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Reverse dispersion relations*

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"Reverse dispersion relations"

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In papers [1, 3] dispersion relations for pion and γ - quantum scattering and for pion photoproduction on nucleons were obtained in the frame of quantum field theory. These relations connect the real part of the amplitude of the process D(E) with the integral of Cauchy's type from the imaginary part of the amplitude A(E) and have the form

$$D(E) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{A(E')}{E'-E} dE' \tag{1}$$

The region of the negative energies is excluded from here by taking into account symmetry properties. The integral over the region of the Unobserved positive energies for forward scattering is calculated in the explicit form. This is due to the fact that in an unobserved region the function A(E) may be presented in the form of the sum of δ - function terms. The result of the integration is expressed by the coupling constants of the corresponding interactions.

Dispersion relations for pion-nucleon scattering were compared with experimental data and gave satisfactory agreement of theory with experiment in the energy region available now. /2/ The verification of dispersion relations at higher energies will be of certain interest. The thing is that the derived dispersion relations are valid only in case of fulfilling the microscopic casuality /3, 5/ and must be modified by a certain way when it fails. [3]

Dispersion relations of the type [1] may, however, appear to be unfavourable for this aim. Such a situation takes place if in the high energy region the real part of the scattering amplitude is considerably less than the imaginary one to that at present there are some indications [4]. In this case one must compare the right-hand side integral of great alternating function with the small quantity in the left-hand side of [1] . It is clear that even small experimental errors in the measurement of A may lead to great summary effect. This circumstance may impede both experimental verification of the dispersion relations of the type [1] in the high emergy region and experimental discovering of elementary length.

In this case reverse dispersion relations of the form

$$A(E) = -\frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{D(E')}{E'-E} dE' \qquad (2)$$

would be much more convenient. They will not possess the above mentioned shortcoming. However, in this case, we face the problem of the determination of the function D(E) in the positive energy unobserved region. This problem has not been solved up to now and the relations of the type [2] have not been considered.

This difficulty may be removed in the following way. As it was shown [5] the initial dispersion relation [1] is valid also for E from the unobserved region. Therefore, the function D under the integral [2] in the unobserved region of positive energies may be determined using [1] that makes it possible to exclude all unobserved quantities from equation [2]. Double integral arising here may be reduced to a single one by interchanging the order of integration.

For pion nucleon forward scattering /in the lab. system/ we obtain for scalar coefficients of scattering amplitude

$$f(E)=\delta_{\rho\rho'}f_1^1+2i(\vec{\sigma}[\vec{q}\times\vec{q}'])\delta_{\rho\rho'}f_2^1+$$
$$+\frac{[\tau_{\rho'},\tau_\rho]}{2}f_2^1+i(\vec{\sigma}[\vec{q}\times\vec{q}'])[\tau_{\rho'},\tau_\rho]f_2^2 \qquad (3)$$

the following reverse dispersion relations

$$A_i^i(E)=\frac{D_i^i(E)-D_i^i(\mu)}{\pi}\ln\frac{E+\mu}{E-\mu}-$$

$$-\frac{2E(E^2-\mu^2)}{\pi}\int_\mu^\infty\frac{dE'}{(E'^2-E^2)(E'^2-\mu)^2}\Big\{D_i^i(E')-D_i^i(\mu)+$$

$$+\frac{A_i^i(E')}{\pi}\ln\frac{E'+\mu}{E'-\mu}\Big\}-C_i\frac{f^2}{\pi}\frac{2E(E^2-\mu^2)}{E^2-(\mu^2/2M)^2}\cdot\frac{\ln\frac{1+\mu/2M}{1-\mu/2M}}{\mu^2-(\mu^2/2M)^2}$$

$$(i=1,2) \qquad (4)$$

$$A_i^K(E)=\Big(D_i^K(E)-\frac{\mu}{E}D_i^K(\mu)\Big)\frac{1}{\pi}\ln\frac{E+\mu}{E-\mu}-$$

$$-\frac{2(E^2-\mu^2)}{\pi}\int_\mu^\infty\frac{dE'E'}{(E'^2-E^2)(E'^2-\mu^2)}\Big\{D_i^K(E')-\frac{\mu}{E'}D_i^K(\mu)+$$

$$+\frac{A_i^K(E')}{\pi E'}\ln\frac{E'+\mu}{E'-\mu}\Big\}-C_i\frac{f^2}{\pi}\frac{\mu}{M}\frac{E^2-\mu^2}{E^2-(\mu^2/2M)^2}\cdot\frac{\ln\frac{1+\mu/2M}{1-\mu/2M}}{\mu^2-(\mu^2/2M)^2}$$

$$(i\neq K;\ i,K=1,2). \qquad (5)$$

Here

$$C_1 = 1, \quad C_2 = -\frac{1}{\mu^2} \tag{6}$$

From [4] and [5] we may obtain in the usual way the most interesting dispersion relations for the scattering of charged pions on protons:

$$\begin{aligned} A_+(E) = & -\frac{E^2-\mu^2}{\pi}\int_\mu^\infty \frac{dE'}{E'^2-\mu^2} P\frac{1}{E'-E}\Big\{ D_+(E') - D_+(\mu) + \\ & + \frac{A_+(E')}{\pi} \ln \frac{E'+\mu}{E'-\mu}\Big\} + \frac{E^2-\mu^2}{\pi}\int_\mu^\infty \frac{dE'}{E'^2-\mu^2} \frac{1}{E'+E}\Big\{ D_-(E') - D_-(\mu) + \\ & + \frac{A_-(E')}{\pi} \ln \frac{E'+\mu}{E'-\mu}\Big\} + \frac{1}{\pi}\Big\{ D_+(E) - \frac{E+\mu}{2E} D_+(\mu) - \\ & - \frac{E-\mu}{2E} D_-(\mu)\Big\} \ln\frac{E+\mu}{E-\mu} + \frac{2f^2}{\pi}\frac{E^2-\mu^2}{E-\mu^2/2M} \cdot \frac{\ln \frac{1-\mu/2M}{1+\mu/2M}}{\mu^2-(\mu^2/2M)^2} \end{aligned} \tag{7}$$

$$\begin{aligned} A_-(E) = & -\frac{E^2-\mu^2}{\pi}\int_\mu^\infty \frac{dE'}{E'^2-\mu^2} P\frac{1}{E'-E}\Big\{ D_-(E') - D_-(\mu) + \\ & \frac{A_-(E')}{\pi} \ln \frac{E'+\mu}{E'-\mu}\Big\} + \frac{E^2-\mu^2}{\pi}\int_\mu^\infty \frac{dE'}{E'^2-\mu^2} \cdot \frac{1}{E'+E}\Big\{ D_+(E') - \\ & - D_+(\mu) + \frac{A_+(E')}{\pi} \ln \frac{E'+\mu}{E'-\mu}\Big\} + \frac{1}{\pi}\Big\{ D_-(E) - \frac{E+\mu}{2E} D_-(\mu) - \\ & - \frac{E'-\mu}{2E} \cdot D_+(\mu)\Big\} \ln\frac{E+\mu}{E-\mu} + \frac{2f^2}{\pi} \cdot \frac{E^2-\mu^2}{E+\mu^2/2M} \cdot \frac{\ln \frac{1-\mu/2M}{1+\mu/2M}}{\mu^2-(\mu^2/2M)^2}. \end{aligned} \tag{8}$$

The "large" function A has entered into the right-hand side integrals of the relations obtained. It might, therefore, be seemed that the problem of obtaining the relations with integrals containing only small quantities has not been completely solved. However, as a matter of fact, the "large" functions A in the integrals are multiplied by a small logarithm that reduces considerably their influence. So, let us assume that total cross sections of the charged pions on protons at high energies are constant and approximately equal to $30 \cdot 10^{-27} sm^2$. Assume also the functions D_+ and D_- at high energies to be constant and approximately equal to $0,3/\mu$ [that is in agreement with [4]] we find that the contribution to the integral from A is approximately five times less than that from D.

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