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(I)

A b s t r a c t

Dispersion relations for the physical amplitudes in the center of mass system have been derived following a method suggested by N.N.Bogolubov for investigating the processes of bremsstrahlung of electrons on a nucleon and pair production by γ -quanta on a nucleon in the lowest approximation in "e".

Introduction

At present one of the most effective methods of consideration of strong interactions is that of dispersion relations.

In its application to the electromagnetic processes such as Compton effect on nucleons, bremsstrahlung or pair production on nucleons et al this method allows under definite assumptions to obtain information about nucleon structure. It should be noted that during the last years one gives much attention to the investigation of the nucleon structure[x/] since this problem is not only important by itself but also connected closely with the limit of applicability of quantum electrodynamics at small distances.

Dispersion relations (D.R.) for the processes of bremsstrahlung of electrons on nucleons and electron-proton pair production by γ - quanta on nucleons allow to consider theoretically strictly the problem of the influence of nucleon structure on the above processes. From this point of view the investigation of dispersion relation for the virtual Compton effect including both above mentioned processes is of quite definite interest.

In this paper one obtains dispersion relation for the virtual Compton effect in the lowest e - approximation by Bogolubov's method [1].

The proof of dispersion relations for the processes of bremsstrahlung and pair production has been made in [2]. Hence, in the present paper the attention is fixed on obtaining dispersion relation, available for practical applications.

Since the method of the deduction of dispersion relation has been expounded in [1-3], in the given paper many intermediate steps in deducing dispersion relation are omitted. In Sections 2,3 dispersion relation are deduced in the most general form. One gives also a minimum number of calculations needed for understanding the deduction of dispersion relation.

Starting from Section 4 our consideration is only referred to the bremsstrahlung process since there is no difference of principle in the dispersion relation derivation for the processes $e+N \rightarrow e+\gamma+N$ and $\gamma+N \rightarrow e^{+}+e^{-}+N$.

The investigation of the unobservable region (Sec. 4) allows to conclude that in bremsstrahlung process there is a finite interval of recoil momenta for which the unobservable region is not present. In Section 5 one calculates an one - nucleon term and shows that the cross section of the processes calculated in one-nucleon approximation coincides with that of the process calculated in the perturbation theory, however, with the difference that the D.R. method allows to introduce in a strict manner Hofstadter form factors into the nucleon vertices

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x/ One can find a detailed bibliographical information on nucleon structure in the review " Nucleon structure " by D.I.Blokhintsev, V.S.Barashenkov and B.M.Barbashov. UFN (in print).

of the Feynman graphs are connected with the virtual photon line. This is one of the grave advantages of the dispersion relation method in comparison with perturbation theory. In Sections 6 – 8 one gives the relativistically invariant structure of the virtual Compton effect and obtains dispersion relations for structure coefficients in the centre – of – mass system. This is an ultimate end of our paper. These relations may be used further, at least for estimation of the contribution of the single -π- meson state to the processes under consideration analogously as it has been made for photoproduction of π – mesons [4,5].

In the paper [6] one has been calculated the processes of bremsstrahlung and pair production in the lowest approximation of perturbation theory taking into account only the Bethe-Heitler graphs with the Hofstadter form factors. If is obviously that for the incident particle energies up to $\sim$ 150 Mev when the contribution from the meson shell of a nucleon and electromagnetic corrections of the lowest order are not important yet, the results of the paper [6] can be used for testing quantum electrodynamics at small distances. The test of quantum electrodynamics implies the test of the locality of interaction of the electromagnetic field with the current of the charged particle. Analogous results have been given in the paper [7]. However, with the energy increase up to 500-600 Mev the contribution from the meson shell of nucleon must become more appreciable and the single – π – meson approximation can change essentially the cross section of the above mentioned processes. Therefore, if we take into account the single π – meson state then we can extend considerably the energy limits of testing quantum electrodynamics at small distances to the photon and electron energies $\sim$ 500-600 Mev. In this case for example, in the bremsstrahlung process for $\sim$ 550 Mev incident electron and emitted photon energy $\sim$ 250 Mev the nucleon recoil reaches $\sim 600 \frac{\text{Mev}}{c}$, this allows to test quantum electrodynamics at the distances up to $\gtrsim 3 \cdot 10^{-14}$ cm.

2. Matrix element of the bremsstrahlung of an electron on nucleon

For the process of bremsstrahlung of electron on nucleon $e + N \to e + N + \gamma$ the matrix element of the S – matrix has the following form:

$$\langle f|S|i\rangle = \langle P,S;\ q,\sigma;\ K\ \nu|S|q_0,\sigma_0;P_0,\sigma_0\rangle = (2\pi)^{9/2}\langle P,S|b^-(\vec{q},\sigma)a^-(\vec{K},\nu)\,S\,\overset{*}{b}{}^+(\vec{q}_0,\sigma_0)|P_0 S_0\rangle \tag{2.1}$$

where P,S (P_0, S_0) is the 4-momentum and spin of the final (initial) state of a nucleon q_0, σ_0 (q, σ) is the 4-momentum and spin of the initial (final) state of an electron, K , ν are the 4-momentum and the photon polarization; $b^-(\vec{q},\sigma)$ and $\overset{*}{b}{}^+(\vec{q}_0,\sigma_0)$ are the operators of creation and annihilation of an electron in states $(\vec{q}, \sigma)$ and $(\vec{q}_0, \sigma_0)$ respectively; $a^-(\vec{K},\nu)$ is the operator of the production of photon with momentum $\vec{K}$ and polarization ν ; $|P_0, S_0\rangle = (2\pi)^{3/2}\overset{*}{C}{}^+(\vec{P}_0, S_0)\,\Phi_0$ is the amplitude of the initial state of nucleon, $\overset{*}{C}{}^+(\vec{P}_0, S_0)$ is the operator of the production of a nucleon in the state with momentum $\vec{P}$ and spin S_0, Φ_0 is the amplitude of the vacuum state.

In the lowest approximation in electric charge e the matrix element (2.1) contains two classes of diagrams (1a) and (1b).

In the given paper we shall only examine the class of diagrams (1b)[x]. Carrying the operators $b^-(\vec{q},\sigma)$ and $a^-(\vec{K},\nu)$ to the right from S, and $b^+(\vec{q}_0,\sigma_0)$ to the left from S and using the generalized Wick theorem for the chronological products [3] we write the expression (2.1) in the form:

$$\langle f|S|i\rangle = -e\frac{m e^n g^{m\ell}}{\sqrt{2k_0\mathcal{E}_0\cdot\mathcal{E}}}\bar{u}(\vec{q},\sigma)\gamma^m u(\vec{q}_0,\sigma_0)\times\int e^{i(q-q_0)x+ikz'}dz'\cdot dx\cdot dz\cdot\mathcal{D}^c(x-z')\langle P,S\left|\frac{\delta^2 S}{\delta A_n(z')\delta A_\ell(z)}\right|P_0,S_0\rangle \qquad (2.2)$$

where k_0, $\mathcal{E}_0$ and $\mathcal{E}$ are the fourth components of the momenta K, q_0 and q accordingly, $g^{m\ell}$ is the metric tensor, $u(\vec{q},\sigma)$ is spinor, describing an electron in the state $(\vec{q},\sigma)$, e^n is the polarization vector of the free photon and $\mathcal{D}^c(x-z)$ is the photon propagator, $A_\ell(z)$ is the ℓ component of the electromagnetic field operator, the spinors are normalized so that $\bar{u}(\vec{q},\sigma)u(\vec{q},\sigma)=1$. Performing the integration in (2.2) over the $\mathcal{D}^c$ function argument we get the final expression for the matrix element of interest

$$\langle f|S|i\rangle = e\frac{m e^n g^{m\ell}}{\sqrt{2k_0\cdot\mathcal{E}\cdot\mathcal{E}_0}}\cdot\frac{\bar{u}(\vec{q},\sigma)\gamma^m u(\vec{q}_0,\sigma_0)}{\varkappa^2}\times\int e^{-i\varkappa z}\cdot e^{ikz'}dz'\,dz\,\langle P,S\left|\frac{\delta^2 S}{\delta A_n(z')\cdot\delta A_\ell(z)}\right|P_0,S_0\rangle \qquad (2.3)$$

where

$$\varkappa = q_0 - q$$

3. Dispersion relations for the virtual Compton effect amplitude

Before to proceed to the direct derivation of dispersion relations let us introduce several notations and obtain some useful relations.

Let us insert the electromagnetic current operators

$$\begin{aligned} j^\ell(z) &= i\frac{\delta S}{\delta A_\ell(z)}S^+ \\ j^n(z') &= i\frac{\delta S}{\delta A_n(z')}S^+ \\ j^\ell &= (j^\ell)^+ \end{aligned} \qquad (3.1)$$

x) The class of diagrams (1a) has been considered in the paper of I.S.Zlatev and P.S.Isaev, JETP, 35, 309 (1958); Nuovo Cimento (in print) and the notations

$$\mathcal{F}^{c}_{n,\ell}(x',x) = i\langle P,\sigma \Big| \frac{\delta^2 S}{\delta A_\ell(x)\cdot\delta A_n(x')} S^+ \Big| P_o\, \sigma_o\rangle \tag{3.2}$$

$$\mathcal{F}^{ret}_{n,\ell}(x',x) = \langle P,\sigma \Big| \frac{\delta j^\ell(x)}{\delta A_n(x')} \Big| P_o, \sigma_o\rangle \tag{3.3}$$

$$\mathcal{F}^{adv}_{n,\ell}(x',x) = \langle P,\sigma \Big| \frac{\delta j^n(x')}{\delta A_\ell(x)} \Big| P_o, \sigma_o\rangle \tag{3.4}$$

(where $\mathcal{F}^{ret}(x',x)=0$ for $x' \lesssim x$, $\mathcal{F}^{adv}(x',x)=0$ for $x \lesssim x'$),

$$\mathcal{F}^{-}_{n,\ell} = i\langle P,\sigma | j^n(x')\cdot j^\ell(x) | P_o, \sigma_o\rangle \tag{3.5}$$

$$\mathcal{F}^{+}_{n,\ell} = i\langle P,\sigma | j^\ell(x)\cdot j^n(x') | P_o, \sigma_o\rangle \tag{3.6}$$

From (3.1) - (3.6) it follows

$$\mathcal{F}^{c} = \mathcal{F}^{ret} - \mathcal{F}^{+} = \mathcal{F}^{adv} - \mathcal{F}^{-} \tag{3.7}$$

$$\mathcal{F}^{ret} - \mathcal{F}^{adv} = \mathcal{F}^{+} - \mathcal{F}^{-} \tag{3.8}$$

Finally let us introduce Fourier transform for the function $\mathcal{F}(x)$:

$$T(k) = \int dx \cdot e^{ikx}\, \mathcal{F}(x) \tag{3.9}$$

Using now the property of stability of one-nucleon state $S^{-}|P_o S_o\rangle = |P_o S_o\rangle$ and that translational invariance and integrating (2.3) over $x + x'$ we get

$$\langle f|S|i\rangle = -ie\frac{m e^n \varepsilon^\ell}{\sqrt{2k_o\, \mathcal{E}\cdot\mathcal{E}_o}} \cdot \frac{(2\pi)^4}{\varkappa^2} \cdot \delta(K+P-\varkappa-P_o)\, T^c_{n\ell}\left(\frac{K+\varkappa}{2}\right) \tag{3.10}$$

where E and E_o are the fourth components of the 4-momenta P and P_o accordingly, $\varepsilon^\ell = \bar{u}(\vec{q},\sigma)\gamma^\ell u(\vec{q}_o,\sigma_o)$ is the virtual photon "polarization"

$$T^c_{n,\ell}\left(\frac{K+\varkappa}{2}\right) = \int e^{i\frac{\varkappa+K}{2}x}\, \mathcal{F}^c_{n\ell}(x)\, dx \tag{3.10a}$$

Further we shall work with the amplitude

$$T^c\left(\frac{K+\varkappa}{2}\right) = e^n \varepsilon^\ell\, T^c_{n\ell}\left(\frac{K+\varkappa}{2}\right)$$

For the purpose of deducing dispersion relations it is important to note that

$$T^c\left(\frac{K+\varkappa}{2}\right) = T^{ret}\left(\frac{K+\varkappa}{2}\right) \tag{3.11}$$

in the region where the law of conservation of energy holds true, and for the positive values of energy, i.e. when $\frac{K_o+\varkappa_o}{2} > 0$.

By analogous means for $\frac{K_o+\varkappa_o}{2} < 0$ the relation

$$T^c\left(\frac{K+\varkappa}{2}\right) = T^{adv}\left(\frac{K+\varkappa}{2}\right) \tag{3.12}$$

takes place. In order to pick out explicitly the independent variables of energy and momentum it is convenient to pass on to the Breit system $\vec{P}_o + \vec{p} = 0$. Let us introduce into this system of coordinates a unit vector $\vec{a} = \vec{\lambda}/|\vec{\lambda}|$ orthogonal to $\vec{p}$: We get such relations:

$$\begin{aligned} \vec{K} &= \vec{a}\lambda + (1-\delta)\vec{P}_o \\ \vec{\varkappa} &= \vec{a}\lambda - (1+\delta)\vec{P}_o \\ K_o &= \varkappa_o \end{aligned} \tag{3.13}$$

$$\lambda = \sqrt{\vec{\varkappa}^2 - (1+\delta)^2\vec{p}^2} = \sqrt{K_o^2 - (1-\delta)^2\vec{p}^2}$$

where

$$\delta = \frac{m_\gamma^2}{4\vec{p}^2}, \quad -m_\gamma^2 = \varkappa_o^2 - \vec{\varkappa}^2$$

Now the relation (3.9) takes the form :

$$T^c(k_o, \vec{a}) = \int dx \cdot e^{i\left(K_o x_o - \vec{a}\vec{x}\sqrt{K_o^2 - (1-\delta)^2\vec{p}^2} + \delta\vec{p}_o\vec{x}\right)} \mathcal{F}^c(x) \tag{3.14}$$

One can see from (3.14) that $T^c(k_o, \vec{a})$ is only determined on the two segments of the real axis:

$$-\infty < k_o < -\sqrt{(1-\delta)^2\vec{p}^2} \qquad (1-\delta)|\vec{p}| < k_o < +\infty$$

The ambiguity in the exponential factor is eliminated by introducing symmetrical and antisymmetrical combinations:

$$\begin{aligned} S_+ T(K_o, \vec{a}) &= \frac{T(K_o, \vec{a}) + T(K_o, -\vec{a})}{2} \\ S_- T(k_o, \vec{a}) &= \frac{T(K_o, \vec{a}) - T(k_o, -\vec{a})}{2\lambda} \end{aligned} \tag{3.15}$$

As regards the questions of the analytical continuation of T^c into the upper and lower half-planes of the complex variabl k_o and determination of its analyticity region they have been considered in details in the paper [2].

Following now the way given in the paper [7] and supposing, that the virtual Compton effect amplitude behaves at the infinity $\sim \frac{1}{K_o}$ [8] we obtain dispersion relation in the following form:

$$\mathcal{D}(K_o) = \frac{\mathcal{P}}{\pi}\int_{E_1}^{\infty} \frac{A(K_o')dK_o}{K_o' - K_o} - \frac{\mathcal{P}}{\pi}\int_{E_1}^{\infty} \frac{A(-K_o')dK_o'}{K_o' + K_o} + \frac{A^{(1)}(-\lambda\vec{a} - \delta\cdot\vec{p})}{E_p - K_o} + \frac{A^{(2)}(+\lambda\vec{a} - \delta\cdot\vec{p})}{E_p + K_o} \tag{3.16}$$

where

$$\mathcal{D}(k_o) = \frac{ST^{ret}(K_o) + ST^{adv}(K_o)}{2}; \quad A(K_o) = \frac{ST^{ret}(K_o) - ST^{adv}(K_o)}{2i};$$

$$A^{(1)}(-\lambda\vec{a} - \delta_i\vec{p}) = (2\pi)^3 \frac{M^2 + \delta_i\vec{p}^2}{M^2 + \vec{p}^2} \varepsilon^{\ell} e^{n} S \sum \langle \vec{P}, \sigma | j^{\ell}(0) | -\lambda\vec{a} - \delta_i\vec{p}, \rho \rangle \langle -\lambda\vec{a} - \delta_i\vec{p}, \rho | j^{n}(0) | \vec{P}_o, \sigma_o \rangle$$

$$A^{(2)}(\lambda\vec{a} - \delta\vec{p}) = (2\pi)^3 \frac{M^2 + \delta\vec{p}^2}{M^2 + \vec{p}^2} \varepsilon^{\ell} e^{n} S \sum_{\rho} \langle \vec{P}, \sigma | j^{n}(0) | \lambda\vec{a} + \delta\vec{p} \rangle \langle \lambda\vec{a} + \delta\vec{p}, \rho | j^{\ell}(0) | \vec{P}_o, \sigma_o \rangle \tag{3.17}$$

For the process of production of pairs by photon on proton dispersion relations are obtained from (2.4) by simple substitution $k \to \varkappa$, $\varkappa \to q + q_o$ and $m_{\gamma}^2 = \varkappa_o^2 - \vec{\varkappa}^2$.

4. Investigation of the unobservable region

When deducing dispersion relations (3.16) we examined the difference

$$S\tilde{T} = ST^{ret} - ST^{adv} = ST^{+} - ST^{-}$$

in the region of the real variables k_o . Then, one made an assumption that one considered only strong interactions. The weak interactions were neglected and the electromagnetic interactions considered in the lowest e approximation.

Expanding now ST^{+} and ST^{-} in a complete system of functions and integrating over three-dimensional momenta of intermediate states, we get:

$$ST^{ret}(K_o,\vec{a}) - ST^{adv}(K_o,\vec{a}) = i(2\pi)^4 S \sum_{\rho} \Big\{ \langle \vec{P}, s | j^{\ell}(0) | -\lambda\vec{a} - \delta_i\vec{p}, \rho \rangle \cdot \langle -\lambda\vec{a} - \delta_i\vec{p}, \rho | j^{n}(0) | \vec{P}_o, \sigma_o \rangle \cdot \delta\Big(\frac{\varkappa_o + k_o - E_o - E}{2} + \ell_o\Big) -$$

$$- \langle \vec{P}, \sigma | j^{n}(0) | \lambda\vec{a} + \delta_i\vec{p}, \rho \rangle \langle \lambda\vec{a} + \delta_i\vec{p}, \rho | j^{\ell}(0) | P_o, \sigma_o \rangle \, \delta\Big(\frac{\varkappa_o + k_o + E_o + E}{2} - \ell_o\Big) \tag{4.1}$$

The first argument of the δ function has a form $K_o - \sqrt{M^2 + \vec{p}^2} + \sqrt{M_{\rho}^2 + \lambda^2 + \delta^2\vec{p}^2}$ and the second: $K_o + \sqrt{M^2 + \vec{p}^2} - \sqrt{M_{\rho}^2 + \lambda^2 + \delta^2\vec{p}^2}$ where M_{ρ} is the mass of intermediate states.

In one-nucleon state ($M_{\rho} = M$) the right first term of Eq. (4.1) is not zero under the condition

$$K_o = E_p = \frac{(1-\delta)\vec{p}^2}{\sqrt{M^2 + \vec{p}^2}} \tag{4.2}$$

and the second - under the condition

$$K_o = -E_p = -\frac{(1-\delta)\vec{p}^2}{\sqrt{M^2 + \vec{p}^2}} \tag{4.3}$$

The continuous spectrum starts with tha values $M_\rho = M+\mu$ i.e. for $|K_0| > E_1$, μ - being the mass of π - meson

$$\mp E_1 = \mp \frac{2M_\mu + \mu^2 - 2(1-\delta)\vec{p}^2}{2\sqrt{M^2+\vec{p}^2}}$$

(the sign $\mp$ are related to the first and second δ - funotions, respectively).

In order that the one-nucleon poles $\pm E_p$ might not lie in the region of the continuous spectrum $|K_0| \geqslant E_1$ i.e. in order that $|E_p| < E_1$, $\vec{p}^2$ has satisfy the condi - tion

$$\vec{p}^2 < \frac{2M_\mu + \mu^2 + m_\gamma^2}{4} \tag{4.4}$$

which imposes some restrictions to the electron energy .

It is important to note that in the considered dispersion relations the unobservable region is absent for the finite momentum transfer $\vec{p}^2$. The interval of the momenta for whioh the unobservable region is absent is defined by the inequality

$$(1-\delta)|\vec{p}| > \frac{2M_\mu + \mu^2 - 2(1-\delta)\vec{p}^2}{2\sqrt{M^2+\vec{p}^2}} \tag{4.5}$$

Solving (4.5) with respeot to $|\vec{P}|$ we obtain

$$|\vec{P}| < \sqrt{\frac{(2M_\mu+\mu^2)^2 + (M+\mu)^2 \cdot m_\gamma^2 + m_\gamma^2 M^2 + (2M_\mu+\mu^2)\sqrt{(2M_\mu+\mu^2+m_\gamma^2)^2 + 4M^2 m_\gamma^2}}{8(M+\mu)^2}} \tag{4.6}$$

For the real Compton effect $m_\gamma^2 = 0$ and we get

$$|\vec{P}| < \frac{2M_\mu + \mu^2}{2(M+\mu)}$$

which coincides with the results given in the paper [9] .

5. Calculation of an one-nucleon term

In order to calculate one-nuoleon terms in (3.23) firstly let us examine the expression

$$\langle \vec{D}, \sigma | j^\ell(0) | -\lambda\vec{a} - \delta\cdot\vec{p}, \rho \rangle \tag{5.1}$$

whioh we write in terms of variational derivative:

$$\langle \vec{P}, \sigma | j^\ell(0) | \vec{P}'', S'' \rangle = i e^{-i(P-P'')x} \langle \vec{P}, \sigma | \frac{\delta S}{\delta A_\ell(x)} | \vec{P}'', S'' \rangle \tag{5.2}$$

where

$$\vec{P}'' = -\lambda\vec{a} - \delta\cdot\vec{p}, \quad \rho = S''$$

Passing on from the variation over $A_\ell(x)$ to that over $A_\ell(K_0)$ we get

$$\delta(P-P''-K) \langle \vec{P}, \sigma | j^\ell(0) | \vec{P}'', S'' \rangle = \frac{i}{(2\pi)^5} \langle \vec{P}, \sigma | \frac{\delta S}{\delta A_\ell(K)} | \vec{P}'', S'' \rangle \tag{5.3}$$

It is easy to verify that $\vec{P}''^2 \geqslant 0$. This allows to work with $|\vec{P}'', S''\rangle$ like with a real state. Performing commutation of the operators of creation and annihilation of nucleons from the amplitudes $\langle \vec{P}, \sigma|$ and $|\vec{P}'', S''\rangle$ with $\frac{\delta S}{\delta A_\ell(K)}$ we obtain that

$$i \langle \vec{P}, \sigma | \frac{\delta S}{\delta A_\ell(K)} | \vec{P}'' S'' \rangle = \frac{i}{(2\pi)^3} \sqrt{\frac{M^2}{E, E''}}\, \bar{w}(\vec{P}, \sigma) \left\langle \left| \frac{\delta^3 S}{\delta \bar{\Psi}(\vec{P}) \cdot \delta A_\ell(K) \cdot \delta \Psi(\vec{P}'')} \right| \right\rangle w(\vec{P}'' S'') \tag{5.4}$$

where $E = \sqrt{\vec{P}^2 + M^2}$ and $E'' = \sqrt{\vec{P}''^2 + M^2}$. According to the relativistic invariance reasons it follows

$$\left\langle \left| \frac{\delta^3 S}{\delta \bar{\Psi}(P) \cdot \delta A_\ell(K) \cdot \delta \Psi(P'')} \right| \right\rangle = \ell \left(\mathcal{F}_1(K^2) \gamma^\ell - \mathcal{F}_2 \frac{\mu [\hat{K}, \gamma^\ell]}{4M} \right) \cdot \delta(P - P'' - k), \quad \hat{k} = \gamma K, \tag{5.5}$$

$\mathcal{F}_i(k^2)$ - are the scalar functions (form factors of a nucleon). Now from (5.5), (5.4), (5.3) and (3.21) we get:

$$A_{n\ell}(\lambda \vec{a}) = -\frac{\ell^2}{(2\pi)^3} \cdot \frac{M^2 + \delta \cdot \vec{P}^2}{M^2 + \vec{P}^2} \cdot \frac{M^2}{\sqrt{E \cdot E_0 \cdot E''}}\, S \sum_{S''} \bar{w}(\vec{P}, \sigma) \left(\mathcal{F}_1(K^2) \gamma^n - \mathcal{F}_2(K^2) \frac{\mu [\hat{K}, \gamma^n]}{4M} \right) w(-\vec{P}'', S'') \times \bar{w}(-\vec{P}'', S'') \left(\mathcal{F}_1(\varkappa^2) \gamma^\ell + \mathcal{F}_2(\varkappa^2) \frac{\mu [\hat{\varkappa}, \gamma^\ell]}{4M} \right) w(\vec{P}_0, \sigma_0) \tag{5.6b}$$

$$A_{\ell n}(-\lambda \vec{a}) = -\frac{\ell^2}{(2\pi)^3} \cdot \frac{M^2 + \delta \cdot \vec{P}^2}{M^2 + \vec{P}^2}\, S \sum_{S''} \frac{M^2}{\sqrt{E \cdot E_0 \cdot E''^2}} \times \bar{w}(\vec{P}, \sigma) \left(\mathcal{F}_1(\varkappa^2) \gamma^\ell + \mathcal{F}_2(\varkappa^2) \frac{\mu [\hat{\varkappa}, \gamma^\ell]}{4M} \right) w(\vec{P}'' S'') \cdot \bar{w}(\vec{P}'' S'') \left(\mathcal{F}_1(K^2) \gamma^n - \mathcal{F}_2(K^2) \frac{\mu [\hat{K}, \gamma^n]}{4M} \right) w(\vec{P}_0, \sigma_0) \tag{5.6a}$$

Taking the sum over S'' and performing the symmetrization operation (see 3.15) we obtain:

$$\begin{aligned} A_{\ell n} &= -\ell^2 \frac{2M(M^2 + \delta \cdot \vec{P}^2)}{2(2\pi)^3 (M^2 + \vec{P}^2) E'' \sqrt{E \cdot E_0}}\, \bar{w}(\vec{P}, \sigma) \left(\mathcal{F}_1(\varkappa^2) \gamma^\ell + \mathcal{F}_2(\varkappa^2) \frac{\mu [\hat{\varkappa}, \gamma^\ell]}{4M} \right) (\hat{P}'' + M) \cdot \left(\mathcal{F}_1(K^2) \gamma^n - \mathcal{F}_2(K^2) \frac{\mu [\hat{K}, \gamma^n]}{4M} \right) w(\vec{P}_0, \sigma_0) \\ A_{n\ell} &= -\ell^2 \frac{M(M^2 + \delta \cdot \vec{P}^2)}{2(2\pi)^3 (M^2 + \vec{P}^2) E'' \cdot \sqrt{E \cdot E_0}}\, \bar{w}(\vec{P}, \sigma) \left(\mathcal{F}_1(K^2) \gamma^n - \mathcal{F}_2(K^2) \frac{\mu [\hat{K}, \gamma^n]}{4M} \right) (\gamma_0 E'' + \vec{\gamma} \vec{P}'' + M) \times \left(\mathcal{F}_1(\varkappa^2) \gamma^\ell + \mathcal{F}_2(\varkappa^2) \frac{\mu [\hat{\varkappa}, \gamma^\ell]}{4M} \right) w(\vec{P}_0, \sigma_0) \end{aligned} \tag{5.7}$$

In the relations (5.7) $\mathcal{F}_1(0) = 1$, $\mathcal{F}_2(0) = 1$, ℓ is the electric charge of a nucleon, and μ is the anomalous magnetic moment of a nucleon measured in nuclear magnetons.

It is important to note, that in one-nucleon approximation, i.e. when

$$\langle f | S | i \rangle = -i \ell \frac{m e^n \varepsilon^\ell}{\sqrt{2K_0 \cdot \mathcal{E} \cdot \mathcal{E}_0}} \cdot \frac{(2\pi)^4}{\varkappa^2} \cdot \delta(K + P - \varkappa_0 - P_0) \left(\frac{A_{\ell n}}{E_p - K_0} + \frac{A_{n\ell}}{E_p + K_0} \right) \tag{5.8}$$

the expression (5.8) coincides with the sum of matrix elements corresponding to the graphs 2a) and 2b), in which at the vertices related to the virtual photon instead of the form factor functions $\Phi_1(\varkappa^2, P''^2)$ and $\Phi_2(\varkappa^2, P''^2)$ there are $\mathcal{F}_1(\varkappa^2)$ and $\mathcal{F}_2(\varkappa^2)$ and the vertices related to the real photon-instead of $\Phi_1(0, P''^2)$ and ($\Phi_2(0, P''^2)$ there are $\mathcal{F}_1(0)$ and $\mathcal{F}_2(0)$.

Thus, the use of the dispersion relation allows to introduce in a strict a manner into the process of bremsstrahlung and pair production in the lowest e approximation the form factors depending on one variable which have been investigated for the negative argument values by Hofstadter [10].

6. Virtual Compton effect amplitude structure

We expand the virtual Compton effect amplitude T^c in relativistically invariant structure obeying gauge invariance requirements. Let us write T^c in the following form

$$T^c = e^n \mathcal{E}^\ell \, T^c_{n\ell} = \bar{w}(P) e^n R_{n\ell} \mathcal{E}^\ell w(P_0) \tag{6.1}$$

where

$$R_{n\ell} = R_{n\ell}(P_0, P, k, \varkappa, \gamma)$$

$T^c_{n\ell}$ must satisfy the conditions $k_n T^c_{n\ell} = 0$ and $\varkappa_\ell T^c_{n\ell} = 0$ (6.2).

Let us introduce, as it has been suggested in [11] $\mathcal{G}$ factors:

$$\mathcal{G}_{n\alpha} = \delta_{n\alpha} - \frac{(P+P_0)_n k_\alpha}{(P+P_0, k)}$$
$$\mathcal{G}_{\ell\beta} = \delta_{\ell\beta} - \frac{(P+P_0)_\ell \varkappa_\beta}{(P+P_0, \varkappa)} \tag{6.3}$$

where $(P+P_0, k)$ denotes scalar product of the two vectors $P+P_0$ and k. Let us write:

$$T_{n\ell} = \mathcal{G}_{n\alpha} \mathcal{G}_{\ell\beta} \, T_{\alpha\beta} \tag{6.4}$$

In the expression (6.4) $\mathcal{G}$ - factors automatically take into account the gauge invariance condition. The choice of $\mathcal{G}$ factors yields:

$$(P+P_0)_\alpha \mathcal{G}_{n\alpha} = 0$$

$$(P+P_0)_\beta \mathcal{G}_{\ell\beta} = 0$$

$$k_\alpha \mathcal{G}_{n\alpha} = k_n$$

Using the general theorem of expansion of the matrix element (see 11) let us represent T^c in the form:

$$T^c = \bar{w}(P) e^n \mathcal{E}^\ell \sum_{s,t} \Omega'_{s,t} \Lambda^{(s)} \mathcal{G}_{n\alpha} \mathcal{G}_{\ell\beta} R^t_{\alpha\beta} w(P_0) \tag{6.5}$$

where $\Omega'_{s,t}$ are some scalar functions, depending only on scalar products of 4 - momenta of initial and final states.

$$\Lambda^{(1)} = 1, \qquad \Lambda^{(2)} = \hat{k},$$

$$R^{(1)}_{\alpha\beta} = \gamma_\alpha \gamma_\beta; \quad R^{(2)}_{\beta\alpha} = \gamma_\beta \gamma_\alpha; \quad R^{(3)}_{\alpha\beta} = \gamma_\alpha k_\beta; \quad R^{(4)}_{\alpha\beta} = \gamma_\alpha \varkappa_\beta;$$
$$R^{(5)}_{\alpha\beta} = \varkappa_\alpha \gamma_\beta; \quad R^{(6)}_{\alpha\beta} = \varkappa_\alpha k_\beta; \quad R^{(7)}_{\alpha\beta} = \varkappa_\alpha \varkappa_\beta \tag{6.6}$$

Thus, from (6.4), (6.5) and (6.6) we obtain

$$T^{c} = \ell^{n}\varepsilon^{\ell} \sum_{\substack{s=1,2\\ t=1,\ldots,7}} T^{s,t}_{\ell,n}\, \Omega'_{s,t}$$

However, the fourteen structures $T^{s,t}_{\ell,n}\,\ell^{n}\varepsilon^{\ell}$ under consideration are not linear - independent ones: the combinations $\frac{1}{2}(T^{1,1}-T^{1,2})$ and $\frac{1}{2}(T^{2,1}-T^{2,2})$ are expressed in terms of other structures. The final form of our twelve linear - independent structures is

$$\begin{aligned}
R^{(1)} &= \tfrac{1}{2}(T^{1,1}+T^{1,2}) = \bar{w}(P)\Big(\ell-\frac{(\ell,P+P_0)K}{(P+P_0,K)}\Big)\Big(\varepsilon-\frac{(\varepsilon,P+P_0)\varkappa}{(P+P_0,\varkappa)}\Big)w(P_0)\\
R^{(2)} &= \bar{w}(P)\Big(\hat{e}-\frac{(\ell,P+P_0)\hat{K}}{(P+P_0,K)}\Big)\Big((\varepsilon K)-\frac{(K\varkappa)(\varepsilon,P+P_0)}{(P+P_0,\varkappa)}\Big)w(P_0)\\
R^{(3)} &= \bar{w}(P)\Big(\hat{e}-\frac{(\ell,P+P_0)\hat{K}}{(P+P_0,K)}\Big)\Big((\varepsilon\varkappa)-\frac{(\varepsilon,P+P_0)\varkappa^2}{(P+P_0,\varkappa)}\Big)w(P_0)\\
R^{(4)} &= \bar{w}(P)\Big((e\varkappa)-\frac{(\ell,P+P_0)(K\varkappa)}{(P+P_0,K)}\Big)\Big(\hat{\varepsilon}-\frac{(\varepsilon,P+P_0)\hat{\varkappa}}{(P+P_0,\varkappa)}\Big)w(P_0)\\
R^{(5)} &= \bar{w}(P)\Big((e\varkappa)-\frac{(\ell,P+P_0)(K\varkappa)}{(P+P_0,K)}\Big)\Big((\varepsilon k)-\frac{(\varepsilon,P+P_0)(K\varkappa)}{(P+P_0,\varkappa)}\Big)w(P_0)\\
R^{(6)} &= \bar{w}(P)\Big((e\varkappa)-\frac{(\ell,P+P_0)(K\varkappa)}{(P+P_0,K)}\Big)\Big((\varepsilon\varkappa)-\frac{(\varepsilon,P+P_0)\varkappa^2}{(P+P_0,\varkappa)}\Big)w(P_0)\\
R^{(7)} &= \tfrac{1}{2}(T^{2,1}+T^{2,2}) = \bar{w}(P)\hat{K}\Big(\ell-\frac{(\ell,P+P_0)K}{(P+P_0,K)}\Big)\Big(\varepsilon-\frac{(\varepsilon,P+P_0)\varkappa}{(P+P_0,\varkappa)}\Big)w(P_0)\\
R^{(8)} &= \bar{w}(P)\hat{K}\Big(\hat{\ell}-\frac{(\ell,P+P_0)\hat{K}}{(P+P_0,K)}\Big)\Big((\varepsilon K)-\frac{(\varepsilon,P+P_0)(K\varkappa)}{(P+P_0,\varkappa)}\Big)w(P_0)\\
R^{(9)} &= \bar{w}(P)\hat{K}\Big(\hat{\ell}-\frac{(\ell,P+P_0)\hat{K}}{(P+P_0,K)}\Big)\Big((\varepsilon\varkappa)-\frac{(\varepsilon,P+P_0)\varkappa^2}{(P+P_0,\varkappa)}\Big)w(P_0)\\
R^{(10)} &= 2T^{(2,5)}-2T^{(1,6)} = \bar{w}(P)\Big[\hat{K}\Big(\hat{\varepsilon}-\frac{(\varepsilon,P+P_0)\hat{\varkappa}}{(P+P_0,\varkappa)}\Big)-\Big(\hat{\varepsilon}-\frac{(\varepsilon,P+P_0)\hat{\varkappa}}{(P+P_0,\varkappa)}\Big)\hat{K}\Big]\Big((e\varkappa)-\frac{(\ell,P+P_0)(K\varkappa)}{(P+P_0,K)}\Big)w(P_0)\\
R^{(11)} &= \bar{w}(P)\hat{K}\Big((e\varkappa)-\frac{(\ell,P+P_0)(K\varkappa)}{(P+P_0,K)}\Big)\Big((\varepsilon K)-\frac{(\varepsilon,P+P_0)(K\varkappa)}{(P+P_0,\varkappa)}\Big)w(P_0)\\
R^{(12)} &= \bar{w}(P)\Big((e\varkappa)-\frac{(\ell,P+P_0)(K\varkappa)}{(P+P_0,K)}\Big)\Big((\varepsilon\varkappa)-\frac{(\varepsilon,P+P_0)\varkappa^2}{(P+P_0,\varkappa)}\Big)w(P_0)
\end{aligned} \tag{6.8}$$

Thus

$$T^{c} = \sum_{i=1}^{12} R^{i}\,\Omega_i \tag{6.9}$$

The structures (6.8) comply with the requirements of relativistic invariance, gauge invariance and conditions of the type of the "crossing symmetry" theorem :

$$R^{i} = \pm (R^{i})^{+} \quad \begin{pmatrix} P\to P_0, & \sigma\to\sigma_0 \\ K\to -K, & \varkappa\to-\varkappa \end{pmatrix}$$

The last condition leads to the following relations for the coefficients Ω_i :

$$\begin{aligned}
\Omega_i &= +\Omega_i^{+} \qquad && i = 1,5,6.\\
\Omega_i &= -\Omega_i^{+} \qquad && i = 2,3,4,7,8,9,10,11,12.
\end{aligned} \tag{6.10}$$

For the real Compton effect the structures R^{3}, R^{6}, R^{9} and R^{12} vanish. Thus, for the real Compton effect there remains eight independent structures. This result coinsides with that of the paper[12].

But dispersion relations can not be written directly for the coefficients Ω_i since the structures of R^i are not symmetrical in $\vec{\lambda}$. Therefore, at first it is necessary to expand T^c in the system $\vec{P}_0+\vec{P}=0$ in twelve independent three-dimensional structures. We choose the following independent structures

$$\begin{array}{lll}
\tau_1=\vec{e}\vec{\varepsilon} & \tau_5=i(\vec{\varepsilon}\vec{\lambda})(\vec{\sigma}[\vec{P}_0\vec{e}]) & \tau_9=i(\vec{e}\vec{\varepsilon})(\vec{\sigma}[\vec{p}\times\vec{\lambda}]) \\
\tau_2=(\vec{e}\vec{p})(\vec{\varepsilon}\vec{\lambda}) & \tau_6=i(\vec{\varepsilon}\vec{p})(\vec{e}\vec{p})(\vec{\sigma}[\vec{p}\times\vec{\lambda}]) & \tau_{10}=i(\vec{\varepsilon}\vec{p})(\vec{\sigma}[\vec{\lambda}\times\vec{e}]) \\
\tau_3=(\vec{e}\vec{p})(\vec{\varepsilon}\vec{p}) & \tau_7=\frac{i}{\lambda^2}(\vec{e}\vec{p})(\vec{\varepsilon}\vec{\lambda})(\vec{\sigma}[\vec{p}\times\vec{\lambda}]) & \tau_{11}=\frac{i}{\lambda^2}(\vec{\varepsilon}\vec{\lambda})(\vec{\sigma}[\vec{\lambda}\times\vec{e}]) \\
\tau_4=i(\vec{\varepsilon}\vec{p})(\vec{\sigma}[\vec{p}\times\vec{e}]) & \tau_8=i(\vec{e}\vec{p})(\vec{\sigma}[\vec{p}\times\vec{\varepsilon}]) & \tau_{12}=i(\vec{e}\vec{p})(\vec{\sigma}[\vec{\lambda}\times\vec{\varepsilon}])
\end{array} \qquad (6.11)$$

Thus, in the system $\vec{P}_0+\vec{p}=0$ the amplitude T^c can be written in the form:

$$T^c=\sum_K \mathcal{L}_K(K_0,\vec{P}^2)\,\tau_K \qquad (6.12)$$

where $\mathcal{L}_K(K_0,\vec{P}^2)$ are the scalar functions of the variable K_0 and recoil $\vec{P}^2$. The symmetrization operation S over $\vec{\lambda}$ is now fulfiled trivially in consequence of explicit dependence of the structures τ_K on the vector $\vec{\lambda}$.

7. Dispersion relations for the Lorentz - invariant coefficients

It is obviously that due to the independence of the structures τ_K dispersion relations (3.16) can be written for each coefficient $\mathcal{L}_K$ taken separately. Each coefficient must conduct itself in the complex K_0 variable plane not worse than the amplitude T^c and decrease at the infinity not slower than $\frac{1}{K_0}$. The symmetrization operation S leads in addition to the fact that the coefficients for the antisymmetrical in $\vec{\lambda}$ structures will decrease not slower than $\frac{1}{K_0^2}$.

$$Re\,\mathcal{L}_K(K_0)=\frac{P}{\pi}\int^{\infty}\left(\frac{1}{K_0'-K_0}+\frac{1}{K_0'+K_0}\right)Jm\,\mathcal{L}_K(K_0')\,dK_0'+\frac{\mathcal{L}_K^{(1)}}{E_p-K_0}+\frac{\mathcal{L}_K^{(2)}}{E_p+K_0};\quad K=1,2,3 \qquad (7.1)$$

$$Re\,\mathcal{L}_K(K_0)=\frac{P}{\pi}\int_{E_1}^{\infty}\left(\frac{1}{K_0'-K_0}-\frac{1}{K_0'+K_0}\right)Jm\,\mathcal{L}_K(K_0')\,dK_0'+\frac{\mathcal{L}_K^{(1)}}{E_p-K_0}+\frac{\mathcal{L}_K^{(2)}}{E_p+K_0};\quad K=4,5,\ldots 12$$

In order to proceed from dispersion relations (7.1) to relations for the coefficients Ω_i, at first it is necessary to establish the connection between Ω_i and $\mathcal{L}_K$ and then analyse the behaviour of $\Omega_i(K_0)$ in the complex plane.

From (6.9) and (6.12) we have

$$T^c=\sum_i R^{(i)}\Omega_i=\sum_K \mathcal{L}_K\,\tau_K$$

In the system $\vec{P}_0 + \vec{P} = 0$ the four-dimensional structures R_i are connected with three-dimensional ones τ_K as follows:

$$R^{(1)} = \frac{1}{M}\left(-E\cdot\tau_1 + \frac{2E}{K_0^2}\cdot\tau_2 + \frac{2E(1+\delta)}{K_0^2}\cdot\tau_3\right)$$

$$R^{(2)} = \frac{1}{M}\left\{\left(-2 + \frac{(1+\delta)(K\varkappa)}{K_0^2}\right)\tau_4 + \frac{(K\varkappa)}{K_0^2}\tau_5\right\}$$

$$R^{(3)} = \frac{1}{M}\left\{\frac{\varkappa^2(1+\delta)}{K_0^2}\tau_4 + \frac{\varkappa^2}{K_0^2}\tau_5\right\}$$

$$R^{(4)} = \frac{2}{M}\left\{-\frac{1+\delta}{K_0^2}\tau_6 - \frac{\lambda^2}{K_0^2}\tau_7 + \tau_8\right\}$$

$$R^{(5)} = \frac{2E}{M}\left\{\frac{(K\varkappa)}{K_0^2}\cdot\tau_2 + \left(-2 + \frac{(1+\delta)(K\varkappa)}{K_0^2}\right)\tau_3\right\}$$

$$R^{(6)} = \frac{2E}{M}\left\{\frac{\varkappa^2}{K_0^2}\tau_2 + \frac{\varkappa^2(1+\delta)}{K_0^2}\tau_3\right\} \qquad (7.2)$$

$$R^{(7)} = \frac{1}{M}\left\{-K_0 M\cdot\tau_1 + \frac{2M}{K_0}\cdot\tau_2 + \frac{2M(1+\delta)}{K_0}\cdot\tau_3 - \frac{2(1+\delta)}{K_0^2}\tau_6 - \frac{2\lambda^2}{K_0^2}\tau_7 + \tau_9\right\}$$

$$R^{(8)} = \frac{1}{M}\left\{-\frac{(K\varkappa)}{K_0}\tau_2 + K_0\left(2 - \frac{(1+\delta)(K\varkappa)}{K_0^2}\right)\tau_3 + M(1-\delta)\left(2 - \frac{(1+\delta)(k\varkappa)}{K_0^2}\right)\tau_4 - \frac{M(1-\delta)(K\varkappa)}{K_0^2}\cdot\tau_5 - \right.$$
$$\left. - \frac{2K_0^2 - (1+\delta)(K\varkappa)}{(E+M)K_0^2}\cdot\tau_6 + \frac{(K\varkappa)\lambda^2}{K_0^2(E+M)}\tau_7 - E\left(2 - \frac{(1+\delta)(K\varkappa)}{K_0^2}\right)\tau_{10} + \frac{E(K\varkappa)\lambda^2}{K_0^2}\cdot\tau_{11}\right\}$$

$$R^{(9)} = \frac{1}{M}\left\{-\frac{\varkappa^2}{K_0}\tau_2 - \frac{\varkappa^2(1+\delta)}{K_0}\tau_3 - \frac{(1-\delta^2)M\varkappa^2}{K_0^2}\tau_4 - \frac{M(1-\delta)\varkappa^2}{K_0^2}\cdot\tau_5 + \right.$$
$$\left. + \frac{(1+\delta)\varkappa^2}{K_0^2(E+M)}\tau_6 + \frac{\varkappa^2\lambda^2}{K_0^2(E+M)}\cdot\tau_7 + \frac{E(1+\delta)\varkappa^2}{K_0^2}\cdot\tau_{10} + \frac{E\varkappa^2\lambda^2}{K_0^2}\cdot\tau_{11}\right\}$$

$$R^{(10)} = \frac{4}{M}\left\{\frac{\vec{P}\vec{\varkappa}}{K_0}\cdot\tau_2 - \frac{\varkappa^2+\lambda^2}{K_0}\cdot\tau_3 + \left(\frac{1}{E+M} + \frac{2M(1+\delta)}{K_0^2}\right)\tau_6 + \frac{2M\lambda^2}{K_0^2}\tau_7 - M(1-\delta)\tau_8 + E\cdot\tau_{12}\right\}$$

$$R^{(11)} = \frac{2}{M}\left\{\frac{M(K\varkappa)}{K_0}\tau_2 - MK_0\left(2 - \frac{(1+\delta)(K\varkappa)}{K_0^2}\right)\tau_3 + \left(2 - \frac{(1+\delta)(K\varkappa)}{K_0^2}\right)\tau_6 - \frac{(K\varkappa)\lambda^2}{K_0^2}\tau_7\right\}$$

$$R^{(12)} = \frac{2}{M}\left\{\frac{M\varkappa^2}{K_0}\tau_2 + \frac{M(1+\delta)\varkappa^2}{K_0}\cdot\tau_3 - \frac{(1+\delta)\varkappa^2}{K_0^2}\tau_6 - \frac{\varkappa^2\lambda^2}{K_0^2}\tau_7\right\}$$

Let us write (7.2) briefly $R_i = \sum a_{iK}\tau_K$. Thus, $\sum_{i,K}\Omega_i a_{iK}\tau_K = \sum_K \mathcal{L}_K\tau_K$ or $\sum_i \Omega_i a_{iK} = \mathcal{L}_K$

Let us write down connections between Ω_i and $\mathcal{L}_K$ in the explicit form:

1) $\Omega_{10}\cdot a_{10,12} = \mathcal{L}_{12}$

2) $\Omega_7\cdot a_{7,9} = \mathcal{L}_9$

3) $\Omega_1\cdot a_{1,1} + \Omega_7 a_{7,1} = \mathcal{L}_1$

$$4)\ \Omega_4 \cdot a_{4,8} + \Omega_{10}\, a_{10,8} = \mathcal{L}_8$$

$$5)\ \Omega_8\, a_{8,10} + \Omega_9\, a_{9,10} = \mathcal{L}_{10}$$

$$6)\ \Omega_8\, a_{8,11} + \Omega_9\, a_{9,11} = \mathcal{L}_{11}$$

$$7)\ \Omega_2 a_{2,4} + \Omega_3 a_{3,4} + \Omega_8 a_{8,4} + \Omega_9 a_{9,4} = \mathcal{L}_4$$

$$8)\ \Omega_2 a_{2,5} + \Omega_3 a_{3,5} + \Omega_8 a_{8,5} + \Omega_9 a_{9,5} = \mathcal{L}_5$$

$$9)\ \Omega_4 a_{4,6} + \Omega_7 a_{7,6} + \Omega_8 a_{8,6} + \Omega_9 a_{9,6} + \Omega_{10} a_{10,6} + \Omega_{11} a_{11,6} + \Omega_{12} a_{12,6} = \mathcal{L}_6 \qquad (7.3)$$

$$10)\ \Omega_4 a_{4,7} + \Omega_7 a_{7,7} + \Omega_8 a_{8,7} + \Omega_9 a_{9,7} + \Omega_{10} a_{10,7} + \Omega_{11} a_{11,7} + \Omega_{12} a_{12,7} = \mathcal{L}_7$$

$$11)\ \Omega_1 a_{1,2} + \Omega_5 a_{5,2} + \Omega_6 a_{6,2} + \Omega_7 a_{7,2} + \Omega_8 a_{8,2} + \Omega_9 a_{9,2} + \Omega_{10} a_{10,2} + \Omega_{11} a_{11,2} + \Omega_{12} a_{12,2} = \mathcal{L}_2$$

$$12)\ \Omega_1 a_{1,3} + \Omega_5 a_{5,3} + \Omega_6 a_{6,3} + \Omega_7 a_{7,3} + \Omega_8 a_{8,3} + \Omega_9 a_{9,3} + \Omega_{10} a_{10,3} + \Omega_{11} a_{11,3} + \Omega_{12} a_{12,3} = \mathcal{L}_3$$

Now we have only to answer the questions: whether Ω_i acquires additional poles in comparison with $\mathcal{L}_K$ and how Ω_i behaves at the infinity. One can obtain the answer from the solution of the system (7.3).

We first make the following remark: for $\lambda = 0$ in T^c all the structures, containing the vector $\vec{\lambda}$ must vanish and, consequently $L_7 = L_{11} = 0$ for this value of λ.

The coefficients L_7 and L_{11} are analytical functions of K_0 and can be expanded in Taylor series at the points $\lambda = 0$:

$$L_K(K_0) = L_K\big((1-\delta)/\vec{P}/\big) + L_K'\big((1-\delta)/\vec{P}/\big)\big(K_0 - (1-\delta)/\vec{P}/\big) + \ldots \qquad (K = 7, 11)$$

Thus, the expression $\frac{L_K}{\lambda^2} = \frac{L_K}{[K_0-(1-\delta)/\vec{P}/][K_0+(1-\delta)/\vec{P}/]}$ contains no additional poles for $\lambda = 0$.

Solving now the system (7.3) with respect to Ω_i and analysing the behaviour of Ω_i with regard to K_0 (taking into account the above remark) we find that all coefficients Ω_i are analytical functions of the variable K_o in the same region where the function ST^c is analytical too. The coefficient Ω_6 at the infinity conducts itself like a constant and, therefore, dispersion relations should be written with one substruction.

Taking into account the condition (6.10) we get the following dispersion relations for the coefficients Ω_i (in the system $\vec{p}+\vec{P}=0$)

$$R_e\Omega_i(K_o)=\frac{\mathcal{P}}{\pi}\int_{E_1}^{\infty}\left(\frac{1}{K_o'-K_o}+\frac{1}{K_o'+K_o}\right)\mathcal{J}_m\Omega_i(K_o')dK_o'+\frac{\Omega_i^{(1)}}{E_p-K_o}+\frac{\Omega_i^{(2)}}{E_p+K_o}; \qquad i=1,5$$

$$R_e\Omega_i(K_o)=\frac{\mathcal{P}}{\pi}\int_{E_1}^{\infty}\left(\frac{1}{K_o'-K_o}-\frac{1}{K_o'+K_o}\right)\mathcal{J}_m\Omega_i(K_o')dK_o'+\frac{\Omega_i^{(1)}}{E_p-K_o}+\frac{\Omega_i^{(2)}}{E_p+K_o}; \qquad i=2,3,4,7,\dots 12$$

$$R_e\Omega_6(K_o)=\frac{2K_o^2}{\pi}\mathcal{P}\int_{E_1}^{\infty}\frac{\mathcal{J}_m\Omega_6(K_o')dK_o'}{K_o'(K_o'^2-K_o^2)}+R_e\Omega_6(0)+\frac{\Omega_6^{(1)}}{E_p-K_o}+\frac{\Omega_6^{(2)}}{E_p+K_o}; \qquad (7.4)$$

$$\Omega_6(0)=\frac{1-\delta}{2\delta}\Omega_5(0)+\frac{1}{4\delta\vec{p}^2}\Omega_1(0)$$

In the relations (7.4) the coefficients $\Omega_i^{(1)}$ and $\Omega_i^{(2)}$ are corresponding parts of the one-nucleon terms $A_{en}(-\lambda\vec{a})$ and $A_{ne}(\lambda\vec{a})$. The explicit form of these coefficients is given by the following expressions:

1) $$\frac{\Omega_1^{(1)}}{E_p-K_o}+\frac{\Omega_1^{(2)}}{E_p+K_o}=-\frac{Me^2}{(2\pi)^3 2E}\cdot\mathcal{J}_1$$

2) $$\frac{\Omega_2^{(1)}}{E_p-K_o}+\frac{\Omega_2^{(2)}}{E_p+K_o}=\frac{Me^2}{(2\pi)^3 2E}\left[\mathcal{F}_2\frac{\mu^2}{M^2}-\frac{2(1+\mu)(PK)}{(P_oK)+(PK)}\mathcal{J}_3-\left(1+\frac{K_oE}{2\vec{p}^2}-\frac{K_oM^2(1-\delta^2)}{2E\lambda^2}\right)\mathcal{J}_2\right]$$

3) $$\frac{\Omega_3^{(1)}}{E_p-K_o}+\frac{\Omega_3^{(2)}}{E_p+K_o}=\frac{Me^2}{(2\pi)^3 2E}\left(\frac{EK_o}{2\vec{p}^2}+\frac{M^2K_o(1-\delta)^2}{2E\lambda^2}\right)\mathcal{J}_2$$

4) $$\frac{\Omega_4^{(1)}}{E_p-K_o}+\frac{\Omega_4^{(2)}}{E_p+K_o}=\frac{Me^2}{(2\pi)^3\cdot 2E}\left(-\mathcal{J}_3-\mathcal{J}_2\left[\frac{EK_o}{2\vec{p}^2}+\frac{M^2K_o(1-\delta)^2}{2E\lambda^2}\right]\right)$$

5) $$\frac{\Omega_5^{(1)}}{E_p-K_o}+\frac{\Omega_5^{(2)}}{E_p+K_o}=\frac{Me^2}{(2\pi)^3 2E}\left(-\frac{\mathcal{J}_1}{2(P_oK)}+\frac{MK_o^2}{2\vec{p}^2\lambda^2}\mathcal{J}_2+\frac{\mu}{2M}\mathcal{J}_3+\mathcal{J}_4\right)$$

$$6)\quad \frac{\Omega_6^{(1)}}{E_p-K_o}+\frac{\Omega_6^{(2)}}{E_p+K_o}=0$$

$$7)\quad \frac{\Omega_7^{(1)}}{E_p-K_o}+\frac{\Omega_7^{(2)}}{E_p+K_o}=(1+\mu)\,J_3\,\frac{Me^2}{(2\pi)^3\cdot 2E}$$

$$8)\quad \frac{\Omega_8^{(1)}}{E_p-K_o}+\frac{\Omega_8^{(2)}}{E_p+K_o}=\frac{Me^2}{(2\pi)^3\cdot 2E}\left[\frac{MK_o(1+\delta)}{2E\lambda^2}J_2-(1+\mu)J_4\right] \qquad (7.5)$$

$$9)\quad \frac{\Omega_9^{(1)}}{E_p-K_o}+\frac{\Omega_9^{(2)}}{E_p+K_o}=\frac{Me^2}{(2\pi)^3\cdot 2E}\left[\frac{MK_o(1-\delta)}{2E\lambda^2}J_2\right]$$

$$10)\quad \frac{\Omega_{10}^{(1)}}{E_p-K_o}+\frac{\Omega_{10}^{(2)}}{E_p+K_o}=\frac{Me^2}{(2\pi)^3\cdot 2E}\left(-\frac{MK_o(1-\delta)}{4E\lambda^2}J_2+\frac{\mu}{4M}J_3\right)$$

$$11)\quad \frac{\Omega_{11}^{(1)}}{E_p-K_o}+\frac{\Omega_{11}^{(2)}}{E_p+K_o}=\frac{Me^2}{(2\pi)^3 2E}\left(-\frac{M^2k_o}{2E\vec{p}\lambda^2}J_2-\frac{\mu}{2M}J_4\right)$$

$$12)\quad \frac{\Omega_{12}^{(1)}}{E_p-K_o}+\frac{\Omega_{12}^{(2)}}{E_p+K_o}=0$$

where

$$J_1=\frac{2\mu}{M}\left(\mathcal{F}_1+(1+\mu)\mathcal{F}_2\right)$$

$$J_2=\frac{\mu^2}{M^2}\mathcal{F}_2+(1+\mu)\left(\mathcal{F}_1+\mu\mathcal{F}_2\right)\left(\frac{1}{pK}-\frac{1}{P_oK}\right) \qquad (7.6)$$

$$J_3=\left(\mathcal{F}_1+\mu\mathcal{F}_2\right)\left(\frac{1}{P_oK}+\frac{1}{pK}\right)$$

$$J_4=\frac{\mu}{2M}\mathcal{F}_2\left(\frac{1}{P_oK}+\frac{1}{pK}\right)$$

In order to proceed to dispersion relations in o.m.s. it is convenient to write down firstly the relations (7.4) in terms of invariant variables τ, t which we choose by the following means

$$\tau=k\,(p+P_o)$$

$$t=(\varkappa-K)^2 \qquad (7.7)$$

For the Breit system we have the connection :

$$\tau=2K_oE=2K_o\sqrt{M^2+\vec{p}^{\,2}}$$

$$(7.8)$$

$$t=-4\vec{p}^{\,2};\qquad \tau=K_o\sqrt{4M^2-t}$$

In o.m.s. as independent variables we choose the full energy W of the system and the angle Cos θ between the directions of incident virtual photon and emitted real photon. In this case the connection is too cumbersome

$$\tau = \frac{W^2-M^2}{4W^2}\left[3W^2+M^2+m_\gamma^2+\sqrt{(W^2-M^2)+2m_\gamma^2(W^2+M^2)+m_\gamma^4}\cdot\cos\theta\right]$$

$$t = -m_\gamma^2+\frac{W^2-M^2}{2W^2}\left[-W^2+M^2+m_\gamma^2+\sqrt{(W^2-M^2)^2+2m_\gamma^2(W^2+M^2)+m_\gamma^4}\cdot\cos\theta\right] \quad (7.9)$$

$$\tau = W^2-M^2+\frac{m_\gamma^2+t}{2}$$

Dispersion relations (7.4) in terms of variables τ, t have now the following form

$$Re\,\Omega_i(\tau,t)=\frac{P}{\pi}\int_{2M\mu+\mu^2+\frac{t+m_\gamma^2}{2}}^{\infty}\left(\frac{1}{\tau'-\tau}+\frac{1}{\tau'+\tau}\right)Im\,\Omega_i(\tau',t)\,d\tau'+\overset{\circ}{\Omega}_i; \qquad i=1,5$$

$$Re\,\Omega_i(\tau,t)=\frac{P}{\pi}\int_{2M\mu+\mu^2+\frac{t+m_\gamma^2}{2}}^{\infty}\left(\frac{1}{\tau'-\tau}-\frac{1}{\tau'+\tau}\right)Im\,\Omega_i(\tau',t)\,d\tau'+\overset{\circ}{\Omega}_i; \qquad i=2,3,4,7,8,9,10,11.$$

$$Re\,\Omega_6(\tau,t)=\frac{2\tau^2}{\pi}\,P\int_{2M\mu+\mu^2+\frac{t+m_\gamma^2}{2}}^{\infty}\frac{Im\,\Omega_6(\tau',t)\,d\tau'}{\tau'(\tau'^2-\tau^2)}+Re\,\Omega_6(0); \quad (7.10)$$

$$Re\,\Omega_6(0)=-\frac{t+m_\gamma^2}{2m_\gamma^2}\,\Omega_5(0)+\frac{1}{m_\gamma^2}\,\Omega_1(0)$$

$$Re\,\Omega_{12}(\tau,t)=\frac{P}{\pi}\int_{2M\mu+\mu^2+\frac{t+m_\gamma^2}{2}}^{\infty}\left(\frac{1}{\tau'-\tau}-\frac{1}{\tau'+\tau}\right)Im\,\Omega_{12}(\tau',t)\,d\tau'$$

In the relations (7.10) Ω_6 and Ω_{12} equal zero. One-nucleon terms in the invariant variables have the form:

$$\overset{\circ}{\Omega}_1=-\frac{Me^2}{(2\pi)^3}\cdot\frac{J_1}{\sqrt{4M^2-t}}$$

$$\overset{\circ}{\Omega}_2=\frac{Me^2}{(2\pi)^3\sqrt{4M^2-t}}\left[J_2\frac{\mu^2}{M^2}-\frac{2(1+\mu)(pK)}{(P_oK)+(pK)}J_3-\left(1-\frac{2}{t}-\frac{4M^2(t^2-m_\gamma^4)\tau}{t[4t\tau^2+(4M^2-t)(t+m_\gamma^2)^2]}\right)J_2\right]$$

$$\overset{\circ}{\Omega}_3=\frac{Me^2}{(2\pi)^3\sqrt{4M^2-t}}\left[-\frac{\tau}{t}+\frac{4M^2\tau(t+m_\gamma^2)^2}{t[4t\tau^2+(t+m_\gamma^2)^2(4M^2-t)]}\right]\cdot J_3$$

$$\overset{\circ}{\Omega}_4=\frac{Me^2}{(2\pi)^3\sqrt{4M^2-t}}\left[-J_3-J_2\left(-\frac{\tau}{t}+\frac{4M^2\tau(t+m_\gamma^2)^2}{t[4t\tau^2+(t+m_\gamma^2)^2(4M^2-t)]}\right)\right] \quad (7.11)$$

$$\overset{\circ}{\Omega}_5=\frac{Me^2}{(2\pi)^3\sqrt{4M^2-t}}\left[-\frac{J_1}{2(P_oK)}-\frac{8M\tau^2}{4t\tau^2+(t+m_\gamma^2)(4M^2-t)}\cdot J_2+\frac{\mu}{2M}\cdot J_3+J_4\right]$$

$$\overset{o}{\Omega}_7 = \frac{Me^2}{(2\pi)^3\sqrt{4M^2-t}}\,(1+\mu)\mathcal{I}_3$$

$$\overset{o}{\Omega}_8 = \frac{Me^2}{(2\pi)^3\sqrt{4M^2-t}}\left[\frac{4t\tau M(t-m_\gamma^2)}{[4t\tau^2+(t+m_\gamma^2)^2(4M^2-t)}\,\mathcal{I}_2-(1+\mu)\mathcal{I}_4\right]$$

$$\overset{o}{\Omega}_9 = \frac{Me^2}{(2\pi)^3\sqrt{4M^2-t}}\cdot\frac{4t\tau M(t+m_\gamma^2)}{4t\tau^2+(t+m_\gamma^2)^2(4M^2-t)}\cdot\mathcal{I}_2$$

$$\Omega_{10} = \frac{Me^2}{(2\pi)^3\sqrt{4M^2-t}}\left[-\frac{\mathcal{I}_2}{2}\cdot\frac{4Mt\tau(t+m_\gamma^2)}{4t\tau^2+(t+m_\gamma^2)^2(4M^2-t)}+\frac{\mu}{4M}\mathcal{I}_3\right]$$

$$\Omega_{11} = \frac{Me^2}{(2\pi)^3\sqrt{4M^2-t}}\left[\frac{16M^2\tau}{4t\tau^2+(t+m_\gamma^2)^2(4M^2-t)}\mathcal{I}_2-\frac{\mu}{2M}\mathcal{I}_4\right]$$

Now dispersion relation for ooeffioients Ω_i in the c.m.s. are obtained simply from (7.9), (7.10) and (7.11) when prooeeding from the variables τ and t to w and Cos. θ .The final form of these dispersion relations is extremely cumbersome and not given here.

8. Dispersion relations for physioal amplitudes in the center-of-mass system

In order to obtain dispersion relation for physical amplitudes in the o.m.s. let us expand the amplitude T^c in independent three-dimensional structures ρ_K in the same system:

$$T^c = \sum_{K=1}^{12} M_K \rho_K \tag{8.1}$$

where M_K - are physical amplitudes depending on W and Cos $\cos\theta$.

$$\begin{aligned}
&\rho_1 = \vec{e}\vec{\mathcal{E}} && \rho_5 = i(\vec{\mathcal{E}}\vec{k})(\vec{\sigma}[\vec{\varkappa}\times\vec{e}]) && \rho_9 = i(\vec{e}\vec{\mathcal{E}})(\vec{\sigma}[\vec{k}\times\vec{\varkappa}])\\
&\rho_2 = (\vec{e}\vec{\varkappa})(\vec{\mathcal{E}}\vec{k}) && \rho_6 = i(\vec{\mathcal{E}}\vec{\varkappa})(\vec{e}\vec{\varkappa})(\vec{\sigma}[\vec{k}\times\vec{\varkappa}]) && \rho_{10} = i(\vec{\mathcal{E}}\vec{\varkappa})(\vec{\sigma}[\vec{k}\times\vec{e}])\\
&\rho_3 = (\vec{e}\vec{\varkappa})(\vec{\mathcal{E}}\vec{\varkappa}) && \rho_7 = i(\vec{e}\vec{\varkappa})(\vec{\mathcal{E}}\vec{k})(\vec{\sigma}[\vec{k}\times\vec{\varkappa}]) && \rho_{11} = i(\vec{\mathcal{E}}\vec{k})(\vec{\sigma}[\vec{k}\times\vec{e}])\\
&\rho_4 = i(\vec{\mathcal{E}}\vec{\varkappa})(\vec{\sigma}[\vec{\varkappa}\times\vec{e}]) && \rho_8 = i(\vec{e}\vec{\varkappa})(\vec{\sigma}[\vec{\varkappa}\times\vec{\mathcal{E}}]) && \rho_{12} = i(\vec{e}\vec{\varkappa})(\vec{\sigma}[\vec{k}\times\vec{\mathcal{E}}])
\end{aligned} \tag{8.2}$$

Let us find the connection between the coefficients $\Omega_i(w,\cos\theta)$ and $M_\kappa(w,\cos\theta)$, substituting the expansion of the relativistically invariant structures (6.8) in ρ_κ :

$$R_i = \sum_\kappa b_{i\kappa}\, \rho_\kappa \tag{8.3}$$

into the relation

$$T^c = \sum \Omega_i R_i = \sum_\kappa M_\kappa \rho_\kappa$$

we obtain

$$M_\kappa = \sum_i \Omega_i\, b_{i\kappa} \tag{8.4}$$

The coefficients $b_{i\kappa}$ are the known functions of the variables W and $\cos\theta$. The matrix $\| C_{i\kappa} \| = \| b_{i\kappa} \|^{-1}$ is obtained from the solution of the system (8.3). The system (8.3) including twelve equations has a cumbersome form. Choosing suitable structures, namely, (8.2) one succeeds in unlinking the system (8.3), and reducing it, thus, to the solution of two system of the second order and two system of the fourth order. Hence, it is easy to find the connection:

$$T^c = \sum_i \Omega_i R_i = \sum_{\kappa, i} M_\kappa C_{i\kappa} R_i$$

or

$$\Omega_i = \sum_\kappa C_{i\kappa} M_\kappa \tag{8.5}$$

Coefficients $C_{i\kappa}$ in the relation (8.5) have a highly cumbersome form and, hence, the connection (8.5) is not given in the explicit form. From (7.10), (7.9) and (8.5) we obtain dispersion relations for the physical amplitudes M_j in the o.m.s.

$$\begin{aligned} Re\, M_j(w,t,m_\gamma^2) = \frac{2\mathcal{P}}{\pi} \int_{M+\mu}^{\infty} \sum_i \xi_i \left(\frac{1}{w'^2 - w^2} + \eta_i \frac{1}{w'^2 + w^2 - 2M^2 + m_\gamma^2 + t} \right) \times \\ \times b_{ji}(w,t,m_\gamma^2) \sum_\kappa C_{i\kappa}(w',t,m_\gamma^2)\, Im\, M_\kappa(w',t,m_\gamma^2)\, w'dw' + \sum_i b_{ji}(w,t,m_\gamma^2)\, \overset{\circ}{\Omega}_i + \\ + b_{j6}(w,t,m_\gamma^2) \sum_\kappa M_\kappa\left(\sqrt{M^2 - \frac{t+m_\gamma^2}{2}}, t, m_\gamma^2\right) \left[\frac{1}{m_\gamma^2} C_{1,\kappa}\left(\sqrt{w^2 - \frac{t+m_\gamma^2}{2}}, t, m_\gamma^2\right) - \frac{m_\gamma^2+t}{2m_\gamma^2} C_{5,\kappa}\left(\sqrt{M^2 - \frac{t+m_\gamma^2}{2}}, m_\gamma^2, t\right) \right] \end{aligned} \tag{8.6}$$

$\xi_6 = \frac{2}{z'}$; $\quad \xi_i = 1$ for $i \neq 6$; $\quad z' = w'^2 - M^2 + \frac{m_\gamma^2 + t}{2}$;

$\eta_i = +1, \quad i = 1,5; \quad \eta_i = -1, \quad i = 2,3,4,6,7 \dots 12.$

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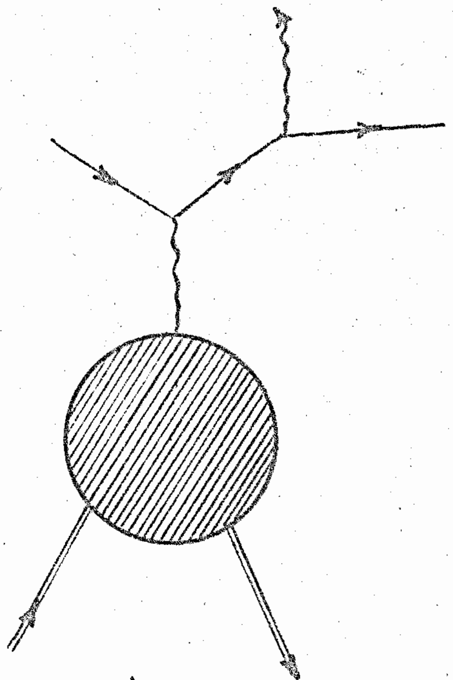

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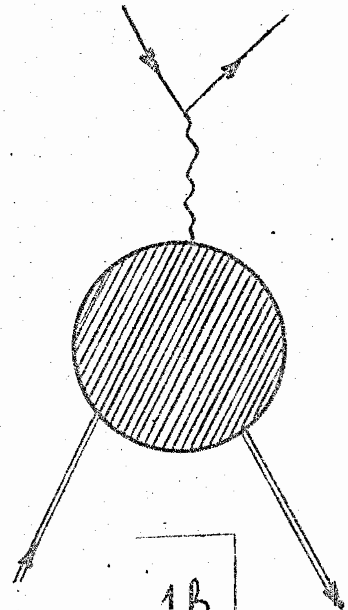

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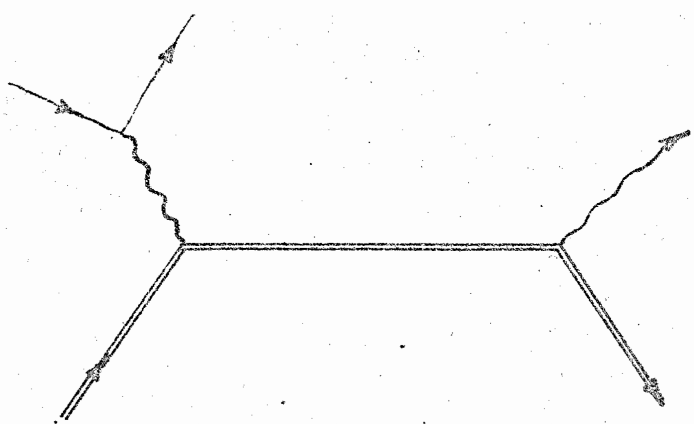

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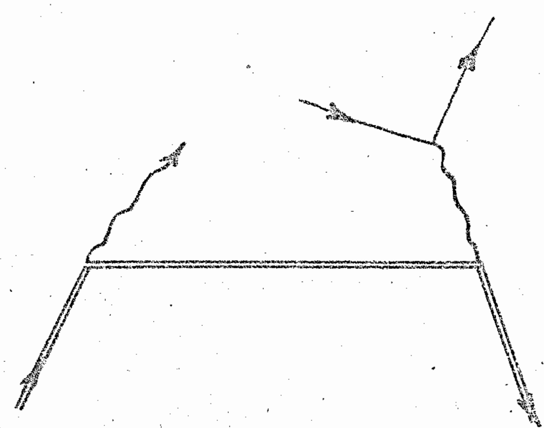

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