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ABSTRACT

The quadruple correlations of nucleons possessing equal energies have been treated on the basis of the effective Hamiltonian describing the pairing interactions between nucleons. It is shown that the appearance of the quadruple correlations in light nuclei is energetically lower if the pairing interaction is attractive. With account of the pairing and quadruple correlations some regularities concerning the binding energy of the last neutron in light nuclei have been explained. A criterion has been obtained which showed what number of neutrons has to exceed that of protons in stable nuclei (due to the extension of Fermi surfaces for neutrons and protons) for explicit α -particle properties of nuclei to disappear.

I. INTRODUCTION

As is well-known in heavy nuclei the pairing correlations of nucleons are observed both in the proton and neutron shells. The account of the pairing correlations by the mathematical technique developed in the theory of superconductivity/1/ made it possible to account for some properties of these nuclei. However, if in heavy nuclei there are no proton-neutron pairing correlations due to a large difference in energies of Fermi surfaces for neutron and proton shells, in light nuclei, on the other hand, the proton-neutron pairing correlations may occur until the Fermi surfaces for neutrons and protons extend. Therefore, in investigating the problem of pairing correlations in light nuclei together with n-n and p-p interactions, n-p interactions were taken into accouns as well/2/.

It has been shown also in /2/ that the account of the residual interactions leads to the formation of superfluid state (which is energetically lower than that of purely degenerate Fermi gas in the central field) of a light nucleus. The energy of the ground state is independent of the way in which the nucleons form pairs: protons with protons, neutrons with neutrons, or protons with neutrons. The account of the pairing correlations in light nuclei made it possible to understand e.g. some regularities concerning binding energies. However, some important properties of light nuclei cannot be explained by considering the pairing nucleon correlations only.

In light nuclei the α -particle- like quadruple correlations between nucleons exhibit themselves fairly explicitly and, when the number of neutrons exceeds that of protons α -particle properties in stable nuclei disappear. The account of the quadruple correlations taken on the basis of α -particle nuclear model enabled us to explain some important properties of light nuclei. However, α -particle model is a rough one, and the range of its application is rather restricted. Therefore, it is of interest to clear up, within the framework of the shell model, the possibility of appearing the nucleon quadruple correlations analogous to pairing correlations in heavy nuclei. It is also interesting to investigate the properties of light nuclei having such quadruple correlations and to obtain the criterion for the disappearance of these correlations. The present paper is devoted to the study of these problems.

II. EFFECTIVE INTERACTION HAMILTONIAN

The possibility of existen ce of the quadruple correlations of nucleons in light nuclei will be investigated on the basis of the shell model, taking into account the residual nucleon interactions after separation of the self-consistent field. The nucleon state in a nucleus will be characterized by quantum numbers σS, where $\sigma = \pm 1$ specifies the projection of the total angular momentum along the symmetry axis of the nucleus.

There is a strong interaction between the nucleons being in the nucleus in the S-state relative to each other. A part of such nucleon interactions leads to the appearance both of pairing and quadruple nucleon correlations. Therefore, the interaction Hamiltonian which takes into account only nucleon interactions with identical quantum numbers S will be used.

Suppose that the quadruple nucleon correlations are the result of the interaction between pairs. In this case the main contribution into the effective pairing interaction Hamiltonian is given by the interactions between nucleons possessing equal energies and having a resonance character. It should be noted that our assumption about the possibility of investigating the quadruple correlations with account of the interactions between pairs is valid in the case when the binding energy inside a pair is much larger than that between pairs. Although, strictly speaking, this condition is not fulfilled, we think that the basic properties of the quadruple correlations in light nuclei may be obtained by means of an effective Hamiltonian describing the interaction between pairs. Note, that it was the effective pairing interaction Hamiltonian we had to use but not the nucleon interaction Hamiltonian since the latter is very difficult to deal with.

When there is a strict isotopic invariance pairs may be constructed in various ways: a pair is formed by a neutron and a proton in the state (σS), or a pair is formed by two protons (neutrons) in the states (+ S), (–S). As far as further we are going to consider the case when the strict isotopic invariance is violated $b_{s\rho}$ is assumed to be the operator of the pair of identical nucleons in the states (+ S), (–S), $\rho = \pm 1$, provided $\rho = +1$ corresponds to a proton pair, whereas $\rho = -1$ to a neutron pair. The operators $b^+_{s\rho}$, $b_{s\rho}$ obey the following commutation relations:

$$b^+_{s\rho} b_{s\rho} + b_{s\rho} b^+_{s\rho} = 1, \quad b_{s\rho} b_{s\rho} = 0$$

$$b^+_{s\rho} b_{s'\rho'} - b_{s'\rho'} b^+_{s\rho} = 0, \quad b_{s\rho} b_{s'\rho'} - b_{s'\rho'} b_{s\rho} = 0, \qquad (1)$$

if either $S \neq S'$ or $\rho \neq \rho'$.

For the case of strict isotopic invariance a model Hamiltonian describing the interactions between pairs may be written as

$$H = \sum_s \{E(s) - \lambda\} \{ b^+_{s+} b_{s+} + b^+_{s-} b_{s-} \} + \sum_{\substack{s,s' \\ s \neq s'}} J(s,s') b^+_{s+} b^+_{s-} b_{s'-} b_{s'+} , \qquad (2)$$

here $E(s)$ is the energy of a pair in the state S, λ is the parameter which plays the role of the chemical potential.

Making use of the method suggested in[3] we introduce an auxiliary Hamiltonian

$$H' = \sum_{s} \{E(s) - \lambda\}\{b_{s+}^{+} b_{s+} + b_{s-}^{+} b_{s-}\} + \\ + \sum_{s,s'} J(s,s')\{B^{*}(s) b_{s'-} b_{s'+} + B(s') b_{s+}^{+} b_{s-}^{+} - B^{*}(s) B(s')\} \quad (3)$$

The functions can be determined from the condition for the minimum of $B(s)$ so that the mean value $\langle H \rangle$ of the operator H in the eigenstates of H' would coincide with $\langle H \rangle$. Then one obtains

$$B(s') = \langle b_{s'-} b_{s'+} \rangle . \quad (4)$$

Performing the transformation

$$b_{s+} = \{1 - 2 a_{s-}^{+} a_{s-}\} a_{s+} , \quad b_{s-} = a_{s-} , \quad (5)$$

we note that in the states with the same S the operators $a_{s\rho}^{+}$, $a_{s\rho}$ behave like the Fermi-amplitudes. An auxiliary Hamiltonian may be written as

$$H' = \sum_{s} \{E(s) - \lambda\}\{a_{s+}^{+} a_{s+} + a_{s-}^{+} a_{s-}\} - \\ - \sum_{\substack{s,s' \\ s \neq s'}} J(s,s')\{B^{*}(s) a_{s'-} a_{s'+} + B(s') a_{s+}^{+} a_{s-}^{+} + B^{*}(s) B(s')\} . \quad (6)$$

III. GROUND STATE

The appearance of the quadruple correlations will be investigated with the help of the Bogolubov variational principle[4], making use of an auxiliary H' in the form (6). Performing the transformation

$$a_{s+} = u_s [1 - 2\nu_s] \beta_{s+} + v_s \beta_{s-}^{+} \\ a_{s-} = u_s \beta_{s-} - v_s [1 - 2\nu_s] \beta_{s+}^{+} , \quad (7)$$

where $\nu_s = \beta_{s-}^{+} \beta_{s-}$, $u_s^2 + v_s^2 = 1$. New operators of the quasi-pairs $\beta_{s\rho}^{+}$, $\beta_{s\rho}$ satisfy the commutation relations (1). Determine a new vacuum state $\beta_{s\rho} \Psi = 0$ and find the mean value H' in this state

$$\langle H' \rangle = 2 \sum_{s} \{E(s) - \lambda\} v_s^2 + \sum_{\substack{s,s' \\ s \neq s'}} J(s,s') u_s v_s u_{s'} v_{s'} . \quad (8)$$

From the condition for the minimum $\langle H' \rangle$ it is possible to obtain the equation

$$\{E(s)-\lambda\}u_s v_s + \frac{u_s^2 - v_s^2}{2}\sum_{s'} J(s,s')u_{s'}v_{s'} = 0 \qquad (9)$$

which has the same form as the equation in[5] taking into account the pairing correlations.

It can be seen from (8) and (9) that the appearance of the quadruplecorrelations is energetically favoured, provided that the effective interaction between pairs is attractive. Note that four nucleons which are relative to each other in the S-state and form a four are related only by the angular variables and are not related, in contrast to α -particle model, by radial ones.

Both the asymptotic solution (9) for $J \to 0$ and the solution in case when the sum is substituted for the integral, provided J and level density ρ to be constant in the outer shell, lead to the conclusion that the energy of the ground state, where all the nucleons are bound in fours, lies lower than that of the state where all the nucleons are bound in pairs. The first excited state due to the breaking-up of a four is separated from the ground state by the energy gap.

$$\langle \beta_{s_0} \beta_{s_0-} H' \beta_{s_0-}^{+} \beta_{s_0+}^{+} \rangle - \langle H' \rangle = 2\varepsilon(s_0), \qquad (10)$$

where

$$\varepsilon(s) = \sqrt{\{E(s)-\lambda\}^2 + C_s^2}\ , \quad C_s = \sum_{s'} J(s,s')u_s v_s\ .$$

It is worth while noting that Eq. (9) may be obtained not only by means of the variational principle given above, but also by using the method of breaking-up the correlations suggested by N.N. Bogolubov[7] on the basis of the Hamiltonian H.

IV. BINDING ENERGY OF THE LAST NEUTRON IN LIGHT NUCLEI

Let us consider the effect of the pairing and quadruplecorrelations of nucleons on the magnitude of the binding energy of the last neutron in light nuclei. This magnitude is, undoubtedly, more sensitive to nucleon correlations than the binding energy for each nucleon.

The energy of the ground state where all the nucleons are bound in fours is determined by the magnitude $\langle H' \rangle$, the energy of the ground state of the nucleus where besides quadruple correlations there is one pair, has the form $\langle \beta_s H' \beta_s^{+} \rangle$. In order to find the binding energy of a pair in the nucleus it is necessary to calculate the difference

$$\langle \beta_s H' \beta_s^{+} \rangle_{N=2n_0 \pm 1} - \langle H' \rangle_{N=2n_0}\ .$$

In a very rough approximation considered earlier in[7] where the integration is replaced for the summation, J and the level density ρ being considered constant all over the shell, we obtain

$$\langle \beta_s H' \beta_s^+ \rangle_{N=2n_0+1} - \langle H' \rangle_{N=2n_0} = -\frac{\Omega-2n_0}{2\rho} - \frac{e^{\frac{2}{\sigma}}+1}{e^{\frac{2}{\sigma}}-1}\cdot\frac{2n_0-\Omega+1}{2\rho},$$

$$\langle \beta_s H' \beta_s^+ \rangle_{N=2n_0-1} - \langle H' \rangle_{N=2n_0} = -\frac{\Omega+2-2n_0}{2\rho} + \frac{e^{\frac{2}{\sigma}}+1}{e^{\frac{2}{\sigma}}-1}\,\frac{2n_0+\Omega-1}{2\rho}, \qquad (11)$$

where $\sigma=-\rho J$, Ω - the number of levels, N - the number of pairs. The formulae taking into account the nucleon pairing correlations are of a similar form

$$\langle \alpha_{+s} H_0 \alpha_{+s}^+ \rangle_{\nu=2\nu_0+1} - \langle H_0 \rangle_{\nu=2\nu_0} = -\frac{\Omega+\frac{3}{2}-\nu_0}{2\rho} - \frac{e^{\frac{2}{\sigma}}+1}{e^{\frac{2}{\sigma}}-1}\,\frac{\nu_0+\frac{5}{4}-2\Omega}{4\rho}, \qquad (12)$$

$$\langle \alpha_{+s} H_0 \alpha_{+s}^+ \rangle_{\nu=2\nu_0-1} - \langle H_0 \rangle_{\nu=2\nu_0} = -\frac{\Omega+\frac{3}{2}-\nu}{2\rho} + \frac{e^{\frac{2}{\sigma}}+1}{e^{\frac{2}{\sigma}}-1}\,\frac{\nu_0-\frac{5}{4}+2\Omega}{4\rho}.$$

It is seen from (11) that an additional pair to the system where all the nucleons are bound in fours affects the energy slightly, and, on the contrary, to remove a pair from the state where all the nucleons are bound in fours, i.e., for breaking up a four a considerable energy is required. By analogy, one additional nucleon to the system consisting of even number of nucleons $\nu=2\nu_0$ does not result in essential change of energy, and, on the contrary, to remove one nucleon from the nucleus where besides quadruple correlations there is one more pair a considerable energy is necessary to break up a pair.

Taking into account the pairing and quadruple correlations it is possible to divide all the light nuclei in the region $16<A<40$ with respect to the binding energy of the last nucleon into three groups: 1) the last neutron participates neither in pairing nor in quadruple correlations; 2) besides the nucleons bound in fours there is one pair. Such a pair may be either (n –n) or (n –p) -pairs; 3) all the neutrons are bound in fours. The binding energy of the last neutron in a nucleus which is referred to the first group depends only upon its binding with the central field. The binding energy of the last neutron in the second group of nuclei depends, roughly speaking, both upon the magnitude of the pairing binding and upon the binding energy of a neutron with the central field. The third group involves α -particle-like nuclei, in which, the binding energy of the last neutron consists of three parts: of the energy necessary for breaking up a four into two pairs, of the binding energy of a pair, and finally, of the binding energy of a neutron with the central field.

The data concerning the binding energy of the last neutron in light nuclei indicate that in the nuclei referred to the first group such as $_{10}Ne^{21}_{11}$, $_{12}Mg^{25}_{13}$, $_{12}Mg^{27}_{15}$, $_{14}Si^{29}_{15}$, $_{14}Si^{31}_{17}$, $_{16}S^{33}_{17}$

$_{16}S^{35}_{19}$, $_{18}Ar^{41}_{23}$, $_{19}K^{42}_{23}$, $_{20}Ca^{43}_{23}$

the binding energy of the last neutron is of the order of (7–8) MeV. In the nuclei referred to the second group $_{10}Ne^{22}_{12}$, $_{11}Na^{23}_{12}$, $_{12}Mg^{26}_{14}$, $_{13}Al^{27}_{14}$, $_{14}Si^{30}_{16}$, $_{15}P^{31}_{16}$, $_{16}S^{34}_{18}$, $_{17}Cl^{35}_{18}$, $_{18}Ar^{38}_{20}$, $_{19}K^{39}_{20}$ /2/ two outside neutrons are bound in a pair and to break it up an additional energy is necessary /2/. The magnitude of the last neutron binding energy being (11–12) MeV gives strong support for this. In view of the pairing correlation of the outside neutron and proton possessing equal energies in the nuclei $_{11}Na^{22}_{11}$, $_{13}Al^{26}_{13}$, $_{15}P^{30}_{15}$, $_{17}Cl^{34}_{17}$, $_{19}K^{38}_{19}$ the magnitude of the binding energy of the last neutron is also of the order of (11–12) MeV. It should be noted that if the outside proton and neutron are in different quantum states there is no correlation between them and the binding energy of the last neutron in such nuclei as $_{11}Na^{24}_{13}$, $_{13}Al^{28}_{15}$, $_{15}P^{32}_{17}$, $_{17}Cl^{36}_{19}$, $_{19}K^{40}_{21}$ is of the order of (7–8) MeV.

In α -particle -like nuclei, such as $_{10}Ne^{20}_{10}$, $_{12}Mg^{24}_{12}$, $_{14}Si^{28}_{14}$, $_{16}S^{32}_{16}$, $_{18}Ar^{36}_{18}$, $_{20}Ca^{40}_{20}$ all the nucleons with equal quantum numbers S are bound in fours, the binding energy of the last neutron in these nuclei is of the order of (15–17) MeV. The difference in the binding energies of the last neutron in the nuclei referred to the third and the second group roughly determines the energy necessary for breaking up a four into two pairs. This enables us to estimate ($-J$) which turns out to be 1 MeV, the energy gap $2\,\varepsilon$ due to the breaking-up of a four into pairs is found to be in this case (4 – 6) MeV.

These data confirm that in light nuclei there are pairing and quadruple correlations of nucleons in the states with equal quantum numbers S . It is to be noted that strong nucleon correlations with different values of the angular momentum projections along the symmetry axis of the nucleus and identical other quantum numbers are likely to be absent. This is supported by the nucleon behaviour in heavy nuclei in which explicit regularities dependent upon the number of outside shell protons (neutrons) multiple to four are not displayed.

It is worth while noting that the statistical method we accepted averages the properties of nuclei referring to the same group with regard to the binding energy of the last neutron. However. the account of quantum characteristics of outside nucleons enables us to determine the individuality of nuclei. Therefore, it is possible to calculate the binding energies of the last neutron for each nucleus in the framework of the scheme given with account of additional factors.

V. THE CONDITION FOR DISAPPEARANCE OF QUADRUPLE CORRELATIONS

Consider the behaviour of the four-fold correlations when the number of neutrons exceeds that of protons. Note, that in this case the magnitude of the effective interaction between the pairs must be much smaller than $J(s, s')$ in (2) which concerns the case of strict isotopic invariance. This is so, because the main contribution to the effective interaction of pairs is made by the resonance interaction of nucleons possessing equal energies. When the number of neutrons exceeds that of protons this resonance will be smeared out to a considerable extent. This will result in sharp weakening of interaction between pairs.

The behaviour of the quadruplecorrelations when the number of neutrons exceeds that of protons will be investigated by the method of advanced and retarded Green functions developed by N.N. Bogolubov and S.V. Tyablikov./9/.

Here are given some general formulae which will be employed in the calculations. The Green functions at zero temperature and the equations for them are written as

$$G_r(t-t') = \langle\langle A(t)B(t')\rangle\rangle_r = -i\theta(t-t')\langle[A(t),B(t')]\rangle,$$
$$G_a(t-t') = \langle\langle A(t)B(t')\rangle\rangle_a = i\theta(t'-t)\langle[A(t),B(t')]\rangle,$$
$$i\frac{d}{dt}\langle\langle A(t)B(t')\rangle\rangle = \delta(t-t')\langle[A(t)B(t')]\rangle + \tag{13}$$
$$+\langle\langle\{A(t)H - HA(t)\}B(t')\rangle\rangle,$$

where $A(t)$, $B(t)$ - the operators in Heisenberg representation. Let us pass to the Fourier -transform of the Green function

$$\langle\langle A(t)B(t')\rangle\rangle = \int_{-\infty}^{\infty}\langle\langle A|B\rangle\rangle_E\, e^{-iE(t-t')}dE$$

and to its spectral representation in case of Fermi statistics

$$\langle\langle A(t)B(t')\rangle\rangle_E = \frac{1}{2\pi}\int_{-\infty}^{\infty} I(\omega)\frac{d\omega}{E-\omega},$$

where

$$iI(E) = \langle\langle A|B\rangle\rangle_{E-i\epsilon} - \langle\langle A|B\rangle\rangle_{E+i\epsilon} \tag{14}$$

The spectral representation of the correlation functions in terms of the function $I(E)$ is expressed as follows:

$$\langle A(t)B(t')\rangle = \int_0^{\infty} I(\omega)e^{-i\omega(t-t')}d\omega$$
$$\langle B(t')A(t)\rangle = \int_{-\infty}^{0} I(\omega)e^{-i\omega(t-t')}d\omega \tag{15}$$

The condition for the disappearance of the quadruple correlations can be found by using the above formulas with an auxiliary Hamiltonian (6) put in the form

$$H' = \sum_s \{ \xi_p(s) a^+_{s+} a_{s+} + \xi_n(s) a^+_{s-} a_{s-} \} - \\ - \sum_{s'} J(s,s') \{ B^*(s) a_{s'-} a_{s'+} + B(s') a^+_{s+} a^+_{s-} + B^*(s) B(s') \}, \tag{16}$$

where

$$\xi_p(s) = E(s) - \lambda_p, \quad \xi_n = E(s) - \lambda_n, \quad \lambda_p \neq \lambda_n .$$

For the Green functions the following equations hold

$$i \frac{d}{dt} \langle\langle a_{sp}(t) | a^+_{sp}(t') \rangle\rangle = \delta(t-t') + \delta_{\rho,+1} \xi_p(s) \langle\langle a_{s+}(t) | a^+_{s+}(t') \rangle\rangle + \\ + \delta_{\rho,-1} \xi_n(s) \langle\langle a_{s-}(t) | a^+_{s-}(t') \rangle\rangle - \delta_{\rho,+1} \sum_{\substack{s' \\ s \neq s}} J'(s,s') B(s') \langle\langle a^+_{s-}(t) | a^+_{s+}(t') \rangle\rangle + \\ + \delta_{\rho,-1} \sum_{\substack{s' \\ s \neq s'}} J(s,s') B(s') \langle\langle a^+_{s+}(t) | a^+_{s-}(t') \rangle\rangle \tag{17}$$

$$i \frac{d}{dt} \langle\langle a^+_{sp}(t) | a^+_{s,-\rho}(t') \rangle\rangle = - \delta_{\rho,+1} \xi_p(s) \langle\langle a^+_{s+}(t) | a^+_{s-}(t') \rangle\rangle - \\ - \delta_{\rho,-1} \xi_n(s) \langle\langle a^+_{s-}(t) | a^+_{s+}(t') \rangle\rangle + \delta_{\rho,+1} \sum_{\substack{s' \\ s' \neq s}} J'(s,s') B^*(s') \langle\langle a_{s-}(t) | a^+_{s-}(t) \rangle\rangle - \\ - \delta_{\rho,-1} \sum J'(s,s') B^*(s') \langle\langle a_{s+}(t) | a^+_{s+}(t') \rangle\rangle . \tag{17'}$$

Passing to the Fourier transforms in time and introducing a new function

$$C_s = \sum_{\substack{s' \\ s' \neq s}} J'(s,s') B(s'), \tag{18}$$

performing some transformations and making use of (14) we obtain

$$\langle\langle a_{s+} | a^+_{s+} \rangle\rangle_{E-i\epsilon} - \langle\langle a_{s+} | a^+_{s+} \rangle\rangle_{E+i\epsilon} = i I_+(E) = \\ = \frac{i}{2} \{ [1 - \frac{\omega(s)}{\varepsilon'(s)}] \delta(E + \gamma + \varepsilon'(s)) + [1 + \frac{\omega(s)}{\varepsilon'(s)}] \delta(E - \gamma - \varepsilon'(s)) \}, \tag{19}$$

$$\langle\langle a_{s-} | a^{+}_{s-} \rangle\rangle_{E-i\epsilon} - \langle\langle a_{s-} | a^{+}_{s-} \rangle\rangle_{E+i\epsilon} = i I_{-}(E) =$$

$$= \frac{i}{2}\left\{\left[1-\frac{\omega(s)}{\varepsilon'(s)}\right]\delta(E-\gamma+\varepsilon'(s)) + \left[1+\frac{\omega(s)}{\varepsilon'(s)}\right]\delta(E-\gamma-\varepsilon'(s))\right\}, \qquad (19')$$

$$\langle\langle a^{+}_{s+} | a^{+}_{s-} \rangle\rangle_{E-i\epsilon} - \langle\langle a^{+}_{s+} | a^{+}_{s-} \rangle\rangle_{E+i\epsilon} = i I(E) =$$

$$= \frac{i}{2}\,\frac{C_s}{\varepsilon'(s)}\left\{\delta(E-\gamma-\varepsilon'(s)) - \delta(E-\gamma+\varepsilon'(s))\right\}, \qquad (20)$$

where

$$\varepsilon'(s) = \sqrt{C_s^2 + \omega(s)^2}\,, \quad \gamma = \frac{1}{2}\left(\xi_n(s) - \xi_p(s)\right),$$

$$\omega(s) = 1/2\left(\xi_n(s) + \xi_p(s)\right).$$

We make use of (15) and (15') in order to obtain the expression for the wave function of a four, as well as for the density both of neutron and proton pairs

$$\langle a^{+}_{s+}(t) | a^{+}_{s-}(t') \rangle = \int_0^\infty dE\, e^{-iE(t-t')} I(E) =$$

$$= \frac{1}{2}\,\frac{C_s}{\varepsilon'(s)} \int_0^\infty dE\, e^{-iE(t-t')}\left\{\delta(E-\gamma-\varepsilon'(s)) - \delta(E-\gamma+\varepsilon'(s))\right\}, \qquad (21)$$

$$\langle a_{s-}(t)\,|\,a^{+}_{s-}(t')\rangle = \int_0^\infty dE\, e^{-iE(t-t')} I_-(E) =$$

$$= \frac{1}{2}\int_0^\infty dE\, e^{-iE(t-t')} \Big\{ \Big[1-\frac{\omega(s)}{\varepsilon'(s)}\Big]\delta\big(E-\gamma+\varepsilon'(s)\big) +$$

$$+ \Big[1+\frac{\omega(s)}{\varepsilon'(s)}\Big]\delta\big(E-\gamma-\varepsilon'(s)\big)\Big\},$$

(22)

$$\langle a_{s+}(t)\,|\,a^{+}_{s+}(t')\rangle = \int_0^\infty dE\, e^{-iE(t-t')} I_+(E) =$$

$$= \frac{1}{2}\int_0^\infty dE\, e^{-iE(t-t')} \Big\{ \Big[1-\frac{\omega(s)}{\varepsilon'(s)}\Big]\delta\big(E+\gamma+\varepsilon'(s)\big) +$$

$$+ \Big[1+\frac{\omega(s)}{\varepsilon'(s)}\Big]\delta\big(E+\gamma-\varepsilon'(s)\big)\Big\}.$$

(22')

Performing the integration and making use of the formulae

$$\langle b_{s+}^{+} b_{s-}^{+} \rangle = -\langle a_{s+}^{+} a_{s-}^{+} \rangle, \quad \langle b_{s\rho}^{+} b_{s\rho} \rangle = \langle a_{s\rho}^{+} a_{s\rho} \rangle,$$

$$\langle a_{s\rho} a_{s\rho}^{+} \rangle = 1 - \langle a_{s\rho}^{+} a_{s\rho} \rangle,$$

we obtain

$$\langle b_{s+}^{+} b_{s-}^{+} \rangle = \begin{cases} -\frac{1}{2}\frac{C_s}{\varepsilon'(s)} & \text{при } |\gamma| < \varepsilon'(s) \\ 0 & \text{при } |\gamma| > \varepsilon'(s) \end{cases} \qquad (23)$$

$$\langle b_{s-}^{+} b_{s-} \rangle = \begin{cases} \frac{1}{2}\left\{1 - \frac{\omega(s)}{\varepsilon'(s)}\right\} & \text{при } |\gamma| < \varepsilon'(s) \\ 0 & \text{при } |\gamma| > \varepsilon'(s), \gamma > 0 \\ 1 & \text{при } |\gamma| > \varepsilon'(s), \gamma < 0 \end{cases} \qquad (24)$$

$$\langle b_{s+}^{+} b_{s+} \rangle = \begin{cases} \frac{1}{2}\left\{1 - \frac{\omega(s)}{\varepsilon'(s)}\right\} & \text{при } |\gamma| < \varepsilon'(s) \\ 1 & \text{при } |\gamma| > \varepsilon'(s), \gamma > 0 \\ 0 & \text{при } |\gamma| > \varepsilon'(s), \gamma < 0. \end{cases} \qquad (24')$$

It the number of neutrons is greater than that of protons then $\gamma < 0$, the function C_s satisfies the following equation

$$C_s + \frac{1}{2}\sum_{\substack{s' \\ s' \neq s}} \tilde{J}(s,s') \frac{C_{s'}}{\sqrt{C_{s'}^2 + \omega(s')^2}} = 0 \qquad (25)$$

Thus, the condition for the disappearance of the quadruple correlations is obtained as

$$\gamma > \varepsilon'(s) \qquad (26)$$

or

$$|\lambda_n - \lambda_p| > 2\,\varepsilon'(s), \qquad (26')$$

i.e., the quadruplecorrelations begin to disappear when the energy difference of neutron and proton Fermi surfaces becomes larger than the energy gap in the spectrum of excited states which is due to the breaking-up of a four into pairs. Using a rough estimate of the energy gap it is possible to find the minimum value $|\lambda_n - \lambda_p|_{min.}$ at which the quadruplecorrelations start to break-up. To estimate $|\lambda_n - \lambda_p|_{min.}$ one can make use of Eq. (25) and obtain

$$|\lambda_n - \lambda_p|_{min} \sim (-J')\Omega .$$

Both estimates give

$$|\lambda_n - \lambda_p|_{min} \sim \eta \cdot 5 \text{ MeV}, \qquad (27)$$

where η is the factor which specifies the decrease of the interaction of pairs.

Let us consider how the quadruplecorrelations are violated with the increase of $|\lambda_n - \lambda_p|$. Let $\lambda_n - \lambda_p = 2\varepsilon'(S_0)$ then

$$\left|E(S_0) - \frac{\lambda_n + \lambda_p}{2}\right| = \sqrt{C_{S_0}^2 + \left(\frac{\lambda_n - \lambda_p}{2}\right)^2} \qquad (28)$$

When $|\lambda_n - \lambda_p| < |\lambda_n - \lambda_p|_{min}$ then all the neutron pairs, lying higher than E_F^p – the Fermi surface energy for the proton shell, take part in forming the quadruplecorrelations, and when $|\lambda_n - \lambda_p| > |\lambda_n - \lambda_p|_{min}$ the outside neutron pairs stop participating in the formation of the quadruple correlations. In the limiting case $\frac{\lambda_n - \lambda_p}{2} \gg C_{S_0}$ and $E(S_0) = \lambda_p$ i.e., all the neutron pairs lying higher than E_F^p fail to participate in forming the quadruple correlations, but the rest neutrons and protons remain to be bound in fours.

Thus, when the number of neutrons becomes greater than that of protons the quadruple correlations are weakening, the outside neutrons do not participate in forming the quadruplecorrelations. This lead to the disappearance of the explicit α particle-like nuclear properties.

It ought to be noted that in stable nuclei in the region $30 < A < 40$ there appear a tendency to increasing the number of neutrons with respect to that of protons due to a difference in the Fermi surface energies for protons and neutrons. However, the quadruplecorrelations between nucleons oppose this tendency. By $A > 40$, $|\lambda_n - \lambda_p|$ seems to reach its critical value $|\lambda_n - \lambda_p|_{min}$. Then the outside neutrons stop participating in the quadruple correlations and the correlations themselves are weakening.

Let us investigate the problem of quadruplecorrelations of the considered type in medium and heavy nuclei. The neutrons possessing the energy greater than E_F^p do not participate in quadruple correlations. They are bound in pairs. The protons, on the contrary, do not only form pairs but also quadruple correlations alongside with neutrons having the energies equal to those of protons. The energy of a nucleus can be written as

$$\langle H'\rangle = \sum_{s<s_0}\left\{\xi_p(s)\langle b^+_{s+} b_{s+}\rangle + \xi_n(s)\langle b^+_{s-} b_{s-}\rangle\right\} + \sum_{s\geq s_0}\xi_n(s) + \sum_{\substack{s<s_0\\ s'<s_0\\ s\neq s'}} J'(s,s')\langle b^+_{s+} b^+_{s-}\rangle\langle b_{s'-} b_{s'+}\rangle . \qquad (29)$$

The density functions and the wave functions of pairs are determined by (23), (24), (24') with $\gamma < 0$ as well as by Eq.

$$C_s + \frac{1}{2}\sum_{\substack{s'<s_0\\ s'\neq s}} J(s,s')\frac{C_{s'}}{\sqrt{C_s^2+\left\{E(s')-\frac{\lambda_p+\lambda_n}{2}\right\}^2}} = 0 . \qquad (25')$$

If for the excitation of a neutron in case of an even shell it is necessary to break the pair up, then for one-particle excitation of a proton not only pairing correlation have to be broken up but also quadruple correlation between two protons and two neutrons having equal energies. It follows from here that the energy of one-particle excitation for protons must be somewhat larger than the corresponding magnitude for neutrons. This conclusion is in agreement with an estimate of this ratio equal to (1.5 ± 0.3) obtained in /10/ from the mass defects.

It should be noted that the effect of the quadruple correlations in complex nuclei must not be great for two reasons: 1) because of decreasing of $J'(s,s')$ due to the extension of Fermi surfaces for neutrons and protons and 2) because the main contribution to (25') is made by the terms for which $E(s') \sim \frac{\lambda_p+\lambda_n}{2}$ whereas in (29) C_s are entering for which $E(s) \ll \frac{\lambda_p+\lambda_n}{2}$ (therefore the interactions J' (s,s') with very different quantum numbers S,s' being less than J' (s,s') possessing similar quantum numbers s,s' are essential).

CONCLUSION

Keeping in mind strong nucleon interactions being in the S-state relative to each other we considered the interaction of nucleon pairs which lead to the formation of quadruple correlations of nucleons in the case when the interactions between pairs are attractive. As a result some regularities concerning light nuclei which are understandable from the standpoint of the α -particle nuclear model are explained in the framework of the shell model by taking into account quadruple correlations. The mathematical procedure devoloped and the main results enable us to calculate the effect of quadruple correlations upon these or those properties of light nuclei.

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