J O I N T I N S T I T U T E F O R N U C L E A R R E S E A R C H

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ELECTROMAGNETIC INTERACTION OF ANOMALOUS PARTICLES WITH SPIN 1/2

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A B S T R A C T:

It is shown in this paper that relativistic wave equations for particles with spin 1/2 which are different from Dirac equations exist. Particles described by these "anomalous" equations can have one or more self masses and the corresponding fields have positive definite charge density. The electromagnetic interaction of particles described by the simplest "anomalous" equations is investigated, magnetic moment, cross-sections for Coulomb and Compton scattering, and self energy are determined. All these quahtities differ from analogous expressions for electrons.

Introduction

Attempts to find relativistic wave equations of known type

$$(\beta^{\nu}\partial_{\nu} - i\mu)\varphi = 0 \tag{1}$$

for particles with spin 1/2 but differnnt from Dirac equations were not successfull. The proofs of Wild [1] and Gelfand and Yaglom [2] about the uniqueness of Dirac equations have spoken against them. But their proofs are not general. They are based on the assumption that the equations for particles with the spin 1/2 can be derived only in the frame of the representation of full Lorentz group for maximum value of spin 1/2.

Because such assumption has no reason we can attempt to look for new equations for particles with spin 1/2 starting from some other representation of full Lorentz group. We have taken the representation D_R for maximum value of spin 3/2 and we have been abhe to show that "anomalous" equations for particles with spin 1/2 do exist.

1. Relativistic form of the anomalous equations

If the equations (1) are covariant, matrices β_ν fulfil following known relations:

$$[\beta_\nu I_{\mu\sigma}] = \cancel{g_{\nu\sigma}} - g_{\nu\sigma}\beta_\mu + g_{\nu\mu}\beta_\sigma \qquad (2a)$$

$$[I_{\mu\nu} I_{\sigma\rho}] = -g_{\mu\sigma}I_{\nu\rho} + g_{\nu\sigma}I_{\mu\rho} + g_{\mu\rho}I_{\nu\sigma} - g_{\nu\rho}I_{\mu\sigma}, \qquad (2b)$$

$$\beta_0 Z - Z\beta_0 = 0, \qquad \beta_k Z + Z\beta_k = 0, \quad k = 1,2,3, \qquad (2c)$$

where $I_{\mu\nu}$ represent infinitesimaly small rotations and Z is the inversion matrix.

With the help of these equations and the known representation of the full Lorentz group i.e. representation of $I_{\mu\nu}$ and Z we can find matrices β_ν. There are three inequivalent representations of $I_{\mu\nu}$ and Z for max. spin 3/2: 12 rows, 8 rows and 4 rows which we denote by τ_3, τ_2, τ_1 respectively.

We have studied more deeply only such cases in which the representations of $I_{\mu\nu}$ are given by direct sums:

$$I_{\mu\nu} = I_{\mu\nu}^{(\tau_3)} \oplus n\, I_{\mu\nu}^{(\tau_1)}, \qquad n = 0,1,2,3,\dots$$

It was shown in [3] that matrices β_ν for energy algebra with maximum spin 3/2 can be decomposed into direct product of Dirac matrices γ_ν and certain adjoint matrices α_ν

$$\beta_\nu = \gamma_\nu \otimes \alpha_\nu \qquad \text{(no sumation!)}$$

The explicit general form of matrices α_ν was also derived in [3] by use of equations (2a,b,c) only. But the coefficients of matrices α_ν are not determined only by invariant condition, it is necessary to impose on them further restrictive conditions: the field must have positive definit charge density and the matrices β_ν

must be irreducibel.

It is not too difficult to show that the case for n = 0 is not permissible and that the case for n = 1 gives only the well-known Pauli-Fierz equations for particles with spin 3/2 and one self mass. The case for n = 2 is more complicated. Now we can choose the coefficients of matrices α_ν in several ways: first we get the equations found by Ginsburg [4] and Bhabha [5] for particles with the spin states 3/2 and 1/2 and two masses $m_{3/2}$ and $m_{1/2}$ resp., second the Harish-Chandra equations for particles with two spin states with one mass only, and third two different cases which lead to the equations for particles with spin 1/2 and one proper mass. Matrices α_ν in the both "anomalous" cases are 5-rows and have the following form:

$$\alpha_0 = \begin{vmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \nu_1 & \nu_2 \\ \cdot & \cdot & \cdot & -\sqrt{\tfrac{1}{2}}\nu_1 & -\sqrt{\tfrac{1}{2}}\nu_2 \\ \cdot & S_1 & -\sqrt{\tfrac{1}{2}}S_1 & \kappa_1 & \cdot \\ \cdot & S_2 & -\sqrt{\tfrac{1}{2}}S_2 & \cdot & \kappa_2 \end{vmatrix} \qquad \alpha_1 = \begin{vmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & -\nu_1 & -\nu_2 \\ \cdot & \cdot & \cdot & -\sqrt{\tfrac{1}{2}}\nu_1 & -\sqrt{\tfrac{1}{2}}\nu_2 \\ \cdot & -S_1 & -\sqrt{\tfrac{1}{2}}S_1 & \kappa_1 & \cdot \\ \cdot & -S_2 & -\sqrt{\tfrac{1}{2}}S_2 & \cdot & \kappa_2 \end{vmatrix}$$

$$\alpha_2 = \begin{vmatrix} \cdot & \cdot & \cdot & \nu_1 & \nu_2 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & -\sqrt{\tfrac{1}{2}}\nu_1 & \sqrt{\tfrac{1}{2}}\nu_2 \\ S_1 & \cdot & -\sqrt{\tfrac{1}{2}}S_1 & \kappa_1 & \cdot \\ S_2 & \cdot & \sqrt{\tfrac{1}{2}}S_2 & \cdot & \kappa_2 \end{vmatrix} \qquad \alpha_3 = \begin{vmatrix} \cdot & \cdot & \cdot & -\nu_1 & -\nu_2 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \sqrt{\tfrac{1}{2}}\nu_1 & -\sqrt{\tfrac{1}{2}}\nu_2 \\ -S_1 & \cdot & \sqrt{\tfrac{1}{2}}S_1 & \kappa_1 & \cdot \\ -S_2 & \cdot & -\sqrt{\tfrac{1}{2}}S_2 & \cdot & \kappa_2 \end{vmatrix}$$

where

$$V_1^2 = \frac{2}{3}\varepsilon_1 K_1^2 \frac{K_2}{K_1-K_2}, \quad V_2^2 = \frac{2}{3}\varepsilon_2 K_2^2 \frac{-K_1}{K_1-K_2}, \quad S_1 = \varepsilon_1 V_1, \quad S_2 = \varepsilon_2 V_2 \qquad (3a)$$

In the first anomalous case we have:

$$K_1 + K_2 = 1, \quad K_1 > K_2 > 0, \quad \varepsilon_1 = -\varepsilon_2 = 1, \qquad (3b)$$

and in the second

$$K_1 + K_2 = 1, \quad K_1 > 0 > K_2, \quad \varepsilon_1 = -\varepsilon_2 = 1. \qquad (3c)$$

The matrices β_μ are irreducible if and only if

$$K_1 \pm K_2 \neq 0, \quad V_1 \neq 0, \quad V_2 \neq 0$$

and they satisfy the relations

$$\sum_P \left(\beta_\mu \beta_\nu \beta_\lambda \beta_\rho - g_{\mu\nu} \beta_\lambda \beta_\rho\right) = 0$$

where $\sum_P$ means the sum over all permutations of indices.

The coefficients of matrices β_μ are not completely determined by the relations (3). One coefficient is free and represents free parameter of the particle. Its role will be clear later.

For higher n, i.e. $n > 2$ we obtain among the various equations for particles with spin 3/2 further equations for particles with spin 1/2. They describe the particles with n-1 mass-states [6] and contain n-1 free parameters.

In the following we restrict ourselfes only to the simplest anomalous wave equations. It is convenient to write down them explicitly in Rarita-Schwinger form:

$$\begin{aligned} &-\left(2\partial^\mu + \gamma^\mu \gamma^\nu \partial_\nu\right)\left(V_1 \chi_1 + V_2 \chi_2\right) = m B^\mu, \\ &\frac{1}{2} S_1 \partial_\mu B^\mu - i K_1 \gamma_\mu \partial^\mu \chi_1 = m \chi_1, \end{aligned} \qquad (4)$$

where

$$V_1^2 = \frac{2}{3}\varepsilon_1 K_1^2 \frac{K_2}{K_1-K_2}, \quad V_2^2 = \frac{2}{3}\varepsilon_2 K_2^2 \frac{-K_1}{K_1-K_2}, \quad S_1 = \varepsilon_1 V_1, \quad S_2 = \varepsilon_2 V_2 \quad (3a)$$

In the first anomalous case we have:

$$K_1 + K_2 = 1, \quad K_1 > K_2 > 0, \quad \varepsilon_1 = \varepsilon_2 = 1 \qquad \textbf{(3b)}$$

and in the second

$$K_1 + K_2 = -1, \quad K_1 > 0 > K_2, \quad \varepsilon_1 = \varepsilon_2 = 1. \qquad \textbf{(3c)}$$

The matrices β_μ are irreducible if and only if

$$K_1 \neq \pm K_2, \quad V_1 \neq 0, \quad V_2 \neq 0$$

and they satisfy the relations

$$\sum_P (\beta_\mu \beta_\nu \beta_\alpha \beta_\rho - g_{\mu\nu} \beta_\alpha \beta_\rho) = 0$$

where $\sum_P$ means the sum over all permutations of indices.

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For higher n, i.e. $n > 2$ we obtain among the various equations for particles with spin 3/2 further equations for particles with spin 1/2. They describe the particles with n-1 mass-states [6] and contain n-1 free parameters.

In the following we restrict ourselfes only to the simplest anomalous wave equations. It is convenient to write down them explicitly in Rarita-Schwinger form:

$$\begin{aligned} -(2\partial^\mu + \gamma^\nu\gamma^\mu\partial_\nu)(V_1\chi_1 + V_2\chi_2) &= m B^\mu, \\ \frac{1}{2} S_1 \partial_\mu B^\mu - iK_1\gamma_\mu\partial^\mu\chi_1 &= m\chi_1, \\ \frac{1}{2} S_2 \partial_\mu B^\mu - iK_2\gamma_\mu\partial^\mu\chi_2 &= m\chi_2 \end{aligned} \qquad \textbf{(4)}$$

Here χ_1 and χ_2 are usual Dirac spinors, B^{μ} is spinvector introduced by Rarita and Schwinger [7] , γ_{ν} are ordinary Dirac matrices.

2. Free anomalous particle

It is easy to verify with the help of the form (4), that the equations describe particles posessing spin 1/2. If we have free particle without any interaction we get from (4) the equation for spinor χ_1

$$\gamma_{\mu}\partial^{\mu}\chi_1 - i\,\frac{M}{K_1+K_2}\,\chi_1 = 0 \qquad (5)$$

and subsidiary conditions

$$K_2^2\,\gamma_1\chi_1 + K_1^2\,\gamma_2\chi_2 = 0$$

$$-\gamma^{\nu}\left(1-\frac{K_2^2}{K_1^2}\right)\left(4\partial^{\kappa} - i\,\frac{M}{K_1+K_2}\,\gamma^{\kappa}\right)\chi_1 = \mu B_1^{\kappa} \quad \kappa=1,2,3, \qquad (6)$$

$$\gamma_{\nu}B^{\nu} = 0$$

by which χ_2 and B^{ν} are determined completely. The equation (5) is usual Dirac equation as we had also tb expect. It is necessary to stress that for anomalous particle in interaction is impossible to derive for spinor χ_1 or χ_2 the Dirac equation. The equations (6)— there are 20 subsidiary conditions there — exclude the superfluous states with spin 3/2 and 1/2.

3. Electromagnetic interaction

The wave equations for free anomalous particle can be derived from Lagrangian because the matrix Λ satisfying the conditions

$$\Lambda \beta_\nu = \beta_\nu^{+} \Lambda ,$$

where $\Lambda = \gamma_0 \otimes \eta$, η = diag $/1,1,1, \varepsilon_1, \varepsilon_2/$ exists. Therefore we have the usual Lagrangian

$$L = i\left(\bar{\psi} B^{\mu} \partial_{\mu} \psi - i \mu \bar{\psi} \psi\right) \text{ with } \bar{\psi} = \psi^{+} \Lambda$$

The "switch on" of the electromagnetic field does not represent any principial difficulty. We can define the interaction term in the classical way by

$$L_{int} = \left(j_\mu A^\mu\right)$$

The electromagnetic interaction does not change the number of subsidiary conditions only their form is different from (6).

To examine the behaviour of anomalous particle interacting with electromagnetic field we have calculated: (a) the intrinsic magnetic moment, differential cross-sections for (b) Coulomb and (c) Compton scattering, and (d) self energy of the particle.

a. Intrinsic magnetic moment

In the non-relativistic approximation we can derive the intrinsic magnetic moment of the anomalous particle (e.g. by the method used in [8]). In both cases the magnetic moment has the value (measured in Bohr's magnetons).

$$m = 1 - 2 g \quad \text{where} \quad g = \frac{4}{3} \ldots$$

For $K_2 > 0$ is

$$\frac{1}{3} < m < 1$$

and for $K_2 < 0$ is

$$m > 1$$

<u>Magnetic moment is anomalous.</u> The anomality is given by the factor g . The values m = 1 or $M \lessgtr \frac{1}{3}$ are excluded, because in these cases the wave equations are reducibal.

b. Coulomb scattering

The Coulomb scattering was calculated in the first approximation of the perturbation theory by Feynman method. Higher approximation and radiative corrections were not considered because mathematical difficulties. We took for the transition amplitude the expression

$$\bar{u}\beta_0 u$$

The differential cross-section for the scattering on fixed center is then given by

$$d\sigma = \pi\left(\frac{e^2 Z}{\mu}\right)\frac{\mu^2}{p^2}\,\frac{d\cos\vartheta}{(1-\cos\vartheta)^2}\left[\frac{2\mu^2}{p^2} + 1 + \cos\vartheta + 4\frac{\varepsilon}{g}\left(1 + g\frac{p^2}{\mu^2}\right)(1-\cos\vartheta)\right].$$

Last term in the square bracket represents just the deviation from similar expression for electron. It is caused by the existence of anomalous magnetic moment. $d\sigma$ differs also from cross-section for ordinary 1/2 spin particle with anomalous electromagnetic interaction as was used in the theory of Pauli.

The main difference against the cross-section for electron is in non-vanishing high energy term. For high energies of the incident particle $d\sigma$ tends to the constant value.

c. Compton effect

The Compton effect was considered by the same method as the Coulomb scattering. Starting from the transition amplitude

$$(\beta e)\left[(\beta_1 p_0 + k_0) - \mu\right]^{-1}(\beta e_0) + (\beta e_0)\left[(\beta_1 p_0 - k) - \mu\right]^{-1}(\beta e)$$

we calculated the cross-section $d\sigma_{\parallel}$ for the light polarised after scattering in the same plane as the incident light and $d\sigma_{\perp}$ for the outgoing light polarixed perpendicularly to the polarisation plane of incident light. The exact expressions are very complicated and therefore we write down only approximate results for non-relativistic and relativistic cases.

In the non-relativistic limit is

$$\frac{d\sigma_{\parallel}}{d\Omega} \approx \left(\frac{d\sigma_{\parallel}}{d\Omega}\right)_{\text{electron}}, \quad \frac{d\sigma_{\perp}}{d\Omega} \sim 4\frac{e^4}{\mu^2} g^4 k_0^2$$

and in extreme relativistic limit:

a . for large scattering angles:

$$\frac{d\sigma_{\parallel}}{d\Omega} \sim \frac{1}{k_0}, \quad \frac{d\sigma_{\perp}}{d\Omega} \sim \frac{1}{k_0}$$

b. for small angles:

$$\frac{d\sigma_{\parallel}}{d\Omega} \approx \left(\frac{d\sigma_{\parallel}}{d\Omega}\right)_{\text{electron}}; \quad \frac{d\sigma_{\perp}}{d\Omega} \approx 4\frac{e^4}{\mu^2} g^4 k_0^2$$

where k_0 is the energy of incident photon measured in the units of mass of the anomalous particle. $d\sigma$ is given by $d\sigma_{\parallel} + d\sigma_{\perp}$. In the extreme relativistic case the total cross-section

$$\sigma_T \approx 4\pi g^4 \frac{e^4}{\mu^2} k_0$$

is increasing linearly with the energy in contrary to the total cross-section for electron. The interesting deviation of $d\sigma_{\perp}$ for non-relativistic case could become the basis of investigation for what kind of particles to apply these anomalous equations.

d. Electromagnetic self-energy

In analogous way as before the electromagnetic self-energy of the anomalous particle was calculated. We obtained

$$W_{sel} = \frac{e^2 m}{\pi}(\bar{u}u)\lim_{\varepsilon\to\infty}\left[\frac{1}{4}g^2\varepsilon^2 + (3-6g+5g^2)\ln\varepsilon - \frac{1}{2}(1-6g+6g$$

where ε is invariant quantity and u is amplitude of the wave function. Although the self-energy diverges quadratically the theory can by renormalised. The utilisation of this fact is limited becau- se complicated mathematic procedure by higher approximations.

Conclusion

The behaviour of the anomalous particle interacting with electromagnetic field is dependent on the parametr g. We can call it as a parametr of anomality. For g = 0 we get the results known from the theory of electron.

It is very difficult to say if the anomalous equations are usefull for description of some particles with spin 1/2 as for example hyperons, nucleons or μ mesons on the basis of results which we obtained. Assuming especially the anomalous equations are convenient for μ meson we must put $g \ll 1$ according to the experiments of Ledermann. Then all deviations are neglible and the electromagnetic interaction does not unable us to distinguish between electron and μ meson. Therefore we have just now begun to study other type of interactions.

It is possible to introduce π mesons interaction without any incompatibility with subsidiary conditions. But in this case the problem is complicated in other manner. For a given type of interaction - for example pseudovector - there are several independent invariants* and it is difficult to choose among them the correct invariant form before calculations.

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* For scalar field Φ with scalar coupling to anomalous particles φ we have following linearly independent invariant forms:

$$\Phi\bar{\varphi}\varphi,\ \Phi\bar{\varphi}\beta^{\mu}\beta_{\mu}\varphi,\ \Phi\bar{\varphi}(\beta^{\mu}\beta_{\mu})^2\varphi,\ \Phi\bar{\varphi}\beta_{\mu}\beta_{\nu}\beta^{\mu}\beta^{\nu}\varphi$$