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A great deal of well-known experimental data point out that high energy ($E \gtrsim 100$ MeV) nucleon-nucleon collision may be reduced in most cases to the nucleon collision with a separate nucleon of the nucleus or to a number of such collisions. When examining this phenomenon one may consider that a) the cross-section of two nucleon interaction is equal to the cross section in free nucleon-nucleon collision; b) the nucleons in the nucleus do not interact at the moment of collision. One can take into account the result of nucleon interaction by assuming the nuclear wave function to be $\Psi_N(r)$ or the nucleon momentum distribution function of the nucleus to be $N(k)$. Such an approach to the problem of nucleon nucleus collision was called the impulse approximation. At present the sufficient conditions of the impulse approximation application are not known. It is necessary, at least, that the wave length of incident nucleon and the effective radius of interaction were less than the distance between nucleons in the nucleus.

The impulse approximation was successful in calculating high-energy nucleon-deutron elastic scattering. This made it possible to make an attempt to apply it for the calculations of the angular dependence of the differential cross sections of elastic nucleon-nucleus scattering. Such calculations are of certain interest from the methodical standpoint and make it possible to draw some physical conclusions which are apparently independent on the use of a number of simplifying assumptions.

If a nucleon characterized by the radius vector $\vec{r}_0$ is incident on the nucleus consisting of A nucleons and the position of each of them is given by the radius vector $\vec{r}_i$ then such a system is convenient to be considered in Jakobi coordinates:

$$\vec{R} = \frac{\vec{r}_0 + \sum_i \vec{r}_i}{A+1}; \quad \vec{R}' = \frac{\sum \vec{r}_i}{A}; \quad \vec{x} = \vec{r}_0 - \vec{R}'; \quad \vec{r}_i' = \vec{r}_i - \vec{R}'$$

The system Hamiltonian is written in the form:

$$H = \frac{P_R^2}{2m(A+1)} + \frac{P_x^2}{2m\frac{A}{A+1}} + \sum_i \frac{P_{r_i}^2}{2m} + \sum_i U_i(r_i') + \sum V_i(\vec{x} - \vec{r}_i) \qquad (1)$$

Here $\vec{p}_R$, $\vec{P}_x$ and $\vec{P}_{r_i}$ are the momenta conjugated with coordinates $\vec{R}$, $\vec{x}$ and $\vec{r}$ respectively; $U_i(r_i)$ is the interaction potential between the nucleons inside the nucleus which determines the form of the nuclear wave function $\psi_N(r)$; $V_i(\vec{x} - \vec{r}_i)$ is the interaction potential of an incident nucleon with i-nucleon of the nucleus. The system of spinless particles is being considered. It may be expected that more exact calculation will lead only to small changes in the region of the application of the impulse approximation. In expression (1) the Coulomb interaction term is absent. It leads to a certain reduction of the calculated cross sections of proton elastic scattering by nuclei in the region of small scattering angles $\lesssim 5 - 10^\circ$.

The matrix element of elastic scattering process is of the form:

$$\mathcal{H} = \Omega^{-1} \sum_i \iint d\vec{x}\, d\vec{r}\, \psi_{NK}^*(\vec{x}\vec{r}_i) V_i(\vec{x} - \vec{r}_i) \psi_{NH}(\vec{x}\vec{r}_i), \qquad (2)$$

where

$$\psi_{NH} = e^{i\vec{p}_H \vec{x}} \psi_N(\vec{r}), \quad \psi_{NK} = e^{ip_K x} \psi_N(r)$$

In accordance with main concepts on which the impulse approximation is based the nuclear wave function may be presented as the product of the wave functions of individual nucleons in the nucleus:

$$\Psi_{Я}(\vec{r}) = \prod_i \psi(\vec{r}_i)$$

After the transformations fully analogous to those given in |1| expression (2) assumes the form

$$\mathcal{H} = \Omega^{-1}\left\{\sum_i \int d\vec{y}\, e^{i\vec{q}\vec{y}} V_i(\vec{y})\right\}\left\{\int d\vec{r}\, e^{i\vec{q}\vec{r}} |\psi(\vec{r}_i)|^2\right\}, \tag{3}$$

if the wave function in the nucleus is normalized for I. The first brackets in expression (3) are (with the accuracy up to a constant factor) the nucleon-nucleon scattering amplitude - a_{NN} with the change of the momentum q. This amplitude would not be calculated under the special assumptions on the potential form, it must be taken directly from the known experimental data on free proton-proton and proton-neutron scatterings. Then the differential nucleon elastic scattering cross section (e.g. a proton one) on the nucleus with the charge Z and the atomic number A is expressed by the formula

$$\frac{d\sigma}{d\Omega}(\Theta^*_{pЯ}) = k\left\{\sigma_{pp}(\Theta^*_{pp})Z + \sigma_{pn}(\Theta^*_{pn})(A-Z)\right\}S(q) + \mathcal{I}, \tag{4}$$

where S(q) equals the square of the second bracket in expression (3), $k = \frac{4A^2}{(A+1)^2}$ is the ratio of solid angles in the two nucleons centre-of-mass-system and in the nucleon and nucleus centre-of-mass-system, I is the interferential term

$$\mathcal{I} = \sum_{k=1}^{A}\sum_{i=1}^{Z}(a_{pn_i}a_{pp_k} + a_{pp_i}a_{pp_k} + a_{pn_i}a_{pn_k})S(q). \tag{5}$$

The angle of proton scattering in the proton and nucleus centre-of-mass-system and the angle of nucleon scatterin on free nucleon $\theta^*_{pp} = \theta^*_{pn}$ in the two nucleon centre-of-mass-system are connected by the relation

$$\sin\frac{\theta^*_{pp}}{2}=\sqrt{K}\sin\frac{\theta^*_{pA}}{2} \qquad (6)$$

Formulas (4)-(6) transit into corresponding formulas for the case of proton deutron elastic scattering obtained in paper [I] if Z = 1, A = 2. When deducing formula (4) the multiple scattering and the absorption of incident nucleons inside the target nucleus were not taken into account. These effects seem not to be very essential for the estimation of the magnitude of the nucleon elastic scattering cross section by ligh nuclei. In order to make numerican calculations one must neglect the interference term about the magnitude of which one may principle judge if compare the results of the calculations with the experiment.

§ 2. The comparison of the results of the calculations with the experiment on proton elastic scattering at 340 MeV on carbon has been made to illustrate the possibility of applying the formular obtained.

The experimental data [2] are given in Fig. 1 The differential proton elastic scattering cross section on carbon as well as on other nuclei is characterized by the rapid diminishing with the increase of the angle of an outgoing proton at high energies of incident protons. As the authors point out (2) the presence of the diffraction minimum and maximum is not excluded which can be clearly seen in proton scattering by the nuclei heavier than carbon. It is possible that in the considered case the diffraction picture is smoothed out at the expense of the incomplete separation of the elastic process from the quasi-elastic proton scattering on individual nucleons of the carbon nuclei as well as at the expense of finite angular resolution of the apparatus.

The calculations of the differential elastic scattering cross section have been made under different assumptions on the nucleon wave function in the carbon nucleus:

1) Bethe-Pearls wave function: $\sim e^{-\alpha r}/r$ In this case the nucleons in the nucleus are distributed by momenta in the following way:

$$N(k) \sim \frac{\alpha^2}{\alpha^2 + k^2} \qquad (7)$$

2) The wave function $\psi \sim e^{-\alpha^2 r^2}$ corresponds to Gaussian momentum distribution:

$$N(k) \sim e^{-\frac{k^2}{\alpha^2}} \qquad (8)$$

where in (7) and (8) $\alpha^2 h^2 = 2 M \cdot 16$ MeV.

3) The wave function is constant inside the nucleus and is equal to zero outside.

4) More real case of the constant nucleon density inside the nucleus and the falling density outside has been considered at the different character of the density change and at different dimensions of the region where it occurs.

It must be noted first of all that the results of the culculations (namely the sticking factor S (q) are strongly dependent on the form of the nucleon wave function in the nucleus.

Fig. 2 illustrates this dependence for the enumerated cases. Curve IV is drawn by assuming the linear density falling from the maximum to 0 at the distance of 1.10^{13} cm. In Fig. 2 the angle of outgoing protons is plotted versus the sticking factor (the latter is done in log. scale).

Sharp difference of the curves makes it possible to do some remarks on the nucleon wave function inside the nucleus. By the way, the known calculations of quasi-elastic scattering [3] are much less critical to the form of $\psi_q(\vec{z})$ or N (k). An appreciable difference can be observed only on the tails of the proton energetic spectrum scattered from the nucleus at the given angle (or on the tails of the corresponding "stripping-curves"), where such a comparison is not, apparently, reasonable. This is due to the violations of necessary conditions of the impulse approximation validity.

The wave functions of the forms (7) and (8) give too great values S(q) for the angles$\gtrsim$10 - 15^{0}. The presentation of the nucleus in the form of a ball with the constant density and with sharp edges leads to the minima of the differential cross sections falling to zero as well as in the case of the diffraction on the absolutely black nucleus. The curve of type IV gives, apparently, better agreement with experiment. The minimum on curve IV has the depth dependent first of all on the relation of the dimensions of the constancy region and the wave function diminshing. If we consider the dimensions of the nucleus edge to be constant when passing to great A, the experimentally observed deepening of the diffraction minima in scattering on intermediate and heavy nuclei becomes qualitatively clear.

We can judge about the agreement of the calculations with experimental data from the curves of Fig. 1. Curve 1 in Fig. 1 corresponds to curve IV in Fig. 2, and curve II is calculated under the assumptions of the linear diminishing of density, beginning from the centre of the nucleus. Here and above the dimensions of the nucleus have been calculated by the formula $R = r_0 \sqrt[3]{A}$

where $r = 1{,}2 \cdot 10^{-13}$ cm.

Further selection of the parameters and the very form of the wave function diminishing is hardly reasonable in the frame of approximate estimation.

§ 3. Thus the results of the given calculations point out, apparently, to the constancy of the nucleon wave function inside the nucleus and its diminishing in the region $\sim 1{,}10^{-13}$ cm on the boundary of the nucleus. This conclusion coincides with the result of the analysis of fast electron scattering by nucleus ($E > 1$ BeV) which points out the constance of charge density inside the nucleus and its slow diminishing on the boundary. It is probable that such density distribution is correct not only for protons possessing the charge, but also for neutrons in the nucleus. Density distribution inside the nucleus estimated in the present note is also in agreement with the form of Sacsen=Wood potential radial dependence which is widely used in the calculations based on the nuclear optical model. The coincidences of the qualitative conclusions which were spoken about in the present note and well-known concepts about the character of nucleon distribution in nuclei make it possible in their turn to extend the region of the impulse approximation application for the case of nucleon elastic scattering by nuclei.

However, the question of the character of nucleon momentum distribution in nuclei is open. In the aforementioned paper [3] devoted to the analysis of experimental data on quasi-elastic proton-proton scattering the Gaussian nucleon momentum distribution is preffered because it is considerably inriched with high momentum components. The authors of the paper [4] who analysed the deutron pickup had to introduce Bether-Pearls distribution on which is still richer with high momentum components. But in the present note the distributions comparatively poor with high momentum components were obtained. The cause of such a discrepancy is, apparently, as follows:the peculari-ties of nucleon motion in nuclei (correlations) connected with the interactions ~~interactions~~ in the groups involving two, three or lar-ger number of nucleons (fragments) play a big role in the process of quasi-elastic scattering and deutron outgoing from the nuclei whereas above we considered the nucleon motion in the average field of all the nucleons of the nucleus.

The authors of the paper [5] arrived at the conclusion about the presence of the correlated nucleon groups in the nucleus on the basis of the analysis of nucleon momentum distributions in the nuclei obtained in different experiments. The calculation of nucle-on elastic scattering by nuclei may contribute to the number of the considered experiments. It seems, however, that it is more rea-sonable to speak not about the nucleon scattering on nucleon pos-sessing high momentum in the nucleus but about the nucleon scatter-ing on the compact nucleon group as a whole.

It is true, one concept is equivalent to another to some ex-tent as high momentum components may be the result only of close nucleon correlation in the groups, but then the concept about the incident nucleon interaction with a individual nucleon in the nuc-leus and the impulse approximation itself turn out to be inapplicab-le. The conclusion about the presence of clusters (fragments) in nuc-lei is confirmed by a great deal of experimental data and parti-cularly the data on the fast fragment knocking out from the nuclei.

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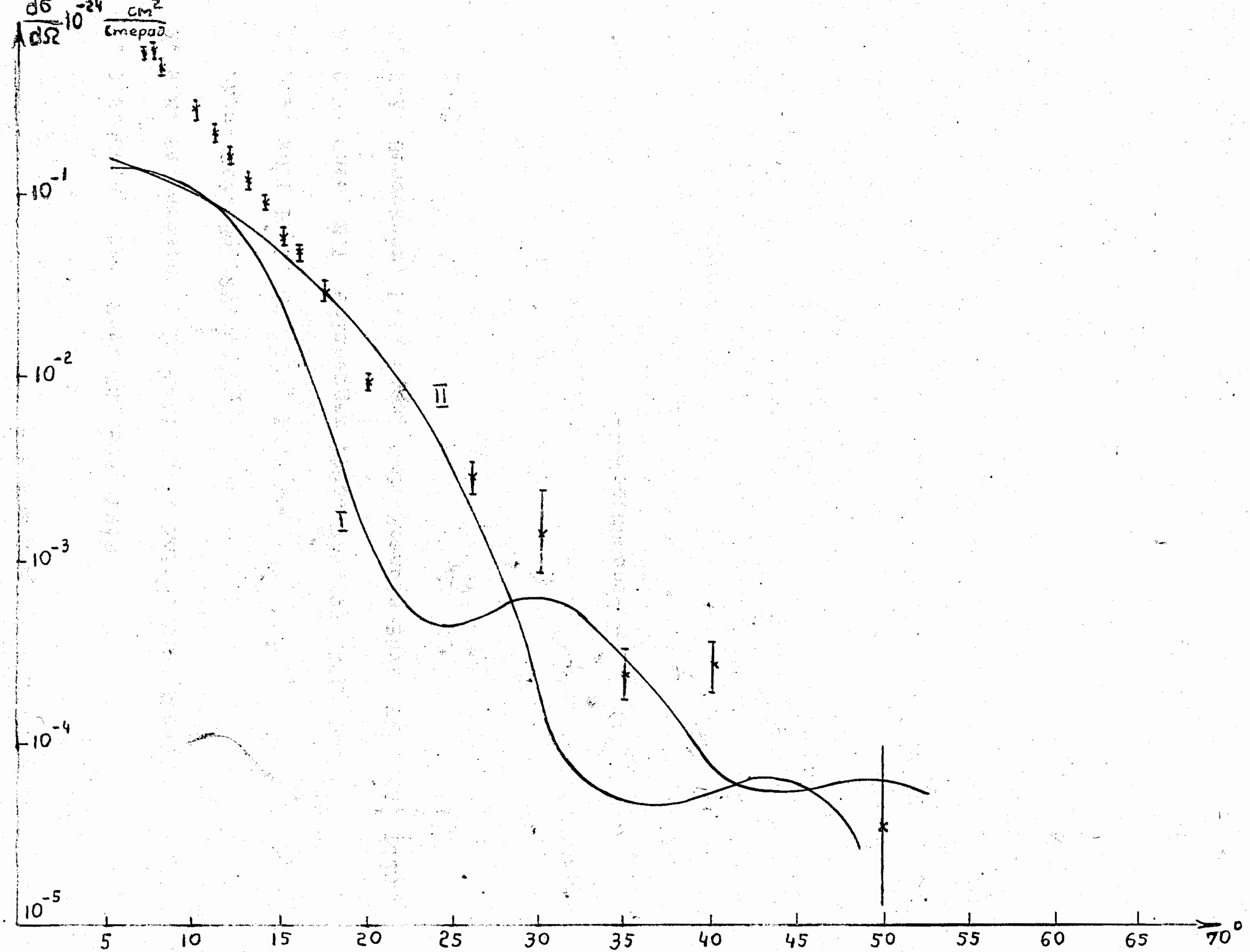

$\frac{d\sigma}{d\Omega} 10^{-24} \frac{см^2}{стерад}$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
I
II
5
10
15
20
25
30
35
40
45
50
55
60
65
70°

Рис. 1

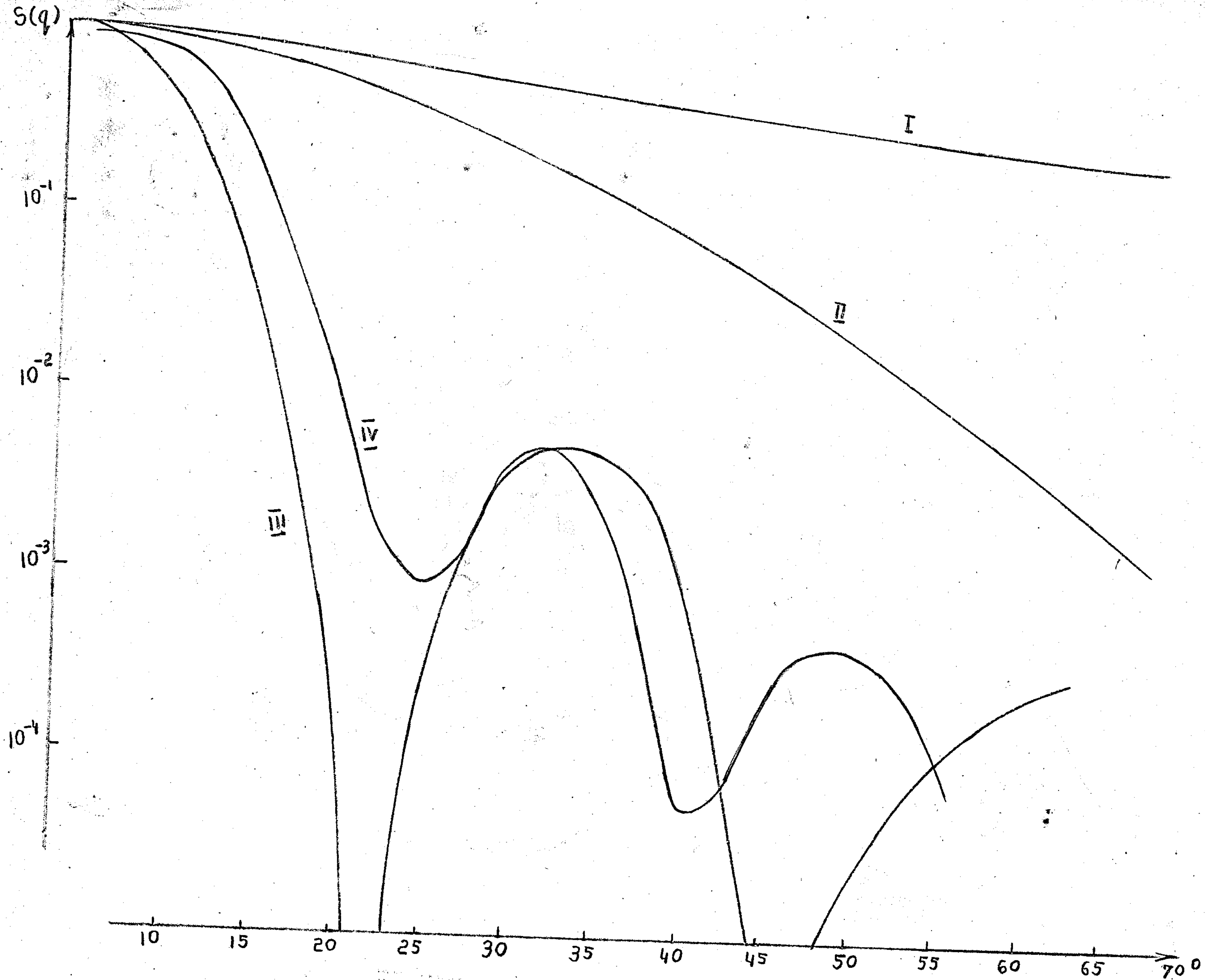

Рис. 2.