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ON THE STATE OF A FERMI-SYSTEM OF PAIRS OF PARTICLES WITH PARALLEL SPINS I

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A b s t r a c t

The solution of a compensation equation of dangerous graphs, for a system of interacting Fermi-particles, as an odd function of wave vectors is investigated. It is shown that this solution leads to the anomalous state the energy of which is smaller than the energy of the normal state. The energy difference is equal to the binding energy of the pairs of fermions with parallel spin-moments. This anomalous state can exist in the presence of the repulsive Coulomb interactions. The elementary excitations are separated from the ground state by a gap the width of which depends on the direction of the wave vector. For the direction perpendicular to the spin direction, energy gap is zero and therefore the state cannot be superconducting. The binding energy of the pairs and the width of the energy gap are of the same order as in the theory based on the creation of pairs with antiparallel spin-moments, where in the expansion of the interaction term in spherical harmonics, the first coefficient is more than three times greater than the zero coefficient. In the absence of the magnetic field the total spin of a system is zero. In the presence of the magnetic field the total spin differs from zero. The paramagnetic susceptibility is calculated.

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I. Introduction

Consider a dynamical system of Fermi-particles with a Hamiltonian

$$H=\sum_{f,f'}T(f,f')a_f^+a_{f'}+\frac{1}{2}\sum_{\substack{f_1,f_2\\f_1',f_2'}}U(f_1,f_2;f_2',f_1')a_{f_1}^+a_{f_2}^+a_{f_2'}a_{f_1'}\quad,\ T(f,f')=I(f,f')-\lambda\delta(f-f') \tag{1}$$

where I - is the Hamiltonian of particle, U-the interaction energy λ - the chemical potential, a_f, a_f^+ the Fermi-amplitudes and f is a set of indices characterising one-particle states. In our case $f=(\vec{p},\sigma_z)$, where $\vec{p}$ is a wave vector and σ_z a spin index.

Similiary as in the paper[1] and also[2,3] we transform the Hamiltonian (1) by means of the transformation

$$a_f=\sum_\nu(u_{f\nu}\alpha_\nu+v_{f\nu}\alpha_\nu^+) \tag{2}$$

To secure the canonical character of the transformation (2) the functions $\{u,v\}$ must be connected by the orthonormality relations

$$\sum_\nu\{u_{f\nu}u^*_{f'\nu}+v_{f\nu}v^*_{f'\nu}\}=\delta(f-f')\ ,$$

$$\sum_\nu\{u_{f\nu}v_{f'\nu}+u_{f'\nu}v_{f\nu}\}=0 \tag{3}$$

We find the functions $\{u,v\}$ from the additional equations obtained from the compensation principle of dangerous graphs[2]

$$\langle\alpha_{\nu_1}\alpha_{\nu_2}H\cdot\rangle_0=0 \tag{4}$$

(the expectation value corresponds to the vacuum state in the α - representation) which are equivalent to the equations for the functions $F(f,f')$, $\phi(f,f')$

$$\sum_f\{\mathfrak{z}(f_1,f)\phi(f,f_2)+\mathfrak{z}(f_2,f)\phi(f_1,f)+S(f_1,f_2)-\sum_f\{F(f,f_1)S(f,f_2)+F(f,f_2)S(f_1,f)\}=0 \tag{5}$$

where

$$F(f,f')=\sum_\nu v^*_{f\nu}v_{f'\nu}\quad,\quad\phi(f,f')=-\phi(f',f)=\sum_\nu u_{f\nu}v_{f'\nu} \tag{6}$$

and

$$S(f_1,f_2)=\sum_{f_1',f_2'}U(f_1,f_2;f_2',f_1')\phi(f_1',f_2')\ ,$$

$$\mathfrak{z}(f_1,f)=T(f_1,f)+\sum_{f',f''}\{U(f_1,f'';f',f)-U(f_1,f'';f,f')\}F(f'',f')\ . \tag{7}$$

In order to represent the functions F, ϕ in the form (6) they must satisfy the additional subsidiary conditions[2/]

$$F(f_1,f_2) = \sum_f \{F(f_1,f)F(f,f_2) + \phi(f,f_1)\phi(f,f_2)\}, \qquad (8a)$$

$$\sum_f \{F(f_1,f)\phi^*(f,f_2) + F(f_2,f)\phi^*(f,f_1)\} = 0 . \qquad (8b)$$

In the formula (1) of the paper[2/] has been put

$$\begin{aligned} &T(f,f') = (E(p)-\lambda)\,\delta(f-f') = \varepsilon(p)\,\delta(f-f'), \\ &U(f_1,f_2;f_2',f_1') = \frac{1}{V}\, J(\vec p_1,\vec p_2;\vec p_2',\vec p_1')\,\delta(\vec p_1+\vec p_2-\vec p_1'-\vec p_2')\,\delta(\sigma_1-\sigma_1')\,\delta(\sigma_2-\sigma_2'), \\ &J(\vec p_1,\vec p_2;\vec p_2',\vec p_1') = J(\vec p_2,\vec p_1;\vec p_1',\vec p_2') = J(-\vec p_1,-\vec p_2;-\vec p_2',-\vec p_1') . \end{aligned} \qquad (9)$$

The solution of the equation (5) with the subsidiary conditions (8 a,b) has been put in the form

$$F(f,f') = F(\vec p)\,\delta(f-f'), \quad \phi(f,f') = \phi(f)\,\delta(f+f'), \quad \phi(\vec p,+) = -\phi(\vec p),\ \phi(\vec p,-) = \phi(\vec p). \qquad (10)$$

The functions $F(\vec p)$ and $\phi(\vec p)$ are assumed to be real and invariant with respect to the transformation of the momentum reflection and satisfy the following equations

$$2\xi(p)\,\phi(\vec p) + \frac{1-2F(\vec p)}{V}\sum_{\vec p'} J(\vec p,-\vec p;-\vec p',\vec p')\,\phi(\vec p') = 0, \quad F(\vec p) = F^2(\vec p) + \phi^2(\vec p) \qquad (11)$$

where

$$\xi(p) = \varepsilon(p) + \frac{1}{V}\sum_{\vec p'}\left[2J(\vec p,\vec p';\vec p',\vec p) - J(\vec p,\vec p';\vec p,\vec p')\right]F(\vec p') . \qquad (12)$$

The solution of this equations leads to the usual formulae of the theory of superconductivity.

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2. The Solution of the Compensation Relation in the Case $\phi(\vec p,\sigma_z)$ on odd function of $\vec p$.

From the form of the equations (11) we see that exist also the solution of this equations if $\phi(\vec p,\sigma_z)$ is an odd function of $\vec p$. So instead of (10) we put

$$\begin{aligned} &F(f,f') = F(\vec p)\,\delta(f-f'), \quad \phi(f,f') = \phi(f)\,\delta(\vec p+\vec p')\,\delta(\sigma-\sigma'), \\ &F(\vec p) = F(-\vec p), \quad \phi(\vec p,\sigma_z) = -\phi(\vec p,-\sigma_z), \quad \phi(\vec p,\sigma_z) = -\phi(-\vec p,\sigma_z) . \end{aligned} \qquad (13)$$

The functions of the form

$$\phi(\vec p,\sigma_z) = e_z\hat\sigma_z \sum_{s_z} S_{\sigma_z}(s_z)\,\varphi(\vec p) = \sigma_z \cos\theta\,\varphi(\vec p), \quad \varphi(\vec p) = \varphi(-\vec p) \qquad (13')$$

satisfy these conditions, where ($\vec{e}=\vec{p}/p$, $S_{\sigma_z}(s_z)$ is a spin function ($S_{\sigma_z}(s_z)=1$, $S_{\sigma_z}(s_z)=0$ for $\sigma_z \neq s_z$, $\hat{\sigma}_z S=\sigma_z S$, $\sigma_z=\pm 1$). θ angle between wave vector and quantization axis of electron spins. Substitute (13) in (5) and (8a,b) we get for the functions $F(\vec{p})$, $\phi(\vec{p},\sigma_z)$ the equations (11) (further we will denote spin index by σ). We define

$$C(\vec{p},\sigma)=-\tfrac{1}{V}\sum_{\vec{p}'} J(\vec{p},-\vec{p};-\vec{p}',\vec{p}')\,\phi(\vec{p}',\sigma) \,. \tag{14}$$

We see that $C(\vec{p},\sigma)$ is an odd function of $\vec{p}$ and σ

$$C(\vec{p},\sigma)=-C(-\vec{p},\sigma)\ ,\quad C(\vec{p},\sigma)=-C(\vec{p},-\sigma)\ ,\quad C^2(\vec{p},\sigma)=C^2(\vec{p}) \,. \tag{15}$$

From the equations (11) we obtain

$$\phi(\vec{p},\sigma)=\frac{C(\vec{p},\sigma)}{2\sqrt{C^2(\vec{p})+\mathfrak{z}^2(p)}}\ ,\qquad F(\vec{p})=\tfrac{1}{2}\left\{1-\frac{\mathfrak{z}(p)}{\sqrt{C^2(\vec{p})+\mathfrak{z}^2(p)}}\right\} \tag{16}$$

hence

$$C(\vec{p},\sigma)=-\frac{1}{2V}\sum_{\vec{p}'} J(\vec{p},-\vec{p};-\vec{p}',\vec{p}')\frac{C(\vec{p}',\sigma)}{\sqrt{C^2(\vec{p}')+\mathfrak{z}^2(p')}} \,. \tag{17}$$

We take

$$J(\vec{p},-\vec{p};-\vec{p}',\vec{p}')=J(|\vec{p}-\vec{p}'|)=\sum_n J_n(p,p')\,P_n(\cos\gamma) \tag{18}$$

where γ is the angle between the vectors $\vec{p}$ and $\vec{p}'$

$$\cos\gamma=\cos\theta\cos\theta'+\cos\varphi\sin\theta\sin\theta' \tag{19}$$

θ,θ' are the angles between the vectors $\vec{p}$ and $\vec{p}'$ and the polar axis, and φ the angle round the polar axis. According to the addition theorem

$$P_n(\cos\gamma)=P_n(\cos\theta)P_n(\cos\theta')+2\sum_{m=1}^{n}\frac{(n-m)!}{(n+m)!}P_n^m(\cos\theta)\,P_n^m(\cos\theta')\cos m\varphi \,. \tag{20}$$

Substituting (20) into (17) and writting them in the integreal form we obtain

$$C(\vec{p},\sigma)=-\frac{1}{2}\frac{1}{(2\pi)^2}\sum_n P_n(\cos\theta)\int_0^\infty\!\!\int_0^\pi J_n(p,p')P_n(\cos\theta')\frac{C(p',\cos\theta',\sigma)\,p'^2dp'}{\sqrt{C^2(p',\cos\theta')+\mathfrak{z}^2(p')}}\sin\theta' d\theta' \tag{21}$$

The solution of (21) we take in the form

$$C(\vec{p},\sigma)=\sigma\cos\theta\,\psi(p) \tag{22}$$

Moreover we consider the case when the interaction is effective only for the momentum near p_F and take into account only $J_1(p,p')\cong J_1(p_F)$ (the contribution from $J_0(p,p')$ vanishes). For the radial part of $C(\vec{p},\sigma)$ we obtain the equation

$$\psi(p)=-\frac{1}{2}\frac{p_F^2}{(2\pi)^2}J_1\int_{-1}^{1}x^2dx\int_{p_F-\Delta}^{p_F+\Delta}\frac{\psi(p')\,dp'}{\sqrt{\psi^2(p')x^2+\mathfrak{z}^2(p')}} \tag{23}$$

Looking for a solution which changes slowly with p near p_F and taking $\zeta(p) \cong E'(p-p_F)$ we get

$$\psi = 2\omega e^{1/3} e^{3/\tilde{\varsigma}_1} = 2\omega e^{1/3} e^{-3/\varsigma_1} \tag{24}$$

where

$$-\tilde{\varsigma}_1 = -\frac{dn}{dE} J_1 = \varsigma_1 > 0 \quad , \quad \frac{dn}{dE} = \frac{p_F^2}{2\pi^2 E'} \quad , \quad \omega = E'\Delta$$

(the interaction of electrons with sound quanta we have to replace by an effective electron-electron interaction, which is different from zero in a narrow layer near the Fermi surface $E_F \pm \omega$). In order that $\psi \to 0$ for $\tilde{\varsigma}_1 \to 0$ the effective interaction must be attractive $(J_1(p_F) < 0)$. Having obtained the solution of (23) we get the function $C(\vec{p},\sigma)$ and so the functions $F(\vec{p})$ and $\phi(\vec{p},\sigma)$. Moreover, as can be seen, we obtain another solution of (11) if we put $\phi(\vec{p},\sigma) = 0$ and $F(\vec{p}) = 1$ for $p < p_F$ and $F(\vec{p}) = 0$ for $p > p_F$.

We examine what special form the transformation (2) takes if the functions F and ϕ have the form (13). Taking into account (6) we see that

$$u_{f\nu} = u(\vec{p})\,\delta(f-\nu) \;,\; v_{f\nu} = v(-\vec{p},\sigma)\,\delta(\vec{p}+\vec{l})\,\delta(\sigma-s) \;,\; f = (\vec{p},\sigma) \;,\; \nu = (\vec{l},s) \;, \tag{25}$$
$$v(\vec{p},\sigma) = -v(-\vec{p},\sigma) \;,\; v(\vec{p},\sigma) = -v(\vec{p},-\sigma) \;,\; v(\vec{p},\sigma) = \sigma\cos\theta\, w(\vec{p}) \;,\; w(\vec{p}) = w(-\vec{p})$$

and

$$F(\vec{p}) = v^2(\vec{p},\sigma) = \cos^2\theta\, w^2(\vec{p}) \quad , \quad \phi(\vec{p},\sigma) = u(\vec{p})\, v(\vec{p},\sigma) \; . \tag{26}$$

Therefore the transformation (2) has now the form

$$a_{\vec{p}\sigma} = u(\vec{p})\,\alpha_{\vec{p}\sigma} - v(\vec{p},\sigma)\,\alpha^+_{-\vec{p}\sigma} \quad , \quad a_{-\vec{p}\sigma} = u(\vec{p})\,\alpha_{-\vec{p}\sigma} + v(\vec{p},\sigma)\,\alpha^+_{\vec{p}\sigma} \; . \tag{27}$$

If by means of (27) we transform the Hamiltonian

$$H = \sum_{\vec{p},\sigma} \varepsilon(p)\, a^+_{\vec{p}\sigma} a_{\vec{p}\sigma} + \frac{1}{2V} \sum_{\substack{\vec{p}_1+\vec{p}_2 = \vec{p}_1'+\vec{p}_2' \\ \sigma_1,\sigma_2}} J(\vec{p}_1,\vec{p}_2;\vec{p}_2',\vec{p}_1')\, a^+_{\vec{p}_1\sigma_1} a^+_{\vec{p}_2\sigma_2} a_{\vec{p}_2'\sigma_2} a_{\vec{p}_1'\sigma_1} \tag{28}$$

(see (1) and (9)), we can see that we must write the compensation equation for the dangerous diagrams for the pairs of fermions with parallel spins and antiparallel momenta.

For the functions $u(\vec{p})$, $v(\vec{p},\sigma)$ we get one solution for the normal state

$$u(\vec{p}) = \theta_G(\vec{p}) = \begin{cases} 1 \;, & E(\vec{p}) > E_F \\ 0 \;, & E(\vec{p}) < E_F \end{cases} \quad , \quad v(\vec{p},\sigma) = \theta_F(\vec{p},\sigma) = \begin{cases} 0 \;, & E(\vec{p}) > E_F \\ \pm 1 \;, & E(\vec{p}) < E_F \end{cases} \tag{29}$$

(where $\theta_F(\vec{p},\sigma) = -\theta_F(-\vec{p},\sigma), ..$), and taking into account (26) and (11) the second solution

$$u^2(\vec{p}) = \frac{1}{2}\left\{1 + \frac{\zeta(p)}{\sqrt{\psi^2\cos^2\theta + \zeta^2(p)}}\right\} \quad , \quad v^2(\vec{p},\sigma) = \frac{1}{2}\left\{1 - \frac{\zeta(p)}{\sqrt{\psi^2\cos^2\theta + \zeta^2(p)}}\right\} . \tag{30}$$

This solution is of the type obtained by Bogolubov[1] but it depends on the direction of a vector $\vec{p}$ and for $\theta = \pi/2$ it goes over into the solution for the normal state.

3. The Energy Difference for Two States. Energy gap.

Now we find the mean energy in the vacuum state in α - representation and evaluate the difference for solutions (29) and (30). The general formula for mean value is

$$\bar{E} = \langle H \rangle_0 = \frac{1}{2} \sum_{f,f'} \left[T(f,f') + \zeta(f,f') \right] F(f,f') + \frac{1}{2} \sum_{\substack{f_1,f_2 \\ f_1',f_2'}} U(f_1,f_2;f_2',f_1') \phi^{+}(f_1,f_2) \phi(f_1',f_2') \quad . \tag{31}$$

Taking into account (9) and (13) we obtain

$$\langle H \rangle_0 = \sum_{\vec{p}} \left[\varepsilon(p) + \zeta(p) \right] F(\vec{p}) + \frac{1}{V} \sum_{\vec{p},\vec{p}'} \mathcal{J}(\vec{p},-\vec{p};-\vec{p}',\vec{p}') \phi(\vec{p},\sigma) \phi(\vec{p}',\sigma) \cong 2 \sum_{\vec{p}} \zeta(p) F(\vec{p}) - \sum_{\vec{p}} \frac{C^2(\vec{p})}{2\sqrt{C^2(\vec{p}) + \zeta^2(p)}} \ . \tag{32}$$

The energy difference for the two solutions we find using the identity

$$\vartheta^2(\vec{p},\sigma) - \theta_F^2(\vec{p}) = \frac{\theta_G(\vec{p})}{2} \left\{ 1 - \frac{\zeta(p)}{\sqrt{\psi^2 \cos^2\theta + \zeta^2(p)}} \right\} - \frac{\theta_F^2(\vec{p})}{2} \left\{ 1 + \frac{\zeta(p)}{\sqrt{\psi^2 \cos^2\theta + \zeta^2(p)}} \right\} \tag{33}$$

Finally we get

$$\frac{\Delta \bar{E}}{V} = \frac{2}{V} \sum_{\vec{p}} \zeta(p) \left[\vartheta^2(\vec{p},\sigma) - \theta_F^2(\vec{p}) \right] - \frac{1}{2V} \sum_{\vec{p}} \frac{C^2(\vec{p})}{\sqrt{C^2(\vec{p}) + \zeta^2(p)}} = -\frac{2}{3} \frac{du}{dE} e^{2/3} \omega^2 \left(e^{-2/g_1} \right)^3 \ . \tag{34}$$

From this it is clear that the solution (30) leads to the energy state which lies lower than the normal state. This is also the anomalous state connected with the pairs of fermions with parallel spin-moments.

Let us compare the ground state for the case of pairs of fermions with antiparallel spins with the ground state for the case of pairs with parallel spins. We see that the binding energy for the pairs of electrons with parallel spin-moments is proportional to $[\exp(-2/g_1)]^3$ while the binding energy for the pairs of electrons with antiparallel spin-moments is proportional to $\exp(-2/g_0)$. When the case $\mathcal{J}_1(p,p') > 3\mathcal{J}_0(p,p')$ is not realized the ground state of the system for pairs of fermions with antiparallel spin-moments has lower energy than the state for pairs with parallel spin-moments.

We shall now find the formula for the elementary excitation in the superconducting state. The general formula is given by[3]

$$\langle \alpha_\nu | H | \alpha_\nu^+ \rangle_0 = \sum_{f,f'} \zeta(f,f') \left(u_{f\nu}^* u_{f'\nu} - v_{f\nu}^* v_{f'\nu} \right) + \tag{35}$$

$$+\frac{1}{2}\sum_{\substack{f_1,f_2\\f_1',f_2'}} U(f_1,f_2;f_2',f_1')\left[\phi(f_2,f_1)(u^*_{f_1'\nu}v^*_{f_2'\nu}-v^*_{f_1'\nu}u^*_{f_2'\nu})+\phi^*(f_2,f_1)(u_{f_1'\nu}v_{f_2'\nu}-u_{f_2'\nu}v_{f_1'\nu})\right] \quad (35)$$

We remark by the way that (35) is identical with the formula for the energy of elementary excitations given by Landau[4].

$$\Omega(\nu) = \frac{\delta \bar{E}}{\delta n_\nu} \quad (36)$$

Here $\bar{E}$ is the diagonal matrix element of the Hamiltonian (1) transformed by means of the transformation (2)[3] if the expectation value of $\alpha^+_\nu\alpha_\nu$ is not zero but n_ν. In that case the mean energy is given by the formula of the form (31) but the functions F and ϕ are now of the form[2]

$$\phi(f_1,f_2)=\sum_\nu\left\{u_{f_1\nu}v_{f_2\nu}(1-n_\nu)+v_{f_1\nu}u_{f_2\nu}n_\nu\right\}, \quad F(f_1,f_2)=\sum_\nu\left\{v^*_{f_1\nu}v_{f_2\nu}(1-n_\nu)+u^*_{f_1\nu}u_{f_2\nu}n_\nu\right\}. \quad (37)$$

For $T = 0$, that is $n_\nu = 0$, the formulae (37) go over into (6). Taking also into account that in (31) F and ϕ are functions of n_ν we get

$$\frac{\delta\bar{E}}{\delta n_\nu}=\frac{1}{2}\sum_{f,f'}\left\{\frac{\delta\mathfrak{z}(f,f')}{\delta n_\nu}F(f,f')+\left[T(f,f')+\mathfrak{z}(f,f')\right]\frac{\delta F(f,f')}{\delta n_\nu}\right\}+\frac{1}{2}\sum_{\substack{f_1,f_2\\f_1',f_2'}}U(f_1,f_2;f_2',f_1')\left[\frac{\delta\phi^*(f_1,f_2)}{\delta n_\nu}+\frac{\delta\phi(f_1',f_2')}{\delta n_\nu}\right]. \quad (38)$$

Finding $\frac{\delta F}{\delta n_\nu}$, $\frac{\delta\phi}{\delta n_\nu}$ from (37) we see that

$$\left(\frac{\delta\bar{E}}{\delta n_\nu}\right)_{n_\nu=0} = \langle\alpha_\nu|H|\alpha^+_\nu\rangle_0 = \Omega(\nu) \quad (39)$$

Introducting (13) into (35) we obtain

$$\Omega(\vec{p}) = \mathfrak{z}(p)\left[u^2(\vec{p})-v^2(\vec{p},\sigma)\right]-\frac{2}{V}u(\vec{p})v(\vec{p},\sigma)\sum_{\vec{p}'}J(\vec{p},-\vec{p};-\vec{p}',\vec{p}')v(\vec{p}',\sigma)u(\vec{p}'). \quad (40)$$

After using (11) and (26)

$$\Omega(\vec{p}) = \sqrt{\psi^2\cos^2\theta+\mathfrak{z}^2(p)} \quad (41)$$

On the Fermi-surface

$$\Omega(p,\cos\theta) = |\cos\theta|\,2\omega e^{1/3}e^{-3/g_1} = \Delta \quad (42)$$

Thus the fermion excitations in the anomalous state are separated from the energy of the ground state by the gap which depends on the direction of the vector $\vec{p}$. Since the energy gap can be equal to zero there will not exist the current-carrying state of system state with respect to weak perturbations. This means the anomalous state which is considered by us is not superconducting.

4. The Influence of the Electron-Electron Coulomb Repulsion

Let us now examine the combined effect of the electron-phonon interaction and the Coulomb interaction between the electrons[1/, 5/]. Similar as in the case of the electron-phonon interaction (which is essential only near the Fermi-surface, $E_F \pm \omega$) we replace the Coulomb interaction by a model interaction. Because of the screening of the Coulomb interaction we replace it by a constant repulsion of electrons (essential only near the Fermi-surface $E_F \pm \omega_1$, $\omega_1 > \omega$). Thus we put

$$\mathcal{J}_1(\zeta,\zeta') = \begin{cases} \mathcal{J}_{1ph}(\zeta,\zeta') + \mathcal{J}_{1c}(\zeta,\zeta') & , \quad |\zeta|, |\zeta'| < \omega \\ \mathcal{J}_{1c}(\zeta,\zeta') & , \quad |\zeta| \text{ or } |\zeta'| > \omega \end{cases} \qquad (43)$$

We obtain the equations analogous to (23), from which we can determine $\psi(\zeta)$ in two intervals($|\zeta| < \omega$ and $\omega < |\zeta| < \omega_1$)

$$\psi(\zeta) = -\frac{1}{4}\frac{dn}{dE}\int_{-1}^{1} x^2 dx \int_{-\omega_1}^{\omega_1} \mathcal{J}_1(\zeta,\zeta') \frac{\psi(\zeta')\,d\zeta'}{\sqrt{\psi^2(\zeta')x^2 + \zeta'^2}} \qquad (44)$$

We denote

$$\psi(\zeta) = \begin{cases} \psi_0 & , \quad |\zeta| < \omega \\ \psi_1 & , \quad \omega < |\zeta| < \omega_1 \end{cases} \qquad (45)$$

From (43) we get

$$\begin{aligned} -\psi_0 &= \frac{1}{3}\left[\rho_{1c}\psi_1 \ln\frac{\omega_1}{\omega} + \psi_0(\tilde{\rho}_{1ph} + \rho_{1c})\ln\frac{2\omega}{\psi_0} + \frac{1}{3}\psi_0(\tilde{\rho}_{1ph} + \rho_{1c})\right] , \\ -\psi_1 &= \frac{1}{3}\left[\rho_{1c}\psi_1 \ln\frac{\omega_1}{\omega} + \psi_0\rho_{1c}\ln\frac{2\omega}{\psi_0} + \frac{1}{3}\psi_0\rho_{1c}\right] . \end{aligned} \qquad (46)$$

where

$$-\tilde{\rho}_{1ph} = -\mathcal{J}_{1ph}\frac{dn}{dE} = \rho_{1ph} > 0 \quad , \quad \rho_{1c} = \mathcal{J}_{1c}\frac{dn}{dE} > 0$$

Hence

$$\frac{1}{3}\left(\ln\frac{2\omega}{\psi_0} + \frac{1}{3}\right)\left(\rho_{1ph} - \frac{\rho_{1c}}{1 + \frac{1}{3}\rho_{1c}\ln\frac{\omega_1}{\omega}}\right) = 1 \qquad (47)$$

So the condition of appearing of an anomalous state has an analogous form as in the paper[1/] for the case of the pairs of fermions with antiparallel spin-moments

$$\varrho_{1ph} > \frac{\varrho_{1c}}{1+\frac{1}{3}\varrho_{1c}\ln\frac{\omega_1}{\omega}} \tag{48}$$

5. Total Spin of a System, Paramagnetic Susceptibility

Spin moment for the unit volume is given by the formula

$$\tilde{S}^z = \frac{1}{V}\sum_{\vec p}\left(a^+_{\vec p+}a_{\vec p+} - a^+_{\vec p-}a_{\vec p-}\right) . \tag{49}$$

We pass to the operators α and find (49) for the state C_0 (vacuum state in the α representation)

$$S^z = \frac{1}{V}\sum_{\vec p}\left\{F(\vec p,+) - F(\vec p,-)\right\} \tag{50}$$

In the absence of the magnetic field $F(\vec p,+) = F(\vec p,-)$ and therefore $S^z = 0$.

Consider now a system of Fermi-particles in the presence of external, weak, constant magnetic field $\delta\mathcal{H}$, applied along the z axis.

In this case to the Hamiltonian (28) is added the term

$$-\frac{e}{m}\delta\mathcal{H}\sum_{\vec p}\left(a^+_{\vec p+}a_{\vec p+} - a^+_{\vec p-}a_{\vec p-}\right) \tag{51}$$

where $+e$ is the charge of the electron and m its mass.

The compensation equations lead to the formulae

$$F_{\delta\mathcal{H}}(\vec p,+) = \frac{1}{2}\left[1 - \frac{\xi(p) - \frac{e}{m}\delta\mathcal{H}}{\sqrt{[\xi(p) - \frac{e}{m}\delta\mathcal{H}]^2 + C^2(\vec p)}}\right], \quad F_{\delta\mathcal{H}}(\vec p,-) = \frac{1}{2}\left[1 - \frac{\xi(p) + \frac{e}{m}\delta\mathcal{H}}{\sqrt{[\xi(p) + \frac{e}{m}\delta\mathcal{H}]^2 + C^2(\vec p)}}\right] \tag{52}$$

$$C(\vec p,+) = C(\vec p,-) \cong C(\vec p)$$

Hence

$$S^z_{\delta\mathcal{H}} = \frac{1}{2}\frac{dn}{dE}\int_{-\omega}^{\omega}\int_{-1}^{1}\frac{\xi + \frac{e}{m}\delta\mathcal{H}}{\sqrt{(\xi + \frac{e}{m}\delta\mathcal{H})^2 + \psi^2 x^2}}\,d\xi\,dx . \tag{53}$$

After performing integration

$$S^z_{\delta\mathcal{H}} = \frac{dn}{dE}\frac{e}{m}\delta\mathcal{H}\left[1 - \exp\left(2/3 - 6/\varrho_1\right)\right] . \tag{54}$$

For the paramagnetic susceptibility we get

$$\chi = \frac{2e^2}{m^2}\frac{dn}{dE}\left[1 - \exp\left(\frac{2}{3} - \frac{c}{g_1}\right)\right] \qquad (55)$$

From (54) we see that in the absence of the magnetic field the total spin of a system equals zero. In the presence of the magnetic field the electron-electron interaction cannot be take into account by means of perturbation theory since we have obtained the non-analytical dependence of (54) and (55) for g_1 near $g_1 = 0$.

In this paper we have considered formally the solution of the compensation equations as an odd function of $\vec{p}$. It is provided that from the point of view of the transformation (2) this leads to the state of system with the pairs of fermions with parallel spins. In order to explain the physical picture of this state one have to examine the thermodynamics of this anomalous state.

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