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A SIX-DIMENSIONAL INTERPRETATION OF ELECTRODYNAMICS AND NUCLEAR INTERACTIONS

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Part I. Electrodynamics

1. A Model of the Univers

Our basic idea is that the traditional 4-dimensional manifold is not sufficient to describe satisfactorily the physical reality. In what follows we shall try to show the usefulness of a six-dimensional approach for interpreting, from a uniform viewpoint, several properties of matter regarded hitherto as primary and heterogeneous.

We share the opinion of Eddington that the fundamental problems of microphysics cannot be solved without taking into account some cosmological aspects. Therefore we start with some considerations concerning the general structure of the univers.

Our working hypothesis is that the physical world is essentially symmetric. If we are faced with an asymmetry, then it is our special position in the univers, or the actual distribution of matter in it, but not the fundamental physical laws that should be blamed for it. Now, one of the most conspicuous asymmetries encountered in physics consists in the fact that the number of space-like dimensions is not equal to that of time - like dimensions. According to our working hypothesis this fact is quite unsatisfactory. Therefore we assume that the manifold underlying the physical phenomena is actually six-dimensional with three space-like and three time-like dimensions. The fact that we overlook the fifth and the sixth dimensions is to be ascribed to the actual distribution of matter and to our actual position in the univers.

We assume that the 6-dimensional manifold is riemannian but singular. The inhomogeneities and singularities of the manifold will find a suitable expression in the metic tensor $g_{\mu\nu}$ (Greek indices run from 1 to 6, latin from 1 to 4). We assume that the singular points within the six-dimensional space form two four-dimensional subspaces (hypersurfaces), one of them being characterised by the signature $+---$, the other by the singature $+++-$.

Introduce a system of coordinates X^{μ} in such a way that the first three of them are space-like, the last three are time-like. Let the equations

$$X^1 = A \quad , \quad X^2 = B \tag{1}$$

define the singular hypersurface (X^3, X^4, X^5, X^6) with the signature $+---$, and the equations

$$X^5 = C \quad , \quad X^6 = D \tag{2}$$

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define the other singular hypersurface (X^1, X^2, X^3, X^4) with the signature +++- .Accordingly we postulate the following metric form

$$ds^2 = g_{\mu\nu} dX^\mu dX^\nu \tag{3}$$

where*

$$g_{11} = g_{22} = 1 + l^a\left[(X^1-A)^2 + (X^2-B)^2\right]^{-\frac{a}{2}}, \qquad g_{33} = 1,$$
$$g_{44} = -1, \quad g_{55} = g_{66} = -1 - l^a\left[(X^5-C)^2 + (X^6-D)^2\right]^{-\frac{a}{2}}, \tag{4}$$

and $g_{\mu\nu}=0$ for $\mu \neq \nu$ is a constant with the dimension of a length, and $a>0$ is a parameter.

The inhomogeneities and singularities of the above described manifold can be interpreted as an averaged background brought about by the existence and special distribution of matter (in a broad sense of this work) within the six-dimensional univers. Next we have to define our position in the univers. We assume to be situated at one of the two singular sibspaces, let say at the hypersurface defined by (2), far apart from the other singular hypersurface defined by (1). Thus, if X^μ means our position in the univers, then X^1-A as well as X^2-B are practically infinite whereas X^5-C , and X^6-D are very small or vanish. Under such circumstances we can simply neglect the terms involving X^1-A , and X^2-B in (4), and shift the origin of the system of coordinates so that $C=D=0$. The new coordinates (whose origin determines our position in the six-dimensional univers) will be called x^μ . Thus, for our special position in the world the metric form reduces practically to

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{3'}$$

where

$$g_{11} = g_{22} = g_{33} = -g_{44} = 1, \quad g_{55} = g_{66} = +b \tag{5}$$

with

$$b = -(1 + l^a r^{-a}) \qquad \text{where} \qquad r = \left[(x^5)^2 + (x^6)^2\right]^{1/2}, \tag{6}$$

and the remaining $g_{\mu\nu}$ vanish. Thus, in spite of a symmetry between space-like and time-like dimensions characterising the whole univers, the two time-like dimensions 5 and 6 appear sharply distinguished from the point of view of human beings.

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*This is an example, what we really need in is only the fact that when approaching the singular hypersurface the metric tensor is singular of the type $g \to r^{-a}$ where r is the distance from the hypersurface. For $r \to \infty$ the metrics should be ps.euclidean.

Of course, the dimensions 1,....4 are to be interpreted as the ordinary space-time. situated far apart from the domain where space-time is singular (or significantly inhomogeneous), the manifold $(x^1,\ldots,x^4)$ appears to us as homogeneous and isotropic. This accounts for the invariance of physical laws under the group of inhomogeneous Lorentz transformations and for the conservation of energy-momentum and angular momentum. On the other hand, since in that part of the univers where we are situated the invariance under translations of the variables x^5 and x^6 is violated, we cannot expect the existence of further conservation laws for a fifth and a sixth components of an "energy-mementum six-vector". Similarly, since the manifold is not isotropic, the groups of rotations in the (x^k, x^5) - and (x^k, x^6) - subspaces oannot play the same role we are accustomed to from considerations of isotropic spaces. However, the subspace (5,6) is isotropic, whence the physical laws have to be strictly invariant under the group of rotations in (5,6) around the point $x^5 = x^6 = 0$. This invariance will constitute the geometrical basis for electrodynamics.

Owing to the symmetry under rotations in (5,6) around the point $x^5 = x^6 = 0$ is will be convenient to introduce polar coordinates

$$x^5 = r\cos\alpha \quad , \quad x^6 = r\sin\alpha \tag{7}$$

whereby the metric form goes over into

$$ds^2 = g_{kl}\,dx^k dx^l + b\left[(dr)^2 + r^2(d\alpha)^2\right] \tag{8}$$

with g_{ik} given by (5) and b given by (6). The contravariant components of the metric tensor are

$$g^{11} = g^{22} = g^{33} = -g^{44} = 1 \quad , \quad g^{\alpha\alpha} = \frac{1}{br^2} \quad , \quad g^{rr} = \frac{1}{b} \tag{9}$$

whereas the remaining $g^{\mu\nu}$ vanish. Now, let us compute the contracted curvature tensor $R_{\mu\nu}$. Its only non-vanishing components are

$$R_{rr} = \frac{1}{2}\left[\left(\frac{b'}{b}\right)' + \frac{1}{r}\frac{b'}{b}\right] \quad , \quad R_{\alpha\alpha} = \frac{1}{2}\left[\left(\frac{r^2 b'}{b}\right)' - r\frac{b'}{b}\right] \tag{10}$$

where prime means a derivative with respect to r. These expressions are singular at the Minkowski hypersurface r = 0

$$R_{rr} \rightarrow \frac{a^2}{2} r^{a-2} l^{-a} \quad , \quad \frac{1}{r^2} R_{\alpha\alpha} \rightarrow \frac{a^2}{2} r^{a-2} l^{-a} \tag{11}$$

provided a $<$ 2. In spite of the singular character of the curvature tensor components, the curvature R itself as well as the quantity $r^{a-2}R$ is regular and vanishes in the limit r = 0,

$$l^{-a} r^{a-2} R = l^{-a} r^{a-2}\left(R_{rr} g^{rr} + R_{\alpha\alpha} g^{\alpha\alpha}\right) \rightarrow -a^2 l^{-3a} r^{3a-4} \tag{12}$$

provided $4/3 < a$. The case

$$4/3 < a < 2 \tag{13}$$

will be called "half-regular". In a "half-regular" univers the curvature tensor is singular which fact enables us to discriminate a four-dimensional space-time in it, but its singularity is so weak that not only the curvature R but even the quantity (12) vanishes at the points r = 0 defining the space-time.

2. A geometrical interpretation of the electromagnetic field*

In order to realise that our procedure of geometrizing the electromagnetic field is quite similar to the well-known procedure of geometrizing the gravitational field, let us recall briefly the essential steps of the latter. First of all one used to extend the Lorentz group to the group of all non-linear transformations

$$x^k = f^k(\overset{\circ}{x}{}^1, \ldots, \overset{\circ}{x}{}^4) \tag{14}$$

where $\overset{\circ}{x}{}^k$ mean the original variables. Under the transformation (14) the metric tensor components cease to be of the simple form $g_{11} = g_{22} = g_{33} = -g_{44} = 1$, $g_{ik} = 0$ for $i \neq k$ but are still subjected to some restrictions whose meaning is that the space-time remained still as it was beforehand, i.e. flat. In this case the g_{kl} describe only finctitious gravitational potentials. Then comes the decisive step: preserving the generalized metric form $ds^2 = g_{kl}\,dx^k dx^l$ we drop the restrictions upon g_{kl}. This means an encroachment upon the geometry of our manifold: the space-time acquires an x^k - dependent curvature and now the g_{kl} characterize the genuine gravitational effects. In the simple case of a gravitational field in vacuo the field is determined from a variational principle

$$\delta W = \delta \int d^4x \sqrt{\operatorname{Det} g_{kl}}\; \mathcal{L} = 0 \tag{15}$$

where **

$$\mathcal{L} = \frac{1}{G} R = \frac{1}{\varkappa^2 l^2} R_{ik} g^{ik} \tag{15'}$$

* We have multiplied the Lagrangian by an inverse of the gravitational coupling constant $G = \varkappa^2 l^2$ in order to have W with a dimension of action so as to be able to add the quantity W describing the gravitational field to the other W's describing the remaining fields occuring in Nature.

The above outlined procedure for the gravitational field can be closely imitated in the case of the electromagnetic field. First of all we have the group of rotations in the subspace (5,6) around the origin $r=0$ that does not influence the form (8). This group can be regarded as analogue to the Lorentz group. Then, quite similarly as was the case with the Lorentz group, we extend the group of rotations in (5,6) to a non-linear group by assuming an x^k-dependence of the angle of rotation

$$\alpha = \overset{\circ}{\alpha} + f(x^k) \tag{16}$$

** Some earlier attempts to geometrize the electromagnetic field in many-dimensional spaces are quoted at the end of this article.

In other words, we allow different rotations in (5,6) for different points in the Minkowski subspace. Owing to the symmetry with respect to rotations in (5,6), physical laws must remain covariant also under the group of non-linear transformations (16). This is nothing else but the group of gauge transformations. The transformation (16) changes the form of the metric tensor components, In order to simplify the computations let us neglect the gravitational effects and use an imaginary time coordinate

$$x^4 = it \tag{17}$$

After the transformation (16) the $g_{\mu\nu}$ become

$$g_{kl} = \delta_{kl} + br^2 f_k f_l\,, \quad g_{k\alpha} = -br^2 f_k\,, \quad g_{\alpha\alpha} = br^2\,, \quad g_{rr} = b\,, \quad g_{k\alpha} = g_{kr} = 0 \tag{18}$$

where f_k is the gradient of the phase f introduced by (16),

$$f_k = \partial_k f \tag{19}$$

and b is again given by (6). Of course, the metric properties of our manifold have not been influenced by the above transformation, and the appearance of f, as defined by (19), does not introduce any new physical contents into the theory, either. However, closely imitating the procedure of general relativity, a decisive step will be made by dropping the restriction (19) but preserving the form (18) of the metric tensor. This constitutes an encroachment upon the geometry. Hereby the manifold will acquire an x^k-dependent curvature and the quantities f_k involved in it will represent the electromagnetic potentials.

The contravariant components of the metric tensor (18) are

$$g^{kl} = \delta^{kl}\,, \quad g^{k\alpha} = f_k\,, \quad g^{\alpha\alpha} = \frac{1}{br^2} + f_k f_k\,, \quad g^{rr} = \frac{1}{b}\,, \quad g^{kr} = g^{k\alpha} = 0\,. \tag{20}$$

From (18) and (20) we compute the Γ symbols (Christoffel sybols)

$$\Gamma^{i}_{jk} = \frac{br^2}{2}\left(f_j f_{ki} + f_k f_{ji}\right), \quad \Gamma^{i}_{j\alpha} = \frac{br^2}{2} f_{ij}\,, \quad \Gamma^{\alpha}_{ik} = -\frac{1}{2}\left(\partial_k f_i + \partial_i f_k\right) + \frac{br^2}{2}\left(f_k f_l f_{il} + f_i f_l f_{kl}\right),$$

$$\Gamma^{r}_{ik} = -\frac{(br^2)'}{2b} f_i f_k\,, \quad \Gamma^{\alpha}_{k\alpha} = \frac{br^2}{2} f_l f_{lk}\,, \quad \Gamma^{r}_{k\alpha} = -\frac{(br^2)'}{2b} f_k\,, \quad \Gamma^{\alpha}_{kr} = -\frac{(br^2)'}{2br^2} f_k\,, \tag{21}$$

$$\Gamma^{r}_{\alpha\alpha} = \frac{(br^2)'}{2b}\,, \quad \Gamma^{\alpha}_{\alpha r} = \frac{(br^2)'}{2br^2}\,, \quad \Gamma^{r}_{rr} = \frac{b'}{2b}\,,$$

while the remaining (essentially different) Γ-symbols vanish. The quantities f_{kl} are defined as

$$f_{kl} = \partial_l f_k - \partial_k f_l \tag{22}$$

With the help of (18), (20) and (21) we compute the curvature R, or rather the expression

$$\mathcal{L} = \lim_{r\to 0} \frac{R}{2l^a r^{2-a}} = \lim_{r\to 0} \frac{1}{2l^a r^{2-a}}\left(R_{ik}g^{ik} + 2R_{kr}g^{kr} + 2R_{k\alpha}g^{k\alpha} + R_{\alpha\alpha}g^{\alpha\alpha} + 2R_{\alpha r}g^{\alpha r} + R_{rr}g^{rr}\right). \tag{23}$$

The result is finite and simple only under the conditions (13), namely: the first term in (23) yields only a divergence

$$\frac{1}{2}\,\partial_j\left(f_k\, f_{kj}\right) \quad , \tag{23'}$$

the socond term yields

$$-\frac{1}{2}\, f_k\, \partial_j\, f_{kj} \quad , \tag{23''}$$

whereas all the remaining terms vanish. Thus, we find

$$\mathcal{L} \equiv \mathcal{L}_{e.m.} = \frac{1}{2}\, f_{kj}\, \partial_j f_k \quad , \tag{24}$$

an expression that is to be reoognized as the Lagrangian for the electromagnetic field in vacuo. The usual variational procedure (variantions δf_k) leads to the Maxwell equations in vacuo

$$\delta W = \delta \int d^4x\, \mathcal{L} = \partial_j f_{jk} = 0 \tag{25}$$

Thus, f_{jk} is actually to be interpreted as the eleotromagnetic field tensor and f_k connoted with f_{jk} by means of (22), as the electromagnetic potential to whioh we can always add a gradient (19) without influencing the physical contents of the theory.

In this way the electromagnetic field has been geometrized on very similar linea to those known from general relativity. The gauge transformations hava acquired as well a geometrical meaning. The main difference between the gravitational and the electromagnetic field consists in the following circumstance: in the case of gravitation only the components R_{ik} of the oontracted curvature tensor contribute to the curvature R whereas in the case of electromagnetism only the mixed components R_{k5} and R_{k6} contribute to the curvature R of the six-dimensional manifold. The comparison of (23) with (15')shows that the gravitational coupling constant $G = \kappa^2 l^2$ can be interpreted as an average of the quantity $l^a r^{2-a}$ over a very small region $0 \leq r \lesssim l$ in the subspace (5,6). The limit transition r $\to$ 0 (whereby $l^a r^{2-a} \to 0$) assumed in this section is in agreement with the fact that the gravitation has been neglected.

3. Interaction with charged fields

Consider a field transforming like a vector under rotations in the subspace (5,6) around the point $x^5 = x^6 = 0$. This field possesses two non-vanishing components to be denoted by φ^5 and φ^6 . (It can be looked upon e.g. as a six-vector whose components φ^k vanish). Let us compute the covariant derivatives of this vector field in the limit r = 0. The metric tensor components expressed in terms of the variables x^1, ... x^6 are

$$g_{kl} = \delta_{kl} + b r^2 f_k f_l \;,\; g_{k5} = b x^6 f_k \;,\; g_{k6} = -b x^5 f_k \;,\; g_{55} = g_{66} = b \;,\; g_{56} = 0 \quad , \tag{26}$$

and

$$g^{kl}=\delta_{kl}\ ,\quad g^{k5}=-x^6 f_k\ ,\quad g^{k6}=x^5 f_k\ ,\quad g^{55}=\frac{1}{b}+(x^6)^2 f_k f_k\ ;$$

$$g^{66}=\frac{1}{b}+(x^5)^2 f_k f_k\ ,\quad g^{56}=-x^5x^6 f_k f_k\ . \tag{27}$$

The Christoffel symbols (computed in the limit r=0) that are of interest for our present purpose, are

$$\Gamma_{kl}^{\ i}=\Gamma_{k5}^{\ 5}=\Gamma_{k6}^{\ 6}=0\ ,\quad \Gamma_{k6}^{\ 5}=\frac{2-a}{2}f_k\ ,\quad \Gamma_{k5}^{\ 6}=-\frac{2-a}{2}f_k\ , \tag{28}$$

whence we get the covariant derivatives

$$\varphi^5_{;k}=\partial_k\varphi^5+\frac{2-a}{2}f_k\varphi^6\ ,\quad \varphi^6_{;k}=\partial_k\varphi^6-\frac{2-a}{2}f_k\varphi^5\ . \tag{29}$$

Combining the components φ^5 and φ^6 (of a real field) into a complex field

$$\varphi=\frac{1}{\sqrt{2}}(\varphi^5+i\varphi^6)\ ,\qquad \varphi^*=\frac{1}{\sqrt{2}}(\varphi^5-i\varphi^6) \tag{30}$$

we find

$$\varphi_{;k}=\left(\partial_k-i\frac{2-a}{2}f_k\right)\varphi\ ,\quad \varphi^*_{;k}=\left(\partial_k+i\frac{2-a}{2}f_k\right)\varphi^* \tag{31}$$

and recognize in them the well known expressions occurring in the theories of electrically charged fields. This discovery enables us to complete a geometrical interpretation of electrodynamics.

First of all we notice that the well-known expressions occurring in the case of electrically charged fields possess a geometrical interpretation in terms of covariant derivatives. Secondly, since a charged (i.e. a complex) field is essentially a vector field, as is seen from (30), it describes particles with a unit spin in the subspace (5,6). In other words, the electric charge can be interpreted as a spin in (5,6). The charged particle possesses an internal angular momentum of absolute value 1 (in the units $c=\hbar=1$) corresponding to an internal rotational motion that takes place in the (5,6)-subspace. However, owing to the geometrical properties of the six-dimensional manifold, this spin does not come forth with its full absolute value 1, but with a strength diminished by the factor $(1/2)(2-a)$. This factor is nothing else but the reciprocal root of the Sommerfeld constant. Its experimental value $e=137^{-1/2}$ lies within the limits (13). In other words, we have found a connection between the electric charge e (expressed in natural units) and the quantity a characteristic for the structure of the underlying manifold,

$$e=\frac{2-a}{2} \tag{32}$$

Expressed in terms of the electric charge the restriction of "half-regularity" (13) becomes

$$0<e<1/3 \tag{13'}$$

Thus, we are unable to predict theoretically the exact experimental value of the constant e but still, we can understand the reasons why there must be an electric coupling constant $e > 0$, and why it must be significantly smaller than unity. If e were zero (if a were equal to 2) then the singularity of the curvature tensor components would disappear, as is seen from (12), and we would not be able to disctiminate sharply the Minkowski space out from the six-dimensional manifold. On the other hand, if e were larger than 1/3, the singularity would be too strong and the part $R_{55}g^{55} + 2R_{56}g^{56} + R_{66}g^{66}$ would contribute infinitely to the action integral W at the points r = 0 forming the usual four-dimensional space-time.

With the aid of (28) we can easily compute the second covariant derivatives and check their commutability. The result is

$$\varphi_{;kl} - \varphi_{;lk} = ie f_{lk} \tag{33}$$

wherefrom we see once more that the existence of the electromagnetic field f_{kl} is equivalent to a change of the geometry. The tensor field $f_{kl}(x^i)$ introduces an x^k-dependent curvature of the 6-dimensional manofold in the immediate neighbourhood of the Minkovski subspace. Thus, the electromagnetic field becomes an intrinsically geometrical quantity.

Let us consider the usual Lagragian for a free complex field φ. According to (30) this Lagrangian $\mathcal{L}(\varphi, \varphi^*, \partial_k\varphi, \partial_k\varphi^*)$ is to be reinterpreted as a Lagrangian for a vector field φ^5, φ^6. Taking the electromagnetic field with the Lagrangian $\mathcal{L}(\partial_k f_l)$ into account we have not only to add the two Lagrangians but, at the same time, to replace the derivatives $\partial_k\varphi$ by the covariant derivatives $\varphi_{;k} = \partial_k\varphi - ief_k\varphi$ because the manifold is no more flat in the presence of the electromagnetic field, Thus, we have to assume the following Lagrangian

$$\mathcal{L} = \mathcal{L}_{e.m.}(\partial_k f_l) + \mathcal{L}(\varphi, \varphi_{;k}) \tag{34}$$

where $\mathcal{L}_{e.m.}(\partial_k f_l)$ is of the form (24). The usual variational procedure applied to (34) yields the usual equations for the complex field φ including its interaction with the electromagnetic field and the Maxwell equations in the presence of a charge and current constructed out from φ, φ^* and $\partial_k\varphi$, $\partial_k\varphi^*$

$$\partial_j f_{jk} = -e j_k \tag{35}$$

If, besides the complex field, we have to do with a real field χ it should be regarded as a scalar under rotations in (5,6). This assumption sufficiently justified the fact that neutral fields are not influenced by the electromagnetic field: indeed, the covariant derivatives of a scalar are identical with the usual derivatives. The same holds for the second covariant derivatives $\chi_{;kl}$, since $\chi_{;k}$ is again a scalar with respect to rotations in the subspace (5,6).

Let us consider the problem of charge conjugation. This transformation is equivalent to the following substitution

$$\varphi \to \varphi^* \quad , \quad \varphi^* \to \varphi \tag{36}$$

This holds obviously for bosons but can be also assumed for fermions (described by four-component spinors) provided the Majorana representation of the γ^k matrices is used. Complex four-component spinors in the Minkovski subspace are vectors in the (5,6)-subspace. Real spinors (describing the Majorana particle) are scalars in the (5,6)-subspace.

From (30) it is seen that the charge conjugation (36) is equivalent to an inversion of the sixth axis. Hereby the axis 6 is by no means privileged as compared with the axis 5 since we can also supplement the inversion by a gauge transformation: a rotation in (5, 6) through the angle π.

In this way also the charge conjugation acquires a geometrical interpretation of an inverstion in the subspace (5,6). The well known transformation CP is a transformation with Det = 1 if looked upon from the six-dimensional viewpoint.

Concluding, it can be stated that - similarly as the theory of gravitation - also electrodynamics can be expressed entirely in a georetrical language. This can be done in a six-dimensional space embedding the Minkowski space. This last is distinguished as a hypersurface where the curvature tensor is singular. The two-dimensional subspace (5,6) is isotropic with respect to rotations around the point r = 0 belonging to the Minkovski subspace. Gauge transformation is identical with a rotation in (5,6) around this point. Electric charge is essentially identical with the angular momentum in (5,6). Charge conjugation is interpretable as inversion in (5,6). The conservation law for the electric charge is a consequence of the isotropic character of the subspace (5,6), quite similarly as the conservation laws for energy-momentum and angular momentum have been consequences of the homogeneous and isotropic dharacter of the Minkovski space. The electromagnetic field is equivalent to an x^k- dependent curvature within the six-eimensional manofld. This curvature is of such a type that only the components R_{k5} and R_{k6} of the curvature tensor matter. The constants l and a involved in the definition of the geometrical "background" are closely connected with the gravitational and electromagnetic coupling constants respectively.

Part II. Mesodynamics

1. The problem of the pion-nucleon interaction

The purpose of Part II is to show that the six-dimensional point of view is useful not only for geometrizing the electrodynamics but also for a better understanding of mesodynamics, in particular of the concept of isospin and the ps-scalar character of nuclear interactions.

Let us start with pointing out a remarkable fact that (in contradistinction to electrodynamics where bosons as well as fermions serve as sources of the electromagnetic field) only fermions (baryons) but never bosons constitute the source of nuclear forces. This gives us a hint that the problem of nuclear interactions mist be intrinsically connected with the theory of spinors.

It is well known (Cartan 1938) that spinors with $2^{n/2}$ components are adequate for the case of n-dimensional spaces (n even). Thus, for the case of a six-dimensional space the eight-component spinors should play a particularly important role (whereas four- or two-component spinors should be regarded rather as exceptional, degenerate cases).

Let us oonsider Dirac matrices Γ^{μ} with eight rows and columns. There exist six independent matrices satisfying the usual relations

$$\Gamma^{\mu}\Gamma^{\nu} + \Gamma^{\nu}\Gamma^{\mu} = 2\,\delta^{\mu\nu} , \tag{37}$$

and the seventh matrix being the product of the ramaining six

$$\Gamma^{7} = -i\,\Gamma^{1}\Gamma^{2}\ldots\Gamma^{6} \tag{38}$$

and satisfying (37) as well. The six matrices Γ^{μ} play the same role in the six-dimensional space as the usual Dirac matrices γ^{k} with four rows and columns do in the four-dimensional space, while Γ^{7} plays the same role as γ^{5}. Let us investigate the following equation

$$(\Gamma^{\mu}\partial_{\mu} + i\,\Gamma^{7}\phi)X = -MX . \tag{39}$$

Since we are situated in the Minkovski subspace of the six-dimensional world, we are interested chiefly in the values of $X(x^{\mu})$ in the subspace $x^{5} = x^{6} = 0$. Denoting

$$X(x^{k},0,0) = \Psi(x^{k}) \tag{40}$$

we satisfy the equation (39) by the following Ansatz

$$X(x^{\mu}) = \Psi(x^{k})\,e^{i(\phi_5 x^5 + \phi_6 x^6)} \tag{41}$$

where ϕ_5 and ϕ_6 are still functions of x^{k}. This

$$\left[\Gamma^k\partial_k + i\left(\Gamma^5\phi_5 + \Gamma^6\phi_6 + \Gamma^7\phi\right)\right]\Psi = -M\Psi \tag{42}$$

valid only in the Minkovski subspace

In order to find out the meaning of this last equation let us introduce a convenient representation of the Γ^μ matrices. It is easily seen that the following eight row and column matrices

$$\Gamma^k = \begin{pmatrix}\gamma^k & \\ & \gamma^k\end{pmatrix}, \quad \Gamma^5 = \begin{pmatrix} & \gamma^5\\ \gamma^5 & \end{pmatrix}, \quad \Gamma^6 = \begin{pmatrix} & -i\gamma^5\\ i\gamma^5 & \end{pmatrix}, \quad \Gamma^7 = \begin{pmatrix}\gamma^5 & \\ & -\gamma^5\end{pmatrix} \tag{43}$$

satisfy all the requirements. Now, we introduce a two-index notation

$$\Psi_1 = \psi_{11}, \quad \Psi_2 = \psi_{21}, \quad \Psi_3 = \psi_{31}, \quad \Psi_4 = \psi_{41},$$
$$\Psi_5 = \psi_{12}, \quad \Psi_6 = \psi_{22}, \quad \Psi_7 = \psi_{32}, \quad \Psi_8 = \psi_{42} \tag{44}$$

and represent the eight row and column matrices (43) in form of a direct product of four row and columns matrices by two row and column matrices

$$\Gamma^k = \gamma^k 1, \quad \Gamma^5 = \gamma^5\tau_1, \quad \Gamma^6 = \gamma^5\tau_2, \quad \Gamma^7 = \gamma^5\tau_3 . \tag{45}$$

In this notation the equation (42) assumes the form

$$\left(\gamma^k\partial_k + i g\gamma^5\vec{\tau}\vec{\varphi}\right)\psi = -M\psi \tag{46}$$

where the components of the "three-vector" $\vec{\varphi}$ are

$$\varphi_1 = \frac{1}{g}\phi_5, \quad \varphi_2 = \frac{1}{g}\phi_6, \quad \varphi_3 = \frac{1}{g}\phi . \tag{47}$$

Now we easily recognize that (46) is nothing else but the equation for the nucleonic field with a ps-scalar coupling to a pion field. In this way it is seen that a ps-scalar coupling to a ps-scalar meson field is more natural than e.g. a scalar coupling to a scalar meson field. At the same time we get a physical interpretation of the concepts of isospace and isospin.

First of all we see that "isospace" is not a genuine three-dimensional space. Only two of its "dimensions" (namely 5 and 6) are genuine whereas the "third" dimension is nothing but an imitation brought about by the fact that, if we have six matrices Γ^μ there exists also the seventh matrix, in form of a product of the remaining six matrices, satisfying (37) too. From the point of view of the Minkowski space three of the matrices Γ^μ namely $\Gamma^5, \Gamma^6, \Gamma^7$ are "redundant" and algebraically equivalent. Therefore it is not surprising that some physical laws (strong interactions) can take advantage of the fact of the existence of three subsidiary Dirac matrices and can be characterized by a higher degree of symmetry: invariance under the group of rotations in a "three-dimensional apace" (5,6,7). However, since it is not a genuine space, the requirement of invariance is not compulsory, and therefore, some other physical laws can as well violate this algebraic symmetry. In particular, since φ_3 is a scalar but φ_1 together with φ_2 form a vector with respect to rotations in the subspace (5,6) (as is seen from (47)), we conclude that $(1/\sqrt{2})(\varphi_1+i\varphi_2)$ should describe charged particles whereas φ_3 is to describe neutral ones in agreement with the experimental evidence. In this way we have found an explanation of the fact why the so called "third component" of isospin is privileged and why it unavoidably contributes to the electric charge. In other words, why the electromagnetic interaction violates the invariance under the group of rotations in "isospace".

Until now we have disregarded, for the sake of simplicity, the fact that our six-dimensional manifold is not flat. Neglecting the electromagnetic interaction we have to put

$$\Gamma^\mu\Gamma^\nu + \Gamma^\nu\Gamma^\mu = 2g^{\mu\nu} \qquad (37')$$

where $g^{55}=g^{66}=1/b$, $g^{56}=0$. To satisfy (37') we have only to replace Γ^5, Γ^6 and Γ^7 by

$$\frac{1}{|b|^{1/2}}\begin{pmatrix} & \gamma^5 \\ \gamma^5 & \end{pmatrix}, \quad \frac{1}{|b|^{1/2}}\begin{pmatrix} & -i\gamma^5 \\ i\gamma^5 & \end{pmatrix}, \quad \frac{1}{|b|^{1/2}}\begin{pmatrix} \gamma^5 & \\ & -\gamma^5 \end{pmatrix} \qquad (43')$$

respectively. However, the factor $|b|^{-1/2}$ can be absorbed into the coupling constant g and we get again the equation (42), or the equivalent equation (46).

7. Strong interactions involving strange particles

The above ideas about nuclear interactions can be extended for the case of strong interactions among "strange" particles. However, once the concept of isospace and isospin has been clarified, we can use the notion of isospin in the traditional way. In order to make a plausible "Ansatz" for the interaction energy density we have to require not only the conservation of isospin and strangeness but, moreover, we have to use, as a guiding principle, the requirement that the interaction energy density should appear as natural as possible from the six-dimensional point of view, i.e. from the point of view of the algebra of Dirac matrices with eight rows and columns. As suggested by (45), this means a restrictive requirement that the τ matrices should appear always multiplied by γ^5 whereas the appearance of $\vec{\tau}$ or γ^5 separately is to be regarded as artificial. This principle restricts the choice of plausible interactions to the following types

$$\text{a) } ig\bar{N}\gamma^5\vec{\tau}\vec{\varphi}N + h.c. \qquad \text{b) } ig\bar{\Lambda}\vec{\varphi}\vec{\Sigma} + h.c. \qquad \text{c) } ig\bar{\Xi}\gamma^5\vec{\tau}\vec{\varphi}\Xi + h.c.$$

$$\text{d) } ig\bar{N}K\Lambda + h.c. \qquad \text{e) } ig\bar{N}\gamma^5\vec{\tau}K\vec{\Sigma} + h.c. \qquad \text{f) } ig\bar{\Xi}\bar{K}\Lambda + h.c. \qquad (48)$$

$$\text{g) } ig\bar{\Xi}\gamma^5\vec{\tau}\bar{K}\vec{\Sigma} + h.c.$$

The above set of interaction terms gives us some information about the relative particles of baryons. Comparing d) and f) we see that the nucleon and the hyperon Ξ have the same relative parities. From b) it follows that Λ and Σ have opposite relative parties. If the K meson is ps-scalar then, from d) it follows that Λ has the opposite parity to that of the remaining baryons. We notice also that, according to our list of strong interactions, not only the process $\Lambda \to \Lambda + \pi$ but also $\Sigma \to \Sigma + \pi$ does not occur.*

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