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It is shown that the mathematical proceeding of Lehmann, Symanzik and Zimmermann leads necessarily to a causal field theory in which the commutators of the field operators vanish for space-like distances. In order to study this fact from a more general point of view we use extensively variational derivatives of the S-matrix with respect to the free-field operators as proposed by Bogolubov and co-workers. The manner in which Lehmann, Symanzik and Zimmermann define and apply the asymptotic condition is investigated in more detail. Concluding we make some general statements about the concept of causality in quantum field theory. It is indicated that only the causality condition in the form used by Bogolubov and co-workers (and not the commutator condition) is sufficient for a general approach to quantum field theory needed, for instance, in the theory of dispersion relations.

I. Introduction

Lehmann, Symanzik and Zimmermann[1/] have recently discussed the concept of a causal S-matrix using retarded multiple commutators of field operators. In their discussion they derived the following commutation relation for the field operator $\Phi(x)$ of a real scalar Bose-field with the destruction operator $a_{in}(\vec{q})$ of the corresponding incoming field*

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* We use a slightly different notation as in[1/]

$$[a_{in}(\vec{q}), \Phi(x)] = \frac{1}{(2\pi)^{3/2}} \int dy \frac{e^{iqy}}{\sqrt{2q^0}} (\Box_y - m^2)\bar{R}(x,y) \tag{1}$$

where $\bar{R}(x,y)$ is the retarded commutator

$$\bar{R}(x,y) = -i\,\theta(x-y)[\Phi(x), \Phi(y)] \tag{2}$$

(they indeed derived expressions for generalized R-products of n field operators, however, for our purposes (1),(2) are sufficient). In their derivation they assumed that $\Phi(x)$ may also be a non-causal field operator which does not necessarily satisfy the causality condition in the commutator form

$$[\Phi(x), \Phi(y)] = 0 \quad \text{if} \quad x \sim y \tag{3}$$

where $x \sim y$ means that $x-y$ is space-like. However, it is easily to show that the operator $\Phi(x)$ in (1),(2) has <u>necessarily</u> to be a <u>causal</u> operator which satisfies (3). For the purposes of a more general and – as far as possibly – complete discussion of this fact we derive in section 2 some commutation relations of the S-matrix with the free-field operators in terms of variational derivatives of the S-matrix with respect to these operators as proposed by Bogolubov and co-workers[2/]. However, we disinguish explicitly between incoming and outgoing fields. In section 3 we show that the application of the asymptotic condition as performed by Lehmann, Symanzik and Zimmermann leads immediately to a causal field theory. This fact is investigated in more detail which leads us to the result that neither their definition nor their application of the asymptotic condition is sufficiently defined. The function $\Theta(x-y)$ in (2),(1), for instance, is quite arbitrary: it has only to fulfill the condition $\Theta(x-y)=1$ if $y^0=-\infty$ and $\Theta(x-y)=0$ if $y^0=+\infty$ with vanishing derivatives at these limits. In section 4 we make some general statements about the concept of causality in quantum field theory. The investigations indicate that only the causality condition in the form used by Bogolubov and co-workers (and not the commutator condition (3)) is sufficient for a general approach to quantum field theory needed, for instance, in the theory of dispersion relations.

2. S-Matrix and Causality Condition

We assume the following structure for the S-matrix[2/]*

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* We remark that the elements of S with respect to states with a finite number of particles are represented by finite sums, so that no problems of convergence arise.

$$S=\sum_{n=0}^{\infty}\int dx_1\ldots dx_n\, f_n(x_1,\ldots,x_n):\phi_{in}(x_1)\ldots\phi_{in}(x_n): \qquad (4)$$

where $\phi_{in}(x)$ describes the incoming particles

$$(\Box-m^2)\phi_{in}(x)=0 \qquad [\phi_{in}(x),\phi_{in}(y)]=i\Delta(x-y) \qquad (5)$$

We write

$$\left.\begin{aligned}&\phi_{in}(x)=\frac{1}{(2\pi)^{3/2}}\int\frac{d\vec{q}}{\sqrt{2q^0}}\left\{e^{iqx}a^*_{in}(\vec{q})+e^{-iqx}a_{in}(\vec{q})\right\}\\&\qquad q^0=+\sqrt{m^2+\vec{q}^{\,2}}\end{aligned}\right\} \qquad (6)$$

where

$$[a_{in}(\vec{q}),a^*_{in}(\vec{q}\,')]=\delta(\vec{q}-\vec{q}\,') \qquad [a_{in}(\vec{q}),a_{in}(\vec{q}\,')]=0 \qquad (7)$$

Then it follows immediately from the assumption (4) and (6) and (7)

$$[a_{in}(\vec{q}),S]=\frac{1}{(2\pi)^{3/2}}\int dx\,\frac{e^{iqx}}{\sqrt{2q^0}}\,\frac{\delta S}{\delta\phi_{in}(x)}\,, \qquad [S,a^*_{in}(\vec{q})]=\frac{1}{(2\pi)^{3/2}}\int dx\,\frac{e^{iqx}}{\sqrt{2q^0}}\,\frac{\delta S}{\delta\phi_{in}(x)} \qquad (8)$$

Of course, we assume

$$SS^{+} = S^{+}S = 1 \tag{9}$$

Further we introduce the field operator $\Phi_{out}(x)$ which describes the outgoing particles

$$\Phi_{out}(x) = S^{+}\Phi_{in}(x)S \tag{10}$$

Because of (9) it is possible to write (4) in the same manner for the outgoing fields

$$S = \sum_{n=0}^{\infty} \int dx_1 \ldots dx_n f_n(x_1, \ldots, x_n) :\Phi_{out}(x_1) \ldots \Phi_{out}(x_n): \tag{11}$$

The relations (5)-(8) are valid also for the outgoing fields without any change (replace only the index in by out).

Following Bogolubov and co-workers we define the current operator by*/**

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*Strictly speaking Bogolubov and co-workers use only the second expression for the outgoing field.

** Another definition for a current would be $j'(x) = i\frac{\delta S}{\delta\Phi_{in}(x)}S^{+}$ however, because of $\frac{\delta S}{\delta\Phi_{in}(x)}S^{+} = S\frac{\delta S}{\delta\Phi_{out}(x)}S^{+}S^{+}$ this definition seems not very useful (compare also (21), where such an expression does not appear).

$$j(x) = i\,S^{+}\frac{\delta S}{\delta\Phi_{in}(x)} = i\,\frac{\delta S}{\delta\Phi_{out}(x)}S^{+} \tag{12}$$

Because of (10) the two expressions on the right define indeed the same j(x) since for the S-matrix we have (4) and (9). The last implies

$$j^{+}(x) = -i\frac{\delta S^{+}}{\delta\Phi_{in}(x)}S = -iS\frac{\delta S^{+}}{\delta\Phi_{out}(x)} = j(x) \tag{13}$$

Then we get

$$\frac{\delta j(x)}{\delta\Phi_{in}(y)} = iS^{+}\frac{\delta^2 S}{\delta\Phi_{in}(y)\delta\Phi_{in}(x)} + i\,j(y)j(x) \tag{14}$$

$$\frac{\delta j(x)}{\delta\Phi_{out}(y)} = i\,\frac{\delta^2 S}{\delta\Phi_{out}(y)\delta\Phi_{out}(x)}S^{+} + i\,j(x)j(y) \tag{14'}$$

Because of

$$\frac{\delta^2 S}{\delta \Phi_{\substack{in\\out}}(y)\,\delta \Phi_{\substack{in\\out}}(x)} = \frac{\delta^2 S}{\delta \Phi_{\substack{in\\out}}(x)\,\delta \Phi_{\substack{in\\out}}(y)} . \tag{15}$$

it follows

$$\frac{\delta j(x)}{\delta \Phi_{\substack{in\\out}}(y)} - \frac{\delta j(y)}{\delta \Phi_{\substack{in\\out}}(x)} = \mp i\,[j(x), j(y)] \tag{16}$$

Following Bogolubov and co-workers we define a causal S-matrix by

$$\frac{\delta j(x)}{\delta \Phi_{in}(y)} = 0 \qquad \text{if} \quad y \gtrsim x \tag{17}$$

$$\frac{\delta j(x)}{\delta \Phi_{out}(y)} = 0 \qquad \text{if} \quad y \lesssim x \tag{17'}$$

where $y \gtrsim x$ respectively $y \lesssim x$ means that y is later respectively earlier than x or $x-y$ is space-like. From (16) the causality condition follows then in the usual commutator form

$$[j(x), j(y)] = 0 \qquad \text{if} \quad x \sim y \tag{18}$$

and also the representation

$$\frac{\delta j(x)}{\delta \Phi_{\substack{in\\out}}(y)} = i \left\{ \begin{matrix} -\Theta(x-y) \\ \theta(y-x) \end{matrix} \right\} [j(x), j(y)] \tag{19}$$

We remark that it is not possible to derive from (18) the condition (17),(17') or the representation (19) (compare (14) and (16), from (16), for instance, it follows only

$$\frac{\delta j(x)}{\delta \Phi_{\substack{in\\out}}(y)} = \frac{\delta j(y)}{\delta \Phi_{\substack{in\\out}}(x)} \qquad \text{for} \qquad x \sim y ;$$

see also the discussion in section 4). On the other hand the representation (19) yields immediately the causality condition in the form (17),(17') since it follows from (19)

$$\frac{\delta j(x)}{\delta \Phi_{\substack{in\\out}}(y)} = 0 \qquad \text{for} \qquad y \gtrless x \tag{20}$$

and for the reason of covariance that has to held also for $x \sim y$. The last statement yields immediately from (19) also the causality condition in the commutator form (18) (without using any further relation). Thus we arrive at the result: <u>the representation (19) defines already a causal field theory.</u>

Concluding this section we derive a further relation needed for the following. From (10), (9), (6) and (8) it follows

$$\Phi_{out}(x) = S^+ \Phi_{in}(x) S = \Phi_{in}(x) + S^+ [\Phi_{in}(x), S] =$$

$$= \Phi_{in}(x) + [\Phi_{out}(x), S] S^+ = \Phi_{in}(x) + \int \Delta(x-y) j(y) dy \tag{21}$$

where $j(x)$ is defined by (both expressions) (12) and $\Delta(x-y)$ as usually. (21) may be used for a reasoning of the definition (12) for the current operator.

3. S-Matrix and Asymptotic Condition

Lehmann, Symanzik and Zimmermann proceed a step further and introduce a field operator $\Phi(x)$ by

$$\Phi(x) = \Phi_{\substack{in \\ out}}(x) - \int \Delta_{\substack{ret \\ adv}}(x-y) j(y) dy \tag{22}$$

$$(\Box - m^2)\Phi(x) = j(x) \tag{22'}$$

which "interpolates" between past and future, i.e. between $\Phi_{in}(x)$ and $\Phi_{out}(x)$ for which we have the connection (10) and (21). They further assume the asymptotic condition*

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* In their mathematically more rigorous treatment they use a discrete orthonormal system $\{f_\alpha(x)\}$ instead of $\left\{\frac{1}{(2\pi)^{3/2}} \frac{e^{-iqx}}{\sqrt{2q^0}}\right\}$ which indeed is necessary in the last step in (27), where an integration by parts is performed. Also the relation (23) is defined in such a manner that the operators stay within a matrix element. However, for our purposes the above treatment is sufficient. We remark that there is a difference in sign in[1] between (18) and the application of the asymptotic condition on page 328.

$$\lim_{t \to \mp\infty} a(\vec{q}, t) = a_{\substack{in \\ out}}(\vec{q}) \tag{23}$$

where

$$\left.\begin{aligned} a(\vec{q},t) &= \frac{-1}{(2\pi)^{3/2}} \; i\int d\vec{x}\, \phi(x) \frac{\overleftrightarrow{\partial}}{\partial x^0} \frac{e^{iqx}}{\sqrt{2q^0}} = \\ &= \frac{-1}{(2\pi)^{3/2}}\, i\int d\vec{x} \left\{ \phi(x) \frac{\partial}{\partial x^0} \frac{e^{iqx}}{\sqrt{2q^0}} - \frac{\partial}{\partial x^0}\phi(x) \frac{e^{iqx}}{\sqrt{2q^0}} \right\} \end{aligned}\right\} \quad (24)$$

$$\phi(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d\vec{q}}{\sqrt{2q^0}} \left\{ e^{iqx} a^*(\vec{q},t) + e^{-iqx} a(\vec{q},t) \right\}; \quad q^0 = +\sqrt{m^2+\vec{q}^{\,2}} \quad (25)$$

Using the condition (23) in connection with (24),(25) they calculate the commutator

$$[a_{in}(\vec{q}), \phi(x)] \quad (26)$$

to the form (1),(2) according to

$$\begin{aligned} [a_{in}(\vec{q}), \phi(x)] &= \lim_{y^0 \to -\infty} [a(\vec{q},y^0), \phi(x)] = \\ &= \lim_{y^0\to-\infty} \frac{1}{(2\pi)^{3/2}}\, i\int d\vec{y}\, [\phi(x),\phi(y)] \frac{\overleftrightarrow{\partial}}{\partial y^0} \frac{e^{iqy}}{\sqrt{2q^0}} \\ &= \lim_{y^0\to-\infty} \frac{1}{(2\pi)^{3/2}} \int d\vec{y}\; i\,\theta(x-y)[\phi(x),\phi(y)] \frac{\overleftrightarrow{\partial}}{\partial y^0} \frac{e^{iqy}}{\sqrt{2q^0}} \\ &= \frac{1}{(2\pi)^{3/2}} \int dy \frac{\partial}{\partial y^0} \left\{ -i\,\theta(x-y)[\phi(x),\phi(y)] \frac{\overleftrightarrow{\partial}}{\partial y^0} \frac{e^{iqy}}{\sqrt{2q^0}} \right\} \\ &= \frac{1}{(2\pi)^{3/2}} \int dy \frac{e^{iqy}}{\sqrt{2q^0}} (\Box_y - m^2) \bar{R}(x,y) \end{aligned} \quad (27)$$

where $\bar{R}(x,y)$ is defined by (2).

Applying $(\Box_x - m^2)$ on (27) and using (22') we get

$$[a_{in}(\vec{q}), j(x)] = \frac{1}{(2\pi)^{3/2}} \int dy \frac{e^{iqy}}{\sqrt{2q^0}} (\Box_x - m^2)(\Box_y - m^2)\bar{R}(x,y) \quad (28)$$

We remark that the commutation relation (28) is not uniquely defined: if we directly replace $\phi(x)$ by $j(x)$ in (27) we get instead of (28)

$$[a_{in}(\vec{q}), j(x)] = \frac{1}{(2\pi)^{3/2}} \int dy \frac{e^{iqy}}{\sqrt{2q^0}} (\Box_y - m^2)\left\{ -i\,\theta(x-y)(\Box_x - m^2)[\phi(x),\phi(y)] \right\} \quad (29)$$

i.e. the operator $(\Box_x - m^2)$ stays now to the right of $\theta(x-y)$. However, it is

$$\left.\begin{aligned} (\Box_x - m^2)\{\theta(x-y)[\phi(x),\phi(y)] - \theta(x-y)(\Box_x - m^2)[\phi(x),\phi(y)] = \\ = P\left(\frac{\partial}{\partial x^0}\right)\delta(x^0-y^0) \end{aligned}\right\} \quad (30)$$

where $P\left(\frac{\partial}{\partial x^0}\right)$ is a polynomial (of first order) in $\frac{\partial}{\partial x^0}$ with coefficients which depend on $\vec{x},\vec{y}$ and x^0. On the other hand $\theta(x-y)$ is only defined for $x^0 \gtrless y^0$ but not for $x^0 = y^0$

so that (30) yields no further indefinitness of the theory*/**. In the same manner we

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* Compare the same situation in the definition of the T-product in[2/] footnote 1 on page 185 in the German translation.

** Also the derivation (27) is only defined in the same uncomplete manner since $\overleftrightarrow{\frac{\partial}{\partial y^0}}$ may operate on $\theta(x-y)$ or may not. We assumed the first case.

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see that (28) or (29) is equivalent to

$$[a_{in}(\vec{q}), j(x)] = \frac{1}{(2\pi)^{3/2}} \int dy \, \frac{e^{iqy}}{\sqrt{2q^0}} \left\{ -i\,\theta(x-y)[j(x), j(y)] \right\} \tag{31}$$

Now the next step is clear: from section 2 we know that the commutator (31) may also be written in the form

$$[a_{in}(\vec{q}), j(x)] = \frac{1}{(2\pi)^{3/2}} \int dy \, \frac{e^{iqy}}{\sqrt{2q^0}} \, \frac{\delta j(x)}{\delta \phi_{in}(y)} \tag{32}$$

Thus we arrive at the result

$$\frac{\delta j(x)}{\delta \phi_{in}(y)} = -i\,\theta(x-y)[j(x), j(y)] \tag{33}$$

and from the remarks made after (20) we conclude that the application of the asymptotic condition (23) leads immediately to a causal field theory.

The situation is now the following: in (22) there was constructed a field operator $\phi(x)$ where j(x) may be assumed as a causal operator or a non-causal one (in the last case we exspect that $\phi(x)$ is also a non-causal one). However, the application of the asymptotic condition (23) in connection with (24),(25) yields the result that in <u>any</u> case the operator j(x) has necessarily to be a causal one. Thus we have to conclude that either tha definition of the asymptotic condition or its application or even both of them are not sufficiently defined.

The last is indeed the case: if we substitute (25) in (24) we get

$$a(\vec{q}, t) = a(\vec{q}, t) + \frac{i}{2q^0} \frac{\partial}{\partial t} a(\vec{q}, t) \tag{34}$$

if we define $\frac{\partial}{\partial x^0}\phi(x)$ in such a manner that we have to differentiate $a(\vec{q},t)$(or $a^*(\vec{q},t)$ respectively) also with respect to the time, i.e. that we have to differentiate with respect to the full time-dependence of $\phi(x)$. Only if we define $\frac{\partial}{\partial x^0}\phi(x)$ in such a manner that only the differentiation with respect to the time-dependence in the exponentials $e^{\mp iqx}$ * is meant the term $\frac{i}{2q^0}\frac{\partial}{\partial t}a(\vec{q},t)$ does not appear in (34) (that we

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* Strictly speaking we have to use wave packets $\{f_\alpha(x)\}$ (compare footnote* on page 7) however, that does not change the situation since (34) may be produced in a continuous manner from the case of wave packets.

have to require). Thus we see: the time-differentiation in (24) is not well defined and correspondingly the asymptotic condition (23) since we assume (24). In the explicit application of the asymptotic condition in (27) we assumed indeed that the differentiation with respect to the full time-dependence in $\Phi(x)$ is meant. However, then we have the contradiction (34).

The following fact seems even more important: the function $\Theta(x-y)$ introduced in (27) is quite arbitrary; it has only to fulfill the condition $\Theta(x-y)=1$ if $y^o=-\infty$ and $\theta(x-y)=0$ if $y^o=+\infty$ and if we assume that $\frac{\overleftrightarrow{\partial}}{\partial y^o}$ acts also on $\Theta(x-y)$, then the time-derivatives of $\Theta(x-y)$ have to vanish correspondingly at these limits (see also the appendix). That expresses the fact that it is well possible to represent a function at a fixed point as an integral over a definite intervall, however, the integrand is not uniquely defined.

Thus we have to conclude: neither the definition nor the application of the asymptotic condition is sufficiently defined in the approach of Lehmann, Symanzik and Zimmermann.

4. The Concept of Causality

We generalize our above considerations and state: a field theory into which expressions like the T-product

$$T(x,y)=Tj(x)j(y) \tag{35}$$

or the R-product

$$R(x,y)=-i\,\Theta(x-y)[j(x),\,j(y)] \tag{2'}$$

enter as scalar quantities (and the S-matrix or - strictly speaking - the T-matrix is expressed by them directly as their Fourier-transform) has to be a causal field theory. The proof for this statement is quite simple (these things are by no means new in principle): the T-product of some scalar operators may be a scalar, i.e. an invariant expression, only and if only these operators commute in space-like regions since time-ordering has a covariant meaning only in time-like regions. The R-product (2') may be also a scalar only and if only the operators $j(x)$, $j(y)$ commute in space-like regions since it vanishes for $x<y$ and for the reason of covariance it has to vanish also in the whole space-like region.

On the other hand if we write (compare (14) and (16))

$$- S^{+}\frac{\delta^2 S}{\delta\Phi_{in}(x)\delta\Phi_{in}(y)} = j(y)j(x) + i\frac{\delta j(x)}{\delta\Phi_{in}(y)} =$$

$$= Tj(x)j(y) + i\,\Theta(x-y)\frac{\delta j(y)}{\delta\Phi_{in}(x)} + i\,\Theta(y-x)\frac{\delta j(x)}{\delta\Phi_{in}(y)} \qquad (36)$$

we cannot conclude that $Tj(x)j(y)$ has to be a scalar (i.e. j(x) has to be a causal operator) since only the whole expression on the right or (36) has to be a scalar. In such a formalism we arrive at no contradiction. If we require causality in the form (17) the last two terms in (36) vanish and $Tj(x)j(y)$ is also a scalar. However, if we assume causality only in the commutator form (18) we may conclude only that

$$i\,\Theta(x-y)\frac{\delta j(y)}{\delta\Phi_{in}(x)} + i\,\Theta(x-y)\frac{\delta j(x)}{\delta\Phi_{in}(y)} \qquad (37)$$

is a scalar which in addition is indefinable. We cannot conclude that (37) has to vanish identically (for the rest it would follow from this the causality condition in the form (17)). Thus we arrive once more at the result: the causality condition in the commutator form (18) is not sufficient to determine (36) as the T-product (that is the same situation as for the representation (19)) which is needed, for instance, in the theory of dispersion relations.

Let us, however, proceed a step further. The essential physical difference between the causality condition (17), (17') and (18) is that the first distinguishes time and also yields a causality condition for time-like regions. The causality condition in the commutator form (18) says nothing about causality for time-like distances and seems therefore uncomplete. Thus the question arises: how it is possible that this condition may be sufficient to define a field theory as a causal one which only uses the S-matrix and field operators but nothing more. Of course, Lehmann, Symanzik and Zimmermann use the wave equation (22') but only as a definition for $\Phi(x)$ according to (22) and it seems very unlikely that the asymptotic condition could be a substitute for a causality condition in time-like regions. In section 3 it was shown that their approach leads indeed to quite arbitrary results. On the other hand it was shown above that the causality condition (17), (17') used by Bogolubov and co-workers is a mathematically sufficient expression for causality in the sense that such a condition leads immediately to a prescribed time-ordering in time-like regions which, of course, is irrelevant in space-like regions: if we define causality only for space-like regions time does not appear as a distinguished quantity (commutator condition (18)). If we add to a relativistic theory the condition of causality we necessarily distinguish time, however, that does not violate the requirement for covariance (that would only be the case if we require a defined time-ordering in space-like regions*). In a theory of dispersion relations we need indeed this

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* In order to be even more strict: causality does not distinguish a time-direction but only prescribes a defined time-ordering. The use of incoming or outgoing fields is completely equivalent.

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form of causality.

The following statement may also be important (which was already mentioned in footnote 8 of[3]): the causality condition (17), (17') has such a form that it also defines causality in a non-relativistic theory (where the commutator condition (18) loses its meaning). We have simply to replace $y \geq x$ by $y > x$ in (17) or $y \leq x$ by $y < x$ in (17') respectively. Then a cut-off meson theory, for instance, which treats the nucleons non-relativistically is necessarily a causal theory (the Hamiltonian be time-independent). Of course, such a theory is not a local one. Further it is not necessary to make a second-quantization procedure to define a current operator in a field-theoretical way: the usual treatment with Schrödinger wave functions is sufficient ($j(x)$ is well defined by (12)). Especially this is valid for the static Chew model[4] which we have to define as a causal but non-local theory* (it is interesting to look at the remarks after equation (61) in[4] from our point of view).

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* A relativistic form factor theory is, of course, non-causal and non-local, however, for a non-relativistic theory these things need not be the same.

I would like to thank Dr. Medvedev for reading the manuscript and a valuable discussino.

Appendix

One can try to examine the proceeding of Lehmann, Symanzik and Zimmermann for the case where $a(\vec{q},t)$ is not given by (24) but by the usual relation

$$a(\vec{q},t)=\frac{1}{(2\pi)^{3/2}}\int d\vec{x}\,\Phi(x)\sqrt{2q^0}\,e^{iqx} \tag{A.1}$$

and require the asymptotic condition as in (23)

$$\lim_{t\to\mp\infty} a(\vec{q},t)=a_{\substack{in\\out}}(\vec{q}) \tag{A.2}$$

Then we write instead of (27)

$$\begin{aligned}[a_{in}(\vec{q}),\Phi(x)] &= \lim_{y^0\to-\infty}[a(\vec{q},y^0),\Phi(x)]\\ &=\lim_{y^0\to-\infty}\frac{1}{(2\pi)^{3/2}}\int d\vec{y}\,[\Phi(y),\Phi(x)]\sqrt{2q^0}\,e^{iqy}\\ &=\lim_{y^0\to-\infty}\frac{1}{(2\pi)^{3/2}}\int d\vec{y}\,\Theta(x-y)[\Phi(y),\Phi(x)]\sqrt{2q^0}\,e^{iqy}\\ &=\frac{1}{(2\pi)^{3/2}}\int dy\,\frac{\partial}{\partial y^0}\left\{\Theta(x-y)\,[\Phi(x),\Phi(y)]\sqrt{2q^0}\,e^{iqy}\right.\end{aligned} \tag{A.3}$$

For the reason of covariance ($\sqrt{2q^0}[a_{in}(\vec{q}),\Phi(x)]$ has to be a scalar) we may then conclude that $\Phi(x)$ has to be a causal operator. However, we have to notice that the introduction of the Θ -function is not well defined: the only requirement is that the function $\Theta(x-y)$ introduced in (A.3) has to satisfy

$$\lim_{y^0\to\mp\infty}\Theta(x-y)=\begin{cases}1\\0\end{cases} \tag{A.4}$$

which yields a great lot of arbitrariness (of course, we assume that (A.4) does not influence the limiting value of its co-factor in (A.3). Nevertheless it may be that formula (A.3) is useful for an approach to quantum field theory which avoids the explicit use of variational derivatives.

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