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JOINT INSTITUTE FOR NUCLEAR RESEARCH

Laboratory of Theoretical Physics

P-307

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Nuovo Cim., 1959, v12, N6, p.602-610.

Dubna, 1959.

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Abstract

The density of a meson cloud in a nucleon is calculated on the basis of the extended source theory. The nucleon periphery is determined as a region of the applicability of the one-meson state. The pion-pion interaction cross section is estimated. The coefficient of pion absorption in a nucleon found experimentally is compared with that calculated by the optical model.

Introduction

Some years ago the analysis of energy losses and of multiple production of mesons in nucleon collisions led us to the conclusion that it is reasonable to distinguish three types of nucleon collisions: a core with a core (KK), a pion cloud with a core ($K\pi$), and finally, the collisions of pion clouds ($\pi\pi$)[1,2]. It was also meant that the collisions of the first type (KK) should be considered by the methods of the Fermi statistical theory, the collisions of the second type ($K\pi$) using the method of the parameter of collision and the meson theory.

The contribution of the collisions of type ($\pi\pi$) is very likely small (see, further §4, as well as[2]). It would have been possible to develop the theory of periphery collisions (πK)only according to the pion cloud theory of a nucleon. The theory of this cloud was developed in papers by G.Chew[3], G. Salzman[4] et.al.

Later on, however, in Hofstadter's experiments (see[5],[6]) on the study of the charge and magnetic moment distribution in the nucleons serious doubts were cast on the correctness of the classical picture of the pion cloud in the real nucleons.

There was found an essential discrepancy between the great magnitude of the proton electric radius and the small magnitude of the proton electric radius and the small magnitude of this quantity for a neutron. These difficulties gave rise to quite different points of view on the nucleon structure[7,8] and even to the doubts concerning the applicability of the electrodynamics at the distances of the order 10^{-13}[6,9] cm.

Meanwhile all these doubts seem to be based on an insufficiently clear understanding that the usual interpretation of R. Hofstadter's experiments

$$F_{1p}(q) = F_{2p}(q) = F_{2n}(q); \quad F_{1n}(q) = 0$$

(Here F_{1p} , F_{1n} are electrical and F_{2p} , F_{2n}-magnetic form-factors for a proton and neutron) is, indeed neither unique nor exact, but only possible.

The contradiction which arises between the density distribution law of the meson charge according to Ykawa theory $\sim e^{-d\tau}/\tau^2$ and the charge distribution $\sim e^{-\delta\tau}$ obtained experimentally is also of no practical importance since the regions of the applicability of these expressions are quite different.

All this made us analyze the <u>spatial</u> picture of the charge and momentum density distribution in the nucleon, which results from the extended source theory and compare it with experimental data. Sections 2,3 are concerned with this.

Section 4 is dealt with the application of the pion cloud theory in the nucleons to the estimate of the ($\pi\pi$) interaction cross section.

§ 2. Nucleon Core and Pion Cloud

We shall suppose that the first approximation is not bare, point nucleon but that distributed over the region $a \simeq \hbar/Mc$(M is the nucleon mass).

This distribution is due to virtual nucleons, antinucleons and strange particles. At the same time it is a pion cloud source in a nucleon*).

If $V(K)$ is the Fourier component of this extended source, then the expressions for the charge density ρ_π and for the density of the magnetic moment in this cloud $\vec{m}_\pi$ yield (see paper[6]):

$$\rho_\pi(r) = -e\mu^3 t_3 \frac{4f^2}{(2\pi)^3}\int \frac{V(K)\,V(K')}{\omega\omega'(\omega+\omega')}(\vec{K}\vec{K}')e^{i(\vec{K}-\vec{K}')\vec{r}}\,d^3(KK') \qquad (1)$$

$$\bar{m}_\pi(r) = -e\mu^3 c\tau_3 \frac{2if^2}{(2\pi)^3}\int \frac{V(K)V(K')}{\omega^2\omega'^2}\bar{K}\left(\bar{\sigma}\left[\bar{\tau}\left[\bar{K}\bar{K}'\right]\right]\right)e^{i(\vec{K}-\vec{K}')\vec{r}}\,d^3(KK') \qquad (2)$$

All the lengths in these formulas are measured in the units of $\frac{1}{\mu} = 1.4\cdot 10^{-13}$cm, the pion mass is put equal to a unit; σ is the Pauli matrix; τ_3 is the matrix of the isotopic spin;

Expressions (1) and (2) are the first approximation which take into account the contribution of only one-pion-state.

The total density of charges and of a magnetic moment in the nucleon is equal to:

$$\rho(r) = \rho_\pi(r) + \rho_K(r)\,; \quad \bar{m}(r) = \bar{m}_\pi(r) + m_K(r) \qquad (3)$$

where the densities of an electric charge and magnetic moment are designated in terms of $\rho_K(r)$ and $\bar{m}_K(r)$. These densities are concentrated in the central part of a nucleon and are due to nucleon and antinucleon pairs and to strange particles (charges and a magnetic moment of a nucleon core), as well as to two-three - and other higher pion states. At

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*) It may be said that Tamm's model (loc.cit.) is applied not to a nucleon as a whole, but only to its central region.

present we know very little about these states and for the time being shall consider them as components of a nucleon core.

Expression (1) can be easily reduced to the form:

$$\rho_{\pi}(r) = e\mu^3\tau_3 \frac{4f^2}{(2\pi)^5} \int_0^\infty d\xi \left(\frac{dI}{dr}\right)^2 \tag{4}$$

where

$$I(r) = \int \frac{v(\kappa)}{\omega} e^{i\kappa r - \xi\omega} d^3\kappa \tag{5}$$

whereas ξ is an auxiliary variable.

Making the integration over angles and designating $v(\kappa) = V(\omega)$ we obtain

$$I(r) = \frac{2\pi}{ir} V\left(-\frac{d}{d\xi}\right) Q(\xi, r) + \text{com.conj.} \tag{6}$$

$$Q(\xi, r) = \int_1^\infty e^{-\xi\omega + ir(\omega^2-1)^{1/2}} d\omega \, . \tag{7}$$

Supposing further $\omega = \text{ch}\,t$ and introducing $\rho = \sqrt{\xi^2 + r^2}$ we obtain

$$I(r) = -\frac{4\pi}{r} V\left(-\frac{d}{d\xi}\right) \frac{dK_0(\rho)}{dr} \tag{8}$$

where $K_0(\rho)$ is an well-known Bessel function.

The function $v(\kappa) = V(\omega)$ serves as a cut-off factor. We choose it in the form

$$V(\omega) = e^{-\beta(\omega-1)} \tag{9}$$

where $1/\beta$ is a cut-off frequency.

Only in such a choice of $V(\omega)$ the operation $V\left(-\frac{d}{d\xi}\right)$ has a simple meaning of displacement $\xi \longrightarrow \xi + \beta$.

Using now (4) and (8) one may easily obtain

$$\rho_{\pi}(r) = e\mu^3\tau_3 \frac{2f^2}{\pi^3} e^{2\beta} r^2 \int_{\sqrt{r^2+\beta^2}}^\infty \frac{d\rho}{\rho^3\sqrt{\rho^2 - r^2}} K_2^2(\rho) \, . \tag{10}$$

Here $K_2(\rho)$ is the Bessel function. Just in a similar way the magnetic moment distribution may be calculated:

$$\bar{m}_{\pi}(r) = e\mu^3 c\tau_3 \frac{f^2}{2(2\pi)^3} e^{2\beta} \left[\bar{r}[\bar{\sigma}\bar{r}]\right] \left\{ \int_{\sqrt{r^2+\beta^2}}^\infty \frac{d\rho\, K_2(\rho)}{\rho\sqrt{\rho^2 - r^2}} \right\}^2 \tag{11}$$

The asymptotic expressions for these magnitudes are independent of the form of a cut-off function are as follows:

$$\rho_{\pi}(r) = e\mu^3\tau_3 \frac{f^2}{2\pi^{3/2}} \frac{e^{-2r}}{r^{5/2}} + \cdots \tag{12}$$

$$\bar{m}_{\pi}(r) = e\mu^3 c\tau_3 \frac{f^2}{16\pi} \left[\bar{r}[\bar{\sigma}\bar{r}]\right] \frac{e^{-2r}}{r^4} \tag{13}$$

§3. Numerical Results

In general the quantities $\rho_\pi(r)$ and $\bar{m}_\pi(r)$ essentially depend upon the form-factor of the source $V(\omega)$, in our choice of $V(\omega)$ they depend upon the magnitude of β.

We have chosen the quantity β so that the calculated phase shift of the P-wave for pion scattering on a nucleon would be in best agreement with experiment in the low energy region. The calculations have shown that $\beta = \frac{1}{7}$. This choice corresponds also to the form-factors accepted in papers [4,10].

The r.m.s. electric and magnetic radii appear to be equalt to $\langle r_e^2 \rangle_\pi = 0{,}19$ and $\langle r_m^2 \rangle_\pi = 0.40$.

For the charge of the pion cloud eQ_π and for the pion magnetic moment $M_\pi \frac{e\hbar}{2Mc}$ we obtain: $Q_\pi = 0.76$ and $M_\pi = 1.25$.

Now we shall be concerned with a more detailed consideration of the nucleon electric radius. According to the definition this radius is equal to:

$$\langle r_e^2 \rangle = \frac{1}{e}\int r^2 \rho(r) d^3 r$$

where ρ is the total density of the pion cloud and nucleon core charges.

Put $\rho_K = Q_K \rho_c$ where Q_K-is the total charge of a core and designate

$$\langle r_e^2 \rangle_c = \frac{1}{e}\int r^2 \rho_c(r) d^3 r. \tag{15}$$

Expanding now the charge of a core Q_K in the scalar and vector parts Q_K^s and Q_K^v respectively we may write (14) as follows:

$$\langle r_e^2 \rangle = \tau_3 \langle r_e^2 \rangle_\pi + (Q_K^s - \tau_3 Q_K^v)\langle r_e^2 \rangle_c \tag{16}$$

The isotopic symmetry of this expression is evident. Since for a neutron $Q_\pi + Q_K = 0$ and $Q_s = 0.5$ then $Q_K^v = 0.26$.

It is well-known experimentally that the root-mean-square radius of a neutron is $\langle r_e^2 \rangle_n \simeq 0$. Therefore, it follows from (16):

$$\langle r_e^2 \rangle_c = \langle r_e^2 \rangle_n / (Q_K^s + Q_K^v) \tag{17}$$

and

$$\langle r_e^2 \rangle_p = \langle r_e^2 \rangle_c \tag{18}$$

Taking into account the values mentioned above Q_K and $\langle r_e^2 \rangle_\pi$ we find that $\langle r_e^2 \rangle_c = (0{,}5)^2_\pi \simeq (0{,}7.10^{-13}\text{cm})^2$.

Thus, assuming the electric radius of a neutron equal to zero we obtain a reasonable value for the proton radius. The form of the charge distribution in the core is arbitrary enough (since only the value Q_κ is known and $< r_e^2 >_c$). We choose $\rho_c(r)$ as follows:

$$\rho_c(r)=\frac{e}{8\pi a^3}\, e^{-r/a} \tag{19}$$

At this

$$< r_e^2 >_c = 12a^2 \tag{20}$$

Now in order to obtain $< r_e^2 >_c = 0{,}25$ it is necessary to take

$$a = \frac{1}{7} \simeq \hbar/MC = 2\cdot 10^{-14}\, cm.$$

Thus, (19) is an example of a core which is characterized by a small length $a \sim \hbar/MC$ At the same time it has a great root-mean-square radius.

Tab.1 presents the values of the densities of an electric charge in the spherical layer $d_\pi(r)=4\pi r^2\rho_\pi(r)$*). In Fig. 1a and 1b are given the charge density distributions in a proton and neutron and their cores.

The curve for a proton coincides practically with that given in Hofstadter's paper[5]. As for the charge dencity in a neutron it is seen that it oscillates near zero. This accounts for a small electric radius of a neutron. In Fig. 1a and 1b the region of the pion "atmosphere" of a nucleon is separated by a vertical line from the région where the core charges are essentially mixed.

As is seen $r > 1.4\cdot 10^{-13}$ cm in the region of an "atmosphere". The region where the asymptotic expansions (12) and (13) are correct lies even further. This is, so to say, the nucleon "stratosphere".

The number of mesons containing in this region is extremely small.

One may analogously consider the magnetic structure of a nucleon. Choosing the distribution of the magnetic moment of a nucleon core in the form

$$m_c(r)=\frac{\mu}{8\pi a^3}\, e^{-r/a} \tag{21}$$

and putting $a=\frac{1}{7}$, from the condition $m=\tau_3\cdot 1{,}85\,\frac{e\hbar}{2MC}$ we obtain for the root-mean-square magnetic radii of a proton and neutron

$$< r_m^2 >_n = < r_m^2 >_p = (0{,}7\cdot 10^{-13}\, cm)^2$$

Thus, the main results of the P. Hofstadter group:

$$< r_e^2 >_n \simeq 0;\quad < r_e^2 >_p \simeq < r_m^2 >_p \simeq < r_m^2 >_n \simeq (0{,}8\cdot 10^{-13}\, cm)^2$$

may be put in agreement with the concepts of the modern meson theory. At the same time the distribution of an electric charge and magnetic moment of a core is defined by a small length $a = \hbar/MC << \frac{\hbar}{m_\pi C}$

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*) The values $\rho_\pi(r)$ we calculated differ from those given in (11). However, as it is shown, (12), the numerical data of paper (11) are not correct.

§4. Pion Cloud and Pion-Pion Interaction

In pion scattering on nucleons with the parameter of the collision $\ell > \frac{\hbar}{m_\pi c}$ one may consider the scattering to be entirely due to the interaction of virtual pions of a nucleon with an incoming pion.

Now we shall draw our attention to the calculation of the pion absorption coefficient in this region.

First of all we evaluate the cross section of pion-pion interaction. Pions may be considered as particles consisting of virtual nucleon-antinucleon pairs (cf.[13]). At this

$$\pi^+ = p\cdot\tilde{n}; \quad \pi^- = \tilde{p}\cdot n; \quad \pi^0 = 2^{-1/2}(p\cdot n + \tilde{p}\tilde{n})$$

and the hypotetic pion $\pi_0^0 = 2^{-1/2}(p\cdot n - \tilde{p}\cdot\tilde{n})$ (cf.[14]). Here p, n are a proton and neutron whereas $\tilde{p}, \tilde{n}$ are an antiproton and antineutron.

This interpretation of a pion as a compound particle makes it possicle to consider the dimension of a pion a as a distance between the particles and antiparticles into which a pion virtually dissociates. Due to strong nucleon interaction the cross section of pion-pion interaction would be:

$$\sigma_{\pi\pi} = \pi a^2 \tag{22}$$

The distance a may be estimated from the mass difference of $\pi^\pm$ and π^0 mesons. It is 9 m_0 (m_0 is an electron mass). At the same time it is a difference between the electromagnetic energies of a charged and neutral pion. It equals

$$\Delta E = \alpha \frac{e^2}{a} + \beta \left(\frac{e\hbar}{2Mc}\right)^2 \frac{m^2}{a^3} \ . \tag{23}$$

Here the first term is an electrostatical energy, the second one is a magnetic energy, the numbers α ; β are of the order of a unit, m is the total magnetic moment of a nucleon ($|m| \simeq 2$). Putting $\Delta E = 9m_0c^2$ we find $a \simeq 2\hbar/Mc$ therefore, $\sigma_{\pi\pi} \simeq 5\cdot 10^{-27} cm^2$.

The coefficient of the pion absorption $K(r)$ may be approximately written

$$K(r) \simeq \sigma_{\pi\pi} \cdot n(r) \tag{24}$$

where $n(r)$ is the pion density in the nucleon "atmosphere". In the region of the one pion state $n(r) \simeq \frac{1}{e}\frac{3}{2}\rho(r)$ (the factor 3/2 takes into account the presence of neutral mesons).

The curve $K(r)$ is plotted in Fig. 2 calculated by the data of Table 1. At this we put $\sigma_{\pi\pi} = 5\cdot 10^{-27} cm^2$.

The curve K (r) calculated by the experimental data for pions with an energy E = 1.3Bev and E = 5 Bev according to the optical model (cf[5]) is also plotted there.

As is seen the agreement is rough.

However, one could hardly expect a better agreement since in the region $r = 0.2 \div 1$ the composition of the nucleon "atmosphere" is not reduced to the one pion state whereas the exact composition $\rho_\pi(z)$ is not known.

In the region $r \gtrsim 1$ the values $k(r)$ obtained by the optical model are very doubtful.

Therefore, for the determination of pion-pion interaction the exact measurements of the diffractional pion scattering on nucleons at small angles seem to be very important and promising.

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Russian variant of this paper was received
by Publishing Department on February 27, 1959.

T a b l e I

$r \cdot 10^{+13}$cm	0	0,05	0,075	0,1	0,125
$d_\pi^{(2)}\left(\frac{\hbar}{m_\pi c}\right)\frac{1}{e}$	0	0,17	0,58	1,13	1,6
$r \cdot 10^{+13}$cm	0,15	0,2	0,3	0,4	0,5
$d_\pi(2)\left(\frac{\hbar}{m_\pi c}\right)\frac{1}{e}$	1,95	2,05	1,46	0,9	0,55
$r \cdot 10^{+13}$cm	0,6	0,7	1,0	1,5	20
$d_\pi(2)\left(\frac{\hbar}{m_\pi c}\right)\frac{1}{e}$	0,35		0,146	0,082	0,0145

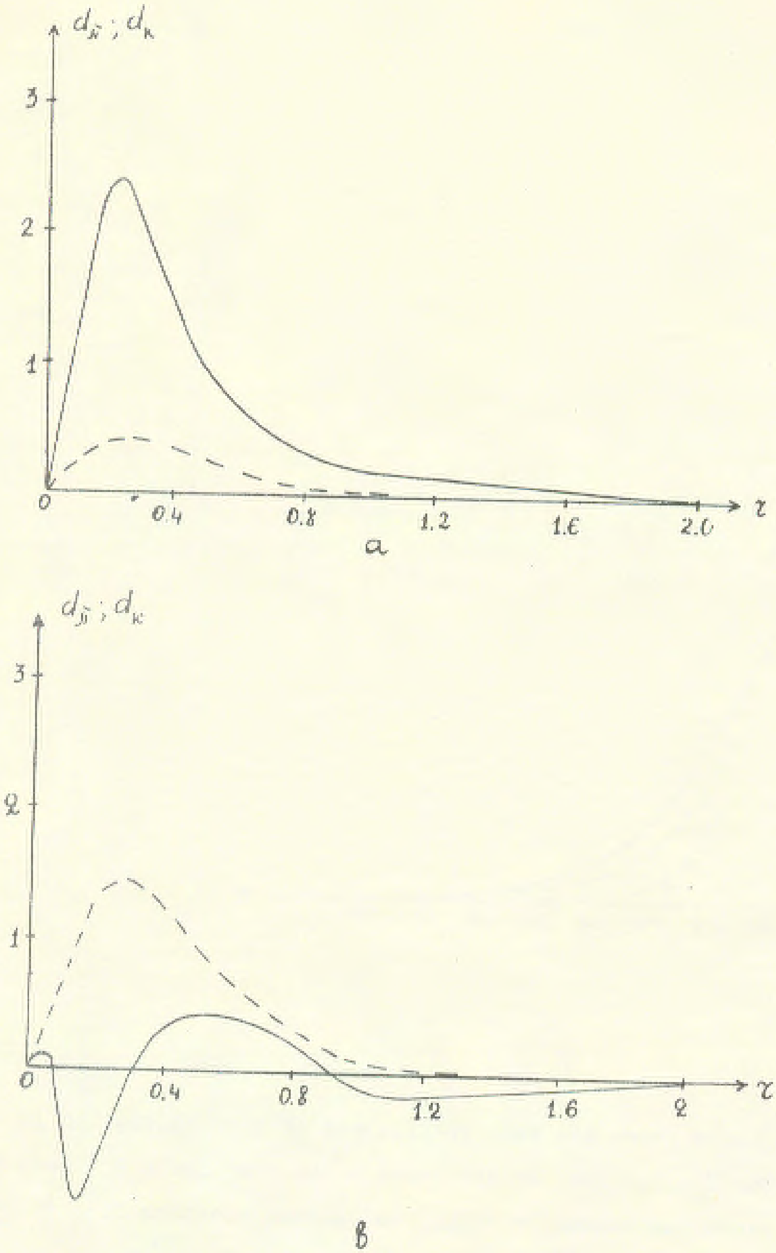

Fig. 1. Electromagnetic structure of a nucleon; a - the structure of a proton; б - the structure of a neutron. The solid curve shows the distribution of an electric charge in a proton and neutron; the dashed line shows the corresponding distribution of an electric charge in cores of a proton and neutron τ in the units of $\hbar/m_{\pi}c = 1.4 \cdot 10^{-13}$cm. $d(\tau)$ and $d_{K}(\tau)$-in the units $e\left(\frac{m_{\pi}c}{\hbar}\right)$

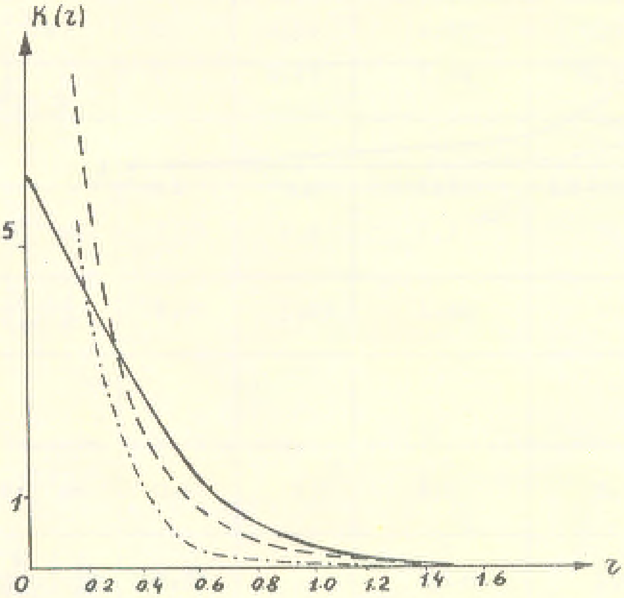

Fig. 2. Solid curve shows the mean coefficient of pion absorption in a nucleon $K=K(z)$ for E = 1,3 Bev. Dashed curve - the same for E = 5 Bev. Point-dash line shows the values $K=K(z)$ calculated starting from $\rho(z)=\rho_p(z)+\rho_n(z)$

z - in the units of 10^{-13}cm; $K(z)$ - in the units of 10^{13}cm.