ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
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МАТЕРИАЛЫ XIII МЕЖДУНАРОДНОЙ КОНФЕРЕНЦИИ ПО ФИЗИКЕ ВЫСОКИХ ЭНЕРГИЙ

Беркли 1966 г.

M. Froissart

FUNDAMENTAL THEORETICAL

QUESTIONS

Introduction

Before reviewing for you the main fields of activity regarding the fundaments of theory, let me show you a map (Fig. 1) of the different logical connections among these fields, because, judging from the attendance to the discussion sessions, I presume that many of you are not quite as up to date as they prehaps might be.

I have drawn a boundary separating the results relevant to off-mass shell theories or field theories in a generalized sense, and the on-mass-

S (cap.)

shell results or ~~s~~-matrix theory results. Ideally the basic axioms should be immediate translations of our experience. A very attractive set would be Lorentz invariance, causality and unitarity. The trouble is that the onl tractable form of causality up to recently was through locality, which had to be formulated in terms of fields. Hence the very prominent role of field theory up to this day in our understanding of the microscopic world.

May I stress also the importance of the success of Quantum Electrodynamics as a reason to stick to field theory. However, in view of the very singular nature of fields, it has been thought wise to dismiss the idea of fields and to consider only integrals of fields, over regions of space time. This led to the theory of local observables.

Most experimentalists are really interested only in knowing what happens on the mass shell. This has put a heavy burden on the theorists. Field theory gave the dispersion relations, and a small domain of analyticity in momentum transfer (Lehmann ellipse). When combined with unitarity, these results furnished bounds and relations on the asymptotic behaviour of the amplitude. From then on, everything was until recently a matter of sophisticated guess. Mandelstam started the game by proposing a certain domain of analyticity compatible with perturbation theory to 4th order and dispersion theory. Then came different hypothes~~i~~es: Maximal analyticity, Regge ~~B~~behaviour, bootstrap.

Field Theory

The main effort in field theory has been devoted to the study of the mathematical structure involved.[1] (Reference) It is interesting to note that a number of tools have been developed, which are useful both for field theory and for statistical mechanics. The reason for this is clear: one has to do in both cases with systems with an infinite number of degrees of freedom. A very interesting example of the mathematical complications involved in the simplest cases has been discussed by Prof. Thirring[2] in connection with the B.C.S. theory of superconductivity. Although the model used is excessively simple and explicitly solvable in the case of a finite number of degrees of freedom, the passage to the limit exhibits a number of unexpected features, among which the fact that some products of operators have a limit different from the product of the limits.

To study this kind of mathematical trouble, Jaffe and Lanford[3] have built model field theories with an artificially limited number of degrees of freedom. The object is to see which quantities have any chance of keeping a meaning as the cut-offs are removed. In an effort to remove the limitations imposed by the assumption of the existence of fields, and considering that the phenomena of ultraviolet divergence might make the fields more singular than they are usually assumed to be, Jaffe[4] has also developed a formalism capable to accomodate some non-renormalizable theories. It appears that all results obtained up to date in conventional field theory hold true in this new version, except for the asymptotic behaviour at high energy. Conventional dispersion relations may be proved, but might need an infinite number of subtractions, if it were not for the results of Martin which we shall discuss.

Much work has been devoted to the structure of the theory of local observables.[5] I understand that only technical difficulties prevent Borches from deriving a concept very close to that of fields, even unobservable charged fields, from the theory of local observables. Epstein[6] has already proved the TCP theorem in this theory.

Analyticity on the mass-shell

A sequence of very important results has broadened considerably the bridge between field theory and ~~$\mathcal{S}$~~ S-matrix theory. The papers of Bros, Epstein, and Glaser[7] have generalized the concept of crossing to all two-body amplitudes, whether dispersion relations are proved or not. Also, they have shown that the two-body amplitudes are all analytic in some small neighborhood of the physical region, this analyticity being in both variables s (energy variable) and t (momentum transfer variable).

Using this information, the known dispersion relations, and unitarity, Martin[8] has been able to prove a very large number of results. In view of the importance of this work, let me go a little into the scheme of the proof.

All s l.c. in formulas

We start from a conventional dispersion relation.

$$A(s,t) = \frac{s^N}{\pi}\int \frac{A_s(s',t)\,ds'}{s'^N(s'-s)} + \frac{u^N}{\pi}\int \frac{A_u(u',t)\,du'}{u'^N(u'-u)} + \text{Subtractions}$$

Unitarity is used then under the following form:

$$A_s(s,t) = \sum_\ell (2\ell+1)\,\mathrm{Im}\,a_\ell(s)\,P_\ell\left(1+\frac{t}{2k^2}\right)$$

with ~~But in~~ $\mathrm{Im}\,a_\ell(s) > 0$, and all derivatives of $P_\ell(Z)$ are positive for $Z > 1$.

l.c. el

This allows to expand $A_s(s, t)$ in a Taylor series in t and to show that the radius of convergence is the same for all s, by interchange of the order of integration and summation of the series. Then the dispersion relation is valid for all values of t for which analyticity has been proved for fixed s. Typical values are:[9]

$\pi\pi$	$\|t\| < 4\, m_\pi^2$
$K\pi$	$\|t\| < 4\, m_\pi^2$
KK	$\|t\| < 4\, m_\pi^2$
πN	$\|t\| < 1.83\, m_\pi^2$

The fact that the dispersion relation stays valid as it is written leads to the consequence that in these regions, the asymptotic behavior is nearly the same as in the forward direction. Hence, by an almost circular argument, using unitarity, one deduces that, even with dispersion relations à la Jaffe--with an infinite number of subtractions--to start with, the behavior of the amplitude is $s \ln^2 s$, therefore the dispersion relations are valid with 2 subtractions. An interesting feature is that the reasoning can be started again from the domain of analyticity just found, and a larger domain is obtained. The result for $\pi\pi$ scattering is represented on Figure 2. The same operation can be performed for πK and πN scattering. The domain obtained in the latter case goes to larger values of $-t$ than was proved in the classical proofs of dispersion relation. More work can undoubtedly be done, by using, in the $\pi\pi$ case, the domains obtained in the three channels, and finding their holomorphy envelope. To date, the real part of the analyticity domain is given approximately by the cuts and the dashed region of Figure 3. That is, it goes to the double spectral region except for a few cases. This gives a region of analyticity for the partial-wave amplitudes indicated on Figure 4. Possible singularities are far enough as compared, say to the mass of the ρ meson, so that calculations neglecting far away singularities may be justified.

On-Mass-Shell Causality

Much work is also devoted to get rid of the circuitous--and not completed-- logical path connecting causality to analyticity. Many people think that an axiom like that of locality is probably as unphysical as an axiom of analyticity, in the sense that it can only be checked indirectly through its consequences. As Blokhintsev[10,11] points out, it is quite feasible to build theories which are non local on the microscopic scale, but which exhibit gross causality. These theories might perhaps only be distinguished by additional singularities. The works of the "Cambridge School" ~~12-19~~ and others[12-19] have shown that an assumption of analyticity of the amplitude in some neighborhood of the physical region, leads to uniquely determined singularities, and hence[20] to the usual description of of successive interactions in multiple scattering. These arguments use only postulates of cluster decomposition[21] and unitarity.

The work of Stapp[22] assumes a weak condition on asymptotic decrease of transition probabilities when particles are taken away. The result is to prove the infinite differentiability of amplitudes on the physical region except at the points where singularities are expected. A condition of exponential decrease of transition probabilities allows Omnès[23] to derive a finite domain of analyticity for the 2-body amplitudes in momentum transfer, apart from some technical difficulties. Some other authors[24 - 26] have also tried to attack this problem, but without any decisive success.

Asymptotic Theorems

We can distinguish between bounds and other theorems such as the Pomeranchuk theorem and the connections between the phase and the modulus of the amplitude. In the first field, no new result has been obtained. However, many bounds are now rigorously derived from field theory, and even from the generalized version due to Jaffe. This is of course due to the work of Martin. I would like to mention specially the bound on form factors $|F(t)| > \exp - b\sqrt{-t}$ derived by Jaffe[27] rigorously, even for the cases where no dispersion relations have been proved for the form factor. Tables I, II, and III have been compiled by Martin, and give an up-to-date account of the situation.[28-43]

In the case of other asymptotic theorems, the situation is still very confused. The Pomeranchuk theorem now holds for all processes, due to B.E.G.,[7] but, as pointed out by Eden[44] no one has yet succeeded in removing the extra hypothesis such as the existence of a limit of the amplitude--or at least some control over possible oscillations--on the one hand, and some control over the growth of the real part with respect to the imaginary part.

To obviate the need for these requirements, Khuri and Kinoshita[45,46] have found a number of inequalities which, unlike ordinary dispersion relations, allow one to test these ideas with measurements of the forward amplitude on a <u>finite</u> real interval. The information used on the behavior at infinity is the positivity of the imaginary part, and the established fact that there are at most 2 subtractions needed.

I would like to end by expressing my apologies to the many people who have been treated unfairly, or whose work I have misrepresented.

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	Assumption	References
Forward scattering $\lvert F(s,\cos\theta=1)\rvert < C\, s \log^2(s/s_0)$ $\sigma_{total} < C' \log^2(s/s_0)$ (arbitrary spins)	Axiomatic	7
$C' < 4\pi/\mu^2$, μ= pion mass for $\pi\pi$, πk, kk		
$C' < 12\pi/\mu^2$ for πN		28
Fixed transfer $\lvert F(s,t)\rvert < s(\log s)^{3/2}$, $t < -\varepsilon < 0$		29
$\lvert F(s,t)\rvert < s^{2-\varepsilon}$ $0 < t < 4\mu^2$	Axiomatic	28
$\lvert F(s,t)\rvert < \text{const}$ $0 < t < 4\mu^2$ if $\lvert F(s,0)\rvert < s^{-\varepsilon}$		30
Fixed angle $\lvert F(s,\cos\theta)\rvert < C(s^{3/4}(\log s)^{3/2})/(\sqrt{\sin\theta})$	Axiomatic	29
$\lvert F(s,\cos\theta)\rvert < C(\log s)^{3/2}/(\sin^2\theta)$	Mandelstam	31
$d\sigma/d\Omega < C(\log s)^3/(\sin^4\theta)\, s)$ (in this form valid for arbitrary spins)		32 - 34

T A B L E I

Upper Bounds

	Assumption	References
$\frac{d}{dt}\log A_s(s,t)\Big\rvert_{t=0} > \frac{1}{9}\left[\frac{(\sigma_{total})^2}{4\pi\sigma_{el}} - \frac{1}{k^2}\right]$	None	35
$\frac{d}{dt}\log A_s(s,t)\Big\rvert_{t=0} < C(\log s)^2$	Axiomatic	30
$\frac{\int_{-1}^{1}\lvert F(s,\cos\theta)\rvert^2 d\cos\theta}{\lvert F(s,\cos=1)\rvert^2} \to 0$ if $A_s(s,\cos\theta=1) > C/\log s$	Axiomatic	36 - 37-

T A B L E II

Diffraction Peak

	Assumption	References
<u>Elastic cross-section</u> $\sigma_{el} > C \frac{(\sigma_{total})^2}{(\log s)^2}$	Axiomatic	38
<u>Forward amplitude</u> (strict for complex s; average sense for real s) a) $\lvert F(s,t=0)\rvert > 1/s^2$ (no unphysical cut) $\sigma_{total} > 1/(s^6 \log^2 s)$	Axiomatic	28
b) if F(threshold) $<$ 0 or is small as compared to an integral over total cross-sections at low energies <u>only</u>	Axiomatic	28, 39
$\lvert F(s,t=0)\rvert >$ const > 0 $\sigma_{total} > 1/(s^2 \log^2 s)$	Axiomatic	
<u>Fixed angle amplitude</u> $\lvert F(s,\cos\theta)\rvert > \exp - \sqrt{s} \log s \, C(\theta)$	Mandelstam	40
$C(\theta)$ for small $\theta \sim \sqrt{\theta}$		41
<u>Form factors</u> $\lvert F(t)\rvert > \exp - C \sqrt{\lvert t\rvert}$	Axiomatic	42, 27
<u>Fixed t amplitude t< o</u> $\lvert F(s,t)\rvert > s^{-N}$	Axiomatic	
<u>Fixed u amplitude u <o</u> $\lvert F(s,u)\rvert > s^{-N}$ if pole at $u=M^2$	Slightly more than axiomatic	43

TABLE III
Lower Bounds

Figure Captions

1. Logical map of "Fundamental" concepts.

   Continuous lines represent inferences, broken lines indications.

   Topics surveyed in this report are indicated by double lines.

2. Values of the momentum transfer variable t for which dispersion relations are proved in π-π scattering.

3. The shaded region is the real trace of the analyticity domain of the π-π scattering amplitude. Within this region the cuts are à la Mandelstam. The dashed curves are the boundaries of the Mandelstam double spectral regions.

4. Analyticity domain for π-π scattering partial waves. The thin neighborhood of the positive real axis is not drawn to scale.

DISCUSSION OF FROISSART'S PAPER: "FUNDAMENTAL THEORETICAL QUESTION"

NAUENBERG (SLAC): Could you give some more details on what results have been obtained about Regge behavior from fundamental assumptions?

FROISSART: From the fundamental side I haven't drawn any continous arrow on the map. There are some inequalities which have been proved to hold asymptotically, and which are consistent with Regge behavior, but I don't think that anything more detailed has been proven.

SUDARSHAN (Syracuse): It was my impression that you had said that Borchers had proved something about the connection between local observables and the existence of a vacuum.

FROISSART: Yes, the proof suffers a little technical difficulty now, but I don't think it's very essential. Modulo such technical difficulties, he proves that in a theory of local observables there exists a vacuum state which has all good properties.

SUDARSHAN: I must have been misunderstood. If you took a free field, a Wick polynomial of second degree as Wightman and other people have done, then the states of this particular system split into two classes, those corresponding to an odd number of particles of the original field, and those corresponding to an even number of particles. If you consider that particular subset of states which consists only of odd numbers of particles of the original free field, the new field we have introduced seems to be able to connect only to other odd numbers of particles. In such a theory there seems to be no vacuum at all, and I don't quite understand how one could have proved the existence of a vacuum. This is a free field theory; there are no divergences of any kind.

FROISSART: The idea of Borchers is very simple. You start by taking any state, and then you essentially average out over the whole space; that is, if there are any particles to start with, ~~when you average out the state over all translations~~ then these particles should not contribute at all to the average state. That's the essence of Borchers' proof. I don't know which of the axioms the phenomena you are mentioning contradict.

SUDARSHAN: This assumes that there exists a discrete mass spectrum of the field?

WIGHTMAN: (Princeton) May I comment on that? The point is that Borchers is using a definition of physical equivalence which is appreciably weaker than that to which many people are accustomed, so that he would not have to assume that there was, a priori, a discrete point in the spectrum. He would count two theories equivalent if they give arbitrarily close results for all measurements in bounded regions of space time. Presumbly he will take your example and prove it equivalent to another example in which there was a vacuum state but only in this weak sense. However, it has to be said that this weak sense is a very physical sense, because it corresponds to measurements in actual laboratories as best we know how to describe them.

TODOROV (Dubna): I would like to make the following remark, concerning the derivation of Pomeranchuk's theorem.

It is not necessary to utilize the far non-trivial results of Bros, Epstein and Glaser about the analyticity domain of any two body scattering amplitude in order to prove a Pomeranchuk type theorem for it. It is sufficient to take into account the analyticity properties of the so called asymptotic amplitude, introduced by Meiman (1964). (The exact conditions under which the physical amplitude has the same high energy behavior as the asymptotic one practically coincide with the smoothness assumption needed for the proof of Pomeranchuk's theorem.) The advantage of such a method results from the fact that it applies to production amplitudes too (cf. Logunov's report at the 12th International Conference on High Energy Physics, Dubna, 1964).

SUCHER (Maryland): Could you extend the remarks you made about going from the local observable theory to field theory, that is, the possible existence of fields once you have the local observable theory?

FROISSART: This is not really a rigorous connection. You have a number of stages of ~~a~~ the theory. You have first the theory of local observables. Then you have fields à la Borchers, that is, things which are defined in the whole space but which are not quite local. They have some finite range, arbitrarily small, but finite. Otherwise, they enjoy all the properties of fields. They you have fields a la Jaffe, where the fields are local but are not distributions. They are generalized functions of a higher order. And then you have fields à la Wightman where the fields are distributions. Now, the step which has been accomplished is to show that local observables imply the existence of fields à la Borchers. Also, Jaffe has proved that fields à la Jaffe are just as good as fields à la

Wightman for all practical purposes. So, we only have to cross now the gap between Borchers and Jaffe.

LICHTENBERG (Indiana): Could you write down some of the asymptotic theorems that Martin has proved recently?

FROISSART: No new asymptotic theorems have been proved since the Dubna Conference. The only refinements which have been proved is that the total cross sections are bounded so that the constant in front of the $(\log)^2$ term is now known for $\pi\pi$, πK, and KK scattering:

$$\sigma_{tot} < \frac{4}{m_\pi^2} \log^2 (s/s_0)$$

lower case

We still don't know s_0. For the case of πN scattering the coefficient is $12\pi/m_\pi^2$. See Tables I through III.

LOGUNOV (Serpukhov): What behavior at infinity of the form factor was assumed in the work of Jaffe in order to get the lower bound?

FROISSART: He did it using his theory, with generalized functions of a higher order. See References 4 and 27.

CHEW (Berkeley): Does the analyticity domain established by Martin suffice to allow the definition of the Froissart-Gribov continuation in angular momentum for large angular momentum?

FROISSART: No, it does not go to infinity.

MARTIN (CERN): The only result on the Froissart-Gribov continuation is that you can do it for any s which is inside the region of analyticity. (see Fig. 2) Of course, that is quite obvious because then you have dispersion relations with two subtractions. But for physical energies I have nothing at all.

EDEN (Cambridge): Could you comment on what has been done on the relation between spin and statistics in S-matrix theory in the past two years?

FROISSART: The relation between spin and statistics in S-matrix theory is largely a matter of semantics because the arguments on analyticity are not very well standardized. The question is better understood now in that we see better how this relation comes about but it is very hard to say what has been derived and what has not been derived in S-matrix theory, in the sense that some authors take as postulates other authors' theorems, and conversely. The connection between spin and statistics has been proved now for four years by Stapp, using quite a number of axioms. Now, one by one these axioms have been removed but the seam is not completely tight, I would say.

STAPP (Berkeley): I think it's been proved now, on the basis of some well-stated assumptions. We know what we're assuming and we know what we can prove. The assumptions, aside from unitarity and cluster decomposition that go into it, are roughly as follows. One assumption is that the pole singularities which are usually associated with one particle exchange diagrams do not lie on top of other poles. We start with the assumption that all the singularities are associated with Landau diagrams, and you assume that there are no accidental singularities lying on top of the singularities associated with one particle exchange diagrams, which are also poles. You furthermore assume that the pôle factorization property, which has been established in the physical region, just on the basis of unitarity and cluster decomposition and the $i\varepsilon$ prescription associated with causal behavior, holds everywhere. Then you can actually derive the analyticity property that you need to prove the connection between spin and statistics completely on the mass shell. Some progress has been made in proving these from analyticity and unitarity but that has not yet been carried out.

HEPP (Princeton): I want to comment on the connection between spin and statistics for local observables. It has not been proved, up to now, for the particles connected with fields "à la Borchers," and it's a challenge for everybody. The TCP theorem has recently been proven by Epstein[6] under the frame work of local observables, but only for the S-matrix.

BLOKHINTSEV (Dubna): Prof. M. Froissart included in his diagram "the Field Theory." This gives me the opportunity to attract your attention to the work concerning non-linear field theory done by B. Barbashev and N. Chernicov (Dubna). About thirty years ago M. Born and L. Infeld developed non-linear field theory. The scalar field of the Born-Infeld Lagrangian is $L = \sqrt{1 - \left(\frac{\partial\varphi}{\partial t}\right)^2 + \left(\frac{\partial\varphi}{\partial x}\right)^2}$ and coincides with the Lagrangian for the "soap film" in Minkowski space. They obtained an explicit and rigorous solution of the quantized non-linear equations corresponding to this Lagrangian. Instead of x and t, they use the variables α, β which only asymptotically coincide with x-t and x + t; but for small x, t both α and β are quantum operators, so that space and time are automatically quantized in the region of high non-linearity. This result seems to be very instructive from the mathematical point of view.

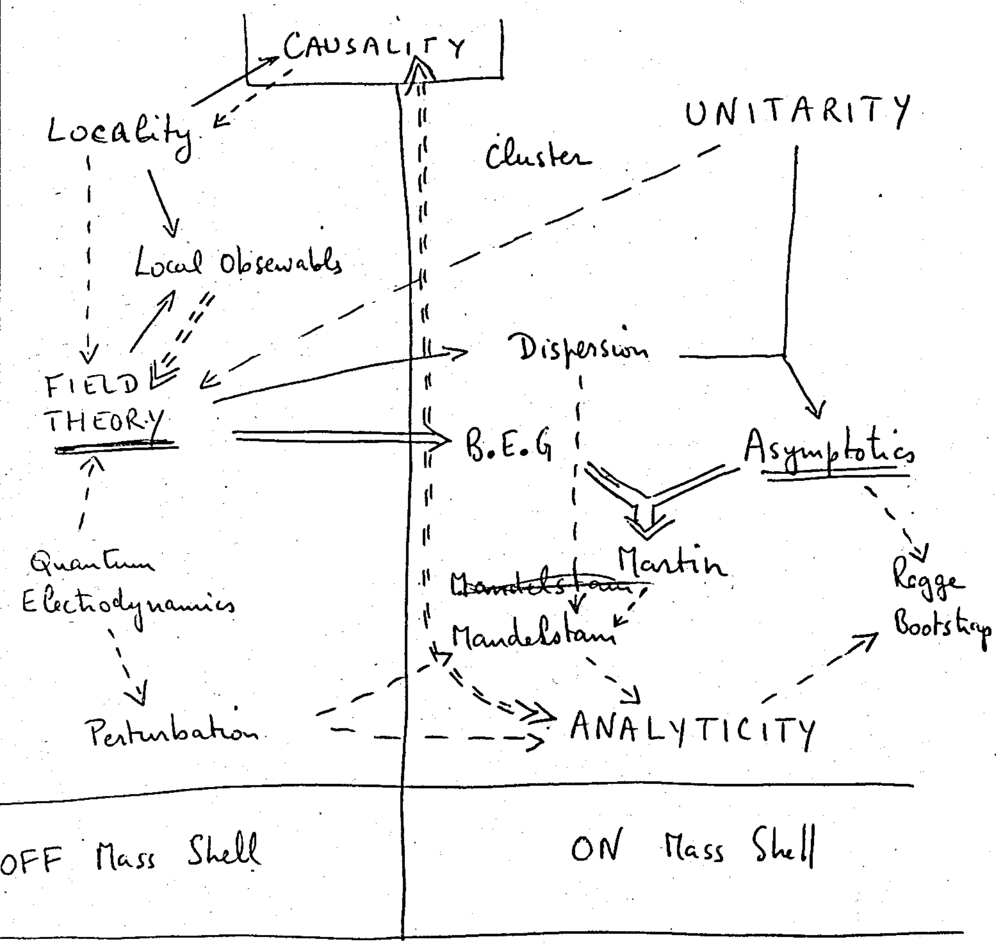
CAUSALITY
Locality
Cluster
UNITARITY
Local Observables
FIELD THEORY
Dispersion
B.E.G
Asymptotics
Quantum Electrodynamics
~~Mandelstam~~
Martin
Regge Bootstrap
Mandelstam
Perturbation
ANALYTICITY
OFF Mass Shell
ON Mass Shell

Fig 1.

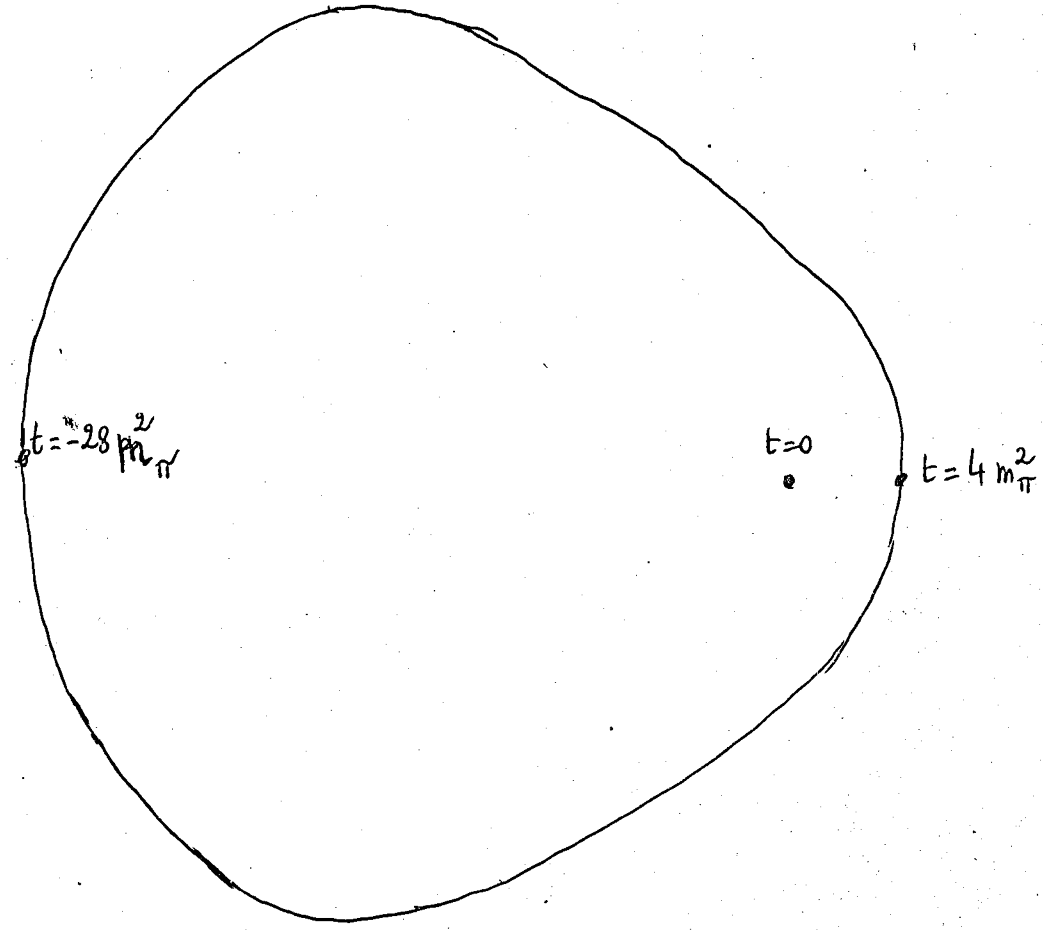
$t=-28\,m_\pi^2$
$t=0$
$t=4\,m_\pi^2$

Fig. 2.

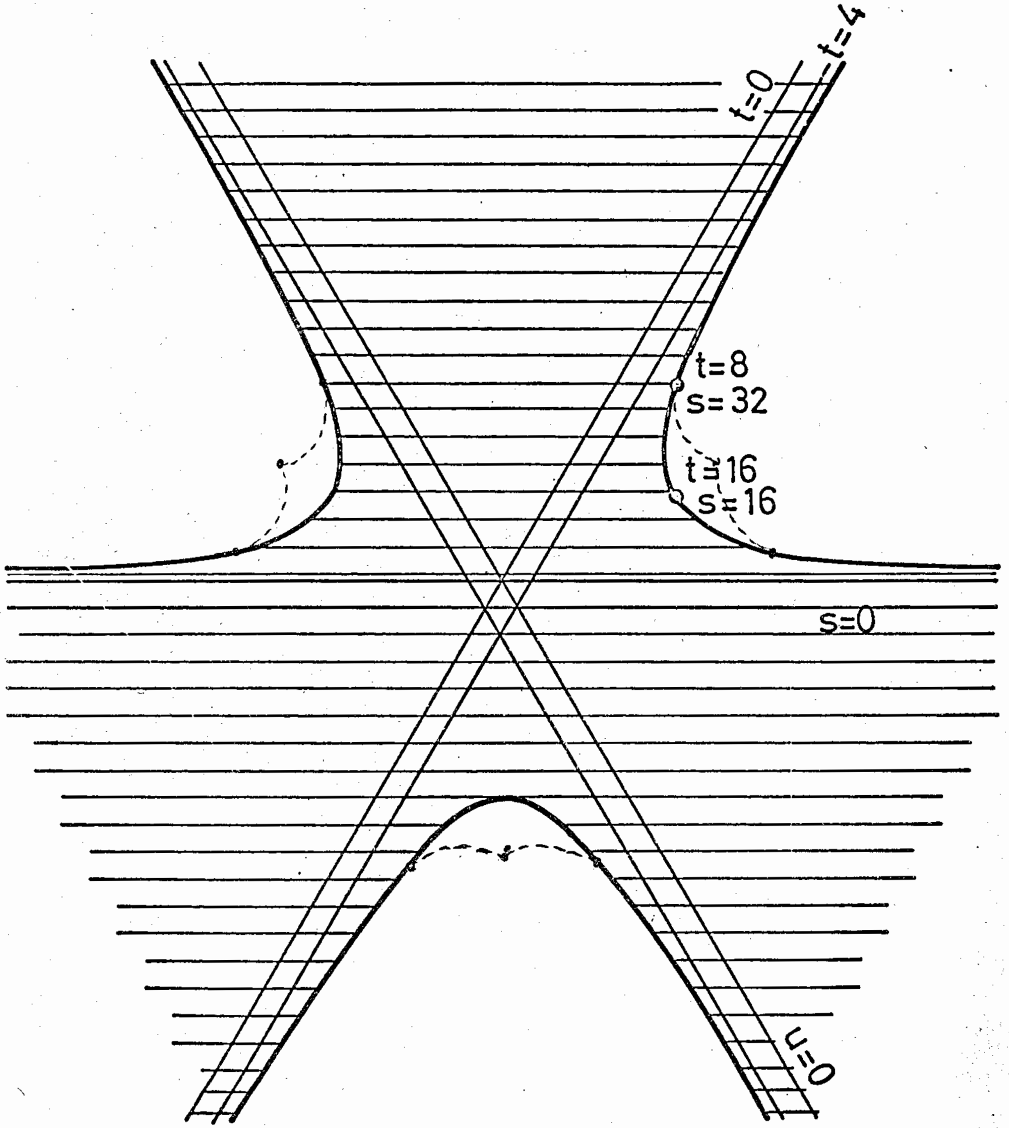
t=0
t=4
t=8
s= 32
t=16
s=16
s=0
u=0

FIG. 3

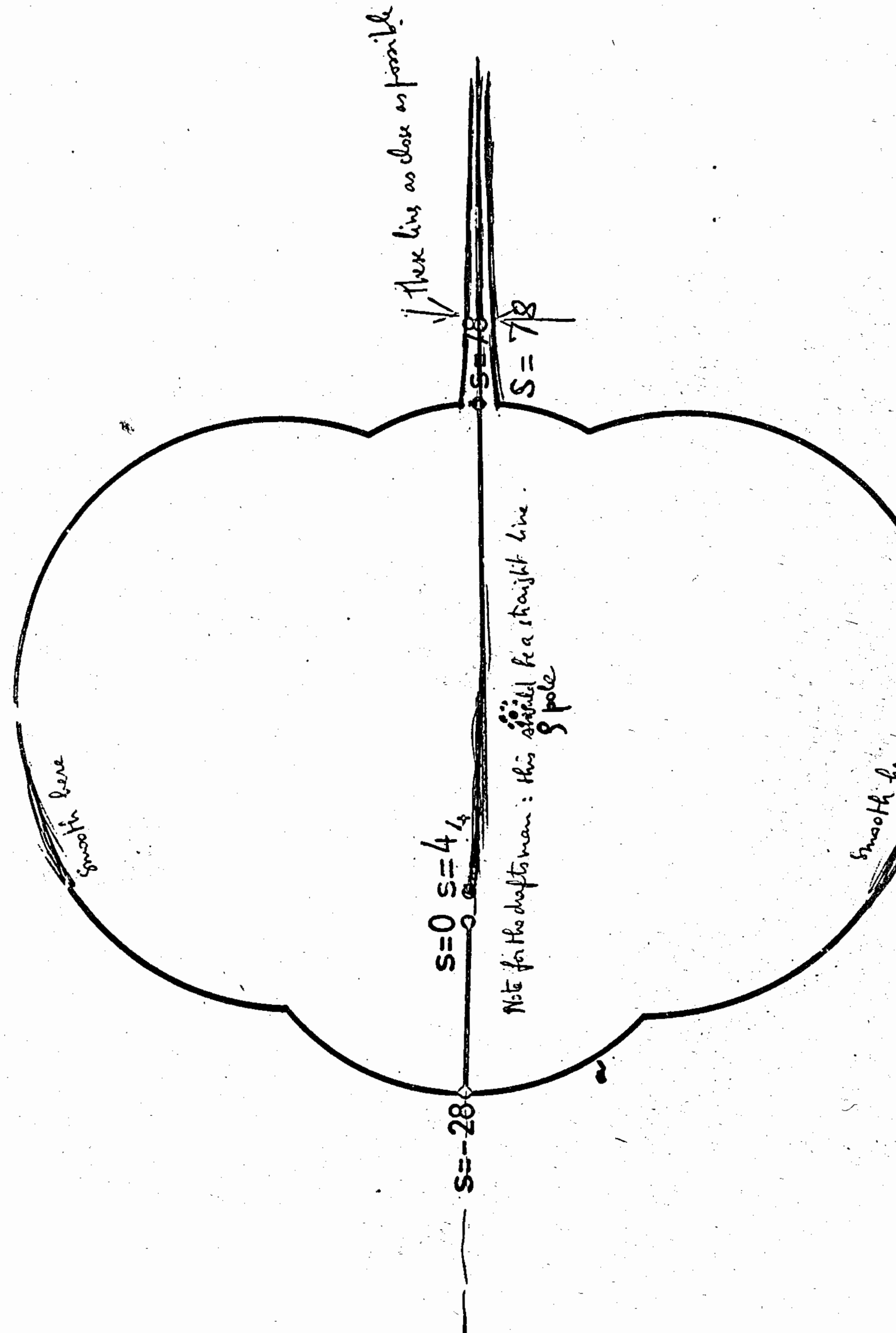
these lines as close as possible
s=8
s = 7/8
Note for the draftsman: this should be a straight line
pole
smooth here
smooth here
s=0
s=4
s=-28

FIG. 4