JOINT INSTITUTE FOR NUCLEAR RESEARCH
Laboratory of Nuclear Problems
Laboratory of Theoretical Physics

L.I.Lapidus, Chzhou Guan-chzhao

P. 314

# ON THE INVESTIGATION OF PION-HYPERON INTERACTION

ЖЭТФ, 1959, т 37, вып 1, с 283-288.

Dubna, 1959

L.I.Lapidus, Chzhou Guan-chzhao

P. 314

# ON THE INVESTIGATION OF PION-HYPERON INTERACTION

It is shown that the unitarity of S matrix makes it possible to obtain some information on pion-hyperon scattering by analysing the data on K meson-nucleon interactions.

The possibility is discussed of the investigation of $\pi$-$\Lambda$ and $\pi$-$\Sigma$ interactions in studying the peripheral collisions of hyperons with nucleons.

The investigation of pion-hyperon interactions is of interest especially for analysing the symmetry properties of pion-baryon interactions.

I. Let us consider the reactions

$$\tilde{K} + N \to \tilde{K} + N \tag{Ia}$$

$$\tilde{K} + N \to \Sigma(\Lambda) + \pi \tag{Ib}$$

$$\Sigma(\Lambda) + \pi \to \Sigma(\pi) + \pi \tag{Ic}$$

in the region of such K meson energies in which the channels where two pions are produced can be neglected. As the elements of S matrix for the reactions (I) are connected by the unitarity condition a question arises: "What information on the amplitudes of scattering $\Sigma(\Lambda)+\pi \to \Sigma(\Lambda)+\pi$ can be obtained from the investigation of cross sections and polarizations in the processes (Ia) and (Ib)?" In the first part of the present paper the attempt is made to answer this question.

In the following we shall assume that the spin of K meson is zero and the spin of a hyperon is equal to ½. We shall consider then that the interactions are invariant under reflection of space, time reversal and rotation in isotopic space.

The reactions (I) are described by the elements of T matrix ( $iT=S-I$) diagonal with respect the the quantum number of the isotopic spin.

$$T^0 = \begin{pmatrix} a_K^0 & a_{K\Sigma}^0 \\ a_{\Sigma K}^0 & a_\Sigma^0 \end{pmatrix} \quad ; \quad T^1 = \begin{pmatrix} a_K^1 & a_{K\Sigma}^1 & a_{K\Lambda} \\ a_{\Sigma K}^1 & a_\Sigma^1 & a_{\Sigma\Lambda} \\ a_{\Lambda K} & a_{\Lambda\Sigma} & a_\Lambda \end{pmatrix} \tag{2}$$

where $a_K^0$ $(a_K^1)$ is the amplitude of scattering $\tilde{K}+N \to \tilde{K}+N$ in the state with the definite isotopic spin value 0/I/; $\tilde{K}+N$ $a_{K\Sigma}^0$ $(a_{K\Sigma}^1)$ is the amplitude of the reaction $\tilde{K}+N \to \Sigma+\pi$ in the state with the isotopic spin 0/I/ and so on.

The spin structure of the scattering amplitude $a_\alpha$ can be represented in the form

$$a_\alpha = A_\alpha + i(\vec{\sigma}[\vec{n}\cdot\vec{n}']) B_\alpha \qquad (3)$$

where $\vec{n}\,(\vec{n}')$ is the unit vector parallel to the particle momentum in the initial (final) state in the barycentric system; $A_\alpha$ and $B_\alpha$ are two complex scalar functions of energy and of $(\vec{n}\cdot\vec{n}')$.

The amplitude of the reaction $a_{\alpha\beta}$ has the form

$$a_{\alpha\beta} = A_{\alpha\beta} + i(\vec{\sigma}[\vec{n}\cdot\vec{n}']) B_{\alpha\beta} \qquad (4)$$

when the product of the intrinsic parities of all four particles in the initial (final) state $\Pi = {}^{+}I$ , or

$$a_{\alpha\beta} = A_{\alpha\beta}\,(\vec{\sigma}\cdot\vec{n}) + B_{\alpha\beta}\,(\vec{\sigma}\cdot\vec{n}') \qquad (4')$$

when $\Pi$ =-I. Here $A_{\alpha\beta}$ and $B_{\alpha\beta}$ are two complex functions of energy and of $(\vec{n}\cdot\vec{n}')$.

We analyse the conditions of the T matrix determination from the experimental data. It is seen from (2),(3),and (4) that the number of real scalar functions in T° and T' matrices is equal to I3x4=52. The invariance of interaction under time reversal leads to the symmetry of S matrix; this decreases the quantity of functions determining T matrix from 52 to 36. One can make sure then that taking account of the unitarity conditions of S matrix it is possible to decrease the number of independent real functions by a factor of 2 and it becomes equal to I8.

The same result is obtained if one use the general formulae [I].

We shall consider now what information can be obtained from the investigation of the processes (Ib) and (Ic). The number of real function characterizing these processes is equal to 5x4=20. They satisfy 4 unitarity relations. Therefore only I6 of them are independent.

Table I presents 8 reactions of the type (Ia) and (Ib) as well as their amplitudes. $K^0_2$/$K^0_I$/denotes the longlived (shortlived) K° meson; $\tilde{a}_{K^0}$ is the amplitude of K° meson scattering which is determined from the analysis of K+ meson scattering on nucleons. We shall assume in the following that the amplitude $\tilde{a}_{K^0}$ is already known.

In fact the reactions (c) and (d) given in Table I are the same process. Studing the dependence of the scattering cross section and polarization after scattering upon the time (that is, upon the distance to the target) one can separately determine the amplitudes of the reactions (d) and (e).

Measuring the differential cross sections and polarization of nucleons in the reactions (a),(b),(c),(d) listed in Table I we shall completely reconstruct the scattering amplitudes $a_K^0$ and $a_K^1$. Experimental data on the cross section and polarization of hyperons in the reactions (e),(f),(g),(h) together with 4 unitarity relations make it possible to determine the amplitudes of the reactions $a_{K\Sigma}^0$, $a_{K\Sigma}^1$ and $a_{K\Lambda}$ with the accuracy up to the common phase factor.

As the expressions for cross sections and polarizations as well as the unitarity relations for the reactions (Ia) and (Ib) are invariant under changing

$$a_{K\Sigma}^0 \to e^{i\delta_0(E)}\, a_{K\Sigma}^0$$
$$a_{K\Sigma}^1 \to e^{i\delta_0(E)}\, a_{K\Sigma}^1$$
$$a_{K\Lambda} \to e^{i\delta_1(E)}\, a_{K\Lambda}$$

we cannot determine these two phase factors $e^{i\delta_0}$ and $e^{i\delta_1}$ which are the functions of the energy only. The latter is obtained from the relations which are valid for the amplitudes (4) and (4') due to unitarity of S matrix.

As the number of independent real functions in T° and T' is equal to I8 and I6 of them are determined with the accuracy up to two phase factors while investigating the processes (Ia) and (Ib), to reconstruct completely the amplitudes of pion scattering by $\Lambda$ and $\Sigma$ hyperons in the states with isotopic spin 0 and I one needs to determine two more real functions of energy and of $(\vec{n}\cdot\vec{n}')$ and two phase factors.

For each state with total momentum $J$ and orbital momentum $\ell = J \pm 1/2$ one can write the matrix T° in the form:

$$-i\begin{pmatrix} \rho_K^0 \exp(2i\delta_K^0) - 1 & i\rho_{K\Sigma}^0 \exp(i\delta_{K\Sigma}^0) \\ i\rho_{K\Sigma}^0 \exp(i\delta_{K\Sigma}^0) & \rho_K^0 \exp(2i\delta_K^0) - 1 \end{pmatrix} \tag{7}$$

where $\rho_\alpha$ is some positive functions of energy and $\delta_\alpha$ is the phases of the corresponding processes.

From the unitarity conditions of S matrix it follows that

$$\delta_{K\Sigma}^0 = \delta_K^0 + \delta_\Sigma^0$$
$$\rho_\Sigma^0 = \rho_K^0 = \{1-(\rho_{K\Sigma}^0)^2\}^{1/2} \tag{8}$$

The values $\rho_K^0, \delta_K^0, \delta_{K\Sigma}^0$ can be calculated with the accuracy up to the common phase factor by studying the processes (Ia) and (Ib). The values $\rho_\Sigma^0$ and $\delta_\Sigma^0$ are determined then with the same accuracy from the relation (8).

Thus for $\pi$-$\Sigma$ scattering the difference of phases in states with zero isotopic spin are completely determined by investigating the reactions with K particles+).

For states with the isotopic spin I we have instead of (7)

$$-i\begin{pmatrix} \rho_K \exp(2i\delta_K)-1 & \rho_{K\Sigma}\exp(i\delta_{K\Sigma}) & \rho_{K\Lambda}\exp(i\delta_{K\Lambda}) \\ \rho_{K\Sigma}\exp(i\delta_{K\Sigma}) & \rho_\Sigma \exp(2i\delta_\Sigma)-1 & \rho_{\Sigma\Lambda}\exp(i\delta_{\Sigma\Lambda}) \\ \rho_{K\Lambda}\exp(i\delta_{K\Lambda}) & \rho_{\Sigma\Lambda}\exp(i\delta_{\Sigma\Lambda}) & \rho_\Lambda\exp(2i\delta_\Lambda)-1 \end{pmatrix} \tag{9}$$

Here instead of $\rho_\alpha^1$ $(\delta_\alpha^1)$ we shall write in the following $\rho_\alpha(\delta_\alpha)$.

From the unitarity conditions obtain:

$$\rho_{K\Sigma}^2+\rho_\Sigma^2+\rho_{\Sigma\Lambda}^2=1 \quad ; \quad \rho_{K\Lambda}^2+\rho_{\Sigma\Lambda}^2+\rho_\Lambda^2=1 \tag{10}$$

$$\cos(2\delta_\Sigma+2\delta_K-2\delta_{K\Sigma}) = \frac{(\rho_K\rho_{K\Sigma})^2+(\rho_{K\Sigma}\rho_\Sigma)^2-(\rho_{K\Lambda}\rho_{\Sigma\Lambda})^2}{2\rho_\Sigma\rho_K(\rho_{K\Sigma})^2} \tag{11}$$

$$\cos(\delta_{\Sigma\Lambda}+2\delta_K-\delta_{K\Lambda}-\delta_{K\Sigma}) = \frac{(\rho_{K\Lambda}\rho_{\Sigma\Lambda})^2+(\rho_K\rho_{K\Sigma})^2-(\rho_{K\Sigma}\rho_\Sigma)^2}{2\rho_{K\Lambda}\rho_{\Sigma\Lambda}\rho_K\rho_{K\Sigma}} \tag{12}$$

$$\cos(2\delta_\Lambda+2\delta_K-2\delta_{K\Lambda}) = \frac{(\rho_K\rho_{K\Lambda})^2+(\rho_{K\Lambda}\rho_\Lambda)^2-(\rho_{\Sigma K}\rho_{\Sigma\Lambda})^2}{2\rho_\Lambda\rho_K(\rho_{K\Lambda})^2} \tag{13}$$

It is easy to make sure that even with known quantities $\rho_K, \rho_{K\Sigma}, \rho_{K\Lambda}, \delta_K, \delta_{K\Sigma}$ и $\delta_{K\Lambda}$ the unitarity relations (10)-(13) are not enough for reconstruction of T'matrix. One must know one more parameter in each state (e.g., $\rho_\Sigma$ ).

We note that relations (10)-(13) lead to some interesting inequalities. Taking into account the fact that $\rho_\alpha > 0$ and $|\cos\theta| < 1$ we obtain from (10) and (11)

$$0<\rho_\Sigma^2<1-\rho_{K\Sigma}^2-\rho_{\Sigma\Lambda}^2<1-\rho_{K\Sigma}^2 \tag{14}$$

$$\left|(\rho_K\rho_{K\Sigma})^2+(\rho_{K\Sigma}\rho_\Sigma)^2-(\rho_{K\Lambda})^2(1-\rho_{K\Sigma}^2-\rho_\Sigma^2)\right|<2\rho_\Sigma\rho_K(\rho_{K\Sigma})^2 \tag{15}$$

We introduce new notations $\rho_{K\Sigma}^2+\rho_{K\Lambda}^2\equiv a$, $\rho_K\rho_{K\Sigma}^2\equiv b$ ; $(\rho_K\rho_{K\Sigma})^2-\rho_{K\Lambda}^2(1-\rho_{K\Sigma}^2)\equiv c$

Then (15) can be written in the form

$$|a\rho_\Sigma^2+c|<2b\rho_\Sigma \tag{15'}$$

---

+) It may turn out that for performing the unambiguity analysis it will be useful to take into account the Coulomb effects and the dependence of S matrix upon energy

It should be noted that for the reactions considered the Minami ambiguity takes place. Some possibilities for determining the parity of Kmeson with respect to hyperons from the analysis of the reaction (I) were discussed recently by Amati and Vitale (2).

From (I5') and (I4) obtain

$$\mathrm{Max}\left\{0;\ \frac{\ell}{a}-\frac{1}{a}\sqrt{\ell^2-ac}\right\} < \rho_\Sigma < \mathrm{Min}\left\{\sqrt{1-\rho_{K\Sigma}^2}\ ;\ \frac{\ell}{a}+\frac{1}{a}\sqrt{\ell^2-ac}\right\} \qquad \text{(I6)}$$

Inequality (I5') takes place only if $\ell^2-ac\geq 0$. Thus the observed values $\rho_K$, $\rho_{K\Lambda}$ and $\rho_{K\Sigma}$ must satisfy also the latter inequality. In analogy we have

$$\mathrm{Max}\left\{0;\ \frac{\ell_1}{a_1}-\frac{1}{a_1}\sqrt{\ell_1^2-a_1c_1}\right\} < \rho_{\Sigma\Lambda} < \mathrm{Min}\left\{\sqrt{1-\rho_{K\Sigma}^2}\ ;\sqrt{1-\rho_{K\Lambda}^2}\ ;\ \frac{\ell_1}{a_1}+\frac{1}{a_1}\sqrt{\ell_1^2-a_1c_1}\right\} \qquad \text{(I7)}$$

$$\mathrm{Max}\left\{0;\ \frac{\ell_2}{a_2}-\frac{1}{a_2}\sqrt{\ell_2^2-a_2c_2}\right\} < \rho_\Lambda < \mathrm{Min}\left\{\sqrt{1-\rho_{K\Lambda}^2}\ ;\ \frac{\ell_2}{a_2}+\frac{1}{a_2}\sqrt{\ell_2^2-a_2c_2}\right\} \qquad \text{(I8)}$$

where

$$a_1 \equiv \rho_{K\Lambda}^2+\rho_{K\Sigma}^2 = a \quad ; \quad \ell_1 \equiv \rho_{K\Lambda}\rho_K\,\rho_{K\Sigma} \quad ; c_1 \equiv (\rho_K\rho_{K\Sigma})^2-\rho_{K\Sigma}^2\,(1-\rho_{K\Sigma}^2)$$

and

$$a_2 \equiv \rho_{K\Lambda}^2+\rho_{K\Sigma}^2 = a \quad ; \ \ell_2 = \rho_K(\rho_{K\Lambda})^2 \quad ; c_2 \equiv (\rho_K\rho_{K\Lambda})^2-\rho_{K\Sigma}^2\,(1-\rho_{K\Lambda}^2)$$

2. Recently Chew and Low, Okun' and Pomeranchuk[3/] proposed to consider the peripheral collisions as a method for studying the interactions between unstable particles. We shall consider that their method can be used for determining the amplitudes of scattering $\Sigma(\Lambda)+\pi\to\Sigma(\Lambda)+\pi$ in the investigation of the processes $\Sigma+N\to\Sigma(\Lambda)+N+\pi$ ; $\Lambda+N\to\Sigma+N+\pi$. The main point of the method is as follows: the amplitude of the reaction $\Sigma+N\to\Sigma(\Lambda)+N+\pi$ which is considered as the function of $(p_N'-p_N)^2$ where $p_N\ (p_N')$ is 4-momentum of nucleons in the initial (final) state, has the pole in nonphysical region $(p_N'-p_N)^2=\mu^2$, where $\mu$ is the pion mass. It is shown that the virtual process

$$N\to N+\pi \quad ; \ \Sigma(\Lambda)+\pi\to\Sigma(\Lambda)+\pi \qquad \text{(I9)}$$

corresponds to the pole term the residue of which is proportional to the amplitude of scattering $\pi-\Sigma(\Lambda)$. Assuming that in the physical region near the pole the reaction $\Sigma+\Lambda\to\Sigma(\Lambda)+$ $+N+\pi$ is determined by the process (I9) one can extrapolate its amplitude into nonphysical region and separate the pole term residue.

To estimate the effect of any other terms in the physical region near the pole we shall phormulate some rules which can take place if in this region the contribution of pole term is predominant.

A. In the region near the pole the amplitude of the reaction $\Sigma^++p\to p+\pi^0+\Sigma^+$ is equal to that of the reaction $\Sigma^-+p\to p+\pi^0+\Sigma^-$. This rule results from the invariance of the virtual process

$$\pi^0 + \Sigma^{\pm} \to \pi^0 + \Sigma^{\pm} \tag{20}$$

under the rotation in isotopic space. Similarly it is possible to prove that the amplitudes of the following processes are equal to each other:

$\Sigma^+ + p \to p + \pi^+ + \Sigma^0$ ; $\Lambda + p \to n + \pi^0 + \Sigma^+$ ; $p + \pi^+ \to p + \pi^0 + \pi^+$

and $\Sigma^- + p \to p + \pi^- + \Sigma^0$ ; $\Lambda + p \to n + \pi^+ + \Sigma^0$ ; $p + \pi^- \to p + \pi^0 + \pi^-$ and so on.

B. Near the pole the amplitude of reactions $\Sigma^{\pm}(\Lambda) + p \to \Sigma^{\pm}(\Lambda) + p + \pi^0$ and $\tilde{\Sigma}^{\pm}(\tilde{\Lambda}) + p \to \tilde{\Sigma}^{\pm}(\tilde{\Lambda}) + p + \pi^0$ are equal to each other. This rule results from the invariance of the virtual process (20) under the charge conjugation. In our case it is of no practical importance but in other cases it can be interesting. For instance, it is possible to prove that the amplitudes of reaction $K^{\pm} + N \to K^{\pm} + N + \pi^0$ are equal. This equality is important for determining the interaction of K meson with a pion and it was noted by Okun and Pomeranchuk.

C. If in the initial states the nucleons are not polarized they remain unpolarized in the final state also.

We shall consider the reaction of the type

$$\Sigma^+ + p \to \Lambda + p + \pi^+ \tag{21}$$

in the region near the pole $(p_\Sigma - p_\Lambda)^2 = \mu^2$. Let us assume that in this region the process

$$\Sigma^+ \to \Lambda + \pi^+ \quad , \quad p + \pi^+ \to p + \pi^+ \tag{22}$$

predominates, the amplitude of which is proportional to

$$\frac{\bar{u}(p_\Lambda)\,\Gamma\, u(p_\Sigma)}{(p_\Sigma - p_\Lambda)^2 - \mu^2}\, a_{\pi p} \tag{23}$$

where $\Gamma = I$, when the relative parity of $\Sigma$ and $\Lambda$ particles is $\Pi = -I$; $\Gamma = \gamma_5$ ; when $\Pi = +I$; $a_{\pi p}$ is the scattering amplitude for $p + \pi^+ \to p + \pi^+$. The amplitude of the process (22) does not contain the dependence of the hyperon spin when $\Pi = -I$, (more exactly, it contains the term proportional to $\vec{\sigma}_Y$ but with small coefficient) and is proportional to $(\vec{\sigma}_Y \cdot \vec{k})$ where $\vec{k}$ is the unit vector parallel to the difference $\frac{\vec{p}_\Sigma}{E_\Sigma + M_\Sigma} - \frac{\vec{p}_\Lambda}{E_\Lambda + M_\Lambda}$ when $\Pi = +I$.

If in the initial state $\Sigma^+$ are polarized (polarization vector is $\vec{P}$ ) then with the aim of (23) it is possible to prove that in final state the polarization vector of $\Lambda$ particles ( $\vec{P}'$ )

$$\vec{P}' = \vec{P} \tag{24}$$

when $\Pi$ =-I and

$$\vec{p}\,' = 2(\vec{p}\vec{k})\vec{k} - \vec{p} \tag{24'}$$

when $\Pi$ =+I.

Thus, if in the region where the pole term is predominant one succeeded in measuring the polarization vector of $\Lambda$ particles produced in the reaction (2I) with polarized $\Sigma$ it would be possible not only to evaluate the effect of non-pole terms but to obtain data on the relative parity of $\Lambda$ and $\Sigma$ hyperons as well.

The investigation of the polarization of products in peripheral collisions can give, in a number of cases, some information on the parity of unstable particles+).

The authors are grateful to Prof.M.A.Markov for valuable discussions.

Table I

| Reaction | Amplitude |
|---|---|
| (a) $K^- + p \to K^- + p$ | $\frac{1}{2}(a_K^1 + a_K^0)$ |
| (b) $K^- + p \to K^0 + n$ | $\frac{1}{2}(a_K^1 - a_K^0)$ |
| (c) $K_2^0 + p \to K_1^0 + p$ | $\frac{1}{2}(a_K^1 - \tilde{a}_K^0)$ |
| (d) $K_2^0 + p \to K_2^0 + p$ | $\frac{1}{2}(a_K^1 + \tilde{a}_K^0)$ |
| (e) $K^- + p \to \Lambda + \pi^0$ | $a_{K\Lambda}$ |
| (f) $K^- + p \to \Sigma^- + \pi^+$ | $-(\frac{1}{\sqrt{6}} a_{K\Sigma}^0 + \frac{1}{2} a_{K\Sigma}^1)$ |
| (g) $K^- + p \to \Sigma^0 + \pi^0$ | $\frac{1}{\sqrt{6}} a_{K\Sigma}^0$ |
| (h) $K^- + p \to \Sigma^+ + \pi^-$ | $-(\frac{1}{\sqrt{6}} a_{K\Sigma}^0 - \frac{1}{2} a_{K\Sigma}^1)$ |

+) The possibility of particle parity determination in studing the peripheral collisions without consideration of polarization was discussed recently by Taylor[4].

References

I. S.M.Bilenjky, L.I.Lapidus, L.D.Puzikov, R.M.Ryndin, JETP 35,959,1958; Nuol.Phys.7, 646,1958.

2. D. Amati, B.Vitale, Nuovo Cim. 9, 895,1958.

3. G.F.Chew, Proc.1958 Annual Conference, CERN,Geneva, p.97.
"A proposal for Determining the Pion-Nucleon Coupling Constant from Nuoleon-Nucleon Soattering" (preprint, 1958)
L.B.Okun, I.Ya.Pomeranchuk, JETP, 36,300,1959.
G.F.Chew, F.E.Low, Unstable particles as targets (preprint, 1958)

4. J.G.Taylor, Nuol.Phys.9,357,1959
"A method for the Determination of Strange Particle Parities and Coupling Constants" (preprint, 1959)

The Russian variant of this paper was received by Publishing Department on February, 26 1959.