ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

R1 - 3129

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Data on Particles and Resonant States

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Data on Particles and Resonant States: Table S, Stable Particles. Rev. Mod. Phys., January 1967

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Particle	$I^G(J^P)C_n$	Mass (MeV)	Mass difference (MeV)	Mean life (sec) $c\tau$ (cm)	Mass² (GeV)²	Decays: Partial mode	Decays: Fraction	Q (MeV)	p or p_{max} (MeV/c)[‡]	Magnetic moment ($e\hbar/2m_pc$)	Decay parameters[†] Measured: α	Measured: Φ (degree)	Derived: γ	Derived: Δ (degree)
γ	$0,1(1^-)^-$	0		stable	0	stable								
ν_e ν_μ	$J=\frac{1}{2}$	0(<0.2 keV)		stable	0	stable								
		0(<2.1 MeV)												
$e^\mp$	$J=\frac{1}{2}$	0.511006		stable	0.000	stable	$\mu_e = 1.001159622 \pm .000000027\ \frac{e\hbar}{2m_ec}$							
		±.000002		$(>2\times10^{21}\,y)$										
$\mu^\mp$	$J=\frac{1}{2}$	105.659		2.199×10^{-6}	0.011	$e\nu\bar\nu$	100 %	105	53					
		±.002		±.001, S=1.3*		$e\gamma\gamma$	(< 1.6)10^{-5}	105	53					
				$c\tau = 6.592\times10^{4}$		3e	(< 1.3)10^{-7}	104	53					
	$\mu_\mu = 1.001164 \pm .000003\ \frac{e\hbar}{2m_\mu c}$					$e\gamma$	(< 6)10^{-9}	105	53					
			-33.920 ±.014											
$\pi^\pm$	$1^-(0^-)^+$	139.579		2.608×10^{-8}	0.019	$\mu\nu$	100 %	34	30					
		±.014		±.015, S=3.5*		$e\nu$	(1.24± 0.03)10^{-4}	139	70					
				$c\tau = 782$		$\mu\nu\gamma$	(1.24± 0.25)10^{-4}	34	30					
			4.6041	$(\tau^+-\tau^-)/\bar\tau = (.4\pm.2)\%$		$\pi^0 e\nu$	(1.01± 0.09)10^{-8}	4	5					
			±.0037	(test of CPT)		$e\nu\gamma$	(3.0 ± 0.5)10^{-8}	139	70					
π^0	$1^-(0^-)^+$	134.975		[b] 0.89×10^{-16}	0.018	$\gamma\gamma$	[c](98.8)%	135	67					
		±.014		±.18, S=1.6*		γe^+e^-	(1.169)%	134	67					
				$c\tau = 2.67\times10^{-6}$		$\gamma\gamma\gamma$	[c](< 5)10^{-6}	135	67					
						$e^+e^-e^+e^-$	(3.47)10^{-5}	133	67					
$K^\pm$	$\frac{1}{2}(0^-)$	493.82		1.235×10^{-8}	0.244	$\mu\nu$	[b](63.4 ± 0.5)% } S=1.5*	388	236					
		±.11		±.006, S=2.4*		$\pi\pi^0$	(21.0 ± 0.3)% }	219	205					
				$c\tau = 3.70$		$\pi\pi^-\pi^+$	(5.6 ± 0.1)% }	75	126					
				$(\tau^+-\tau^-)/\bar\tau = (.09\pm.08)\%$		$\pi\pi^0\pi^0$	(1.71± 0.08)%, S=1.5*	84	133					
				(test of CPT)		$\mu\pi^0\nu$	(3.41± 0.22)%, S=1.9*	253	215					
						$e\pi^0\nu$	(4.79± 0.18)%, S=2.1*	358	229					
						$\pi\pi^\mp e^\pm\nu$	(3.8 ± 0.8)10^{-5}	214	204					
			-4.05			$\pi\pi^\pm e^\mp\nu$	(< 2)10^{-6}	214	204					
			±.12			$\pi\pi^\mp\mu^\pm\nu$	(≤ 1.4)10^{-5}	109	151					
						$\pi\pi^\pm\mu^\mp\nu$	(< 3)10^{-6}	109	151					
						$e\nu$	(1.9 ± 1.2)10^{-5}	493	247					
						$\pi\pi^0\gamma$	(2.2 ± 0.7)10^{-4}	219	205					
						$\pi\pi^+\pi^-\gamma$	(10 ± 4)10^{-5}	75	126					
						πe^+e^-	(< 1.1)10^{-6}	353	227					
						$\pi\mu^+\mu^-$	(< 3)10^{-6}	143	172					
K^0	$\frac{1}{2}(0^-)$	497.87		50% K_{Short}, 50% K_{Long}										
		±0.16												
K_{Short}	$\frac{1}{2}(0^-)$			0.87×10^{-10}	0.248	$\pi^+\pi^-$	(69.3 ± 1.2)% S=1.25*	219	206					
			$-0.48\times\frac{1}{\tau_S}$	±.009, S=1.3*		$\pi^0\pi^0$	(30.7)%	228	209					
			±.02	$c\tau = 2.61$										
K_{Long}	$\frac{1}{2}(0^-)$			5.68×10^{-8}	0.248	$\pi^0\pi^0\pi^0$	(23.5 ± 2.1)%	93	139					
				±.26		$\pi^+\pi^-\pi^0$	(11.5 ± .4)%	84	133					
				$c\tau = 1703$		$\pi\mu\nu$	(27.5 ± 1.8)%	253	216					
						$\pi e\nu$	(37.4 ± 1.8)%	358	229					
						$\pi^+\pi^-$	(.153 ±.007)%	219	206					
						$\pi^+\pi^-\gamma$	(< 0.3)%	219	206					
						$\pi^0\pi^0$	(< 2.[illegible])10^{-3}	228	209					
						$e\mu$	(< 4)10^{-5}	392	238					
						$\gamma\gamma$	(1.3 ± 0.6)10^{-4}	498	249					
						$\mu^+\mu^-$	(< 4)10^{-6}	287	225					
						e^+e^-	(< 4)10^{-5}	497	249					
η	$0^+(0^-)^+$	548.6		1<Γ<10 keV		Neutral decays 72.9%: $\gamma\gamma$	(31.4 ± 2.2)%	549	274					
		±0.4		$(2<c\tau<20)10^{-10}$		$\pi^0\gamma\gamma$	(20.5 ± 3.5)% S=1.3*	414	258					
						$3\pi^0$	(21.0 ± 3.2)% S=1.5*	144	179					
						Charged decays 27.1%: $\pi^+\pi^-\pi^0$	(22.4 ± 1.8)%	135	174					
						$\pi^+\pi^-\gamma$	(4.6 ± 0.8)% S=1.1*	269	236					
						$\pi^0e^+e^-$	(< 0.2)%	413	258					
						$\pi^+\pi^-e^+e^-$	(0.1 ± 0.1)%	268	236					
p	$\frac{1}{2}(\frac{1}{2}^+)$	938.256		stable	0.880					2.792763				
		±0.005	-1.2933	$(>6\times10^{27}\,y)$						±.000030				
n	$\frac{1}{2}(\frac{1}{2}^+)$	939.550	±.0001	$(1.01\pm.03)10^{3}$	0.882	$pe^-\nu$	100 %	1	1	-1.913148				
		±0.005		$c\tau = 3.03\times10^{13}$						±.000066				
Λ	$0(\frac{1}{2}^+)$	1115.58		2.51×10^{-10}	1.245	$p\pi^-$	(66.4 ± 1.1)% S=1.1*	38	100	-0.73	.663±.022	(7±7)°	.74	(-8±8)°
		±0.10		±.04, S=1.4*		$n\pi^0$	(33.6)%	41	104	±.16	.73 ±.18			
				$c\tau = 7.52$		$pe\nu$	(0.88 ± 0.15)10^{-3}	177	163		.06 ±.19			
						$p\mu\nu$	(1.35 ± 0.60)10^{-4}	72	131					
Σ^+	$1(\frac{1}{2}^+)$	1189.47		0.810×10^{-10}	1.412	$p\pi^0$	(52.8 ± 1.5)%	116	189	2.3	-.960±.067			
		±0.08		±.013		$n\pi^+$	(47.2)%	110	185	±.6	+.008±.037	(180±30)°	-1.0	(0±85)°
				$c\tau = 2.43$		$p\gamma$	(1.9 ± 0.4)10^{-3}	251	225					
						$n\pi^+\gamma$	(≈ 0.2)10^{-4}	110	185					
						$\Lambda e^+\nu$	(1.5 ± 0.9)10^{-5}	73	72					
						$n\mu^+\nu$	(< 1.1)10^{-4}	144	202					
			-7.97			$ne^+\nu$	(< 5)10^{-5}	249	224					
Σ^0	$1(\frac{1}{2}^+)$	1192.56	±.11	$<1.0\times10^{-14}$	1.422	$\Lambda\gamma$	[c] 100 %	77	75					
		±0.11	-4.88	$c\tau<3\times10^{-4}$		Λe^+e^-	(5.45)10^{-3}							
Σ^-	$1(\frac{1}{2}^+)$	1197.44	±.06	1.65×10^{-10}	1.434	$n\pi^-$	100 %	118	193		-.010±.043			
		±0.09		±.03, S=1.4*		$ne^-\nu$	(1.25 ± 0.17)10^{-3}	257	230					
				$c\tau = 4.95$		$n\mu^-\nu$	(0.62 ± 0.12)10^{-3}	152	210					
						$\Lambda e^-\nu$	(0.61 ± 0.16)10^{-4}	81	79					
						$n\pi^-\gamma$	(≈ 1)10^{-5}	118	193					
Ξ^0	$\frac{1}{2}(\frac{1}{2}^+)$?	1314.7		3.0×10^{-10}	1.728	$\Lambda\pi^0$	100 %	64	135		-.33 ±.10			
		±1.0		±.5, S=1.3*		$p\pi^-$	(< .5)%	237	299					
				$c\tau = 8.99$		$pe^-\nu$	(< .6)%	376	323					
						$\Sigma^+e^-\nu$	(< .7)%	125	119					
						$\Sigma^-e^+\nu$	(< .6)%	117	112					
			-6.5			$\Sigma^+\mu^-\nu$	(< .7)%	20	64					
			±.2			$\Sigma^-\mu^+\nu$	(< .6)%	12	49					
						$p\mu^-\nu$	(< .6)%	271	309					
Ξ^-	$\frac{1}{2}(\frac{1}{2}^+)$?	1321.2		1.74×10^{-10}	1.746	$\Lambda\pi^-$	100 %	66	139		-.391±.032	(6±8)°	.92	(14±15)°
		±0.2		±.05		$\Lambda e^-\nu$	(2.5 ± 1.8)10^{-3}	205	190					
				$c\tau = 5.22$		$n\pi^-$	(< 5)10^{-3}	242	303					
						$\Lambda\mu^-\nu$	(< 1.2)%	100	163					
						$\Sigma^0e^-\nu$	(< 0.3)%	128	122					
						$\Sigma^0\mu^-\nu$	(< 0.5)%	23	70					
						$ne^-\nu$	(< 1)%	381	327					
Ω^-	$0(3/2^+)$? ?	1674		1.5×10^{-10}	2.802	$\Xi\pi$	(~50)%	221	296					
		±3		±.5, $c\tau = 4.5$		$\Lambda\bar K$	(~50)%	66	216					

General Atomic and Nuclear Constants[a]

N = 6.02252 $\times10^{23}$ mole^{-1} (based on $A_{C12} = 12$)
c = 2.997925 $\times10^{10}$ cm sec^{-1}
e = 4.80298 $\times10^{-10}$ esu = 1.60210×10^{-19} coulomb
1 MeV = 1.60210 $\times10^{-6}$ erg
ħ = 6.5819 $\times10^{-22}$ MeV sec
= 1.05449 $\times10^{-27}$ erg sec
ħc = 1.9732 $\times10^{-11}$ MeV cm = 197.32 MeV fermi
k = 8.6171 $\times10^{-11}$ MeV deg^{-1} (Boltzmann const.)
α = $e^2/\hbar c$ = 1/137.0388
m_e = 0.511006 MeV/c² = 1/1836.10 m_p
m_p = 938.256 MeV/c² = 1836.10 m_e = 6.721 $m_{\pi^\pm}$
= 1.00727663 m_1 (where m_1 = 1 amu = $\frac{1}{12}\,m_{C12}$ = 931.478 MeV/c²)
r_e = e^2/m_ec^2 = 2.81777 fermi (1 fermi = 10^{-13} cm)
$\lambda\!\!\!^-_e$ = $\hbar/m_ec$ = $r_e\alpha^{-1}$ = 3.86144×10^{-11} cm
a_∞ Bohr = $\hbar^2/m_ee^2$ = $r_e\alpha^{-2}$ = 0.529167 A (1 A = 10^{-8} cm)
$\sigma_{Thompson}$ = $\frac{8}{3}\pi r_e^2$ = 0.66516×10^{-24} cm² = 0.66516 barn
R_∞ = $m_ee^4/2\hbar^2$ = $m_ec^2\alpha^2/2$ = 13.60535 eV (Rydberg)

Hydrogen-like atom (non-rel., μ = reduced mass)

$$E_n = \frac{\mu z^2e^4}{2(n\hbar)^2}\,;\quad a_{n=1} = \frac{\hbar^2}{\mu ze^2}\,;\quad \left.\frac{v}{c}\right)_{rms} = \frac{ze^2}{n\hbar c}$$

μ_{Bohr} = $e\hbar/2m_ec$ = 0.578817×10^{-14} MeV gauss^{-1}
μ_{nucl} = $e\hbar/2m_pc$ = 3.1524 $\times10^{-18}$ MeV gauss^{-1}
$\frac{1}{2}\omega_{cyclotron}$ = $e/2m_ec$ = 8.79404 $\times10^{6}$ rad sec^{-1} gauss^{-1}
= $e/2m_pc$ = 4.7895 $\times10^{3}$ rad sec^{-1} gauss^{-1}
$\sigma_{natural}$ = $\pi(\hbar/m_{\pi^\pm}c)^2$ = 62.768 mb

Other Physical Constants

1 year = 3.1536×10^7 sec (≈ $\pi\times10^7$ sec)
density of air = 1.205 mg cm^{-3} (at 20°C)
acceleration by gravity = 980.67 cm sec^{-2}
gravitational constant = 6.670×10^{-8} cm³g^{-1} sec^{-2}
1 calorie = 4.184 joules
1 atmosphere = 1033.2 g cm^{-2}
1 eV per particle = 11604.9°K (from E = kT)

Numerical Constants

1 rad	= 57.29578 deg	e	= 2.71828
C	= 0.577216	1/e	= 0.367879
ln 2	= 0.69315	$\log_{10}e$	= 0.43429
ln 10	= 2.30259	$\log_{10}2$	= 0.30103

[a] Based mainly on E. R. Cohen and J. W. M. DuMond, Rev. Mod. Phys. 37, 537 (1965).

[†] The definition of these quantities is as follows

$$\alpha = \frac{2\,\mathrm{Re}(S^*P)}{|S|^2+|P|^2}\,;\quad \beta = \frac{2\,\mathrm{Im}(S^*P)}{|S|^2+|P|^2}\,;\quad \gamma = \frac{|S|^2-|P|^2}{|S|^2+|P|^2}$$

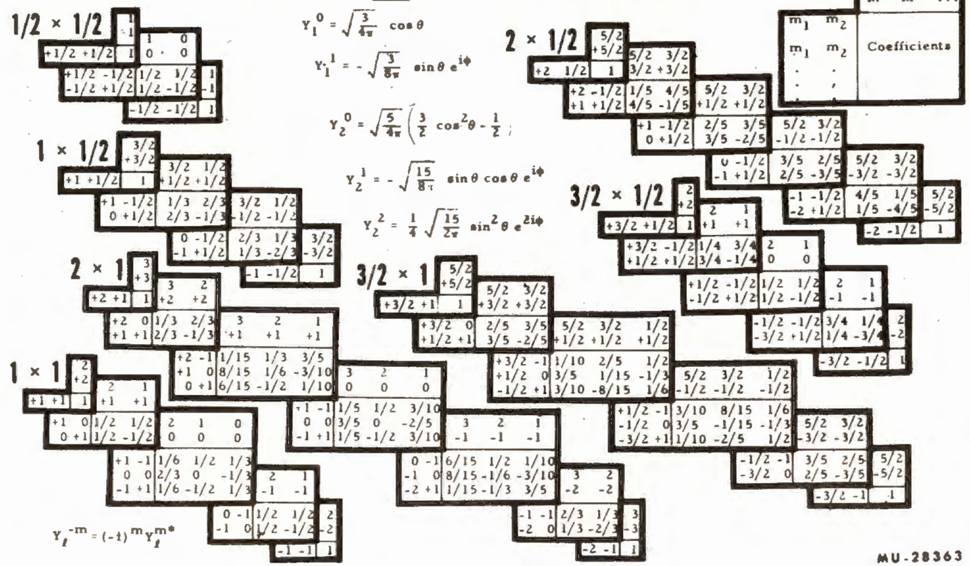
$$\tan\Phi = \frac{\beta}{\gamma}\,,\quad \tan\Delta = \frac{-\beta}{\alpha}\,.$$

* S = Scale factor = $\sqrt{\chi^2/(N-1)}$ where N ≈ number of experiments. S should be ≈ 1. If S > 1, we have enlarged the error of the mean, δx, i.e., δx → S δx. This new convention, is still inadequate, since if S > 1, the real uncertainty is probably even greater than Sδx. See text.
a. See notes on Stable Particles in text. b. See notes in data card listings. c. Theoretical value. See also data card listings.
‡ In decays with more than two bodies, $\bar p_{max}$ is the maximum momentum that any particle can have.

MUB 13977

CLEBSCH-GORDAN COEFFICIENTS AND SPHERICAL HARMONICS

Note: A $\sqrt{\ }$ is to be understood over every coefficient; e.g., for -8/15 read $-\sqrt{8/15}$.



MU-28363

Atomic and Nuclear Properties of Materials

Material	Z	A	Cross Section σ^a barns	Collision Length L_{coll} [b] g cm^{-2}	Collision Length cm	Minimum $-dE/dx$ [c] MeV g^{-1} cm^2	Minimum $-dE/dx$ MeV cm^{-1}	Radiation Length L_{rad} [d] g cm^{-2}	Radiation Length cm	Density ρ g cm^{-3}
H_2	1	1.01	0.063	26.5	374	4.13	0.292	58.0	819	0.0708[e]
D_2	1	2.01	0.100	33.4	202	2.07	0.342	116	703	0.165[e]
He	2	4.00	0.16	42.0	336	1.94	0.242	85.4	683	0.125[e]
Li	3	6.94	0.23	50.4	94.3	1.69	0.902	78.7	148	0.534
Be	4	9.01	0.28	55.0	29.9	1.60	2.96	63.7	34.7	1.848
C	6	12.01	0.33	60.4	f	1.78	f	42.4	f	≈1.55[f]
N_2	7	14.01	0.36	63.6	78.9	1.81	1.46	37.8	46.7	0.808[e]
Al	13	26.98	0.57	79.2	29.3	1.62	4.37	24.0	8.9	2.70
Fe	26	55.85	0.92	101.2	12.8	1.48	11.6	13.9	1.8	7.87
Cu	29	63.54	1.00	105.4	11.8	1.44	12.9	12.0	1.34	8.96
Sn	50	118.69	1.55	129.7	17.8	1.28	9.4	8.89	1.22	7.31
W	74	183.85	2.02	150.8	7.80	1.17	22.6	6.89	0.36	19.3
Pb	82	207.19	2.20	156.2	13.8	1.13	12.8	6.52	0.58	11.35
U	92	238.03	2.42	163.6	≈8.63	1.09	≈20.6	6.13	≈0.32	≈18.95
Air				64.6	53620	1.81	0.0022	36.5	30290	0.001205[g]
Freon (CF_3Br)				87.1	≈58.0	1.52	≈2.3	16.6	≈11	≈1.5
H_2 (bubble chamber, 27°K)				26.5	442	4.13	0.248	58.0	970	≈0.060[h]
H_2O				57.2	57.2	2.03	2.03	35.7	35.7	1.00
Ilford Emulsion				103.0	27.0		5.49	11.2	2.91	3.815
LiF				63.8	24.2	1.69	4.46	39.0	14.8	2.64
Mylar ($C_5H_4O_2$)				59.1	42.8	1.91	2.64	39.6	28.7	1.38
NaI				119.0	32.4	1.32	4.84	9.58	2.61	3.67
Polyethylene (CH_2)				51.0	≈55.5	2.09	≈1.92	44.1	≈48	≈0.92
Polystyrene (CH)				54.9	≈52.3	2.03	≈2.14	43.4	≈41.3	≈1.05
Propane (C_3H_8, bubble chamber)				48.9	119.3	2.28	0.935	44.6	109	0.41

a. $\sigma = \sigma_{natural} = \pi(\hbar/m_\pi c)^2 \times A^{2/3} = 62.8 \text{ mb} \times A^{2/3}$

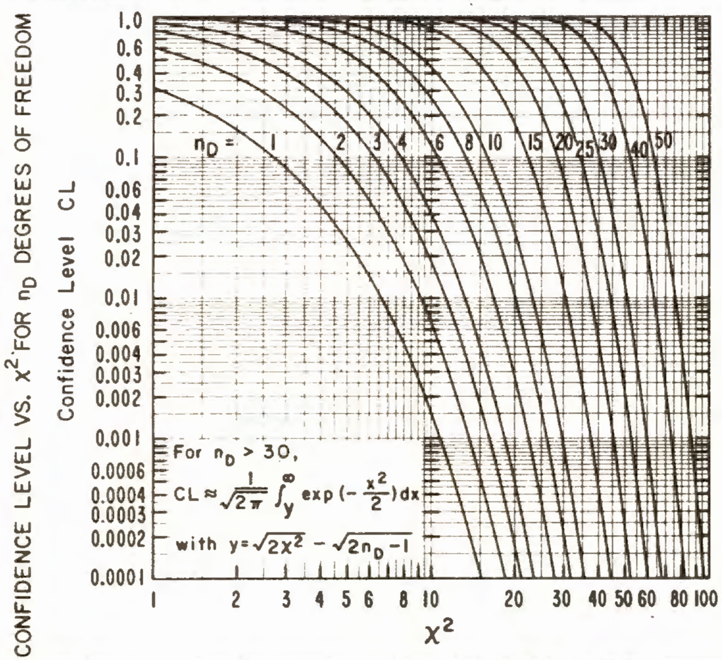
b. $L_{coll} = A/(N\sigma_{natural}) = 26.5 \text{ g cm}^{-2} \times A^{1/3}$

c. From W. H. Barkas and M. J. Berger, Tables of Energy Losses and Ranges of Heavy Charged Particles. NASA SP-3013 (1964)

d. Mainly from High Energy and Nuclear Physics Data Handbook, W. Galbraith and W. S. C. Williams, Ed. (N.I.R.N.S., Rutherford Lab., Chilton, Didcot, Berks.) 1964

e. boiling at 1 atmosphere f. density variable g. at 20°C

h. May vary by about ± 3%, depending on operation conditions



STIRLING'S APPROXIMATION

$$\sqrt{2\pi n}\left(\frac{n}{e}\right)^n < n! < \sqrt{2\pi n}\left(\frac{n}{e}\right)^n\left(1 + \frac{1}{12n-1}\right)$$

Gaussianlike Distributions

For $n > -1$ but not necessarily integral:

$$\int_0^\infty x^{2n+1} \exp\left[-\frac{x^2}{2\sigma^2}\right] dx = 2^n\, n!\, \sigma^{2n+2};\ \left(\frac{1}{2}\right)! = \sqrt{\pi}/2$$

Relation between standard deviation σ and mean deviation α:

$2\sigma^2 = \pi\alpha^2$; σ = 1.4826 probable error.

Odds against exceeding one standard deviation = 2.15:1; two, 21:1; three, 370:1; four, 16,000:1; five, 1,700,000:1

MULTIPLE COULOMB SCATTERING

The rms projected angle θ due to multiple Coulomb scattering (only) of a particle of charge z, momentum P, velocity V is

$$\theta_{proj} = z\,\frac{15(\text{MeV})}{PV(\text{MeV})}\sqrt{\frac{L}{L_{(rad)}}}\,(1+\epsilon) \text{ radians;}$$

L = Length in scatterer;
For $L \geq 1/10\ L(rad)$ ϵ is generally < 1/10. The distribution of θ is not truly Gaussian. The rms projected displacement y on traversing an absorber of thickness L is

$$y_{rms} = L\theta_{proj}/\sqrt{3}$$

RADIOACTIVITY

1 curie = 3.7×10^{10} disintegrations/sec

1 R = 87.8 ergs/g air = 5.49×10^7 MeV/g air

Fluxes (per cm^2) to liberate 1 R in carbon:

3×10^7 minimum ionizing singly charged particles
0.9×10^9 photons of 1 MeV energy.

(These fluxes are actually correct to within a factor of two for all materials.)

Natural background: 100 mR/year

"Tolerance" 100 millirem/week [Note, 1 R may produce up to 10 "Rem" (R equivalent for man), depending on type of radiation.]

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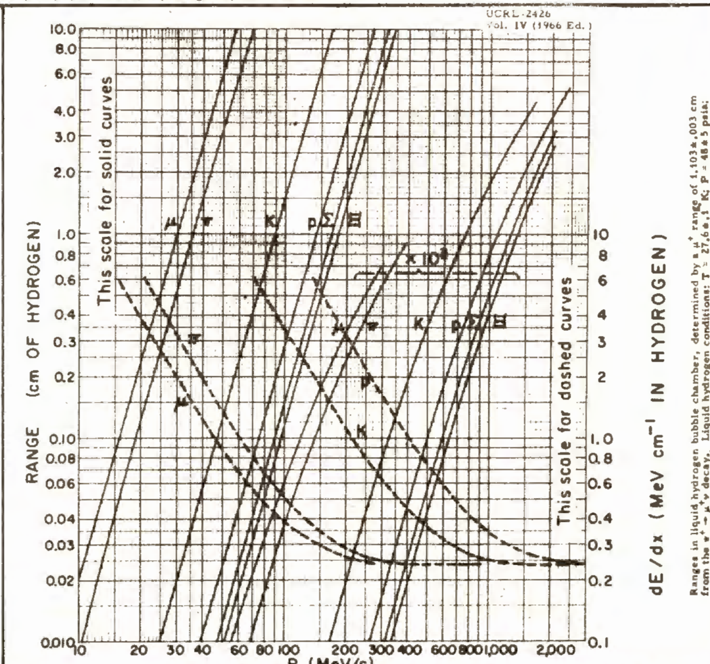


M E S O N S, November 1966

Particle	Symbol(J^P)	$I^G(J^P)C_n$ (underline = estab.)	Mass M (MeV)	Width Γ (MeV)	$M^2 \pm \Gamma M$(a) $(GeV)^2$	Partial Decay Modes: Mode	Fraction (%)	Q (MeV)	p or p_{max}(b) (MeV/c)	Nonet
η(549)	η(0^-)	$0^+(0^-)+$	548.6 ±0.4	<0.01	0.301 <.000005	all neutral	73	See Table S		η (0^-)
σ, ε§						$\pi^+\pi^-\pi^0+\pi^+\pi^-\gamma$	27			
ω(783)	φ'(1^-)	$0^-(1^-)-$	783.4 ±0.7* S=1.8	11.9 ±1.5	0.614 ±.009	$\pi^+\pi^-\pi^0$	≈ 90	369	328	ω (1^-)
						$\pi^+\pi^-$	seen (c)	504	366	
						$\pi^0\gamma$	9.7±0.8	648	380	
						η+ neutral	< 1.5	234	199	
						$\pi^+\pi^-\gamma$	< 5	504	366	
						e^+e^-	0.012±.003	782	392	
						$\mu^+\mu^-$	< 0.10	572	377	
η'(958) or X^0	η'(0^-)	$0^+(0^-)+$	958.3 ±0.8	<4	0.918 <.004	$\eta\pi\pi$	75 ± 3 S=1.8*	131	232	η' (0^-)
						$\pi^+\pi^-\gamma$(incl. $\rho^0\gamma$)	25 ± 3	679	458	
						for upper limits see footnote (f)				
H§										
φ(1019)	φ(1^-)	$0^-(1^-)-$	1018.6 ±0.5	4.0 ±1.0 S=1.2*	1.039 ±.004	K^+K^-	48 ± 3	31	125	φ (1^-)
						K_LK_S	40 ± 3	23	107	
						$\pi^+\pi^-\pi^0$(incl. ρπ)	12 ± 4	604	461	
						for upper limits see footnote (g)				
η_V(1050) → K_SK_S	η(0^+)	$0^+(0^+)+$	1050	50	1.10 ±.05	ππ	< 70	780	507	η_V (0^+)
Some data still favor large scattering length						$K\bar{K}$	> 30	54	167	
f(1250)	η'(2^+)	$0^+(2^+)+$	1254 ±12	117 ±15	1.57 ±.15	ππ	large	975	611	f (2^+)
						$2\pi^+2\pi^-$	< 4	696	547	
						$K\bar{K}$	2.3±0.6	258	381	
D(1285)	η(A)	$0^+(1^+)+$	1285 ±4	32 ±8	1.65 ±.04	$K\bar{K}\pi$ (mainly π_V(1003)π) only mode seen		154	304	D (1^+)
						$K^*\bar{K}+\bar{K}^*K$		-100		
						ππρ	not seen	256	356	
E(1420)	η(A)	$0^+(0^-)+$	1424 ±7	76 ±9	2.03 ±.11	$K^*\bar{K}+\bar{K}^*K$	50 ±10	38	157	? (0^-); E? (1^+)
						π_V(1003)π	50 ±10	284	338	
						ππρ	not seen	395	462	
K_sK_s§ ρρ										
f'(1500)	η(2^+)	$0^+(2^+)+$	1514 ±16	86 ±23	2.29 ±.13	ππ	< 14	1235	744	f' (2^+)
						$K\bar{K}$	> 60	518	570	
						$K\bar{K}^*+\bar{K}K^*$	< 40	128	294	
						ηη	not seen	417	522	
$\pi^\pm$(140) π^0(135)	π(0^-)	$1^-(0^-)+$	139.58 134.98		0.019 0.018	See Table S				π (0^-)
$\rho^\pm$(760)	ρ(1^-)	$1^+(1^-)-$	778 (h)	160 (h)	0.605 ±.124	ππ	≈100	480	353	ρ (1^-)
						$\pi^\pm\pi^+\pi^-\pi^0$	< 0.2	206	243	
						$\pi^\pm\pi^+\pi^-\pi^-$	< 0.6	199	238	
						$\pi^0\gamma$	< 0.4	619	367	
ρ^0(760)			770 (h)	140 (h)	0.593 ±.108	$\eta\pi^\pm$	< 0.8	71	135	
						e^+e^-	.0065 +.011 −.005	759	380	
						$\mu^+\mu^-$	.0033 +.0016 −.0007	549	365	
δ(965)	?( )	1( )	963.1 ±4.2	<5	0.927 <.005	$\delta^\pm$ → 1 charged+neutral(s)	≈ 60			
						$\delta^\pm$ → ≥3 charged+neutral(s)	≈ 40			
π_V(1003) → $K\bar{K}$	π(0^+)	$1^-(0^+)+$	1003	70 ±15	1.006 ±.057	$K^\pm K^0$	large	11	75	π_V (0^+)
may also be interpreted as due to large scattering length						ηπ	see note in data listings	315	333	
A1(1080)	π(1^+)	$1^-(1^+)+$	1079 ±8	130 ±40	1.16 ±.14	ρπ	≈100	181	245	A_1 (1^+)
unresolved mixture of resonance and "Deck effect"						$K\bar{K}$	< 0.25, $G=(-1)^{\ell+I}$ forbids this (Eq. 5)			
						ηπ	< 1.5	391	385	
						η'π	< 1.5	-19		
B(1210)	ρ(A ?)	$1^+(1^+)-$? ?	1208 ±12	119 ±24 S=1.6*	1.46 ±.14	ωπ	≈100	297	339	
						ππ	< 30	941	594	
						$K\bar{K}$	< 2	232	358	
						4π	< 50	662	528	
						φπ	< 1.5	66	137	
A2(1300)	π(2^+)	$1^-(2^+)+$ (d)	1306 ±8 S=2.6*	81 ±8 S=1.4*	1.70 ±.11	ρπ	93 ±3	408	417	A_2 (2^+)
						$K\bar{K}$	3.8±1.3	314	425	
						ηπ	2.9±2.4 S=1.5*	618	527	
						η'π	< 1.5	208	276	
						$\pi^+\pi^-\pi^0$(excl. ρπ)	< 17	892	616	
π(1640) → 3π	π(A) ?	≥1⁻(A)+ ?	1640 ±20	100 ±20	2.69 ±.16	3π	appears dominant	1235	792	
						ρπ	< 40	746	636	
						fπ	?	251	319	
						$K\bar{K}$	< 40	644	652	
ρ(1650) g→2π	ρ(V)	$1^+(V)-$ S=1.4*	1637 ±23	150 ±50	2.68 ±.24	2π	observed	1358	807	
$R_1R_2R_3$ bumps suggest more structure in this peak						4π	probably observed	1079	758	
						ρππ	probably observed	599	605	
$R_1R_2R_3$§										
S(1930) X^-	?(V) ? (e)	≥1⁺( ) ?	1929 ±14	≤35	3.72 ≤.07	1 charged + neutral(s)	6(+15/−6)			
						3 charged + neutral(s)	92(+ 8/−20)			
						>3 charged + neutral(s)	2(+13/−2)			
T(2200) X^-	?( )	≥1	2195 ±15	≤13	4.82 ≤.03	1 charged + neutral(s)	4(+11/−4)			
						3 charged + neutral(s)	94(+ 6/−19)			
						>3 charged + neutral(s)	2(+13/−2)			
U(2380) X^-	?( )	≥1	2382 ±24	≤30	5.67 ≤.07	1 charged + neutral(s)	30 ±10			
						3 charged + neutral(s)	45 ±15			
						>3 charged + neutral(s)	25 ±10			
K^+(494) K^0(498)	K(0^-)	$1/2(0^-)$	493.78 497.7		0.244 0.248	See Table S				K (0^-)
K^*(890)	K(1^-)	$1/2(1^-)$	892.4 ±0.8	49.8 ±1.7 S=1.1	0.796 ±.044*	Kπ	≈100	259	288	K^* (1^-)
κ(725)§ K_V(1080)§	$m_0-m_\pm$ = 3.5±1.8 S=1.2*					Kππ	< 0.2	119	216	
K_C(1215)§ K_A(1320)	K(A) ?	$1/2(A)$?	1320 ±10	80 ± 20	1.742 ±.106	$K^*\pi$ } overlap	large	288	338	K_A (1^+)
						Kρ		63	198	
m and Γ values taken from Shen+. Appreciable discrepancies with other experiments. See note in data listings.						Kω	probably seen, < 10	39	155	
						Kπ	< 30	687	558	
						Kη	< 10	278	405	
K_V(1420)	K(2^+)	$1/2(2^+)$	1411 ±5 S=1.8*	92 ±7 S=1.2*	1.991 ±.130	Kπ	52 ± 5 (S=2.2*)	778	610	K_V (2^+)
						$K^*\pi$	36 ± 6	379	407	
						Kρ	9 ± 5	158	319	
						Kω	1.0±1.7	134	293	
						Kη	2.1±3.0	368	475	
K_A(1800)	K(A)	$1/2(A)$	1789 ±10	80 ±20	3.20 ±.14	Kπ	< 10	1156	819	
						$K^*\pi$	35 ±12	762	664	
						K_V(1420)π	8 ± 5	243	315	
$K^*_{3/2}$(1175)§						Kρ	7 ± 5	532	630	
$K^*_?$(1270)§						Remaining Kππ	40 ±15	1021	801	
						Kω	10 ± 3	508	616	

Nonets: CP=−1 (0^-); CP=+1 (0^+) tentative, (1^-), (1^+) tentative, (2^+).

	(0^-)	(0^+)	(1^-)	(1^+)	(2^+)
$m_8 = \sqrt{K^2 + \frac{K^2-\pi^2}{3}}$ =	566.8 ±0.2		928.4 (i)±3.0	1391. ±13.	1444.3 ±6.9
$\sin^2\theta = \frac{\eta - m_8}{\eta - \eta'}$ =	0.033 ±0.001		0.414 (i)±0.013	0.25 ±0.10	0.29 ±0.06
θ =	10.4°		40.1°	29.7°	32.4°

§ The following bumps, excluded above, are listed among the data cards: σ (410), ε (700), H (975), K_sK_s (1440) and ρρ (1410), R_1, R_2, R_3 (≈1700), κ (725) K_V(1080), K_c(1215), $K^*_{3/2}$(1175), $K^*_?$(1270).

* Quoted error includes scale factor $S = \sqrt{\chi^2/(N-1)}$. See footnote to Table S.

Footnotes continued in right margin.

(a) ΓM is the half-width of the resonance when plotted against M^2.
(b) For decay modes into ≥ 3 particles p_{max} is the maximum momentum that any of the particles in the final state can have. The momenta have been calculated using the averaged central mass values, without taking into account the widths of the resonances.
(c) Reported values range between 1% and 10%, and depend on assumptions on ρ-ω interference.
(d) If A2 → both ρπ and $K\bar{K}$, then $J^P = 2^+$.
(e) S is ρ(V) if identified with $\pi^+\pi^-$ bump at 1910 MeV. See note on mesons.
(f) Empirical limits on fractions for other decay modes of η'(958): $\pi^+\pi^- < 7\%$, $3\pi < 7\%$, $4\pi < 1\%$, $6\pi \lesssim 1\%$, $\pi^+\pi^-e^+e^- < 0.6\%$, $\pi^0e^+e^- < 1.3\%$, $\eta e^+e^- < 1.1\%$, $\pi^0\rho^0 < 4\%$, $\pi^0\omega < 8\%$.
(g) Empirical limits on fractions for other decay modes of φ(1019): $\pi^+\pi^- < 20\%$, $\eta\gamma < 8\%$, η + neutrals < 15%, $\pi^+\pi^-\gamma < 4\%$, $e^+e^- < 0.2\%$, $\mu^+\mu^- < 0.5\%$, $\omega\gamma < 5\%$, $\rho\gamma < 2\%$.
(h) m_ρ, Γ_ρ from p-wave fit to compiled spectrum of 2-4 GeV/c $\pi^+p \to \Delta^{++}\rho^0$, $|t| < 10\ m_\pi^2$, and comparison of $\rho^\pm$ - ρ^0 in similar reactions. Results depend on background and t-cut, hence real errors unknown, but larger than statistical errors ~ 5 MeV. (See Matts Roos, Phys. Letters, to be publ.) Note contrast between Roos' p-wave fit vs. weighted average of published results (see listings). m(±) 778 vs. 757; m(0) 770 vs. 760, Γ(±) 160 vs. 132, Γ(0) 140 vs. 116. This demonstrates present uncertainty.
(i) Error on m_ρ taken to be 10 MeV.

MUB 13978

BARYONS - January 1967

	Particle or resonance	$I(J^P)$ ⊢⊣ = estab.	Beam π, K (BeV) (BeV/c)	Mass (MeV)	Γ (MeV)	$M^2 \pm \Gamma M$ (BeV²)	Partial decay modes: Mode	Fraction (%)	Q (MeV)	p or p_{max}[‡] (MeV/c)	$4\pi\lambda\!\!\!^{-2}$ (mb)
$N^*_{1/2}$	p n	1/2(1/2⁺)		938.3 939.6		0.880 0.883	See Table S				
	N'(1400)	1/2(1/2⁺) P_{11}	T=0.43πp p=0.55	~1400[a]	~200	1.96 ±0.28	Nπ	70	322	367	36.3
	N(1525)	1/2(3/2⁻) D_{13}	T=0.62 p=0.75	1525[a]	105	2.33 ±0.16	Nπ Nππ [Δ(1236)π][e]	65 35 [~ 20]	447 308 149	460 414 229	23.2
	N(1570)	1/2(1/2⁻) S_{11}	T=0.69 p=0.82	1570[a]	130	2.46 ±0.20	Nπ Nη	~ 30 ~ 70	492 82	491 242	20.3
	N(1670)	1/2(5/2⁻) D_{15}	T=0.87 p=1.00	1670[a]	140	2.79 ±0.23	Nπ Nππ [Δ(1236)π][e] ΛK Nη	40 dominant[a] [?] small small	592 453 294 57 182	560 526 357 200 368	15.6
	N(1688)	1/2(5/2⁺) F_{15}	T=0.90 p=1.03	1688[a]	110	2.85 ±0.19	Nπ Nππ [Δ(1236)π][e] ΛK Nη	65 dominant[a] [?] small small	610 471 312 75 200	572 538 372 231 388	14.9
	N'(1700)[c]	1/2(1/2⁻) S_{11}	T=0.92 p=1.05	1700[a]	240	2.89 ±0.41	Nπ	100	622	580	14.5
	N(2190)	1/2(7/2⁻)	T=1.94 p=2.07	2190	200	4.80 ±0.14	Nπ ΛK	30 ?	1112 577	888 710	6.21
	N(2650)	1/2(11/2⁻)[b]	T=3.12 p=3.26	2650 ± 10	~300	7.02 ±0.80	Nπ ΛK	7 ?	1572 1037	1154 1022	3.67
	→ N(3030)[c]	1/2(15/2⁻)[b]	T=4.26 p=4.40	3030	400	9.18 ±1.21	Nπ	0.7	1972	1377	2.62
$N^*_{3/2}$	Δ(1236)	3/2(3/2⁺) P_{33}	T=0.195 p=0.304	(++) 1236.0 ±0.6	120 ±2	1.53 ±0.15	Nπ Nπ⁺π⁻	100 0	158 18	231 89	91.9
		$m_0 - m_{++}$ = 0.45 ± 0.85		$m_- - m_{++}$ = 7.9 ± 6.8							
	Δ(1670)	3/2(1/2⁻) S_{31}	T=0.87 p=1.00	1670[a]	~180	2.79 ±0.30	Nπ Nππ	40 ?	592 453	560 526	15.6
	Δ(1920)	3/2(7/2⁺)	T=1.35 p=1.48	1920	200	3.69 ±0.38	Nπ ΣK	50 seen	842 229	722 423	9.37
	Δ(2420)	3/2(11/2⁺)[b]	T=2.51 p=2.65	2423 ±10	~275	5.87 ±0.67	Nπ ΣK	10 ?	1345 732	1024 830	4.66
	Δ(2850)	3/2(15/2⁺)[b]	T=3.71 p=3.85	2850 ±12	~300	8.12 ±0.86	Nπ	3	1772	1266	3.05
	→ Δ(3230)[c]	3/2(19/2⁺)[b]	T=4.94 p=5.08	3230	440	10.4 ±1.4	Nπ	0.6	2152	1475	2.24
Z^*_0	→ Z_0(1865)[c]	0(?)	p=1.15 K⁺p	1863	150	3.47 ±0.28	NK	55 (if J = 1/2)	432	579	14.6
Y^*_0	Λ	0(1/2⁺)		1115.6		1.24	See Table S				
	Λ(1405)[d]	0(1/2⁻)	p<0 K⁻p	1405	35	1.97 ±0.05	Σπ	100	68	142	
	Λ(1520)	0(3/2⁻)	p=0.392	1518.8 ±1.5	16 ±2	2.31 ±0.02	NK̄ Σπ Λππ	S=1.7* → {39±5 51±6 10±2	81 182 124	235 258 251	83.6
	Λ'(1670)[a]	0(1/2⁻)	p=0.74	1670	18	2.79 ±0.03	Λη NK̄	K⁻p→Λη seen	6 233	66 410	 28.5
	Λ'(1700)	0(3/2⁻)	p=0.80	1700 ±10	40 ±10	2.89 ±0.07	NK̄ Σπ	20 seen	263 363	439 411	25.0
	Λ(1820)	0(5/2⁺)	p=1.06	1819.5 ±3.5	83 ± 8	3.31 ±0.15	NK̄ Σπ Σ(1385)π Λη	70 11 18 ~ 1	382 482 295 155	541 502 362 349	16.5
	Λ(2100)	0(7/2⁻)	p=1.68	2100	160	4.41 ±0.34	NK̄ Σπ	29 seen	663 763	748 699	8.68
	Λ(2340)	0(?)	p=2.27	2340 ±20	105	5.48 ±0.25	NK̄ seen in σ(total)	10 ↳ if J = 9/2	903	907	5.92
Y^*_1	Σ	1(1/2⁺)		(+)1189.5 (0)1192.6 (-)1197.4		1.41 1.42 1.43	See Table S				
	Σ(1385)	1(3/2⁺)	p<0 K⁻p	(+)1382.2±0.9 S=1.6* S=4.8* ← (-)1388.0±3.0	(+)37±3 S=2.1* (-)38±4 S=3.7*	1.92 ±0.05	Λπ Σπ	91±3 9±3 S=1.4*	130 48	208 117	
	Σ(1660)[a]	1(3/2⁻)	p=0.72	1660	50	2.76 ±0.08	Λ(1405)π Σπ Λπ NK̄	large ? ? small	115 323 405 223	197 379 439 400	 29.9
		Large contradictions among measured branching ratios.									
	Σ(1770)	1(5/2⁻)	p=0.95	1768 ±4 S=1.5*	89 ±12 S=2.0*	3.13 ±0.16	NK̄ Λπ Λ(1520)π Σ(1385)π Ση Σπ	49 17 19 12 2 < 1	331 517 110 243 27 431	498 520 192 318 143 463	19.4
	→ Σ(1910)[c]	1(5/2⁺)	p=1.25	1910 ±10	60	3.65 ±0.11	NK̄ Λπ Σπ	8 10 3	473 655 573	612 619 568	12.9
	Σ(2035)	1(7/2⁺)	p=1.53	2035 ±15	160	4.14 ±0.33	NK̄ Λπ Σπ	16 25 seen	598 784 698	703 703 655	9.83
	→ Σ(2260)[c]	1(?)	p=2.06	2260 ±20	180	5.11 ±0.41	NK̄ seen in σ(total)	14 ↳ if J = 9/2	823	855	6.66
$Ξ^*_{1/2}$	Ξ	1/2(1/2⁺)		(0)1314.7 (-)1321.2		1.73 1.75	See Table S				
	→ Ξ(1530)	1/2(3/2⁺) p-wave		(0)1528.9±1.1 (-)1533.8±1.9	7.3 ±1.7	2.34 ±0.01	Ξπ	100	69	145	
	Ξ(1815)	1/2(?)		1815 ±3	16 ±8 S=2.2*	3.29 ±0.03	ΛK̄ Ξπ Ξππ [Ξ(1530)π][e]	~ 65 ~ 10 ~ 25 [~ 20]	202 354 215 145	391 409 351 229	
		Branching ratios poorly known									
	→ Ξ(1930)	1/2(?)		1933 ±16	140 ±35	3.74 ±0.27	Ξπ ΛK̄	seen seen	472 320	501 504	
$Ω^-$	Ω⁻	0(3/2⁺)		1674		2.80	See Table S				

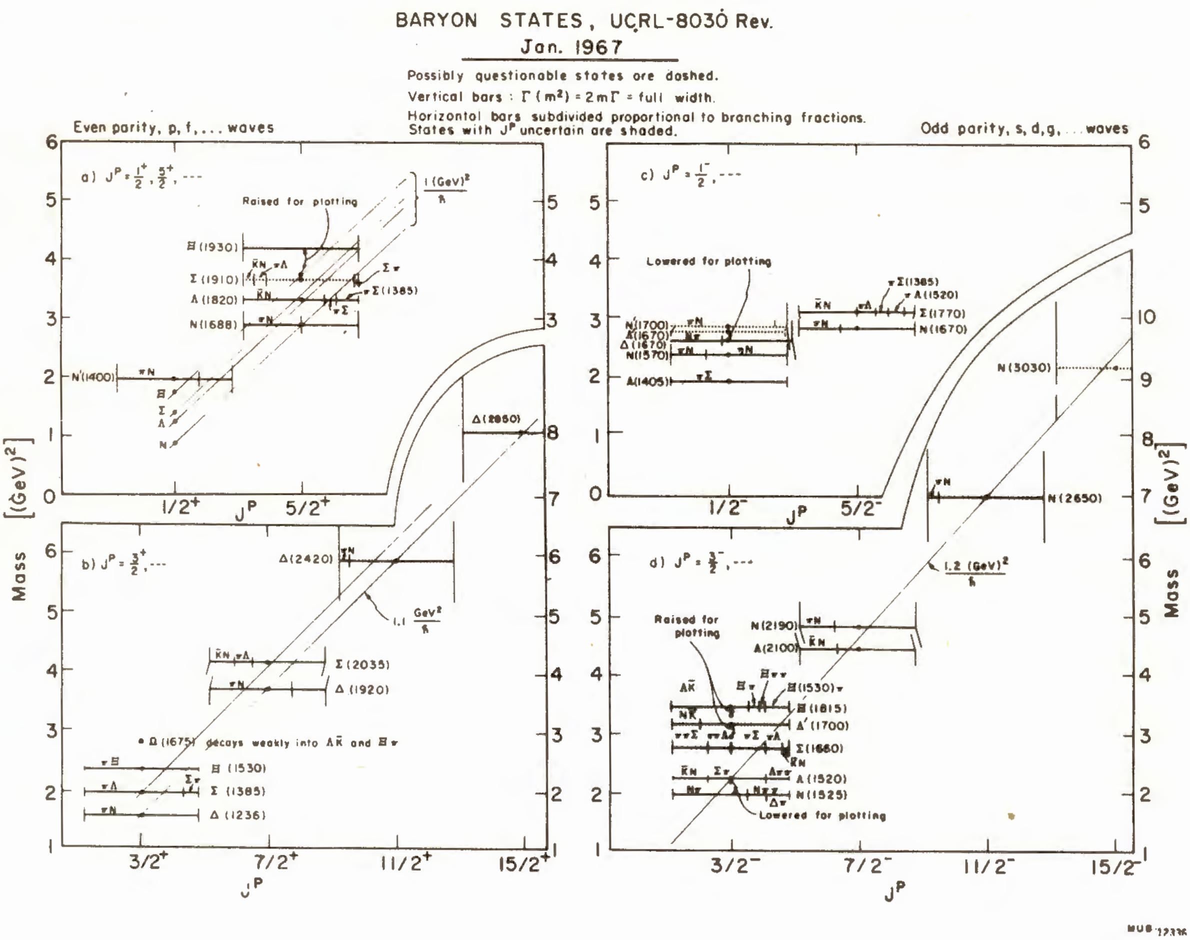
a. See note in data listings.
b. J^P assignment based on straight-line Regge-trajectory-recurrence hypothesis and supported by fits to πp elastic scattering at 180°. See note following data listings.
c. Evidence for the existence of the effect and/or for its interpretation as a resonance is open to some question.
d. A virtual bound state of the K̄N system with negative scattering length [$a_0 = (-1.6 + 0.6i)F$]; i.e., a pole in the S matrix below the elastic threshold. See notes in main text and data listings.
e. Square brackets indicate a sub-reaction of the previous unbracketed decay mode.

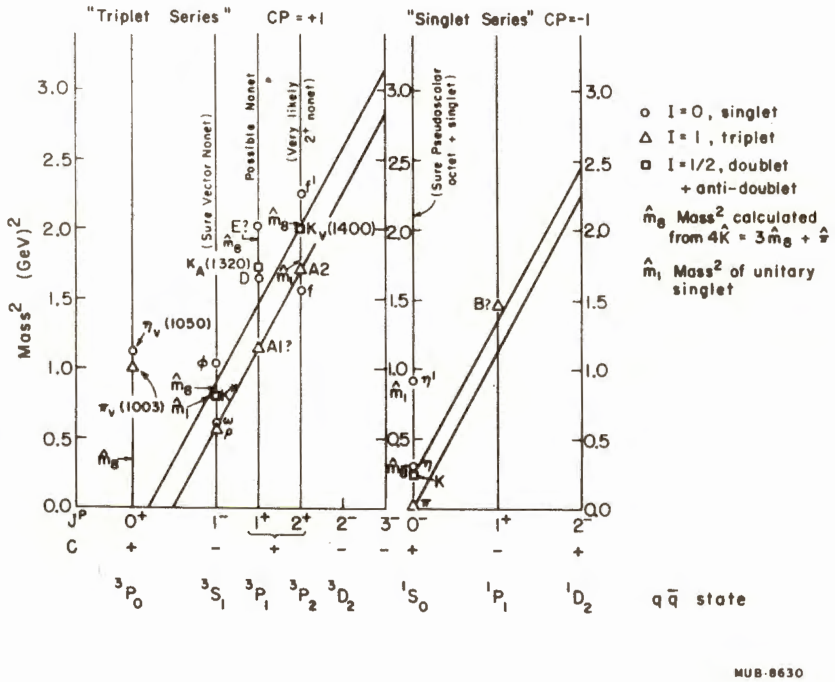
→ at left of Table indicates a candidate that has been omitted because the evidence for the existence of the effect and/or for its interpretation as a resonance is open to considerable question. See listings for information on the following: $N_{3/2}$(3245), N(3695), $N^*_{5/2}$(1560), Z^*_1(1910), Σ(1780), Σ(3000), Ξ(1705), and $Ξ_2$(2270).

* Quoted error includes an S (scale) factor. See footnote to Table S.

‡ For decay modes into ≥3 particles p_{max} is the maximum momentum that any of the particles in the final state can have. The momenta have been calculated using the averaged central mass values, without taking into account the widths of the resonances.

MUB 13979


BARYON STATES, UCRL-8030 Rev.
Jan. 1967
Possibly questionable states are dashed.
Vertical bars : Γ (m²) = 2mΓ = full width.
Horizontal bars subdivided proportional to branching fractions.
States with J^P uncertain are shaded.
Even parity, p, f, ... waves
Odd parity, s, d, g, ... waves
Mass [(GeV)²]
a) J^P = 1/2+, 5/2+, ---
b) J^P = 3/2+, ---
c) J^P = 1/2-, ---
d) J^P = 3/2-, ---
Raised for plotting
Lowered for plotting
Ω (1675) decays weakly into ΛK̄ and Ξπ
MUB-12336


"Triplet Series" CP = +1
"Singlet Series" CP=-1
Mass² (GeV)²
(Sure Vector Nonet)
Possible Nonet
(Very likely 2+ nonet)
(Sure Pseudoscalar octet + singlet)
o I=0, singlet
Δ I=1, triplet
□ I=1/2, doublet + anti-doublet
m̂8 Mass² calculated from 4K̂ = 3m̂8 + π̂
m̂1 Mass² of unitary singlet
qq̄ state
MUB-8630

Special Relativity

Notation. 4-vector in c.m. $p = (w, \vec{p})$; in lab $P = (W, \vec{P})$, $T = W - m$.
Solid-angle element $d\omega = 2\pi d\cos\theta$; $d\Omega = 2\pi d\cos\Theta$.
$p^2 = w^2 - \vec{p}^2 = m^2$ is an invariant. Cross section σ is invariant.

Lorentz Transformation

$$\begin{pmatrix} w \\ p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} \bar\gamma & -\bar\eta & 0 & 0 \\ -\bar\eta & \bar\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} W \\ P_x \\ P_y \\ P_z \end{pmatrix}$$

If θ and Θ are measured with respect to the transformation axis x,

$$\frac{p_\perp}{p_x} = \tan\theta = \frac{P\sin\Theta}{-\bar\eta W + \bar\gamma P\cos\Theta}. \qquad (1)$$

If particle 1 is beam, 2 is target, $(W_2, \vec{P}_2) = (m_2, \vec{0})$ and

$$\bar\gamma = (W_1 + m_2)/\sqrt{s}, \quad \bar\eta = \bar\gamma\bar\beta = |\vec{P}_1|/\sqrt{s}, \quad |\vec{p}_1| = |\vec{p}_2| = \bar\eta m_2 = |\vec{P}_1| m_2/\sqrt{s}. \qquad (2)$$

For $m_1 = m_2$, $\bar\gamma^2 = 1 + T_1/2m_1$.

Invariants. Notation: $1 + 2 \to 1' + 2'$.

$$s = (p_1 + p_2)^2 = m_1^2 + m_2^2 + 2(w_1 w_2 - \vec{p}_1 \cdot \vec{p}_2) \qquad (3)$$

$$t = (p'_i - p_i)^2 = m_i^2 + m_i'^2 - 2(w_i w'_i - \vec{p}_i \cdot \vec{p}'_i) \quad (i = 1, 2) \qquad (4)$$

$$u = (p'_1 - p_2)^2 = (p'_2 - p_1)^2 \qquad (5)$$

General relation: $s + t + u = m_1^2 + m_1'^2 + m_2^2 + m_2'^2$. (6)

In lab system $P_2 = (m_2, \vec{0})$, and writing $W = m + T$

$$s = m_1^2 + m_2^2 + 2W_1 m_2 = (m_1 + m_2)^2 + 2T_1 m_2 \qquad (3, \text{lab})$$

$$t = m_2^2 + m_2'^2 - 2W'_2 m_2 = (m_2 - m'_2)^2 - 2T'_2 m_2 \qquad (4, \text{lab})$$

In c.m. system $dt = +2|\vec{p}_i|\,|\vec{p}'_i|\, d\cos\theta$ (4, cm)

For elastic scattering ($m_1 = m'_1$, $m_2 = m'_2$), (4) in c.m. simplifies to

$$\left.\begin{matrix} t \\ u \end{matrix}\right\} = -2\vec{p}^2(1 \mp \cos\theta) = -4\vec{p}^2 \begin{Bmatrix} \sin^2\theta/2 \\ \cos^2\theta/2 \end{Bmatrix} \qquad (4, \text{el})$$

For elastic scattering, using (4, lab), (4, el), and (2),

$$T'_2 = \frac{2\vec{P}_1^2 m_2}{s} \sin^2\left(\frac{\theta}{2}\right) \quad \text{(useful for calculating } \delta\text{-ray energies).} \qquad (7)$$

Two-Body States. Energies and momenta in c.m.

$$w_1 = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}}, \quad \vec{p}_1^2 = \vec{p}_2^2 = \frac{1}{4s}[s - (m_1 + m_2)^2][s - (m_1 - m_2)^2]. \qquad (8)$$

3- and 4-Body States. Let $m_{ij}^2 = (p_i + p_j)^2$, etc., then

$$\sum_{i<j} m_{ij}^2 = \Sigma m_i^2 + m_{123}^2 = \text{const.} \quad (i, j = 1, 2, 3) \text{ (follows from (6)).} \qquad (9)$$

$$\left.\begin{aligned} &= 2\Sigma m_i^2 + m_{1234}^2 = \text{const.} \\ \sum_{i<j<k} m_{ijk}^2 &= \Sigma m_i^2 + 2m_{1234}^2 = \text{const.} \end{aligned}\right\} (i, j, k = 1, 2, 3, 4.) \qquad (10)$$

R_n, Invariant Volume in n-Body Momentum Space

A useful invariant is $\int d^4p\, \delta(p^2 - m^2) = \frac{d^3\vec{p}}{2w} = \frac{\vec{p}^2 d|\vec{p}| d\omega}{2w} = \frac{1}{2}|\vec{p}|\, dw\, d\omega$.

$R_2 = \pi|\vec{p}_1|/\sqrt{s}$, $R_3 = \pi^2 \int dw_1 dw_2 = (\pi^2/4s)\int dm_{12}^2\, dm_{23}^2$ (using c.m. quantities)

Recurrence Relation for Factoring R_n (see e.g., Hagedorn, p. 93[a])

Write $N \to 1, 2, \ldots k, k+1, \cdots n$ (R_n), then
as $N \to K, k+1, \cdots n$ (R_{n-k+1}); $K \to 1, 2, \cdots k$ (R_k); $R_n = \int dM_K^2 R_k R_{n-k+1}$

Cross Section (or Decay Rate)

For $1 + 2 \to n$ particles (or $1 \to n$ particles)

$\sigma F = |M|^2 R_n$ (or $\Gamma w_1 = |M|^2 R_n$)

where M is an invariant matrix element.

F is (Møller's) invariant flux factor, $F^2 = (p_1 p_2)^2 - m_1^2 m_2^2$.

In every system where $\vec{p}_1$ and $\vec{p}_2$ are collinear, $F = w_1 w_2 |\vec{v}_1 - \vec{v}_2|$ $(\vec{v} = \vec{p}/w)$.

If particle 1 is beam, 2 is target $(\vec{P}_2 = 0)$, $F = |\vec{P}_1| m_2 = |\vec{p}_1|\sqrt{s}$ [cf. (2)].

The rate (= number per unit 4-dimensional volume $d^4r = dt\, d^3\vec{r}$) is

$d^4N/d^4r = \sigma\, \rho_1 \rho_2 |\vec{v}_1 - \vec{v}_2|$, ρ_i = volume density of particles (i = 1, 2).

a. R. Hagedorn, Relativistic Kinematics, (W. A. Benjamin, New York, 1964).

C. M. Energy and Momentum vs Beam Momentum of e, π, K, or p on p

$\mu d\mu = m_p dE = m_p v_{beam,\,lab}\, dp \approx m_p dp$

P(LAB) (MEV/C)	INVARIANT MASS (MEV) ep	πp	Kp	pp	P(CMS) (MEV/C) ep	πp	Kp	pp
0	939	1078	1432	1877	0	0	0	0
20	958	1079	1432	1877	20	17	13	10
40	977	1083	1433	1877	38	35	26	20
60	996	1089	1434	1877	56	52	39	30
80	1015	1096	1436	1878	74	68	52	40
T(PI) = PLAB - 59 MEV								
100	1033	1105	1439	1875	91	85	65	50
120	1051	1116	1441	1880	107	101	78	60
140	1069	1127	1445	1882	123	117	91	70
160	1087	1139	1449	1883	138	132	104	80
180	1104	1152	1453	1885	153	147	116	90
T(PI) = PLAB - 92 MEV								
200	1121	1165	1457	1887	167	161	129	99
220	1137	1178	1462	1889	182	175	141	109
240	1154	1192	1468	1892	195	189	153	119
260	1170	1206	1474	1894	209	202	166	129
280	1186	1219	1480	1897	222	215	178	138
T(PI) = PLAB - 107 MEV								
300	1201	1233	1486	1900	234	228	189	148
320	1217	1247	1493	1903	247	241	201	158
340	1232	1261	1500	1906	259	253	213	167
360	1247	1274	1507	1910	271	265	224	177
380	1262	1288	1514	1913	282	277	235	186
T(PI) = PLAB - 115 MEV								
400	1277	1302	1522	1917	294	288	247	196
420	1292	1315	1530	1921	305	300	258	205
440	1306	1329	1538	1925	316	311	268	214
460	1320	1342	1546	1929	327	322	279	224
480	1335	1356	1554	1933	337	332	290	233
T(PI) = PLAB - 120 MEV								
500	1349	1369	1563	1938	348	343	300	242
520	1362	1382	1572	1943	358	353	310	251
540	1376	1395	1580	1947	368	363	321	260
560	1390	1408	1589	1952	378	373	331	269
580	1403	1421	1598	1957	388	383	341	278
T(PI) = PLAB - 123 MEV								
600	1416	1434	1607	1962	397	393	350	287
620	1430	1446	1616	1967	407	402	360	296
640	1443	1459	1625	1973	416	412	370	304
660	1456	1472	1634	1978	425	421	379	313
680	1468	1484	1644	1984	434	430	388	322
T(PI) = PLAB - 125 MEV								
700	1481	1496	1653	1989	443	439	397	330
720	1494	1509	1662	1995	452	448	406	339
740	1506	1521	1671	2001	461	457	415	347
760	1519	1533	1681	2007	470	465	424	355
780	1531	1545	1690	2013	478	474	433	364
T(PI) = PLAB - 127 MEV								
800	1543	1557	1699	2019	486	482	442	372
820	1555	1569	1709	2025	495	490	450	380
840	1567	1580	1718	2031	503	499	459	388
860	1579	1592	1728	2037	511	507	467	396
880	1591	1604	1737	2043	519	515	475	404
T(PI) = PLAB - 129 MEV								
900	1603	1615	1747	2049	527	523	483	412
920	1615	1627	1756	2056	535	531	492	420
940	1626	1638	1765	2062	542	538	500	428
960	1638	1649	1775	2069	550	546	508	435
980	1649	1661	1784	2075	558	554	515	443
T(PI) = PLAB - 130 MEV								
1000	1660	1672	1794	2082	565	561	523	451
1020	1672	1683	1803	2088	573	569	531	458
1040	1683	1694	1812	2095	580	576	538	466
1060	1694	1705	1822	2102	587	583	546	473
1080	1705	1716	1831	2108	594	591	553	481
T(PI) = PLAB - 131 MEV								
1100	1716	1726	1840	2115	601	598	561	488
1120	1727	1737	1850	2122	609	605	568	495
1140	1738	1748	1859	2129	616	612	575	502
1160	1748	1758	1868	2135	622	619	583	510
1180	1759	1769	1877	2142	629	626	590	517
T(PI) = PLAB - 131 MEV								
1200	1770	1780	1887	2149	636	633	597	524
1220	1780	1790	1896	2156	643	639	604	531
1240	1791	1800	1905	2163	650	646	611	538
1260	1801	1811	1914	2170	656	653	618	545
1280	1812	1821	1923	2177	663	660	624	552
T(PI) = PLAB - 132 MEV								
1300	1822	1831	1932	2184	669	666	631	559
1320	1832	1841	1941	2191	676	673	638	565
1340	1843	1851	1950	2198	682	679	645	572
1360	1853	1862	1959	2205	689	685	651	579
1380	1863	1872	1968	2212	695	692	658	585
T(PI) = PLAB - 133 MEV								

P(LAB) (MEV/C)	INVARIANT MASS (MEV) ep	πp	Kp	pp	P(CMS) (MEV/C) ep	πp	Kp	pp
1400	1873	1881	1977	2219	701	698	664	592
1420	1883	1891	1986	2226	708	704	671	599
1440	1893	1901	1995	2233	714	711	677	605
1460	1903	1911	2004	2240	720	717	684	612
1480	1912	1921	2013	2247	726	723	690	618
T(PI) = PLAB - 133 MEV								
1500	1922	1930	2022	2254	732	729	696	624
1520	1932	1940	2031	2261	738	735	702	631
1540	1942	1950	2039	2268	744	741	709	637
1560	1951	1959	2048	2275	750	747	715	643
1580	1961	1969	2057	2282	756	753	721	650
T(PI) = PLAB - .133 MEV								
1600	1970	1978	2065	2289	762	759	727	656
1620	1980	1988	2074	2296	768	765	733	662
1640	1989	1997	2083	2304	773	770	739	668
1660	1999	2006	2091	2311	779	776	745	674
1680	2008	2016	2100	2318	785	782	751	680
T(PI) = PLAB - 134 MEV								
1700	2018	2025	2109	2325	791	788	756	686
1720	2027	2034	2117	2332	796	793	762	692
1740	2036	2043	2126	2339	802	799	768	698
1760	2045	2053	2134	2346	807	805	774	704
1780	2054	2062	2143	2353	813	810	779	710
T(PI) = PLAB - 134 MEV								
1800	2064	2071	2151	2360	818	816	785	716
1820	2073	2080	2159	2367	824	821	791	721
1840	2082	2089	2168	2374	829	827	796	727
1860	2091	2098	2176	2381	835	832	802	733
1880	2100	2107	2184	2388	840	837	808	739
T(PI) = PLAB - 134 MEV								
1900	2108	2115	2193	2395	845	843	813	744
1920	2117	2124	2201	2402	851	848	818	750
1940	2126	2133	2209	2409	856	853	824	756
1960	2135	2142	2217	2416	861	859	829	761
1980	2144	2150	2226	2423	867	864	835	767
T(PI) = PLAB - 135 MEV								
2000	2153	2159	2234	2430	872	869	840	772
2020	2161	2168	2242	2437	877	874	845	778
2040	2170	2176	2250	2444	882	879	851	783
2060	2179	2185	2258	2451	887	885	856	789
2080	2187	2194	2266	2458	892	890	861	794
T(PI) = PLAB - 135 MEV								
2100	2196	2202	2274	2465	897	895	866	799
2120	2204	2211	2282	2472	902	900	872	805
2140	2213	2219	2290	2479	907	905	877	810
2160	2221	2227	2298	2486	912	910	882	815
2180	2230	2236	2306	2493	917	915	887	821
T(PI) = PLAB - 135 MEV								
2200	2238	2244	2314	2500	922	920	892	826
2220	2246	2253	2322	2507	927	925	897	831
2240	2255	2261	2330	2514	932	930	902	836
2260	2263	2269	2338	2520	937	934	907	841
2280	2271	2277	2346	2527	942	939	912	846
T(PI) = PLAB - 135 MEV								
2300	2280	2286	2353	2534	947	944	917	852
2320	2288	2294	2361	2541	951	949	922	857
2340	2296	2302	2369	2548	956	954	927	862
2360	2304	2310	2377	2555	961	959	932	867
2380	2312	2318	2384	2561	966	963	937	872
T(PI) = PLAB - 135 MEV								
2400	2320	2326	2392	2568	970	968	941	877
2420	2328	2334	2400	2575	975	973	946	882
2440	2336	2342	2407	2582	980	977	951	887
2460	2344	2350	2415	2589	984	982	956	892
2480	2352	2358	2423	2595	989	987	960	897
T(PI) = PLAB - 136 MEV								
2500	2360	2366	2430	2602	994	991	965	901
2520	2368	2374	2438	2609	998	996	970	906
2540	2376	2382	2445	2616	1003	1001	975	911
2560	2384	2390	2453	2622	1007	1005	979	916
2580	2392	2398	2460	2629	1012	1010	984	921
T(PI) = PLAB - 136 MEV								
2600	2400	2405	2468	2636	1017	1014	988	926
2620	2408	2413	2475	2643	1021	1019	993	930
2640	2415	2421	2483	2649	1025	1023	998	935
2660	2423	2429	2490	2656	1030	1028	1002	940
2680	2431	2436	2498	2663	1034	1032	1007	944
T(PI) = PLAB - 136 MEV								
2700	2439	2444	2505	2669	1039	1037	1011	949
2720	2446	2452	2512	2676	1043	1041	1016	954
2740	2454	2459	2520	2682	1048	1045	1020	958
2760	2462	2467	2527	2689	1052	1050	1025	963
2780	2469	2474	2534	2696	1056	1054	1029	968
T(PI) = PLAB - 136 MEV								

P(LAB) (GEV/C)	INVARIANT MASS (GEV) πp ep	Kp	pp	P(CMS) (GEV/C) πp ep	Kp	pp
3.0	2.56	2.61	2.77	1.10	1.08	1.02
3.2	2.63	2.68	2.83	1.14	1.12	1.06
3.4	2.70	2.75	2.89	1.18	1.16	1.10
3.6	2.77	2.82	2.96	1.22	1.20	1.14
3.8	2.83	2.88	3.02	1.26	1.24	1.18
T(PI) = PLAB - .137 GEV						
4.0	2.90	2.95	3.08	1.29	1.27	1.22
4.2	2.96	3.01	3.14	1.33	1.31	1.26
4.4	3.03	3.07	3.19	1.36	1.34	1.29
4.6	3.09	3.13	3.25	1.40	1.38	1.33
4.8	3.15	3.19	3.31	1.43	1.41	1.36
T(PI) = PLAB - .138 GEV						
5.0	3.21	3.25	3.36	1.46	1.44	1.40
5.2	3.27	3.31	3.42	1.49	1.48	1.43
5.4	3.32	3.36	3.47	1.53	1.51	1.46
5.6	3.38	3.42	3.52	1.56	1.54	1.49
5.8	3.43	3.47	3.58	1.59	1.57	1.52
T(PI) = PLAB - .138 GEV						
6.0	3.49	3.52	3.63	1.61	1.60	1.55
6.2	3.54	3.58	3.68	1.64	1.63	1.58
6.4	3.59	3.63	3.73	1.67	1.65	1.61
6.6	3.65	3.68	3.78	1.70	1.68	1.64
6.8	3.70	3.73	3.83	1.73	1.71	1.67
T(PI) = PLAB - .138 GEV						
7.0	3.75	3.78	3.87	1.75	1.74	1.70
7.2	3.80	3.83	3.92	1.78	1.76	1.72
7.4	3.85	3.88	3.97	1.81	1.79	1.75
7.6	3.89	3.93	4.02	1.83	1.82	1.78
7.8	3.94	3.97	4.06	1.86	1.84	1.80
T(PI) = PLAB - .138 GEV						
8.0	3.99	4.02	4.11	1.88	1.87	1.83
8.2	4.04	4.07	4.15	1.91	1.89	1.85
8.4	4.08	4.11	4.20	1.93	1.92	1.88
8.6	4.13	4.16	4.24	1.95	1.94	1.90
8.8	4.17	4.20	4.29	1.98	1.96	1.93
T(PI) = PLAB - 138 GEV						
9.0	4.22	4.25	4.33	2.00	1.99	1.95
9.2	4.26	4.29	4.37	2.03	2.01	1.97
9.4	4.31	4.33	4.41	2.05	2.03	2.00
9.6	4.35	4.38	4.46	2.07	2.06	2.02
9.8	4.39	4.42	4.50	2.09	2.08	2.04
T(PI) = PLAB - .139 GEV						
10.0	4.43	4.46	4.54	2.12	2.10	2.07
10.5	4.54	4.57	4.64	2.17	2.16	2.12
11.0	4.64	4.67	4.74	2.22	2.21	2.18
11.5	4.74	4.77	4.84	2.28	2.26	2.23
12.0	4.84	4.86	4.93	2.33	2.31	2.28
T(PI) = PLAB - .139 GEV						
12.5	4.94	4.96	5.03	2.38	2.36	2.33
13.0	5.03	5.05	5.12	2.43	2.41	2.38
13.5	5.12	5.15	5.21	2.47	2.46	2.43
14.0	5.21	5.24	5.30	2.52	2.51	2.48
14.5	5.30	5.32	5.39	2.57	2.56	2.53
T(PI) = PLAB - .139 GEV						
15.0	5.39	5.41	5.47	2.61	2.60	2.57
15.5	5.48	5.50	5.56	2.66	2.65	2.62
16.0	5.56	5.58	5.64	2.70	2.69	2.66
16.5	5.64	5.67	5.72	2.74	2.73	2.70
17.0	5.73	5.75	5.81	2.78	2.77	2.75
T(PI) = PLAB - .139 GEV						
17.5	5.81	5.83	5.89	2.83	2.82	2.79
18.0	5.89	5.91	5.97	2.87	2.86	2.83
18.5	5.97	5.99	6.04	2.91	2.90	2.87
19.0	6.05	6.07	6.12	2.95	2.94	2.91
19.5	6.12	6.14	6.20	2.99	2.98	2.95
T(PI) = PLAB - .139 GEV						
20.0	6.20	6.22	6.27	3.03	3.02	2.99
21.0	6.35	6.37	6.42	3.10	3.09	3.07
22.0	6.49	6.51	6.56	3.18	3.17	3.14
23.0	6.64	6.66	6.70	3.25	3.24	3.22
24.0	6.78	6.79	6.84	3.32	3.31	3.29
T(PI) = PLAB - .139 GEV						
25.0	6.91	6.93	6.98	3.39	3.38	3.36
26.0	7.05	7.07	7.11	3.46	3.45	3.43
27.0	7.18	7.20	7.24	3.53	3.52	3.50
28.0	7.31	7.33	7.37	3.59	3.59	3.56
29.0	7.44	7.45	7.50	3.66	3.65	3.63
T(PI) = PLAB - .139 GEV						
30.0	7.56	7.58	7.62	3.72	3.71	3.69
35.0	8.16	8.17	8.21	4.02	4.02	4.00
40.0	8.72	8.73	8.77	4.31	4.30	4.28
45.0	9.24	9.25	9.29	4.57	4.56	4.55
50.0	9.73	9.74	9.78	4.82	4.81	4.80
T(PI) = PLAB - .139 GEV						