}{(2\pi)^3} \left[|p_4|\ (1-\cos\theta) - G\, \frac{p^2}{|p_4|^2} \left(\sin\theta - \frac{m}{p}\cos\theta\right)^2 \right]\ . \tag{18}$$

contrary to the NJL model there is no need to introduce any cut off. The equation for calculating the optimal angle θ becomes the transcendental one

$$|p_4|^3\ \sin\theta - 2G\ \ (p\cos\theta + m\sin\theta)\,(p\sin\theta - m\cos\theta) = 0\ , \tag{19}$$

and, clearly, it is rather difficult to get its solution in a general form. Fortunately, it is much easier and quite informative to analyse the model in the chiral limit $m = 0$. There exist one trivial solution $\theta = 0$ and two nontrivial ones (for the positive and negative angles) which obey the equation

$$\cos\theta = \frac{p}{2G}\ . \tag{20}$$

Obviously, these solutions are reasonable if the momentum is limited by $p < 2G$. Then for the mean energy we have $W_\pm(0) = -\frac{G^4}{15\pi^2}$ if the quark condensate defined as $\langle\sigma|\bar{q}q|\sigma\rangle(0) = \frac{i\,N_c\,G^3}{2\pi}$. For the trivial solution the mean energy equals to zero together with the quark condensate $W_0(0) = 0$, $\langle\sigma|\bar{q}q|\sigma\rangle_0(0) = 0$. Introducing the practical designation $\sin\theta = \frac{M_\theta}{(p^2+M_\theta^2)^{1/2}}$ which characterizes the pairing strength by the parameter M_θ we have, for example, for the nontrivial solution $M_\theta = (4G^2 - p^2)^{1/2}$. In order to compare the results with the NJL model we fixed the value of four-fermion interaction constant as $M_\theta(0) = 2G = 335$ MeV. It is interesting to notice that the respective energy becomes constant $E(p) = \sqrt{p^2 + M_\theta^2}$, $E(p) = 2G$.

After having done the analysis in the chiral limit which is shown by the dotted line in Fig.1 we would like to comment the situation beyond this limit. The evolution of corresponding branches is available on the same plot 1. One solution denoted by A is developing in the local vicinity of coordinate origin and for small values of quark mass this domain is practically indistinguishable. In order to make it noticeable (to have a reasonable resolution on the plot) the quark mass was put as $m = 50$ MeV. Besides, there are two solutions a and b in the domain denoted by I, three solutions denoted by 1, 2, 3 in the domain II and one solution B in the domain III. The minimum of mean energy functional can be realized with the piecewise continuous functions. At the local vicinity of coordinate origin

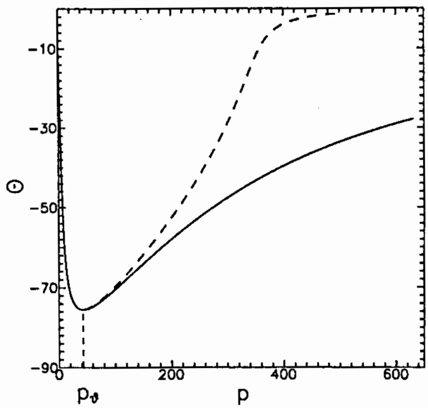

Figure 2: The optimal angle θ as a function of momentum p(MeV). The solid line corresponds to the NJL model and the dashed one to the Keldysh model. The current quark mass is $m = 5.5$ MeV and $p_\theta \sim 40$ MeV.

we start with the solution branch A, then relevant solution passes to the branch a or b interchanging its position from a to b in any subinterval. But in any case there is only one way to continue the solution at streaming to the infinite limit and it is related with the branch B where the angle is going to the zero value. As to the functional (18) the contribution of the term proportional to the cosine in the second parenthesis is divergent even if the angle θ is zero. It means the mean energy out of chiral limit goes to an infinity at any nonzero value of quark mass. The same conclusion is valid for the chiral condensate. In principle this functional could be regularized and corresponding continuation might be done but it is out of this presentation scope. It is not difficult to demonstrate the similar discontinuities of functional are present, for example, for Gaussian $I(x) = G\ \exp{(-a^2\ x^2)}$, and exponential $I(x) = G\ \exp{(-a\ |x|)}$, formfactors and they are present even in the NJL model but this fact is masked by the cut off parameter.

Comparing the optimal angles in the NJL and Keldysh models (see Fig. 2) it is interesting to notice that the formation of quasiparticles becomes significant at some momentum value close to the origin $p_\theta \sim 40$ MeV but not directly at the zero value. It is clear the inverse value of this parameter determines the characteristic size of quasiparticle. Parameter M_θ as a function of momentum p corresponding to the best fit to the NJL data $M_q = 335$ MeV, $\langle\sigma|\bar{q}q|\sigma\rangle = -i\ (245\ \text{MeV})^3$ is shown in Fig.3. The solid line corresponds to the Gaussian formfactor in the chiral limit and the dashed one shows the same dependence for the current quark mass $m = 5.5$ MeV. This dependence for exponential behaviour of formfactor is presented by the dotted lines on the same plot (the characteristic angle is $p_\theta \sim 150$ MeV in this case). Analysing the discontinuity of mean energy functional and quark condensate we face some troubles at fitting the quark condensate, for example. However, the dynamical quark mass and quark condensate are nonobservable quantities and it is curious to remark here that although the mean energy of the quark system is minus infinity the meson observables are finite and even in Keldysh model the mesons are recognizable with reasonable scale and we can in principle make a fit for this observables.

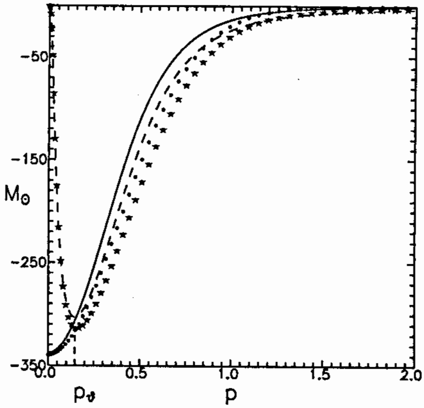

Figure 3: The parameter M_θ(MeV) as a function of momentum p(GeV) corresponding to the best fit of the NJL data M_q = 335 MeV, $\langle\sigma|\bar{q}q|\sigma\rangle = -i$ (245 MeV)^3. The solid line corresponds to the Gaussian formfactor in chiral limit and the dashed line corresponds to the magnitude of current quark mass $m = 5.5$ MeV. The exponentially behaving formfactor is represented by the dotted lines and $p_\theta \sim 150$ MeV.

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Heavy baryons in the field correlator method

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Abstract

We use the field correlator method in QCD to calculate the masses of Σ_c, Ξ_c and recently observed Σ_b, Ξ_b baryons and their orbital excitations.

The spectroscopy of c and b baryons has undergone a great renaissance in recent years. New results have been appearing in abundance as a result of improved experimental techniques including information on states made of both light (u, d, s) and heavy (c, b) quarks. Before 2007, the only one baryon with a b quark, the isospin-zero Λ_b^0, was known. Now, we have the isospin one Σ_b, Σ_b^* baryons and Ξ_b. The CDF Collaboration has seen the states $\Sigma_b^{\pm}$ and $\Sigma_b^{*\pm}$, while DØ and CDF have observed the Ξ_b^- [1].

On theoretical side there are many results on heavy baryon masses from different approaches. In the present paper we use the field correlator method (FCM) [2] to calculate the masses of the S wave baryons containing c and b quarks and orbitally excited states that will be experimentally accessible in the future.

The key ingredient of the FCM is the use of the auxiliary fields (AF) initially introduced in order to get rid of the square roots appearing in the relativistic Hamiltonian, see [3] and references therein. Using the AF formalism allows to write a simple local form of the Effective Hamiltonian (EH) for the three quark system

$$H = \sum_{i=1}^{3}\left(\frac{m_i^2}{2\mu_i} + \frac{\mu_i}{2}\right) + H_0 + V, \tag{1}$$

where H_0 is the kinetic energy operator, V is the sum of the string potential and a one gluon exchange potential, m_i are the bare quark masses, and μ_i are the *constant* AF which are eventually treated as variational parameters. They have the meaning of quark constituent masses. The string potential is $V_Y(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3) = \sigma\, r_{min}$, where σ is the string tension and r_{min} is the minimal length corresponding to the Y–shaped string configuration

The mass M_B of a baryon is given by $M = M_0 + \Delta E_{HF}$, where ΔE_{HF} is the spin correction, and

$$M_0 = \sum_{i=1}^{3}\left(\frac{m_i^2}{2\mu_i} + \frac{\mu_i}{2}\right) + E_0(\mu_i) + C, \tag{2}$$

$E_0(\mu_i)$ being the energy eigenvalue of the Shrödinger operator $H_0 + V$. The constant C in (2) is the calculable quark self-energy correction [4] which is created by the color magnetic moment of a quark propagating through the vacuum background field. This correction adds an overall negative constant to the hadron masses.

Using the hyperspherical method we calculate the constituent quark masses μ_i and the zero-order baryon masses M_0. Then we estimate HF splittings from the perturbative color-magnetic interaction with account of the wave function corrections. We calculate the

Table 1: Heavy Baryons with $L = 0$. The underlined masses have been used to fix m_c and m_b. The experimental baryon masses are for the isospin averaged states. All masses are in units of MeV.

Baryon	μ_n	μ_s	μ_h	M_0	$\Delta E_{HF}^{(p)}$	$\Delta E_{HF}^{(np)}$	M	M_{exp}
Σ_c	470		1455	2479	-19	-6	$\underline{2454}$	2455
Σ_c^*	470		1455	2479	30	13	2522	2520
Ξ_c	476	522	1458	2519	-39	-20	2460	2471
Σ_b	509		4749	5806	0	2	$\underline{5808}$	5810
Σ_b^*	509		4749	5806	+19	8	5833	5830
Ξ_b	514	615	4751	5844	-36	-17	5791	5790

hyperfine splitting with account of the both perturbative and nonperturbative spin-spin forces between quarks in a baryon.

We employ $m_n = 7$ MeV (with n standing for either u or d) and the strange quark mass $m_s = 185$ MeV found previously from the fit to D_s spectra. However, our predictions need an additional input for the bare quark masses m_c and m_b. These were fixed from the masses of Σ_c and Σ_b, respectively, $m_c = 1359$ MeV and $m_b = 4712$ MeV.

The result of the calculation of the S wave states is given in Table 1. In this Table we also present the dynamical quark masses μ_n, μ_s and μ_Q for various baryons (Q standing for either c or b). The latter are computed solely in terms of the bare quark masses, σ and α_s and marginally depend on a baryon. We also display the results obtained without the HF corrections. The result show good agreement between data and theoretical predictions. In particular, the hyperfine splitting between Ξ_c^* and Ξ_c' is found to be 69 MeV that agrees with the experimental value ($\sim$ 70 MeV), while the predicted mass difference $\Xi_b^* - \Xi_b' =$ 26 MeV agrees with the finding of Ref. [5]. However, our perturbative calculations do not reproduce the observed Ξ_c' - Ξ_c mass difference. The large hyperfine splitting between axial and scalar ns diquarks is usually described by the smeared δ-function that requires additional model-dependent assumptions about the structure of interquark forces.

A similar calculations were performed for the P-wave orbitally-excited states, see Table 2. Our basis states diagonalize the confinement problem with eigenfunctions that correspond to separate excitations of the light and heavy quarks (ρ - and λ - excitations, respectively). Excitation of the λ variable unlike excitation in ρ involves the excitation of the "odd" heavy quark. For states with one unit of orbital angular momentum between Q quark and the two light quarks we obtain $M(\Sigma_c) = 2832$ MeV, $M(\Xi_c) = 2867$ MeV, $M(\Sigma_b) = 6132$ MeV, and $M(\Xi_b) = 6164$ MeV, while the states with one units of orbital momentum between the two light quarks are typically $\sim$ 100 MeV heavier. Note that zero order results of Table 2 do not include the spin corrections and the (negative) string corrections contributing into the masses of the orbitally excited baryons. Our preliminary

Table 2: Heavy Baryons. $L = 1$. The bare quark masses are the same as in Table 1.

Baryon	$\mathbf{L}_\alpha$	μ_n	μ_s	μ_h	E_0	M
nnc	1_ρ	536		1452	1397	2920
nnc	1_λ	495		1491	1377	2832
nsc	1_ρ	542	582	1455	1372	2954
nsc	1_λ	497	544	1494	1353	2867
nnb	1_ρ	570		4746	1294	6240
nnb	1_λ	540		4764	1234	6132
nsb	1_ρ	574	615	4748	1271	6272
nsb	1_λ	542	588	4765	1211	6164

analysis of the latter shows that the string corrections tend to decrease the masses of the P-wave states by ~ 50 MeV. A more complete analysis will be given elsewhere.

In conclusion, we have calculated the masses of heavy baryons systematically using the FCM and the perturbative color-magnetic interaction. There are two main points in which we differ from other approaches to the same problem based on various relativistic Hamiltonians and equations with local potentials. The first point is that we do not introduce the constituent mass by hand. On the contrary, starting from the bare quark mass we arrive to the dynamical quark mass that appears due to the interaction. The second point is that for the first time we calculate the hyperfine splitting with account of the nonperturbative spin-spin forces between quarks in a baryon.

This work was supported by the RFBR grants 06-02-17120, 08-02-00657, 08-02-00677.

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Impact of eight-quark interactions in chiral phase transitions I: Secondary magnetic catalysis

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Abstract

The influence of a constant magnetic field on the order parameter of the four-dimensional Nambu and Jona-Lasinio model extended by the 't Hooft six-quark term and eight-quark interactions is considered. It is shown that the multi-quark interactions cause the order parameter to increase sharply (secondary magnetic catalysis) with increasing strength of the field at the characteristic scale $H \sim 10^{14}\Lambda^2$ G/MeV^2.

It has been shown in a series of papers [1]-[3] that in 2+1 and 3+1 dimensions a constant magnetic field $H \neq 0$ catalyzes dynamical symmetry breaking leading to a fermion mass even at the weakest attractive four-fermion interaction between massless particles, and the symmetry is not restored at any arbitrarily large H.

It is known, however, that the QCD motivated effective lagrangian for the light quarks ($N_f = 3$) contains also the six-fermion term: the $U(1)_A$ breaking 't Hooft interaction, and probably eight-quark terms. These extensions of the Nambu and Jona-Lasinio model are well-known, for instance, the four-quark $U(3)_L \times U(3)_R$ chiral symmetric lagrangian together with the 't Hooft six-quark interactions has been extensively studied at the mean-field level [4]-[7]. Recently it has been also shown [8, 9] that the eight-quark interactions are of vital importance to stabilize the multi-quark vacuum.

The additional multi-quark forces can affect the result which is obtained when only four-fermion interactions are considered. We argue here that the 't Hooft and eight-quark interactions can modify the theory in such a way that the local minimum, catalyzed by the constant magnetic field, is smoothed out by increasing the strength of the field. This is an alternative regime to the known one in which the strong magnetic field cannot change the ground state of the system. For the first scenario to become possible it is sufficient that the couplings of multi-quark interactions are chosen such that the system displays more than one solution of the gap equation at $H = 0$. However, the above condition is not a requirement. Even if the gap equation has only one nontrivial solution at small H, an increase in the magnetic field can induce the formation of a second minimum. Starting from some critical value H_c the second minimum is becoming a new ground state. We call this phenomenon a secondary magnetic catalysis. To see the details we need the effective potential of the theory, $V(m, |QH|) = V_{st} + V_S$, which is the sum of two terms. The first contribution results from the many-fermion vertices, after reducing them to a bilinear form with help of bosonic auxiliary fields, and subsequent integration over these fields, using the stationary phase method. This part does not depend on the magnetic field. The specific details of these calculations are given in our recent work [8]. In the $SU(3)_f$ symmetric case the result is

$$V_{st} = \frac{1}{16}\left(12Gh^2 + \kappa h^3 + \frac{27}{2}\lambda h^4\right). \tag{1}$$

The function h is a solution of the stationary phase equation $12\lambda h^3+\kappa h^2+16(Gh+m)=0$, where G, κ, λ are couplings of four, six and eight-quark interactions correspondingly. This cubic equation has one real root, if $G/\lambda > (\kappa/24\lambda)^2$. Assuming that the couplings fulfill the inequality, one finds the single valued function $h(m, G, \kappa, \lambda)$.

The second term, V_S, derives from the integration over the quark bilinears in the functional integral of the theory in presence of a constant magnetic field H. As has been calculated by Schwinger a long time ago [10] $V_S = \sum_{i=u,d,s} V_S(m_i, |Q_i H|)$, where

$$V_S(m, |QH|) = \frac{N_c}{8\pi^2} \int_0^\infty \frac{ds}{s^2} e^{-sm^2} \rho(s, \Lambda^2)|QH| \coth(s|QH|) + \text{const.} \tag{2}$$

Here the cutoff Λ has been introduced by subtracting off suitable counterterms to regularize the integral at the lower limit: $\rho(s,\Lambda^2) = 1-(1+s\Lambda^2)e^{-s\Lambda^2}$. The unessential constant is chosen to have $V_S(0, |QH|) = 0$. We ignore in the remaining the charge difference of u and d, s quarks: the averaged common charge $|Q| = |4e/9|$ will be used.

One sees that the gap equation, $\mathrm{d}V(m)/\mathrm{d}m = 0$, has always a trivial solution $m = 0$, which corresponds to the point where the potential reaches its local maximum, if $H \neq 0$. This phenomenon is known as magnetic catalysis of dynamical chiral symmetry breaking. The nontrivial solution is contained in the equation

$$-\frac{2\pi^2 h(m)}{\Lambda^2 N_c m} = \psi\left(\frac{\Lambda^2+m^2}{2|QH|}\right) - \frac{|QH|}{\Lambda^2}\left[\ln\left(1+\frac{\Lambda^2}{m^2}\right) - \frac{\Lambda^2}{\Lambda^2+m^2} + 2\ln\frac{\Gamma\left(\frac{\Lambda^2+m^2}{2|QH|}\right)}{\Gamma\left(\frac{m^2}{2|QH|}\right)}\right], \tag{3}$$

where $\psi(x) = \mathrm{d}\ln\Gamma(x)/\mathrm{d}x$ is the Euler dilogarithmic function. Here the l.h.s. originates from V_{st} and the r.h.s. from V_S.

Let us consider first the standard case with $\kappa, \lambda = 0$ and $h = -m/G$. Then the l.h.s. is a constant $\tau^{-1} = 2\pi^2/G\Lambda^2 N_c$. Fig. 1 (left panel) illustrates this pattern. One sees that at $H = 0$ the system is in the subcritical regime of dynamical symmetry breaking. The introduction of a constant magnetic field, however small it might be, changes radically the dynamical symmetry breaking pattern: due to the singular behaviour of the r.h.s. of Eq. (3) close to the origin the curves corresponding to the r.h.s. and l.h.s. will always intersect and the value of m where this happens is a minimum of $V(m)$. One concludes that in the theory with just four-fermion interactions the effective potential has only one minimum at $m > 0$, and this property does not depend on the strength of the field H.

In the theory with four-, six-, and eight-quark interactions one can find either one or two local minima at $m > 0$. We illustrate these two cases in the central panel of fig. 1. Namely, the upper full curve f (r.h.s. of Eq. (3) for $|QH|\Lambda^{-2} = 0.5$) has only one intersection point with the bell-shaped curve u (l.h.s. of Eq. (3) for $G\Lambda^2 = 3$, $\kappa\Lambda^5 = -10^3$, $\lambda\Lambda^8 = 3670$). This point corresponds to a single vacuum state of the theory. The other full curve f for $|QH|\Lambda^{-2} = 0.1$ has three intersections with the same curve u. These intersections, successively, correspond to a local minimum, a local maximum and a further local minimum of the potential. The first minimum catalyzed by a constant magnetic field (that is, a slowly varying field) is then smoothed out with increasing H. It ceases to exist at some critical value of $|QH|\Lambda^{-2}$, from which on only the large M_{dyn} solution survives. This is shown in the right panel of fig. 1. The new phenomenon might be a clear signature of eight-quark interactions.

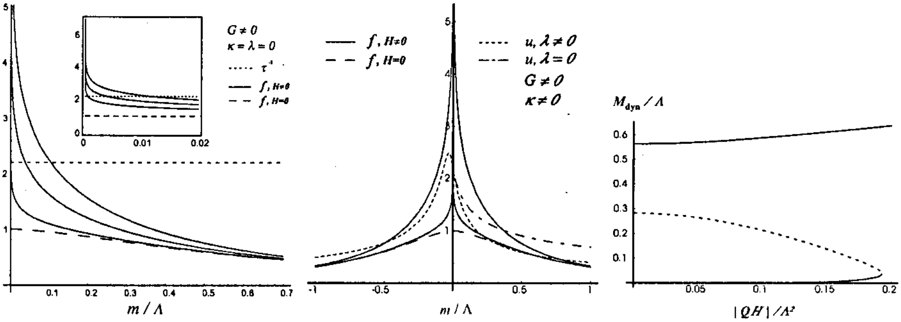

Figure 1: Left: The l.h.s. (straight short-dashed line) and the r.h.s. of Eq. (3) at $\kappa, \lambda = 0$ and $G\Lambda^2 = 3$ as functions of m/Λ for four different values of H: full curves (top to bottom) correspond to $|QH|\Lambda^{-2} = 0.5$; 0.3; 0.1, and the dashed curve to $H = 0$. Box insert: close-up of region around origin with solid lines for $|QH|\Lambda^{-2} = 0.2$; 0.15; 0.1. Centre: The l.h.s. (short-dashed line) and the r.h.s. of Eq. (3) at $\kappa\Lambda^5 = -10^3\,, \lambda\Lambda^8 = 3670$ (or $\lambda = 0$), and $|QH|\Lambda^{-2} = 0.5$; 0.1; 0. Right: The dimensionless dynamical mass $M_{\rm dyn}/\Lambda$ as a function of the dimensionless magnetic field $|QH|\Lambda^{-2}$. The full lines are minima, the dashed line maxima. Up to $|QH|\Lambda^{-2} = 0.084$ the smaller $M_{\rm dyn}/\Lambda$ correponds to the deeper minimum of the potential; from this value on the larger solution becomes the stable configuration.

This work has been supported in part by grants provided by FCT: POCI/2010, FEDER, POCI/FP/63930/2005 and POCI/FP/81926/2007.

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Next-to-Leading order Analysis of the Gluon Distribution at low x

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Abstract

We present solutions of the gluon distribution function by the Taylor expansion as a function of $F_2(x, Q^2)$ and its derivative with respect to LnQ^2 in the next- to-leading order of the perturbative theory at low x. The obtained values are in the range $10^{-4} \leq x \leq 10^{-2}$ at $Q^2 = 20GeV^2$.

The DGLAP evolution equation for the singlet quark structure function has the form:

$$\frac{d\Sigma(x,Q^2)}{dlnQ^2} = \frac{\alpha_s}{2\pi}\int_0^{1-x} dz[P_{qq}^{LO+NLO}(1-z)\Sigma(\frac{x}{1-z},Q^2) + 2n_f P_{qg}^{LO+NLO}(1-z)G(\frac{x}{1-z},Q^2)] \tag{1}$$

where the splitting functions are the leading order (LO) and the next- to- leading order (NLO) Altarelli- Parisi splitting kernels [1]. To find an analytic solution, we note that the splitting kernels have the following forms as $z \rightarrow 0$ [2,3]:

$$\begin{aligned} P_{qq}^{LO+NLO}(z) &= \frac{\alpha_s}{2\pi}\frac{40C_F N_f T_R}{9z}, \\ P_{qg}^{LO+NLO}(z) &= \frac{\alpha_s}{2\pi}\frac{40C_A N_f T_R}{9z}. \end{aligned} \tag{2}$$

For an SU(N) gauge group we have $C_A = N$, $C_F = (N^2-1)/2N$, $T_F = N_f T_R$, and $T_R = 1/2$, that C_F and C_A are the color Cassimir operators.

We introduce the standard parameterization of the singlet distribution functions and the gluon distribution as

$$\begin{aligned} \Sigma(x,Q^2) = A_S x^{-\delta_S}(1-x)^{\nu_S}(1+\epsilon_S\sqrt{x}+\gamma_S x) \equiv \widetilde{\Sigma}(x,Q^2)x^{-\delta_S}, \\ G(x,Q^2) = A_g x^{-\delta_g}(1-x)^{\nu_g}(1+\epsilon_g\sqrt{x}+\gamma_g x) \equiv \widetilde{G}(x,Q^2)x^{-\delta_g}. \end{aligned} \tag{3}$$

where, the usual assumption is that $\delta_{i(=S,g)} = 0$. However, the small x behavior could well be more singular. Note that the behavior of Eq.(3) with a Q^2 independent value for $\delta_{i(=S,g)}$ obeys the DGLAP equations when $x^{-\delta_{i(=S,g)}} >> 1$[4]. According to Regge theory, the high energy (low x) behavior of both gluons and sea quarks is controlled by the same singularity factor in the complex angular momentum plane, and so we would expect $\delta_S = \delta_g = \delta$, where δ is taken as a constant factor throughout the calculation. For the structure functions we take $\tilde{f}(x,Q^2) = x^\delta f(x,Q^2)$ to be finite at $x=0$ with δ satisfying $0 \leq \delta \leq \frac{1}{2}$, i.e. $\tilde{G}(x) = x^\delta G(x)$ and $\tilde{\Sigma}(x) = x^\delta \Sigma(x)$. Expanding $\tilde{G}(x/1-z)$ and $\tilde{\Sigma}(x/1-z)$ about $x=0$, we obtain

$$\begin{aligned}
\tilde{G}(\frac{x}{1-z}) &= \tilde{G}(0) + \frac{x}{1-z}\tilde{G}'(0), \\
\tilde{\Sigma}(\frac{x}{1-z}) &= \tilde{\Sigma}(0) + \frac{x}{1-z}\tilde{\Sigma}'(0).
\end{aligned} \quad (4)$$

The assumptions in these equations are the convergence and the possibility to neglect $O(x^2)$ terms.

Inserting Eqs.(2) and (4) in Eq.(1) we will have the DGLAP equations for the singlet evolution at small x :

$$\begin{aligned}
\frac{d\Sigma}{dlnQ^2} &= \frac{\alpha_s}{2\pi}\int_0^{1-x} dz(\frac{\alpha_s}{2\pi}\frac{\zeta}{9(1-z)})(\frac{x}{1-z})^{-\delta} \times (\tilde{\Sigma}(0) + \frac{x}{1-z}\tilde{\Sigma}'(0)] \\
&+ \frac{\alpha_s}{2\pi}\int_0^{1-x} dz(2n_f)(\frac{\alpha_s}{2\pi}\frac{\xi}{9(1-z)})(\frac{x}{1-z})^{-\delta} \times (\tilde{G}(0) + \frac{x}{1-z}\tilde{G}'(0)]
\end{aligned} \quad (5)$$

where $\zeta = 40 C_F N_f T_R$ and $\xi = 40 C_A N_f T_R$.

Solving this equation and taking all these considerations into account, we find:

$$\begin{aligned}
\frac{d\Sigma}{dlnQ^2} &= V_I[\frac{\delta^{\delta-1}}{|\delta-1|^\delta}\Sigma(x\frac{\delta}{|\delta-1|}) - \frac{1}{\delta}\tilde{\Sigma}(\frac{\delta}{|\delta-1|})] \\
&+ V_{II}[\frac{\delta^{\delta-1}}{|\delta-1|^\delta}G(x\frac{\delta}{|\delta-1|}) - \frac{1}{\delta}\tilde{G}(\frac{\delta}{|\delta-1|})],
\end{aligned} \quad (6)$$

where $V_I = (\frac{\alpha_s}{2\pi})^2\frac{\zeta}{9}$ and $V_{II} = (\frac{\alpha_s}{2\pi})^2(2n_f)\frac{\xi}{9}$. The function $\tilde{f}(\frac{\delta}{|\delta-1|})$ $(f = G, \Sigma)$ is a small constant at $x=0$. At low- x, this constant can be neglected in the Eq.(6) due to the singular behavior of the gluon distribution. We therefore have

$$\frac{d\Sigma}{dlnQ^2} = \tau[V_I\Sigma(\mu x) + V_{II}G(\mu x)], \quad (7)$$

where $\tau = \frac{\delta^{\delta-1}}{|\delta-1|^\delta}$, $\mu = \frac{\delta}{|\delta-1|}$. This equation is a formula to extract the gluon distribution function from singlet structure function and its derivative $d\Sigma/dlnQ^2$ at small x in the next-to- leading order of perturbation theory.

So that, we can arrive of the gluon distribution function from the F_2 proton structure function and its scaling violation at low x as

$$xg(x,Q^2) = \frac{18}{5V_{II}}[\frac{1}{2}\frac{dF_2}{dlnQ^2} - V_I F_2]. \quad (8)$$

By means of this equation we have extracted the gluon distribution from HERA data, using the slopes $dF_2/dlnQ^2$ determined in Ref.[10]. Figure 1 shows the extracted values of the gluon distribution compared to the Kotikov-Parente (KP) model [4], the Ellis-Kunszt-Levien (EKL) model [5] and MRST [6,7] parameterization. This result indicate that our calculations [8,9], based upon the available structure functions and its derivative [10], are of the same form as the one predicted by the QCD theory. The formulae used to generate the parton distribution is in agreement with the rise observed by H1 experiments. We observed a continuous rise towards low x.

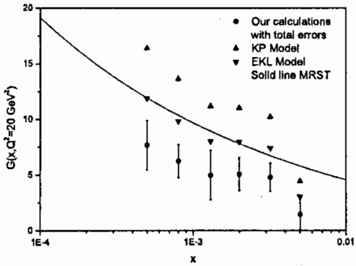

Figure 1: The solid circles represent our gluon prediction (Eq.8) using the structure function F_2 and $dF_2/dlnQ^2$ are taken by the H1 [10] collaboration for a range of x values at $Q^2 = 20GeV^2$. The error bar show total errors to H1 data. We compared our results with KP model [4], EKL model [5] and MRST fit[6,7](Solid line).

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Gravimagnetic nucleon form-factors in the impact parameter representation

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Abstract

In the framework of the new t-dependence of the General Parton Distributions (GPDs), which reproduce the electromagnetic form factors of the proton and neutron at small and large momentum transfer, the gravitational form factors of the nucleons and a separate contribution of the quarks to them are obtained.

As a basis, it is assumed that the form factor is dominated by a soft mechanism and the Generalized Parton distributions (GPDs)-handbag approach [1] is utilized. GPDs for $\xi = 0$ provide information about the distribution of the parton in impact parameter space [4]. It is connected with t-dependence of $GPDs$.

In [3], a simple ansatz was proposed which will be good for describing the form factors of the proton and neutron by taking into account a number of new data that have appeared in the last years. We choose the t-dependence of GPDs in the form

$$\mathcal{H}^u(x,t) \ = u(x)\ exp[a_+ \ \frac{(1-x)^2}{x^m}\ t]; \quad \mathcal{H}^d(x,t) \ = d(x)\ exp[a_+ \ \frac{(1-x)^2}{x^m}\ t]. \tag{1}$$

The size of the parameter $m = 0.4$ was determined by the low t experimental data; the free parameters $a_\pm$ (a_+ - for $\mathcal{H}$ and a_- - for $\mathcal{E}$) were chosen to reproduce the experimental data in a wide t region. The $q(x)$ was taken from the MRST2002 global fit [5] wth the scale $\mu^2 = 1$ GeV^2. In all our calculations we restrict ourselves, as in other works, only to the contributions of u and d quarks and the terms in $\mathcal{H}^q$ and $\mathcal{E}^q$. Correspondingly, for $\mathcal{E}^u(x)$, as for example [2], we have

$$\mathcal{E}^u(x) \ = \frac{k_u}{N_u}(1-x)^{\kappa_1}\ u(x), \qquad \mathcal{E}^d(x) \ = \frac{k_d}{N_d}(1-x)^{\kappa_2}\ d(x), \tag{2}$$

where $\kappa_1 = 1.53$ and $\kappa_2 = 0.31$ [2]. With standard normalization of the form factors, we have $k_u = 1.673$, $k_d = -2.033$, $N_u = 1.53$, $N_d = 0.946$. The parameters $a_+ = 1.1$ and a_- were chosen to obtain two possible forms of the ratio of the Pauli and Dirac form factors.

1 Proton and neutron electromagnetic form factors

The proton Dirac form factor calculated in [3] reproduces sufficiently well the behavior of experimental data not only at high t but also at low t. Our description of the ratio of the Pauli to the Dirac proton form factors and the ratio of G_E^p/G_M^p shows that in our model we can obtain the results of both the methods (Rosenbluth and Polarization) by changing the slope of $\mathcal{E}$. Based on the model developed for proton the neutron form factors are calculated

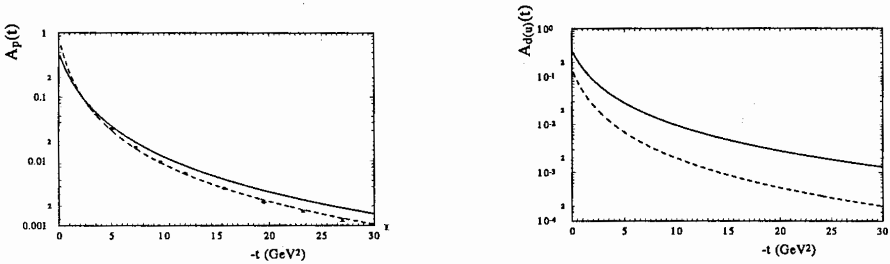


Figure 1: a)[left] Gravitation form factor A_q (hard line) and Proton Dirac form factor (dashed line), both calculated in this work; the data for F_1^p are from [7]. b)[right] Contributions of the u (dashed line) and d (hard line) to the gravitation form factor A_q

too. To do this the isotopic invariance can be used to change the proton GPDs to neutron GPDs. Hence, we do not change any parameters and conserve the same t-dependence of GPDs as in the case of proton. Our calculation of G_E^n shows that the variant which describes the polarization data is in better agreement with the experimental data. The calculation of G_M^n more clearly shows that this variant much better describes the experimental data especially at low momentum transfer.

2 Gravitational form factors

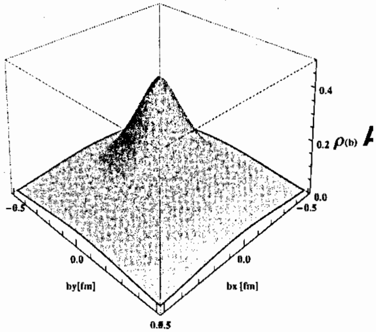
As was shown in [6], the gravitational form factor for fermions is determined as

$$\int_{-1}^{1} dx\, x[H(x,\Delta^2,\xi) + E(x,\Delta^2,\xi)] = A_q(\Delta^2) + B_q(\Delta^2). \qquad (3)$$

Using this representation we can calculate the gravitational form factor for the nucleon. Our result for $A_q(t)$ is shown in Fig.1a. Separate contributions of the u and d- quark distribution are shown in Fig.1b. At $t=0$ these contributions equal $A_u(t=0)=0.35$ and $A_d(t=0)=0.14$; and $B_u(t=0)=0.22$, $B_d(t=0)=-0.27$. The sum of B_q will be near zero $B_q(t=0)=-0.05$. In accuracy of our approximations this result coincides with zero. In fig.1a we compare gravitational form factors with our calculations of electromagnetic form factors. It can be seen that at large momentum transfer they have the same t-dependence. Of course, they essentially differ in size.

3 Conclusion

We introduced a simple new form of the t-dependence of GPDs. It satisfies the conditions of the non-factorization, introduced by Radushkin, and the Burkhardt condition on the power of $(1-x)^n$ in the exponential form of the t-dependence. With this simple form we



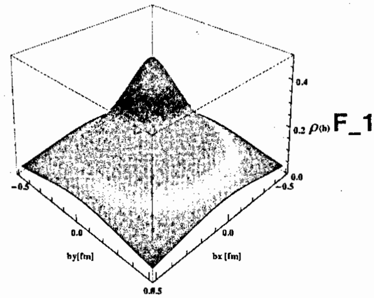


Figure 2: Densities of a)[left] the gravitational form factor A and b)[right] the electromagnetic form factor F_1.

obtained a good description of the proton electromagnetic Sachs form factors. Using the isotopic invariance we obtained good descriptions of the neutron Sachs form factors without changing any parameters.

On the basis of our results we calculated the contribution of the u and d quarks to the gravitational form factor of the nucleons. The cancellation of these contributions at $t = 0$ shows that the gravimagnetic form factor is zero for separate contributions, gluons and quarks, which confirms the result of [8].

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Spectroscopic implications from the combined analysis of processes with pseudoscalar mesons

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Abstract

The results of analyses of experimental data on the isovector P-wave of $\pi\pi$ scattering and on the isoscalar S- and D-waves of processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta, \eta\eta'$ in approaches, based on analyticity and unitarity, are given. Some spectroscopic implications of these analyses are discussed.

We present results of the combined analysis of data on processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta, \eta\eta'$ in the $I^G J^{PC} = 0^+0^{++}$ and 0^+2^{++} channels and on the $\pi\pi$ scattering in the 1^+1^{--} channel. The analysis adds to clarifying a QCD nature of the observed mesonic states and their assignment to the quark-model configurations, which is problematic up to now. Parameters of the resonances (and even the status of some of them) are not well known yet [1].

Our method of analysis is based on analyticity and unitarity of the S-matrix utilizing the uniformizing variable [2] (the model-independent approach – MIA) and on the multichannel Breit-Wigner (BW) forms. In MIA a resonance (depending on its nature) is represented by three and seven types of pairs of complex-conjugate clusters (of poles and zeros on the Riemann surfaces) in the 2- and 3-channel cases, respectively. MIA being very sensitive to data is applicable, however, only in the 2- and 3-channel considerations.

Some results of the analysis in the scalar and vector channels were already published in Refs.[3] and [4], respectively. Here we summarize the results and add some new results for the tensor channel.

Analysis of the isoscalar-scalar sector. Considering processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta, \eta\eta'$ in MIA, we performed separately the 3-channel analyses of $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ and $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta'$. Influence of the remaining channels is taken into account via the background [3]. We got a satisfactory description in both analyses: the total χ^2/NDF is $345.603/(301-40) \approx 1.32$ in the former and $282.682/(293-38) \approx 1.11$ in the latter.

In the analyses, an additional confirmation of the σ-meson with mass 835 MeV is obtained. This mass value accords rather well with prediction ($m_\sigma \approx m_\rho$) on the basis of mended symmetry by S. Weinberg [5].

A model-independent indication for $f_0(980)$ to be the $\eta\eta$ bound state is obtained.

The $f_0(1370)$ and $f_0(1710)$ have the dominant $s\bar{s}$ component. Our conclusion about the $f_0(1370)$ agrees well with the one drawn in Ref.[6] where the $f_0(1370)$ was identified as $\eta\eta$ resonance in the $\pi^0\eta\eta$ final state of the $\bar{p}p$ annihilation at rest. Conclusion about the $f_0(1710)$ is quite consistent with the experimental facts that this state is observed in $\gamma\gamma \to K_S\bar{K}_S$ [7] but not in $\gamma\gamma \to \pi^+\pi^-$ [8].

The $f_0(1500)$ is almost the 8th component of octet mixed with a glueball dominant in this state. Its largest width among the enclosing states also suggests its glueball nature [9].

We can assign the scalar mesons to lower nonets, excluding the $f_0(980)$ as the $\eta\eta$ bound state. The ground nonet: the isovector $a_0(980)$, the isodoublet $K_0^*(900)$, and $f_0(600)$ and $f_0(1370)$ as mixtures of the 8th component of octet and the SU(3) singlet. The Gell-Mann–Okubo (GM-O) formula $3m_{f_8}^2 = 4m_{K_0^*}^2 - m_{a_0}^2$ gives $m_{f_8} = 872$ MeV. In the relation for masses of nonet $m_\sigma + m_{f_0(1370)} = 2m_{K_0^*}$ the left side is by 25 % larger than the right one. The next nonet: $a_0(1450)$, $K_0^*(1450)$, and $f_0(1500)$ and $f_0(1710)$. From the GM-O formula, $m_{f_8} \approx 1450$ MeV. In the relation, $m_{f_0(1500)} + m_{f_0(1710)} = 2m_{K_0^*(1450)}$, the left side is by 12 % larger than the right one. Now an adequate mixing scheme should be found.

Analysis of the isovector P-wave of $\pi\pi$ scattering. Here we applied both MIA and the multichannel BW forms [4]. In MIA, the thresholds of the $\pi\pi$ and $\omega\pi$ channels and the left branch-point at $s = 0$ were included into the uniformizing variable whereas influence of other channels was accounted via the background. In the BW method, four inelastic channels, $\pi^+\pi^-2\pi^0$, $2\pi^+2\pi^-$, $\eta 2\pi$, and $\omega\pi^0$, were assumed. We analyzed the energy behaviour of the phase shift and inelasticity parameter of the $\pi\pi$-scattering amplitude taking three, four, and five resonances respectively from the $\rho(770)$, $\rho(1250)$, $\rho(1600)$, $\rho(1910)$ and $\rho(1450)$.

As a result, the 1st ρ-like meson has the mass 1257.8±11 MeV in MIA and 1249.8±15.6 MeV in the BW one unlike the mass 1459±11 MeV of the 1st ρ-like meson, cited in the PDG tables [1]. If the $\rho(1250)$ is interpreted as the 1st radial excitation of the 1^+1^{--} $q\bar{q}$ state, then it lies down well on the corresponding linear trajectory with an universal slope on the (n, M^2) plane (n is the radial quantum number of the $q\bar{q}$ state) [9, 12], whereas the $\rho(1450)$ is considerably higher than this trajectory. The $\rho(1250)$ and the isodoublet $K^*(1410)$ are well located to the octet of 1st radial excitations. Then the GM-O formula, $3m_{\omega_8'}^2 = 4m_{K^{*\prime}}^2 - m_{\rho'}^2$, gives $m_{\omega_8'} = 1460$ MeV compatible fairly good with the mass of the 1st ω-like meson $\omega(1420)$, for which one obtains the values in range 1350-1460 MeV [1].

Existence of the $\rho(1450)$ together with $\rho(1250)$ does not contradict to the analyzed data. In the $q\bar{q}$ picture, the former might be the first $^3D_1$ state with, possibly, the isodoublet $K^*(1680)$ in the corresponding octet. From the GM-O formula, we obtain the value 1750 MeV for the mass of the 8th component of this octet. This corresponds to one of the observations of the 2nd ω-like meson cited in the PDG tables under the $\omega(1650)$ [1].

The 3rd ρ-like meson has the mass 1600 MeV rather than 1720 MeV cited in the PDG tables. As to the $\rho(1900)$, in its energy region there are practically no data on the P-wave of $\pi\pi$ scattering. The MIA analysis testifies in favour of this state existence but the BW one gives equivalent description with and without it.

The suggested picture for the first two ρ-like mesons is consistent with predictions of some quark models [10]. However, if existence of the $\rho(1250)$ is confirmed, some of the quark potential models, *e.g.* in Ref.[11], will require substantial revisions, because, in these models, the 1st ρ-like meson is usually predicted about 200 MeV higher than this state. To the point, the 1st K^*-like meson is obtained in the indicated quark model at 1580 MeV, whereas the corresponding very well established resonance has the mass of 1410 MeV.

Analysis of the isoscalar-tensor sector. In analysis of processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$, we considered explicitly also the channel $(2\pi)(2\pi)$. Here, we generated the resonance poles and zeroes in the S-matrix by the 4-channel BW forms.

We obtained a satisfactory description with ten resonances $f_2(1270)$, $f_2(1450)$, $f_2'(1525)$, $f_2(1580)$, $f_2(1730)$, $f_2(1810)$, $f_2(1960)$, $f_2(2000)$, $f_2(2240)$ and $f_2(2410)$ and with eleven states adding the $f_2(2020)$ needed in the analysis of processes $p\bar{p} \to \pi\pi, \eta\eta, \eta\eta'$ [12]. The total χ^2/n.d.f. is $161.147/(168-65) \approx 1.56$ in the 1st case and $156.617/(168-69) \approx 1.58$

in the 2nd one. We do not obtain the $f_2(1640)$, $f_2(1910)$ and $f_2(2150)$, however, we see $f_2(1450)$ and $f_2(1730)$ which are related to the statistically-valued experimental points.

Usually one puts the $f_2(1270)$ and $f_2'(1525)$ into the ground tensor nonet. One can assign $f_2(1600)$ and $f_2(1760)$ to the 2nd nonet though the isodoublet member is not discovered yet. If $a_2(1730)$ is the isovector of this octet and if $f_2(1600)$ is almost its 8th component, then the GM-O formula gives this isodoublet mass at 1633 MeV. Then the relation for masses of nonet will be fulfilled with a 3% accuracy. In Ref.[13] the strange isodoublet was observed with yet indefinite remaining quantum numbers and with mass 1629 ± 7 MeV in the mode $K_s^0\pi^+\pi^-$. This state can be the tensor isodoublet of the 2nd nonet.

The $f_2(1963)$ and $f_2(2207)$ together with the isodoublet $K_2^*(1980)$ could be put into the 3rd nonet. Then in the relation $M_{f_2(1963)} + M_{f_2(2207)} = 2M_{K_2^*(1980)}$, the left side is only by 5.3 % larger than the right one. If $f_2(1963)$ is almost the 8th component of octet, $M_{a_2}^2 = 4M_{K_2^*(1980)}^2 - 3M_{f_2(1963)}^2$ gives $M_{a_2} = 2030$ MeV that coincides with the a_2-meson mass obtained in Ref.[14]. This state is interpreted as the 2nd radial excitation of the 1^-2^{++} state considering the a_2 trajectory on the (n, M^2) plane [12].

Presence of the $f_2(2020)$ in analysis with eleven resonances makes it possible to interpret $f_2(2000)$ as a glueball. In the case of ten resonances, the ratio of the $\pi\pi$ and $\eta\eta$ widths is in the limits obtained in Ref.[12] for the tensor glueball. However, the $K\overline{K}$ width is too large for the glueball. When passing from the ten-states description to the eleven-states one, the resonance parameters have varied a little, except for $f_2(2000)$ and $f_2(2410)$. Mass of the latter has decreased by 40 MeV but the $K\overline{K}$ width of the former has changed significantly. Then all ratios of the partial widths are in the limits corresponding to the glueball.

Interpretation of the $f_2(1450)$, $f_2(1730)$, $f_2(2020)$ and $f_2(2410)$ remains open.

Yu.S. acknowledges support provided by the Votruba-Blokhintsev Program. P.B. thanks the Grant Agency of the Czech Republic, Grant No.202/08/0984. Yu.S. and R.K. acknowledge support provided by the Bogoliubov–Infeld Program.

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Wandzura - Wilczek Approximations and Transverse Momentum Dependent Parton Distributions

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Abstract

Certain *exact* relations among transverse momentum dependent parton distributions due to QCD equations of motion turn into *approximate* ones upon the neglect of pure twist-3 terms. On the basis of available data from HERMES we test the usefulness of one such "Wandzura-Wilczek-type approximation" connecting $h_{1L}^{\perp(1)}$ to h_1.

1 Introduction

Semi-inclusive deep inelastic lepton nucleon scattering (SIDIS) allows one to access information on transverse momentum dependent parton distributions (TMDs) and fragmentation functions, where altogether eight twist-2 and sixteen twist-3 TMDs exist [1, 2]. Integrating over transverse parton momenta one is left with just six independent "collinear" parton distribution functions (PDFs) [3, 4]. In view of the numerous TMDs we ask whether there are useful approximations among the various, independent TMDs. In the present note we will briefly summarize our results presented in [5].

We start from the QCD equations of motion (EOMs) which provide the *exact* relation [1]

$$h_{1L}^{\perp(1)}(x) \overset{\text{EOM}}{=} -\frac{x}{2}\, h_L(x) + \frac{x}{2}\, \tilde{h}_L(x)\,, \tag{1}$$

with $h_{1L}^{\perp(1)}(x)$ denoting the second $\mathbf{p}_T$-moment of the TMD $h_{1L}^{\perp}(x,\mathbf{p}_T^2)$, $h_L(x)$ a "collinear" twist-3 PDF, and $\tilde{h}_L(x)$ a pure twist-3 quark-gluon-quark contribution. Next we exploit the "Wandzura-Wilczek (WW) approximation" [4, 6]

$$h_L(x) = 2x\int_x^1 \frac{\mathrm{d}y}{y^2}\, h_1(y) + \tilde{h}'_L(x) \overset{\text{WW}}{\approx} 2x\int_x^1 \frac{\mathrm{d}y}{y^2}\, h_1(y)\,, \tag{2}$$

where in the second step in (2) the pure twist-3 term $\tilde{h}'_L(x)$ is neglected.

Neglecting also $\tilde{h}_L(x)$ in the EOM (1) and taking the WW-approximation (2) into account one obtains the new WW-type approximation

$$h_{1L}^{\perp(1)}(x) \overset{!?}{\approx} -x^2\int_x^1 \frac{\mathrm{d}y}{y^2}\, h_1(y)\,. \tag{3}$$

This approximation will be tested by the following single spin asymmetry (SSA) in SIDIS,

$$A_{UL}^{\sin 2\phi} \propto h_{1L}^{\perp(1)}\, H_1^{\perp}\,, \tag{4}$$

for which final HERMES [7] and preliminary CLAS [8] data are available.

2 $A_{UL}^{\sin 2\phi}$ in WW-type approximation

The expression for the SSA (4) in terms of structure functions is given by [1]

$$A_{UL}^{\sin 2\phi}(x) = \frac{\int \mathrm{d}y \, [\cos\theta_\gamma (1-y)/Q^4] \, F_{UL}^{\sin 2\phi}}{\int \mathrm{d}y \, [(1-y+\frac{y^2}{2})/Q^4] \, F_{UU,T}}, \tag{5}$$

where the structure function $F_{UU,T}$ in the parton model reads

$$F_{UU,T}(x) = \sum_a e_a^2 \, x \, f_1^a(x) \, \langle D_1^a \rangle . \tag{6}$$

Since our purpose is to test the approximation (3), we are interested in the x-dependence of the SSA and denote here and in the following averages over z by $\langle \dots \rangle = \int \mathrm{d}z \, (\dots)$.

The tree-level expression for the structure function $F_{UL}^{\sin 2\phi}$ in the numerator is given in terms of a convolution integral in transverse momenta containing $h_{1L}^{\perp}(x, \mathbf{p}_T^2)$ and the Collins function $H_1^{\perp}(z, \mathbf{K}_T^2)$ [2]. Assuming a Gaussian model for the distribution of transverse parton momenta in $h_{1L}^{\perp}(x, \mathbf{p}_T^2)$ and the Collins function one finds

$$F_{UL}^{\sin 2\phi}(x) = \sum_a e_a^2 \, x \, h_{1L}^{\perp(1)a}(x) \, \langle C_{\text{Gauss}} \, H_1^{\perp(1/2)a} \rangle . \tag{7}$$

Here $H_1^{\perp(1/2)}(z) = \int d^2\mathbf{K}_T \, \frac{|\mathbf{K}_T|}{2\,z\,m_h} \, H_1^{\perp}(z, \mathbf{K}_T^2)$. The factor $C_{\text{Gauss}}(z)$ contains the dependence on the Gauss model parameters and is defined in [5].

In order to evaluate the SSA we approximate $h_{1L}^{\perp(1)}(x)$ by means of (3), and use predictions from the chiral quark-soliton model for the transversity $h_1(x)$ [9]. We use information on the Collins effect from [10, 11] and the parameterizations of [12, 13] for $f_1(x)$ and $D_1(z)$.

Fig. 1 shows our results in comparison to the HERMES data [7], which do not exclude the usefullness of the approximation (3), but they do not prove it, either.

3 Conclusions and outlook

In this work we asked whether there are useful approximations among the various, independent TMDs. With the help of the EOMs, the WW-approximation and the neglect of pure twist-3 terms we obtained a new WW-type approximation connecting $h_{1L}^{\perp(1)}(x)$ to $h_1(x)$. To test this approximation we analyzed the SSA $A_{UL}^{\sin 2\phi}$ which is sensitive to $h_{1L}^{\perp(1)}(x)$. The HERMES data on this SSA are compatible with such a WW-type approximation.

More stringent insights on the actual usefulness of this approximation will be provided by COMPASS and CLAS. The former will constrain the small-x region, where the WW-type approximation predicts a small asymmetry $A_{UL}^{\sin 2\phi}$. CLAS will provide complementary information on the valence-x region, where the effect is predicted to be largest [5].

If the usefullness of the WW-type approximation is further confirmed by COMPASS and CLAS, this would imply that in principle it is possible to extract information on transversity from a longitudinally polarized target.

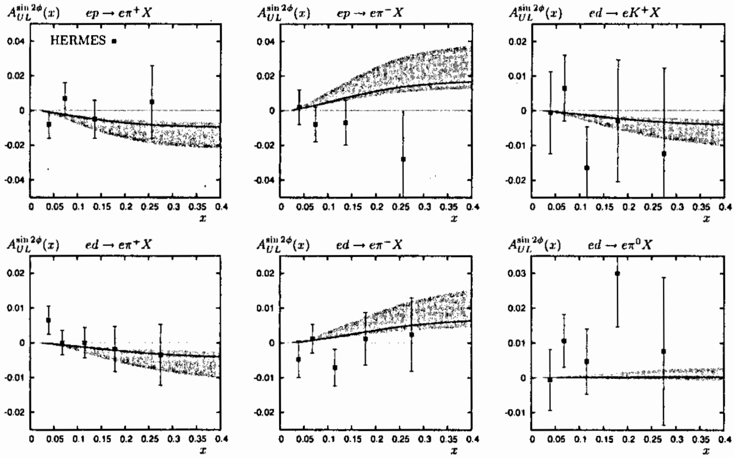

Figure 1: Prediction for azimuthal SSA $A_{UL}^{\sin 2\phi}$ as a function of x on the basis of the WW-type approximation (3). The data are from HERMES [7].

Acknowledgments

The work is partially supported by BMBF (Verbundforschung), German-Russian collaboration (DFG-RFFI) and is part of the EIIIHP project. T. T. is supported by the Cusanuswerk.

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Quantum Field Theory

Effect of a Strong Laser on Spin Precession

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Abstract

The semiclassical dynamics of a charged spin-1/2 particle in an intense electromagnetic plane wave is analyzed beyond the electric dipole approximation and taking into account the leading relativistic corrections to the Pauli equation. It is argued that the adiabatic spin evolution driven by a low intensity radiation changes its character drastically as an intensity of a laser is increasing. Particularly, it is shown that a charged particle exposes a spin flip resonance at a certain pick value of a laser field strength, which is determined by a particle's gyromagnetic ratio.

The problem and result. A good deal of a considerable knowledge on a charged spinning particle interaction with a low intensity laser has been gleaned from the extensive use of the electric dipole approximation [1]. This approximation works perfectly to describe the particle's classical trajectory as well as to understand the adiabatic evolution of spin, represented by the intrinsic angular momentum [2]. With the growing intensity of a radiation different relativistic corrections to the charge motion become relevant [3], [4]. This demands to refuse the electric dipole approximation and to take into account the influence of the magnetic part of the Heaviside-Lorentz force. Entering to this non-dipole region a new physics become tangible. In this context, the present talk aims to report on the manifestation of a such non-dipole physics: *a charged particle's spin flip resonance induced by a strong laser field.*

It is arguably the best to describe the spin-flip resonance in the so-called average rest frame, frame where the mean particle's velocity vanishes. In this frame, as our calculations show, the probability to flip for a spin, that is initially polarised along the direction of propagation of the circularly polarised monochromatic plane wave, is given by an analog of the well-known formula from the Rabi magnetic resonance problem [5]:

$$\mathcal{P}_{\downarrow\uparrow} = \mathcal{A}_{\downarrow\uparrow}(\eta)\,\sin^2(\omega_S\, t)\,, \qquad \omega_S := \frac{\omega_L|1-g|}{8}\sqrt{\kappa^2\eta^2+(\eta^2-\eta_*^2)^2}\,, \tag{1}$$

The frequency ω_S differs from a laser circular frequency ω_L and depends nonlinearly on a particle's gyromagnetic ratio g, as well as on a laser field strength parameter [6]:

$$\eta^2 = -2\,\frac{e^2}{m^2c^4}\,\langle A^2\rangle\,, \qquad \langle\cdots\rangle - \text{time average}, \quad A - \text{a laser gauge potential}. \tag{2}$$

The flipping amplitude $\mathcal{A}_{\uparrow\downarrow}(\eta)$ in (1) has the following resonance form

$$\mathcal{A}_{\downarrow\uparrow}(\eta) = \frac{\kappa^2\eta^2}{\kappa^2\eta^2+(\eta^2-\eta_*^2)^2}\,, \qquad \kappa^2 := \frac{2\,g^2}{(1-g)^2}\,, \qquad \eta_*^2 := \frac{4}{g-1}\,. \tag{3}$$

Sketch of the calculations. To get the above results the recently elaborated method [7] is extended from the classical to the quantum case. The conventional semiclassical approach attitude, when a charged particle motion in a given electromagnetic background is studied classically within the non-relativistic Hamilton-Jacobi theory, is adopted. At the same time, the spin evolution is treated quantum mechanically, as required by the spin nature, using the Pauli equations with the leading relativistic corrections. The spin-radiation interaction is encoded in the effective spatially homogeneous magnetic field configuration, which is determined by the geometry of a particle's classical trajectory. The described approximation is formulated mathematically as follows. The laser radiation is modelled by the *elliptically polarized monochromatic plane wave* propagated along the z-axis

$$A^{\mu} := a\Big(0\,,\ \varepsilon\cos(\omega_L\xi)\,,\ \sqrt{1-\varepsilon^2}\sin(\omega_L\xi)\,,\ 0\Big)\,, \qquad \xi = t - \frac{z}{c}\,, \tag{4}$$

where $0 \leq \varepsilon \leq 1$ is the light polarisation parameter, and the constant a is related to the laser field strength (2) , $\eta^2 = e^2a^2/m^2c^4$. A charged spin-1/2 particle is in a pure quantum state Ψ admitting the semiclassical charge & spin decomposition,

$$|\Psi\rangle = \sum_{i=0,1}\sum_{\alpha=\pm} c_{\alpha,i}\,|\psi_\alpha\rangle \otimes |\chi_i\rangle\,. \tag{5}$$

Two states, $|\psi_\pm\rangle$, [1] are the linearly independent WKB solutions to the Schrödinger equation for a charged spinless particle moving in the background (4). According to the semiclassical calculations the spin state vectors $|\chi\rangle$ in the decomposition (5) satisfy the spin evolution equation:

$$i\frac{\mathrm{d}}{\mathrm{d}t}\,|\chi\rangle = -\frac{ge}{4mc}\,\mathbf{B}'(t)\cdot\boldsymbol{\sigma}\,|\chi\rangle\,, \tag{6}$$

where the spatially homogeneous magnetic field $\mathbf{B}'(t)$ is accounted for a laser filed coupling to a spin moving in the laboratory frame with the velocity $\boldsymbol{v}$ and acceleration $\boldsymbol{a}$,

$$\mathbf{B}'(t) := \left(\boldsymbol{B} - \frac{1}{c}\,\boldsymbol{v}\times\boldsymbol{E}\right) + \frac{m}{egc}\,\boldsymbol{v}\times\boldsymbol{a}\,. \tag{7}$$

Here the term in parenthesis is magnetic field seen in the particle's instantaneous rest frame and evaluated along the particle's classical orbit. The last contribution in (7) corresponds to the leading part of the so-called Thomas precession correction due to the non-vanishing curvature of a particle's trajectory [2].

Analysis of the equations begins with the derivation of the exact solution to the classical non-relativistic Hamilton-Jacobi equation for spinless particle moving in the electromagnetic background (4). Solving this Hamilton-Jacobi problem [7] one determine both, the WKB solution to the Schrödinger equation and the effective magnetic field (7). We assume below that the particle's classical trajectory $\boldsymbol{x}(t)$ fulfills the initial condition $\boldsymbol{x}(0) = 0$ and fix also the frame, where the average of a particle's velocity component, orthogonal to the wave propagation direction vanishes, $< \boldsymbol{v}_\perp >= 0$. To find the effective magnetic field we owe from [7] the expression for a particle's velocity

$$\boldsymbol{v} = \Big[-c\eta\varepsilon\,\mathrm{cn}\,(\omega_L' t,\,\mu)\,,\ -c\eta\sqrt{1-\varepsilon^2}\,\mathrm{sn}\,(\omega_L' t,\,\mu)\,,\ c - c(1-\beta_z)\,\mathrm{dn}\,(\omega_L' t,\,\mu)\Big]\,, \tag{8}$$

[1] To simplify expressions, the initial state is assumed to have only one nonzero coefficient, $c_{+,0}$. Note that, the unit normalization condition on the WKB wave function fixes this coefficient $\pi c_{+,0}^2 = 2m\,\omega_{\mathrm{P}}$, ω_{p} – a particle's fundamental frequency.

as well as the expression for its z-coordinate: $z(t) = ct - \frac{c}{\omega_L} \mathrm{am}(\omega'_L t,\, \mu)$. The argument of the elliptic Jacobian functions and the Jacobian amplitude function, $\mathrm{am}(u,\, \mu)$, is the laboratory frame time t, scaled by the non-relativistically Doppler shifted laser frequency $\omega'_L = \gamma_z \omega_L$, $\gamma_z = 1 - v_z(0)/c$ and the modulus μ is $\gamma_z^2 \mu^2 = \eta^2(1 - 2\,\varepsilon^2)$.[2] Using this solution the exact expression for the effective magnetic field $\mathbf{B}'(t)$ can be found:

$$\mathbf{B}' = \frac{a\omega'_L}{gc}\left(\sqrt{1-\varepsilon^2}\left[(g+1)\mathrm{dn} - \gamma_z\right]\mathrm{cn}, \varepsilon\left[(g+1)\mathrm{dn} - \gamma_z(1-\mu^2)\right]\mathrm{sn}, \varepsilon\sqrt{1-\varepsilon^2}\left[\mathrm{dn} - g\gamma_z^{-1}\right]\right).$$

The resonant oscillations.– In the average rest frame, $<\boldsymbol{v}> = 0$, when laser beam is circularly polarised, $\varepsilon^2 = 1/2$, the expression for the effective magnetic field $\mathbf{B}'$ simplifies to the constant magnitude field:

$$\mathbf{B}'(t) = |\mathbf{B}'|\, \boldsymbol{n}(t)\,, \qquad |\mathbf{B}'| := \frac{a\omega_L|1-g|}{2gc}\sqrt{\kappa^2 + \eta^2}\,, \tag{9}$$

aligned the unit time dependent vector $\boldsymbol{n}(t) := (\sin\theta\cos\omega_L t,\ \sin\theta\sin\omega_L t,\ \cos\theta)$. The effective magnetic field (9) rotates with the frequency ω_L about the axis inclined with respect to the field. The inclination angle θ is determined from the relation: $\eta\tan\theta = \kappa$.

Therefore, *the effective laser-spin interaction for the circular polarised radiation is precisely the famous rotated magnetic field describing the nuclear magnetic resonance phenomenon !* Having in mind this observation one can use the well-known exact Rabi solution [5] to find the semiclassical evolution of spin-1/2 particle. Particularly, a straightforward calculations lead to the expressions (1) and (3) for the spin flipping probability announced at the beginning of this report.

Conclusion. In the present talk the non-dipole effect of a strong laser on the spin of a charged particle was described quantum mechanically, while the evolution of position and momentum of a particle itself were treated according to the classical Newton equations with complete Heaviside-Lorentz force. The derived results indicate a very different spin physics in a high intensity laser field versus to a low intensity adiabatic regime.

Acknowledgment.–The research was supported in part by the Georgian National Science Foundation under Grant No. GNSF/ST06/4-050, the Russian Foundation for Basic Research Grant No. 08-01-00660 and by the Ministry of Education and Science of the Russian Federation Grant No. 1027.2008.2.

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Cayley - Klein Topological Quantum Field Theory

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Abstract

New applications of categorical methods are connected with new additional structures on categories. One of such structures, the double category, is considered in this article. It is shown that double categories exist in the Cayley-Klein topological quantum field theories.

1 Double Category

A double category D consists of the following:

(1) A category D_0 of objects $Obj(D_0)$ and morphisms $Mor(D_0)$ of 0-level.

(2) A category D_1 of objects $Obj(D_1)$ of 1-level and morphisms $Mor(D_1)$ of 2-level.

(3) Two functors $d, r : D_1 \rightrightarrows D_0$.

(4) A composition functor

$$* : D_1 \times_{D_0} D_1 \to D_1$$

where the bundle product is defined by commutative diagram

$$\begin{array}{ccc} D_1 \times_{D_0} D_1 & \xrightarrow{\pi_2} & D_1 \\ \pi_1 \downarrow & & \downarrow d \\ D_1 & \xrightarrow{r} & D_0 \end{array}$$

(5) A unit functor $ID : D_0 \to D_1$, which is a section of d, r.

The above data is subject to **Associativity Axiom** and **Unit Axiom.** If both of them are fulfilled only up to equivalence then the double category is called a **weak** double category, if they are fulfilled strictly then it is a **strong** double category.

Here we see that for two objects $A, B \in Obj(D_0)$ there are 0-level morphisms $D_0(A, B)$ which we note by ordinary arrows $f : A \to B$, and 1-level morphisms $D_{(1)}(A, B)$ which we note by the arrows $\xi : A \Rightarrow B$, for $A = d(\xi)$ and $B = r(\xi)$. So with a 2-level morphism $\alpha : \xi \to \xi'$, where $\xi : A \Rightarrow B$ and $\xi' : A' \Rightarrow B'$ we can associate the following diagram

$$\begin{array}{ccccc} A & \overset{\xi}{\Rightarrow} & B & & \xi \\ d(\alpha) \downarrow & & \downarrow r(\alpha) & \longmapsto & \downarrow \alpha \\ A' & \overset{\xi'}{\Rightarrow} & B' & & \xi' \end{array}$$

and arrow $\alpha : d(\alpha) \Rightarrow r(\alpha)$

The composition on 2-level associated with the diagram

$$\begin{array}{ccccc} A & \overset{\xi}{\Rightarrow} & B & & \xi \\ d(\alpha) \downarrow & & \downarrow r(\alpha) & & \downarrow \alpha \\ A' & \overset{\xi'}{\Rightarrow} & B' & \longmapsto & \xi' \\ d(\alpha') \downarrow & & \downarrow r(\alpha') & & \downarrow \alpha' \\ A'' & \overset{\xi''}{\Rightarrow} & B'' & & \xi'' \end{array}$$

2 Cobordism and Double Categories

Let M_d be the category of oriented compact d-dimensional smooth manifolds (with boundary) and piecewise smooth maps (the sense of the condition we do not define more exactly here; this may be such continuous maps $f : M \to Y$ that are smooth on a dense open subset $U_f \subset M$), let CM_d be its subcategory of closed (with empty boundary) manifolds and smooth maps, $CM_d \subset M_d$.

There are the following functors:

(1) Disjoint union

$$\sqcup : M_d \times M_d \to M_d : (X, Y) \mapsto X \sqcup Y.$$

(2) Changing of the orientation of manifolds on opposite

$$(-) : M_d \to M_d : X \mapsto -X.$$

(3) Boundary operator

$$\partial : M_{d+1} \to CM_d : X \mapsto \partial X.$$

(4) Multiplication on the unit segment $I = [0, 1]$

$$I \times \ . : CM_d \to M_{d+1} : X \mapsto I \times X.$$

Now we define a double category $\mathbf{C(d)}$ with

(1) $\mathbf{C(d)_0 = CM_d}$.

(2) 1-level morphisms $\mathbf{C(d)_{(1)}(X, X')}$ is a set of pairs (Y, f) where Z is oriented compact $(d+1)$-dimensional smooth manifold with the boundary ∂Y and f is an diffeomorphism

$$f : (-X) \sqcup X' \to \partial Y,$$

where $\sqcup$ notes the disjoint union of $-X$ and X'. Thus we write $(Y, f) : X \Rightarrow X'$.

(3) The composition of $(Y, f) : X \Rightarrow X'$ and $(Y', f') : X' \Rightarrow X''$ is the morphism

$$(Y \sqcup_{X'} Y', (f|_X) \sqcup (f'|_{X'})) : X \Rightarrow X'',$$

where $(Y \sqcup_{X'} Y')$ denotes the union $(Y \sqcup Y')$ after identification of each point $f(y) \in f(Y)$ with the point $f'(y) \in f'(Y)$ for all $y \in Y$ and smoothing this topological manifold.

(4) The 1-level identical morphism ID_X is $(X \times [0;1], id_{(-X)\sqcup X})$, because $\partial(X \times [0;1]) = (-X) \sqcup X$.

(5) 2-level morphisms of $\mathbf{C(d)_1}(\xi, \xi')$ from $\xi = (Y, f : X' \sqcup (-X) \to \partial Y) : X \Rightarrow X'$ to $\xi' = (Y', f' : X'' \sqcup (-X') \to \partial Y') : X' \Rightarrow X''$ are such triples of smooth maps (f_1, f_2, f_3) that the following diagram is commutative

$$\begin{array}{ccccc} (-X) \sqcup X' & \xrightarrow{f} & \partial Y & \subset Y \\ \downarrow f_1 \sqcup f_2 & & & \downarrow f_3 \\ (-X') \sqcup X'' & \xrightarrow{f'} & \partial Y' & \subset Y' \end{array}$$

It easy to see that functors $\cup$ and $(-)$ may be expanded to double category functors

$$\cup : \mathbf{C(d)} \to \mathbf{C(d)}, \quad (-) : \mathbf{C(d)} \to \mathbf{C(d)}^\circ$$

and $(-)$ is an equivalence of the double categories.

3 Cayley-Klein Topological Quantum Field Theory

Cayley-Klein topological quantum field theory is a functor Z from the category $CM(d)$ of d-dimensional manifolds to the category of CK of (usually Hermitian) finite dimensional Cayley-Klein spaces. Really, the **functor** Z is a functor between double categories.

Thus, Cayley-Klein topological quantum field theory in dimension d is a functor

$$Z : \mathbf{C(d)} \to \mathbf{Morph(CK)},$$

between double categories such that:

(1) the disjoint union in $\mathbf{C(d)}$ go to the tensor product

$$\cup \quad \mapsto \otimes,$$

where $(\_)^* : CK \to CK^\circ$ is dualization of Cayley-Klein spaces.

(2) changing of orientation in $\mathbf{C(d)}_0$ go to dualization

$$(-) \mapsto (.)^*$$

Thus, as consequence of double categorical functorial properties, we get

(1) for each compact closed oriented smooth d-dimensional manifold $X \in Obj(\mathbf{C(d)}_0)$ the value of the functor $Z(X)$ is a finite dimensional vector space over the field $\mathbf{C}$ of the complex numbers (usually with Hermitian metric),

(2) for each $(Y, f) : X \Rightarrow X'$ from $Obj(\mathbf{C(d)}_1)$ the value of the functor $Z(Y, f)$ is a homomorphism $Z(X) \to Z(X')$ of (Hermitian) vector spaces,

and the following well known axioms of topological quantum field theory are satisfied:

A(1) (involutivity) $Z(-X) = Z(X)^*$, where $-X$ denotes the manifold with opposite orientation, and $*$ denotes the dual vector space.

A(2) (multiplicativity) $Z(X \cup X') = Z(X) \otimes Z(X')$, where $\cup$ denotes disconnected union of manifolds.

A(3) (associativity) For the composition $(Y'', f'') = (Y, f) * (Y', f')$ of cobordisms must be

$$Z(Y'', f'') = Z(Y', f') \circ Z(Y, f) \in Hom_{\mathbf{C}}(Z(X), Z(X'')).$$

(Usually the identifications

$$Z(X' - X) \cong Z(X)^* \otimes Z(X') \cong Hom_{\mathbf{C}}(Z(X), Z(X'))$$

allow us to identify $Z(Y, f)$ with the element $Z(Y, f) \in Z(\partial Y)$.

A(4) For the initial object $\emptyset \in Obj(\mathbf{C(d)}_0)$ $\quad \mathbf{Z}(\emptyset) = \mathbf{C}$.

A(5) (trivial homotopy condition) $Z(X \times [0, 1]) = id_{Z(X)}$.

Physics at the Planck Scales and Unification of Interactions

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1 Introduction

The main problems of modern physics include in particular:
1. Emergence of quantum mechanics.
2. Ultraviolet divergences in local quantum field theories (QFT).
3. Quantum theory of gravitation.
4. Unification of all interactions (including gravity).
5. Emergence of supersymmetry (SUSY).
6. Dark matter.
7. Dark energy.

There is a considerable activity in direction of the first problem [1-5]. Supersymmetry softens the problem of divergences. Quantization of gravity is still a very serious problem. The problem of unification of all interactions also is not yet solved. The problem of emergence of SUSY is open. Recently there appeared new difficult problems: dark matter and dark energy (their nature, theory). We give no references to the points 2 – 7 because they either are well known, or even not formulated (the fifth point).

The superstring theory claims to become a theory of everything. But it meets difficulties, e.g. incorporation of strings into QFT [6]. Indeed, excitations of strings contain graviton (excitation of space), but strings move in curved space. Does it mean that there are two kinds of gravitons? The same question about photons: there is electromagnetic field and there are massless spin 1 excitations of strings.

2 Planck scales. Model

It turns out that modelling of 3d-space by a bosonic strings network put into a thermal bath [3-5] solves most of these problems. In the large scale limit one obtains the 3d-space. Excitations of the structure manifest themselves as matter. They are described by fields. The latter satisfy relativistic equations of motion (appearance of relativity, the Minkowski space M^4). Evolution of the nonequilibrium distributions of a harmonic oscillator (and a bosonic string) in a thermal bath is described by probability amplitudes [4,5]. In this approach there automatically appear the Planck constant $\hbar$, the Fock space, the Schroedinger equation, operators. A bosonic string in a thermal bath becomes a helix which models the Ramond-Neveu-Schwarz (RNS) superstring [4], and in effect we obtain a 3d-network made of superstrings. Its excitations are described by quantum mechanics and contain all the known fields. The bosonic strings are taken as discrete ordered sets of harmonic oscillators, so there is no divergences in this theory. Besides, it solves the problem of emergence of space [6]. Thus, in this approach both the 3d-space and relativistic quantum mechanics emerge in the natural way. Because of (global) supersymmetry the ground state energy is zero and there is no problem of vacuum energy (one of cosmological problems: in present theory the vacuum density of fields exceeds the observed value approximately by 120 orders of magnitude). At the same time this theory inevitably leads to non-zero cosmological constant: $2\Lambda = 1/t_r^2$, t_r being the relaxation time of the nonequilibrium distribution [4].

In principle, there is also place for dark matter. We identify the "normal" matter with the "second level excitations" $< X^\mu X^\nu >$ (gravitational and vector gauge fields, see Sec. 3) and fermions $< S^\mu >$ of the RNS string, but there are a lot of other excitations, for example, $< X^\mu >$. We conclude that this approach looks very promising and gives clue to solutions of the most serious problems of modern physics.

3 The Kaluza-Klein-Mandel-Fock model

The proposed model of the 3d-space leads to unification of gravitation and Yang-Mills theory in the framework of the generalized Kaluza-Klein-Mandel-Fock (KKMF) approach [7-10]. It is remarkable that there appears no problem of compactification.

The subtle point is the problem of dimension of the embedding space D. Bosonic string theory is formulated in the space-time with $D = 26$. There are two unphysical variables among X^μ (X^0 and $\vec{X}_{||}$), thus we have 24 physical components. This is exactly the dimension of the group $SU(5)$ adjoint representation. But in a thermal bath the bosonic string becomes a helix [4], i.e. (approximately) the superstring with the critical dimension $D = 10$ and 8 (=10 - 2) physical degrees of freedom (just the dimension of the group $SU(3)$ adjoint representation). Of course, this is true only for "ideal" RNS-string. Here we have a helix which needs extra two dimensions (taken from 16=26 - 10 ones). Thus, the model is effectively formulated in (11+1) space-time. Now application of the KKMF theory leads to the 4d gravitational equations together with those for the $SU(3)$ gauge vector fields ("colour"). Notice that application of the KKMF approach to strings (not to network in (11+1) space-time) gives "gravitation" in (1+1).

Now, one should define the metric tensor. In M^D we introduce the coordinates $X^\mu(x^i, x^a)$, $i = 0, 1, 2, 3, a = 4, ..., D - 1$. For any cell in the network (e.g. a cube) one always can find links composing a closed string. It is known that spin 2 massless excitations of the latter correspond to the tensor $X^\mu(\tau,\sigma)X^\nu(\tau,\sigma)$ (σ parametrizes the string). We define the metric tensor associated with the cell (which represents a "point" in the emerging space) with the center $\mathbf{n}(n_1, n_2, n_3)$, n_i — entire numbers, by the average

$$g^{\mu\nu}_{(\tau,\mathbf{n})} = < X^\mu X^\nu > \equiv \frac{1}{L_c l^2} \oint_{\mathbf{n}} d\sigma X^\mu(\tau,\sigma) X^\nu(\tau,\sigma), \tag{1}$$

where l characterizes the size of the cell, L_c is the length of the integration contour. It is a nonlocal object in the string theory. But in the continuous limit $n_i l_i \rightarrow x^i$ (l_i is the distance between the centers of the neighbor cells along the axis i, $n_i \rightarrow \infty$, $l_i \rightarrow 0$, $i = 1, 2, 3$) we obtain the local tensor $g^{\mu\nu}(x)$, $x = (x^0, x^1, x^2, x^3), x^0 = \tau$. Evidently, outside the 4d pseudoeuclidean manifold it is defined by the equations

$$\frac{\partial g_{\mu\nu}}{\partial x^a} = 0, \quad \frac{\partial g_{ab}}{\partial x^\mu} = 0, \quad a, b = 4, ..., D - 1. \tag{2}$$

The covariant second order equations of motion for $g_{\mu\nu}$ follow from the action (in proper units)

$$S = \frac{-1}{2\kappa^2}\int d^D x\sqrt{|g_D|}R_D, \qquad g_D = detg_{\mu\nu}, \qquad \kappa^2 = 8\pi G, \tag{3}$$

where R_D is the scalar curvature of D-dimensional space-time, G is the gravitational constant, $l \sim \kappa$, and

$$g_{\mu\nu} = \begin{pmatrix} g_{ij} + g_{ab}A_i^a A_j^b & A_i^a g_{ab} \\ A_j^b g_{ba} & g_{ab} \end{pmatrix}. \tag{4}$$

Conditions (2) for $D = 12$ give in the standard way [11,12] the action

$$S = \frac{-V^8}{2\kappa^2}\int d^4 x\sqrt{|g|}(R - \frac{1}{4}F^2). \tag{5}$$

Here V^8 is the (infinite) volume of 8-dimensional co-space, $g = detg_{ij}$, R is the scalar curvature of (3+1) physical space-time in the large scale limit, $\hat{F}^2 = g_{ab}F^a_{ki}F^b_{lj}g^{kl}g^{ij}$, $\hat{F}_{ki} = [D_k, D_i]$, D_k is covariant derivative in the $SU(3)$ Yang-Mills theory.

4 Dark matter

There are left the cell excitations $< S^\mu >$ and $< X^\mu >$ (notice that $< 1 >= 1/l^2$, and $dim < X^\mu >= diml^{-1}$). The former present fermions. Their incorporation into the formalism is a separate difficult problem [13]. As for $< X^\mu >$, they give vector $A^i =< X^i >$ and scalar $\phi^a =< X^a >$ fields. These fields are candidates for dark matter. It is useful to "disentangle" operators X^μ and $< X^\mu >$. Introducing $\tilde{X}^\mu = X^\mu - l^2 < X^\mu >$, $< \tilde{X}^\mu >= 0$, we define instead of (1)

$$\tilde{g}^{\mu\nu} =< \tilde{X}^\mu >< \tilde{X}^\nu >= g^{\mu\nu} - l^2 < X^\mu >< X^\nu >\equiv g^{\mu\nu} - l^2 A^\mu A^\nu. \tag{6}$$

The tensor and vector fields $g^{\mu\nu}$, A^μ appear in the way specific for the Bekenstein $TeVeS$ theory [14]. The latter was invented to describe the dark matter effects. It leads to modification of the Newton potential what is of importance in connection with the recent astrophysical observations [15] and Pioneer spacecraft [16] measurements.

We conclude that the proposed model allows to solve a number of the most difficult problems of modern theoretical physics.

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Radiative Fermion Masses at MSSM

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Abstract

We consider the mass generation for the usual quarks and leptons in the Minimal Supersimmetric Standard Model (MSSM). The masses of the top, the bottom, the charm, the tau and the muon are given at the tree level. All the other quarks and the electron get their masses at the one loop level. This symmetry and the assumption of alignment between fermions and sfermions allow us to avoid FCNC problems. Here a more general hypothesis of flavor mixing in the sfermion sector of MSSM is considered and we show that the s quark is heavier than u, d quarks due to different content of sfermions contributions. Our results are in agreement with the experimental constraint on the values of sfermions masses.

1 Introduction

The fact that the lighter fermions may arise only as higher-order radiative effect was suggest by S. Weinberg [1]. Later L. Ibañez show that, if supersymmetry is spountaneously broken we can generate tiny fermion masses radiatively [2]. In order to restrict this mechanism to the first family a discrete symmetry is applied into SUSY models in Refs. [3, 4]. From the analysis performed by Ferrandis [5] the radiative mechanism for the fermions masses is allowed through sfermion–gaugino loops and the observed flavor physics is obtained if "the supersymmetry breaking terms receive small corrections, which violate the symmetry of the superpotential" and are responsible for the observed flavor physics. Later, the presence of soft supersymmetry breaking terms allows for the radiative generation of quark and charged lepton masses throught sfermion-gaugino loops.

The outline of this work is as follows. The section 2 describes how the additional discrete symmetry $\mathcal{Z}_2'$ is introduced into the framework of MSSM in order to prevent the light quarks and the electron to acquire mass at tree level. The radiative mechanism is described in section 3, where we remove the assumption of alignment between quark and squarks sectors. From these results, we can explain why the quarks u and d are lighter than the s quark. Our conclusions are found in the last section.

2 MSSM and $\mathcal{Z}_2'$ Symmetry.

The fermion mass comes from the following terms of the superpotential

$$W = -\left(y^l_{ab} L_a H_1 l^c_b + y^d_{ij} Q_i H_1 d^c_j + y^u_{ij} Q_i H_2 u^c_j + h.c.\right) + \dots, \quad (1)$$

where y^l_{ab}, y^d_{ij} and y^u_{ij} are the yukawa couplings of Higgs with leptons families, "down" sector quarks and "up" sector quarks respectively and ...stands for other terms which we are not concerned here. The family indices a and i run over e, μ, τ and $1, 2, 3$, respectively.

The key feature of this kind of mechanism is to allow only the quarks c, b, t, and the leptons μ and τ have Yukawa couplings to the Higgs bosons. It means to prevent u, d, s and e from picking up tree-level masses, all one needs to do at this stage is to impose the following $\mathcal{Z}_2'$ symmetry on the Lagrangian

$$\widehat{d}^c_2 \longrightarrow -\widehat{d}^c_2,\ \widehat{d}^c_3 \longrightarrow -\widehat{d}^c_3,\ \widehat{u}^c_3 \longrightarrow -\widehat{u}^c_3,\ \widehat{l}^c_3 \longrightarrow -\widehat{l}^c_3, \tag{2}$$

the others superfields are even under this symmetry.

Thus the quarks u, d, s and the electron come about be massless due to $\mathcal{Z}_2'$ symmetry [6]. This point of view has implications for nuclear physics. Due to u, d and s quarks are lights one is allowed to build an effective field theory as an expansion on masses of light quarks of the underlying theory. The Chiral Perturbation Theory (ChPT) is the prototype of this approach. It respects all principles of the underlying theory but with effective degrees of freedom instead of quarks degrees of freedom.

Using the $\mathcal{Z}_2'$ symmetry we can also, get an expression to the CKM matrix as follows [6]:

$$V_{CKM} = E^{u\dagger}_L E^d_L = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{3}$$

We can conclude that the $\mathcal{Z}_2'$ symmetry in the MSSM can explain the lower masses of the u, d and s quarks and also gives a hint about the mixing angles of quarks.

3 Radiative Mechanism to the fermions masses

The discrete symmetry $\mathcal{Z}_2'$ has to be broken in order to allow the generation of fermions masses by radiative corrections. By another hand, Supersymmetric non-renormalization theorem guarantee that corrections to the relations arising from these constraints are very small, even if the discrete symmetry, defined at Eq.(2), is broken.

The behavior of scalar components of chiral superfields under $\mathcal{Z}_2'$ symmetry are given by:

$$\begin{aligned} \tilde{d}^c_2 &\longrightarrow -\tilde{d}^c_2,\ d^c_2 \longrightarrow -d^c_2,\ \tilde{d}^c_3 \longrightarrow -\tilde{d}^c_3,\ d^c_3 \longrightarrow -d^c_3, \\ \tilde{u}^c_3 &\longrightarrow -\tilde{u}^c_3,\ u^c_3 \longrightarrow -u^c_3,\ \tilde{l}^c_3 \longrightarrow -\tilde{l}^c_3,\ l^c_3 \longrightarrow -l^c_3, \end{aligned} \tag{4}$$

while all other fields of the model are even. It worth noting that the $\mathcal{Z}_2'$ symmetry has the same role as in the fermion sector: it forbids the flavor mixing between the third family and the other two families of sfermions [7].

Therefore the $\mathcal{Z}_2'$ symmetry help us to keep under control the dangerous FCNC problems and we still obtain the mass hierarchy pattern without additional assumptions. We can rewrite the light fermion mass expressions as follow [7]:

$$M_u = \frac{g_s^2 m_{\tilde{g}} \sin(2\theta_{\tilde{u}})}{16\pi^4} \sum_{i=1}^{2} \frac{m^2_{\tilde{u}_i}}{(m^2_{\tilde{u}_i} - m^2_{\tilde{g}})} \ln\left(\frac{m^2_{\tilde{u}_i}}{m^2_{\tilde{g}}}\right), M_d = \frac{g_s^2 m_{\tilde{g}} \sin(2\theta_{\tilde{d}})}{16\pi^4} \sum_{i=1}^{2} \frac{m^2_{\tilde{d}_i}}{(m^2_{\tilde{d}_i} - m^2_{\tilde{g}})} \ln\left(\frac{m^2_{\tilde{d}_i}}{m^2_{\tilde{g}}}\right),$$

$$M_e = \frac{g'^2 m' \sin(2\theta_{\tilde{e}})}{16\pi^4} \sum_{\imath=1}^{2} \frac{m^2_{\tilde{e}_\imath}}{(m^2_{\tilde{e}_\imath} - m'^2)} \ln\left(\frac{m^2_{\tilde{e}_\imath}}{m'^2}\right),$$

$$M_s = \frac{g_s^2 m_{\tilde{g}}}{16\pi^4} \sum_{\alpha=1}^{2} \left\{ R^{(d)}_{1\alpha} R^{(d)}_{2\alpha} \frac{m^2_{\tilde{g}}}{(m^2_{\tilde{g}} - m^2_{\tilde{d}_\alpha})} \ln\left(\frac{m^2_{\tilde{g}}}{m^2_{\tilde{d}_\alpha}}\right) + R^{(d)}_{1\alpha+2} R^{(d)}_{2\alpha+2} \frac{m^2_{\tilde{g}}}{(m^2_{\tilde{g}} - m^2_{\tilde{d}_{\alpha+2}})} \ln\left(\frac{m^2_{\tilde{g}}}{m^2_{\tilde{d}_{\alpha+2}}}\right) \right.$$

$$+ \frac{R^{(d)}_{1\alpha} R^{(d)}_{2\alpha+2}}{(m^2_{\tilde{d}_\alpha} - m^2_{\tilde{d}_{\alpha+2}})(m^2_{\tilde{g}} - m^2_{\tilde{d}_\alpha})(m^2_{\tilde{d}_{\alpha+2}} - m^2_{\tilde{g}})} \left(\delta^d_{\alpha\alpha+2}\right)_{LR} M^2_{SUSY} \left[m^2_{\tilde{d}_\alpha} m^2_{\tilde{d}_{\alpha+2}} \ln\left(\frac{m^2_{\tilde{d}_\alpha}}{m^2_{\tilde{d}_{\alpha+2}}}\right) \right.$$

$$\left.\left. + \; m^2_{\tilde{d}_\alpha} m^2_{\tilde{g}} \ln\left(\frac{m^2_{\tilde{g}}}{m^2_{\tilde{d}_\alpha}}\right) + m^2_{\tilde{d}_{\alpha+2}} m^2_{\tilde{g}} \ln\left(\frac{m^2_{\tilde{d}_{\alpha+2}}}{m^2_{\tilde{g}}}\right) \right] \right\}. \qquad (5)$$

It is important to emphasize that the first two contribution to the mass of c quark are the same as those in the mass expressions of u and d quarks. The third contribution came from the flavor non-diagonal sfermion mass matrix contribution.

4 Conclusions

We show that we can introduce a discrete symmetry $\mathcal{Z}'_2$ in MSSM and in both SUSYLR in order to explain the lower masses of the quarks u, d and s and of the electron while a consistent picture with experimental data of CKM matrix is obtained.

We, also, showed that the extension of $\mathcal{Z}'_2$ symmetry to the squarks sector provide us with a natural mechanism for explaining the chiral mass hierarchy pattern and also the mass gap between strange and non-strange quarks. It means, we can give a reasonable explanation of the mass gap between s quark and non strange quarks, even in the case of all squarks are degenerate in mass at low energy. It is due to the fact that the s quark can couple with two families of squarks while the u and d quarks can couple only with one family.

Acknowledgments

This work is supported by Conselho Nacional de Ciência e Tecnologia (CNPq) under the contract number 309564/2006-9. The participation on the conference was supported by CNPq under the contract number 450263/2008-8.

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Quantum effects in QED under the condition of Lorentz and CPT invariance violation

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Abstract

Working within the context of Lorentz-violating *extended electrodynamics* (a part of the *Standard Model Extension, SME*), we investigate the following physical problems: Chern-Simons term (CS) generation by the fermion loop in the Lorentz-violating background, electron bound state in the Coulomb potential (hydrogen-like atom), electron in a constant homogeneous magnetic field and synchrotron radiation (SR).

1 Introduction

1.1 The Standard Model Extension (SME)

The SME stems from a spontaneous breaking in a covariant FT, hence Lorentz-violating terms in the *Lagrangian of the SME* are singlets under observer Lorentz transformations. So, the field indices should match those of the coupling coefficients $\sim$ some power of $m_W/M_P = 10^{-17}$. There emerges the *extended quantum electrodynamics* from the lepton-gauge sector of the SME. Its Lagrangian can be written in the following general form:

$$\mathcal{L}^{\mathrm{QEDExtension}} = \mathcal{L}^{\mathrm{QED}} + \mathcal{L}_{\mathrm{lepton}} + \mathcal{L}_{\mathrm{photon}}, \tag{1}$$

where $\mathcal{L}^{\mathrm{QED}}$ is the well-known *QED Lagrangian*:

$$\mathcal{L}^{\mathrm{QED}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\overline{\psi}i\gamma^{\mu}\overleftrightarrow{D}_{\mu}\psi - \overline{\psi}m\psi, \tag{2}$$

$\mathcal{L}_{\mathrm{lepton}}$, $\mathcal{L}_{\mathrm{photon}}$ are the terms describing the Lorentz non-invariant interactions in the electron and the photon sectors of the theory respectively. They can be decomposed into a sum of CPT-even and odd parts, among which the following are of interest in our investigation:

$$\mathcal{L}^{\mathrm{CPT-odd}}_{\mathrm{lepton}} = -\overline{\psi}\gamma^{\mu}a_{\mu}\psi - \overline{\psi}\gamma^{5}\gamma^{\mu}b_{\mu}\psi, \tag{3}$$

$$\mathcal{L}^{\mathrm{CPT-odd}}_{\mathrm{photon}} = \frac{1}{2}(k_{AF})^{\alpha}\varepsilon_{\alpha\beta\mu\nu}A^{\beta}F^{\mu\nu}. \tag{4}$$

The SME action is covariant under observer Lorentz transformations. Particle Lorentz transformations leave the coupling coefficients (condensates) unaffected. This provides a 4-dimensional directionality to the spacetime in any fixed inertial frame.

The aim of the investigations is to establish *possible signals of Lorentz violation*, place *bounds on the associated couplings.* High-precision tests have been carried out including those using anomalous magnetic moments, charge-to-mass ratios for particles and antiparticles confined in a Penning trap, etc., that have shown no evidence for the existence of Lorentz violation; all SME-couplings are thus tightly constrained ($\lesssim 10^{-22} - 10^{-33}$GeV), except for the few ones, e.g. the zero component of the axial vector b_μ:

$$|b_0| \lesssim 10^{-2}\text{eV}, \quad |\boldsymbol{b}| \lesssim 10^{-19}\text{eV}. \tag{5}$$

1.2 Extended electrodynamics

We use the minimal CPT-odd form of the extended QED with electrons and photons:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\left(i\gamma^\mu D_\mu - m_e - b_\mu\gamma_5\gamma^\mu\right)\psi. \tag{6}$$

Electron charge $q_e = -e < 0$, $\alpha = e^2/4\pi$; $D_\mu = \partial_\mu - ieA_\mu$, $\gamma_5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$, and b^μ is a constant axial vector condensate that introduces a Lorentz-violating CPT-odd interaction.

The possible dynamical origin of b_μ still remains a subject of an open question. In particular, it can be related to some background torsion in the large-scale Universe, $b_\mu \sim \varepsilon_{\mu\nu\lambda\delta}T^{\nu\lambda\delta}$, and it could also be generated by chiral fermions [1].

2 Chern-Simons term generation by the fermion loop in the b_0-background

Consider a Lorentz- and CPT-violating axial-vector term in the fermion Lagrangian

$$\mathcal{L}_l = \bar{\psi}(i\gamma\partial - \gamma A - m_e - (\gamma b)\gamma_5)\psi \tag{7}$$

with $b^\mu = \{b_0, \boldsymbol{b}\}$. In *perturbative treatment*, we have two one-loop diagrams with axial-vector fermion-line insertions. As a result, the photon Lagrangian should be added by the CS term linear in b^μ

$$\mathcal{L}_{\text{CS}} = \xi b_\alpha \varepsilon^{\alpha\beta\gamma\delta}F_{\beta\gamma}A_\delta. \tag{8}$$

However, the final result depends on the *regularization scheme*, i.e. ξ is a numerical constant *depending on the method of calculation* [2]. Therefore, a *nonperturbative method* of calculation should be applied, where such ambiguity is eliminated.

The one-loop effective action has the form

$$\begin{aligned}\Gamma_{\text{eff}}(A,b) &= -i\text{Tr}\ln\left(i\gamma\partial - \gamma A + (\gamma b)\gamma^5 - m_e\right) = \\ &= -i\text{Tr}\ln\left|i\gamma\partial - \gamma A + (\gamma b)\gamma^5 - m_e\right| - \text{Tr}\ \arctan\frac{\gamma\partial - i\gamma A + i(\gamma b)\gamma^5}{m_e}.\end{aligned} \tag{9}$$

The second term vanishes and we have

$$\Gamma_{\text{eff}}(A,b) = -\frac{i}{2}\text{Tr}\ln\left(-(i\gamma\partial - \gamma A + (\gamma b)\gamma^5)^2 + m_e^2\right) = \frac{i}{2}\int_0^\infty \frac{dz}{z}\text{Tr}e^{-zH}, \tag{10}$$

$$H = -\left(\pi^\mu\pi_\mu + 2i\sigma^{\mu\nu}b_\mu\pi_\nu\gamma^5 - \frac{1}{2}\sigma^{\mu\nu}F_{\mu\nu} - b_\mu b^\nu - m^2\right),$$

where $\pi_\mu = i\partial_\mu - eA_\mu$. In the linear in b approximation we obtain

$$\mathrm{Tr}(e^{-zH}) = -16(b_\alpha g^\mu_\beta + b_\nu R^{\mu\nu}{}_{\alpha\beta})(C_2\tilde{F}^{\alpha\beta} + C_4 F^{\alpha\beta})e^{-zm^2}\times$$
$$\times\left(g^\xi_\eta - \sum_{n=0}^{\infty}\frac{(-2izF^\xi_\eta)^n}{(n+1)!}\right)^{-1}\frac{\partial}{\partial\lambda^\eta}\mathrm{Tr}_x \exp\left(z(\pi^\mu\pi_\mu + \lambda_\rho\pi^\rho)\right)|_{\lambda=0}, \qquad (11)$$

where $R^{\mu\nu}{}_{\alpha\beta}$ is a certain function of $F^{\mu\nu}$ and

$$\mathrm{Tr}_x \exp\left(z(\pi^\mu\pi_\mu + \lambda_\rho\pi^\rho)\right) = \int d^4x \,\langle x|\exp\left(z(\pi^\mu\pi_\mu + \lambda_\rho\pi^\rho)\right)|x\rangle\,. \qquad (12)$$

In the weak field limit, we finally obtain the linear in b term in action

$$\Gamma_{\mathrm{eff}}(A,b) = ib_\mu\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}\int_0^\infty dz z\mathrm{Tr}_x\pi_\nu\exp(-zm^2 + z\pi^2) =$$
$$= \frac{i}{2}b_\mu\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}\int_0^\infty dz\frac{\partial}{\partial\lambda^\nu}e^{-zm^2 - z\lambda^2}\mathrm{Tr}_x\exp(z(\pi^\rho + \lambda^\rho)^2)\,|_{\lambda=0} = 0 \qquad (13)$$

Due to gauge invariance of the theory, the above expression vanishes after differentiating over λ^ν and putting $\lambda = 0$. *The CS term*, which should be *linear in* b^μ, thus vanishes in our approach. This lies in agreement with earlier calculations made by different methods [3].

3 Hydrogen-like bound state

The one-particle bound states of the electron in the external electromagnetic field $A_\mu(x)$ are described with the modified Dirac equation, the Lagrangian (6) leads to:

$$i\hbar\partial\psi/\partial t = \hat{H}_{\mathrm{D}}(t)\psi, \quad \|\psi\|^2 = 1; \qquad (14)$$

$$\hat{H}_{\mathrm{D}} = c\boldsymbol{\alpha}\cdot\hat{\boldsymbol{P}} + \gamma^0 m_e c^2 - eA_0 - b_0\gamma_5 - \boldsymbol{b}\cdot\boldsymbol{\Sigma}, \qquad \boldsymbol{\alpha} = \gamma^0\boldsymbol{\gamma} = -\boldsymbol{\Sigma}\gamma_5, \hat{\boldsymbol{P}} = \hat{\boldsymbol{p}} + \frac{e}{c}\boldsymbol{A}. \quad (15)$$

We first examine the quasirelativistic regime of our model, making an expansion into a series with respect to $1/c$ up to the second order assuming $\hbar, c \neq 1$, $b^\mu = \{cb_t, \boldsymbol{b}\}$. This method is analogous to that proposed by Landau within QED. The wavefunction ψ of a positive-energy bound electron, which fulfills (14), can be approximately expressed in terms of a 2-component *quasi-relativistic wavefunction* Φ so that $1 = \|\psi\|^2 = \|\Phi\|^2 + O(1/c^3)$, if we assume $\hat{\boldsymbol{P}}, \boldsymbol{E}, \boldsymbol{H} = O(c^0)$, when acting upon ψ, and $b_t, \boldsymbol{b} = O(c^0)$. As a result, the equations of motion for Φ lead to the expression for the *quasi-relativistic Hamiltonian*

$$\hat{h} = \hat{K}\left(1 - \frac{\hat{K}}{2m_e c^2}\right) + \frac{e\hbar}{2m_e c}\boldsymbol{\sigma}\boldsymbol{H} - \boldsymbol{\sigma}\boldsymbol{b} - eA_0 + \frac{e\hbar}{4m_e^2c^2}\boldsymbol{\sigma}[\boldsymbol{E}\hat{\boldsymbol{P}}] + \frac{e\hbar^2}{8m_e^2c^2}\boldsymbol{\nabla}\boldsymbol{E} + \frac{\boldsymbol{\sigma}[\hat{\boldsymbol{P}}[\boldsymbol{b}\hat{\boldsymbol{P}}]]}{2m_e^2c^2}, \quad (16)$$

where $\hat{K} \equiv \frac{1}{2m_e}((\hat{\boldsymbol{P}}^2 - b_t\boldsymbol{\sigma})^2 - 2b_t^2)$, in agreement with [6] in the corresponding special cases.

In the Coulomb field, $eA^\mu = \{\frac{Z\alpha\hbar c}{r}, 0\}$ $(Z\alpha \ll 1)$, for $\boldsymbol{b} = 0$, the unitary transformation

$$\Phi(x) = \hat{U}\tilde{\Phi}(x) \equiv \exp\left\{\frac{ib_t}{\hbar}\left(1 + \frac{Zr_e}{2r}\right)\boldsymbol{\sigma}\cdot\boldsymbol{r}\right\}\tilde{\Phi}(x), \qquad r_e = \frac{\alpha\hbar}{m_e c}, \qquad (17)$$

where $x = \{ct, \boldsymbol{r}\}$, gives $\hat{\tilde{h}} \equiv \hat{U}^\dagger \hat{h} \hat{U} = \hat{h}|_{b_0=0} + O(b_0^2)$, so the $b_0 \neq 0$ solutions are readily found to be $\Phi_{nljm_j} = \hat{U}\Phi_{nljm_j}|_{b_0=0} + O(b_0^2)$ (for the explicit expression, see [4]). The wavefunctions do not possess a definite parity, and l quantum number responds to an integral of motion other than $\hat{\boldsymbol{l}}^2$. Within the first order in b_0 and the second order in $1/c$, the energy spectrum is unaffected and is given by the conventional Dirac formula [7, §34].

We further resort to a fully relativistic description of a bound electron, taking $\hbar = c = 1$ and using the quadratic approximation in $b^\mu = \{b_0, \boldsymbol{0}\}$. *We consider an electron in an arbitrary spherically-symmetric potential $\phi(r)$, and in a weak 'external' field $A_\mu^{(e)}(x)$, so that $A^\mu(x) = \{\phi(r) + A_0^{(e)}(x), \boldsymbol{A}^{(e)}\}$.* Perform a *gauge-covariant* unitary transformation:

$$\psi(x) = \exp\left\{-ib_0\left[\boldsymbol{\Sigma}\hat{\boldsymbol{x}} - \frac{i}{m_e}\left(\boldsymbol{\Sigma}[\boldsymbol{r}\hat{\boldsymbol{P}}] + 1\right)\gamma^0\gamma_5\right]\right\}\tilde{\psi}(x), \tag{18}$$

$$\hat{\tilde{H}}_{\mathrm{D}} = \hat{H}_{\mathrm{D}}|_{b_0=0} - \frac{b_0^2}{m_e}\hat{f}\gamma^0 - \hat{\boldsymbol{d}}_A\boldsymbol{E}^{(\mathrm{e})} - \hat{\boldsymbol{\mu}}_A\boldsymbol{H}^{(\mathrm{e})} + \hat{H}_{\mathrm{int}}^{(2)}[A^{(\mathrm{e})}] + O(b_0^3), \tag{19}$$

where $\hat{\boldsymbol{\mu}}_A = \frac{eb_0}{m_e}\gamma^0[\boldsymbol{\Sigma}\hat{\boldsymbol{r}}]$, $\hat{\boldsymbol{d}}_A = i\gamma_5\hat{\boldsymbol{\mu}}_A$, $\hat{f} \equiv \boldsymbol{\Sigma}\hat{\boldsymbol{l}} + 1$, and $\hat{H}_{\mathrm{int}}^{(2)}[A^{(\mathrm{e})}] = O(b_0^2)$. We have shown that the parity-odd correction $\hat{\boldsymbol{\mu}}_A$ generates a nonzero *anapole moment* of the electron orbital. When $A_\mu^{(\mathrm{e})} = 0$, i.e. in a spherically-symmetric potential, $\hat{\tilde{H}}_{\mathrm{D}}$ is parity-even (as does $\hat{H}_{\mathrm{D}}$ in the $b_0 = 0$ case). Moreover, $\hat{f}\gamma^0 = \varkappa(j + 1/2)$ at the eigenfunctions $\psi_{nljm_j}^{(0)}$ of $\hat{H}_{\mathrm{D}}|_{b_0=0}$, so we took them as the $b_0 \neq 0$ solutions in the transformed representation [4]. *After the inverse transformation* is performed, we found the desired eigenfunctions

$$\psi_{nljm_j} = e^{-iE_{nlj}t}e^{-\frac{b_0^2}{2}\left(r^2+(j+1/2)^2/m_e^2\right)}\begin{pmatrix} R_{nlj}^{(1)}Y_{jm_j}^{l} - b_0\left(\frac{j+1/2}{m_e}R_{nlj}^{(2)} - \varkappa r R_{nlj}^{(1)}\right)Y_{jm_j}^{l'} \\ \varkappa R_{nlj}^{(2)}Y_{jm_j}^{l'} - b_0\varkappa\left(\frac{j+1/2}{m_e}R_{nlj}^{(1)} + \varkappa r R_{nlj}^{(2)}\right)Y_{jm_j}^{l} \end{pmatrix}, \tag{20}$$

$$E_{nlj} = E_{nlj}^{(0)} - \varkappa b_0^2(j + 1/2)/m_e, \tag{21}$$

where $j = \frac{1}{2}, \frac{3}{2}, \ldots$, $m_j = -j, ..., j$; $l = j - \frac{\varkappa}{2}$ determines the parity $\tilde{P} \equiv (-1)^l$ of the function $\tilde{\psi}$, *and not the orbital moment.* $\{E_{nlj}^{(0)}\}$ and $R_{nlj}^{(1,2)}$ are the spectrum and the radial wavefunctions in the $b_0 = 0$ case, which are known *in the Coulomb potential* [7, §36]. A nonzero b_0 removes the degeneracy over l typical for this case, the splitting being $\lesssim 10^5$Hz when $|b_0| \lesssim 10^{-2}$eV though it increases linearly with j.

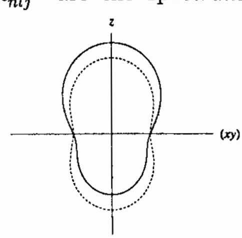

Figure 1: An effect of b_0 on the radiation angular distribution.

The parity of a hydrogen atom ($Z = 1$) broken in the background of $b^\mu = \{b_0, \boldsymbol{0}\}$ affects its radiative transition properties. We found that the first-order b_0-induced corrections to the angular radiation distribution appear *only for polarized states* (i.e. without a summation over m_j quantum numbers). Within the $1/c$-approximation, after the transformation (17), *the signature of Lorentz violation is provided by the modified electron magnetic moment operator* $\hat{\boldsymbol{\mu}} = -\frac{e\hbar}{2m_ec}(\hat{\boldsymbol{l}} + \boldsymbol{\sigma}) + \hat{\boldsymbol{\mu}}_A$, $\hat{\boldsymbol{\mu}}_A = \frac{eb_0}{m_ec^2}[\boldsymbol{\sigma}\hat{\boldsymbol{r}}]$. $\hat{\boldsymbol{\mu}}_A$ introduces a parity-nonconserving correction to the $M1$ term of the transition matrix element, that mixes $E1$ and $M1$ transitions, the radation distribution acquiring corrections of the order $|b_0|/m_ec^2 \lesssim 10^{-8}$. Within the linear order in b_0, the total rates are unaffected. For instance, the angular distribution (summed over the photon polarizations) for the transition $|2p_{1/2},\, m_j = 1/2\rangle \to |1s_{1/2},\, m_j' = -1/2\rangle$ was shown to lose its symmetry:

$$\frac{dw_{fi}}{d\Omega_k} = \frac{256\alpha^3 R}{6561\pi}\left\{1 + \cos^2\theta - \frac{8b_0}{m_e c^2}\cos\theta\right\}, \tag{22}$$

where $R = \alpha^2 m_e c^2/2\hbar$, θ is the angle between the photon momentum $\hbar\boldsymbol{k}$ and the axis of the angular moment quantization (z). It is shown in fig.1, with the dashed line demonstrating the $b_0 = 0$ case and $b_0/m_e c^2 = 0.05$ chosen for vividness.

4 Synchrotron radiation

In this section we present a *quantum treatment* of the radiation problem within the extended QED for an electron moving in a classical constant homogeneous *external magnetic field* $\boldsymbol{H} = H\boldsymbol{e}_z$ (with 4-potential $A^\mu = \{0, \frac{1}{2}[\boldsymbol{H}\boldsymbol{r}]\}$). We assume $b^\mu = \{b_0, \boldsymbol{0}\}$ in the observer frame; and we also take into account the particle *anomalous magnetic moment* $\mu \approx \frac{\alpha}{2\pi}\frac{e}{2m_e}$ (since $H \ll H_c = m_e^2/e \approx 4.41 \times 10^{13}$ Gauss). The one-particle Hamiltonian then reads:

$$\hat{H}_{\rm D} = \hat{H}_{\rm D}(b_0, \hat{p}_z, m_e, \mu) = \boldsymbol{\alpha}\hat{\boldsymbol{P}} + \gamma^0 m_e + \mu H\gamma^0\Sigma_3 - b_0\gamma_5, \quad \hat{\boldsymbol{P}} \equiv \{\hat{P}_1, \hat{P}_2, \hat{p}_z\}. \tag{23}$$

In the subspace with definite p_z where $\psi(\boldsymbol{r}) \sim e^{ip_z z}$, using the unitary transformation

$$\psi = e^{-\vartheta\gamma^3/2}\,\tilde{\psi}, \qquad \hat{\tilde{H}}_{\rm D} = \hat{H}_{\rm D}(0, \tilde{p}_z, \tilde{m}_e, \tilde{\mu}), \tag{24}$$

$$\tilde{\mu}H = \sqrt{(\mu H)^2 + b_0^2}, \quad \begin{pmatrix}\tilde{m}_e\\ \tilde{p}_z\end{pmatrix} = \begin{pmatrix}\cos\vartheta & \sin\vartheta\\ -\sin\vartheta & \cos\vartheta\end{pmatrix}\begin{pmatrix}m_e\\ p_z\end{pmatrix}, \quad \vartheta = \arctan\frac{b_0}{\mu H}, \tag{25}$$

we reduce the problem to that without Lorentz violation solved in [8], except for the change $(m_e, p_z, \mu) \to (\tilde{m}_e, \tilde{p}_z, \tilde{\mu})$. Taking $\hat{j}_z$ and $\hat{\tilde{\Pi}}_\perp \equiv \tilde{m}_e\Sigma_3 + i\gamma^0\gamma_5[\boldsymbol{\Sigma}\hat{\tilde{\boldsymbol{P}}}]_z$ as the integrals of motion with the eigenvalues $n - s - 1/2$ and $\Pi \equiv \zeta\sqrt{\tilde{m}_e^2 + 2eHn}$ ($n, s = 0, 1, \ldots,$ $\zeta = \pm 1$) and going back to the initial representation, we find that the electron *transversal* spin polarization operator $\hat{\tilde{\Pi}}_\perp$ transforms to a *mixed (partially longitudinal)* one:

$$\hat{\Pi} = \hat{\Pi}_\perp\cos\vartheta + \hat{\Pi}_\parallel\sin\vartheta, \qquad \hat{\Pi}_\parallel \equiv \boldsymbol{\Sigma}\hat{\boldsymbol{P}}, \quad \hat{\Pi}_\perp \equiv m_e\Sigma_3 + i\gamma^0\gamma_5[\boldsymbol{\Sigma}\hat{\boldsymbol{P}}]_z. \tag{26}$$

The solutions for ψ and the spectrum have the form (see details in [5]):

$$\begin{aligned}\psi_{\epsilon,p_z,n,s,\zeta}(\boldsymbol{r}; b_0, m_e, \mu) &= e^{-\frac{\vartheta}{2}\gamma^3}e^{i(p_z-\tilde{p}_z)z}\psi_{\epsilon,\tilde{p}_z,n,s,\zeta}(\boldsymbol{r}; 0, \tilde{m}_e, \tilde{\mu}), &(27)\\ E_{\epsilon,p_z,n,\zeta}(b_0, m_e, \mu) &= E_{\epsilon,\tilde{p}_z,n,\zeta}(0, \tilde{m}_e, \tilde{\mu}) = \epsilon\sqrt{(\Pi + \tilde{\mu}H)^2 + \tilde{p}_z^2}, &(28)\end{aligned}$$

where $\epsilon = \pm 1$ is the sign of the energy and $\hat{\Pi}\psi_{\epsilon,p_z,n,s,\zeta} = \Pi\psi_{\epsilon,p_z,n,s,\zeta}$.

Using these solutions, we apply the quasiclassical theory of synchrotron radiation quite analogous to the $b_0 = 0$ case, so we discuss it only briefly (see [5] for details). The electron orbital motion becomes quasiclassical for $H \ll H_c$ and $m_e/E \equiv \lambda \ll 1$. We neglect the ratio $\tilde{\mu}H/E$ under typical laboratory conditions ($E \sim 1\,$GeV, $H \sim 10^4$ Gauss), since $\tilde{\mu}H/E \ll 1$ if $b_0 \gg 10^{-20}\,$eV, and we take the initial momentum $p_z = 0$. However, ϑ *is not assumed* *to be small.* In spherical coordinates with the polar angle θ measured from the z-axis, the total radiation power is

$$W = W_{cl} \int dy \, \sin\theta \, d\theta \, \frac{27}{64\pi^2} \frac{y^2}{\lambda^5(1+\xi y)^4} \, \Phi(\theta), \quad W_{cl} = \frac{8}{27} (e m_e \xi)^2, \tag{29}$$

where $y > 0$ is a dimensionless variable related to the photon energy k: $\frac{k}{E} = \frac{\xi y}{1+\xi y}$, and $\xi = \frac{3}{2} \frac{H}{H_c} \frac{1}{\lambda}$ estimates *the role of quantum effects.* One of the angular distributions $\Phi^{\pm}_{\tau}(\theta)$ for polarization τ and spin $\zeta' = \pm\zeta$ is shown in fig.2. The distribution asymmetry relative to the orbit plane $\theta = \pi/2$ caused by the longitudinal admixture to its spin polarization (when $b_0 \neq 0$) also maintains for unpolarized electrons. Since experiment confirms the 'transversality' of electron states, we can conclude that $\vartheta \ll 1$. Taken under laboratory conditions, this implies: $|b_0| \ll \mu H \sim 10^{-6}\,$eV.

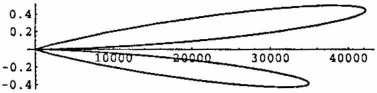

Figure 2: Normalized angular distribution $\Phi^{-}_{\sigma}(\theta)$ for $\zeta = -1$, $k = 1\,$MeV, $E = 1\,$GeV, $H = 10^4\,$Gauss, $b_0 = 10^{-9}\,$eV.

5 Conclusion

We have shown that CS term linear in b_μ does not appear in the one-loop effective action within the SME. At the same time, for an electron being in a bound state, the Lorentz-violating coupling b_0 manifests itself in the modified electron spectrum and integrals of motion (parity or polarization), in the nonperturbative interaction with its anomalous magnetic moment, and in the asymmetries of the radiation distributions, especially for polarized electrons. The results obtained gave us stringent constraints on b_0 and seem promising in suggesting new experiments.

Acknowledgements

The authors are grateful to A.V.Borisov, D.Ebert, and A.E.Lobanov for helpful discussions.

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Quantum Theory

Quantum Channel Decoherence in Optics

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Abstract

The decoherence effects of distribution of an optical quantum channel produced by the Bell-like states are considered. The degrees of entanglement of quantum channels of Φ and Ψ types depending on conditions of their propagation along optical paths are given.

The quantum information schemes for quantum cryptography, quantum teleportation and quantum computation use entangled states as a basis for a quantum channel. In optics such quantum channels are realized by means of dielectric four-port devices like beam splitters, mirrors and fibres through which entangled photons are distributed. This dielectric environment arouses some dispersion and absorption that destroys the initial quantum coherence of the photon state.

A quantum theory of state transformations of the electromagnetic field at dispersing and absorbing four-port devices based on the quantization of the Maxwell equations was given in [1]. In this theory, the effect of the dielectric device on the incoming radiation can be described by introducing in quantum input-output relations bosonic fields playing the role of the device excitations which are associated with absorption [2]. So that the density matrix of the electromagnetic field states in output ports of a dispersive and absorbing four-port device of an arbitrary frequency linear response is related to the initial density matrix by the following expression

$$\hat{\varrho}_{\rm out}^{(F)} = {\rm Tr}^{(D)}\left\{\hat{\varrho}_{\rm in}\left[\mathbf{\Lambda}^{\dagger}(\omega)\hat{\vec{\alpha}}\right]\right\}, \tag{1}$$

where the $SU(4)$ matrix $\mathbf{\Lambda}$ is completely determined by the *complex* refractive-index profile of the device, and the four-dimensional vector operator $\hat{\vec{\alpha}}$ is formed by two input field operators and two other bosonic operators of the device excitations over which the trace is carried out. Relation (1) allows one to estimate the state of the electromagnetic field after propagation along an optical path by representing it as a four-port device with a given refractive-index profile.

The proposed formalism is suited for an analysis of entanglement degradation of a quantum channel. As quantum channels which can be used for optical schemes on quantum information, let us consider states of the Bell type

$$|\Phi(\lambda)\rangle = \frac{|00\rangle + \lambda|11\rangle}{\sqrt{1+|\lambda|^2}}, \tag{2}$$

$$|\Psi(\lambda)\rangle = \frac{|01\rangle + \lambda|10\rangle}{\sqrt{1+|\lambda|^2}}, \tag{3}$$

where λ stands for an arbitrary complex parameter. States (2) and (3), whose density matrices have the forms

$$\hat{\varrho}_{\text{in}}^{\Phi} = \frac{1}{1+|\lambda|^2}\left\{|00\rangle\langle 00| + \lambda|11\rangle\langle 00| + \lambda^*|00\rangle\langle 11| + |\lambda|^2|11\rangle\langle 11|\right\}, \tag{4}$$

$$\hat{\varrho}_{\text{in}}^{\Psi} = \frac{1}{1+|\lambda|^2}\left\{|01\rangle\langle 01| + \lambda|10\rangle\langle 01| + \lambda^*|01\rangle\langle 10| + |\lambda|^2|10\rangle\langle 10|\right\}, \tag{5}$$

are maximally entangled states at $|\lambda| = 1$.

Let us assume that the quantum channel made up by the state (2) or (3) transmits along two communication paths described by some transmission coefficients $T_1(\omega)$ and $T_2(\omega)$. As it results from the transformation relation (1), the states of the quantum channels (4) and (5) after propagation become (see also [3])

$$\begin{aligned}\hat{\varrho}_{\text{out}}^{\Phi} = \frac{1}{1+|\lambda|^2}\Big\{&\left[1+|\lambda|^2(1-|T_1|^2)(1-|T_2|^2)\right]|00\rangle\langle 00| \\ &+\lambda T_1T_2|11\rangle\langle 00| + \lambda^*T_1^*T_2^*|00\rangle\langle 11| + |\lambda|^2|T_1|^2(1-|T_2|^2)|10\rangle\langle 10| \\ &+|\lambda|^2(1-|T_1|^2)|T_2|^2|01\rangle\langle 01| + |\lambda T_1T_2|^2|11\rangle\langle 11|\Big\},\end{aligned} \tag{6}$$

$$\begin{aligned}\hat{\varrho}_{\text{out}}^{\Psi} = \frac{1}{1+|\lambda|^2}\Big\{&(1+|\lambda|^2-|\lambda T_1|^2-|T_2|^2)|00\rangle\langle 00| + |T_2|^2|01\rangle\langle 01| + |\lambda T_1|^2|10\rangle\langle 10| \\ &+\lambda T_1T_2^*|10\rangle\langle 01| + \lambda^*T_1^*T_2|01\rangle\langle 10|\Big\},\end{aligned} \tag{7}$$

correspondingly. Eqs. (6) and (7) show that the initial pure quantum states convert into the mixed ones because of the interaction with dielectric matter of the paths. Such decoherence processes reduce the original degree of entanglement of the quantum channel.

The effect of entanglement degradation can be estimated with the help of a measure of entanglement based on the eigenvalue problem of the partial transpose of the density matrix of the bipartite mixed state [4]. Since for any entangled state the partial transpose $\hat{\varrho}^{\text{PT}}$ of its density matrix $\hat{\varrho}$ defined as

$$\langle i,j|\hat{\varrho}^{\text{PT}}|k,l\rangle = \langle k,j|\hat{\varrho}|i,l\rangle \tag{8}$$

has necessarily negative eigenvalues $\mu_i < 0$, the sum of their absolute values

$$\mathcal{N}(\hat{\varrho}) = \left|\sum_i \mu_i\right| \tag{9}$$

which is called *negativity* determines the degree of entanglement in terms of the *logarithmic negativity*

$$E_{\mathcal{N}}(\hat{\varrho}) = \log_2\left[1+2\mathcal{N}(\hat{\varrho})\right]. \tag{10}$$

This method can be directly applied to the density matrices (6) and (7) to evaluate their degrees of entanglement. The straightforward calculations of eigenvalues of the partial transpose of these matrices give the following expressions for the negativities

$$\mathcal{N}_{\Phi} = \frac{|\lambda|}{2(1+|\lambda|^2)}\left\{\sqrt{4|T_1T_2|^2+|\lambda|^2(|T_1|^2-|T_2|^2)^2} - |\lambda|(|T_1|^2+|T_2|^2-2|T_1T_2|^2)\right\}, \tag{11}$$

provided that $|\lambda| \le (1+|T_1T_2|^2-|T_1|^2-|T_2|^2)^{-1/2}$, and $\mathcal{N}_{\Phi} = 0$ otherwise;

$$\begin{aligned}\mathcal{N}_{\Psi} = \frac{1}{2(1+|\lambda|^2)}\Big\{&\sqrt{[|\lambda|^2(1-|T_1|^2)+1-|T_2|^2]^2+4|\lambda T_1T_2|^2} \\ &-|\lambda|^2(1-|T_1|^2)+1-|T_2|^2\Big\}.\end{aligned} \tag{12}$$

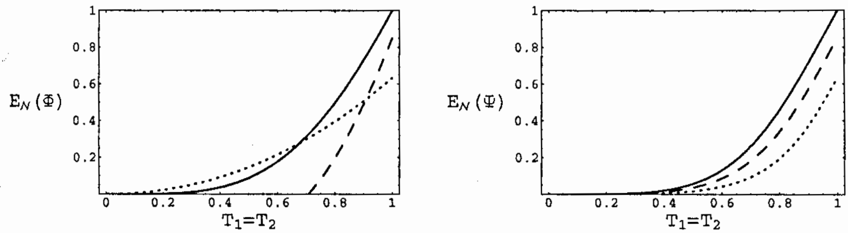

Figure 1: Logarithmic negativities of the Φ-channel (left) and Ψ-channel (right) for the symmetric communication paths with transmission coefficients $T_1 = T_2$ and $|\lambda|$ being equal to 1 (solid line), 2 (dashed line) and 0.3 (dotted line).

The behavior of the logarithmic negativities (10) which correspond to expressions (11) and (12) depending on the transmission coefficients of the distribution paths in the symmetric scheme $T_1(\omega) = T_2(\omega)$ and on the parameter λ are shown in Fig. 1. It is clearly seen a rapid decrease of the logarithmic negativities from their initial values as the transmissivity of the communication paths is reduced. This signifies the entanglement degradation of the quantum channel on account of unavoidable losses during its distribution. Moreover, the Φ-channel can become completely disentangled, $E_\mathcal{N}(\Phi) = 0$, at certain values of the transmission coefficients and the parameter $|\lambda|$ (> 1) in accordance with the relation in eq. (11), whereas the Ψ-channel keeps a residual part of the initial entanglement even at very low transmissivity of optical paths.

The stated approach for estimation of entanglement degradation can be also applied to the case of continuous-variable quantum channels, in particular based on two-mode squeezed vacuum states [5]-[8]. Thus, a quantum theory of state transformations of the electromagnetic field at dispersing and absorbing four-port devices is a convenient tool in consideration of specific problems of quantum information processing.

This work is partly supported by the RFBR Grant No. 08-02-00118.

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New Concept of Time and Spin

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Abstract

The development of Theory of Self-Organization was originated by the creation of the natural, argued, and novel concept of time. Time by itself exists in the form of the scalar temporal field and is the cornerstone of any dynamical theory and, hence, the dynamical theory of spin (spindynamics) as well. The equations of spindynamics are represented in the most symmetrical and obviously dynamical form, using the formalism of vector algebra and vector analysis in the 4-dimensional and general covariant form. Through this, we recognize the operator of evolution in spindynamics as the covariant derivative along the stream of time. It differs from the operator of evolution for the (general) electromagnetic field and the gravitational field, where it is the Lie derivative along the stream of time.

1 Introduction

Theory of Self-Organization is the new, self-consistent, and integral structure in whicl geometry, symmetries, and fields are tightly connected and kept inseparable providing the adequate solution of the most difficult conceptual problems [1], [2], [3]. In the dynamica Theory of Self-Organization all manifolds of phenomena are projected on the set of the four fundamental fields: the gravitational field, the temporal field, the general electromagneti field, the spinning field. In the static theory of Self-Organization the temporal field i absent. The first principles and laws of Self-Organization of this defining system of field are established and on this basis the novel theory of spin phenomena is developed. It i shown that spin is the diverse phenomenon which involves the concepts of the spinning field spin symmetry with its bipolar structure, the equations of spinstatics and spindynamics a the natural and fundamental manifestation of bipolar structure, new understanding of the time reversal, unbroken spin symmetry with the concepts of internal spin and the spir current. Since the Theory of Self-Organization is integral structure, spindynamics involve all phenomena connected with spin. Hence, having in our disposal the bipolar structur of spin symmetry we can maintain that observer does not need to consider the artificia concept of weak or strong isotopic spin. In this case, the geometrical and physical natur of the strong interactions can be understood only in the framework of the nontrivial causa structure defining by the temporal field. We can consider in the first approximation tha physical space is 4-dimensional Euclidean space. It is easy to find that in this case the basi equation of temporal field has general solution and singular solution. The first solutior defines the causal structure that corresponds to the special relativity. The second solutior is considered as the starting point for understanding the strong interactions on the basis of the new causal structure tightly connected with rotations.

For comparison, let us look at the historical development of Quantum Mechanics. From the geometrical point of view, in the Schrodinger theory the two real scalar fields are introduced and internal symmetry appears at first in the form of the complex scalar field. Here the principle of sufficient cause is substituted by the experiment but the question remain open. In the Dirac theory already the four complex scalar fields are introduced and, hence, it can be considered as the theory of the Higgs fields with nontrivial internal symmetry defined by the Dirac spin matrices. In the electroweak theory and quantum chromodynamics the number of the scalar fields increases again and again and thus, the artificial internal symmetry is extended. This way of development of the theory looks like artificial and oriented on the explanation of the artificial phenomena since it is impossible to derive the theory of elementary particles from the first principles without understanding the essence of time. But nevertheless the final judgment should be leaved to the future development of the theory and experiment.

2 Equations of Spindynamics in Hamiltonian Form

In the Theory of Self-Organization the properties of time and physical space are not defined by the properties of devices and by the methods of measurements. A temporal field (together with other fields) designs physical space, but it has also other very important functions in spindynamics. The temporal field with respect to the coordinate system u^1, u^2, u^3, u^4 is denoted by $f(u) = f(u^1, u^2, u^3, u^4)$. The gradient of the temporal field (the stream of time) is the vector field $\mathbf{t}$ with the components

$$t^i = g^{ij}\frac{\partial f}{\partial u^j} = g^{ij}\partial_j f = g^{ij}t_j, \quad D_{\mathbf{t}} f = g^{ij}\frac{\partial f}{\partial u^i}\frac{\partial f}{\partial u^j} = 1.$$

where g^{ij} are the contravariant components of the positive-definite Riemann metric $ds^2 = g_{ij}du^i du^j$. The rate of change with time of some quantity is the Lie derivative with respect to the stream of time $\mathbf{t}$ and the symbol $D_{\mathbf{t}}$ denotes this operation. The other possible operator of evolution has the form of covariant derivative in the direction of the stream of time. This operator is denoted by $\nabla_{\mathbf{t}} = t^i\nabla_i$, where ∇_i is a covariant derivative with respect to the connection that belongs to the Riemann metric g_{ij}.

We remind some definitions of the vector algebra and vector analysis in the four-dimensional and general covariant form [1]. The operator rot is defined for the vector fields as follows:

$$(\text{rot}\mathbf{M})^i = e^{ijkl}t_j\partial_k \mathrm{M}_l = \frac{1}{2}e^{ijkl}t_j(\partial_k \mathrm{M}_l - \partial_l \mathrm{M}_k),$$

where e_{ijkl} is the orientation of physical space ("element of volume"). It is easy to show that

$$(\mathbf{M}, \text{rot}\mathbf{N}) + \text{div}[\mathbf{MN}] = (\text{rot}\mathbf{M}, \mathbf{N}),$$

where

$$[\mathrm{MN}]^i = e^{ijkl}t_j \mathrm{M}_k \mathrm{N}_l$$

is a vector product of two vector fields $\mathbf{M}$ and $\mathbf{N}$, $\text{div}\,\mathbf{M} = \nabla_i \mathrm{M}^i$. Thus, the operator rot is self-adjoint. We also mention that $(\text{grad}\varphi)_i = \triangle_i\varphi$, $\triangle_i = \nabla_i - t_i\nabla_{\mathbf{t}}$ and rot grad =

0, $\operatorname{div}\operatorname{rot} = 0$. To apply vector analysis to equations of spindynamics in the geometrical form [3]

$$\frac{1}{2}(\tilde{Q}_{\mathbf{t}}\Lambda + \Lambda\tilde{Q}_{\mathbf{t}})\Psi = m\tilde{Q}_{\mathbf{t}}\Psi, \quad \frac{1}{2}(\tilde{Q}_{\mathbf{t}}\Lambda + \Lambda\tilde{Q}_{\mathbf{t}}) \overset{*}{\Psi} = m\tilde{Q}_{\mathbf{t}} \overset{*}{\Psi}, \tag{1}$$

we consider the mapping of the spinning field

$$\Psi = (\psi, \psi_i, \psi_{ij}, \psi_{ijk}, \psi_{ijkl})$$

onto two scalars κ and μ, two pseudoscalars λ and ν, two vectors $\mathbf{K}$ and $\mathbf{M}$, two pseudovectors $\mathbf{L}$ and $\mathbf{N}$, all being orthogonal to the stream of time, $(\mathbf{t},\mathbf{K}) = (\mathbf{t},\mathbf{L}) = (\mathbf{t},\mathbf{M}) = (\mathbf{t},\mathbf{N}) = 0$. This mapping is defined as follows:

$$\kappa = t^i\psi_i, \quad \mu = \psi, \quad \lambda = \frac{1}{3!}t_l e^{ijkl}\psi_{ijk}, \quad \nu = \frac{1}{4!}e^{ijkl}\psi_{ijkl},$$

$$\mathrm{K}^i = h^i_m\psi^m, \quad \mathrm{M}^i = t_k\psi^{ki}, \quad \mathrm{L}^i = \frac{1}{3!}h^i_m e^{mjkl}\psi_{jkl}, \quad \mathrm{N}^i = t_k\tilde{\psi}^{ki},$$

where $h^i_j = \delta^i_j - t^i t_j$, $\tilde{\psi}^{ki} = \frac{1}{2}e^{kijl}\psi_{jl}$. The inverse mapping has the form

$$\psi = \mu, \quad \psi_i = \mathrm{K}_i + \kappa t_i, \quad \psi_{ij} = t_i\mathrm{M}_j - t_j\mathrm{M}_i + e_{ijkl}t^k\mathrm{N}^l,$$

$$\psi_{ijk} = e_{mijk}\mathrm{L}^m + \lambda\, t^m e_{ijkm}, \quad \psi_{ijkl} = \nu\, e_{ijkl}.$$

Further, we write down the equations (1) in the component form and after some transformations arrive at the following symmetric system of equations of spindynamics which include four scalar and four vector equations:

$$\begin{aligned} (\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\kappa &= \operatorname{div}\mathbf{K} - m\,\mu \\ (\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\lambda &= \operatorname{div}\mathbf{L} - m\,\nu \\ (\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mu &= \operatorname{div}\mathbf{M} + m\,\kappa \\ (\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\nu &= \operatorname{div}\mathbf{N} + m\,\lambda \end{aligned} \tag{2}$$

$$\begin{aligned} (\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mathbf{K} &= -\operatorname{rot}\mathbf{L} + \operatorname{grad}\kappa + m\,\mathbf{M} \\ (\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mathbf{L} &= \operatorname{rot}\mathbf{K} + \operatorname{grad}\lambda + m\,\mathbf{N} \\ (\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mathbf{M} &= \operatorname{rot}\mathbf{N} + \operatorname{grad}\mu - m\,\mathbf{K} \\ (\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mathbf{N} &= -\operatorname{rot}\mathbf{M} + \operatorname{grad}\nu - m\,\mathbf{L}, \end{aligned} \tag{3}$$

where $\nabla_{\mathbf{t}} = t^i\nabla_i$, $\varphi = \nabla_i t^i$. ¿From the first principles of Self-Organization it follows that Equations (2) and (3) describe all phenomena connected with spin symmetry and spin.

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Locality and Nonlocality in the Quantum Theory

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Abstract

In last years investigations in the area of quantum telecommunication intensively develop. In this connection the problem of a locality has got a special urgency. In proposed work the quantum-theoretical scheme which is based on the theory of algebras and classical probability theory is described. This scheme possesses the following distinctive peculiarities. Firstly, for quantum ensembles it reproduces the standard mathematical formalism of quantum mechanics. Second, it provides performance of locality condition for each individual event, including the act of measurement. Thirdly, it allows to establish that the quantum state does not possess property of a locality. As an example double-slit experiment is discussed.

1 Introduction

Since the famous debates between Einstein [1] and Bohr [2], specialists have argued about locality in quantum mechanics. Einstein claimed that the locality principle is violated in quantum mechanics, which is therefore inconsistent or at least incomplete. The locality property occupies the central place in quantum field theory. The locality property is not denied in nonrelativistic quantum mechanics when considering interactions of quantum objects. But the situation is drastically changed when discussing the problem of interaction between quantum objects and classical measuring devices.

The main constructive tool describing such an interaction is the projection principle. In the most cases, the projection principle had indeed proved its usefulness when describing the influence of a measuring device on a quantum object in the framework of the standard mathematical apparatus of quantum mechanics. But the physical mechanism for realizing this principle as well as its consistency with the locality condition is still missing.

Instead, people vaguely reason that the human brain has an experience of describing only classical objects and therefore cannot provide a concrete image of how a quantum object interacts with a classical device.

The locality problem has become more and more relevant in recent years. This is because it passes more and more from the domain of theoretical reasonings and Gedankenexperiments into the domain of actual experiments. Moreover, the first attempts are being made to construct prototypes of engineering constructions in the domain of so-called quantum telecommunication. The quantum locality problem plays a key role in this domain.

We mention here that the results of modern experiments are interpreted in most cases as evidence that a "local physical reality" does not exist in quantum physics. The locality of the quantum theory then acquires a somewhat metaphysical status removed from material reality.

Here, we attempt to demonstrate a quantum theory formulation that, on one hand, preserves the mathematical apparatus of the standard quantum mechanics and, on the other hand, ensures the satisfaction of the locality condition for each individual event including the measurement procedure (see also [3]).

The basic notions of the proposed approach to quantum mechanics were presented in [4]. We only briefly review these notions here. The entire construction is performed in the framework of the algebraic approach. We therefore do not assume that a physical system state is described by a vector of a Hilbert space (or by a density matrix) and that observables are described by operators in this space.

We take the following postulates as basic.

POSTULATE 1. Observables of a physical system are described by Hermitian elements of some C^*-algebra $\mathfrak{A}$.

Elements of the algebra $\mathfrak{A}$ are called dynamical variables. We let $\mathfrak{A}_+$ denote the set of observables. We let $\mathfrak{Q}_\xi$ denote maximum commutative subalgebras of the algebra $\mathfrak{A}$ belonging to $\mathfrak{A}_+$. We use the subscript $\xi \in \Xi$ to distinguish among these subalgebras.

POSTULATE 2. For observables $\hat{A}$ and $\hat{B}$ to be compatible (simultaneously measurable), it is necessary and sufficient for them both to belong to some subalgebra $\mathfrak{Q}_\xi$.

We let $\varphi_\xi(\cdot)$ denote a character of the subalgebra $\mathfrak{Q}_\xi$, i.e., $\hat{A} \xrightarrow{\varphi_\xi} \varphi_\xi(\hat{A})$ is a homomorphic map of the algebra $\mathfrak{Q}_\xi \quad (\hat{A} \in \mathfrak{Q}_\xi)$ into the algebra of real numbers.

We call the collection $\varphi = [\varphi_\xi] \quad (\xi \in \Xi)$ of functionals $\varphi_\xi(\cdot)$, each of which is a character of the corresponding subalgebra $\mathfrak{Q}_\xi$, an elementary state of a physical system.

POSTULATE 3. The result of any individual experiment on measuring physical system observables is determined by an elementary state of this system.

Because compatible measurements are possible only for compatible observables, we cannot unambiguously fix the elementary state φ in an experiment. The maximum that we can do is to determine the reduction of the elementary state to a subalgebra $\mathfrak{Q}_\xi$ only for one ξ $(\xi = \eta \in \Xi)$, i.e., we can fix the functional $\varphi_\eta(\cdot)$. We say that elementary states φ are φ_η-equivalent if they have the same reduction $\varphi_\eta(\cdot)$ on the subalgebra $\mathfrak{Q}_\eta$.

We define the purely quantum state Ψ_{φ_η}, to be the class $\{\varphi\}_{\varphi_\eta}$ of φ_η-equivalent elementary states. We can therefore experimentally determine only whether a system under investigation belongs to a definite quantum state.

The collection of physical systems whose elementary states constitute the equivalence class $\{\varphi\}_{\varphi_\eta}$ is called a pure quantum ensemble.

POSTULATE 4. A quantum ensemble admits the structure of the probability space.

We recall that the probability space is the fundamental object in the classic probability theory (see, e.g. [5, 6]. The probability space is a triple $(\Omega, \mathcal{F}, P)$. The first term in the triple, Ω, is a set of elementary events. In our case, the elementary state φ plays the role of an elementary event. The second term of the triple, $\mathcal{F}$, is the Boolean σ-algebra of the set

Ω. Elements of the algebra $\mathcal{F}$ are subsets of the set Ω. These subsets are called (probability) events. The third term in the triple is a probability measure P. This is a map of the set $\mathcal{F}$ into the set of real numbers: each $F \in \mathcal{F}$ is sent to a number P(F). This mapping must satisfy the conditions $0 \leq P(F) \leq 1$ for all $F \in \mathcal{F}$, $P(\Omega) = 1$, and $P(\sum_j F_j) = \sum_j P(F_j)$ for any denumerable union $\sum_j F_j$ of nonintersecting subsets $F_j \in \mathcal{F}$.

We recall that the probability measure is defined only for the events $F \in \mathcal{F}$. For elementary events, the probability measure may not exist in general.

The mathematical representation of a physical system is the algebra of its dynamical variables; vice versa, the physical representation of the algebra of dynamical variables is some physical system. We can therefore consider the physical representation of a subalgebra to be the corresponding physical subsystem. This subsystem is by no means isolated from the rest of the system, i.e., it can be an open system and not have its own dynamics. But in most cases, the conclusions of the probability theory are not related to the dynamics. In particular, we can treat the subalgebra $\mathfrak{Q}_\xi$ as an algebra of observables of a classical subsystem of the quantum system under investigation. Because we can confine ourself to the measurements compatible with the measurements of observables from the subalgebra $\mathfrak{Q}_\xi$ in order to find the mean $\langle \hat{A} \rangle$ of an observable $\hat{A} \in \mathfrak{Q}_\xi$, the classical probability theory suffices for calculating such a mean. The formula

$$\langle \hat{A} \rangle = \int_{\varphi \in \Psi} P_{\hat{A}}(d\varphi)\, A_\xi(\varphi) \equiv \int_{\varphi \in \Psi} P_{\hat{A}}(d\varphi)\, \varphi_\xi(\hat{A}) \equiv \Psi(\hat{A}). \tag{1}$$

then holds. Here,

$$P_{\hat{A}}(d\varphi) = P(\varphi : \varphi_\xi(\hat{A}) \leq A + dA) - P(\varphi : \varphi_\xi(\hat{A}) \leq A), \qquad A_\xi(\varphi) \equiv \varphi_\xi(\hat{A}). \tag{2}$$

Experiment proves that the following statement holds.

POSTULATE 6. The quantity $\Psi(\hat{A})$ is a linear functional of observables, i.e.,

$$\Psi(\hat{A}) + \Psi(\hat{B}) = \Psi(\hat{A} + \hat{B}) \text{ for all } \hat{A}, \hat{B} \in \mathfrak{A}_+.$$

This functional can be unambiguously extended to the algebra $\mathfrak{A}$ using the formula $\Psi(\hat{A} + i\hat{B}) = \Psi(\hat{A}) + i\Psi(\hat{B})$, where $\hat{A}, \hat{B} \in \mathfrak{A}_+$.

Every C^*-algebra $\mathfrak{A}$ is isometrically isomorphic to a subalgebra $\mathfrak{B}(\mathfrak{H})$ of bounded linear functionals in a Hilbert space $\mathfrak{H}$ (see, e.g., [7]), i.e.,

$$\hat{A} \leftrightarrow \Pi(\hat{A}), \quad \hat{A} \in \mathfrak{A}, \quad \Pi(\hat{A}) \in \mathfrak{B}(\mathfrak{H}).$$

It can be shown (see [4]) that the mean $\langle \hat{A} \rangle$ of the observable $\hat{A}$ with respect to the quantum ensemble Ψ defined by formula (1) can be represented as the expectation of the operator $\Pi(\hat{A})$:

$$\langle \hat{A} \rangle = \langle \Psi | \Pi(\hat{A}) | \Psi \rangle, \tag{3}$$

where $|\Psi\rangle \in \mathfrak{H}$ is the corresponding vector in the Hilbert space.

2 Double-slit experiment and EPR-paradox

Formula (1) and (3) indicate that, on one hand, we can use the mathematical tools of the standard quantum mechanics to calculate quantum means $\Psi(\hat{A})$ and, on the other hand, we can interpret a quantum state as an equivalence class of elementary states. An equivalence class is a mathematical notion existing out of time and space. Speaking about a localization of a quantum state is therefore absurd.

To discuss the locality problem in more detail, we consider how we can describe particle scattering by two slits a and b using the idea of the elementary state. An interference pattern is vividly observed in this experiment. This picture is clearly determined by the probability distribution of particle momenta after scattering. Three events are essential in the experiment under consideration: the event F_a, which means that the particle hits a domain of slit a, the event F_b, which means that the particle hits a domain of slit b, and the event F_k, which means that the scattered particle momentum falls into a fixed small solid angle around the direction K.

The problem under consideration can be formulated in these terms as a typical problem of calculating conditional probability. We must calculate the probability of the event F_k under the condition of realization of either event F_a or event F_b. Classical probability theory provides a standard formula, but we cannot apply it directly in the quantum case because it involves the probability of simultaneous realization of the events F_k and $F_a + F_b$. But a probability measure does not exist for this event because the events F_k and $F_a + F_b$ are incompatible because of the incompatibility of simultaneous measurements of the coordinate and the momentum.

But we can propose a detour for calculating such a conditional probability. For this, it suffices to consider the first stage of scattering in which the particle hits either the domain of slit a or the domain of slit b as the preparation of a quantum state. When using this quantum state as the new probability space, we can consider the event F_k as an unconditional one.

We can set the observable $\hat{p}_a$, which takes the value $p_a = 1$ if the particle hits the domain of slit a and value $p_a = 0$ if the particle misses this domain, into correspondence with the event F_a. We set the analogous observable $\hat{p}_b$ into correspondence with the event F_b. Only those particles whose elementary states correspond to the value of the observable $\hat{p}_a + \hat{p}_b$ equal to one contribute to the interference pattern. Such elementary states constitute an equivalence class, denoted by Ψ_{a+b}. Because the observable $\hat{p}_a + \hat{p}_b$ is not the only independent generator of the maximum subalgebra of compatible observables in the general case, the quantum state corresponding to the equivalence class Ψ_{a+b} can be mixed. But even in this case, the functional describing the means of observables with respect to this quantum state is positive definite, linear, and normalized to unity. We let $\Psi_{a+b}(\cdot)$ denote this functional. For all the elementary states in this quantum ensemble, we have

$$\varphi_\xi(\hat{I}) = \varphi_\xi(\hat{p}_a + \hat{p}_b) = 1,$$

where $\hat{I}$ is the unit element of the algebra $\mathfrak{A}$. Therefore, the functional $\Psi_{a+b}(\cdot)$ by virtue of formula (1) satisfies the condition

$$\Psi_{a+b}(\hat{I}) = \Psi_{a+b}(\hat{p}_a + \hat{p}_b) = 1. \tag{4}$$

Because the functional $\Psi_{a+b}(\cdot)$ is positive definite, the Cauchy-Buniakowski-Schwarz inequality holds for it,

$$\left|\Psi_{a+b}\left(\hat{A}(\hat{I}-\hat{p}_a-\hat{p}_b)\right)\right|^2 \leq \Psi_{a+b}(\hat{A}^*\hat{A})\Psi_{a+b}(\hat{I}-\hat{p}_a-\hat{p}_b). \tag{5}$$

By virtue of equalities (4), the right-hand side of inequality (5) is zero. Therefore,

$$\Psi_{a+b}(\hat{A}) = \Psi_{a+b}\left(\hat{A}(\hat{p}_a+\hat{p}_b)\right). \tag{6}$$

Analogously,

$$\Psi_{a+b}(\hat{A}) = \Psi_{a+b}\left((\hat{p}_a+\hat{p}_b)\hat{A}\right). \tag{7}$$

Replacing $\hat{A} \longrightarrow \hat{A}(\hat{p}_a+\hat{p}_b)$ in (7) and taking (6) into account, we obtain

$$\Psi_{a+b}(\hat{A}) = \Psi_{a+b}\left((\hat{p}_a+\hat{p}_b)\hat{A}(\hat{p}_a+\hat{p}_b)\right). \tag{8}$$

We set the observable $\hat{K}$ into correspondence with the event F_k. Using formula (8), we obtain the expression for the mean of this observable

$$\langle\hat{K}\rangle = \Psi_{a+b}(\hat{K}) = \Psi_{a+b}(\hat{p}_a\hat{K}\hat{p}_a) + \Psi_{a+b}(\hat{p}_b\hat{K}\hat{p}_b) + \Psi_{a+b}(\hat{p}_a\hat{K}\hat{p}_b+\hat{p}_b\hat{K}\hat{p}_a). \tag{9}$$

The first and second terms in the right-hand side of (9) describe the scattering from the respective slits a and b. The third term describes the interference. Because $\hat{p}_a\hat{p}_b = \hat{p}_b\hat{p}_a = 0$, in the case where $[\hat{p}_a, \hat{K}] = 0$ or $[\hat{p}_b, \hat{K}] = 0$, the interference term disappears.

The interference pattern is purely determined by the structure of the abstract equivalence class Ψ_{a+b} would therefore organize our experiment as follows. We can prepare many copies of the same experimental device and distribute it over the globe. At each device, we perform one scattering act at random time instants. We then put together all the screens on which we have spots from hits of the scattered particles and put all these screens in one stack. For a sufficiently large number of screens, we must obtain a pattern close to that described by formula (9).

We note that in contrast to considering the same experiment in the standard quantum mechanics, we consider that the scattered particle hit either the domain of slit a or the domain of slit b in each separate case, not passing in a mysterious way through both slits simultaneously. This means that we consider a particle well localized in each separate act.

The quantum correlation problem is closely related to the nonlocality problem. This is because these correlations often look like a distant action. A typical example is the Einstein-Podolsky-Rosen (EPR) paradox. For example, in the variant proposed by Bohm [8], the result of measuring the projection of a spin of one particle from the singlet pair of particles with the spins 1/2 on one direction instantly and unambiguously predicts the result of measuring the projection of the other particle spin on the same direction even if the particles are separated by a large distance in space. It seems that this result contradicts the locality principle. But this contradiction arises only if we assume that the correlation results from the interaction between the particles at the instant of the measurement.

The notion that a correlation between separate elements of a physical system is always due to interaction between these elements is a deeply rooted delusion. It is even reflected in the terminology used in quantum mechanics. We can often hear the terms "exchange interaction," "nonforce quantum action," or reasonings about "strong quantum correlations."

In fact, quantum correlations are not caused by features specific to quantum interactions. For example, in the Einstein-Podolsky-Rosen paradox, the correlations between spin projections of two particles arise because these particles were created as a singlet pair for which the law of conservation of the proper angular momentum is satisfied. But the angular momentum conservation law also holds in classical physics.

In most cases, quantum correlations are due to the structure of the physical system ensemble participating in the quantum experiments. This structure is fixed by the procedure for preparing the ensemble under consideration, and the preparation procedure is in turn determined by the properties of the classical device used. As a rule, quantum correlations are therefore caused by the interaction between each separate constituent of the quantum ensemble and the classical device (or devices) preparing this quantum ensemble, not by the interaction between quantum objects. This interaction can be smeared both in time and space for separate constituents of the ensemble. It is therefore not amazing that it often seems that correlations contradict the principle of the locality of interaction. In fact, the locality principle is always satisfied for correlations. But this correlation must be verified not from the standpoint of interaction between different constituents of the quantum ensemble but from the standpoint of interaction of separate constituents of this ensemble with the devices preparing this ensemble.

3 Conclusion

Summarizing, we can draw the following conclusions.

Quantum theory, both relativistic and nonrelativistic, can be formulated such that it does not contradict the locality condition accepted in quantum field theory. The measurement process also does not contradict this condition.

The incompleteness of quantum mechanics noted by Einstein can be removed by introducing a new notion of the "elementary state," which is to be attributed to an individual physical system

The elementary state is the mathematical representation of the material carrier of the wave properties of the physical system.

A quantum state is an equivalence class in the set of elementary states and plays the role of the mathematical representation of the (quantum) ensemble of physical systems under investigation. The quantum state does not have the property of locality in the Minkowski space.

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SOME QUANTUM GENERALIZATION SEQUENCES OF EQUILIBRIUM STATISTICAL THERMODYNAMICS *

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1. GENERAL OUTLOOK

We present our generalization of statistical thermodynamics in which quantum effects are taken into account on the macrolevel without using the operator formalism while traditional relations between the macroparameters are preserved.

In formulated by us a generalized thermostat model, thermal equilibrium is characterized by an effective temperature bounded from below [1-3]. We introduce for the first time fundamental macroparameters of the theory [4]: the effective entropy and the effective action.

>From our theory it follows the effective entropy is nonzero at low temperatures. So we can write the third law of thermodynamics in the form postulated by Nernst. The effective action is also bounded from below.

As a very important result, we establish that the ratio of the effective action to the effective entropy in the low-temperature limit is determined by an universal stochastic-action constant depending on the Planck and Boltzmann constants simultaneously .

We study the discrepancy between the behavior of the action-to-entropy ratio in the low-temperature limit in the quantum equilibrium statistical thermodynamics proposed by us and one in quantum equilibrium statistical mechanics, which can be verified experimentally [5]. We demonstrate that the more early theory is the quasi classical limit of our theory.

2. STATEMENT OF PROBLEM

In recent years an increased interest in equilibrium thermodynamics as an autonomous macro-theory is observed. This tendency can be explained by at least two facts. From a fundamental standpoint, thermodynamics gives a universal macro-description of nature in which specific micro-models of objects are unnecessary. The role of an universal description without micro-models increases sharply under contemporary conditions when our knowledge about structure of matter is not seemed very complete one.

>From a pragmatic standpoint there is an obviously demand for thermodynamics using to describe the behavior of relatively small objects (nanoparticles, nuclear spins, etc.) in thermal equilibrium at low temperatures; to study high-energy physics problems (including

* The paper is done by the support of Fundamental Research Russian Fond (project 07-06-00239)

the quark-gluon plasma) and models of the early Universe, and to study fundamental sense of phenomena of viscosity, superfluidity etc. in micro- and macro-world.

3. THERMODYNAMICS DESCRIBING NOW

Now we have three versions of thermodynamic describing.
a). *Classical thermodynamics.*
In this theory the Zeroth principle has the form

$$T = T_0, \tag{1}$$

where T is the object temperature and T_0 is the thermostat temperature. We note that in this case fluctuations of any macro-parameters miss.

b). *Non-quantum version of statistical thermodynamics.*
In the contrary we have the Zeroth principle in the form

$$T = T_0 \pm \delta T, \tag{2}$$

where $T_0 = \langle T \rangle$ and δT is the temperature fluctuation of object. Here we suppose that

$$\frac{(\delta T)^2}{T_0^2} \leqslant 1. \tag{3}$$

i.e. the temperature fluctuation is less or equal the average temperature of object. Correspondingly there is the canonical distribution in the macro-parameters space:

$$dw(\mathcal{E}) = \exp\left\{\frac{F - \mathcal{E}}{\Theta}\right\} d\mathcal{E}. \tag{4}$$

where $\mathcal{E} = \mathcal{E}(V, T)$ is the object energy, $\theta = k_B T_0$ is the distribution modulus.

At last it is accepted to think that once more version of thermodynamics describing follows from

c). *Quantum statistical mechanics.*
In this case the canonical distribution in the micro-parameters space is

$$w_n = \exp\left\{\frac{F - \varepsilon_n}{k_B T}\right\}, \tag{5}$$

where $T = T_0$ and ε_n is a quantized energy spectra of object. Many people are sure that from quantum statistical mechanics they can get the most complete thermodynamics version. But we will show that it is not true.

4. PRINCIPAL DIFFICULTIES OF QUANTUM STATISTICAL MECHANICS

It is important to say that:
* The contribution of the energy $\frac{\hbar\omega}{2}$ to canonical distribution vanish because of the normalization condition. Hence the canonical distribution (5) in this theory is initially quasi-classical.

* Temperature fluctuations are absent from the very beginning.
* All results are valid only in thermodynamics limit. So all other fluctuations vanish too.

Our point of view is such:

1. Any generalization of quantum statistical mechanics is not effective.
2. An alternative way is generalization of statistical thermodynamics (in the quantum version).

5. OUR MAIN IDEA IS EFFECTIVE TEMPERATURE [1] (1998)

We propose to pass from the classical thermostat model to a more general quantum one. Correspondingly, instead

$$\langle \varepsilon_{cl.} \rangle = \Theta \equiv k_B T, \tag{6}$$

we get

$$\langle \varepsilon_{qu.} \rangle = \Theta \equiv k_B T^*. \tag{7}$$

Then the generalized canonical distribution [4] in macro-parameters space is

$$dw(\mathcal{E}) = \rho(\mathcal{E}) d\mathcal{E} = \frac{1}{k_B T^*} \exp\left\{ -\frac{\mathcal{E}}{k_B T^*} \right\} d\mathcal{E}, \tag{8}$$

where $\mathcal{E}$ - a random c-number quantity.

In this formula effective temperature as a characteristic of quantum and thermal stochastic actions together is a function of two variables

$$T^* \equiv \frac{\langle \varepsilon_{qu} \rangle}{k_B} = \frac{\hbar\omega}{2k_B} \coth \frac{\hbar\omega}{2k_B T}. \tag{9}$$

Correspondingly the condition of generalized equilibrium is

$$T^* = T_0^*. \tag{10}$$

But at the same time $T = T_0$ and $\omega = \omega_0$ simultaneously where T_0 and ω_0 are characteristics of a quantum thermostat normal mode.

Limit cases of effective temperature are:

* if $T \to \infty$, then $T^* \to T$;
* if $T \to 0$, then

$$T^* \to T^*_{min.} \equiv \frac{\hbar\omega}{2k_B T}. \tag{11}$$

SO we can give the definitions for concepts of low and high temperatures starting from the criteria:

Low temperatures: $T \ll T^*_{min.}$.
High temperatures: $T \gg T^*_{min.}$.

6. EFFECTIVE ENTROPY

Now we can calculate effective entropy using the generalized canonical distribution (8) with effective temperature (9). We apply our theory for a concrete model. For any object characterized by frequency ω we choose the model of a harmonic oscillator. To make calculations we pass to the dimensionless probability density

$$\widetilde{\rho}(\mathcal{E}) = \rho(\mathcal{E})\hbar\omega/2. \tag{12}$$

Then effective entropy is

$$S^* = -k_B \int \widetilde{\rho}(\mathcal{E}) \ln \widetilde{\rho}(\mathcal{E}) \left(\frac{\hbar\omega}{2}\right)^{-1} d\mathcal{E} = k_B \left[1 + \ln(\coth \frac{\hbar\omega}{2k_B T})\right] = $$
$$= k_B \left[1 + \ln(\coth \frac{T^*_{min}}{T})\right]. \tag{13}$$

Limit cases for effective entropy are:
* if $T \to \infty$, then $S^* \to k_B \ln T + const$;
* if $T \to 0$, then $S^* \to S^*_{min} = k_B$.

This result corresponds to the initial formulation of the Nernst theorem with $S_{min} \neq 0$.

7. EFFECTIVE ACTION AS MACROPARAMETER

Our new idea is to introduce a notion of action into quantum statistical thermodynamics. For the classical oscillator with energy ε we suppose to pass to new variables action - angle j, φ instead of variables p, q . Then the action in classical mechanics is

$$j = \frac{\varepsilon}{\omega} \tag{14}$$

and this quantity is an adiabatic invariant.

According to Boltzmann (1904), the action as a macroparameter for the classical oscillator in a classical thermostat is determined as

$$J \equiv \langle j \rangle = \frac{\langle \varepsilon_{cl} \rangle}{\omega} = \frac{k_B T}{\omega}. \tag{15}$$

So we determine effective action for quantum oscillator in the quantum thermostat analogically:

$$J^* = \langle j \rangle = \frac{\langle \varepsilon_{qu} \rangle}{\omega} = \frac{\mathcal{E}^*}{\omega} = \frac{k_B T^*}{\omega}. \tag{16}$$

We have limit cases:
* if $T \to \infty$, then $J^* \to J$;
* if $T \to 0$, then $J^*_{min} = \frac{\hbar}{2} \neq 0$. Here $J^*_{min.}$ has a sense intrinsic, or own, action of object.

There is a fundamental interconnection between effective action and effective entropy:

$$S^* = S^*_{min}\left[1 + \ln\frac{J^*}{J^*_{min}}\right] = S^*_{min}\left[1 + \ln\coth\frac{T^*_{min}}{T}\right]. \tag{17}$$

Effective action and effective entropy are very important characteristics of objects in a thermostat. In general case action-to-entropy ratio has the form

$$\frac{J^*}{S^*} = \frac{J^*_{min}}{S^*_{min}} \cdot \frac{\coth T^*_{min}/T}{1 + \ln\coth T^*_{min}/T} = \varkappa \cdot \frac{\coth \varkappa\omega/T}{1 + \ln\coth\varkappa\omega/T}. \tag{18}$$

Here

$$\varkappa \equiv \frac{J^*_{min}}{S^*_{min}} = \frac{\hbar}{2k_B} = 3,82 \cdot 10^{-12}\ \text{K} \cdot \text{c} \tag{19}$$

is an universal constant of joint quantum and thermal stochastic action.

8. THE ACTION AND THE ENTROPY IN QUANTUM STATISTICAL MECHANICS

For comparison with our results above let us consider analogical quantities in quantum statistical mechanics. According to Planck, in this quasi-classical theory we have

$$J_{quasi} = \frac{\mathcal{E}_{quasi}}{\omega} = \frac{\hbar}{\exp(\frac{\hbar\omega}{k_B T}) - 1} = J^* - \frac{\hbar}{2}, \tag{20}$$

$$S_{quasi} = -k_B\left\{\frac{\hbar\omega}{k_B T}\left(1 - \exp\frac{\hbar\omega}{k_B T}\right)^{-1} + \ln\left(1 - \exp\left(-\frac{\hbar\omega}{k_B T}\right)\right)\right\}. \tag{21}$$

The subject of our especially interest is the limiting behavior of the corresponding quantities in both theories. At $T \to 0$ we have in our theory

$$J^* \to \frac{\hbar}{2}; \quad S^* \to k_B; \tag{22}$$

$$\frac{J^*}{S^*} \to \varkappa \neq 0. \tag{23}$$

In quantum statistical mechanics at $T \to 0$ we have not only

$$J_{quasi} \to 0; \quad S_{quasi} \to 0, \tag{24}$$

but the ratio

$$\frac{J_{quasi}}{S_{quasi}} \to \frac{\hbar\exp(-\hbar\omega/k_B T)}{k_B(\hbar\omega/k_B T)\exp(-\hbar\omega/k_B T)} = \frac{T}{\omega} \to 0. \tag{25}$$

Which of the two theories is right we will soon know from experiments.

9. THE COMPARISON WITH THEORIES [5] CONSIDERING THE GRAVITY DUALS AT $T \neq 0$

In these theories it was shown that shear viscosity η -to-entropy density $s = \partial S/\partial V$ ratio is bounded below at $T \to 0$:

$$\frac{\eta}{s} \to \varkappa. \tag{26}$$

Because according to our theory

$$\varkappa \equiv \frac{J^*_{min}}{S^*_{min}} = \frac{\hbar}{2k_B}, \tag{27}$$

this limit is universal .

Now the experimental situation is such. In experiments with quark-gluon plasma (RHIC, 2005) and liquid He-4 (Andronikashvili, 1948) it was shown that the ratio is independent on the temperature and is not equal to zero, limited below by a constant close to universal constant $\varkappa$. This fact follows automatic from our theory and contrary to quantum statistical mechanics. We hope that validity of our theory can be confirmed soon in new experiments as in high energy Physics (LHC) so in hydrodynamics of superfluids.

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100"

Научное издание

XIII International Conference on Selected Problems of Modern Physics, dedicated to the 100th anniversary of the birth of D. I. Blokhintsev (1908–1979)

Proceedings of the Conference

XIII Международная конференция по избранным проблемам современной физики, посвященная 100-летию со дня рождения Д. И. Блохинцева (1908–1979)

Труды конференции

E1,2-2009-36

Ответственный за подготовку сборника к печати
С. М. Елисеев.

Сборник отпечатан методом прямого репродуцирования
с оригиналов, предоставленных оргкомитетом.

Подписано в печать 07.04.2009.
Формат 60 × 90/16. Бумага офсетная. Печать офсетная.
Усл. печ. л. 25,88. Уч.-изд. л. 44,64. Тираж 350 экз. Заказ № 56527.

Издательский отдел Объединенного института ядерных исследований
141980, г. Дубна, Московская обл., ул. Жолио-Кюри, 6.
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www.jinr.ru/publish/