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Some authors considered the dispersion relations for nucleon-nucleon scattering [1-3]. The relations which they obtained could not be compared with the experiment, for they involved non-observed magnitudes and also unknown magnitudes connected with antinucleon-nucleon scattering.

In the present paper we investigated the approximate dispersion relations. The nucleon-nucleon scattering in the absence of anti-nucleons is considered. We are preceeding from the dispersion relation obtained in the usual way [4].

$$D(E)-\frac{1}{2}\left(1+\frac{E}{M}\right)D(M)-\frac{1}{2}\left(1-\frac{E}{M}\right)D(-M)= \tag{1}$$

$$=\frac{E^2-M^2}{\pi}P\int_0^\infty\frac{A(E')\,dE'}{(E'-E)(E'^2-M^2)}-\frac{E^2-M^2}{\pi}P\int_0^\infty\frac{A(-E')\,dE'}{(E'+E)(E'^2-M^2)}\ .$$

Here D and A are the real and imaginary parts of the nucleon-nucleon scattering amplitudes respectively, E is the scattering nucleon energy in the coordinate system where the sum of the momenta of the scatterer before and after the collision equals zero. ($\vec{P}+\vec{P}'=0$). The nucleon-nucleon scattering amplitudes $D(-M)$ and $A(-E')$ are linearly expressed by antinucleon-nucleon scattering amplitudes with positive energy. So the dispersion relation bounds the amplitudes of nucleon-nucleon and antinucleon-nucleon scatterings.

In the considered approximation (1) has the form:

$$D(E) - \frac{1}{2}\left(1+\frac{E}{M}\right) D(M) =$$

$$= \frac{E^2-M^2}{\pi} P \int\limits_{\frac{M^2-\vec{p}^2}{\sqrt{M^2+\vec{p}^2}}}^{\infty} \frac{A(E')\,dE'}{(E'-E)(E'^2-M^2)} + C\,\frac{(E^2-M^2)\,\delta_{0T}\,\delta_{1S}}{E - \frac{M^2-\vec{p}^2-2M\varepsilon}{\sqrt{M^2+\vec{p}^2}}}, \tag{2}$$

where the last term is the contribution of deutron intermediate state, δ_{0T} and δ_{1S} — Croneker symbols different from zero if the total isotopic spin T = 0 and the total usual spin S = 1, C — the constant independent on energy which must be determined from the comparison with the experiment, ε — the energy of deutron coupling. Such an approximation is satisfactory in the energy region where the magnitude of the integral is determined mainly by the behavior of the integrand if $E' \sim E$. For this the integrand $A(E')$ must possess a great derivative for the integration is being made in the sense of principal part. The scattering amplitude $A(E')$ which is an integrand in (2) is not an observed magnitude in the region

$$\sqrt{M^2+\vec{p}^2} > E' > \frac{M^2-\vec{p}^2}{\sqrt{M^2+\vec{p}^2}} \tag{3}$$

For forward scattering $\vec{p}^2 = 0$, the non-observed region is vanishing.

In nucleon-nucleon scattering it is necessary to take into account the identity of particles and consider not the scattering amplitude $f(\theta)$ for a certain angle θ, but a linear combination of the form

$$f(\theta) \pm f(\pi-\theta) \tag{4}$$

where θ is the scattering angle in the centre of mass system. The non-observed region (3) remains at any θ for this amplitude. With the aim of conducting the symmetrization we write down (2) in the centre of mass system:

$$D(W,\vec{p}^{\,2}) - \frac{1}{2}\left(1+\frac{W^2-2M^2-2\vec{p}^{\,2}}{2M\sqrt{M^2+\vec{p}^{\,2}}}\right) D(2M,\vec{p}^{\,2}) =$$

$$= \frac{W^4+4(\vec{p}^{\,2}-W^2)(M^2+\vec{p}^{\,2})}{\pi}\, P\int_{2M}^{\infty} \frac{2W'A(W',\vec{p}^{\,2})\,dW'}{(W'^2-W^2)[W'^4+4(\vec{p}^{\,2}-W'^2)(M^2+\vec{p}^{\,2})]} +$$

$$+ C'\,\frac{W^4-4(W^2-\vec{p}^{\,2})(M^2+\vec{p}^{\,2})}{W^2-4M(M-\varepsilon)} \tag{5}$$

where W is the total energy of two nucleon system,

$$C' = \frac{C}{2\sqrt{M^2+\vec{p}^{\,2}}} \quad \text{and} \quad \vec{p}^{\,2} = \frac{W^2-4M^2}{8}(1-\cos\theta).$$

Non-observed region in relation (5) is in the limits

$$2M \le W' < 2\sqrt{M^2+\vec{p}^{\,2}}$$

In this region $\cos\theta' < -1$. The transition, the $\theta \longrightarrow \pi-\theta$ is equivalent in (5) to the substitution:

$$\vec{p}^{\,2} \to \vec{p}'^{\,2} = \frac{W^2-4M^2}{4} - \vec{p}^{\,2}.$$

To fulfil the symmetrization it is essential that

$$\frac{W^4+4(\vec{p}^{\,2}-W^2)(\vec{p}^{\,2}+M^2)}{W'^4+4(\vec{p}^{\,2}-W'^2)(\vec{p}^{\,2}+M)} - \frac{W^4+4(\vec{p}'^{\,2}-W^2)(\vec{p}'^{\,2}+M^2)}{W'^4+4(\vec{p}'^{\,2}-W'^2)(\vec{p}'^{\,2}+M)^2)} \sim W'^2 - W^2 \tag{6}$$

Besides, $\cos\theta$ and $\cos(\pi-\theta)'$ incorporated into (5) under the integral are equal to:

$$\cos\theta' = 1 - \frac{8\vec{p}^{\,2}}{w'^2 - 4M^2}$$

and $$\cos(\pi-\theta)' = -1 + \frac{8\vec{p}^{\,2}}{w'^2 - 4M^2} + 2\frac{w'^2 - w^2}{w'^2 - 4M^2} \qquad (7)$$

Therefore if the function A(W') in the region $W' \sim w$ changes rapidly enough in accordance with the assumption made earlier the integral involving difference (6) may be neglected in comparison with the integral involving an anologous sum and the last term in expression (7) for Cos. (π - θ)' may be neglected too. Then:

$$\cos(\pi - \theta)' = \cos(\pi - \theta') = -\cos\theta';$$

and the performance of operation (4) turns out to be possible. In this appriximation the non-observed region fully vanishes for forward scattering ($\vec{p}^{\,2} = 0$) and we obtain

$$D(k) - \frac{1}{2}\left(\frac{3}{2} + \frac{k^2}{M^2} + \frac{\sqrt{k^2+M^2}}{2M}\right)D(0) = \frac{k^2(k^2+M^2)}{2\pi^2}\int_0^\infty \frac{\sigma(k')\,dk'}{(k'^2-k^2)(k'^2+M^2)} + C'\frac{3k^2(k^2+M^2)}{k^2+M^2} \qquad (8)$$

where $k^2 = \frac{w^2-4M^2}{4}$. We used the optical theorem for forward scattering. The experimental verification of the written dispersion relation (8) is of interest. The existing experimental data on the total cross-section and the angular distribution make it possible to calculate the integral of the total cross-section and to compare thus obtained D(k) with the one experimentally measured. This verification will give an answer to the problem about the limits of the application of dispersion relation (8).

In the very low energy region (up to 6 MeV) where the scattering is described well by S-wave we can use the expansion

$$k \cdot \mathrm{ctg}\, \delta = -\frac{1}{a} + \frac{1}{2} r k^2 \tag{9}$$

and to perform the verification of the dispersion relation (8). In this case it assumes the form:

$$\frac{1}{k} \sin 2\delta_{0,1}(k) + \left(\frac{3}{2} + \frac{k^2}{M^2} + \frac{\sqrt{k^2+M^2}}{2M}\right) a_{0,1} =$$

$$= \frac{4k^2(k^2+M^2)}{\pi} \mathcal{P} \int_0^\infty \frac{\sin^2 \delta_{0,1}(k')\, dk'}{k'^2(k'^2-k^2)(k'^2+M^2)} + C' \frac{3k^2(k^2+M^2)}{k^2+M\varepsilon}, \tag{10}$$

where $a_{0,1} = \lim_{k \to 0} \left[\frac{1}{k} \sin 2\delta_{0,1}(k)\right]$, δ_0 and δ_1 phase-shifts of singlet and triplet S-scattering, respectively. Substituting expansion (9) into (10), differentiating by K^2 and assuming $K^2 = 0$ we get the relation involving r_0 and a_0. These magnitudes are well-known and equal [5] to: $r_0 = 3 \cdot 10^{-13}\,\mathrm{cm}$, $a_0 = -23{,}69 \cdot 10^{-13}\,\mathrm{cm}$; $r_1 = 1.704 \cdot 10^{-13}\,\mathrm{cm}$, $a_1 = 5.38 \cdot 10^{-13}\,\mathrm{cm}$. The deutron term is absent for singlet scattering and the left-hand side of (10). coincides with the right hand one with the accuracy up to 0,01%.

One fails to solve the problem of the determination of r_0 through a_0. It is due to the fact that r_0 is very sensitive to the change of total cross-section $\sigma_{\pm}(k)$. The substitution of expansion (9) instead of the total cross-section is the rough verification of the dispersion relations.

For triplet scattering if we do not take into account the deutron term, the left-hand side of (10) coincides with the right-hand side with the accuracy up to 3%.

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