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On the Production of Mesons in Nucleon-Nucleon Collisions and the Nucleon Structure

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A B S T R A C T

An effective cross section for the production of mesons and the relative losses of energy in inelastic peripheral collisions of nucleons with the energy T = 1 - 10 BeV are calculated on ground of the conception of the nucleon structure.

It is essential that the presence of a "core" within a nucleon leads to two types of collisions with small and large energy losses.

INTRODUCTION

The experimental data on the production of mesons by the collision of high energy nucleons are compared at present with the results of calculations based on the Fermi and Landau theories. However the Fermi-Landau theory evidently refers to very high energies of nucleons only and in particular does not explain the fact that the mean energy losses typical for the interactions of the cosmic ray nucleons with light nuclei are small[1].*

Small mean energy losses of the fast nucleons can be explained assuming that a nucleon has a complex structure (see also [2]). The available experimental data make it possible to concretize this assumption. The pion-proton scattering at T = 1,5 BeV [3] indicates that inside a nucleon there is a region of a very strong interaction (with $r_c = 5.10^{-14}$ cm.) - a "core", surrounded by the weaker interaction region with $R = 12.10^{-14}$ cm. As soon as the dimentions of the core is near to the Compton wave length of K-mesons with $m = 965\, m_e \approx M/2$ ($\lambda_K \sim 2\frac{\hbar}{Mc} = 4^{-14}.10$ cm) M is the mass of a nucleon) one may think that the nucleon core is evidently connected with K-mesons**.

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* See also D.I. Blockhintsev's note [2] (in the press) about the validity of hydrodynamic discription of the generation of mesons in collisions of high energy nucleons.

** The presence of the "core" inside a nucleon surrounded by virtual pion shell as well as the strong interaction of nucleons at very small distance follows for example from Blochintsev's hypothesis about the indirect interaction of a nucleon with pions through K-meson field.[4]

Then at the energy T $\geqslant$ 1 BeV when the nucleon wavelength is

$$\lambda \approx \hbar c / \sqrt{E^2 - (Mc^2)^2} < 1{,}0 \cdot 10^{-14}\,cm \ll b, \quad \text{where } r_c < b < R$$

it is possible to distinguish two types of nucleon collisions: peripheral with the collision parameter $b > r_c$ and central with $b < r_c$.

In peripheral collisions nucleons interact through the field of pions and this interaction is not strong. That is why the energy losses are small and a few mesons are produced.

We suggest that in the central collisions the interaction is very strong so the energy losses may reach 100 per cent and many mesons including K-mesons must be produced.

Both types of collisions - are quantum phenomena and hence their treatment requires a consistent meson theory. As there is no such a theory we can try to apply the approximate methods. Namely, to calculate the peripheral collisions we use the semiclassical Weizsäcker-Williams method. However, unlike the old calculations of Heitler et. al. [5] we do not use the perturbation theory for calculating the cross section for the scattering of virtual mesons on nucleons but take it from the experiment. So only a flux of virtual mesons accompaning a fast nucleon is calculated from the theory.

As for the central collisions it is possible now to indicate only the order of magnitude of their effective cross section $\sigma_{ц} \sim \pi r_c^2$

INELASTIC PERIPHERAL COLLISIONS OF NUCLEONS

According to the Weizsäcker-Williams method the virtual meson shell of a fast nucleon is replaced by a free pion flux hiving the same energy as the shell (see [5]).

$$E^{\pi} = \int_0^{\infty} \varepsilon q(\varepsilon)\, d\varepsilon = 2\pi \int_{r_c}^{\infty} b\, db \int_{-\infty}^{\infty} S_x(b,t)\, dt \qquad (1)$$

where $S_x(b,t)$ is a longitudinal component of the energy flux density of the meson field of a nucleon describing the energy propagation by this field and r_c is a radius of a nucleon core, so that the integration in this formula extends only to the shell.

Mesons of the flux scattered on a nucleon at rest are considered to be produced by the collisions of nucleons. The cross section of the meson production and the energy flux lost in this collision by the fast nucleon are equal respectively:

$$\sigma_1 = \int_{mc^2}^{\varkappa\varepsilon_{max}} \sigma(\varepsilon) q(\varepsilon)\, d\varepsilon \quad \text{and} \quad \mathcal{J}_1 = \int_{mc^2}^{\varkappa\varepsilon_{max}} \sigma(\varepsilon) q(\varepsilon)\, \varepsilon\, d\varepsilon \qquad (2)$$

where $\sigma(\varepsilon)$ is the total cross section for the scattering of mesons on nucleons; $\varkappa\varepsilon_{max} = E - \sqrt{(E + Mc^2)\, Mc^2/2}$ - the upper limit of the integration defined from the consideration that a fast nucleon in the center of mass system of the colliding nucleons can lose all its kinetic energy. The relative energy losses of a fast nucleon in the process is

$$\delta_1 = \frac{\mathcal{J}_1}{\sigma_1 E} \qquad (3)$$

Conditions of the validity of the Weizsäcker-Williams method [5] $\lambda \ll b_{min}$, where $b_{min} > \frac{\hbar}{Mc}$, and $\Delta E \ll E$ are fulfilled for the peripheral collisions at 1-10 BeV. Indeed, $\lambda = (1-0{,}1)\cdot 10^{-14}$ cm, there is $\lambda \ll b_{min} = r_c \approx 4\cdot 10^{-14}$ cm and mean relative losses $\delta = \frac{\Delta E}{E} \lesssim 0{,}3$. (see [1]).

VIRTUAL MESON FLUX

It follows from the isotopic invariance hypothethis that virtual charged and neutral π-mesons are in the nucleon shell with the relative probability of 2/3 and 1/3. Therefore, the longitudinal component of energy flux density of neutral π-meson field of a nucleon in the point on the plane x = 0 is equal to

$$S_x^0(b,t) = \frac{1}{3} v \left[\nabla_x \varphi(b,x,t)\right]^2_{x=0}, \tag{4}$$

where

$$\varphi(b,x,t) = \frac{\gamma g}{\sqrt{4\pi}} \left(\frac{m}{2M}\right)(\sigma\nabla) \frac{\exp\left\{-\frac{1}{R}\sqrt{b^2+\gamma^2(x-vt)^2}\right\}}{\sqrt{b^2+\gamma^2(x-vt)^2}}$$

is the function of fast nucleon meson field in the non-relativistic approximation for the nucleon; m and M - the masses of π-meson and nucleon, $R = \frac{\hbar}{mc}$, $\gamma = 1/\sqrt{1-\beta^2}$, $\beta = v/c$.

Substituting (4) in the formula (1) and decomposing $\nabla_x \varphi(b,x,t)\Big|_{x=0}$ into Furie integral [5] we obtain

$$E^0 = E \frac{m}{M} g^2 \int_0^\infty Q^0(\tau, z)\, z\, dz,$$

where $$Q^{0}(\tau, z) = \frac{1}{3}\,\frac{z}{2\pi}\left\{2u K_0(u) K_1(u) + z^2\left[K_0^2(u) - K_1^2(u)\right]\right\}^{(*)}$$

is interpreted as a spectrum of virtual π^0-mesons accompanying fast nucleon. $K_0(u), K_1(u)$ are the modified Bessel functions,

$$U = \frac{\tau}{R}\sqrt{1 + \left(\frac{M}{m} z\right)^2}, \quad z = \frac{\varepsilon}{\beta E},$$

where E - the total nucleon energy in the lab. system, ε - the energy of virtual π-meson. $Q^0(\tau, z)$ is essentially depended on the radius of nucleon core τ (Fig. 1).

Spectra of virtual $\pi^{+,-}$-mesons accompanying proton and neutron respectively are equal to $Q^+ = Q^- = 2Q^0$.

TWO CONTRIBUTION TO THE PROCESS OF THE MESON PRODUCTION IN PERIPHERAL COLLISIONS OF NUCLEONS

The used solution of the equation of the meson field with a source corresponds to the first approximation of the perturbation theory. So virtual mesons can be emitted by a fast nuc-

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* In the paper by W. Heitler [5] when calculating $Q(\tau, z)$ the factor 2 arising from $\int_{-\infty}^{\infty} e^{-i(\varepsilon-\varepsilon')t}\,dt = 2\pi\,\delta(\varepsilon-\varepsilon')$ was lost.

leon or by a nucleon at rest. Thus, the process of the meson production involvs two contributions. The first contribution is the scattering of mesons from the virtual shell of a fast nucleon on a nucleon at rest. The second contribution is the same process in the coordinate system where the fast nucleon is at rest. (in L* system)*. Therefore the total cross section for the meson production in the peripheral collisions of nucleons equals the double cross section for the mesons production in one contribution

$$\sigma = 2\sigma_1 = 2g^2 \frac{m}{M} I_1 ;$$

$$I_1 = \frac{1}{\beta}\int_0^{z_{max}} [\sigma^{\pm}(\beta E z) Q^{\pm}(\tau, z) + \sigma^{0}(\beta E z) Q^{0}(\tau, z)] dz . \qquad (6)$$

Mean relative energy losses of fast nucleon

$$\delta = \frac{1}{2}(\delta_1 + \delta_2)$$

Relative losses in the first contribution are equal to

$$\delta_1 = \frac{\mathcal{Y}_1}{\sigma_1 E} = \frac{I_2}{I_1} ,$$

$$I_2 = \int_0^{z_{max}} [\sigma^{\pm}(\beta E z) Q^{\pm}(\tau, z) + \sigma^{0}(\beta E z) Q^{0}(\tau, z)] z\, dz , \qquad (7)$$

where $\sigma^{+,-,0}(\varepsilon)$ -the total cross sections of $\pi^{+,-,0}$ meson scattering on nucleon (proton or neutron), where $\sigma^0(\varepsilon) = \frac{1}{2}[\sigma^+(\varepsilon) + \sigma^-(\varepsilon)]$

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* For the second contribution the condition of the validity of the Weizsäcker-Williams method $\Delta E^* \ll E$ transfering to $T_{rec} \ll Mc^2$ in the laboratory system is fulfilled, since from the experiment $T_{recoil} \lesssim 150$ MeV.

in accordance with the isotopic invariance hypothethis; $\sigma^{+}(\varepsilon)$ and $\sigma^{-}(\varepsilon)$ are taken from the experiment.* The use of cross sections from the experiment as $\sigma(\varepsilon)$ is possible only by assuming the smallness of nucleon shell interaction, that is, probably, correct for the nucleons with the energy $E \leq 10$ Bev, as the cross sections of such nucleons $\sigma_{NN} \sim 30 \cdot 10^{-27} cm < \pi (2R)^2$.

The cut off parameter of meson spectrums

$$Z_{max} = \frac{\varepsilon_{max}}{\beta E} = \frac{1}{\beta}\left[1 - \frac{1}{E}\sqrt{\frac{1}{2}(E + Mc^2)Mc^2}\right].$$

To obtain the relative energy losses in the second contribution it is nucessary to pass from Λ^* - system into lab. system

$$\delta_2 = \frac{1}{Mc^2}(v\bar{P}_{\parallel}^{*} - \bar{T}^{*}),$$

$\bar{P}^*$ and $\bar{T}^*$ are the longitudinal momentum and the kinetic energy of the recoil-nucleon in the L* system averaged over the angular distribution and energy spectrum of the virtual mesons. In the case of an elastic scattering of virtual mesons with the angular distribution $\frac{d\sigma}{d\omega} = a + b\cos\theta + c\cos^2\theta$ in the center of mass system of the colliding meson and nucleon

$$\delta_2 = \frac{1}{\beta I_1}\int_0^{z_{max}} Q(\tau, z)\left[\sigma(\beta E z) - \frac{4\pi}{3} b(\beta E z)\right]\frac{vv_c - v_c^2}{c^2 - v_c^2}\,dz \qquad (8)$$

where V is the velocity of the fast nucleon, $V_c = \frac{\sqrt{\varepsilon^2 - (mc^2)^2}}{Mc + \varepsilon/c}$

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* For the energies $4,5 < \varepsilon < 7$ BeV assume that $\sigma(\varepsilon) = const = 30 \cdot 10^{-27} cm$.

- the velocity of the c.m. system of a virtual meson and a nucleon, $\varepsilon = \beta E z$.

Available data on the differential cross sections for $\pi - P$ scattering [8] make it possible to determine δ_2 from the recoil nucleon energy in the L* system only for the energy of the fast nucleon T ~ 0,7 BeV where $\delta_2 \approx 0{,}1$.

For high energies ($\gamma \gg 1$) δ_2 can be evaluated from the considerations that the energy lost by the fast nucleon in the L* system - ΔE^* is taken up mainly by mesons. As $\Delta E^* \gg mc^2$ this energy in the laboratory system is

$$(\Delta E)_2^{\pi} \approx \gamma(1-\beta)\Delta E^* \approx \frac{\delta_1}{2\gamma} E \qquad (\Delta E^* = \delta_1 E)$$

The recoil-nucleon energy in the L -system determined from the losses of the fast nucleon in the L* - system

$$T_2 = \gamma (V \Delta P^* - \Delta E^*)$$

is equal at $\gamma \gg 1$

$$T_2 = \frac{\delta_1^2}{2(1-\delta_1)} Mc^2$$

and

$$\delta_2 = \frac{1}{E}\left[(\Delta E)_2^{\pi} + T_2\right] = \frac{1}{2\gamma}\,\frac{\delta_1}{1-\delta_1} \qquad (9)$$

Then the mean relative energy losses of the fast nucleon are equal

$$\delta = \frac{\delta_1}{2}\left[1 + \frac{1}{2\gamma(1-\delta_1)}\right] \qquad \text{at } \gamma \gg 1. \quad (10)$$

DISCUSSION OF THE RESULTS

In the observed energy interval T = 1 - 10 BeV the effective cross section for the inelastic peripheral collisions of nucleons (8) is practically constant (fig. 2) /more than the mean value for p-p collisions and less for n-p collisions/. But it depends very much upon the magnitude of the coupling constant of the meson field with the nucleon and upon the radius of the nucleon "core".

If we use $g^2 = 15$ the best agreement with the experiment will be reached at $r_0 = 2\frac{\hbar}{Mc}$. This is essential for understanding the entire pattern of the inelastic nucleon-nucleon collisions because in that case the cross section for the central collisions $\sigma \sim \pi r^2 \approx 5,6 \cdot 10^{-27}$ cm.^{2} amounts to about 20% of the total inelastic collision cross section.

The mean relative losses of a free nucleon in inelastic peripheral collisions are 15-20% and the energy of a recoil-nucleon evaluated from the second contribution is equal to 40-110 MeV (fig.3). It is essential that both quantities absolutely do not depend upon the coupling constant g^2.

As for the average number of mesons produced in peripheral collisions it is more than the unity, as the virtual mesons are scattered on the nucleon both elastically and inelastically, and must increase with energy.

The observed pattern of the meson production by nucleon-

nucleon collisions is certainly very approximate. However it is in a good agreement with the experiments of Yu. A. Smorodin and G.B. Zhdanov [9], performed in a cloud chamber, which show that the nucleons with the same energy (10^{10} - 10^{11} ev) form showers of the essentially different multiplicity. 80 per cnet of showers correspond to the production of 1-2 meson, the initial nucleon losing 20% of its energy and a recoil-nucleon taking up energy of 20-50 MeV but not more than 150 MeV.

Showers of the second type contain more than five particles, the energy losses of the initial nucleon being 80%.

Thus the existence of two types of showers confirms the presence inside the nucleon of a "core" with the radius $r = 4.10^{-14}$ cm. surrounded with virtual pion shell of a radius $R = 14.10^{-14}$ cm.

In conclusion I wish to thank greatly prof. D.I. Blokhintsev for some valuable advice and great interest to this work.

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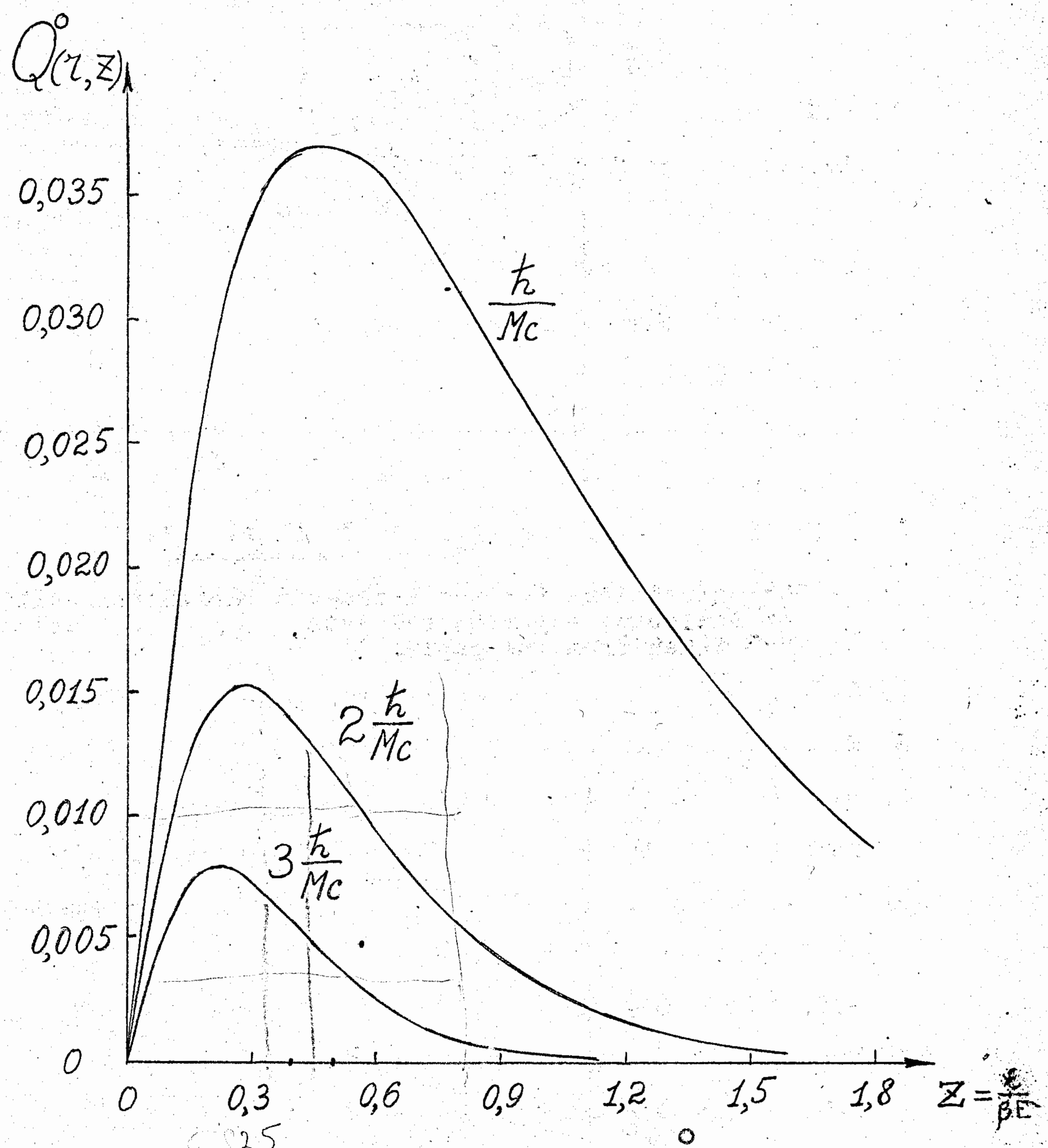
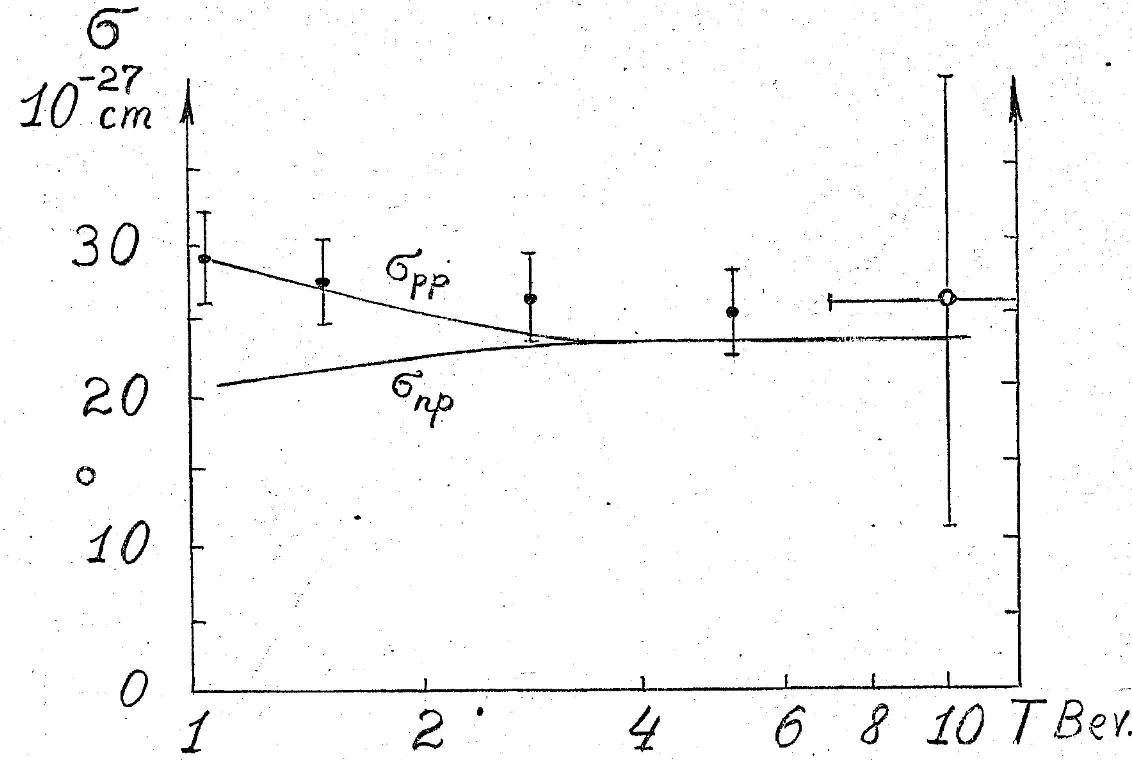


F i g. 1.

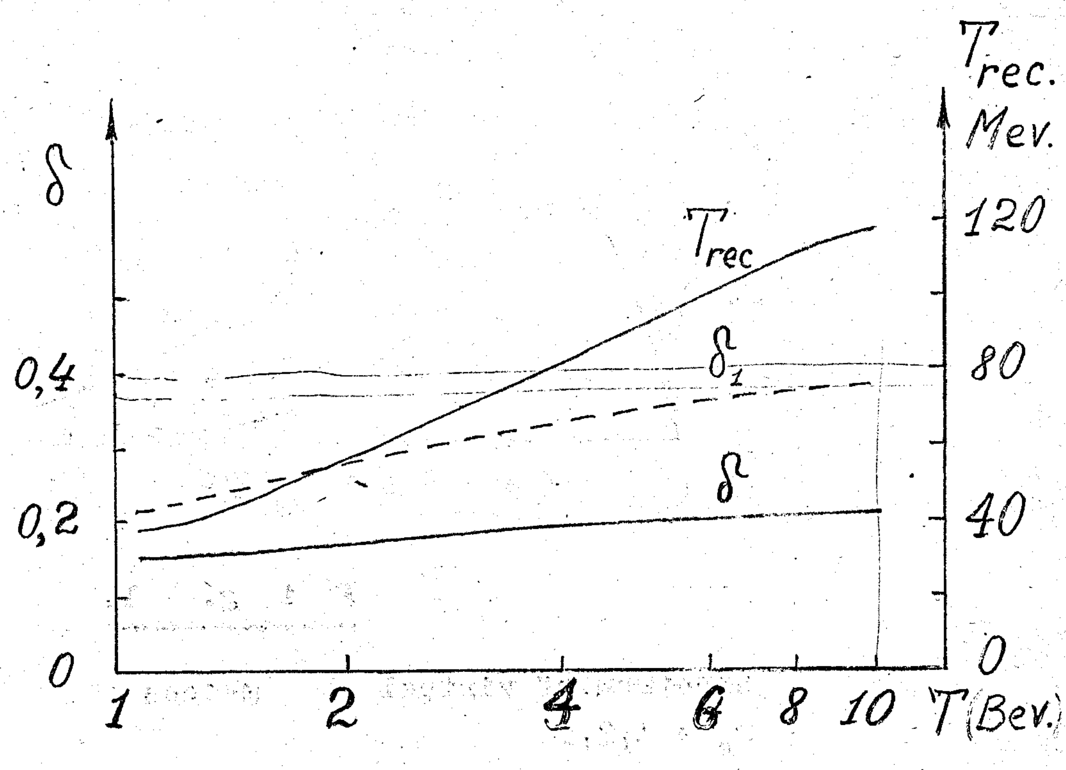
Spectrum of virtual π^0 mesons $Q(\tau, z)$ at $r_0 = 1,2,3\ \frac{\hbar}{Mc}$

(M - is the nucleon mass).



F i g. 2.

Cross sections for the inelastic peripheral collisions of nucleons; experimental data σ_{pp} – and σ_{NN} – are taken from the papers[10].



F i g. 3.

Recoil-nucleon energy - Trec. and relative energy losses in inelastic peripheral collisions of nucleons: δ – in the first contribution and δ – mean energy losses.