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P-400

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ЖЭТФ, 1959, т37, в6, с1758-1763.

Dubna 1959

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I. Introduction

Recent theoretical and experimental study resulted in the almost unambigious proof of the $V-A$ form of a weak interaction[1-3]. Basing upon the approximate equality of μ - decay constant G_{μ} and the Fermi constant G_F of β - decay Feynman and Gell-Mann have pointed out a fundamental analogy between the nuclear matrix element in the vector part of the weak interaction and electromagnetic nucleon current. As Gell-Mann showed in his later [4] work this lead to the additional term in the effective interaction Hamiltonian for β decay ("weak magnetism"). The experimental confirmation of the corrections connected with such a term would constitute a proof of the hypothesis. The corrections to the β - decay of nuclei which are connected with the Gell-Mann and Feynman hypothesis have been discussed in other papers[5-6] as well.

In the present paper the effect of strong interactions upon the most elementary example of β - decay, i.e. neutron decay is considered. The phenomenological approach to inclusion of strong interactions results as is well known[7], in the case of the $V-A$ interaction in the effective Hamiltonian described by six form-factors which depend (due to the locality of the weak interactions) only on the square of the four-dimensional momentum transfer. If from all these terms we take into account only terms of the order of $\frac{m}{M}$ (m and M being masses of electron and nucleon respectively) and neglect terms of higher order it turns out that

1) the dependence of the form-factors upon the four-momentum transfer is disregarded and we shall treat them as constants.

2) in addition to the ordinary constants of β - decay G and λ the theory involves only a oonstant corresponding to the "weak magnetism". The remaining terms appearing in the amplitude turn out to be of the second order and should be omitted.

It is essential that the numerical value of a new constant is determined by the anomalous magnetic moments of nucleons. Therefore, the final formulae in this approximation does not contain any unknown parameters.

The final modifications of the usual formula include the effect of the "weak magnetism" as well as the proton recoil. Although these corrections are not high (about 0,1 percent of isotropic part of the decay probability) in certain cases (electron-neutrino correlation and up-down asymmetry for electron) they reach a few percent of the unisotropic parts and can be measured, as long as the accuracy of the experiments will be improved by several times. Such measurements are of great importance since they provide the test of the universal theory of weak interactions free from any assumptions on the nucleus properties. It is worth to emphasize the difference between the test of the hypothesis of $V-A$ interaction and Feynman-Gell-Mann electrodynamic analogy. The latter gives the numerical value of the effect of the "weak magnetism" while the first defines only the form of the amplitude. The investigation of the neutron decay gives a sufficient information for testing both hypothesis.

It should be emphasized that the experimental study of the effects discussed here is at present difficult since the exact value of the ratio of two constants of β - interaction λ is unknown ($\lambda = 1,25 \pm 0,04$ according to data available up to now). Therefore, the quantitative study of weak magnetism corrections may be performed either simultaneously with more accurate measurement of λ (for example, by measuring simultaneously two effects - electron-neutrino correlation and decay asymmetry of a polarized neutron) or by measuring the dependence of the above effects upon the electron energy.

The experimental proof of the theory gives us the means for the investigation of more intimate features of the weak interaction, in particular, it will be possible to get information on the ratio of the constants of the interaction for μ- decay and for β - decay including different corrections due to the interaction with the radiation and π - mesons fields. This problem was discussed in the work of Berman[8] ; .

It should be noted that until the influence of the above mentioned effects on the β - decay will be investigated theoretically in detail it is impossible to say about a rigirous test of $V-A$ theory by studying the β - decay of nuclei.

2. Matrix element

Following the Feynman and Gell-Mann theory the β - decay Hamiltonian is represented by the product of leptonic and nucleonic "weak currents". The effect of strong interactions is described by means of form-factors upon the four-momentum transfer. The most general expression for the matrix element is of the form

$$S = (2\pi)^4 i\,\delta(n-p-e-\nu)\left(\frac{m\, m_\nu\, M_p\, M_n}{e_0\, \nu_0\, p_0\, n_0}\right)^{1/2} R \qquad (1)$$

where

$$R=\frac{G}{\sqrt{2}}(\bar{u}_e\gamma_\mu(1+\gamma_5)u_\nu)(\bar{u}_p(a_1\gamma_\mu+a_2\sigma_{\mu\nu}K_\nu+ia_3K_\mu+b_1\gamma_\mu\gamma_5+b_2\sigma_{\mu\nu}\gamma_5K_\nu+ib_3\gamma_5K_\mu)u_n)$$

Here n and p are the 4-momenta of neutron and proton, ν and e are the 4-momenta of antineutrino and electron, a_i and b_i are some functions of the square of momentum transfer $K=p-n,\ p_o=-ip_4,\ n_o=-in_4$ etc and $\sigma_{\mu\nu}=\frac{1}{2i}(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)$. We use the invariant normalization of spinors $\bar{u}u=\pm 1$ introducing thus appropriate factors in (1). Note also that for the invariant normalization of the neutrino spinor u_ν we introduced the mass of a neutrino m_ν which was put equal to zero in the final formulae.

The invariance of the interaction under time reversal results in the relation

$$R_{\alpha\beta}=R_{-\beta\,-\alpha} \tag{2}$$

where the states $(-\beta)$ and $(-\alpha)$ are related to β and α by inversing the sign of spins and of space components of momenta. The unitary conditions for the S matrix in the first order of the weak interaction constant yields

$$R_{\alpha\beta}=R^*_{\beta\alpha} \tag{3}$$

From (2) and (3) follows that the form factors appearing in the amplitude (1) are real

If we restrict ourselves to the consideration of the corrections of order $\sim\frac{m}{M}$ then the expression (1) is essentially simplified. The reason may be explained as follows:

1. G - invariance [9-10] of the strong interactions gives

$$a_3=b_2=0 \tag{4}$$

If we do not put as in the usual theory $\Delta=M_n=M_p=0$ then $\frac{a_3}{a_1}$ and $\frac{b_2}{b_1}$ are not really zero as it is given by (4), but are of the order of $\frac{\Delta}{M}$ (this is also confirmed by perturbation calculations). However these ooefficients appear in the matrix elements together with quantities of the same order Δ, therefore they give the contribution $\left(\frac{\Delta}{M}\right)^2$ and may be omitted.

2. The term with b_3 is $\sim\left(\frac{\Delta}{M}\right)^2$ as well because of the vector K_μ and the matrix element of γ_5.

3. One may neglect the K^2 -dependence of the form-factors (see [7]). It is seen from the expansion of the form factors (for example, for a_1) in the form $a_1(K^2) = 1 - \frac{1}{6} r^2 K^2$ which differs from unity only by the second order term. (r - characterizes the nucleon size).

4. Hence, in the approximation linear in $\frac{\Delta}{M}$, the matrix element contains only one additional term caused by strong interactions (a_2). In accordance with the Feynman and Gell-Mann hypothesis a_2 is to be replaced by $-\frac{\mu^0}{2M}$, μ_0 stands for the difference between anomalous magnetic moments of proton and neutron.

The final expression for R becomes

$$R = \frac{G}{\sqrt{2}} (\bar{u}_e \gamma_\mu (1+\gamma_5) u_\nu)(\bar{u}_p (\gamma_\mu - \frac{\mu_0}{2M} \sigma_{\mu\nu} K_\nu + \lambda \gamma_\mu \gamma_5) u_n) \tag{5}$$

where λ is the ratio of $G.T.$ and F constants.

In our calculations we use the covariant density matrix formalism /11-12/. The initial density matrix is

$$\rho_0 = \frac{1}{2} (1 + i \gamma_5 \gamma_\mu \xi_\mu^n) \Lambda_+ (n) \tag{6}$$

Here $\Lambda_+(n) = \frac{n_\mu \gamma_\mu + i M_n}{2 i M_n}$ is the projection operator and $\xi_\mu^n = S_p\, i \gamma_5 \gamma_\mu \rho_0$ is the 4-pseudovector of the neutron polarization satisfying the condition

$$\xi_\mu^n \cdot n_\mu = 0 \tag{7}$$

The degree of polarization is defined as $\sqrt{\xi_\mu^n \xi_\mu^n}$.

The density matrix of the final state equals

$$\rho = \rho_{\alpha\beta}^L \rho_{\alpha\beta}^N \tag{8}$$

where

$$\rho_{\alpha\beta}^L = -\Lambda_+(e) O_\alpha \Lambda_-(\nu) \bar{O}_\beta \Lambda_+(e)$$

$$\rho_{\alpha\beta}^N = \Lambda_+(p) N_\alpha \rho_0 \bar{N}_\beta \Lambda_+(p)$$

The notations are

$$\bar{O}_\beta = \gamma_4 O_\beta^+ \gamma_4 ; \quad \bar{N}_\beta = \gamma_4 N_\beta^+ \gamma_4$$

and

$$O_\alpha = \gamma_\alpha (1+\gamma_5) ; \quad N_\alpha = \frac{G}{\sqrt{2}} (\gamma_\alpha - \frac{\mu_0}{2M} \sigma_{\alpha\rho} K_\rho + \lambda \gamma_\alpha \gamma_5)$$

It is worth to note that the matrices appearing in $\rho^{L}_{\alpha\beta}$ affect the spin variables of leptons while the matrices appearing in $\rho^{N}_{\alpha\beta}$ affect the spin variables of nucleons. We temporarly have omitted in the expression (8) for the final density matrix the common factor written explicity in (1). We shall add this to final expressions .

We conclude with the remark that our definition of polarization differs from the usual one by the factor $\frac{m}{E}$ connected with the covariant normalization of spinors. With this normalization we get for the electron polarization $-\vec{\beta}$, instead of the usual $-\frac{\vec{p}}{m}$.

3. Electron-Neutrino Correlation

Let us discuss the observable effects. We start with the electron-neutrino correlation in the decay of a unpolarized neutron. This problem reduces to the calculation of $S_{\rho\rho}$ for $\xi^{n}_{\mu}=0$ and the density of the final states in the approximation mentioned above. Dropping all the terms $\left(\frac{\Delta}{m}\right)^2$ and higher we get for the density of final states the following expression

$$\frac{2}{(2\pi)^5}\left(1+\frac{3(E-p\cos\theta)-\Delta}{M}\right)pE(E_o-E)^2\,dE\,d\Omega \tag{9}$$

where $d\Omega=2\pi\sin\theta\,d\theta$ θ - is the angle between the directions of the emission of electron and neutrino, E and p are the total energy and the momentum of the electron, E_o is the maximum energy of the electron which is

$$E^{o}=\Delta-\frac{\Delta^2-m^2}{2M}$$

The calculation of the expression for the electron-neutrino correlation function rather tedious and we give only the final result

$$dw(E,\cos\theta)=A(E)(1+\beta a(E)\cos\theta+\beta^2 b(E)\cos^2\theta)\,dE\,d\Omega \tag{10}$$

where β is the electron velocity

$$A(E)=\frac{2G^2}{(2\pi)^4}pE(E_o-E)^2\Big\{1+3\lambda^2+\frac{E}{M}[3(1+3\lambda^2)+4\lambda\mu]$$
$$-\frac{m^2}{ME}[1+\lambda^2+2\lambda\mu]-2\frac{\Delta}{M}\lambda(\lambda+\mu)\Big\}.$$

$$a(E) = \frac{1-\lambda^2}{1+3\lambda^2} + \frac{1}{(1+3\lambda^2)^2}\Big\{\frac{\Delta}{M}4\lambda(1+\lambda^2)(\lambda+\mu) + \frac{m^2}{ME}(1-\lambda^2)(1+\lambda^2+2\lambda\mu)$$
$$-\frac{E}{M}\left[8\lambda(1+\lambda^2)\mu + 3(1+6\lambda^2+9\lambda^4)\right]\Big\}$$
$$b(E) = \frac{3E}{M}\,\frac{\lambda^2-1}{1+3\lambda^2};$$

and μ the difference between the total magnetic moments of proton and neutron. It is of interest to notice the appearance of the $\cos^2\theta$ term absent in the usual equations for $e-\nu$ correlation. This term is of kinematical nature and is due to the recoil. The values for the corrections to the main unisotropic term are given in the Table I. It is seen that the corrections amounts up to a several percent (see also [4]). The main reason is rather small value of the principle term $1-\lambda^2$ with λ being close to unity.

4. Electron spectrum and the decay probability

Integrating (10) over the angle between electron and neutrino we get the following expression for the spectrum of electrons :

$$dw(E) = \frac{4G^2}{(2\pi)^3}\,pE\,(E_0-E)^2\Big[1+3\lambda^2 - \frac{\Delta}{M}2\lambda(\lambda+\mu) + \frac{E}{M}2(5\lambda^2+2\lambda\mu+1) - \frac{m^2}{ME}2\lambda(\lambda+\mu)\Big]dE \qquad (11)$$

The corrections to the spectrum turn out to be small (the principle effect is given by the term $1+3\lambda^2$), not greater than few tenth of percent. For example, at the electron energies 0,91 Mev ; 1, 11 Mev and 1.29 Mev the total corrections equal respectively 0,3 %, 0,4 % and 0,6 %.

From (11) we obtain the following expression for the total probability of the decay:

$$W = \frac{4G^2}{(2\pi)^3}\Big\{\frac{m^4}{4}\ln\frac{E_0+\sqrt{E_0^2-m^2}}{m}\Big[E_0\times\Big(1+3\lambda^2-2\frac{\Delta}{M}\lambda(\lambda+\mu)\Big) -$$
$$-\frac{1}{2}(1+3\lambda^2)\frac{m^2}{M} - (1+\lambda^2-2\lambda\mu)\frac{E_0^2}{M}\Big] + \sqrt{E_0^2-m^2}\Big[\frac{E_0^5}{30M}(5\lambda^2+2\lambda\mu+1) +$$
$$+\frac{1}{5}\Big(\frac{1}{6}E_0^4-\frac{3}{4}m^2E_0^2-\frac{2}{3}m^4\Big)\times\Big(1+3\lambda^2-2\lambda\frac{\Delta}{M}(\lambda+\mu)\Big)\Big) - \frac{m^2E_0^3}{M}\times \qquad (12)$$
$$\times\Big(\frac{3}{10}\lambda\mu+\frac{1}{15}+\frac{1}{2}\lambda^2\Big) + \frac{m^4E_0}{M}\Big(\frac{23}{24}\lambda^2+\frac{49}{120}-\frac{4}{15}\lambda\mu\Big).$$

The correction to the probability is small and amounts to 0,2 %.

5. Up-down Asymmetry and the Electron Polarization

We get the following equation for the angular distribution of electrons :

$$dW(E,\cos\theta_e) = \frac{1}{4\pi}(\text{Spectrum})\left[1+P\beta\, c(E)\cos\theta_e\right]d\Omega_e$$

where

$$c(E) = \frac{2\lambda(1-\lambda)}{1+3\lambda^2} + \frac{2(\lambda+\mu)}{(1+3\lambda^2)^2}\Big[\frac{\Delta}{3M}(3\lambda^2+2\lambda-1)$$

$$-\frac{E}{3M}(3\lambda^3+9\lambda^2+5\lambda-1) - 2\frac{m^2}{ME}\lambda^2(\lambda-1)$$

p is the neutron polarization and θ_e is the angle between the direction of the neutron polarization and of the electron momentum. From the table 2 one can see that the corrections change the anisotropic terms by several percent which as in the case of electron-neutrino correlation is rather small. Accordingly the relative corrections to the anisotropy of the neutrino emission where the principle term is large turn out to be small.

In conclusion we give the expression for the electron polarization P_e calculated from the amplitude (5) (in the usual normalization) :

$$P_e = -\beta\Big(1+\frac{m^2}{ME}2\lambda\,\frac{\lambda+\mu}{1+3\lambda^2}\Big) \qquad (14)$$

The corrections to the polarization are small and at energies 0,71 Mev, 0,91 Mev and 1,11 Mev amount to 0,14 %, 0,08 % and 0,07 % respectively.

The authors express their thanks to Prof. V.Telegdi for very usefull discussions of the results of this paper.

Received by Publishing Department
on Jule 11,1959.

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Corrections to $(e-\nu)$ - correlation in %

Total electron energy	From the "weak magnetism"	From the recoil
0,71	0,4	1,9
0,91	1,4	2,7
1,11	2,4	3,5
1,29	3,3	4,2

T A B L E   II

Correction to the up down asymmetry for the electron in %

Total electron energy	From the "weak magnetism"	From the recoil
0,71	1,3	0,3
0,91	1,8	0,5
1,11	2,4	0,6
1,29	3,0	0,8

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