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ABSTRACT

In this paper the results of phase shift analysis of experimental data on π-meson-nucleon scattering in the energy region of 240-330 MeV are given. Information on phase shifts of π-meson interactions with isotopic spin $T = 1/2$ are obtained with a satisfactory accuracy.

I. FORMULAS OF PHASE SHIFT ANALYSIS

When performing the phase shift analysis of experimental data/1/, we used isotopic spin formalism based on charge independence hopothesis of nuclear forces. At ~300 MeV one can expect an appreciable contribution into the scattering of states with orbital moments $l \gg 1$ and, first of all, with $l = 2$. If we assume that phase shifts are real values, the differential cross sections of π-meson scattering on hydrogen can be written as follows/2/:

$$K^2 \left(\frac{d\sigma}{d\omega}\right)_{\pi^+\to\pi^+} = \Big[\sin^2\alpha_3 + (\sin^2\alpha_{31} + 2\sin^2\alpha_{33})\cos\vartheta + \tag{1}$$

$$+ (2\sin^2\delta_{33} + 3\sin^2\delta_{35})\, P_2(\cos\vartheta)\Big]^2 + \frac{1}{4}\Big[\sin 2\alpha_3 +$$

$$+ \sin 2\alpha_{31} + 2\sin 2\alpha_{33})\cos\vartheta + (3\sin 2\delta_{35} + 2\sin 2\delta_{33})\, P_2(\cos\vartheta)\Big]^2 +$$

$$+ \sin^2 \Big\{\Big[\sin^2\alpha_{33} - \sin^2\alpha_{31} + 3\cos\vartheta(\sin^2\delta_{35} - \sin^2\delta_{33})\Big]^2 +$$

$$+ \frac{1}{4}\Big[\sin 2\alpha_{33} - \sin 2\alpha_{31} + 3\cos\vartheta(\sin 2\delta_{35} - \sin 2\delta_{33})\Big]^2\Big\}.$$

$$9k^2 \left(\frac{d\sigma}{d\omega}\right)_{\pi^-\to\pi^-} = \Big[\sin^2\alpha_3 + 2\sin^2\alpha_1 + \tag{2}$$

$$+ (\sin^2\alpha_{31} + 2\sin^2\alpha_{33} + 2\sin^2\alpha_{11} + 4\sin^2\alpha_{13})\cos\vartheta +$$

$$+ (2\sin^2\delta_{33} + 3\sin^2\delta_{35} + 4\sin^2\delta_{13} + 6\sin^2\delta_{15})\, P_2(\cos\vartheta)\Big]^2 +$$

$$+\frac{1}{4}\Big[\sin 2\alpha_3 + 2\sin 2\alpha_1 + (\sin 2\alpha_{31} + 2\sin 2\alpha_{33} +$$

$$+ 2\sin 2\alpha_{11} + 4\sin 2\alpha_{13})\cos\vartheta + (3\sin 2\delta_{35} +$$

$$+ 6\sin 2\delta_{15} + 2\sin 2\delta_{33} + 4\sin 2\delta_{13}) P_2(\cos\vartheta)\Big]^2 +$$

$$+ \sin^2\vartheta\Big\{\Big[\sin^2\alpha_{33} + 2\sin^2\alpha_{13} - \sin^2\alpha_{31} - 2\sin^2\alpha_{11} +$$

$$+ 3\cos\vartheta(\sin^2\delta_{35} + 2\sin^2\delta_{15} - \sin^2\delta_{33} - 2\sin^2\delta_{13})\Big]^2 +$$

$$+ \frac{1}{4}\Big[\sin 2\alpha_{33} + 2\sin 2\alpha_{13} - \sin 2\alpha_{31} - 2\sin 2\alpha_{11} +$$

$$+ 3\cos\vartheta(\sin 2\delta_{35} + 2\sin 2\delta_{15} - \sin 2\delta_{33} - 2\sin 2\delta_{13})\Big]^2\Big\}.$$

(3)

$$\frac{9}{2}k^2\left(\frac{d\sigma}{d\omega}\right)_{\pi\to\pi^0} = \Big[\sin^2\alpha_3 - \sin^2\alpha_1 + (2\sin^2\alpha_{33} - 2\sin^2\alpha_{13} +$$

$$+ \sin^2\alpha_{31} - \sin^2\alpha_{11})\cos\vartheta + (3\sin^2\delta_{35} - 3\sin^2\delta_{15} + 2\sin^2\delta_{15} -$$

$$- 2\sin^2\delta_{13}) P_2(\cos\vartheta)\Big]^2 + \frac{1}{4}\Big[\sin 2\alpha_3 - \sin 2\alpha_1 + (2\sin 2\alpha_{33} -$$

$$- 2\sin 2\alpha_{13} + \sin 2\alpha_{31} - \sin 2\alpha_{11})\cos\vartheta + (3\sin 2\delta_{35} -$$

$$- 3\sin 2\delta_{15} + 2\sin 2\delta_{33} - 2\sin 2\delta_{13}) P_2(\cos\vartheta)\Big]^2 +$$

$$+ \sin^2\vartheta\Big\{\Big[\sin^2\alpha_{33} - \sin^2\alpha_{13} - \sin^2\alpha_{31} + \sin^2\alpha_{11} + 3\cos\vartheta(\sin^2\delta_{35} -$$

$$- \sin^2\delta_{15} - \sin^2\delta_{33} + \sin^2\delta_{13})\Big]^2 + \frac{1}{4}\Big[\sin 2\alpha_{33} - \sin 2\alpha_{13} -$$

$$- \sin 2\alpha_{31} + \sin 2\alpha_{11} + 3\cos\vartheta(\sin 2\delta_{35} - \sin 2\delta_{15} - \sin 2\delta_{33} +$$

$$+ \sin 2\delta_{13})\Big]^2\Big\}.$$

The total cross sections (after integrating over 1-3) are written in the following way:

for π^+-mesons

$$\sigma_t^+ = \frac{4\pi}{k^2}\left(\sin^2\alpha_3 + 2\sin^2\alpha_{33} + \sin^2\alpha_{31} + 3\sin^2\delta_{35} + 2\sin^2\delta_{33}\right), \tag{4}$$

for π^--mesons

$$\sigma_t^- = \frac{4\pi}{3k^2}\left(\sin^2\alpha_3 + 2\sin^2\alpha_1 + 2\sin^2\alpha_{33} + 4\sin^2\alpha_{13} + \sin^2\alpha_{31} + 2\sin^2\alpha_{11} + 3\sin^2\delta_{35} + 6\sin^2\delta_{15} + 2\sin^2\delta_{33} + 4\sin^2\delta_{13}\right) \tag{5}$$

The phase shift notations are tabulated in Table 1.

Table I.

	S	P	P	D	D
T = 3/2	α_3	α_{31}	α_{33}	δ_{33}	δ_{35}
T = 1/2	α_1	α_{11}	α_{31}	δ_{13}	δ_{15}

II. METHOD FOR CALCULATING PHASE SHIFTS AND THEIR ERRORS

According to the least squares method a set of phase shifts was chosen which would give the minimum value of the weighted least squares sum

$$M = \sum_i \left(\frac{\sigma_{i\,calc} - \sigma_{i\,exp}}{\varepsilon_i}\right)^2 \tag{6}$$

where $\sigma_{i\,calc}$ are differential and total cross sections, calculated from phase shifts by means of formulas (1–3), $\sigma_{i\,exp}$ are the corresponding experimental values of cross sections, ε_i are experimental errors in the measured cross sections.

All the calculations were carried out with the help of the high-speed electronic computer 'Strela'.

To decrease the time necessary for performing the calculations the deduction of the minimum value of M was carried out by the gradient method. In the hyperspace of phase shifts a definite point corresponds to the minimum value of $M = M_{min}$. For all $M > M_{min}$ the functions $M = const$ form the system of hypersurfaces which are level surfaces of the scalar field $M(\alpha)$. Apparently, moving along the gradient of the field, one can quickly approach the minimum point. In this connection at the initial point of phase shift hyperspace the value M_1 and the normal to the hypersurface $M_1 = const$ at this point were determined. Then all the phase shifts got such an increment as to move along in the direction of this normal approximately by 1^o. At a new point the calculation $M = M_2$ was made, and a new value of M_2 was compared with the initial one. When $M_2 < M_1$, the movement along the initial gradient was repeated until a new value of M turned out to be larger than the previous one. Then at the last point but one a new direction of the gradient was determined, and further movement along this direction was performed at the same pace. It was repeated until the first pace led to the increase of M, after the gradient had been calculated. In this case the size of the pace was reduced twice and further the whole procedure of the calculation was analogous to the previous one. Then the pace was reduced twice again, etc. The calculations were stopped when the required pace was 2^{-4} degree.

On finding the solution $(M = M_{min})$ the errors of phase shifts were calculated. The Tailor expansion of the function $M(\alpha_1, \alpha_2 \ldots)$ at the point M_o yields:

$$M = M_o + \sum_i \frac{\partial M}{\partial \alpha_i} \Delta\alpha_i + \frac{1}{2} \sum_i \sum_j \frac{\partial^2 M}{\partial \alpha_i \partial \alpha_j} \Delta\alpha_i \Delta\alpha_j \ldots \qquad (7)$$

At the minimum point the second term vanishes and

$$M - M_{min} \cong \sum_i \sum_j S_{ij} \Delta\alpha_i \Delta\alpha_j \qquad (8)$$

where $$S_{ij} = \frac{1}{2} \frac{\partial^2 M}{\partial \alpha_i \partial \alpha_j}$$

Equation (8) determines hyperellipsoid at the minimum region, i.e. the coefficient matrix $\langle S_{ij} \rangle$ defines the M_{min} sensitivity to the phase shift deviations from optimum values. Respectively, the terms of reciprocal matrix $\langle S_{ij} \rangle^{-1}$ determine phase shift errors (with $i = j$) and the correlations of phase shifts with each other (when $i \neq j$)/3/.

The coefficients S_{ij} can be calculated directly from formula (8), but it is too complicated. We used the approximated method of final differences. To determine the diagonal elements S_{ij} the phase shift α_i was given an increment equal to $\Delta\alpha_i$, and the value of M was determined. Then according to (8)

$$S_{ii} = \frac{M - M_{min}}{(\Delta\alpha_i)^2} \tag{9}$$

Calculations were carried out both for positive and negative increments of the phase shift α_i; the result was averaged.

When defining the non-diagonal elements both the phase shifts were given an increment. All the possible signs of the increments were taken and the result was averaged. If one denotes the values of M according to possible combinations of the increment signs $\Delta\alpha_i$ and $\Delta\alpha_j$ $(++,--,+-,-+)$ as M_{++}, M_{--}, M_{+-}, M_{-+} respectively, one has:

$$\begin{aligned}
M_{++} - M_{min} &= S_{ii}(\Delta\alpha_i)^2 + S_{jj}(\Delta\alpha_j)^2 + 2S_{ij}(\Delta\alpha_i)\cdot(\Delta\alpha_j) \\
M_{--} - M_{min} &= S_{ii}(-\Delta\alpha_i)^2 + S_{jj}(-\Delta\alpha_j)^2 + 2S_{ij}(-\Delta\alpha_i)\cdot(-\Delta\alpha_j) \\
M_{+-} - M_{min} &= S_{ii}(\Delta\alpha_i)^2 + S_{jj}(-\Delta\alpha_j)^2 + 2S_{ij}(\Delta\alpha_i)(-\Delta\alpha_j) \\
M_{-+} - M_{min} &= S_{ii}(-\Delta\alpha_i)^2 + S_{jj}(\Delta\alpha_j)^2 + 2S_{ij}(-\Delta\alpha_i)\cdot(\Delta\alpha_j)
\end{aligned} \tag{10}$$

Subtracting from the sum of the first two equations (10) the third and the fourth ones, we obtain (when $\Delta\alpha_i = \Delta\alpha_j = \Delta\alpha$)

$$S_{ij} = \frac{(M_{++} + M_{--}) - (M_{+-} + M_{-+})}{8(\Delta\alpha)^2} \tag{11}$$

In all the cases the increment $\Delta\alpha$ was 1°.

III. PHASE SHIFT ANALYSIS WHEN S AND P-WAVES ARE TAKEN INTO ACCOUNT

While performing phase shift analysis for each π-meson energy value, 25 experimental points were used: eight differential cross sections of elastic π^+-meson scattering by protons, seven differential cross sections of elastic π^--meson scattering, eight differential cross sections of charge-exchange π^--meson scattering on hydrogen and total cross sections of π^+ and π^- meson scattering on hydrogen.

Differential cross section values of elastic π^+-meson scattering on hydrogen and total cross sections of $\pi^{\pm}$-meson interaction with hydrogen as well as the initial values of phase shifts for the states with T 3/2 were taken from Mukhin and colleagues papers:/4,5,6,7/.

Differential cross sections of π^+-meson scattering on hydrogen at 333 MeV were determined by means of interpolation of the data from Mukhin, Ozerov, Pontecorvo/6/, and Grigoriev and Mitin/8/. *In connection with this the result of the phase shift analysis at 333 MeV cannot be considered entirely independent.*

The initial values of phase shifts for the states with $T = 1/2$ were obtained with the help of Ashkin graphical method/9/. This method provides two possible sets of phase shifts (Variant 1,3. Tables 2–6). Together with them at each energy as initials two sets of phases were employed, differing by the opposite signs of a phase (variants 2,4) from the obtained ones by the gradient method, and two sets with the negative phase shifts of P-waves with $T=1/2$ (variants 6,7). Such set were to be more consistent with some data on photoproduction/10/.

Besides the data from/1/, the informations obtained by Ashkin et. al./II/ at 220 MeV were treated.

The initial and the calculated values of phase shifts are given in Tables 2–6.

From Tables 2–6 it is seen that there are only two stable variants of solution which are observed at all the energies. In each of these variants the same phase shifts do not differ greatly at various energies. These are solutions 'a' and 'b'. Other variants either coincide with the abovementioned solutions, or give the values of phase shifts observed only at some energies (for instance, variant 4 at 240 and 270 MeV). Apparently, these last solutions are bound with the mathematical side of the problem but not with the physical one, i.e. they are accidental.

From solutions 'a' and 'b' only solution 'a' describes experimental data well enough. Indeed, the expected value of M at SP-analysis is equal to 19 (25 experimental points minus 6 phase shifts). In case of solution 'b' the value of M has the magnitude ~30 nearly at all the energies. The probability of obtaining such a value of M is equal to ~5%/12/. At 307 MeV the value of M for solution 'b' reaches the magnitude as high as 71. The probability of observing such a value of M is negligibly small. At the same time the values of M, obtained in solution 'a', are well consistent with the expected values.

The angular distributions of π^--mesons and γ-quanta of elastic and charge exchange scattering computed from phase shifts of solution 'a', are shown in fig. 1-4.

It is necessary to mark one more feature of solution 'b', namely, that the signs of phases α_1 and α_{11} change into opposite ones when passing from 240 MeV to 270 MeV. It seems to be hardly probable that the phase α_1 which has a positive sign at small energies, should change it twice in the energy range from 150 MeV up to 270 MeV. The experimental results on investigation of recoil proton polarisation, when π-mesons interact with hydrogen, can serve as an additional criterion permitting to choose between various solutions. These experiments were carried out by Ashkin et al./13/ At π^--meson energy of 223 MeV, by Vasilevski and Vishniakov/14/ (preliminary data) at π^--meson energy of 300 MeV. From these experiments it follows that solution 'a' is more probable.

The aforesaid statements allow to conclude that π^--meson scattering on protons in the region above 'resonance' is described with a better probability by solution 'a'. The error matrixes for this solution are tabulated in Tables 7-11. The phase shift errors of solution 'a' listed in Tables 2-6, are diagonal elements of the corresponding matrixes.

Phase shifts at states with isotopic spin $T = 3/2$, given by solution 'a', do not practically differ from those which are obtained from π^+-meson experiments/7/.

The following can be said about the phase shifts at states with isotopic spin $T = 1/2$. The phase shifts of S-state are positive. The impulse dependence of α_1 is shown in fig. 5. It is seen that the behaviour of the phase shift α_1 , within errors in the determination of its value, does not contradict the linear dependence which is observed at smaller energies ($\alpha_1 = 9.9^{\circ}\eta$).

The phase shift α_{11} can be said to have the value not exceeding 10°. The sign of this phase appears to be positive.

As it is seen from Tables 2-6, the absolute value of the phase α_{13} is small (1° -5°). Probably, it has a negative sign. It must be noted here that according to Chew theory /15,16,17/ the phase α_{13} must be negative and is approximately equal to the phase α_{31} . The phase shifts of solution 'a ', satisfy this condition to within errors.

In the published paper of Chiu and Lomon /18/ the phase shift analysis of Ashkin et al data and our preliminary data at 307 MeV are given. Since the solution, which is considered by them to be most true, does not coincide with our solution 'a', some additional calculations were made. The solution phases, given by Chiu and Lomon were taken as initial ones in these calculations . They have led to solution 'b' at all the energies (except 240 MeV).

The cross sections of inelastic processes have a noticeable value at 307 and 333 MeV. Hence, scattering phase shifts cannot be considered as real numbers, strictly speaking. In a general form the differential cross sections for π-meson scattering on nucleons at states with definite isotopic spin can be written as follows:

$$\left(\frac{d\sigma}{d\omega}\right)_{\pi^+\to\pi^+} = \left|\sum_{\ell=0}^{\infty} \frac{a_\ell}{2ik} P_\ell(\cos\vartheta)\right|^2 + \left|\sum_{\ell=0}^{\infty} \frac{b_\ell}{2ik} P_{\ell 1}(\cos\vartheta)\right|^2 \qquad (12)$$

where k-impulse of π-meson in the centre-of-mass system, and a_ℓ and b_ℓ are determined by equations:

$$a_\ell = (\ell+1)\, e^{2i\alpha^{\ell}_{3,2\ell+1}} + \ell\, e^{2i\alpha^{\ell}_{3,2\ell+1}} - (2\ell+1)$$

$$b_\ell = e^{2i\alpha^{\ell}_{3,2\ell+1}} - e^{2i\alpha^{\ell}_{3,2\ell+1}} \qquad (13)$$

The presence of imaginary part of the phase shifts leads to the fact that in (13) expressions of the form $e^{2i\alpha_\ell}$ appear instead of expressions of the form $\beta_\ell e^{2i\alpha_\ell}$, where β_ℓ and α_ℓ are real values. Using the obtained experimental cross sections of inelastic processses /19/, one can evaluate β_ℓ , basing on the general quantum mechanics theory of inelastic scattering /20/. It turns out that even at 333 MeV for any state the value β_ℓ differs from I not more than (2–3)%, which corresponds to the value of imaginary part not more than 0.5°. A small difference of β_ℓ from I leads to the fact that the maximum possible change of phases, connected with the account of inelastic processes, due to the performed evaluations, will not exceed $\sim (\pm\ 3^{\circ})$, i.e. it does not exceed phase determination accuracy. Thus, for energies up to 333 MeV the phase shift analysis made on the assumption, that phase shits are real numbers, is apparently, rather a good approximation.

IV. PHASE SHIFT ANALYSIS WITH S–P– AND D–WAVES TAKEN INTO ACCOUNT (SPD–ANALYSIS)

At π-meson energy of ~ 300 MeV an appreciable contribution of D-waves (1 = 2) into scattering is expected.

In connection with this for all the π-meson energies a phase shift analysis was performed, when S- P- and D-waves were taken into account. The phase shifts of solution 'a' and 'b' were employed as the initial ones for S- and P-waves. The initial values of D-wave phase shifts at states with T = 3/2 in accordance with the results, obtained in π^+-meson experiments/7/, were taken to be equal to $\delta_{33} = 6^0$ and $\delta_{35} = -6^0$. The absolute values of initial phase shifts of D-waves at states with isotopic spin T = ½ were taken to be equal to 8^0. The signs of these phase shifts were taken in all possible combinations, i.e. (++), (– –), (+ –), and (– +). Thus, for each energy eight variants of initial phase shifts were obtained: four for solution 'a' and four for solution 'b'.

As a result of the performed calculations the initial variants, differing only by signs of the phase shifts δ_{13} and δ_{15} have led to one and the same solution. In other words, two final variants of solution were obtained: one for initial solution 'a' and the other for solution 'b'. Phase shift values, corresponding to solutions 'a' and 'b' are given in Tables 12 and 13 . These two variants of solution are called further: solution 'a_{SPD} ' and solution 'b_{SPD} '.

For the 333 MeV energy, three different variants of solution turned out to be in solution 'a'. Two of them are given in Table 12. These solutions provide phase shift values, corresponding to those which are obtained at smaller energies.

From Table 12 one can see that phase shifts α_1 , α_{11} and α_{13} of solution 'a' change only by some degrees, not more, when D-waves are taken into account, i.e. they remain within experimental errors. This result could be expected because just S- and P-phase shifts alone had described experimental data well enough.

Phase shifts δ_{33} and δ_{35} of D-waves at states with isotopic spin T = 3/2 do not differ practically from those which were obtained earlier in Mukhin and Pontecorvo's paper/7/.

Phase shifts δ_{13} and δ_{15} of D-waves at states with isotopic spin T = ½, as is seen from Table 12, are small. The accuracy of their determination does not allow to make a definite conclusion, concerning these phase shifts signs.

It is difficult to say something definite about solution 'b_{SPD} ' . We cannot omit it on the ground that the corresponding solution 'b' in SP-analysis is not in agreement with the experimental data. There appears a possibility that poor agreement, given by solution 'b', is caused by a necessity to take D-waves into consideration in this analysis. However, it seems to be hardly probable that already at energies as high as 240-307 MeV such great values of D-phases (up to 15^0), which are obtained in solution 'b_{SPD}', take place. Dispersion relations (see next sect.). indicate also that solution 'b_{SPD}' is probably incorrect.

V. COMPARISON OF PHASE SHIFT RESULTS WITH THE DISPERSION RELATIONS

As is known, dispersion relations allow to determine the energy dependence of the real part of the pion forward scattering amplitude D(0). The expression for D (0) can be obtained from formula (2) for the cross section of -meson elastic scattering in hydrogen which is a sum of squares of real and imaginary parts of the scattering amplitude. We have:

$$D_{-}(0) = \frac{1}{6k}\Big(\sin 2\alpha_3 + \sin 2\alpha_{31} + 2\sin 2\alpha_{33} + 2\sin 2\delta_{33} + 3\sin 2\delta_{35} + 2\sin 2\alpha_1 + 2\sin 2\alpha_{11} + 4\sin 2\alpha_{13} + 4\sin 2\alpha + 6\sin 2\delta_{15}\Big) \qquad (13)$$

The values of D (0), calculated from phase shifts, are shown in fig. 6. It is seen that solution 'a' provides quite a satisfactory agreement with the curve, obtained in[14] with the coupling constant value of meson-nucleon interaction equal to $f^2 \simeq 0.08$.

In conclusion the authors consider their pleasant duty to express their sincere gratitude to B. Pontecorvo for constant attention and help in work, to A.I. Mukhin. L.I. Lapidus, S.N. Sokolov, N.P. Klepiknov for numerous and fruitfull discussions, to I.V. Popova and L.A. Chudov for valuable assistance in compiling the programme for an electric computer.

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Table 2.

Phase shifts (in degrees) at π-meson energy of 220 MeV (Ashkin et al data /11/).

Variant		3	6	I	2	4	7
Initial values	α_3	-I2,0	-I9,0	-I2,0	-I2,0	-I2,0	-I9,0
	α_{31}	0,0	- 5,0	0,0	0,0	0,0	-5,0
	α_{33}	I07,0	I25,0	I07,0	I07,0	I07,0	I25,0
	α_1	I0,0	I0,0	I0,0	-I0,0	-I0,0	-I0,0
	α_{11}	8,0	-I0,0	- 6,0	- 6,0	8,0	-I0,0
	α_{13}	-3,0	-I0,0	5,0	5,0	-3,0	-I0,0
	M	36,0	687,0	I09,0	88,0	I22,0	I559,0
Solution		a		B			
Values at the minimum point	α_3	-I5,8 ±I,5		- I7,0			
	α_{31}	- 2,0 ±2,9		- 0,I			
	α_{33}	III,0 ±I,8		II2,0			
	α_1	I4,0 ±4,3		- 7,0			
	α_{11}	6,2 ±3,3		I6,I			
	α_{13}	- 5,2 ±0,8		3,6			
	M	I6,I		I4,5			

Table 3.

Phase shifts (in degrees) at π-meson energy of 240 MeV

Variant		1	3	5	4	2	6	7
Initial values	α_3	-I8,I	-I8,I	-I4,7	-I8,I	-I8,I	-I9,0	-I9,0
	α_{31}	- 2,6	- 2,6	- 3,2	- 2,6	- 2,6	- 5,0	-5,0
	α_{33}	II4,7	II4,7	I09,5	II4,7	II4,7	I25,0	I25,0
	α_1	I0,0	I0,0	8,0	-I0,0	-I0,0	I0,0	-I0,0
	α_{11}	- 6,5	8,0	-5,4	8,0	-6,5	-I0,0	-I0,0
	α_{13}	4,5	-2,5	+3,3	-2,5	4,5	-I0,0	-I0,0
	M	60,0	20,0	85,0	I40,0	I2I,0	468,0	832,0
Solution		a			C	B		
Values at the minimum point	α_3	-I8,I ± I,3			-I8,5	-I9,6		
	α_{31}	- 3,0 ± 2,5			- 0,5	- I,4		
	α_{33}	II5,0 ±I,0			II5,0	II5,0		
	α_1	II,2 ±4,4			4,8	- 0,3		
	α_{11}	I0,0 ±5,4			I6,7	- 2,6		
	α_{13}	-2,4 ±I,4			I,7	I4,0		
	M	I8,8			I8,8	29,7		

Table 4.

Phase shifts (in degrees) at π-meson energy of 270 MeV

Variant		3	6	7	4	I	2	5
Initial values	α_3	-20,2	-I9,0	-I9,0	-20,2	-20,2	-20,2	-24,I
	α_{31}	- 6,7	- 5,0	- 5,0	- 6,7	- 6,7	- 6,7	-I0,3
	α_{33}	I29,3	I25,0	I25,0	I29,3	I29,3	I29,3	I32,8
	α_1	II,0	I0,0	-I0,0	-II,0	II,0	-II,0	9,6
	α_{11}	I0,5	-I0,0	-I0,0	I0,5	-I7,5	-I7,5	-I0,0
	α_{13}	- 7,5	-I0,0	-I0,0	- 7,5	I0,5	I0,5	I0,0
	$\mathcal{M}$	76,0	404,0	727,0	410,0	I04,0	I80,0	2I6,0
Solution		a			c	B		
Values at the minimum point	α_3	-20,I±I,3			-I9,9	-20,3		
	α_{31}	- 7,0±I,8			- 6,I	- 8,2		
	α_{33}	I29,0±0,9			I29,2	I29,7		
	α_1	25,7±2,5			3,0	0,2		
	α_{11}	5,3±3,0			27,3	- 9,4		
	α_{13}	- I,2±I,2			4,I	20,I		
	$\mathcal{M}$	9,6			I5,3	27,0		

Table 5.

Phase shifts (in degrees) at π-meson energy of 307 MeV

Variant		3	4	6	7	I	2	5
Initial values	α_3	-25,0	-25,0	-25,0	-25,0	-25,0	-25,0	-24,0
	α_{31}	-I0,0	-I0,0	-I2,0	-I2,0	-I0,0	-I0,0	-I0,3
	α_{33}	I33,0	I33,0	I35,0	I35,0	I33,0	I33,0	I32,8
	α_1	9,0	- 9,0	I0,0	-I0,0	9,0	- 9,0	9,6
	α_{11}	I0,0	I0,0	-I0,0	-I0,0	-I8,5	-I8,5	-I0,0
	α_{13}	- 9,0	- 9,0	-I0,0	-I0,0	I0,0	I0,0	I0,0
	$\mathcal{M}$	56,0	2I7,0	379,0	505,0	I5I,0	I59,0	228,0
Solution		a				B		
Values at the minimum point	α_3	-23,9±I,2				-23,6		
	α_{31}	-I0,0±2,0				-I5,4		
	α_{33}	I32,4±0,9				I35,2		
	α_1	I7,I±5,2				4,I		
	α_{11}	II,4±3,3				-22,4		
	α_{13}	- 5,0±I,2				I4,6		
	$\mathcal{M}$	29,0				7I,0		

Table 6.

Phase shifts (in degrees) at π-meson energy of 333 MeV

Variant		3	4	I	2	5	6	7
Initial values	α_3	-30,0	-30,0	-30,0	-30,0	-24,I	-25,0	-25,0
	α_{31}	-I2,0	-I2,0	-I2,0	-I2,0	-I0,3	-I2,0	-I2,0
	α_{33}	I39,0	I39,0	I39,0	I39,0	I32,8	I35,0	I35,0
	α_1	I0,0	-I0,0	I0,0	-I0,0	9,6	I0,0	-I0,0
	α_{11}	I0,0	I0,0	-I8,0	-I8,0	-I0,0	-I0,0	-I0,0
	α_{13}	-9,0	-9,0	I0,0	I0,0	I0,0	-I0,0	-I0,0
	$\mathcal{M}$	87	324	I79	224	25I	3875	6706
Solution		a		б			d	
Values at the minimum point	α_3	-26,5±I,4		-25,0			-28,0	
	α_{31}	-I0,6±2,I		-I5,7			-I0,7	
	α_{33}	I37,2±I,I		I40,I			I39,5	
	α_1	29,2±I,8		II,7			3I,4	
	α_{11}	8,I±2,9		-24,0			-I6,3	
	α_{13}	-2,0±I,3		I6,0			I0,0	
	$\mathcal{M}$	22,6		33,I			94,4	

Table 7.

Phase shift error matrix (radian^2). E_π = 220 MeV.

	α_3	α_{31}	α_{33}	α_1	α_{11}	α_{13}	.
α_3	0,0006I6	-0,000540	-0,000008	0,000068	0,000746	0,000I28	
α_{31}	0,	0,002234	-0,000056	0,000380	-0,00I266	0,000I36	
α_{33}			0,000854	0,00I638	-0,0005I6	0,000II4	
α_1				0,004938	-0,002I22	0,000388	
α_{11}					0,002984	-0,000032	
α_{13}						0,000I70	

Table 8.

Phase shift error matrix (radian^2). $E_\pi = 240$ MeV.

	α_3	α_{31}	α_{33}	α_1	α_{11}	α_{13}
α_3	0,000492	-0,000I80	-0,000042	-0,0003I6	0,000724	0,000I76
α_{31}		0,00I754	-0,000038	0,000256	-0,00I090	0,000080
α_{33}			0,000256	0,000446	-0,000252	-0,000082
α_1				0,005234	-0,0047I0	-0,000852
α_{11}					0,007972	0,00I464
α_{13}						0,000532

Table 9.

Phase shift error matrix (radian^2). $E_\pi = 270$ MeV.

	α_3	α_{31}	α_{33}	α_1	α_{11}	α_{13}
α_3	0,000448	-0,000028	0,0000I0	0,000I00	0,000I52	0,000I26
α_{31}		0,000862	-0,000028	0,000068	-0,000528	0,000082
α_{33}			0,0002I6	0,000340	-0,000I98	0,000I08
α_1				0,00I640	-0,00I360	0,000628
α_{11}					0,002436	-0,000588
α_{13}						0,000394

Table 10.

Phase shift error matrix (radian^2). $E_\pi = 307$ MeV.

	α_3	α_{31}	α_{33}	α_1	α_{11}	α_{13}
α_3	0,000370	-0,000020	-0,000040	-0,000I86	0,000238	0,000044
α_{31}		0,00I028	-0,000042	0,00II98	-0,00II40	0,000274
α_{33}			0,000I94	0,000520	-0,000I04	0,000080
α_1				0,007304	-0,002346	0,00I364
α_{11}					0,002896	-0,000478
α_{13}						0,000398

Table 11.

Phase shift error matrix (radian^2). $E_\pi = 333$ MeV.

	α_3	α_{31}	α_{33}	α_1	α_{11}	α_{13}
α_3	0,000556	-0,000238	0,000I22	0,000048	0,000562	-0,000004
α_{31}		0,00II88	-0,000286	0,000366	-0,00I066	0,0005I8
α_{33}			0,000344	0,000I84	0,000240	0,000006
α_1				0,000868	-0,000840	0,000494
α_{11}					0,002268	-0,000766
α_{13}						0,000460

Phase shifts (in degrees) at π-meson energy of 240–333 MeV.

T a b l e 12.

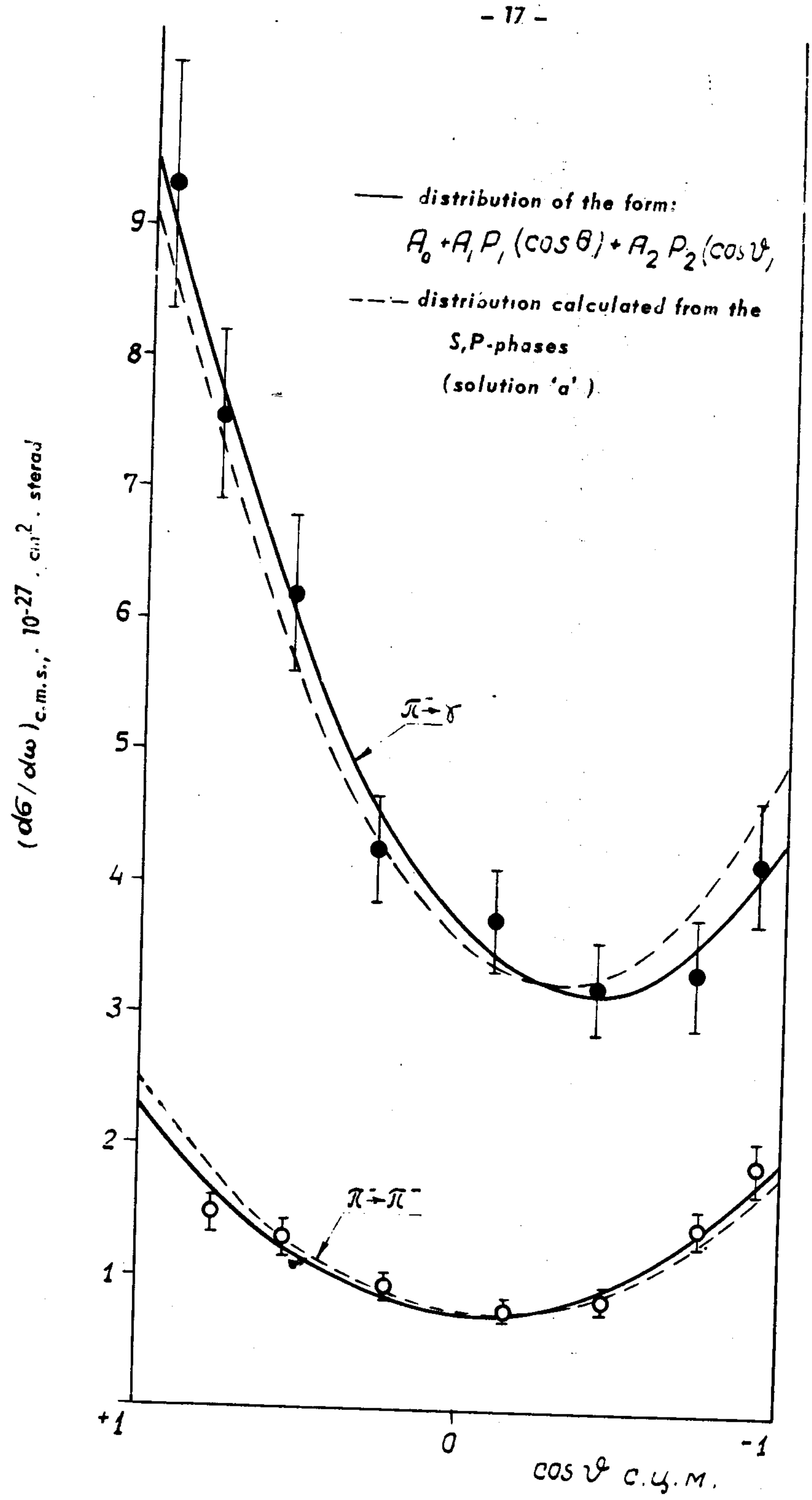
Solution „a_{SPD}"

Meson energy in MeV	240	270	307	333(I)	333(2)
α_3	-20,4±I,9	-I5,0±6,2	-I0,2±3,7	-22,0±I,6	-I0,5±I,I
α_{31}	- 4,2±3,6	- 4,0±3,I	- 3,6±2,2	- 6,3±2,I	- 2,8±I,7
α_{33}	II6,7±2,8	I28,4±I,0	I34,7±I,9	I36,2±I,I	I39,7±I,5
δ_{33}	- 0,9±3,4	3,9±6,6	II,I±3,0	2,4±I,3	I2,I±I,8
δ_{35}	4,9±5,5	-5,3±5,I	-I2,0±2,I	-4,3±I,0	-I3,2±I,0
α_1	8,I±3,5	25,8±2,2	23,8±3,3	28,9±2,0	3I,8±I,6
α_{11}	II,2±3,2	4,2±3,8	8,0±4,0	9,4±3,4	4,3±3,5
α_{13}	- I,4±I,45	-0,4 ±I,5	- 2,2±I,9	- 2,2±I,0	I,0 ±I,9
δ_{13}	3,0±2,I	0,8±2,2	I,3±I,4	2,0±I,I	0,8±I,5
δ_{15}	2,3±2,5	I,3±2,2	I,9±2,0	I,0±I,0	I,8±I,2
M_{min}	9,7	5,9	7,0	I6,2	9,6

T a b l e 13.

Solution „b_{SPD}"

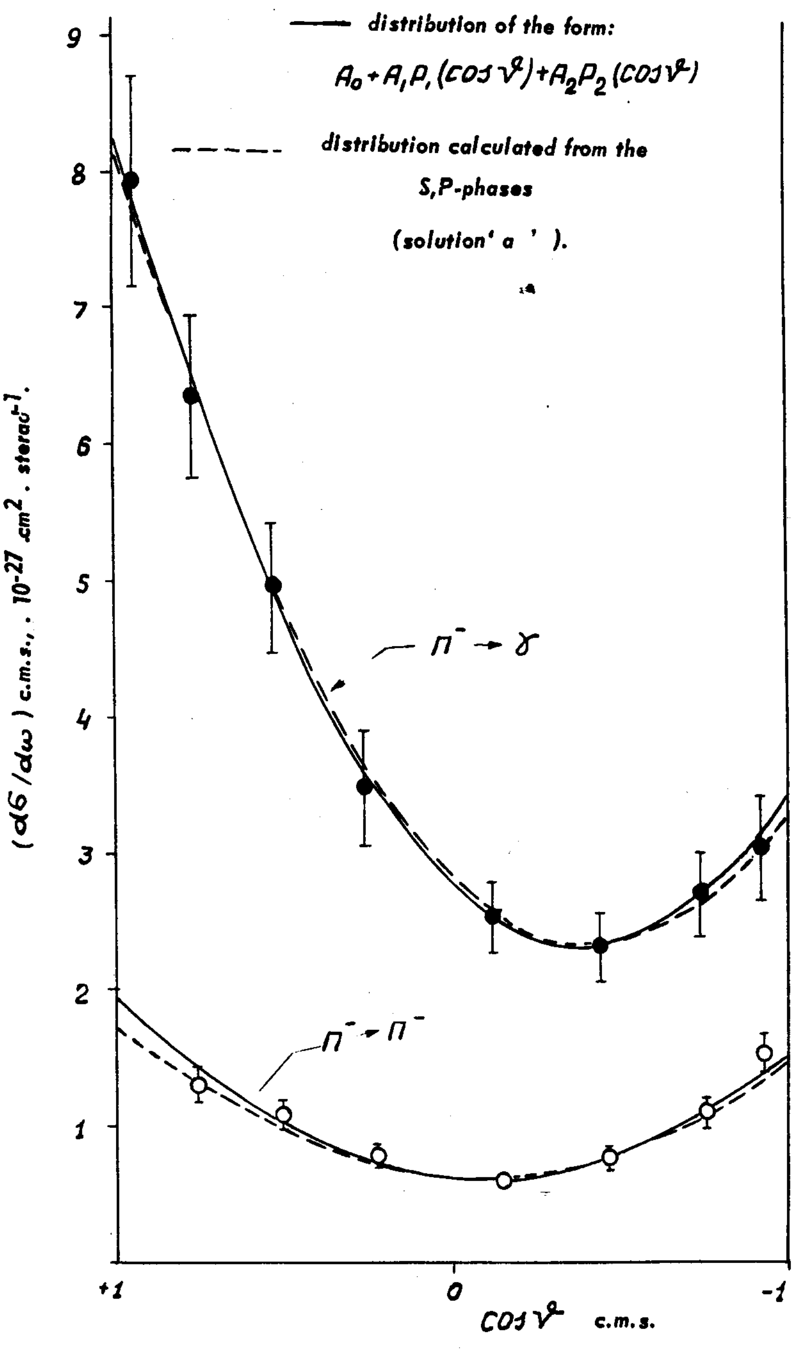
Meson energy in MeV	240	270	307	333
α_3	-I6,8	-I6,0	-I0,I	-I9,8
α_{31}	- I,7	- 6,3	- 3,2	- 9,6
α_{33}	II5,5	I28,0	I34,7	I36,3
δ_{33}	7,I	3,6	II,3	3,6
δ_{35}	- 2,4	- 5,7	-II,9	- 6,I
α_1	- 0,3	- 4,8	- 3,5	- 3,0
α_{11}	- 6,9	-I2,9	0,0	-2I,9
α_{13}	7,2	9,2	6,6	I5,2
δ_{13}	6,2	9,0	I5,0	2,7
δ_{15}	3,8	3,6	4,2	4,5
M_{min}	I3,9	I0,I	5,3	I7,5



S 38/8 Fz.

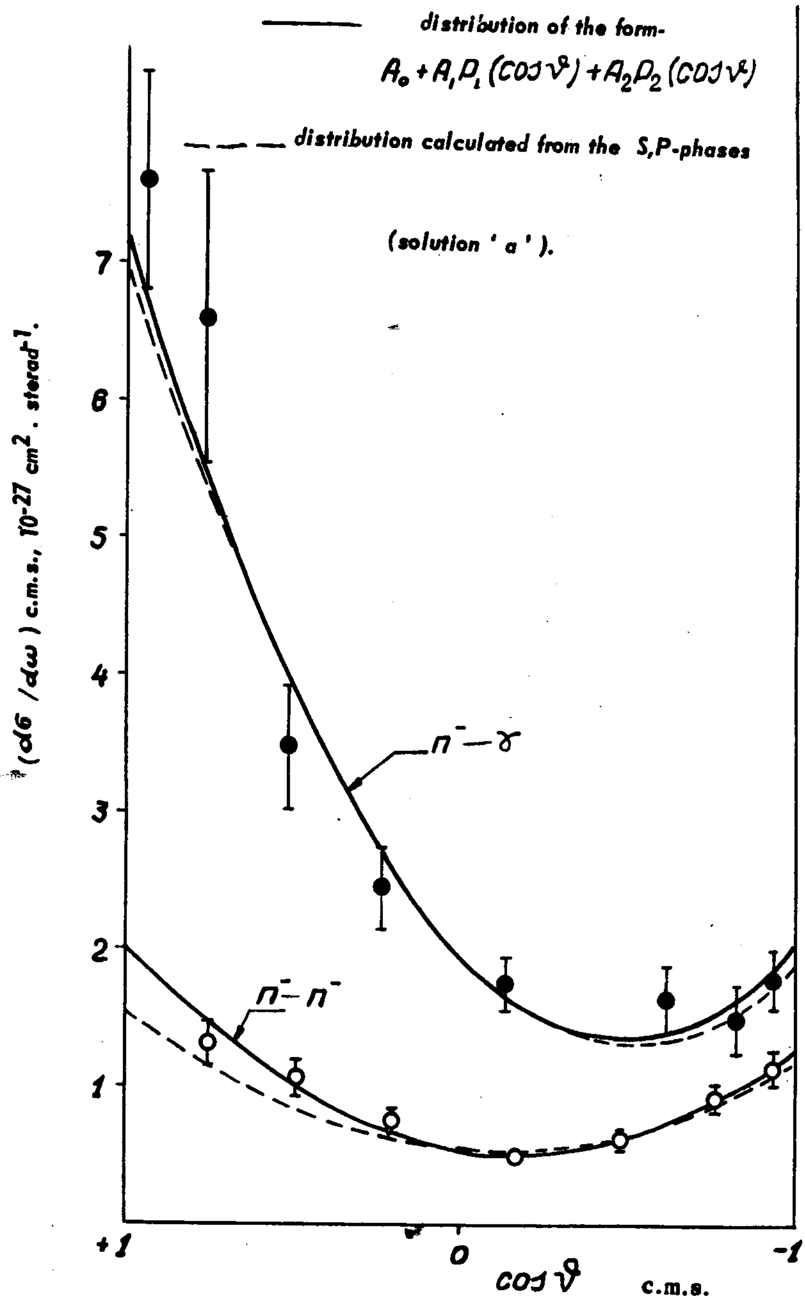
Differential cross sections of elastic and charge exchange π^--meson scattering in hydrogen at 240 MeV.

Fig 1.



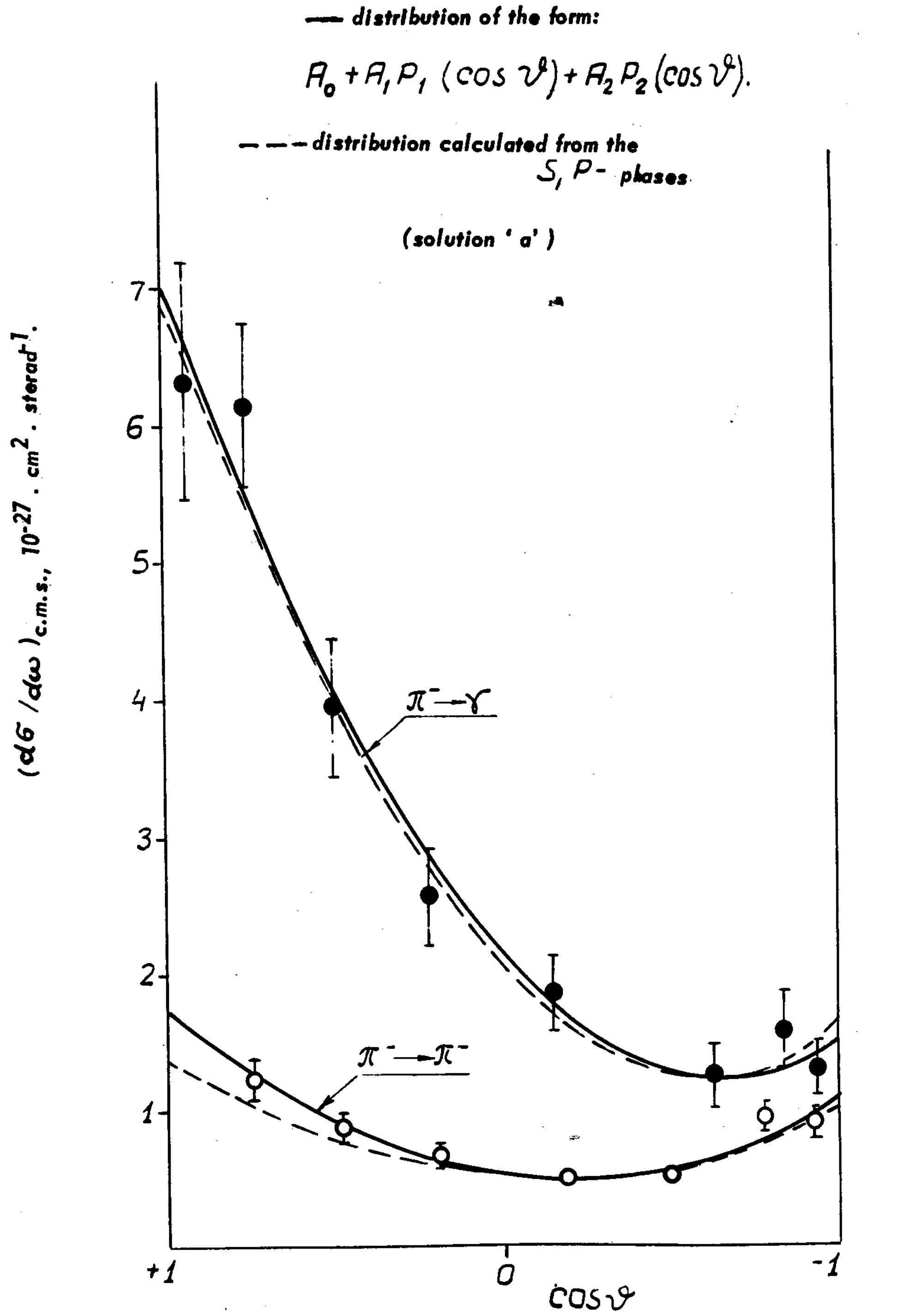
Differential cross sections of elastic and charge exchange π^--meson scattering in hydrogen at 270 MeV.

Fig. 2.



Differential cross sections of elastic and charge exchange π^--meson scattering in hydrogen at 307 MeV.

Fig. 3.



Differential cross sections of elastic and charge exchange π^--meson scattering in hydrogen at 333 MeV.

Fig. 4.

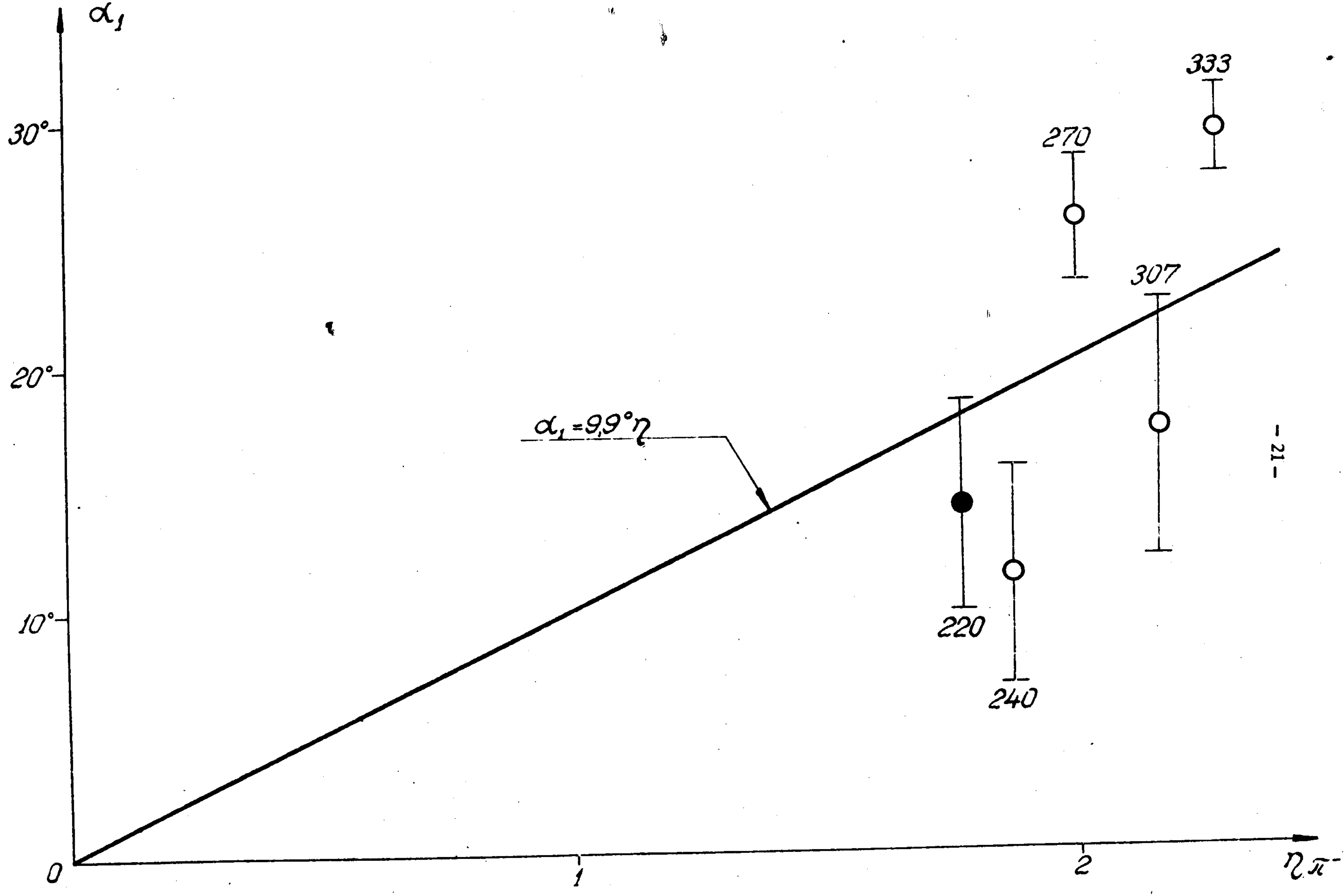
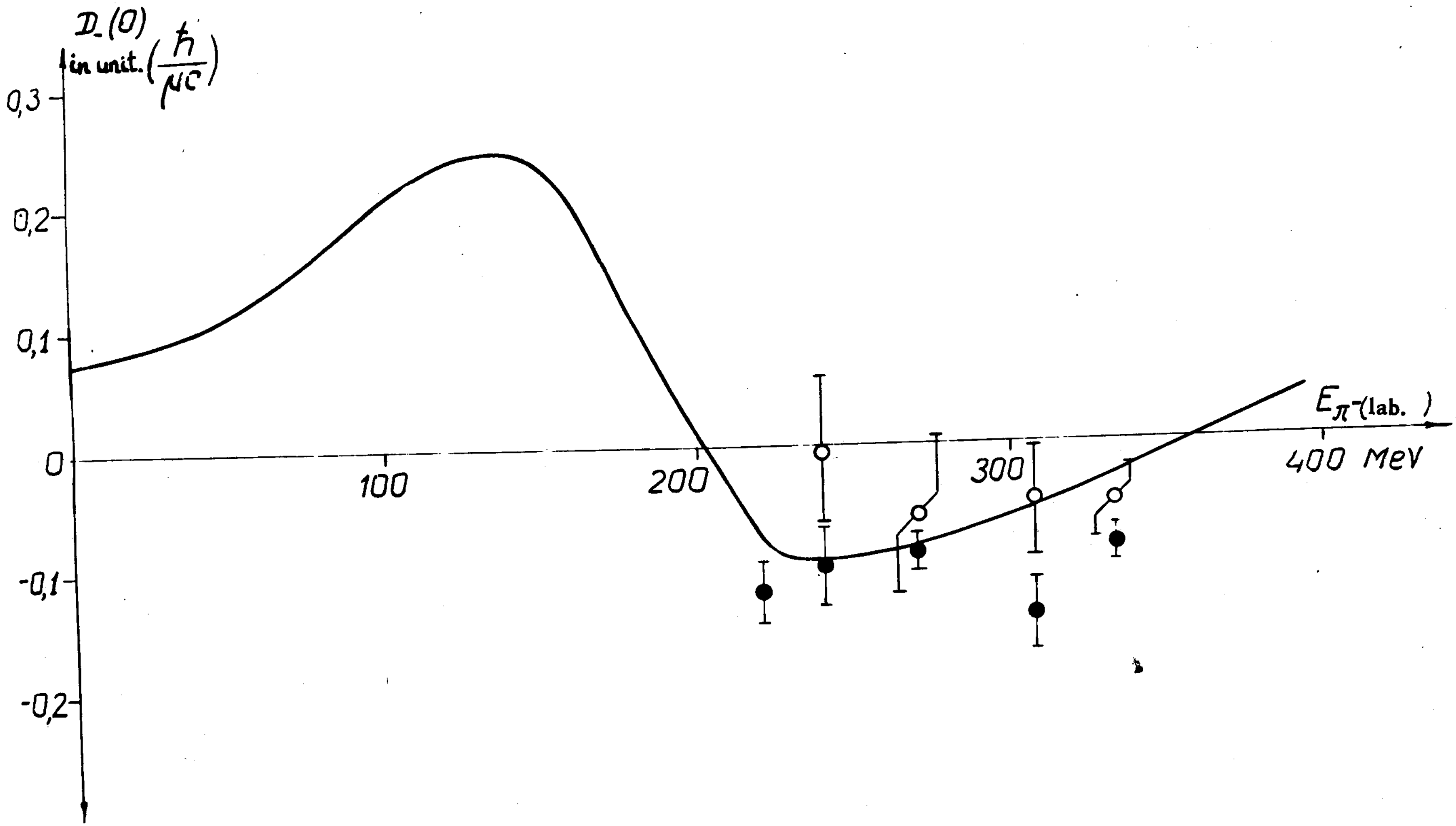


Fig. 5. The dependence of phase shift α_1 magnitude on π-meson impulse in the centre-of-mass system.



Energy dependence of the real part of the π-meson forward scattering amplitude $D_-(0)$. Continuous curve is taken from/14/.

Fig. 6.

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