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Laboratory of Theoretical Physics

V.G. Soloviev

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A B S T R A C T

Variational principle, proposed by N.N. Bogolubov, physical ideas and mathematical methods, developed in the theory of superconductivity were applied to the study of heavy nucleus properties. Basing on the nuclear shell model there were considered residual interactions of nucleons being on the outer shell, that lead to appearing of a superfluid state of a nucleus. The energies of the ground superfluid state and of a set of excited states in case of an even as well as of an odd number of nucleons on the shell were calculated. Some conformities with a law were revealed in a spectrum of levels of even-even nuclei and of nuclei with an odd A. Changes of the ground nuclear state energy were calculated in case of changes by a unity of the number of the outer shell nucleons. That made it possible to conclude of greater stability of even-even nuclei than of odd-odd nuclei as to β -decay, that numerically conforms to Weizsäcker semi-empirical formula.

The obtained results weakly depend on the chosen nuclear model, they are also valid for highly deformed nuclei.

Some similarity of properties of nuclear and metal Fermi-systems allows to apply physical ideas and mathematical methods developed in the theory of superconductivity[I] to the study of properties of the nuclear matter and a finite nucleus[2-7]. Basing on the nuclear shell model, it is shown in[5-7], that interactions of the outer shell nucleons, lead to the superfluid (energetically lower state than that of the completely degerated Fermi-gas) nuclear state.

In this work further investigation of the superfluid nuclear state will be carried on, that is the energies of excited states of the shell with an even as well as with an odd nucleon number will be calculated and stability of nuclear isobars as to β -decay will be considered.

1. Ground Superfluid State

We shall investigate the superfluid state of the atomic nucleus with the help of the new variational principle, proposed by Bogolubov[8], which is the generalization of Fock well-known method[9].

We accept the following model of a heavy nucleus: nucleons, forming inner shells, create the central-symmetrical field, which is somewhat distorted by the outer shell nucleons. Let us consider residual interactions of the nucleons (protons or neutrons), situating on the outer nuclear shells. We shall characterize the nucleon state by a set of quantum numbers s determining the shell, and by the quantum numbers j and m total angular momentum and of its projection along the symmetry axis of the nucleus. We think that the nuclear field slightly deviates from the central-symmetric field, therefore there will be no energetical degeneration over m .

Let us consider interactions of the nucleons with identical s, but with any quantum numbers j and m being in close proximity to Fermi surface energy.
The Hamiltonian describing such interactions will be written in the following form:

$$H_s = \sum_{j,m,\rho} \{E(j,\rho m) - \lambda\} a_{m\rho}(j)^+ a_{m\rho}(j) +$$

$$+ \frac{1}{4} \sum_{\substack{j_1,j_2,j_1',j_2' \\ j_1+j_2=j_1'+j_2'}} \sum_{\substack{m_1,m_2,m_1',m_2' \\ \rho_1,\rho_2,\rho_1',\rho_2' \\ \rho_1 m_1+\rho_2 m_2=\rho_1' m_1'+\rho_2' m_2' \\ \rho_1 m_1 \neq \rho_1' m_1'}} J(j_1,j_2;j_1',j_2' | \rho_1 m_1,\rho_2 m_2;\rho_1' m_1',\rho_2' m_2') \cdot$$

$$\cdot a_{m_1\rho_1}(j_1)^+ a_{m_2\rho_2}(j_2)^+ a_{m_2'\rho_2'}(j_2') a_{m_1'\rho_1'}(j_1'), \qquad \text{(I)}$$

here summation is being carried on over positive values of m and over ρ, $\rho = \pm 1$ characterizing the sign of m; $a_{m\rho}(j)^+, a_{m\rho}(j)$ are operators of nucleon production and absorbtion $E(j,m)$ is the energy of the nucleon, which is on the outer shell in j, m state, λ is the parameter playing the role of chemical potential, which is determined from the condition:

$$n_j = \sum_{m,\rho} \overline{a_{m\rho}(j)^+ a_{m\rho}(j)} . \qquad (2)$$

The function $J(j_1, j_2; j_1', j_2' | \rho_1 m_1, \rho_2 m_2; \rho_1' m_1', \rho_2' m_2')$ is a real one and possesses the following properties:

$$J(j_1, j_2; j_1', j_2' | \rho_1 m_1, \rho_2 m_2; \rho_1' m_1', \rho_2' m_2') = J(j_1', j_2'; j_1, j_2 | \rho_1' m_1', \rho_2' m_2'; \rho_1 m_1 \rho_2 m_2),$$

$$J(j_1, j_2; j_1', j_2' | \rho_1 m_1, \rho_2 m_2; \rho_1' m_1', \rho_2' m_2') = J(j_2, j_1; j_2', j_1' | \rho_2 m_2, \rho_1 m_1; \rho_2' m_2' \rho_1' m_1'), \qquad (3)$$

$$J(j_1, j_2; j_1', j_2' | m_1, m_2; m_1', m_2') = J(j_1, j_2; j_1', j_2' | -m_1, -m_2, -m_1', -m_2').$$

Let us perform linear canonical transformation of the Fermi-amplitudes

$$a_{m\rho}(j) = u_{mj} \alpha_{m,-\rho}(j) + \rho v_{mj} \alpha_{m\rho}(j)^+ , \qquad (4)$$

In order that it might not violate their commutation properties it is necessary that the following condition would be fulfilled:

$$\eta_m(j) = u_{mj}^2 + v_{mj}^2 - 1 = 0 . \qquad (5)$$

Let us define a new vacuum state Ψ

$$\alpha_{m\rho}(j)\Psi = 0$$

and find the mean value of H_s by the new vacuum state

$$\bar{H}_s = \langle \Psi^* H_s \Psi \rangle \equiv \langle H_s \rangle =$$

$$= 2\sum_{j,m} \{E(j,m) - \lambda\} V_{mj}^2 +$$

$$+ \sum_{j,m,m'} J(j|m,-m;m',-m') U_{mj} V_{mj} U_{m'j} V_{m'j} - \qquad (6)$$

$$- \sum_{j,j',m,m'} \frac{1}{2}\{J(j,j'|m,m';m',m) + J(j,j'|m,-m';-m',m)\} V_{mj}^2 V_{m'j'}^2,$$

where

$$J(j,j;j,j|\ldots) \equiv J(j|\ldots), \quad J(j,j';j',j|\ldots) \equiv J(j,j'|\ldots).$$

Let us define U_{mj} , V_{mj} from the minimum condition of $\bar{H}_s$ in the presence of an additional condition (5):

$$\delta\{\bar{H}_s + \sum_{j,m} \lambda_{mj} \eta_m(j)\} = 0, \qquad (7)$$

here λ_{mj} is the Lagrangian factor. Considering the variations of δU_{mj}, δV_{mj} as independent ones, we shall receive

$$2\tilde{\xi}(j,m) U_{mj} V_{mj} + (U_{mj}^2 - V_{mj}^2) \sum_{m'} J(j|m,-m;m',-m') U_{m'j} V_{m'j} = 0 \quad (8)$$

where

$$\tilde{\xi}(j,m) \equiv E(j,m) - \lambda - \sum_{j',m'} \frac{1}{2}\{J(j,j'|m,m';m',m) + J(j,j'|m,-m';-m',m)\} V_{m'j'}^2. \quad (9)$$

For $\mathcal{U}_{mj}$, $\mathcal{V}_{mj}$ satisfying (5) and (8), $\bar{H}_s$ gives the ground state energy. The equation (8) permits a trivial solution

$$\mathcal{U}_{mj} = 1 - \theta_F(j,m), \quad \mathcal{V}_{mj} = \theta_F(j,m),$$

which corresponds to the state of completely degenerated Fermi-gas ($\theta_F(j,m)=1$ if $E(j,m) < E_F$ and $\theta_F(j,m)=0$ if $E(j,m) > E_F$).

In[4,7] is shown, that for appearance of non-trivial solution (8) it is required that

$$J(j|m,-m;m',-m') < 0 \qquad (10)$$

if energies are in the neighbourhood of the Fermi surfaces energy.

To solve (8) we introduce a new unknown function

$$C_{mj} = \sum_{m'} J(j|m,-m;m',-m')\,\mathcal{U}_{m'j}\mathcal{V}_{m'j},$$

connected with $\mathcal{U}_{mj}$ and $\mathcal{V}_{mj}$ in the following way:

$$\mathcal{U}_{mj}^2 = \frac{1}{2}\left[1 + \frac{\xi(j,m)}{\tilde{\varepsilon}(j,m)}\right], \quad \mathcal{V}_{mj}^2 = \frac{1}{2}\left[1 - \frac{\xi(j,m)}{\tilde{\varepsilon}(j,m)}\right],$$

$$\mathcal{U}_{mj}\mathcal{V}_{mj} = -\frac{1}{2}\,\frac{C_{mj}}{\tilde{\varepsilon}_{mj}}, \quad \tilde{\varepsilon}(j,m) = \sqrt{C_{mj}^2 + \xi(j,m)^2}, \qquad (11)$$

and satisfying the equation

$$C_{mj} = -\frac{1}{2}\sum_{m'} J(j|m,-m;m',-m')\frac{C_{m'j}}{\sqrt{C_{mj}^2+\xi(j,m)^2}} \tag{12}$$

There is obtained the asymptotic (if $J \longrightarrow 0$) solution (12) in[5], and shown that the interactions of nucleons with equal and opposite angular momentum projections along the symmetry axis of the nucleus play the leading part. In[6,7] solutions (12) are found in supposition that $J(j|m,-m;m',-m')$ does not depend on m , m' and it is shown that in this crude approximation the main properties of the asymptotic solution are conservated.

Let us consider the approximation: $J(j|m,-m;m',-m')$ does not depend on m , m' and $\rho_j(E)$ does not depend on E, i.e.

$$J(j) = Const\,, \quad \rho_j = Const\,. \tag{13}$$

We ignore the interactions of nucleons with various m, j because as is shown in[5], they will give a small correction, that can be calculated by the method of the perturbation theory. In this case $\xi(j,m)$ and $\hat{\varepsilon}(j,m)$ ought to be replaced by

$$\xi(j,m) = E(j,m) - \lambda\,, \quad \varepsilon(j,m) = \sqrt{C_{mj}^2 + \xi(j,m)^2}$$

We examined the interaction of the nucleons which were situating on the nuclear outer shell, characterized by the quantum number j (unlike the case considered in[7]) being able to take various values. The energy of the nucleons, being on this shell was in the neighbourhood of the Fermi surface i.e.

$$E_F - \delta_j \leq E(j,m) \leq E_F + \Delta_j ,$$

where all the levels with data j were filled, if $\delta_j > 0$, $\Delta_j < 0$ and free if $\delta_j < 0$, $\Delta_j > 0$.

Let us now pass in (12) from the sum to the integral and obtain

$$C_{mj} = -\frac{1}{2} \int_{E_F-\delta_j}^{E_F+\Delta_j} dE' \rho_j(E') J(j|m,-m;m',-m') \frac{C_{mj}}{\sqrt{C_{mj}^2 + (E'-\lambda)^2}} \tag{12'}$$

in the approximation (13) it will take the form

$$1 = -\frac{1}{2}\rho_j J(j) \int_{E_F-\delta_j}^{E_F+\Delta_j} \frac{dE'}{\sqrt{C_j^2 + (E'-\lambda)^2}} . \tag{14}$$

Let us obtain the equation to find λ , namely:

$$n_j = \rho_j \int_{E_F-\delta_j}^{E_F+\Delta_j} dE \left\{ 1 - \frac{dE'}{\sqrt{C_j^2 + (E'-\lambda)^2}} \right\} . \tag{15}$$

The solutions (14) and (15) will be written in the following form:

$$C_j = \frac{\sqrt{(2\Omega_j - n_j) n_j}}{\rho_j (e^{\frac{2}{G(j)}} - 1)} e^{\frac{1}{G(j)}} , \tag{16}$$

$$E_F - \lambda = \frac{\Omega_j - n_j}{\rho_j (e^{\frac{2}{G(j)}} - 1)} + \delta_j - \frac{n_j}{2\rho_j} , \tag{17}$$

where $G(j)=-\beta_j J(j)$, n_j is the nucleon number, and Ω_j is the levels number with the given values of j

In cases when $\delta_j \Delta_j < 0$ $c_j = 0$ and in case when $\Delta_j > 0$, $\delta_j > 0$

$$E_F - \lambda = \frac{\Omega_j - n_j}{\beta_j\left(e^{\frac{2}{G(j)}} - 1\right)} .$$

In the approximation

$$G \gg 1 \quad , \quad |E_{max}(j) - E_{min}(j)| << \frac{\Omega_j G(j)}{2\beta_j} , \tag{18}$$

$\mathcal{E}(j,m)$ function has for all cases $\delta_j \Delta_j \lessgtr 0$ the same value, that is:

$$\mathcal{E}(j,m) \approx \frac{\Omega_j G(j)}{2\beta_j} \tag{19}$$

2. Excited States

Let us calculate the energy of the nuclear excited states, taking into account the interaction of the nucleons, being on the outer shell. It is known, that the wave function Ψ of the new vacuum state is the superposition of the states with an even nucleons number, i.e.

$$\Psi = \prod_{s,j,m} \left(U_{mj}(s) + V_{mj}(s) a_{m+}(j)^+ a_{m-}(j) \right) \Psi_0 , \tag{20}$$

where $a_{m\rho}(j)\Psi_0 = 0$. In this connection the wave function of the ground state of the system with an odd particles number is $\alpha_{m\rho}(j)^+\Psi$.

Let us calculate the energy of the excited states of the shell with even nucleons number on it. The difference ΔE^{I}_{mj} between the energies of the first excited state $\alpha_{m+}(j)^+\alpha_{m-}(j)^+\Psi$ and of the ground superfluid state Ψ is of the following form:

$$\Delta E^{I}_{mj} = \langle \alpha_{m+}(j)\alpha_{m-}(j)H_s\alpha_{m-}(j)^+\alpha_{m+}(j)^+\rangle - \langle H_s\rangle = 2\,\hat{\varepsilon}(j,m) . \tag{21}$$

Let us calculate the difference $\Delta E^{I}_{mj\,m'j'}$ between the energies of the excited state of the $\alpha_{m_1\rho_1}(j_1)^+\alpha_{m_2\rho_2}(j_2)^+\Psi$ form and of the ground state Ψ . It will be as a result:

$$\begin{aligned}\Delta E^{I}_{mj,m'j'} &= \langle \alpha_{m_1\rho_1}(j_1)\alpha_{m_2\rho_2}(j_2)H_s\alpha_{m_2\rho_2}(j_2)^+\alpha_{m_1\rho_1}(j_1)^+\rangle - \\ &- \langle H_s\rangle = \hat{\varepsilon}(j_1,m_1) + \hat{\varepsilon}(j_2,m_2) + \\ &+ \frac{1}{4}\left\{J(j_1,j_2|\rho_1m_1,-\rho_2m_2;-\rho_2m_2,\rho_1m_1) - J(j_1,j_2|\rho_1m_1,\rho_2m_2;\rho_2m_2,\rho_1m_1)\right\} \\ &- \frac{1}{4}\,\frac{\tilde{\varepsilon}(j_1,m_1)\tilde{\varepsilon}(j_2,m_2)}{\hat{\varepsilon}(j_1,m_1)\hat{\varepsilon}(j_2,m_2)}\left\{J(j_1,j_2|m_1,-m_2;-m_2,m_1) + J(j_1,j_2|m_1,m_2;m_2,m_1)\right\}\end{aligned} \tag{21'}$$

In the approximation (13), (18) the addition in (21') comparing with (21) is rather small for any values of j_1, m_1, j_2, m_2

(including for the values of j, which correspond to the completely occupied levels of the outer shell). Thus, there is an energetic gap of the order of $2\mathcal{E}(j,m)$ between the ground and first excited states, having appeared as a result of the interactions of the outer shell nucleons.

In the even-even nuclei first excited states both of neutron and proton shells are separated from ground states by a gap, hence a gap in excited states spectra is to appear, that is proved by experimental data.

The first excited state in the shells with an even nucleons number is separated from the ground one by a gap, further follows a set of levels with the successively increasing values of spins. These levels are energetically situated in the neighbourhood of one to another. Let us investigate the problem of further sequence of the excited states of the type x)

$$\alpha_{m_1\rho_1}(j_1)^+ \alpha_{m_2\rho_2}(j_2)^+ \alpha_{m_3\rho_3}(j_3)^+ \alpha_{m_4\rho_4}(j_4)^+ \Psi ,$$

therefore we shall calculate the ΔE_j^{II} -difference between the excited states of such a type and of $\alpha_{m-}(j)^+ \alpha_{m+}(j)^+ \Psi$ namely:

$$\Delta E_j^{II} = \langle \alpha_{m+}(j)\alpha_{m-}(j)\alpha_{m'+}(j)\alpha_{m'-}(j) H_s \alpha_{m'-}(j)^+\alpha_{m'+}(j)^+\alpha_{m-}(j)^+\alpha_{m+}(j)^+ \rangle$$

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x) Prof. B.S. Dzhelepov directed my attention to the possibility of appearance of the second gap, whom I express my deepest thanks.

$$-\langle \alpha_{m+}(j)\alpha_{m-}(j) H_s \alpha_{m-}(j)^+ \alpha_{m+}(j)^+ \rangle =$$

$$= 2\hat{\varepsilon}(j,m) + 2\frac{C_{m'j}C_{mj}}{\hat{\varepsilon}(j,m')\,\hat{\varepsilon}(j,m)} J(j|m,-m;m',-m') -$$

$$-\frac{\tilde{\varepsilon}(j,m')\,\tilde{\varepsilon}(j,m)}{\hat{\varepsilon}(j,m')\,\hat{\varepsilon}(j,m)}\left[J(j|m,m';m',m) + J(j|m,-m';-m',m)\right]. \tag{22}$$

In the approximation (13), (18) the expression (22) is considerably simplified

$$\Delta E_j^{II} \approx \frac{G(j)}{\beta_j}\left(\Omega_j - 2\,\frac{(2\Omega_j - n_j)\,n_j}{\Omega_j^2}\right) + \tag{22'}$$

$$+\left\{-J(j|m,m';m',m) - J(j|m,-m';-m',m)\right\}$$

The terms, which lead to decrease of ΔE^{II} in the approximation (13), (18), are relatively small, though larger than in case (20'), the greatest decrease of ΔE^{II} being achieved with the levels with given j half-filled. It should be noted that in case of different values of j

$$\Delta E^{II}_{jj'} \gtrsim \Delta E^{II}_j \quad .$$

Thus we have the following picture of the excited states of even-even nuclei: the first excited state is separated from the ground one by the distinctly expressed gap, followed then by a series of excited states with the successively increasing spin values, separated from the levels with the corresponding spin values by ΔE^{π}. As the levels with high spin values may experimentally not show that is why the semblance of appearance of the second gap arises. Undoubtedly, the "second gap" in some nuclei may be expressed rather weekly. It is connected firstly with the decrease of ΔE^{π} in a proton or neutron shell, if the levels with any j are half filled, and secondly with somewhat different energy of the first and the following excited states in proton and neutron shells.

Let us calculate the energies of the excited states of the shell with an odd nucleon number. The wave function of the ground state is $\alpha_{m_0\rho_0}(j_0)^+\Psi$ if $E(j_0,m_0)<E(j,m)$. While exciting, the system will be turning from the $\alpha_{m_0\rho_0}(j_0)^+\Psi$ state into the $\alpha_{m_1\rho_1}(j_1)^+\Psi$ -state, where $E(j,m)>E(j_0,m_0)$ etc. spins of the states also increasing. The more excited states will be of the

$$\alpha_{m_3\rho_3}(j_3)^+\alpha_{m_2\rho_2}(j_2)^+\alpha_{m_1\rho_1}(j_1)^+\Psi .$$

form. The ΔE_j difference between the energy of the excited state of such type with a small spin value and the ground state energy will be obtained in the following form:

$$\Delta E_j = \langle \alpha_{m+}(j)\,\alpha_{m-}(j)\,\alpha_{m'+}(j) H_s \alpha_{m'+}(j)^+\alpha_{m-}(j)^+\alpha_{m+}(j)^+ \rangle -$$

$$-\langle \alpha_{m'+}(j) H_s \alpha_{m'+}(j)^+ \rangle =$$

$$= 2\hat{\mathcal{E}}(j,m) + \frac{C_{m'j}\, C_{mj}}{\hat{\mathcal{E}}(j,m')\,\hat{\mathcal{E}}(j,m)}\, J(j|m,-m;m',-m') -$$

$$- \frac{\hat{\xi}(j,m')\,\hat{\xi}(j,m)}{\hat{\mathcal{E}}(j,m')\,\hat{\mathcal{E}}(j,m)} \left[J(j|m,m';m',m) + J(j|m,-m';-m',m) \right] . \qquad (23)$$

In the approximation (13), (18)

$$\Delta E_j = \frac{G(j)}{\rho_j}\left(\Omega_j - \frac{(2\Omega_j - n_j)\, n_j}{\Omega_j^{\,2}}\right) +$$

$$+ \left\{ - J(j|m,m';m',m) - J(j|m,-m';-m',m) \right\}, \qquad (23')$$

i.e. the additions, decreasing ΔE_j are small, being not increasing in case of different j.

Thus we have the following picture in case of odd nuclei: there is a set of excited states with successively increasing spin values which are in the close neighbourhood of the ground state. If the states with high spins do not arise, the energetical "gap" may appear below the excited states of an even shell and the excited states with small spins of an odd shell.

It should be noted that in Bogolubov new variational principle(8) a type of canonical transformation distinguishes the corresponding part of an interaction in the Hamiltonian of the system. In the case under review the canonical transformation distinguishes the nucleon interactions with equal and opposite values of the angular momentum projection along the symmetry axis of the nucleus, if the other quantum numbers are equal, the results weakly depending on concrete definition of other quantum numbers and therefore on the details of the nuclear model. Indeed, in(7) we have considered the interactions of the outer shell nucleons with different values of m however with the fixed other quantum numbers; here we consider interactions with different quantum numbers j and m ; in Belyayev's work(6) the set of the quantum numbers if still less concretized, nevertheless in all the three cases the same gap value in even-even nuclei has been obtained.

Thus, both for the previously obtained results and for the results of the following section, distinguishing of the quantum number of m from the whole set of the quantum numbers of the nucleons situating in the neighbourhood of the Fermi surface is substantial. By this the details of the nuclear model are not substantial and hence the obtained results are also applicable to the case of highly deformed nuclei.

3. Stability of Nuclear Isobars as to β -Decay

The fact of greater stability of even-even nuclei than of odd-odd nuclei as to β decay has found its reflection in Weiz-säcker semi-empirical formula for nuclear masses by means of introduction of term $\pm \frac{\delta}{2A}$ plus being referred to an odd-odd isobar and minus to and even-even one[10]. Let us investigate the problem - what will be the influence on stability of the isobars with respect to β -decay, if the interactions of the outer nuclear shell nucleons are taken into account. For this purpose let us calculate the change of the ground state energy, if the number of the outer shell nucleons is changed by a unity. The difference between the energies of the shells, containing $(2n_0 \pm 1)$ and $2n_0$ of nucleons, will be expressed as follows:

$$\mathcal{E}_s(2n_0 \pm 1) - \mathcal{E}_s(2n_0) = \langle \alpha_{m\rho}(j) H_s \alpha_{m\rho}(j)^+ \rangle_{n=2n_0 \pm 1} - \\ - \langle H_s \rangle_{n=2n_0} = \bar{H}_s(n=2n_0 \pm 1) - \bar{H}_s(n=2n_0) + \mathcal{E}(j,m) \,. \tag{24}$$

In the approximation (13), (18) this expression will of rather a simple form:

$$\mathcal{E}_s(2n_0+1) - \mathcal{E}_s(2n_0) = \frac{\Omega_j - 2n_0 - 1/2}{\rho_j \left(e^{\frac{2}{G(j)}} - 1\right)} \,, \tag{25}$$

$$\mathcal{E}_s(2n-1) - \mathcal{E}_s(2n_0) = \frac{\Omega_j + 2n_0 - 1/2}{\rho_j \left(e^{\frac{2}{G(j)}} - 1\right)} \,. \tag{25'}$$

One can see from (25), (25'), that addition of one nucleon to the even shell does not give use to a substantial change of the shell energy, and vica versa one needs a considerable energy to remove one nucleon from the even shell. Addition of one nucleon to the odd shell causes a considerable release of energy, however one requires little energy to remove one nucleon from the odd shell.

If we roughly define G from the gap value in even-even nuclei and basing on (25), (25'), calculate the difference of the even-even and odd-odd isobar masses, we shall obtain rough numerical agreement with the experimental data, that found their reflection in Weizsäcker formula.

One can easily show, that from (25), (25') no conclusions of relative stability of odd nuclei follow.

We have considered rather an idealized nuclear model and obtained a number of conformities with a law which automatically arise from the properties of nuclear superfluidity. These conformities with a law should be specified and concretized by means of complication of the nuclear model and improvement of the approximation and compared with experimental data.

Investigations, carried out in this work as well as in[3-7], prove fruitfulness of application of physical ideas and mathematical methods that were developed in the theory of superconductivity to the study of the atomic nucleus properties.

Closing, I express deep thanks to N.N. Bogolubov and D.F. Zaretskyi for highly interesting discussions.

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