JOINT INSTITUTE FOR NUCLEAR RESEARCH

Laboratory of Theoretical Physics

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P-428

DISPERSION RELATIONS FOR PHOTOPRODUCTION OF π^0-MESONS ON DEUTERONS

Dubna 1959

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ABSTRACT

The dispersion relations for the process of the photoproduction of neutral mesons on deuterons are derived with the help of impulse approximation. The unobservable region, beginning with the state of a free proton and neutron gives continuous spectrum at which no other poles exist. In this two-nucleon approximation, valid up to a maximum photon energy of 236 MeV the calculations including the recoil in the Breit's system for the nonforward scattering with the absolute value of the momentum $\vec{\pi}^2 = \frac{\mu^2}{4}\frac{M}{M+\mu}$ are carried out. The specialization of the results is given for the case of deuterons perpendicularly polarized to the direction of the propagation of mesons without the spin-flip part.

1. INTRODUCTION

The existing papers on the photoproduction of π -mesons on deuterons $\gamma + d \to d + \pi^0$ (Chew and Lewis, Lax and Feshbach [1]) which use the impulse approximation and give good results, as well as the papers on the dispersion relations for scattering of π -mesons on deuterons (Kaschluhn[2]) make us believe that it is possible to design the dispersion relations for photoproduction of π^0-mesons on deuterons.

We are compelled to treat the bound state-deuteron - with the aid of the impulse approximation because so far there exists no satisfactory theory of general use in the theory of wave fields applicable also to the vertex parts of the bound states. In the present case the impulse approximation is characterized: 1) by the interactions with the separate nucleons as if they were unbounded, 2) by the fact that the interactions of the separate free nucleons in the intermediate state are in comparison with the other interactions negligible.

From the general point of view of wave fields and from the causality condition it is necessary to point out that we must regard the interaction of wave fields with separate nucleons separately - locally - which leads through the composition of the local interactions for both nucleons to a certain 'unlocality'.

It is evident that in this case the dispersion relations for nonforward scattering [3] will be considered.

1/ G.F. Chew, H.W. Lewis Phys.Rev. 84, 779, (1951), H. Feshbach, M. Lax, Phys.Rev. 88, 509, (1952).

2/ F. Kaschluhn, Zeitschr.f.Naturforschung 13 a, 183, (1958); Nucl.Phys. 5, 303, (1958).

3/ N.N. Bogolyubov, B.V. Medvedev and M.L. Polivanov, Problems of the Theory of Dispersion Relations, Gosizdat. Phys.Mat.Lit., Moscow, 1958 (in Russian); abridged translation in German by F. Kaschluhn, Fortschr.d.Phys. 6, 169, (1958).

In addition the dispersion relations for the case of non-forward scattering were used for the photoproduction of π -mesons on nucleons in the paper by Logunov, Tavchelidze and Solovyev [4].

In order to design the dispersion relations in our process of the photoproduction of π -mesons on deuterons, we must rederive all the quantities. Let us remark, that deuteron as a spin triplet and as an isotopic singlet has no other states.

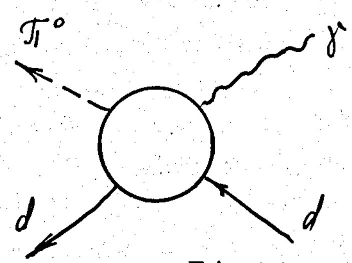
In this way in the 'two-nucleon' approximation we get two free nucleons in the intermediate state - the beginning of the unobservable region - to which we assign as to the system of two nucleons the relative momentum $\vec{p}_k$.

A continuous spectrum belongs to the state of two free nucleons; there and in the unobservable region there are no other poles.

With regard to the complicated general results for the photoproduction amplitude the dispersion relations are referred to the deuterons polarized perpendicularly to the direction of the propagation of mesons and are given in the Breit's system.

2. PHOTOPRODUCTION AMPLITUDE

Let us consider the process of photoproduction of neutral mesons on deuterons $\gamma + d \to d + \pi^o$ (see fig. 1).



F i g. 1.

We denote the final and initial momenta of the system of nucleons by $\vec{p}'$ and $\vec{p}$; – their quantum numbers by m' and m ; momenta of π -meson and photon $\vec{q}$ and $\vec{k}$ -; – their quantum numbers are denoted together: ρ' for π^o -meson and μ for photon, respectively.

We shall use the following labels for the field operators: $A_\mu(x)$ for the electromagnetic field operator, $\varphi_{\rho'}(x)$ for the pseudoscalar meson field operator (in which ρ' means the isospin index), $\psi(x)$ for the spinor of the nucleon field in Heisenberg's representation.

Then we write our matrix element using S -matrix to obtain the photoproduction amplitude,

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4/ A.A. Logunov, A.N. Tavchelidze, L.D. Solovyov, Nucl.Phys. 4, 427, (1957), see also E.Corinaldesi,NuovoCim.4, 1384,(1956).

which we shall need further in the dispersion relations

$$S\left(\alpha, \vec{q}\,'; \omega, \vec{k}\right) = \langle \vec{p}\,', m'; \vec{q}\,', \rho' / S / \vec{k}, \mu; \vec{p}, m \rangle \quad (1)$$

To include the recoil we choose the Breit's coordinate system in which

$$\vec{p} + \vec{p}\,' = 0 \quad (2)$$

With respect to the laws of conservation of momentum and energy this gives us the possibility of expressing the momentum of the photons $\vec{k}$ and of the meson $\vec{q}$ in the form

$$\begin{aligned} \vec{k} &= \lambda \vec{e} - (1+t)\vec{p} \\ \vec{q} &= \lambda \vec{e} + (1-t)\vec{p} \end{aligned} \quad (3)$$

where $t = \mu^2 / 4\vec{p}^{\,2}$ and λ is an arbitrary parameter and the unit vector $\vec{e}$ is orthogonal to the momentum $\vec{p}$. If we have the fixed momentum $\vec{p}$ we get in this way the energy of π - meson ($E = q^0$),

$$E^2 = \lambda^2 + (1+t)^2 \vec{p}^{\,2} \quad (4)$$

Turning back to the matrix element (1) by using the creation and annihilation operators of meson and electromagnetic field with their commutation relations with S -matrix, we can rewrite the matrix element in the form

$$S\left(\alpha, \vec{q}\,'; \omega, \vec{k}\right) = \sum_{\mu} e_{\mu}^{\lambda}(\vec{k})\, S_{\mu\rho'} \quad (5)$$

$$S_{\mu\rho'} = \frac{1}{2\sqrt{k^0 q^0}} \int e^{i(qx - ky)} \langle \vec{p}\,', m' / \frac{\delta^2 S}{\delta \varphi_\rho(x)\, \delta A_\mu(y)} S^+ / \vec{p}, m \rangle dx\, dy \quad (5a)$$

The radiation operator of the second order H_2 between bra- and ketvector in the matrix element in (5 a) can be expressed by the hermitian current operators of meson $j^{\rho'}(x)$ and the electromagnetic $i^{\mu}(x)$ field. Usually denoted by the current operators being

$$j^{\rho'}(x) = i \frac{\delta S}{\delta \varphi_{\rho'}(x)} S^+ \qquad i^{\mu}(x) = i \frac{\delta S}{\delta A_\mu(x)} S^+ \quad (6)$$

we get the radiation operator expressed simply by

$$H_2 = -j^{\rho'}(x)\, i^{\mu}(y) - i\,\frac{\delta j^{\rho'}(x)}{\delta A_{\mu}(y)} = -i^{\mu}(y)\, j^{\rho'}(x) - i\,\frac{\delta i^{\mu}(y)}{\delta \rho_{\rho'}(x)} \quad . \tag{7}$$

In the same manner as in the case of photoproduction of π^--mesons on nucleons[5] in our case of deuterons in the plane of complex energies in the physical region we can go from the causal matrix element in (5a) (we label this $V^{\rho'\mu}_{\vec{p}',m';\vec{p},m}(x)$; see [4]) to the retarded (which we label $W^{\rho'\mu}_{\vec{p}',m';\vec{p},m}(x)$) and advanced matrix elements which are indentical with $V^{\rho'\mu}_{\vec{k}',m';\vec{k},m}(x)$ in upper or lower half-plane respectively and on the real axis (with the exception of the finite number of points and lines of discontinuity on the real axis).

Using the Fourier transformation we pass from the quantities $V(x)$, $W(x)$ to the quantities $T(k)$, $M(k)$. For example for $M(k)$ we have

$$M^{\rho'\mu}_{\vec{p}',m';\vec{p},m} = \int dx\, e^{ikx}\, W^{\rho'\mu}_{\vec{k}',m';\vec{p},m}(x) \tag{8}$$

where instead of $M(k)$ and $W(x)$ the causal $T(k)$ and $V(x)$ either advanced quantities, or product of currents from 7 can be substituted as well.

As usual we can consider only the retarded parts thanks to quite analogous properties of the advanced parts in the lower half plane.

In the place of the variations of the first order in the function $W^{\rho'\mu}_{\vec{k}',m';\vec{p},m}(x)$ we use with the aid of 7 terms which are formed as a product of meson and electromagnetic current and which can be split spectrally[6] as a product of two matrix elements. Otherwise we split the function $M^{\rho'\mu}_{\vec{p},m';\vec{p},m}(k)$ as obvious into a dispersive part D and an absorptive one A which can be defined again either by means of our functions $W(x)$ or of the product of currents from (7) .

In this way we get the matrix element (5a) expressed in the upper half plane of complex energies and on the real axis in the form of

$$S_{\mu\rho'} = \frac{i\,(2\pi)^4}{\sqrt{4k^0 q^0}}\,\delta(p'+q-p-k)\left\{ D^{\rho'\mu}_{\vec{p}',m';\vec{p},m}\left(\frac{k+2}{2}\right) + i\,A^{\rho'm}_{\vec{p}',m';\vec{p},m^2}\left(\frac{k+2}{2}\right)\right\} \tag{9}$$

Owing to the fact that only the absorptive part – forming the photoproduction amplitude of π-mesons on deuterons with (5) - is necessary for determining dispersion relations, we in-

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5/ See ref. 4/.

6/ See N.N. Bogolyubov, D.V. Shirkov, Introduction to the Quantum Field Theory, GITTL, M o s c o w 1957, p. 383 (in Russian) . See also G.F. Chew, F.F. Low, M.L. Goldberger, Y. Nambu Phys.Rev. 106, 1345, (1957) and ref. 4/.

troduce only this part explicitly. So after the indicated integrations we have A in our coordinate system (2)

$$A^{\rho' \mu}_{\vec{p}', m'; \vec{p}, m}(E, \lambda \vec{e}) = \pi \sum_n \langle \varphi^*_{-\vec{p}, m'} \, J^{\rho'}(0) \, \varphi_{n, \lambda\vec{e} - t\vec{p}} \rangle \langle \varphi^*_{n, \lambda\vec{e} - t\vec{p}} \, J^{\mu}(0) \, \varphi_{\vec{p}, m} \rangle \times$$
$$\times \, \delta\left(\sqrt{M^2 + \vec{p}^{\,2}} + E - \sqrt{M_n^2 + \lambda^2 + t^2 \vec{p}^{\,2}} \right) - \pi \sum_n \langle \varphi^*_{-\vec{p}, m'} \, J^{\mu}(0) \, \varphi_{n, -\lambda\vec{e} + t\vec{p}} \rangle$$
$$\times \langle \varphi^*_{n, -\lambda\vec{e} + t\vec{p}} \, J^{\rho'}(0) \, \varphi_{\vec{p}, m} \rangle \, \delta\left(M^2 + \vec{p}^{\,2} - E - \sqrt{M_n^2 + \lambda^2 + t^2 \vec{p}^{\,2}} \right) \qquad (10).$$

where all quantities in matrix elements are in Heisenberg's picture.

3. ISOTOPIC STRUCTURE

On the basis of isotopic invariance the isotopic structure of the process of photoproduction of π^0 -mesons on deuterons is reduced to a particularly simple case – vector in the isotopical space- that is to its third component.

As the only possible way of the process over the intermediate state according to the change of the isotopic spin by 1 there may be taken into account only the intermediate states with isotopical spin 1 – i.e. in our case a free neutron and proton.

For the process $\gamma + d \to n + p$ with the initial state $|I, I_z\rangle$ given by $|0, 0\rangle$ and with the final state either $\langle 0, 0|$ for a scalar wave function in the isotopic space, or $\langle 1, I_z'|$ for $I_z' = 0$ for a vector wave function S -matrix may be written in the form of a scalar $\varphi^{(2)}$ and of the third component of a vector V_3^2

$$S^{(2)} = \varphi^{(2)} + V_3^{(2)} = C_s + C_+ V_3 \qquad C_s^2 + C_+^2 = 1 \qquad (11)$$

Analogous for the process $p + n \to d + \pi^0$ with the initial state $|0, 0\rangle$ or $|1, I_z\rangle$ for $I_z = 0$ and with the final state $\langle 1, I_z'$ for $I_z' = 0$ the $S^{(3)}$ matrix is a scalar $\varphi^{(3)}$

$$S^{(3)} = \varphi^{(3)} = (\vec{V} \vec{\pi}) \qquad (12)$$

The resulting wave function will be the third component of a vector only

$$S = S^{(2)} S^{(3)} = C_+ V_3 (\vec{V} \vec{\pi}) \qquad C_+^2 = 1 \qquad (13)$$

4. TWO-NUCLEON APPROXIMATION

We direct our endeavour to the possible contribution of various terms in the expansion (10).

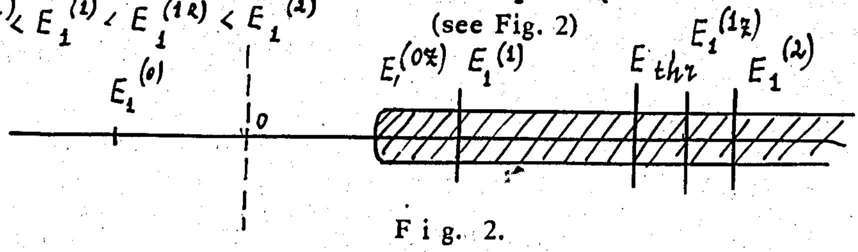
From the laws of conservation of energy (in our coordinate system $k^0 = g^0$) and from the condition that the quantity $\lambda^2 \geq 0$ we obtain the energy of the photoproduction threshold E_{thr} in the form

$$E_{thr} = (1+t)\,|\vec{p}\,| \qquad t = \frac{\mu^2}{4\vec{p}^{\,2}} \tag{14}$$

When we consider in (10) the higher quantum states for the corresponding intermediate ones namely 0) the deuteron or two nucleons respectively 1) the deuteron and π -meson (or two nucleons and π -meson respectively) 2) the deuteron and two π -mesons, we get the expression of the energies of these states

$$E_i^{(n)} = \pm \frac{\frac{1}{2}\{M_n^2 - M^2 - \mu^2\} - \vec{p}\,\vec{g}}{\sqrt{M^2 + \vec{p}^{\,2}}} \qquad \begin{matrix} n = 0, 1, 2 \\ i = 1, 2 \end{matrix} \tag{15}$$

where M_n is equal to 0) M_0 to the mass of deuteron M or the mass of two nucleons M_R 1) $M_1 = M + \mu$, $(M_{1R} = M_R + \mu)$ 2) $M_2 = M + 2\mu$. The corresponding energies to the two δ – functions are symmetrically placed on the real axis $E_1^{(n)} = - E_2^{(n)}$ and it is valid for these $E_1^{(0)} < E_1^{(0R)} < E_1^{(1)} < E_1^{(1R)} < E_1^{(2)}$ (see Fig. 2)



F i g. 2.

We find out through gradual comparison with the energy of the threshold E_{thr} of the photoproduction process that it is always $E_1^{(1)} \leq E_{thr}$ and that for energies of γ -quants in the interval from 10 MeV up to 236 , 25 MeV it is always $E_1^{(2)} > E_{thr}$. We obtain the equality $E_1^{(1)} = E_{thr}$ for the value of momentum

$$\vec{p}^{\,2} = \frac{M\mu^2}{4\,(M+\mu)} \tag{16}$$

which is a nonforward scattering ($|\vec{p}\,| \neq 0$).

In this way we see that at least for one value of the momentum (given by /16/) the contributions of the terms in the expansion (10) belonging to the higher quantum states are negligible.

From this point of view we limit ourselves to the first term in the expansion (10) which includes the deuteron or two free nucleons in the intermediate state respectively.

With regard to the relations given by the isotopic structure in our process, which permit the lowest intermediate states with isotopic spin 1 - in virtue of the emission of neutral π -mesons - we see that the deuteron which is the isotopic spin singlet can not exist as an intermediate state.

We obtain in this way – one neutron and one proton like the only intermediate state – which we include into the expansion (10).

Further we shall use the relative momentum of these two free nucleons $\vec{p}_r$ so that for relative mass $M_\mu = M_{0R}$ in (15) we may simply have

$$M_{0R}^2 = M_R^2 + 4\vec{p}_\mu^{\,2} \tag{17}$$

In virtue of $M = M_R - \varepsilon$ from the formula (15) for beginning of the continuous spectra we get the energy (with the aid of (3)) equal to:

$$E_1^{0R} = E_p = -\frac{M\varepsilon + \beta' - \vec{p}^{\,2} + 2\vec{p}_\mu^{\,2}}{\sqrt{M^2 + \vec{p}^{\,2}}} \qquad \beta' = \frac{\varepsilon^2}{2} - \frac{\mu^2}{4}; \tag{18}$$

For these two-nucleon terms the δ -functions can be expressed in the form:

$$\delta\left(\sqrt{M^2 + \vec{p}^{\,2}} \mp E - \sqrt{M_{0R}^2 + \lambda^2 + t^2\vec{p}^{\,2}}\right) = \left|\frac{MM_R + \beta' + 2\vec{p}_\mu^{\,2}}{M^2 + \vec{p}^{\,2}}\right| \delta\left(E \pm \frac{M\varepsilon + \beta' - \vec{p}^{\,2} + 2\vec{p}_r^{\,2}}{\sqrt{M^2 + \vec{p}^{\,2}}}\right) \tag{19}$$

For our photoproduction amplitude with the polarization of photons we obtain in two-nucleon approximation the following result

$$Y^{\rho'}_{\vec{p}_1' m'; \vec{p}, m}(E, \lambda\vec{e}) = \sum_\mu e^\lambda_\mu(k) A^{\rho'\mu}_{\vec{p}_1' m'; \vec{p}, m}(E, \lambda\vec{e})$$

$$= \pi\left|1 - \frac{E_p}{\sqrt{M^2 + \vec{p}^{\,2}}}\right| \sum_\mu \Big\{ \langle \varphi^*_{-\vec{p}_1, m'} J^{\rho'}(0) \varphi_{0R, \lambda\vec{e} - t\vec{p}} \rangle \langle \varphi^*_{0R, \lambda\vec{e} - t\vec{p}} J^\mu(0) \varphi_{\vec{p}, m} \rangle \tag{20}$$

$$\times \delta(E + E_p) - \langle \varphi^*_{-p, m} J^\mu(0) \varphi_{0R, -\lambda\vec{e} + t\vec{p}} J^{\rho'}(0) \varphi_{\vec{p}, m} \rangle \delta(E - E_p)$$

where for $\lambda\vec{e} - t\vec{p}$ the expressions

$$\lambda\vec{e} - t\vec{p} = \vec{q} - \vec{p} = \vec{k} + \vec{p}' \tag{21}$$

are valid.

We express the separate matrix elements in the photoproduction amplitude (20). Going over from Heisenberg's representation to Schrödinger's we rewrite the expression (20) briefly in the form

$$Y^{\rho'\lambda}_{\vec{p}_1' m_1' \vec{p}_1 m}\left(E, \vec{q}-\vec{p}\right) = \pi \left| 1 - \frac{E_p}{\sqrt{M^2+\vec{p}^2}} \right| \cdot \sum_{\mu} \left\{ \langle -\vec{p}_1 m' | j^{\rho'}(x) | \vec{g}-\vec{p}_1 \ell \rangle \times \right.$$

$$\times \langle \vec{g}-\vec{p}_1 \ell | i^{\mu}(x) | \vec{p}_1 m \rangle \delta(E+E_p) - \langle -\vec{p}_1 m' | i^{\mu}(x) | \vec{p}-\vec{g}_1 \ell \rangle \times$$

$$\left. \times \langle \vec{p}-\vec{g}_1 \ell | j^{\rho'}(x) | \vec{p}_1 m \rangle \delta(E-E_p) \right\} \qquad (22)$$

where $j^{\rho'}(x)$ means the meson current and $i^{\mu}(x)$ the components of electromagnetic current.

We derive the following formula as a first approximation for matrix elements containing the meson current, using the impulse approximation from the definition of S -matrix[7]

$$\langle \vec{g}_1'' \ell | j(x) | \vec{p}, p \rangle = -i \frac{g\sqrt{2}}{M_R} \left\{ \int d\vec{x}_2 \, \chi^*_{\beta}(\vec{x}_1 \vec{x}_2, t) \, Y_e X_L \tau^1 \vec{\sigma}_1 \vec{g}' \chi_0(\vec{x}_1 \vec{x}_2; t) \times \right.$$

$$\left. \times Y_{\beta} X_5 + \int d\vec{x}_1 \, \chi^*_R (\vec{x}_1, \vec{x}_1, t) \, Y_e X_L \tau^1 \vec{\sigma}_2 \vec{g}' \chi_0(\vec{x}_1, \vec{x}_1, t) Y_{\beta} X_s \right\} \qquad (23)$$

where

$$\vec{g}' = \vec{g}'' - \vec{p}$$

$$\chi_D(\vec{x}_1, \vec{x}_2, t) = e^{i \vec{R} x_{\beta}} \varphi_D(\vec{x}_{\mu}) e^{-i p_0 t} \qquad p_0 = \sqrt{\left(\frac{M_R}{2}\right)^2 + \vec{p}^2}$$

$$\chi_R(\vec{x}_1, \vec{x}_2, t) = e^{i \vec{g}'' \vec{x}_{\beta}} \varphi_{\beta}(\vec{x}_{\mu}) e^{-i g_0'' t} \qquad g_0'' = \sqrt{M^2_{0R} + \vec{g}''^2} \qquad (24)$$

$$\vec{x}_{\beta} = \frac{1}{2}(\vec{x}_1 + \vec{x}_2); \qquad \vec{x}_{\mu} = \vec{x}_1 - x_2$$

The wave functions of intermediate state symmetrical or antisymmetrical respectively are given by the expression

$$\varphi_R(\vec{x}_R) = \frac{1}{\sqrt{2}} \left\{ \varphi_R'(\vec{x}_{\mu}) \pm \varphi_R'(-\vec{x}_{\mu}) \right\} = \frac{1}{\sqrt{2}(2\pi)^{3/2}} \left(e^{-i \vec{p}_{\mu} \vec{x}_{\mu}} \pm e^{i \vec{p}_{\mu} \vec{x}_{\mu}} \right) \qquad (25)$$

We introduce the wave function of the deuteron in the reliable form

$$\varphi_D(\vec{x}_{\mu}) = \left(\frac{\alpha\beta(\alpha+\beta)}{2\pi(\alpha-\beta)^2} \right)^{1/2} \frac{1}{|\vec{x}_{\mu}|} \left(e^{-\alpha |\vec{x}_{\mu}|} - e^{-\beta |\vec{x}_{\mu}|} \right) \qquad (26)$$

which appears to be a good approximation for the choice of constants $\beta = 6\alpha$

The individual symbols in the expressions (23), (24), (25) and (26) mean: $\vec{x}_{\mu}$ the relative coordinate, $\vec{x}_p$ the coordinate of centre of mass, R the incides of the intermediate state, 1 the first nucleon, L the second nucleon, ℓ the quantum numbers of the inter-

7/ See ref. 2).

mediate state, s the quantum numbers of the final state; $\vec{q}''$ the momentum of the intermediate state, Y the spin-functions of the state, X the isospin-functions of the state M_R the mass of two-nucleons and $\lambda = 0.32\,\mu_0$ (μ_0 is the mass of π^0)

For the matrix elements with electromagnetic current we obtain on the basis of impulse approximation with the aid of S- matrix as a first approximation analogically the formula[8/]

$$\langle \vec{q}, \ell | i^{\mu}(x) | \vec{p}, \beta \rangle = \langle \vec{q}, \ell | i^{j}(x) | \vec{p}, p \rangle =$$
$$= i \int d\vec{x}_2\, \chi^{*}(\vec{x}, \vec{x}_2, t)\, \chi_{B}(\vec{x}, \vec{x}_2, t) \Big\{ Y_e X_L \frac{\partial}{\partial x'_j} Y_\beta X_S \frac{2e}{M_R} \left(\tfrac{1}{2} - t_{\varphi_1}\right) + Y_e X_L \times$$
$$\times [\vec{\sigma}_1 \times \vec{k}]_j\, Y_\beta \mu_1 X_S \Big\}\, i\, d\vec{x}_1\, \chi_R^{*}(\vec{x}_1, \vec{x}, t)\, \chi_D(\vec{x}_1, x, t) \times \tag{27}$$
$$\Big\{ Y_e X_L \frac{\partial}{\partial x_j^2} Y_\beta X_S \frac{2e}{M_R} \left(\tfrac{1}{2} - t_{\varphi_2}\right) + Y_e X_L [\vec{\sigma}_2 \times \vec{k}]\, Y_{\beta\mu_2} X_S \Big\}$$

where

$$\mu_1 = \mu_P \left(\tfrac{1}{2} - t_{\varphi_1}\right) + \mu_N \left(\tfrac{1}{2} + t_{\varphi_1}\right)$$
$$\mu_2 = \mu_P \left(\tfrac{1}{2} - t_{\varphi_2}\right) + \mu_N \left(\tfrac{1}{2} + t_{\varphi_2}\right) \tag{28}$$

$$t_{\varphi_i} = \begin{cases} \tfrac{1}{2} & \text{for a neutron} \\ -\tfrac{1}{2} & \text{for a proton} \end{cases}$$

and t_φ expresses the eigenvalue of the zet-component of the isotopic spin of the i-th nucleon, $j = 1,2,3$, μ_P the magnetic moment of a proton, μ_N the magnetic moment of a neutron and other meaning of symbols given by the formula (23). Using the formulas (23) and (27) in the photoproduction amplitude (22) for $x = 0$ we sum, after performing operations of spin matrices $\vec{\sigma}(\sigma_+, \sigma_-, \sigma_0)$

$$\sigma_0^{1,2}\, Y_{1,0} = \pm Y_{0,0}, \ldots \tag{29}$$

and matrices of isotopic spin

8/ The nonrelativistic interaction's Hamiltonian as a starting point for the derivation of the formula see W.M.Mac-Donald, Phys.Rev. 98, 60, (1955). For the possibility of derivation of the formula (27) in this form the author is indebted very much to Dr. F. Kaschluhn.

$$\tau_0^{1,2}\, X'_{0,0} = \pm\, X_{1,0} \tag{30}$$

over the spin-functions Y and isospin-functions X.

We obtain simpler results, to which we restrict ourselves in this case, in the case of deuterons with spin perpendicularly polarized to the direction of the propagation of meson ($\vec{g}$). The first expressions in braces of the formula (27) in this case vanish after summing over spin – and isospin-function. Physicaly this case is realizable by inserting a target of deuterons into a magnetic field. We shall not take into account the polarization of the spins of deuterons in the direction of propagation of π^0 -mesons.

Then the photoproduction amplitude of deuterons with the spin polarized perpendicularly to the direction of the propagation of mesons we obtain after carrying out the above operations and after summing it over

$$Y^{\rho'\lambda}_{\vec{p},\,m';\,\vec{p},m}(E,\lambda\vec{e}) = \pi\Big/1-\frac{E_p}{\sqrt{M^2+\vec{p}^{\,2}}}\Big/ B^0(E,\vec{p}-\vec{g})\big(\delta(E+E_p)- -\delta(E-E_p)\big) \tag{31}$$

where

$$B^0(E,\vec{p}-\vec{g}) = \frac{2g\sqrt{2}}{M_R}\,\vec{g}\,(-i\,k_y+\vec{j}\,\vec{k})(\mu_p-\mu_N)\, I(\vec{g})\, I(\vec{k})$$

and

$$I(\vec{n}) = \int \varphi_D(\vec{x}_\mu)\,\varphi_{\kappa p y m}(\vec{x}_\mu)\cos\left(\tfrac{1}{2}\vec{n}\,\vec{x}_\mu\right) d\vec{x}_\mu \tag{32}$$

where the meaning of separate symbols is: E_p the energy given by the expression (18) only with the difference that β' is now equal to

$$\beta' = \varepsilon^2/2 - \tau/4 \tag{33}$$

with analytical continuation on the real axis ($\tau \to \mu^2$). The argument in B^0 (where ^{0} means the value of projection of the spin on the direction of the propagation of meson), $\vec{p}-\vec{g}$ will go over into $t\vec{p}-\lambda\vec{e}$.

The value of the integral (31) is given by the expression

$$I(\vec{n}) = \frac{(1/2\,\alpha\beta\,(\alpha+\beta))^{1/2}}{\pi\,(\alpha-\beta)}\left\{\frac{1}{\alpha^2+g_+^2}-\frac{1}{\beta^2+g_+^2}+\frac{1}{\alpha^2+g_-^2}-\frac{1}{\beta^2+g_-^2}\right\} \tag{34}$$

$\beta = 6\alpha$

$$g_\pm^2 = \left|\vec{p}_\mu \pm \vec{n}/2\right|^2 \tag{35}$$

5. DISPERSION RELATIONS

In order to derive[9/] the dispersion relations we pay closer attention to the photoproduction amplitude (31) for deuterons perpendicularly polarized to the direction of mesons, which is an odd function and which we shall use in the following.

For negative values of E_p only the first δ-function contributes to the expression (31) and for positive values only the second δ-function. According to the varying value of the relative momentum E_p reaches its maximum for the value $\vec{p}_\mu = 0$

$$E_p^{max} = \frac{M_\varepsilon + \beta - \vec{p}^{\,2}}{\sqrt{M^2 + p^2}} \qquad (36)$$

For the value of $\vec{p}_\mu^{\,2}$

$$\vec{p}_\mu^{\,2} = 1/2 \left(M_\varepsilon + \beta - \vec{p} \right) = p'^2 \qquad (37)$$

is E_p equal to zero.

For the values of $\vec{p}_\mu^{\,2}$ in the interval

$$0 < p_\mu^2 < p'^2$$

E_p runs through positive values of energies in the interval

$$E_p^{max} > E_p > 0$$

and therefore only the second δ-function makes contribution. For the values $\vec{p}_\mu^{\,2} > p'^2$, E_p is negative and therefore the first δ-function contributes only to the whole region

$$E_{thr} < E_p < 0$$

meanwhile the relative momenta p_μ^2 run through values of the interval

$$p'^2 < \vec{p}_\mu^{\,2} < p^2$$

where p^2 is the value of the relative momentum $\vec{p}_\mu^{\,2}$ when

$$E_{thr} = - E_p \qquad (38)$$

For the special case of the momentum of deuterons

9/ See reference 3), chapter VI Construction of the dispersion relations.

$$\vec{p}^{\,2} = \frac{1}{4}\mu^2 \frac{M}{M+\mu}$$

we obtain the above values of momenta P and P' in the form

$$P'^2 = -\frac{\mu^2}{4}\,\frac{2M+\mu}{M+\mu} + \frac{\varepsilon}{2}(M+\varepsilon) \tag{39}$$

$$P^2 = \frac{\mu-\varepsilon}{2}\left(M+\frac{\mu+\varepsilon}{2}\right) = 7{,}08\,\mu_0^2$$

With respect to the fact that there always exists an interval of definite length on the boundary of the region ($\mathrm{Im}\,E = 0$) – on the real axis – we can regard the function Y as the only one with the branch line on the real axis. The dispersion relations for the photoproduction of π^0 -mesons on deuterons perpendicularly polarized to the direction of the propagation of mesons can then be written

$$D^0(E,\tau) - D^0(E_0,\tau) = \frac{2}{\pi}\left(E^2-E_0^2\right) \times$$

$$P\left[\int_{|E_{min}(\tau)|}^{\infty} dE' \frac{E'Y(E',\tau)}{(E'^2-E^2)(E'^2-E_0^2)} + \int_{|E_{max}(\tau)|}^{|E_{min}(\tau)|} dE' \frac{E'Y(E',\tau)}{(E'^2-E^2)(E'^2-E_0^2)}\right] \tag{40}$$

where

$$E_p = -\frac{M\varepsilon + \beta' - \vec{p}^{\,2} + 2\vec{p}_\perp^{\,2}}{\sqrt{M^2+\vec{p}^{\,2}}} \qquad \beta' = \frac{\varepsilon^2}{2} - \frac{\tau}{4} \tag{41a}$$

$$E_{min}(\tau) = \frac{\tau-\mu^2}{4\sqrt{M^2+\vec{p}^{\,2}}} - \frac{\vec{p}^{\,2}+\frac{\mu^2}{4}}{|\vec{p}|} \tag{41b}$$

$$E_{max}(\tau) = -\frac{M\varepsilon + \beta' - \vec{p}^{\,2}}{\sqrt{M^2+\vec{p}^{\,2}}} \tag{41c}$$

and where the other symbols are: D^0 the Hermitian part of the amplitude for deuterons, Y the antihermitian part (the odd function), P the principal value of the integral, E_0 the energy which we choose equal to the mass of π^0 meson μ.

The first integral is related to the observable region from the energy E_{min} to infinity.

The second integral falls into the unobservable region with imaginary momenta and may be expressed by means of Bogolyubov method. This integral is the function of the energy E only and can be determined separately.

For special value of the momentum of deuteron, given by eq. (16), for which the expression of photoproduction amplitude in the two-nucleon approximation comes to be true, the dispersion relations for photoproduction of π -mesons on deuterons perpendicularly polarized to the direction of propagating π^{o} -mesons take the form

$$D(E)-D(\mu)=\frac{2}{\pi}\vec{g}^{2}P\int dE'\frac{E'Y_{\vec{p}',m;\vec{p},m}(E')}{(E'^{2}-E^{2})(E'^{2}-\mu^{2})}+\mathcal{Y}(E) \qquad (42)$$

where
$$\vec{g}^{2}=E^{2}-\mu^{2},\quad \vec{g}'^{2}=E_{p}(\tau)-\mu^{2}$$

and where $Y^{\rho'\lambda}_{\vec{p}'m;\vec{p}m}(E')$ is given by the expression (31).

The expression $\mathcal{Y}(E)$ which comes from the unobservable region is given by the integral

$$\mathcal{Y}(E)=\frac{4g\sqrt{2}}{M_{p}}(\mu_{p}-\mu_{N})\vec{g}^{2}P\int_{0}^{p}\left(-\vec{i}k_{x}+\vec{j}k_{y}\right)\Big/1-\frac{E_{p}(\tau)}{\sqrt{M^{2}+\vec{p}^{2}}}\Big/\frac{\vec{g}''}{\vec{g}'^{2}}\,\frac{E_{p}(\tau)}{E^{2}-E_{p}^{2}(\tau)} \qquad (43)$$
$$\times I(\vec{g})\,I(\vec{k})\,d^{3}\vec{p}$$

where g is the coupling constant, P the principal value of the integral;

$$\vec{g}''^{2}=E_{p}^{2}(\tau)-\tau>0 \qquad (44)$$

$p=2{,}6608\,\mu$ and $I(\vec{g})$ and $I(\vec{k})$ are given by the expression (34).

A C K N O W L E D G M E N T S

In conclusion the author would like to thank Professor N.N. Bogolyubov for his interest throughout the work, Dr. A.A. Logunov and J. Fischer for numerous helpful conversations. The author is especially indebted to Dr. F. Kaschluhn for many discussions and his criticizm.

Received by Publishing Department
on November 3, 1959.