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C o n t e n t s

§ I. Introduction

In 1935 Yukawa suggested a hypothesis according to which the nucleons interact by a certain intermidiate meson field. The particles of this field-mesons have the mass μ less than that of the nucleon M [1] approximately by an order. Under this hypothesis the real "physical nucleon" consists of a "bare" point nucleon surrounded by a meson field like a point electric charge is surrounded by an electric field. In terms of a corpuscular theory it can be said that there is a cloud of virtual mesons around a nucleon as well as there is a cloud of virtual photons and electron-positron pairs around an electric charge.

The probability of a virtual meson production becomes appreciably less when its momentum becomes more than μc 1). Therefore, anambiguity in the meson cloud energy is about $\Delta E \sim \mu c^2$ whereas an ambiguity in the momentum - $\Delta p \sim \mu c$. It follows from here that the lifetime of a virtual meson $\Delta t \sim \hbar/\mu c$, the dimension of a meson field $\Delta x \sim \hbar/\mu c$.

The discovery of charged and neutral π -mesons (pions) with a mass $m_\pi = 1/6{,}5 M$ confirmed the main postulate of Yukawa. As for the mathematical formulation of Yukawa's idea recently suggested by Chew and Low for the nonrelativistic energies of particles ($E \ll Mc^2 \approx 0{,}94$ BeV) it yielded, on the one hand, a satisfactory quantitative discription of the experiments on scattering and photoproduction of π mesons and a correct order of the magnitude of the proton and neutron magnetic moment.[2]

These results of the nonrelativistic theory allow to think that the virtual cloud of π -mesons is a reality. However, the nucleon may dissociate not only into a nucleon and a pion, but also into a K-meson and a hyperon, into a pair nucleon-antinucleon etc:

$$N \rightleftarrows N + \pi \tag{1}$$

$$N \rightleftarrows Y + K \tag{2}$$

$$N \rightleftarrows N + N + \tilde{N} \tag{3}$$

Speculations for process (1) analogous to those given above lead to the conclusion that besides a pion cloud with the characteristic dimension $\hbar/m_\pi c$ there must also exist a cloud of virtual K-mesons with a characteristic dimension $\hbar/m_K c$ (m_k is the mass of a K-meson; $m_K \simeq N/2$ 2), and the cloud of virtual pairs ($N, \tilde{N}$) with a characteristic dimension $\hbar/Mc$ (M is the nucleon mass).

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1) The operator of meson produotion is proportional to $\frac{1}{\sqrt{\omega_K}}; \hbar\omega_K = \sqrt{(\hbar c k)^2 + (\mu c^2)^2}$, k - is a meson wave vector.

2) The coupling between K-mesons and a "bare" nucleon is likely to be weaker than that between a pion and a nucleon. Therefore, the K-meson oloud is less intensive than the pion one.

May be a somewhat naive picture of the physical nucleon structure is given in Fig.1 as it follows from the concept about possible virtual processes inside a nucleon.

This physical picture leads to the conception about the existence of two different regions inside a nucleon-the outer pion cloud and the central part, "core" where K-mesons, nucleon-antinucleon pairs and hyperons are essentially important. A characteristic dimension of a "core" is $\sim \frac{\hbar}{MC} = 2,1 \cdot 10^{-14}$ cm that is some times less than the dimesions of a pion cloud[3].

At present our knowledge of a nucleon "core" is very poor. But we may hope that we know much more of the nucleon pion cloud.

At any rate it is clear that if the contemporary theoretical concepts are able to describe even roughly the structure of such "elementary" particles as a nucleon, then charged virtual π-mesons should be considered to distribute an electrical charge and currents in the nucleon pion cloud. In particular, it is these currents which must be a cause of the anomalous magnetic moment of the nucleons.

Later on we shall consider some experimental evidence in favour of the described picture of the nucleon structure.

We shall see that a mosaic of physical ideas, experimental data and calculations is far from being a harmonious and complete theory. However, one may console oneself that no longer than ten years ago the very word "structure" of a nucleon would have seemed criminal to a majority of physicists.

§ 2. Methods for Studying Particle Structure

The best way for studying the structure of an object is to see it. Since the dimensions of microparticles are small it is evident that this possibility is excluded. However, there remains something analogous - this is an elastic scattering by the particle under consideration of any rays with the wave length λ less than the dimensions of the particle a.

When observing such a scattering in a particle ensemble one obtains a mean optical image of a particle from which under some conditions it is possible to obtain a space-time

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3) Jastrov was likely to be first who arrived at the conclusion that there exists a core of a nucleon-a region of great repulsive potential. |3| from the analysis of nucleon-nucleon scattering . One of the authors |4|,|5| came to the same conclusion starting from quite other considerations.

picture of particle structure with respect to the chosen rays. Experiments on electron or γ -quanta scattering on nucleons may serve as an example of such measurements.

The more detailed information on particle structure we want to obtain the shorter waves should be used. If we restrict ourselves to the wave lengths λ, more than the Compton wave length $\frac{\hbar}{\mu c}$ of the lightest particles associated with a nucleon (π -mesons are such particles), then the structural nucleon effects will not be significant. For example, the scattering of light with the wave length $\lambda \gg \frac{\hbar}{m_\pi c}$ will yield the information only about the magnitude of the total nucleon charge. At smaller values of λ the scattering already depends upon an anomalous magnetic moment of a nucleon, and by $\lambda < \frac{\hbar}{m_\pi c}$ the detailed dynamic structure of a nucleon is important. In this case the scattering cross section is essentially different from the scattering cross section on a point particle.

It should be borne in mind that with the decrease of the wave length applied for studying the structure of ray particles there occur inevitably new effects complicating a simple "vision" picture of this structure.

This is, firstly, the recoil effect; secondly, inelastic processes. Let us consider them in succession.

A. Recoil Effect

If the wave length of the rays $\lambda < \frac{\hbar}{Mc}$, where M is the mass of the particle under consideration, then in the interaction with a ray particle, this particle changes essentially its motion due to the momentum transferred to it. To get an idea about the importance of such an effect to obtain a particle image let us suppose that the intrinsic structure of a particle may be described by a wave function $\varphi_1(K, \xi)$, where K is the vector of the energy-momentum of a particle before its interaction with a ray, ξ are intrinsic coordinates of a particle. Let further $\varphi_2(K', \xi)$ be the same function after the particle interaction with a ray K' is the vector of energy-mementum after the interaction). The product $\varphi_1(K, \xi)$ and $\varphi_2(K', \xi)$ incorporates into an experimentally determined quantity (the form-factor of a particle). Therefore, the optical image will be inevitably a certain interference of pictures of the initial and final states which differ by a direction and magnitude of a Lorentz compression.

The Fourier component of the form-factor $\mathcal{F}$ due to relativistic considerations will be the function

$$(K'-K)^2 = \Delta\bar{K}^2 - \Delta\mathcal{E}^2; \quad (\Delta\bar{K} = \bar{K}' - \bar{K}; \ \Delta\mathcal{E} = \mathcal{E}' - \mathcal{E}),$$

where $\bar{K}$ is spatial and $\mathcal{E}$ is time-like parts of four-fermion vector K/ i.e.,

$$\mathcal{F} = \mathcal{F}(\Delta \vec{K}^2 - \Delta \mathcal{E}^2). \tag{4}$$

In the centre of gravity system the energy transfer $\Delta \mathcal{E} = 0$, and the form-factor allows to interpret the image as a certain spatial distribution

$$\rho(\vec{\xi}) = \int \mathcal{F}(\Delta \vec{K}^2) e^{i \Delta \vec{K} \vec{\xi}} d(\Delta \vec{K}). \tag{5}$$

However, this distribution will be a interference of two distributions "before the experiment" and "after the experiment". If we agree to consider such a generalized distribution as an image of the particle structure then by a sufficiently great number of scattering acts ("rich statistics", as it is accepted) and at a sufficiently short wavelength λ it is possible to obtain whatever exact picture of similar optical image of a particle.

B. Inelastic Processes

With the decrease of the wave length λ i.e. with the increase of the scattered particles energy the inelastic processes become of greater importance. Their role is especially great in case of strong interactions. The available experimental data on pion-nucleon and nucleon-nucleon interaction indicate that interaction cross sections rather remain constant than decrease with the energy increase. It can be seen, e.g., from Table I, where are given experimental values of the cross sections of inelastic proton and neutron interactions with ferrum nuclei at great energies [5,6] together with the statistical errors of measurements. The mean energy for each interval in Table 1 is calculated with account of the proton energy spectrum in an atmosphere. The measurements made by Grigorov et al. [6] and Begzhanov et al. [7] lead to analogous conclusions. Less clear are the tendencies in case of electromagnetic interactions. At high energies, however, the electromagnetic interaction may hardly be considered separately from other ones; in particular, from the interactions producing a μ -meson and neutrino. Theoretical estimates show that at very high energies these processes become strong in the sense that the cross sections become more than $\pi \lambda^2$ (λ is the photon wavelength [8]).

It seems quite probable that at high energies the constancy of the inelastic processes cross sections may be general. This circumstance is very likely to have a principle significance for space-time description of the phenomena within a small range.

Indeed, with the energy increase the number of possible channels of inelastic processes will increase. This means that the cross section of inelastic nondiffractional scattering with the energy increase will tend to zero. But it is this elastic scattering which is a source of information about a space-time structure of particles. Meanwhile with the energy increase all the elastic scattering will gradually be reduced to a diffractional one which is entirely due to inelastic processes.

These considerations may be illustrated by the conclusions of the statistical theory of multiple particle production. According to this theory for (π, p) collisions at $E_o = 5$ BeV the cross section of elastic nondiffractional scattering $\sigma_{nd} = 6.10^{-3}\,\sigma_{in}$ (σ_{in} - is the cross section of inelastic processes), and at $E_o = 7$ BeV, $\sigma_{nd} = 1{,}5.10^{-3}\,\sigma_{in}$; while the cross section of diffractional scattering $\sigma_d = 0{,}3\,\sigma_{in}$ [9]. Analogously, for (pp)-collisions $\sigma_{nd} = 5.10^{-3};\ 1.10^{-3};\ 2{,}5.10^{-4}$ from σ_{in} at $E_o = 5, 7$ Bev while $\sigma_d = 0{,}3\,\sigma_{in}$ [8,9].

The constancy of diffractional scattering is likely to indicate that a certain small inner region of a particle is able to scatter like a "black sphere" of radius R. It is clear that the maximum information obtained from the scattering picture will be restricted by the data about an outer dimension of this "black sphere". From this point of view the magnitude R will be that scale of length which determines a real nonlocality (see Fig.2). This dimension is not any universal length, but depends upon the kind of interaction[4].

At present we have not got any experimental roof that there exists a "black" region inside the nucleons. The analysis of experimental data by (π, p) and (p,p)-scattering points out, however, that there is actually a tendency fowards the appearance of "blackness" in the central region of a nucleon while its periphery regions remain halftransparent("grey") |10||14|. Scattering in these regions is also purely diffractional, i.e. $\sigma_{nd} = 0$. It may be used, however, to obtain the information about the distribution of π-meson and nucleon absorption inside a nucleon if these particles have a sufficiently small wave length. Thus, the study of diffractional scattering of particles should be also considered as a means for investigating the nucleon structure. In this connection it must be emphasized that further measurements of the interaction cross sections with different nuclei of ultra-high energy cosmic rays are of principle importance. Making use of the optical model one may obtain from these measurements the interaction cross sections of elementary particles at ultra-high energies |7|,|15|.

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4) Note that the dimension of a "black sphere" R determining the scale of nonlocality may be introduced into the theory by the relativistically-invariant way |10|.

C. Other Methods for Studying Nucleon Structure

Besides scattering processes the study of bound states will yield some data on nucleon structure: superfine stripping of hydrogen spectrum lines, a spectrum of μ -meson atoms [16]. All these methods, however, give only rough mean characteristics of nucleon structure which cannot be compared with the results of the scattering experiments of fast particles (electrons, π -mesons, nucleons).

Therefore, in conclusion of this Section it would be appropriate to give those wave lengths which are now available to the experimentalists and to show some perspective possibilities. These data are presented in Table II[5)].

§ 3. Electromagnetic Structure of a Nucleon

A supposition that there exists a cloud of charged mesons in a nucleon makes the study of quick electron scattering on nucleons[6)] extremely interesting.

Theoretically one may calculate the density of an electric charge $\rho_\pi(r)$ and the magnetic moment m(r) of a pion cloud in a nucleon making use of Chew-Low theory.

According to this theory the nucleon is regarded as an extended source of a meson field. At the same time it is considered infinitely heavy so that the recoil in virtual processes is not taken into account.

The extension of a nucleon as a source of a meson field may be understood as a result of the production of nucleon pairs, antinucleons and hyperons in the central region of a nucleon. In the first nonvanishing approximation Salzman has obtained: [18]

$$\rho_\pi(r) = -\frac{4e\tau_3}{(2\pi)^5}\left(\frac{f}{\mu}\right)^2 \int \frac{v(k)\,v(k')(\bar{k}\bar{k}')}{\omega(k)\,\omega(k')[\omega(k)+\omega(k')]}\, e^{i(\bar{k}-\bar{k}')r}\, d(kk') \tag{6}$$

$$\bar{m}_\pi(r) = -\frac{2ie\tau_3}{(2\pi)^5}\left(\frac{f}{\mu}\right)^2 \frac{v(k)\,v(k')(\bar{k}\bar{\sigma})}{\omega^2(k)\,\omega^2(k')}\left[\bar{r}[\bar{k}\bar{k}']\right] e^{i(\bar{k}-\bar{k}')r} d^3(kk') \tag{7}$$

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5) For reference let us give an expression for the wave length λ of the ray particle in the center of mass system

$$\lambda = \lambda_0 \frac{\sqrt{1-V^2}}{\sqrt{T_2(T_2+2M)} - V(T_2+M)}, \quad \text{where } V = \frac{\sqrt{T_2(T_2+2M)} - \sqrt{T_1(T_1+2)}}{T_2+T_1+M+1}$$

$T_2; T_1$ - are kinetic energies of the ray particle and the particle which is considered as a target in the lab.system respectively; M is the mass of a ray particle. T_2, T_1, M are expressed in the units of the mass of a target particle; $\lambda_0 = \frac{\hbar}{mc}$ is the Compton wave length of the target particle.

6) The problem related was first set in Saakjans dissertation [17].

where

$$\omega(k)=\sqrt{k^2+\mu^2}; \quad \mu=m_\pi\frac{c}{\hbar}; \quad v(k)$$ is

a cut-off function describing the distribution of π-meson field sources:

$$S(r)=\frac{1}{(2\pi)^3}\int v(k)e^{ikr}d^3k \tag{8}$$

The form of the function S(r) for great values r ($r > \frac{\hbar}{m_\pi c}$) weakly effect the conclusions of the theory as the density of the sources described by it at $r > \frac{\hbar}{m_\pi c}$ is noticeably less than their density in the central regions of a nucleon. With a sufficient accuracy one may consider $S(r) \simeq 0$ at $r > \frac{\hbar}{m_\pi c}$ (Gauss theorem in electrodynamics).

By $\xi = \frac{r}{\hbar/\mu c} > 1$ the density of charge (6) independently of the choice of the form-function V(k) may be rewritten as follows

$$\rho_\pi(r) = -\tau_3 e\mu^3 f^2 \frac{2}{(2\pi)^{3/2}} \frac{e^{-2\xi}}{\xi^2}\left\{\frac{1}{\sqrt{2\xi+4}} + \frac{4}{\xi\sqrt{2\xi+5}} + \frac{13}{2\xi^2\sqrt{2\xi+6}} + \dots\right\} \tag{9}$$

Analogously by $\xi > 1$ the expression for the magnetic moment density (7) may be presented as

$$m_\pi(r)=\tau_3 e\mu^2 c\frac{f^2}{4\pi}\left[\vec{n}\left[\vec{n}\vec{\sigma}\right]\right]\frac{e^{-2\xi}}{\xi^2}\left\{1+\frac{2}{\xi}+\frac{1}{\xi^2}\right\}. \tag{10}$$

Here $\quad \vec{n} = \frac{\vec{r}}{|r|}$.

These asymptotic expressions are essentially independent of the form of the meson field source (i.e. of the form of the function V(k)). However, as is seen from what will be said (see Fig.6) only a small part of the meson cloud (this is, so to say, the nucleon stratosphere) is concentrated in the region $r > \frac{1}{\mu}$. The expression [6] for $\rho_\pi(r)$ can be easily reduced to:

$$\rho_\pi(r)=e\mu^3\tau_3\frac{4f^2}{(2\pi)^5}\int_0^\infty d\xi\left(\frac{dI}{dr}\right)^2, \tag{11}$$

where

$$I(r)=\int\frac{v(k)}{\omega(k)}e^{ikr-\xi\omega(k)}d^3k \tag{12}$$

ξ is a new auxiliary variable.

Integrating in (12) over angles and denoting $V(k) \equiv V(w)$ we obtain:

$$I(r)\ \frac{2\pi}{ir}\ V\left(-\frac{d}{d\xi}\right)Q(\xi,r) \qquad + \text{comp. conj.} \tag{13}$$

$$Q(\xi,r)=\int_1^\infty e^{-\xi\omega+ir(\omega^2-1)^{\frac{1}{2}}}d\omega \tag{14}$$

Supposing further W =cht and introducing $\rho=\sqrt{\xi^2+r^2}$ we obtain

$$I(r)=-\frac{4\pi}{r}V\left(-\frac{d}{d\xi}\right)dK_0(\rho)/dr, \tag{15}$$

where $K_0(\rho)$ is a well-known Bessel function. Let us choose a cut-off form-factor V(w) as follows

$$V(\omega)=e^{-\beta(\omega-1)}, \tag{16}$$

where β is the cut-off parameter. In this case the operator V_β in (15) becomes simply a displacement operator $\xi\rightarrow\xi+\beta$ and the charge density (11) is written in the form convenient for numerical calculations:

$$\rho_\pi(r)=e\mu^3\tau_3\ \frac{2f}{\pi^3}\ e^{2\beta}r^2\int_{\sqrt{\beta^2+r^2}}^\infty K_2^2(\rho)\frac{d\rho}{\rho^3\sqrt{\rho^2-r^2}} \tag{17}$$

Similarly the expression for the magnetic moment density (7) may be transformed:

$$\bar{m}_\pi(r)=e\mu^3c\tau_3\ \frac{f^2}{2(2\pi)^2}\ e^{2\beta}\left[\bar{r}[\bar{\sigma}\bar{r}]\right]\left(\int_{\sqrt{\beta^2+r^2}}^\infty K_2(\rho)\frac{d\rho}{\beta\sqrt{\rho^2-r^2}}\right)^2 \tag{18}$$

Generally speaking, the densities $\rho_\pi(r)$ and $\bar{m}_\pi(r)$ essentially depend upon the form of the form-factor V(w), whereas by our choice of V(w)-upon the magnitude of the cut-off parameter β. In Fig. 3 and 4 are given the values of the electrical charge and magnetic moment of the pion cloud depending on the magnitude of the parameter β,

$$Q_\pi=\int\rho_\pi(r)\,d^3x \tag{19}$$

and

$$\mathcal{H}_\pi=\int m_\pi(r)\,d^3x, \tag{20}$$

as well as the values of the corresponding r.m.s. radii

$$\langle r_e^2\rangle=\frac{1}{e}\int\rho_\pi(r)r^2\,d^3x \tag{21}$$

and

$$\langle r_m^2 \rangle_\pi = \frac{0,6}{\varkappa} \int m_\pi(r) r^2 d^3 x . \qquad (22)$$

Here e =1 is the proton charge; $\varkappa$ =1,85 $\frac{e\hbar}{2MC}$ is an anomalous magnetic moment of a proton. The coefficient 0.6 is introduced so that the values $\langle r_m^2 \rangle_\pi$ would brought in agreement with formula (30).

It is seen from these Figures that both the charge and magnetic moment are sensitive to the choice of β while the values of r.m.s. radii remain almost constant within the wide limit of the change of the parameter β .

We choose the value β so that the theoretical $\mathcal{P}$ -phase for π -meson scattering on nucleons would be in best agreement with experiment in the low energy region. (As is known in the low energy region the $\mathcal{P}$-phase is a determinant). The calculations have shown that $\beta \simeq \frac{1}{7}$.

In this case (see Fig. 3 and 4).

Q = 0.76 (the proton charge is assumed to be a unit);

$\varkappa_\pi$ = 1.2 (in the units $\frac{e\hbar}{2MC}$) 2MC;

$\langle r_e^2 \rangle_\pi = (0.62.10^{-13}\mathrm{cm})^2$;

$\langle r_m^2 \rangle_\pi = (0.62,10^{-13}\mathrm{cm})^2$;

For the above-mentioned choice of the form-factor and the cut-off parameter we calculated the density of an electric charge confined in a spherical layer, $d_\pi(r) = 4\pi r^2 \rho_\pi(r)$ for a wide interval of the values r[7]. The results of these calculations are given in Table III. As it is seen from this Table the electric charge confined in the periphery region $r > \frac{\hbar}{m_\pi c}$ is insignificant compared with the charge of the pion cloud in the central regions of a nucleon.

The starting up of the electron linear accelerator at Stanford made it possible to check the conclusions of the theory when Hofstadter's group measured the electron scattering in hydrogen and deuterium at E =100-650 MeV.

We shall not be concerned with the detailed description of these experiments since it is given in |21|-|23| and restrict ourselves to the most essential items necessary for further analysis.

The main idea of the experiments was to establish the deviations from an electron

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7) The calculated values of $\rho_\pi(r)$ are essentially different from the values given in Zachariasens paper |19|. However, according to |20| Zachariasens results are not correct.

scattering with an energy E on a point nucleon, with an anomalous magnetic moment $\varkappa$. This scattering is described by the Rozenbluth's formula |24|

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathcal{F}=1} = \sigma_{NS}\left\{1+\frac{\hbar q^2}{4M^2c^2}\left[2(1+\varkappa)^2 tg\frac{Q}{2}+\varkappa^2\right]\right\}, \tag{24}$$

where

$$\sigma_{NS} = \left(q\frac{e^2\hbar c}{4E^2}\right)^2 \frac{\cos^2\frac{Q}{2}}{\sin^4\frac{Q}{2}} \tag{25}$$

is the Mott's formula for electron scattering on a point charge. $q = \frac{2E}{\hbar c}\sin\frac{Q}{2}\Big/\sqrt{1+\frac{2E}{MC^2}\sin\frac{Q}{2}}$ is a transferred momentum; Θ - is the angle of electron scattering in the lab.system.

The account of the charge and momenta distribution inside a nucleon leads to the appearance of the form-factor $\mathcal{F}_e(q)$ and $\mathcal{F}_m(q)$ corresponding for the charge and magnetic moment which are the functions of the transferred momentum q.

$$\frac{d\sigma}{d\Omega} = \sigma_{NS}\left\{\mathcal{F}_e^2(q)+\frac{\hbar^2q^2}{4M^2c^2}\left[2(\mathcal{F}_e(q)+\varkappa\mathcal{F}_m(q))^2 tg^2\frac{Q}{2}+\varkappa^2\mathcal{F}_m^2(q)\right]\right\}. \tag{26}$$

Under the restrictions which have been discussed above (see § 2A) the form-factors are the Fourier transforms of the spatial distributions of an electric charge $\rho(r)$ and magnetic moment $\vec{m}(r)$.

The analysis of Hofstadter's experiments is based upon Rozenbluth's formula |24|. This formula is only the first Bohr's approximation in the scattering problem (see Fig. 5a). The second approximation (see 5a) for a nucleon with a point magnetic moment, i.e. by $\mathcal{F}_m(q) = 1$ is divergent. The magnitudes of the form-factors $\mathcal{F}_e(q)$ and $\mathcal{F}_m(q)$ may be determined from an experiment by an approximate formula (26). A priori $\mathcal{F}_e(q)$ and $\mathcal{F}_m(q)$ cannot be said to differ noticeably from 1. Therefore, strictly speaking, to justify the interpretation of experimental results obtained by Hofstadter it is necessary to show that the second Bohr approximation is much less than the first one for the values $\mathcal{F}_e$ and $\mathcal{F}_m$ obtained by (26). The calculations made recently by Drell and Fubini[36] have actually shown that the contribution of the second Bohr approximation to the scattering cross section does not exceed 1% up to $E \leqslant 1$ BeV. Thereby the application of Rozenbluth's formula (24) to the analysis of Hofstadter's experiments is quite justified.

Consider now the main experimental results for a proton and neutron.

A. Proton

A detailed list of experimental results for angular distributions of elastically scattered electrons is given in[21-24]. As for experimental curves of angular distributions they are drawn considerably lower than it is predicted by the formula for scattering on a point electrical and magnetic charges[24].

The experimental and theoretical curves may be compared if one assumes that there occurs scattering on an extended electrical charge and a magnetic moment. The best agreement with experiment may be obtained if one puts for a proton

$$\mathcal{F}_e(q) = \mathcal{F}_m(q) \tag{27}$$

and the corresponding spatial distribution $\rho(r)$ /of (5)/ is chosen as follows:

$$\rho(r) = \frac{1}{8\pi a^3} e^{-\frac{r}{a}}; \quad a = 2,3 \cdot 10^{-14} cm., \tag{28}$$

i.e.

$$a \sim \frac{\hbar}{Mc}$$

The agreement of this formula with theory will be considered below.

For small values of q the form-factors may be expanded in series:

$$\mathcal{F}_e(q) = 1 - \frac{1}{6} \langle r_e^2 \rangle_p q^2 + \ldots \tag{29}$$

$$\mathcal{F}_m(q) = 1 - \frac{1}{6} \langle r_m^2 \rangle_p q^2, \tag{30}$$

where

$$\langle r_e^2 \rangle_p = \frac{1}{e}\int r^2 \rho(r)\, d^3x \tag{31}$$

is a root-mean-square "electric" proton radius; e = 1 is a proton charge;

$$\langle r_m^2 \rangle_p = \frac{1}{\varkappa}\int r^2 m(r)\, d^3x \tag{32}$$

- a root- mean- square "magnetic" proton radius;

$\varkappa$ = 1.85 is an anomalous magnetic moment of a proton.

This definition $\langle r_e^2 \rangle$ and $\langle r_m^2 \rangle$ is taken for a neutron as well. (An anomalous magnetic moment for a neutron is $\varkappa_n \simeq -\varkappa$).

In accordance with (28)

$$\sqrt{\langle r_e^2 \rangle_p} = (0{,}80 \pm 0{,}04) \cdot 10^{-13}\ cm \tag{33}$$

Approximately the same is the value of $\sqrt{\langle r^2 m \rangle_p}$.

It should be noted that the equality of an electric and magnetic form-factors is established only with an accuracy of about 20% [20], [25].

B. Neutron

To investigate the electromagnetic structure of a neutron the Hofstadter's group performed some experiments on quick electron scattering on deuterons. The results obtained from these experiments as well as from those on scattering of slow thermal neutrons on atoms (see further) were found to be quite unexpected, even paradoxal.

It followed from an experiment that the angular distribution of elastically scattered electrons on a deuteron calculated theoretically may be best brought into an agreement with an experiment if we assume the spatial distribution of the proton and neutron magnetic moments to coincide

$$\begin{gathered} \mathcal{F}_{mp}(q) = \mathcal{F}_{mn}(q) \\ \langle r_m^2 \rangle_p = \langle r_m^2 \rangle_n , \end{gathered} \tag{34}$$

whereas an electric form-factor of a neutron does not differ from zero. The latter circumstance means that an electric radius of a neutron in contrast to that of a proton is very small

$$\langle r_e^2 \rangle_n \simeq 0. \qquad (35)$$

The evidence on the electric structure of a neutron may be also obtained from experiments on inelastic electron scattering on deuterons.

Since a deuteron is a veakly bound system then for great momentum transfers a proton and neutron may be considered independent with a great accuracy. In this case the differential cross section of electron scattering on a deuteron $\left(\frac{d\sigma}{d\Omega}\right)_d$ is equal approximately to a sum of differential cross sections of electron scattering on a proton and neutron.

$$\left(\frac{d\sigma(\theta)}{d\Omega}\right)_d = (1+\Delta)\left[\left(\frac{d\sigma}{d\Omega}\right)_n + \left(\frac{d\sigma}{d\Omega}\right)_p\right], \qquad (36)$$

where Δ is a correction of the order of some percents taking into account the kinematic effects of nucleon motion in a deuteron, the nucleon interaction in the final state etc.|23|,|26|

Making use of the experimental values $\left(\frac{d\sigma}{d\Omega}\right)_d$ and $\left(\frac{d\sigma}{d\Omega}\right)_p$ and putting the magnetic form-factors for a proton and neutron to be equal, Hofstadter obtained a good agreement of the cross section $\left(\frac{d\sigma}{d\Omega}\right)_{n'}$ caloulated theoretically with an experiment, the electric formfactor of a neutron $\mathcal{F}_{en}(q)$ is being chosen equal to zero.

Thus, two independent experiments-elastic and inelastic (ed)-scattering lead to the same conclusion about the distribution of charges and currents in a neutron.

The conclusion about the smallness of a neutron electric radius is also confirmed by scattering experiments of very slow "thermal" neutron on atoms 8/.

These experiments require a great accuracy since a slow neutron is scattered both on a nucleus atom and on its electrons. The most accurate results in these experiments were obtained by Havens, Rabi, Rainwater|27|.

The scattering amplitude in this oase may be written in the form $a_t = a_n + Z a_e$, where a_n is an amplitude of neutron scattering on a nucleus, a_n is a scattering amplitude on an electron on Z, is a number of electrons in an atom. It was found out that $a_n \sim 10^{-12}$ cm; $a_e \sim 1.5 \cdot 10^{-16}$ cm. It is seen from these figures that the cross section on an atom with great Z is different from that on a nucleus only by some percent. At low energies the electron interaction with a neutron is expressed by the value for the effective potential V_0, which is connected with a scattering amplitude by the relation

$$V_0 \left(\frac{e^2}{m_e c^2}\right)^3 = \frac{3\hbar^2}{2M} a_e. \qquad (37)$$

8/ The atoms of noble gases with zero magnetic moment of an electron shell are used to eliminate the additional magnetic interaction with a neutron.

The calculation of V_o in the framework of the relativistic theory [25] yields:

$$V_o = 3e\left(\frac{m_e c^2}{e^2}\right)^3 \left[\langle r_e^2 \rangle_n + \frac{\hbar}{2MC}\varkappa_n\right]. \tag{38}$$

As is seen V_o consists of two parts: the first is due to the distribution of an electric charge in a neutron (the part is proportional to $\langle r_e^2 \rangle_n$), the second is determined by the magnitude of an anomalous magnetic moment $\varkappa_n$.

The experimental value $V_o = -(3860 \pm 370)$ev, whereas the contribution from a neutron magnetic moment $\varkappa_n = -1.91$ is equal to -4080 ev. Thus, (30 ± 200) ev is for an electric interaction. The corresponding value for the "electric" radius of a neutron allowed by an experimental error cannot exceed 10% of the "electric" radius of a proton

$$\sqrt{\langle r_e^2 \rangle_n} \leqslant 0{,}1 \cdot \sqrt{\langle r_e^2 \rangle_p}. \tag{39}$$

Thus, the analysis of experimental data concerning a neutron seems not to agree with a pion model according to which due to considerations of the charge independence of pion interaction with nucleons the charge of the pion cloud in a neutron must be equal and opposite in sign to the charge of this cloud in a proton

$$\rho_{\pi e}(r) = -\rho_{\pi n}(r) \tag{40}$$

and, therefore, the root-mean-square radii of these distributions must be also equal.

Just here one can see the paradoxicality of the experimental results on electron scattering on neutrons.

As it is shown further this paradoxicality proves to be a seeming one. It is based on an unjustified application of the Ukawa fundamental theory to the central regions of a nucleon.

§ 4. Critical Remarks and Analysis of Hofstadter's Experiments

A descripancy between the theory and experimental data on charge distribution in a neutron makes us analyze all experimental conditions and theoretical investigations on electron scattering on nucleons in more detail.

A. Limits of Applicability of Electrodynamics

The physicists at Stanford like to emplhasize that a theoretical analysis of electron scattering on nucleons is based upon the asumption about the possibility of applying electrodynamics up to very small distances. What if there is a "break-down" of electrodynamics?

The research recently carried out by V. Panofsky and Richter [25] as well as Drell's theoretical investigations [28] are devoted to the study of this problem.

In case of elastic scattering of electrons on protons it may be supposed that all the deviation from the point nucleon or its considerable part are due not to the nucleon structure but to the "break-down" of electrodynamics in the region of the scale Λ_γ. The only conclusion which so far can be made from this supposition is as follows: if there takes place such a "break-down" of electrodynamics" then $\Lambda_\gamma \lesssim 0.30.10^{-13}$cm.

The study of pair photoproduction process $\gamma + p \rightarrow p + e^+ + e^-$ may present in this respect interacting possibilities. (see Fig. 5b and 5b').

In this case the deviations from the electron interaction with a photon are being studied. The corresponding experiments are in their initial stage.

The accuracy of the performed experiments up to now yields still higher limit for possible deviations from the well-known laws of electrodynamics:

$$\Lambda_e \leqslant 0.6.10^{-13}\text{cm}.$$

Theoretically one may expect the "break-down" of electrodynamics involving μ -mesons and neutrinos(for instance, $e + e \rightarrow \mu + \mu$; $\gamma + e \rightarrow \mu + \nu + \nu'$) in spatial regions $\lambda < 10^{-16}$cm. [6].

The experimental investigation of such small spatial regions is not possible yet. (see Table II).

Thus, no experimental data are available so far which would allow to think that deviations from the laws of quantum electrodynamics are essential in the phenomena we are interested in.

B. Role of Inelastic Processes

Besides the elastic scattering of electrons on nucleons on nucleons there take place inelastic processes, e.g., for a proton the reactions hold:

(41) $$e + p \rightarrow e + p + \gamma \qquad \text{(bremsstrahlung)} \qquad (41)$$

(42) $$e + p \rightarrow e + p + e^+ + e^- \qquad \text{(bremsstrahlung)} \qquad (42)$$

$$e+p \longrightarrow \begin{cases} e+n+\pi^{+} \\ e+p+\pi^{0} \end{cases} \qquad \text{(meson production)} \quad (43)$$

These inelastic processes could, in principle, have contributed to elastic scattering due to a possible appearance of diffractional scattering.

However, in all these processes the number of the involved phase-shifts is great, whereas these phase-shifts themselves are small. In similar cases the inelastic one must exceed many times the elastic one so that the diffractional scattering would be noticeable. Indeed, the cross section for elastic scattering is equal (in usual notations).

$$\sigma_{e} = \sum_{\ell} \frac{\pi(2\ell+1)}{K^{2}} \left| 1-\beta_{\ell} e^{2i\delta_{\ell}} \right|^{2} = \sum_{\ell} \frac{\pi(2\ell+1)}{K^{2}} \left[4\beta_{\ell} \sin^{2}\delta_{\ell} + (1-\beta_{\ell})^{2} \right] \qquad (44)$$

The cross sections for inelastic processes

$$\sigma_{in} = \sum_{\ell} \frac{\pi(2\ell+1)}{K^{2}} \left(1-\beta_{\ell}^{2}\right). \qquad (45)$$

If the phase-shifts of inelastic processes are small, i.e.,

$$\beta_{\ell} = 1-\varepsilon_{\ell}; \quad \varepsilon_{\ell} \ll 1, \qquad (46)$$

then

$$\sigma_{in} = \frac{\pi}{K^{2}} \sum_{\ell} (2\ell+1) 2\varepsilon_{\ell} + o(\varepsilon_{\ell}^{2})$$

$$\sigma_{d} = \frac{\pi}{K^{2}} \sum_{\ell} (2\ell+1) \left\{ 4\sigma_{\ell}^{2} + \varepsilon_{\ell}^{2} \right\}. \qquad (47)$$

In order the inelastic process to give a noticeable contribution to the elastic scattering it is necessary that $\varepsilon_{\ell} \sim \delta_{\ell}$. Therefore,

$$\sigma_{in} \sim a\bar{\varepsilon}; \qquad \sigma_{d} = a\bar{\varepsilon}^{2},$$

i.e.

$$\frac{\sigma_{in}}{\sigma_{el}} \sim \frac{1}{\bar{\varepsilon}} \qquad \text{or} \qquad \sigma_{in} \gg \sigma_{el} \qquad (48)$$

This is the condition we have looked for.

The estimates made on the basis of this inequality have shown that diffractional scattering due to pion production (reaction (43)), is not essential. Apparently, this holds for the processes (41) and (42), (bremsstrahlung is great mainly in the small angle region.

It is difficult to expect that it will yield essential diffractional effects at the scattering angles $>30^{\circ}$ when the Hofstadter's group made the measurements see (29)). The cross section for pair production is approximately two orders $(1\sim e^2/\hbar c)$ less than that for bremstrahlung.

C. Analysis of Electron Scattering on Protons and Neutrons

It is usually emphasized that the meson theory does not agree with an experiment when the distributions of electric charge and magnetic moment are concerned. However, this assertion is very categorical.

Indeed, distributions (6) and (7) may be compared with an experiment. But these distributions are obtained for the meson field formed by an infinitely heavy nucleon. They are correct only in the periphery ($r > \frac{\hbar}{m_\pi c}$) regions of a nucleon.

There are no grounds to hope that these distributions are correct over all the region of the values r. On the contrary, considerable deviations from them ought to be expected in the regions $r < \frac{\hbar}{m_\pi c}$ (i.e. by the momenta $q > m_\pi c$), where besides the recoil the properties of internal regions of a nucleon must essentially display themselves.

It seems more consistent to write down the distribution of an electric charge of a nucleon $\rho_N(r)$ and the magnetic moment $m_N(r)$ as a sum

$$\rho_N(r) = \tau_3 \rho_\pi(r) + \rho_{KN}(r) \tag{49}$$

$$\bar{m}_N(r) = \tau_3 \bar{m}_\pi(r) + \bar{m}_{KN}(r), \tag{50}$$

where by $\rho_{KN}(r)$ and $\bar{m}_{KN}(r)$ are denoted the densities of an electric charge and magnetic moment concentrated in the central part of a nucleon and due to nucleon and antinucleon pairs, strange particles as well as to two-three and other higher pion states.

At present we know very little about these states and shall so far refer them to the nucleon core.

It is natural to expect that $\rho_{KN}(r)$ and $\bar{m}_{KN}(r)$ will decrease with the increase of r more rapidly than $\rho_\pi(r)$ and $\bar{m}_\pi(r)$.

Consider the electric radius of a nucleon in more detail. According to the definitions (31) and (49)

$$\langle r_e^2 \rangle_N = \langle r_e^2 \rangle_{KN} + \tau_3 \langle r_e^2 \rangle_\pi, \tag{51}$$

where $\langle r_e^2\rangle_{KN}$ is the r.m.s. radius of a nucleon core. See § 6 C on an isotopic symmetry of these expressions.

Putting $\rho_{KN}(r) = Q_{KN}\rho_c(r)$ (52), where Q_{KN} is the total charge of a core and denoting

$$\langle r_e^2\rangle = \frac{1}{e}\int r^2 \rho_c(r)d^3x, \tag{53}$$

we may rewrite (51) as follows:

$$\langle r_e^2\rangle_N = Q_{KN}\langle r_e^2\rangle_c + \tau_3\langle r_e^2\rangle_\pi . \tag{54}$$

It is well-known experimentally that the electric radius of a neutron is very small. $\langle r_e^2\rangle_n \simeq 0$ it follows from here, that

$$\langle r_e^2\rangle_c = \frac{1}{Q_{Kn}}\langle r_{ie}^2\rangle_\pi \tag{55}$$

$$\langle r_e^2\rangle_p = \langle r_e^2\rangle_c . \tag{56}$$

Taking into account the numerical values of (23) we obtain that

$$Q_{Kn} = +0{,}76\ ;\ \langle r_e^2\rangle_p = (0{,}7\cdot 10^{-13}\,cm)^2. \tag{57}$$

Thus, putting an electric radius of a neutron equal to zero we obtain the value for an electric radius of a proton very close to the experimental one (cf.(33)).

The form of the charge distribution in a core remains still sufficiently arbitrary (since only integral magnitudes are known). We choose fc(r) as follows

$$\rho_c(r) = \frac{e}{8\pi a^3}\, e^{-\frac{r}{a}}. \tag{58}$$

In this case as it is easy to make sure

$$\langle r_e^2\rangle_c = 12a^2. \tag{59}$$

Now, in order to obtain $\langle r_e^2 \rangle_c$ $(0.7.10^{-13}\text{cm})^2$ it is necessary to choose

$$a = \frac{1}{?}\frac{\hbar}{m_\pi c} \simeq \frac{\hbar}{MC} = 2\cdot 10^{-14}\ \text{cm}. \tag{60}$$

Thus, (58) may serve as an example of a core, which is characterized by small length a. At the same time it has a great r.m.s. radius.

In Fig. 6a and 6b are plotted the curves for the charge density distribution in a proton and neutron and in their cores $d_p(r) = d_\pi(r) + d_{\kappa p}(r)$

$$d_n(r) = -d_\pi(r) + d_{\kappa n}(r) \text{ and } d_{kp}(r) = Q_{\kappa p} d_e(r); \quad d_{\kappa n}(r) = Q_{\kappa n} d_c(r).$$

The curve for a proton $d_p(r)$ coincides practically with that obtained after the treatment of the experimental data in Hofstadter's papers |21|-|23|.

As for the charge density in a neutron it is seen that it oscillates near zero. This accounts for a small electric radius of a neutron. In Fig. 6a and 6b the region of the one-pion "atmosphere" of a nucleon is separated by a vertical line from the region where the core charges are essentially mixed. As is seen in the one pion region $r > 1.4\cdot 10^{-13}$cm.

The region, where the asymptotic expansions (9), (10) involving one or two terms are correct, contains an extremely small number of mesons and may be referred, so to say, to the nucleon "stratosphere".

One may analogously consider the magnetic structure of nucleons. Choosing the distribution of a magnetic in a nucleon core in the form of an exponent

$$m_c(r) = \frac{\varkappa}{8\pi a^3}\, e^{-\frac{r}{a}} \tag{61}$$

and putting for a the value (60) we obtain from the condition $\varkappa_p = -\varkappa_n = 1{,}85\frac{e\hbar}{2M_c}$ for the r.m.s. magnetic radii of a proton and neutron

$$\langle r_m^2 \rangle_p = \langle r_m^2 \rangle_n = (0{,}?\cdot 10^{-13}\ \text{cm})^2. \tag{62}$$

This is in good agreement with the experimental values obtained by Hofstadter's group.

* * *

Thus, assuming that there exists a core in the nucleon one may bring into agreement all the experimental data about quick electron scattering on protons and deuterons and about slow neutron scattering on atoms with the main concepts of the modern meson theory.

At this the distribution of a charge and magnetic moment of a core is determined by a small length $a \simeq \frac{\hbar}{MC} \ll \frac{\hbar}{m_\pi c}$.

§ 5. Some Effects of Nucleon Structure

Here we shall be concerned with two more problems connected with the electromagnetic structure of a nucleon. These are, on the one hand, the effect of the meson cloud polarizabitlity in a nucleon and, on the other, the problem of electromagnetic mass of nucleon.

A. Electrical Polarizability of a Meson Cloud in a Nucleon

To clear up the properties of an electromagnetic structure of a nucleon it is very important to consider besides quick electron scattering other effects in which the electromagnetic structure of a nucleon may display itself. One of such effects is the scattering of slow nucleons in an inhomogeneous electric field which extends the cloud of different charges in a nucleon and turns the nucleon into an electric dipole with an induced moment $\bar{p} = \alpha E$. The electrical polarizability of charges in a nucleon may display itself in the Compton-effect and in the photoproduction of pions on nucleons |30|, as well as in scattering of slow neutrons on nuclei |31-33|.

In the first approximation of Chew theory (see Fig. 7.)

$$\alpha = \frac{2}{3\pi} e^2 f^2 \frac{1}{\mu^3} \int \frac{\kappa^2}{\omega(\kappa)} \left\{ v^2(\kappa)\left[27\kappa^4 - 34\kappa^2\omega^2(\kappa)\right] - \left(\frac{dv(\kappa^2)}{d\kappa}\right) 4\kappa^2\omega^4(\kappa) \right\} d\kappa \qquad (63)$$

The energy is $E = \frac{1}{2}\alpha E^2$. The coefficient $\frac{1}{2}$ takes into account the energy losses for the dipole extention. The same notations as in formulae (6),(7) are used here.

It should be expected that in the calculation of the nucleon magnetic moments and the interaction potential of a neutron with an electron the higher terms of the expansion by the coupling constant will not change the result essentially.

It follows from formula (63) that

$\alpha = 1.6 \cdot 10^{-42}$ cm^3, if $v(\kappa) = \frac{1}{\{1+(\frac{\kappa}{5,6})^2\}}$;

$\alpha = 1.8 \cdot 10^{-42}$ cm, if $v(\kappa) = \exp\{-\frac{\kappa^2}{2}/(5,6)^2\}$.

These values have been obtained for $f^2/\hbar c = 0.08$. The results of the calculations are slightly sensitive to the choice of the form $v(k)$ |34,35| within the wide limit.

The calculated value α is close to that obtained by A.M. Baldin from the analysis of the experiments on photoproduction and Compton effect on a nucleon |35|

$$4 \cdot 10^{-43}\,\text{cm}^3 \lesssim \alpha \leq 1.4 \cdot 10^{-42}\,\text{cm}^3.$$

However, it is considerably less than the value $\alpha \simeq 8 \cdot 10^{-41}$ cm^3 obtained by Yu.A. Aleksandrov from the experiments on slow neutron scattering on heavy nuclei |32|. A more rigorous theoretical analysis of Yu.A. Aleksandrov's experiments is necessary. It is possible that there occur effects of neutron interaction with an electron shell of heavy nuclei[9].

The experimental result which shows that the neutron polarizability $d \neq 0$ may be considered as a direct proof that there exist charged "clouds" in a neutron.

B. Electromagnetic Mass of Nucleons and Proton Stability

A small difference between proton and neutron masses (Δ (MC^2) = 2.5 electron masses) as well as the charge independence of "nuclear interaction" of nucleons allow to suppose that this difference is of a purely electromagnetic origin.

As a first approximation one may assume that the charge of a core Q_{kN} and its magnetic moment $\vec{\varkappa}_{KN}$ are concentrated in the nucleon centre.

Then the electromagnetic energy of a nucleon may be written as follows:

$$E_N = Q_{KN}\varphi_\pi(0) - \vec{\varkappa}_{KN}\vec{H}_\pi(0), \tag{64}$$

where

$$\varphi_\pi(r) = \int \rho_\pi(r)\frac{d^3x}{r}; \quad \vec{H}_\pi(r) = \int \frac{d^3x}{r^3}\vec{\jmath}_\pi(r) \tag{65}$$

the electrostatical potential and the magnetic field formed in the nucleon centre by a cloud of charged pions, $\vec{J}_\pi$ is an electric current of a meson field.

Since

$$Q_{Kn} + Q_{Kp} = e \tag{66}$$

and

$$\varkappa_{Kp} - \varkappa_{Kn} = e\hbar/2MC \tag{67}$$

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9) One of the authors V.S. Barashenkov is grateful to Yu.A. Aleksandrov, A.N.Baldin, I.P. Stakhanov and L.N. Usachev for useful discussions.

(Here the Dirac magnetic moment of a "bare" nucleon is involved in a magnetic moment of a core),

then

$$\Delta(Mc^2) = E_n - E_p = \frac{e\hbar}{2Mc} H_n(0) - e\varphi_\pi(0). \tag{68}$$

As is seen from this formula the stability of a proton is entirely due to the interaction energy of the magnetic moment of a proton core and the currents of its pion cloud. The electrostatic energy of a proton is, on the contrary, greater than that of a neutron. When substituting the numerical values for the parameters formula (68) gives the value for the mass difference Δ (Mc^2) which agrees with an experiment by the order of magnitude; however, the sign $\Delta(Mc^2)$ is essentially dependent upon the form of the cut-off form-factor V(k).

§ 6. Theoretical Attempts to Interpret the Electromagnetic Structure of Nucleon Central Regions

The electromagnetic properties of the periphery regions in a nucleon and their theoretical interpretation were considered in detail in the previous Sections. Let us now be concerned with the consideration of some theoretical attempts to interpret the properties of the central regions of a nucleon.

A. The Influence of Strange Particles

Sandri [37] and independently one of the authors noticed that the dissociation of a nucleon into a hyperon and a K-meson leads to the production of positively charged K-meson cloud both in case of a neutron and in case of a proton.

This effect must lead to the increase of $\langle r_e^2 \rangle_{Kp}$ and to the decrease of $\langle r_e^2 \rangle_{Kn}$ in accordance with the scheme of nucleon structure given in Fig.6.

Indeed, according to the laws of the conservation of strangeness and baryon number the following processes are possible:

$$p \rightleftarrows \Lambda^0 + K^+; \quad \Xi^0 + K^0 + K^+; \quad \Xi^- + K^+ + K^+$$

and

$$n \rightleftarrows \Lambda^0 + K^0; \quad \Xi^0 + K^0 + K^0; \quad \Xi^- + K^0 + K^+;$$

$$\Sigma^0 + K^0 \quad ; \quad \Sigma^- + K^+.$$

For an estimate of this effect we used the first approximation of the Chew theory and obtained that the mean-root-square "electric" radius for a neutron is equal to[10)]

$$\langle r_e^2 \rangle_n = \langle r_e^2 \rangle_{n(\pi+K)} = \frac{e}{2\pi}\left(\frac{Mc^2}{e^2}\right)\left(\frac{f^2}{m_\pi}\right)\left\{5{,}98 + \left(\frac{g}{f}\right)^2 0{,}07\right\}, \tag{69}$$

i.e. $\langle r_e^2 \rangle_n \simeq 0$, if the coupling constant of K^+-mesons with a nucleon is $g^2/\hbar c \simeq 8$ However, in this case it is difficult to explain the experiments on scattering and photoproduction of K-mesons. For more probable values of $g^2/\hbar c$ the contribution of K-mesons to $\langle r_e^2 \rangle_n$ both in case of scalar and pseudoscalar variants of the theory does not exceed 10% by the order of a magnitude.

Thus, for present there are no grounds to think that strange particles are capable to change the charge distribution essentially in the central regions of a nucleon.

B. Contribution of Nucleon-Antinucleon Pairs

I.E. Tamm suggested to explain zero electric radius for a neutron $\langle r_e^2 \rangle_n$ taking into account the dissociation of virtical π -mesons into nucleon pairs |38|.

If a π-meson dissociates into a pair a nucleon plus an antinucleon: $\pi \rightleftarrows N + \tilde{N}$, then the arising antinucleon may annihilate with the original "bare" nucleon which is in the centre of a physical nucleon, whereas the remaining nucleon turns out to be displaced if compared with the former centre by the magnitude of the order $\frac{\hbar}{m_\pi c}$. As a result the positive charge of a neutron core will spread over the region of the radius $\sim \frac{\hbar}{m_\pi c}$. Figuratively speaking, the core and π -meson exchange their places (see Fig. 8). The mean density of an electric charge in a neutron during its interaction with an electron and, therefore, in this case $\langle r_e^2 \rangle_n$ will be close to zero. The spreading of a neutral proton core over the region of the radius $\sim \frac{\hbar}{m_\pi c}$ cannot essentially change the value of $\langle r_e^2 \rangle_p$

This hypothesis, however, was not confirmed by the calculations. The estimates show that the role of the pairs ($N\tilde{N}$) is not great. The change of $\langle r_e^2 \rangle_n$ which is due to these pairs is negligibly small |40|.

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10) Since at present there are no experimental grounds for introducing strong interaction of pions and K-mesons|59| that would essentially increase spatial dimensions of the K-meson source then the form-factor for the density of the K-meson sources in a nucleon was chosen as for π -mesons.

There is one more objection against Tamm's hypothesis it is difficult to be brought into agreement with the relation of uncertanties $\Delta P \Delta x \sim \hbar$. Indeed, the ambiguity in the coordinate of nucleon inertia centre will be of the order $\frac{\hbar}{m_\pi c}$ (it is just such an ambiguity Δx of the point in which the nucleon-antinucleon pair is produced), whereas the ambiguity in the momentum $\Delta p \sim \frac{\Delta E}{c} \sim Mc$. All what as been said above concerns hyperon pairs ($Y, \tilde{Y}$) as well.

C. Application of Dispersion Relations

A number of attempts have been recently made to calculate the distribution of the charge and magnetic moment in a nucleon, as well as to take into account the contribution of central regions using not only Chew-Low method but also that of dispersion relations[40-43]. Here we restrict ourselves to the consideration of only some qualitative results. Let us consider the isotopic structure of the form-factors.

In view of charge symmetry considerations it is reasonably to suppose that the electric and magnetic form-factors of a nucleon may be presented as a combination of isotopic vectors and scalars:

$$\left.\begin{aligned} e\mathcal{F}_{ep}(q) &= G_e^s(q) + G_e^v(q); & \varkappa_p \mathcal{F}_{mp}(q) &= G_m^s(q) + G_m^v(q) \\ e\mathcal{F}_{en}(q) &= G_e^s(q) - G_e^v(q); & \varkappa_n \mathcal{F}_{mn}(q) &= G_m^s(q) - G_m^v(q) \end{aligned}\right\} \tag{70}$$

Here G_e^v and C_m^v are the vector parts of the form-factors determining the behaviour of the electric and magnetic density in the periphery regions of a nucleon. In these regions the condition[40] is fulfilled.

Evidently,

$$G_e^v(0) = \frac{e}{2}; \qquad G_m^v(0) = \varkappa_p. \tag{71}$$

G_e^s and G_m^s are the scalar parts of the form-factors affecting the properties of the central regions in a nucleon. These parts of the form-factors satisfy the normalization conditions.

$$G_e^s(0) = G_e^v(0) = \frac{e}{2}; \quad G_m^s(0) = 0. \tag{72}$$

Taking into account that $\varkappa_p \simeq -\varkappa_n$ from the experimental relation $\mathcal{F}_{mp} \simeq \mathcal{F}_{mn}$ we obtain

$$G_m^v \gg G_m^s \tag{73}$$

i.e. the "magnetic form-factor" of a nucleon is, in general, a vector in the isotopic space.

This fact leads to an important conclusion that the main part of the magnetic structure of a nucleon is generally determined by a virtual process in which two mesons of charges opposite in sign are involved (see Fig. 9a)[11].

This can be understood if we take into account that a meson pair ($\pi^+\pi^-$) forms a vector in the isotopic space. Besides, the intermediate state in this case is a vector state, the lowest one by energy |40-42|. The next intermidiate vector state will be a state involving four virtual mesons etc. It is possible to show that the meson energy in the main process with two intermidiate mesons $E_\pi < 1$ BeV .

As for the "electric form-factors" $\mathcal{F}_{ep}$ and $\mathcal{F}_{en}$, they are determined to a considerable extent by a scalar form-factor G_e^s . The process involving three virtual mesons is a determinant in this case. One of such virtual processes is plotted in Fig. 9b.

To evaluate the contribution of the process involving three mesons is extremely difficult. All the attempts to do it where so far unsuccessful.

Since, apparently, $\langle r_e^2\rangle_{pair} \lesssim 0.1\langle r_e^2\rangle_p$ and $\langle r_e^2\rangle_{K\cdot mes.} \lesssim 0.1\cdot\langle r_e^2\rangle_p$ then it ought pair to be expected that

$$\langle r_e^2\rangle_{3\pi} \lesssim 0.3\langle r_e^2\rangle_p . \qquad (74)$$

All these results agree with the state above-considered picture of the nucleon structure according to the statical theory.

It should be emphasized that the analysis of π -meson cloud of a nucleon and the calculation of the form-factors G^s and G^v are made by means of the so-called "truncated" dispersion relations which are not grounded now since the cut-off of the used expansions on this or that number of π -mesons cannot be justified.

§ 7. Nuclear Structure of Nucleons

In the preceding Sections we have considered the problem of nucleon structure from

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11) In the notations accepted in the theory of dispersion relations the state involving a virtual π -meson interacting before its absorption (cf. Fig. 9a) is called two-pion state.

the standpoint of electromagnetic interactions.

However, interactions of nucleons between each other, interactions with pions, K-mesons, antinucleons etc may yield also the information on the nucleon structure. So, some years ago the analysis of the experiments on nucleon-nucleon scattering in the energy range of some hundred MeV led to the conclusion that nucleon in these collisions may be considered as a "black sphere" of the radius $\Lambda \simeq 5.10^{-14}$cm. On the other hand, the meson field in a nucleon is extended over a considerably great distance. This circumstance has led one of the authors to the idea of a nucleon as consisting of a dense core - the region of extremely strong nuclear interactions and a π-meson cloud |5|.

A. Core of a Nucleon

At present our knowledge of a nucleon core is very poor and reduces in general to the following facts:

1. In the experiments on 9 BeV nucleon interaction with photoemulsion nucleons performed recently with the Joint Institute synchrophasotron two essentially different types of collisions with great of small multiplicity of secondary particle production for one act of nucleon-nucleon collision are observed |44|.

In most of the collisions a great multiplicity of secondary particles ($n \geqslant 3 \div 4$) is observed. The energy losses of a primary nucleon in one act of a nucleon-nucleon collision is 40% ± 10% of their initial energy. In the center of mass system the angular distribution of particles produced in these collisions by $n > 3 \div 4$ turns out to be isotropic (see Fig. 10). The collisions with small intensity of secondary particles are observed in a considerably less number of events. The energy losses in these collisions are likely to be smaller than in the collisions of the first type. The angular distribution of the produced particles is essentially anisotropic (in the centre of mass system) (see Fig. 10).

The collisions of the first type (with great multiplicity) may be considered as nucleon core collisions; but the greater part of the collisions of the second type (with small multiplicity) - as collisions of the core of one nucleon with the periphery cloud of another nucleon.

It should be emphasized, however, that at present such a classification may be done only with great accuracy since a part of the stars of the second type is likely to be also produced in the collisions of nucleon cores, but the number of the produced particles is not great. At present it is not also clear what part of the angular distribution anisotropy observed in the experiment is due to the law of momentum conservation in the collision of a core with a core.

The calculation of energy losses and angular asymmetry (c.m.s.) made under the assumption that about 80% of all the collisions are at $E \approx 10$ BeV those of the cores is in good agreement with an experiment.[45]

In the laboratory system the particles produced as a result of periphery collisions must emerge at small angles. In Table IV are given the experimental and theoretical ratios of particle number emitting at the angles $\theta \leq 10^{\circ}$[47]. It is seen from this Table that in small angle region the contribution of particles produced due to the mechanism not described by the statistical theory of multiple production is great. (The distortions introduced by the momentum conservation law were not again taken into account).

2. The cross sections for stange particle production calculated by the statistical theory of multiple production may be put into agreement with an experiment only assuming that K-mesons are concentrated in the regions of a nucleon $\sim \frac{\hbar}{m_K c}$ (m_K is the mass of a K-meson)[39].

3. The optical analysis of the elastic scattering of nucleons and pions on nucleons at an energy $E = 1 \div 10$ BeV leads to the conclusion that the absorption coefficient in the region $\sim \frac{\hbar}{Mc}$ essentially increases (see further).

4. The comparison of numerous calculations with an experiment made in the framework of Chew-Low theory has shown that the agreement with an experiment may be obtained only if we assume that the dimensions of the pion field source in a nucleon $a \sim \frac{\hbar}{Mc}$. (From a mathematical point of view the form-factor with the cut-off frequency $\omega \sim \frac{\hbar}{a}$). corresponds to this fact).

5. The analysis of Hofstadter's experiments and (ne)-interaction made above leads to the conclusion about specific properties of the nucleon central region.

6. Various meson theories (see, e.g.[42]) lead to the qualitative conclusion that there exists a core in a nucleon.

It is this poor evidence on a nucleon core which is available at present.

B. Nucleon Optical Model

The investigation of nucleon structure by means of nucleon, pion and strange particle scattering is quite different from that of photons and electrons due to a great magnitude of interaction.

The Bohr approximation in these cases is unapplicable altogether; but it is this approximation only that makes it possible by a simple Fourier transform to find energy interaction operator and structural form-factors by the scattered wave amplitude. In the general case we are not aware of the ways which would enable to solve the so-called "inverse problem" - i.e. the problem of finding the particle interaction law by the scattered wave phase shifts.

We cannot make use here of interesting and important investigations performed in this connection by I.I. Gelfand[48], Marchenko[49], Krein[50] et al[51] as in the high energy region the notion of interaction potential becomes meaningless (see further).

However, the fact that the wave length at high energies of interacting particles becomes considerably less than the dimensions of nucleons (see Table II) makes it possible to think that the quasiclassical approximation is applicable. In this case the motion of particles inside a nucleon may be considered as such taking place along the trajectory (along the ray) in a medium with the given refraction and absorptioncoefficients $n=n(r)$ and $k=k(r)$ respectively.

If the variations of these magnitudes are small over the length of the particle wave λ then the nucleon may be considered as an optically inhomogeneous medium with sufficiently smooth variations of its optical constants and it is possible to apply the laws of geometrical optics to the particle scattering.

The problem may be simplified in the respect that at high energies of pions and nucleons the cross section for the diffractional scattering σ_d is much more than the cross section for the nondiffractional scattering σ_{nd} so that the real part of the elastic scattering amplitude

$$R_e F(\theta) \ll I_m F(\theta). \tag{75}$$

This means that the nucleon refraction factor is purely imaginary and reduces to the absorption coefficient. The phase-shifts of the scattered waves η become in this case also purely imaginary[13]

$$n_\ell \ll k_\ell . \tag{76}$$

This enables to determine the amplitude of the scattered wave

$$f(k;\theta) = \frac{\lambda}{2i} \sum_{\ell=0}^{\infty} (2\ell+1)(1-e^{2i\eta_\ell}) P_\ell(\cos\theta), \tag{77}$$

(where θ is the scattering angle) directly through the cross section of elastic diffractional scattering

$$f(k;\theta)\simeq\left[\frac{d\sigma_d(k;\theta)}{d\Omega}\right]^{\frac{1}{2}} \tag{78}$$

Expanding this expression by Legendre polynomials $P_\ell(\cos\theta)$ it is possible to find the phase shifts η_ℓ :

$$\eta_\ell=\frac{1}{2}\ln\left\{1-\frac{1}{\lambda}\int_0^\infty\sqrt{d\sigma_d(k\theta)/d\Omega}\,P_\ell(\cos\theta)\sin\theta\,d\theta\right\} \tag{79}$$

As the calculations have shown the conditions (75), (76) for pion-nucleon interactions are fulfilled satisfactorily at the energies E > 1 BeV |10-14|, |52|.

In the quasi-classical approximation the phase-shift η_ℓ , corresponding to the parameter of the collision $\rho=\lambda\sqrt{\ell(\ell+\frac{1}{2})}\approx\lambda\ell$ is entirely determined by pion absorption on the way inside a nucleon (see Fig. 11). Therefore, if we denote by k(r) the pion absorption coefficient at the distance of r $=\sqrt{\rho^2+s^2}$ from the nucleon centre then it is possible to write

$$\eta_\ell=\eta(\rho)=2i\int_0^\infty k(\sqrt{\rho^2+s^2})ds \tag{80}$$

or

$$\eta(\rho)=2i\int_\rho^\infty k(r)\frac{dr}{\sqrt{r^2-\rho^2}}. \tag{81}$$

If one knows $\eta(\rho)$ from the experiment it is possible to determine (by numerical method) k(r) for each value of the parameter of collision. Practically it turned out that for an energy of some BeV it is sufficient to know 6-7 phase-shifts (ℓ = 0,1,2,...).

Thus calculated pion absorption coefficient for pion energy E = 1.3 BeV and E = 5 BeV is given in Fig. 12 (see (10) and (11)). As is seen there is a tendency for the appearance of "blackness" near the centre of a nucleon. According to these data the r.m.s. "pion" radius of a nucleon is found to be

$$\langle r_\pi^2\rangle=(0{,}82\cdot10^{-13}cm)^2, \tag{82}$$

that is in agreement with the value for the "electromagnetic" radius of a proton (33) .

It should be emphasized, however, that it is seen from the obtained data that in the central regions of a nucleon the conditions of the applicability of an optical model are fulfilled badly.

At great values for $r(r \gtrsim \frac{\hbar}{m_\pi c})$ the accuracy of the experimental data leads, as is seen from the curves of Fig. 12, to a great spread of possible values k(r).

Thus, the most reliable are the values in the intermidiate interval of the distances r.

For nucleon-nucleon collisions the conditions (44), (45) become applicable at considerably greater energies than for (π N) - collisions. This is seen from Table V, where are given the values for the coefficient ratios n(r) and k(r) for the energies E = 1 $\div$ 10BeV calculated by the experimental data. The conditions (44), (45) are fulfilled with sufficient accuracy only at the energies E = 5BeV. In the papers by Huang-Niang-Ning and one of the authors [13,14] a way for calculating the absorption coefficient k(r) and refraction coefficient n (r) through the relative coefficients K = K(r)/K*(r) and n=n(r)k*(r) where the function K*(r) is determined by the known experimental data at E > 5 BeV. The calculations in this case are also considerably simplified.

As the calculations have shown (see [12,14]) the spatial behaviour of k(r) and n(r) for nucleon-nucleon collisions is analogous to that given in Fig. 12 for pion-nucleon interaction. The r.m.s. radius $\langle r^2 \rangle$ coincides with (82).

The knowledge of the pion absorption coefficient in the nucleon cloud and its pion composition makes it possible to determine in principle the cross section for pion interaction $\sigma_{\pi\pi}$. Let us consider the related problems.

C. Pion-Pion Interaction

Pions may be considered as particles consisting of virtual nucleon-antinucleon pairs (see [53]). The detailed bibliography is also given there).

The "structure" of a pion may be expressed by the formulas:

$$\pi^+ = p\cdot\tilde{n}; \quad \pi^- = \tilde{p}\cdot n; \quad \pi^\circ = 2^{-\frac{1}{2}}(p\cdot n + \tilde{p}\cdot\tilde{n}). \tag{83}$$

a hypothetical pion [54]

$$\pi_0^\circ = 2^{-\frac{1}{2}}(p\cdot n - \tilde{p}\cdot\tilde{n}) \tag{83^{I}}$$

Here p,n are a proton and a neutron, whereas $\tilde{p}$ and $\tilde{n}$ are an antiproton and antineutron.

This conception of a pion as a complex particle enables to consider the dimention of the pion a as a distance between the particles and antiparticles into which a pion virtually dissociates. Due to strong interaction of nucleons it ought to be considered that the cross section for pion-pion interaction will be

$$\sigma_{\pi\pi} \simeq \pi a^2. \tag{84}$$

On the other hand, the distance a may be estimated from the mass difference $\pi^{\pm}$ and π° -mesons which is 9 electron masses and is a difference of electromagnetic energies of the charged and neutral pions. This difference is equal to:

$$\Delta E = \alpha \frac{e^2}{a} + \beta \left(\frac{e\hbar}{2MC} \right)^2 \frac{m^2}{a^2} \tag{85}$$

Here the first term is an electrostatical energy; the second term is a magnetic energy, the numbers α, β are of the order of a unit; m is the total magnetic moment of a nucleon ($| m | \simeq 2$). Assuming $\Delta E = 9\, m_0 c^2$ one may find by the order of the magnitude and $5.10^{-27} cm^2$. The pion absorption coefficient in a nucleon k(r) may be approximately written as follows

$$K(r) = \sigma_{\pi\pi} \cdot n(r), \tag{86}$$

where n (r) is the pion density in a nucleon "atmosphere". In the region of the one-pion state

$$n(r) \simeq \frac{1}{e} \frac{3}{2} \rho(r), \tag{87}$$

where $\rho(r) = \rho_{\pi}(r) = \rho_{\varkappa}(r)$ is taken from Table III as well as from (58). (The factor 3/2 also takes into account the presence of neutral mesons).

The curve k(r) calculated by formula (86) for 5.10^{-27} cm^2 is plotted in Fig. 12. As is seen from this Figure the agreement is only rough.

However, one could hardly expect a better agreement since in the region $r = 0.2 \div 1$ the composition of the pion atmosphere is not reduced to the one-pion state whereas the exact composition $\rho_{\varkappa}$ (r) is not known.

In the region r = 1 the values for k(r) obtained by the optical model are very doubtful.

Therefore, to determine pion-pion interaction more accurate measurements of the diffractional pion scattering on nucleons in the small angle region seem to be very important and promising[13].

§ 8. Theory of Nucleon Optical Model

The optical theory of particle scattering on a nucleon considered above is simple and visual. At the same time it is rather naive.

At any rate the nucleon model as a refracting and absorbing medium must be theoretically and experimentally grounded.

Two problems must be considered in this connection:

1) What are the conditions for the optical model of a nucleon to be correct?

2) If the optical model is correct what are the conditions for the formula (81) be applicable for the calculation of the phase shift ρ_ℓ ?

Let us consider these problems.

A. Equation for Pion Scattering on a Nucleon

At present we have no general theory of π -meson and nucleon interaction. Therefore, there is no well-grounded theoretical base to analyze the importance of any approximate methods, in particular, the nucleon optical model. Nevertheless, rather general conclusions concerning the conditions of the applicability of the optical model or, in other words, of the complex interaction potential are possible. Therefore, let us write down the Schrödinger equation for the pion and nucleon system in the momentum space as a chain of relativistically invariant equations |55|.

$$(E-H_0)\Phi_s = \sum_{S=S'} W_{ss'}\,\Phi_{s'}\,, \tag{88}$$

where $\Phi_s \equiv \Phi(\bar{k}_1;\bar{k}_2;\dots\bar{k}_s)$ is the wave function describing the real and virtual state of particles in pion interaction with a nucleon; $\bar{k}_s$ are the momenta of particles; E is the total energy of the system, H_0 is the Hamiltonian of the noninteracting particles, $W_{ss'}$ is the interaction operator, describing the production (or annihilation) of particles in the transition $S \rightarrow S'$.

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13) One may arrive at important conclusions about ($\pi\pi$) interaction by studying angular distributions of particles produced in (π N)-interactions. In particular, the deviation from the isotropy (in the c.m.s.) for the stars with a small multiplicity of the produced particles is an indication to ($\pi\pi$) - interaction.

When considering the optical model of all the functions ϕ_s we are interested only in the function $\phi_s = \psi(\bar{k}; \bar{p})$ which describes the elastic scattering of a pion with the moment $\bar{k}_1 \equiv \bar{k}$ on a nucleon with the momentum $\bar{k}_2 \equiv \bar{p}$.

To distinguish this component $\phi_2 = \psi$ we rewrite the equation system (88) as follows:

$$\left.\begin{aligned} (E - H_0)\psi &= \sum_{s'\neq 2} W_{2s'}\, \phi_s' \\ (E - H_0)\phi_{s'} &= \sum_{s'\neq 2} W_{s's''}\phi_{s''} + W_{s's}\psi \end{aligned}\right\} \tag{89}$$

or, dividing the second equation into ($E - H_0$), with the help of the "elementary scattering matrix" τ , equal to

$$\tau = \delta^{+}(E - H_0)W, \tag{90}$$

(see, in more detail[55]) we may rewrite the equations (58) in the operator form

$$\left.\begin{aligned} (E - H)\psi &= W\phi \\ \phi &= \tau\psi + \tau\phi \end{aligned}\right\}. \tag{91}$$

From here one may obtain the equation for the function ψ by integrating

$$(E - H_0)\psi = W\frac{\tau}{1-\tau}\psi + (W\tau^N\phi)_{N\to\infty} \tag{92}$$

If we assume that

a) all the divergent terms from this equation are excluded.

b) the residual term by $N \to \infty$ is tending to zero (cf.[57]), then this equation may be written as follows:

$$\left\{E - E(\bar{k}, \bar{p})\right\}\psi(\bar{k}, \bar{p})/\bar{k}'\bar{p}') = \int \omega(\bar{k}, \bar{p}/\bar{k}', \bar{p}')\psi(\bar{k}'\bar{p}')d(kp), \tag{92}$$

where the operator

$$\omega = W\frac{\tau}{1-\tau}$$

(93)

For the real processes $E = H_0$ the matrix τ will also contribute to the imaginary part of the operator in view of the function being imaginary; thus, the operator G is complex and this complexity is due to the real, inelastic processes.

Let us pass now to the coordinate system where $\bar{K} + \bar{p} = 0$. Then Eq.(92) may be written as follows

$$\left\{E - E_N(q) - E_\pi(q)\right\}\psi(q) = \int G(q/q')\psi(q')d^3q \tag{94}$$

or in the coordinate representation

$$\left\{E - H_N(x) - H_\pi(x)\right\}\psi(x) = \int \mathcal{F}\left(\frac{x}{x'}\right)\psi(x')d^3x'. \tag{95}$$

Here $H_n(x)$ and $H(x)$ are the Hamiltonians of the free motion of a pion and a nucleon. As it can be seen the interaction in which we take an interest is, generally speaking, determined by the nonlocal operator $\mathcal{F}\left(\frac{x}{x'}\right)$. However, if the initial equation system (51) is local, the nonlocality in (95) is only a result of the writing down in the coordinate representation of the fact that the interaction is dependent upon the velocities:

$$\int \mathcal{F}\left(\frac{x}{x'}\right)\psi(x')d^3x' = V(x, E, \nabla^2)\psi(x), \tag{96}$$

where V is a complex interaction operator dependent upon the velocities of particles:

$$V(x; E; \nabla^2) = \sum_K \frac{1}{K!}\int \mathcal{F}(\bar{x}/\bar{x}', E)\left[(x' - x)\frac{\partial}{\partial\bar{n}(x)}\right]^n d^3x' = \sum_K \frac{1}{K!}V_K(x E)\frac{\partial^K}{\partial\bar{n}(x)^K}$$

or

$$V(x; E; \nabla^2) = \sum \frac{1}{K!}V_K(\bar{x}; E)\sqrt{\nabla^{2K}} \tag{97}$$

<u>B. Conditions for the Existence of Complex Potential</u>

It can be seen that there exists no complex potential which could be interpreted as an optical refraction and absorption factor of the pion waves inside a nucleon since we obtain a nonlocal operator instead of a potential.

However, in case of the applicability of geometry optics

$$\varphi(x) = \varphi_0(x)\, e^{i\bar{k}\bar{x}} \qquad \text{and} \qquad \frac{\partial \varphi_0}{\partial x} \simeq 0.$$

In this case the operator $V(x,E,\nabla^2)$ is reduced to the complex potential

$$V(x,E,\nabla^2) \longrightarrow V(x,k), \qquad (99)$$

dependent upon a particle momentum. In other words, the conditions for the geometric optics being applicable coincide with the condition when the nonlocal operator F or V may be replaced for complex potential (99).

In those regions of a nucleon where the absorption becomes so strong that the amplitude of the pion wave changes noticeably over the wavelength $\lambda\!\!\!{}^{-}$ more general relations should be used for the calculation of the phase η_ℓ. Let us remind also that the applicability of the optical model is improved when passing to smaller wave lengths.

§ 9. Conclusion

The study of the structure of elementary particles and of nucleons seems to be at a very early stage of its development. A great progress was made in the investigation of the electromagnetic structure of a proton and a neutron. The obtained data on the charge and magnetic moment distribution in nucleons are fitted with the main theoretical concepts of the pion theory. However, the electromagnetic properties of the nucleon central regions still remain very obscure.

The study of nucleon structure by means of nuclear interactions is in embryo.

The simplest approach to the meson structure of a nucleon is the analysis of pion scattering on the basis of the nucleon optical model. In its present form this model has a physical meaning for sufficiently energetic pions (nucleons) and not for too deep layers of nucleon structure.

Table I

Energy interval (BeV)	Mean energy (BeV)	$\sigma_{in}(n+Fe)$ $/\times 10^{24}\ cm^2/$	$\sigma_{in}(p+Fe)$ $/\times 10^{24}\ cm^2/$
28-58	37	0,60 ±0,04	0,59 ± 0,05
58-121	77	0,62 ± 0,05	0,61 ± 0,06
121 - 387	178	0,67 ± 0,13	0,79 ± 0,25
28 - 387	50	0,61 ± 0,03	0,61 ± 0,04

Table II

Wavelengths of Varions Rays

Source of rays	Nature of rays	Wavelengths of a nucleon and scattered particle (ray)(c.m.s.) in the units $f = 10^{-13}$ cm.
6 BeV Berkeley Bevatron	protons (6.15 BeV)	0.12
	pions (5 BeV)	0.13
Joint Institute Synchrophasotron	protons (10 BeV)	0.09
	pions (8 BeV)	0.10
50 BeV Synchrophasotron	protons (50 BeV)	0.04
	pions (50 BeV)	0.04
Stanford electron linear accelerator	electrons(1BeV)	0.35
10 BeV Proton Beams (perspective)	protons + protons (10 BeV)	0.018
10 BeV proton and electron beams (perspective)	proton+electrons (10 BeV)	0.019
500 MeV electron beams (perspective)	electrons+electrons (500 MeV)	0.4

Table III

Distribution of an Electric Charge of a Pion Cloud in a Nucleon

$r\cdot\left(\frac{m_\pi c}{\hbar}\right)$	0	0,05	0,075	0,0875	0,1	0,125
$d_\pi(r)\,\hbar/m_\pi c\cdot 1/e$	0	0,17	0,58	0,88	1,13	1,64
$r\cdot\left(\frac{m_\pi c}{\hbar}\right)$	0,15	0,2	0,3	0,4	0,5	0,6
$d_\pi(r)\,\hbar/m_\pi c\cdot 1/e$	1,95	2,05	1,46	0,9	0,55	0,35
$r\cdot\left(\frac{m_\pi c}{\hbar}\right)$	1,0	1,5	2,0			
$d_\pi(r)\,\hbar/m_\pi c\ 1/e$	0,146	0,082	0,0145			

Table IV

Angular Distribution of Rays in |NN|-collisions /lab.system/

Angle interval	The number of particles (experimental) /Calculated/
0 - 3	2,2
0 - 5	1,9
0 - 10	1,4

Table V

Ratio of Refraction and Absorption Optical Coefficients

for (p-p) - Collisions

E Бэв	1,5	2,24	2,75	4,40	6,15	9,0
$n(\tau)/K(\tau)$	1,2	1,2	0,95	0,51	0,1	0,05

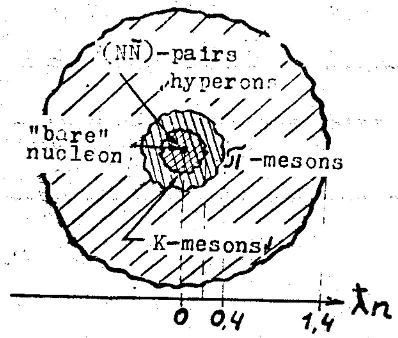

Fig.1. π -meson, K-meson etc clouds in a nucleon.

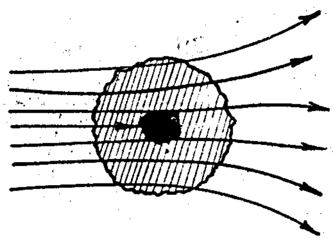

Fig.2. "Grey" and "black" regions inside a nucleon. Elastic scattering becomes by $\lambda \to 0$ mainly diffractional. The "black" region is a possible region of the "nonlocality".

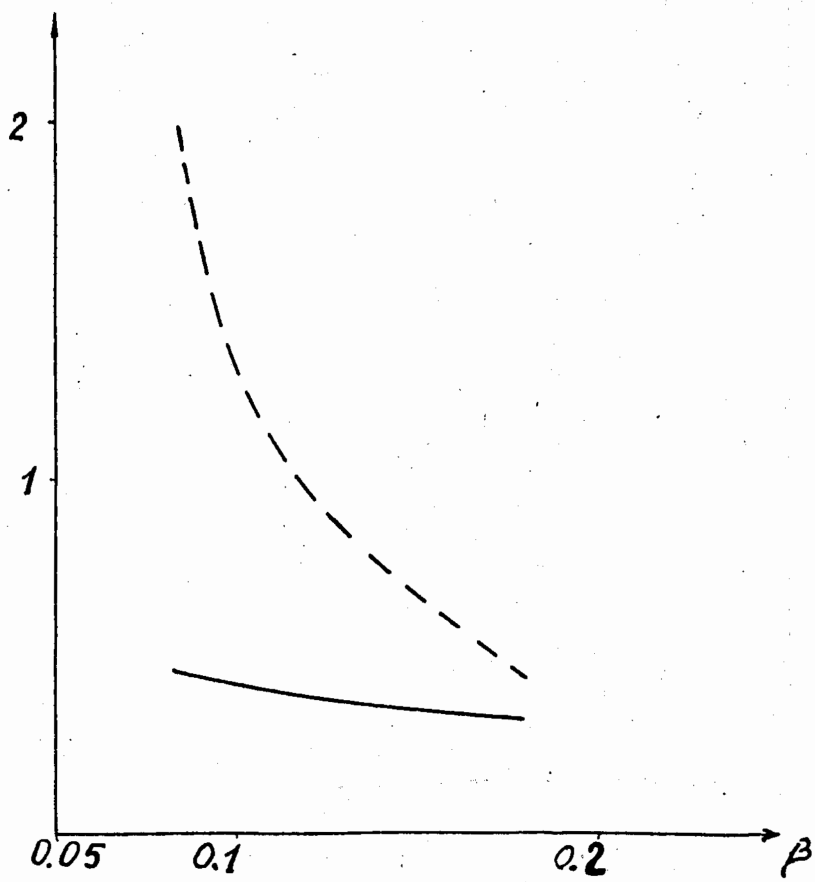

Fig.3. Dashed line shows a dependence of an electric charge of a nucleon pion cloud Q_π upon a magnitude of the cut-off parameter β . Solid line indicates the same for the electric radius $\langle r_e^2\rangle_\pi$.

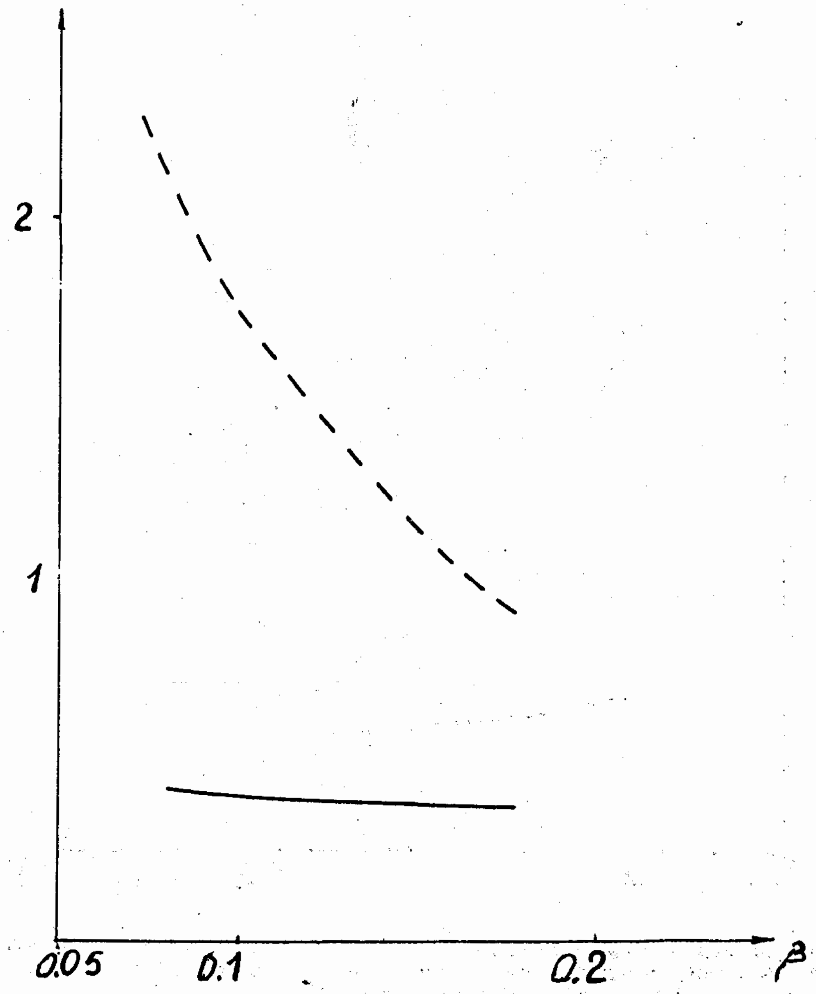

__Fig.4.__ Dashed line shows the dependence of the magnetic moment of a pion cloud χ_{π} upon the magnitude of the cut-off parameter β. Solid line shows the same for the magnetic radius $\langle r_m^2 \rangle_{\pi}$.

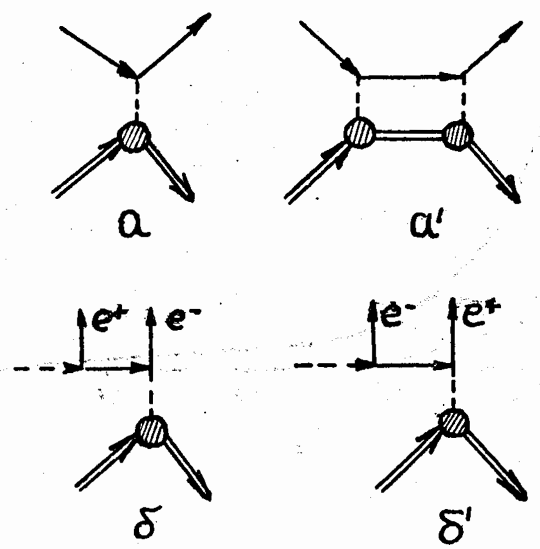

Fig.5. Feynmann diagram for electron scattering |a|, |a'| and for the production of electron-positron pairs | δ |, | δ' | on a nucleon. A solid double line indicates a proton, a solid single line - an electron or a positron, the wave line a σ -meson, the dashed line a γ - quantum, the shaded regions show a form-factor.

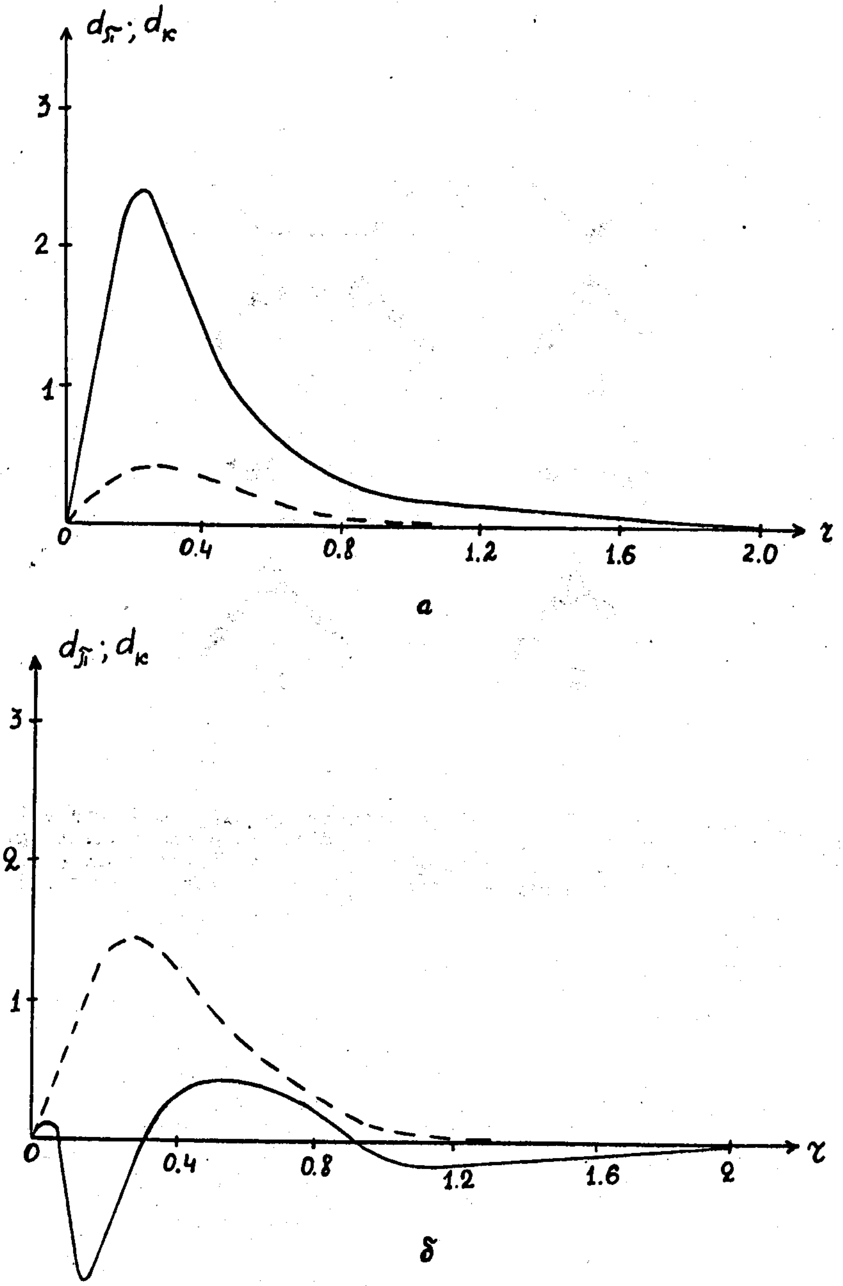

Fig.6. Electromagnetic structure of a nucleon. a is the structure of a proton, б is the structure of a neutron. The solid curve indicates the distribution of an electric charge in a proton and neutron; the dashed line - the corresponding distributions of an electric charge in the cores of a proton and a neutron.

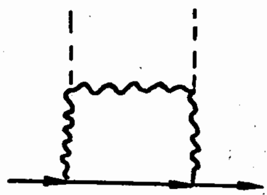

Fig.7. Feynmann diagram for an electric polarization of a meson cloud in a nucleon The solid curve is a nucleon, the wave-like - a pion, the dashed line- γ- quantum.

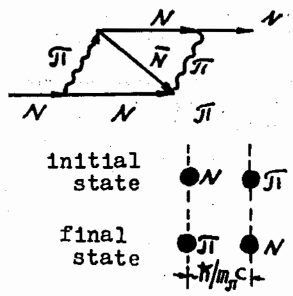

Fig.8. Graphic picture of Tamm hypothesis.

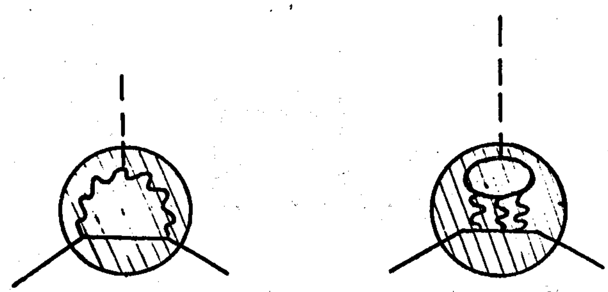

Fig. 9. a is the Feynmann diagram for the vector part of the electromagnetic form-factor of a nucleon.
$б$ is one of the Feynmann diagrams for the scalar part of this form-factor. The solid line clows a nucleon, the wave-like - a pion, the dashed line - γ - quantum.

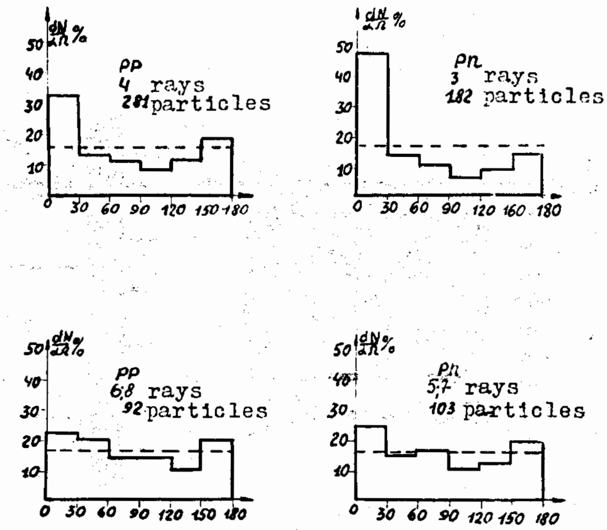

Fig.10. Histograms of the distribution of a number of charged particles produced in proton-proton and proton-neutron collisions at 9BeV. The values of the particle emission angle are put along the absicca axis in the c.m.s. |44|.

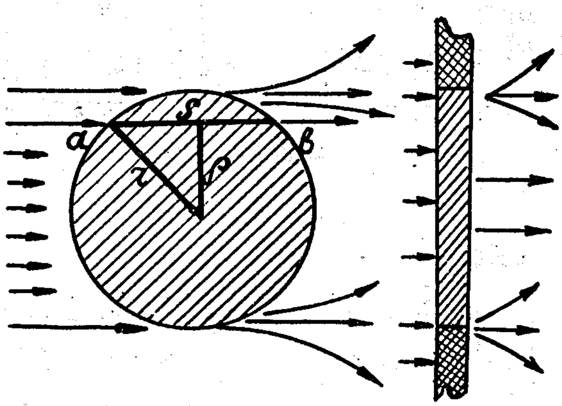

Fig.11. The range of a pion inside a nucleons, is a collision parameter; r is the distance to the centre of a nucleon. The mesons are absorbed along the length ab = 2S. The transparency is $D(\rho) = e^{-2\eta(\rho)}$ $D(\rho) = 1$.
The diffraction due to this limited transparency of a nucleon is exactly the same as the diffraction from the hole in the screen with the transparency $\tilde{D}(\rho) = 1 - e^{-2\eta(\rho)}$.

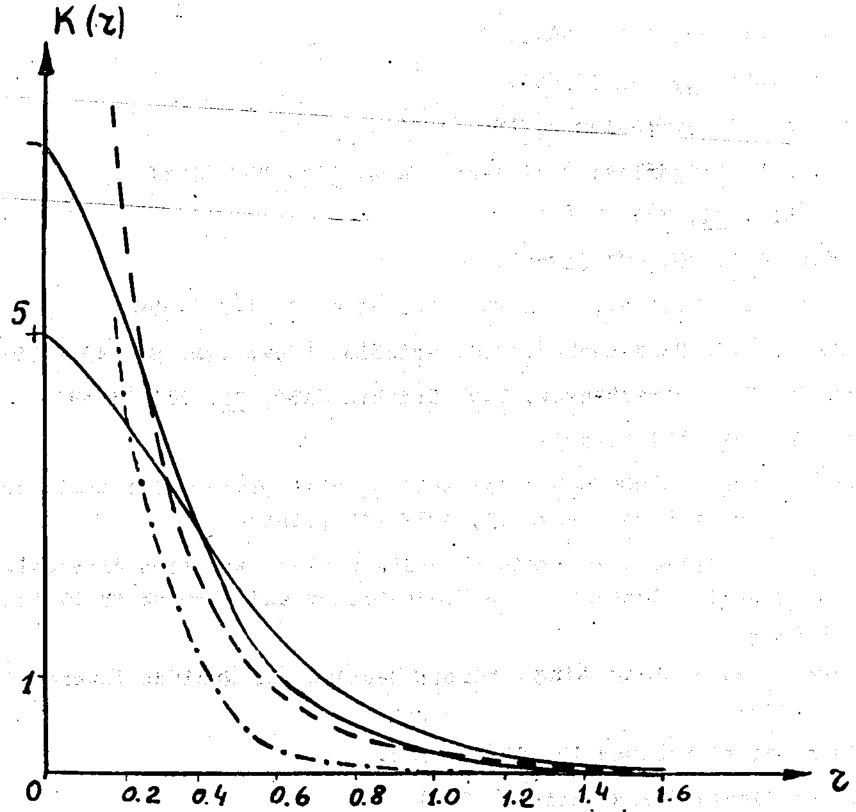

Fig.12. The coefficient of pion absorption in a nucleon K-k(r) as a function of the distance r from the nucleon centre. The values in the units 10^{-13}cm, the values k(r)- in the units 10^{+13}cm. Solid curves show the values k(r) constructed by the extreme experimental values for E = 13B. The dashed line shows the mean values k(r) for E = 5 BeV. The point-dash line indicates the dependence $K(r) = \sigma_{\pi\pi} \rho(r)$ constructed with account of the theoretical values for the density $\rho(r)$.

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Note Added in Proof

1. The estimate of $\sigma_{\pi\pi}$ in Seo. 7 B is a lower limit of the possible values of $\sigma_{\pi\pi}$, when the nucleon and antinucleon into which a pion dissociated is considered to be point ("bare") particles. If we take into account their effective dimensions then the effective dimension of a pion and the oross section $\sigma_{\pi\pi}$, corespondingly, may increase some times. The curve $K(r)=\sigma_{\pi\pi}\frac{4}{3}\rho_{\pi}(r)$ may be brought into agreement with the curve $K(r)$, obtained from the optical analysis.

2. As recent experiments of E. Tzyganov with the Joint Institute synchrophasotron have shown (in print) the angular distributions of the eleastic (p-p)- scattering as a small angles) $\theta < 10^{\circ}$ in c.m.s. cannot be accounted for in the framework of a purely absorbing nucleon ("black" or "grey"). It is possible that even at high energies in the small angle region $\theta < 10^{\circ}$ the potential soattering still plays a noticeable role, i.e.,

$$\frac{d\sigma_{nd}(\theta)}{d\Omega} \neq 0.$$