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SCATTERING OF γ -RAYS BY NUCLEONS NEAR THE PRODUCTION THRESHOLD OF MESONS

A b s t r a c t

The elastic scattering of γ-rays by nucleons near the production threshold of a single meson is considered with the help of dispersion relations.

It is shown, that the production of mesons in the S-state leads to a cusp dependence of the scattering amplitude, the cross section and other observable quantities near the threshold.

For forward γ-N-scattering 6 - dispersion relations are obtained which do not contain infrared divergence or arbitrary constants.

With some definite assumptions on the analysis of photoproduction data, the scattering amplitude, differential and total scattering cross section with polarized and unpolarized γ-rays, and also the polarization of the recoil nucleon above the threshold are calculated as functions of energy up to 220 MeV.

* * *

I. It is particularly interesting to study the scattering of γ-rays from nucleons near the threshold of meson production.

As is well known, in the low energy region scattering of γ-quanta from a particle with spin 1/2 and magnetic moment μ is described by the amplitude obtained by Low [1] and Gell-Mann-Goldberger [2].

A study of the scattering near the threshold of photoproduction may be of interest not only because it can be compared with the theoretical prediction of the dispersion relations but also because it is connected with the study [3] of the cusp energy dependence of the cross-section (or polarization) in this region. From this last point of view the scattering of γ-quanta from nucleons or nucleons near the threshold of production of mesons is of great interest for it can serve as an example of the process, which has a rather small cross-section and is strongly perturbed by the intensive production of mesons above the threshold. Therefore, one can expect a large effect in the region near the threshold. It is clear that an experimental study of the threshold anomaly with sufficient accuracy may help the study of photoproduction of mesons in this region.

The basic aim of the present work is to give a detailed analysis of the influence of meson production on the cross-section, polarization of recoil nucleon and polarization of γ-quanta in the compton scattering near the threshold.

It is shown that the polarization effect is very sensitive to the parameters describing the photoproduction.

In obtaining useful formula for the analysis of experimental data the phenomenological analysis and the dispersion relations are used. The numerical results based on definite assumption in the analysis of photoproduction must be considered as prelimenary. In out the numerical estimation the small effect in connection with the mass difference of mesons (and nucleons) has been completely neglected.

There are already many works, in which the scattering of γ- quanta from nucleon has been considered with different methods. In the present work an effort has been made in order to retain a minimum number of assumptions and avoid those approximation methods which are hard to justify.

II.

The general expression for the scattering amplitude of γ -quanta from particles with spin 1/2 has the form[4,5,6]

$$T = R_{1c}\,(\vec{e}\cdot\vec{e}\,') + R_{2c}\,(\vec{S}_c\cdot\vec{S}_c') + iR_{3c}\,(\vec{\sigma}\,[\vec{e}\,'\cdot\vec{e}]) + iR_{4c}\,(\vec{\sigma}\,[\vec{S}_c'\cdot\vec{S}_c])$$
$$+ iR_{5c}\left[(\vec{\sigma}\cdot\vec{k}_c)(\vec{S}_c'\cdot\vec{e}) - (\vec{\sigma}\cdot\vec{k}_c')(\vec{S}_c\cdot\vec{e}\,')\right] + iR_{6c}\left[(\vec{\sigma}\cdot\vec{k}_c)(\vec{S}_c'\cdot\vec{e}) - (\vec{\sigma}\cdot\vec{k}_c)(\vec{e}\,'\cdot\vec{S}_c)\right] \quad (1)$$

where R_1, R_3 and R_5 describe the electric transition, while R_2, R_4 and R_6 - the magnetic transition; $\vec{e}$ and $\vec{e}\,'$ are polarization vectors before and after the scattering; $\vec{S} = [\vec{k}\cdot\vec{e}]$, $\vec{S}\,' = [\vec{k}\,'\cdot\vec{e}\,']$, $\vec{k} = \frac{\vec{\varkappa}}{|\varkappa|}$ и $\vec{k}\,' = \frac{\vec{\varkappa}\,'}{|\varkappa|}$ are unit vectors along the direction of the impulse of γ quanta before and after the scattering; respectively; symbol "C" denotes the quantities in the c - m - s.

In the low energy region with terms not higher than the linear dependence on energy of γ -rays the expression T can be written in the form[1,2].

$$T = -\frac{e^2}{M}(\vec{e}\cdot\vec{e}\,') + \frac{ie}{M}\left(2\mu - \frac{e}{2M}\right)\nu_c\,(\vec{\sigma}\,[\vec{e}\,'\cdot\vec{e}]) + 2\mu^2\nu_c\,(\vec{\sigma}\,[\vec{S}_c\cdot\vec{S}_c']) +$$
$$+ \frac{ie}{M}\mu\nu_c\left[(\vec{e}\cdot\vec{k}_c')(\vec{\sigma}\cdot\vec{S}_c') - (\vec{e}\,'\cdot\vec{k}_c)(\vec{\sigma}\cdot\vec{S}_c)\right] \quad (2)$$

With the help of

$$(\vec{\sigma}\cdot\vec{S}\,')(\vec{e}\cdot\vec{k}\,') - (\vec{\sigma}\cdot\vec{S})(\vec{e}\,'\cdot\vec{k}) = -2\,(\vec{\sigma}\,[\vec{e}\,'\cdot\vec{e}]) + (\vec{\sigma}\cdot\vec{k}\,')(\vec{e}\cdot\vec{S}\,') - (\vec{\sigma}\cdot\vec{k})(\vec{e}\,'\vec{S}) \quad (3)$$

expression (2) can be reduced to the form of (1). Then

$$R_1^o = -\frac{e^2}{M} \; ; \; R_2^o = 0 \; ; \; R_3^o = -2\left(\frac{e}{2M}\right)^2 \nu_c \; ; \; R_4^o = -2\mu^2 \nu_c \; ; \; R_5^o = 0 \; ; \; R_6^o = \frac{e}{M}\mu\nu_c$$

3\.

Denote the transition matrix by (4)

$$T = \sum_{\mu\nu} e'_\mu \cdot N_{\mu\nu} \cdot e_\nu = e' \cdot N \cdot e$$

Choose two such coordinate systems x', Y', z' and x, y, z, in which the axes z and z' are parallel to the initial and final impulse of the photon respectively while axes y and y' have the same direction. In these coordinate systems the eigenstates of the photon the spin with eigenvalue $S_z = \pm 1$ have the following form

$$\vec{\xi}_1 = -\frac{1}{\sqrt{2}}(\vec{h} - i\vec{j}) \quad ; \quad \vec{\xi}_{-1} = \frac{1}{\sqrt{2}}(\vec{h} + i\vec{j})$$

$$\vec{\xi}'_1 = -\frac{1}{\sqrt{2}}(\vec{h}' - i\vec{j}) \quad ; \quad \vec{\xi}'_{-1} = \frac{1}{\sqrt{2}}(\vec{h}' + i\vec{j}) \tag{5}$$

where $\vec{h}$, $\vec{j}$, $\vec{k}$ are unit basic vectors along the coordinate axis. In general case the polarization state of photon will be a mixed state, i.e.

$$\vec{e} = c_1 \vec{\xi}_1 + c_{-1} \vec{\xi}_{-1} \tag{6}$$

where $|c_1|^2$ and $|c_{-1}|^2$ are the probability of finding the photon in states with $S_z = +1$ and $S_z = -1$ respectively.

Using the eigenstate of spin as the basis for the representation, transition matrix will have the form

$$T = \begin{pmatrix} \vec{\xi}_1^{\prime *} \cdot N \vec{\xi}_1 & 0 & \vec{\xi}_1^{\prime *} \cdot N \cdot \vec{\xi}_{-1} \\ 0 & 0 & 0 \\ \vec{\xi}_{-1}^{\prime *} \cdot N \cdot \vec{\xi}_1 & 0 & \vec{\xi}_{-1}^{\prime *} \cdot N \cdot \vec{\xi}_{-1} \end{pmatrix} \tag{7}$$

Introduce the density matrix of photon in the form

$$\rho = \begin{pmatrix} c_1 c_1^* & 0 & c_1 c_{-1}^* \\ 0 & 0 & 0 \\ c_{-1} c_1^* & 0 & c_{-1} c_{-1}^* \end{pmatrix} \tag{8}$$

The density matrix in the final state ρ_f is connected with the density matrix in the initial state ρ_{in} by the relation

$$\rho_f = T \cdot \rho_{in} \cdot T^{+} \tag{9}$$

Although in (7) and (8) the transition matrix and the density matrix are represented by three dimensional matrix, they have only four nonvanishing independent matrix. Therefore we can represent them with the help of two dimensional matrix and apply the well known technique of Pauli matrix [7]

$$T = \begin{pmatrix} \vec{\zeta}_1'^{*}\cdot N \cdot \vec{\zeta}_1 & \vec{\zeta}_1'^{*}\cdot N \cdot \vec{\zeta}_{-1} \\ \vec{\zeta}_1'^{*}\cdot N \cdot \vec{\zeta}_1 & \vec{\zeta}_{-1}'^{*}\cdot N \cdot \vec{\zeta}_{-1} \end{pmatrix} = A + \vec{\sigma}_\gamma \cdot \vec{B} \tag{10}$$

$$\rho = \begin{pmatrix} c_1 c_1^{*} & c_1 c_{-1}^{*} \\ c_{-1} c_1^{*} & c_{-1} c_{-1}^{*} \end{pmatrix} = \frac{1}{2}\left(1 + \vec{\sigma}_\gamma \cdot \vec{P}\right) \tag{11}$$

where P_x , P_y and P_z are Stokes parameters; P_x and P_y represent the linear polarization of photon along the x and y axis, while $P_z \neq 0$ represents the circular polarization of the photon.

From (10) it is easy to obtain

$$\begin{aligned} 2A &= S_p T = \vec{\zeta}_1'^{*}\cdot N \cdot \vec{\zeta}_1 + \vec{\zeta}_{-1}'^{*}\cdot N \cdot \vec{\zeta}_{-1} ; \\ 2B_z &= S_p(\sigma_\gamma^z T) = \vec{\zeta}_1'^{*}\cdot N \cdot \vec{\zeta}_1 - \vec{\zeta}_{-1}'^{*}\cdot N \cdot \vec{\zeta}_{-1} \\ 2B_x &= S_p(\sigma_\gamma^x T) = \vec{\zeta}_1'^{*}\cdot N \cdot \vec{\zeta}_{-1} + \vec{\zeta}_{-1}'^{*}\cdot N \cdot \vec{\zeta}_1 \\ 2iB_y &= \vec{\zeta}_1'^{*}\cdot N \cdot \vec{\zeta}_{-1} - \vec{\zeta}_{-1}'^{*}\cdot N \cdot \vec{\zeta}_{-1} \end{aligned} \tag{12}$$

where the trace is taken with respect to the variables of γ-quanta.

Quantities A and B can be written in terms of $R_1 \ldots R_6$ defined in (I)

$$2A = (R_1+R_2)(1+\cos\theta) - i(R_3+R_4)\sin\theta\,(\vec{\sigma}\cdot\vec{n})$$
$$2B_z = (R_3+R_4)(\vec{\sigma};\vec{k}+\vec{k}') + (1+\cos\theta)(R_5+R_6)(\vec{\sigma},\vec{k}+\vec{k}')$$
$$2iB_y = [R_3-R_4-(1-\cos\theta)(R_5-R_6)]\,(\vec{\sigma},\vec{k}-\vec{k}')$$
$$2B_x = (R_1-R_2)(1-\cos\theta) + i(R_3+R_4)\sin\theta\,(\vec{\sigma}\cdot\vec{n}) \tag{13}$$

where $\vec{n}\sin\theta = [\vec{k}\cdot\vec{k}']$, $\cos\theta = (\vec{k}\cdot\vec{k}')$

It is easy to calculate the density matrix in the final state

$$\rho_f = \frac{1}{2}(A+\vec{\sigma}_\gamma\cdot\vec{B})(1+\vec{\sigma}_\gamma\cdot\vec{P})(A^+ + \vec{\sigma}_\gamma\cdot\vec{B}^+) = \frac{1}{2}\Big\{AA^+ + BB^+ + (A\vec{B}^+ + \vec{B}A^+)\vec{P}$$
$$-i([\vec{B}\cdot\vec{B}^+]\vec{P})\Big\} + \frac{1}{2}\vec{\sigma}_\gamma\cdot\Big\{A\vec{B}^+ + \vec{B}A^+ + i[\vec{B}\cdot\vec{B}^+] + (AA^+ - \vec{B}\vec{B}^+)\cdot\vec{P} + \tag{14}$$
$$+\vec{B}\cdot\vec{P}\vec{B}^+ + \vec{B}\,\vec{P}\cdot\vec{B}^+ + iA[\vec{P}\cdot\vec{B}^+] - i[\vec{P}\cdot\vec{B}]A^+\Big\}$$

Using (14) one can calculate all the observable quantities. For unpolarized γ - quanta and unpolarised nucleons the differential cross section will have the form

$$\frac{d\sigma}{do} \equiv I_0(\theta) = \frac{1}{2}Sp\,(AA^+ + \vec{B}\cdot\vec{B}^+) \tag{15}$$

where the trace is taken with respect to the nucleon variables. Substituting (13) in (15)

$$4I_0(\theta) = |R_1+R_2|^2(1+\cos^2\theta) + |R_1-R_2|^2(1-\cos\theta)^2 + |(R_3+R_4)|^2(3-\cos^2\theta+2\cos\theta) +$$
$$+|R_3-R_4|^2(3-\cos^2\theta-2\cos\theta) + 2|R_5+R_6|^2(1+\cos\theta)^3 +$$
$$+2|R_5-R_6|^2(1-\cos\theta)^3 + 4\,\mathrm{Re}\,(R_3+R_4)^*(R_5+R_6)(1+\cos\theta)^2 - \tag{16}$$
$$-4\,\mathrm{Re}\,(R_3-R_4)^*(R_5-R_6)(1-\cos\theta)^2$$

or

$$2I_0(\theta) = (1+\cos^2\theta)\Big[|R_1|^2+|R_2|^2+4\,\mathrm{Re}\,(R_3^*R_6+R_4^*R_5)\Big] + (3-\cos^2\theta)(|R_3|^2+|R_4|^2) +$$
$$+2(1+3\cos^2\theta)(|R_5|^2+|R_6|^2) + 4\,\mathrm{Re}\Big[R_1^*R_2 + R_3^*R_4 +$$
$$+2(R_3^*R_5+R_4^*R_6) + (3+\cos^2\theta)R_5^*\cdot R_6\Big]\cos\theta \tag{16'}$$

Expression for the polarization of recoil nucleon has the form and

$$2I_0(\theta)\langle\vec{\sigma}\rangle_f = Sp\,(A\cdot A^+ + \vec{B}\cdot\vec{B}^+)\,\vec{\sigma}$$

$$I_0(\theta)\langle\vec{\sigma}\rangle_f = \frac{\vec{n}}{2}\sin\theta\, \mathrm{Im}\left[(R_3+R_4)(R_1+R_2)^*(1+\cos\theta) - (R_3-R_4)(R_1-R_2)^*(1-\cos\theta)\right] =$$

$$= i[\vec{k}\cdot\vec{k}']\Big\{R_1R_4^* - R_1^*R_4 + R_2R_3^* - R_2^*R_3 +$$

$$+ \left[R_1R_3^* - R_1^*R_3 + R_2R_4^* - R_2^*R_4\right]\cos\theta\Big\} \tag{17}$$

The well known theorem that cross section will not change under the interchange of electric and magnetic transition is shown in the fact, that (16) is invariant under the simultaneous substitution

$$R_1 \rightleftarrows R_2 \quad , \quad R_3 \rightleftarrows R_4 \quad , \quad R_5 \rightleftarrows R_6 \tag{18}$$

From (17) it is clear, that the polarization of the recoil nucleon is also invariant under this transformation.

4.

Now we shall establish the relations between Stokes parameters and the statistical tensor moments are defined by the following well known equations

$$T_{00} = \frac{1}{\sqrt{3}} \qquad T_{10} = \frac{1}{\sqrt{2}} S_z \qquad T_{20} = \sqrt{\frac{2}{3}}\left(\frac{3}{2}S_z^2 - 1\right)$$

$$T_{22} = \frac{1}{2}\left[S_x^2 - S_y^2 + i\left(S_xS_y + S_yS_x\right)\right] \tag{19}$$

$$T_{2-2} = \frac{1}{2}\left[S_x^2 - S_y^2 - i\left(S_xS_y + S_yS_x\right)\right]$$

They are normalized in such a way, that

$$Sp\; T_{JM}\, T^{+}_{J'M'} = \delta_{JJ'}\delta_{MM'} \tag{20}$$

The density matrix can be written in terms of these tensor moments

$$\rho_f = \rho_{00}T_{00} + \rho_{10}T_{10} + \rho_{20}T_{20} + \rho_{22}T_{22} + \rho_{2-2}T_{2-2} \tag{21}$$

with

$$\rho_{00} = \sqrt{2}\;\rho_{20} = 1/\sqrt{3}$$

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* Formula (19), (23) and (24) in [5] contain some mistakes

Parameters ρ_{JM} are connected with the Stokes parameters

$$\rho_{10} = \sqrt{2}\, P_z \quad ; \quad \rho_{22} = P_x - i P_y \quad , \quad \rho_{2-2} = P_x + i P_y \tag{22}$$

On account of the invariance under the time reversal the expression for the scattering cross-section $I(\theta,\varphi)$ of polarized γ -quanta with unpolarized nucleons may be written in the form /8,5/

$$I(\theta,\varphi) = I_0(\theta)\left[1 + 2\langle T_{22}\rangle_i \langle T_{22}\rangle_f \cos 2\varphi\right] \tag{23}$$

where

$$2 I_0(\theta)\langle T_{22}\rangle_f = \sin^2\theta \left(|R_1|^2 + |R_4|^2 - |R_3|^2 - |R_2|^2\right), \tag{24}$$

$\langle T_{22}\rangle_i$ is the initial polarization of the γ -beam. It is noted, that (24) changes its sign under the transformation (18).

5.

For energies of γ-quanta below the threshold of pion photoproduction the imaginary parts of $R_1 \ldots R_6$ are small. Above the threshold the imaginary parts of $R_1 \ldots R_6$ are determined by the unitarity condition of S-matrix, which takes the following form the terms quadratic in electromagnetic interaction are neglected.

$$\begin{aligned} i\left[T^+(-\vec{k}', -\vec{e}', -\vec{k}, -\vec{e}, -\vec{\sigma}) - T(\vec{k}', \vec{e}', \vec{k}, \vec{e}, \vec{\sigma})\right] = \\ = \frac{\nu_c}{2\pi}\int d\Omega(q_+)\left[T^+_{\gamma\to\pi^+}(\vec{q}_+, \vec{k}', \vec{e}', \vec{\sigma})\, T_{\gamma\to\pi^+}(\vec{q}_+, \vec{k}, \vec{e}, \vec{\sigma})\right] + \\ + \frac{\nu_c}{2\pi}\int d\Omega(q_0)\left[T^+_{\gamma\to\pi^0}(\vec{q}_0, \vec{k}', \vec{e}'; \vec{\sigma})\, T_{\gamma\to\pi^0}(\vec{q}_0, \vec{k}, \vec{e}, \vec{\sigma})\right] \end{aligned} \tag{25}$$

where

$$\begin{aligned} T_{\gamma\to\pi}(\vec{q}, \vec{k}, \vec{e}, \vec{\sigma}) = i E_1 (\vec{\sigma}\cdot\vec{e}) - M_1\left[(\vec{q}[\vec{k}\cdot\vec{e}]) - i(\vec{\sigma}[[\vec{k}\cdot\vec{e}]\vec{q}])\right] - \\ - M_3\left[2([\vec{k}\cdot\vec{e}]\vec{q}) + i(\vec{\sigma}[[\vec{k}\cdot\vec{e}]\vec{q}])\right] + \frac{iE_2}{2}\left[(\vec{\sigma}\cdot\vec{k})(\vec{e}\vec{q}) + (\vec{\sigma}\cdot\vec{e})(\vec{k}\cdot\vec{q})\right] \end{aligned} \tag{26}$$

is the amplitude of photoproduction of pions on a proton .

In (26) only the lowest states are taken into consideration. E_1 corresponds to the transition from state $\frac{1}{2}^-$ and negative parity to meson state $S_{1/2}$, M_1 - transition from $1/2^+$ to $P_{1/2}$ state, M_3 and E_3 are transition from $3/2^+$ to the meson resonance state $P_{3/2}$ From (25) it follows, that above the threshold

$$\mathrm{Im}\, R_{1c} = \nu_c \left\{ |E_1|^2 + \frac{1}{3}|E_2|^2 \cos\theta \right\} = \nu_c A_1 \qquad ; \quad \mathrm{Im}\, R_{3c} = \mathrm{Im}\, R_{1c}$$
$$\mathrm{Im}\, R_{2c} = \nu_c \left\{ |M_1|^2 + 2|M_3|^2 - \frac{1}{6}|E_2|^2 \right\} = \nu_c A_2$$
$$\mathrm{Im}\, R_{4c} = \nu_c \left\{ |M_1|^2 - |M_3|^2 + \frac{1}{12}|E_2|^2 + \frac{1}{2}(E_2^* M_3 + E_2 M_3^*) \right\} = \nu_c A_4$$
$$\mathrm{Im}\, R_{5c} = -\nu_c \left\{ \frac{1}{6}|E_2|^2 + \frac{1}{2}(E_2^* M_3 + E_2 M_3^*) \right\} = \nu_c A_5 \; ; \quad \mathrm{Im}\, R_{6c} = 0 \qquad (27)$$

with the help of (27) it is easy to become convinced that the total cross section of -rays

$$\sigma_t = \frac{4\pi}{\nu_c} \mathrm{Im}\left[R_{1c}(0^\circ) + R_{2c}(0^\circ)\right] = 4\pi \left\{ |E_1|^2 + |M_1|^2 + 2|M_3|^2 - \frac{1}{6}|E_2|^2 \right\} \qquad (28)$$

coincides with the total cross section of photoproduction[9/] as it should be.

The threshold for production of π° meson $\nu_0(\pi^\circ)$ equals 144,7 MeV, while that of a π^+ meson $\nu_0(\pi^+)$ equals 150,5 MeV. For energies above $\nu_0(\pi^+)$ the right hand side of (27) contains both the quantitites characterizing the production of π° meson ($E_1^\circ, M_1^\circ, \ldots, A_1^\circ, A_2^\circ \ldots$) and those for production of π^+ meson ($E_1^+, M_1^+ \ldots A_1^+, A_2^+$). The effects connected with the mass difference in the energy $\nu_0(\pi^+) > \nu > \nu_0(\pi^\circ)$ are not considered in the present work.

Cusp dependence of scattering cross section of γ -quanta in the neighbourhood of the threshold is caused by the production of mesons in the S-state. According to the existing experimental data, cross section for the production of π^+ meson in the S-state is much greater than that of π° meson. It is a difficult experimental task to establish the existince of production of π° meson in the S-state. Evidently, an experimental study of scattering of γ-rays in the energy region $\nu_0(\pi^+) > \nu > \nu_0(\pi^\circ)$ will provide additional information on this problem.

The imaginary parts of $R_1 \ldots R_6$ are calculated with the help of unitarity condition. For the calculation of the real parts we have used the dispersion relations discussed in a number of works.

6.

Turning to the study of dispersion relations for scattering of γ -rays from nucleons we begin with a detailed consideration of the kinematics.

Let the four vectors κ and κ' denote the impulse of the incident and scattered photon, p and p' those of a nucleon. They satisfy the law of conservation

$$\kappa + p = \kappa' + p' \tag{29}$$

Introduce $\mathcal{P} = \frac{1}{2}(p + p')$

Following Prange [10] we will choose the following 4 orthogonal vectors as the basic vectors

$$K = \frac{1}{2}(\kappa + \kappa') \quad , \quad Q = \frac{1}{2}(\kappa - \kappa') = \frac{1}{2}(p - p') \quad , \quad \mathcal{P}' = \mathcal{P} - \frac{(\mathcal{P}\cdot K)}{K^2} K$$

$$N_\mu = i\, \varepsilon_{\mu\nu\lambda\sigma}\, P_\nu{}'\, K_\lambda\, Q_\sigma \tag{30}$$

Scattering amplitude can be written in the form

$$T = \bar{u}(p')\, e'_\mu\, N^\circ_{\mu\nu}\, e_\nu\, u(p) \tag{31}$$

while $N^\circ_{\mu\nu}$ can be written in terms of the invariant functions

$$N^\circ_{\mu\nu} = \sum_{\sigma\sigma'} \eta^\sigma_\mu\, C_{\sigma\sigma'}\, \eta^{\sigma'}_\nu \tag{32}$$

where η^σ are four basic vectors introduced in (30).

Gauge invariance reqires that $e'\cdot\kappa' = 0$, $e\cdot\kappa = 0$ and $\kappa'_\mu N^\circ_{\mu\nu} = 0$, $N^\circ_{\mu\nu}\kappa_\nu = 0$. As a consequence of these relations $N^\circ_{\mu\nu}$ can be reduced to a system with eight invariant functions $T_1 \dots T_8$ of two invariant variables $M\nu = -P\cdot K$ and Q^2

$$\begin{aligned}
e'_\mu N^\circ_{\mu\nu} e_\nu = {} & \frac{Q^2}{M^2\nu^2 - Q^2(Q^2+M^2)} (e'\mathcal{P}')(e\mathcal{P}) \left[T_1 + i\hat{k}\, T_2 \right] + \\
& + \frac{1}{Q^2\left[M^2\nu^2 - Q^2(Q^2+M^2)\right]} (e'N)(eN) \left[T_3 + i\hat{k}\, T_4 \right] - \\
& - \frac{i}{M^2\nu^2 - Q^2(Q^2+M^2)} \left[(e'\mathcal{P}')(eN) - (e'N)(eP') \right] \gamma_5 \left[T_5 + i\hat{k}\, T_7 \right] - \\
& - \frac{i}{M^2\nu^2 - Q^2(Q^2+M^2)} \left[(e'\mathcal{P}')(eN) + (e'N)(eP') \right] \gamma_5 \left[T_8 + i\hat{k}\, T_6 \right] .
\end{aligned} \tag{33}$$

Normalization factors $\frac{Q^2}{M^2\nu^2 - Q^2(Q^2+M^2)}$ etc are introduced for convenience.

One can show, that in any system (including, in particular, the Breit system and the center of mass system) the following equations hold

$$\frac{Q^2}{M^2\nu^2 - Q^2(Q^2+M^2)}(e'P')(eP') = \frac{(\vec{e}\,'\vec{\varkappa})(\vec{e}\vec{\varkappa}\,')}{|\vec{\varkappa}||\vec{\varkappa}\,'|\sin^2\theta} = \frac{(\vec{e}\,'\cdot\vec{k})(\vec{e}\cdot\vec{k}\,')}{\sin^2\theta}$$

$$\frac{(e'N)(eN)}{M^2\nu^2 - Q^2(Q^2+M^2)} = \frac{(\vec{e}\,'[\vec{k}\cdot\vec{k}\,'])(\vec{e}[\vec{k}\cdot\vec{k}\,'])}{\sin^2\theta} = \frac{(\vec{e}\,'\vec{\rho})(\vec{e}\cdot\vec{\rho})}{\sin^2\theta} \tag{34}$$

$$-\frac{(e'P')(eN) \mp (e'N)(eP')}{M^2\nu^2 - Q^2(Q^2+M^2)} = \frac{(\vec{e}\,'\vec{k})(\vec{e}\cdot\vec{\rho}) \mp (\vec{e}\,'\vec{\rho})(\vec{e}\cdot\vec{k}\,')}{\sin^2\theta}$$

where θ is the angle between $\vec{k}$ and $\vec{k}\,'$; $\vec{k}$ and $\vec{k}\,'$ are unit vectors along $\vec{\varkappa}$ and $\vec{\varkappa}\,'$, $\vec{\rho} = [\vec{k}\cdot\vec{k}\,']$. We shall prove (34) in the Breit system, where

$$\vec{P} = 0 \quad , \quad \vec{P}\,' = -\frac{P\cdot k}{k^2}\vec{k} = -\frac{M\nu}{Q^2}\vec{k} \tag{35}$$

It is easy to justify the following formula

$$|\vec{\varkappa}| = k_0 = \frac{M\nu}{\sqrt{Q^2+M^2}} \tag{36}$$

$$2Q^2 = k_0^2(1-\cos\theta) \tag{37}$$

$$k_0^2 - Q^2 = \frac{M^2\nu^2 - Q^2(Q^2+M^2)}{Q^2+M^2} = \frac{k_0^2}{2}(1+\cos\theta) \tag{38}$$

Multiplying (37) by (38) we obtain

$$\frac{k_0^4}{4}\sin^2\theta = \frac{Q^2[M^2\nu^2 - Q^2(Q^2+M^2)]}{Q^2+M^2} \tag{39}$$

With the help of (36), (39) can be transformed into the form

$$k_0^2\sin^2\theta = 4Q^2\frac{M^2\nu^2 - Q^2(Q^2+M^2)}{M^2\nu^2} \tag{40}$$

from (33) and (35) we obtain finally

$$\frac{Q^2}{M^2\nu^2 - Q^2(Q^2+M^2)}\cdot\frac{M^2\nu^2}{Q^4}\cdot\frac{1}{4}\cdot(\vec{e}\,'\vec{\varkappa})(\vec{e}\vec{\varkappa}\,') = \frac{(\vec{e}\,'\vec{\varkappa})(\vec{e}\vec{\varkappa}\,')}{k_0^2\sin^2\theta} = \frac{(\vec{e}\,'\vec{k})(\vec{e}\cdot\vec{k}\,')}{\sin^2\theta} \tag{41}$$

Using formula

$$\vec{N} = -\sqrt{Q^2+M^2}\,\frac{1}{2}[\vec{k}\cdot\vec{k}\,'] \tag{42}$$

one can justify other equations in (34) in a similar way. As is well known, the invariance under time reversal (or as one can prove it below, the requirement of crossing symmetry)reduces the number of independent invariant functions to six ($T_7 = T_8 = 0$). If we write the scattering amplitude in Breit system in the form (1), then we obtain

$$R_1 \sin^2\theta = \frac{E}{M}\left(T_1\cos\theta + T_3\right) - k_0\left(T_2\cos\theta + T_4\right) \quad ; \quad R_3 = \frac{k_0^2}{2M} T_2$$
$$R_2 \sin^2\theta = -\frac{E}{M}\left(T_1 + T_3\cos\theta\right) + k_0\left(T_2 + T_4\cos\theta\right) \quad ; \quad R_4 = \frac{k_0^2}{2M} T_4$$
$$R_5 \sin^2\theta = \frac{k_0^2}{2M}\left(T_2\cos\theta + T_4\right) - \frac{k_0}{2M}\left(1+\cos\theta\right) T_5 - \frac{k_0 E}{2M}\left(1-\cos\theta\right) T_6 \tag{43}$$
$$R_6 \sin^2\theta = -\frac{k_0^2}{2M}\left(T_2 + T_4\cos\theta\right) + \frac{k_0}{2M}\left(1+\cos\theta\right) T_5 - \frac{k_0 E}{2M}\left(1-\cos\theta\right) T_6$$

It is easy to become convinced, that as $(E = M\ ,\ \nu = k_0)$

$$R_1 + R_2 \Big|_{\theta=0} = \frac{1}{2}\left[T_1 - T_3 - \nu\left(T_2 - T_4\right)\right]$$
$$R_5 + R_6 \Big|_{\theta=0} = \frac{1}{4M}\left[\nu^2\left(T_2 - T_4\right) + M\nu T_6\right] \tag{44}$$
$$R_3 \Big|_{\theta=0} = \frac{\nu^2}{2M} T_2$$
$$R_4 \Big|_{\theta=0} = \frac{\nu^2}{2M} T_4$$

Based on the first principle of quantum field theory it is proved /11-15/ that T_i are analytic functions of ν both for $\theta = 0^\circ (Q^2 = 0)$ and for

$$Q^2 < Q^2_{max} = \frac{(2M + m_\pi)(6M^2 + 9Mm_\pi + 4m_\pi^2)}{4M(M + m_\pi)^2} m_\pi^2 \approx 3m_\pi^2$$

where m_π is the mass of a π meson.

Usually, only two dispersion relations for the forward scattering amplitude $R_1 + R_2$ and $R_3 + R_4 + 2R_5 + 2R_6$ are considered. From (44) it is shown, that at $\theta = 0^\circ$ they are really four dispersion relations for the scattering amplitudes $R_1 + R_2$, R_3, R_4 and $R_5 + R_6$

7.

Retarded causal amplitude for the scattering of proton can be written in the form

$$\bar{u}(p')\, N_{\mu\nu}^{ret}\, u(p) = -2\pi^2 i \left(\frac{p_0 p_0'}{M^2}\right)^{1/2} \int d^4z\, e^{-ikz} \langle p' | \theta(z_0) \left[j_\mu\left(\tfrac{z}{2}\right), j_\nu\left(-\tfrac{z}{2}\right)\right] | p \rangle \tag{45}$$

Similarly we have for the advanced causal amplitude

$$\bar{u}(p')\, N_{\mu\nu}^{adv}\, u(p) = -2\pi^2 i \left(\frac{p_0 p_0'}{M^2}\right)^{1/2} \int d^4z\, e^{-ikz} \langle p' | \theta(-z_0) \left[j_\mu\left(\tfrac{z}{2}\right), j_\nu\left(-\tfrac{z}{2}\right)\right] | p \rangle \tag{46}$$

Define the dispersive part $D_{\mu\nu}$ and the absorptive part $A_{\mu\nu}$ in the following way.

$$\mathcal{D}_{\mu\nu} = \frac{1}{2}\left(N_{\mu\nu}^{ret} + N_{\mu\nu}^{adv}\right) \qquad ; \; A_{\mu\nu} = \frac{1}{2i}\left(N_{\mu\nu}^{ret} - N_{\mu\nu}^{adv}\right) \tag{47}$$

Taking the complex conjugate on both sides of (45) and remembering that j_μ are hermitian operators, we obtain

$$\beta \overset{+}{N}{}_{\mu\nu}^{ret}(p'k'pk)\beta = N_{\mu\nu}^{ret}(p-k'\,p-k) \tag{48}$$

Changing the order of j_μ and j_ν in the commutator in (45) and using $-z$ instead of z as the variable of integration, we have

$$N_{\mu\nu}^{ret}(p'k'pk) = N_{\mu\nu}^{adv}(p'-k\;p-k') \tag{49}$$

Substituting (33) in (48) and (49) we obtain

$$\begin{aligned} T^{*}_{1,3,5,6}(-\nu, Q^2) &= +T_{1,3,5,6}(\nu, Q^2) \\ T^{*}_{2,4}(-\nu, Q^2) &= -T_{2,4}(\nu, Q^2) \\ T_7 = T_8 &= 0 \end{aligned} \tag{50}$$

With the help of (50) one can easily write down the dispersion relation

$$\begin{aligned} \mathcal{D}_{1,3,5,6}(\nu, Q^2) &= \frac{2}{\pi} P\int_0^\infty \frac{\nu' A_{1,3,5,6}(\nu', Q^2)}{\nu'^2 - \nu^2}\,d\nu' \\ \mathcal{D}_{2,4}(\nu, Q^2) &= \frac{2\nu}{\pi} P\int_0^\infty \frac{A_{2,4}(\nu', Q^2)}{\nu'^2 - \nu^2}\,d\nu' \end{aligned} \tag{51}$$

Consider the dispersion relation at $Q^2=0$ when the Breit system coincides with the laboratory system and ν becomes the energy of γ-quanta in the laboratory system.

Since the dispersion relations for $T_{2,4}$ in e^2-approximation contain infrared divergence of the form

$$a\left(\frac{1}{\nu} - \frac{1}{\nu_0}\right)$$

at $Q^2=0$ it is possible, in fact, to use only the combination

$$R_1 + R_2 = L_1$$

and

$$R_5 + R_6 = L_2$$

also the quantities

$$R_3 = L_3 \quad , R_4 = L_4$$

which do not contain divergent term as $\nu \to 0$. At $Q^2=0$, $L = L(\nu)$ it can be shown, that

$$L_1(-\nu) = L_1(\nu)$$
$$L_{2,3,4}(-\nu) = -L_{2,3,4}(\nu) \tag{52}$$

Therefore, for these quantities we may write the dispersion relations.

$$\mathrm{Re}\, L_1(\nu) - \mathrm{Re}\, L_1(0) = \frac{2\nu^2}{\pi} P \int_{\nu_0}^{\infty} \frac{\mathrm{Im}\, L_1(\nu')}{\nu'(\nu'^2-\nu^2)} d\nu'$$
$$\mathrm{Re}\, L_{2,3,4}(\nu) - \nu\, \mathrm{Re}\, L'_{2,3,4}(0) = \frac{2\nu^3}{\pi} P \int_{\nu_0}^{\infty} \frac{\mathrm{Im}\, L_{2,3,4}(\nu')}{\nu'^2(\nu'^2-\nu^2)} d\nu' \tag{53}$$

Two other dispersion relations are obtained in the following section.

From (4)

$$\mathrm{Re}\, L_1(0) = -\frac{e^2}{M} \quad ; \quad \nu\, \mathrm{Re}\, L_2'(0) = \frac{e}{M}\mu\nu = \frac{e^2}{M}\cdot\frac{\lambda\nu_0}{2M}\cdot\frac{\nu}{\nu_0} \quad ; \quad \mu = \frac{e\lambda}{2M} \tag{54}$$
$$\nu\, \mathrm{Re}\, L_3'(0) = -2\left(\frac{e}{2M}\right)^2 \nu = -\frac{1}{2}\,\frac{e^2}{M}\cdot\frac{\nu_0}{M}\cdot\frac{\nu}{\nu_0}$$
$$\nu\, \mathrm{Re}\, L_4'(0) = -2\mu^2\nu = -\frac{1}{2}\,\frac{e^2}{M}\cdot\lambda^2\cdot\frac{\nu_0}{M}\cdot\frac{\nu}{\nu_0}$$

First expression can be reduced with the help of the optical theorem to the usual form

$$\mathrm{Re}\left[R_1(\nu) + R_2(\nu)\right] = -\frac{e^2}{M} + \frac{\nu^2}{2\pi^2} P \int_{\nu_0}^{\infty} \frac{\sigma(\nu')\, d\nu'}{\nu'^2-\nu^2} \tag{53'}$$

i.e. coincides with the dispersion relation first obtained by Gell-Mann, Goldberger and Thirring.

8.

In the center-of-mass system $R_{1c} \dots R_{6c}$ can be expressed in terms of the scattering amplitude with definite angular momentum and parity. Let us denote the electric dipole transition amplitude with total angular momentum 1/2 and 3/2 by $\mathcal{E}_1$ and $\mathcal{E}_3$ respectively; the electric quadrupole transition amplitude with angular momentum 3/2 by $\mathcal{E}_2$. Similarly the magnetic dipole and quadrupole transition amplitudes are denoted by $\mathcal{M}_1, \mathcal{M}_3$ and $\mathcal{M}_2$. Besides these it is necessary to introduce amplitudes $C'(\mathcal{M}_3\mathcal{E}_2)$, $C'(\mathcal{E}_2, \mathcal{M}_3)$ $C'(\mathcal{E}_3\ \mathcal{M}_2)$ and $C'(\mathcal{M}_2\mathcal{E}_3)$ which correspond to the transition from the state m_i $(\mathcal{E}_i)$ to $\mathcal{E}_k$ (m_k) Invariance under time reversal leads to the equation

$$C'(m_3\,\mathcal{E}_2) = C'(\mathcal{E}_2\,m_3) \quad ; \; C'(\mathcal{E}_3\,m_2) = C'(m_2\,\mathcal{E}_3) \tag{55}$$

Finally, using the technique of projecting operator and restricting to the states with $J \leq 3/2$, we obtain

$$\begin{aligned}
R_{1c} &= \mathcal{E}_1 + 2\mathcal{E}_3 + 2\mathcal{E}_2\cos\theta - m_2 \\
R_{2c} &= m_1 + 2m_3 + 2m_2\cos\theta - \mathcal{E}_2 \\
R_{3c} &= \mathcal{E}_1 - \mathcal{E}_3 + 2\mathcal{E}_2\cos\theta + \tfrac{1}{2}m_2 + \sqrt{6}\,C'(\mathcal{E}_3\,m_2) \\
R_{4c} &= m_1 - m_3 + 2m_2\cos\theta + \tfrac{1}{2}\mathcal{E}_2 + \sqrt{6}\,C'(m_3\,\mathcal{E}_2) \\
R_{5c} &= -\mathcal{E}_2 - \sqrt{6}\,C'(m_3\,\mathcal{E}_2) \equiv -\mathcal{E}_2 - C(m_3,\mathcal{E}_2) \\
R_{6c} &= -m_2 - \sqrt{6}\,C'(\mathcal{E}_3\,m_2) \equiv m_2 - C(\mathcal{E}_3, m_2)
\end{aligned} \tag{56}$$

Using (2) and (4) we can determine the dependence of energy of these quantities at $\nu_c \to 0$

$$\mathcal{E}_1^\circ + 2\,\mathcal{E}_3^\circ = -\frac{e^2}{M} \quad ; \; \mathcal{E}_1^\circ - \mathcal{E}_3^\circ = -\left[2\left(\frac{e}{2M}\right)^2 - \frac{e}{M}\mu\right]\nu_c \quad ; \; \mathcal{E}_2^\circ = 0 \quad ; \; m_2^\circ = 0$$

$$m_1^\circ = -\frac{4}{3}\mu^2\nu_c \quad ; \; m_3^\circ = \frac{2}{3}\mu^2\nu_c \quad ; \quad C^\circ(\mathcal{E}_2\,m_3) = 0 \quad ; \; C^\circ(\mathcal{E}_3\,m_2) = -\frac{e}{M}\mu\nu_c \tag{57}$$

In addition to (53) one can obtain two other dispersion relations by differentiation

$$R_3 = \frac{k_0^2}{2M}T_2 \qquad \text{and} \qquad R_4 = \frac{k_0^2}{2M}T_4$$

with respect to Q^2 and then putting $Q^2 = 0$. The factor k_0^2 containing in R_3 and R_4 at $Q^2 \to 0$ tends to ν^2 which compensate the possible infrared divergence in T_2 and T_4.

Consider

$$R_3 = \frac{k_0^2}{2M}T_2 = \frac{M\nu^2}{2}\cdot\frac{T_2(\nu, Q^2)}{\sqrt{Q^2+M^2}}$$

If T_2 and T_4 is an analytic function ν at $Q^2 < Q^2_{max}$ then R_3 and $\partial R_3/\partial Q^2$ will also be analytic functions of ν. Since the pole terms of T_2 and T_4 have the form

$$\frac{\mathcal{D}\,\nu\,\nu_B^2}{\nu_B^2 - \nu^2}$$

where $\mathcal{D}$ is some constant, while $\nu_B = \frac{Q^2}{M}$, then it follows

$$\mathcal{D}\frac{\partial}{\partial Q^2}\left.\frac{\nu\,\nu_B^2}{\nu_B^2 - \nu^2}\right|_{Q^2=0} = 0$$

i.e. the pole terms in dispersion relation for $\partial R_3/\partial Q^2$ and $\partial R_4/\partial Q^2$ vanish

Restricting the number of states as has been shown in (56)

$$\frac{\partial R_3}{\partial Q^2} = -2 \; \frac{\mathcal{E}_2(\nu)}{\nu^2} \tag{58}$$

then $\mathcal{E}_2(\nu)/\nu^2$ will be analytic function of ν whose behavior under crossing symmetry coincides with that of R_3. Therefore the dispersion relation for $\mathcal{E}_2/\nu^2$ has the form

$$Re \; \frac{\mathcal{E}_2(\nu)}{\nu^2} = \frac{2\nu}{\pi}\int_0^\infty \frac{Im \; \mathcal{E}_2(\nu')\,d\nu'}{\nu'^2(\nu'^2-\nu^2)} = \frac{2\nu}{\pi}\int_{\nu_0}^\infty \frac{Im \; \mathcal{E}_2(\nu')\,d\nu'}{\nu'^2(\nu'^2-\nu^2)}$$

or finally

$$Re \; \mathcal{E}_2(\nu) = \frac{2\nu^3}{\pi} P\int_{\nu_0}^\infty \frac{Im \; \mathcal{E}_2(\nu')}{\nu'^2(\nu'^2-\nu^2)}\,d\nu' \tag{59}$$

Combining (57) with the dispersion relation for R_5+R_6 we see that $\mathcal{E}_2(\nu)$ and $C(\mathcal{E}_2\,\mathcal{M}_3)$ satisfy the dispersion relation (59) separately.

Similarly, differentiating R_4 over Q^2 one can show that $\mathcal{M}_2(\nu)$ also satisfies the dispersion relation (59). Finally, it is possible to construct 6 dispersion relations for the 8 quantities characterizing the scattering amplitude of γ -quanta under the restriction given by (56). We did not succeed in obtaining more dispersion relations without infrared divergence. The differentiation over Q^2 for many times leads to unknown constants, which are determined by perturbation theory in some papers. If the experimental data are sufficient, one can also determine these unknown constants from experimental data similar to the case of π-N scattering. In the present work we do not adopt this kind of approach.

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In carrying out the calculation with dispersion relation for the scattering of γ -quanta from nucleons it requires a quite detailed analysis of the experimental data on the photoproduction. From the existing data can conclude that $Im\; \mathcal{M}_2 = 0$. If we also take into consideration $\mathcal{M}_2^0 = 0$ then

$$\mathcal{M}_2(\nu) = 0 \tag{60}$$

The calculation is carried out under the assumptions

$$\begin{aligned} &C(\mathcal{M}_2\,\mathcal{E}_3) = C(\mathcal{E}_3\,\mathcal{M}_2) = C^0(\mathcal{E}_3\,\mathcal{M}_2) = -\frac{e}{M}\mu\nu \\ &\mathcal{M}_1(\nu) = \mathcal{M}_1^0(\nu) = -\frac{4}{3}\mu^2\nu \end{aligned} \tag{61}$$

For the remaining quantities we can write the following dispersion relations

$$\mathrm{Re}\,\mathcal{E}_2(\nu) = \frac{1}{6}\cdot\frac{2\nu^3}{\pi}P\int_{\nu_0}^{\infty}\frac{d\nu'}{\nu'}\cdot\frac{|E_2^+|^2+|E_2^0|^2}{\nu'^2-\nu^2}$$

$$\mathrm{Re}\,\mathcal{M}_3(\nu) = \frac{2}{3}\mu^2\nu + \frac{2\nu^3}{\pi}P\int_{\nu_0}^{\infty}\frac{d\nu'}{\nu'}\cdot\frac{|M_3^+|^2+|M_3^0|^2}{\nu'^2-\nu^2}$$

$$\mathrm{Re}\left[\mathcal{E}_1(\nu)+2\,\mathcal{E}_3(\nu)\right] = -\frac{e^2}{M}+\frac{\nu^2}{2\pi^2}P\int_{\nu_0}^{\infty}\frac{\sigma^0+\sigma^+}{\nu'^2-\nu^2}d\nu' - \mathrm{Re}\,\mathcal{E}_2(\nu) - 2\,\mathrm{Re}\left[\mathcal{M}_3(\nu)-\mathcal{M}_3^0\right]$$

$$\mathrm{Re}\left[\mathcal{E}_1(\nu)-\mathcal{E}_3(\nu)\right] = -\left[2\left(\frac{e}{2M}\right)^2-\frac{e}{M}\mu\right]\nu + \frac{2\nu^3}{\pi}P\int_{\nu_0}\frac{d\nu'}{\nu'}\cdot\frac{|E_1^+|^2+|E_1^0|^2}{\nu'^2-\nu^2} \qquad (62)$$

$$\mathrm{Re}\;C(\mathcal{M}_3\mathcal{E}_2) = \mathrm{Re}\,C(\mathcal{E}_2\mathcal{M}_3) = \frac{2\nu^3}{\pi}P\int_{\nu_0}^{\infty}\frac{d\nu'}{\nu'}\;\frac{(\mathrm{Re}\,E_2^*M_3)^+ + (\mathrm{Re}\,E_2^*M_3)^0}{\nu'^2-\nu^2}$$

Since the considered energy region is near the production threshold of a single meson, the processes in which more than one meson or other particles are created, are neglected.

If the future analysis indicates that $\mathrm{Im}\,\mathcal{M}_2 \neq 0$ it is not difficult to take it into consideration.

9.

The energy of γ -rays in the laboratory system and that in the center-of-mass system ν_c are connected by the relation

$$\nu_c = \frac{\nu}{\sqrt{1+\frac{2\nu}{M}}}$$

With the help of the expression for the total energy meson in the center-of-mass system

$$\omega_c = \frac{\nu + m_\pi^2/2M}{\sqrt{1+2\nu/M}}$$

it is easy to obtain the expression for the quadratic impulse of the meson produced.

$$q_c^2 = \omega_c^2 - m_\pi^2 = \frac{(\nu-\nu_0)\,(\nu+\nu_0-m_\pi^2/2M)}{1+\frac{2\nu}{M}} \quad ; \quad \nu_0 = m_\pi\left(1+\frac{m_\pi}{2M}\right)$$

With an accuracy (better than 7.5%) one may put it equal to

$$q_c^2 \cong \frac{\nu^2-\nu_0^2}{1+\frac{2\nu}{M}} \qquad (63)$$

then

$$\frac{q_c}{\nu_c} = \frac{\sqrt{\nu^2-\nu_0^2}}{\nu}$$

On account of the mass difference between proton and neutron, π^+ and π^0 meson, the scattering amplitude near threshold has some fine structure. For a reliable numerical calculation it is necessary to have a more detailed analysis of the photoproduction data than that we have at the present time. Since we hope only to obtain the order of magnitude of the anomaly near the threshold, we restrict ourselves by taking into consideration the production mainly of π^+ meson. Quantities E_1, M_3 and E_2 were taken from the analysis of Watson and others |16|. It is assumed that π^0 are produced only in the resonant state $J = 3/2$ (through E_2, M_3). The connection between the amplitude of photoproduction and the phase shift in π-N scattering is now well-known.* [See, for example |16,17|] In the enerty region, where one can neglect the mass difference of and after summing over the contribution of π^0 and π^+ production, the terms containing the phase shifts cancel each other. For example

$$|M_3^+|^2 + |M_3^0|^2 = 6\,|M_{33}|^2 + \frac{3}{4}\,|M_{13}^{(1)} - 2\,\delta M_{13}^{(1)}|^2 \approx 6\,|M_{33}|^2$$

Dispersion integrals are performad analytically after approximating A by some simple expressions.

Let the energy be measured in the units $\nu_0 = 150$ Мэв We approximate $|E_1|^2$ in the energy region $1 \leq \nu \leq \nu_L = 2{,}20$ by the following expression

$$|E_1|^2 \approx |E_1^+|^2 = A\,\frac{q_c}{K_c} = A\,\frac{\sqrt{\nu^2-1}}{\nu} \quad ; \quad A = (3{,}3 \cdot 10^{-15})^2\,\frac{\text{см}^2}{\text{стер.}}\,\nu_0 = 0{,}54\,\frac{e^2}{M} \tag{64}$$

It is just the contribution of $|E_1|^2$ in the dispersion integral which leads to the cusp dependence of the real part of amplitude on energy. As can be seen from (62) the contribution of $|E_1|^2$ is characterized by two integrals

$$\frac{2}{\pi}\,\nu^2\,P\int_1^{\nu_L} \frac{|E_1|^2}{\nu'^2-\nu^2}\,d\nu' \qquad \text{and} \qquad \frac{2}{\pi}\,\nu^3\,P\int_1^{\nu_L} \frac{|E_1|^2\,d\nu'}{\nu'(\nu'^2-\nu^2)} \tag{65}$$

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*In the general case it requires to parametrize the 3 x 3 S matrix, which describes[5] both the π-N scattering, the photoproduction and the scattering of γ-rays by nucleons. The part of phase shift in π-N scattering due to the destroy of isotopic spin invariance may cause a change in the scattering of γ -rays.

Substituting (64) into(65) gives

$$\frac{2}{\pi}\nu^2 P\int_1^{\nu_1}\frac{|E_1|^2}{\nu'^2-\nu^2}d\nu' = \frac{2A}{\pi}\begin{cases}\operatorname{arctg}\sqrt{\nu_1^2-1}-\frac{1}{2}\sqrt{\nu^2-1}\,\ln\left|\frac{\sqrt{\nu_1^2-1}+\sqrt{\nu^2-1}}{\sqrt{\nu_1^2-1}-\sqrt{\nu^2-1}}\right| & (\nu>1)\\ \operatorname{arctg}\sqrt{\nu_1^2-1}-\sqrt{1-\nu^2}\,\operatorname{arctg}\sqrt{\frac{\nu_1^2-1}{1-\nu^2}} & (\nu<1)\end{cases} \tag{66}$$

and

$$\frac{2\nu^3}{\pi}\int_1^{\nu_1}\frac{|E_1|^2}{\nu'(\nu'^2-\nu^2)}d\nu' = \frac{2A\nu}{\pi}\begin{cases}\sqrt{\frac{\nu_1^2-1}{\nu_1^2}}-\frac{\sqrt{\nu^2-1}}{2\nu}\ln\left|\frac{\nu\sqrt{\nu_1^2-1}+\nu_1\sqrt{\nu^2-1}}{\nu\sqrt{\nu_1^2-1}-\nu_1\sqrt{\nu^2-1}}\right| & (\nu>1)\\ \sqrt{\frac{\nu_1^2-1}{\nu_1^2}}-\sqrt{\frac{1-\nu^2}{\nu^2}}\operatorname{arctg}\sqrt{\frac{\nu^2(\nu_1^2-1)}{\nu_1^2(1-\nu^2)}} & (\nu<1)\end{cases} \tag{67}$$

From (66), (67), (64), (62) and (56) it can be seen that both the derivative of the imaginary part of the quantities R_1 and R_3 (from the side $\nu>1$) and that of their real parts (from the side $\nu<1$) turn out to be infinity at the threshold. At the same time their derivatives from the opposite side are finite. This is a very general conclusion. Therefore, dispersion relations automatically lead to the threshold anomaly which has been studied carefully with the R-matrix formalism by Wigner, Baz', Breit, Okun', Adair, Newton and many others*.

The application of the dispersion relation permits a more detailed analysis of the influence of the inelastic processes on elastic scattering (or reaction) in some energy region. Furthermore, the anomaly in the neighbourhood of the threshold (Such a "local effect" is the only effect which can be obtained by a direct analytic continuation without using dispersion relation) is shown to be only a part of the general influence of inelastic process on the energy dependence of the elastic scattering amplitude.

From the example of scattering of γ quanta by nucleon one may see how the existence of the inelastic process photoproduction in the energy region $\nu>\nu_0$ can influence the characteristics of elastic scattering in the region with $\nu<\nu_0$ (the deviation from the Powell formula, or from (2) at $\nu<\nu_0$).

The form of the nonmonotonic dependence in (66) and (67) is characterized by a steep drop of the value of the function in the region $\nu<\nu_0$ (with infinity derivati-

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* Authors will return to the application of dispersion relation to this general problem in a subsequent communication.

ve at $\nu = \nu_{0-}$) and a slow increase in the region $\nu > \nu_0$ (with a finite derivative at $\nu = \nu_{0+}$)

10.

In the energy region 330-550 MeV ($2.2 < \nu < 3.34$) the quantity $|E_1|^2$ is approximated by

$$|E_1|^2 = 1{,}27\,\frac{e^2}{M}\,(1-0{,}175\,\nu)^2 \tag{68}$$

The contribution of photoproduction in this energy region to the real part of the scattering amplitude near or below the threshold is shown to be small.

A previous analysis of photoproduction and, in particular, the result of Akiba and Sato indicates that

$$|M_3|^2 \cong |E_2|^2 \cong Re\,(E_2^* M_3) \cong 6\,|M_{33}|^2 \tag{69}$$

For our estimates we shall use (69). The polarization of the recoil nucleon is especially sensitive to this assumption. In the energy region $1<\nu<2{,}0$ the quantity $|M_{33}|^2$ is approximated by

$$|M_{33}|^2 = B_0\,\nu\,(\nu^2-1)^{3/2} \quad ; \quad B_0 = 0{,}009\,\frac{e^2}{M} \tag{70}$$

It follows then

$$|M_3|^2 = 6\,|M_{33}|^2 = B\,\nu\,(\nu^2-1)^{3/2} \quad ; \quad B = 0{,}054\,\frac{e^2}{M}$$

The contribution of this term corresponding to the photoproduotion of meson in $P_{3/2}$ state is given by the following dispersion integrals

$$\frac{2\nu^2}{\pi}\,P\int_1^{\nu_1} \frac{|E_2|^2}{\nu'^2-\nu^2}\,d\nu' = \frac{2B\nu^2}{\pi}\left[\frac{1}{3}(\nu_1^2-1)^{3/2} + (\nu_1^2-1)^{1/2}(\nu^2-1) + \begin{cases} -\frac{1}{2}(\nu^2-1)^{3/2}\ln\left|\frac{\sqrt{\nu_1^2-1}+\sqrt{\nu^2-1}}{\sqrt{\nu_1^2-1}-\sqrt{\nu^2-1}}\right| & (\nu>1) \\ (1-\nu^2)^{3/2}\,\mathrm{arctg}\sqrt{\frac{\nu_1^2-1}{1-\nu^2}} & (\nu<1) \end{cases}\right] \tag{71}$$

and

$$\frac{2\nu^3}{\pi} P\int_1^{\nu_1} \frac{d\nu'}{\nu'} \frac{|E_2|^2}{\nu'^2-\nu^2} = \frac{B}{\pi}\nu^3\left[\nu_1(\nu_1^2-1)^{3/2} + (\nu^2-\tfrac{3}{2})\ln\left|\frac{\nu_1+\sqrt{\nu_1^2-1}}{\nu_1-\sqrt{\nu_1^2-1}}\right| - \right.$$

$$\left. -(\nu^2-1)\begin{cases} 2\sqrt{\frac{1-\nu^2}{\nu^2}}\,\mathrm{arctg}\sqrt{\frac{\nu_1^2-1}{1-\nu^2}\cdot\frac{\nu^2}{\nu_1^2}} & (\nu<1) \\ \sqrt{\frac{\nu^2-1}{\nu^2}}\,\ln\left|\frac{\nu\sqrt{\nu_1^2-1}+\nu_1\sqrt{\nu^2-1}}{\nu\sqrt{\nu_1^2-1}-\nu_1\sqrt{\nu^2-1}}\right| & (\nu>1)\end{cases}\right] \tag{72}$$

in which the second derivative with respect to energy tends to infinity at $\nu=1$ (from the side $\nu<1$)

In the energy region $2<\nu<3.34$

$$6|M_{33}|^2 = 2.17\frac{e^2}{M}(1-0.244\,\nu)^2 \tag{73}$$

contribution from (70) and (73) are given by integrals of the form

$$J_1(\nu) = \frac{2\nu^3}{\pi}P\int_{\nu_1}^{\nu_2}\frac{\alpha+\beta\nu'+\gamma\nu'^2}{\nu'(\nu'^2-\nu^2)}d\nu' =$$

$$= \frac{\nu}{\pi}\ln\left\{\left(\frac{\nu_2-\nu}{\nu_1-\nu}\right)^{\alpha+\beta\nu+\gamma\nu^2}\left(\frac{\nu_2+\nu}{\nu_1+\nu}\right)^{\alpha-\beta\nu+\gamma\nu^2}\left(\frac{\nu_1}{\nu_2}\right)^{2\alpha}\right\} \tag{74}$$

and

$$J_2(\nu) = \frac{2\nu^2}{\pi}P\int_{\nu_1}^{\nu_2}\frac{d\nu'(\alpha+\beta\nu'+\gamma\nu'^2)}{\nu'^2-\nu^2} = \frac{\nu}{\pi}\left\{2\gamma\nu(\nu_2-\nu_1)+\ln\left[\left(\frac{\nu_2-\nu}{\nu_1-\nu}\cdot\frac{\nu_1+\nu}{\nu_2+\nu}\right)^{\alpha+\gamma\nu^2}\left(\frac{\nu_2^2-\nu^2}{\nu_1^2-\nu^2}\right)^{\beta\nu}\right]\right\} \tag{75}$$

11.

The energy dependence of the real part of the amplitude $R_1 \dots R_6$ (in laboratory system), calculated with the help of the dispersion relations is shown in fig. 1-3. The half widths of R_1 and R_3 are equal to $\frac{1}{10}\nu_0$ and $\frac{1}{20}\nu_0$, respectively, which are determined mainly by the quadratic ratio of the real part of the amplitude to the coefficient A in (64)

$$\mathcal{E} = 1-\nu = \frac{1}{8}\left(\frac{\mathrm{Re}\,R}{A}\right)^2 \tag{76}$$

In the general analysis of the nonmonotonic dependence near threshold Baz' gave a restriction for the width of the peak $kR_0 \ll 1$ (where R_0 is the radius of interaction). In the present paper a more accurate criterion (76) follows in a natural way.

The influence of an inelastic process on $Re\,R_3$ is very strong but the contribution of $Re\,R_3$ in the observable quantities is rather small, so that the experimental study of the energy dependence of $Re\,R_3$ seems to be quite difficult.

The energy dependence of $Re\,R_4$ and $Re\,R_6$ are determined with great accuracy by the formula (4).

The nonvanishing values of $Re\,R_2$ and $Re\,R_5$ are due completely to the inelastic process, although the photoproduction in the S-state does not contribute to these quantities.

Differential cross-section (in the center-of-mass system)(12)can be written in the form

$$I_0(\theta,\nu) = A_0(\nu) + A_1(\nu)\cos\theta + A_2(\nu)\cos^2\theta + A_3(\nu)\cos^3\theta \qquad (77)$$

The numerical results for the energy dependence of differential cross section at 90° and 0° are presented in figs. 4 and 5, in which $I_0(90°,\nu)$ is calculated in the laboratory system while $I_0(0°,\nu)$ is calculated both in the laboratory system and in the center of mass system.

The energy dependence $I_0(0°,\nu)$ has been calculated by Cini and Strofollini. Our result gives a agreement near the threshold region. Outside this region the agreement is very good.

Our result for $I_0(90°,\nu)$ in the energy region near 200 MeV also agrees with the other published results |13,18|. In the present work the energy region near the threshold is considered more carefully.

On fig.4. the total scattering cross-section $\sigma_s/4\pi$ is presented. The cross section according to formula (16) and (18) is also presented for comparison. The local effect in the neighbourhood of the threshold is practically nonobservable, but is seen more clearly in the difference

$$\sigma_s/_{4\pi} - I_0(90°)$$

or in the energy dependence of $A_2(\nu)$ (fig.6 and 7). In order to obtain the experimental data of $A_2(\nu)$ it is sufficient to measure the cross section $I_0(\theta,\nu)$ at $\theta=45°, 90°$ and $135°$ with such an accuracy that it is possible to study the energy dependence of the difference

$$I_0(45°) + I_0(135°) - 2I_0(90°)$$

It is interesting to note the energy dependence of the polarization of a recoil nucleon. Below the threshold the imaginary parts of $R_1 \dots R_6$ in the e^2-approximation vanish, then the righthand side of (13) becomes zero and the polarization of the recoil proton also vanishes. On account of the invariance under time reversal the cross section for scattering of γ- quanta from polarized proton will not differ from $I_0(\theta)$ below the threshold.

Above the threshold for production of π- meson, polarization of the recoil proton differs from zero Numerical result for the energy dependence (energy in l.s.) of polarization at $\theta = 90^0$ (angle in c.m.s.) is shown in fig. 9. It is seen, that in a rather large energy region from 180-220 MeV the polarization reaches $20 \div 25\,\%$.

The magnitude of polarization is sensitive to the assumptions made in the analysis of the photoproduction, and especially to the assumption (69). It follows that the experimental study of the polarization of the recoil nucleon might give useful information on the photoproduction of mesons.

In the expression (20), the contribution of the term $|R_4|^2$ is decreased in comparison with that in $I_0(\theta)$ and the term containing $|R_3|^2$ has a minus sign, therefore the energy dependence of $\langle T_{22}(90^0)\rangle$ near threshold has a deep pit (Fig.7).

12.

A detailed study of the scattering of γ-rays by nucleon near the threshold of meson production with the help of dispersion relation permits us to see how the photoproduction in the S-state gives rize to the anomaly in the neighbourhood of the threshold. The scattering of γ-rays by nucleon and nucleons, the photodisintegration of deuteron and other nucleons are examples of processes, in which the inelastic process has a very large influence on the energy dependence of the elastic amplitude in a wide energy region. Although the local effect for a series of observable quantities in the case of γ-N scattering is shown to be great, however in the experimental study of the local anomaly it is necessary to have a good experimental condition. Especially this refers to the high energy resolution since the halfwidth of the corresponding cusp equals approximately $5 \div 10$ MeV.

The experience obtained in the numerical evaluation shows that the contribution of other states may smear out the sharp energy dependence of the observable quantities near the threshold. Therefore, for the experimental study of such an effect it is more favourable to have cases not at very high energy and with particles of low spin.

The scattering of γ rays by deuteron near the threshold of photodisintegration will be reported in other place. The local effect in this case seems to be not very small.

From the point of view of general influence of one process on another it is of interest to study the photodisintegration of deuteron in the energy region near and below the threshold of meson production. It seems to be possible to study the well-known "resonance" dependence of the cross-section in photodisintegration by the method of dispersion relation.

Local effect in photodisintegration of deuteron might arise also from the reaction

$$\gamma + d \rightarrow d + \pi^0$$

It is usually adopted, that at very high energy the γN scattering is completely determined by the inelastic process (i.e. the imaginary parts of the amplitude). On account of this it may be very interesting to study the γN scattering and especially the polarization of the recoil nucleon near the production threshold of new particles like

$$\gamma + N \rightarrow Y + K$$

and many other processes.

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* J. Smorodinsky kindly informed us that G. Ustinova had considered cusp problem in γ-N scattering near the threshold by Baz' method (in press).

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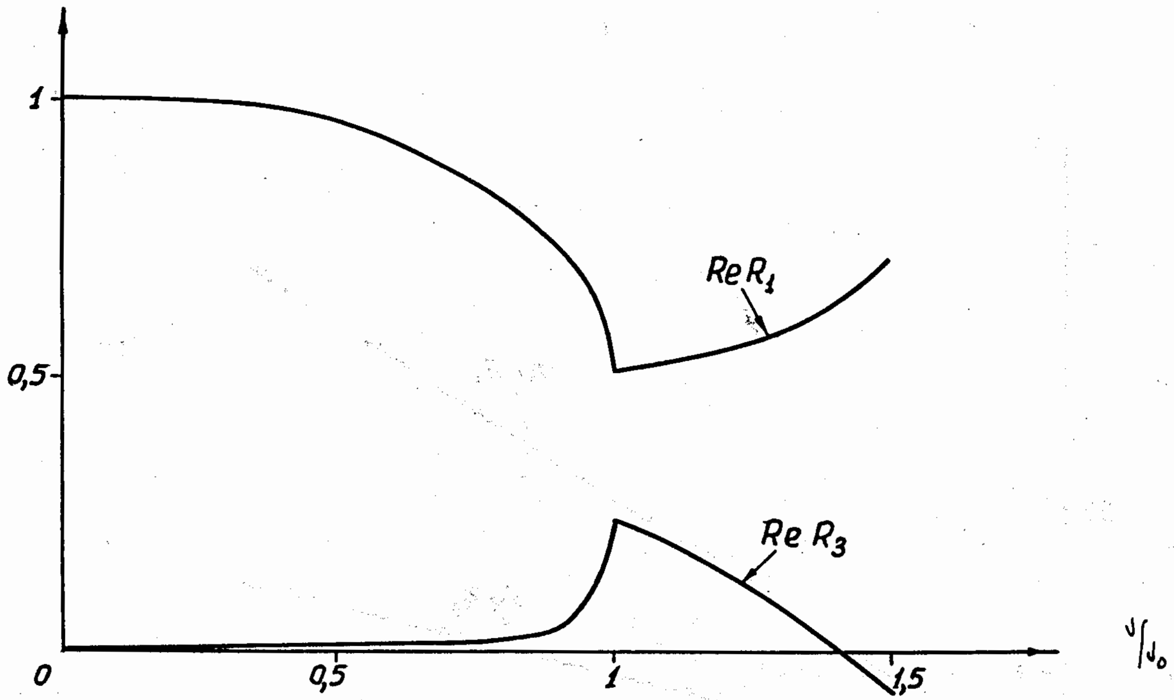

Fig. 1.

The dependence of $Re\,R_1$ (the upper curve) and of $Re\,R_3$ upon energy. The values of the functions are expressed in the fractions of e^2/M.

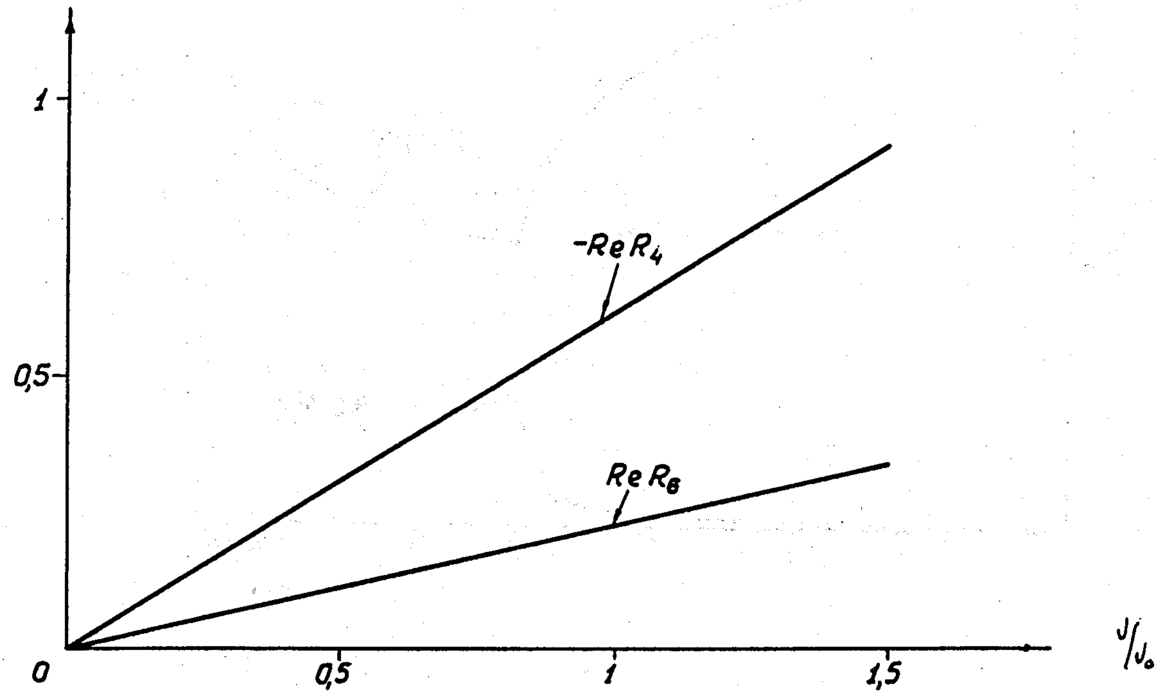

Fig.2

The dependence of $Re\,R_4$ (the upper curve) and of $Re\,R_6$ upon energy. The values of the functions are expressed in the fractions of e^2/M.

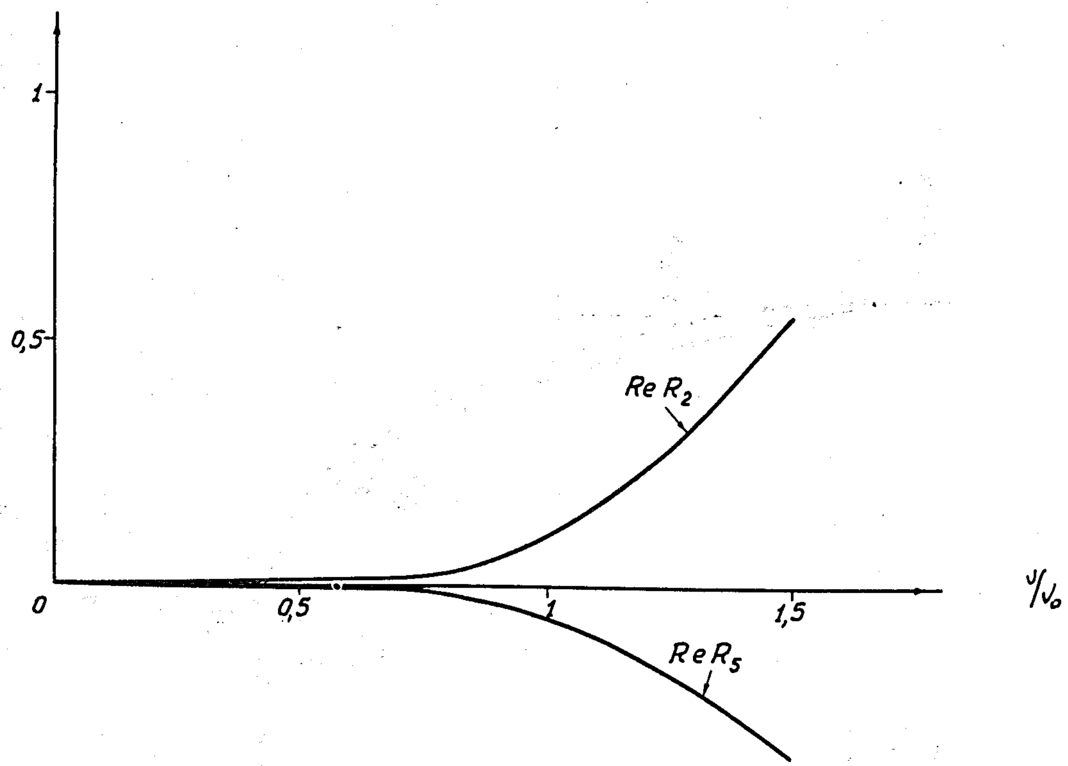

Fig. 3.

The dependence of $Re\, R_2$ (the upper curve) and of $Re\, R_5$ upon energy. The values of the functions are expressed in the fractions of e^2/M.

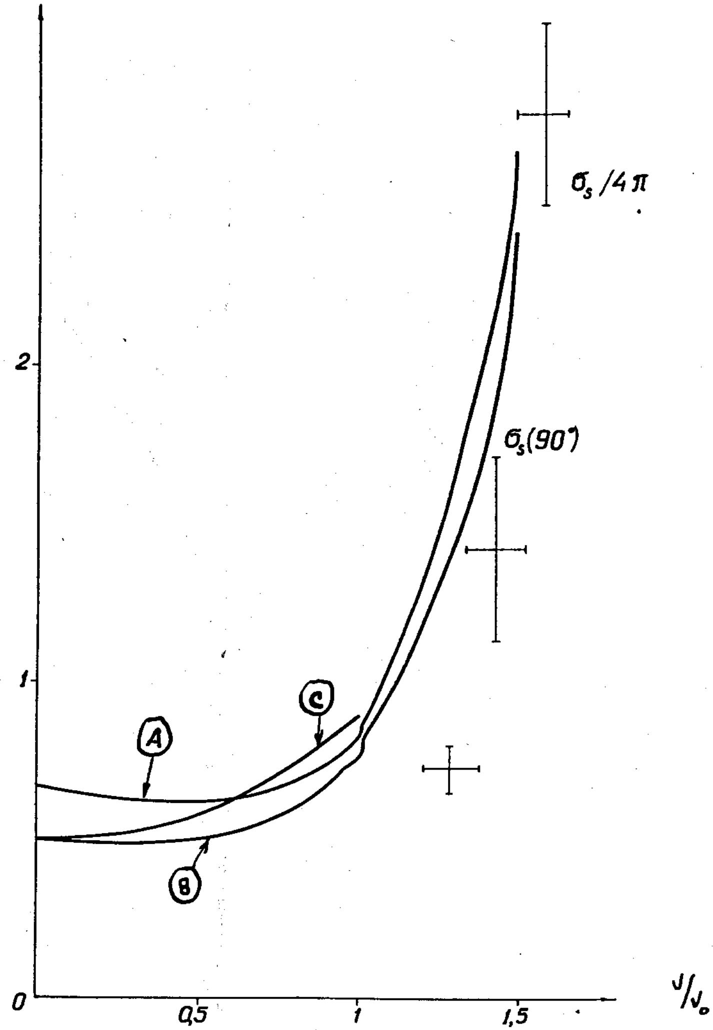

Fig. 4.

The dependence of the differential cross section I_0 (90^0) upon energy (the curve A), of the total scattering cross section $\sigma_s/4\pi$ (the curve B), of the differential scattering cross section without the account of the dispersion part (the curve C). The values of the functions are expressed in the fractions of $(e^2/M)^2$.

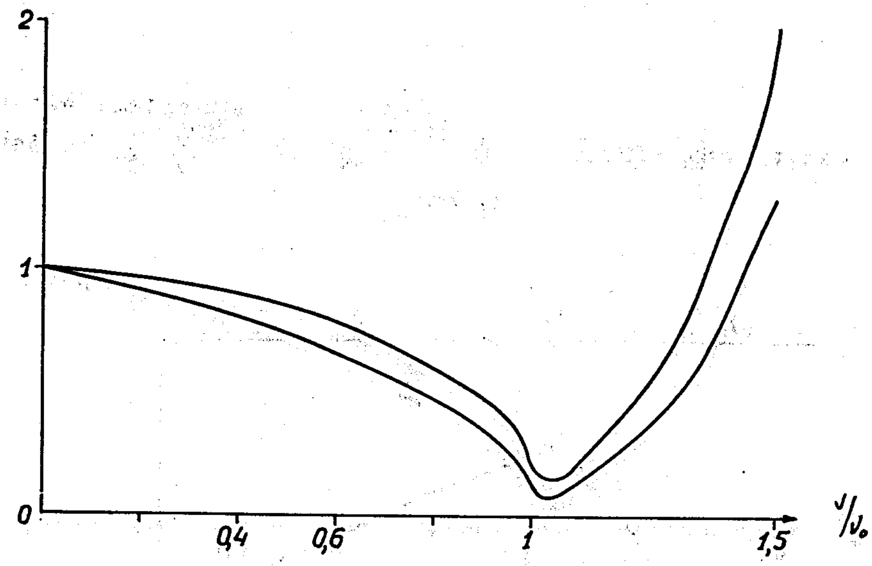

Fig. 5.

The dependence of the differential cross section $I_0\ (0^\circ)$ upon energy in the lab. system. The upper curve shows the cross section in the lab. system, the lower one – in the centre of mass system. The experimental data are taken from [18]. The values of the functions are expressed in the fractions of $\left(\frac{e^2}{M}\right)^2$.

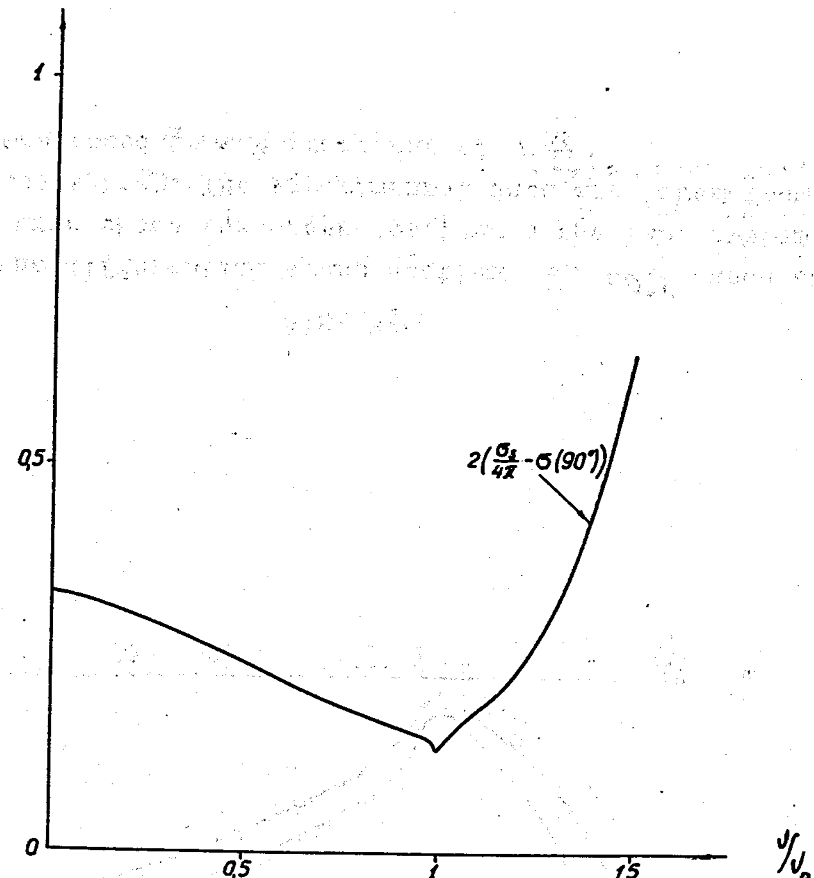

Fig. 6.

The dependence of $\sigma_s/4\pi - I_o(90^\circ)$ upon energy. The values of the function are expressed in the fractions of $(\frac{e^2}{M})^2$.

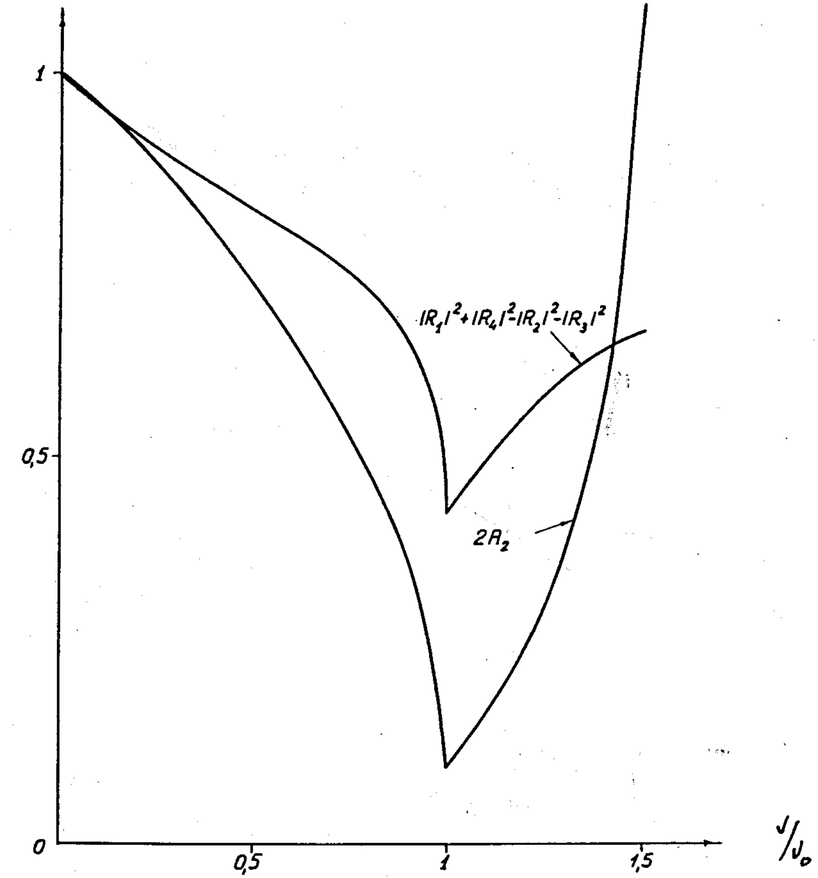

Fig. 7.

The lower curve shows the dependence upon energy of the $2A_2(\nu)$ coefficient at $\cos^2\theta$ in the cross section. The upper curve shows the dependence upon energy of the photon polarization $2\langle T_{22}(90^\circ)\rangle$. The values of the functions are expressed in the fractions of $\left(\frac{e^2}{M}\right)^2$.

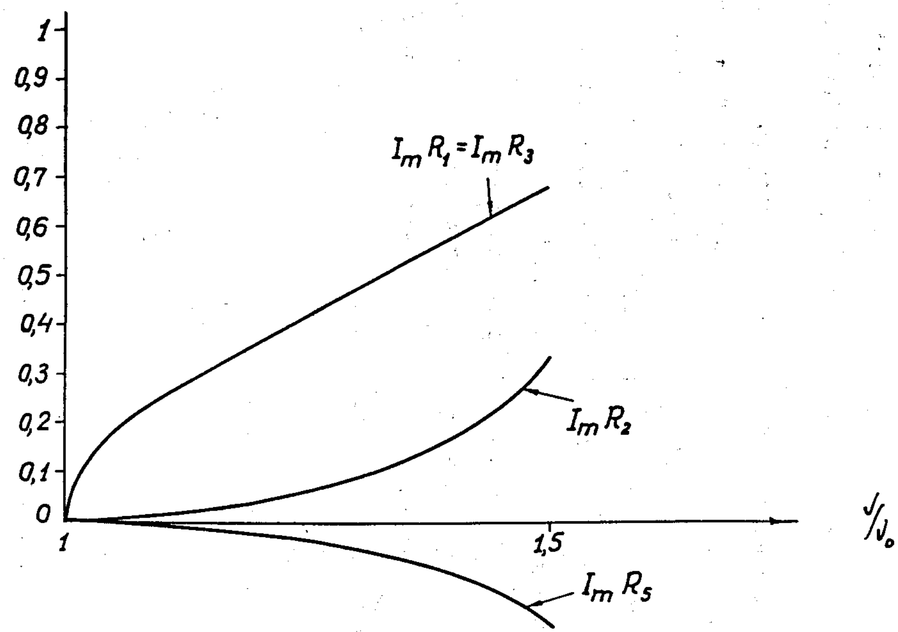

Fig. 8.

The dependence of the imaginary parts of the amplitudes upon energy. The values of the functions are expressed in the fractions of $\left(\frac{e^2}{M}\right)^2$.

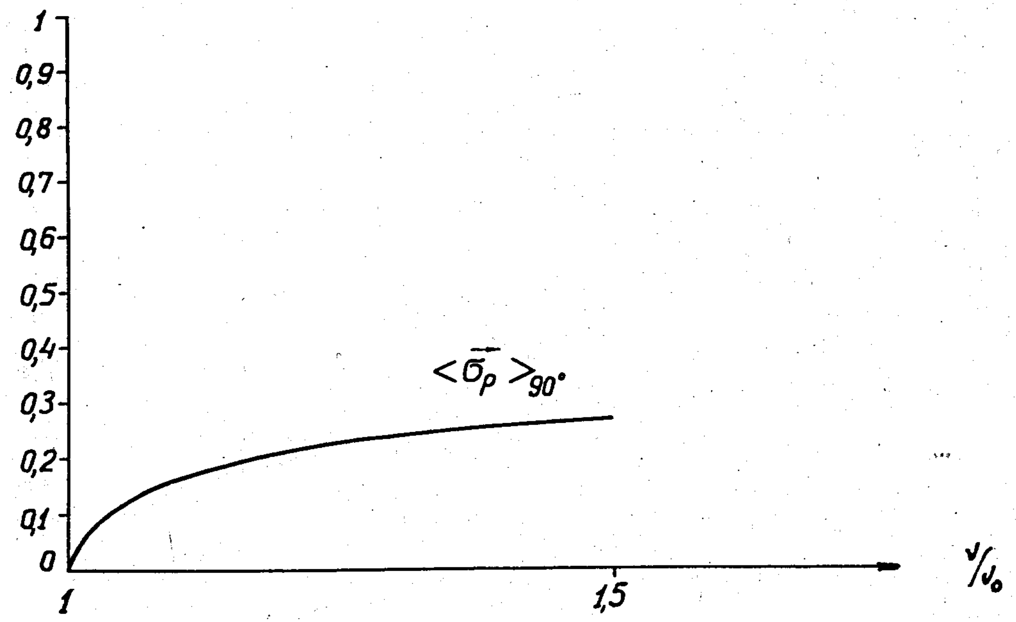

Fig. 9.

The dependence of the recoil proton polarization at $\Theta = 90^0$ upon energy.