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# PION PRODUCTION IN p-d COLLISIONS AND INTRANUCLEAR MOTION OF NUCLEONS*

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# Abstract

It is shown that the energy dependence of the reaction cross sections for pion production in nucleon-deuteron collisions and the energy spectra of pions may be calculated rather accurately on the basis of the data on free nucleon-nucleon collisions. In the considered energy region from the threshold for meson production to $\approx$ 700 MeV the influence of the nucleon binding in a deuteron is essentially the change of the magnitudes of the cross sections due to the intranuclear motion of nucleons. There was found an effective momentum distribution of nucleons in a deuteron.

## 1. Introduction

The investigation of the reactions which occur in collision of protons with deuterons is a convenient method for studying the proton interaction with a neutron. In the energy range $\gtrsim$ 1000 MeV which is considerably higher than the pion production threshold (280 MeV) this method was successfully applied in measuring the total cross sections for p-n interaction[1] as well as in obtaining the information on pion production reactions[2]. The influence of nucleon binding in a deuteron at such high energies is not great and may be easily taken into account by introducing a small correction to the cross sections being measured[1,2].

On approaching the threshold the nucleon binding in a deuteron even more essentially effects the pion production processes. The main effect of those caused by this binding is the change of the magnitudes of the cross sections because of the intranuclear motion of nucleons. Among other effects which are due to the presence of the "odd" nucleon it should be noted: a mutual screening of nucleons (which is small[1-3] because of a large radius of a deuteron), reabsorption of the produced pion by a nucleon pair, the prohibition of some final states due to the Pauli principle, a possible influence of the interference of nucleon states, the contribution from the reactions occuring without a deuteron disintegration (the cross sections of these processes are very small[4]).

In order the obtain the information on the pion production in p-n collisions from the data on p-d collisions it is necessary, at least approximately, to take into account the influence of the nucleon binding in a deuteron and first of all to make an attempt to estimate the magnitude of the change of the cross sections due to the intranuclear motion (these changes are especially considerable near the threshold, where the corresponding factor of the increase of the cross section grows to infinity). This problem will be considered below according to the impulse approximation, the reaction for neutral pion production in p-d collisions being taken as an example

$$p + d \rightarrow \pi^0 + \text{nucleons} \qquad (1)$$

which has been studied in detail in[5].

## 2. Momentum Distribution of Nucleons in a Deuteron

If a deuteron is regarded as a set of two nucleons moving with respect to each other, then the total cross section of reaction (1) $\sigma_{pd}$ may be represented as follows:

$$\sigma_{pd} = \int \left\{ \sigma_{pn}\left[\eta_m(p_1,\vec{p}_2),\vec{p}_2\right] + \sigma_{pp}\left[\eta_m(p_1,\vec{p}_2),\vec{p}_2\right] \right\} F(\vec{p}_2)\, d\vec{p}_2 \, . \tag{2}$$

Here $F(\vec{p}_2)$ is the momentum distribution of nucleons in a deuteron, $\vec{p}_2$ is their momentum in the center-of-mass system, $\eta_m$ is the maximum momentum of the generating neutral pion, $p_1$ is the momentum of an incident proton, $\sigma_{pn}$ and $\sigma_{pp}$ are the cross sections for neutral pion production in the collision of an incident proton with the neutron and proton of a deuteron. When the values of $p_1$ are small the dependence $\eta_m(p_1,\vec{p}_2)$ is the main in the functions $\sigma_{pn}$ and $\sigma_{pp}$ ; that allows to simplify these functions by presenting them as $K\sigma_{pn}[\eta_m(p_1,\vec{p}_2)]$ (analogously $\sigma_{pp}$ ). The factor $K$ thus introduced takes into account all the binding effects except the intranuclear motion. At the same time it is supposed that $K$ changes little with energy.

Integrating in (2) over the unit vector $\vec{p}_2/p_2$ we obtain

$$\sigma_{pd} = \int K \left\{ \sigma_{pn}(p_1,p_2) + \sigma_{pp}(p_1,p_2) \right\} F(p_2)\, p_2^2\, dp_2 \tag{3}$$

Here the cross sections $\sigma_{pn}(p_1,p_2)$ and $\sigma_{pp}(p_1,p_2)$ correspond to the reactions

$$pn \rightarrow pn\pi^0 \quad \text{and} \tag{4}$$

$$pp \rightarrow pp\pi^0 \tag{5}$$

which occur on the moving nucleons of a deuteron. Since the reaction (5) cross section is comparatively small[6] the contribution from the second term to sum (3) is not great. The functions $\sigma_{pn}(p_1,p_2)$ and $\sigma_{pp}(p_1,p_2)$ entering into (3) were calculated for a wide range of the values $p_1$ and $p_2$ by means of an electronic computer "Urals". When determining the function $\sigma_{pp}(p_1,p_2)$ the experimental data[6] have been used. According to the phenomenological theory[7], near the threshold the cross section $\sigma_{pn}(\eta_m)$ must have the form $\eta_m^{\delta}$, where $3 < \delta < 4$. The calculations of $\sigma_{pn}(p_1, p_2)$ have been made for $\delta = 3$ and $\delta = 4$. The integration in (3) has been performed for several types of momentum distribution $F(p_2)$ (some of these distributions are given in Fig. 1). The dependences of $\sigma_{pd}$ upon the incident proton energy obtained are given in Fig.2, where they are compared with the experimental energy dependence of the reaction (1) cross section[5]. The curve 4b in this Figure has been calculated for the momentum distribution of the Chew-Goldberger type (the so-called improved):

$$F(p_2) \sim (\alpha^2 + p_2^2)^{-2}(\beta^2 + p_2^2)^{-2}, \quad \beta = 2{,}5\alpha, \quad \alpha = 190\ \text{MeV}/c, \tag{6}$$

for which the long "tail" is characteristic (see Fig.1).

Together with the concideration of the intranuclear motion an attempt has been made to take approximately into account the influence of the Pauli principle by excluding from reaction (1) the contribution from the collisions in which the secondary nucleons remain inside the Fermi sphere. The change of the magnitudes of the cross sections due to the Pauli principle turned out to be insignificant in the energy region under consideration. This can be seen from the comparison of the curve 4b in Fig.2 with the curve 4, which, in contrast to 4b, has been calculated with account of the Pauli principle.

It has been already pointed out above that the character of the dependence of the cross section $\sigma_{pd}$ upon energy near the threshold is mainly determined by the form of the momentum distribution and is little sensitive to what energy dependence of the cross sections of reactions (4) and (5) is. One can become convinced in this if curves 4 and 4a in Fig. 2 calculated for the cases $\sigma_{pn} \sim \eta_m^3$ and $\sim \eta_m^4$ are compared.

Curves 5 and 6 drawn in Fig. 2 have been calculated for the case of the Gaussian distribution

$$F(p_2) \sim \exp(-p_2^2/\bar{p}_2^2), \tag{7}$$

which describes the momentum distribution in complex nuclei rather well. The distribution of this type, in contrast to that of Chew-Goldberger, has a comparatively small number of high momentum nucleons (see Fig.1). As is seen from Fig.2 both Gaussian and Chew-Goldberger distributions are in bad agreement with the experimental data[5] on reaction (1).

A good agreement with the measured cross sections is obtained if one makes use of the momentum distributions for the deuteron found by Salpeter and Goldstein[8] (see Fig.1). In the region of small momenta these distributions are approximately described by the dependence:

$$F(p_2) \sim (\gamma^2 + p_2^2)^{-2}, \tag{8}$$

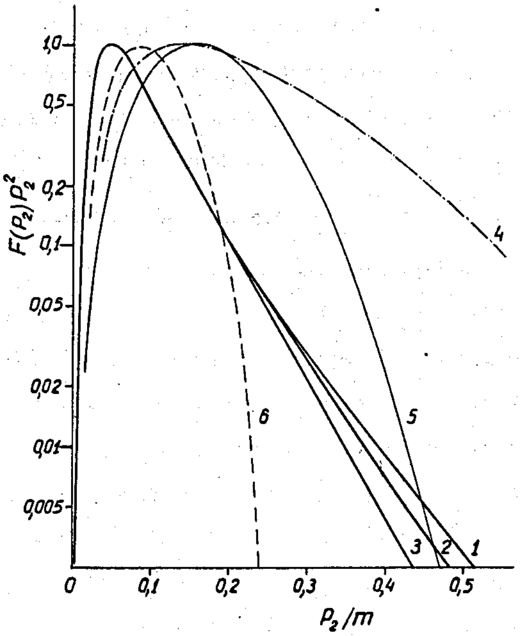

**Fig.1.** Momentum distributions (arbitrary units).
1,2,3 - Salpeter-Goldstein distributions for the potentials of the Yukawa, exponential and Gaussian shapes; 4 - Chew-Goldberger distribution (an improved one); 5,6 - Gaussian distributions with the dispersions $\sqrt{\overline{p_2^2}}/m$ = 0.11 and 0.06; m - the nucleon mass.

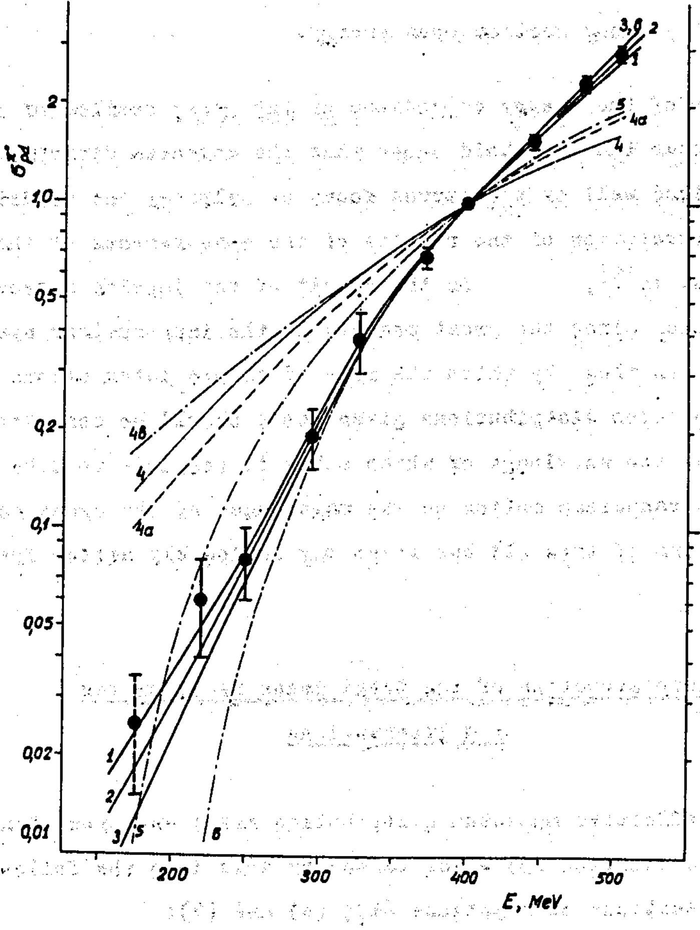

**Fig.2.** Energy dependence of reaction (1) total cross section.
● – measured in [5]. 1-6 – energy distributions calculated for the momentum distributions given in Fig.1. 4a and 4b – see the text. All the data are normalized to unity at the incident proton energy $E = 400$ MeV.

where $\gamma = 46$ MeV/c. Distributions 1-3 calculated in[8] for the cases of the potentials of the Yukawa, exponential and the Gaussian shapes differ, as is seen from Fig.1, only in the region of very large momenta. All of them agree equally well with the experimental dependence of the reaction (1) cross section upon energy.

Thus, the analysis of the energy dependence of the cross section of the reaction $p+d \to \pi^0$ + nucleons near the threshold shows that the momentum distribution of nucleons in a deuteron is described well by the curves found by Salpeter and Goldstein. It should be noted that the interpretation of the results of the measurements of the energy dependences, similar to those in[5], in the spirit of the impulse approximation encounter essential difficulties since the great momenta of the intranuclear nucleons correspond to small distances between them by which the role of triple interactions becomes important. Therefore, the momentum distributions given above should be considered as certain effective distributions, the knowledge of which makes it possible to take into account the influence of the intranuclear motion on the magnitudes of the cross sections of the meson production reactions of type (1) but which may noticeably differ from the real momentum distribution.

## 3. Reconstruction of the Total Cross Sections for $p$-$n$ Interactions

Making use of the effective momentum distribution for a deuteron obtained above, it is possible to integrate equation (3) which passes by this into the following relationship between the cross sections of reactions (1), (4) and (5):

$$\sigma_{pd} = \kappa(g_{pn}\sigma_{pn} + g_{pp}\sigma_{pp}). \qquad (9)$$

Here $\sigma_{pn}=\sigma_{pn}(p_1,0)$ and $\sigma_{pp}=\sigma_{pp}(p_1,0)$ are the "usual" cross sections, whereas $g_{pn}$ and $g_{pp}$ are the magnitudes characterizing the change of the cross sections due to the intranuclear motion and depending only on $p_1$. To make a similar integration it is necessary to know the energy dependence of the cross sections of reactions (4) and (5). This problem must be solved by the method of subsequent approximations. However, since the momentum distribution $F(p_2)$ is not wide, for the determination of $g_{pn}$ a sufficient accuracy may be provided by the first approximation for the cross section, $\sigma_{pn}^{(1)}$. As the latter the dependence $\eta_m^3$ near the threshold was used. In the high energy region it can be assumed $\sigma_{pn}^{(1)}=\sigma_{pd}-\sigma_{pp}$. Above 600 MeV the rise of $\sigma_{pn}^{(1)}$ slows down and at the energies $\gtrsim$ 1000 MeV $\sigma_{pn}^{(1)} \approx$ const. The obtained functions $\sigma_{pn}^{(1)}(p_1,p_2)$ which were used for the determination of $g_{pn}$ are given in Fig.3. The functions $\sigma_{pp}(p_1,p_2)$ have an analogous form. The coefficients $g_{pn}$ and $g_{pp}$ found by the method described above are presented in Fig.4.

To calculate the coefficient $\kappa$ entering into (9) is practically impossible because of the imperfection of the modern theory of a nucleus. This coefficient may be only said to be close to unity at high energies, where $\sigma_{pd} \approx \sigma_{pn} + \sigma_{pp}$. The only factor entering into the coefficient which can be calculated is the decrease of the cross sections due to the mutual screening of nucleons in a deuteron[3]. The corresponding correction is found to be not great (some percent).

The coefficient $\kappa$ may be found experimentally by comparing the magnitudes of the cross sections measured at the beams of protons and neutrons with an equal mean energy. In case when the incident particle is a neutron, the cross section for a deuteron is of the form

$$\sigma_{nd} = \kappa(g_{np}\sigma_{np} + g_{nn}\sigma_{nn}), \qquad (9)^1$$

that is analogous to (9), as due to the charge symmetry of nuclear forces $\sigma_{np} = \sigma_{pn}$, $\sigma_{nn} = \sigma_{pp}$ $g_{np} = g_{pn}$ and $g_{nn} = g_{pp}$. For the determination of the magnitude of the coefficient $\kappa$ it is convenient to use not the cross sections the measurement accuracy of which is not great but the ratios of the cross sections $\alpha_p = \sigma_{pd}/\sigma_{pp}$ and $\alpha_n = \sigma_{nd}/\sigma_{np}$ measured more exactly. Under these notations

$$1/\kappa = g_{pp}/\alpha_p + g_{pn}/\alpha_n \qquad (10)$$

The values $\alpha_p$ and $\alpha_n$ were determined at 590 MeV[5,6,9]: $\alpha_p = 3.00\pm0.15$, $\alpha_n = 1.30\pm0.04$. From here:

$$k(590) = 0.89\pm0.03.$$

The magnitude of $\kappa$ may be also determined in one more point, at 380 MeV, where the magnitudes of the cross sections of reactions (1), (4), and (5)[5,6,10] are known:

$$k(380) = 0.72\pm0.16.$$

The comparison of the obtained magnitudes allows to suppose that the coefficient $\kappa$ is constant in the whole energy region from the threshold to 600 MeV.

Making use of the found values $g_{pn}$, $g_{pp}$ and k one can reconstruct the cross section for neutral pion production in p-n collisions by the experimental data on the cross sections $\sigma_{pd}$ and $\sigma_{pp}$:

$$\sigma_{pn} = \sigma_{pd}/\kappa g_{pn} - \sigma_{pp} g_{pp}/g_{pn} \qquad (11)$$

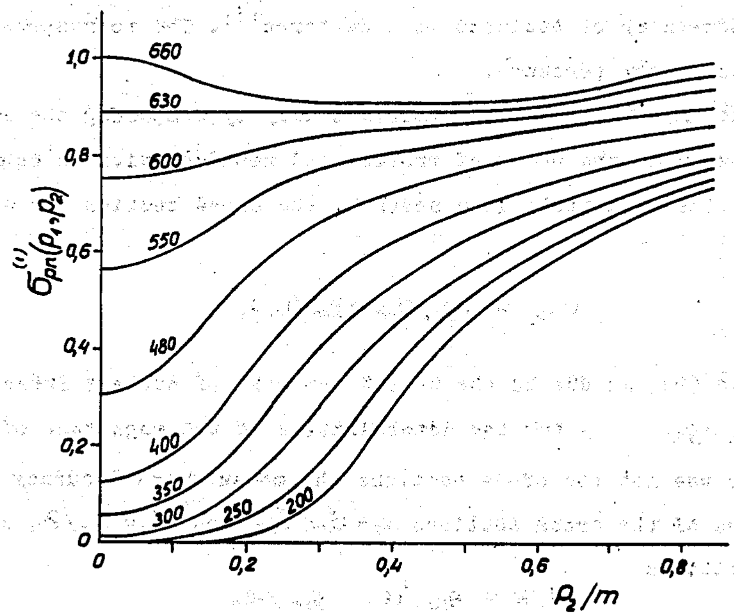

**Fig.3.** The functions $\sigma^{(1)}_{pn}(p_1,p_2)$.
The figures on the curves indicate the corresponding values of the kinetic energy $E = \sqrt{p_1^2+m^2}-m$.

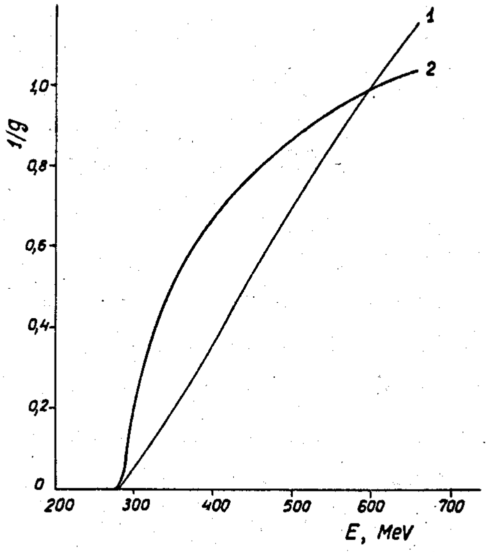

**Fig.4.** The coefficients $g$ at various energies.
1 — $1/g_{pp}$ , 2 — $1/g_{pn}$

## 4. Pion Spectra

The energy spectra of pions produced in the reactions of type (1) are subject to the nucleon binding in a greater extent than the magnitudes of the total cross sections $\sigma$ Even at high energies they are different by their form from the spectra of pions produced in the collisions of free nucleons[11,12] (see Fig.5,6). In particular, one can see at once that the "peak" so characteristic for free proton-proton collisions is absent in these spectra. Assuming as earlier that the influence of the binding mainly reduced to the change of the magnitudes of the differential cross sections $d^2\sigma/d\Omega\, dE$ due to the intranuclear motion of neutrons in a deuteron it is possible to calculate the change of the spectrum form by the same method which was used in previous paragraphs when solving the problem about the total cross sections. This calculation was made for the spectra of positive pions produced in p-d collisions at 655 MeV to compare the results of the calculations with the experimental data * on the pion spectra in the reaction $p+d \to \pi^+ +$ nucleons[12]. The calculations were made separately for the reactions $pp \to d\pi^+$ and $pp \to pn\pi^+$ (the parts of the spectra corresponding to these reactions are shown in Fig.5). The functions $d^2\sigma/d\Omega\, dE(P_1,P_2)$ analogous to those entering into (3) have been calculated using the energy dependence of the cross section for positive pion production[13] and the spectra obtained in[12,13]. The integration over the nucleon momenta in a deuteron $P_2$ has been made for the case of Salpeter-Goldstein momentum distribution.

The calculated spectra (see Fig.6) differ essentially from those of positive pions produced in p-p collisions. Most of all this refers to the "peak" corresponding to the reaction $pp \to d\pi^+$. The width of this "peak" increases under the influence of the intranuclear motion up to 50%. This makes it practically unobservable in the spectrum of pions produced in p-d collisions. At the same time the relative contribution of the "peak" somewhat decreases (by 15%) as the energy dependence of the reaction $pp \to d\pi^+$ cross section has a resonance character.

The comparison of the spectra of pions produced in p-p collisions in a deuteron[12] with the calculated ones shows (Fig.6) that the form of the spectra for p-d collisions may be rather accurately predicted on the basis of the data on free p-p collisions.

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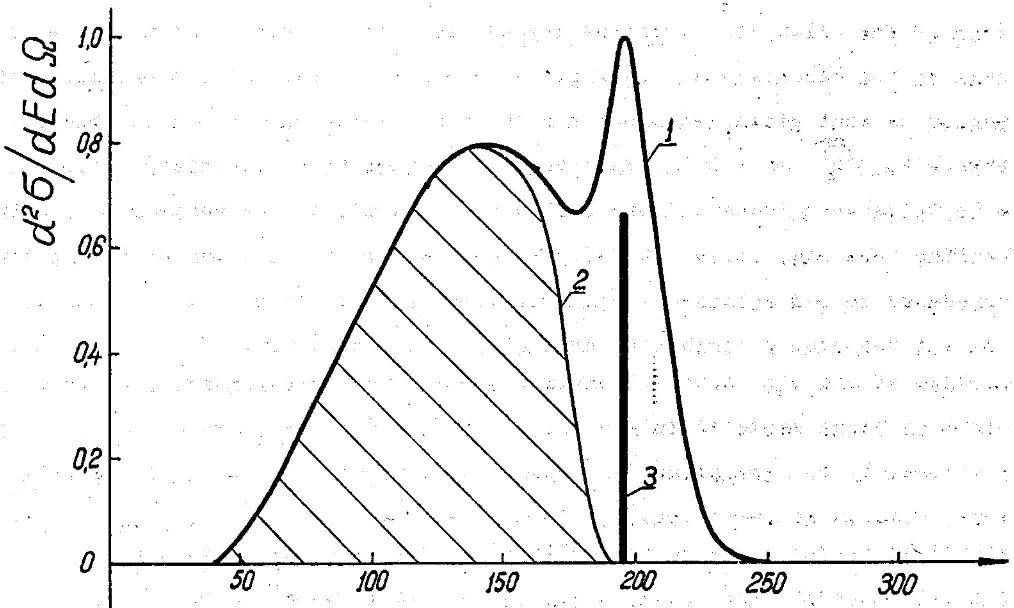

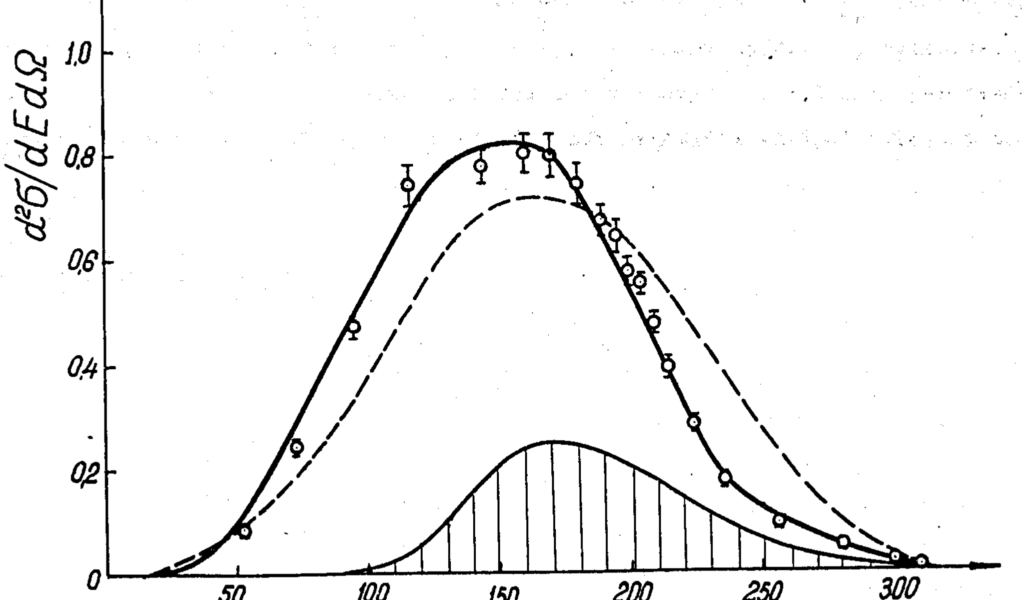

Fig.5.

Energy spectrum of positive pions produced in p-p collisions (in relative units). I- measured by G.Gelfer et al|12| at an angle ≈ 90° in the c.m.s. at the proton energy 655 MeV; 2- the positive pions spectrum from the reaction pp → $pn\pi^+$ ; 3- position of the peak corresponding to the reaction pp → $d\pi^+$ .

Fig.6.

Spectrum of positive pions produced in p-d collisions (in rel.units). o -the measured spectrum of positive pions produced in p-p collisions in a deuteron |I2| (it was obtained as a difference of positive and negative pion spectra). The thick solid curve shows the same spectrum calculated with the use of Salpeter- Goldstein momentum distribution. The dashed curve indicates the spectrum calculated for the distribution of the Gaussian type with the dispersion $\sqrt{\overline{p^2}}/m$ = 0.06. The thin curve represents the spectrum of positive pions from the reaction pp → $d\pi^+$ , transformed as a result of intranuclear motion. In calculating the spectra there was taken into account the resolution of the spectrometer|12|(this changes the form of the spectra only slightly due to their large width).

The form of the calculated spectrum depends essentially upon what momentum distribution was used in the calculations. In Fig.6 is shown the spectrum of pions calculated in the same manner as that given above but for the case of the momentum distribution of the Gaussian type with $\sqrt{\overline{p_z^2}}/m = 0.06$. In spite of the fact that this distribution is rather close to Salpeter-Goldstein distribution (see Fig.1), the corresponding spectra considerably differ from each other. It follows from here that the study of the spectra of the pions produced in p-d collisions makes it also possible to obtain the quantitative information on the momentum distribution of nucleons in a deuteron.

The problem of the pion spectra considered above may be reversed, i.e. from the measured spectrum of pions produced in p-d collisions it is possible when the latter is measured very accurately to reconstruct the spectrum of pions produced in the collisions of free nucleons. This is of great interest in case when a direct investigation of the corresponding reactions taking place in the collisions of free nucleons is very difficult from the experimental point of view (such, e.g., is the reaction $pn \rightarrow pp\pi^-$ ).

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