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# STRONG AND WEAK INTERACTIONS INVOLVING HYPERONS

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Gell-Mann's theory of universal strong interaction is extended to include weak interactions. The result is found to be consistent with experiment.

Gell-Mann[1] has recently proposed that the interactions of pions with nucleons and hyperons are universal with an interaction constant about fifteen times larger than those of K-mesons with baryons. According to his scheme the various baryons may be considered as forming four degenerate doublets which cannot be distinguished from one another if the interaction with K-mesons were absent. In this note we shall correlate this symmetry property with the charge independent nature of the pion-baryon interactions. This same symmetry is then extended to included weak interactions and definite forms of interaction functions are obtained. These interaction functions lead in very natural way to the well-known selection rule $\Delta T = 1/2$.

We have not considered the possibility that the weak interactions may be parity non-conserving. However, to extend the investigation to cover this possibility will not be difficult.

We shall start by expressing meson and baryon wave functions in spinor notation. Using following representation for Pauli matrices

$$\tau_1^{\dot\alpha\beta} = -\tau_{1\alpha\dot\beta} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2^{\dot\alpha\beta} = -\tau_{2\alpha\dot\beta} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \quad \tau_3^{\dot\alpha\beta} = -\tau_{3\alpha\dot\beta} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{1}$$

We may write

$$\pi^{\dot\alpha\beta} = -\pi_{\alpha\dot\beta} = \underset{\sim}{\tau}^{\dot\alpha\beta}\cdot\underset{\sim}{\pi} = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$$

$$N_\alpha = \begin{pmatrix} N^+ \\ N^0 \end{pmatrix}, \quad \Xi^{\dot\alpha} = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix}, \quad K_\alpha = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}$$

$$\psi_{\alpha\dot\beta} = \Lambda - \underset{\sim}{\tau}_{\alpha\dot\beta}\cdot\underset{\sim}{\Sigma} = \begin{pmatrix} Z^0 & \Sigma^+ \\ \Sigma^- & Y^0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \Lambda+\Sigma^0 & \Sigma_1+i\Sigma_2 \\ \Sigma_1-i\Sigma_2 & \Lambda-\Sigma^0 \end{pmatrix} \tag{2}$$

$$\psi^{\dot\alpha\beta} = \Lambda - \underset{\sim}{\tau}^{\dot\alpha\beta}\cdot\underset{\sim}{\Sigma} = \begin{pmatrix} Y^0 & -\Sigma^+ \\ -\Sigma^- & Z^0 \end{pmatrix}$$

where the dotted indices are transformed as the complex conjugate of the undotted indices, and $\pi^{0}$, $\pi$, $N^{+}$, $N^{0}$, $\Sigma$, $\Lambda$, etc. represent wave functions of the corresponding particles. Since we shall only consider rotations in three dimensional isobaric space, the dotted indices will also transform contravariantly to the undotted indices. Thus the following expressione are invariant under the transformation:

$$C_{\dot\alpha}C_{\alpha},\quad C^{\dot\alpha}C_{\alpha},\quad C^{\dot\alpha}C^{\alpha},\quad C^{\alpha}C_{\dot\alpha},\quad C^{\alpha*}C^{\alpha},\quad C^{\dot\alpha*}C^{\dot\alpha}$$
$$C^{\alpha*}C_{\dot\alpha},\quad \text{etc.} \tag{3}$$

where repeated indices means summation over $\alpha$ = 1,2.

According to d'Espagnat and Frentki[2] the strong interactions of pions with nucleons and cascade particles are given by

$$H_1 = g_1\bar{N}_{\dot\alpha}\, i\gamma_5\pi^{\dot\alpha\beta}N_\beta\,,\qquad H_2 = g_2\bar{\Xi}^{\dot\alpha}\, i\gamma_5\pi^{\dot\alpha\beta}\Xi^{\beta} \tag{4}$$

The interaction function of pions with $\Sigma$ and $\Lambda$ particles are likewise given by

$$H_3' = g_3\bar{\Psi}_{\dot\alpha\dot\beta}\, i\gamma_5\pi^{\dot\alpha\lambda}\Psi_{\lambda\dot\beta} + g_4\bar{\Psi}^{\dot\alpha\beta}\, i\gamma_5\pi^{\dot\alpha\lambda}\Psi^{\dot\lambda\beta} \tag{5}$$

In usual notation this may be weitten as

$$H_3' = (g_3 - g_4)\bar{\Lambda}\, i\gamma_5 \ldots + (g_3 + g_4)(\bar{\Sigma}\, i\gamma_5 \times \Sigma)\ldots + \text{c.c.} \tag{5a}$$

It is no8 essential in Gell-Mamm's scheme to consider only two cases: (i) $g_4$ = o, and (ii) $g$ since in all other cases, as it can be seen from (5a), the mass increment of $\Lambda$ and $\Sigma$ due to interaction with pions will notibe the same and therefore the inclusion of $\Lambda$ and $\Sigma$ in one wave function would have little significance. We see that the above two casses only lead to different signs for the first term of (5a). Remembering that all wave functions are defined only up to an arbitrary unimodular factor, these two cases will become identical if we replace $\Lambda$ by $-\Lambda$. Therefore the above two cases are essentially the same and in the following we shall only consider the case (1). Then (5) becomes

$$H_3 = g_3 \bar{\Psi}_{\alpha\dot{\beta}}\, i\gamma_5 \pi^{\dot{\alpha}\lambda} \psi_{\lambda\dot{\beta}} \tag{6}$$

Using (2) we see that (3) and (6) are invariant under rotation in isobaric space. This invariant property is however more stringent than the requirement that the interactions of pions with baryons are independent of the charge of the meson. To ensure the latter property it would be sufficient for (6) to be invariant under transformation w with respect to those indices which are contained in $\pi^{\dot{\alpha}\lambda}$. To illustrate this poiht we write (6) as sum of two terms:

$$H_3 = g_3 \bar{\Psi}_{\dot{\alpha}1}\, i\gamma_5 \pi^{\dot{\alpha}\lambda} \psi_{\lambda 1} + g_3 \bar{\Psi}_{\dot{\alpha}2}\, i\gamma_5 \pi^{\dot{\alpha}\lambda} \psi_{\lambda 2} \tag{7}$$

The interaction will be independent of the charge of the meson if the two terms in (7) are invariant separately under spinor transformation with respect to the indices $\dot{\alpha}$ and $\lambda$, i.e., if $\psi_{\lambda}$ and $\bar{\psi}_{\dot{\alpha}}$ ~~and~~ are transformed as two spinors of the first rank. The above argument can also be put more precisely as follows: Write down (6) explicitly we obtain

$$\begin{aligned} H_3 = g_3 \{ & (\bar{\Sigma}^+ i\gamma_5 \Sigma^+ - \bar{Y}^0 i\gamma_5 Y^0)\pi^0 + \sqrt{2}(\bar{\Sigma}^+ i\gamma_5 Y^0 \pi^+ + \bar{Y}^0 i\gamma_5 \Sigma^+ \pi^-) \\ & + (\bar{Z}^0 i\gamma_5 Z^0 - \bar{\Sigma}^- i\gamma_5 \Sigma^-)\pi^0 + \sqrt{2}(\bar{\Sigma}^- i\gamma_5 Z^0 \pi^- + \bar{Z}^0 i\gamma_5 \Sigma^- \pi^+) \} \end{aligned} \tag{8}$$

We see from (8) that the interaction causes only transitions between $\Sigma^+$ and $Y^0$ and also between $\Sigma^-$ and $Z^0$, but no transition from the doublet $\Sigma^+$, $Y^0$ to the doublet $\Sigma^-$, $Z^0$. This already shows that as far as interaction with pions is conserned, these two doublets are as distinguished from each other as the doublets $\Xi^0, \Xi^-$ and $N^+$, $N^0$. Now the hypothesis of charge independence says that in interaction with hyperons the pion cause no difference between $\Sigma^+$ and $Y^0$ and also no difference between $\Sigma^-$ and $Z^0$. Thus the interaction will be the same if the hyperon state is an arbitrary superposition of $\Sigma^+$ and $Y^0$ or of $\Sigma^-$ and $Z^0$. It will not be necessary to consider superposition of $\Sigma^+$, $Y^0$, $\Sigma^-$ and $Z^0$ since the interaction does not cause any transition from the $\Sigma^+$, $Y^0$ state to the $\Sigma^-$, $Z^0$ state. In mathematical lagguage this means that as far as interactions with pions are concerned, $\Sigma^+$, $Y^0$ and $\Sigma^-$, $Z^0$ should be considered as two spinors and not as one spinor-matrix.

Following Gell-Mann we may assume that the interaction constants $g_1$, $g_2$ and $g_3$ in (3) and (7) are the same. Then the strong interactions of pions with verious hyperons can be weitten in the following univerdal form:

$$H_i = g\,\bar{\psi}^{(i)}_\alpha\, i\gamma_5\, \pi^{\alpha\beta}\, \psi^{(i)}_\beta \qquad (i=1,2,3,4) \tag{9}$$

with

$$\psi^{(1)} = \begin{pmatrix} N^+ \\ N^0 \end{pmatrix},\quad \psi^{(2)} = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix},\quad \psi^{(3)} = \begin{pmatrix} Z^0 \\ \Sigma^- \end{pmatrix},\quad \psi^{(4)} = \begin{pmatrix} \Sigma^+ \\ Y^0 \end{pmatrix} \tag{9a}$$

If we assume further that the unrenormalized masses of N, $\Xi$ and $\Sigma$

particles are originally the same (the aquality of masses of $\Lambda$ and $\Sigma$ has been assumed before), then Gell-Mann's "global symmetry" is obtained: The baryons form four pairs of degenerate doublets which cannot be distinguished from one another if interactions with K-mesons were absent. From the foregoing derivation we see that the equality of all baryon masses is not necessary for the universality of strong interactions with pions, the latter only requires the equality of unrenormalized masses of $\Lambda$ and $\Sigma$ particles.

Following the same spirit we may also propose an universal weak interactions of pions with baryons. Unlike the strong interactions in which the pions interact only with one baryon field $\psi^{(i)}$ in an elementary act, in weak interactions the pions must interact simultaneously with two different baryon fields. We assume that the universal weak interaction is given by

$$H_{wij} = G\bar{\psi}^{(i)}_{\alpha} i\gamma_5 \pi^{\dot{\alpha}\beta} \psi^{(j)}_{\beta} + c.c. \qquad (i \neq j) \tag{10}$$

where G is the universal constant for the weak interactions we note that the behaviour of (10) under rotations in isobaric space is the same as the strong interaction(9).

The interaction functions (10) with different values of i and j, as they stand, do not all conserve the electric charge of the system. We see essily that electric dharge is conserved only for the following two interactions

$$H_{W_1} = -G\bar{N}_{\alpha} \pi^{\dot{\alpha}\beta} \psi^{\beta 2} + c.c.$$

$$H_{W_2} = -G\bar{\Xi}^{\dot{\alpha}} \pi^{\dot{\alpha}\beta} \psi^{\dot{\beta} 1} + c.c. \tag{11}$$

Before we proceed further, it should be pointed out that an alternative form of weak interaction functions which satisfy all requirements satisfied by (11) also exists:

$$
\begin{aligned}
H_{W_1} &= -G \bar{N}_{\alpha} \pi^{\dot{\alpha}\beta} \psi^{\dot{\beta}2} + c.c. \\
H_{W_2} &= -G \bar{\Xi}^{\dot{\alpha}} \pi^{\dot{\alpha}\beta} \psi^{\dot{\beta}1} + c.c.
\end{aligned}
\qquad (12)
$$

As in the case considered before (12) reduces to (11) if we replace $\Lambda$ by $-\Lambda$. Since the ambiguity in the sign of the wave function $\Lambda$ has already been removed by the choice of (6), therefore (11) $\neq$ (12) must now be different from each other. Writing (11) and (12) in usual notations we obtain

$$
\begin{aligned}
H_{W_1} &= G\{(\bar{N}^{+} i\gamma_5 \Sigma^{+} \mp \tfrac{1}{\sqrt{2}} \bar{N}^{+} i\gamma_5 \Lambda + \tfrac{1}{\sqrt{2}} \bar{N}^{0} i\gamma_5 \Sigma^{0})\pi^{0} \\
&\quad + (\pm \bar{N}^{+} i\gamma_5 \Lambda - \bar{N}^{+} i\gamma_5 \Sigma^{0})\pi^{+} + \sqrt{2}\, \bar{N}^{0} i\gamma_5 \Sigma^{-} \pi^{-} + c.c.\} \\
H_{W_2} &= G\{(-\bar{\Xi}^{-} i\gamma_5 \Sigma^{-} \mp \tfrac{1}{\sqrt{2}} \bar{\Xi}^{0} i\gamma_5 \Lambda + \tfrac{1}{\sqrt{2}} \bar{\Xi}^{0} i\gamma_5 \Sigma^{0})\pi^{0} \\
&\quad + (\mp \bar{\Xi}^{-} i\gamma_5 \Lambda + \bar{\Xi}^{-} i\gamma_5 \Sigma^{0})\pi^{-} + \sqrt{2}\, \bar{\Xi}^{0} i\gamma_5 \Sigma^{-} \pi^{+} + c.c.\}
\end{aligned}
\qquad (13)
$$

where the upper and lower signs before $\Lambda$ correspond to interaction (11) and (12) respectively. It will be noted that (11) and (12) are components of a spinor in isobaric space. That the ~~weak~~ Hamiltonians for weak interactions are spinors in isobaric space has already been proposed and investigated by several authors[3]. Such Hamiltonians will ensure the change by 1/2 of the isotopic spin T. We notice that no term in (13) will cause the transition $\Sigma^{-} \longrightarrow N^{0} + \pi^{-}$. We see also from the form of (13) that $\Sigma$ particles have only transition to the state T = + 1/2 (i.e., $\Delta$ T = - 1/2). At first sight this would mean that (13) is in disagreement with the experiment since we know that the decay process $\Sigma^- \rightarrow N^0 + \pi^-$ has comparable life time as the decay process of $\Sigma^+$. We shall show that under the influeince of strong interactions our weak interactions (13) can still lead to $\Sigma^-$ decay*. The decay process may take place via strong interactions in the following manner:

$$\Sigma^- \rightarrow \pi^- + \Lambda \rightarrow \pi^- + \pi^- + N^+ \rightarrow \pi^- + N^0 \tag{14}$$

$$\Sigma^- \rightarrow \pi^- + \Sigma^0 \rightarrow \pi^- + \pi^- + N^+ \rightarrow \pi^- + N^0 \tag{15}$$

If we take (11) as the Hamiltonian for weak interactions and (3) and (6) as the Hamiltonians for strong interactions, then after some straight

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* It should be stressed that owing to strong interaction, the selection rule for $\Delta$ T as read from the Hamiltonian for weak interactions is in general not the same as the observed one. For instance, the unstable strange particle under consideration may be converted by strong interaction into another unstable strange particle for which the selection rule given by the interaction Hamiltonian is different. If the decay of the original particle takes place through this second particle, the selection rule for the process must be modified. Even when we do not consider another unstable particle, the strange particle under consideration may still omit virtual mesons so that the isotopic spin $\tilde{T}'$ of the particle in this virtual state as a vector in the space-quantization diagram is opposite to the direction of total $\tilde{T}$. Thus we have $\Delta$ T = $-\Delta$ T'. If

the interaction Hamiltonian only allows, say, $\Delta T = +1/2$, we see immediately that if decay takes place trough this virtual state, the change of T is given by $\Delta T = -1/2$. Detailed considerations will be given elsewhere.

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forward calculation we find that the transition amplitude for (14) is just equal and opposite to that of (15). We can easily see that for more complicated paths the contribution from virtual par- ticles and similar contributions from virtual particles always cancel one another and therefore the net transition amplitude for $\Sigma^-$ decay is always zero. However, if we take (12) as the interaction Hamiltonian, we find that the corresponding amplitudes always reenforce each other. Consequently we obtain a non-vanishing transition amplitude for the $\Sigma^-$ decay, the life time of which is comperable with the of $\Sigma^+$ decay.

We conclude from the above result that (12), not (11), should be taken as the Hamiltonian for universal weak interactions.

In the following we shall show that (12) also lead to decay of $K^0$ meson. The most general form of interaction Hamiltonian of K-mesons with baryons are given by

$$H' = f_1 \bar{\Xi}^{\alpha} \psi_{\alpha\dot{\beta}} K^{\dot{\beta}} + f_2 \bar{\Xi}^{\dot{\alpha}} \psi^{\dot{\alpha}\beta} K^{\beta} + f_3 \bar{N}_{\alpha} \psi_{\alpha\dot{\beta}} K_{\dot{\beta}} + f_4 \bar{N}_{\alpha} \psi^{\dot{\alpha}\beta} K_{\beta} + c.c. \qquad (16)$$

In usual notation this is

$$H' = (f_1 + f_2)\, \bar{\Xi}\, \tau_2 K^{*} \Lambda + (f_1 - f_2)\, \bar{\Xi}\, \underset{\sim}{\tau} \cdot \underset{\sim}{\Sigma}\, K^{*} +$$

$$+(f_1 + f_2)\bar{N} K \Lambda + (f_3 - f_4)\bar{N}\, \underset{\sim}{\tau} \cdot \underset{\sim}{\Sigma}\, K + c.c.$$

According to Gell-Mann the interaction constants $f_i$ are one order smaller than $g$ in (9), and (16) is responsible for the mass differences of nucleon, $\Sigma$-, $\Lambda$- and $\Xi$ - particles. The decay of $K^0$ into $\pi^+ + \pi^-$ and $\pi^0 + \pi^0$ can take place through the strong interactions (3) and (6), the moderate strong interaction (16) and the weak interaction (12). The simplest diagrams for such processes consist of three vertices, one for each kind of interaction. The smallness of $f_i$ in comparison with $g$ may have the effect of making the decay life time of $K^0$-mesons longer than those of $\Lambda$ and $\Sigma$, but this effect is compensated by the situation that the number of virtual paths is rather large (about ten). Another compensating factor is the large available phase space of the final states. Therefore we may expect that the life time of $K^0$ -decay will come out to be of the same order as that of $\Lambda$ and $\Sigma$ particles, in consistent with the experiment.

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