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Hing HU

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THE PROPER MESON FIELD OF A PHYSICAL NUCLEON

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A B S T R A C T

The distribution of mesons in the proper field of a nucleon is calculated based on a model of physical nucleon consisting of a finite core surrounded by a cloud of π-mesons. An asymptotic solution which holds when the number of mesons is large is obtained. This solution is expected to account for the region very near to the core. From the theoretical estimation of Friedman, Lee and Christian and also from the evidence that the orientation of the core in the isotopic spin space and the ordinary spin space may be quite random, it follows that the renormalization factor for the interaction constant may be quite large. If we accept the value obtained by Friedman et al and use the conventional cut-off value $K \approx 6$ then the number of mesons at small distances from the core will be of the order ten. This result is consistent with the view that the production of meson shower by high energy nucleon-nucleon collision is due to the shaking off of meson clouds during collision. No inconsistency with the low energy experiments is introduced by the present theory.

I. Introduction

Recent discoveries of various types of strange particles serve to indicate that the structure of physical nucleons cannot be as simple as it was thought before. Owing to the strong interactions of these particles with nucleons, a physical nucleon would inevitably involve these particles in its virtual states, as a consequence the structure in the central portion of a single nucleon system would be greatly complicated. This may be the reason why relativistic treatment of the system always leads to results which are not in agreement with the experiment. With the further realization that the present form of quantum mechanics may not even be adequate for the description of the central portion of physical nucleons, attempts have been made to replace this central portion by a fixed core,[2/] and consider the outer portion as consisting only of mesons. Success of these attempts shows that virtual nucleon pairs and virtual strange particles indeed do not interfere with the outer part of meson cloud of a nucleon.

The question arises as to whether the above picture can still be used at small distances from the core. As we shall see below, at small distances from the core there might be large number of mesons, so evidences about the situation there must come from experiments on production of meson showers by high energy nucleon-nucleon collisions. Observation shows that the shower particles are mainly mesons. This may be considered as an evidence supporting the view that the effect of virtual nucleon pairs and virtual strange particles may still be unimportant at small distances from the core.

It will be of interest in any case to find out the distribution of meson cloud just outside the core under the assumption that states involving virtual nucleon pairs and strange particles can be neglected. We have mentioned above that the number of mesons at small distances from the core may be very large. One important consequence of this is that the interaction constant g and the nucleon rest mass M appearing in formulas can no longer be equal to the observed values. They should rather be considered as the intrinsic interaction constant and intrinsic mass of the core.

This difference in observed and intrinsic quantities is very important in our case. In usual calculations this difference is either neglected entirely or taken into account only up to the lowest order, say those arisen from one or two meson states.

It is well known according to the accepted form of meson-nucleon interaction that the intrinsic mass and interaction constant must be larger than the observed ones. This has two important consequencies: Firstly, the strength of interaction between the core and the meson field surrounding it is increased in the region very close to the core, so that according to the criteria put torward by Blokhintsev[1/] one may expect that the strong coupling approximation would be much more accurate there than in the fringe outer regeon where a sort of intermediate coupling approximation such as proposed by Tomonaga[2/], Chew[3/] or Tamm and Dancoff[4/] would be more suitable. Secondly, the largeness of the intrinsic mass of the core tends to reduce the recoil effect of the core. Since the average kinetic energy of the meson field increases very rapidly as the distance from the core decreases, one would expect that the neglection of the core recoil will not be justified in the region very close to the core. However, the following consideration will show that the situation is not really so bad. The recoil momentum of the core is equal to minus the sum of momenta of the mesons. At small distances from the core although the momentum of each meson is very large, since the number of mesons is also very large, the sum of these momenta may turn out to be not vely large. The largeness of intrinsic mass mentioned above is an additional factor which reduces the effect of recoil.

Our treatment of the region of meson field close to the core is very similar to the strong coupling treatment of Pauli and Dancoff, the main difference being that Fock's representation is now used to express the solution directly as distribution of mesons in momentum space. The connection of the present results with those obtained by Bloch-Nordsieck approximation is also discussed.

Application of the present theory to the problem of production of meson shower by high energy nucleon-nucleon collision is given in Section 3.

2. Meson field outside the core of a physical nucleon at rest

We shall neglect the recoil of the core. The Hamiltonian operator for a nucleon at rest at origin in space coordinate system is given by

$$H = \sum_{\alpha=1}^{3} \sum_{\underset{\sim}{k}} \left\{ \varepsilon \varphi_\alpha^*(\underset{\sim}{k}) \varphi_\alpha(\underset{\sim}{k}) + \frac{ig}{\sqrt{2\varepsilon}} U(k)(\underset{\sim}{\sigma} \cdot \underset{\sim}{k}) \tau_\alpha \left[\varphi_\alpha^*(\underset{\sim}{k}) - \varphi_\alpha(\underset{\sim}{k}) \right] \right\} \tag{1}$$

where α represents polarization of the meson in charge space, $\underset{\sim}{k}$ is the momentum vector of the meson, $\varepsilon = (\mu^2 + k^2)^{1/2}$, μ being the meson rest mass. The natural unit $\hbar = c = \mu = 1$ is used so that $\varepsilon = (1 + k^2)^{1/2}$. σ_i and τ_i (i = 1,2,3) are respectively the spin and isotopic spin matrices of the nucleon. $\varphi_\alpha^*(\underset{\sim}{k})$ and $\varphi_\alpha(\underset{\sim}{k})$ are respectively the emission and absorption operators for mesons of momentum $\underset{\sim}{k}$ and charge α. They satisfy the following commutation relation

$$\left[\varphi_\alpha(\underset{\sim}{k}), \varphi_\beta^*(\underset{\sim}{k}') \right] = \delta_{\alpha\beta} \delta_{\underset{\sim}{k}k'} \tag{2}$$

We may choose a set of real functions $F_n(k)$ satisfying the following orthonormal conditions

$$\sum_{\underset{\sim}{k}} k_i k_j F_n(k) F_m(k) = \delta_{ij}\delta_{nm} \qquad \begin{pmatrix} n,m = 0,1,2,\ldots \\ i,j = 1,2,3 \end{pmatrix} \tag{3}$$

We may express $\varphi_\alpha(\underset{\sim}{k})$ in the following form

$$\varphi_\alpha(\underset{\sim}{k}) = \sum_{i=1}^{3}\sum_{n=0}^{\infty} k_i F_n(k)\phi_{\alpha i}^{(n)} \tag{4}$$

where

$$\phi_{\alpha i}^{(n)} = \sum_{\underset{\sim}{k}} F_n(k) k_i \varphi_\alpha(\underset{\sim}{k}) . \tag{5}$$

From (2),(4),(5) we can easily verify that $\phi_{\alpha i}^{(n)}$ satisfies the following commutation relation

$$[\phi_{\alpha i}^{(n)}, \phi_{\beta j}^{(m)*}] = \delta_{ij}\delta_{\alpha\beta}\delta_{nm} . \tag{6}$$

Inserting (4) into (1) gives

$$H = \sum_{i=1}^{3}\sum_{\alpha=1}^{3}\sum_{n=0}^{\infty}\Big\{A_n \phi_{\alpha i}^{(n)*}\phi_{\alpha i}^{(n)} + \sum_{m>n} B_{nm}[\phi_{\alpha i}^{(n)*}\phi_{\alpha i}^{(m)} + \phi_{\alpha i}^{(m)*}\phi_{\alpha i}^{(n)}]$$

$$+\frac{ig}{\sqrt{2}} C_n \tau_\alpha \sigma_i [\phi_{\alpha i}^{(n)} - \phi_{\alpha i}^{(n)*}]\Big\} \tag{7}$$

where

$$A_n = \frac{1}{3}\sum_{\underset{\sim}{k}} k^2 F_n(k)^2 \varepsilon , \quad B_{nm} = \frac{1}{3}\sum_{\underset{\sim}{k}} k^2 F_n(k) F_m(k)\varepsilon$$

$$C_n = \frac{1}{3}\sum_{\underset{\sim}{k}} \frac{U(k)}{\varepsilon^{1/2}} k^2 F_n(k) \tag{8}$$

Sorting out terms containing $\phi_{\alpha i}^{(0)}$ we obtain

$$H=\sum_{\alpha=1}^{3}\sum_{i=1}^{3}\left\{A_0\phi_{\alpha i}^{(0)*}\phi_{\alpha i}^{(0)}+\frac{ig}{\sqrt{2}}C_0\tau_\alpha\sigma_i\left[\phi_{\alpha i}^{(0)}-\phi_{\alpha i}^{(0)*}\right]\right\}$$

$$+\sum_{\alpha=1}^{3}\sum_{i=1}^{3}\sum_{n=1}^{\infty}\phi_{\alpha i}^{(n)*}\left[B_{0n}\phi_{\alpha i}^{(0)}-\frac{ig}{\sqrt{2}}C_n\tau_\alpha\sigma_i\right]$$

$$+\sum_{\alpha=1}^{3}\sum_{i=1}^{3}\sum_{n=1}^{\infty}\phi_{\alpha i}^{(n)}\left[B_{0n}\phi_{\alpha i}^{(0)*}+\frac{ig}{\sqrt{2}}C_n\tau_\alpha\sigma_i\right]$$

$$+\sum_{\alpha=1}^{3}\sum_{i=1}^{3}\sum_{n=1}^{\infty}\left\{A_n\phi_{\alpha i}^{(n)*}\phi_{\alpha i}^{(n)}+\sum_{m>n}B_{nm}\left[\phi_{\alpha i}^{(n)*}\phi_{\alpha i}^{(m)}+\phi_{\alpha i}^{(m)*}\phi_{\alpha i}^{(n)}\right]\right\}. \tag{9}$$

The Schroedinger equation of the present problem is

$$H\Psi = E\Psi \tag{10}$$

There existed in the past two systematic approach for solving the problem. One is the strong coupling approximation proposed by Wentzel[8] and the other is the intermediate coupling approximation proposed by Tomonaga[2]. Both these two authors made use of the single wave function approximation which in our notation means that Ψ should be a function of $\phi_{\alpha i}^{(0)*}$ only. In other words

$$\phi_{\alpha i}^{(n)}\Psi \cong 0 \qquad (n \neq 1). \tag{11}$$

From (9) and (11) we see that in Fock's representation in which $\phi_{\alpha i}^{(n)}$ is replaced by $\partial/\partial\phi_{\alpha i}^{(n)*}$, Ψ must satisfy the following equations

$$\sum_{\alpha=1}^{3}\sum_{i=1}^{3}\left\{A_0\phi_{\alpha i}^{(0)*}\frac{\partial}{\partial\phi_{\alpha i}^{(0)*}}+\frac{ig}{\sqrt{2}}C_0\tau_\alpha\sigma_i\left[\phi_{\alpha i}^{(0)*}-\frac{\partial}{\partial\phi_{\alpha i}^{(0)*}}\right]\right\}\Psi=E\Psi \tag{12}$$

and

$$\left\{B_{0n}\frac{\partial}{\partial\phi_{\alpha i}^{(0)*}}+\frac{ig}{\sqrt{2}}C_n\tau_\alpha\sigma_i\right\}\Psi\cong 0 \qquad (n\neq 1) \tag{13}$$

(13) follows from the requirement that the emission operator $\phi_{\alpha i}^{(n)*}$ should be absent in (10). Tomonaga put

$$F_0(k)=\frac{U(k)}{\varepsilon^{3/2}I_1}, \qquad I_1^2=\frac{1}{3}\sum_{k}\frac{U(k)^2k^2}{\varepsilon^3} \tag{14}$$

It follows from (8)

$$C_0=I_1A_0, \qquad C_n=B_{0n}I_1$$

$$A_0=\frac{1}{3}\sum_{k}\frac{U(k)^2k^2}{\varepsilon^2I_1^2}, \qquad B_{0n}=\frac{1}{3}\sum_{k}\frac{U(k)k^2}{\varepsilon^{1/2}I_1}F_n(k) \tag{15}$$

(12) and (13) then become

$$\sum_{\alpha=1}^{3}\sum_{i=1}^{3}\left\{\phi_{\alpha i}^{(0)*}\frac{\partial}{\partial\phi_{\alpha i}^{(0)*}}+\frac{ig}{\sqrt{2}}I_1\tau_\alpha\sigma_i\left[\phi_{\alpha i}^{(0)*}-\frac{\partial}{\partial\phi_{\alpha i}^{0(*)}}\right]\right\}\Psi=\lambda\Psi \tag{16}$$

$$\lambda_1=E/A_0$$

$$\left\{\frac{\partial}{\partial\phi_{\alpha i}^{(0)*}}+\frac{ig}{\sqrt{2}}I_1\tau_\alpha\sigma_i\right\}\Psi\cong 0 \tag{17}$$

(16) is just the equation obtained by Tomonaga using his intermediate coupling approximation. The extent to which (17) is satisfied may be used as a measure of the accuracy of his approximation.

It will be noted that the solution of (12) is independent of any particular choice of $F_0(k)$. The concrete choice of $F_0(k)$ is important only when we want to satisfy the condition (11). Therefore in the following we shall proceed to solve (12) without fixing the function $F_0(k)$.

In the present investigation we shall develop a new method to solve (9) and (10). First we notice that τ_α and σ_i are Pauli matrices with two rows and two columns. Following Pauli and Dancoff we may introduce orthogonal transformations to the ordinary space and the isotopic spin space. Let τ'_ν and σ'_k be components of $\underset{\sim}{\tau}$ and $\underset{\sim}{\sigma}$ after the transformations, we may write

$$\tau_\alpha = \sum_{r=1}^{3} B_{r\alpha} \tau'_r \ , \quad \sigma_i = \sum_{i=1}^{3} A_{is} \sigma'_s \tag{18}$$

where $A_{r\alpha}$ and B_{is} satisfy the following conditions for orthogonal transformations:

$$\begin{aligned} &\sum_{\alpha=1}^{3} B_{r\alpha} B_{s\alpha} = \delta_{rs} \ , \quad \sum_{r=1}^{3} B_{r\alpha} B_{r\beta} = \delta_{\alpha\beta} \\ &\sum_{\alpha=1}^{3} A_{\alpha r} A_{\alpha s} = \delta_{rs} \ , \quad \sum_{r=1}^{3} A_{\alpha r} A_{\beta r} = \delta_{\alpha\beta} \end{aligned} \tag{19}$$

If we interprete above transformations as rotations of coordinates as was done by Pauli and Dancoff, $A_{\alpha\beta}$ and B_{is} must be real numbers. In Appendix I we shall show that if $A_{\alpha\beta}$ and B_{is} are allowed to be complex numbers the following results still hold true as when they are real numbers.

We may choose $A_{i s}$ and $B_{r\alpha}$ satisfying the following condition

$$\sum_{\alpha}\sum_{i} B_{r\alpha}\phi_{\alpha i}^{(o)*} A_{is} = \delta_{rs} Q_r . \tag{20}$$

Using (19) we can also write

$$\phi_{\alpha i}^{(o)*} = \sum_{r} A_{ir} B_{r\alpha} Q_r . \tag{21}$$

We notice that $\phi_{\alpha i}^{(o)}$ and $\phi_{\alpha i}^{(o)*}$ should be considered as a pair of conjugated complex numbers. In Appendix I we have shown that A_{is} and $B_{r\alpha}$ which satisfy (20) and (21) must also be complex numbers. There we have also shown that $\pi_{\alpha i}^{(o)*}$ which is the conjugate dynamical variable to $\phi_{\alpha i}^{(o)*}$ is given by

$$\pi_{\alpha i}^{(o)*} = -i\frac{\partial}{\partial\phi_{\alpha i}^{(o)*}} = \sum_{r}\sum_{k} A_{ir} B_{k\alpha}\left\{P_r\delta_{rk} + \frac{1}{2}\left[\frac{L^{rk}+T^{rk}}{Q_r - Q_k} - \frac{L^{rk}-T^{rk}}{Q_r + Q_k}\right](1-\delta_{rk})\right\} \tag{22}$$

where P_r is the momentum variable corresponding to Q_r, L^{rk} and T^{rk} are angular momentum operators in ordinary space and isotopic spin space respectively. They satisfy the following commutation relations

$$i\left[P_r , Q_s\right] = \delta_{rs} \tag{23}$$

$$i[L^{rs}, A_{kt}] = \delta_{rt} A_{ks} - \delta_{st} A_{kr} \tag{24}$$

$$i[T^{rs}, B_{t\alpha}] = \delta_{rt} B_{s\alpha} - \delta_{st} B_{r\alpha} . \tag{25}$$

The same expression (22) has also been obtained by Pauli and Dancoff using the concept of rotations in ordinary and isotopic spin spaces. It is now obtained by algebraic method as in our case the transformation (18) cannot be interpreted directly as rotations.

Inserting (21), (22) and (23) into (12) we obtain

$$\sum_r \Big\{ i A_0 Q_r P_r + \frac{g}{\sqrt{2}} C_0 \tau_r' \sigma_r' Q_r + i \frac{g}{\sqrt{2}} C_0 \tau_r' \sigma_r' P_r$$

$$+ \sum_{k \neq r} \frac{g C_0}{2\sqrt{2}} \tau_r' \sigma_r' R_{rk} \Big\} \Psi = E \Psi \tag{26}$$

where

$$R_{kr} = \frac{L^{kr} + T^{kr}}{Q_k - Q_r} - \frac{L^{kr} - T^{kr}}{Q_k + Q_r} . \tag{27}$$

We can easily verify the following relations

$$[R_{12}, \tau_1' \sigma_1' Q + \tau_2' \sigma_2' Q_2] = 2 \tau_2 \sigma_1$$

$$[R_{12}, \tau_3' \sigma_3' Q_3] = 0 \tag{28}$$

and similar relations obtained by cyclic permutations of the indices. These relations will be useful in calculations in Appendix II.

It will be sufficient for our purpose to look for the asymptotic solution of (26) which holds when the number of mesons is large. We shall show below that the eigen value λ is determined already by the asymptotic solution. After the asymptotic solution is obtained, (26) is needed only for continuing Ψ to the region where the number of mesons is small.

It is shown in Appendix II that when the number of meson is very large, the term containing $R_{k\gamma}$ in (26) can be neglected (this can also be seen qualitatively from the fact that P_γ and Q_γ are both proportional to the square root of the number of mesons n. This makes the term containing $R_{k\gamma}$ n times smaller than the other terms), we obtain

$$\sum_{\gamma}\left\{iQ_\gamma P_\gamma + \frac{ig}{\sqrt{2}} I \tau_\gamma' \sigma_\gamma' Q_\gamma + \frac{g}{\sqrt{2}} I \tau_\gamma' \sigma_\gamma' P_\gamma\right\}\psi = \lambda\psi$$

$$\lambda = E/A^{0}, \quad I = C_0/A_0 . \tag{29}$$

We notice that the following four quantities

$$1, \quad \tau_1'\sigma_1', \quad \tau_2'\sigma_2', \quad \tau_3'\sigma_3'$$

commute with one another. Consequently although they are rather complicated matrices, they can still be handeled as ordinary numbers in (29). Replacing iP_γ by $\partial/\partial Q_\gamma$, we have

$$\sum_{\gamma=1}^{3}\left\{Q_\gamma \frac{\partial}{\partial Q_\gamma} + \frac{ig}{\sqrt{2}} I \tau_\gamma' \sigma_\gamma' Q_\gamma - \frac{ig}{\sqrt{2}} I \tau_\gamma' \sigma_\gamma' \frac{\partial}{\partial Q_\gamma}\right\}\psi = \lambda\psi \tag{30}$$

Put

$$\Psi = \Psi_1(Q_1)\Psi_2(Q_2)\Psi_3(Q_3) \tag{31}$$

(30) becomes

$$\left\{Q_r\frac{d}{dQ_r}+\frac{ig}{\sqrt{2}}I\tau_r'\sigma_r'Q_r-\frac{ig}{\sqrt{2}}I\tau_r'\sigma_r'\frac{d}{dQ_r}\right\}\psi_r=\lambda_r\psi_r$$

$$(r=1,2,3) \tag{32}$$

and

$$\lambda=\lambda_1+\lambda_2+\lambda_3 \tag{33}$$

(32) are three total differential equations of the first order, the general solutions of which are found immediately to be

$$\psi_r=A_r\left[Q_r-\frac{ig}{\sqrt{2}}I\tau_r'\sigma_r'\right]^{\left(\frac{g^2}{4}I^2+\lambda_r\right)}\exp\left\{-\frac{ig}{\sqrt{2}}I\tau_r'\sigma_r'Q_r\right\} \tag{34}$$

where A_r is the normalization constant. By definition ψ_r must be a single valued power series of Q_r, this requires that

$$n_r=\frac{g^2}{4}I^2+\lambda_r \tag{35}$$

must be an integer. It will easily be seen that n_r cannot be a negative integer. If n_r is negative then when $Q\rightarrow-\frac{g}{\sqrt{2}}I\tau_r'\sigma_r'$, ψ_r will approach to infinity. Since infinite value cannot have any physical meaning, we conclude that n_r can only be a positive integer. From (35) we obtain

$$E=A_0(\lambda_1+\lambda_2+\lambda_3)=A_0(n_1+n_2+n_3-\frac{3g^2}{4}I^2) \tag{36}$$

The ground state of the system is given by n =0(r=1,2,3). The solution of (32) for the ground state is given by

$$\psi_0=A\exp\left\{-\frac{ig}{\sqrt{2}}I\sum_{r=1}^{3}\tau_r'\sigma_r'Q_r\right\}$$

$$E=A_0\lambda_0=\frac{-3}{4}g^2I^2A_0\,. \tag{37}$$

Since

$$\sum_{r=1}^{3} \tau_r' \sigma_r' q_r = \sum_\alpha \sum_i \tau_\alpha \sigma_i \phi_{\alpha i}^{(0)*} \tag{38}$$

(37) can also be written as

$$\Psi_0 = A \exp\left\{-\frac{ig}{\sqrt{2}} I \sum_\alpha \sum_i \tau_\alpha \sigma_i \phi_{\alpha i}^{(0)*}\right\}. \tag{39}$$

Using (5) and (8) we obtain further

$$\Psi_0 = A \exp\left\{-\frac{ig}{\sqrt{2}}\left[\sum_{\underset{\sim}{k}} U(k)\varepsilon^{-1/2} k^2 F_0(k)\right]\left[\sum_{\underset{\sim}{k}} k^2 F_0(k)^2 \varepsilon\right]^{-1} \sum_\alpha \sum_i \sum_{\underset{\sim}{k}} \tau_\alpha \sigma_i k_i F_0(k) \varphi_\alpha^*(\underset{\sim}{k})\right\}. \tag{40}$$

Inserting Tomonaga's assumption for $F_0(k)$ given by (14) into (40), we obtain

$$\Psi_0 = A_1 \exp\left\{-\frac{ig}{\sqrt{2}} \sum_\alpha \sum_i \sum_{\underset{\sim}{k}} \tau_\alpha \sigma_i k_i U(k) \varepsilon^{-3/2} \varphi_\alpha(k)^*\right\}. \tag{41}$$

It will be pointed out however, that Tomonaga's expression for $F_0(k)$, namely (14), does not satisfy (11) very well. To see this we insert (21) and (22) into equation (13) which is equivalent to (11), we obtain

$$\text{Bon}\left\{iP_r + \frac{g}{\sqrt{2}} I_1^{1/2} \tau_r' \sigma_r'\right\}\Psi \cong 0 \quad (\text{when } r = s) \tag{43}$$

$$\text{Bon}\left\{\frac{i}{2} R_{rs} + \frac{g}{\sqrt{2}} I^{1/2} \tau_s' \sigma_r'\right\}\Psi \cong 0 \quad (\text{when } r \neq s) \tag{44}$$

It can easily be verified by inserting (41) that the left-hand side of (43) vanishes identically, but the left-hand side of (44) is different from zero. It is difficult to assert from this how far this would violate the one wave equation assumption. To find a better $F_0(k)$ various authors have resorted to variational methods. For our calculation in the following section it is sufficient to leave $F_0(k)$ undeterminate. (40) can also be written in the following form:

$$\Psi_0 = A\exp\left\{-\frac{ig'}{\sqrt{2}}\sum_{\alpha}\sum_{i}\sum_{k}\tau_\alpha\sigma_i k_i F_0(k)\varphi_\alpha^*(k)\right\} \qquad (45)$$

where

$$g' = g\left[\sum_{k} U(k)\varepsilon^{-1/2}k^2F_0(k)\right] / \left[\sum_{k} k^2F_0(k)^2\varepsilon\right] \qquad (46)$$

It will be noted that (41) is just the solution of (1) if we from the beginning consider τ_α and σ_i as two classical unit vectors. Approximation of similar nature has been used before by Oppenheimer, Lewis and Wouthuyson[6] in treating the problem of multiple production of mesons by nucleon-nucleon collisions. It may be considered as the generalization of Bloch-Nordsieck approximation in quantum electro-dynamics in which the Dirac matrix α_i is treated also as a classical unit vector. This approximation has also been used by the present author [7] several years ago to obtain the same solution (41).

3. The average number of mesons in the proper field of a nucleon

From the asymptotic solution (45) for large n the probability P_n of finding n mesons in the proper field of a nucleon can be obtained immediately as

$$P_n = \frac{1}{n!}|A|^2\,\frac{g'^{2n}}{2^n}\left\{3\sum_{k} F(k)^2k^2\right\}^n \qquad (47)$$

replacing summation by integration, we obtain

$$P_n = \frac{1}{n!}|A|^2\,\frac{g'^{2n}}{2^n}\left\{\frac{3}{(2\pi)^3}\int_0^\infty F_0(k)^2k^2\cdot 4\pi k^2dk\right\}^n \qquad (48)$$

(48) gives the distribution in number of mesons outside the core. It should be remembered that n in (48) should be sufficiently

large to warrant the use of asymptotic solution (45). When n is very large (48) will decrease monotonously as n increases. An interesting situation may arise in which P_n first increases with n and then after passing through a maximum decreases monotonously. From (48) we find that this maximum occurs at n = N where

$$N = \frac{g'^2}{2} \cdot \frac{3}{2\pi^2} \int_0^\infty F_0(k)^2 k^4 dk \tag{49}$$

As we have mentioned before N must be sufficiently large to assure that the maximium derived from the asumptotic solution represents a real maximium. If (49) is the only absolute maximium, then N will also represent the average number of mesons in the field. In the following we shall see that this will be the case if the interaction is sufficiently strong.

(49) shows that the most probable number of meson N is determined by the function $F_0(k)$ and the constant g'. In the last section we have considered Tomonaga's expression for $F_0(k)$

$$F_0(k) = \frac{U(k)}{\varepsilon^{3/2} I_1}, \quad I_1^2 = \frac{1}{3} \frac{1}{(2\pi)^3} \int_0^\infty \frac{U(k)^2 k^2}{\varepsilon^3} \cdot 4\pi k^2 dk \tag{50a}$$

Using variational method Friedman, Lee and Christian[10/] obtained the following expression

$$F_0(k) = \frac{U(k)}{\varepsilon^{1/2}(\varepsilon + a) I_2}, \quad I_2^2 = \frac{1}{3} \frac{1}{(2\pi)^3} \int_0^\infty \frac{U(k)^2 k^2}{\varepsilon(\varepsilon + a)^2} \cdot 4\pi k^2 dk \tag{50b}$$

where a is a constant. When a is very large, (50b) becomes

$$F_0(k) = \frac{U(k)}{\varepsilon^{1/2} I_3} \quad , I_3^2 = \frac{1}{3}\frac{1}{(2\pi)^3}\int_0^\infty \frac{U(k)k^2}{\varepsilon} \cdot 4\pi k^2 dk \, . \tag{50c}$$

From the result of last section we have

$$g'^2 = g\int_0^\infty U(k)F_0(k)\frac{k^4}{\varepsilon^{1/2}}dk \Big/ \int_0^\infty F_0(k)^2 k^4 dk \, . \tag{51}$$

In the following we shall estimate the value of N using Tomonaga's expression (50a) and also the extreme case (50c). The expression (50b) of Friedman et al must lead to a result intermediate between these above two cases. In addition we shall use the following three types of cut-off function U(k):

(i) Straight cut-off

$$\begin{aligned} U(k) &= 1 \qquad k \leq K \\ U(k) &= 0 \qquad k > K \end{aligned} \tag{52a}$$

(ii) Gaussian cut-off

$$U(k) = e^{-k^2/2K^2} \tag{52b}$$

(iii) exponential cut-off

$$U(k) = e^{-k/K} \tag{52c}$$

For K we may use the value used by Chew and also by Friedman et al: K = 6.

Before we can proceed to calculate N we must also know the intrinsic interaction constant g . This g is different from the observed or renormalized interaction constant g_r by a renormalization factor R, i.e., $g^2 = Rg_r^2$. According to the calculation of Friedman et al, R is found to be as large as 7. On the other hand, a recent estimation by Fubini and Thirring[11] gives a much smaller value 2.2. The descripancy between these two values can be explaned as follows. As pointed out by Strof-folini[12] the solution of Friedman et al describes more accurately the situation at small distances from the core where the energy of the meson is high, while the solution of Fubini and Thirring, which correlates the experimental results at low energy region, describes more accurately the situation in the outer region of the meson cloud. Consequently the effect of charge renormalization is likely to be underestimated in the calculation of Fubini and Thirring owing to the neglection of the contribution from high energy mesons at small distances from the core. Since in our calculation we are mainly concerned with the situation at small distances from the core, we think that the value obtained by Friedman et al is a better one.

That the renormalization factor could be quite large can also be seen from the following consideration. It should be noted that in many fixed source theories, the renormalization factor R for g^2 is finite. This factor is equal to the ratio of the probability of the bare nucleon to emit or absorb a meson of a given charge to the same probability of the physical nucleon. Thus for the neutral scalar theory this renormalization factor is 1, since the core of the nucleon does not loss any ability to emit or absorb a meson owing to the presence of the meson cloud. For the charge scalar theory the upper limit of this factor is 2 which corresponds to the strong coupling limit when the core has equal probability of being a bare proton or a bare neutron. By similar reason for the charge-symmetrical scalar theory, the strong coupling limit gives the value 3. In our case of charge-symmetrical pseudo-scalar theory it can easily be seen that the upper limit for the renormalization factor is 9 (the isotopic spin space and the ordinary spin space each contributing a factor 3). The value 7 obtained by Friedman et al corresponds to the situation that the orientation of the core is 80% at random in the isotopic and ordinary spin spaces.

It will further be pointed out that the largeness of R is also a neccesary condition for our calculation to be self-consistent. In deriving (49) we have required that N should be sufficiently large. It can easily be seen that when N is large, the orientation of the core in isotopic and ordinary spin spaces must be quite random. From the above consideration it follows that R cannot be much less than the limiting value 9.

If we accept the value R = 7 and use for the renormalized coupling constant determined from experiments $g_r/4\pi$ = 0.1, then inserting (50) - (52) into (49), we can calculate the numerical values of N for different choices of $F_0(k)$ and U(k). The results are tabulated in Table I.

T a b l e I.

*

$F_0(k)$	U(k)/[illegible] I			U(k)/[illegible] II		
U (k)	straight cut-off	Gaussian cut-off	expon. cut-off	straight cut-off	Gaussian cut-off	expon. cut-off
N	10	7	19	13	13	26

We see from this table that N is indeed a large number. We see further than the largeness of N is quite independent of the form of $F_0(k)$ and U(k). It depends rather sensitively on the cut-off momentum K and the value of renormalization factor R, In our calculation we have assumed that R should be large, but the value of K has been fixed by Chew and many other authors before. In calculating N we have replace ξ by K in all integrals since the contribution to these integrals comes mainly from large values of k. The above result shows that for a wide range of $F_0(k)$ and $U(k)$, the most probable or average number of mesons in the vicinity of the core is of the order ten.

Based on our above picture of physical nucleon, the "S-star" observed by Lord, Fainberg and Schein[13/] may be explained immediately as formed by shaking off of the meson cloud outside the cores of two colliding nucleons. In center of mass system, the shower particles shaken off from each nucleon form a narrow cone containing about ten mesons moving in the direction of the initial nucleon.

From the above picture we see also that the average transverse component of momentum of the shower particles perpendicular to the line of collision should be $\approx K$. This is also in agreement with the obsergation of the S-star. We therefore conclude that the phenomena of meson shower production by nucleon collisions at energy 10^{17} ev fit quite well into our picture of nucleon structure. To explain the phenomena there is no urgent need to go to Fermi's statistical theory[14/] or even to hydrodynamical[15/] or non-linear theories[16/]. One decisive defect of all these theories is that they cannot give satisfactory explanation why anti-nucleons and strange particles are not produced in large quantities alongside with the shower of mesons.

The question now arises as to whether the ground state solution obtained above is consistent with experimental results at low energies. Since the elecromagnetic properties of the core are yet unknown, no definite statements on the magnetic moments and cross-sections for photo-production of mesons can be made by any cut-off theory unless the coupling is extremely strong, in which case the contribution of the core will vanish on the average. The nuclear potential cannot give any indication as to the number of mesons in the field since it is well known that in neutral scalar theory Yukawa potential is always obtained independent of the number of virtual mesons in the field. The same situation also happens in the strong coupling limit for all types of meson theories. As has been stressed by Fubini and Thirring, the only experimental material which can be used for comparison is the p-wave scattering. The results of Strof-folini and Friedman et al have shown that the experiment can always be explained by scattering integral equation involving only zero and one meson states <u>irrespective</u> of the form of solution of the ground state of the nucleon. In this sense we may conclude that the present theory is not inconsistent with any existing experimental result.

The results of last two sections may be summarized as follows: We have accepted Chew's non-relativistic model of physical nucleon more seriously and investigated the situation at small distances from the core. We found that if the renormalization factor constant for coupling is large, then the asymptotic solution for the ground state will be very useful for estimating the distribution of mesons at small distances from the core. Since the intrinsic coupling constant is large the coupling between the core and the field is quite strong in the vicinity of the core and the number of mesons there will be quite large. This gives a satisfactory explanation of the S-star observed by Lord et al. As the distance from the core increases, the effective coupling constant will decrease and the coupling between the core and the field will become weaker. In the outer region of the field, the renormalization factor will be equal to that given by Fubini and Ihirring. There the theory of Chew must be relied on to give correct descriptions of low energy phenomena.

Two further remarks are now in order. Firstly, when we accept (49) as giving the maximium in the region of large n, we have assumed that there is no other maximium in the region of small n. If the latter is the case, then the absolute maximium may not be given by (49) and the value N will no longer represent the <u>average</u> number of meson in the field. We know however from the result of Pauli and Dancoff that in the limit when the coupling is very strong, the absolute maximium is given by (49), and we have reasons to expect that this is still true when the coupling is moderately strong. (i.e., when the renormalization factor is 7 instead of 9). Since it is yet impossible to obtain the rigorous solution for the ground state, at the moment this expectation can only be an assumption. Secondly, although our assumption that the intrinsic coupling constant is large is consistent with know experimental results, the possibility that it is small cannot be overlooked. Fubini and Thirring have given some evidence for preferring a model of physical nucleon with a smaller value of intrinsic coupling constans. Further experimental as well as theoretical investigation is therefore need before definite conclusion can be drawn

Appendix I

A large part of the following results has been obtained by Pauli and Dancoff before. But in their calculations the meaning of $\Phi_{\alpha s}^{(o)}$ is entirely different. In their case $\Phi_{\alpha s}^{(o)}$, Q_r are all real quantities, whereas our $\Phi_{\alpha s}^{(o)}$ and Q_r must be conside-red as complex quantities. Therefore in our calculations we shall avoid to use the geometrical meaning of A_{sk} and $B_{r\alpha}$ and their representation in terms of Euler angles.

We introduce the following transformation in the ordinary space and the isotopic spin space:

$$\sigma_s = \sum_{k=1}^{3} A_{sk}\sigma_k' \tag{1'}$$

$$\tau_\alpha = \sum_{r=1}^{3} B_{r\alpha}\tau_r' \tag{2'}$$

A_{sk} and $B_{r\alpha}$ are all complex numbers but satisfy the follow-ing conditions of orthogonal transformation.

$$\begin{aligned} &\sum_\alpha B_{r\alpha}B_{s\alpha} = \delta_{rs}, \quad \sum_r B_{r\alpha}B_{r\beta} = \delta_{\alpha\beta} \\ &\sum_\alpha A_{\alpha r}A_{\alpha s} = \delta_{rs}, \quad \sum_r A_{\alpha r}A_{\beta r} = \delta_{\alpha\beta} \end{aligned} \tag{3'}$$

We may choose A_{ks} and $B_{r\alpha}$ such that

$$\sum_{\alpha,s} B_{r\alpha}\Phi_{\alpha s}A_{sk} = \delta_{rk}Q_r$$

In order to prove that this is possible, we consider the fol-lowing two matrices

$$C_{rs} = C_{sr} = \sum_\alpha \Phi_{\alpha r}\Phi_{\alpha s}, \quad C'_{\alpha\beta} = C'_{\beta\alpha} = \sum_r \Phi_{\alpha r}\Phi_{\beta r}$$

C_{rs} and $C'_{\alpha\beta}$ should be considered as complex numbers.

Since any symmetrical complex matrix can be transformed into diagonal matrix by a complex orthogonal transformation, we may choose A_{sk} and $B_{r\alpha}$ to satisfy

$$\sum_{r,s} A_{r\alpha} C_{rs} A_{s\beta} = \delta_{\alpha\beta} W_\alpha \tag{6'}$$

$$\sum_{\alpha\beta} B_{r\alpha} C'_{\alpha\beta} B_{s\beta} = \delta_{rs} W'_r \tag{7'}$$

Using (5) and (3) above two relations can be written

$$\sum_t \Omega_{t\alpha} \Omega_{t\beta} = \delta_{\alpha\beta} W_\alpha \tag{8'}$$

$$\sum_t \Omega_{st} \Omega_{rt} = \delta_{rs} W'_r \tag{9'}$$

where

$$\Omega_{t\beta} = \sum_{\tau,s} B_{t\tau} \Phi_{\tau s} A_{s\beta} \tag{10'}$$

(6) and (7) can also be written as

$$\sum_s C_{rs} A_{s\beta} = A_{r\beta} W_\beta \tag{11'}$$

$$\sum_\alpha B_{r\alpha} C_{\alpha\beta} = B_{r\beta} W'_r \tag{12'}$$

Since

$$\sum_{r,\alpha} \varphi_{\rho r} \varphi_{\alpha r} \varphi_{\alpha s} = \sum_r \varphi_{\rho r} C_{rs} = \sum_\alpha C'_{\rho\alpha} \varphi_{\alpha s} \tag{13'}$$

multiply (11') from the right by $\varphi_{\rho r}$ we obtain

$$\sum_{r,s} \varphi_{\rho r} C_{rs} A_{s\beta} = \sum_{\alpha,s} C'_{\rho\alpha} \varphi_{\alpha s} A_{s\beta} = \sum_{r} \varphi_{\rho r} A_{r\beta} W_\beta .$$

Multiply again by $B_{n\rho}$

$$\sum_{\alpha,s,\rho} B_{n\rho} C'_{\rho\alpha} \varphi_{\alpha s} A_{s\beta} = \sum_{\rho,r} B_{n\rho} \varphi_{\rho r} A_{r\beta} W_\beta .$$

From (12') we obtain

$$\sum_{\alpha s} B_{n\alpha} W'_n \varphi_{\alpha s} A_{s\beta} = \sum_{r\rho} B_{n\rho} \varphi_{\rho r} A_{r\beta} W_\beta$$

or

$$W'_n \Omega_{n\beta} = \Omega_{n\beta} W_\beta . \tag{14'}$$

If W'_1, W'_2 W'_3 are all distinct, (14') gives immediately

$$W'_n = W_n \tag{15'}$$

and that $\Omega_{n\beta}$ is a diagonal matrix. If W'_1 W'_2, W'_3 are not distinct, e.g., $W'_1 = W'_2$, we may consider the case as the limiting case of $W'_1 \rightarrow W'_2$ and obtain the same result. From (8'), (9') we have

$$\Omega_{r\alpha} = \sum_{\tau,s} B_{r\tau} \Phi_{\tau s} A_{s\alpha} = \pm \delta_{r\alpha} \sqrt{W_r} \tag{16'}$$

(16') proves that (4') is possible and $Q_r = \pm\sqrt{W_r}$

In the following we shall use (3') to derive some important relations. (i). The angular momentum operator of the meson field is

$$L_{ik} = \sum_{\alpha} (\varphi_{\alpha i} \pi_{\alpha k} - \varphi_{\alpha k} \pi_{\alpha i}) \tag{19'}$$

Using commutation relations between $\pi_{\alpha k}$ and $\varphi_{\alpha i}$ we prove easily

$$i\left[L_{ik}, \sum_{s} \varphi_{\alpha s} \varphi_{\beta s}\right] = 0$$

$$i\left[L_{ik}, \sum_{\alpha} \varphi_{\alpha s} \varphi_{\alpha s}\right] = 2\sum_{\alpha} \varphi_{\alpha i} \varphi_{\alpha s} \delta_{ks} - 2\sum_{\alpha} \varphi_{\alpha k} \varphi_{\alpha s} \delta_{is}$$

Above two relations can also be written as

$$i\left[L_{ik}, C'_{\alpha\beta}\right] = 0 \tag{20'}$$

$$i\left[L_{ik}, C_{ss}\right] = 2C_{is}\delta_{ks} - 2C_{ks}\delta_{is} \tag{21'}$$

From (7') and (15'), we obtain

$$C'_{\alpha\beta} = \sum_{r} B_{r\alpha} W_r B_{r\beta} \tag{22'}$$

Inserting (22') into (20') we obtain

$$i\left[L_{ik}, W_r\right] = 0 \tag{23'}$$

From (6') we have

$$C_{st} = \sum_{r} A_{sr} W_r A_{tr} \tag{24'}$$

Because of (23'),

$$i\left[L_{ik}, C_{ss}\right] = 2\sum_{r} A_{ir} W_r A_{sr} \delta_{sk} - 2\sum_{r} A_{kr} W_r A_{sr} \delta_{is}. \tag{25'}$$

Inserting (24) into (21') and comparing with (25'), We obtain finally

$$i\left[L_{ik}, A_{sr}\right] = A_{ir}\delta_{ks} - A_{kr}\delta_{is} \tag{26'}$$

(ii) The angular momentum operator in isotopic spin space is given by

$$T_{\alpha\beta} = \sum_{r} \left(\varphi_{\alpha r} \pi_{\beta r} - \varphi_{\beta r} \pi_{\alpha r}\right) \tag{27'}$$

By similar calculation we can prove

$$i\left[T_{\alpha\beta}\, B_{mr}\right] = \delta_{\beta m} B_{\alpha r} - \delta_{\alpha m} B_{\beta r} \tag{28'}$$

(iii) Let

$$L^{rs} = \sum_{i,k} A_{ir} A_{ks} L_{ik} \; ; \tag{29'}$$

$$T^{rs} = \sum_{\alpha,\beta} B_{r\alpha} B_{s\beta} T_{\alpha\beta} \; . \tag{30'}$$

Using (26') we obtain

$$i\left[L^{rs}, A_{et}\right] = \sum_{i,k} A_{ir} A_{ks} \left[L_{ik}, A_{et}\right] = \delta_{rt} A_{es} - \delta_{st} A_{er} . \tag{31'}$$

Similarly from (28') we obtain

$$i\left[T^{rs}, B_{t\alpha}\right] = \sum_{\alpha,\beta} B_{r\alpha} B_{s\beta} \left[T_{\alpha\beta}, B_{kt}\right] = \delta_{rt} B_{s\alpha} - \delta_{st} B_{r\alpha} . \tag{32'}$$

From (19'), (27'), (29') and (30') we obtain

$$L^{rs} = \sum_{k,\alpha} \left(Q_r A_{ks} B_{r\alpha} \overset{(o)}{\pi}_{\alpha k} - Q_s A_{kr} B_{s\alpha} \overset{(o)}{\pi}_{\alpha k}\right)$$

$$T^{rs} = \sum_{k,\alpha} \left(Q_r A_{kr} B_{s\alpha} \overset{(o)}{\pi}_{\alpha k} - Q_s A_{ks} B_{r\alpha} \overset{(o)}{\pi}_{\alpha k}\right.$$

which give when $r \neq s$,

$$\sum_{\alpha,k} A_{kr} B_{s\alpha} \overset{(o)}{\pi}_{\alpha k} = \frac{1}{2}\left\{\frac{L^{rs} + T^{rs}}{Q_r - Q_s} - \frac{L^{rs} - T^{rs}}{Q_r + Q_s}\right\} \tag{33'}$$

and when $r = s$, we may define

$$\sum_{\alpha,k} A_{kr} B_{r\alpha} \overset{(o)}{\pi}_{\alpha k} = P_r \; . \tag{34'}$$

From (33') and (34') we obtain finally:

$$\overset{(o)}{\pi}_{\alpha s} = \sum_{r,k} A_{sr} B_{k\alpha} \left\{P_r \delta_{rk} + \frac{1}{2}\left(\frac{L^{rk} + T^{rk}}{Q_r - Q_k} - \frac{L^{rk} - T^{rk}}{Q_r + Q_k}\right)\left(1 - \delta_{rk}\right)\right\} \tag{35'}$$

Appendix II.

In the following we shall investigate the effect of neglecting terms containing R_{kr} in equation (26). We consider the term proportional to $(\sum_r \tau_r' \sigma_r' Q_r)^n$ of (37):

$$T_n = A\frac{1}{n!}(-g)^n(\tfrac{1}{2}I)^{n/2}(\sum_{r=1}^{3}\tau_r'\sigma_r' Q_r)^n \tag{36'}$$

We first evaluate the result of the terms containing R_{12} applying to (36'). Expanding (36') in the following form

$$T_n = A(-g)^n(\tfrac{1}{2}I)^{n/2}\sum_{n_3=0}^{n}\frac{1}{(n-n_3)!n_3!}(\tau_1'\sigma_1'Q_1+\tau_2'\sigma_2 Q_2)^{n-n_3}(\tau_3'\sigma_3'Q_3)^{n_3} \tag{37'}$$

and using (28) we obtain

$$\frac{gI^{1/2}}{2\sqrt{2}}(\tau_2'\sigma_1')R_{12}T_n = -A(-g)^{n+1}(\tfrac{1}{2}I)^{(n+1)/2}(\tau_2'\sigma_1')\sum_{n_3=0}^{n-1}\sum_{\ell=0}^{n-n_3-1}\frac{1}{(n-n_3)!n_3!} \tag{38'}$$

$$(\tau_1'\sigma_1'Q_1+\tau_2'\sigma_2'Q_2)^{n-n_3-\ell+1}(\tau_2'\sigma_1')(\tau_1'\sigma_1'Q_1+\tau_2'\sigma_2 Q_2)^{\ell}(\tau_3'\sigma_3'Q_3)^{n_3}.$$

Since $\tau_2'\sigma_1'$ anti-communties with $(\tau_1'\sigma_1'Q_1+\tau_2'\sigma_2'Q_2)$ we may displace the factor $(\tau_2'\sigma_1')$ by using this anti-commutation relation to the right. After adding the result obtained by interchanging 1 and 2, we obtain

$$\frac{g}{2\sqrt{2}}I^{r/2}\left[t_2'\sigma_1'R_{12}+\tau_2'\sigma_2'R_{21}\right]T_n = -A(-g)^{n+1}(\tfrac{1}{2}I)^{\frac{n+1}{2}}\sum_{n_3=0}^{n-1}\left[1-(-1)^{n-n_3}\right]\frac{1}{(n-n_3)!n_3!}$$
$$(\tau_1'\sigma_1'Q_1+\tau_2'\sigma_2'Q_2)^{n-n_3-1}(\tau_3'\sigma_3'Q_3)^{n_3}. \tag{39'}$$

On the other hand, inserting (37) into (30), we see that the first two terms of (30) operating on (37) give rise to zero. The part containg $\tau_3'\sigma_3'$ in the third term of (30) when applied to (36') gives

$$\frac{g}{\sqrt{2}} I^{1/2} \tau_3' \sigma_3' \frac{\partial}{\partial Q} T_n = A \frac{1}{(n-1)!} (-g)^{n+1} \left(\tfrac{1}{2} I\right)^{\frac{n+1}{2}} \left(\sum_{r=1}^{3} \tau_r' \sigma_r' Q_r \right)^{n-1}$$

$$A(-g)^{n+1} \left(\tfrac{1}{2} I\right)^{\frac{n+1}{2}} \sum_{n_3=0}^{n-1} \frac{1}{(n-n_3-1)!\,n_3!} (\tau_1' \sigma_1' Q_1 + \tau_2' \sigma_2' Q_2)^{n-n_3-1} (\tau_3' \sigma_3' Q_3)^{n_3}$$

(40')

We see that the coefficient of $(\tau_1' \sigma_1' Q_1 + \tau_2' \sigma_2' Q_2)^{n-n_3-1} (\tau_3' \sigma_3' Q_3)^{n_3}$ in (39') is smaller than the corresponding coefficient in (40') by a factor $[1-(-1)^{n-n_3}]/[n-n_3]$. When $n - n_3$ is even this factor is zero; when $n - n_3$ is odd this factor is $2/(n - n_3)$. When n is very large, the terms in (39') and (40') which represent the largest probability are given by $n_3 \approx n - n_3 \approx n/2$. Therefore the ratio of terms in (39') and (40') corresponding to the largest probability is equal to zero or $4/n$. This suffices to show that when n is very large (39') may be neglected in comparison to (40'). In other words the term containing R_{rk} in (26) may be neglected when n is very large.

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