JOINT INSTITUTE FOR NUCLEAR RESEARCH

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LABORATORY OF THEORETICAL PHYSICS

J.A. Smorodinsky

"ON THE ISOTROPIC MODELS OF THE UNIVERSE"*

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* The report at the VII Cosmogony Meeting, June 5-7, 1957.

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these measurements up to the velocities 0,4 (120.000 km/sec.) and analysed the old data anew.

As a result the authors of both papers conclude that the closed model or, at least, the flat one (with zero curvature) may be put into agreement with the observation. This conclusion, however, seems to be not convincing. The topic of the present report is to discuss this problem.

§ 2. Principal Formulas

Let us give the main formulas of isotropic and homogeneous model | cf[3] |. The metrics of such a model is defined by a linear element

$$ds^2 = d\tau^2 - g^2(\tau)\left[d\chi^2 + S^2(\chi)\,d\Omega\right] \tag{2.1}$$

Here $d\Omega$ is the element of three dimensional solid angle, the velocity of light is assumed to be equal to unity and the function $S(\chi)$ is:

$$S(\chi) = \sin\chi \qquad \text{(closed model)}$$

$$\chi \qquad \text{(flat model)}$$

$$\mathrm{Sh}\,\chi \qquad \text{(open model)}$$

Thus, $S^2(\chi)\,d\Omega$ -is the element of the spherical surface. Instead of the time τ the variable η is introduced by the formula

$$d\tau = g(\eta)\,d\eta \tag{3.2}$$

Then the linear element is rewritten in the symmetrical form

$$ds^2 = g^2(\eta)\left[d\eta^2 - d\chi^2 - S^2(\chi)\,d\Omega\right] \tag{2.4}$$

The function $g(\tau)$ satisfies the equation (cf[4]/

$$2gg + g^2 = -\zeta$$

$$g \quad \frac{1}{6}\varkappa\rho g \qquad (2.5)$$

Here $\zeta = +1, 0, -1$ for closed, flat and open models, respectively. The point means the differentiation by τ, ρ the mean density, $\varkappa$ is the gravitation constant (6,67.10^{-8}.8)

$$\varkappa = 1{,}67 \cdot 10^{-6} \quad \frac{sm^3}{sec^2} \qquad (2.7)$$

Introduce "Huble constant" (function of τ).

$$h \quad g/g = \frac{1}{g^2}\frac{dg}{d\eta} \qquad (2.8)$$

Then the type of the model is determined by the sign of the quantity

$$\frac{1}{3}\frac{\varkappa\rho}{h^2} - 1 \lessgtr 0 \qquad (2.9)$$

The equality sign corresponds to the flat model. Let us introduce the quantity

$$\rho_{cr} \quad \frac{3h^2}{\varkappa} \qquad (2.10)$$

The type of the model is determined by the ratio between ρ and ρ_{cr}:

$$\begin{array}{ll} \rho > \rho_{cr} & \text{(closed)} \\ \rho = \rho_{cr} & \text{(flat)} \\ \rho < \rho_{cr} & \text{(open)} \end{array} \qquad (2.11)$$

At the present value h^2 *

$$h = 150 \frac{km/sec}{MPS} \quad 0{,}18 \quad 10^{-3} \quad \text{years} \qquad (2.12)$$

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* Humason's et al [1] give h = 180 ± 20%.

$$\rho = 5{,}5 \cdot 10^{-29} \frac{g}{Sm^3} \tag{2.13}$$

The solutions of equations (2.5) may be written in the parametric form[3]: for the closed model

$$\begin{aligned} g &= g_0 (1 - \cos \eta) \\ \tau &= \tau (\eta - \sin \eta) \end{aligned} \tag{2.14}$$

and for the open model

$$\begin{aligned} g &= g_0 (ch \eta - 1) \\ \tau &= \tau (sh \eta - \eta) \end{aligned} \tag{2.15}$$

g_0 is the constant of integration.

From formulae (2.14) and (2.15) one may calculate the value of Huble's constant(2.8):

$$h = \frac{1}{g_0} \frac{\sin \eta}{(1 - \cos \eta)^2}, \tag{2.16}$$

or

$$h = \frac{1}{g_0} \frac{sh \eta}{(ch \eta - 1)^2}, \tag{2.17}$$

for two models respectively. From the formulae we get the relation between time τ and h

$$\tau = a \frac{1}{h}, \tag{2.18}$$

where

$$a = \frac{\sin\eta(\eta - \sin\eta)}{(1-\cos\eta)^2} \qquad (2.19)$$

or

$$a = \frac{sh\eta\,(sh\eta - \eta)}{(ch\eta - 1)^2}\,; \qquad (2.20)$$

If $\eta \ll 1$ we get

$$\tau = \frac{2}{3}h \qquad (2.21)$$

independent upon a model. For the closed model $a < 3/2$, for the open one $> 2/3$. For the old closed model ($\eta \gg 1$)

$a \longrightarrow 1$ and

$$\tau \approx \frac{1}{h}\,; \qquad (2.22)$$

It can be seen from here that the old closed model give tne longest time scale. If $h^{-1} = 5.10^9$ years it seems that such a model may be brought to agreement with other astrophysical and geological data. The closed model gives $\tau < 3.10^9$ years, which appears to be too short. It is useful for the further consideration to introduce the quantity:

$$q = \frac{1}{2}\,\frac{\rho}{\rho_{cr}} = -g\ddot{g}/\dot{g}^2 \qquad (2.23)$$

It is easy to see that

$$|1-2q| = (gh)^2 \qquad (2.24)$$

If q and h are determined from the observations then it will be possible to calculate g and thus all the parameters. Let us denote (for all the models)

$$\kappa = |1-2q|^{1/2} \tag{2.55}$$

Then

$$g = (h\kappa)^{-1} \tag{2.26}$$

$$g_0 = h^{-1}\kappa^{-3} \tag{2.27}$$

Further

$$(1-q)/q = \operatorname{ch}\eta \quad \text{or} \quad \cos\eta \tag{2.28}$$

~~(because of this (1 - q) / q > 1 or < 1)~~ and

$$T = \frac{1}{h}\left(\kappa^{-2} - q \ln\frac{1+\kappa}{1-\kappa}\right). \tag{2.29}$$

§ 3. R e d S h i f t

Huble effect may be determined from the relation between visual magnitude of the clusters and red shift. The energy flux from the distant luminous object is defined by the formula:

$$\mathcal{Y} = \mathrm{const}\,\frac{g^2(\eta-\chi)}{g^4(\eta)S^2(\chi)}; \tag{3.1}$$

It is supposed that the light is emitted at the moment corresponding to $\eta-\chi$ and is detected by an observed at the moment η. $S(\chi)$ is determined by formula (2.2). Red shift is determined by the formulae:

$$z = \frac{\lambda-\lambda_0}{\lambda_0} \tag{3.2}$$

$$\frac{1}{\lambda} = \frac{1}{\lambda_0}\,\frac{g(\eta-\chi)}{g(\eta)} \tag{3.3}$$

where λ_0 is the observed wave length, λ is the wave length of the same line for near object. The formula necessary for comparison with the experiment may be obtained by exluding the variable χ from (3.1)-(3.3) and expanding $\mathcal{Y}$ into power series of z. Such an expression is obtained by Robertson up to the terms 1/Z. It is called Robertson formula.[5] In the calculation we use the formula

$$\frac{dg}{d\eta} = gg \tag{34}$$

Besides it is necessary to use the formulae:

$$g = gh;\; g = -gqh^2;\; g = 2gqh^3. \tag{3.5}$$

The last formula follows from (2.5)*.

Being expressed through h and q all the formulas become applicable both for closed and open (and for flat models). For z we obtain: $z = h\chi + \frac{1}{2}(1+q)(h\chi)^2 + \frac{1}{6}(1+4q)^2(h\chi)^2$

Solving for χ and using (2.24)

$$\chi = z(1-2q)^{1/2}\Big\{1 - \frac{1}{2}(1+q)z +$$

$$+\Big[\frac{1}{2}(1+q)^2 - \frac{1}{6}(1+4q)\Big]z^2 + \dots\Big\}$$

inserting into (3.1) we find

$$y = \text{Const}\ \frac{h^2}{z^2}\Big[1-(1-q)z + \frac{1}{4}(3-2q-q^2)z^2 + \dots\Big]$$

It can be seen from this formula that for $z \sim 0{,}4$, the term with z^2 is important. If we put $M = -2{,}5 \log_{10} y + \text{const}$, then we shall get the usual form of this formula.

Let us dwell upon experimental data.

The authors mentioned above point out that their data may be presented by the curve described by two term formula (without the term with z^2) with the following value of q:

$$\begin{array}{ll} \text{HMS}^{(1)} & q \simeq 2{,}6 \\ \text{Baum}^{|2|} & q \simeq 1 \pm 0{,}5 \end{array} \qquad (3.9)$$

As it was pointed by Baum HMS values are, apparently, too high. Professor D.I. Martynov who reports at this meeting agrees with it as well.

When interpreting the data (3.9) it is worth to point out |1| that the value q is obtained by assuming that the absolute bright-

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* The derivation of similar formula was given already in the literature (compare 6). However, since this derivation makes no use of (3.5) it does not give the final formula (3.8). I do not know whether the simple formula (3.8) has ever been published.

ness of the object does not change with time and that there is no considerable absorption of radiation in intergalaxical space (and that, besides, the calculation of the bolometrical magnitudes was correct).

It is evident that at present there is no data which would make it possible to estimate these errors. We note only that this change may be neglected if the absolute brightness changes so that $| \gamma | \gamma | \ll h$.

The assumption that $\gamma > 0$ is equivalemt to the increase of q ("closing" of the models) $\gamma < 0$ is equivalent to the decrease of q ("opening" of the models).

The absorption also leads to the increase of q.

The Baum' resuls (3.9) may be formulated in the following way. If we represent the values of $\frac{\gamma z^2}{h^2}$ as the function of z (in arbitrary units) then all the experimental points in the interval $0 < z < 0,4$ fall between two straight lines

$$y_1 = 1 - 1/2\, z$$

and

$$y_2 = 1 + 1/2\, z$$

Baum makes a conclusion from here that this experiment declares for the closed model. However, if we take into account the term proportional to z^2 then the low line lies in the region of the open mofrl. ~~Yhid-ven~~ This can be seen from Fig. 1. Therefore from Baum data one cannot draw a conclusion that the model is a closed one.

If we take into account the considerations on the time scale which we have spoken about at the end of § 2 then the open model turns out to be more probable.

The data about density ρ * give evidence for an open

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* Unfortunately the magnitude of density and the value of the parameter cannot be estimated from (3.9) due to low accuracy of the observations.

model.

As the discussion at this Meeting has shown almost all the estimations of the density give the value $\rho < \rho_{cr}$

Thus, one may conclude that the existing experimental data may be brought to agreement in the frame of the open model. The question of the application of isotropic and homogeneous model itself requires considerably larger number of observations for the analysis.

References

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Figure I.

Dependence of visual magnitude on red shift. Curves are drawn under assumptions that the decrease of magnitude is due only to the motic of the object |formula (3.8)|. The dotted lines show the region in which (according to Baum) the experimental curve lies. The curve $q = 0$ corresponds to the old ($\eta \gg 1$) open world. The curve $q = 1/2$ separates the regions of closed and open models.