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SCATTERING OF PARTICLES AND QUANTUM MECHANICS

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Abstract

As is known one needs not to do all possible polarization experiments for a phase-shift analysis of π-p or p-p scattering. There exists a so-called necessary and complete set of experiments |1,2|. It is shown that the "redundancy" of some remaining experiments is a consequence of the main features of the quantum mechanics formalism (and, principally, is due to the description of the states with wave functions). The establishment of the equality between the calculated (with the help of phase-shifts) and the measured results of the "redundant" experiments will thus constitute a check of the fundamental quantal postulates. Experiments of such a kind have already been performed but their accuracy is to be considerably improved for our purpose.

Introduction

The paper is devoted to the question of testing the fundamental quantal postulates:

$|\alpha|$ Primary and complete characteristic of the physical system state is a wave function which may be represented by a vector in some space. Particularly, the time-development of the characteristics of the system (mean values and so on) is due to the change of its wave function[1/].

$|\beta|$ In the above space there are <u>complete</u> sets of <u>mutually orthogonal</u> vectors-wave functions in which one can expand the wave function of any state. In the case of a spinless particle one of such systems is a system of wave functions describing states with all possible definite momenta.

The above mentioned assumptions are formulated in more details in |3| as "General properties" (properties A-G).

It is known that the Heisenberg uncertainty principle and the wave-particle dualism follow from this formalism. Although the indicated description is not a formalism deduced inevitably from fundamental experiments |4| at present one assumes that the particularities of any experiment can be explained by means of either particular assumptions (for example, on the form of the potential in the Schrödinger equation) within the framework of this formalism. Without suggesting an another mechanics we indicate means for testing the validity of the main properties $|\alpha|$ and $|\beta|$, more accurately, the properties

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1/ If a state is described by the density matrix the weights of separate sharp states under free development of the system do not change. Only the sharp states themselves change.

A-G in [3] (but not the correctness of one or another type of interaction). One may believe such a testing to be reasonable at high energies of scattered particles.

Since this test deals with the most general assumptions of the quantum theory it must, strictly speaking, precede the test of dispersion relations, for example, (which use the so-called "local properties", see [3]).

The idea of this paper may be summarized as follows (see also the abstract): "If the phase-shift analysis is not possible the quantum mechanics is not correct". This assertion is illustrated in §1, in §2 concrete experiments are discussed.

§ 1. Scattering of spinless particles

From | α |, | β | and from the laws of conservation follows that the angular distribution $\sigma(\vartheta)$ of scattered particles has to be equal to the square of the module of a some complex function $f(\cos\vartheta)$ possessing the unitarity property [5]

$$Jm\, f(\cos\vartheta_0) = \frac{K}{4\pi}\iint d\cos\vartheta\, d\varphi\; f^*(\cos\vartheta)\, f(\cos\vartheta\cos\vartheta_0 + \sin\vartheta\sin\vartheta_0\cos\varphi) \qquad |1.1|$$

if there are no other channels of the reaction $a+b\to$ besides the elastic scattering $a+b\to a+b$.

The two equations $\sigma(\vartheta) = |f(\vartheta)|^2$ and the non-linear integral equation (1.1) (for the function $f(\vartheta)$) may have no solution for any kinds of function $\sigma(\vartheta)$. For example, one of the conditions of its solvability is

$$\sigma(0^\circ) \geqslant K^2/(4\pi)^2 \cdot \sigma^2 \qquad |1.2|$$

following from |1.1| when $\vartheta_0 = 0$ (optical theorem): $Jm\, f(0^\circ) = \frac{K\sigma}{4\pi}$ and from $\sigma(0^\circ) \geq [Jm\, f(0^\circ)]^2$. Here σ is the total cross section of the elastic scattering. It is clear that |1.2| is not fulfiled for any function $\sigma(\vartheta)$ *.

The general considerations of the conditions of solvability of Eq. |1.1| and $\sigma(\vartheta) = |f(\vartheta)|^2$ can be performed by using the way of solution of Eq. |1.1| which is based on the well-known expansion of $f(\vartheta)$ in Legendre polinomials (phase-shift analysis).

The function $\sigma(\vartheta)$ m[illegible] be viewed as a representation (description) of experimental data. The fact that it depends on only one angle ϑ (and on the energy of scattered particles) follows from the law of conservation of total momentum and angular momentum (but does not follows from the quantum-mechanical approach, see appendix A). $\sigma(\vartheta)$ as any

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* Of course, it is not possible to measure the cross section accurately at the angle $\vartheta = 0^\circ$. But if the particles are identical (for example, in the scattering reactions $He + He \to He + He$ or $\pi^+ + \pi^+ \to \pi^+ + \pi^+$) then $\sigma(0^\circ) = \sigma(180^\circ)$.

function of ϑ, can be expanded in the complete set of functions of ϑ, for example, in Legendre polinomials:

$$\sigma(\vartheta) = \sum_{L=0}^{\infty} P_L(\cos\vartheta)\, B_L \tag{1.3}$$

If the electromagnetic interactions can be neglected $\sigma(\vartheta)$ is represented in the form of a finite sum over L (consequence of short-rangeness of the interaction). This may be assumed as a generalization of experimental data. Let us neglect all B_L for $L \geq 3$ *. If the scattering may be described by the quantum mechanics, then (see for example [6]):

$$B_L = \frac{1}{K^2} \sum_{\ell=0}^{\infty} \sum_{\ell'=|\ell-L|}^{\ell+L} (2\ell+1)(2\ell'+1) \left[C^{L0}_{\ell 0 \ell' 0} \right]^2 \sin\delta_\ell \sin\delta_{\ell'} \cos(\delta_\ell - \delta_{\ell'}) \qquad |1.4|$$

From $0 = B_3 = B_4 = \ldots$ follows that all the phaseshifts are equal to zero excepting δ_0 and δ_1. The three coefficients B_0, B_1, B_2 are expressed in terms of this two real phases. Using the known values of B_0 and B_2, for example, we can find two possible values for B_1. If the experimental value of B_1 coincides with no one of them, $|\alpha|$ and $|\beta|$ are not correct.

We do not suggest concrete experiments on the spinless particle scattering. The reason may be explained as follows: one might to carry out for the present only the reactions such as scattering of π-meson by helium, by carbon and so on and such as $He + He \to He + He$. However the scattering of the first type at any energies have without fail another channels (for example, $\pi^- + He \to T + n$) and the unitarity condition |1.1| becomes more complicated. The combined study of several channels is not a simple experiment (note that |1.2| holds true in the presence of other channels if σ means the sum of total cross sections of all the channels, including the scattering). The reaction $He + He \to$ has no other channels until the energy of incident α-particle exceeds 30 MeV (in the laboratory system). However, the majority of the facts confirming the quantum mechanics relates to the small energy region. It is worth to seek discrepancies at energies much higher than the binding energy of the nucleon in the nucleus, for instance.

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* In other words three coefficients B_0, B_1, B_2 calculated with help of the values of $\sigma(\vartheta)$ for three values of the angle ϑ give correct values of $\sigma(\vartheta)$ for several other angles.

§ 2. Scattering of Particles with Spin 1/2

The cross section of scattering of particles with spin 1/2 on spinless particles may be also represented in the form of Eq. |1.3| (see appendix B) but in this case the number of coefficients B is equal to the number of phase-shifts |6|.

$$B_L = \frac{1}{8k^2} \sum_{\ell_1, \ell_2, J_1, J_2} Z^2(\ell_1 J_1 \ell_2 J_2; \tfrac{1}{2} L) \, Re\left[(1-e^{-2i\delta_{J_1\ell_1}})(1-e^{2i\delta_{J_2\ell_2}})\right] \qquad |2.1|$$

in view of the fact that to each orbital momentum ℓ there correspond now two phases with $J=\ell \pm \frac{1}{2}$ (space parity is assumed to be conserved). Therefore $\sigma(\vartheta)$ may be arbitrary (non-negative) function of ϑ .

Let other channels be absent. For instance, let us take the scattering $\pi^+ + p \to \pi^+ + p$ at π^+-meson energies up to 170 MeV (in the laboratory system)*. In fact the contribution of inelastic interaction is small at 300 MeV as well. Then from the unitarity condition follows that the phases $\delta_{J\ell}$ are real and to the measured coefficients B_0, B_1 etc. there correspond several possible sets of phase-shifts, several solutions of the system |2.1|. With their aid one can calculate several possible values for the recoil proton polarization (at some angles ϑ) and compare with the polarization measured. If not a single calculated polarization value $P(\vartheta)$ coincides with the experimental one this would mean that the quantum - mechanical approach to scattering problems was not true**. The accuracy of experiments already made may be characterized by the fact that one can not yet choose definitively one set of phase-shifts (see |7|, the energy of π^+ mesons is 307 MeV and |8| the energy is 312 MeV).

If other channels are essential the phases $\delta_{J\ell}$ are complex. In order to choose either solution we have to compare the measured and calculated values of the asymmetry $e_{3S}(\vartheta)$ in the triple scattering, see |1| ("R - experiment"). However, if there are no polarized hydrogen targets it is impossible to carry out experiment of such a type for the reac-

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* Of course, there is always the channel $\pi^+ + p \to \pi^+ + p + \gamma$. However, its contribution does not exceed 1% at energies up to 300 MeV(see, for ex. |14|). This means that if the deviations from quantum mechanics does not exceed 1% the process is described by quantum mechanics.

** Of course, if this occured, at first it would be necessary to attempt to make a phase-shift analysis with greater number of phase-shifts, i.e. the non-coincidence should be "steady" with respect to various ways of phase shift analysis. The phase-shift analysis may be substituted by any other way of investigation of the solvability of Eqs. |3.2| and |3.4| in |2| analogous to the equations $\sigma(\vartheta)=|f(\vartheta)|^2$ and |1.1| for scattering of the spinless particles.

tion $\pi^{\pm}+p\to\pi^{\pm}+p$. But one may suggest a triple scattering of protons on helium.

For such a type of triple scattering one may propose one more "redundant" for any energies experiment which does not require a phase-shift analysis. We have in mind the asymmetry e_{3n} , see |1|. All the three scattering are performed in the same plane. Let us denote the number of particles scattered in the second scattering to the left and in the third one to the right by $\sigma_{LR}(\vartheta)$, ϑ being the polar angle of the second scattering. The function

$$\frac{1}{4P_1P_3}\left[\sigma_{LL}(\vartheta)+\sigma_{RR}(\vartheta)-\sigma_{LR}(\vartheta)-\sigma_{RL}(\vartheta)\right]. \tag{2.2}$$

must coincide with cross section $\sigma(\vartheta)$ when the beam is unpolarized* (depolarization coefficient D is to be equal to unity, see |1| and also appendix B). P_1 is the magnitude of polarization after the first scattering, P_3 characterizes the third scatterer.

The investigation of the PP scattering is also at present in that stage when only the insufficient accuracy prevents from testing | α | and | β | (yet it is not possible, for example, to choose definitely either set of phase-shifts, see |9,10|). We note that the measured value of the coefficient C_{nn} (correlation of polarizations of the scattered and the recoil protons, all the scattering planes coincide) and the calculated value closist to it differs almost by two standard errors ($0,75 \pm 0,11$ and $0,61 \pm 0,06$ respectively |10|; an another set of phase-shifts yields $C_{nn} = 0,38 \pm 0,08$). If the difference of such a type maintained after improving the accuracy and if it turned out to be steady with respect to various ways of phase-shift analysis this would provide evidence that | α | and | β | fail.

Appendix A. Scattering of Spinless Particles

The conditions and the results of the experiment may be described by the function $\varphi(\vec{p}_1',\vec{p}_2';\vec{p}_1,\vec{p}_2)$ giving the number of particles scattered in the direction of the momentum $\vec{p}_1'$ (corresponding recoil particles have the momentum $\vec{p}_2'$) under the condition that the beam particle momentum was definite and was equal to $\vec{p}_1$ (target particle momentum was $\vec{p}_2$). As regards the coordinates they are in any case not fixed (by accelerator) and are not measured (by counter) with such an accuracy that one may speak about the observation of scattering of a certain beam particle marked by indication of its trajectory on the marked (by the indication of its coordinate) target particle. The experiment makes sense for beam with arbitrarily large section.

In classical, quantum and any another mechanics there ought take place the law of con-

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* As prof. Markov observed this result follows from | α | used for the description of the spin state only (but not the coordinate-momentum one).

servation of total linear and angular momenta and of energy since they follow from the fact that the space (and the time) are expected to be homogeneous and isotropic. Let us introduce instead of $\vec{P}_1$ and $\vec{P}_2$ the total momentum $\vec{P}$ and the relative momentum $\vec{p}$. The law of conservation of total momentum means that $\varphi(\vec{p}', \vec{P}'; \vec{p}, \vec{P})$ is equal to zero, provided $\vec{P}' \neq \vec{P}$. Therefore we omit the indices $\vec{P}'$ and $\vec{P}$ and in the following we assume the process is described in the center of mass system where $\vec{P}' = \vec{P} = 0$. Then, if in the space there are no prefered directions the number of particles scattered in the direction $\vec{p}'$ under the condition that the initial relative momentum was $\vec{p}$ is equal to the number of particles scattered in the direction of $\hat{g}\vec{p}'$ ($\hat{g}\vec{p}'$ is a momentum $\vec{p}'$ rotated around a certain axis about a certain angle) under the condition that the initial relative momentum was $\hat{g}\vec{p}$:

$$\varphi(\hat{g}\vec{p}'; \hat{g}\vec{p}) = \varphi(\vec{p}', \vec{p}) \tag{A.1}$$

(indeed, the second experiment differ from the first one only by the position with respect to the frame of reference). From here follows that φ depends only on the modules of $\vec{p}'$ and $\vec{p}$ (which are equal in virtue of the energy conservation) and on the angle ϑ between them.

If there are no other channels then $\iint d\Omega\, \varphi(\vartheta) = 1$ as a consequence of conservation of number of particles. By subtracting from $\varphi(\vartheta)$ the function describing initial reduced beam (such as $\alpha\, \delta(\vec{p}' - \vec{p})$, $0<\alpha<1$) we obtain the cross section $\sigma(\vartheta)$.

Appendix B. Scattering of Particles with Spin 1/2 by Spinless Particles

The quantum - mechanical description of the spin state is taken. Since our purpose is to verify if the dynamical parameters introduced by quantum mechanics are sufficient for describing experiment or a greater number of parameters is required then this adoption from quantum mechanics simply restricts perhaps the possibilities for testing $|\alpha|$ and $|\beta|$ (the difference from quantum mechanics will consist only in rejecting the assumption that the weights of sharp states do not change with time, see remark [1]).

The arbitrary spin state of the particle with spin 1/2 is described by the Hermitean density matrix involving four elements or so-called polarization tensors, see for ex. |11|. For example, scattered particles are described by the function $\sigma(\vec{p}', \vec{p})$ (which represents their angular distribution) and by functions $P_x'(\vec{p}', \vec{p})$, P_y', P_z' (the projections of their polarization vector). The spin state may be also given by indicating the fraction P of totally polarized particles (1-P will be then a fraction of unpolarized particles). Note that P turns out to be equal to the magnitude of the polarization vector $P = \sqrt{P_x^2 + P_y^2 + P_z^2}$, see [11]. Let us direct the axis z of the coordinate system along the beam polarization vector $\vec{P}$. Introduce the function $W(\vec{p}', \vec{p})$ which represents the angular distribution from unpolarized beam (P=0) and the function $\widetilde{W}_z(\vec{p}', \vec{p})$ which represents the angular distribution in the case of totally polarized beam (P=1). If $0<P<1$ we have;

$$\begin{aligned} \sigma(\vec{p}', \vec{p}) &= W(\vec{p}', \vec{p})(1-P) + \widetilde{W}_z(\vec{p}', \vec{p})P \equiv \\ &\equiv W(\vec{p}', \vec{p})\cdot 1 + W_z(\vec{p}', \vec{p})\cdot P \end{aligned} \qquad |B.1|$$

The linear dependence of $\sigma(\vec{p}', \vec{p})$ on the beam polarization vector magnitude is a consequence of the obvious fact that the number of scattered particles is proportional to the number of incident particles of either kind (polarized or unpolarized).

Let us denote the values of x, y and z - components of the polarization vector of scattered particles when the beam is unpolarized, by $W^x(\vec{p}', \vec{p})$, W^y and W^z correspondingly. Analogously $\widetilde{W}_z^x(\vec{p}', \vec{p})$, $\widetilde{W}_z^y$ and $\widetilde{W}_z^z$ denote the corresponding components in the case when P = 1. If $0<P<1$ we have

$$P_k' = W^k(1-P) + \widetilde{W}_z^k P \equiv W^k\cdot 1 + W_z^k\cdot P \; ; \quad k = x, y, z \qquad |B.2|$$

In the other coordinate system (the axis z is usually taken along initial momentum $\vec{p}$) the beam polarization vector will have the projections P_i : $P\cdot\delta_{j,z} = \sum_i g_{j,i} P_i$ where $g_{j,i}$ is the matrix of the corresponding space rotation of the coordinate system. We introduce new functions $W_i(\vec{p}', \vec{p}) = W_z(\vec{p}', \vec{p}) g_{z,i}$ and $W_i^k = W_z^k g_{z,i}$ Let us introduce also instead of the Decart projections of polarization vector the cyclic ones

$$\rho_{1,-1} = (P_x + iP_y)/\sqrt{2} \quad , \quad \rho_{1,0} = P_z \quad , \quad \rho_{1,+1} = -(P_x - iP_y)/\sqrt{2} \qquad |B.3|$$

Now |B.1| and |B.2| can be combined into the relation

$$\rho'_{q'\tau'}(\vec{p}', \vec{p}) = \sum_{q,\tau} W_{q\tau}^{q'\tau'}(\vec{p}', \vec{p})\rho_{q\tau}(\vec{p}) \quad , \quad \tau' = -1, 0, +1 . \qquad |B.4|$$

where $\rho'_{00}(\vec{p}', p) \equiv \sigma(\vec{p}', \vec{p})$, $\rho'_{1\tau'}$ are cyclic projections of $\vec{P}'$; $\rho_{00}(\vec{p}) = 1$ if the flux density of incident particles equals 1 particle/sm²sec. and there is one target particle.

From the hermiticity of the density matrix (or from the reality of Decart projections of the polarization vector) follows that $\rho_{q,\tau}^* = (-1)^\tau \rho_{q,-\tau}$. Therefore,

$$\left[W_{q,\tau}^{q',\tau'}(\vec{p}',\vec{p})\right]^{*}=(-1)^{\tau'+\tau}\,W_{q,-\tau}^{q',-\tau'}(\vec{p}',\vec{p}) \qquad |B.5|$$

In quantum mechanics
$$W_{1\tau}^{1\tau'}=\tfrac{1}{2}\,S_p\,\sigma_{\tau'}\,R\,\tilde{\sigma}_{\tau}\,R^{\dagger} \qquad |B.6|$$
where R is the transition matrix (see for example, [12] and also [1]).

We have showed that |B.4| take place also when $W_{q\tau}^{q'\tau'}$ is not represented in such a form.

We agree that futher the projections τ are referred to the system of axis A with the axis $z_A \parallel \vec{p}$ and the axis y_A directed along that beam polarization vector component which is perpendicular to $\vec{p}$, while the projections τ' are referred to the system of axis C with the axis $z_c \parallel \vec{p}'$ and $y_c \parallel [\vec{p}\times\vec{p}']$. If there are no prefered directions in the spaoe then $W_{q\tau}^{q'\tau'}(\vec{p}',\vec{p})$ is to be of the form

$$W_{q\tau}^{q'\tau'}(\vec{p}',\vec{p})=\sum_{J,L',L}\mathcal{D}_{\tau',\tau}^{J}(-\pi,\vartheta,\pi-\varphi)\,C_{q'\tau'L'0}^{J\tau'}\times \qquad |B.7|$$
$$\times(q'L'|W^{J}|qL)\,C_{q\tau L0}^{J\tau}\equiv W_{q\tau}^{q'\tau'}(\vartheta,\varphi)$$

Here $(-\pi,\vartheta,\pi-\varphi)$ are Euler angles of rotation carrying the axis A into the axis C; ϑ and φ are spherioal angles of the vector $\vec{p}'$ with respect to the system A.

If the parity conserves, then (see [12])
$$R(\vartheta,0)\equiv\begin{pmatrix}(+\frac{1}{2}|R(\vartheta,0)|+\frac{1}{2}) & (\frac{1}{2}|R|-\frac{1}{2})\\ (-\frac{1}{2}|R|+\frac{1}{2}) & (-\frac{1}{2}|R|-\frac{1}{2})\end{pmatrix}=\begin{pmatrix}a(\vartheta) & b(\vartheta)\\ -b(\vartheta) & a(\vartheta)\end{pmatrix}$$
where the projections m' and m appearing in the elements $(m'|R(\vartheta,0)|m)$ are related to the system C and A respectively.

The angular distribution from the unpolarized beam is defined by the function $W_{00}^{00}(\vartheta,0)=(|a|^2+|b|^2)$ and the function |2.2| is defined by the sum $W_{1,-1}^{1,-1}(\vartheta,0)+W_{1,+1}^{1,-1}(\vartheta,0)$ (see Eq. |31| in [12]), equal to $|a|^2+|b|^2$

Let us note the fact that the equation $\mathcal{D}\equiv\left(W_{1-1}^{1-1}(\vartheta,0)+W_{1,+1}^{1,-1}(\vartheta,0)\right)/W_{00}^{00}(\vartheta,0)=1$. does not follow from the invariance under spaoe rotarions. Indeed, $W_{1\tau}^{1\tau'}$ is expressed in terms of coefficients $(1L'|W^J|1L)$ while W_{00}^{00} is expressed in terms of other coefficients $(0L'|W^J|0L)=B_L\,\delta_{L',L}\,\delta_{J,L}$, see (B.7).

Let us note that the law of parity conservation may be formulated in terms of polarization tensors (without the help of the transition matrix)as it has been made in [13]. We obtain the same selection rules as in quantum mechanics, see [12] (of course, in quantum mechanics there are some additional stronger parity selection rules). We emphasize that the test of parity conservation suggested in [12] does not depend on the fact whether or not quantum mechanics is consistent.

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