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Progr. theor. Phys., 1960, v 23, n 2, p 197.

D u b n a 1959

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A b s t r a c t

The Hamiltonian of a dynamical system of Fermi-particles is transformed by means of the general unitary transformation proposed by N.N. Bogolubov. From this Hamiltonian we obtain the approximate second quantization (a.s.q.) Hamiltonian introducing the Bose - amplitudes with two indices. This Hamiltonian is diagonalized and the collective oscillations considered, especially for the pairs of particles with parallel spins. For the forced collective oscillations the paramagnetic term in the Hamiltonian is also taken into account. This term leads to the additional "spin" current, connected with the elementary excitations with spin-moment $\pm$ 1. If the transfer momentum tends to zero, this current vanishes as the terms omitted in obtaining the Meissner-Ochsenfeld effect.

I. Introduction

In the paper of N.N. Bogolubov[1)] using the self-consistent field method, among others, the problem of collective oscillations of a dynamical system of Fermi-particles, and the problem of electrodynamics of superconducting state are studied. It is proved that in the approximation used, the collective oscillations split in two independent branches: a) for the elementary excitations with spin moment 0, b) for the elementary excitations with spin moment $\pm$ 1. The collective oscillations of the first branch are investigated (see also the monography of the theory of superconductivity N.N. Bogolubov, V.V. Tolmachov, D.V. Shirkov |2|), and the results are applied to the electrodynamics of superconducting state, obtaining the Meissner effect.

In this paper the problems investigated in the paper quoted above, are considered with the approximate second quantization method[3),2)]. The collective oscillations of second branch, connected with the elementary excitations with spin moment $\pm$ 1 are studied. Moreover, the additional term in the Hamiltonian which gives the energy of magnetic spin moment in a magnetic field is taken into account. This leads to the additional "spin" current.

In order to use the approximate second quantization method, the products of Fermi-amplitudes $\alpha_\nu \alpha_\mu$ are replaced by the Bose-amplitudes $\beta_{\nu\mu}$. It is proved that the secular equation for the energy of collective oscillations obtained by means of this method is identical with the equation obtained in paper |1|. Furthemore are considered the forced collective oscillations for the case of weak external electromagnetic field. In the Hamiltonian besides the terms which depend linearly on the vector-potential of the external field (see also |1|) is also taken into account the paramagnetic term proportional to the produot $\vec{s}.\vec{\mathcal{H}}$ (where $\vec{s}$ is the spin vector of the particle, and $\vec{\mathcal{H}}$ the magnetic field vector).

This additional term of the Hamiltonian depend linearly on the operators $\beta_{\nu\mu}$. So we change to new operators $\beta_{\nu\mu}+C(\nu,\mu)$, where $C(\nu,\mu)$ are c-numbers. We determine them from the condition that in the transformed Hamiltonian, the terms linear in the operators $\beta_{\nu\mu}$ must vanish. It is shown that current and particle density are expressed by the functions $C(\nu,\mu)$ Thereby, from the approximate second quantization method like in the self-consistent field method [1] we obtain the electrodynamics of superconducting state and especially the Meissner cefect. - The paramagnetic term in the Hamiltonian leads to the "spin" current, connected with the elementary excitations with spin moment $\pm$ 1.

2. Approximate Second Quantization Hamiltonian

Consider a dynamical system of Fermi-particles with a Hamiltonian

$$H=\sum_{f,f'}T(f,f')a^{+}_{f}a_{f'}+\frac{1}{2}\sum_{\substack{f_1,f_2\\ f_1',f_2'}}U(f_1,f_2;f_2',f_1')a^{+}_{f_1}a^{+}_{f_2}a_{f_2'}a_{f_1'}=H_1+H_2\quad,$$

$$T(f,f')=I(f,f')-\lambda\delta(f-f') \tag{1}$$

where I is the Hamiltonian of the particle, U - the interaction energy, λ - the chemical potential, a_f, a^{+}_{f} the Fermi-amplitudes and f is a set of indices characterising one particle states; in particular $f=(\vec{p},\sigma)$, where $\vec{p}$ is a wave vector and σ a spin index.

Similiary as in the paper [2] we transform the Hamiltonian changing to new amplitudes

$$a_f=\sum_{\nu}(u_{f\nu}\alpha_{\nu}+v_{f\nu}\alpha^{+}_{\nu})\quad. \tag{2}$$

To secure the canonical character of transformation (2) the functions $\{u,v\}$ must be connected by orthonormality relations

$$\sum_{\nu}\{u_{f\nu}u^{*}_{f'\nu}+v_{f\nu}v^{*}_{f'\nu}\}=\delta(f-f')\quad,$$

$$\sum_{\nu}\{u_{f\nu}v_{f'\nu}+u_{f'\nu}v_{f\nu}\}=0\quad. \tag{3}$$

The functions $\{u,v\}$ we find from the additional equations obtained from the compensation principle of dangerous graphs [1]

$$\langle\alpha_{\nu_1}\alpha_{\nu_2}H\rangle_0=0\quad. \tag{4}$$

The expectaction value corresponds to the vacuum state C_0 in the α-representation

$$\alpha_{\nu}C_0=0\quad,\quad C^{*}_{0}\alpha^{+}_{\nu}=0\quad. \tag{5}$$

In the paper [1] it has been shown that the equations (4) are equivalent to the equation

$$\sum_{f}\left\{z(f_1,f)\phi(f,f_2)+z(f_2,f)\phi(f_1,f)\right\}+S(f_1,f_2)-\sum_{f}\left\{F(f,f_1)S(f,f_2)+F(f,f_2)S(f_1,f)\right\}=0 \tag{6}$$

where

$$z(f_1,f)=T(f_1,f)+\sum_{f',f''}\left\{U(f_1,f'';f',f)-U(f_1,f'';f,f')\right\}F(f'',f')\ ,$$

$$S(f_1,f_2)=\sum_{f_1',f_2'}U(f_1,f_2;f_2',f_1')\,\phi(f_1',f_2')\ ,\qquad F(f,f')=\sum_{\nu}v^*_{f\nu}v_{f'\nu}\ ,\qquad \phi(f_1,f_2)=\sum_{\nu}u_{f_1\nu}v_{f_2\nu}\ . \tag{7}$$

The transformed Hamiltonian has the form

$$H=H_1+H_2^{(A)}+H_2^{(B)}+H_2' \tag{8}$$

where

$$H_1=\sum_{f,f'}T(f,f')\sum_{\mu,\nu}\left(u^*_{f\mu}u_{f'\nu}\alpha^+_\mu\alpha_\nu+v^*_{f\mu}v_{f'\nu}\alpha_\mu\alpha^+_\nu+u^*_{f\mu}v_{f'\nu}\alpha^+_\mu\alpha^+_\nu+v^*_{f\mu}u_{f'\nu}\alpha_\mu\alpha_\nu\right),$$

$$H_2^{(A)}=\frac{1}{2}\sum_{\substack{f_1,f_2\\f_1',f_2'}}U(f_1,f_2;f_2',f_1')\sum_{\substack{\mu,\nu\\\rho,\delta}}\left\{u^*_{f_1\mu}u^*_{f_2\nu}u_{f_2'\rho}u_{f_1'\delta}\alpha^+_\mu\alpha^+_\nu\alpha_\rho\alpha_\delta+v^*_{f_1\mu}v^*_{f_2\nu}v_{f_2'\rho}v_{f_1'\delta}\alpha_\mu\alpha_\nu\alpha^+_\rho\alpha^+_\delta+\right.$$

$$\left.+u^*_{f_1\mu}v^*_{f_2\nu}v_{f_2'\rho}u_{f_1'\delta}\alpha^+_\mu\alpha_\nu\alpha^+_\rho\alpha_\delta+v^*_{f_1\mu}u^*_{f_2\nu}u_{f_2'\rho}v_{f_1'\delta}\alpha_\mu\alpha^+_\nu\alpha_\rho\alpha^+_\delta+u^*_{f_1\mu}v^*_{f_2\nu}u_{f_2'\rho}v_{f_1'\delta}\alpha^+_\mu\alpha_\nu\alpha_\rho\alpha^+_\delta+v^*_{f_1\mu}u^*_{f_2\nu}v_{f_2'\rho}u_{f_1'\delta}\alpha_\mu\alpha^+_\nu\alpha^+_\rho\alpha_\delta\right\}\ , \tag{9}$$

$$H_2^{(B)}=\frac{1}{2}\sum_{\substack{f_1,f_2\\f_1',f_2'}}U(f_1,f_2;f_2',f_1')\sum_{\substack{\mu,\nu\\\rho,\delta}}\left\{u^*_{f_1\mu}u^*_{f_2\nu}v_{f_2'\rho}v_{f_1'\delta}\alpha^+_\mu\alpha^+_\nu\alpha^+_\rho\alpha^+_\delta+v^*_{f_1\mu}v^*_{f_2\nu}u_{f_2'\rho}u_{f_1'\delta}\alpha_\mu\alpha_\nu\alpha_\rho\alpha_\delta\right\}\ ,$$

$$H_2'=\frac{1}{2}\sum_{\substack{f_1,f_2\\f_1',f_2'}}U(f_1,f_2;f_2',f_1')\sum_{\substack{\mu,\nu\\\rho,\delta}}\left\{u^*_{f_1\mu}u^*_{f_2\nu}v_{f_2'\rho}v_{f_1'\delta}\alpha^+_\mu\alpha^+_\nu\alpha_\rho\alpha^+_\delta+u^*_{f_1\mu}v^*_{f_2\nu}v_{f_2'\rho}v_{f_1'\delta}\alpha^+_\mu\alpha_\nu\alpha^+_\rho\alpha^+_\delta+\right.$$

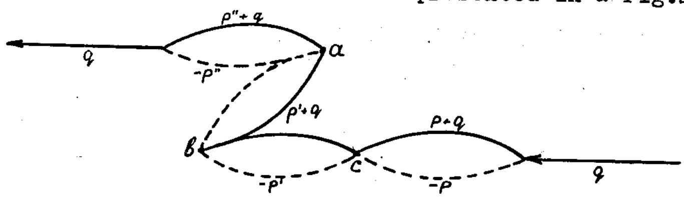
$$+v^*_{f_1\mu}u^*_{f_2\nu}v_{f_2'\rho}v_{f_1'\delta}\alpha_\mu\alpha^+_\nu\alpha^+_\rho\alpha^+_\delta+u^*_{f_1\mu}u^*_{f_2\nu}v_{f_2'\rho}u_{f_1'\delta}\alpha^+_\mu\alpha^+_\nu\alpha^+_\rho\alpha_\delta+$$

$$\left.+v^*_{f_1\mu}v^*_{f_2\nu}v_{f_2'\rho}u_{f_1'\delta}\alpha_\mu\alpha_\nu\alpha^+_\rho\alpha_\delta+u^*_{f_1\mu}v_{f_2\nu}u_{f_2'\rho}u_{f_1'\delta}\alpha^+_\mu\alpha_\nu\alpha_\rho\alpha_\delta+v^*_{f_1\mu}u^*_{f_2\nu}u_{f_2'\rho}u_{f_1'\delta}\alpha_\mu\alpha^+_\nu\alpha_\rho\alpha_\delta+v^*_{f_1\mu}v_{f_2\nu}u_{f_2'\rho}v_{f_1'\delta}\alpha_\mu\alpha_\nu\alpha_\rho\alpha^+_\delta\right\}.$$

Now we want to obtain from the Hamiltonian (8) the approximate second quantization Hamiltonian(a.s.q. Hamiltonian). Similiary as in the particular case |2| we make use of the results of papers of Gell-Mann, Brueckner, Savada, Brout and Fukuda |4|. This means in order to obtain the fundamental approximation, one may restrict oneself to the summation of only those graphs of the form of the complex, given in the Fig. 1.

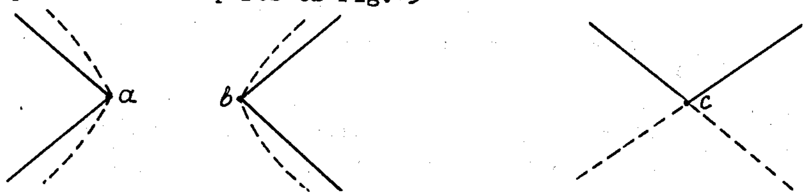
F i g. 1

All more complicated graphs are of the form represented in a Fig.2 (see also |2|)



F i g. 2

These graphs contain the vertex parts of Fig. 3



F i g. 3

In the exact Hamiltonian (8) to this vertex parts correspond the terms grouped in $H_2^{(A)}$ and $H_2^{(B)}$. In the approximate Hamiltonian thus obtained we omit the contribution from H_2'.

To the vertex parts a) (Fig.3) corresponds the second term of $H_2^{(B)}$ to the vertex parts b) the first term of $H_2^{(B)}$ and to the vertex parts c) the terms of $H_2^{(A)}$. To obtain the a.s.q. Hamiltonian we consider only $H_2^{(A)}$ and $H_2^{(B)}$. How, instead of the products $\alpha_{\nu_1}\alpha_{\nu_2}$ of Fermi-amplitudes, describing the particle-hole complexses, we introduce Bose-amplitudes $\beta_{\nu_1\nu_2}$ $(\beta_{\nu_1\nu_2}=-\beta_{\nu_2\nu_1})$. This means that from Hamiltonians $H_2^{(A)}$ and $H_2^{(B)}$ we must take out pairs of operators $\alpha_{\nu_1}\alpha_{\nu_2}$ and $(\alpha_{\nu_3}\alpha_{\nu_4})^+$ and substitute them by Bose-amplitudes $\beta_{\nu_1\nu_2}$ and $\beta^+_{\nu_3\nu_4}$. Also we must find in H_2 the coefficients of the products $(\alpha_{\nu_1}\alpha_{\nu_2})(\alpha_{\nu_3}\alpha_{\nu_4})$ and $(\alpha_{\nu_3}\alpha_{\nu_4})^+(\alpha_{\nu_1}\alpha_{\nu_2})$. This coefficients are given by formulae

$$\langle \alpha^+_{\nu_4}\alpha^+_{\nu_3}\alpha^+_{\nu_2}\alpha^+_{\nu_1} H_2 \rangle_0 \quad , \quad \langle \alpha_{\nu_3}\alpha_{\nu_4} H_2 \alpha^+_{\nu_2}\alpha^+_{\nu_1} \rangle_0$$

So finaly we get from H_2

$$\tilde{H}_2 = \sum_{\substack{\nu_1,\nu_2\\ \nu_3,\nu_4}} A(\nu_3,\nu_4;\nu_1,\nu_2)\beta^+_{\nu_3\nu_4}\beta_{\nu_1\nu_2} + \frac{1}{2}\sum_{\substack{\nu_1,\nu_2\\ \nu_3,\nu_4}} B(\nu_1,\nu_2;\nu_3,\nu_4)\beta^+_{\nu_3\nu_4}\beta^+_{\nu_1\nu_2} + \frac{1}{2}\sum_{\substack{\nu_1,\nu_2\\ \nu_3,\nu_4}} B^*(\nu_1,\nu_2;\nu_3,\nu_4)\beta_{\nu_1\nu_2}\beta_{\nu_3\nu_4} \tag{10}$$

where

$$A(\nu_3,\nu_4;\nu_1,\nu_2) = \frac{1}{4}\langle \beta_{\nu_3\nu_4} H_2 \beta^+_{\nu_1\nu_2}\rangle_0 = \frac{1}{4}\langle \alpha_{\nu_3}\alpha_{\nu_4} H_2^{(A)} \alpha^+_{\nu_2}\alpha^+_{\nu_1}\rangle_0 = \frac{1}{8}\sum_{\substack{f_1,f_2\\ f_1',f_2'}} U(f_1,f_2;f_2',f_1') \times$$

$$\times\Big\{ (u^*_{f_1\nu_4}u^*_{f_2\nu_3} - u^*_{f_1\nu_3}u^*_{f_2\nu_4})(u_{f_2'\nu_1}u_{f_1'\nu_2} - u_{f_2'\nu_2}u_{f_1'\nu_1}) + (v_{f_1'\nu_3}v_{f_2'\nu_4} - v_{f_1'\nu_4}v_{f_2'\nu_3})(v^*_{f_1\nu_1}v^*_{f_2\nu_2} - v^*_{f_1\nu_2}v^*_{f_2\nu_1}) +$$

$$+ (u^*_{f_2\nu_3}v^*_{f_1\nu_1} - u^*_{f_1\nu_3}v^*_{f_2\nu_1})(v_{f_1'\nu_4}u_{f_2'\nu_2} - v_{f_2'\nu_4}u_{f_1'\nu_2}) + (u^*_{f_2\nu_4}v^*_{f_1\nu_2} - u^*_{f_1\nu_4}v^*_{f_2\nu_2})(v_{f_1'\nu_3}u_{f_2'\nu_1} - v_{f_2'\nu_3}u_{f_1'\nu_1}) +$$

$$+ (u^*_{f_2\nu_3}v^*_{f_1\nu_2} - u^*_{f_1\nu_3}v^*_{f_2\nu_2})(v_{f_2'\nu_4}u_{f_1'\nu_1} - v_{f_1'\nu_4}u_{f_2'\nu_1}) + (u^*_{f_2\nu_4}v^*_{f_1\nu_1} - u^*_{f_1\nu_4}v^*_{f_2\nu_1})(v_{f_2'\nu_3}u_{f_1'\nu_2} - v_{f_1'\nu_3}u_{f_2'\nu_2})\Big\},$$

$$B(\nu_1,\nu_2;\nu_3,\nu_4) = \frac{1}{4}\langle \beta_{\nu_3\nu_4}\beta_{\nu_1\nu_2} H_2\rangle_0 = \frac{1}{4}\langle \alpha_{\nu_4}\alpha_{\nu_3}\alpha_{\nu_2}\alpha_{\nu_1} H_2^{(B)}\rangle_0 = \frac{1}{8}\sum_{\substack{f_1,f_2\\ f_1',f_2'}} U(f_1,f_2;f_2';f_1') \times \tag{11}$$

$$\times\Big\{ (u^*_{f_1\nu_2}u^*_{f_2\nu_1} - u^*_{f_1\nu_1}u^*_{f_2\nu_2})(v_{f_2'\nu_4}v_{f_1'\nu_3} - v_{f_2'\nu_3}v_{f_1'\nu_4}) + (u^*_{f_1\nu_4}u^*_{f_2\nu_3} - u^*_{f_1\nu_3}u_{f_2\nu_4})(v_{f_2'\nu_2}v_{f_1'\nu_1} - v_{f_2'\nu_1}v_{f_1'\nu_2}) +$$

$$+ (u^*_{f_1\nu_3}u^*_{f_2\nu_2} - u^*_{f_1\nu_2}u^*_{f_2\nu_3})(v_{f_2'\nu_4}v_{f_1'\nu_1} - v_{f_2'\nu_1}v_{f_1'\nu_4}) + (u^*_{f_1\nu_4}u^*_{f_2\nu_1} - u^*_{f_1\nu_1}u^*_{f_2\nu_4})(v_{f_2'\nu_3}v_{f_1'\nu_2} - v_{f_2'\nu_2}v_{f_1'\nu_3}) +$$

$$+ (u^*_{f_1\nu_1}u^*_{f_2\nu_3} - u^*_{f_1\nu_3}u^*_{f_2\nu_1})(v_{f_2'\nu_4}v_{f_1'\nu_2} - v_{f_2'\nu_2}v_{f_1'\nu_4}) + (u^*_{f_1\nu_4}u^*_{f_2\nu_2} - u^*_{f_1\nu_2}u^*_{f_2\nu_4})(v_{f_2'\nu_3}v_{f_1'\nu_1} - v_{f_2'\nu_1}v_{f_1'\nu_3})\Big\}.$$

The expectation value corresponds to the vacuum state C_0 defined by (5). The matrix elements A and B have the following properties

$$A(\nu_3,\nu_4;\nu_1,\nu_2) = A^*(\nu_1,\nu_2;\nu_3,\nu_4), \qquad A(\nu_3,\nu_4;\nu_1,\nu_2) = A(\nu_4,\nu_3;\nu_2,\nu_1) = -A(\nu_4,\nu_3;\nu_1,\nu_2),$$

$$B(\nu_1,\nu_2;\nu_3,\nu_4) = B(\nu_3,\nu_4;\nu_1,\nu_2), \qquad B(\nu_1,\nu_2;\nu_3,\nu_4) = B(\nu_2,\nu_1;\nu_4,\nu_3) = -B(\nu_2,\nu_1;\nu_3,\nu_4). \tag{12}$$

(Owing to these properties and to the fact that $\beta_{\nu_1\nu_2} = -\beta_{\nu_2\nu_1}$ in the definition of A and B should have written the coefficient 1/4).

How to obtain the complete Hamiltonian H we must add the part $\tilde{H}_1$ -self-energy of the particle-hole complex. In order to obtain the correct energy denominators, the same as in the correct Hamiltonian, it is necessary to choose

$$\tilde{H}_1 = \frac{1}{2} \sum_{\nu_1,\nu_2} \left[\Omega(\nu_1) + \Omega(\nu_2) \right] \beta^+_{\nu_1\nu_2} \beta_{\nu_1\nu_2} \qquad (13)$$

where

$$\Omega(\nu_1) = \langle \alpha_{\nu_1} H \alpha^+_{\nu_1} \rangle_0 = \sum_{f,f'} \mathfrak{z}(f,f') \left(u^*_{f\nu_1} u_{f'\nu_1} - v^*_{f\nu_1} v_{f'\nu_1} \right) + \frac{1}{2} \sum_{\substack{f_1,f_2\\ f'_1,f'_2}} U(f_1,f_2;f'_2,f'_1) \times$$

$$\times \left[\phi(f_2,f_1)\left(u^*_{f'_1\nu_1} v^*_{f'_2\nu_1} - v^*_{f'_1\nu_1} u^*_{f'_2\nu_1} \right) + \phi^*(f_2,f_1)\left(u_{f_1\nu_1} v_{f'_2\nu_1} - u_{f'_2\nu_1} v_{f'_1\nu_1} \right) \right] + + \, ,$$

$$\mathfrak{z}(f,f') = T(f,f') + \sum_{f_1,f'_1} \left[U(f',f'_1;f_1,f) - U(f',f'_1;f,f'_1) \right] F(f_1,f'_1) \, . \qquad (14)$$

Terms neglected in (14) do not depend on ν_1 .

Thus the complete Hamiltonian in the method of approximate second quantization has the form

$$\tilde{H} = \frac{1}{2} \sum_{\nu_1,\nu_2} \left[\Omega(\nu_1) + \Omega(\nu_2) \right] \beta^+_{\nu_1\nu_2} \beta_{\nu_1\nu_2} + \sum_{\substack{\nu_1,\nu_2\\ \nu_3,\nu_4}} A(\nu_3,\nu_4;\nu_1,\nu_2) \beta^+_{\nu_3\nu_4} \beta_{\nu_1\nu_2} +$$

$$\frac{1}{2} \sum_{\substack{\nu_1,\nu_2\\ \nu_3,\nu_4}} B(\nu_3,\nu_4;\nu_1,\nu_2) \beta^+_{\nu_3\nu_4} \beta^+_{\nu_1\nu_2} + \frac{1}{2} \sum_{\substack{\nu_1,\nu_2\\ \nu_3,\nu_4}} B^*(\nu_3,\nu_4;\nu_1,\nu_2) \beta_{\nu_3\nu_4} \beta_{\nu_1\nu_2} \, . \qquad (15)$$

This Hamiltonian is a quadratic form in the Bose-operators β .

The Diagonalization of H and the Collective Oscillations

In the monography [3] was proved that the diagonalization of a quadratic form of the type (15) may be reduced to solving a system of linear homogeneous equations with respect to the c-number quantities $\xi(\nu_1,\nu_2)$, $\eta(\nu_1,\nu_2)$

$$E\,\xi(\nu_1,\nu_2) = \left[\Omega(\nu_1) + \Omega(\nu_2) \right] \xi(\nu_1,\nu_2) + 2 \sum_{\nu_3,\nu_4} A(\nu_1,\nu_2;\nu_3,\nu_4)\, \xi(\nu_3,\nu_4) + 2 \sum_{\nu_3,\nu_4} B(\nu_1,\nu_2;\nu_3,\nu_4)\, \eta(\nu_3,\nu_4) \, ,$$

$$-E\,\eta(\nu_1,\nu_2) = \left[\Omega(\nu_1) + \Omega(\nu_2) \right] \eta(\nu_1,\nu_2) + 2 \sum_{\nu_3,\nu_4} A^*(\nu_1,\nu_2;\nu_3,\nu_4)\, \eta(\nu_3,\nu_4) + 2 \sum_{\nu_3,\nu_4} B^*(\nu_1,\nu_2;\nu_3,\nu_4)\, \xi(\nu_3,\nu_4) \qquad (16)$$

with the normalization condition

$$\sum_{\nu_1,\nu_2} \left\{ |\xi(\nu_1,\nu_2)|^2 - |\eta(\nu_1,\nu_2)|^2 \right\} = 1 \, . \qquad (17)$$

The homogeneous equations (16) lead to the secular equation for determining E . When E_n are the roots of this equation and $\xi_n(\nu_1,\nu_2)$, $\eta_n(\nu_1,\nu_2)$ the corresponding functions, we can perform the diagonalization of Hamiltonian (15) changing to new Bose-amplitudes b_n, b^+_n by means of the canonical transformation

$$\beta(\nu_1,\nu_2) = \sum_n \left\{ \xi_n(\nu_1,\nu_2)\, b_n + \eta_n(\nu_1,\nu_2)\, b^+_n \right\} \, . \qquad (18)$$

We obtain the Hamiltonian (15) in the form

$$\tilde{H} = -\sum_n E_n \sum_{\nu_1,\nu_2} \eta^*(\nu_1,\nu_2)\,\eta(\nu_1,\nu_2) + \sum_n E_n b_n^+ b_n \tag{19}$$

The functions ξ_n, η_n must obey the following normalization condition

$$\sum_n \left\{ |\xi_n|^2 - |\eta_n|^2 \right\} = 1 \tag{20}$$

When in (16) and (11) we denote the coefficients $2A = X$, $-2B = Y$ formulae (16) and (11) are identical with the formulae obtained in paper |1| with the self-consistent field method. The normalization condition for functions ξ, η gives the positive sign of E. This is the advantage of the method of approximate second quantization over the self-consistent field method.

From the equations (16) we obtain

$$\begin{aligned} E(\xi(\nu_1,\nu_2)-\eta(\nu_1,\nu_2)) &= [\Omega(\nu_1)+\Omega(\nu_2)](\xi(\nu_1,\nu_2)+\eta(\nu_1,\nu_2)) + 2\sum_{\nu_3,\nu_4}[A(\nu_1,\nu_2;\nu_3,\nu_4)+B^*(\nu_1,\nu_2;\nu_3,\nu_4)]\,\xi(\nu_3,\nu_4) + \\ &+ 2\sum_{\nu_3,\nu_4}[A^*(\nu_1,\nu_2;\nu_3,\nu_4)+B(\nu_1,\nu_2;\nu_3,\nu_4)]\,\eta(\nu_3,\nu_4), \\ E(\xi(\nu_1,\nu_2)+\eta(\nu_1,\nu_2)) &= [\Omega(\nu_1)+\Omega(\nu_2)](\xi(\nu_1,\nu_2)-\eta(\nu_1,\nu_2)) + 2\sum_{\nu_3,\nu_4}[A(\nu_1,\nu_2;\nu_3,\nu_4)-B^*(\nu_1,\nu_2;\nu_3,\nu_4)]\,\xi(\nu_3,\nu_4) - \\ &- 2\sum_{\nu_3,\nu_4}[A^*(\nu_1,\nu_2;\nu_3,\nu_4)-B(\nu_1,\nu_2;\nu_3,\nu_4)]\,\eta(\nu_3,\nu_4) . \end{aligned} \tag{21}$$

We assume that the matrices A and B are real. For the types of interactions investigated, this assumption leads to the requirement, that the functions $\{\mu,\vartheta\}$ must be real, which does not violate the conditions (3). After these assumptions we can change to new functions

$$\vartheta(\nu_1,\nu_2) = \xi(\nu_1,\nu_2)+\eta(\nu_1,\nu_2) \quad , \quad \theta(\nu_1,\nu_2) = \xi(\nu_1,\nu_2)-\eta(\nu_1,\nu_2) . \tag{22}$$

For the functions θ, ϑ we obtain the equations

$$\begin{aligned} E\,\theta(\nu_1,\nu_2) &= [\Omega(\nu_1)+\Omega(\nu_2)]\,\vartheta(\nu_1,\nu_2) + 2\sum_{\nu_3,\nu_4}[A(\nu_1,\nu_2;\nu_3,\nu_4)+B(\nu_1,\nu_2;\nu_3,\nu_4)]\,\vartheta(\nu_3,\nu_4) , \\ E\,\vartheta(\nu_1,\nu_2) &= [\Omega(\nu_1)+\Omega(\nu_2)]\,\theta(\nu_1,\nu_2) + 2\sum_{\nu_3,\nu_4}[A(\nu_1,\nu_2;\nu_3,\nu_4)+B(\nu_1,\nu_2;\nu_3,\nu_4)]\,\theta(\nu_3,\nu_4) . \end{aligned} \tag{23}$$

These equations are more convenient than the equations (16), because the right hand side depends either on functions ϑ or on functions θ . Equations of this form are obtained in a form more general than in paper[1].

Let us consider the equations (23) in the case of a simple form of the transformation (2) , used in the theory of supperconductivity [2),1)]. One may determine the corresponding functions $\{\mu,\vartheta\}$ putting

$$u_{f\nu} = u(p)\,\delta(\nu - f) \quad , \quad v_{f\nu} = v(p,\sigma)\,\delta(\nu + f) \quad ; \quad v(p,+) = v(p) \quad , \quad v(p,-) = -v(p) \quad . \tag{24}$$

Moreover we take

$$I(f,f') = E(p)\,\delta(f - f') \quad , \quad U(f_1,f_2;f_2',f_1') = \frac{1}{V}\,\mathcal{J}(p_1,p_2;p_2',p_1')\,\delta(p_1 + p_2 - p_1' - p_2')\,\delta(\sigma_1 - \sigma_1')\,\delta(\sigma_2 - \sigma_2') \quad . \tag{25}$$

As one can see the nonvanishing matrix elements of (15) are of the form

$$\begin{aligned} &A_{-+,-+}(p_1,p_2;p_1',p_2') = A_{-+,+-}(..) \quad , \quad A_{++,++}(..) = A_{--,--}(..) \quad , \\ &B_{-+,-+}(p_1,p_2;p_1',p_2') = B_{-+,+-}(..) \quad , \quad B_{++,--}(..) = B_{--,++}(..) \quad . \end{aligned} \tag{26}$$

The spectrum of collective oscillations is divided also into two branches. For the first one the oscillations take place for pairs of particles having opposite spins (elementary excitations with spin 0), for the second one for the pairs of particles having parallel spins (elementary excitations with spin $\pm$ 1). Let us note that the compensation equations (6), according to (24) and (25) go over into the equations of compensation for pairs of particles having opposite spins. These compensation equations are used to determine the lowest superconducting energy state in the theory of superconductivity [2),1)]. The collective oscillations of the first branch are investigated in the paper [|1|]. Now let us consider the spectrum of elementary excitations for the second branch. From (11), (24) and (25) we get

$$E\,\xi_{++}(p_1,p_2) = [\Omega(p_1) + \Omega(p_2)]\,\xi_{++}(p_1,p_2) + 2\sum_{p_1',p_2'} A_{++,++}(p_1,p_2;p_1',p_2')\,\xi_{++}(p_1',p_2') - 2\sum_{p_1',p_2'} B_{++,--}(p_1,p_2;p_1',p_2')\,\eta_{--}(-p_2',-p_1') , \tag{27,a}$$

$$-E\,\eta_{++}(-p_2,-p_1) = [\Omega(p_1) + \Omega(p_2)]\,\eta_{++}(-p_2,-p_1) + 2\sum_{p_1',p_2'} A_{++,++}(p_1,p_2;p_1',p_2')\,\eta_{++}(-p_2',-p_1') - 2\sum_{p_1',p_2'} B_{++,--}(p_1,p_2;p_1',p_2')\,\xi_{--}(p_1',p_2') , \tag{27,b}$$

$$E\,\xi_{--}(p_1,p_2) = [\Omega(p_1) + \Omega(p_2)]\,\xi_{--}(p_1,p_2) + 2\sum_{p_1',p_2'} A_{--,--}(p_1,p_2;p_1',p_2')\,\xi_{--}(p_1',p_2') - 2\sum_{p_1',p_2'} B_{--,++}(p_1,p_2;p_1',p_2')\,\eta_{++}(-p_2',-p_1') , \tag{28,a}$$

$$-E\,\eta_{--}(-p_2,-p_1) = [\Omega(p_1) + \Omega(p_2)]\,\eta_{--}(-p_2,-p_1) + 2\sum_{p_1',p_2'} A_{--,--}(p_1,p_2;p_1',p_2')\,\eta_{--}(-p_2',-p_1') - 2\sum_{p_1',p_2'} B_{--,++}(p_1,p_2;p_1',p_2')\,\xi_{++}(p_1',p_2') \tag{28,b}$$

where

$$A_{++,++}(p_1,p_2;p_1',p_2') = A_{--,--}(..) = A(..) =$$

$$= \frac{1}{4V}\,\delta(p_1+p_2-p_1'-p_2')\Big\{\big[J(p_2,p_1;p_1',p_2') - J(p_1,p_2;p_1',p_2')\big]\big[u(p_1)u(p_2)u(p_1')u(p_2') + v(p_1)v(p_2)v(p_1')v(p_2')\big] +$$

$$+ J(p_2,-p_2';-p_1,p_1')\big[v(p_1)u(p_2)u(p_1')v(p_2') + u(p_1)v(p_2)v(p_1')u(p_2')\big] -$$

$$- J(p_2,-p_1';-p_1,p_2')\big[v(p_1)u(p_2)v(p_1')u(p_2') + u(p_1)v(p_2)u(p_1')v(p_2')\big]\Big\},$$

$$B_{++,--}(p_1,p_2;p_1',p_2') = B_{--,++}(..) = B(..) =$$

$$= \frac{1}{4V}\,\delta(p_1+p_2-p_1'-p_2')\Big\{\big[J(p_1,p_2;p_1',p_2') - J(p_2,p_1;p_1',p_2')\big]\big[u(p_1)u(p_2)v(p_1')v(p_2') + v(p_1)v(p_2)u(p_1')u(p_2')\big] +$$

$$+ J(p_2,-p_2';-p_1,p_1')\big[v(p_1)u(p_2)v(p_1')u(p_2') + u(p_1)v(p_2)u(p_1')v(p_2')\big] - \qquad (29)$$

$$- J(p_2,-p_1';-p_1,p_2')\big[v(p_1)u(p_2)u(p_1')v(p_2') + u(p_1)v(p_2)v(p_1')u(p_2')\big]\Big\}.$$

$\Omega(p)$ is the same as in [1]

$$\Omega(p) = \sqrt{\xi^2(p) + C^2(p)}\,, \qquad \xi(p) = E(p) - \lambda + \frac{1}{V}\sum_{p'}\big\{2J(p',p;p,p') - J(p,p';p,p')\big\}v^2(p'). \qquad (30)$$

The function $C(p)$ satisfies the equation

$$C(p) + \frac{1}{V}\sum_{p'} J(p,-p;-p',p')\,\frac{C(p')}{2\,\Omega(p')} = 0$$

For a change of the spin direction for (+) to (-) the equation (27) go into the equations (28) and vice versa. These equations do not separate into equations for functions ξ_{++}, η_{++} and for functions ξ_{--}, η_{--}. Moreover, these equations connect only functions with fixed $\vec{p}_1 + \vec{p}_2$. Therefore we may put

$$p_1 = p\,, \quad p_2 = -p+q\,, \quad p_1' = p'\,, \quad p_2' = -p'+q\,.$$

Then we obtain (29) in the form

$$A(p,-p+q;p',-p'+q)=\frac{1}{4V}\Big\{\big[J(-p+q,p;p',-p'+q)-J(p,-p+q;p',-p'+q)\big]\big[u(p)u(p-q)u(p')u(p'-q)+v(p)v(p-q)v(p')v(p'-q)\big]+$$
$$+J(-p+q,p'-q;-p,p')\big[v(p)u(p-q)u(p')v(p'-q)+u(p)v(p-q)v(p')u(p'-q)\big]-$$
$$-J(-p+q,-p';-p,-p'+q)\big[v(p)u(p-q)v(p')u(p'-q)+u(p)v(p-q)u(p')v(p'-q)\big]\Big\}, \tag{31}$$

$$B(p,-p+q;p',-p'+q)=\frac{1}{4V}\Big\{\big[J(p,-p+q;p',-p'+q)-J(-p+q,p;p',-p'+q)\big]\big[u(p)u(p-q)v(p')v(p'-q)+v(p)v(p-q)u(p')u(p'-q)\big]+$$
$$+J(-p+q,p'-q;-p,p')\big[v(p)u(p-q)v(p')u(p'-q)+u(p)v(p-q)u(p')v(p'-q)\big]-$$
$$-J(-p+q,-p';-p,-p'+q)\big[v(p)u(p-q)u(p')v(p'-q)+u(p)v(p-q)v(p')u(p'-q)\big]\Big\}.$$

The solution of the equations (27), (28) leads to the diagonalization of the Hamilto-

$$\tilde{H}=\frac{1}{2}\sum_{p_1,p_2}\big[\Omega(p_1)+\Omega(p_2)\big]\beta^+_{++}(p_1,p_2)\beta_{++}(p_1,p_2)+\sum_{\substack{p_1,p_2\\p_1',p_2'}}A(p_1,p_2;p_1',p_2')\beta^+_{++}(p_1,p_2)\beta_{++}(p_1',p_2')+\frac{1}{2}\sum_{p_1,p_2}\big[\Omega(p_1)+\Omega(p_2)\big]\beta^+_{--}(p_1,p_2)\beta_{--}(p_1,p_2)+$$
$$+\sum_{\substack{p_1,p_2\\p_1',p_2'}}A(p_1,p_2;p_1',p_2')\beta^+_{--}(p_1,p_2)\beta_{--}(p_1',p_2')+\sum_{\substack{p_1,p_2\\p_1',p_2'}}B(p_1,p_2;p_1',p_2')\big[\beta^+_{++}(p_1,p_2)\beta^+_{--}(-p_1',-p_2')+\beta_{++}(p_1,p_2)\beta_{--}(-p_1',-p_2')\big]. \tag{32}$$

Hamiltonian (32) describes the elementary excitations with spin moment $\pm$ 1 which interact with each other. Therefore the equations (28 a,b) do not split into the equations for the two independent branches. If we group the equation (27b) with (28a) and (27a) with (28b) we obtain two independent systems of equations, but for the functions with sign (+) and (-) simultaneously. Then we define new functions

$$\xi_{--}(p_1,p_2)-\eta_{++}(-p_2,-p_1)=\Theta_q^{(-+)}(p)=\Theta_q(p),$$
$$\xi_{--}(p_1,p_2)+\eta_{++}(-p_2,-p_1)=\vartheta_q^{(-+)}(p)=\vartheta_q(p), \qquad (p_1+p_2=p_1'+p_2'=q). \tag{33}$$

For the functions Θ_q and ϑ_q we obtain equations in the some convenient form as (23)

$$E\,\Theta_q(p)=\big[\Omega(p)+\Omega(p-q)\big]\vartheta_q(p)+2\sum_{p'}\big[A(p,-p+q;p',-p'+q)+B(p,-p+q;p',-p'+q)\big]\vartheta_q(p'),$$
$$E\,\vartheta_q(p)=\big[\Omega(p)+\Omega(p-q)\big]\Theta_q(p)+2\sum_{p'}\big[A(p,-p+q;p',-p'+q)+B(p,-p+q;p',-p'+q)\big]\Theta_q(p'). \tag{34}$$

We note that the functions ξ,η and consequently the functions Θ, ϑ are add functions of p_1,p_2 variables, this means for $q = 0$

$$\Theta_0(\vec{p})=\Theta(\vec{p})=-\Theta(-\vec{p}), \qquad \vartheta_0(\vec{p})=\vartheta(\vec{p})=-\vartheta(-\vec{p}) \tag{35}$$

(we now introduce arrows to distinguish vector $\vec{p}$ from $p=|\vec{p}|$).

We consider the equation (34) for q = 0. We get from (31)

$$A(p,-p;p',-p') = \frac{1}{4V}\Big\{\big[\mathcal{J}(-p,p;p',-p') - \mathcal{J}(p,-p;p',-p')\big]\big[u^2(p)u^2(p') + v^2(p)v^2(p')\big] + \big[\mathcal{J}(-p,p';-p,p') - \mathcal{J}(p,p';p,p')\big]v(p)u(p)v(p')u(p')\Big\},$$

$$B(p,-p;p',-p') = \frac{1}{4V}\Big\{\big[\mathcal{J}(p,-p;p',-p') - \mathcal{J}(-p,p;p',-p')\big]\big[u^2(p)v^2(p') + v^2(p)u^2(p')\big] + \big[\mathcal{J}(-p,p';-p,p') - \mathcal{J}(p,p';p,p')\big]v(p)u(p)v(p')u(p')\Big\}. \tag{36}$$

If we restrict ourselves to the case when the interaction may be replaced by a constant inside a thin layer in the neighbourhood of the Fermi-sphere ($E_F \pm \omega$), and by zero outside it, we get

$$A = B = 0. \tag{37}$$

(Hamiltonian (32) is obtained also in the paper[5] where it is stated that in the case of interaction which is constant in a thin layer, we do not obtain the collective oscillations with reversed spin. Then the temperature dependence of paramagnetic susceptibility obtained in the paper |6| is not altered and remains inconsistent with Reif's |7| experiment).

For small q from (27) using (37) we get

$$E\theta_q(p) = \left[\Omega(p) + \Omega(p-q)\right]\vartheta_q(p) \quad , \quad E\vartheta_q(p) = \left[\Omega(p) + \Omega(p-q)\right]\theta_q(p) \quad . \tag{38}$$

To show that $E > 0$ as solutions of (38) we take the antisymmetrical functions

$$\theta_q(p) = S\left[\delta(p-p_0) - \delta(p-q+p_0)\right] \quad , \quad \vartheta_q(p) = S\left[\delta(p-p_0) - \delta(p-q+p_0)\right] \quad . \tag{39}$$

Thus we get the continuous spectrum

$$E = \Omega(p_0) + \Omega(p_0 - q) \tag{40}$$

separated by a gap. With the given q the energy E depends continuously on the momentum p_0. Hence the simple model of interaction of particles, leads to elementary excitations which have a continuous energy spectrum. We next consider the case when the interaction $\mathcal{J}$ is effective only in a narrow layer near the Fermi-surface ($E_F \pm \omega$) and has the form

$$\mathcal{J}(p_1,p_2;p_2',p_1') = -g^2(p_1 - p_1') \quad , \quad p_1 + p_2 = p_1' + p_2' \quad . \tag{41}$$

After introducing (41) to (36) and taking into account (35) we obtain

$$E\vartheta(\vec{p}) = 2\,\Omega(p)\,\theta(\vec{p}) - \frac{1}{V}\sum_{\vec{p}'} g^2(|\vec{p}' - \vec{p}|)\,\theta(\vec{p}') \quad ,$$

$$E\theta(\vec{p}) = 2\,\Omega(p)\vartheta(\vec{p}) + \frac{1}{V}\sum_{\vec{p}'} g^2(|\vec{p}'-\vec{p}|)\frac{C(p)C(p') - \xi(p)\xi(p')}{\Omega(p)\,\Omega(p')} \tag{42}$$

These equations are identical with the equations obtained in [2], [1] by considering the collective oscillations of the first branch. But in our case the solution must be the odd function of $\vec{p}$ whereas in the former case, following the compensation relations, it had to be the even function.

We change to spherical variables taking for the spherical axis the direction of the $\vec{p}$ vector. In this case

$$g^2(|\vec{p}'-\vec{p}|) \equiv g^2(p, p', p_F\sqrt{2(1-\cos\alpha)})\,, \quad \vartheta(\vec{p}) = \vartheta(p,\cos\alpha)\,, \quad \theta(\vec{p}) = \theta(p,\cos\alpha) \tag{43}$$

where α is the angle between the vectors $\vec{p}$ and $\vec{p}'$. If the angle α takes on the value π, the functions ϑ and θ change the sign. Then in the expansions of the functions ϑ, θ we have only the terms with odd spherical harmonical functions

$$\theta = \sum_{n=0}^{\infty} a_{2n+1}(p)P_{2n+1}(\cos\alpha)\,, \quad \vartheta = \sum_{n=0}^{\infty} b_{2n+1}(p)P_{2n+1}(\cos\alpha)\,, \quad g^2 = \sum_{n=0}^{\infty} g_n(p,p')P_n(\cos\alpha)\,. \tag{44}$$

Putting (44) in (42) we obtain the equations for the separate terms of the expansions. We consider the equations for the first term only, writing them in the integral form

$$\begin{aligned} E b_1(\xi) &= 2\Omega(\xi)a_1(\xi) - \frac{1}{3}\frac{dn}{dE}\int_{-\infty}^{+\infty} g_1(\xi,\xi')a_1(\xi')\,d\xi'\,, \\ E a_1(\xi) &= 2\Omega(\xi)b_1(\xi) + \frac{1}{3}\frac{dn}{dE}\int_{-\infty}^{+\infty}\frac{C(\xi)C(\xi') - \xi\xi'}{\Omega(\xi)\,\Omega(\xi')}g_1(\xi,\xi')\,d\xi' \end{aligned} \tag{45}$$

where

$$\frac{dn}{dE} = \frac{1}{2\pi^2}\frac{p_F^2}{E'(p_F)}$$

At first we consider the collective oscillations in the superconducting state when C is a constant different from zero in a narrow layer near the Fermi-surface. We define

$$C_1(\xi) = \frac{1}{3}\frac{dn}{dE}\int_{-\infty}^{+\infty} g_1(\xi,\xi')a_1(\xi')\,d\xi'\,, \quad C_2(\xi) = \frac{1}{3}\frac{dn}{dE}\int_{-\infty}^{+\infty}\frac{\xi' g_1(\xi,\xi')}{\Omega(\xi')}b_1(\xi')\,d\xi'\,,$$

$$C_3(\xi) = \frac{1}{3}\frac{dn}{dE}C\int_{-\infty}^{+\infty}\frac{g_1(\xi,\xi')}{\Omega(\xi')}b_1(\xi')\,d\xi'\,. \tag{46}$$

From (45) and (46) we obtain

$$\begin{aligned} a_1(\xi) &= \frac{1}{4\Omega^2(\xi) - E^2}\left[2\Omega(\xi)C_1(\xi) - \frac{CE}{\Omega(\xi)}C_3(\xi) + \frac{\xi}{\Omega(\xi)}EC_2(\xi)\right]\,, \\ b_1(\xi) &= \frac{1}{4\Omega^2(\xi) - E^2}\left[EC_1(\xi) - 2CC_3(\xi) + 2\xi C_2(\xi)\right] \end{aligned} \tag{47}$$

The first and second term on the right hand side of (47) are even functions of z, the third term an odd function. Then we get the following dependence

$$E C_3(z) = 2CC_1(z) - C\,\frac{1}{3}\frac{dn}{dE}\int_{-\infty}^{+\infty}\frac{g_1(z,z')}{\Omega(z')}\,C_1(z')\,dz'$$

and the following equations for the determination of E

$$C_1(z) = \frac{1}{3}\frac{dn}{dE}\int_{-\infty}^{+\infty}\frac{2g_1(z,z')z'^2}{\Omega(z')[4\Omega^2(z')-E^2]}\,C_1(z')\,dz' \;+$$

$$+\,C^2\left(\frac{1}{3}\frac{dn}{dE}\right)^2\int_{-\infty}^{+\infty}\frac{g_1(z,z')}{\Omega(z')[4\Omega^2(z')-E^2]}\left[\int_{-\infty}^{+\infty}\frac{g_1(z',z'')}{\Omega(z'')}\,C_1(z'')\,dz''\right]dz'\,,$$

$$C_2(z) = \frac{1}{3}\frac{dn}{dE}\int_{-\infty}^{+\infty}\frac{2g_1(z,z')z'^2}{\Omega(z')[4\Omega^2(z')-E^2]}\,C_2(z')\,dz' \qquad (48)$$

For $g_1(z,z')\cong g_1(0,0)=g_1(p_F)$ the equations (48) lead to the expressions

$$\left\{\tilde\varrho\ln\frac{2\omega}{C} - \tilde\varrho\,\frac{\sqrt{1-\varepsilon^2}}{\varepsilon}\,\mathrm{arctg}\,\frac{\varepsilon}{\sqrt{1-\varepsilon^2}} - 1\right\} = 0$$

$$\left\{\left(\tilde\varrho\ln\frac{2\omega}{C} - 1 + \varepsilon^2\right)\left(\frac{\tilde\varrho}{\varepsilon\sqrt{1-\varepsilon^2}}\,\mathrm{arctg}\,\frac{\varepsilon}{\sqrt{1-\varepsilon^2}} + 1\right) - \varepsilon^2\right\} = 0 \qquad (49)$$

where

$$\tilde\varrho = \frac{1}{3}\,g_1(p_F)\,\frac{dn}{dE}\,, \qquad \varepsilon = \frac{E}{2C}\,.$$

From (49) we see that the stability conditions for the collective excitations of the second branch in the superconducting state are the same as the stability conditions for the first branch for transversed waves, obtained in [2]. Eq. (49) has a single root, for $\tilde\varrho$ in the interval

$$-1 < \tilde\varrho < \frac{1}{\ln\frac{2\omega}{C}}\,. \qquad (50)$$

That is, the interaction (41), different from the interaction considered formerly, leads to the spin-wave like elementary excitations. These excitations are stable if the interaction is not too strong.

Let us now consider the collective excitations in the normal state. Putting in (45) C = 0, we obtain finally

$$C_1(z) = \frac{1}{3}\frac{dn}{dE}\int_0^{\infty} g_1(z,z')\,\frac{4z'}{4z'^2-E^2}\,C_1(z')\,dz' \qquad (51)$$

which leads to the asymptotic formula

$$-E^2 \sim e^{-2/g_1}$$

which means that the normal state is instable.

A.S.Q. Hamiltonian in the Case of Weak External Fields

Let us consider the dynamical system of Fermi-particles under the action of a weak external fields. We assume that this gives only rise to a variation $I(f,f')$. Thus instead of Hamiltonian (1) we have

$$H + \delta H = H + \sum_{f,f'} \delta I(f,f')\, a_f^+ a_{f'} \qquad (52)$$

Applying the transformation (2) to the additional term in (52) and - in the same way as earlier - to the Bose-amplitudes $\beta_{\nu\mu}$ we get in the approximate Hamiltonian (15) the additional term $\delta\tilde{H}$

$$\tilde{H}' = \tilde{H} + \delta\tilde{H} = \tilde{H} + \frac{1}{2}\sum_{f,f'} \delta I(f,f') \sum_{\nu_1,\nu_2} (u^*_{f\nu_2} v_{f'\nu_1} - u^*_{f\nu_1} v_{f'\nu_2}) \beta^+_{\nu_1\nu_2} + \frac{1}{2}\sum_{f,f'} \delta I^*(f,f') \sum_{\nu_1,\nu_2} (u_{f\nu_2} v^*_{f'\nu_1} - u_{f\nu_1} v^*_{f'\nu_2}) \beta_{\nu_1\nu_2} \,. \qquad (53)$$

It is seen that the a.s.q. Hamiltonian now contains terms linear in the operators $\beta_{\nu\mu}$ which leads to the forced collective oscillations. In the Hamiltonian (15) we do not have terms linear in the operators $\beta_{\nu\mu}$ due to the compensation relation (4).

In order to remove the terms linear in $\beta_{\nu\mu}$ in the Hamiltonian (53) we can use the method of translation of the Bose-amplitudes

$$\beta_{\nu\mu} \rightarrow \beta_{\nu\mu} + C(\nu,\mu) \quad , \quad \beta^+_{\nu\mu} \rightarrow \beta^+_{\nu\mu} + C^*(\nu,\mu) \qquad (54)$$

where C and C* are c-numbers. They are be to be determined from the condition of the vanishing of the linear forms

$$\frac{\partial \tilde{H}'}{\partial \beta^+_{\nu\mu}} = 0 \quad , \quad \frac{\partial \tilde{H}'}{\partial \beta_{\nu\mu}} = 0 \,. \qquad (55)$$

We now write the Hamiltonian (53) when the transformation (2) is defined by the formulae (24) and the interaction may be replaced by a constant. In the approximation used, we have the sum of three terms, according to the spin of quasiparticles

$$\tilde{H}' = \tilde{H}'(-+) + \tilde{H}'(++) + \tilde{H}'(--) \,. \qquad (56)$$

The perturbation terms caused by the external weak fields we write in the form

$$-\frac{e}{2m}\left\{(\vec{p}\vec{A}(r)) + (\vec{A}(r)\vec{p})\right\} - \frac{e}{2m}\vec{\sigma}\,\delta\vec{\mathcal{H}}(r) \qquad (57)$$

where +e, m are charge and mass of electron, $\delta\vec{\mathcal{H}}$ the magnetic field vector, $\vec{A}$ the vector-potential of this field, $\vec{p}$ momentum operator of particle, $\vec{\sigma}$ spin vector, the components of which are the Pauli matrices.

By means of the functions

$$\Psi(r,s_z) = \frac{1}{\sqrt{V}} \sum_{p,\sigma} a_{p\sigma}\, e^{ipr} S_\sigma(s_z) \qquad (58)$$

and using the formulae

$$\sigma_x S_\alpha = S_{-\alpha} \;,\quad \sigma_y S_\alpha = i\, S_{-\alpha}\, \mathrm{sgn}\,\alpha \;,\quad \sigma_z S_\alpha = S_\alpha\, \mathrm{sgn}\,\alpha \tag{59}$$

we change from (57) to the Hamiltonian of second quantization

$$\delta H_{\mathcal{A}} = -\frac{e}{2m} \sum_{p_1+p_2=q} (\vec{p}_2 - \vec{p}_1)\, \vec{\mathcal{A}}(p_1+p_2) \left[a^+_{-p_1+}\, a_{p_2+} + a^+_{-p_1-}\, a_{p_2-} \right] , \tag{60}$$

$$\delta H_{\mathcal{H}} = -\frac{e}{2m} \sum_{p_1+p_2=q} \Big\{ \left[\delta\mathcal{H}_x(p_1+p_2) - i\,\delta\mathcal{H}_y(p_1+p_2) \right] a^+_{-p_1+}\, a_{p_2-} + \left[\delta\mathcal{H}_x(p_1+p_2) + i\,\delta\mathcal{H}_y(p_1+p_2) \right] a^+_{-p_1-}\, a_{p_2+} + \delta\mathcal{H}_z(p_1+p_2) \left[a^+_{-p_1+}\, a_{p_2+} - a^+_{-p_1-}\, a_{p_2-} \right] \Big\} . \tag{60b}$$

(The Hamiltonian (60b) for $\delta\mathcal{H} = \delta\mathcal{H}_x$ has been used in the paper [6] to obtain the temperature dependence of paramagnetic susceptibility in the B.C.S. theory [8]).

Finally we get in the approximate Hamiltonian (56) the following terms linear in the β operators

$$\delta\tilde{H}_{\mathcal{A}}(-+) = -\frac{e}{2m} \sum_{p_1+p_2=q} (\vec{p}_2 - \vec{p}_1)\, \vec{\mathcal{A}}(p_1+p_2) \left[v(p_1)\, u(p_2) - v(p_2)\, u(p_1) \right] \left[\beta_{-+}(p_1,p_2) - \beta^+_{-+}(-p_2,-p_1) \right] ,$$

$$\delta\tilde{H}_{\mathcal{H}}(-+) = -\frac{e}{2m} \sum_{p_1+p_2=q} \delta\mathcal{H}_z(p_1+p_2) \left[v(p_1)\, u(p_2) - v(p_2)\, u(p_1) \right] \left[\beta_{-+}(p_1,p_2) - \beta^+_{-+}(-p_2,-p_1) \right] ,$$

$$\delta\tilde{H}(++) = -\frac{e}{4m} \sum_{p_1+p_2=q} \left[v(p_2)\, u(p_1) - v(p_1)\, u(p_2) \right] \Big\{ \left[\delta\mathcal{H}_x(p_1+p_2) + i\,\delta\mathcal{H}_y(p_1+p_2) \right] \beta_{++}(p_1,p_2) - \left[\delta\mathcal{H}_x(p_1+p_2) - i\,\delta\mathcal{H}_y(p_1+p_2) \right] \beta^+_{++}(-p_2,-p_1) \Big\} ,$$

$$\delta\tilde{H}(--) = \frac{e}{4m} \sum_{p_1+p_2=q} \left[v(p_2)\, u(p_1) - v(p_1)\, u(p_2) \right] \Big\{ \left[\delta\mathcal{H}_x(p_1+p_2) - i\,\delta\mathcal{H}_y(p_1+p_2) \right] \beta_{--}(p_1,p_2) - \left[\delta\mathcal{H}_x(p_1+p_2) + i\,\delta\mathcal{H}_y(p_1+p_2) \right] \beta^+_{--}(-p_2,-p_1) \Big\} . \tag{61}$$

Thus the Hamiltonian (56) is the sum of the Hamiltonians

$$\tilde{H}'(-+) = \sum_{p_1,p_2} \left[\Omega(p_1) + \Omega(p_2) \right] \beta^+_{-+}(p_1,p_2)\, \beta_{-+}(p_1,p_2) + \sum_{\substack{p_1,p_2\\ p_1',p_2'}} A_{-+}(p_1,p_2;p_1',p_2')\, \beta^+_{-+}(p_1,p_2)\, \beta_{-+}(p_1,p_2) +$$
$$+ \sum_{\substack{p_1,p_2\\ p_1',p_2'}} B_{-+}(p_1,p_2;p_1',p_2') \left[\beta^+_{-+}(p_1,p_2)\, \beta^+_{-+}(-p_2',-p_1') + \beta_{-+}(p_1,p_2)\, \beta_{-+}(-p_2',-p_1') \right] + \delta\tilde{H}_{\mathcal{A}}(-+) + \delta\tilde{H}_{\mathcal{H}}(-+) , \tag{62}$$

where

$$A_{-+}(p_1,p_2;p_1',p_2') = \frac{1}{V}\, \delta(p_1+p_2-p_1'-p_2') \Big\{ \mathcal{J}(p_1,p_2;p_2',p_1') \left[u(p_1)\, u(p_2)\, u(p_1')\, u(p_2') + v(p_1)\, v(p_2)\, v(p_1')\, v(p_2') \right] +$$
$$+ \mathcal{J}(p_1,-p_1';p_2',-p_2) \left[u(p_1)\, v(p_2)\, v(p_1')\, u(p_2') + v(p_1)\, u(p_2)\, u(p_1')\, v(p_2') \right] +$$
$$+ \left[\mathcal{J}(-p_1,p_2';-p_1',p_2) - \mathcal{J}(p_2',-p_1;-p_1',p_2) \right] \left[v(p_1)\, u(p_2)\, v(p_1')\, u(p_2') + u(p_1)\, v(p_2)\, u(p_1')\, v(p_2') \right] \Big\} ,$$

$$B_{-+}(p_1,p_2;p_1',p_2') = \frac{1}{V}\delta(p_1+p_2-p_1'-p_2')\Big\{-\mathcal{J}(p_1,p_2;p_1',p_2')\big[u(p_1)u(p_2)v(p_1')v(p_2')+v(p_1)v(p_2)u(p_1')u(p_2')\big]+$$
$$+\mathcal{J}(p_1,-p_1';p_2',-p_2)\big[v(p_1)u(p_2)v(p_1')u(p_2')+u(p_1)v(p_2)u(p_1')v(p_2')\big]+ \tag{63}$$
$$+\big[\mathcal{J}(-p_1,p_2';-p_1',p_2)-\mathcal{J}(p_2',-p_1;-p_1',p_2)\big]\big[u(p_1)v(p_2)v(p_1')v(p_2')+v(p_1)u(p_2)u(p_1')v(p_2')\big]\Big\},$$

$$\widetilde{H}'(++) = \frac{1}{2}\sum_{p_1,p_2}\big[\Omega(p_1)+\Omega(p_2)\big]\beta^+_{++}(p_1,p_2)\beta_{++}(p_1,p_2)+\delta\widetilde{H}(++), \tag{64}$$

$$\widetilde{H}'(--) = \frac{1}{2}\sum_{p_1,p_2}\big[\Omega(p_1)+\Omega(p_2)\big]\beta^+_{--}(p_1,p_2)\beta_{--}(p_1,p_2)+\delta\widetilde{H}(--). \tag{65}$$

(The first terms in (62) lead to the secular equation for the collective oscillations of first branch, investigated in |1|,|2|).

The perturbation in (62), (64), (65) linear in β are given by (61). Thus we performe the translation of β amplitudes

$$\beta_{-+}(p_1,p_2)\to\beta_{-+}(p_1,p_2)+C_{-+}(p_1,p_2),\quad \beta_{++}(p_1,p_2)\to\beta_{++}(p_1,p_2)+C_{++}(p_1,p_2),\ \dots \tag{66}$$

and according to (55) the equations for the function C are

$$\big[\Omega(p_1)+\Omega(p_2)\big]C^*_{-+}(p_1,p_2)+\sum_{p_1',p_2'}A_{-+}(p_1,p_2;p_1',p_2')C^*_{-+}(p_1',p_2')+\sum_{p_1',p_2'}B_{-+}(p_1,p_2;p_1',p_2')C_{-+}(-p_2',-p_1')+\frac{e}{2m}\vec{A}(p_1+p_2)(\vec{p}_2-\vec{p}_1)\big[v(p_2)u(p_1)-v(p_1)u(p_2)\big]=0,$$
$$\tag{67}$$
$$\big[\Omega(p_1)+\Omega(p_2)\big]C_{-+}(-p_2,-p_1)+\sum_{p_1',p_2'}A_{-+}(p_1,p_2;p_1',p_2')C_{-+}(-p_2',-p_1')+\sum_{p_1',p_2'}B_{-+}(p_1,p_2;p_1',p_2')C^*_{-+}(p_1',p_2')-\frac{e}{2m}\vec{A}^*(-p_1-p_2)(\vec{p}_2-\vec{p}_1)\big[v(p_2)u(p_1)-v(p_1)u(p_2)\big]=0$$

or

$$\big[\Omega(p_1)+\Omega(p_2)\big]\big[C^*_{-+}(p_1,p_2)-C_{-+}(-p_2,-p_1)\big]+\sum_{p_1',p_2'}\big[A_{-+}(p_1,p_2;p_1',p_2')-B_{-+}(p_1,p_2;p_1',p_2')\big]\big[C^*_{-+}(p_1',p_2')-C_{-+}(-p_2',-p_1')\big]=-\frac{e}{m}(\vec{p}_2-\vec{p}_1)\vec{A}(p_1+p_2)\big[v(p_2)u(p_1)-v(p_1)u(p_2)\big],$$

$$\big[\Omega(p_1)+\Omega(p_2)\big]\big[C^*_{-+}(p_1,p_2)+C_{-+}(-p_2,-p_1)\big]+\sum_{p_1',p_2'}\big[A(p_1,p_2;p_1',p_2')+B(p_1,p_2;p_1',p_2')\big]\big[C^*_{-+}(p_1',p_2')+C_{-+}(-p_2',-p_1')\big]=0. \tag{68}$$

Similiary we get

$$C_{++}(p_1,p_2)+C^*_{++}(-p_2,-p_1)=C_{--}(p_1,p_2)+C^*_{--}(-p_2,-p_1)=-\frac{ei}{m}\,\delta\mathcal{H}_y(-p_1-p_2)f(p_1,p_2)$$

$$C_{++}(p_1,p_2)-C^*_{++}(-p_2,-p_1)=-\left[C_{--}(p_1,p_2)-C^*_{--}(-p_2,-p_1)\right]=\frac{e}{m}\,\delta\mathcal{H}_x(-p_1-p_2)f(p_1,p_2) \tag{69}$$

where

$$f(p_1,p_2)=\frac{1}{\Omega(p_1)+\Omega(p_2)}\left[v(p_2)u(p_1)-v(p_1)u(p_2)\right].$$

The equations (68) connect the functions C, C^* only with fixed $\vec{p}_1+\vec{p}_2$. In these equations, for the time being, we do not take into account the term $\delta\tilde{H}_{\mathcal{H}}$. When in (68) we denote the functions

$$\left[C^*_{-+}(p_1,p_2)-C_{-+}(-p_2',-p_1')\right]\to\theta_q(p)\quad,\ \left[C^*_{-+}(p_1,p_2)+C_{-+}(-p_2,-p_1)\right]\to\vartheta_q(p) \tag{70}$$

the equations (68) are identical with the equations (81) and (117) solved in paper |1|. We shall apply the results of this paragraph to the investigation of the electrodynamics of superconducting state.

Electrodynamics of Superconducting State

Let us consider the change of lowest superconducting state due to the weak external electromagnetic field. We want to obtain a current as a function of vector-potential $\vec{A}$ and a magnetic field $\delta\mathcal{H}$.

Considering the Hamiltonian whose mean value is given by

$$\bar{H}=\int\Psi^+(r,s_z)\left[\frac{1}{2m}(\vec{p}-e\vec{A}(r))-\frac{e}{2m}\vec{\sigma}\,\delta\vec{\mathcal{H}}(r)\right]\Psi(r,s_z)\,dV \tag{71}$$

we obtain the current as a coefficient of the variation of $\vec{A}$ in the formula

$$\delta\bar{H}=-\int\vec{j}'\delta\vec{A}\,dV \tag{72}$$

Thus the current averaged in addition over the spin variables

$$\vec{j}'(r)=\sum_{s_z}\Big\{\frac{ie}{m}\left(\nabla\Psi^+(r,s_z)\Psi(r,s_z)-\Psi^+(r,s_z)\nabla\Psi(r,s_z)\right)-\frac{e^2}{m}\vec{A}(r)\Psi^+(r,s_z)\Psi(r,s_z)-$$
$$-\frac{e}{2m}\,\mathrm{curl}\,\Psi^+(r,s_z)\vec{\sigma}\Psi(r,s_z)\Big\}=\vec{j}_1'(r)+\vec{j}_2'(r) \tag{73}$$

$\vec{j}_2'(r)$ denotes the last term of (73). In the Hamiltonian (71) we do not write the interaction energy of particles because this term give no contribution to the current.

Let us consider the Fourier-representation for the current $\vec{j}_1'(r)$

$$\vec{j}_1'(r)=\frac{e}{2m}\frac{1}{V}\sum_{p_1,p_2}(\vec{p}_2-\vec{p}_1)e^{i(p_1+p_2)r}\left(a^+_{-p_1+}a_{p_2+}+a^+_{-p_1-}a_{p_2-}\right)-\frac{e^2}{m}\vec{A}(r)\Big[\varrho_0+\sum_{p_1+p_2=q\neq0}e^{i(p_1+p_2)r}\left(a^+_{-p_1+}a_{p_2+}+a^+_{-p_1-}a_{p_2-}\right)\Big] \tag{74}$$

where $\varrho_0=\frac{2}{V}\sum_p v^2(p)$.

In the same approximation in which we have obtained the Hamiltonian (62),(64),(65) we express the current $\vec{j}_1'$ as a linear function of β_{-+} and β_{-+}^{+}. Then we change to the new translated Bose-amplitudes $\beta_{-+}+C_{-+}$. The functions C_{-+} are given by (68).

Let us consider the current

$$\vec{j}_1(r) = \langle \vec{j}_1'(r) \rangle_0 \tag{75}$$

Taking into account that

$$\langle \beta_{-+} + C_{-+} \rangle_0 = C_{-+} \tag{76}$$

we obtain $\vec{j}_1(r)$ as a function of C_{-+}

$$\vec{j}_1(r) = \vec{j}_1(-+) = \frac{e}{2m}\frac{1}{V}\sum_{p_1+p_2=q}(\vec{p}_2-\vec{p}_1)e^{i(p_1+p_2)r}\left[v(p_1)u(p_2)-v(p_2)u(p_1)\right]\left[C_{-+}(p_1,p_2)-C^{*}_{-+}(-p_2,-p_1)\right] -$$

$$-\frac{e^2}{m}\left[\sum_q \vec{A}(q)e^{iqr}\right]\left\{\varrho_0+\sum_{p_1+p_2=q\neq 0}\left(v(p_1)u(p_2)+v(p_2)u(p_1)\right)\left[C_{-+}(p_1,p_2)+\overset{*}{C}(-p_2,-p_1)\right]e^{i(p_1+p_2)r}\right\}. \tag{77}$$

Similiary we obtain $\vec{j}_2(r)$ as a function of C_{-+}, C_{++}, C_{--}. In the formula (77) we can use the notation of (70) and make direct use of results of paper[1]

$$\vartheta_q(p) = 0 \quad , \quad \Theta_q(p) = e\,\mathcal{O\!l}(q)\,\tau(p,q) \tag{78}$$

where $\mathcal{O\!l}(q)$ is the Fourier-component of the transverse part of the vector-potential $\vec{A}$, the q-dependence of the function $\tau(p,q)$ can be emphasized by writing $\tau(p,q)=q\,\tilde{\tau}(p,q)$. The Fourier-components of (77) are

$$\vec{j}_1(q) = \frac{e}{2m}\frac{1}{V}\sum_p (2\vec{p}-\vec{q})\left[v(p-q)u(p)-v(p)u(p-q)\right]\Theta_q(p) - \frac{2e^2}{mV}\vec{A}(q)\sum_p v^2(p) \tag{79}$$

Following[1], for sufficiently small q

$$j_1(q) = -\frac{e^2\varrho_0}{m}\mathcal{O\!l} \tag{80}$$

we obtain the Meissner effect[5],[6]. To obtain (80) we consider the collective excitations of the Bose-type only. In the papers[1],[9] is proved that in order to satisfy the Buckingham relations (or, what is the same, the gauge invariance of Meissner-Ochsenfeld effect), for small q is essential only the contribution from the Bose-type collective excitations.

Now, let us consider the last term of (73)

$$\vec{j}_2'(r) = -\frac{e}{2m}\,\mathrm{curl}\sum_{s_2}\Psi^{+}(r,s_2)\vec{\sigma}\,\Psi(r,s_2) \quad , \tag{81}$$

$\vec{j}_1'(r)$ is the current of elementary excitations with spin 0, and $\vec{j}_2'(r)$ is the current of elementary exoitations with spin 0 ± 1. This means that we obtain the additional term to $\vec{j}_1$ and moreover the "spin" current, current of elementary excitations with spin $\neq 0$. The components of (81) in the Fourier-representation are

$$j'_{2x}(-+) = \frac{ie}{2m}\frac{1}{V}\sum_{p_1+p_2=q}e^{i(p_1+p_2)r}(p_1+p_2)_y\left(a^{+}_{-p_1+}a_{p_2+}-a^{+}_{-p_1-}a_{p_2-}\right), \tag{82}$$

$$j'_{2y}(-+) = -\frac{ie}{2m}\frac{1}{V}\sum_{p_1+p_2=q}e^{i(p_1+p_2)r}(p_1+p_2)_x\left(a^{+}_{-p_1+}a_{p_2+}-a^{+}_{-p_1-}a_{p_2-}\right),$$

$$\left[j_2'(++)+j_2'(--)\right]_x=-\frac{e}{2m}\frac{1}{V}\sum_{p_1+p_2=q}e^{i(p_1+p_2)r}(p_1+p_2)_z\left(a^+_{-p_1+}a_{p_2-}-a^+_{-p_1-}a_{p_2+}\right),$$

$$\left[j_2'(++)+j_2'(--)\right]_y=\frac{ie}{2m}\frac{1}{V}\sum_{p_1+p_2=q}e^{i(p_1+p_2)r}(p_1+p_2)_z\left(a^+_{-p_1+}a_{p_2-}+a^+_{-p_1-}a_{p_2+}\right),$$

$$\left[j_2'(++)+j_2'(--)\right]_z=-\frac{ie}{2m}\frac{1}{V}\sum_{p_1+p_2=q}e^{i(p_1+p_2)r}\left\{i(p_1+p_2)_x\left(a^+_{-p_1+}a_{p_2-}-a_{-p_1-}a_{p_2+}\right)+(p_1+p_2)_y\left(a^+_{-p_1+}a_{p_2-}+a^+_{-p_1-}a_{p_2+}\right)\right\}. \qquad (83)$$

In a similiar way as the formula (77) we get

$$j_{2x}(-+)=\frac{ie}{2m}\frac{1}{V}\sum_{p_1+p_2=q}e^{i(p_1+p_2)r}(p_1+p_2)_y\left[v(p_1)u(p_2)-v(p_2)u(p_1)\right]\left[C_{-+}(p_1,p_2)-C^*_{-+}(-p_2,-p_1)\right],$$

$$j_{2y}(-+)=-\frac{ie}{2m}\frac{1}{V}\sum_{p_1+p_2=q}e^{i(p_1+p_2)r}(p_1+p_2)_x\left[v(p_1)u(p_2)-v(p_2)u(p_1)\right]\left[C_{-+}(p_1,p_2)-C^*_{-+}(-p_2,-p_1)\right], \qquad (84)$$

$$j_{2x}(++)=\frac{e}{4m}\frac{1}{V}\sum_{p_1+p_2=q}e^{i(p_1+p_2)r}(p_1+p_2)_z\left[v(p_2)u(p_1)-v(p_1)u(p_2)\right]\left[C_{++}(p_1,p_2)+C^*_{++}(-p_2,-p_1)\right],$$

$$j_{2y}(++)=\frac{ie}{4m}\frac{1}{V}\sum_{p_1+p_2=q}e^{i(p_1+p_2)r}(p_1+p_2)_z\left[v(p_2)u(p_1)-v(p_1)u(p_2)\right]\left[C_{++}(p_1,p_2)-C^*_{++}(-p_2,-p_1)\right],$$

$$j_{2z}(++)=\frac{ie}{4m}\frac{1}{V}\sum_{p_1+p_2=q}e^{i(p_1+p_2)r}\left\{i(p_1+p_2)_x\left[v(p_2)u(p_1)-v(p_1)u(p_2)\right]\left[C_{++}(p_1,p_2)+C^*_{++}(-p_2,-p_1)\right]-\right.$$

$$\left.-(p_1+p_2)_y\left[v(p_2)u(p_1)-v(p_1)u(p_2)\right]\left[C_{++}(p_1,p_2)-C^*_{++}(-p_2,-p_1)\right]\right\}. \qquad (85)$$

The components of $\vec{j}_2(--)$ we do not write they turn out to be equal to the components $\vec{j}_2(++)$.

Let us consider the current (84) representing an additional term to $\vec{j}_1(-+)$ which arises in the (X, Y) plane. In order to obtain the exact functions ($C_{-+}-C^*_{-+}$) one must in the equations (68) take into account the term $\delta\tilde{H}_{\mathcal{H}}$ too. This gives also a small correction to $\vec{j}_1$. We want to investigate the current $\vec{j}_2(-+)$ in the zero approximation only. We take the equation (68) as our basic equation and consider the term arising from $\delta\tilde{H}_{\mathcal{H}}$ as a perturbation. According to [1] the equation without perturbation has the solution of the form

$$C_{-+}(p_1,p_2)-C^*_{-+}(-p_2,-p_1)=e\,\mathcal{O}\!l_z(q)\,q\,\tilde{\tau}(p,q) \qquad (86)$$

Putting this solution in (84) we obtain $\vec{j}_2(-+)$ in the zero approximation

$$j_{2x}(-+) = \frac{ie^2}{2m}\frac{1}{V}\sum_q e^{iqr} q_y \widetilde{\mathfrak{A}}_z(q) = \frac{e^2}{2m}\frac{\partial}{\partial y}\widetilde{\mathfrak{A}}_z(r),$$

$$j_{2y}(-+) = -\frac{ie^2}{2m}\frac{1}{V}\sum_q e^{iqr} q_x \widetilde{\mathfrak{A}}_z(q) = -\frac{e^2}{2m}\frac{\partial}{\partial x}\widetilde{\mathfrak{A}}_z(r) \tag{87}$$

where

$$\widetilde{\mathfrak{A}}_z(q) = \mathfrak{A}_z(q)\, q \sum_p \widetilde{\tau}(p,q)\left[v(p)u(p-q) - v(p-q)u(p)\right], \qquad \widetilde{\mathfrak{A}}_z(r) \equiv \sum_q \widetilde{\mathfrak{A}}_z(q) e^{iqr}.$$

Introducing the vector $\vec{\widetilde{\mathfrak{A}}} = (0,0,\widetilde{\mathfrak{A}})$ we get (87) in the form

$$\vec{j}_2(-+) = \frac{e^2}{2m}\frac{1}{V}\,\mathrm{curl}\,\vec{\widetilde{\mathfrak{A}}} = \frac{e^2}{2m}\frac{1}{V}\vec{\widetilde{\mathcal{H}}}. \tag{88}$$

As we see from (87), for small q $\vec{j}_2(-+)$ vanishes like q^2, that is like the terms neglected in obtaining the formula (80). Thus $\vec{j}_2(-+)$ gives not rise to the Meissner effect

Let us consider the "spin" current of elementary excitations with spin moment + 1. From (85) and (69) we get

$$j_{2x}(++) = \frac{ie^2}{4m^2}\frac{1}{V}\sum_q e^{-iqr} q_z \overline{\mathcal{H}}_y(q) = -\frac{e^2}{4m^2}\frac{1}{V}\frac{\partial}{\partial z}\overline{\mathcal{H}}_y(r),$$

$$j_{2y}(++) = -\frac{ie^2}{4m^2}\frac{1}{V}\sum_q e^{-iqr} q_z \overline{\mathcal{H}}_x(q) = \frac{e^2}{4m^2}\frac{1}{V}\frac{\partial}{\partial z}\overline{\mathcal{H}}_x(r), \tag{89}$$

$$j_{2z}(++) = -\frac{ie^2}{4m^2}\frac{1}{V}\sum_q e^{-iqr}\left[q_x\overline{\mathcal{H}}_y(q) - q_y\overline{\mathcal{H}}_x(q)\right] = \frac{e^2}{4m^2}\frac{1}{V}\left[\frac{\partial}{\partial x}\overline{\mathcal{H}}_y(r) - \frac{\partial}{\partial y}\overline{\mathcal{H}}_x(r)\right]$$

where

$$\vec{\overline{\mathcal{H}}} = (\overline{\mathcal{H}}_x, \overline{\mathcal{H}}_y, 0), \qquad \vec{\overline{\mathcal{H}}}(r) \equiv \sum_q \vec{\overline{\mathcal{H}}}(q) e^{-iqr}, \qquad \overline{\mathcal{H}}_x(q) = \mathcal{H}_x(q)\sum_p \frac{\left[v(p)u(p+q) - v(p+q)u(p)\right]^2}{\Omega(p) + \Omega(p+q)} = \mathcal{H}_x(q)\frac{V}{2}F(q). \tag{90}$$

The factor $F(q)$ is obtained in paper [2] in connection with screening of Coulomb interaction

$$F(q) = \frac{4}{V}\sum_{k'-k=q}\frac{\theta_G(k)\,\theta_F(k)}{\widetilde{E}(k) - \widetilde{E}(k')}$$

where $\widetilde{E}(k)$ is the energy of the electron elementary excitation and θ_F, θ_G the solution of the compensation equations for the normal state.

From (89) we see that the x-component of "screened" magnetic field $\vec{\overline{\mathcal{H}}}$ produces the currents in the (Y,Z) -plane and the y-component in the (X,Z) -plane. After introducing the vector $\vec{\overline{\mathcal{H}}}$ by (90) we get (89) in the form

$$\vec{j}_2(++) = \frac{1}{V}\frac{e^2}{4m^2}\,\mathrm{curl}\,\vec{\overline{\mathcal{H}}}(r) = \frac{1}{V}\frac{e^2}{4m^2}\,\mathrm{curl}\,\mathrm{curl}\,\vec{\overline{A}}(r). \tag{91}$$

The same formula we obtain for $\vec{j}_2(--)$. After changing in (91) to Fourier expansion we obtain the Fourier coefficients vanishing like q^2. Then the "spin" current gives not rise to the Meissner effect.

It is a pleasent duty to thank prof. N.N. Bogolubov for proposing this problem and helpful advice and D.V. Shirkov, V.V. Tolmachov and V.G. Soloviev for valuable discussions.

References

1. N.N. Bogolubov (preprint) "On the principle of compensation and method of the self-consistent field".
2. N.N. Bogolubov, V.V. Tolmachov, D.V. Shirkov"A new method in the theory of superconductivity", Izdat.Akad.Nauk SSSR (1958), English copy in Forts.d.Phys. (1959) - in print.
3. N.N. Bogolubov, Lectures on quantum statistics (in Ucrainian) Kiev, 1947.
4. M. Gell-Mann, K.A. Brueckner, Phys.Rev. 106, 364 (1957);
   K. Sawada, Phys.Rev. 106, 372 (1957); K. Sawada, K.A. Brueckner, N. Fukuda, R. Brout, Phys.Rev. 108, 507 (1957), R. Brout, Phys.Rev. 108, 515 (1957).
5. K. Yosida, (preprint) "Collective excitations in superconductors".
6. K. Yosida, Phys.Rev. 110, 769 (1958).
7. F. Reif, Phys.Rev. 106, 208 (1957).
8. J. Bardeen, L. Cooper, J. Schrieffer, Phys.Rev. 108, 1175 (1957).
9. J.M. Blatt, T. Matsubara, Prog.Theor.Phys. 20, 781 (1958);
   (preprint) "The Meissner-Ochsenfeld effect in the Bogolubov theory".

Received by Publication Department on May 5, 1959.