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Abstract

It is shown that the strong interaction introduces not only phase shifts representing final-state interaction but also new parameters into the transition matrix for hyperon decay processes. These parameters must be determined by solving problems of physical hyperons under strong interaction with the π-meson and K-meson fields. As a consequence the selection rule read off from the Hamiltonian is in general different from that given by the transition matrix. The general consideration is also applied to the case of universal Fermi interactions including hyperons. Consistency with the selection rule $\Delta T = \pm \frac{1}{2}$ cannot be achieved unless the interaction also includes the charge-retention pairs such as (N^+, N^+), (N^o, N^o) and (N^o, Λ^o). The theoretical difficulties of including these pairs are also discussed.

I. Introduction

It has been suggested that the decay of hyperons into nucleons and mesons satisfies the seleotion rule $\Delta T = \pm \frac{1}{2}$ [1], where T is the total isotopic spin of the hyperon. Further developments of this suggestion proceed along two directions. On the one side the transition matrix for various decay processes obeying this rule has been determined phenomenologically from the observed branching ratios and ratios of life-times, on the other side interaction Hamiltonian for week interactions has been searched which will lead most naturally to this selection rule. It seems however to have escaped attention that owing to the effect of strong interaction, the connection between the transition matrix and the interaction Hamiltonian is in general not very simple, and it is the aim of the present paper to give a general account of this conneotion. It will first be pointed out that the effect of strong interaction is twofold. Firstly, it introduces phase shifts into wave functions for various final states. This effect has been taken into account properly in literature and will not be dwelt with in the present paper. The second effect arises from the fact that the initial hyperon is already a complicated physical system owing to the existence of strong interaction. It will be shown below that the general expression for transition matrix contains a number of unknown parameters which must be determined by strong interaction. From this we shall see that the selection rule read off from the interaction Hamiltonian is in general different from that given by the transition matrix. In order to determine these parameters we must solve the Schrödinger equation for the physical hyperons under π - and K-interactions. In Section 3, the consideration is extended to the case of universal Fermi interaction.

2. The transition matrix for weak interaction of the Yukawa type

In the following consideration we shall assume that the strong interaction is invariant under rotations in isotopic spin space. The state vector of a physical Σ^+-particle may be written in the following form:

$$\begin{aligned}\Psi(\Sigma^+) = {} & \alpha Z_0^0 \bar{\Sigma}^+ + \beta X_1^1 \bar{\Lambda}^0 + \gamma\left(\frac{1}{\sqrt{2}} Z_1^1 \bar{\Sigma}^0 - \frac{1}{\sqrt{2}} Z_1^0 \bar{\Sigma}^+\right) \\ & + \delta\left(\sqrt{\frac{3}{5}} Z_2^2 \bar{\Sigma}^- - \sqrt{\frac{3}{10}} Z_2^1 \bar{\Sigma}^0 + \sqrt{\frac{1}{10}} Z_2^0 \bar{\Sigma}^+\right) \\ & + \epsilon Z_{1/2}^{1/2} \bar{\Xi}^0 + \lambda\left(\sqrt{\frac{3}{4}} Z_{3/2}^{3/2} \bar{\Xi}^- - \sqrt{\frac{1}{4}} Z_{3/2}^{1/2} \bar{\Xi}^0\right)\end{aligned} \qquad (1)$$

where $\bar{\Sigma}^+$, $\bar{\Sigma}^o$, $\bar{\Sigma}^-$, $\bar{\Lambda}^o$, $\bar{\Xi}^o$ and $\bar{\Xi}^-$ are emission operators for the corresponding hyperons*, $X_T^{T_3}$ and $Z_T^{T_3}$ represent the normalized states of the meson cloud surrounding the hyperon with isotopic spin equal to T and the z-component T_3. The number of mesons (π and $\widetilde{K}$) contained in these states is indefinite as it should be; in fact $X_T^{T_3}$ and $Z_T^{T_3}$ are ascending series of products of emission operators for $\widetilde{K}$- and π -mesons. The whole expression (1) should be operated on the vacuum state vector which we have omitted for the sake of simplicity. In writing (1) we have also neglected the virtual hyperon pairs. It can easily be seen that the six terms containing unknown parameters α , β , γ , δ , ϵ and λ each represent a state of $T = 1$ and $T_3 = 1$. We see also from (1) that the core of the hyperon is in a superposition of Σ^+ , Σ^o , Σ^-, Λ^o , Ξ^o and Ξ^- states. These states will pass into one another by emitting or absorbing a π -meson or a $\widetilde{K}$-meson. The core can also pass into N^+ and N^o states by emitting a π -meson or a $\widetilde{K}$-meson. However, these latter states will not lead to hyperon decay and therefore have been dropped.

Similarly we have for the physical Σ^- and Λ^o particles the following state vectors

$$\begin{aligned}\Psi(\Sigma^-) = &\alpha Z_0^o \bar{\Sigma}^- + \beta X_1^{-1} \bar{\Lambda}^o + \gamma \left(\tfrac{1}{\sqrt{2}} Z_1^o \bar{\Sigma}^- - \tfrac{1}{\sqrt{2}} Z_1^{-1} \bar{\Sigma}^o \right) \\ &+ \delta \left(\sqrt{\tfrac{1}{10}}\, Z_2^o \bar{\Sigma}^- - \sqrt{\tfrac{3}{10}}\, Z_2^{-1} \bar{\Sigma}^o + \sqrt{\tfrac{3}{5}}\, Z_2^{-2} \bar{\Sigma}^+ \right) \\ &+ \epsilon\, Z_{1/2}^{-1/2}\, \bar{\Xi}^- + \lambda \left(\sqrt{\tfrac{1}{4}}\, Z_{3/2}^{-1/2}\, \bar{\Xi}^- - \sqrt{\tfrac{3}{4}}\, Z_{3/2}^{-3/2}\, \bar{\Xi}^o \right)\end{aligned} \tag{2}$$

$$\begin{aligned}\Psi(\Lambda^o) = &\alpha' X_o^o \bar{\Lambda}^o + \beta' \left(\sqrt{\tfrac{1}{3}}\, R_1^1 \bar{\Sigma}^- - \sqrt{\tfrac{1}{3}}\, R_1^o \bar{\Sigma}^o + \sqrt{\tfrac{1}{3}}\, R_1^{-1} \bar{\Sigma}^+ \right) \\ &+ \epsilon' \left(\tfrac{1}{\sqrt{2}}\, S_{1/2}^{1/2}\, \bar{\Xi}^- - \tfrac{1}{\sqrt{2}}\, S_{1/2}^{-1/2}\, \bar{\Xi}^o \right)\end{aligned} \tag{3}$$

where $R_T^{T_3}$, $S_T^{T_3}$ has similar meaning as $Z_T^{T_3}$. We have not written down the state vector for the Σ^o particle which decays into a Λ^o particle and a γ -quantum before any weak interaction can take place and therefore is not of interest in our consideration. In writting (2), use has been made of the invariance under the charge-symmetry substitution $Y_T^{T_3} \longrightarrow (-1)^{T-T_3} Y_T^{-T_3}$.

In the present investigation, we shall not consider the decay of physical Ξ

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* Under rotation of isotopic spin space, $\bar{\Sigma}^+$, $\bar{\Sigma}^o$ and $\bar{\Sigma}^-$ behave like spherical harmonics Y_1^1 , Y_1^o and Y_1^{-1} . Our $\bar{\Sigma}^+ \equiv \frac{1}{\sqrt{2}}(-\bar{\Sigma}_1 - i\,\bar{\Sigma}_2)$ differs by a sign from the conventional definition.

-particles, since these particles can only decay through the channel $\Xi^{0,-} \longrightarrow \Lambda^0 + \pi^{0,-}$ therefore the selection rule $\Delta T = \pm\frac{1}{2}$ is always satisfied by the transition matrix.

Interaction Hamiltonian for weak interaction has been obtained by many authors using the concept of fictitious spurons[1/] which is equivalent to the requirement that the interaction Hamiltonian should be an iso-spinor instead of an iso-scalar. The part of the Hamiltonian which satisfies the selection rule $\Delta T = -1/2$ is given by*

$$H_1 = g_1\Big\{\sqrt{\tfrac{2}{3}}\,\Big(\sqrt{\tfrac{2}{3}}\,\bar{N}^0 O_1 \Sigma^+ \pi^- + \sqrt{\tfrac{1}{3}}\,\bar{N}^+ O_1 \Sigma^+ \pi^0\Big) + \sqrt{\tfrac{1}{3}}\,\Big(\sqrt{\tfrac{1}{3}}\,\bar{N}^0 O_1 \Sigma^0 \pi^0 - \sqrt{\tfrac{2}{3}}\,\bar{N}^+ O_1 \Sigma^0 \pi^+\Big)\Big\} + h.c \tag{5a}$$

and

$$H_2 = g_2\Big\{-\sqrt{\tfrac{1}{3}}\,\bar{\Sigma}^+ O_2 \Xi^0 \pi^+ - \sqrt{\tfrac{1}{3}}\,\bar{\Sigma}^0 O_2 \Xi^0 \pi^0 - \sqrt{\tfrac{1}{3}}\,\bar{\Sigma}^- O_2 \Xi^0 \pi^-\Big\} + h.c \tag{5b}$$

The other part satisfying the selection rule $\Delta T = +1/2$ is given by

$$H_3 = g_3\Big\{\sqrt{\tfrac{1}{6}}\,\Big(-\sqrt{\tfrac{1}{3}}\,\bar{N}^0 O_3 \Sigma^+ \pi^- + \sqrt{\tfrac{2}{3}}\,\bar{N}^+ O_3 \Sigma^+ \pi^0\Big) + \sqrt{\tfrac{1}{3}}\,\Big(\sqrt{\tfrac{2}{3}}\,\bar{N}^0 O_3 \Sigma^0 \pi^0 - \sqrt{\tfrac{1}{3}}\,\bar{N}^+ O_3 \Sigma^0 \pi^+\Big) + \sqrt{\tfrac{1}{2}}\,\Big(-\bar{N}^0 O_3 \Sigma^- \pi^+\Big)\Big\} + h.c \tag{6a}$$

$$H_4 = g_4\Big\{\sqrt{\tfrac{1}{3}}\,\Big(-\sqrt{\tfrac{1}{2}}\,\bar{\Sigma}^0 O_4 \Xi^0 \pi^0 + \sqrt{2}\,\bar{\Sigma}^- O_4 \Xi^0 \pi^-\Big) + \sqrt{\tfrac{2}{3}}\,\Big(\sqrt{\tfrac{1}{2}}\,\bar{\Sigma}^0 O_4 \Xi^- \pi^+ - \sqrt{\tfrac{1}{2}}\,\bar{\Sigma}^- O_4 \Xi^- \pi^0\Big)\Big\} + h.c \tag{6b}$$

$$H_5 = g_5\Big\{-\sqrt{\tfrac{2}{3}}\,\bar{N}^+ O_5 \Lambda^0 \pi^+ - \sqrt{\tfrac{1}{3}}\,\bar{N}^0 O_5 \Lambda^0 \pi^0\Big\} + h.c \tag{6c}$$

$$H_6 = g_6\Big\{\sqrt{\tfrac{1}{3}}\,\big(\bar{\Lambda}^0 O_6 \Xi^0 \pi^0\big) + \sqrt{\tfrac{2}{3}}\,\big(-\bar{\Lambda}^0 O_6 \Xi^- \pi^+\big)\Big\} + h.c \tag{6d}$$

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* Notice that in our notation the emission and absorption operators for Σ^+ and π^+ differ by a sign from the conventional ones.

where $N^{+}_{;}$ N^{0} and $\pi^{\pm,0}$ are field operators for protons, neutrons and π -mesons respectively. g_i's are coupling constants and O_i's are operators which may give rise to parity non-conservation.

We shall now proceed to evaluate the physical transition matrix for the decay of hyperons using (5). Since the coupling constants g_i are very small we shall consider only the first order perturbation with respect to the weak interaction. To simplify the calculation we shall consider only the case $g_2=g_4=g_6=0$. This means that the decay of particles via the bare Ξ^{o} and Ξ^{-} states is not to be considered. We shall see later that this simplification does not influence our final conclusion. The transition matrix representing observed physical decay processes is thus obtained by introducing in (1), (2) and (3) the following substitutions

$$\begin{aligned}
\bar{\Sigma}^{+} &\longrightarrow F_1(t)\left\{\sqrt{\tfrac{2}{3}}\, g_1 \mathfrak{Z}^{1/2}_{1/2}(1) + \sqrt{\tfrac{1}{6}}\, g_3 \mathfrak{Z}^{1/2}_{3/2}(3)\right\} \\
\bar{\Sigma}^{o} &\longrightarrow F_2(t)\left\{\sqrt{\tfrac{1}{3}}\, g_1 \mathfrak{Z}^{-1/2}_{1/2}(1) + \sqrt{\tfrac{1}{3}}\, g_3 \mathfrak{Z}^{-1/2}_{3/2}(3)\right\} \\
\bar{\Sigma}^{-} &\longrightarrow F_3(t)\sqrt{\tfrac{1}{2}}\, g_3 \mathfrak{Z}^{-3/2}_{3/2}(3) \\
\bar{\Lambda}^{o} &\longrightarrow F_4(t)\, g_5\, \mathfrak{Z}^{-1/2}_{1/2}(5)
\end{aligned} \tag{7}$$

where $\mathfrak{Z}^{T_3}_{T}(i)$ represents a normalized wave function for one meson and one bare nucleon with total isotopic spin T, the z-component T_3 and parity violation determined by O_i . $F_i(t)$ are the time dependent amplitudes obtained by perturbation calculations*.
After the substitution, (3) becomes**

$$\begin{aligned}
\Phi(\Lambda^{o}) = F(t)\Big\{ &\alpha' X^{o}_{o}\, g_5 \mathfrak{Z}^{-1/2}_{1/2}(5) + \beta'\sqrt{\tfrac{1}{3}}\, R^{1}_{1}\sqrt{\tfrac{1}{2}}\, g_3 \mathfrak{Z}^{-3/2}_{3/2}(3) \\
&- \beta'\sqrt{\tfrac{1}{3}}\, R^{o}_{1}\left(\sqrt{\tfrac{1}{3}}\, g_1 \mathfrak{Z}^{-1/2}_{1/2}(1) + \sqrt{\tfrac{1}{3}}\, g_3 \mathfrak{Z}^{-1/2}_{3/2}(3)\right) \\
&+ \beta'\sqrt{\tfrac{1}{3}}\, R^{-1}_{1}\left(\sqrt{\tfrac{2}{3}}\, g_1 \mathfrak{Z}^{1/2}_{1/2}(1) + \sqrt{\tfrac{1}{6}}\, g_3 \mathfrak{Z}^{1/2}_{3/2}(3)\right)\Big\}
\end{aligned} \tag{8}$$

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* In the following we shall overlook the differences of these functions and denote them by the same function F(t). This is probably justified when the mass differences of these particles are neglected according to an argument of Pais[2]. We shall see in the following that this approximation is essential for the cancellation of $\Delta T = 3/2$ and $5/2$ terms in the decay matrix.

** It should be noted that our perturbation must involve a renormalization of state vectors so that it is $X^{o}_{o}\mathfrak{Z}^{-1/2}_{1/2}$, $R^{1}_{1}\mathfrak{Z}^{-3/2}_{3/2}$ etc. and not X^{o}_{o} , R^{1}_{1} , etc. which are normalized to unity after the perturbation.

At the moment just after the weak interaction has taken place and before the strong interaction among the final states has set in, the system is already no longer a pure state with definite values of T and T_3. With the help of Clebsh-Gordon formulae we can write (8) in the following form

$$\Phi(\Lambda) = F(t)\left[\alpha' g_5 \mathcal{Y}^{-1/2}_{1/2}(5) + \sqrt{\tfrac{1}{3}}\,\beta' g_3 \mathcal{Y}^{-1/2}_{1/2}(3)\right] - F(t)\sqrt{\tfrac{1}{3}}\,\beta' g_1 \mathcal{Y}^{-1/2}_{1/2}(1) \qquad (9a)$$

where $\mathcal{Y}^{T_3}_{T}(i)$ represents the state of the meson-nucleon system with isotopic spin T and the z-component T_3 and with degree of parity violation determined by O_1.

By similar calculation we obtain for the decays of Σ^+ and Σ^- particles.

$$\Phi(\Sigma^+) = F(t)\Big[\big(\sqrt{\tfrac{2}{3}}\,\alpha + \tfrac{2}{3}\gamma\big) g_1 \mathcal{Y}^{1/2}_{1/2}(1) + \big(\tfrac{1}{3}\gamma + \tfrac{1}{\sqrt{3}}\delta\big) g_3 \mathcal{Y}^{1/2}_{1/2}(3) + \sqrt{\tfrac{2}{3}}\,\beta g_5 \mathcal{Y}^{1/2}_{1/2}(5)\Big]$$

$$+ F(t)\Big[\big(-\tfrac{1}{3\sqrt{2}}\gamma - \tfrac{1}{\sqrt{6}}\delta\big) g_1 \mathcal{Y}^{1/2}_{3/2}(1) + \big(\sqrt{\tfrac{1}{6}}\,\alpha + \tfrac{\sqrt{5}}{6}\gamma + \tfrac{1}{2\sqrt{3}}\delta\big) g_3 \mathcal{Y}^{1/2}_{3/2}(3) + \sqrt{\tfrac{1}{3}}\,\beta g_5 \mathcal{Y}^{1/2}_{3/2}(5)\Big] \qquad (9b)$$

$$\Phi(\Sigma^-) = \sqrt{3}\,F(t)\Big[\big(-\tfrac{1}{3\sqrt{2}}\gamma - \tfrac{1}{\sqrt{6}}\delta\big) g_1 \mathcal{Y}^{-3/2}_{3/2}(1) + \big(\sqrt{\tfrac{1}{6}}\,\alpha + \tfrac{\sqrt{5}}{6}\gamma + \tfrac{1}{2\sqrt{3}}\delta\big) g_3 \mathcal{Y}^{-3/2}_{3/2}(3) + \sqrt{\tfrac{1}{3}}\,\beta g_5 \mathcal{Y}^{1/2}_{3/2}(5)\Big] \qquad (9c)$$

as soon as the state (9) is reached, the system will enter into the so-called "final-state" interaction.Since this interaction cannot change the eigen values T and T_3,we see already from (9) that the transition matrix contains only transitions with $\Delta T = \pm 1/2$. We shall come back to the effect of final-state interaction later on. We see also from the above result that the ratio of transition probability for the decay of Σ^- into T = 3/2 state is three times larger than that of Σ^+ into the same isotopic spin state. This is a well-known result for the transition matrix for hyperon decays.

From the above result the following points will be noted. Firstly if we assume that all g_i's in (5) and (6) except g_1 are equal to zero then the interaction Hamiltonian contains only the matrix elements for which $\Delta T = -1/2$. However from (8)-(9) we see immediately that the resulting transition matrix contains matrix elements for both $\Delta T = -1/2$ and $\Delta T = +1/2$. Similarly if we assume for the case of Ξ -decay that all g_i's except g_2 to be equal to zero, then again the interaction Hamiltonian contains only matrix elements with $\Delta T = -1/2$. On the other hand we find that the resulting transition matrix contains only the transitions $\Delta T = +1/2$.

The above difference in selection rules between the interaction Hamiltonian and the resulting transition matrix is due to the situation that the total iso-spin of the physical hyperon is equal to the vector sum of the iso-spin of the bare hyperon and the iso-spin of the meson cloud surrounding it. Therefore in the general case the change of iso-spin of the physical hyperon need not be the same as that of the core.

The following interaction Hamiltonian for the decay of Σ and Λ has been obtained by us recently[3] without using the concept of spurons

$$H_{weak} = g_1 H_1 + g_5 H_5 \tag{12}$$

It is seen that our interaction Hamiltonian contains no transition for the process $\Sigma^- \rightarrow N^0 + \pi^-$, while from (9) we see by putting all g_i's except g_1 and g_5 equal to zero that this process is not forbidden in the transition matrix. The conclusion of our foregoing discussions is very important since it provides a larger room for the searching of interaction Hamiltonian for the weak interaction, accordingly no prejudice should be taken against the Hamiltonian like (12).

It is well-known that in the strong-coupling limit the core of a physical nucleon has equal probabilities of being a proton and a neutron. One may infer from this result that the unknown parameters α, β, γ, δ, etc. might be determined from the condition that the core of the hyperon should have equal probabilities of being in various bare hyperon states. We have shown by a model in which the K^--interaction is absent and the π-meson is coupled to the hyperons by a scalar interaction that the above expectation is true only for Ξ-particles. For Σ and Λ particles, the parameters α, β, α', β', etc. are complicated functions of the coupling constants and do not lead in the general case to equal probabilities for the core to be in different bare hyperon states in the strong coupling limit. There is no more reason for the above expertation to be true in the actual case.

As soon as the weak interaction has taken place, the transition probability of the system to be in a given channel with angular momentum ℓ and the isotopic spin T cannot be altered by the strong interaction in the final state. This transition probability can therefore be determined directly from the Φ's given by (9). The final-state interaction can only introduce a phase factor to the state vectors for each channel. As an example, we find by using the results of Fermi[6] and Taketa[1] that the transition matrix element for the decay of Λ into the channel T and ℓ is given by

$$\langle T, \ell | R | \Lambda \rangle = \pm [P(T, \ell)]^{1/2} e^{i\delta_{2T+1,2\ell+1}}$$

where $\delta_{2T+1,2\ell+1}$ is the phase shift for the baryon-meson system, and $P(T, \ell)$ is the transition probability as determined from (9a)

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* I.e. the same π-interaction as given by d'Espagnat and Prentki[5] that $i\gamma_5$ is replaced by 1.

3. The interaction matrix for the case of universal Fermi interaction

It has been shown recently by Feynman and Gell-Mann[7] and also by Sudarshan and Marshak[8] that a universal Fermi interaction with parity violating V − A coupling gives a satisfactory account of decay processes not involving hyperons. These authors also pointed out that if two new pairs, (Λ^o , N^+) and (Λ^o , Ξ^-), are added to the already known pairs (μ^- , ν), (e^- , ν) and (N^o, N^+) which constitute the well-known Puppi triangles, then the decay of hyperons into baryons and mesons can also be accounted for by the universal Fermi interactions. In the last section the general relations between the interaction Hamiltonian for the weak interaction of the Yukawa type and the observed transition matrix are given. It is the aim of this section to extend the consideration to the case of universal Fermi interaction including hyperons. In the following calculation we shall show that the selection rule $\Delta T = \pm 1/2$ cannot be satisfied unless "charge-retention" pairs such as (N^o, Λ^o),(N^+,N^+),(N^+,N^o) which consist of two particles with the same charge are also included in addition to the conventional "charge-exchange" pairs.

In our consideration we shall assume that the number of heavy particles is conserved and that the decay of hyperons into baryons and mosons satisfies the selection rule $\Delta T = \pm 1/2$. This selection rule gives satisfactory explanation of the stability of charged K-mesons and the observed branching ratio of Λ -decay,but this is not the only possible explanation*. On the other hand, owing to the difficulty in carrying through the actual calculation, we cannot see any clue as to what dynamical effect could be there which simulates the rule $\Delta T = \pm 1/2$. Therefore if one does not consider the experimental results as accidental, the assumption of $\Delta T = \pm 1/2$ seems to be the most natural one. In any case it will be of interest to find out whether the universal Fermi interaction is consistent with the selection rule $\Delta T = \pm 1/2$.

Following Feynman and Gell-Mann we shall further assume that the parity violating V − A interaction exists between any two pairs that are included in the interaction. The whole family of possible "charge exchange" pairs that can participate in the interaction are as follows

$$\begin{array}{l} (e^-,\nu),\ (\mu^-\ \nu),\ (N^o,N^+) \\ (\Sigma^+,N^o),\ (\Sigma^-,N^o),\ (\Lambda^o,N^+),\ (\Sigma^o,N^+) \\ (\Xi^-,\Lambda^o),(\Xi^-,\Sigma^o),\ (\Xi^o,\Sigma^+),\ (\Xi^o\ \Sigma^-) \end{array} \qquad (13)$$

Out of any two of (13) say (Σ^+ ,N^o) and (Ξ^o , Σ^-), the following interaction Ha-

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* For instance, when $\Delta T = + 1/2$ rule holds, the branching ratio of Λ -decay is described correctly by the wave function $\sqrt{\tfrac{1}{3}}N^+\pi^o-\sqrt{\tfrac{2}{3}}N^o\pi^+$. But the same branching ratio is also obtained if the wave function is replaced by $\sqrt{\tfrac{1}{3}}N^+\pi^o+\sqrt{\tfrac{2}{3}}N^o\pi^+$. This latter wave function is a superposition of $T = 3/2$ and $T = 1/2$ states. We are indebted to Professor Votruba for drawing our attention to this point.

miltonian can be constructed

$$(\Sigma^+, N^o)(\Sigma^-, \Xi^o) \equiv g(\bar{\Sigma}^+(1+\gamma_5)\gamma_\nu N^o)(\bar{\Sigma}^-(1+\gamma_5)\gamma_\nu \Xi^o) \tag{14}$$

where g is the universal constant. The pairs such as (Ξ^-, N^0) and (Ξ^o, N^+) have not been included since they would evidently lead to the processes $\Xi^{o,-} \longrightarrow N^{+,o} + \pi^-$ which violate the selection rule $\Delta T_3 = \pm 1/2$.

However, (13) also contains other pairs which will lead to processes with $\Delta T_3 \neq \pm 1/2$ by annihilation of virtual Σ and Λ with their anti-particles. For instance we have for the interactions (Ξ^-, Σ^o) (Σ^+, N^0) and (Σ^+, N^0)(Σ^o, N^+) the following processes.

$$\Xi^- \longrightarrow N^o + \tilde{\Sigma}^+ + \Sigma^o \longrightarrow N^o + \pi^- \tag{15}$$

and

$$\Xi^- \longrightarrow K^- + \Sigma^o \longrightarrow K^- + N^+ + N^o + \tilde{\Sigma}^+ \longrightarrow N^o + \pi^-$$

It can easily be seen that in order to exclude all processes of these types, the universal Fermi interaction can only exist among one of the following two sets of pairs:

$$(i) \quad (N^o, \Sigma^+), (\Sigma^-, \Xi^o), (N^+ N^o), (e^- \nu), (\mu^- \nu)$$

$$(ii) \quad (N^o, \Sigma^-), (\Sigma^+ \Xi^o), (N^+ \Sigma^o), (N^+ \Lambda^o), (\Lambda^o \Xi^-), (\Sigma^o, \Xi^-) \tag{16}$$

$$(N^+ N^o), (e^- \nu), (\mu^- \nu)$$

It should be noted that the interactions (Σ^+, N^0), (Σ^-, Ξ^o), (Σ^-, N^0), (Σ^+, Ξ^o) and (Σ^o, N^+)(Σ^o, Ξ^-) also contain $\Delta T_3 = 0$ transition of the following type

$$\Xi^o \longrightarrow \Sigma^- + \Sigma^+ + \tilde{N}^o$$

However, since these transitions cannot conserve the energy of the system, they do not give rise to any real decay process. The sets (i), (ii) represent those which contain maximium number of pairs. We may, for instance, obtain new sets from (i) by dropping the pair (Σ^- , Ξ^o) or (N^0, Σ^+). When (N^0, Σ^+) is dropped, the decay of physical Σ^+-particle will take place via the bare Ξ^o -state with the emission of a virtual K-meson. Similarly when (Σ^- , Ξ^o) is dropped, the decay of physical Ξ^o-particle will take place via the bare Σ^+ state.

It can further be verified that apart from those transition which cannot cause any real decay process, the universal Fermi interactions for the case (i) also satisfy the stronger rule $\Delta T = \pm 1/2$ and that when the pair (Λ^0 , N^+) is excluded from the (ii) the universal Fermi interaotion for the resulting set

(iia) $(N^0\Sigma^-),(\Sigma^0\Xi^-),(\Sigma^+\Xi^0),(N^+,\Sigma^0),\ (\Lambda^0,\Xi^-),\ (N^+,N^0),\ (e^-,\nu),\ (\mu^-,\nu)$

also satisfies the selection rule $\Delta T = \pm 1/2$. The reason for excluding (Λ^0 , N^+) is that the following process

$$\Lambda^0 \longrightarrow N^+ + \tilde{N}^+ + N^0 \begin{cases} N^+ + \pi^- \\ N^0 + \pi^0 \end{cases} \tag{17}$$

resulted from the interaction (N^0,N^+) (N^+, Λ^0) leads to T = 3/2 (Δ T = 3/2)as well as T = 1/2 (Δ T = 1/2) states.

One might oonclude from the foregoing result that the universal Fermi interaction for any one of these two mutually exolusive sets may be the true weak interaction for all decay processes. However, we have shown in the last seotion that owing to the influence of strong interaction, the selection rule with respect to isotopic spin whioh we read off from the interaction Hamiltonian are in general not the same as those observed experimentally. We shall show in the following that the selection rule $\Delta T = \pm 1/2$ is not satisfied by the transition matrix deduced from the interaction Hamiltonian for both cases (i) and (iia).[9]

We shall oonfine our attention to the decay of Λ -partiole since it is for this decay process that the seleotion rule $\Delta T = \pm 1/2$ has been inferred from the experiment. The state vector for the physical Λ -particle may be written

$$\Psi(\Lambda^0) = \alpha' X_0^0 \bar{\Lambda}^0 + \beta'(\sqrt{\tfrac{1}{3}} R_1^1 \bar{\Sigma}^- - \sqrt{\tfrac{1}{3}} R_1^0 \bar{\Sigma}^0 + \sqrt{\tfrac{1}{3}} R_1^{-1} \bar{\Sigma}^+) + \epsilon'(\tfrac{1}{\sqrt{2}} S_{1/2}^{1/2} \bar{\Xi}^- - \tfrac{1}{\sqrt{2}} S_{1/2}^{-1/2} \bar{\Xi}^0) \tag{18}$$

If we choose the set (i) as participants of universal Fermi interaotion and oonsider only the first order perturbation with respect to this interaction, we find that the interaction term$(N^+,N^0)(N^0,\Sigma^+)$will lead to virtual transition of the bare Σ^+ state into two nucleons and one anti-nucleon. With the help of Glebsh-Gordan formulae we can easily see that the final state is a superposition of T = 1/2 and T = 3/2 states.
One of the decay channels has the form* $\sqrt{\tfrac{2}{3}}\, \mathfrak{z}_{1/2}^{1/2} + \sqrt{\tfrac{1}{3}}\, \mathfrak{z}_{3/2}^{1/2}$ where $\mathfrak{z}_T^{T_3}$ denotes the state of meson cloud plus a nucleon with isotopic spin T and the z-oomponent T_3. The effect of first order perturbation is to introduce the following replacement in (18)

$$\bar{\Sigma}^+ \longrightarrow F(t)\, g\, (\sqrt{\tfrac{2}{3}}\, \mathfrak{z}_{1/2}^{1/2} + \sqrt{\tfrac{1}{3}}\, \mathfrak{z}_{3/2}^{1/2}),\ \ \bar{\Sigma}^{-,0} \longrightarrow 0\ \ \bar{\Lambda}^0 \longrightarrow 0 \tag{19}$$

where F(t) is the time dependent transition amplitude as defined in Section 2. Therefore the state vector for the final state is given by

$$\Phi(\Lambda^0) = F(t)\, \beta' \sqrt{\tfrac{1}{3}}\, R_1^{-1}\, g\, (\sqrt{\tfrac{2}{3}}\, \mathfrak{z}_{1/2}^{1/2} + \sqrt{\tfrac{1}{3}}\, \mathfrak{z}_{3/2}^{1/2}) \tag{20}$$

* On similiar way, the same results may be obtained from the other channels, which may include the hypothetical neutral pi-meson with I=0.

R_1^{-1} and $\mathfrak{z}_T^{T_3}$ may be expressed as a linear combination of eigen functions of T and T_3 by Clebsh-Gordan formulae as we have done in Section 2. We have then

$$\Phi(\Lambda^0) = F(t)\beta'\sqrt{\tfrac{1}{3}}\, g\left(\sqrt{\tfrac{1}{10}}\, \mathcal{Y}_{5/2}^{-1/2} + \frac{\sqrt{2}}{3}\left(1-\frac{2}{\sqrt{5}}\right)\mathcal{Y}_{3/2}^{-1/2} - \frac{1}{3}\left(2-\frac{1}{2}\right)\mathcal{Y}_{1/2}^{-1/2}\right) \tag{21}$$

where $\mathcal{Y}_T^{T_3}$ has been defined in Section 2. We see immediately that the transition matrix contains transitions with $\Delta T = 5/2, 3/2$ as well as $1/2$. This result illustrates as an extreme case how far the transition matrix can be different from the interaction Hamiltonian. We conclude therefore that the universal Fermi interaction involving the pair (N^0, Σ^+) is in disagreement with the selection rule $\Delta T = \pm 1/2$.

Next we consider the interaction (N^+,N^0) (Σ^-, Ξ^0). This interaction will lead to virtual transition of a bare Ξ^0 to $\Sigma^- + N^+ + \tilde{N}^0$ in a mixture of T = 0, 1 and 2 states. From the last term of (18) we see immediately that the resulting transition will contain $\Delta T > 1/2$. Thus we conclude that the pairs (N^0, Σ^+) and (Σ^-, Ξ^0) and consequently the set (i) are inconsistent with the selection rule $\Delta T = \pm 1/2$.

By similar consideration we can show that the universal Fermi interactions for the set (iia) is also inconsistent with the selection rule $\Delta T = \pm 1/2$.

From (18) it is clear that if the decay of physical Λ -particle takes place via the $\Delta T = \pm 1/2$ transition of <u>any one</u> of the states Σ^+, Σ^0, Σ^-, Ξ^0 and Ξ^-, then transitions with $\Delta T > 1/2$ will always appear in the transition matrix. For the weak interactions considered in Section 2, the transitions via Σ^+, Σ^0 and Σ^- and via Ξ^0 and Ξ^- are in such a proportion that the transitions with $\Delta T > 1/2$ just cancel themselves. This explains why all the pairs containing Σ -particles and Ξ -particles of (16) leads to violation of $\Delta T = \pm 1/2$. On the other hand, we see from (18) that if the decay takes place via the $\Delta T = \pm 1/2$ transition of Λ, then the resulting transition matrix will always satisfy $\Delta T = \pm 1/2$. From (16) we find that the only pair which contains only Λ is (N^+, Λ). However, as we have pointed out before, the interaction Hamiltonian (N^0,N^+)(N^+, Λ) contains also transitions with $\Delta T = 3/2$ unless it is modified to the following form

$$H_\Lambda = g\left\{(N^0; N^0)(N^0\Lambda^0) + (N^+N^+)(N^0\Lambda^0) - 2(N^0N^+)(N^+\Lambda^0)\right\} \tag{22}$$

For the V-A interaction the second term is actually equivalent to the third term. The first term contains only neutral pairs and is a new feature which does not exist for β -decays. That the appearance of the first term is objectionable in the universal Fermi interaction is seen from the fact that we would have no reason to reject the terms $(N^0,N^0)(\nu,\nu)$, $(N^0,N^0)(e^-,e^-)$, etc. representing double β-decay processes which would be possible for an excited nucleus. We see from (22) that if the equivalent second and third terms are combined together, the resulting terms $(N^0,N^0)(N^0\ \Lambda^0)$ and $(N^0,N^+)(N^+,\ \Lambda^0)$ both have the same coefficient, except that the signs are different, which means that the coupling strength is still universal.

If the interaction (22) is accepted in spite of the difficulties with the neutral pairs, then the universal Fermi interaction of the V-A type

(iii) (Λ^0,N^+), (Λ^0,N^0), (N^0,N^0) (N^+,N^0), $(e^-\nu)$, $(\mu^-\nu)$

will give a description of all decay processes which is consistent with the selection rule $\Delta T = \pm 1/2$ for decays of hyperons into nucleons and mesons.

Finally, the authors note that the β-(μ)-decay of hyperons which have been recently discovered may be considered as the decay through neutron and $(\bar{K},\pi)$ meson cloud which we do not consider in this paper.

Note added in proof:

In the foregoing we have only considered the decay of physical Λ-particles via bare Λ-state, and decay of physical Ξ^--particles via bake Ξ^--state, etc. the following, we have to consider the decay of physical particle via bare $\Sigma^+,\Sigma^-,\Sigma^0,\Xi^-$ and Ξ^0 states.

References

1. G. Wentzel, Phys.Rev. 101 1214 (1956).
   G. Taketa, Phys.Rev. 101 1547 (1956).
   d'Espagnat and Prentki, Nuovo Cim., 3 1045 (1956).
   R. Gatto, Nuovo Cim., 3 318 (1956).
   L. Okun, JETF 31 333 (1956).
   Iso and Kawaguchi, Prog.Theor.Phys. 16 177 (1956)
   R.H. Dalitz, Proc.Phys.Soc. A 69 527 (1956)
2. A. Pais, Phys.Rev. 110 574 (1958).
3. N. Hu, JETF 34 646 (1958), Padova-Venice Conference 1957.
4. N. Hu, "On the Interaction of Strange Particles" Report from Joint Institute for Nuclear Research P-168 also to be published in Nuclear Physics.
5. d'Espagnat and Prentki, Nuclear Phys. 1 33 (1956).
6. E. Fermi, Nuovo Cimento, Supplemento al 2 1 (1955).
7. Feynmann and Gell-Mann, Phys.Rev. 109 193 (1958).
8. Sudarshan and Marshak, Phys.Rev. 109 1860 (1958).

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