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WEAK INTERACTIONS

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Appendix A: K Decay Experiments -- A Summary of Results

by U. Camerini and C. T. Murphy

Appendix B: Hyperon Non-Leptonic Decay Status

by J. P. Berge

In this report I will discuss the present situation in the study of weak interactions.[1]

Among the new experimental results are new data on the Σ non-leptonic decays, which are in good agreement with the $\Delta I = 1/2$ selection rule, and solve the $S - P$ ambiguity in the Σ triangle in the sense suggested by recent theoretical investigations. Not less interesting is the flow of more accurate data on the leptonic and non-leptonic decays of K mesons, and accurate determinations of sign and magnitude of the $K_L - K_S$ mass difference.

On the theoretical side the theory based on octet hadron currents has met with some spectacular success with the application of new techniques (algebra of currents). For the first time some real progress has been made on the understanding of non-leptonic decays.

Fig. 1 comments on the general good situation of the field.

The problems of CP violation are still unsolved and we will hear more both about its general aspects, and its effects on the neutral kaon system, in the talks by Prof. V. Fitch and Prof. T. D. Lee.

Our basic assumption is that weak interactions are due to a coupling between currents. All of the existing experimental evidence is in agreement with a simple local coupling, although it is very possible that the coupling is not really local, but mediated by vector mesons, in which case the structure of weak interactions would resemble that of the strong and electromagnetic ones. The present limit on the mass of the vector boson coupled with the lepton current, $M_w > 2M_p$, corresponds to a range of possible non-locality of $r \lesssim 10^{-14}$ cm.

Weak interactions seem then to be described in terms of a simple phenomenological lagrangian:

$$L_W = \frac{G}{\sqrt{2}} \left\{ \ell_\lambda \ell_\lambda^* + J_\lambda \ell_\lambda + J_\lambda^* \ell_\lambda^* \right\} + L_{NL} \quad . \tag{1}$$

L_{NL} describes non-leptonic weak interaction, and we will discuss later the question of its possible structure. G is the Fermi coupling constant, and ℓ_λ a lepton current, which has the simple form:

$$\ell_\lambda = (\bar{\mu}\, \gamma_\lambda (1 + \gamma_5) \nu_\mu) + (\bar{e}\, \gamma_\lambda (1 + \gamma_5) \nu_e) \tag{2}$$

J_λ is the hadron current.

Leptonic Interactions

The first term in the bracket is a pure lepton-lepton interaction; it is responsible for muon decay, but would also give rise to other processes like e-ν_e scattering or electron-positron annihilation into neutrino pairs

$$e^\pm + \nu_e \longrightarrow e^\pm + \nu_e \quad ,$$

$$e^+ + e^- \longrightarrow \nu_e + \bar{\nu}_e \quad .$$

Although these have not been directly observed, indirect evidence for the existence of the second process is found in astrophysics, since it is claimed that this process plays an important and necessary role as a star-cooling mechanism.[2] If muon decay is due to the exchange of a W^+ meson between the $(\mu\nu_\mu)$ and $(e\nu_e)$ parts of ℓ_μ, the same meson would also couple each of these terms to itself, giving rise to the above mentioned processes.

The form of the muon decay amplitude implied by the above equations has been checked by two recent measurements of the Michel parameter ρ. These precise results,[3,4] 0.750 ± 0.003 and 0.760 ± 0.009 are in excellent agreement with the predicted value of $3/4$. It has been emphasized by Prof. Telegdi that it would be very important to measure also the Michel parameter η, relevant in the very low energy end of the spectrum. This is especially important since the accurate ρ-values quoted above are obtained by assuming $\eta = 0$ (Chicago) or $\eta < 0.04$ (Columbia) and a larger error would be obtained in an unconstrained fit.

The lifetime of the muon is given, including lowest order radiative corrections,[5] by:

$$\frac{1}{\tau_\mu} = \frac{G^2 m_\mu^{\ 5}}{192\pi^3} \left[1 - \frac{\alpha}{2\pi}\left(\pi^2 - \frac{25}{4}\right)\right] \tag{3}$$

Using the CERN value[6] $\tau_\mu = (2.198 \pm 0.001)\, 10^{-6}$s C. S. Wu obtains[7] $G = (1.4350 \pm 0.0011)\, 10^{-49}$ erg cm^3.

Semileptonic Interactions

The second and third terms in Eq. (1) describe weak interactions of leptons with hadrons. The central problem here is that of defining the structure of $J_\lambda(x)$, i.e. of identifying its selection rules, (ΔS, ΔI, G parity, parity, C, etc.) and its dynamical properties (conserved and partially conserved currents, and so on). In the current-current scheme, the amplitude of any semileptonic process, such as

$$A \longrightarrow B + \ell^- + \bar{\nu}_\ell \ , \quad A' \longrightarrow B' + \ell^+ + \nu_\ell \qquad (\ell^\pm = e^\pm \text{ or } \mu^\pm)$$

is proportional to the matrix element of $J_\lambda(x)$ or of $J^*_\lambda(x)$ between the initial and the final baryon:

$$\langle B|J_\lambda(x)|A\rangle$$
$$\langle B'|J^*_\lambda(x)|A'\rangle \quad .$$

The theoretical problem (apart from that of radiative corrections) is thus reduced to the computation of such matrix elements, and conversely the experimental results can be interpreted as giving information on their structure.

It has been proposed[8] that the hadron current $J_\lambda(x)$ has a simple structure, being a member of an SU(3) octet of currents, $J^i_\lambda(x)$, and that precisely

$$J_\lambda(x) = \cos\theta\,(J^1_\lambda + iJ^2_\lambda) + \sin\theta\,(J^4_\lambda + iJ^5_\lambda) \quad . \tag{4}$$

The octet $J^i_\lambda(x)$ has mixed parity, each member being the sum of a vector current $j^i_\lambda(x)$ and an axial current $g^i_\lambda(x)$: $J^i_\lambda = j^i_\lambda(x) + g^i_\lambda(x)$.

$(J^1_\lambda + iJ^2_\lambda)$ and $(J^4_\lambda + iJ^5_\lambda)$ are the $\Delta Q = +1$ members of this octet with $\Delta S = 0$ and $\Delta S = 1$, respectively. In the octet they thus occupy the same positions as π^+ and K^+ . Let me also write down the vector and axial vector parts of J_λ :

$$\begin{aligned} V_\lambda(x) &= \cos\theta\,(j^1_\lambda + ij^2_\lambda) + \sin\theta\,(j^4_\lambda + ij^5_\lambda) \ , \\ A_\lambda(x) &= \cos\theta\,(g^1_\lambda + ig^2_\lambda) + \sin\theta\,(g^4_\lambda + ig^5_\lambda) \ . \end{aligned} \tag{5}$$

Let me first discuss the selection rules implied by this scheme, and the related problem of what to do with the non-leptonic interaction. According to Eq. (4) J_λ is made of two pieces, a strangeness conserving current with $\Delta I = 1$ and a strangeness violating one with $\Delta S = \Delta Q = 1$ and $\Delta I = 1/2$. The existence of these two pieces is well established. The evidence for the absence of other pieces is not overwhelming, but neither is the evidence for their presence, so that we can feel justified in using the scheme as a good working hypothesis.

Let us discuss briefly this evidence, starting from that relating to a possible $\Delta S = -\Delta Q$ term in the current. The limits on the $\Delta S = -\Delta Q$ current come from two types of experiments.

a) The ratio between rates of similar $\Delta S = -\Delta Q$ and $\Delta S = \Delta Q$ decays.

1) $$\frac{K^+ \to \pi^+ + \pi^+ + e^- + \bar{\nu}_e}{K^+ \to \pi^+ + \pi^- + e^+ + \nu_e} < 0.02 \qquad \text{(Reference 9)}$$

2) $$\frac{\Sigma^+ \to n + e^+ + \nu_e}{\Sigma^- \to n + e^- + \bar{\nu}_e} < 0.12 \qquad \text{(Reference 10)}$$

The evidence from 1) relates to the absence of <u>axial</u> $\Delta S = -\Delta Q$ currents only, while the evidence from 2) is mixed. Since beta decay rates are roughly proportional to the combination $3|G_A|^2 + |G_V|^2$, the limit on the axial component with $\Delta S = -\Delta Q$ implied by 2) is stronger than that on the vector components.

b) Due to the peculiar properties of the neutral K system it is possible to measure the ratio of the <u>amplitudes</u> of $\Delta S = -\Delta Q$ versus $\Delta S = \Delta Q$ $K^\circ_{\ell 3}$ decays

$$x = \frac{a(\overline{K^\circ} \to \pi^- + \ell^+ + \nu)}{a(K^\circ \to \pi^- + \ell^+ + \nu)} .$$

Two new experiments on this subject have been presented at the conference, one by a University of Pennsylvania group,[11] the other by a Carnegie Tech - Brookhaven group.[12] Figure 2 displays the results of these groups as well as previous results.[13-15] Time reversal invariance requires x to be real, $\Delta S = \Delta Q$ would require $x = 0$. The experimental situation is not extremely clear, but collecting all these data one could tentatively conclude that $|x| < 0.5$ and that $x = 0$ is certainly <u>not</u> excluded. If the $\Delta S = \Delta Q = 1$ current obeys a $\Delta I = 1/2$ rule, it follows that

$$\langle\pi^-|J^*_\lambda(x)|K^\circ\rangle = \sqrt{2}\,\langle\pi^\circ|J^*_\lambda(x)|K^+\rangle .$$

This, together with $\Delta S = \Delta Q$ implies a relation between $K^+_{\ell 3}$ and $K^\circ_{L\ell 3}$ decays:

$$\Gamma(K^\circ_L \to \ell^+ + \nu + \pi^-) + \Gamma(K^\circ_L \to \ell^- + \bar{\nu} + \pi^+) = 2\Gamma(K^+ \to \ell^+ + \nu + \pi^\circ) .$$

This relation holds even in the presence of CP violation; furthermore the spectra of pions and leptons, and in general the Dalitz plot, should be the same in K^+ and K°_L decays. All data existing at the moment agree with these conditions within the experimental errors.[1]

<u>Non-Leptonic Interactions</u>

In the W meson picture, the semileptonic processes are mediated by a charged W^+ meson coupled to both the lepton current $\ell_\lambda(x)$ and the hadron current $J_\lambda(x)$. This boson would then couple ℓ_λ with itself, giving rise to the leptonic decays, as already discussed, and also couple J_λ to itself. This suggests that the non-leptonic lagrangian contains at least a term of the form

$$\frac{G}{\sqrt{2}} J_\lambda(x) J^*_\lambda(x) . \tag{6}$$

This term gives rise to non-leptonic interactions with $\Delta S = 0$ and $\Delta S = \pm 1$. Recent nuclear physics experiments have searched for the former. They test for parity mixing in nuclear levels[16] or for parity violating angular correlations in (n,γ) reactions with polarized neutrons.[17] The upper limits now placed on this term are roughly at the level where one expects the term to appear.

If $\Delta S = -\Delta Q$ terms are added to the current this would give rise to $\Delta S = 0$ non-leptonic transitions, such as would cause a $K^\circ_S - K^\circ_L$ mass difference at the lowest order in weak interactions, of the order of 1 eV. This is in contrast with the now well established value[18] $m_L - m_S = +0.5\,\Gamma_S \approx 4 \times 10^{-6}$ eV which is fully compatible with second order weak interaction. However, Eq. (6) gives rise to $\Delta S = 1$ transitions with both $\Delta I = 1/2$ and $\Delta I = 3/2$. Now one of the most striking facts about the non-leptonic transitions is the validity of a $\Delta I = 1/2$ selection rule. Bangerter, et al.[19] have presented the results of an experiment in which the asymmetry parameters of the three non-leptonic Σ decays have been measured. The previously existing data were in mild disagreement with $\Delta I = 1/2$. The new results, together with the established values of the three decay rates, agree very well with the $\Delta I = 1/2$ prediction. Berley, et al.[20] presented results of an experiment in which γ^+ has been found to be near -1. This resolves the S - P ambiguity of the Σ triangle, Σ_+^+ being an essentially pure P wave.

Figure 3 displays the well known Σ triangle, that results from an average of new and old data.

The evidence for a $\Delta I = 1/2$ selection rule in non-leptonic K decays has recently been discussed by Trilling in his report to the Argonne conference. The evidence is very good. The only solid evidence for a $\Delta I = 3/2$ amplitude comes from the $K^+ \rightarrow \pi^+ + \pi^\circ$ decay, which requires a $\Delta I = 3/2$ amplitude that is $\approx 5\%$ of the $\Delta I = 1/2$ amplitude observed in $K_S \rightarrow 2\pi$. So we have a dilemma: on one side it would be desirable to identify the non-leptonic lagrangian with the self-coupling of $J_\lambda(x)$, as in Eq. 6. In this case we have to assume that the $\Delta I = 3/2$ piece of it is suppressed for some dynamical reason. This is indeed a possibility, and the algebra of currents approach suggests mechanisms which could cause such a suppression. This has been shown for some -- but not all -- of the relevant cases, as will be discussed later.

Alternatively one can assume that the non-leptonic lagrangian obeys explicitly the $\Delta I = 1/2$ selection rule. Within the scheme of current-current interactions this can be obtained by adding an appropriate neutral hadron current, $N_\lambda(x)$ and writing

$$L_{NL}(x) = \frac{G}{\sqrt{2}} [J_\lambda(x)J_\lambda^*(x) + N_\lambda(x)N_\lambda^*(x)] \tag{7}$$

The $\Delta I = 3/2$ amplitudes, as seen in $K^+ \rightarrow \pi^+ + \pi^\circ$, would then arise through an electromagnetic interaction. Although the observed amplitude seems an order of magnitude too large to be thus explained, we note that our record in predicting the orders of magnitude of electromagnetic effects has not been very brilliant (example: $\eta \rightarrow 3\pi$ versus $\eta \rightarrow 2\pi + \gamma$ and $\eta \rightarrow \gamma + \gamma$).

A more unpleasant feature of neutral hadron currents is that they are known to have no lepton counterparts. Table I summarizes some of the experimental limits on possible processes which would arise from neutral lepton currents.

The question of a dynamical versus explicit $\Delta I = 1/2$ selection rule may well remain with us for a long time.

Universality of Vector Currents

Let us go into more detailed properties of the weak current. These follow from the assumption that the octet of currents to which $J_\lambda(x)$ belongs has very special properties. The first is that the vector octet, j_λ^i, is the same to which the electromagnetic current belongs,

$$j_\lambda^{e.m.} = j_\lambda^3 + \frac{1}{\sqrt{3}} j_\lambda^8 \quad ,$$

i.e. the octet of vector currents associated with the generators, F^i, of the approximate symmetry SU(3) , where

$$F^i = \int d\vec{x}\; j_o^i(x) \quad .$$

In particular the $\Delta S = 0$ vector current is associated with the raising operator of isotopic spin:

$$I^+ = F^1 + iF^2 = \int d\vec{x}\; (j_o^1 + ij_o^2) \quad .$$

The hypothesis that the vector part of J_λ is in the same octet as the electromagnetic currents is therefore the natural extension of the CVC hypothesis, which has received good experimental confirmation (weak magnetism, beta decay of the pion, etc.).[7] The presence of a factor $\cos\theta$ in front of the $\Delta S = 0$ current is a small but experimentally visible correction to beta decay. This factor (as the $\sin\theta$ in the $\Delta S = 1$ current) arises from the requirement that the hadron current $J_\lambda(x)$ has the same strength as the lepton current $\ell_\lambda(x)$. The strength of $J_\lambda(x)$ is then shared between $\Delta S = 0$ and $\Delta S = 1$ transitions according to the factors $\cos\theta$ and $\sin\theta$. We can compare the strengths of the vector parts of $J_\lambda(x)$ and $\ell_\lambda(x)$ by considering the "charge" operators associated with these currents:

$$Q^V = \int V_o(x)\,d\vec{x} = \cos\theta\,(F^1 + iF^2) + \sin\theta\,(F^4 + iF^5) \quad ,$$
$$L^V = \int \left\{(e^-\gamma_4\,\nu_e) + (\mu^-\gamma_4\,\nu_\mu)\right\} d\vec{x} \quad .$$

Then we build the commutators of L^V and Q^V with their hermitian conjugates:

$$2L^3 = [L^V, L^{V\dagger}] \quad ,$$
$$2Q^3 = [Q^V, Q^{V\dagger}] \quad .$$

Both can be explicitly computed, the first from the explicit expression of L^V , and the second from the SU(3) commutation relations. L^3 is a counting operator, analogous to the third component of isotopic spin: $L^3 = \frac{1}{2}(N_{e^-} + N_{\mu^-} - N_{\nu_e} - N_{\nu_\mu})$. In fact L^V, $L^{V\dagger}$ and L^3 obey the same commutation relations as the I-spin operators I^+, I^-, I^3, and the same is true of Q^V, $Q^{V\dagger}$, Q^3 . This means that Q^3 has the same spectrum of eigenvalues as L^3, or that L^V and Q^V are both well normalized raising operators, and have the same strength.

The operators Q^V, $Q^{V\dagger}$, and Q^3 generate an SU(2) subgroup entirely equivalent to that of isospin rotations generated by $\vec{I}$. The two groups are related by a rotation of an angle 2θ around the 7th axis in SU(3) space. We note that it is the SU(3) breaking interaction which chooses the isospin $\vec{I}$ among the possible SU(2) algebras (including the weak isospin $\vec{Q}$) so the angle θ should not be considered as a renormalization effect, but as a measure of the difference between the SU(2) subgroup of SU(3) generated by the weak current, and the SU(2) subgroup which remains good after SU(3) breaking. Even a very small breaking of SU(3) would define such an angle, which is, a priori, arbitrary. In fact, any hope of <u>computing</u> θ is going to fail, unless it postulates an up to now unknown relation between weak and SU(3) breaking interactions. It is only through a dynamical interplay of the two interactions that a definite value for the angle would arise.

Fermi Beta Decay and K_{e3} Decays -- A Check of the Theory

The vector coupling constant in beta decay is predicted to be

$$G_V = G \cos \theta$$

A measurement of G_V is obtained from the ft values of pure Fermi beta decays, like $O^{14} \rightarrow N^{14} + e^+ + \nu$. In order to obtain the accuracy needed, it is necessary to apply different corrections, mainly screening corrections and radiative corrections. Although there is a certain consensus among the experts on the use of Rose's screening corrections, there have been serious concern and disagreement about the radiative corrections computed by Berman and Kinoshita and Sirlin[3].

The main reason is that for a pointlike $V - 1.18A$ interaction the corrections diverge. (They would only be finite for $V + A$.) The situation does not seem to be improved by the introduction of nucleon form factors if one uses the diagrams a, b, and c in Fig. 4 and also takes into account the soft-photon emission (diagrams d, e). In fact the logarithmic divergence in diagram b -- the self energy of the electron -- is not removed by the introduction of form factors of the nucleon.

It has been noted by G. Källen,[21] that although the overall corrections must clearly be gauge invariant, the contribution of each particular graph depends on the choice of gauge for the electro-magnetic field. In particular one can choose a gauge where the divergence of the electron self energy is made to disappear. In this particular gauge the overall radiative corrections may turn out to be finite if the nucleon form factors vanish fast enough at high momentum transfer. A first rough evaluation by Källen yields corrections which are finite and agree with the ones computed by Kinoshita and Sirlin with a cutoff of 1.3 M_p. The situation is not very clear, still, because one would expect that if the corrections are finite in the gauge chosen by Källen they should be finite in any gauge. It would probably be worthwhile to go over the problem again in the normal gauge. Foldy noted in a discussion at the Seattle Summer Institute of Theoretical Physics, 1966, that at the weak vertex one has a flow of charge from a pointlike particle (the electron) to a diffused one (the proton). This could force the introduction of a $p\, n\, e\, \nu\, \gamma$ vertex and of diagrams like diagram f in Fig. 4, and could perhaps remove the divergence of the electron self energy. The suggestion is interesting and should be looked into.

If one uses the Berman-Kinoshita-Sirlin radiative corrections one finds[22]

$$G_V = (1.4034 \pm 0.0016) \times 10^{-49} \text{ erg cm}^3 .$$

Comparing this with the value of G from muon decay one obtains

$$1 - \cos \theta = \frac{G - G_\beta}{G} = (2.2 \pm .2 \pm .5)\ \% .$$

The $\pm.2$ is the experimental error, the $\pm.5$ the evaluated uncertainty in the radiative corrections.

Thus one obtains from the Fermi beta decays a value of sin θ

$$\sin\theta = .210 \pm .016 \qquad \text{(from beta decay)} \quad .$$

The most direct measurement of $\sin\theta$ for a $\Delta S = 1$ process is from the decay $K^+ \to \pi^\circ + e^+ + \nu$ (or $K^\circ \to \pi^- + e^+ + \nu$) . The matrix element in K_{e_3} decays is parameterized as

$$\langle\pi^\circ|J_\lambda(x)|K^+\rangle = \sin\theta\,[f_+(q^2)(p^K + p^\pi)_\lambda + f_-(q^2)(p^K - p^\pi)_\lambda] \tag{8}$$

where q^2 is the momentum transfer to the leptons, $q^2 = (p^K - p^\pi)^2$. The matrix element describes both K_{e_3} and $K_{\mu 3}$ ($\mu - e$ universality). Only $f_+(q^2)$ enters in the $K^+ \to \pi^\circ + e^+ + \nu$ decay. Furthermore SU(3) predicts that

$$f_+(0) = \frac{1}{\sqrt{2}} ,$$

and the Ademollo-Gatto theorem insures that this prediction is accurate up to second order in SU(3) breaking.

If we use a parameterization

$$\begin{aligned} f_+(q^2) &\approx f_+(0)\left(1 + \frac{q^2}{M_+^2}\right) , \\ f_-(q^2) &\approx f_-(0)\left(1 + \frac{q^2}{M_-^2}\right) \end{aligned} \tag{9}$$

one gets

$$\Gamma(K^+ \to \pi^\circ + e^+ + \nu) = \sin^2\theta\,[2f_+^2(0)]\left(1 + 0.277\left(\frac{M_K}{M_+}\right)^2\right) 7.42 \times 10^7\ \text{sec}^{-1}$$

From Trilling's report to the Argonne conference[1] one has

$$\Gamma(K^+ \to \pi^\circ + e^+ + \nu) = (3.61 \pm 0.20) \times 10^6\ \text{sec}^{-1}$$

Negelcting SU(3) breaking and assuming <u>no</u> q^2 dependence of the f_+ form factor (i.e. $M_+ = \infty$) this gives $\sin\theta = 0.222 \pm 0.006$. The agreement is very good. If q^2 dependence is introduced with a positive mass,[23] $\sin\theta$ decreases. As an example, for $M_+ = M_{K^*} = 890$ MeV $\sin\theta = 0.21$, also in excellent agreement.

<u>The Experimental Situation in $K_{\ell 3}$ Decays</u>

It is appropriate at this moment to discuss briefly the general situation of $K_{\ell 3}$ decays:

$$K_{\mu 3} \begin{cases} K^+ \to \pi^\circ + \mu^+ + \nu \\ K_2^\circ \to \pi^\pm + \mu^\mp + \nu \end{cases}$$

$$K_e \begin{cases} K^+ \to \pi^\circ + e^+ + \nu \\ K_2^\circ \to \pi^\pm + e^\mp + \nu \end{cases}$$

The problem at hand is that of determining the form factors $f_+(q^2)$ and $f_-(q^2)$ defined in Eq. 8. The K_{e3} decays can be used to study the q^2 dependence of $f_+(q^2)$. Table A5 in Appendix A gives a list of experimental results for the radius associated with f_+ . The parameter used is

$$\lambda_+ = \frac{m_\pi{}^2}{M_+{}^2}$$

The situation is not yet such as to have a definite and clear cut answer. Let us note that some of the more accurate results favor a low value of M_+, in the region 500-600 MeV. This should not be surprising, since the proton radius corresponds to a mass of $\sim$600 MeV, lower than that of the ρ, ω, and ϕ mesons. Up to recent times most $K_{\mu 3}$ experiments were analyzed on the assumption of constant form factors, with the intent of obtaining a value of ξ, defined as

$$\xi = \frac{f_-}{f_+}$$

Different kinds of data were used, mainly of two kinds

1) Ratio $K_{\mu 3}/K_{e3}$, Dalitz plot, etc.,
2) Longitudinal and total polarization data.

The total polarization measurement is based on the idea that in a two component neutrino theory, if one fixes the kinematics, the muon is totally polarized along some direction (i.e. it is in a pure state). The direction of polarization is very sensitive to the value of ξ.[24] As emerges from Table A1 in Appendix A, data of the first kind do not agree with polarization data if analyzed under the assumption of constant form factors. The polarization data favor a negative ξ, $(\xi \approx -1)$. The Dalitz plot analysis agrees with the polarization data[25] if a q^2 dependence for f_+ is assumed with a mass of about 500 MeV. As an example of how the problem of ξ is related to that of M_+ one can consider the ratio $K_{\mu 3}$ to K_{e3}. Keeping terms linear in $1/M_\pm^2$ one gets[26]

$$\frac{K_{\mu 3}}{K_{e3}} = 0.6487 + 0.1045\,\frac{M_k^2}{M_+^2} + 0.1269\,\mathrm{Re}\xi + 0.0193\xi^2 + 0.0006\,\frac{M_k^2}{M_+^2}\,\mathrm{Re}\xi$$
$$+\ 0.0358\,\frac{M_k^2}{M_-^2}\,\mathrm{Re}\xi + 0.0127\,\frac{M_k^2}{M_-^2}\,\xi^2 - 0.0535\,\frac{M_k^2}{M_+^2}\,\xi^2 \ . \qquad (10)$$

If $M_+ \sim 500$ MeV rather than infinity, $\mathrm{Re}\xi$ decreases by about 0.8, and a negative ξ is possible. It emerged from the discussion that although the situation is not completely satisfactory at the moment, a general clarification is in view, which should yield a fairly accurate value of ξ as well as of M_+, to be compared with the different theoretical models. The indications available at the moment suggest a negative ξ with a substantial dependence of f_+ on the momentum transfer.

Other Tests -- Fit to Hyperon Decays, $K_{\mu\nu}/\pi_{\mu\nu}$

A new fit to hyperon decays has been carried out by Brene, Cronström, Roos and Veje.[27] In this fit allowance is made for some violation of SU(3) by introducing a renormalized value for θ in the axial current, θ_A. In accordance with the Ademollo-Gatto theorem, no renormalization is introduced for vector currents.

The fit is rather good, for

$$\sin\theta = 0.21 \quad (\text{from } K_{e3} \text{ and } G_V \text{ as discussed above}),$$

$$\sin\theta_A = 0.28 \quad (\text{from fit to hyperon decays}),$$

$$\alpha = D/(D+F) = 0.665 \pm 0.018 \ (D/F = 2),$$

$$g_A = (G_A/G_V)_{\beta\ \text{decay}} = 1.18 \ .$$

Predictions and experimental values for the rates are compared in Table II. Note that the introduction of a renormalized θ_A is not the only possible way in which SU(3) predictions can be broken. It would be of great interest to have more experimental data on these decays to see at which point the theory breaks down, and in which way. New values for the $\Sigma^\pm \to \Lambda + e^\pm + \nu$ branching ratios have been presented at this conference by the Maryland group:[28]

$$\frac{(\Sigma^- \to \Lambda + e^- + \bar\nu)}{(\Sigma^- \to N + \pi^-)} = (0.61 \pm 0.16) \times 10^{-4} \ ,$$

$$\frac{(\Sigma^- \to \Lambda + e^- + \bar\nu)}{(\Sigma^+ \to \Lambda + e^+ + \nu)} = 2.0 \pm 1.1 \ .$$

They are not included in the fit, but they agree with the predicted values. It is interesting to note that the value of $\theta_A = 0.268$ agrees fairly well with what one obtains from a comparison of $K \to \mu + \nu$ to $\pi \to \mu + \nu$. One can parametrize the amplitudes for these dacays in two equivalent ways:

$$\begin{aligned} \langle 0|A_\lambda|K^+\rangle &= \sin\theta\, f_K p_\lambda^K = \sin\theta_A\, f_\lambda^K \ , \\ \langle 0|A_\lambda|\pi^+\rangle &= \cos\theta\, f_\pi p_\lambda^\pi = \cos\theta_A\, f p_\lambda^\pi \ . \end{aligned} \tag{11}$$

From the ratio of the rates one then derives $\tan^2\theta_A = 0.075$ or $\sin\theta_A = 0.28$, which agrees with the value obtained by Brene, et al. from hyperon data. The reason for the coincidence is not understood, as is the case with many effects which relate to symmetry breaking. Using the alternative parametrization one obtains

$$\frac{f_K}{f_\pi} = \frac{\tan\theta_A}{\tan\theta} \approx 1.28 \ . \tag{12}$$

Some Algebra of Currents -- Universality of Axial Currents

The idea of universality can be extended to axial currents if we assume, as suggested by Gell Mann, that the charges associated with the axial octet

$$F_5^i = \int d\vec{x}\, g_0^i(x)$$

enlarge the algebra of SU(3) from that of SU(3) to that of SU(3) $\otimes$ SU(3). The following set of commutation relations is postulated:

$$\begin{aligned} [F^i, F^j] &= [F_5^i, F_5^j] = i\, f_{ijk} F^k \ , \\ [F^i, F_5^j] &= [F_5^i, F^j] = i\, f_{ijk} F_5^k \ . \end{aligned} \tag{13}$$

These commutation relations can be seen to correspond to the algebra of SU(3) $\otimes$ SU(3) by introducing linear combinations of F^i and F_5^i:

$$F_\pm^i = \frac{1}{2}(F^i \pm F_5^i) \ .$$

The F_+^i and F_-^i commute with each other, and each set forms the basis of an SU(3) algebra. The currents themselves are supposed to obey commutation relations

$$[F^i, j_\lambda^j(x)] = [F_5^i, g_\lambda^j(x)] = i f_{ijk} j_\lambda^k(x) ,$$
$$[F^i, g_\lambda^j(x)] = [F_5^i, j^j(x)] = i f_{ijk} g_\lambda^k(x) . \tag{14}$$

An important consequence of these is that the octet $J_\lambda^i(x) = j_\lambda^i(x) + g_\lambda^i(x)$ behaves in the same way when commuted with the F_5^i or with the F^i :

$$[F_5^i, J^j(x)] = [F^i, J_\lambda^j(x)] = i f_{ijk} J_\lambda^k(x) . \tag{15}$$

The same is true of the non-leptonic lagrangian L_{NL} , if it is built with the charged hadron current $J_\lambda(x)$ (as in Eq. (6)):

$$[F_5^i, L_{NL}] = [F^i, L_{NL}] . \tag{16}$$

This equation is true if L_{NL} is given by Eq. (6), and also if its $\Delta I = 3/2$ piece is eliminated by the addition of neutral currents, also belonging to the octet J_λ^i as in Eq. (7). Under the above hypothesis, the charge associated with the axial weak current $A_\lambda(x)$,

$$Q^A = \int A_o(x)\, d\vec{x} = \cos\theta\,(F_5^1 + iF_5^2) + \sin\theta\,(F_5^4 + iF_5^5)$$

has the property that

$$[Q^A, Q^{A\dagger}] = [Q^V, Q^{V\dagger}] = 2Q^3$$

so that we can conclude that the axial and the vector weak currents have the same strength.

PCAC and the Adler - Weisberger Relations

The algebra of currents has been used with great success in the last two years to obtain relations between different weak interaction processes and between weak and strong interactions. In order to obtain useful results, the algebra of currents is used in conjunction with the PCAC hypothesis (partially conserved axial current), which relates the divergence of the $\Delta S = 0$ axial currents to the field of the pion:

$$\partial_\lambda g_\lambda^i = m_\pi^2 f_\pi \phi^i \qquad i = 1, 2, 3$$

All matrix elements of the two sides of this equation have a pion-pole contribution, and the equation is an identity at the position of the pole. Away from the pole the equation could be taken as a definition of ϕ_π .

The PCAC equation does not, however, tell the whole story. In practical use it is meant to signify that all matrix elements of the divergence of the axial current are dominated by the pion pole contribution, not only at the position of the pole, $q^2 = -m_\pi^2$, where this is obviously true, but also in the neighborhood including $q^2 = 0$.

So the name PDDAC (pole dominance of the divergence of the axial current) has been proposed as more appropriate.

The first interesting and very successful result of the current algebra in conjunction with PCAC has been the Adler-Weisberger sum rule,[29] which allows a computation of the axial coupling constant in beta decay, $g_A = G_A/G_V$ in terms of $\pi^{\pm}p$ total cross sections:

$$1 - \frac{1}{g_A^2} = \left(\frac{2M}{g_{\pi N}}\right)^2 \frac{1}{2\pi} \int \frac{d\omega}{\omega} \left(\sigma^+(\omega) - \sigma^-(\omega)\right)$$

where $\sigma^{\pm}$ are total $\pi^{\pm}p$ cross sections at energy ω for a pion of zero mass. The relation works with great accuracy, and completes, for beta decay, at least, the program of expressing all parameters of weak interactions in terms of the two parameters G and θ, and quantities defined and measurable in other kinds of interactions.

The relation has then been extended to $\Delta S = 1$ decays by many authors,[30] with results which are on the whole good, but not as accurate due to the uncertainties in the $K - p$ scattering amplitudes below threshold.

Zero Energy Theorems

A new class of applications of the algebra of currents comes from the use of a zero energy theorem[31] which states that the amplitude of a weak process involving a pion, $A \to B + \pi$, is, in the limit of vanishing four momentum of the pion, related to that of the simpler process $A \to B$. I will here discuss briefly two such chains of relations, one for leptonic decays

$$(K \to \pi + \pi + \text{leptons}) \to (K \to \pi + \text{leptons}) \to (K \to \text{leptons})$$

and one for non-leptonic decays

$$(K \to \pi + \pi + \pi) \to (K \to \pi + \pi) .$$

Both of them have given results in excellent agreement with experimental data.

Zero energy theorems have also been applied to hyperon non-leptonic decays. The situation here is more complicated and not completely understood, and I will comment on it later. Let me start with a simple derivation of the zero energy theorem.[32]

Consider the matrix element of a local operator (to be identified with either a current or the non-leptonic lagrangian itself) between two states, one of which contains a pion of momentum q_λ and charge state i :

$$M = \langle \beta + \pi^i | d(0) | \alpha \rangle .$$

We can apply the standard reduction formalism, and rewrite our matrix element in terms of a retarded commutator:

$$M = -i \int d^4x \, e^{-iqx} \, (\Box^2 - m^2) \, \langle\beta | [\phi^i(x), d(0)] | \alpha\rangle \, \theta(x) .$$

With an integration by parts[33] we can rewrite this as

$$M = +i \int d^4 x \, e^{-iqx} (q^2 + m^2) \langle\beta|[\phi^i(x), d(0)]|\alpha\rangle \, \theta(x) .$$

We can now introduce the PCAC hypothesis and express the pion field in terms of the divergence of an axial current; it is, however, more convenient to make a fresh start by introducing a new quantity defined in terms of the retarded commutator of the operator $d(0)$ and the axial current itself:

$$T_\mu = \int d^4 x \, e^{-iqx} \langle\beta|[g_\mu^i(x), d(0)]|\alpha\rangle \, \theta(x) .$$

We now form the quantity $T_\mu q_\mu$. This can be computed by noting that $q_\mu \exp(-iqx) = i \, \partial/\partial x_\mu \exp(-iqx)$, and performing an integration by parts:

$$q_\mu T_\mu = i \int d^4 x \, e^{-iqx} \langle\beta|[\partial_\mu g_\mu^i(x), d(0)]|\alpha\rangle \, \theta(x)$$
$$+ \; i \int d^4 x \, e^{-iqx} \langle\beta|[g_\circ^i(x), d(0)]|\alpha\rangle \, \delta(x_\circ) .$$

We now consider the limit in which the four components of q_λ all vanish. In this limit $q_\mu T_\mu$ vanishes, unless T has a singularity at that point. If no such singularity exists, we get, in the limit, an equation for the two terms on the r.h.s. which we can write, by using PCAC, as:

$$\lim_{q\to 0} \frac{i f_\pi}{\sqrt{2}} m_\pi^2 \int d^4 x \, e^{-iqx} \langle\beta|[\phi^i(x), d(0)]|\alpha\rangle \, \theta(x)$$
$$= \int d^3 x \, \langle\beta|[g_\circ^i(\vec{x}, 0), d(0)]|\alpha\rangle = \langle\beta|[F_5^i, d(0)]|\alpha\rangle .$$

The limit of M as q vanishes is seen to be identical apart from a factor, to the limit in the l.h.s., so that we get to a final result:

$$\frac{f_\pi}{\sqrt{2}} \lim_{q\to 0} \langle\beta + \pi^i|d(0)|\alpha\rangle = \langle\beta|[F_5^i, d(0)]|\alpha\rangle . \qquad (17)$$

A matrix element of $d(0)$ between two states $\beta + \pi$ and α is thus reduced, in the limit of vanishing pion energy and momentum, to the matrix element of a related operator between β and α. We are thus able to connect a process in which N pions are involved to others in which $N-1, N-2, \ldots\ldots 0$ pions are involved.

The $K \to (n\pi) +$ Leptons Chain

The zero energy theorem has been applied to relate the leptonic decays of K mesons:

K_{e4} (ex. $K^+ \to \pi^+ + \pi^- + e^+ + \nu_e$)
K_{e3} (ex. $K^+ \to \pi^\circ + \mu^+ + \nu_\mu$)
K_{e2} (ex. $K^+ \to \mu^+ + \nu_\mu$)

These decays are produced by the $\Delta S = \Delta Q = -1$ part of the weak hadron current, $\sin\theta \, (J_\lambda^4 - iJ_\lambda^5)$.

Let us start from the last step in the ladder, $K_{e3} \to K_{e2}$. The zero energy theorem (Eq. 17) says that

$$\lim_{q_\lambda \to 0} \langle \pi^0, q | J^4_\lambda - iJ^5_\lambda | K \rangle = \frac{\sqrt{2}}{f_\pi} \langle 0 | [F^3_5, (J^4_\lambda - iJ^5_\lambda)] | K \rangle$$
$$= -\frac{1}{\sqrt{2}\, f_\pi} \langle 0 | (J^4_\lambda - iJ^5_\lambda) | K \rangle$$

The last step is obtained by remembering that J^i_λ have the same commutators with F^i_5 as with F^i, and that $(J^4 - iJ^5)$ has $\Delta I^3 = -1/2$. In terms of the parametrization defined in Eqs. 8 and 11 this means that

$$f^+ (M^2_K, 0) + f^- (M^2_K, 0) = \frac{1}{\sqrt{2}} \frac{f_K}{f_\pi} , \qquad (18)$$

where $f^\pm (M^2_K, 0)$ indicate the form factors at momentum transfer $q^2 = M^2_K$ and extrapolated to zero pion mass. This formula has been obtained by C. Callan and S. Treiman,[32] and by V. S. Mathur, S. Okubo and L. K. Pandit.[34] In order to compare this to experimental data we remember that the SU(3) prediction $f_+ (0, m^2_\pi) = \frac{1}{\sqrt{2}}$ is well supported by experimental data. We will also assume that the extrapolation to $M_\pi = 0$ does not have any effect, and rewrite Eq. 18 as:

$$\frac{f_+ (M^2_K) + f_- (M^2_K)}{f_+ (0)} = \frac{f_K}{f_\pi}$$

or (see Eq. 12)

$$\frac{f_+ (M^2_K)}{f_+(0)} \; [1 + \xi (M^2_K)] = 1.28 .$$

If the f_+ form factor has a q^2 dependence corresponding to $M_+ \approx 600$ MeV, this equation can be satisfied with a negative ξ, which is suggested by the recent trend in $K_{\mu 3}$ experiments. It is not now possible to say much more than this, and we have to wait for a further clarification of the $K_{\mu 3}$ data. An important property of Eq. 18 is that it gives a relation between SU(3) breaking in axial and in vector transitions.

Let us go to the next step in the ladder. The theorem applied to K_{e4} decays says that

$$\lim_{q^+_\lambda \to 0} \langle \pi^+, q^+; \pi^-, q^- | (J^4_\lambda - iJ^5_\lambda) | K^+ \rangle = \frac{\sqrt{2}}{f_\pi} \langle \pi^-, q^- | [F^-_5, (J^4 - iJ^5)] | K^+ \rangle = 0$$

$$\lim_{q^-_\lambda \to 0} \langle \pi^+, q^+; \pi^-, q^- | (J^4_\lambda - iJ^5_\lambda) | K^+ \rangle = \frac{\sqrt{2}}{f_\pi} \langle \pi^+, q^+ | [F^+_5, (J^4 - iJ^5)] | K^+ \rangle$$
$$= \frac{\sqrt{2}}{f_\pi} \langle \pi^+, q^+ | (J^6_\lambda - iJ^5_\lambda) | K^+ \rangle = \frac{2}{f_\pi} \langle \pi^0, q^+ | (J^4_\lambda - iJ^5_\lambda) | K^+ \rangle .$$

Let us parametrize the K_{e4} amplitude as

$$\langle \pi^+, q^+; \pi^-, q^- | (J^4_\lambda - iJ^5_\lambda) | K^+ \rangle = f_1 (q^+ + q^-)_\lambda + f_2 (q^+ - q^-)_\lambda + f_3 (p^K - q^+ - q^-)_\lambda$$
$$+ h \varepsilon^{\lambda\mu\nu\rho} p_\mu^K q_\nu^+ q_\rho^- .$$

The form factors can depend upon the different kinematical variables in the process: $(q^+ q^-)$, $(q^\pm p^K)$.

The two conditions read:

$$\text{at } q^+_\lambda = 0 \quad \begin{cases} f_1 = f_2 \\ f_3 = 0 \end{cases} \qquad (19)$$

$$\text{at}\ q_\lambda^- = 0 \quad \begin{cases} f_1 + f_2 = 4f_+/f_\pi \\ f_3 = 2(f_+ + f_-)/f_\pi \end{cases} \quad . \tag{20}$$

These equations have been obtained by Callan and Treiman.[32] It is clear that f_3 has to have a sizeable dependence upon the kinematical variables if both sets of equations are to be satisfied at the same time. The situation has been clarified by Weinberg[35] who proposed a more symmetrical way of treating more than one pion. What happens is that f_3 has a K^+ pole contribution as in Fig. 5. Weinberg shows that the pole contribution gives:

$$f_3 \propto \frac{(p^K q^+)}{(p^K, q^+ + q^-)} \begin{cases} \nearrow = 1 \text{ if } q^- = 0 \\ \searrow = 0 \text{ if } q^+ = 0 \end{cases}$$

so that there is no disagreement between the two limits. The importance of these pole diagrams in the case of $K \to 2\pi$ had been noted earlier by C. Bouchiat and P. Meyer.[36] The experimental results of the Berkeley, University College (London), Wisconsin group,[9] give, for the ratio of the phase-space averages of f_1 and f_2 : $\langle f_2 \rangle / \langle f_1 \rangle = 0.9 \pm 0.25$, in excellent agreement with Eq. 19. Assuming $f_1 = f_2$, from the experimental value[37] for the rate, $\Gamma(K^+ \to \pi^+ + \pi^- + e^+ + \nu) = 2.9 \pm 0.6 \times 10^3 s^{-1}$, Weinberg obtains a value of f_1 which is slightly smaller than the theoretical prediction of Eqs. 19 and 20. The difference between the two numbers is about $(20 \pm 20)\%$ which indicates a fair agreement, in view of the approximations involved in the argument, one of which is the neglect of the momentum dependence of the $K_{\ell 3}$ form factor f_+ .

The striking success of these predictions, as well as of those on $K \to 3\pi$ which we will discuss next, would hardly be understood in the presence of a strong π-π interaction which would make the extrapolation procedure very doubtful. These successes can be taken as an indication of a rather weak π-π interaction at low energies.

The π-π scattering phase shift in the $I = 0$, S wave state can be directly measured in the K_{e4} decay. At this conference the Berkeley - University College - Wisconsin group[9] quoted a scattering length

$$a_0 = 0.6 {+0.6 \atop -0.5} m_\pi^{-1}$$

which agrees with Weinberg's prediction of $a_0 = 0.20\ m_\pi^{-1}$. The K_{e4} experiment is a valuable tool for the study of strong interactions, and should be pursued further.

The Non-Leptonic K Decays

The zero energy theorem has been applied to K non-leptonic decays by S. Suzuki,[38] who found a relation between the $K \to 3\pi$ and the $K \to 2\pi$ rates, and then by many others: Hara and Nambu,[39] Elias and Taylor,[40] Bose and Biswas,[41] and Nefkens.[42]

Assuming a linear dependence of the amplitude of $K \to 3\pi$ decays on the energy of the "odd" pion these authors were also able to determine the value of the slope. Recently Abarbanel[43] reconsidered the problem applying the method of S. Weinberg, and reached the same conclusions as the previous authors in a more transparent way. The experimental data on the Dalitz plot of $K \to 3\pi$ decays have been analyzed, according to a suggestion of Weinberg,[44] by assuming a linear dependence of the transition probability on the energy of the odd pion. In terms of the variables (T_3 refers to the odd pion)

$$Y = (3T_3/Q) - 1$$
$$X = \sqrt{3}\,(T_1 - T_2)/Q \ .$$

This means an approximation of the form

$$|S|^2 = 1 + aY \cdot$$

To see the accuracy of this approximation, we can consider an experiment of a Rutgers group[45] (presented at this conference) on $K^+ \to \pi^+\pi^+\pi^-$. If they try to fit to a quadratic dependence in Y of the form $1 + aY + cY^2$, they obtain $a = 0.260 \pm 0.027$, $c = -0.068 \pm 0.058$, which shows that the linear approximation is indeed adequate.

I will discuss the application of the zero energy theorem to $K \to 3\pi$ following the derivation given by J. S. Bell.[46]

Assume that the lagrangian L_{NL} is of the current-current type and includes neutral currents so as to yield the $\Delta I = 1/2$ selection rule. If we are interested in $\Delta S = -1$ decays the interesting part of L_{NL} will behave as the $-1/2$ component of a doublet L_λ. In terms of a "spurion" $s = \binom{0}{1}$ we can write

$$L_{NL} = (sL) \ .$$

The zero energy theorem says that

$$\lim_{q^3 \to 0} \langle \pi^1, q^1; \pi^2 q^2; \pi^3, q^3 | (sL) | K \rangle =$$
$$= \frac{\sqrt{2}}{f_\pi} \langle \pi^1, q^1; \pi^2, q^2 | [F^{(3)}, (sL)] | K \rangle$$

where we made use of the algebra of currents result that $[F_5^i, L_\lambda] = [F^i, L_\lambda]$, $(\lambda = \pm 1/2)$. Introducing vectors in isospace corresponding to the three pions, $\vec{\pi}_1$, $\vec{\pi}_2$, $\vec{\pi}_3$ the most general form of the $K \to 3\pi$ matrix element, consistent with Bose statistics and linear in E_1, E_2, E_3 is:

$$3\pi | (sL) | K\rangle = (\vec{\pi}_1 \cdot \vec{\pi}_2)[\vec{\pi}_3 \cdot (s\vec{\tau}K)](\alpha + \beta E_3)$$
$$+ \text{(cyclic permutations of 1, 2, 3)} \ .$$

We also parametrize the $K \to 2\pi$ matrix element as:

$$\langle 2\pi | (sL) | K\rangle = \gamma \vec{\pi}_1 \cdot \vec{\pi}_2 (sK)$$

From the zero energy theorem one then finds, in the limit where

$$E_1 = E_2 = \frac{M_K}{2}, \qquad E_3 = 0 :$$

$$(\vec{\pi}_1 \cdot \vec{\pi}_2)[\vec{\pi}_3 \cdot (s\vec{\tau}K)]\alpha + [(\vec{\pi}_2 \cdot \vec{\pi}_3)\vec{\pi}_1 + (\vec{\pi}_3 \cdot \vec{\pi}_1)\vec{\pi}_2] \cdot (s\vec{\tau}K)(\alpha + \tfrac{1}{2}\beta M_K)$$
$$= \frac{\sqrt{2}}{f_\pi} \langle \pi^1, \pi^2 | (s\vec{\pi}_3 \cdot \vec{\tau}K) | K \rangle$$
$$= \frac{\sqrt{2}\gamma}{f_\pi} (\vec{\pi}_1 \cdot \vec{\pi}_2)(s\vec{\pi}_3 \cdot \vec{\tau}K)$$

it follows

$$\alpha = \gamma$$
$$\alpha + \frac{1}{2}\beta M_K = 0 ,$$

in terms of the single parameter γ, determined by $K \to 2\pi$, which fixes rates and slopes of all $K \to 3\pi$ decays. Writing

$$|s|^2 = A + sY$$

Hara and Nambu[40] give the following table of predictions compared with experimentsl results:

The upper sign is that of the K meson and the lower signs those of the decay products:

	exp × 10^6	Prediction (from $(K_1 \to \pi^+\pi^-) \times 10^6$)
A^-_{0+-}	0.89 ± 0.01	0.81 ± 0.01
A^-_{000}	0.92 ± 0.05	
A^+_{-++}	0.96 ± 0.02	
A^+_{+00}	0.94 ± 0.01	

	Exp.	Pred.
S^-_{0+-}	-0.67 ± 0.06	-0.67
S^+_{++-}	0.23 ± 0.03	0.30
S^+_{+00}	-0.70 ± 0.06	-0.68

These results are very agreeable.

If we assume only charged currents $\mathcal{L}_{NL}$ contains both $\Delta I = 1/2$ and $\Delta I = 3/2$. It is easy to prove that if $K \to 2\pi$ are assumed to obey $\Delta I = 1/2$, the $\Delta I = 3/2$ part in $K \to 3\pi$ is highly suppressed. In fact in the limit in which any of the three pions has zero four-momentum the zero energy theorem connects the amplitude to a $K \to 2\pi$ $\Delta I = 3/2$ amplitude (zero by hypothesis). The constraint of having the amplitude vanishing at three different points is rather strong, and as pointed out of J. Bell, is enough to exclude $\Delta I = 3/2$ amplitudes which are either constant (as already noted by Suzuki) or linear in the energy.

In effect one can go further, as proposed by Suzuki[38] and prove that the $\Delta I = 3/2$ amplitude in $K \to 2\pi$ is suppressed. Applying the zero energy theorem to $K \to 2\pi$, we can reduce it to a $K \to \pi$ and into a $K \to$ vacuum $\Delta I = 3/2$ amplitude and the latter is forbidden by I-spin conservation. We note that the $\Delta I = 3/2$ $K \to 2\pi$ does not suffer from the K pole already encountered in K_{e4}. The extrapolation of both pions to zero is, however, a very long one (by about M_K as opposed to M_π in the $K_{3\pi}$ to $K_{2\pi}$ link).

Non-Leptonic Hyperon Decays

The extrapolation method has been applied to hyperon decays with a mixed degree of success. Suzuki and Sugawara[31] derived some interesting relations for the S waves, assuming a simple a $J_\lambda^*(x)\ J_\lambda(x)$ lagrangian (no built-in $\Delta I = 1/2$):

$$\begin{aligned}
\sqrt{2}\,\Xi^\circ_\circ + \Xi^-_- &= 0 \\
\sqrt{2}\,\Lambda^\circ_\circ + \Lambda^-_\circ &= 0 \\
\Sigma^-_- - \sqrt{2}\,\Sigma^+_\circ &= \Sigma^+_+ \\
2\Xi^-_- + \Lambda^\circ_- - \sqrt{2}\,\Sigma^+_\circ &= \sqrt{\tfrac{3}{2}}\,\Sigma^+_+ \quad .
\end{aligned}$$

The first two coincide with $\Delta I = 1/2$ predictions. The third is not the $\Delta I = 1/2$ prediction, because the sign of Σ^+_+ is wrong. If one requires $\Delta I = 1/2$, one has the further condition

$$S(\Sigma^+_+) = 0$$

which agrees with the findings of the Yale group.[18] With this equation the fourth condition is then equivalent with the Lee-Sugawara[47] relation.

The situation in respect to the P waves is more complicated, because of pole diagrams, and not yet solved. On the experimental side, Fig. 6 shows the agreement of experimental data with the Lee-Sugawara relation, which is quite good for both S waves, where we understand it, and P waves, where we do not.

Appendix B, prepared by J. Peter Berge, contains a new analysis of the present status of hyperon non-leptonics. A warning for theoreticians who want to try their hand at fitting these data: the errors on many of them are correlated quite strongly to one another, as indicated by the correlation parameter $\zeta_{\alpha\beta}$.

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Table I

EXPERIMENTAL DATA ON NEUTRAL LEPTON CURRENTS

Decay Mode	Branching Ratio to all Modes (90% Confidence)	Reference
$K^\circ_L \to \mu^+ + \mu^-$	$< 2.5 \times 10^{-6}$	W. Vernon, et al. This conference. (Oral communication.)
	$< 8 \times 10^{-6}$	M. Bott-Bodenhausen, et al. This conference. (Oral communication.)
	$< 5 \times 10^{-5}$	A. Abashian, et al. This conference. (University of Illinois report.)
$K^\circ_L \to e^+ + e^-$	$< 5 \times 10^{-6}$	M. Bott-Bodenhausen, et al. This conference. (Oral communication.)
	$< 5 \times 10^{-5}$	A. Abashian, et al. This conference. (University of Illinois report.)
$K^\circ_L \to e^\pm + \mu^\pm$	$< 1 \times 10^{-4}$	D. W. Carpenter, et al., Phys. Rev. 142, 871 (1966).
$K^+ \to \pi^+ + e^+ + e^-$	$< 1.1 \times 10^{-6}$	U. Camerini, et al., Phys. Rev. Letters 13, 318 (1964).
$K^+ \to \pi^+ + \mu^+ + \mu^-$	$< 3 \times 10^{-6}$	U. Camerini, et al., Nuovo Cimento 37, 1795 (1965).

Table II

LEPTONIC DECAYS OF BARYONS *

$\sin\theta = 0.21$; $\sin\theta_A = 0.28$; D/F = 2

Decay	Best Fit			Experimental Branching Ratio
	Relative Coupling Constants		Branching Ratio	
	V_{ch} (0)	A (0)		
$n \to p\, e^-\bar{\nu}$	0.978	-1.138 ± 0.026	1	1
$\Lambda^\circ \to p\, e^-\bar{\nu}$	($-$ 0.258	($-$ 0.213	$(0.87 \pm 0.03)\ 10^{-3}$	$(0.88 \pm 0.08)\ 10^{-3}$
$\Lambda^\circ \to p\, \mu^-\bar{\nu}$	$\pm$ 0.005)	$\pm$ 0.007)	$(1.48 \pm 0.05)\ 10^{-4}$	$(1.35 \pm 0.6)\ 10^{-4}$
$\Sigma^- \to n\, e^-\bar{\nu}$	($-$ 0.211	(+ 0.103	$(1.29 \pm 0.13)\ 10^{-3}$	$(1.28 \pm 0.16)\ 10^{-3}$
$\Sigma^- \to n\, \mu^-\bar{\nu}$	$\pm$ 0.004)	$\pm$ 0.022)	$(0.62 \pm 0.06)\ 10^{-3}$	$(0.62 \pm 0.12)\ 10^{-3}$
$\Sigma^- \to \Lambda^\circ e^-\bar{\nu}$	0	(+ 0.618	$(0.70 \pm 0.04)\ 10^{-4}$	$(0.75 \pm 0.28)\ 10^{-4}$
$\Sigma^+ \to \Lambda^\circ e^+\nu$		$\pm$ 0.022)	$(0.21 \pm 0.01)\ 10^{-4}$	$(0.13 \pm 0.13)\ 10^{-4}$
$\Xi^- \to \Lambda^\circ e^-\bar{\nu}$	(+ 0.258	(+ 0.043	$(0.43 \pm 0.03)\ 10^{-3}$	$(1.2 \pm 0.8)\ 10^{-3}$
$\Xi^- \to \Lambda^\circ \mu^-\bar{\nu}$	$\pm$ 0.005)	$\pm$ 0.005)	$(0.12 \pm 0.01)\ 10^{-3}$	
$\Xi^\circ \to \Sigma^+ e^-\bar{\nu}$	(+ 0.211	(+ 0.312	$(0.31 \pm 0.04)\ 10^{-3}$	
$\Xi^\circ \to \Sigma^+ \mu^-\bar{\nu}$	$\pm$ 0.004)	$\pm$ 0.007)	$(0.24 \pm 0.04\ \ 10^{-5}$	

* N. Brene, L. Veje, M. Roos and C. Cronstrom, Phys. Rev. (to be published).

FIGURE CAPTIONS

Fig. 1 Where we stand.

Fig. 2 Results of five recent K°_{e3} experiments plotted in the complex x plane. The errors are those quoted in the experimental papers; x defined as ratio of amplitudes:

$$x = \frac{a(\overline{K}^\circ \rightarrow \pi^- + \ell^+ + \nu)}{a(K^\circ \rightarrow \pi^- + \ell^+ + \nu)}$$

Fig. 3 The Σ triangle.

Fig. 4 Some diagrams for the radiative corrections to beta decay.

Fig. 5 Pole diagram for $K \rightarrow \pi^+ + \pi^- +$ leptons which contributes to the form factor f_3 .

Fig. 6 Lee-Sugawara triangle.

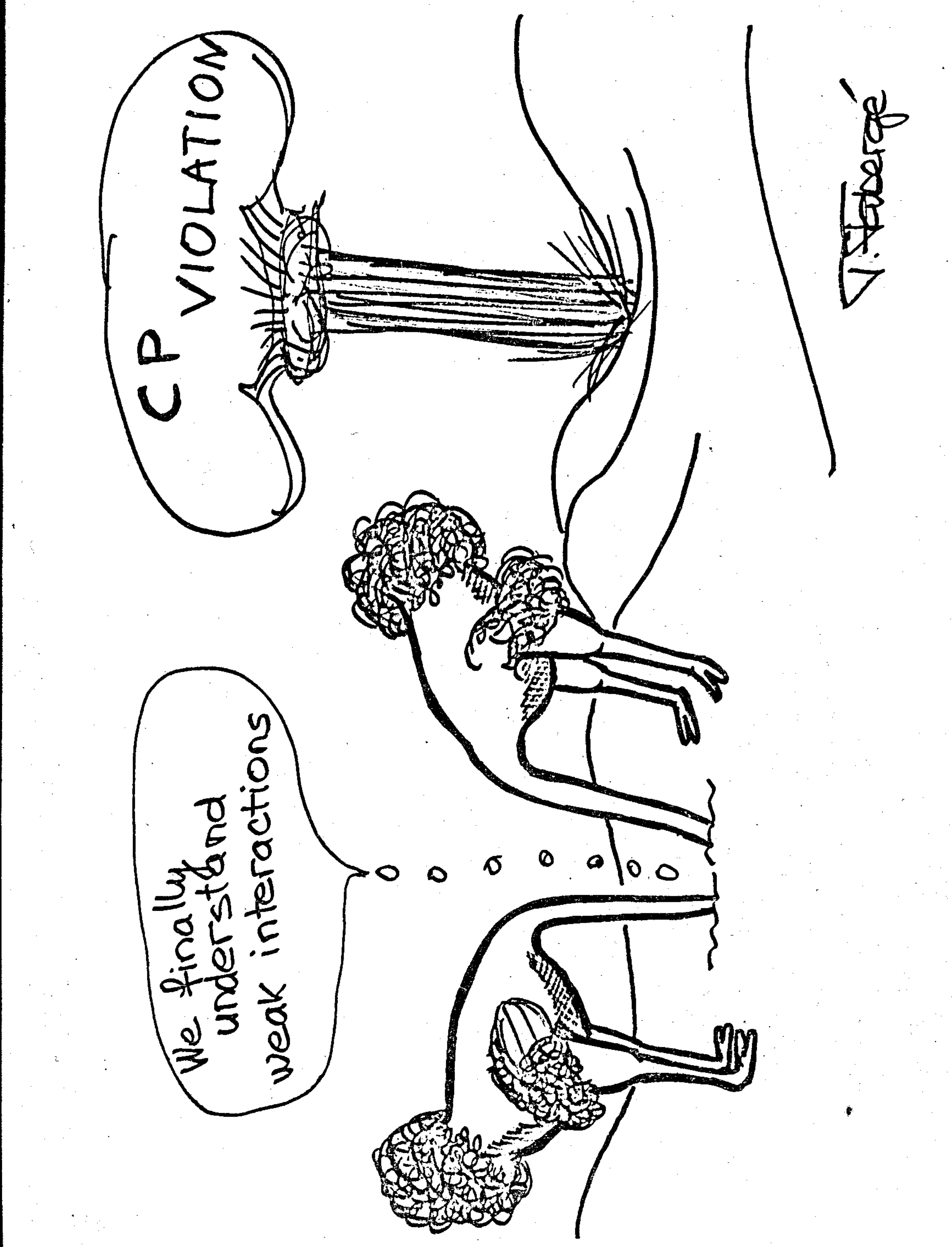
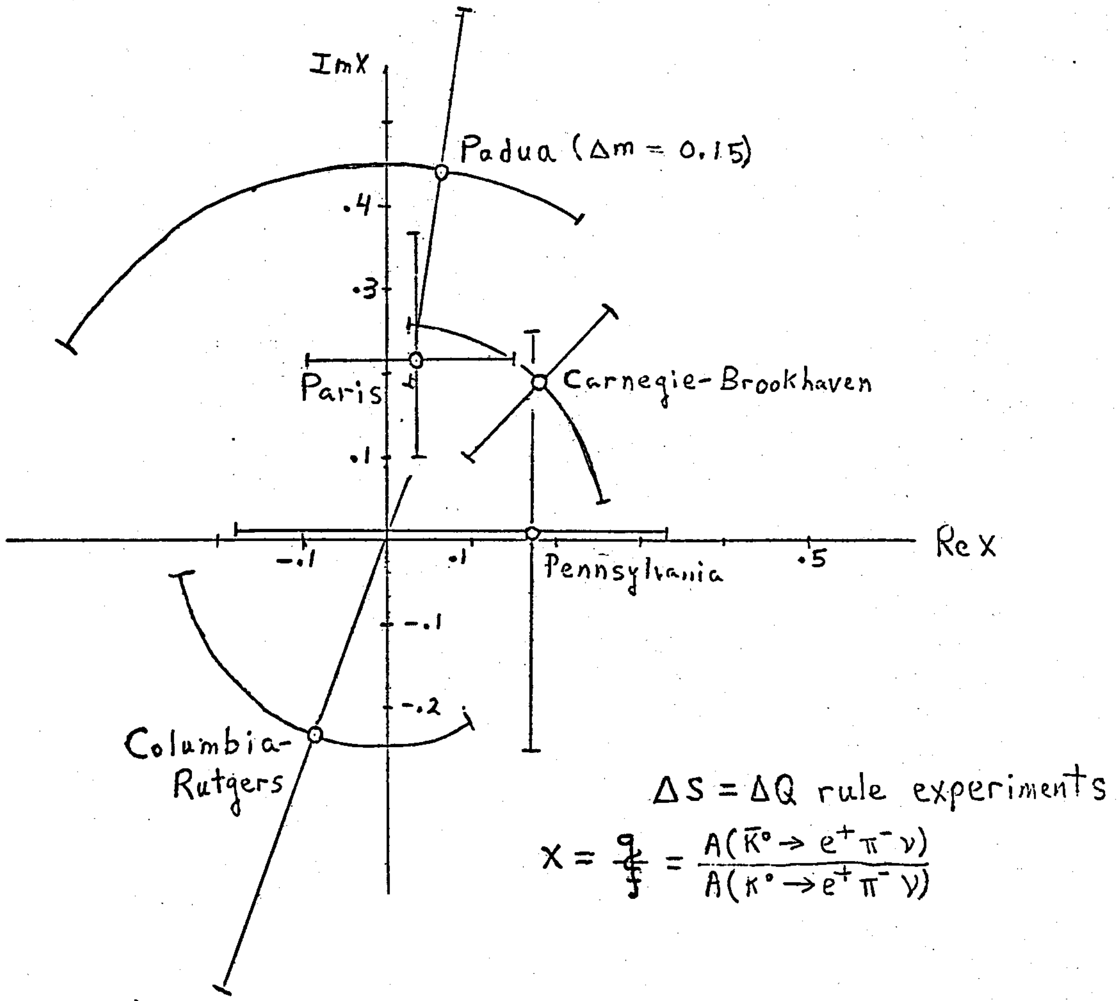
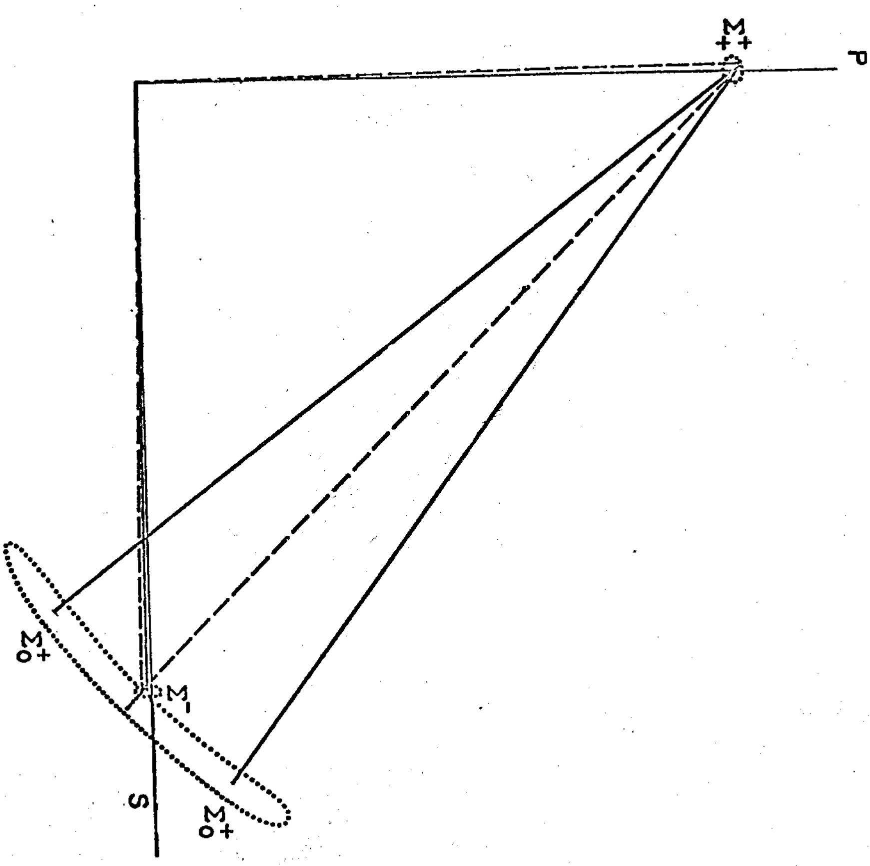


Fig. 2



Im X
Re X
.4
.3
.1
-.1
-.2
.1
.5
-.1
Padua ($\Delta m = 0.15$)
Paris
Carnegie-Brookhaven
Pennsylvania
Columbia-Rutgers
$\Delta S = \Delta Q$ rule experiments
$X = \frac{g}{f} = \frac{A(\bar{K}^{\circ} \to e^{+}\pi^{-}\nu)}{A(K^{\circ} \to e^{+}\pi^{-}\nu)}$



P
S
Σ_{+}^{+}
Σ_{-}
Σ_{0}^{+}
Σ_{0}^{+}

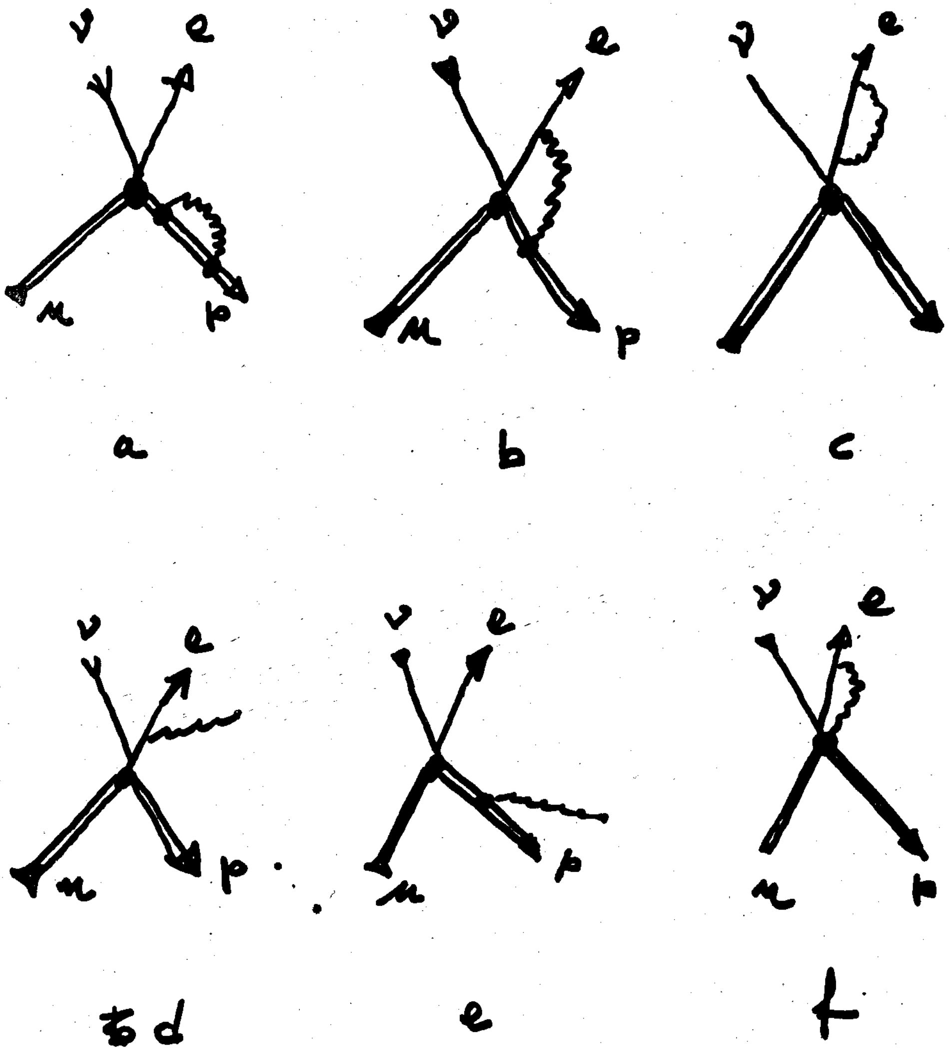


fig 4

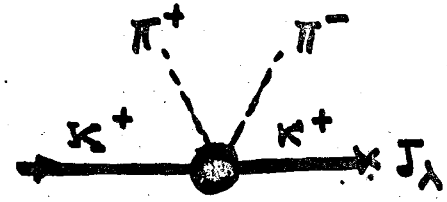
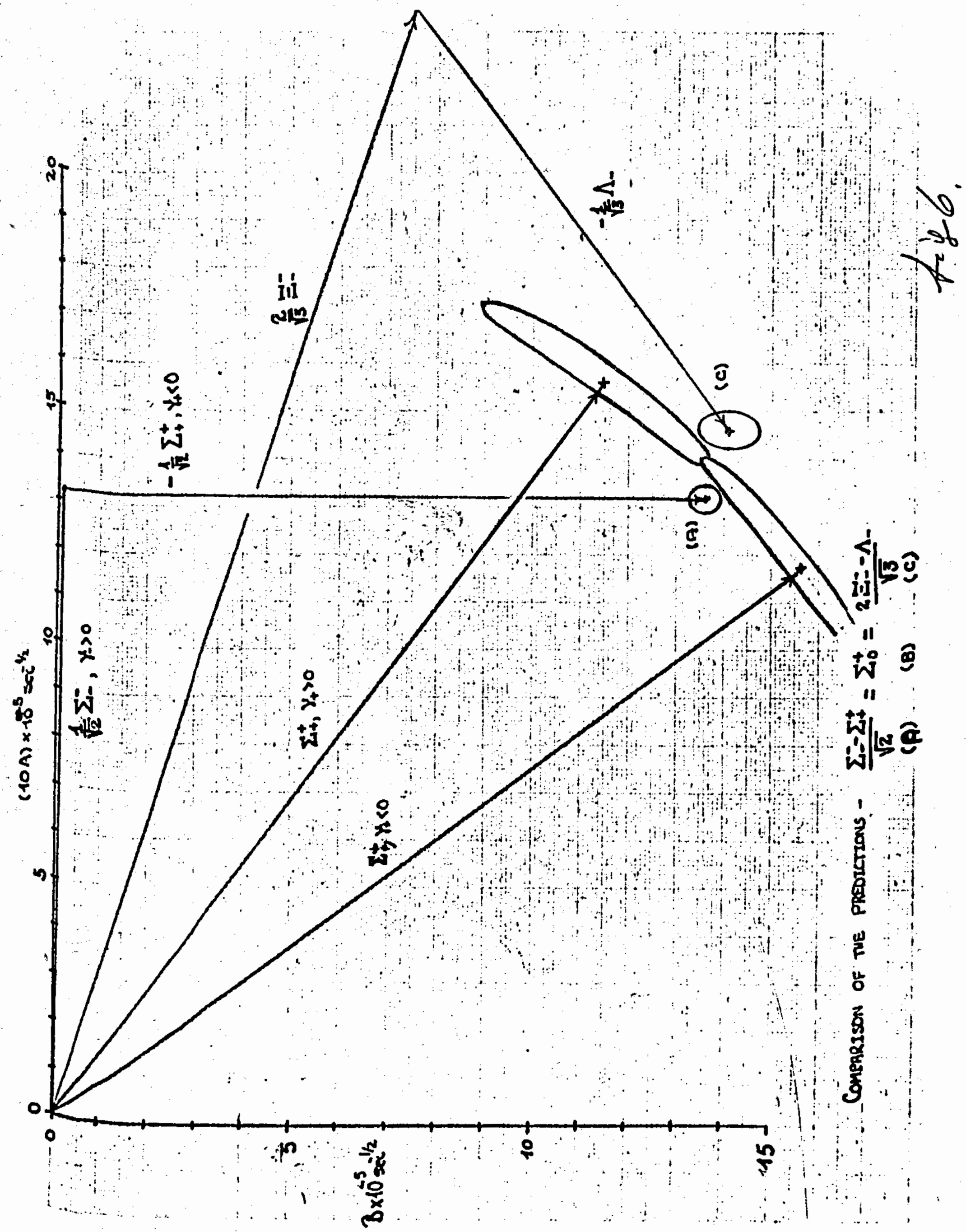


fig 5



Appendix A

K DECAY EXPERIMENTS, A SUMMARY OF RESULTS

prepared by U. Camerini and C. T. Murphy

The tables which follow were prepared for Discussion Group 2b and were discussed at that session.

$K_{\mu 3}$ DECAY EXPERIMENTS

Experiments are grouped by method used. Whenever a parameter is assumed rather than determined in the analysis, it appears, as for example, " $\equiv 0.06$".

The following trends should be noted in the data:

(a) The total polarization experiments, one with K^+ (X2, Aachen) and two with K° (Abrams, Auerbach) yield values of $\mathrm{Re}\,\xi$ near -1.0 and are inconsistent with $\mathrm{Re}\,\xi > 0$. These experiments are relatively insensistive to a q^2 dependence of f_+.

(b) Two Dalitz plot analyses (Callahan with K^+, Carpenter with K°) yield $\mathrm{Re}\,\xi > 0$, if f_+ is assumed to have no q^2 dependence and are inconsistent with the polarization results.

However, in both these experiments and a third one (Jensen), the results are compatible with the polarization experiments if an energy dependence of f_+ of order $\lambda_+ = .08$ is assumed. (See, for instance, the second entry in the Callahan experiment in the " $\mu^+ + \pi^\circ$ spec. plus angular correl." section.)

Definition of notation:

$M = \langle\pi|J_V|K\rangle\langle\nu|j_V|\mu\rangle$ where $j_V = \gamma_\mu(1 + \gamma_5)$ and $J_V = f_+(q_K + q) + f_-(q_K - q)$

$\xi = f_-/f_+$ $\qquad f_+ = f_{+\circ}(1 + \lambda q^2/m_\pi^2)$

Table A1

$K_{\mu 3}$ DECAY EXPERIMENTS

Reference	Techn.	Exp.	Re ξ	Im ξ	λ
K^+ Experiments					
*Callahan	FrBC	μ^+ Spectrum	$0.0 {+1.1 \atop -0.9}$	0.0 ± 1.0	$\equiv 0$
Bisi PRL 12, 490 (64)	PrBC	μ^+ Spec. + Branch.R.	-0.75 ± 0.5	$\pm 3.5 \pm 0.5$	$\equiv 0$
Giacomelli NC 34, 6182 (64)		μ^+ Spec.	$+0.7 \pm 0.5$	$\equiv 0$	$\equiv 0$
*Callahan	FrBC	$\mu^+ + \pi^\circ$ Spec. + Angular Correl.	$+0.72 \pm 0.37$ -0.58	$\equiv 0$ $\equiv 0$	$\equiv 0$ $\equiv 0.08$
Jensen PR 146, B1436 (65)	XeBC	$\mu^+ + \pi^\circ$ Spec. + Angular Correl.	-1.2 ± 1.0 $+0.6 \pm 2.0$ -1.8 ± 1.0	$\|\mathrm{Im}\,\xi\| < 2.4$ $\equiv 0$ $\equiv 0$	$\equiv 0.014$ $-.05 \pm .07$ $\equiv 0.06$
Brown PRL 8, 450 (62)	XeBC	$\mu^+ + \pi^\circ$ Spec. + Angular Correl.	$+1.8 \pm 1.6$	$\equiv 0$	$\equiv 0$
*Callahan	FrBC	Total Polarization	-1.4 ± 1.8	$+1.6 \pm 1.3$	$\equiv 0$
*X2-Aachen, etc.	FrBC	Total Polarization	$-1.4 {+0.6 \atop -0.4}$ -1.32 ± 0.33	$0.1 {+0.4 \atop -0.3}$ $\equiv 0$	$\equiv 0$ $\equiv 0$
Borreani PR 140, B1686 (65)	PrBC	Long. Polarization	$+1.2 {+2.4 \atop -1.8}$	$\equiv 0$	$\equiv 0$
*Callahan	FrBC	Long. Polarization	$-0.7 {+0.9 \atop -3.3}$	$0.5 \pm {1.4 \atop 0.5}$	$\equiv 0$
Cutts PR 133, B969 (65)	Sprk Sprk	Long. Polarization Branch.R., $E_\mu > 100$	$-4.0 < \mathrm{Re}\,\xi < +1.7$ $0.2 < \mathrm{Re}\,\xi < 1.4$	$\equiv 0$ $\equiv 0$	$\equiv 0$ $\equiv 0$
K^0 Experiments					
Carpenter PR 142, 871 (66)	Sprk	Spectra + Ang.Corr.	$+1.2 \pm 0.8$ -0.82 ± 0.6 d)	$\equiv 0$ $\equiv 0$	$\equiv 0$ a) $.07 \pm .011$
*Abrams	Sprk	Total Polarization	-1.1 ± 0.5	c) $\equiv 0$	$\equiv 0$
*Auerbach	Sprk	Total Polarization	$-1.25 {+0.47 \atop -0.41}$	b) $\equiv 0$	$\equiv 0$
*Kulyukina	Cloud	Spectra	$-0.2 {+1.0 \atop -1.7}$	$\equiv 0$	$\equiv 0$
*Longo	Counter	Perp. Polarization	-----	$.036 \pm .072$	$\equiv 0$
Bartlett PRL 16, 282 (66)	Counter	Perp. Polarization	-----	0.11 ± 0.35	$\equiv 0$

a) Based on fit to J = 1 intermediate state
b) Up Down asymmetry measured to be $-.067 \pm .025$.
c) $\alpha\bar{P}$ measured to be $-.1 \pm .03$; $\mathrm{Im}\,\xi = 0.1 \pm 0.5$.
* Presented at this conference.

d) Includes systematic error estimate. Original paper includes only statistical error.

Table A2

MEASURED VALUE OF THE K_1-K_2 MASS DIFFERENCE

(Units of τ_S^{-1})

Ref.		Value	Method
Fitch	NC 22, 1160 (1961)	1.9 ± 0.3	
Camerini	PR 128, 352 (1962) and erratum to be published	0.88 ± 0.20	Strangeness
Meisner	PRL 16, 278 (1965)	0.65 ± 0.30	vs.
Camerini	PR (to be published)	0.50 ± 0.16	Time
*Hill		0.62 ± 0.16	
Good	PR 124, 1223 (1961)	$0.84 \, ^{+0.29}_{-0.22}$	
Fujii	PRL 13, 253 (1964)	0.72 ± 0.15††	
Christenson-	PR 140, B74 (1965)	0.55 ± 0.10	Regeneration
*Vishnevsky-	PL 18, 339 (1965) Revised version this conf.	0.52 ± 0.15	
*Alff-Steinberger-	PL 21, 595 (1966)	0.445 ± 0.034	Interference of K_L^o and regenerated K_S^o decay into $\pi^+\pi^-$.
*Bott-Bodenhausen		0.480 ± 0.024	
*Melhop		0.44 ± 0.06	Sign of mass difference experiments†.
*Canter		0.55 ± 0.15	
*Jovanovich		0.35 ± 0.15	

†All three experiments, and also Meisner et. al, obtain $M_{K_L} > M_{K_S}$.

*Papers presented to this conference.

††Value revised downward from published value (private communication from the authors).

Table A3

ΔS = ΔQ RULE

K^o_{e3} time dependence:

$$\Gamma^{\pm} = |1+\chi|^2 e^{-\gamma_1 t} + |1-\chi|^2 e^{-\gamma_2 t} + \left[\pm 2(1-\chi^2)\cos\Delta mt + 4|\chi| \sin\theta \sin\Delta mt\right] e^{-(\gamma_1+\gamma_2)t/2}$$

$\chi \equiv g/f = |\chi| e^{i\theta}$

$g \equiv A(\bar{K}^o \longrightarrow e^+\pi^-\nu)$, $f \equiv A(K^o \longrightarrow e^+\pi^-\nu)$

$\Delta m \equiv m_S - m_L$ (taken to be positive in summary of results for θ)

$\pm$ indicates charge of lepton

χ is real if CP conserved

$\chi = 0$ if $\Delta S = \Delta Q$

A new limit for the validity of the $\Delta S = \Delta Q$ rule for the axial vector current has been established, based on the absence of the decay mode $K^+ \rightarrow e^-\pi^+\pi^+\nu$. The upper limit on χ thereby determined is 0.13 (90% confidence). (Birge et al, "Low Energy π-π Interaction Studies from Ke4 Decays", presented to this session.)

(for Δm positive)

Reference	Technique	$\lvert\Delta m\rvert$	Re χ	Im χ	$\lvert\chi\rvert$	θ
Paris PL 17, 59 (65)	FrBC: $K^+n \rightarrow K^op$	0.47 ± 0.21	$0.035 \, ^{+0.11}_{-0.13}$	$0.215 \, ^{+0.15}_{-0.11}$	$0.22 \, ^{+0.16}_{-0.11}$	$+79^o \, ^{+37}_{-27}$
Padua NC 38, 684 (65)	FrBC: $K^+n \rightarrow K^op$	$0.15 \pm ^{0.50}_{0.35}$			$0.44 \, ^{+0.19}_{-0.25}$	$+82 \, ^{+68}_{-26}$
Columbia PR 140, 127 (65)	HBC: $\bar{p}p \rightarrow K^o\bar{K}^o$	$\equiv 0.5$			$0.24 \, ^{+0.33}_{-0.24}$	$-109 \, ^{+70}_{-43}$
*Pennsylvania	SprK: $\pi^-p \rightarrow \Lambda K^o$	?	$0.187 \, ^{+0.16}_{-0.35}$	0.0 ± 0.25		
*Carnegie Tech.	D_2BC: $K^+n \rightarrow K^op$	$\equiv 0.58$	$0.18 \, ^{+0.10}_{-0.14}$	$0.19 \, ^{+0.14}_{-0.12}$	0.27 ± 0.12	$+43^o \pm 33^o$
*LRL	HBC: $\pi^-p \rightarrow \Lambda K^o$	$\equiv 0.54$	$\left\lvert\frac{1+\chi}{1-\chi}\right\rvert^2 = -0.7 \, ^{+1.2}_{-0.8}$; $\frac{2\,\mathrm{Im}\chi}{\lvert 1-\chi\rvert^2} = +0.6 \, ^{+0.5}_{+0.7}$			

*Paper presented to this session.

Table A4

BRANCHING RATIOS K^+ (in %)

Reference	Techn.	$\mu 2$	μ^3	$\pi 2$	τ	τ'	e3
Shaklee PR 136 B1423 (64)	XeBC		3 ± .5		5.1 ± .2	1.8 ± .2	4.7 ± .3
Roe PRL 7 346 (61)	XeBC				5.7 ± .3	1.7 ± .2	5.0 ± .5
Borreani PL 12 123 (64)	PrBC						5.12 ± .36
*Callahan			2.82 ± .19	21.8 ± 0.9			4.04 ± .24
Callahan PR 136 B1463 (64)	FrBC				5.54 ± .12		
De Marco PR 136 B1423 (64)					5.71 ± .15		
Taylor PR 114 359 (59)						1.5 + .2	
Bisi NC 35 768 (65)	PrBC					1.7 + .07	
Bisi			3.52 ± .2				
Trilling UCRL 16473 (65)		63.5 ± 0.7	3.17 ± .35	21.6 ± 0.6	5.59 ± .11	1.68 ± .05	4.49 ± .25
A.T.							
Poh Shen Young UCRL 16362	Emul	61.8 ± 2.9	5.4 ± .9	19.3 ± 1.6	6.0 ± .4	2.3 ± .6	5.3 ± .9
*Auerbach	Sprk	63.29 ± 0.44	3.81 ± .26	20.59 ± 0.32			5.04 ± .33
*Tsipis	Sprk		3.74 ± .25				

*Presented to this session. See reference chart.

Table A5

K_{e3} DECAY

$$f_+ = f_+(0)\left(1 + \lambda \frac{q^2}{m_\pi^2}\right) \quad \text{or} \quad f_+ = f_+(0)\frac{m^2}{m^2 - q^2}$$

Ref.	Technique	λ	M (MeV)
J. Brown UCRL-10205 (1962)	XeBC	0.038 ± 0.045	
G. Jensen PR 136, B1431 (1964)	XeBC	-0.01 ± 0.029	
G. Borreani PL 12, 123 (1964)	PBC	-0.04 ± 0.05	M ≥ 1000 - No confidence level quoted.
*G. Kalmus	FrBC	+0.09 ± 0.05	540 ± 90 (> 380, 95% Confidence level)
*R. Imlay	Sprk	0.025 ± 0.020	

$K^o_{e_3}$ Decay

Ref.	Technique	λ	M (MeV)
D. Luers PR 133, B1276 (1965)	HBC	0.07 ± 0.06	≳400
G. Fisher Argonne Conf. on Weak Int. (1965)	Sprk	0.15 ± 0.08+	430 < M < 660 1% in likelihood
*A. Firestone	HBC	0.024 ± 0.021	>500 - 95% Conf. level.
*L. Kulyukina	Cloud Chbr.	0.0	

+Includes systematic error not included in published paper.
*Papers presented to this session.

Papers Presented to Discussion Group 2b †

$K^{\pm}_{e3}$ ENERGY DEPENDENCE OF FORM FACTOR

2) Imlay, Eschstruth, Franklin, Hughes, Reading, Bowen, Mann, McFarlane
"Measurement of the Energy Dependence of the Form Factor in Ke3 Decays"

21) Kalmus, Kernan
"Determination of the Energy Dependence of the Form Factor in K^+e3 Decays"

16) Firestone, Kim, Lach, Sandweiss, Taft
"Experimental Study of the $K_L \rightarrow \pi e \nu$ Decay"

1) Kulyukine, Mestvirishvili, Neagu, Petrov, Rusakov, Wutsun, Fan
"K_2^o Meson Decay Properties"

$K^{\pm}_{\mu 3}$ ξ DETERMINATION

10) Aachen, Bari, Bergen, CERN, Ecole Polytechnique, Nijmegen, Orsay, Padua, Turin
"Measurement of the Total Muon Polarization in $K\mu3$ Decay"

20) Callahan, Camerini, Hantman, March, Murphree, Gidal, Kalmus, Powell, Sandler, Pu, Natali, Vallani
"Measurement of $K\mu3$ Decay Parameters"

23) Tsipis, Garland, Devons, Tycko, Meyer, Rosen, Pondrom
"Study of $K^+ \longrightarrow \mu^+ \nu \pi^o$ Decays"

18) Abrams, Abashian, Carpenter, Nefkens, Smith, Thatcher
"The Polarization of the Mu Meson in the Decay
$K_L^o \longrightarrow \pi^- + \mu^+ + \nu$

11) Carpenter, Abashian, Abrams, Fisher, Nefkens, Smith
"$K\mu3$ Decay Spectrum and Current Algebra"

19) Auerbach, Mann, McFarlane, Sciulli
"A Value of ξ From Measurement of a Transverse Component of Muon Polarization from $K_L^o \longrightarrow \pi^- + \mu^+ + \nu$".

sion 4A) Longo, Young, Helland, "Time Reversal Invariance in the Reaction $K_2^o \rightarrow \pi^- + \mu^+ + \nu$"

† Numbering corresponds to numbering system of the primary list of papers (see

BRANCHING RATIOS

4) Auerbach, ~~Mace~~, Dobbs, Mann, McFarlane, White, Cester, Eschstruth, O'Neill, Yount
"Measurement of the Branching Ratios of $K^+_{\mu 2}$, $K^+_{\pi 2}$, K^+_{e3}, and $K^+_{\mu 3}$"

23) Tsipis, Garland, Devons, Meyer, Rosen, Pondrom
"Study of $K^+ \longrightarrow \mu^+ \nu \pi^o$ Decays"

3) Bowen, Mann, McFarlane, Franklin, Hughes, Imlay, O'Neill, Reading
"Measurement of the K^+_{e2} Branching Ratio"

8) Abashian, Abrams, Carpenter, Mischke, Nefkens, Smith, Thatcher, Verhey, Wattenberg
"Search for $K_L^o \longrightarrow \mu^+ + \mu^-$ and $K_L^o \longrightarrow e^+ + e^-$ Decays"

1) Kulyukine, Mestvirishvili, Neagu, Petrov, Rusakov, Wutsun, Fan
"K_2^o Meson Decay Properties"

26) Bott-Bodenhausen, et al., "Interference of K_L and K_S Amplitudes in the Decay Mode $K^o \rightarrow \pi^+ + \pi^-$."

ΔM

7) Hill, Robinson, Sakitt, Canter, Cho, Engler, Fisk, Kraemer, Melzer
"Magnitude of the $K_L^o - K_S^o$ Mass Difference

6) Canter, Cho, Engler, Fisk, Kraemer, Melzer, Hill, Robinson, Sakitt
"Sign of the $K_L - K_S$ Mass Difference"

14) Mehlhop, Good, Piccioni, Swanson, Murty, Burnett, Holland, Bowles
"Interference Effects of Neutral Kaons and Sign of their Mass Difference"

9) Meisner, Crawford, Crawford
"Sign of the $K_1^o - K_2^o$ Mass Difference"

26) Bott-Bodenhausen, et al., see above.

24) J. Jovanovich, et al., "An Experiment on the Sign and Magnitude of the $K_L^o - K_S^o$ Mass Difference.

27) M. J. Balatz, et al., "Investigation of K_S^o and K_L^o Interference in the $K_L^o \rightarrow \pi^+ + \pi^-$ Decay Mode."

$\Delta Q = \Delta S$

5) Cho, Canter, Dralle, Engler, Fisk, Kraemer, Meltzer, Hill, Robinson, Sakitt, Bacon, Hopkins
"Experimental Test of $\Delta S = \Delta Q$ Rule in Neutral Leptonic K Decays"

17) Feldman, Frankel, Highland, Sloan, Van Dyck, Wales, Winston, Wolfe
"Time Dependence of K^o_{e3} Decays"

22) Birge, Ely, Gidal, Hagopian, Kalmus, Powell, Billing, Bullock, Esten, Govan, Henderson, Knight, Miller, Stannard, Tovey, Treutler, Camerini, Cline, Fry, Haggerty, March, Singleton
"Low Energy $\pi - \pi$ Interaction Studies From K_{e_4} Decays"

[illegible]) Golden, Crawford, Stern
"Three Body Leptonic Decays of Neutral K Mesons"

τ DECAYS

12) Fung, Goldberg, Meyer, Plano, Zinchenko
"Study of a Sample of Kinematically Determined K^+ Decays from Rest into $\pi^+ \pi^o \pi^o$"

13) Fung, Goldberg, Meyer, Plano, Zinchenko
"Study of a Sample of K^+ Tau Decays from Rest"

15) Behr, Brisson, Petiau, Bellotti, Pullia, Baldo Ceolin, Calimani, Ciampolillo, Huzita, Sconze, Aubert, Chounet, Lowys, Pascaud
"Decay of the K^o Meson into Three Pions: Test of CP Conservation and $\Delta I = 1/2$ Non Leptonic Selection Rule"

Appendix B

HYPERON NON-LEPTONIC DECAY STATUS *

by J. Peter Berge

Consider the decay $M \to m + \mu$:

$$\Sigma^-_- \to n + \pi^- \quad (1)$$

$$\Sigma^+_+ \to n + \pi^+ \quad (2)$$

$$\Sigma^+_0 \to p + \pi^o \quad (3)$$

$$\Lambda \to n + \pi^- \quad (4)$$

$$\Xi^-_- \to \Lambda + \pi^- \quad (5)$$

These are characterized by a decay rate, branching fraction into a particular channel, and the standard non-leptonic decay asymmetry parameters α , β , γ .

The decay rate = branching fraction x total rate is written

$$b\Gamma = \left[\frac{1}{8\pi}\frac{q}{\mu^\pm}\frac{(M+m)^2-\mu^2}{M^2}\right]\left\{|A|^2 + \frac{(M-m)^2-\mu^2}{(M+m)^2-\mu^2}|B|^2\right\} = \frac{1}{C_1}\left\{|A|^2 + C_2|B|^2\right\} \quad (6)$$

$$C_1 = \frac{1}{8\pi}\frac{q}{\mu^\pm}\frac{(M+m)^2-\mu^2}{M^2} \qquad C_2 = \frac{(M-m)^2-\mu^2}{(M+m)^2-\mu^2} \quad (7)$$

q = c.m. decay momentum

$\mu^\pm$ = mass of charged pion

The decay parameters are defined as

$$\alpha = \frac{2\mathrm{Re}A^*(\sqrt{C_2}B)}{|A|^2 + C_2|B|^2} \qquad \beta = \frac{2\mathrm{Im}A^*(\sqrt{C_2}B)}{|A|^2 + C_2|B|^2} \qquad \gamma = \frac{|A|^2 - C_2|B|^2}{|A|^2 + C_2|B|^2} \tag{8}$$

$$\Phi = \tan^{-1}\left(\frac{\beta}{\gamma}\right)$$

The sign convention is taken so that the helicity of the decay baryon agrees in sign with the parameter α. The definition of Φ is such that a value less than $\pi/2$ yields $\gamma > 0$ and $\Phi > \pi/2$ yields $\gamma < 0$; e.g., $\Phi = \pi$ corresponds to pure P wave decay.

In the calculations that follow it is assumed that $\beta = 0$. The measurements on the phase parameter Φ that exist for the Λ (references 8 and 9), the Σ^+_+ (reference 5) and the Ξ (reference 7) are all consistent with this assumption.

All masses, decay rates, and branching fractions are as given in UCRL-8030 -- revision of August 1966.

The decay parameter measurements are summarized in Table 1. The decay amplitudes as calculated from these world average quantities are presented in Table 2.

From the experimental data of Table 2 we obtain the amplitudes

		A	B	
Σ^+_0 ($\gamma_0 > 0$)	=	1.558 ± 0.142	-11.71 ± 1.88	(9)
($\gamma_0 < 0$)	=	1.168 ± 0.187	-15.61 ± 1.42	

We compare these to

$$\left(\frac{\Sigma^-_- - \Sigma^+_+}{\sqrt{2}}\right) = 1.310 \pm 0.027 \qquad -13.600 \pm 0.367 \tag{10}$$

and

$$\left(\frac{2\Xi^-_- - \Xi_-}{\sqrt{3}}\right) = 1.440 \pm 0.037 \qquad -14.030 \pm 0.657 \tag{11}$$

(In the Σ result, the Σ^+_+ is known[5] to be $\gamma_+ < 0$: We have assumed for the Σ^-_-, $\gamma_- > 0$).

We may perform the Σ triangle closure fit -- the fit is good ($\chi^2_{2c} = 1.7$) and gives as result,

			A	B	
Σ^+_0	=	$\left(\frac{\Sigma^-_- - \Sigma^+_+}{\sqrt{2}}\right)$ =	1.322 ± 0.25	-13.90 ± 0.34	(12)

This result may then be compared to the Lee-Sugawara relation, Eq. 11. The p wave parts are seen to be in excellent agreement. The S wave parts are equal to about 8% ; they fail <u>equality</u> by about 2.6 standard deviations.

Consideration of the Λ^0_0 and Ξ^0_0 decays has been excluded from this note: the known precision of α_{Λ_0} and of λ_{Ξ_0} is so low as to make such calculations pointless. There is no significant violation of the well known $\Delta I = 1/2$ predictions for these decays.

Table I

DECAY PARAMETERS

Decay	Result	Reference	Average
Σ_-^-	$\alpha_- = -0.010 \pm 0.043$	6	
	-0.16 ± 0.21	1	$\bar{\alpha}_- = 0.017 \pm 0.042$
Σ_+^+	$\alpha_+ = 0.014 \pm 0.052$	6	
	0.047 ± 0.07	5	
	-0.03 ± 0.08	3	
	-0.20 ± 0.24	1	$\bar{\alpha}_+ = 0.008 \pm 0.037$
	$\Phi_+ = 180° \pm 30°$	5	
Σ_0^+	$\alpha_0 = -0.986 \pm 0.072$	6	
	-0.80 ± 0.18	2	$\bar{\alpha}_0 = -0.960 \pm 0.067$
Λ_-	$\alpha_\Lambda = 0.655 \pm 0.025$	8	
	0.62 ± 0.07	9	$(\alpha_\Lambda = 0.651 \pm 0.024)$[a]
Ξ_-^-	$\alpha_\Lambda = 0.747 \pm 0.086$		
	$\alpha_\Xi = -0.342 \pm 0.044$	7	$\alpha_\Lambda = 0.663 \pm 0.022$
	$C_{\alpha_\Lambda \alpha_{\Xi^-}} = 0.626$[b]		$\alpha_{\Xi^-} = -0.391 \pm 0.032$
			$C_{\alpha_\Lambda \alpha_{\Xi^-}} = 0.240$[b]
	$(\alpha_\Lambda \alpha_{\Xi^-}) = -0.321 \pm 0.048$	4	
	$\Phi_{\Xi^-} = 1.2° \pm 7.5°$	7	

[a] See reference 10.

[b] See reference 11.

Table II

DECAY AMPLITUDES

Decay →	Σ^-_-	Σ^+_+	Σ^+_0	Λ_-	Ξ^-_-
M	1197.2	1189.4	1189.4	1115.4	1321.0
m	939.6	939.6	938.2	938.2	1115.4
μ	139.6	139.6	135.0	139.6	139.6
C_1	5.73609	5.94340	5.82167	10.37365	7.44651
C_2	0.01032	0.00951	0.00996	0.00284	0.00385
$\Gamma \times 10^{10}\text{sec}^{-1}$	0.6040 ± 0.011	1.234 ± 0.020	1.234 ± 0.020	0.400 ± 0.007	0.572 ± 0.016
b	1.0	0.4723 ± 0.0154	0.5277 ± 0.0154	0.663 ± 0.014	1.0
α	−0.017 ± 0.042	0.008 ± 0.036	0.960 ± 0.067	0.663 ± 0.023	−0.391 ± 0.032
$A \times 10^5\text{sec}^{-1/2}$	1.861 ± 0.017	0.008 ± 0.034	1.558 ± 0.142 [a] 1.168 ± 0.187 [b]	1.551 ± 0.024	2.022 ± 0.029
$B \times 10^5\text{sec}^{-1/2}$	−0.152 ± 0.386	19.081 ± 0.347	−11.71 ± 1.88 [a] −15.61 ± 1.42	11.045 ± 0.475	−6.628 ± 0.574
C_{AB} [c]	0.016	0.001	0.959	−0.367	0.085
fit to $\Sigma^+_0 = 1/\sqrt{2}\,(\Sigma^-_- - \Sigma^+_+)$					
$A \times 10^5\text{sec}^{-1/2}$	1.864	−0.005	1.322		
$B \times 10^5\text{sec}^{-1/2}$	−0.311	19.348	−13.901		

[a] $\gamma_o > 0$ solution.

[b] $\gamma_o < 0$ solution.

[c] See reference 10.

REFERENCES TO APPENDIX B

* Formula and reference numbers refer only to Appendix B. Figures referred to are in the main text.

1. R. D. Tripp, M. Ferro-Luzzi, and M. Watson, Phys. Rev. Letters 9, 66 (1962).

2. E. F. Beal, B. Cork, D. Keefe, W. C. Murphy, and W. A. Wenzel, Phys. Rev. Letters 8, 75 (1962).
The value reported there has recently undergone revision. A more restricted and reliable sample of events was used to obtain the value of α_o listed. (D. Keefe, LRL, private communication).

3. B. Cork, L. T. Kerth, W. A. Wenzel, J. W. Cronin, and R. L. Cool, Phys. Rev. 120, 1000 (1960).

4. H. K. Ticho, invited talk at Int'l. Conference on the Fundamental Aspects of Weak Interactions, Brookhaven Nat'l. Lab., Sept. 1963. The Berkeley contributions have been extracted from his compilation.

5. D. Berley, S. Herzbach, R. Kofler, S. Yamamoto, W. Heitzelman, M. Schiff, J. Thompson, and W. Willis, report submitted to this conference.

6. Roger B. Bangerter, Angela Barbaro-Galtieri, J. Peter Berge, Joseph J. Murray, Frank T. Solmitz, M. Lynn Stevenson, and Robert D. Tripp, Phys. Rev. Letters, 17, 495 (1966).

7. Deane W. Merril, Janice Button-Shafer, and J. Peter Berge, report submitted to this conference. The data given there have been reanalyzed together with the result of Overseth and Roth (reference 8 on α_Λ to give the results listed in the compendium).

8. O. E. Overseth and R. F. Roth, report submitted to this conference.

9. J. W. Cronin and O. E. Overseth, Phys. Rev. 129, 1795 (1963).

10. The value obtained for the parameter α_Λ from references 8 and 9, together with the independent measurement of $(\alpha_\Lambda \alpha_{\Xi^-})$, was combined in a least squares manner with the results from the Ξ^- analysis of ref. 7 to obtain the final value of the parameters α_Λ and α_{Ξ^-} used in this note.

11. The number $C_{AB} = \frac{\langle \delta A \; \delta B \rangle}{\sqrt{\langle \delta A^2 \rangle} \sqrt{\langle \delta B^2 \rangle}}$ is the statistical correlation coefficient.

The large figure listed for the Σ_0^+ demonstrates the poor angular precision in the determination of these decay amplitudes. The length of the amplitude vector is known to good accuracy through the decay rate measurement (see figure 3).