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-9 RESEARCH

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POLARIZED PARTICLES REACTIONS[x/]

by

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A B S T R A C T

Statistical tensors of particles which are produced in reactions of the $a+b \to c+d$ or $a \to c+d$ type in the most general case when the incident beam and the target particles are in certain spin states, have been obtained. The work is based on using diagonality of the S-matrix of such reactions with respect to the total enegry, total angular momentum and its projection and on applying consistently Dirac's transformation theory. New "rules of selection" completing the Simon and Welton general rules of selection [1,2] are derived. The first may be considered as generalization of the rule which reads: polarization vector of particles which are produced in a reaction is perpendicular to the reaction plane if the incident beam and the target are unpolarized.

The second reads that statistical tensors of a particle which is produced in a reaction determined with respect to its momentum direction are either purely real or purely imaginary if the incident beam and the target are unpolarized. As a specific case, the decay of an unstable particle into particles with spins 1/2 or 0 has been considered and it has been demonstrated that polarization and angular distribution of these particles depend only upon the spin and the spin state of the decaying particles. — x x x — o

The general theory of nuclear reactions has already been largely extended in [1,2]. This paper puts forward a new method for obtaining statistical tensors (see for definition Appendix I) of particles produced in a reaction. It has been presented in a short and somewhat different manner in [3] (see a congenial method in [4]). This approach allows relativistic generalization

as well as direct generalization for nuclear reactions with more than two particles in final state. Similarly, diagonality of the S-matrix with respect to other physical constants of motion may be used to obtain a number of general properties of quantum-mechanical processes.

The article makes large use of Dirac's formalism [5] (see chapters I-IV, esp. § 17).

§ 1. Reactions of the $a+b \to c+d$ and $a \to c+d$ type are discussed. a, b, c, d denote either "elementary" particles or nuclei with spins i_a, i_b, i_c, i_d respectively. The spin is treated in the Pauli approximation and in this sense the consideration will be non-relativistic (in particular, particles with rest mass equal to zero are not discussed).

Knowing the initial (before the reaction) state of the system and presuming that the elements of the S-matrix in corresponding representation are known, we may obtain the wave function of the final state [1]:

$$\psi'_{\eta\xi_0} = (\eta' | S | \xi)\, \psi_{\xi\xi_0} \qquad (1.I)$$

Integration on summation over ξ is implied. The indices ξ and η mean a complete set of quantities describing the state of the system (species of particle, their momenta, etc.; see a complete set for a two-particle system below). $\psi'_{\eta\xi_0}$ is the amplitude of the probability that in final state quantities η take values η' if the system was originally in ξ_0-state. [5]

However, the inital and final states of the reaction are

1/ See the next page for this footnote

generally statistical mixtures (statistical ensembles or mixed ensembles) of pure spin states and so should be described by a density matrix and not by wave functions.

For example, an unpolarized beam of particles striking the target is described by an assembly with equal probabilities of all possible orientations of the spin. We define the density matrix as follows:

$$(\xi_1|\rho|\xi_2) = \sum_\alpha P_\alpha \, {}_{\xi_1}\psi_\alpha \cdot {}_{\xi_2}\psi_\alpha^* \tag{1.2}$$

where P_α denotes weights of separate pure states.

With the help of (1.1) we will find that the density matrix of the final state is equal to:

$$(\eta_1'|\rho'|\eta_2') = (\eta_1'|S|\xi_1)(\eta_2'|S|\xi_2)^*(\xi_1|\rho|\xi_2) = (\eta_1'|S\rho S^+|\eta_2') \tag{1.3}$$

1/ The relationship (1.1) is usually written as applied to reactions of the $a+b\to c+d$ type. In this case $\hat{S} = \hat{u}(\infty, -\infty)$ where the operator $\hat{u}(t, t_0)$ for example, satisfies the Schrödinger equation (in the interaction representation) $ih\frac{\partial}{\partial t}\hat{u}(t,t_0) = \hat{H}_{int.}\hat{u}(t,t_0)$ [6]. But if the particle a is long-lived enough to enable us to speak roughly about a definite (quazi-stationary) initial state (with definite energy, etc.), (1.1) may also be written for the reaction $a\to c+d$, with $\hat{S} = \hat{u}(\infty, 0)$. The time counting begins from the moment of unstable particle creation.

The diagonal elements $(\eta' | \rho' | \eta')$ provide the probability that after reaction the system will be in the η'- state if the initial state was characterized by the density matrix (1.2).

Further we shall use instead of the S -matrix the scattering matrix $\hat{R} = \hat{S} - \hat{1}$, all elements of which coincide with those of the S -matrix excepting transitions without change of state.

As is known, the total angular momentum J , its projection M , the total momentum $\vec{P}$ and the total energy E of an isolated system are conserved. This fact expresses itself in diagonality of the system S -(or R -) matrix with respect to these quantities. Generally speaking, $J, M, \vec{P}$ and E cannot, however, belong to the same complete set. It is necessary therefore, in a general case, after expressing diagonality of the R -matrix elements with respect to J, M, E in the representation of a set including J, M, E to express then through these elements the matrix elements of the R-matrix in the representation of another complete set including $\vec{P}$ and to write out diagonality with respect to $\vec{P}$. Rather cumbersome unitary transformations would be required to carry out practically these operations.

But there is a coordinate system in which we can utilize diagonality of the R-matrix with respect to J, M, E and $\vec{P}$ at a time. Namely, if we choose a Lorentz frame of reference with $P^2 = 0$.

to describe a two-particle system , the set including J, M and E may also include: i_c and i_d – spins of the two particles; their sum s; l – summarized orbital momentum of the particles with respect to the centre-of-gravity [2]; L – orbital momentum of the whole system with

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2/ In the rest system $\hat{l} = [(\hat{r}_c - \hat{R}_c) \cdot \hat{p}_c] + [(\hat{r}_d - \hat{R}_c) \cdot \hat{p}_d] = [(\hat{r}_c - \hat{R}_c) \cdot \hat{p}_c] + [(\hat{r}_d - \hat{R}_c) \cdot (-\hat{p}_c)] = [(\hat{r}_c - \hat{r}_d) \cdot \hat{p}_c]$. $\hat{p}_c = -\hat{p}_d$ are momentum operators of the particles c and d in the rest system.

respect to the origin of the coordinate system (see below), total orbital momentum $\mathcal{L}$ ($\hat{\mathcal{L}} = \hat{L} + \hat{\ell}$), module of the total momentum $\vec{P}$ (equal to zero) of the system [3]. As the operator $\hat{L}$ can be expressed as $[\hat{R}_c \cdot \hat{P}]$ where $\hat{R}_c$ is a center-of-gravity operator, in states where $\vec{P} = 0$ we have too $L = 0$ (we consider just such states of the system in which $|\vec{P}| = 0$ and, consequently, $\vec{P} = 0$)

As P^2 is conserved, L is always zero and the total orbital moment of the system is equal to ℓ.

In the representation of this complete set for a reaction, $a + b \to c + d$, for example, we have

$$(i_c\, i_d\, s'\ell'\mathcal{L}\mathcal{L}P'J'M'E'\alpha' \,|\, R \,|\, i_a\, i_b\, s\ell L\mathcal{L}0JME\alpha) =$$

$$= (i_c\, i_d\, s'\ell'0\ell'\alpha' \,|\, R^{0JE} \,|\, i_a\, i_b\, s\ell 0\ell\alpha)\, \delta(\vec{P}' - \vec{0})\, \delta_{J'J}\, \delta_{M'M}\, \delta(E' - E) \qquad (1.4)$$

The superscripts denote dependence of the diagonal elements upon P, J and E; it may be shown that they do not depend upon M. Further we will not write in corresponding places the indices of the spins of the particles i_c, i_d, etc. as well as those of the total momentum and quantities L and $\mathcal{L}$ either in the R-matrix elements or in the density matrix (in other words, the motion of the system as a whole, i.e. the fact that it is at rest in a chosen Lorentz system, will not be described).

In representation of the compete set s, ℓ, J, M, E, α (13) may be written, if (1.4) is used, as follows:

3/ Note that a complete set must also include, generally speaking, particle masses. For short, they are not written in a manifest way. Besides, the complete set may include a number of variables, as internal parties of the two particles. All of them are denoted with α.

it is convenient to retain them in order to carry out the summation over thise indices. For this purpose let us use the following formula:

$$\sum_{m_1, m_2, \mu_1, \mu_2} C^{s_1 \sigma_1}_{j_1 m_1 \ell_1 \mu_1} \cdot C^{s_2 \sigma_2}_{j_2 m_2 \ell_2 \mu_2} C^{j m_j}_{j_1 m_j j_2 - m_2} \cdot C^{\ell m_\ell}_{\ell_1 \mu_1 \ell_2 - \mu_2} =$$

$$= (-1)^{s_2 - j_2 - \ell_2} [(2s_1+1)(2s_2+1)(2j+1)(2\ell+1)]^{1/2} \cdot$$

$$\cdot \sum_{g, m_g} C^{g m_g}_{s_1 \sigma_1 s_2 - \sigma_2} \cdot C^{g m_g}_{j m_j \ell m_\ell} \cdot X(j_1 j j_2;\ s_1 g s_2;\ \ell_1 \ell \ell_2) \qquad (2.3)$$

(2.3) may be derived with the help of relations (1), (18) and (19) in [9] and also from formula (3) in [10], the coefficients X are defined in [10] and [1]. With the help of (2.3) we may carry out in (2.1) at once the sum over m_e, m_c', m_d, m_d' of the product of the first, second, third and the last Clebsch-Gordan coefficients (note that $-m_c' - m_d' = -m_2'$). After that we may carry out in (2.1) the sum over $m_1', m_2', \mu_1', \mu_2'$ (again a sum of the (2.3) type) and, finally, we obtain from (2.1):

$$\rho'(\vec{n}_e, p_e, \alpha';\ q_e \nu_e q_d \nu_d) = N_1' \cdot N_2' (4\pi)^{-1/2} \cdot [(2i_e+1)(2i_d+1)]^{1/2} \cdot$$

$$\times \sum (\vec{n}_e | L' m_L') \cdot C^{J'M'}_{q'\nu' L' m_L'} \cdot C^{q'\nu'}_{q_e \nu_e q_d \nu_d} \cdot C^{L'0}_{\ell_1' 0 \ell_2' 0} \cdot (-1)^{J_2' - M_2'} \cdot C^{J'M'}_{J_1' M_1' J_2' - M_2'} \times$$

$$\times [(2s_1'+1)(2s_2'+1)(2q_e+1)(2q_d+1)]^{1/2} \cdot X(i_e q_e i_e';\ s_1' q' s_2';\ i_d q_d i_d') \times$$

$$\times [(2\ell_1'+1)(2\ell_2'+1)(2J_1'+1)(2J_2'+1)(2q'+1)]^{1/2} \cdot X(s_1' q' s_2';\ J_1' J' J_2';\ \ell_1' L' \ell_2') \times$$

$$\times i^{-\ell_1' - \ell_2'} \cdot (s_1' \ell_1' J_1' M_1' E_1' \alpha' | \rho' | s_2' \ell_2' J_2' M_2' E_2' \alpha') \qquad (2.4)$$

The sum $\sum$ is over $s_1', s_2', \ell_1', \ell_2';\ J_1', J_2', q', L', J'$ and over $\nu', m_L', M_1', M_2', M'$. $N_{1,2}' = 2\sqrt{2}\pi h \cdot [R/V]^{1/2} \cdot p_e^{-1} (p_e | E_{1,2}')$ (see Appendix II).

Formulae (2.1) and (2.4) are written in the rest system, the directions of the axes have been chosen arbitrarily so far. For a beam of particles the most evident distinguished direction in space is the direct-

ion of the beam. Let us introduce the s.tensors $\rho'(\vec{n}_c, p_c, \alpha'; q_c \tau_c q_d \tau_d)$ with indices τ related to the $\vec{n}_c$ as to a quantization axis. Their relation to the former s. Tensors may be written as follows[4]:

$$\rho'(\vec{n}_c, p_c, \alpha'; q_c \tau_c q_d \tau_d) = \sum_{\nu_c, \nu_d} \mathcal{D}^{q_c}_{\tau_c \nu_c}(-\pi, \vartheta_c, \pi - \varphi_c) \times$$

$$\times \mathcal{D}^{q_d}_{\tau_d \nu_d}(-\pi, \vartheta_c, \pi - \varphi_c) \cdot \rho'(\vec{n}_c, p_c, \alpha'; q_c \nu_c q_d \nu_d) \qquad (2.5)$$

$\pi - \varphi_c, \vartheta_c, -\pi$ are the Euler angles[4] of such a rotation g_c of the zyx axes that the axis z should coincide with the unit vector $\vec{n}_c$ and the axis y should be perpendicular to the former direction of the axis z and to the $\vec{n}_c$. The rotation inverse to $g_c = \{-\pi, \vartheta_c, \pi - \varphi_c\}$ is written as $g_c^{-1} = \{\varphi_c, \vartheta_c, 0\}$. Note that the spherical angles are defined with respect to the former axes zyx as before, only a new quantization axis for the spin indices has been introduced.

4/ The indices ν (or τ) of the s.tensors are bra indices (see Appendix I). $\rho(q,\nu)$ are therefore transformed under three-dimensional rotations as $Y^*_{q\nu}(\theta, \phi) = (q\nu | \theta\phi)$. Formula (2.5) is simply derived by means of complex conjugation of the formula for spherical function transformation which we write as follows

$$Y_{q\tau}(\vec{n}') = \sum_{\nu} Y_{q\nu}(\vec{n}) \mathcal{D}^{q}_{\nu,\tau}(g^{-1})$$

if the unit vector transformation under a rotation g is written as $\vec{n}' = \hat{g}\vec{n}$. If the rotation g is interpreted as a rotation of the (right-hand) coordinate system, it may be given by the Euler angles φ_1 (rotation about the axes z), ϑ (rotation about the axis y') and φ_2 (rotation about the axis z'). All rotations clockwise. Let us emphasize that the Euler angle ϑ is usually defined as a rotation about the axis x'; but only with our definition the above formula will be true. $\mathcal{D}^{\ell}_{m,n}(\varphi_2, \vartheta, \varphi_1) = \exp(-im\varphi_2) i^{n-m} P^{\ell}_{mn}(\cos\vartheta) \exp(-in\varphi_1)$ the functions $P^{\ell}_{m,n}$ are computed in [11]. Ibid. see for a definition of

Substituting in (2.5) for $\rho'(\vec{n}_c, p_c, \alpha'; q_c \nu_c q_d \nu_d)$ its expression (2.4), we may carry out, first of all, the summation over ν_c and ν_d

$$\sum_{\nu_c, \nu_d} \mathcal{D}^{q_c}_{\tau_c \nu_c}(g) \cdot \mathcal{D}^{q_d}_{\tau_d \nu_d}(g) \cdot C^{q' \nu'}_{q_c \nu_c q_d \nu_d} = \mathcal{D}^{q'}_{\tau_c + \tau_d, \nu'}(g) \cdot C^{q' \tau_c + \tau_d}_{q_c \tau_c \, q_d \tau_d} \qquad (2.6)$$

Let us introduce a notation $\tau' = \tau_c + \tau_d$. Using the same formula (2.6) [1], we can carry out the sum

$$\sum_{\nu', m'_L} Y_{L' m'_L}(\vec{n}_c) \cdot \mathcal{D}^{q'}_{\tau', \nu'}(-\pi, \vartheta_c, \pi - \varphi_c) \cdot (q' L' \nu' m'_L \mid q' L' J' M')$$

If we use the relation $Y_{\ell n}(\vartheta, \pi - \varphi_\ell) = [2\ell + 1/4\pi]^{1/2} \cdot \mathcal{D}^{\ell}_{0, n}(\varphi_2, \vartheta, \varphi_1)$

The final result will be the following:

$$\rho'(\vec{n}_c, p_c, \alpha'; q_c \tau_c q_d \tau_d) = N'_1 \cdot N'_2 \, (4\pi)^{-1} [(2i'_c + 1)(2i'_d + 1)]^{1/2} \times$$

$$\times \sum \mathcal{D}^{J'}_{\tau', M'}(-\pi, \vartheta_c, \pi - \varphi_c) \cdot C^{q' \tau'}_{q_c \tau_c q_d \tau_d} (-1)^{q' + \tau'} (-1)^{J'_2 - M'_2} \cdot C^{J' M'}_{J'_1 M'_1 J'_2 - M'_2}$$

$$\cdot [(2q_c + 1)(2q_d + 1)]^{1/2} \cdot X(i_c q_c i'_c ; s'_1 q' s'_2 ; i_d q_d i_d) \times$$

$$\times G^{*}_{\tau'}(J'_1 \ell'_1 s'_1 ; J' \; q' ; J'_2 \ell'_2 s'_2) \cdot (s'_1 \ell'_1 J'_1 M'_1 E'_1 \alpha' \mid \rho' \mid s'_2 \ell'_2 J'_2 M'_2 E'_2 \alpha') \qquad (2.7)$$

where the coefficient $G_{\tau'}$ has been defined in [2] formula (3.3); the sum is over $s'_1, s'_2, \ell'_1, \ell'_2, q', J'_1, J'_2, J'$ and over M'_1, M'_2, M'

The expressions of the $(s_1 \ell_1 J_1 M_1 E_1 \alpha_1 \mid \rho \mid s_2 \ell_2 J_2 M_2 E_2 \alpha_2)$ through $\rho_{\alpha_1 \alpha_2}(\vec{n}_a, p_a ; q_a \nu_a q_b \nu_b)$ or $\rho_{\alpha_1, \alpha_2}(\vec{n}_a, p_a ; q_a \tau_a q_b \tau_b)$ are similarly derived. We will call them formulae (2.8) and (2.9), respectively. Leaving out of account the change of notation, let us note that corresponding transformation functions are simply obtained by means of complex conjugation of the transformation functions in (2.4) and (2.7) (i.e. by means of complex conjugation of all coefficients preceding $(s'_1 \ell'_1 J'_1 M'_1 E'_1 \alpha' \mid \rho' \mid s'_2 \ell'_2 J'_2 M'_2 E'_2 \alpha')$ in (2.4) and (2.7)) and substitution of $[(2i_a + 1)(2i_b + 1)]^{-1/2}$ for $[(2i'_c + 1)(2i'_d + 1)]^{1/2}$ (see (I.3) and (I.4)).

Combining (2.4), (1.5) and (2.8), we obtain a first final formula (we will call it formula (2.10): all spin indices and spherical angles relate to an abritrarily chosen system of axes. Since $J_1' = J_1$; $J_2' = J_2$, $M_1' = M_1$, $M_2' = M_2$ the sum over M_1 and M_2 is easily performed: $\sum_{M_1, M_2} (J_1 J_2 M_1 -M_2 | J_1 J_2 J'M')(J_1 J_2 M_1 -M_2 | J_1 J_2 JM) = \delta_{J'J} \delta_{M'M}$ and that is why $J' = J$, $M' = M$ too. Besides, $(p_c | E_1)(E_1 | p_a) = (p_c | E_2)(E_2 | p_a) = (p_c ; p_a)$ where the symbol $(p_c ; p_a)$ expresses the law of energy conservation in the momentum representation. With this we exhaust all transformation and formula (2.10) may easily be written in its final form; it is very bulky (see page 26)

Combining (2.7), (1.5) and (2.9) results in a second final formula - (2.11). If we substitute in formula (8) in [3] the normalization factor $N/(4\pi)^2$ (see Appendix II) for $\chi_a^2/4$ and $\sum_M \mathcal{D}^J_{\tau' M}(-\pi, \vartheta_c, \pi - \varphi_c) \cdot \mathcal{D}^{J*}_{\tau M}(-\pi, \vartheta_a, \pi - \varphi_a) = \mathcal{D}^J_{\tau', \tau}(g_c g_a^{-1})$ for $\mathcal{D}^J_{\chi, \chi'}(\varphi_n, \vartheta_n, 0)$ (this is necessary because of changing the definition of s.tensors: cf. formulae (5) in [3] and (II.3)) then the grouping of formulae (7), (8)' and (9) in [3] will give exactly the formula (2.11).*/

It will follow from (2.12), as may be shown, that the s. tensors of the final state depend, in fact, upon the parameters of the rotation $g_c g_a^{-1}$ of the system of $z_a y_a x_a$ axes which are defined by the initial state (the axis $z_a \parallel \vec{n}_a$, the direction of the y_a axis may be defined by the spin state; for example, it may be directed along that polarization vector component which is perpendicular to the $\vec{n}_a$) towards and up to the system of axes $z_c y_c x_c$ (the axis $z_c \parallel \vec{n}_c$, the axis $y_c \parallel [\vec{n}_a \times \vec{n}_c]$). Thus, all quantities in (2.11) may be defined with respect to several physical directions of the reaction so that for (2.11) we need not introduce any auxiliary coordinate system. In the coordinate

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*/ There is a misprint in (8) [3] : the square bracket $[(2s_1'+1)(2s_2'+1)(2s_1+1)(2s_2+1)]$ must be raised to the $-1/2$ power, not in the $+1/2$ one.

system currently adopted which coincides with the system of axes $z_a\, y_a\, x_a$ $\quad q_c q_a^{-1} = \{-\pi, \vartheta, \pi-\varphi\}$ where ϑ, φ are spherical angles of the unit vector $\vec{n}_c$ in such a coordinate system.

If the spin state of the incident beam a and the target particles b is completely unpolarized or has the axial symmetry

$$\rho(q_a \tau_a q_b \tau_b) = \rho(q_a 0\, q_b 0) \cdot \delta_{\tau_a 0} \cdot \delta_{\tau_b 0}$$

then after performing the sum over τ in (2.11) there will remain only $\mathcal{D}^J_{\tau' 0}(-\pi, \vartheta, \pi-\varphi) = P^J_{\tau' 0}(\cos\vartheta)$ and so statistical tensors of the final state (including the angular distribution $\rho'(\vec{n}_c, p_c, \alpha'; 0,0,0,0)$)will not depend upon φ. This is natural as in this case the choice of the axis y_a is quite arbitrary and no physical quantity can depend upon such a choice.

By means of a procedure analogous with that used for formula (2.11) we may get a general formula for the reaction of the $a \to c + d$ type:

$$\rho'(\vec{n}_c, p_c, \alpha'; q_c \tau_c q_d \tau_d) = N_0^2 (2\pi)^{-1} [(2i_c+1)(2i_d+1)]^{1/2} \cdot (2s+1)^{-1/2} \cdot$$
$$\cdot \sum (-1)^{q'+\tau'} \cdot C^{q'\tau'}_{q_c \tau_c q_d \tau_d} \cdot [(2q_c+1)(2q_d+1)]^{1/2} \cdot \chi(i_c q_c i_c; s_1' q' s_2'; i_d q_d i_d) \cdot$$
$$\cdot G_{\tau'}(s l_1' s_1'; q \quad q'; s l_2' s_2') \cdot (s_1' l_1' \alpha' | R^{SE} | \alpha_1) \cdot (s_2' l_2' \alpha' | R^{SE} | \alpha_2)^* \cdot$$
$$\cdot \mathcal{D}^q_{\tau', \nu}(-\pi, \vartheta_c, \pi-\varphi_c) \cdot \rho_{\alpha_1, \alpha_2}(q, \nu) \qquad (2.13)$$

The sum is over $q', \tau', s_1', s_2', l_1', l_2', q, \nu$. $N_0 = 2\pi h [R/V]^{1/2} \cdot p_c^{-1} \cdot (p_c | E)$. $\rho(q,\nu)$ are s.tensors of the particle a, S is its spin, E its total energy.

The symbol $(p_c | E)$ equals a unity if p_c is the root of the equation $\sqrt{p_c^2 c^2 + m_c^2 c^4} + \sqrt{p_c^2 c^2 + m_d^2 c^4} = E = m_a \cdot c^2$

and is zero if ρ_c does not satisfy this equation (see Appendix II)

Let us examine a case when the particles a and b (or/and c and d) are identical. Let the momenta $\vec{p}_1$ and $\vec{p}_2$ of the particles and their spin projections m_1 and m_2 be variables of the complete set ξ_1 (and ξ_2) in the definition (1.2) of the density matrix. It is necessary then that

$$(\xi_1|\rho|\vec{p}_1, p_2, m_1, m_2) = (-1)^{2i}(\xi_1|\rho|\vec{p}_2, \vec{p}_1, m_2, m_1) \tag{2.14}$$

i.e. the density matrix elements with a fixed ξ_1 should either change their sign, if the identical particles have a half-integral spin, or should not change at all for Bose-particles, depending on the fact whether we ascribe to the "first" particle (a) a momentum $\vec{p}_1$ and a spin projection m_1 and to the "second" particle (a) a momentum $\vec{p}_2$ and projection m_2 or we ascribe to the "first" particle $\vec{p}_2$ and m_2 and to the "second" $\vec{p}_1$, m_1 . Independently, the same must hold for the indices ξ_1 if ξ_2 are fixed. Substituting the total momentum $\vec{P}$ and the rest system momentum $\vec{p}$ for $\vec{p}_1$ and $\vec{p}_2$, (2.14) may be re-written as follows: $(\xi_1|\rho|\vec{P}, \vec{p}, m_1, m_2) = (-1)^{2i}(\xi_1|\rho|\vec{P}, -\vec{p}, m_2, m_1)$

Taking into account that $(\xi_1|\rho|-\vec{p}, i, i, m_2, m_1) = (\xi_1|\rho|\ell u\, iism)_{\pi} \cdot (\ell\mu p|\pi-\vartheta, \varphi+\pi, p)\cdot(iism|iim_2 m_1) = (\xi_1|\rho|\ell\mu p\, iism)(-1)^{\ell}\cdot(\ell\mu p|\vartheta,\varphi,p)\cdot (-1)^{s-2i}(iism|iim_1 m_2)$

(the index $\vec{P}$ is omitted), we obtain that the density matrix element symmetrized with respect to the righthand indices (i.e. satisfying (2.14)) may be written in this way:

$$(\xi_1|\rho|\vec{p}, m_1, m_2)_{symm.} = 1/\sqrt{2}\,[(\xi_1|\rho|\vec{p}, m_1, m_2) + (-1)^{2i}(\xi_1|\rho|-\vec{p}, m_2, m_1)] =$$
$$= 1/\sqrt{2}\,(\xi_1|\rho|\ell\mu p s m)[1 + (-1)^{\ell+s}]\cdot(\ell\mu p|\vartheta,\varphi,p)\cdot(iism|iim_1 m_2) \tag{2.15}$$

That is why, if the particles a and b are identical, in formulae (2.10), (2.11), besides equalizing appropriate indices ($i_a = i_b = i$ etc), it is necessary to insert in the sum the factor

$$1/2\,[1 + (-1)^{\ell_1+s_1}]\cdot[1 + (-1)^{\ell_2+s_2}]$$

An analogous factor (with ℓ and S prime) should be inserted into (2.4), (2.7), (2.11), etc., if c and d are identical.

§ 3. In general the final state of a $a+b\to c+d$ reaction is described by $(2i_c+1)+(2i_d+1)$ s.tensors (the total number of their components is $(2i_c+1)^2+(2i_d+1)^2$). In turns out, however, that an appropriate choice of the quantization axis in case of a completely unpolarized initial state considerably reduces the number of final state s.tensor components requiring knowledge of the matrix elements (1.4) for their computation.

As usual, the axis Z of the coordinate system in which a formula of the (2.10) type is written is directed along the $\vec{n}_a$. Let us direct it perpendicularly to the reaction plane, i.e. along the vector $[\vec{n}_a\times\vec{n}_c]$. In other words, for each particular case of reaction an axis Z of its own is chosen, the axis X being directed along the $\vec{n}_a$ so that the direction of the incident beam will always be described in the same way $\vartheta_a=\pi/2$, $\varphi_a=0$ 5/. In this case the angle φ_c is one between the direction of the momentum $\vec{p}_c$ and the incident beam direction.

For what will ensue we shall need the following factors in the summand of the sum of Formula (2.10) in the chosen coordinate system:

$$C^{JM}_{q'v'L'm'_L}\cdot C^{JM}_{qvLm_L}\cdot C^{L'0}_{e'_1 0 e'_2 0}\cdot C^{L0}_{e_1 0 e_2 0}\cdot Y_{L'_1 m'_L}(\pi/2,\varphi_c)\cdot Y^{*}_{L_1 m_L}(\pi/2,0) \tag{3.1}$$

5/ S. tensors of the spin state of the beam and the target, however, will generally be dependent upon the choice of Z axes. They may be expressed through s.tensors related, for instance, to the $\vec{n}_a$, as to a quantization axis, according to the formulae of § 2 (of the (2.5) type).

It follows from the properties of the coefficients $C^{L'0}_{\ell'0\ell'_2 0}$ and $C^{L0}_{\ell_1 0 \ell_2 0}$ and from the law of conservation of total space parity that $\ell_1' + \ell_2' + L'$, $\ell_1 + \ell_2 + L$ and $\ell_1' + \ell_2' + \ell_1 + \ell_2$ should be even numbers, Consequently, $L' + L$ is even too. (This fact is a certain rule of selection independent of the choice of coordinate system).

Owing to the properties of the associated Legendre polynomials of the first kind, the functions $Y_{\ell\mu}(\pi/2, \varphi)$ do not vanish only when $\ell + \mu$ is even. In (2.10) therefore only those terms do not vanish in which $L' + m_{L'}$, $L + m_L$ and, consequently, $L' + L + m_{L'} + m_L$ are even. So $m_L + m_{L'}$ must be even too. Owing to the properties of the first two Clebsch-Gordan coefficients we have $\nu' + m_{L'} = \nu + m_L$, hence $\nu' - \nu = m_L - m_{L'}$ As ν', ν, m_L and $m_{L'}$ are integers, the rule of selection we have found out may be formulated as follows: "$\nu' + \nu$ must be an even number, if the quantization axis is chosen perpendicular to the reaction plane". In an important particular case when the incident beam and the target are completely unpolarized (in all coordinate systemes all $\rho(\vec{n}_a, \rho_a; q_a \nu_a\ q_b \nu_b)$ are zero, excepting $\rho(\vec{n}_a, \rho_a; 0, 0, 0, 0)$) this rule reads that only $\rho'(\vec{n}_c, \rho_c; q_c \nu_c q_d \nu_d)$ with even $\nu_c + \nu_d$ do not vanish.

In particular, the fact that $\rho'(\vec{n}_c, \rho_c; 1, \pm 1, 0, 0)$ and $\rho'(\vec{n}_c, \rho_c; 0, 0, 1 \pm 1)$ are equal to zero means that the polarization vector of each of the particles c and d is perpendicular to the reaction plane if the incident beam and the target are unpolarized. [1].

If the initial state in formula (2.11) is considered to be unpolarized and the usual coordinate system (axis $Z \parallel \vec{n}_a$) is chosen, we may derive one more rule of selection. In this case $\rho'(\vec{n}, \rho; q_c \tau_c 00)$ (or $\rho'(\vec{n}, \rho; 00 q_d \tau_d)$) dependence upon τ_c is determined

by the following factors in the summand of the sum (see (8)*/ in [3] with $\mathcal{D}^{J}_{\tau_c,0}(-\pi,\vartheta,\pi-\varphi)$ substituted for $\mathcal{D}^{J}_{\lambda,\lambda'}(\varphi_1,\vartheta_1,0)$, see § 2):

$$(-1)^{\tau_c} G_{\tau_c}(J_1 l_1' s_1'; J \quad q_c; \; J_2 l_2' s_2') \cdot \mathcal{D}^{J}_{\tau_c,0}(-\pi,\vartheta,\pi-\varphi)$$

Taking into account the expression (3.3) in [2] for G_{τ_c} and the formula $\mathcal{D}^{J}_{\tau,0}(\varphi_2,\vartheta,\varphi_1)=[4\pi/2J+1]^{1/2}\cdot(-1)^{\tau}\cdot Y_{J,\tau}(\vartheta,\pi-\varphi_2)$ we get that this dependence upon τ_c is finally due to the factors $(q_c J \tau_c -\tau_c | q_c J L' 0)\cdot Y_{J,\tau_c}(\vartheta,0)$

Similarly, $\rho'(\vec{n}_c,\rho_c;\, q_c -\tau_c, 0, 0)$ depends upon $-\tau_c$ as do the factors $(q_c J -\tau_c \tau_c | q_c J L' 0)\cdot Y_{J,-\tau_c}(\vartheta,0)$.

The law of conservation of space parity of a system and presence of the coefficients $(l_1' l_2' 0 0 | l_1' l_2' L' 0)$, $(l_1 l_2 0 0 | l_1 l_2 L 0)$ which are contained in G_{τ_c} and G_0^* entail that the

$l_1+l_2+l_1'+l_2'$, $l_1'+l_2'+L'$; l_1+l_2+J

are even numbers. Hence $J+L'$ (and, of course, $J-L'$) must be even too. Consequently, $(q_c J -\tau_c \tau_c | q_c J L' 0) = (-1)^{q_c+J-L'}\cdot(q_c J \tau_c -\tau_c | q_c J L' 0) = (-1)^{q_c}\cdot(q_c J \tau_c -\tau_c | q_c J L' 0)$.

Considering that $Y_{J,-\tau_c}(\vartheta,0)=(-1)^{\tau_c}\cdot Y_{J,\tau_c}(\vartheta,0)$

we obtain $\rho'(\vec{n},\rho;\, q_c,-\tau_c,0,0)=(-1)^{q_c+\tau_c}\cdot\rho'(\vec{n},\rho;\, q_c,\tau_c,0,0)$

Taking into account also the Hermitian property of s.tensors[6], we finally infer that if the initial state is completely unpolarized, $\rho'(\vec{n},\rho;\, q_c,\tau_c,0,0)$ are real when q_c is even and purely imaginary when q_c is odd.[7] This fact simplifies the problem of find-

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6/ It is easy to make out that the Hermitian property of the density matrix $(m_1|\rho|m_2)^*=(m_2|\rho|m_1)$ corresponds to the following property of s.tensors: $\rho^*(q,\nu)=(-1)^{\nu}\rho(q,-\nu)$ (see (I.3))

7/ This selection rule may be generalized to a certain extent: contribution to $\rho'(\vec{\rho};q,\tau,0,0)$ from each s.tensor $\rho(\vec{n}_a,\rho;\, q_a,0,q_b,0)$ is real, if $q+q_a+q_b$ is even, and purely imaginary if $q+q_a+q_b$ is odd.

ing out the spin state of the system c, d in almost the same degree as the preceding selection rule does.

In particular, if the particle c is unstable, the above rules of selection enable us to simplify to a certain extent the description of the initial state of its decay reaction without hypothesizing the mechanism of its creation.

Analogous rules of selection may be obtained for the reactions directing the axis Z perpendicularly to the $\vec{n}_c$ and along the $\vec{n}_c$, respectively, and considering the particles a as completely unpolarized.

§ 4. Let us obtain some consequences following from formula (2.13) for a $a \to c + d$ reaction in a particular case when $i_c = 1/2$, $i_d = 0$, of which Λ^0-decay is an important example: $\Lambda^0 \to p + \pi^-$.

First, when $i_d = 0$ $X(i_c q_c i_c; s_1' q' s_2'; 0\, q_d\, 0) = (2i_c+1)^{-1} \cdot (2q_c+1)^{-1/2} \cdot \delta_{q_d 0} \cdot \delta_{q_c q'} \cdot \delta_{s_1' i_c} \cdot \delta_{s_2' i_c}$. Second, if $i_c = 1/2$, then from the triads of the $G_{\tau'}(s, l_1', 1/2; q\ \ q_c; s l_2' 1/2)$ in (2.13) we get $l_1' = s \pm 1/2$, $l_2' = s \pm 1/2$, i.e. the difference between l_1' and l_2' can not be more than a unity. As the R-matrix elements in (2.13) do not vanish only for transitions with conservation of space parity, l_1' and l_2' must have the same parity. We find that $l_1' = l_2' = l' = s + 1/2$ or $l_1' = l_2' = l' = s - 1/2$ depending upon the unknown parity of the particle a and the parities of c and d. It is assumed hereafter that the state of an assembly of particles with respect to the variables α[3] is pure one (i.e. $\rho_{\alpha_1, \alpha_2}(q, \nu) = \rho(\alpha; q\, \nu) \cdot \delta_{\alpha, \alpha_1} \cdot \delta_{\alpha, \alpha_2}$).

Then only one element of the R-matrix in (2.13) does not vanish and if there is no other alternative way for decay, from the S-matrix unitarity follows that:

$$(\tfrac{1}{2}0\tfrac{1}{2}\ell'\alpha'|R^{sE}|\alpha)\cdot(\tfrac{1}{2}0\tfrac{1}{2}\ell'\alpha'|R^{sE}|\alpha)^{*}=1 \qquad (4.1)$$

If there are other schemes of decay, ($a\to c'+d'$) total probability w ($0<w<1$) of the decay by the scheme $a\to c+d$ should be put in place of the unity on the right side of equation (4.1).

Now we integrate $\rho'(\vec{n}_c, p_c, \alpha'; q_c, \tau_c, 0, 0)$ with the weight factor $V/(2\pi h)^3$ (see Appendix II) over the momentum region $(\vec{p}_c, \vec{p}_c+\Delta\vec{p})$. It is implied that this region contains momenta $\vec{p}_c$ for which $(p_c/E)=1$. Denoting the intergarated quantities divided by the solid angle of this region by $T^{q_c}_{\tau_c}(\vartheta,\varphi)$ (index α' is omitted) we get from (2.13):

$$T^{q_c}_{\tau_c}(\vartheta,\varphi)=\frac{w(2s+1)^{-1/2}}{4\sqrt{2}\cdot\pi}\cdot\sum(-1)^{q_c+\tau_c}\cdot G_{\tau_c}(s\ell'1/2;q\ \ q_c;s\ell'1/2)\cdot\mathcal{D}^{q}_{\tau_c,\nu}(-\pi,\vartheta,\pi-\varphi)\cdot\rho(\alpha;q\nu) \qquad (4.2)$$

The interchange of the upper and the lower rows of the arguments of the coefficient $G_{\tau_c}(s\ell'1/2;q\ \ q_c;s\ell'1/2)$ multiplies this coefficient by the factor $(-1)^{2s+1+q+q_c}$ (see [2] p. 1060). But since

these rows are identical, this does not affect G_{τ_c} in any way and $2s+1+q+q_c$ must be even. As the spin of the particle a is half-integral, $2s+1$ is even and $q+q_c$ must therefore be even. So the decay products angular distribution; $T_0^0(\vartheta,\varphi)$ is determined only by even s.tensors of the initial state of the particle $\overbrace{a}$ and the decay proton polarization (i.e. s.tensors T_0^1, $T_{\pm 1}^1$) only by odd ones.

By measureing the angular distribution and polarization of the particle c we obtain information about $\rho(q,\nu)$; about the spin and parity of the particle a and, vice versa, if the latter are known, we can predict all s.tensors, $T_{\tau_c}^{q_c}(\vartheta,\varphi)$.

By integrating the angular distribution $T_0^0(\vartheta,\varphi)$ with respect to ϑ or φ, we may obtain general formulae for distributions wich are measured in experiment (e.g. by integrating with respect to ϑ we get a distribution in an angle between the plane of Λ^0-creation and the plane of its decay) [16].

In conclusion, the author believes it his duty to emphasize that a considerable part of the subject-matter has been worked out in collaboration with A.M. Baldin (see [3]) to whom the author owes deep gratitude for the discussion of some other questions treated in this paper as well. The author takes pleasure in thanking professor M.A.Markov for his sustained interest and L.G. Zastavenko for discussion of a number of problems relative to the rotation group representation theory.

A p p e n d i x I.

A quantum-mechanical state may be described not only by wave functions or density matrices but by giving mean values of a set of some operators [12]. In particular, we will use this procedure to describe the spin state of a particle (or of a system of particles). Note that in experiment as well it is the mean value of the spin vector operator (the so-called polarization of particle), for instance, that is measured and not the probabilities of some values of spin projection.

Let $\hat{A}$ be the operator acting on the spin variables of a particle with spin i; the state of the particle is described by the density matrix $(m_1 \xi_1 | \rho | m_2 \xi_2)$, where m_1 and m_2 are magnetic quantum numbers; ξ the other variables of a representation. It is possible to find the mean value of $\hat{A}$ in this state, if the matrix elements of $\hat{A}$ in the same m, ξ-representation are known:

$$\overline{A} = \sum_{m_1, m_2, \xi_1, \xi_2} (m_2 \xi_2 | \hat{A} | m_1 \xi_1)(m_1 \xi_1 | \rho | m_2 \xi_2) \equiv Sp.(A\rho) \qquad (1.1)$$

Let us form with spin vector operator projections $\underbrace{\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z}$ a tensor of the rank q which transforms under three-dimensional rotations by the irreducible representation of weight q and denote its components by $\hat{A}^{qv}$. Note that by its structure this operator has the same covariance as $\hat{\vec{\sigma}}$, in the sense that under rotations it transforms like $Y^*_{qv}(\vec{n})$ and not like $Y_{qv}(\vec{n})$

For example, $\hat{A}^{1v}$ are simply cyclical (or canonical [11]) projections of $\hat{\vec{\sigma}}$ and transform like the vector $\vec{n}$,

i.e. like $Y_{1\nu}(\vec{n}) \equiv (1\,\nu\,|\,\vec{n})$

Dependence of the matrix elements of $\hat{A}^{q\nu}$ upon m_2 and m_1 according to the Wigner- Eckart theorem [8] coresponding-ly modified (ν is a "contravariant" index in the above sense!) may be written as follows:

$$(m_2\,\xi_2|\hat{A}^{q\nu}|\,m_1\,\xi_1) = \sqrt{2i+1}\,(i\,\xi_2\|\hat{A}^{q}\|i\,\xi_1)(\xi_1|\xi_2)(-1)^{i-m_2}(i\,i\,m_1-m_2\,|\,i\,i\,q\,\nu) \tag{I.2}$$

(if the spin variables and variables ξ separate, $(i\xi_1\|\hat{A}^{q}\|i\xi_1)$ does not depend upon ξ_1). Inserting (I.2) into (I.1), we see that to know $\overline{A^{q\nu}}$ we need to know directly not the density matrix but rather the quantity

$$\rho_{\xi_1,\xi_2}(q,\nu) = \sqrt{2i+1}\cdot\sum_{m_1,m_2}(-1)^{i-m_2}(i\,i\,m_1-m_2\,|\,i\,i\,q\,\nu)(m_1\,\xi_1|\rho|\,m_2\,\xi_2) \tag{I.3}$$

which is proportional to $\overline{A^{q\nu}}$ (see also [12]). It follows from (I.3) that the quantities $\rho(q,\nu)$ $\quad q=0,1\ldots 2i\,,\ \nu=-q,-q+1,\ldots+q$ may characterize the spin state of the particle with the same success as the density matrix. The inverse transformation from q,ν -representation to m_1, m_2 -represenation is the fol-lowing

$$(m_1\,\xi_1|\rho|\,m_2\,\xi_2) = (2i+1)^{-1/2}\cdot\sum_{q,\nu}(-1)^{-i+m_2}\cdot(i\,i\,m_1-m_2\,|\,i\,i\,q\,\nu)\cdot\rho_{\xi_1,\xi_2}(q,\nu)$$

(I.4)

Note that $\rho_{\xi_1,\xi_2}(q,\nu)$ is the same density matrix with res-pect to ξ as $(m_1\,\xi_1|\rho|m_2\,\xi_2)$.

By origin $\rho_{\xi_1,\xi_2}(q,\nu)$ may be called mean values or statisti-cal mean values of the irreducible tensor spin operators. There-fore we will call them statistical tensors (abbr.s. tensors) in accordance with [13] (p. 735).

Suppose we are interested only in distribution of probabi-

Presence of the factor $i^{-\ell}$ in (II.2) assures invariance of the time-reversal operator action on wave functions with definite ℓ and μ under addition of angular momenta [15] (i.e. the application of this operator to $\psi_{\ell\mu}$ may be written in the same way as to $\psi_{\ell_1\mu_1}$ and $\psi_{\ell_2\mu_2}$ if $\vec{\ell}=\vec{\ell}_1+\vec{\ell}_2$) Note that

$$(\ell\mu E|\vartheta\varphi p)=(\vartheta\varphi p|\ell\mu E)^*=(-1)^{-\ell+\mu}(\vartheta\varphi p|\ell,-\mu,E) \qquad (II.3)$$

It may be established in a similar way that $(\vec{p}|\vec{p}')$ takes the form of (17) in $[^{14}_{\circ}]$ (i.e. $(\vec{p}|\vec{p}')=1$ when $\vec{p}=\vec{p}'$ and $(\vec{p}|\vec{p}')=0$ when $\vec{p}\neq\vec{p}'$)

If $(\vec{p}''|\vec{p})(\vec{p}|\vec{p}')$ is required to be equal to $(\vec{p}''|\vec{p}')$ according to the Dirac formalism [5], it turns out that normalization (II.1) requires that the integration with respect to $\vec{p}$ implied in the notation $(\vec{p}''|\vec{p})(\vec{p}|\vec{p}')$ should be performed with the weight factor $V/(2\pi h)^3$ (which is equivalent to a certain summation). According to Dirac (see [5], § 24), this factor must be introduced into every formula in which integration with respect to momenta is being performed.

In connection with normalization (II.1), there arises a problem of obtaining with the help of $\rho'(\vec{n}_c,p_c;q_c\nu_c q_d\nu_d)$ quantities which can be directly compared with results of the experimental study of $a+b\to c+d$ reaction. What does $\rho'(\vec{p}_c;q_c\nu_c q_d\nu_d)$ mean? There is a physical system which originally consists (at the instant of time $-T$ where $T=R/v$ and v is the module of the relative velocity of the particles a and b: $v=|v_a|+|v_b|$) of particles a and b enclosed in volume V, its state is described by the quantities $\rho(\vec{p}_a;q_a\nu_a q_b\nu_b)(\vec{p}_a'|\vec{p}_a)\cdot(\vec{p}_a''|\vec{p}_a)$ Then $\rho'(\vec{p}_c;0,0,0,0)$ is a probability that by the instant of time $+T$ particles c and d with momentum $\vec{p}_c$ will appear in the volume V. The other ρ' with q_c, q_d, ν_c, are quantities proportional to the mean values of corresponding spin operators (see Appendix I) in the assembly of particles with such a momentum.

For comparison with the experiment it is necessary, first of all, to know how many particles c and d with momenta in the interval

$(\vec{p}_c, \vec{p}_c+\Delta\vec{p})$ will appear per 1 sec., if given are the constant flux of incident particles and target particles density (and not the number of particles a and b in some volume). For this $\rho'(\vec{p}_c; 0,0,0,0)$ must be first divided by $2T$ and integrated (with weight $V/(2\pi h)^3$) over the interval $(\vec{p}_c, \vec{p}_c+\Delta\vec{p})$. If the result is multiplied then by V/v (renormalization to the flux), we shall get the differential cross section $\Delta\sigma$ for the reaction:

$$\Delta\sigma(\vec{n}_c) = {}^{h^2}/_{4p_a^2} \cdot {}^{(4\pi)^2}/_{N} \cdot \rho(\vec{n}_c, p_c'; 0,0,0,0) \cdot \Delta\Omega \qquad \text{(II.4)}$$

$N = (2\pi h)^4 \cdot (p_c; p_a)^2 \cdot (2R)^2 \cdot [V^2 \cdot p_a^2 \cdot p_c^2]^{-1}$, p_c' is those module of the momentum $\vec{p}_c$, which follows from the law of energy conservation. The interval $(\vec{p}_c, \vec{p}_c+\Delta\vec{p})$ is assumed to contain momenta with the module p_c'. $\Delta\Omega$ is the solid angle of this interval.

We could obtain quantities $\rho'_{fl.}(\vec{n}_c, p_c'; q_c\nu_c\, q_d\nu_d)$ normalized to the flux which would be related with $\rho'(\vec{n}_c, p_c; q_c\nu_c\, q_d\nu_d)$ in just the same way as $\Delta\sigma(\vec{n}_c)$ is related with $\rho'(\vec{n}_c, p_c; 0,0,0,0)$.

They would be mean values of certan spin operators in an assembly of particles, with momentum in the interval $(\vec{p}_c, \vec{p}_c+\Delta\vec{p})$ The mean values thus obtained would depend, however, not only upon the nature of the spin state but upon the number of particle in such an assembly as well.

It is therefore the mean value of the operator $\hat{A}^{q\nu}$ (for the particle c, for instance) computed per <u>one</u> particle that is characteristic of the spin state:

$$\overline{A^{q\nu}(\vec{p}_c)} = (i_c \| \hat{A}^q \| i_c) \cdot \rho'_{fl.}(\vec{n}_c, p_c'; q, \nu, 0,0) \cdot \frac{1}{\Delta\sigma(\vec{n}_c)} =$$

$$= (i_c \| \hat{A}^q \| i_c) \cdot \rho'(\vec{n}_c, p_c; q, \nu, 0,0) \cdot \frac{1}{\rho(\vec{n}_c, p_c; 0,0,0,0)} \qquad \text{(II.5)}$$

F o r m u l a (2.10)

$$\rho'(\vec{n}_c, p_c, \alpha'; q_c \nu_c q_d \nu_d) =$$

$$= N/4\pi \cdot [(2i_c+1)(2i_d+1)]^{1/2} \cdot [(2i_a+1)(2i_b+1)]^{1/2} \times$$

$$\times \sum C^{q'\nu'}_{q_c \nu_c q_d \nu_d} Y_{L'm_L'}(\vec{n}_c) \cdot i^{-\ell_1'-\ell_2'} \cdot C^{L'0}_{\ell_1'0\,\ell_2'0} [(2\ell_1'+1)(2\ell_2'+1)]^{1/2} \times$$

$$\times [(2s_1'+1)(2s_2'+1)(2q_c+1)(2q_d+1)(2q'+1)(2J_1+1)(2J_2+1)]^{1/2} \times$$

$$\times X(i_c q_c i_c; s_1' q' s_2'; i_d q_d i_d) \cdot X(s_1' q' s_2'; J_1 J J_2; \ell_1' L' \ell_2') \cdot C^{JM}_{q'\nu' L' m_L'} \times$$

$$\times (s_1' \ell_1' \alpha' | R^{J_1 E(p_a)} | s_1 \ell_1 \alpha_1) \cdot (s_2' \ell_2' \alpha' | R^{J_2 E(p_a)} | s_2 \ell_2 \alpha_2)^* \times$$

$$\times C^{JM}_{q\nu L m_L} \cdot X(s_1 q s_2; J_1 J J_2; \ell_1 L \ell_2) \cdot X(i_a q_a i_a; s_1 q s_2; i_b q_b i_b) \times$$

$$\times [(2s_1+1)(2s_2+1)(2q_a+1)(2q_b+1)(2q+1)(2J_1+1)(2J_2+1)]^{1/2} \times$$

$$\times [(2\ell_1+1)(2\ell_2+1)]^{1/2} \cdot C^{L0}_{\ell_1 0\,\ell_2 0} \cdot i^{\ell_1+\ell_2} \cdot Y^*_{Lm_L}(\vec{n}_a) \cdot C^{q\nu}_{q_a \nu_a q_b \nu_b} \times$$

$$\times \rho_{\alpha_1, \alpha_2}(\vec{n}_a, p_a; q_a \nu_a q_b \nu_b)$$

$\sum$ denote a summation over $s_1', s_2', \ell_1', \ell_2', q', L', J_1, J, J_2$, $L, q, \ell_1, \ell_2, s_1, s_2, q_a, q_b$ and over $\nu', m_L', M, m_L, \nu, \nu_a, \nu_b$.

$N = (2\pi h)^4 \cdot (2R)^2 \cdot (p_c; p_a)^2 \cdot [V^2 \cdot p_c^2 \cdot p_a^2]^{-1}$.

The symbol $(p_c; p_a) = (E'(p_c) | E(p_a))$ equals unity, if p_c take such a value that the law of energy conservation is assured, and vanishes otherwise.

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