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A b s t r a c t

Polarization experiments in the reaction $p+p\to d+\pi^+$ on a longitudinally-polarized proton beam have been considered here. An interest in similar experiments was taken after S.B. Nurushev[1] had proposed a method for obtaining a beam of longitudinally-polarized protons.

Polarization effects in the reaction $p+p\to d+\pi^+$ on an unpolarized and transversely-polarized proton beam have been already considered.[2]

§ 1.

The proton polarization vector of the longitudinally-polarized beam is directed along the axis OZ and $\vartheta=0^\circ$, $\delta=0^\circ$ so that

$$q_1=\cos\frac{\vartheta}{2}e^{-\frac{i}{2}\delta}=1;\quad q_2=\sin\frac{\vartheta}{2}e^{\frac{i}{2}\delta}=0.$$

The components of the finite wave functions $\vec{F}_1$ and $\vec{F}_2$ are in this case of the following form:

$$\left.\begin{aligned}
\alpha_1&=\frac{1}{2}\left[\sqrt{\frac{3}{2}}c_s+D_2(3\cos^2\theta-1)\right],\\
\beta_1&=-\frac{1}{2}\sin\theta\cos\theta\, e^{i\varphi}D_0,\\
\gamma_1&=\frac{1}{2}\sin^2\theta\, e^{i2\varphi}D_1,\\
\alpha_2&=-\frac{1}{4}\sin\theta\, e^{-i\varphi}(c_-+\sqrt{15}\cos\theta\cdot d_+),\\
\beta_2&=\frac{1}{2\sqrt{2}}\cos\theta\cdot c_+,\\
\gamma_2&=\frac{1}{4}\sin\theta\, e^{i\varphi}(c_--\sqrt{15}\cos\theta\, d_+).
\end{aligned}\right\}\tag{1}$$

It a relation between the amplitudes of d-transitions in the reaction $p+p\to d+\pi^+$ is taken into account which is in agreement with experimental data at the proton energy of about 600 Mev[3] then $C_{d_3}=C_{d_4}=0$; $d_+=C_{d_2}$; $d_-=\sqrt{\frac{3}{2}}C_{d_2}$;

$$D_0=\frac{1}{2}\sqrt{\frac{3}{2}}(3c_{d_1}+\sqrt{5}c_{d_2});\; D_1=\frac{\sqrt{3}}{4}(-3c_{d_1}+\sqrt{5}c_{d_2});\; D_2=-\frac{\sqrt{3}}{4}(c_{d_1}+\sqrt{5}c_{d_2}).$$

The components of the final wave functions are equal to:

$$\alpha_1'=\frac{1}{2}\left[\sqrt{\frac{3}{2}}c_s-\frac{\sqrt{3}}{4}(c_{d_1}+\sqrt{5}c_{d_2})(3\cos^2\theta-1)\right];$$

$$\beta_1'=-\frac{1}{2}\sin\theta\cos\theta\, e^{i\varphi}\frac{1}{2}\sqrt{\frac{3}{2}}(3c_{d_1}+\sqrt{5}c_{d_2});$$

$$\left.\begin{aligned}
\gamma_1' &= \frac{1}{2}\sin^2\theta\, e^{i2\varphi}\frac{\sqrt{3}}{4}\left(-3C_{d_1}+\sqrt{5}\,C_{d_2}\right);\\
\alpha_2' &= -\frac{1}{4}\sin\theta\, e^{-i\varphi}\left(C_{p_0}-\sqrt{\frac{5}{2}}\,C_{p_2}+\sqrt{15}\cos\theta\cdot C_{d_2}\right);\\
\beta_2' &= \frac{1}{2\sqrt{2}}\cos\theta\left(C_{p_0}+\sqrt{10}\,C_{p_2}\right);\\
\gamma_2' &= \frac{1}{4}\sin\theta\, e^{i\varphi}\left(C_{p_0}-\sqrt{\frac{5}{2}}\,C_{p_2}-\sqrt{15}\cos\theta\cdot C_{d_2}\right).
\end{aligned}\right\} \qquad (2)$$

when calculating the mean values of the spin-tensors one may neglect the squares and products of any amplitudes except the terms proportional to C_{p2} and $|C_{p2}|^2$.

Then

$$|\alpha_1|^2 = |\beta_1|^2 = |\gamma_1|^2 = 0;$$

$$|\alpha_2|^2 = \frac{1}{16}\sin^2\theta\left(\frac{5}{2}|C_{p_2}|^2-\sqrt{10}\,\mathrm{Re}\,C_{p_2}^*C_{p_0}-\cos\theta\cdot\sqrt{150}\,\mathrm{Re}\,C_{p_2}^*C_{d_2}\right);$$

$$|\beta_2|^2 = \frac{1}{8}\cos^2\theta\left(10|C_{p_2}|^2+2\sqrt{10}\,\mathrm{Re}\,C_{p_2}^*C_{p_0}\right);$$

$$|\gamma_2|^2 = \frac{1}{16}\sin^2\theta\left(\frac{5}{2}|C_{p_2}|^2-\sqrt{10}\,\mathrm{Re}\,C_{p_2}^*C_{p_0}+\cos\theta\cdot\sqrt{150}\,\mathrm{Re}\,C_{p_2}^*C_{d_2}\right).$$

We find, further, that

$$\left.\begin{aligned}
&\langle T_{00}\rangle = \gamma_0+\gamma_2\cos^2\theta;\\
&\text{where}\\
&\gamma_0 = \frac{5}{16}|C_{p_2}|^2-\frac{1}{8}\sqrt{10}\,\mathrm{Re}\,C_{p_2}^*C_{d_2};\\
&\gamma_2 = \frac{15}{16}|C_{p_2}|^2+\frac{3}{8}\sqrt{10}\,\mathrm{Re}\,C_{p_2}^*C_{d_2}.
\end{aligned}\right\} \qquad (3)$$

As it ought to be expected the differential cross section determined by $\langle T_{00}\rangle$ is independent of the longitudinal polarization of the beam. Therefore, in this case experiments on the measurement of deuteron polarization states are of interest.

§ 2.

Consider the vector polarization of a deuteron determined by $\langle T_{11}\rangle$ and $\langle T_{10}\rangle$. On a longitudinally-polarized proton beam arises an additional effect proportional to the magnitude of longitudinal polarization. For the polarization equal to 100%, we have:

$$\left.\begin{aligned}
\alpha_2^*\beta_2 &= -\frac{1}{8\sqrt{2}}\sin\theta\cos\theta\, e^{i\varphi}\left[\sqrt{10}\, C_{p_0}^* C_{p_2} - \sqrt{\tfrac{5}{2}}\, C_{p_2}^* C_{p_0} - 5|C_{p_2}|^2 + 5\sqrt{6}\cos\theta\; C_{d_2}^* C_{p_2}\right];\\
\beta_2^*\gamma_2 &= \frac{1}{8\sqrt{2}}\sin\theta\cos\theta\, e^{i\varphi}\left[\sqrt{10}\, C_{p_0}^* C_{p_2} - \sqrt{\tfrac{5}{2}}\, C_{p_2}^* C_{p_0} - 5|C_{p_2}|^2 - 5\sqrt{6}\cos\theta\; C_{d_2}^* C_{p_2}\right];\\
&\alpha_1^*\beta_1 = \beta_1^*\gamma_1 = 0 .
\end{aligned}\right\} \quad (4)$$

As a result:

$$i\langle T_{11}\rangle_{\rightarrow} = \frac{\sqrt{3}}{16}\sin\theta\cos\theta\, e^{i\varphi}\left\{6\sqrt{\tfrac{5}{2}}\,\mathrm{Im}\,(C_{p_2}^* C_{p_0}) + i\cos\theta\cdot 10\sqrt{6}\cdot \mathrm{Re}\,(C_{p_2}^* C_{d_2})\right\}. \quad (5)$$

It follows, from (5) that the measurement of the imaginary part of $i\langle T_{11}\rangle_{\rightarrow}$ yields new important data about the reaction $p+p\rightarrow d+\pi^+$. At the same time the real part of $i\langle T_{11}\rangle_{\rightarrow}$ determined by $\mathrm{Im}(C_{p_2}^* C_{p_0})$ will be known from deuteron polarization on an unpolarized proton beam [4].

The angular dependence of the scattering cross section of the polarized deuteron on a second target is usually described by the relation [5]:

$$\left.\begin{aligned}
&I = I_0\left[1+\alpha + e\cos\phi + B\cos 2\phi\right],\\
\text{where}\quad &e = 2\left[-\langle T_{21}\rangle_1\langle T_{21}\rangle_2 + i\langle T_{11}\rangle_1\cdot i\langle T_{11}\rangle_2\right];\\
&\alpha = \langle T_{20}\rangle_1\langle T_{20}\rangle_2; \quad B = 2\langle T_{22}\rangle_1\langle T_{22}\rangle_2 .
\end{aligned}\right\} \quad (6)$$

The quantities $\langle T_{20}\rangle_1$, $i\langle T_{11}\rangle_1$, $\langle T_{21}\rangle_1$ and $\langle T_{22}\rangle_1$ are always real in case of a transversely polarized proton beam. In our case the quantity $i\langle T_{11}\rangle_{\rightarrow}$ which determines the polarization state of a deuteron is complex. Therefore, relation (6) must be generalized [6]:

$$\left.\begin{aligned}
&I = I_0\left[1+\alpha + e_1\cos\phi + e_2\sin\phi + B_1\cos 2\phi + B_2\sin 2\phi\right],\\
\text{where}\quad &e_1 = 2\left[i\langle T_{11}\rangle_2 \mathrm{Re}\, i\langle T_{11}\rangle_1 - \langle T_{21}\rangle_2\, \mathrm{Re}\langle T_{21}\rangle_1\right];\\
&e_2 = 2\left[i\langle T_{11}\rangle_2\, \mathrm{Im}\, i\langle T_{11}\rangle_1 - \langle T_{21}\rangle_2\, \mathrm{Im}\langle T_{21}\rangle_1\right];\\
&B_1 = 2\langle T_{22}\rangle \mathrm{Re}\langle T_{22}\rangle_1; \quad B_2 = 2\langle T_{22}\rangle_2\, \mathrm{Im}\langle T_{22}\rangle_1 .
\end{aligned}\right\} \quad (7)$$

If one makes use of a usual frame of reference [2] and chooses $\varphi = 0^\circ$ then

$$\left.\begin{aligned}
\mathrm{Re}\, i\langle T_{11}\rangle_1 &= \sin\theta\cos\theta\,\frac{3}{8}\sqrt{\frac{15}{2}}\,\mathrm{Im}\,(C_{p_2}^* C_{p_0});\\
\mathrm{Im}\, i\langle T_{11}\rangle_1 &= \sin\theta\cos^2\theta\,\frac{15}{8}\sqrt{2}\,\mathrm{Re}(C_{p_2}^* C_{d_2});
\end{aligned}\right\} \quad (8)$$

It is seen from (7) and (8) that to determine $\mathrm{Re}\,(c^{*}_{p_2} c_{d_2})$ it is necessary to measure the "upward-downward" asymmetry in deuteron scattering on the second target.

Let us consider the scheme of the experiment (Fig.1)

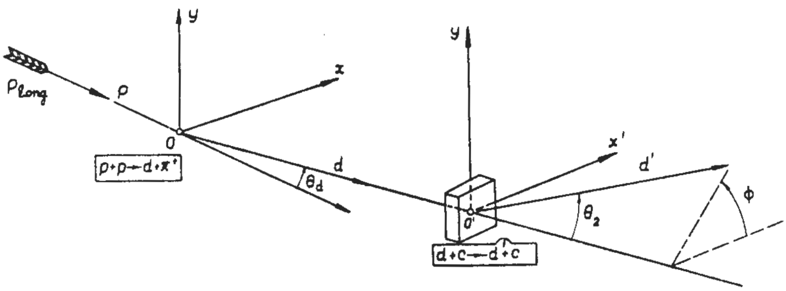

Fig. 1. The scheme of the experiment on a longitudinal polarized proton beam

The second scattering takes place in the shaded frame of reference turned with respect to an unshaded frame of reference at the angle $\theta_d^{lab.}$ Since $\theta_d^{lab.}$ is small then in the new frame of reference $(\mathrm{Im}\, i\langle T_{11}\rangle)'$ and $\mathrm{Im}\, i\langle T_{11}\rangle$ equal approximately to each other. Indeed, if we make use of the transformation formulas [2]

$$(\mathrm{Im}\, i\langle T_{11}\rangle)'_{\rightarrow} = (\mathrm{Im}\, i\langle T_{11}\rangle)_{\rightarrow} \cos\varphi + \frac{\theta_d^{lab}}{\sqrt{2}}\langle T_{10}\rangle_{\rightarrow},$$

and take into account that

$$\langle T_{10}\rangle_{\rightarrow} = \sqrt{\frac{3}{2}}\sum_{i=1,2}\left(|\alpha_i|^2 - |\gamma_i|^2\right) \sim \sin^2\theta_d \cos\theta \cdot \mathrm{Re}\,(c^{*}_{p_2} c_{d_2})$$

is equal to $\mathrm{Im}\, i\langle T_{11}\rangle_{\rightarrow}$ by the order of a magnitude, then the difference between $(\mathrm{Im}\, i\langle T_{11}\rangle)'$ and $\mathrm{Im}\, i\langle T_{11}\rangle$, due to the factor $\theta_d^{lab.}$ will be very small.

Besides the experiments on the azimuthal "upward-downward" asymmetry in deuteron scattering on a second target no additional experiments on an unpolarized beam are necessary, since the asymmetry determined by $\sin\phi$ is absent on an unpolarized beam.

§ 3.

An additional asymmetry effect determined by the coefficients B_1 and B_2 and due to the second rank spin-tensor $\langle T_{22} \rangle$ is equal to

$$\langle T_{22} \rangle_{\rightarrow} = \sqrt{3} \sum_{i=1,2} \alpha_i^* \gamma_i = -\frac{\sqrt{3}}{16} \sin^2\theta \, e^{i2\varphi} \left\{ \left(\frac{5}{2} |C_{p2}|^2 - \sqrt{10}\, \mathrm{Re}\, C_{p0}^* C_{p2} \right) + i \cos\theta \cdot 10 \sqrt{\frac{3}{2}}\, \mathrm{Im}\, C_{p2}^* C_{d2} \right\} \qquad (9)$$

on a longitudinally-polarized beam.

It follows from (7) and (9) that in order to determine $\mathrm{Im} \langle T_{22} \rangle$ it is necessary to measure the azimuthal asymmetry $\sim \sin 2\phi$. As in case of vector polarization additional experiments with an unpolarized beam are not necessary.

At the energy of a deuteron 160 MeV the quantity $\langle T_{21} \rangle_2$ is equal approximately to zero[7], however, these data are not sufficiently accurate. In the course of the measurements of the azimuthal asymmetry determined by $\sim \sin\phi$ it is neoessary that $\langle T_{21} \rangle_2$ would be smaller than $i \langle T_{11} \rangle_2$ since $\mathrm{Im} \langle T_{21} \rangle_{d\pi^+}$ and $\mathrm{Im}\, i \langle T_{11} \rangle_{d\pi^+}$ are the magnitudes of the same order.

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