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ELASTIC SCATTERING OF NEGATIVE PIONS

BY PROTONS AT ENERGIES 128 AND 162 MEV *

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ABSTRACT

The elastic scattering angular distribution of negative pions by protons has been measured by means of a hydrogen-filled diffusion cloud chamber in the magnetic field. In a set of exposition 344 and 941 elastic scattering events were recorded at the angles θ (c.m.s.) $> 10^{o}$. The total cross sections for elastic scattering σ_{el} (128 MeV) $= (12.8 \pm 1.0)\ 10^{-27}$ cm^2 and σ_{el} (162 MeV) $(21.4 \pm 1.2)\ 10^{-27}$ cm^2 were determined by calculating the total length of negative pion tracks in the chamber. The elastic $\pi^- p$-scattering angular distributions were obtained in the form (the SP - analysis):

$$\frac{d\sigma}{d\Omega}(128\ \text{MeV}) = (1.00 \pm 0.08)\left[(0.55 \pm 0.07) + (0.34 \pm 0.12)\ \text{Cos}\ \theta + (1.30 \pm 0.24)\ \text{Cos}^2\ \theta\right] 10^{-27}\ \text{cm}^2/\text{sterad}$$

$$\frac{d\sigma}{d\Omega}(162\ \text{MeV}) = (1.00 \pm 0.06)\left[(0.93 \pm 0.07) + (0.51 \pm 0.12)\text{Cos}\ \theta + (2.28 \pm 0.22)\text{Cos}^2\theta\right] 10^{-27}\text{cm}^2/\text{sterad}$$

The differential cross sections for scattering in the forward direction are equal to $(2.20 \pm 0.32)\ 10^{-27}$cm^2/sterad and $(3.73 \pm 0.32)\ 10^{-27}$cm^2/sterad respectively. The real parts of the forward scattering amplitude (in the c.m.s.) have been calculated in the units of $\hbar/m_\pi c$:

$\mathcal{D}_-^{\ell}$ (128 MeV) $= 0.261 \pm 0.031$ and $\mathcal{D}_-^{\ell}$ (162 MeV) $= 0.216 \pm 0.038$.

These quantities agree with those calculated from dispersion relations with the coupling constant $f^2 = 0.08$.

INTRODUCTION

During the past few years a considerable attention was paid to an experimental check of dispersion relations. For scattering of charged pions by protons they were first obtained by Goldberger, Miyazawa, and Oehme(1). These relations connect the real parts of the forward scattering amplitudes with the lengths of the - S-scattering and with the integral over the total cross sections for $\pi^+ - p$ and $\pi^- - p$-interactions in the whole energy range. In the integration region $0 \leq \omega < \mu$ there is a contribution from the 'bound state', i.e., that from a neutron as a possible intermediate state of the system in scattering. This contribution may be expressed in terms of a renormalized coupling constant f^2, characteristic of the pseudovector interaction. General physical principles such as the microscopic causality principle (the requirement of the absence of the signals propagating with the velocity greater than that of light) and the hypothesis of the charge independence of nuclear forces form the basis for the derivation of dispersion relations. The comparison with the experimental results enables us to check the validity of these fundamental principles. On the other hand, an independent determination of the coupling constant and the lengths of the

S-scattering becomes possible.

The comparison of the dispersion relations with an experiment performed for the first time by Anderson, Davidon, and Kruse[2] has shown a good agreement between them. However, in 1957 Puppi and Stanghellini[3] paid their attention to the fact that for negative pions there is a discrepancy between the experimental and theoretical data. If for positive pions the real parts of the forward scattering amplitude determined experimentally in the energy range to 400 MeV have been found in agreement with those calculated from dispersion relations with the coupling constant $f^2 = 0.09 \pm 0.0.1$ then for negative pions in the energy interval to 200 MeV a better agreement was obtained if the coupling constant is assumed to be 0.04. At the same time Agodi, Cini and Vitale[4] have shown that the effects due to the charge, the electromagnetic corrections and the contribution from the hyperons and K-mesons cannot account for the observed discrepancy. On the other hand, as is seen from the original paper [3] this discrepancy is mainly based on the results of only one experiment – the measurements of elastic π^-- p -scattering by a group of the Carnegie Institute [5] at an energy of negative pions 150 and 170 MeV since the rest experimental data in this energy range have a comparatively low accuracy. In this connection new experimental data on elastic $\pi^- p$- scattering in the energy region 100 – 200 MeV and the comparison of them with the predictions of dispersion relations are, undoubtedly , of an interest.

In the present paper are given the results of measurements of negative pion elastic scattering by protons at an energy of 128 and 162 MeV obtained with the help of a hydrogen-filled diffusion cloud chamber. The application of such methods enabled us to avoid some specific difficulties due to the use of electronics methods of measurements as well as made it possible to go over to the region of smaller scattering angles that proved to be very essential for determining the magnitude of the differential cross section at 0°.

II. EXPERIMENTAL DETAILS AND APPARATUS

I. EXPERIMENTAL ARRANGEMENT

A scheme of an experimental arrangement is given in Fig. 1. A berillium target (3) 40 mm thick was placed in an 670 MeV internal proton beam (2) from the synchrocyclotron of the Laboratory of Nuclear Problems. Negative pions (4) generated in the target were extracted outside from the vacuum chamber (1) by the magnetic field of the accelerator, and after passing through an additional concrete shielding (5) they were collimated by the collimator (6), (3.6 m long, the inside diameter being 50 mm), placed into the shielding wall (7). Then the beam of negative pions was deflected by 40° by a clearing magnet (8) and struck the diffusion chamber (11) placed inside the electromagnet (10). Before the chamber there was an additional lead collimator (9) with a rectangular hole 350 x 50 mm^2.

The beam of negative pions was traced by a usual method of current carrying thread. As a whole about 90000 stereophotographs were obtained. Together with elastic $\pi^- p$-scattering this material enabled us to detect a β -decay of negative pions ($\pi^- \to e^- + \tilde{\nu}$) [6] and to obtain some data on decays of neutral pions by the schemes $\pi^0 \rightarrow e^- + e^+ + \gamma$ (7) and $\pi^0 \rightarrow 2e^- + 2e^+$ (8).

2. DIFFUSION CLOUD CHAMBER

A diffusion cloud chamber was constructed for work with light gases at the pressure up to 25 atm.(9) A scheme of the diffusion cloud chamber placed in the magnet is given in Fig.2. The chamber is reservoir made of stainless steel and consisting of three parts. In the lower,main part of the chamber a required temperature gradient is created by means of a system of heaters and a copper coil, through which the colled acetone is pumped.

In the lower flange of the middle part of the chamber there is a small tray for methyl – alcohol which is a working liquid of the chamber. At the temperature of the bottom – 70° and that of the tray + 10° an approximately linear temperature distribution with a gradient of 7 degrees/cm is being established in the sensitive volume of the chamber. At the same time the height of the sensitive layer amounts to 6–7 cm. The temperature in various parts of the chamber is controlled and measured by means of copper-constantan thermocouples.

The chamber was illuminated at an angle of 90° to the photographing axis through the side windows closed by plexiglass plates 30 mm thick. These windows were placed at the ends of the rectangular extension ports 150 mm long weld on the lower cylindrical part of the chamber. This made possible to increase to the maximum working diameter of the chamber equal to 380 mm, the diameter of the hole in the upper pole of the magnet being 460 mm.

The chamber was lighted by two xenon flash tubes ИФП–500 through which at the moment of a flash the capacities of 200 μF, the voltage being 2000 в, were discharged. The light from each tube was formed into a parallel beam by means of two parabolic reflectors.

The photographs were taken by a stereophotocamera with two objectives 'Гелиос-37', (f = 62 mm) on Панхром -X 35 mm film of sensitivity equal to 1000 units ГОСТ. The objectives were corrected for the distortion arising in photographing through the upper glass windows 25 mm thick. The resolution of the objectives is 50 lines/mm in the center of the field of vision. The base of the stereophotocamera is equal to 120 mm, the object distance is about 1 meter. A requided depth of volume we take photos of was achieved at the aperture of 5.6.

The operation cycle of the chamber was 8 sec. The time cycle was operated by an electronic circuit which was meant for: a) switching on a definite number of acceleration cycles (as usual 2-4) of a high-frequency voltage to the synchrocyclotron dee; b) firing the lights with delay with respect to the particle pulse 0.2 –0.3 sec; c) switching of the camera film advance; d) switching on and out of the electric clearing field. Required time delays between the operations were realized by one-shot multivibrator circuits. By regulating the number of the acceleration cycles the intensity of the beam was kept such that on the average about 30 tracks of negative pions were recorded in each picture.

3. ELECTROMAGNET

The permanent magnetic field of 9000 gauss in the working volume of the chamber was created by a magnet-solenoid of МС-4A type* meant for work both in the permanent and in the pulsed supply. The magnetic field variation by the height of sensitive volume of the chamber was not more than 3.5%, whereas by the radius - not more than 2.5%. The magnetization curve and the decrease of the magnetic field by the height and radius of the sensitive volume was obtained with the help of a magnetometer operating of the principle of the Hall effect**. The device was calibrated by the proton resonance method.

4. NEGATIVE PION BEAMS

The mean energy of mesons in a beam and their spread in energies have been determined directly by measuring the radii of the track curvatures of the photographs. With the aim to decrease the distortions of the tracks due to the convection currents in the chamber the measurements were made of the films exposed with delay of the firing the lights which is twice as short as usual. The energies thus determined are equal to (128 ± 8) MeV and (162 ± 10) MeV where the uncertainties indicated are the halfwidth of the energy distribution of negative pions in the chamber.

To determine the contamination of negative muons and electrons in the beam as well as to control the energy the absorption curves in copper have been obtained using scintillation counters. The mean energies of the beam determined by these two methods coincided with a good accuracy. The total contamination of negative muons and electrons in the beams was (16 ± 2)%.

III. SCANNING AND ANALYSIS

The photographs obtained were scanned with the help of stereoscopes. All the films were scanned twice by several scanners independently from each other. A part of the films was scanned for the third time especially thoroughly in order to estimate the efficiency of the double scanning which turned out to be 97%. The investigation of the angular distribution of the missed events has shown that this efficiency does not depend upon the concrete form of the scattering event (the scattering angle or the azimuthal angle).

As a result of double scanning 379 events of scattering were found at an energy of 128 MeV

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* The magnet-solenoid MC-4A is a modification of the magnet MC-4. These magnets were constructed in the Research Institute of Electrophysical Apparatus by N.S. Streltzov, A.V. Ugamm, N.N. Indjukov, Yu.P. Semenov, V.I. Sergeeva, A.G. Studennikova.

** The authors are grateful to D.P. Vasilevskaja and Yu.N. Denisov for the permission to use this device.

and 1113 events at an energy of 162 MeV. The analysis of these events was made by the reprojection method. Both pictures of the stereopair were reprojected through the same optical system with the help of which the photos were taken upon the screen having a scaler for measuring the angles. The screen is mounted in gimbal rings fixed on the frame which can move in two perpendicular directions (X and Y). The coordinates Z were measured by displacing of the optical system itself in the vertical direction. The reprojector is constructed analogously to that described in Ref.(10) For each scattering event there were measured: a) the coordinates of the interaction points /x,y,z/ and the coordinate z_0 of the reper mark situated on the bottom of the chamber; b) the scattering angles of the meson θ_π and the recoil proton ϑ_p /with an accuracy of about 1^o/; c) the azimuthal angle φ of the scattering plane with an accuracy of about 2^o / $\varphi = 0$ corresponds to the event the plane of which is horizontal/; d) the range of the recoil proton /in those events when this was possible/.

The correction for film shrinkage which usually amounted to 5–7% was introduced into the values of the coordinates x,y,z. For each case this correction was determined as a ratio of a real distance from the bottom of the chamber up to the main point of the objective to the measured one. The influence of the film shrinkage on the measurements of the angles θ and φ is very small and was neglected.

Simultaneously with the analysis of the scatteringevent the z- coordinate of an accidental negative pion track was measured on the same picture. The comparison of the distributions by the height of the sensitive volume h of the scattering events with the corresponding distributions of the same number of accidental tracks shows that by scanning there were no noticeable missing of the scattering events situated at the bottom of the chamber and at the upper boundary of the sensitive layer. (Fig. 3).

For a further analysis the events satisfying the following criteria were selected:

1. The particle tracks must be coplanar with an accuracy of 2^o, i.e. the angle between any track and the plane formed by the two others must not exceed 2^o.

2. The scattering angles of a negative meson and a recoil proton must satisfy the kinematic requirements with an accuracy of 2^o; the track curvatures, the ionization density and the range of the recoil proton /in those events when it is possible to measure it/ must also correspond to the kinematics.

3. The scattering angle of a negative pion must be more than 8^o lab.system (10^o c.m.s.), i.e., the maximum angle by $\pi^- - \mu^-$- decay.

4. The length of each of three tracks must be not less than 5 mm.

5. The interaction point must not be situated in the non-sensitive region with a dimension of more than 5 mm or darkened by the gathering of drops of more than 5 mm.

6. The interaction point must be situated at a distance of more than 1 cm from the walls of the chamber.

7. The incident negative pion must not deviate from the main direction of the beam by more

than 5^{o}.

8. The events on the pictures having any defects and on those with a very high intensity were neglected.

344 scattering events at 128 MeV and 941 events at 162 MeV satisfy the selection criteria above-mentioned. All further results on angular distributions are based on this statistical material.

IV. TOTAL CROSS SECTIONS FOR ELASTIC SCATTERING

The total cross sections for elastic $\pi^- - p$ -scattering were determined by counting the total length of the negative pion tracks. This calculation was made in the rectangular region S separated in the central part of the chamber (Fig.4). All scattering events satisfying the selection criteria above-mentioned were counted in this region. The distributions by the azimuthal angle φ, constructed for the above-mentioned cases (Fig. 5) allowed to determine the coefficients β, taking into account the inefficiency of recording the events the scattering plane of which is close to the vertical one.

The whole number of tracks in the region S was determined by counting the tracks in all the pictures multiple to 25; the number obtained was multiplied by 25 and corrected for the number of tracks situated at the end of each film. The total length of the tracks L was determined by the formula:

$$L = 15.36 \frac{T\delta}{\cos\alpha_m} \text{ (cm)} \qquad (1)$$

where T is the total number of tracks, 15.36 cm is the width of the region S, α_m is the mean angle of track with respect to the edges of the region S(Fig.4), δ is the coefficient taking into account the replacement of the length of the arc by a chord (in our case it was assumed that $\delta = 1$). The correction for the gaps of the tracks the length of which was measured directly in calculating the number of tracks was introduced into the total length of the tracks. This correction does not exceed 1%. The total cross sections were determined by the formula:

$$\sigma_{exp} = \frac{N\beta}{L\, n_{eff}\,(1-q)\tau} \qquad (2)$$

where N and L is the number of scattering events and the total length of tracks; n_{eff} is the effective number of hydrogen nuclei in 1 cm^3; β is the coefficient taking into account the inefficiency of recording the events with φ close to 90^o; q is the contamination of negative muons and electrons in a beam; τ is the efficiency of film scanning. In Table 1 are given the values used for calculating the total cross sections. The root-mean-square errors are indicated in per cents. After subtracting from the cross sections calculated by the formula (2) the Coulomb corrections (the method for calculating these corrections see in Sec.Vb) and introducing the correction for scattering in the angle interval 0–8^o (lab.system) (Table 1) there were obtained the total cross sections for elastic $\pi^- p$-scattering at the energies of 128 and 162 MeV.

T a b l e I

DETERMINATION OF TOTAL CROSS SECTIONS FOE ELASTIC SCATTERING

	128 MeV		162 MeV	
	M a g n i t u d e	Absolute root-mean-square error	M a g n i t u d e	Absolute root-mean-square error
The number of scattering events in the region S $\mathcal{N}$	190	7.2%	449	4.7%
Correction for φ-inefficiency β	1.13	2.5%	1.10	1.5%
The total number of tracks T	$0.991.10^6$		$1.446.10^6$	
The total length of tracks L	$15.3.10^6$ cm	1%	$22.3.10^6$ cm	1%
The effective pressure in the chamber P_{eff}	22.5 atm.		23.0 atm.	
The number of nuclei in cm^3 at the pressure of P_{eff} n_{eff}	$1.22.10^{21}$ cm^{-3}	2%	$1.24.10^{21}$ cn^{-3}	2%
The comtamination of μ^--mesons and electrons in the beam q	0.16	2%	0.16	2%
The scanning efficiency τ	0.97	1%	0.97	1%
The total cross section σ_{exp}	$14.2.10^{-27}$	8.2%	$22.2.10^{-27} cm^2$	5.8%
The total cross section for Coulomb scattering at the angles $\theta_{lab} > 8^\circ$	$1.6.10^{-27}$ cm^2		$1.1.10^{-27} cm^2$	
The correction for scattering in the angle interval $\theta_{lab} < 8^\circ$	$0.24.10^{-27}$ cm^2		$0.35.10^{-27}$ cm^2	
The total cross section for elastic scattering σ_{el}	$12.8.10^{-27} cm^2$	8.2%	$21.4.10^{-27} cm^2$	5.8%

As is seen from Table II, the cross sections determined in the present paper are in good agreement with other experimental data.

V. ANGULAR DISTRIBUTION OF ELASTIC SCATTERING

The scattering angles of negative mesons were recounted into the center-of-mass-system. Then all the events were divided into 8 angular intervals by 20^o from $\theta = 10^o$ up to $\theta = 170^o$ (in the center-of-mass-system) The scattering events found in the interval $\Delta\theta = 0^o - 10^o$ (one event at an energy of 128 MeV and 9 events at an energy of 162 MeV) were not taken into account (See the selection criteria, Sec III) due to an extremely low efficiency of observation since the recoil protons in meson scattering at the angles $\theta < 10^o$ have a very small range, and the scattering cases cannot be separated from $\pi^- - \mu^-$-decays. Besides, in this angular interval the cross section for Coulomb scattering is very great. The interval $\Delta\theta = 170^o - 180^o$ was not also taken into account due to a low efficiency (only one event $\theta = 173^o$ was found by the number 3–5 expected).

In Colomns 1 and 2 of Tables III and IV are given the angles θ, corresponding to the middles of the angular intervals, as well as the numbers of scattering events ΔN in each interval. In order to obtain the differential cross sections from these data it is necessary to take into account the inefficiency of recording the events with the angles φ close to 90^o, the Coulomb corrections and the corrections for the averaging over the angular interval.

a/ Corrections for φ-inefficiency. The scattering events the planes of which are close to a vertical one ($\varphi \sim 90^o$), may be missed in scanning with a greater probability than those lying in the planes close to a horizontal one. At this, evidently, the efficiency must be lower for small and for very large scattering angles θ. In Fig. 6 and 7 are given the distributions of scattering events by the azimuthal angle φ for different angular intervals $\Delta\theta$. The efficiency of recording the events in the interval $\Delta\theta = 45^o - 135^o$ found from these distributions is 92–93%, whereas for the angular intervals $\Delta\theta = 10^o - 30^o$ and $\Delta\theta = 150^o - 170^o$ it falls to 80–83%. The coefficients α, taking into account the correction for the φ inefficiency, are given in Colomns 3 of Tables III and IV.

b/ Coulomb Corrections. The Coulomb corrections were calculated using the relativistic amplitudes of Coulomb scattering obtained by Solmitz[19]. The differential scattering cross section which is due to the Coulomb interaction may be written as

$$\left(\frac{d\sigma}{d\Omega}\right)_c = \frac{\Phi_1}{\sin^4\frac{\theta}{2}} + \frac{\Phi_2}{\sin^2\frac{\theta}{2}} + \Phi_3 + \Phi_4 \sin^2\frac{\theta}{2} \qquad (3)$$

where θ is the scattering angle in the c.m.s. The first term in expression (3) is the cross section of a purely Coulomb scattering, while the rest are due to the interference of the Coulomb and nuclear scattering. The coefficients Φ are calculated by the following formulas:

Table II

TOTAL CROSS SECTIONS FOR ELASTIC $\pi^- p$ SCATTERING IN THE ENERGY RANGE 100–200 MeV

E (MeV)	σ_{el} (10^{-27} cm^2)	Reference
98	6.15 ± 0.22	11
118	9.6 ± 2.0	12
120	11.3 ± 1.6	13
128	12.8 ± 1.0	This experiment
130	12.0	14
144	17.0 ± 2.4	13
150	$20,0 \pm 1.0$	5
152	18.8	14
162	21.4 ± 1.2	This experiment
165	22.5 ± 1.5	15
169	21.2 ± 2.0	16
170	23.5 ± 1.0	5
187	22.5 ± 1.3	17
189	23.0 ± 1.4	18
194	26.4 ± 2.7	16
210	28.7 ± 3.1	16

$$\left.\begin{aligned}
\Phi_1 &= \mathcal{A}^2(\mathcal{B}+\mathcal{C})^2 \\
\Phi_2 &= 4\mathcal{A}^2\mathcal{D}^2 - 4\mathcal{A}^2\mathcal{C}(\mathcal{B}+\mathcal{C}) + \mathcal{A}(\mathcal{B}+\mathcal{C})(\mathcal{E}+\mathcal{F}) \\
\Phi_3 &= 4\mathcal{A}^2(\mathcal{C}^2-\mathcal{D}^2) - 2\mathcal{A}\mathcal{C}(\mathcal{E}+\mathcal{F}) - 2\mathcal{A}\mathcal{F}(\mathcal{B}+\mathcal{C}) - 4\mathcal{A}\mathcal{D}\mathcal{G} \\
\Phi_4 &= 4\mathcal{A}(\mathcal{C}\mathcal{F}+\mathcal{D}\mathcal{G})
\end{aligned}\right\} \quad (4)$$

where

$$\left.\begin{aligned}
\mathcal{A} &= \frac{e^2}{2pc(\beta_\pi+\beta_p)} \\
\mathcal{B} &= 1 + \tfrac{1}{2}\beta_\pi\beta_p - \tfrac{1}{4}\beta_p^2(2\mu_p-1) \\
\mathcal{C} &= \tfrac{1}{2}\beta_\pi\beta_p + \tfrac{1}{4}\beta_p^2(2\mu_p-1) \\
\mathcal{D} &= \tfrac{1}{2}\mu_p\beta_\pi\beta_p + \tfrac{1}{4}\beta_p^2(2\mu_p-1) \\
\mathcal{E} &= \tfrac{1}{3K}(2\sin 2\alpha_1 + \sin 2\alpha_3) \\
\mathcal{F} &= \tfrac{1}{3K}(2\sin 2\alpha_{11} + \sin 2\alpha_{31} + 4\sin 2\alpha_{13} + 2\sin 2\alpha_{33}) \\
\mathcal{G} &= \tfrac{1}{3K}(2\sin 2\alpha_{13} + \sin 2\alpha_{33} - 2\sin 2\alpha_{11} - \sin 2\alpha_{31})
\end{aligned}\right\} \quad (5)$$

In these formulas $p = \hbar K$ is the momentum in the c.m.s., β_π and β_p are the velocities of a pion and a proton in the c.m.s., μ_p is the magnetic moments of a proton in nuclear magnetons. The magnitudes of the coefficients Φ for the pion energies 128 and 162 MeV calculated by formulas (4) and (5) are presented in Table V, and for calculating $\mathcal{E}$, $\mathcal{F}$ and $\mathcal{G}$ the phases of meson-nucleon interaction obtained by Chiu and Lomon were used[20]. The integration of the expression (3) within the limits $10^\circ < \theta < 180^\circ$ gives the corrections for the total cross sections for elastic scattering (see Table 1):

$$\sigma_c = 2\pi \int_{10^\circ}^{180^\circ} \left(\frac{d\sigma}{d\Omega}\right)_c \sin\theta\, d\theta = \begin{cases} 1{,}6 \cdot 10^{-27}\ \text{cm}^2 & \text{for 128 MeV} \\ 1{,}1 \cdot 10^{-27}\ \text{cm}^2 & \text{for 162 MeV} \end{cases}$$

To obtain the differential cross sections for the purely nuclear scattering the cross sections $(d\sigma)_c = \int_{\Delta\theta} \left(\frac{d\sigma}{d\Omega}\right)_c \sin\theta d\theta$ were subtracted from the magnitudes $(d\sigma)_{exp}$ determined experimentally. The errors in the cross sections $(d\sigma)_c$ were determined by varying the phase-shifts entering into expressions (5) within reasonable limits.

c/ Correction for averaging over the angular interval $\Delta\theta$. Since the value of the cross section which is averaged over the final angular interval $\Delta\theta$ is assigned to the middle of the interval θ then the real value of the cross section for the angle θ will be different from the averaged one by the magnitude

$$d(\theta) = -\frac{(\Delta\theta)^2}{24}\left[\frac{d\sigma}{d\Omega}(\theta)\right]'', \qquad (6)$$

where $\left[\frac{d\sigma}{d\Omega}(\theta)\right]''$ is the second derivative of the differential cross section. As a first approximation for calculating $d(\theta)$ the dependence $\frac{d\sigma}{d\Omega}(\theta) = a' + b'\cos\theta + c'\cos^2\theta$ found by the least squares by the experimental differential cross sections corrected for the Coulomb scattering was used. Then

$$d(\theta) = \frac{(\Delta\theta)^2}{24}\left(b'\cos\theta + 2c'\cos 2\theta\right) \qquad (7)$$

The maximum value of the correction calculated by formula (7) does not exceed 2.5%.

d/ Differential Cross Sections for Elastic Scattering. The differential cross sections $\left(\frac{d\sigma}{d\Omega}\right)_{exp}$ (Colomns 4 of Tables III and IV) were obtained from the angular distributions $\frac{\Delta N}{\Delta\Omega}$ by normalizing to the total cross sections σ exp.(Table I) with account of the coefficients α. In Colomns 5 of Table III and IV are given the differential cross sections for elastic $\pi^- p$-scattering which are due to a purely nuclear interaction. They are obtained by subtracting the cross sections for Coulomb scattering from $\left(\frac{d\sigma}{d\Omega}\right)_{exp}$ and by introducing the corrections for averaging over angular intervals. The errors indicated are the root-mean-square errors which are mainly due to the statistical errors and to the small uncertainties from introducing the corrections. The common factor before the cross sections is an error in the normalizing total cross section. A least squares fit of the form $\frac{d\sigma}{d\Omega} = a + b\cos\theta + c\cos^2\theta$ was made to the differential cross sections experimentally obtained. In Table VI are given the values of the coefficients a, b and c, as well as $M = \sum_i \left(\frac{\varepsilon_i}{\Delta\sigma_i}\right)^2$ which is the sum of the squares of the deviations ε_i of the calculated cross sections from the experimental points expressed in the units of the experimental errors $\Delta\sigma_i$, and the number of degrees of freedom $M_0 = n - m$ (n is the number of experimental points, m is the number of the parameters of the curve). The closeness M to M_0 points to a good agreement between the curves and the experimental cross sections.

In the righthand side of Table VI are given the matrices of the errors G^{-1}_{ij} calculated by the method stated in [21]. The errors of the coefficients a, b, and c are the square roots of the diagonal elements of the matrices. The off-diagonal elements are the product of the corresponding standard deviations multiplied by the correlation coefficient. In one half of the matrices on the

Table III

DIFFERENTIAL CROSS SECTIONS FOR ELASTIC π^-p-SCATTERING AT 128 MeV

θ (c.m.s.)	ΔN	α	$\left(\frac{d\sigma}{d\Omega}\right)_{exp}$ $\left(10^{-27}\frac{cm^2}{sterad}\right)$		$\left(\frac{d\sigma}{d\Omega}\right)_{nucl}$ $\left(10^{-27}\frac{cm^2}{sterad}\right)$	
20°	61	1.22 ± 0.06	1.00 ± 0.08	3.63 ± 0.50	1.00 ± 0.08	2.01 ± 0.54
40°	78	1.14 ± 0.05		2.31 ± 0.28		2.07 ± 0.28
60°	44	1.08 ± 0.02		0.91 ± 0.14		0.85 ± 0.14
80°	37	1.08 ± 0.02		0.68 ± 0.11		0.66 ± 0.11
100°	33	1.08 ± 0.02		0.60 ± 0.10		0.60 ± 0.10
120°	33	1.08 ± 0.02		0.69 ± 0.12		0.70 ± 0.12
140°	34	1.08 ± 0.06		0.95 ± 0.17		0.98 ± 0.17
160°	23	1.25 ± 0.10		1.40 ± 0.31		1.43 ± 0.31

Table IV

DIFFERENTIAL CROSS SECTIONS FOR ELASTIC π^-p SCATTERING AT 162 MeV

θ (c.m.s.)	ΔN	α	$\left(\frac{d\sigma}{d\Omega}\right)_{exp}$ $\left(10^{-27}\frac{cm^2}{sterad}\right)$		$\left(\frac{d\sigma}{d\Omega}\right)_{nucl}$ $\left(10^{-27}\frac{cm^2}{sterad}\right)$	
20°	139	1.20 ± 0.04	1.00 ± 0.06	4.76 ± 0.43	1.00 ± 0.06	3.60 ± 0.46
40°	175	1.02 ± 0.01		2.71 ± 0.21		2.55 ± 0.21
60°	159	1.09 ± 0.01		1.96 ± 0.16		1.91 ± 0.16
80°	96	1.09 ± 0.01		1.04 ± 0.11		1.02 ± 0.11
100°	86	1.09 ± 0.01		0.93 ± 0.10		0.92 ± 0.10
120°	102	1.09 ± 0.01		1.26 ± 0.12		1.26 ± 0.12
140°	124	1.12 ± 0.03		2.11 ± 0.20		2.13 ± 0.20
160°	60	1.20 ± 0.06		2.06 ± 0.23		2.09 ± 0.28

Table V

COEFFICIENTS Φ OF FORMULA (3) FOR ENERGIES OF 128 and 162 MeV

E (MeV)	Φ_1	Φ_2	Φ_3	Φ_4
	$\left(10^{-27}\frac{cm^2}{sterad}\right)$			
128	$2.18.10^{-4}$	$3.61.10^{-2}$	$-9.33.10^{-2}$	$3.21.10^{-2}$
162	$1.54.10^{-4}$	$2.57.10^{-2}$	$-7.26.10^{-2}$	$2.96.10^{-2}$

place of the off-diagonal elements are given the coefficients of the correlation between a, b and c (in %). The obtained curves together with the experimental points are presented in Fig. 8. They are in agreement with the angular distributions of the elastic $\pi^- p$ -scattering measured by Ashkin et al.(5) at 150 and 170 MeV, as well as with the results of the paper by Kruse and Arnold(14) at energies of negative pions 130 and 152 MeV.

T a b l e VI.

The Expansion Coefficients $a + b\cos\theta + c\cos^2\theta$ and the Errors Matrices

Energy in MeV	Expansion coefficients $a + b\cos\theta + c\cos^2\theta$ $\left(10^{-27} \frac{cm^2}{sterad}\right)$	Matrices of errors $G_{ij}^{-1} \cdot 10^{-4}$ $\left(10^{-27} \frac{cm^2}{sterad}\right)^2$
128	1.00 ± 0.08 $\left\{ \begin{matrix} a = 0.55 \pm 0.07 \\ b = 0.34 \pm 0.12 \\ c = 1.30 \pm 0.24 \end{matrix} \right.$ $M = 5.8$ $M_0 = 5$	$\begin{matrix} 53.53 & -3.73 & -118.15 \\ -4.1\% & 153.65 & 86.48 \\ -66.3\% & 28.6\% & 593.87 \end{matrix}$
162	1.00 ± 0.06 $\left\{ \begin{matrix} a = 0.93 \pm 0.07 \\ b = 0.51 \pm 0.12 \\ c = 2.28 \pm 0.22 \end{matrix} \right.$ $M = 5.2$ $M_0 = 5$	$\begin{matrix} 49.78 & 0.76 & -101.65 \\ 0.9\% & 140.28 & 39.35 \\ -64.8\% & 14.9\% & 494.60 \end{matrix}$

e/ Determination of the real part of the forward scattering amplitude. The differential cross sections for scattering in the forward direction $\frac{d\sigma}{d\Omega}(0) = a + b + c$ at energies of 128 and 162 MeV are equal correspondingly to $(2{,}20 \pm 0{,}32)\, 10^{-27}$ cm²/sterad. and $(3{,}73 \pm 0{,}32) \cdot 10^{-27}$ cm²/sterad. The indicated errors besides the mean square deviations of the coefficients a, b, and c take into account the correlation between the coefficients (the off-diagonal elements of the error matrices). The magnitude of the real part of the forward scattering amplitude $\mathrm{Re} f(0,\omega)$ can be easily obtained if we make use of the optical theorem

$$\mathrm{Im}\, f(0,\omega) = \frac{k}{4\pi}\sigma_t , \tag{8}$$

where σ_t is the total cross section for $\pi^- p$ -interaction.
Then

$$\mathcal{D}_-^{\ell} = \mathrm{Re} f(0,\omega) = \sqrt{\frac{d\sigma}{d\Omega}(0) - \frac{k^2}{16\pi^2}\sigma_t^2} \tag{9}$$

The quantities σ_t were taken from the curve of the energy dependence of the total cross sections which was obtained by Klepikov, Mescheryakov and Sokolov (22) by means of an analysis of all the experimental data of the total cross sections for $\pi^{\pm}p$ and $\pi^{-}p$-interaction in a wide energy interval:

$$\sigma_t\ (128\ \text{MeV}) = (39{,}7 \pm 1{,}0)\, 10^{-27}\ \text{cm}^2$$

$$\sigma_t\ (162\ \text{MeV}) = (63{,}3 \pm 1{,}0)\, 10^{-27}\ \text{cm}^2$$

Then the real parts of the forward scattering amplitude in the center-of-mass-system (in the units of $\hbar/m_\pi c$) are equal to:

$$\mathcal{D}_-^{b}\ (128\ \text{MeV}) = 0{,}261 \pm 0{,}031$$

$$\mathcal{D}_-^{b}\ (162\ \text{MeV}) = 0{,}216 \pm 0{,}038$$

A greater relative error in the magnitude $\mathcal{D}_-^{b}$ (162 MeV) appears because the formula for calculating the error in D contains the quantity D in the dominator and, therefore, by approaching the energy to a resonance one when $\mathcal{D} \to 0$ the relative error increases infinitely.

VI. DISCUSSION

In the course of performing the present experiment some theoretical papers appeared(23 – 31) in which possible causes of discrepancy discovered by Puppi and Stanghellini were analysed. Zaidi and Lomon(23) have shown a sharp dependence of the quantity $\mathcal{D}_-^{b}$ upon the shope of the curve showing the total cross sections for π^-p-interaction near the resonance. A detailed analysis of the errors arising when the dispersion relations are applied to π^-p-scattering was made by Hamilton(25) and Schnitzer and Salzman(28). These authors have shown that by using the dependence of the total cross sections upon the energy obtained by Anderson(32) one succeed in decreasing the discrepancy between the experimental and theoretical values of $\mathcal{D}_-^{b}$ as much as approximately two times. Apart from this, an account of the normalization factor in the differential cross sections and of the correlation between the coefficients a, b, and c leads to the increase of the errors in the experimental values of $\mathcal{D}_-^{b}$ at 150 and 170 MeV approximately two times if compared with that indicated by Puppi and Stanghellini. In spite of this there still remained a small discrepancy. To clear up this discrepancy it was necessary to make the experimental data on the total cross sections near the resonance more exact and to obtain new values of $\mathcal{D}_-^{b}$ (28–30).

Most recently Klepikov, Mescheryakov and Sokolov(22) have calculated the curve $\mathcal{D}_-^{b}(\omega)$ for $f^2 = 0.08$ using new data on the total cross sections in the wide energy range including the results of the measurements $\sigma_t(\pi^-p)$ with an accuracy of 2-3% made by Zinov, Konin, Korenchenko and Pontecorvo(33) in the energy range of negative pions 160-330 MeV. In Fig. 9 the curve obtained by Klepikov et al. is a solid curve, a dashed curve is that by Schnitzer and Salzman(28) for $f^2=0.08$. The quantities $\mathcal{D}_-^{b}$ obtained in the present paper are denoted by black points. Besides, in Figure are given the results of the latest papers by Barnes et al.(34) (41.5 MeV), Edwards et al.(11)

(98 MeV) and Kruse and Arnold[14] (130 and 152 MeV), as well as the values $\mathcal{D}_-^b$ * at 150 and 170 MeV[3], the magnitudes of the errors of which were recalculated by Schnitzer and Salzman[28]. As is seen from Fig. 9 the results of our experiments and the data of all, the latest experimental papers on elastic $\pi^- - p$ -scattering in the energy range up to the resonance are in a quite good agreement with the new theoretical curve calculated with the coupling constant f^2=0.08.

Thus, at present one may consider that the experimental data on elastic scattering of negative pions by protons also agree with the dispersion relations at f^2=0.08 as it was established earlier for scattering of positive pions..

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* It should be noted, that the magnitudes of $\mathcal{D}_-^b$ at 150 and 170 MeV were calculated[3] with somewhat underestimated total cross sections which were measured in[5]. The use of new data for $\sigma_+(\pi^- p)$ will lead to the decrease of these magnitudes.

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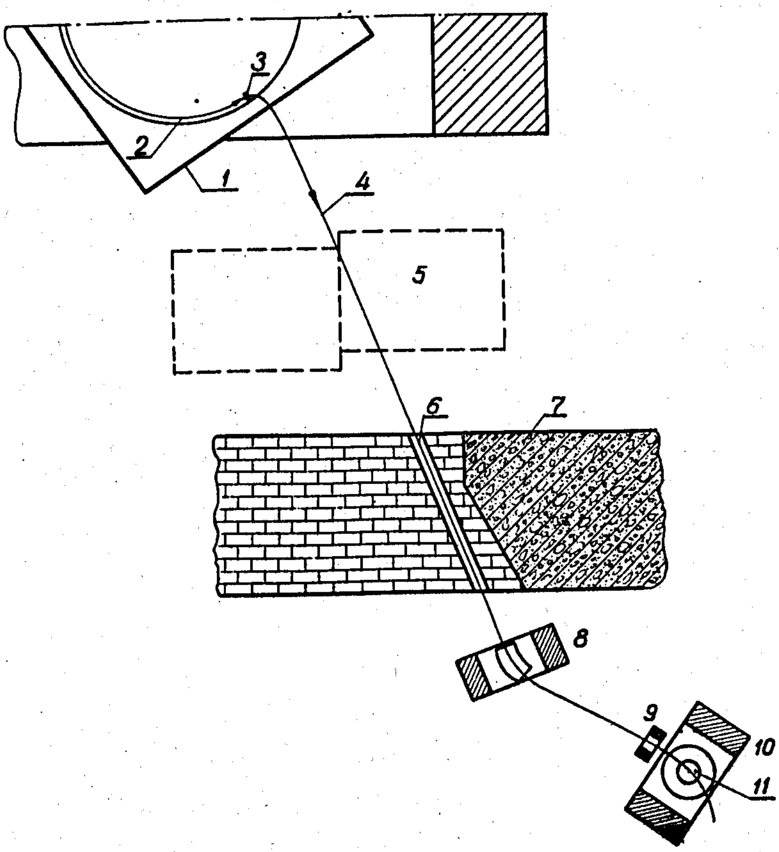

Fig. 1. An experimental arrangement.

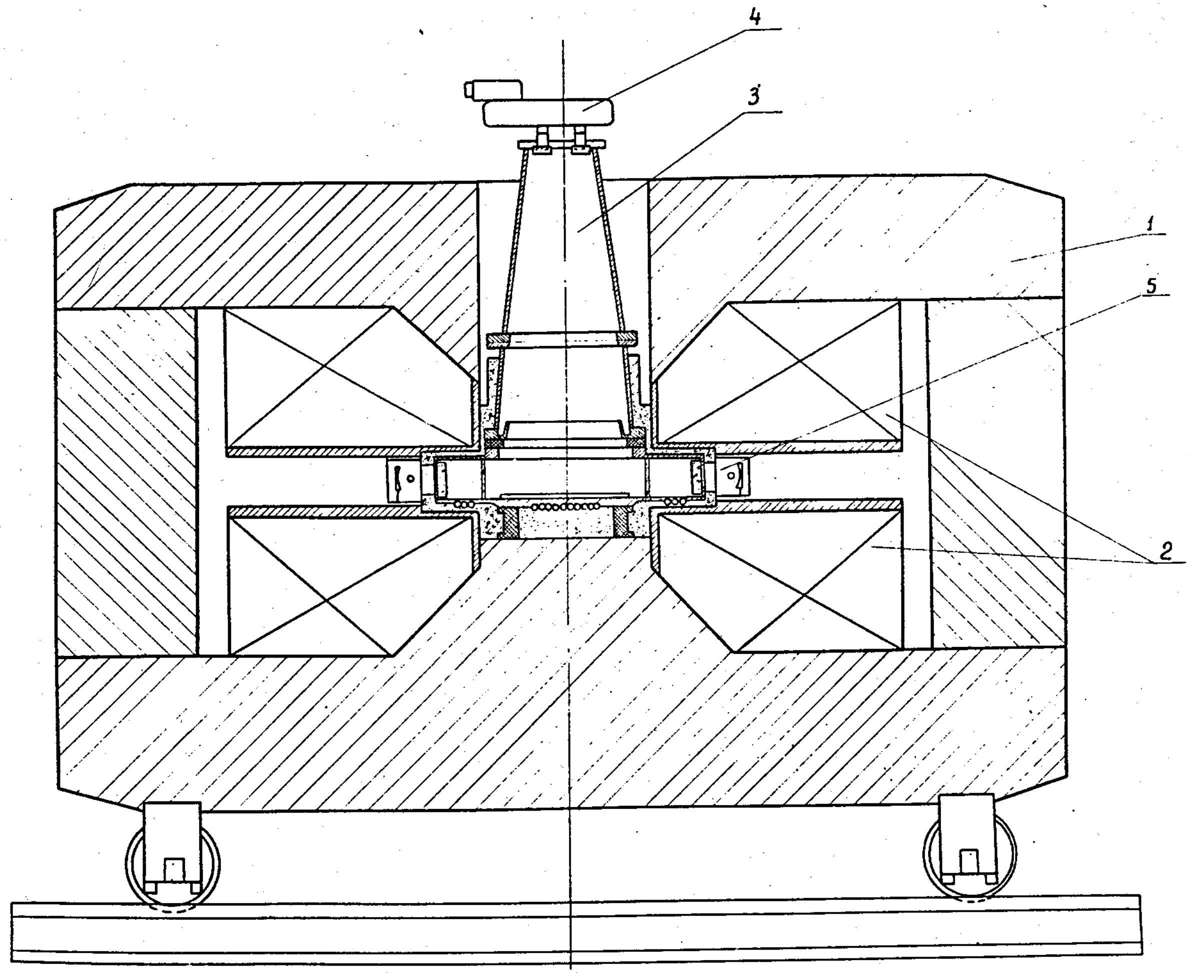

Fig. 2. A scheme of diffusion cloud chamber situated in the magnet-solenoid MC-4A.
1- an electromagnet yoke, 2- coils, 3- diffusion cloud chamber, 4- stereophotocamera, 5 illuminators.

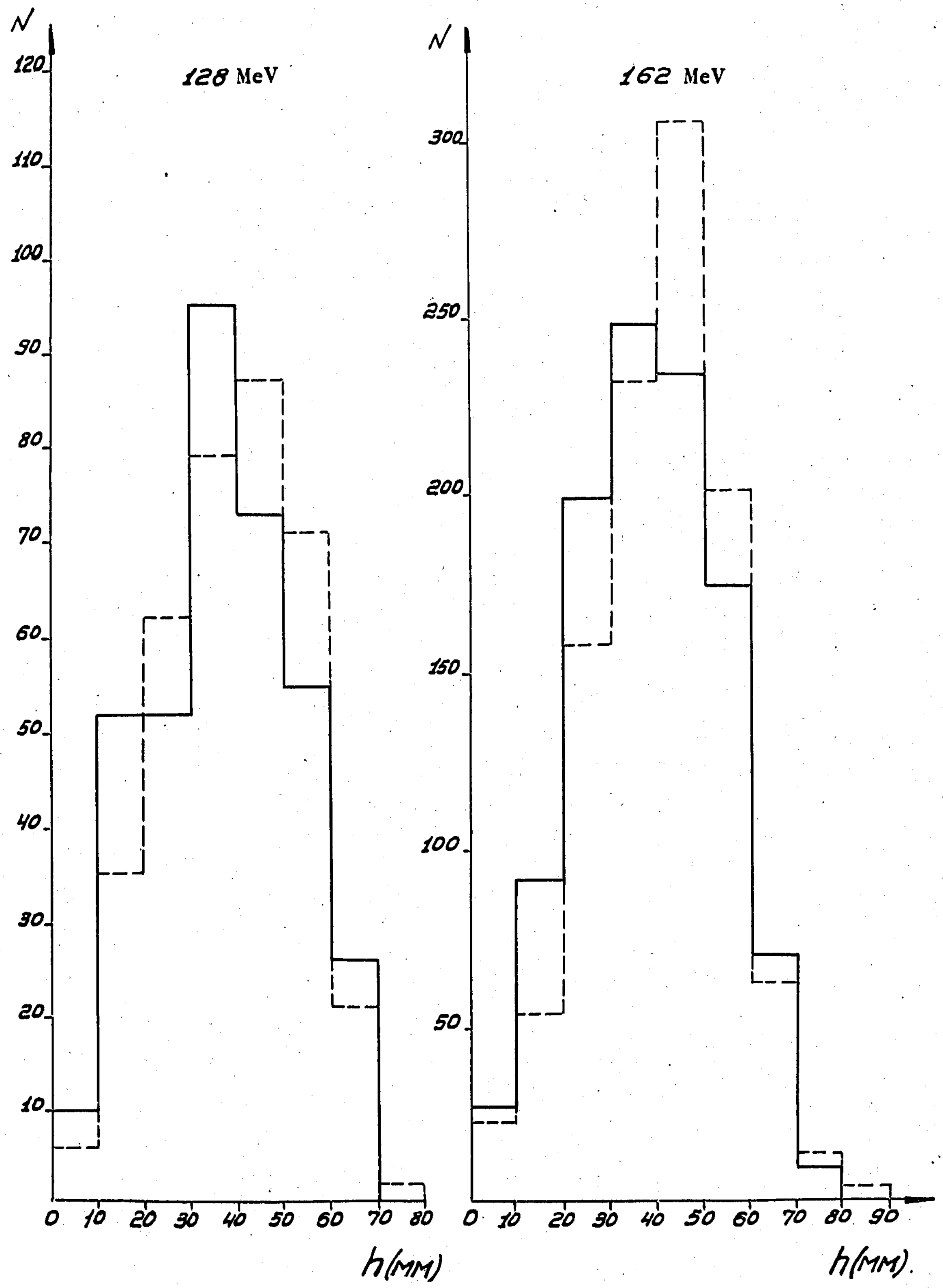

Fig. 3. Distributions by the height of the sensitive layer of elastic scattering events (solid curve) and of the same number of negative pion accidental tracks (a dashed line).

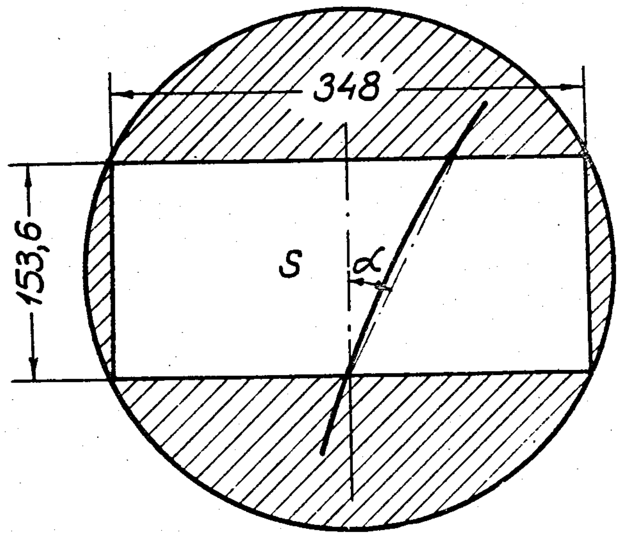

Fig. 4. The region S separated in the chamber for counting the total cross section of elastic scattering.

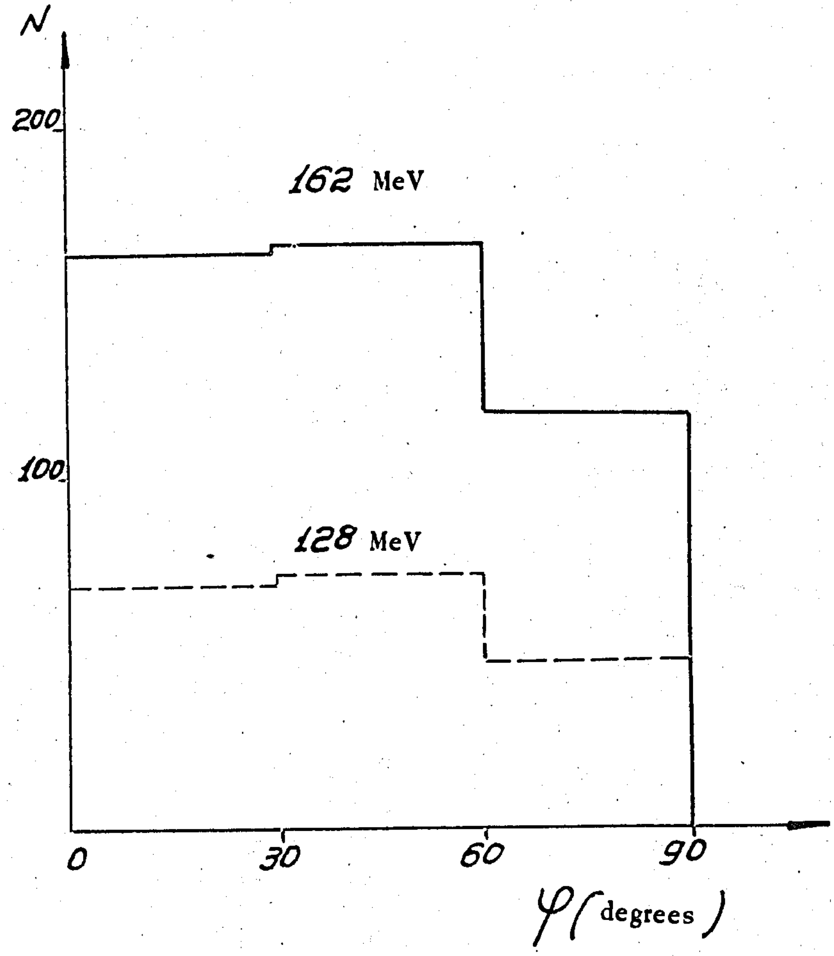

Fig. 5. Distribution by the azimuthal angle φ of the scattering events situated in the region S.

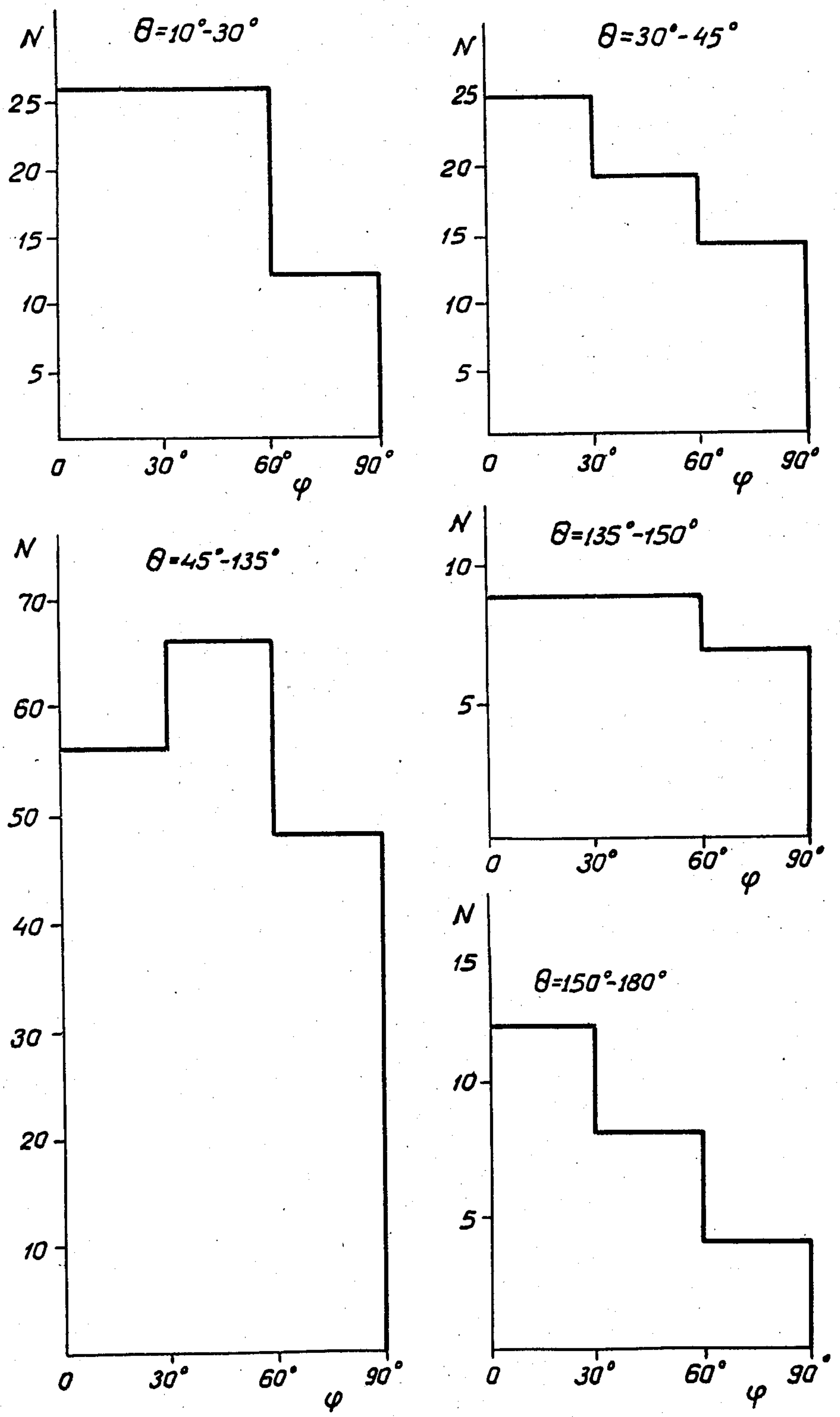

Fig. 6. Distributions by the azimuthal angle φ of the scattering events at 128 MeV for different angular intervals $\Delta\Theta$

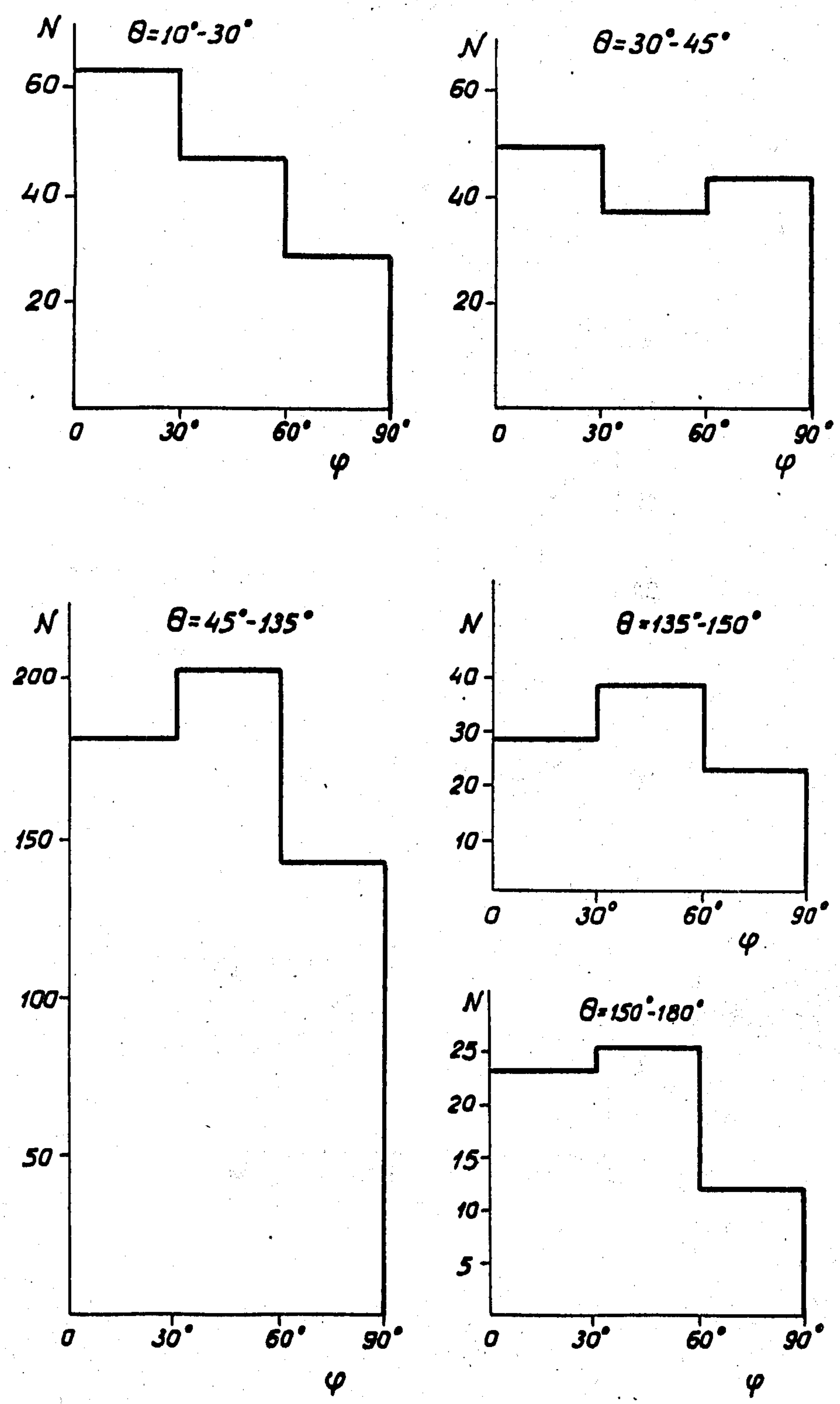

Fig. 7. Distributions by the azimuthal angle φ of the scattering events at 162 MeV for different angular intervals $\Delta\theta$

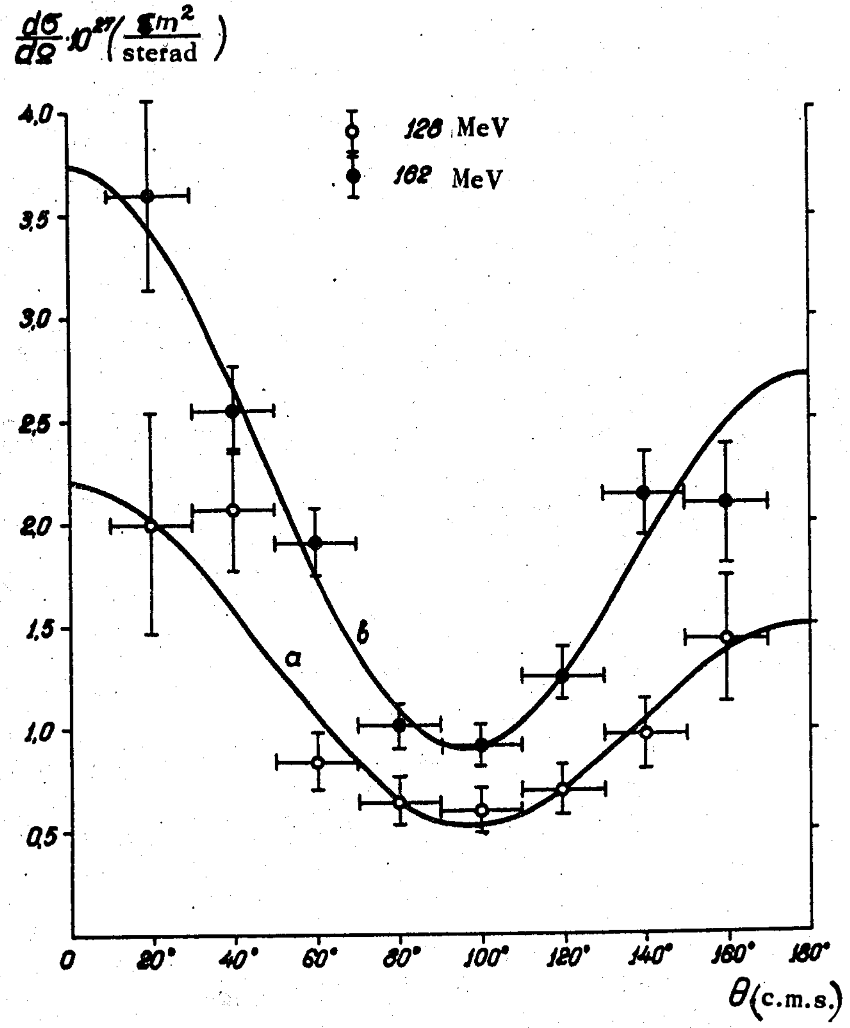

Fig. 8. Angular distributions of elastic $\pi^- p$-scattering at 128 and 162 MeV. The curves a and b are obtained by the least squares with account of the S and P - waves only.

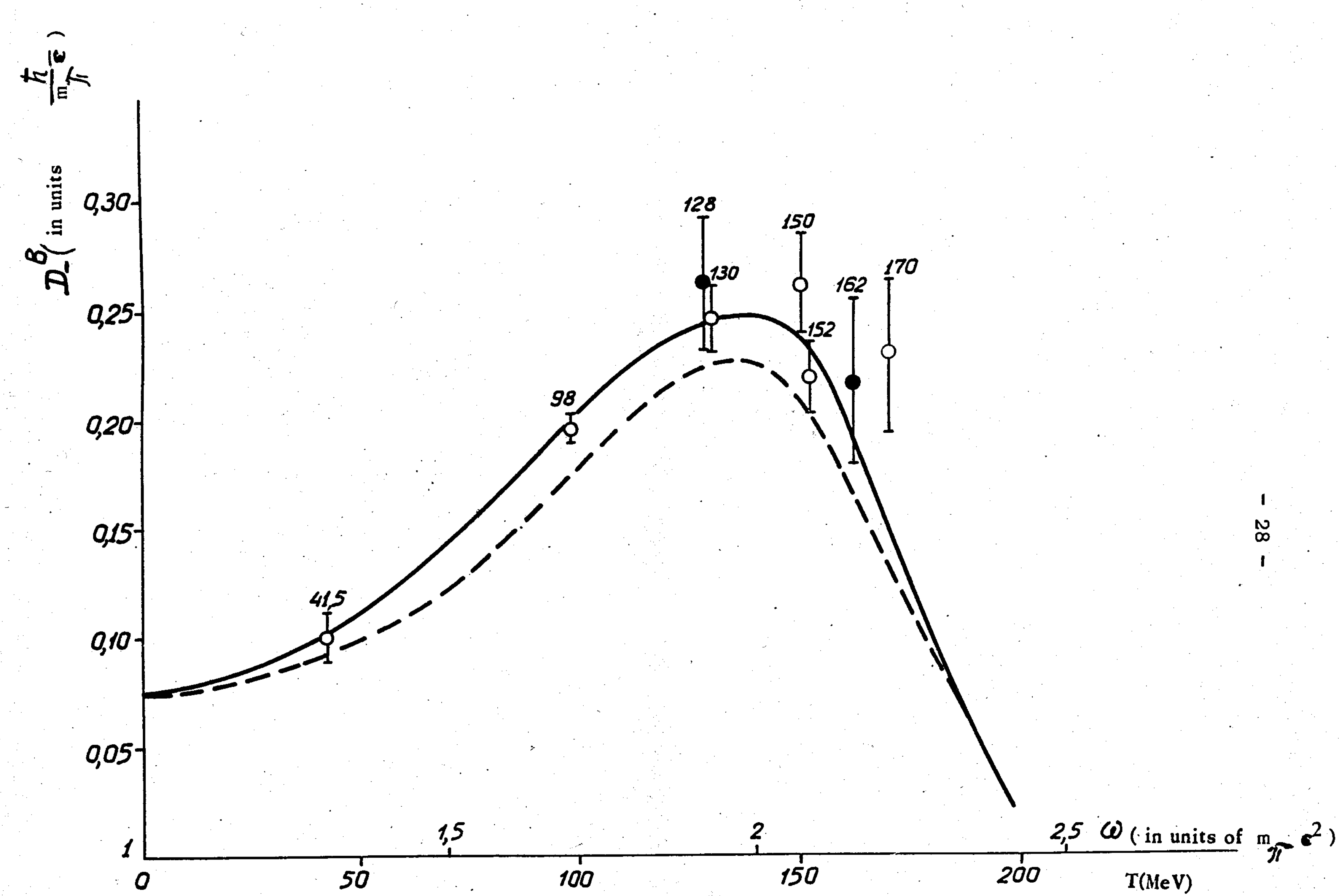

Fig. 9. The dependence of the real part of the forward scattering amplitude $\mathcal{D}_-^{b}$ upon the energy (ω is the total energy of negative pions in the units $m_\pi c^2$, T is the kinetic energy of negative pions in MeV). The solid curve is that obtained by Klepikov et al[22] ($f^2 = 0.08$), a dashed line is that obtained by Schnitzer and Salzman[28]($f^2 = 0.08$).