JOINT INSTITUTE FOR NUCLEAR RESEARCH

Laboratory of Theoretical Physics

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ЖЭТФ, 1959, т37, в4, с1139-1142.

Dubna 1959

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MINIMUM ERROR IN THE EXPERIMENTAL DETECTION OF AN ASYMMETRY

Many experiments with elementary particles aiming at the proof or disproval of the conservation of spatial parity or at the proof of the existence of spin a particle come to the detection of a specific asymmetry in the distribution of particles emerging from some reaction. It seems worth while to estimate the probability of an error made in the deduction of a conclusion about the presence or absence of an asymmetry from such an experiment.

In the observation of an asymmetry all particles are devided (during the experimental itself or during the treatment of data) into such two groups that in the absence of the asymmetry of the considered process a particle has equal probabilities to get into each group. Usually the probability of the detection of a particle in one of the groups does not depend on the numbers of particles that are already piled in these groups. Therefore, if the data are corrected for possible systematic errors, the numbers of particles in the two groups n_+ and n_- have Poisson distribution with means $n(\frac{1}{2} \pm \frac{F}{2})$, where $n = n_+ + n_-$ and F is a constant characterising the intensity of the interaction that causes the violation of the symmetry.

It may be shown that for $n_+ > n_- \gg 1$ the ratio

$$t = \frac{n_+ - n_- - 1}{\sqrt{n}} \tag{1}$$

has the Student's t–distribution with f degrees of freedom, where

$$\frac{1}{f} = \left(\frac{n_+}{n}\right)^2 \frac{1}{n_+ - 1} + \left(\frac{n_-}{n}\right)^2 \frac{1}{n_- - 1} \approx \frac{1 + 2\,\frac{(n_+ - n_-)^2}{n^3}}{n - 2} \tag{2}$$

For $n \gg 1$ the ratio t tends to $F \cdot \sqrt{n}$.

At $F = 0$ the ratio (1) satisfies within the probability $1 - \alpha$ the inequality

$$t < t_{1-\frac{\alpha}{2}}(f) \tag{3}$$

where $t_P(f)$ is a fractile of the t–distribution, that is, such a number that the probability of the fulfillment of the inequality $t < t_P$ is equal to P. If the experimentally found value of t does not satisfy the inequality (3), the difference of F from zero is considered as significant, and the hypotheses about a casual asymmetry is rejected.

But there remains the problem of the choice of the level of significance α that is equal to the probability of the error of the first kind when a casual deviation of t from $t = 0$ is taken for a deviation caused by the fact that $F \neq 0$. In order to solve this problem one should take into account the probability of an error of the second kind, β, when $F \neq 0$, but a small value of t is considered as an indication that $F = 0$. The values of β are given in special tables [1]. For $f \gg 1$

$$\beta = 1 - \Phi\left[u_{\alpha/2} + \lambda\left(1 - \frac{1+u^2_{\alpha/2}}{4f}\right)\right] - \Phi\left[u_{\alpha/2} - \lambda\left(1 - \frac{1+u^2_{\alpha/2}}{4f}\right)\right] \tag{4}$$

where $\lambda = F\sqrt{n}$ and $\Phi(X) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\{-\tfrac{1}{2}u^2\}du$ is the cumulative distribution function of the normal distribution.

Now we choose α in such a way as to make the sum of the probabilities of the errors of the two kinds $\alpha+\beta$ minimum. It may be easily proven that for the approximation (4)

$$(\alpha+\beta)_{min} = 1 - \Phi\left[u_{\alpha/2} + \lambda\left(1 - \frac{1+u^2_{\alpha/2}}{4f}\right)\right] - \Phi\left[u_{\alpha/2} - \lambda\left(1 - \frac{1+u^2_{\alpha/2}}{4f}\right)\right] + \tag{5}$$
$$+ 2\,\Phi(u_{\alpha/2}),$$

where

$$u_{\alpha/2} = u + \frac{1}{4f}\left[u^3 - u + \frac{1+u^2}{\sqrt{1-e^{-\lambda^2}}}\right], \qquad u = -\frac{1}{\lambda}\,\mathrm{Arch}\, e^{\frac{\lambda^2}{2}}. \tag{6}$$

The last term of (5) is equal to the optinium value of α.

Fig. 1 shows schematically the origin of the errors of the two kinds (before the minimization of $\alpha + \beta$). In the upper half of fig. 2. the dependence of α and $(\alpha + \beta)_{min}$ on $\lambda = F \cdot \sqrt{n}$ and f is shown; in the lower part the corresponding values of $t_{\alpha/2} = t_{1-\alpha/2}$ are given. By means of these plots it is possible to determine the minimum value of the probability of the errors of the first and second kinds if n_+ and n_- are known. This probability turns out to be a function of that value of F which the experimenter tries to distinguish from the value $F=0$.

On the contrary, having fixed some value of F and the upper bound of the probability of an error, one may find the number of observation $n = \left(\frac{\lambda}{F}\right)^2$ that is necessary in order to distinguish between F and zero.

An example: if $n = 100$, $F = 0.1$ is considered as present for $t > 1.098$ and as absent for $t < 1.098$, the probability to make an error being equal to 82%; if $n = 6400$, $F = 0.1$ is rejected for $t < 4.087$ and the probability of an error is equal to 0.018%. Another example: in order to decide whether is an asymmetric interaction with an intensity $F = 0.01$ and to guarantee that the probability of an erroneous decision be less than 1%,, one should make $n = \left(\frac{5.30}{0.01}\right)^2 = 280\,000$ observations. The third example: it is found experimentally that $n_+ = 5080$ and $n_- = 4920$; we find $t = 1.59$ and $f = 9998$; whence we conclude that the values $F > 0.026$ are rejected, the probability of an error in the statement about the presence of $F = 0.02$ being equal to 45%, about the presence of $F=0.002$ - equal to 99% and about the absence of $F=0.05$ - equal to 1.5%. If $n_f=5200$ and $n_-=4800$, $t=3.99$ and $f=9998$; the values $F>0.078$ are rejected, the probability of an error in the decision about the presence of $F=0.07$ being equal to 0.08% and about the presence of $F=0.01$ - equal to 80%.

The author is indepted to R.M. Ryndin who drew his attention to the usefulness of the considered problem.

Reference

1. G.J. Resnikoff and G.J. Lieberman, Tables of the non-central t - distribution. Stanford, 1957.

Received by Publishing Department
on 9 June, 1959.

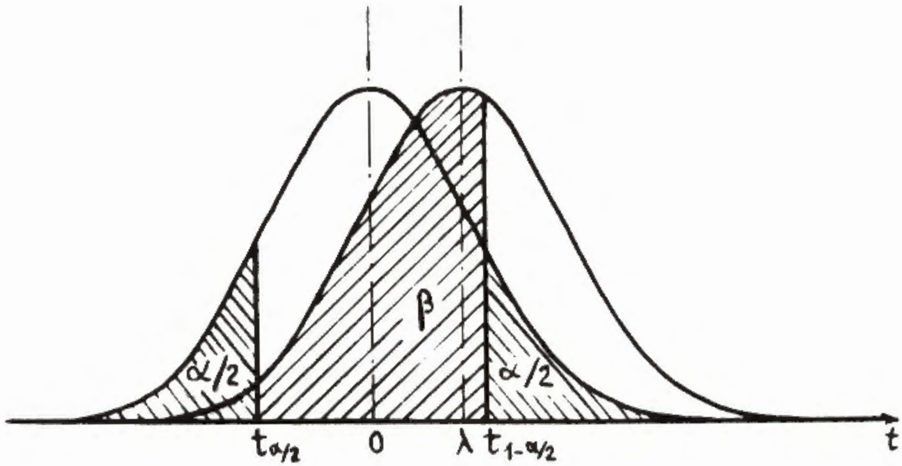

Fig. 1.

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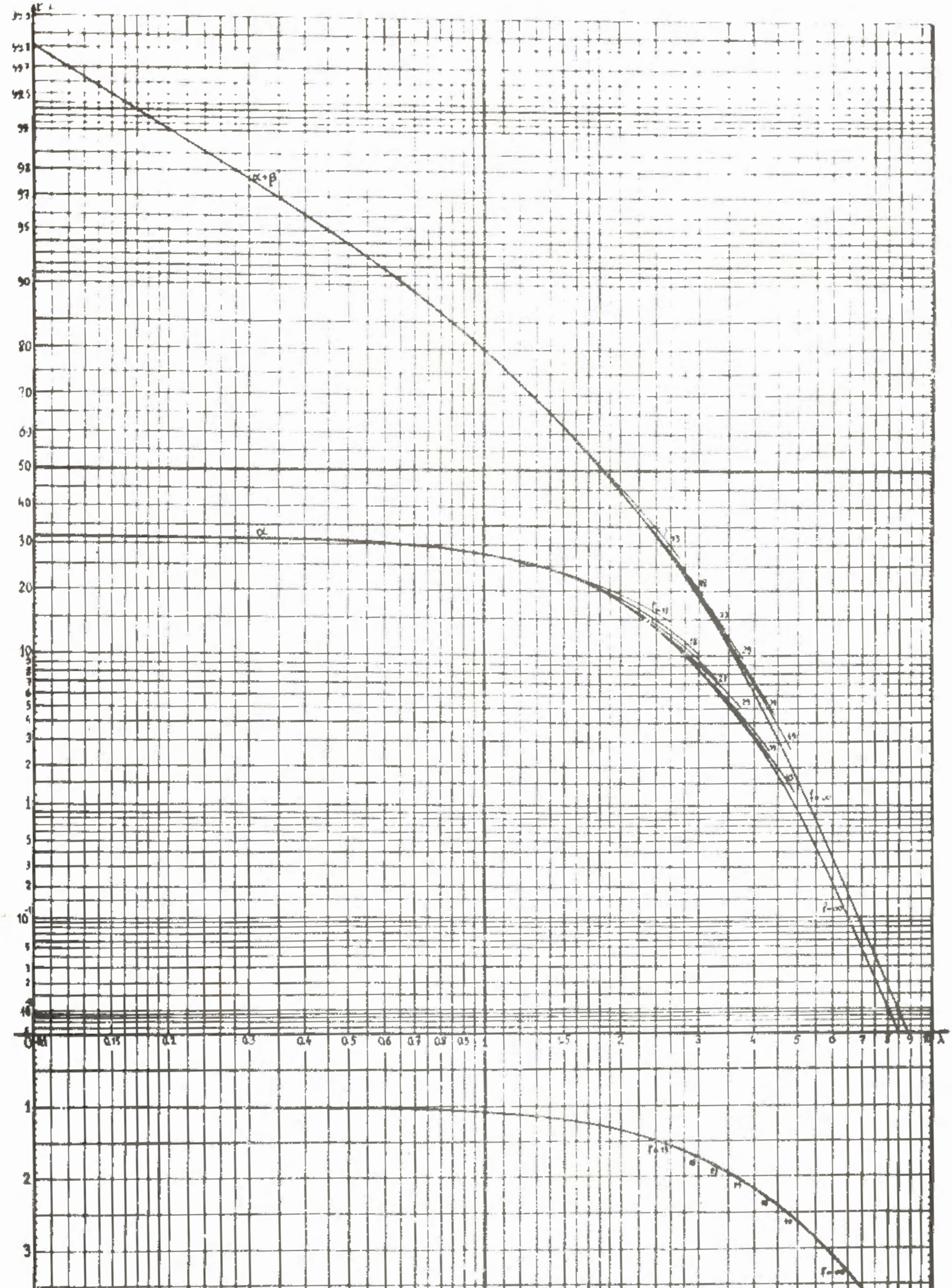

Fig. 2.