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A b s t r a c t

Some angular azimuthal symmetries resulting from parity conservation have been obtained for the cascades of reactions of the type of triple proton scattering. It is shown that the experimental evidence of the simplest, well-known symmetry of secondarily scattered particles with respect to the plane of primary scattering is not an exhausting check of parity conservation. The proposed experiments may be a more fundamental verification of this law and in some cases even a complete one.

The following azimuthal symmetries are obtained in the present paper.

|1| Consider triple scattering on unpolarized targets, (an incident beam in the first reaction is also unpolarized). If the parity is conserved in all these reactions[1/] then the number of particles secondarily scattered in the direction (ϑ, φ)and scattered in the direction (ϑ', φ') for the third time equals that of the particles secondarily scattered at the angles (ϑ, $-\varphi$) and scattered once more (on a target placed properly) at the angles (ϑ', $-\varphi'$)

$$\sigma_{\vartheta,\varphi}(\vartheta',\varphi') = \sigma_{\vartheta,-\varphi}(\vartheta',-\varphi'). \qquad (1)$$

The angles ϑ, φ refer to the following system of coordinate orths: the orth z is parallel to the direction of the first scattering, the orth y is perpendicular to the plane of the first reaction. For the angles ϑ', φ' the orth Z is parallel to the direction (ϑ, φ), the orth Y is perpendicular to the plane of the second reaction (see details in §3).

Symmetry (1) is correct for any cascade of the type

$$a+b \to c+d \quad (\vartheta_0,\varphi_0), \quad c+e \to f+g \quad (\vartheta,\varphi), \quad f+h \to i+j \quad (\vartheta',\varphi') \qquad (2)$$

if the particles a and the targets b, e, h are not polarized. a, b, c etc. may be nuclei or "elementary" particles (involving γ-quanta) with arbitrary spins.

|2| If the parity is conserved in the reactions of a cascade

$$a+b \to a+b \qquad \begin{matrix} a+e_1 \to a+e_1 & (\vartheta_1,\varphi_1) \\ b+e_2 \to b+e_2 & (\vartheta_2,\varphi_2) \end{matrix} \qquad (3)$$

with an unpolarized beam a and the targets b, e_1, e_2, then the number of coincidences when the counters of secondarily scattered particles a and b are placed in the directions.

1/ For brevity we mean that "parity is conserved" if 1) a real three-dimensional space have not any handedness, i.e., it is neither "right" nor "left". 2) all the particles involved in the reaction have a definite parity. The "parity is not conserved" if at least one of these assumptions is not correct .

(ϑ_1, φ_1) and (ϑ_2, φ_2) must equal the number of coincidences when the counters are placed in the position $(\vartheta_1, -\varphi_1)$ and $(\vartheta_2, -\varphi_2)$

Some reactions in (2) and (3) may be replaced by those of particle decay of the type $a \to c + d$. For example, (1) holds also for the cascade $K^- + p \to \Xi^- + K^+$, $\Xi^- \to \Lambda + \pi$, $\Lambda \to p + \pi$ (4)-also for the cascade $\pi^- + p \to \Sigma^- + K^+$, $\Sigma^- \to n + \pi$, $K^+ \to \pi + \pi$.

The proposed experiments complement that set of experiments which is necessary for the reconstruction of the transition matrix (the S-matrix) of the reaction which is suggested under the assumption that parity is conserved.

Of course, even if one of the proposed symmetries (or well-known already) is violated then "the parity is not conserved".

§ 1. General Formulae

There are well-known formulas expressing the angular distribution and polarization vector (as well as polarization tensors, if any) of the reaction products in terms of the transition matrix elements when a beam and target are polarized.

Simple formulae for the scattering of a particle with spin 1/2 are given, e.g. in Dalitz paper.[1] (see, also[2]). Introducing instead of Decart projections of the spin vector the cyclic ones- $\sigma_{-1} = \frac{1}{\sqrt{2}}(\sigma_x + i\sigma_y)$; $\sigma_0 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$; $\sigma_{+1} = -\frac{1}{\sqrt{2}}(\sigma_x - i\sigma_y)$ these formulas for the scattering of particle with spin 1/2 on a spinless particle may be written as follows:

$$\dot{I} = \tfrac{1}{2}\{ Sp\, RR^+ + \sum_{\tau=-1}^{+1} P_\tau^{in}\, Sp\, R\tilde{\sigma}_\tau R^+ \} \tag{5}$$

- angular distribution from a polarized incident beam;

$$P'_{\tau'} \cdot \dot{I} = \tfrac{1}{2}\{ Sp\, \sigma_{\tau'} RR^+ + \sum_{\tau=-1}^{+1} P_\tau^{in}\, Sp\, \sigma_{\tau'} R\tilde{\sigma}_\tau R^+ \tag{6}$$

- cyclic projections of the polarization vector of the scattered particles when the incident beam is polarized.

Write (5) and (6) in the concise form:

$$\rho'(\vartheta,\varphi; q'\tau') = \sum_{q=0}^{1} \sum_{\tau=-q}^{+q} (q'\tau'|W(\vartheta,\varphi)|q\tau)\rho(q,\tau) \tag{7}$$

where $\rho(0,0) = 1$ (if there is one particle in a target and the flux density of incident particles is equal to $1 \frac{\text{particle}}{\text{cm}^2 \text{ sec}}$), $\rho(1,\tau) \equiv P_\tau^{in}$ - are cyclic components of the polarization vector of the beam, $\rho'(\vartheta,\varphi; 00)$ is the angular distribution, $\rho'(\vartheta,\varphi; 1\tau') \equiv P'_{\tau'}(\vartheta,\varphi)\dot{I}(\vartheta,\varphi)$

$$(1\tau'|W(\vartheta,\varphi)|1\tau) \equiv \tfrac{1}{2} Sp\, \sigma_{\tau'} R(\vartheta,\varphi)\, \tilde{\sigma}_\tau R^+(\vartheta,\varphi) \tag{8}$$

Formula of such a form also holds for any reaction a + b → c + d (spins are arbitrary); see[3] as well as papers[4],[5] and [6].

$$\rho'(\vartheta,\varphi;\, q_c\tau_c q_d\tau_d) =$$
$$= \sum_{q_a\tau_a q_b\tau_b} (q_c\tau_c q_d\tau_d | W(\vartheta,\varphi) | q_a\tau_a q_b\tau_b)\, \rho(q_a\tau_a q_b\tau_b) \tag{9}$$

$$(q_c\tau_c q_d\tau_d | W(\vartheta,\varphi) | q_a\tau_a q_b\tau_b) = \left[(2i_c+1)(2i_d+1)\right]^{1/2} \left[(2i_a+1)(2i_b+1)\right]^{-1/2} \cdot$$
$$\cdot \sum_{m_c m_c' m_d m_d'} (-1)^{i_c-m_c'} (i_c i_c m_c - m_c' | q_c\tau_c)(-1)^{i_d-m_d'} (i_d i_d m_d - m_d' | q_d\tau_d) \cdot \tag{10}$$
$$\cdot \sum_{m_a m_a' m_b m_b'} (m_c m_d | R(\vartheta,\varphi) | m_a m_b)(m_c' m_d' | R(\vartheta,\varphi) | m_a' m_b')^* \cdot$$
$$\cdot (-1)^{i_a-m_a'} (i_a i_a m_a - m_a' | q_a\tau_a)(-1)^{i_b-m_b'} (i_b i_b m_b - m_b' | q_b\tau_b)$$

The letters i designate the particle spins, m their projections, q is the rank of polarization tensors (see their definition in[4]). The Clebsh-Gordan coefficients $(i\ i\ m{-}m'|q\ \tau)$ replace in this general case the elements of the matrices σ_τ in formulae (5),(6),(8).

For the decay reaction $a \to c + d$ the formula is analogous (see[4]):

$$\rho'(\vartheta,\varphi;\, q_c\tau_c q_d\tau_d) = \sum_{q_a\tau_a} (q_c\tau_c q_d\tau_d | W(\vartheta,\varphi) | q_a\tau_a)\, \rho(q_a\tau_a)$$

§2. Parity Conservation Selection Rule

The space reflection invariance may be expressed in the form of commutability of the S-matrix or of the physical process with the reflection operator $\dot{I}$: $S\dot{I} - \dot{I}S = 0$ or $\dot{I}^{-1} S \dot{I} = S$, $\dot{I}^{-1} = \dot{I}^{\dagger}$.

Therefore, the S-matrix elements between the states $\psi_{\vec{p}\, m_1 m_2}$ of the two-particle system with a relative momentum $\vec{p}$ and definite spin projections m_1 and m_2 must have the property:

$$(\dot{I}\psi_{\vec{p}_c m_c m_d},\, S\dot{I}\psi_{\vec{p}_a m_a m_b}) = (\psi_{\vec{p}_c m_c m_d},\, S\psi_{\vec{p}_a m_a m_b}) \tag{11}$$

or

$$\pi_c^* \pi_d^* \pi_a \pi_b (-\vec{p}_c, m_c m_d | S | -\vec{p}_a m_a m_b) = (\vec{p}_c m_c m_d | S | \vec{p}_a m_a m_b) \tag{12}$$

since according to the definition $\dot{I}\psi_{\vec{p}\, m_1 m_2} = \pi_1 \pi_2 \psi_{-\vec{p}\, m_1 m_2}$ for the particles with definite intrinsic parities π_1 and π_2. If there takes place a rotational invariance and if a coordinate system with an axis $z \parallel [\vec{p}_a \times \vec{p}_c]$ (i.e. perpendicular to the plane of the reaction) is chosen then by rotation 180° about this axis we obtain

$$(-\vec{p}_c\, m_c \,|\, S \,|\, -\vec{p}_a\, m_a) = (-1)^{-m_c + m_a} (\vec{p}_c\, m_c \,|\, S \,|\, \vec{p}_a\, m_a)$$

Here and sometimes further for the sake of brevity we omit the indices of the particles b and d). Therefore in the chosen coordinate system,

$$\left.\begin{array}{ll} (\vec{p}_c\, m_c\, m_d \,|\, S \,|\, \vec{p}_a\, m_a\, m_b) = 0 & \text{if} \\ \pi_c^* \pi_d^* \pi_a \pi_b (-1)^{-m_c - m_d + m_a + m_b} & \text{is not equal to } + 1 \end{array}\right\} \quad (13)$$

This selection rule in terms of the polarization tensors states that $V_c + V_d + V_a + V_b$ must be even, otherwise the corresponding coefficients $(q_c V_c\, q_d V_d \,|\, W \,|\, q_a V_a\, q_b V_b)$ vanish (see another derivation of this selection rule in[4]).

Further we assume that spin projections of the particles a and b are referred to the system of three orths A with an orth $z_a \parallel \vec{p}_a$ (the orth y_a is chosen e.g., in the direction of the polarization vectors of a or b), whereas m_c and m_d - to the system of orths C : $z_c \parallel \vec{p}_c$, $y_c \parallel [\vec{p}_a \times \vec{p}_c]$ 2/.

Space reflection does not change the spin projections. But we agreed to consider as an axis of quantization the direction of the relative momentum which changes its sign by reflection. Therefore, the reflected state spin projections which do not really change have to be referred to new systems of orths: the system C' with an orth $z_c' \parallel -\vec{p}_c$ and an axis $y_c' \parallel y_c$ (since $[-\vec{p}_a \times -\vec{p}_c] = [\vec{p}_a \times \vec{p}_c]$). Analogously only the direction of the orth Z of the reference system A is changed. Note that C' and A' are obtained from C and A symply by rotation 180° around the y -axes. The wave function ψ_m of the state having a definite spin projection m on the old z -axis is expressed in terms of the wave functions having difinite projections m' on the new Z-axis:

2/ Then it is possible to show, as it was done in[4] for the coefficients W that the S-matrix elements will depend neither upon $\vec{p}_c$ nor upon $\vec{p}_a$ separately but upon the Euler angles $\{-\pi, \vartheta, \pi-\varphi\}$ of the rotation which carries a system of the orths A into the system C (ϑ and φ are spherical angles of the vector $\vec{p}_c$ with respect to the system A. Besides, the depenence upon φ is known:

$$(m_c\, m_d \,|\, S(\vartheta, \varphi) \,|\, m_a\, m_b) = (m_c\, m_d \,|\, S(\vartheta, 0) \,|\, m_a\, m_b) \exp i(m_a + m_b)\varphi .$$

$$\psi_m = \sum_{m'=-i}^{+i} \varphi_{m'} \mathcal{D}^{i}_{m'm}(0,\pi,0) = \sum_{m'} \varphi_{m'} (-1)^{\pm i \pm m'} \delta_{m',-m} = (-1)^{\pm i \mp m} \varphi_{-m}$$

The functions $\mathcal{D}^{i}_{m',m}$ are defined in[4]. Therefore,

$$\left(\dot{I}\psi_{\vec{p}_c m_c}, S\dot{I}\psi_{\vec{p}_a m_a}\right) = (-1)^{i_c+i_a-m_c-m_a}(-\vec{p}_c, -m_c | S | -\vec{p}_a, -m_a)$$

In the right-hand side of this equality the projections m are defined now in the same way as in the non-reflected element of the S-matrix in the left-hand side: as projections on the direction of the relative momentum.

Reminding that S is dependent not upon $\vec{p}_c$ and $\vec{p}_a$ separately 2/ and making sure (e.g., on the space model) that the spherical angles of the momentum $-\vec{p}_c$ in the system of the orths A' equal $\vartheta, -\varphi$, finally we obtain from (11):

$$\pi_c^* \pi_d^* \pi_a \pi_b (-1)^{i_a+i_b+i_c+i_d-m_a-m_b-m_c-m_d}(-m_c\, -m_d | S(\vartheta,-\varphi) | -m_a\, -m_b) = \tag{14}$$
$$= (m_c\, m_d | S(\vartheta,\varphi) | m_a\, m_b)$$

This relation is proved in two ways. In particular it may be obtained from (13) by space rotations.

Just in the same manner as in §1[5] we obtain the corresponding relation for the coefficients W:

$$(q_c \tau_c q_d \tau_d | W(\vartheta,\varphi) | q_a \tau_a q_b \tau_b) =$$
$$= (-1)^{q_a+q_b+q_c+q_d+\tau_a+\tau_b+\tau_c+\tau_d}(q_c\, -\tau_c\, q_d\, -\tau_d | W(\vartheta,-\varphi) | q_a\, -\tau_a\, q_b\, -\tau_b) \tag{15}$$

The projections τ are referred to the directions of the corresponding relative momenta.

This "selection rule" in other formulation was first obtained by Chou Kuang Chao[7].

L.G. Zastavenko pointed out that it must be entirely equivalent to the rule "$V_c + V_d + V_a + V_b$ even"[4] /since (14) may be obtained from (13)/. For the decay reactions $a \to c + d$ we have analogous relation: we had only to remove the indices q_b, τ_b in (15).

Since the dependence W upon φ is known (see[5] formula (6)) then both parts of (15) may be divided by $\exp i\varphi(\tau_a + \tau_b)$. That is, we may put $\varphi = 0$ in (15) and (14) (see note 2) without any loss of generality.

Conjugating complexly both parts of (10), taking into account further that $(lim\text{-}m'|q\tau)=$

The ρ' subindices $-\vartheta,\varphi$ designate that those particles f which emerged at the angles ϑ,φ in the second reaction are incident on h. The angles ϑ',φ' are counted off with respect to the orths $\widetilde{F}$: $\widetilde{z}_f$ is parallel to the $\vec{p}_f^{\,\ell}$, i.e. to the momentum of the particle f in the laboratory system, $\widetilde{y}_f \parallel [\vec{p}_c^{\,\ell} \times \vec{p}_f^{\,\ell}]$. In particular, for the angular distribution in the third reaction we have $\sigma_{\vartheta,\varphi}(\vartheta',\varphi') = \sigma_{\vartheta,-\varphi}(\vartheta',-\varphi')$. The generalization for the cascades with any number of reactions is evident (all the azimuthal angles φ in the right-hand side of the equalities are substituted by $-\varphi$).

To establish (1) of all relations (15) only those of the form

$$(q_c\tau_c 00 | W(\vartheta,\varphi) | q_a\tau_a 00) = (-1)^{q_c+q_a+\tau_c+\tau_a}(q_c -\tau_c 00 | W(\vartheta,-\varphi) | q_a -\tau_a 00) \tag{21}$$

were used (q_c, q_a assume the values $0,1\ldots 2i_c$ and $0,1\ldots\ 2i_a$ respectively).

To prove (4) some other relations from (15) are used. In the previous consideration we were not at all interested in the second product of the reaction. However, the common origin of the reaction products a + b $\rightarrow$ a + b yields that the angular distributions of secondarily scattered particles a and b display a certain interrelation / on the polarization correlation see, e.g. [8]/. Namely, let us select of all the primary scattered particles a only those which emerged together with the particles b, scattered further in the direction ϑ_2,φ_2. The angular distribution $\sigma(\vartheta_1,\varphi_1)$ of secondary scattering of such a subset of particles a depends upon ϑ_2 and φ_2 as upon the parameters. The selection is being performed using a usual coincidence method. For this combined angular distribution of secondary scatterings we obtain with the help of (15)

$$\sigma(\vartheta_1,\varphi_1;\vartheta_2,\varphi_2) = \sum_{q_a\tau_a q_b\tau_b} (|W_1(\vartheta_1,\varphi_1)|q_a\tau_a)(|W_2(\vartheta_2,\varphi_2)|q_b\tau_b)\cdot$$

$$\cdot (q_a\tau_a q_b\tau_b | W_0(\vartheta_0,0)|) = \sigma(\vartheta_1,-\varphi_1;\vartheta_2,-\varphi_2)$$

In the same way for the cascade

$$a+b\rightarrow c+d\,,\qquad c+e\rightarrow f+g\quad(\vartheta,\varphi)\qquad \begin{matrix} f+h_1\rightarrow i_1+j_1 & (\vartheta_1',\varphi_1') \\ g+h_2\rightarrow i_2+j_2 & (\vartheta_2',\varphi_2') \end{matrix}$$

with unpolarized a, b, e, h_1, h_2 we obtain

$$\sigma_{\vartheta,\varphi}(\vartheta_1',\varphi_1';\vartheta_2',\varphi_2') = \sigma_{\vartheta,-\varphi}(\vartheta_1',-\varphi_1';\vartheta_2',-\varphi_2') \tag{22}$$

/the numbers of coincidences in the last cascade reactions are compared/. The azimuthal angles of the momenta do not change when momenta are transfered from the c.m. systems to the lab. system. Therefore, we may consider the φ's in (1), (4), (22) to be azimuthal angles of the momenta in the lab.system. Similarly we may substitute ϑ and ϑ' by the polar angles of $\vec{p}_f^{\,\ell}$ and $\vec{p}_i^{\,\ell}$ (referred to the same axes $\widetilde{z}$ and $\widetilde{z}_f$). I.e. one may substitute other numbers ϑ_ℓ and ϑ_ℓ' for numbers ϑ and ϑ'. So we may consider ϑ,φ and

ϑ', φ' in (1), (4), (22) as spherical angles of the particle tracks which are observable in a cloud chamber or emulsion (of course, these angles must be counted off with respect to $\tilde{C}$ and $\tilde{F}$ respectively).

Symmetries (1), (4), (22) for the cascade involving γ-quanta or for those involving the reactions of particles decay into two particles are proved just in the same manner (all the formulas and relations necessary for a proof if a neutrino and γ-quanta are present are contained in [6]).

§4. Parity Conservation Check and Azimuthal Symmetries

Let us now set a reverse task: how to check experimentally "the parity conservation" in the given reaction. Strictly speaking, it is necessary to verify all the relations (14)[3/]. The correctness of only the part of them may be either accidental or due to some other symmetrical property of the interaction (see further).

If it is established experimentally that $\sigma(\vartheta,\varphi)=\sigma(\vartheta,-\varphi)$ for $2i_c$ values of the angle φ then the following $2i_c$ equalities will hold:

$$\sum_{q_c=\tau}^{2i_c}(|W(\vartheta,0)|q_c\tau)(q_c\tau|W_0(\vartheta_0,0)|) = \sum_{q_c=\tau}^{2i_c}(|W(\vartheta,0)|q_c-\tau)(q_c-\tau|W_0(\vartheta_0,0)|) \tag{23}$$

for $\tau = 1,2,\dots 2i_c$. Let us assume for the sake of simplicity that the first reaction is a polarizator in the sense that (15) holds for the coefficients W_0. Even then a verification of $\sigma(\vartheta,\varphi)=\sigma(\vartheta,-\varphi)$ allows to check only $2i_c$ Eqs. of all Eqs. (15). The number of Eqs. (21) is much more. Therefore, one may expect that the check of (1) is a more fundamental verification of parity conservation. However, one may object that since not all of eqs. (15) are independent (see footnote[3/]), then the possibility that (23) involves as many independent Eqs. as (21) is not excluded. Our assertion that (1) verifies the parity conservation more throughly than the symmetry with respect to the plane of the first

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3/ All these relations are independent. If all the spins i_a, i_b, i_c, i_d or two of them are halfinteger then there are $Q=1/2\,(2i_a+1)(2i_b+1)(2i_c+1)(2i_d+1)$ complex relations (14) or two times as much equivalent real ones. If all the spins are integer then the number of real relations is $2Q-1$ if $\pi_c^*\pi_d^*\pi_a\pi_b = (-1)^{-i_c-i_d+i_a+i_b}$ and $2Q-1$ if this equality is not fulfilled. The number of relations (15) is greater than that of relations (14). Therefore, not all of (15) are independent.

reaction will be proved by an analysis of the simplest cascade[4/]: a particle with spin 1/2 scatters three times on a spinless particle. (For instance, proton scattering on helium targets).

The scattring matrix (m'|R| m)(m' and m are referred to different systems of orths - see § 2) has in this case only four elements

$$\begin{pmatrix} (\tfrac12|R(\vartheta,0)|\tfrac12) & (\tfrac12|R(\vartheta,0)|-\tfrac12) \\ (-\tfrac12|R(\vartheta,0)|+\tfrac12) & (-\tfrac12|R(\vartheta,0)|-\tfrac12) \end{pmatrix} \equiv \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \begin{pmatrix} \mathcal{A}e^{i\alpha} & Be^{i\beta} \\ Ce^{i\gamma} & \mathcal{D}e^{i\delta} \end{pmatrix} \tag{24}$$

The relations (14) (φ is set equal to zero) are reduced in this case only to two complex equalitities a = d, b = -c if the product of the intrinsic parities does not change (as in elastic reaction) or a = - d, b = c if it changes its sign. Further we shall be in need of the following expressions of W (ϑ , 0) coefficients in terms of elements (24) (they are obtained by formulae (5)-(8) of §1):

$$(1\,\text{-}1\,|W|\,00) \equiv (-1\,|W|\,) = 1/\sqrt{2}\,[\mathcal{A}C\exp i(-\alpha+\gamma) + B\mathcal{D}\exp i(-\beta+\delta)] \tag{25}$$

$$(00\,|W|\,1\,\text{-}1) \equiv (\,|W|\,-1) = 1/\sqrt{2}\,[\mathcal{A}B\exp i(-\alpha+\beta) + C\mathcal{D}\exp i(-\gamma+\delta)] \tag{26}$$

$$(1\,\text{-}1\,|W|\,1\,\text{-}1) \equiv (-1\,|W|\,-1) = \mathcal{A}\mathcal{D}\exp i(-\alpha+\delta) \tag{27}$$

$$(1\,\text{-}1\,|W|\,1{+}1) \equiv (-1\,|W|\,+1) = -BC\exp i(-\beta+\gamma) \tag{28}$$

$$(1\,\text{-}1\,|W|\,1\,0) = 1/\sqrt{2}\,[\mathcal{A}C\exp i(-\alpha+\gamma) - B\mathcal{D}\exp i(-\beta+\delta)] \tag{29}$$

Making use of (16) the angular distribution of the second scattering may be written as follows

$$\sigma(\vartheta,\varphi) = \sum_{q=0}^{1} (\,|W(\vartheta,0)|\,q\,0)\,\rho_0(q,0) +$$

$$+ 2\,\mathrm{Re}\,(\,|W(\vartheta,0)|\,1\,\text{-}1)\,\rho_0(1,-1)\cos\varphi + 2\,\mathrm{Jm}\,(\,|W(\vartheta,0)|\,1\,\text{-}1)\,\rho_0(1,-1)\sin\varphi \tag{30}$$

4/ Note that it is very difficult to verify the parity conservation in the reactions involving spinless particles. The reaction is forbidden if the product of the parities of all involved particles is not equal to + 1. But there is not a single example of a reaction in which the parities of all the particles would be known. For instance, the reaction $\pi + He \to H_\Lambda^4 + K$ will be used, first of all, to establish the unknown parities of the hyperfragment and the K-meson (supposing, of course, that parity is conserved). Besides, the "prohibition of the reaction" means that the cross section is less than the one which can be measured in the experiment. However, besides the prohibition by parity other reasons for the smallness of the cross section are possible.

To simplify the further analysis we consider that (15) holds for the coefficients W_0 of the first reaction and W' of the third one, (i.e. in this sense the first reaction is a polarizator, the third one is an analyzator[5/]). Then if for one value of $\varphi = \varphi_0 \neq 0$ $\sigma(\vartheta, \varphi)$ is equal $\sigma(\vartheta, -\varphi)$, then

$$Jm\,(1W1\,1\text{-}1)\,\rho_0\,(1,-1) = 0$$

from where $Re\,(1W1\,1\text{-}1) = 0$ since $\rho_0(1,-1) = (1\text{-}1|W_0|)$ is imaginary according to the above assumption. As is seen from (26) the equality $Re\,(1W1\,1\text{-}1) = 0$ means simply that there is one limiting equation between the matrix elements R[6/]: $Re(a^*b + c^*d) = 0$ from which four relations a = d, b = -c cannot follow. Indeed, it is possible to imagine the following simple symmetrical properties of the interaction, which are of the same character as parity conservation law and "imitate" it in the sense that they also have vanishing Re (a* b + c*·d):

1) The probability amplitude of the transition from the state with spin projection + 1/2 on the direction of the initial momentum to the state with spin projection + 1/2 on the final momentum is equal (with a plus or minus sign) to the transition amplitude from + 1/2 to - 1/2. That is a = ± c. And the transition amplitudes + 1/2 ← - 1/2 and -1/2 ← -1/2 are also equal (b = ± d). In these terms parity conservation law is expressed as equality (with an accuracy up to a sign) of the transition amplitudes + 1/2 ← + 1/2 and -1/2 ← -1/2 (a = ± d) and +1/2 ← -1/2 and -1/2 ← + 1/2 (b = ∓ c).

2) a = ± id, b == ± id - the probabilities of the corresponding transitions are equal but in contrast to the parity conservation law their amplitudes differ by a phase factor $\pi/2$.

A doubt concerning the parity conservation implies that parity conservation should be considered as one of possible symmetrical properties (and on the same footing). Only additional experiments may show which of these properties takes place indeed. A check of the azimuthal symmetry (1) may serve as such an experiment. Let us write $\sigma_{\vartheta,\varphi}(\vartheta', \varphi')$ in the form analogous to (30) (see (20))

$$\sigma_{\vartheta,\varphi}(\vartheta',\varphi') = f(\vartheta', \vartheta, \vartheta_0) + 2\rho w_1' \{-Jm\,(-1|W|)\cos\varphi' + Re\,(-1|W|)\sin\varphi'\} +$$

$$+ 2w'\rho_1 \{-Jm\,(1W|-1)\cos\varphi + Re\,(1W|-1)\sin\varphi\} - \tag{31}$$

5/More complicated experiments (with spin rotation between subsequent scattering)are very likely to make it possible to check the parity conservation without this simplification (and provide us with polarizators and analyzators).

6/ Or between the coefficients K, L, M, N in the expression

$$R = K + L(\vec{\sigma}\cdot[\vec{p}\times\vec{p}\,']) + M(\vec{\sigma}\cdot\vec{p}) + N(\vec{\sigma}\cdot\vec{p}\,')$$

/see[1,2]. That is, the symmetry $\sigma(\vartheta,\varphi) = \sigma(\vartheta,-\varphi)$ holds when R has both scalar and pseudoscalar terms.

$$-2w_1'\rho_1\{Re(-1|W|-1)\cos(\varphi'+\varphi)+Jm(-1|W|-1)\sin(\varphi'+\varphi)+$$
$$+Re(-1|W|+1)\cos(\varphi'-\varphi)+Jm(-1|W|+1)\sin(\varphi'-\varphi)\}$$

where $\rho\equiv\rho_0(\vartheta_0;00)$; $i\rho_1\equiv\rho_0(\vartheta_0;1-1)$; $w'\equiv(00|W'(\vartheta',0)|00)$ and $iw_1'\equiv(00|W'(\vartheta',0)|1-1)$. Establishing that (1) holds at the four points (φ',φ) for instance $(0,\pi/2)$, $(\pi/2,0)$, $(\pi/2,\pi/2)$, $(\pi/2,-\pi/2)$, we obtain that Re $(-1|W|)$ = Re$(|W|-1)=0$ and $Jm(-1|W|-1)=Jm(-1|W|+1)=0$ or (see (25) - (28)))

$$\mathcal{A}\mathcal{D}\sin(\alpha-\delta)=0\ ;\qquad \mathcal{B}\mathcal{C}\sin(\beta-\gamma)=0\ ;$$
$$\mathcal{A}\mathcal{C}\cos(\alpha-\gamma)+\mathcal{B}\mathcal{D}\cos(\beta-\delta)=0\ ; \tag{32}$$
$$\mathcal{A}\mathcal{B}\cos(\alpha-\beta)+\mathcal{C}\mathcal{D}\cos(\gamma-\delta)=0\ .$$

Besides the solutions of "parity conservation" a = d, b = -c and a = -d, b = c (32) has some other solutions more (for instance, when one or two of the parameters A,B,C,D are equal to zero).

Strictly speaking, to reject them an additional experiment is necessary. All of them turn the real part of the coefficient (29) into zero (that does not follow from the parity conservation at all). If after the first reaction the polarization vector is additionally rotated (by a magnetic field, e.g.,) around $[[\vec{p}_a\times\vec{p}_c^{\,e}]\times\vec{p}_c^{\,e}]$ so, that its component in the direction of the first scattering $-\rho_0(\vartheta_0,10)$ would become different from zero then the following term

$$2\rho_0(\vartheta_0,10)\,w_1'\{-Jm(1-1|W|10)\cos\varphi'+Re(1-1|W|10)\sin\varphi'\}$$

will be added to (31).

If now the symmetry (1) is violated then $Re(1-1|W|10)\neq 0$ and there remains two solutions of "parity conservation". Since in (15) the intrinsic parities of particles are not involved the azimuthal symmetries do not allow to find out whether the product of the particle parities changes in the reaction or not.

Note, that the establishment of the parity conservation in elastic scattering of spin 1/2 particle on a spinless particle means that time reversal invariance holds for this reaction as well.

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