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XII International Conference

Selected Problems of Modern Physics

Proceedings of the Conference

Joint Institute for Nuclear Research
Bogoliubov Laboratory of Theoretical Physics

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XII International Conference on Selected Problems of Modern Physics

ЭКЗ. ЧИТ. ЗАЛА

Dedicated to the 95th anniversary of the birth
of D. I. Blokhintsev (1908–1979)

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Dubna, June 8–11, 2003

Section I
Problems of Quantum Field Theory
Proceedings of the Conference

Edited by B. M. Barbashov, G. V. Efimov, A. V. Efremov, S. M. Eliseev,
V. V. Nesterenko and M. K. Volkov

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The collection of papers includes the talks given at the XII International Conference on Selected Problems of Modern Physics, dedicated to the 95th anniversary of the birth of D. I. Blokhintsev (1908–1979), the organizer and first director of the Joint Institute for Nuclear Research. This collection contains the papers reflecting scientific interests of Dmitrii Ivanovich and reminiscences of some theorists of the important events of his many-sided activity.

Международная конференция по избранным проблемам современной физики (12; 2003; Дубна).
Труды XII Международной конференции по избранным проблемам современной физики, посвященной 95-й годовщине со дня рождения Д. И. Блохинцева (1908–1979), Дубна, 8–11 июня 2003 г. — Дубна: ОИЯИ, 2003. — 379 с., 24 с. фото.

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В сборник вошли избранные доклады, представленные на XII Международной научной конференции по избранным проблемам современной физики, посвященной 95-й годовщине со дня рождения Д. И. Блохинцева (1908–1979) — организатора и первого директора ОИЯИ. Включены работы, отражающие научные интересы Дмитрия Ивановича Блохинцева, а также воспоминания сотрудников о важных этапах его многогранной деятельности.

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Дмитрий Иванович Блохинцев

Proceedings of XII International Conference on Selected Problems of Modern Physics, Section I, Dubna, June 8–11, 2003.

Preface

The 12th International Conference on Selected Problems of Modern Physics dedicated to the 95th anniversary of the birth of the outstanding scientist Professor Dmitrii Ivanovich Blokhintsev (1908-1979), the first supervisor and director of the first atomic station in the world and the founder of the Joint Institute for Nuclear Research, was held at Dubna on June 8-11, 2003.

This series of conferences was initiated by D.I. Blokhintsev as meetings on fundamental problems of quantum field theory. Thirty-five years passed since the first meeting enriched the methods of field theory and opened many new areas. The present conference was the 12th of this kind organized by the Joint Institute for Nuclear Research. The conference was opened by the memorial session. Further work of the conference proceeded in two parallel parts – "Problems of Quantum Field Theory" and "Physical Investigations at Pulsed Reactors".

The topic of the conference reflects the current status of many fundamental problems in modern physics (Quantum Mechanics; Quantum Field Theory, QCD; Hadron Physics; Gravitation and Cosmology) and those areas of physics to which Blokhintsev has made significant contributions. These Proceedings collect the talks presented at the part "Problems of Quantum Field Theory".

The total number of participants of the conference was the following: twenty scientists came to Dubna from "remote abroad", one hundred and twenty from "neighboring abroad" (Community of Independent States and Russia). We had twenty-three plenary talks and seventy-five sectional talks. This volume compiles the plenary and sectional talks and contains the main part of presented reports. We hope this will give an idea of the scientific content of the conference.

The organization of the conference would not be possible without the sponsorship of the Russian Foundation for Basic Research (grant 02-2-2002/19), the Ministry of Industry, Science and Technology, the Ministry of Atomic Energy of the Russian Federation and JINR through the Heisenberg-Landau, Votruba-Blokhintsev and Bogoliubov-Infeld Programs. On behalf of the Organizing Committee we gratefully acknowledge this support.

The Editors.

Dmitrii Ivanovich Blokhintsev
(11.01.1908 - 27.01.1979)
(Essay of scientific activity)

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Abstract — A brief survey is given on the scientific activity of the Corresponding Member of the USSR AS Dmitrii Ivanovich Blokhintsev, one of the pioneers of atomic science and technology in USSR, the organizer and the first director of the Joint Institute for Nuclear Research in Dubna.

"None in the world will wake
The soul, who left to the rest,
But on the Earth, you stranger,
Your songs will wander be."
(D.I. Blokhintsev, Coll. "Muza in the temple of science", M., 1982.)

There is a good tradition in our country to name the streets of cities after their famous citizens. Among them are Blokhintsev streets in Russian little towns of Dubna and Obninsk, named in honor of the outstanding Soviet physicist, distinguished organizer of science, a colleague of Kurchatov in creation, formation and development of nuclear science, technology and nuclear power engineering in our country and countries of East Europe. "The name of Dmitrii Ivanovich Blokhintsev ranks row with the names of Sechenov, Timiryazev, Umov, Lebedev, Vernadsky, Vavilov, Khokhlov and many others, who make the pride of our nation" ("Pravda", 23 January, 1980).

Dmitrii Blokhintsev enriched the world's science by fundamental works in solid state and statistical physics, acoustics, physics of reactors and atomic power engineering, quantum mechanics, quantum field theory and quantum electrodynamics, high-energy and nuclear physics, philosophy and methodology of science. His role in education of physicists and engineers in our and East Europe countries is widely known and received the deserved acknowledgement. He was lucky enough to be the founder of many directions in science but first of all he was the personality - phenomenally versatile and many-sided person, scientist, engineer, inventor, teacher, artist, poet, state and public figure, the contact with whom was a great pleasure.

Strong influence on the outlook of Dmitrii Ivanovich was made by the acquaintance with the works by K.E. Tsiolkovski and personal correspondence with him. DI perceived from Tsiolkovskiy that spirit of the Russian science of the beginning of the XX-th century, which was not just aspiration to achievement of specific scientific results, but rather the creation of integral harmonic world outlook. Admiration of beauty and harmony of the world, and also the highest degree of respect to Nature and Man was inherent in the mentality of Tsiolkovskiy. Just for this reason – loved to stress DI – Tsiolkovskiy never used such word combinations as "conquest" of Space, but he always spoked about its "exploration". Blokhintsev succeeded in preserving these youthful ideals of the world perception till the end of his life. This tendency toward the truth of his initial period, ever increasing with the years, forced him not only to be up to date in all basic scientific achievements in both physics and other fields of knowledge, mathematics, philosophy, biology, economy, etc. but also to develop his own original viewpoints and judgments.

Dmitrii Ivanovich believed that it is rather easy to learn solving already formulated problems in any fashionable field of contemporary physics. Almost any person with a sufficiently regulated mind, can become a not bad theoretician. More difficult is to pose problems by himself. Physicists whose interests are determined by their own outlook are considerably less numerous, but just they most frequently become the authors of those "fashionable" directions in science which give food and work to minds of many others.

The famous experiments of Rutherford in splitting an atom forced young Dmitrii Blokhintsev, a graduate of Moscow industrial-economical technical school, to focus his attention on those enviable possibilities which nuclear energy promises and this determined his further way. In 1926 he entered the Physics Department of Moscow State university (MSU), where distinguished scientists as L.I. Mandel'shtam, S.I. Vavilov, N.I. Luzin, D.F. Egorov and I.E. Tamm were his teachers.

There were the years of quantum mechanics formation and of explanation with its aid of many mysterious physical phenomena. DI's early works were already noted by large skill and depth of physical thought. For his graduate work he was recognized worthy the Doctor of Sciences degree (1934).

DI calculated the work function of electrons from the metal and based on it offered an explanation of the anomalous magnetic properties of bivalence metals. This was the reason that energy of electron in these metals depended not only on the absolute value of its momentum but also on its direction with respect to the crystal axes. He generalized Bloch's theory to the case of overlapping zones. His formula for the energy of the overlapping zones was of special importance.

At the same time, DI opened a nonlinear dependence of the radiated light (in Stark effect) on the intensity of the falling one (1933). This work was the first study on the nonlinear optics, which is now being developed substantially.

During the subsequent years he gave the first explanation of the mechanism of the mysterious phosphorescence phenomenon. The basic idea of this work wonderfully illustrates figurativeness of thinking of the scientist. He noted that the presence of local impurities in phosphorouses led to the appearance of local levels between the lower zone and the conduction band. Therefore, the electron which fell on this level and "hole" in the lower zone are space divided so that the probability of their recombination substantially decreases and leads to the anomalously long recombination time.

In his subsequent works DI developed this basic idea and, in particular, investigated the kinetics of phosphorescence and he was the first to explain the experimentally observed luminescence behaviour with time.

Further he turned to the effect of the rectification of current by semiconductors and found a simple and correct explanation of this phenomenon. The essence of his explanation is based on the fact that near the contact of two semiconductors the gradient of electric field leads to the appearance of a space charge and, therefore, to a change in the electrical conductivity. However, the sign of this change depends on the direction of current that causes the rectifying action of the system.

These and subsequent basic works of DI, in particular, the development of the theory of heteropolar and colored crystals, and the theory of electrical breakdown of dielectrics played an important role in the development of studies in quantum solid state theory and in practical use of their results.

Already in his early works he showed up a deep understanding of the essence of quantum mechanics and originality of thinking anticipating a further development of physics. Especially characteristic in this respect is the work on calculation of "spectral line shift" caused by a reverse action of radiation. The work actually contained the theory of the Lamb shift, opened only ten years later and served as the beginning of quantum electrodynamics. The formula obtained by DI for the Lamb shift differs from the famous one of H. Bethe only by a numerical coefficient, which appeared as a result of the ultraviolet cut. The work was reported at the seminar in the Lebedev Physical Institute in 1938. Unfortunately, this very important discovery was not understood by contemporaries and the article was rejected by the editorial staff of JETF. It

was published only in 1958 in the collection of Blokhintsev's works, although its results were cited earlier in the review by Ya.A. Smorodinsky (Uspekhi Fizicheskikh Nauk, 1949, v. 39, p. 325).

In 1935 D.I. Blokhintsev was elected professor of the Department of Theoretical Physics of MSU. From that time and to his last days of life his activity was tidily connected with the Physical Faculty of MSU, where he managed the chair of Nuclear Physics and prepared many generations of specialists. DI was one of the organizers of the Department of Nuclear Physics of the Physical Faculty of MSU and creator of the Dubna branches of MSU and of the Moscow Institute of Radio Engineering, Electronics and Automation, whose tasks were to approach the student audience to research laboratories.

DI liked the student audience. He was a frequent guest in a student hostel. Among his students there are many well-known scientists who produce a worthy contribution to the development of science. "Science – he said – is a matter of talent and vocation, and now science is also a matter of a team. Nevertheless, among scientists, regardless titles and the fact who they are – graduate or PhD students – is a special category of people obsessed by passion to science, scientists whose great talent only seldom gives gladness to them but often causes a constant flour of dissatisfaction to achievements. Just on these rare brittle people entire success of an Institute is based. These people are usually unpractical, they are easily hurtled and vulnerable, they must be taken care of, they are necessary to be guarded, they are white cranes".

Dmitrii Ivanovich created and read many fundamental theoretical courses, among which especially should be mentioned the course of quantum mechanics, which composed the basis of the first in the world university textbook. It has withstood 22 publications since 1944 – six in our country and 16 in other countries in the world in nine languages. Many generations of student-physicists were brought up on it. For his successes in quantum mechanics D.I. Blokhintsev was awarded the State Prize.

From 1935 through 1950 DI, besides the teaching activity in MSU, worked in the Lebedev Physical Institute. In the same years he was the member of the Scientific Council of the Physical Institute in Kiev (Ukraine), where he supervised works of young Ukrainian physicists. After liberation of Kiev from fascists occupation he took active part in the restoration of science in the Ukraine.

Blokhintsev's attention in the pre-war years was concentrated on fundamental questions of quantum mechanics. This activity continued also in the postwar period. He established a relation between the quantum description of a system of particles in the phase space and its classical distribution function. In particular, he revealed the impossibility of direct transfer into classics of quantum condition of the indistinguishability of identical particles. He first introduced the concept of "quasi-probability" (1940) at which Dirac arrived much later.

He established that the diffraction pattern not always gives a possibility for unambiguous judgment about the form of the observed object. Objects of various forms can give similar diffraction patterns. He showed also that under some conditions it is possible to see an atom with the aid of an electron microscope. He showed that the "detailed balance principle" can be broken in spite of time reversibility.

D.I. Blokhintsev is the author of the quantum ensemble concept. On the basis of this concept he gave an objective treatment of the wave function. This approach, being of large heuristic value, helps removing a number of internal contradictions in the interpretation of quantum mechanics and established a close connection between quantum mechanics and statistical physics. This concept of the "Moscow school" gives the modest role to an observer and emphasizes everywhere the objective nature of the quantum ensembles and regularities controlling them. He was the first to realize the special role of a classical instrument in quantum mechanics as an unstable state of a macroscopic system. It was an important step in overcoming the barrier, set by Nils Bohr authority, who considered that there is no sense in uniting a measuring instrument with a microscopic system, since then another classical instrument will be required for studying the integrated system.

The works by D. Blokhintsev played an important role in formulation of methodological basis of contemporary quantum theory. In the preface to his book "Fundamentals of quantum mechanics" (fifth edition, 1976) he wrote: "I always gave much importance to the correct methodology without which even the most outstanding mind acquires the nuance of handicraft. Therefore, the materialist methodology, sometimes clearly sometimes less clearly pierces the entire book". In more detail these questions were considered in his monographs "Fundamental Questions of Quantum Mechanics" (1966) and "Quantum Mechanics (Lectures on selected questions)", 1981.

During the years of the second world war DI almost completely turned to work on the military subjects in the field of acoustics and soon became the leading specialist in this field creating the acoustics of inhomogeneous and moving media. On the basis of the gas-hydrodynamics equations he obtained the equations of acoustics for the most general case ("Blokhintsev's equation")

of which he derived a number of acoustic laws, explained and calculated diverse acoustic phenomena in the moving and inhomogeneous media (including the turbulentenes) which concern, on the one hand, the mechanism of generation of noise and, on the other hand, methods and means of its reception. In particular, the sound produced by propellers, the excitation of resonators by a gas flow and methods of reduction in this excitation, protection of the sound receivers from the large- and small-scale fluctuation of the incident flow and a number of others problems forming the basis of the theory of acoustic location of aircraft and submarines. He formulated the equations of geometric acoustics.

He introduced an extremely fruitful concept of the pseudo-sound as a phenomenon which possesses the formal criteria of sound, but which is not an acoustic process. In some manifestations the pseudo-sound is identified with the Rayleigh waves or with the Fresnel zones of radiation in the electrodynamics (although does not reduce to these phenomena). He formulated the theorem which determines a necessary and sufficient condition for generation of sound during the motion of a body in the liquid or during the motion of liquid. Further development of this question led him to the conclusion that the basis of any emission, including acoustics, are the phenomena analogous to the Vavilov-Čerenkov effect. He emphasized high fruitfulness of this acoustic and electrodynamic analogy.

For these works D.I. Blokhintsev was awarded the Order of Lenin (1946). Subsequently they were united in the monograph "Acoustics of an Inhomogeneous Moving Medium" (1946), published twice in USSR and abroad which is now the classics of a large intensively developing branch of physics. Almost every work in physics of noise in the turbulent boundary layer which arose on fuselages of modern liners or noise of exhaust jets of their engines refer the Blokhintsev's book as a basis of new acoustics.

In the last years of the war and in the postwar years the task of usage of atomic energy became vitally important for our country. Beginning in 1947, Dmitrii Ivanovich actively worked on the development of Soviet atomic science and technology headed by I.V. Kurchatov. Igor Kurchatov made a great impact on his formation as a leader of large scientific and technical projects capable of uniting a collective and to inspire it for reaching a result.

Kurchatov saw in an outstanding theoretician the talent of a big organizer and research engineer. Since then the name of D.I. Blokhintsev is inseparably connected with the history of the peaceful atom. Together with Kurchatov, he became the initiator of the creation of the world first atomic power station (Obninsk). In his book "Birth of the peaceful atom" DI wrote that "... he was lucky to participate in the great epic of the creation of Soviet atomic power engineering".

In 1950 he was appointed the fist director of the Institute of Physics and Power Engineering in Obninsk, and also the scientific supervisor of creation and putting into operation of the the world first atomic power station ("Atomnaya energiya", v. 44, no. 6, 1979). To him belong the physical and design calculations of the reactors of this first APS. In the middle of 1954 the first APS gave current. A long-standing successful operation of the station confirmed the correctness of choosing the reactor type and basic parameters of the first APS. For this work D.I. Blokhintsev was awarded the Lenin prize (1955). His talk on the first APS in Obninsk was the main one at the first International Conference on Peaceful Use of Atomic Energy in Geneva (1955).

In the subsequent years he calculated and supervised the development of design and construction of a new type of reactors – the promising, in industrial sense, fast-neutron reactors with the liquid-metal heat-transfer agent. Now such reactors are exploited at other APS. He also developed the effective methods of calculation of slow and intermediate neutrons reactors. For the fulfillment of important State tasks in creation of atomic power engineering D.I. Blokhintsev was awarded the title of Hero of Socialist Labor (1956).

Reactors attracted Blokhintsev's attention not only as the basis of power plants, but also as an intensive neutron source for diverse scientific studies. He is the author of the remarkable invention (1955) – the fast pulsed reactors (IBR-1 and IBR-2), the pulsed power of which at a very small mean power is competitive with the most powerful reactors of a constant action. The first reactor of this type, IBR-1, was built and put into operation in Dubna in the Laboratory of Neutron Physics under the scientific leadership and with the direct participation of DI (1960). (He frequently called it his "dowry"). After many years of work this reactor proved to be a remarkable tool for studies in nuclear physics, physics of liquid and solid states and elementary particles physics. For this work D.I. Blokhintsev was awarded the State Prize (1971). During the subsequent years he was the scientific leader of the design and construction of a more advanced and powerful reactor IBR-2. He led its physical start (1977) and to the last days of his life the preparation of its power start. Now this last engineering creation of DI gives interesting physical results.

DI initiated the creation of the Joint Institute for Nuclear Research. In 1956 the Committee of Plenipotentiaries of eleven countries unanimously elected him the first Director of this Institute. The leading scientists of the Soviet

Union and other JINR Member States were drawn in the work at the Institute. In addition to the two existed in Dubna Laboratories – Laboratory of Nuclear Problems and Laboratory of High Energies, three new Laboratories were created: Laboratory of Nuclear Reactions, Laboratory of Neutron Physics and Laboratory of Theoretical Physics. The last two – on the initiative of Blokhintsev. During the period of his stay as Director (1956-1965) the Institute was finally organized and converted into the largest scientific research center which won high authority and international recognition for his research. It became the smithy of the scientific staff of the Member States. During the subsequent years (1965-1979) Blokhintsev headed the Laboratory of Theoretical Physics. He also made a noticeable personal contribution to the world scientific authority of Dubna.

Fundamental problems of theoretical physics always drew DI's attention. In 1957, based on the "deuteron peaks" in the reactions of quasielastic high-energy proton scattering on nuclei, discovered by the group of M.G. Meshcheryakov, he proposed and developed the idea of fluctuations of nuclear density, capable as a whole to receive a large momentum transfer. The idea of "Blokhintsev's fluctons" best manifested itself 20 years latter when in reactions with relativistic nuclei the so-called "cumulative" particles were discovered. Later on DI participated in the development of the multi-quark interpretation of fluctons. Just they were the subject of the last DI's report at the Tokyo Conference on High Energy Physics in the fall of 1978. These studies grew now in the new promising direction – relativistic nuclear physics. In particular, just the presence of multi-quark states explains the "core" of nuclear forces. The remarkable confirmation of the flucton idea was obtained in experiments at CERN for deeply inelastic scattering of muons on nuclei and in the production of cumulative protons by a neutrino beam at Serpukhov.

In the same years he investigated (on the basis of the optical "eikonal" model) the structure of nucleons, established its division into the central and peripheral parts and came to the conclusion about the dominant role of peripheral interactions. He showed the contradiction of the hydrodynamic approach to the multi-particle production processes with the basic principles of quantum mechanics (1957). The force of this criticism increasingly more begins to appear now as more correlation and spin measurements are carried out.

Dmitrii Ivanovich proposed the idea of existence of several vacua in quantum field theory and spontaneous transition between them (1960). This idea is intensively used in contemporary unified theories of elementary particles. He was the first to point out the possibility of existence of the so-called "unitary limit" in weak interactions (1957) and the limit of applicability of quantum electrodynamics.

A large and important cycle of his works was dedicated to quantum field theory, nonlinear and nonlocal theories, non-Hamiltonian approach, and stochastic space-time geometry. In particular, he showed the possibility of breaking the finite time signal propagation "in the small" without the essential breakdown of this fundamental law in the macrocosm. D.I. Blokhintsev proposed a new approach to nonlocal fields based on the hypothesis of stochastic fluctuations of the space-time metrics.

Investigating substantially nonlinear fields, DI came to the conclusion that the concept of point-like coordinates becomes meaningless and requires a change in the geometry of microcosm if the mass spectrum of particles is bounded from above. These questions found their reflection in the book by D.I. Blokhintsev "Space and Time in the Microworld", published in 1970 and in 1982 in our country and repeatedly republished abroad.

Considerable time was spent to searches for a non-Hamiltonian S-matrix method in the field theory which would replace the traditional Hamiltonian formalism. Blokhintsev proposed the specific version of the mathematical apparatus of this method (1947) based on the introduction of the "elementary scattering matrix". This apparatus gave the results which coincided with the approaches of usual relativistic invariant perturbation theory.

The creative activity of Dmitrii Ivanovich did not fade to his last days. He investigated the problem of anomalously short time of the ultra-cold neutrons (UCN) storage and proposed the simple mechanism of explanation of this effect – heating of UCN by hydrogen adsorbed by surface. He worked at one of the most complex problems, confinement of quarks, and proposed the original hypothesis for the nature of this phenomenon. In the last few years his thoughts repeatedly returned to the "Big Bang" in cosmology. Analyzing Friedman's model he arrived at the conclusion that the visible part of our universe could not be formed within the limits of four-dimensional space-time and proposed his original hypothesis of the existence of extra space dimensions, meta-space, in which meta-bodies and antibodies collide. Our Universe could be formed as a result of such a collision.

DI always took a great interest in the philosophy and methodologies of science. He repeatedly had to defend in discussions the idea of materialism from both his opponents and primitive defenders. Much attention was given by him to defence of the energy conservation law as the basis of materialistic natural science and the correct understanding of the theory of relativity and

atomic theory. In his first book "What is the theory of relativity?" he gave not only comprehensible exposition of this theory, but also its first correct interpretation. Special importance he gave to his last work "On the Relationship of Applied and Basic research" where, being based on the specific features of man as a biological form – inquisitiveness, extended transmission of information from one generation to the next one, which caused the detachment of man from the remaining living world, the needs for the emotional contact with the external world – he came to the conclusion on inevitability of increase of the activity of people in the production of ideas. Very interesting are his unpublished "Science and Skill" and "Essays on the Materialistic Philosophy".

His gift of foresight appeared not only in his scientific and philosophical works, but also in the organization of conferences, in particular, conferences on nonlocal quantum field theory (which, actually, were conferences on fundamental problems of field theory) in the period of its almost complete refusion when it was necessary to have courage to foresee its subsequent renaissance. He was the permanent chairman of these unique conferences during 1964-1979. In accordance with his understanding of creative activity, DI proposed such organization of conferences, which would give to its participants as much the leisure as possible (not the rest, but the leisure – in that sense of this word, what ancient Greeks put into it, and which is so small in the contemporary life). He considered it useful not only to listen to reports, but even more useful to converse with interesting men. The conferences and the workshops organized under his leadership, thoroughly planned, gave participants the possibility of maximum of self-realization. This was one of the reasons for a constant increase of their popularity.

To him belongs a big role in the establishment of the first scientific exchanges between CERN (Geneva) and JINR, in the organization of many international conferences and symposia, including the so-called Rochester conferences - the largest conferences on high-energy physics.

D.I. Blokhintsev was the outstanding public figure: the member of the Soviet Peace Committee, the scientific adviser of Secretary General of the UN, Vice President (1963-1966) and President (1966-1969) of the Union of Pure and Applied Physics (IUPAP), the member of the USSR State and Lenin Prizes Committee and a large number of commissions, scientific councils and editorial boards.

Blokhintsev's merits were recognized by the highest Soviet and foreign rewards – the title of Hero of Socialist Labor, Lenin and two State prizes winner, four Orders of Lenin, the order of the October Revolution, the Order of the

Red Banner of Labor, the nominal gold medal of the Academy of Sciences of Czechia, the order of Cyril and Methody of 1-st degree (Bulgaria), the highest orders of Romania, Mongolia, and many other orders and medals of the USSR and other countries.

The public activity D.I. Blokhintsev was marked by the honorable certificate of the World Council for Peace (1969). He was chosen member of the Academies of Sciences of many countries and the honorable doctor of a number of universities. Scientist, citizen he in his articles and presentations repeatedly emphasized that the scientist must not be locked in the professional shell: "...Our duty, great duty of scientists and engineers of our time, and no one must escape from this, is to explain to all people what threat will hang over the world and let then the wrath of entire humanity stop the madmen of atomic warfare".

The many-sidedness of D.I. Blokhintsev, his universality appeared not only in the scientific but also in the aesthetical perception of the world. He was original poet and artist whose pictures were repeatedly demonstrated at exhibitions and their reproductions were published in periodicals and newspapers. Through entire life he carried love for the poetry. His many verses were published in the periodicals and in the collection "Muza in the Temple of Science" (1982). But the main part of his verses still awaits for publication. In his pictures and verses he is a fine psychologist, attentive observer, deep philosopher. He deeply understood the process of creative thinking directed for the creation of a new in science and skill. "Creation – he said – is not volitional event but the special state of spirit and intellect which implicates into the process of thinking rich aesthetical experiences".

The personal charm of ingenious collocutor, the unique combination of calmness and boiling creative energy which he always generously shared left lasting impression. The essence of his personality is possible to express briefly – the creation. Personal contacts with him enriched the collocutor. He began to feel himself as a creative personality and acquired the belief in his own forces.

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Memorial Session

Proceedings of XII International Conference on Selected Problems of Modern Physics,
Section I, Dubna, June 8–11, 2003.

Дмитрий Иванович Блохинцев - организатор и первый директор ОИЯИ

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Открывая эту конференцию, я хотел бы прежде всего отметить что Дмитрий Иванович сделал очень много еще до того, как организовал и возглавил ОИЯИ. Он был талантливым ученым и крупным специалистом как в фундаментальных, так и в прикладных областях физики, блестящим инженером, выдающимся организаторим, осуществлявшим масштабные научно-технические проекты. Он был одим из организаторов и первым директором физико-энергетического института в Обнинске, где была спроектирована и построена первая в мире атомная электростанция. Это было событие поистине всемирного значения, ибо впервые на практике была показана возможность и экономическая целесообразность получения злектрической знергии на основе использования энерги расщепления атомного ядра.

Вспоминая о Дмитрии Ивановиче, невозможно следовать заранее подготовленному тексту и, с вашего разрешения, я отвлекусь. Вспоминаю газеты июня 1954 года пестрящие, сообщениями о запуске первой в мире атомной станции в СССР. В то время я, окончив Суворовское училище, находился а военном лагере и планировал продолжить свое высшее образование. Но эта информация так подействовала на меня, что я понял:следует поступать только в Университет.

Позже на физфаке МГУ я посещал лекции Дмитрия Ивановnatural, которые всегда привлекали многих слушателей. В 1956 году, когда был организован ОИЯИ, один из преподавателей физфака сказал мне:"Попробуйте попасть в Дубну, Москва-это уже научная провинция". И я внял этому совету.

Попав в ОИЯИ, я общался с Д.И. Блохинцевым уже в начале своей работы, несмотря на то, что был тогда всего лишь молодым научным сотрудником. Изумляла широта научных интересов Дмитрия Ивановича: мировой авторитет в области акустики, квантовой механики, физики реакторов... Его знаменитый и популярный учебник по квантовой механике,

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изданный во время войны (1944 год) на плохой бумаге выдержал затем 22 издания на 9 языках. Недавно побывавший в ОИЯИ прфессор В. Блюм, зять знаменитого В. Гайзенберга, рассказывал мне, что Гайзенберг оценивал книгу Дмитрия Ивановича как один из лучших учебников по квантовой механике. Для многих физиков эта книга до сих пор остается настольной: так много в ней ценного и важного для постижения квантовуй механики.

Имя Блохинцева неразрывно связано с созданием в нашем Институте первых в мире импульсных реакторов (ИБР). Сегодня на его детище реакторе ИБР-2 ведутся интереснейшие эксперименты учеными из многих стран мира.

Велика заслуга Дмитрия Ивановича в подготовке научных кадров. Преподавая с 1935 года в МГУ, он был неразрывно связан с этим Университетом, где прочитал ряд фундаментальных курсов. Он же явился одним из инициаторов и организаторов филиала НИИЯФ МГУ в городе Дубне, где была освоена новая форма обучения студентов в тесном контакте с научными исследованиями.

Давайте попробуем на этой конференции воссоздать образ Дмитрия Ивановича во всей его полноте. Ведь такие люди встречаются крайне редко и память о них всегда живет с нами.

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Д.И.Блохинцев и Лаборатория теоретической физики

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Уважаемые коллеги, друзья.

Позвольте мне начать свое короткое выступление с представления вам нескольких фотографий конца 50-х годов, сохранивших образ Дмитрия Ивановича Блохинцева вместе со всемирно известными физиками 20-го столетия.

Владимир Георгиевич в своем выступлении дал яркую характеристику деятельности Дмитрия Ивановича Блохинцева как первого директора ОИЯИ (1956-1965 гг.).

В 1965 г., оставив пост директора Института, он был избран директором Лаборатории теоретической физики, которую возглавлял до конца своей жизни (1979 г.). ЛТФ (также как и Лаборатория нейтронной физики) была создана при учреждении Объединенного института по личной инициативе Дмитрия Ивановича, руководить которой он тогда пригласил академика Н.Н.Боголюбова.

В соответствии со своим пониманием творческой деятельности ученого и роли науки в обществе, которые сформировались у него на огромном опыте работы над фундаментальными и прикладными проблемами физики, а еще в юные годы под влиянием личной переписки с К.Э.Циолковским, от которого он воспринял дух российской науки начала XX века, Д.И.Блохинцев стремился быть всегда в курсе всех научных достижений в физике, философии, других разделов науки, поощрять все новые и подчас нестандартные идеи и начинания, активно поддерживать творчество молодых исследователей.

Из личных научных достижений Дмитрия Ивановича напомню, прежде всего, получившие мировую известность работы по акустике неоднородной и движущейся среды. Дмитрий Иванович является автором концепции квантовых ансамблей, на основе которой им была дана объективная трактовка волновой функции и объяснение роли наблюдателя в квантовой механике. Впервые им была предложена идея учета взаимодействия электрона с собственным электромагнитным полем и качественное вычисление сдвига электронных уровней в атомах (1938 г.) - Лэмбовский сдвиг, экспериментальное обнаружение которого и последовательный расчет были осуществлены лишь 10 лет спустя. В области прикладных исследований ему принадлежит идея создания импульсных реакторов и ее практическое осуществление - импульсные реакторы на быстрых нейтронах в ОИЯИ.

В 60-х-70-х годах вместе с Дмитрием Ивановичем активно работали в ЛТФ такие известные ученые как М.А.Марков (кстати - однокурсник Дмитрия Ивановича), А.А.Логунов, А.Н.Тавхелидзе, Д.В.Ширков, В.Г.Соловьев, ученый из Китайской Народной Республики Чжоу Гуан Чжао, И.Тодоров (Народная Республика Болгария), Нгуен Ван Хьеу (Вьетнам), А.М.Балдин, Я.А.Смородинский, молодые в то время: В.Г.Кадышевский, В.А.Матвеев, Р.М.Мурадян, С.С.Герштейн, Л.Д.Соловьев, В.И.Огиевецкий, С.М.Биленький, В.А.Мещеряков, Н.А.Черников и др. В начале 60-х годов, благодаря личным научным и дружеским контактам Дмитрия Ивановича с Виктором Вайскопфом, Генеральным директором ЦЕРН в то время, был установлен научный обмен между учеными ОИЯИ и ЦЕРН, что явилось важной вехой с развитием наших контактов с Европейской организацией ядерных исследований в Женеве.

Возглавляя Лабораторию теоретической физики, Дмитрий Иванович создал научную школу из молодых тогда его учеников (Б.М.Барбашов, Г.В.Ефимов, А.В.Ефремов, М.К.Волков, Г.И.Колеров, В.Н.Первушин, В.В.Нестеренко и др.), успешно работающую и поныне в области нелинейной и нелокальной квантовой теории поля и теории элементарных частиц.

К концу 50-х годов относятся два крупных достижения Дмитрия Ивановича Блохинцева. Во-первых, им было введено понятие унитарного предела. Во-вторых, в области ядерной физики им была выдвинута и разработана концепция флуктуации плотности ядерного вещества - флуктоны Блохинцева, позволившая объяснить ряд загадочных тогда процессов при соударении протонов высоких энергий с ядром, например, обнаруженные еще в 1957 году группой М.Г.Мещерякова "дейтронные пики" в реакции квазиупругого рассеяния протонов на ядрах.

Понятие флуктонов нашло наиболее яркое подтверждение через 20 лет, когда в реакциях с релятивистскими ядрами были зафиксированы, так называемые, кумулятивные частицы. Еще одно подтверждение этой идеи было получено в эксперименте НА-4 в ЦЕРНе по глубоко-неупругому рассеянию мюонов на ядрах и в рождении кумулятивных протонов нейтринным пучком в Институте физики высоких энергий в Протвино. Это новое направление исследований - релятивистская ядерная физика, сейчас успешно развивается как у нас в Институте, так и в других ядерных центрах мира, но уже на основе многокварковой интерпретации флуктонов (С.Б.Герасимов, А.В.Ефремов, В.К.Лукьянов, А.И.Титов, В.Д.Тонеев, С.М.Елисеев и др.).

В эти же годы Д.И.Блохинцев исследует на основе оптической эйкональной модели структуру нуклонов и приходит к необходимости ее разделения на центральную и периферическую части, делает вывод о доминирующей роли в процессах рассеяния периферических соударений.

В теории множественного рождения частиц он показывает явные противоречия между гидродинамическим подходом и основными принципами квантовой механики. Эта несовместимость гидродинамики с квантовыми законами микромира все больше проявляется сейчас по мере расширения корреляционных и спиновых измерений в физике высоких энергий (А.В.Ефремов, О.В.Теряев, С.В.Голоскоков и др.). Я хотел бы с благодарностью отметить, что и в своей научной деятельности, начиная с конца 60-х годов, я испытывал влияние идей Дмитрия Ивановича, в т.ч., в его воззрениях на процессы множественного рождения частиц.

Творческая активность Дмитрия Ивановича, охватывавшая широкий диапазон проблем физики и философии, не угасала до самых последних дней его жизни. Так одним из его последних исследований была задача объяснения аномально малого времени удержания ультра-холодных нейтронов (УХН), для решения которой он предложил простой физически ясный механизм нагревания ультра-холодных нейтронов водородом, адсорбированным поверхностью сосуда, в котором находятся эти УХН. Этот механизм получил свое экспериментальное подтверждение в опытах по температурной зависимости времени хранения УХН.

Особо следует отметить исключительную заслугу Д.И.Блохинцева в деле подготовки и воспитания молодого поколения ученых-физиков во многих странах-участницах ОИЯИ и, прежде всего в Советском Союзе. Будучи с 1935 года профессором физического факультета Московского университета, он за время своей долгой преподавательской деятельности прочитал целый ряд фундаментальных теоретических курсов и среди них особо следует выделить курс квантовой механики, начатый им еще в 1933 году. Созданный на основе этого курса университетский учебник "Основы квантовой механики" выдержал с 1944 года 22 издания у нас в стране и за рубежом. Как известно, В.Гейзенберг высоко ценил этот учебник, хотя и не разделял философскую трактовку квантовой механики, изложенную в книге. Дмитрий Иванович Блохинцев стоял у истоков создания ядерного отделения на физическом факультете МГУ. Первые три кафедры возглавили И.М.Франк, В.И.Векслер и Дмитрий Иванович. Д.И.Блохинцев и В.И.Векслер в 1960 году открывают в Дубне две новые кафедры физического факультета - "Теории атомного ядра" - зав.кафедрой Д.И.Блохинцев, и "Физики элементарных частиц", зав.кафедрой В.И.Векслер. Целью создания новых учебных кафедр в Дубне было обучение студентов старших курсов на базе ОИЯИ, привлечение в Дубну лучших студентов из вузов многих городов страны, подготовка кадров и для бывших социалистических стран. Эта идея Дмитрия Ивановича была активно поддержана Б.М.Понтекорво, который в 1966 году возглавил кафедру "Физики элементарных частиц" и был ее заведующим на протяжении почти 20 лет. Таким образом, Дмитрий Иванович продолжал заниматься учебным процессом в Дубне, в филиале НИИЯФ МГУ, наряду с созданием своей научной школы.

Здесь необходимо отметить интересную деталь: московская кафедра Блохинцева -"Физика атомного ядра" - некоторое время существовала одновременно с дубненской кафедрой "Теория атомного ядра", а затем эти кафедры объединились, и осталась лишь дубненская. В 1973 году кафедра Д.И.Блохинцева меняет название и становится кафедрой "Теоретической ядерной физики".

Формально филиал НИИЯФ МГУ был открыт в 1961 году. Но если посмотреть на документы, то станет ясно, что идея создания филиала пришла в Дубну вместе с создателями Объединенного института, потому как уже в 1956 году вышло распоряжение Совета министров об организации филиала физического факультета МГУ. В 1959 году распоряжение о строительстве здания филиала в Дубне. Весной 1961 года здание было сдано в эксплуатацию. В это же время приказом по Министерству высшего и среднего специального образования РСФСР и приказом по МГУ утверждена структура и определены задачи филиала. Созданию филиала активно способствовали ректор МГУ Иван Георгиевич Петровский, директор НИИЯФ Дмитрий Владимирович Скобельцин. Но практическими создателями филиала являются, прежде всего, Дмитрий Иванович Блохинцев и Сергей Николаевич Вернов, который с 1960 года стал директором НИИЯФ и заведующим отделением, которое в 1960 году было переименовано в Отделение ядерной физики. Ясно, что для них создание филиала не было случайным и обособленным событием, а было совершенно естественное развитие процесса интеграции науки и образования.

Первого октября 1961 года в Дубне в филиале НИИЯФ МГУ начались занятия, приехали первые студенты. Д.И.Блохинцев и Б.М.Понтекорво часто встречались со студентами, приходили к ним в общежитие, принимали самое деятельное участие в устройстве студентов-выпускников на работу, в их дальнейшей научной карьере. Вместе с собственной увлеченностью наукой, вместе с такими человеческими качествами как честность, доброжелательность и чувство юмора, все это создавало неповторимый светлый образ кафедр (в филиале НИИЯФ МГУ), который, можно сказать, остался в памяти многих, если не всех выпускников.

Таким образом, созданием филиала было объединено получение образования в МГУ и научная деятельность в крупнейшем институте, ОИЯИ. Многие выпускники кафедры Блохинцева пополнили коллектив Лаборатории теоретической физики и успешно продолжают работать в ней, внося большой вклад в ее научные достижения. Среди них такие уже известные ученые, как В.Н.Первушин, Е.А.Иванов, Д.Ю.Бардин, В.В.Нестеренко, М.А.Иванов, В.В.Воронов, А.И.Вдовин и другие. Отмечу, что идея создания Университета в Дубне, а также Учебно-научного центра (УНЦ) в ОИЯИ - это тоже является развитием наследия Дмитрия Ивановича. Союз науки и университетского образования - идея, которая активно пропагандировалась в 50-е годы рядом выдающихся ученых, и в их числе Д.И.Блохинцевым, М.А.Лаврентьевым и другими.

Дмитрию Ивановичу был присущ дар предвидения в развитии науки. Он поддерживал и поощрял в ЛТФ ряд исследований, которые в то время нельзя было отнести к "модным" направлениям. Так, например, по его инициативе и активном участии многие годы организовывались международные конференции по нелокальной и нелинейной квантовой теории поля, и сегодняшняя конференция - это продолжение традиции. Это был период, когда квантово-полевые методы в физике элементарных частиц были почти преданы забвению. Теперь же они являются доминирующими, и современные достижения в этой области получены в рамках этих подходов. Можно сказать, что этот акцент в исследованиях позволяет Лаборатории теоретической физики быть на передовых рубежах теоретико-полевых подходов в изучении квантовых закономерностей микромира и вносить достойный вклад в такие разделы теории как квантовая хромодинамика, теория релятивистских струн, суперсимметрия, космология и др.

Дмитрий Иванович Блохинцев и Николай Николаевич Боголюбов оказали и продолжают оказывать влияние на дух научного демократизма не только в ЛТФ, но и во всем Институте и связанных с ним научных центрах.

Универсальность Дмитрия Ивановича проявлялась не только в научной деятельности, но и в эстетическом восприятии мира - он был оригинальный художник и поэт. Его картины неоднократно демонстрировались на выставках, а их репродукции печатались в журналах; его философская концепция выражена им в предисловии к книге "Основы квантовой механики", где он писал: "Я всегда придавал большое значение методологии, без владения которой даже самый отличный ум приобретает оттенок ремесленничества", а в статье "Две ветви познания мира" ("Техника молодежи", 1982 г.) он писал: "Я верю в силу разума и возможность гармонии между ним и Природой. Нам нужна вера в благонамеренность Будущего, творимого человеком и природой, потеря такой веры означало бы увядание человеческого рода".

"Никто на свете не разбудит
Души, ушедшей на покой,
Но на Земле; тебе чужой,
Твои скитаться песни будут..."

- незадолго до внезапного ухода из жизни написал Дмитрий Иванович.

Сегодня действующее поколение теоретиков с благодарностью ощущает на себе влияние яркой личности Д.И.Блохинцева - замечательного ученого, творца.

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Н. Н. Боголюбов и Д. И. Блохинцев

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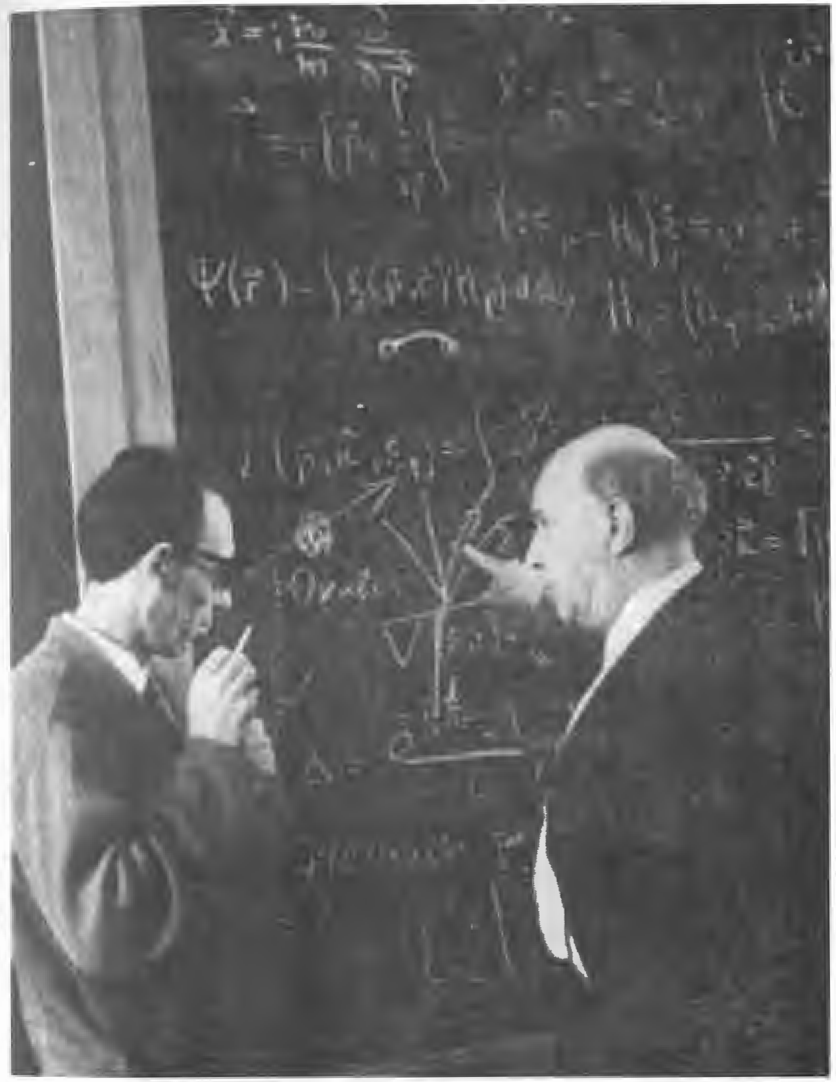

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Д.И. Блохинцев - первый научный директор Лаборатории "В".

А.В. Зродников, Ю.В. Фролов

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Abstract — Воспоминания о работе Д.И. Блохинцев в качестве первого научнго директора Лаборатории "В".

О Д.И. Блохинцеве часто пишут, что в 1950 г. он был назначен первым директором ФЭИ, или как тогда назывался наш научный центр - Лаборатории "В" Первого главного управления (ПГУ) при Совмине СССР. На самом деле до Дмитрия Ивановича были уже два начальника Лаборатории "В" (тогда не было должности директора). Это были полковники инженерно-технической службы МВД СССР, при которых научное руководство осуществляли научный руководитель (немецкий профессор Ханс Позе) и зам. начальника Лаборатории по научной части (Андрей Капитонович Красин). Важное отличие статуса Дмитрия Ивановича в том, что он стал первым научным директором института.

Дмитрий Иванович был зачислен в штат Лаборатории "В" с 16 марта 1950г. как начальник теоретического отдела, а с 21 июля того же года назначен ее директором [1]. Однако, с Лабораторией "В" он был связан гораздо раньше.

К работам по атомной проблеме Блохинцев был привлечен не позднее 1946г., когда стал сотрудником 9-го Управления МВД СССР. (Другое его название: Управление специальных институтов.) Еще ни разу в литературе не была правильно указана должность, которую занимал Блохинцев в 9-м Управлении. Как следует из приказа начальника 9-го Управления о назначении Блохинцева в Лабораторию "В" последняя его должность в этом Управлении - "начальник отделения".

С его работой в 9-м Управлении связана одна из самых загадочных легенд о нем, согласно которой Д.И. Блохинцев был назначен "дублером"

И.В. Курчатова на случай неудачи с первой бомбой. Мы говорим ”легенд” потому, что эта информация проходит только в воспоминаниях мемуаристов. Никаких архивных документов, подтверждающих эту версию, в научном обороте пока не появилось.

9-е Управление МВД СССР руководило институтами, которые были созданы для организации работы немецких специалистов, приглашенных в СССР в 1945-1946гг. Одним из институтов, образованных в системе 9-го Управления, была Лаборатория ”В”. Работая в 9-м Управлении Блохинцев принимал непосредственное участие в создании Лаборатории ”В” и формировании ее научных планов.

Став директором Лаборатории ”В” он совместил в одном лице административное и научное руководство институтом: ”За собой оставляю общее руководство объектом, научное руководство лабораториями и теоретическим отделом”. (При этом была сохранена должность зам. директора по научной части (А.К. Красин), за которым было оставлено: ”руководство научным сектором № 2, составление производственных планов и отчетов по науке, контроль за выполнением планов Лабораторией, руководство повышением квалификации нс и итр”.) 2)

Д.И.Блохинцев возглавил институт на переломном этапе его истории: заканчивался немецкий этап и начинался новый этап развития ФЭИ. Этот этап - 1950-1956гг. - ветераны иногда называют Блохинцевским. Именно в этот период были сформированы основные научные направления в исследованиях ФЭИ и начаты работы, во многом определяющие его лицо и в настоящее время.

При директоре Блохинцеве были: * собраны первые в институте физические сборки уран-графитовых реакторов;

* спроектирована, построена и пущена Первая в мире АЭС (1951-1954); начаты работы по:

• созданию атомных реакторов для подводных лодок;

• созданию ядерных энергетических установок для космических аппаратов;

• созданию реакторов на быстрых нейтронах;

• выполнены расчетно-теоретические исследования по термоядерному взрывному устройству (1951-1955 гг.);

• открыто при институте вечернее отделение (затем филиал) МИФИ - сегодня Обнинский государственный технический университет атомной энергетики.

Первая АЭС

Наибольшую известность Д.И. Блохинцеву принесла Первая в мире АЭС.

16 мая 1950 г. вышло Постановление Совмина СССР о сооружении на площадке Лаборатории "В" трех транспортных реакторных установок (объект "В-10")3). В действительности наиболее подготовленным был проект реактора АМ, опиравшийся на опыт уран-графитовых реакторов. Однако по своим размерам АМ не умещался в отсек подводной лодки, и было решено реализовать его в виде первой АЭС. (Так - АМ - "Атом морской" стал "Атомом мирным".)

Первоначально все научные исследования по установке АМ проводились в Лаборатории измерительных приборов АН СССР (ЛИПАН, так с 1949г. стала называться Лаборатория № 2, ныне РКНЦ КИ). В июне 1951г., согласно постановлению Совмина и приказу ПГУ, ответственными за сооружение АЭС назначаются руководители Лаборатории "В" Д.И.Блохинцев и П.И. Захаров. Тогда же все проектные материалы по АМ передаются из ЛИПАН в Лабораторию "В"4). Таким образом с этого времени Лаборатория "В" являлась и заказчиком, и научным руководителем всех последующих разработок по проекту Первой АЭС. Главным конструктором оставался НИИХИММАШ (Н.А.Доллежаль) [5].

Здесь необходимо пояснить, что проектные материалы по реактору АМ были переданы Лаборатории "В" в июне 1951г. без технических решений по целому ряду важнейших проблем, в частности, - по твэлам. Видимо поэтому на письме 1-го зам. директора ЛИПАН И.Н.Головина о передаче документов ("Пересылаю Вам все имеющиеся у нас проектные материалы по АМ") над словом "все" стоит знак вопроса, выражающий недоумение Д.И.Блохинцева. Потому, окончательный проект АЭС существенно отличался от первоначального и основная разработка его была проведена в Лаборатории "В".

"В ходе дальнейшей разработки проекта, - вспоминал Д.И.Блохинцев - продолжавшейся параллельно с сооружением здания, возникло множество проблем. ...Большое значение имела проверка различных предварительных данных на экспериментальном стенде..."[6]).

Этот стенд - критическая сборка-макет активной зоны реактора АМ из графита, урана и воды, - названная впоследствии "физ. стендом АМФ", собирался прямо под кабинетом Д.И.Блохинцева А.К.Красиным и Б.Г.Дубовским. Целью осуществления "физ. стенда аппарата АМ" являлось получение экспериментальных данных, позволяющих проверить правильность методики расчета и выбора параметров, используемых при расчетах аппарата АМ. АМФ достиг критического состояния 3 марта 1954г. и стал первым реактором в Обнинске, на котором была осуществлена цепная реакция деления урана. Проведенные на нем эксперименты показали, что больших ошибок, по крайней мере на начало кампании Первой АЭС, не будет [7]).

Основная идея проекта АМ состояла в применении трубчатого твэла, в котором поток воды для теплосъема движется внутри трубки, а уран находится снаружи и должен иметь надежный тепловой контакт со стенкой трубки. Создание такого твэла было одной из самых труднейших проблем. К началу проектирования способ изготовления трубчатых твэлов не был известен. Многочисленные попытки ряда институтов (ЛИПАН, НИИ-9, НИИ-13) изготовить опытные образцы трубчатых твэлов, которые выдержали бы проектные тепловые нагрузки с термоциклированием, заканчивались неудачами. "Решающий успех, - отмечает Д.И.Блохинцев в своих воспоминаниях, - выпал на долю технологического отдела в Обнинске, руководимого В.А.Малыхом"[8]). В конце 1952г. они разработали твэл, конструкция которого допускала осуществление многих термоциклов и выдерживала нагрузки в три с лишним раза превышающие проектные.

Подчеркивая роль Д.И.Блохинцева и сотрудников Лаборатории "В" в создании Первой АЭС мы ни в коей мере не хотим принизить роль руководителей ПГУ и вклад в эту работу опытных ученых и специалистов других институтов и предприятий. С начала монтажа оборудования на станции почти безотлучно находился Е.П.Славский, приезжали И.В.Курчатов, А.П.Александров, главный конструктор реактора Н.А.Доллежаль.

Славский фактически взял на себя руководство монтажными работами, Курчатов - больше занимался физикой реактора, Александров - дополнял Курчатова в части инженерно-производственных вопросов, Доллежаль - четко представлял себе картину развития энергетического реакторостроения, о чем свидетельствует выбор трубчатой конструкции твэл и канала реактора АЭС9).

Д.И.Блохинцев, как научный руководитель проекта, занимался не только вопросами сооружения реактора, но и вопросами создания твэлов, и всеми инженерными проблемами. Его рабочий день, как вспоминают ветераны института, продолжался не менее 15 часов и вряд ли он имел выходные. Такое напряжение мог выдержать только спортсмен, каковым он и являлся (как известно, он был альпинистом [10]).

Те трудности, с которыми столкнулись Д.И.Блохинцев и его соратники в этот период, очень хорошо отражены в письме Дмитрия Ивановича в ПГУ в декабре 1951г.:

"...сообщаю, что в Лаборатории "В" имеются только три лица, способных осуществлять научное руководство: Лейпунский А.И., Красин А.К. и Блохинцев Д.И. В остальном Лаборатория состоит из малоопытной молодежи. Между тем, если учесть обязанности, возложенные на названных лиц правительственным решением, относящиеся к строительству и проектированию различных агрегатов, и к расчетам по специальной проблеме, то объем уже полученной работы далеко выходит за пределы сколько-нибудь нормальной нагрузки, при которой могло обеспечиваться надежное повседневное руководство"[11]).

Пуск и успешная работа Первой АЭС имели и государственное, и политическое значение, потому как в такой закрытой области как атомная СССР еще мало что мог открыто показать миру. А здесь - мирный атом.

Д.И.Блохинцев выступает в открытой печати со статьями о мирном применении атомной энергии. В нашем архиве сохранились интересные документы о подготовке Дмитрием Ивановичем первой статьи почти сразу после пуска АЭС в июле 1954г. для газеты "Известия", которая так и называлась "Первый шаг по пути мирного применения атомной энергии". Обсуждение статьи "в отношении степени информации, которая может в ней содержаться" происходило на высшем министерском уровне: В.А.Малышев отписал статью на отзыв Б.Л.Ванникову, Е.П.Славскому, Б.С.Позднякову и А.П.Александрову. Отзывы разделились. Поздняков и Малышев потребовали ее переработки. Причем Поздняков написал развернутые замечания, которые, касались только научно-технических вопросов.

Дмитрию Ивановичу, судя по его пометам на полях замечаний красным карандашом: "Верно", "Это все понимают", "Путаница", "Что же верно?" и т. д. - критика не очень понравилась. Тем более начальники, видимо, сами не знали, чего хотели, ибо Славский тогда же в своем резюме констатировал: "Статья хорошая и не требует корректировки", а Александров: "Считаю статью очень хорошей, и написана хорошо, и нечего из нее высосать..."[12]).

Примечательно, что в июле 1954г. Блохинцев просил министра

В.А.Малышева перед публикацией этой первой статьи "решить вопрос относительно псевдонима, т.к. обозначение моей [т. е. Блохинцева] фамилии может способствовать локализации объекта, не обозначенной в сообщении правительства"[13]).

Проблема "локализации объекта", не обозначенного местонахождением в сообщении ТАСС о пуске Первой в мире АЭС, решилась очень быстро: уже в 1955г. слава Лаборатории "В" перешагнула границы Советского Союза, а на Первой АЭС стали принимать иностранные делегации.

Так, например, в ноябре 1955 г. в Москву прибыла делегация английских ученых. Делегация побывала и ФЭИ, где ознакомилась с атомной электростанцией. Здесь английским ученым был преподнесен в подарок фильм "Первая в мире" об атомной электростанции Академии наук СССР, как тогда называли Первую АЭС.

Почти 50-летний период успешной работы Первой АЭС подтвердил правильность принятых тогда решений. За эту работу Дмитрий Иванович в числе других создателей Первой АЭС был удостоен Ленинской премии (1955). Доклад Блохинцева о Первой АЭС был основным докладом на Первой международной конференции по мирному использованию атомной энергии в Женеве (1955).

Первая АЭС особыми нитями братства связала ее создателей на все последующие годы. Эти связи сохранились до сих пор. При жизни Дмитрий Иванович не пропустил ни одной юбилейной встречи в Обнинске, посвященной пуску Первой АЭС.

Быстрые реакторы

Работы по реакторам на быстрых нейтронах в СССР инициировались с конца 40-х гг. и проводились под научным руководством А.И.Лейпунского. Блохинцев, в начале 50-х годах совместно с Лейпунским осуществлял идеологическое руководство разработкой теории и проекта первого в Европе реактора на быстрых нейтронах с жидкометаллическим теплоносителем, который стал предшественником ряда реакторов-размножителей.

В те времена слово "руководить" не имело того смысла, в котором оно часто используется сегодня. Руководить физрасчетом реактора означало, что человек должен был сам просчитать, что относилось к физике реактора. Д.И.Блохинцев лично занимался расчетно-теоретическими исследованиями по физике быстрых реакторов. В 1950г. он выполнил работу "Кинетические уравнения для быстрых нулевых точек" по теории быстрых реакторов, в которой была поставлена задача нахождения пространственно-энергетического распределения нейтронов с учетом всех основных существенных эффектов взаимодействия нейтронов с ядрами. В этой работе было дано кинетическое уравнение для функции распределения нейтронов и предложен ряд методов решения этого уравнения [14]).

В другой работе "К теории кинетических уравнений" им были сформулированы основы теории расчетов критических масс и воспроизводства в реакторах на быстрых и промежуточных нейтронах. К 1953г. основы созданной им лично теории расчета реакторов были значительно развиты его учениками - молодыми сотрудниками, руководимого Блохинцевым теоретического отдела, и применены к практически осуществляемым системам [15]). Первыми такими системами стали реакторы БНТ и БНТФ (БР-1 и БР-2) малого размера с плутониевой активной зоной.

И, наконец, нельзя не отметить, что именно в Обнинске в 1955г. Дмитрий Иванович предложил идею ИБР и стал главным идеологом создания импульсного реактора периодического действия.

ЯЭУ для летательных аппаратов

Первые проработки ядерных установок для летательных аппаратов космического и авиационного назначения были начаты в Лаборатории "В" в 1953г. Первые нейтронно-физические, теплотехнические и термодинамические расчеты, выполненные тогда, показали возможность создания ядерного ракетного двигателя (ЯРД) с прямым нагревом водорода в качестве рабочего тела.

Этот проект его авторы неоднократно обсуждали с С.П.Королевым, В.П.Глушко, М.В.Келдышем, А.М.Люлькой. В 1955г. проект был оформлен в виде научного отчета, авторами которого были Д.И.Блохинцев, И.И.Бондаренко, В.Я.Пупко и др.[16]).

Несмотря на то, что конструкторы летательных аппаратов не спешили воплощать идеи Лаборатории "В", достаточно быстро за ней закрепился некий приоритет в этой новой области исследований. Так И.В.Курчатов, получая материалы по созданию атомных ракетных двигателей, просит Б.Л.Ванникова направлять их "для получения заключения тов. Блохинцеву Д.И." [17]).

В 1955 г. Лаборатория "В" предложила проект баллистической ракеты с "твердым реактором". С.П. Королев и В.П. Глушко считали, что такая ракета будет не конкурентоспособной в сравнении с ракетой на химическом жидком топливе, потребует больших экспериментальных и исследовательских работ, направление которых на тот период представлялось им слишком неопределенным.

Блохинцев по этому вопросу имел особое мнение, которое в декабре 1955г. выразил в записке, адресованной А.П.Завенягину: "Я думаю, что область применения атомных ракет - это сверхдальние ракеты, позволяющие перебрасывать большой груз (не менее 10 тонн) в любую точку земного шара. ... ракета с твердым реактором является сейчас единственным вариантом, техническое осуществление которого является вполне мыслимым. Поэтому он предлагал развернуть в Лаборатории "В" не только расчетные, но и экспериментальные работы по атомной ракете, "ориентировать их на создание малой опытной атомной ракеты, которая должна явиться прототипом будущих больших атомных ракет" [18]).

Корабельные реакторы

9 сентября 1952г. вышло Постановление СМ СССР за подписью И.В.Сталина о создании атомной подводной лодки. Общее руководство научно-исследовательскими работами и работами по проектированию объекта возлагалось на ПГУ при СМ СССР (Б.Л.Ванников, А.П.Завенягин, И.В.Курчатов), строительство же, а также разработка корабельной части и вооружения объекта - на Министерство судостроительной промышленности (В.А.Малышев, Б.Г.Чиликин). Научным руководителем работ по осуществлению объекта был назначен А.П.Александров, главным конструктором комплексной энергетической установки - Н.А.Доллежаль, главным конструктором объекта - В.Н.Перегудов. Для руководства работами и рассмотрения научных и конструкторских вопросов, связанных с постройкой лодки, при Научно-техническом совете ПГУ была организована секция № 8, которую возглавил В.А.Малышев. Выполнение основных работ по атомной установке поручалось Лаборатории "В", о чем 20 сентября 1952г. Завенягин в письме сообщил Блохинцеву: "Вы утверждены заместителем по центросистемным расчетам и исследованиям научного руководителя объекта № 627 т. Александрова А.П."19)

А.П.Завенягин писал также, что Постановлением Совмина на Лабораторию "В" одновременно возложено выполнение расчетно-теоретических работ, разработка твэлов, сооружение и испытание опытного реактора подводной лодки.

Первой и важнейшей задачей явился выбор типа реактора в качестве основного источника энергии, а также общего облика энергетической установки. Сначала это были реакторы на графитовом и бериллиевом замедлителе с тепловыделяющими трубами, несущими давление. По типу они оказались близки к строящейся тогда первой АЭС. Несколько позднее возникли установки, у которых замедлителем была тяжелая вода. И только потом (а по тем темпам это был один месяц) появился корпусной водо-водяной реактор.

В октябре 1952г. Блохинцев докладывал в Секцию № 8 НТС ПГУ о проведенных в Лаборатории "В" работах: "В результате проведенной предварительной работы мы считаем возможным предложить для обсуждения ... следующие варианты:

а) Технологическую схему, на основе реактора АМ с перегревом пара внутри реактора, разработанную в отделе тов. А.К.Красина и

б) Схемы с применением металлического охлаждения, разработанные в отделе тов. Лейпунского А.И.

Уже при проектировании реакторной установки для ПЛА выявилось множество трудностей. Сложным оказалось создание биологической защиты, которая, с одной стороны, обеспечивала бы хорошую защиту личного состава от излучения реакторной установки, а с другой стороны, не должна была по весу утопить подводную лодку. Вот как пишет об этом в 1952г. Блохинцев:

"Нетривиальность этой задачи [подразумевается задача разработки лодочного реактора] видна из того, что мощнейший из до сих пор разработанных энергетических реакторов [в документе везде проходит как "кристаллизатор"] имеет тепловую мощность [30] 000 квт и вес вместе с защитой [в документе везде - "изоляция"] - 5000 тон. При этом главный вес сосредоточен в защите27.

Трудности удалось преодолеть, и в апреле-мае 1953г. был выпущен эскизный проект энергетической установки и несколько позже всей подводной лодки. Он показал, что лодка и энергетика для нее могут быть созданы в весьма компактном виде и с умеренными массами, что открыло дорогу вперед.

Однако руководители проекта, особенно А.П.Александров, Д.И.Блохинцев, А.И.Лейпунский, отлично понимали, что поставленная серьезная задача может быть решена только при наличии крупномасштабных экспериментальных стендов, на которых оборудование отрабатывалось бы в условиях, близких к натурным. Поэтому в 1953г. на базе Лаборатории "В" приступили к строительству полномасштабных стендов-прототипов энергетических установок реакторов ПЛА с водяным охлаждением и жидкометаллическим охлаждением (первый из которых был введен в действие при Дмитрии Ивановиче в начале 1956г.). Они представляли собой полностью реакторный отсек и турбинную установку ПЛА. На этих стендах затем длительное время отрабатывались реакторы новых типов и проходили обучение экипажи подводных лодок.

РДС-6т

26 апреля 1950 года Совет Министров СССР принял Постановление "О работах по созданию РДС-6". В этом постановлении предусматривалась организация расчетно-теоретических, экспериментальных и конструкторских работ по созданию изделий РДС-6с ("Слойка") и РДС-6т ("Труба")20). (Шифр "РДС" по одной легенде расшифровывается как "Реактивный двигатель Сталина", по другой - "Россия делает сама".)

К расчетно-теоретическим исследованиям по термоядерному взрывному устройству (тема РДС-6т) Постановлением СМ СССР от 9 мая 1951г. наряду с группой Я.Б.Зельдовича в КБ-11 (ВНИИЭФ) была привлечена и Лаборатория "В". В приказе ПГУ от 16 мая 1951г. №167сс говорилось: "Принять к руководству, что Совет министров СССР данным постановлением разрешил Главку организовать в Лаборатории "В" отдел прикладной теоретической физики в количестве 15 человек укомплектовав его квалифицированными физиками, теоретиками, математиками и расчетчиками" [21]).

В соответствии с этими решениями приказом Блохинцева от 13 октября 1951г. для выполнения работ по термоядерной тематике в Лаборатории "В" был создан новый отдел № 6 прикладной теоретической физики в составе двух лабораторий. Научное руководство отделом и всеми работами Лаборатории "В" по этой проблеме согласно указанию ПГУ возлагалось на Д.И.Блохинцева [22]).

Хронологический интервал между принятием решения (май) и созданием отдела №6 (октябрь) связан с комплектованием отдела и оформлением допусков выпускникам вузов. Блохинцеву были даны очень широкие полномочия набирать специалистов в НИИ и учебных заведениях. Но квалифицированные специалисты, о которых говорилось в постановлении Совмина и приказе ПГУ, уже были заняты в других институтах. Потому исследованиями по теме РДС-6т (как, впрочем, и по всем другим темам Лаборатории "В") вместе с Блохинцевым занимались, в основном, молодые физики и математики только что окончившие институты. Блохинцев, остававшийся профессором МГУ, имел возможность выбирать и приглашать своих учеников. (Так в Лабораторию "В" попали Б.Б.Кадомцев, Ю.П.Райзер, В.С.Имшенник, Л.Н.Усачев, А.С.Романович и др., ставшие потом известными учеными.)

За период 1951-1955гг. под руководством Дмитрия Ивановича по теме РДС-6т были проведены серьезные расчетно-теоретические исследования по термоядерному взрывному устройству, начало которым положил отчет Д.И.Блохинцева "Газодинамика вещества при высоких температурах" (1951г., часть 1).

Всего по результатам работ было выпущено более 20 научных отчетов Лаборатории "В", в которых сформулирована физико-математическая модель всех основных процессов, протекающих во взрывном устройстве. Среди них такой фундаментальный отчет, как: "О непрозрачности воздуха в ударной волне и минимуме яркости огненного шара" (1954г.). Авторы: Д.И.Блохинцев, А.С.Давыдов, Ю.П.Райзер.

Надо отметить, что в исследованиях Лаборатории "В", проводимых под руководством Блохинцева, рассматривались не отдельные физические проблемы, а конкретная конструкция термоядерного взрывного устройства. Физическая схема предложенного группой Блохинцева устройства отличалась от аналогичных разработок ВНИИЭФ, с которыми сотрудники Лаборатории "В" знакомы не были. Идея состояла в воспламенении большой массы дейтерия в виде сферы.

Эти исследования повлияли на оценку перспектив разработок по "трубе". В начале 1954г. в Министерстве состоялось совещание, которое вел И.В.Курчатов; участвовали В.А.Малышев, Д.И.Блохинцев, И.Е.Тамм, А.Д.Сахаров, Я.Б.Зельдович, Л.Д.Ландау, И.Я.Померанчук, Ю.Б.Харитон и др. На этом совещании от Лаборатории "В" вначале выступил Д.И.Блохинцев, а затем сотрудники его отдела. Главным был доклад Б.Б.Кадомцева о переносе нейтронов в дейтерии. В своем выступлении Кадомцев показал, что в результате протяженного в пространстве переноса энергии и импульса быстрыми нейтронами, а также из-за эффекта комптонизации в дейтерии наблюдается пространственное энерговыделение на больших расстояниях. Поэтому, задача получения детонации в "трубе" не имеет гарантированного положительного решения [23]).

В 1955г. исследования по термоядерной тематике в Лаборатории "В" прекращены. Американцы убедились в бесперспективности этого метода еще в 1950г.

"Физический идеализм" и развитие теоретической физики

Время работы Д.И.Блохинцева в Лаборатории "В" совпало с окончанием так называемой дискуссии о "физическом идеализме". Начатая В.И.Лениным еще в "Материализме и эмпириокритицизме" борьба с физическим идеализмом продолжалась почти на протяжении всей истории советской философии и физики. В послевоенные годы она получила новое развитие и велась вокруг основных положений теории относительности и квантовой механики. Дискутирующие стороны представляли с одной стороны физики Академии наук, с другой - часть философов и физиков МГУ.

Ортодоксальные партийные философы (еще их называли "механисты") доказывали, что теория относительности и квантовая механика построены на идеалистической философии, придуманы за рубежом и тормозят развитие советской физики. На основе развенчания новейших теорий "идеалистов" и "космополитов" они хотели радикальным образом перестроить преподавание и подготовку научных кадров.

Между тем, большинство физиков, обвинявшихся в идеализме и космополитизме (еще их называли - "западники"), были ведущими теоретиками, признанными специалистами в теории относительности, квантовой механике и ядерной физике (Я.И.Френкель, В.А.Фок, И.Е.Тамм, Л.Д.Ландау и др.).

Эта идеологическая дискуссия достигла апогея в 1949-1952гг., при этом возникла парадоксальная ситуация: создание атомного оружия опиралось на ядерную физику, которая немыслима без теории относительности и квантовой механики. Принято считать, что И.В.Сталин вмешался и отменил подготовку так называемого "философско-космополитического погрома" 1949г. после того, как с ним переговорили обеспокоенные Л.П.Берия или И.В.Курчатов. Бомба была важнее философии. (Правда, этой красивой версии нет документального подтверждения). Однако дискуссия на этом не прекратилась, и значительную роль в ней на всем протяжении играл Блохинцев.

При этом положение его было в известной степени не простым: Дмитрий Иванович был одинаково близок к обеим группам ученых, так как длительное время одновременно работал и в МГУ, и в ФИАН, т.е. в системе Академии наук.

С другой стороны, Блохинцев несомненно пользовался особым доверием руководителей советского атомного проекта, и во время дискуссии по физике неоднократно выступал в роли консультанта руководителей ПГУ, а затем и Министерства. Можно предположить, что после провала совещания по физике 1949г. и смерти президента АН СССР С.И.Вавилова (который в значительной мере сдерживал развитие негативных процессов), руководители атомного проекта решили взять ситуацию под контроль. Вероятно поэтому не позднее февраля 1951г. Курчатов предложил, чтобы Блохинцев возглавил следующую возможную дискуссию по физике [24]).

Коснемся кратко взглядов Блохинцева на положение в области советской теоретической физики в тот период.

В справке, подготовленной для А.П.Завенягина в феврале 1951г. Блохинцев пишет, что "теоретическая физика XX столетия опирается на две фундаментальные теории - теорию относительности и квантовую механику". Однако тут же он отмечает, что "обе эти фундаментальные физические теории возникли на почве зарубежной, буржуазной науки и это не могло не отразиться отрицательно на развитии советской теоретической мысли, вместе с этими теориями к нам были занесены враждебные революционному материализму философские концепции и дух низкопоклонства перед представителями зарубежной науки [25]).

Читая тексты тех лет надо хорошо понимать проблему взаимоотношения науки и идеологии в то время, представлять, каким образом удавалось сочетать философскую теорию марксизма и философские принципы научного реализма, и на уровне практики разграничивать сферы компетенции ученых, специалистов и идеологов. Блохинцев, используя сакраментальную фразеологию тех лет, говорит о необходимости развития теоретической физики. Но об этом же говорил и академик Фок - один из главных объектов атак партийный философов в тех дискуссиях, - Фок говорил, что "никто из наших физиков не считает квантовую теорию потолком физической теории".

"Для всех теоретиков, - пишет далее Блохинцев - совершенно ясна ... и понятна недостаточность квантовой механики, а может быть и теории относительности в области ультрамалых масштабов, характерных для элементарных частиц. Переход к новым масштабам, как учит диалектический материализм, должен сопровождаться, принципиальными, качественными изменениями теории.

Такое положение дел, когда у нас практически не работают над принципами теории, является тем более недопустимым, все мыслящие физики понимают, что их наука стоит сейчас накануне нового великого перелома и советские теоретики страстно желали бы осуществить этот перелом" [26]).

Большой интерес представляет анализ состояния и развития советской теоретической физики, подготовленный Блохинцевым в октябре 1953г. для отдела среднего машиностроения Совмина СССР. Этот анализ был сделан в связи с работой комиссии ЦК КПСС по проверке физфака МГУ. Возглавлял комиссию министр среднего машиностроения СССР В.А.Малышев. Инициировало эту проверку обращение группы ведущих ученых (И.В.Курчатов, М.А.Леонтович, И.Е.Тамм, Л.А.Арцимович, Д.И.Блохинцев и др. - почти все участники атомного проекта) в Академию наук и Министерство среднего машиностроения.

В своем анализе Блохинцев делит историю развития советской теоретической физики на три этапа и кратко их характеризует.

Его беспокоит отставание нашей теоретической физики, и он предлагает ряд мер (из 12 пунктов), необходимых для ее развития. Среди них, в частности, такие предложения:

• вернуть массовость нашей науке; • существенно поощрять работу над проблемами теоретической физики; • снять режим секретности с работ, не имеющих отношения к технике и производству; • считать обязательным для руководящих научных работников, занятых в промышленности, участие в научных семинарах и личное ими руководство.

Многие из предложений, высказанных в этой справке Д.И.Блохинцевым, вошли в той или иной редакции, в заключение комиссии и в последующее затем Постановление ЦК "О мерах по улучшению подготовки кадров физиков в МГУ".

Насколько сильно волновало Дмитрия Ивановича состояние советской теоретической физики видно и из другой справки, составленной почти в то же время (ноябрь 1953г.) для Научно-технического управления Министерства среднего машиностроения. Справка называется "К совещанию по теоретической физике". В ней Блохинцев пишет:

В последние годы в Америке сделаны существенные успехи по квантовой электродинамике ("метод перенормировки") и по теории ядра ("ядерные оболочки"), а в Дании разработана "коллективная модель ядра", представляющая существенный прогресс по сравнению "капельной" моделью.

В нашей же стране, в настоящее время, работа по теории протекает, в основном, в рамках дальнейшего развития названных зарубежных идей и по разработке теории отдельных частных явлений".

В этих обстоятельствах он предлагает избежать на совещании обсуждения различных мелких достижений отдельных лиц, и обсудить основные вопросы: Как дошли до существующего неудовлетворительного положения? Что делать, чтобы выйти из него? Обсудить эти вопросы с полной откровенностью, проанализировать ошибки прошлого, развернув беспощадную критику тех руководителей, которые не поддерживали новых физических идей в нашей стране, глушили их развитие [27]).

Последняя справка написана в то время, когда еще не окончилась борьба с физическим идеализмом, и еще свежи были отголоски борьбы с космополитизмом. Но здесь уже нет упоминаний об отрицательном влиянии буржуазной науки на развитие советской теоретической физики и прочих подобных вещей. С другой стороны, его слова об отставании советской физики звучат диссонансом тому "звездопаду", который обрушился на физиков после первых испытаний ядерного оружия.

Д.И.Блохинцев в воспоминаниях

Д.И.Блохинцев, как всякий талантливый человек был многогранной личностью. Архивные документы, легшие в основу этого рассказа, сухи и официальны. Для того чтобы полнее раскрыть характер и лучше узнать этого человека обратимся к воспоминаниям тех, кто работал с Дмитрием Ивановичем. Вот как писал о нем участник создания Первой АЭС Михаил Егорович Минашин:

"Несмотря на занятость, Дмитрий Иванович сохранял свежесть ума, молниеносную реакцию, никогда не утрачивал чувство юмора и, главное, железным образом не допускал расстройства общественно-научной жизни института. В то время казалось, что пропуск физического семинара в Институте был равноценен взрыву бомбы".

Любопытно, что эти семинары, как писал Блохинцев в одном из приказов по институту, он проводил для "поднятия научного тонуса производственной жизни Лаборатории "В" и дальнейшего развертывания критического обсуждения ... выполненных работ" [28]).

Он не любил скороспелых решений, требовал всестороннего обдумывания, обсуждения со специалистами смежных проблем. Он никогда не упускал случая, чтобы воодушевить своего подчиненного собеседника. В таких случаях он говорил: "Ах, если бы не эта текучка, я бы обязательно попробовал решить эту задачу, уж очень она интересна. Но это мне не суждено. Времени нет. Единственное мое преимущество, как директора, состоит в том, что за моим кабинетом есть еще одна комната и мне не приходится далеко ходить и тратить время".

Дмитрий Иванович обладал удивительной способностью быстро решать задачи, получать численные результаты, вероятно, по этой мерке он оценивал и других людей. Думаю, что именно поэтому он загружал расчетчиков таким объемом расчетных работ, который он сам искренне считал еще не исчерпывающим возможностей, но на выполнение которого не хватало и 16-ти часов рабочего дня.

И все же работа вместе с Дмитрием Ивановичем казалась легкой и веселой! В случае, когда у нас что-либо не получалось, мы шли к Дмитрию Ивановичу и всегда получали ответ, что проще всего это делать так-то, или "а эту задачу и решать не надо", потому-то и потому-то.

Несмотря на то, что по мере продвижения проекта Первой АЭС, физик Д.И.Блохинцев все более втягивался в технику, он все же остался физиком. До последних дней пребывания в Лаборатории "В" он иногда возмущался многими техническими терминами, а в первое время посмеивался над "фамильной алгеброй" критериальных уравнений теплотехники и гидравлики, возмущался по поводу применения таких единиц как килокалория в час": "В часах можно измерять только время сна, а не количество переданной энергии. Ну, скажите, зачем Вы используете такую единицу, когда она сама напрашивается сделать из нее хотя бы "калорию в секунду?". Порядок один и тот же!".

Превосходство знаний и умения у Дмитрия Ивановича в сравнении с нами, его подчиненными, не позволяло нам спорить и, кроме того, оправдывать эти единицы в то время мы еще не могли. Однако превосходство в знаниях Дмитрия Ивановича не оставляло неприятного чувства, т. к. он всегда использовал его в доброжелательном направлении для своего собеседника. Переход Дмитрия Ивановича из Лаборатории "В" на новую работу в течение длительного времени поддерживал чувство сожаления у очень многих инженеров, особенно у расчетчиков" [29]).

Рассказывая об обнинском периоде жизни Д.И.Блохинцева нельзя не вспомнить о его жене Серафиме Иосифовне Драбкиной. При Дмитрии Ивановиче в Лаборатории "В" было открыто вечернее отделение МИФИ, и Серафима Иосифовна стала одним из первых преподавателей вновь созданного института. Она преподавала математику и квантовую физику. С большой теплотой и любовью ее до сих пор вспоминают все бывшие студенты. В 1957г. состоялся первый выпуск Обнинского филиала МИФИ. И первый выпуск пригласил Дмитрия Ивановича и Серафиму Иосифовну (которые жили и работали уже в Дубне) на праздничное торжество по этому поводу в Обнинск.

Заключение

Таким образом, короткий период деятельности Дмитрия Ивановича в Обнинске - важная страница в истории нашего научного Центра. Как уже говорилось в начале, Блохинцев возглавил институт на переломном этапе его истории и оставил в ней неизгладимый след. При Блохинцеве численность института выросла в 3,5 раза. Но важнее то, что к середине 50-х годов (как отмечал сам Дмитрий Иванович в своем выступлении перед коллективом Лаборатории "В" в феврале 1956г., за три месяца до его перехода в ОИЯИ), институт "из группы новичков, озирающихся на то, что делается у более опытных и могучих соседей, ... превратился в мощный, квалифицированный коллектив, за плечами у которого немалый опыт в самостоятельном решении больших задач"30).

Наряду с основными направлениями деятельности Лаборатории "В", в развитие которых внес большой вклад Дмитрий Иванович, особой его любовью оставалась теоретическая физика. Дмитрий Иванович приложил все усилия, чтобы создать в институте сильный теоретический отдел, ставший гордостью ФЭИ, его мозговым центром. Всем известны прекрасные ученые, выросшие в этом отделе: Л.Н.Усачев, Г.Н.Смиренкин, В.Золотухин, Г.И.Марчук, А.В.Игнатюк, А.С.Романович, Н.С.Работнов и другие, много сделавшие для развития теоретической физики и практических ее приложений.

Столь же большую роль сыграл и ФЭИ в жизни Дмитрия Ивановича. Здесь он приобрел первый опыт руководства крупным научным коллективом, здесь к нему пришла настоящая мировая слава. И вполне обосновано в характеристике по выдвижению Дмитрия Ивановича в Академию наук СССР А.К.Красин писал в 1953г.: "В лице Д.И.Блохинцева страна имеет выдающегося ученого и организатора, смело развивающего научные знания и практически направляющего эти знания на развитие новых областей техники и народного хозяйства" [31]).

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Д.И.Блохинцев и его работа в Саратовском университете.

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Abstract — Воспоминание о работе Д.И. Блохинцева в Саратовском университете. Выступление на продолжении этой конференции в Саратовском университете.

...Потерял голубую звезду,
Пусто в темном замерзшем саду.
О потере ревниво молчу...
...Все себе же ревниво шепчу:
Я её отыщу, отыщу.
Д.И. Блохинцев

I. После окончания аспирантуры и защиты диссертации (в1934 г.) Д.И. Блохинцеву на ее основе была присуждена в 1935 г. ученая степень доктора физики. В том же году он был приглашен на работу в Саратовский государственный университет им. Н.Г. Чернышевского на должность профессора - руководителя создаваемой кафедры Теоретической физики физико-математического факультета. Собственно вопрос о ее создании возник гораздо раньше: еще в конце 1919 начале 1920 гг. на базе существующего кабинета теоретической физики Физического института СГУ, руководимого Богуславским С.А. Организация кафедры была признана весьма желательной и в решении VI Всесоюзного съезда физиков (1928г.) и постановлении правления университета. Однако реальные возможности организации такой кафедры появились к середине 30 -х годов XX в. Вот что писала многотиражка университета "за научные кадры" (от 23 июня 1935г.): "В1934г. ... были открыты исторические факультеты при Московском и Ленинградском университетах. В этом году открывается теоретический факультет в Саратовском университете. Это свидетельствует, что СГУ после столичных университетов занимает одно из первых мест по своему научно-учебному оборудованию, библиотеке и своим научным кадрам. Особенно решительно можно будет об этом заявить после осуществления широкой программы, которая намечена в развитии СГУ с 1935-36 учебного года. С осени 1935 года развертываются новые кафедры и лаборатории по математике, физике, химии, биологии геологии и т.д. По количеству кафедр СГУ занимает первое место среди университетов РСФСР после университетов Москвы и Ленинграда. Для руководства новыми кафедрами приглашаются университетом крупные специалисты Советского Союза, профессоры: Хинчин, Петровский, Курош, Вагнер, Блохинцев и целый ряд других ученых". Месяцем позже та же газета писала (в ноябре от 29 числа 1935 г.) в статье ректора Хворостина: "Количество кафедр в СГУ в этом году вырастет с 16 до 34. На занятие их приглашены профессора Московского университета А. Хинчин, И. Петровский, А. Курош и В. Вагнер по математическим кафедрам и Д. Блохинцев по кафедре теоретической физики, Кабак по кафедре динамики развития т.д. " Наконец, приведем свидетельство областной газеты "Коммунист "в корреспонденции по Саратовскому университету (№ 234 (521) от10 октября 1935 г., с. 4): "... Для руководства кафедры геометрии прибыл доктор математических наук профессор Вагнер и для руководства теоретической физики - доктор физических наук профессор Блохинцев. Кафедрой динамики развития (биологический факультет) заведует приехавший в Саратов профессор Кабак и кафедрой физической географии - профессор Пиотровский". Мы специально так подробно остановились на документальном обоосновании факта о создании кафедры Теоретической фтзики СГУ и ее первом руководителе Блохинцеве Д.И. в связи с тем, что создание кафедры до настоящего времени относили к 1946 году, а работа профессора Блохинцева в СГУ вообще во всех официальных материалах (юбилейных, некрологе и др.) даже не упоминается. Так что, по видимому, настоящая публикация в этом плане является первой. А это обязывает нас привести еще несколько документальных данных. Официально кафедра Теоретической Физики была утверждена Наркомпросом летом 1935 г. Тогда же на кафедру, наряду с Блохинцевым, были приглашены Ю.Б. Руммер и С. И. Драбкина. Должность заведующего кафедрой утверждена с !5 XII 1935 г. (Приказ по университету от 19 марта 1936 г.) С октября месяца 1936 г. на кафедру зачислен был кандидат физических наук Ш.Ш. Шехтер - ученик Д.И. Блохинцева (приказ № от 11 XII 1936 г.). Небезынтересно, что в приказах по университету (от 9 IX и 4 XI 1935 г.) определялось число выполненных Д.И. Блохинцевым учебных часов (лекционных и семинаров). При этом во втором приказе ему дополнительно засчитывалось

50 лекционных часов и 36 семинаров (сверх ранее запланированных). Небезынтересно также, что в должностных перечнях "на36/37 учебный год "устанавливались (в соответствии с приказом Накромпроса РСФСР - дан в приказе по университету № 5 от 11 января 1937 г.) персональные ставки профессорско-преподавательскому составу, в том числе профессору Блохинцеву (1000 рублей в месяц) и доцентам Драбкиной и Шехтеру (с 1 XII 1936 г.). И что симптоматично - так это весьма не типичное канцелярски-административное распоряжение за подписью профессора Голуба (ВРИО ректора СГУ): "... 9. Прекратить выплату персональных ставок профессору Блохинцеву и доценту Драбкиной с IV 1937 г в виду прекращения их работы в университете " (?), т.е. еще до конца семестра. И в завершение первой части нашего сообщения о становлении кафедры Теоретической физики СГУ отметим, что Постановлением ВАКК ВКВШ (от 23 V 1938) Шехтер Ш.Ш. был утвержден в ученом звании доцента по кафедре теоретической физики, а затем приказом ВКВШ при СНК СССР утвержден "в должности Вр. И.О. зав. кафедрой Теоретической физики (приказ от 9 июня 1938 г.)

II. Учебно-педагогическая работа по теоретической физике с приходом Блохинцева и др. естественно повысила необходимые и уровень, и систематичность и последовательность, поскольку выполнялась специалистами данного профиля в рамках университетской структурной ячейки - кафедры Теоретической физики читались курсы квантовой механики, теории поля и электронной теории, статистической физики и термодинамики, теории относительности и курс дополнительные главы современной физики - элементы теории атома и радиоактивности и др. Для общего представления об уровне лекционных занятий и семинаров целесообразно привести названия некоторых научных работ Блохинцева, опубликованных в "Саратовские" и первые годы после работы в СГУ (несомненно, задуманных в Саратове) период: Время жизни частиц в адсорбированном состоянии (совм. с Шеятером). Атомистика в современной физике (совм. с Ф. Гальпериным). Материя, масса и энергия (совм. с Ф. Гальпериным). Эфир (совм. с Ф. Гальпериным в журнале "Фронт науки и техники"). В чем заключаются основные особенности квантовой механики. Дискуссия о структуре атомного ядра (в Успехах физических наук) Что такое теория относительности ОНТИ. М.-Л. 1936 г. Повторно оговоримся, что перечень не включает как ранние работы, так и все последующие публикации Блохинцева, ровно как и не подразумевает какую либо их характеристику. Весьма важно, однако, отметить, что работа профессора Блохинцева в СГУ не ограничивалась только одной академической нагрузкой. Им сразу же была развита систематическая, вне должностной сетки, научно популярная и просветительская деятельность. Вот что писала областная газета "Коммунист" (в номере от 22 октября 1935 г., № 244 (531), с 4.): "Прибывший из Москвы для работы в Саратовском университете профессор Блохинцев Д.И. прочтет цикл лекций для научных работников Саратова на тему "Проблемы атомного ядра". Первая лекция состоится сегодня в 8 ч. 10 мин. вечера в физической аудитории (III корпус университета). Сегодня же профессор Хинчин приступает к чтению университетского курса теории действительного переменного. Лекции будит читаться по четвертым дням шестидневки с 9 ч. 20 мин. вечера в физической аудитории университета". Замечу, что эти первые лекции (а за тем и последующие) я - школьник 9 класса средней школы посетил, к сожалению не проводя каких-либо, хотя бы кратких, записей. В памяти сохраняются только отдельные частные факты. Отчетливо помню, например, элементы из лекций о гипотезе нейтрино и законе сохранения энергии и некоторые другие (по теории относительности и эфире). Я стал студентом физмата саратовского университета в сентябре 1937 г., когда Дмитрий Иванович в СГУ уже не работал. Однако свою роль его лекции сыграли в выборе моей специальности. Школяром в соответствии с духом того времени, я нацеливался на специализацию по кафедре аэрогидромеханики (т.е. "авиационному" направлению университетского образования). Лекции профессора Блохинцева в корне изменили мое представление о соотношении отдельных научных дисциплин. Я в меру возрастных возможностей стал осозновать что есть физика как фундаментальная университетская наука и ее положение в системе знаний. Возвращаясь к теме "Блохинцев и его работа в СГУ", отмечу, что в последние предвоенные и военные годы (1938 - 1945) в университете и на кафедре Теоретической физики СГУ каких-либо письменных материалов о контактах и связях с Д.И. Блохинцевым нет. Нет и у меня таковых, ровно и личных воспоминаний. По-видимому, он в этот период с университетом не общался и в Саратов не приезжал. Вновь с Д.И. Блохинцевым я встретился уже в 1946 г. во время совместной командировки с зав. кафедрой А.С. Шехтером в Москву (по делам НИИМФ СГУ, пом. директора которого по административно-технической части я был, будучи ст. преподавателем кафедры Общей физики, и одновременно по научным вопросам моей н/и работы - проблеме разделения тяжелых изотопов ВЧ электромагнитным полем с фазовой фокусировкой - ИЗОТРОН). Встреча с Д.И. Блохинцевым и его содействие обеспечили первые мои контакты с НИФИ-2 (на Соколе) и знакомство с профессором С.Н. Верновым, а так же другими профессорами физического факультета МГУ (Спиваковым и Тимирязевым). Затем , в начале уже 50-х годов, содействие Дмитрия Ивановича привело к контактам с Т.Т.Л. в связи с проблемой создания циклотронной лаборатории в Саратовском университете и, наконец, в конце 50-х годов, после организации в СГУ проблемной лаборатории ядерной физики (зав. лаб. Игонин В.В.), благодаря поддержке Дмитрия Ивановича Блохинцева и активным действиям зав. кафедрой теоретической Физики А.С. Шехтера, было установлено научное содружество с ОИЯИ (г. Дубна), переросшее в создание филиала кафедры в Объединенном институте (мы здесь не останавливаемся на выходящим за рамки данного сообщения, содружестве кафедры и ЛЯФ СГУ с физическим институтом АН СССР, поддержанного академиком Скобелевым В.Д. - отдел "Питомник", зав отделом академик В.И. Векслер лабораторией - Л.Е. Лазарева) Среди первых студентов, окончивших СГУ по специальности физика атомного ядра и космических лучей Г.И. Колеров, Н.Н. Слонов, В.К. Лукьянов, В.Г. Серяпин, О.Г. Боков, И.А. Грищенко, О.И. Логинов, А.Б. Фенюк, Н.Б. Скачков, В.П. Гердт, Г.Н. Лыкасов, Ю.Р. Тюхтяев, В.П. Жигунов, М.Б. Шертель, Ю.И. Иваньшин, Л.Ш. Шехтер и др. (за десятилетие 1955 - 1965 гг. число студентов по данной специализации теоретической и экспериментальной возросло до полной группы университетского плана подготовки специалистов). Среди первых руководителей дипломных работ по ОИЯИ были Р.М.Рындин, С.М. Биленький, В.Б. Беляев, М.И. Широков, Нгуен Ван Хьеу, Б.Н. Захарьев, Б.А. Арбузов, В.А. Мещеряков, А.В. Ефремов, В.Г. Кадышевский, С.С. Герштейн. Н.А. Черников, Н.И. Таранкин, В.В. Балашов и др. Все руководители (и рецензенты) дипломных работ широко известные ученые, безотказно принимали на себя весомый груз руководства дипломными работами, обеспечивая их высокий научный уровень, предъявляемым к выпускникам университетов. С другой стороны, имена руководителей свидетельствуют о степени предварительной подготовки студентов, специализировавшихся по кафедре теоретической и ядерной физики СГУ, способных работать на уровне требований высококвалифицированных руководителей. На этом можно было бы поставить точку в рассмотрении вопроса о Д.И. Блохинцеве и его связях с Саратовским университетом и роли в становлении и развитии направлении физика ядра и элементарных частиц на физфаке СГУ. Однако, нам представляется целесообразным привести некоторые фрагменты из поэтического творчества

Дмитрия Ивановича, объектизирующее его внутреннее состояние души в годы о которых здесь шла речь. Вот эти строки (из стихотворения " Мечте моей"): "...Моя надежда и мечта!

Нас с тобою били, изранили, измяли.
...Под ноги бросали бревна недоверия!
...В лицо нахально ржало лицемерье.
Но мы остались на ногах.
...Но как забыть. Все помним.
...Били: нелепостью сверху,
дикостью в пах,
Ножом предательства в спину.
А мы с тобой остались на ногах.
Но крылья смяты!
Теперь лететь уже другим,
Поверим в них, они хорошие ребята".

Д. И. Блохинцев и А. К. Красин на первой в мире АЭС. Обнинск, 1954 г.

Д. И. Блохинцев с рабочими на АЭС в Обнинске. 1954 г.

Д. И. Блохинцев на праздновании 10-летия первой в мире АЭС.
Обнинск, 1964 г.

20-летний юбилей первой в мире АЭС. 1974 г.

Секретно.

Товарищу МАЛЫШЕВУ В.А.

В соответствии с Вашим указанием направляю Вам проект статьи для "Известий".

Прошу Вас, до решения о её опубликовании, дать её на авторитетное заключение в отношении секретной информации, которая может в ней содержаться.

Прошу Вас также решить вопрос относительно псевдонима, т.к. обозначение моей фамилии может способствовать локализации объекта, не обозначенной в сообщении Правительства.

ПРИЛОЖЕНИЕ: на "20" листах, несекретно, в 2-х экз. [illegible] 20.

Д. БЛОХИНЦЕВ.

"9" июля 1954 года.

Вх. № ИС [illegible]
[illegible] 7 54

92

Сов. секретно

Экз. №____

Тов. Блохинцеву Д.И.
Разрешаю работать на стенде "БФ"
Славский
2-IV-55

Товарищу СЛАВСКОМУ Е.П.

Сообщаю, что в Лаборатории "В" подготовлен к пуску стенд БФ – аппарат на быстрых нейтронах, рассчитанный на мощность до 10 ватт, установленный в здании № 168.

Комиссия, назначенная моим приказом в составе председатель Дубовский Б.Г., члены Маталин Л.А., Кормиленко И.И., Матющенкова З.Ф., Черевань Г.Ф. произвела приемку установки и вынесла заключение о готовности аппарата к запуску.

В соответствии с приказом Министра от 24 июня 1954 г. прошу Вашего разрешения на запуск стенда БФ.

52

Для сведения т.т. Лейпунскому А.И., Казачковскому О.Д.
19.4.55

Д.БЛОХИНЦЕВ

"29" марта 1955 г.

22.04.55

17.40

Вх. № Ис-712сс

3543

6 ЭБ

Применительно к такому металлическому был рассмотрен более сложный вариант изоляции :

I. Слой свинца	150 мм		
II. Слой вода и карбид бора	450 мм		
III. Слой свинца	200 мм		
IY. Слой вода и карбид бора	500 мм	700 г/см^2	
Y. Слой свинца	50 мм		
YI. Слой вода и карбид бора	850 мм		
	1700 мм		

Общий вес изоляции составляет 250 тонн, из них 160 тонн составляет свинец /при изоляции на 45 /. Меньший вес изоляции в случае металлического варианта обусловливается меньшими размерами активной зоны.

Рассмотренные здесь варианты изоляции являются предварительными. Однако можно думать, что сам выбор веществ для изоляции, именно свинца, карбида бора и вода является весьма обоснованным, как в отношении физических свойств этих веществ, так и в отношении их доступности и технологичности.

Другие тяжелые вещества, напр. вольфрам, олово не имеют очевидных преимуществ перед свинцом. Поэтому вопрос о целесообразности их применения требует более детального рассмотрения.

Более целесообразного поглотителя [illegible] нежели бор... или, если установить не удалось.

Если же остановиться на комбинации Pb, $B_4C + H_2O$, то проблема рационального выбора изоляции сводится к задаче о наивыгоднейшем распределении этих веществ, что может быть сделано сравнением различных вариантов или применением расчета на минимум веса. Работы в этом направлении продолжаются и дальнейшее сокращение веса изоляции не является исключением.

ПРИЛОЖЕНИЕ
к вх. № [illegible]

Товарищу ЗАВЕНЯГИНУ А.П.

В соответствии с Вашим указанием направляю Вам справку "О положении в области теоретической физики".

Пользуюсь также случаем обратить Ваше внимание на то, что я, ввиду возложенных на меня задач и коротких сроков, предоставленных на их решение, не могу согласиться с предложением И.В. Курчатова возглавить дискуссию по физике, /если она будет признана в той или иной форме целесообразной/ и буду вынужден ограничиться ролью рядового участника

Приложение: по тексту на 5 листах, маш.№ 3/1188.

п/п Д.Блохинцев

верно: Царькова

" " февраля 1951 г.

... Н 75

н/с.

[redacted]

Отдел „В" МВД СССР
Вх. № С01503-оп
„28" 6 195 1 г.

П Р И К А З

№ ...сс-оп

г.Москва 18 июня 1951 года

Во исполнение Постановления Совета Министров СССР от 12 июня 1951 года № 1965-939оп о сооружении опытной установки " В-10 ",-

П Р И К А З Ы В А Ю:

I.

I. Тов.тов.Павлову Н.И., Петросьянц А.М., Поздникову В.С., Стахирову С.И., Блохинцеву Д.И., Захарову П.И. принят к руководству и исполнению, что:

а) Главное Управление обязано построить на территории Лаборатории _В_ опытную электрическую станцию (установку _В-10_) на _обогащенном олове_, с вводом в действие опытного _оловяно_ - _керамического кристаллизатора_ с водяным охлаждением (агрегат _АМ_) во II квартале _1952 г._ г., опытного _оловяно_ - _керамического кристаллизатора_ с _неоновым_ охлаждением (агрегат _ШГ_) и опытного _оловяно_ - _алюминиевого кристаллизатора_ с жидкометаллическим охлаждением (агрегат _БТ_) во втором полугодии _1952_ г.;

б) обеспечить окончание составления рабочих чертежей строительства первой очереди установки " _В-10_ " до I октября 1951 года;

Сл. 11-19
Гриф снят на основании решения № 16/156 от ... 94

Нс 16

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Экз. № 1

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Вх. 578 с/оп
2.VII.51

Товарищу В Л О Д И Н Ц Е В У Д.И.

Пересылаю Вам все имеющиеся у нас проектные материалы по «ЯРД».

ПРИЛОЖЕНИЕ:- упомянутое инв. №№ /п/м/ 1636 – 1 экз. на 26-ти листах он же печ. № 854-СП; 1669 – 1 экз. на 14-ти листах он же печ. 858-ОП.

Инв. № 1498 – 1 экз. на 18-ти листах к н/вх. № 1195 сс/оп от 2.ХП-50 г.

Чертёж – синьки в 1-м экз. инв. №№ 1800, 1801, 1802, 1803, 1804, 1805, 1806 и 1 синька н/о к н/вх. № 1195 сс/оп от 2.ХП-1950 года.

Всё приложение – только в адрес.

/М.Н. ГОЛОВИН/

"27" июня 1951 года

№ 864/СП

Приложение: взято по описи дела №№ 134, 135, 136, чертежи в инв. – 136.

Группа «С» МВД СССР
Вх. № 00/80-оп
2.7 1951 г.

1086

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П Р И К А З
МИНИСТРА СРЕДНЕГО МАШИНОСТРОЕНИЯ СССР
№ 286сс

г.Москва — 26 марта 1954 г.

В связи с предстоящим завершением работ по пуску электростанции В-10, П Р И К А З Ы В А Ю:

1. Назначить, для проверки готовности электростанции В-10 к пуску и эксплоатации, комиссию в следующем составе:

1. Славский Е.П. — председатель комиссии
2. Поздняков Б.С. — зам.председателя комиссии
3. Блохинцев Д.И.
4. Доллежаль Н.А.
5. Григорьянц А.Н.
6. Николаев Н.А.

2. Поручить указанной комиссии произвести проверку готовности электростанции В-10 к пуску и последующей эксплоатации, в том числе:

а/ произвести длительное испытание совместной работы всех механизмов реактора АМ /включая циркуляцию воды в контуре высокого давления/, а также совместную работу устройств реактора и ТВЦ при разных температурах, включая все

К СВЕДЕНИЮ

Категорически запрещается снятие копий и выписок из настоящего приказа

Вх. №

Вх. №

Quantum Mechanics

Proceedings of XII International Conference on Selected Problems of Modern Physics, Section I, Dubna, June 8–11, 2003.

Conditional Probabilities in Quantum Mechanics

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The famous book "An Introduction to Quantum Mechanics" by Prof. D.I. Blokhintsev was one of the first textbooks on quantum mechanics for universities published in Russia. Dmitriy Ivanovich succeeded to explain very delicate and intricate problems in a strict and comprehensible manner.

The notion of measurement in quantum mechanics and probabilistic character of the theory were among them. These problems were important in the early years of quantum mechanics. They attract a great interest now as well, particularly in connection with the present development of quantum communication.

Here we present a detailed description of a new quantum mechanical notion — Conditional Density Matrix — proposed by the authors [1], [2], [3]. This notion is a natural generalization of von Neumann density matrix for such processes as divisions of quantum systems into subsystems and reunifications of subsystems into new joint systems. As we will see, conditional density matrix assigns a quantum state to a subsystem of a composite system under condition that another part of the composite system is in some pure state.

First, let us recall some elements of the general scheme of quantum mechanics proposed by von Neumann in 1927 [4].

According to von Neumann, to evaluate the mean value of a physical variable $\mathcal{F}$ one should calculate the trace

$$< \mathcal{F} > \quad = \quad Tr(\hat{F}\hat{\rho}).$$

Here operator $\hat{\rho}$ satisfies three conditions:

$$1) \quad \hat{\rho}^{+} \quad = \quad \hat{\rho},$$

$$2) \quad Tr\hat{\rho} \quad = \quad 1,$$

$$3) \quad \forall \psi \in \mathcal{H} \quad < \psi|\hat{\rho}\psi > \quad \geq 0.$$

By the formula for mean values von Neumann found out the correspondence between linear operators $\hat{\rho}$ and states of quantum systems:

$$\textit{state of a system } \rho \iff \textit{linear operator } \hat{\rho}.$$

In this way, the formula for mean values becomes quantum mechanical definition of the notion "a state of a system". The operator $\hat{\rho}$ is called **Density Matrix.**

From the relation

$$(<\mathcal{F}>)^* = Tr(\hat{F}^+\hat{\rho})$$

one can conclude that Hermitian-conjugate operators correspond to complex-conjugate variables and Hermitian operators correspond to real variables.

$$\mathcal{F} \leftrightarrow \hat{F} \iff \mathcal{F}^* \leftrightarrow \hat{F}^+,$$

$$\mathcal{F} = \mathcal{F}^* \iff \hat{F} = \hat{F}^+.$$

The real variables are called *observables.*

From the properties of density matrix and the definition of positively definite operators:

$$\hat{F}^+ = \hat{F}, \qquad \forall\psi \in \mathcal{H} \quad <\psi|\hat{F}\psi> \ \geq 0,$$

it follows that the mean value of a nonnegative variable is nonnegative. Moreover, the mean value of nonnegative variable is equal to zero if and only if this variable equals zero. Now it is easy to give the following definition:

variable $\mathcal{F}$ has a definite value in the state ρ if and only if its dispersion in the state ρ is equal to zero.

In accordance to general definition of the dispersion of an arbitrary variable

$$\mathcal{D}(A) = <A^2> - (<A>)^2,$$

the expression for dispersion of a quantum variable $\mathcal{F}$ in the state ρ has the form:

$$\mathcal{D}_\rho(\mathcal{F}) = Tr(\hat{Q}^2\hat{\rho}),$$

where $\hat{Q}$ is an operator:

$$\hat{Q} = \hat{F} - <\mathcal{F}> \hat{E}.$$

If $\mathcal{F}$ is observable then Q^2 is a positive definite variable. It follows that the dispersion of $\mathcal{F}$ is nonnegative. And all this makes clear the above-given definition.

Since density matrix is a positive definite operator and its trace equals 1, we see that its spectrum is pure discrete and it can be written in the form

$$\hat{\rho} = \sum_n p_n \hat{P}_n,$$

where $\hat{P}_n$ is a complete set of self-conjugate projective operators:

$$\hat{P}_n^+ = \hat{P}_n, \quad \hat{P}_m \hat{P}_n = \delta_{mn} \hat{P}_m, \quad \sum_n \hat{P}_n = \hat{E}.$$

Numbers $\{p_n\}$ satisfy the condition

$$p_n^* = p_n, \quad 0 \leq p_n, \quad \sum_n p_n Tr \hat{P}_n = 1.$$

It follows that $\hat{\rho}$ acts according to the formula

$$\hat{\rho}\Psi = \sum_n p_n \sum_{\alpha \in \Delta_n} \phi_{n\alpha} \langle \phi_{n\alpha} | \Psi \rangle.$$

The vectors $\phi_{n\alpha}$ form an orthonormal basis in the space $\mathcal{H}$. Sets $\Delta_n = \{1, ..., k_n\}$ are defined by degeneration multiplicities k_n of eigenvalues p_n.

Now the dispersion of the observable $\mathcal{F}$ in the state ρ is given by the equation

$$\mathcal{D}_\rho(\mathcal{F}) = \sum_n p_n \sum_{\alpha \in \Delta_n} ||\hat{Q}\phi_{n\alpha}||^2.$$

All terms in this sum are nonnegative. Hence, if the dispersion is equal to zero, then

$$if \quad p_n \neq 0, \quad then \quad \hat{Q}\phi_{n\alpha} = 0.$$

Using the definition of the operator $\hat{Q}$, we obtain

$$if \quad p_n \neq 0, \quad then \quad \hat{F}\phi_{n\alpha} = \phi_{n\alpha}\langle F \rangle.$$

In other words, *if an observable $\mathcal{F}$ has a definite value in the given state ρ, then this value is equal to one of the eigenvalues of the operator $\hat{F}$.*

In this case we have

$$\hat{\rho}\hat{F}\phi_{n\alpha} = \phi_{n\alpha} p_n \langle \mathcal{F} \rangle,$$

$$\hat{F}\hat{\rho}\phi_{n\alpha} = \phi_{n\alpha} \langle \mathcal{F} \rangle p_n,$$

that proves the commutativity of operators $\hat{F}$ and $\hat{\rho}$.

It is well known, that if $\hat{A}$ and $\hat{B}$ are commutative self-conjugate operators, then there exists self-conjugate operator $\hat{T}$ with non-degenerate spectrum such that $\hat{A}$ and $\hat{B}$ are functions of $\hat{T}$:

$$\hat{T}\Psi = \sum_{n\alpha} \phi_{n\alpha} t_{n\alpha} \langle \phi_{n\alpha} | \Psi \rangle,$$

$$t^*_{n\alpha} = t_{n\alpha}, \quad t_{n\alpha} \neq t_{n'\alpha'}, \quad if \quad (n, \alpha) \neq (n', \alpha').$$

$$\hat{F}\Psi = \sum_{n\alpha} \phi_{n\alpha} f_1(t_{n\alpha}) \langle \phi_{n\alpha} | \Psi \rangle,$$

$$\hat{\rho}\Psi = \sum_{n\alpha} \phi_{n\alpha} f_2(t_{n\alpha}) \langle \phi_{n\alpha} | \Psi \rangle,$$

Suppose that $\hat{F}$ is an operator with non-degenerate spectrum; then

if the observable $\mathcal{F}$ with non-degenerate spectrum has a definite value in the state ρ, then it is possible to represent the density matrix of this state as a function of the operator $\hat{F}$.

The operator $\hat{F}$ can be written in the form

$$\hat{F} = \sum_n f_n \hat{P}_n,$$

$$\hat{P}^+_n = \hat{P}_n, \quad \hat{P}_m \hat{P}_n = \delta_{mn} \hat{P}_m, \quad tr(\hat{P}_n) = 1, \quad \sum_n \hat{P}_n = \hat{E}.$$

The numbers $\{f_n\}$ satisfy the conditions

$$f^*_n = f_n, \quad f_n \neq f_{n'}, \quad if \quad n \neq n'.$$

We obviously have

$$\hat{F} = \sum_n f_n \hat{P}_n.$$

From

$$\langle F \rangle = \sum_n p_n f_n = f_N,$$

$$\langle F^2 \rangle = \sum_n p_n f^2_n = f^2_N$$

we get

$$p_n = \delta_{nN}.$$

In this case density matrix is a projective operator satisfying the condition

$$\hat{\rho}^2 = \hat{\rho}.$$

It acts as

$$\hat{\rho}\Psi = \Psi_N \langle \Psi_N | \Psi \rangle,$$

where $|\Psi\rangle$ is a vector in Hilbert space.

The mean value of an arbitrary variable in this state is equal to

$$\langle \mathcal{A} \rangle = \langle \Psi_N | \hat{A} \Psi_N \rangle.$$

It is so-called *PURE* state. If the state is not pure it is known as *mixed.*

Suppose that every vector in $\mathcal{H}$ is a square integrable function $\Psi(x)$, where x is a set of continuous and discrete variables. Scalar product is defined by the formula

$$\langle \Psi | \Phi \rangle = \int dx \Psi^*(x) \Phi(x).$$

For simplicity we assume that every operator $\hat{F}$ in $\mathcal{H}$ acts as follows .

$$(\hat{F}\Psi)(x) = \int F(x, x') dx' \Psi(x').$$

That is for any operator $\hat{F}$ there is an integral kernel $F(x, x')$ associated with this operator

$$\hat{F} \Longleftrightarrow F(x, x').$$

Certainly, we may use δ-function if necessary.

Now the mean value of the variable $\mathcal{F}$ in the state ρ is given by equation

$$\langle \mathcal{F} \rangle_\rho = \int F(x, x') dx' \rho(x', x) dx.$$

Here the kernel $\rho(x, x')$ satisfies the conditions

$$\rho^*(x, x') = \rho(x', x),$$

$$\int \rho(x, x) dx = 1,$$

$$\forall \Psi \in \mathcal{H} \quad \int \Psi(x) dx \rho(x, x') dx' \Psi(x') \geq 0.$$

Suppose the variables x are divided into two parts: $x = \{y, z\}$. Suppose also that the space $\mathcal{H}$ is a direct product of two spaces $\mathcal{H}_1$, $\mathcal{H}_2$:

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2.$$

So, there is a basis in the space that can be written in the form

$$\phi_{an}(y,z) \quad = \quad f_a(y)v_n(z) \, .$$

The kernel of operator $\hat{F}$ in this basis looks like

$$\hat{F} \quad \Longleftrightarrow \quad F(y,z;y',z') \, .$$

In quantum mechanics it means that the system S is a unification of two subsystems S_1 and S_2:

$$S \quad = \quad S_1 \cup S_2 \, .$$

The Hilbert space $\mathcal{H}$ corresponds to the system S and the spaces $\mathcal{H}_1$ and $\mathcal{H}_2$ correspond to the subsystems S_1 and S_2.

Now suppose that a physical variable $\mathcal{F}_1$ depends on variables y only. The operator that corresponds to $\mathcal{F}_1$ has a kernel

$$F_1(y,z;y',z') \quad = \quad F_1(y,y')\delta(z-z') \, .$$

The mean value of F_1 in the state ρ is equal to

$$\langle F_1 \rangle_\rho \quad = \quad \int F(y,y')dy'\rho_1(y',y)dy \, ,$$

where the kernel ρ_1 is defined by the formula

$$\rho_1(y,y') \quad = \quad \int \rho(y,z;y',z)dz \, .$$

The operator $\hat{\rho}_1$ satisfies all the properties of Density Matrix in S_1. Indeed, we have

$$\rho_1{}^*(y,y') \quad = \quad \rho_1(y',y) \, ,$$

$$\int \rho_1(y,y)dy \quad = \quad 1 \, ,$$

$$\forall \Psi_1 \in \mathcal{H}_\infty \quad \int \Psi_1(y)dy\rho_1(y,y')dy'\Psi_1(y') \geq 0 \, .$$

The operator

$$\hat{\rho}_1 \quad = \quad Tr_2\hat{\rho}_{1+2} \, ,$$

is called **Reduced Density Matrix** . Thus, the state of the subsystem S_1 is defined by reduced density matrix.

The reduced density matrix for the subsystem S_2 is defined analogously.

$$\hat{\rho}_2 = Tr_1 \hat{\rho}_{1+2}.$$

Quantum states ρ_1 and ρ_2 of subsystems are defined uniquely by the state ρ_{1+2} of the composite system.

Suppose the system S is in a pure state then a quantum state of the subsystem S_1 is defined by the kernel

$$\rho_1(y, y') = \int \Psi(y, z) dz \Psi^*(y', z).$$

If the function $\Psi(y, z)$ is the product

$$\Psi(y, z) = f(y)w(z), \quad \int |w(z)|^2 dz = 1,$$

then subsystem S_1 is a pure state , too

$$\rho_1(y, y') = f(y)f^*(y'), \quad \int |f(y)|^2 dy = 1.$$

As it was proved by von Neumann, it is the only case when purity of composite system is inherited by its subsystems.

Let us consider an example of a system in a pure state having subsystems in mixed states. Let the wave function of composite system be

$$\Psi(y, z) = \frac{1}{\sqrt{2}}(f(y)w(z) \pm f(z)w(y)),$$

where $< f|w >= 0$ and $< f|f >=< w|w >= 1$. The density matrix of the subsystem S_1 has the kernel

$$\rho_1(y, y') = \frac{1}{2}(f(y)f^*(y') + w(y)w^*(y')).$$

The kernel of the operator $\hat{\rho}_1^2$ has the form

$${\rho_1}^2(y, y') = \frac{1}{4}(f(y)f^*(y') + w(y)w^*(y')).$$

Therefore, the subsystem S_1 is in the mixed state. Moreover, its density matrix is proportional to unity operator.

The mean value of a variable with the kernel

$$F^c(x,x') = F_1(y,y')u(z)u^*(z'), \quad \int |u(z)|^2 dz = 1,$$

is equal to

$$\langle F^c \rangle_\rho \quad = \quad p \int F_1(y,y')dy' \rho^c(y',y)dy,$$

where

$$\rho^c(y,y') = \frac{1}{p} \int u^*(z)dz\, \rho(y,z;y',z')\, u(z')dz',$$

$$p \quad = \quad \int u^*(z)dz\, \rho(y,z;y,z')\, u(z')dz'dy.$$

Since we can represent p in the form

$$p \quad = \quad \int P(z,z')dz'\, \rho_2(z';z)dz,$$

$$P(z,z') \quad = \quad u(z)u^*(z'),$$

we see that p is an mean value of a variable P of the subsystem S_2. Operator $\hat{P}$ is a projector ($\hat{P}^2 = \hat{P}$). Therefore it is possible to consider the value p as a probability.

It is easy to demonstrate that the operator $\hat{\rho}^c$ satisfies all the properties of density matrix. So the kernel $\rho^c(y,y')$ defines some state of the subsystem S_1. What is this state?

According to the decomposition of δ-function

$$\delta(z-z') = \sum_n \phi_n(z)\phi_n{}^*(z'),$$

$\{\phi_n(z)\}$ being a basis in the space H_2, the reduced density matrix is represented in the form of the sum

$$\rho_1(y,y') = \sum p_n \rho_n^c(y,y').$$

Here

$$\rho_n^c(y,y') = \frac{1}{p_n} \int \phi_n{}^*(z)dz\, \rho(y,z;y',z')\, \phi_n(z')dz'$$

and

$$p_n \quad = \quad \int \phi_n{}^*(z)dz\, \rho(y,z;y,z')\, \phi_n(z')dz'dy$$

$$= \quad \int \hat{P}_n(z,z')dz'\, \rho_2(z',z)dz.$$

The numbers p_n satisfy the conditions

$$p_n{}^* = p_n, \qquad p_n \geq 0, \qquad \sum_n p_n = 1.$$

and are connected with a probability distribution.

The basis $\{\phi_n\}$ in the space H_2 corresponds to some observable $\hat{G}_2$ of the subsystem S_2 with discrete non-degenerate spectrum. It is determined by the kernel

$$G_2(z,z') = \sum_n g_n \phi_n \phi *_n, \quad g_n = g*_n; \quad g_n \neq g_{n1} \quad if \quad n \neq n1.$$

The mean value of G_2 in the state ρ_2 is equal to

$$\int dy \rho_2(z,z') dz' G(z',z) =$$

$$= \sum_n g_n \int dy \rho_2(z,z') dz' \phi_n(z') \phi_n{}^*(z') = \sum_n p_n g_n.$$

Thus number p_n defines the chance that the observable $\hat{G}_2$ has the value g_n in the state ρ_2. Obviously, the kernel $\rho_n^c(y,y')$ in this case defines the state of system S_1 under condition that the value of variable G_2 is equal to g_n. Hence it is natural to call operator $\hat{\rho}_n^c$ as **Conditional Density Matrix**

$$\hat{\rho}_{c1|2} = \frac{Tr_2(\hat{P}_2 \hat{\rho})}{Tr(\hat{P}_2 \hat{\rho})}.$$

It is (*conditional*) density matrix for the subsystem S_1 under the condition that the subsystem S_2 is selected in a pure state $\hat{\rho}_2 = \hat{P}_2$. It is the most important case for quantum communication. We can see that conditional density matrix satisfies all the properties of density matrix.

As density matrix is an operator with a pure discrete spectrum, that is

$$\hat{\rho} \quad = \quad \sum_n q_n \hat{Q}_n \, ,$$

and projective operators satisfy the condition

$$\sum_n q_n Tr \hat{Q}_n \quad = \quad 1 \, ,$$

then the number

$$w_n \quad = \quad q_n Tr(\hat{Q}_n)$$

is the probability for the system to be in the state given by the projective operator $\hat{Q}_n$.

Conditional density matrix determines conditional probability in the same manner as usual density matrix gives probability of a measurement.

To make this statement clear let us recall the notion of **conditional probability**.

Let H be an event with the positive probability $P(H)$. Then the probability of the event AH is presented in the form

$$P\{AH\} = P\{A|H\}P\{H\}$$

Here $P\{A|H\}$ is *the conditional probability* of the event A under the condition that the event H takes place.

Obviously for a given H all the properties of the probabilities are valid for conditional probabilities, e.g.,

$$P\{A \cup B|H\} = P\{A|H\} + P\{B|H\} - P\{AB|H\}.$$

Consider the set of mutually excluded events $\{H_1, H_2, ...\}$. And let one of them takes place inevitably. Then any event A can be fulfilled only with one of the H_j simultaneously. Since the event A is represented in the form

$$A = AH_1 \cup AH_2 \cup \dots,$$

the probability of the event A can be written in terms of the probabilities of the events $\{H_j\}$ and corresponding conditional probabilities:

$$P\{A\} = \sum_j P\{A|H_j\}P\{H_j\}.$$

For a system $S = S_1 \cup S_2$ matrix elements of density matrix are of the form $\rho(y, z;\, y', z')$.

The elements of the reduced density matrix are

$$\rho^{(1)}(y;\, y') = \sum_z \sum_{z'} \delta(z, z')\, \rho(y, z;\, y', z').$$

Let a set of operators $\hat{P}_a$ give a decomposition of the identity operator in the space $\mathcal{H}_2$. Then

$$\delta(z, z') = \sum_a P_a(z, z').$$

Now

$$\rho^{(1)}(y;\,y') = \sum_a \sum_z \sum_{z'} P_a(z,z')\,\rho(y,z;\,y',z') = \sum_a p_a\,\rho_a^{c(1)}(y;\,y')\,.$$

Here

$$\rho_a^{c(1)}(y;\,y') = \frac{1}{p_a}\sum_z \sum_{z'} P_a(z,z')\,\rho(y,z;\,y',z')\,,$$

and

$$p_a = \sum_z \sum_{z'} P_a(z,z') \sum_y \rho(y,z;\,y,z')\,, = \sum_z \sum_{z'} P_a(z,z')\,\rho^{(2)}(z;\,z')\,.$$

Thus, p_a is the probability for the subsystem S_2 to be in the state given by the projective operator $\hat{P}_a$.

$\rho_a^{c(1)}(y;\,y')$ is a **density matrix** of the subsystem S_1.

The number

$$w_y = \sum_a p_a\,\rho_a^{c(1)}(y;\,y)$$

is the probability for the subsystem S_1 to be in the state $|y\rangle$.

The above equation is nothing else but the representation of **total probability** in terms of **conditional probabilities.**

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[4] J. von Neumann. Gött. Nach. pp. 1–57, 245 – 272, **1927**. See, also, J. von Neumann, *Mathematische Grundlagen der Quantenmechanik*, Berlin, 1932.

Proceedings of XII International Conference on Selected Problems of Modern Physics, Section I, Dubna, June 8-11, 2003.

D. I. Blokhintsev microworld spacetime: a modern outlook

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Abstract — The Blokhintsev investigations of the structure of microworld spacetime are reviewed. Some new proposals are made.

D. I. Blokhintsev was a man with unusual wide personality. He was a scientist and a poet, a painter and a philosopher. He was also an outstanding animator and manager of science. His personal charm was wonderful.

As a physicist D. I. Blokhintsev equally well was interested in applied physics and in fundamental problems. One of his main subject of investigation in the field of fundamental physics was the problem of spacetime in microworld [1]. After a profound and detailed analysis of all aspects of the concept of space and time in macrophysics he performed a critical review of all fundamental spacetime notions and tried to define them from the scratch. The goal was to find such definitions of spacetime concepts which, as particular cases, will include the standard macrophysical models of spacetime from one side and will give adequate models of spacetime in microworld from the other side.

He started from a very general definition of geometry as a science of ordering physical events. Similarly, the fundamental notion of causality he defined as a genetical relation between physical events.

The understanding of spacetime as a specific physical object usually is attributed to general relativity. It is generally believed that before the era of general relativity all physicists treated space as empty arena for all physical processes.As a matter of fact, it is not a true statement because already Helmholtz paid attention to the fact that Euclidean geometry is tightly related to the notion of physical rigid bodies. All our ability to check geometrical relations dictated by Euclidean geometry depends on the existence of physical bodies which more or less accurately posses the properties of ideal rigid bodies. In a modern language the Helmholtz analysis allows to look at the Euclidean geometry as a representation of the group of symmetries of rigid bodies. Therefore, it may be said that the intrinsic relation between geometry of spacetime and the class of physical bodies and processes was always present and the fact that some time ago this relation was not properly understood does not mean that this relation was absent. The discovery of new physical objects, new interactions between them and new kind of processes in which they participate must always be accompanied by critical reviews of fundamental spacetime notions in order to clarify which of them may still be applied to the new physical situation and which of them must be replaced by new and/or more general notions. The relation between geometry and physical laws was also stressed by B. Riemann.

In his general analysis of spacetime concepts D. I. Blokhintsev realized that, contrary to the wide believe, in some coordinate systems the natural properties of spacetime and physical processes may be described simpler than in other ones. In this way he admitted the possibility of formulating physical theories in privileged reference systems. This statement does not contradict the relativity principles. As a matter of fact, the physical theories with preferred reference systems are now intensively developed [2]. It seems that the preferred systems are necessary for relativistic quantum mechanics as well as in all classical theories in which the space properties are described in terms of generalized functions instead of ordinary functions.

¿From the general Blokhintsev's analysis of spacetime concepts it follows that he was close to formulate the statement that the understanding of these concepts depends on [3]:

1. The class of observers.
2. The class of elementary events.
3. The kind of agents which are used to connect the observers with the elementary events.
4. The assumed way of interaction of the agents with the events.
5. Some conventions.

These items needs some explanation.

Ad. 1: In the standard approach to special relativity it is assumed that the observers have emitters and detectors of light signals. The emission times and detection times are registered by ordinary clocks. These facts limit the applicability of models constructed by such observers to classical models. In microworld the emission and detection times are connected with some uncertainties and neglecting that is possible only in classical physics.

Ad. 2: It is not so obvious how to select the class of elementary events.

Here already the very definition of elementary events is a complicated problem. Again, as a rule, we apply here only classical intuitional concepts.

Ad. 3: As a rule, it is uncritically assumed that only light signals may serve as agents to connect observers and events. The light is treated classically as objects moving uniformly with invariant velocity. It is however easy to develop special relativity assuming that light is moving with constant acceleration [4]. It is also an interesting exercise to repeat the Einstein-Bondi radiolocation method with different from light signals agents. In each case we end up with different models of spacetime. Simultaneously we see that light is not the only tool of communication between observers and events. Such possibilities were already been considered by D. I. Blokhintsev.

Ad. 4: This point is the most difficult item in the construction of models of spacetime. Here we have to include as one of fundamental facts the knowledge of the interaction between agents and elementary events. However most of such knowledge is connected with some models of spacetime. We fall therefore in a closed circle of reasoning. The only way to get out from this circle is to agree that all our constructions should be treated as approximate and we must perform repeated applications of model dependent knowledge to construct better models. The final goal is to get (in some limit) a universal model of spacetime adequate for microworld physics.

The standard radiolocation method assumes that light behaves according to geometrical optics. In particular, the light signals immediately reflect from ideal "mirrors" located at elementary events. The real light is behaving according to quantum electrodynamics and therefore any model which disregards this fact must be treated as very approximate one with a restricted domain of applicability. It is almost obvious that disregarding the quantum character of interaction of light with matter located at elementary events excludes models based on such an assumption as invalid in microworld.

Ad. 5: The conventional character of all models of spacetime was extensively discussed by H. Reichenbach [5]. In this line D. I. Blokhintsev discussed other "non spatial" models of spacetime. For example, he discussed models based not on numbered coordinates but on different colors ascribed to different points of space. This is an example rather of a qualitative picture of spacetime. Another line of reasoning consists in investigation the influence of "pseudo events" taken as real physical events. Again, as an example, D. I. Blokhintsev considered a model based on the replacement of the space coordinate by the value of force exerted by a mass point at the given position. He observed that the standard Hook's law

$$\vec{x} = -\frac{1}{k}\vec{F}$$

allows to replace the concept of position by the concept of force. This method seems to be more general because the force is directly measurable in a very wide class of events. Unfortunately, the Hook's law is invalid for large values of the coordinate and this restricts its validity at large scale distances. A better example seems to be the method based on a Coulomb law

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{\vec{r}}{r^3}$$

which is valid for large distances. For small distances we meet a limitation connected with the fact that the maximal value of measurable intensities of electric field is of the order of $10^5 V/m$. This definitely shows that for small distances our intuition breaks down. The applicability of the Hook's law for small distances is however artificial because as a matter of fact we do not know whether in microworld such law is reasonable or not.

D. I. Blokhintsev extensively investigated the possibility of replacement the standard classical spatial coordinatization of spacetime by the values of physical fields. Unfortunately, he never considered the real electromagnetic field as a fundamental object on which we may base our understanding the concept of spacetime. As is well known, Maxwell equations possess definite symmetry properties [6] which in the simplest case is the $SO(3,3)$ symmetry. The standard Minkowski spacetime posses the $SO(1,3)$ symmetry and all physical objects are described by various irreducible representations of this symmetry group. Here we get a possibility to reverse the way. Instead of looking for the representations of the $SO(1,3)$ group we may look for the various representations of the $SO(3,3)$ group and base the construction of the spacetime model on one such representation which adequately will describe the regularities in the microworld. In particular, this approach in natural way introduces the notion of a three-dimensional time, a concept also extensively investigated at the present time [7].

D. I. Blokhintsev came to the conclusion that each way of ascribing space and time coordinates to elementary events is in fact an arbitrary procedure which requires to perform some definite manipulations with physical objects. Since our knowledge increases in time also the known constructions of spacetime models time to time must be revised and, if necessary, improved. In this way we may achieve extensions of the applicability domains for different existing models and improve their degree of accuracy. This applies not only to microworld models but also to macroworld models, especially for non stationary worlds.

According to D. I. Blokhintsev there is rather a little doubt that we adequately know precise models of macroworld spacetime. It is not so for microworld where we shall agree that adequate models of microworld spacetime must be more abstract and less intuitive than the macroworld concepts are. He came to the conclusion that we should also be ready to reject the concepts of spacetime coordinates at all and base our understanding of space order and time relations on other more abstract notions. The full understanding of this new abstract notions will be achieved gradually after deriving sufficient number of conclusions which can experimentally be checked.

D. I Blokhintsev deeply believed that for investigating the microworld spacetime structures "we need to find new physical concepts and, consequently, a new language which will be more adapted for description of internal nature of elementary particles". He hoped that for finding this new language "we need to find only two or three new words which will allow to express that new physical idea which is necessary to understand all microworld events". He appealed to the coming generations of physicists to find these new words having in mind that "these new words should be as revolutionary as those words which led to formulate quantum mechanics or the theory of relativity".

In my opinion, it is rather the opposite case: **we do not need to add some new elements to the spacetime physics but we have first to remove from it some standard concepts which obscure the new looks into microworld spacetime physics.** Let me remind that quantum physics removed from physics the classical notion of the trajectory and relativity theory rejected the absolute character of time. Since all essential troubles of microworld spacetime physics are connected either with finite accuracy of experimental devices or with fundamental uncertainty relations I would like to propose **to replace the standard mathematics used in analyzing physical phenomena by new, more modern mathematical methods.** In particular, I propose to use mathematical methods based on:

1. Non-standard analysis.
2. The theory of fuzzy sets.
3. The topos theory, the categorial approach to logic.

These proposal are indeed revolutionary. The immediate gains from them are the following:

1. The non-standard analysis [8] allows to avoid all questions connected with fundamental uncertainties. The physical uncertainties simply define the

ıathematical "monads" which surround all standard numbers used in stanard analysis.

2. The fuzzy sets [9] in a more fundamental way introduce into physics ıe quantum mechanical languages. In particular, the quantum mechanical robabilities define the degree of inclusion the elements into a given sets. The ew physics will be simply a fuzzy version of ordinary physics.

3. The topos theory [10] is the most radical change in mathematics which llows to realize different logical deduction schemes. For physics, the most ttractive feature of topos theory is to have a possibility to analyze structures f complex systems without specifying their elements. This allows to avoid ıe question where the physical limit of divisibility of matter is. According ɔ Niels Bohr and Werner Heisenberg the theory which do not allow to ask nanswerable questions is a good physical theory.

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Lifetime and path-length of the virtual particle

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Abstract — The concepts of the lifetime and path-length of a virtual particle are introduced. It is shown that near the mass surface of the real particle these quantities constitute a 4-vector. At very high energies the virtual particle can propagate over considerable (even macroscopic) distances. The formulae for the lifetime and path-length of an ultrarelativistic virtual electron in the process of bremsstrahlung in the Coulomb field of a nucleus are obtained. The lifetime and path-length of the virtual photon at its conversion into the electron-positron pair is discussed. The connection between the path-length of the virtual particle and the coherence length (formation length) has been analyzed.

1 Localization of a virtual particle in time and space

It is known that internal lines of Feynman diagrams play the role of intermediate states, or virtual particles [1]. The definite 4-momentum $P = \{E, \mathbf{P}\}$ outside the mass surface of the real particle corresponds to the virtual particle. Here E is the energy of the virtual particle, $\mathbf{P}$ is its 3-momentum (we use the unit system, in which $\hbar = c = 1$). The magnitude

$$M = \sqrt{P^2} \equiv \sqrt{E^2 - \mathbf{P}^2} \tag{1}$$

has the meaning of the mass of the virtual particle. When P is the time-like 4-momentum, then the virtual particle has the positive mass as well as the real particle. In so doing $M \neq m$, where m is the mass of the real particle. When P is the space-like 4-momentum, then the imaginary mass ($M^2 < 0$) formally corresponds to the virtual particle.

The lifetime T of the virtual particle, characterizing the time scale of the considered process, can be defined on the basis of the uncertainty principle for energy and time: $T = |\Delta E|^{-1}$, where

$$\Delta E = \sqrt{\mathbf{P}^2 + m^2} - \sqrt{\mathbf{P}^2 + M^2}$$

is the difference of energies of the real and virtual particles with <u>the same 3-momentum $\mathbf{P}$</u>. As a result, we obtain:

$$T = \frac{E + \tilde{E}}{|m^2 - M^2|}. \tag{2}$$

Here E is the energy of the virtual particle, $\tilde{E} = \sqrt{E^2 + m^2 - M^2}$ is the energy of the real particle. Relation (2) can be rewritten in the form:

$$T = \frac{E}{|m^2 - M^2|}\left(\sqrt{1 + \frac{m^2 - M^2}{E^2}} + 1\right). \tag{3}$$

Analogously, the space dimensions L of the region of propagation of the virtual particle, characterizing the space scale of the given process, can be defined on the basis of the uncertainty principle for momenta and coordinates: $L = |\Delta\mathbf{P}|^{-1}$, where

$$|\Delta\mathbf{P}| = \sqrt{E^2 - m^2} - \sqrt{E^2 - M^2}$$

is the difference of momenta of the real and virtual particles moving along the direction of the momentum vector $\mathbf{P}$ with the same energy E. It is easy to see that

$$L = \frac{|\mathbf{P}| + |\tilde{\mathbf{P}}|}{|m^2 - M^2|}. \tag{4}$$

Here $|\tilde{\mathbf{P}}| = \sqrt{\mathbf{P}^2 - m^2 + M^2}$ is the absolute value of momentum of the real particle. We can also write

$$L = \frac{|\mathbf{P}|}{|m^2 - M^2|}\left(\sqrt{1 - \frac{m^2 - M^2}{\mathbf{P}^2}} + 1\right). \tag{5}$$

At small differences of masses of the virtual and real particles, when

$$|m^2 - M^2| \ll E^2, \qquad |m^2 - M^2| \ll |\mathbf{P}|^2,$$

it follows from Eqs. (2) and (3) that

$$T = \frac{2E}{|m^2 - M^2|}, \qquad L = \frac{2|\mathbf{P}|}{|m^2 - M^2|}. \tag{6}$$

Under these conditions the quantities T and $\mathbf{L} = 2\mathbf{P}/|m^2 - M^2|$ constitute the 4-vector [2]

$$\{T, \mathbf{L}\} = \frac{2P}{|m^2 - M^2|}. \tag{7}$$

All the relations presented above are valid for any 4-momentum of the virtual particle (both time-like and space-like). In the case of the time-like 4-momentum P the quantity $\mathbf{L}$ can be clearly interpreted as the vector of path-length of the virtual particle moving with the velocity $\mathbf{v} = \mathbf{P}/E < 1$; in so doing, the path-length and lifetime of the virtual particle are connected by the standard equation $\mathbf{L} = \mathbf{v}T$ [1] $=$).

When the mass M of the virtual particle is close to the mass m of the real particle, the lifetime T of the virtual particle strongly increases and its path-length $L = |\mathbf{L}|$ may considerably exceed the interatomic distances. The same takes place also at ultrarelativistic energies $E \gg |M|$, $E \gg m$, when the velocity of the virtual partucle approaches the velocity of light. In the limit of very high energies the path-length of the virtual particle can reach even macroscopic values. That leads to the coherent effects observed in the interaction of very high-energy particles with matter [3-6].

2 Lifetime and path-length of the relativistic virtual electron at the electron bremsstrahlung

Using the Born approximation, the bremsstrahlung of the electron in the Coulomb field of a heavy nucleus is described by two Feynman diagrams (see Fig.1). Let us denote the 4-momenta of the initial electron, final electron and photon, respectively, as
$p_1 = \{E_1, \mathbf{p}_1\}, p_2 = \{E_2, \mathbf{p}_2\}$ and $k = \{\omega, \mathbf{k}\}$. Taking into account that the energy transfer to the heavy nucleus is absent, the equality $E_2 = E_1 - \omega$ is valid, and the energy component of the space-like 4-momentum of the virtual photon $q = \{0, \mathbf{q}\}$ is equal to zero (here $\mathbf{q}$ is the 3-momentum transferred to the nucleus)[1]. Then the 4-momentum of the virtual electron, corresponding to the diagram I, is

$$P_I = p_2 + k = \{E_1, \mathbf{p}_2 + \mathbf{k}\},$$

and the 4-momentum of the virtual electron, corresponding to the diagram II,

[1] In the usual unit system the formulae (3), (5), (6) and (7) have the following form:

$$T = \frac{\hbar E}{|m^2 - M^2|c^4}\left(\sqrt{1 + \frac{(m^2 - M^2)c^4}{E^2}} + 1\right); \tag{3a}$$

$$L = \frac{\hbar|\mathbf{P}|}{|m^2 - M^2|c^2}\left(\sqrt{1 - \frac{(m^2 - M^2)c^2}{\mathbf{P}^2}} + 1\right); \tag{5a}$$

$$T = \frac{2\hbar E}{|m^2 - M^2|c^4}, \qquad L = \frac{2\hbar = |\mathbf{P}|}{|m^2 - M^2|c^2}; \tag{6a}$$

$$\{cT, \mathbf{L}\} = \frac{2\hbar P}{|m^2 - M^2|c^2}. \tag{7a}$$

is

$$P_{II} = p_1 - k = \{E_2, \mathbf{p}_1 - = \mathbf{k}\}.$$

It is evident that the square of mass of the virtual electron for the diagram I has the form:

$$M_I^2 = (p_2 + k)^2 = m^2 + 2p_2k = m^2 + 2E_2\omega(1 - v_2 \cos\theta),$$

where m is the electron mass, θ is the angle between the final electron momentum $\mathbf{p}_2$ and the photon momentum $\mathbf{k}$, $v_2 = |\mathbf{p}_2|/E_2$ is the velocity of the final electron. According to Eqs.(6), the lifetime and path-length of the virtual electron under the condition
$E_1^2 \gg E_2\omega(1 - v_2 \cos\theta)$ are determined by the following formulae:

$$T_I = \frac{E_I}{E_2\omega(1 - v_2 \cos\theta)}; \quad L_I = \frac{|\mathbf{p}_2 + \mathbf{k}|}{E_2\omega(1 - v_2 \cos\theta)}. \tag{8}$$

Analogously, in the case of the diagram II we have:

$$M_{II}^2 = (p_1 - k)^2 = m^2 - 2p_1k = m^2 - 2E_1\omega(1 - v_1 \cos\tilde{\theta}),$$

where $\tilde{\theta}$ is the angle between the initial electron momentum $\mathbf{p}_1$ and the photon momentum $\mathbf{k}$, $v_1 = |\mathbf{p}_1|/E_1$ is the velocity of the initial electron. In accordance with Eqs.(6), we obtain, under the condition $E_2^2 \gg E_1\omega(1 - v_1 \cos\tilde{\theta})$, the following expression for the lifetime and path-length of the virtual electron:

$$T_{II} = \frac{E_2}{E_1\omega(1 - v_1 \cos\widetilde{\theta})}; \quad L_{II} = \frac{|\mathbf{p}_1 - \mathbf{k}|}{E_1\omega(1 - v_1 \cos\widetilde{\theta})}. \tag{9}$$

At ultrarelativistic energies $E_1 \gg m$, $E_2 \gg m$ and small angles $\theta \ll 1$, $\tilde{\theta} \ll 1$ one can write:

$$1 - v_2 \cos\theta \approx 1 - v_2 + v_2\frac{\theta^2}{2} \approx \frac{1}{2\gamma_2^2}(1 + \gamma_2^2\theta^2); \qquad 1 - v_1 \cos\tilde{\theta} \approx 1 - v_1 + v_1\frac{\widetilde{\theta^2}}{2} \approx \frac{1}{2\gamma_1^2}(1 + \gamma_1^2\widetilde{\theta^2}),$$

where $\gamma_1 = E_1/m$ and $\gamma_2 = E_2/m$ are the Lorentz-factors of the initial and final electrons, respectively ($\gamma_1 \gg 1$, $\gamma_2 \gg 1$). Under these conditions, taking into account the approximate equalities $|\mathbf{p}_2 + \mathbf{k}| \approx E_1$ and $|\mathbf{p}_1 - \mathbf{k}| \approx E_2$, the relations (8) and (9) give:

$$T_I \approx L_I \approx \frac{2\gamma_1\gamma_2}{\omega(1 + \gamma_2^2\theta^2)}, \qquad T_{II} \approx L_{II} \approx \frac{2\gamma_1\gamma_2}{\omega(1 + = \gamma_1^2\tilde{\theta}^2)}. \tag{10}$$

The main contribution into the process of the bremsstrahlung of the ultrarelativistic electron in the Coulomb field of a heavy nucleus arises from the region of small angles $\theta <\sim 1/\gamma_2$, $\tilde{\theta} <\sim 1/\gamma_1$. As a result, the effective path-length of the ultrarelelativistic electron at the bremsstrahlung in the Coulomb field has the magnitude of the order of

$$L \sim L_I \sim L_{II} \sim \frac{\gamma_1 \gamma_2}{\omega}. \tag{11}$$

In the usual unit system $L_I \sim L_{II} \sim \hbar c \gamma_1 \gamma_2/(E_1 - E_2)$. For example, for the electron with the energy $E_1 \approx E_2 = 100$ GeV and the photon energy $\omega = 10$ MeV the effective path-length L of the virtual electron amounts to $\sim 8 \cdot 10^{-2}$ cm ~ 1 mm (the virtual electron lifetime T equals $\sim 3 \cdot 10^{-12}$ sec).

3 Conversion of the virtual photon into the electron-positron pair

Any process with the emission of a photon may be accompanied by the other process in which the virtual photon is converted into the e^+e^--pair (see Fig.2). In this case the virtual photon has the time-like 4-momentum $P = \{E_+ + E_-, \mathbf{p}_+ + \mathbf{p}_-\}$, and its mass is given by the formula

$$M^2 = 2m^2 + 2E_+E_-(1 - v_-v_+ \cos\phi),$$

where m is the mass of the positron and electron, $\mathbf{p}_+, E_+, v_+$ – the momentum, energy and velocity of the positron, respectively, $\mathbf{p}_-, E_-, v_-$ – the same for the electron, ϕ is the angle between the electron and positron momenta. According to Eqs.(6), under the condition $E^2 = (E_+ + E_-)^2 \gg M^2$ the lifetime and path-length of the virtual photon at the conversion into the e_+e_--pair amount to

$$T \approx L = \frac{2E}{M^2}. \tag{12}$$

For narrow electron-positron pairs, when $\phi \approx 0$, $M^2 \approx 4m^2$, $E_+ \approx E_-$, we obtain:

$$T \approx L \approx \frac{E_+ + E_-}{2m^2} \approx \frac{\gamma}{m},$$

where γ is the Lorentz-factor of the positron and electron. In the usual unit system $T \approx \hbar\gamma/mc^2$, $L \approx \hbar\gamma/mc$. For example, at $\gamma = 10^6$ (the energies of the ultrarelativistic electron and positron amount to $E_+ \approx E_- \approx 500$ GeV), the lifetime and the path-length of the virtual photon reach the values $T \approx 1.3 \cdot 10^{-15}$ sec , $L \approx 3.8 \cdot 10^{-5}$ cm, respectively. These magnitudes are not nacroscopic but they, = nevertheless, strongly exceed the characteristic atomic cale.

4 Connection between the path-length of the virtual particle and the coherence length

For the process of the ultrarelativistic electron bremsstrahlung, the coherence length (formation length) is inversely proportional to the minimal momentum $q_{\min}$ transferred to the nucleus [1,3-6]:

$$L_{\text{coh}} \sim \frac{1}{q_{\min}}.$$

The minimal transferred momentum corresponds to the forward direction, when the momenta of both the final electron and photon are parallel to the momentum of the initial electron. It is clear that

$$q_{\min} = \sqrt{E_1^2 - m^2} - \sqrt{E_2^2 - m^2} - \omega = \sqrt{E_1^2 - m^2} - \sqrt{E_2^2 - m^2} - (E_1 - E_2) \approx$$

$$\approx \frac{m^2}{2E_2} - \frac{m^2}{2E_1} = \frac{m^2}{2} = \frac{E_1 - E_2}{E_1 E_2} = \frac{\omega}{2\gamma_1\gamma_2}$$

(all the notations are the same as in Section 2). So, we obtain:

$$L_{\text{coh}} \sim \frac{2\gamma_1\gamma_2}{\omega}. \tag{13}$$

Thus, the path-length L of the virtual ultrarelativistic electron (see Eq. (11)) coincides, by the order of magnitude, with the coherence length L_{coh}.

The relation $L \sim L_{\text{coh}}$ is valid for any process, in which the transition of a real particle into the virtual one, or the reverse transition, takes place with the very low momentum transfer to a nucleus.

5 Summary

1. The concepts of the lifetime T and path-length L of a virtual particle are introduced. On the basis of the uncertainty principle for energy and time and the uncertainty principle for momenta and coordinates, the general expressions for T and L are obtained.

2. Using the example of the ultrarelativistic electron bremsstrahlung in the Coulomb field of a heavy nucleus, it is shown that at very high energies the virtual particle can propagate over considerable (even macroscopic) distances.

3. The connection between the path-length of a virtual particle and the coherence length at the interaction of ultrarelativistic particles with nuclei is analyzed.

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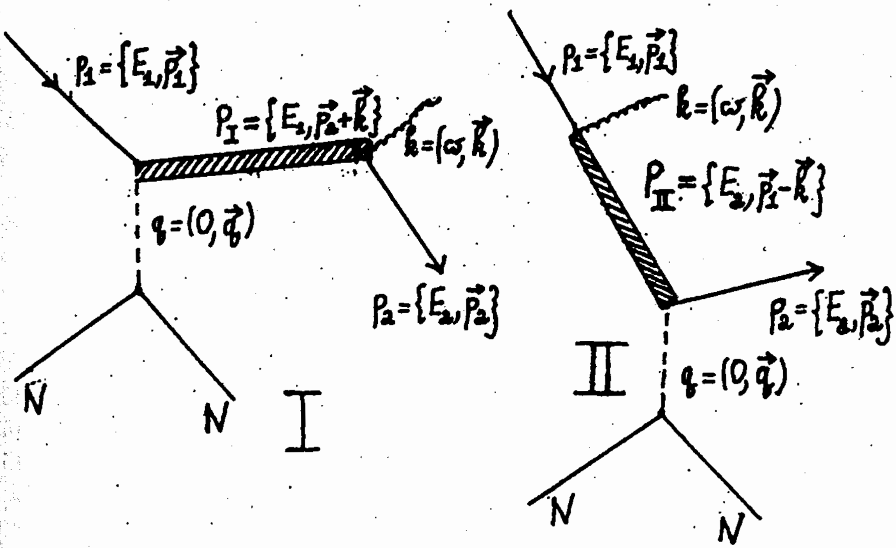
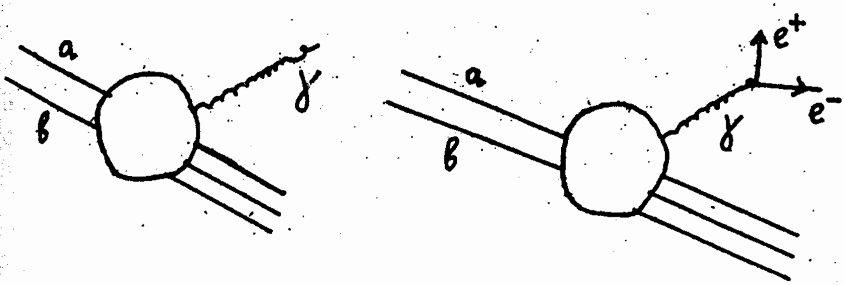


'igure 1. Feynman diagrams for the electron bremsstrahlung in the Coulomb field of a nucleus.



gure 2. Conversion of the virtual photon into the electron-positron pair.

Particles and wave functions

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Abstract — Problems of particles, wave functions and measurement are considered. It is shown that there are superselection rules for macroscopic objects.

1 Quantum mechanics: Questions

The non-relativistic quantum mechanics (QM) rises obvious questions:
1) What is particle?
2) What is wave function?
3) What is measurement?
Different "interpretations" of QM give different answers to the questions.
The Copenhagen interpretation (CI). A particle has no certain position or momentum; ψ-functions describe individual systems (e.g. electron) and contain full information about particles; measurement cannot be described in details ("uncontrolled interaction" of a microscopic object with a classical apparatus). It is tacitly assumed that particles are pointlike.
The statistical interpretation (SI). "... a momentum eigenstate (plane wave in configuration space) represents the ensemble whose members are single electrons each having the same momentum but distributed uniformly over all positions." [1. p. 361]. Thus QM is not complete: particles can be characterized by both positions and momenta. There is no problem of decoherence of macroscopic apparatuses in the act of measurement (ψ-function describes a statistical ensemble!). Particles are pointlike.
The Everett interpretation (EI). The interpretation was invented to solve the orthodox QM problem of decoherence of macroscopic objects [2]. In the process of measurement the initial state $|o\rangle|A\rangle$ of an object $|o\rangle$ and of an apparatus $|A\rangle$ becomes entangled:

$$|o\rangle|A\rangle \to \sum_k c_k|k\rangle|A_k\rangle.$$

The process is described by the Schroedinger equation. Both initial and final states are pure. Thus the classical apparatus is described by superposition of macroscopically different states. Assumption: the world branches. Only the

›served state $|A_o\rangle$ is in "this" world. All the other states also exist, but in ›ther" worlds. The Everett interpretation respects both the orthodox QM ıd the principle of macroscopic certainty of classical bodies.

It is significant that even in XXI century quantum problems are discussed ı terms appeared some 70-75 years ago.

D.I. Blokhintsev: Answers

It is usually assumed that D.I. Blokhintsev confessed the statistical inter- retation [1, p. 360]. Actually it is not the case [3]. As is easily seen from book] he adhered to the orthodox QM, though keeping door open for the other ›ssibilities. D.I. Blokhintsev acknowledged wave function though invented ew term "quantum ensemble" in order to stress stochastic nature of QM and eculiarity of its mathematical apparatus (probability amplitudes instead of robability densities).

1. PARTICLES. Blokhintsev pointed out that particles are not pointlike ›jects [5,6]; they are excitations (quanta) of fields (see also [7]). Actually this the key to the main problem of QM. It follows from it that particles are non- ›cal objects. They can be registered at any point where the field is excited. the field is treated as an ordered set of oscillators then it is natural to call ıe set "quantum ensemble". But Blokhintsev understood under "quantum nsemble" an ensemble of macroscopic experimental devices plus microobjects].

2. WAVE FUNCTIONS. Blokhintsev was not certain about nature of wave ınctions. He discussed, in particular, the hypothesis that wave function is a diary of physicist" [4, p. 121]. Indirectly he even admitted the possibility ıat there exists more detailed description of microobjects ("Who prevents ou to set the Thames on fire?" [4, p. 157]; the Russian equivalent sounds less :rong). It is amazing that he did not identified wave function with the field ›r functional) describing the one-particle field excitations. In [5, p. 81] he ısisted: "Of course one should not identify field $\psi(x,t)$ with the wave function ". Even more amazing that sixteen years later in his book [4] he even did not ıention this idea on nature of particles as quanta of fields. Of course he had rious reasons for that, 1) QM is an essentially linear theory while the field juations are non-linear, 2) one has to explain why the excitation behaves like particle.

3. MEASUREMENT. One of the main problems of QM is description of the easurement process. Microobjects are described by QM while experimental evices are classical objects. Question: what mechanics should be used? This

was the source of many vague ideas like uncontrolled interaction of a particle with the measuring apparatus. Later on it became clear that macroscopic objects are also described by QM. The problem was to explain nonexistence of superpositions of macroscopically different states of macroscopic objects.

Blokhintsev has given a model of measurement permitting its full description. Of course, the model didn't solve the general problem but it allowed to understand some essential features of the procedure.

3 Particles and wave functions: Quantum mechanics and quantum field theory

At present time the idea that particles are quanta of fields is universally accepted. Gradually it becomes clear that for free fields one should identify the wave function of a particle with the function describing excitation of the corresponding field. Indeed, in case of a scalar field state vectors $|\mathbf{k}\rangle = \hat{a}^+(\mathbf{k})|0\rangle$, $|x\rangle = \hat{\varphi}(x)|0\rangle$ describe the one-particle states with momentum $\mathbf{k}$ and coordinate x correspondingly (here $|0\rangle$ is the vacuum vector, $\hat{\varphi}(x)$ — the field operator; in case of $|x\rangle$ the field is excited only at the point x). By definition the scalar product

$$\langle x|\mathbf{k}\rangle = \langle 0|\hat{\varphi}(x)|\mathbf{k}\rangle = e^{-ikx}$$

is the spinless particle wave function. Taking $\hat{\varphi}(f) = -i\int d^3x f(x)\partial_0\hat{\varphi}(x) - \partial_0 f(x)\hat{\varphi}(x)$, we observe that the state vector $\hat{\varphi}(f)|0\rangle$ also describes the one-particle state, but the excitation of the field depends on the function $f(x)$; the field $\varphi(x)$ is excited only at the area where $f(x) \neq 0$. On the other hand, $f(x)$ is the particle wave function by definition

$$\langle 0|\hat{\varphi}(f)^+|\mathbf{k}\rangle = i\int d^3x f^*(x) \overleftrightarrow{\partial}_0 e^{-ikx}.$$

We see: wave functions describe excitations of fields.

There are two obvious difficulties in this approach. First, the integrity of particles (in particular, photons). One should explain why such excitations behave like unbreakable objects. The reason is simple. Dividing a quantum of a field into, say, two parts one introduces discontinuity of the field, thus getting a field configuration with infinite energy: Hamiltonian of any bosonic field contains the term $(\nabla\varphi)^2$ leading in this case to the non-integrable singularity $(\delta(x))^2$. Thus, the integrity of particles in QFT is guaranteed by the continuity of fields. In case of discrete set of oscillators jump of a field leads to finite, but very large energy $\sim \Delta x(\Delta\varphi/\Delta x)^2 \sim l_P^{-1}(\Delta\varphi)^2$ where $l_P = 1.6 \cdot 10^{-33}$ cm.

The second difficulty comes from the non-linearity of quantum field theory (QFT). The resolution of the problem is simple. In QFT the particle number is not conserved, and one should consider more general object — a vector in the Fock space or the Fock functional. These functionals also describe excitations of fields, and their evolution in time is given by linear equations (both in quantum and classical theories). Thus, QFT allows to "materialize" the wave function.

We conclude: at low energies particles are a one-particle excitations of fields, and wave functions are functions describing these excitations. At higher energies one should use the functionals, i.e. the state vectors in the Fock space.

4 Measurement: Macroscopic objects and the superselection rules

The main problem of measurement is description of the process in the framework of QM. Both the measuring apparatus and the microobject are described by some state vector. Of course, the former is a macroscopic object, but all the objects in the world are more or less complex excitations of fields, and the latter are quantum ones. In this case one should explain the main property of classical bodies important for experimenters: the macroscopic objects cannot be in the states which are superpositions of their macroscopically different states.

The mentioned model of measurement considered by Blokhintsev though takes into account the essential features of the procedure, deals whith classical objects. Therefore, this consideration does not solve the problem.

It is easy to see that the superpositions of macroscopically different quantum states of macroscopic objects are indeed forbidden due to the superselection rules. These rules was first discussed in [8]: the state $\psi = \psi_b + \psi_f$, where ψ_b, ψ_f are correspondingly bosonic and fermionic states, is forbidden. Under 2π-rotation of the coordinate system the wave function ψ_f changes the sign ($\psi_f \to -\psi_f$), and $\psi \to \psi' = \psi_b - \psi_f$. But the 2π-rotation is an identical transformation and ψ cannot be changed, i.e. the vector ψ cannot be realized. Analogous statement is valid for macroscopic objects.

Definition. The macroscopic object is that having all the properties of a compact stable system of N particles retained in the limit $N \to \infty$.

Superselection rule. If there is an operator $\hat{S}$ commuting with all the physical operators of a system (Hermitian polynomials of canonical variables) then the liner combinations of eigenfunctions of $\hat{S}$ are forbidden.

THEOREM. Superpositions of wave functions of macroscopic objects with different centers of mass are forbidden.

PROOF. Let $\{\mathbf{x}_i, \mathbf{p}_i\}$, $i = 1, 2, ..., N$ be the canonical variables of a compact stable system of N particles in the $3D$ space. Then

$$\hat{\mathbf{X}} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{x}_i$$

is the operator of center of mass, and the limit $N \to \infty$ obviously exist. In this limit $\hat{\mathbf{X}}$ commutes with all the physical operators $\hat{P}_n(\hat{\mathbf{x}}_i, \hat{\mathbf{p}}_i)$, $n < \infty$

$$[\hat{\mathbf{X}}, \hat{P}_n] \to 0, \quad N \to \infty.$$

Thus, $\hat{\mathbf{X}}$ is an operator of the type $\hat{\mathcal{S}}$, and superposition of states corresponding to different proper values of $\hat{\mathbf{X}}$ are forbidden [9].

The physics behind this phenomenon is simple. In the limit $N \to \infty$ the body is described by the quantum field theory. Bodies with different centers of mass has orthogonal Hilbert spaces and the evolution operator cannot transform a vector from one space to the other because it is a physical operator.

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Vhat is semiclassical field theory?

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Abstract — The main objects of the semiclassical field theory are clarified.

It seems to be evident that classical and quantum field theories are quite lifferent. Main objects — states — are presented as classical field configu-ations in classical field theory and Hilbert space vectors in quantum. The 'oincare group transformations are symplectic in classical theory and unitary n quantum. The classical observables are functions of the field configuration, vhile in quantum theory the obsrvables are operators. Moreover, classical the-ry is much simpler and can be defined mathematically, while quantum theory as not been rigorously formulated yet.

However, classical and quantum models are applied to the same object. 'his means that the results of classical and quantum theories should not con-radict each other, so that classical results should be reproduced from the uantum theory. Therefore, we come to the problem of correspondence be-ween "exact" (quantum) and "approximate" but simpler (classical) theories f the same object.

Different approaches to this problem has been developed for systems of a inite number of degrees of freedom. Various semiclassical methods are known. ome of them (the Ehrenfest approach) are based on investigation of the av-rage values of "semiclassical" observables; other use the path integral tech-ique. However, from the mathematical viewpoint, it is much more suitable o use approaches based on substituting the approximate wave function to the chrodinger equation, since the accuracy of the approximation can be esti-nated.

Consider the Schrodinger equiation of a general form

$$ih\frac{\partial\psi(x,t)}{\partial t} = H(-ih\frac{\partial}{\partial x}, x)\psi(x,t), \qquad x \in \mathbf{R}^n, t \in \mathbf{R}. \tag{1}$$

\ first step to apply any of the semiclassical techniques is to choose such a sys-em of units that the Planck constant is dimensionless and small (it is denoted s h in (1)), while all other parameters of the quantum system (parameters of the potential, particle mass, distances etc.) are also dimensionless and of the order $O(1)$. If such a system of units exists, one comes to a *purely mathematical* problem of solving eq.(1) as $h \to 0$.

Historically, the first known semiclassical substitution (satisfying eq.(1) as $h \to 0$) was the WKB wave fucntion of the form

$$\psi(x,t) \simeq \varphi(x,t) e^{\frac{i}{h}S(x,t)}. \tag{2}$$

Nowadays, a much wider class of semiclassical solutions of eq.(1) is known. Maslov [1,2] has classified them. The simplest example is given by the Maslov theory of a complex germ in a point. It is the semiclassical wave packet

$$\psi(x,t) \simeq const e^{\frac{i}{h}S(t)} e^{\frac{i}{h}P(t)(x-Q(t))} f(t, \frac{x-Q(t)}{\sqrt{h}}) \equiv K^h_{S(t),P(t),Q(t)} f(t). \tag{3}$$

One can show that the wave function (3) is indeed an approximate solution of eq.(1), provided that the following conditions are satisfied:

(a) "classical" equations for S, P, Q:

$$\frac{dS(t)}{dt} = P(t)\frac{dQ(t)}{dt} - H(P(t),Q(t)); \tag{4}$$

$$\frac{dP(t)}{dt} = -\frac{\partial H}{\partial Q}(P(t),Q(t)); \qquad \frac{dQ(t)}{dt} = \frac{\partial H}{\partial P}(P(t),Q(t)); \tag{5}$$

(b) Schrodinger equiation for f with quadratic Hamiltonian:

$$i\frac{\partial f(t,\xi)}{\partial t} = \left[\frac{1}{2}\frac{1}{i}\frac{\partial}{\partial \xi}\frac{\partial^2 H}{\partial P \partial P}\frac{1}{i}\frac{\partial}{\partial \xi} + \frac{1}{2}\xi\frac{\partial^2 H}{\partial Q \partial P}\frac{1}{i}\frac{\partial}{\partial \xi} + \frac{1}{2}\frac{1}{i}\frac{\partial}{\partial \xi}\frac{\partial^2 H}{\partial P \partial Q}\xi + \frac{1}{2}\xi\frac{\partial^2 H}{\partial Q \partial Q}\xi\right] f(t,\xi). \tag{6}$$

Let us compare the wave functions of the types (2) and (3). The uncertanties of coordinates and momenta for the WKB wave function are usually of the order O(1) (not small as $h \to 0$). Thus, the wave function (2) cannot correspond to a point classical particle. On the other hand, for the Maslov wave function (3), the uncertanties of coordinates and momenta are of the order $O(\sqrt{h})$, so that the state (3) corresponding to the minimal value of product $\delta p \delta q \sim O(h)$ can be interpreted as a point particle with momenta $P(t)$ and coordinates $Q(t)$. Substitution (3) confirms the heuristic Ehrenfest theorem then.

Let us discuss the geometric interpretation of the semiclassical state (3) and its evolution. At fixed moment of time, it is specified by classical variables $X = (S,P,Q) \in \mathbf{R}^{2n+1}$ and quantum function $f(\xi)$. Therefore, a semiclassical wave packet state can be viewed as a point of a bundle ("semiclassical bundle"

[4]) with the base $\mathcal{X} = \mathbf{R}^{2n+1} = \{(S,P,Q)\}$ being a classical space and fibres $L^2(\mathbf{R}^n) = \{f\}$ being Hilbert spaces $\mathcal{F}_X$ of quantum states in the external classical background X.

The semiclassical evolution transformation (taking initial conditions for eqs.(4),(5),(6) to the solution) can be viewed as an authomorphism of the semiclassical bundle. The initial state $(X^0 \equiv (S(0),P(0),Q(0)), f^0 = f(0,\xi))$ is taken to $(X^t \equiv (S(t),P(t),Q(t)), f^t = f(t,\xi))$: $(X^0,f^0) \mapsto (X^t,f^t)$ with

$$X^t = u_t X^0, \qquad f^t = U_t(u_t X^0 \leftarrow X^0) f^0;$$

here u_t are transformations of the base, while $U_t(u_t X^0 \leftarrow X^0) : \mathcal{F}_{X^0} \to \mathcal{F}_{u_t X^0}$ are unitary operators.

Geometric structures of the classical mechanics can be derived from the semiclassical point of view [4]. If one shifts the classical variables by the quantities of the order $O(h)$, $S(t) \to S(t) + h\delta S(t)$, $P(t) \to P(t) + h\delta P(t)$, $Q(t) \to Q(t) + h\delta Q(t)$, the wave packet wave function (3) will be (in a leading order in h) multiplied by a c-number

$$\psi \sim e^{-i(P(t)\delta Q(t) - \delta S(t))}\psi. \tag{7}$$

Since both wave functions (7) are approximate solutions of eq.(1), one comes to the conclusion that $P(t)\delta Q(t) - \delta S(t) = const$, provided that $(\delta S(t), \delta P(t), \delta Q(t))$ is a solution of the variation system for (4), (5). Thus, the differential form

$$\omega = PdQ - dS$$

on the base of the bundle should conserve under time evolution.

Analogosly, under the shift $X \to X + \sqrt{h}\delta X$ of the order $O(\sqrt{h})$, the wave packet (3) transforms as $K^h_{X+\sqrt{h}\delta X} f \simeq const K^h_X e^{i\Omega[\delta X]} f$ with the operator-valued 1-form

$$(\Omega[\delta X] f)(\xi) = (\delta P \xi - \delta Q \frac{1}{i}\frac{\partial}{\partial \xi}) f(\xi),$$

or

$$\Omega = dP \cdot \xi - dQ \cdot \frac{1}{i}\frac{\partial}{\partial \xi},$$

which should also conserve under time evolution

$$\Omega_{X^t}[\delta X^t] U_t(X^t \leftarrow X^0) = U_t(X^t \leftarrow X^0) \Omega_{X^0}[\delta X^0].$$

The introduced 1-forms satisfies the important commutation relation

$$[\Omega[\delta X_1]; \Omega[\delta X_2]] = i d\omega(\delta X_1, \delta X_2) \tag{8}$$

which can be also interpreted geometrically.

It happens that other semiclassical substitutions (including WKB) of the wave packets (3) can be presented as a superposition [3]

$$\psi(x,t) = const \int d\alpha K^h_{S(t,\alpha),P(t,\alpha),Q(t,\alpha)} f(t,\alpha), \quad \alpha \in \mathbf{R}^k. \tag{9}$$

One can show [3] that the superposition (9) is not exponentially small only under the Maslov isotropic condition

$$\omega[\frac{\partial X}{\partial \alpha_a}] = 0, \tag{10}$$

while the inner product is then as follows

$$(\psi,\psi) \simeq \int d\alpha (f(\alpha), \prod_a \delta(\Omega[\frac{\partial X}{\partial \alpha_a}]) f(\alpha)). \tag{11}$$

Thus, the semiclassical approximation shows that quantum mechanics (with Hilbert space of states) leads not to the calssical mechanics (with phase space) directly but to the *semiclassical mechanics* with the *semiclassical bundle as the main object* (analog of the space of states); the classical space is the base of the bundle. Wave packet semiclassical states (3) are points of the bundle; more complicated semiclassical states (9) are identified with surfaces $(X(\alpha), f(\alpha))$ on the bundle. Symmetry transformations of the semiclassical mechanics are authomorphisms of the semiclassical bundle. The c-number 1-form ω and operator-valued 1-form Ω are also important objects: they should conserve under any symmetry transformation; they also enter to important relations (8), (10), (11).

Let us now clarify the main objects of the semiclassical field theory for the simplest scalar theory case with the Lagrangian $\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{h}V(\sqrt{h}\varphi)$, h being a "Planck constant". In the canonical approach, the operators $\sqrt{h}\hat{\varphi}(\mathbf{x})$ and $\sqrt{h}\hat{\pi}(\mathbf{x})$ are analogs of quantum mechanical coordinate and momenta operators. One can formally develop the semiclassical theory in the Schrodinger functional representation analogously to finite dimensional case. The main objects of the semiclassical field theory will be [5]

- the semiclassical bundle with base $\mathcal{X} = \{X\} = \{S, \Pi(\mathbf{x}), \Phi(\mathbf{x})\}$ and fibres $\mathcal{F}_X$ being quantum field state spaces in the given classical background;

- the c-number form $\omega[\delta X] = \int d\mathbf{x}\Pi(\mathbf{x})\delta\Phi(\mathbf{x}) - \delta S$ and operator-valued 1-form $\Omega[\delta X]$ satisfying commutation relation (8);

- the semiclassical wave packet states — points of the semiclassical bundle; the "composed" semiclassical states — surfaces $\{X(\alpha), f(\alpha)\}$ obeying the conditions (10) (the inner product is of the form (11));

- semiclassical Poincare transformations: to each element g of the Poincare group one assigns classical transformations $X \mapsto u_g X$ and unitary operators $U_g(u_g X \leftarrow X) : \mathcal{F}_X \to \mathcal{F}_{u_g X}$; the group property

$$u_{g_1} u_{g_2} = u_{g_1 g_2};$$
$$U_{g_1}(u_{g_1 g_2} X \leftarrow u_{g_2} X) U_{g_2}(u_{g_2} X \leftarrow X) = U_{g_1 g_2}(u_{g_1 g_2} X \leftarrow X)$$

  should be satisfied; the 1-forms ω and Ω should conserve under Poincare transformations:

$$\omega_{u_g X}[\delta X_g] = \omega_X[\delta X_0];$$
$$\Omega_{u_g X}[\delta X_g] U_g(u_g X \leftarrow X) = U_g(u_g X \leftarrow X)\Omega_X[\delta X_0], \tag{12}$$

  with $u_g(X + \delta X_0) \simeq u_g X + \delta X_g$; the properties (12) provide unitarity of Poincare transformations of states (9).

- field operators in the semiclassical mechanics (can be expressed via Ω; their Poincare invariance is a corollary of (12)).

he formulated properties are checked in [5] with the help of results of [6].

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D.I. Blokhintsev and Fundamental Problems of Quantum Mechanics

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1 The Statement of the Problem

Foundations of Quantum Mechanics have been formed more than 75 years ago. Its creation opened up new horizons for the development of thinking. But the process of the new world-outlook familiarization was rather slow and contradictory. In these circumstances a great deal has been done by those scientists, who along with the creators of the new theory contributed to the propagation and adequate interpretation of fundamental ideas of this theory.

Among those who made a significant contribution to this matter a particular position belongs to representatives of the Moscow school, headed by academician L.I. Mandelshtam [1]. One of them was K.V.Nikol'skii, in his book "Quantum Processes" (1940) there have been presented for the first time ideas of this school [2]. Starting from the middle of 1940th it was D.I. Blokhintsev who became the leader of this school, and to whose memory this conference is devoted. The Moscow's interpretation of the Quantum Mechanics, developed by him, was in strong opposition to the initial version of the Copenhagen's interpretation from the Bohr's school [3].

Still at 1944 he created the first in the world textbook "Foundations of Quantum Mechanics" (6th edition in 1983), which still remains to be one of the best and has been translated to nine languages [4]. During almost 40 years Blokhintsev systematically gave an account of his point of view in the papers, devoted to methodological problems of Quantum Mechanics [5,6]. They have been collected later on in his well-known monograph "Principle Questions of Quantum Mechanics" (1966, 1987) [7] and in Lectures on Quantum Mechanics for young scientists (1981, 1988) [8] delivered at Dubna in 1970th years. Theses repeatedly advocated by him were not always accepted by adherents of other views.

But Blokhintsev firmly persisted and, as a result, the positions of many of his opponents has been changed. At the same time, when analyzing views of his opponents, he always yearned to find a rational pearl inside of them, in order to fill up, as he noticed by himself "... all gaps in that "Moscow" understanding of Quantum Mechanics..." (1981). In relation with this still sound actual his words, which have been said at 1968 [5]: "... Now, when already many things were thought over and lots of things were written, it looks more reasonable to consider a good many of those "alternative" points of view as just a various aspects of the one and the same scientific problem... Evidently, that the deal is not as much in the setting off the different points of view, but instead in the successive development and in more thoughtful understanding of the problem".

In the presented report we do not pursue the aim to embrace all methodological problems of Quantum Mechanics, which were discussed by Blokhintsev. The aim of this talk is to show that

- the main Blokhintsev's methodological ideas definitely passed the so-called time-test;
- their development allows one to get an approach to a new understanding of the Complementarity Principle;
- they might be spread out on the other physical theories with the probabilistic description;
- these ideas open up the perspectives for creation of an integral non-classical (non-deterministic) physical theory.

The main tool of our analysis would be the generalized uncertainty relations (UR), introduced in Quantum Mechanics by Shroedinger still at 1930 [9]. As it became clear nowadays, their successive application allows one to make more precise and to extend the Moscow interpretation of Quantum Mechanics. More than that, an extension of similar UR on the other physical theories with the probabilistic description demonstrates an intrinsic proximity of several theories, which are traditionally considered as standing far from each other.

2 Some methodological ideas by D.I. Blokhintsev

Without saying, the probabilistic character of the description of Nature in Quantum Mechanics does not have any opponents already for a long time. The questions arise when one tries to make more precise the physical meaning of these probabilities. In this direction today has been achieved a substantial progress, initiated in much by the Moscow school ideas, developed by Blokhintsev. Their last versions might be in short given in the form of several thesises, based on Blokhintsev's sayings (1976):

" The principal quarrels are concentrated around the understanding of the wave function. Does the wave function provides one with the objective and complete description of the physical reality or it is just a "notebook" of an observer...? Does the wave function describes the state of the particle, or that of the system of particles?

We restrict ourselves to an explanation of the putted above questions, starting from the concept of quantum ensembles... This concept... differs from .. the concept of Copenhagen school to those, that it gives a more modest role to an observer, while underlines the objective character of quantum ensembles and that of patterns, which control them.

The concept of quantum ensembles is a very close one to the concept of Gibbs' classical ensemble. In Gibbs' ensemble a microsystem is considered in the interaction with a macroscopic thermal bath, having the temperature Θ. The probability W_Θ of one or another result of the measurement... is related with the ensemble, formed by unlimited reiterations of the microsystem μ at one and the same macroscopic conditions, given by the thermal bath with the temperature Θ.

In the full analogy with a classical Gibbs' ensemble a quantum ensemble is formed by unlimited reiterations of situations, made up by one and the same microsystem μ, which is imbedded into one and the same macroscopic environmentz M. Macrosituation might be artificially formed at the laboratory... , as well as to arise by itself in natural conditions... . The wave function Ψ_M (or the density matrix ρ_M) is an objective characteristic of the quantum ensemble and, in principle, might be found by measurement".

In the above expressions there are posed several fundamental problems, which would be commented in what follows. The main problem, which was under discussion, that is whether the probabilistic description of Nature in Quantum Mechanics is the primary, fundamental one or it is a secondary one, as it has been traditionally considered in the Classical (deterministic) Physics.

As it is known, in Classical Physics there is initially presupposed the univalent predeterminance of the course of events. Therefore the impossibility of an univalent prediction of all events, which is encountered in practice, one usually refers to an incompleteness of the initial data, that is to consider as a secondary effect. However, "... the real collapse of determinism happened along with the development of Quantum Mechanics, starting from the Einstein's work (1916) on the radiation theory, were there have been introduced a priori probabilities for the first time in physics" (V.A. Fock, 1957) [10]. Now it is possible to affirm, that the a priory, a primer character of the probability description in Quantum Mechanics is accepted by majority among investigators.

A substantial progress in this question has been achieved only after elucidation of the meaning of probability, as used in physics as a whole. In relation with that Blokhintsev (1977) [6] wrote: "Probability is the numerical measure of the potential possibility of that or another one draw of events. The draw happens in a some statistical ensemble of events, which should be defined by clearly formulated material conditions. ... Probability is not a characteristic of an individual mechanical system as itself. It belongs to a particular system to that extent, to what extent this system is a member of some definite statistical ensemble." Let us underline, that the use by Blokhintsev and by some other well-known physicists of the term "quantum ensemble" caused in the past obdurate discussions.

Evidently, that just a refusal of the term "ensemble" and its change by another similar terms – statistical population (von Neumann) [11], statistical collective (Mandelshtam, Fock) etc. - does not make any essential changes. It is needed to define clearly, what kind of statistical collective (ensemble) is subtended in Quantum Mechanics. In relation with this Fock (1957) [10] wrote: " Elements of statistical collectives, considered in Quantum Mechanics, are not micro-objects by themselves, but results of experiments over these objects. Moreover, a definite setting of the experience corresponds to a definite collective... Probability is related to a particular object and characterizes its potential possibilities; at the same time for an experimental definition of its numerical value it is needed the statistics of realization of these possibilities, that is a multiple reiteration of the experiment. Hence it is clear, that the probabilistic character of the theory does not exclude, that it might be related with an individual object".

It is not difficult to see, that Blokhintsev's and Fock's positions on these questions finally became rather close. From these positions it follows that the probability is a characteristic of an individual object, but not of it as itself, but rather as that of a member of some statistical collective (ensemble), defined by the same external conditions.

In the clearest form this statement might be expressed when using the concepts of observables and states, introduced by P. Dirac [12]. According to L.D. Faddeev (1980) [13], "... the conditions of experiment do determine the state of a system, if for a multiple reiteration of the experiment under these conditions there arise probabilistic distributions for all observables". Hence it follows, that each of similar systems under invariable exterior macro-conditions appears to be at one and the same state, independently of that whether they will be putted under these conditions sequentially one by one or all together simultaneously. In this case it is not to any extent essential are these states pure or mixed.

One should say, that Blokhintsev in the course of development of Moscow school ideas always persisted for the consistent consideration of pure (wave function) and mixed (density matrix) states in Quantum Mechanics. The latter were traditionally associated with mixed states of Classical Statistical Mechanics, which (in contradiction to Gibbs view) were related with an ensemble of particles, but not with an individual particle at all. Therefore Blokhintsev's point of view on this problem was not usually accepted.

Now the position of scientific society on this question has been drastically changed. Due to the efforts of I. Segal [14], G. Mackey [15] and Faddeev [13] the description of Nature on the language of observables and states became possible to extend from quantum mechanics to other physical theories, up to the traditional classical mechanics. The only peculiarity, which is inherent for classical (deterministic) theories, is that to their pure states does correspond degenerate (delta-like) distributions of probabilities. It is possible to look at this position as on the development of Moscow school ideas.

Let us recall two more important methodological problems to which Blokhinsev paid a considerable attention. These are "Probability and Objectiveness of Knowledge" and "Probability and Perceptuality of Nature".

The first problem deals with the role of an instrument and an observer in Quantum Mechanics. As it is known these questions were among the most important in the initial version of Copenhagen interpretation. That led to the exaggeration of the role of a measuring instrument and an observer, and gave rise to some subjective interpretations up to the perception on the defining role of an observer consciousness. Our point is that there is no any base for any doubts on the possibility of an objective description of micro-world. In this relation Fock (1957) [10] wrote: "We can call "an instrument" such a device, which on the one side might interact with a micro-object and react on its effect, and on the other side provides a classical (deterministic) description... In this definition of an instrument it is completely inessential whether this is human hand made one or it presents by himself a natural.. combination of exterior conditions, where the micro-object was putted in. It is important only, that these conditions should be described classically".

In his turn Blokhintsev (1968) [5] noted: "An observer does not take part in events, therefore he must be excluded from the game. Paradoxes, incident to an understanding of wave function as a collection of information, or as a notebook of an observer, are cleared up when one performs an analysis of a microsystem action on the instrument". Such an analysis has been successfully performed by Blokhintsev in 1968.

In the second problem one deals with the role of an uncontrollable quantum influence (interaction) of a microobject on the system, which forms its state. There were different opinions expressed. Let us give Fock's (1958) [16] statement: "Actually we are speaking here not about an interaction as itself, but instead about a logical interrelation between quantum and classical ways of description at the junction of that part of system, which is described quantum-mechanically, and that one, which is described classically. The term "uncontrollable interaction" as it sounds literally, causes perplexity: since any physical process is perceptual one, and therefore available for a control".

The thesis "about a logical interrelation between classical and quantum ways of description" has a subjective inflection and there-fore was rejected by Moscow school. There were some reasons for that. In Quantum Mechanics there are considered two qualitatively different ways of changing the state-either an evolution in accord with Schrodinger equation (the controlled influence), either a reduction as a result of changes in macro-environment (the uncontrolled influence). Minimal uncontrolled quantum influence consists in an interchange by an elementary quantum of action. The fact that his numerical value differs from zero demonstrates an uncontrolled influence inherent in the very Nature is unremovable. Hence, the presence of an elementary quantum of action restricts the perception of Nature not more, than the existence of the limiting velocity of motion. As it was noticed by Blokhintsev (1963) in this respect " the discovered in quantum region finiteness of interactions does not set any boundaries for perception."

3 Development of D.I.Blokhintsev's ideas and the modern outlook on the Complementarity Principle.

Blokhintsev's ideas an quantum ensembles originated prerequisites for the contemporary interpretation of such statements as the wave-particle duality concept and the Complementary Principle. Blokhintsev (1963) wrote: "according to Bohr, a quantum description of phenomena falls into two alternative classes, which are complementary to each other in that very sense, that their totality in classical physics provides with complete description. In other words, in the first row are not objective peculiarities of micro-world, but instead the abilities of an observer, who manipulates with macroscopic quantities and concepts".

In essence, position of the Moscow school is different. The basis for this position is the perception about a self-dependent role of the state of system as a qualitatively new feature of physical reality. The fundamental property of a state is its entirety, which gives rise to a substantial interdependency of conjugate characteristics of a system. The task raises to demonstrate in the most visual form the necessity of an entire opinion of the Nature, which could not be reduced to a collection of complementary mutually exclusive pictures. Its resolution appeared to be possible on the base of investigation of the most general properties of the state spaces, which are of use in Quantum Mechanics and elsewhere.

At the description of the Nature in terms of observables and states in different physical theories these spaces can be vastly different from each other, but their "geometrical" properties are similar in many respects. They are determined by the Cauchy-Bunyakovskii-Shwarz's inequalities which, as a rule, lead to non-trivial UR. These relations have sense of the fundamental restrictions determining the basic features and interrelations in corresponding space of states.

Let's analyze this thesis by the example of the complex Hilbert's space used in quantum mechanics [17,18]. Opportunities for such analyzes have appeared after Shroedinger [9] has received the most general UR for any pair of conjugate observables and in any states which are not eigenvectors of operators $\widehat{A}$ and $\widehat{B}$. Further this conclusion has been propagated on the any mixed states of quantum mechanics.

Let us restrict ourselves with the consideration of the conjugate observables - coordinate and momentum - for the micro-particle, which is subjected to the one-dimensional motion. In this case Shroedinger's UR "coordinate - momentum" has the form

$$\Delta q^2 \cdot \Delta p^2 \geq |\tilde{R}_{qp}|^2. \tag{1}$$

Here Δq^2 and Δp^2 are dispersions (variances) of observables q and p, which characterize their fluctuations in the state $|\Psi\rangle$, and

$$|\tilde{R}_{qp}|^2 = \sigma_{qp}^2 + c_{qp}^2 = \frac{1}{4}\langle\Psi|\{\Delta\hat{q},\Delta\hat{p}\}|\Psi\rangle^2 + \frac{\hbar^2}{4} \tag{2}$$

is the square of generalized complex correlator (covariance) of these fluctuations.

Let us note some features of Shroedinger's UR (1):

- in the generalized correlator $|\tilde{R}_{qp}|$ on the equal rights enter the same footing average values of the commutator $c_{qp} = \frac{1}{2}|[\Delta\hat{q},\Delta\hat{p}]|$ and the anticommutator σ_{qp} for the operators $\Delta\hat{q} = \hat{q}-\bar{q}$ and $\Delta\hat{p} = \hat{p}-\bar{p}$ accordingly with $\bar{q}$ and $\bar{p}$ being average values of observables.

- it goes over it's special version, namely the Heisenberg's UR only by the condition $\sigma_{qp} = 0$;
- in the quasiclassical limit $c_{qp} \to 0$ whereas σ_{qp} don't go to zero, but goes over into the correlator $\overline{(\Delta q \cdot \Delta p)}$ from the classical probability theory.

For more detailed analysis let us present the first term in the Eq. (2) in the following form [19]:

$$\sigma_{qp} = m \int dq(q - \bar{q}) j_{pr}(q), \tag{3}$$

where

$$j_{pr}(q) = \frac{\hbar}{m} \rho(q) \frac{\partial \varphi}{\partial q}, \tag{4}$$

is the density of the probability current in the state, described by the wave function $\Psi(q) = \sqrt{\rho_{pr}(q)} \exp\{i\varphi(q)\}$. This denotes, that the quantity σ_{qp} turns equal to zero for all the real wave functions and also for the infinite harmonic de Broglie wave, when $j_{pr} = const.$ In order the quantity to be different from zero ($\sigma_{qp} \neq 0$), the corresponding wave function have to be complex and non-stationary (with $\Delta q(t) \neq 0$).

As an example of realistic model for which $\sigma_{qp} \neq 0$ let us consider the free motion of micro-particle with the mass m in the state of Gauss wave packet with initial width $\Delta q(0) = b$ and the velocity of the packet center $V_0 = const.$ In an arbitrary instant of time we have:

$$\Delta q^2(t) = b^2 + V_{qu}^2 t^2; \quad \Delta p^2 = (mV_{qu})^2 \tag{5}$$

$$\sigma_{qp} = mV_{qu}^2 t, \tag{6}$$

where $V_{qu} = \frac{\Delta p}{m} = \frac{\hbar}{2mb}$ is the rate of the spreading of the Gauss packet.

In this case the Shroedinger's UR obtains the form [19]

$$\Delta q^2(t) \cdot \Delta p^2 = |\tilde{R}_{qp}|^2 = (mV_{qu}^2 t)^2 + \frac{\hbar}{4}, \tag{7}$$

so the Gauss packet minimizes Shroedinger's UR (1), turning it into equality at any value of t. We note, that the rate of spreading is deter-mined by the dispersion of the momentum in the initial instant of time ($\Delta p^2 = \hbar / 4\Delta q_0^2$), when both UR (Shroedinger's and Heisenberg's) coincide .

As is known the starting version of complementarity principle [3] was tightly bound with the analysis of the Heisenberg's UR. It follows from this one that in Nature there are no states , for which the dispersions of coordinate and momentum equal to zero together. But the illusion remained that there may be experimentally realized the states for which the coordinate and momentum dispersions go to zero individually. But it does not seem reasonable to say that this illusions are valid. The example with the Gauss packet for which shows $\Delta p^2 = const \neq 0$, that it never can spread enough to transform into an infinite de Broglie harmonic wave with $\Delta p^2 = 0$. Thus the states which illustrate the complementarity principle are only mathematical abstractions of a kind of the δ- functions or an infinite harmonic de Broglie wave and not more.

Moreover, one cannot ever say formally that the quantity $\Delta p \to 0$ by $\Delta q \to \infty$ or vice versa because the square of generalized correlator $|\tilde{R}_{qp}|$ in the form (7) has two terms playing different roles on the divers stages of the packet's spreading. As a threshold time one may define $\tau_{qu} = \frac{b}{V_{qu}}$. It characterizes the time interval which is necessary for doubling of initial packet width. Then the second term which corresponds to the traditional contribution into the Heisenberg's UR can be ignored at $t \gg \tau_{qu}$. In this limit it comes out that the dispersions Δq^2 and Δp^2 are directly proportional each other $\Delta q^2 \approx (\frac{t}{m})^2 \Delta p^2$. Thus the traditional statement that these quantities are inversely proportional each other $(\Delta q^2 \approx \frac{\hbar^2}{4\Delta p^2})$ has only some meaning at the time $t \ll \tau_{qu}$.

Thus the widely used perceptions about the particle-wave dualism and about the complementarity of these two alternative types of micro- particles description are seen now as somewhat conventional. In the Nature there are no micro-objects in states, in which they have quite definite values of the coordinate or the momentum. Moreover in many cases the initial mutual inverse proportionality of the values Δq^2 and Δp^2 with the time evolution turns into a direct proportionality. This means that the concept of the two complementary (alternative) ways of the micro-object description is the echo of classical representations. It may be realized only approximately. For an adequate description of the micro-world it is only convenient the holistic approach by which both conjugate observables - the coordinate and the momentum - posses no definite values in any state. They are characterized by average values, dispersions and generalized correlator. Furthermore the type of their fluctuations correlation is changed from anti-phase one $(\Delta q^2 \sim \frac{1}{\Delta p^2})$ to the in-phase one $(\Delta q^2 \sim \Delta p^2)$.

Summing up we are dealing in fact with the same observables in the classical and in the quantum mechanics. The only difference consists in the following: it the first (classical) case the conjugate observables are considered as independent primary, whereas in the other (quantum) case - as interdependent ones. The interdependence between coordinate and momentum in the quantum mechanics one may demonstrate explicitly [19] in its version named Nelson's stochastic mechanics [20].

4 The propagation of D.I. Blokhintsev's ideas beyond the frameworks of Quantum Mechanics.

The ideas of Moscow school about the unified treatment of pure and mixed states in quantum mechanics initiated the problem of interrelations between the probability description in quantum mechanics and outside it. This problem was interesting for Blokhintsev for a long time. The last paper on this subject under the title "Classical statistical physics and quantum mechanics" was published by him in 1977 [6] (a year and a half before his death).

His ensemble approach opens up some possibilities to a more wide interpretation of probabilistic type theories. Now it is possible to say that there are the theories beyond frameworks of the quantum mechanics also. In them the probability description has also a primary fundamental character. It would be quite natural to expect that corresponding limitations in their state spaces will appear in these theories,which would be analogous to the Schroedinger's UR (1).

For convenience of comparison with behavior of one free micro-particle let us consider one-dimensional generalized diffusion [21], described by the Fokker-Planck equation in the configurational space

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial q}(\rho v), \tag{8}$$

where

$$v = w + u \tag{9}$$

is the velocity of flow or the current velocity, proportional to the particle flow density $v = \frac{1}{\rho} j$.

This quantity is made up of two qualitatively different terms. Addend w means an average velocity of the motion forward of separate particle or drift velocity. It is defined by Newton's dynamics, that is initial conditions and known forces working on a particle (controllable influence).

The second term in the formulae (9)

$$u = -D_T \frac{1}{\rho}\frac{\partial \rho}{\partial q} \tag{10}$$

refers to osmotic (or diffusion) velocity. It is meaningful to velocity of displacement of particles of a flow as a result of diffusion according to Fick's law (uncontrollable thermal influence) with coefficient of diffusion

$$D_T = \frac{k_B T}{m}\tau \tag{11}$$

Here T - is temperature of the environment,m - is mass of a particle, τ - is the relaxation time and - is the fundamental Boltzmann's constant. Let's notice, that by $w = 0$ the equation (8) turns to the ordinary equation of diffusion. On the contrary, at the same equation turns to Liouville's equation concerning to the mixed states of classical (deterministic) mechanics.

In the case of the generalized diffusion, as the momentum conjugate to coordinate it is accepted to choose [22,23,21] the current momentum

$$\rho \equiv mV = \frac{m}{\rho}j = m(w - D_T\frac{1}{\rho}\frac{\partial\rho}{\partial q}) \tag{12}$$

The corresponding UR "coordinate-momentum" which follows from Cauchy -Bunyakovskii-Schwartz inequality has the following form

$$\Delta q^2 \cdot \Delta p^2 \geq \overline{(\Delta q \cdot \Delta p)}^2 = m^2[\overline{(\Delta q \cdot \Delta w)} + \overline{(\Delta q \cdot \Delta u)}]^2, \tag{13}$$

where $\overline{(\Delta q \cdot \Delta u)} = D_T$. As well as in quantum mechanics, right -hand side UR of a kind (13) at adequate definition conjugated coordinate and a momentum is distinct from zero only when velocity w (or u) and coordinate q are interdependent. As to study sources of such interdependence outside of quantum mechanics let us analyze two extreme cases.

First we shall consider a case when $D_T = 0$, i.e. diffusion is absent (osmotic velocity $u = 0$). This case corresponds to Classical (deterministic) Mechanics of the particle which is taking place in the mixed state owing to disorder of the initial data. The elementary example of similar type has been described in known Blokhintsev's monograph (1966) [7] in which was considered one-dimensional movement of a free particle with initial distribution function for coordinate and momentum of Gauss' type

$$\rho_0(q,p) = \frac{1}{2\pi\sqrt{\Delta q_0^2 \Delta p_0^2}}\exp\{-\frac{(p-\bar{p})^2}{2\Delta p_0^2} - \frac{(q-\bar{q})^2}{2\Delta q_0^2}\} \tag{14}$$

In this case the Fokker-Planck equation (8) transforms to the special case of the Lioville's equation where the current velocity $V = w = P/m = const$:

$$\frac{\partial\rho}{\partial t} = -\frac{\partial}{\partial q}(\rho w) = -\frac{p}{m}\frac{\partial\rho}{\partial q} \tag{15}$$

According to Faddeev's terminology [13] such kind of description corresponds to the Liouville's picture of motion in classical (deterministic) mechanics. The solution of Eq. (15) at an arbitrary moment of time is obtained from the Eq. (14) by the change of q on $(q - \frac{p}{m} \cdot T)$.

The simplest calculation shows that in this case

$$\Delta q^2 = \Delta q_0^2 + V_{cl}^2 t^2; \Delta p^2 = (m \cdot V_{cl})^2 = \Delta p_0^2 = const, \tag{16}$$

where $V_{cl} = \Delta p_0/m$- the velocity of spreading of Gauss initial distribution of classical mechanics. Continuing the analysis of the space states properties we obtain the expression for the correlator

$$\sigma_{qp} = m\overline{(\Delta q \cdot \Delta w)} = mV_{cl}^2 t. \tag{17}$$

As a result the UR "coordinate-momentum" (13) in considered special case takes the form

$$\Delta q^2 \cdot \Delta p^2 \geq \sigma_{qp}^2 = (mV_{cl}^2 t)^2 \tag{18}$$

Let us now analyze another special case of UR (13) corresponding to the Brownian motion by the absence of external forces when the drift velocity $w = 0$. In the limit $t \gg \tau$ it has been examined by Furth [23] still in 1933 [23] when instead of (14) he has received the formula

$$\Delta q^2 - \Delta p^2 = \sigma_{qp}^2, \tag{19}$$

where the correlator

$$\sigma_{qp} = mD_T = k_B T\tau. \tag{20}$$

It is easily to see that the non-trivial UR "coordinate-momentum" is the consequence of the non-controlled thermal influence which is determined by the Bolzmann's constant.

The generalization of the UR (19) on an arbitrary time intervals [24] in the frameworks of the Ornstein- Uhlenbeck theory [25,26] leads to the expression

$$\Delta q^2 \cdot \Delta p^2 \geq \sigma_{qp}^2, \tag{21}$$

where the correlator has form

$$\sigma_{qp} = mD_T(t) = mD_T(1 - \exp -t/\tau) \qquad (22)$$

By $t \gg \tau$ it obviously transforms to Eq. (20), but by $t \ll \tau$ it gives :

$$\sigma_{qp} = mD_T \cdot \frac{t}{\tau} = k_B T t = mV_T^2 t. \qquad (23)$$

Here $V_T^2 = \frac{k_B T}{m}$ -is the dispersion of velocity in the thermal equilibrium state determined by one-dimensional Maxwell's distribution .

Thus, in the Brownian's motion theory the state of a particle is formed due to uncontrollable thermal influence of the thermal bath. Let us emphasize that it also is unremovable. In this state both the conjugate observables q and p also are interdependent, that results in essential correlation of their fluctuations. The formalism of this theory is qualitatively distinct from a formalism of quantum mechanics. However in the methodological attitude they are close enough, whereas both ones being conceptually different from that in Classical Statistical Mechanics.

5 The conclusion.

So, the approach of Moscow school (Mandelshtam, Nikolskii, Blokhintsev and their successors) to any physical theories on a basis of probability descriptions shows the efficiency. The further development of these ideas opens, in our opinion, prospects for creation of the holistic non-classical (non-deterministic) physical theory.

As a first step there is an opportunity to enter the new criterion, allowing to divide the all physical theories on classical (deterministic) and non-classical (non-deterministic). Such criterion, in our opinion, is presence or absence of fundamental uncontrollable influence of this or that type resulting to formation of a state of system in certain macro-enviroment.

From this point of view qualitative differencies between the mixed states in classical (deterministic) mechanics and states in the Brownian motion theory are distinctly visible as two special types of the Fokker's-Planck's equation decisions. In both theories takes place a correlation of fluctuations of coordinate and momentum. However in classical mechanics the state of system is determined to two observable q and p which primary are independent. Interdependence of these observables in mixed states can arise, but only during movement ($t \neq 0$), in other words it is secondary. On the contrary, in the Brownian motion theory interdependence of observables q and p arises initially at forming of a state of system due to uncontrollable (thermal) influence, in other words it is primary.

It is interesting, that the certain analogy between the Brownian motion theory and quantum mechanics is observed to this attribute. In last one interdependence between observables q and p, as well as in the Brownian motion theory, arises initially at forming of a system state due to uncontrollable (quantum) influence, in other words, it is primary. It gives the ground to approve, that both these theories concern to non-classical (non-deterministic) physical theories. Furthermore, Brownian motion theory appears to be more close to Quantum Mechanics than to Classical Statistical Mechanics.

Thus, the search of the holistic theory in which both types of uncontrollable influence would be viewed as conjoint and equal in rights, is not deprived sense [27-30]. As an illustration of this thesis let us address to Brownian motion of a free micro-particle. Recently we have [30] shown, that the generalized correlator $|\tilde{R}_{qp}|$ at any arbitrary moment of time rather complicate de-pends from the both fundamental constants $\hbar$ and k_B. This dependence becomes simpler at $t \sim \tau_{cr} = \hbar/2k_BT$, where r_{cr}- the moment of time dividing two regimes at which prevails either quantum, or thermal influence. In these conditions

$$|\tilde{R}_{qp}|^2 \approx \{(k_BT\tau)^2 + \frac{\hbar^2}{4}\} \tag{24}$$

so "thermal" and "quantum" contributions to the generalized correlator prove to be additive. The commensurability of them might be reached at certain values of macro-parameters (temperature, and relaxation time, related with the viscosity of medium), and, along with an increase of these macro-parameters, "thermal" contribution might became a dominant one. Under these conditions the correlation of the coordinate and momentum fluctuations for micro-particle is determined not by the Planck constant, but mostly by the Boltzman constant.

Thus, ideas of the Moscow school play an essential role not only at interpretation of quantum mechanics. They can be used as a com-pass by searches of the complete non-classical physical theory based on the joint and equal in rights account of all types of uncontrollable influence. In this connection Blokhintsev's researches on fundamental problems of quantum mechanics are actual and today. They, undoubtedly, represent a subject of pride of the Russian and world science.

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Positronium Bose condensate as gamma laser

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Abstract — The rate of self-stimulated emission of photons produced by the decay of Bose condensate of para-positronium atoms has been calculated. This rate exceeds the rate of spontaneous two-photon decay at plausible density values of positronium gas, thus opening, in principle, the way to the positronium Bose condensate gamma laser.

1 Introduction

We present the estimation of the main parameter upon which the principal possibility of one of the earliest gamma ray laser idea crucially depends. This parameter is the density — number of particles per unit volume — of positronium gas in a state of deep quantum degeneracy. The choice of exotic substance is suggested by the decay property of a free para-positronium atom that emits two monochromatic recoilless gamma quanta in opposite directions. So the acute in *Mev* range Doppler shift difficulty simply does not exist for the stimulated two-photon decay of condensate particles.

2 Rate of self-stimulated emission of photon pairs from Bose condensate of pseudoscalar field

For our aim we accept the phenomenological quantum field theory formalism, the most straightforward one for the primary investigation of the problem. As the only on-mass-shell treatment is needed at the first stage, the description of ground state para-positronium atoms as point-like particles — quanta of the local pseudoscalar field $\varphi(x)$ — will do no harm.

So let us begin with the phenomenological Lagrangian density of interacting electromagnetic and neutral pseudoscalar field:

$$\mathcal{L} = \frac{\mathbf{E}^2 - \mathbf{H}^2}{2} + \frac{\dot{\varphi}^2 - (\nabla\varphi)^2 - m^2\varphi^2}{2} + g\,\varphi\,\mathbf{E}\,\mathbf{H}. \tag{1}$$

The radiation gauge is the most appropriate for our case:

$$A_0 = 0, \qquad \nabla\mathbf{A} = 0, \qquad \mathbf{E} = -\dot{\mathbf{A}}, \qquad \mathbf{H} = \nabla\times\mathbf{A}. \tag{2}$$

Canonical momenta are defined as:

$$\mathbf{\Pi} = \dot{\mathbf{A}} - g\,\varphi\,\mathbf{H}, \qquad \pi = \dot{\varphi}. \tag{3}$$

All previous formulas lead to the Hamiltonian density:

$$\begin{aligned}\mathcal{H} = \quad & \frac{\mathbf{E}^2+\mathbf{H}^2}{2}+\frac{\dot{\varphi}^2+(\nabla\varphi)^2+m^2\varphi^2}{2} = \\ & \frac{(\mathbf{\Pi}+g\,\varphi\,\mathbf{H})^2+\mathbf{H}^2}{2}+\frac{\pi^2+(\nabla\varphi)^2+m^2\varphi^2}{2},\end{aligned} \tag{4}$$

that consists of two positive contributions from each field. The interaction manifests itself through the non-commutativity of these two energy densities.

The canonical commutation relations (nontrivial) are:

$$\begin{aligned}[\pi(\mathbf{x},t),\varphi(\mathbf{x}',t)] \quad & = -i\delta^{(3)}(\mathbf{x}-\mathbf{x}'), \\ [\Pi_i(\mathbf{x},t),A_j(\mathbf{x}',t)] \quad & = -i\delta_{ij}\,\delta^{(3)}(\mathbf{x}-\mathbf{x}')-i\frac{\partial^2}{\partial_i\partial_j}\frac{1}{4\pi|\mathbf{x}-\mathbf{x}'|}.\end{aligned} \tag{5}$$

They are consistent with the radiation gauge conditions: $\mathrm{div}\mathbf{A}=\mathrm{div}\mathbf{\Pi}=0$.

With the help of the canonical commutators we obtain the rate of energy exchange between the two fields:

$$\begin{aligned}\frac{d}{dt}\int d^3\mathbf{x}\frac{\mathbf{E}^2+\mathbf{H}^2}{2} = \quad & i\left[\mathcal{H},\int d^3\mathbf{x}\frac{\mathbf{E}^2+\mathbf{H}^2}{2}\right] = \\ = g\int d^3\mathbf{x}\frac{\pi\mathbf{H}\mathbf{E}+\mathbf{E}\mathbf{H}\pi}{2} = \quad & -\frac{d}{dt}\int d^3\mathbf{x}\frac{\pi^2+(\nabla\varphi)^2+m^2\varphi^2}{2}.\end{aligned} \tag{6}$$

So in a closed conservative system of the two fields the flow of energy between them oscillates with a characteristic frequency of order $T^{-1} = \langle g\,\dot{\varphi}\rangle$. For a real open system in thermal environment the rate of energy exchange can be enhanced through the Bose-Einstein condensation of the φ field.

We have to consider this possibility in detail. The canonical variables $\varphi(\mathbf{x},t)$ and $\pi(\mathbf{x},t)$ can be expressed by the plane-wave decomposition:

$$\begin{aligned}\varphi(\mathbf{x},t) \quad & = \quad \sum_{\mathbf{p}}\frac{1}{\sqrt{2VE_{\mathbf{p}}}}\left(b(\mathbf{p},t)e^{i\mathbf{p}\mathbf{x}}+b^\dagger(\mathbf{p},t)e^{-i\mathbf{p}\mathbf{x}}\right), \\ \pi(\mathbf{x},t) \quad & = \quad \sum_{\mathbf{p}}\sqrt{\frac{E_{\mathbf{p}}}{2V}}\left(-ib(\mathbf{p},t)e^{i\mathbf{p}\mathbf{x}}+ib^\dagger(\mathbf{p},t)e^{-i\mathbf{p}\mathbf{x}}\right), \\ & E_{\mathbf{p}}=\sqrt{\mathbf{p}^2+m^2}, \qquad \sum_{\mathbf{p}}\dots=\int\frac{Vd^3\mathbf{p}}{(2\pi)^3}\dots \\ & \left[b(\mathbf{p},t),b^\dagger(\mathbf{p}',t)\right]=\frac{(2\pi)^3}{V}\delta^{(3)}(\mathbf{p}-\mathbf{p}')\xrightarrow[\mathbf{p}'\to\mathbf{p}]{}1.\end{aligned} \tag{7}$$

For the electromagnetic field the decomposition into plane waves with circular polarization is:

$$
\begin{aligned}
\mathbf{A}(\mathbf{x},t) &= \sum_{\mathbf{k}} \frac{1}{\sqrt{2V\omega}} \left[\left(a^R(\mathbf{k},t)\mathbf{e}(\mathbf{k}) + a^L(\mathbf{k},t)\mathbf{e}^*(\mathbf{k}) \right) e^{i\mathbf{kx}} + H.c. \right], \\
& \qquad \omega = |\mathbf{k}|,\ \mathbf{e}(\mathbf{k}) = \frac{1}{\sqrt{2}} \left(\mathbf{e}_\perp + i\frac{\mathbf{k}}{\omega} \times \mathbf{e}_\perp \right),\ \mathbf{e}_\perp \mathbf{k} = 0, \mathbf{e}_\perp^2 = 1, \\
& \qquad \mathbf{e}(-\mathbf{k}) = \mathbf{e}^*(\mathbf{k}),\ \mathbf{e}^2 = \mathbf{e}^{*2} = 0,\ \mathbf{e}\mathbf{e}^* = 1,\ \mathbf{e}\mathbf{k} = \mathbf{e}^*\mathbf{k} = 0, \qquad (8) \\
\mathbf{\Pi}(\mathbf{x},t) &= \sum_{\mathbf{k}} \sqrt{\frac{\omega}{2V}} \left[-i \left(a^R(\mathbf{k},t)\mathbf{e}(\mathbf{k}) + a^L(\mathbf{k},t)\mathbf{e}^*(\mathbf{k}) \right) e^{i\mathbf{kx}} + H.c. \right], \\
\mathbf{H}(\mathbf{x},t) &= \sum_{\mathbf{k}} \sqrt{\frac{\omega}{2V}} \left[\left(a^R(\mathbf{k},t)\mathbf{e}(\mathbf{k}) - a^L(\mathbf{k},t)\mathbf{e}^*(\mathbf{k}) \right) e^{i\mathbf{kx}} + H.c. \right],
\end{aligned}
$$

as

$$
i\mathbf{k} \times \mathbf{e}(\mathbf{k}) = \omega\, \mathbf{e}(\mathbf{k}), \qquad i\mathbf{k} \times \mathbf{e}^*(\mathbf{k}) = -\omega\, \mathbf{e}^*(\mathbf{k}).
$$

The commutation relations for the right (analogous for the left) polarized modes are:

$$
\left[a^R(\mathbf{k},t),\, a^{\dagger R}(\mathbf{k}',t) \right] = \frac{(2\pi)^3}{V} \delta^{(3)}(\mathbf{k}-\mathbf{k}') \xrightarrow[\mathbf{k}'\to\mathbf{k}]{} 1. \qquad (9)
$$

For the condensate field $\varphi(\mathbf{x},t) \equiv \varphi(t)$:

$$
\begin{aligned}
\varphi(t) &= \frac{1}{\sqrt{2mV}} \left(b(t) + b^\dagger(t) \right), \\
\pi(t) &= \sqrt{\frac{m}{2V}} \left(-ib(t) + ib^\dagger(t) \right), \qquad (10) \\
& \left[b(t),\, b^\dagger(t) \right] = 1, \quad [\pi(t), \varphi(t)] = -\frac{i}{V}.
\end{aligned}
$$

When we take into account only the condensate mode for physical reasons, the total Hamiltonian becomes:

$$
\begin{aligned}
\mathcal{H} &= \int_V d^3\mathbf{x} \frac{\mathbf{\Pi}^2 + \mathbf{H}^2}{2} + V\frac{\pi^2 + m^2\varphi^2}{2} + \\
& g\varphi \int_V d^3\mathbf{x} \frac{\mathbf{\Pi}\mathbf{H} + \mathbf{H}\mathbf{\Pi}}{2} + g^2\varphi^2 \int_V d^3\mathbf{x} \frac{\mathbf{H}^2}{2}. \qquad (11)
\end{aligned}
$$

'or reasonable fields $g^2\langle\varphi^2\rangle \ll 1$ and the last term in (11) is negligible. The otal Hamiltonian without it takes the form in the momentum space:

$$\mathcal{H} = \sum_{\mathbf{k}} \omega \left(a^{\dagger R}(\mathbf{k},t)a^R(\mathbf{k},t) + a^{\dagger L}(\mathbf{k},t)a^L(\mathbf{k},t) + 1\right) +$$

$$m\left(b^\dagger(t)b(t) + \frac{1}{2}\right) + \frac{g}{\sqrt{2mV}}\left(b(t) + b^\dagger(t)\right) \times$$

$$\sum_{\mathbf{k}} \frac{\omega}{2}\left[-i\left(a^R(\mathbf{k},t)a^R(-\mathbf{k},t) - a^L(\mathbf{k},t)a^L(-\mathbf{k},t)\right) + H.c.\right] \tag{12}$$

t is convenient to separate the fast energy dependence by going to slow varying ilde-operators:

$$a^R(\mathbf{k},t) = e^{-i\omega t}\tilde{a}^R(\mathbf{k},t), \ldots, \qquad b(t) = e^{-imt}\tilde{b}(t), \ldots \tag{13}$$

'heir time dependence is determined by the interaction Hamiltonian:

$$\dot{\tilde{b}}(t) = i\left[\mathcal{H}_{int}(t), \tilde{b}(t)\right], \ldots, \dot{\tilde{a}}^R(\mathbf{k},t) = i\left[\mathcal{H}_{int}(t), \tilde{a}^R(\mathbf{k},t)\right], \ldots \tag{14}$$

n which we retain only the resonant ($\omega = m/2$) terms

$$\mathcal{H}_{int}(t) = \frac{g}{\sqrt{2mV}} \times$$

$$\sum_{\mathbf{k},\,\omega=m/2} \frac{\omega}{2}\left[-i\tilde{b}^\dagger(t)\left(a^R(\mathbf{k},t)a^R(-\mathbf{k},t) - a^L(\mathbf{k},t)a^L(-\mathbf{k},t)\right) + H.c.\right] \tag{15}$$

nd omit the fast varying nonresonant terms like $\tilde{b}\,\tilde{a}\,\tilde{a}\,e^{-i(m+2\omega)t}$.

For every fixed direction and circular polarization of a plane electromag- etic wave there are two coupled equations, *e.g.* :

$$\begin{aligned} \dot{\tilde{a}}^R(\mathbf{k},t) &= \frac{g\,\omega}{\sqrt{2mV}}\,\tilde{b}(t)\,\tilde{a}^{\dagger R}(-\mathbf{k},t), \\ \dot{\tilde{a}}^{\dagger R}(-\mathbf{k},t) &= \frac{g\,\omega}{\sqrt{2mV}}\,\tilde{b}^\dagger(t)\,\tilde{a}^R(\mathbf{k},t). \end{aligned} \tag{16}$$

et the annihilation decay of the condensate be compensated by a new delivery o it. For this case, when one could consider $b^\dagger b = N \gg 1$ as a constant c- umber, the system (16) is linear and easily solvable. It appears that the time volution produces the Bogolyubov transformation:

$$\tilde{a}^R(\mathbf{k},t) =$$

$$\cosh\left(\frac{g}{2}\sqrt{\frac{mN}{2V}}\,t\right)\tilde{a}^R(\mathbf{k},0) + \sinh\left(\frac{g}{2}\sqrt{\frac{mN}{2V}}\,t\right)\frac{b}{\sqrt{N}}\,\tilde{a}^{\dagger R}(-\mathbf{k},0). \tag{17}$$

This is a clear case of quantum parametric resonance [1]. The given condensat field plays the role of a time dependent external field — the energy source c constant intensity pumping the coherent electromagnetic field. The opposit case of coherent creation of scalar boson pairs by a time dependent externa electric field was considered in [2] with a similar result.

The photon number increment according to (17) and agreeing with (6 tends to

$$\frac{\dot{N}_{ph}}{N_{ph}} = |g|\sqrt{\frac{mN}{2V}}. \quad (18$$

The dependence of the photon emission rate on the density of emitters is simila to other collective phenomena. Another peculiar feature of the result (18) i that a measurable effect comes out with the first rather than the second powe of the small coupling constant, that is more favorable.

3 The feasibility of positronium Bose condensate gamma laser

Formula (18) is applicable for the self-stimulated two-photon decay of an pseudoscalar particles condensate, *e.g.*, for π^0-mesons or hypothetical axion As our priority is the para-positronium condensate, we substitute the parame ters m and g by m_e and α :

$$\frac{1}{\tau_{para}} = \alpha^5\frac{m_e}{2} = \frac{g^2}{64\pi}m^3, \quad m = 2m_e, \quad \frac{\dot{N}_{ph}}{N_{ph}} = \sqrt{\frac{4\pi\alpha^5}{m_e}\frac{N}{V}}. \quad (19$$

It follows from (19) that the photon number increment dominates ove spontaneous two-photon decay at the condensate densities $N/V \geq 10^{19} cm^{-}$ this should be feasible at the liquid hydrogen temperatures.

All above results were obtained a decade ago and published in [3]. Toda the whole topic of Bose-Einstein condensation is very promising physics, an this fact gives new impetus to the problem considered.

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Quantum Field Theory

The scalar $O(N)$ model beyond the Hartree approximation, in and out of equilibrium

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Abstract — We discuss recent results obtained in the two-loop approximation of the two-particle point-irreducible (2PPI) effective action applied to the scalar $O(N)$ model at finite temperature and out of equilibrium.

1 Introduction

The fundamental theories and phenomenological models of elementary particle physics contain, as a basic constructive element, the Higgs mechanism. This involves the physical phenomena of spontaneous symmetry breaking, phase transitions and symmetry restoration at high temperatures and energy densities, particle production in time dependent background fields via the mass term provided by the Higgs field. The question of thermalization of particles produced in this way arises in inflationary cosmology. Most of these concepts are not based on exact results in quantum field theory but on certain approximations to it. Beyond the tree level approximation as defined by the basic Lagrangian typical approaches are: resummation schemes like 1PI, 2PI, 2PPI, the $1/N$ expansion, the loop expansion, the semiclassical approximation, the Hartree approximation and variational approaches based on Gaussian and other wave functionals. A simple model which displays spontaneous symmetry breaking and in which such approximations can be studied is the $O(N)$ sigma model with a Mexican hat potential

$$\mathcal{L} = \frac{1}{2}\partial_\mu\Phi_i\partial^\mu\Phi_i - \frac{\lambda}{4}\left(\Phi_i\Phi_i - v^2\right)^2 . \quad (1)$$

The model appears, e.g., in the Higgs sector of some fundamental theories (Coleman-Glashow model, $SO(10)$ GUT etc.), as an effective low energy theory for π and σ mesons, and as a model for particle creation after inflation. Many results are known in leading order (1-loop, large N, Hartree) in thermal equilibrium and out of equilibrium. Next-to-leading order calculations are technically very demanding, both analytically and numerically. Not even renormalization has been achieved in some resummation schemes. In out-of-equilibrium quantum field theory an outstanding problem is the approach to thermalization after copious particle production at the end of inflation. In leading order approximations the systems do not thermalize.

In this context we have recently investigated some aspects of the model, we will present here some of our results for the $O(N)$ model in the 2PPI scheme in the two-loop approximation: at finite temperature in $3+1$ dimensions, and out of equilibrium in the $O(1)$ model in $1+1$ dimensions. We will discuss the different resummation schemes in section 2, and present our results in section 3. In section 4 we will draw some conclusions.

2 Effective action formalisms

2.1 The effective action, 1PI

Consider the action S of a quantum field theory with external source $J(x)$. Then the effective action $\Gamma^{1PI}[\phi]$ generates the one-particle irreducible (1PI) Feynman graphs. Vice versa, Γ^{1PI} is the sum of all 1PI Feynman graphs with external lines representing $\phi(x)$. We have

$$\Gamma[\phi] = S_{\rm cl}[\phi] + \Gamma^{1PI}[\phi]$$

The summation of all orders in the external field can be obtained by writing all vacuum Feynman graphs, but with the free Green function $G^{(0)}(x,x')$ replaced by the Green function in the external field defined by

$$\left[\Box + m^2 + 3\lambda\phi^2(x)\right] G(x,x') = -\delta^4(x-x') \tag{2}$$

In this way one resums all seagull diagrams. In the lowest order approximation one just takes account of the simple bubble diagram and obtains the *one-loop effective action.*

2.2 The 2PI effective action (CJT)

One gets a more powerful resummation [1] by introducing in addition to an external field $\phi(x)$ the Green function $G(x,x')$ via Legendre transformation of a bilocal source term $K(x,x')\phi(x)\phi(x')$.

$$\begin{aligned}\Gamma[\phi,G] &= S_{\rm cl}[\phi] + \frac{1}{2}i\mathrm{Tr}\ln G^{-1} + \frac{1}{2}i\mathrm{Tr}\left(D^{-1}G\right) \\ &\quad + \Gamma^{2PI}[\phi,G]\,,\end{aligned} \tag{3}$$

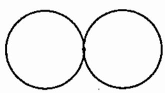
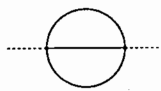
Here $\Gamma^{2PI}[\phi,G]$ is the sum of all *two-particle irreducible* (2PI) vacuum Feynman graphs, Feynman diagrams that do not fall apart if one cuts any two interior lines. Here the Greens function G is a solution of the Schwinger-Dyson equation

$$(\Box + m^2 + 3\lambda\phi^2)G(x,x') + \frac{\delta\Gamma^{2PI}[\phi,G]}{\delta G(x,x')} = -\delta(x-x') \tag{4}$$

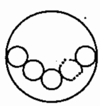
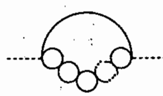
The simple bubble is already contained in the "Trace-log" term, the lowest nontrivial contribution is the double-bubble graph, giving rise to the daisy and superdaisy resummation. This reproduces the exact theory in the limit $N \to \infty$. The next term in a loop expansion is the sunset diagram. Here the insertions

$$\Delta(x,x') = \frac{\delta\Gamma^{2PI}[\phi,G]}{\delta G(x,x')} \tag{5}$$

are no longer local. If we truncate Γ_2 including two-loop order terms we have

$\Gamma^{(2)}[\phi, G] =$  +  .

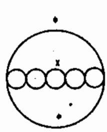
In order to obtain the next-to-leading order approximation (2PI-NLO-1/N) in the $1/N$ expansion [2] one has to resum all necklace diagrams:

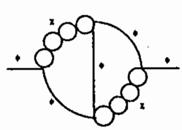
(a)  (b) 

This approximation has been investigated in nonequilibrium quantum field theory in $1+1$ and $3+1$ dimensions by Berges and collaborators, and at finite temperature by Bordag and Skalozub [3].

2.3 The Bare-Vertex-Approximation (BVA)

The bare vertex approximation (BVA) has been introduced in nonequilibrium quantum field theory by Blagoev, Cooper, Dawson and Mihaila (see, e.g. [4]). It is based on the $2PI$ scheme. One introduces an auxiliary field $\chi(x) = \mu^2 + \frac{\lambda}{2i}\mathrm{Tr}G$ and replaces the exact vertex $\langle\Phi(x)\Phi(x')\chi(x'')\rangle$ by the bare one. By this approximation the set of Schwinger-Dyson-equations closes; the expansion ends at the two-loop-level of graphs containing internal ϕ and χ lines.





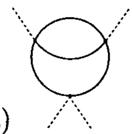
It differs from NLO-1/N by inclusion of the second graph, so it becomes identical to the NLO-1/N approximation if $\phi = 0$.

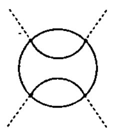
2.4 The 2PPI formalism

The 2PPI formalism in its original form [5] is based on the action

$$\Gamma[\phi, \Delta] = S_{\rm cl}[\phi] + \Gamma^{2\rm PPI}[\phi, \mathcal{M}^2] + \frac{3\lambda}{4}\int d^D x \Delta^2(x) \,. \tag{6}$$

It differs from the 2PI formalism by introducing, in the functional integral, only a *local* source term $\int d^D x K(x)\phi^2(x)$. Here $\Gamma^{2\rm PPI}[\phi, \mathcal{M}^2]$ is the sum of all *two-particle point-irreducible* (2PPI) graphs; these are defined as graphs which do not decay into two parts if two lines *joining at a point* are cut. So among the following diagrams


(a)


(b)

the diagram (a) is both two-particle reducible (2PR) and and two-particle point reducible (2PPR) while diagram (b) is 2PR but 2PPI.

In $\Gamma^{2\rm PPI}[\phi, \mathcal{M}^2]$ the Feynman propagators are replaced by the solutions of

$$(\Box + \mathcal{M}^2(x))G(x, x') = -\delta(x - x') \tag{7}$$

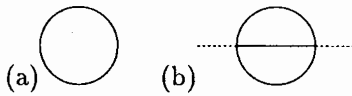
and the gap equation is

$$\mathcal{M}^2(x) - m^2 - 3\lambda\phi^2(x) = 3\lambda\Delta(x) = -6\lambda\frac{\delta\Gamma^{2\rm PPI}[\phi, \mathcal{M}^2]}{\delta\mathcal{M}^2(x)} \,. \tag{8}$$

So Δ contains all *local* insertions into the propagator. If one uses this in order to eliminate Δ, the action becomes a function of ϕ and $\mathcal{M}^2$. By variation with respect to ϕ and $\mathcal{M}^2$ one reobtains the gap equation and the equation of motion for the mean field

$$0 = \Box\phi + \mathcal{M}^2(x)\phi(x) - 2\lambda\phi^3(x) - \frac{\delta\Gamma^{2\rm PPI}[\phi, \mathcal{M}^2]}{\delta\phi(x)} \,, \tag{9}$$

the equation of motion for the mean field and the gap equation. Unlike in the 2PI formalism the Green function itself is a functional of $\mathcal{M}^2$ and therefore not a variational variable. In the lowest order this approach reduces to the Hartree approximation. Here we go beyond the Hartree approximation by including, besides the bubble diagram (a) the sunset diagram (b) which constitutes the entire two-loop contribution.



Renormalization in the Hartree approximation leads to well known inconsistencies of the counter terms with those of standard perturbation theory; this is due to the lack of crossing symmetry in the subgraphs taken into account in a certain order. This problem has been analyzed thoroughly by Verschelde [6]. He decomposes the counter-terms into 2PPI ones and the remainders and formulates the bookkeeping to all orders. He thereby shows the overall consistency of the procedure. In praxi his approach reduces to renormalizing on the level of the gap equations.

2.5 Comparison of the different formalisms

In general the three schemes resum large classes of Feynman graphs; this is not immediately evident from the graphs displayed when formulating a certain approximation, as by resummation the meaning if the lines in these graphs is different in the three schemes. If one "counts" the graphs actually included in the resummation in next-to-leading order one can roughly rank:

$$\mathrm{BVA} > \mathrm{NLO} - 1/\mathrm{N} > \mathrm{2PPI(2-loop)} \tag{10}$$

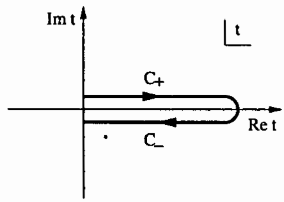
It is unclear, however, whether these resummations correctly reproduce the expected features of the model even qualitatively: order of phase transition, late-time behavior, thermalization (or equilibration), Goldstone particles etc.. It is unclear, furthermore, how small $1/N$ has to be in the first two schemes or if there is another small parameter ($\hbar$). So we would plead for taking the pragmatic approach: to apply these schemes, and to compare them with each other and with known features of the exact theory, whenever possible.

3 Numerical results

3.6 The $O(1)$ model in $1+1$ dimensions, out of equilibrium

The simulation of the simultaneous time evolution of the classical field and the fluctuations (as encoded in the Green functions) is a formidable task, once one goes beyond the Hartree approximation. As the leading order large-N or Hartree approximation is often used e.g. in cosmological models of particle production, it is important to see whether and how the qualitative and quantitative features survive in higher orders, either in a loop or a $1/N$ expansion. This is very difficult in $3+1$ dimensions [7], where in the 2PI formalism the renormalization has not been performed even in equilibrium for nonzero external fields. We have chosen to consider simulations in $1+1$ dimensions [8], where Cooper, Dawson and Mihaila [4] have already compared the Hartree, NLO-1/N and BVA approximations.

The time evolution of a quantum system in the Heisenberg picture has been formulated by Schwinger and Keldysh [9]. The perturbation theory in the so-called Schwinger-Keldysh or closed-time-path (CTP) formalism is similar to the Feynman graph expansion of the S-matrix with the following modifications: in all Feynman graphs the integrations over p_0 are replaced by integrations over $x^0 = t$, and this time integration is not from $-\infty$ to $+\infty$, but along a closed time path.



The vertices have different signs on the two parts of the path; thereby the time integration decays into parts, whose number proliferates as 2^{n_i} where n_i is the number of interior lines. These combine in such a way that the kernels of the time integration become causal. The spatial momentum integrals are those of the usual Feynman graphs. They are regularized in dimensional regularization using $d^{3-\epsilon}p$.

How does the calculation [8] proceed in practice? If we consider a spatially homogenous classical field $\phi(t)$ the variational masses likewise depend only on time. So the x dependence can be formulated in a plane wave basis. The Green function for momentum p satisfies

$$\left[\frac{\partial^2}{\partial t^2} + p^2 + \mathcal{M}^2(t)\right] G(t,t',p) = -\delta(t-t') \tag{11}$$

As in the 2PPI formalism used by us the mass term is local in time the solution can be written in factorized form

$$G(t,t',p) = \Theta(t-t')f(p,t)f^*(p,t') + \Theta(t'-t)f(p,t')f^*(p,t) \tag{12}$$

Having solved the differential equations for the mode functions

$$\left[\frac{\partial^2}{\partial t^2} + p^2 + \mathcal{M}^2(t)\right] f(p,t) = 0 \tag{13}$$

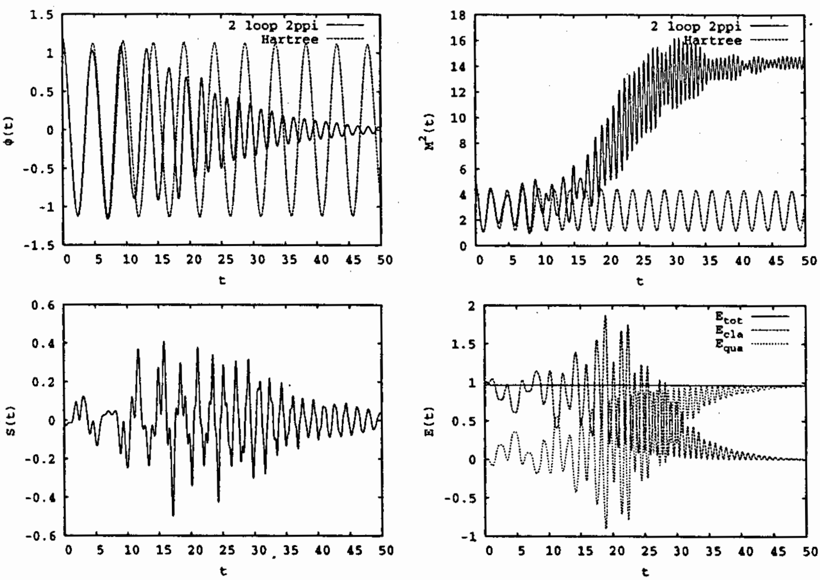
he Greens function can be computed and used to compute all relevant Feyn-nan graphs. The factorized form represents a tremendous computational sim-lification. We do not have to store the memory of the past, and the compu-ation of the Feynman graphs is also greatly simplified.

The mode functions $f(p,t)$ can be used as a basis for a Fock space, and serve o formulate particle production via Bogoliubov coefficients. This Fock space s also used in formulating the initial state or density matrix. Note however, hat these Green functions do not converge to the exact Green functions, and herefore, unlike in leading order, the particle interpretation becomes rather uestionable.

The initial conditions of course depend on the physics of the system under onsideration. In inflationary models one of the initial conditions specifies he initial inflation amplitude $\phi(0)$ and reheating after inflation starts with an 'empty" vacuum, so the Fock vacuum is widely used as initial condition for he quantum state.

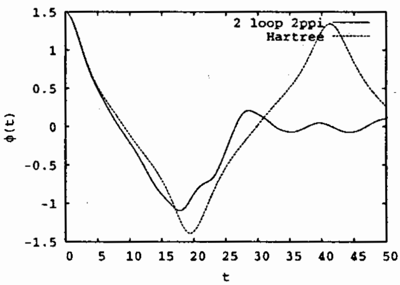
We here consider both the case of a double well potential as also the case vithout spontaneous symmetry breaking, i.e., a potential $V(\Phi) = m^2\Phi^2/2 +$ $\lambda\Phi^4/4$ with $m^2 > 0$.

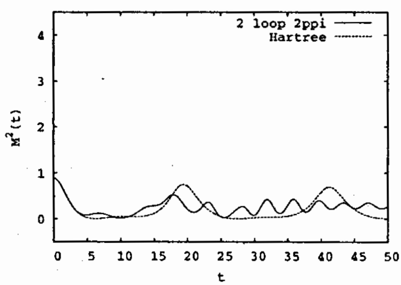
We first display the results for the latter case, with $m^2 = 1$, $\lambda = 1$ and $\phi(0) = 1.2$.



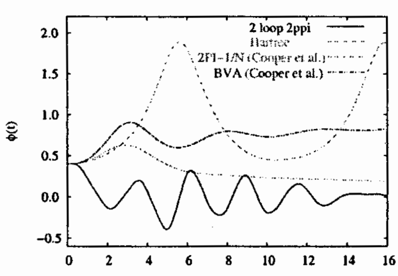
The four figures show the time dependence of the mean field $\phi(t)$, the effective mass squared $\mathcal{M}^2(t)$, the sunset contribution $\mathcal{S}(t)$ to the equation of motion of $\phi(t)$, and the energy. While dissipation is negligible in the Hartree approximation, the amplitude $\phi(t)$ shows strong dissipation when the sunset diagram is included. Indeed the sunset contribution is large in the region $10 < t < 35$ where the dissipation is strong.

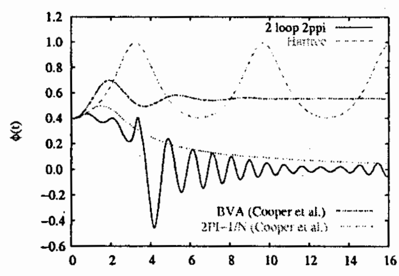
We now consider the case of broken symmetry (on the tree level). For for $v = 1$ and $\lambda = 1/6$ we display the mean field amplitude $\phi(t)$ and the effective mass $\mathcal{M}^2(t)$ for the Hartree and the two-loop approximations. With the initial amplitude $\phi(0) = 1.5$





the total energy is larger than the height of the potential barrier. Again we find strong dissipation in the two-loop approximation, while there is practically no dissipation in the Hartree approximation. While in this case the mean field oscillates around $\phi = 0$ at late times the case becomes more interesting if the total energy is lower than the height of the barrier. In the Hartree approximation the amplitude then oscillates around the minimum of one of the wells. In the following figures the results are shown for the Hartree, the NLO-1/N, the BVA and the 2PPI two-loop approximations, for the parameter sets $v = 2, \lambda = 0.5$, and $v = 0.523$, $\lambda = 3.65$ used by Cooper et al. [4].



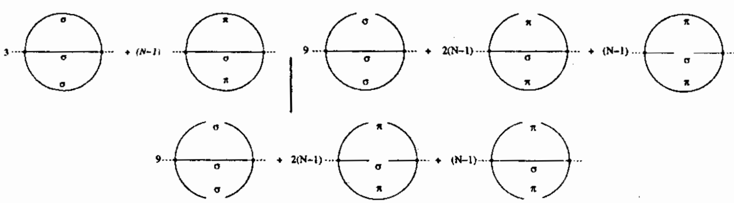


)ne sees that the 2PI-NLO-1/N approximation tends towards the symmetric onfiguration $\phi = 0$ at late times, and so does our two-loop 2PPI simulation. 'or the BVA, as in the leading Hartree approximation, the mean field tends owards the broken symmetry minimum at late times.

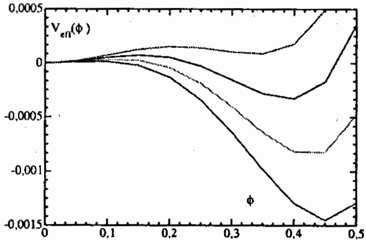
In summary the inclusion of the sunset diagram (i) leads to dissipation in he symmetric case; (ii) causes the system to end up in the symmetric phase for he broken symmetry case. No thermalization is observed in the momentum pectra at late times.

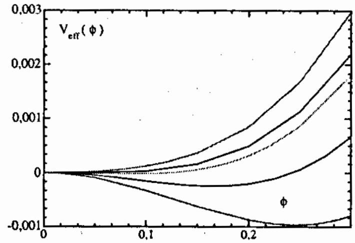
3.7 The $O(N)$ model in $3+1$ dimensions, in thermal equilibrium

The $O(N)$ model in $3+1$ dimensions with spontaneous symmetry breaking n the tree level has a *first-order* phase transition in the Hartree approximation vhile there is evidence that is has a *second-order* phase transition in the exact heory. We recently have investigated the model beyond the Hartree approxi-nation by going to the next-to-leading order in the 2PPI formalism [11]. This mplies the inclusion of sunset diagrams at finite temperature.

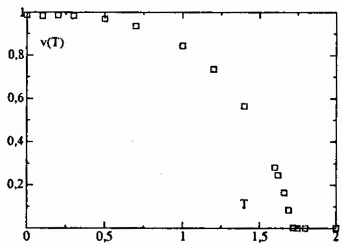


The next-to-leading order in the 2PPI formalism has been done for the $O(1)$ nodel by Smet et al. [10]. We have performed the calculations for the $O(N)$ nodel. An $O(N)$ invariant decomposition of the Green function contains two nass parameters $\mathcal{M}_\sigma$ and $\mathcal{M}_\pi$ and leads to a coupled set of nonlinear gap quations. Instead of solving these we compute the free energy (finite tem-erature action) and look for maximum as a function of the two masses $\mathcal{M}_\sigma$ nd $\mathcal{M}_\pi$. This then leads to the effective potential $V(\phi)$. The behavior of the ffective potential is displayed in the following figures for several values of the emperature in the vicinity of the phase transition:

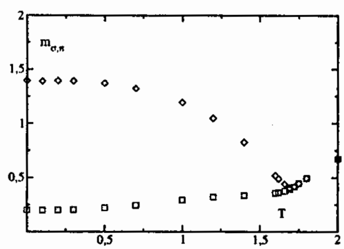


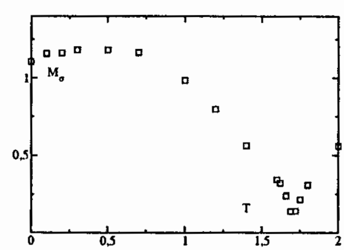


The figure on the left hand side shows the results for the Hartree approximation, by inclusion of the new diagrams the phase transition becomes second order as seen in the graph on the right hand side. A second-order phase transition is also signalled by the behavior of $v(T)$:



The values m_σ (diamonds) and m_π (squares) of the effective masses $\mathcal{M}_\sigma$ and $\mathcal{M}_\pi$ at the equilibrium point, as well as the sigma mass M_σ defined by the curvature of the potential at the minimum are displayed in the following figures:





Note that in 2PPI, as also in 2PI, the propagator G is not, and does not converge to the exact propagator of the theory. Therefore the effective masses m_σ, m_π displayed in the graph on the left hand side are *not* the real sigma and pion masses but simply variational parameters. So if the "pion mass" is not zero below the phase transition, this does not signal a violation of the Goldstone theorem. One has to note that the resummation schemes violate the crossing relations of the included four-point amplitudes. The effective potential is $O(N)$-symmetric, however. The sigma mass as defined by the curvature at the minimum, displayed on the right hand side, it goes to zero at the phase transition.

The 2PI approach to this model in NLO-1/N has not been performed even in equilibrium. Renormalization is an obstacle, the problem is solved in principle [7], but only for $\phi = 0$.

In conclusion we find for the $O(N)$ model in $3+1$ dimensions in thermal equilibrium: (i) the renormalization à la Verschelde is found to work in this nontrivial case; (ii) the phase transition towards the symmetric phase becomes second order; (iii) besides the instabilities (imaginary part) due to negative $\mathcal{M}_\pi^2$ and $\mathcal{M}_\sigma^2$ we find a new instability: the decay, in the heat bath, of $\sigma \rightarrow 2\pi$ with one external (constant) σ field. Thereby the region where the effective potential makes sense is further reduced.

4 Conclusions and Outlook

As we have seen the inclusion of next-to-leading order diagrams leads to new features as well in thermal equilibrium as out of equilibrium:

1. The naive association of modes with particles is no longer convincing, as the effective masses are not those of real particles (poles or the exact Green function) but simply variational parameters; this puts a question mark on popular models of particle production in external fields. This is even more apparent in the 2PI formalism where beyond the leading order the Green function does not factorize anymore into mode functions.

2. While the effective potential already gets an imaginary part in the Hartree approximation, due to negative squared masses (spinodal instability), when including the sunset diagram we find at finite temperature new imaginary parts associated with the presence of unstable particles (the Higgs) in the heat bath.

3. In the 1+1 dimensional out-of-equilibrium simulations we have seen that the various approximations lead to controversial results, so one is led to

the question whether summing more diagrams automatically implies a better approximation to physics.

These topics have to be investigated in the near future. Likewise it is important to see what happens if one goes to more realistic theories by including gauge fields.

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Analyticity in Noncommutative Quantum Field Theory

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Abstract — We derive the analytical properties of the elastic forward scattering amplitude of two scalar particles from the axioms of the noncommutative quantum field theory for the case of only space-space noncommutativity.

The proof of the analytical properties of scattering amplitudes is one of the most remarkable achievements of the axiomatic approach to quantum field theory [1]- [5]. The implications of the modern ideas of noncommutative geometry [6] in physics have been lately of great interest (see review [7]). The task of establishing the analytical properties of noncommutative field theory is very interesting, but highly nontrivial. In passing from a usual space-time manifold to a space on which the coordinate operators do not commute, i.e. $[x_\mu, x_\nu] = i\theta_{\mu\nu}$, where $\theta_{\mu\nu}$ is an antisymmetric constant matrix, the interactions acquire a nonlocal character, which gives rise to a novel behaviour of the NC QFT. For the derivation of analyticity, of crucial importance is the microcausality, which is affected by the noncommutativity of space-time. The effect is drastic when time does not commute with the spacial coordinates ($\theta_{0i} \neq 0$), in the sense that microcausality is completely lost [8,9]. In the case of theories with commutative time ($\theta_{0i} = 0$) microcausality survives, but as a weaker condition than in the commutative case [8] (see eq. (2)). For this reason one may hope that analyticity can still be obtained in field theories with only space-space noncommutativity.

The first step in this direction was made in [10]. The essential difference between the analytical properties of the scattering amplitude in commutative and noncommutative cases found in this work was related to a specific way of continuation of the scattering amplitude to the complex plane.

In the present work we aim at deriving analyticity first for forward elastic scattering of two spinless particles with masses m and M. We prove that if the noncommutativity affects only the space variables, then analyticity similar to the commutative case is valid.

In the case of space-space noncommutativity we can choose the coordinates in such a way that only $\theta_{12} = -\theta_{21} \neq 0$. Then the usual condition of local commutativity can be substituted by its analog containing only the x_0 and x_3 coordinates [8] (see eqs. (1) and (2)). Our proof is valid under the condition (7) on scattering amplitude , which is weaker than the usually used polynomial boundedness. We have proven the analyticity of the elastic scattering amplitudes on the basis of LSZ reduction formulas [11].

In the commutative case we admit the condition of local commutativity:

$$[j(x), j(y)] = 0, \quad \text{if} \quad (x-y)^2 < 0, \tag{1}$$

where $j(x)$ is the current of interacting fields.

In the noncommutative case with $\theta_{0i} = 0$ on the same basis as in the usual case, we assume the local commutativity condition to be [8]

$$[j(x), j(y)] = 0 \quad \text{if} \quad (x_0 - y_0)^2 - (x_3 - y_3)^2 < 0. \tag{2}$$

This condition was shown to be valid [9] for the cases when $j(x)$ is any power of field operators with $\star$-product.

If in the noncommutative case "in" and "out" fields can be constructed in the same way as in usual theory then the standard LSZ reduction formulas are valid and the scattering amplitude is:

$$F(E, \vec{q}) = \int d^4x \, e^{i(E x_0 - \vec{q}\cdot\vec{x})} \, \tau(x_0) \, F(x), \tag{3}$$

where $F(x) = < M \,|[j\,(x/2)\,, j\,(-x/2)]|\, M >$. We omit in (3) numerical factor which is irrelevant to the analytical properties of $F(E, \vec{q})$ and the term, which is some polynomial of E. Eq. (3) is written in the reference frame in which the particle with the mass M is at rest. E and $\vec{q}$ are the energy and momentum of the particle with mass m.

In order to extend $F(E, \vec{q})$ to the upper complex E-plane $(Im\, E > 0)$ we integrate (3) over x_1 and x_2 (similarly as in [10]). Then $F(E, \vec{q})$ is represented in the form:

$$F(E, |\vec{q}|, \vec{e}) = \int_0^\infty e^{iEx_0} \, d\,x_0 \int_{-x_0}^{x_0} e^{-i e_3 x_3 \sqrt{E^2 - E_0^2}} \, \Phi(x_0, x_3) \, d\,x_3, \tag{4}$$

where $\Phi(x_0, x_3) = \int F(x)\, e^{-i(q_1 x_1 + q_2 x_2)}\, dx_1\, dx_2$.
We admit that as in the commutative case one has from the energy-momentum relation $E_0^2 \equiv E^2 - q_3^2 = m^2 + q_1^2 + q_2^2$. In order to exclude the singularity at $\sqrt{E^2 - E_0^2}$, we make the substitution

$$F(E, |\vec{q}|, \vec{e}) \to \frac{1}{2}\,(F(E, |\vec{q}|, \vec{e}) + F(E, |\vec{q}|, -\vec{e}) \equiv F(E),$$

$\vec{q} = \vec{e}\,|\vec{q}|$, $|\vec{e}| = 1$. A direct extension of $F(E)$ into the complex E-plane is impossible since $Im\sqrt{E^2 - E_0^2} > Im\, E$ (see [12], chapter 10). To overcome this obstacle, following [12], we substitute $F(E)$ by the regularized amplitude $F_\varepsilon(E)$:

$$F_\varepsilon(E) = \int_0^\infty e^{iEx_0}\, dx_0 \int_{-x_0}^{x_0} \cos\left(e_3\, x_3 \sqrt{E^2 - E_0^2}\right) e^{-\varepsilon\,(x_0^2 + x_3^2)}\, \Phi(x_0, x_3)\, dx_3. \quad (5)$$

$F_\varepsilon(E)$ is an analytical function in the upper half-plane, where the integral in (5) converges.

The main problem is to prove the existence of analytical function $F(E) = \lim_{\varepsilon\to 0} F_\varepsilon(E)$. To this end let's use the analytical properties of $F_\varepsilon(E)$. Our goal is to represent $F_\varepsilon(E)$ at complex E as integral over real axe only and then go to $\varepsilon = 0$. But it is impossible to do this directly as $F_\varepsilon(E) \not\to 0$ at $E \to \infty$. So first we have to construct such a function. In commutative case to this end the polynomial boundedness of $F(E)$ is used. Here we use the weaker bound. Namely we suppose that there exists α, $0 < \alpha < 1$ such that

$$|\,F(E)\,| < \exp(E^\alpha), \qquad E \to \infty. \quad (6)$$

Evidently inequality (6) is fulfilled also for $F_\varepsilon(E)$. Condition (6) is valid also at $E \to -\infty$ as

$$F(-E + i\,0) = F^*(E + i\,0), \quad F_\varepsilon(-E + i\,0) = F_\varepsilon^*(E + i\,0). \quad (7)$$

Eq. (7) is a standard crossing symmetry condition. Evidently, function $\psi_\varepsilon(E) = F_\varepsilon(E)\,\xi(E)$, where $\xi(E) = \exp\left[-\left(\sqrt{m^2 - E^2}\right)^\beta \exp(-i\,\pi\,\beta)\right]$, $0 < \beta < 1$, $\alpha < \beta$ satisfies the necessary condition $\psi_\varepsilon(E) \to 0$, $\quad E \to \pm\infty$. It is easy to see that $\xi(E)$ is an analytical function in the whole E-plane with cuts (m, ∞), $(-\infty, -m)$ satisfying the conditions

$$\xi(-E + i\,0) = \xi^*(E + i\,0) = \xi(E - i\,0) \quad (8)$$

Using the Cauchy formula we see that

$$\psi_\varepsilon(E) = \frac{1}{2\pi i}\int_C \frac{\psi_\varepsilon(E')\,dE'}{E'-E}, \quad Im\,E > 0. \tag{9}$$

Contour C consists of interval $(-R, R)$ and semicircle in an upper half-plane.

Now let's demonstrate that owing to cond.(2) $\psi_\varepsilon(R\,e^{i\varphi}) \to 0$ if $R \to \infty$, $0 \le \varphi \le \pi$. Owing to the factor $\exp(-\varepsilon\, {x_3}^2)$ integral over x_3 converges when $x_0 \to \infty$, so the integration is really taken over some finite interval. Thus the growing factor in exponential in eq. (5) disappears at $|E| \to \infty$ and we can put $R = \infty$ in eq. (9). So

$$\psi_\varepsilon(E) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} \frac{\psi_\varepsilon(E')\,dE'}{E'-E}, \qquad Im\,E > 0. \tag{10}$$

Eq. (10) is valid at any fixed ε. Now let's go to $\varepsilon = 0$. First let's consider the interval (m, ∞). If $E' > E_0$, we can go to the limit $\varepsilon = 0$ without any problem as in this interval $\lim_{\varepsilon\to 0} F_\varepsilon(E') = F(E')$ (see eq. (5)). In the interval (m, E_0) we can't use directly eq.(5). But $F_\varepsilon(E)$ is the function of E only. Let's consider two functions $F_\varepsilon^{(1)}(E)$ and $F_\varepsilon^{(2)}(E)$ with different $q_1^2 + q_2^2$ and so with different $E_0^{(1)}$ and $E_0^{(2)}$. If e.g. $E_0^{(2)} > E_0^{(1)}$, then $F_\varepsilon^{(1)}(E) = F_\varepsilon^{(2)}(E)$ at $E > E_0^{(2)}$. Thus analytical in upper half-plane functions $F_\varepsilon^{(1)}(E)$ and $F_\varepsilon^{(2)}(E)$ coincide everywhere. The interval $(E_0^{(1)}, E_0^{(2)})$ is a physical one for function $F_\varepsilon^{(1)}(E)$. As $\lim_{\varepsilon\to 0} F_\varepsilon^{(1)}(E)$ exists in this interval $\lim_{\varepsilon\to 0} F_\varepsilon^{(2)}(E)$ exists in it as well. Continuing this process we come to $E_0 = m$. The interval $(-\infty, -m)$ can be treated similarly in accordance with eqs. (7) and (8).

The remained interval can be considered as well as in commutative case. Let's construct the analytical function in the lower half-plane and prove that this function is an analytical continuation of $F_\varepsilon(E)$. To this end we use the function $\tilde F(E, \vec q)$, which is determined by eq. (3) with the standard substitution: $\tau(x_0)\,F(x) \to \tau(-x_0)\,F(-x)$. As before we substitute $\tilde F(E, \vec q)$ by $\tilde F_\varepsilon(E)$. The function $\tilde\psi_\varepsilon(E) = \tilde F_\varepsilon(E)\,\xi(E)$ is an analytical function in a lower half-plane and $\tilde\psi_\varepsilon(E) \to 0$, $|E| \to \infty$. Let's suppose that vectors $|p, n>$ form the complete system of basis vectors, p is a momentum, n denotes all other quantum numbers and that masses of intermediate states satisfy the condition $M_n \ge M + m$ (excluding one M particle intermediate state). After usual calculations [12] we see that in the interval $(-m, m)$ $\lim_{\varepsilon=0}(\psi_\varepsilon(E) - \tilde\psi_\varepsilon(E)) = 0$, excluding 2 points: $\pm m^2/2M$. Thus function $F(E)$ is an analytical function in the whole E-plane excluding cuts $(-\infty, -m)$, (m, ∞) and poles $\pm\frac{m^2}{2M}$.

Based on the analytical properties obtained, we can derive the formal dispersion relations analogous to the usual case of quantum field theory [13].

Conclusion: We have derived the analytical properties for the forward elastic scattering amplitude of two spinless particles in noncommutative quantum field theory with space-space noncommutativity.

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Quartic Field Interaction and the Strong-coupling Problem

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Abstract — The strong-coupling problem is considered in the scalar super-renormalizable field theory $g\phi^4$. We have found an optimal representation where the exact strong-coupling behaviour of the free energy is obtained. The interaction becomes weaker as the bare coupling constant grows, so the higher-order corrections can be systematically estimated. The new regularization procedure regroups the initial counter terms so that the divergences are exactly removed in the final expressions.

1 Introduction

The perturbation method is the basic, well established and reliably working instrument in QFT. However, various physical tasks require going beyond the weak-coupling region and the strong-coupling problem is one of the main and unsolved tasks in QFT. Therefore, a number of investigations have been devoted to the strong-coupling resummation techniques, particularly, the Pàde approximants and the Borel transform [1], the Heaviside transformation [2], a modified Laplace transformation [3], the renorm-group methods [4], the variational interpolation method [5] and the strong-coupling expansion [6].

Along the low convergence, the above methods are not optimal for theories with divergences - the structure of the counter terms is not transformed correctly and, the strong-coupling behaviour differs from the exact solution. Besides, variational estimates may contain errors of infinite order.

The model ϕ^4 plays a significant role in theoretical physics by providing a simple but nontrivial testing ground for new methods and techniques [7].

In our earlier investigations [8, 9] an effective *approximate strong-coupling solution* has been obtained in the theory φ^4_2 by combining the canonical transformations and normal ordering procedure. This technique ensures the canonical structure of the theory [10].

Now we demonstrate the *exact strong-coupling solution* in the scalar field theory $g\phi^4_d$ in d-dimensional Euclidean space. For simplicity, we consider the case $d = 2$, because up to $d < 4$ the theory remains superrenormalizable, only minor modifications will be required for extension. In higher dimensions ($d > 4$) the theory becomes either instable, or trivial [11, 12].

2 Scalar field model in two dimensions

Consider the ground-state energy in the ϕ_2^4-theory. The Lagrangian reads:

$$\mathcal{L} = \frac{1}{2}(\phi[-\partial^2 - m^2]\phi) - \frac{g^2}{2}(:\phi^4:). \tag{1}$$

All divergences are removed by introducing the normal form of the interaction:

$$:\phi^4(x) := \phi^4(x) - 6D_0\phi^2(x) + 3D_0^2 = (\phi^2(x) - 3D_0)^2 - 6D_0^2, \tag{2}$$

$$D_0 = D(0), \qquad D(x) = \int \frac{dk}{(2\pi)^2}\frac{e^{ikx}}{k^2 + m^2} = \int_0^\infty \frac{d\beta}{4\pi\beta}\, e^{-\frac{\beta}{2}m^2 - \frac{x^2}{2\beta}}.$$

1. For intermediate calculations we use a dimensional regularization [13]:

$$D_{\mathrm{reg}}(x) = \frac{1}{2}\int_0^\infty \frac{d\beta}{(2\pi\beta)^{d/2}}\, e^{-\frac{\beta}{2}m^2 - \frac{x^2}{2\beta}}, \quad D_0 = \frac{\Gamma\left(1 - \frac{d}{2}\right)}{2(2\pi)^{d/2}}\left(\frac{2}{m^2}\right)^{1-\frac{d}{2}},$$

i.e. consider $d < 2$ and then, all auxiliary divergences are explicitly and accurately removed out. For final formulae we restore the true value $d = 2$.

2. The normalized partition function in the Gaussian representation is:

$$\begin{aligned} Z_\Omega(g) &= \sqrt{\det(-\partial^2 + m^2)}\int \delta\phi\; e^{-\frac{1}{2}(\phi[-\partial^2+m^2]\phi) - \frac{g^2}{2}\left([\phi^2 - 3D_0]^2\right) + 3g^2D_0^2\Omega} \\ &= \int \delta\Phi\; e^{-\frac{1}{2}(\Phi\Phi)}\sqrt{\det(-\partial^2 + m^2)}\int \delta\phi\; e^{-\frac{1}{2}(\phi[-\partial^2+m^2]\phi) - ig\left(\Phi\left[\phi^2 - 3D_0\right]\right) + 3g^2D_0^2\Omega} \\ &= e^{-g^2D_0^2\Omega}\int \delta\Phi\; e^{-\frac{1}{2}(\Phi\Phi) + igD_0(\Phi) - \frac{1}{2}\mathrm{Tr}\ln(1 - 4g^2D_0D + i2g\Phi D)}. \end{aligned} \tag{3}$$

In (3) we have obtained a "logarithmic" interaction, quite different from the initial "quartic" one and slowly increasing at $g \to \infty$. It is *important to trace and provide the correct regularization scheme with new counter terms to eliminate all divergences in the new representation.* The new interaction reads:

$$U[\Phi] = \mathrm{Tr}\ln\left\{\frac{D^{-1} - 4g^2D_0 + i2g\Phi}{D^{-1}}\right\} = \int dx\int d\mu_\alpha\int d\sigma[\xi]\;\left\{1 - e^{2\alpha g^2D_0 - ig(\Phi B_\xi)}\right\},$$

where

$$\int d\mu_\alpha(...) = \int\limits_0^\infty \frac{d\alpha}{(2\pi\alpha)^{d/2}\alpha}\; e^{-\frac{\alpha m^2}{2}} (...) \int d\sigma[\xi]\; e^{-\frac{1}{2}\int\limits_0^\alpha d\beta\; \xi^2(\beta)}$$

$$B_\xi(y) = \int\limits_0^\alpha d\beta\; \delta(y - x - \xi(\beta))\,, \quad (\Phi B_\xi) = \int dy\; \Phi(y) B_\xi(y) = \int\limits_0^\alpha d\beta\; \Phi\,(x + \xi(\beta))\,.$$

with $\xi(\beta) \in \mathbf{R}^d$, $\xi(0) = \xi(\alpha) = 0$ and $\int d\sigma[\xi] = 1$.

3. To extract the main contribution from interaction functional $U[\Phi]$ we substitute $\Phi(x) \to \Phi(x) - igA$, where $A = A(g)$ is a shift value. Then,

$$Z_\Omega[g] = e^{g^2\left[\frac{1}{2}A^2 + D_0 A - D_0^2\right]\Omega} \cdot \int \delta\Phi\; e^{ig(D_0+A)(\Phi) - \frac{1}{2}U[\Phi - igA]} \tag{4}$$

4. The normal-ordered forms with respect to the Gaussian measures read:

$$e^{ik(\xi(\beta_1)-\xi(\beta_2))} =: e^{ik(\xi(\beta_1)-\xi(\beta_2))} :_\xi\; e^{-\frac{k^2}{2}F_\alpha(\beta_1-\beta_2)}\,, \quad F_\alpha(t) = |t| - t^2/\alpha\,,$$

$$e^{-ig(\Phi B_\xi)} =: e^{-ig(\Phi B_\xi)} :_\Phi\; e^{-\frac{g^2}{2}\left[\langle (B_\xi B_\xi)\rangle_\xi + :W_\alpha[\xi]:_\xi\right]}\,,$$

where

$$\langle (B_\xi B_\xi)\rangle_\xi = \int d\sigma[\xi] (B_\xi B_\xi) = \frac{2\alpha^{2-\frac{d}{2}}}{(2\pi)^{d/2}} B\left(2 - \frac{d}{2}, 1 - \frac{d}{2}\right),$$

$$: W_\alpha[\xi] :_\xi = \iint\limits_0^\alpha d\beta_1 d\beta_2 \int \frac{d^d k}{(2\pi)^d} : e_0^{ik(\xi(\beta_1)-\xi(\beta_2))} :_\xi\; e^{-\frac{k^2}{2}F_\alpha(\beta_1-\beta_2)}\,,$$

$$\int d\sigma[\xi] : W_\alpha[\xi] :_\xi = 0\,, \qquad e_k^z = e^z - \sum_{s=0}^{k} \frac{z^s}{s!}\,.$$

Then,

$$U[\Phi - igA] = \Omega U_0 + ig(\Phi) U_1 + U_I[\Phi] \tag{5}$$

where

$$U_0 = \int d\mu_\alpha \left[1 - e^{Q(g^2\alpha,\, A)} R\left(g^2\alpha\right)\right], \qquad U_1 = \int d\mu_\alpha\; \alpha\; e^{Q(g^2\alpha,\, A)} R\left(g^2\alpha\right),$$

$$U_I[\Phi] = -\int dx \int d\mu_\alpha\; e^{Q(g^2\alpha,\, A)} \int d\sigma[\xi]\; e^{-\frac{g^2}{2}:W_\alpha[\xi]:_\xi} : e_1^{-ig(\Phi B_\xi)} :_\Phi\,, \tag{6}$$

$$Q(g^2\alpha,\, A) = 2\alpha g^2 D_0 - \frac{g^2}{2}\langle (B_\xi B_\xi)\rangle_\xi - g^2\alpha A\,, \quad R\left(g^2\alpha\right) = \int d\sigma[\xi]\; e^{-\frac{g^2}{2}:W_\alpha[\xi]:_\xi}\,.$$

The optimal value of the shift A should be defined from the condition eliminating the total linear terms over Φ in (4) and is obtained

$$A = -\frac{\alpha}{2}\int d\mu_\alpha \left[1 - e^{Q(g^2\alpha,\ A)} R\left(g^2\alpha\right)\right].$$

The intermediate divergences are completely removed because of the equality:

$$-g^2 D_0 A + g^2 D_0^2 = \frac{1}{2}\int d\mu_\alpha\ Q(g^2\alpha,\ A).$$

After removal of all divergences we put $d = 2$. Going to new scale:

$$\xi(\beta) = \sqrt{\alpha}\ \eta(\tau), \qquad B = \frac{A}{h}, \qquad \alpha = \frac{2}{m^2}\,t, \qquad h = \frac{g^2}{\pi m^2}$$

we write the final representation for (3) as follows:

$$Z[g] = e^{-\Omega E} = e^{-\Omega E_0}\int \delta\Phi\ e^{-\frac{1}{2}(\Phi\Phi)-\frac{1}{2}U_I[\Phi]} = e^{-\Omega(E_0+E_{corr})}. \tag{7}$$

The leading-order term for the vacuum energy is

$$E_0 = -\frac{m^2}{8\pi}\left\{hB^2 + \int\limits_0^\infty \frac{dt}{t^2}\ e^{-t}\left[e^{Q(t)} - 1 - Q(t) + e^{Q(t)}\ (R(ht)-1)\right]\right\} \tag{8}$$

$$B = \frac{1}{2}\int\limits_0^\infty \frac{dt}{t}\ e^{-t}\left[e^{Q(ht,B)}\ R(ht) - 1\right],$$

$$Q(t) = -h\ t\ [B + \mathbf{C} + \ln(t)], \qquad R(ht) = \int \delta\eta\cdot e^{-\frac{1}{2}\int_0^1 d\tau\ \dot\eta^2(\tau) - ht:W[\eta]:_\sigma},$$

$$:W[\eta]:_\sigma = \iint\limits_0^1 d\tau_1 d\tau_2 \int \frac{dq}{(2\pi)^2}\ :e_0^{iq(\eta(\tau_1)-\eta(\tau_2))}:_\sigma\ e^{-\frac{q^2}{2}\left(|\tau_1-\tau_2|-(\tau_1-\tau_2)^2\right)}.$$

The higher-order correction reads

$$E_{corr} = -\lim_{\Omega\to\mathbf{R}^2}\frac{1}{\Omega}\ln\int\delta\Phi\ e^{-\frac{1}{2}(\Phi\Phi)-\frac{1}{2}U_I[\Phi]}. \tag{9}$$

Eqs. (7)-(9) define the ground-state energy of the system at arbitrary coupling h. For large h we estimate $R(ht) \propto e^{O(\ln(h)/h)}$. Therefore, for $h \to \infty$ we obtain

$$E_0 = \left(\frac{m^2}{8\pi}\right)\left[-\frac{3}{2}h\ln^2(h) + 3(\mathbf{C}+2)h\ln(h)\right] + O(h), \tag{10}$$

$$B = \ln(h) - \mathbf{C} - 2, \qquad \mathbf{C} = 0.577215...$$

This coincides with the **exact strong-coupling solution** [8] and it has been reached within the leading-order approximation only.

By concluding, we have represented a path-integral technique effective for the strong-coupling regime. It has been applied to the problem of the ground-state energy in the superrenormalizable scalar theory ϕ^4 in two dimensions. Hereby,

• the initial renormalization scheme of elimination of the divergences is correctly transformed into a new re-summation scheme,

• the exact strong-coupling solution has been achieved in the leading-order approximation,

• there are no artificial infinite terms in the final results,

• any appearance of auxiliary complex functionals do not represent any difficulties for this technique.

• this technique is extendable to higher dimensions up to $d < 4$ because the theory remains superrenormalizable, only minor modifications are required.

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NS bremsstrahlung under collision of bosonic strings in D dimensions.

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Abstract — We calculate classically NS radiation emitted in the collision of two bosonic strings in $D \geq 4$ space-time dimensions. Radiation arises when the point of minimal separation between two crossed straight strings moving in parallel planes separated by finite distance moves faster than light. Radiation has typical Cherenkov nature and is emitted at the Cherenkov's angle with respect to the direction of motion of this point. We analyze the radiation spectrum in arbitrary dimensions. In four dimensions the spectrum contains an infrared divergence which is absent in all higher dimensions.

1 Introduction

Radiation emitted by strings is interesting both in the context of the cosmic string theory [1–3] and in the fundamental superstring theory. So far generation of radiation of different nature (gravitational, dilaton, axion) was considered only in four space-time dimensions, the main mechanism being the radiation from the oscillating string loops [4,5]. Radiation from an oscillating string can be treated within the classical theory as an effect of the first order in the interaction constant [6–11]. The equations of motion contain an infinite self-force . It was shown that in four dimensions the equations of motion of radiating strings can be consistently renormalized to absorb the associated infinite self-action terms into the redefinition of the string tension and the coupling constant [7,8]. The finite part of radiation can then be easily calculated within the linearized theory for any given string excitation mode.

Here we investigate another mechanism of the string axion radiation — the bremsstrahlung, which should arise under collision of two strings due to their mutual interaction. To separate the bremsstrahlung from the other radiation mechanisms we assume the strings to be in unexcited states as the infinite straight strings (i.e. endless open strings). We assume that they are moving in the parallel planes in space avoiding the direct contact. Contrary to the case of oscillating loops, the bremsstrahlung radiation is the effect of the second order in the interaction constant, like the usual electromagnetic radiation of colliding charges in the perturbative treatment within classical electrodynamics. For strings similar perturbative technique was suggested before to calculate the gravitational radiation [12], however in that case the actual computation has led to zero result. The reason for vanishing of gravitational radiation in the collision of straight strings can be explained by the absence of gravitons in the $1+2$ gravity theory which can be applied in this case. In contrary, other types of string bremsstrahlung such as the electromagnetic one in the case of superconducting strings [13] and the axion radiation under collision of global strings in four dimensions [14] do exist.

Here we consider the NS radiation in the collision of two bosonic strings an arbitrary space-time dimensions assuming essentially the same geometry of collision as in [12]. When two straight strings propagate with respect to each other at some inclination angle α with the relative velocity v, the point of their minimal separation can move with any velocity, from zero value for orthogonal strings to an infinite value for parallel strings. It turns out that radiation arises once this point, which corresponds to the localization of an effective radiation source, moves faster than light. Radiation exhibits Cherenkov nature being concentrated on the cone with respect to the direction of motion of the superluminal point. Its properties depend on the dimensionality of space-time basically due to the different phase volume of emitted quanta. Our calculations are perturbative and involve two subsequent iterations in the strings equations of motion and the NS field equations.

2 Strings interacting via NS field

Consider a pair of relativistic strings $x^\mu = x^\mu_n(\sigma^a)$, $\mu = 0, ..., D-1$, $\sigma_a = (\tau, \sigma)$, for $a = 0, 1$ and $n = 1, 2$ is the index labelling the strings (in the following its position up and down will be arbitrarily used for compactness of the formulas). The D-dimensional space-time is flat and the signature is mostly minus (and $(+,-)$ for the string world-sheets). Strings interact via the NS two-form field $B_{\mu\nu}$ as described by the action

$$S = S_{st} + S_B, \tag{1}$$

where the strings term is

$$S_{st} = \frac{1}{2} \sum_{n=1,2} \int \left(\mu_n \sqrt{-\gamma} \gamma^{ab} \partial_a x^\mu_n \partial_b x^\nu_n \eta_{\mu\nu} - \lambda_n B_{\mu\nu} \epsilon^{ab} \partial_a x^\mu_n \partial_b x^\nu_n \right) d^2\sigma_n, \tag{2}$$

and the field term is

$$S_f = \frac{1}{192\pi} \int H_{\mu\nu\lambda} H^{\mu\nu\lambda} d^D x. \tag{3}$$

Here μ_n, λ_n are the string tension and coupling parameters (their n-labelling helps to control the perturbation expansion), Levy-Civita symbol is chosen so that $\epsilon_{01} = 1$, γ_{ab} is the metric on the world-sheet, and the NS field strength is $H_{\mu\nu\lambda} = \partial_\mu B_{\nu\lambda} + \partial_\nu B_{\lambda\mu} + \partial_\lambda B_{\mu\nu}$.

Using them, we fix the gauge $\gamma_{ab} = \eta_{ab}$ so that the constraint equations read $\dot{x}^2 + x'^2 = 0$, $\dot{x}^\mu x'^\nu \eta_{\mu\nu} = 0$ for each string, where dots and primes denote the derivatives over τ and σ as usual. It will be necessary to distinguish the NS fields generated by each string

$$\Box B_n^{\mu\nu} = 4\pi J_n^{\mu\nu}, \tag{4}$$

where the gauge $\partial_\lambda B^{\nu\lambda} = 0$ is assumed and the D-dimensional D'Alembert operator is understood $\Box = -\eta^{\mu\nu}\partial_\mu\partial_\nu$, and the source term

$$J_n^{\mu\nu} = 4\lambda_n \int V_n^{\mu\nu}\, \delta^D(x - x_n(\sigma_n))\, d^2\sigma_n, \tag{5}$$

with $V_n^{\mu\nu}$ being a covariant version of the tensor $V_n^{\alpha\beta} = \epsilon^{ab}\partial_a x_n^\alpha \partial_b x_n^\beta$.

In the gauge $\gamma_{ab} = \eta_{ab}$ the strings equation of motion reads

$$\mu_n \left(\partial_\tau^2 - \partial_\sigma^2\right) x_n^\mu = \frac{\lambda_n}{2} H_m^{\mu\nu\lambda} V_{\nu\lambda}^m. \tag{6}$$

The self-action presumably can be renormalized , in four dimensions this was done in [8], in higher dimensions this issue is worth to be clarified further. However, here we do not intend to focus on the renormalization problem and include only the mutual interaction terms ($m \neq n$, no sum over m, n). The retarded solution to the wave equation can be presented in the standard form.

Our strategy consists in solving the system of equations (4,6) iteratively, expanding the solution in powers of the coupling constants λ_n:

$$B_{\mu\nu} = \sum_{l=1}^{\infty} \overset{(l)}{B}{}^{\mu\nu}, \quad x_n^\mu(\sigma_n) = \sum_{l=0}^{\infty} \overset{(1)}{x}{}_n^\mu (\sigma_n). \tag{7}$$

In the zero order under mutual interaction the strings world-sheets are defined by the linear functions

$$x_n^{(0)\mu} = d_n^\mu + u_n^\mu \tau + \Sigma_n^\mu \sigma , \quad n = 1, 2 . \tag{8}$$

The constraint equations imply the orthogonality and normalization conditions (choosing the time-like D-velocity with the unit norm) $u^\mu \Sigma_\mu = 0$,

$u^\mu u_\mu = 1 = -\Sigma^\mu \Sigma_\mu$ for each string. We choose the reference frame so that the first string is at rest and is stretched along the axis x^3 while the second string is assumed to move in the plane x^2, x^3 , with the velocity v orthogonal to the string itself and inclined at the angle α with respect to he first string:

$$u_1^\mu = (1, 0, ...0), \quad \Sigma_1^\mu = (0, 0, 0, 1, 0, ...0),$$

$$u_2^\mu = \gamma(1, 0, -v\cos\alpha, v\sin\alpha, 0...0), \quad \Sigma_2^\mu = (0, 0, \sin\alpha, \cos\alpha, 0, ...0), \tag{9}$$

where $\gamma = (1 - v^2)^{-\frac{1}{2}}$. We also choose both impact parameters d_n^μ orthogonal to u^μ and Σ^μ and aligned with the axis x^1. With this choice the point of minimal separation between the strings moves with the velocity $v_p = v/\sin\alpha$ along the axis x^3.

According to the expansion (7), the first order field $\overset{(1)}{B}{}^{\mu\nu}$ in the equation (4) is determined by the zero order source term (5)

$$\overset{(0)}{J}{}_n^{\mu\nu} = 4\lambda_n \int \overset{(0)}{\dot{V}}{}_n^{\mu\nu}\, \delta^D(x - \overset{(0)}{x}_n(\sigma))\, d^2\sigma, \tag{10}$$

where $\overset{(0)}{V}{}_n^{\mu\nu} = (u_n^\mu \Sigma_n^\nu - \Sigma_n^\mu u_n^\nu)$. In the zero order the strings are freely moving, so the lowest order NS field $\overset{(1)}{B}{}_n^{\mu\nu}$ does not contain the radiative part. Inserting the field terms into the right hand side of the string equations (6) we obtain the small deviations $\overset{(1)}{x}{}_n^\mu(\tau, \sigma)$ of the string's trajectories due to mutual interaction. In the second approximation the axion field $\overset{(2)}{B}{}_n^{\mu\nu}$ may be obtained by the substitution to the right-hand side of Eq.(4) the first-order expression for the source $\Box\, \overset{(2)}{B}{}^{\mu\nu} = -4\pi\, \overset{(1)}{J}{}^{\mu\nu}$. In the first-order source of energy $\overset{(1)}{J}{}^{\mu\nu}$ has the form

$$\overset{(1)}{J}{}_n^{\mu\nu} = 4\lambda_n \int \left(2(\overset{(0)}{\dot{x}}{}_n^{[\mu} \overset{(1)}{\acute{x}}{}_n^{\nu]} + \overset{(1)}{\dot{x}}{}_n^{[\mu} \overset{(0)}{\acute{x}}{}_n^{\nu]}) - \overset{(0)}{V}{}_n^{\mu\nu} \overset{(1)}{x}{}_n^\lambda\, \partial_\lambda \right) \delta^D(x - \overset{(0)}{x}_n(\sigma)) d^2\sigma. \tag{11}$$

The square brackets denote anti-symmetrization $(a^{[\mu}b^{\nu]} = \frac{1}{2}(a^\mu b^\nu - a^\nu b^\mu))$.

3 Radiation loss

Evaluating the source terms in the D'Alembert equations (4) with account for these corrections we can calculate the second order NS field $\overset{(2)}{B}{}_n^{\mu\nu}$, which already has the radiative component. The radiation power can be computed as the reaction work produced by the half sum of the retarded and advanced fields upon the source. The radiation energy with frequency ω emitted during an infinite time of collision in the direction $\mathbf{n}$ ($D-1$ dimensional unit vector) is given by the following expression

$$P^{\mu} = \frac{\lambda}{2(2\pi)^{D-1}} \int k^{\mu} |J_{\alpha\beta}(k)|^2 \frac{k^0}{|k^0|} \delta(k^2) d^D k \,, \tag{12}$$

where the null D-dimensional wave vector k^{μ} is introduced as $k^{\mu} = \omega(1, n^i \cos\psi, N^a \sin\psi)$, where n^i, $i = 1,2,3$ is the unit vector in the string sector $n^i n^i = 1$, $n^i = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ and N^a is the unit vector in the remaining space, $a = 1, ..., D-4$.

Here for simplicity we consider the ultrarelativistic collision $\gamma \gg 1$. After some calculations one obtains for the spectral distribution of the radiated energy per unit string length in $D = 4$:

$$\frac{dP}{d\omega d\Omega_2 dl} \simeq 128\pi^2 \frac{\lambda^6}{\mu^2} \frac{\exp(-\frac{2\omega d\xi}{v})}{\omega} \delta(\cos\theta - \frac{\sin\alpha}{v}) f_4(\phi,\theta,\alpha), \tag{13}$$

where $\xi = \cos\alpha + v\sin\theta\sin\phi$ and $f_4(\phi,\theta,\alpha) = \frac{1}{(\gamma^2\cos\alpha^2-1)} \frac{1}{\gamma^4\xi^4}((\gamma^2\xi + \sin\phi^2)^2 + \cos\phi^2)$. This expression contains an infrared divergence, so to integrate over ω one has to introduce a cutoff parameter Δ. This leads to the logarithmic dependence of the total radiated energy $\ln|\gamma\Delta/2d|$, where $d = d_1 - d_2$ is the impact parameter.

It is easy to recognize the Cherenkov nature of radiation which arises once the point of minimal separation moves with the faster-than-light velocity ($v/\sin\alpha > 1$) and is concentrated on the cone with respect to the direction of motion of the superluminal point ($\cos\theta = \frac{\sin\alpha}{v}$).

The structure of the angular dependence through the function $f_4(\phi,\theta,\alpha)$ indicates that the main part of the radiation is emitted within the narrow interval of the angle ϕ around $\phi \simeq \frac{3}{2}\pi$.

In higher dimensions $D > 4$ there is no infrared divergence in the spectral distribution

$$\frac{dP}{d\omega d\Omega_2 d\psi dl} \simeq \frac{\lambda^6}{\mu^2} F_n \, \delta(\cos\theta\cos\psi - \frac{\sin\alpha}{v}) f_D(\phi,\theta,\psi), \tag{14}$$

where

$$F_n = \frac{1}{d^{2n+2}} \frac{\pi^{\frac{n}{2}}}{(2\pi)^{3n}} \frac{\Gamma^2(\frac{n+1}{2})}{\Gamma(\frac{n}{2})} \left\{ \begin{array}{rl} \frac{\pi}{2}\left(\frac{d\zeta}{v}\right)^{n+1} \omega^{2n+1} \exp\left(-\frac{2\omega d\zeta}{v}\right), & \omega \to \infty, \\ \Gamma^2(\frac{n+1}{2})(2\omega)^n, & \omega \to 0, \end{array} \right\}$$

$\zeta = \cos\alpha + v\sin\theta\sin\phi\cos\psi$ and $f_D = \frac{1}{(\gamma^2\cos\alpha^2-1)}\frac{\cos\alpha^2}{\zeta^4 v^2}(2\cos\alpha\zeta - \cos\phi^2\cos\alpha^2 + v^2\cos\psi^2 - 1)$.

In $D \geq 5$ dimensions the frequency region for the radiation energy is $0 < \omega < \gamma v/d$, in the lower limit ($\omega \to 0$) the spectral distribution tending to a constant at low frequencies in 5-dimensions and to zero in higher $D > 5$ dimensions. Note that the argument of the delta-function, which indicates the shape of the Cherenkov cone, now depends on the angle ψ showing the propagation of radiation into the "extra" space: $\cos\theta\cos\psi = \sin\alpha/v)$. Again the main part of radiation is concentrated around the angle $\phi \simeq \frac{3}{2}\pi$. More detailed description of radiation will be given elsewhere.

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Some facts and fictions in subnuclear realm

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Abstract — Some well-known but poorly understood phenomenological facts in subnuclear realm, which may be of paramount importance for the future theory, together with some widespread mythic concepts are briefly reviewed.

1 Facts and their interpretation

• *Lack of isolated quarks.* For whatever reason colored particles can not be isolated and directly observed. This fact culminates in the idea of confinement, including the color confinement (kinematic aspect) and the permanent confinement of quarks in hadrons due to a linearly rising potential of stringy color forces between quarks (dynamical aspect), for a review see, e. g., Prokhorov (1994). The nuclear forces, that keep colorless hadrons in nuclei, are often viewed as a residue of these direct inter-quark interactions, similar to the van der Waals forces collecting electro-neutral H_2O molecules of water. However, assuming all the gluon lines of force to be squeezed in a string-like bunch, the binding of hadrons in nuclei are prevented from being patterned after the van der Waals multipole scheme: the residue of the color field outside such string-like hadrons is zero.

• *Fewness of species of quark clusters.* Only baryons and mesons (QCD-atoms) and about 300 types of different nuclei (QCD-molecules) completing the periodic table of elements are experimentally observed, as opposed to the great diversity of usual atoms, molecules and their aggregates making up condensed substances. Relatively stable are only hadrons, that is, clusters composed of two or three quarks, while multi-quark clusters are evanescing, even though they are crucial for the cumulative effect in nuclear collisions (see Baldin 1977), which was originally pointed out by D. I. Blokhintsev. Furthermore, the predicted glueballs still remain to be experimentally discovered.

• *The Okubo-Zweig-Iizuka rule*: Strong interactions show a mechanism of the suppression of creations and annihilations of quarks and anti-quarks belonging to the same meson, see, e. g., Ogawa *et al.* (1980). To illustrate, experimental data on the ϕ-meson decay evidence that the probability of the $\phi \to K^+ + K^-$ mode is far beyond that of $\phi \to \rho + \pi$, see Figure ??.

Thus, the OZI rule makes it clear that quarks should not be taken as ordinary quantum-mechanical objects since a quark and an anti-quark with opposite quantum numbers, constituting some meson, defy their mutual annihilation (even at a sacrifice of the identity principle, see Figure ??). Such persistence of quarks is characteristic for classical objects. Recall, classical particles are immune to creations and annihilations.

• *The quark sea suppression.* A medium of the normal nuclear density in a tiny volume occupied by a hadron (about 2 fm in diameter) would offer a fertile ground for creations and annihilations of quark-anti-quark pairs. Nevertheless, the quark-anti-quark sea is largely suppressed in hadrons. Undisturbed hadrons are systems with a fixed number of constituents, namely two or three valence quarks. This suppression of virtual pairs appears quite enigmatic from the uncertainty principle standpoint; yet it is a further evidence in favor of classicality of the valence quarks.

• *Witten's hadron phenomenology.* The bulk of the low energy hadron phenomenology is grasped by the Harari–Rosner planar diagrams which are tree diagrams containing no intersections of the quark lines. The classicality of constituent quarks is readily seen in such diagrams where most of quark world lines are timelike smooth curves. Planar diagrams describing collisions and decays of hadrons involve the Λ- and V-shaped world lines of valence quarks related to annihilations and creations of quark-anti-quark pairs, but the contribution of these diagrams is suppressed as the number of such lines increases. If we substitute the QCD gauge group SU(3) by the N-color group SU(N) and go to the limit $N \to \infty$, the planar diagrams become dominating among all relevant Feynman diagrams. The real hadronic world is qualitatively displayed even in the zeroth approximation of the $1/N$ expansion, that is at $1/N = 0$, (for a review see Yaffe 1982 and Das 1987).

• *Quantum fluctuations of gauge invariant quantities disappear as $N \to \infty$,* see Witten (1979). Thus, QCD mutates to a classical theory in the large-N limit. Were we actually capable to reach this classical limit of QCD, and then quantize the resulted classical theory in the ordinary way, where should the color-SU(3) Yang-Mills theory come from? Is the desired SU(3) gauge group a relic of the $N = \infty$ limiting classical theory?

• *The puzzle of nucleon spin.* Results of recent experiments show that only about 30% of the proton's spin is produced by quark's spin. The missing 70% of the spin may come from gluon spins and the orbital angular momentum due to the motion of quarks and gluons within the nucleon. In any case, a spinless classical particle is a good first approximation to the description of the real valence quark. However, in hadron collisions with momentum transfer greater than the characteristic QCD scale $Q \approx 200$ MeV, quarks appear to regain their quantum status.

• *Two phases of subnuclear matter.* At temperature $T \approx 200$ MeV and/or density of some units of the proton energy density (0.5 GeV/fm^3) nuclear matter melts converting to a quark-gluon plasma. The lattice calculations show that the order of the phase transition is sensitive to the involved approximation. The real deconfinement may well be the second-order phase transition, see, e. g., Meyer-Ortmanns, (1996). Should this be the case, the question arises of whether we are aware of the hadronic phase symmetry? We reasonably assume that the symmetry of the plasma phase is the color-SU(3). Recall that the gauge invariance cannot be spontaneously broken. Meanwhile some symmetry is certain to change in the second-order transition, and there is no suitable symmetry except for the gauge symmetry to be subject to changing (the chiral symmetry-restoring transition seems to be distinct from the deconfinement, and the conformal symmetry is immaterial).

• *The Regge spectrum of hadrons.* $J = \alpha_0 + \alpha' M^2$. Hadrons belonging to some Regge trajectory are separated by intervals $\Delta J = 2$. Ne'eman and Šijački related this Regge arrangement to the SL$(4, R)$ classification of hadrons. All the Regge sequences are thus infinite multiplets of the noncompact group SL$(4, R)$, see Dothan *et al.* (1965), and Ne'eman and Šijački (1988). Thus SL$(4, R)$ appears to be an appropriate gauge group of the hadronic phase. Where may this SL$(4, R)$ come from?

2 Model

2.1 Basic principles and specific assumptions

The classical limit of QCD is related to the limit of large number of colors. Substitute the color group SU(3) by SU(N) and go to infinite N. Then planar diagrams dominate, and vacuum fluctuations of gauge invariant operators disappear. I guess: the classical limit of QCD is related to the SU(N) Yang–Mills–Wong theory with large N.

Consider classical spinless point particles interacting with classical SU(N) gauge field. Those are called quarks and labelled by index I. Quarks with color charges Q_I^a are moving along timelike world lines $z_I^\mu(\tau_I)$. This gives rise to the current

$$j_\mu(x) = \sum_{I=1}^{K} \int d\tau_I \, Q_I(\tau_I) \, v_\mu^I(\tau_I) \, \delta^4\Big(x - z_I(\tau_I)\Big),$$

where $Q_I = Q_I^a T_a$, T_a are generators of SU(N), $v_\mu^I \equiv dz_\mu^I/d\tau_I$ is the four-velocity of Ith quark. The action (see Balachandran *et al.* 1978) is

$$S = -\sum_{I=1}^{K} \int d\tau_I \,(m_0^I \sqrt{v_\mu^I v_I^\mu} + \mathrm{tr}\, Z_I \lambda_I^{-1} \dot{\lambda}_I) - \int d^4x \,\mathrm{tr}\left(j_\mu A^\mu + \frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu}\right),$$

where $\lambda_I = \lambda_I(\tau_I)$ are time-dependent elements of SU(N), $Z_I = e_I^a T_a$, e_I^a being some constants whereby the color charge is specified, $Q_I = \lambda_I Z_I \lambda_I^{-1}$.

The Euler–Lagrange equations are the Yang–Mills equations

$$D^\mu F_{\mu\nu} = 4\pi j_\nu,$$

the equations of motion of bare quarks

$$m_0^I a_I^\lambda = v_\mu^I \,\mathrm{tr}\left(Q_I F^{\lambda\mu}(z_I)\right),$$

$a_I^\lambda \equiv \dot{v}_I^\lambda$ is the four-acceleration, and the Wong equations

$$\dot{Q}_I = -ig\,[Q_I,\, v_\mu^I A^\mu(z_I)]$$

describing the evolution of the quark color charges, see Wong (1970).

2.2 Solutions to the classical Yang–Mills equations

The invariance of a given quantum field system is the invariance of its classical background fields (Coleman 1966). There are two classes of exact retarded solutions to the Yang–Mills equations with point-like sources, describing background fields of two phases of the gluon vacuum (see Kosyakov 1998). Solutions of the first class are complex-valued with respect to the Cartan basis of su(N), but it is possible to convert them to the real form, then those become invariant under SL(N, R) or its subgroups.

The Cartan–Weyl basis of su(N) consists of N elements H_n of Cartan subalgebra obeying the relation $\sum_{n=1}^{N} H_n = 0$, and $N^2 - N$ raising and lowering elements E_{mn}^{+} and E_{mn}^{-}, $n > m$. Nontrivial commutators are:

$$[H_m,\, E_{mn}^{\pm}] = \pm\, E_{mn}^{\pm}, \quad [E_{mn}^{+},\, E_{mn}^{-}] = H_m - H_n, \quad [E_{kl}^{\pm},\, E_{lm}^{\pm}] = \pm\, E_{km}^{\pm}.$$

An example of such solutions:

$$A_\mu = \mp\frac{2i}{g}\sum_{I=1}^{K}\left(H_I\,\frac{v_\mu^I}{\rho_I} + g\,\kappa\, E_{I\,K+1}^{\pm}\, R_\mu^I \prod_{I=1}^{K-1} \delta(R^K \cdot R^I)\right).$$

It is interpreted as the field generated by a K-quark cluster in the cold phase. This solution is invariant under the gauge group SL(K+1, R). The background generated by a three-quark cluster is invariant under SL(4, R), and that due to a two-quark cluster is invariant under SL(3, R). This symmetry is independent of N and is retained in the limit $N \to \infty$. Thus the Yang–Mills backgrounds of all hadrons are ivariant under SL(4, R).

Solutions of the second class are real-valued and invariant under SU(N). For $\kappa = 0$, the Yang–Mills equations linearize. Coulomb-like solutions describing the background field in the hot phase are

$$A_\mu = \sum_{I=1}^{K} \sum_{n=1}^{N} e_I^n \, H_n \, \frac{v_\mu^I}{\rho_I}.$$

For dimension different from 4, there is no exact retarded solutions other than Coulomb-like (Kosyakov, 1999). Only for 4D, both differentiation and multiplication by A_μ raise the power of a singularity by 1, hence both terms of covariant derivative $\partial_\mu - igA_\mu$ act coherently.

3 Discussion

Any Yang-Mills background generated by a cold quark occupies individually some color sl(2, R) cell. Backgrounds generated by different quarks cannot be contained in the same sl(2, R). This is similar to the Pauli blocking principle. Just as a cell of volume h^3 in the phase space might be occupied by at most one fermion with a definite spin projection, so any color sl(2, R) cell is intended for the background of only one quark. This color blocking guarantees the totality of color cells against shrinkage. We deal actually with the large-N situation. We thus arrived at a plausible classical 4D picture of the cold phase.

By contrast, the most energetically advantageous field configuration in the hot phase is such that the color charges of quarks are lined up into a fixed direction, thereby reducing SU(N) to SU(2). This bears resemblance to the Bose–Einstein condensation in the color space. We are dealing with the effectively low-N situation. Thus the hot phase is described by a 4D quantum picture, the usual QCD picture.

In the cold phase, a dressed quark possesses the 4-momentum

$$p^\mu = m\,(v^\mu + \tau_0\, a^\mu)$$

where

$$\tau_0 = \frac{2}{3m}\,|\,\mathrm{tr}\, Q^2\,| = \frac{8}{3mg^2}\left(1 - \frac{1}{N}\right).$$

It follows that

$$p^2 = m^2\,(1 + \tau_0^2\, a^2).$$

The dressed quark should turn to a tachyonic state with $p^2 < 0$ if its acceleration exceeds the critical value $|a| = 1/\tau_0$. Such accelerations are within the scope of validity of classical description. Indeed, let two light quarks be separated by distance ρ. The critical acceleration due to the Coulomb-like color force is achieved at a distance $\rho \approx |\mathrm{tr}\, Q^2|/m$ which is about $2g^{-2}$ times greater than than the Compton wave length of the quarks. Rather than turn to the tachyonic state, the cold quark plunges into the hot phase for a period of order of the inverse of its mass $1/m$. The cold phase melts whenever the acceleration of any cold quark exceeds $1/\tau_0$, i. e., the energy transferred to it is about its constituent mass m, and subsequently the hot phase freezes up again.

4 Conclusions

Two central ideas advocated here are:

(1) The hadronic phase can be accounted for by a four-dimensional classical picture, while the plasma phase is described by a four-dimensional quantum picture, the usual QCD picture. Deconfinement occurs whenever the acceleration of a valence quark exceeds $1/\tau_0$, that is, the critical momentum transfer Q is about its constituent mass m; for u and d constituent quarks, we have $Q \sim 200$ MeV.

The classical character of the subnuclear realm in the hadronic phase must be verified experimentally in a direct way, e. g., through the use of the Bell inequalities.

(2) The color is not confined in the hadronic phase. The color gauge symmetry of the cold phase is SL$(4, R)$, the Ne'eman and Šijački symmetry group. It manifests itself in infinite multiplets that constitute Regge sequences.

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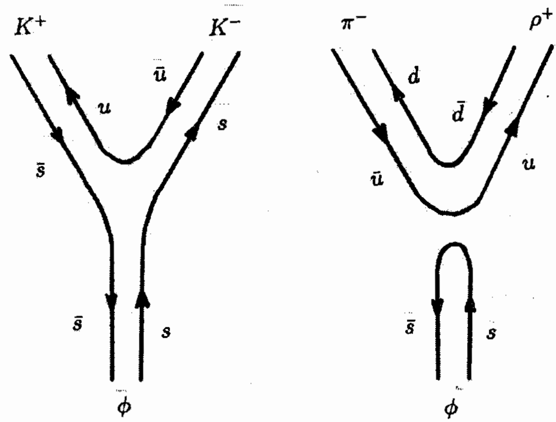

Figure 1: Allowed (left) and forbidden (right) channels of the ϕ-meson decay

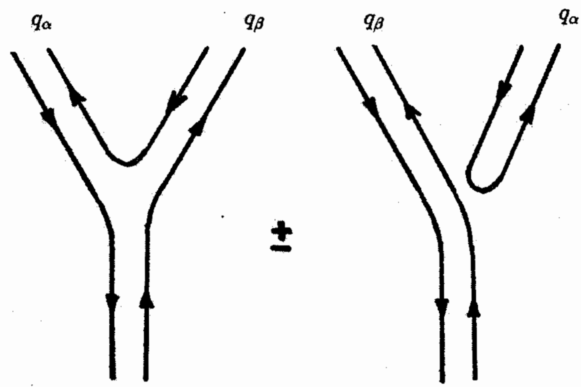

Figure 2: While QM requires a superposition of these two amplitudes (the diagram on the right is derived form that on the left by interchanging identical quarks q_β), the OZI rule forbids the diagram on the right

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On the faster-than-light motions in electrodynamics

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Abstract — A variant of electrodynamics is constructed in which the faster-than-light motions are possible.

The existence of faster-than-light motions (with velocities $v > 3 \cdot 10^{10}$ cm/sec) is the subject of discussion in modern physics.

It is in 1946-1948 when Blokhintsev already [1] paid attention to the possibility of formulating a field theory that permits the propagation of faster-than-light (superluminal) interactions outside the light cone. Kirzhnits (1954) [2] showed that a particle possessing the tensor of mass $M^i{}_k = diag(m_0, m_1, m_1, m_1)$ may move with the faster-than-light velocity if $m_0 > m_1$. Terletsky (1960) [3] introduced into theoretical physics the particles with imaginary masses moving faster-than-light. Feinberg (1967) [4] named these particles tachyons and described their main properties. Research on the superluminal tachyon motions opened up additional opportunities which were studied by many authors (hundreds publications), for example by Bilaniuk and Sudarshan (Sb. [4]), Kirzhnits and Sazonov (Sb. [4]), Recami [5], Mignani (Mon. [5]), Corben (Mon. [5]).

The publications are also known in which the violation of invariance of the speed of light is considered [6] - [12]. One can note, for example, Pauli monograph [6] with the elements of Ritz and Abraham theories; Logunov lections on Relativity Theory [7]; Glashow paper [8] on the violation of Lorentz-invariance in astrophysics; publications [9] - [12] considering the violation of invariance of the speed of light in SR.

Below a version of the theory permitting faster-than-light motions of electromagnetic fields and charged particles with real masses is proposed as the continuation of such investigations.

Let us introduce space-time R^4 with the metric

$$\begin{gathered} ds^2 = ({c_0}^2 + v^2)dt^2 - dx^2 - dy^2 - dz^2 = \\ ({c_0'}^2 + v'^2)dt'^2 - dx'^2 - dy'^2 - dz'^2 - invariant, \end{gathered} \tag{1}$$

where t is the time, x, y, z are the spatial variables, v is the velocity of a particle being investigated, c_0 is the proper value of the speed of light. Due to homogeneity and isotropy of space, the velocity v does not depend on space-time variables. Let the proper value of the speed of light be invariant:

$$c_0 = c'_0 = 3 \cdot 10^{10} cm/sec. \tag{2}$$

As a result, the common time may be introduced on the trajectory of movement of frame K' and the velocity of light c, corresponding to the velocity v: $dt = dt'_0 \to c = c_0\sqrt{1 + v^2/c_0{}^2}$ [1]. Let us introduce also the new variables

$$x^0 = \int_0^t c d\tau = c_0 \int_0^t \sqrt{1 + v^2/c_0{}^2} d\tau, \quad x^\alpha = (x, y, z), \ \alpha = 1, 2, 3, \tag{3}$$

and turn to Minkowski space M^4 with the metric

$$ds^2 = (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2 - invariant. \tag{4}$$

The infinitesimal transformations, retaining the invariance of the form ds^2 are $dx'^i = L^i{}_k dx^k$, $i, k = 0, 1, 2, 3$, where $L^i{}_k$ is the matrix of Lorentz group [14]. The corresponding integral transformations are

$$x'^0 = \frac{x^0 - \beta x^1}{\sqrt{1-\beta^2}}; \ x'^1 = \frac{x^1 - \beta x^0}{\sqrt{1-\beta^2}}; \ x'^{2,3} = x^{2,3}; \ c' = c\frac{1 - \beta u^1}{\sqrt{1-\beta^2}}. \tag{5}$$

They belong to the group of direct product $L_6 X S_1$, where L_6 is the Lorentz group, S_1 is the scale transformations group $c' = \gamma c$. One can say that these transformations act in the 5-space $V^5 = M^4 X V^1$, where V^1 is a subspace of the velocities of light c. The relationship between the partial derivatives of variables (t, x, y, z) and (x^0, x^1, x^2, x^3) are as follows:

$$\begin{gathered} \frac{\partial}{\partial t} = \frac{\partial x^0}{\partial t}\frac{\partial}{\partial x^0} + \sum_\alpha \frac{\partial x^\alpha}{\partial t}\frac{\partial}{\partial x^\alpha} = c\frac{\partial}{\partial x^0}; \\ \frac{\partial}{\partial x} = \frac{\partial x^0}{\partial x}\frac{\partial}{\partial x^0} + \sum_\alpha \frac{\partial x^\alpha}{\partial x}\frac{\partial}{\partial x^\alpha} = \Big(\int_0^t \frac{\partial c}{\partial x} d\tau\Big)\frac{\partial}{\partial x^0} + \frac{\partial}{\partial x^1}. \end{gathered} \tag{6}$$

The expressions for $\partial/\partial y$ and $\partial/\partial z$ are analogous to the expression $\partial/\partial x$. Below we restrict our study by the case, when the velocity of light in the range of interaction does not depend on the space variables. Then

$$\nabla c(x^0) = 0 \to \nabla c(t) = 0. \tag{7}$$

[1] In the form of $c' = c(1 - \beta^2)^{1/2}$ this formula was obtained by Abraham before [6].

Some features of this study are:
1. As in SR, parameter $\beta = V/c$ in the present work is in the range $0 \leq \beta < 1$.
2. As in SR, the value dx^0 is the exact differential.
3. The time $x^0 = \int cdt$ is the functional of $c(t)$ in general case.
4. The property β=const is compatible with $V(t)$, $c(t)$.

Keeping this in mind, let us construct a theory in M^4 and reflect it on the space R^4. Following [13], we start from the expression for integral of action:

$$S = S_m + S_{mf} + S_f = -mc_0 \int ds - \frac{e}{c_0} \int A_i dx^i - \frac{1}{16\pi c_0} \int F_{ik} F^{ik} d^4x. \quad (8)$$

Here S_m is the action for a free particle; S_f is the action for a free electromagnetic field; S_{mf} is the action for interaction between a charge e and electromagnetic field; m is the mass of a particle, $j^i = (\rho, \rho \mathbf{u}^\alpha)$ [6] is the 4-vector of current density; $\mathbf{u}^\alpha = \mathbf{v}^\alpha / c$ is the 3-velocity of a particle. The meaning of the other values is standard. Accordingly to the construction, the action is Lorentz invariant and does not depend on the velocity of light c. As a result the action is also invariant with respect to the group of direct product $L_6 X S_1$. Lagrangian takes the form:

$$L = -mc_0\sqrt{1-u^2} + \frac{e}{c_0}(\mathbf{A} \cdot \mathbf{u} - \phi). \quad (9)$$

The generalized momentum $\mathbf{P}$ and generalized energy $\mathcal{H}$ are:

$$\mathbf{P} = \frac{\partial L}{\partial \mathbf{u}} = \frac{mc_0 \mathbf{u}}{\sqrt{1-u^2}} + \frac{e}{c_0}\mathbf{A} = \mathbf{p} + \frac{e}{c_0}\mathbf{A} = m\mathbf{v} + \frac{e}{c_0}\mathbf{A},$$
$$\mathcal{H} = \mathbf{P} \cdot \mathbf{u} - L = (mc_0 c + e\phi)/c_0. \quad (10)$$

Here $\mathbf{p} = m\mathbf{v}$ is the momentum, $mc_0 c = m{c_0}^2(1 + v^2/{c_0}^2)^{1/2} = \mathcal{E}$ is the energy, $T = m{c_0}^2(c/c_0 - 1)$ is the kinetic energy of a particle. For a free particle they are integrals of motion, $\mathcal{E}$ and $\mathbf{p}$ may be united into the 4-momentum p^i (as in SR)

$$p^i = mc_0 u^i = \left(\frac{mc_0 c}{c_0}, mcu^\alpha\right) = \left(\frac{\mathcal{E}}{c_0}, m\mathbf{v}\right). \quad (11)$$

The components of the 4-momentum are related by the expressions:

$$p_i p^i = \frac{\mathcal{E}^2}{{c_0}^2} - \mathbf{p}^2 = m^2 {c_0}^2; \quad \mathbf{p} = \frac{\mathcal{E}}{c_0 c}\mathbf{v}; \ \mathbf{p} = \frac{\mathcal{E}}{c_0 c}\mathbf{c}, \ if \ m = 0, \ \mathbf{v} = \mathbf{c}. \quad (12)$$

One can see from here that the momentum of a particle with the zero mass $m = 0$ does not depend from the particle velocity $v = c$ and is only determined by the particle energy in accordance with $\mathbf{p} = \mathbf{n}\mathcal{E}/c_0$, $\mathbf{n} = \mathbf{c}/c$.

Next we start from the mechanical [13] and field [13,14] Lagrange equations:

$$\frac{d}{dx^0}\frac{\partial L}{\partial \mathbf{u}} - \frac{\partial L}{\partial \mathbf{x}} = 0; \ \frac{\partial}{\partial x^k}\frac{\partial \mathcal{L}}{\partial(\partial A_i/\partial x^k)} - \frac{\partial \mathcal{L}}{\partial A_i} = 0. \tag{13}$$

Here L is the Lagrangian; $\mathcal{L} = -(1/c_0)A_i j^i - (1/16\pi c_0)F_{ik}F^{ik}$ is the density of the Lagrange function; $\partial(F_{ik}F^{ik})/\partial(\partial A_i/\partial x^k) = -4F^{ik}$ [13]. As a result, we find the equations of motion for a charged particle and electromagnetic field:

$$\begin{aligned} \frac{d\mathbf{p}}{dt} &= m_0\frac{d\mathbf{v}}{dt} = \frac{c}{c_0}e\mathbf{E} + \frac{e}{c_0}\mathbf{v}\mathrm{x}\mathbf{H}; \\ \frac{d\mathcal{E}}{dt} &= e\mathbf{E}\cdot\mathbf{v} \rightarrow m_0\frac{dc}{dt} = \frac{e}{c_0}\mathbf{v}\cdot\mathbf{E}. \end{aligned} \tag{14}$$

$$\begin{aligned} \nabla \mathrm{X}\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} &= 0; \qquad \nabla\cdot\mathbf{E} = 4\pi\rho; \\ \nabla \mathrm{X}\mathbf{H} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} &= 4\pi\rho\frac{\mathbf{v}}{c}; \quad \nabla\cdot\mathbf{H} = 0, \end{aligned} \tag{15}$$

where $c(t) = c_0(1 + v^2/c_0{}^2)^{1/2} = c(0)[1 + (e/mc_0c(0))\int_0^t \mathbf{v}\cdot\mathbf{E}d\tau]$, $\nabla c = 0$. These equations, considered as the whole, form the set of nonlinear equations of electrodynamics. They permit the existence of faster-than-light motion of a particle with the real mass m, rest energy $\mathcal{E}_0 = mc_0{}^2$ and velocity

$$v = \sqrt{\mathcal{E}^2 - m^2c_0{}^4}/mc_0 > c_0, \tag{16}$$

if the particle energy is $\mathcal{E} > \sqrt{2}\mathcal{E}_0$ [12]. For example, for electron $\sqrt{2}\mathcal{E}_0$ is 723 keV, the velocity of 1 GeV electron is $\sim 2000\ c_0$. The $\mathcal{E}$ and $\mathbf{p}$ variations with time determine a new dynamics which goes to Newton one if $v^2/c_0{}^2 \ll 1$.

Explained in the framework of the constructed theory may be the Michelson experiment, Fizeau experiment, aberration of light, dilatation of life time of atmospheric μ - mesons, Doppler effect, known tests to check independence of the speed of light from the velocity of the light source, Compton effect, decay of unstable particles, creation of new particles in nuclear reactions, possible faster-than-light motions of nuclear reaction products. For example, in the case of Compton effect [15] we find from the law of energy-momentum conservation: $\hbar\omega = \hbar\omega' + mc_0{}^2\left[\left(1+v^2/c_0{}^2\right)^{1/2} - 1\right]$; $(\hbar\omega/c_0) = (\hbar\omega'/c_0)cos\theta + mvcos\alpha$, $0 = (\hbar\omega'/c_0)sin\theta - mvsin\alpha$. Here $\hbar\omega$, $\hbar\omega'$ are the energies of incident and scattered γ quanta, $mc_0{}^2$ is the rest energy of electron, α and θ are the angles of scattering the electron and γ quantum respectively. As a result the angular distribution of scattered γ quanta is $\omega' = \omega/[1 + \hbar\omega(1 - cos\theta)/mc_0{}^2]$. (As in SR [15]). But the velocity of forward-scattered electron may exceed the speed of light c_0: $v_e(\alpha = 0) = \hbar\omega/mc_0 - 0.5c_0 > c_0\ if\ \hbar\omega > 1.5mc_0{}^2$, which is in accordance with the formula (16) and differs from SR. The velocity of scattered γ quantum does not depend on the angle θ and is determined by mechanism of scattering (immediately, or in the act of absorption-emission by scattered electron).

As a result the theory has been constructed which is invariant with respect to the group of direct product L_6XS_1. In accordance with [1] we may assume that the proposed theory may be useful in the field of quantum physics of extended particles, where the property of elementary nature should not contradict to the existence of the internal structure of a particle.

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Non-commutative Differential Calculus and q-oscillators

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Abstract — The factorization method for the noncommutative differential calculus over the commutative algebra of functions is applied to construct the generalized ladder operators. It is shown that on this way all known types of the q-oscillators can be obtained in a systematic way. As the algebra of functions is defined on the spectrum of the unitary irreducible representations of the Lorentz group, these q-oscillators are the direct relativistic analogs of standard harmonic oscillator problem in quantum mechanics

For more than one decade the kit of ideas of algebraic geometry and topology under the general name "non-commutative" or "quantum" differential geometry have been considered as a possible mathematical arena for comprehensive Quantum Field Theory incorporating gravity mostly in the framework of String theory [1]. As it often happens in the history of science, these ideas has already been expressed and cultivated in physics for many years may be in somewhat different context[1].

Non-commutative differential calculus is introduced [9]- [11] as a deformation of the theory of differential forms. Here, we outline the notion of differential calculus on a commutative algebra A [11] generated by the coordinate functions x. The differential calculus on A is a $\mathbb{Z}$-graded associative algebra over $\mathbb{C}$:

$$\Omega(A) = \bigoplus_{r\in\mathbb{Z}} \Omega^r(A), \quad \Omega^0(A) = A, \quad \Omega^r(A) = \{0\} \quad \forall r < 0 \tag{1}$$

The elements of $\Omega^r(A)$ are r-forms. There exists an exterior derivative ($\mathbb{C}$ – linear) operator $d : \Omega^r(A) \to \Omega^{r+1}(A)$ which satisfies the following conditions:

$$d^2 = 0, \quad d(\omega\omega') = (d\omega)\,\omega' + (-1)^r \omega d\omega' \tag{2}$$

where ω and ω' are r – and r' – forms, respectively. In the standard differential calculus over usual manifolds, differentials commute with functions:

[1] See the references to old works of H.Snyder, W.Pauli, C.N.Yang, I.E.Tamm, Yu.A.Golfand, V.G.Kadyshevsky, and others in [2]- [7] and also [12].

$[x^i,\, dx^j] = 0$. For us it is essential that this relation can be generalised (deformed) in different ways with (1, 2) still being true. Let us consider in more detail the deformation of the form (in what follows we consider the 1-dimensional case).

$$[x,\, dx] \;=\; a\, dx \tag{3}$$

where a is a (complex) constant which is constrained by the requirement of a consistency of the differential calculus. Operator d is defined as

$$d\omega \;=\; [\hat{F},\, \omega]_r \;=\; \hat{F}\omega \;-\; (-1)^r\, \omega\hat{F} \tag{4}$$

where $\hat{F}$ is the operator valued 1-form. It is easily seen that conditions (2) are satisfied We introduce the limitations.

Limitation I. Algebra A is a commutative algebra generated by the coordinate functions $x \in \mathbb{C}$:

$$\omega^0 \;=\; f(x) \qquad df \;=\; \left[\hat{F},\, f\right] \qquad dx \;=\; \left[\hat{F},\, x\right] \tag{5}$$

Limitation II.

$$\left[\hat{F},\, x\right] \;=\; -a\hat{F} \tag{6}$$

We arrive at the differential calculus in which (3) holds:

$$[x,\, dx] \;=\; \left[x,\, \left[\hat{F},\, x\right]\right] \;=\; -a\left[x,\, \hat{F}\right] \;=\; a\, dx \tag{7}$$

Noncommutative Quantum Mechanics. Let us introduce the translation operators $\overrightarrow{T}$ and $\overleftarrow{T}$.

$$\overrightarrow{T} = a\overrightarrow{\ast}d + 1,\; \overleftarrow{T} = a\overleftarrow{\ast}d + 1, \quad \left[\overrightarrow{T}, \overleftarrow{T}\right] = 0, \quad \left(\overrightarrow{T}\right)^{-1} = \overleftarrow{T} \tag{8}$$

In terms of the "left and right momentum operators" [2]

$$p_{\rightarrow} \;=\; 1 \;-\; \overrightarrow{T} \;=\; -\,a\overrightarrow{\ast}\,d \qquad\qquad p_{\leftarrow} \;=\; \overleftarrow{T} \;-\; 1 \;=\; -\,a\overleftarrow{\ast}\,d \tag{9}$$

the physical momentum operator and free hamiltonian have the form

$$p \;=\; p_{\rightarrow} + p_{\leftarrow}, \quad H_0 \;=\; \frac{p^2}{2} \;=\; \frac{a^2}{2}\,(\overrightarrow{\ast}\,d \;+\; \overleftarrow{\ast}\,d)^2 \;=\; \frac{a^2}{2}\left(\overrightarrow{\delta} + \overleftarrow{\delta}\right) d \tag{10}$$

Note that operators p and H_0 act on the function i.e. the zero order form ω_0: $\overrightarrow{\delta}\,\omega_0 \;=\; 0$, $\overleftarrow{\delta}\,\omega_0 \;=\; 0$ and the relation $\overrightarrow{\ast}\,\overleftarrow{\ast} \;=\; 0$ holds. $\overrightarrow{\delta}$ and $\overleftarrow{\delta}$ are right and left [2] Hodge operators. For clarity we recall the corresponding nonrelativistic expression for the free hamiltonian operator in terms of (commutative) differential calculus: $H_0 = -\frac{\hbar^2 \Delta}{2} = -\frac{\hbar^2}{2}\,\delta\, d$. The free hamiltonian (10) can be factorized in different ways, giving us the different q-oscillators [7, ?] α and β are arbitrary numbers

$$H_0 = \frac{a^2}{2}\left(\overrightarrow{T} - \overleftarrow{T}\right)\left(\overrightarrow{T} - \overleftarrow{T}\right) = \frac{a^2}{2}\overrightarrow{T}^{\alpha}\left(\overrightarrow{T}^{1-\alpha} - \overrightarrow{T}^{-\alpha}\overleftarrow{T}\right)\cdot$$
$$\overleftarrow{T}^{\beta}\left(\overrightarrow{T}\overleftarrow{T}^{-\beta} - \overleftarrow{T}^{1-\beta}\right) = \frac{a^2}{2}\overrightarrow{T}^{\alpha-\beta}\left(\overrightarrow{T}^{1-\alpha} - \overleftarrow{T}^{1+\alpha}\right)\left(\overrightarrow{T}^{1+\beta} - \overleftarrow{T}^{1-\beta}\right) \tag{11}$$

Remember that the nonrelativistic hamiltonian can be factorized only as

$$H_0 = \frac{p^2}{2} = -\frac{\hbar^2}{2}\,\delta\, d = -\frac{\hbar^2}{2}(*\,d)(*\,d) = -\frac{\hbar^2}{2}\left(\frac{d}{dx}\right)\left(\frac{d}{dx}\right) \tag{12}$$

if we avoid the fractional derivatives. We can single out the simplest cases: 1) $\alpha = \beta = 0$, 2) $\alpha = -\beta = 1$, 3) $-\alpha = \beta = 1$, 4) $\alpha = \beta = 1$
Now let us apply the factorization method in the noncommutative case. Let us concentrate on the case 2). We introduce the ladder operators

$$\eta^{+} = -\frac{1}{\sqrt{2}}e^{\rho_1}\left(1 - \overleftarrow{T}^{2}\right)e^{-\rho_2},\ \eta^{-} = \frac{1}{\sqrt{2}}e^{-\rho_3}\left(1 - \overleftarrow{T}^{2}\right)e^{-\rho_4},\ \left(\eta^{\pm}\right)^{\dagger} = \eta^{\mp} \tag{13}$$

It can be shown that the modification of the Leibnitz rule [2] for the noncommutative interior derivatives leads in a natural way to the notion of $q(x)$-mutators

$$\left[\eta^{-}, \eta^{+}\right]_{q(x)} = \eta^{-}\, q(x)\eta^{+} - \eta^{+}\, q(x)\eta^{-} \qquad q(x) = e^{\alpha(x)} \tag{14}$$

function $\alpha(x)$ is unknown quantity (deformation parameter) which must be evaluated. Inseting here (13) we come to the complicated expression

$$\left[\eta^{-}, \eta^{+}\right]_{q(x)} = \frac{1}{2}\left\{\begin{array}{l} e^{\rho_1+\rho_4-\rho_2-\rho_3}\left(e^{-\alpha} - e^{\alpha}\right) - \\ -\left(e^{-\rho_3}\overleftarrow{T}^{2}e^{\rho_1+\alpha} - e^{\rho_1}\overleftarrow{T}^{2}e^{-\rho_3-\alpha}\right)e^{\rho_4-\rho_2} \\ -e^{\rho_1-\rho_3}\left(e^{-\rho_4}\overleftarrow{T}^{2}e^{-\rho_2+\alpha} - e^{-\rho_2}\overleftarrow{T}^{2}e^{\rho_4-\alpha}\right) + \\ + - e^{-\rho_3}\overleftarrow{T}^{2}e^{\rho_1+\rho_4+\alpha}\overleftarrow{T}^{2}e^{-\rho_2} - \\ -e^{\rho_1}\overleftarrow{T}^{2}e^{-\rho_3-\rho_4-\alpha}\overleftarrow{T}^{2}e^{\rho_4} \end{array}\right\} \tag{15}$$

The noncommutative analog of the oscillator corresponds to the case when the right hand side of (15) is a *const*. This means that terms containing equal powers of $\overleftarrow{T}$ must cancel each other. The solution of this problem can be found using important identies $\overrightarrow{T} e^{\rho(x)} = e^{\overrightarrow{T}\rho(x)}$ $\quad \overleftarrow{T} e^{\rho(x)} = e^{\overleftarrow{T}\rho(x)}$. We have $\rho_i(x) = \frac{\omega x^2}{2} + i\,\zeta_i x$. The first term in this expression coincides with the nonrelativistic $\rho_i(x) = \frac{\omega x^2}{2}$. After applying the condition (13) we conclude that the imaginary parts ζ_i are fixed up to one arbitrary parameter ξ:

$$\zeta_1 = -\omega\,(2+\xi)\,,\ \zeta_2 = -\omega\xi, \zeta_3 = \omega\xi,\ \zeta_4 = \omega\,(2+\xi)\,,\ -\infty < \xi < \infty$$

$$\begin{aligned} \eta^+ &= \tfrac{a}{\sqrt{2}} e^{-a\omega x(\xi+2)} \left(1 - e^{\frac{a\omega x}{2}} \overleftarrow{T}^2 e^{\frac{a\omega x}{2}}\right) e^{a\omega x\xi} \\ \eta^- &= -\tfrac{a}{\sqrt{2}} e^{-a\omega x\xi} \left(1 - e^{-\frac{a\omega x}{2}} \overleftarrow{T}^2 e^{-\frac{a\omega x}{2}}\right) e^{a\omega x(\xi+2)} \end{aligned} \tag{16}$$

In the oscillator case

$$\alpha(x) = const = \omega, \quad q_\eta = e^\omega, \quad [\eta^-, \eta^+]_{q_\eta} = \sinh\omega \tag{17}$$

Another couple of the creation and annihilation operators $\kappa^\pm$ comes out if we start with the case 4)

$$\begin{aligned} \kappa^+ &= -\tfrac{a}{\sqrt{2}} \left(\overleftarrow{T}\right)^{2a(\xi+2)} \left(1 - e^{-\frac{a\omega x}{2}} \overrightarrow{T}^2 e^{-\frac{a\omega x}{2}}\right) \left(\overrightarrow{T}\right)^{a\xi} \\ \kappa^- &= \tfrac{a}{\sqrt{2}} \left(\overrightarrow{T}\right)^{a\xi} \left(1 - e^{-\frac{a\omega x}{2}} \overleftarrow{T}^2 e^{-\frac{a\omega x}{2}}\right) \left(\overleftarrow{T}\right)^{a(2-\xi)} \\ q_\kappa &= e^{-\omega}, \quad [\kappa^-, \kappa^+]_{q_\kappa} = \sinh\omega \end{aligned} \tag{18}$$

DIMENSIONAL QUANTITIES Trying to understand how deformations are related to physical objects, it is very instructive to analyse the dimensional quantities entering into the theory. The deformation parameter q is dimensionless. At the same time, we expect it to have physical meaning, i.e. q must depend on the dimensional parameters of the theory considered. This means that different parameters (at least two) of the same dimensionality (we choose the length) must be inherent in the theory, entering into the operators, wave functions etc. In the case of nonrelativistic oscillator we have only one quantity of the dimension of length: $l = \sqrt{\frac{\hbar}{m\omega}}$Transferring to the relativistic theory we aquire the second length, the Compton wave length of the particle:

$$\lambda_0 = \frac{\hbar}{mc} \tag{19}$$

and obviously an infinite number of quantities of the same dimensionality of length which could be constructed from ( ) and (19). For us important will be the quantity

$$\lambda = \frac{l^2}{\lambda_0} = \frac{c}{\omega} \tag{20}$$

It can be shown [7] that

$$q = e^{-\frac{\lambda_0}{4\lambda}} = e^{-\frac{\omega}{4mc^2}} \tag{21}$$

So the q – plane structure is inherent in the noncommutative theory from the beginning. Using the q-mutator relations as (17)(cf. [7]) we obtain the different energy spectra corresponding It is instructive to write down the energy spectra corresponding to cases 1), 2), 4):

$$e_n^{(1)} = 2mc^2 \left(e^{\frac{2n+1}{4}\frac{\omega}{mc^2}} - \cosh \frac{\omega\hbar}{4mc^2} \right) \tag{22}$$

$$e_n^{(2)} = 2mc^2 \left(-e^{-\frac{2n+1}{4}\frac{\omega}{mc^2}} + \cosh \frac{\omega\hbar}{4mc^2} \right) \tag{23}$$

$$e^{(4)}g_n = 2\omega\hbar \frac{\sinh \frac{\omega\hbar}{4mc^2}(n+1/2)}{\sinh \frac{\omega\hbar}{8mc^2}} \tag{24}$$

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Quantum Chromodynamics

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Large-N_c QCD, Operator Product Expansion and string-like meson spectra

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Abstract — The relations for spectra of masses and decay constants for variety of non-strange meson resonances in the energy range 0–2.5 GeV are analyzed which follow from the string-like, linear mass spectrum for vector, axial-vector, scalar, and pseudoscalar mesons with an universal slope. The possible corrections to the linear trajectories of radial excitations are discussed. The way to match the universality with the Operator Product Expansion (OPE) is proposed. From the ensuing sum rules and normalizing on the ground states, the whole spectrum for meson masses and residues is obtained in a good agreement with phenomenology.

1 Introduction

As follows from phenomenology [1,2], the masses squared of mesons with given quantum numbers form linear trajectories with respect to the number of radial excitation n, which may signify that QCD is a string-like theory. As follows from the bosonic string model, the slope of all trajectories must be equal since this quantity is proportional to the string tension depending on gluedynamics only (we call this universality). On the other hand, there exist sizeable deviations from the string picture which are not understood yet. In the present analysis we examine possible corrections to the linear trajectories in the vector (V), axial-vector (A), scalar (S), and pseudoscalar (P) channels [3]. Our method is based on the consideration of the two-point correlators of V,A,S,P quark currents in the large-N_c limit of QCD [4]. On one hand, by virtue of confinement they are saturated by an infinite set of narrow meson resonances, that is, up to subtraction constants, they can be represented by the sum of related meson poles in Euclidean space:

$$\Pi^J(Q^2) = \int d^4x \exp(iQx)\langle\bar{q}\Gamma q(x)\bar{q}\Gamma q(0)\rangle_{\text{planar}} = \sum_n \frac{2F_J^2(n)}{Q^2+m_J^2(n)}, \qquad (1)$$

$$J \equiv S, P, V, A; \qquad \Gamma = i, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5.$$

On the other hand, their high-energy asymptotics is provided by the perturbation theory and the OPE with condensates [5]. Matching these two approaches results in some Chiral Symmetry Restoration (CSR) sum rules representing a set of constraints on meson mass parameters.

In Sect. 2 we give our results and we conclude in Sect. 3.

2 Matching and results

Let us consider the linear ansatz for the meson mass spectra with a non-linear correction δ:

$$m_J^2(n) = m_{0;J}^2 + a\,n + \delta_J(n), \qquad J \equiv V, A, S, P. \tag{2}$$

The last term in Eq. (2) simulates a possible deviation from the string picture in QCD. The condition of convergence for the generalized Weinberg sum rules imposes the equality of slopes and exponential decrease of non-linear correction.

The asymptotic freedom leads to the relation between residues and masses: $F^2(n) \sim \frac{dm^2(n)}{dn}$. Our analysis showed that the analytical structure of the OPE admits, however, exponentially small deviations from this relation (compare to [6]). We can argue also that D-wave vector meson must asymptotically drop out the CSR sum rules. This results in the exponential decrease of the corresponding residues (we denote them as $F_{V_D}^2(n)$). Our final results are as follows:

$$m_J^2(n) = m_0^2 + an + A_m^J e^{-B_m n}, \tag{3}$$

$$F_{V,A}^2(n) = \left(C_{V,A} + A_F^{V,A} e^{-B_F^{V,A} n}\right)\left(a - A_m^{V,A} B_m e^{-B_m n}\right), \tag{4}$$

$$F_{V_D}^2(n) = a A_F^D e^{-B_m n}, \tag{5}$$

$$F_{S,P}^2(n) = \left(C_{S,P} + A_F^{S,P} e^{-B_F^{S,P} n}\right)\left(1 - \frac{3\alpha_s}{4\pi^3 C_{S,P}} \frac{\lambda^2}{m_{S,P}^2(n)}\right)\frac{dm_{S,P}^2(n)}{dn}, \tag{6}$$

$$C_{V,A} = \frac{1}{8\pi^2}\left(1 + \frac{\alpha_s}{\pi}\right), \; C_{S,P} = \frac{3}{16\pi^2}\left(1 + \frac{11\alpha_s}{3\pi}\right), \; B_m > 0, \; B_F^J > 0, \tag{7}$$

where λ represents the possible dimension two condensate. In principle, there can be several exponents in each case, but we considered only the simplest case — with one exponent. We do not know the underlying dynamics responsible for the appearance of those exponential corrections. But it seems for us reasonable to suppose that, for masses, this dynamics does not depend on flavor. Thus, we keep the exponent B_m the same for all channels.

We have carried out numerical calculations with and without D-wave V-nesons. The former case let us call Set I (see Tables 1,3 of Appendix), the atter case let be Set II (see Tables 2,3 of Appendix). In the V,A,P sectors our its are compared with experiment [2,7]. The scalar sector is quite involved ecause of strong mixing effects and one should relate the masses of scalar nesons to the corresponding bare masses [2].

Given an ansatz for mass spectrum $m_J^2(n)$ and decay constants F_J^2, one an calculate the electromagnetic pion mass difference $\Delta m_\pi \equiv m_{\pi^+} - m_{\pi^0}$ and he chiral constant L_{10} [8] (parametrizing the decay $\pi \to e\nu\gamma$). The example f such a calculation was demonstrated in [9]. Besides, in the scalar sector ve can calculate the chiral constant L_8 [8] (parametrizing the ratio of current uark masses). Below we give the results of numerical calculations.

$L_{10} = -5.7 \cdot 10^{-3}$ (for the Set I and II) to be compared with the accepted stimate from phenomenology: $L_{10}|_{\text{phen}} = (-5.5 \pm 0.7) \cdot 10^{-3}$ [10].

$\Delta m_\pi = 3.5$ MeV (Set I) and $\Delta m_\pi = 4.2$ MeV (Set II). The experimental value s [7]: $\Delta m_\pi|_{\text{expt}} = 4.5936 \pm 0.0005$ MeV.

$L_8 = 0.8 \cdot 10^{-3}$. The accepted estimate for L_8 is $L_8|_{\text{phen}} = (0.8 \pm 0.3) \cdot 10^{-3}$ [10].

It is interesting to note that the first two chiral pairs of resonances saturate he chiral constants L_8 and L_{10} almost completely, but this is not the case for he quantity Δm_π. In the latter case the contribution of D-wave vector mesons urns out to be substantial.

3 Summary

In the present work we have considered the problem of matching the vector, xial-vector, scalar, and pseudoscalar meson mass spectra $m^2(n)$ (n is the umber of radial excitation) with universal slope to the Operator Product Expansion. The analysis was carried out for the light non-strange mesons in he large-N_c and chiral limits. The main result is that, given the masses of he ground states, the chiral symmetry restoration sum rules (following from his matching) allow to restore the whole spectrum for the V,A,S,P masses and esidues. Let us summarize the results our analysis:

- The matching to the OPE can not be achieved by a simple linear parametrization of the mass spectrum.

- The convergence of the generalized Weinberg sum rules requires the universality of intercepts for masses of chiral partners. The tree-level limit implies universality of intercepts for all V,A,S,P spectra. Thus, there must exist deviations from the linear spectra (bosonic string picture) parametrizing the chiral symmetry breaking. These deviations must decrease at least exponentially.

- There are also deviations from the relation for the meson residues (decay constants) $F^2(n) \sim \frac{dm^2(n)}{dn}$. The analytic structure of OPE imposes the exponential decrease on these deviations (or faster).

- At high energies, the D-wave vector mesons have to decouple the CSR sum rules. This fact implies the exponential (or faster) decreasing the corresponding decay constants $F_D^2(n)$. However, at small n the contribution of D-wave vector mesons may be sizeable and important.

We have proposed the simplest ansatz for the V,A,S,P meson masses and residues meeting the all general principles above. The numerical solutions of the CSR sum rules for these spectra were obtained and compared with the experimental data. Our fits agree well with the existing phenomenology.

Unfortunately, the underlying dynamics, which generates the non-linear contributions to the spectra of meson masses and residues, is not known. We can say only that these deviations from the string picture seem to parametrize the chiral symmetry breaking in QCD and, consequently, might be proportional to the chiral condensate $\langle \bar{q}q \rangle$. Developing a theory of these non-linear contributions is an interesting task for future.

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Appendix

In this Appendix we give an example of meson mass spectra proposed in our work. The inputs general for all tables (if any) are: $a = (1200\,\text{MeV})^2$, $\langle\bar{q}q\rangle = -(240\,\text{MeV})^3$, $\frac{\alpha_s}{\pi}\langle(G^a_{\mu\nu})^2\rangle = (360\,\text{MeV})^4$, $f_\pi = 87\,\text{MeV}$, $F_\pi = \frac{\langle\bar{q}q\rangle^2}{f_\pi^2}$, $\alpha_s = 0.3$. The units are: $m(n)$, $F(n)$ — MeV; A_m — MeV^2; A_F, $B_{F,m}$ — MeV^0.

Table 1: Set I. An example of parameters for the mass spectra of our work without D-wave vector mesons. The corresponding experimental values [2, 7] (if any) are displayed in brackets.

Case	Inputs	Predictions
For all *VASP*		$m_0 = 1240$, $B_m = 0.8$
V	$m(0) = 770\,(769.3 \pm 0.8)$	$A_m = -970^2$, $A_F = -0.004$, $B_F = 1.7 + i\pi$
A	$m(0) = 1130\,(1230 \pm 40)$	$A_m = -500^2$, $A_F = -2 \cdot 10^{-7}$, $B_F = 0.1$
S	$m(0) = 1230$	$A_m = -165^2$, $A_F = -0.019$, $B_F = 0.6 + i\pi$ $m_S^{\text{out}} = 770$
P	$m(0) = 1400\,(1300 \pm 100)$	$A_m = 650^2$, $A_F = 0.004$, $B_F = 0.48$

Table 2: Set II. The account of D-wave vector mesons.

Case	Inputs	Predictions
A, S, P	The same as in Table 1	The same as in Table 1
V_S	$m(0) = 770\,(769.3 \pm 0.8)$ $F(0) = 145\,(154 \pm 8)$	$A_m = -970^2$, $A_F = -0.003$, $B_F = 0.6$
V_D	$m(0) = 1700\,(1700 \pm 20)$ $m_D = 1500$, $F(0) = 59$	$A_m = 800^2$, $A_F = 0.003$, $B_F = 0.8$

Table 3: The mass spectrum and residues for the parameter sets of previous Tables The known experimental values [2, 7] are displayed in brackets.

n	out	0	1	2	3
$m_V(n)$		770(769.3±0.8)	1540 (1465 ± 25)	1960	2280 (2149 ± 17)
$F_V(n)$		143(154±8)	152	139	135
$m_A(n)$		1130(1230±40)	1640 (1640 ± 40)	2000 (1971 ± 15)	2300 (2270 ± 50)
$F_A(n)$		142(123±25)	137	134	133
$m_S(n)$	770	1230	1670	2010	2300
$F_S(n)$	207	95	213	160	190
$m_P(n)$	0	1400	1730 (1801 ± 13)	2030 (2070 ± 35)	2310 (2360 ± 30)
$F_P(n)$	–	155	169	175	178
$m_{V_S}(n)$		770(769.3±0.8)	1540 (1465 ± 25)	1960	2280 (2149 ± 17)
$F_{V_S}(n)$		145(154±8)	139	135	133
$m_{V_D}(n)$		1700(1700±20)	1950 (1980 ± 30)	2210 (2285±?)	2460
$F_{V_D}(n)$		59	40	27	18

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Pion distribution amplitude and QCD vacuum: recent results from CLEO and E791 experiments

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Abstract — The CLEO experimental data on the $\pi\gamma$ transition are analyzed to NLO in QCD perturbation theory using light-cone QCD sum rules. We obtain new constraints for the Gegenbauer coefficients a_2 and a_4, as well as for the inverse moment x^{-1} of the pion distribution amplitude (πDA) and conclude that the data confirm the shape of the πDA previously obtained by us using QCD sum rules with nonlocal condensates, while the exclusion of the asymptotic and the Chernyak–Zhitnitsky DAs is reinforced. We also check our πDA against the di-jets data of the E791 experiment, providing credible evidence for our results far more broadly.

The recent high-precision CLEO results [1] for the $\pi\gamma$ transition form factor gave rise to dedicated theoretical investigations [2–9]. These experimental data are of particular importance because they can provide crucial quantitative information on nonperturbative parameters of the πDA and – as we pointed out in [9] – on the QCD vacuum nonlocality parameter λ_q^2, which specifies the average virtuality of the vacuum quarks. In the absence of a direct solution of the nonperturbative sector of QCD, we are actually forced to extract related information from the data, relying upon a theoretical analysis as complete and as accurate as currently possible.

It was shown by Khodjamirian [5] that the most appropriate tool to analyze the CLEO data is provided by the light-cone QCD sum-rule (LCSR) method. Schmedding and Yakovlev (SY) [6] applied these LCSRs to the NLO of QCD perturbation theory. More recently [9], we have taken up this sort of data processing in an attempt to (i) account for a correct ERBL [10] evolution of the πDA to each measured momentum scale, (ii) estimate more precisely the contribution of the (next) twist-4 term, and (iii) improve the error estimates in determining the 1- and 2-σ error contours.

The main outcome of these theoretical analyses can be summarized as follows: (i) the asymptotic πDA [10] and the Chernyak–Zhitnitsky (CZ) [11] one are both outside the 2-σ error regions; (ii) the extracted parameters a_2 and

a_4 are rather sensitive to the strong radiative corrections and to the size of the twist-4 contribution; (iii) the CLEO data allow us to estimate the correlation scale in the QCD vacuum, λ_q^2, to be $\lesssim 0.4$ GeV^2.

The present note gives a summary of our lengthy analysis [9] extending it a step further in an attempt to obtain from the CLEO data a direct estimate for the inverse moment of the πDA that plays a crucial role in electromagnetic/transition form factors of the pion. Moreover, we take into account the variation of the twist-4 contribution and treat the threshold effects in the strong running coupling more accurately. The predictive power of our refined analysis lies in the fact that the value of the inverse moment obtained from an *independent* sum rule is compatible with that extracted from the CLEO data, referring in both cases to the same low momentum scale. As a result, the πDA obtained from QCD sum rules with nonlocal condensates is within the 1-σ error region, while the asymptotic and the CZ πDAs are clearly excluded, as being well outside the 2-σ region. Our prediction for the πDA is confirmed by the analysis of Fermilab E791 data [12].

Below, we sketch the improved NLO procedure for the data processing, developed in [9]. Let us recall that this procedure is based on LCSRs for the transition form factor $F^{\gamma^*\gamma\pi}(Q^2, q^2 \approx 0)$ [5,6]. Accordingly, the main LCSR expression for the form factor

$$F_{\text{LCSR}}^{\gamma^*\gamma\pi}(Q^2) = \frac{1}{\pi}\int_0^{s_0}\frac{ds}{m_\rho^2}\,\rho(Q^2,s;\mu^2)\,e^{(m_\rho^2-s)/M^2} + \frac{1}{\pi}\int_{s_0}^{\infty}\frac{ds}{s}\,\rho(Q^2,s;\mu^2) \quad (1)$$

follows from dispersion relation. Corresponding spectral density $\rho(Q^2, s; \mu^2) \equiv \mathbf{Im}\left[F_{\text{QCD}}^{\gamma^*\gamma^*\pi}(Q^2, q^2 = -s; \mu^2)\right]$ is calculated by virtue of the factorization theorem for the form factor at Euclidean photon virtualities $q_1^2 = -Q^2 < 0$, $q_2^2 = -q^2 \leq 0$ [10,13], with $M^2 \approx 0.7$ GeV^2 being the Borel parameter, whereas m_ρ is the ρ-meson mass, and $s_0 = 1.5$ GeV^2 denotes the effective threshold in the ρ-meson channel. The factorization scale μ^2 was fixed by SY at $\mu^2 = \mu_{\text{SY}}^2 = 5.76$ GeV^2. Moreover, $F_{\text{QCD}}^{\gamma^*\gamma^*\pi}(Q^2, q^2; \mu^2)$ contains a twist-4 contribution, which is proportional to the coupling $\delta^2(\mu^2)$. This contribution has been calculated for the asymptotic twist-4 DAs of the pion [5].

We set $\mu^2 = Q^2$ in $F_{\text{QCD}}^{\gamma^*\gamma^*\pi}(Q^2, q^2; \mu^2)$ and use the complete 2-loop expression for the form factor, absorbing the logarithms into the coupling constant and the πDA evolution at the NLO [9] so that $\alpha_s(\mu^2) \xrightarrow{\text{RG}} \alpha_s(Q^2)$ (RG denotes the renormalization group) and $\varphi_\pi(x; \mu^2) \xrightarrow{\text{ERBL}} \varphi_\pi(x; Q^2) = U(\mu^2 \to Q^2)\varphi_\pi(x; \mu^2)$. Then, we use the spectral density $\rho(Q^2, s, \mu^2 = Q^2)$, derived in [6] and used there at $\mu^2 = \mu_{\text{SY}}^2$, in Eq. (1) to obtain $F^{\gamma^*\gamma\pi}(Q^2)$ and fit the CLEO data over the probed momentum range, denoted by $\{Q^2_{\text{exp}}\}$. In our recent analysis [9] the evolution $\varphi_\pi(x;Q^2) = U(\mu^2_{\text{SY}\to Q^2)\varphi_\pi(x;\mu^2_{\text{SY}})}$ was performed *for every point* Q^2_{exp}, with the aim to return to the normalization scale μ^2_{SY} and to extract the DA parameters (a_2, a_4) at this reference scale for the sake of comparison with the previous SY results [6]. Stated differently, for every measurement, $\{Q_{\text{exp}^2,F^{\gamma^*\gamma\pi}(Q_{\text{exp}^2})\}}$, its own factorization/renormalization scheme has been used so that the NLO radiative corrections were taken into account in a complete way. The accuracy of this procedure is still limited mainly by the uncertainties of the twist-4 scale parameter [9], $k\cdot\delta^2$, where the factor k expresses the deviation of the twist-4 DAs from their asymptotic shapes.

To summarize, the focal points of our procedure of the CLEO data processing are (i) $\alpha_s(Q^2)$ is the exact solution of the 2-loop RG equation with the threshold $M_q = m_q$ taken at the quark mass m_q, rather than adopting the approximate popular expression in [14] that was used in the SY analysis. This is particularly important in the low-energy region $Q^2 \sim 1$ GeV^2, where the difference between these two couplings reaches about 20%. (ii) All logarithms $\ln(Q^2/\mu^2)$ appearing in the coefficient function are absorbed into the evolution of the πDA, performed separately at each experimental point Q^2_{exp}. (iii) The value of the parameter δ^2 has been re-estimated in [9] to read $\delta^2(1\text{GeV}^2) = 0.19\pm0.02$ GeV^2. The present study differs from the SY approach in all these points and extends our recent analysis [9] with respect to points (i) and (iii) yielding to significant improvements of the results. In the absence of reliable information on higher twists, one may assume that the uncertainty due to the shapes of the twist-4 DAs is of the same order as that for the leading twist case. Therefore we set $k = 1 \pm 0.1$. As a result, the final (rather conservative) accuracy estimate for the twist-4 scale parameter can be expressed in terms of $k\cdot\delta^2(1\text{GeV}^2) = 0.19 \pm 0.04$ GeV^2. To produce the complete 2σ- and 1σ-contours, corresponding to these uncertainties, we need to unite a number of contours, resulting from the processing of the CLEO data at different values of the scale parameter $k\cdot\delta^2$ within this admissible range. This is discussed in technical detail in [9]. Here we only want to emphasize that our contours are more stretched then the SY ones. The obtained results for the asymptotic DA (u), the BMS model (6) [7], the CZ DA (n), the SY best-fit point (l) [6], a recent transverse lattice result (t) [15], and three instanton-based models, viz., (H) [16], (s) [17], and (F) (using in this latter case $m_q = 325$ MeV, $n = 2$, and $\Lambda = 1$ GeV) [18], are compiled in Table 1 for the maximal and the minimal twist-4 scale parameter. For the middle $k\cdot\delta^2$ value (0.19 GeV^2) – discussed

Table 1: Models/fits for different values of $k \cdot \delta^2$ (see text).

$k \cdot \delta^2$		0.23 GeV^2		0.15 GeV^2	
Models/fits	Symbol	$(a_2, a_4)\big\vert_{\mu_{\text{SY}^2}}$	χ^2	$(a_2, a_4)\big\vert_{\mu_{\text{SY}^2}}$	χ^2
Best fit (b.f.)		$(+0.28, -0.29)$	0.47	$(+0.16, -0.16)$	0.47
SY b.f.	1	$(+0.19, -0.14)$	1.0	$(+0.19, -0.14)$	0.57
BMS	6	$(+0.14, -0.09)$	1.7	$(+0.14, -0.09)$	0.52
Asymp.	u	$(-0.003, +0.00)$	5.9	$(-0.003, +0.00)$	2.2
CZ	n	$(+0.40, -0.004)$	4.0	$(+0.40, -0.004)$	7.0
Lattice$^\perp$	t	$(+0.06, +0.01)$	3.8	$(+0.06, +0.01)$	1.2
PPRWK	H	$(+0.03, +0.005)$	4.7	$(+0.03, +0.005)$	1.6
ADT	s	$(+0.03, -0.03)$	4.6	$(+0.03, -0.03)$	1.6
PR	F	$(+0.06, -0.01)$	3.6	$(+0.06, -0.01)$	1.1

in [9] – the corresponding values of the best-fit point (:) are $a_2(\mu_{\text{SY}^2})_{=+0.22}$, $a_4(\mu_{\text{SY}^2})_{=-0.22}$, and $\chi^2 = 0.47$.

We turn now to the important topic of whether or not the set of CLEO data is consistent with the non-local QCD SR results for φ_π. We present in Fig. 1 the results of the data analysis for the twist-4 scale parameter $k \cdot \delta^2$ varied in the interval $[0.15 \le k \cdot \delta^2 \le 0.23]$ GeV^2. We have established in [7] that a two-parameter model $\varphi_\pi(x; a_2, a_4)$ factually enables us to fit all the moment constraints that result from NLC QCD SRs (see [19] for more details). The only parameter entering the NLC SRs is the correlation scale λ_q^2 in the QCD vacuum, known from nonperturbative calculations and lattice simulations [20, 21]. A whole bunch of admissible πDAs resulting from the NLC QCD SR analysis associated with $\lambda_q^2 = 0.4$ GeV^2 at $\mu_0^2 \approx 1$ GeV^2 [7] was determined, with the optimal one given analytically by

$$\varphi_\pi^{\text{BMS}}(x) = \varphi_\pi^{\text{as}}(x)\left[1 + a_2^{\text{opt}} \cdot C_2^{3/2}(2x-1) + a_4^{\text{opt}} \cdot C_4^{3/2}(2x-1)\right], \qquad (2)$$

where $\varphi_\pi^{\text{as}}(x) = 6x(1-x)$ and $a_2^{\text{opt}} = 0.188$, $a_4^{\text{opt}} = -0.13$ are the corresponding Gegenbauer coefficients. From Fig. 1 we observe that the NLC QCD SR constraints encoded in the slanted shaded rectangle are in rather good overall agreement with the CLEO data at the 1σ-level. This agreement could

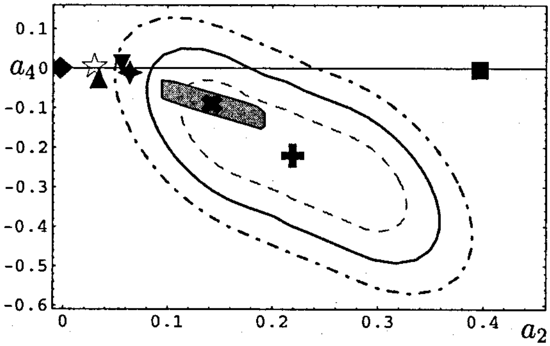

Figure 1: Analysis of the CLEO data on $F_{\pi\gamma^*\gamma}(Q^2)$ in terms of error ellipses in the (a_2,a_4) plane contrasted with various theoretical models explained in the text. The solid line denotes the 2σ-contour; the broken line stands for the 1σ-contour. The slanted shaded rectangle represents the constraints on $(a_2,\ a_4)$ posed by the NLC QCD SRs [7] for the value $\lambda_q^2 = 0.4$ GeV^2. All constraints are evaluated at $\mu^2_{\text{SY}=5.76\ \text{GeV}^2}$ after NLO ERBL evolution.

eventually be further improved adopting smaller values of λ_q^2, say, 0.3 GeV^2, which however are not supported by the QCD SR method and lattice calculations [21]. On the other hand, as it was demonstrated in [9], the agreement between QCD SRs and CLEO data fails for larger values of λ_q^2, e. g., 0.5 GeV^2.

In the present study we have processed the CLEO data in such a way as to obtain an experimental constraint on the value of $\langle x^{-1}\rangle_\pi = \int_0^1 \varphi_\pi(x)x^{-1}dx$ that appears in different perturbative calculations of pion form factors. This is illustrated in Fig. 2(a). A "daughter SR" has been previously constructed directly for this quantity by integrating the r.h.s. of the SR for $\varphi_\pi(x)$ with the weight x^{-1}, (for details, see [22, 7]). Due to the smooth behavior of the NLC at the end points $x = 0, 1$, this integral is well defined, supplying us with an *independent* SR, with a rather good stability behavior of $\langle x^{-1}\rangle_\pi^{\text{SR}(M^2)}$, as one sees from Fig. 2(b). We have estimated $\langle x^{-1}\rangle_\pi^{\text{SR}(\mu_0^2\approx 1\ \text{GeV}^2)=3.28\pm 0.31}$ at the value $\lambda_q^2 = 0.4$ GeV^2 of the non-locality parameter. It should be emphasized that this estimate is not related to the model of πDA, $\varphi^{\text{BMS}_\pi(x;a_2,a_4)}$, constructed within the same framework. Nevertheless, the value obtained with the "daughter" QCD SR and those calculated using the bunch of πDAs, mentioned above, match each other.

On the other hand, from the CLEO data one obtains a constraint on the value of $a_2 + a_4 = \langle x^{-1}\rangle^{\exp_\pi/3-1}$ at the low point $\mu_0^2 \approx 1$ GeV^2 that complies with the NLC SR estimate. In Fig. 2(a) we demonstrate the united regions,

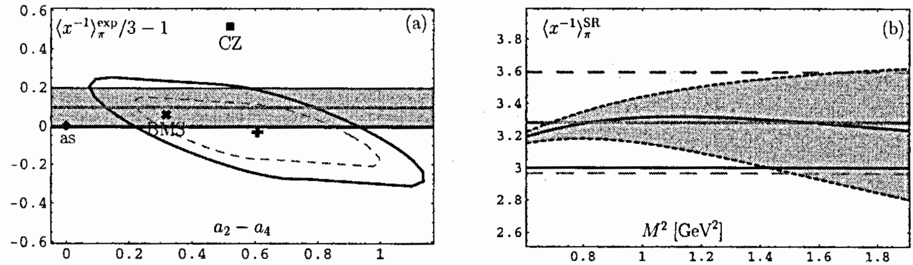

Figure 2: (a) The result of the CLEO data processing for the quantity $\langle x^{-1}\rangle^{\exp_\pi/3-1}$ at the scale $\mu_0^2 \approx 1$ GeV^2 in comparison with three theoretical models. The thick solid-line contour corresponds to the union of 2σ-contours, while the thin dashed-line contour denotes the union of 1σ-contours. The light solid line with the hatched band indicates the mean value of $\langle x^{-1}\rangle^{\text{SR}_\pi/3-1}$ and its error bars in the second part of the Figure. (b) The inverse moment $\langle x^{-1}\rangle^{\text{SR}_\pi}$ shown as a function of the Borel parameter M^2 from the NLC SR at the same scale μ_0^2 [7]; the light solid line is the estimate for $\langle x^{-1}\rangle^{\text{SR}_\pi}$; the dashed lines correspond to its error-bars.

corresponding to the merger of the 2σ-contours (solid thick line) and the 1σ-contours (thin dashed line), which have been obtained for values of the twist-4 scale parameter within the determined range. This resulting admissible region is strongly stretched along the $(a_2 - a_4)$ axis, demonstrating the poor accuracy for this combination of DA parameters, while more restrictive constraints are obtained for $\langle x^{-1}\rangle^{\exp_\pi}$. One appreciates that the NLC SR result, $\langle x^{-1}\rangle^{\text{SR}_\pi}$, with its error bars appears in good agreement with the constraints to $\langle x^{-1}\rangle^{\exp_\pi}$ at the 1σ-level, as one sees from the light solid line within the hatched band in Fig. 2(a). In particular, the 1σ-constraint obtained at the central value $k \cdot \delta^2 = 0.19$ GeV^2 exhibits the same good agreement with the corresponding SR estimate because the theoretical uncertainty of the twist-4 scale parameter affects only the $(a_2 - a_4)$ constraint. Moreover, the estimate $\langle x^{-1}\rangle^{\text{SR}_\pi}$ practically coincides with that obtained in the data analysis on the electromagnetic pion form factor in the framework of a different LCSR method in [23]. These three independent estimates are in good agreement to each other, giving robust support that the CLEO data processing and the theoretical calculations are mutually consistent.

More importantly, the end-point contributions to the $\langle x^{-1}\rangle^{\rm SR}_\pi$ are suppressed, the range of suppression being controlled by the value of the parameter λ_q^2. The larger this parameter, at fixed resolution scale $M^2 > \lambda_q^2$, the stronger the suppression of the NLC contribution. Similarly, an excess of the value of

$\langle x^{-1}\rangle_\pi$ over 3 (asymptotic DA) is also controlled by the value of λ_q^2, becoming smaller with increasing λ_q^2. Therefore, to match the value $\langle x^{-1}\rangle_\pi^{\text{SR}}$ to the CLEO best-fit point (:) in Fig. 2(a), would ask to use larger values of λ_q^2 than 0.4 GeV^2. But this is in breach of the (a_2, a_4) error ellipses. A window of about 0.05 GeV^2 exists to vary λ_q^2: any smaller and one is at the odds with QCD SRs and lattice calculations [21]; any larger and the NLC QCD SRs rectangle can tumble out of the CLEO data region.

Before we come to the Fermilab experiment [12], let us summarize our findings. They have been obtained by refining the CLEO data analysis in the following points. We corrected the mass thresholds in the running strong coupling and incorporated the variation of the twist-4 contribution more properly. In addition, the CLEO data were used to extract a direct constraint on the inverse moment $\langle x^{-1}\rangle_\pi(\mu_0^2)$ of the πDA – at the core of form-factor calculations. This has relegated the asymptotic and CZ πDAs beyond the 2σ level (95%), with the SY best-fit point still belonging to the 1σ deviation region (68%) in the parameter space of (a_2, a_4), while providing compelling argument in favor of our model [7].

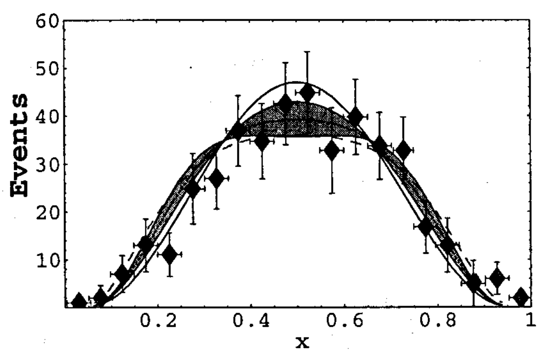

Figure 3: Comparison of φ^{as} (solid line), φ^{CZ} (dashed line), and the BMS bunch of πDAs (strip, [7]) with the E791 data [12].

To compare our model DA for the pion [7] with the E791 di-jet events, we adopt the convolution approach developed in [24] having also recourse to [25]. The results are displayed in Fig. 3 making evident that, though the data from E791 are not that sensitive as to exclude other shapes for the πDA, also displayed for comparison, they are in good agreement with our prediction.

As a conclusion, both analyzed experimental data sets (CLEO [1] and Fermilab E791 [12]) converge to the conclusion that the πDA is not everywhere a convex function, like the asymptotic one, but has instead two maxima with the end points ($x = 0, 1$) strongly suppressed – in contrast to the CZ DA. These two key features are controlled by the QCD vacuum correlation length λ_q^2, whose value suggested by the CLEO data analysis here and in [9] is approximately 0.4 GeV^2 in good compliance with the QCD SR estimates and lattice computations [21].

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Singlet axial vector current correlator and topological susceptibility

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Abstract — The behavior of nonperturbative part of isosinglet axial-vector correlator at Euclidean momenta is studied in the framework of a covariant chiral quark model with nonlocal quark-quark interactions.

It is well known that due to $U_A(1)$ axial Adler-Bell-Jackiw anomaly the singlet axial-vector current is not conserved even in the chiral limit,

$$\partial_\mu J^{50}_\mu(x) = 2N_f Q_5(x), \tag{1}$$

where $Q_5(x) = (\alpha_s/8\pi)G^a_{\mu\nu}(x)\tilde{G}^a_{\mu\nu}(x)$ is topological charge density. The correlator of singlet axial-vector currents, $J^{50}_\mu = \sum_{f=(u,d)} \overline{q}_f \gamma_\mu \gamma_5 q_f$, is defined as

$$\Pi^{A,0}_{\mu\nu}(q) = i\int d^4x\, e^{iqx}\langle 0\left|T\left\{J_\mu(x)J_\nu(0)^\dagger\right\}\right|0\rangle = \tag{2}$$

$$= \left(q_\mu q_\nu - g_{\mu\nu}q^2\right)\Pi^{A,0}_T(Q^2) + q_\mu q_\nu \Pi^{A,0}_L(Q^2). \tag{3}$$

In the chiral limit its longitudinal part is related to the topological susceptibility, the correlator of the topological charge densities $Q_5(x)$,

$$\chi\left(Q^2\right) = i\int d^4x\, e^{iqx}\langle 0\left|T\left\{Q_5(x)Q_5(0)\right\}\right|0\rangle, \tag{4}$$

by the relation (see, *e.g.*, [1])

$$\Pi^{A,0}_L\left(Q^2\right) = \frac{(2N_f)^2}{Q^2}\chi\left(Q^2\right). \tag{5}$$

At high Q^2 the operator product expansion (OPE) predicts for $\chi(Q^2)$, [2],

$$\chi\left(Q^2 \to \infty\right) = -\frac{\alpha_s}{16\pi}\left\langle \frac{\alpha_s}{\pi}\left(G^a_{\mu\nu}\right)^2\right\rangle + \mathcal{O}(Q^{-2}) + \mathcal{O}(e^{-Q\rho}), \tag{6}$$

where the perturbative contribution has been subtracted, and the exponential corrections are due to nonlocal instanton interactions.

At low Q^2 the topological susceptibility $\chi(Q^2)$ can be represented as sum of contributions coming purely from QCD and from hadronic resonances, [1]. Crewther proved theorem [4] that $\chi(0) = 0$ in any gauge theory where at least one massless quark exists (the dependence of $\chi(0)$ on current quark masses has been found in [3]). Also, the contribution of nonsinglet hadron resonances to χ is absent in the chiral limit. Thus, at low-Q^2 for massless current quarks one has

$$\chi\left(Q^2 \to 0\right) = -Q^2\chi'(0) + \mathcal{O}(Q^4). \tag{7}$$

The estimates of $\chi'(0)$ existing in the literature are rather controversial:

$$\chi'(0) = (48 \pm 6\ \mathrm{MeV})^2 \qquad [5], \qquad \chi'(0) = (26 \pm 4\ \mathrm{MeV})^2 \qquad [6]. \tag{8}$$

This makes further model estimates valuable.

Within the nonlocal chiral quark model [7] the full isosinglet axial-vector current becomes [8]

$$\begin{aligned}\Gamma^{50}_{\mu}(k,q,k'=k+q) &= \gamma_\mu\gamma_5 - \gamma_5(k+k')_\mu \frac{\left(\sqrt{M(k')}-\sqrt{M(k)}\right)^2}{k'^2-k^2} \\ &- \gamma_5\frac{q_\mu}{q^2}2\sqrt{M(k')M(k)}\frac{G'}{G}\frac{1-GJ_{PP}(q^2)}{1-G'J_{PP}(q^2)}.\end{aligned} \tag{9}$$

where $M(k)$ is dynamical, momentum dependent quark mass, G and G' are 4-quark couplings in isotriplet and isosinglet channels, correspondingly, and

$$J_{PP}(q^2)\delta_{ab} = -\frac{i}{M_q^2}\int\frac{d^4k}{(2\pi)^4}M(k)\,M(k+q)\,Tr\left[S(k)\gamma_5\tau^a S(k+q)\,\gamma_5\tau^b\right]. \tag{10}$$

In (10) the (inverse) quark propagator is $S^{-1}(p) = \hat{p} - M(p)$. Because of axial anomaly the singlet current does not contain massless pole, since as $q^2 \to 0$ one has:

$$\frac{1-GJ_{PP}(q^2)}{-q^2} = G\frac{f_\pi^2}{M_q^2}, \tag{11}$$

where f_π is pion weak decay constant and $M_q = M(0)$. Instead, the current develops a pole at the η'– meson mass, $1-G'J_{PP}(q^2 = -m^2_{\eta'}) = 0$, thus solving the $U_A(1)$ problem. The vertice (9) satisfies the anomalous Ward-Takahashi identity:

$$q_\mu\Gamma^{50}_{\mu}(k,q,k'=k+q) = \hat{q}\gamma_5 - \gamma_5\left[M(k')+M(k)\right] + \gamma_5\frac{2\sqrt{M(k')M(k)}}{1-G'J_{PP}(q^2)}\left(1-\frac{G'}{G}\right), \tag{12}$$

where the last term is due to anomaly. Thus, the QCD pseudoscalar gluonium operator is interpolated by the pseudoscalar effective quark field operator with coefficient expressed in terms of dynamical quark mass. This is a consequence of the fact that in the effective quark model the connection between quark and integrated gluon degrees of freedom is fixed by the gap equation.

Full model calculations lead to the following expression for the topological susceptibility [8]

$$\begin{aligned}
& -\left(2N_f\right)^2\chi\left(Q^2\right) = \\
= \;& 2N_f\left(1-\frac{G'}{G}\right)\left\{Q^2 J_{\pi A}\left(Q^2\right)\left[1-\frac{G' J_{AP}(Q^2)}{M_q^2}+\frac{1}{1-G' J_{PP}(Q^2)}\right]\right. \\
+ \;& M_q^2 J_{PP}\left(Q^2\right)\left(1-\frac{G'}{M_q^2}J_{AP}(Q^2)\right)\left[\frac{G J_{AP}(Q^2)}{M_q^2}-\frac{G-G'}{G\left[1-G' J_{PP}(Q^2)\right]}\right] \\
+ \;& \left.\frac{G}{M_q^2}\left[4N_cN_f\int\frac{d^4k}{(2\pi)^4}\frac{M(k)}{D(k)}\left[M(k)-\sqrt{M(k+Q)\,M(k)}\right]\right]^2\right\},
\end{aligned} \tag{13}$$

where $D(k) = k^2 + M^2(k)$ and the integrals $J_{AP}(q^2)$ and $J_{\pi A}(q^2)$ are defined by

$$\begin{aligned}
J_{AP}(q^2) &= 4N_cN_f\int\frac{d^4l}{(2\pi)^4}\frac{M(l)}{D(l)}\sqrt{M(l+q)\,M(l)}, \\
J_{\pi A}\left(q^2\right)\delta_{ab} &= \frac{q_\mu}{q^2}\int\frac{d^4k}{(2\pi)^4}Tr\left[S(k)\tilde{\Gamma}^{5a}_\mu(k,q,k+q)S(k+q)\,\Gamma^a_\pi(k+q,k)\right].
\end{aligned}$$

At large Q^2 one obtains the power-like behavior consistent with the OPE prediction (6), namely

$$-\left(2N_f\right)^2\chi\left(Q^2\to\infty\right) = \frac{2N_f M_q^2}{G}\left(1-\frac{G'}{G}\right). \tag{14}$$

At zero momentum the topological susceptibility is zero

$$\chi(0) = 0, \tag{15}$$

in accordance with the Crewther theorem. For the first moment of the topological susceptibility we obtain

$$\chi'(0) = \frac{1}{2N_f}\left\{f_\pi^2\left(2-\frac{G'}{G}\right)+\left(1-\frac{G'}{G}\right)^2 J'_{AP}(0)\right\}. \tag{16}$$

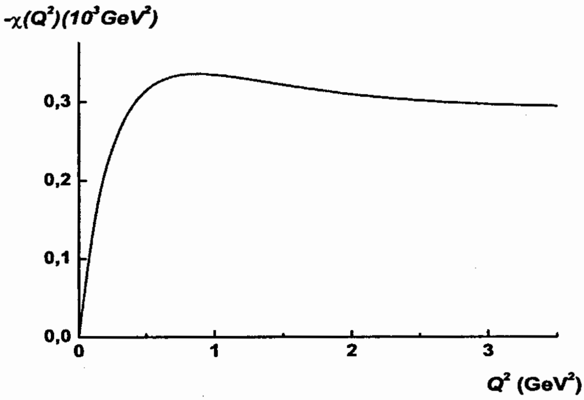

Figure 1: Topological susceptibility versus Q^2 predicted by the model with $G' = 0.1\ G$, Eq. (13), (solid line).

The constants G and G' are fixed with the help of the meson spectrum. Approximately one has $G' = 0.1\ G$. Then the estimate for the first moment of the topological susceptibility is:

$$\chi'(0) = (50\ \text{MeV})^2 . \tag{17}$$

For the above estimate we have taken $N_f = 3$. We can see that the model gives the value of $\chi'(0)$ which is close to the estimate of Ref. [5]. In Fig. 1 we present the model prediction for the topological susceptibility at low and moderate values of Q^2 for the full model calculations (13). In the region of small and intermediate momenta our result is quantitatively close to the prediction of the QCD sum rules with the instanton effects included [1].

In the present talk we have analyzed the nonperturbative part of the Euclidean-momentum correlation functions of the singlet axial-vector currents within an effective nonlocal chiral quark model. By considering this correlator the topological susceptibility has been found as a function of the momentum, and its first moment is estimated. In addition, the fulfillment of the Crewther theorem has been demonstrated. It would be interesting to verify the predictions given in Fig. 1 by modern lattice simulations.

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Some problems of the perturbative QCD

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Abstract — The perturbative quantum chromodynamics (pQCD) has been extremely successful in the prediction and description of main properties of quark and gluon jets. There are, however, some problems of the jet calculus with the higher order corrections of the modified perturbative expansion which should be resolved to get more precise statements. Some of them are discussed here. The property of the asymptotic freedom of QCD is also questioned in this talk.

The perturbative quantum chromodynamics (pQCD) has been extremely successful in the prediction and description of main properties of quark and gluon jets. There are, however, some problems of the jet calculus with the higher order corrections of the modified perturbative expansion which should be resolved to get more precise statements. Some of them are discussed here. The property of the asymptotic freedom of QCD is also questioned in this talk.

The numerous achievements of pQCD in the jet calculus are well known and described in many review papers (see, e.g., [1–5]). The leading approximation is perfect and only high order terms need more care. In this talk I would like to describe some problems related to these calculations and rarely discussed. The Figures demonstrating the comparison with experiment are omitted to shorten the presentation. They can be found in the abovecited review papers.

I mention briefly the following six problems:

1. Different characteristics of jets are differently sensitive to higher order corrections. Therefore, for the comparison with experiment, one should choose those which are not overshadowed by the leading terms of the perturbative expansion and help most efficiently elucidate these corrections.

2. The correction terms are proportional to higher powers of the coupling strength but can get the large numerical coefficients in front of them. Thus, even though in asymptotics this expansion is valid due to the running nature of the coupling strength, at present energies it fails to provide small corrections. Therefore one should find such characteristics where these coefficients are also small enough for corrections to be trusted.

3. It is very desirable to get the physical interpretation and motivation for the value and nature of the higher order corrections (especially, for cumulant moments).

4. The QCD equations or approximations used in the jet calculus are sometimes not completely precise themselves. Their modifications can be considered or the influence of the omitted terms estimated. This can lie at the origin of the problems discussed above.

5. Some shortcomings of the analytic approach are discussed.

6. The property of the asymptotical freedom of QCD is briefly described and, if violated, the typical energy scales are found and possible tests discussed.

I will be mainly concerned with jet characteristics in some sense related to the jet multiplicity distributions that are closer to my personal interests. First, let me remind some simplest definitions [6, 1] concerning jet multiplicities in QCD. The generating function G is defined by the formula

$$G(y,u) = \sum_{n=0}^{\infty} P_n(y) u^n, \tag{1}$$

where $P_n(y)$ is the multiplicity distribution at the scale $y = \ln(p\Theta/Q_0) = \ln(2Q/Q_0)$, p is the initial momentum, Θ is the angle of the divergence of the jet (jet opening angle), assumed here to be fixed, Q is the jet virtuality, $Q_0 =$ const, u is an auxiliary variable which is often omitted to shorten notations. The analytic properties of the generating functions in u are of the special interest (see [1, 3]) in view of some analogies with the statistical physics, but we will not consider them here.

The moments of the distribution are defined as

$$F_q = \frac{\sum_n P_n n(n-1)...(n-q+1)}{(\sum_n P_n n)^q} = \frac{1}{\langle n \rangle^q} \cdot \frac{d^q G(y,u)}{du^q}\Big|_{u=1}, \tag{2}$$

$$K_q = \frac{1}{\langle n \rangle^q} \cdot \frac{d^q \ln G(y,u)}{du^q}\Big|_{u=1}. \tag{3}$$

Here, F_q are the factorial moments, and K_q are the cumulant moments, responsible for total and genuine (irreducible to lower ranks) correlations, correspondingly. These moments are not independent. They are connected by definite relations which can easily be derived from moments definitions in terms of the generating function:

$$F_q = \sum_{m=0}^{q-1} C_{q-1}^m K_{q-m} F_m. \tag{4}$$

The QCD equations for the generating functions are[1]:

$$G'_G = \int_0^1 dx K_G^G(x)\gamma_0^2[G_G(y+\ln x)G_G(y+\ln(1-x)) - G_G(y)]$$
$$+ \quad n_f \int_0^1 dx K_G^F(x)\gamma_0^2[G_F(y+\ln x)G_F(y+\ln(1-x)) - G_G(y)], \quad (5)$$

$$G'_F = \int_0^1 dx K_F^G(x)\gamma_0^2[G_G(y+\ln x)G_F(y+\ln(1-x)) - G_F(y)]. \quad (6)$$

Here $G'(y) = dG/dy$, n_f is the number of active flavours,

$$\gamma_0^2 = \frac{2N_c\alpha_S}{\pi}. \quad (7)$$

The running coupling constant in the two-loop approximation is

$$\alpha_S(y) = \frac{2\pi}{\beta_0 y}\left(1 - \frac{\beta_1}{\beta_0^2}\cdot\frac{\ln 2y}{y}\right) + O(y^{-3}), \quad (8)$$

where

$$\beta_0 = \frac{11N_c - 2n_f}{3}, \qquad \beta_1 = \frac{17N_c^2 - n_f(5N_c + 3C_F)}{3}. \quad (9)$$

The labels G and F correspond to gluons and quarks, and the kernels of the equations are

$$K_G^G(x) = \frac{1}{x} - (1-x)[2 - x(1-x)], \quad (10)$$

$$K_G^F(x) = \frac{1}{4N_c}[x^2 + (1-x)^2], \quad (11)$$

$$K_F^G(x) = \frac{C_F}{N_c}\left[\frac{1}{x} - 1 + \frac{x}{2}\right], \quad (12)$$

N_c=3 is the number of colours, and $C_F = (N_c^2-1)/2N_c = 4/3$ in QCD.

Herefrom, one can get equations for any moment of the multiplicity distribution both for quark and gluon jets. One should just equate the terms with the same powers of u in both sides of the equations. In particular, the equations for average multiplicities read

$$\langle n_G(y)\rangle' = \int dx\gamma_0^2[K_G^G(x)(\langle n_G(y+\ln x)\rangle + \langle n_G(y+\ln(1-x))\rangle - \langle n_G(y)\rangle)$$
$$+ n_f K_G^F(x)(\langle n_F(y+\ln x)\rangle + \langle n_F(y+\ln(1-x))\rangle - \langle n_G(y)\rangle)], \quad (13)$$

[1]To exclude the nonperturbative region from further consideration, the limits of integration in these equations are often chosen as $\exp(-y)$ and 1- $\exp(-y)$ which tend to 0 and 1 at high energy y.

$$\langle n_F(y)\rangle' = \int dx\gamma_0^2 K_F^G(x)(\langle n_G(y+\ln x)\rangle + \langle n_F(y+\ln(1-x))\rangle - \langle n_F(y)\rangle). \quad (14)$$

Their solutions can be looked for as

$$\langle n_{G,F}\rangle \propto \exp(\int^y \gamma_{G,F}(y\prime)dy\prime). \quad (15)$$

The lower limit of integration has not been fixed because its variation results in the substitution of a new normalization constant which is not shown in the above relation but is in practice considered as a fitted parameter which depends on the nonperturbative component of the underlying dynamics of a process.

Using the perturbative expansion of the exponent in (15)

$$\gamma_G \equiv \gamma = \gamma_0(1 - a_1\gamma_0 - a_2\gamma_0^2 - a_3\gamma_0^3) + O(\gamma_0^5), \quad (16)$$

one arrives to the so-called modified perturbative expansion of QCD. This means that the perturbative expansion has been used in the exponent of the expression for a physical quantity, i.e., even the first term includes higher power corrections of the ordinary perturbative formulas. Moreover, the expansion parameter is the coupling strength itself and not its squared value α_S as usually happens. The structure of the equations (5), (6) dictates such series. Let me stress that the ordinary perturbative expansion for mean multiplicity, if boldly attempted, would surely fail because the coupling strength decreases with energy while multiplicities increase mainly due to the enlarged phase space volume. The coefficients a_i are calculable from the eqs (13), (14). This method was first proposed and used for higher order calculations in gluodynamics in [7].

Let us briefly mention that the equations (5), (6) can be exactly solved [8, 9] for fixed coupling strength, i.e., if γ_0 is set constant. Then the mean multiplicities increase like a power of energy.

For the running coupling strength the multiplicities increase [6,10,11] slower than power-like but stronger than logarithmically, namely

$$\langle n_{G,F}\rangle = A_{G,F}y^{-a_1c^2}\exp(2c\sqrt{y} + \delta_{G,F}(y)), \quad (17)$$

where $c = (4N_c/\beta_0)^{1/2}$,

$$\delta_G(y) = \frac{c}{\sqrt{y}}[2a_2c^2 + \frac{\beta_1}{\beta_0^2}(\ln 2y+2)] + \frac{c^2}{y}[a_3c^2 - \frac{a_1\beta_1}{\beta_0^2}(\ln 2y+1)]) + O(y^{-3/2}). \quad (18)$$

The corresponding expression for $\delta_F(y)$ can be easily obtained from the formulas for γ_F. The relation between the anomalous dimensions γ of gluon and quark jets is

$$\gamma_F = \gamma - \frac{r'}{r}, \tag{19}$$

where

$$r' \equiv dr/dy = Br_0 r_1 \gamma_0^3 [1 + \frac{2r_2}{r_1}\gamma_0 + (\frac{3r_3}{r_1} + B_1)\gamma_0^2 + O(\gamma_0^3)] \tag{20}$$

with $r_0 = \frac{N_c}{C_F} = 9/4$; $B = \beta_0/8N_c$; $B_1 = \beta_1/4N_c\beta_0$.

Thus

$$\gamma_F = \gamma_0[1-a_1\gamma_0-(a_2+Br_1)\gamma_0^2-(a_3+2Br_2+Br_1^2)\gamma_0^3-(a_4+B(3r_3+3r_2r_1+B_1r_1+r_1^3))\gamma_0^4]. \tag{21}$$

Usually, in place of γ_F the ratio of average multiplicities in gluon and quark jets

$$r = \frac{\langle n_G \rangle}{\langle n_F \rangle} = \frac{A_G}{A_F}\exp(\delta_G(y) - \delta_F(y)) \tag{22}$$

is introduced, and its perturbative expansion

$$r = r_0(1 - r_1\gamma_0 - r_2\gamma_0^2 - r_3\gamma_0^3) + O(\gamma_0^4) \tag{23}$$

is used. The analytic expressions and numerical values of the parameters a_i, r_i for all $i \leq 3$ have been calculated from the perturbative solutions of the above equations. All of them are at least twice less than 1 (the review is given in [3]).

1. **Sensitivity to high order terms.** Within these approximations the experimental data about the energy dependence of mean multiplicity in e^+e^--annihilation are well described. The two leading terms in expressions (17) completely determine it. They are the same for quark and gluon jets. That is why gluodynamics can be used for their estimate as was done in early years. The higher order corrections given by $\delta_{G,F}$ are almost unnoticeable there. Thus mean multiplicities are not sensitive to these corrections by themselves.

However, if one considers their ratio r, it happens to be really sensitive. This is because in the ratio the two leading terms corresponding to leading order (LO) and next-to-leading order (NLO) cancel since they are the same for both quark and gluon jets. Therefore, only higher order corrections determine the energy behaviour of the ratio r. The first term $r_0 = C_A/C_F = 9/4$ is given by the relative strengths of gluon and quark forces. It is the same both in LO and NLO approximations. The next term is proportional to γ_0 and will be called NLO_r-correction in distinction to common NLO-terms. Actually, NLO_r corresponds to 2NLO-terms (like a_2) because of the cancellation of the NLO (power-like in y) terms in the ratio of the expressions for multiplicities. In the same sense the "r_3"-term in r corresponds to 4NLO contribution in γ even though it is proportional to γ_0^3 etc (see [11]). This leads to shift and misuse of the terminology for the anomalous dimensions γ's and for the ratio r.

Thus we have found the characteristics which is more sensitive to higher order corrections than the original formulas for mean multiplicities. The experimental data about the ratio r are described with much lower accuracy about 15% in such an analytic approach. Even though each subsequent perturbative term in r improves the agreement, no precise fit has been achieved yet. This is one of the problems.

However, one should mention here that the computer solution of the equations [12, 13] provides the quantitative fit. This poses the question about the accuracy of perturbative approximations for this particular characteristics and indicates that the higher order uncalculated corrections are still comparatively large for this ratio up to the highest presently available energies.

Another very sensitive characteristics is the behaviour of the factorial moments (2) as functions of the size of the phase space bins in which they are measured. Now one has to deal with a part of the phase space and the above equations are not applicable directly. One has to use the Feynman diagram technique for the treatment of these small bins [14–16]. This complicates the matter. It was impossible to pass to high order corrections. Some NLO-terms have only been taken into account [15]. From comparison with experiment (see, e.g., [17–19]) it is seen that the qualitative behaviour is described but quantitatively the disagreement becomes stronger at smaller bins. This poses the problem of the proper account of higher order corrections. Possible flow of partons from small bins should be considered more precisely. The newly developed technique of the so-called non-global logarithms [20] can be helpful in this respect.

2. **High order coefficients.** Fortunately, the coefficients a_i and r_i happened to be small enough (see the Table in [3]) so that the subsequent terms in the expansions can be trusted even at the rather large values of the expansion parameter $\gamma_0 \approx 0.4 - 0.5$ at present energies. This is not always the case. If the high order terms become larger than 1, the expansion can not be trusted. Thus the next problem is to find such characteristics in which it does not happen. Only these features can be reliably compared with experiment.

This criterium becomes crucial, e.g., for the slope r' of the ratio r. The cancellation of two leading terms in the ratio r reveals itself in the proportionality of the scale (energy) derivative r' to γ_0^3. Therefore it can be calculated up to the terms $O(\gamma_0^5)$. The leading term is very small (about 0.02 at the Z^0-resonance). Asymptotically, all corrections vanish. However, at present energies of Z^0, they are so large that calculations become unreliable. The second term in the brackets in (20) is larger than 1 since $2r_2/r_1 \approx 4.9$ and $\gamma \approx 0.45 - 0.5$. Even the third term is approximately about 0.4. The problem of convergence of the series at Z^0-energies and below becomes crucial.

Therefore, it is desirable to use at present energies such characteristics which are sensitive to these corrections and do not possess large coefficients in front of the expansion parameter. In particular, it has been shown in [11] that these coefficients are smaller in the ratio of derivatives (slopes)

$$r^{(1)} = \frac{\langle n_G \rangle'}{\langle n_F \rangle'}. \tag{24}$$

This ratio should be slightly larger than r

$$r^{(1)} \approx r(1 + Br_1\gamma_0^2) \approx r(1 + 0.07\gamma_0^2). \tag{25}$$

The same is true for the ratio of curvatures (or second derivatives)

$$r^{(2)} = \frac{\langle n_G \rangle''}{\langle n_F \rangle''}. \tag{26}$$

It is even closer to the asymptotics

$$r^{(2)} \approx r(1 + 2Br_1\gamma_0^2) \approx r(1 + 0.14\gamma_0^2). \tag{27}$$

The QCD predictions for them

$$r < r^{(1)} < r^{(2)} < 2.25 \tag{28}$$

have been confirmed in experiment.

The present experimental accuracy does not allow, unfortunately, to measure these values more accurately. As one sees, in expressions for $r^{(1)}, r^{(2)}$ the coefficients in front of γ_0^2 are slightly decreased compared with r but not in front of γ_0. The last ones cancel in their ratios to r so that the second order terms are left. However, these ratios $r^{(1)}/r$ and $r^{(2)}/r$ have not yet been accurately measured. Further search for such characteristics is needed.

3). **Interpretation.** Another question I'd like to raise concerns physical interpretation of the high order effects. First of all I mean the oscillations of cumulant moments as functions of their rank in QCD. They have not yet been completely clarified. They were predicted [7,21] as the effect of the high order terms of the modified perturbative expansion. First experimental confirmation was found in [22,23].

Usually exploited phenomenological distributions of the probability theory do not possess any oscillations. E.g., all cumulant moments of the Poisson distribution are identically zero. One interprets this as the absence of genuine correlations irreducible to the lower-rank correlations. For the negative binomial distribution one easily gets

$$H_q = \frac{K_q}{F_q} = \frac{2}{q(q+1)} > 0. \tag{29}$$

Since F_q are always positive according to their definition, this inequality implies the positive values of K_q.

In the leading order approximation, the gluodynamics equation for the generating function

$$[\ln G(y)]'' = \gamma_0^2(G(y) - 1) \tag{30}$$

transforms in the relation

$$q^2 K_q = F_q \quad or \quad H_q = \frac{1}{q^2}. \tag{31}$$

However already in the next-to-leading order H_q-moments become negative with a minimum at the rank $q_{min} \approx \frac{24}{11\gamma_0} + 0.5 \approx 5$ [7]. This minimum is rather stable. Nevertheless this is a purely preasymptotic feature. The minimum slowly moves to higher ranks with energy increase and disappears in asymptotics as is required according to the formula (31). At higher orders of the perturbative expansion, the oscillations of higher rank cumulant moments show up [21]. They have been confirmed in experiment.

Both the role of conservation laws and the changing character of the genuine correlations (described by the cumulants) can be blamed as originating these oscillations. If the latter factor is important it would imply that attraction (clustering) is replaced by repulsion (and vice versa) in particle systems with different number of particles. It would be exciting to find other examples of such a behaviour in hadronic systems.

We are also interested to get from experiment the data about the energy behaviour of the ratios H_q or, better, of the asymptotically normalized ratios

$D_q = q^2 H_q$ which should tend to 1 in asymptotics independently of q. It would ask for high precision data at different energies.

4). **Generalization.** Finally, there exists the problem of possible generalization of the equations for the generating functions. As such, the Eqs (5), (6) have only been proved (see, e.g., [6]) up to the NLO-approximation. In principle, their high order treatment is unjustifued. Nevertheless, one can assume that these equations have the status of the kinetic equations of QCD and studied in this approach.

From one side, we understand that even if treated as kinetic equations these equations are limited by our ignorance of the four-gluon interaction and non-perturbative effects, by the simplified treatment of conservation laws etc. Actually, the energy conservation is only approximately accounted by the $\ln x$ and $\ln(1-x)$ terms in the equations. In the perturbative expansion we consider the Taylor expansions of the generating functions. Thus we approximate further the energy conservation. Namely this reveals itself in factorial moments behaviour for small bins and in the oscillations of cumulant moments. In the computer solutions [12, 13] the kinematic restrictions are precisely considered and the results also show better precision. Thus, probably the inaccuracies of the analytic approach are connected just with the improper treatment of the kinematic boundaries.

Some phenomenological approaches to avoid these limitations were attempted from the very beginning [24–26]. In [24] it was proposed to treat hadronization of partons at the final stage of jet evolution in analogy with the ionization in electromagnetic cascades where it results in their saturation and in the finite length of the shower. This leads to some modified equations if the analogy between ionization losses in QED and deconfinement in QCD is imposed. Three different stages of the cascade were considered in the modified kinetic equations proposed in [25, 26]. No quantitative results were, however, obtained.

The more successful modification of above equations was proposed [27] in the framework of the dipole approach to QCD with more accurate kinematic bounds accounting for the transverse momenta as well. It has been shown that the ratio r can be obtained in good agreement with experimental data. Nevertheless, further study [28] of higher rank moments of the multiplicity distribution predicted by the modified equations has shown their extremely high sensitivity to higher orders of the perturbative expansion. The results become inconclusive.

Thus no successful generalization is at work nowadays. Rather, the general theoretical trend has shifted to the direct calculation of non-perturbative effects in some jet characteristics (see, e.g., [29,30]) and to understanding effects described by the non-global logarithms [20].

From another side, the success of numerical solutions of the existing equations [12, 13] raises the question if the generalization will give any other noticable contribution. Probably, our failure to describe more precisely the ratio r could be just due some defects of the purely perturbative expansion at available energies. More rigorous treatment of the numerical solutions of the equations should be done. Moreover, it was claimed recently [31] that the renormalization group improvement of the perturbative results gives rise to good description of experimental data.

5). **The shortcomings of the analytic approach.** Apart from the non-perturbative corrections there are terms which are not directly taken into account in the analytic approach. The high order terms considered above correspond to corrections only due to more accurate treatment of the energy conservation and of the two-loop expression for the coupling strength (the term with β_1 in (8), (18) considered). It is still impossible to consider in a proper way the transverse momenta. No high order terms have been added to the kernels (10), (11), (12). The four-gluon vertex has been also ignored. All these shortcomings provide the problems for further studies.

6). **The asymptotical freedom of QCD.** In principle, the asymptotical value of the QCD coupling strength at infinite momentum transfer could be not equal to zero as it follows from Eq. (8) but a finite value. Fortunately, the upper bound on it is low enough for jet calculations at present energies to be safe enough when using the expression (8). Some problems can arise in jet calculus only at unachivable superhigh energies (see [32]). However, the typical scales of possible violation of the asymptotic freedom are at the TeV energy region available at the next generation of electron colliders.

In conclusion, I'd say that the practical accuracy of the pQCD calculations is high enough. This is somewhat surprising in view of the rather large value of the expansion parameter at present energies. They can serve as a good estimate of the background in searches for new physics effects. However some principal questions concerning the calculation of several properties of quark-gluon jets and the validity of QCD equations for the generating functions at higher orders has not yet been resolved.

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String representation of the dual Ginzburg-Landau theory and confinement

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Abstract — The effective string action of the color-electric flux tube in the dual Ginzburg-Landau (DGL) theory is studied by performing a path-integral analysis and taking into account the finite thickness of the flux tube. A modified Yukawa interaction appears as a boundary contribution and is reduced into the ordinary Yukawa interaction in the London Limit.

1 Introduction

The construction of a realistic low-energy Lagrangian for hadrons based on quantum chromodynamics (QCD) clearly requires a deeper understanding of the mechanism of confinement. A very useful concept for an analytical description of this phenomenon is the dynamical scheme of a dual superconductor proposed more than twenty years ago by 't Hooft and Mandelstam [1,2]. This approach emphasizes, in particular, the role of magnetic monopoles for confinement. The condensation of monopoles squeezes the chromoelectric flux into (open) Abrikosov-Nielsen-Olesen (ANO) type vortices which then confine the quark and antiquark sitting at their ends. Recent studies in lattice QCD in the maximally Abelian gauge indeed suggest remarkable properties of the QCD vacuum, such as Abelian dominance and monopole condensation. This confirms the dual superconductor picture, which is suitably described by the dual Ginzburg-Landau (DGL) theory [3,4]. The DGL theory is here obtained by Abelian projection which is a crucial step in order to find the relevant IR degrees of freedom of QCD.

In the present talk we review recent analytical studies of the confinement mechanism leading to a path-integral derivation of effective string actions from the U(1) and U(1)×U(1) DGL theory [5,6]. As has been shown earlier (see e.g. [7–10] and references therein), the corresponding path-integral approach is essentially simplified in the so-called London limit of large monopole self-coupling $\lambda \to \infty$ (large monopole mass $m_\chi \to \infty$), where the modulus of the magnetic monopole field is frozen to its v.e.v., and ANO vortices become infinitely thin (core radius $\rho = (m_\chi)^{-1} \to 0$). In this limit the DGL theory can be suitably reformulated as a theory of a massive antisymmetric Kalb-Ramond (KR) field interacting with surface elements of the world-sheet swept out by the ANO vortex (Dirac string). By performing finally a derivative expansion of the resulting effective action, it was possible to derive the Nambu-Goto (NG) action including a correction (rigidity) term, and to estimate field strength correlators of the DGL theory. Obviously, the London limit picture is only appropriate for large transverse distances from the vortex (string), where the thickness of the core and the corresponding contribution to the field energy per unit length (string tension) are neglected. Moreover, since the monopole field in this approximation is nowhere vanishing, one gets a massive dual gluon leading to a Yukawa interaction term in addition to the confining potential. Clearly, it is a challenge to go beyond the London limit in the sense of taking into account the finite thickness of vortices and the vanishing of the monopole field inside the vortex core. One might then naturally ask, whether one gets besides of the confining potential a Coulomb potential (as used in quarkonium spectroscopy) instead of the Yukawa one, or possibly something between them.

The main goal of this talk is to show how to extend the usual path-integral approach as much as possible beyond the London limit, paying special attention to the treatment of boundary terms related to the nonconfining (shorter range) part of the potential.

Finally, in the second part of the talk, we are going to generalize the discussion of the U(1) model by investigating the U(1)×U(1) DGL theory beyond the London limit. Note that the Weyl symmetric formulation reduces the U(1)×U(1) DGL theory here into the sum of three types of the U(1) theory, which was quite useful to simplify the treatment of the U(1)×U(1) case and the investigation of the flux-tube solution [11,12].

2 U(1) DGL theory

In this section, we shall review the derivation of the effective string action of the U(1) DGL theory [5] which, in distinction to earlier work [7–10], does not employ the London limit. We first briefly explain how one obtains the flux-tube solution and then discuss how one arrives at its effective action *with taking into account the finite thickness* by performing the path integral transformation.

2.1 Classical flux-tube solution

The U(1) DGL action in Euclidean space-time has the form

$$S = \frac{\beta_g}{2}(F)^2 + (d\phi)^2 + ((B + d\eta)\phi)^2 + \lambda(\phi^2 - v^2)^2, \tag{1}$$

Table 1: Ingredients of the U(1) DGL theory in the differential-form notation.

dual gauge field	1-form	$B \equiv B_\mu dx_\mu$
monopole field	0-form	$\chi \equiv \phi \exp(i\eta)$
electric Dirac string	2-form	$\Sigma \equiv \frac{1}{2}\Sigma_{\mu\nu} dx_\mu \wedge dx_\nu$
electric current	1-form	$j \equiv j_\mu dx_\mu$

Table 2: Definitions in differential forms in $\mathbb{R}^4$.

r-form $(0 \le r \le 4)$	ω_r	$\omega_r \equiv \frac{1}{r!}\omega_{\mu_1 \dots \mu_r} dx_{\mu_1} \wedge \cdots \wedge dx_{\mu_r}$
exterior derivative	d	r-form $\mapsto$ $(r+1)$-form
Hodge star	$*$	r-form $\mapsto$ $(4-r)$-form
	$**$	multiply a factor $(-1)^r$ for r-form
codifferential	$\delta \equiv - * d *$	r-form $\mapsto$ $(r-1)$-form
Laplacian	$\Delta \equiv d\delta + \delta d$	r-form $\mapsto$ r-form
Inner product	$(\omega_r, \eta_r) \equiv \int \omega_r \wedge * \eta_r$	$(\omega_r, \eta_r) = \frac{1}{r!}\int d^4x \omega_{\mu_1 \dots \mu_r} \eta_{\mu_1 \dots \mu_r}$ $(\omega_r)^2 \equiv (\omega_r, \omega_r)$

where the dual field strength F is expressed as

$$F = dB - 2\pi * \Sigma. \tag{2}$$

We summarize the ingredients of this theory in Table 1 and useful differential form notations in Table 2. In the dual formulation external quark sources, corresponding to the q-$\bar{q}$ system, have been introduced as an *open* electric Dirac string Σ, the boundary of which describes the trajectories of the quarks. In fact, the presence of the electric charge is seen from the violation of the dual Bianchi identity as

$$dF = -2\pi d * \Sigma = -2\pi * \delta\Sigma = 2\pi * j \neq 0, \tag{3}$$

where the relation $\delta\Sigma = -j$ is used. Clearly, if there is no external electric current, one must set $\Sigma = 0$. The inverse of the dual gauge coupling is denoted by $\beta_g = 1/g^2$, the strength of the self-interaction of the monopole field by λ, and the monopole condensate by v. These couplings are related to the mass of the dual gauge boson and the monopole mass as $m_B \equiv \sqrt{2/\beta_g} v = \sqrt{2} g v$ and $m_\chi = 2\sqrt{\lambda} v$. The ratio of these masses is the Ginzburg-Landau (GL) parameter $\kappa = m_\chi / m_B$, which determines the type of the dual superconductivity. The value $\kappa < 1$ ($\kappa > 1$) describes the type-I (type-II) vacuum.

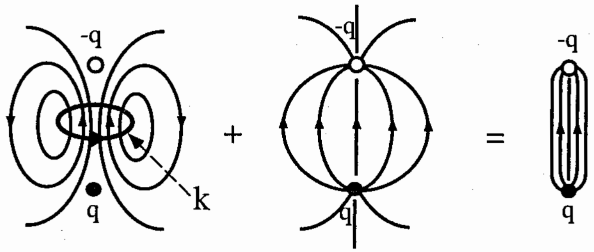

Figure 1: The composed structure of the electric field inside a flux tube, which is represented by $dB^{\rm reg}$ (left) $+\,2\pi\Delta^{-1}\delta * j$ (middle) $= F$ (right)

To find the flux-tube solution associated with Σ, it is convenient to decompose the dual gauge field as

$$B = B^{\rm reg} + B^{\rm sing}, \tag{4}$$

where the singular part has the explicit form

$$B^{\rm sing} \equiv 2\pi\Delta^{-1}\delta * \Sigma. \tag{5}$$

By using the relation $d\Delta^{-1}\delta + \delta\Delta^{-1}d = 1$ and the equation $\delta\Sigma = -j$, we find

$$dB^{\rm sing} = 2\pi * \Sigma + 2\pi\Delta^{-1}\delta * j = 2\pi(*\Sigma + *C), \tag{6}$$

where $*C = \Delta^{-1}\delta * j$ is the 2-form field. The dual field strength is then written as

$$F = dB^{\rm reg} + 2\pi * C. \tag{7}$$

The physical meaning of $*C$ will be clear if one computes its contribution to the action. Using $\delta * C = 0$, one obtains $(F)^2 = (dB^{\rm reg})^2 + 4\pi^2(*C)^2$, where the second term is further evaluated as $(*C)^2 = (j, \Delta^{-1}j)$, which is nothing but the Coulombic interaction of electric currents. This means that $*C$ is the *Coulomb electric field* originating from the electric quark charges [1].

The field equations are given by

$$\beta_g\partial_\mu(\partial_\mu B_\nu^{\rm reg} - \partial_\nu B_\mu^{\rm reg}) = 2(B_\nu^{\rm reg} + B_\nu^{\rm sing})\phi^2 = k_\nu, \tag{8}$$

$$\partial_\mu\partial_\mu\phi + (B_\mu^{\rm reg} + B_\mu^{\rm sing})^2\phi = 2\lambda\phi(\phi^2 - v^2), \tag{9}$$

[1] Note that the equality of F in Eqs. (2), (7) does not at all mean that the Dirac string world sheet is simply replaced by the Coulombic electric field of the quark charges, since both objects are combined with different fields B and $B^{\rm reg}$ having different boundary conditions.

where the phase η of the monopole field is absorbed into $B^{\rm reg}$. The boundary conditions of $B^{\rm reg}$ and ϕ are determined so as to make the energy of the system finite: just on Σ, $B^{\rm reg}=0$ and $\phi=0$, whereas at large distances from the string, $B^{\rm reg}=-B^{\rm sing}$ and $\phi=v$. The r.h.s. of Eq. (8) is identified as the monopole supercurrent, which circulates around the azimuthal direction of the q-$\bar{q}$ axis. It then leads to the *solenoidal electric field* $(\partial\wedge B^{\rm reg})_{\mu\nu}$, which plays an important role to cancel the Coulombic field $^*C_{\mu\nu}$ at distant place from the string. The total electric field thus forms a flux-tube structure (see Fig. 1). Clearly, the width of the flux tube depends on the strength of $(\partial\wedge B^{\rm reg})_{\mu\nu}$ and hence on the profile function of ϕ near the string. These two widths are characterized by the inverse of masses of the dual gauge boson m_B^{-1} and the monopole m_χ^{-1}, respectively. The standard London limit approximation, considered in [7,9,10] corresponds to taking $\phi=v$ everywhere, which will be realized when $\lambda\to\infty$ ($m_\chi\to\infty$).

2.2 Path integration towards the effective string action

Let us next perform the path integration of fields with taking into account the finite thickness (width) of the flux tube. The starting partition function is given by

$$\mathcal{Z}(\Sigma) \;=\; \int \mathcal{D}B\mathcal{D}\chi\mathcal{D}\chi^*\exp\left[-S(B,\chi,\chi^*,\Sigma)\right]. \tag{10}$$

The gauge fixing term and corresponding Faddeev-Popov determinant is omitted here for simplicity.

We start from the linearization of the square term $((B^{\rm reg}+B^{\rm sing}+d\eta)\phi)^2$ by means of a 1-form auxiliary field E as

$$\begin{aligned}&\exp\left[-\left((B^{\rm reg}+B^{\rm sing}+d\eta^{\rm reg})\phi\right)^2\right]\\&=\phi^{-4}\int\mathcal{D}E\exp\left[-\left\{(E,\frac{1}{4\phi^2}E)-i(E,B^{\rm reg}+B^{\rm sing}+d\eta^{\rm reg})\right\}\right].\end{aligned} \tag{11}$$

Then, based on the relation $(E,d\eta)=(\delta E,\eta)$, we can integrate over the regular part of the phase η, which leads to the delta functional $\delta[\delta E]$ in the integration measure. The constraint on E can be resolved by introducing the 2-form KR field h as

$$\delta[\delta E]=\int\mathcal{D}h\delta[\delta h-f_h]\delta[E-\delta * h], \tag{12}$$

where the first delta functional is to avoid the overcounting in the integration over h, which is due to the hyper-gauge invariance $h\mapsto h+d\Lambda$ with 1-form field Λ. Now, we can immediately perform the integration over the auxiliary field E and then $B^{\rm reg}$:

$$\mathcal{Z}(\Sigma)=\int \phi^{-3}\mathcal{D}\phi\mathcal{D}h\delta[\delta h-f_h]\exp\Big[-\Big\{2\pi^2\beta_g(j,\Delta^{-1}j)+(d\phi)^2+(dh,\frac{1}{4\phi^2}dh)$$
$$+\frac{1}{2\beta_g}(h)^2-\frac{1}{2\beta_g}(\delta h,\Delta^{-1}\delta h)-2\pi i(h,\Sigma)-2\pi i(\delta h,\Delta^{-1}j)+\lambda(\phi^2-v^2)^2\Big\}\Big].\quad(13)$$

For further evaluation, we divide the action into three parts as

$$S=S^{(1)}+S^{(2)}+S^{(3)},\quad(14)$$

where each action is defined by

$$S^{(1)}=2\pi^2\beta_g(j,\Delta^{-1}j),\quad(15)$$
$$S^{(2)}=(d\phi)^2+(dh,\frac{1}{4}\{\frac{1}{\phi^2}-\frac{1}{v^2}\}dh)+\lambda(\phi^2-v^2)^2,\quad(16)$$
$$S^{(3)}=\frac{1}{4v^2}(dh)^2+\frac{1}{2\beta_g}(h)^2-\frac{1}{2\beta_g}(\delta h,\Delta^{-1}\delta h)-2\pi i(h,\Sigma)-2\pi i(\delta h,\Delta^{-1}j).\quad(17)$$

The first term $S^{(1)}$ leads to the pure Coulomb potential (pure boundary contribution) for the static q-$\bar{q}$ system. The second term $S^{(2)}$ is defined so as to give a zero contribution to the effective string action in the case that $\phi=v$, which usually corresponds to taking the London limit. In other words, this term leads for finite λ to a nonvanishing contribution due to the finite thickness of the the flux tube, inside which the monopole field smoothly becomes zero, $\phi=0$. The third term $S^{(3)}$ is mainly responsible for the field contributions outside the core, near the surface of the flux tube, which remains even in the case that the monopole field has a constant value, $\phi=v$.

In order to discuss the effective string action let us consider here the case that $m_B<m_\chi$, but m_χ is finite. For the rough space-time structure, whose mesh size is larger than m_χ^{-1}, the London limit picture based on a dominating expression $S^{(3)}_{>m_\chi^{-1}}$ is valid, where the subscript means that one has to integrate over transverse distances from the string, $\rho>m_\chi^{-1}$, with the monopole mass m_χ chosen as an effective cut-off $\Lambda_{\rm eff}$. Clearly, in this case, we cannot see the inside structure of the flux tube. On the other hand, to see the finer structure of the flux tube, variations of the monopole field in $S^{(2)}$ should be taken into account. The effective action of the vortex "core" contribution is described by $S^{(2)} + S^{(3)}_{<m_\chi^{-1}}$, and its leading term contains the NG action with the string tension $\sigma_{\rm core}$ and a current term $S_{\rm core}(j)$,

$$S_{\rm core} = S_{\rm core}(j) + \sigma_{\rm core} \int d^2\xi \sqrt{g(\xi)}, \tag{18}$$

where ξ^a $(a = 1, 2)$ parametrize the string world sheet described by the coordinate $\tilde{x}_\mu(\xi)$, and $g(\xi)$ is the determinant of the induced metric, $g_{ab}(\xi) \equiv \frac{\partial \tilde{x}_\mu(\xi)}{\partial \xi^a}\frac{\partial \tilde{x}_\mu(\xi)}{\partial \xi^b}$. The string tension $\sigma_{\rm core}$ is controlled by the solution of field equations derived from the action in the core region. Note that $S_{\rm core}(j)$ results only from $S^{(3)}_{<m_\chi^{-1}}$ [2].

Let us evaluate the "surface" contribution described by $S^{(3)}_{>m_\chi^{-1}}$. To do this, we first integrate out the KR field, and then extract the surface contribution from it by taking into account a suitable regularization in transverse variables. Since the action $S^{(3)}$ does not depend on the monopole modulus ϕ, and it is gaussian with respect to h, we can easily integrate out h. The resulting effective action from the surface contribution is

$$S^{(3)}_{>m_\chi^{-1}} = 4\pi^2 v^2 \left(\Sigma, D\Sigma\right)\Big|_{>m_\chi^{-1}} + 2\pi^2 \beta_g \left(j, \left\{D - \Delta^{-1}\right\} j\right)\Big|_{>m_\chi^{-1}}, \tag{19}$$

where we have defined the propagator of the massive KR field $D \equiv (\Delta + m_B^2)^{-1}$. Note that "$|_{>m_\chi^{-1}}$" means that a corresponding effective cutoff (mesh size) should be taken into account. One finds that the first term represents the interaction between world sheet elements of the electric Dirac string via the propagator of the massive KR field.

By the Taylor (derivative) expansion of the propagator of the KR field, with respect to the Laplacian on the string world sheet

$$\Delta_\xi = -\frac{1}{\sqrt{g(\xi)}} \partial_a g^{ab}(\xi) \sqrt{g(\xi)} \partial_b, \tag{20}$$

we obtain from the first term of Eq. (19) the explicit form

$$\begin{aligned} 4\pi^2 v^2 \left(\Sigma, D\Sigma\right)\Big|_{>m_\chi^{-1}} &= \sigma_{\rm surf} \int d^2\xi \sqrt{g(\xi)} \\ &+ \alpha_{\rm surf} \int d^2\xi \, \sqrt{g(\xi)} g^{ab}(\xi) \left(\partial_a t_{\mu\nu}(\xi)\right) \left(\partial_b t_{\mu\nu}(\xi)\right) + \mathcal{O}(\Delta_\xi^2). \end{aligned} \tag{21}$$

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[2]If one approximately considers a vortex core with radius m_χ^{-1} in which $\phi = 0$ and $dh = 0$ everywhere as on the string world sheet, one finds $S_{\rm core}(j) = 0$, since $S^{(3)}_{<m_\chi^{-1}} = 0$.

The first term of the r.h.s. represents the NG action with the string tension

$$\sigma_{\text{surf}} = \pi v^2 \ln \frac{m_B^2 + m_\chi^2}{m_B^2} = \pi v^2 \ln\left(1+\kappa^2\right), \tag{22}$$

and the second term is the so-called rigidity term with the negative coefficient

$$\alpha_{\text{surf}} = \frac{\pi v^2}{2}\left(\frac{1}{m_\chi^2 + m_B^2} - \frac{1}{m_B^2}\right) = -\frac{\pi \beta_g}{4}\frac{\kappa^2}{1+\kappa^2} \qquad (<0), \tag{23}$$

where $\kappa = m_\chi/m_B$ is the GL parameter. Note that the rigidity term appears as a first order contribution in the derivative expansion with the Laplacian Δ_ξ.

The second term of Eq. (19), which is induced from the boundary of the string world sheet, is evaluated in a similar "effective regularization" scheme. Finally, by combining it with the pure Coulomb term $S^{(1)}$, we may write the effective string action

$$\begin{aligned} S_{\text{eff}}(\Sigma) &= S(j) + (\sigma_{\text{core}} + \sigma_{\text{surf}}) \int d^2\xi \sqrt{g(\xi)} \\ &\quad + \alpha_{\text{surf}} \int d^2\xi \, \sqrt{g(\xi)} g^{ab}(\xi) \left(\partial_a t_{\mu\nu}(\xi)\right) \left(\partial_b t_{\mu\nu}(\xi)\right) + \mathcal{O}(\Delta_\xi^2), \end{aligned} \tag{24}$$

where the boundary (electric current) contribution is given by

$$S(j) = S_{\text{core}}(j) + \frac{1}{2\beta_e}(j, \Delta^{-1} j) + \frac{1}{2\beta_e}\left(j, \left\{D - \Delta^{-1}\right\} j\right)\Big|_{>m_\chi^{-1}}. \tag{25}$$

Here, $\beta_e = 4/e^2$ is introduced instead of β_g through the Dirac quantization condition $eg = 4\pi$. Clearly, only in the London limit $m_\chi \to \infty$, $S_{\text{core}}(j)$ vanishes and the Coulomb contribution is completely cancelled, so that $S(j) = \frac{1}{2\beta_e}(j, Dj)$. This means that the non-confining part of the potential reproduces the usual Yukawa one (cf. [9,10]). The complete potential includes, of course, the confining potential $V_{\text{conf}} = (\sigma_{\text{core}} + \sigma_{\text{surf}})r$ arising from the NG action in Eq. (24).

3 U(1)×U(1) DGL theory

In this section, we extend the discussion of the U(1) theory of the previous section to the U(1)×U(1) DGL theory. However, this requires no further techniques of the path integration if one realizes the Weyl permutation symmetry of the U(1)×U(1) DGL theory [11,12].

3.3 Weyl symmetric formulation

The U(1)×U(1) DGL action can be written as the sum of three types of the U(1) DGL theory as

$$S_{\rm DGL} = \sum_{i=1}^{3} \left[\frac{\beta_g}{2} F_i^2 + (d\phi_i)^2 + ((B_i + d\eta_i)\phi_i)^2 + \lambda(|\chi_i|^2 - v^2)^2 \right], \tag{26}$$

where the ingredients of this action are the same as in the U(1) DGL theory except the additional labels of the color, indicated by $i = 1, 2, 3$. The dual field strength is expressed as

$$F_i = dB_i - 2\pi * \Sigma_i^{(m)}, \tag{27}$$

with

$$*\Sigma_i^{(m)} \equiv \sum_{j=1}^{3} m_{ij} * \Sigma_j^{(e)}, \tag{28}$$

Here, $\Sigma_i^{(m)}$ $(i = 1, 2, 3)$ denotes the "color-magnetic" (Weyl symmetric) representation of the open color-electric Dirac string $\Sigma_j^{(e)}$ $(j = 1, 2, 3)$. The factor m_{ij} in front of the color-electric Dirac string appears as a result of the extended Dirac quantization condition between the color-magnetic charges $\vec{Q}_i^{(m)} \equiv g\vec{\epsilon}_i$ $(i = 1, 2, 3)$ and color-electric charges $\vec{Q}_j^{(e)} \equiv e\vec{w}_j$ $(j = 1, 2, 3)$:

$$\vec{Q}_i^{(m)} \cdot \vec{Q}_j^{(e)} = g\vec{\epsilon}_i \cdot e\vec{w}_j = 2\pi m_{ij} \qquad (eg = 4\pi), \tag{29}$$

where $\vec{\epsilon}_i$ and $\vec{w}_j$ are the root and weight vectors of the SU(3) Lie algebra, respectively. Explicitly, these vectors are defined as $\vec{\epsilon}_1 = (-1/2, \sqrt{3}/2)$, $\vec{\epsilon}_2 = (-1/2, -\sqrt{3}/2)$, $\vec{\epsilon}_3 = (1, 0)$ and $\vec{w}_1 = (1/2, \sqrt{3}/6)$, $\vec{w}_2 = (-1/2, \sqrt{3}/6)$, $\vec{w}_3 = (0, -1/\sqrt{3})$. The labels $j = 1, 2, 3$ describe, for instance, red (R), blue (B), and green (G) color-electric charges, respectively. The matrix

$$m_{ij} \equiv 2\vec{\epsilon}_i \cdot \vec{w}_j = \sum_{k=1}^{3} \varepsilon_{ijk} \tag{30}$$

takes the integer values ± 1 or 0. Based on these definitions of charges, the R-$\bar{R}$ system is defined by $\Sigma_{j=1}^{(e)} = \Sigma \neq 0$, $\Sigma_{j=2}^{(e)} = \Sigma_{j=3}^{(e)} = 0$, which is also expressed as $\Sigma_{i=1}^{(m)} = 0$, $\Sigma_{i=2}^{(m)} = -\Sigma$, and $\Sigma_{i=3}^{(m)} = \Sigma$ in the color-magnetic representation. The color-electric Dirac string in the color-magnetic representation satisfies the relation $\sum_{i=1}^{3} \Sigma_i^{(m)} = 0$.

3.4 The effective string action

The effective string action $S_{\text{eff}}(\Sigma_i^{(m)})$, written in terms of the color-magnetic representation of the color-electric Dirac string, will be obtained after the path integration over both the dual gauge fields and the monopole fields:

$$\mathcal{Z}(\Sigma_i^{(m)}) = \int \prod_{i=1}^{3} \mathcal{D}B_i \mathcal{D}\chi_i \mathcal{D}\chi_i^* \delta[\sum_{i=1}^{3} \delta B_i] \exp[-S_{\text{DGL}}(B_i, \chi_i, \chi_i^*, \Sigma_i^{(m)})]. \quad (31)$$

It is important to notice that the Weyl symmetric formulation requires to take into account the (Lorentz invariant) delta function constraint among the dual gauge fields. As a result, the dual gauge symmetry of the DGL theory is kept in U(1)×U(1). In fact, although the DGL action (26) itself has the $[\text{U}(1)]^3$ dual gauge symmetry

$$\chi_i \mapsto \chi_i \exp(i\alpha_i), \quad B_i \mapsto B_i - d\alpha_i \quad (i = 1, 2, 3), \quad (32)$$

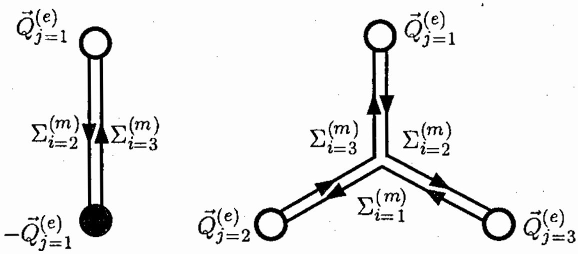
this is finally reduced to U(1)×U(1) by the relation $\sum_{i=1}^{3} d\alpha_i = 0$, which is implied by the constraint used.

The evaluation of the constraint is not difficult, since it is written as

$$\delta[\sum_{i=1}^{3} \delta B_i] = \int \mathcal{D}s \exp[i \sum_{i=1}^{3} (\delta B_i, s)], \quad (33)$$

where s is a 1-form auxiliary field. The same path integration techniques as in the U(1) case, and the integration over s, finally lead to the similar expressions as in Eqs. (15), (16), and (17). The corresponding expressions can be found in [6].

We mention here only the difference between the U(1) and U(1)×U(1) cases. The explicit form of the effective string action depends on the quark system, that is to say, the combination of the color-electric charges. In the SU(3) case, one can consider not only the mesonic string, described by R-$\bar{R}$, B-$\bar{B}$ and G-$\bar{G}$ systems, but also the *baryonic* string, as given by the combination of three color-electric charges R-B-G (see, Fig. 2). Remarkably, since the form of the action is now the Weyl symmetric one, which means invariance under the exchange of the color labels i, one can immediately conclude that all mesonic strings do not depend on the color. The baryonic system is constructed by connecting the three-colored mesonic strings, R-$\bar{R}$, B-$\bar{B}$, and G-$\bar{G}$, at a certain *junction*, forming a Y-shaped structure [6,8]. This is possible because the sum of all three color charges at the same point turns out to be zero.



'igure 2: The structure of the color-electric Dirac string in the color-magnetic epresentation for the mesonic (left) and the baryonic (right) quark systems.

It is also interesting to note that we can refer to the interaction among tring world sheets of various colors without solving their dynamics. Since the um of two types of color charges reduces to another type of *anti*-color charge, or instance as $R + B \to \bar{G}$, one finds that if two strings composed of R-$\bar{R}$ and B-$\bar{B}$ interact with each other, this will be equivalent to a $\bar{G}$-G string. Then, t is easy to imagine that the force between them must be attractive.

4 Summary and conclusions

We have reviewed the effective string action of the color-electric flux tube n the dual Ginzburg-Landau (DGL) theory using the path-integral analysis in lifferential forms, both in U(1) and in U(1)×U(1) cases. Important findings re summarized as follows.

(i) The effect of the finite thickness of the flux tube leads to a modified Yukawa interaction as the boundary contribution of the open string, which urns over to the ordinary Yukawa one only in the London limit.

(ii) The derivative expansion of the Kalb-Ramond propagator with respect o the covariant Laplacian (20) leads to the Nambu-Goto action and the rigidity erm as the leading and next-to-leading parts of the action. Such an expansion s clearly justified when the fluctuation of the string world sheet can be treated erturbatively. If not, one has to handle the expression of the first term in Eq. (19) directly.

In the U(1)×U(1) case, thanks to the Weyl symmetric formulation, where he symmetry is naturally extended to $[\mathrm{U}(1)]^3$, we have found that the treatment of the U(1)×U(1) DGL theory becomes rather simple. The resulting orm is the sum of three types of the U(1) DGL theory, which allows us to ise the same technique to derive almost the same effective string action. So the interpretation of the action was straightforward. The difference is only the number of color-electric charges, which then allows us to describe not only the mesonic system but also the baryonic system.

Note finally that the extension to $[\mathrm{U}(1)]^{N-1}$, corresponding to the SU(N) gauge theory in Abelian projection, is also straightforward.

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ONFINEMENT OF COLOR: A REVIEW

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Abstract — The status of our understanding of the mechanisms of color confinement is reviewed, in particular the results of numerical simulations on the lattice.

Introduction

Quarks and gluons are visible at short distances. They have never been served as free particles. Search for quarks started in 1963, when first Gell-ann introduced them as fundamental constituents of hadrons: the signature their fractional charge $q = \pm 1/3, \pm 2/3$.

The upper limit to the ratio of quark abundance to proton abundance is $/n_p < 10^{-27}$ [?], to be compared with the expectation in the Standard Cos-ological Model $n_q/n_p = 10^{-12}$ [?]. The experimental limit on the production oss section in nucleon collisions is [?] $\sigma < 10^{-40}\mathrm{cm}^2$ to be compared with the pected value $\sigma \sim 10^{-25} cm^2$ in the absence of confinement. The only natural planation of these small numbers is that these ratios are exacly zero, or that nfinement is an absolute property due to some symmetry.

A transition, however, can occur at high temperature to a phase in which arks and gluons are deconfined and form a quark-gluon plasma. [?].

Big experiments at heavy ions colliders aim to detect such a phase tran-ion, even if no clear signature for it is known. [?] A number of theoretical eas exist on the mechanism of confinement, which we shall briefly review low.

Lattice is a unique tool to investigate the existence of the deconfining phase ansition, to check the theoretical ideas and possibly to give indications on at to look at in experiments.

Lattice investigations

The partition function of a field theory at temperature T is equal to the clidean Feynman integral extending in time from 0 to $1/T$, with periodic undary conditions in time for bosons, antiperiodic for fermions.

$$Z \equiv Trexp(-H/T) = \int [d\phi] \exp\left[-\int_0^{1/T} dt \int d^3x L(\vec{x}, t)\right] \quad (1)$$

Finite temperature QCD is simulated on a lattice $N_s^3 \times N_t$ with $N_s \gg N_t$ The temperature T is given by

$$T = \frac{1}{N_t a(\beta)} \qquad (2)$$

where a is the lattice spacing in physical units , which by renormalization group arguments in the weak coupling regime is given by

$$a(\beta) = \frac{1}{\Lambda}\exp(-\beta/b_0) \qquad (3)$$

with $\beta = 2N/g^2$; $-b_0$ the lowest order coefficient of the beta function, which is negative because of asymptotic freedom. It follows then from eq.(2) that the strong coupling region corresponds to low temperatures, weak coupling to high temperatures.

If a deconfining phase transition exists at some temperature, how can it be detected, or what is the criterion for confinement?

For pure gauge theories a reasonable answer exists, which consists in looking at the static potential acting between a quark and an antiquark at large distances: if it is positive and diverging there is confinement by definition. In principle this criterion does not insure that no colored particles exist as an asymptotic state, but certainly means that heavy quarks are confined. The static potential is related to the correlator $D(\vec{x})$ of Polyakov lines

$$D(\vec{x}) = \langle L(\vec{x})L(0)\rangle \qquad (4)$$

as follows

$$V(x) = -T\ln D(\vec{x}) \qquad (5)$$

It can be shown by use of the cluster property that at large distances

$$D(\vec{x}) \underset{|\vec{x}|\to\infty}{\simeq} c\,\exp\left(-\frac{\sigma}{T}|\vec{x}|\right) + |\langle L\rangle|^2 \qquad (6)$$

A temperature T_c is found in numerical simulations such that

for $T < T_c$ $\langle L\rangle = 0$ and hence $V(r) = \sigma \cdot r$ (confinement)

for $T > T_c$ $\langle L\rangle \neq 0$, $V(r) \sim const$ (deconfinement) both for $SU(2)$ and for SU(3) pure gauge theories. $\langle L\rangle$ is an order parameter for confinement, and Z_3 is the corresponding symmetry.

Of course no real phase transition can take place on a finite lattice [?] , so that the transition from 0 to 1 of $\langle L\rangle$ is smooth on a finite lattice , and becomes steeper and steeper as the size goes large. The steepness is measured by the susceptibility χ_L,

$$\chi_L = \int d^3x \langle L(x)L^\dagger(0) - L(0)L^\dagger(0)\rangle \tag{7}$$

which diverges with some critical index γ at the critical point

$$\chi_L \underset{\tau\to 0}{\simeq} \tau^{-\gamma} \qquad \tau \equiv \left(1 - \frac{T}{T_c}\right) \tag{8}$$

Other relevant critical indices are the index ν of the correlation length ξ of the order parameter

$$\xi \propto \tau^{-\nu} \tag{9}$$

and the index α of the specific heat

$$C_v - C_v^0 \propto \tau^{-\alpha} \tag{10}$$

α,γ and ν identify the universality class and/or the order of the phase transition. A weak first order transition is a limiting case $\alpha = 1$, γ =1 and ν= 1/d (d the number of spacial dimensions i.e. 3).

The critical indices are determined from the dependence of susceptibilities on the spacial size of the system, by use of a technique known as finite size scaling [?]. The result is that for quenched SU(2) the transition is second order and belongs to the universality class of the 3d ising model [?], for SU(3) it is weak 1rst order [?].

In the presence of dynamical quarks Z_3 is not a symmetry and therefore the Polyakov line cannot be an order parameter.Moreover the string breaks due to the instability for production of dynamical quark pairs and the potential at large distances is not growing with the distance, even if there is confinement. Another symmetry exists at zero quark masses, the chiral symmetry.At T=0 it is spontaneously broken , the pseudoscalar bosons being the Goldstone particles , but it is restored at $T \simeq 170$ Mev. The corresponding order parameter is $\langle\bar\psi\psi\rangle$. It is not clear what exacly chiral symmetry has to do with confinement: in any case it is explicitely broken by quark masses , and therefore it cannot be the symmetry responsible for confinement discussed in sect 1. For [scale=0.4]fig1.eps

Figure 1: The phase diagram of $N_f = 2$ QCD.

a theory with $N_f = 2$, $m_u = m_d = m$, which is a model approximation of reality, the situation is schematically represented in fig 1. The critical line $T_c(m)$ is defined by the maxima of the susceptibilities $\chi_L, \chi_{\bar{\psi}\psi}, \chi_{C_v}$, which coincide within errors [?, ?], and as an empirical definition the region below the line is assumed to be confined, the region above it to be deconfined. Theoretical ideas are needed to understand the symmetry pattern of the system.

As for the chiral transition a renormalization group analysis and the assumption that that the pions are the relevant degrees of freedom gives the following predictions [?]. If the U(1) axial symmetry is restored below the chiral transition, the transition is first order and such is the critical line at $m \neq 0$. If instead the anomaly persists below T_c the transition is second order and the critical line at $m \neq 0$ is a crossover.

3 Theoretical ideas

A number of theoretical models of deconfinement exist in the literature.

There is a Gribov model, which is a clever picture of the chiral phase transition [?]. It does not apply, however, to quenched theory. In the spirit of the $N_c \to \infty$ approach the mechanism of confinement should be the same for quenched and unquenched.

Confinement could be produced by the condensation of vortices [?] The model corresponds to a well defined symmetry in 2+1 dimensions and quenched theory , but in any case the Z_3 symmetry does not survive the introduction of dynamical quarks.

A most appealing idea is dual superconductivity of the vacuum [?]: chromoelectric charges are confined by dual Meissner effect, which sqeezes the chromoelectric field acting between colored particles into Abrikosov flux tubes, in the same way as magnetic charges are confined in ordinary superconductors. A number of pioneering papers on this mechanism were based on the definition and the counting of monopoles. [?] We shall instead concentrate on the symmetry patterns involved. Dual superconductivity means that the vacuum is a Bogolubov-Valatin superposition of states with different monopole charge (monopole condensation).

In order to define monopoles a magnetic U(1) gauge symmetry must be identified in QCD , which has to be a color singlet if monopoles condense without breaking the color symmetry.

The procedure to identify such magnetic U(1) is known as Abelian Projection [?]. We shall present the abelian projection in a form which will prove useful for what follows [?]. Let $G_{\mu\nu} = T^i G^i_{\mu\nu}$ be the gauge field strength with T^i the gauge group generators in the fundamental representation, and $\Phi = T^i \Phi^i$ any operator in the adjoint representation. Define

$$F_{\mu\nu} = Tr\{\phi G_{\mu\nu}\} - \frac{i}{g} Tr\{\phi [D_\mu \phi, D_\nu \phi]\} \tag{11}$$

$F_{\mu\nu}$ is gauge invariant and color singlet, and such are separately the two terms in its definition.

Theorem [?]. A necessary and sufficient condition for the cancellation of bilinear terms $A_\mu A_\nu$ between the two terms in the right hand side of eq.(11) is that

$$\Phi = \Phi^a = U(x) \Phi^a_{diag} U^\dagger(x) \tag{12}$$

with $U(x)$ an arbitrary gauge transformation and

$$\Phi^a_{diag} = diag \left(\overbrace{\frac{N-a}{N}, .., \frac{N-a}{N}}^{a}, \overbrace{-\frac{a}{N}, .., -\frac{a}{N}}^{N-a} \right) \tag{13}$$

For any choice of the form eq(12) $F_{\mu\nu}$ obeys Bianchi identities and the identity holds

$$F^a_{\mu\nu} = \partial_\mu Tr\{\phi^a A_\nu\} - \partial_\nu Tr\{\phi^a A_\mu\} - \frac{i}{g} Tr\{\phi^a [\partial_\mu \phi^a, \partial_\nu \phi^a]\} \tag{14}$$

$F_{\mu\nu}$ is gauge invariant and can be computed in the gauge in which Φ^a is diagonal. In that gauge

$$F^a_{\mu\nu} = \partial_\mu Tr(\Phi^a A_\nu) - \partial_\nu Tr(\Phi^a A_\mu) \tag{15}$$

has an abelian form. By developing A^μ_{diag} in terms of roots $A_\mu = \alpha^i A^i_\mu$

$$\alpha^i = diag(0,0,0 \ldots \overset{i}{1}, \overset{i+1}{-1}, 0 \ldots 0) \tag{16}$$

with

$$Tr(\alpha^i \Phi^j) = \delta^{ij}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu \tag{17}$$

The gauge transformation $U(x)$ which brings to the unitary gauge is called abelian projection.

A magnetic current can be defined as

$$J^a_\nu = \partial_\mu F^{a*}_{\mu\nu} \tag{18}$$

This current is identically zero due to Bianchi identities, but can be non zero in compact formulations like lattice regularization , in which Dirac strings are invisible. In any case J^a_μ is identically conserved

$$\partial_\mu J^a_\mu = 0 \tag{19}$$

and defines a magnetic U(1) conserved charge. This magnetic U(1) symmetry can either be Higgs broken, and then the system is a magnetic (dual) superconductor, or it can be realized à la Wigner,and then magnetic charge is superselected. For any choice of the field Φ^a in eq (12) a magnetic U(1) symmetry is defined.

To detect dual superconductivity the vev of a magnetically charged operator can be used as an order parameter. Such an operator has been constructed [?],and is magnetically charged and U(1) gauge invariant [?] . The continuum version of the construction goes as follows. Define

$$\mu^a(\vec{x},t) = e^{i\int d^3\vec{y}\, Tr\left(\phi^a \vec{E}(\vec{y},t)\right)\vec{b}_\perp(\vec{x}-\vec{y})} \tag{20}$$

where Φ^a is defined by eq(12) and $\vec{E}(\vec{x},t)$ is the chromoelectric field operator $E_i = G_{0i}$ and

$$\vec{\nabla}\vec{b}_\perp = 0 \quad , \vec{\nabla} \wedge \vec{b}_\perp = \frac{2\pi}{g}\frac{\vec{r}}{r^3} + \text{Dirac string} \tag{21}$$

μ^a is gauge invariant by construction if Φ^a transforms in the adjoint representation. In the abelian projected gauge, where $\Phi^a = \Phi^a_{diag}$ it assumes the form

$$\mu^a(\vec{x},t) = \exp\left\{i\int d^3\vec{y}\, \vec{E}^a_\perp(\vec{y},t)\vec{b}_\perp(\vec{x}-\vec{y})\right\} \tag{22}$$

where $\vec{E}^a_\perp$ is the component of the electric field along the residual $U(1)$ direction as defined by eq.(16),(17), and only the transverse part survives in the convolution with $\vec{b}_\perp$. In any quantization procedure $\vec{E}^a_\perp$ is the conjugate momentum to $\vec{A}^a_\perp$ so that μ^a is nothing but the translation operator of $\vec{A}^a_\perp$ and

$$\mu^a(\vec{x},t)|\vec{A}^a_\perp(\vec{y},t)\rangle = |\vec{A}^a_\perp(\vec{y},t) + \vec{b}_\perp(\vec{x}-\vec{y})\rangle \tag{23}$$

^{a} creates a magnetic monopole.

In the confined phase , in which monopoles condense and the ground state not an eigenstate of the magnetic charge $\langle\mu_a\rangle \neq 0$ can signal dual superconuctivity. In the deconfined phase $\langle\mu_a\rangle = 0$ All this refers to a given choice of ne abelian projection, i.e. of the gauge transfomation U(x) defining $F^a_{\mu\nu}$.

To explore [?] [?] how physics depends on the choice of the abelian projecon let us go back to eq (20). By use of the cyclic invariance of the trace μ^a an be rewritten

$$\mu^a(\vec{x},t) = e^{i\int d^3\vec{y}\, Tr\left(\phi^a_{diag}U^\dagger(\vec{y},t)\vec{E}(\vec{y},t)U(\vec{y},t)\right)\vec{b}_\perp(\vec{x}-\vec{y})} \qquad (24)$$

ı computing the correlation functions of μ^a's a change of variables can be erformed in the Feynman integral corresponding to a gauge transformation enerated by U(x). If U(x) is independent of the field configuration the jaobian of the transformation is 1 , the operator μ^aassumes to all effects the orm eq.(24) so that the correlators , and in particular the one point funcion $\langle\mu^a\rangle$ are independent of $U(x)$. If $U(x)$ depends on the field configuration as happens e.g. for the max abelian gauge or for any gauge in which a pecific field dependent operator is diagonalized, then the jacobian can be diferent from 1 and the correlators depend on the abelian projection. However, the number density of monopoles is finite ,the gauge transformation which onnects two abelian projections is continuous everywhere except in a finite umber of points and preserves topology: the operator μ^a defined by eq(22) vill then create a monopole in all abelian projections. If $\langle\mu^a\rangle \neq 0$ it signals ual superconductivity in all abelian projections.

An extensive investigation of the density of monopoles in different abelian rojections has been performed, and indeed the number density of monopoles finite. Fig 2 illustrates the method , and refers specifically to the abelian rojection in which the Polyakov line operator is diagonal. The eigenvalues of ıat (unitary) operator have the form

$$L_i = e^{i\phi_i} \qquad i = 1,2,3 \qquad (25)$$

nd in defining the abelian projection are ordered in decreasing order of ϕ_i. monopole singularity in a point x implies that two eigenvalues are equal , ϕ_1 and ϕ_2. Fig 2 shows the distribution of the difference of the first two genvalues on the lattice sites of 1000 field configurations of quenched SU(3) ı a 16^4 lattice . In no site there is a monopole. Repeating the determination ı a finer lattice gives similar results. As a consequence one can state that the [angle=270,scale=0.4]histocl.eps

Figure 2: An example of probability distribution of the difference of the two highest eigenvalues of the phase Φ of the Polyakov line $e^{i\Phi}$, at the lattice sites. $SU(3)$ gauge group, $\beta = 6.4$, lattice 16^4, 10^3 configurations.

number density of monopoles is finite and the dual superconductivity (or non) is an intrinsic property , independent of the abelian projection which defines the monopoles.

An extensive analysis on the lattice [?] shows that the vacuum is indeed a dual superconductor in the confined phase, and goes to normal in the deconfined one. A finite size scaling analysis of the susceptibility ρ defined as

$$\rho^a = \frac{d}{d\beta} \ln\langle\mu^a\rangle \tag{26}$$

gives that in the quenched case $\langle\mu^a\rangle$ is strictly zero above the critical temperature defined by the Polyakov line order parameter, is different from zero below it. The behaviour around T_c allows to determine the critical indices , which are consistent with those determined by use of the Polyakov line. This is clear evidence that dual superconductivity is a mechanism for confinement [?].

4 The case of full QCD

The order parameters $\langle\mu^a\rangle$ can equally well be defined in the presence of dynamical quarks (Full QCD) [?] and have the same physical meaning of creators of monopoles. One can then ask if a criterion for confinement could be provided by $\langle\mu^a\rangle$'s , i.e. by dual superconductivity (or absence of) . One should prove that in the dual superconducting phase no colored asymptotic states exist, which is of course non trivial. However this is not much different from the situation in quenched theory with the Polyakov line criterion , as discussed in sect 2. It has in fact been checked [?] that QCD vacuum is a dual superconductor in the phase below the critical line of fig1, ($\langle\mu^a\rangle \neq 0$), and is normal in the region above it ($\langle\mu^a\rangle = 0$). The finite size scaling analysis in this case goes as follows. For dimensional reasons the order parameter $\langle\mu^a\rangle$ has the form

$$\langle\mu^a\rangle = \Phi(\frac{a}{\xi}, \frac{N_s}{\xi}, mL^{y_h}) \tag{27}$$

where ξ is the correlation length, a the lattice spacing, m the quark mass and y_h the corresponding anomalous dimension . Near the critical line ξ goes large ompared to a and the dependence on a/ξ can be neglected. (Scaling) The roblem has two scales. If y_h is known one can choose different values of the ass and of the spacial size N_s such that mN_s is constant, and then

$$\langle\mu^a\rangle \underset{T\to T_c}{\simeq} f(\frac{N_s}{\xi}) \qquad (28)$$

y use of eq(9) the variable $N_s\xi$ can be traded with $\tau N_s^{1/\nu}$ and the scaling law llows

$$\rho^a = N_s^{1/\nu} f(\tau N_s^{1/\nu}), \qquad \tau = (1 - \frac{T}{T_c}) \qquad (29)$$

hence ν can be extracted and the order of the transition can be determined. he result is $\nu = .33$ compatible with a first order transition.

A cross check is obtained by studying the scaling of the maximum of the ecific heat, which for the same choices of m and N_s should scale as

$$C_v - C_v^0 \sim Ns^{\alpha/\nu} \qquad (30)$$

the critical indices determined through $\langle\mu^a\rangle$ coincide with those resulting om the analysis of the specific heat, this would be additional evidence for ual superconductivity as a mechanism of confinement, implying that $\langle\mu^a\rangle$ an be the order parameter.

The situation is described in ref [?] ,and is presently at the stage of indiation that this is indeed the case. Numerical work is on the way which will efinitely clarify the problem.

In conclusion Confinement is a fundamental but difficult problem. Some nderstanding has been reached on the symmetry patterns involved. Lattice a unique tool to address the problem.

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Instanton vacuum QCD effective action beyond chiral limit

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1 Introduction

Instantons represent a very important component of the QCD vacuum. Their properties are described by the average instanton size $\rho \sim 1/3\,\mathrm{fm}$ and inter-instanton distance $R \sim 1\,\mathrm{fm}$ [1–3]. The Lee&Bardeen's fermionic determinant $\det_N$ (in the field of N_+ instantons and N_- antiinstantons) is [4]:

$$\det_N = \det B,\ B_{ij} = im\delta_{ij} + a_{ji},\ a_{-+} = - < \Phi_{-,0}|i\partial\!\!\!/|\Phi_{+,0} > . \qquad (1)$$

Here $\Phi_{\pm,n}$ are defined by Eq. $(i\partial\!\!\!/ + gA\!\!\!/^{I(\bar{I})})|\Phi_{\pm,n}\rangle = \lambda_n|\Phi_{\pm,n}\rangle$ (with the convention $\lambda_0 = 0$). $\Phi_{\pm,0}$ are quark zero-modes, generated by instanton(antiinstanton) $I(\bar{I})$. $\det_N$ averaged over instanton/anti-instanton positions, orientations and sizes is a partition function of light quarks Z_N. A small packing parameter $\pi^2(\frac{\rho}{R})^4 \sim 0.1$ provides independent averaging over instanton collective coordinates. Then in the chiral limit Diakonov-Petrov effective action [5, 6] is reproduced [7,8].

We have to calculate a similar quark determinant but in the presence also of the external fields (we demonstrate this one on the example of the external electromagnetic field v_μ) and average it over the collective coordinates of the instantons to find the partition function with the quark current mass m contribution taken into account.

2 Light quarks in the instanton background and external electromagnetic field v_μ

The extended zero-modes $\tilde{\Phi}_0$ for the quark placed into the instanton field A^I_μ, with his position $z = 0$, and electromagnetic field v_μ are obeying to the equation:

$$(i\partial\!\!\!/_x + gA\!\!\!/^I(x) + ev\!\!\!/(x))\tilde{\Phi}_0(x) = 0 \qquad (2)$$

We apply the U(1) gauge transformation

$$\tilde{\Phi}_0 = L\Phi'_0,\ \ L(x) = \exp\left(iex_\mu \int_0^1 v_\mu(\alpha x)d\alpha\right), \qquad (3)$$

where L is simply a path-ordered exponent $P\exp(i\int_0^x v_\mu(s)ds_\mu)$ with a straight path connecting the points 0 to x (see for example [9]). Then Eq. (2) is reduced to

$$(i\partial\!\!\!/_x + gA\!\!\!/^I(x) + ev\!\!\!/'(x))\Phi_0'(x) = 0, \tag{4}$$

where $v'_\mu(x) = x_\rho\int_0^1 F_{\rho\mu}(\alpha x)\alpha d\alpha$ has the explicit gauge invariant form. An approximate solution of Eq. (4) is

$$\Phi_0' = \Phi_0 - S^I ev\!\!\!/'\Phi_0, \tag{5}$$

which can be expanded to any desired order of ev'. Here $S^I = (i\partial\!\!\!/ + gA\!\!\!/^I + im)^{-1}$ is a quark propagator (with a small mass m) in the field of a single instanton. The non-zero mode quark propagator S_{NZ} in the single instanton field was derived in an exact closed form (see a review [10]). It is clear from this formula that this propagator is reduced to the free one at a short distance as well as a long one. We are following the argumentations of [5] and will assume that $S_{NZ}(x,y) \simeq S_0(x-y)$, where $S_0 \doteq (i\partial\!\!\!/ + im)^{-1}$ is a free quark propagator. This formula together with Eqs.(3,4,5) leads to a desired extended zero mode $\tilde\Phi_0 = L\Phi_0'$. Now we consider the quark propagator $\tilde S$ in the fields of the single instanton A^I_μ and v_μ with the same assumption $S_{NZ} \simeq S_0$. Then it is easy to show that

$$\tilde S^I \simeq \tilde S_0 + \frac{|\tilde\Phi_0\rangle\langle\tilde\Phi_0|}{im}, \tag{6}$$

where $\tilde S_0 = (i\partial\!\!\!/ + ev\!\!\!/ + im)^{-1}$.

Now we have to consider an ensemble of the instantons $A_\mu = \sum_I A_{I\mu}$ representing the instanton liquid model of the QCD vacuum. This sum consists of both instantons and antiinstantons. The quarks are moving in the presence of the instanton ensemble A_μ and external electromagnetic field v. We will derive an extended zero-mode approximation for the quark determinant.

The quark propagator $\tilde S$ can be expanded with respect to an single instanton [5]. Then, by using Eq. (6) the $\tilde S$ becomes

$$\begin{aligned}\tilde S(x,y) &\simeq \tilde S_0(x,y) + \sum_I \frac{\tilde\Phi_{I,0}(x)\tilde\Phi^\dagger_{I,0}(y)}{im} \qquad (7)\\ &+ \sum_{I\neq J}\frac{\tilde\Phi_{I,0}(x)}{im}\left(\int d^4r\,\tilde\Phi^\dagger_{I,0}(r)(i\partial\!\!\!/ + ev\!\!\!/ + im)\tilde\Phi_{J,0}(r)\right)\frac{\tilde\Phi^\dagger_{J,0}(y)}{im} + \cdots.\end{aligned}$$

Here, $I, J, \cdots$ refer to both instantons and anti-instantons. Let us to define overlap integrals

$$\tilde{a}_{IJ} = -\int d^4r\, \tilde{\Phi}^{\dagger}_{I,0}(r)(i\partial\!\!\!/ + e v\!\!\!/)\tilde{\Phi}_{J,0}(r) \tag{8}$$

Since, the expansion (7) is a geometrical progression, it can be easily summed to give

$$\tilde{S}(x,y) \simeq \tilde{S}_0(x,y) + \sum_{I,J} \tilde{\Phi}_{I,0}(x)\left(\frac{1}{\tilde{a}+im}\right)_{IJ} \tilde{\Phi}^{\dagger}_{J,0}(y). \tag{9}$$

Accordingly, the total quark determinant should be splitted into low and high frequencies (with respect to a free mass parameter $M_1 \sim \rho^{-1}$): $\mathrm{Det} = \mathrm{Det}_{\mathrm{low}} \times \mathrm{Det}_{\mathrm{high}}$ [5]. $\mathrm{Det}_{\mathrm{high}}$ can be written as a product of the determinants calculated in the field of individual instantons. However, $\mathrm{Det}_{\mathrm{low}}$ is treated approximately, would-be zero modes being taken into account only. Then

$$\ln \mathrm{Det}_{\mathrm{low}} = \sum_f \int_{M_1}^{m_f} idm[\mathrm{Tr}(\tilde{S} - \tilde{S}_0) - \mathrm{Tr}(\tilde{S}_0 - S_0)]. \tag{10}$$

As it is clear from Eq.(9)

$$\mathrm{Tr}(\tilde{S} - \tilde{S}_0) = \mathrm{Tr}\frac{1}{\tilde{a}+im}. \tag{11}$$

The second factor in Eq.(10) is included into the normalization factor. These formulae without the external field v correspond exactly to Lee and Bardeen's quark determinant $\det_N$ and lead to their extension to the case with the external field v switched on:

$$\mathrm{Det}_{\mathrm{low}} = \det{}_{N,v} = \det \tilde{B}, \quad \tilde{B}_{IJ} = \tilde{a}_{JI} + im\delta_{JI} \tag{12}$$

Fermionization of Eq. (12) with account of the current masses m_f [11, 12] leads to the gauged quark determinant in terms of the constituent quarks ψ_f:

$$\begin{aligned} \det{}_{N,v} &= \int D\psi D\psi^{\dagger} \exp\Big(\int d^4x \sum_f \psi^{\dagger}_f(i\partial\!\!\!/ + e_f v\!\!\!/ + im_f)\psi_f\Big) \\ &\times \prod_f \Big(\prod_{+}^{N_+} \tilde{V}_+[\psi^{\dagger}_f, \psi_f]\Big)\Big(\prod_{-}^{N_-} \tilde{V}_-[\psi^{\dagger}_f, \psi_f]\Big)\Big), \end{aligned} \tag{13}$$

where

$$\tilde{V}_\pm[\psi_f^\dagger,\psi_f] = \int d^4x(\psi_f^\dagger(x)L_f(x)i\partial\!\!\!/\Phi_{\pm,0}(x;\xi_\pm))\int d^4y(\Phi_{\pm,0}^\dagger(y;\xi_\pm)i\partial\!\!\!/L_f^\dagger(y)\psi_f(y)). \tag{14}$$

and

$$L_f(x) = \exp(ie_f(x-z)_\mu\int_0^1 d\alpha v_\mu(z+(x-z)\alpha)) \simeq \exp(ie_f(x-z)_\mu v_\mu(z)),$$

assuming $q\rho << 1$. The averaging over instanton collective coordinates (with the essential usage of the smallness of the packing parameter $\pi^2(\frac{\rho}{R})^4 \sim 0.1$) leads to the gauged partition function (with account of quark current mass m):

$$Z_N[v,m] = \int D\psi D\psi^\dagger \exp\left(\int d^4x \sum_f \psi_f^\dagger(i\partial\!\!\!/ + e_f v\!\!\!/ + im_f)\psi_f\right) \tilde{W}_+^{N_+}\tilde{W}_-^{N_-}, \tag{15}$$

where

$$\tilde{W}_\pm = \int d\xi_\pm \prod_f(\tilde{V}_\pm[\psi_f^\dagger\,\psi_f]) = (i)^{N_f}\left(\frac{4\pi^2\rho^2}{N_c}\right)^{N_f}\int\frac{d^4z}{V}\det_f i\tilde{J}_\pm(z) \tag{16}$$

and

$$\tilde{J}_\pm(z)_{fg} = \int\frac{d^4kd^4l}{(2\pi)^8}\exp(-i(k-l)z)F((k+e_fv(z))^2)F((l+e_gv(z))^2)\,\psi_f^\dagger(k)\frac{1\pm\gamma_5}{2} \tag{17}$$

$F(k^2)$ is related to the Fourie-transform of the zero-mode $\Phi_\pm(k;\xi_\pm)$ and its explicit content is

$$F(k^2) = -t\frac{d}{dt}\left[I_0(t)K_0(t) - I_1(t)K_1(t)\right],\ t = \frac{1}{2}\sqrt{k^2}\rho,$$

This action was succesfuly tested by the axial anomaly low energy theorems [7,8,13] in the chiral limit and we may conclude that we found a proper way of gauging the effective action.

As a first application of this effective action we have calculated the terms of order $O(m)$ and $O(m^2)$ of Gasser and Leutwyler phenomenological chiral lagrangian [14] and found that

$$\bar{l}_3^{theor} = 1.39\,(\bar{l}_3^{phenom} = 2.9\pm 2.4),\ \bar{l}_4^{theor} = 3.57\ (\bar{l}_4^{phenom} = 4.3\pm 0.9).$$

We see the good correspondence of the theory with phenomenology at least for these quantities. The calculations of other couplings are in the progress.

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On the Nature of f_0 Mesons below 1.9 GeV and Lower Scalar Nonets

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Abstract — The combined 3-channel analysis of experimental data on the processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ is carried out in the channel with the vacuum quantum numbers. An approach, using only first principles (analyticity and unitarity) and the uniformizing variable, is applied. Definite indications of the QCD nature of the f_0 resonances below 1.9 GeV are obtained. An assignment of the scalar mesons below 1.9 GeV to lower nonets is proposed.

1. Introduction. The problem of scalar mesons draws permanently an attention of investigators in view of an important role played by these mesons (especially σ-meson) in the hadronic dynamics. [1]

The f_0 mesons are most direct carriers of information about the QCD vacuum. Therefore, every step in understanding nature of the f_0 mesons is especially important.

Note a situation with the σ-meson. Now, there are evidences of the existence of the σ-meson both on the basis of phenomenologic analyses of experimental data [1] and the theoretical explanation of phenomena and quantities. The latter is (1) $a_0^0(\pi\pi)$ (see, *e.g.*, [2]); (2) rather big experimental value of the $\pi - N$ sigma term (see, e.g., [3]); (3) the enhancement of the $\Delta I = 1/2$ processes in the $K^0 \to \pi^+\pi^-, \pi^0\pi^0$ decays [4]; (4) the phase shift analyses of the $N - N$ scattering in the $^1S_0$ channel have discovered an attraction in the intermediate range ($1 \sim 2$ fm), which is indispensable for the binding of a nucleus and is stipulated by the light σ-meson exchange [5].

However, a number of physicists questions till now the existence of the σ-meson (see, *e.g.*, [6]). Therefore, a further combined analysis of the coupled processes (with adding other channels) is needed for studying scalar mesons.

2. Three-coupled-channel formalism. We consider the processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ in the 3-channel approach. The S-matrix is determined on the 8-sheeted Riemann surface. The elements $S_{\alpha\beta}$, where $\alpha, \beta = 1(\pi\pi), 2(K\overline{K}), 3(\eta\eta)$

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have the right-hand cuts along the real axis of the s complex plane, starting with $4m_\pi^2$, $4m_K^2$, and $4m_\eta^2$, and the left-hand cuts. The Riemann-surface sheets are numbered according to the signs of analytic continuations of the channel momenta $k_1 = (s/4 - m_\pi^2)^{1/2}$, $k_2 = (s/4 - m_K^2)^{1/2}$, $k_1 = (s/4 - m_\pi^2)^{1/2}$ as follows: signs $(\text{Im}k_1, \text{Im}k_2, \text{Im}k_3) = +++, -++, --+, +-+, +--, ---, -+-, ++-$ correspond to sheets I, II,$\cdots$, VIII.

The resonance representations on the Riemann surface are obtained with the help of the formulae (see, ref. [7], Table 1), expressing analytic continuations of the matrix elements to unphysical sheets in terms of those on sheet I that have only zeros (beyond the real axis) corresponding to resonances. Starting from resonance zeros on sheet I, from these 3-channel formulae, we obtain 7 types of resonances corresponding to conjugate resonance zeros on sheet I of (**a**) S_{11}; (**b**) S_{22}; (**c**) S_{33}; (**d**) S_{11} and S_{22}; (**e**) S_{22} and S_{33}; (**f**) S_{11} and S_{33}; and (**g**) S_{11}, S_{22}, and S_{33}. For example, the arrangement of poles corresponding to a (**g**) resonance is: each sheet II, IV, and VIII contains a pair of conjugate poles at the points that are zeros on sheet I; each sheet III, V, and VII contains two pairs of conjugate poles; and sheet VI contains three pairs of poles.

A resonance of every type is represented by a pair of complex-conjugate clusters (of poles and zeros on the Riemann surface). The cluster kind is related to the state nature. The resonance coupled relatively more strongly to the $\pi\pi$ channel than to the $K\overline{K}$ and $(\eta\eta)$ ones is described by the cluster of type (**a**); if the resonance is coupled more strongly to the $K\overline{K}$ and $(\eta\eta)$ channels than to the $\pi\pi$, it is represented by the cluster of type (**e**) (say, the state with dominant $s\bar{s}$ component); the flavour singlet (*e.g.*, glueball) must be represented by the cluster of type (**g**) as a necessary condition for the ideal case.

We can distinguish, in a model-independent way, a bound state of colourless particles (*e.g.*, $K\overline{K}$ molecule) and a $q\bar{q}$ bound state [7,8].

It is convenient to use the Le Couteur-Newton relations [9] expressing the S-matrix elements of all coupled processes in terms of the Jost matrix determinant $d(k_1, k_2, k_3)$ that is the real analytic function with the only square-root branch-points at $k_i = 0$. Now we must find a proper uniformizing variable for the 3-channel case. However, it is impossible to map the 8-sheeted Riemann surface onto a plane with the help of a simple function. Therefore, we neglect the influence of the $\pi\pi$-threshold (however, unitarity on the $\pi\pi$ cut is taken into account). This approximation means the consideration of the nearest to the physical region semi-sheets of the Riemann surface. The uniformizing variable can be chosen as $w = (k_2 + k_3)/(m_\eta^2 - m_K^2)^{1/2}$. It maps our model of the

8-sheeted Riemann surface onto the w-plane divided into two parts by a unit circle centered at the origin. The sheets I (III), II (IV), V (VII) and VI (VIII) are mapped onto the exterior (interior) of the unit disk in the 1st, 2nd, 3rd and 4th quadrants, respectively. The physical region extends from the point w_π on the imaginary axis ($\pi\pi$ threshold, $|w_\pi| > 1$) down this axis to the point i on the unit circle ($K\overline{K}$ threshold), further along the unit circle clockwise in the 1st quadrant to point 1 on the real axis ($\eta\eta$ threshold) and then along the real axis to ∞. The type (**a**) resonance is represented in S_{11} by the pole on the image of the sheet II, of the sheet III, VI and VII and by zeros, symmetric to these poles with respect to the imaginary axis. Here the left-hand cuts are neglected in the Riemann-surface structure, and contributions on these cuts will be taken into account in the background.

On the w-plane, the Le Couteur–Newton relations are

$$S_{11} = d^*(-w^*)/d(w), \qquad S_{22} = d(-w^{-1})/d(w), \qquad S_{33} = d(w^{-1})/d(w), \quad (1)$$
$$S_{11}S_{22} - S_{12}^2 = d^*(w^{*-1})/d(w), \qquad S_{11}S_{33} - S_{13}^2 = d^*(-w^{*-1})/d(w).$$

The d-function is $d = d_B d_{res}$ with $d_B = \exp[-i\sum_{n=1}^{3} k_n(\alpha_n + i\beta_n)]$ describing the background, and $d_{res}(w) = w^{-\frac{M}{2}}\prod_{r=1}^{M}(w + w_r^*)$ being the resonance part (M is the number of resonance zeros).

3. Combined analysis of experimental data. We analyzed data on the processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ in the channel with $I^G J^{PC} = 0^+0^{++}$. For the $\pi\pi$-scattering, the data from the threshold to 1.89 GeV are taken from ref. [10]; below 1 GeV, from many works [2]. For $\pi\pi \to K\overline{K}$, practically all the accessible data are used [2]. The $|S_{13}|^2$ data for $\pi\pi \to \eta\eta$ from the threshold to 1.72 GeV are taken from ref. [11].

For the $\pi\pi$-scattering, we obtain a satisfactory description from ~ 0.4 GeV to 1.89 GeV (χ^2/ndf ≈ 1.5). Taking into account the the $\eta\eta$-threshold branch-point, we have considerably improved the description of the phase shift for the process $\pi\pi \to K\overline{K}$ (χ^2/ndf ≈ 2.2) in comparison with the 2-cannel case. The total χ^2/ndf for all three processes is 1.74; the number of adjusted parameters is 40. The background parameters (in GeV^{-1} units) are $\alpha_1 = 1.37, \beta_1 = 0, \alpha_2 = -1.43, \beta_2 = 0.2, \beta_3 = 0.928$. In Table 2, the obtained pole clusters for resonances are shown (poles on sheets V and VI, corresponding to the $f_0(1370)$ and $f_0(1710)$, are of the 2nd order; the $f_0(1500)$ cluster has a pole of the 3rd order on sheet VI and a pole of the 2nd order on each of sheets III, V and VII).

Table 2: Resonance pole-clusters ($\sqrt{s_r} = E_r - i\Gamma_r$, MeV).

Sheet	$\sqrt{s_r}$	$f_0(600)$	$f_0(980)$	$f_0(1370)$	$f_0(1500)$	$f_0(1710)$
II	E	683±21	1004±6		1505±14	
	Γ	600±22	39±5		326±34	
III	E	672±25	963±12	1380±22	1505±20	1702±18
	Γ	600±30	76±9	116±15	260±39	30±5
IV	E			1380±19	1505±15	1702±14
	Γ			134±15	250±30	150±7
V	E			1367±30	1501±12	1711±15
	Γ			260±35	127±35	145±18
VI	E	627±23		1361±30	1515±23	1688±17
	Γ	600±23		253±30	172±25	162±24
VII	E	638±40		1380±25	1491±22	1702±19
	Γ	600±35		134±23	116±18	150±20
VIII	E			1380±15	1505±12	1702±10
	Γ			116±13	110±10	30±4

For now, we did not calculate coupling constants in the 3-channel approach. Therefore, for subsequent conclusions, let us mention the results for coupling constants from our previous 2-channel analysis (Table 3, g_1 is the coupling constant with $\pi\pi$, g_2 – with $K\overline{K}$) [2].

Table 3.

	$f_0(665)$	$f_0(980)$	$f_0(1370)$	$f_0(1500)$
g_1, GeV	0.652 ± 0.065	0.167 ± 0.05	0.116 ± 0.03	0.657 ± 0.113
g_2, GeV	0.724 ± 0.1	0.445 ± 0.031	0.99 ± 0.05	0.666 ± 0.15

¿From Table 3, the $f_0(980)$ and the $f_0(1370)$ are coupled essentially more strongly to the $K\overline{K}$ system than to the $\pi\pi$ one, *i.e.*, they have a dominant $s\bar{s}$ component. The $f_0(1500)$ has the approximately equal coupling constants with the $\pi\pi$ and $K\overline{K}$, which apparently could point up to its dominant glueball component. In the 2-channel case, $f_0(1710)$ is represented by the cluster corresponding to a state with the dominant $s\bar{s}$ component.

Our 3-channel conclusions on the basis of resonance cluster types generally confirm the ones drawn in the 2-channel analysis, besides the surprising conclusion about the $f_0(980)$ nature. It turns out that this state lies slightly above the $K\overline{K}$ threshold and is described by a pole on sheet II and by a shifted pole on sheet III under the $\eta\eta$ threshold without an accompaniment of the corresponding poles on sheets VI and VII, as it was expected for standard clusters.

This corresponds to the description of the $\eta\eta$ bound state.

4. Lower scalar 0^{++} nonets. Now we can propose a following assignment of scalar mesons below 1.9 GeV to lower nonets. We exclude from this consideration the $f_0(980)$ as the $\eta\eta$ bound state. Then one can include to the lowest nonet the $f_0(600)$ as the eighth component of octet, the isodoublet $K_0^*(900)$ (or $\kappa(800)$), the isovector $a_0(980)$ and the $f_0(1370)$ as the SU(3) singlet. We consider the $K_0^*(900)$ (or κ), observed at analyzing the $K-\pi$ scattering (S. Ishida *et al.*) [12] and at studying the decay $D^+ \to K^-\pi^+\pi^+$ (Fermilab experiment E791, C. Gobel *et al.*) [12]. Then the Gell-Mann–Okubo formula $3m_{f_8}^2 = 4m_{K^*}^2 - m_{a_0}^2$ gives $m_{f_8} = 0.87$ GeV. Our result for the σ-meson mass is $m_\sigma \approx 0.85 \pm 0.02$ GeV, if to take the resonance part of $\pi\pi$ amplitude in the form $T^{res} = \sqrt{s}\Gamma_{el}/(m_\sigma^2 - s - i\sqrt{s}\Gamma_{tot})$.

The second relation for masses of nonet, which is obtained only on basis of the quark contents of the nonet members and somehow restricts mass of the SU(3) singlet, is $m_\sigma + m_{f_0(1370)} = 2m_\kappa$. The left-hand side of this relation is ~ 20 % bigger than the right-hand one. That is, the $\sigma - f_0(1370)$ system gets an additional contribution absent in the κ.

The next nonet could be formed of the $f_0(1500)$ (the eighth component of octet mixed with a glueball), the isodoublet $K_0^*(1430-1460)$, the isovector $a_0(1490)$ and the $f_0(1710)$ (SU(3) singlet). From the Gell-Mann–Okubo formula we obtain $m_{f_8} \approx 1.45$ GeV. In second formula $m_{f_0(1500)} + m_{f_0(1710)} = 2m_{K^*(1460)}$, the left-hand side is ~ 10 % bigger than the right-hand one.

Though the Gell-Mann–Okubo formula is fulfilled for both nonets almost as for an ideally-mixed nonet, however, the not identical mass splitting of the isoscalar of octet, the isodoublet and the SU(3) singlet and the breaking of the second relation (especially for the lowest nonet) yet tell us about non-ideal mixing.

5. Conclusions

1. In a combined model-independent analysis of data on the $\pi\pi$ scattering (from 0.4 to 1.89 GeV), processes $\pi\pi \to K\overline{K}$ (from the threshold to 1.48 GeV) and $\pi\pi \to \eta\eta$ (from the threshold to 1.7 GeV), a confirmation of the σ-meson with mass 0.85 GeV is obtained once more. This mass value rather accords with prediction ($m_\sigma = m_\rho$) on the basis of mended symmetry by Weinberg [13].
2. Consideration of the $\eta\eta$ channel is necessary for a cosistent and reasonable representation of the obtained resonances.
3. The $f_0(980)$, $f_0(1370)$ and $f_0(1710)$ have the dominant $s\bar{s}$ component.

Moreover, we obtain an additional indication for $f_0(980)$ to be the $\eta\eta$ bound state. Remembering a dispute [14] whether the $f_0(980)$ is narrow or not, we agree rather with the former.
4. The $f_0(1500)$ has the dominant flavour-singlet (*e.g.*, glueball) component.
5. An assignment of the scalar mesons below 1.9 GeV to lower nonets is proposed. This assignment is not a solution of the scalar meson problem. However, it moves a number of questions and does not put the new ones. Therefore, this is probably a way to solve this problem.

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Hadron Physics

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Neutron polarizability

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Abstract — In the paper the problem of introducing the notion of neutron polarizability and the role of Prof. D.I.Blokhintsev in the development of this problem are discussed. The influence of neutron polarizability on neutron scattering by heavy nuclei is considered. The results of estimations of electric neutron polarizability in the megaelectronvolt energy region and in the energy region of less than 300 keV are correlated. The reasons are given in favor of the opinion that neutron polarizability was observed for the first time in neutron experiments, namely, in megaelectronvolt small angle neutron scattering, i.e. in 1957.

I would like to tell you about neutron polarizability - a problem, which appeared about 50 years ago, and about the influence of neutron polarizability on neutron scattering by heavy nuclei at relatively low energies of neutrons (less than 10 MeV).

The problem of neutron polarizability is closely connected with the name of D.I.Blokhintsev. Among other things, first experimental search of the neutron polarizability influence on the character of neutron scattering was initiated in the Institute of Physics and Power Engineering (IPPE), Obninsk, where Prof. Blokhintsev was director in the 1950s.

One of the great successes in physics of the 1950s were famous experiments by Hofstadter carried out on the electron accelerator of Stanford University (USA). Hofstadter was the first to show experimentally that proton was not a point particle. In this connection physicists were wondering if there are other natural phenomena indicative of nucleon space structure. In the middle of the 50s this question was considered independently by three groups of physicists: in the USA by Klein (1955) [1], in Russia by Baldin (unfortunately first paper in 1960 [2]) and by Alexandrov and Bondarenko (1956) [3]. In all of the mentioned papers the notion of nucleon polarizability was introduced (independently) similar to a certain degree, to the existing notions of atom polarizabilities. Since the influence of neutron polarizability on neutron scattering was considered only by the third group, I would like to tell you about the research initiated by this group in Obninsk.

The phenomenon of polarizability implies a deformation of spatially extended nucleon in electric or magnetic field. In case of electric field E neutron acquires electric dipole moment d = αE, where α is a coefficient of electric polarizability, and neutron obtains additional potential energy

$$V(r) = -dE = -1/2(\alpha E^2) \tag{1}$$

I discussed with Prof. Blokhintsev the possibility of such phenomenon inherent exactly in neutron as far back as 1954, after that he sanctioned the experimental search of this effect in neutron scattering.

At this time the first fast reactor in Europe was started in Obninsk. Together with Dr.Bondarenko we made a decision to search the influence of neutron polarizability on small angle neutron scattering. The angles of scattering can be estimated using the correlation $\theta \leq \lambda/R$, where R is the radius of interaction. At $\lambda \simeq 4.6 \times 10^{-13} cm$ (E = 1 MeV) and at R exceeding the radius of nuclear interaction ($R \simeq 2 \times 10^{-11} cm$) it is possible to obtain $\theta \simeq 1.5^0$.

As a result of first measurements in 1955 [3], carried out in Obninsk on lead at neutron energies of 2-3 MeV, so-called Schwinger scattering was discovered. This phenomenon was predicted by Schwinger in 1948 but was not found till the mid-50s, despite the efforts of physicists from USA, Canada and other countries. It is the result of interaction between the magnetic moment of moving neutron and the Coulomb field of nucleus.

The following data [4,5], obtained by me in Obninsk at small angle megaelectronvolt neutron scattering by the nuclei of Pu, U, Bi, Pb, Sn and Cu, were processed using the optical model of nucleus supplemented by the Schwinger potential. The results are shown on Fig.1 [5], where dashed curves represent purely nuclear scattering, dot-and-dashed curves represent nuclear scattering supplemented by the Schwinger potential. Thus, as long ago as 1957 the additional scattering for the plutonium and uranium nuclei in the region of small angles was observed, which could not be explained by nuclear and the Schwinger potentials only. Later similar additional scattering of megaelectronvolt neutrons was observed in many works (Obninsk - up to 1989, Gatchina, USA, Italy, etc.).

It was natural to explain the obtained results by the contribution of scattering caused by neutron polarizability using potential (1). In case of the Coulomb field it will be

$$V(r) = -1/2(\alpha E^2) = -\alpha(Ze^2)/(2r^4) \tag{2}$$

and the value of polarizability coefficient α obtained during experimental data processing will be about $\alpha \approx 10^{-40} cm^3$ [5].

In 1960 the Goldansky's group measured the α value for proton in the experiment of γ-quantum scattering with the energy from 40 to 70 MeV on hydrogen. It proved to be in the region of $10^{-42}cm^3$, that is 100 times less than Obninsk's value. By that time theoretical evaluations of the nucleon α appeared, primarily, in the works by Prof. Baldin. However, all of them led to the value $\alpha \approx 10^{-42}cm^3$ and, thus, were at variance with the Obninsk's value.

The information about the value of α can be also obtained by studying neutron scattering on heavy nuclei at the energies less than 300 keV. Such kind of experiments were started since 1960. Among them the experiment of 1966 should be mentioned. It was performed in FLNP (Dubna) using the time-of-flight method on lead at the pulsed reactor IBR in the neutron energy region from 0.6 to 26 keV. As a result, the value $\alpha \leq 6 \times 10^{-42}cm3$ was obtained [6]. This value remained record-breaking up to 1986, which is about 20 years.

Later the measurements of angular distribution of neutrons and of total cross sections were performed by FLNP in cooperation with Garching (Germany), in Gatchina, as well as in Austria - USA, in England and other countries. However, in all this works the estimations of α neutron value are less by a factor of 100 than the value $10^{-40}cm^3$, namely, they are in the region $10^{-42}cm3$. Thus, there was a serious deviation between the results obtained in the megaelectronvolt neutron region and those obtained in the region of energies lower than 300 keV. This contradiction remained unexplained for 45 years. I think, the explanation was found due to two more factors, apart from the factor of time, of course. The first one was pointed out in 1959 by Blokhintsev, Barashenkov and Barbashov in the article [7] : ...perhaps there are effects of interaction between the neutron and the electron shell of heavy nuclei. The second one is a possible existence of long-range action (forces of the van der Waals' type, r^{-6} in hadron interactions, to which Sawada (Japan) [8] paid and pays special attention (in 2000 one of the paper by Sawada was entitled as Proposal to observe the strong van der Waals force in the low energy neutron - Pb scattering). Apropos, as I know, the first work discussing a possible existence of long-range hadron interactions belongs to Prof. Wilkinson (1961, The Rutherford Jubilee International Conference). In 1999 it was shown in the work by Pokotilovsky [9] and in his work made in cooperation with his colleagues [10] that at the neutron energies 0.5 - 10 MeV the changes in differential cross section of neutron scattering on the isotope ^{208}Pb in the region of small angles caused by the potential (2) with the value ($\alpha = 1.5 \times 10^{-40}cm^3$ will be the same as those caused by the van der Waals potential

$$V(r) = -U_R(R/r)^6 \tag{3}$$

(R is the radius of nucleus) when choosing constant $U_R \approx 300$ keV. At the same time the potential (3) will not practically appear in the region of lower energies. The constant UR was not calculated in the works by Pokotilovsky's group but was defined by selection, by trial method to achieve the best agreement between the calculations and the experiment. The constant UR was not related to neutron polarizability spatially.

However, it was desirable to calculate the constant U_R and see how it was related to neutron polarizability that is to the value α. These calculations were performed by me, your obedient servant, in 2001. I will not tell you about these calculations in detail. I told about these calculations in Dubna (ISINN-10) and in Sarov on International Conferences [11, 12] last year (see also [13]). It should be noted that they are not undoubtedly precise yet. However, at present, it is enough to be sure that there are no considerable errors that would change the neutron value (by the factor 100, since this is the difference between the values $10^{-42} cm^3$ and $10^{-40} cm^3$ which were obtained at low (less than 300 keV) and higher (0.5 - 10 MeV) energies.

As a result of calculations in the second approximation of perturbation theory one can obtain:

$$U(\theta) = \frac{3\alpha}{2R^6} \sum \Delta E_i \alpha_i(\theta) \tag{4}$$

where $\Delta E_i \approx E_i$ is the binding energy of the i-th electron in atom, α_i is the polarizability of the i-th electron in atom.

Equation (4), which shows that the van der Waals interaction energy is proportional to the product of polarizabilities of two systems, is universal, i.e. it does not depend on the internal structure of the interacting systems and it holds for atoms, hadrons, and elementary particles of other types. It only depends on the validity of general principles, such as the Lorentz invariance, electromagnetic current conservation, analyticity and unitarity (see, e.g. [14]). We will continue using eq.(4) below.

The sought constant U_R can be obtained by the operation of averaging over the scattering angles from 3^0 to 15^0 (this small angle range was considered in paper[10]):

$$U_R = \int U_R(\theta) sin(\theta) d\theta / \int sin(\theta) d\theta \tag{5}$$

It is necessary to know (i of electrons in a complex atom. At present, however, there exists no strict theory of their calculation. In the first approximation we can accept the model of atom as a linear oscillator and, thus, we have from the book by Blokhintsev [15]:

$$\alpha_i(\theta) = N_i(\theta)\frac{e^2\hbar^2}{mE_i^2} \tag{6}$$

where m is electron mass, $N_i(\theta)$ is the number of i-th electron in atom.

It is possible to verify eq (6) on the examples of some atoms. It is known, e.g., that for hydrogen atom $\alpha_H = 6.66 \times 10^{25} cm^3$. From eq (6) one can obtain $\alpha_H = 6.06 \times 10^{25} cm^3$, if E =13.5 eV . From eq (6) one can obtain for tin atoms the value of α_{Sn}, which will be approximately equal to the value obtained as the result of the Thomas-Fermi model calculations [16]. For uranium atoms the difference between the results of analogous calculations does not exceed 1.5 times [16].

The spatial distribution of electrons in the atom can be determined from the angular distribution of small angle scattered neutrons. In the first approximation the distance of the neutron trajectory going through the atom from the nucleus, ΔR, is related to the scattering angle as $\Delta R \approx \lambda/\theta$. Knowing ΔR and using the Thomas-Fermi model for the atom, it is possible to determine the number of electrons N_i participating in the investigated process, i.e. of those which are at a distance smaller than ΔR from the nucleus.

Carrying out the calculations numerically, it is possible to obtain the U_R = 210 keV for uranium, for the neutron energy 1 MeV and for the neutron polarizability $\alpha = 1.5 \times 10^{-42} cm^3$. Thus, the neutron polarizability was first detected in small angle neutron scattering experiment as far back as 1957 in Obninsk, i.e. earlier than proton polarizability was observed in the $\gamma - p$ scattering experiment (1960).

In conclusion, it should be emphasized, that similar to the Hofstadter experiments that proved the nucleon to have a spatial structure, the notion of deformation (polarizability) of the nucleon and its discovery in the experiment do not only lead to a new important physical property but are also of fundamental philosophic importance.

The important role in solution of this problem belongs to Prof. D. I. Blokhintsev and I want to emphasize it in my report.

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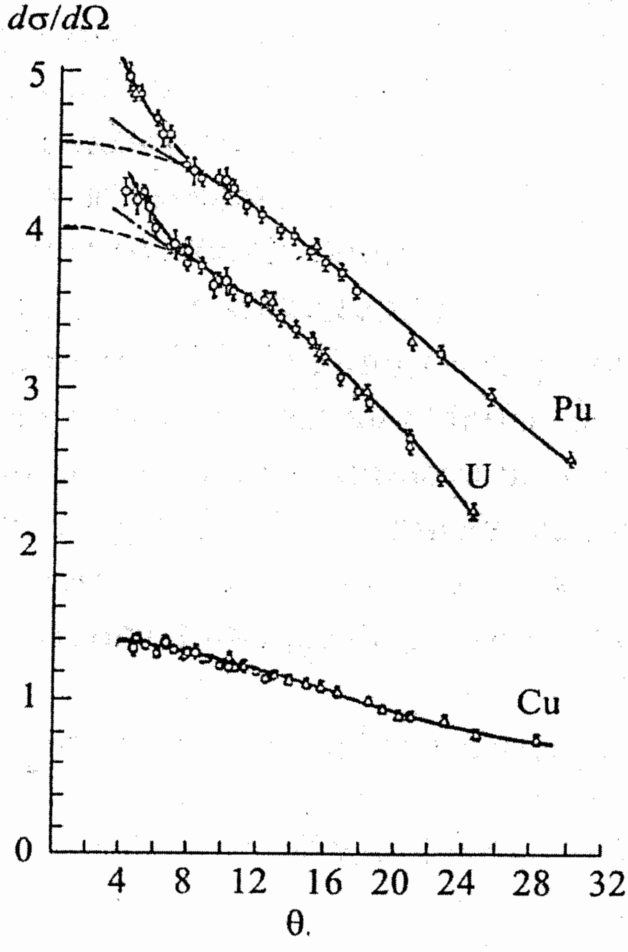

Figure 1:

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Domain wall generation by fermion self-interaction

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Abstract — A possibility for light fermions and Higgs bosons to be localized on the four-dimensional domain wall in five-dimensional world is examined. The mechanism of light particle trapping is accounted for by a strong self-interaction of five-dimensional pre-quarks. The fluctuation of the brane gives rise to a nearly sterile scalar particles, branons, which may be candidates for the dark matter.

1 Introduction

The possibility of location of the observable world on a four-dimensional surface – a domain wall or a 3-brane – in a space-time with dimension higher than four has been recently set forth as a theoretical concept [1]– [4] for solutions to the problems of the Planck mass scale, symmetry breaking scales and fermion mass hierarchy. Respectively, an experimental program has been posed for the forthcoming collider and non-collider physics research to discover new particles signaling on the existence of extra dimensions. The extensive literature on those subjects and their applications is now covered in few review articles [5]- [8]. Among other options the domain wall formation and the trapping of low-energy particles in its layer might be triggered [9]- [11] by a number of background fields living in the multi-dimensional bulk. Certainly, the dynamical origin of such background fields and the way of how they induce spontaneous symmetry breaking is to be explored.

In our talk we describe the non-compact *4 + 1*-dimensional fermion model with strong local four-fermion interaction that leads to the discrete symmetry breaking and to a domain wall pattern of the vacuum state [12]. As a consequence light massive Dirac particles and light scalar bosons, Higgs-like particles and "branons" [13], live in four dimensions whereas very heavy states may leave a brane, *i.e.* disappear from our world.

The main ingredients of the model are made of the five-dimensional fermion bi-spinors $\psi(X)$ coupled to a scalar field $\Phi(X)$. The extra-dimension coordinate is assumed to be space-like, $(X_\alpha) = (x_\mu, z)$, $\mu = (0,1,2,3)$, and the subspace of x_μ eventually corresponds to the four-dimensional Minkowski space.

The extra-dimension size is supposed to be large enough. The fermion wave function obeys then the Dirac equation

$$[i\gamma_\alpha\partial^\alpha - \Phi(X)]\psi(X) = 0\ , \quad \gamma_\alpha = (\gamma_\mu, -i\gamma_5)\ , \quad \{\gamma_\alpha, \gamma_\beta\} = 2g_{\alpha\beta}\ ,$$

with γ_α being a set of the Dirac matrices.

The trapping of light fermions on a four-dimensional hyper-plane – the domain wall – located in the fifth dimension at $z = 0$ can be generated by a certain *topological* background configuration of the scalar field, for instance, by $\langle\Phi(X)\rangle_0 = M\tanh(Mz)$ as it induces a z-localized zero-mode in the four-dimensional fermion spectrum, i.e. an essentially four-dimensional massless fermion with a given chirality (left or right).

Meanwhile the real quarks and leptons of our world are mainly massive Dirac fermions. Therefore, for each light fermion on a brane one needs at least two five-dimensional proto-fermions $\psi_1(X), \psi_2(X)$, to obtain left- and right-handed parts of a four-dimensional Dirac bi-spinor as zero modes. In order to produce zero modes with different chiralities they have to couple to a background scalar field with opposite charges,

$$[i\,\hat{\not\partial} - \tau_3\Phi(X)]\Psi(X) = 0\ , \quad \hat{\not\partial} \equiv \hat\gamma_\alpha\partial^\alpha\ , \quad \Psi(X) = \begin{pmatrix} \psi_1(X) \\ \psi_2(X) \end{pmatrix}\ , \qquad (1)$$

where $\hat\gamma_\alpha \equiv \gamma_\alpha \otimes \mathbf{1}_2$ are Dirac matrices and $\tau_a \equiv \mathbf{1}_4 \otimes \sigma_a$, $a = 1, 2, 3$ are Pauli matrices acting on the bi-spinor components $\psi_i(X)$.

The next task is to supply these fermions with light masses. As the mass operator mixes left- and right-handed components of the four-dimensional fermion it is embedded in the Dirac operator (1) with the mixing matrix $\tau_1 m_f$ of the fields $\psi_1(X)$ and $\psi_2(X)$. Following the Standard Model fermion mass generation by means of Higgs scalars, one may introduce the second scalar field $H(x)$ replacing the bare mass $\tau_1 m_f$ by v.e.v. $\tau_1 < H(x) >$. Both scalar fields might be dynamical indeed and their self-interaction should justify the spontaneous symmetry breaking by certain classical configurations allocating light massive fermions on the domain wall.

2 Fermion model with self-interaction in 5D

After the preliminary motivation we formulate our model with the following Lagrange density

$$\mathcal{L}^{(5)}(\overline{\Psi}, \Psi) = \overline{\Psi}\, i\,\hat{\not\partial}\Psi + \frac{g_1}{4N\Lambda^3}\left(\overline{\Psi}\tau_3\Psi\right)^2 + \frac{g_2}{4N\Lambda^3}\left(\overline{\Psi}\tau_1\Psi\right)^2,$$

$$\Longrightarrow \overline{\Psi}(i\,\partial\!\!\!/ - \tau_3\Phi - \tau_1 H)\Psi - \frac{N\Lambda^3}{g_1}\Phi^2 - \frac{N\Lambda^3}{g_2}H^2 .$$

where $\Psi(X)$ is an eight-component five-dimensional fermion field with the total number $N = N_f N_c$ of color and flavor states. The ultraviolet cut-off scale Λ bounds fermion momenta and g_1 and g_2 are suitable dimensionless effective couplings. In the second line the bosonization with the help of a pair of auxiliary scalar fields $\Phi(X)$ and $H(X)$ is performed to get the required trilinear coupling of fermions and scalars.

In this Lagrangian the discrete τ-symmetry: $\Psi \to \tau_1\Psi$, $\Phi \to -\Phi$ and $\Psi \to \tau_3\Psi$, $H \to -H$, does not allow the fermions to acquire a mass and prevents a breaking of translational invariance in the perturbation theory.

On the other hand, for sufficiently strong couplings, this system undergoes spontaneous breaking of the τ-symmetry. In order to describe it, the effective low-energy Lagrange density is derived with the help of FMR [14],

$$\begin{aligned}\mathcal{L}^{(5)}_{\rm low} &= \overline{\Psi}_l(X)[\,i\,\partial\!\!\!/ - \tau_3\Phi(X) - \tau_1 H(X)\,]\Psi_l(X) \\ &+\frac{N\Lambda}{4\pi^3}\,\{\partial_\alpha\Phi(X)\partial^\alpha\Phi(X) + \partial_\alpha H(X)\partial^\alpha H(X) \\ &+2\Delta_1\Phi^2(X) + 2\Delta_2 H^2(X) - [\,\Phi^2(X) + H^2(X)\,]^2\Big\} . \qquad (2)\end{aligned}$$

where the two mass scales Δ_i are introduced in order to parameterize the deviations from the critical point $g_i^{\rm cr} = 9\pi^3$, $\quad i = 1, 2$:

$$\Delta_i(g_i) = \frac{2\Lambda^2}{9g_i}\,(g_i - g_i^{\rm cr}) \qquad \Delta_1(g_1) > \Delta_2(g_2) .$$

If $\Delta_1(g_1) > 0$, then the true minima appear at a non-vanishing vacuum expectation value of the scalar field $\Phi(X)$: namely,

$$(I) \qquad \Phi_I \equiv \langle\Phi(X)\rangle_0 = \pm\sqrt{\Delta_1(g_1)} \equiv \pm M\ , \quad H_I \equiv \langle H(X)\rangle_0 = 0\ .$$

This follows from the stationary point conditions,

$$2\left[M^2 - \Phi^2 - H^2\right]\Phi = \partial^\alpha\partial_\alpha\Phi\ , \quad 2\left[\Delta_2 - H^2 - \Phi^2\right]H = \partial^\alpha\partial_\alpha H\ ,$$

and from the positive definiteness of the second variation of the boson effective action. From the latter one we find the masses of composite scalars: $M_1 = 2M$ (the Nambu relation) and $M_2 = \sqrt{2\Delta_1 - 2\Delta_2}$. Respectively, the generation of a dynamical fermion mass M occurs that breaks the τ_1-symmetry. Around this minimum the particle physics is entirely five-dimensional.

3 Domain walls

The existence of two minima in the potential gives rise to another set of vacuum solutions which connect smoothly the minima.

$$(J) \qquad \langle\Phi(X)\rangle_0 = \pm M\tanh(Mz) \ , \quad \langle H(X)\rangle_0 = 0 \ ;$$

$$(K) \qquad \langle\Phi(X)\rangle_0 = \pm M\tanh(\beta z) \ , \quad \langle H(X)\rangle_0 = \pm\mu\,\mathrm{sech}(\beta z) \ .$$

where $\beta = \sqrt{M^2-\mu^2} = \sqrt{2(\Delta_1-\Delta_2)}$. The solution (K) exists only for $0 < 2\Delta_2 - M^2 \equiv \mu^2$. For such a range of parameters Δ_i this solution delivers a minimum (it can be found out of the second variation around it) whereas the solution (J) lies on a saddle point.

In both phases v.e.v. of the scalar field $\langle\Phi(X)\rangle$ has a kink shape and hence its coupling to fermions induces the trapping of the lightest, massless fermion state on the domain wall. However, only the solution (K) supplies this light fermion with a mass due to a non-zero v.e.v. of the field $\langle H(X)\rangle$. Let us focus on this phase.

At ultra-low energies much smaller than M, the physics on the vacuum (K) is essentially four-dimensional. It is described by the Dirac fermion with the mass

$$m_f \equiv \int_{-\infty}^{+\infty} dz\, \overline{\psi}_0(z)\, H_K(z)\, \psi_0(z) = \frac{\pi}{4}\,\mu.$$

As well one has two localized ultra-light modes in the spectrum of the second-variation operator for the scalar fields $\Phi(X)$ and $H(X)$. They produce two four-dimensional scalar bosons, a massless ϕ and a massive h. When assuming that $\mu \ll M$ one finds,

$$m_h = \mu\sqrt{2} + \mathcal{O}(\frac{\mu^2}{M}).$$

The ultra-low-energy effective four-dimensional Lagrangian density for the light states reads:

$$\mathcal{L}^{(4)} = \overline{\psi}(x)(i\gamma^\mu\partial_\mu - m_f)\psi(x) + \frac{1}{2}\Big(\partial_\mu\phi(x)\Big)^2 + \frac{1}{2}\Big(\partial_\mu h(x)\Big)^2 - \frac{1}{2}m_h^2 h^2(x)$$

$$-g_f\overline{\psi}(x)\psi(x)h(x) - \lambda_1\phi^4(x) - \lambda_2\phi^2(x)h^2(x) - \lambda_3 h^4(x) - \lambda_4\, h^3(x) \ ,$$

with the ultra-low energy effective couplings given by

$$g_f = \frac{\pi}{4}\sqrt{\frac{M\pi^3}{\Lambda N}} \ , \ \lambda_1 = \frac{18M\pi^3}{35\Lambda N} \ , \ \lambda_2 = \frac{2M\pi^3}{5\Lambda N} \ , \ \lambda_3 = \frac{M\pi^3}{3\Lambda N} \ , \lambda_4 = \mu\pi\sqrt{\frac{M\pi}{\Lambda N}} \ .$$

Herein the heavy scalars and fermions with masses $\sim M$ have been decoupled.

Quite remarkably, the domain wall Lagrange density has a non-trivial large cut-off limit in the vicinity of the scaling point $\mu \ll M$, provided that the ratio $M/\Lambda < 1$ is fixed. The four-dimensional ultra-low-energy theory happens to be interacting.

We stress that *a priori* possible 3-point vertex $\phi^2(x)\,h(x)$ does not emerge and the coupling of light fermions to the massless scalar $\overline{\psi}(x)\psi(x)\phi(x)$ cannot appear in principle. Thereby the direct decay of the massive Higgs-like boson h into a pair of massless branons [13] is suppressed and the low-energy Standard Model matter turns out to be stable.

4 Manifest breaking of translational invariance

One can conceive that in reality the translational invariance in five dimensions is broken not only spontaneously but also manifestly due to the presence of a gravitational background, of other branes etc. In a full analogy with the Pseudo-Goldstone boson physics one can expect [12,13] that the small manifest breaking of translational symmetry supplies the branons with a small mass.

In the model presented here the natural realization of the translational symmetry breaking can be implemented by the inhomogeneous scalar background fields coupling to the lowest-dimensional fermion currents. Let us restrict ourselves to the scenario of the type (K) and introduce two scalar defects with the help of the background fields $\mathrm{F}_\Phi(z)$ and $\mathrm{F}_H(z)$, which are supposed to be quite small. These scalar defect fields catalyze the translational symmetry breaking and the domain wall formation by means of their interactions with the fermion currents,

$$\mathcal{L}_{\mathrm{F}}^{(5)} = \; - \,\mathrm{F}_\Phi(z)\,\overline{\Psi}(X)\tau_3\Psi(X) \; - \; \mathrm{F}_H(z)\,\overline{\Psi}(X)\tau_1\Psi(X)\,.$$

In units comparable with the low energy effective action (2),

$$\mathrm{F}_\Phi(z) \equiv \frac{g_1\mu^3}{4\pi^3\Lambda^2}\,\mathrm{f}_\Phi(z)\;, \qquad \mathrm{F}_H(z) \equiv \frac{g_2\mu^3}{4\pi^3\Lambda^2}\,\mathrm{f}_H(z)\;,$$

Let us introduce a defect of topological type:

$$\mu\,\mathrm{f}_\Phi(z) = M\,\gamma\tanh(\bar\beta z); \quad \mathrm{f}_H(z) = \kappa\,\mathrm{sech}(\bar\beta z)\;,$$

where γ, κ are dimensionless parameters. For this particular ansatz the solution of the modified equations for stationary configurations of scalar fields can be found analytically [12] in a similar form,

$$\langle\Phi(X)\rangle_0 = M\,a\tanh(\bar\beta z)\;, \quad \langle H(X)\rangle_0 = \mu\,(1+\xi)\,\mathrm{sech}(\bar\beta z)\;.$$

where for $\mu/M \ll 1, \kappa \ll 1, \xi \ll 1$ one obtains approximately $a \approx 1$, $\bar{\beta} \approx \beta$ and $\kappa \approx 2\xi + (\gamma/2)$. Further on we use ξ as an input parameter instead of κ.

The branon mass happens to be triggered entirely by the topological defect,

$$(m_\phi)^2 \approx \gamma\mu^2 .$$

One can see a strong polarization effect induced by a topological defect: the local minimum is guaranteed only for asymptotics at infinities which are coherent in their signs, *i.e.*, for positive γ.

The Higgs mass encodes both topological and non-topological vacuum perturbations,

$$(m_h)^2 \approx \mu^2 \left(2 + 6\xi + \frac{1}{2}\gamma\right) .$$

As $\gamma > 0$ the topological defect makes the Higgs particle heavier. However the sign of ξ is not fixed by the requirement to provide a local minimum. Therefore the Higgs mass ratio to the fermion ($\sim$ top-quark) mass can be substantially reduced with an appropriate choice of a non-topological part of the defect $\sim \xi$. In particular, the Higgs masses may be well adjusted to a phenomenologically acceptable value $\sim$ 135 GeV for a reasonably small value of a defect $\xi \sim 0.4$. The induced coupling constants $\lambda_1, \lambda_2, \lambda_3$ appear to be insensitive to a very small background defect whereas the fermion mass and the constant λ_4 are subject to rescaling $m_f \approx m_f^{(0)}(1+\xi)$, $\lambda_4 \approx \lambda_4^{(0)}(1+\xi)$.

Thus a very small topological defect does not change the main dynamics of domain wall trapping of light fermions and scalars. In particular, the Standard Model matter remains stable and the "branons", being now massive, yet do not decay directly into a pair of fermion and anti-fermion. It makes then difficult to register them in the collider experiments. Nonetheless, for top-quarks (in the fusion production) there might be a room to discover branon pair signals for sufficiently light branons.

Anyhow, the nearly sterile, massive branons seem to be good candidates for saturation of the dark matter of our Universe [13].

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Coupled channel analysis of f_0 mesons in $p\bar{p}$ annihilation and $\pi\pi$ scattering

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Abstract — The properties of scalar $f_0(980)$, $f_0(1370)$, $f_0(1500)$ mesons are studied in $\pi\pi$ scattering and $p\bar{p}$ annihilation at rest with unitarized multi-state Breit-Wigner (BW) formulae for scattering amplitudes and production processes.

1 Unitarized multi-state BW approach

We consider a situation with several overlapping resonances with the same quantum numbers coupling to various hadron states and develop unitarized multi-resonance BW formula.

The use of the K-matrix rather than T-matrix is a simple way to satisfy unitarity. The actual physics states (the most often point of view is that they are closely related to BW description) and the K matrix states are different. Thus, a writing down the S matrix as a sum of resonance terms and then to solve the constraints imposed by unitarity can give a direct way to extract resonances parameters from data.

Lets write the scattering matrix S in a resonance form:

$$\mathrm{S}(E) = \mathrm{I} - i\sqrt{\rho(E)}\,\mathrm{T}(E)\,\sqrt{\rho(E)}, \quad \mathrm{T}(E) = \sum_{r=1}^{N} \frac{\vec{g}_r \vec{g}_r}{E - \varepsilon_r(E)}. \tag{1}$$

Here $\varepsilon_r(E) \equiv \varepsilon_r^x(E) + i\varepsilon_r^y(E)$. Complex energy independent vectors of partial widths $\vec{g}_r \equiv \vec{g}_r^x + i\vec{g}_r^y$ have M-components (M is the number of channels). The scattering matrix T is free of threshold singularities that are included in the diagonal $M \times M$ matrix $\rho(E)$. Below the k-th threshold $\rho_k(E) => i|(\rho_k(E)|$ (variable E is used instead of $s = E^2$ to simplify formulae). In further development of work [1] the resonances widths are energy dependent and we can study the case when resonances lie close to thresholds. A way to include background is described in [1].

A complicity of the vectors $\vec{g}_r$ is another way to take into account the relative phases between different BW terms, thus

$$T_{ij}(E) = \sum_{r=1}^{N} \mathrm{e}^{i\varphi_{ij}^{(r)}} \frac{|g_{ir}| \cdot |g_{rj}|}{E - \varepsilon_r(E)}, \tag{2}$$

where φ^r_{ij} are real (constant) relative phases.

Find the conditions that should be imposed on vectors $\vec{g}_r$ to keep matrix S unitary and T invariant, i.e. $S^+(E)S(E)= I$ and $S_{ij}(E) = S_{ji}(E)$ identically on E. To satisfy these relations, vectors $\vec{g}_r$ are constructed in the way such that their imaginary parts $\vec{g}^y_r$ are combinations of the real parts $\vec{g}^x_r$:

$$\vec{g}^y_r = u_{r1}\vec{g}^x_1 + u_{r2}\vec{g}^x_2 + ... + u_{rN}\vec{g}^x_N, \qquad (r = 1, ..., N), \tag{3}$$

where matrix $U = \{u_{rk}\}^N_{r,k=1}$ is a real antisymmetric matrix ($u_{rr} = 0$, $u_{rk} = -u_{kr}$) and vectors $\vec{g}_r$ satisfy the relations

$$\sum_{k=1}^{M} \rho_k(E)\theta_k(E)\,|g_{rk}|^2 = -\frac{2}{S}[S + 2Q_r]\varepsilon^y_r, \tag{4}$$

$$\sum_{k=1}^{M} \rho_k(E)\theta_k(E)\mathrm{Re}(g^*_{qk}g_{rk}) = -\frac{2}{S}[F_{qr}(\varepsilon^x_q - \varepsilon^x_r) + G_{qr}(\varepsilon^y_q + \varepsilon^y_r)], \tag{5}$$

$$\sum_{k=1}^{M} \rho_k(E)\theta_k(E)\mathrm{Im}(g^*_{qk}g_{rk}) = -\frac{2}{S}[G_{qr}(\varepsilon^x_q - \varepsilon^x_r) - F_{qr}(\varepsilon^y_q + \varepsilon^y_r)]. \tag{6}$$

Here $r = 1, ..., N$, $q = r + 1, ..., N$. The constant coefficients S, Q_r, F_{qr} G_{qr} are determined via the elements of matrix U.

The matrix U gives a measure of overlapping of resonances. If the resonances do not overlap, $|E_i - E_j| \gg \Gamma_i + \Gamma_j$, the matrix elements $u_{rk} \to 0$, and vectors $\vec{g}_r$ are getting real and orthogonal: $g_r = g^x_r$, $(\vec{g}_r\vec{g}_q) = 0$. In this case $\varepsilon^x_r(E)$ can be arbitrary real functions of E (constants, when $\rho(E) \approx const$) and the resonance widths are

$$\Gamma_r(E) = -2\varepsilon^y_1(E) = \sum_{k=1}^{M} \rho_k(E)\,\theta_k(E)|g_{rk}|^2. \tag{7}$$

Taking, for instance, $\varepsilon_r(E) = m_r(E) - \frac{i}{2}\sum_{k=1}^{M} \rho_k(E)(g_{rk})^2$, where $m_r(E)$ are real functions (or constants), we obtain Flatte kind formula for several resonances:

$$T_{ij} = \sum_{r=1}^{N} \frac{g_{ir}g_{rj}}{E - m_r + \frac{i}{2}\sum_{k=1}^{M} \rho_k(g_{rk})^2}. \tag{8}$$

For overlapping resonances vectors $\vec{g}_r$ are complex and the expressions for masses and widths can be obtained from relations (4) - (6):

$$\varepsilon^y_r(E) = -\frac{S}{2(S + 2Q_r)}\sum_{k=1}^{M} \rho_k(E)\theta_k(E)\,|g_{rk}|^2, \qquad r = 1, ..., N.$$

If function $\varepsilon_1^x(E)$ is chosen as an arbitrary real function then functions $\varepsilon_r^x(E)$ ($r = 2, ..., N$) are determined with

$$\varepsilon_r^x(E) = \varepsilon_1^x(E) + \frac{S}{2(F_{1r}^2 + G_{1r}^2)} \times \\ \times \sum_{k=1}^{M} \rho_k(E)\theta_k(E) \left[F_{1r}\mathrm{Re}(g_{1k}^* g_{rk}) + G_{1r}\mathrm{Im}(g_{1k}^* g_{rk})\right]. \tag{9}$$

The remaining $(N-1)(N-2)/2$ equations (5)-(6) imply the limitations on the vectors $\vec{g}_r$ (when the number of resonances $N > 2$) [1]. For three resonances ($N = 3$) these limitations are just the quadratic equations for each $k = 1, ..., M$. When $N > 3$, the corresponding equations become nonlinear and practical usage of this method (when $\rho_k(E)$ are not constants) is not simple.

The solution of equation $E - \varepsilon_r^x(E) = 0$ gives the mass of the r-th resonance, $\hat{\epsilon}_r$ (generally there might be several roots, but for f_0 resonances we obtained one root for each r).

Equation (7) gives the width of the r-th resonance:

$$\hat{\Gamma}_r = \Gamma_r(\hat{\epsilon}_r) = \frac{S}{S + 2Q_r} \sum_{k=1}^{M} \rho_k(\hat{\epsilon}_r)\theta_k(\hat{\epsilon}_r)\, |g_{rk}|^2. \tag{10}$$

Correspondingly, partial widths are given by

$$\hat{\Gamma}_{rk} = \frac{S}{S + 2Q_r} \rho_k(\hat{\epsilon}_r)\theta_k(\hat{\epsilon}_r)\, |g_{rk}|^2. \tag{11}$$

Branching ratio of the r-th resonance into the k-th channel is

$$B_{rk} = \frac{\hat{\Gamma}_{rk}}{\hat{\Gamma}_r} = \frac{\rho_k(\hat{\epsilon}_r)\theta_k(\hat{\epsilon}_r)\, |g_{rk}|^2}{\sum_{k=1}^{M} \rho_k(\hat{\epsilon}_r)\theta_k(\hat{\epsilon}_r)\, |g_{rk}|^2}. \tag{12}$$

It is instructive to show two resonances case (for two overlapping resonances with constant widths the unitarized BW formula was derived first in [2]):

$$S_{ij} = \delta_{ij} - i\sqrt{\rho_i \rho_j}\left[\frac{g_{i1}g_{1j}}{E - \varepsilon_1(E)} + \frac{g_{i2}g_{2j}}{E - \varepsilon_2(E)}\right], \quad i, j = 1, ..., M. \tag{13}$$

In this case $\mathbf{U} = \begin{pmatrix} 0 & -\alpha \\ \alpha & 0 \end{pmatrix}$, i.e. $\vec{g}_1^y = -\alpha \vec{g}_2^x$, $\vec{g}_2^y = \alpha \vec{g}_1^x$. Parameter α is not entirely free but limited by the condition $0 \le \alpha < 1$. The coefficients in the unitary constraints (4) - (6) are: $S = 1 - \alpha^2$, $Q_1 = Q_2 = \alpha^2$, $F_{12} = -\alpha$, $G_{12} = 0$. According to the unitarity relation, $\varepsilon_1^y(E)$ and $\varepsilon_2^y(E)$ should obey the relation (4)

$$\varepsilon_r^y = -\frac{1-\alpha^2}{2(1+\alpha^2)} \sum_{k=1}^{M} \rho_k(E)\theta(E)\,|g_{rk}|^2, \quad r = 1,2.$$

Real part of "energy" of the first resonance $\varepsilon_1^x(E)$ is an arbitrary real function of E. Lets choose $\varepsilon_1(E)$ in a form similar to the Flatte formula:

$$\varepsilon_1(E) = m_1 - i\,\frac{1-\alpha^2}{2(1+\alpha^2)} \sum_{k=1}^{M} \rho_k |g_{1k}|^2. \tag{14}$$

Function $\varepsilon_2^x(E)$ is defined by relation (9):

$$\varepsilon_2^x(E) = \varepsilon_1^x - \frac{1-\alpha^2}{2\alpha} \sum_{k=1}^{M} \rho_k(E)\theta_k(E)\, g_{1k}^x g_{2k}^x.$$

Therefore, the complex "energy" of the 2-nd resonance can be given as

$$\begin{aligned}
\varepsilon_2(E) &= D(E) - i\,\frac{1-\alpha^2}{2(1+\alpha^2)} \sum_{k=1}^{M} \rho_k |g_{2k}|^2, \\
D(E) &= m_1 + \frac{1-\alpha^2}{2(1+\alpha^2)} \sum_{k=1}^{M} \rho_k (1-\theta_k(E))(|g_{1k}|^2 - |g_{2k}|^2) - \\
&\quad - \frac{1-\alpha^2}{2\alpha} \sum_{k=1}^{M} \rho_k(E)\theta_k(E)\, g_{1k}^x g_{2k}^x.
\end{aligned}$$

The widths of the resonances are $\Gamma_r(\hat{\epsilon}_r) = \frac{1-\alpha^2}{1+\alpha^2} \sum_{k=1}^{M} \rho_k(\hat{\epsilon}_r)\,|g_{rk}|^2$. Free parameters are m_1, α and coordinates of vectors $\vec{g}_1^x$, $\vec{g}_2^x$.

2 Production processes

We have discussed so far the case of transitions between the same states i, j as occur in matrix T. Similarly to [3] it is simple to consider the slightly more general case in which g_{ir} is replaced by f_{pr} where p, the production channel, does not occur in the sums in $\Gamma_r(E)$ and $\varepsilon_r(E)$. For the transition from production state p to final state j we have

$$F_{pj} = \sum_{r=1}^{N} \frac{f_{pr} g_{rj}}{E - \varepsilon_r^x(E) + \frac{i}{2}\Gamma_r(E)}. \tag{15}$$

Vectors $\vec{f}_r$ like vectors $\vec{g}_r$ are complex. The amplitudes F and T have the same poles.

3 Analysis of $f_0(980), f_0(1370), f_0(1500)$ states

The model includes four channels: $\pi\pi$, $K\bar{K}$, $\eta\eta$, 4π. The phase states factors $\rho_k(s)$ are:

$$\begin{aligned}
\rho_k(s) &= \sqrt{(s-s_k)/s}, \quad \sqrt{s_k} = 2m_k, \quad k = 1,2,3, \\
\rho_4(s) &= \sqrt{(s-s_4)/s}/\left(1+\exp\left[\Lambda \cdot (s_0 - s)\right]\right), \\
& \quad \sqrt{s_4} = 4m_\pi, \quad s_0 = 1.9\ (GeV^2), \quad \Lambda = 6.0\ (GeV^{-2}).
\end{aligned}$$

Such a choice of ρ_4 approximates either the $\rho\rho$ or $\sigma\sigma$ phase space [4].

We describe well the data on the S-wave $\pi\pi$ scattering and available (see [5]) mass spectra of the processes $p\bar{p} \to \pi^0(\eta\eta)$, $\pi^0(4\pi)$ (due to lack of space we do not present figures). The tables show f_0 parameters.

Table 1. Parameters of f_0 mesons (in GeV)

Meson	Mass	Width
$f_0(980)$	0.987	0.117
$f_0(1370)$	1.356	0.269
$f_0(1500)$	1.549	0.151

Table 2. Branching ratios of f_0 mesons (in %) and relative (to $f_0(980)$) phases of $f_0(1370)$ and $f_0(1500)$ (in degrees).

State	$f_0(980)$	$f_0(1370$	$\varphi_{f_0(1370)}$	$f_0(1500)$	$\varphi_{f_0(1500)}$
$\pi\pi$	77.29	8.19	−28.9	4.03	28.4
KK	20.81	5.83	−38.0	3.37	18.2
$\eta\eta$	1.89	52.66	68.2	57.76	18.6
4π	0.01	33.31	88.7	34.84	26.5

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Coexistence of color superconductivity and chiral symmetry breaking

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Abstract — The phase diagram for quark matter is investigated within a simple Nambu–Jona-Lasinio model without vector correlations for two parameter sets. In one of the two parametrization schemes a coexistence phase of broken chiral symmetry with color superconductivity is found.

1 Introduction

The phenomenon of color superconductivity in quark matter [1] is of general interest nowadays due to its relevance for the physics of compact stars [2] and potential precursor phenomenona in heavy-ion collisions [3]. Different aspects have been investigated so far, whereby models of the NJL type have been widely employed [4–7] in studies of the phase structure in the vicinity of the hadronization transition. It has been shown in these investigations that for low temperatures T and not too large chemical potentials μ the two-flavor Color Superconductivity (2SC) phase is favored over alternative color superconducting phases [8–10] (for $\mu \gtrsim 430$ MeV according to Ref. [10]). This finding is in contrast to studies which do not solve the gap equations but rather assume approximate SU(3) flavor symmetry [11].

It is generally agreed that at low temperatures the transition of the matter from the phase with broken chiral symmetry to the 2SC phase is of the first order (see e. g. [12]). The conclusion about the first order phase transition was drawn within models without vector interaction channels taken into account [13, 14]; the vector interaction has been considered in few papers [15, 16]. It has been demonstrated in Ref. [17] that the critical line of first order phase transition in the $\mu - T$ plane can have a second end-point at low temperatures, besides the well known one at high temperatures, as a consequence of the presence of interaction in the vector channel. In the present contribution (see [18] for more details) we demonstrate that in the absence of the vector channel interaction the resulting phase transition as well the phase diagram is sensitive to the choice of parametrization strategy and is not necessarily of the first order.

2 NJL model with the scalar diquark channel

To study the 2SC phase, a new order parameter, the scalar diquark condensate $\langle\psi\psi\rangle$ for u and d quarks in the form

$$\delta = \left\langle \bar{\psi} i\gamma_5\tau_2\lambda_2 C\bar{\psi}^T \right\rangle , \tag{1}$$

is introduced in addition to the chiral quark condensate $\langle\bar{q}q\rangle$. The new order parameter characterizes the domain of spontaneous breaking of color symmetry. In (1) the matrix $C = i\gamma_0\gamma_2$ is the charge conjugation matrix operator for fermions. The matrices τ_2 and λ_2 are Pauli and Gell-Mann matrices, acting on the flavor indices of spinors and in the color space, respectively.

We use here a simple and tractable nonperturbative model of quark interaction, the NJL model [4–7, 19, 20], which has been extensively exploited for the description of the properties of the light meson sector of QCD (also to describe the color superconductivity phase [8, 21, 22]).

Leaving the strange quark and effects related to it beyond our consideration, we start with the $SU(2)_L \times SU(2)_R$-symmetric Lagrangian density for scalar (σ), pseudoscalar triplet ($\vec{\pi}$), and diquark (Δ, Δ^*) fields

$$\begin{aligned}\tilde{\mathcal{L}} &= \bar{\psi}(i\,\partial\!\!\!/ - m^0 + \sigma + i\gamma_5\vec{\tau}\vec{\pi})\psi - \frac{1}{2}\Delta^*\psi^T C\gamma_5\tau_2\lambda_2\psi \\ &+ \frac{1}{2}\Delta\bar{\psi}\gamma_5\tau_2\lambda_2 C\bar{\psi}^T - \frac{\sigma^2+\vec{\pi}^2}{2G} - \frac{|\Delta|^2}{2H} .\end{aligned} \tag{2}$$

where $\hat{m}^0$ is the diagonal current quark mass matrix $\hat{m}^0 = \mathrm{diag}(m_u, m_d)$, G and H are interaction constants. The isospin symmetry ($m_u^0 = m_d^0 \equiv m^0$) is assumed hereafter.

Using the Nambu–Gorkov bispinor representation $q(x) = \begin{pmatrix} \psi(x) \\ C\bar{\psi}^T(x)\end{pmatrix}$, and integrating over the quark fields, one obtains

$$\mathcal{L}_{\text{eff}} = -\frac{\sigma^2+\vec{\pi}^2}{2G} - \frac{|\Delta|^2}{2H} - i\int\frac{d^4p}{(2\pi)^4}\frac{1}{2}\mathrm{Tr}\ln\left(S^{-1}(p)\right). \tag{3}$$

with

$$S^{-1}(p) = \begin{pmatrix} p\!\!\!/ - \hat{M} & \Delta\gamma_5\tau_2\lambda_2 \\ -\Delta^*\gamma_5\tau_2\lambda_2 & p\!\!\!/ - \hat{M} \end{pmatrix} \tag{4}$$

being the inverse quark propagator, $\hat{M} = (m^0-\sigma)\mathbf{1} - i\gamma_5\vec{\tau}\vec{\pi}$, and $\mathbf{1} = \mathbf{1}_c\cdot\mathbf{1}_f\cdot\mathbf{1}_D$. The trace in 3 is taken in the color, flavor, and Dirac spaces.

The minimum of the effective potential $V_{\text{eff}} = -\lim_{v_4\to\infty}\frac{1}{v_4}\int_{v_4} d^4x\mathcal{L}_{\text{eff}}$, where v_4 is the 4-volume, is given by a set of gap equations

$$\frac{\partial V_{\text{eff}}}{\partial\sigma} = \frac{\partial V_{\text{eff}}}{\partial\Delta} = \frac{\partial V_{\text{eff}}}{\partial\Delta^*} = 0\ . \tag{5}$$

Having solved Eq. (5) for σ one obtains the gap equation $\langle\sigma\rangle = 2G\langle\bar\psi\psi\rangle$ and introduces the constituent quark mass $m = -\langle\sigma\rangle + m_0$. In the NJL model the quark condensate is $\langle\bar\psi\psi\rangle = -4mI_1^\Lambda(m)$, where $I_1^\Lambda(m)$ is a divergent integral regularized by a sharp ultraviolet cutoff at the scale Λ: $I_1^\Lambda(m) = \frac{-iN_c}{(2\pi)^4}\int\theta(\Lambda^2-\vec p^{\,2})\frac{d^4p}{m^2-p^2}$.

In our model we have to fix four parameters: the four-quark interaction constants G and H, cut-off Λ, and the current quark mass m^0. Without diquarks, there are only three: G, Λ, and m^0. They are fixed by the following relations: 1) the Goldberger-Treiman relation: $m = g_\pi F_\pi$, where $F_\pi^{\text{exp}} \approx 93$ MeV is the pion weak coupling constant and g_π describes the coupling of a pion with quarks $g_\pi\vec\pi\bar\psi\vec\tau\psi$, $g_\pi^{-2} = 4I_2^\Lambda(m)$, $I_2^\Lambda(m) = \frac{-iN_c}{(2\pi)^2}\int\frac{\theta(\Lambda^2-\vec p^{\,2})d^4p}{(m^2-p^2)^2}$; 2a) the quark condensate from QCD sum rules $\langle\bar\psi\psi\rangle_{\text{QCDSR}} = -4mI_1^\Lambda(m) \approx (-240\text{ MeV})^3$; 2b) the decay constant g_ρ for the $\rho\to 2\pi$ process $g_\rho = \sqrt{6}g_\pi$, $g_\rho^{\text{exp}} \approx 6.1$; 3) the current quark mass m^0 is fixed from the GMOR relation: $F_\pi^2M_\pi^2 = -2m^0\left\langle\bar\psi\psi\right\rangle$, $M_\pi^{\text{exp}} \approx 140$ MeV or $M_\pi = 0$ in the chiral limit. With the diquark channel included, there is an additional parameter H which can be fixed with 4) the relation $H = 3/4G$ from Fierz symmetry (as e. g. in [8]) [1].

In the item 2, we have given two alternatives: a) one can either use the value of the quark condensate taken from QCD sum rule estimates or b) demand from the model that it should describe the $\rho\to 2\pi$ decay. The latter is well observable in experiment, contrary to the quark condensate.

For simplicity, we perform all calculations in the chiral limit $m^0 = 0$. In this case, when investigating the hot and dense quark matter, the borders between phases turn out to be sharp and the critical temperature and chemical potential are well defined. As a result, one obtains two different parameter sets shown in Table 1. In the Type I parameter set the interaction of quarks is stronger, the UV cut-off is smaller, and the constituent quark mass is greater. As we will see further, these two parametrizations result in qualitatively different phase diagrams.

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[1]Some authors use $H = 1/2G$. It turned out that within our model the resulting phase diagram is not much affected if one makes the choice in favor of $H = 1/2G$. However, it would be preferable to fix the constant H from some observable, e. g. from the nucleon mass.

	Λ [GeV]	G [GeV^{-2}]	H [GeV^{-2}]	m [MeV]	$\sqrt[3]{-\langle\bar{\psi}\psi\rangle}$ [MeV]	g_ρ
type I	600	12.8	9.6	350	240	9.2
type II	856	5.1	3.8	233	284	6.1

Table 1: The model parameters for two different schemes of parameter fixing.

We extend the NJL model to the case of finite temperatures T and chemical potential μ (without isospin asymmetry), applying the Matsubara formalism. The thermodynamical potential per volume is

$$\Omega(T,\mu) = -T\sum_n \int \frac{d^3p}{(2\pi)^3}\, \frac{1}{2}\mathrm{Tr}\ln\left(\frac{1}{T}\tilde{S}^{-1}(i\omega_n,\vec{p})\right) + \frac{\sigma^2}{2G} + \frac{|\Delta|^2}{2H}, \quad (6)$$

where $\omega_n = (2n+1)\pi T$ are Matsubara frequencies for fermions, and the chemical potential is included into the definition of inverse quark propagator

$$\tilde{S}^{-1}(p_0,\vec{p}) = \begin{pmatrix} \not{p} - \hat{M} + \mu\gamma_0 & \Delta\gamma_5\tau_2\lambda_2 \\ -\Delta^*\gamma_5\tau_2\lambda_2 & \not{p} - \hat{M} - \mu\gamma_0 \end{pmatrix}. \quad (7)$$

3 Numerical results

In the case of type I parameter set, we have for $T = \mu = 0$ a nonzero constituent quark mass (quark condensate) corresponding to the absolute minimum of the thermodynamical potential at $m \sim 350$ MeV and $\langle|\Delta|\rangle = 0$. At a certain chemical potential, a new local minimum related to the diquark condensate near $\langle|\Delta|\rangle \sim 110$ MeV and $m = 0$, but it does not yet give the absolute minimum. There is also a local maximum around $m \sim 200$ MeV and $\langle|\Delta|\rangle = 0$. As the matter becomes more dense, the second minimum lowers until it becomes degenerate with the first one

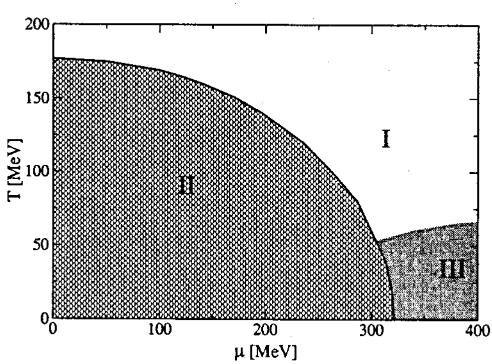

Figure 1: The quark matter phase diagram from the NJL model with the Type I parameter set.

while the average value of σ (or $-m$) remains almost unchanged. At $\mu > \mu_c \approx$ 321 MeV, the second minimum becomes the absolute one and a first order transition occurs, during which $\langle\sigma\rangle$ discontinuously changes to zero while the diquark condensate acquires nonzero value breaking the color symmetry of the strong interaction. This characterizes the color superconducting phase transition in quark matter. Further, at higher μ, only the local minimum at $|\Delta| \sim$ 130 MeV and $m = 0$ remains.

At $\mu > \mu_c$ and $T > 0$, the average value of $|\Delta|$ decreases until it reaches zero at $T_c \approx 0.57 \langle|\Delta|\rangle_{T=0}$ (BCS formula holds here!). Above T_c the quark matter is in the symmetric phase [2] where the chiral and color symmetries are restored. The corresponding phase diagram is shown on Fig. 1 with three phases: the symmetric phase (I), hadron phase (II), and 2SC phase (III). All three phases coexist at the triple point: $T_t \approx 55$ MeV and $\mu_t \approx 305$ MeV.

For the type II parameter set we have four phases shown in Fig. 2: the symmetric (I), hadronic (II), 2SC (III), and the phase of massive superconducting matter (IV). The cases of dilute ($\mu = 0$) and very dense matter ($\mu = 400$ MeV) are qualitatively analogous to the type I scheme, only the absolute values of m and $|\Delta|$ are noticeably lower. At intermediate densities, however, there is a qualitative difference.

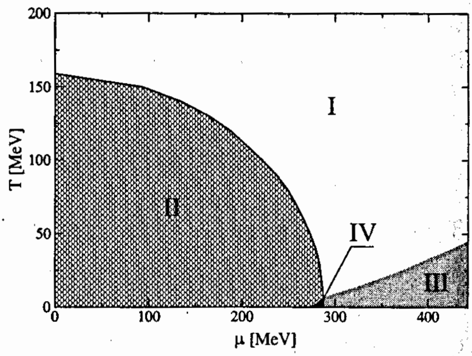

Figure 2: The quark matter phase diagram from the NJL model with the Type II parameter set.

Within a very narrow range of values of the chemical potential (about $\mu = 286$ MeV), there exists a new phase of massive superconducting matter. At higher μ the chiral symmetry is restored and the quark matter is in the pure superconducting phase. [3]

Thus, for the Type II parameter set, the transition from the hadronic to the superconducting phase is smooth. This behavior is unlike to what is commonly expected for a cold and dense matter but it parallels the findings of Ref. [17]

[2] According to recent investigations [3,25], a so-called pseudo-gap phase as a precursor of color superconductivity can occur in this region.

[3] A possibility of the chiral diquark condensates to coexist at certain condition has been already noticed in Ref. [23]

where vector interactions are responsible for this behavior. As a consequence, the border between 2SC and the symmetric phases of quark matter lies at noticeably lower temperatures. (see Fig. 2).

4 Conclusion

In the framework of the simple NJL model for two flavors, a phase diagram is obtained for T = 0–200 MeV and μ = 0–450 MeV. Three phases are found for the Type I parameter set and four phases for the Type II parameter set. The critical temperature and chemical potential obtained in the Type I scheme differ from those obtained with the Type II parameter set. At $T = 0$, $\mu_c \approx 320$ MeV for the Type I parameter set and $\mu_c \approx 288$ MeV for the Type II. The corresponding quark densities differ by a factor 1.5 – 1.7. The critical temperature for the Type II parameter set is as low as 7 MeV and thus much closer to critical temperatures for the paring instability in nuclear matter systems (see [24]) whereas for the Type I parameter set the critical temperatures are an order of magnitude larger. This striking difference in the critical parameters obtained within the same model calls for a more detailed investigation of the question of model parametrization.

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The Search of tensor interaction for pion and kaon decays.

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Abstract — Two ways to experimental studying of rare decays are described. The abvantages and the disadvantages are regarded. The deviations from Standart Theory of weak interactions in some experimental results of different setups are discussed.

1 Introduction

This note is devoted possible deviation from Standard Model of weak interaction. The deviation appear from recent experiment data on $\pi \to e\nu\gamma$ and $K \to e\nu\pi^0$. In the middle 80th were made several experiments on study these processes. Main goal was to measure ratio $\gamma = F_A/F_V$, axial-vector (F_A) and vector (F_V) weak currents [1] in radiative pion decay. The amplitude of the radiative

$$\pi \to e\nu\gamma \tag{1}$$

decay is traditionally described in two terms corresponding to the inner bremsstrahlung (IB) and structure-dependent (SD) radiation.

Two experimental methods decay measuring: one used decays of stopped pions and other - decay in flight. Typical layouts are shown in two figures: in fig.1 the layout PIBETA for stopped pions and in fig.2 the ISTRA setup for pion decays in flight. PIBETA and ISTRA setups had been described in details in papers [21] and [15].

All the previous experimental studies of the decay (1) were made with stopped pions. Such measurements are sensitive to the SD^+ contribution mainly and, thus, yield two different values for γ [6] [7]. The high statistics measurements at SIN (now PSI) give the values [8] $\gamma = 0.52 \pm 0.06$ and $\gamma = -2.48 \pm 0.06$ the positive value being more likely than the negative one. The positive sign of γ was confirmed by the LAMPF experiment [9], where the unique (3.5 standard deviations) value $\gamma = 0.25 \pm 0.12$ was found. These results have been confirmed additionally by the study of the $\pi^+ \to e^+\nu e^+e^-$ decay [10]. Nevertheless, the experimental study of the $\pi \to e\nu\gamma$ decay could not have been considered as completed since (i) there was a discrepancy (approximately 2 st. dev.) between the values obtained at SIN and at LAMPF, (ii) the selection of the sign of γ in the $\pi \to e\nu\gamma$ experiments was not sufficiently reliable, and (iii) the studied kinematical region was relatively small (Fig.3).

The experimental results of study of $\pi \to e\nu\gamma$ decay are summarized in table (W – the ratio of probability of positive and negative value γ.)

The high energy of the decaying particles allows to overcome the principal difficulties encountering in experiments with stopped pions. INR experiment was performed at the ISTRA set up at the end of the 80^{th} with 17 GeV pion beam produced by the IHEP U-70 accelerator. The admixture of kaons and muons in the beam was respectively 3% and 2% respectively. Special attention was paid to the purity of the pion isolation. Pions decayed in 10 m long decay volume. Pion and electron tracks were respectively measured by the scintillation hodoscopes [11] and by the proportional chambers with an induced charge readout [12]. The electrons and photons from the decays were detected in 20x24 array of the lead glass EM calorimeter [13]. The experimental setup is described in detail in our previous papers on the kaon decays [14] and in the complete review of ISTRA detector [15](Fig2). Main results of the study of radiative pion decay were published in ref [16]. From the analysis of 1 one gets: $\gamma = F_A/F_V = 0.41 \pm 0.23$. The value $\gamma = -2.4$ is suppressed by factor of $W = 5 \cdot 10^9$, that corresponds to 6.7 standard deviations.

The wide range of the kinematical variables in INR experiment has enabled us to determine the value of F_V without using the CVC hypothesis. Considering the value of F_V to be a free parameter in the fit, one gets $|F_V| = 0.014 \pm 0.009$. The obtained result agrees both with the CVC prediction and with the SINDRUM value $|F_V| = 0.023 \pm 0.015$ [10]. The discrepancy for the total branching ratio (more than 3 standard deviations) is due to the negative (unphysical) value of the SD contribution. Authors cannot explain this result by a systematic error due to the specific features of event detection and/or their processing. So, authors [16] found some discrepancy for the total decay probability, and the kinematical distributions for missing events being similar to that of SD^- radiation [16]. It should be mentioned [17] that decay (1) may be sensitive to the search for deviations from the Standard Model since a $\pi \to e\nu\gamma$ decay is strongly suppressed. In particular, the negative value for a_{SD} may be simulated by adding tensor radiation term to the structure dependent amplitude:

$$M_T = i(eG_F V_{ud}/\sqrt{2})\varepsilon^\mu q^\nu F_T u(p_e)\sigma_{\mu\nu}(1+\gamma^5)\nu(p_\nu) \qquad (2)$$

The decay rate densities for the SD^- radiation and the interference term between the inner bremsstrahlung and the tensor radiation are similar, so destructive interference may reproduce the results of fit, giving $F_T = (-5.6 \pm 1.7) \cdot 10^{-3}$. This value does not contradict the listed constraints on a tensor coupling from nuclear beta decay as well as from muon decay (if universality is supposed) [17]. This result also does not contradict the previous experiments carried out with stopped pions either [18].

Several works [19] were devoted to the study of possible deviation from SM in radiative pion decay. In one of them the involving of antisymmetric tensor fields into the standard electroweak theory allows to explain results of this work as well as [20]. It is evident that additional experimental and theoretical investigation of this problem should be carried out.

Recently a report on preliminary results performed by the PIBETA Collaboration from PSI meson factory has appeared [21]. The fits were made in two-demensional kinematic space of $x = 2E_\gamma/m_\pi$ and $\lambda = (x + y - 1)/x = y\sin^2(\Theta_{e\gamma}/2)$ on a very large statistical material (60k events $\pi \to e\nu\gamma$ decays) Fig.4. Fitting experimental data requires $F_T \neq 0 (F_T \cong 0.0017 \pm 0.0001)$. Here is the quotation from PIBETA Annual Progress Report: " Thus, like the ISTRA data, our data appear to call for a destructive interference between the IB term and a small negative tensor amplitude."

The results from ISTRA and PIBETA setups don't solve the problem. That study should be continued.

The Lorentz invariant form of the matrix element for decay $K^- \to l^-\nu\pi^0$, is [23]:

$$M = \frac{G_F sin\theta_C}{\sqrt{2}}\bar{u}(p_\nu)(1+\gamma^5)[m_K f_S - \frac{1}{2}[(P_K + P_\pi)_\alpha f_+ + (P_K - P_\pi)_\alpha f_-]\gamma^\alpha + i\frac{f_T}{m_K}\sigma_{\alpha\beta}P_K^\alpha P_\pi^\beta]v(p_l) \qquad (3)$$

It consists of scalar, vector and tensor terms $f_S, f_T, f_\pm$ are the functions of $t = (P_K - P_\pi)^2$.

Assuming linear dependence of $f_+ = f_+(0)(1 + \lambda_+ t/m_\pi{}^2)$ and real constant f_S, f_T it is getting the Dalitz plot density expression. From experimental Dalitz plot (Fig. 5) there are extracted of the parameters λ_+, f_S, f_T after the subtraction normalized MC estimated background. The result of the fit are summarized in Table 2.

The comparison of these results with the most recent $K^\pm$ data [20] [24] [25] shows that our statistics, at present, is the highest in the world and the errors smaller than in [20] [24] and comparable with [25].

These results do not confirm the observation of a significant f_S and f_T in [20]. They are in a good agreement with [24] [25] [27] and with the theoretical calculation for $\lambda_+ = 0.031$ [26].

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Table 1.

Experiment	$\gamma = F_A/F_V$	W	F_V	**Br.ratio** (10^7)
CERN [6] (63)	+0.26 -1.98	–	–	0.3 $(E_e, E_\gamma > 48MeV)$
LBL [7] (78)	$+0.48 \pm 0.12$ -2.42 ± 0.12	0.7	–	0.56 ± 0.07 $E_e > 56MeV, \Theta_{e\gamma} > 132^o$
SIN [8] (86)	$+0.52 \pm 0.06$ -2.48 ± 0.06	8.5	–	–
LAMPF [9] (86)	$+0.25 \pm 0.12$	2175	–	–
SINDR [17] (84)	$+0,75 \pm 0.31$	$4 \cdot 10^7$	$0.023^{+0.015}_{-0.013}$	–
ISTRA [16] (90)	$+0.41 \pm 0.23$	$4 \cdot 10^9$	$+0.014 \pm 0.009$	$+1.61 \pm 0.23$ $(E_\gamma > 21MeV$ $E_e > 70MeV - 0.8E_\gamma)$ $Br(IB) = 1.62 \pm 0.20$ $Br(SD^+) = 0.56 \pm 0.21$ $Br(SD^-) < 0.3(95\%CL)$

Table 2. $K^- \to e^- \nu \pi^0$.

Experimental results	Experiment or the firrst author of work	Year of issue	Events
$\lambda_+ = 0.0293 \pm 0.0015$ $F_T = f_T/f_+(0) = 0.045 \pm 0.060$ $F_S = f_S/f_+(0) = 0.019 \pm 0.025$	ISTRA [28]	(02)	133k
λ_+ 0.0278 ± 0.0026	Shimizu[24]	(00)	41k
0.0284 ± 0.0027	Akimenko[20]	(91)	32k
0.0290 ± 0.0040	Bolotov[27]	(88)	62k
F_S $0.002 \pm 0.026 \pm 0.014$	Shimizu[24]	(00)	41k
$0.07 \pm 0.016 \pm 0.016$	Akimenko[20] saa	(91)	32k
F_T $-0.01 \pm 0.14 \pm 0.09$	Shimizu[24]	(00)	41k
$0.53 \pm 0.010 \pm 0.1$	Akimenko[20]	(91)	32k

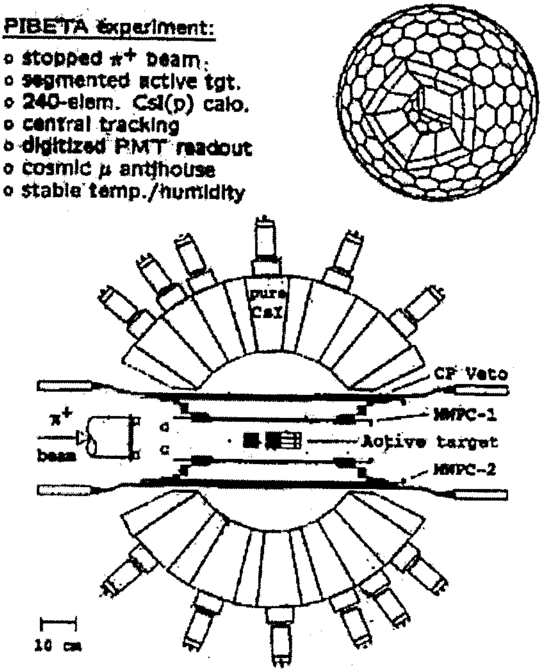

Figure 1: The layout of PIBETA setup.

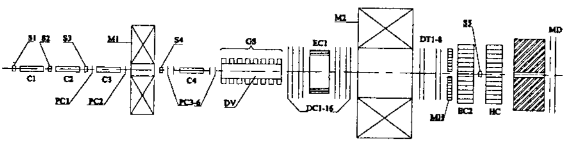

Figure 2: The layout of ISTRA setup.

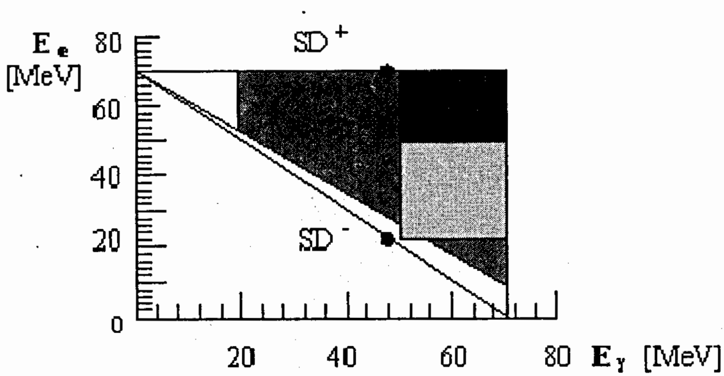

Figure 3: The kinematic regions of the $\pi \to e\nu\gamma$ decay at the ISTRA setup (grey), in the stopped pion experiments (black) and at the PIBETA (black and light grey). Black points correspond to the maximum values of the $SD^{\pm}$ terms.

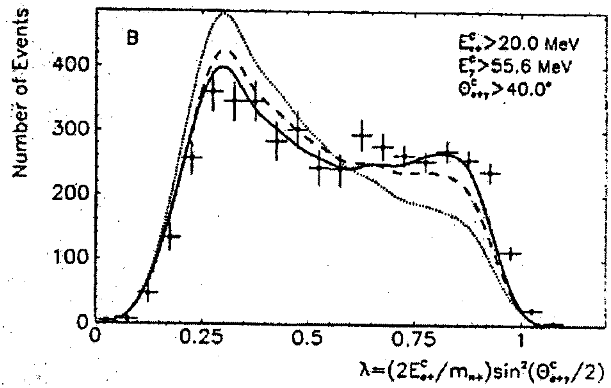

Figure 4: Measured spectrum of the kinematical variables $\lambda = (x+y-1)/x = y\sin^2(\Theta_{e\gamma}/2)$ in $\pi \to e\nu\gamma$ decay. Dotted curve: fit with F_V fixed by CVC hypothesis, and F_A taken from PDG 2002 compilation. Dashed curve fit with F_V constrained by CVC, and F_A and F_T unconstrained. The resulting value for F_T is -0.0017 ± 0.0001. Preliminary results - work in progress.

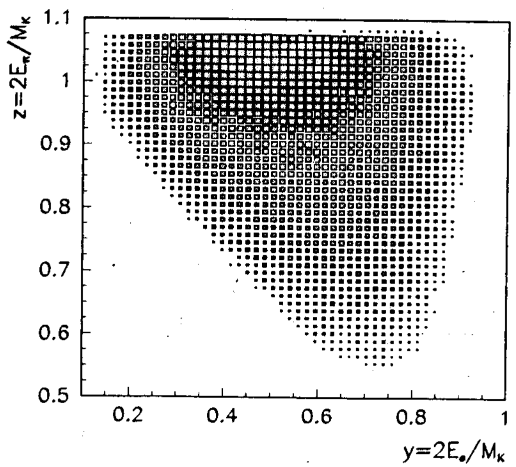

Figure 5: Dalitz plot ($y = 2E_e$ / M_K; $Z = 2E_\pi$ / M_K) for the selected $K^- \to e^-\nu\pi^0$ events after the 2C fit.

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Whether the Abelian Z' boson could be detected with the LEP2 data

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Abstract — The preliminary LEP data on the $e^+e^- \to l^+l^-$ scattering are analysed to establish a model-independent search for the signals of virtual states of the Abelian Z' boson. The recently introduced observables give a possibility to pick up uniquely the Abelian Z' signals in these processes. The mean values of the observables are in accordance with the Z' existence at the 1σ confidence level.

1. Introduction

The recently stopped LEP2 experiments have accumulated a huge amount of data on four-fermion processes at the center-of-mass energies $\sqrt{s} \sim 130-207$ GeV [1]. Besides the precision tests of the Standard Model (SM) of elementary particles these data allow the estimation of the energy scale of a new physics beyond the SM.

Various model-dependent and model-independent approaches to detect manifestations of physics beyond the SM have been proposed in the literature. It seems to us that it is reasonable to develop the model-independent searches for the manifestations of heavy particles with specific quantum numbers. Such an approach is intended to detect the signal of some heavy particle by means of the experimental data without specifying a model beyond the SM. In this way, it is also possible to derive model-independent constraints on the mass and the couplings of the considered heavy particle. To develop this approach one has to take into account some model-independent relations between the couplings of the heavy particle as well as some features of the kinematics of the considered scattering processes.

In the present report we focus on the problem of model-independent searches for signals of the heavy Abelian Z' boson [2] by means of the analysis of the LEP2 data on $e^+e^- \to \mu^+\mu^-, \tau^+\tau^-$. This particle is a necessary element of different models extending the SM. In the previous papers [3] we argued that the low-energy couplings of the Abelian Z' boson to the SM particles satisfy some model-independent relations, which are the consequences of renormalizability of a theory beyond the SM remaining in other respects unspecified. These relations reduce the number of unknown Z' couplings. They give a possibility to introduce sign-definite observables, which uniquely pick up the Z' virtual state in the process $e^+e^- \to l^+l^-$ [3].

The presented analysis has to answer whether or not one could detect the model-independent signal of the Abelian Z' boson by treating the LEP2 data. As it will be shown, the LEP2 data on the scattering into μ and τ pairs lead to the Abelian Z' signal at no more than 1σ confidence level.

2 The observable

The Abelian Z' boson can be introduced in a model-independent way by defining its effective low-energy couplings to the SM left-handed fermion doublets, f_L, the right-handed fermion singlets, f_R, and the scalar doublet, ϕ [2]. Such a definition usually means that the covariant derivatives in the Lagrangian contain the additional terms $-i\tilde{g}\tilde{Y}\tilde{B}_\mu/2$, where $\tilde{B}_\mu$ denotes the massive Z' field before the spontaneous breaking of the electroweak symmetry, and $\tilde{g}$ is the charge corresponding to the Z' gauge group. Diagonal matrices $\tilde{Y}(\phi) = \mathrm{diag}(\tilde{Y}_{\phi,1}, \tilde{Y}_{\phi,2})$, $\tilde{Y}(f_L) = \mathrm{diag}(\tilde{Y}_{L,f_u}, \tilde{Y}_{L,f_d})$ and numbers $\tilde{Y}(f_R) = \tilde{Y}_{R,f}$ mean the unknown Z' generators characterizing the model beyond the SM. Such a parameterization involves the Z' interactions of renormalizable type, since the non-renormalizable interactions are generated at higher energies due to radiation corrections and suppressed by $m_{Z'}^{-1}$. The corresponding Lagrangian generally leads to the Z–Z' mixing of order $m_Z^2/m_{Z'}^2$ which is proportional to $\tilde{Y}_{\phi,2}$ and originated from the diagonalization of the neutral vector boson states. The mixing contributes to the scattering amplitudes and cannot be neglected at the LEP2 energies.

In what follows we will use the couplings to the vector and axial-vector fermion currents $v^f_{Z'}, a^f_{Z'} = (\tilde{Y}_{R,f} \pm \tilde{Y}_{L,f})/2$. These two numbers parameterize the Abelian Z' couplings to a fermion f. Assuming an arbitrary underlying theory, one usually supposes that a_f and v_f are independent couplings. However, if a theory beyond the SM is renormalizable these parameters satisfy some relations [3]:

$$v_f - a_f = v_{f^\star} - a_{f^\star}, \quad a_f = T_{3,f}\tilde{Y}_\phi, \quad \tilde{Y}_{\phi,1} = \tilde{Y}_{\phi,2} \equiv \tilde{Y}_\phi, \tag{1}$$

where $l^\star = \nu_l, \nu_l^\star = l, \ldots$, and T_f^3 is the third component of the fermion weak isospin. In what follows we will use the short notation $a = a_l = -\tilde{Y}_\phi/2$. Note also that the Z–Z' mixing is expressed in terms of the axial-vector coupling a.

We investigate the processes $e^+e^- \to l^+l^-$ $(l = \mu, \tau)$ with the non-polarized initial- and final-state fermions. To take into account the correlations (1) we introduce the observable $\sigma_l(z)$ defined as the difference of cross sections integrated in some ranges of the scattering angle θ [3]:

$$\sigma_l(z) \equiv \int_z^1 \frac{d\sigma_l}{d\cos\theta} d\cos\theta - \int_{-1}^z \frac{d\sigma_l}{d\cos\theta} d\cos\theta, \qquad (2)$$

where z stands for the cosine of the boundary angle. The idea of introducing the z-dependent observable is to choose the value of the kinematic parameter z in a way to pick up the characteristic features of the Abelian Z' signals.

The lower-order diagrams for the process describe the exchange of photon, Z or Z' boson in the s-channel. Two classes of the one-loop corrections are taken into account. The first one includes the pure SM mass operators, vertex corrections, and boxes. The second set improves the Born-level Z'-exchange amplitude by 'dressing' the Z' propagator and the Z'–fermion vertices. We assume that Z' states are not excited inside loops (the Z'-boson is completely decoupled). The differential cross-section consists of the squared tree-level amplitude and the term from the interference of the tree-level and the one-loop amplitudes. To obtain an infrared-finite result, we also take into account the processes with the soft-photon emission in the initial and final states.

In the lower order in $m_{Z'}^{-2}$ the Z' contributions to the observable are expressed in terms of four-fermion contact couplings, only. If one takes into consideration the higher-order corrections in $m_{Z'}^{-2}$, it becomes possible to estimate separately the Z'-induced contact couplings and the Z' mass. In the present analysis we keep the terms of order $O(m_{Z'}^{-4})$ to fit both of these parameters.

After the expansion in $m_{Z'}^{-2}$ the observable $\Delta\sigma_l(z) = \sigma_l(z) - \sigma_l^{\rm SM}(z)$ consists of various products $a_i a_j$, $a_i a_j \zeta$, and $a_i a_j a_k a_n$, where

$$a_i = \sqrt{\tilde{g}^2 m_Z^2/(4\pi m_{Z'}^2)}(a, v_e, v_\mu, v_\tau, v_d, v_s, v_b), \quad \zeta = m_Z^2/m_{Z'}^2.$$

The coefficients at these products are determined by $\sqrt{s}$, z, and the SM couplings and masses, only. They are evaluated with FEYNARTS, FORMCALC and LOOPTOOLS software within the $\overline{\rm MS}$ renormalization scheme.

Among the mentioned products of the unknown Abelian Z' couplings one can find the sign-definite quantities $\epsilon = \tilde{g}^2 m_Z^2 a^2/(4\pi m_{Z'}^2)$, $\epsilon\zeta$, and ϵ^2. There is an interval of values of the boundary angle z, at which the factors at these products contribute more than 95% of the observable value. To estimate the contributions from the sign-definite factors at a given value of the boundary angle z we define some quantitative criterion [4]. Maximizing the criterion, we found the cosine of the boundary angle $z^*(s)$, which provides the sign-definite observable (with the accuracy 4–5%):

$$\Delta\sigma_l(z^*) = \left[\tilde{A}^l_{11}(s,z^*) + \zeta \tilde{B}^l_{11}(s,z^*)\right]\epsilon + \tilde{C}^l_{1111}(s,z^*)\epsilon^2 < 0, \qquad (3)$$

where $\tilde{A}^l_{11}(s,z^*)$, $\tilde{B}^l_{11}(s,z^*)$, and $\tilde{C}^l_{1111}(s,z^*)$ are calculable coefficients. The cosine of the boundary angle $z^*(s)$ decreases with the growth of the center-of-mass energy from 0.45 at $\sqrt{s} = 130$GeV to 0.37 at 207GeV.

The introduced observable $\Delta\sigma_l(z^*)$ selects the model-independent signal of the Abelian Z' boson in the processes $e^+e^- \to l^+l^-$. It allows to use the data on scattering into $\mu\mu$ and $\tau\tau$ pairs in order to estimate the Abelian Z' coupling to the axial-vector lepton currents.

Although the observable can be computed from the differential cross-sections directly, it is also possible to recalculate it approximately from the total cross-sections and the forward-backward asymmetries [4]:

$$\Delta\sigma_l(z^*) = \left[A^{\rm FB}_l\left(1 - z^{*2}\right) - \frac{z^*}{4}\left(3 + z^{*2}\right)\right]\Delta\sigma^{\rm T}_l + \left(1 - z^{*2}\right)\sigma^{\rm T,SM}_l \Delta A^{\rm FB}_l. \quad (4)$$

As computations show, the theoretical error of the approximation is of order 0.003pb, whereas the corresponding statistical uncertainties on the observable are larger than 0.06pb. Thus, the introduced approximation can be successfully used to obtain more accurate experimental values of the observable, because the published data on the total cross-sections and the forward-backward asymmetries are still more precise than the data on the differential cross-sections.

3 Data fit and conclusions

In the lower order in $m^{-2}_{Z'}$ the observable (3) depends on the one flavor-independent parameter ϵ, which can be fitted from the experimental values of $\Delta\sigma_{\mu,\tau}(z^*)$. The sign of the fitted parameter ($\epsilon > 0$) is by construction a signal of the Abelian Z'. To derive the central value of ϵ and the corresponding 1σ confidence-level interval we apply the usual fit method based on the likelihood function $\mathcal{L}(\epsilon) \propto \exp(-\chi^2(\epsilon)/2)$ [4]. We also introduce the probability of the Abelian Z' signal, P, as the integral of $\mathcal{L}(\epsilon)$ over the positive values of ϵ.

Actually, the fitted value of the contact coupling ϵ originates mainly from the leading-order term in $m^{-2}_{Z'}$ in Eq. (3). The analysis of the higher-order terms allows to constrain the Z' mass. Substituting ϵ in the observable (3) by its fitted central value, $\bar{\epsilon}$, one obtains the expression, which depends on the parameter $\zeta = m^2_Z/m^2_{Z'}$. Then, the central value of ζ and the corresponding 1σ confidence level interval are derived in the same way as those for ϵ.

The contact coupling ϵ with the 1σ confidence-level uncertainty, the probability of the Z' signal, P, and the value of $\zeta = m_Z^2/m_{Z'}^2$ are fitted from the observable recalculated from the experimental values of the total cross-sections and forward-backward asymmetries [1]. We assume several data sets, including the $\mu\mu$, $\tau\tau$, and the complete $\mu\mu$ and $\tau\tau$ data:

$$\begin{aligned}
\mu\mu: &\quad \epsilon = 0.0000366^{+0.0000489}_{-0.0000486}, \quad P = 0.77, \quad \zeta = 0.009 \pm 0.278;\\
\tau\tau: &\quad \epsilon = -0.0000266^{+0.0000643}_{-0.0000639}, \quad P = 0.34, \quad \zeta = -0.001 \pm 0.501;\\
\mu\mu + \tau\tau: &\quad \epsilon = 0.0000133^{+0.0000389}_{-0.0000387}, \quad P = 0.63, \quad \zeta = 0.017 \pm 0.609.
\end{aligned}$$

The more precise $\mu\mu$ data demonstrate the signal of about 1σ level. It corresponds to the Abelian Z'-boson with the mass of order 1.2–1.5TeV, if one assumes $\tilde{g}^2/4\pi \sim$0.01–0.02. No signal is found by the analysis of the $\tau\tau$ cross-sections. The combined fit of the $\mu\mu$ and $\tau\tau$ data leads to the signal below the 1σ confidence level. Being governed by the next-to-leading contributions in $m_{Z'}^{-2}$, the fitted values of ζ are characterized by significant errors. The $\mu\mu$ data set gives the central value which corresponds to $m_{Z'} \simeq 1.1$ TeV.

We also perform a separate fit of the parameters based on the direct calculation of the observable from the differential cross-sections. The complete set of the available data is used. Three of the LEP2 Collaborations demonstrate positive values of ϵ. The combined value $\epsilon = 0.00012 \pm 0.0003$ is also positive.

As it follows from the present analysis, the Abelian Z' boson has to be light enough to be discovered at the LHC. On the other hand, the LEP2 data on the processes $e^+e^- \to \mu^+\mu^-, \tau^+\tau^-$ do not provide the necessary statistics for the detection of the model-independent signal of the Abelian Z' boson at more than 1σ confidence level. So, it is of interest to find observables for other scattering processes.

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Deep Inelastic Scattering of Leptons on Nuclei: Hadronization Process, Cumulative Particles Production and Blokhintsev's Fluctons

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Abstract — We introduce and briefly discuss a model for propagation of quark and gluon jets in nuclear matter, taking into account the Landau–Pomeranchuk–Migdal effect and cascading of soft particles in a nucleus. Calculations were performed by Monte Carlo method. The role of Blokhintsev's fluctons in the generation of energetic cumulative nucleons produced in neutrino interactions with nuclei is investigated and the formation zone length is obtained.

1 Introduction

The main motivation for the study ultrarelativistic heavy-ion collision is their role in the testing quantum chromodynamics (QCD) and their possible to manifest the predicted the quark-gluon plasma (QGP). QGP scenarios should be tested against the background of particle yields from elementary collisions, which can be described by perturbative QCD. Thus, to confirm the production of QGP, we require accurate knowledge the 'partonic background' of nuclear collision and its formation time in the dense medium. In addition, the study of the structure of particle jets in hard collisions continue to be an active field of research, in the strong interaction physics. The interest is directed in explore predictions of peturbative QCD on the parton cascade evolution and to investigate the hadronization process which cannot be treated perturbatively. In order to investigate the possible emergence of guark–gluon plasma, it is necessary to understand the properties of ordinary multiparticle productions mechanisms in more simple conditions than in the relativistic collisions of heavy ions. The particle–nuclear reaction offers the unique possibility of studding the space–time development of production process: the system formed as a consequence of a primary $h - N$ or $l - N$ interaction will interact again with other nucleons long before it achieves the asymptotic state that we see in free $h - -N$ or $l - N$ collisions. In this sense the nucleus is really used as a very dense hydrogen bubble chamber. An advantage of lepton induced interactions with respect to hadronic processes is the fact that the projectile interacts only once and the interaction can practically take place *everywhere* in the nucleus. In the case of hadron–nucleus collisions, the interaction takes place essentially on the surface of the nucleus. As a consequence not all the nucleons participate in the process. At present, effects of such type are investigated in different scientific centers, e.g., by HERMES and NOMAD collaborations [1,2]. Besides backward proton production (according to A.M. Baldin [3]– *cumulative production*) has been studied in lepton interactions with nuclei. In absence of nuclear effects their production is forbidden. The analogous effect was observed in Dubna for a long time (see [4] and references therein). In 1957, Dmitry Ivanovich Blokhintsev suggested the interpretation of Dubna's experimental data on backward elastic *pd*-scattering (and other nuclear reactions.) He proposed that proton interacts with a narrow group of nucleons (with a flucton) of the dimension less than wave length of the projectile. At present, the ideas (for the first time established by D.I. Blokhintsev) about the peculiarity of nuclear structure at short distances is widely used in the study of various new effects: cumulative production, chance of the creation of QGP etc. In this presentation, we analyse the data of semi-inclusive DIS of neutrino on nuclei to find the space–time property of partons hadronization and to include the idea of fluctons (or multiquarks bags) in the mechanism of cumulative nucleons production (see, e.g., [1,5,6]).

2 The model

We developed a cascade model of multiproduction of neutrino–nuclei interaction. On the phenomenological standard, the model describes a Markov branching process of the evolution of parton's jet (up to hadronization) in the atomic nucleus. We assume that the interaction between incident an lepton and a target nucleus takes place in a lepton–nucleon interaction. (More detail description of the model along with the full list of the references about experimental data can be found in [5,6].) The nucleus is excited by a series of collisions between secondaries (produced in the first lepton–nucleon interaction) and the intranuclear nucleons. At high energies the secondaries traverse the nucleus in such a short time that nucleons cannot rearrange themselves until the probe has left. In the main the target nucleons are just static spectators,so that the scattering problem is in the first approximation a sequence of two-body one. This process continues until all secondaries escape target nucleus. A part of the energy is spread through the nucleus to produce a fully–equilibrated nucleus which then decays statistically. The process of generation of particles is simulated by the Monte Carlo (MC) method. The characteristics of the partons from neutrino–nucleon interaction [1] and of the produced particles with nucleons in nucleus are taken from experiments with free nucleons, see, e.g., [5,6].

The space–time characteristics of lepton–nucleon interactions inside the target nucleus were taken into consideration. The cross section for the next collision of a secondary particle with a nucleon inside the nucleus is given by

$$\sigma_{hN} = \sigma_{hN}^{exp}(1 - e^{-\tau/\tau_0}), \tag{1}$$

where τ is the time from the moment of production of this particle in the previous collision and σ_{hN}^{exp} is the experimentally determined total interaction cross section of a hadron with a free nucleon at the corresponding energy of the secondary particle produced. Thus, only after a relatively long time τ does the cross section of intranuclear interaction reach the value σ_{hN}^{exp} (hadronic Landau–Pomeranchuk–Migdal effect).

In our equation, the parameter τ_0 is a certain characteristic corresponding to the formation time of the secondary generated hadron. The equation for σ_{hN} can be rewritten in the form with the formation length parameter L_f (in the System of a Moving Parton, SMP). In the present model, at a finite value of L_f, secondary particles–pions–are formed not instantly but after a certain time.

3 Results & discussion

The average multiplicities of secondary relativistic particles (mostly pions) and nonrelatisistic one (mostly nucleons), produced in charged–current ν_μ–emulsion interaction, vs. corresponding quantities calculated in our model at different values of L_f are presented in Table 1. It is to be noted that when $L_f \to 0$, our model will reduce to the old fashioned cascade model in which secondary hadrons are produced instantly in the intranuclear interactions. In the present model, at a finite value of L_f, secondary particles (e.g. pions) are not formed instantly but after a certain time. The cross section of the intranuclear interaction is calculated as a function of the mean free path L_f (in the SMP) and the velocity of parton (at fixed L_f). The slow secondary particles interact inside the target nucleus more frequently than the fast ones. The most rapid secondaries may fly out without interaction with the subsequent

[1]The parton spectra is assumed to be the same as hadronic one. That approach is based on the concept of "Local Parton Hadron Duality" [5,6].

nucleon inside the target nucleus. As the model prediction are sensitive to the formation lenght L_f, in Table 1. we compare our MC theoretical computation of the multiplicities vs. L_f with the corresponding experimental value. Table 1. argue the accordance of experimental and calculated data at $L_f = 0.5$ fm.

Table 1. The average multiplicities of relativistic N_{re} and nonrelativistic N_{nr} particle produced in charged-current ν_μ-emulsion interactions obtained in the experiment compared with the values calculated according to our model.

Experimental data	Calculations of the model			
	$L \to 0$	$L_f = 0.2$ fm	$L_f = 0.5$ fm	$L_f = 1$ fm
N_{re} 5.28±0.26	6.45±0.06	5.60±0.04	5.12±0.03	4.08±0.02
N_{nr} 1.33±0.15	2.08±0.03	1.71±0.02	1.35±0.02	0.82±0.01

Figure 1. represents the multiplisity distribution of relativistic particle. The dashed line correspond to the prediction with the value of the parameter $L_f \to 0$ (clasical cascade model, i.e. momentary particles production in the intranuclear interactions). The solid line demonstrate agreement of the model with data at L_f=0.5 fm.

Figure 2. display the spectrum of protons emitted backwards, with respect to the beam direction, which have energies not allowed by the kinematics of collisions on a free and stationary nucleon. The solid curve on the Figure show our theoretical results. The left part of the curve (with *big slope*) manifests the of proton with $P^2 \leq 0.2(GeV/c)^2$). The mechanism of such slow proton production is a process of the evaporation of the residual nuclei excited in the stage of the propagaton of jets in the nuclei. It is worth to remark that in our approach the evolution of quark–gluon jets in nuclei in the framework of our model is accompained by a nucleon emission at backward angles and momentum $\geq$ 300 MeV/c [1,6] (cumulative nucleons). The spectrum of such protons is depicted by the right part of the solid curve on Figure 2 (with smaller slope than the left one). In our model the underlying mechanism responsible for energetic protons production was the ordinary *quasideuteron* intranuclear absorption process. Such Cumulative Protons(CP) were observed in deep inelastic charged–current neutrino–emulsion interactions [1,6] . The experimental multillicity of CP 0.33±0.07 are in good agreement with the calculated one equal to 0.29. The effect of intranuclear absorption of particles (pions) by intranuclear 'deuterons' is well known in the theory of nuclear reactions at intermediate energy. This process is essential only for the slow pions (of energy $\leq 1GeV$). At high energy beams this effect makes all only some percents. As a result, the *existing* experimental data on CP from semi–inclusive DIS of

neutrino on nuclei can be interpreted without new nuclear 'exotic' of the type of 'fluctons', multiquar bags etc. [2]

Further more, we insert the *Three Nucleons 'Fluctons'*, TNF, in our model (in addition to quasideuteron, see above) to explore the chance to catch more energetic nucleons (than it is observed in the present day experiment). The cross sections of interactions with TNF is assumed to be ten times less then the cross section of quasideuteron intranuclear absorption process. In such a way, we predicted the production of more energetic nucleons (see dashed line in Figure 2.). This forecast can be tested in the new class of experiment with much more statistics [1,2]. In conclusion, effect of the formation of particle is essential in many high energy phenomenon, e.g., in the study of "jet quenching" as a potentual signature for QGP formation [8].

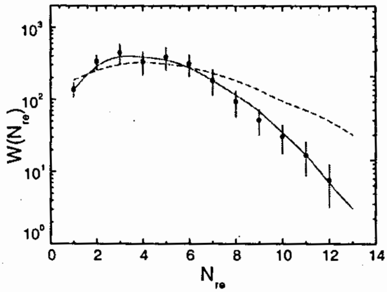

Fig.2. Pseudorapidity distribution of relativistic particles.

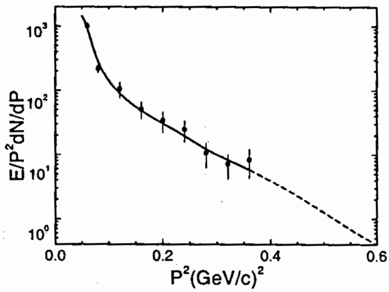

Fig.3. Cumulative proton production.

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On radiative widths and a pattern of quark-diquark-gluon configuration mixing in low-lying scalar mesons

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Abstract — The earlier suggested and developed idea of quark-hadron duality, underlying "bremsstrahlung-weighted" sum rules for total or polarized photon interaction cross sections, is applied to the description of excitation of light scalar mesons in gamma-gamma interactions. The emphasis is put on the discussion of a role of the scalar diquark cluster degrees of freedom in the radiative formation of light scalar mesons.

1 Introduction

The (constituent) quark hadron duality sum rules, used here, follow from the assumed equivalence of two complete sets of state vectors, saturating certain integral sum rules, one of the sets being the solution of the bound state problem with colour-confining interaction, while the other describes free partons. The sum rules satisfying the assumed duality condition have been chosen to be those related to fluctuation of the relativistic electric dipole moment (EDM) operator in the configuration space of valence partons in a given system taken in the "infinite momentum" frame. The relevance of these sum rules has been tested in some models of quantum field theory [1] and used to derive a number of seemingly successful relations for the hadron electromagnetic radii [2] and two-photon decay widths of the lowest spin meson resonances [3]. The two-photon-meson couplings give rather direct information on the flavour content of considered states. Moreover, the salient feature of our sum rule approach enables one to put forward most distinctly the flavour content of the states at hand and, at the same time, somehow to circumvent many of model-dependent aspects of the detailed structure and dynamics of multi-component bound quark-gluon states. Therefore they could be especially useful in the case of hadrons with the complex and poorly understood constituent structure.

In what follows, we only mention briefly some technical details. Varying the polarizations of colliding photons, one can show that a linear combination of certain $\gamma\gamma \to q\bar{q}$ cross-sections will dominantly collect the $q\bar{q}$- states with definite spin-parity and hence the low-mass meson resonances with the same quantum numbers. The polarization structure of the transition matrix element $M(J^{PC} \leftrightarrow 2\gamma)$ for the scalar meson resonance

$$M(0^{++} \leftrightarrow 2\gamma) = G[(\epsilon_1\epsilon_2)(k_1k_2) - (\epsilon_1 k_2)(\epsilon_2 k_1)] \qquad (1)$$

where k_i^μ are the momenta of photons, ϵ_i^ν are polarization vectors of photons, G is a constant proportional to the coherent sum of amplitudes describing the two-photon annihilation of partons, composing a given meson. By definition

$$|G| = 64\pi\Gamma_{\gamma\gamma}/m^3, \qquad (2)$$

where m is the meson mass and $\Gamma_{\gamma\gamma}$ is the two-photon width of a given meson. Introducing the $\gamma\gamma$ - cross-sections $\sigma_{\perp(\parallel)}$ (and the integrals thereof) that refer to colliding plane-polarized photons with the perpendicular (parallel) polarizations, and σ_p corresponding to circularly polarized photons with parallel spins, one can then show that the combinations of the integrals over the bremsstrahlung-weighted and polarized $\gamma\gamma \to q\bar{q}$ cross-sections, $I_\perp - (1/2)I_p$, $I_\parallel - (1/2)I_p$, I_p will be related to low-mass meson resonances having spatial quantum numbers $J^{PC} = 0^{-+}$ and 2^{-+}, 0^{++} and $2^{++}(\lambda = 0)$, $2^{++}(\lambda = 2)$, if we confine ourselves to the mesons with spins $J \leq 2$ ($\lambda = 0$ or 2 being the z-projection of the total angular momentum of the tensor mesons).

In what follows, we focus mainly on scalar meson sum rules in the light quark sector. As is known, the long-lasting experimental efforts have presently resulted in identification of a few scalar states with masses below $2GeV$, labelled by isospin [4,5]:

$$I=0: f_0(600) \text{ or } \sigma(500), f_0(980), f_0(1200 \div 1500),$$
$$f_0(1506), f_0(1710);$$
$$I=1/2: \kappa(800), K_0^*(1430);$$
$$I=1: a_0(980), a_0(1450).$$

Following [6,7] we assume that the listed resonances are interpreted as two meson nonets and a scalar glueball with the mixed valence quark and gluon configurations. In the states lying above 1 GeV the dominant configuration is, presumably, a conventional $q\bar{q}$ nonet mixed with the glueball of (quenched) lattice QCD. Below 1 GeV the states also form a nonet, where the central binding role is played by the $SU(3)_{c(f)}$ -triplet diquark clusters $(qq)_{\bar{3}}(\bar{q}\bar{q})_3$ in

S-wave with some $q\bar{q}$ admixtures in P-wave, and maybe less important glueball part in their state vectors.

We begin with consideration of only constituent quark and scalar diquark as the basic degrees of freedom in the first stage of $\gamma\gamma$ -reactions: $\gamma + \gamma \to q + \bar{q}, (qq) + (\bar{q}\bar{q})$ where the quark and diquark are treated as elementary structureless spinor and scalar massive particles with "minimal" electromagnetic interaction.

Evaluating cross-sections and elementary integrals we get the sum rules for radiative widths of resonances with $J^{PC} = 0^{++}$

$$\sum_i \frac{\Gamma(S_i \to 2\gamma)}{m_{S_i}^3} \simeq \sum_q I_S(q) + \sum_{qq} I_S(qq), \tag{3}$$

where

$$I_S(q) = \frac{3}{16\pi^2} \langle Q(q)^2 \rangle^2 \frac{5\pi\alpha^2}{9m_q^2}, \tag{4}$$

$$I_S(qq) = \frac{3}{16\pi^2} \langle Q(qq)^2 \rangle^2 \frac{2\pi\alpha^2}{9m_{qq}^2}. \tag{5}$$

All the integrals over the parton (that is the quark and diquark) production cross sections are rapidly converging and all the resonance cross sections are taken in the narrow width approximation so that for the wide scalar mesons the masses in Eq.(3) have rather the meaning of the "mean value"-masses.

The term $I_S(qq)$ in Eqs.(3) and (5) corresponds to ascribing a possible role to scalar diquarks as a constituent triplet $(\bar{d}\bar{s}), (\bar{u}\bar{s}), (\bar{u}\bar{d})$ of "partons" with respective masses and electric charges composing, at least in part, the scalar meson nonets.

Assuming now for $a_0(980)$ either of two limiting options: (a)- the isovector quark-antiquark $\bar{q}q$-structure, or (b)- the isovector diquark-antidiquark $(\bar{q}\bar{s})(qs)$ -configuration, and using Eq.(3)-(5), one gets

$$\Gamma_{\gamma\gamma}(a_0(980)) \simeq 1.6\ (.12)\ keV \tag{6}$$

The lower value of the width in (6) is obtained if, following [8], we accept for the diquark masses $m_{qs} \simeq 560$ MeV and $m_{ud} \simeq 320$ MeV (those seem to be of minimal value as compared to fitted mass values of many other models, thus stressing maximally the role of the diquark configurations in various hadrons), while the masses of light quarks are taken to be $m_{u,d} \simeq 240$ MeV and $m_s \simeq 350$ MeV, according to [2].

Both values in Eq.(6) are different from $\Gamma_{\gamma\gamma}(a_0(980)) = .30 \pm .10$ keV [9], that is in between two. Another evidence against the interpretation of $a_0(980)$ - meson as a usual $\bar{q}q$-state is the quenched LQCD evaluation of the scalar, isovector quarkonium mass $m(0^{++}, I^G = 1^-) = 1.330(50)\ GeV$ [10]. Therefore, it is quite natural to suppose the $a_0(980)$- meson to have a mixed structure (together with its higher-lying partner)

$$|a_0(1474)\rangle = cos\theta|(1/\sqrt{2})(u\bar{u} - d\bar{d})\rangle + \\ +sin\theta|(1/\sqrt{2})((\bar{d}\bar{s})(ds) - (\bar{u}\bar{s})(us))\rangle, \quad (7)$$

$$|a_0(985)\rangle = -sin\theta|(1/\sqrt{2})(u\bar{u} - d\bar{d})\rangle + \\ +cos\theta|(1/\sqrt{2})((\bar{d}\bar{s})(ds) - (\bar{u}\bar{s})(us))\rangle. \quad (8)$$

By convention, we take here the phases of $\sqrt{I_S(q)}$ and $\sqrt{I_S(qq)}$ as +1 and−1, respectively. Further, with the fit $\theta \simeq 10^o$ to reproduce $\Gamma_{\gamma\gamma}(a_0(980)) \simeq .3$ keV, one gets also $\Gamma_{\gamma\gamma}(a_0(1474)) \simeq 4.6$ keV, which should be tested experimentally yet.

2 A model of mixing matrices for light scalar mesons

We turn now to a "reconstruction" of the bare masses of two (finally) mixed scalar nonets. Taking for granted the physical masses 1474 MeV - for higher-mass, $q\bar{q}$-dominant isovector meson and 985 MeV - for lower-mass $(qq)(\bar{q}\bar{q})$ one, and having defined $\theta \simeq 10^o$, the diagonal and nondiagonal elements in the 2×2 mass-matrix of the isovector states are easily derived to be $M(I = 1; (q\bar{q})) =$ 1434 MeV, $M(I = 1; ((qs)(\bar{q}\bar{s}))) =$ 1005 MeV, and the universal, with the tentatively assumed $SU(3)$-symmetry, nondiagonal "mass" $h =$ 95 MeV. In fact, it represents the transition coupling between states of two multiplets.

The "bare" masses of the isospinor states are $M(I = 1/2; (q\bar{s})) \simeq 1435$ MeV and $M(I = 1/2; ((ud)(\bar{q}\bar{s}))) \simeq 812$ MeV. They correspond to "physical" masses $m \simeq 1450$ MeV and $m \simeq 790$ MeV, according to latest data [?,5].

At last, to define the mass of the lightest, isoscalar "bare" state we invoke the mass formula of the ideal-mixing-form

$$M((ud)(\bar{u}\bar{d})) = 2M((ud)(\bar{u}\bar{s})) - \\ -M((qs)(\bar{q}\bar{s})) \simeq 620\ MeV. \quad (9)$$

The mixing of a glueball and 2 pairs of isoscalar mesons is described by the following mass matrix, which is diagonalized by the masses of 5 physical states:

$$\begin{pmatrix} M_G & f & f\sqrt{2} & g & g\sqrt{2} \\ f & M_{S_1} & 0 & h\sqrt{2} & 0 \\ f\sqrt{2} & 0 & M_{N_1} & h & h\sqrt{2} \\ g & h\sqrt{2} & h & M_{S_2} & 0 \\ g\sqrt{2} & 0 & h\sqrt{2} & 0 & M_{N_2} \end{pmatrix} \tag{10}$$

$$\Longrightarrow \text{diag}\,(m_1, m_2, m_3 m_4, m_5).$$

M_G and M_{S_1,N_1} (or M_{S_2,N_2}) stand for the mass of the primitive glueball, and $S_1 = s\bar{s}$ and $N_1 = n\bar{n} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$ (or $S_2 = ((\bar{n}\bar{s})(ns) \equiv ((\bar{d}\bar{s})(ds) + (\bar{u}\bar{s})(us))/\sqrt{2}$ and $N_2 = (\bar{u}\bar{d})(ud)))$ mesons, respectively, the subscripts 1 or 2 indicating the quark (or diquark) composition of the nonet the state belongs to; m_i stand for the masses of the physical states; f (or g) is the glueball–$q\bar{q}(or(\bar{q}\bar{q})(qq))$–meson coupling and h is the nondiagonal quark-to-diquark pair transition coupling. Following [11], we take all couplings having dimensionality $(mass)$, in accord with the dimensionality of the diagonal entries of (10). All quantities in (10) are considered to be real numbers.

The mixing between the glueball and the low-lying $(\bar{q}\bar{q})(qq)$-states can be less important also due to relative smallness of the lowest order gg-to-$(\overline{qq})(qq)$ transition amplitude as compared to the gg-to-$\bar{q}q$ transition. The relevance of these arguments is illustrated also by the (approximate) validity of mass-formulae, Eq.(9), which could be strongly violated if the annihilation-induced mixing of different flavours would take place. Therefore, we neglect, as a first approximation, the coupling g in the general 5×5 mass-matrix.

Defining the relation between the physical and bare states

$$\begin{pmatrix} \underline{f_0(1710)} \\ \underline{f_0(1506)} \\ f_0(m_3) \\ \underline{f_0(980)} \\ f_0(m_\sigma) \end{pmatrix} = U(5) \begin{pmatrix} G \\ S_1 \\ \underline{N_1} \\ \underline{S_2} \\ \underline{N_2}, \end{pmatrix}, \tag{11}$$

where masses of the underlined states are considered to be defined and

$$U(5) = \begin{pmatrix} x_1 & y_1 & z_1 & u_1 & v_1 \\ x_2 & y_2 & z_2 & u_2 & v_2 \\ x_3 & y_3 & z_3 & u_3 & v_3 \\ x_4 & y_4 & z_4 & u_4 & v_4 \\ x_5 & y_5 & z_5 & u_5 & v_5 \end{pmatrix}, \tag{12}$$

we obtain the expression for the individual two-photon width of a scalar meson in the form

$$\begin{aligned}\Gamma_{\gamma\gamma}(f_0(m_i)) &= m_i^3(125\alpha^2/7776\pi) \times \\ &\times(y_i A_{S_1} + z_i A_{N_1} + u_i A_{S_2} + v_i A_{N_2})^2, \quad (13)\\ A_{S_1} &= \sqrt{2}/(5m_s), A_{N_1} = 1/m_q, \quad (14)\\ A_{S_2} &= -\sqrt{2}/(\sqrt{5}m_{qs}), A_{N_2} = -2/(5\sqrt{5}m_{ud}), \quad (15)\end{aligned}$$

the coefficients $y_i, .., v_i$ being the probability amplitudes to find the quark configurations S_1, N_1, S_2, N_2 in the state vector of the (iso)scalar meson $f_0(m_i)$ with mass m_i. The minus signs in front of A_{N_2,S_2} in Eq.(15) is the reflection of our convention about opposite signs of the square-roots $\sqrt{I_{S,N}(q)}$ and $\sqrt{I_{S,N}(qq)}$, defined in Eq.(5) and effectively representing the fermion-quark and boson-diquark loops in the meson-two-photon transition diagrams.

The orthogonality of the matrix U in Eq.(12) provides the "inclusive" sum rule to be fulfilled

$$\sum_i \frac{\Gamma_{\gamma\gamma}(f_0(m_i))}{m_i^3} = |A_{S_1}|^2 + |A_{N_1}|^2 + \\ +|A_{S_2}|^2 + |A_{N_2}|^2 \quad (16)$$

With assumed $g \simeq 0$, the unknown elements in the mass-mixing-matrix (10) are the coupling f and masses M_G and M_{S_1}, while among the physical masses the essentially unknown is a mass $1.2 \leq m(3) \leq 1.5$ GeV [4]. It seems worthwhile to mention that starting with the evident constraint $f^2 \geq 0$, we have obtained the reasonable bounds for these three quantities (in units of GeV)

$$1.31 \leq m(3) \leq 1.55, 1.47 \leq M_G \leq 1.51, \\ 1.49 \leq M_{S_1} \leq 1.69, \quad (17)$$

just from basic secular equations with using the known masses of the physical mesons. The lower-bound of M_G and upper-bound of M_{S_1} in (17) are close to the values of respective masses found in [11] while their $m(3) = 1.26$ GeV is somewhat beyond of our more general bound.

3 Concluding remarks

The main results of this work are the following. We applied the idea of the $(q\bar{q}) - (\bar{q}\bar{q})(qq)$-configuration-mixing to a simpler isovector sector of two low-lying scalar nonets to fit the two-photon width of $a_0(980)$ and extract thereby the nondiagonal element in the mass-mixing-matrix. From the resulting more general (iso)scalar-mass 5×5-matrix, we derived the bounds on missing masses M_G and M_{S_1} of the "bare" glueball and scalar strangeonium states and poor defined mass of the so-called $f_0(1370)$-resonance.

Under the assumptions that the bare glueball-two-diquark and also higher-lying "strangeonium"($s\bar{s}$)-two-diquark mixing can be neglected we obtained [12] masses of lowest $f_0(590)$,$f_(986)$ and $f_0(1470)$ (iso)scalar resonances in reasonable agreement with latest data of the E-791 and FOCUS Collaborations [5]. The preliminary estimates of the two-photon decay widths can be confronted only with the experimental value $.56 \pm .11$ keV for $f_0(986)$-meson and, with stated reservations [4], for the $f_0(m_3)$-resonance, where it is in the range of $3.8 \pm 1.5 \div 5.4 \pm 2.3$ keV. The theoretical estimates via dual sum rules [12] are, roughly, two times larger than the cited data. Clearly, for more quantitative statements we have to have new and more accurate $\gamma\gamma$ data.

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Quark-Hadron Duality in the CLAS Data on the Proton Structure Function

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Abstract — The recent experimental data on the proton structure function F_2 collected at the JLab by the CLAS collaboration are analyzed within a Regge-dual model suggested earlier [1,2]. In this paper the new improved fits are obtained and their physical implications are discussed.

Dedicated to the memory of M. Kotsky

1 Introduction

The recently measured inclusive electron-proton cross section in the nucleon resonance region, performed with the CLAS detector at the Thomas Jefferson Laboratory, has provided new data [3] for the nucleon structure function F_2 (SF) with with previously unavailable precision. A preliminary analyses of these data within a Regge-dual model was presented at this conference and subsequently published in Refs. [1,2]. The present contribution contains improved fits to the data [3] within the same model. Let us remind the reader that the basic idea behind the model [2] is to combine in a single explicit expression resonance-Regge (or Veneziano) duality with quark-hadron or (Bloom-Gilman) duality. The basis for this unification is a dual amplitude with Mandelstam analyticity, *properlycontinuesofmassshell.* The resulting scattering amplitude, in principle is applicable for all values of its Mandelstam variables as well as the virtualities of the external particles and thus can be applied to Compton scattering, whose forward imaginary part is related to the DIS structure functions as well as to the electroproduction of vector mesons of real photons, the latter being called deeply virtual Compton scattering. The model is dual in two dimensions, one in the hadron sector (Veneziano duality), the other one relating the quark-parton and the hadron degrees of freedom. Until now we considered the high-energy (low-x) Regge domain separately from the low-energy (large-x) resonance region. The two should match since both are limiting cases of the basic dual amplitude (see [4]). To understand better the complicated structure of the direct-channel resonances the low-energy behaviour of the amplitude was analyzed [2] with special emphasis on the form of the nonlinear, complex baryonic Regge trajectories and the form of the transition form factors, both being constrained by the dual model. Details of the model can be found in [2] and earlier references therein. A similar analysis using earlier data [5] was carried out in our previous paper [6], and in the Ref. [1] we tried to perform a fit over both datasets from [5] and [3]. Here, for brevity, we consider only the pole decomposition of the whole dual amplitude, appropriate in the resonance region of the CLAS data.

2 Regge-dual model

The main starting point [6, 1, 2] is the inclusion of the three prominent resonances, $N^*(1520)$, $N^*(1680)$ and $\Delta(1232)$ plus a background, dual to the Pomeron exchange. The SF is still let to depend on effective trajectories, whose parameters are fitted to the data. This approach is justified "a posteriori" in the sense that the parameters of the effective trajectories are found to be close to those fitted to the spectrum of the baryon resonances.

The imaginary part of the scattering amplitude is written as a sum of the contributions from the resonances plus the background,

$$\mathcal{I}m\ A(s,Q^2) = N\left\{[\mathcal{I}m\ A(s,Q^2)]_R + [\mathcal{I}m\ A(s,Q^2)]_{BG}\right\}\ . \tag{1}$$

Accordingly, the resonance contribution takes the following form:

$$[\mathcal{I}m\ A(s,Q^2)]_R = \sum_{j=\Delta,\ N_1,\ N_2} f_j^2(Q^2)\frac{\mathcal{I}m\ \alpha_j}{(n_j - \mathcal{R}e\ \alpha_j)^2 + \mathcal{I}m\ \alpha_j^2}\ , \tag{2}$$

where α_j and $\mathcal{I}m\ \alpha_j$ is the relevant Regge trajectory, and the form factors are calculated as described in details in previous papers [1, 2]. For example, the form factor for the Δ resonance can be written as

$$f_\Delta^2(Q^2) = q^2c^2(Q_0')\left(c^3(Q_0)|G_+(0)|^2 + c^5(Q_0)|G_-(0)|^2\right)\ , \tag{3}$$

where $c(z) = \frac{z^2}{Q^2+z^2}$ and $|G_{+,-}(0)| = \frac{1}{\sqrt{4\pi\alpha}}\sqrt{\frac{M}{M-m}}|A_{1/2,3/2}|$; $A_{1/2}$ and $A_{3/2}$ are the helicity photoproduction amplitudes. Similar expressions can be cast for other contributions.

The imaginary part of the forward scattering amplitude coming from the background is written as

$$[\mathcal{I}m\ A(s,Q^2)]_{BG} = \sum_{j=E,E'} G_j\ c^4(Q_j)\ \frac{\mathcal{I}m_j}{(n_j^{min} - \mathcal{R}e_j)^2 + \mathcal{I}m_j^2}\ . \tag{4}$$

Here n_j^{min} is the lowest integer, larger than Max $[\mathcal{R}e_j]$, ensuring that no resonances will appear on the exotic trajectory. The advantage of such choice is that the two terms of the background depend on two different scales, Q_E^2 and $Q_{E'}^2$, so they will dominate in different regions.

The model constructed in this way, has 23 free parameters: each resonance is characterized by three (the intercept is kept fixed) coefficients describing the relevant Regge trajectory plus the two helicity photoproduction amplitudes. The form factors leave only two free parameters, Q_0 and Q_0'. Finally, the background, contains 8 free parameters: 4 for the two exotic trajectories, 2 energy scales Q_E and $Q_{E'}$ and two amplitudes G_E and $G_{E'}$. With the overall normalization factor, N this gives a total of 23 free parameters. The resulting fits to the CLAS data are presented in Table 1.

3 Description of the fit

Initially, the fit has been made by keeping some of the parameters fixed, close to their physical values, particularly those of the Regge trajectories and of the photoproduction amplitudes. Also, a single-term background was used. The resulting fit (fit 1) is shown in Table 1. Subsequently (fit 2) some of the parameters of the Regge trajectory were varied. Consequently the χ^2 was improved, although still remaining unsatisfactory. Actually in this fit 2 we obtain recently a substantial improvement: from $\chi^2 = 4.69$ [6] to $\chi^2 = 2.88$. We can see even bigger improvement, comparing to the result of the similar fit in [1] leading to $\chi_{d.o.f.} = 9.4$, but this should be taken with care, since in [1] the used dataset included both data from [5] and [3]. Finally, we let all the parameters vary (fit 3) with the result reported in the Table. This final fit is quite good, with $\chi_{d.o.f.} = 1.30$.

The progress in the fits is shown in Fig. 1, where the experimental data are plotted against the structure functions for four different values of Q^2 with the parameters from the three different fits.

4 Moments and duality ratio

Another important quantity to evaluate is given by the moments of the structure functions, as they can be used to estimate the role of the non-perturbative effects (higher twists). In order to study higher twists effects, it is essential to have a complete knowledge of the F_2 covering the entire $x-$range for each fixed Q^2. Higher twists can be well established only with higher moments ($n > 2$), meanwhile for M_2 their contribution is small even at $Q^2 \sim 1$ GeV^2. Therefore the most interesting kinematical region lies between 0 and 5 GeV^2 and large values of x, where the higher moments dominate. The JLab data and relevant calculations in [3] cover most of this region. Therefore, we have evaluated, using the explicit expressions and parameters fitted in the previous section, the Nachtmann (N) and Cornwall-Norton (CN) moments within our Regge-dual model and compared them with the data of the CLAS collaboration [3] as well as with those from Ref. [7]. The relevant moments are defined as

$$M_n^I(Q^2) = \int_0^1 dx p_n^I(x) F_2(x, Q^2) \tag{5}$$

where

$$p_n^I(x) = \begin{cases} \frac{\xi^{n+1}}{x^3} P(x, Q^2) & \text{for I= N} \\ x^{n-2}, & \text{for I = CN} \end{cases}$$

$$P(x, Q^2) = \left[\frac{3 + 3(n+1)r + n(n+2)r^2}{(n+2)(n+3)}\right],$$

$$r = \sqrt{1 + 4M^2x^2/Q^2}, \qquad \xi = 2x/(1+r).$$

We have used the parameters of fit 3 from Table 1. In Fig. 3 we plot the Nachtmann moments for $n = 2,\ 4,\ 6,\ 8$ together with the results from [3]. In Fig. 4, the calculated N- and CN-moments are compared with those from [7]. On this second set of figures the errors in the momenta are not displayed; according to [7] they should be less than 5%.

As seen from the figures, the agreement between our model and the data is quite good in the region $Q^2 < 5$ GeV^2, where the SFs were fitted to the data. The discrepancies increase with Q^2, away from the measurements. Also, note that in our calculations the elastic part of the SF (for $x = 1$) was not taken into account (see section III.G in Ref. [3]).

Duality Ratio. The validity of the parton-hadron duality for our Regge-dual model can be investigated by looking at the so-called 'duality ratio'

$$I(Q^2) = \int_{s_{min}}^{s_{max}} ds\ F_2^{Res} \Big/ \int_{s_{min}}^{s_{max}} ds\ F_2^{scaling}, \tag{6}$$

where we have fixed the lower integration limit $s_{min} = s_0$, varying the upper limit s_{max} varying between 5 GeV ^{2} and 10 GeV^2. For fixed Q^2 the integration variable can be either s (as in our case), x or any of its modifications (x', $\xi, ...$) with properly scaled integration limits. The difference may be noticeable at small values of Q^2 due to the target mass corrections (for details see e.g. [3]). These effects are typically non-perturbative and, apart from the choice of the variables, depend on detail of the model.

In choosing the smooth "scaling curve" $F_2^{scaling}$ (actually, it contains scaling violation, in accord with the DGLAP evolution) we rely on a model developed in [8] and based on a soft non-perturbative Regge pole input with subsequent evolution in Q^2, calculated [8] from the DGLAP equation.

The function F_2^{Res} is our SF with the parameters of fit 3 (see Table 1). The results of the calculations for different values of s_{max} are shown in Fig. 2. However, we note that the duality ratio may depend on the details of the different parameterizations, therefore any interpretation has to be taken with care.

5 Conclusions and discussions

The main objective of the present study was a phenomenological analyses of the CLAS data in a model within the low-energy decomposition of our Regge-dual model. In perspective, there is the possibility to link low-energy, resonance physics (and the JLab data) with the high-energy (or low x) physics (from HERA) by "Veneziano duality" (apart from parton-hadron duality), inherent in the model [4].

One of our main observations is the important role of the little known background. The existing parameterizations of the background are based on purely phenomenological models, whereas we treat the background on the same footing as the resonances, both corresponding to Regge trajectories within the analytical $S-$ matrix theory.

This bold idea was put forward in paper [9], where a new concept, called super-broad resonance approximation to dual amplitude was suggested. As suggested by its name, this approximation is opposite to the well-known narrow-resonance approximation of Veneziano. The difference is that while the shift of narrow resonances to physical ones destroys the attractive features of the narrow-resonance dual amplitude, a similar procedure, i.e. shifting the super-broad exotic "resonances" to the physical sheet of the amplitude is harmless for the model (for more details see [9]).

The difference between resonances poles and the background, is qualitative rather than quantitative: both correspond to definite singularities of the scattering amplitude. The same is true for the high-energy limit of the scattering amplitude, containing two components ordinary Regge pole exchanges and the Pomeron, responsible for diffraction. Moreover, since the the background is dual to the Pomeron, we expect that its high-energy (or low-x) behaviour matches that of the Pomeron, producing non-decreasing cross sections (structure functions). Actually, this is not the case for a limited (small!) number of poles. As we know, within Veneziano duality, the high energy Regge (including that of the Pomeron) behavior is built up from a large number of direct channel resonances. We anticipate that this shortcoming can be resolved by a relevant choice of the exotic trajectory by still keeping a limited, or even small number of exotic poles. This option will be studied in a forthcoming paper [10].

6 Acknowledgments

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	parameters	Fit 1	Fit 2	Fit 3
N_1^*	α_0	-0.8377°	-0.8377°	-0.8377°
	α_1 [GeV^{-2}]	0.9500°	0.9551	0.9825
	α_2 [GeV^{-1}]	0.1473°	0.1500	0.0920
	$A^2(1/2)$ [GeV^{-1}]	0.0484E-2°	0.0484E-2°	0.8647E-2
	$A^2(3/2)$ [GeV^{-1}]	0.2789E-1°	0.2789E-1°	0.9634E-2
N_2^*	α_0	-0.3700°	-0.3700°	-0.3700°
	α_1 [GeV^{-2}]	0.9500°	0.9646	0.9551
	α_2 [GeV^{-1}]	0.1471°	0.0699	0.0949
	$A^2(1/2)$ [GeV^{-1}]	0.0289E-2°	0.0289E-2°	0.9724E-2
	$A^2(3/2)$ [GeV^{-1}]	0.1613°	0.1613°	5.1973E-11
Δ	α_0	0.0038°	0.0038°	0.0038°
	α_1 [GeV^{-2}]	0.8500°	0.8815	0.8605
	α_2 [GeV^{-1}]	0.1969°	0.1622	0.2005
	$A^2(1/2)$ [GeV^{-1}]	0.0199°	0.0199°	5.3432E-08
	$A^2(3/2)$ [GeV^{-1}]	0.0666°	0.0666°	0.0866
E_1	G_{E_1}	6.5488	2.8001	3.6049
	α_0	0.3635	0.7499	0.3883
	α_2 [GeV^{-1}]	0.1755	0.1699	0.3246
	$Q^2_{E_1}$ [GeV^2]	5.2645	4.3000	3.9774
	s_{E_1} [GeV^2]	1.14°	1.1740	1.14°
E_2	G_{E_2}	—	—	-0.6520
	α_0	—	—	-0.8929
	α_2 [GeV^{-1}]	—	—	1.7729
	$Q^2_{E_2}$ [GeV^2]	—	—	2.4634
	s_{E_2} [GeV^2]	—	—	1.14°
	s_0 [GeV^2]	1.14°	1.14°	1.14°
	$Q_0'^2$ [GeV^2]	0.4089	0.4699	0.9998
	Q_0^2 [GeV^2]	3.1709	2.5499	1.8926
	N [GeV^{-2}]	0.0408	0.05593	0.0567
	$\chi^2_{d.o.f.}$	12.92	2.8824	1.3005

Table 1. Parameters of the fits. The symbol ° refers to the fixed parameters.

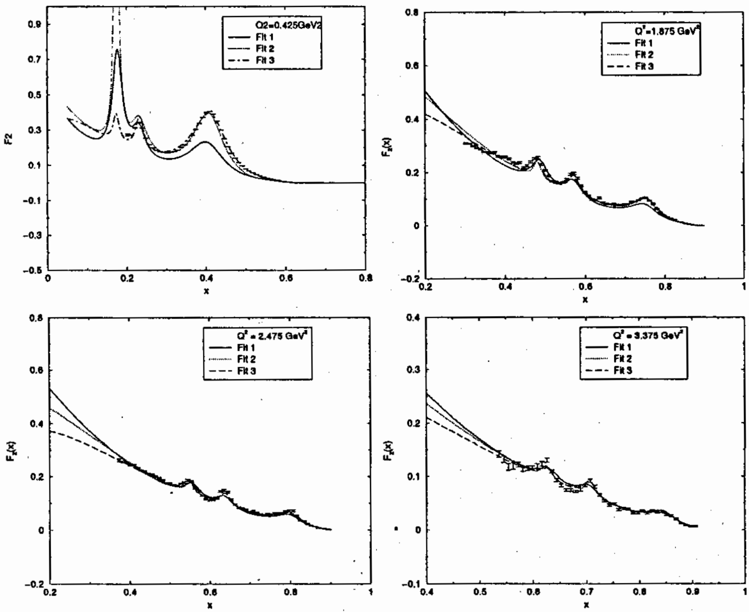

Figure 1: Comparison between three different fits performed in the present model (see text).

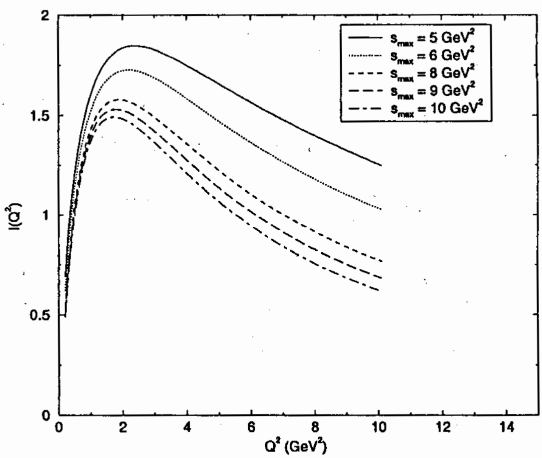

Figure 2: Global parton-hadron duality test for different values of s_{max}.

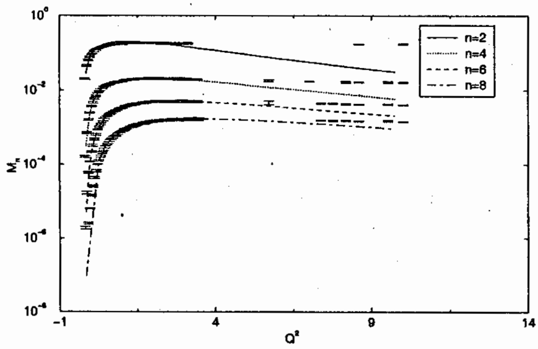

Figure 3: Nachtmann moments, M_n^N for $n = 2, 4, 6, 8$. The plot compares the moments calculated from the Regge-dual with those extracted from the data and reported in [3] (inelastic part).

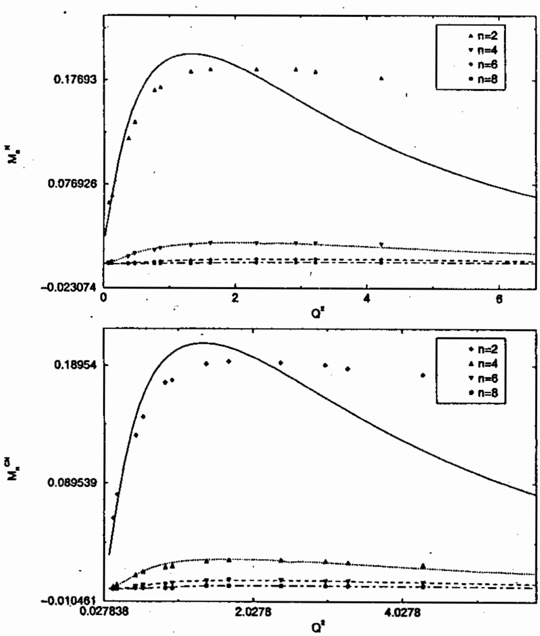

Figure 4: Nachtmann moments, M_n^N, and Cornwall-Norton moments, M_n^{CN}, for $n = 2, 4, 6, 8$. These plots show the comparison between the moments evaluated according to our Regge-dual model and the values extracted from the electron-proton scattering data reported in [7] (inelastic part).

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The Time Structure of Anomalous J/ψ Suppression in Nuclear Collisions

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Abstract — The data for $\langle p_t^2\rangle(E_t)$ of J/ψ produced in Pb-Pb collisions at the CERN-SPS contain information about when anomalous suppression is active. A general transport equation which describes transverse motion of J/ψ in the absorptive medium is proposed and solved for a QGP and a comover scenario of suppression. While inconsistencies appear in the QGP, the comover approach accounts for the data fairly well. For central collisions the bulk of anomalous suppression happens rather late, 3-4 fm/c after the nuclear overlap.

The discovery in 1996 of anomalous J/ψ suppression in Pb-Pb collisions at the SPS has been one of the highlights of the research with ultrarelativistic heavy ions at CERN [1]. Does it point to the discovery of the predicted quark gluon plasma (QGP)? Six years later the situation is still confused, since several models - with and without the assumption of a QGP - describe the observed suppression, after at least one parameter is adjusted. The data on the mean squared transverse momentum $\langle p_t^2\rangle(E_t)$ [2] for the ψ (this symbol stands for J/ψ and ψ') in the regime of anomalous suppression and as a function of transverse energy E_t have received less attention - for no good reason. We claim: $\langle p_t^2\rangle(E_t)$ contains additional information, namely about the time structure of anomalous suppression. This idea has already been considered more than 10 years ago [3] (c.f. also more recent works [4,5]) and is based on the following phenomenon: Anomalous suppression is not an instantaneous process, but takes a certain time t_A. During this time ψ's produced with high transverse momenta may leak out of the parton/hadron plasma and escape suppression. As a consequence, low p_t ψ's are absorbed preferentially. The $\langle p_t^2\rangle$ of the surviving (observed) ψ's shows an increase $\delta\langle p_t^2\rangle$, which grows monotonically with t_A [5]. In this letter we propose a general formalism how to incorporate the effect of leakage into the various models, which have been proposed to describe anomalous suppression. We extract information about the time t_A from a comparison with experiment.

It has become customary to distinguish between normal and anomalous values of suppression $S(E_t) = \sigma^{\psi}(E_t)/\sigma^{DY}(E_t)$ and values of $\langle p_t^2\rangle(E_t)$ for ψ's produced in nuclear collisions, c.f. reviews [6, 7]. Here, σ^ψ is the production cross section of a ψ and σ^{DY} is the Drell-Yan cross section in an AB collision. By definition, ψ's produced in pA collisions show normal suppression via inelastic ψN collisions in the final state and normal increase of $\langle p_t^2 \rangle$ (above $\langle p_t^2 \rangle_{NN}$ in NN collisions) via gluon rescattering in the initial state. These normal effects are also present in nucleus-nucleus collisions and happen, while projectile and target nuclei overlap. Anomalous values of S and $\langle p_t^2 \rangle$ are attributed to the action on the ψ by the mostly baryon free phase of partons and/or hadrons (we call it parton/hadron plasma) which is formed after the nuclear overlap. It may lead to deconfinement of the ψ via colour screening, dissociation via gluon absorption or inelastic collisions by the comovers during the later period of the plasma evolution. In this letter we describe anomalous ψ suppression within a transport theory and apply it to two rather different scenarios: (I) Absorption involving a threshold in the energy density (QGP), (II) continuous absorption via comovers.

We denote by $d\sigma^\psi/d\vec{p}_t(\vec{p}_t, E_t)$ the cross section for the production of a ψ with given p_t and in an event with fixed transverse energy E_t. It can be related to the phase space density f^ψ via

$$\frac{d\sigma^\psi}{d\vec{p}_t}(\vec{p}_t, E_t) = \lim_{t\to\infty} \int d\vec{b} P(E_t; b) \int d\vec{s}\, f^\psi(\vec{s}, \vec{p}_t, t; \vec{b}). \qquad (1)$$

Here, $P(E_t; b)$ describes the distribution of transverse energy E_t in events with a given impact parameter $\vec{b}$ between projectile A and target B. We follow ref. [8] in notation for $P(E_t; \vec{b})$ and the values of the numerical constants. The function $f^\psi(\vec{s}, \vec{p}_t, t; \vec{b})$ is the distribution of ψ's in the transverse phase space $(\vec{s}, \vec{p}_t)$ at time t for given $\vec{b}$.

We define $t = 0$ as the time, when the process of normal suppression and normal generation of $\langle p_t^2 \rangle$ has ceased and denote by $f_N^\psi(\vec{s}, \vec{p}_t; \vec{b})$ the distribution of ψ's at this time. We take f_N^ψ as initial condition for the motion and absorption of the ψ's due to anomalous interactions. The evolution of the ψ is described by a transport equation

$$\frac{\partial}{\partial t} f^\psi + \vec{v}_t \cdot \vec{\nabla}_s f^\psi = -\alpha f^\psi. \qquad (2)$$

The time dependence arises from the free streaming of the ψ with transverse velocity $\vec{v}_t = \vec{p}_t / \sqrt{m_\psi^2 + p_t^2}$ (l.h.s.) and an absorptive term on the r.h.s., where the function $\alpha(\vec{s}, \vec{p}_t, t; \vec{b})$ contains all details about the surrounding matter and the absorption process. We have left out effects from a mean field, because the elastic ψN cross section is very small and have also neglected a gain term on the r.h.s., because recombination processes $c\bar{c}+N \to \psi+X$ seem unimportant at SPS energies since at most one $c\bar{c}$ is created per event.

Eq. (2) can be solved analytically with the result

$$f^{\psi}(\vec{s},\vec{p}_t,t;\vec{b}) = \exp\left(-\int_0^t dt'\alpha(\vec{s}+\vec{v}_t(t-t'),\vec{p}_t,t';\vec{b})\right) f_N^{\psi}(\vec{s}+\vec{v}_t t,\vec{p}_t;\vec{b}), \quad (3)$$

which for $t = 0$ reduces to $f^{\psi} = f_N^{\psi}$. If we denote by t_f the time when anomalous suppression has ceased, $\alpha(\vec{s},\vec{p}_t,t;\vec{b}) = 0$ for $t > t_f$, the limit $t \to \infty$ in eq. (1) can be replaced by setting $t = t_f$, since the distribution in p_t does not change for larger t's.

There is little controversy about ψ production and suppression in the normal phase: The gluons, which fuse to the $c\bar{c}$, collide with nucleons before fusion and gain additional p_t^2. The ψ on its way out is suppressed by inelastic ψN collisions. Neglecting effects of formation time [9], one has

$$\begin{aligned} f_N^{\psi}(\vec{s},\vec{p}_t;\vec{b}) &= \sigma_{NN}^{\psi}\int dz_A\, dz_B\, \rho_A(\vec{s},z_A)\,\rho_B(\vec{b}-\vec{s},z_B)\,\langle p_t^2\rangle_N^{-1}\exp\left(-p_t^2/\langle p_t^2\rangle_N\right) \\ &\times \exp\left(-\sigma_{abs}^{\psi}\left[T_A(\vec{s},z_A,\infty)+T_B(\vec{b}-\vec{s},-\infty,z_B)\right]\right), \end{aligned} \quad (4)$$

where

$$\langle p_t^2\rangle_N(\vec{b},\vec{s},z_A,z_B) = \langle p_t^2\rangle_{NN}^{\psi} + a_{gN}\rho_0^{-1}\left[T_A(\vec{s},-\infty,z_A)+T_B(\vec{b}-\vec{s},z_B,+\infty)\right] \quad (5)$$

with the thickness function $T(\vec{s},z_1,z_2) = \int_{z_1}^{z_2} dz\,\rho(\vec{s},z)$. All densities ρ_A,ρ_B are normalized to the number of nucleons (ρ_0 is the nuclear matter density). We shortly explain eqs. (4) and (5): For given values $\vec{b}$ and $\vec{s}$ in the transverse plane the ψ is produced at coordinates z_A and z_B in nuclei A and B, respectively. On its way out, the ψ experiences the thicknesses $T_A(\vec{s},z_A,\infty)$ and $T_B(\vec{b}-\vec{s},-\infty,z_B)$ in nuclei A and B, respectively and is suppressed with an effective absorption cross section σ_{abs}^{ψ}. The two gluons which fuse carry transverse momentum from two sources. (i) Intrinsic p_t, because they had been confined to a nucleon. The intrinsic part is observable in $NN \to \psi$ collisions and leads to $\langle p_t^2\rangle_{NN}^{\psi}$ in eq. (5). (ii) In a nuclear collision, the gluons traverse thicknesses $T_A(\vec{s},-\infty,z_A)$ and $T_B(\vec{b}-\vec{s},z_B,+\infty)$ of nuclear matter in A and B, respectively, and acquire additional transverse momentum via gN collisions. This is the origin of the second term in eq. (5).

The constants $\sigma_{abs}^{J/\psi}$ and a_{gN} are usually adjusted to the data from pA collisions, before one investigates anomalous suppression. Fig. 1 shows (dashed curves) the results for normal suppression $S^{\psi}(E_t)$ and normal $\langle p_t^2\rangle^{\psi}(E_t)$ calculated with f_N^{ψ} eq. (4) in eq. (1). While the difference between calculation and data is enormous for the suppression, it is rather small for $\langle p_t^2\rangle^{\psi}(E_t)$.

Since the physical origin of anomalous suppression is not yet settled, we investigate suppression $S^{\psi}(E_t)$ and $\langle p_t^2\rangle^{\psi}(E_t)$ for two models, which have rather contradictory assumptions.

I. QGP scenario: ψ's is totally and rapidly destroyed, when they are in a medium whose energy density above a critical one, and nothing happens elsewhere. As a representative model we use the approach by Blaizot *et al.* [8].

II. Comover scenario: The plasma of comovers (partons and/or hadrons) leads to a continuous absorption of long duration due to inelastic collisions with the comoving particles. As a representative model, we use the approaches by Capella *et al.* [10] and Kharzeev *et al.* [11].

We begin with model I: In their schematic approach Blaizot *et al.* [8] include anomalous suppression via

$$f^{\psi}(\vec{s},\vec{p}_t;\vec{b}) = \theta(n_c - n_p(\vec{b},\vec{s}))\, f_N^{\psi}(\vec{s},\vec{p}_t;\vec{b}). \tag{6}$$

Here n_c is a critical density and $n_p(\vec{b},\vec{s})$ is the density of participant nucleons

$$n_p(\vec{b},\vec{s}) = T_A(\vec{s},-\infty,+\infty)\left[1-\exp\left(-\sigma_{in}^{NN} T_B(\vec{b}-\vec{s},-\infty,+\infty)\right)\right] + (A \leftrightarrow B). \tag{7}$$

According to eq. (6) all ψ's are destroyed if the energy density (which is directly proportional to the participant density) at the location $\vec{s}$ of the ψ is larger than a critical density n_c. The other ψ's survive. While the prescription eq.(6) successfully describes the data in the full E_t range of anomalous suppression after the only one free parameter, n_c, is adjusted, the predictions for $\langle p_t^2\rangle(E_t)$ are significantly below the data, especially at large E_t (see below).

The expression eq. (6) for the phase space distribution f^{ψ} including anomalous suppression within the QGP model is recovered within our transport approach eq. (3) by setting

$$\alpha(\vec{s},\vec{p},t;\vec{b}) = \alpha_0 \theta(n_p(\vec{b},\vec{s}) - n_c)\, \delta(t) \tag{8}$$

and taking the limit $\alpha_0 \to \infty$. The delta function $\delta(t)$ has to be included in order to recover Eq. (6), but it reflects our preconception that the energy density is highest at $t = 0$ and absorption therefore most likely.

There are various ways to introduce another time structure into the absorption term. In this paper we investigate the generalization

$$\alpha(\vec{s}, \vec{p}_t, t; \vec{b}) = \alpha_0 \, \theta\left(n_p(\vec{b}, \vec{s}) - n_c\right) \delta(t - t_A). \tag{9}$$

The idea of a threshold density is kept, but it acts at a later time t_A, which time is then determined from a comparison with the data. For $\alpha_0 \to \infty$ and a time $t_A^+ = t_A + \epsilon$, one finds from the general solution eq. (3)

$$f^{\psi}(\vec{s}, \vec{p}_t, t_A^+) = \theta\left(n_c - n_p(\vec{b}, \vec{s})\right) f_N^{\psi}(\vec{s} + \vec{v}_t t_A, \vec{p}_t), \tag{10}$$

which differs from the expression (6), by the motion in phase space of f_N^{ψ} during the time $0 \le t \le t_A$. For $t > t_A$ the momentum distribution derived from f^{ψ} does not change any more. The physical interpretation of t_A is a kind of mean value replacing a time structure of longer duration.

We calculate the suppression $\sigma^{\psi}(E_t, t_A)/\sigma^{DY}(E_t)$ and $\langle p_t^2 \rangle^{\psi}(E_t, t_A)$ with the distribution f^{ψ} from eq. (10). We use the parameters of ref. [8] where available, i.e. for J/ψ: $\sigma_{abs}^{J/\psi} = 6.4$ mb, $n_c = 3.75$ fm^{-2}. The parameters for the generation of $\langle p_t^2 \rangle$ by gluon rescattering are taken from a fit to the pA data [2]: $\langle p_t^2 \rangle_{NN} = 1.11$ (GeV/c)^2 and $a_{gN} = 0.081$ (GeV/c^2) fm^{-1}. We also account for the transverse energy fluctuations [8] which have been shown to be significant for the explanation of the sharp drop of J/ψ suppression in the domain of very large E_t values, by replacing n_p by $\frac{E_t}{\langle E_t \rangle} n_p$ where $\langle E_t \rangle$ is the mean transverse energy at given b. We then calculate $\sigma^{J/\psi}/\sigma^{DY}$ as a function of t_A. Since the critical density for J/ψ is quite high, the leakage affects only the very high momentum J/ψ's, and $\sigma^{J/\psi}/\sigma^{DY}$ increases only slightly as t_A increases (Fig. 1 upper left).

We turn to a discussion of $\langle p_t^2 \rangle^{J/\psi}(E_t)$. Fig. 1 (upper right) shows calculated curves for values of $t_A = 0$ to 4 fm/c. The dotted line ($t_A = 0$) is the result of the original threshold model with immediate anomalous suppression (and has been predicted in [12]). It fails badly at large values of E_t. Also no other curve with a given t_A describes the data for all values of E_t. We have to conclude that t_A depends on E_t: $t_A(E_t)$. The larger the values E_t the later anomalous suppression acts. From a comparison with data we have

The results for the extracted values t_A, when anomalous suppression acts within the QGP model, are strange: Can we believe that the J/ψ is absorbed only at a mean time t_A =3-4 fm/c after the two nuclei have separated? In the sence of the mean value, only one half of anomalous suppression acts for $t < t_A$ and one half for $t > t_A$.

Before we answer this questin we proceed to model II, the scenario of comovers: Partons and/or hadrons which move with the ψ may destroy the ψ with a cross section σ^{ψ}_{co}. The comover density $n_{co}(t)$ depends on time, for which the Bjorken expansion scenario predicts t^{-1}. The absorptive term α in eq. (2) then takes the form

$$\alpha(\vec{s},\vec{p}_t,t;\vec{b}) = \sigma^{\psi}_{co}\,\frac{n_{co}(\vec{b},\vec{s})}{t}\theta\left(\frac{n_{co}(\vec{b},\vec{s})}{n_f}t_0 - t\right)\,\theta(t-t_0), \tag{11}$$

i.e. absorption by comovers starts at $t = t_0$ and ends at t_1, when the comover density $n_{co}(\vec{b},\vec{s}) \cdot t_0/t_1$ has reached a value n_f independent of $\vec{b}$ and $\vec{s}$. The comover approach contains a definite time structure for anomalous suppression and we have not changed it. In the choice of parameters we have followed [10, 11]: $n_{co}(\vec{b},\vec{s}) = 1.5\,n_p(\vec{b},\vec{s})$ with the participant density from eq. (7), $t_0 =$ 1 fm/c, $n_f = 1$ fm^{-2}, $\sigma^{J/\psi}_{abs} = 4.5$ mb and $\sigma^{J/\psi}_{co} = 1$ mb.

The calculated E_t dependence of the suppression shown in Fig. 1 (lower left) fits the data acceptably but less well than model I. For $\langle p_t^2\rangle$, we compare the results of two calculations with the data. The solid curves represent the result of the full calculation, including leakage, i.e. expression f^{ψ} eq.(3). It fits the data reasonably well for the full range of transverse energies E_t. The dashed lines in Fig. 1 (lower right) show the calculation leaving out leakage. Formally this limit (which has been calculated already in [15]) is obtained from eq. (3) by setting $\vec{v}_t = 0$ in the exponent and in f^{ψ}_N. The case without leakage fits the data less well, through the difference between the two curves is not very big. We stress: the calculation of $\langle p_t^2\rangle(E_t)$ in the comover model is a true prediction in the sense that no parameter is adjusted above those which are fitted to the suppression $S^{\psi}(E_t)$. We have also calculated the mean time $\langle t^{\psi}_A\rangle$ for comover action by studying the suppression $S^{\psi}(E_t)$ as a function of time and taking the mean of t with the weight $dS^{\psi}(E_t)/dt$ and find

$$\langle t^{J/\psi}_A\rangle = 3.5\,\text{fm}/c, \tag{12}$$

which values include the time t_0, eq. (11), between the end of normal suppression and the beginning of comover action. The values eq. (12) are found to be rather independent of E_t.

We summarize: we have described a method to derive information about the time structure of anomalous charmonium suppression from the data on $\langle p_t^2\rangle(E_t)$ measured for $Pb-Pb$ collisions at SPS-energies, where anomalous suppression has been discovered. The time evolution of the ψ during anomalous suppression including leakage is described within a general transport equation. The formalism is applied to two models with rather contradictory underling physical assumptions, the QGP and the comover models with the following results:

(i) Calculations within the original models, where leakage is left out, do not describe the data for $\langle p_t^2\rangle^{J/\psi}(E_t)$, the discrepancy being particularly strong for the QGP model at high values of E_t.

(ii) Including leakage into the comover model, without changing its structure nor its parameters leads to a good agreement with the data for $\langle p_t^2\rangle(E_t)$ for J/ψ over the full range of values E_t.

(iii) The assumption in the threshold model that anomalous suppression acts instantaneously at $t_A = 0$, i.e. right after normal suppression is not supported by experiment, but a mean value $t_A \approx 3.5$ fm fits the data for central events.

We conclude: According to the present study, anomalous suppression of charmonia is a process which takes a certain time, ≈ 2 fm/c in peripheral collisions and ≈ 4 fm/c in central collisions. Unless the partonic phase with its high density and rapid suppression has an unusual time structure [4, 16], the results of our paper favor the comover scenario with its rather weak but continuous suppression over a comparably long time.

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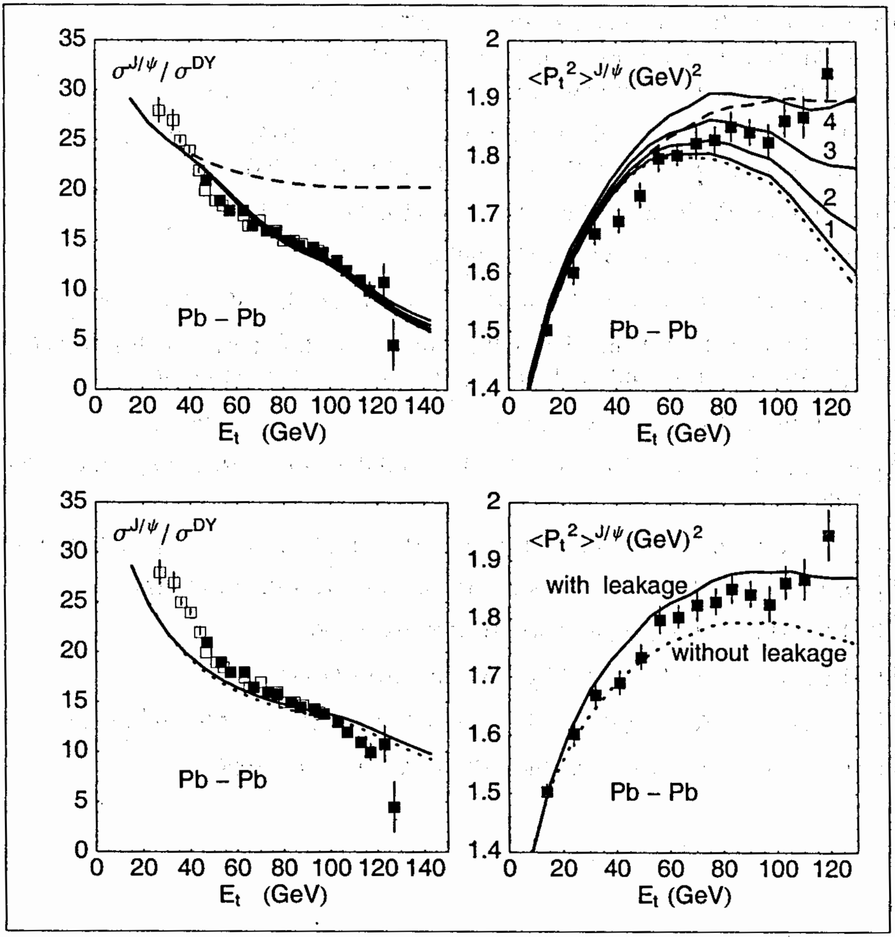

Figure 1: Nuclear suppression $\sigma^{J/\psi}/\sigma^{DY}$ and $\langle p_t^2\rangle^{J/\psi}$ as a function of transverse energy E_t. Data are from [1] for $\sigma^{J/\psi}/\sigma^{DY}$ and from [2] for $\langle p_t^2\rangle^{J/\psi}$. Upper two curves: Dashed curves show the result of normal suppression alone. The other curves include anomalous suppression within the QGP model eqs. (6) and (10), where anomalous suppression is assumed to act at time t_A. The curves are labled by the values $t_A = 1, 2, 3, 4$ fm/c. Also the curves in $\sigma^{J/\psi}/\sigma^{DY}$ carry these lables. Lower two figures: Dotted and solid lines are calculated in the comover model. Dotted line from ref. [15], which in our case corresponds to eq. (3) but setting $\vec{\nu}_t = 0$. Solid line full calculations including $\vec{\nu}_t$.

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Meson model with nonlocal four-quark interaction

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Abstract — The nonlocal version of the $SU(2) \times SU(2)$ symmetric four-quark interaction of the NJL type is considered. Each of the quark lines contains the form factors. These form factors remove the ultraviolet divergences in quark loops and ensures the absence of the poles in the quark propagator(quark confinement). The constituent quark mass $m(0)$ is expressed thought the cut-off parameter Λ, $m(0) = \Lambda = 340$ MeV in the chiral limit. These parameters are fixed by the experimental value of the weak pion decay and allow us to describe the mass of the light scalar meson and strong decays $\rho \to \pi\pi$ and $a_1 \to \rho\pi$ in the qualitative agreement with experimental data.

The effective meson Lagrangians obtained on the basis of the local four-quark interaction of the Nambu–Jona-Lasinio(NJL) type satisfactorily describe low-energy meson physics [1,2]. However, these models contain ultraviolet(UV) divergences and do not describe quark confinement. Satisfactory results in these models can be obtained only for light mesons and interactions at low energies in the range of 1 GeV. In order to overcome these restrictions, it is necessary to consider nonlocal versions of these models which allow us to remove UV divergences and describe the quark confinement.

A lot of models of this type were proposed in the last few years. Unfortunately, we cannot give here the full list of references concerning this activity. Therefore, we will concentrate only on the direction connected with the nonlocal quark interaction motivated by the instanton theories [3, 4].Recently, a few nonlocal models of the this type were proposed [5, 6]. In these models the nonlocal kernel is taken in the separable form where each quark line contain form factor following from instanton theories. These form factors naturally remove UV divergences in quark loops. Thus, in [5] a nonlocal form factor was chosen in the Gaussian form $f(p) = \exp(-p^2/\Lambda^2)$ where Λ is the cut-off parameter[1]. In [6] it was proposed to use an additional condition for the form factor $f(p)$ (quark mass function $m(p)$, respectively) which lead to the absence

[1]Here and further all expressions are given in Euclidean domain.

of the poles in the quark propagator. Namely, it is supposed that the scalar part of the quark propagator is expressed through the entire function

$$\frac{m(p)}{p^2+m^2(p)} = \frac{1}{\mu}\exp\left(-p^2/\Lambda^2\right), \tag{1}$$

where μ is an additional arbitrary parameter.

In this work an analogous condition will be used . However, we will take into account that each quark line contain square of the form factor which is expressed through the quark mass

$$\frac{m(p)f^2(p)}{p^2+m^2(p)} \to \frac{m^2(p)}{p^2+m^2(p)} = \exp\left(-p^2/\Lambda^2\right). \tag{2}$$

As a result, we obtain a simpler solution for the mass function than in [6]. In our model $m(0)$ and the cut-off parameter Λ have a simple connection in the chiral limit $m(0) = \Lambda$; the function $m(p)$ contains only one arbitrary parameter. We fix this parameter by weak pion decay. Then, for $F_\pi = 93$ MeV we have $m(0) = \Lambda = 340$ MeV. This leads to reasonable predictions for the scalar meson mass and widths of the decays $\rho \to \pi\pi$, $a_1 \to \rho\pi$.

Our model is based on the $SU(2) \times SU(2)$ symmetric action

$$\begin{aligned} \mathcal{S}(\bar{q},q) = \int d^4x \, \Big\{ &\bar{q}(x)(i\hat{\partial}_x - m_c)q(x) + \frac{G_\pi}{2}\left(J^a_\pi(x)J^a_\pi(x) + J_\sigma(x)J_\sigma(x)\right) \\ &-\frac{G_\rho}{2}J^{\mu\,a}_\rho(x)J^{\mu\,a}_\rho(x) - \frac{G_{a_1}}{2}J^{\mu\,a}_{a_1}(x)J^{\mu\,a}_{a_1}(x)\Big\}, \end{aligned} \tag{3}$$

where $\bar{q}(x) = (\bar{u}(x), \bar{d}(x))$ are the u and d quark fields, m_c is the diagonal matrix of the current quark masses. The nonlocal quark currents $J_I(x)$ are expressed as

$$J_I(x) = \int d^4x_1 d^4x_2 \, f(x_1)f(x_2)\, \bar{q}(x-x_1)\,\Gamma_I\, q(x+x_2). \tag{4}$$

where $f(x)$ is formfactor. The matrices Γ_I are defined as

$$\Gamma_\sigma = \mathbf{1}, \Gamma^a_\pi = i\gamma^5\tau^a, \Gamma^{\mu\,a}_\rho = \gamma^\mu\tau^a, \Gamma^{\mu\,a}_{a_1} = \gamma^5\gamma^\mu\tau^a,$$

where τ^a are the Pauli matrices and γ^μ, γ^5 are the Dirac matrices.

In this report, we mainly consider the strong interactions. The electroweak fields may be introduced by gauging the quark field by the Schwinger phase factors (see, c.f. [4,5]).

After bosonization the action becomes

$$\mathcal{S}(q,\bar{q},\sigma,\pi,\rho,a) = \int d^4x \Big\{\bar{q}(x)(i\hat{\partial}_x - m_c)q(x)$$
$$-\frac{1}{2G_\pi}\left(\pi^a(x)^2 + \tilde{\sigma}(x)^2\right) + \frac{1}{2G_\rho}(\rho^{\mu a}(x))^2 + \frac{1}{2G_{a_1}}(a_1^{\mu a}(x))^2$$
$$+\int d^4x_1 d^4x_2\, f(x-x_1)f(x_2-x)\,\bar{q}(x_1)$$
$$\times\left(\tilde{\sigma}(x) + \pi^a(x)i\gamma^5\tau^a + \rho^{\mu a}(x)\gamma^\mu\tau^a + a_1^{\mu a}(x)\gamma^5\gamma^\mu\tau^a\right) q(x_2)\Big\}, \quad (5)$$

where $\tilde{\sigma},\pi,\rho,a$ are the σ, π, ρ, a_1 meson fields, respectively. The field $\tilde{\sigma}$ has a nonzero vacuum expectation value $<\tilde{\sigma}>_0 = \sigma_0 \neq 0$. In order to obtain a physical scalar field with zero vacuum expectation value, it is necessary to shift the scalar field as $\tilde{\sigma} = \sigma + \sigma_0$. This leads to the appearance of the quark mass function $m(p)$ instead of the current quark mass m_c

$$m(p) = m_c + m_{dyn}(p), \quad (6)$$

where $m_{dyn}(p) = -\sigma_0 f^2(p)$ is the dynamical quark mass. From the action, eq. (5), by using $\langle\delta S/\delta\sigma\rangle_0 = 0$, one can obtain the gap equation for the dynamical quark mass

$$m_{dyn}(p) = G_\pi \frac{8N_c}{(2\pi)^4} f^2(p) \int d_E^4k f^2(k)\frac{m(k)}{k^2+m^2(k)}. \quad (7)$$

Equations (6),(7) have the following solution:

$$m(p) = m_c + (m_q - m_c)f^2(p), \quad (8)$$

where $m_q = m(0)$.

In order to provide quark confinement we propose the anzatz given by eq.(2) for the quark mass function $m(p)$(in the chiral limit). It worth noticing, the form of the left-hand side of eq.(2) coincides with the integrand in the gap equation (7). From eq.(2) we obtain the following solution

$$m(p) = \left(\frac{p^2}{\exp(p^2/\Lambda^2) - 1}\right)^{1/2}; \quad (9)$$

here we have only one free parameter Λ; $m(p)$ does not have any singularities in the whole real axis and exponentially drops as $p^2 \to \infty$ in the Euclidean domain. From eq.(8) follows that the form factors have a similar behavior that provides the absence of UV divergences in our model. At $p^2 = 0$ the mass function is equal to the cut-off parameter Λ, $m(0) = \Lambda$. The pole part of the quark propagator also does not contain singularities that provide quark confinement

$$\frac{1}{m^2(p) + p^2} = \frac{1 - \exp(-p^2/\Lambda^2)}{p^2}. \tag{10}$$

Let us consider the scalar and pseudoscalar mesons. The meson propagators are given by

$$D_{\sigma,\pi}(p^2) = \frac{1}{-G_\pi^{-1} + \Pi_{\sigma,\pi}(p^2)} = \frac{g^2_{\sigma,\pi}(p^2)}{p^2 - M^2_{\sigma,\pi}}, \tag{11}$$

where $M_{\sigma,\pi}$ are the meson masses, $g_{\sigma,\pi}(p^2)$ are the functions describing renormalization of the meson fields and $\Pi_{\sigma,\pi}(p^2)$ are the polarization operators. The meson masses $M_{\sigma,\pi}$ are found from the position of the pole in the meson propagator $\Pi_{\sigma,\pi}(M^2_{\sigma,\pi}) = G_\pi^{-1}$ and the constants $g_{\sigma,\pi}(M^2_{\sigma,\pi})$ are given by

$$g^{-2}_{\sigma,\pi}(M^2_{\sigma,\pi}) = \frac{d\Pi_{\sigma,\pi}(p^2)}{dp^2}|_{p^2=M^2_{\sigma,\pi}}. \tag{12}$$

Let us consider this model in the chiral limit. The pion constant is not depend on parameter Λ and takes the form

$$g_\pi^{-2}(0) = \frac{N_c}{4\pi^2}\left(\frac{3}{8} + \frac{\zeta(3)}{2}\right), \; g_\pi(0) \approx 3.7 \tag{13}$$

here ζ is the Riemann zeta function.

The gap equation and quark condensate take the forms

$$G_\pi \Lambda^2 = \frac{2\pi^2}{N_c}, \quad \langle \bar{q}q\rangle_0 = -\frac{N_c}{4\pi^2}\int\limits_0^\infty du\, u \frac{m(u)}{u + m^2(u)}. \tag{14}$$

The Goldberger-Treiman relation is fullfield in the model of this kind [4–6]

$$F_\pi = \frac{m_q}{g_\pi}. \tag{15}$$

From eq.(15) the value $\Lambda = m_q = 340$ MeV is obtained for $F_\pi = 93$ MeV. Then, from eq.(14) we obtain $G_\pi = 56.6\,\mathrm{GeV}^{-2}$, $\langle \bar{q}q\rangle_0 = -(190\,\mathrm{MeV})^3$.

We get for sigma meson $M_\sigma = 420$ MeV and $g_\sigma(M_\sigma) = 3.85$. The total decay width is $\Gamma_{(\sigma\to\pi\pi)} = 150$ MeV. Comparing these results with experimental data one finds that M_σ is in satisfactory agreement with experiment; however, the decay width is very small. This situation can be explained by the fact that for a correct description of the scalar meson it is necessary to take into account the mixing with the four-quark state [8] and the scalar glueball [9]. Moreover, it has recently been shown that the $1/N_c$ corrections in this channel are rather big [10].

Similary we can describe the vector and axial-vector mesons. The constants G_{ρ,a_1} are fixed by physical meson masses and numerically equal $G_\rho = 6.5$ GeV^{-2}, $G_{a_1} = 0.67$ GeV^{-2}.

The amplitude for the process $\rho \to \pi\pi$ is $A^\mu_{(\rho\to\pi\pi)} = g_{(\rho\to\pi\pi)}(q_1 - q_2)^\mu$ where q_i are momenta of the pions. We obtain $g_{(\rho\to\pi\pi)} = 5.7$ and the decay width $\Gamma_{(\rho\to\pi\pi)} = 135$ MeV which is in qualitative agreement with the experimental value 149.2±0.7 MeV [7]. The amplitude for the process $a_1 \to \rho\pi$ is $A^{\mu\nu}_{(a_1\to\rho\pi)} = a_{(a_1\to\rho\pi)}g^{\mu\nu} + b_{(a_1\to\rho\pi)}p^\nu q^\mu$ where p, q are momenta of a_1,ρ mesons, respectively. We obtain $a_{(a_1\to\rho\pi)} = -1.26$ GeV, $b_{(a_1\to\rho\pi)} = 26.8$ GeV^{-1}. As a result the decay width is equal to $\Gamma_{(a_1\to\rho\pi)} = 170$ MeV. This value has the same order as experimental data $250 - 600$ MeV [7]. Note that the width of the decay $a_1 \to \rho\pi$ strongly depends on mass of the a_1 meson. Indeed, for $M_{a_1} = 1.3$ GeV we have $\Gamma_{(a_1\to\rho\pi)} = 260$ MeV.

In our model like in all models of this kind $\pi - a_1$ transitions can be neglected.

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Three-Fluid Simulations of Relativistic Heavy-Ion Collisions

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Abstract — A relativistic 3-fluid 3D hydrodynamic model has been developed for describing heavy-ion collisions at incident energies between few and $\sim$ 200 A·GeV. In addition to two baryon-rich fluids which simulate mutually decelerating counterflows of target and projectile nucleons, the new model incorporates evolution of a third, retarded baryon-free fluid created by this decelerated baryonic matter. Different equations of state, including those with the deconfinement phase transition, are treated. A reasonable agreement with experiment is illustrated by proton rapidity spectra, their dependence on collision centrality and beam energy.

1 Introduction

During past twenty years, hydrodynamics proved to be quite a reasonable tool for describing heavy-ion collisions at moderate energies (see for example references in [1]). From general point of view this application of hydrodynamics seemed to be more successful at higher energies when the number of produced particles gets larger. However, as it has been noted by D.I. Blokhintsev [2] in the beginning of the multiple-production era, the uncertainty principle introduces certain restrictions to this extension. Though a general conclusion of Ref. [2], that the very beginning of the expansion stage should be considered quantum–mechanically, is valid till now, the modern quark–gluon picture of interactions shifts the results of Blokhintsev's simplified estimate to far ultra-relativistic energies. On other hand, it became clear that non-equilibrium processes are very important at this stage and that only a part of the total collision energy can be thermalized. In this respect the standard one-fluid description assuming instantaneous local equilibrium seems to be quite limited.

In the present paper we demonstrate first results of a relativistic 3-fluid 3-dimensional hydrodynamic code developed for describing highly relativistic nucleus–nucleus collisions, i.e. up to energies reached at the CERN SPS where a partial deconfinement of hadrons may occur. Below, basic features of the hydrodynamic model will be presented and a numerical solution of 3-fluid hydrodynamics will first be compared with observables.

2 3-Fluid Hydrodynamic Model

A specific feature of the dynamic 3-fluid description is a finite stopping power resulting in a counter-streaming regime of leading baryon-rich matter. This counter-streaming behavior is supported by experimental rapidity distributions in nucleus–nucleus collisions and simulated by introducing the multi–fluid concept. The basic idea of a 3-fluid approximation to heavy-ion collisions [3, 4] is that at each space-time point $x = (t, \mathbf{x})$ the distribution function of baryon-rich matter, $f_{\rm b}(x,p)$, can be represented by a sum of two distinct contributions

$$f_{\rm b}(x,p) = f_{\rm p}(x,p) + f_{\rm t}(x,p), \tag{1}$$

initially associated with constituent nucleons of the projectile (p) and target (t) nuclei. In addition, newly produced particles, populating the mid-rapidity region, are associated with a fireball (f) fluid described by the distribution function $f_{\rm f}(x,p)$. Note that both the baryon-rich and fireball fluids may consist of any type of hadrons and/or partons (quarks and gluons), rather then only nucleons and pions. However, here and below we suppress the species label at the distribution functions for the sake of transparency of the equations.

Using the standard procedure for deriving hydrodynamic equations from the coupled set of relativistic Boltzmann equations with the above-introduced distribution functions f_α (α =p, t, f), we arrive at equations for the baryon charge conservation

$$\partial_\mu J^\mu_\alpha(x) \;=\; 0, \tag{2}$$

(for α =p and t) and the energy–momentum conservation of the fluids

$$\begin{aligned}
\partial_\mu T^{\mu\nu}_{\rm p}(x) &= -F^\nu_{\rm p}(x) + F^\nu_{\rm fp}(x), && (3)\\
\partial_\mu T^{\mu\nu}_{\rm t}(x) &= -F^\nu_{\rm t}(x) + F^\nu_{\rm ft}(x), && (4)\\
\partial_\mu T^{\mu\nu}_{\rm f}(x) &= F^\nu_{\rm p}(x) + F^\nu_{\rm t}(x) - F^\nu_{\rm fp}(x) - F^\nu_{\rm ft}(x). && (5)
\end{aligned}$$

Here $J^\mu_\alpha = n_\alpha u^\mu_\alpha$ is the baryon current defined in terms of proper baryon density n_α and hydrodynamic 4-velocity u^μ_α normalized as $u_{\alpha\mu} u^\mu_\alpha = 1$. Eq. (2) implies that there is no baryon-charge exchange between p- and t-fluids, as well as that the baryon current of the fireball fluid is identically zero, $J_{\mathrm{f}}^{\mu} = 0$. The energy–momentum tensors $T_{\alpha}^{\mu\nu}$ in Eqs. (3)–(4) are affected by friction forces F_{p}^{ν}, F_{t}^{ν}, F_{fp}^{ν} and F_{ft}^{ν}. Friction forces between baryon-rich fluids, F_{p}^{ν} and F_{t}^{ν}, partially transform the collective incident energy into thermal excitation of these fluids ($|F_{\mathrm{p}}^{\nu} - F_{\mathrm{t}}^{\nu}|$) and give rise to particle production into the fireball fluid ($F_{\mathrm{p}}^{\nu} + F_{\mathrm{t}}^{\nu}$), see Eq. (5). F_{fp}^{ν} and F_{ft}^{ν} are associated with friction of the fireball fluid with the p- and t-fluids, respectively. Note that Eqs. (3)–(5) satisfy the total energy–momentum conservation

$$\partial_{\mu}(T_{\mathrm{p}}^{\mu\nu} + T_{\mathrm{t}}^{\mu\nu} + T_{\mathrm{f}}^{\mu\nu}) = 0. \tag{6}$$

In terms of the proper energy density, ε_{α}, and pressure, P_{α}, the energy–momentum tensors of the baryon-rich fluids ($\alpha =$p and t) take the conventional hydrodynamic form

$$T_{\alpha}^{\mu\nu} = (\varepsilon_{\alpha} + P_{\alpha})\, u_{\alpha}^{\mu}\, u_{\alpha}^{\nu} - g^{\mu\nu} P_{\alpha}. \tag{7}$$

For the fireball, however, only the thermalized part of the energy–momentum tensor is described by this hydrodynamic form

$$T_{\mathrm{f}}^{(\mathrm{eq})\mu\nu} = (\varepsilon_{\mathrm{f}} + P_{\mathrm{f}})\, u_{\mathrm{f}}^{\mu}\, u_{\mathrm{f}}^{\nu} - g^{\mu\nu} P_{\mathrm{f}}. \tag{8}$$

Its evolution is defined by the Euler equation with a retarded source term

$$\begin{aligned} \partial_{\mu} T_{\mathrm{f}}^{(\mathrm{eq})\mu\nu}(x) &= \int d^4x' \delta^4\left(x - x' - U_F(x')\tau\right)\left[F_{\mathrm{p}}^{\nu}(x') + F_{\mathrm{t}}^{\nu}(x')\right] \\ &- F_{\mathrm{fp}}^{\nu}(x) - F_{\mathrm{ft}}^{\nu}(x), \end{aligned} \tag{9}$$

where τ is the formation time, and

$$U_F^{\nu}(x') = \frac{F_{\mathrm{p}}^{\nu}(x') + F_{\mathrm{t}}^{\nu}(x')}{|F_{\mathrm{p}}(x') + F_{\mathrm{t}}(x')|} \tag{10}$$

is a free-streaming 4-velocity of the produced fireball matter. In fact, this is the velocity at the moment of production of the fireball matter. According to Eq. (9), the energy and momentum of this matter appear as a source in the Euler equation only later, at the time $U_F^0\tau$ after production, and in different space point $\mathbf{x}' - \mathbf{U}_F(x')\tau$, as compared to the production point $\mathbf{x}'$.

The residual part of $T_{\mathrm{f}}^{\mu\nu}$ (the free-streaming one) is defined as

$$T_{\mathrm{f}}^{(\mathrm{fs})\mu\nu} = T_{\mathrm{f}}^{\mu\nu} - T_{\mathrm{f}}^{(\mathrm{eq})\mu\nu}. \tag{11}$$

The equation for $T_{\rm f}^{(\rm eq)\mu\nu}$ can be easily obtained by taking the difference between Eqs. (5) and (9). If all the fireball matter turns out to be formed before freeze-out (which is the case, in fact), then this equation is not needed. Thus, the 3-fluid model introduced here contains both the original 2-fluid model with pion radiation [3, 5, 6] and the (2+1)-fluid model [7, 8] as limiting cases for $\tau \to \infty$ and $\tau = 0$, respectively.

The nucleon–nucleon cross sections at high energies are strongly forward-backward peaked. Since the involved 4-momentum transfer is small, the Boltzmann collision term can be essentially simplified and in this case the friction forces, $F_{\rm p}^{\nu}$ and $F_{\rm t}^{\nu}$, are estimated as

$$F_{\alpha}^{\nu} = \rho_{\rm p}\rho_{\rm t}\left[\left(u_{\alpha}^{\nu} - u_{\bar{\alpha}}^{\nu}\right) D_P + \left(u_{\rm p}^{\nu} + u_{\rm t}^{\nu}\right) D_E\right], \tag{12}$$

$\alpha =$p and t, $\bar{\rm p} =$t and $\bar{\rm t} =$p. Here, ρ_{α} denotes the scalar densities of the p- and t-fluids,

$$D_{P/E} = m_N \; V_{\rm rel}^{\rm pt} \; \sigma_{P/E}(s_{\rm pt}), \tag{13}$$

where m_N is the nucleon mass, $s_{\rm pt} = m_N^2 \left(u_{\rm p}^{\nu} + u_{\rm t}^{\nu}\right)^2$ is the mean invariant energy squared of two colliding nucleons from the p- and t-fluids, $V_{\rm rel}^{\rm pt} = [s_{\rm pt}(s_{\rm pt} - 4m_N^2)]^{1/2}/2m_N^2$ is their mean relative velocity, and $\sigma_{P/E}(s_{\rm pt})$ are determined in terms of nucleon-nucleon cross sections integrated with certain weights (see [3, 5, 9] for details).

Eqs. (2)–(4) and (9), supplemented by a certain equation of state (EoS) and expressions for friction forces F^{ν}, form a full set of equations of the relativistic 3-fluid hydrodynamic model. To make this set closed, we still need to define the friction of the fireball fluid with the p- and t-fluids, $F_{\rm fp}^{\nu}$ and $F_{\rm ft}^{\nu}$, in terms of hydrodynamic quantities and some cross sections.

To estimate the scale of the friction force between the fireball and baryon-rich fluids, similar to that done above for baryon-rich fluids, we consider a simplified system, where all baryon-rich fluids consist only of nucleons, as the most abundant component of these fluids, and the fireball fluid contains only pions. At incident energies from 10 (AGS) to 200 A·GeV (SPS) the relative nucleon-pion energies prove to be in the resonance range dominated by the Δ-resonance. The resonance-dominated interaction implies that the essential process is absorption of a fireball pion by a p- or t-fluid nucleon with formation of an R-resonance (most probably Δ). As a consequence, only the loss term contributes to the kinetic equation for the fireball fluid. After momentum integrating this collision term weighted with the 4-momentum p^ν, we get [1]

$$F^\nu_{f\alpha}(x) = D_{f\alpha} \frac{T_f^{(eq)0\nu}}{u_f^0} \rho_\alpha , \quad (14)$$

where transport coefficients take the form

$$D_{f\alpha} = W^{N\pi\to R}(s_{f\alpha})/(m_N m_\pi) = V_{rel}^{f\alpha} \sigma_{tot}^{N\pi\to R}(s_{f\alpha}). \quad (15)$$

Here, $V_{rel}^{f\alpha} = [(s_{f\alpha} - m_N^2 - m_\pi^2)^2 - 4m_N^2 m_\pi^2]^{1/2}/(2m_N m_\pi)$ denotes the mean invariant relative velocity between the fireball and the α-fluids, $s_{f\alpha} = (m_\pi u_f + m_N u_\alpha)^2$, and $\sigma_{tot}^{N\pi\to R}(s)$ is the parameterization of experimental pion–nucleon cross-sections. Thus, we have expressed the friction $F^\nu_{f\alpha}$ in terms of the fireball-fluid energy-momentum density $T_f^{0\nu}$, the scalar density ρ_α of the α fluid, and a transport coefficient $D_{f\alpha}$. Note that this friction is zero until the fireball pions are formed, since $T_f^{(eq)0\nu} = 0$ during the formation time τ.

In fact, the above treatment is an estimate of the friction terms rather than their strict derivation. As it is seen from Eq. (13) for the excited matter of baryon-rich fluids, a great number of hadrons and/or deconfined quarks and gluons may contribute into this friction. Furthermore, these quantities may be modified by in-medium effects. In this respect, $D_{P/E}$ and $D_{f\alpha}$ should be understood as quantities that give a scale of this interaction.

3 Simulations of Nucleus–Nucleus Collisions

The relativistic 3D code for the above described 3-fluid model was constructed by means of modifying the existing 2-fluid 3D code of Refs. [3, 5, 6]. In actual calculations we used the mixed-phase EoS developed in [10, 11]. This phenomenological EoS takes into account a possible deconfinement phase transition of nuclear matter. The underlying assumption of this EoS is that unbound quarks and gluons may coexist with hadrons in the nuclear environment. In accordance with lattice QCD data, the statistical mixed-phase model describes the first-order deconfinement phase transition for pure gluon matter and crossover for that with quarks [10, 11]. A detail of all the used EoS's can be found in [12].

In Fig. 1 global dynamics of heavy-ion collisions is illustrated by the energy-density evolution of the baryon-rich fluids ($\varepsilon_b = \varepsilon_p + \varepsilon_t$) in the reaction plane of the Pb+Pb collision at $E_{lab} = 158$ A·GeV. Different stages of interaction at relativistic energies are clearly seen in this example: Two Lorentz-contracted nuclei (note the different scales along the x- and z-axes in Fig.1) start to interpenetrate through each other, reach a maximal energy density by the time ~ 1.1 fm/c and then expand predominantly in longitudinal direction forming a "sausage-like" freeze-out system. At this and lower incident energies the baryon-rich dynamics is not really disturbed by the fireball fluid and hence the cases $\tau = 0$ and 1 fm/c turned to be indistinguishable in terms of ε_b.

Time–evolution of ε_b in Fig.1 is calculated for the mixed phase model. Topologically results for EoS of pure hadronic and that of two-phase models look very similar. Due to essential softening the equation of state near the deconfinement phase transition, in the last case the system evolves noticeably slower what may have observable consequences [10, 11].

The energy released in the fireball fluids is of an order of magnitude smaller than that stored in baryon–rich fluids and depends on the formation time. At realistic values of the formation times, $\tau \sim 1$ fm/c, the effect of the interaction is substantially reduced. It happens because the fireball fluid starts to interact only near the end of the interpenetration stage. As a result, by the end of the collision process it looses only 10% of its available energy at $E_{\rm lab} = 158$ A·GeV and 30%, at $E_{\rm lab} = 10.5$ A·GeV. Certainly, this effect should be observable in mesonic quantities, in particular, in such fine observables as directed and elliptic flows. The global baryonic quantities stay practically unchanged at finite τ [1].

To calculate observables, hydrodynamic calculations should be stopped at some freeze-out point. In our model it is assumed that a fluid element decouples from the hydrodynamic regime, when its energy density ε and densities in the eight surrounding cells become smaller than a fixed value ε_{fr}. A value $\varepsilon_{fr} = 0.15\ GeV/fm^3$ was used for this local freeze-out density which corresponds to the actual energy density of the freeze-out fluid element of $\sim 0.12\ GeV/fm^3$. To proceed to observable free hadron gas, the shock-like freeze-out [14] is assumed, conserving energy and baryon charge.

Proton rapidity spectra calculated for $Au + Au$ collisions are presented in Fig.2 for E_{beam} =6, 8 and 10.5 $A \cdot GeV$. One should note that hydrodynamics does not make difference between bounded into fragments and free nucleons. In the results presented the contribution from light fragments (d, t, ^{3}He and ^{4}He) has been subtracted using a simple coalescence model [15]. This procedure allows one to reproduce reasonably evolution of the spectra shape with changing the impact parameter b for all the energies considered. Some discrepancy observed for peripheral collisions near the target and projectile rapidity are mainly due to a not-subtacted contribution of heavier fragments. As seen, the rapidity spectra are only slightly sensitive to the EoS used. The same con-

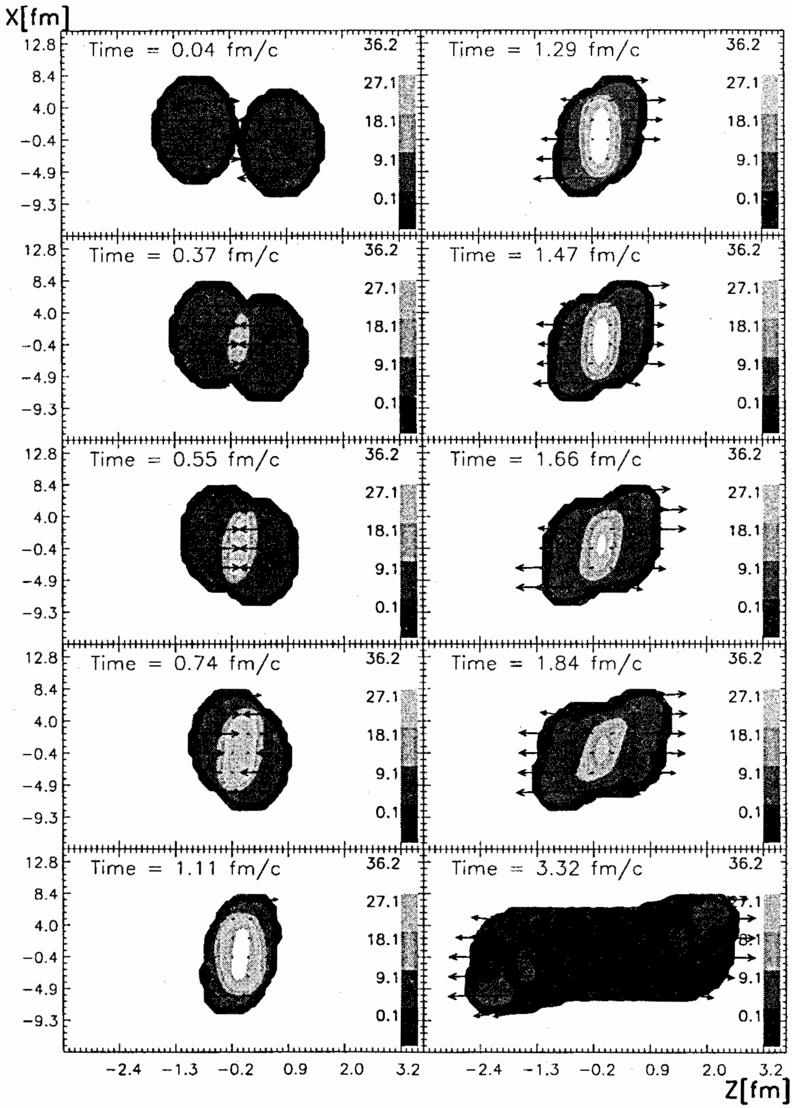

Figure 1: Time evolution of the energy density, $\varepsilon_b = \varepsilon_p + \varepsilon_t$, for the baryon-rich fluids in the reaction plane (xz plane) for the Pb+Pb collision (E_{lab} = 158 A·GeV) at impact parameter $b = 2$ fm. Shades of gray represent different levels of ε_b as indicated at the right side of each panel. Numbers at this palette show the ε_b values (in GeV/fm^3) at which the shades change. Arrows indicate the hydrodynamic velocities of the fluids.

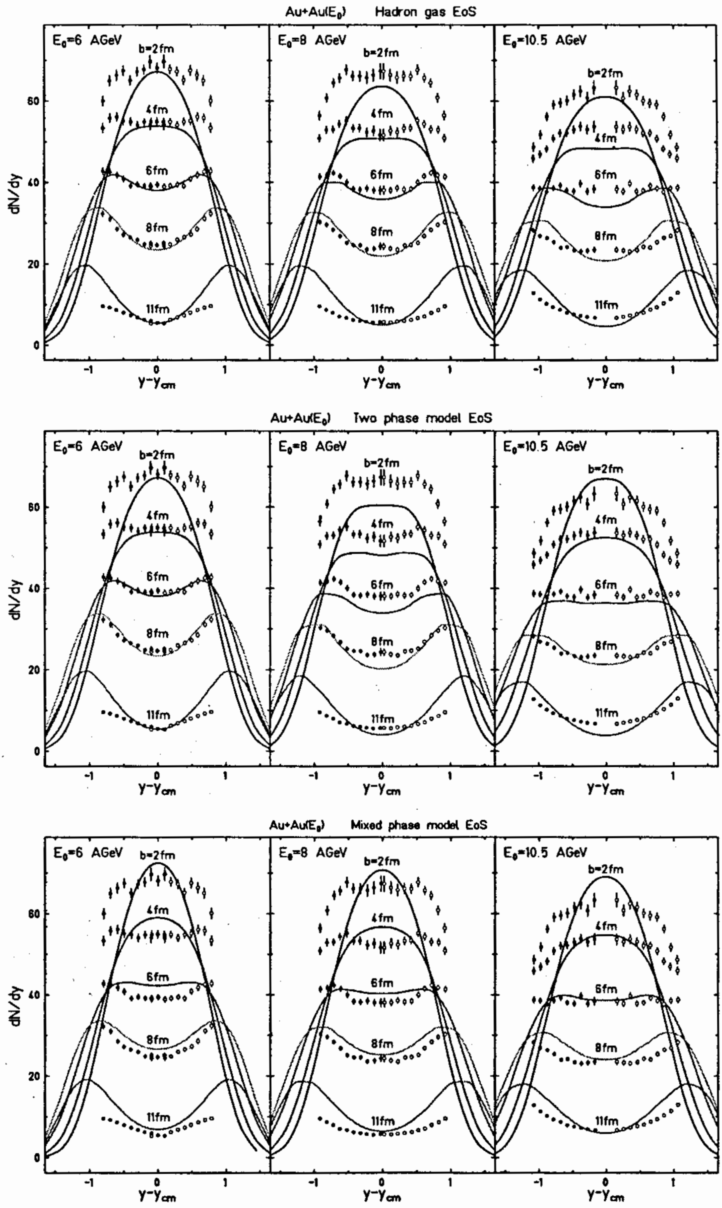

Figure 2: Proton rapidity spectra from $Au + Au$ collisions at three bombarding energies and different impact parameters b(given in fm) Three panels correspond to different equation of state. Experimental points are from [13]

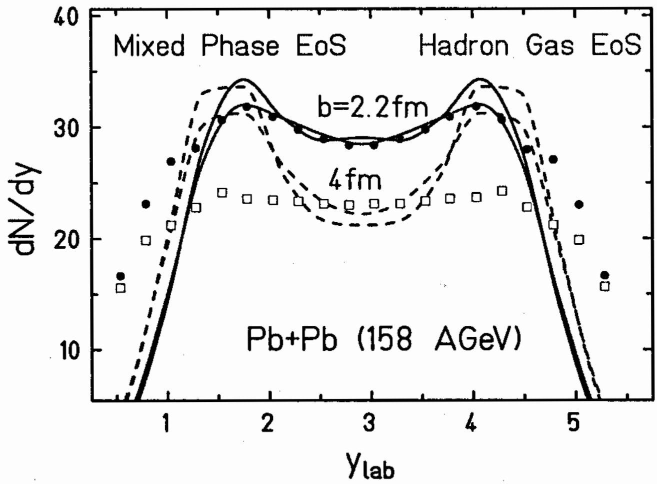

Figure 3: Rapidity spectra of protons from $Pb + Pb$ collisions for the mixed phase and hadron gas EoS. Solid and dashed lines are calculated for impact parameters 2 and 4 fm, respectively. Experimental points are from [16].

clusions can be drawn from Fig.3 for higher CERN SPS energies. One should notice that a general agreement here for central collisions is much better than that reached in any other 2-fluid hydrodynamic model. This originates mainly from larger energy-momentum transfer in (12) rather than from the account for the third fireball fluid.

4 Conclusions

In this paper we have developed a 3-fluid model for simulating heavy-ion collisions in the range of incident energies between few and about 200 A·GeV. In addition to two baryon-rich fluids, which constitute the 2-fluid model, a delayed evolution of the produced baryon-free (fireball) fluid is incorporated. This delay is governed by a formation time, during which the fireball fluid neither thermalizes nor interacts with the baryon-rich fluids. After the formation, it thermalizes and comes into interaction with the baryon-rich fluids. This interaction is estimated from elementary pion-nucleon cross-sections. Implementation of different EoS, including those with the deconfinement phase transition, may open great opportunities for analysis of collective effects in relativistic heavy-ion collisions. Unfortunately, in spite of reasonable reproduction of observable proton rapidity spectra in the wide range of bombarding energies and centrality parameters we are unable to favor any of considered EoS. Analysis of more delicate characteristics is needed. This work is in progress now.

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Gravitation

Astrophysics as conformal theory

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Abstract — The astrophysical data are considered as an evidence of conformal—invariant unified theory, where the dilaton module w plays the role of the Higgs mass in the Higgs potential of Standard Model of elementary particles. The identification of conformal variables and coordinates (including conformal time, conformal temperature, running masses) with observational quantities in the field space of events helps us to solve problems of homogeneity, horizon, cosmic initial data, singularity, particles, and their cosmological creation from a physical vacuum and to describe the latest astrophysical data on the Supernova luminosity—distance — redshift relations $H_0 l(z) = z + z^2/2$, primordial nucleosynthesis $m(\eta) = m_0(1+2H_0(\eta-\eta_0))^{1/2}$ value of Cosmic Microwave Background-temperature $(M_W^2 H_0)^{1/3}$ 2.7 K, baryon asymmetry $n_b/n_\gamma \sim X_{CP}\ 10^{-9}$ and energy budget of the universe, including the visual baryon matter $\Omega_b \sim \alpha/\sin^2\theta_W$. The cosmic evolution of the dilaton masses forms galaxies and eliminates the need to assume the hidden masses in galaxies.

1 Introduction

New data [1, 2] for distent Supernovae at large redshifts $z \sim 1$, $z = 1.7$ testify that our universe is mainly filled with mysterious substance called Quintessence [3, 4]. The primordial nucleosynthesis and the chemical evolution of the matter in the universe [5] point out to a definite equation of state of the matter in the universe. This equation helps us to determine a kind of matter taking part in the cosmic evolution of the redshift.

The visible baryon matter gives only 0.03 part of the critical density $\rho_{\rm cr}$ of the observational cosmic evolution [6]. The data on the Cosmic Microwave Background radiation with the temperature 2.7 K and its fluctuations [7] give us information about the evolution of the early universe.

Beginning with the pioneer papers by Friedmann [8] and ending with the last papers on inflationary model [9] of the Hot Universe Scenario [10], all observational data are interpreted in the theoretical cosmology as some evidence of the expanding universe.

In the papers [11–16] these new data are considered as evidence of conformal symmetry of the laws of nature that do not depend not only on initial data but also on the units of their measurement. In conformal—invariant theory all measurable quantities are identified with the conformal variables and coordinates (including conformal time η, temperature T_c, running masses $m(\eta)$, coordinate distance), so that the z-history of temperature becomes the z–history of masses.

The purpose of the present paper is the description of the results and consequences of the relative measurement standards which expand together with the universe.

2 Astrophysical data in the relative units

The interval [8]

$$ds^2_{\text{THEORY}} = dt^2 - a^2(t)\left[(dx^1)^2 + (dx^2)^2 + (dx^3)^2\right], \tag{1}$$

plays important role in theoretical cosmology. Here $a(t)$ is a scale factor while x^i are space and time coordinates of 4-D manifold. It uses the measurement standard that is not expanding together with the universe there are comoving coordinates. Observational conformal cosmology uses with the conformal interval of the space—time

$$ds^2_{\text{OBSERVATION}} = ds^2_{\text{THEORY}}/a^2(t) = (d\eta)^2 - (dx^i)^2 \tag{2}$$

of the cosmic photons flying on the light cone to an observer. This interval is given in terms of the conformal time $d\eta = dt/a(t)$ and coordinate distance.

In terms of the relative standards and the conformal interval (2) we reveal that the measurable spatial volume of the universe is a constant V_c, while all masses including the Planck mass are scaled by the cosmic scale factor.

It was shown [11–13] that the relative units give a completely different physical picture of the evolution of the universe than the absolute units of the standard cosmology. The spectrum of photons emitted by atoms from distant stars billion years ago remains unchanged during the propagation and is determined by the mass of the constituents at the moment of emission. When this spectrum is compared with the spectrum of similar atoms on the Earth which, at the present time, have larger masses, then a redshift is obtained. The temperature history of the expanding universe in the relative units looks like the history of evolution of masses of elementary particles in the cold universe with a constant temperature of the cosmic microwave background.

As it was shown in [12], in the case of the *relative* units, both the epoch of chemical evolution and the recent experimental data for distant supernovae [1,2] are described by the square root dependence of the cosmic factor:

$$a(\eta) = (1 + 2H_0(\eta - \eta_0))^{1/2}. \tag{3}$$

Other consequence of the relative standard of measurement is the redshift independence of the cosmic microwave background temperature [13,14]. This is at the first glance in a striking contradiction with the observation [17] of $6.0\ \text{K} < T_{\text{CMBR}}(z = 23371) < 14\ \text{K}$. However, the relative population of different energy levels EI from which the temperature has been inferred in this experiment follows basically the Boltzmann statistics with the same z–dependence of the Boltzmann factors for both the absolute standards and relative one [12]. Therefore, the experimental finding can equally well be interpreted as a measurement of the z–dependence of energy levels (masses) at constant temperature. The abundances of nuclear species is also mainly governed by the Boltzmann factors with the z–dependence which is invariant with respect to the theoretical interpretation.

Thus, one more argument in favor of the relative units is the sharp simplification of the scenario of the evolution of the universe. Astrophysical data in the relative units can be described by a single epoch with the dominance of the Scalar Quintessence, while the same data in the absolute units require the scenario with three different epoches (inflation, radiation, and inflation with the dark matter). And the astrophysical data in the relative units testify to the hidden conformal symmetry of the Einstein general relativity and Standard Model [18,19].

We have shown [11–16] that the conformal—invariant version of general relativity and Standard Model with geometrization of constraint and frame—fixing with the primordial initial data $\varphi_I = 10^4\text{GeV}$, $H_I = 2.7\ \text{K} = 10^{29} H_0$ (determined by a free homogeneous motion of the Scalar Quintessence, i.e., its electric tension) can describe the following events: creation of the "empty" universe ($\eta = 0$), creation of vector bosons with a temperature $T = (M^2 H_0)^{1/3} \sim 2.7\ \text{K}$ ($0 < \eta \sim 10^{-12} s$), formation of baryon asymmetry with $n_b/n_\gamma \sim X_{\text{CP}} \sim 10^{-9}$ ($10^{-12} s < \eta < 10^{-11} \div 10^{-10} s$), decays of vector bosons into CMB radiation ($\eta \sim 10^{-10} s$), primordial chemical evolution of matter ($10^{-10} s < \eta < 10^{11} s$), recombination or separation of CMB ($\eta \sim 10^{11} s$), formation of galaxies by cosmic evolution of the dilaton ($\eta \sim 10^{15} s$), hep experiments and Supernova evolution, $H_0 l(z) = z + z^2/2$ ($10^{17} s < \eta$), experiments and Supernova evolution ($10^{17} s < \eta$), see [20].

3 Cosmic evolution as a superfluid motion of the dilaton masses

The fixation of the dilaton $w(x^0, x^i) = M_{\text{Planck}}(3/8\pi)^{1/2}$ leads to the conventional GR and SM with the running volume $V = V_f(t)$ and to all problems that are solved by the inflationary scenario or the reference on the future M-theory [21]. The fixation of the volume $V = V_c$ with the running masses $w(x^0, x^i) = \varphi(x^0)$ corresponds to the choice of the relative units and conformal variables of the observational cosmology and quantum field theory. The identification of these conformal variables with observables is just the new point proposing in the present paper. It was shown [11, 12, 18, 19] that the identification of the dilaton with the evolution parameter in the field world space $[\varphi|F]$ solves key problems of cosmology (energy, time, homogeneity, horizon, cosmic initial data, singularity, particles, and their cosmological creation from vacuum) by analogy with the solving the similar problems for a relativistic particle (see Fig. 2 from [20]), without any additional hypothesizes of the type of inflation [13].

Averaging exact equations over the spatial volume gives the conformal version of the well—known cosmological equations [19], so that the homogeneity is the consequence of the averaging. Flatness is a consequence of a choice of the initial data. Horizon is a consequence of a superfluid motion of the dilaton masses $\varphi(\eta)$. The corresponding variable plays the role of the field time in the field space of events $[\varphi|F]$, and its canonical momentum is the energy of the universe. The causal quantization of the superfluid dilaton motion describes the creation of the Universe with a positive energy in the field space of events, where the universe moves along its world hypersurface. The conformal time is the measure of the geometric interval on the hypersurface. Therefore the time measured by an observer was created with the universe.

4 Conclusion

The relative measurement standard opens out the well—known truth that the universe is an ordinary physical object with a finite volume and finite lifetime.

Results of theoretical description of the finite universe depend on the choice of a frame of reference and initial data like the results of solution of the Newton equations depend on initial positions and initial velocities of a particle. Creation of the universe has taken place in a particular frame of reference which was remembered by the cosmic microwave radiation. We remind that the "frame of reference" is identified with a set of the physical instruments for measuring the initial data needed for unambiguous solving differential equations of theoretical physics. These differential equations are invariant structural relations of the whole manifold of all measurable quantities with respect to their transformations.

The relative units reveal that a symmetry group of the whole manifold of measurable physical quantities in the world includes conformal symmetry of the Faraday—Maxwell electrodynamics, and the field nature of matter should also be supplemented by the field nature of space and time.

The relative units lead us to the "kingdom of freedom" of initial data. This "kingdom" includes also last dimensional absolute of modern quantum field theory and those initial data of creation of the universe, for which an observer does not carry any responsibility, as he at this moment existed only as an intention. Who has carried out this experiment of creation of the universe? Who has determined the initial data of this creation? Whose notebook is the wave function of the universe?

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Account of Light Velocity Constancy in the Galilean Problem on the Free Fall of a Particle onto the Ground

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Abstract — The well known fact of the light velocity constancy is taken into account in the Galilean problem about the movement of a particle free from non-gravitational forces and its fall onto the ground.

The question about taking into account the light velocity constancy in the Galilean problem has been raised by Einstein in [1]. Here we shall provide an answer to this question. In the Galilean problem the earth surface look like a fixed euclidean plane, the tension of the gravitation field above the earth surface is characterized by a positive constant, called the free fall acceleration, usually denoted by g. Concretely this is the same problem, considered in the school program. As in each problem, the consideration of the light velocity constancy in the Galilean problem is reduced to the replacement of the Euclidean geometry in the velocity space with the Lobachevsky's geometry, since the light vlocity c, entering the Lorentz trasformations, turns out to be the Lobachevsky's parameter in the velocity space. The notion of velocity space for the case of Euclidean geometry in this space was introduced by Hamilton in [2]. The notion of velocity space for the case of Lobachevsky's geometry in this space was intpoduced by Kotelnikov in [3]. The transition from the Galilean to the Loretnz transformations leads to the transition from Euclidean to Lobachevsky's geometry in the velocity space (on this subjet see [4]). According to [5], in the 4-dimensional spacetime M of events $m \in M$ there exist two affine connections: the field connection Γ^a_{mn} and the background connection $\check{\Gamma}^a_{mn}$. Their difference

$$P^a_{mn} = \check{\Gamma}^a_{mn} - \Gamma^a_{mn} \tag{1}$$

is called the tensor of gravitational field. Only in the case of the gravitational vacuum the tensor (1) equals to zero at each point $m \in M$.

Further we develop our results, published in [6 – 8]. In the considered problem the space of events in regard to the background connection is the

4-dimentional affine space with map (x^1, x^2, x^3, x^4), in which all $\check{\Gamma}^a_{mn}$ equal to zero. The notion of a fixed plane (let's denote it by E_2) demands the [ntroduction in the space of events of the structure of a direct multiplication $M = P \times T$ of the 3-dimensional affine space P with the affine time line T. The coordinates $x^1 = x, x^2 = y, x^3 = z$ are the affine coordinates in P, the coordinate $x^4 = t$ is the affine coordinate in T. The history of the fixed plane E_2 is represented by the hyperplane $z = 0$ in M with the following metric

$$c^2 d\check{\tau}^2 = c^2 dt^2 - dx^2 - dy^2 \,. \tag{2}$$

The gravitational field is considered in the domain $z > 0$, where the field metric equals

$$c^2 d\tau^2 = c^2\Big(1 + \frac{gz}{c^2}\Big)^2 dt^2 - dx^2 - dy^2 - dz^2 \,. \tag{3}$$

The field connection Γ^a_{mn} is the Christoffell connection for this metric. The particle's equations of motion are the geodesic equations for this connection. In the considered case they can be transformed to the following form

$$\frac{d}{d\tau}\frac{dx}{d\tau} = 0\,, \quad \frac{d}{d\tau}\frac{dy}{d\tau} = 0\,, \tag{4}$$

$$\frac{d}{d\tau}\frac{dz}{d\tau} + g\Big(1 + \frac{gz}{c^2}\Big)\Big(\frac{dt}{d\tau}\Big)\Big(\frac{dt}{d\tau}\Big) = 0\,, \tag{5}$$

$$\frac{d}{d\tau}\Big[\Big(1 + \frac{gz}{c^2}\Big)\frac{dt}{d\tau}\Big] + \frac{g}{c^2}\frac{dz}{d\tau}\frac{dt}{d\tau} = 0\,. \tag{6}$$

On the particle's world line the equation

$$c^2\Big(1 + \frac{gz}{c^2}\Big)^2\Big(\frac{dt}{d\tau}\Big)^2 - \Big(\frac{dx}{d\tau}\Big)^2 - \Big(\frac{dy}{d\tau}\Big)^2 - \Big(\frac{dz}{d\tau}\Big)^2 = c^2 \tag{7}$$

is fulfilled. From there follows that

$$\frac{dt}{d\tau} = \frac{1}{\sqrt{V^2 - v^2/c^2}}\,, \tag{8}$$

where

$$V = 1 + \frac{gz}{c^2}\,, \quad v^2 = \Big(\frac{dx}{dt}\Big)^2 + \Big(\frac{dy}{dt}\Big)^2 + \Big(\frac{dz}{dt}\Big)^2 . \tag{9}$$

The homogeneous Lagrangian equals

$$L = \left[\dot{t} - \sqrt{(V\dot{t})^2 - \frac{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}{c^2}}\right] c^2 \,, \text{ where } \dot{x} = \frac{dx}{d\tau} \,. \tag{10}$$

As a consequation of (6) the energy E (energy of particle, divided by its rest mass)

$$E = \Big(V^2\frac{dt}{d\tau} - 1\Big)c^2 = \Big(\frac{V^2}{\sqrt{V^2 - v^2/c^2}} - 1\Big)c^2 = -\frac{\partial L}{\partial \dot{t}} \,. \tag{11}$$

is conserved.

The second order system of equations (4)—(6) splits up into the following couple of first order equations

$$\frac{du^1}{dt} = 0, \ \frac{du^2}{dt} = 0, \ \frac{du^3}{dt} = -gu^4, \ \frac{du^4}{dt} = -\frac{g}{c^2}u^3, \tag{12}$$

$$u^1 = \Big(\frac{dx}{d\tau}\Big), \ u^2 = \Big(\frac{dy}{d\tau}\Big), \ u^3 = \Big(\frac{dz}{d\tau}\Big), \ u^4 = \Big(1 + \frac{gz}{c^2}\Big)\Big(\frac{dt}{d\tau}\Big). \tag{13}$$

Integrating (12) we find

$$u^1 = U^1 \,, \quad u^2 = U^2 \,, \tag{14}$$

$$u^3 = U^3 \cosh\frac{s}{c} - cU^4 \sinh\frac{s}{c} \,,$$

$$cu^4 = cU^4 \cosh\frac{s}{c} - U^3 \sinh\frac{s}{c} \,,$$

where U^a is the value of u^a at $t = 0$, and $s = gt$ is rapidity of the particle. It is important that the rapidity is proportional to the time t with a coefficient of proportionality equal to g.

From here it is not difficult to find the world line of the particle. Indeed, from (13) and (14) it follows that

$$U^1 = \frac{dx}{d\tau} \,, \quad U^2 = \frac{dy}{d\tau} \,, \tag{15}$$

$$U^3 = u^3 \cosh\frac{s}{c} + cu^4 \sinh\frac{s}{c} = \frac{d}{d\tau}\Big[\Big(\frac{c^2}{g} + z\Big)\cosh\frac{s}{c}\Big] \,,$$

$$cU^4 = u^3 \sinh\frac{s}{c} + cu^4 \cosh\frac{s}{c} = \frac{d}{d\tau}\Big[\Big(\frac{c^2}{g} + z\Big) \sinh\frac{s}{c}\Big] .$$

Assuming that at $\tau = 0$ coordinate t of the particle takes zero value, and also the coordinates x, y, z take values X, Y, Z, from (15) we find

$$x = X + U^1\tau , \quad y = Y + U^2\tau , \tag{16}$$

$$\Big(\frac{c^2}{g} + z\Big) \cosh\frac{s}{c} = \frac{c^2}{g} + Z + U^3\tau , \quad \Big(\frac{c^2}{g} + z\Big) \sinh\frac{s}{c} = cU^4\tau . \tag{17}$$

These formulaes give the world line of the particle.

From (17) it follows that

$$\tanh\frac{gt}{c} = \frac{cU^4 g\tau}{c^2 + gZ + U^3 g\tau} , \tag{18}$$

$$c^2 + gz = \sqrt{(c^2 + gZ + g\tau U^3)^2 - (g\tau cU^4)^2} . \tag{19}$$

Taking into consideration that according to (7), (13) and (14)

$$cU^4 = \sqrt{c^2 + U^1U^1 + U^2U^2 + U^3U^3} , \tag{20}$$

from (19) we find the dependence of the particle's coordinate z on its time τ in the following form

$$c^2 + gz = \tag{21}$$

$$= \sqrt{(c^2 + gZ)^2 + 2U^3(c^2 + gZ)g\tau - (c^2 + U^1U^1 + U^2U^2)(g\tau)^2} .$$

The function (21) reaches its maximal value H at $\tau = \tau_0$, where

$$g\tau_0 = \frac{(c^2 + gZ)U^3}{c^2 + U^1U^1 + U^2U^2} , \tag{22}$$

$$c^2 + gH = \frac{(c^2 + gZ)cU^4}{\sqrt{^2 + U^1U^1 + U^2U^2}} . \tag{23}$$

At $\tau = \tau_0$ the function (18) takes the following value

$$\tanh\frac{g\tilde{t}}{c} = \frac{U^3}{cU^4} . \tag{24}$$

According to (14), in this case $u^3 = 0$.

In accordance to (23), for the energy E we have

$$c^2 + E = (c^2 + gz)u^4 = (c^2 + gZ)U^4 \,, \tag{25}$$

$$c^2 + E = (c^2 + gH)\sqrt{1 + \frac{U^1U^1 + U^2U^2}{c^2}} \,. \tag{26}$$

From the condition $z \geq 0$ it can be found that the time τ of the particle's flight above the ground is restricted by the roots of the following quadratic equation

$$g(c^2 + U^1U^1 + U^2U^2)\tau^2 = 2U^3(c^2 + gZ)\tau + 2c^2Z + gZ^2 \,. \tag{27}$$

This equation is obtained by setting up $z = 0$ in (21). With the help (20), (22) and (23) it can be derived

$$g\tau_0 - \sqrt{\Delta} \leq g\tau \leq g\tau_0 + \sqrt{\Delta} \,, \tag{28}$$

where

$$\Delta = \frac{(c^2 + gH)^2 - c^4}{c^2 + U^1U^1 + U^2U^2} \,. \tag{29}$$

Let us now clarify the geometrical meaning of the dependence of the 4-velocity of the partice on the time t. Poincare found in [9] that on the hyperboloid the Lobachevsky's geometry is realized. In our case the role of the Poincare's hyperboloid is played by the hypersurface (20) in the Minkovski world. Its interior geometry is given by the following metric

$$dS^2 = dU^1dU^1 + dU^2dU^2 + dU^3dU^3 - c^2dU^4dU^4 \tag{30}$$

the Lobachevsky space, where

$$c^2dU^4dU^4 = \frac{(U^1dU^1 + U^2dU^2 + U^3dU^3)^2}{c^2 + U^1U^1 + U^2U^2 + U^3U^3} \,.$$

The equation $U^1 = 0$ determines on the upper half of the hyperboloid (20) the Lobachevsky plane, and the couple of equations $U^1 = 0, U^2 = 0$ determines the Lobachevsky straight line. According to (14), the particle's velocity moves steadily on a equidistant, remaining from this straight line at a distance σ, determined by the expression

$$\cosh\frac{\sigma}{c} = \sqrt{1 + \frac{U^1U^1 + U^2U^2}{c^2}} = \frac{c^2 + E}{c^2 + gH} \,. \tag{31}$$

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New Interpretation of the Hubble Law

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There are two assertions in the modern cosmological science which are beyond question.

First. The Universe expands, the expansion accelerated, the expansion obeys the Hubble law

$$cz = Hr$$

Here c is the light velocity, $z = \frac{\Delta\lambda}{\lambda}$ is the quantity of the redshift of observed spectral lines, H is the Hubble constant equal to $55-65\frac{km}{Mpc.sec}$, r-is the distance from the Earth to an observed object of the Universe. The redshift is connected with the Doppler effect. Though it is well known that the Doppler effect functionally depends on the velocity of objects $z = \frac{\Delta\lambda}{\lambda} \sim \frac{v}{c}$ (this relation valid for nonrelativistic velocities v), where v is the radial velocity of the observed object, and for the Hubble law $\frac{\Delta\lambda}{\lambda} \sim r$,(but not v), nevertheless it is stated that if we use the right value of the coefficient of the proportionality H, the Hubble formulae "may be used for definition of the distance r if we measure the redshift of galaxy or if we calculate (from this redshift) the velocity of the galaxy remove $v = cz$" [1]. The same statement was made in review paper by A.D. Chernin:"At each moment of the world history the recession velocity of an object, the distance of which from us is r, is proportional to r: $v = Hr$, where H is the constant coefficient. This dependence is only a consequence of the similarity and the isotropy of the Universe..." [2]. An analogous statement was made by I.D. Karachentsev and D.I. Makarov : "...Distribution of these galaxies relative to distances and radial velocities is given in Fig.1... The line of regression $v = HD$, given in Fig.1, through the origin of the coordinate system corresponds to the local value of the Hubble "constant" $H = (64 \pm 2)\frac{km}{Mpc.s}$..." [3]. Karachentsev and Makarov found out nonisotropic field of velocities of the local Group of galaxies. "...peculiar velocity is found to be described with a tensor of local Hubble parameter having the principal axis 81:62:48 $\frac{km}{Mpc.s}$ with a standard error $\sim 4\frac{km}{Mpc.s}$ each" [4].

Apparently, due to the anisotropy of the Hubble stream, in the phrase "Hubble constant" the word "constant" was taken by Karachentsev and Makaro in the quotation mark.

The presence of anisotropy of the local Hubble stream requires a correction of the definition of similarity and isotropy of the Universe and a correction of using the Hubble law to definition of distances to distant galaxies.

The main conclusion from the above mentioned passages consists in that exchanging the dependence of redshift on the velocity of an observed object by the dependence on the distance of observed objects the authors not only distort the physical contents of the Doppler effect, but also make from this distortion unfaithful conclusion about the expanding of the Universe and this conclusion leads to a pill of hypotheses about possible properties of cosmological vacuum.

The remark about of the pile of hypotheses do not refer to the papers of Chernin, and Karachentsev and Makarov - it mostly refers to the established views on modern science which have roots stretching back into the 20th century when the special theory of relativity, quantum mechanics and relativistic field theory arose.

The second statement accepted in the modern cosmology can be formulated as follows: vacuum is not an emptiness. The hypothesis of the Big Bang in the emptiness would lead to the delay of the expansion of the Universe, but the delay is not observed as it follows from the unfaithful interpretation of the Hubble law. And then it was supposed that vacuum is not emptiness, and a number of unusual properties was assigned to it to solve the problem of expanding the Universe. One appeals to using the idea of virtual processes which it was used in the Lamb shift calculations of the levels of hydrogen atom or the anomalous magnetic moment of the electron. Of course, the agreement of the theory with the experiments in these calculations is fantastic, but it was obtained because some mathematical artifices like the renormalization of mass, charge and others were used. Very prominent scientists did not trust the renormalization procedure and, for instance, Dirac wrote:" J am inclined to suspect that renormalization theory is something that will not survive in the future, and that the remarkable agreement between its and the experiments should be looked on as a fluke" [5].

In the modern field theory the vacuum is defined by the expression $a^-|0\rangle = 0$, where a^--an operator of annihilation of quantum of any field. The operator a^-, acting on the vacuum state $|0\rangle$ gives zero and, therefore, there are no any particles in the vacuum, the vacuum is emptiness. The quantum numbers of the vacuum such as momentum, energy, electrical charge and others - all equal zero [6].

If somebody obtains nonzero energy, momentum, charge and mass from "nothing" using virtual physical processes and mathematical artifices, then this somebody deceives us.

The fundamental scientific philosophical statement that it is impossible to introduce the unity of the metric system in the emptiness is forgotten now. Here it is appropriate to remember the quotation of Feynman:"... When you follow of our physics too far, you find that it always gets in some kind of trouble..." [7] and farther:"...There are so many things about elementary particles we still don't understand..."[8].

Today the so-called international community of science about all the above mentioned "some kind of trouble" "turn a blind eye", because we have not another theory.

In this work I advocate that the emptiness does not exist but there is real physical medium which uniformly fills all the world space - the Ψ-ether [9]. Thus postulate homogeneity and isotropy of the space. The Ψ-ether being a homogeneous medium ensures the unity of measure and number in the Universe, the Maxwell spoke about at the end of the 19th century.

The Ψ-ether is defined as the Bose-Einstein condensate of neutrino- antineutrino pairs of Cooper type and it is a carrier of the electromagnetic field. This definition of the Ψ-ether can easily be seen from a simple chain of the well-known formulae.

The vector $\vec{A}$ and scalar φ potentials are related with the strengths of the electric $\vec{E}$ and magnetic $\vec{H}$ fields by [10]

$$\vec{H} = rot\vec{A}$$

$$\vec{E} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} - grad\varphi$$

No attention is paid in textbooks or monographs to the fact that thus introduced auxiliary electromagnetic potentials $\vec{A}$ and φ are equated to the physical observables $\vec{H}$ and $\vec{E}$. This is impermissible in physics. If we make use of the Lorentz condition

$$div\vec{A} + \frac{1}{c}\frac{\partial \varphi}{\partial t} = 0$$

and combined the vector $\vec{A}$ and scalar φ potentials into one four-dimensional vector Ψ_i,the Maxwell equation can be written in the form

$$\sqcup \Psi_i(x,t) = 0$$

I define these equations as equations for the real Ψ-ether.

The Klein-Gordon operator $\sqcup$ can be represented as a product of two commuting matrix operators of the first order, each acting on spinor function of neutrino. Thus, we establish the connection of the electromagnetic field with the spinor particles-neutrinos. Neutrinos and antineutrinos can form a combination with the spin 1 (electromagnetic field) and spin 0 (it is natural to suppose that there may exist combinations with higher values of the spin).

We conclude that the electromagnetic potentials $\vec{A}$ and φ are not auxiliary functions, as it is written in all textbook for University students, but they one connected by some definite relations with physical reality - the Ψ-ether.

It can be assumed that the scalar component of the Ψ-ether takes the properties of a carrier of gravitational forces.

So, I state that the light is not a substance but a process going on in a substance, as was affirmed by Maxwell, Lorenz, Poincare at the end of the XIX - beginning XX centuries.

The whole visible known Universe is immersed in the all-penetrating Ψ-ether, and it lives and develops according to its laws. The Ψ-ether is an abyss in which the known physical world negligible as compared to the ether is immersed. The world of the Ψ-ether has no neither top, nor bottom, nor left, nor right.

The relict radiation can now be interpreted as age-long luminescence of the weakly excited world Ψ-ether, or in other words, as eternal oscillations of $\nu\bar{\nu}$-pairs forming the either. Naturally, the relict radiation has to by isotropic, it goes to the Earth with equal intensity and its spectrum influence the radiation of the spectrum of the black body. In the world Ψ-ether gigantic magnetic fields and axes can naturally arise, and the light which comes to the Earth from Universe may have a definite kind of polarization.

The Ψ-ether is a single candidate for its definition as an inertial coordinate system. "...It is self-evident (and this was always supposed even before the creation of the theory of relativity) that there exists at least one reference frame, which is inertial as concerns mechanics and in which at the same time the Maxwell equations is valid..." [11].

Sometimes the relict radiation is connected with the inertial system of coordinates. However, as has been mentioned above, the relict radiation is considered here as eternal oscillation of the weakly excited world Ψ- ether. The inertial system is defined by the distribution of matter in the Universe [12] and the Ψ-ether is thus the only candidate for the definition of the inertial system of coordinates, or the privileges system of coordinates in the Universe, as it is defined by V.A. Fock [13].

It was justified above that the electromagnetic field has the discrete structure - it consists of neutrino-antineutrino pairs. Therefore, as philosophers say,"bad infinity" is interrupt when constant properties of any physical object, including the electromagnetic field, are preserved, even under unlimited division of its size up to zero.

Let us call the cooper neutrino-antineutrino pairs, of which the electro magnetic field consists, psions (a derivative of the word Ψ-ether). There is a limit for the existence of electromagnetic waves of a small size when the length of a free path of psions becomes smaller than the distance between them. The smallest length may be defined from maximal energy of γ-quanta coming from cosmos to the Earth. This energy approximately equals $\leq 10^{+23}$eV. The frequency ν of these quanta approximately equals $\sim 10^{38}\frac{1}{sec}$, and the wavelength $\lambda \sim 10^{-28}$ cm.

The density of psions in $1cm.^3$ determines the reliability of information accepted and transferred by the Ψ-ether. If two or more impulses come to a psion, gained information will by distorted in further transmission.

As concerns large wave lengths, the limit for the existence of electromagnetic waves sets in when large wavelengths of the Ψ-ether lose their wave configuration becoming a chaotic motion of huge masses of the Ψ- ether ("noises").The largest lengths of radiowaves amounts to $\sim 10^{10}$ meter and may be even larger, and the frequency of such waves (the lowest frequency) is $\nu \sim 3.10^{-2}\frac{1}{sec}$.

So, when the wave properties of the Ψ-ether come to an end, our cognition of secrets of the Universe by means of optical instruments and radiotelescopes ends to. The man does not "hear" the full voice of the Universe, the man becomes "blind" and "deaf" in the Universe.

Evidently, there exist phenomena whose description requires studies of the microscopic properties of constituents of the Ψ-ether, psions.

So, the light considered by us as the vibration of the Ψ-ether in the process of propagation in the Universe will lose the energy. The propagation of light can be considered as propagation of waves in an elastic medium. Therefore, the loss of the energy in process of propagation of the light in the Universe will be extremely small.

Two models of the loss of the energy of the light wave are considered here.

Let us an energy of light quanta equal $h\nu$.

The first model. Suppose that the relative loss of γ-quantum energy is proportional to the distance r passed by γ-quantum in the Ψ- ether. The initial frequency of γ-quantum equals ν_0 and the final one equals ν $(\nu < \nu_0)$. Therefore, $\delta\nu = \nu - \nu_0 < 0$

$$-\frac{\delta\nu}{\nu_0} = \alpha \cdot r; -\frac{\delta\nu}{\nu_0} = \frac{\delta\lambda}{\lambda} = z$$

So:

$$z = \alpha \cdot r$$

The coefficient α is determined from experiment. It is natural propose that $\alpha = \frac{H}{c}$, where H - is the Hubble constant in $\frac{km}{Mpc \cdot sec}$, and r is the distance in Mpc. Hence, we get the known expression for the Hubble law

$$cz = Hr$$

Thus, the redshift is related to the distance r only.

The second model is the model of exponential loss energy of γ-quanta.

$$d\nu = \alpha \cdot \nu \cdot dr$$

After integration we get

$$\nu = \nu_0 \cdot e^{-\alpha \cdot r}$$

Under the condition $\alpha \cdot r \ll 1$ we have $\lambda \approx \lambda_0 \cdot (1 + \alpha \cdot r)$ and

$$\frac{\lambda - \lambda_0}{\lambda_0} = \frac{\Delta\lambda}{\lambda_0} = z = \alpha r$$

and in this case we put $\alpha = \frac{H}{c}$ and get the same expression for the energy loss of the light wave

$$cz = Hr$$

The law of energy loss of light quanta in both the model is the same as in the region $\alpha r < 1$, $r < \frac{1}{\alpha}$. This value r is defined from the inequality

$$r < \frac{c}{H} \approx \frac{300000 \frac{km}{sec}}{60 \frac{km}{Mpc sec}}$$

Up to $5000 Mpc$ we cannot distinguish between the linear law of energy loss and the exponential one. But if $r \sim 6500 Mpc$, exponential loss is approximately twice larger then the linear one.

The critical value of r, at which the linear law of energetic loss begins to differ from the exponential one is defined by the Hubble constant.

Now the law of redshiff has to be written in the form

$$z_{full} = z_{Doppler} + z_{\Psi} = (\frac{v}{c})_{Doppler} + (\frac{H}{c}r)_{Psi}$$

The correct Doppler redshift has the Usual form

$$z_{full} - z_{\Psi} = (\frac{v}{c})_{Doppler}$$

The redshift in the Hubble diagram from the work by Karachentsev and Makarov (see Fig. 1) will now have the form (see Fig. 2)

From Figure 2 we can see that of 145 galaxies of the local Group approximately one half has a redshiff ($\frac{\Delta\lambda}{\lambda} > 0$), and the other half has a blue shift ($\frac{\Delta\lambda}{\lambda} < 0$). One half of Galaxies moves away from the Centre of the Local Group of Galaxies and other half of Galaxies comes to the Centre of the Local Group of Galaxies. So, we observe streams of Galaxies in some directions with respect to the Centre of the Local Group approximately with the velocity $\sim 100\frac{km}{sec}$.

We have no grounds to think that the law of motion of Galaxies in the Universe at distances far from the Centre of the Local Group of Galaxies differs from the law of motion of Galaxies of the Local Group and, therefore, a general picture of the motion of Galaxies in the Universe do not look like the expansion of the Universe.

So, we have a different point of view on the problem of the evolution of the Universe. But it is clear that an unfaithful interpretation of the redshift in the Hubble law led to many false problems which are now very actively discussed in many scientific papers.

And if we admit the fault in the interpretation of the Hubble law then the problem of the monotonous expanding of the Universe changes to the problem of studying vivid dynamics of the nonrelativistic motion of Galaxies in the Universe, and modern cosmological science turns to real problems of the evolution of the Universe.

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Weakly interacting particles and the Universe evolution

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Abstract — On the basis of hypotheses, that a density of weakly interacting particles in the Universe has an order of nuclear matter density or more the Lagrangian is offered, through which one can be obtained a propagator of a vector boson with a non-zero rest-mass. The dependence of vector bosons masses on the time allows to explain the availability of the hot stage of the Universe evolution, not using to the hypothesis of the Universe expansion.

Following Blokhintsev [1] which considered that it is impossible to describe physical systems in the deterministic manner let's consider the Boltzmann hypothesis of the Universe birth owing to a gigantic fluctuation not in a empty space but in medium which consists of weakly interacting particles characterized by zero temperature and forming the Bose condensate. Certainly, if the particles are fermions they should be in the coupled state. For the description of such state of the Universe matter (this state we shall consider pure one) it is necessary to introduce an amplitude of probability $\mathcal{B}$ with components $\mathcal{B}_a^b(\omega)$ $(a, b, c, d, e, f, g, h = 1, 2..., r)$ dependent from points ω of a manifold M_r (not excepting a limiting case $r \to \infty$). In this case we can not define the metric but for its definition we need a density matrix $\rho(\mathcal{B})$ (the rank of which equals to 1 for a pure state), determining its standard mode $\mathcal{B}\mathcal{B}^+ = \rho(\mathcal{B}\mathcal{B}^+)$ $(\rho = 1,$ $\rho^+ = \rho$, the top index "+" is the symbol of the Hermitian conjugation).

Let as a result of a fluctuation the disintegration of the Bose condensate will begin with formation of fermions (for their description we shall introduce an amplitude of probability Ψ) and with an increase of pressure in some local area of Universe (in addition some time the temperature of the Universe particles could remain equal or close to zero point — so-called the inflation period). As a result the rank of the density matrix ρ will begin to grow that characterizes the appearance of mixed states. An inverse process of relaxation (characterized by the formation of the Bose condensate and by the pressure decline) should go with an energy release which will go on heating of the Fermi liquid with formation of excited states — of known charged fermions (quarks and leptons). From this moment it is possible to introduce the metric and use the results obtained for the hot model of Universe interpreting the Universe evolution as the process characterized by the increase of the entropy $S = -(\rho \ln \rho)$. Now Universe is at that stage of an evolution when the dominating number of particles has returned to the Bose condensate state. They are also displayed only at a weak interaction with particles of a visible matter.

For the description of a matter it is convenient to use differentiable fields given in a differentiable manifold M_n which one we shall call as a space-time and the points x of which one will have coordinates x^i $(i, j, k, l... = 1, 2..., n)$. Probably the rank of the density matrix ρ equals n, but it is impossible to eliminate that the generally given equality is satisfied only approximately when some components of a density matrix can be neglected. In any case we shall consider that among fields $\mathcal{B}$ the mixtures Π^i_a were formed with non-zero vacuum means h^i_a which determine differentiable vector fields $\xi^i_a(x)$ as:

$$\Pi^i_a = \mathcal{B}^b_a \; \xi^i_b \tag{1}$$

for considered area $\Omega_n \subset M_n$ (field $\xi^i_a(x)$ determine a differential of a projection $d\pi$ from $\Omega_r \subset M_r$ in Ω_n). It allows to define a space–time M_n as a Riemannian manifold, the basic tensor $g_{ij}(x)$ of which we shall introduce through a reduced density matrix $\rho'(x)$.

So let components ρ^j_i of a reduced density matrix $\rho'(x)$ are determined by the way:

$$\rho^j_i = \xi^{+}{}^a_i \; \rho^b_a \; \xi^j_b \; / \; (\xi^{+}{}^c_k \rho^d_c \xi^k_d). \tag{2}$$

and let fields

$$g^{ij} = \eta^{k(i} \rho^{j)}_k \; (g^{lm} \eta_{lm}) \tag{3}$$

are components of a tensor of a converse to the basic tensor of the space–time M_n. By this components $g_{ij}(x)$ of the basic tensor must be the solutions of following equations: $g^{ij} g_{ik} = \delta^j_k$. (Hereinafter η_{ij} are metric tensor components of a tangent space to M_n and η^{ik} are determined as the solution of equations: $\eta^{ij} \eta_{ik} = \delta^j_k$.)

The influencing of the macroscopic observer will display in an approximation of the transition operator T by the differential operators ∂_i. First of all we shall require that for fermion fields deviations $X_a(\Psi) = T_a(\Psi) - \xi^i_a \partial_i \Psi$ were minimum in the "mean" [2]. For this purpose we shall consider a following integral

$$\mathcal{A} = \int\limits_{\Omega_n} \mathcal{L}(\Psi) d_n V = \int\limits_{\Omega_n} \kappa \; \overline{X}^a(\Psi) \; \rho^b_a(x) \; X_b(\Psi) \; d_n V \tag{4}$$

(κ is a constant, $\mathcal{L}(\Psi)$ is a Lagrangian; the bar means the Dirac conjugation that is to be the superposition of Hermitian conjugation and the spatial inversion) which is the action. Let the action $\mathcal{A}$ is quasi-invariant at infinitesimal substitutions $x^i \to x^i + \delta x^i = x^i + \delta\omega^a \xi^i_a(x)$, $\Psi \to \Psi + \delta\Psi = \Psi + \delta\omega^a T_a(\Psi)$ of Lie local loop G_r, the structural tensor components C^c_{ab} of which satisfy to Jacobi generalized identity [2]. The given requirement causes to introduce the full Lagrangian (instead of a Lagrangian $\mathcal{L}(\Psi)$) recording it as

$$\mathcal{L}_t = \mathcal{L}(\Psi) + \kappa'_o \mathcal{F}^c_{ab} \mathcal{F}^f_{de} [t^{ad}(s^e_c s^b_f - v s^b_c s^e_f) + t^{be}(s^a_f s^d_c - v s^a_c s^d_f) + u_{cf}(t^{ad} t^{be} - v t^{ab} t^{de})]/4 \tag{5}$$

(κ'_o, v are constants). In addition intensities $\mathcal{F}^c_{ab}(B)$ of the boson (gauge) fields $B^c_a(x)$ will look like

$$\mathcal{F}^c_{ab} = \Theta^c_d \, (\Pi^i_a \, \partial_i B^d_b - \Pi^i_b \, \partial_i B^d_a + \Xi^d_{ab}), \tag{6}$$

where

$$\Theta^c_b = \delta^c_b - \xi^i_b \, \Pi^d_i \, (B^c_d - \beta^c_d), \quad \Xi^b_{ad} = (B^c_a \, T^{\ e}_{cd} - B^c_d \, T^{\ e}_{ca}) \, B^b_e - B^c_a \, B^e_d \, C^b_{ce}. \tag{7}$$

Hereinafter a selection of fields $\Pi^a_i()$ and β^a_c are limited by the relations: $\Pi^a_j \, \Pi^i_a = \delta^i_j$, $\beta^a_c \, \xi^i_a = h^i_c$ (δ^i_j are Kronecker deltas).

If $s^{\ b}_a = \delta^b_a$, $t^{ab} = \eta^{ab}$, $u_{ab} = \eta_{ab}$ (η_{ab} are metric tensor components of the flat space and η^{ab} are tensor components of a converse to basic one) then the given Lagrangian is most suitable one at the description of the hot stage of the Universe evolution because it is most symmetrical one concerning intensities of the gauge fields $\mathcal{F}^c_{ab}$. What is more we shall require the realization of the correlations: $T_a^{\ b} \, \eta^{cd} + T_{ac}^{\ d} \, \eta^{cb} = 0$, that the transition operators $T_a^{\ b}$ generate the symmetry, which follows from the made assumptions. In absence of fields $\Pi^i_a(x)$ and $\Psi(x)$ at earlier stage of the Universe evolution the Lagrangian (5) becomes even more symmetrical ($\mathcal{L}_t \propto B^4$), so that the formation of fermions (the appearance of fields Ψ in a full Lagrangian $\mathcal{L}_t$) from primary bosons is a necessary condition (though not a sufficient one) of the transition of Universe to the modern stage of its development with a spontaneous symmetry breaking. Only the formation of the Bose condensate from pairs of some class of fermions (the neutrinos of different flavors) has resulted in a noticeable growth of rest–masses of those vector bosons (W^+, W^-, Z^o), which interact with this class of fermions. In parallel there could be a growth of rest–masses and other fundamental particles, though and not all (photon, directly with a neutrino not interacting, has not a rest-mass).

Let's connect non-zero vacuum means β^b_a of gauge fields B^b_a with a spontaneous violation of a symmetry, which has taken place in the early Universe and which is a phase transition with a formation of Bose condensate from fermion pairs. The transition to the modern stage of the Universe evolution for which it is possible to suspect the presence of cluster states of weakly interacting particles will be expressed in following formula for tensors s_a^b, t^{ab}, u_{ab} and h_i^a:

$$s_a^b = s\ \xi_a^i\ h_i^b + \xi_a^{\underline{c}}\ \epsilon_{\underline{c}}^b, \quad t^{ab} = t\ \epsilon_{(i)}^a\ \epsilon_{(j)}^b\ \eta^{(i)(j)} + \epsilon_{\underline{c}}^a\ \epsilon_{\underline{d}}^b\ \eta^{\underline{cd}},$$

$$u_{ab} = u\ \xi_a^i\ \xi_b^j\ h_i^c\ h_j^d\ \eta_{cd} + \xi_a^{\underline{c}}\ \xi_b^{\underline{d}}\ \eta_{\underline{cd}}, \quad h_i^a = h_i^{(j)}\ \epsilon_{(j)}^a \tag{8}$$

$((i),(j),(k),(l),... = 1,2,...,n;\ \underline{a},\underline{b},\underline{c},\underline{d},\underline{e} = n+1, n+2, ..., n+\underline{r};\ \underline{r}/r \ll 1)$, where fields $h_i^{(j)}(x)$, taking into account the relations (8), are determinated uniquely from equations: $h_k^a\ h_a^i = \delta_k^i$. Similarly tensors $\eta^{(i)(j)}, \eta^{\underline{ab}}$ are determined from equations: $\eta^{(i)(k)}\eta_{(j)(k)} = \delta_{(j)}^{(i)}, \eta^{\underline{ab}}\eta_{\underline{cb}} = \delta_{\underline{c}}^{\underline{a}}$, while tensors $\eta_{(i)(j)}, \eta_{\underline{ab}}$ are determined as follows $\eta_{(i)(k)} = \eta_{ab}\ \epsilon_{(i)}^a\ \epsilon_{(k)}^b$, $\eta_{\underline{ab}} = \eta_{cd}\ \epsilon_{\underline{a}}^c\ \epsilon_{\underline{b}}^d$. We shall connect constants $\epsilon_{(i)}^a$, $\epsilon_{\underline{b}}^a$ with a selection of the gauge fields $\Pi_i^a(x)$ recording them by in the form $\Pi_i^a = \Phi_i^{(j)}\ \epsilon_{(j)}^a + \Phi_i^{\underline{b}}\ \epsilon_{\underline{b}}^a$ and let $\epsilon_{\underline{b}}^a = 0$. Besides we shall apply the decomposition of fields $B_b^a(x)$ in the form $B_c^a = \zeta_i^a\ \Pi_c^i + \zeta_{\underline{b}}^a\ A_c^{\underline{b}}$, where $A_b^{\underline{a}} = B_b^c\ \xi_c^{\underline{a}}$. In addition components of intermediate tensor fields $\xi_a^i(x)$, $\xi_a^{\underline{b}}(x)$, $\zeta_i^a(x)$, $\zeta_{\underline{b}}^a(x)$ should be connected by the relations: $\zeta_i^a\xi_a^j = \delta_i^j$, $\zeta_i^a\xi_a^{\underline{b}} = 0$, $\zeta_{\underline{b}}^a\xi_a^j = 0$, $\xi_a^{\underline{c}}\zeta_{\underline{b}}^a = \delta_{\underline{b}}^{\underline{c}}$.

Let $n = 4$, $\underline{r} = 1$, $v = 2$, $tu = s^2$, $C_{i\underline{a}}^{\underline{c}} = (\zeta_i^b\zeta_{\underline{a}}^d C_{bd}^e + \partial_i\zeta_{\underline{a}}^e)\xi_e^{\underline{c}}$, $T_{c\ (k)}^{\ a} = T_{cb}^{\ a}\epsilon_{(k)}^b = T_{c(k)}^{(i)}\epsilon_{(i)}^a$, $T_{i(j)}^{(k)} = \zeta_i^a T_{a(j)}^{(k)}$, $T_{\underline{b}(j)}^{(k)} = \zeta_{\underline{b}}^a T_{a(j)}^{(k)}$ and

$$T_{a(k)}^{(i)}\ \eta^{(j)(k)} + T_{a(k)}^{(j)}\ \eta^{(i)(k)} = \epsilon_a^{\underline{b}}\ t_{\underline{b}}\ \eta^{(i)(j)}. \tag{9}$$

As a result we can receive the vector boson propagator in the form

$$D_{ij}(p) = -g_{ij}/(p^k p_k - m^2) \tag{10}$$

(we use the Feynman calibration), where p^k is the 4-momentum, and m is the mass of the vector boson, which is defined by the formula [3]

$$m^2 = 3\kappa_1 t_{\underline{a}}^2/(\kappa_o\eta_{\underline{aa}}) - g^{jk}C_{j\underline{a}}^{\underline{a}}C_{k\underline{a}}^{\underline{a}}\,. \tag{11}$$

So, the transition to the hot state of Universe was connected with the destruction of the Bose condensate and the increase of the Fermi gas pressure accordingly. In addition some time a temperature of background particles of Universe could remain equal or close to zero (the stage of the inflation). As a result the rest–mass of W^+, W^-, Z^o bosons have decreased ($t_{\underline{a}} = 0$) so, that the weak interaction has stopped to be weak and all (or nearly so all) particles from a ground (vacuum) state started to participate in an installation of a thermodynamic equilibrium. The given phenomenon also has become the cause of an apparent increase of a density of particles in the early Universe. Suggesting, that mean density n_o of particles in the Universe did not vary at the same time and the script of the hot model in general is correct, we come to its following estimation $n_o \sim m_\pi^3 \sim 10^{-3} GeV^3$ (m_π is a mass of a π meson; hereinafter the system of units are used $h/(2\pi) = c = 1$, where h is the Planck constant, c is the speed of light). This result allows to give explanation to a known ratio [4] $H_o/G_N \approx m_\pi^3$, if to consider, that the Hubble constant H_o gives an estimation $1/H_o$ to the length $l \sim 1/(n_o\sigma_\nu)$ of a free run of particle in "vacuum" on the modern stages of the Universe evolution (σ_ν is a scattering cross–section of neutrinos on a charged particle) and to take into account the estimation given earlier [5] for the gravitational constant G_N ($G_N \sim \sigma_\nu$). Thus the gravitational constant G_N is inversely proportional to the time of a free run of a charged particle in the neutrinos medium characterizing a kinetic phase of a relaxation process in the Universe.

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Spacetime Bag Model

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Abstract — The relativistic spacetime bag model is formulated. The Dirac and the Maxwell equations for the particles in the spacetime bag are presented and some consequences are analyzed. In particular, it is shown that the electrostatic potential for the charged particles in the bag has a form of the strong potential, which is in agreement with experiment.

1 Introduction

The quark bag model is well-known and its great positive role in the hadron physics and quantum chromodynamics is widely accepted (see for example review articles [1], [2] and references therein.) It is shown here that a bag model has a new reading due to rotations. In general case particle is doomed to rotates if constraint

$$x^2 + y^2 + z^2 + u^2 = a^2 \tag{1}$$

hold valid. In the physics of relativity we have additional time variable. In view of this, we introduce the five-dimensional Minkowski spacetime $M^5_{1,4}$ with Cartesian coordinates x^i ($i = 0, 1, 2, 3, 4$) and metrics

$$ds^2 = \eta_{ij}dx^i dx^j = (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2 - (dx^4)^2,$$

where as usually, $x^0 = ct$, $x^1 = x$, $x^2 = y$, $x^3 = z$, $x^4 = u$. After that we accept that spacetime background for strongly interacting particles is defined by the $M^5_{1,4}$ and constraint

$$\eta_{ij}x^i x^j = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2 - (x^4)^2 = -a^2, \tag{2}$$

where a is the radius of gyration. Thus, we put forward the idea that the form of physical laws at subnucleus realm is defined by the representation of rotations in given form. In this report we consider one Dirac field and electromagnetic field in the spacetime bag (2) since other interacting fields poses no new problems once the vector and spinor theories are understood.

2 The Dirac field in the spacetime bag

We will use the scalar product $(X,Y) = \eta_{ij}U^iV^j$ for any vector fields $X = U^i\partial_i$ and $Y = V^i\partial_i$ in the five-dimensional Minkowski spacetime. The vector fields

$$P_i = \delta_i^k\partial_k, \quad M_{ij} = (x_i\delta_j^k - x_j\delta_i^k)\partial_k,$$

where $x_i = \eta_{ij}x^j$, are generators of the Poincare group of the five-dimensional Minkowski spacetime. All vectors fields M_{ij} are orthogonal to the radius-vector $R = x^k\partial_k$, but this is not the case for the vector fields P_i. Representing P_i as the sum of the component aligned with the direction of the radius vector R and the component orthogonal to this direction, we obtain the vector fields

$$M_i = aP_i + \frac{1}{a}(R,P_i)R = (a\delta_i^k + \frac{1}{a}x_ix^k)\partial_k,$$

which are tangent to surface (2) (relativistic bag), because from (2), it follows that $(R,M_i) = 0$ at each point. The vector fields M_i and M_{ij} are generators of the group of spacetime symmetry of a bag because we have

$$[M_i, M_j] = -M_{ij}, \quad [M_i, M_{jk}] = \eta_{ij}M_k - \eta_{ik}M_j. \tag{3}$$

Let us now introduce the operators (vector fields)

$$X_0 = M_0, \quad X_1 = M_{14} + M_{23}, \quad X_2 = M_{24} + M_{31}, \quad X_3 = M_{34} + M_{12}. \tag{4}$$

It is straightforward to see that the vector fields X_0, X_1, X_2, and X_3 are continuous and do not vanish at any point of the spacetime bag. Because $(X_a, X_b) = 0$ for $a \neq b$, $a,b = 0,1,2,3$ and

$$(X_0,X_0) = -(X_1,X_1) = -(X_2,X_2) = -(X_3,X_3) = a^2 + x_0^2,$$

the vector fields X_0, X_1, X_2, and X_3 are linearly independent at each point of the bag. Thus, it is shown that the spacetime bag in question has a spinor structure.

¿From (3), it follows that

$$[X_0, X_\mu] = 0, \quad [X_\mu, X_\nu] = 2e_{\mu\nu\lambda}X_\lambda, \quad \mu,\nu,\lambda = 1,2,3,$$

where $e_{\mu\nu\lambda}$ is the completely antisymmetric Levi-Civita symbol specified by the equality $e_{123} = 1$. In this way, we have proven that the our bag admits a simply transitive group of transformations whose generators are given by (4) and which has only the following nonzero structure constants:

$$f^1_{23} = f^2_{31} = f^3_{12} = 2. \tag{5}$$

In our case this group play the role of the group of translation in familiar Minkowski spacetime.

In accordance with the original Dirac equation and consideration given above, we write the Dirac equation for the rotating particles in the form

$$\gamma^c P_c \psi = \mu\psi, \tag{6}$$

where

$$\gamma^a\gamma^b + \gamma^b\gamma^a = 2\eta^{ab}, \quad P_c = X_c + \frac{iqa}{\hbar c}A_c, \quad \eta^{ab} = (1, -1, -1, -1).$$

Here q is the charge of a particle, and A_c are the components of the vector potential of the electromagnetic field in the basis X_a. In the spacetime bag we have

$$\mu = mca/\hbar,$$

since operators (4) are dimensionless.

In general, $[X_a, X_b] = f^c_{ab}X_c$. Thus, in the spacetime bag we have

$$[P_a, P_b] = f^c_{ab}P_c + \frac{iqa}{\hbar c}F_{ab},$$

where

$$F_{ab} = X_aA_b - X_bA_a - f^c_{ab}A_c \tag{7}$$

are the components of the strength tensor of the electromagnetic field in the basis X_a and f^a_{bc} are given by (5).

When the wave equation is established it is not difficult to derive the equations of electromagnetic field. The Jacobi identity $[P_a[P_b, P_c]] + [P_b[P_c, P_a]] + [P_c[P_a, P_b]] = 0$ results in the first four Maxwell equations

$$X_a\tilde{F}^{ab} + \frac{1}{2}f^b_{ad}\tilde{F}^{ad} = 0. \tag{8}$$

where f^a_{bc} take the values (5) and $\tilde{F}^{ab} = \frac{1}{2}e^{abcd}F_{cd}$, where e^{abcd} are components of the antisymmetric Levi-Civita unit tensor in the basis X_a. In view of the known dual symmetry of the Maxwell equations, from (8) it follows that the remaining Maxwell equations are of the form

$$X_a F^{ab} + \frac{1}{2} f^b_{ad} F^{ad} = \frac{4\pi a}{c} j^b, \tag{9}$$

where j^b are components of the current vector in the basis X_a.

Now we will write the Maxwell equations in the spacetime bag in the three-dimensional vector form. Before doing so it is useful to introduce an intrinsic coordinate system in the spacetime bag, which is defined by the vector field X_0. In accordance with the definition of X_0 we have the following system of ordinary differential equations for integral curvatures of this vector field

$$\frac{dx^0}{cd\tau} = 1 + (x^0)^2/a^2, \quad \frac{dx^b}{cd\tau} = x^0 x^b/a^2, \quad b = 1,2,3,4.$$

Under the condition $x^0(0) = 0$, we have the solution in the following form

$$x^0 = a \tan \frac{c\tau}{a}, \quad x^b = \frac{au^b}{\cos(c\tau/a)},$$

where u^b, $b = 1,2,3,4$ are constants of integration. It is easy to see that intrinsic time variable τ and u^b define intrinsic coordinate system in the spacetime bag. In this coordinate system we have for the operator X_0

$$X_0 = \frac{a}{c}\frac{\partial}{\partial \tau}.$$

To write the Maxwell equations for $\vec{E}$ and $\vec{H}$, we put as usual

$$j^a = (c\rho, \vec{\mathrm{j}}), \quad A_a = (\varphi, \quad -\vec{\mathrm{A}}),$$

$$E_\mu = F_{0\mu}, \quad H_\mu = \frac{1}{2} e_{\mu\nu\lambda} F^{\nu\lambda}, \quad \mu, \nu, \lambda = 1,2,3.$$

Then from (7) we obtain

$$\vec{\mathrm{E}} = -\frac{\mathrm{a}}{\mathrm{c}}\frac{\partial}{\partial \tau}\vec{\mathrm{A}} - \nabla\varphi, \quad \vec{\mathrm{H}} = \mathrm{rot}\vec{\mathrm{A}} = \nabla \times \vec{\mathrm{A}} - 2\vec{\mathrm{A}}. \tag{10}$$

where $\nabla = (\nabla_1, \quad \nabla_2, \quad \nabla_3)$, $\quad \nabla_\mu = X_\mu$, $\quad \mu = 1,2,3$.

Considering that $\mathrm{div}\vec{A} = \sum_{\mu=1}^{3} \nabla_\mu A_\mu$, we can recast the Maxwell equations (8) and (9) into the familiar vector form

$$-\frac{\mathrm{a}}{\mathrm{c}}\frac{\partial}{\partial\tau}\vec{\mathrm{H}} = \mathrm{rot}\vec{\mathrm{E}}, \quad \mathrm{div}\vec{\mathrm{H}} = 0, \quad \mathrm{rot}\vec{\mathrm{H}} = \frac{\mathrm{a}}{\mathrm{c}}\frac{\partial}{\partial\tau}\vec{\mathrm{E}} + \frac{4\pi\mathrm{a}}{\mathrm{c}}\vec{\mathrm{j}}, \quad \mathrm{div}\vec{\mathrm{E}} = 4\pi\mathrm{a}\rho. \quad (11)$$

To have more concrete representation about spacetime bag it important to derive eigenvalues E of the Dirac Hamiltonian of the free particle in the bag and to find also electrostatic potential as a solution of the Maxwell equations (10), (11). Consider the first problem. Squaring equation (6) and using (5), we obtain the following equation for E

$$E^2\psi = m^2c^4\psi - \frac{c^2\hbar^2}{a^2}(\Delta + P)\psi,$$

where $P = \Sigma_1\nabla_1 + \Sigma_2\nabla_2 + \Sigma_3\nabla_3$ and $\Sigma_\mu = \frac{1}{2}e_{\mu\nu\lambda}\gamma^\nu\gamma^\lambda$. Since $\Delta + P = -P(P+1)$, where Δ is the Laplacian on a three-dimensional sphere, then

$$E^2 = m^2c^4 + p(p+1)\frac{c^2\hbar^2}{a^2},$$

where p is an eigenvalue of the operator P. It can be shown that $p = \pm 3, \pm 4, \ldots$. For the energy, we then have

$$E^2 = m^2c^4 + n(n+1)\frac{c^2\hbar^2}{a^2} = m^2c^4(1 + n(n+1)\frac{\lambda^2}{a^2}), \quad (12)$$

where $n = 2, 3, \ldots$ and $\lambda = \hbar/mc$. Formula (12) is nothing else but the energy of rotation which gives the quantum-mechanical value of the energy of the relativistic rotating particles. Consider it in more details. At large a, the moment of inertia $I = ma^2$ is also large, so that the angular velocity is small. Therefore, the nonrelativistic limit can be found from the condition $a \gg \lambda$. In this limit it follows from (12) that $E = mc^2 + L^2/2I$, where $L^2 = n(n+1)\hbar^2$ and $I = ma^2$. The last relation corresponds to the formula $E = mc^2 + P^2/2m$ for the kinetic energy of the free particles in the limit $v \gg c$. We also see that **in the spacetime bag, separation of the energy of a particle to the internal and rotational parts has no strict sense as it is evident from the relation (12), $E \to mc^2$ only when $a \to \infty$ or $I \to \infty$ (no rotation).**

3 On the Coulomb law in the spacetime bag

Let us now consider the Coulomb law for the rotating particles. The electrostatic potential can be derived as a solution of the equations of electrostatics, which is invariant under the group of Euclidean motions, including rotations and translations. In the case being considered, we will seek a Coulomb potential in an analogous manner. From (10) and (11), it follows that for a electrostatic field $\mathrm{div}\vec{\mathrm{E}} = 4\pi a\rho$, $\vec{\mathrm{E}} = -\nabla\varphi$, and consequently, φ obeys the Poisson equation

$$\Delta\varphi = -4\pi a^2\rho. \tag{13}$$

As it is well known, the electron Coulomb potential

$$\phi_e(r) = \frac{e}{r}$$

is the fundamental solution to the Laplace equation $\Delta\phi = \mathrm{div}\ \mathrm{grad}\,\phi = o$. In framework of our consideration the Coulomb potential for the charge particle can be derived as follows. Consider the stereographic projection S^3 from point (0,0,0,-a) onto the ball $r^2 = x^2 + y^2 + z^2 \leq a^2$:

$$x^1 = fx, \quad x^2 = fy, \quad x^3 = fz, \quad x^4 = a(1-f),$$

where $f = 2a^2/(a^2 + r^2)$. Then, it follows that the element of length on the three-dimensional sphere can be represented in the form

$$ds^2 = f^2(dx^2 + dy^2 + dz^2)$$

and hence the Laplace equation on S^3 can be written as follows

$$\Delta\phi = f^{-3}\mathrm{div}(f\mathrm{grad}\ \ \phi) = 0.$$

We seek the solution to this equation that is invariant under the transformation of the group $SO(3)$. This subgroup of the $SO(4)$ group is determined by fixing the point $(0, 0, 0, -a)$. Let us put

$$\psi = f\frac{1}{r}\frac{d\phi}{dr}.$$

Since

$$\Delta\phi = f^{-3}(r\frac{d\psi}{dr} + 3\psi) = r^{-2}f^{-3}\frac{d}{dr}(r^3\psi),$$

then $r^3\psi = c_1 = constant$. Thus, we have

$$\frac{d\phi}{dr} = c_1\frac{a^2 + r^2}{2a^2r^2} = c_1(\frac{1}{2r^2} + \frac{1}{2a^2})$$

and hence

$$\phi_q = c_1(-\frac{1}{2r} + \frac{r}{2a^2}) + c_2. \quad (14)$$

Now we have the electrostatic potentials ϕ_e outside the bag and electrostatic potential (14) inside the bag. We should demand that electrostatic potential is continuous function and hence put $\phi_e(a) = \phi_q(a)$, so that $c_2 = e/a$ and the Coulomb potential for particles in the spacetime bag can be presented as follows

$$\phi_q(r) = q(\frac{1}{2r} - \frac{r}{2a^2}) + \frac{e}{a}, \quad (15)$$

where q is the charge of particle.

It should be noted that our expression (15) coincides with the strong potential [3](which is in agreement with experiment) and exhibits clearly its electrodynamical nature. We also see that the boundary condition $\phi_e(a) = \phi_q(a)$ does not fix the charge q of particle in the spacetime bag (leaving it as a free parameter) and hence our consideration does not contradict the standard QCD nomenclature.

In conclusion it is useful to put together the Hamiltonian for the electron and for the particle in the spacetime bag. The electron Hamiltonian has as usual the following form:

$$\mathbf{H}_e = c(\alpha, \mathbf{P}_e) + \rho_3 m_e c^2,$$

where $\mathbf{P}_e = i\hbar\nabla$. For the particle in the bag we have

$$\mathbf{H}_q = c(\alpha, \mathbf{P}_q) + \rho_3 m_q c^2,$$

where

$$\mathbf{P}_q = \frac{i\hbar}{a}(\mathbf{r} \times \nabla + \frac{a^2 - r^2}{2a}\nabla + \frac{\mathbf{r}}{a}(\mathbf{r}, \nabla)).$$

Here,

$$\mathbf{P}_q = \frac{i\hbar}{a}\mathbf{X}, \quad \mathbf{X} = (X_1, X_2, X_3)$$

and X_1, X_2, X_3 are expressed through the stereographic coordinates.

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Проблемы квантования на примере простой нелинейной системы

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Abstract — Дано краткое, но доступное представление о проблемах, которые возникают при асимптотическом шредингеровском квантования нелинейных гамильтоновых систем и состоят в ограничениях на топологию конфигурационного пространства, неинвариантности квантового гамильтониана относительно преобразований координат и произвола правила упорядочения при подстановке операторов канонически сопряженных координат и импульсов в полиномиальные наблюдаемые. В случае простейшей нелинейной системы - частицы в классическом смысле движущейся по геодезической линии риманова пространства (в статическом гравитационном поле), описано, как можно преодолеть последние две из этих трудностей.

1. Введение Среди научных интересов Д.И. Блохинцева видное место занимали вопросы связи геометрии пространства-времени и квантовой теории. Автору настоящей статьи запомнилось также, как он ориентировал своих учеников и сотрудников на то, чтобы "разобраться" (именно это слово он употреблял) в какой–либо из значительных научных проблем, т.е. понять глубинные корни проблемы и изучить в деталях ее современный статус и перспективы решения. В частности, в бытность Д.И. директором Лаборатории теоретической физики ОИЯИ им были поддержаны экзотические для того времени попытки (с участием автора данной статьи) разобраться в проблеме сочетания квантовой теории и общей теории относительности. В результате эти исследования дали заметный начальный толчок тому впечатляющему прогрессу в сочетании космологии и теории элементарных частиц, который наблюдается сейчас. Настоящая статья является доступным очерком некоторых последних работ автора [1,2], продолжающих упомянутые попытки "разобраться" с несколько другой точки зрения. Речь пойдет о приложении современых представлений о квантовании систем с конечным числом степеней свободы к "вечной" проблеме построения квантовой механики в n–мерном римановом конфигурационном пространстве V_n , связанной с именами Дирака, Шредингера, Подольского, ДеВитта, а также многих других, не столь известных исследователей.

Как известно, квантованием называются математические построения, которые ставят в соответствие классическим динамическим системам и их обобщениям квантовые аналоги, характеристическим свойством которых является некоммутативность (физических) наблюдаемых. К сожалению, квантование встречается со значительными трудностями, как математическими, так и в части соответствия некоторым установившимся физическим принципам за исключением случая линейных гамильтоновых систем. (В этом аспекте о статусе стандартной квантовой механики частицы в потенциальном поле см. ниже в разд.3). Приведем вначале самые необходимые общие сведения о классических динамических системах и их шредингеровском квантовании.

2. Исходные предположения о классических системах. Простейшую классическую "стартовую площадку" для квантования составляют симплектическое фазовое пространство P_{2n}, $\{x^A\} \in P_{2n}$, $A, B, \ldots = 1, 2, \ldots, = 2n$, с заданной невырожденной симплектической формой $\epsilon = \epsilon_{AB}(x)dx^A \wedge dx^B$ и некоторая алгебра $\mathbf{sp}_{2n}$ подходящих действительных функций на P_{2n} с умножением, определяемым скобкой Пуассона

$$\{f, g\} \stackrel{def}{=} \epsilon^{AB}\partial_A f \partial_B g, \quad f, g \in \mathbf{sp}_{2n}; \quad \|\epsilon^{AB}\| \stackrel{def}{=} \|\epsilon_{AB}\|^{-1}. \tag{1}$$

Такая алгебра называется алгеброй Пуассона. Для нашего рассмотрения будет важна следующая цепочка вложенных в одна в другую алгебр Пуассона: $\mathbf{h}_{2n} \subset \mathbf{sp}_{2n}^{(2)} \subset \mathbf{sp}_{2n}^{(\infty)} \subset \mathbf{sp}_{2n}(\mathrm{C}^\infty)$, где $\mathbf{sp}_{2n}(\mathrm{C}^\infty)$ — алгебра бесконечно-дифференцируемых функций на P_{2n}, $\mathbf{sp}_{2n}^{(\infty)}$ — алгебра полиномов из определенных ниже координат Дарбу $\{x^A\} = \{q^i, \ p_j\}$, $i, j, \ldots = 1, \ldots, n$ на P_{2n}, $\mathbf{sp}_{2n}^{(2)}$ — алгебра полиномов не выше второй степени по q^i, p_j и $\mathbf{h}_{2n} \stackrel{def}{=} \mathrm{span}\{1, \ q^i, \ p_j\}$ (т.е. линейная оболочка указанных элементов) — алгебра Гейзенберга. Кроме того, не будем терять из виду и абелеву алгебру Ли $\mathbf{lp_{2n}}(C^\infty)$ с обычным поточечным умножением C^∞–функций на P_{2n}.

Всякая функция $f(x) \in \mathbf{sp}_{2n}$ порождает систему уравнений Гамильтона

$$\frac{d}{ds}x^A(s) = \{x^A, \ f(x)\}(s) \tag{2}$$

и в этом смысле является классическим гамильтонианом. В то же время для каждой системы, определенной конкретным гамильтонианом $f(x)$, все действительные функции на P_{2n} считаются физическими наблюдаемыми. В достаточно малой области $P' \subset P_{2n}$ можно ввести упомянутые координаты Дарбу (их называют также канонически–сопряженными) q^i, p_j, $i,j,... = 1,...,n$, в которых $\epsilon = dp_i \wedge dq^i$.

Для дальнейшего обсуждения проблемы квантования мы сильно сузим нашу "стартовую площадку" до $P_{2n} \sim R_n \times R_n^*$, (тривиальное кокасательное расслоение с топологически тривиальной базой), что, в свою очередь, означает, что можно ввести кординаты Дарбу так, что координаты q^i покрывают все пространство R_n, а координаты p_j пробегают при этом двойственное пространство R_n^*. При этом координатные линии q^i открыты и полны; в этом смысле q^i очень похожи на декартовы координаты. Пока еще совсем не обязательно, но для последующего удобно предположить, что на базе R_n введено положительно–определенное симметричное тензорное поле $\omega_{ij}(q)$, играющее роль метрического тензора и определяющее способ измерения пространственных длин и объемов. Таким образом R_n превращается в топологически тривиальное риманово пространство V_n с ненулевой кривизной. Для квантовой механики это означает, что считаем существенными только локальные проявления кривизны

3. Шредингеровское квантование гамильтоновых систем. Пусть $L^2(V_n; C; \sqrt{\omega}\, d^nq)$ - гильбертово пространство квадратично-интегрируемых по введенной римановой мере комплексных функций на V_n . Тогда в рамках принятых ограничений шредингеровское квантование есть отображение [1]:

$\mathcal{Q}$: $\mathrm{sp}_{2n} \ni f \xrightarrow{\mathcal{Q}} \hat{f}$ (эрмитовы операторы в), такое, что
(Q1) $(1)^\wedge = \hat{1}$ (единичный оператор в $L^2(V_n; C; \sqrt{\omega}\, d^nq)$);
(Q2) $(f+g)^\wedge = \hat{f} + \hat{g}$;
(Q3) $\{f,g\}^\wedge = \mathrm{i}\hbar^{-1}[\hat{f},\hat{g}] + \hat{O}(\hbar; f, g)$; где $\hat{O}(\hbar; f, g)$ - неопределенный пока асимптотический по $\hbar$ оператор в $L^2(V_n; C; \sqrt{\omega}\, d^nq)$; он антисимметричен и билинеен по f и g и исчезает при $\hbar \to 0$;
(Q4) $\hat{q}^i = q^i \cdot \hat{1}$, $\hat{p}_j = -\mathrm{i}\hbar(\partial_j + \frac{1}{4}\partial_j \lg \omega) \equiv \omega^{-\frac{1}{4}}\partial_j \cdot \omega^{\frac{1}{4}}$.

Следуя классическим работам по квантованию (см., например, [3], стр 425 - 475, и ссылки, приведенные там), нужно было бы в условии (Q3) положить $\hat{O}(\hbar; f, g) \equiv 0$, и такое $\mathcal{Q}$ было бы тем, что называют *полным (full) шредингеровским квантованием*. Полное оно в том смысле, что искомое

[1] Приводимая формулировка является более простой версией формулировки, данной в работах [1, 2], которая, в свою очередь, следует более общей деформационной формулировке в [4].

$\mathcal{Q}$ определяет по единому правилу квантовые аналоги для всей категории классических систем, образующих алгебру P_{2n}.

К сожалению, довольно легко показать , см., например, [3], стр 435 - 439, что полное в этом смысле квантование, вообще говоря, *не существует*. Исключение составляет только случаи, когда $\mathbf{sp}_{2n} \sim \mathbf{h}_{2n}$ и $\mathbf{sp}_{2n} \sim \mathbf{sp}_{2n}^{(2)}$, т.е. гамильтонианы не выше второй степени по координатам Дарбу q^i, p_j, и, следовательно, соответствующие уравнения Гамильтона линейны. В этом случае полное квантование достаточно продемонстрировать на примере базиса квадратичных мономов для $n = 1$: $(q^2)^\wedge = \hat{q}^2$, $(p^2)^\wedge = \hat{p}^2$, $(qp)^\wedge = \frac{1}{2}(\hat{q}\hat{p} + \hat{p}\hat{q})$. Эти выражения являются единственно возможными эрмитизациями с действительными коэффициентами при подстановке операторов $\hat{q}, \hat{p}$ в соответствующие мономы. После того как квадратичные квантовые гамильтонианы таким образом построены, можно предположить (но это будет волевой акт!), что абелева калибровочная инвариантность одинаково реализуется в классической и квантовой теории:

$$\left(p_i - \frac{\mathrm{i}\hbar}{mc}A_i(q)\right)^\wedge = \hat{p}_i - \frac{\mathrm{i}\hbar}{mc}A_i(\hat{q}). \tag{3}$$

Тогда мы приходим к стандартной квантовой механике заряда во внешнем электромагнитном поле и, в частном случае, в потенциальном поле, но в основе последней, как мы видим, должна быть калибровочная симметрия.

4. Асимптотическое шредингеровское квантование. Именно для того, чтобы иметь возможность квантовать в изложенном смысле категории гамильтонианов, более общих, чем квадратичные, мы ввели в условии (Q3) асимптотический произвол в виде члена $\hat{O}(\hbar; f, g)$. Он дает дополнительную свободу при построении $\mathcal{Q}$, но, как выясняется, чрезмерную.

Автору настоящей заметки известен только один способ, как этой свободой распорядиться для $\mathbf{sp}_{2n}(C^\infty)$:

(CQ1). Обратим внимание, что для q^i, p_j и $\hat{q}^i, \hat{p}_j$, которые будем называть теперь *первичными наблюдаемыми*, условие (Q3) выполняется точно при $O(\hbar; q, p) = 0$ вследствие условия (Q1), т.е. предполагается, что шредингеровское квантование дает операторное представление алгебры Гейзенберга $\mathbf{h}_{2n}$. [2]

[2]Замечательно, что этот важный факт находит подтверждение в альтернативном подходе [11] к построению квантовой механики в V_n как квазинерелятивистской асим-

(CQ2). Любому гамильтониану $f(q,p) \in \mathbf{sp}_{2n}^{(\infty)}$ (т.е. являющемуся полиномом по q^i и p_j) поставим в соответствие квантовый гамильтониан $\hat{f}$, полученный путем подстановки $q^i \to \hat{q}^i$, $p_j \to \hat{p}_j$ и последующей эрмитизации с действительными коэффициентами (*упорядочения операторов первичных наблюдаемых*) каждого монома в полиноме.

(CQ3). Пользуясь плотностью множества полиномов произвольной степени в C^∞, обобщим последнее правило на $\mathbf{sp}_{2n}(C^\infty)$, см. [8]. Для этого-то и нужны ограничения на топологию P_{2n} и на выбор координат q^i, введенные разделе 2. Таким образом будет построено отображение $\mathcal{Q}$ для достаточно широкой категории гамильтонианов, которое уместно называть *асимптотическим шредингеровским квантованием.* При этом нам не нужно знать явно вид операторов $\hat{O}(\hbar; f, g)$ в условии (Q3), а наоборот их можно вычислить, зная операторы $\hat{f}, \hat{g}$ и $\{f,g\}^\wedge$. Это означает, что вместо конкретизации условия (Q3) мы задаем отображение алгебры $\mathbf{lp_{2n}}(C^\infty)$ на операторы в $L^2(V_n; C; \sqrt{\omega}\; d^n q)$.

Однако имеется бесконечное множество процедур упорядочения, см., например, [6–8]. В математических работах, ориентированных, в основном, на установление существования отображения $\mathcal{Q}$, обычно используют упорядочение Вейля, (см. определение, например, в [8], стр. 207); все прочие возможности рассматриваются как в определенном смысле эквивалентные. Однако получаемые разными упорядочениями квантовые гамильтонианы имеют разные спектры и потому физически неэквивалентны. По-видимому, внутри любого известного формализма квантования, имеющего дело с достаточно широким классом систем нет способа выделить единственное "физически правильное" квантование, и всегда остаются какая-либо физическая неоднозначность или неточность.

5. Каноническое квантование геодезического движения. Чтобы лучше оценить эту ситуацию применим изложенную схему квантования к простейшей нелинейной системе, физическая и геометрическая интерпетациия которой совершеннно прозрачна. А именно, рассмотрим гамильтониан вида

$$H_\omega(q,p) = \frac{1}{2m}\omega^{ij}(q)p_ip_j. \tag{4}$$

птотики одночастичного сектора квантовой теории скалярного поля: здесь условие (Q3) выполняется в нерелятивистской теории ($c^{-1} = 0$). Однако оно нарушается для первичных наблюдаемых $\hat{q}^i, \hat{p}_j$ в первом порядке по c^{-2} и таким образом шредингеровское квантование должно быть пересмотрено при учете релятивистских поправок.

Он описывыет классическое движение точечной нейтральной частицы во внешнем статическом гравитационном поле в виде метрического тензора $\omega_{ij}(q)$ или, что тоже самое, движение по геодезической линии в V_n . При применении к H_ω схемы (CQ1) — (CQ2) оказывается [1,2], что все возможные эрмитизации с действительными коэффициентами составляют однопараметрическое семейство:

$$\hat{H}_\omega^{(\kappa)} = \frac{\kappa}{8m}\omega^{ij}(\hat{q})\hat{p}_i\hat{p}_j + \frac{2-\kappa}{4m}\hat{p}_i\omega^{ij}(\hat{q})\hat{p}_j + \frac{\kappa}{8m}\hat{p}_i\hat{p}_j\omega^{ij}(\hat{q}) \equiv -\frac{\hbar^2}{2m}\Delta_{(\omega)}(q) + V_{\mathrm{q}}^{(\kappa)}(q), \tag{5}$$

где κ, $-\infty < \kappa < \infty$, — произвольный параметр, $\Delta_{(\omega)}$ – оператор Лапласа-Бельтрами на V_n , а $V_{\mathrm{q}}^{(\kappa)}(q)$ - так называемый квантовый потенциал, зависящий только от ω_{ij} и его частных производных и, что очень важно, *неинвариантный относительно преобразований координат* q^i (в отличие от $\Delta_{(\omega)}$). Таким образом, шредингеровское квантование приводит к противоречию с общерелятивистской догмой общековариантности уравнений физики и к описываемому параметром κ произволу. [3]

6. Оправдание нековариантности квантования. Нековариантность квантования геодезического движения аналогична известной конформной аномалии в квантовой теории поля и ее уместно называть *квантовой аномалией пространственных диффеморфизмов* (C^∞–преобразований координат). Она может быть объяснена на основе идей К. Ровелли [9] об относительности квантовой механики (*relational quantum mechanics*) в смысле неизбежного (хотя, как правило, неявного) присутствия в квантовом описании системы информации о "наблюдающей" классической системе, влияющей на квантовую динамику. Соотношение информации о "наблюдаемой" и "наблюдающей" системах, зависит от выбора последней. Выбор той или иной системы координат $\{q^i\}$ *в качестве физических наблюдаемых* есть формализация выбора системы, измеряющей положение точечной частицы [11]. Координатные линии можно рассматривать как траектории микроскопических часов, арифметизующих V_n . Эти траектории сформированы действием на часы некоторой внешней силы, равной вектору первой нормали траектории, см, например, [12], стр. 18. Информация об этой силе включается в гамильтониан через $V_{\mathrm{q}}^{(\kappa)}(q)$, поскольку

[3] В работе [10] утверждается, что формализм, основанный на применении алгебры Клиффорда приводит в рассматриваемом частном случае к единственному гамильтониану, а именно к (5) с $V_{\mathrm{q}}^{(\kappa)}(q) \equiv 0$. На наш взгляд, это утверждение является следствием элементарной ошибки .

последний изменяется при изменении системы координат.

Есть только один класс систем координат, для которых первая кривизна и, соответственно, первая нормаль равны нулю. Это — *нормальные римановы координаты* (их называют также *квазидекартовыми*, см., например, [12], стр. 74). В них координатными линиями являются геодезические, выходящие из одной точки (начала) $\{q_0^i\}$, и координатами точки X в V_n являются направление касательной к геодезической линии OX в $\{q_0^i\}$ и расстояние вдоль нее. Таким образом, квазидекартовы координаты определяются только геометрией V_n и выбором начала $\{q_0^i\}$ и в этом смысле являются привелигированными. Можно ожидать (и было бы важно доказать), что именно при использовании таких наблюдаемых пространственного положения извлекается максимум информации о квантовом движении частицы.

7. Устранение неоднозначности шредингеровского квантования геодезического движения. Полагая невозможным выделение единственного "правильного" упорядочения первичных операторов внутри сформулированного формализма (что, вообще говоря, еще не факт), сравним полученный гамильтониан с тем, к какому гамильтониану для той же системы приводят другие формализмы квантования. Естественно ожидать, что *разные формализмы для одной и той же физической системы, должны приводить, хотя бы и приблизительно, к совпадающим результатам.* В качестве такого альтернативного формализма в [1,2] рассматривается квазиклассическое квантование ДеВитта [13]. Он использует следующий нерелятивистский пропагатор для частицы [4] в V_n:

$$< q,t|q_0,t_0 > \; = \; \omega^{-1/4}(q)D^{1/2}(q,t|q_0,t_0)\omega^{-1/4}(q)\; \exp\left(-\frac{i}{\hbar}S(q,t|q_0,t_0)\right) \quad (6)$$

$$S(q,t|q_0,t_0) \stackrel{def}{=} \int_{t_0}^{t}\frac{1}{2}\omega_{ij}(q,t)\dot{q}^i\dot{q}^j; \quad q_0 \stackrel{def}{=} q(t_0); \quad (7)$$

$$D(q,t|q_0,t_0) \stackrel{def}{=} \det\left(-\frac{\partial^2 S(q,t|q_0,t_0)}{\partial q^i\partial q_0^j}\right) \quad \text{(детерминантВанФлека)}$$

Рассматривая предельный переход $t \to t_0$ $(q \to q_0)$ *вдоль геодезической*

[4]Фактически формализм Де Витта можно считать упрощением формализма Блаттнера–Костанта–Стернберга в геометрическом квантовании, см., например, [14], при применении к нашей частной системе.

линии, соединяющей q и q_0, ДеВитт приходит к квантовому гамильтониану

$$\hat{H}_\omega^{(\mathrm{DW})}(q) = -\frac{\hbar^2}{2m}\left(\Delta_\omega(q) - \frac{1}{6}R_\omega(q) + o(q-q_0)\right), \qquad (8)$$

где $R_{(\omega)}$ - скалярная кривизна по метрике ω_{ij}.

Квазиклассический квантовый потенциал (*в нулевом приближении*) $V_{\mathrm{q}}^{\mathrm{DW}} \stackrel{def}{=} -(\hbar^2/(12m))R_\omega(q)$ выглядит как скаляр[5], однако вследствие построения, содержащего предельный переход именно вдоль геодезической линии, он получен именно для квазидекартовых координат (это отмечено также в [15]). Потребуем теперь, чтобы $V_{\mathrm{q}}^{(\kappa)} \equiv V^{\mathrm{DW}}$ *для квазидекартовых координат в окрестности их начала* $\{q_0\}$. Это имеет место при $\kappa = 0$, т.е. канонический квантовый гамильтониан для частицы в V_n равен замечательно симметричному выражению:

$$\hat{H}_\omega^{(0)} = \frac{1}{2m}\hat{p}_i\omega^{ij}(\hat{q})\hat{p}_j. \qquad (9)$$

Подчеркнем однако, что таким способом мы устранили неоднозначность пока лишь в гамильтониане. Этого, казалось бы, достаточно, поскольку операторы $\hat{q}^i$, $\hat{p}_j$ и $\hat{H}$ дают всю информацию о движении квантовой частицы. Однако, если бы мы захотели знать оператор $\hat{f}$, соответствующий какой-то иной классической наблюдаемой $f(q,p)$ но для той же системы (т.е. точечной частицы), то, по-видимому, пришлось бы рассматривать $f(q,p)$ в качестве гамильтониана некоторой физической системы и заново заниматься проблемой выбора упорядочения первичных наблюдаемых, поскольку для получения результата (9) был существенно использован квазиклассический пропагатор для геодезического движения. Ситуация складывается довольно странная, хотя формализм Блаттнера-Костанта–Стернберга, который является, на наш взгляд, обобщением подхода ДеВитта, возможно позволит применить представленную здесь схему для построения $\hat{f}$.

Отметим также, что установленное нами правило упорядочения ($\kappa = 0$), дает однозначный ответ на давний вопрос в формализме интегрирования по путям: в какой из точек ребра решетки интегрирования — на каком-то из концов или где-то между ними — надо брать значение подинтегрального выражения? Ответ, полученный в [1,2], таков: точно в середине.

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[5]Отметим замечательное совпадение этого выражения по форме с известным членом $(1/6)R$ в общерелятивистской теории скалярного поля с конформной связью [16]

8. Заключение. Изложенный материал представляется интересным не только тем, что здесь при определенных ограничениях решена фундаментальная проблема построения нерелятивистской квантовой механики в статическом внешнем гравитационном поле, но и продемонстрировано, какие серьезные вопросы о структуре квантовой теории возникают при последовательном рассмотрении конкретных нелинейных физических систем, имеющих ясный физический смысл.

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Dirac-type operators on curved spaces

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Abstract — The role of the Killing-Yano tensors in the construction of the Dirac-type operators is pointed out. The general results are applied to the case of the four-dimensional Euclidean Taub-Newman-Unti-Tamburino space. Three new Dirac-type operators, equivalent to the standard Dirac operator, are constructed from the covariantly constant Killing-Yano tensors of this space. Finally the Runge-Lenz operator for the Dirac equation in this background is expressed in terms of the forth Killing-Yano tensor which is not covariantly constant.

1 Introduction

In the study of the Dirac equation in curved spaces, it has been proved that the Killing-Yano tensors play an essential role in the construction of new Dirac-type operators. The Dirac-type operators constructed with the aid of covariantly constant Killing-Yano tensors are equivalent with the standard Dirac operator. The non-covariantly constant Killing-Yano tensors generates non-standard Dirac operators which are not equivalent to the standard Dirac operator and they are associated with the hidden symmetries of the space.

The general results are applied to the case of the four-dimensional Euclidean Taub-Newman-Unti-Tamburino (Taub-NUT) space. The motivation to carry out this example is twofold. First of all, in the Taub-NUT geometry there are known to exist four Killing-Yano tensors [1]. From this point of view the Taub-NUT manifold is an exceedingly interesting space to exemplify the effective construction of the conserved quantities in terms of geometric ones. On the other hand, the Taub-NUT geometry is involved in many modern studies in physics.

2 Dirac equation on a curved background

In what follows we shall consider the Dirac operator on a curved background which has the form

$$D_s = \gamma^\mu \hat{\nabla}_\mu . \tag{1}$$

In this expression the Dirac matrices γ_μ are defined in local coordinates by the anticommutation relations $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} I$ and $\hat{\nabla}_\mu$ denotes the canonical covariant derivative for spinors.

Carter and McLenaghan showed that in the theory of Dirac fermions for any isometry with Killing vector R_μ there is an appropriate operator [2]:

$$X_k = -i(R^\mu \hat{\nabla}_\mu - \frac{1}{4}\gamma^\mu\gamma^\nu R_{\mu;\nu}) \tag{2}$$

which commutes with the *standard* Dirac operator (1).

Moreover each Killing-Yano tensor $f_{\mu\nu}$ produces a *non-standard* Dirac operator of the form

$$D_f = -i\gamma^\mu(f_\mu{}^\nu \hat{\nabla}_\nu - \frac{1}{6}\gamma^\nu\gamma^\rho f_{\mu\nu;\rho}) \tag{3}$$

which anticommutes with the standard Dirac operator D_s.

3 Euclidean Taub-NUT space

Let us consider the Taub-NUT space and the chart with Cartesian coordinates $x^\mu(\mu,\nu = 1,2,3,4)$ having the line element

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = f(r)(d\vec{x})^2 + \frac{g(r)}{16m^2}(dx^4 + A_i dx^i)^2 \tag{4}$$

where $\vec{x}$ denotes the three-vector $\vec{x} = (r,\theta,\varphi)$, $(d\vec{x})^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2$ and $\vec{A}$ is the gauge field of a monopole div$\vec{A} = 0$, $\vec{B} = \text{rot}\,\vec{A} = 4m\frac{\vec{x}}{r^3}$. The real number m is the parameter of the theory which enter in the form of the functions $f(r) = g^{-1}(r) = V^{-1}(r) = \frac{4m+r}{r}$.

In the Taub-NUT geometry there are four Killing vectors [1,3]. On the other hand in the Taub-NUT geometry there are known to exist four Killing-Yano tensors of valence 2. The first three are covariantly constant

$$\begin{aligned} f_i &= 8m(d\chi + \cos\theta d\varphi) \wedge dx_i - \epsilon_{ijk}(1 + \frac{4m}{r})dx_j \wedge dx_k, \\ D_\mu f^\nu_{i\lambda} &= 0\,, \quad i,j,k = 1,2,3. \end{aligned} \tag{5}$$

The fourth Killing-Yano tensor is

$$f_Y = 8m(d\chi + \cos\theta d\varphi) \wedge dr + 4r(r+2m)(1 + \frac{r}{4m})\sin\theta d\theta \wedge d\varphi \tag{6}$$

having a non-vanishing covariant derivative

$$f_{Y\,r\theta;\varphi} = 2(1 + \frac{r}{4m})r\sin\theta. \tag{7}$$

In Taub-NUT space there is a conserved vector analogous to the Runge-Lenz vector of the Kepler-type problem [1,3]

$$\vec{K} = \frac{1}{2}\vec{K}_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu} = \vec{p}\times\vec{j} + \left(\frac{q^2}{4m} - 4mE\right)\frac{\vec{r}}{r} \tag{8}$$

where the conserved energy is $E = \frac{1}{2}g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}$. The components $K_{i\mu\nu}$ involved with the Runge-Lenz vector (8) are Stäckel-Killing tensors satisfying the equations $K_{i(\mu\nu;\lambda)} = 0$, $K_{i\mu\nu} = K_{i\nu\mu}$ and they can be expressed as symmetrized products of the Killing-Yano tensors f_i, f_Y and Killing vectors [4,5].

4 Dirac equation in the Taub-NUT space

When one uses Cartesian charts in the Taub-NUT geometry it is useful to consider the local frames given by tetrad fields $\hat{e}(x)$, such that $g_{\mu\nu} = \delta_{\hat{\alpha}\hat{\beta}}\hat{e}^{\hat{\alpha}}_{\mu}\hat{e}^{\hat{\beta}}_{\nu}$. The four Dirac matrices $\hat{\gamma}^{\hat{\alpha}}$ satisfy $\{\hat{\gamma}^{\hat{\alpha}}, \hat{\gamma}^{\hat{\beta}}\} = 2\delta^{\hat{\alpha}\hat{\beta}}$ and the standard Dirac operator is

$$D_s = \hat{\gamma}^{\hat{\alpha}}\hat{\nabla}_{\hat{\alpha}} = i\sqrt{V}\vec{\hat{\gamma}}\cdot\vec{P} + \frac{i}{\sqrt{V}}\hat{\gamma}^4 P_4 + \frac{i}{2}V\sqrt{V}\hat{\gamma}^4\vec{\Sigma}^*\cdot\vec{B} \tag{9}$$

where $\hat{\nabla}_{\hat{\alpha}}$ are the components of the spin covariant derivatives with local indices

$$\hat{\nabla}_i = i\sqrt{V}P_i + \frac{i}{2}V\sqrt{V}\varepsilon_{ijk}\Sigma^*_j B_k\,, \quad \hat{\nabla}_4 = \frac{i}{\sqrt{V}}P_4 - \frac{i}{2}V\sqrt{V}\vec{\Sigma}^*\cdot\vec{B}\,. \tag{10}$$

These depend on the momentum operators $P_i = -i(\partial_i - A_i\partial_4)$, $P_4 = -i\partial_4$ which obey the commutation rules $[P_i, P_j] = i\varepsilon_{ijk}B_k P_4$ and $[P_i, P_4] = 0$.

The Hamiltonian operator of the *massless* Dirac field reads [6–8]:

$$H = \hat{\gamma}^5 D_s = \begin{pmatrix} 0 & V\pi^*\frac{1}{\sqrt{V}} \\ \sqrt{V}\pi & 0 \end{pmatrix}. \tag{11}$$

This is expressed in terms of the operators

$$\pi = \sigma_P - \frac{iP_4}{V}, \quad \pi^* = \sigma_P + \frac{iP_4}{V}, \quad \sigma_P = \vec{\sigma}\cdot\vec{P} \tag{12}$$

and the Klein-Gordon operator has the form $\Delta = -\nabla_\mu g^{\mu\nu}\nabla_\nu = V\pi^*\pi = V\vec{P}^2 + \frac{1}{V}P_4^2$.

The conserved observables can be found among the operators which commute or anticommute with D_s and $\hat{\gamma}^5$ [6, 9]. That is the case of the total angular momentum $\vec{J} = \vec{L} + \vec{S}$ where the orbital angular momentum is $\vec{L} = \vec{x} \times \vec{P} - 4m\frac{\vec{x}}{r}P_4$.

Dirac-type operators are constructed from the Killing-Yano tensors $f_i (i = 1,2,3)$ and f_Y using eq.(3). In the quantum Dirac theory these operators replace the supercharges from the pseudo-classical approach. The first three

$$Q_i = -i f_{i\,\hat{\alpha}\hat{\beta}} \hat{\gamma}^{\hat{\alpha}} \hat{\nabla}^{\hat{\beta}} \tag{13}$$

anticommute with D_s and $\hat{\gamma}^5$, commute with H and obey the $N = 4$ superalgebra, including $Q_0 = iD_s = i\hat{\gamma}^5 H$ [10]:

$$\{Q_A, Q_B\} = 2\delta_{AB} H^2, \quad A, B, ... = 0, 1, 2, 3 \tag{14}$$

linked to the hyper-Kähler geometry of the Taub-NUT space.

The concrete form of these operators depends on the representation of the Dirac matrices which can be changed at any time with the help of a non singular operator T such that all of the 4×4 matrix operators of the Dirac theory transform as $X \to X' = TXT^{-1}$. In this way one obtains an *equivalent* representation which preserves the commutation and the anticommutation relations. In [8] we have used such transformations for pointing out that the convenient representations where we work are equivalent to an unitary one.

Finally, using eq.(3), from the fourth Killing-Yano tensor f_Y of the Taub-NUT space we can construct the Dirac-type operator [6, 11–13]

$$Q_Y = \frac{r}{4m}\left\{H\,, \begin{pmatrix} \sigma_r & 0 \\ 0 & -\sigma_r V^{-1} \end{pmatrix}\right\} = i\frac{r}{4m}\left[Q_0\,, \begin{pmatrix} \sigma_r & 0 \\ 0 & \sigma_r V^{-1} \end{pmatrix}\right]. \tag{15}$$

The hidden symmetries of the Taub-NUT geometry are encapsulated in the non-trivial Stäckel-Killing tensors $K_{i\mu\nu}, (i = 1, 2, 3)$. For the Dirac theory the construction of the Runge-Lenz operator can be done using products among the Dirac-type operators Q_Y and Q_i.

Let us define the operator [6]:

$$N_i = m\,\{Q_Y, Q_i\} - J_i P_4. \tag{16}$$

The components of the operator $\vec{N}$ commutes with H and satisfy the following commutation relations

$$\begin{aligned} [N_i, P_4] &= 0, \quad [N_i, J_j] = i\varepsilon_{ijk} N_k\,, \\ [N_i, Q_0] &= 0, \quad [N_i, Q_j] = i\varepsilon_{ijk} Q_k P_4\,, \\ [N_i, N_j] &= i\varepsilon_{ijk} J_k F^2 + \frac{i}{2}\varepsilon_{ijk} Q_i H \end{aligned} \tag{17}$$

where $F^2 = {P_4}^2 - H^2$. In order to put the last commutator in a form close to that from the scalar case [1,3], we can redefine the components of the Runge-Lenz operator, $\vec{\mathcal{K}}$, as follows:

$$\mathcal{K}_i = N_i + \frac{1}{2}H^{-1}(F - P_4)Q_i \tag{18}$$

having the desired commutation relation [6]:

$$[\mathcal{K}_i, \mathcal{K}_j] = i\varepsilon_{ijk}J_kF^2 \,. \tag{19}$$

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Fermion Self-Energy during Inflation

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Abstract — I report on work done with Tomislav Prokopec (Heidelberg, CERN). We computed the one loop self-energy of a massless fermion during inflation. When the fermion is free it experiences only a small amount of inflationary particle production owing to the conformal invariance of its classical action. However, when the fermion is Yukawa coupled to a massless, minimally coupled scalar, there is copious production of fermions. In a more complicated model this effect might generate baryon asymmetry during inflation.

1 Introduction

Tomislav Prokopec and I have recently computed the one loop self-energy, during de Sitter inflation, of a massless fermion which is Yukawa coupled to a massless, minimally coupled scalar. The diagrammatic representation of what we did is given in Fig. 1. This would be a trivial exercise in flat space, and a fairly uninteresting one. The inflationary background geometry makes the calculation both nontrivial and physically interesting. What we found is that inflation engenders copious production of scalars which then decay into fermion-anti-fermion pairs. The first fact was known but the second is novel and potentially quite significant. In a more complicated theory it may provide a mechanism for generating a baryon asymmetry *during* inflation.

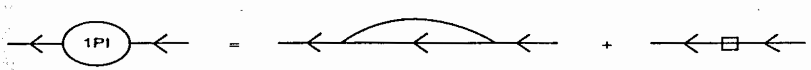

Figure 1: *The one loop fermion self-energy.* Fermion lines have an arrow, scalar lines do not. The final diagram gives the contribution of field strength renormalization.

Most of the theoretical technology we used was developed in a previous computation, with Ola Törnkvist, of the one loop vacuum polariztion induced by massless scalar QED during inflation [1, 2]. However, the result is very different. Whereas the photon develops a mass, which highly *suppresses* the production of photons during inflation, we find copious production of Yukawa coupled fermions. I will therefore devote most of this talk to explaining the physics in very simple terms. In Section 2 I give a brief review of inflation. The largest part of the talk is Section 3. In it I explain why the direct production of massless, minimally coupled scalars occurs during inflation, and why there is no significant direct production of fermions. Section 4 presents the result, without giving the derivation, and shows how it implies fermion production.

2 Inflation

On the largest scales the universe is amazingly homogeneous and isotropic [3]. It also seems to be devoid of spatial curvature [4]. The spacetime geometry consistent with these three features is characterized by the following simple invariant element,

$$ds^2 = -dt^2 + a^2(t) d\vec{x} \cdot d\vec{x} \; . \tag{1}$$

The coordinate t represents physical time, the same as it does in flat space. However, the physical distance between $\vec{x}$ and $\vec{y}$ is not given by their Euclidean norm, $\|\vec{x} - \vec{y}\|$, but rather by $a(t)\|\vec{x} - \vec{y}\|$. Because it converts coordinate distance into physical distance $a(t)$ is known as the *scale factor*.

Although the scale factor is not directly measurable, three simple observable quantities can be constructed from it,

$$z \equiv \frac{a_0}{a(t)} - 1 \quad , \quad H(t) \equiv \frac{\dot{a}}{a} \quad , \quad q(t) \equiv -\frac{a\ddot{a}}{\dot{a}^2} = -1 - \frac{\dot{H}}{H^2} \; . \tag{2}$$

The *redshift* z gives the proportional increase in the wavelength of light emitted at time t and received at the current time, t_0. Redshift is often used to measure cosmological time, even for epochs from which we detect no radiation. The *Hubble parameter* $H(t)$ gives the rate at which the universe is expanding. It's current value is, $H_0 = (71{}^{+4}_{-3}) \frac{\text{km}}{\text{s Mpc}} \simeq 2.3 \times 10^{-18}$ Hz [4]. The *deceleration parameter* $q(t)$ is less well measured. Observations of Type Ia supernova are consistent with a current value of $q_0 \simeq -.6$ [5].

Inflation is defined as a phase of accelerated expansion, that is, $q(t) < 0$ with $H(t) > 0$. From the current values of the cosmological parameters one can see that the universe seems to be in such a phase now. However, I wish to discuss *primordial inflation*, which is conjectured to have occurred at something like 10^{-37} seconds after the beginning of the universe with a Hubble parameter 55 orders of magnitude larger than it is today. There are many reasons for believing that the very early universe underwent such a phase [6]. I will confine myself to reviewing how inflation resolves the *smoothness problem.* This can be summed up in the question, why does the large scale universe possess such a simple geometry (1)?

To understand the problem we need to compare the distance light can travel from the beginning of the universe to the time of some observable event, with the distance it can travel from then to the present. From the invariant element (1) we see that the radial position of a light ray obeys, $dr = \pm dt/a(t)$. The minus sign gives the *past light-cone* of the point $x^\mu = (t_0, \vec{0})$, whereas the plus sign gives the *future light-cone* of a point $x^\mu = (t_i, \vec{0})$ at the beginning of the universe,

$$R_{\rm past} = \int_{t_{obs}}^{t_0} \frac{dt}{a(t)} \quad , \qquad R_{\rm future} = \int_{t_i}^{t_{obs}} \frac{dt}{a(t)} \; . \tag{3}$$

We can observe thermal radiation from the time of decoupling ($z_{dec} \simeq 1089$) whose temperature is isotropic to one part in 10^5. Unless the universe simply began this way — which seems unlikely — equilibrium must have been established by causal processes. In other words, we must have $R_{\rm future} > R_{\rm past}$.

Suppose that, during the period $t_1 \leq t \leq t_2$, the deceleration parameter is constant $q(t) = q_1$. In that case we can obtain explicit expressions for the Hubble parameter and the scale factor in terms of their values at $t = t_1$,

$$H(t) = \frac{H_1}{1 + (1{+}q_1)H_1(t{-}t_1)} \quad \text{and} \quad a(t) = a_1\Big[1 + (1{+}q_1)H_1(t{-}t_1)\Big]^{\frac{1}{1+q_1}} \; . \tag{4}$$

These expressions permit us to evaluate the fundamental integral involved in the past and future light-cones (3),

$$\int_{t_1}^{t_2} \frac{dt}{a(t)} = \frac{1}{a_1 H_1 q_1}\Big[1{+}(1{+}q_1)H_1(t{-}t_1)\Big]^{\frac{q_1}{1+q_1}}\Big|_{t_1}^{t_2} = \frac{1}{q_1}\left\{\frac{1}{a_2 H_2} - \frac{1}{a_1 H_1}\right\} . \tag{5}$$

Although q_0 is negative, this is a recent event ($z \simeq 1$) which followed a long period of nearly perfect matter domination with $q = +\frac{1}{2}$. Much before the time of matter-radiation equality ($z_{eq} \simeq 3200$) the universe was almost perfectly radiation-dominated, which corresponds to $q = +1$. To simplify the computation we will ignore the recent phase of acceleration and also the transition periods,

$$a(t)H(t) = a_0 H_0 \begin{cases} \sqrt{1+z} & \forall \, z \leq z_{eq} \\ \frac{1+z}{\sqrt{1+z_{eq}}} & \forall \, z \geq z_{eq} \end{cases} \; . \tag{6}$$

The cosmic microwave radiation was emitted within about a hundred redshifts of $z_{dec} < z_{eq}$, so the past light-cone is,

$$R_{\rm past} = \frac{2}{a_0 H_0} \left\{ \frac{1}{\sqrt{1+0}} - \frac{1}{\sqrt{1+z_{dec}}} \right\} \simeq \frac{2}{a_0 H_0} \; . \tag{7}$$

The future light-cone derives from both epochs and it depends slightly upon the beginning redshift, z_{beg},

$$R_{\rm future} = \frac{2}{a_0 H_0} \left\{ \frac{1}{\sqrt{1+z_{dec}}} - \frac{1}{\sqrt{1+z_{eq}}} \right\} + \frac{1}{a_0 H_0} \left\{ \frac{\sqrt{1+z_{eq}}}{1+z_{eq}} - \frac{\sqrt{1+z_{eq}}}{1+z_{beg}} \right\} . \tag{8}$$

One maximizes $R_{\rm future}$ by taking $z_{beg} \to \infty$, but it isn't enough. Under the assumption of $q = +1$ before z_{eq} we are forced to conclude that the 2-dimensional surface we can see from the time of decoupling consists of $(R_{\rm past}/R_{\rm future})^2 \simeq 2200$ regions which cannot have exchanged even a photon since the beginning of time! So how did they reach equilibrium?

This embarrassment resulted from the fact that the upper limit of integration dominates $R_{\rm future}$ for positive deceleration. Inflation solves the problem by positing a very early epoch of negative deceleration. This makes the lower limit of $R_{\rm future}$ dominate, so the future light-cone can be made as large as necessary by increasing z_{beg}.

3 Inflationary Particle Production

3.1 Virtual particles in flat space

To understand why certain kinds of virtual particles can be ripped out of the vacuum during inflation it is instructive to review what constrains the lifetimes of virtual particles in flat space. A particle with wave vector $\vec{k}$ and mass m has energy,

$$E(\vec{k}) = \sqrt{m^2 + \|\vec{k}\|^2} \; . \tag{9}$$

According to quantum field theory, virtual particles are continually emerging from the vacuum with all different momenta. This process can conserve momentum if a virtual pair emerges with opposite 3-momenta, but it cannot conserve energy. Before the pair came into existence the energy was zero, afterwards it was $2E(\vec{k})$. The energy-time uncertainty principle asserts that we cannot detect this violation provided the pair annihilates in less than a time Δt given by the inequality,

$$\int_t^{t+\Delta t} dt' \, 2E(\vec{k}) \lesssim 1 \qquad \Longrightarrow \qquad \Delta t \lesssim \frac{1}{2E(\vec{k})} \; . \tag{10}$$

Of course the formalism of quantum field theory automatically incorporates these effects, without the need to invoke additional principles. However, the energy-time uncertainty principle allows one to understand many things. For example, virtual electron-positron pairs polarize the vacuum the most because they are the lightest charged particles and hence have the longest time to align themselves with applied fields. Similarly, it is long wave length virtual particles that survive the longest, which is one way of understanding long range forces.

3.2 Virtual particles in cosmology

In a homogeneous and isotropic geometry (1) particles are still labeled by their constant, comoving wave vectors. However, because $\vec{k}$ involves an inverse wave length it must be divided by the scale factor in computing physical quantities such as the energy,

$$E(t, \vec{k}) = \sqrt{m^2 + \|\vec{k}\|^2/a^2(t)} \; . \tag{11}$$

Let us now reconsider the emergence of a virtual pair of such particles with wave vectors $\pm\vec{k}$. If they emerge at time t, the energy-time uncertainty principle implies we will detect no violation of energy conservation provided they annihilate within a time Δt such that,

$$\int_t^{t+\Delta t} dt' \, 2E(t', \vec{k}) \lesssim 1 \; . \tag{12}$$

What can increase Δt? Obviously anything that reduces $E(t', \vec{k})$. At fixed wave vector $\vec{k}$ and time t' this is accomplished by taking the mass to zero. Zero mass simplifies the integrand in (12) to $2\|\vec{k}\|/a(t)$. Up to a constant this is the same expression as in the light-cones (3)! We have already seen that negative deceleration (and hence inflation) causes these integrals to be dominated by their lower limits. Hence they cannot become arbitrary large as Δt goes to infinity. The most negative deceleration consistent with stability is $q_i = -1$, for which the uncertainty bound gives,

$$m = 0 \text{ and } a(t) = a_i e^{H_i t} \quad \Longrightarrow \quad \int_t^{t+\Delta t} dt' \, 2E(t, \vec{k}) = \frac{2\|\vec{k}\|}{H_i a(t)} \left[1 - e^{-H_i \Delta t}\right] . \tag{13}$$

We therefore conclude that any virtual particle pair which emerges with $\|\vec{k}\| \lesssim H_i a(t)$ can survive for ever.

3.3 Conformal invariance is the kiss of death

One might imagine that the big obstacle to inflationary particle production is nonzero mass, but this is not so. At the enormous energy scales envisaged for primordial inflation all the known particles are effectively massless. The reason most of them still do not experience inflationary particle production is that they possess a symmetry suppressing the rate at which virtual particles emerge from the vacuum.

The killer symmetry is known as *conformal invariance.* It rescales the fields which carry scalars (ϕ), spin $\frac{1}{2}$ fermions (ψ), vectors (A_μ) and gravitons ($g_{\mu\nu}$) by powers of an arbitrary function of space and time $\Omega(x)$,

$$\phi(x) \rightarrow \Omega^{-1}(x)\phi(x) \quad , \quad A_\mu(x) \rightarrow A_\mu(x) \; , \tag{14}$$

$$\psi(x) \rightarrow \Omega^{-\frac{3}{2}}(x)\psi(x) \quad , \quad g_{\mu\nu}(x) \rightarrow \Omega^2(x) g_{\mu\nu}(x) \; . \tag{15}$$

A simple, conformally invariant theory is electromagnetism,

$$\mathcal{L}_{\rm EM} = -\frac{1}{4} F_{\alpha\beta} F_{\rho\sigma} g^{\alpha\rho} g^{\beta\sigma} \sqrt{-g} \; , \tag{16}$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$. Since the vector potential is unaffected by a conformal transformation, the Lagrangian's invariance follows from the combination of inverse metrics and the square root of the determinant,

$$g^{\alpha\rho} g^{\beta\sigma} \sqrt{-g} \rightarrow \Omega^{-2} g^{\alpha\rho} \cdot \Omega^{-2} g^{\beta\sigma} \cdot \Omega^4 \sqrt{-g} = g^{\alpha\rho} g^{\beta\sigma} \sqrt{-g} \; . \tag{17}$$

The connection between conformal invariance and cosmology derives from the existence of a coordinate system in which the general homogeneous and isotropic metric (1) becomes just $g_{\mu\nu} = a^2 \eta_{\mu\nu}$,

$$dt = a d\eta \qquad \Longrightarrow \qquad ds^2 = a^2 \Big(-d\eta^2 + d\vec{x} \cdot d\vec{x} \Big) = a^2 \eta_{\mu\nu} dx^\mu dx^\nu \; . \tag{18}$$

If we take a conformally invariant Lagrangian and express it in terms of the conformally rescaled fields with $\Omega = 1/a$, it is just as if the theory was in flat space! Some examples are the conformally coupled scalar, massless Dirac fermions, and electromagnetism,

$$\mathcal{L}_{\rm CCS} = -\frac{1}{2} \partial_\mu \phi \partial_\nu \phi g^{\mu\nu} \sqrt{-g} - \frac{1}{12} \phi^2 R \sqrt{-g} \;\; = \;\; -\frac{1}{2} \partial_\mu (a\phi) \partial_\nu (a\phi) \eta^{\mu\nu} \; , \tag{19}$$

$$\mathcal{L}_{\rm Dirac} = \overline{\psi} e^\mu{}_b \gamma^b \Big(i\partial_\mu - \frac{1}{2} A_{\mu cd} J^{cd} \Big) \psi \sqrt{-g} \;\; = \;\; (a^{\frac{3}{2}} \overline{\psi}) \gamma^\mu i \partial_\mu (a^{\frac{3}{2}} \psi) \; , \tag{20}$$

$$\mathcal{L}_{\rm EM} = -\frac{1}{4} F_{\alpha\beta} F_{\rho\sigma} g^{\alpha\rho} g^{\beta\sigma} \sqrt{-g} \;\; = \;\; -\frac{1}{4} F_{\alpha\beta} F_{\rho\sigma} \eta^{\alpha\rho} \eta^{\beta\sigma} \; . \tag{21}$$

Physics does not depend upon local, invertible field redefinitions, so a conformally invariant theory cannot be different, in conformal coordinates, than it is in flat space. This applies to all physical quantities, including the rate at which virtual particles emerge from the vacuum, per unit conformal time. Converting back to physical time gives the stated suppression,

$$\frac{dn}{dt} = \frac{d\eta}{dt}\frac{dn}{d\eta} = \frac{1}{a}\Big(\text{flat space rate}\Big) . \tag{22}$$

Therefore any conformally invariant, massless virtual particles which happen to emerge with $\|\vec{k}\| \lesssim H_i a(t)$ will become real, but not many will emerge.

3.4 Massless minimally coupled scalars

Most familiar particles become (classically) conformally invariant when their masses are taken to zero. The exceptions are gravitons and massless, minimally coupled scalars. The Lagrangian density for the later is,

$$\mathcal{L}_{\rm MMCS} = -\frac{1}{2}\partial_\mu\phi\partial_\nu\phi g^{\mu\nu}\sqrt{-g} = -\frac{1}{2}a^2\partial_\mu\phi\partial_\nu\phi\eta^{\mu\nu} . \tag{23}$$

Integrating over space to obtain the Lagrangian, then using Parseval's theorem to convert to Fourier space, gives a form we can recognize,

$$L_{\rm MMCS} \equiv \int d^3x \mathcal{L}_{\rm MMCS} = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\Big\{a^3(t)|\dot{\widetilde{\phi}}(t,\vec{k})|^2 - a(t)\|\vec{k}\|^2|\widetilde{\phi}(t,\vec{k})|^2\Big\}. \tag{24}$$

Each wave vector $\vec{k}$ represents an independent harmonic oscillator with a time dependent mass, $m(t) \sim a^3(t)$, and frequency, $\omega(t) = \|\vec{k}\|/a(t)$!

For the case of de Sitter inflation ($a(t) = a_i e^{H_i t}$) the mode functions take a simple form,

$$\widetilde{\phi}(t,\vec{k}) = u(t,k)a(\vec{k}) + u^*(t,k)a^\dagger(-\vec{k}) \text{ where } u(t,k) \equiv \frac{H_i}{\sqrt{2k^3}}\Big(1 - \frac{ik}{aH_i}\Big)e^{\frac{ik}{aH_i}}. \tag{25}$$

To understand the meaning of the operators $a(\vec{k})$ and $a^\dagger(\vec{k})$, note that the minimum energy in wave vector $\vec{k}$ at time t is $\omega(t)$. However, the state with this energy does not evolve onto itself. Indeed, this theory has no stationary states! A reasonable "vacuum" is the state that was minimum energy in the distant past. This is known as *Bunch-Davies vacuum* and it is defined by $a(\vec{k})|\Omega\rangle = 0$.

For Bunch-Davies vacuum the 0-point energy in wave vector $\vec{k}$ is,

$$E_0(t,\vec{k}) = \frac{1}{2}a^3(t)|\dot{u}(t,k)|^2 + \frac{1}{2}a(t)\|\vec{k}\|^2|u(t,k)|^2 = \frac{\|\vec{k}\|}{2a(t)} + \frac{H_i^2 a(t)}{4\|\vec{k}\|} . \tag{26}$$

The first term on the far right represents the irreducible, minimum energy. The second term gives the extra energy due to inflationary particle production. Since the energy of a particle of wave number $\vec{k}$ is $\|\vec{k}\|/a(t)$ we can easily compute the number of particles,

$$N(t,\vec{k}) = \left(\frac{H_i a(t)}{2\|\vec{k}\|}\right)^2 . \tag{27}$$

As one might expect from the preceding discussion, this is much less than one at very early times, and it becomes order one when the wave number just begins to satisfy the uncertainty bound (13).

3.5 Scalar decay to fermions

Although we have seen that inflation produces enormous numbers of massless, minimally coupled scalars, the conformal invariance of the Dirac Lagrangian (20) implies that there can be no comparable, direct production of fermions. However, it is still possible to make lots of fermions during inflation by allowing the scalars to decay into them. This can be accomplished by making scalars and fermions interact through a Yukawa coupling,

$$\mathcal{L}_{\rm Yukawa} = \mathcal{L}_{\rm MMCS} + \mathcal{L}_{\rm Dirac} - \Gamma\phi\overline{\psi}\psi\sqrt{-g}, \tag{28}$$

$$= -\frac{1}{2}a^2\partial_\mu\phi\partial_\nu\phi\eta^{\mu\nu} + (a^{\frac{3}{2}}\overline{\psi})\gamma^\mu i\partial_\mu(a^{\frac{3}{2}}\psi) - \Gamma a^4\phi\overline{\psi}\psi. \tag{29}$$

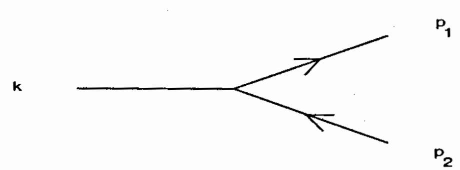

Figure 2: *Scalar decay into a fermion-anti-fermion pair.* The initial state scalar has wave vector $\vec{k}$. The final state fermions have wave vectors $\vec{p}_1 = \vec{p}$ and $\vec{p}_2 = \vec{k} - \vec{p}$.

The decay process is depicted in Fig. 2. Just as with inflationary particle production, 3-momentum is conserved but energy is generally not,

$$\vec{k} = \vec{p} + (\vec{k} - \vec{p}) \qquad \Longrightarrow \qquad \Delta E(t) = \Big[\|\vec{p}\| + \|\vec{k} - \vec{p}\| - \|\vec{k}\|\Big] \frac{1}{a(t)} \; . \tag{30}$$

In flat space the decay would be only be allowed for the infinitesimal phase space in which three momenta are co-linear. However, just as for particle production, the inflationary expansion weakens the constraint imposed by the energy-time uncertainty principle so as to permit the decay over a large volume of phase space,

$$\int_t^{t+\Delta t} dt' \Delta E(t') \lesssim 1 \qquad \Longrightarrow \qquad \Big[\|\vec{p}\| + \|\vec{k} - \vec{p}\| - \|\vec{k}\|\Big] \lesssim H a(t) \, . \tag{31}$$

This is the physics behind our result.

4 The Calculation

I now specialize to the de Sitter scale factor, $a(t) = e^{Ht} = -1/H\eta$. It is useful to express the propagators in terms of the following conformal coordinate interval,

$$\Delta x^2(x;x') \equiv \|\vec{x} - \vec{x}'\|^2 - (|\eta - \eta'| - i\epsilon)^2 \, . \tag{32}$$

Owing to the conformal invariance of the free Lagrangian (20), the fermion propagator is a trivial rescaling of the flat space result,

$$iS_{ij}(x;x') = (aa')^{-\frac{3}{2}} \gamma^\mu_{ij} \, i\partial_\mu \left\{ \frac{1}{4\pi^2} \frac{1}{\Delta x^2(x;x')} \right\} , \tag{33}$$

where $a \equiv a(t)$ and $a' \equiv a(t')$ are the scale factors evaluated at the two points. The free scalar is not conformally invariant so its propagator has an additional term,

$$i\Delta(x;x') = \frac{1}{4\pi^2} \left\{ \frac{(aa')^{-1}}{\Delta x^2(x;x')} - \frac{1}{2} H^2 \ln\Big[H^2 \Delta x^2(x;x')\Big] \right\} . \tag{34}$$

The $\phi\overline{\psi}_i \psi_j$ vertex and the fermion field strength renormalization are,

$$-i\Gamma a^4 \delta_{ij} \qquad \text{and} \qquad i\delta Z_2 (aa')^{\frac{3}{2}} \gamma^\mu_{ij} \, i\partial_\mu \delta^4(x - x') \, . \tag{35}$$

Computing one loop diagrams in position space is easy. One simply multiplies the various vertices and propagators. There are no integrations. The two diagrams of Fig. 1 give,

$$\begin{aligned} -i\Big[{}_i\Sigma_j\Big](x;x') \;&=\; \Big(-i\Gamma a^4 \delta_{ik}\Big) iS_{k\ell}(x;x') \Big(-i\Gamma a'^4 \delta_{\ell j}\Big) i\Delta(x;x') \\ &\qquad + i\delta Z_2 (aa')^{\frac{3}{2}} \gamma^\mu_{ij} \, i\partial_\mu \delta^4(x - x') \, . \end{aligned} \tag{36}$$

Well, it isn't *quite* that easy! The actual calculation was done using the Schwinger-Keldysh formalism, which involves summing diagrams with slight variations in the $i\epsilon$ prescription of expression (32) [7,8].

One must also account for the ultraviolet divergences which manifest as products that are not integrable functions of x^μ and x'^μ. One deals with these by partially integrating to reduce negative powers of $\Delta x^2(x;x')$ until the result is integrable. This procedure can be implemented so as to segregate the divergence to a delta function that can then be absorbed by δZ_2. Doing this rigorously requires the use of a regulator. We employed dimensional regularization, which makes slight changes in the fermion propagator (33) and the vertices (35), and major changes in the scalar propagator (34) [9].

Most of the formalism has been discussed in print [2] so I will confine myself to quoting the fully renormalized result,

$$\begin{aligned}\Big[{}_i\Sigma_j\Big](x;x') = \frac{-\Gamma^2}{2^8\pi^3}(aa')^{\frac{3}{2}}\gamma^\mu_{ij}\, i\partial_\mu\partial^4 \Big\{\theta(\Delta\eta)\theta(\Delta\eta-\Delta x)\Big(\ln\Big[\mu^2(\Delta\eta^2-\Delta x^2)\Big]-1\Big)\Big\} \\ -\frac{\Gamma^2}{2^5\pi^2}\ln(aa')(aa')^{\frac{3}{2}}\gamma^\mu_{ij}\, i\partial_\mu\delta^4(x-x') \\ -\frac{\Gamma^2 H^2}{2^6\pi^3}(aa')^{\frac{5}{2}}\gamma^\mu_{ij}\, i\partial_\mu\partial^2\Big\{\theta(\Delta\eta)\theta(\Delta\eta-\Delta x)\ln\Big[H^2(\Delta\eta^2-\Delta x^2)\Big]\Big\}. \quad (37)\end{aligned}$$

The first line is the conformally rescaled flat space result. Note that it involves a renormalization scale μ. The second line is the well-known contribution from the conformal anomaly. The intrinsic de Sitter result is on the third line. It is distinguished from the other two terms by its extra factor of aa'.

The self-energy gives quantum corrections to the Dirac equation,

$$a^{\frac{3}{2}}\gamma^\mu_{ij} i\partial_\mu\Big(a^{\frac{3}{2}}\psi_i(x)\Big) + \int d^4x'\Big[{}_i\Sigma_j\Big](x;x')\psi_j(x') = 0\ . \quad (38)$$

Note that the flat space part of the one loop self-energy (37) has the same number of scale factors as the tree order differential operator. Perturbation theory only makes sense if the coupling constant is small, so we may assume $\Gamma^2 \ll 1$. This means that the flat space part of the one loop self-energy can be ignored. So too can the conformal anomaly, which is only enhanced by a factor of $\ln(a)$. However, we cannot necessarily ignore the de Sitter correction. Although it is also down by Γ^2, it is enhanced by a scale factor, which becomes enormously large during inflation. It therefore makes sense to keep only the de Sitter contribution of (37).

Because the background is spatially translation invariant it also makes sense to look for plane wave solutions,

$$\psi_i(\eta, \vec{x}) = a^{-\frac{3}{2}} \chi_i(\eta) e^{i\vec{k}\cdot\vec{x}} , \tag{39}$$

Because the self-energy conserves helicity, we may as well also specialize to 2-component helicity eigenstates,

$$\chi_i(\eta) = \begin{pmatrix} \chi_L(\eta) \\ \chi_R(\eta) \end{pmatrix} \qquad \text{with} \qquad \vec{k}\cdot\vec{\sigma}\,\chi_{L,R} = \pm k\,\chi_{L,R} \,. \tag{40}$$

For the left-handed spinor the result of performing the spatial integrations is,

$$0 = (i\partial_0 \pm k)\chi_L(\eta) + \frac{i\Gamma^2 H^2}{8\pi^2} a \int_{\eta_i}^{\eta} d\eta' a' \chi_L(\eta') e^{\mp ik\Delta\eta} \left[2\ln(2H\Delta\eta) + 1 + \int_0^{2k\Delta\eta} d\tau \frac{e^{\pm i\tau} - 1}{\tau} \right] . \tag{41}$$

To obtain the result for a right-handed spinor with $\pm$ helicity, simply change the signs of $\pm k$, $\mp ik\Delta\eta$ and $\pm i\tau$.

Recovering the full time evolution of this sort of nonlocal mode solution requires numerical integration. However, it is straightforward to get the asymptotic behavior for late times. In this regime the physical 3-momentum has redshifted to essentially zero, so we can simplify (41) by setting $k = 0$. This obliterates the distinctions between left and right handedness, and between different helicities. The equation can be further simplified by transforming from conformal time η to co-moving time t,

$$\partial_t \chi + \frac{\Gamma^2 H^2}{4\pi^2} \int_0^t dt' \chi \left\{ \ln\left[2e^{-Ht'} - 2e^{-Ht}\right] + \frac{1}{2} \right\} \approx 0 \,. \tag{42}$$

Now factor the leading exponential out of the logarithm,

$$\ln\left[2e^{-Ht'} - 2e^{-Ht}\right] = -Ht' + \ln(2) + \ln\left[1 - e^{-H\Delta t}\right] . \tag{43}$$

It is an excellent approximation to retain only the term $-Ht'$. Acting another derivative results in a local equation,

$$\partial_t^2 \chi - \frac{\Gamma^2 H^2}{4\pi^2} Ht\, \chi \approx 0 \,, \tag{44}$$

whose approximate solution can be obtained by the WKB method,

$$\chi \longrightarrow (Ht)^{-\frac{3}{4}} \exp\left[\frac{\Gamma}{3\pi} (Ht)^{\frac{3}{2}}\right] . \tag{45}$$

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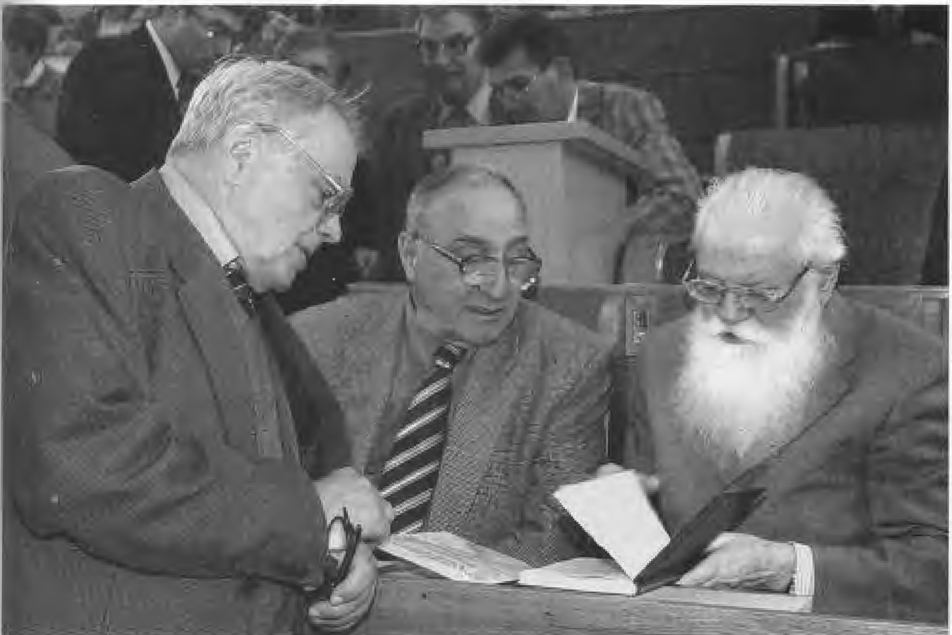

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M.V. Yurova and G.A. Alekseev (NPI Moscow State Univ., Russia)
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