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Abstract

Dispersion relations (spectral representations) for the amplitudes describing the decay of K - mesons and hyperons into two strongly interaoting particles are de - rived. The singular integral equations for the amplitudes obtained in the " single - meson " approximation are here solved following the method suggested by Muskhelishvili.

The works of Källen and Lehmann [1] on spectral representation of the one-particle Green's functions and, especially, the development of the method of proving dispersion relations by Bogolubov [2] drew attention of many physicists to the problem of obtaining spectral representations for the vertex parts (see [3]).

The existance of such representations for complete vertices (and not for some vertex diagrams of perturbation theory) is not evident and in order to prove their existence non-trivial mathematical methods are necessary. Jost ' s example [4] shows that it is impossible to obtain (basing on local commutation relations and the assumptions about the spectre masses) the spectral representation for the nucleon-meson vertex. Analogous difficulties arise in investigating analytio properties of the nucleon-photon vertex (photon and meson are virtual).

It is of interest that one easily succeeds in proving dispersion relations for the vertices which correspond to the real processes of decay into two particles (if the mass of the decaying particle is regarded as the energy variable). This fact was noticed by Goldberger and Treiman [5] who obtained and applied dispersion relations for the ($\pi \to \mu + \nu$) decay.

In the present paper spectral representations for the amplitudes describing decay of K-mesons and hyperons into two strongly interacting particles are derived . The strict proof of dispersion relations following the method suggested by Bogolubov [2] is in this case performed simply since the unobservable region is absent.

Dispersion relations in the one meson approximation are solved exactly by using the techniques developed by Muskhelishvili [9] (the amplitude of the prooess is determined within the accuracy of a constant real factor). One obtains in this case results (for arbitrary virtual energies of the decaying particle) analogous to those given by Takeda [6].

In the recent work of Okubo, Marshak and Sudarshan [11] [x)] in which one investigates the decay of the Λ - hyperon by using the theory of the axial-vector four-fermion interaction, one applies in particular dispersion relations for the Λ - hyperon decay too. The authors of the above paper examine the auxiliary problem when the squares of the all four momenta (namely, of hyperon, nucleon and π - meson) equal the oorresponding squares of masses, but when the four-momentum conservation low does not take place, and they assume that the Λ - hyperon is " totally uncoupled ". On the contrary in our work the conservation low (1.8) holds and the square of the four-momentum of the hyperon q^2 is regarded as variable quantity. Because of this the spin structure is in our case determined by two independent functions Ω_ℓ while in [11] four functions M_i are introduced and hence both dispersion relations in the one meson approximation and their solutions become considerably complicated. In view of this the final results are different although they do not oontradict one another [x)].

1. Analytical properties of the decay amplitude. Derivation of the spectral representations.

At first we consider the case when a hyperon decays into a nucleon and a π - meson. At the close of this Section we indicate what changes we have to do in order to obtain dispersion relations for the decay of the K-meson into two π - mesons.

Let us denote the four-momenta of hyperon, nucleon and π - meson by q, p and K respectively. We shall assume that only four - momenta of the decay products obey the usual relation

$$p^2 = M^2, \qquad K^2 = \mu^2 \qquad / M \approx 6,8\mu / \tag{1.1}$$

while

$$q^2 = \omega^2 \tag{1.2}$$

is considered as a variable quantity (if all three masses are oonsidered to be fixed then in virtue of the conservation law there will be no invariant variable with respect to which one might consider analytic properties of the decay amplitude).

Introduoe the current operators

$$\mathcal{J}(x) = i\frac{\delta S}{\delta \Psi(x)}\overset{+}{S}, \qquad j(x) = i\frac{\delta S}{\delta \varphi(x)}\overset{+}{S}, \tag{1.3}$$

where $\Psi(x)$ and $\varphi(x)$ are the operators of the hyperon and the meson fields accordingly.

The matrix element of the process $Y \longrightarrow N + \pi$ may be written in the form

$$\langle p, K | S | q \rangle = i\frac{(2\pi)^{5/2}}{\sqrt{2K_0}}\delta(p+K-q)\,T^{rel}(K)\,u_Y(\vec{q}), \tag{1.4}$$

where

x) Comp. the foot-note to the Eq. (2.8).

$$T^{ret}(K) = -\frac{1}{(2\pi)^{3/2}}\int\langle p|\frac{\delta J(0)}{\delta\varphi(x)}|0\rangle e^{iKx}d_4x = -\sqrt{2K_0}\langle p,K|J(0)|0\rangle \tag{1.5}$$

is the retarded amplitude ($d_4x = dx_0dx_1dx_2dx_3$, $Kx = K_0x_0 - \vec{K}\vec{x} = K_0x_0 - K_1x_1 - K_2x_2 - K_3x_3$, $\delta(\tau) = \delta(\tau_0)\delta(\tau_1)\delta(\tau_2)\delta(\tau_3)$). We assume that the spinor $u_y(\vec{q})$ satisfies the Dirac equation describing particle with mass ω (see Eq. (1.2)).

$$\gamma q\, u_y(\vec{q}) = \begin{pmatrix} q_0 & -\vec{\sigma}\vec{q} \\ \vec{\sigma}\vec{q} & -q_0 \end{pmatrix} u_y(\vec{q}) = \omega u_y(\vec{q}). \tag{1.6}$$

The quantity hermitian conjugated to (1.5) which is equal to the advanced amplitude may be written in the form

$$T^{adv}(K) = -\frac{1}{(2\pi)^{3/2}}\int\langle p|\frac{\delta j(x)}{\delta\psi(0)}|0\rangle e^{iKx}d_4x. \tag{1.7}$$

In virtue of the conservation law (following from (1.4)) :

$$q = p + K \tag{1.8}$$

and according with the Dirac equation the amplitude posses in usual space a structure of the following type

$$T^{ret}(K)u_y(\vec{q}) = \bar{u}_N(\vec{p})\{\Omega_0^{\tau} + \Omega_1^{\tau}\gamma_5\}u_y(\vec{q}). \tag{1.9}$$

A similar structure may be written for the amplitude T ^{adv} (1.7) (the indices in the isotopic space are omitted). Here Ω_0 and Ω_1 are invariant functions ; as invariant variable it is convenient to choose instead of ω the quantity :

$$E \equiv \frac{1}{M}pK, \qquad /2ME = \omega^2 - M^2 - \mu^2/. \tag{1.10}$$

The matrix γ_5 will be considered as an Hermitian one:

$$\gamma_5 = \gamma_1\gamma_2\gamma_3\, i\gamma_0 = -\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}.$$

Let us introduce a frame of reference in which the nucleon rests :

$$\vec{P} = 0 \tag{1.11}$$

and

$$P_0 = M, \quad K_0 = E, \quad \vec{K} = \vec{q} = \sqrt{E^2 - \mu^2}\,\vec{e} \quad /|\vec{e}| = 1/.$$

In order to prove dispersion relations let us consider firstly a fictitious case when

$$k^2 = \tau < 0.$$

In this case the retarded amplitude (1.5) takes the form:

$$T^{ret}(E,\tau) = -\frac{1}{(2\pi)^{3/2}} \int \exp\{i(Ex_0 - \sqrt{E^2-\tau}\,\vec{e}\,\vec{x})\} \langle p| \frac{\delta J(0)}{\delta\varphi(x)} |0\rangle d_4 x$$

(from (1.7) the similar expression is obtained also for T^{adv}).

In consequence of the causality condition [2] the amplitudes T^{ret} (respectively T^{adv}) are analytic functions in the upper (accordingly, lower) half-plane of the complex variable E. We show that their difference vanishes along some interval of the real axis. With this aim we shall investigate the anti-Hermitian part of the amplitude:

$$A(E,\tau) \equiv \frac{1}{2i}\{T^{ret}(E,\tau) - T^{adv}(E,\tau)\}. \tag{1.12}$$

Let us use the equality

$$\frac{\delta j(x)}{\delta\psi(0)} - \frac{\delta J(0)}{\delta\varphi(x)} = i[j(x), J(0)] \tag{1.13}$$

and expand (1.12) in the complete system of the eigenfunctions of the energy-momentum operator :

$$A(E,\tau) = \pi(2\pi)^{3/2} \sum_n \{ \langle p|j(0)|p_n\rangle\langle p_n|J(0)|0\rangle \delta(M+E-\sqrt{M_n^2+E^2-\tau}) - \tag{1.14}$$

$$- \langle p|J(0)|p'_n\rangle\langle p'_n|j(0)|0\rangle \,\delta(E+\sqrt{M_n'^2+E^2-\tau})\}$$

($\sum_n$ implies integration over the continuous part of the spectrum and summation over the discrete characteristics of intermediate states. As is well known from the theory of spectral representations of the meson Green's function in virtue of considerations of stability of one-particle states and invariance with respect to space reflections the matrix element

$$\langle p'_n|j(0)|0\rangle = 0$$

for $M_n'^2 < (3\mu)^2$. Since p'_{n0} is positive we conclude that the second term of the right hand side in (1.14) vanishes when

$$\tau < (3\mu)^2. \tag{1.15}$$

The singularities of the first δ - function of Eq. (1.14) lie at points

$$E_n = \frac{1}{2M}\left(M_n^2 - M^2 - \tau\right). \qquad (1.16)$$

One can see from (1.14) that, generally speaking, there exists one discrete pole of the "one-nucleon" term for $M_n = M$

$$E = -E_1(\tau) = -\frac{\tau}{2M}. \qquad (1.17)$$

The residue at this pole is proportional to the product of the strong and weak coupling constants and has the same spin structure like the amplitude (1.9).

The continuous spectrum for the first term in (1.14) starts with the value $M_n = M + \mu$:

$$E_n \geq \frac{2M\mu + \mu^2 - \tau}{2M} \equiv E_c(\tau). \qquad (1.18)$$

Thus one can see that the functions $(E + E_1)\, T^{ret}(E, \tau)$ and $(E + E_1)\, T^{adv}(E, \tau)$ coincide for $\tau < (3\mu)^2$ and

$$-\infty < E < E_c(\tau). \qquad (1.19)$$

Therefore, according to the well-known theorem of Bogolubov [2] (see also [7], where one proves more precise theorem) there exists a single function $T(E, \tau)$ analytic with respect to E and τ in the region

$$\left| \mathrm{Im}\sqrt{E^2 - \tau} \right| < \left| \mathrm{Im}\sqrt{(E + 3\mu)(E - E_c(\tau))} \right|, \qquad (1.20)$$

which coincides with $T^{ret}(E, \tau)$ and $T^{adv}(E, \tau)$ for real E and τ which satisfy (1.19). It is easy to see that the inequality (1.20) holds for $\mathrm{Im}\, E \neq 0$ or $E = \mathrm{Re}E < \mu$ for $\tau = \mu^2$. Consequently the function $T(E, \tau)$ can be continued analytically with respect to τ up to $\tau = \mu^2$. For the real case of interest to us $\tau = \mu^2$ the boundary of the continuous spectrum

$$E_c(\mu^2) = \mu$$

coincides with the treshold of the reaction under consideration. It is well known [2] that the functions T(E) increase at infinity not faster than a polynomial. We suppose (for the sake of simplicity) that T(E) is bounded at infinity. By using the Cauchy integral theorem for the function $\frac{1}{E}\, T(E)$, after performing transition to the limit $\mathrm{Im}E \to 0$ we get the following dispersion relations

$$Re\,T(E)=\frac{E}{\pi}\mathcal{P}\int_{\mu}^{\infty}\frac{Im\,T(E')}{E'(E'-E)}dE'+\frac{c}{\pi}\frac{E}{E_1}\frac{1}{E+E_1}+T(o).$$

According to (1.9) and using the fact the anti-Hermitean part $A(E,\tau)$ has the same structure like the amplitude :

$$A(E,\tau)u_y(\vec{q})=\pi(2\pi)^{3/2}\frac{M+E}{M}\sum_n\langle P|j(o)|P_n\rangle\langle P_n|J(o)|o\rangle\delta(E-E_n(\tau))u_y(\vec{q})= \tag{1.21}$$

$$=\bar{U}_N(\vec{p})(A_o(E)+A_1(E)\gamma_5)\,u_y(\vec{q})$$

one can easily see that the invariant functions Ω_ℓ (E), $\ell=0,1$ obey the same dis - persion relations. They can be written in the form:

$$\Omega_\ell^r(E)=\frac{E}{\pi}\int_{\mu}^{\infty}\frac{Im\,\Omega_\ell^r(E')}{E'(E'-E-i\varepsilon)}dE'+\frac{E}{E_1}\frac{a_\ell}{E+E_1+i\varepsilon}+\Omega_\ell(o) \tag{1.22}$$

where a_ℓ and $\Omega_\ell(o)$ are real constant quantities.

The relations (1.22) and (1.21) give spectral representations for the invariant fun - otions Ω_ℓ(E) (the variable E is also determined in an invariant manner (1.10)).

An approaoh to the process (K $\rightarrow$ 2 π) is much simpler since its amplitude has no spin structure. The use of the stability of states with one π - meson yields an error of the seoond-order smallness with respect to the constant of the weak interaction. In the (K $\rightarrow$ 2 π) decay a discrete pole in the imaginary part of the amplitude does not appear by pseudoscalarity.

In this case dispersion relations assume the form

$$T^{ret}(E)=\frac{E}{\pi}\int_{\mu}^{\infty}\frac{Im\,T^{ret}}{E'(E'-E-i\varepsilon)}dE'+T(o), \tag{1.23}$$

where

$$Im\,T^{ret}(E)=\pi(2\pi)^{3/2}\frac{\mu+E}{\mu}\sum_{\substack{n\\M_n\geq 2\mu}}\langle P|j(o)|P_n\rangle\langle P_n|J_K(o)|o\rangle\delta\left(E-\frac{M_n^2-2\mu^2}{2\mu}\right). \tag{1.24}$$

J_K - is the K-meson current which is determined in an analogous manner as the current J (1.3).

2. Approximate solution of dispersion relations.

A. The decay of K-mesons.

In using dispersion relations obtained here we restrict ourselves in the expansions (1.21), (1.24) for anti-Hermitean parts of corresponding amplitudes to the "one meson state".

Let us start with the decay of K - mesons. In this case the "one meson" approximation corresponds to an intermediate state with two π - mesons. For the sake of definiteness we restrict ourselves to the investigation of the K^0 - meson decay. The decay of charged K - mesons into two π - mesons may be considered in a simpler manner since this reaction goes in one channel only.

As is well known [8] from CP - invariance it follows that only the K^0_1 - meson decays into two π- mesons where

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right).$$

We write dispersion relations for the amplitude T_I which corresponds to a definite value of the total isotopic spin I of the system of two π - mesons. In decay of K^0_1-meson I equals 0 and 2, the amplitudes T_I are related to the amplitudes for the decay into particles with definite charge by means of the Clebsh-Gordan's coefficients.

$$T_0 = \sqrt{\tfrac{2}{3}}\,T_{+-} - \frac{1}{\sqrt{3}}\,T_{00} \qquad (2.1)$$
$$T_2 = \frac{1}{\sqrt{3}}\,T_{+-} + \sqrt{\tfrac{2}{3}}\,T_{00},$$

where

$$T_{+-} \equiv T(\pi^+,\pi^-|K_1^0), \qquad T_{00} \equiv T(\pi^0,\pi^0|K_1^0).$$

By restricting in the expansion (1.24) for the imaginary part of the amplitude to intermediate states with two π - mesons and in virtue of the isotopic spin invariance of scattering of decay products we get

$$\mathrm{Im}\,T_I^{ret}(E) = e^{-i\delta_I(E)}\sin\delta_I(E)\,T_I^{ret}(E) \qquad (2.2)$$

where δ_I is the phase shift for the S - wave with isotopic spin I of the π-π scattering amplitude (since K-meson is scalar only s-waves contribute in the scattering amplitude of the decay products).

From (2.2) one obtains

$$T_I^{ret}(E) = f_I(E)\, e^{i\delta_I(E)} \tag{2.3}$$

where $f_I(E)$ is a real function of E . In order to determine this function we make use of the dispersion relation.

Using (2.2) one can represent dispersion relation (1.23) as follows

$$T_I^{ret}(E) = \frac{E}{\pi}\int_{\mu}^{\infty} \frac{h_I(E')T_I^{ret}(E')}{E'(E'-E-i\varepsilon)}\,dE' + T_I(0). \tag{2.4}$$

We have here introduced the notation

$$h_I(x) = e^{-i\delta_I(x)}\sin\delta_I(x)$$

The singular integral equation is easily solved by Muskhelishvili's method [9] (see also the paper of Omnes where the author examines specially the equations of the type (2.4) only without the factor $\frac{E}{E'}$, under the integral sign). Let us consider with this aim the function $T_I(z)$ of the complex variable z :

$$T_I(z) = \frac{z}{\pi}\int_{\mu}^{\infty} \frac{h_I(E')T_I^{ret}(E')}{E'(E'-z)}\,dE' + T_I(0).$$

From the definition of $T_I(z)$ follow the relations (for real values of E):

$$T_I(E_+) = T_I^{ret}(E)$$

$$T_I(E_+) - T_I(E_-) = \left(1 - e^{-2i\delta_I(E)}\right)\theta(E-\mu)\,T_I^{ret}(E)$$

where $E_\pm = E \pm i\varepsilon, \quad \varepsilon > 0, \quad \theta(x) = \begin{cases} 0 & x < 0 \\ 1 & x > 0. \end{cases}$

Thus, the function $T_I(z)$ is the solution of Hilbert's boundary problem

$$T_I(E_+) = T_I(E_-)\exp\left\{2i\delta_I(E)\,\theta(E-\mu)\right\}.$$

Under our assumption that $T_I(z)$ does not increase at infinity the solution of this problem has the form (for real values of E)

$$T_I^{ret}(E) = C_I \exp\left\{\frac{E}{\pi}\mathcal{P}\int_{\mu}^{\infty}\frac{\delta_I(x)}{x(x-E)}\,dx + i\delta_I(E)\,\theta(E-\mu)\right\} \tag{2.5}$$

where

$$C_I = T_I(0)$$

is a real constant. It may be defined (if we consider the phase shifts of the $\pi-\pi$ scattering to be known) by the experimental values of the decay time or theoretically (following the perturbation theory); for this purpose we have to make hypotheses about the concrete form of the interaction responsible for the decay under consideration.

In order to deal with measurable physical quantities we make use of the relations (2.1). For the real decay

$$\omega = \sqrt{q^2} = m_K \approx 494 \text{ Mev},$$

$$E = E_0 \equiv \frac{m_K^2 - 2\mu^2}{2\mu} \approx 5{,}5\mu \approx 750 \text{ Mev}$$

which corresponds to the total kinetic energy in the center-of-mass system:

$$\omega_K = m_K - 2\mu \approx 220 \text{ MeV}.$$

According to (2.1) and (2.3) for the probabilities of decays into $\pi^+ + \pi^-$ and $\pi^0 + \pi^0$ we obtain :

$$|T_{+-}|^2 = \frac{1}{3} f_2^2(E_0) + \frac{2}{3} f_0^2(E_0) + 2\frac{\sqrt{2}}{3} f_0(E_0) f_2(E_0) \cos(\delta_2 - \delta_0)$$

$$|T_{00}|^2 = \frac{2}{3} f_2^2(E_0) + \frac{1}{3} f_0^2(E_0) - 2\frac{\sqrt{2}}{3} f_0(E_0) f_2(E_0) \cos(\delta_2 - \delta_0). \tag{2.6}$$

Introduce the notation:

$$\lambda = \frac{f_2(E_0)}{f_0(E_0)}.$$

For the ratio of these probabilities we obtain thus the well-known result [6]

$$\frac{W(K_1^0 \to \pi^0 + \pi^0)}{W(K_1^0 \to \pi^+ + \pi^-)} = \frac{2\lambda^2 + 1 - 2\sqrt{2}\,\lambda \cos(\delta_2 - \delta_0)}{\lambda^2 + 2 + 2\sqrt{2}\,\lambda \cos(\delta_2 - \delta_0)}.$$

It is evident that this ratio equals 1/2 not only for $\lambda = 0$ (which corresponds to the selection rule with respect to the isotopic spin $|\Delta I| = \frac{1}{2}$) but for one more definite value λ which depends on $\delta_2 - \delta_0$.

B. <u>The decay of hyperons</u>

The solution of dispersion relations for the hyperon decay is carried out in an analogous manner but with some insignificiant complication because of the spin structure and the "one-nucleon" pole in amplitude. Therefore, the details of solution are not repeated.

Since in π - meson-nucleon scattering the parity is conserved then in scattering of products of the hyperon decay together with total angular momentum j of the system the orbital momentum $\ell = j \pm \frac{1}{2}$ is conserved too. It is not difficult to see that the invariant functions Ω_0 and Ω_1 introduced in (1.9) are eigen - functions of the orbital moment operator which correspond to the eigenvalues $\ell = 0$ and $\ell = 1$. For this purpose it is quite sufficient to note that in the center-of-mass system $/\vec{q} = \vec{p} + \vec{k} = 0/$ the decay amplitude has the form

$$T_{s's}(E)u_y(\vec{q}) = \sqrt{\frac{M+p_0}{2M}}\left\{\Omega_0(E)\delta_{s's} - \Omega_1(E)\frac{(\vec{\sigma}\vec{p})_{s's}}{M+p_0}\right\}, \qquad (2.7)$$

where

$$p_0 = \frac{\omega^2+M^2+\mu^2}{2\omega} = M\frac{M+E}{\sqrt{M^2+\mu^2+2ME}}$$

is the energy of the nucleon in the center-of-mass system.

We rewrite dispersion relations for the amplitudes of the state with definite iso - topic spin of the system of decay products $\Omega_{I\ell}$ $/\ell = 0,1;\ \ I = \frac{1}{2}, \frac{3}{2}/$. After performing a phase shift analysis for the meson-nucleon scattering amplitude in the expression for the imaginary part (1.21) we obtain x)

$$\mathcal{I}m\,\Omega^{\tau}_{I\ell}(E) = \pi a_{I\ell}\ \delta(E+E_1) + h_{I\ell}(E)\Omega^{\tau}_{I\ell}(E)\theta(E-\mu), \qquad (2.8)$$

where

$$E_1 = \frac{\mu^2}{2M},$$

$$h_{I\ell}(E) = e^{-i\delta_{I\ell}(E)}\sin\delta_{I\ell}(E), \qquad (2.9)$$

($a_{I\ell}$ are real constant quantities, $\delta_{I\ell}(E)$ are the phase shifts of the meson - nucleon scattering). From (2.8) follows that for $E > \mu$

$$\Omega^{\tau}_{I\ell}(E) = f_{I\ell}(E)\,e^{i\delta_{I\ell}(E)}, \qquad (2.10)$$

$f_{I\ell}(E)$ being a real function. In order to determine this function we make use again of the dispersion relations which can be written in the form

$$\Omega^{\tau}_{I\ell}(E) = \frac{E}{E_1}\frac{a_{I\ell}}{E+E_1+i\epsilon} + \frac{E}{\pi}\int_{\mu}^{\infty}\frac{h_{I\ell}(E')\Omega^{\tau}_{I\ell}(E')}{E'(E'-E-i\epsilon)}\,dE' + \Omega_{I\ell}(0) \qquad (2.11)$$

---

x) The relations (2.8),(2.9) are obtained easily from Eqs. (25),(26) in the work of Okubo et. al [11] under the assumption that the spinor $u_y(\vec{q})$ satisfies the Dirac equation with a mass ω . Between the amplitudes M^I_i of the above-mentioned paper and our amplitudes $\Omega_{I\ell}$ there exists the simple connection :

$$\Omega_{I0} = M^I_1 - (\omega - M)M^I_3 \qquad \Omega_{I1} = M^I_2 - M^I_4(\omega - M)$$

As was already pointed out, in the paper[11] one assumes that $u_y(\vec{q})$ obeys the Dirac equation with the real hyperon mass. Therefore, here and further the authors[11] obtain considerably more complicated relations.

The solution of this equation which does not increase at infinity for a complex argument z has the form:

$$\Omega_{Ie}(z)=\left\{\frac{z}{E_1}\frac{a_{Ie}}{z+E_1}e^{-\frac{E_1}{\pi}\int_{\mu}^{\infty}\frac{\delta_{Ie}(x)}{x(x+E_1)}dx}+\Omega_{Ie}(0)\right\}exp\left\{\frac{z}{\pi}\int_{\mu}^{\infty}\frac{\delta_{Ie}(x)}{x(x-z)}dx\right\}.$$

For real $E>\mu$ it acquire the following form

$$\Omega_{Ie}^{?}(E)=\Omega_{Ie}(E+i\varepsilon)=$$

$$=\left\{\frac{E}{E_1}\frac{a_{Ie}}{E+E_1}exp\left[-\frac{E_1}{\pi}\int_{\mu}^{\infty}\frac{\delta_{Ie}(x)}{x(x+E_1)}dx\right]+\Omega_{Ie}(0)\right\}exp\left[\frac{E}{\pi}P\int_{\mu}^{\infty}\frac{\delta_{Ie}(x)}{x(x-E)}dx+i\delta_{Ie}(E)\right]. \tag{2.12}$$

It is interesting to apply the obtained results to the processes of the decay of Σ^+ and Λ - hyperons which go in two channels with respect to the charge. With this aim it is necessary to proceed from the amplitudes with definite isotopic spin to the amplitudes of the deacy into definite particles by using the Clebsh - Gordan coefficients (see for instance [8]) and to put ω to be equal to the real mass of the corresponding hyperon. This yields :

$$E=\frac{m_{\Lambda}^2-M^2-\mu^2}{2M}\approx 183\ MeV$$

(the total kinetic energy which releases in the center-of-mass system is $\omega_K\approx 38$ Mev) when decaying Λ - hyperon.

$$E=\frac{m_{\Sigma}^2-M^2-\mu^2}{2M}\approx 275\ MeV \qquad /\omega_K\approx\begin{cases}110\\116\end{cases} MeV/$$

when decaying Σ^+ - hyperon.

It appears to be quite interesting to compare the formulas obtained with the experimental data on the decay and the meson-nucleon scattering (after determining the constants entering (2.12) by using the perturbation theory, starting with a con - crete kind of the interaction (comp. [11])).

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