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ПО ФИЗИКЕ ВЫСОКИХ ЭНЕРГИЙ

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F. Low

DYNAMICS OF STRONG INTERACTIONS

My report today is meant to cover the material submitted to four parallel sessions. These were:

Session 11a	Analytic structure of partial waves
Session 11b	Off-mass shell dynamical questions
Session 11c	S-matrix dynamics
Session 11d	Boot-strap models

There were about thirty papers submitted to these four sessions, many of them very interesting. Rather than attempt to tell you about them in two minutes apiece, I will briefly list them at the end of my report and devote the present time to a general discussion of the state of the field, together with a report on a small number of those developments which I believe are of most interest to the general particle physics community. This does not mean that those papers which I omit are of less interest, but only that they are of less general interest. For example, several are essentially calibrations: compare a new instrument with a standard, say, an N/D calculation with the Schroedinger equation used to solve the same problem.

If one defines dynamical theory to mean the calculation from first principles and a <u>small</u> number of parameters of scattering and production amplitudes (which, of course, includes by implication the calculation of the residues and locations of poles, i.e. the coupling constants and masses of stable, unstable and virtual states), then I am afraid that I must report that in spite of a great deal of intelligent and diligent work we have at present (or at least I have at present) very little understanding of dynamics. We do know many "truths" (which may later turn out to be only partial truths, or even falsehoods) which we can apply to the analysis of data and whose consistency with the data we can check. Many of these "truths" are at the level of conjectures, others are more firmly established, and our tool-kit of "truths" is growing in an exciting way. For example, in the former category are the local current algebras; in the latter, fixed small momentum transfer (best: zero) dispersion relations for pion nucleon scattering. Nevertheless I do not believe that any of these methods can be called dynamics since the essential ingredient "small number of parameters" is missing. The reasons for this belief should emerge in the course of my talk.

The historical basis for all our present day methods of theorizing about particles is the Lagrangian quatum theory of fields. Thus, TCP, analyticity properties, current algebras and more have been abstracted from this framework. During the last ten years or so two deviations have grown up among the practitioners of field theory. For purposes of avoiding confusion, they may be called left and right wing deviations, although the deviation space is somewhat curved so that they are in many cases closer to each other than either is to the center. Each has abstracted and kept ~~lent~~ certain aspects of Lagrangian field theory and rejected others. The right wing retains the concept of the local field, but rejects all detailed dynamical assumptions such as equations of motion and commutation relations. The left wing rejects the concept of the local field and insists that the analytic and unitary ~~analysticity~~ properties of the ~~unitary~~ S-matrix are sufficient for all calculations. The central position is that there might well be no contradiction between the two sets of axioms ~~axeims~~, and that they might both be right (or, of course, both wrong); further that it pays to put enough detail into a model so that one can, for example, carry out perturbation or perhaps other calculations as a testing ground for general hypotheses. This does not imply belief in the validity of perturbation theory (or any other existing approximation method) for the strong interactions, or even the validity of the analytically continued sum of perturbation terms.

This brings me to the first contribution on which I wish to report. The contribution is by Weinberg and~~, as you may guess,~~ is a new approach to perturbation theory calculations. He points out that since the S-matrix is Lorentz ~~is~~ invariant, one may calculate it in the most convenient co-ordinate system. If one calculates in a manfiestly co-variant manner, say with Feynman diagrams, then this availeth naught. If one uses a non-covariant method of calculation, then the co-ordinate system will make a difference in the ease of calculation. Weinberg suggests that there are many advantages to old fashioned energy-denominator perturbation theory combined with a co-ordinate system that is moving with an infinite momentum. This is the same co-ordinate system that has been so useful in deriving Adler-Weissberger-like consequences of current algebras. I will briefly describe the method since it may turn out to be useful.

The T matrix consists of sums of terms each of which is characterized by a time ordered diagram, with momentum conservation at each vertex, one energy denominator $E_I - E_i$ between every vertex and, for scalar particles, a wave-function normalization $1/2\omega_n$ $\cancel{\frac{1}{2\varsigma_n}}$ for every internal line.

The momenta are parametrized by

$$\vec{p}_n = \eta_n \vec{P} + \vec{q}_n \qquad (1)$$

where $\vec{P}$ will go to infinity and $\vec{q}_n \cdot \vec{P} = 0$.

We may choose $\sum \vec{q}_n = 0$, so that, if $\vec{P}$ represents the total momentum, $\sum \eta_n = 1$

It is easy to see that in the initial state all η's are positive, since

$(p_n)_z = \left((p_n^{(0)})_z + v E_n^0\right)/\sqrt{1-v^2}$ (where $p_n^{(0)}$ represents the rest system vector)

so that, as $v \to 1$, $(p_n)_z > 0$.

Let us now calculate the energy, En:

$$E_n = \sqrt{\eta_n^2 P^2 + m_n^2 + q_n^2} \simeq |\eta_n| P + \frac{m_n^2 + q_n^2}{2|\eta_n| P} . \qquad (2)$$

The energy denominator is thus

$$E_I - E = \sum |\eta_n^I| P + \sum \frac{m_n^2 + \vec{q}_n^{I\,2}}{2P|\eta_n^I|} - \left(\sum |\eta_n| P + \sum \frac{m_n^2 + \vec{q}_n^{\,2}}{2P|\eta_n|} \right). \tag{3}$$

If all η^I are positive, then $\sum |\eta_n^I| = 1$, and, since the initial η's are positive, the P terms cancel. Furthermore, for this condition, clearly

$$s = E^2 - P^2 = \sum \frac{m_n^2 + \vec{q}_n^{\,2}}{\eta_n} \tag{4}$$

so that

$$E_I - E = \frac{s_I - s}{2P}, \quad \text{all } \eta_n^I > 0. \tag{5}$$

Thus energy denominators become covariant. If some $\eta^I < 0$, then $\sum |\eta_n^I| > 1$ and $E_I - E \sim P$. Such terms give no contribution (apart from possible divergence difficulties which we will not discuss) since we have at each vertex

$$\prod_n \frac{d^3\vec{p}_n}{(2\omega_n)} \, \delta\left(\sum \vec{p}_n - \vec{P}\right) = \frac{1}{P} \prod_n d\vec{q}_n \, d\eta_n \, \delta\left(\sum \eta_n - 1\right) \delta\left(\sum \vec{q}_n\right). \tag{6}$$

Thus for energy denominator of type (5) we get a finite result as $P \to \infty$. The others go to zero. Thus, η's are integrated only over positive values. Clearly $\eta < 0$ corresponds to a particle moving with infinite momentum in the initial co-ordinate system. Suppressing $\eta < 0$ permits us to leave out all diagrams containing spontaneous vacuum fluctuations. Thus, in $\pi - \pi$ scattering, we would retain the diagram Fig I(a), but reject Fig I(b).

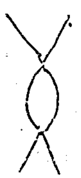

Fig. I(a)

Fig. I(b)

Weinberg further shows that in this case the integration variable η plays the role of a Feynman parameter, although only one energy denominator (as opposed to two Feynman propagators) enters.

The case of spin $\frac{1}{2}$ and higher may well present extra difficulties which will have to be studied.

I return now to more general questions, in particular to the role of current algebras and of quark models in helping us formulate the dynamics of strong interactions. First, current algebras. Should it become clear (which I believe is not yet the case) that local commutation relations do hold between components of the beta-decay and electromagnetic currents, it will be hard to avoid imagining that there is some truth to the simplest model that provides such relations: quantum field theory with more or less canonical commutation relations between the fields and currents constructed from these fields. There is at present no compelling reason to suppose that these fields must have quark quantum numbers, since none of the existing checks of the current algebras can distinguish between a quark algebra and, say, an algebra constructed with an octet of elementary fields. The quark hypothesis is simply the most economical. Should these fields exist, Nature could still conceal their existence from us in a variety of ways. The simplest would be not to let the corresponding particles exist. That is to say, the mass operator, acting on state having the quantum numbers of the field, would have no discrete eigenvalue, but only a continuum starting at some lower limit, so that there would be no asymptotic states with those quantum numbers. Put differently, the propagation function of the field would not have a pole. I am not talking here of instability since I include the case of quark fields, with absolutely conserved quantum numbers. A second way for Nature to conceal the existence of such elementary fields would be to put the corresponding particles on a Regge trajectory by means of an elementary vector field, as discussed in detail by Mandelstam and others. The particle corresponding to the vector field might itself be put on a trajectory by the mechanism discussed by Johnson and others for gauge fields. In the end there would be no particles corresponding to the fields, or at least no distinguished particles (not to use the word elementary). However, the current algebra would presumably remain. It may, of course, be the case that current algebras hold without the existence of corresponding fields. However, I want to emphasize that a proof that S-matrix equations are compatible with current algebras does not show the lack of need for local fields, since the S-matrix must be compatible with whatever the composition of the world happens to be.

Although the subject probably belongs to another session, I would like to ask a question about the physical quark models, which have had considerable success in correlating high energy scattering and low energy properties of particles. The question is: is it possible for particles to be bound so deeply that they loose a large fraction of their rest mass, and still behave non relativistically? I would ask professor Charles Schwartz what happens, for example, in a Bethe-Salpeter equation in which one adjusts an attractive long range and a repulsive short range scalar exchange to simulate the state of affairs I have just described. Are the energy levels, for example, correctly calculated from the non-relativistic Schroedinger equation with the equivalent Yukawa potentials? Can the mass, for example, go smoothly through zero with the particle remaining non-relativistic?

I turn now to relatively rigorous applications of possible candidates for truths. These applications are quite impartial with respect to the underlying philosophical truths we have just finished discussing. First Goebel has shown that from forward dispersion relations for $\pi-\pi$ scattering one may find a lower bound for the $\pi_0-\pi_0$ scattering length:

$$a > -I_0$$

where $I_0 = \dfrac{2}{\pi}\displaystyle\int dk\, \mu\frac{(\mu^2+2k^2)}{\omega^3}\,\frac{\sigma}{4\pi|f|^2}\,,$ (7)

one gets easily, retaining only S waves up to an energy less than D wave excitation,

$$a_0 + 2a_2 > -1/\mu\,.$$

I wish next to discuss two new kinds of sum-rules which are based on conjectured analyticity and high energy properties of scattering amplitudes. The first is due to Wanders, and is based on the Mandelstam representation (in fact a little less: that analyticity which can be derived from field theory) for processes in which at least two of the three Mandelstamm channels (s, t, and u) represent the same physical process. Wanders applies the method to pion-pion scattering. Unfortunately, it does not apply to pion-nucleon or nucleon-anti nucleon scattering where the comparison with data might be easier. The method goes as follows:

Imagine an amplitude which has the symmetry

$$T(s,t;u) = T(s,u,t) \qquad 8)$$

for example, as shown in Fig. II

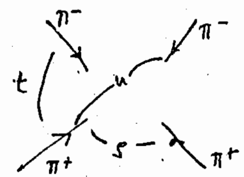

Fig. II

the process $\pi^+ + \pi^+ \to \pi^+ + \pi^+$ in the s channel becomes

$\pi^+ + \pi^- \to \pi^+ + \pi^-$ in both the u and t channels.

New variables which embody the symmetry (8) are then introduced:

$$x = \tfrac{1}{4}(t+u) = \tfrac{1}{4}(4-s)$$

and $$y = \frac{-1}{16}\, t.u$$

and the Mandelstam representation used to study analyticity in the variable x when $y = ax + b$, a and b constant.

In particular, the two cases

$$y = ax$$

and $$y = a(x - 1)$$

lead to integrals over physical values of ImT for $a \to o$.

The straightforward limit $a \to o$ leads back to $t = o$ dispersion relations. Five new relations follow by differentiating with respect to (a) before taking the limit. These relations express the P and D wave scattering lengths as well as S wave effective ranges as integrals over I = 0,1 and 2 cross sections and first derivatives with respect to t of absorptive parts at t = o. The convergence is so rapid that the ~~regional~~ resonance region contributes negligibly. Thus ~~the~~ comparison with experiment is not yet feasible.

The second kind of sum rule is based on the idea of taking seriously all our present information on high energy behavior and applying it to the number of subtractions in dispersion relations. This was first suggested by Drell and Hearn. They considered the dispersion relation for the forward spin-flip scattering of light:

$$f_2 = -\frac{1}{2}\frac{\alpha}{M_p^2}k_p^2 + \frac{\nu^2}{\pi}\int \frac{d\nu'^2}{\nu'^2}\,\frac{Im\, f_2(\nu')}{\nu'^2-\nu^2} \qquad (9)$$

where $K_p = 1.79$ in the anomalous proton moment, $\alpha = 1/137$ and ν is the laboratory energy. The first term on the right follows from the low energy theorem.

The amplitude f_2 is defined by the equation

$$f = f_1\,\vec{e}_2\cdot\vec{e}_1 + \nu\, f_2\, i\,\vec{\sigma}\cdot\vec{e}_2\times\vec{e}_1$$

and since the leading Pomerantschuk trajectory cannot flip the spin in a forward scattering, they assume

$$|\nu f_2| < \nu^{\alpha}, \qquad \alpha < 1$$

or

$$|f_2| < \nu^{\alpha-1} \quad \text{as } \nu \to \infty.$$

Then (9) leads to the sum rule:

$$\frac{1}{2}\frac{\alpha}{M_p^2}k_p^2 = \frac{1}{\pi}\int_0 \frac{d\nu'^2}{\nu'^2}\, Im\, f_2(\nu'). \qquad (10)$$

Since $$Im\, f = \frac{\nu\sigma}{4\pi},$$

we have

$$Im\, f_2 = \frac{\sigma_p - \sigma_A}{8\pi}$$

where σ_p is the total cross-section for scattering of circularly polarized photons parallel to the proton-spin, σ_A anti-parallel. An approximate evaluation of (10), appears to be in agreement with the da

An extension and generalization to the case where there is no zero energy theorem to help has been suggested by de Alfaro, Fubini, Furlan and Rossetti (hereinafter called AFFR)

1.5 They consider an ~~t~~ s channel process at small t. Its high ~~s~~ s behavior is ~~demonstrated~~ dominated by the leading Regge trajectory in the t channel, according to

$$f \sim P_\alpha(\cos\theta_t) \sim s^{\alpha(t)} \tag{11}$$

for spin zero particles.

However, for particles with spin, invariant amplitudes corresponding to spin flip in the t channel are governed by derivatives of P_α, and thus asymptotically go like $s^{\alpha-n}$ where n is the total crossed channel spin flip.

This can also be shown in a somewhat less Regge-istic way. Consider, for example the scattering of spin one particles by spin zero particles. Then, if ε_1 and ε_2 are the initial and final polarization vectors of the spin one particle, the invariant amplitude may be written

$$M = \varepsilon_{2\mu}\, M_{\mu\nu}\, \varepsilon_{1\nu} \tag{12}$$

where $M_{\mu\nu}$ is a 2nd rank tensor constructed from the momenta of the particles and the $\delta_{\mu\nu}$ symbol . $M_{\mu\nu}$ in turn can be expanded (assuming time reversal ~~resonances~~ invariance) in terms of four independent tensors:

$$M_{\mu\nu} = P_\mu P_\nu A + (Q_\mu P_\nu + Q_\nu P_\mu) B + Q_\mu Q_\nu C + \delta_{\mu\nu} D \tag{13}$$

where $P = p_1 + p_2$, $Q = q_1 + q_2$, and the

p's are spin-zero momenta, q s spin one momenta, and where A, B, C, D are scalar functions satisfying simple dispersion

relations. Now ~~once~~ since the tensor coefficients are ~~similar~~ bilinear in the momenta of the particles, for $M_{\mu\nu}$ not to grow too rapidly as $\nu \to \infty$; some of the invariants must go to zero more rapidly than for scalar particles. In particular, it is found that A must vanish like $1/\nu^2$ and B like $1/\nu$ compared to scalar particle behavior for fixed energy dependence, say $\nu^{\alpha-1}$, of the elastic cross-section. This corresponds precisely to the t channel spin-flip analysis above: A is double helicity flip and B single helicity flip in the t channel.

The authors next assume Regge-istic behavior for α. That is, $\alpha = 1$ for the crossed I=0 channel, $\alpha < 1$ for I=1 and $\alpha < 0$ for I=2. It then follows that

$$A^{I=2} < \frac{1}{\nu^2}, \quad A^{I=1} < \frac{1}{\nu}$$

$$\text{and} \quad B^{I=2} < \frac{1}{\nu}. \tag{1}$$

It turns out that $A^{I=1}$ and $B^{I=2}$ are odd under crossing, so that they satisfy unsubtracted dispersion relations of the form

$$\begin{pmatrix} A^{I=1} \\ B^{I=2} \end{pmatrix} = \frac{2\nu}{\pi} \int_0^\infty d\nu' \, \frac{\mathrm{Im} \begin{pmatrix} A^{I}(\nu') \\ B^{I}(\nu') \end{pmatrix}}{\nu'^2 - \nu^2}. \tag{1}$$

Evidently, for $\begin{pmatrix} A^{I=1} \\ B^{I=2} \end{pmatrix} < \frac{1}{\nu}$, we must have

$$\int_0^\infty d\nu' \, \mathrm{Im} \begin{pmatrix} A^{I=1} \\ B^{I=2} \end{pmatrix} = 0 . \tag{16}$$

One might call this a negatively subtracted dispersion relation. These are the de A-F-F-R. sum rules. They are supposed to be exact, but they have very charming approximate consequences. Suppose one considers $\rho - \pi$ scattering, keeps only intermediate π, ϕ and ω states, and neglects the widths of the resonances. One finds two equations:

$$\left(g^2_{\omega\rho\pi} + g^2_{\phi\rho\pi} \right) = 4 g^2_{\rho\pi\pi} / m_\rho^2 \tag{17}$$

and

$$\nu_\omega \, g^2_{\omega\rho\pi} + \nu_\phi \, g^2_{\phi\rho\pi} = 0 \tag{18}$$

where $\nu_{\phi,\omega} = \frac{1}{2}\left(m^2_{\phi,\omega} - m_\rho^2 - m_\pi^2 \right)$.

The second relation gives $g^2_{\phi\rho\pi} \sim 0$ (19)

since $m_\omega \sim m_\rho$.

A-F-F-R limit themselves to these two sum rules at t = 0. Now Reggeism tells us that $f \sim s^{\alpha(t) - n}$; thus $\frac{\partial f}{\partial t} \sim \log s \, \alpha'(t) s^{\alpha(t)-n}$ and also vanishes rapidly. Since spin one intermediate states are involved, the $\mathrm{Im}\, A$, etc., are linear functions of t. Also, and the coefficients of t must also satisfy superconvergence relations. the amplitude $A^{I=2}(\nu)$ satisfies an unsubtracted relation like

$$A^{I=2} = \frac{1}{\pi} \int \frac{d\nu'^2}{\nu'^2 - \nu^2} \, \mathrm{Im}\, A^{I=2} . \tag{20}$$

Since $A^{I=2} < \frac{1}{\nu^2}$, we should have, with equal validity to Eq. (16),

$$\int d\nu'^2 \, \mathrm{Im}\, A^{II}(\nu', t) = 0 . \tag{21}$$

This now gives, in all, six equations ~~during~~ among three coupling constants and three masses. Of course, only the first two are satisfied. We are therefore warned to be careful about asymptotic behavior; in particular not to truncate our ~~saws~~ sums too soon. I, therefore, believe we must regard the success of (18) as somehwat fort~~ia~~uitous. Nevertheless, the basic idea I believe is correct, important, and will be very fruitful. It can be applied to many processes: pion plus nucleon going to rho plus nucleon, or delta plus pion and so on, including photo-pion production on nucleons.

The AFFR idea also has implications for dynamical calculations. In particular, coupling constants in one channel (or weighted sums of coupling constants in one channel) are expressed as weighted sums of coupling constants in the same and other channels through a crossing matrix. You will recognize that these are equations of the type that are typically called boot-strap equations. That is, according to the boot-strap philosophy, they would be taken to imply in this case that the pi and omega mesons were bound states of a pi and a rho (I leave out the phi, since its coupling constant is so small). Clearly, the dispersion sum rules just written down have no such implication. Either, neither, or both of the particles could be composite (or elementary). The sum rule (correctly applied) is simply a true relation between coupling constants which is enforced by the high spin of the rho. To clarify this point further, since I am sure it is very controversial, let me go back to a more familiar example: pion-nucleon scattering with static kinematics.

The static P-wave dispersion relations are

$$h_\alpha(\omega) = \int \frac{\operatorname{Im} h_\alpha(\omega')d\omega'}{\omega'-\omega} + \sum_\beta A_{\alpha\beta} \int \frac{d\omega' \operatorname{Im} h_\beta(\omega')}{\omega'+\omega} \tag{22}$$

where

$$h_\alpha = \frac{e^{i\delta_\alpha} \sin\delta_\alpha}{q^3}. \tag{23}$$

δ_α is the α'th phase-shift, and ω and q are the pion energy and momentum. The "boot-strap" language applied to this equation goes as follows: replace the crossed channel by its approximate poles. Then, if there is a state in the β'th channel at $\omega = \omega_\beta$, with $\lambda_\beta = \int \operatorname{Im} h_\beta(\omega')d\omega'$, we get a sum of poles replacing the left hand cut:

$$\text{left hand cut} \longrightarrow \sum_\beta A_{\alpha\beta} \frac{\lambda_\beta}{\omega_\beta+\omega}. \tag{24}$$

Now if you do N/D, assume D_α vanishes at $\omega = \omega_\alpha$, and is linear (effective range approximation) you find

$$\lambda_\alpha = \sum_\beta A_{\alpha\beta} \lambda_\beta. \tag{25}$$

Applied to the (1,1) and (3,3) states, these are the reciprocal boot-strap equations of Chew. They predict $\lambda_{11}/\lambda_{33}$ in agreement with experiment. Because it appears that the vanishing D function is essential to the argument, one <u>can</u> interpret them as implying that <u>both</u> ω_{11} and ω_{33} are zeros of D and hence bound states.

There is however a much simpler derivation of (25), I believe first given (like many other ~~decays~~ things) by Nambu and published by him and others, which makes it clear that (25) is simply a consistancy condition on the residues, very ~~avalogem~~ analogous to the new AFFR sum rules: observe that as $\omega \to \infty$, $h_\alpha \to 0$ faster than $1/\omega$. Thus the coefficient of $1/\omega$ on the r.h.s. of (22) must vanish. Hence

$$-\frac{1}{\omega}\int \operatorname{Im} h_\alpha(\omega')d\omega' + \frac{1}{\omega}\sum_\beta A_{\alpha\beta}\int \operatorname{Im} h_\beta(\omega')d\omega' = 0 \tag{26}$$

or again

$$\lambda_\alpha = \sum_\beta A_{\alpha\beta}\lambda_\beta. \tag{25}$$

This is clearly in the nature of an AFFR sum rule. Although the π-n system does not strictly have an AFFR condition for the whole amplitude, the consideration of P states has ~~apparently~~ promoted one. In any case, the moral appears to me to be that so-called boot-strap relations between coupling constants, if correct, are consequences of AFFR type conditions. They thus verify that nature is consistant with reasonably truncated dispersion relations, and do not <u>necessarily</u> have dynamical content.

For example, to show that such equations permit symmetric solutions (as in SU(6)) is not to derive the symmetry, but simply to show that nature permits it consistantly.

I wish to emphasize two things I am not saying:

1) I am not saying that either the N(940) or N*(1238) is elementary. (I apologize for the use of the word.) On the contrary, there appears to be evidence from backward π-p scattering that both lie on trajectories. What I said is that Eq. (25) gives us no clue as to whether or not they lie on trajections, and if so, who put them there.

2) I am not saying that it is impossible to do N/D calculations which will provide information as to the underlying dynamics of the system. Of course that is in principle possible, with moderately weak, non-singular potentials. Unfortunately, these are precisely not the type which appear to dominate in nature.

Can one formulate a clean criterion for believability of a dynamical model? Probably not. It is a question of individual judgement, of what in law might be called the judgement of the man of ordinary prudence. As a minimum, one should be able to calculate masses as well as coupling constants. The calculation should also close, in the sense that one should not have to integrate over an energy region where channels that have not been considered make an important contribution. I do not believe these conditions have really been met by any existing calculations on systems of baryon number one or less. Baryon number two or more I leave for the moment to our friends in radio-chemistry.

One final remark: the AFFR sum rules get more powerful the higher the spins of the particles involved. Dynamical calculations become more reliable (in the above sense) the lower the spins.

In summary, then, we have not learned very much dynamics, but mainly "truths" with which nature is consistant. It is of course very important to check that these "truths" are indeed true. One of the games people play is the following: try to understand state A in the s channel by the process B + C goes to D + E with exchange of F in the u channel and G in the t channel. It is called, sometimes, "showing that the forces have the right sign to account for the particles". This game is valuable when taken as a test of the above type, including, for example, the complicated question of behavior of phase shifts in many channel ~~positions~~ problems. Presumably it is in just those amplitudes that satisfy AFFR conditions that one can legitimately ignore the effect of higher mass states as distorters of force, or sources of effective constants.

Insert II.

We turn now to a new class of candidates for "truths": daughter trajectories, discussed by Freedman and Wang. Like so many other truths, this arises in a rather Hegelian way. Thus, with Regge poles, one might have said, Thesis: unitarity bound on cross-sections. Antithesis: particles in the crossed channels with spin larger than one. Synthesis: Regge trajectories, ie., spin a function of ~~recovery~~ energy. One can give a similar dialectic for ~~Similarly with~~ the just discussed AFFR sum rules. In the case of daughter trajectories ~~This time~~ we might say, Thesis: Regge pole behavior. Antithesis: particles of unequal mass Synthesis: daughter trajectories. The point is the following:

Consider, say, s channel π-N scattering near the backward direction. Then one easily calculates

$$u = \frac{(m_N^2 - m_\pi^2)^2}{s} - 2q^2(1+\cos\theta_s) \tag{27}$$

so that at $\theta_s = \pi$, $u = \dfrac{(m_N^2 - m_\pi^2)^2}{s}$

and as θ_s moves forward from π, u goes through zero and becomes negative.

On the other hand, the u channel $\cos\theta_u$ is given by

$$\cos\theta_u = -\left[1 + \frac{2\left(su - (m_N^2 - m_\pi^2)^2\right)}{u^2 - 2u(m_N^2 + m_\pi^2) + (m_N^2 - m_\pi^2)^2}\right] \tag{28}$$

Thus for $0 \leq u \leq (m_N^2 - m_\pi^2)^2/s$,

$\cos\theta_u$ is between $\pm$ 1, irrespective of s. The question, then, is whether the Regge asymptotic form

$$f \sim s^{\alpha(u)} \quad \text{for } s \to \infty, \quad u \sim 0 \tag{29}$$

holds, even though $\cos\theta_u$ is not large.

Freedman and Wang show that the form (29) can hold, but only if for every trajectory $\alpha(u)$ there exists a sequence of daughter trajectories of alternating signature which satisfy

$$\alpha_k(0) = \alpha(0) - k \, , \quad k = 1, \cdots n \tag{30}$$

where $$\tfrac{1}{2} > \alpha(0) - n > -\tfrac{1}{2} \tag{31}$$

if one assumes a background $\sim s^{-1/2}$. Roughly speaking, these poles are there to compensate the singularity at u = 0 of the coefficients of the 1/s expansion of

$$(q_{su}^2)^\alpha \, P_\alpha(\cos\theta_u).$$

Freedman and Wang also explicitly solve a Bethe-Salpeter equation at u = 0 and show the existance of the daughters.

The daughters of the known mesons would be of particular interest, since they would have opposite C to the usual one. Thus the Pomeranchuk daughter would have P = Y = I = 0, C = +, but negative signature, so $J^P = 1^-, 3^-$, etc., would be physical. We would thus have a neutral vector with even C. Similarly the rho daughter would be I = 1 $J^P = 0^+$, and ~~add~~ odd C for the neutral component.

I come now to the last contribution I wish to discuss, by Dashen and Frautschi. These authors observe that in a process like

$$\pi + N \rightarrow \pi + \pi + N$$

the two final π's may be grouped together and considered as a system of definite mass and spin. For a fixed value of the squared mass, M^2, there will be a singularity at $s = \alpha_i(M^2)$ where α_i is the i'th Regge trajectory that couples to the $\pi + \pi$ system.

Dashen and Frautschi then treat the Regge trajectory as a final particle, whose mass they can keep small at will. In fact, the interesting experimental region is just small M^2 (which is usually called t if another particle is attached) for example as shown in Fig. III,

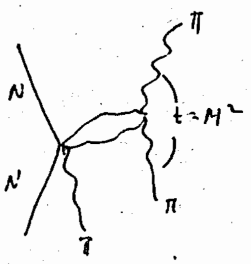

Fig. III

which represents a physical process. The low value of M^2 is useful, since one can use static kinematics, for which the crossing matrix is simple and independent of s. Thus relations like

$$\beta_i(M^2) \cong \sum A_{ij}\, \beta_j(M^2)$$

where A is the relevant crossing matrix,

are written down as before by (a) solving static dispersion relations or (b) better, using super-convergence sum rules on different amplitudes. The β's that are found are not the coupling constants which directly give the decay width for a heavy meson, but the coupling constant of the meson's trajectory at masses such that $M^2 << (1\ \mathrm{BeV})^2$. Specific results are found for spin flip and non-flip couplings of P, V and T trajectories which can be compared directly with experiment. Example: non spin-flip V and T couple to $\bar{B}B$ like F + 1/4 D. ~~non~~ spin-flip: P, V, and T couple to $\bar{B}B$ like D + 2/3 F, and equally to $\bar{B}\Delta$.

Δ good, delta

These results give agreement with Johnson-Treiman relations along the lines of Sawyer's explanation (vector exchange with F coupling.)

In closing let me make two comments: First, these interesting results of Dashen and Frautschi should lead to exact sum rules for inelastic Regge processes in complete analogy with the AFFR super-convergence relations. Second, relevant again to the question of dynamical explanation: the Johnson-Treiman relations, which follow, for example, from $SU(6)_W$, have not been dynamically explained by the above calculation. What has been shown is that the appropriate trajectories, couplings, low lying masses, etc all conspire to exist so that in fact these Johnson-Treiman relations, which are experimentally true, will be consistent with everything else. Why all this happens, we don't yet know. I hope we will know more soon.

We give here a short summary of all the papers which were submitted in advance to the different discussion groups. First, 11.a Analytical Structure of Partial Waves. Warnock has considered the question of the existence of N/D representations for many-channel problems and has proved a theorem analogous to the usual one for the one-channel problem. That is to say, for a T-matrix which is sufficiently bounded at infinite energy, he has shown that there exists an N/D decomposition.

A paper by Franklin studies the different kinematic singularities of partial wave scattering amplitudes. The particular points discussed are threshold zeros, especially the crossed threshold, behavior at s = 0 as related to backward asymptotic behavior as well as spin effects, especially square root of s singularities for bosons and fermions. This paper has some connections with the paper submitted to session 11.c by Freedman and Wang dealing with Regge poles and unequal mass scattering processes. I will discuss the Freedman and Wang paper later.

Next, a paper by Ciulli, Ghika, Stihi, and Visinecu deals with integral equations for one-channel scattering for the generalized Jost function rather than the more usual N/D functions.

Next, there is a paper by Jin which uses analytical properties of the scattering amplitude in the cos θ plane to derive the usually assumed threshold behavior of elastic phase shifts, i.e., Presumably the assumption of no zero energy resonance is implicitly made.

Finally, a paper of Jin and Kang considers the zeros of the partial wave scattering amplitude which is related to the high energy behavior of the amplitude itself.

We come now to session 11.b, Off-Shell Dynamical Equations. Weinberg has reformulated perturbation theory for the S-matrix in a reference frame moving with infinite total momentum. The infinite momentum system appears to have many advantages. Many undesirable diagrams disappear and the contribution of the remaining diagrams can be described by a new set of rules with properties intermediate between those of Feynman diagrams and old-fashioned energy denominator diagrams. The new energy denominators become covariant, Feynman parameters appear naturally. Bogolyubov has considered the question of magnetic moments of bound quarks and has shown that in the Dirac equation with a scalar potential the magnetic moment is characterized, not by the quark mass, but by the quark mass minus the binding energy of the quark. This is apparently not necessarily true for other potentials. Meshcheryakov has considered the question of extrapolating observed S-wave scattering to zero energy in order to obtain the S-wave scattering lengths. He uses an effective range theory based on static kinematic dispersion relations and finds $a_1 - a_3 = 0.305$, $a_1 + 2a_3 = 0$. Wyld has considered a model of S-wave pseudo-scalar meson baryon scattering in which the forces are vector meson exchange. The potential is approximated by a static Yukawa potential and the kinematics is simplified. Actual masses were used so that SU(3) symmetry is broken. The resulting equations are solved on a computer. The model yields a Y_0^* (1405) as a virtual bound state of $\overline{K}$,N and an $N^*_{1/2}$ ~~1/2~~ (1510) as a virtual bound of K, Σ.

Two related papers come next. The first, by Marchesini and Wong, compares on-shell and off-shell methods for the two-body T-matrix in the case of a Yukawa potential where the off-shell equation, that is to say the Schroediger equation, is modified to be consistent with relativistic kinematics. The on-shell equations do not correct the left-hand cut, but simply take the potential itself to give the left-hand cut. The potential is taken to be the sum of an attractive and a repulsive potential with the repulsive range shorter by a factor of two than the attractive range and the repulsive coupling constants somewhat larger. It is found that to reproduce the singlet S_0 scattering length, effective range, and energy at which the phase shift goes to zero, coupling constants that differ by less than 10% in the two methods suffice. On the other hand, in the $N\overline{N}$ system, without the short range repulsion, the two methods give quite different results. The second of these two papers by Son and Sucher compares the results for a simple relativistic two-body Hamiltonian with the numerical results previously obtained by Schwartz for the "corresponding" Bethe-Salpeter equation. They conclude that even in the strong binding limit the pair, multi-meson and retardation effects taken into account by the ladder approximation B-S equation are not very important, at least as far as the relation between coupling constant and binding energy is concerned. A paper by Connille considers the connection between the Marchenko formalism and N/D equations. Finally, Nambu ~~Namda~~ has considered a class of wave equations derivable from Lagrangians which couple an infinite number of tensors or spinors of all ranks. Such a system of equations naturally possesses an infinite number of mass levels and each Eigen-function contains implicitly a built-in form factor.

Next: Session 11.c, S-Matrix Dynamics. Auberson and Wanders have studied nonrelativistic, purely elastic S and P wave two-pole models. They have found that the location of the poles of the partial wave amplitudes is an extremely sensitive function of the input parameters and that certain instabilities exist in this model which for slight differences of input parameters produce unphysical results in the pole location. Wanders has

←— derived new sum rules for pi-pi scattering based on the Mandelstam representation and explicitly on the symmetry between the different channels in the pi-pi problem. These sum rules are physical in the sense that they involve integrals over physically measurable amplitudes. One interesting consequence is that the d-wave scattering length in $\pi_0 \pi_0$ scattering must be positive. Caruthers and Nieto have considered mechanisms for Regge recurrences of inelastic resonances. In particular, transitions of the type $\pi N_i \to \rho N_j$ where i and j represent isobars of the nucleon are shown to give natural dynamical mechanisms for Regge recurrences of inelastic resonances. The pairs of expected resonances D_{13}-G_{17}, D_{15}-G_{19}, S_{11}-D_{15}, are discussed as well as possible resonances in G_{33}-G_{37} and D_{35}-D_{39}. Pasquier and Aitcheson have considered the Khuri-Treiman type amplitudes for three-body production or decay processes with final state interactions. They show that such amplitudes satisfy a form of three-body unitarity in which the three bodies interact in pairs only. Domokos and Palmer have used a dispersion theoretic approach to study systems with large binding energies. In particular, they construct a relativistic degenerate quark model of mesons. The model is unsatisfactory in the sense that while yielding a sufficiently strong binding, the symmetry required for SU(6) invariant vertices in the static limit is destroyed by the relativistic motion of quarks. They conjecture that a realistic model necessarily requires the existence of long-range, super-strong interactions.

Freedman and Wang have studied the question of the proper Regge-representation for high energy backward scattering in a process involving unequal mass particles, for example, backward pion nucleon scattering at high energy. They show that the naively expected power law should be valid even though the cosine of the angle in the cross channel is never large in the immediate neighborhood of the backward direction. The mechanism that makes this possible is the existence of daughter trajectories, of signature $(-1)^k$ relative to the parent, satisfying $\alpha_k = \alpha_0 - k$ at the point u = 0. They verify that the Bethe-Salpeter equation permits such daughter trajectories to exist at the claimed point. They have also found that the behavior of partial wave amplitudes in unequal mass scattering at u = 0 has a singularity $u^{-\alpha}$, where alpha is the leading u channel Regge trajectory. This last result is also contained in Franklin's paper which I discussed earlier. Tiggi has attempted in a practical calculation to take into account the short range potential from distant singularities which arise from some parts of the direct channel strips. He has shown that under some circumstances such effects can be quite important. A paper of Van der Spuy introduces the idea that a field theoretical equation may possess basically nonperturbitive solutions which correspond to a strongly interacting family. Finally, Lovelace has considered the question of uniqueness and symmetry breaking in S-matrix theory and has applied the method of the Frechet differential to this question. The hypothesis of dynamical generation of symmetries can be put to experimental test within the framework of partial wave dispersion relations and fails.

We come now to session 11.d, Bootstrap Models. Cutkosky and Jacobs have studied the connection between two bootstrap models which they call Fermi-Yang and Chew-Low. The connection should exist by virtue of the vertex symmetry but it is hard to exploit this fact within common S-matrix calculational schemes. These authors find, however, that when they look at the Bethe-Salpeter amplitude for a meson as a bound state of a baryon-anti-baryon pair, the singularities which govern the large distant shape of the amplitude lie very close to the baryon mass and are described by the same Landau graphs which pertain to the Chew-Low model. With an off-shell method, therefore, it is possible to exploit the relation between the models. Udgaonkar has constructed a simple closed bootstrap of mesons and baryons in an SU(6) model. This model is based on an Eigen-value equation which not Eig only includes baryon-baryon-meson coupling constants. It is pointed

It thus appears quite possible that local fields are needed for a complete dynamics, but that they are too deeply buried for the S matrix to see them. Let me remind you that in perturbation theory, to every field there corresponds a particle, which is in general <u>not</u> on a trajectory but appears as a pole in one angular momentum state. In contrast, a compound state is supposed to appear on a trajectory. The bootstrap hypothesis is roughly that all particles lie on trajectories. We should not forget that when we leave perturbation theory, there may be no contradiction at all between field theory and the bootstrap hypothesis.

One last word on all this: the bootstrap hypothesis is very beautiful, since it appears to determine all strong interaction parameters. However, it is at present a hypothesis which can be checked only be making sure that <u>all</u> particles are on trajectories, or by satisfying ourselves theoretically that the dynamics is of the bootstrap variety. We are very far from either of these goals.

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Insert II

F. Low
September 6, 1966

One can perhaps crudely characterize the degree of dynamical understanding we have achieved by such model calculations in the following way: They give information about the long range part of the wave-function in the Schroedeyer equation language). They do not usually tell us about the short range potentials which actually control the dynamics.

Session IIa Analytic Structure of Partial Waves

S. Ciulli, Gh. Ghika, M. Stihi and M. Visinescu; The General Solution of the One-Channel Scattering Singular Integral Equations.

Jerrold Franklin: Kinematic Singularities of Partial Wave Scattering Amplitudes

Y.S. Jin and Kyungsik Kang: Zeros of the Partial-Wave Scattering Amplitude

Y.S. Jin: Threshold Behavior of Partial-Wave Scattering Amplitude

M.L. Goldberger and C.E. Jones: Behavior of Eigenphases in the multi-channel Problem

Robert L. Warnock: Existence of many-channel N/D Representation

Session IIb Off-Shell Dynamical Equations

P.N. Bogolubov: Magnetic moment of Bound Quark and model of mesons

Henri Cornille: Connection Between Marchenko Formalism and N/D Equations: Regular Interactions

D.A. Kirzhnits: Differential Equations in the Coupling Constant and Non-renormalizable Interactions

G. Marchesini and D.Y. Wong: Comparisons Between On-shell and Off-shell Methods for the Two-body T-matrix.

V.A. Meshcheryakov: Effective Range Theory of s-wave πN Scattering Based on a General Solution of Chew-Low Equations.

Y. Nambu: Relativistic Equations for Particles with Internal Structure and Mass Spectrum.

J.M. Namyslowski: Relativistic 3-Dimensional 2-Body Integral Equations, an On-shell and Off-shell Formalism

Session IIb (cont.)

Nguyen D. Son and J. Sucher: Bound States of a Relativistic Two-Body Hamiltonian; Comparison with the Bethe-Salpeter Equation

Steven Weinberg: Dynamics at Infinite Momenta

H. W. Wyld, Jr.: S Wave Pseudoscalar Meson Baryon Scattering in a Simple Model with Broken SU(3) Symmetry.

Session IIc S. Matrix Dynamics

I. J. R. Aitchison and R. Pasquier: Three-Body Unitarity and Khuri-Treiman Amplitudes

G. Auberson and G. Wanders. Possible Instability of Partial Wave Amplitudes with Respect to the Left-Hand Discontinuity

P. Carruthers and M. M. Nieto: Dynamical Mechanisms for Regge Recurrences of Inelastic Resonances

G. F. Chew: Survey, mainly of Low Energy Two-Particle Dynamics

G. Domokos and W. Palmer: Dynamics of Strongly Bound Systems

V. Ya. Fainberg: Dispersion Relations

Daniel Z. Freedman and Jiunn-Ming Wang: Regge Poles and Unequal Mass Scattering Processes

→ Keiji Igi: The S-matrix with a Generalized Potential

– C. J. Goebel: Sum Rule for $\pi\pi$ Forward Scattering Amplitude

Session IIc (cont.)

F. Kaschluhn: Dynamical N/D Solutions with Applications to the Chew-Low Theory.

C. Lovelace: Uniqueness and Symmetry Breaking in S-Matrix Theory

S. Mandelstam: Qualitative Discussion of Phase Shifts and Regge Parameters at High Energy

R. Pasquier: Some Analytic Properties in the Total Energy Variable of Khuri-Treiman Type amplitudes

E. Van der Spuy: The Spin Family Solution of a Fundamental Equation

G. Wanders: Sum Rules for π-π Scattering

H. J. Kaiser and F. Kaschluhn: Dynamical Solutions to the Chew-Low Theory. Numerical Results.

Session IId Bootstrap Models

J. Bender, D. Heis, E. Tränkle: Two channel N/D Calculation of the P_{11} Partial Wave of πN Scattering at Low Energies

R. Blankenbecler, D. D. Coon, and S. M. Roy: S-Matrix Approach to Internal Symmetries and Bootstraps

Richard H. Capps: $SU(6)_W$, The Bootstrap, and Tensor Potentials

R.E. Cutkosky and M.A. Jacobs: The Fermi-Yang Model

Roger Dashen and Steven Frautschi: Chew-Low Model for Regge Pole Couplings

C.J. Goebel: Representations of Strong Coupling Groups

C.J. Goebel: Strong Coupling of ~~Pseudo-Scalar~~ Pseudoscalar-meson Octet

C.J. Goebel: Static Model Strong Coupling and Bootstrapping

Virendra Singh: Strong Coupling Theory of Chew-Low Model

B.M. Udgaonkar: A Simple Closed Bootstrap of Mesons and Baryons in an SU(6) Model